Title: Fun from none: deformed Fock space and hidden entanglement

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Abstract: Attempts to go beyond the framework of local quantum field theory include scenarios in which the action of external symmetries on the quantum fields Hilbert space is deformed. A common feature of these models is that the quantum group symmetry of their Hilbert spaces induces additional structure in the multiparticle states which in turns reflects a non-trivial momentum-dependent statistics. In certain particular models which might be relevant for quantum gravity the richer structure of the deformed Fock space allows for the possibility of entanglement between the field modes and certain "planckian" degrees of freedom invisible to an observer that cannot probe the Planck scale.

Fun from none: deformed Fock space and hidden entanglement

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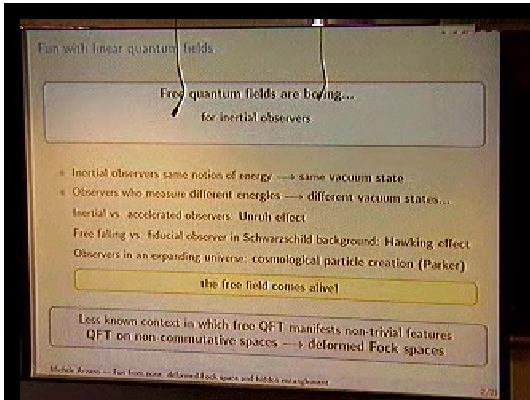

Free quantum fields are boring...

for inertial observers

- **Inertial observers** same notion of energy $\longrightarrow$ same **vacuum state**
- Observers who measure **different energies** $\longrightarrow$ **different vacuum states...**

Inertial vs. accelerated observers: **Unruh effect**

Free falling vs. fiducial observer in Schwarzschild background: **Hawking effect**

Observers in an expanding universe: **cosmological particle creation (Parker)**

the free field comes alive!

Less known context in which **free** QFT manifests non-trivial features
QFT on non-commutative spaces $\longrightarrow$ deformed Fock spaces

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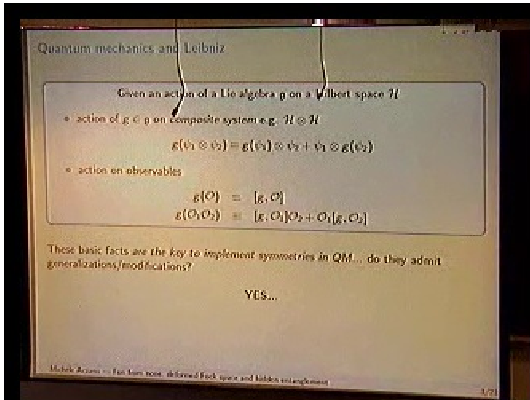

uantum mechanics and Leibniz

Given an action of a Lie algebra $\mathfrak{g}$ on a Hilbert space $\mathcal{H}$

- action of $g \in \mathfrak{g}$ on *composite system* e.g. $\mathcal{H} \otimes \mathcal{H}$

$$g(\psi_1 \otimes \psi_2) \equiv g(\psi_1) \otimes \psi_2 + \psi_1 \otimes g(\psi_2)$$

- action on observables

$$\begin{aligned} g(\mathcal{O}) &\equiv [g, \mathcal{O}] \\ g(\mathcal{O}_1\mathcal{O}_2) &\equiv [g, \mathcal{O}_1]\mathcal{O}_2 + \mathcal{O}_1[g, \mathcal{O}_2] \end{aligned}$$

hese basic facts *are the key to implement symmetries in QM...* do they admit eneralizations/modifications?

YES...

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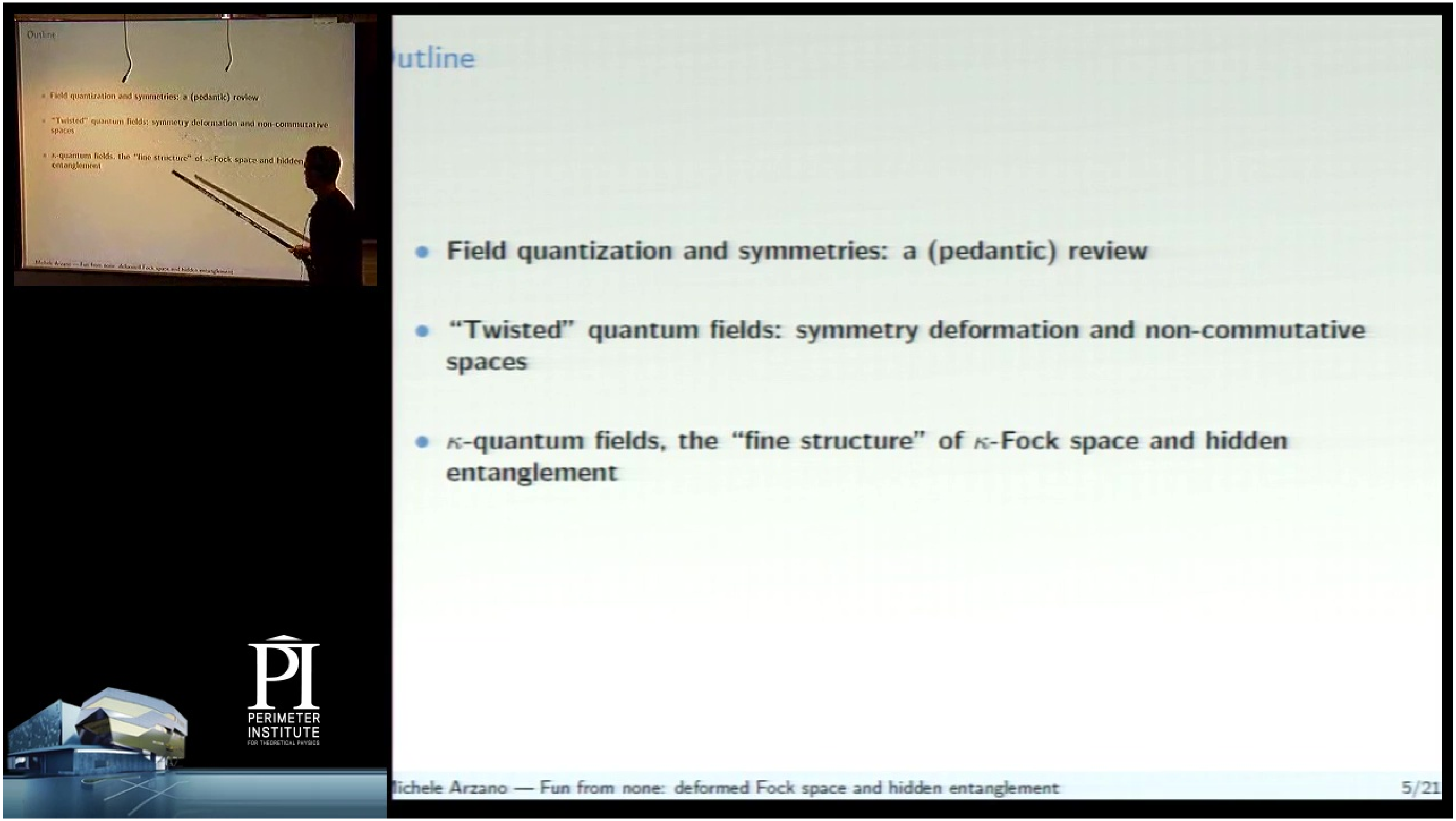
utline
Field quantization and symmetries: a (pedantic) review
"Twisted" quantum fields: symmetry deformation and non-commutative spaces
κ-quantum fields, the "fine structure" of κ-Fock space and hidden entanglement
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ack to the origins: field quantization

Classical fields	Quantum fields
state = *point in phase space* $\{\varphi, \pi\} \in \Gamma_\Sigma$	**state** = *ray in complex Hilbert space* $\mathcal{H}$
observable = *function* on Γ_Σ	**observable** = *self-adjoint operator* on $\mathcal{H}$
joint system = $\Gamma^A_\Sigma \oplus \Gamma^B_\Sigma$	**joint system** = $\mathcal{H}^A \otimes \mathcal{H}^B$

Quantization: *"Recipe for going from the left to the right"*

ovariant phase space formalism $\Gamma_\Sigma \simeq \mathcal{S}=$ space of solutions of KG equation

- *complexify* the space of *real* solutions $\mathcal{S}^{\mathbb{C}} \simeq \mathcal{S} \otimes \mathbb{C}$
- define an *inner product* $< \phi_1, \phi_2 > \equiv -i\omega(\bar{\phi_1}, \phi_2)$, ω symplectic product on $\mathcal{S}$

$$\omega(\phi_1, \phi_2) = \int_\Sigma (\phi_2 \nabla_\mu \phi_1 - \phi_1 \nabla_\mu \phi_2) d\Sigma^\mu$$

- "One-particle" Hilbert space $\mathcal{H} \equiv (\mathcal{S}^{\mathbb{C}+}, < \cdot, \cdot >)$
- "n-particle" Hilbert space $\mathcal{H}^{\otimes n} = \underbrace{\mathcal{H} \otimes \mathcal{H} ... \otimes \mathcal{H}}_{n-times}$;

  for *n-identical* particles $S_n \mathcal{H}^{\otimes n}$ with $S_n = \frac{1}{n!} \sum_{\sigma \in P_n} \sigma$

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bservables and symmetries

lassical observables = *functions on phase space*

uantization:

- to each classical observable ψ associate an operator $\mathcal{O}_\psi$ on $\mathcal{H}$
- "2nd quantization" of a 1-particle operator $\mathcal{O}$ (Cook 1953)

$$d\Gamma(\mathcal{O}) \equiv 1 + \mathcal{O} + (\mathcal{O}\otimes 1 + 1\otimes\mathcal{O}) + (\mathcal{O}\otimes 1\otimes 1 + 1\otimes\mathcal{O}\otimes 1 + 1\otimes 1\otimes\mathcal{O}) + ...$$

such construction naturally leads to the notion of **coproduct** $\Delta\mathcal{O} = \mathcal{O}\otimes 1 + 1\otimes\mathcal{O}$

$$d\Gamma(\mathcal{O}) \equiv 1 + \mathcal{O} + \Delta\mathcal{O} + \Delta_2\mathcal{O} + ... + \Delta_n\mathcal{O} + ...$$

with $\Delta_n\mathcal{O} = (\Delta\otimes 1)\circ\Delta_{n-1}$, $\Delta_1 \equiv \Delta$ and $n \geq 2$

pace-time symmetry generators are **special** observables

- $\mathcal{H}$ constructed from $\mathcal{S}$ (solutions of K-G equation) $\longrightarrow$ $\mathcal{H}$ is a unitary irreps of the Poincaré algebra $\mathcal{P}$
- We have a natural action of the generators of $\mathcal{P}$ as *one-particle operators*
- A commuting set such operators used to label one-particle states (e.g. $\mathsf{P} \rightarrow |\mathbf{p}\rangle$)

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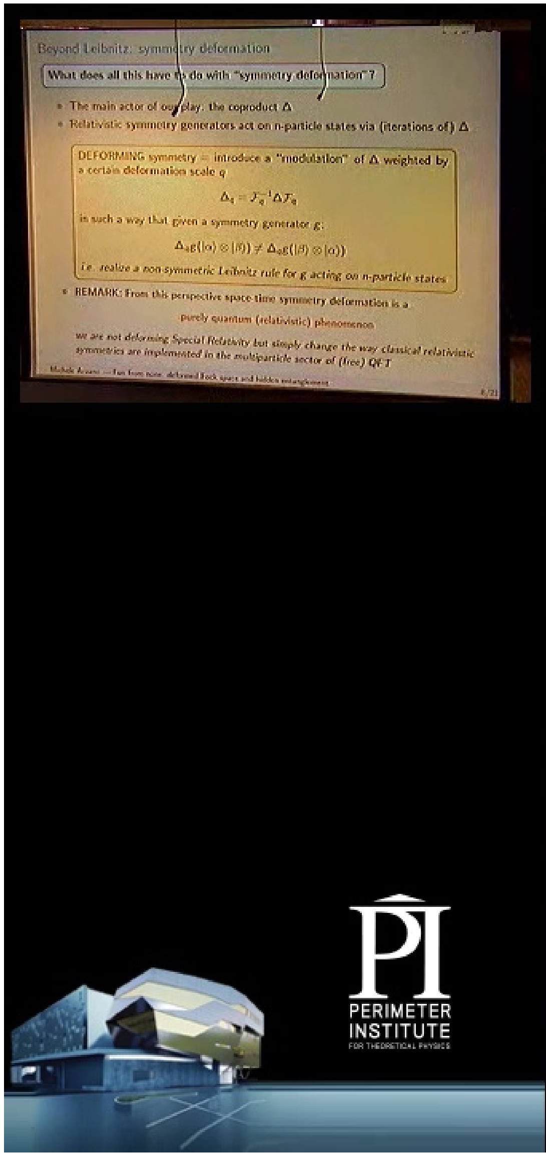


eyond Leibnitz: symmetry deformation

What does all this have to do with "symmetry deformation"?

- The main actor of our play: the coproduct Δ
- Relativistic **symmetry generators** act on n-particle states via (iterations of) Δ

DEFORMING symmetry = introduce a **"modulation"** of Δ weighted by a certain **deformation scale** q

$$\Delta_q = \mathcal{F}_q^{-1} \Delta \mathcal{F}_q$$

in such a way that given a symmetry generator g:

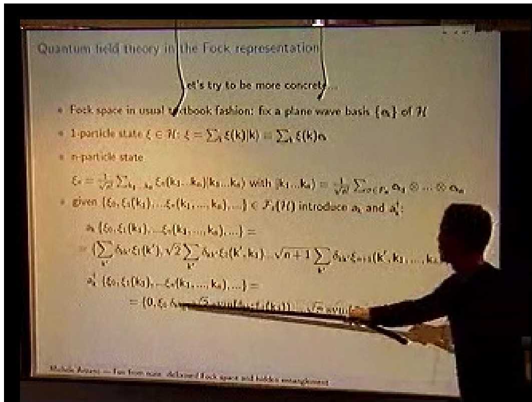
$$\Delta_q g(|\alpha\rangle \otimes |\beta\rangle) \neq \Delta_q g(|\beta\rangle \otimes |\alpha\rangle)$$

i.e. realize a non-symmetric Leibnitz rule for g acting on n-particle states

- **REMARK:** From this perspective space-time symmetry deformation is a

purely quantum (relativistic) phenomenon

we are not deforming Special Relativity but simply change the way classical relativistic symmetries are implemented in the multiparticle sector of (free) QFT



uantum field theory in the Fock representation

Let's try to be more concrete...

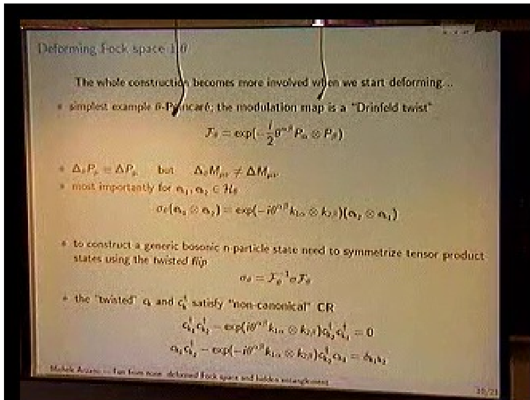
- Fock space in usual textbook fashion: fix a plane wave basis $\{e_{\mathbf{k}}\}$ of $\mathcal{H}$
- 1-particle state $\xi \in \mathcal{H}$: $\xi = \sum_{\mathbf{k}} \xi(\mathbf{k})|\mathbf{k}\rangle \equiv \sum_{\mathbf{k}} \xi(\mathbf{k}) e_{\mathbf{k}}$
- n-particle state

$$\xi_n = \frac{1}{\sqrt{n!}} \sum_{\mathbf{k_1}...\mathbf{k_n}} \xi_n(\mathbf{k_1}...\mathbf{k_n})|\mathbf{k_1}...\mathbf{k_n}\rangle \text{ with } |\mathbf{k_1}...\mathbf{k_n}\rangle = \frac{1}{\sqrt{n!}} \sum_{\sigma \in P_n} e_{\mathbf{k_1}} \otimes ... \otimes e_{\mathbf{k_n}}$$

- given $\{\xi_0, \xi_1(\mathbf{k_1}), ...\xi_n(\mathbf{k_1}, ..., \mathbf{k_n}), ...\} \in \mathcal{F}_s(\mathcal{H})$ introduce $a_{\mathbf{k}}$ and $a^{\dagger}_{\mathbf{k}}$:

$$a_{\mathbf{k}} \{\xi_0, \xi_1(\mathbf{k_1}), ...\xi_n(\mathbf{k_1}, ..., \mathbf{k_n}), ...\} =$$
$$= \{\sum_{\mathbf{k}'} \delta_{\mathbf{kk}'} \xi_1(\mathbf{k}'), \sqrt{2} \sum_{\mathbf{k}'} \delta_{\mathbf{kk}'} \xi_2(\mathbf{k}', \mathbf{k_1}) ... \sqrt{n+1} \sum_{\mathbf{k}'} \delta_{\mathbf{kk}'} \xi_{n+1}(\mathbf{k}', \mathbf{k_1}, ..., \mathbf{k_{n+1}}), ...\}$$

$$a^{\dagger}_{\mathbf{k}} \{\xi_0, \xi_1(\mathbf{k_1}), ...\xi_n(\mathbf{k_1}, ..., \mathbf{k_n}), ...\} =$$
$$= \{0, \xi_0 \, \delta_{\mathbf{kk_1}}, \sqrt{2} \, \mathrm{sym}(\delta_{\mathbf{kk_1}} \xi_1(\mathbf{k_2})), ... \sqrt{n} \, \mathrm{sym}(\delta_{\mathbf{kk_1}} \xi_{n-1}(\mathbf{k_2}, ..., \mathbf{k_n})), ...\}$$



eforming Fock space 1.θ

The whole construction becomes more involved when we start deforming...

- simplest example θ-**Poincaré**; the **modulation map** is a "Drinfeld twist"

$$\mathcal{F}_\theta = \exp(-\frac{i}{2}\theta^{\alpha\beta} P_\alpha \otimes P_\beta)$$

- $\Delta_\theta P_\mu \equiv \Delta P_\mu$ but $\Delta_\theta M_{\mu\nu} \neq \Delta M_{\mu\nu}$
- most importantly for $e_{k_1}, e_{k_2} \in \mathcal{H}_\theta$

$$\sigma_\theta(e_{k_1} \otimes e_{k_2}) = \exp(-i\theta^{\alpha\beta} k_{1\alpha} \otimes k_{2\beta})(e_{k_2} \otimes e_{k_1})$$

- to construct a generic bosonic n-particle state need to symmetrize tensor product states using the *twisted flip*

$$\sigma_\theta = \mathcal{F}_\theta^{-1} \sigma \mathcal{F}_\theta$$

- the "twisted" c_k and $c_k^\dagger$ satisfy **"non-canonical" CR**

$$c_{k_1}^\dagger c_{k_2}^\dagger - \exp(i\theta^{\alpha\beta} k_{1\alpha} \otimes k_{2\beta}) c_{k_2}^\dagger c_{k_1}^\dagger = 0$$

$$c_{k_1} c_{k_2}^\dagger - \exp(-i\theta^{\alpha\beta} k_{1\alpha} \otimes k_{2\beta}) c_{k_2}^\dagger c_{k_1} = \delta_{k_1 k_2}$$

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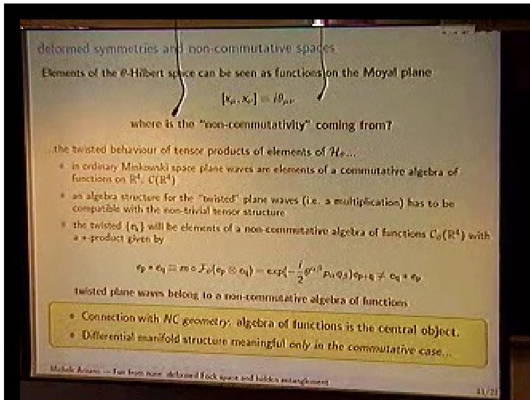


eformed symmetries and non-commutative spaces

lements of the θ-Hilbert space can be seen as functions on the **Moyal plane**

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}$$

where is the "non-commutativity" coming from?

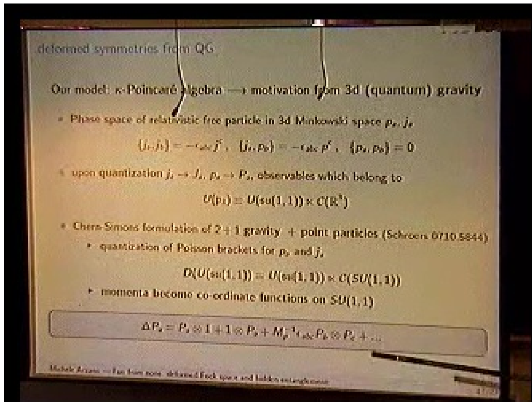
.the twisted behaviour of tensor products of elements of $\mathcal{H}_\theta$...

- in ordinary Minkowski space plane waves are elements of a **commutative** algebra of functions on $\mathbb{R}^4$: $\mathcal{C}(\mathbb{R}^4)$
- an algebra structure for the "twisted" plane waves (i.e. a multiplication) has to be compatible with the non-trivial tensor structure
- the twisted $\{e_{\mathbf{k}}\}$ will be elements of a **non-commutative** algebra of functions $\mathcal{C}_\theta(\mathbb{R}^4)$ with a $*$-product given by

$$e_{\mathbf{p}} * e_{\mathbf{q}} \equiv m \circ \mathcal{F}_\theta(e_{\mathbf{p}} \otimes e_{\mathbf{q}}) = \exp(-\frac{i}{2}\theta^{\alpha\beta} p_\alpha q_\beta) e_{\mathbf{p+q}} \neq e_{\mathbf{q}} * e_{\mathbf{p}}$$

twisted plane waves belong to a non-commutative algebra of functions

- Connection with *NC geometry*: **algebra of functions** is the **central object.**
- Differential manifold structure meaningful *only in the commutative case...*



eformed symmetries from QG

Our model: κ-**Poincaré algebra** $\longrightarrow$ motivation from **3d (quantum) gravity**

- **Phase space** of relativistic free particle in 3d Minkowski space p_a, j_a

$$\{j_a, j_b\} = -\epsilon_{abc}\, j^c\,, \quad \{j_a, p_b\} = -\epsilon_{abc}\, p^c\,, \quad \{p_a, p_b\} = 0$$

- upon **quantization** $j_a \to J_a$, $p_a \to P_a$, observables which belong to

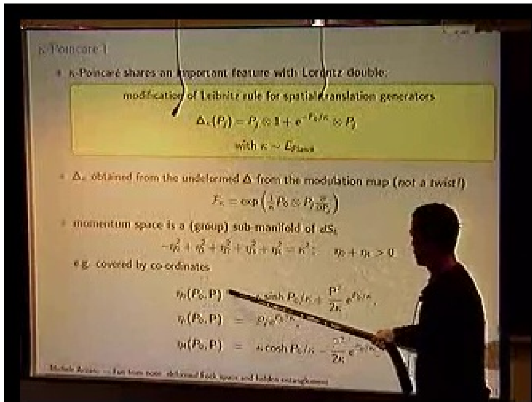
$$U(\mathfrak{p}_3) \equiv U(\mathfrak{su}(1,1)) \ltimes \mathcal{C}(\mathbb{R}^3)$$

- Chern-Simons formulation of $2+1$ **gravity** $+$ **point particles** (Schroers 0710.5844)
  - quantization of Poisson brackets for p_a and j_a

$$D(U(\mathfrak{su}(1,1)) \equiv U(\mathfrak{su}(1,1)) \ltimes \mathcal{C}(SU(1,1))$$

  - **momenta become co-ordinate functions on** $SU(1,1)$

$$\Delta P_a = P_a \otimes 1 + 1 \otimes P_a + M_p^{-1} \epsilon_{abc} P_b \otimes P_c + \ldots$$



-Poincaré I

- κ-Poincaré shares an important feature with Lorentz double:

> modification of Leibnitz rule for spatial translation generators
>
> $$\Delta_\kappa(P_j) = P_j \otimes 1 + e^{-P_0/\kappa} \otimes P_j$$
>
> with $\kappa \sim E_{Planck}$

- Δ_κ obtained from the undeformed Δ from the modulation map (*not a twist!*)

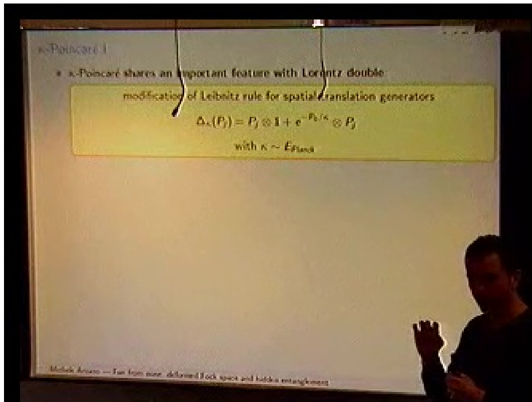
$$\mathcal{F}_\kappa = \exp\left(\frac{1}{\kappa} P_0 \otimes P_j \frac{\partial}{\partial P_j}\right)$$

- momentum space is a (group) sub-manifold of dS_4

$$-\eta_0^2 + \eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 = \kappa^2; \quad \eta_0 + \eta_4 > 0$$

e.g. covered by co-ordinates

$$\begin{aligned}
\eta_0(P_0, \mathbf{P}) &= \kappa \sinh P_0/\kappa + \frac{\mathbf{P}^2}{2\kappa} e^{P_0/\kappa}, \\
\eta_i(P_0, \mathbf{P}) &= P_i \, e^{P_0/\kappa}, \\
\eta_4(P_0, \mathbf{P}) &= \kappa \cosh P_0/\kappa - \frac{\mathbf{P}^2}{2\kappa} e^{P_0/\kappa}.
\end{aligned}$$



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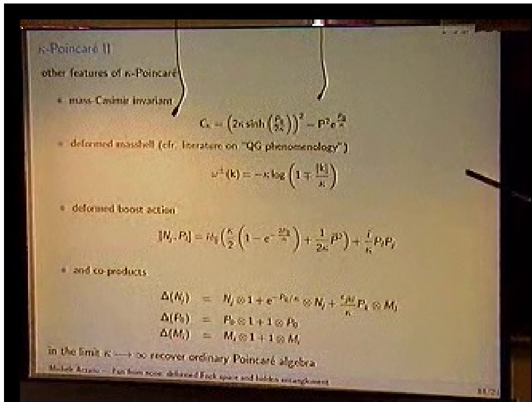

-Poincaré II

ther features of κ-Poincaré

- **mass-Casimir invariant**

$$C_\kappa = \left(2\kappa \sinh\left(\frac{P_0}{2\kappa}\right)\right)^2 - \mathbf{P}^2 e^{\frac{P_0}{\kappa}}$$

- **deformed masshell** (cfr. literature on "QG phenomenology")

$$\omega^{\pm}(\mathbf{k}) = -\kappa \log\left(1 \mp \frac{|\mathbf{k}|}{\kappa}\right)$$

- **deformed boost action**

$$[N_j, P_l] = i\delta_{lj}\left(\frac{\kappa}{2}\left(1 - e^{-\frac{2P_0}{\kappa}}\right) + \frac{1}{2\kappa}\vec{P}^2\right) + \frac{i}{\kappa}P_l P_j$$

- **and co-products**

$$\begin{aligned} \Delta(N_j) &= N_j \otimes 1 + e^{-P_0/\kappa} \otimes N_j + \frac{\epsilon_{jkl}}{\kappa} P_k \otimes M_l \\ \Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0 \\ \Delta(M_i) &= M_i \otimes 1 + 1 \otimes M_i \end{aligned}$$

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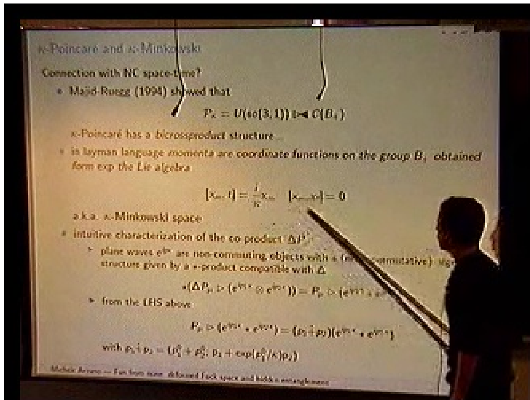

-Poincaré and κ-Minkowski

Connection with NC space-time?

- Majid-Ruegg (1994) showed that

$$\mathcal{P}_\kappa = U(so(3,1)) \bowtie \mathcal{C}(B_+)$$

κ-Poincaré has a *bicrossproduct* structure...

- in layman language *momenta are coordinate functions on the group* B_+ *obtained form* exp *the Lie algebra*

$$[x_m, t] = \frac{i}{\kappa} x_m \quad [x_m, x_l] = 0$$

a.k.a. κ-**Minkowski space**

- intuitive characterization of the co-product ΔP_μ:
  - plane waves e^{ipx} are non-commuting objects with a (non-commutative) algebra structure given by a $\star$-product compatible with Δ

$$\star(\Delta P_\mu \triangleright (e^{ip_1 x} \otimes e^{ip_2 x})) = P_\mu \triangleright (e^{ip_1 x} \star e^{ip_2 x})$$

  - from the LHS above

$$P_\mu \triangleright (e^{ip_1 x} \star e^{ip_2 x}) = (p_1 \dot{+} p_2)(e^{ip_1 x} \star e^{ip_2 x})$$

with $p_1 \dot{+} p_2 = (p_1^0 + p_2^0; \mathbf{p}_1 + \exp(p_1^0/\kappa)\mathbf{p}_2)$

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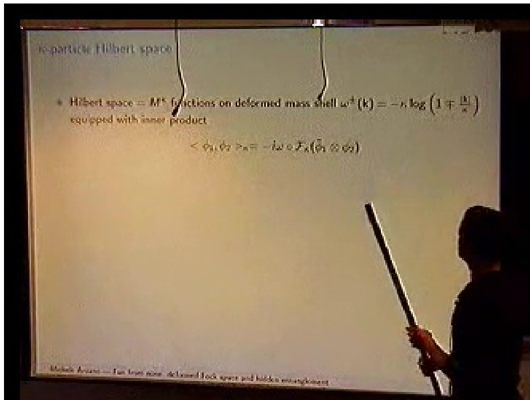
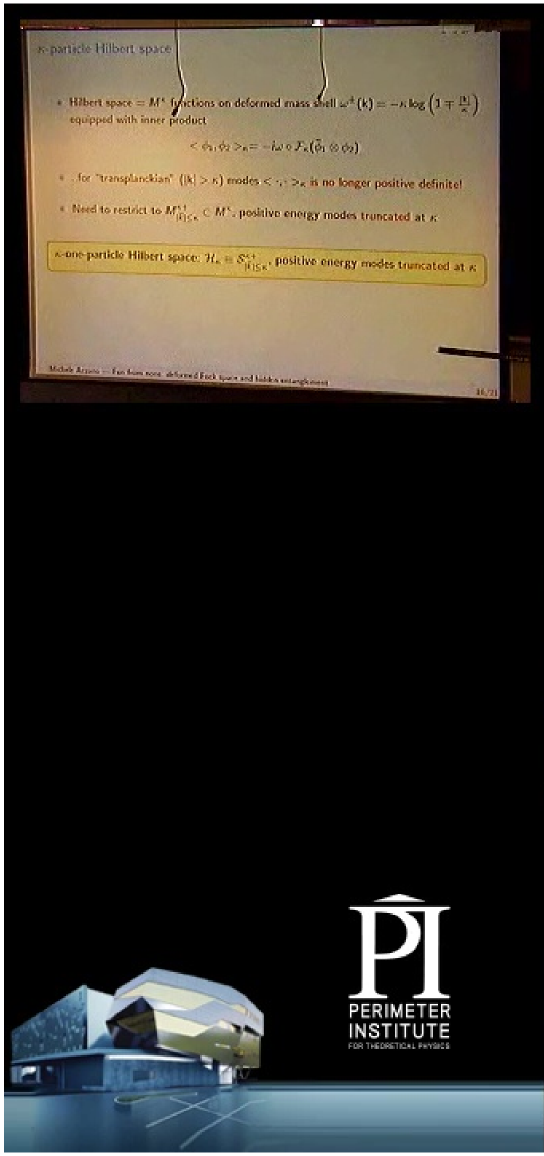


-particle Hilbert space

- Hilbert space $= M^{\kappa}$ functions on deformed mass shell $\omega^{\pm}(\mathbf{k}) = -\kappa \log\left(1 \mp \frac{|\mathbf{k}|}{\kappa}\right)$ equipped with inner product

$$< \phi_1, \phi_2 >_{\kappa} = -i\omega \circ \mathcal{F}_{\kappa}(\bar{\phi}_1 \otimes \phi_2)$$



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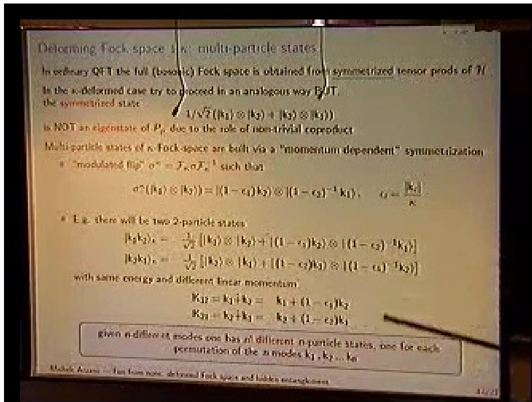
$$< \phi_1, \phi_2 >_\kappa = -i\omega \circ \mathcal{F}_\kappa(\bar{\phi}_1 \otimes \phi_2)$$

- ..for "transplanckian" ($|\mathbf{k}| > \kappa$) modes $< \cdot, \cdot >_\kappa$ is no longer positive definite!
- Need to restrict to $M^{\kappa+}_{|\vec{k}|\leq\kappa} \subset M^\kappa$, **positive energy** modes truncated at κ

κ-**one-particle Hilbert space**: $\mathcal{H}_\kappa \equiv \mathcal{S}^{\kappa+}_{|\vec{k}|\leq\kappa}$, **positive energy** modes truncated at κ



eforming Fock space 1.κ: multi-particle states

n ordinary QFT the full (bosonic) Fock space is obtained from symmetrized tensor prods of $\mathcal{H}$

n the κ-deformed case try to proceed in an analogous way BUT...
he symmetrized state

$$1/\sqrt{2}(|\mathbf{k}_1\rangle \otimes |\mathbf{k}_2\rangle + |\mathbf{k}_2\rangle \otimes |\mathbf{k}_1\rangle)$$

NOT an eigenstate of P_μ due to the role of non-trivial coproduct

Iulti-particle states of κ-Fock-space are built via a "momentum dependent" symmetrization

- "modulated flip" $\sigma^\kappa = \mathcal{F}_\kappa \sigma \mathcal{F}_\kappa^{-1}$ such that

$$\sigma^\kappa(|\mathbf{k}_1\rangle \otimes |\mathbf{k}_2\rangle) = |(1-\epsilon_1)\,\mathbf{k}_2\rangle \otimes |(1-\epsilon_2)^{-1}\,\mathbf{k}_1\rangle\,, \qquad \epsilon_i = \frac{|\mathbf{k}_i|}{\kappa}$$

- E.g. there will be two 2-particle states

$$|\mathbf{k}_1\mathbf{k}_2\rangle_\kappa = \frac{1}{\sqrt{2}}\left[|\,\mathbf{k}_1\rangle \otimes |\,\mathbf{k}_2\rangle + |\,(1-\epsilon_1)\mathbf{k}_2\rangle \otimes |\,(1-\epsilon_2)^{-1}\mathbf{k}_1\rangle\right]$$

$$|\mathbf{k}_2\mathbf{k}_1\rangle_\kappa = \frac{1}{\sqrt{2}}\left[|\,\mathbf{k}_2\rangle \otimes |\,\mathbf{k}_1\rangle + |\,(1-\epsilon_2)\mathbf{k}_1\rangle \otimes |\,(1-\epsilon_1)^{-1}\mathbf{k}_2\rangle\right]$$

with same energy and different linear momentum

$$\mathbf{K}_{12} = \mathbf{k}_1 \dot{+} \mathbf{k}_2 = \quad \mathbf{k}_1 + (1-\epsilon_1)\mathbf{k}_2$$

$$\mathbf{K}_{21} = \mathbf{k}_2 \dot{+} \mathbf{k}_1 = \quad \mathbf{k}_2 + (1-\epsilon_2)\mathbf{k}_1$$

> given n-different modes one has $n!$ different n-particle states, one for each permutation of the n modes $\mathbf{k}_1\,, \mathbf{k}_2\,...\,\mathbf{k}_n$

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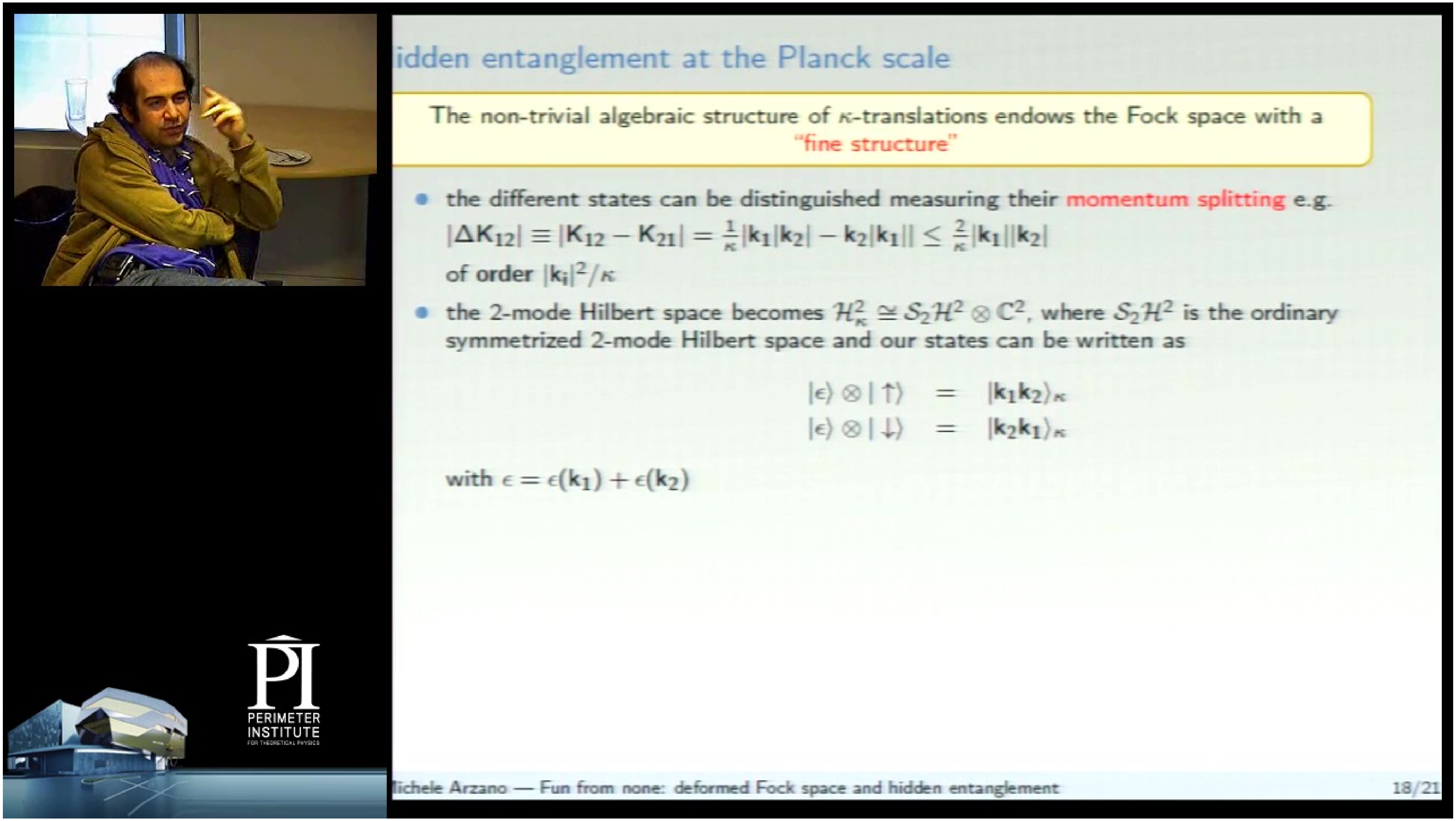
idden entanglement at the Planck scale
The non-trivial algebraic structure of κ-translations endows the Fock space with a "fine structure"
the different states can be distinguished measuring their momentum splitting e.g.
$|\Delta \mathsf{K}_{12}| \equiv |\mathsf{K}_{12} - \mathsf{K}_{21}| = \frac{1}{\kappa}|\mathbf{k}_1|\mathbf{k}_2| - \mathbf{k}_2|\mathbf{k}_1|| \leq \frac{2}{\kappa}|\mathbf{k}_1||\mathbf{k}_2|$
of order $|\mathbf{k}_i|^2/\kappa$
the 2-mode Hilbert space becomes $\mathcal{H}^2_\kappa \cong \mathcal{S}_2\mathcal{H}^2 \otimes \mathbb{C}^2$, where $\mathcal{S}_2\mathcal{H}^2$ is the ordinary symmetrized 2-mode Hilbert space and our states can be written as
$|\epsilon\rangle \otimes |\uparrow\rangle = |\mathbf{k}_1\mathbf{k}_2\rangle_\kappa$
$|\epsilon\rangle \otimes |\downarrow\rangle = |\mathbf{k}_2\mathbf{k}_1\rangle_\kappa$
with $\epsilon = \epsilon(\mathbf{k}_1) + \epsilon(\mathbf{k}_2)$
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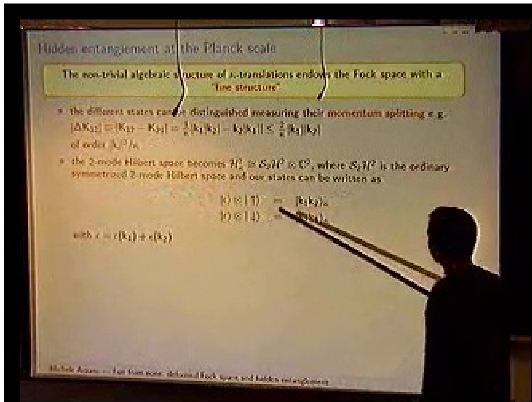

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The non-trivial algebraic structure of κ-translations endows the Fock space with a "fine structure"

- the different states can be distinguished measuring their momentum splitting e.g. $|\Delta \mathbf{K}_{12}| \equiv |\mathbf{K}_{12} - \mathbf{K}_{21}| = \frac{1}{\kappa}|\mathbf{k}_1|\mathbf{k}_2| - \mathbf{k}_2|\mathbf{k}_1|| \leq \frac{2}{\kappa}|\mathbf{k}_1||\mathbf{k}_2|$ of order $|\mathbf{k_i}|^2/\kappa$
- the 2-mode Hilbert space becomes $\mathcal{H}^2_\kappa \cong \mathcal{S}_2\mathcal{H}^2 \otimes \mathbb{C}^2$, where $\mathcal{S}_2\mathcal{H}^2$ is the ordinary symmetrized 2-mode Hilbert space and our states can be written as

$$
\begin{aligned}
|\epsilon\rangle \otimes |\uparrow\rangle &= |\mathbf{k}_1\mathbf{k}_2\rangle_\kappa \\
|\epsilon\rangle \otimes |\downarrow\rangle &= |\mathbf{k}_2\mathbf{k}_1\rangle_\kappa
\end{aligned}
$$

with $\epsilon = \epsilon(\mathbf{k}_1) + \epsilon(\mathbf{k}_2)$

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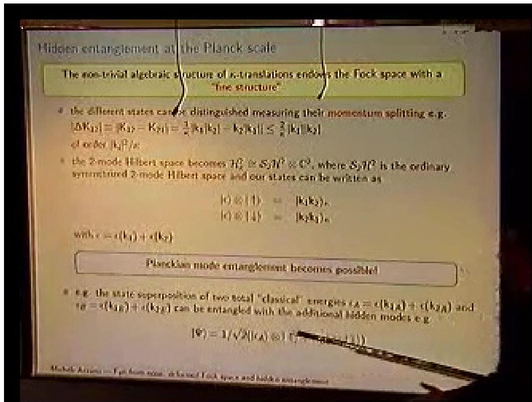

idden entanglement at the Planck scale

The non-trivial algebraic structure of κ-translations endows the Fock space with a "fine structure"

- the different states can be distinguished measuring their momentum splitting e.g. $|\Delta \mathbf{K}_{12}| \equiv |\mathbf{K}_{12} - \mathbf{K}_{21}| = \frac{1}{\kappa}|\mathbf{k}_1|\mathbf{k}_2| - \mathbf{k}_2|\mathbf{k}_1|| \leq \frac{2}{\kappa}|\mathbf{k}_1||\mathbf{k}_2|$ of order $|\mathbf{k}_i|^2/\kappa$
- the 2-mode Hilbert space becomes $\mathcal{H}^2_\kappa \cong \mathcal{S}_2\mathcal{H}^2 \otimes \mathbb{C}^2$, where $\mathcal{S}_2\mathcal{H}^2$ is the ordinary symmetrized 2-mode Hilbert space and our states can be written as

$$
\begin{aligned}
|\epsilon\rangle \otimes |\uparrow\rangle &= |\mathbf{k}_1\mathbf{k}_2\rangle_\kappa \\
|\epsilon\rangle \otimes |\downarrow\rangle &= |\mathbf{k}_2\mathbf{k}_1\rangle_\kappa
\end{aligned}
$$

with $\epsilon = \epsilon(\mathbf{k}_1) + \epsilon(\mathbf{k}_2)$

Planckian mode entanglement becomes possible!

- e.g. the state superposition of two total "classical" energies $\epsilon_A = \epsilon(\mathbf{k}_{1A}) + \epsilon(\mathbf{k}_{2A})$ and $\epsilon_B = \epsilon(\mathbf{k}_{1B}) + \epsilon(\mathbf{k}_{2B})$ can be entangled with the additional hidden modes e.g.

$$|\Psi\rangle = 1/\sqrt{2}(|\epsilon_A\rangle \otimes |\uparrow\rangle + |\epsilon_B\rangle \otimes |\downarrow\rangle)$$

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lanckian degrees of freedom and decoherence

- consider a quantum system evolving unitarily

$$\rho(t) = U(t)\rho(0)U^{\dagger}(t)$$

- start with a pure state $\rho(0)$ factorized with respect to the bipartition in $\mathcal{H}^n_\kappa \cong \mathcal{S}_n\mathcal{H}^n \otimes \mathbb{C}^n$
- If $U(t)$ acts as an "entangling gate", the state $\rho(t)$ will be entangled
- A macroscopic observer who is not able to resolve the planckian degrees of freedom at the beginning will see the reduced system in a pure state

$$\rho_{obs}(0) = \mathrm{Tr}_{Pl}\rho(0)$$

- As the system evolves she will see the mixed state

$$\rho_{obs}(t) = \mathrm{Tr}_{Pl}\rho(t) = \mathrm{Tr}_{Pl}\left[U(t)\rho(0)U^{\dagger}(t)\right].$$

For the *macroscopic* observer, the evolution is not unitary!

A *simple model* which exhibits decoherence due to presence of planckian d.o.f...

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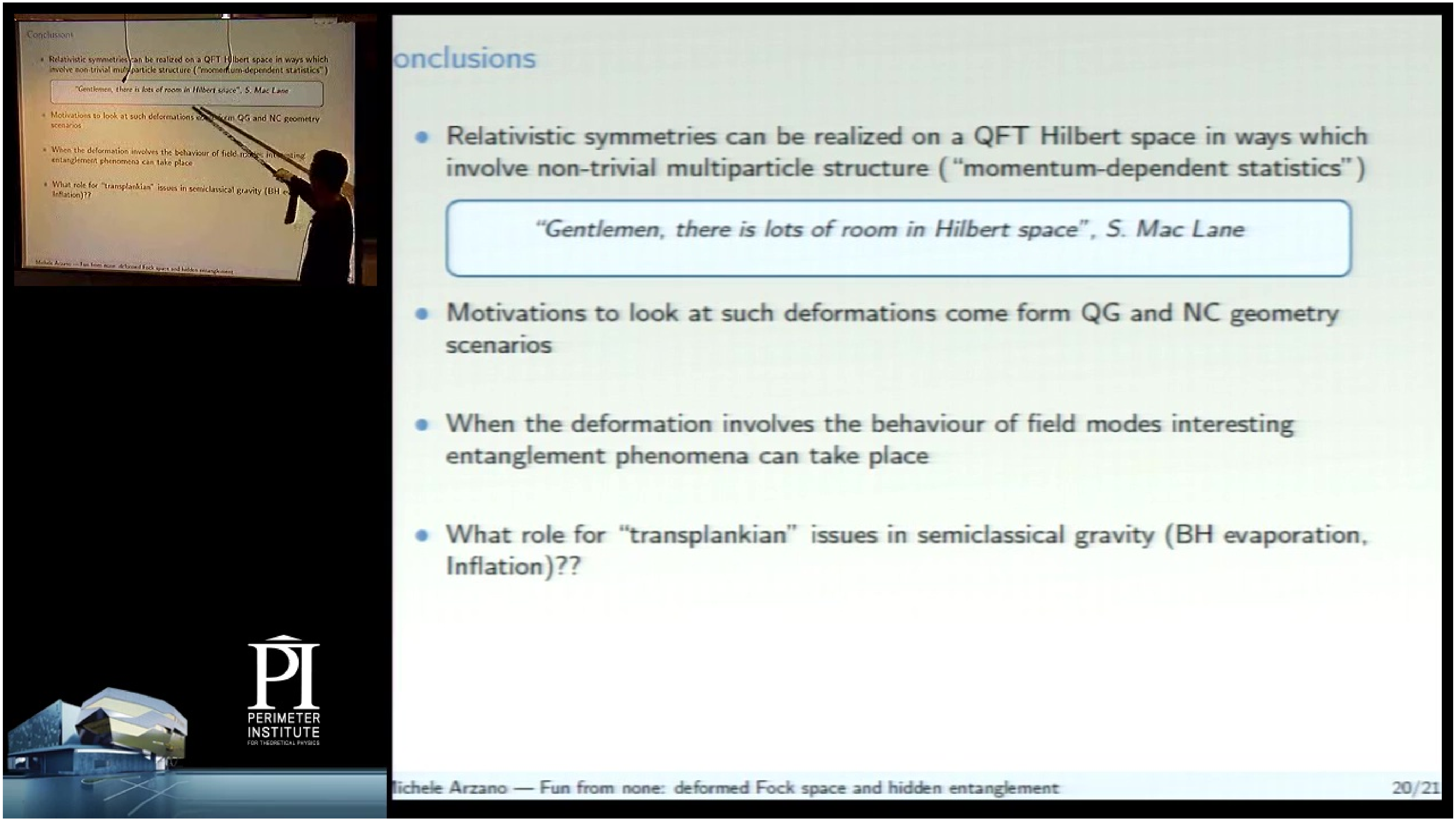
a new window to phenomenological effects??


onclusions
Relativistic symmetries can be realized on a QFT Hilbert space in ways which involve non-trivial multiparticle structure ("momentum-dependent statistics")
"Gentlemen, there is lots of room in Hilbert space", S. Mac Lane
Motivations to look at such deformations come form QG and NC geometry scenarios
When the deformation involves the behaviour of field modes interesting entanglement phenomena can take place
What role for "transplankian" issues in semiclassical gravity (BH evaporation, Inflation)??
lichele Arzano — Fun from none: deformed Fock space and hidden entanglement
20/21
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