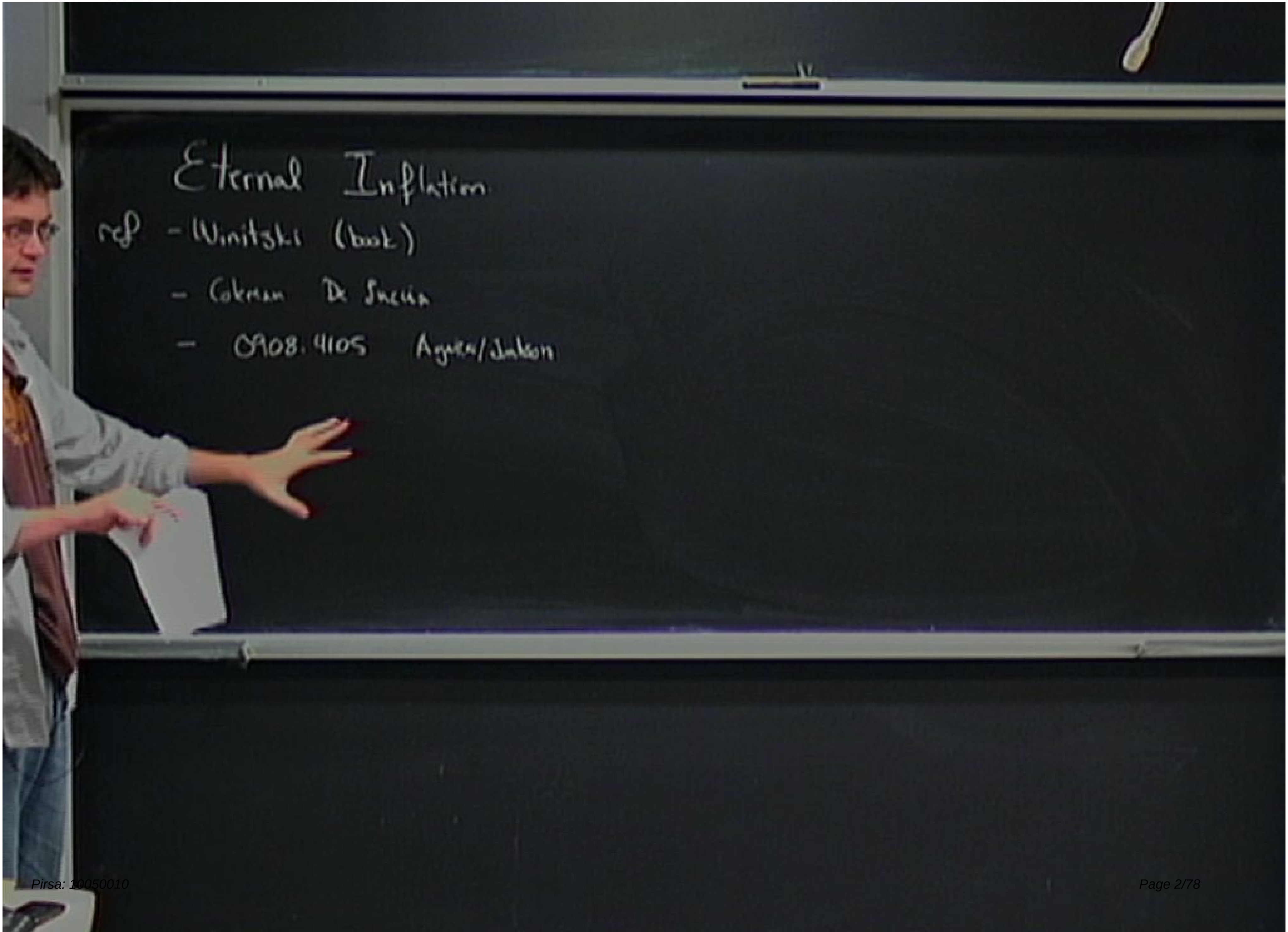
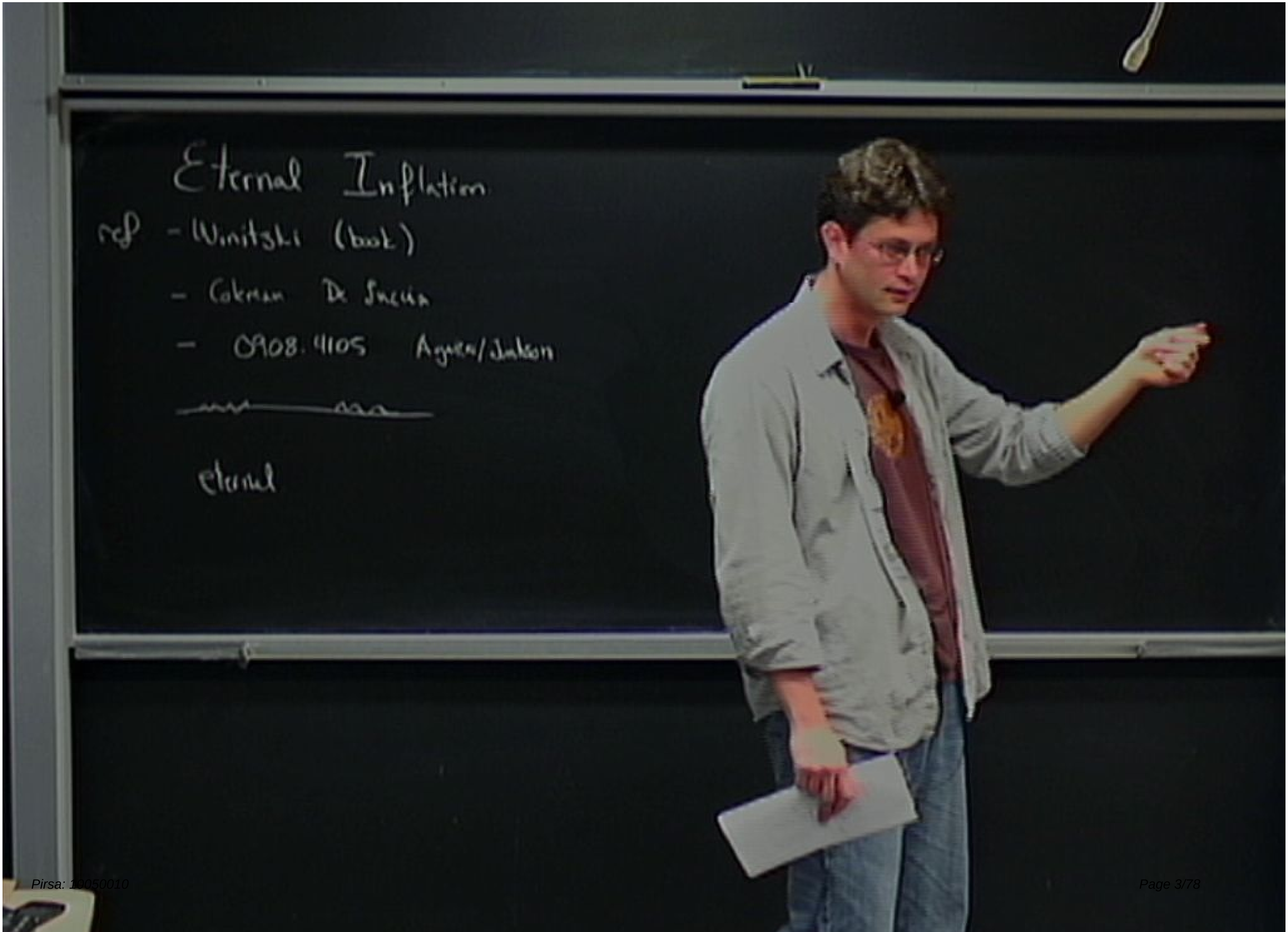
Title: Eternal Inflation

Date: May 03, 2010 11:00 AM

URL: http://pirsa.org/10050010

Abstract:


Eternal Inflation
ref - Winitzki (book)
- Coleman De Luccia
- 0908.4105 Aguirre/Johnson


Eternal Inflation
ref - Winitzki (book)
- Coleman De Luccia
- 0908.4105 Aguirre/Johnson
eternal

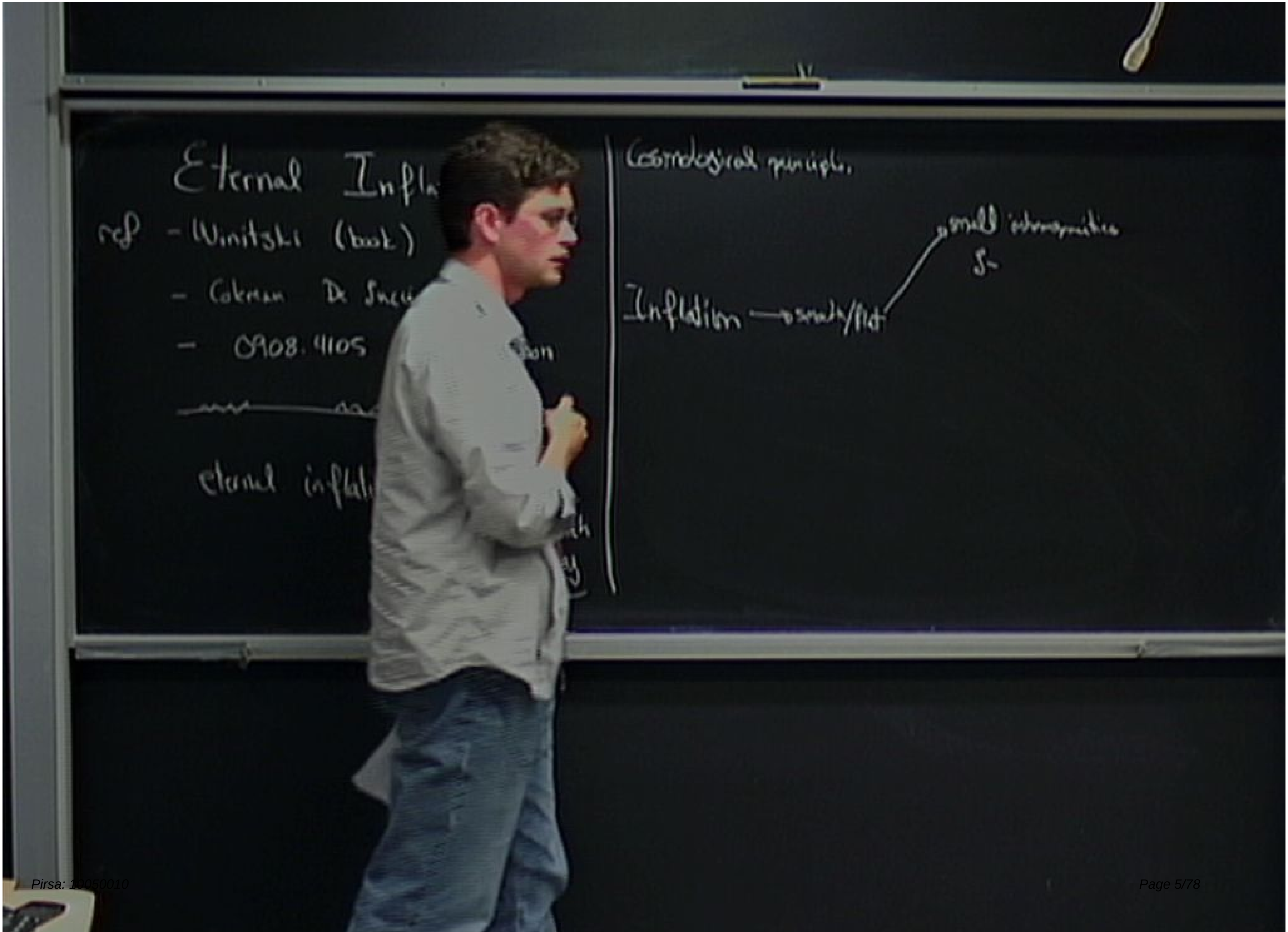
Eternal Inflation

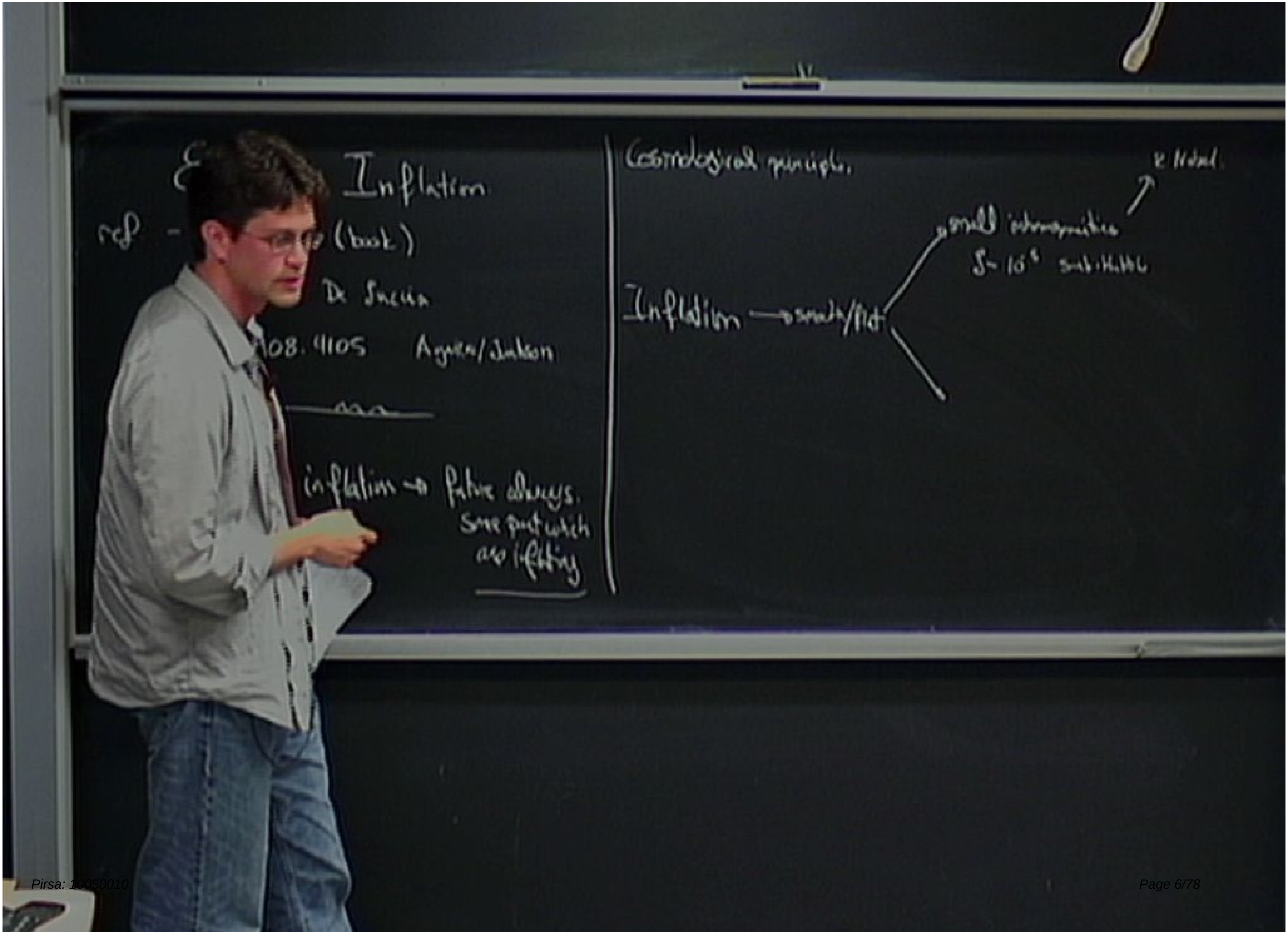
ref - Winitzki (book)

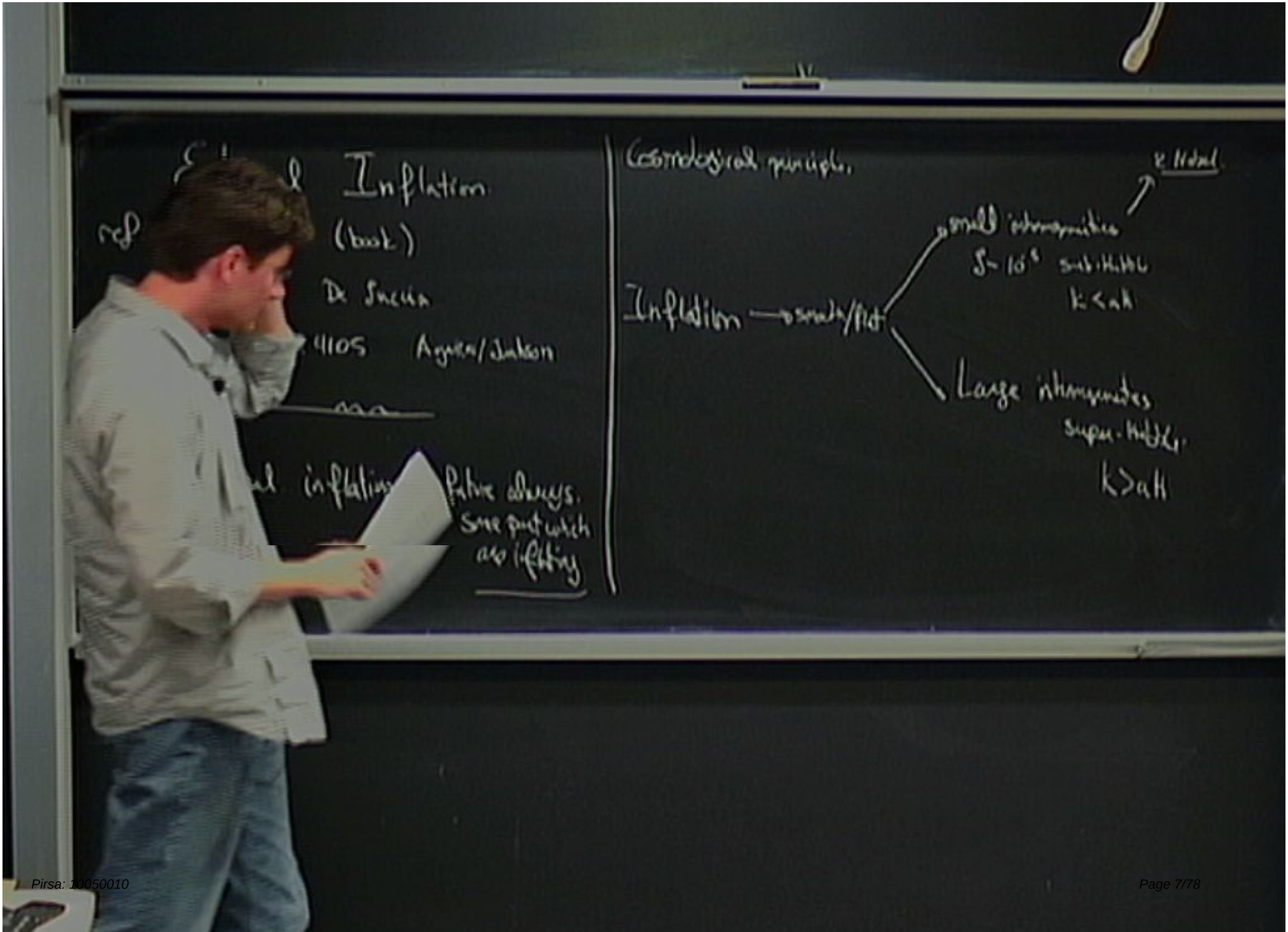
- Coleman De Luccia

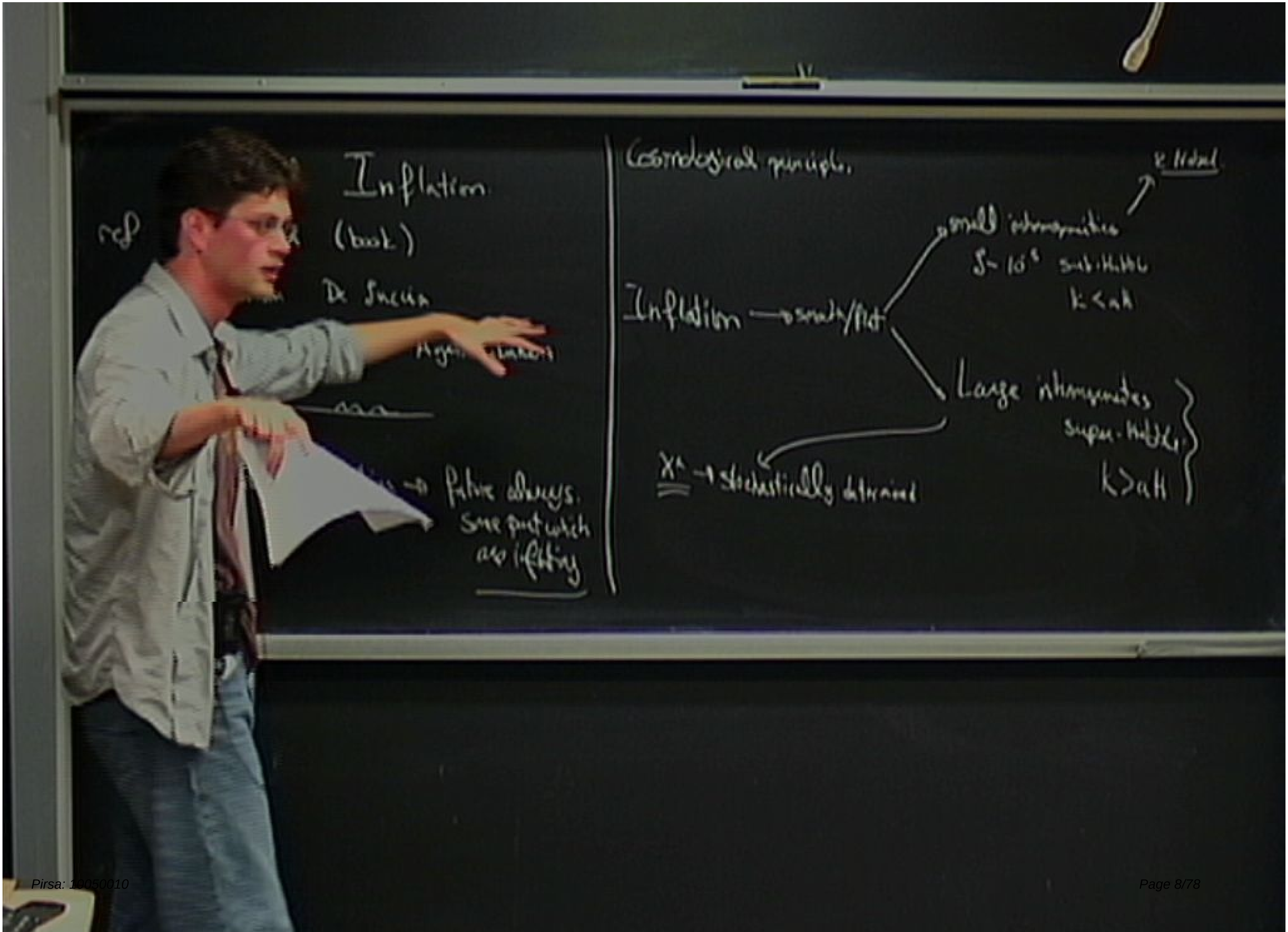
- 0908.4105 Aguirre/Johnson

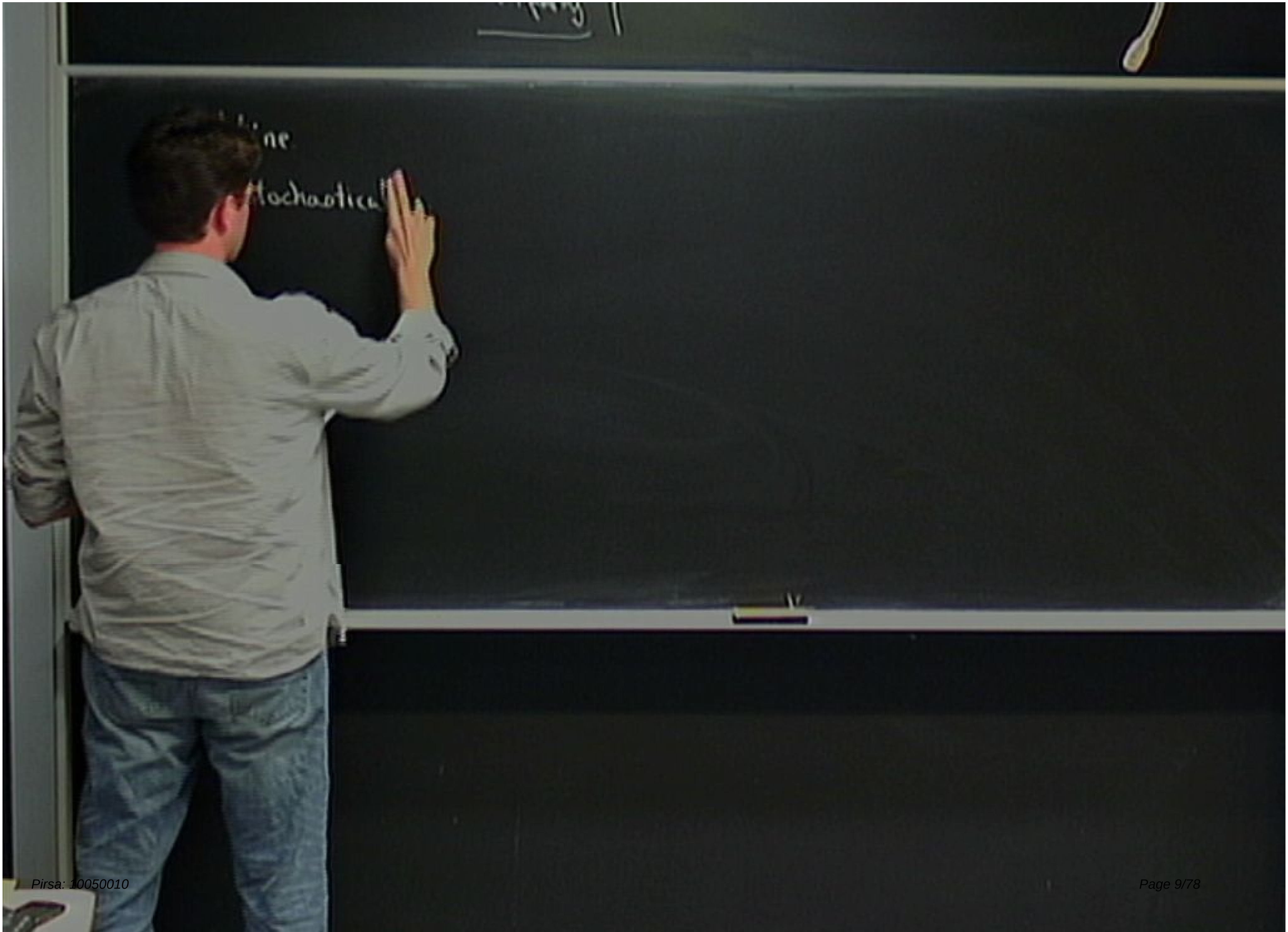
eternal inflation → future always
some part which
are inflating

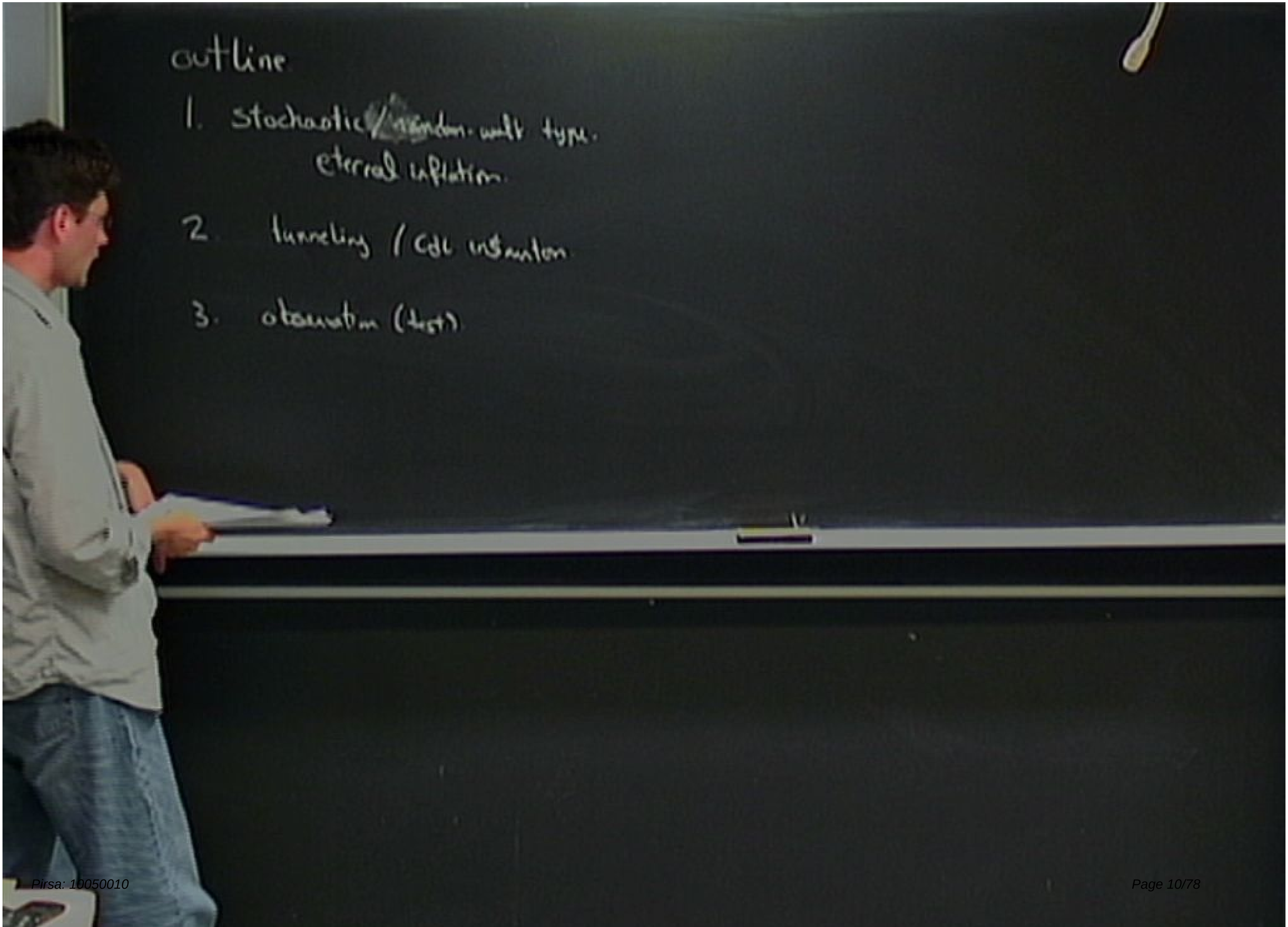

Eternal
ref - Winitski (book)
0908.4105
Cosmological principle.
Inflation → smooth/flat
small inhomogeneities

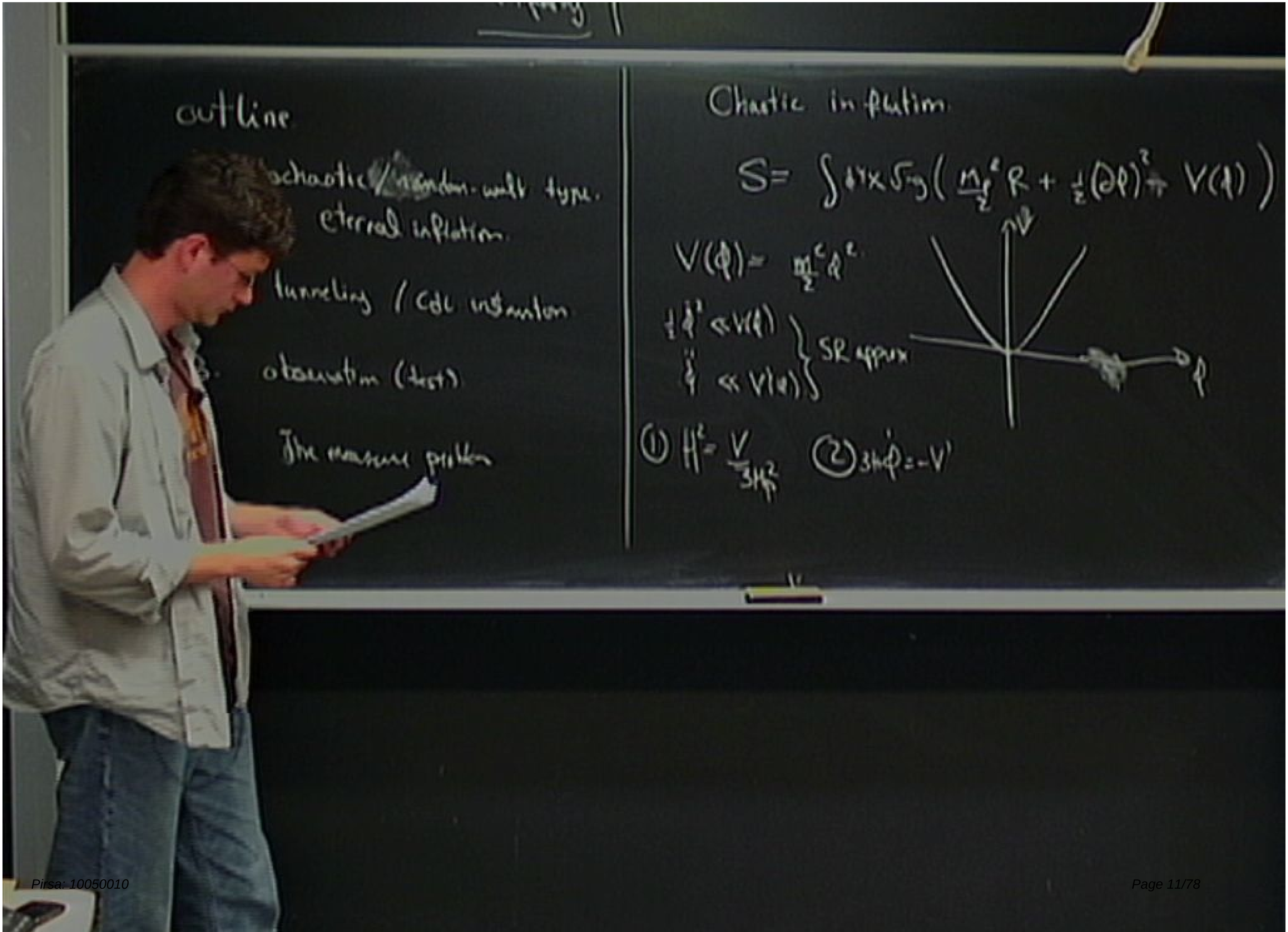

Inflation
ref
(book)
Aguirre/Johnson
Cosmological principle.
Inflation → smooth/flat
small inhomogeneities
sub-Hubble
inflation → future always.

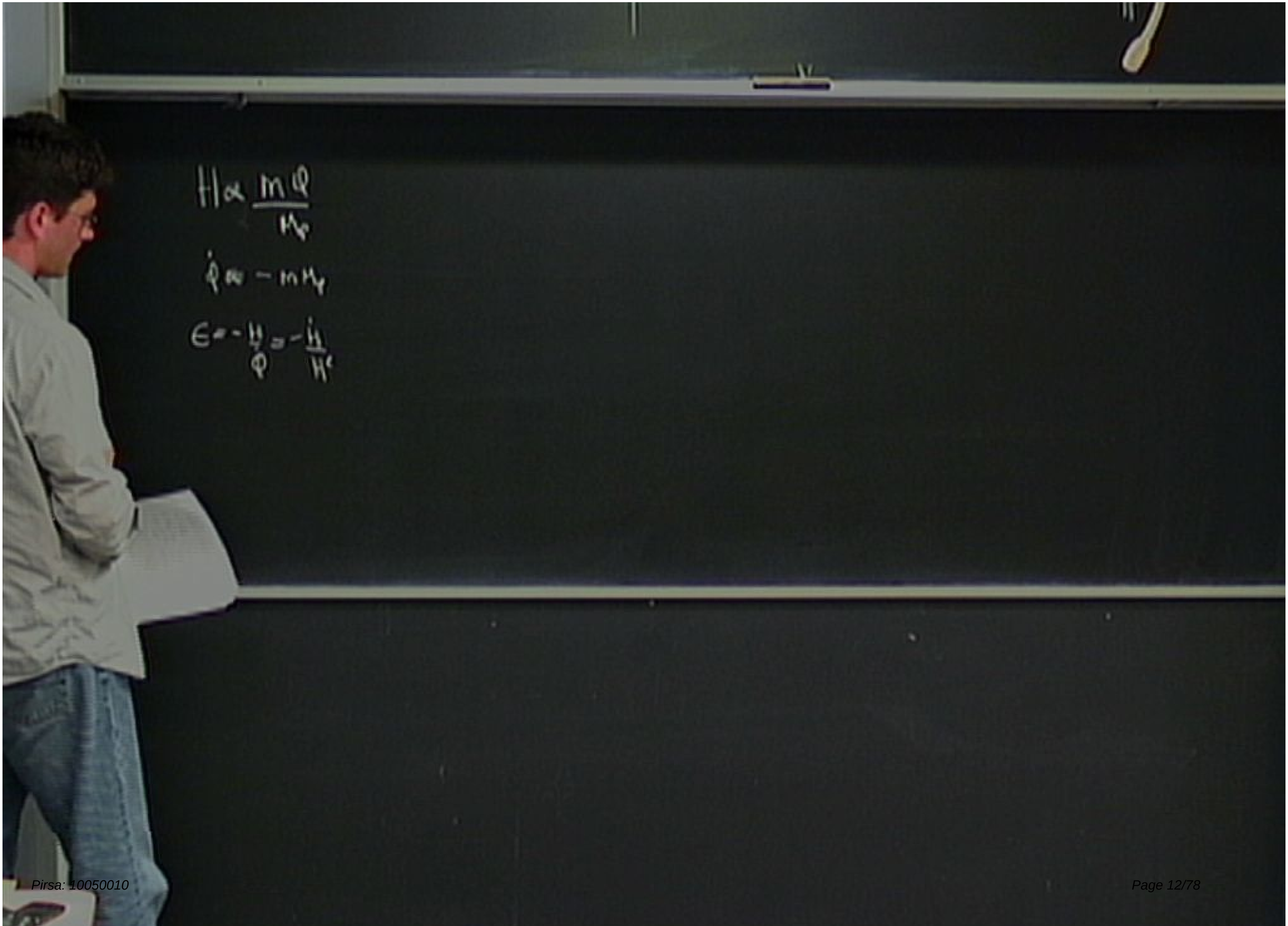

Inflation
(book)
Cosmological principle.
Inflation
small inhomogeneities
k<aH
Large inhomogeneities
k>aH

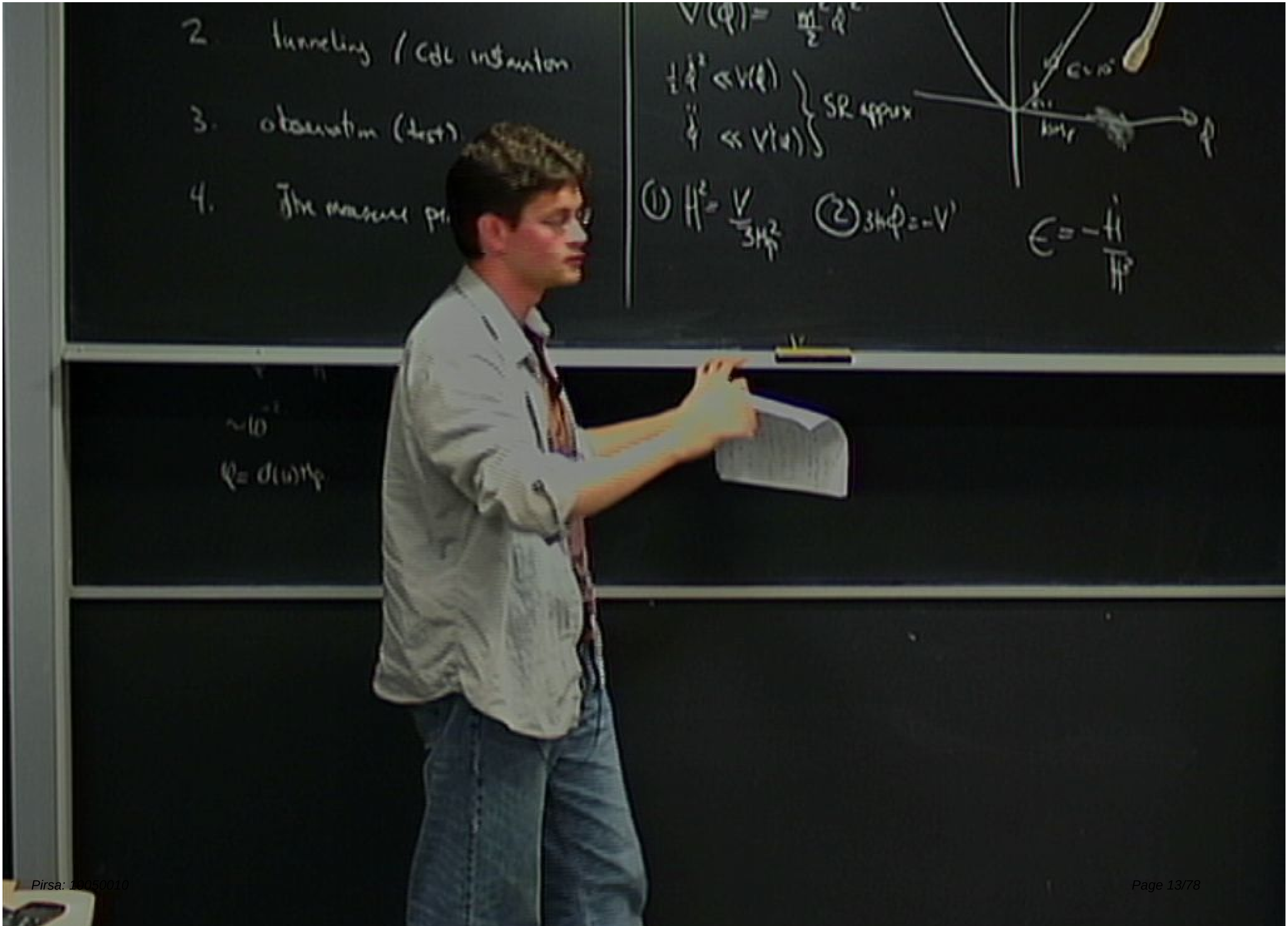

Inflation
(book)
Cosmological principle.
Inflation → smooth/flat
small inhomogeneities
Large inhomogeneities
k<aH
super-Hubble
k>aH
stochastically determined
future always.

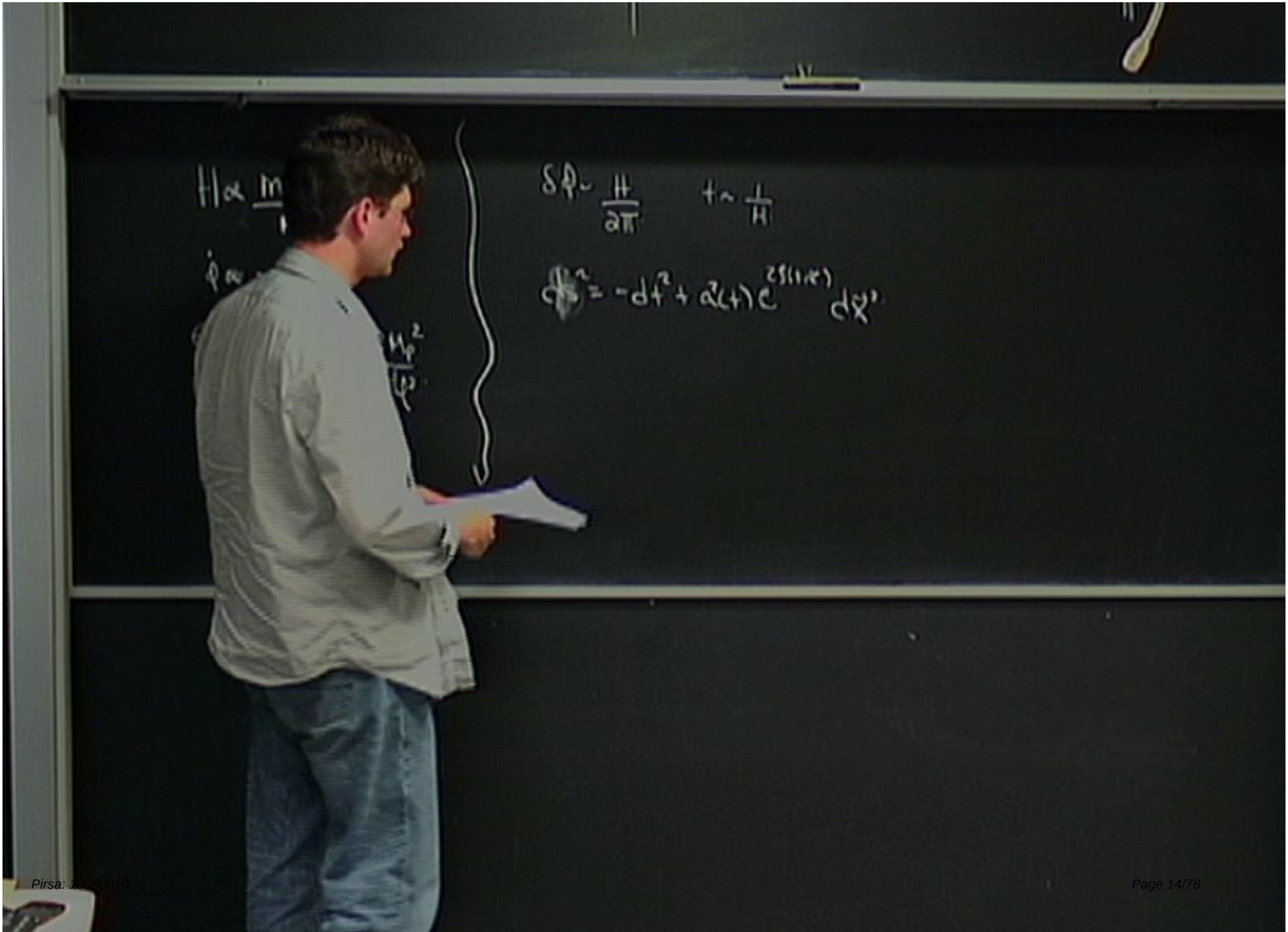


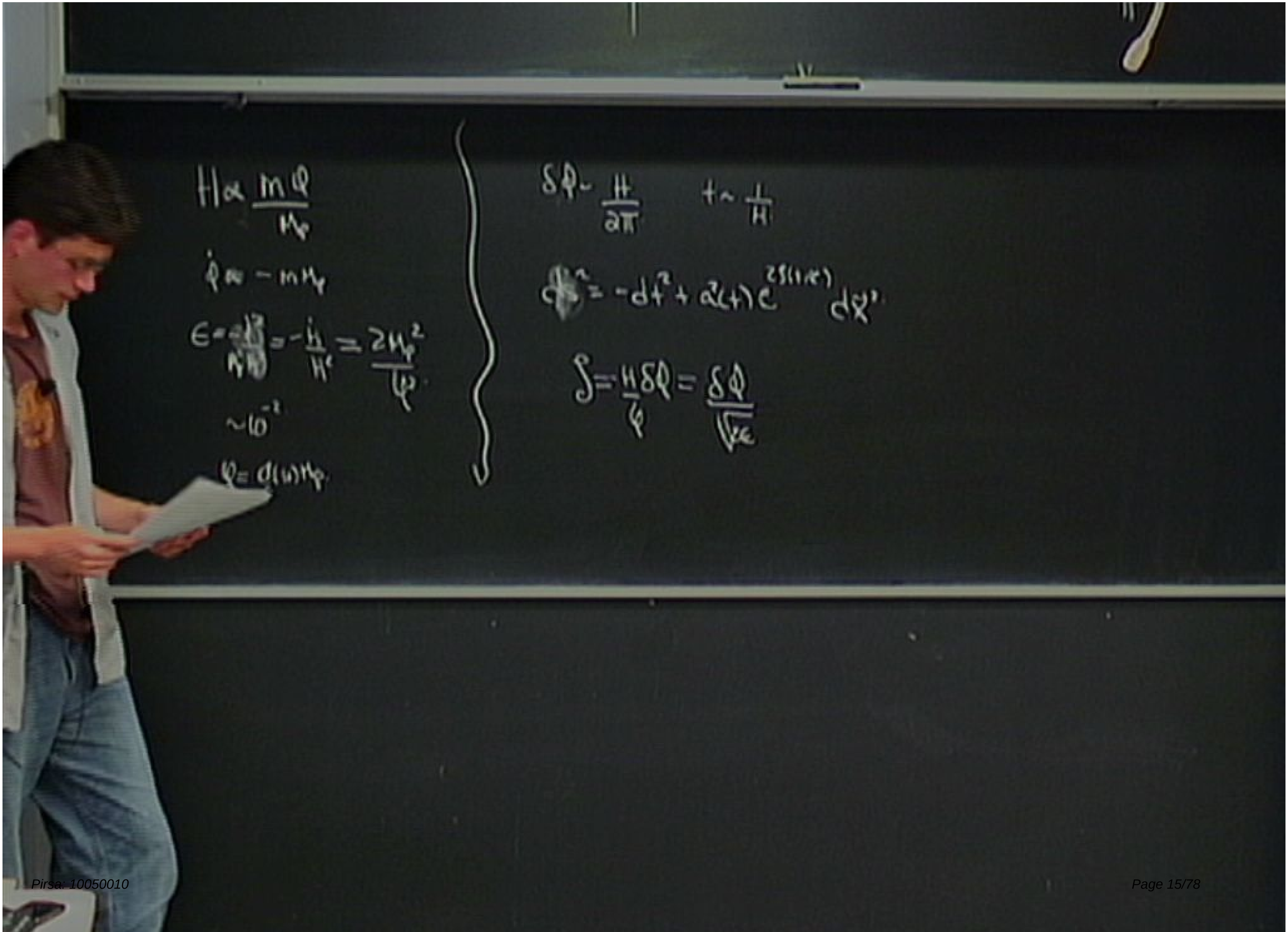

outline
1. stochastic / random-walk type.
eternal inflation
2. tunneling / CdL instanton
3. observation (test)

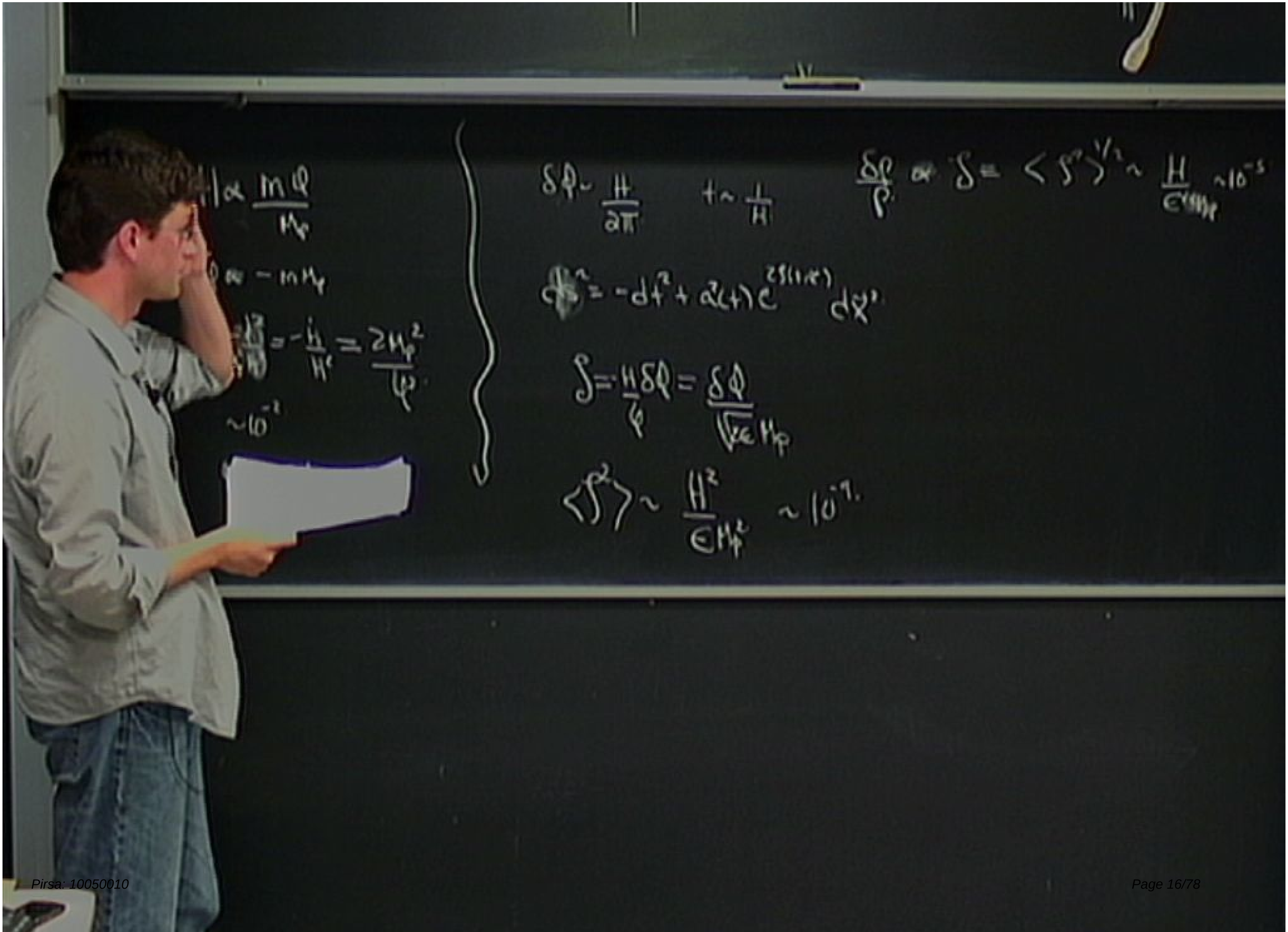

outline
chaotic / random-walk type.
eternal inflation
tunneling / CdL instanton
observation (test)
The measure problem
Chaotic inflation
SR approx


$H \propto \frac{m\varphi}{M_p}$
$\dot{\varphi} \approx - m M_p$
$\epsilon \approx - \frac{H}{\varphi} = - \frac{\dot{H}}{H^2}$

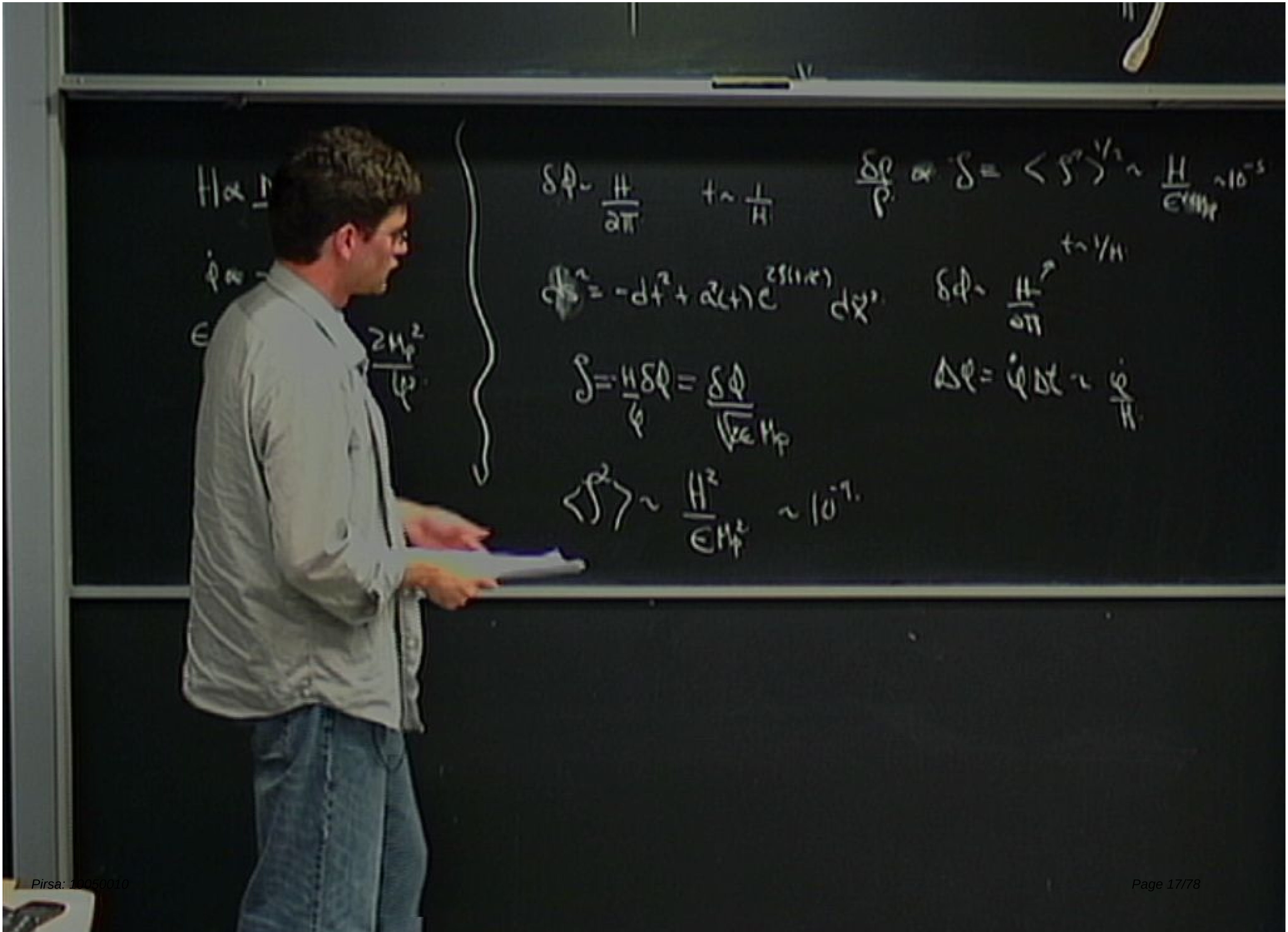

2. tunneling / CdL instanton
3. observation (test)
4. The measure pr
SR approx
① $H^2 = \frac{V}{3M_p^2}$
② $3H\dot{\phi} = -V'$
$\epsilon = -\frac{\dot{H}}{H^2}$


δφ ~ H/2π
t ~ 1/H
ds² = −dt² + a²(t) e^{2ζ(t,x)} dx²


H ∝ mφ / Mp
φ̇ ≈ − m Mp
ε = − Ḣ / H² = 2Mp² / φ²
~ 10⁻²
φ = 𝒪(10) Mp
δφ ~ H / 2π
t ~ 1 / H
= − dt² + a²(t) e^{2ζ(x,t)} dx²
ζ = H/φ̇ δφ = δφ / √2ε



$\delta\phi \sim \frac{H}{2\pi}$
$t \sim \frac{1}{H}$
$\frac{\delta\rho}{\rho} \propto \zeta = \langle \zeta^2 \rangle^{1/2} \sim \frac{H}{\epsilon^{1/2} M_p} \sim 10^{-5}$
$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,x)}\, dx^2$
$\zeta = -\frac{H}{\dot\phi}\delta\phi = \frac{\delta\phi}{\sqrt{2\epsilon}\, M_p}$
$\langle \zeta^2 \rangle \sim \frac{H^2}{\epsilon M_p^2} \sim 10^{-9}$
$\sim 10^{-2}$


$\delta\phi \sim \frac{H}{2\pi}$
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$\frac{\delta\rho}{\rho} \propto \zeta = \langle \zeta^2 \rangle^{1/2} \sim \frac{H}{\sqrt{\epsilon} M_p} \sim 10^{-5}$
$ds^2 = -dt^2 + a^2(t) e^{2\zeta(t,x)} dx^2$
$t \sim 1/H$
$\delta\phi \sim \frac{H}{2\pi}$
$\zeta = -\frac{H}{\dot\phi}\delta\phi = \frac{\delta\phi}{\sqrt{2\epsilon} M_p}$
$\Delta\phi = \dot\phi \Delta t \sim \frac{\dot\phi}{H}$
$\langle \zeta^2 \rangle \sim \frac{H^2}{\epsilon M_p^2} \sim 10^{-9}$

$$H \approx \frac{m\varphi}{M_p}$$

$$\dot\varphi \approx -m M_p$$

$$\epsilon = -\frac{\dot H}{H^2} = \frac{2M_p^2}{\varphi^2}$$

$$\sim 10^{-2}$$

$$\varphi = O(10) M_p$$

$$\delta\varphi \sim \frac{H}{2\pi} \qquad t \sim \frac{1}{H} \qquad \frac{\delta\rho}{\rho} \text{ or } \zeta = \langle \zeta^2 \rangle^{1/2} \sim \frac{H}{\sqrt{\epsilon} M_p} \sim 10^{-5}$$

$$ds^2 = -dt^2 + a^2(t) e^{2\zeta(t,x)} dx^2 \qquad \delta\varphi \sim \frac{H}{2\pi} \quad t \sim 1/H$$

$$\zeta = -\frac{H}{\dot\varphi}\delta\varphi = \frac{\delta\varphi}{\sqrt{2\epsilon} M_p} \qquad \Delta\varphi = \dot\varphi \Delta t \sim \frac{\dot\varphi}{H}$$

$$\langle \zeta^2 \rangle \sim \frac{H^2}{\epsilon M_p^2} \sim 10^{-9} \qquad \left| \frac{\delta\varphi}{\Delta\varphi} \right| = \zeta$$

eternal inflation

$\delta\phi \sim \dot{\phi}\Delta t$

$\underline{\underline{\zeta \sim 1}}$

eternal inflation.

$\delta\phi \sim \dot{\phi}\Delta t.$

$\boxed{\zeta \sim 1}$

eternal inflation

$\delta\phi \sim \dot{\phi}\Delta t$

$\boxed{\delta \sim 1}$

$$S \sim \overset{S'}{S_0} + S_1 + S_2 + \overset{(\delta^3)}{S_3}$$

$V(\phi) \sim H^2 M_p^2$

eternal inflation

$\delta\phi \sim \dot{\phi}\,\Delta t$

$\boxed{\zeta \sim 1}$

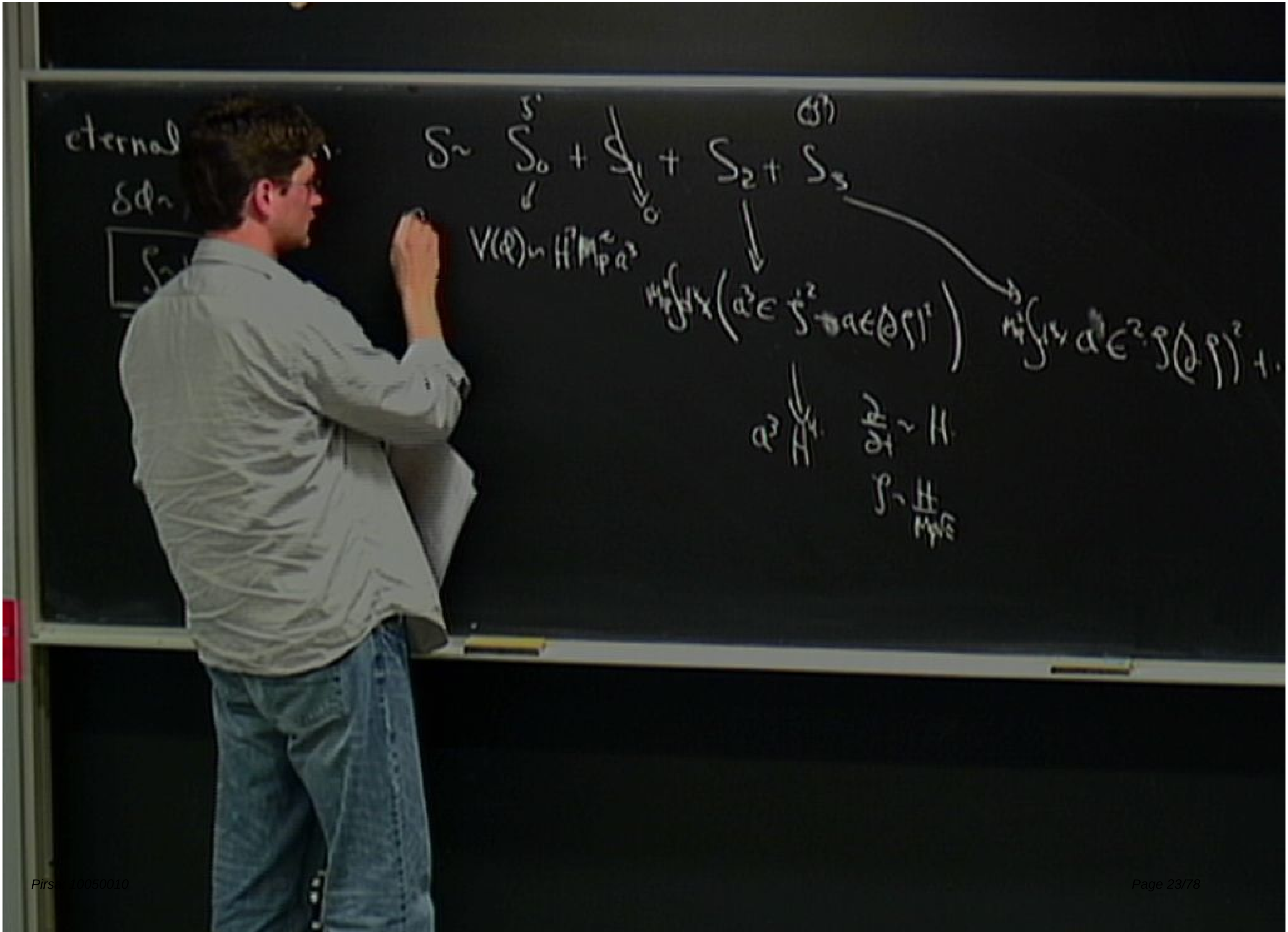
$$S \sim \overset{\zeta^0}{S_0} + \cancel{S_1} + S_2 + \overset{(\zeta^3)}{S_3}$$

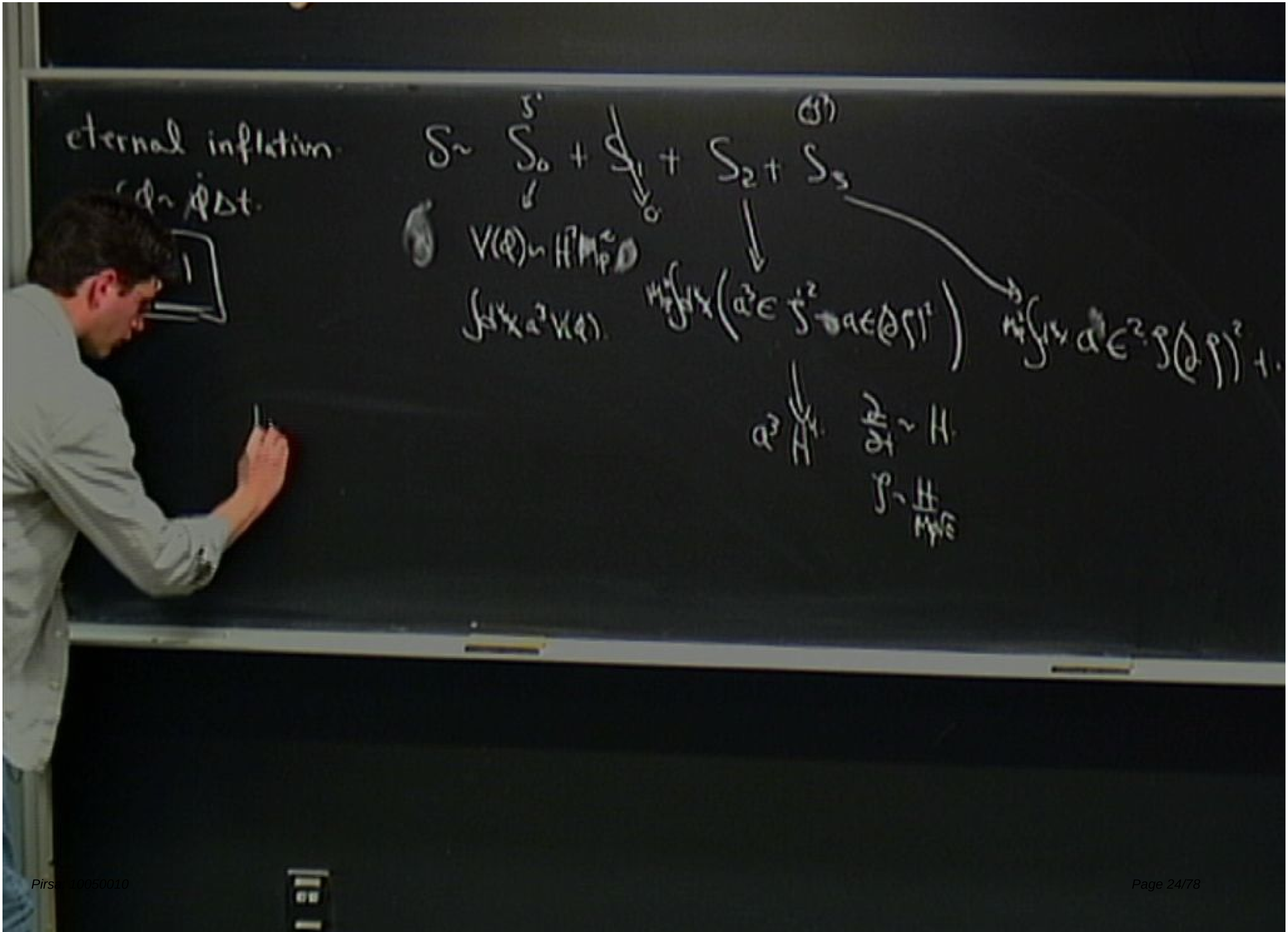
$S_0 \to V(\phi) \sim H^2 M_p^2$

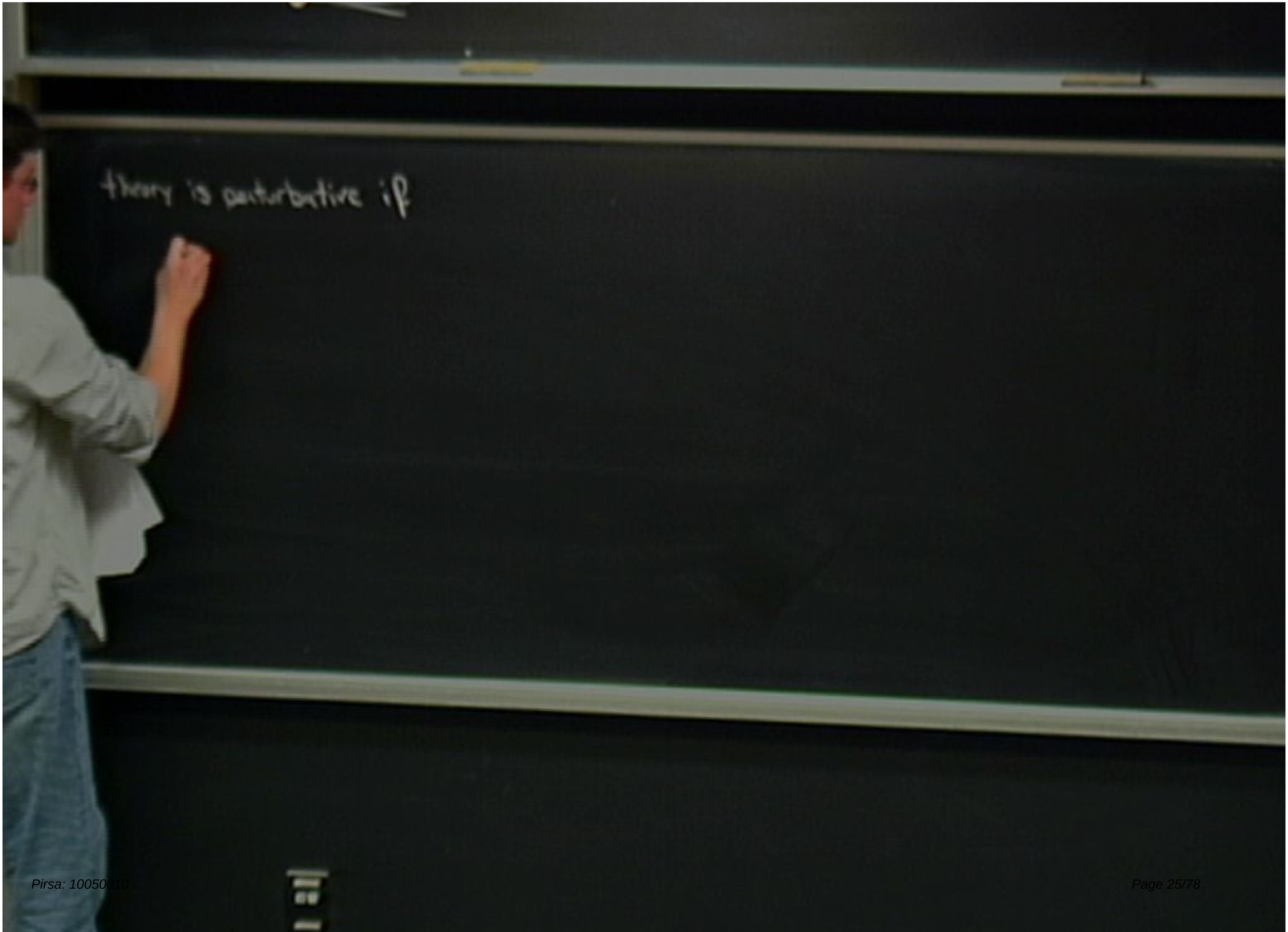
$S_1 \to 0$

$S_2 \to \int d^4x \left(a^3 \epsilon \dot{\zeta}^2 - a \epsilon (\partial\zeta)^2 \right)$

$S_3 \to \int d^4x\, a^3 \epsilon^2\, \zeta (\partial \zeta)^2 + \ldots$


eternal
S ~ S_0 + S_1 + S_2 + S_3
V(φ) ~ H^2 M_p^2 a^3
a^3 H^4
∂/∂t ~ H
ζ ~ H/(M_p √ε)


eternal inflation
$\delta\phi \sim \dot{\phi}\Delta t$
$S \sim S_0 + S_1 + S_2 + S_3$
$V(\phi) \sim H^2 M_p^2$
$\int d^4x\, a^3 V(\phi)$
$M_p^2 \int d^4x \left(a^3 \epsilon \dot{\zeta}^2 - a \epsilon (\partial\zeta)^2\right)$
$M_p^2 \int d^4x\, a^3 \epsilon^2 \zeta (\partial\zeta)^2 + \dots$
$a^3 H^4$
$\frac{\partial}{\partial t} \sim H$
$\zeta \sim \frac{H}{M_p\sqrt{\epsilon}}$


theory is perturbative if

theory is perturbative if

$$\epsilon^{1/2} \frac{H^5}{M_p} < H^4$$

$$\frac{H}{M_p\sqrt{\epsilon}} \ll \frac{1}{\epsilon}$$

$$\boxed{\zeta \ll \frac{1}{\epsilon}}$$

theory is perturbative if

$$\epsilon^{1/2} \frac{H^5}{M_p} < H^4$$

$$\frac{H}{M_p \sqrt{\epsilon}} \ll \frac{1}{\epsilon}$$

$$\boxed{\rho \ll \frac{1}{\epsilon}}$$

Description of stochastic eternal inflation

(1)

theory is perturbative if

$$\frac{\epsilon^{1/2} H^5}{M_P} < H^4$$

$$\frac{H}{M_P\sqrt{\epsilon}} \ll \frac{1}{\epsilon}$$

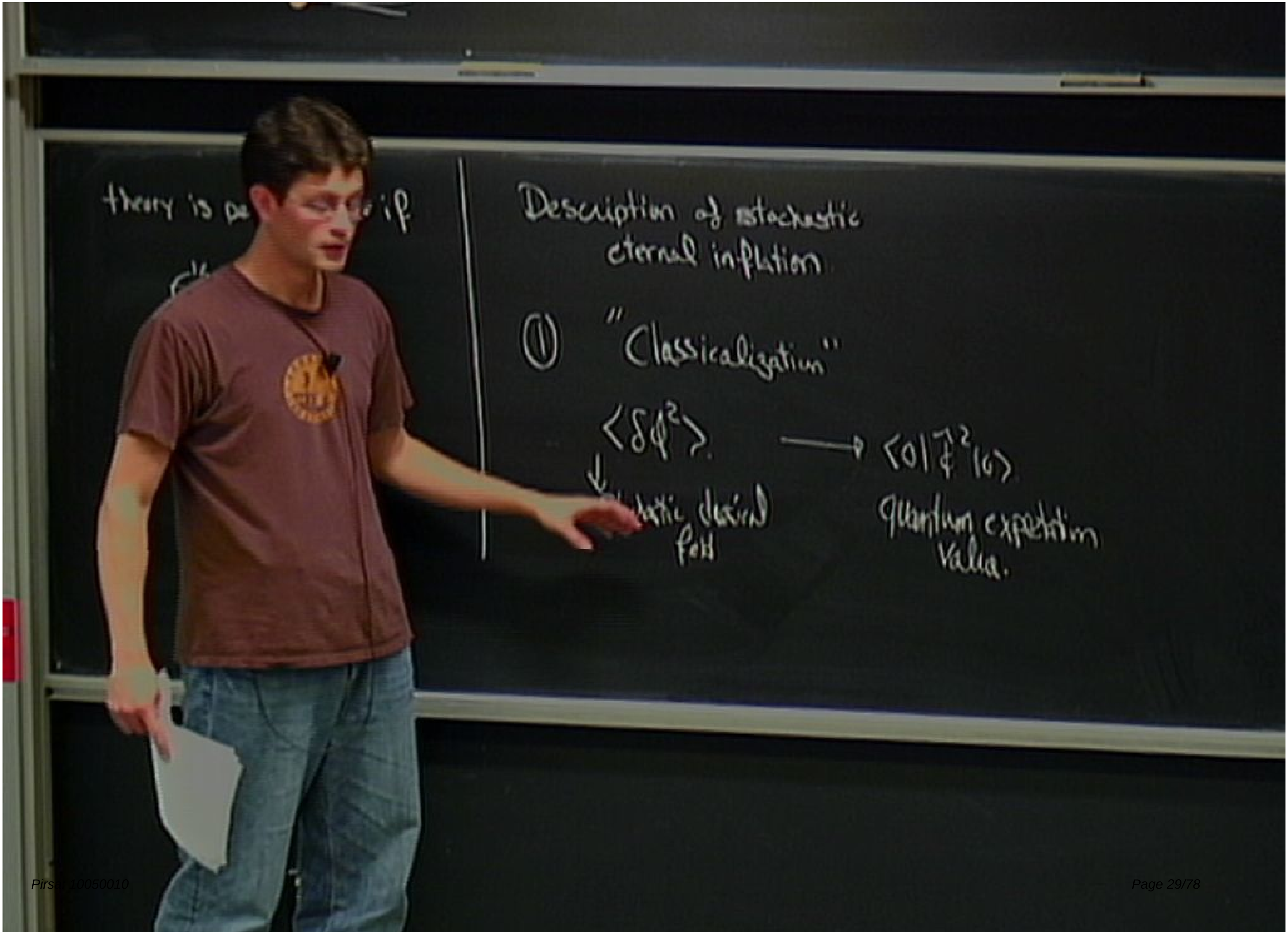
$$\boxed{\zeta \ll \frac{1}{\epsilon}}$$

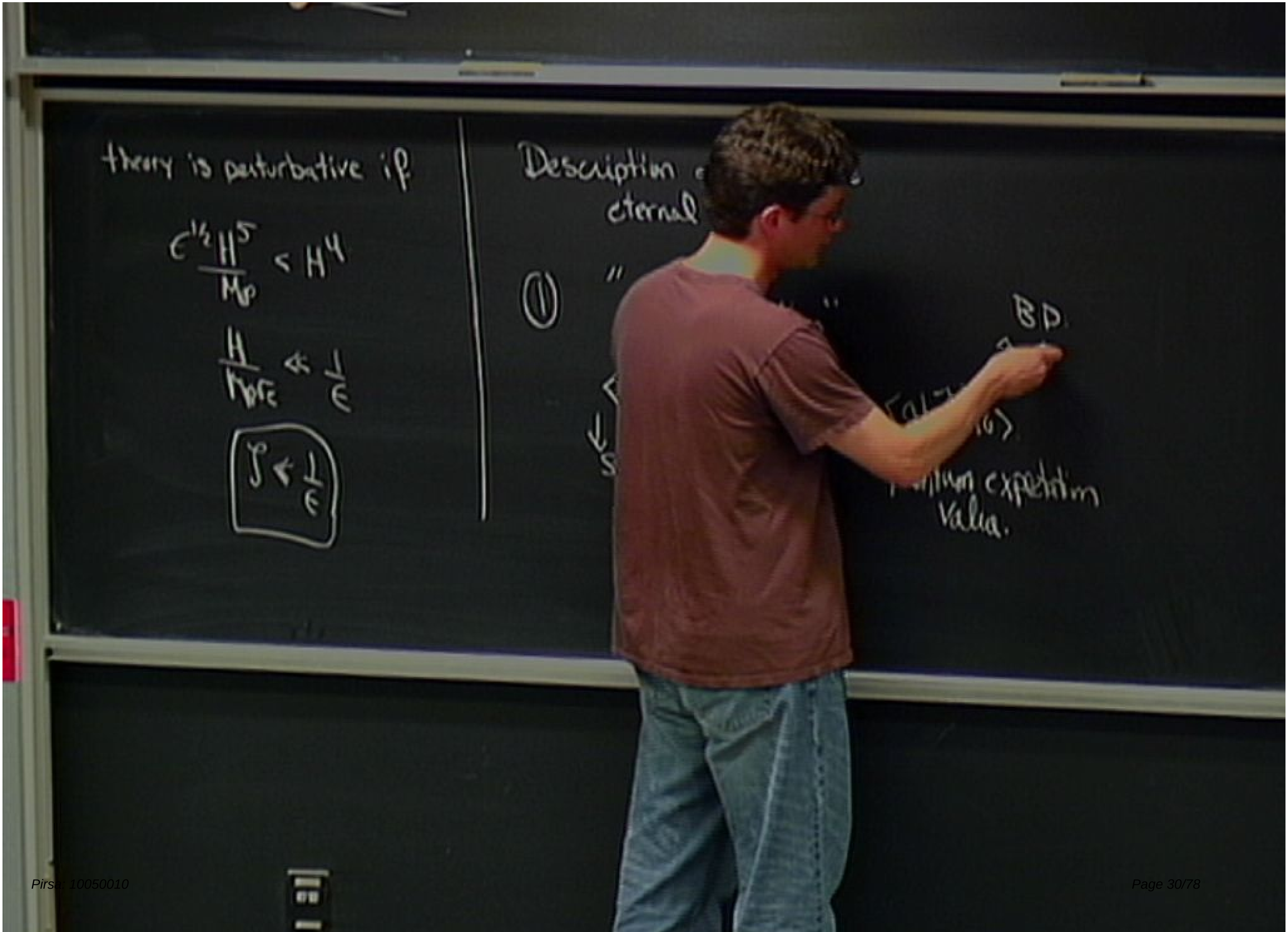
Description of stochastic eternal inflation

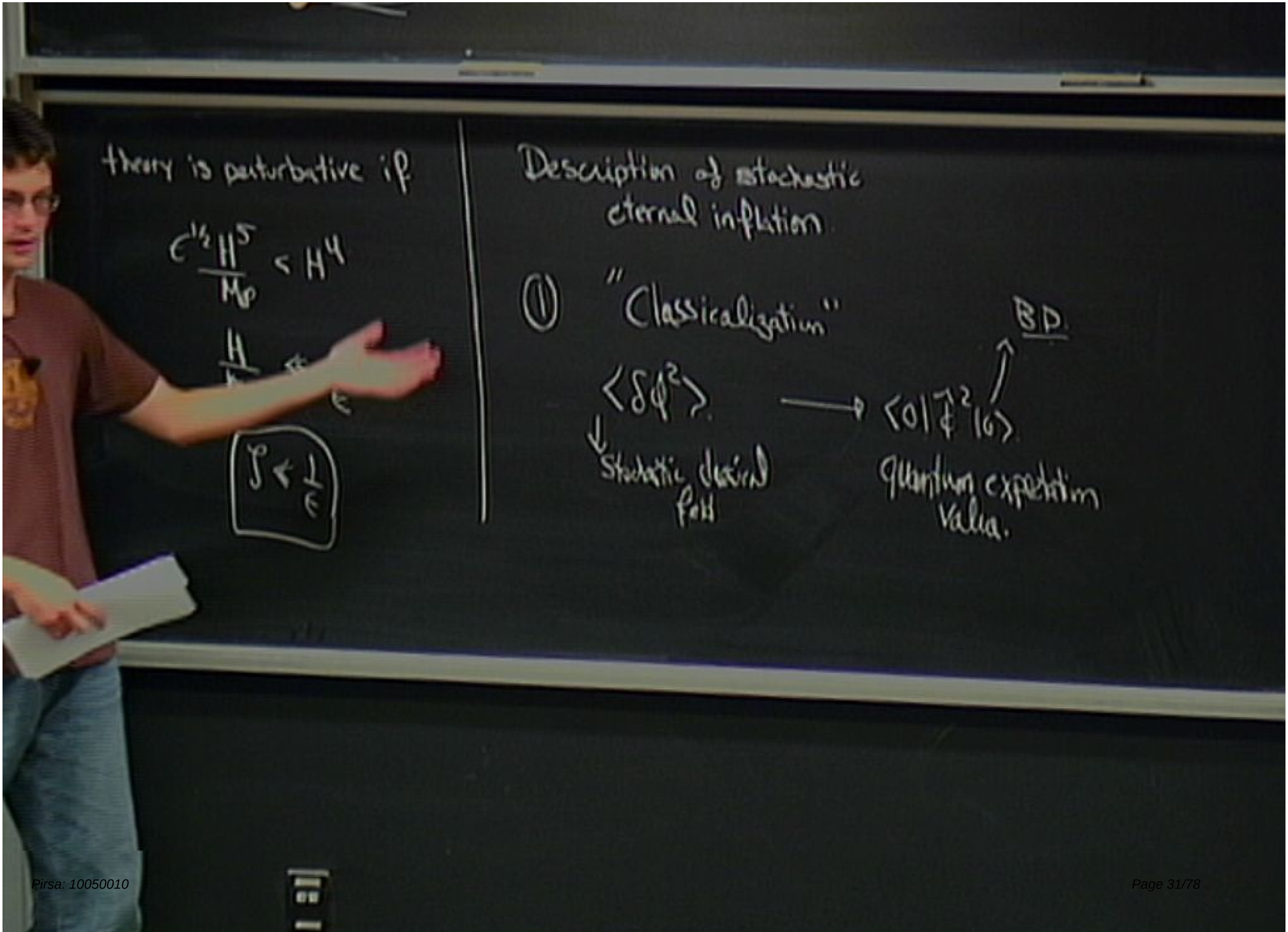
① "Classicalization"

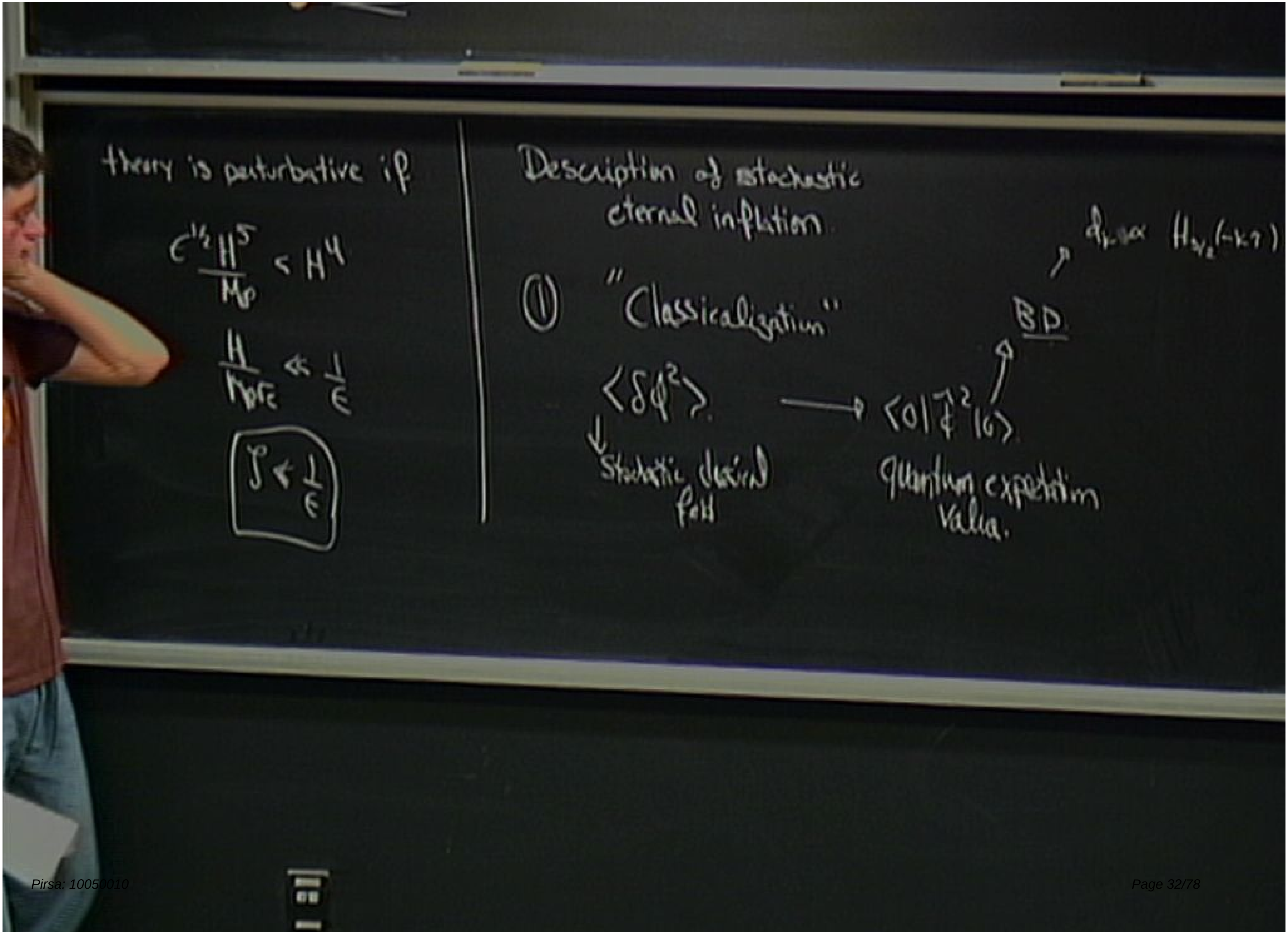
$\langle \delta\phi^2 \rangle \longrightarrow \langle 0 | \hat{\phi}^2 | 0 \rangle$

$\downarrow$ Stochastic classical field

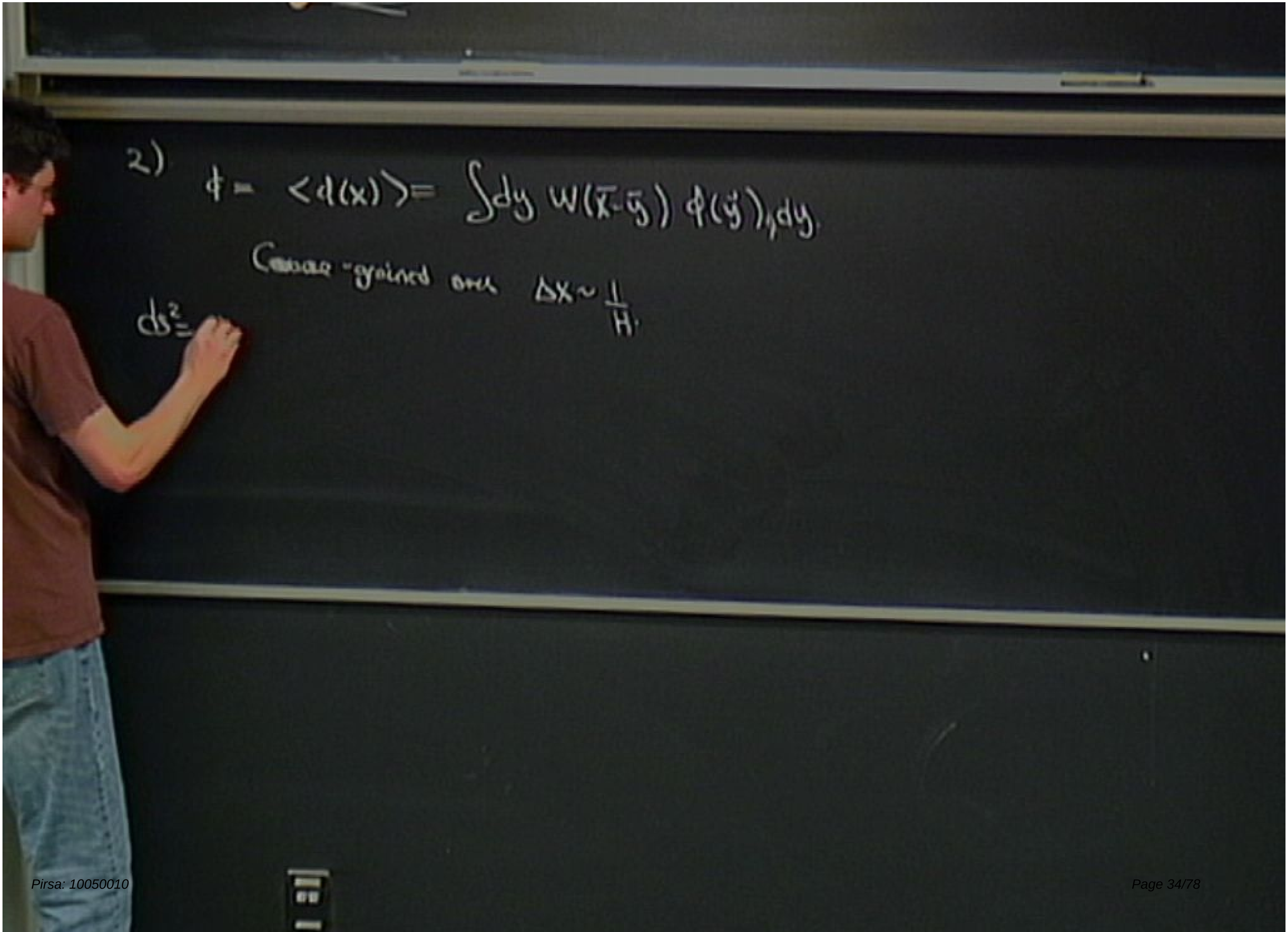

Description of stochastic
eternal inflation
① "Classicalization"
⟨δφ²⟩
⟨0|φ̂²|0⟩
classical field
quantum expectation value.

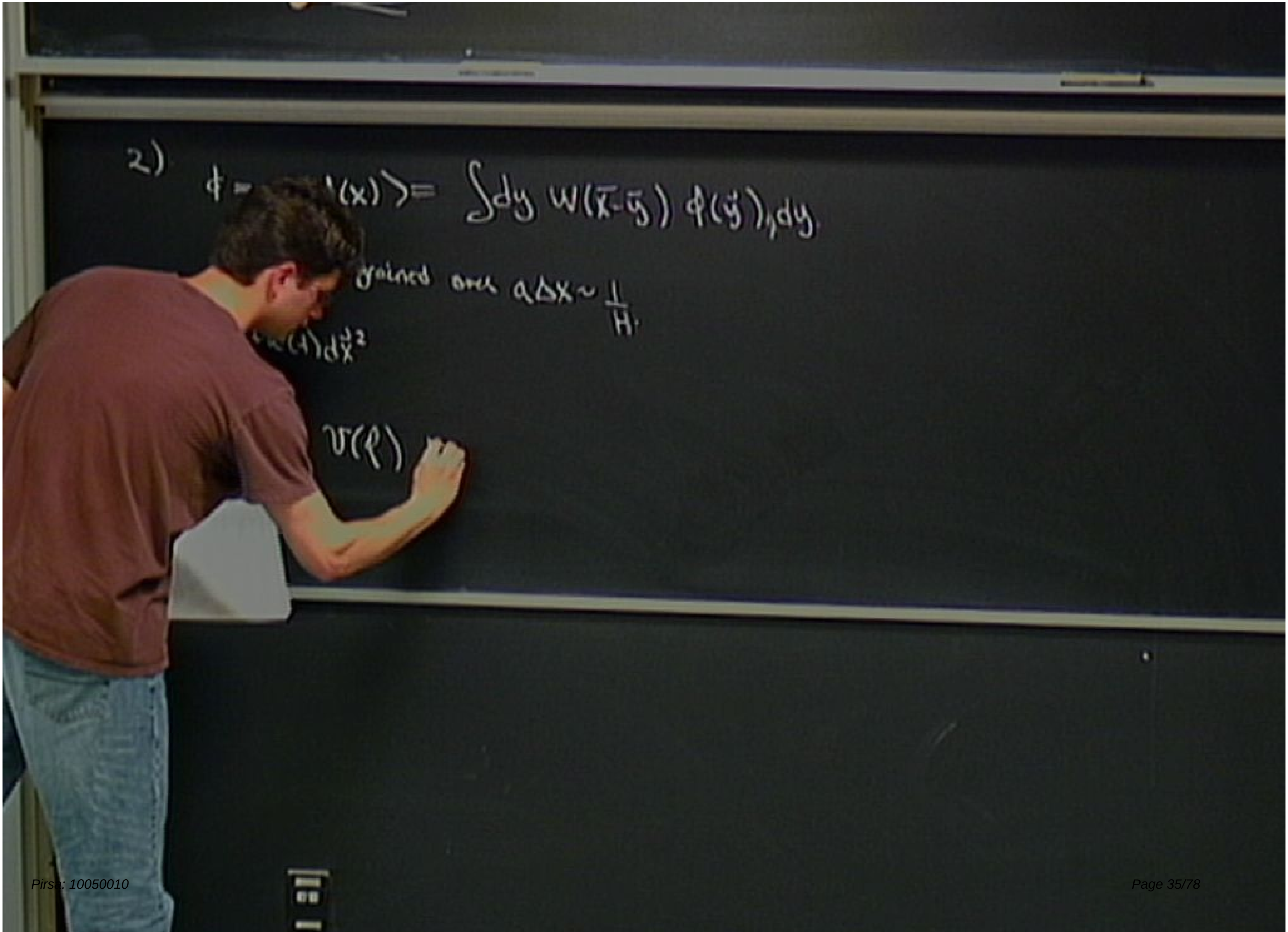

theory is perturbative if
$\epsilon^{1/2} \frac{H^5}{M_p} < H^4$
$\frac{H}{M_{pl}\epsilon} \ll \frac{1}{\epsilon}$
$\boxed{\rho \ll \frac{1}{\epsilon}}$
Description
eternal
BD
expectation
value.


theory is perturbative if
$\epsilon^{1/2} \frac{H^5}{M_p} < H^4$
$\boxed{\rho < \frac{1}{\epsilon}}$
Description of stochastic eternal inflation
① "Classicalization"
B.D.
$\langle \delta\phi^2 \rangle \longrightarrow \langle 0 | \hat{\phi}^2 | 0 \rangle$
Stochastic classical field
quantum expectation value.


theory is perturbative if
$\epsilon^{1/2}\frac{H^5}{M_p} < H^4$
$\frac{H}{M_p\sqrt{\epsilon}} \ll \frac{1}{\epsilon}$
$\boxed{\rho \ll \frac{1}{\epsilon}}$
Description of stochastic eternal inflation
① "Classicalization"
$\langle \delta\phi^2 \rangle \longrightarrow \langle 0|\hat{\phi}^2|0\rangle$
Stochastic classical field
quantum expectation value.
B.D.
$\phi_k \propto H_{3/2}(-k\tau)$

2) $\phi = \langle \phi(x) \rangle = \int dy\, W(\bar{x}-\bar{y})\, \phi(\vec{y}),\, dy$


2)
φ = <φ(x)> = ∫dy W(x̄ - ȳ) φ(ȳ), dy
Coarse-grained over Δx ~ 1/H
ds² =


2)
(x)⟩= ∫dy W(x̄-ȳ) φ(ȳ),dy
grained over aΔx ~ 1/H.
(t) dx²
V(φ)

2) $\phi = \langle \phi(x) \rangle = \int dy\, W(\vec{x}-\vec{y})\, \phi(\vec{y}), dy$

Coarse-grained over $a\Delta x \sim \frac{1}{H}$

$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$

$\dot{\phi} = v(\phi) +$

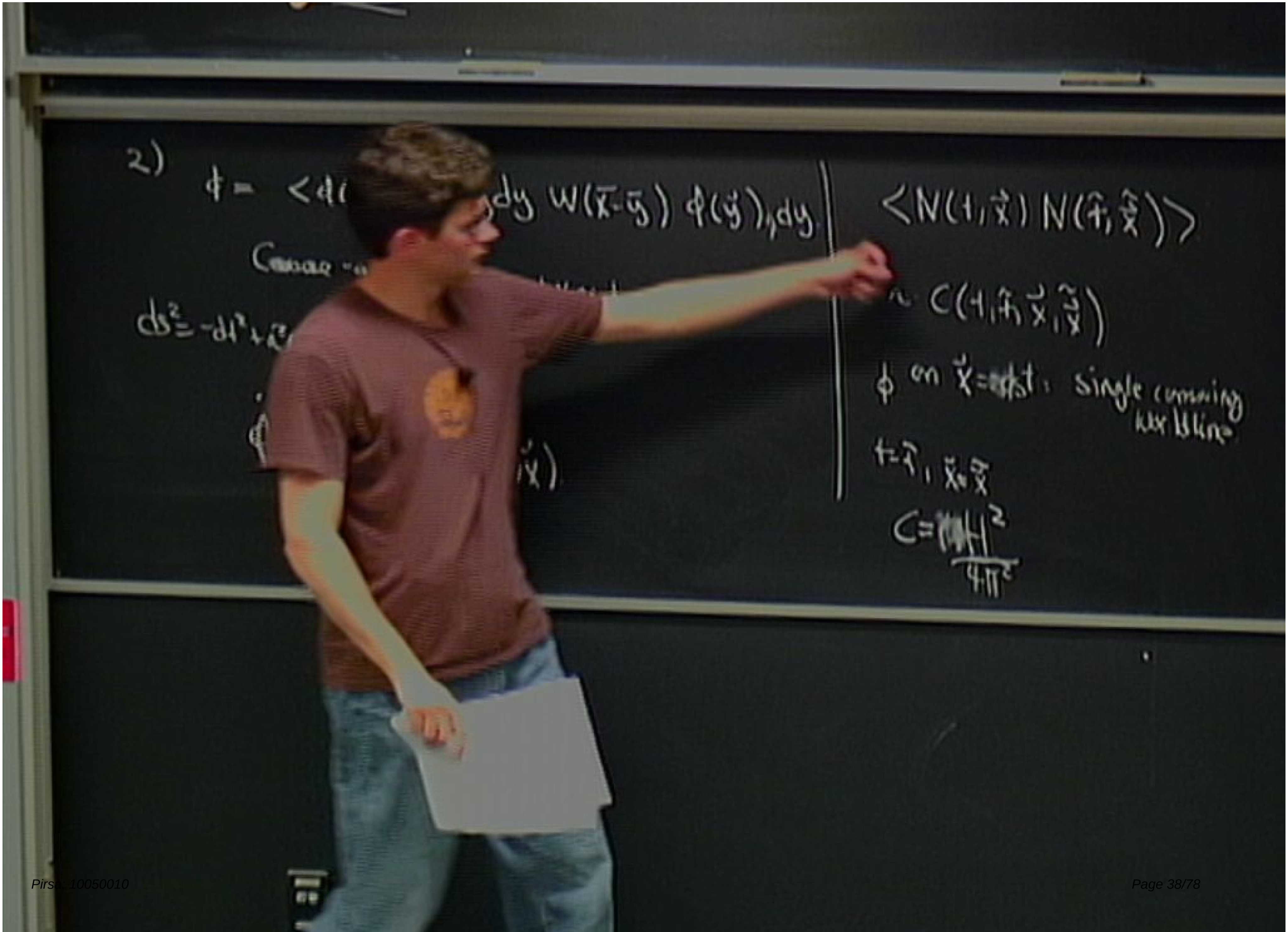
2) $\phi = \langle \phi(x) \rangle = \int dy\, W(\vec{x}-\vec{y})\, \phi(\vec{y}), dy$

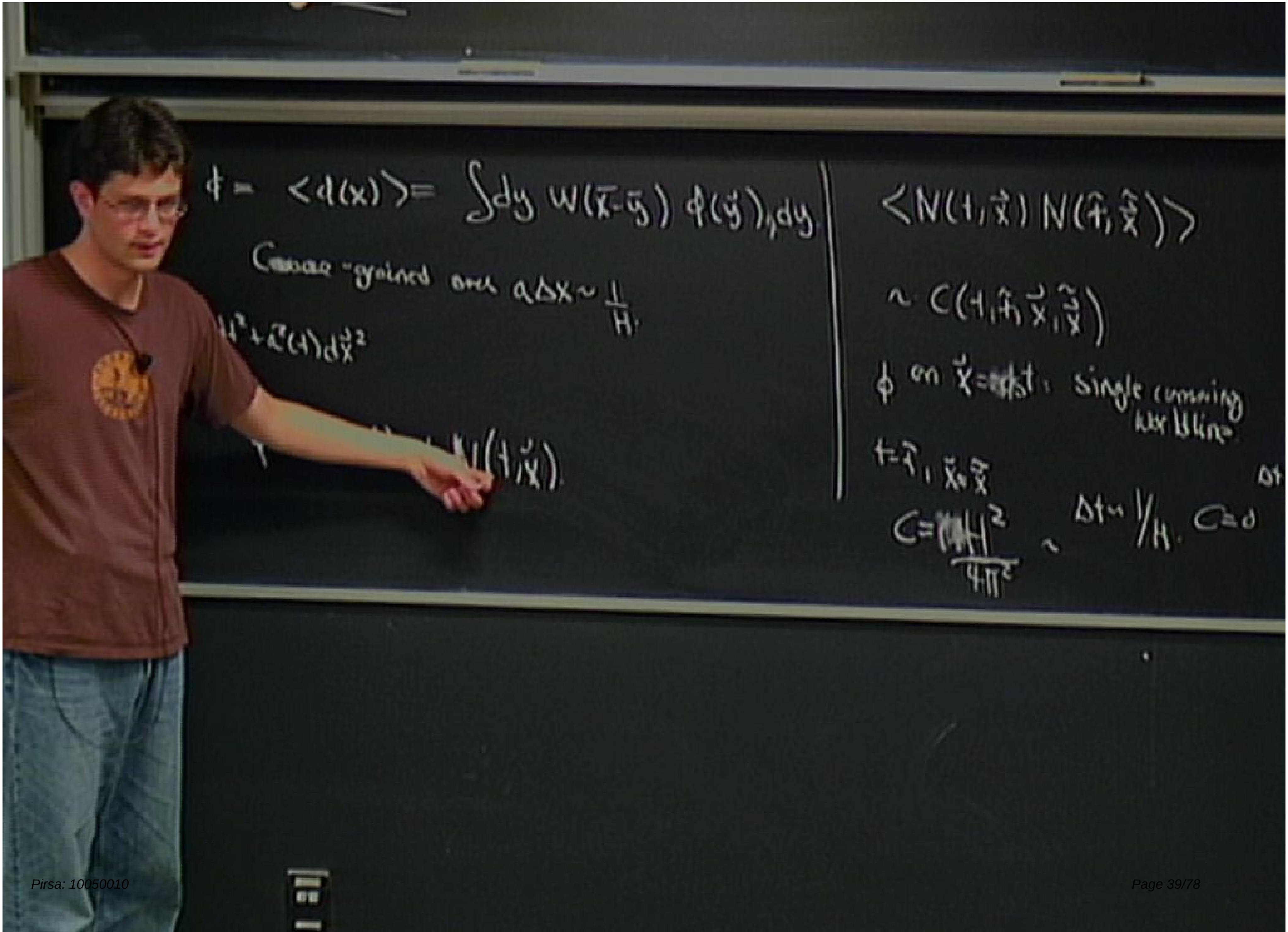
Coarse-grained over $a\Delta x \sim \frac{1}{H}$

$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$

$\dot{\phi} = v(\phi) + N(t, \vec{x})$

$\langle N(t, \vec{x})\, N(\hat{t}, \hat{\vec{x}}) \rangle$


2)
$\langle N(t,\vec{x})\, N(\tilde{t},\tilde{\vec{x}}) \rangle$
$\sim C(t,\tilde{t},\vec{x},\tilde{\vec{x}})$
single
$t=\tilde{t},\ \vec{x}=\tilde{\vec{x}}$



Coarse-grained over a Δx ~ 1/H
<N(t, x) N(t̃, x̃)>
~ C(t, t̃, x, x̃)
ϕ on x = const : single comoving worldline
C = H²/4π²
Δt ~ 1/H. C = 0

theory is perturbative if

$$\epsilon^{1/2}\frac{H^5}{M_p} < H^4$$

$$\frac{H}{M_{Pl}} \ll \frac{1}{\epsilon}$$

$$\boxed{\mathcal{P} \ll \frac{1}{\epsilon}}$$

Description of stochastic eternal inflation

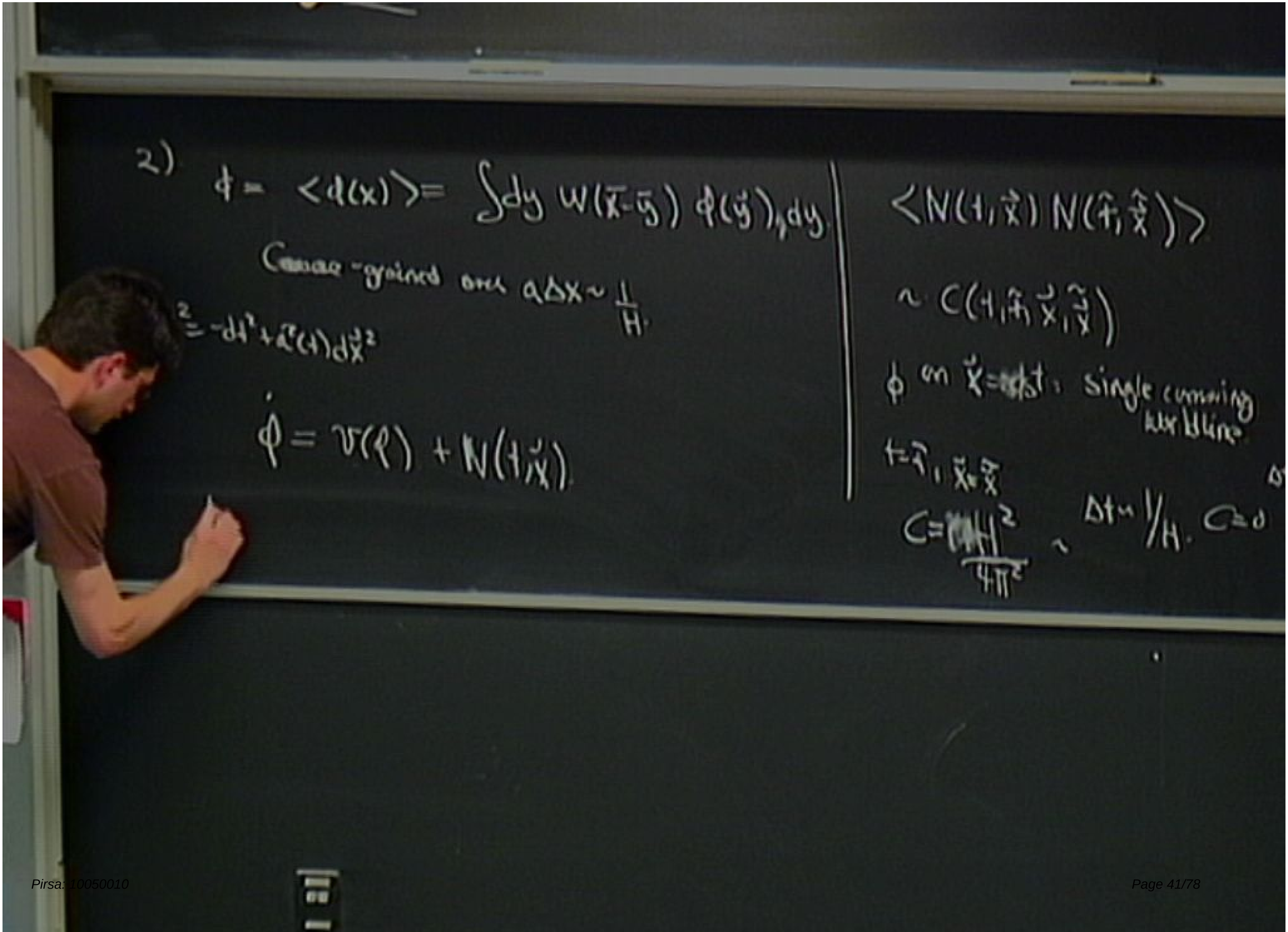
(1) "Classicalization"

$\langle \delta\phi^2 \rangle \longrightarrow \langle 0 | \hat{\phi}^2 | 0 \rangle \to$ B.D. $\to \phi_k \propto H_{3/2}(-k\tau)$

stochastic classical field

quantum expectation value.

$$\delta\phi_k \sim \frac{H^2}{(2\pi)^2}$$


2) φ = ⟨φ(x)⟩ = ∫dy W(x⃗ − y⃗) φ(y⃗), dy
Coarse-grained over aΔx ~ 1/H
ds² = −dt² + a²(t) dx⃗²
φ̇ = V(φ) + N(t, x⃗)
⟨N(t, x⃗) N(t̂, x̂⃗)⟩
~ C(t, t̂, x⃗, x̂⃗)
φ on x⃗ = const: single comoving worldline
t = t̂, x⃗ = x̂⃗
C = H²/4π²
Δt ~ 1/H. C = 0

2) $\phi = \langle \phi(x) \rangle = \int dy\, W(\vec{x}-\vec{y})\, \phi(\vec{y}), dy$

Coarse-grained over $a\Delta x \sim \frac{1}{H}$

$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$

$\dot{\phi} = v(\phi) + N(t, \vec{x})$ $\quad \langle N^2 \rangle \sim 1$

diffusion

$\Delta\phi = \phi(t+\Delta t) - \phi(t) = v(\phi)\Delta t + \sqrt{2D(\phi)\Delta t}\, \xi(t)$

drift

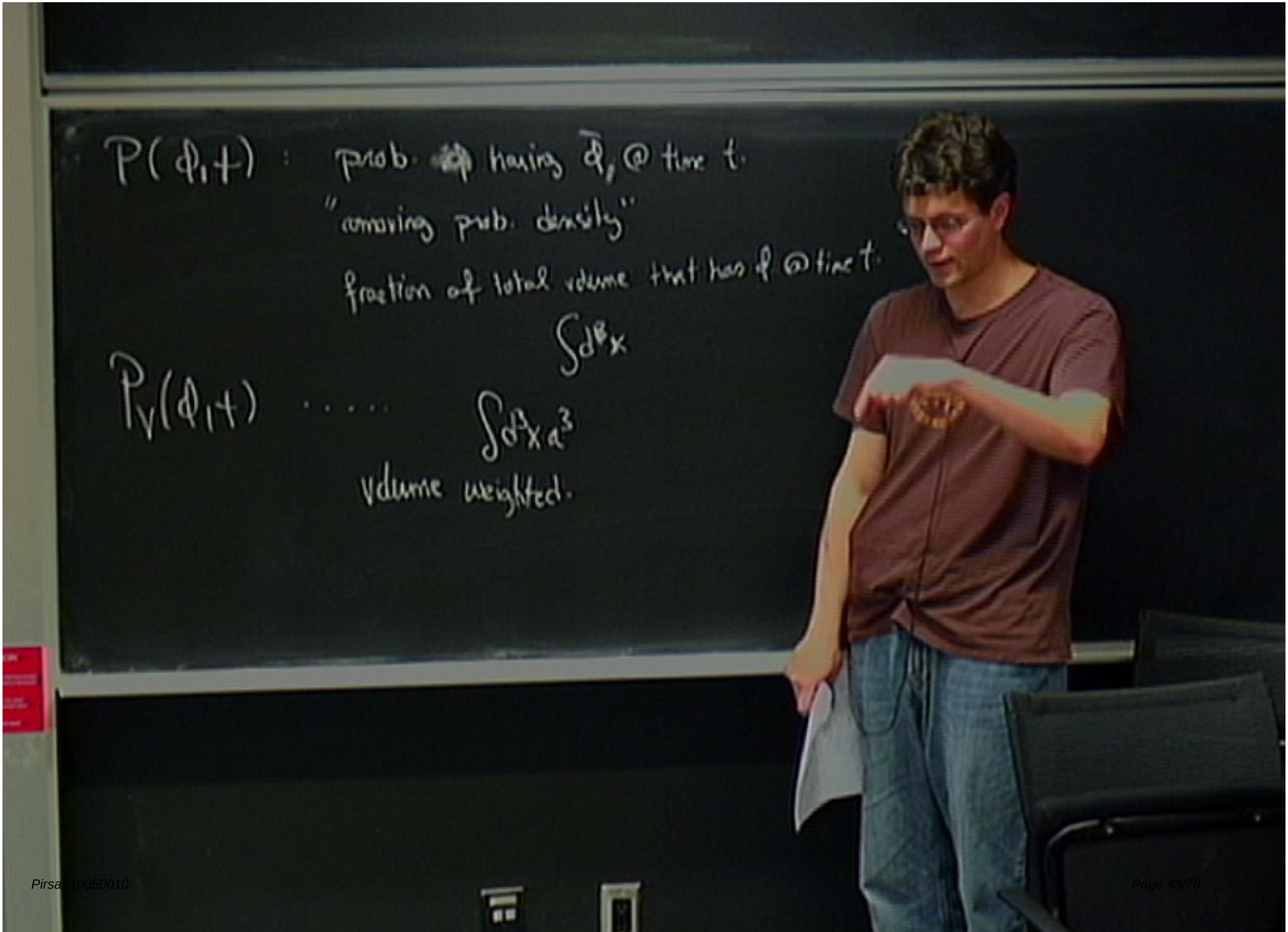
$\langle N(t, \vec{x})\, N(\hat{t}, \hat{\vec{x}}) \rangle$

$\sim C(t, \hat{t}, \vec{x}, \tilde{\vec{x}})$

ϕ on $\vec{x} = \text{const}$: single comoving worldline

$t = \tilde{t},\ \vec{x} = \tilde{\vec{x}}$

$C = \frac{H^2}{4\pi^2} \sim$ $\quad \Delta t \sim 1/H$. $\quad C = 0$


P(φ,t) : prob. having φ, @ time t.
"comoving prob. density"
fraction of total volume that has φ @ time t.
∫d³x
P_V(φ,t) . . . . .
∫d³x a³
volume weighted.

$P(\phi, t)$: prob. having $\bar{\phi}$ @ time t.

"comoving prob. density"

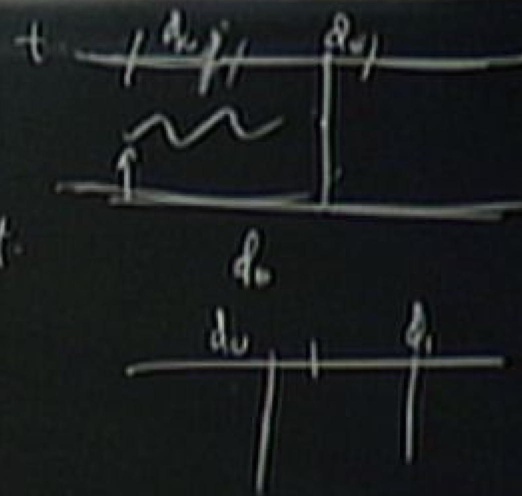
fraction of total volume that has ϕ @ time t.

$$\int d^3x$$

$P_V(\phi, t)$ . . . . .

$$\int d^3x \, a^3$$

volume weighted.



$P(\phi, t)$: prob. having $\bar{\phi}$, @ time t.

"comoving prob. density"

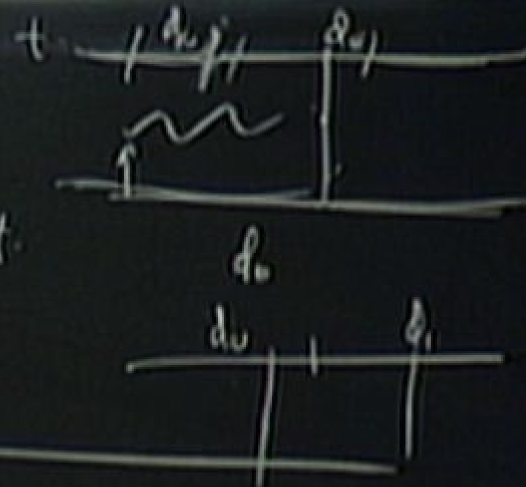
fraction of total volume that has ϕ @ time t.

$\int d^3x$

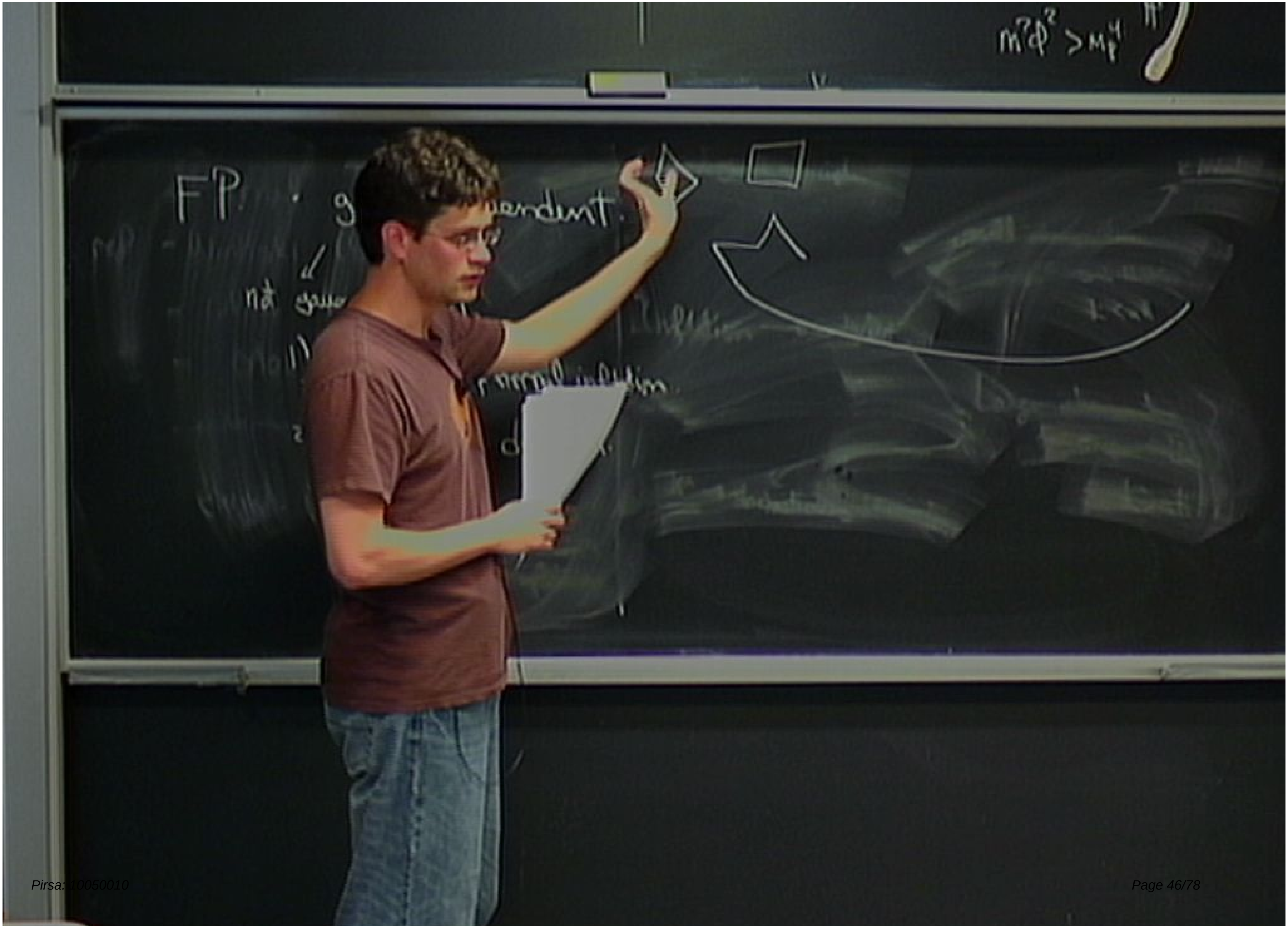
$P_V(\phi, t)$ . . . . .

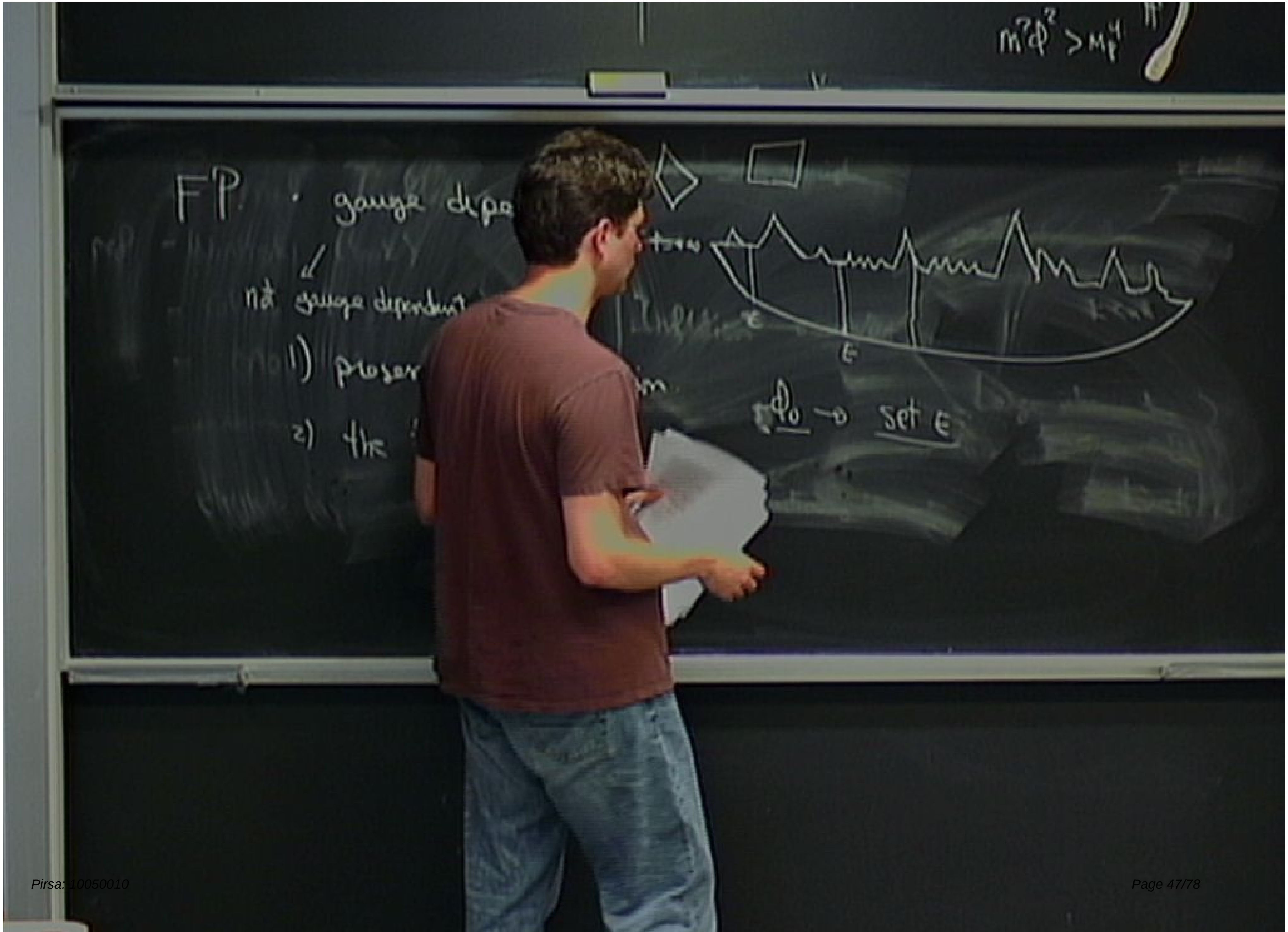
$\int d^3x\, a^3$

volume weighted.



$$\partial_t P(\phi, t) = \partial_\phi\left(-v'(\phi) P + \partial_\phi (\Gamma(\phi) P)\right)$$


FP


m²φ² > Mp⁴
FP · gauge depe
not gauge dependent
1) preser
2) the
φ0 → set ε

$m^2\phi^2 > M_p^4$

FP · gauge dependent.

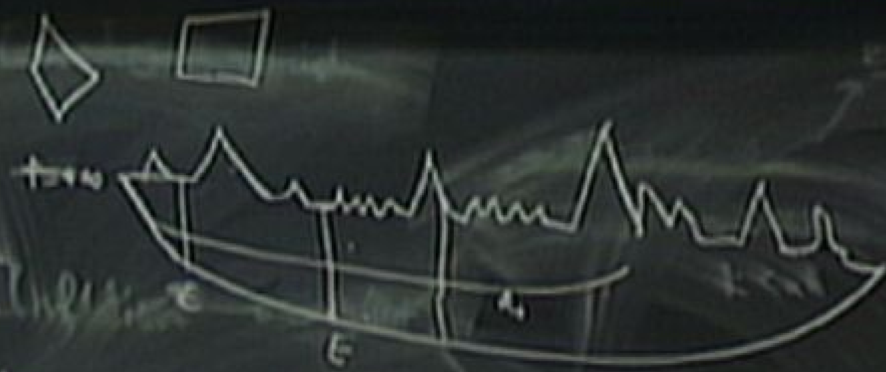
⇓

not gauge dependent

1) presence of eternal inflation

2) the fractal dimension

3)



$\phi_0 \to$ set E

⇓

$\langle \phi_0 \rangle \to$ set F

$P(\phi,t)$: prob. having ϕ, @ time t.

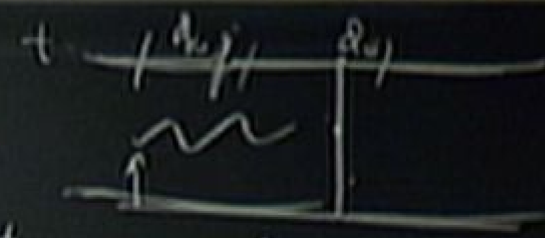
"comoving prob. density"

fraction of total volume that has ϕ @ time t.

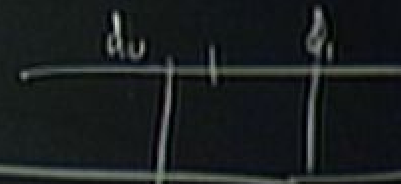
$\int d^3x$

$P_V(\phi,t)$ . . . . . . $\int d^3x\, a^3$

volume weighted.



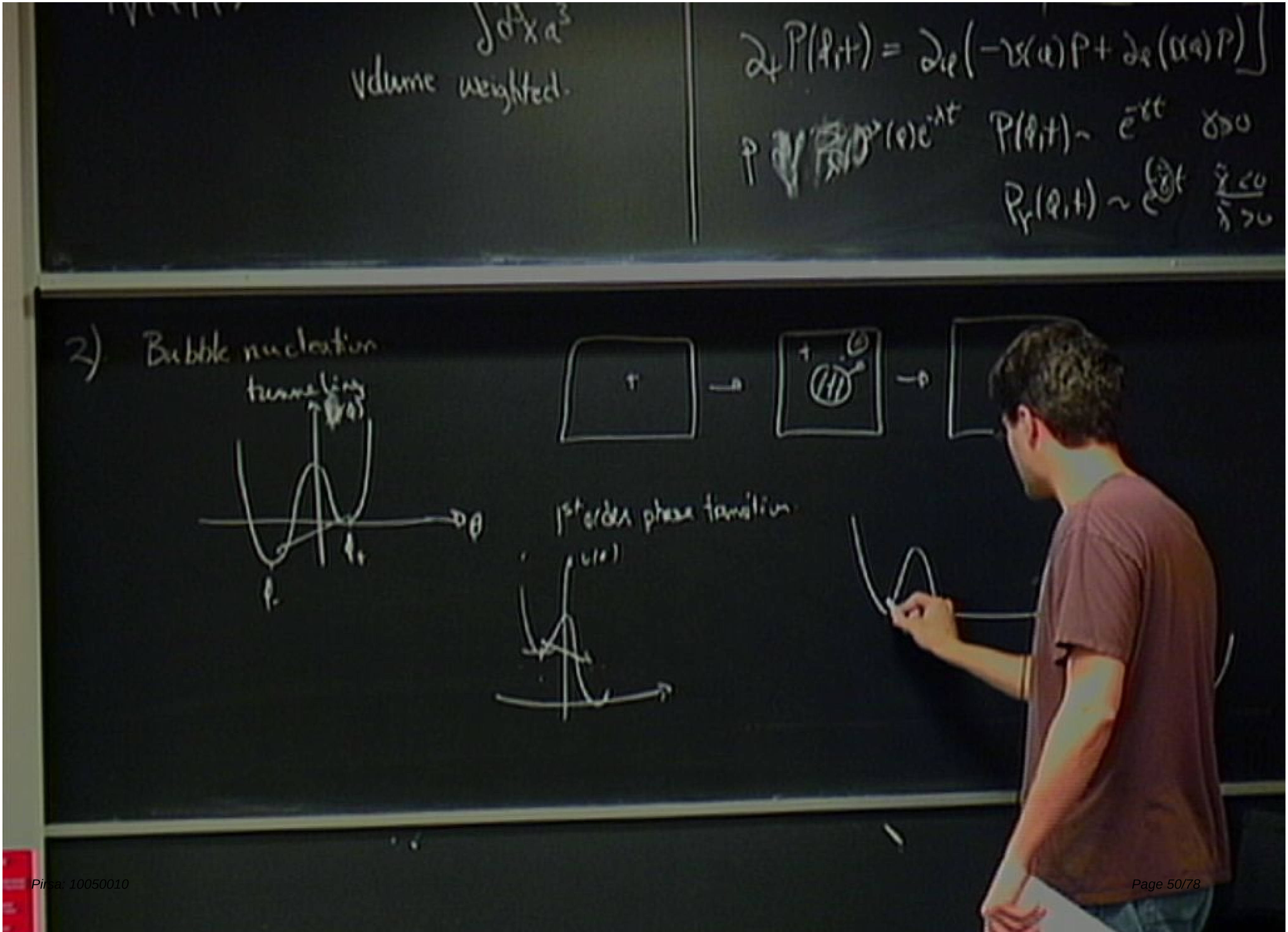
ϕ_0



$$\boxed{\partial_t P(\phi,t) = \partial_\phi\left[-v(\phi)P + \partial_\phi(D(\phi)P)\right]}$$

$P \ldots (\phi) e^{-\lambda t}$ $\quad P(\phi,t) \sim e^{\gamma t}$ $\quad \gamma > 0$

$P_V(\phi,t) \sim e^{\tilde{\gamma} t}$ $\quad \tilde{\gamma} < 0$, $\tilde{\gamma} > 0$


volume weighted.
2) Bubble nucleation
tunneling
1st order phase transition

$\int d^3x\, a^3$

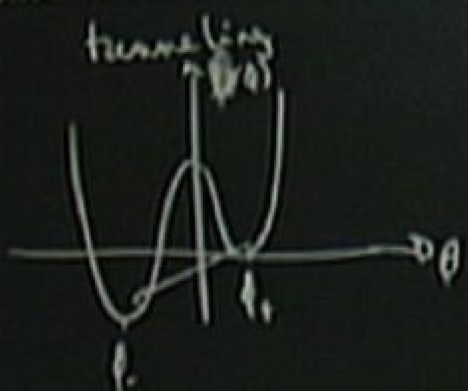
volume weighted.

$$\partial_t P(\phi,t) = \partial_\phi\left(-v(\phi)P + \partial_\phi(D(\phi)P)\right)$$

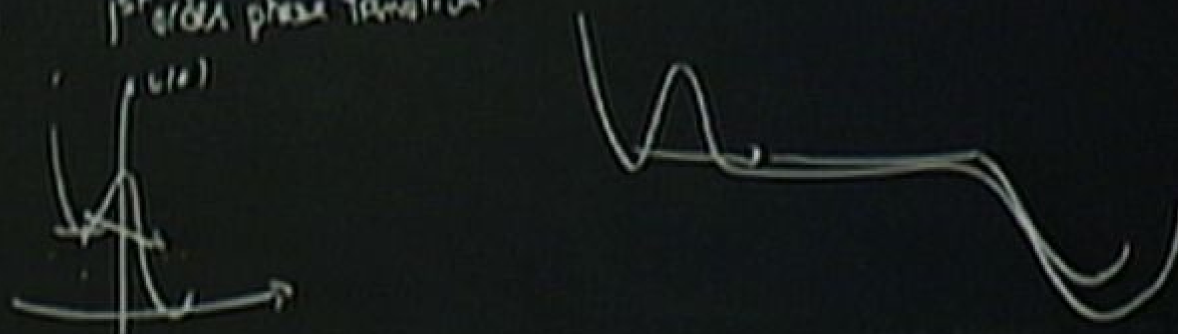
$P(\phi,t) \sim e^{-\gamma t}$ $\gamma > 0$

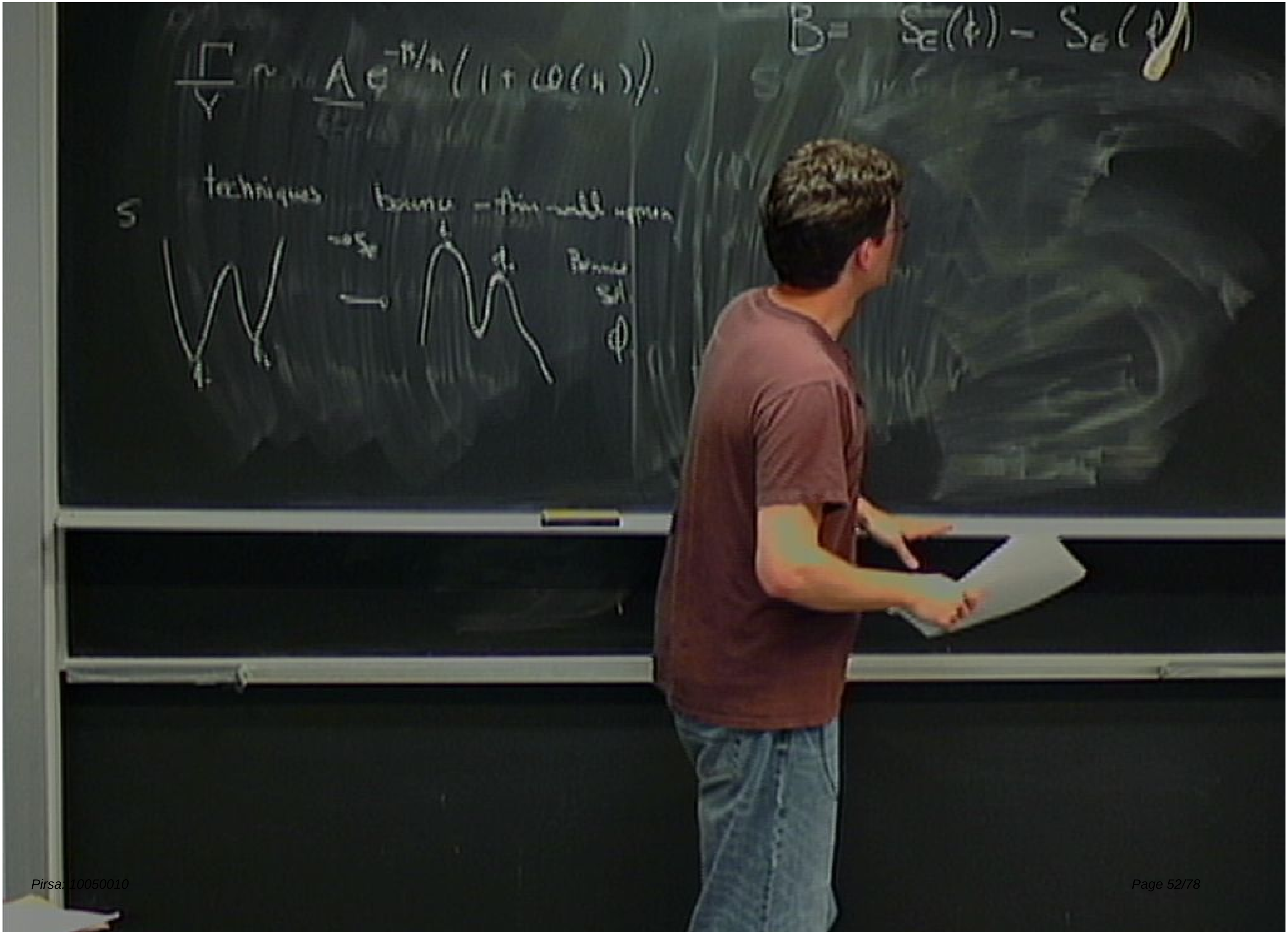
$P_V(\phi,t) \sim e^{\tilde{\gamma} t}$ $\tilde{\gamma} < 0$, $\tilde{\gamma} > 0$

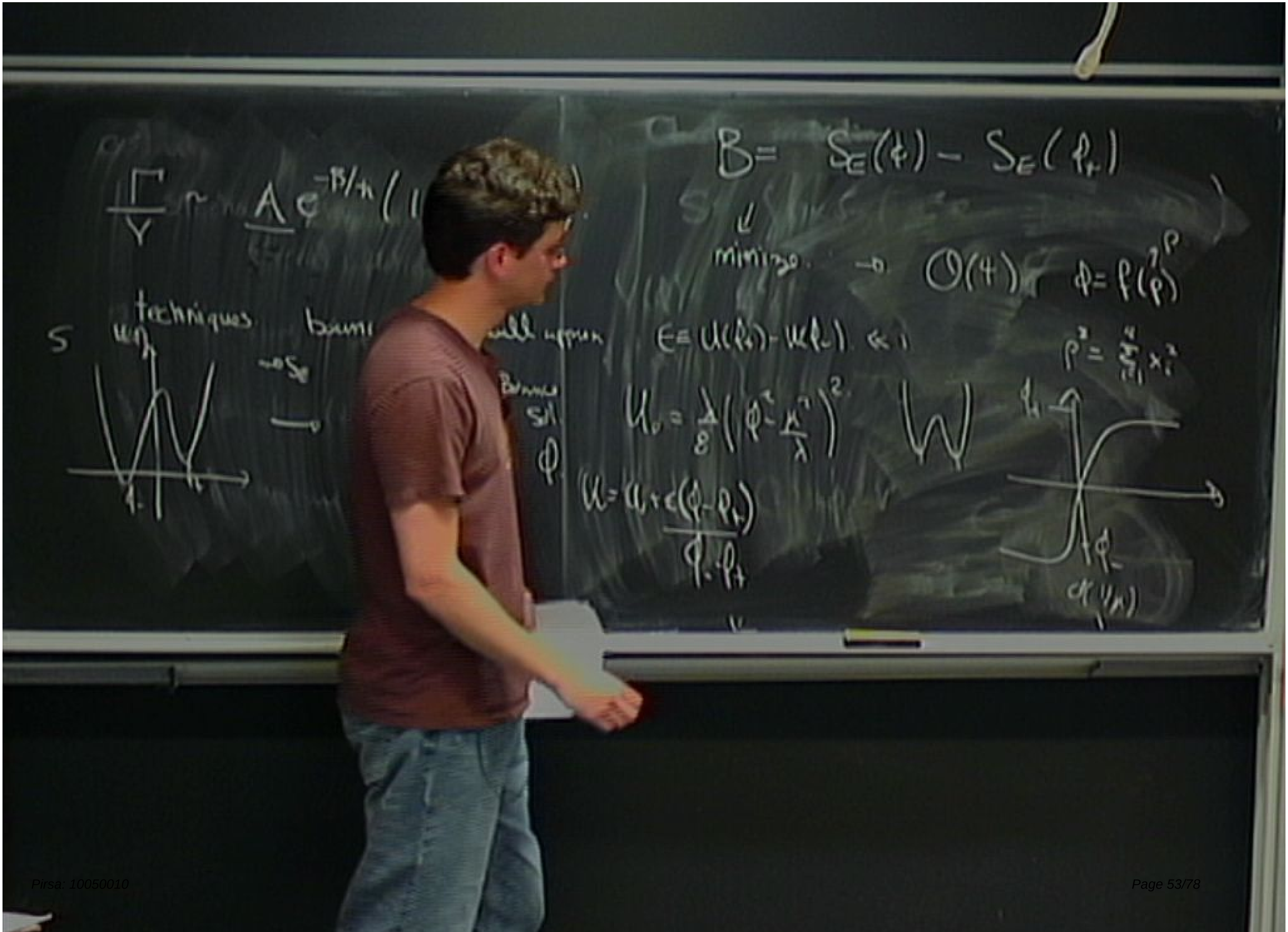
2) Bubble nucleation

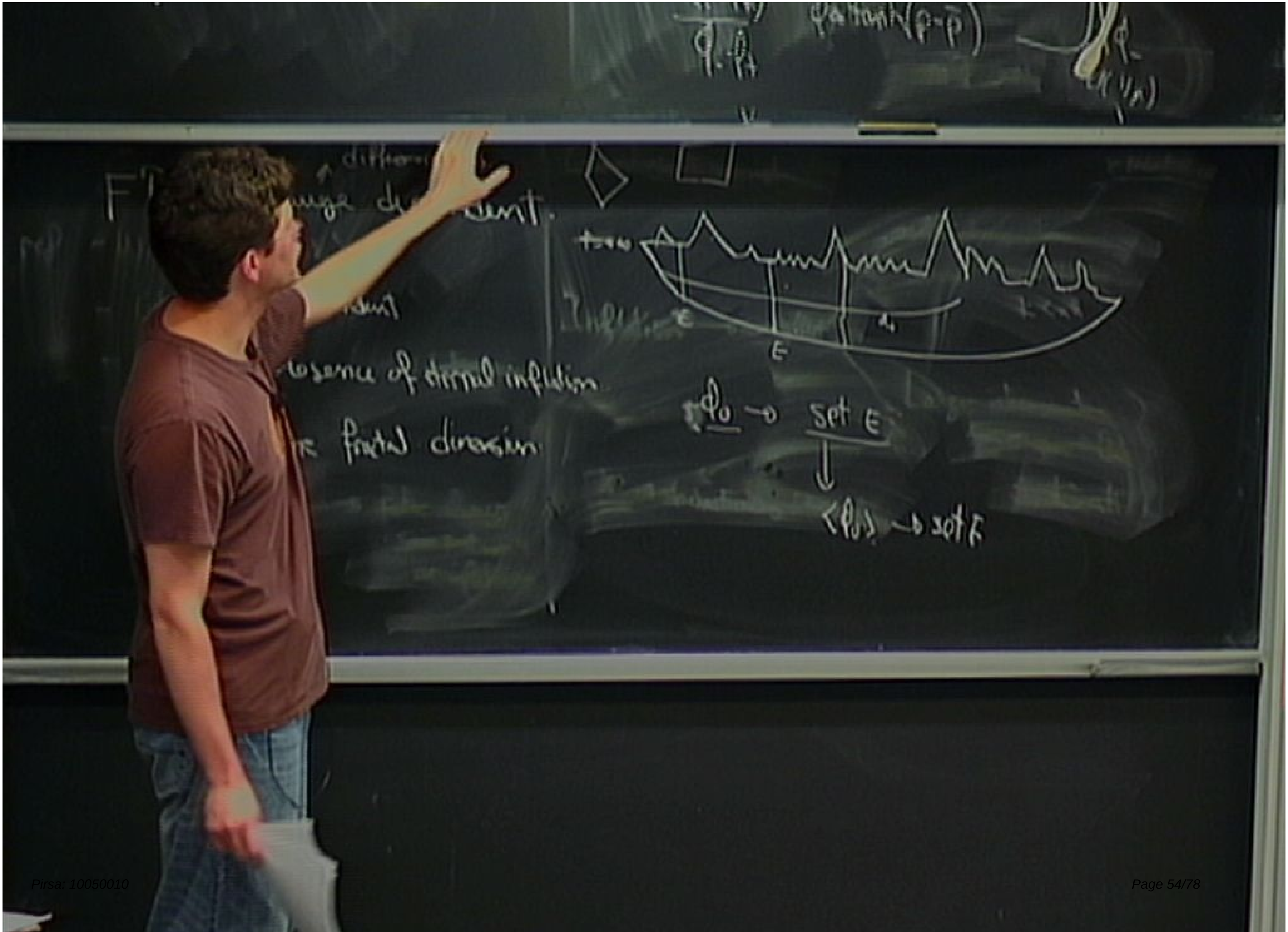


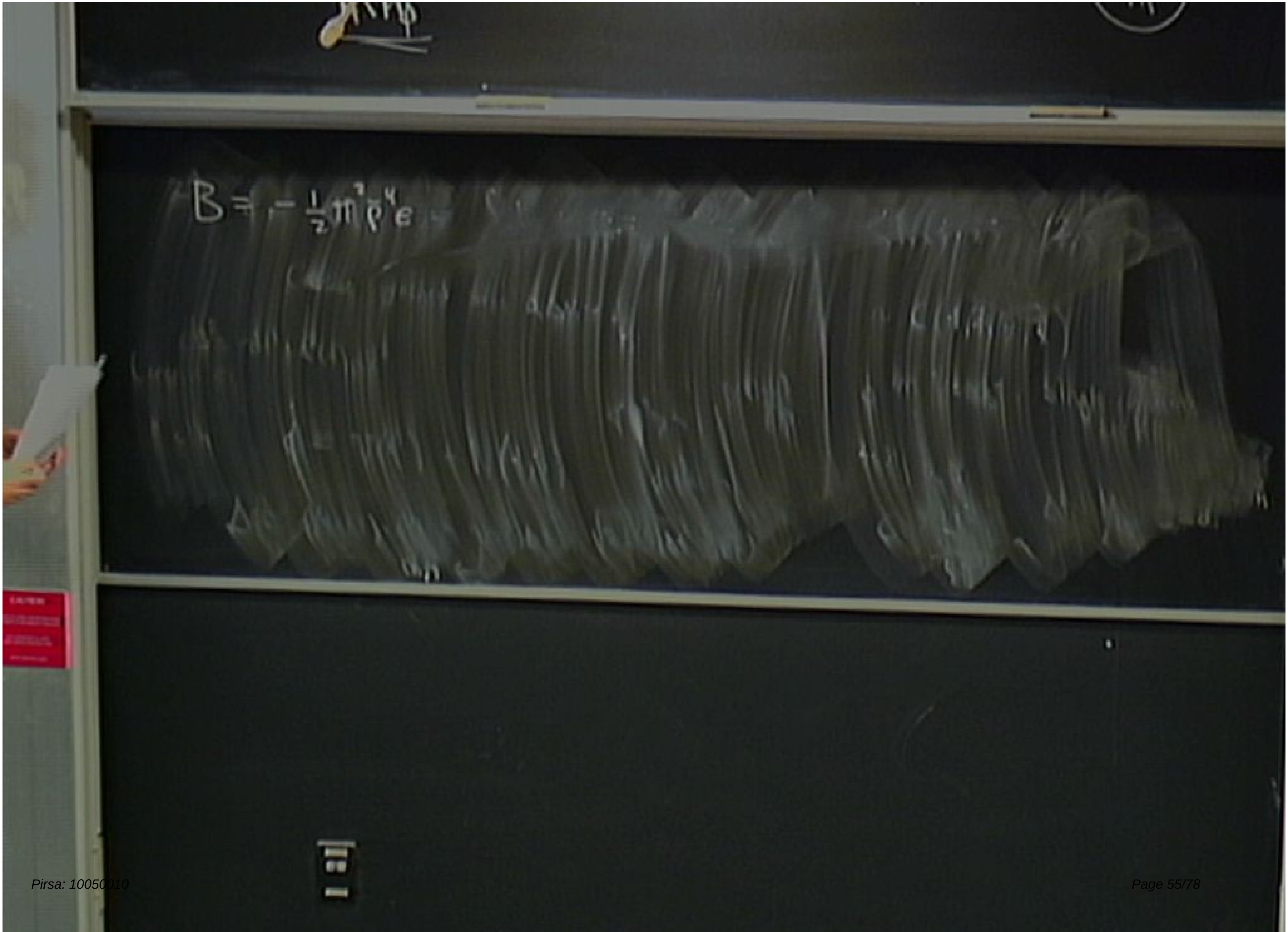
1st order phase transition




techniques
bounce


B= S_E(φ) − S_E(φ+)
minimize
techniques
φ = f(ρ)
ε = U(φ+) − U(φ−) ≪ 1



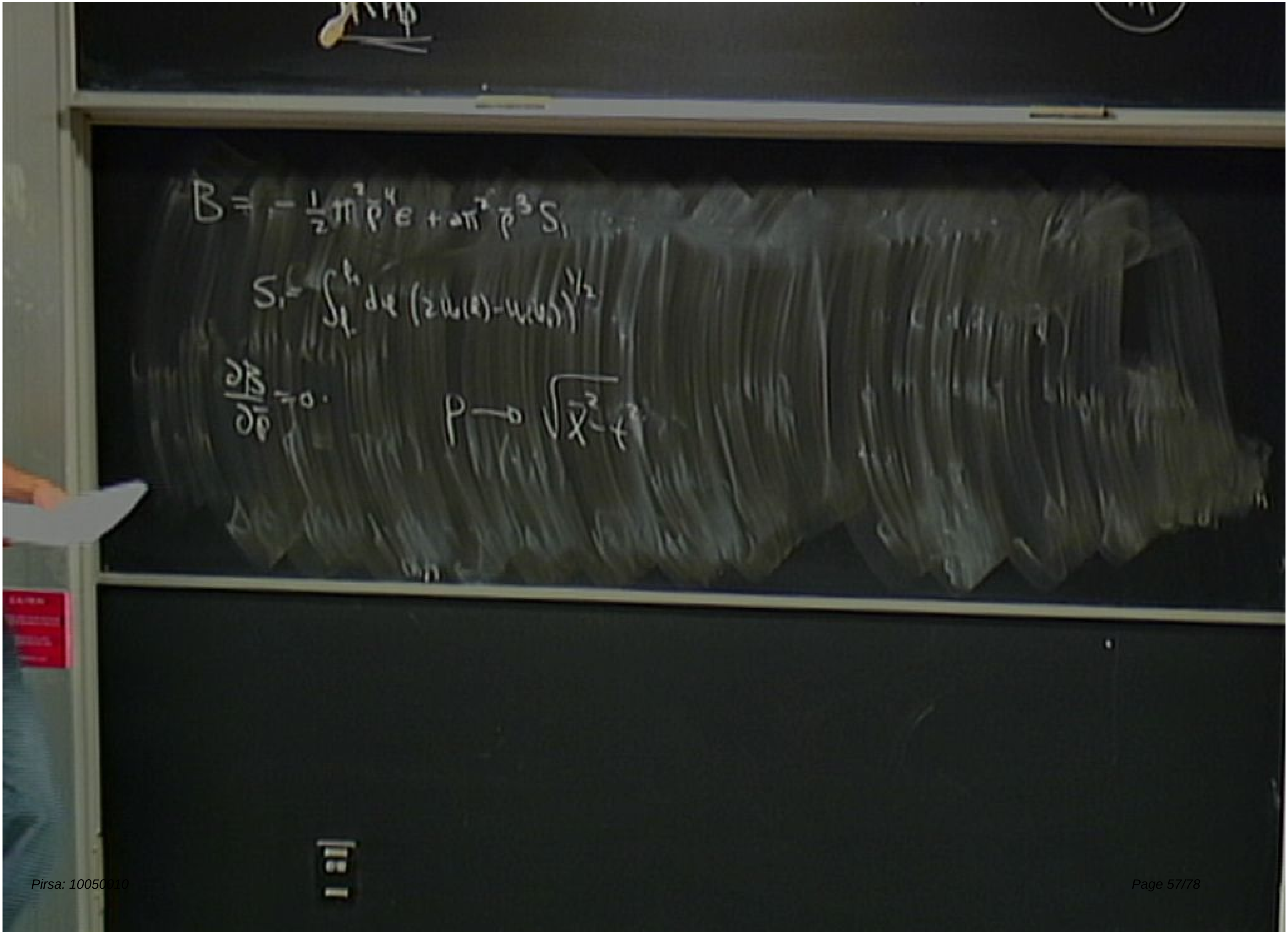

$B = -\frac{1}{2} m^2 \rho^4 e$

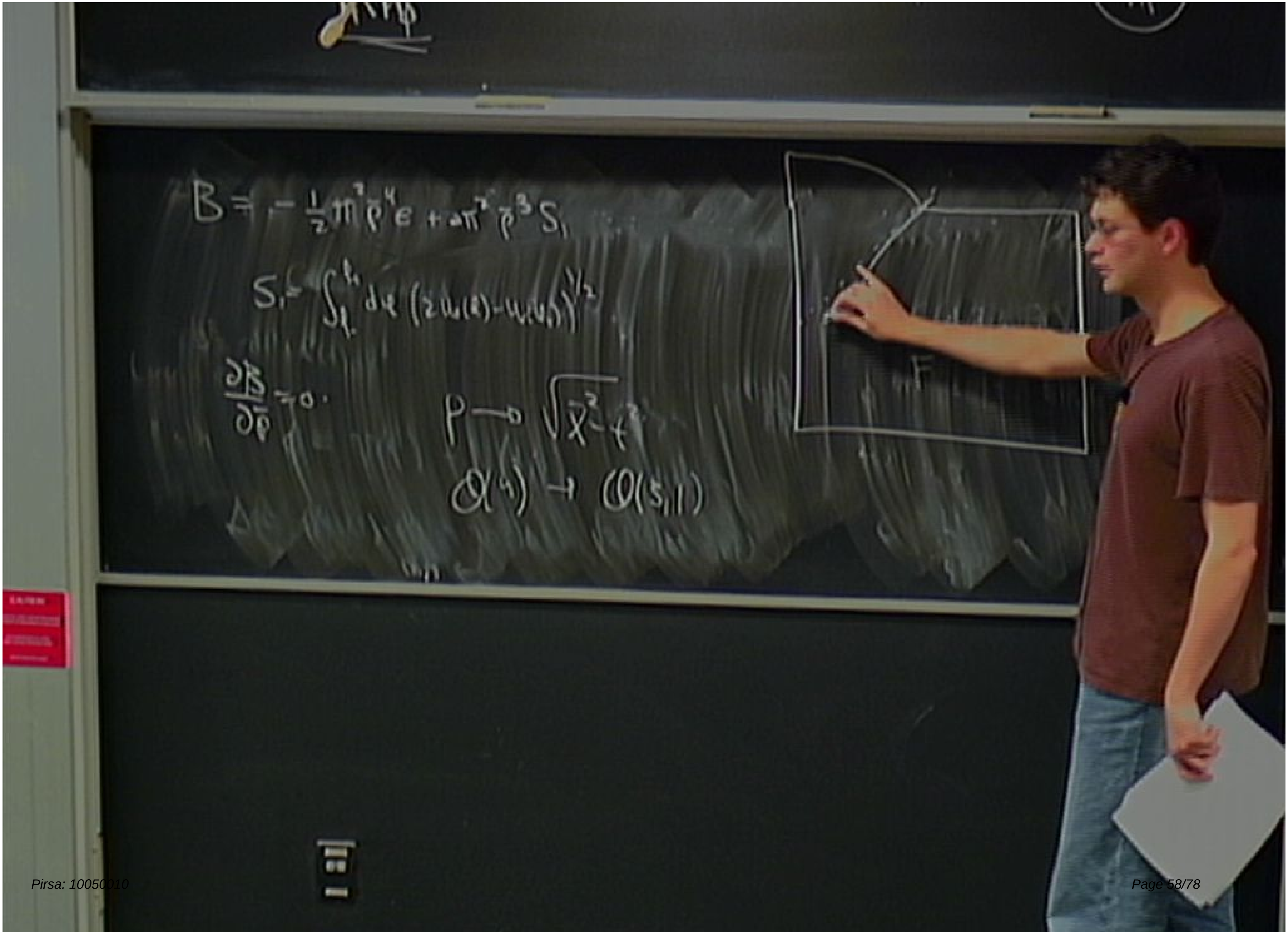
$$B = -\frac{1}{2}\pi^2 \bar{\rho}^4 \epsilon + 2\pi^2 \bar{\rho}^3 S_1$$

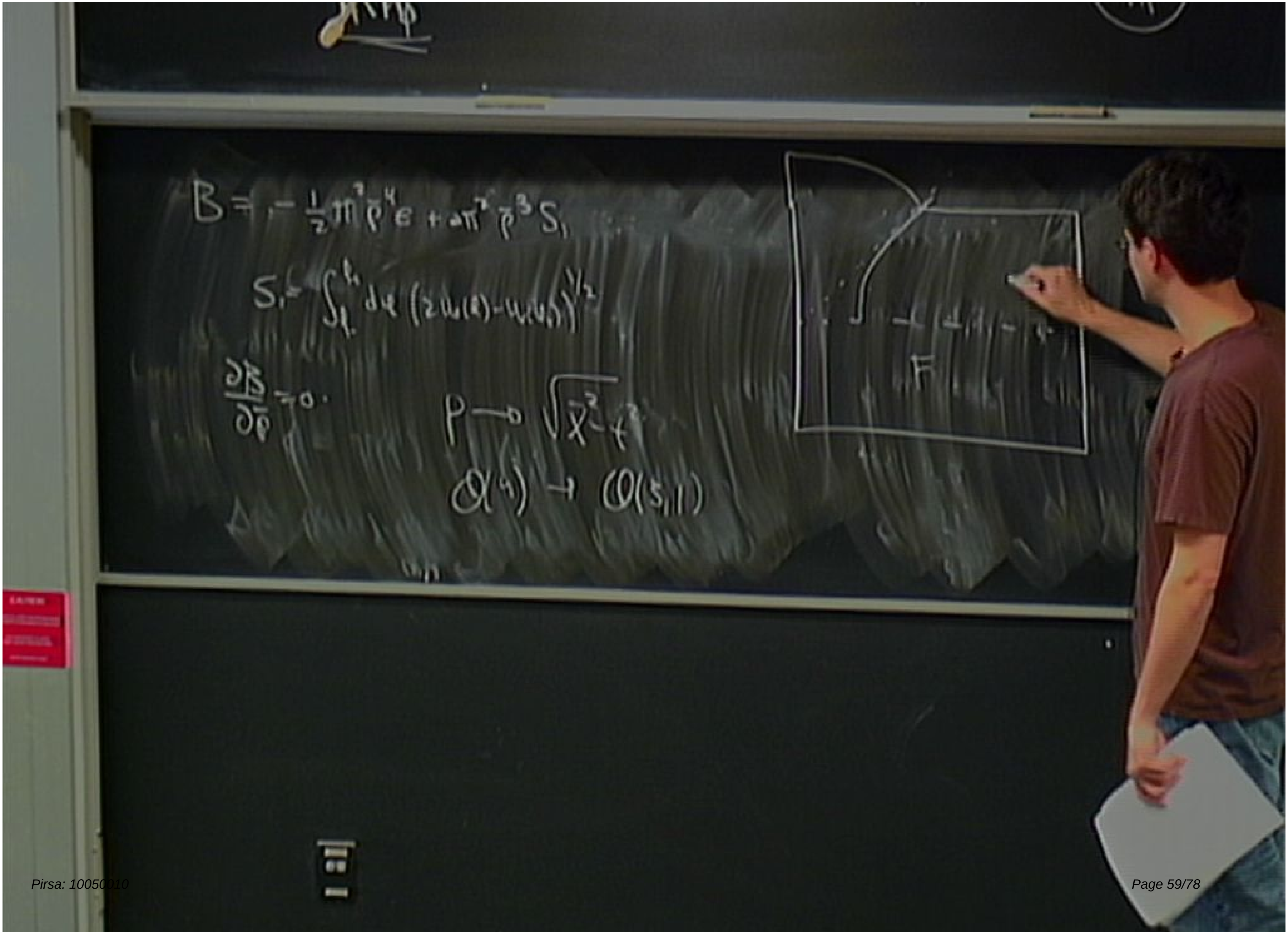
$$S_1 = \int_{\varphi_1}^{\varphi_2} d\varphi \, \left(2V_0(\varphi) - V_0(\varphi_1)\right)^{1/2}$$

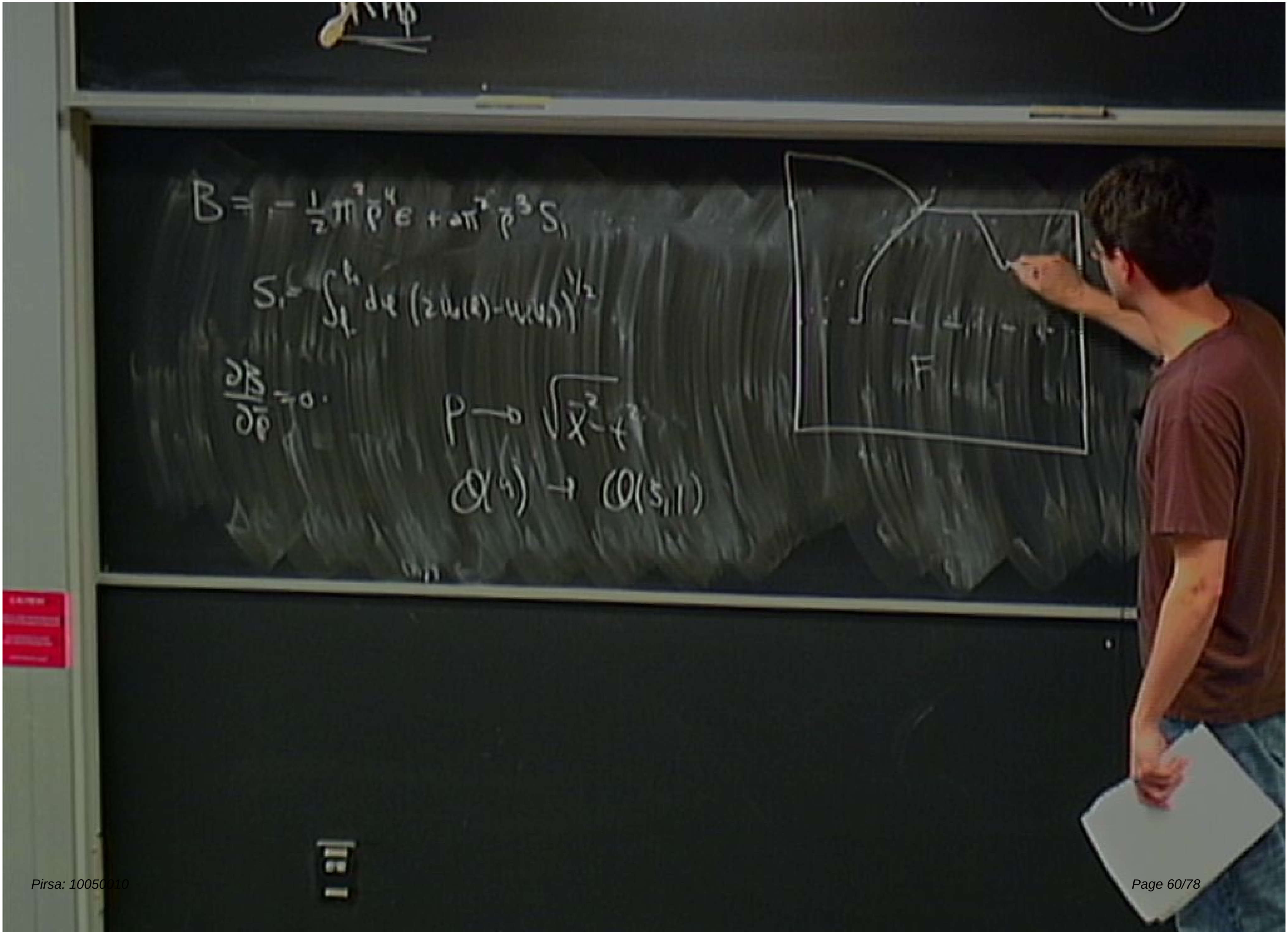
$$\frac{\partial B}{\partial \bar{\rho}} = 0.$$

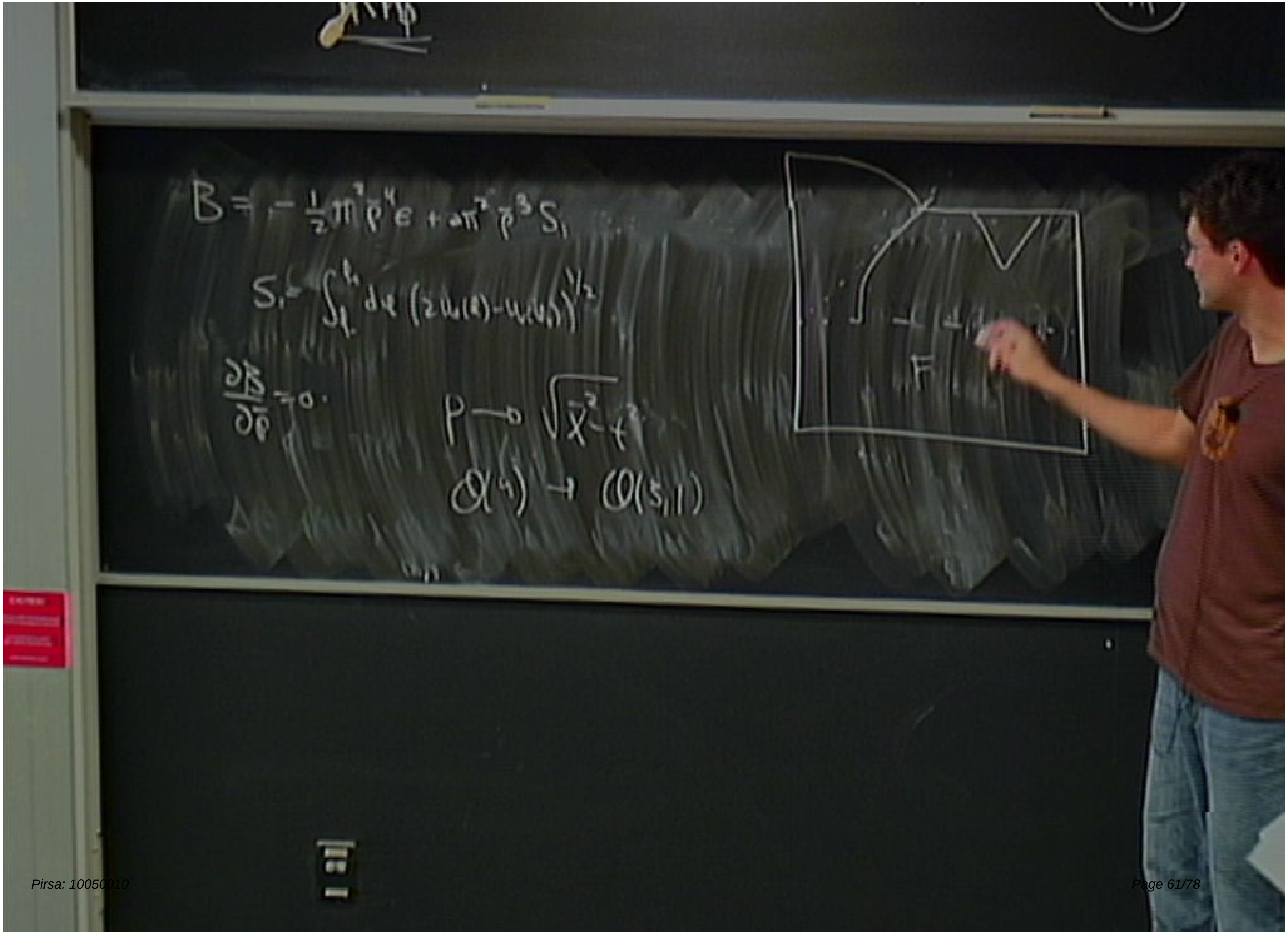
$$\rho \to \sqrt{\vec{x}^2 + t^2}$$


$B = -\frac{1}{2}\pi^2\bar{\rho}^4\epsilon + 2\pi^2\bar{\rho}^3 S_1$
$S_1 = \int d\varphi \, (2V_0(\varphi) - V_0(\varphi_0))^{1/2}$
$\frac{\partial B}{\partial \bar{\rho}} = 0.$
$\rho \to \sqrt{\vec{x}^2 - t^2}$

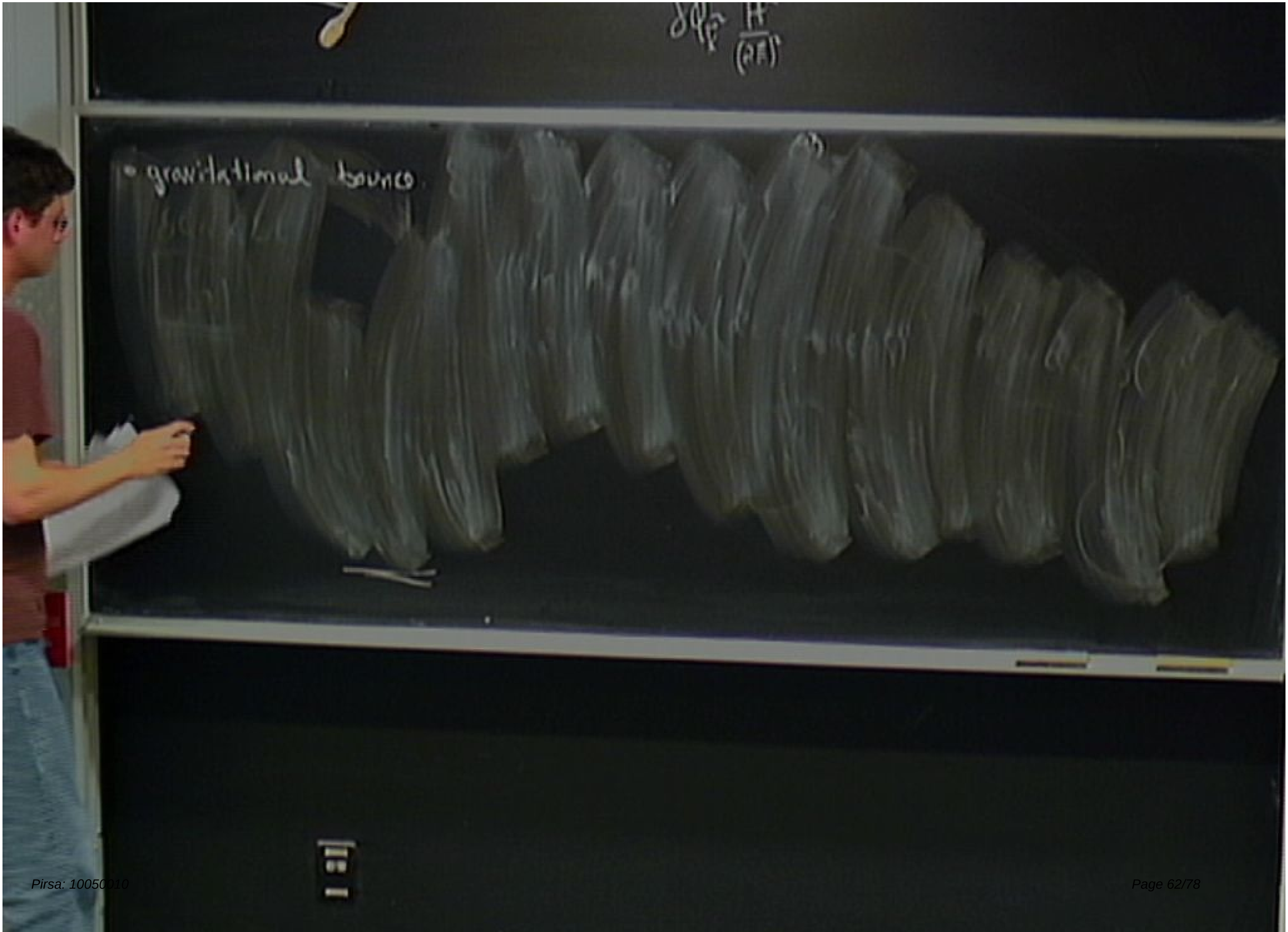

B = - ½ π² ρ̄⁴ e + 2π² ρ̄³ S₁
∂B/∂ρ̄ = 0
ρ → √(x² + t²)
O(4) → O(3,1)
F

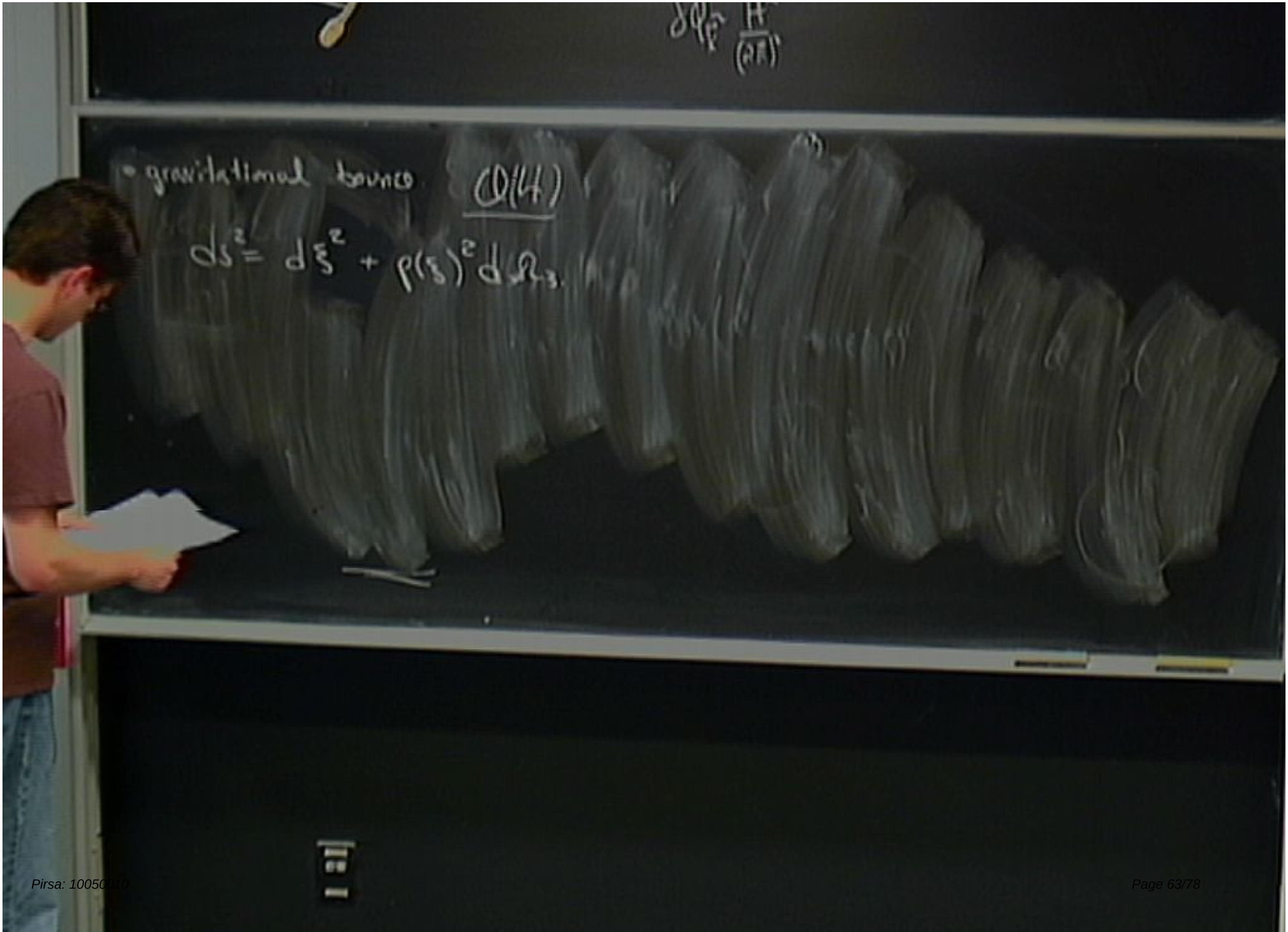

B = - 1/2 π² ρ⁴ ε + 2π² ρ³ S₁
∂B/∂ρ = 0
ρ → √(x² + t²)
O(4) → O(3,1)
F

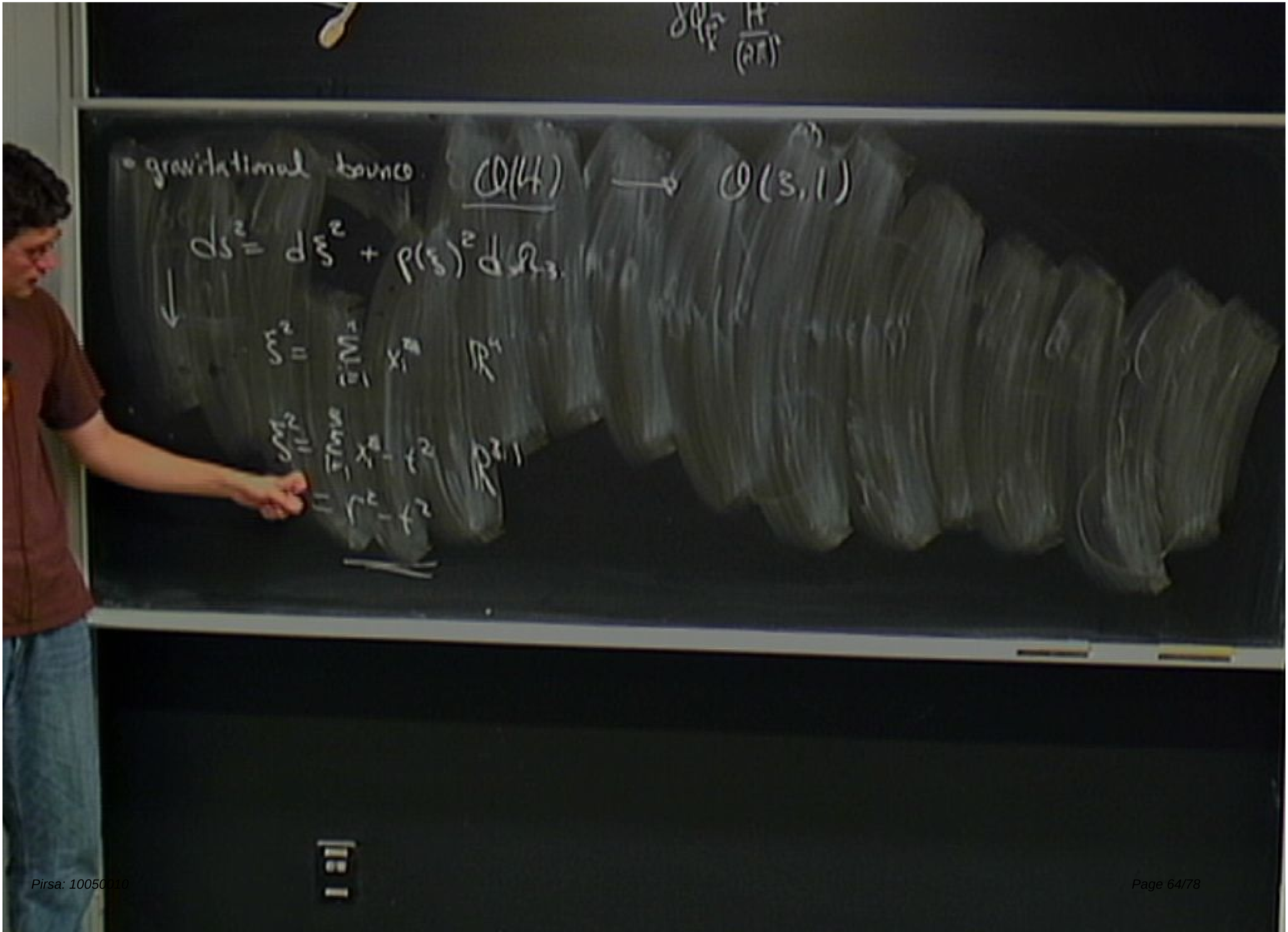




$B = -\frac{1}{2}\pi^2 \rho^4 \epsilon + 2\pi^2 \rho^3 S_1$
$S_1 = \int_{\phi_t}^{\phi_f} d\phi \, (2V_0(\phi) - V_0(\phi_t))^{1/2}$
$\frac{\partial B}{\partial \rho} = 0$
$\rho \to \sqrt{\vec{x}^2 - t^2}$
$O(4) \to O(3,1)$
F


• gravitational bounce


• gravitational bounce O(4)
ds² = dξ² + ρ(ξ)² dΩ₃


• gravitational bounce $O(4) \longrightarrow O(3,1)$
$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3$
$\xi^2 = \sum_{i=1}^{4} x_i^2 \quad \mathbb{R}^4$
$\xi^2 = \sum_{i=1}^{3} x_i^2 - t^2 \quad \mathbb{R}^{3,1}$
$= r^2 - t^2$

• gravitational bounce $\underline{O(4)} \longrightarrow O(3,1)$

$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3$ inside $\xi = i\tau$

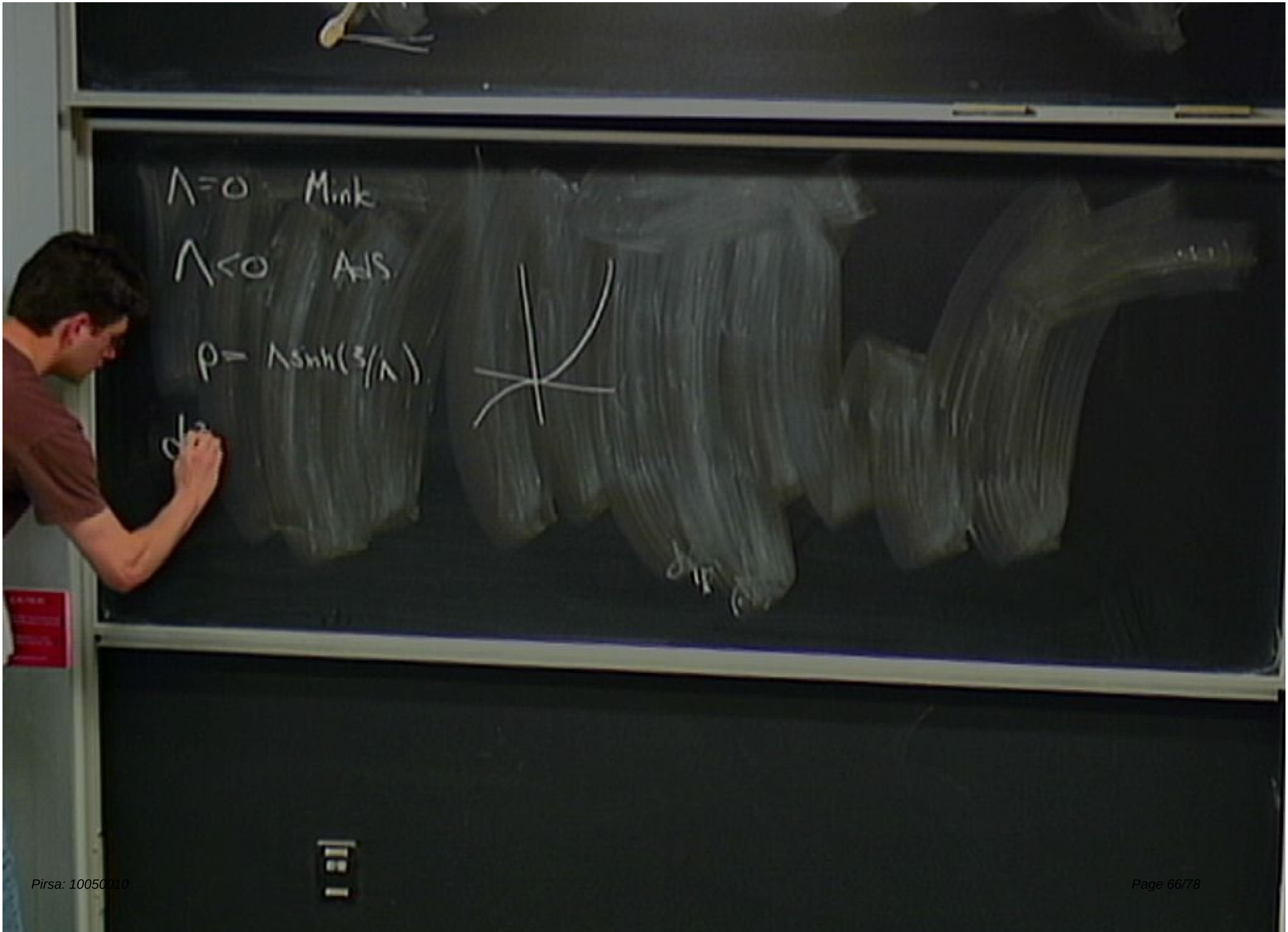
$ds^2 = -d\tau^2 + \rho(\tau)^2 d\Omega_{H_3}$

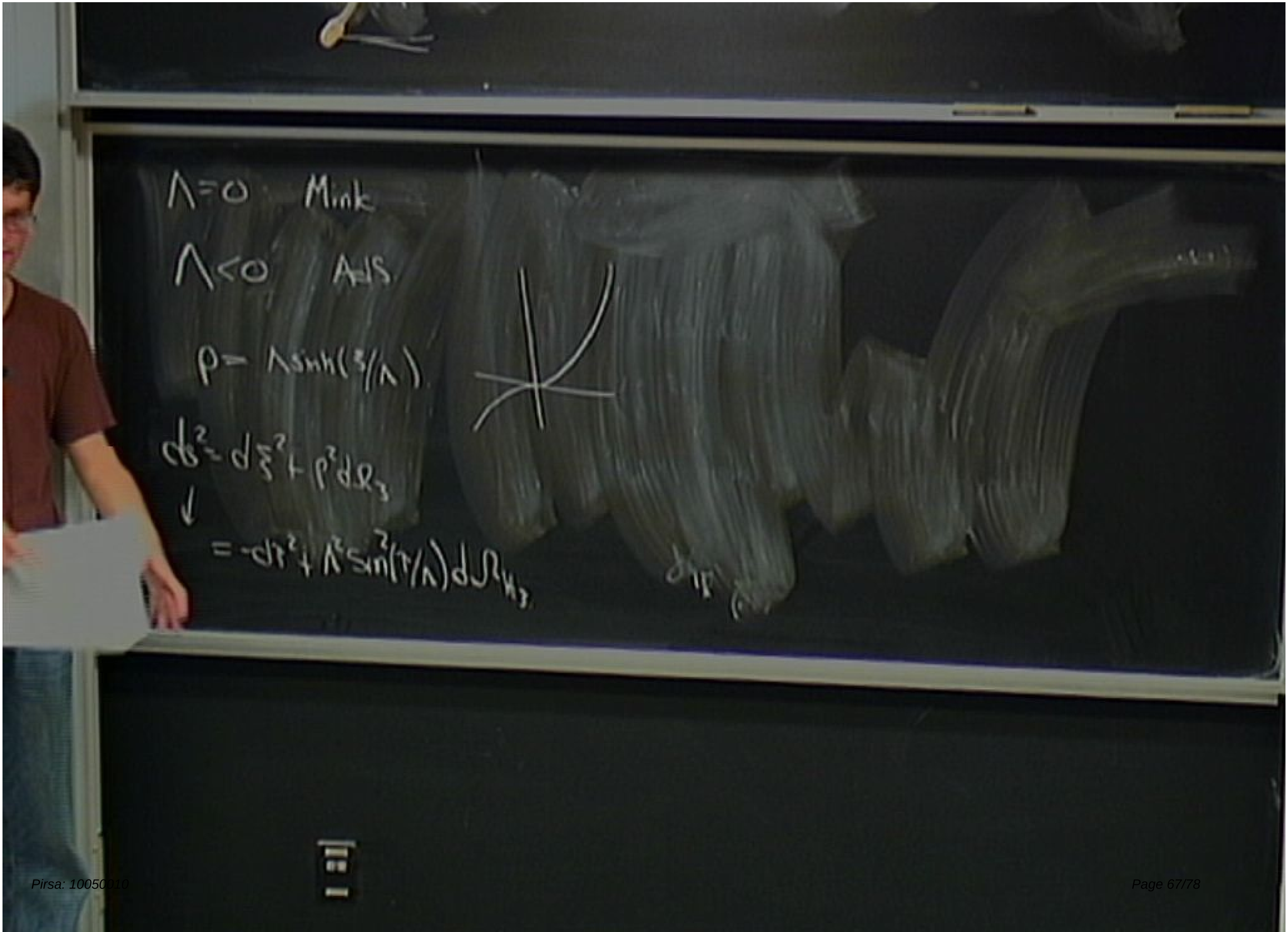
$\downarrow$

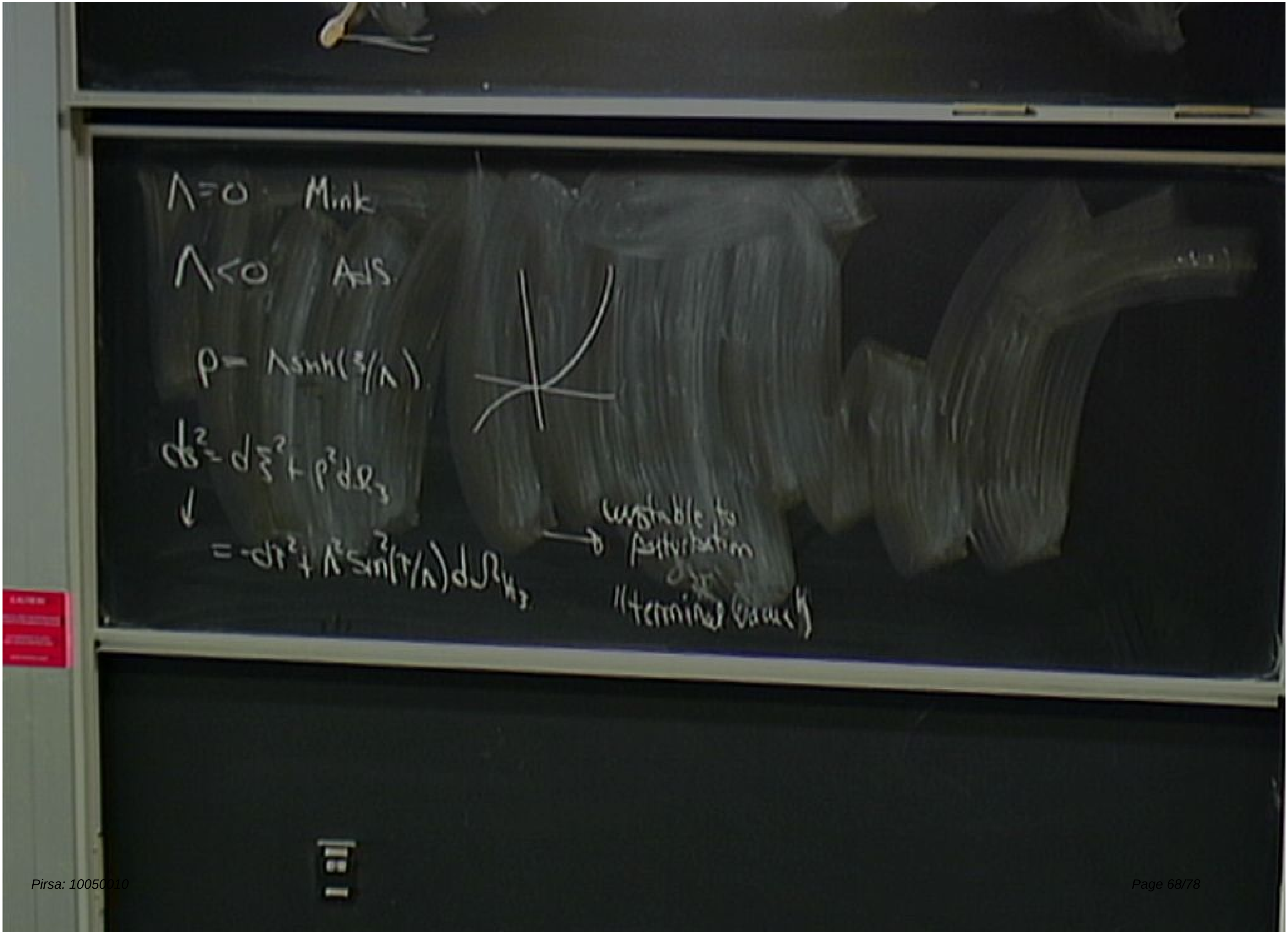
$\xi^2 = \sum_{i=1}^{4} x_i^2 \quad \mathbb{R}^4$

$\xi^2 = \sum_{i=1}^{3} x_i^2 - t^2 \quad \mathbb{R}^{3,1}$

$= r^2 - t^2$


Λ=0 Mink
Λ<0 AdS
ρ= Λ sinh(s/Λ)


$\Lambda = 0$ Mink
$\Lambda < 0$ AdS
$\rho = \Lambda \sinh(\xi/\Lambda)$
$ds^2 = d\xi^2 + \rho^2 d\Omega_3$
$= -d\tau^2 + \Lambda^2 \sin^2(\tau/\Lambda) d\Omega_{H_3}$


$\Lambda = 0$ Mink
$\Lambda < 0$ AdS
$\rho = \Lambda \sinh(\xi/\Lambda)$
$ds^2 = d\xi^2 + \rho^2 d\Omega_3$
$\downarrow$
$= -d\tau^2 + \Lambda^2 \sin^2(\tau/\Lambda) d\Omega_{H_3}$
unstable to perturbation
or
"terminal bounce"

$\Lambda = 0$ Mink

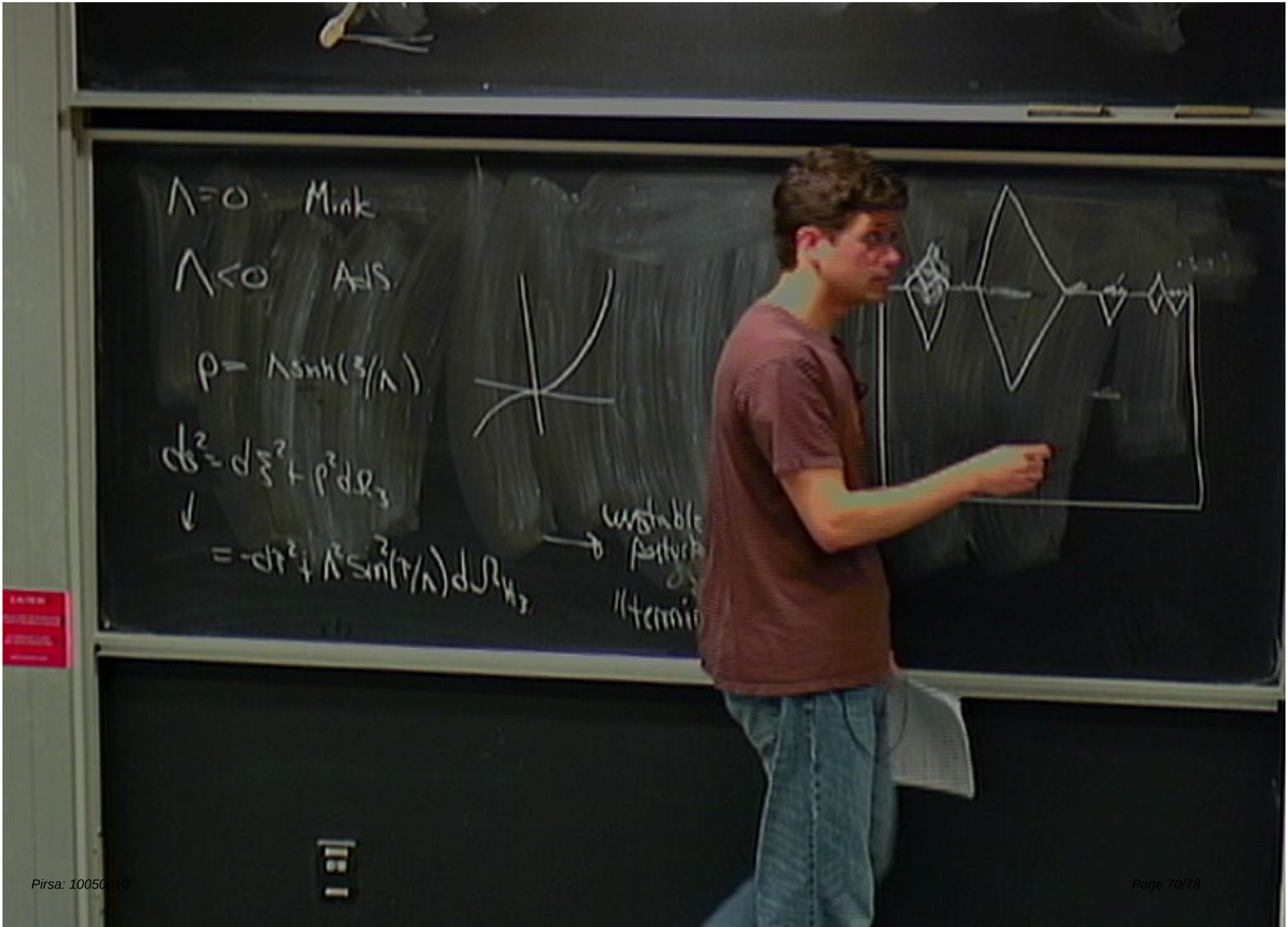
$\Lambda < 0$ AdS

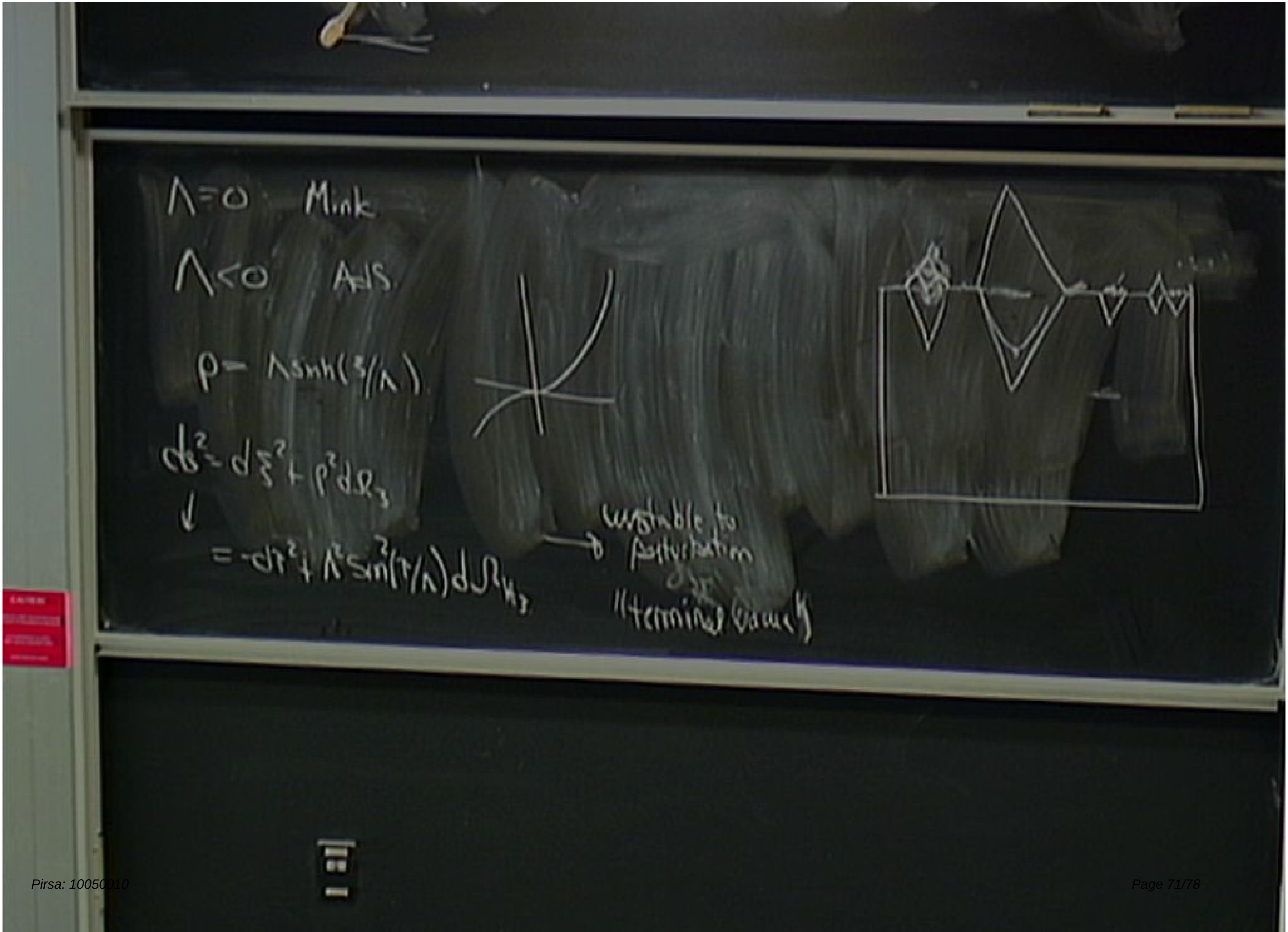
$\rho = \Lambda \sinh(\xi/\Lambda)$

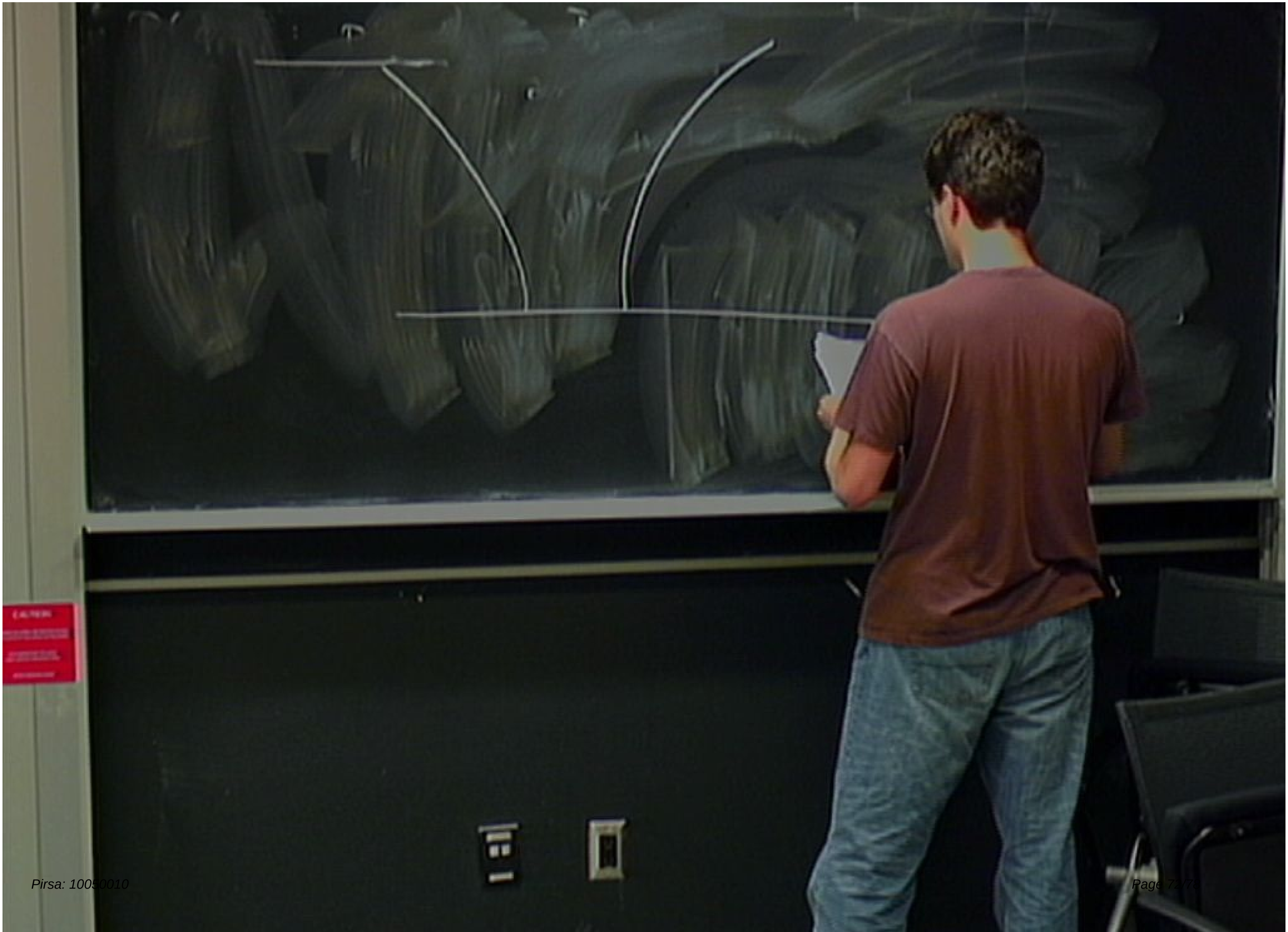
$ds^2 = d\xi^2 + \rho^2 d\Omega_3$

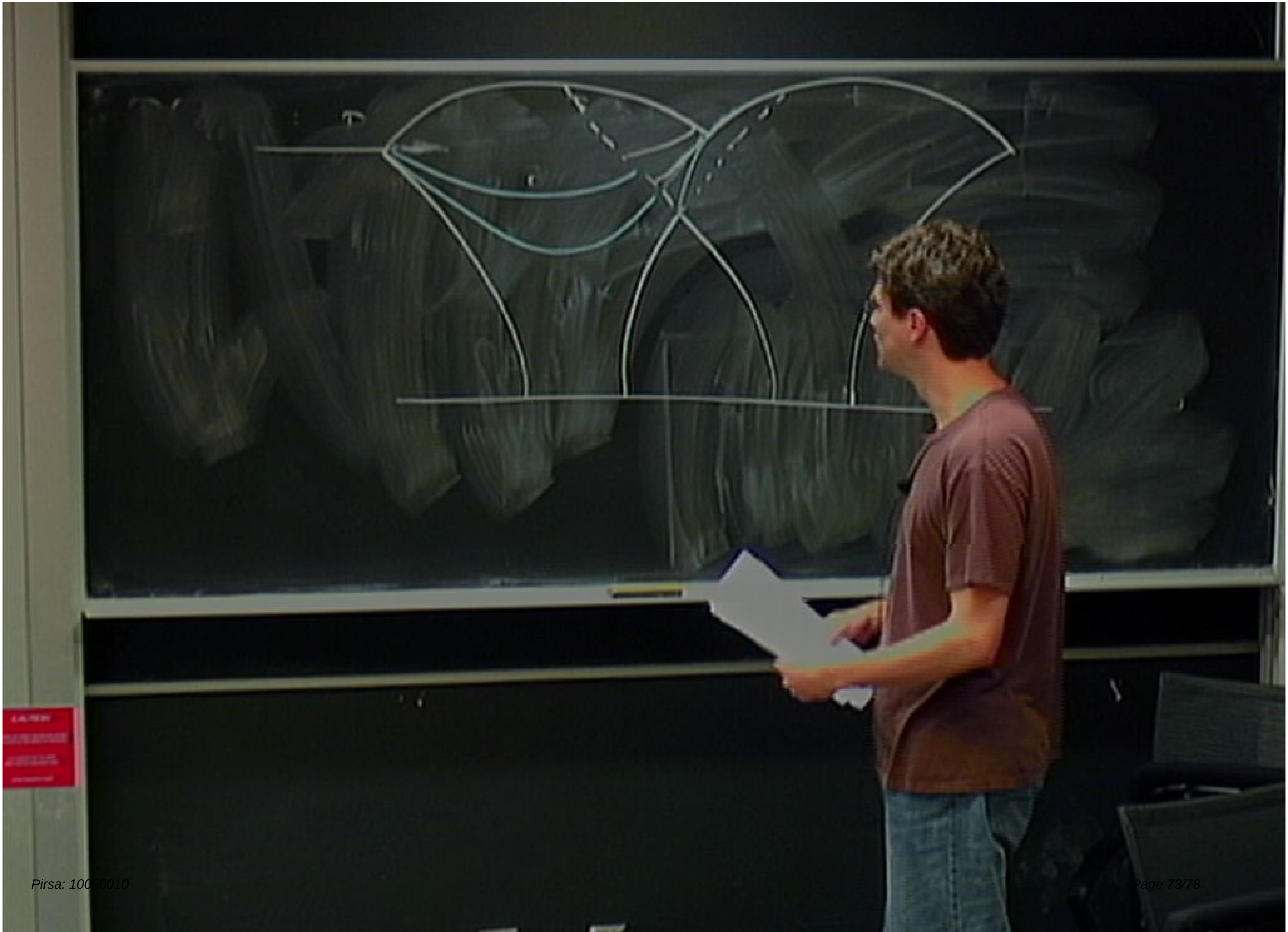
$\downarrow$

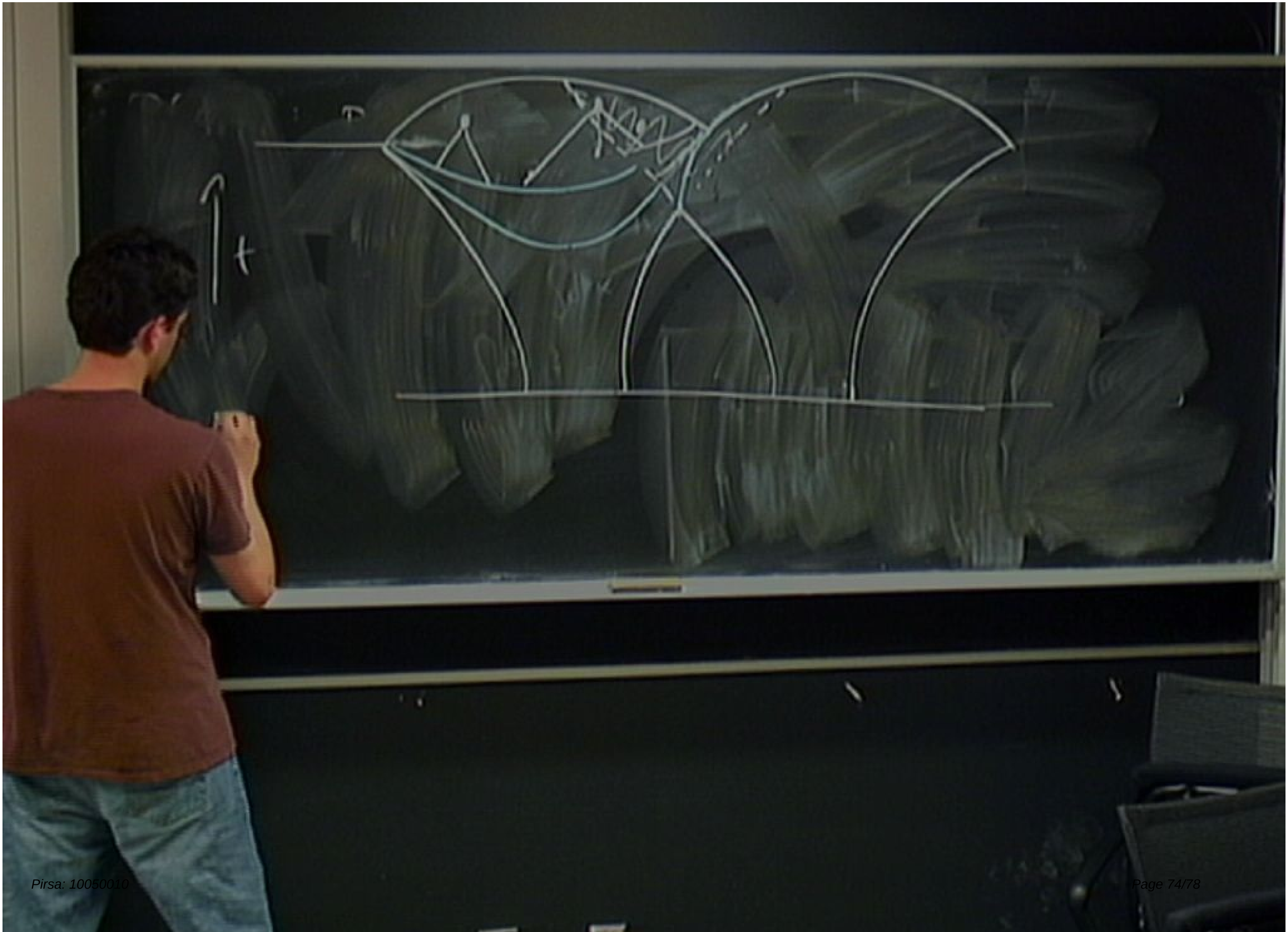
$= -d\tau^2 + \Lambda^2 \sin^2(\tau/\Lambda)\, d\Omega_{H_3}$ $\rightarrow$ unstable to perturbation of "terminal crunch"

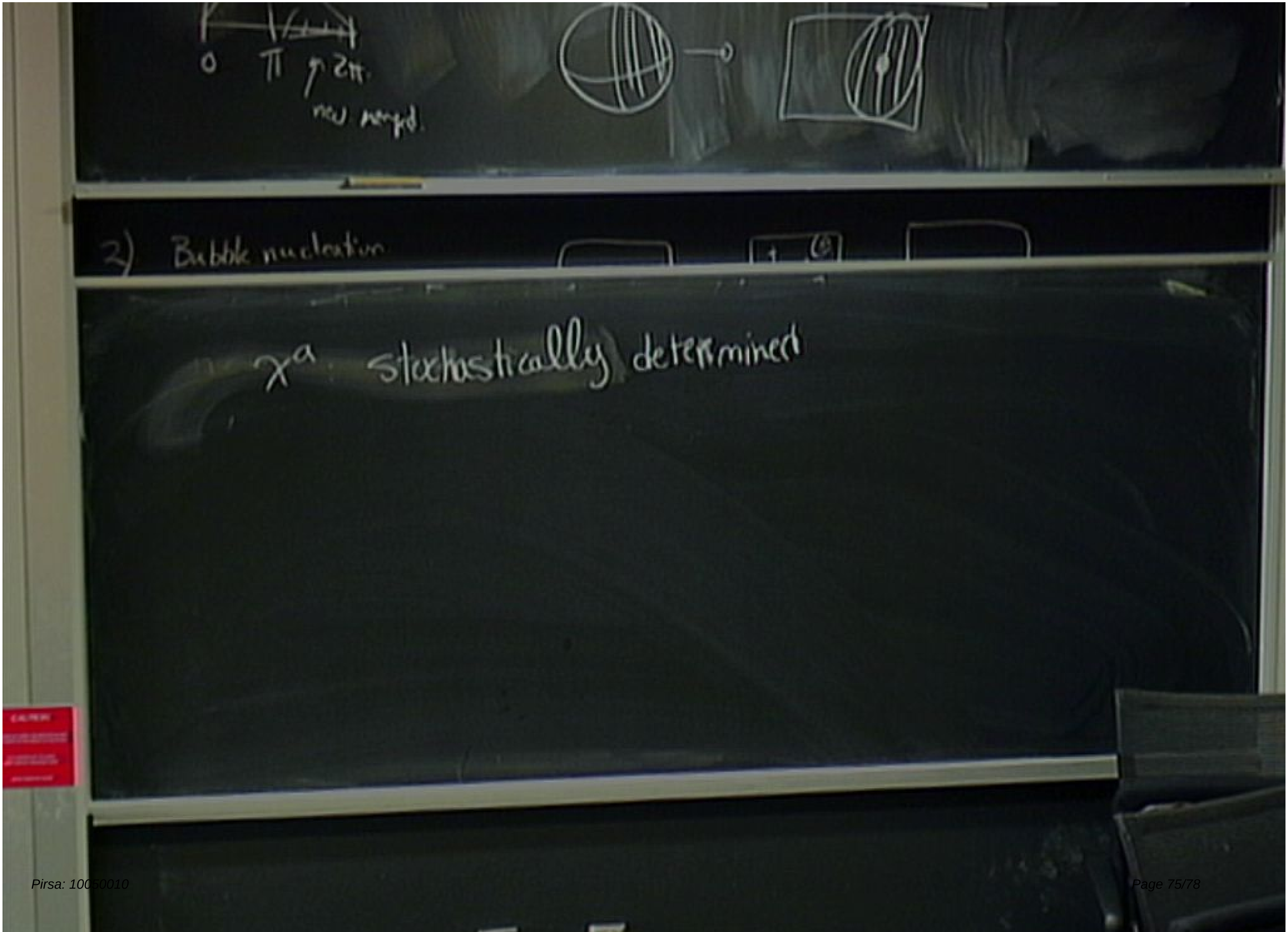

Λ=0 Mink
Λ<0 AdS
ρ= Λ sinh(ξ/Λ)
ds² = dξ² + ρ² dΩ₃
= -dτ² + Λ² sin²(τ/Λ) dΩ_H₃

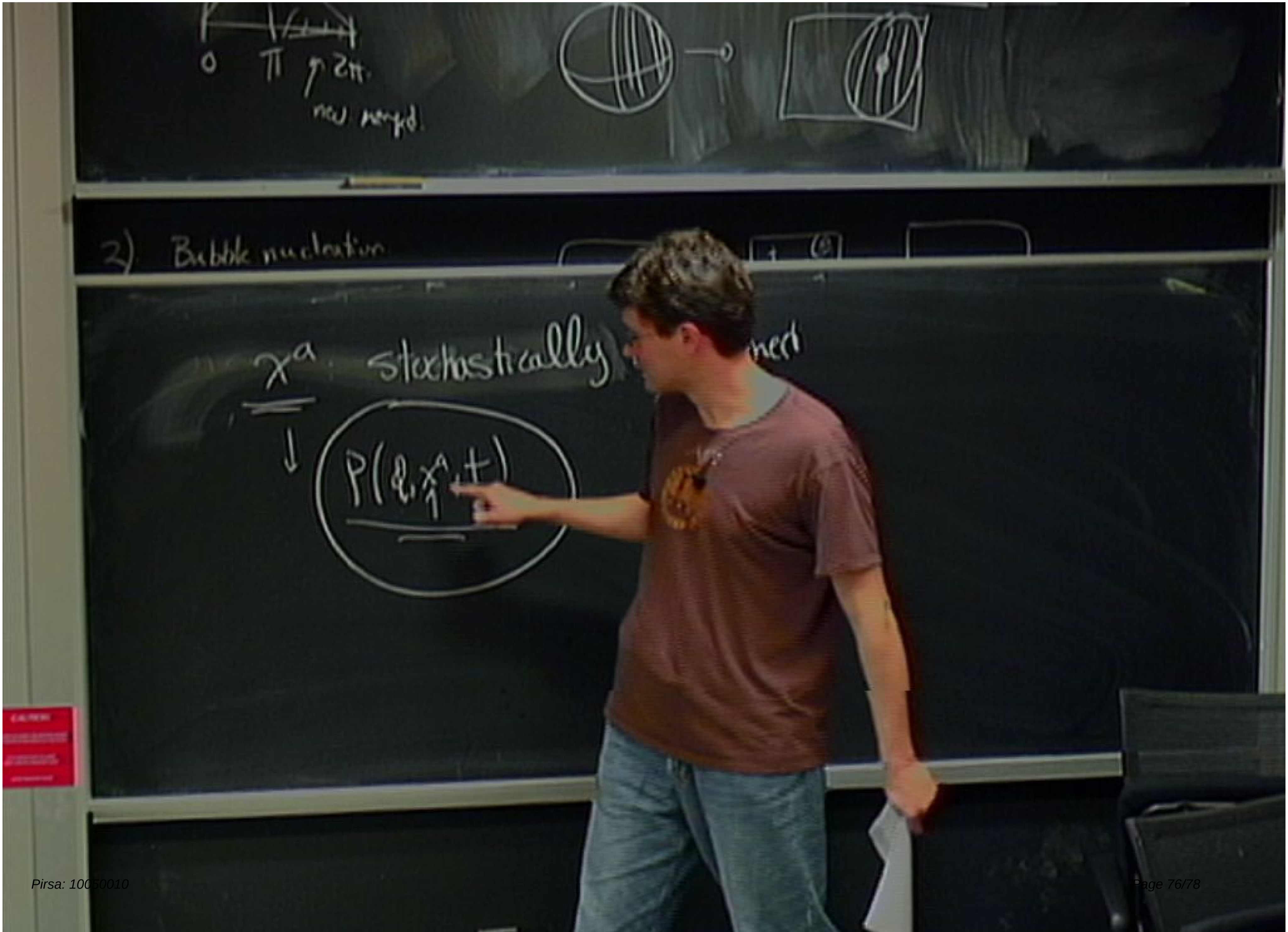

Λ=0 Mink
Λ<0 AdS
ρ = Λ sinh(ξ/Λ)
ds² = dξ² + ρ² dΩ₃
= -dτ² + Λ² sin²(τ/Λ) dΩ_H₃
unstable to perturbation
or
"terminal bang"

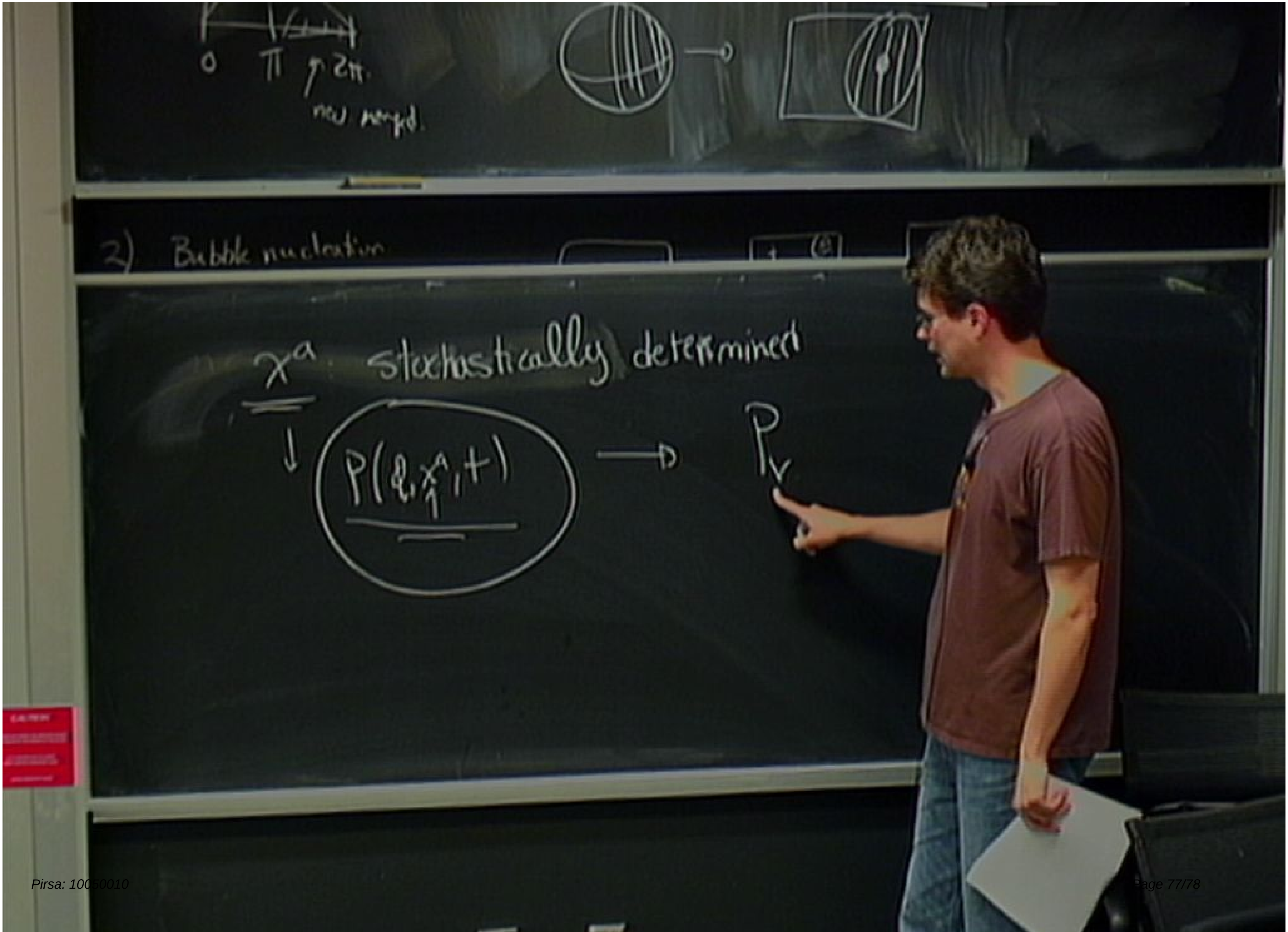


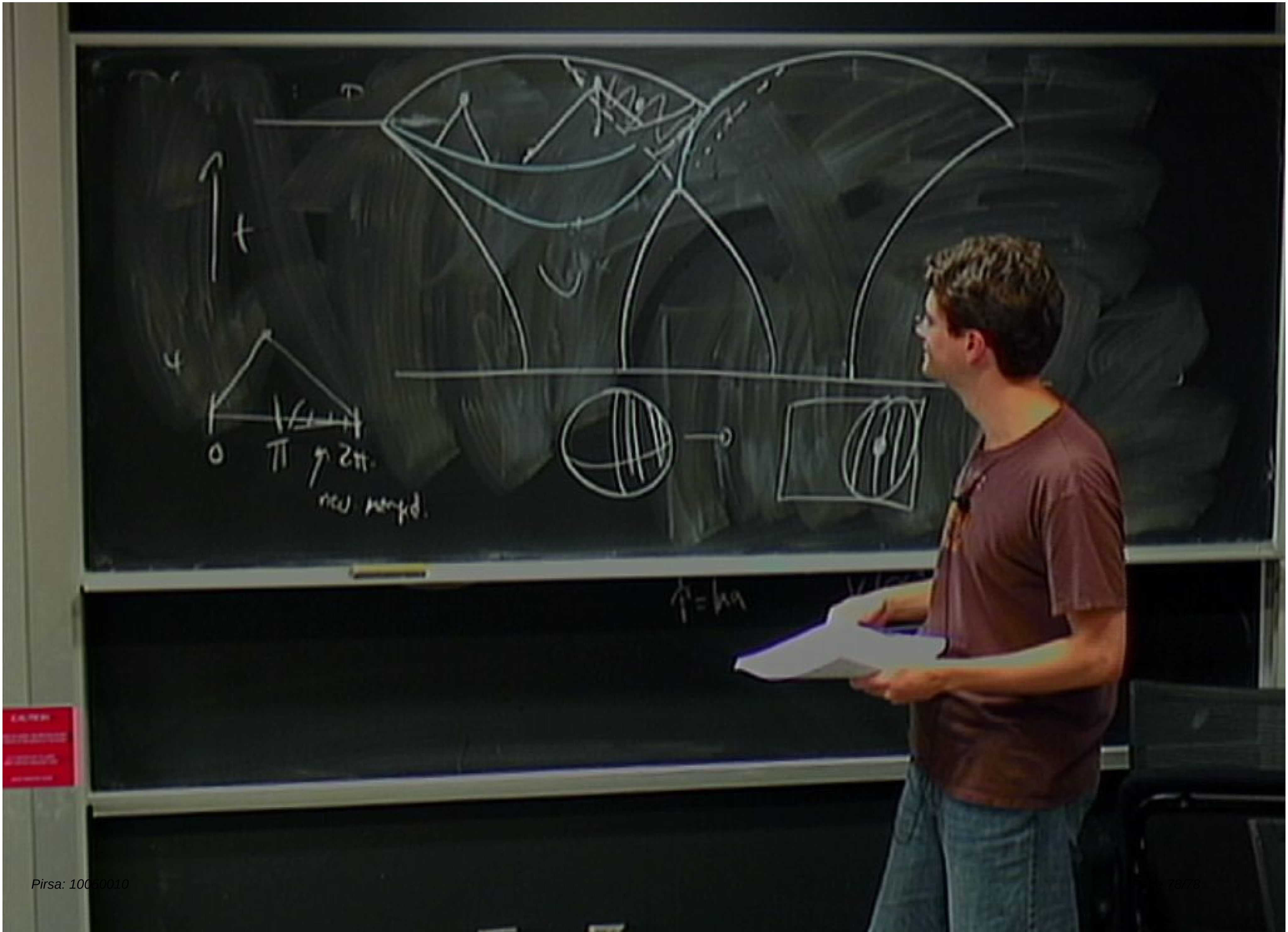





0 π 2π
2) Bubble nucleation
χ^a stochastically determined


2) Bubble nucleation
stochastically


0 π 2π
2) Bubble nucleation
x^a stochastically determined
P(φ, x^a, t)
P_v


t
0
π
2π