

Title: Bouncing alternatives to inflation

Date: May 11, 2010 02:45 PM

URL: <http://pirsa.org/10050009>

Abstract: Although inflation is, by far, the best known mechanism to explain the observed properties of our Universe, there is still some room for alternative models, most of which implying a contracting phase preceding the current expanding one. Both phases are connected by a bounce at which the expansion rate must vanish. General relativity can only produce such a phase provided the spatial curvature is positive, in contradiction with the current observations. I will discuss the lines along which one can modify either the matter or the gravity sector (or both) in order to implement a bounce, and show the generic observable cosmological consequences it can induce, in particular in the microwave background.

Problems with standard model:

Singularity

Horizon

Flatness

Homogeneity

Perturbations

Dark matter

Dark energy / cosmological constant

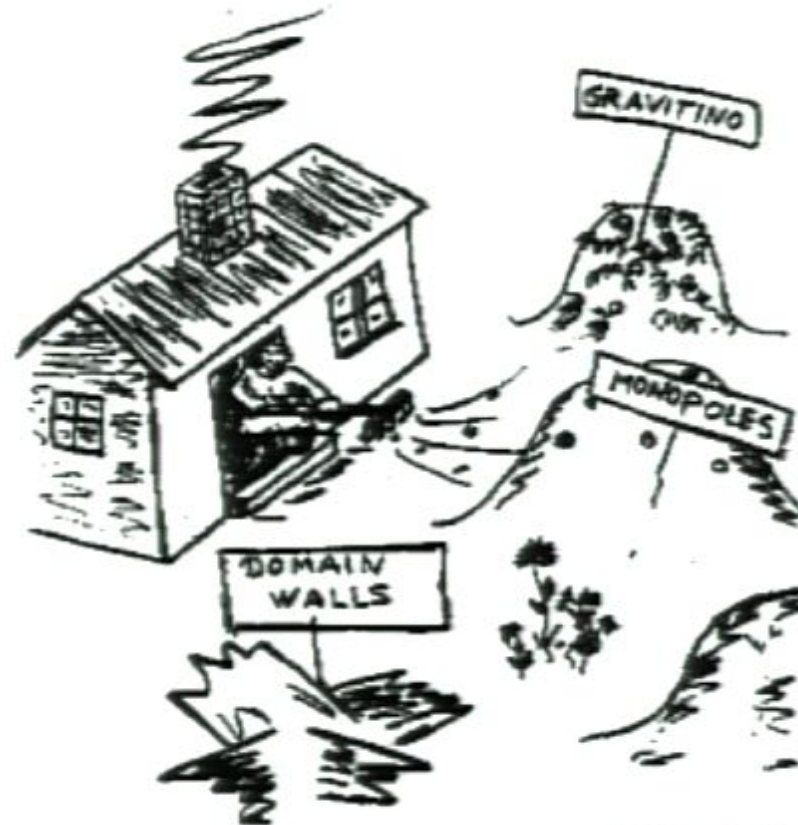
Baryogenesis

...

Accepted solution = INFLATION

Topological defects (monopoles)

THE MAIN IDEA OF THE
INFLATIONARY UNIVERSE SCENARIO



(Linde's book)

Inflation

- ☺ **solves cosmological puzzles**
- ☺ **uses GR + scalar fields [(semi-)classical]**
- ☺ **can be implemented in high energy theories**
- ☺ **makes falsifiable predictions ...**
- ☺ **... consistent with all known observations**
- ☺ **string based ideas (brane inflation, ...)**

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Alternative model???

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Alternative model???

- string based ideas (PBB, other brane models, string gas, ...)

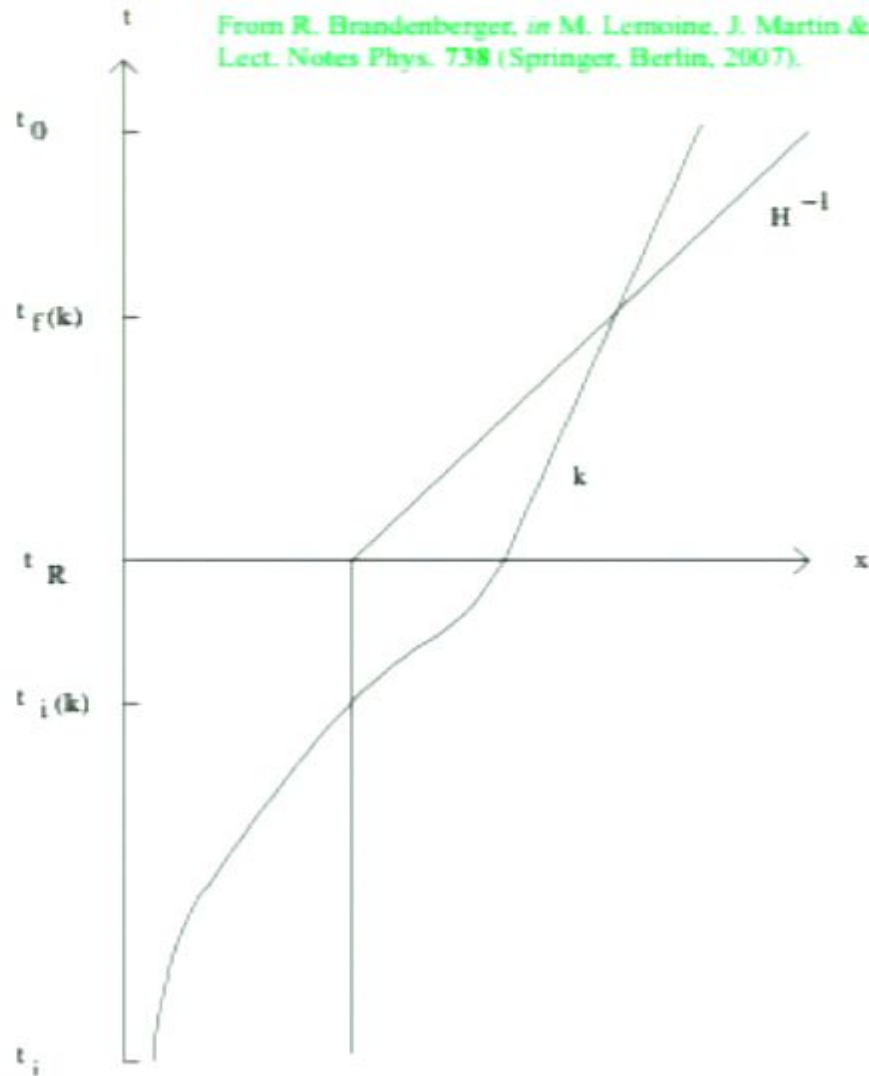
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Alternative model???

- string based ideas (PBB, other brane models, string gas, ...)
- Quantum gravity / cosmology
- singularity, initial conditions & homogeneity

From R. Brandenberger, in M. Lemoine, J. Martin & P. P. (Eds.), "Inflationary cosmology",
Lect. Notes Phys. 738 (Springer, Berlin, 2007).



💡 Scalar field origin?

💡 Trans-Planckian

$$\exists t; \ell(t) = \ell_0 \frac{a(t)}{a_0} \leq \ell_{\text{Pl}}$$

💡 Hierarchy (amplitude)

$$\frac{V(\varphi)}{\Delta\varphi^4} \leq 10^{-12}$$

💡 Singularity

$$\exists t_{(\pm\infty)}; a(t) \rightarrow 0$$

💡 Validity of GR?

$$E_{\text{inf}} \simeq 10^{-3} M_{\text{Pl}}$$

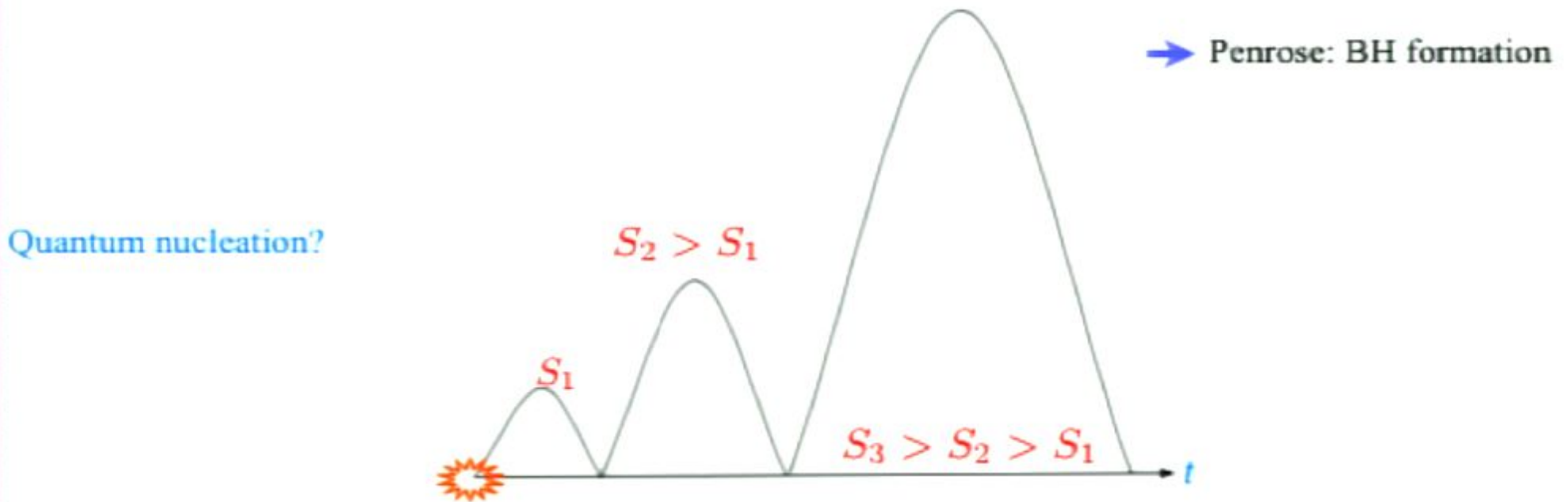
A brief history of bouncing cosmology

R. C. Tolman, "On the Theoretical Requirements for a Periodic Behaviour of the Universe", PRD 38, 1758 (1931)

G. Lemaitre, "L'Univers en expansion", Ann. Soc. Sci. Bruxelles (1933)

A. A. Starobinsky, "On one non-singular isotropic cosmological model", Sov. Astron. Lett. 4, 82 (1978)

R. Durrer & J. Laukerman, "The oscillating Universe: an alternative to inflation", Class. Quantum Grav. 13, 1069 (1996)

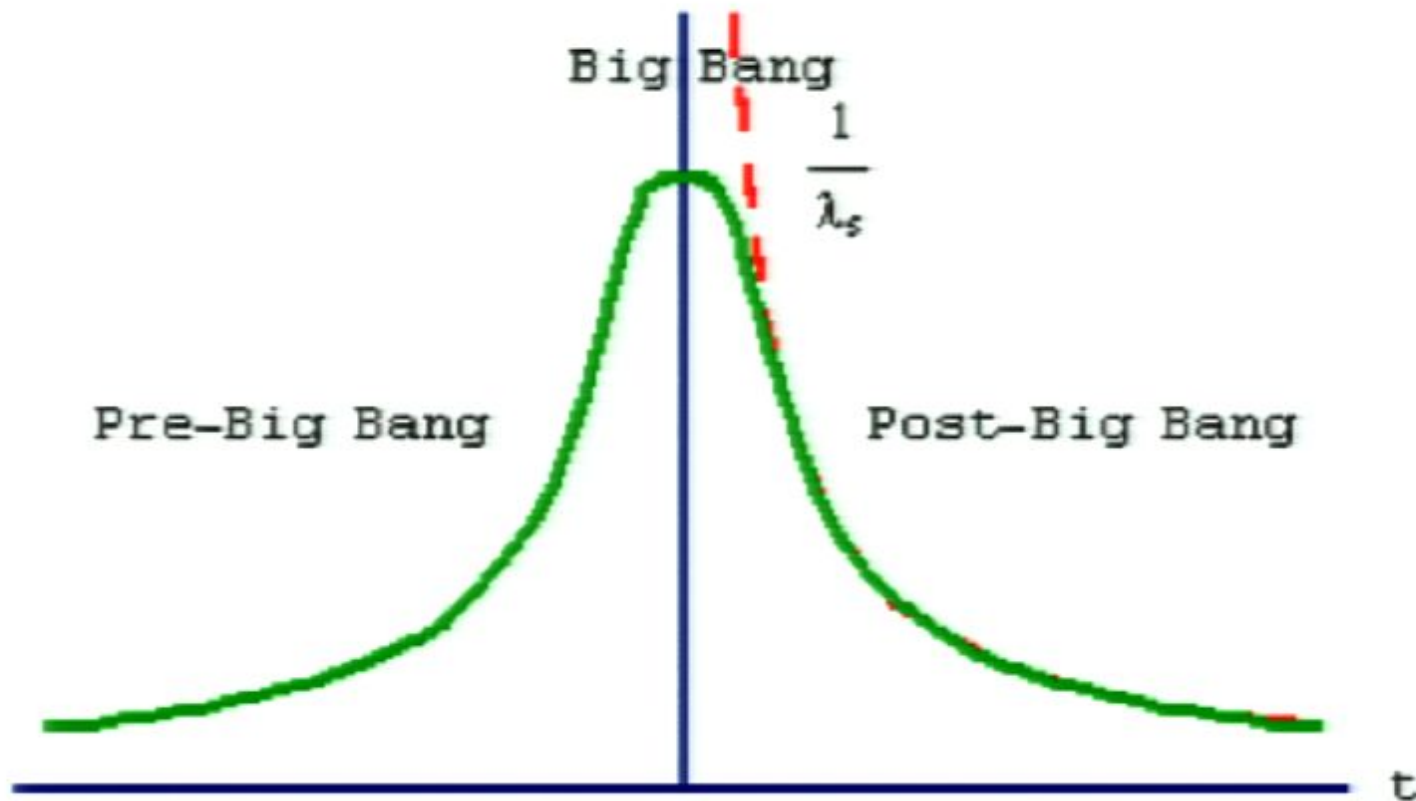


→ PBB - Ekpyrotic - Modified gravity - Quantum cosmology - Quintom - Horava-Lifshitz - Lee-Wick - ...

Perimeter Institute - 11th May 2010

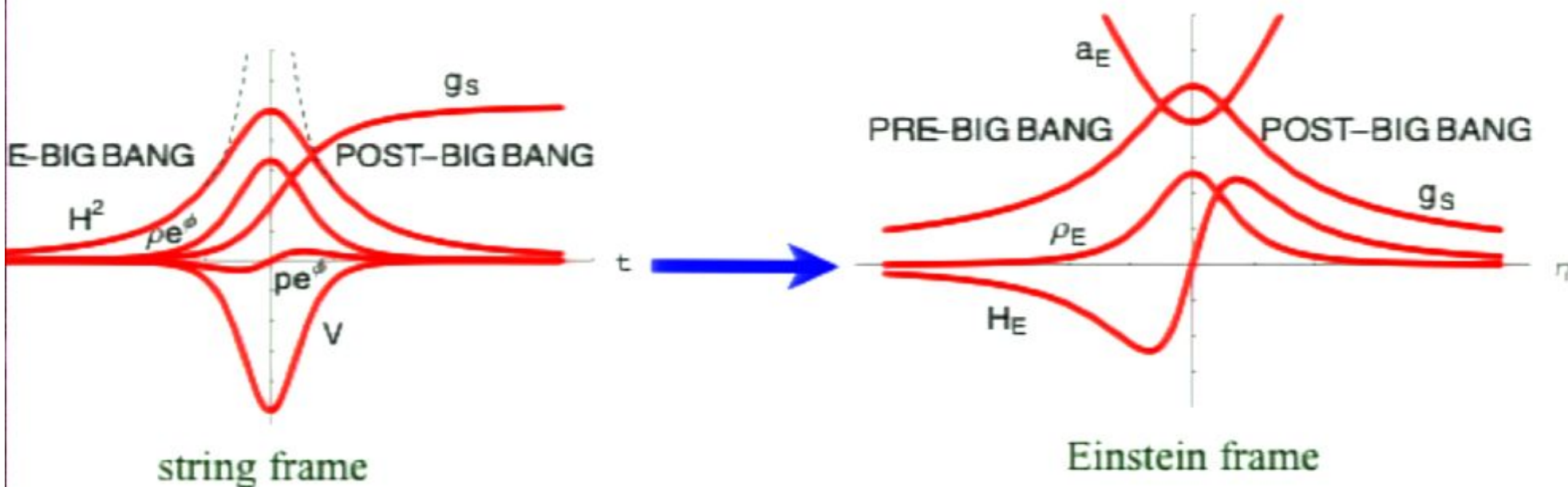
Pre Big Bang scenario:

(cf. M. Gasperini & G. Veneziano, arXiv: hep-th/0703055)

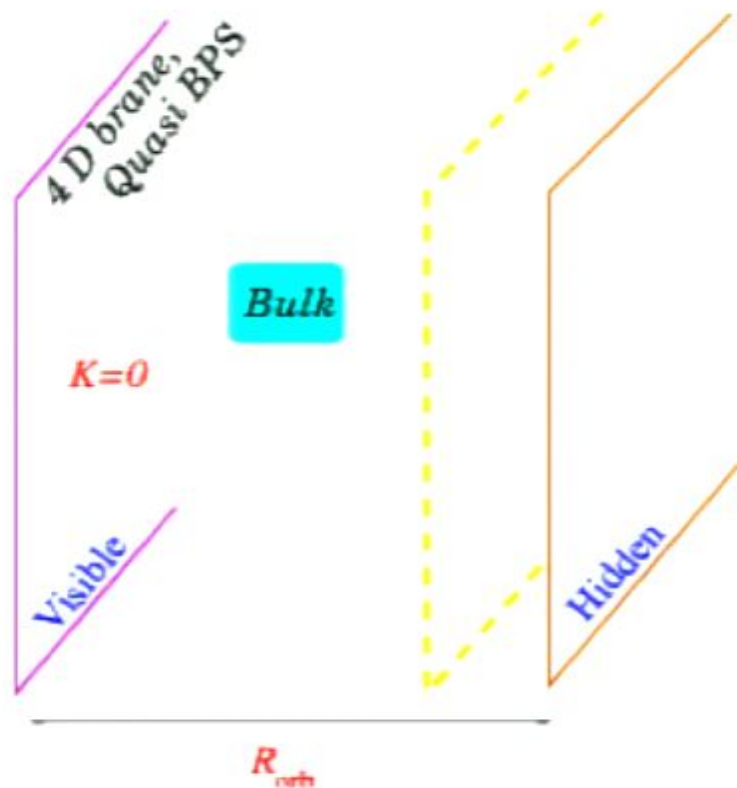


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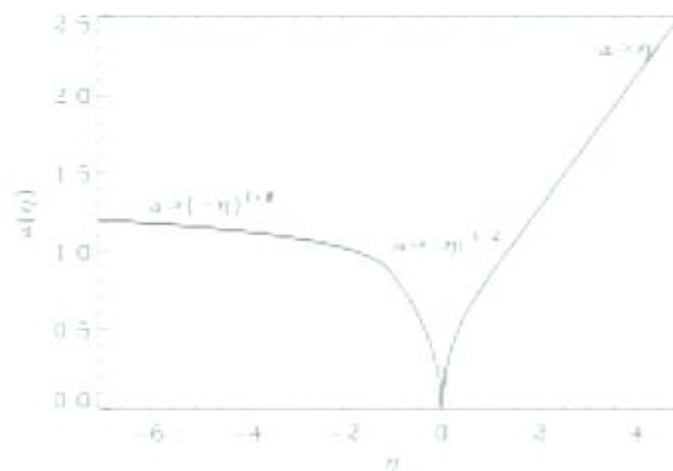
Ekpyrotic scenario:



$$S_5 \propto \int_{\mathcal{M}_5} d^5x \sqrt{-g_5} \left[R_{(5)} - \frac{1}{2} (\partial\varphi)^2 - \frac{3}{2} \frac{e^{2\varphi} \mathcal{F}^2}{5!} \right],$$

$$S_4 = \int_{\mathcal{M}_4} d^4x \sqrt{-g_4} \left[\frac{R_{(4)}}{2\kappa} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right].$$

$$V(\varphi) = -V_1 \exp \left[-\frac{4\sqrt{\pi\gamma}}{m_{\text{Pl}}} (\varphi - \varphi_1) \right].$$



Standard Failures and some solutions

- Singularity
- Horizon
- Flatness
- Homogeneity
- $t_{\text{dissipation}}$
- Perturbations
- Others

Standard Failures and some solutions

● Singularity Merely a non issue in the bounce case!



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$d_H \equiv a(t) \int_{t_i}^t \frac{d\tau}{a(\tau)}$ can be made divergent easily if $t_i \rightarrow -\infty$

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accelerated expansion (**inflation**) or decelerated contraction (**bounce**)





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 $\frac{t_{\text{dissipation}}}{t_{\text{Hubble}}} \propto \frac{\lambda}{R_H^{1/3}} \left(1 + \frac{\lambda}{AR_H^2} \right) \implies$ enough time to dissipate any wavelength

 \implies vacuum state!
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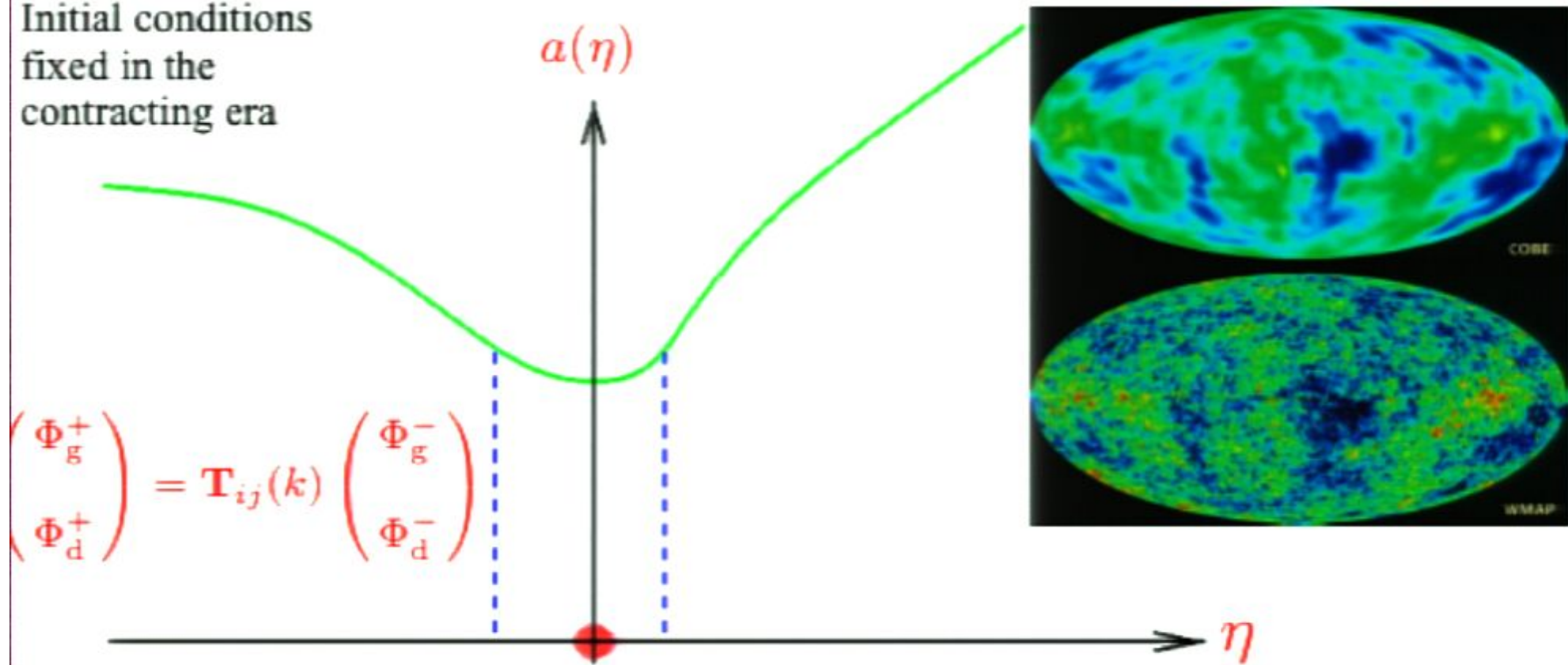
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Initial conditions
fixed in the
contracting era



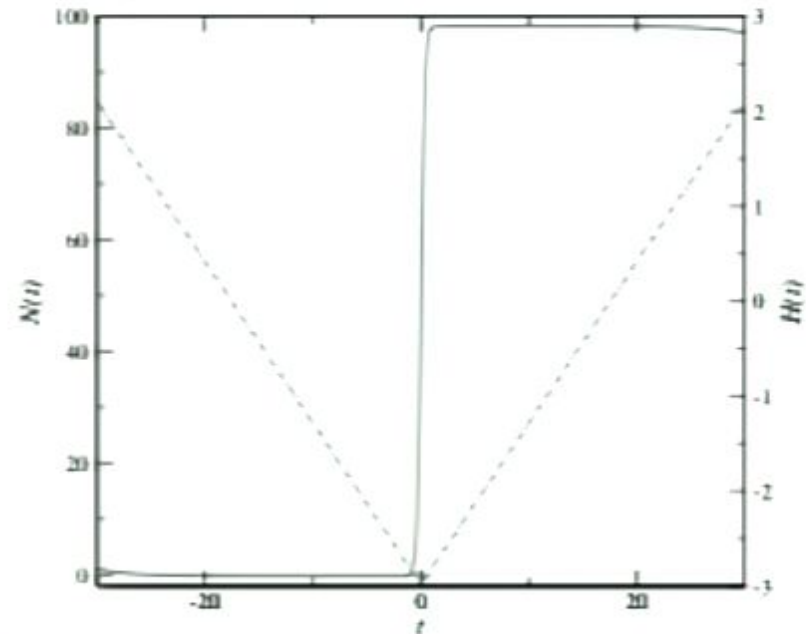
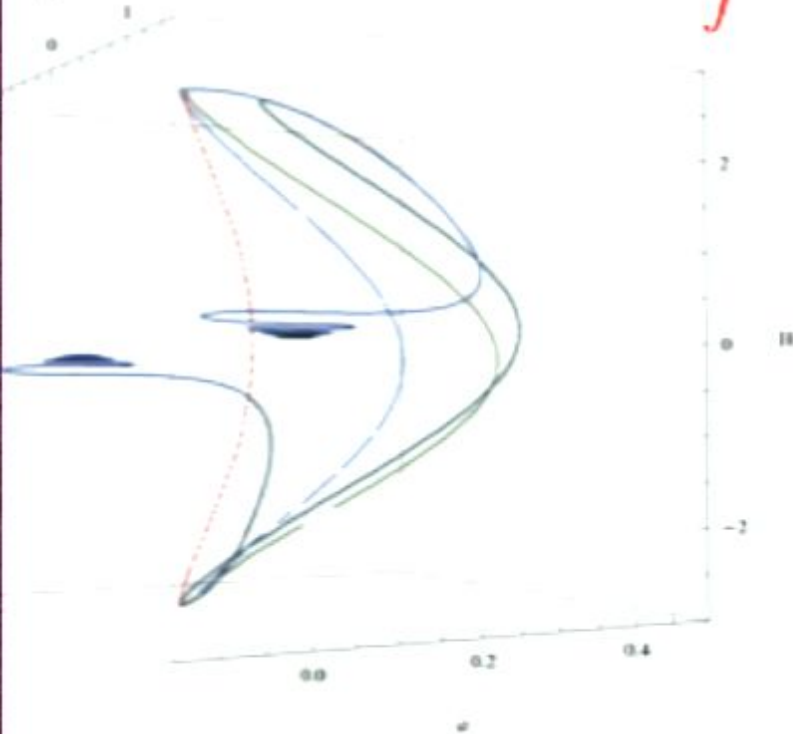
Self consistent bounce:

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2 \right)$$

→ One d.o.f. + 4 dimensions G.R.

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{6\ell_{Pl}^2} - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right]$$

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\varphi}^2 + V \right) - \frac{\kappa}{a^2} \quad \text{Positive spatial curvature}$$

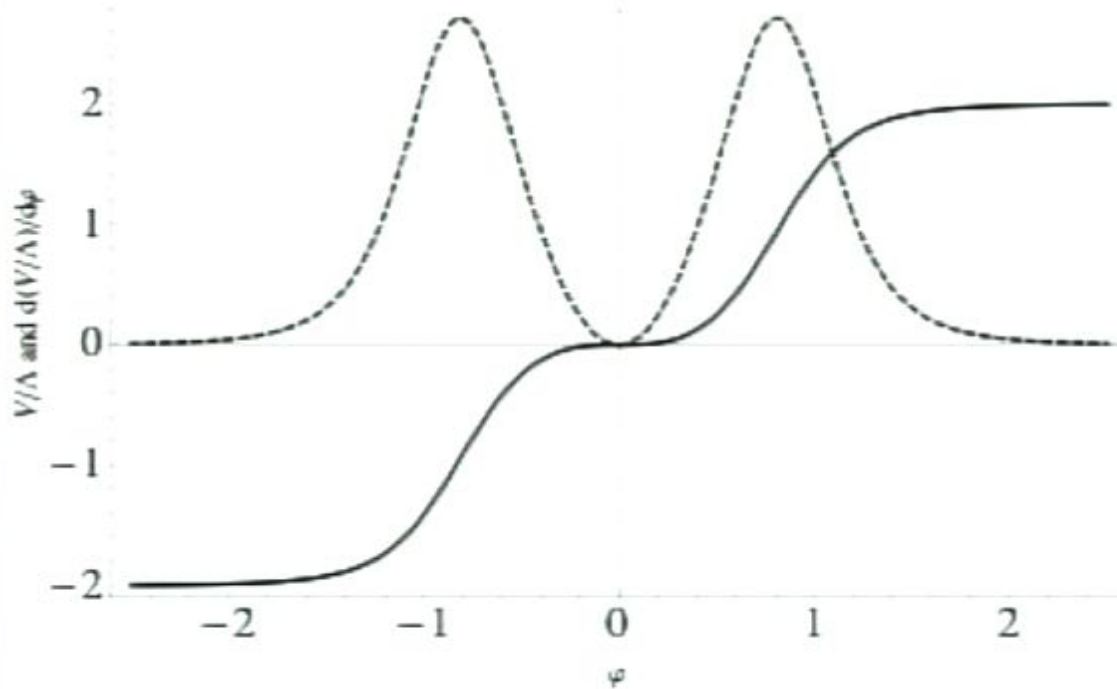


de Sitter, M. Lilley & P. P. Avez. Rev. D77, 083513 (2008)

fluence of the spatial curvature? $ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2 \right) \Rightarrow \eta_0 > 1$

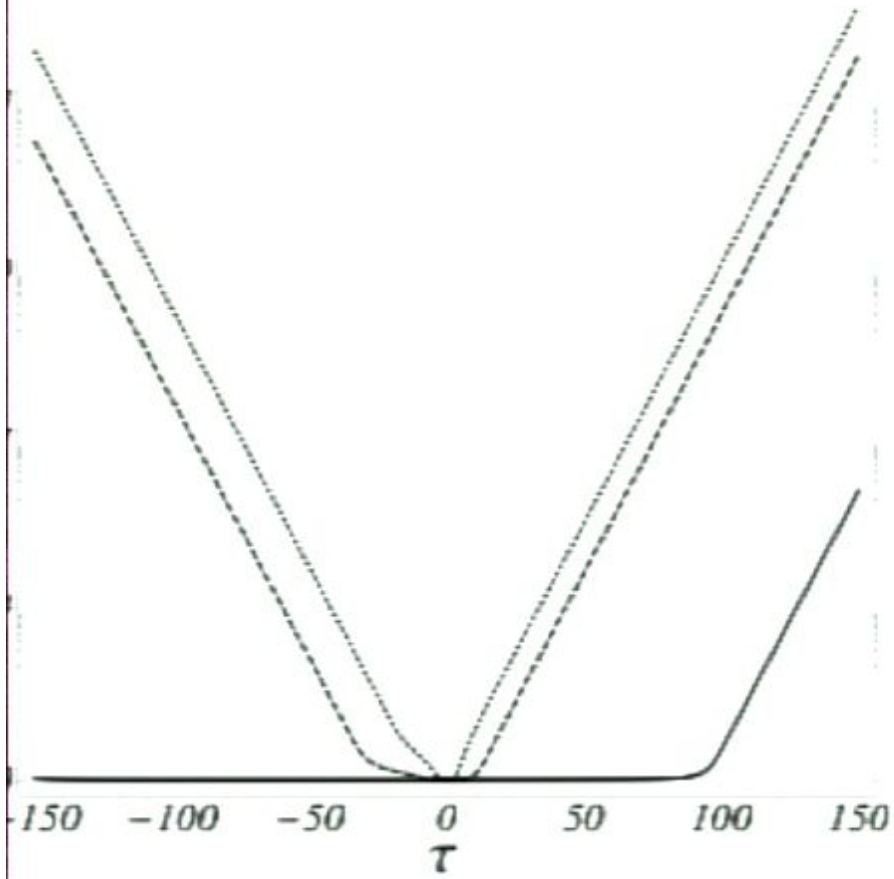
→ Modify GR to non singular theories (curvature invariants)

●
$$\mathcal{S} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[R + \sum_{i=1}^N \varphi_i I^{(i)} - V(\varphi) \right] \quad \Rightarrow \quad \frac{dV}{d\varphi} = I$$

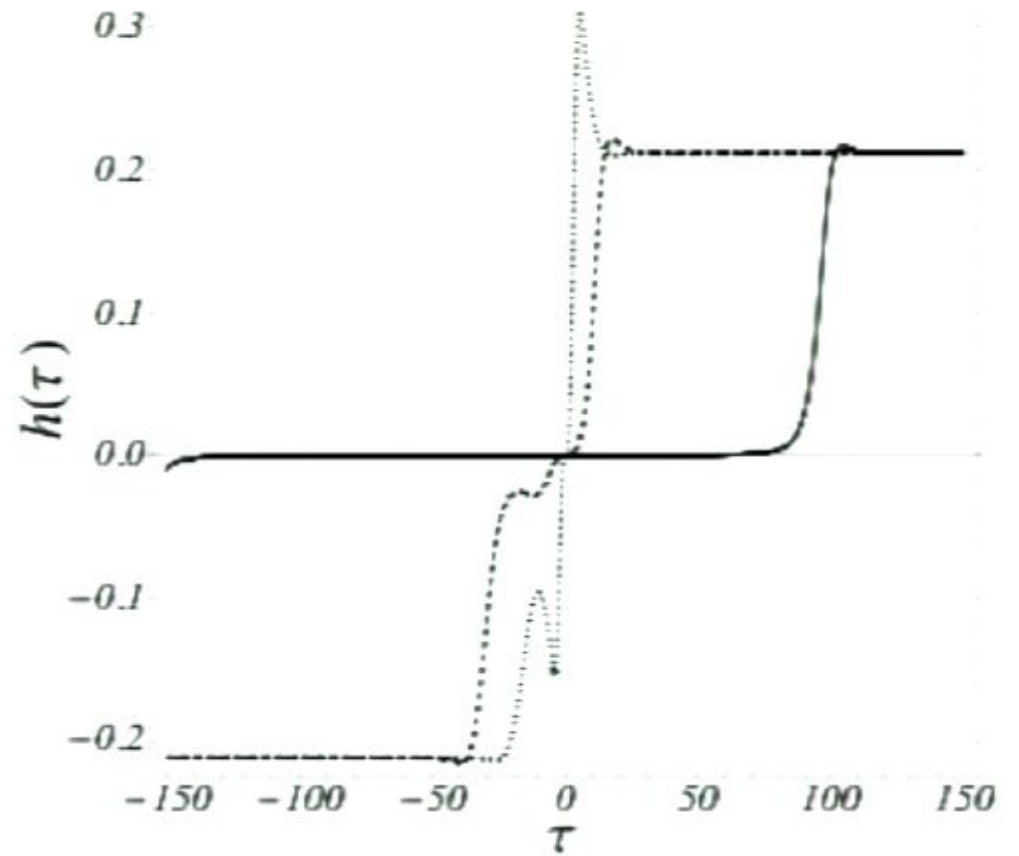


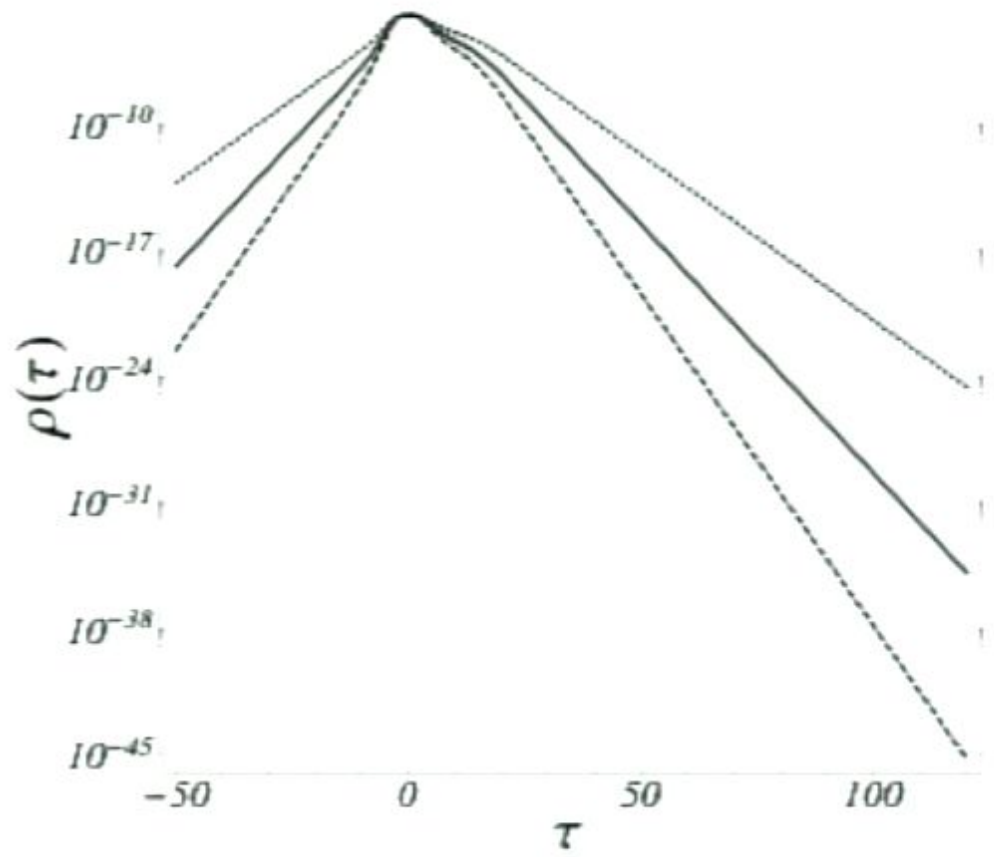
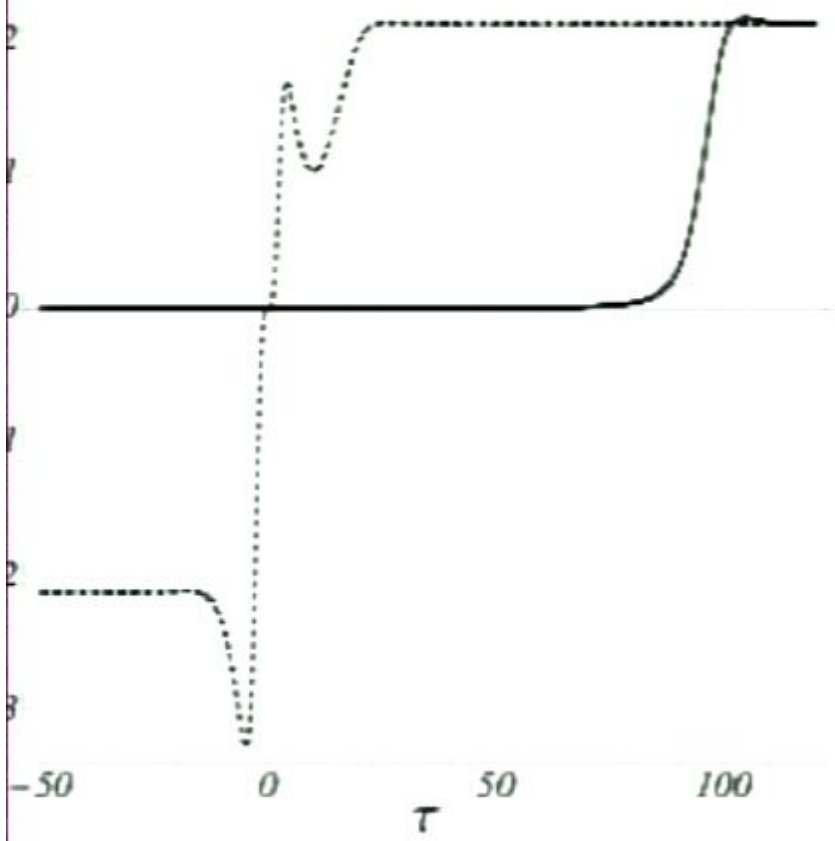
$$I = R - \sqrt{3(4R_{\mu\nu}R^{\mu\nu} - R^2)}$$

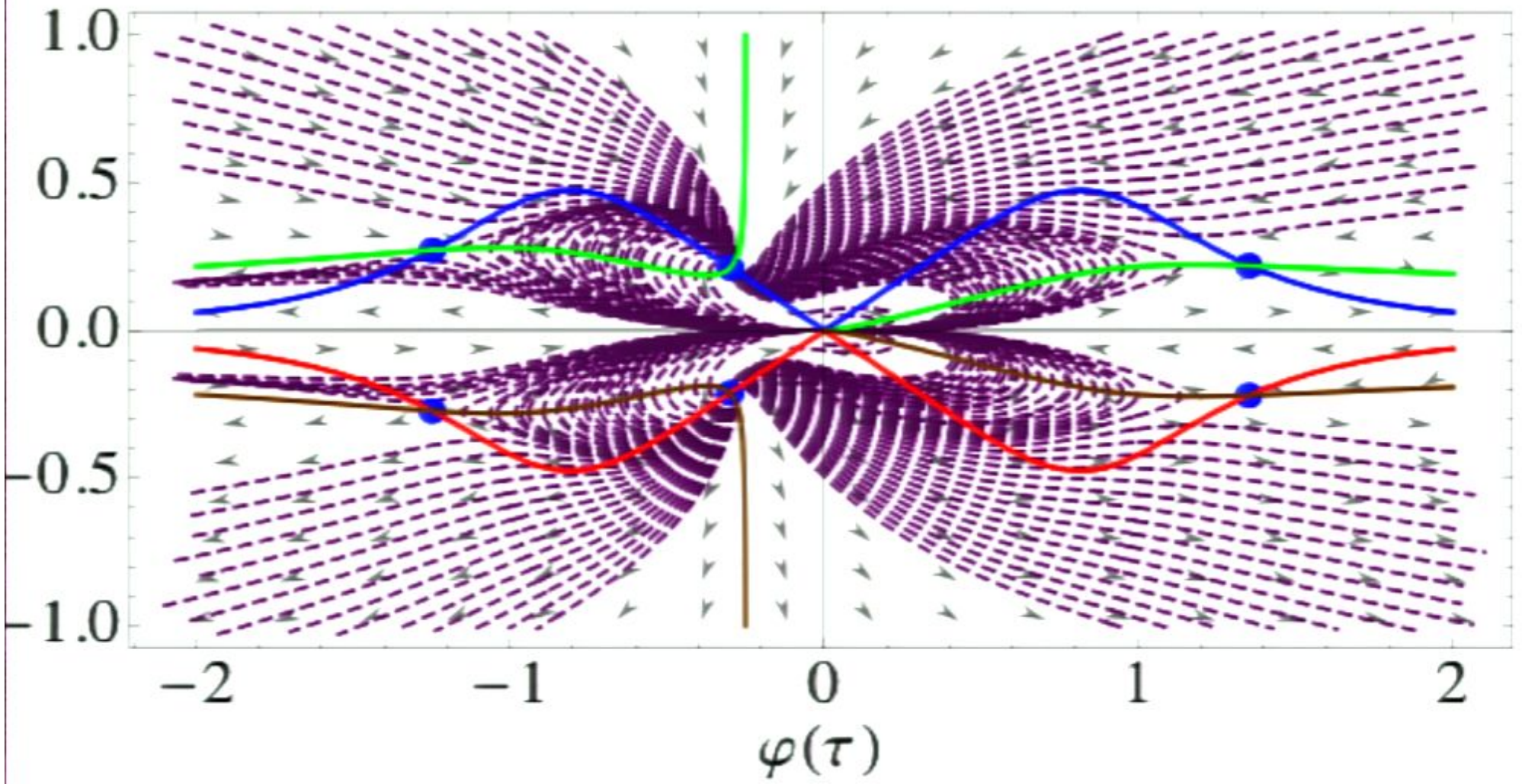
R. Abramo, P. P. & I. Yassda, *Phys. Rev. D* **81**, 023511 (2010)



η_0 arbitrary







perturbations:

$$ds^2 = a^2(\eta) \left\{ (1 + 2\Phi) d\eta^2 - [(1 - 2\Phi) \gamma_{ij} + h_{ij}] dx^i dx^j \right\}$$

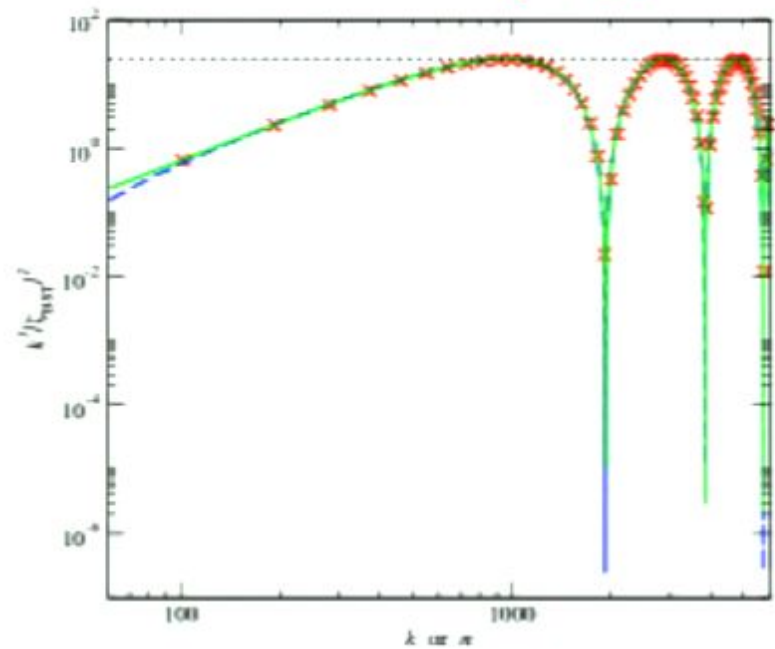
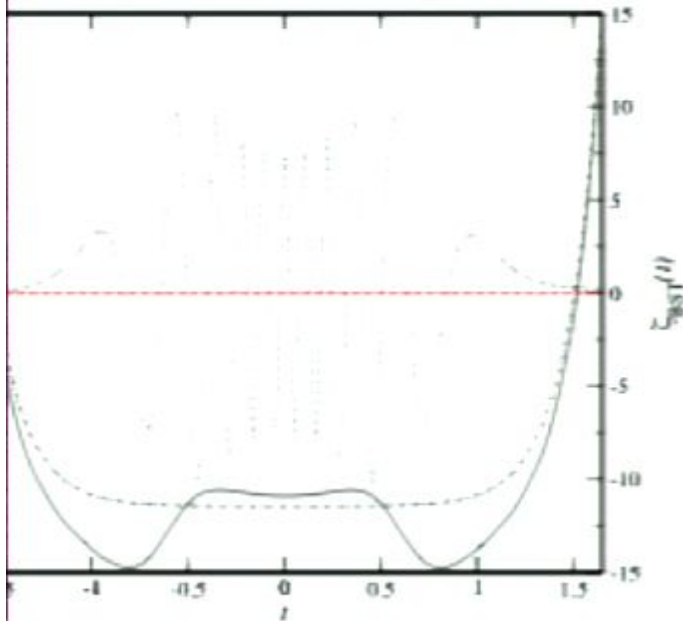


$$\Phi = \frac{3\mathcal{H}u}{2a^2\theta}$$

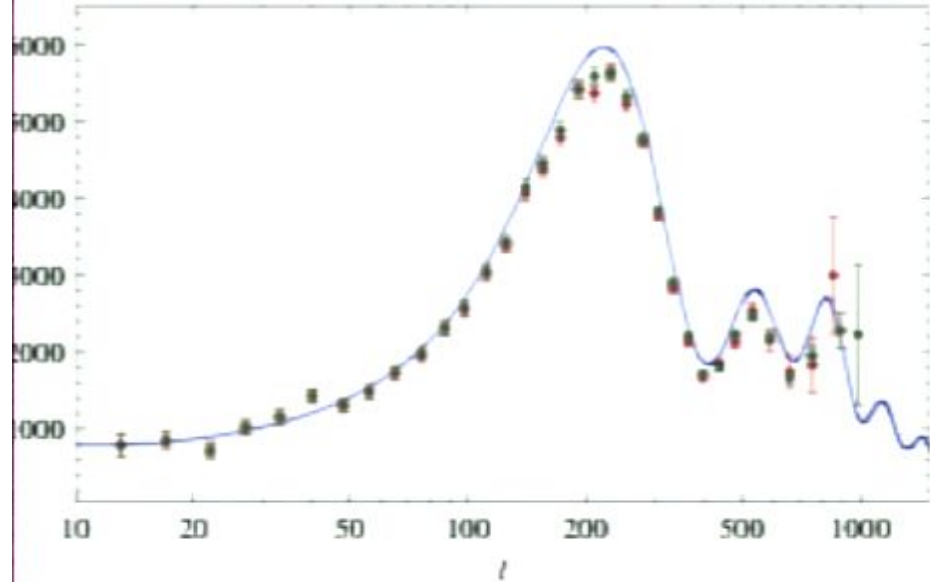
$$\theta \equiv \frac{1}{a} \sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi} \left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2} \right)}$$

$$u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K} (1 - c_s^2) \right] u = 0$$

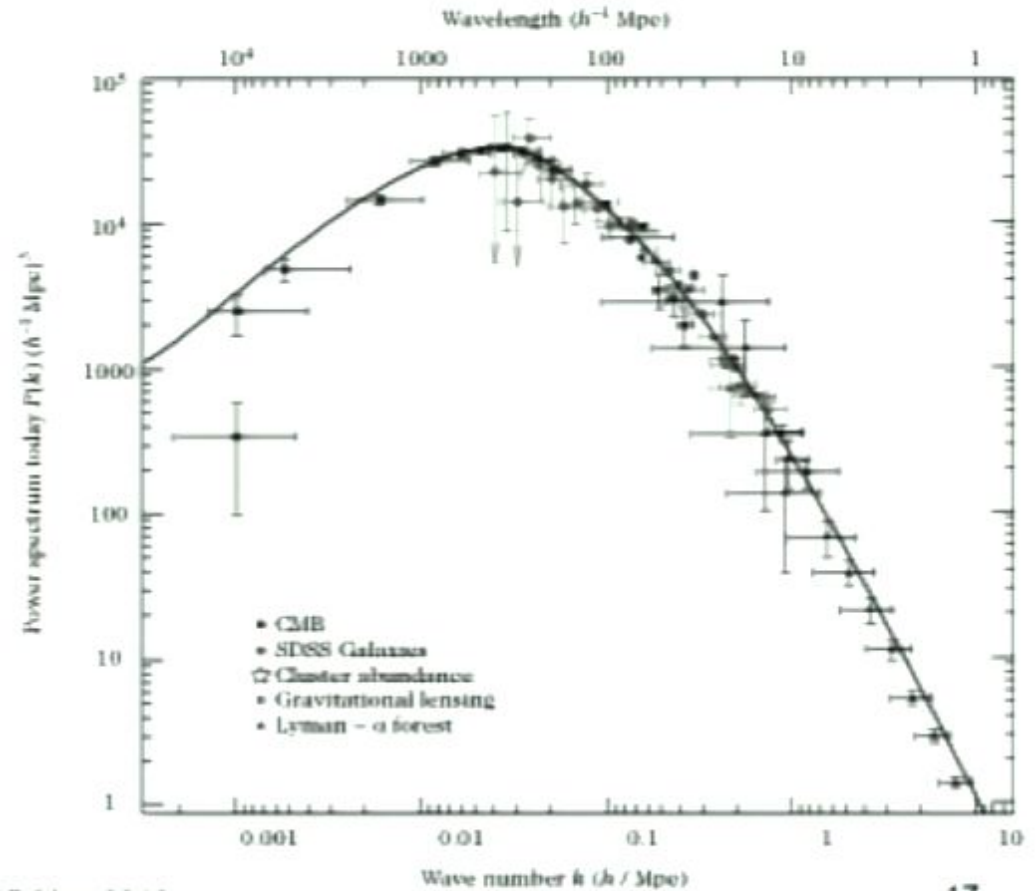
$$\mathcal{P}_\zeta = \mathcal{A} k^{n_s - 1} \cos^2 \left(\omega \frac{k_{\text{ph}}}{k_*} + \psi \right)$$

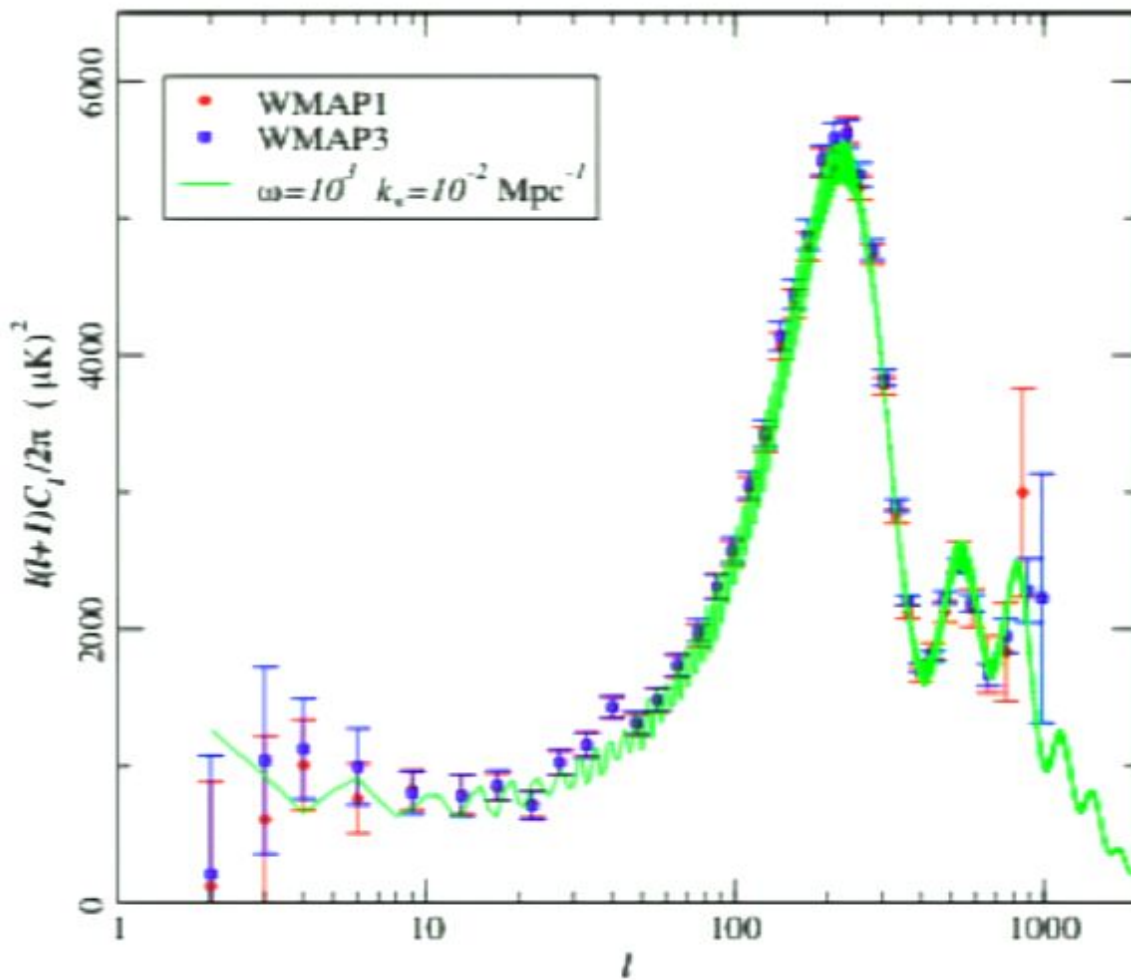


Data!

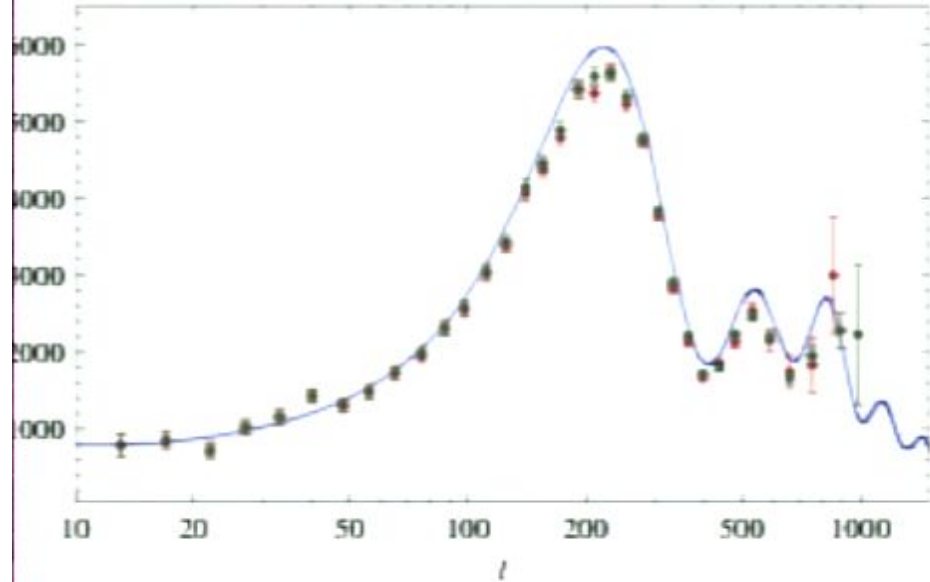


No obvious oscillations ...

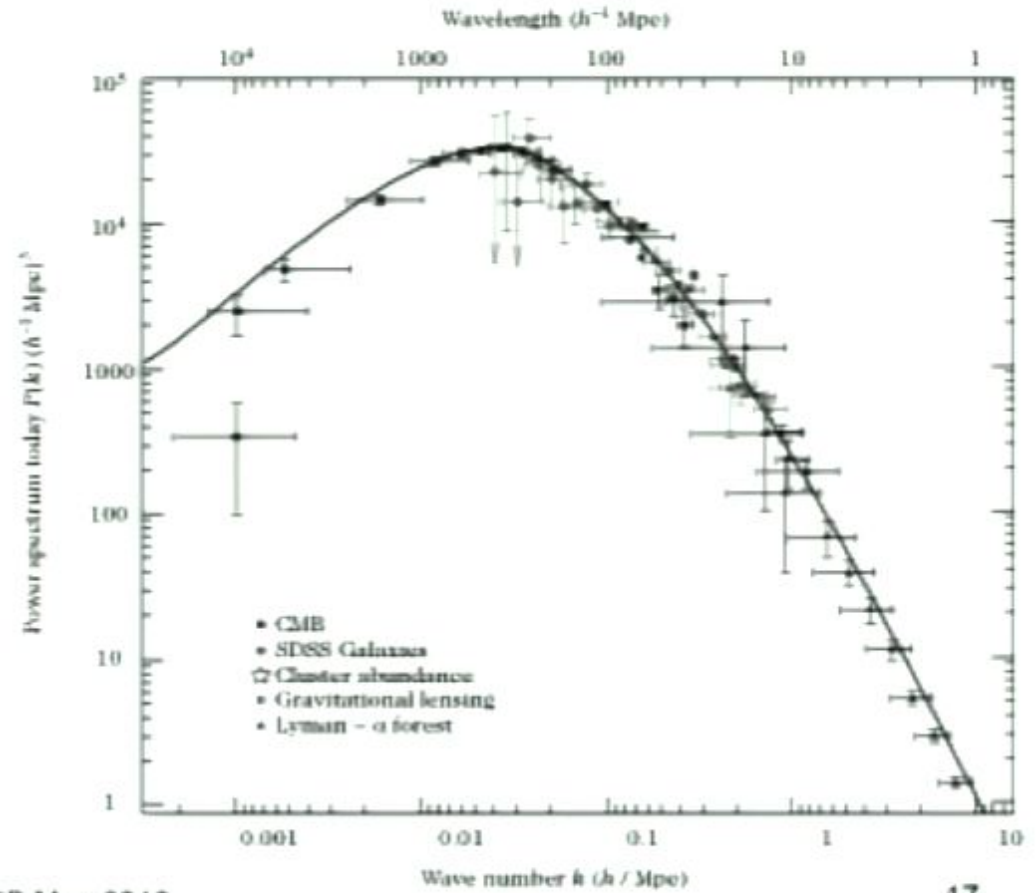




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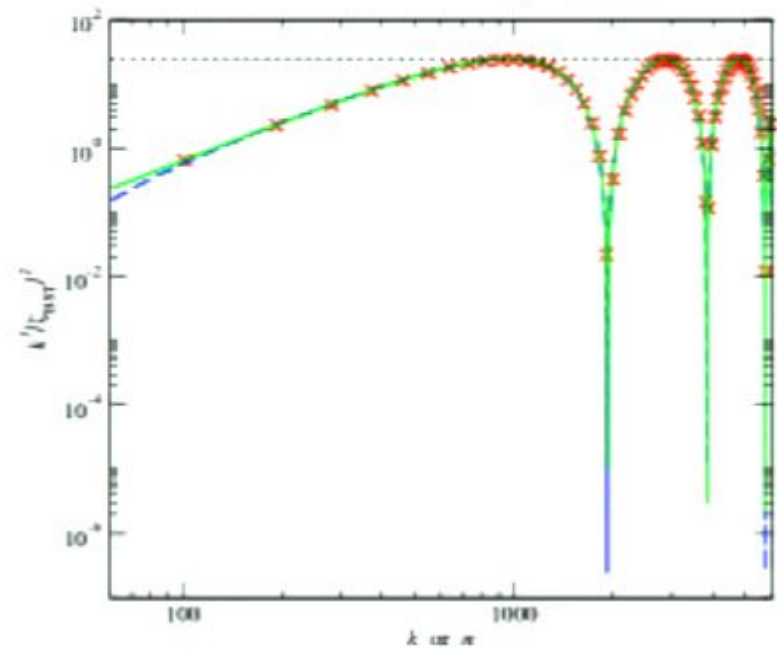
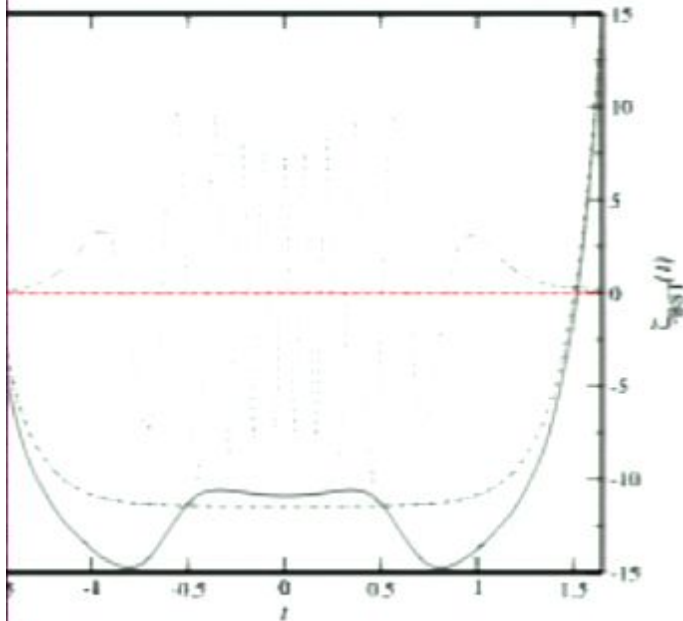


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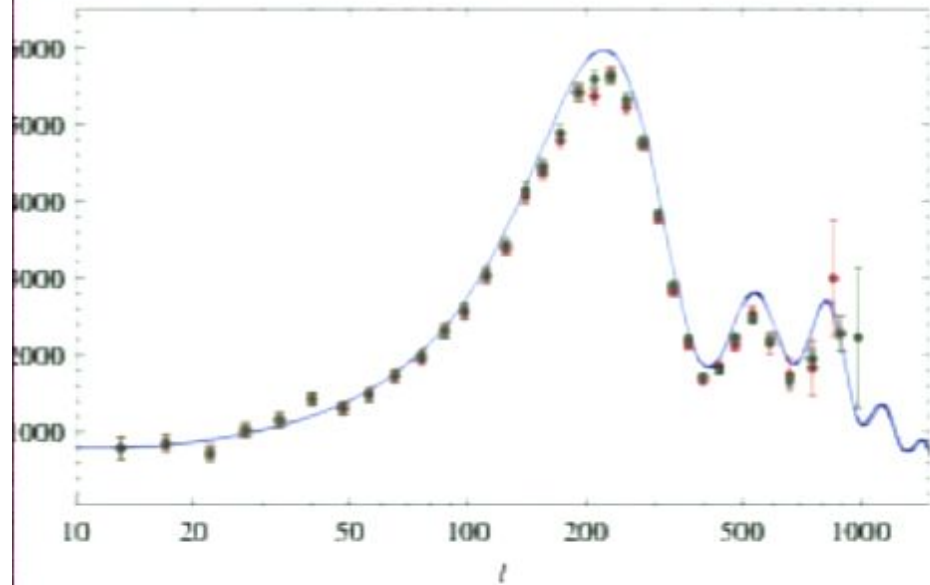
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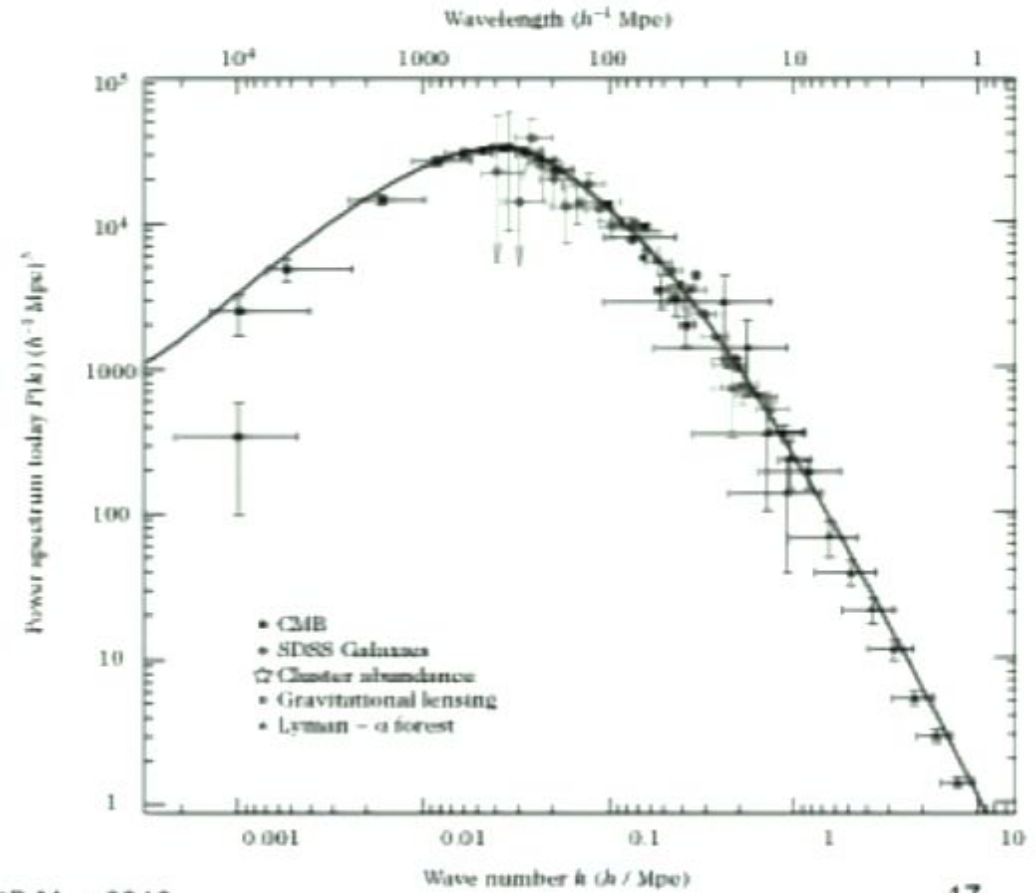
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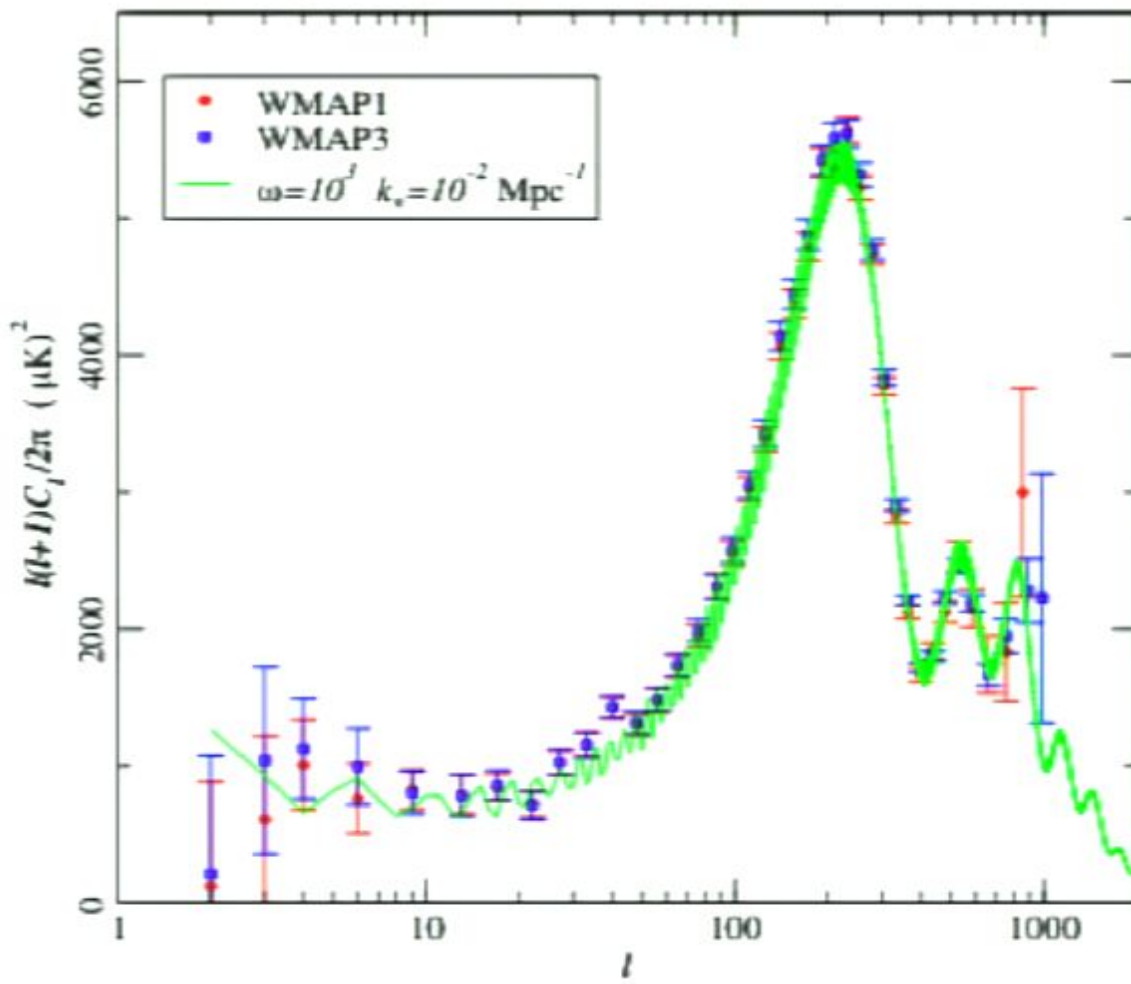


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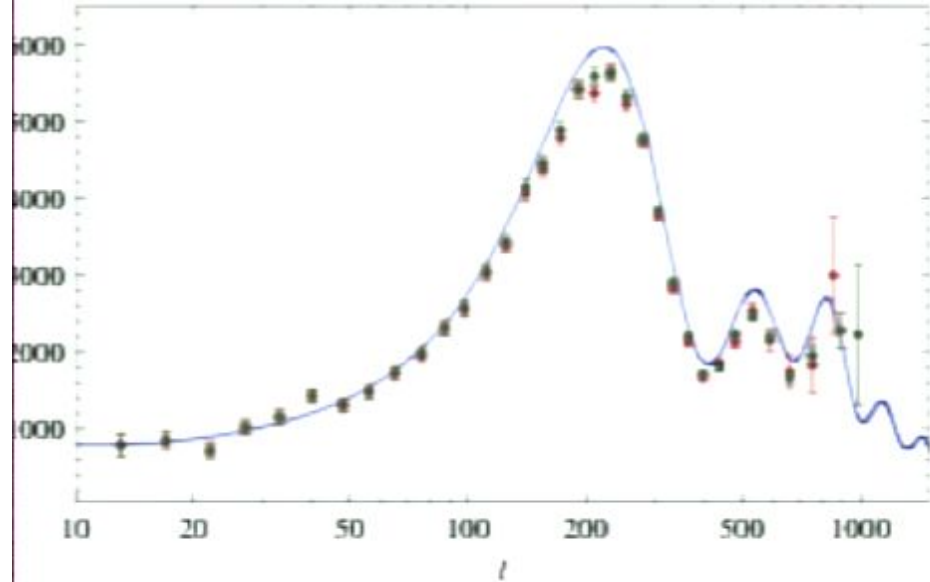


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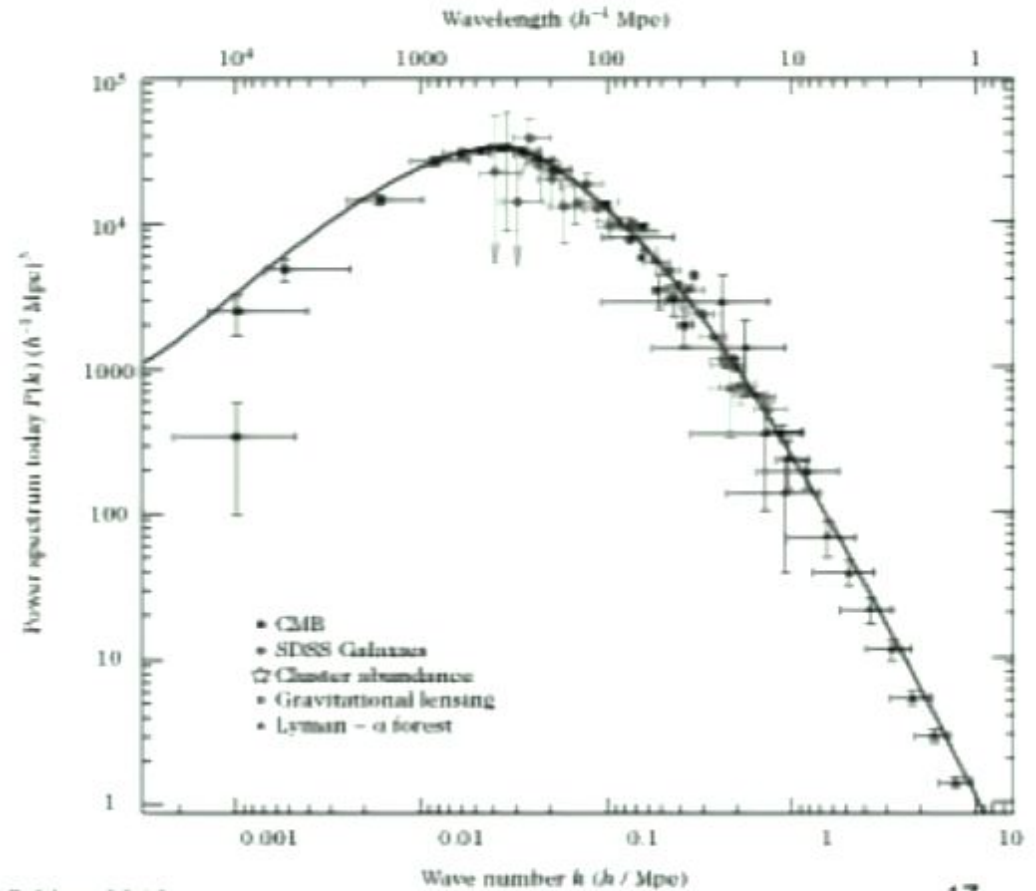




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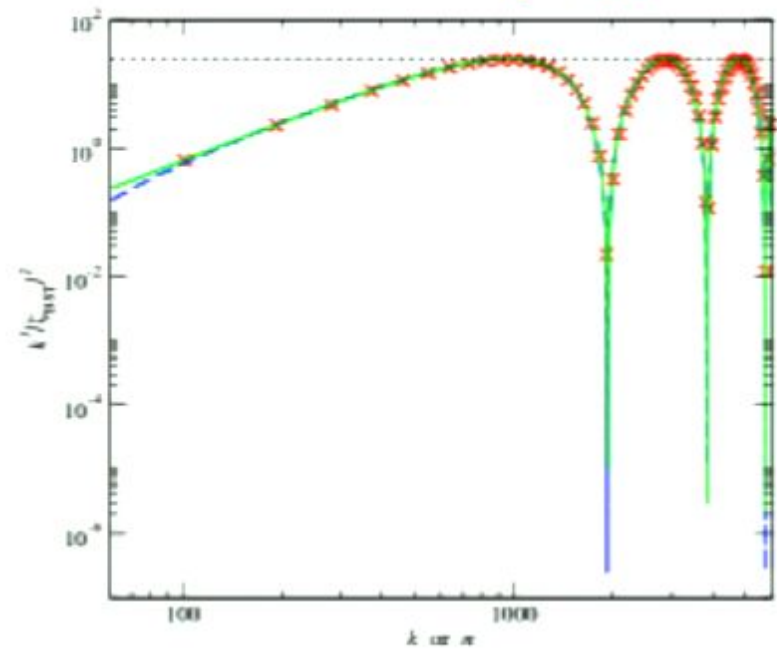
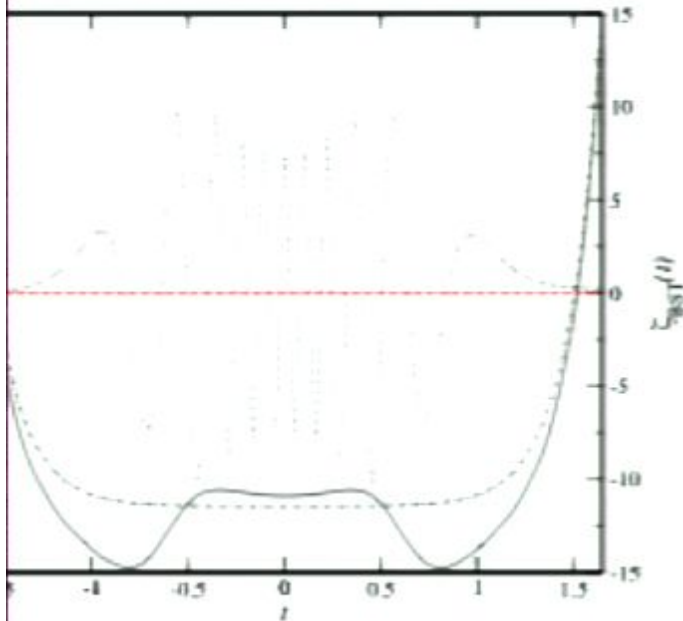


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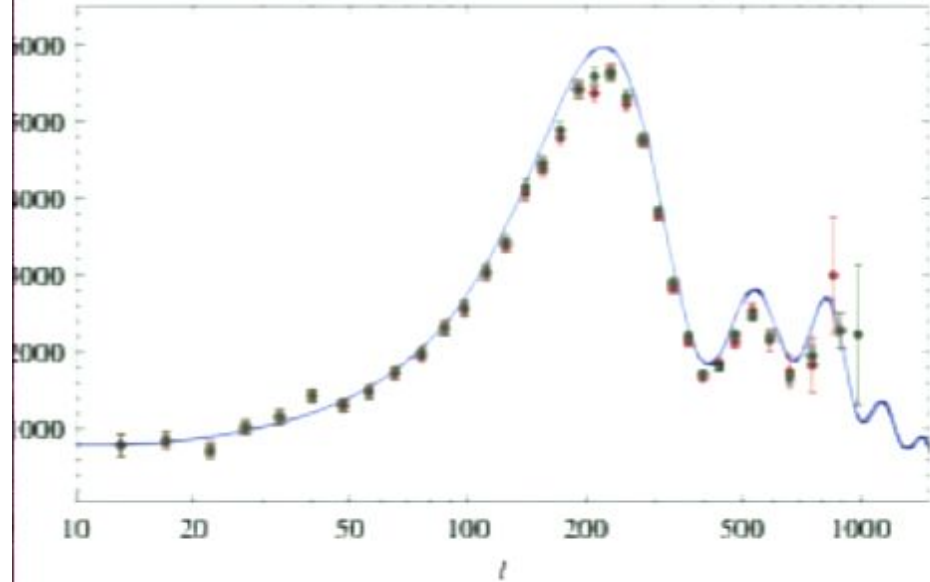
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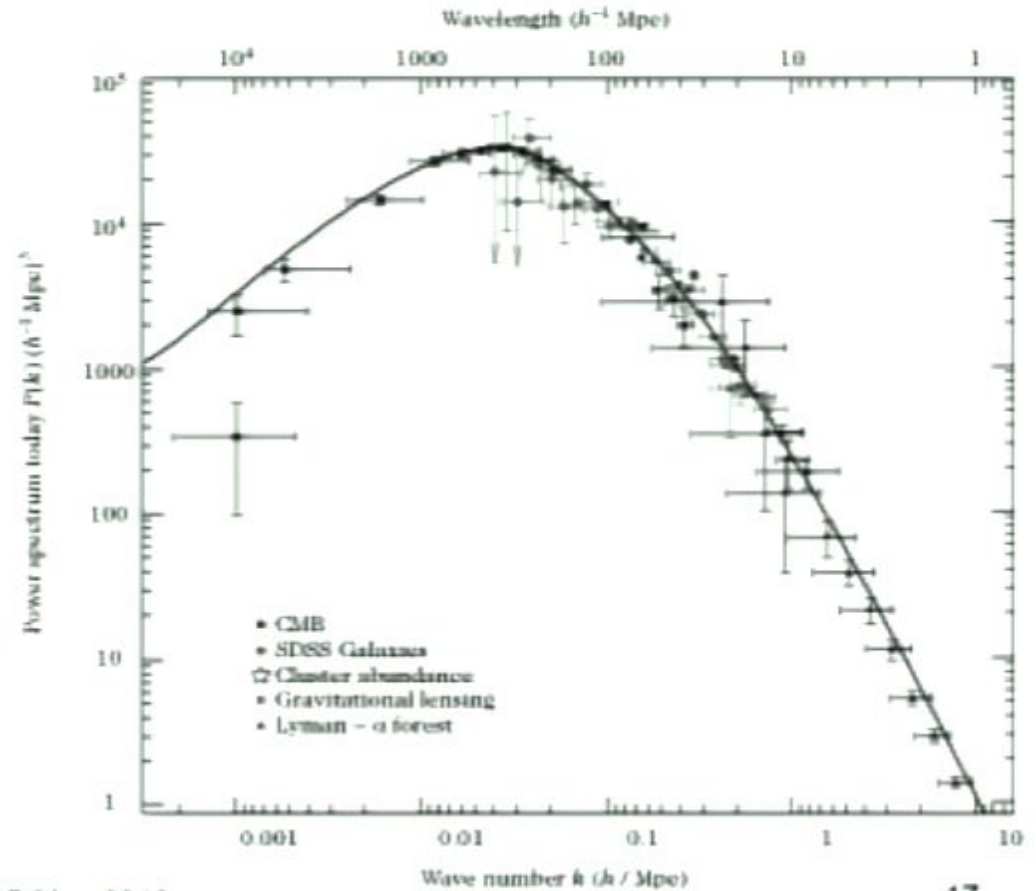
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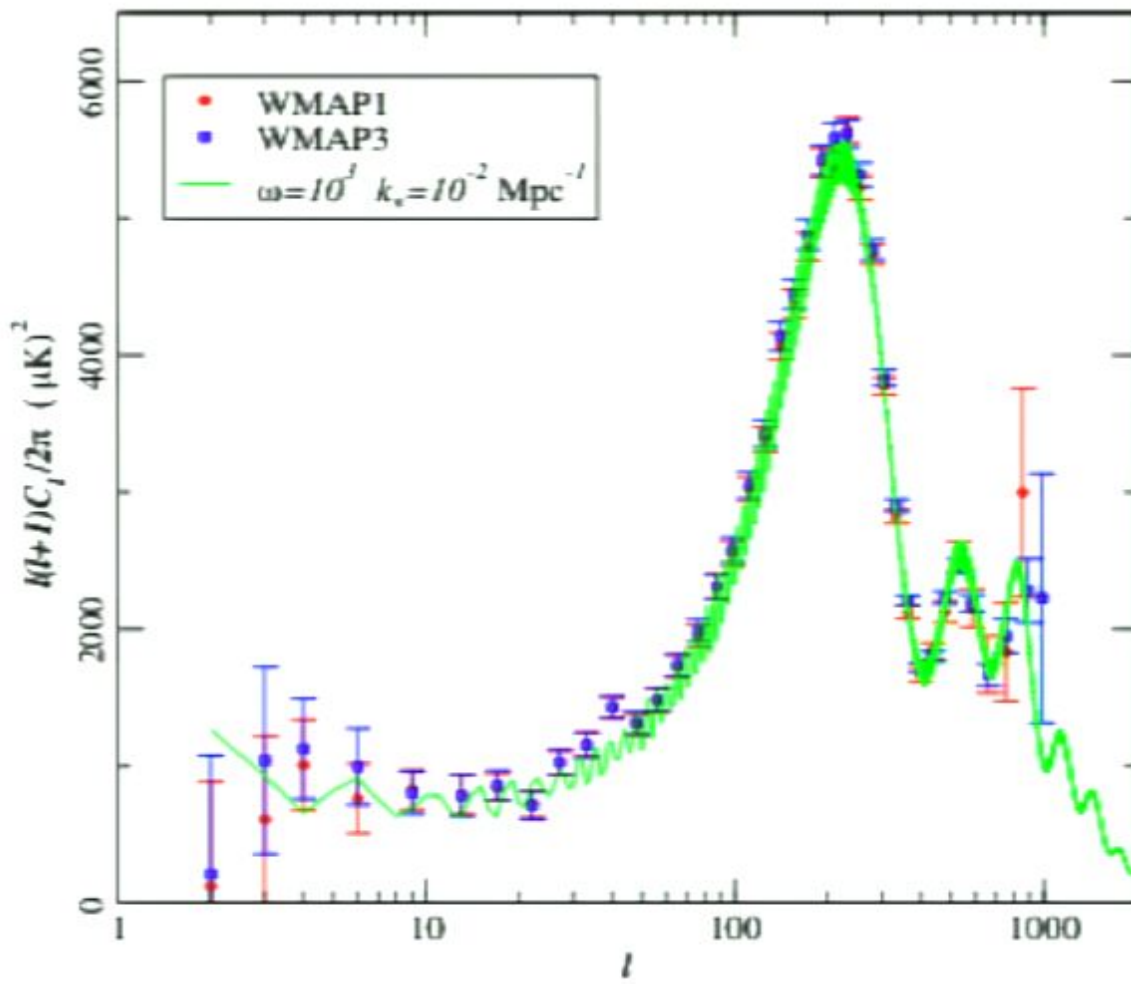


Data!



No obvious oscillations ...





K-bounce: $\mathcal{L} = p(X, \varphi)$

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

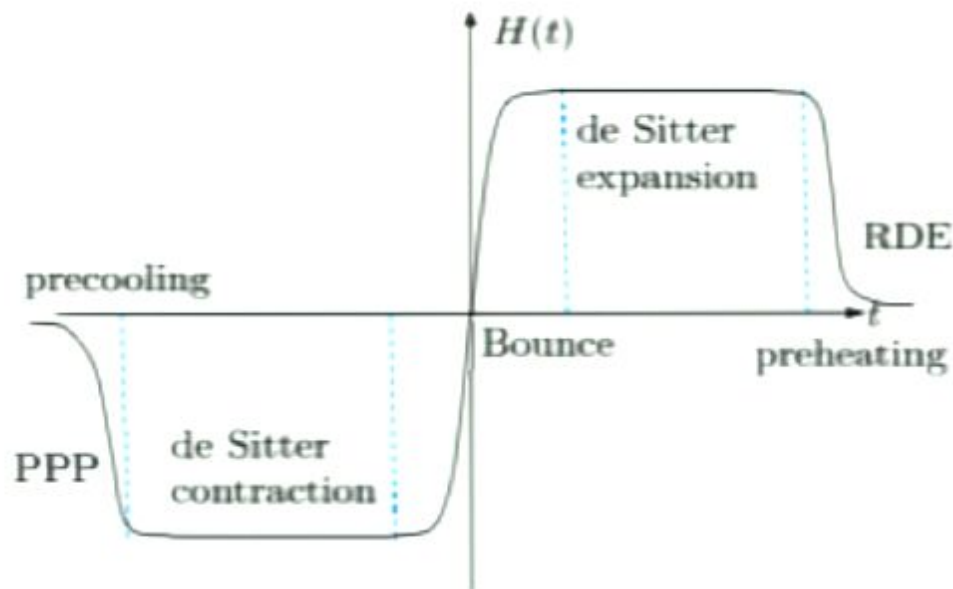
$$\Rightarrow T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$\rho \equiv 2X \frac{\partial p}{\partial X} - p$$

vanishing spatial curvature possible in 4 dimensions G.R.?

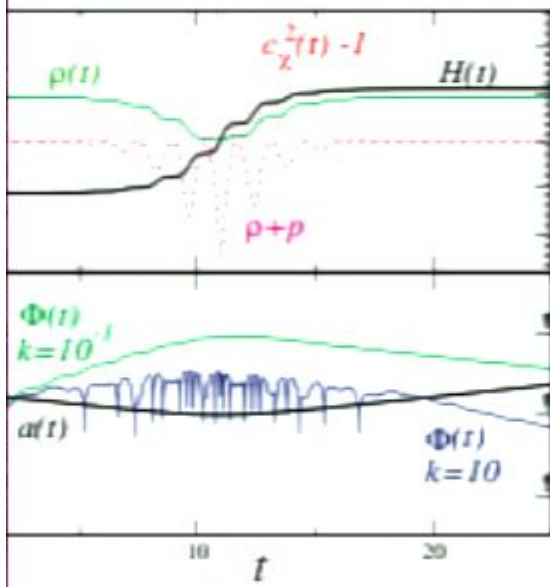
$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{2X}}$$

$$\rho(t_{\text{bounce}}) = 0 \implies p(t_{\text{bounce}}) < 0$$

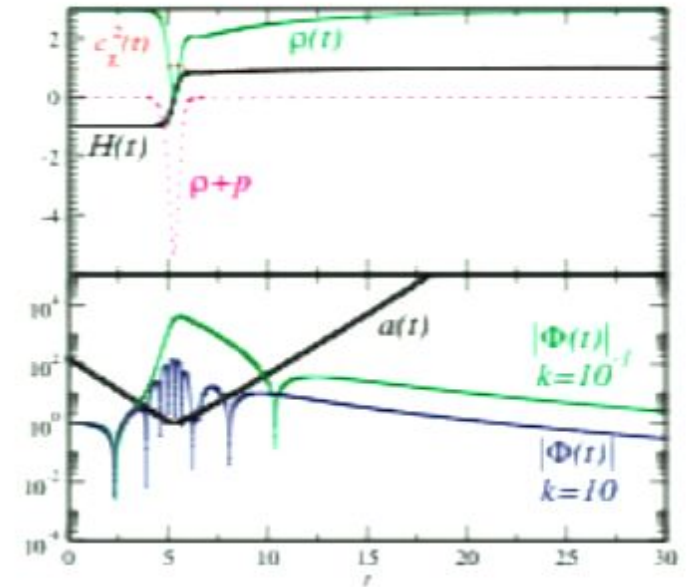
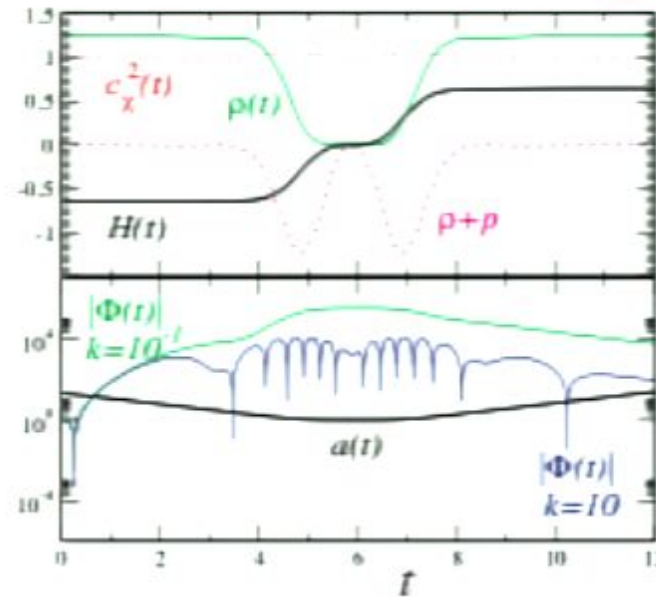


Model for the bounce phase only:

$$= p_0 + p_X(X - X_0) + p_\varphi\varphi + p_{X\varphi}\varphi(X - X_0) + \frac{1}{2}p_{XX}(X - X_0)^2 + \frac{1}{2}p_{\varphi\varphi}\varphi^2 + \dots$$

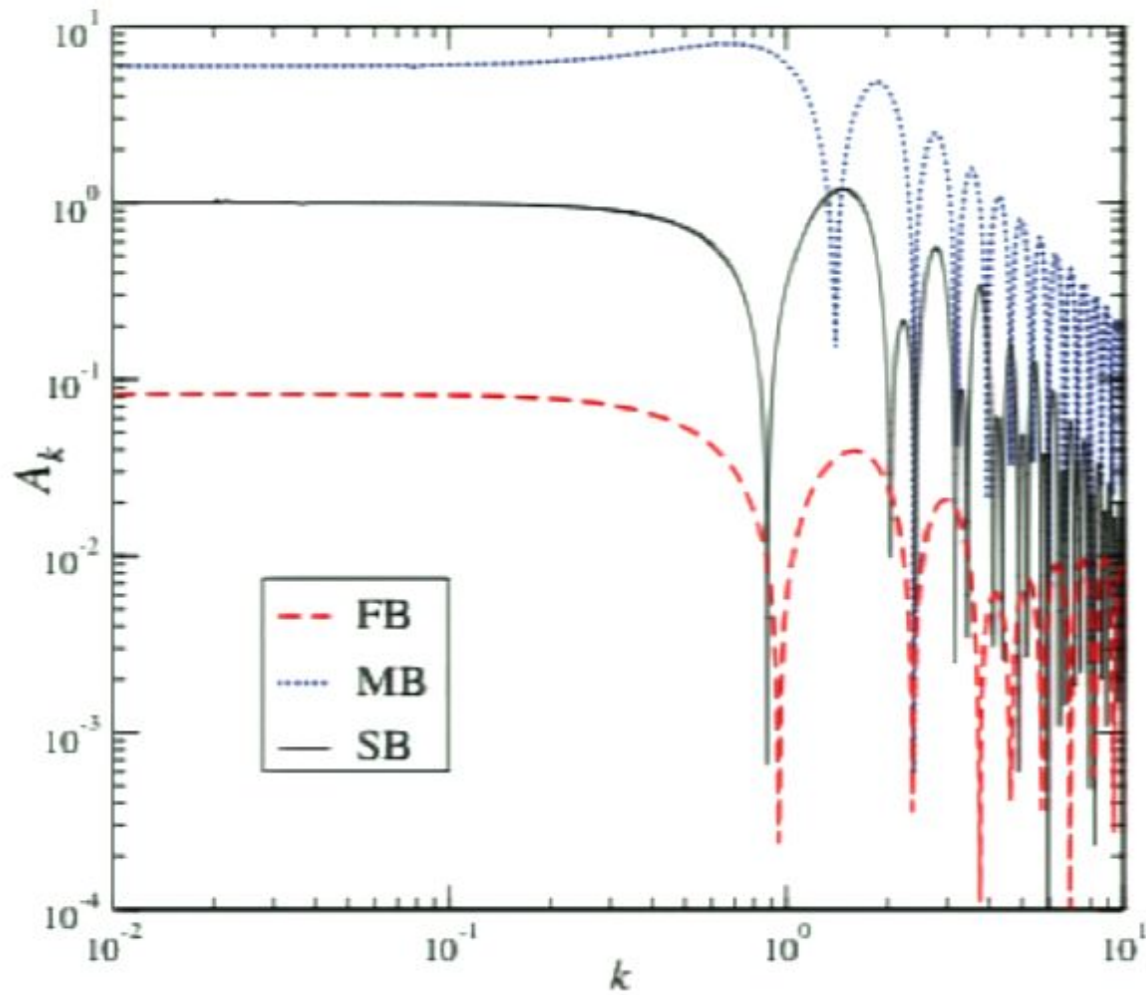


Slow



Fast

Oscillations + ζ conserved

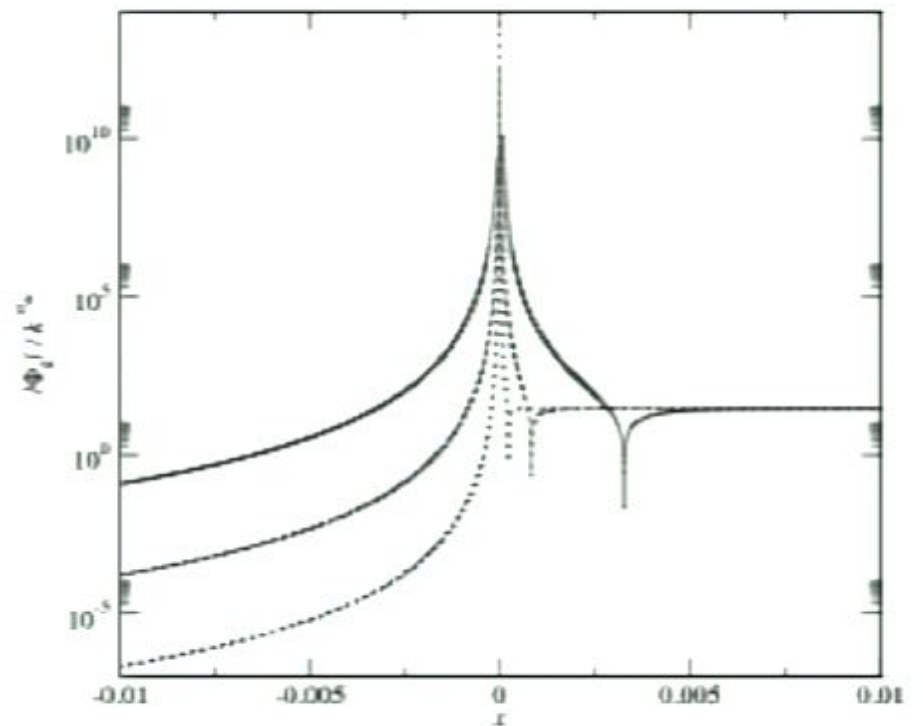
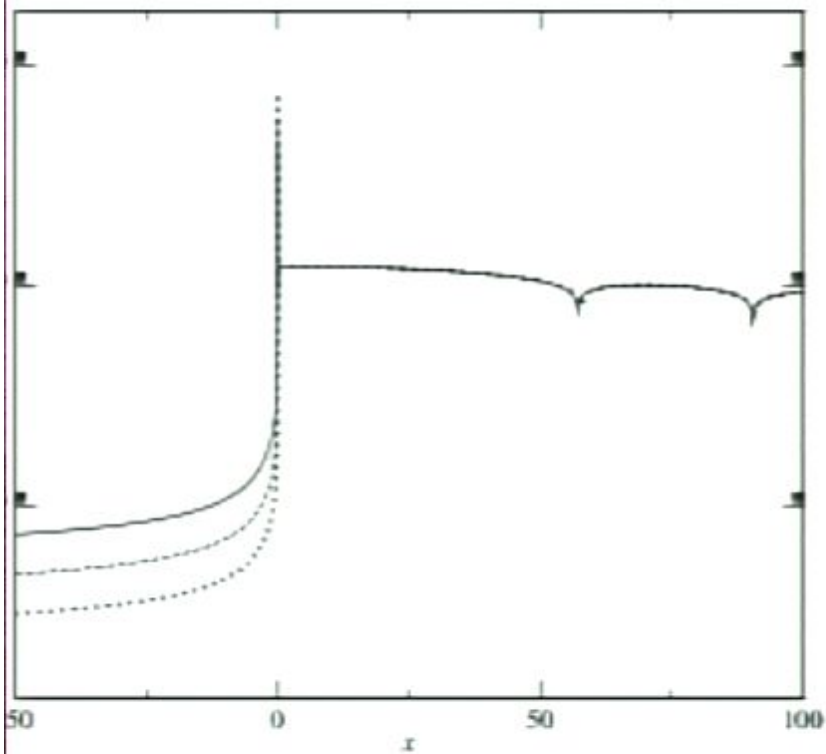


k -mode mixing ...



2 d.o.f. + 4 dim G.R.

$$H^2 = \ell_{\text{Pl}}^2 \left[\frac{\rho_+}{a^{3(1+w_+)}} - \frac{\rho_-}{a^{3(1+w_-)}} \right]$$



k-mode inversion ...

F. Finelli, P. P. & N. Pinto-Neto, *Phys. Rev. D* **77**, 103508 (2008)

A specific model: 4D Quantum cosmology

$$S = \int \sqrt{-g} \left(-\frac{R}{6\ell_p^2} + p \right) d^4x$$

Perfect fluid:

$$p = \omega\rho$$



bounce

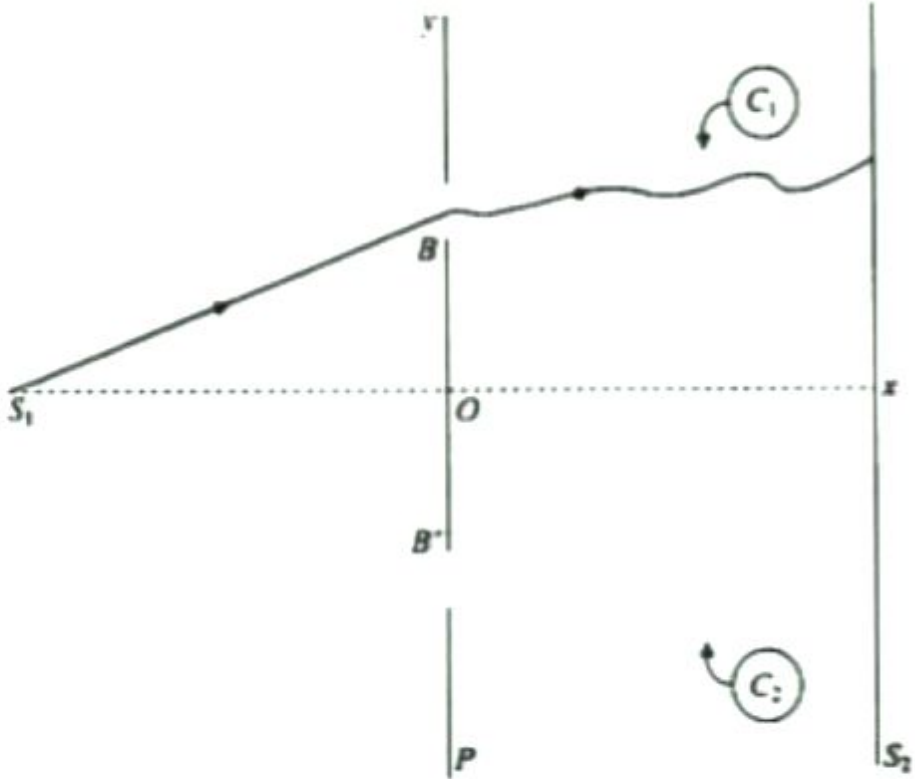
☹️ no horizon problem if $\omega > -\frac{1}{3}$ 💡

Results:

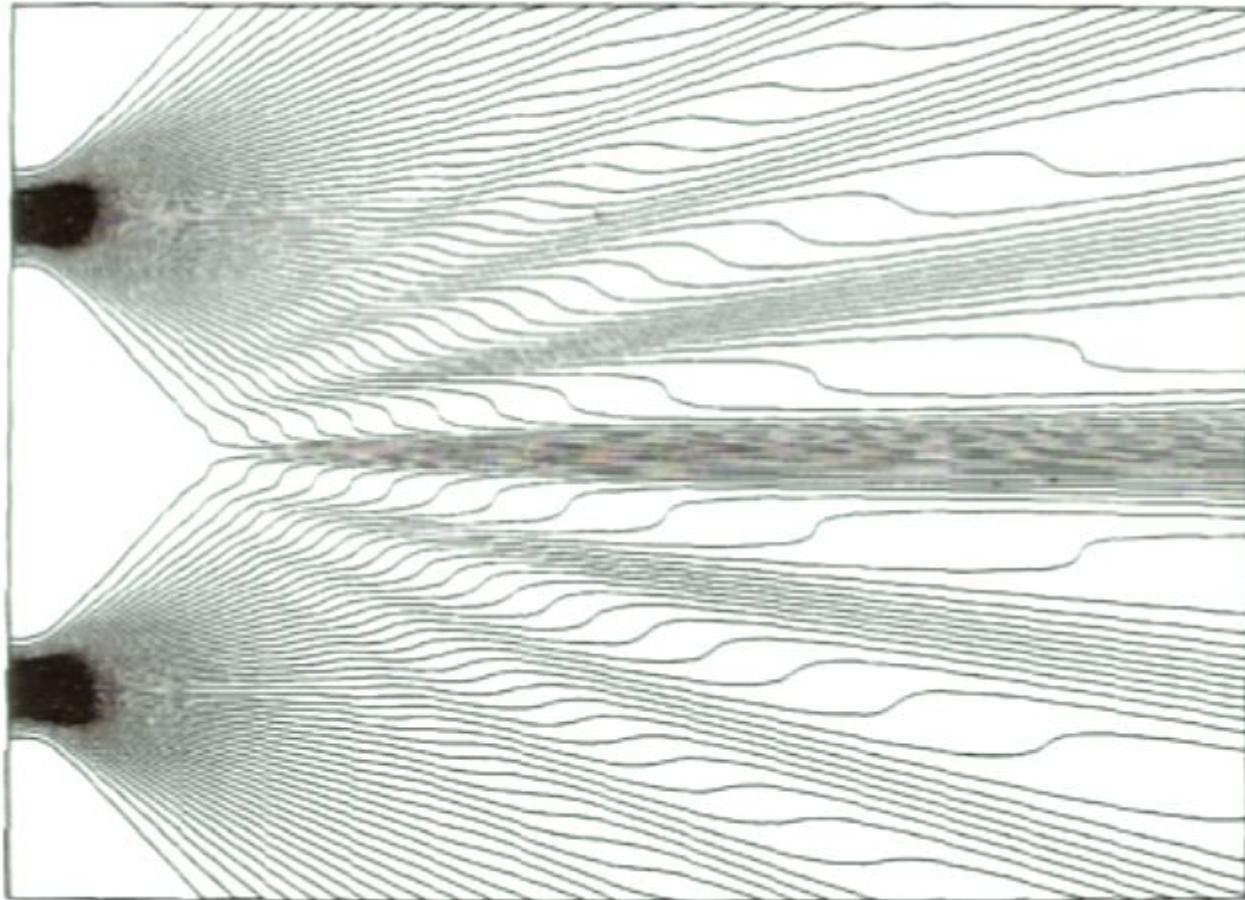
$$n_T = n_S - 1 = \frac{12\omega}{1 + 3\omega}$$

$$\frac{T}{S} \simeq 4 \times 10^{-2} \sqrt{n_S - 1}$$

The two-slit experiment:



Trajectories in the two-slit experiment



quantum cosmology $ds^2 = N^2(\tau)d\tau - a^2(\tau)\gamma_{ij}dx^i dx^j$

canonical transformation

rescaling (volume ...) = a simple Hamiltonian:

units

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}} \right) N$$

$a^{3\omega}$

Wheeler-De Witt

$$H\Psi = 0$$

+ Technical trick: $\mathcal{K} = 0 \implies \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \implies i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}$

space defined by $\chi > 0$ \longrightarrow constraint

$$\bar{\Psi} \frac{\partial\Psi}{\partial\chi} = \Psi \frac{\partial\bar{\Psi}}{\partial\chi}$$

alternative way of getting the solution:

WKB exact superposition: $\Psi = \int e^{iET} \rho(E) \psi_E(T) dE$

Gaussian wave packet $\propto e^{-(ET_0)^2}$

$\Psi = \left[\frac{8T_0}{\pi (T_0^2 + T^2)^2} \right]^{\frac{1}{4}} \exp\left(-\frac{T_0 \chi^2}{T_0^2 + T^2}\right) e^{-iS(x,T)}$

phase $S = \frac{T \chi^2}{T_0^2 + T^2} + \frac{1}{2} \arctan \frac{T_0}{T} - \frac{\pi}{4}$

Bohmian trajectory $\dot{a} = \{a, H\}$

$a = a_0 \left[1 + \left(\frac{T}{T_0} \right)^2 \right]^{\frac{1}{3(1-\omega)}}$

quantum cosmology $ds^2 = N^2(\tau)d\tau - a^2(\tau)\gamma_{ij}dx^i dx^j$

canonical transformation

rescaling (volume ...) = a simple Hamiltonian:

units

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}} \right) N$$

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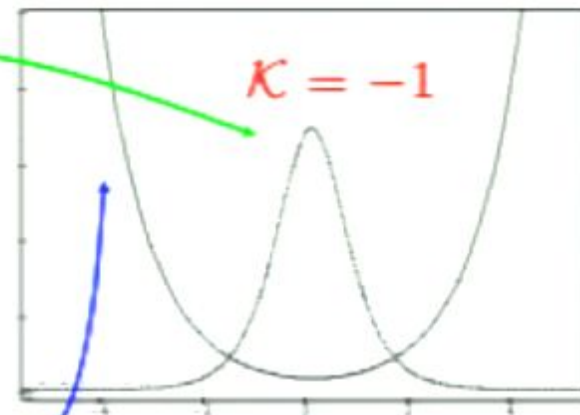
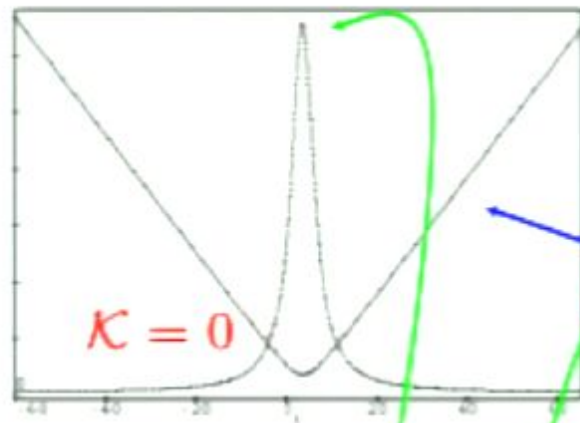
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$\Psi = \left[\frac{8T_0}{\pi (T_0^2 + T^2)^2} \right]^{\frac{1}{4}} \exp\left(-\frac{T_0 \chi^2}{T_0^2 + T^2}\right) e^{-iS(x,T)}$

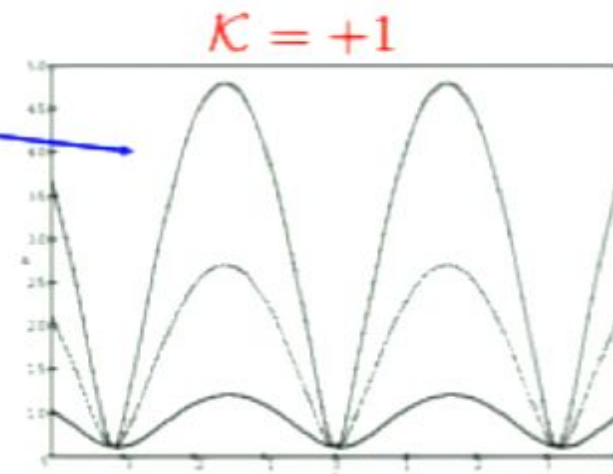
phase $S = \frac{T \chi^2}{T_0^2 + T^2} + \frac{1}{2} \arctan \frac{T_0}{T} - \frac{\pi}{4}$

Bohmian trajectory $\dot{a} = \{a, H\}$

$a = a_0 \left[1 + \left(\frac{T}{T_0} \right)^2 \right]^{\frac{1}{3(1-\omega)}}$



$a(T)$



$Q(T)$

quantum potential

What about the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \dots$

factorization of the wave function

$$\Psi = \Psi_{(0)}(a, T) \Psi_{(2)}[v, T; a(T)]$$

comes from 0th order

$$\Delta\Phi = -\frac{3\ell_{\text{Pl}}^2}{2} \sqrt{\frac{\rho + p}{\omega}} a \frac{d}{d\eta} \left(\frac{v}{a} \right)$$

Bardeen (Newton) gravitational potential

$$ds^2 = a^2(\eta) \{ (1 + 2\Phi) d\eta^2 - [(1 - 2\Phi) \gamma_{ij} + h_{ij}] dx^i dx^j \}$$

conformal time $d\eta = a^{3\omega-1} dT$

+ **canonical transformations:**

$$i \frac{\partial \Psi_{(2)}}{\partial \eta} = \int d^3x \left(-\frac{1}{2} \frac{\delta^2}{\delta v^2} + \frac{\omega}{2} v_{,i} v^{,i} - \frac{a''}{a} \right) \Psi_{(2)}$$

Fourier mode

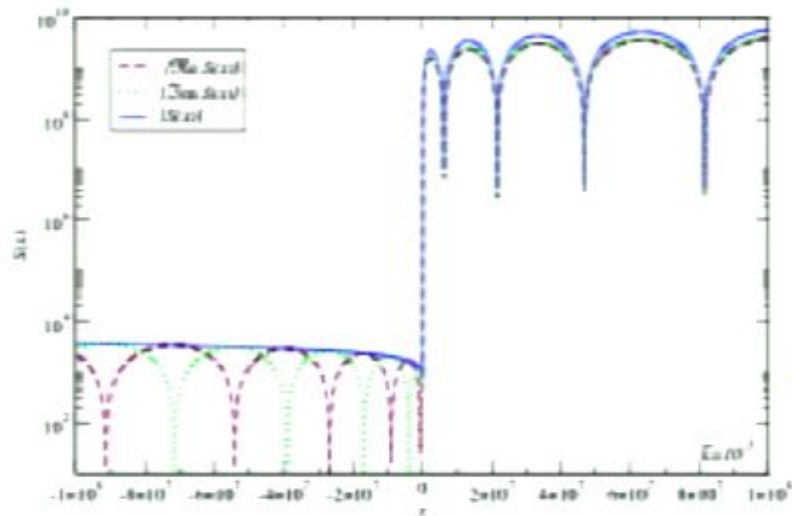
$$v_k'' + \left(c_s^2 k^2 - \frac{a''}{a} \right) v_k = 0$$

$$c_s^2 = \sqrt{\omega} \neq 0$$

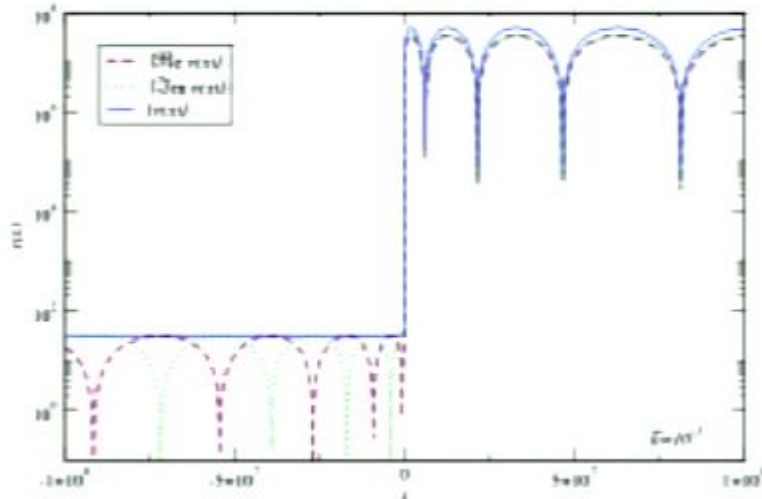
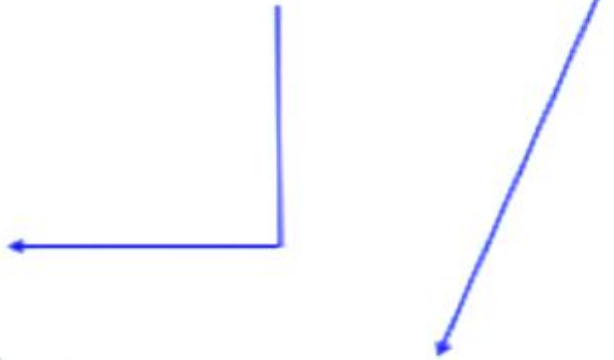
vacuum initial conditions

$$v_k \propto \frac{e^{-ic_s k \eta}}{\sqrt{2c_s k}}$$

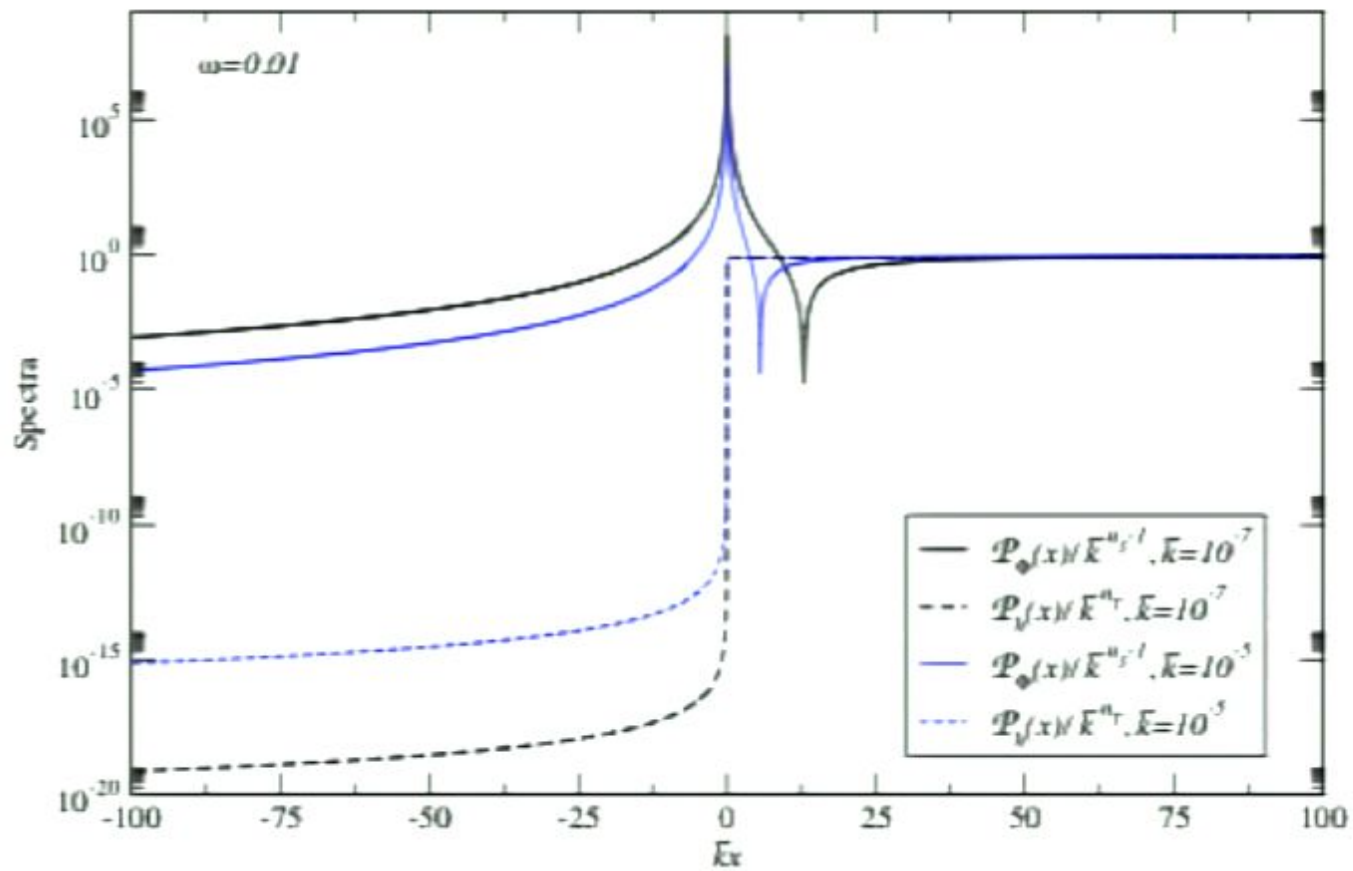
+ **evolution** (matchings and/or numerics)



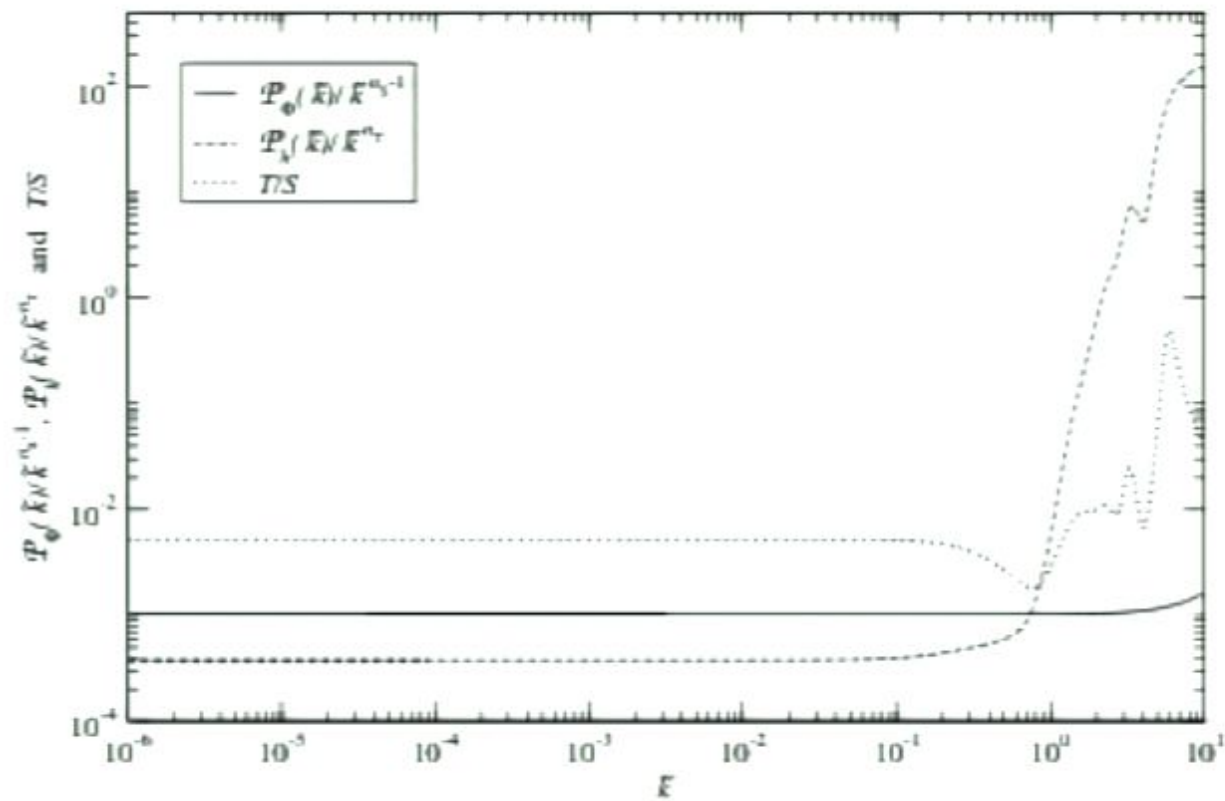
$$s(x) = a^{\frac{1}{2}}(1-3\omega) \frac{v(x)}{\sqrt{T_0}}$$



$$x \equiv \frac{T}{T_0}$$



Full numerical spectrum



spectrum $\mathcal{P}_\Phi \propto k^3 |\Phi_k|^2 \propto A_s^2 k^{n_s-1}$

id. grav. waves: $\mu'' + \left(k^2 - \frac{a''}{a}\right) \mu = 0 \quad \mu \equiv \frac{h}{a}$

$\mu_{\text{ini}} \propto \frac{\exp(-ik\eta)}{\sqrt{k\eta}} \quad \mathcal{P}_h \propto k^3 |h_k|^2 \propto A_T^2 k^{n_T}$

same dynamics + initial conditions \implies **same spectrum**

$$n_T = n_s - 1 = \frac{12\omega}{1 + 3\omega}$$

CMB normalisation $A_s^2 = 2.08 \times 10^{-10}$

\implies **bounce curvature** $T_0 a_0^{3\omega} \simeq 1500 \ell_{\text{Pl}}$

WMAP constraint

$$n_s = 0.96 \pm 0.02 \implies w \lesssim 8 \times 10^{-4}$$

predictions

■ spectrum slightly blue

power-law + concordance

$$\frac{T}{S} = \frac{C_{10}^{(T)}}{C_{10}^{(S)}} = \mathcal{F}(\Omega, \dots) \frac{A_T^2}{A_S^2} \propto \sqrt{w}$$

$\simeq 0.62$

$$\frac{T}{S} \simeq 4 \times 10^{-2} \sqrt{n_s - 1}$$