Title: Bouncing alternatives to inflation

Date: May 11, 2010 02:45 PM

URL: http://pirsa.org/10050009

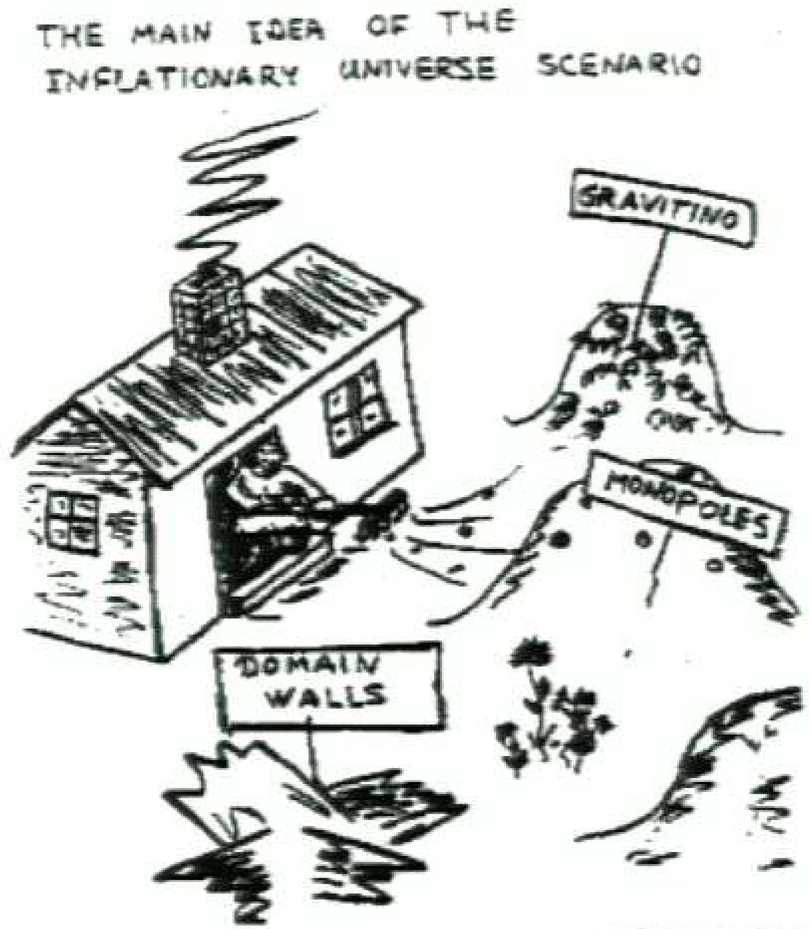
Abstract: Although inflation is, by far, the best known mechanism to explain the observed properties of our Universe, there is still some room for alternative models, most of which implying a contracting phase preceding the current expanding one. Both phases are connected by a bounce at which the expansion rate must vanish. General relativity can only produce such a phase provided the spatial curvature is positive, in contradiction with the current observations. I will discuss the lines along which one can modify either the matter or the gravity sector (or both) in order to implement a bounce, and show the generic observable cosmological consequences it can induce, in particular in the microwave background.

Problems with standard model:

- *Singularity*
- *Horizon*
- *Flatness*
- *Homogeneity*
- *Perturbations*
- *Dark matter*
- *Dark energy / cosmological constant*
- *Baryogenesis*
- *...*

Accepted solution = INFLATION

Topological defects (monopoles)



(Linde's book)

Inflation

- solves cosmological puzzles
- uses GR + scalar fields [(semi-)classical]
- can be implemented in high energy theories
- makes falsifiable predictions ...
- ... consistent with all known observations
- string based ideas (brane inflation, ...)

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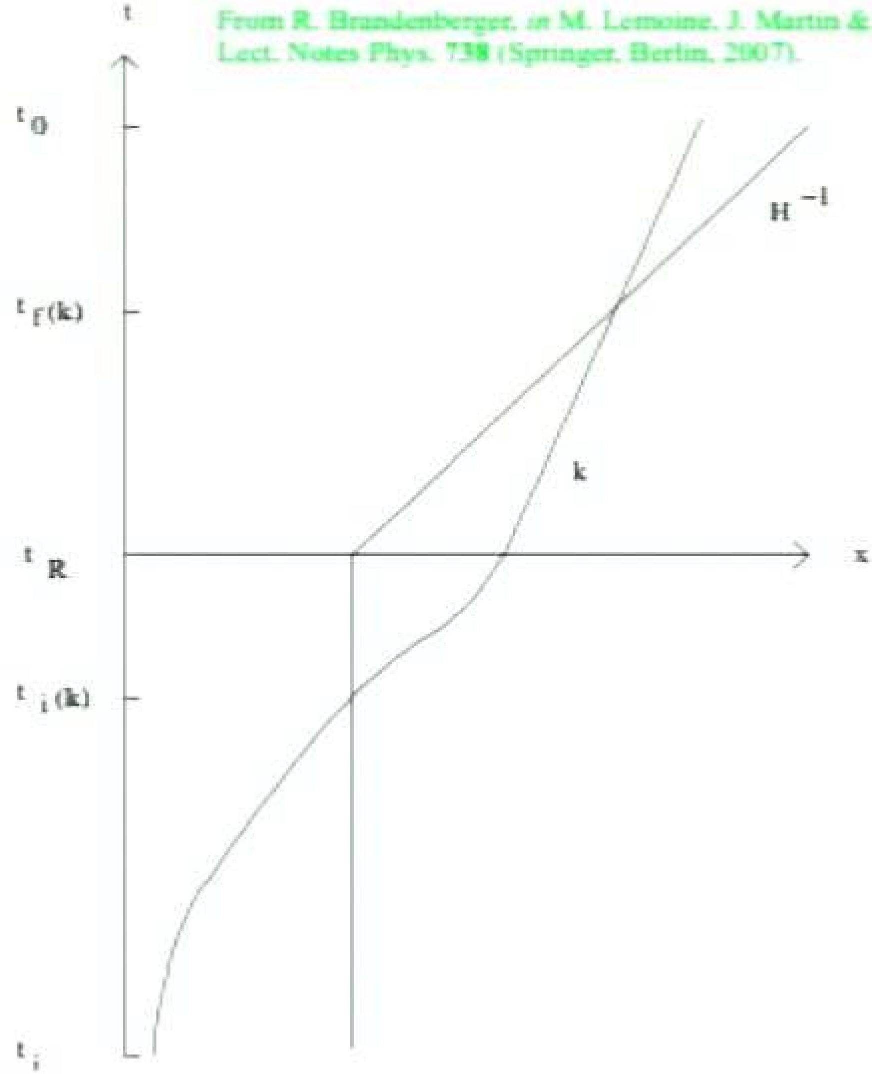
Alternative model???

- string based ideas (PBB, other brane models, string gas, …)
- Quantum gravity / cosmology
- singularity, initial conditions & homogeneity



- Scalar field origin?
- Trans-Planckian

$$\exists t; \ell(t) = \ell_0 \frac{a(t)}{a_0} \leq \ell_{\rm Pl}$$

- Hierarchy (amplitude)

$$\frac{V(\varphi)}{\Delta\varphi^4} \leq 10^{-12}$$

- Singularity

$$\exists t_{(\pm\infty)}; a(t) \to 0$$

- Validity of GR?

$$E_{\rm inf} \simeq 10^{-3} M_{\rm Pl}$$

A brief history of bouncing cosmology

R. C. Tolman, "*On the Theoretical Requirements for a Periodic Behaviour of the Universe*", PRD 38, 1758 (1931)

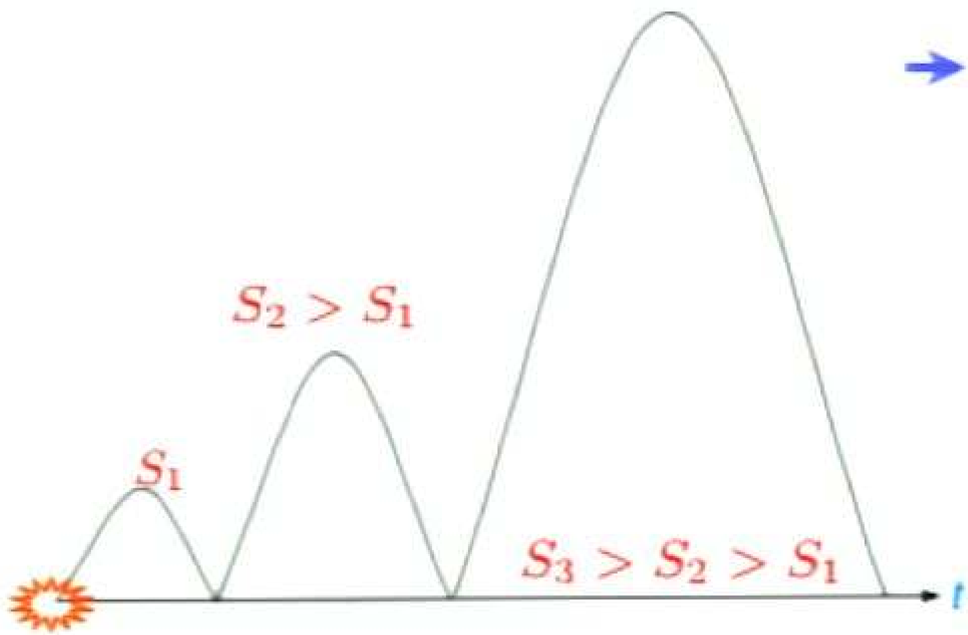
G. Lemaître, "*L'Univers en expansion*", Ann. Soc. Sci. Bruxelles (1933)

A. A. Starobinsky, "*On one non-singular isotropic cosmological model*", Sov. Astron. Lett. 4, 82 (1978)

R. Durrer & J. Laukerman, "*The oscillating Universe: an alternative to inflation*", Class. Quantum Grav. 13, 1069 (1996)

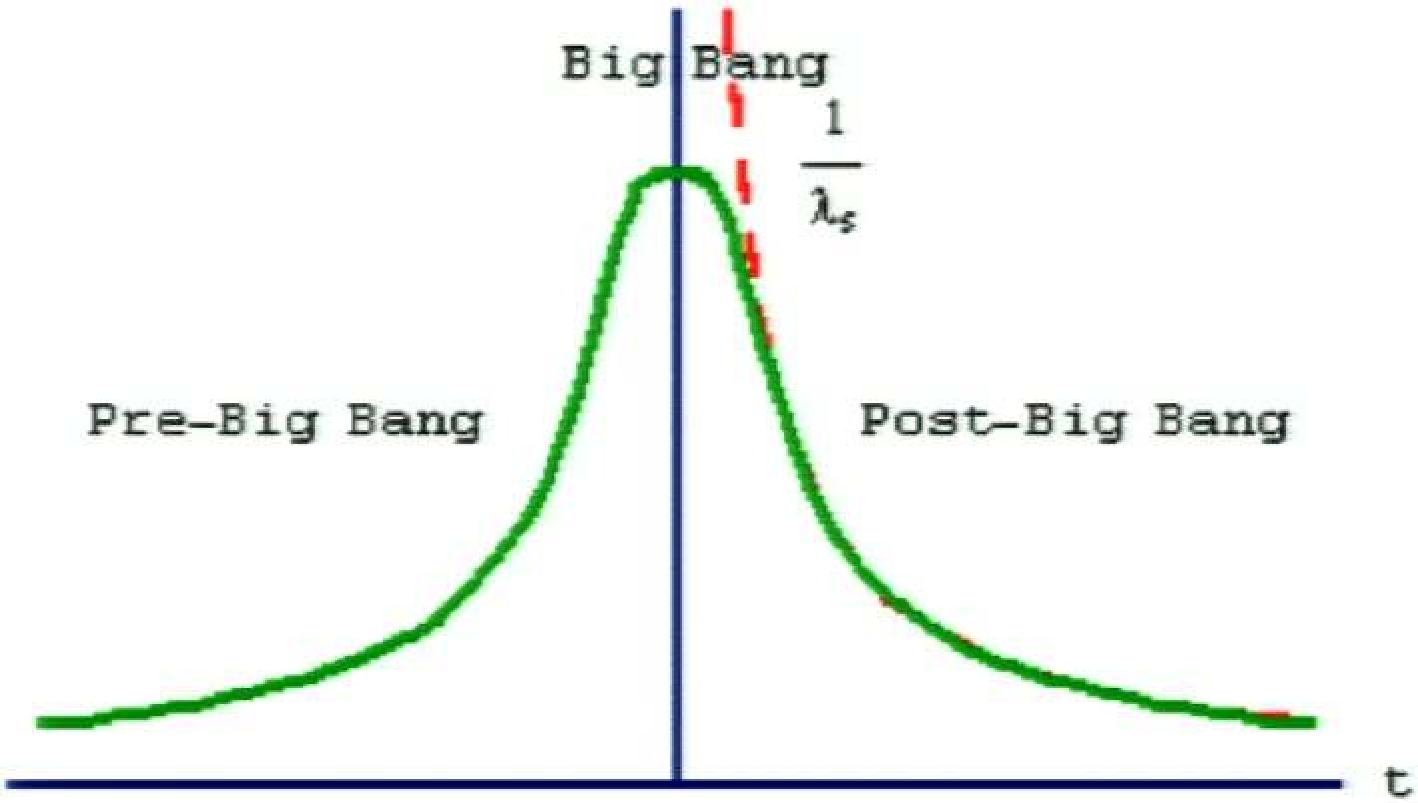
→ Penrose: BH formation

Quantum nucleation?



→ PBB - Ekpyrotic - Modified gravity - Quantum cosmology - Quintom - Horava-Lifshitz - Lee-Wick - ...

Pre Big Bang scenario: (cf. M.Gasperini & G. Veneziano, arXiv: hep-th/0703055)



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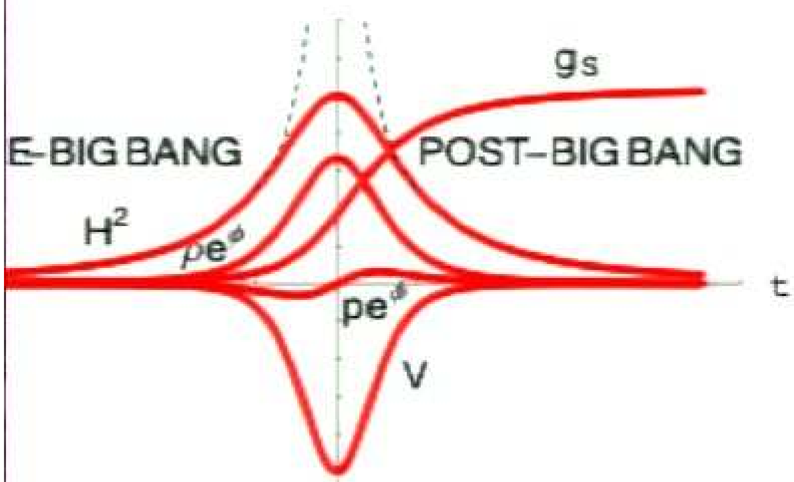
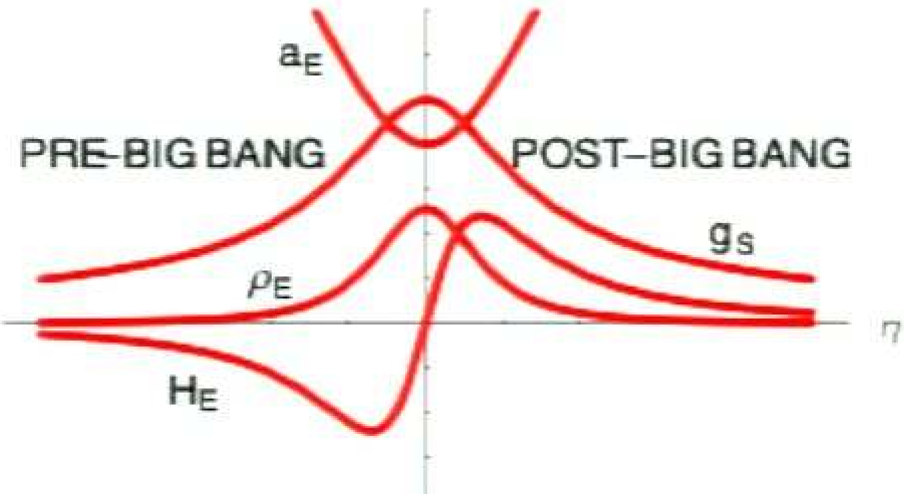


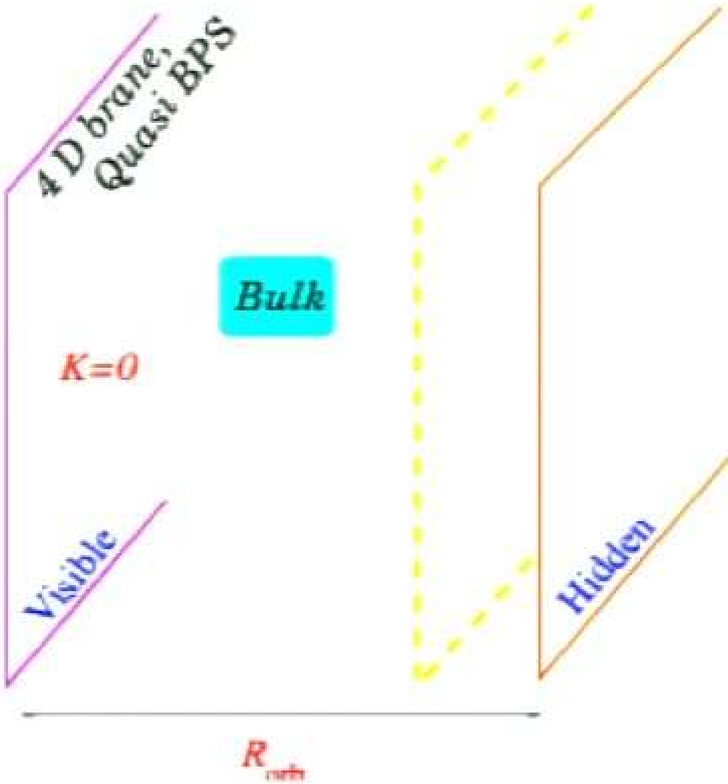
string frame



Einstein frame

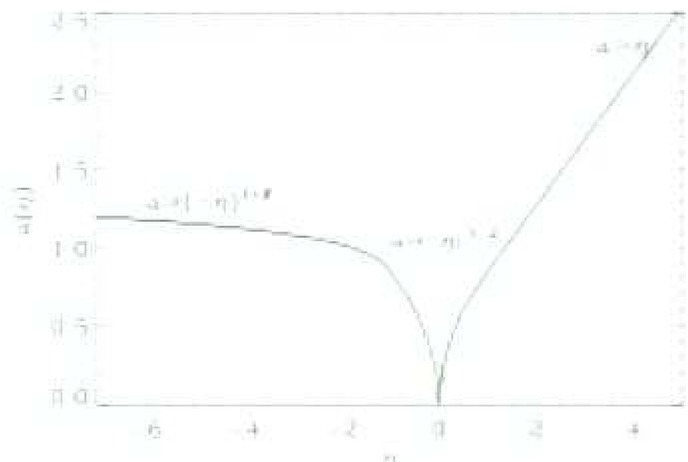
Ekpyrotic scenario:

$$\mathcal{S}_5 \propto \int_{\mathcal{M}_5} \mathrm{d}^5 x \sqrt{-g_5} \left[R_{(5)} - \frac{1}{2} (\partial \varphi)^2 - \frac{3}{2} \frac{\mathrm{e}^{2\varphi} \mathcal{F}^2}{5\,!} \right],$$



$$\mathcal{S}_4 = \int_{\mathcal{M}_4} \mathrm{d}^4 x \sqrt{-g_4} \left[\frac{R_{(4)}}{2\kappa} - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right].$$

$$V(\varphi) = -V_\mathrm{i} \exp\left[-\frac{4\sqrt{\pi\gamma}}{m_\mathrm{Pl}} (\varphi - \varphi_\mathrm{i}) \right].$$



Standard Failures and some solutions

- Singularity
- Horizon
- Flatness

- Homogeneity

  $t_{\text{dissipation}}$

- Perturbations
- Others

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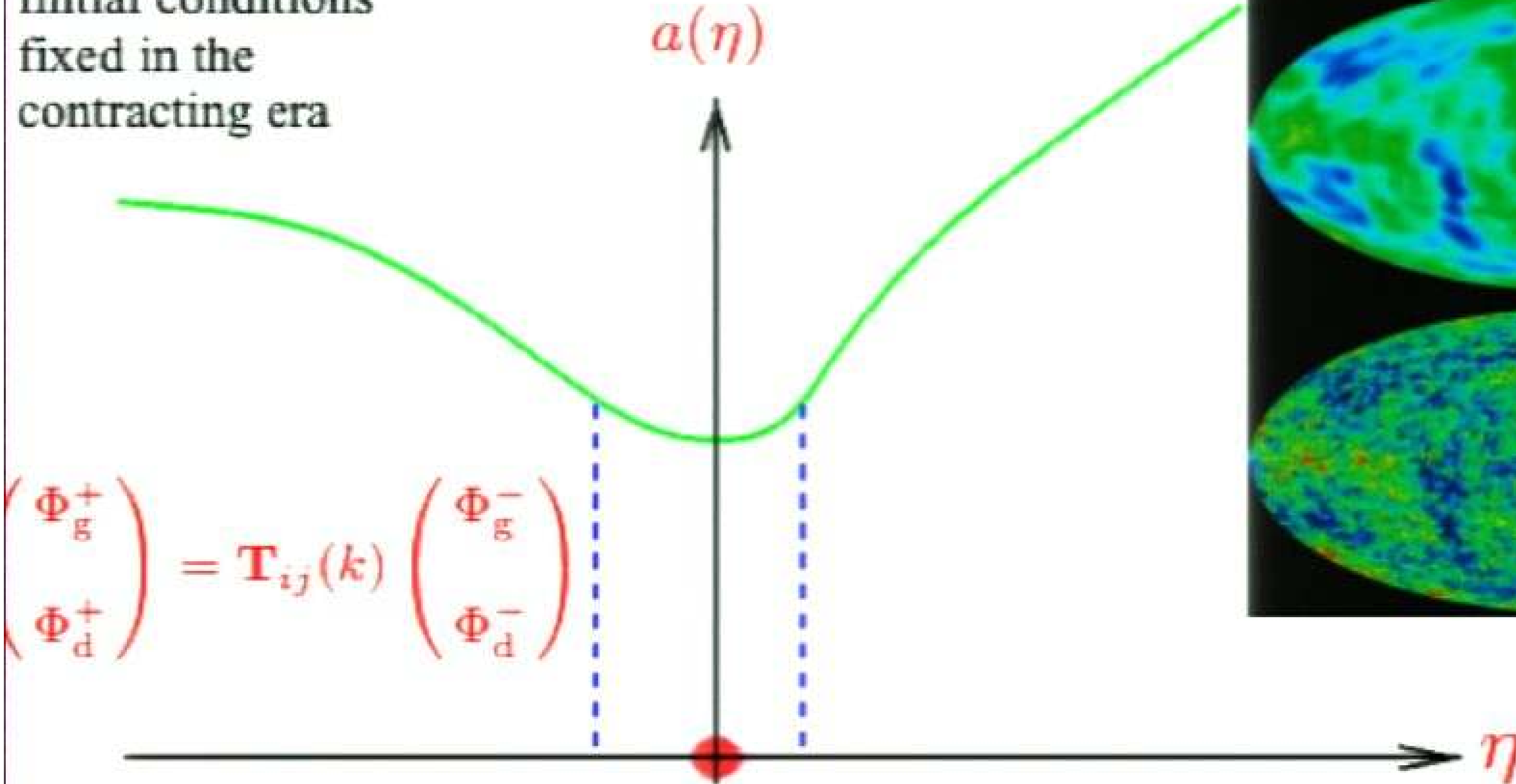
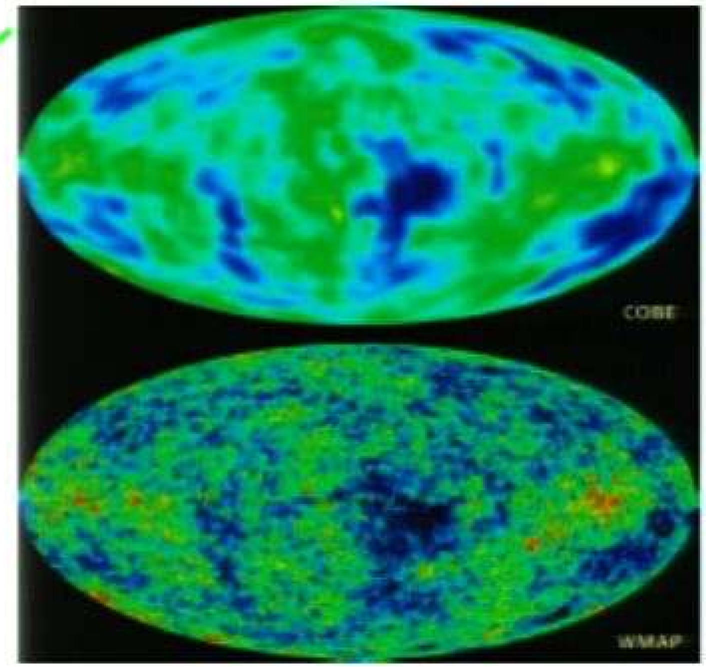
Initial conditions fixed in the contracting era





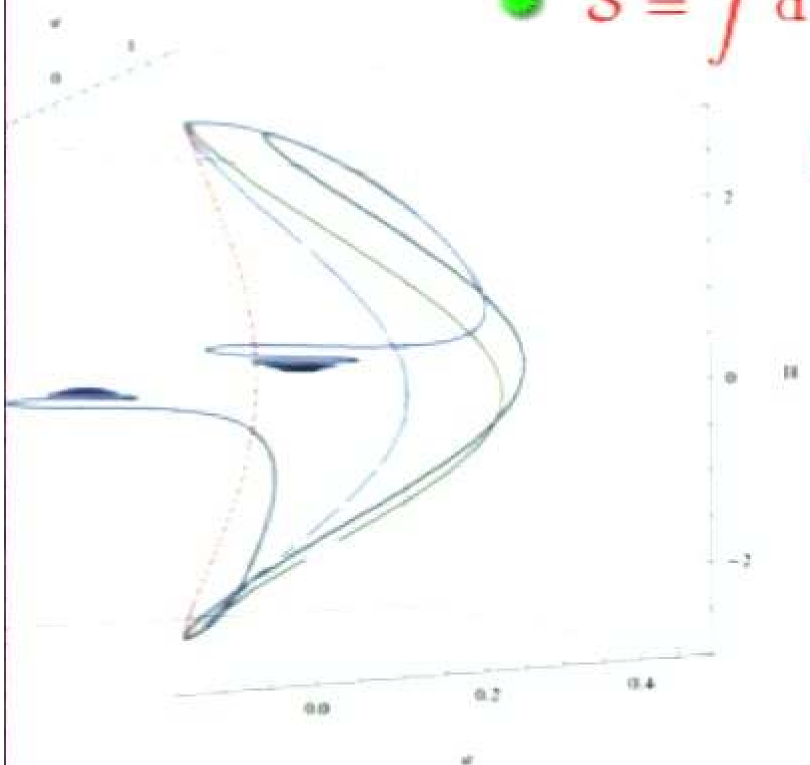
Self consistent bounce:

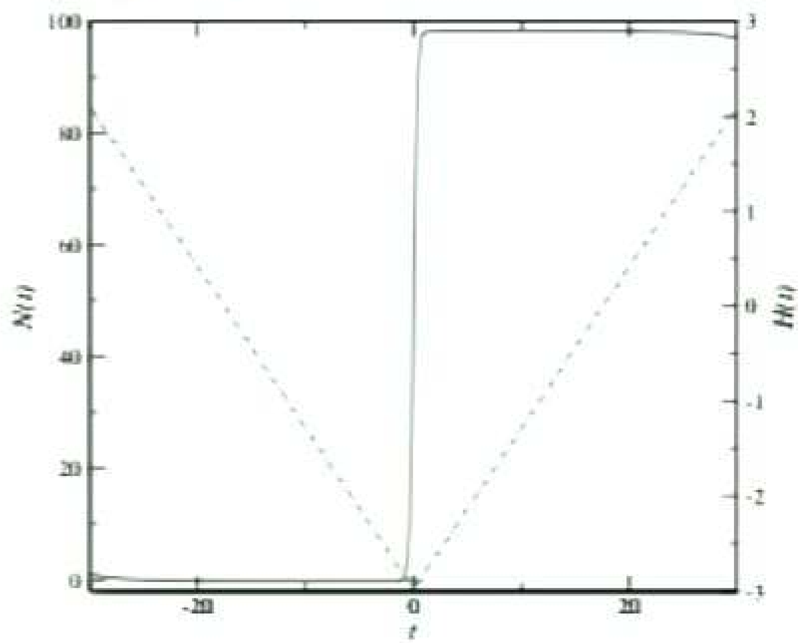
$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\mathcal{K}r^2} + r^2 d\Omega^2\right)$$

→ One d.o.f. + 4 dimensions G.R.

- $$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{6\ell_{\rm Pl}^2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right]$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) - \frac{\mathcal{K}}{a^2}$$ Positive spatial curvature

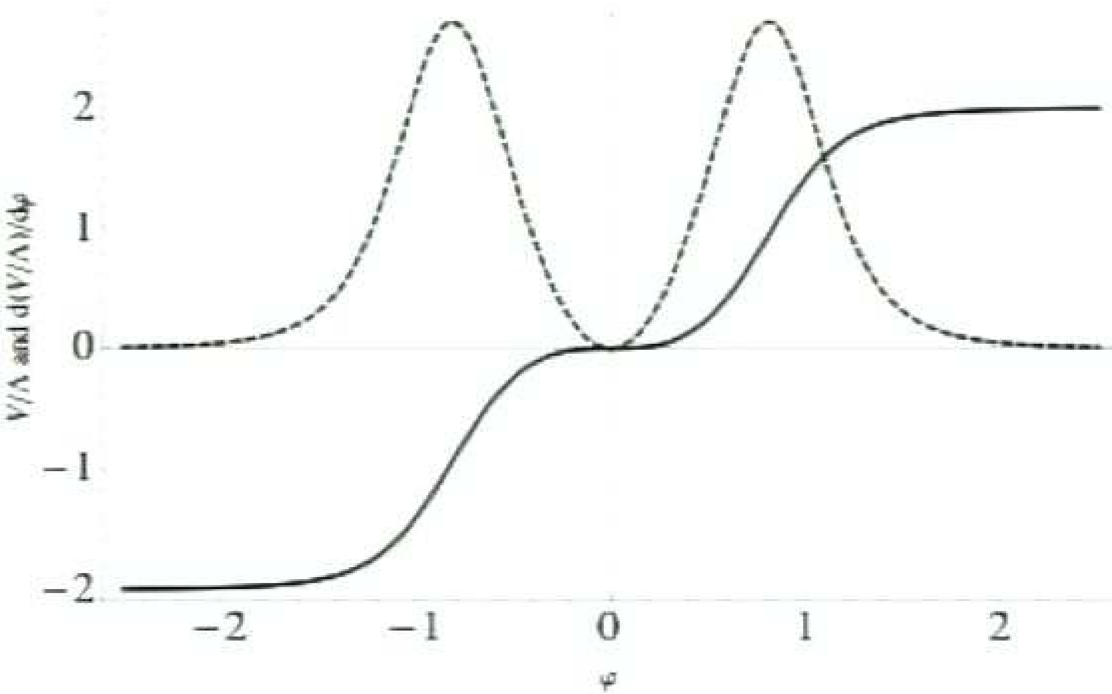




Falcarno, M. Lilley & P. P., *Phys. Rev.* D77, 083513 (2008) fluence of the spatial curvature? $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\mathrm{d}\Omega^2\right) \Longrightarrow \eta_0 > 1$

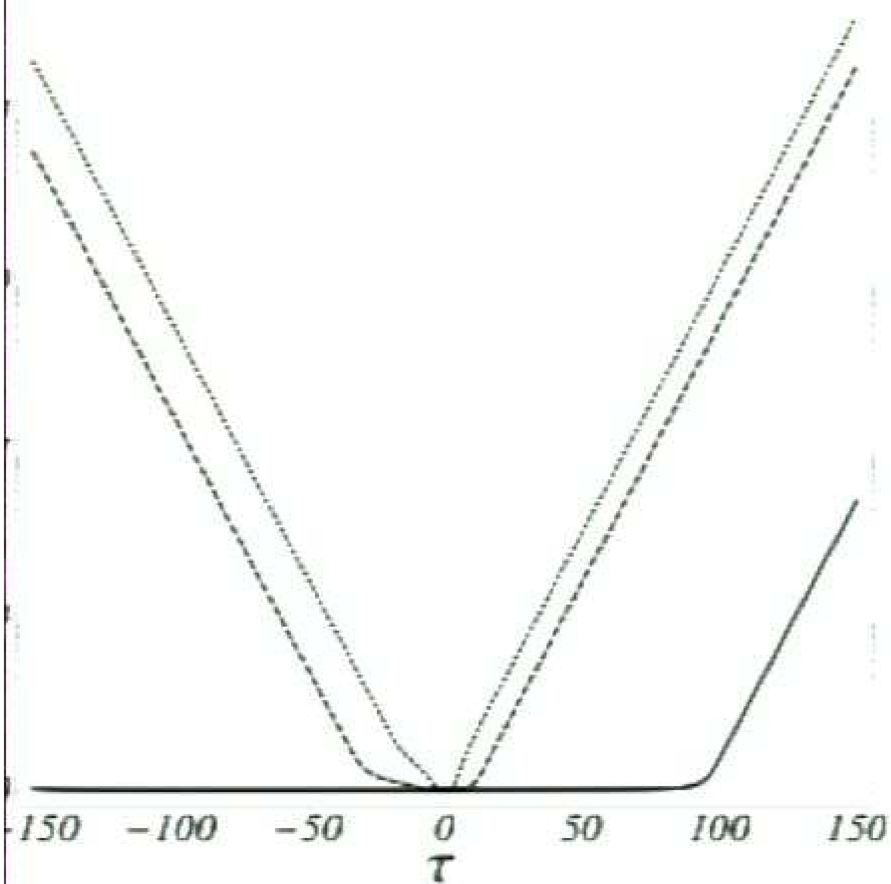
→ Modify GR to non singular theories (curvature invariants)

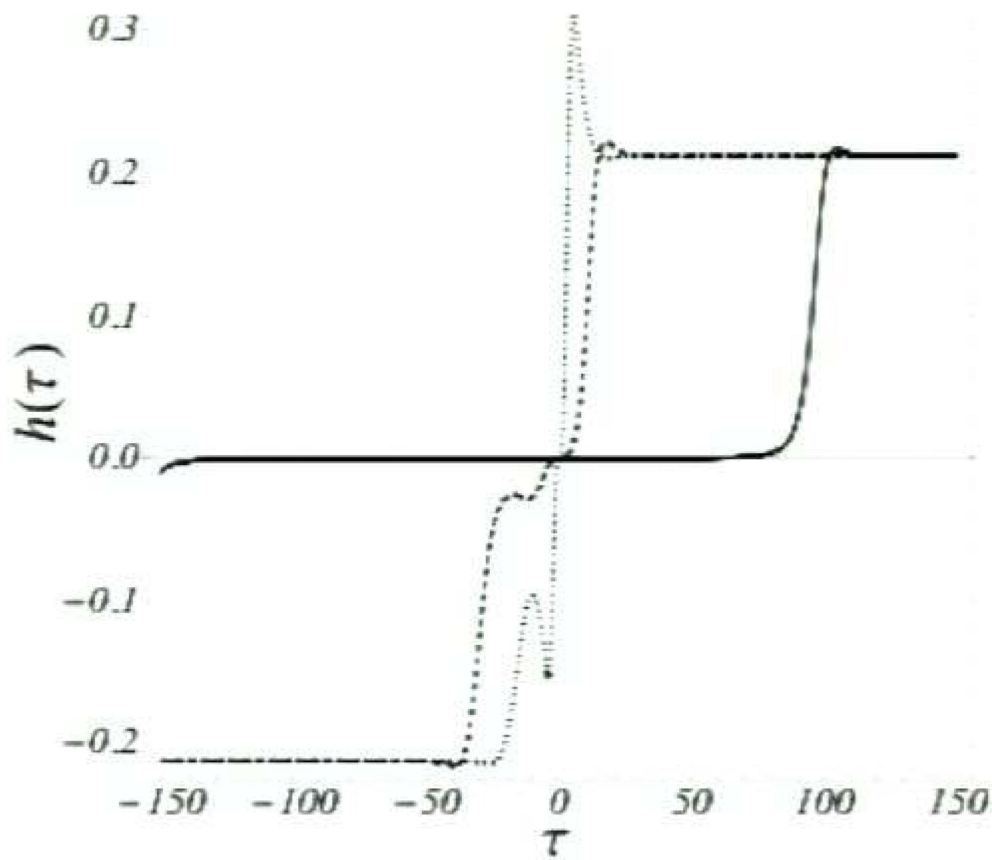
- $$\mathcal{S} = \frac{1}{16\pi G_N}\int \mathrm{d}^4x\sqrt{-g}\left[R + \sum_{i=1}^{N}\varphi_i I^{(i)} - V(\varphi)\right] \Longrightarrow \frac{\mathrm{d}V}{\mathrm{d}\varphi} = I$$



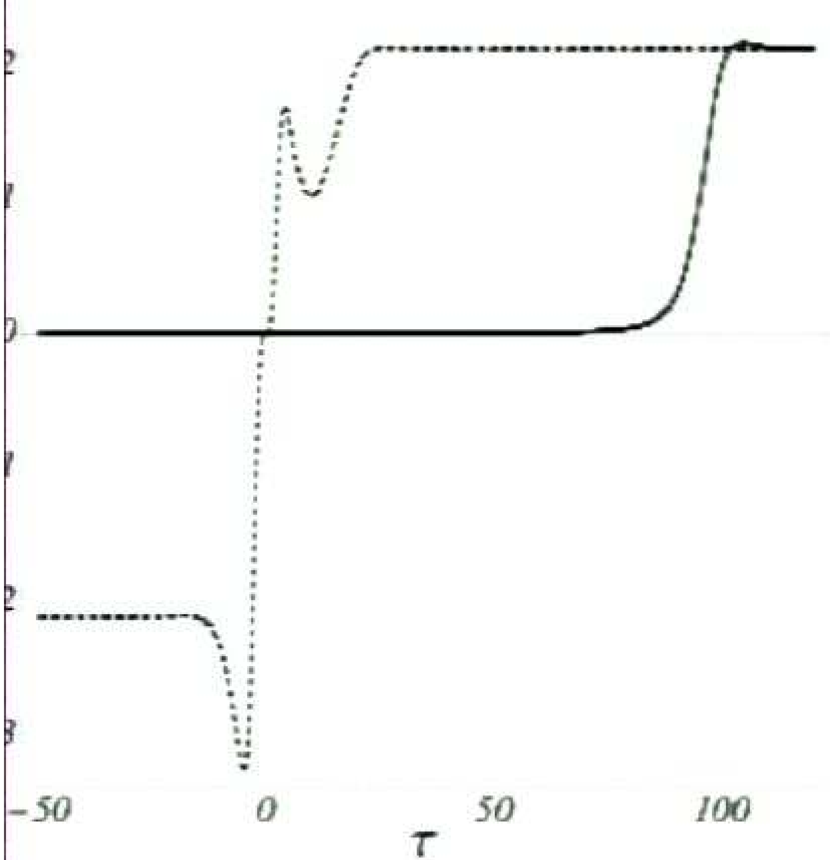
$$I = R - \sqrt{3\left(4R_{\mu\nu}R^{\mu\nu} - R^2\right)}$$

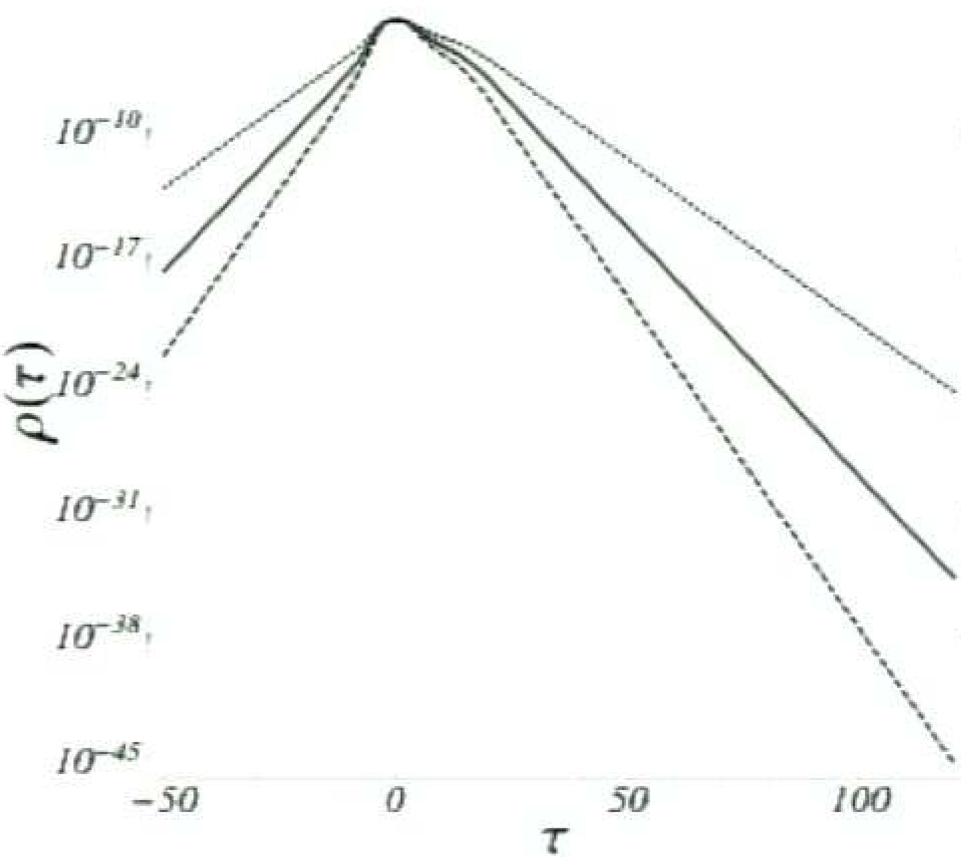
R. Abramo, P. P. & I. Yasuda, *Phys. Rev.* **D81**, 023511 (2010)





η_0 arbitrary


2
1
0
1
2
3
−50
0
50
100
τ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$\rho(\tau)$
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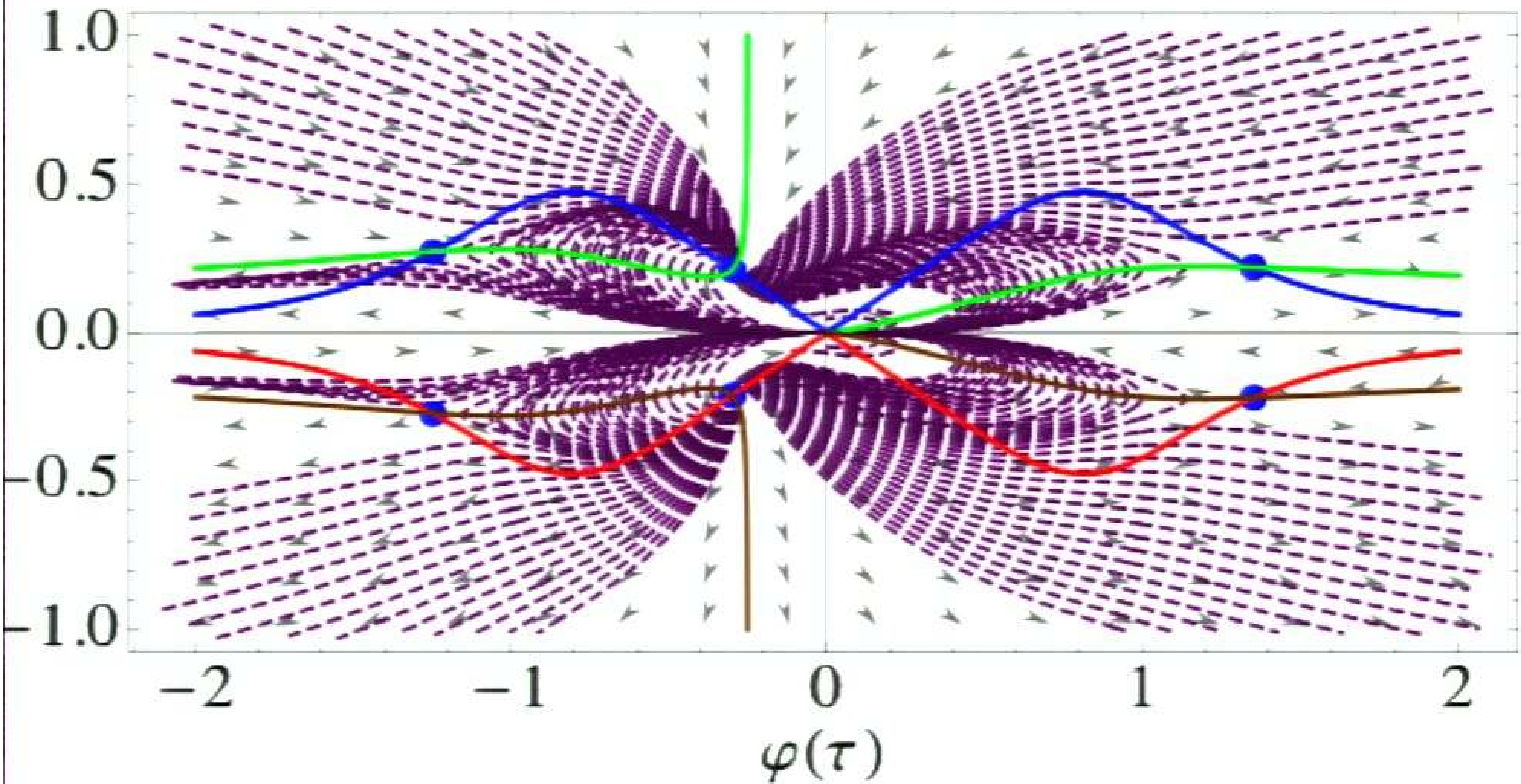
1.0
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-2
-1
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1
2
φ(τ)

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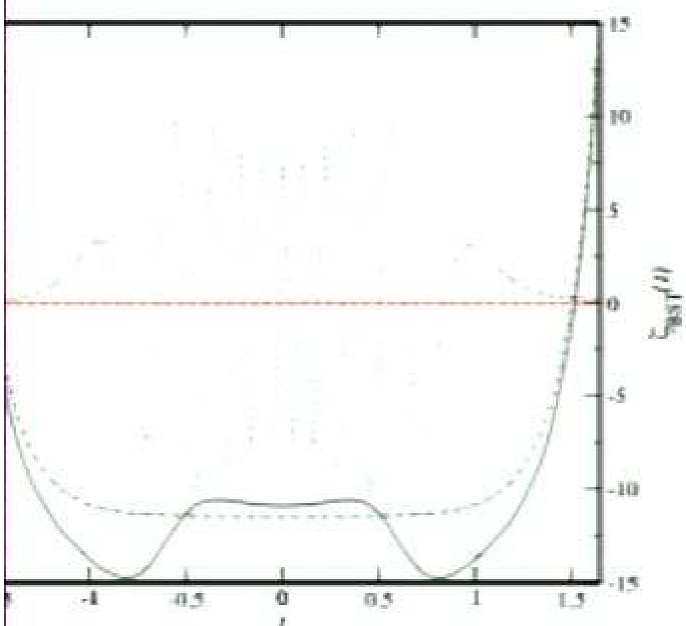
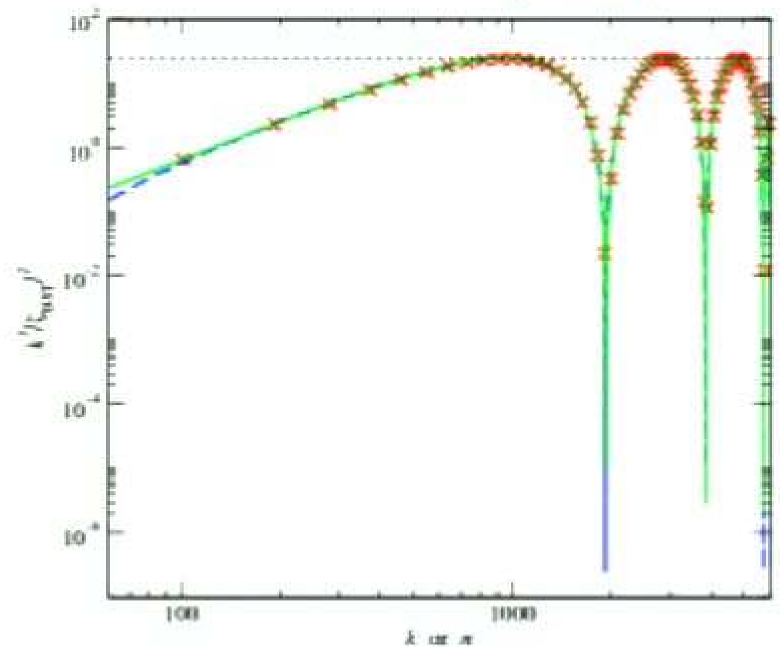
$$\Longleftrightarrow \quad \Phi = \frac{3\mathcal{H}u}{2a^2\theta} \qquad \theta \equiv \frac{1}{a}\sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi}\left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2}\right)}$$

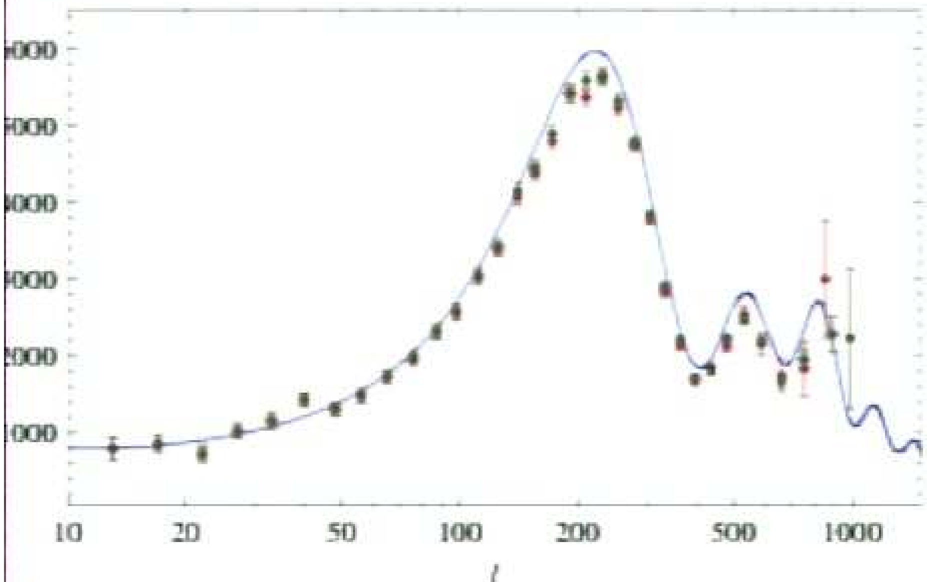
$$u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_s^2\right)\right]u = 0$$

$$\mathcal{P}_\zeta = \mathcal{A}k^{n_\mathrm{s}-1}\cos^2\left(\omega\frac{k_\mathrm{ph}}{k_\star} + \psi\right)$$

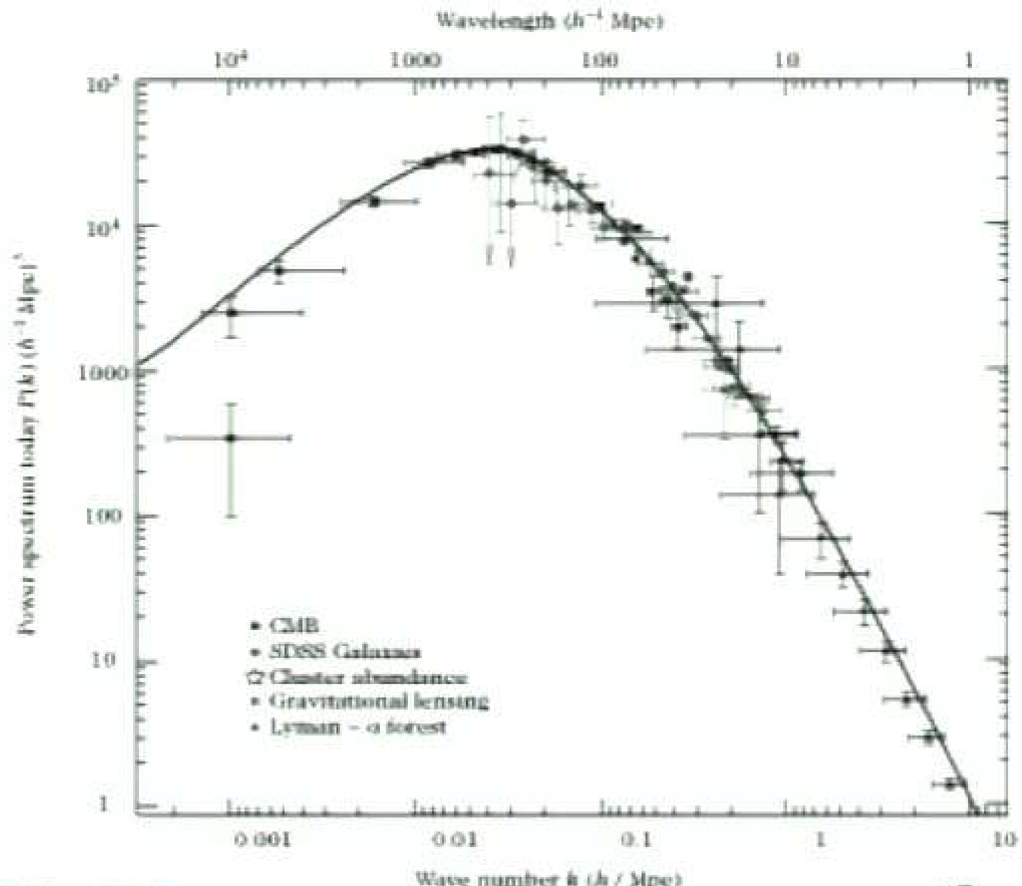




Data!



No obvious oscillations ...



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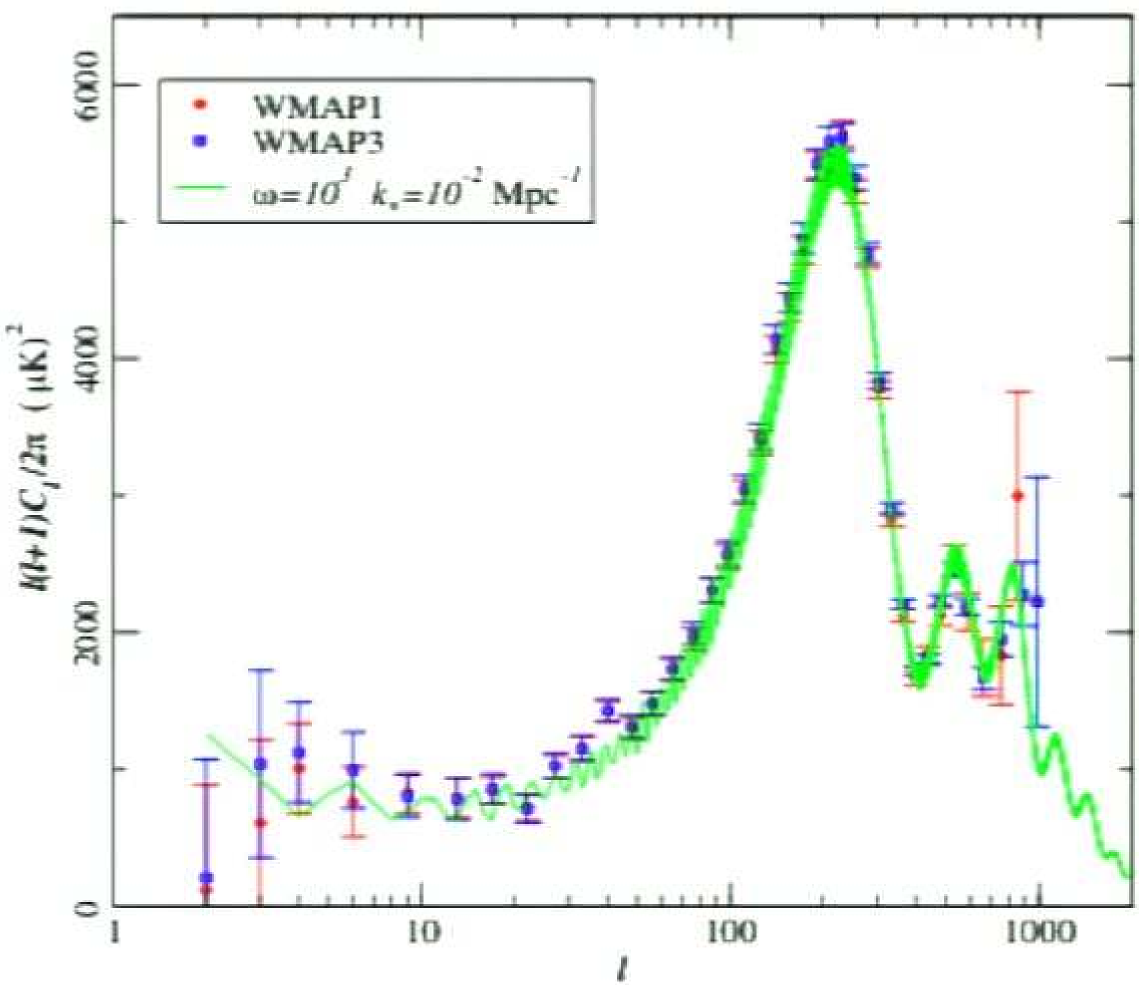

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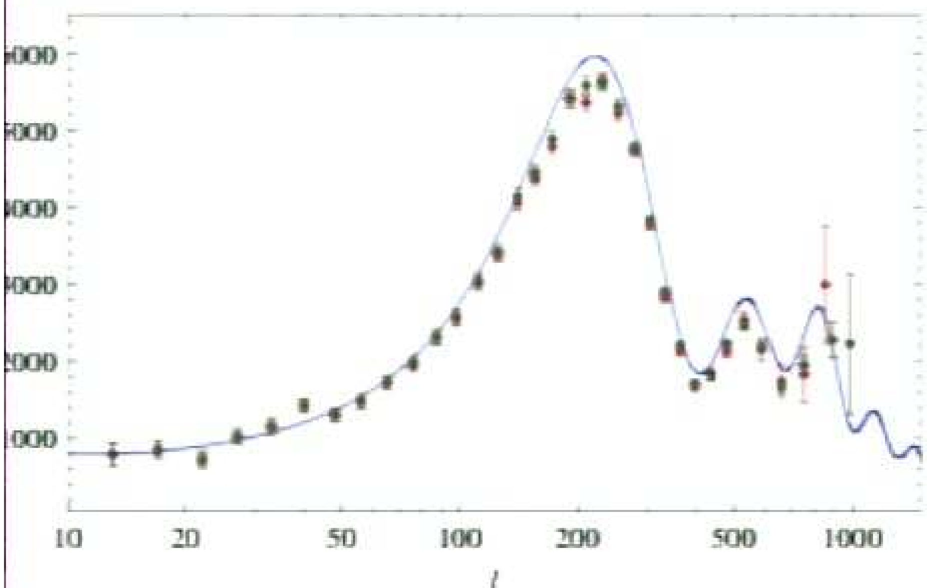

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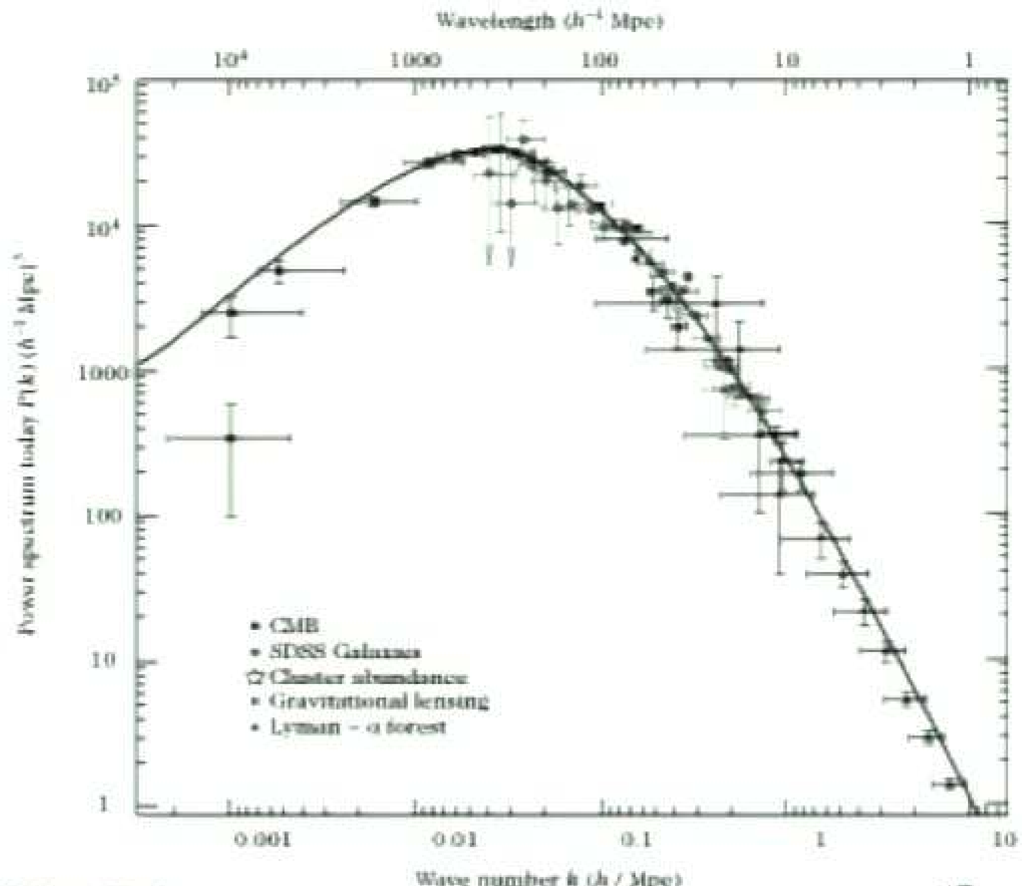

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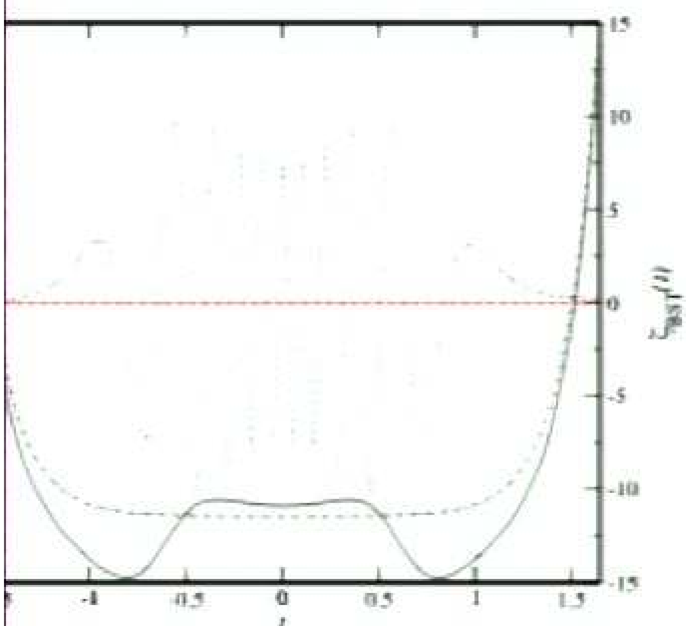

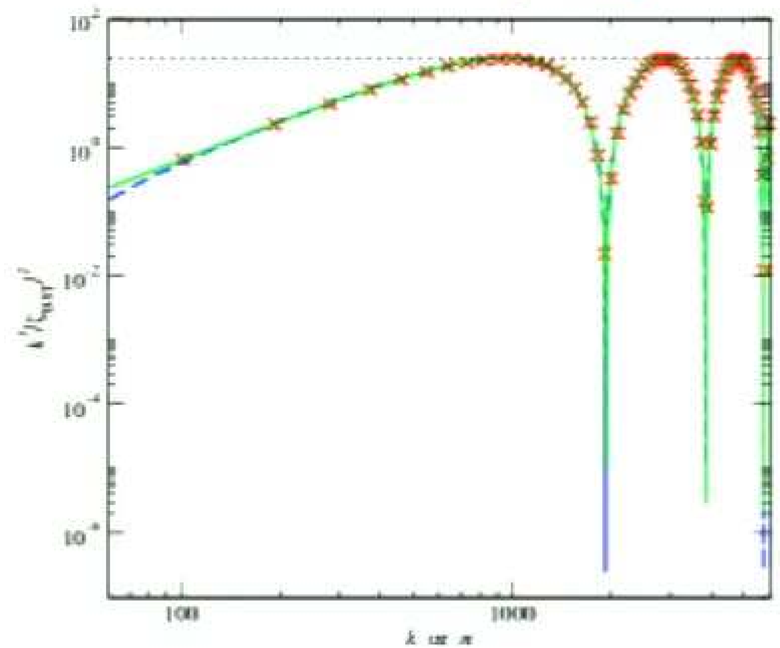

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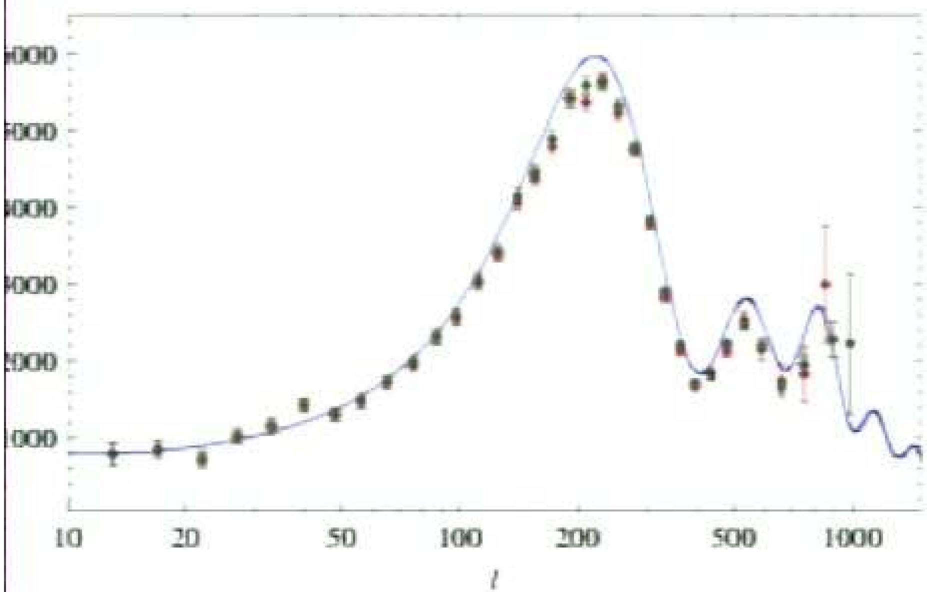

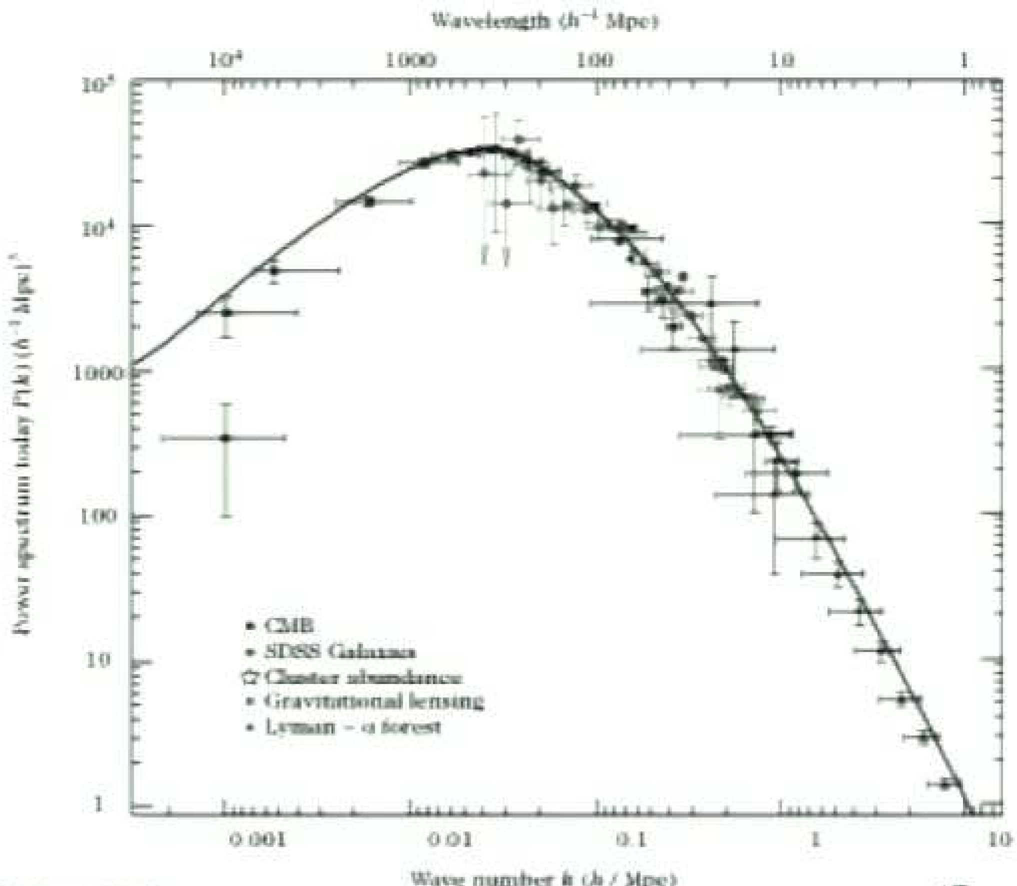

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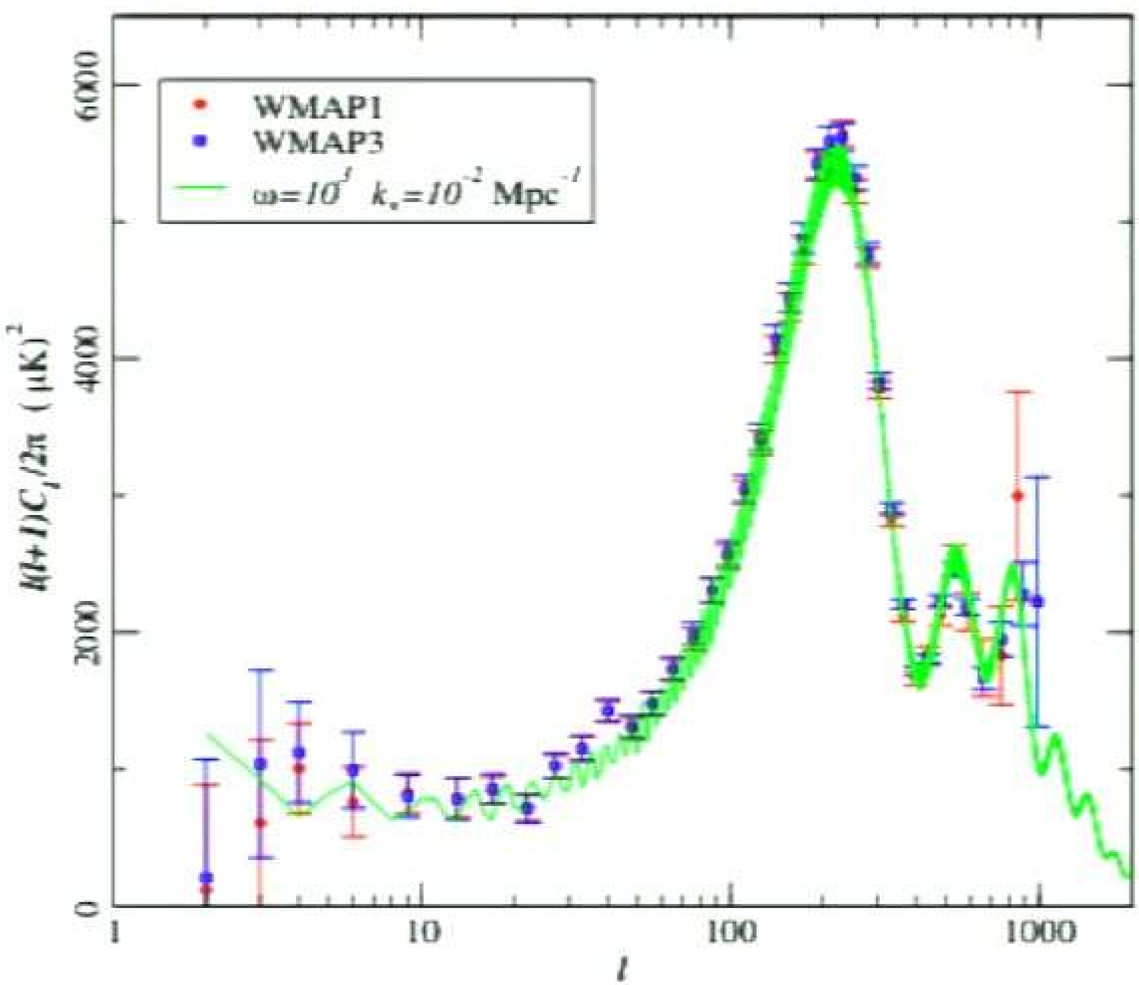
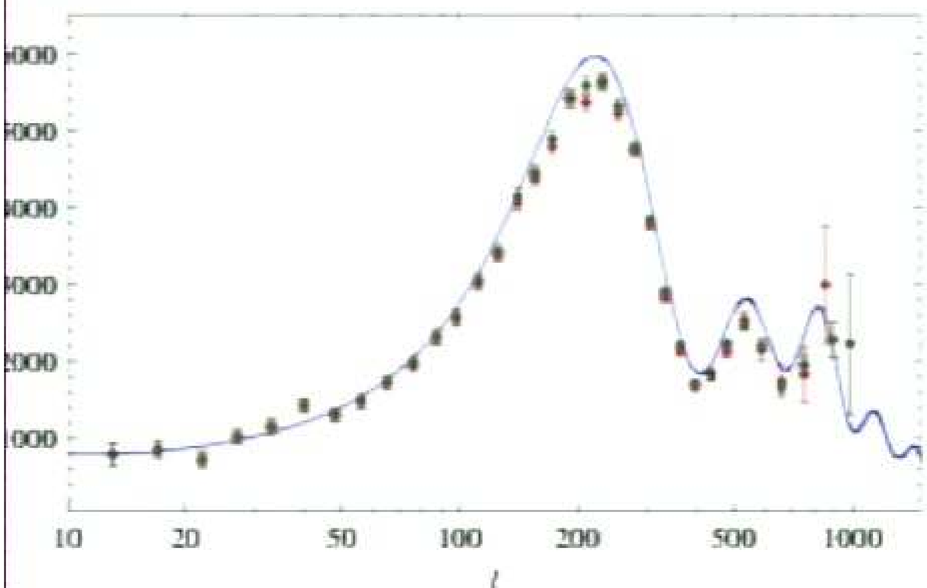
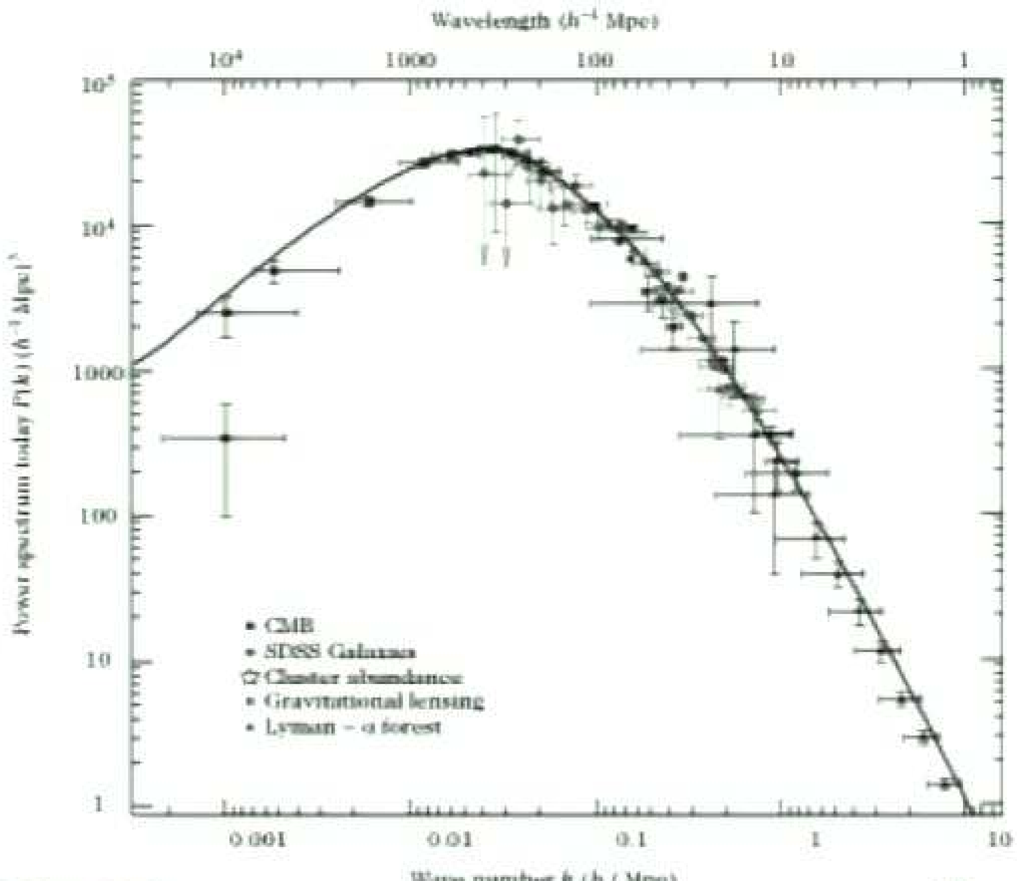

WMAP1
WMAP3
ω=10^-1 k_*=10^-2 Mpc^-1
l(l+1)C_l/2π (μK)^2
l
0
2000
4000
6000
1
10
100
1000

Data!



No obvious oscillations ...



Perimeter Institute - 11th May 2010

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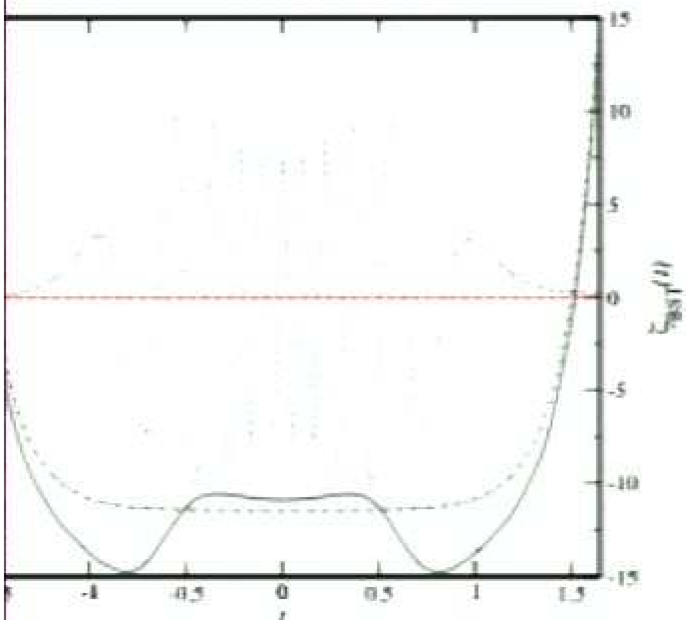
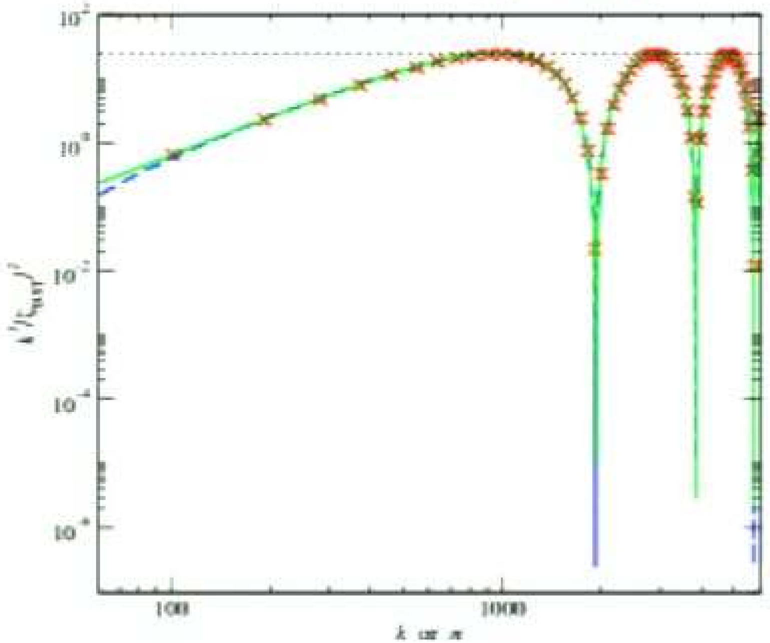
rturbations: $$ds^2 = a^2(\eta)\left\{(1+2\Phi)\,d\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]dx^i dx^j\right\}$$

$$\Longleftrightarrow \qquad \Phi = \frac{3\mathcal{H}u}{2a^2\theta} \qquad \theta \equiv \frac{1}{a}\sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi}\left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2}\right)}$$

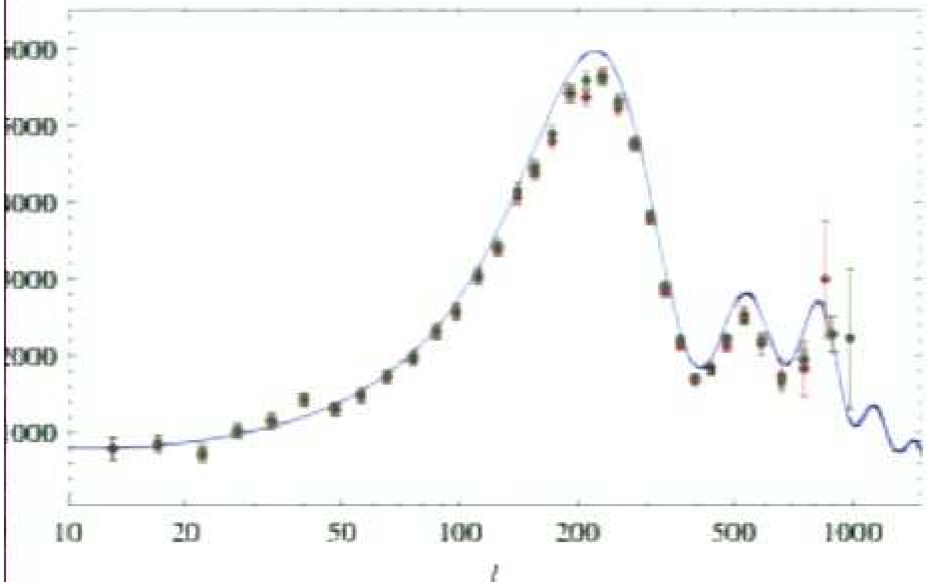
$$u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_s^2\right)\right]u = 0$$

$$\mathcal{P}_\zeta = \mathcal{A}k^{n_s - 1}\cos^2\left(\omega\frac{k_{\rm ph}}{k_\star} + \psi\right)$$

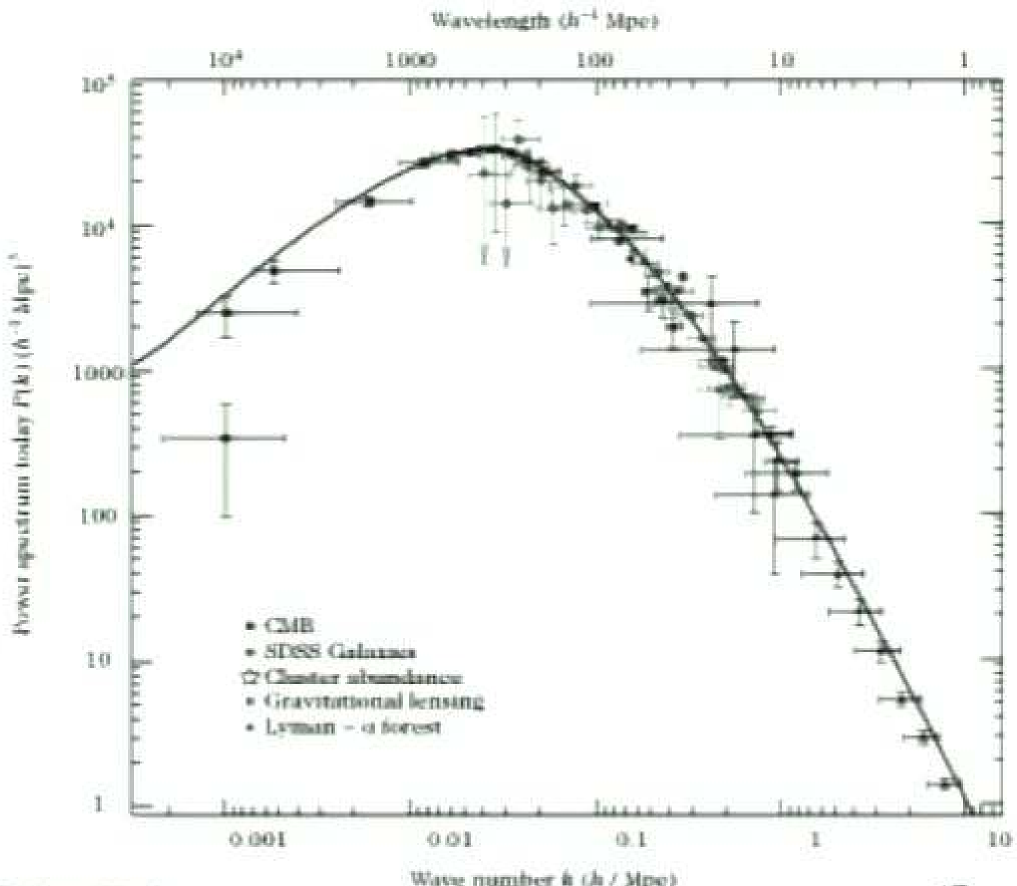


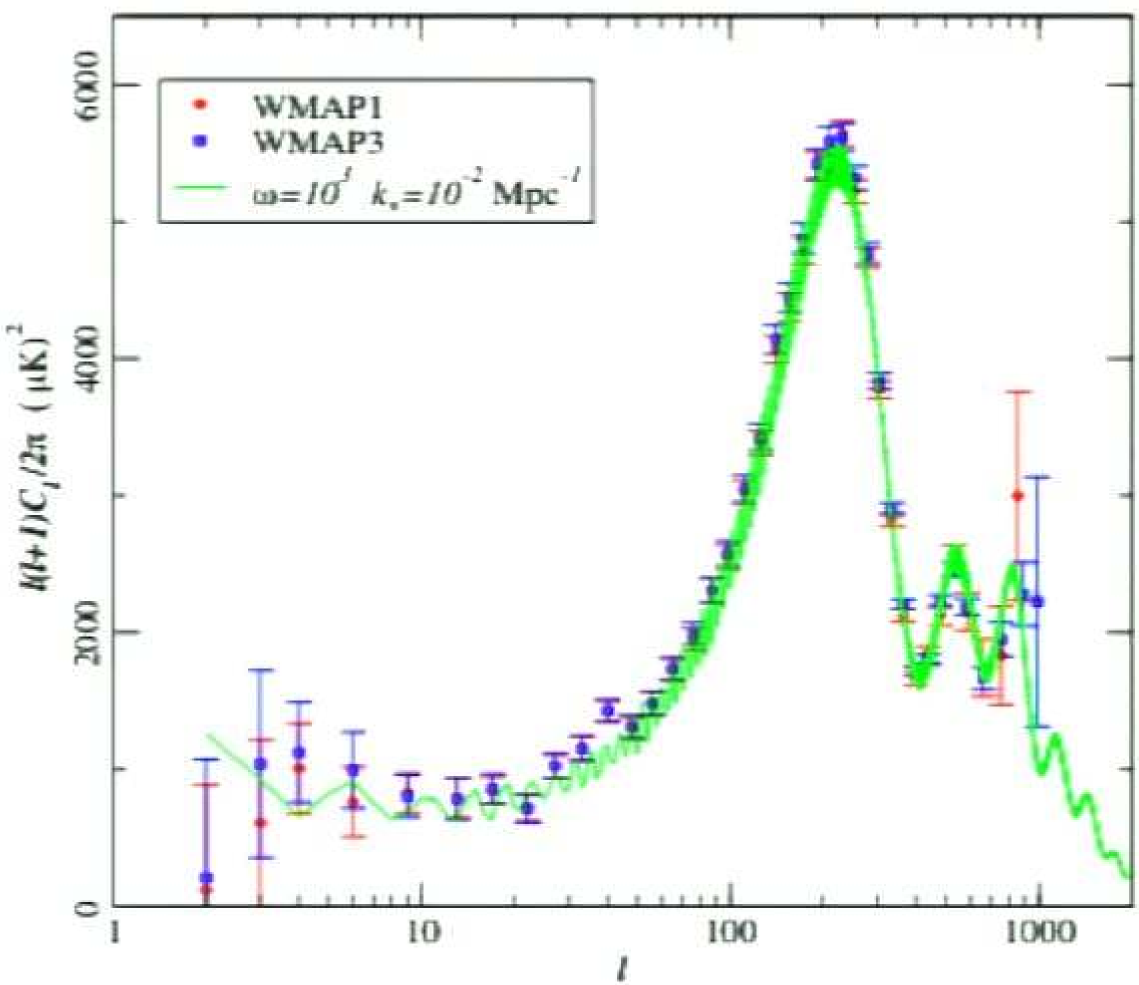


Data!



No obvious oscillations ...




WMAP1
WMAP3
ω=10^-1 k_*=10^-2 Mpc^-1
l(l+1)C_l/2π (μK)^2
l

- *K*-bounce: $\mathcal{L} = p(X, \varphi)$

$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$

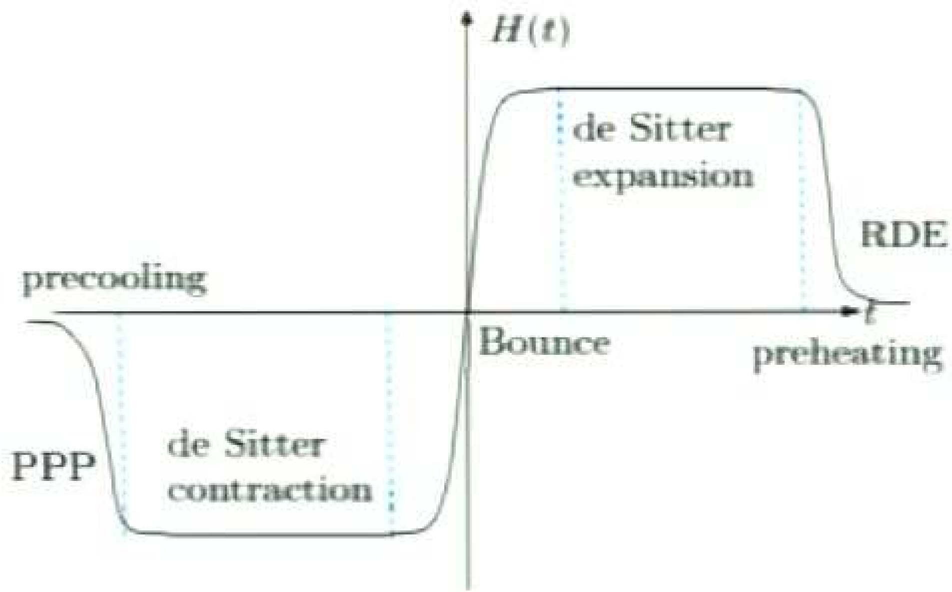
$\Rightarrow T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu}$

$\rho \equiv 2X \frac{\partial p}{\partial X} - p$

vanishing spatial curvature possible in 4 dimensions G.R.?

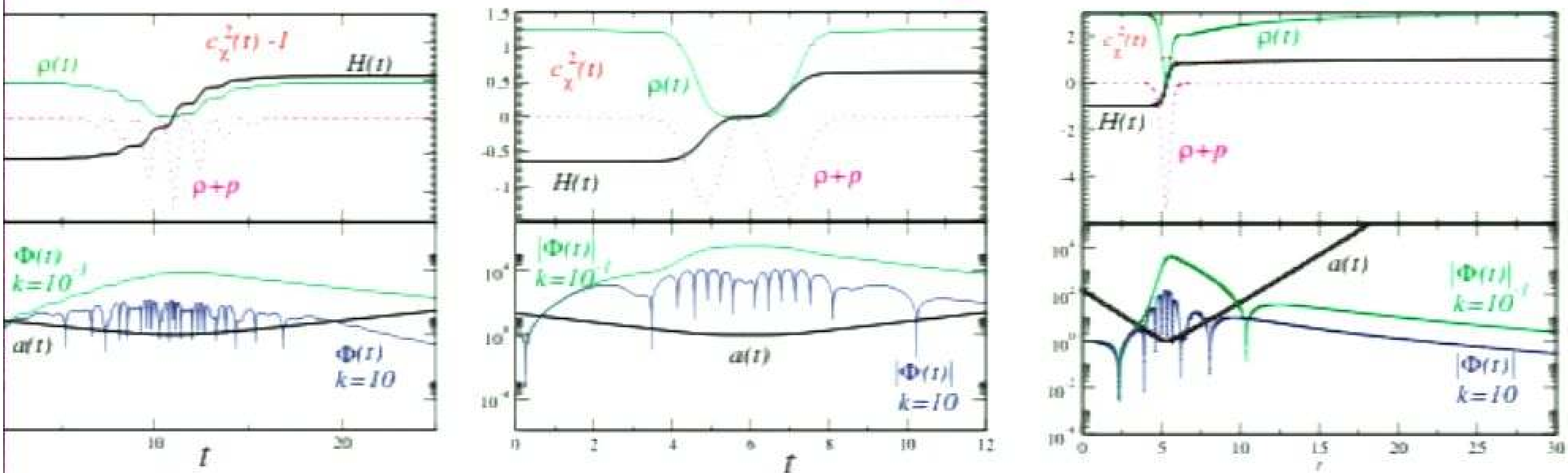
$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{2X}}$

$\rho\,(t_{\mathrm{bounce}}) = 0 \Longrightarrow p\,(t_{\mathrm{bounce}}) < 0$



Model for the bounce phase only:

$$= p_0 + p_X(X - X_0) + p_\varphi \varphi + p_{X\varphi}\varphi(X - X0) + \frac{1}{2}p_{XX}(X - X_0)^2 + \frac{1}{2}p_{\varphi\varphi}\varphi^2 + \cdots$$

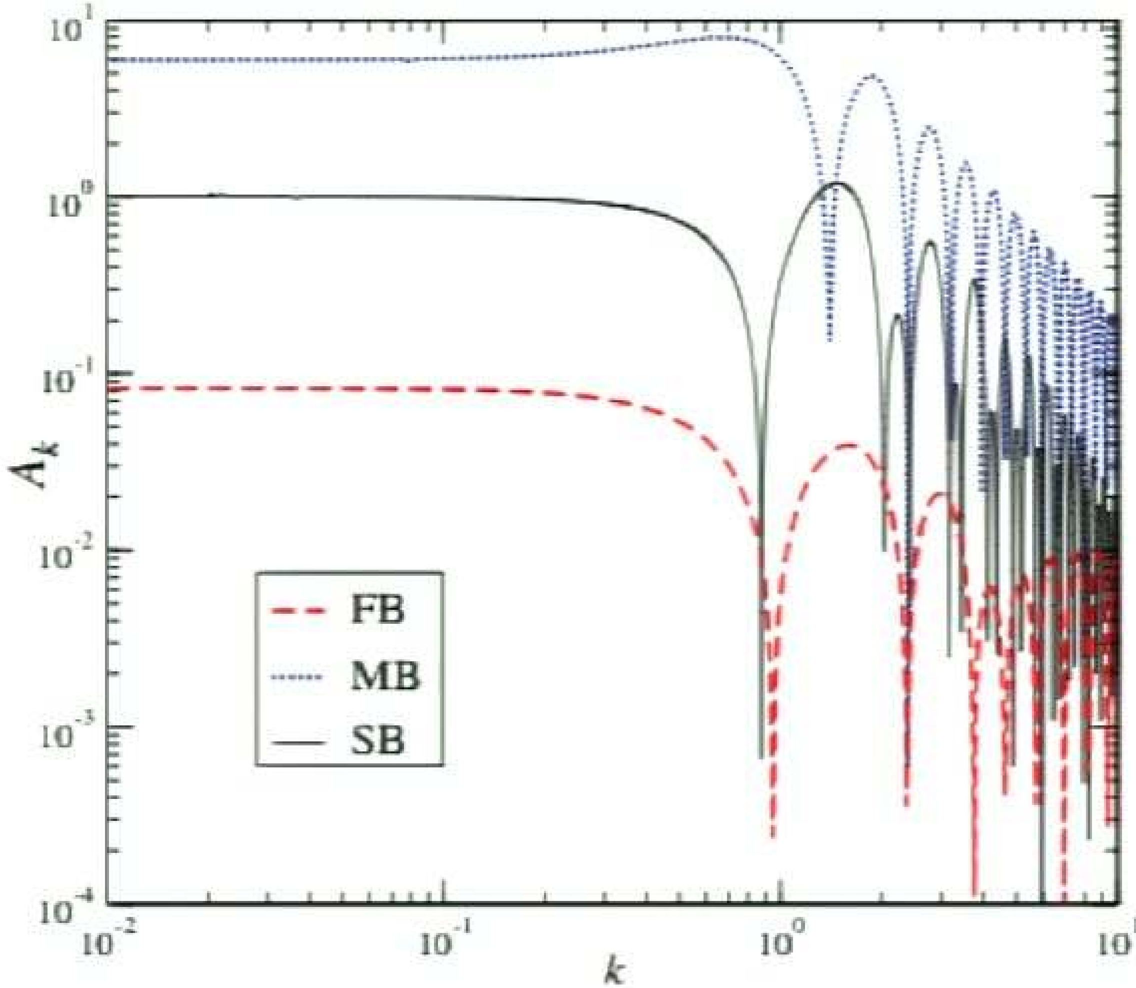


Slow

Fast

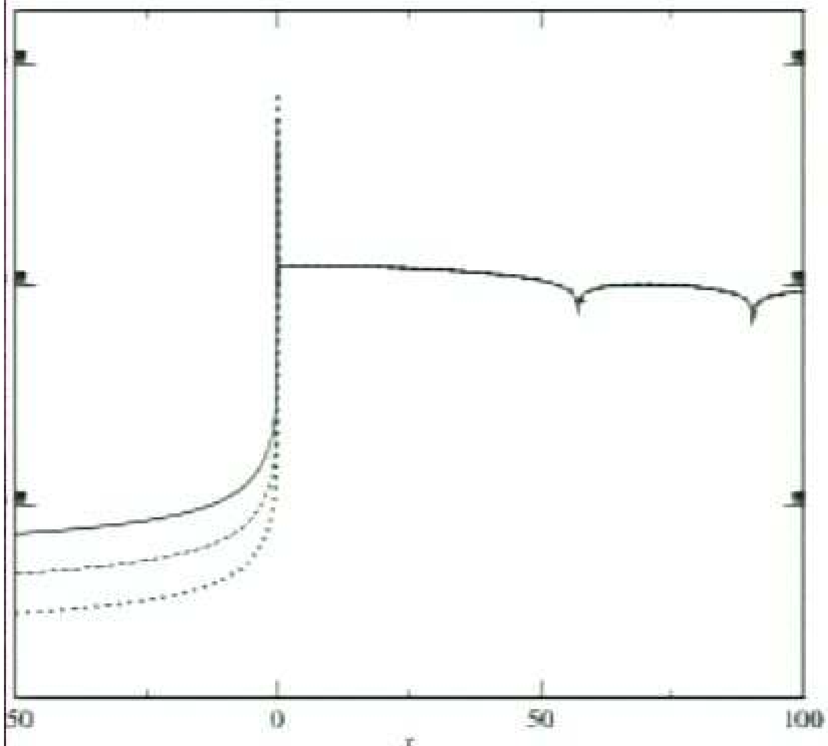
Oscillations + ζ conserved

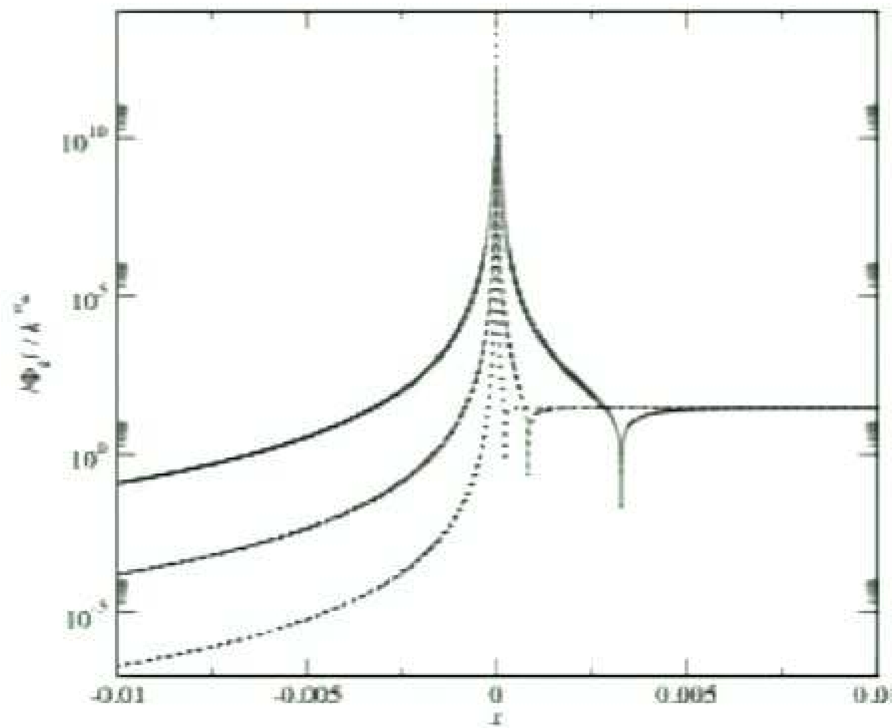
...bramo & P. P., JCAP 09, 001 (2007)



k-mode mixing ...

→ 2 d.o.f. + 4 dim G.R. $H^2 = \ell_{Pl}^2 \left[\frac{\rho_+}{a^{3(1+w_+)}} - \frac{\rho_-}{a^{3(1+w_-)}} \right]$





k-mode inversion ...

F. Finelli, P. P. & N. Pinto-Neto, *Phys. Rev.* D77, 103508 (2008)

A specific model: 4D Quantum cosmology

$$\mathcal{S} = \int \sqrt{-g}\left(-\frac{R}{6\ell_{\rm P}^2} + p\right) \mathrm{d}^4 x$$

Perfect fluid: $p = \omega\rho$ ➡ **bounce**

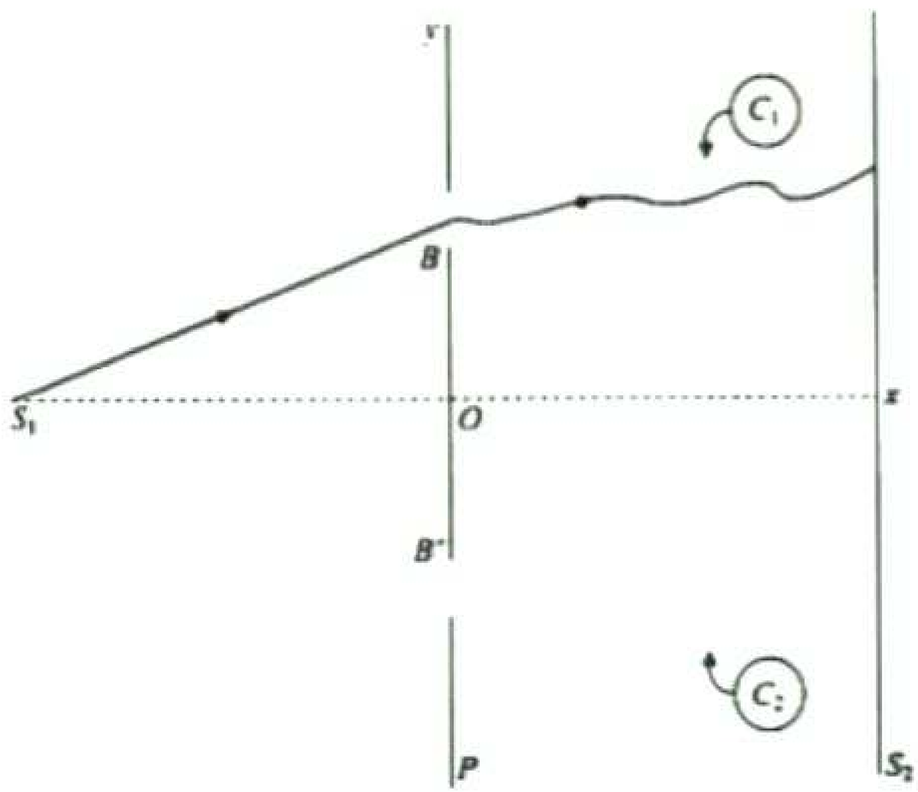
- **no horizon problem if** $\omega > -\frac{1}{3}$

Results:

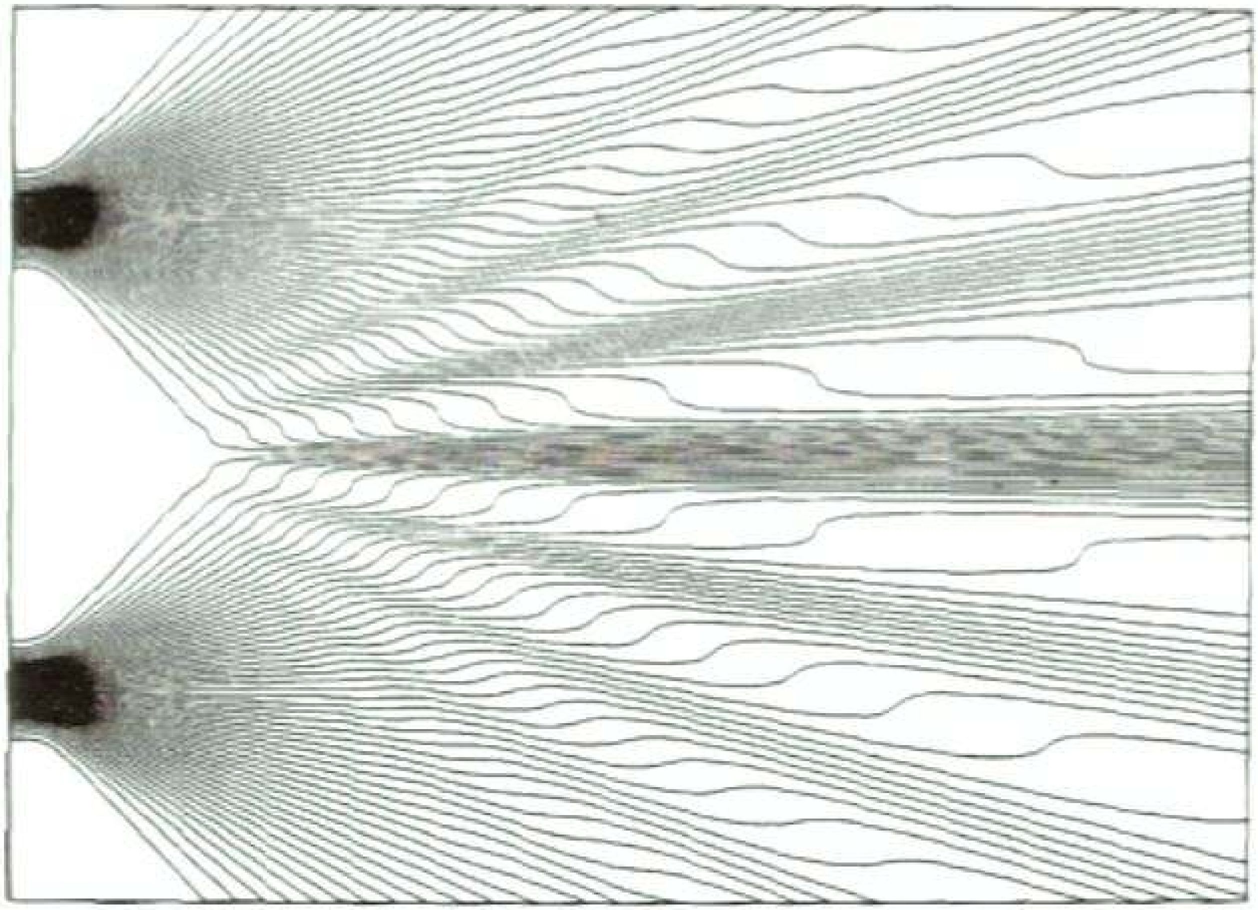
$$n_{\rm T} = n_{\rm s} - 1 = \frac{12\omega}{1+3\omega}$$

$$\frac{T}{S} \simeq 4 \times 10^{-2}\sqrt{n_{\rm s} - 1}$$

The two-slit experiment:



Trajectories in the two-slit experiment



uantum cosmology $ds^2 = N^2(\tau)d\tau - a^2(\tau)\gamma_{ij}dx^i dx^j$

canonical transformation

rescaling (volume ...) = a simple Hamiltonian:

units

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

$+$ Technical trick: $\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}$

space defined by $\chi > 0$ → **constraint** $\bar{\Psi}\frac{\partial\Psi}{\partial\chi} = \Psi\frac{\partial\bar{\Psi}}{\partial\chi}$

alternative way of getting the solution:

WKB exact superposition: $\Psi = \int e^{iET} \rho(E) \psi_E(T) dE$

Gaussian wave packet $\propto e^{-(ET_0)^2}$

$$\Psi = \left[\frac{8T_0}{\pi (T_0^2 + T^2)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0 \chi^2}{T_0^2 + T^2}\right) e^{-iS(\chi,T)}$$

phase $S = \frac{T\chi^2}{T_0^2 + T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$

Bohmian trajectory $\dot{a} = \{a, H\}$

$$a = a_0 \left[1 + \left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$

uantum cosmology $ds^2 = N^2(\tau)d\tau - a^2(\tau)\gamma_{ij}dx^i dx^j$

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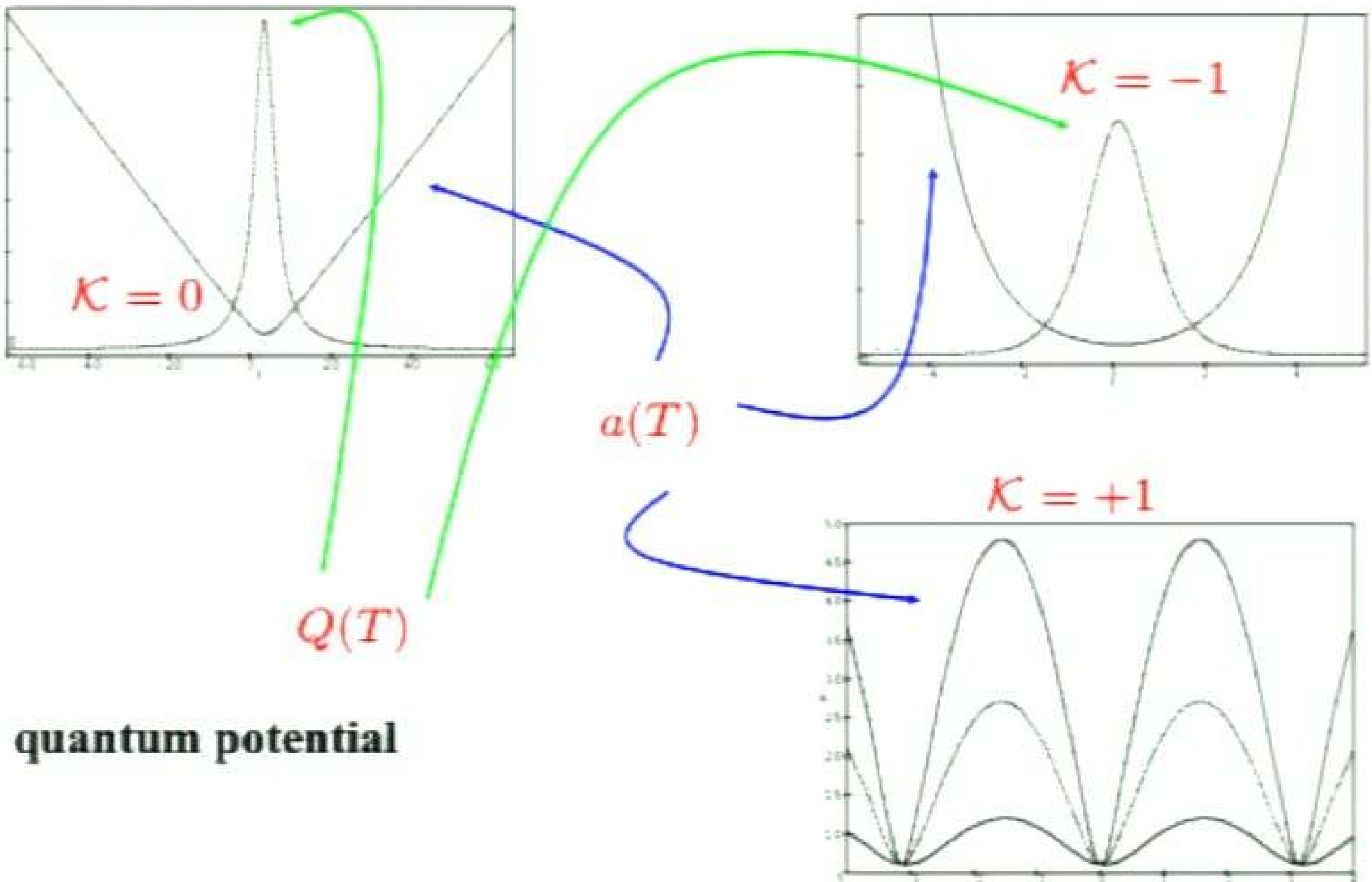


quantum potential

What about the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a, T\right)\Psi_{(2)}\left[v, T; a\left(T\right)\right]$$

comes from 0th order

$$\Delta\Phi = -\frac{3\ell_{\rm Pl}^2}{2}\sqrt{\frac{\rho+p}{\omega}}a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a^{3\omega-1}\mathrm{d}T$

+ **canonical transformations:**

$$i\frac{\partial\Psi_{(2)}}{\partial\eta} = \int d^3x\left(-\frac{1}{2}\frac{\delta^2}{\delta v^2} + \frac{\omega}{2}v_{,i}v^{,i} - \frac{a''}{a}\right)\Psi_{(2)}$$

Fourier mode

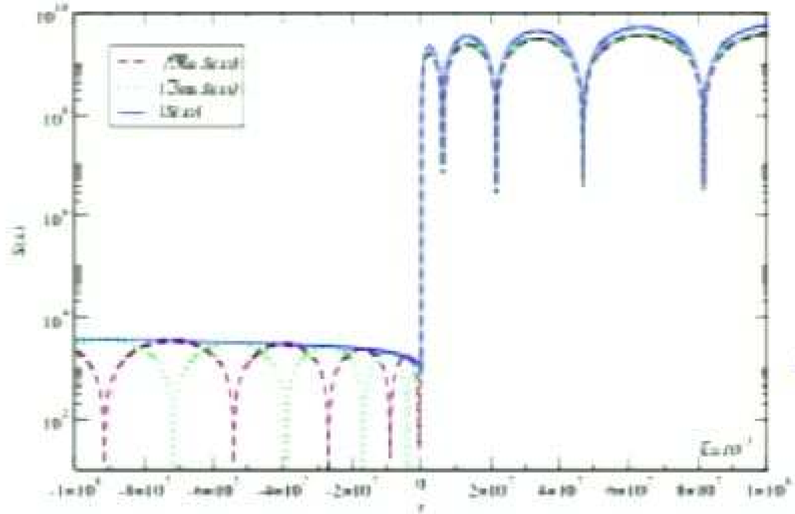
$$v_k'' + \left(c_{\rm S}^2 k^2 - \frac{a''}{a}\right)v_k = 0$$

$$c_{\rm S}^2 = \sqrt{\omega} \neq 0$$

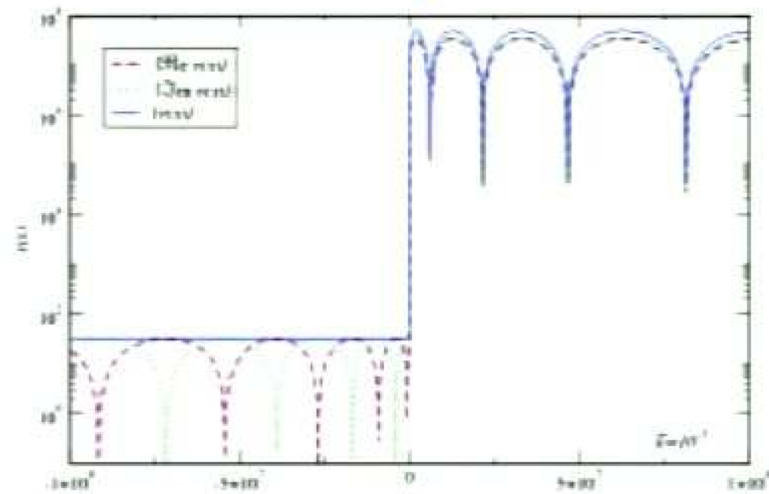
vacuum initial conditions

$$v_k \propto \frac{e^{-ic_{\rm S}k\eta}}{\sqrt{2c_{\rm S}k}}$$

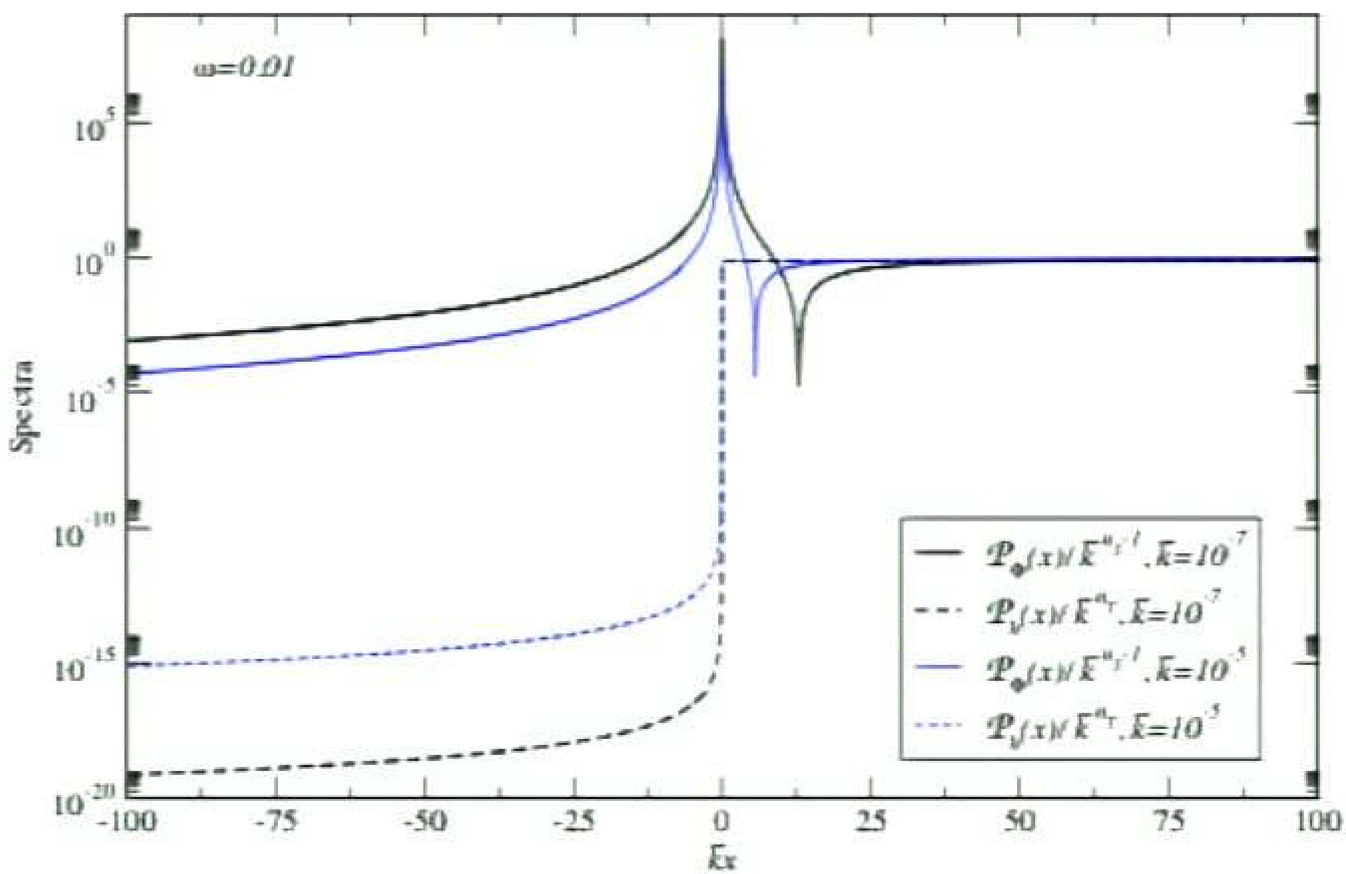
+ **evolution** (matchings and/or numerics)



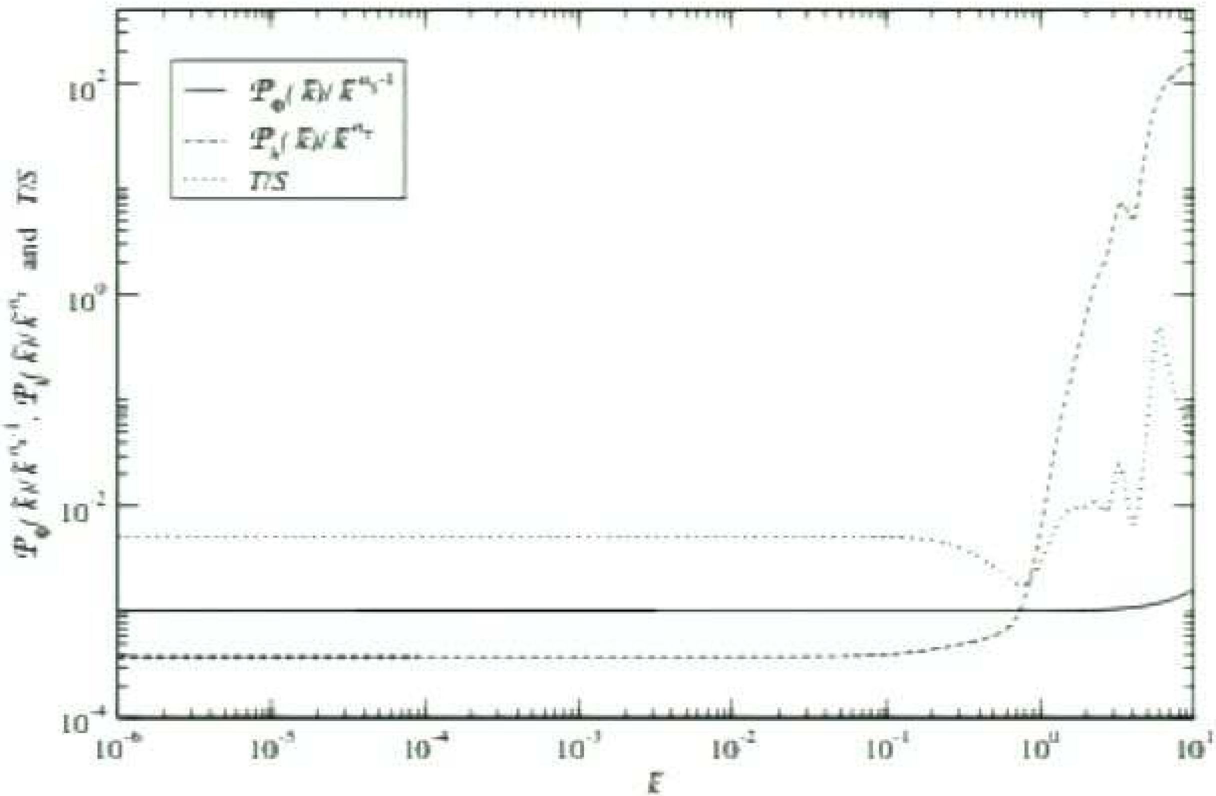
$$s(x) = a^{\frac{1}{2}(1-3\omega)}\,\frac{v(x)}{\sqrt{T_0}}$$



$$x \equiv \frac{T}{T_0}$$


ω=0.01
Spectra
$\bar{k}x$
$\mathcal{P}_\Phi(x)/\bar{k}^{n_s-1}, \bar{k}=10^{-7}$
$\mathcal{P}_h(x)/\bar{k}^{n_T}, \bar{k}=10^{-7}$
$\mathcal{P}_\Phi(x)/\bar{k}^{n_s-1}, \bar{k}=10^{-5}$
$\mathcal{P}_h(x)/\bar{k}^{n_T}, \bar{k}=10^{-5}$

Full numerical spectrum



spectrum $\mathcal{P}_\Phi \propto k^3 |\Phi_k|^2 \propto A_S^2 k^{n_S - 1}$

id. grav. waves: $\mu'' + \left(k^2 - \frac{a''}{a}\right)\mu = 0$ $\mu \equiv \frac{h}{a}$

$\mu_{\rm ini} \propto \frac{\exp(-ik\eta)}{\sqrt{k\eta}}$ $\mathcal{P}_h \propto k^3 |h_k|^2 \propto A_T^2 k^{n_T}$

same dynamics + initial conditions ⇨ **same spectrum**

$$n_T = n_S - 1 = \frac{12\omega}{1+3\omega}$$

CMB normalisation $A_S^2 = 2.08 \times 10^{-10}$

⇨ **bounce curvature** $T_0 a_0^{3\omega} \simeq 1500 \ell_{Pl}$

WMAP constraint

$$n_s = 0.96 \pm 0.02 \Longrightarrow w \lesssim 8 \times 10^{-4}$$

predictions

- **spectrum slightly blue**

power-law + concordance

$$\frac{T}{S} = \frac{C_{10}^{(T)}}{C_{10}^{(S)}} = \mathcal{F}(\Omega, \cdots) \frac{A_T^2}{A_S^2} \propto \sqrt{w}$$

$\mathcal{F}(\Omega, \cdots) \simeq 0.62$

$$\boxed{\frac{T}{S} \simeq 4 \times 10^{-2} \sqrt{n_s - 1}}$$

P. P. & N. Pinto-Neto, *Phys. Rev.* **D78**, [illegible] (2008)