Title: Dressing factor in the integrable AdS/CFT system

Date: May 04, 2010 11:00 AM

URL: http://pirsa.org/10050006

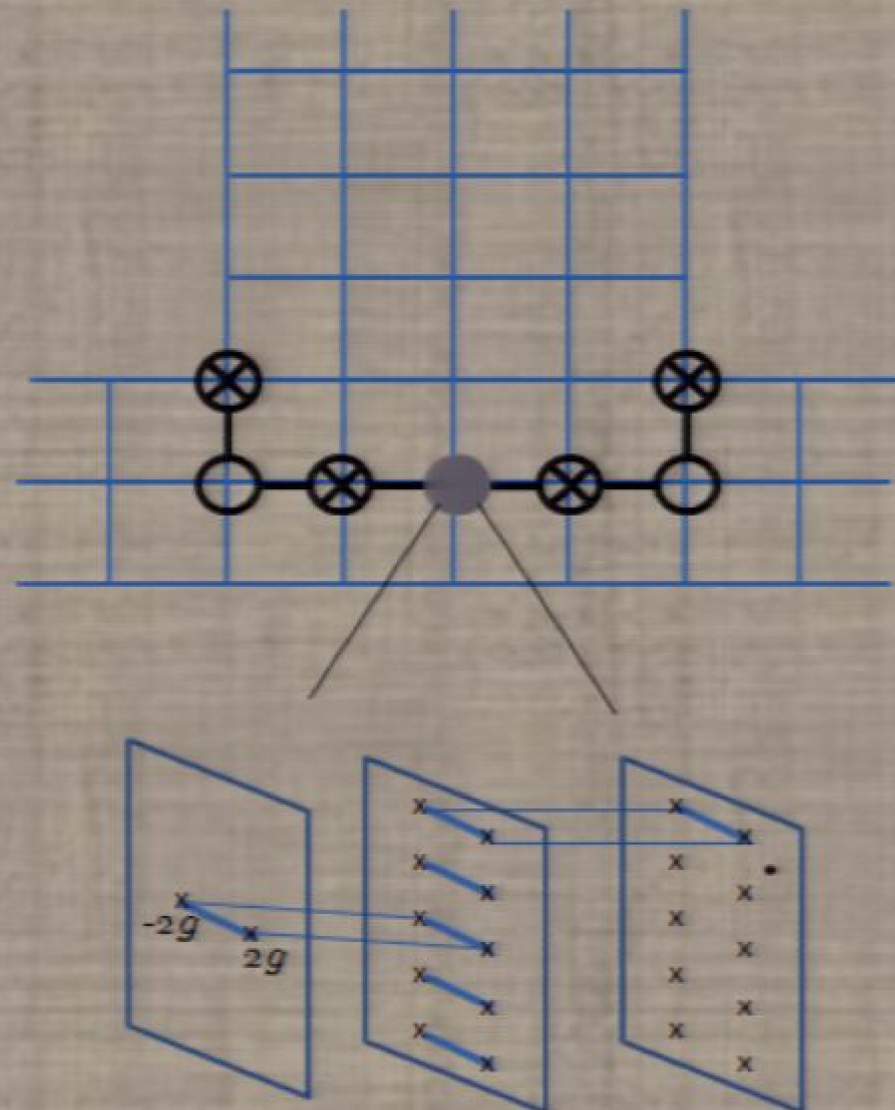
Abstract: The seminar is devoted to the solution of the AdS/CFT spectral problem, both for infinite and finite volume cases, using integrability. The basic constructions will be explained using an analogy with the relativistic O(4) sigma model. We devote a special attention to the study of the so called dressing factor. This is a scalar factor of the scattering matrix fixed using discrete crossing symmetry. We will show how to explicitly solve the equations dictated by the crossing symmetry, discuss essential properties of the dressing factor and give a shorthand and suggestive notation for the Bethe Ansatz of AdS/CFT. Then we will sketch a shorthand way to pass from the Bethe Ansatz to the Y-system.

Dressing factor in integrable AdS/CFT system

Dmytro Volin

arXiv:0904.4929
arXiv:1003.4725

Review with P.Vieira, 2010



Perimeter Institute 04 May 2010

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$=4$ SYM = IIB, $AdS_5 x S^5$

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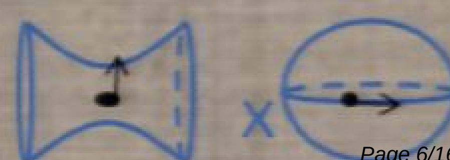
Local operators	=	String states
Conformal dimension	=	Energy

$\Phi_1 + i\Phi_2$

$\Phi_3 + i\Phi_4$

$\mathrm{Tr} XZZZXZ \ldots XZZ$



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1) AdS/CFT duality

2) Integrability

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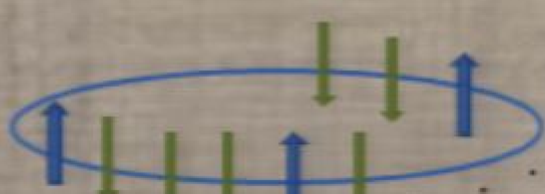

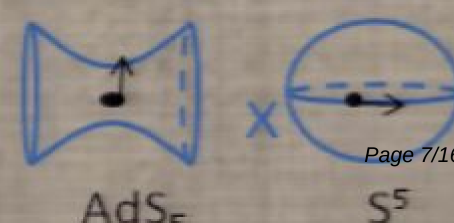

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ɿ anomalous dimension

$D^{n_1} Z D^{n_2} Z$

$+ n_2 = S$

$= D_0 - D_1$

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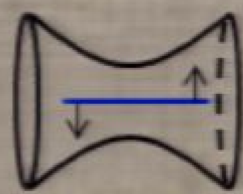

[Beisert, Staudacher, 03]
[Beisert, 03-04]

$- S = f[g] \log S + \ldots, \quad S \to \infty$

[Moch, Vermaseren, Vogt, 04]
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[Bern et al., 06]
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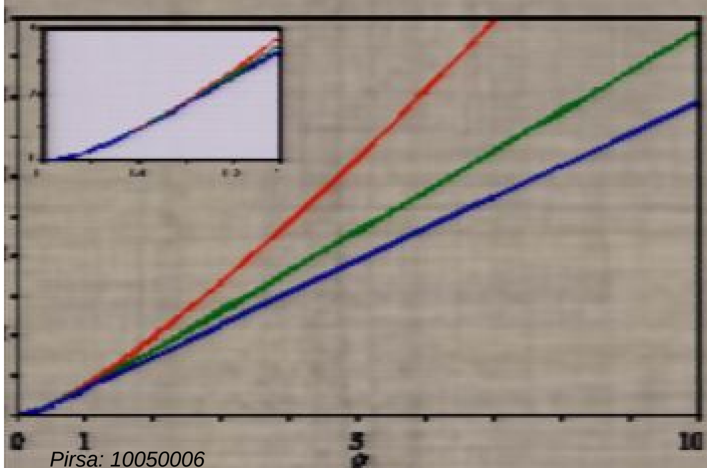
$$f[g] = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - \left(\frac{584}{315}\pi^6 + 64\zeta(3)^2\right) g^8 + \ldots$$

[Beisert, Eden, Staudacher, 06]



[Benna, Benvenuti, Klebanov, Scardicchio, 06]

[Gubser, Klebanov, Polyakov, 02]
[Frolov, Tseytlin, 02]
[Roiban, Tseytlin, 07]

$$f[g] = 4g - \frac{3\log 2}{\pi} - \frac{1}{4g}\frac{K}{\pi^2} - \ldots$$

[Klebanov et al, 06]
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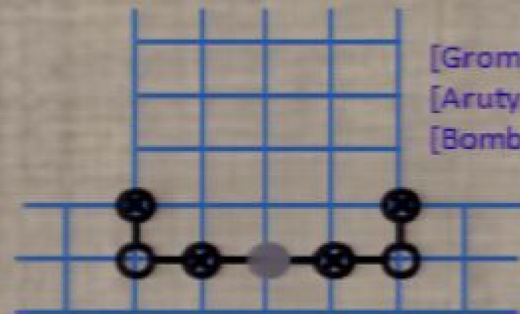
$\mathrm{Tr}XZXZ - \mathrm{Tr}XXZZ$



[Gromov, Kazakov, Kozak, Vieira, 09]
[Arutyunov, Frolov, 09]
[Bombardelli, Fioravanti, Tateo, 09]

[Fiamberti, Santambrogio, Sieg , Zanon,,'08]

$$\Delta[g] = 4 + 12g^2 - 48g^4 + 336g^6 + c_1 g^8 + c_2 g^{10}$$

[Bajnok, Janik,'08]
[Bajnok, Hegedus, Janik, Lukowski'09]
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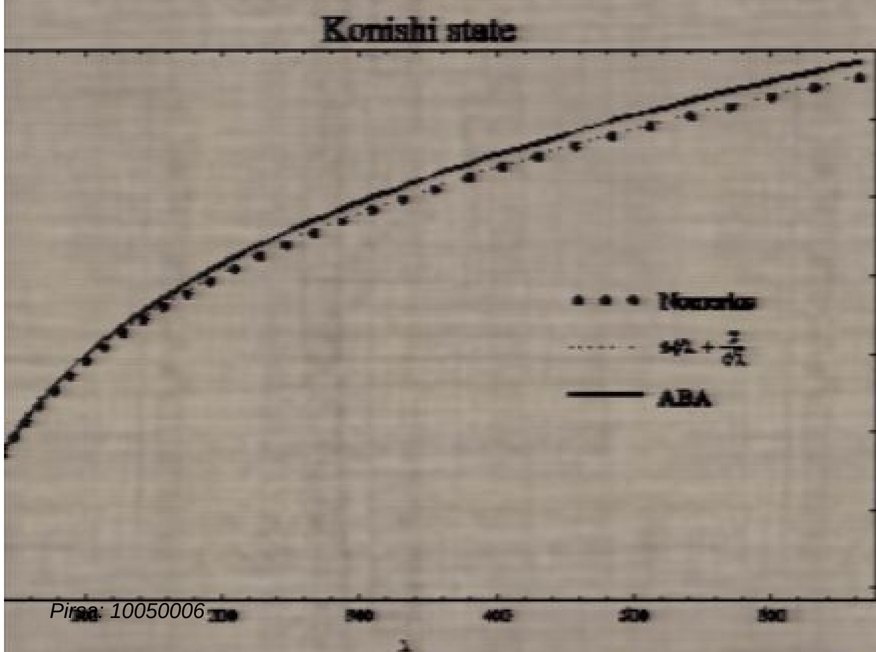

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Only numerics and discrepancy with string

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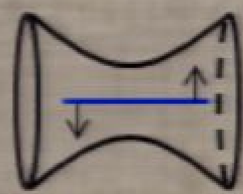

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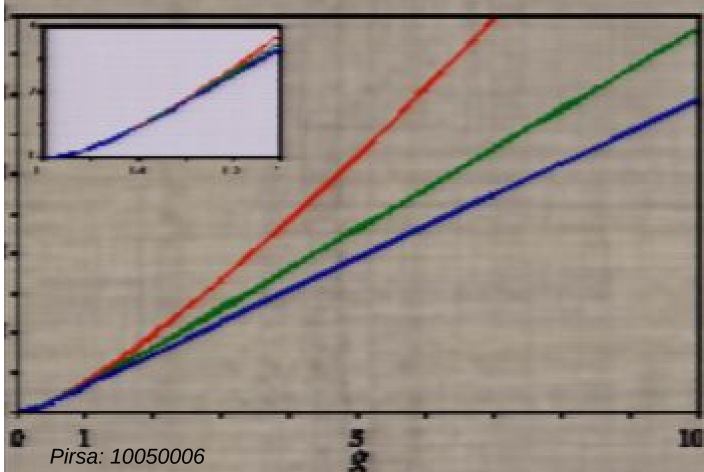

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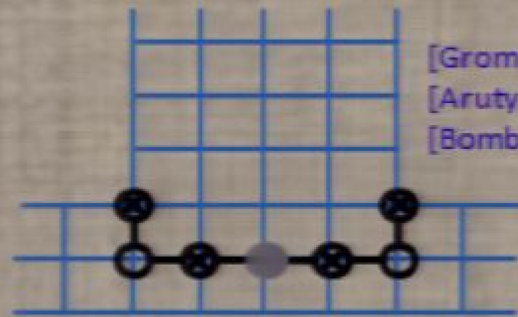

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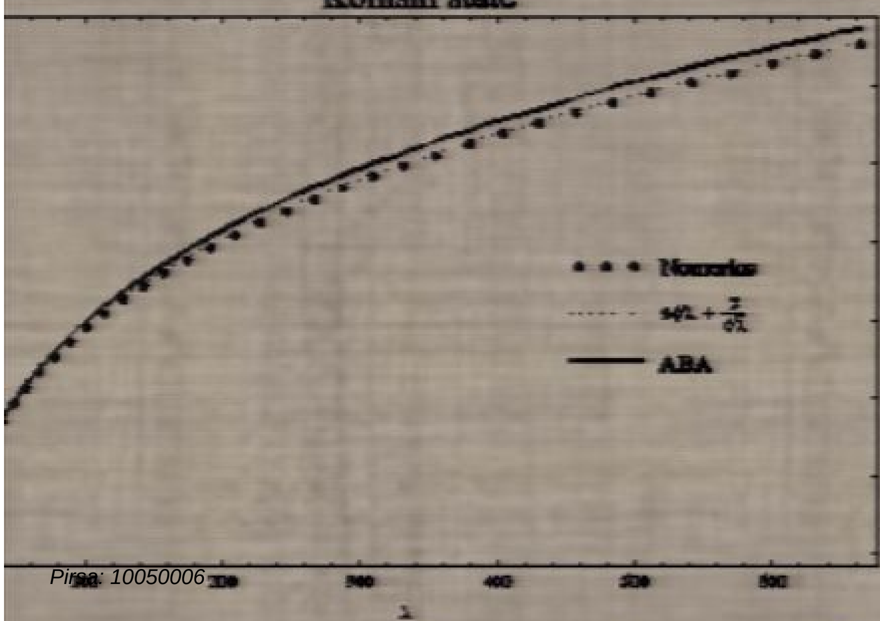

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SU(2)× SU(2) PCF

Asymptotic Bethe Ansatz for
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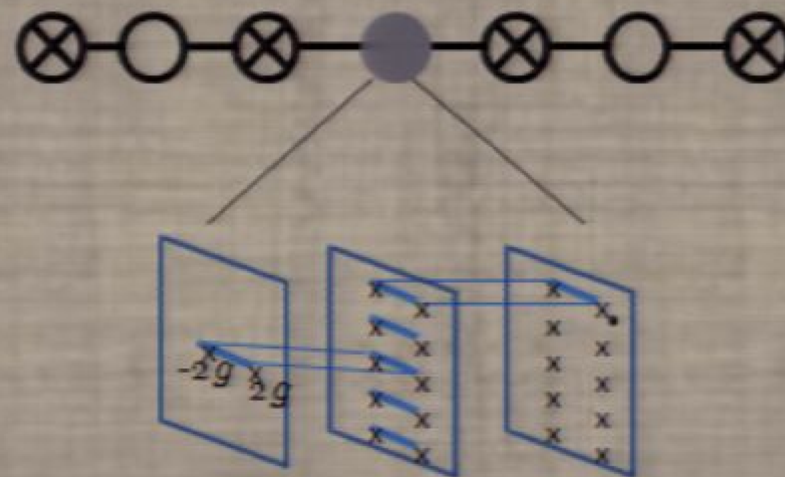
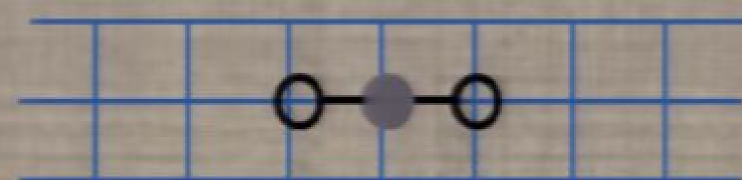


→ Dressing phase and
analytical structure

Thermodynamic BA for
SU(N)× SU(N) PCF



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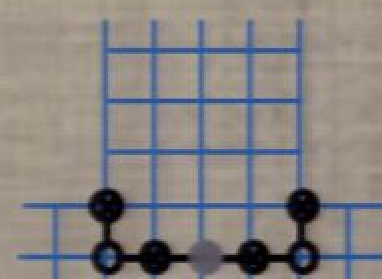

Part I

Asymptotic Bethe Ansatz for
SU(2) × SU(2) PCF

(2)× SU(2) PCF is equivalent to the O(4) vector sigma model

$$S = \frac{1}{2\alpha^2}\int d^2\sigma(\partial_\mu \vec{n})^2, \qquad \vec{n}^2 = 1$$

Target space is

$$S^3 \simeq \frac{SU(2)\times SU(2)}{SU(2)}$$

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ere is a dynamically generated mass scale

rticle content of the theory: massive vector multiplet of O(4).

Polyakov showed presence of infinitely many conserved charges

[Polyakov '75]

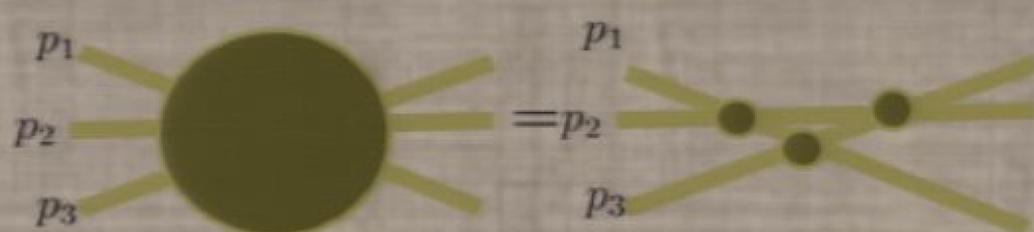
↓

No particle production

$$Q_n = \sum_i (p_i)^n = \sum_i (p'_i)^n$$

Only permutation of the momenta

Factorization of scattering



Completely know scattering process if the scattering matrix is
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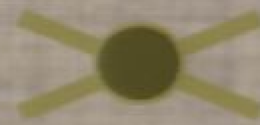
otstrap approach

olodchikov, Zamolodchikov '77]



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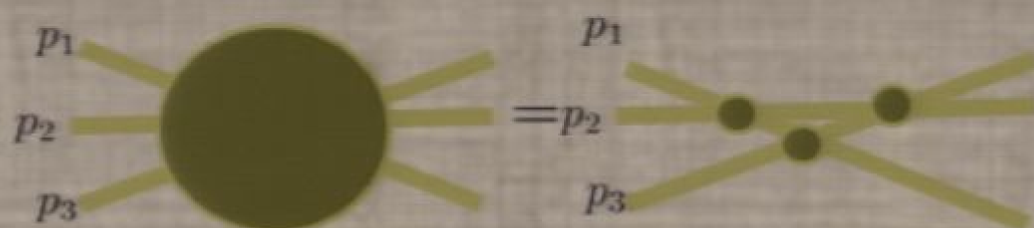

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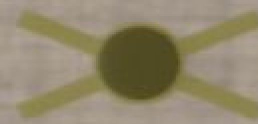

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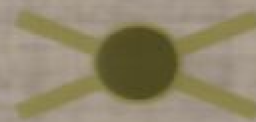

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renz invariance

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$$\widehat{S}[\theta] = \left(\frac{\theta \times - f[\theta]\,)(}{\theta - i}\right)_L \otimes \left(\frac{\theta \times - f[\theta]\,)(}{\theta - i}\right)_R \times S_0^2[\theta]$$

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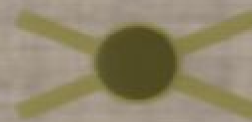

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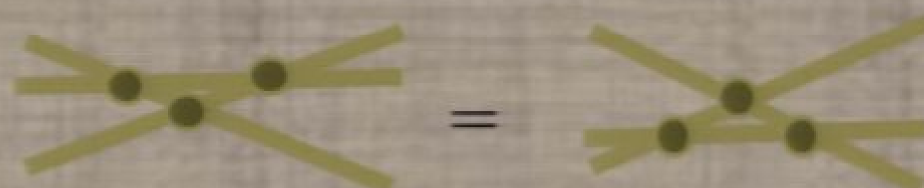
ng-Baxter equation



$$f[\theta] = i$$

$$\widehat{S}_{12}\,\widehat{S}_{23}\,\widehat{S}_{23} = \widehat{S}_{23}\,\widehat{S}_{13}\,\widehat{S}_{12}$$

ymptotic Bethe Ansatz

$$\widehat{S} = \left(\frac{\theta - i\mathcal{P}}{\theta - i}\right)_L \otimes \left(\frac{\theta - i\mathcal{P}}{\theta - i}\right)_R \times S_0^2$$

umber of particles is conserved. Therefore we can use a first ntization language and describe scattering in terms of wave function.

$$= \int_{x_1<x_2} dx_1 dx_2 \, e^{ip_1x_1+ip_2x_2}|(x_1,a_1),(x_2,a_2)> + S_{a_1a_2}^{b_1b_2}[p_2,p_1] e^{ip_2x_1+ip_1x_2}|(x_1,b_2),(x_2,b_1)>$$

riodicity condition is realized as:

$$e^{-ip_kL}\Psi = -\prod_{j\neq k} \widehat{S}[\theta_k - \theta_j] \cdot \Psi$$

gebraic part of S-matrix, $\theta - i\mathcal{P}$, is the same as R-matrix of senberg XXX spin chain. Diagonalization of periodicity condition – the e as albraic Bethe Ansatz in XXX.

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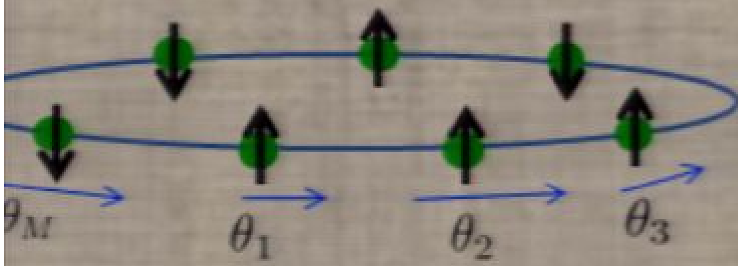

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$$\prod_{k=1}^{M}\frac{\lambda_{L,j}-\theta_k+\frac{i}{2}}{\lambda_{L,j}-\theta_k-\frac{i}{2}} = -\prod_{j'=1}^{K_L}\frac{\lambda_{L,j}-\lambda_{L,j'}+i}{\lambda_{L,j}-\lambda_{L,j'}-i},$$

$$e^{-imL\sinh[\pi\theta_k]} = -\prod_{k'=1}^{M} S_0[\theta_k-\theta_{k'}]\prod_{j=1}^{K_L}\frac{\theta_k-\lambda_{L,j}+\frac{i}{2}}{\theta_k-\lambda_{L,j}-\frac{i}{2}}\prod_{j=1}^{K_R}\frac{\theta_k-\lambda_{R,j}+\frac{i}{2}}{\theta_k-\lambda_{R,j}-\frac{i}{2}},$$

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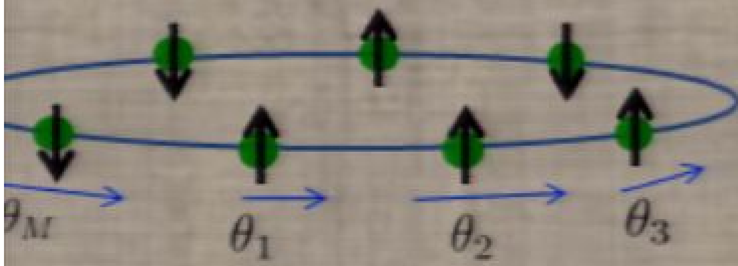

$$e^{-ip_k L}\Psi = -\prod_{j\neq k} \widehat{S}[\theta_k - \theta_j] \cdot \Psi$$

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$$D \equiv e^{\frac{i}{2}\partial_\theta}$$

$$f[\theta]^D \equiv e^{D \log f[\theta]}$$

$$S_0^{D + D^{-1}} = \theta^{-(D - D^{-1})}$$

...ing the scalar factor

$$\widehat{S} = \left(\tfrac{\theta - i\mathcal{P}}{\theta - i}\right)_L \otimes \left(\tfrac{\theta - i\mathcal{P}}{\theta - i}\right)_R \times S_0^2$$

...itarity and crossing conditions require: $S_0[\theta + \frac{i}{2}]S_0[\theta - \frac{i}{2}] = \frac{\theta - \frac{i}{2}}{\theta + \frac{i}{2}}$

$$D \equiv e^{\frac{i}{2}\partial_\theta}$$

$$S_0^{D+D^{-1}} = \theta^{-(D-D^{-1})}$$

$$f[\theta]^D \equiv e^{D \log f[\theta]}$$

...lution of crossing: $S_0 \simeq \theta^{-\frac{D-D^{-1}}{D+D^{-1}}}$

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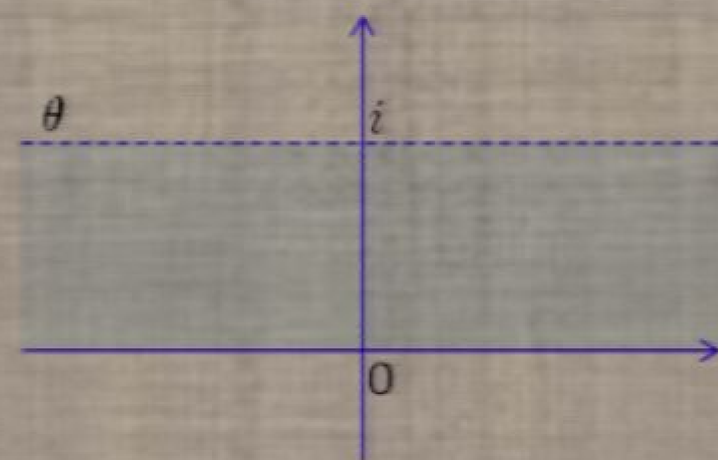
...rticle content $\rightarrow$ analytical structure in ...physical strip



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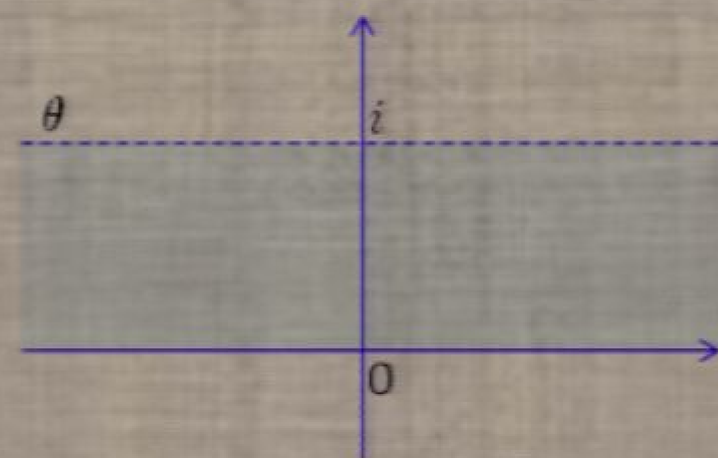

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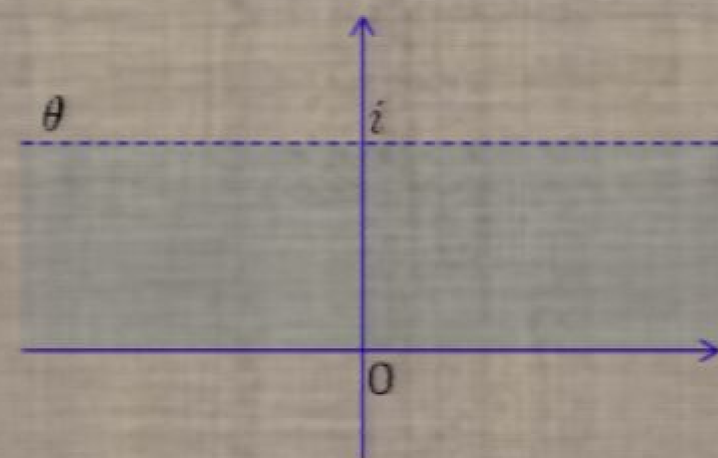

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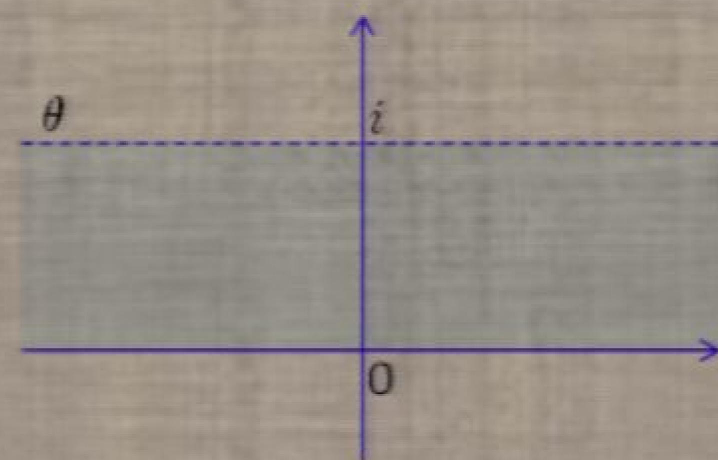

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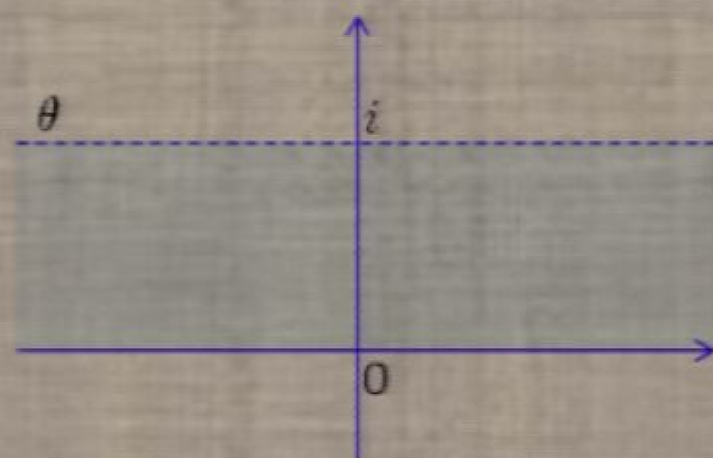

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S-matrix is completely fixed!

Summary

Can find the spectrum of O(4) in the infinite volume

rticle content

rticle content

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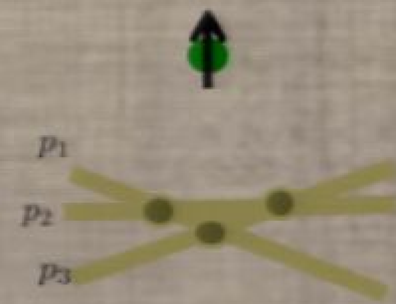

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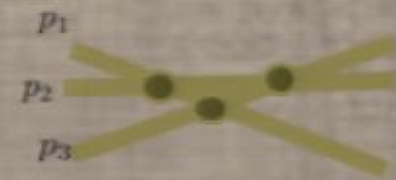

ed to fix only 2-particle S-matrix

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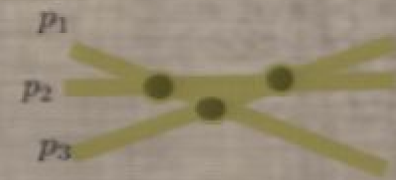

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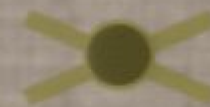

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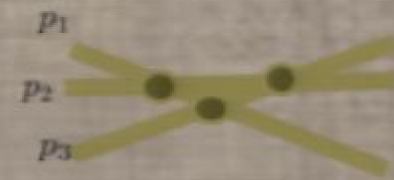

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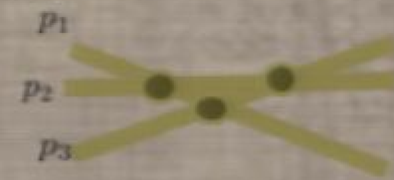

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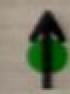

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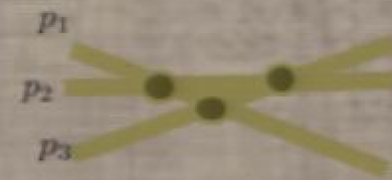

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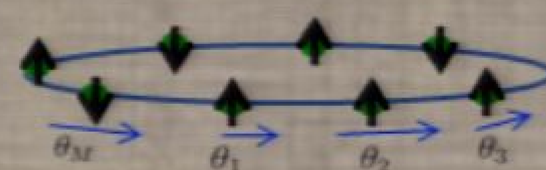

agonalization by Bethe Ansatz

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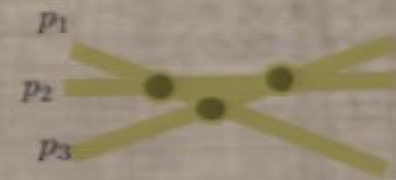

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$$E = m \sum_i \cosh[\pi \theta_i]$$

Part II

Asymptotic Bethe Ansatz in spectral problem of AdS/CFT

rticle content

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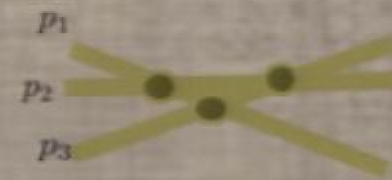

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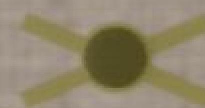

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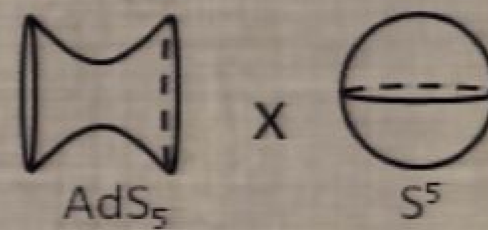
× SU(2) PCF is a sigma model on a coset

$$\frac{SU(2)\times SU(2)}{SU(2)}$$

Type IIB string theory (1st quantized only) is described by a coset sigma model

$$\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$$



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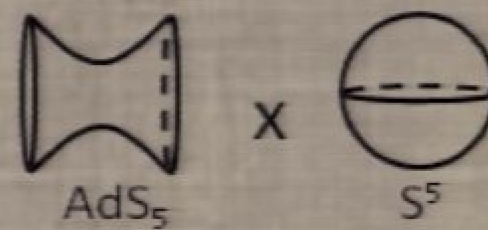

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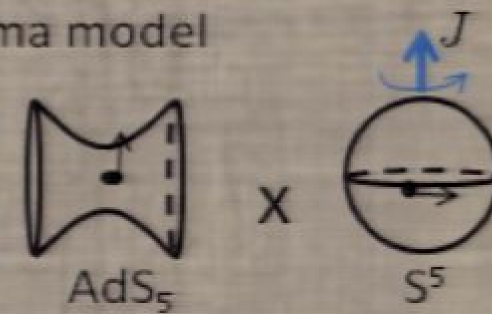

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need to pick a nontrivial string solution from the beginning

standard choice: BMN string: a point-like string encircling the equator of S^5 with angular omentum J.

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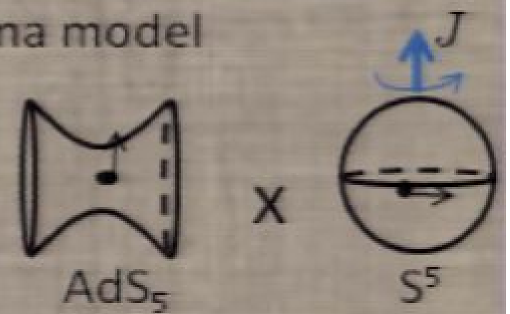

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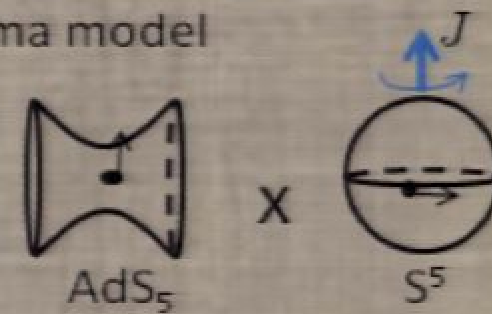

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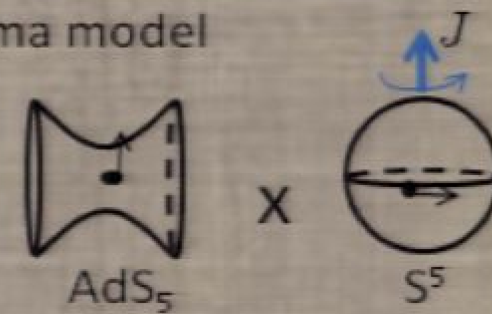

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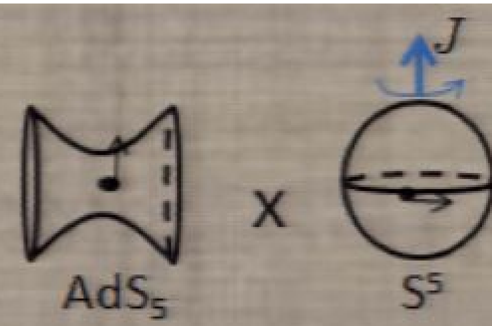


$$(PSU(2|2) \times PSU(2|2)) \ltimes \mathbb{R}^3$$

$$g^2 = \frac{g_{YM}^2 N_c}{16\pi^2} = \frac{\lambda}{16\pi^2}$$

egrability [Staudacher, 04]

❑ was observed

- classically on the string side (g is large) [Bena, Polchinski, Roiban, 04]
- at one-loop and partially up to three loops on the gauge side (g is small) [Minahan, Zarembo, 02] [Beisert, 04]

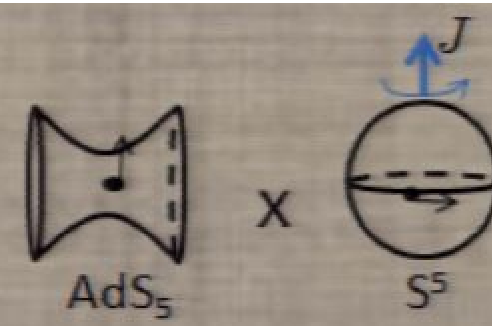
❑ was conjectured to hold on the quantum level [Beisert, Kristjansen, Staudacher 03]

❑ has nontrivial checks of validity up to

- 2 loops on the string side [.........................]
- 5 loops on the gauge side [.........................]
- some nonperturbative checks [.........................]

egrability in AdS/CFT

$$(PSU(2|2) \times PSU(2|2)) \ltimes \mathbb{R}^3$$



If integrability holds on the quantum level, let us apply bootstrap approach

[Staudacher'04]

Algebraic part of 2-particle S-matrix is fixed using $(PSU(2|2) \times PSU(2|2)) \ltimes \mathbb{R}^3$

[Beisert'04]

$$\widehat{S} = (S_{PSU(2|2)})_L \otimes (S_{PSU(2|2)})_R \times \sigma^2$$

Can then apply Bethe Ansatz technis.

he Ansatz in AdS/CFT (Beisert-Staudacher Bethe Ansatz)

[Beisert, Staudacher, 03]
[Beisert, 03-04]
[Arutyunov, Frolov, Zamaklar, 06]

- The symmetry fixes the form of the Bethe equations up to a scalar factor (dressing factor):

$$\left(\frac{x^+_{4,k}}{x^-_{4,k}}\right)^L = \prod_{j\neq k} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\sigma[u_{4,k},u_{4,j}]^2$$
$$\prod_{j=1}^{K_3}\frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}}\prod_{j=1}^{K_5}\frac{x^-_{4,k}-x_{5,j}}{x^+_{4,k}-x_{5,j}}\prod_{j=1}^{K_1}\frac{1-\frac{1}{x^-_{4,k}x_{1,j}}}{1-\frac{1}{x^+_{4,k}x_{1,j}}}\prod_{j=1}^{K_7}\frac{1-\frac{1}{x^-_{4,k}x_{7,j}}}{1-\frac{1}{x^+_{4,k}x_{7,j}}}$$

me history...

olution up to the dressing factor	[Beisert, Staudacher, 03] [Beisert, 03-04]
ressing factor is not trivial	[Arutyunov, Frolov, Staudacher, 04] [Hernandez, Lopez, 06]
he dressing factor is constrained by the rossing equations	[Janik, 06]
symptotic strong coupling solution for crossing .	[Beisert,Hernandez, Lopez 06]
xact expression (BES/BHL proposal)	[Beisert,Eden, Staudacher 06]
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$$\prod_{j=1}^{K_3}\frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}}\prod_{j=1}^{K_5}\frac{x^-_{4,k}-x_{5,j}}{x^+_{4,k}-x_{5,j}}\prod_{j=1}^{K_1}\frac{1-\frac{1}{x^-_{4,k}x_{1,j}}}{1-\frac{1}{x^+_{4,k}x_{1,j}}}\prod_{j=1}^{K_7}\frac{1-\frac{1}{x^-_{4,k}x_{7,j}}}{1-\frac{1}{x^+_{4,k}x_{7,j}}}$$

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Dispersion relation $E = \sqrt{1 - 16g^2 \sin^2 \frac{p}{2}}$

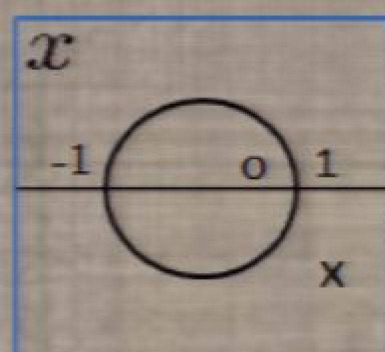
Zhukovsky parametrization $E = ig\left(x^- - \frac{1}{x^-} - x^+ + \frac{1}{x^+}\right)$

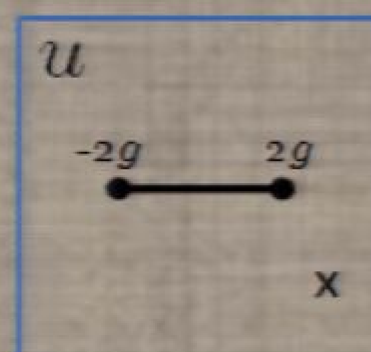
$$ip = \log \frac{x^+}{x^-}$$

$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}$$

$$\frac{u}{g} = x + \frac{1}{x}$$

$$x^\pm = x[u \pm i/2]$$





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he dressing factor is constrained by the rossing equations — [Janik, 06]

symptotic strong coupling solution for crossing. — [Beisert,Hernandez, Lopez 06]

xact expression (BES/BHL proposal) — [Beisert,Eden, Staudacher 06]

Iseful Integral representations — [Kostov, Serban, D.V. 07] [Dorey, Hofman, Maldacena, 07]

..... getting experience ......

heck that BES/BHL satisfy crossing — [Arutyunov, Frolov, 09]
irect solution of crossing equations — [D.V. 09]

he Ansatz in AdS/CFT (Beisert-Staudacher Bethe Ansatz)

[Beisert, Staudacher, 03]
[Beisert, 03-04]
[Arutyunov, Frolov, Zamaklar, 06]

• The symmetry fixes the form of the Bethe equations up to a scalar factor (dressing factor):

$$\left(\frac{x^+_{4,k}}{x^-_{4,k}}\right)^L = \prod_{j\neq k} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\,\sigma[u_{4,k},u_{4,j}]^2$$

$$\prod_{j=1}^{K_3}\frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}} \prod_{j=1}^{K_5}\frac{x^-_{4,k}-x_{5,j}}{x^+_{4,k}-x_{5,j}} \prod_{j=1}^{K_1}\frac{1-\frac{1}{x^-_{4,k}x_{1,j}}}{1-\frac{1}{x^+_{4,k}x_{1,j}}} \prod_{j=1}^{K_7}\frac{1-\frac{1}{x^-_{4,k}x_{7,j}}}{1-\frac{1}{x^+_{4,k}x_{7,j}}}$$

Dispersion relation $E = \sqrt{1 - 16g^2 \sin^2 \frac{p}{2}}$

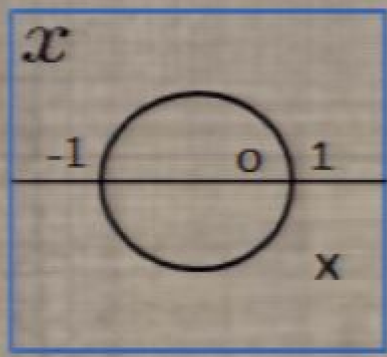
Zhukovsky parametrization $E = ig\left(x^- - \frac{1}{x^-} - x^+ + \frac{1}{x^+}\right)$

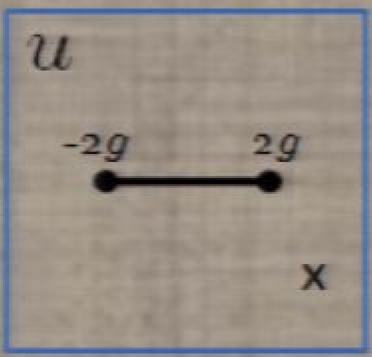
$$ip = \log \frac{x^+}{x^-}$$

$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}$$

$$\frac{u}{g} = x + \frac{1}{x}$$

$$x^\pm = x[u \pm i/2]$$





›ssing equations

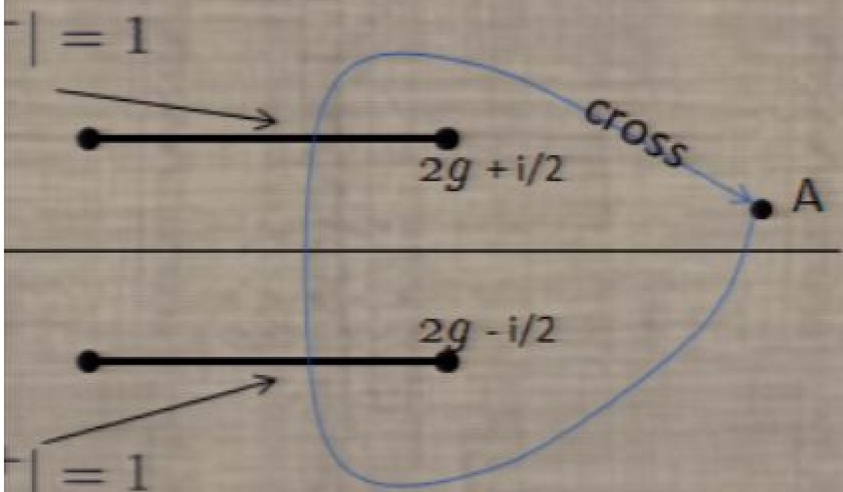
Relativistic case: $$S[\theta + \tfrac{i}{2}]S[\theta - \tfrac{i}{2}] = \frac{\theta - \frac{i}{2}}{\theta + \frac{i}{2}}$$

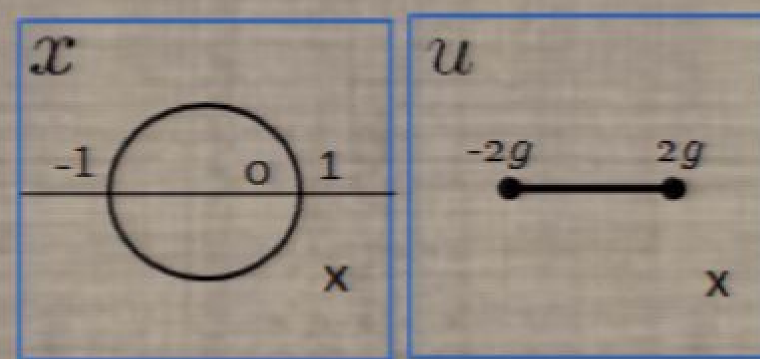
Shift by i changes sign of E and p

$$p = m\cosh[\pi\theta]$$
$$\varepsilon = m\cosh[\pi\theta]$$

ossing equations

AdS/CFT case: $x^{\pm} \to \frac{1}{x^{\pm}}$





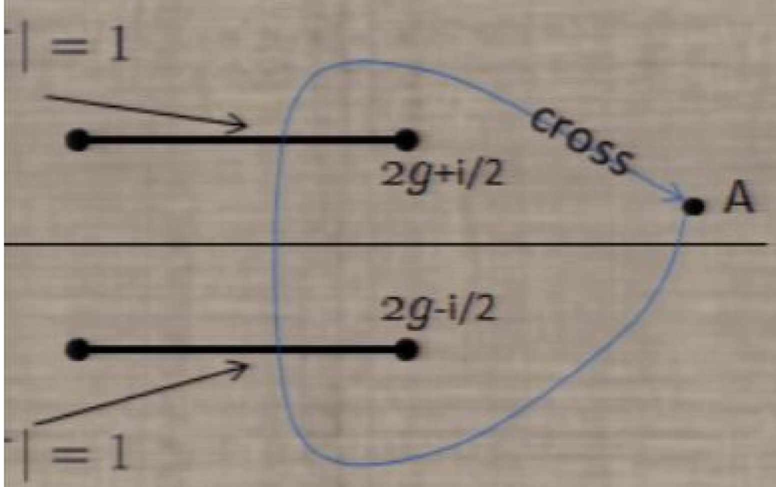
$$x^{\pm} = x[u \pm i/2]$$

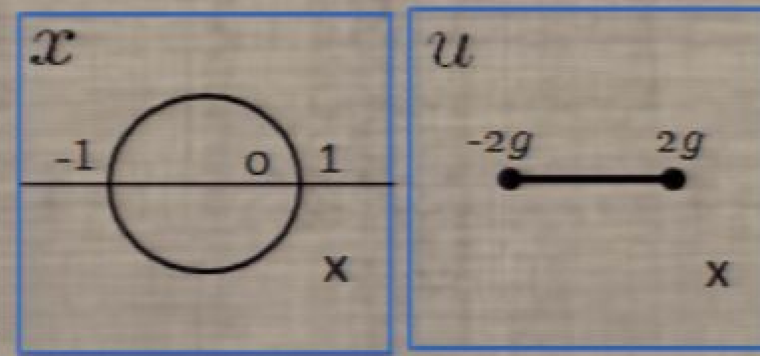
$$y^{\pm} = y[v \pm i/2]$$

[Janik, 06]

$$\sigma[u,v]\sigma_{cross}[u,v] = \frac{y^-}{y^+}\frac{x^- - y^+}{x^+ - y^+}\frac{1-\frac{1}{x^- y^-}}{1-\frac{1}{x^+ y^-}},$$

lution of crossing equations





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[Arutyunov, Frolov, 09]
[D.V. 09]
[Vieira,D.V.,10]

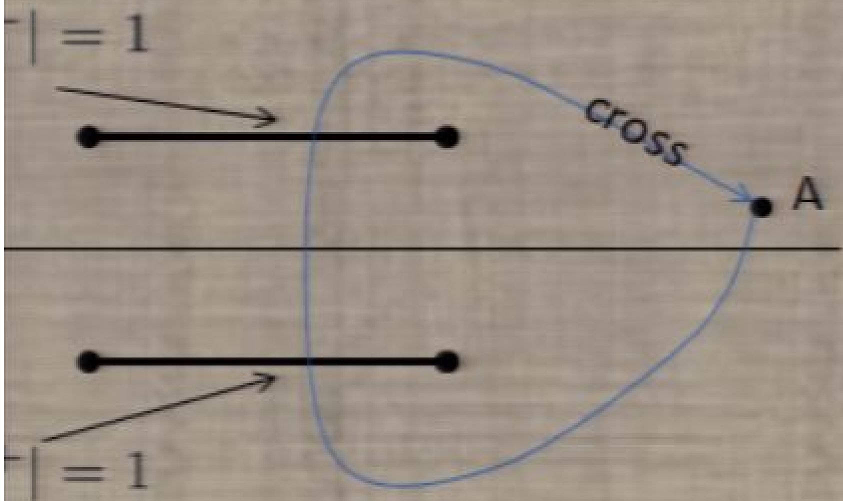
sumptions on the structure of the dressing factor:

ecomposition in terms of χ: $$\sigma[u,v] = e^{i\theta[u,v]}$$

$$\theta[u,v] = \chi[x^+,y^-] - \chi[x^-,y^-] - \chi[x^+,y^+] + \chi[x^-,y^+]$$
$$\chi[x,y] = -\chi[y,x]$$

is analytic for |x|>1
nly DHM poles for |x|<1
ll branch points for |x|<1 that are explicitly required by crossing.
→const x→∞

lution of crossing equations



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an we make equation more symmetrical?

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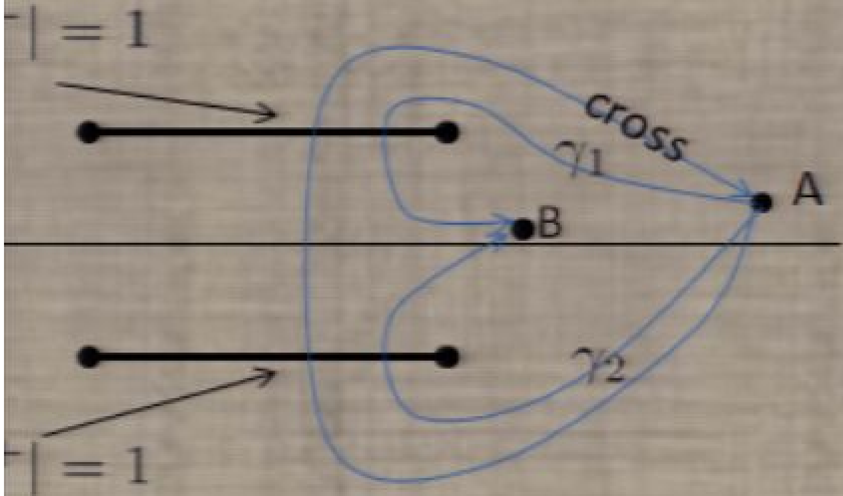

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es: analytically continue the equation through the contour γ_1

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$$\sigma_{\gamma_1}[u,v]\sigma_{\gamma_2}[u,v] = \frac{1-\frac{1}{x^+ y^+}}{1-\frac{1}{x^- y^-}}\frac{1-\frac{1}{x^- y^+}}{1-\frac{1}{x^+ y^-}}$$

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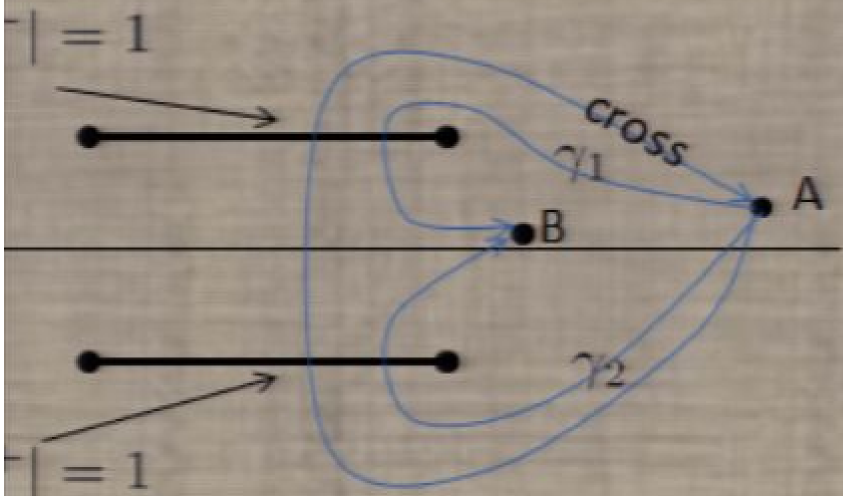

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$$\sigma[u,v] = \frac{\sigma_1[x^+,v]}{\sigma_1[x^-,v]}$$

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$$(\ \sigma_1[x,v]\sigma_1[1/x,v]\ )^{D-D^{-1}} = \left(\frac{x-\frac{1}{y^+}}{x-\frac{1}{y^-}}\right)^{D+D^{-1}}$$

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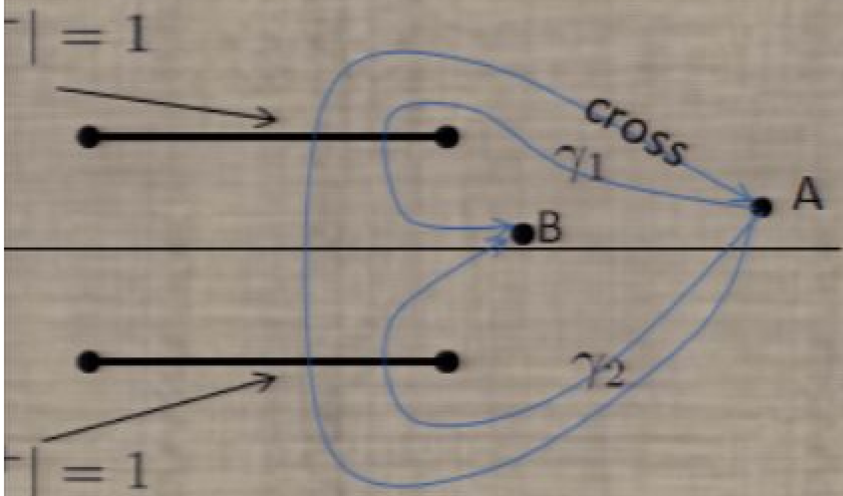

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$$S[\theta]S[\theta+i] = \frac{\theta}{\theta+i}$$

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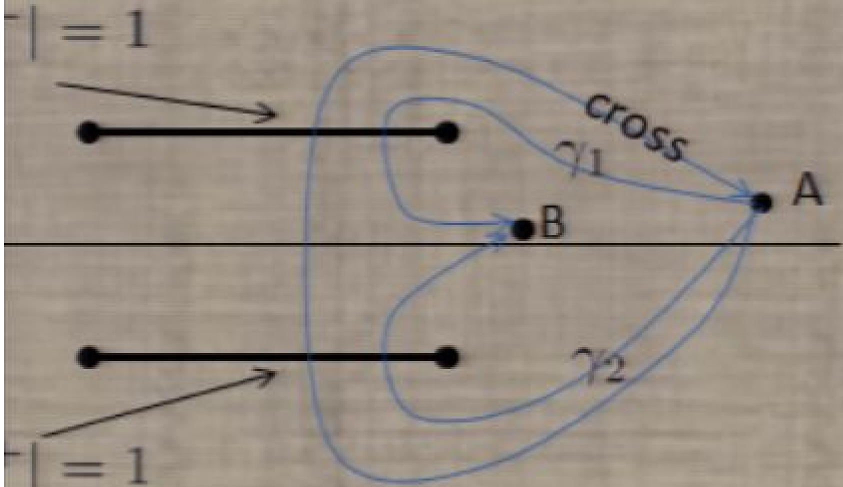

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$$\theta[u,v] = \chi[x^+,y^-] - \chi[x^-,y^-] - \chi[x^+,y^+] + \chi[x^-,y^+]$$

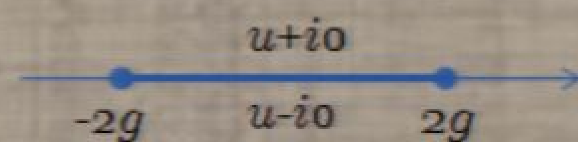
$$\left(\, \sigma_1[x,v]\sigma_1[1/x,v] \,\right)^{D-D^{-1}} = \left(\frac{x-\frac{1}{y^+}}{x-\frac{1}{y^-}}\right)^{D+D^{-1}}$$

$$\sigma[u,v] = (u-v)^{(D-D^{-1})\tilde{K}\left(\frac{D^2}{1-D^2}-\frac{D^{-2}}{1-D^{-2}}\right)\tilde{K}(D-D^{-1})}$$

$$(\tilde{K}\cdot F)[u] = \int_{-2g+i0}^{2g+i0} \frac{dw}{2\pi i}\frac{\sqrt{u^2-4g^2}}{\sqrt{w^2-4g^2}}\frac{1}{w-u}F[w]$$

Kernel creates Jukowsky cut. The main property of the Kernel:

$$(\tilde{K}\cdot F)[u+i0] + (\tilde{K}\cdot F)[u-i0] = F[u], \;\; u^2 < 4g^2.$$



alytical structure of the dressing factor

$$\sigma[u,v] = (u-v)^{(D-D^{-1})\tilde{K}\left(\frac{D^2}{1-D^2}-\frac{D^{-2}}{1-D^{-2}}\right)\tilde{K}(D-D^{-1})}$$

$$i\chi[u,v] = \tilde{K}\left(\frac{D^2}{1-D^2}-\frac{D^{-2}}{1-D^{-2}}\right)\tilde{K}\log[u-v]$$

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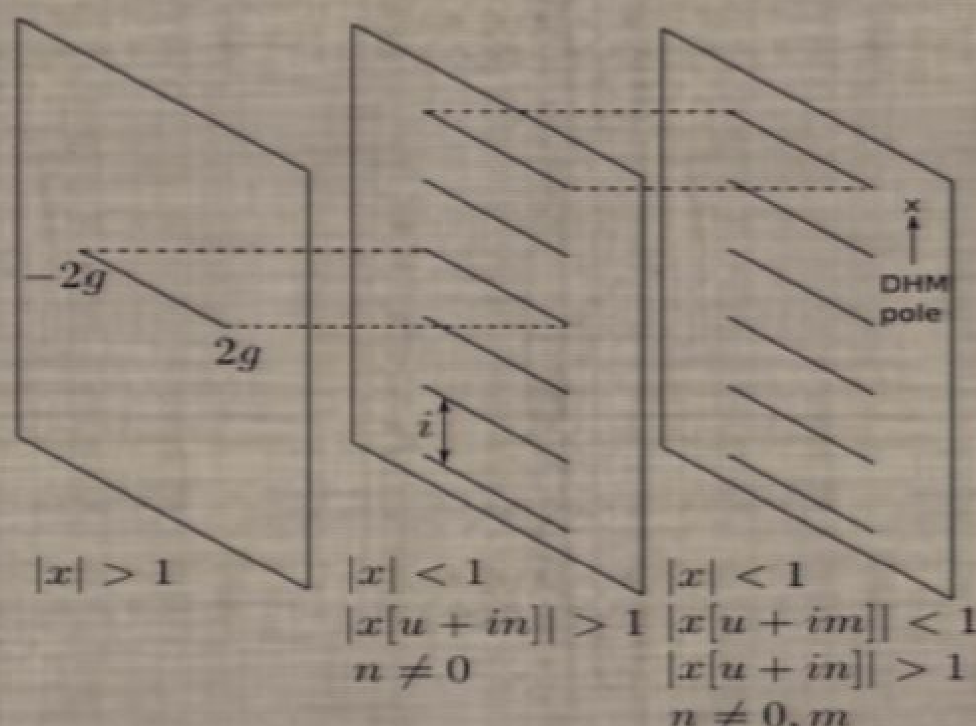

nplified form of Bethe Ansatz equations

$$\left(\frac{\;\,_{,k}}{\;\,_{,k}}\right)^L = \prod_{j\neq k} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\sigma[u_{4,k},u_{4,j}]^2$$

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can write these equations in a more suggestive form using the properties:

$$\prod \frac{x^- - y_k}{x^+ - y_k} = \prod_k (u-v_k)^{(D-D^{-1})(1-\tilde{K})} \qquad \prod_k \frac{1-\frac{1}{x^- y_k}}{1-\frac{1}{x^+ y_k}} = \prod_k (u-v_k)^{(D-D^{-1})\tilde{K}}$$

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$$S_0[\theta] = \theta^{-\frac{D^2}{1+D^2}+\frac{D^{-2}}{1+D^{-2}}}$$

e Bethe equations in the Beisert-Staudacher Bethe Ansatz
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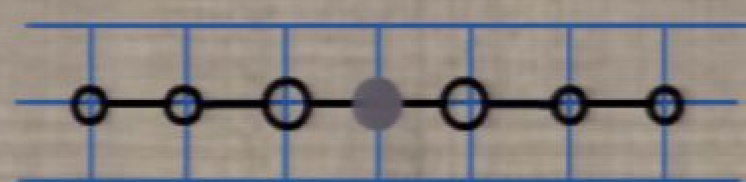
Part III

Thermodynamic Bethe Ansatz (TBA) for $SU(N) \times SU(N)$ PCF



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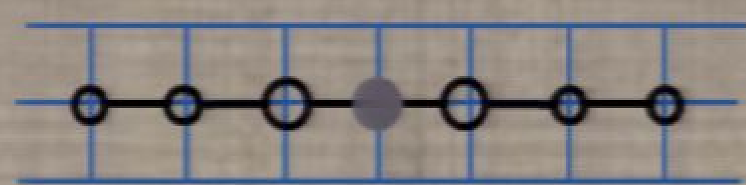
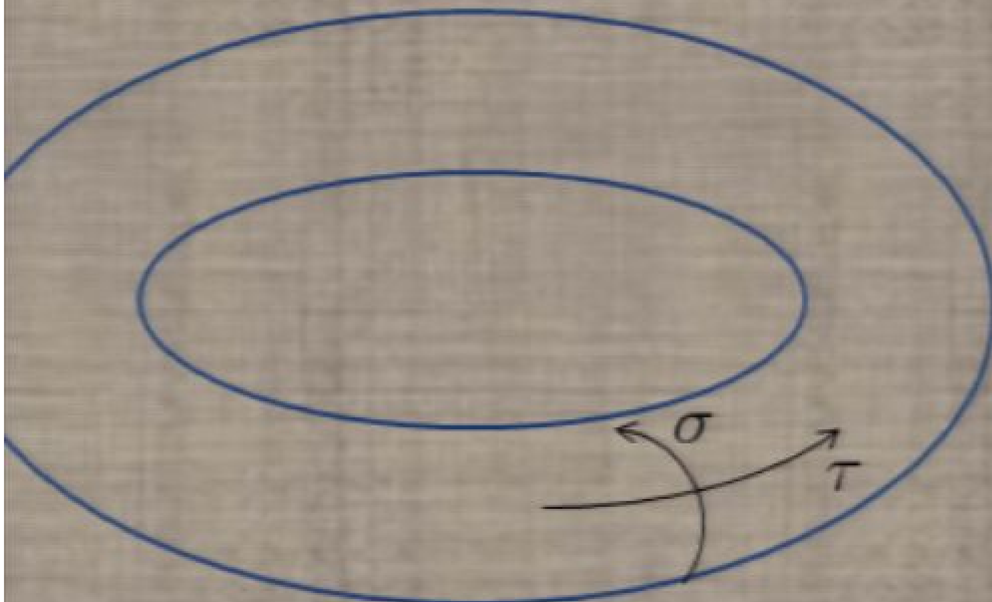


sic idea of TBA



$$\sigma \simeq \sigma + L$$
$$\tau \simeq \tau + R$$

$$Z[R, L] = Tre^{-R\ H[L]} = Tre^{-L\ H[R]}$$

$$E_0[L] = -\lim_{R\to\infty} \frac{\log Z}{R} = \mathcal{F}[T = 1/L]$$

Part III

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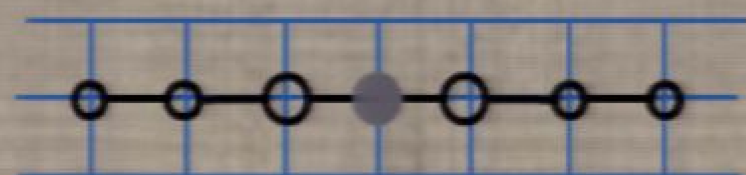

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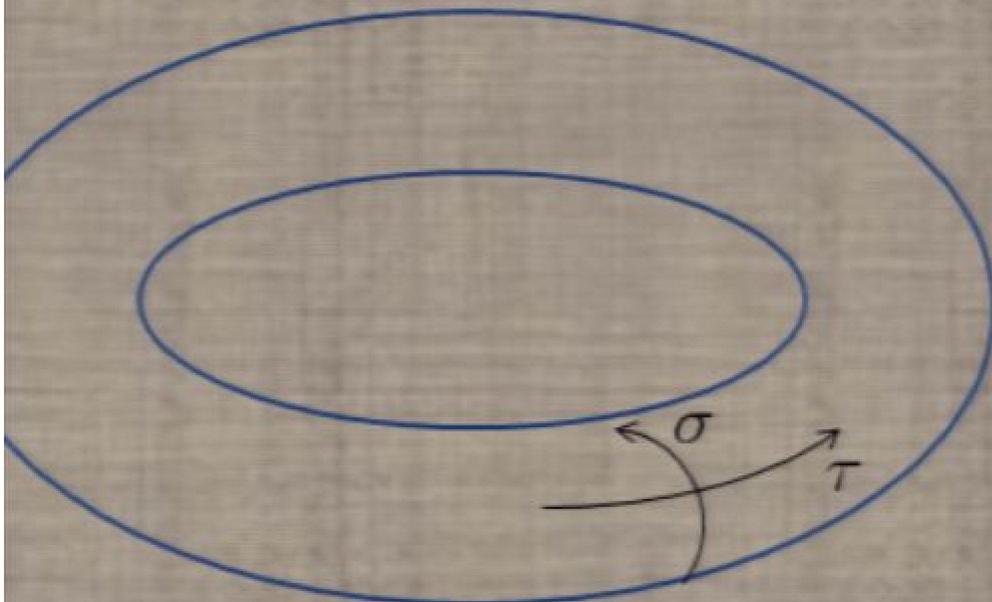

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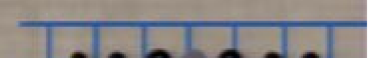

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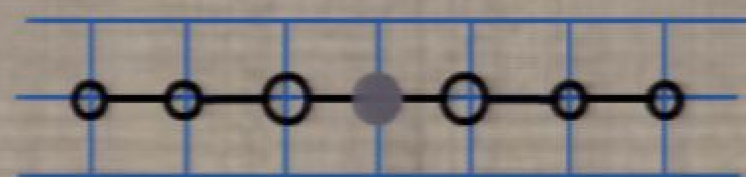

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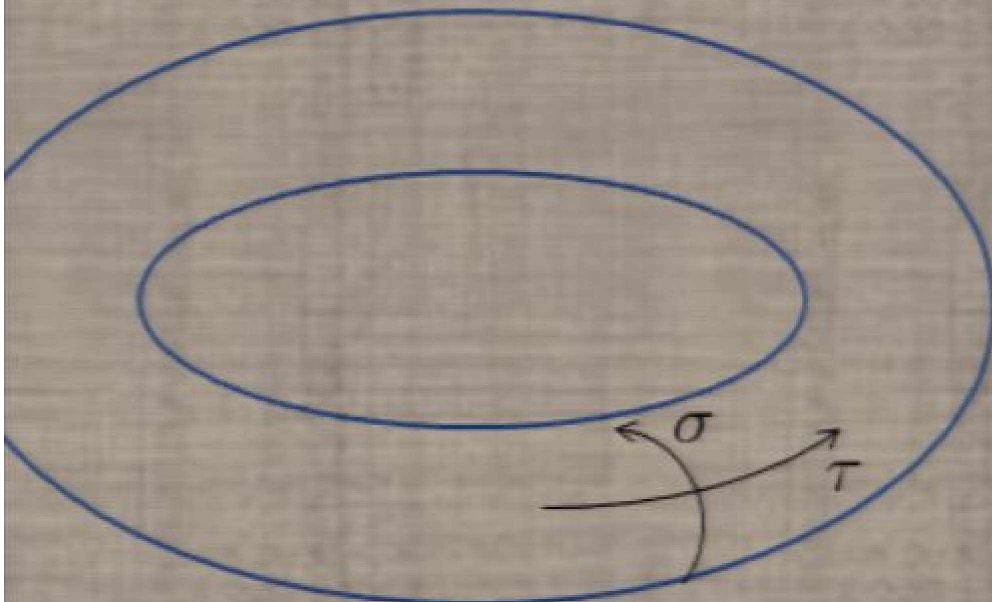

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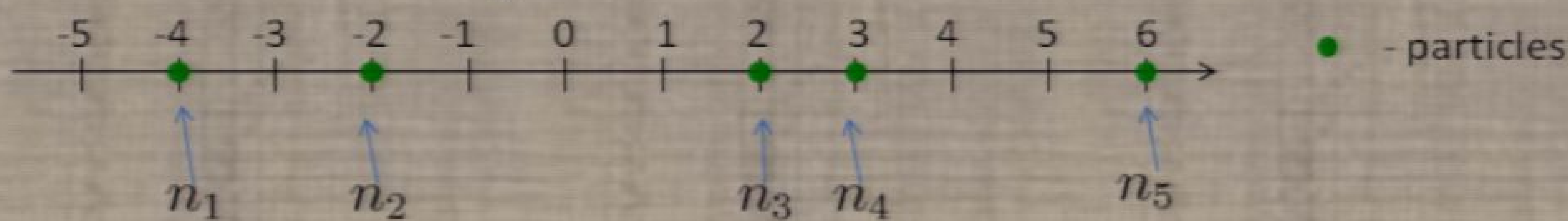

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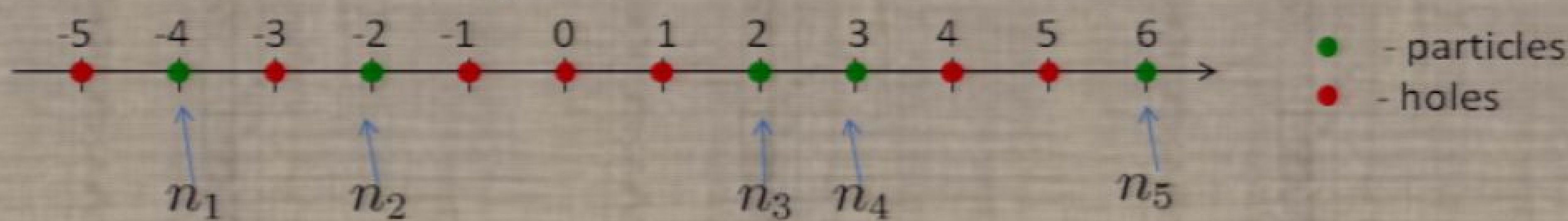

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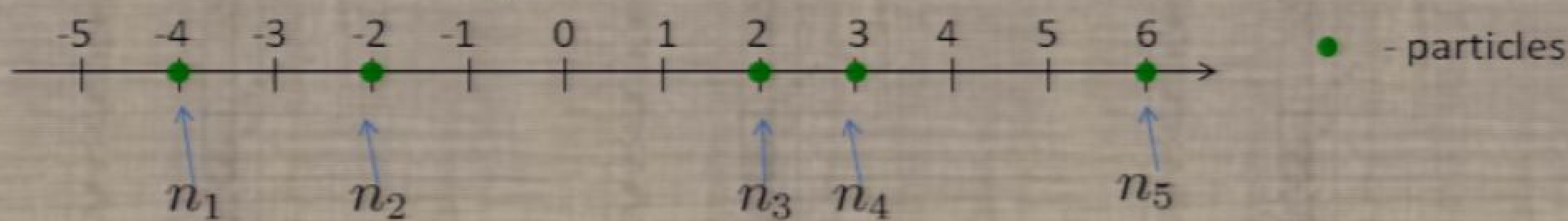

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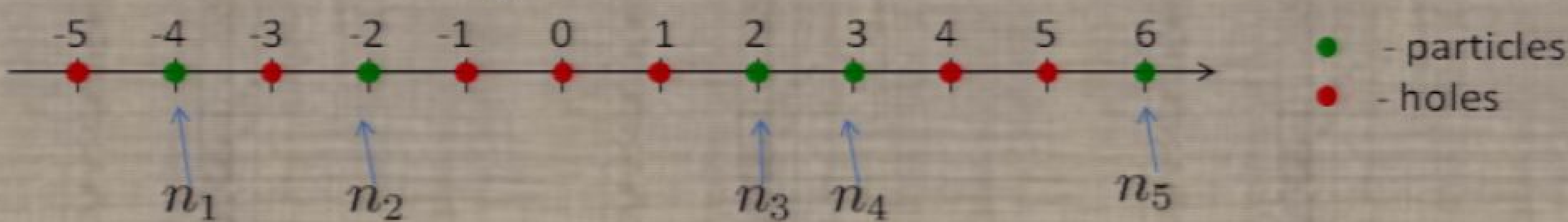

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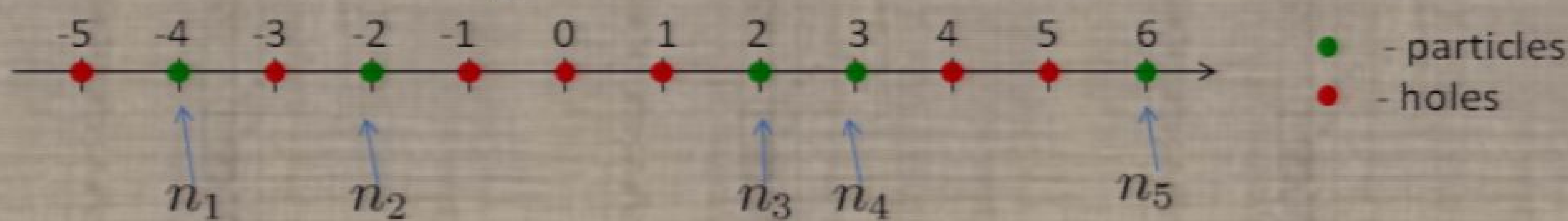

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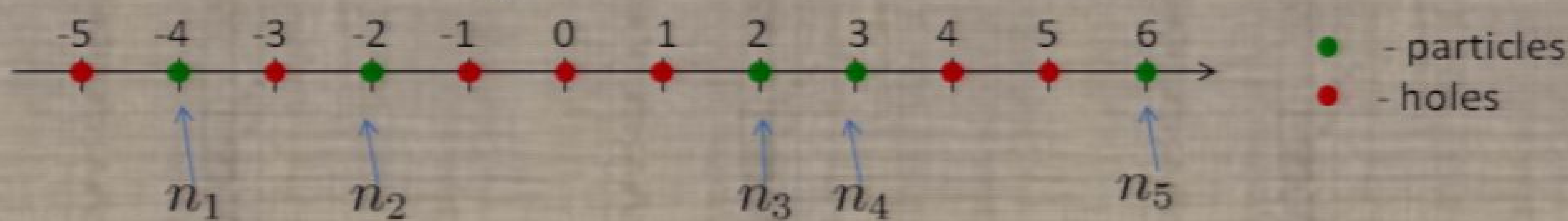

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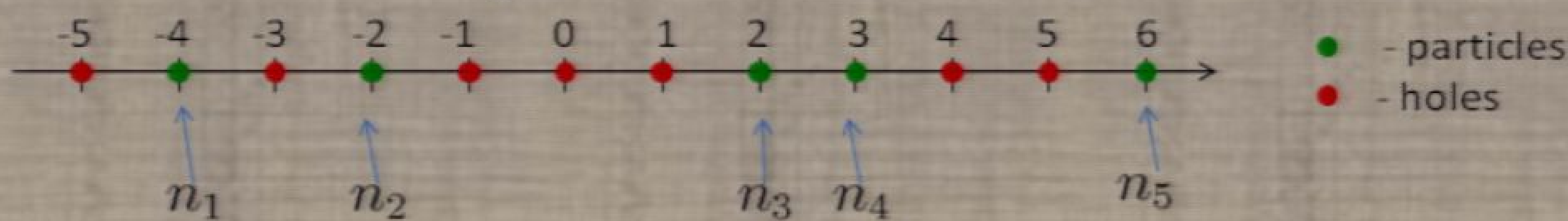

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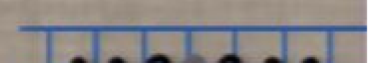

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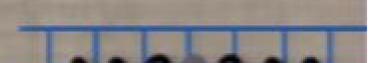

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Where did we see such formulas?

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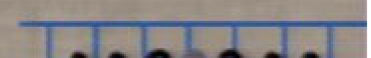

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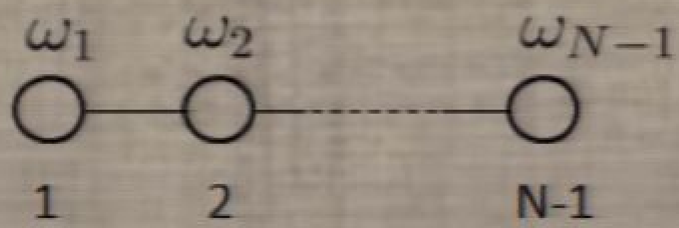
neral situation: SU(N) XXX spin chain



$$\left(\frac{u_{a,j}+\frac{i\omega_a}{2}}{u_{a,j}-\frac{i\omega_a}{2}}\right)^L = -\prod_{b=1}^{N-1}\prod_{j=1}^{K_b}\frac{u_{a,j}-u_{b,k}+i\frac{c_{ab}}{2}}{u_{a,j}-u_{b,k}-i\frac{c_{ab}}{2}}, \qquad a=\overline{1,N-1}$$

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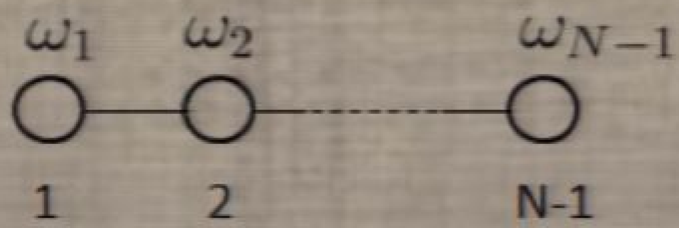

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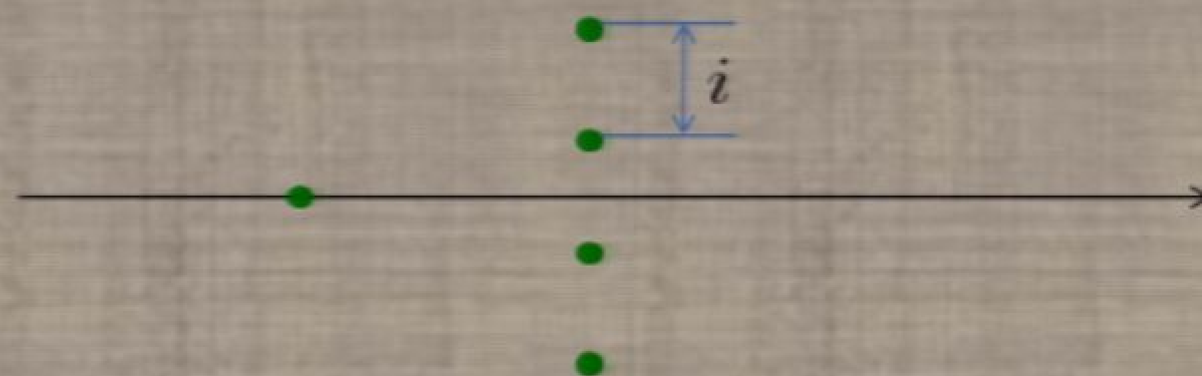

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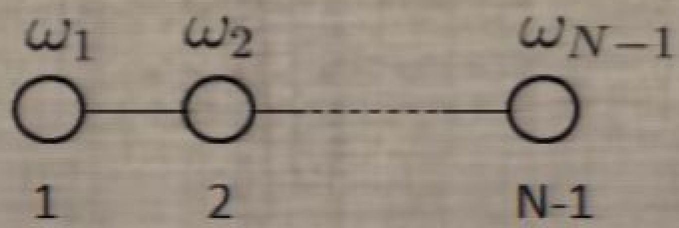

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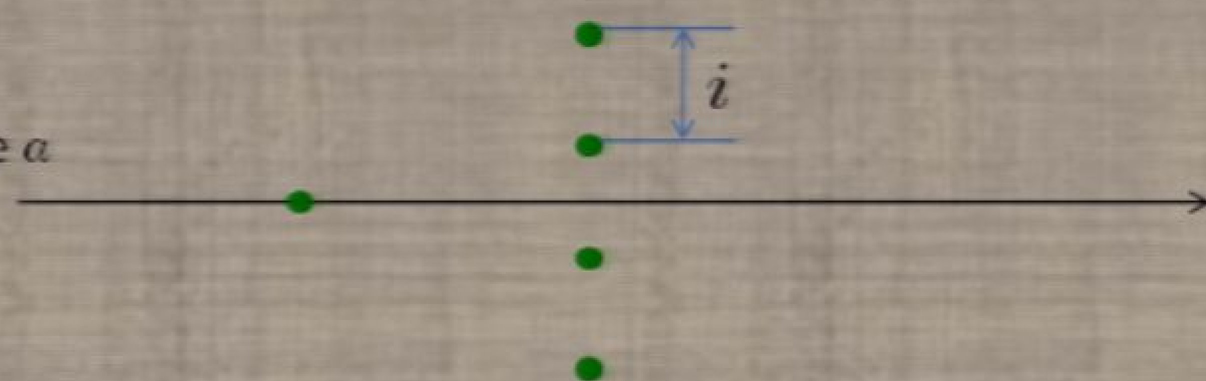
$_{,s}$ - density of strings of length s formed from Bethe roots of type a



$_{a,s}$ -- corresponding resolvent

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egral equations can be rewritten as:

$$\sum_{s'=1}^{\infty} C^{\infty}_{s,s'} R^{*}_{a,s'} + \sum_{a'=1}^{N-1} C^{N-1}_{a,a'} R_{a',s} = J_{a,s}$$

$$C_{ss'} = \begin{pmatrix} D+D^{-1} & -1 & 0 & 0 & \dots \\ -1 & D+D^{-1} & -1 & 0 & \dots \\ 0 & -1 & D+D^{-1} & -1 & 0 \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

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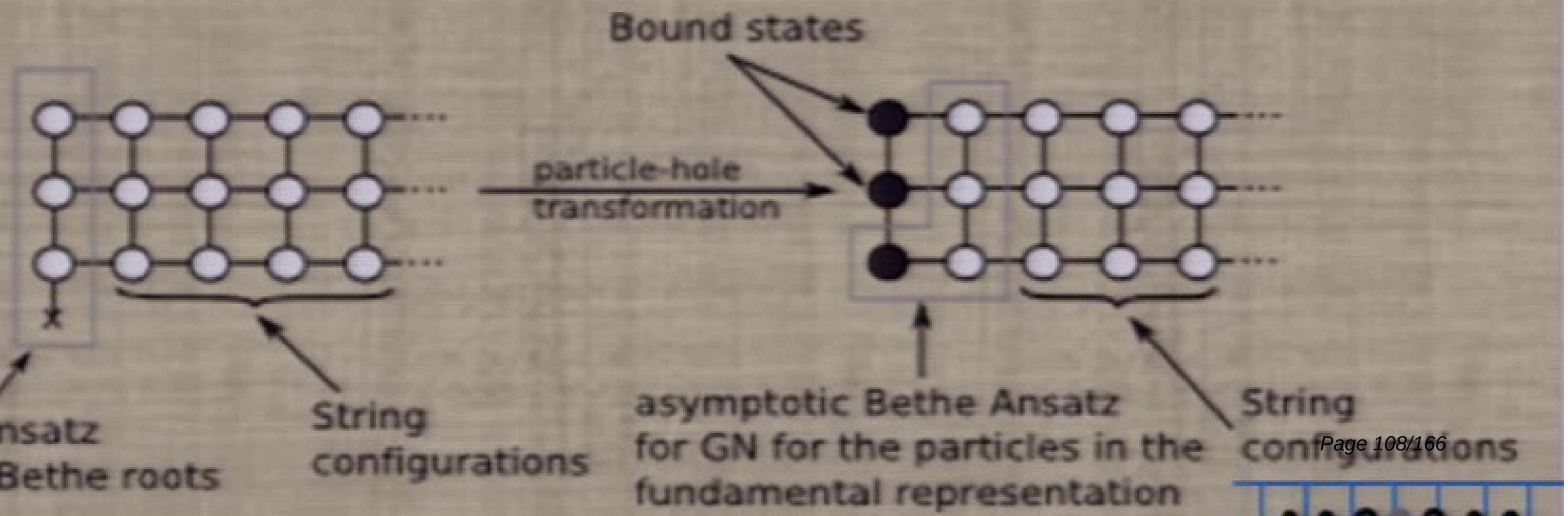
Case of GN model: $J_{a,s} = \delta_{a,1}\delta_{s,1} J$



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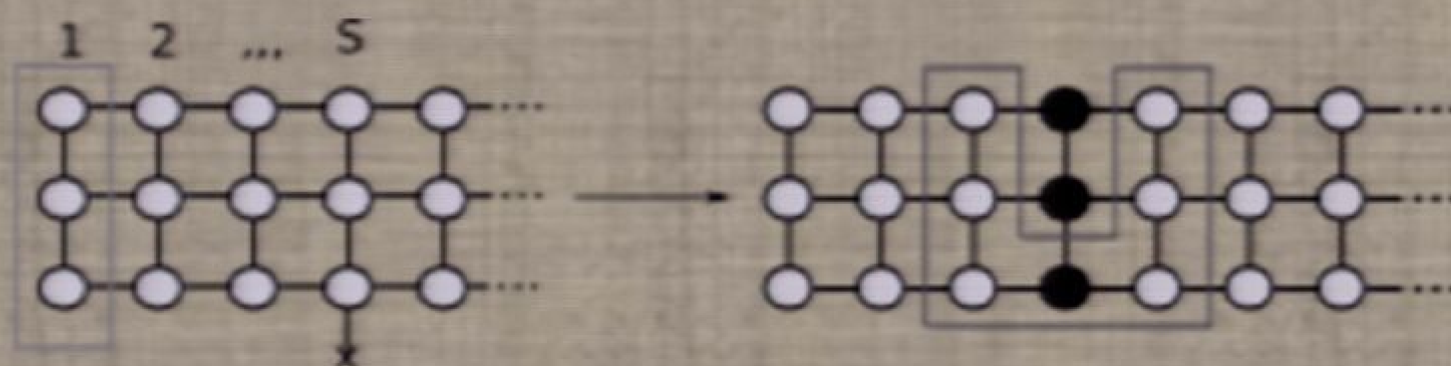

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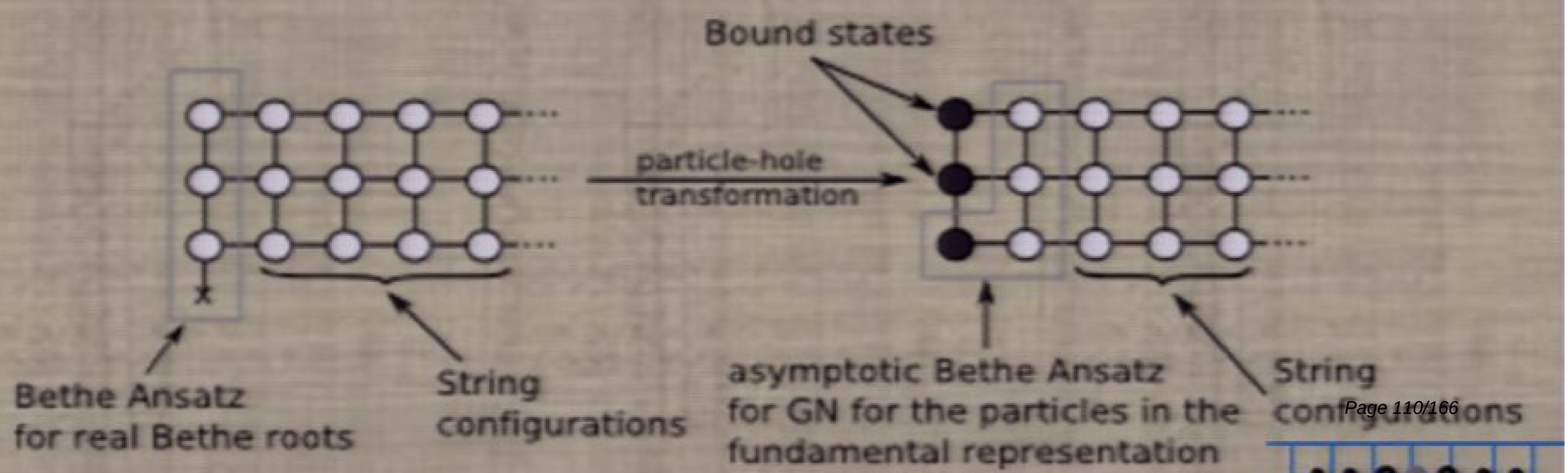

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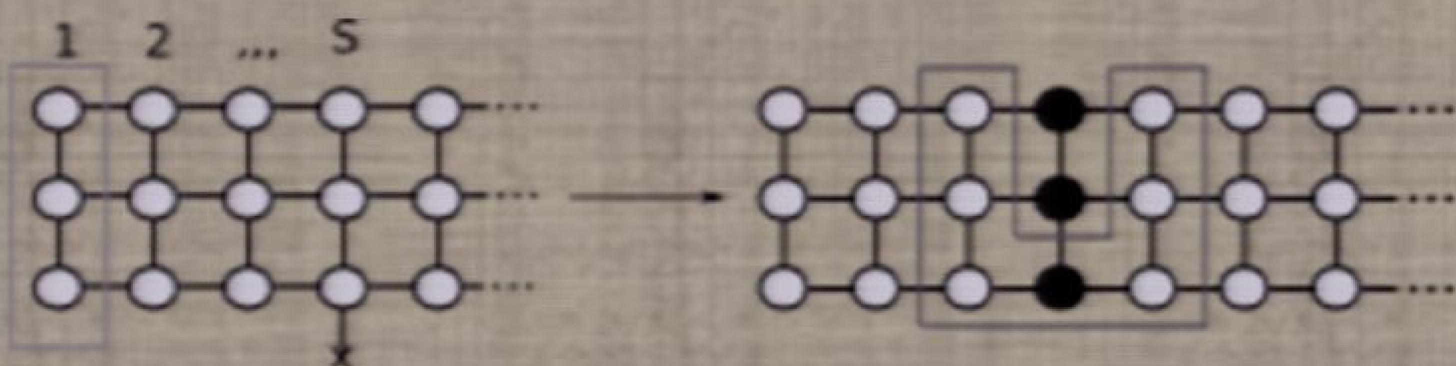

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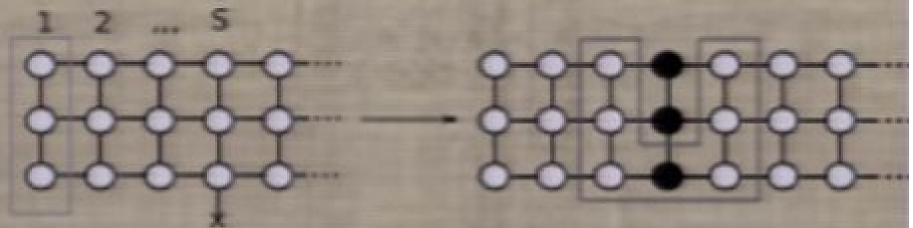

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TBA

$$Y_{a,s} = \frac{\rho^*_{a,s}}{\rho_{a,s}}$$

$$(1 + Y_{a,s'})^{C_{ss'}} = \left(1 + \frac{1}{Y_{a',s}}\right)^{C_{aa'}}$$

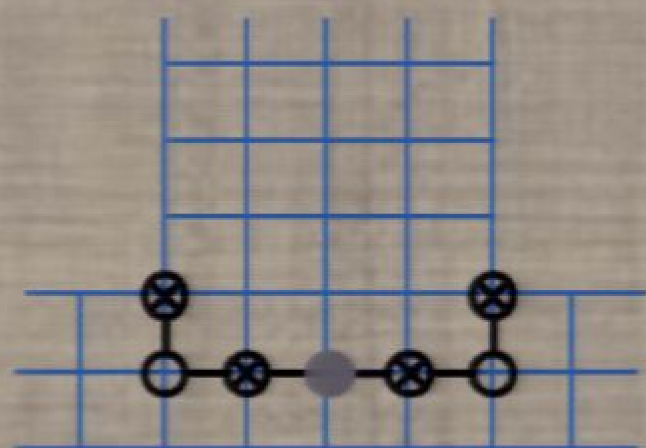
$$E_0[L] =$$

$$= -\sum \int dv J_{a,s}[v] \log [1 + Y_{a,s}]$$

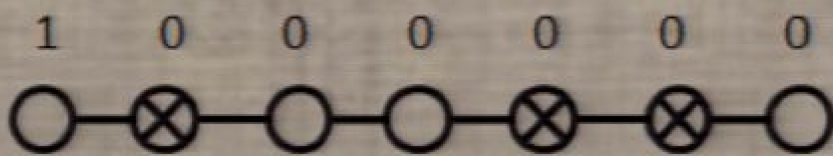
$$\frac{Y^+_{a,s} Y^-_{a,s}}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

Part IV

Thermodynamic Bethe Ansatz (TBA) in spectral problem of AdS/CFT

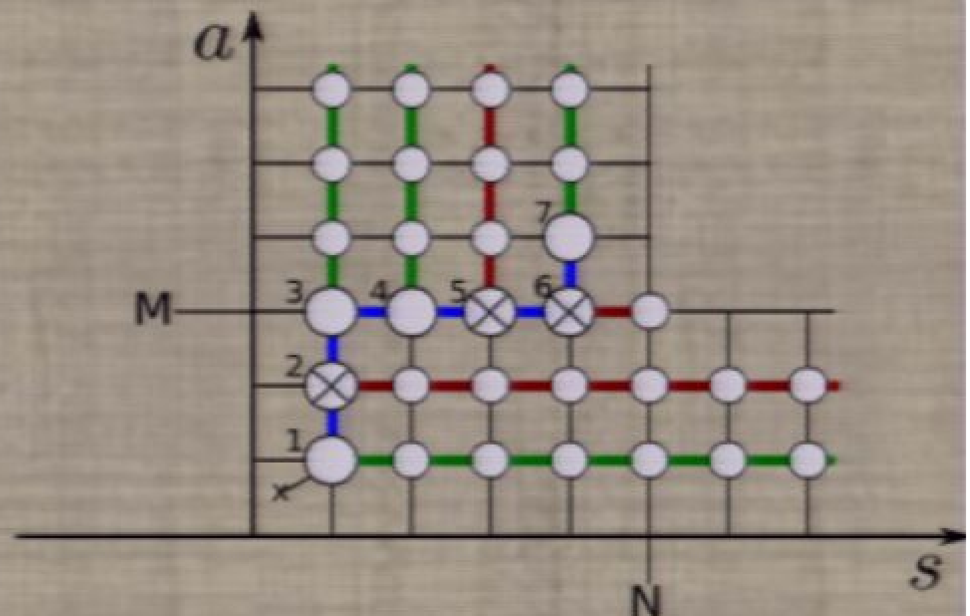


neral situation: rational Gl(N|M) spin chain



[Saleur, 99]
[Gromov, Kazakov, Kozak, Vieira, 09]
[D.V., 09]

$$\left(\frac{\ldots_{,j} + \frac{i\omega_a}{2}}{\ldots_{,j} - \frac{i\omega_a}{2}}\right)^L = (-)^{\cdots} \prod_{b=1}^{N-1} \prod_{j} \frac{u_{a,j} - u_{b,k} + i\frac{c_{ab}}{2}}{u_{a,j} - u_{b,k} - i\frac{c_{ab}}{2}}$$



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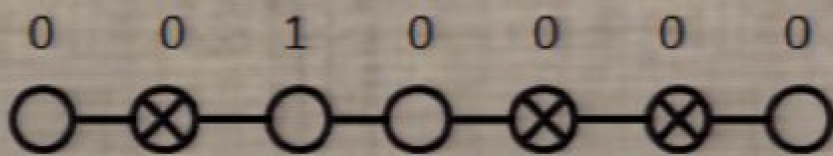

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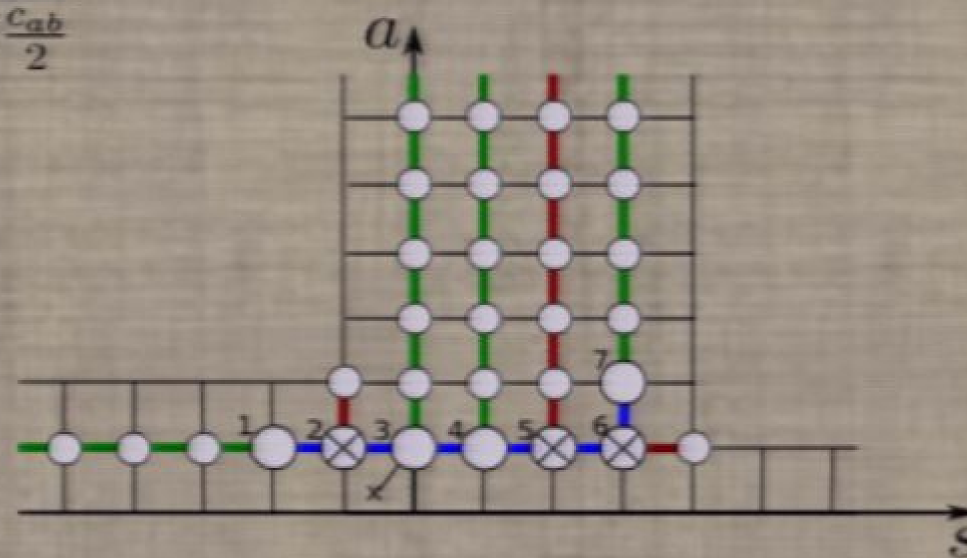

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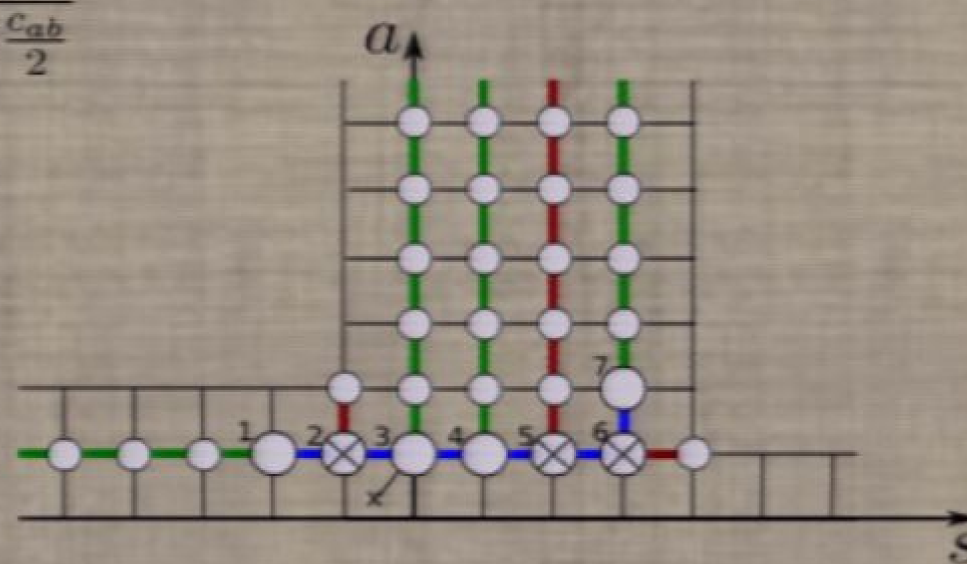

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TBA

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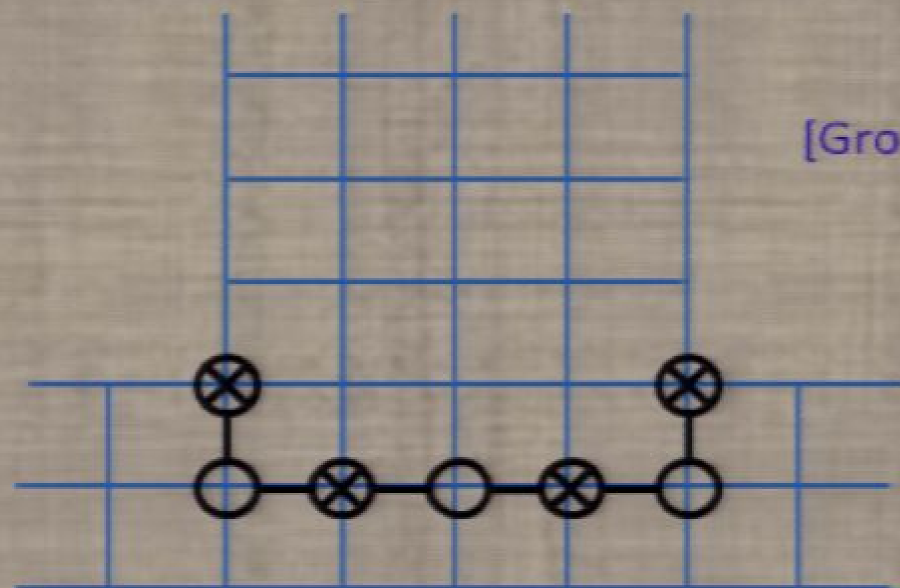
S/CFT case



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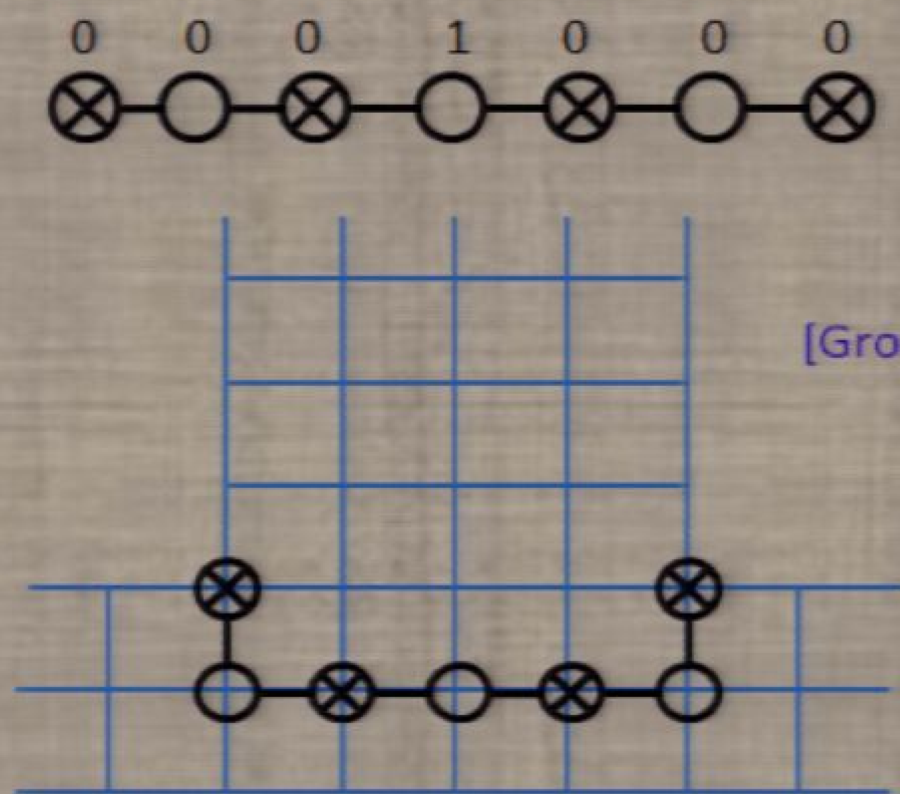

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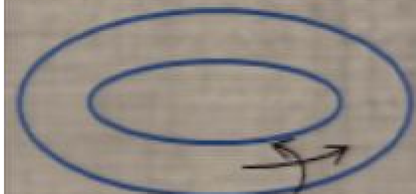

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Dispersion relation is reversed

$$p_{mirr} = -iH$$
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Dispersion relation in terms of x is the same : $x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}$

But different branches of x^+ and x^- are chosen:

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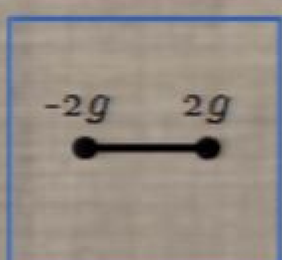

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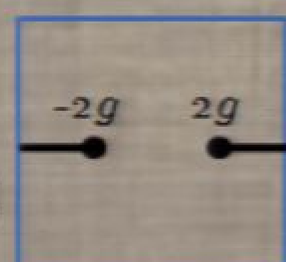
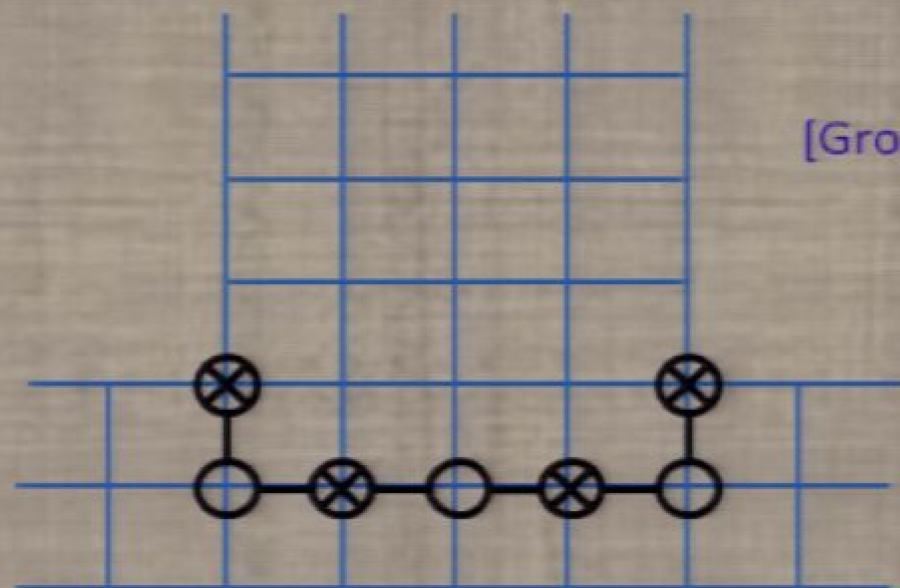


S/CFT case



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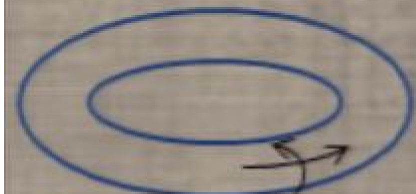

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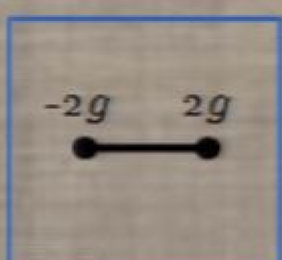

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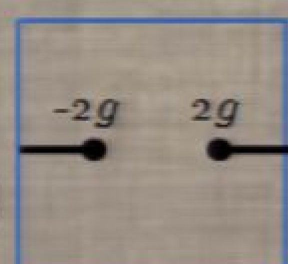

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$$F = \int_{-2g+i0}^{2g+i0} \frac{dv}{2\pi i} \frac{\sqrt{u^2-4g^2}}{\sqrt{v^2-4g^2}} \frac{1}{v-u} F[v]. \qquad \widetilde{K}_m\, F = \int_{\mathbb{R}+i0\setminus[-2g,2g]} \frac{dv}{2\pi i} \frac{\sqrt{4g^2-u^2}}{\sqrt{4g^2-v^2}} \frac{1}{v-u} F[v].$$

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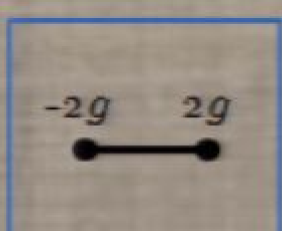

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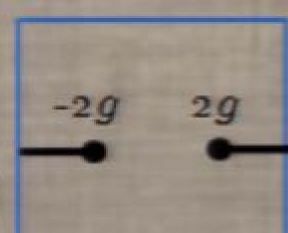

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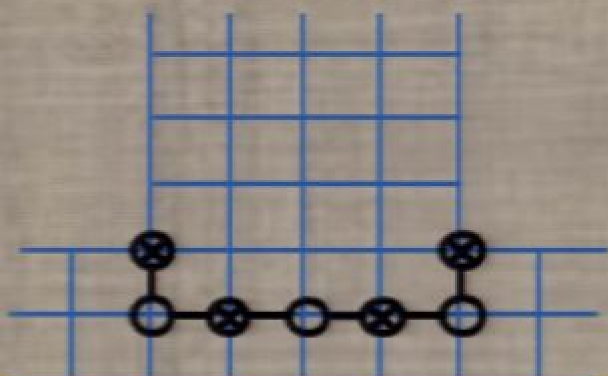

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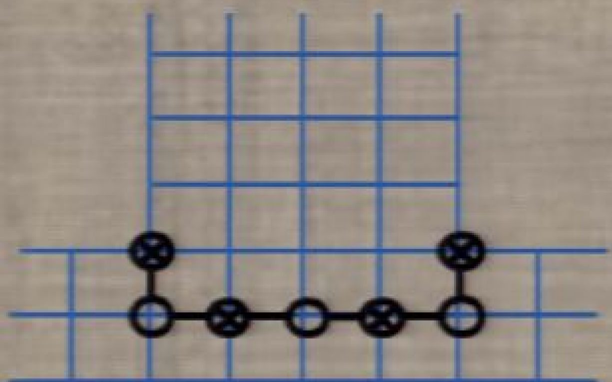

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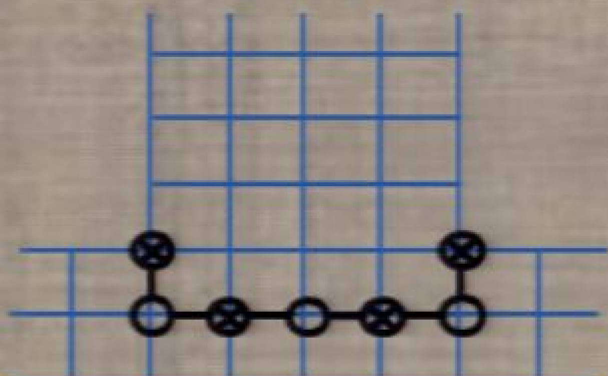

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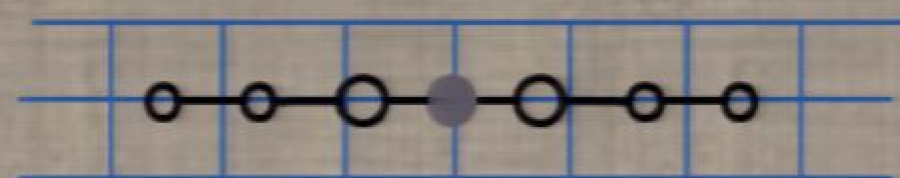

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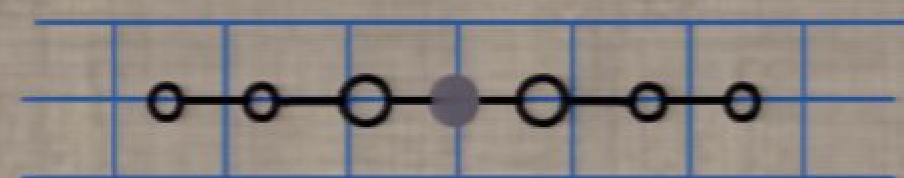

AdS/CFT integrable system is solved similarly to the relativistic case.
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Differences to the relativistic case

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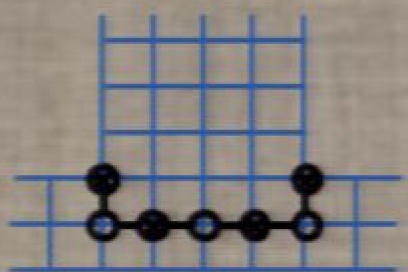
- Possible options:



  - No underlying spin chain, everything as is.
  - Condensation of roots on the hidden level
  - Spin chain exists (Hubbard-like models, noncompact models)

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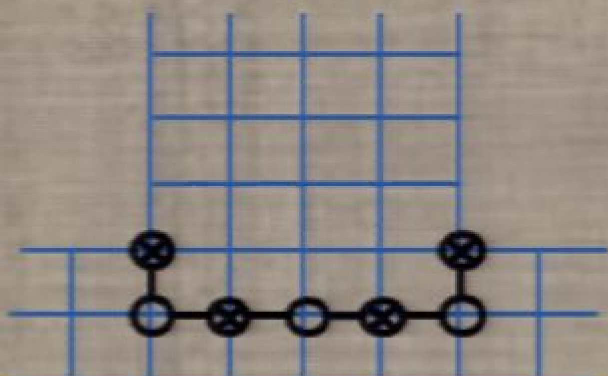

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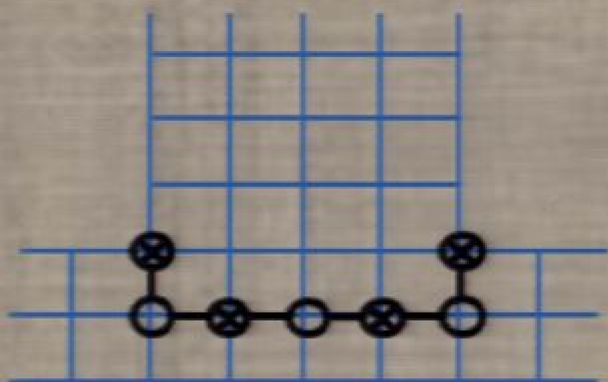

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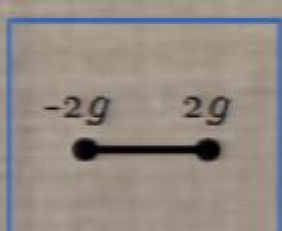

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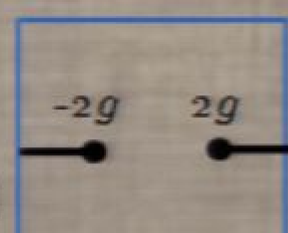

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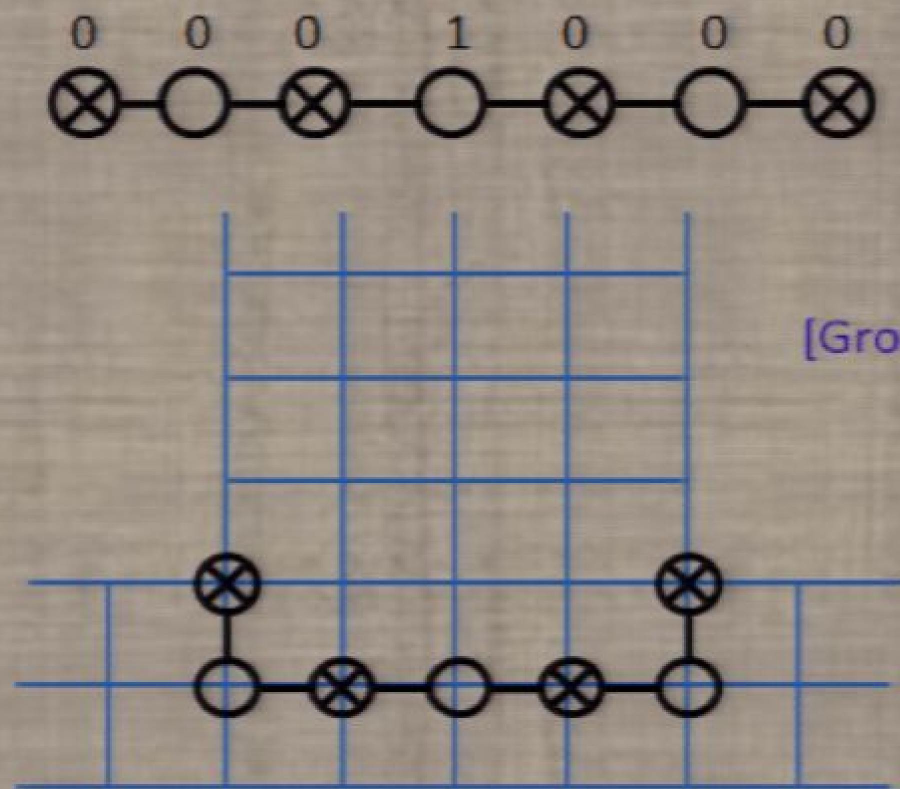

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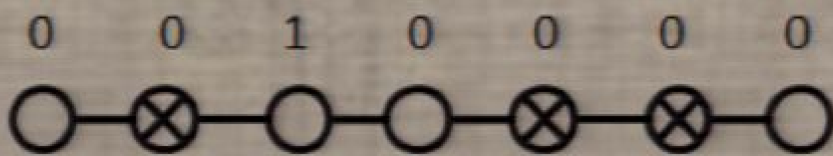
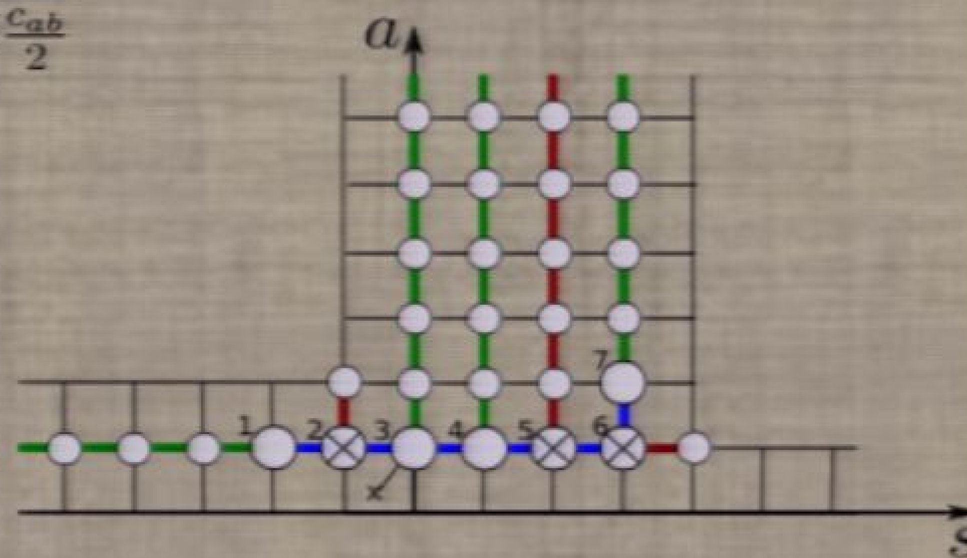


[Saleur, 99]
[Gromov, Kazakov, Kozak, Vieira, 09]
[D.V., 09]

$$\left(\frac{\ldots_{,j}+\frac{i\omega_a}{2}}{\ldots_{,j}-\frac{i\omega_a}{2}}\right)^L = (-)^{\cdots} \prod_{b=1}^{N-1}\prod_{j} \frac{u_{a,j}-u_{b,k}+i\frac{c_{ab}}{2}}{u_{a,j}-u_{b,k}-i\frac{c_{ab}}{2}}$$



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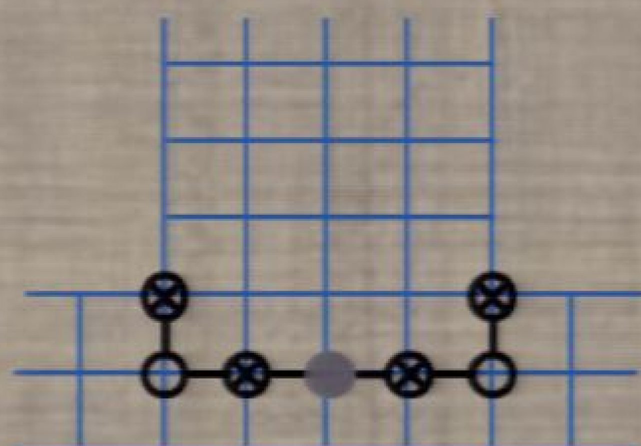
Part IV

Thermodynamic Bethe Ansatz (TBA) in spectral problem of AdS/CFT



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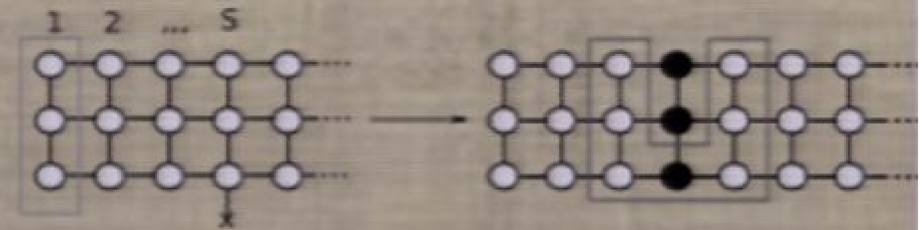

$$E_0[L] = - \lim_{R\to\infty} \frac{\log Z}{R} = \mathcal{F}[T = 1/L]$$

TBA

$$Y_{a,s} = \frac{\rho^{*}_{a,s}}{\rho_{a,s}}$$

$$(1 + Y_{a,s'})^{C_{ss'}} = \left(1 + \frac{1}{Y_{a',s}}\right)^{C_{aa'}}$$

$$E_0[L] = - \sum \int dv J_{a,s}[v] \log [1 + Y_{a,s}]$$

$$\frac{Y^{+}_{a,s} Y^{-}_{a,s}}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

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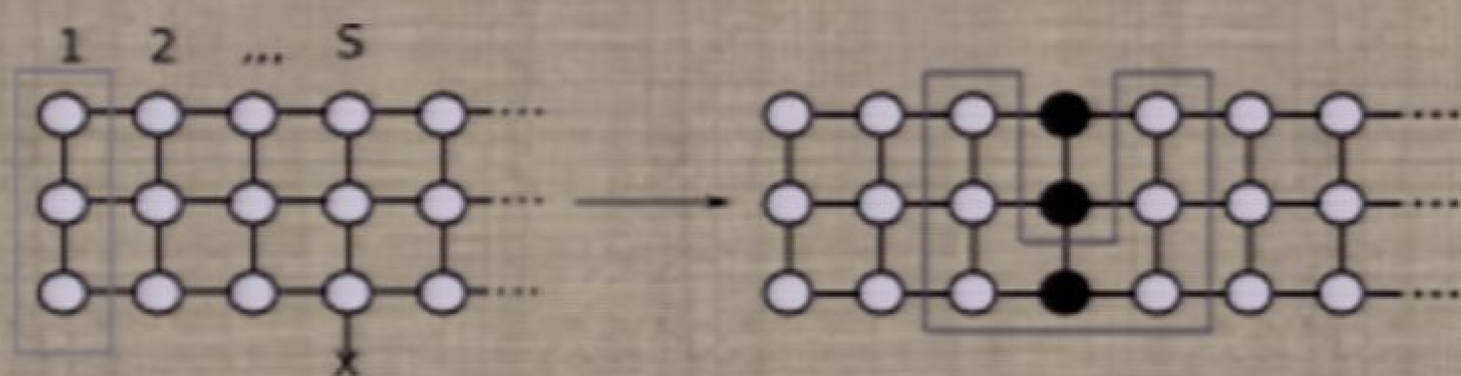
egral equations can be rewritten as:

$$\sum_{s'=1}^{\infty} C^{\infty}_{s,s'} R^{*}_{a,s'} + \sum_{a'=1}^{N-1} C^{N-1}_{a,a'} R_{a',s} = J_{a,s}$$

$$C_{ss'} = \begin{pmatrix} D+D^{-1} & -1 & 0 & 0 & \dots \\ -1 & D+D^{-1} & -1 & 0 & \dots \\ 0 & -1 & D+D^{-1} & -1 & 0 \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

Case of PCF model: $J_{a,s} = \delta_{a,1}\delta_{s,S} J, \quad S \to \infty$



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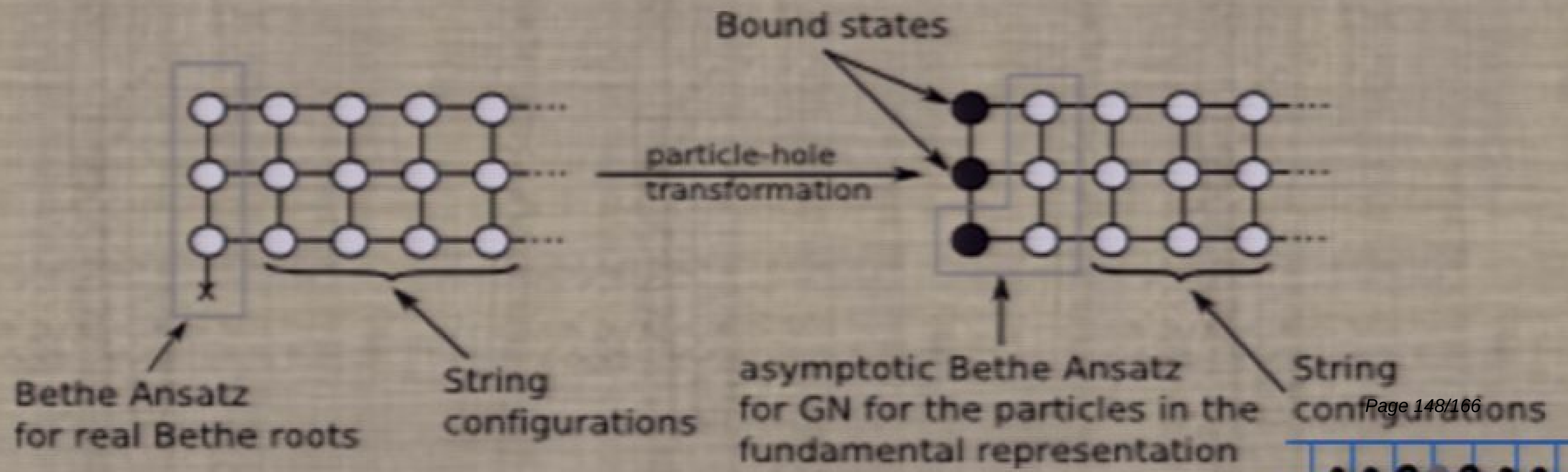

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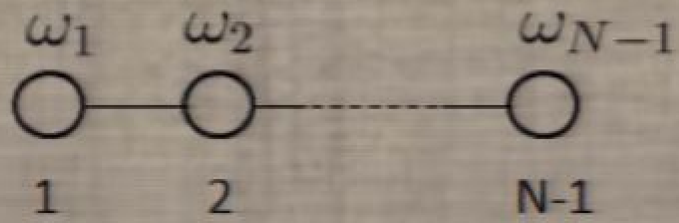

$$\left(\frac{u_{a,j}+\frac{i\omega_a}{2}}{u_{a,j}-\frac{i\omega_a}{2}}\right)^L = -\prod_{b=1}^{N-1}\prod_{j=1}^{K_b}\frac{u_{a,j}-u_{b,k}+i\frac{c_{ab}}{2}}{u_{a,j}-u_{b,k}-i\frac{c_{ab}}{2}}, \qquad a=\overline{1,N-1}$$

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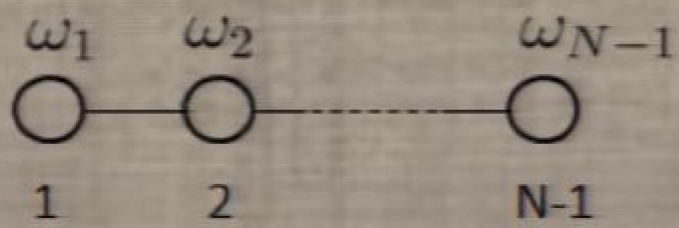

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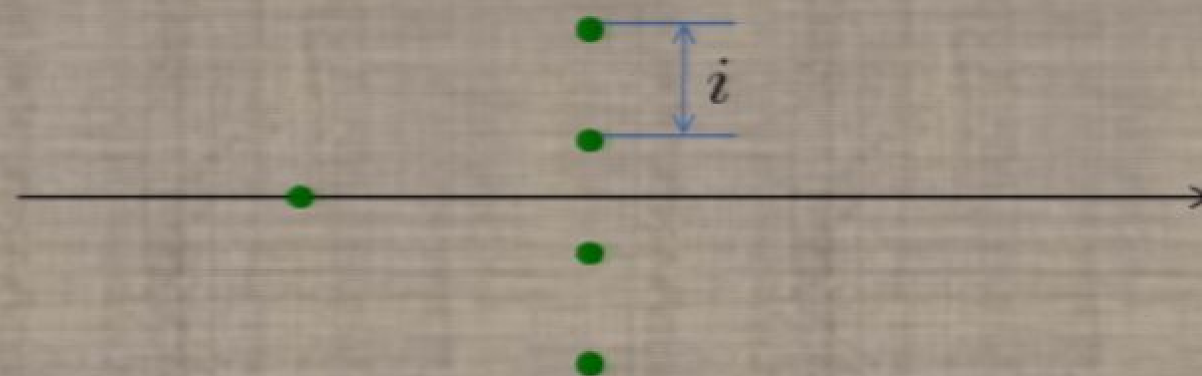
large L, each type of Bethe root can be real or form a string combination



ample: XXX spin chain

$$\left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}}\right)^L = -\prod_k \frac{u_j - u_k + i}{u_j - u_k - i}$$

$$\frac{L}{2\pi}\frac{dp[\theta]}{d\theta} = \rho[\theta] + \rho_h[\theta] + \int d\theta' K[\theta - \theta']\rho[\theta']$$

$$\frac{L}{2\pi}\frac{1}{u^2+\frac{1}{4}} = \rho[u] + \rho_h[u] + \int \frac{dv}{\pi}\frac{1}{(u-v)^2+1}\rho[v]$$

nplified form of Bethe Ansatz equations

$$\ldots_{,k} \Big)^L = \prod_{j\neq k} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\sigma[u_{4,k},u_{4,j}]^2$$

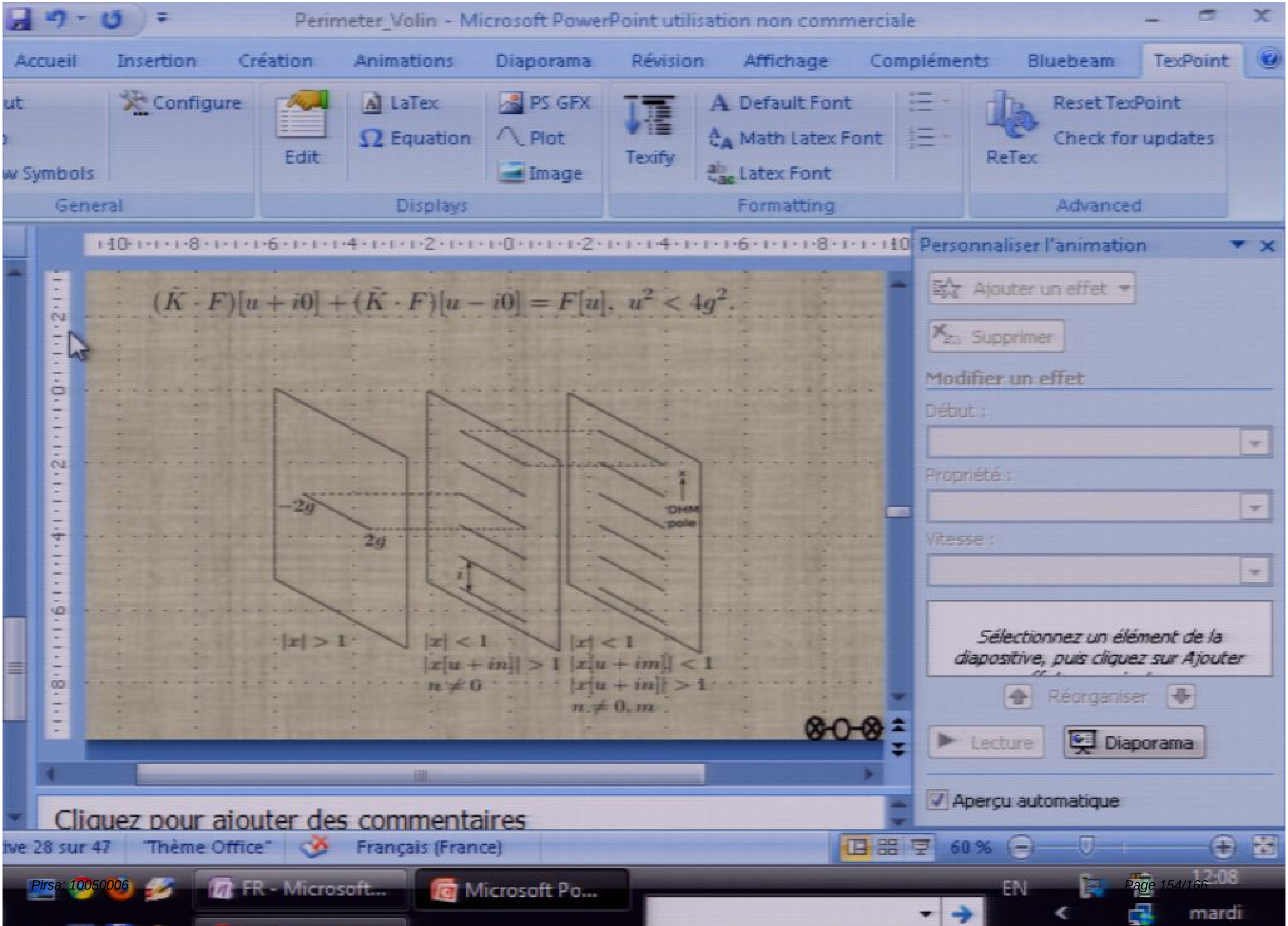
$$\prod_{j=1}^{K_3}\frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}}\prod_{j=1}^{K_5}\frac{x^-_{4,k}-x_{5,j}}{x^+_{4,k}-x_{5,j}}\quad\prod_{j=1}^{K_1}\frac{1-\frac{1}{x^-_{4,k}x_{1,j}}}{1-\frac{1}{x^+_{4,k}x_{1,j}}}\prod_{j=1}^{K_7}\frac{1-\frac{1}{x^-_{4,k}x_{7,j}}}{1-\frac{1}{x^+_{4,k}x_{7,j}}}$$

e can write these equations in a more suggestive form using the properties:

$$\prod \frac{x^- - y_k}{x^+ - y_k} = \prod_k (u-v_k)^{(D-D^{-1})(1-\tilde{K})} \qquad \prod_k \frac{1-\frac{1}{x^- y_k}}{1-\frac{1}{x^+ y_k}} = \prod_k (u-v_k)^{(D-D^{-1})\tilde{K}}$$

$$\sigma[u,v] = (u-v)^{(D-D^{-1})\tilde{K}\left(\frac{D^2}{1-D^2}-\frac{D^{-2}}{1-D^{-2}}\right)\tilde{K}(D-D^{-1})}$$

e Bethe equations in the Beisert-Staudacher Bethe Ansatz
n be written in terms of difference function (u-v) in the
ower of a rational combination of the operators D and $\tilde{K}$.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Aperçu automatique
$(\bar{K}\cdot F)[u+i0]+(\bar{K}\cdot F)[u-i0]=F[u],\ u^2<4g^2.$
$-2g$
$2g$
i
DHM pole
$|x|>1$
$|x|<1$
$|x[u+in]|>1$
$n\neq 0$
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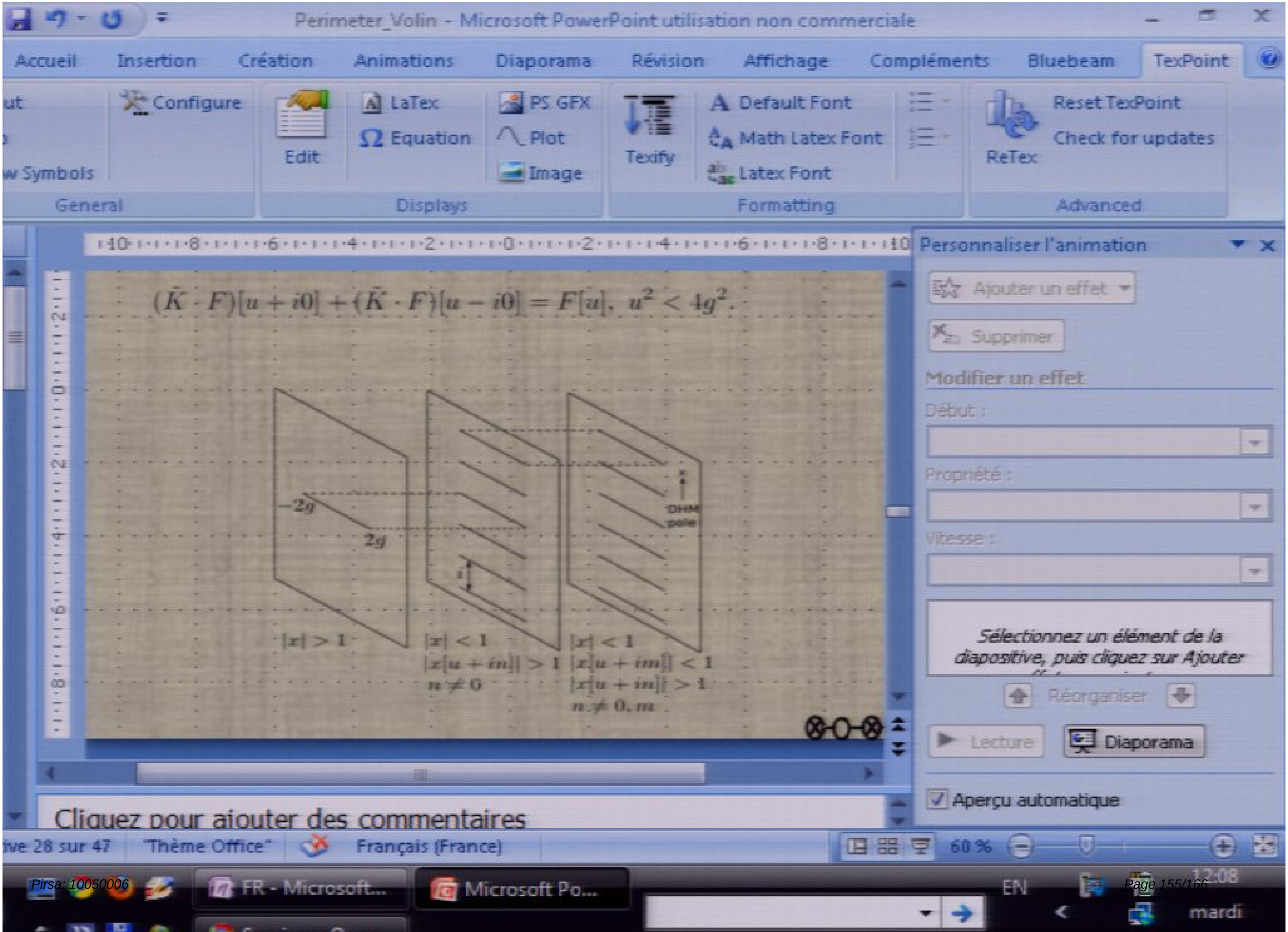
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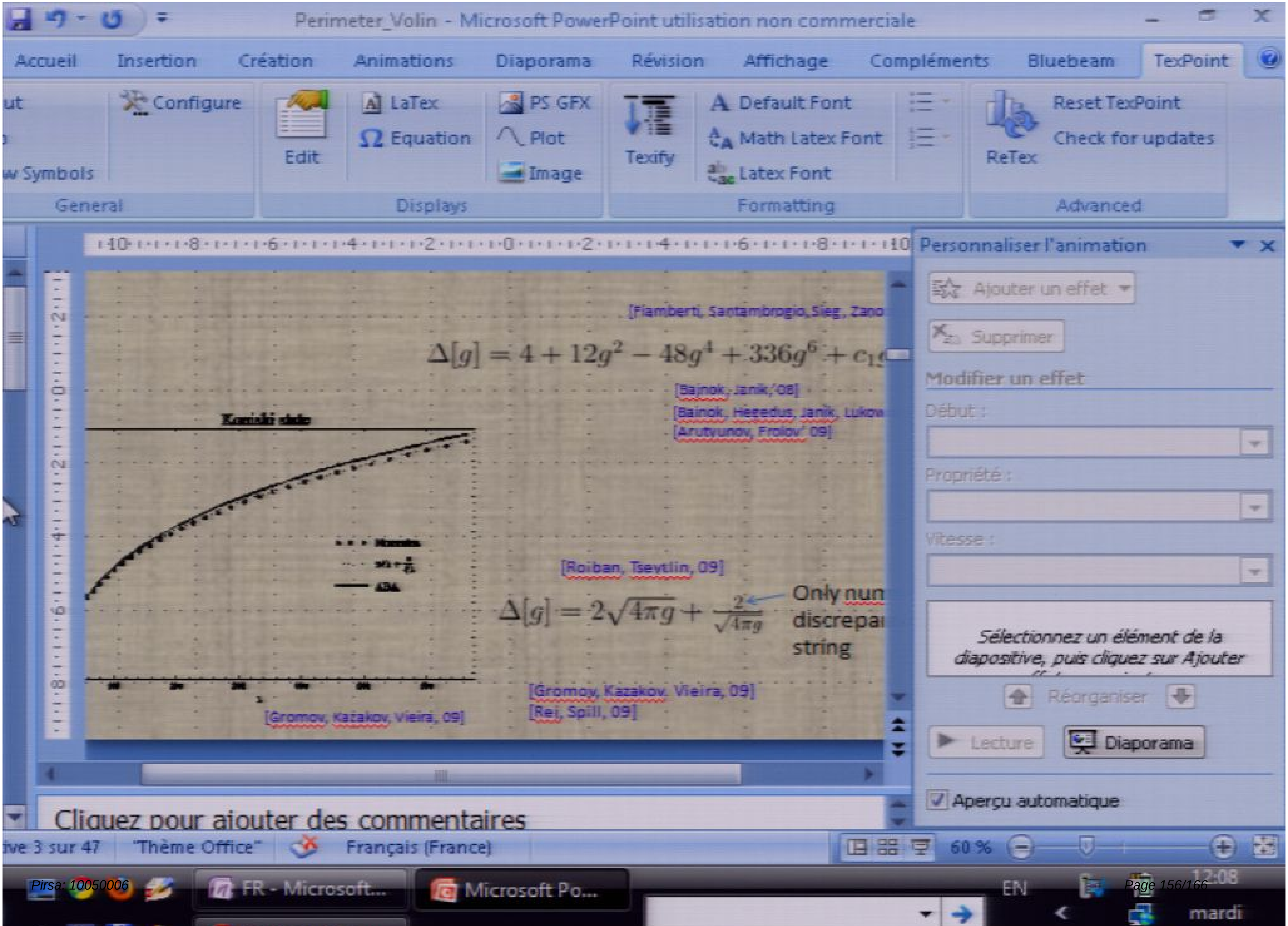
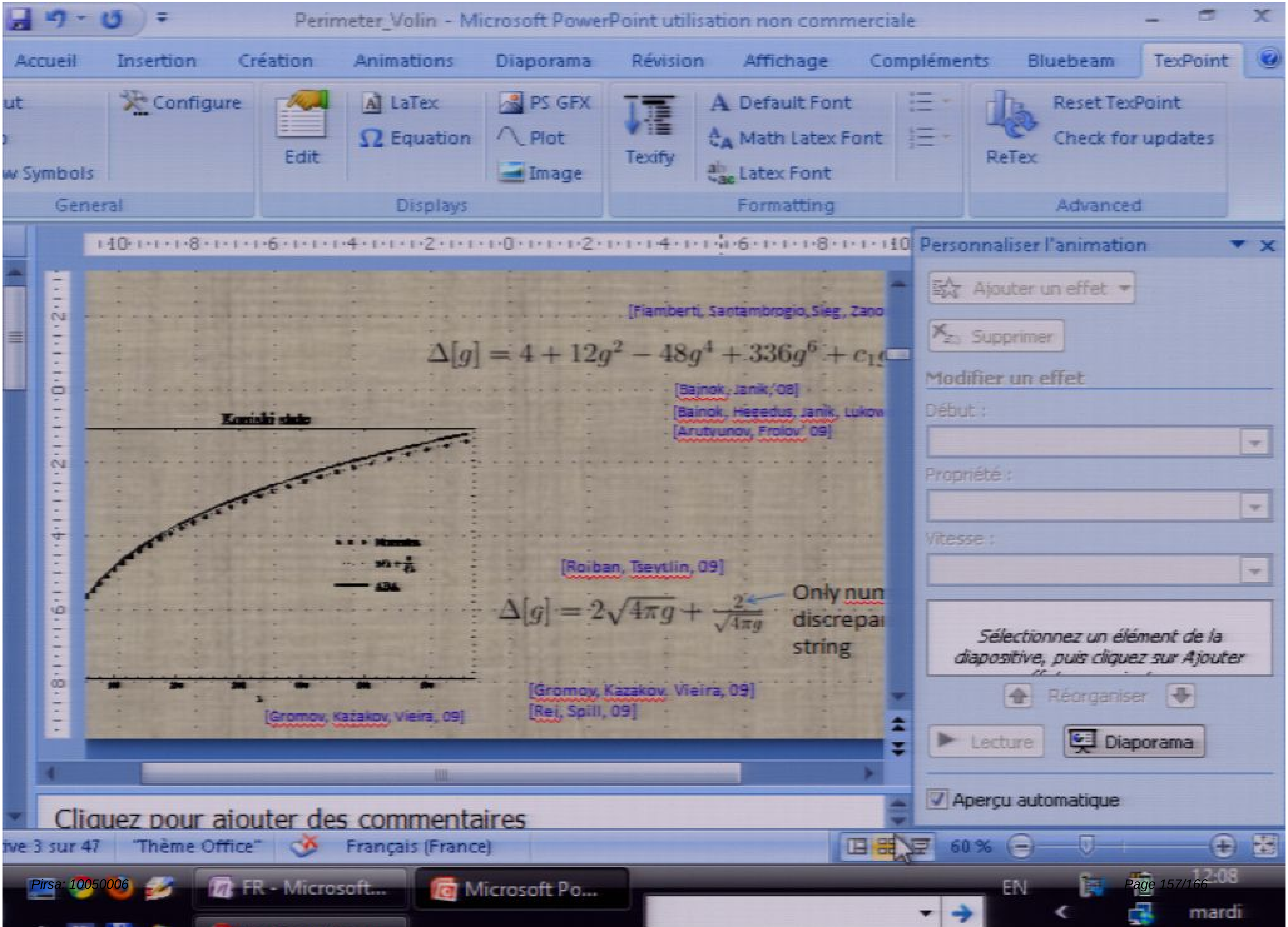


$$\Delta[g] = 4 + 12g^2 - 48g^4 + 336g^6 + c_{1\ldots}$$

$$\Delta[g] = 2\sqrt{4\pi g} + \frac{2}{\sqrt{4\pi g}}$$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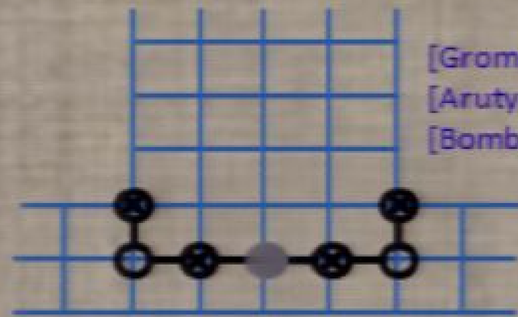
nple 2:
nalous dimension of Konishi state

$\mathrm{Tr}XZXZ - \mathrm{Tr}XXZZ$



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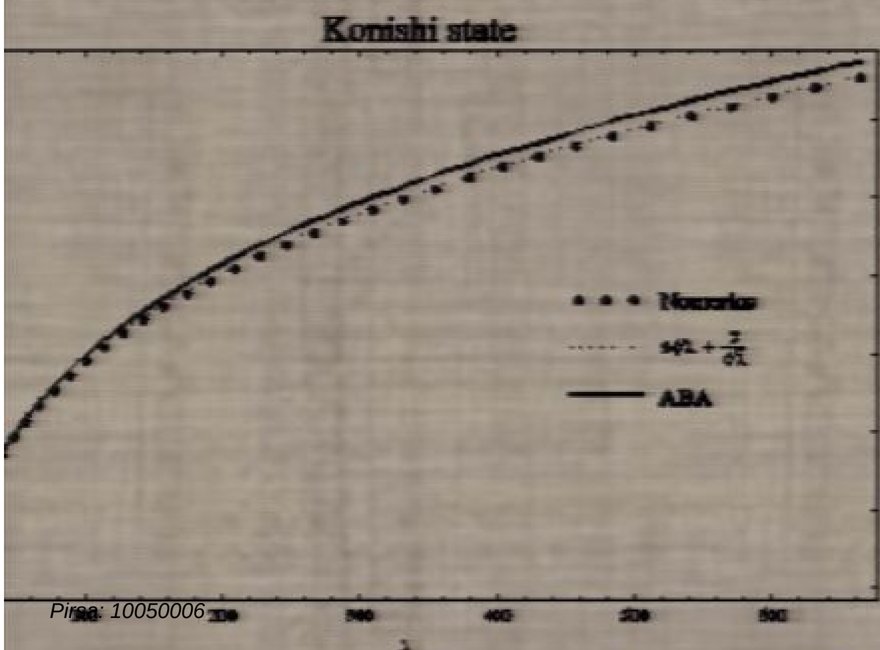

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Only numerics and discrepancy with string

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is theory is $\mathcal{N}=4$ SYM

ow we learned this?

1) AdS/CFT duality

$=4$ SYM = IIB, $AdS_5 x S^5$

$g=0$ $\quad g^2 = \frac{g_{YM}^2 N_c}{16\pi^2} \quad$ $g=\infty$

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Conformal dimension	=	Energy

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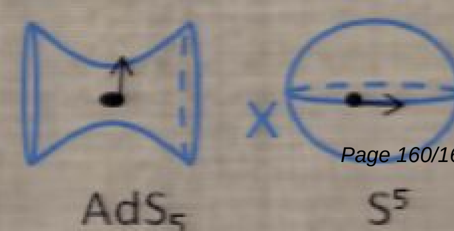
$\Phi_3 + i\Phi_4$

$\mathrm{Tr} XZZZXZ \ldots XZZ$



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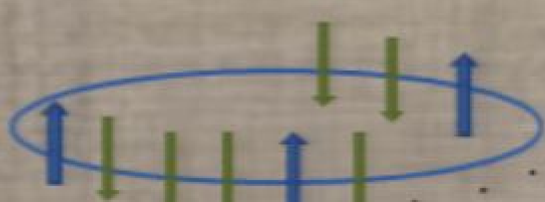

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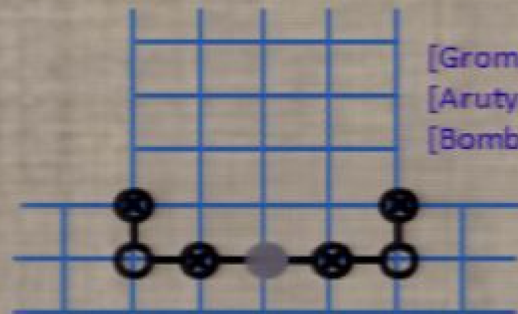

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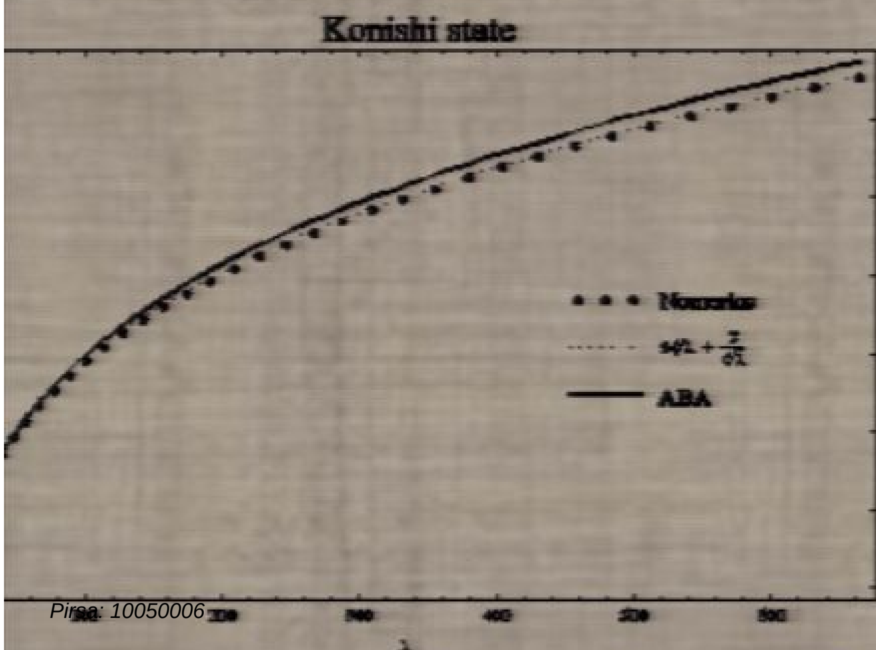

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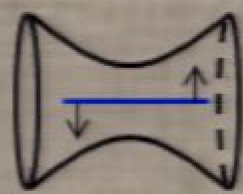

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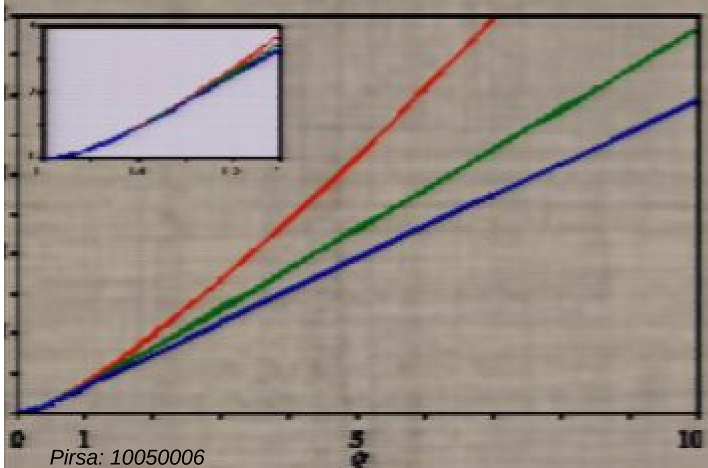

[Benna, Benvenuti, Klebanov, Scardicchio, 06]

[Gubser, Klebanov, Polyakov, 02]
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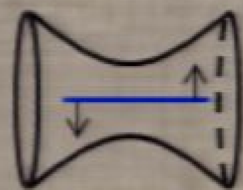

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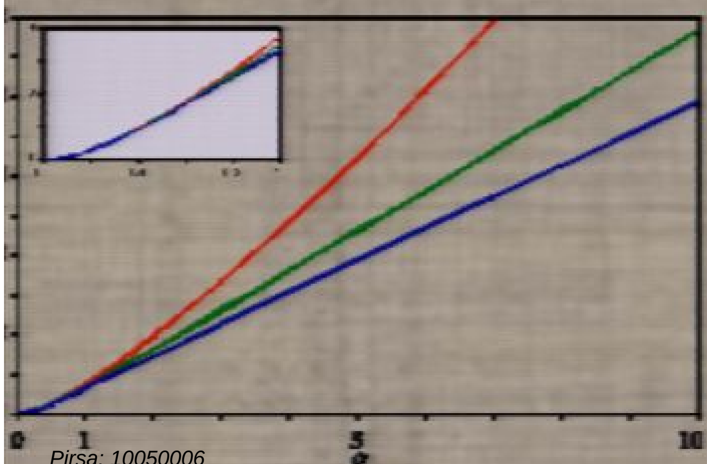

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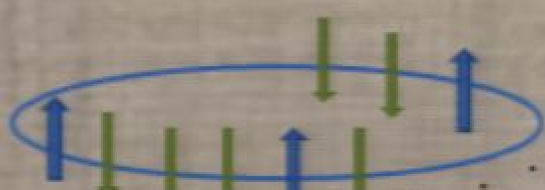

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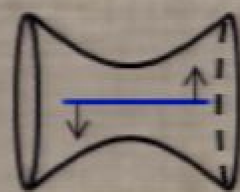

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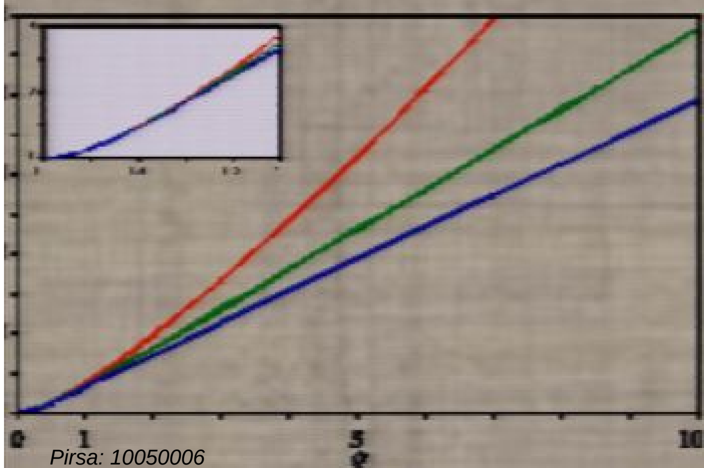

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