

Title: Topological Insulators and Fractionalization

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Abstract: An entirely new kind of band insulator was discovered recently. These new electronic states - called "topological insulators" - are fundamentally different from standard band insulators. They are distinguished by the fact that their edges (in the 2D case) or surfaces (in the 3D case) support gapless transport which is extremely robust. In the two dimensional case, topological insulators can be thought of as time reversal invariant analogues of integer quantum Hall states. This analogy is intriguing since integer quantum Hall states are a special case of the far richer class of fractional quantum Hall states. It is natural to wonder: can topological insulators also be generalized? In this talk, I will investigate this question. I will show that, just as in quantum Hall systems, electron interactions allow for a whole new class of states - which we call "fractional topological insulators." These states have excitations with fractional charge and statistics, in addition to protected edge modes.



# Topological insulators and fractionalization

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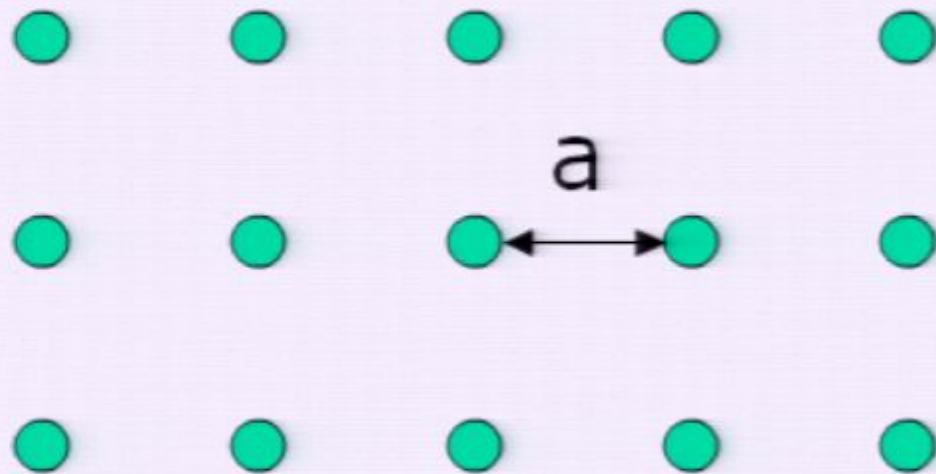
Michael Levin

*Harvard University*

Ady Stern

*Weizmann Institute*

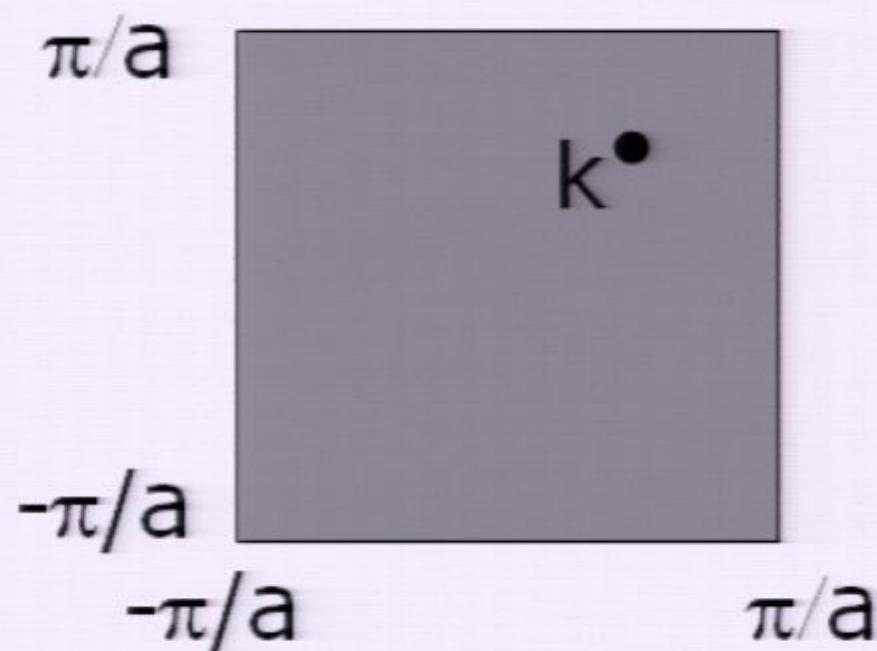
# Model



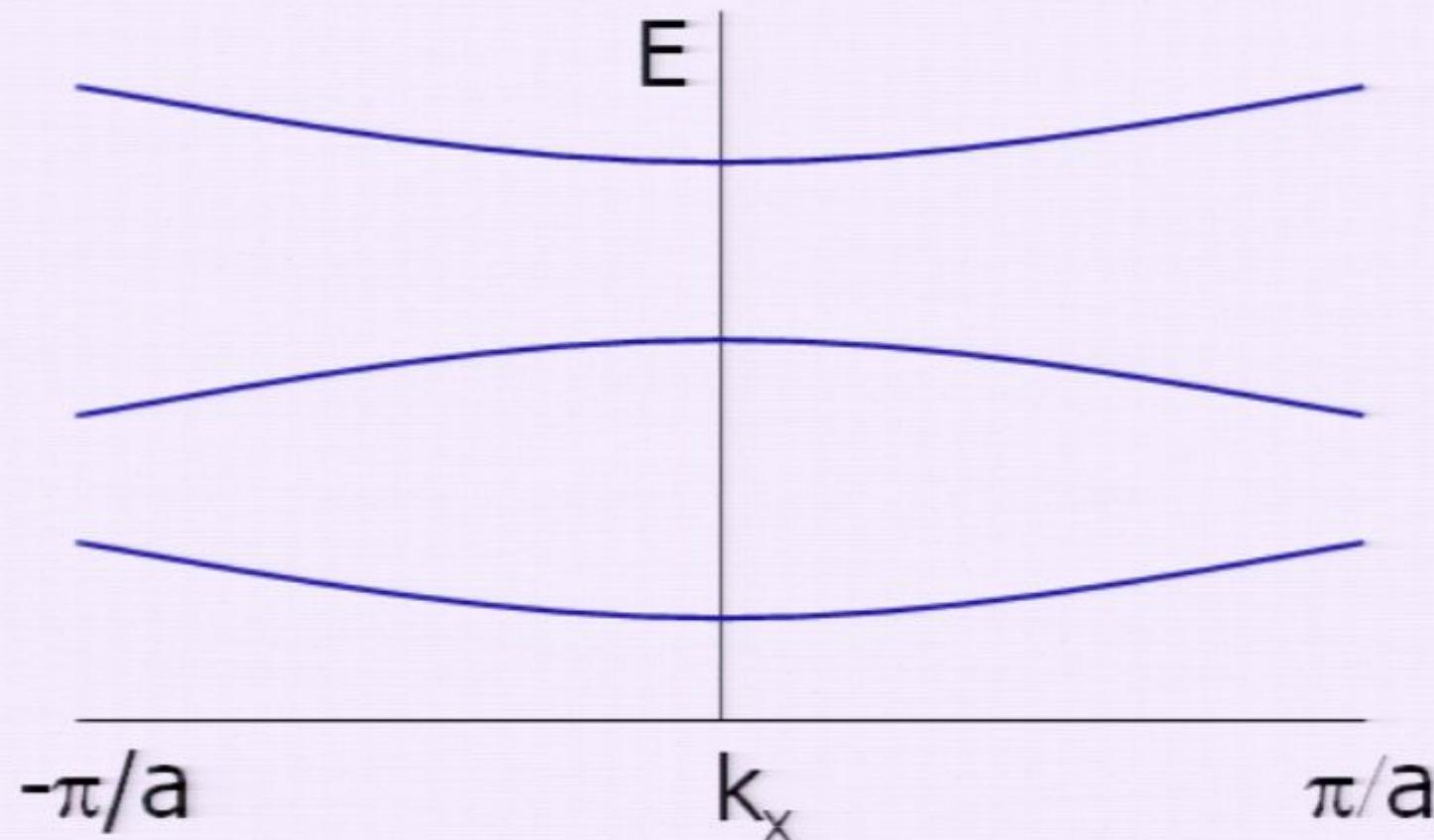
Non-interacting electrons  
in a periodic potential

# Solving the model

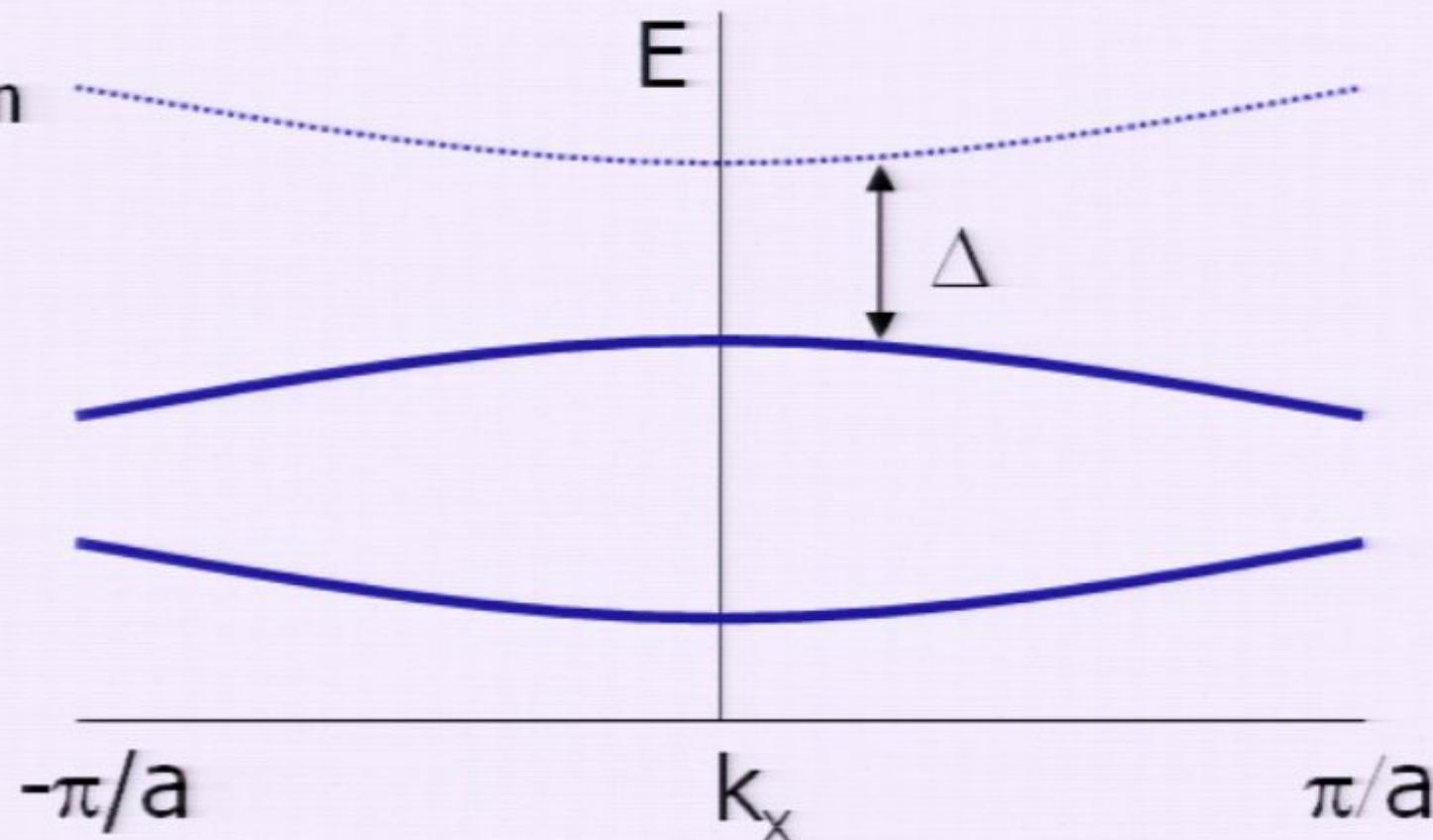
Discrete translational symmetry  $\Rightarrow k_x, k_y$   
are good quantum numbers ( $\text{mod } 2\pi/a$ )



# Energy spectrum



# Energy spectrum





# Are all band insulators the same?



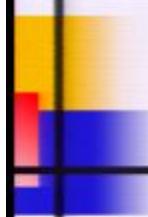
# Are all band insulators the same?

NO!

Two distinct kinds of band insulators in  
time reversal invariant 2D systems:

1. Conventional insulators
2. "Topological insulators"

(Kane, Mele,  
PRL, 2005)



# Outline

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## I. Review

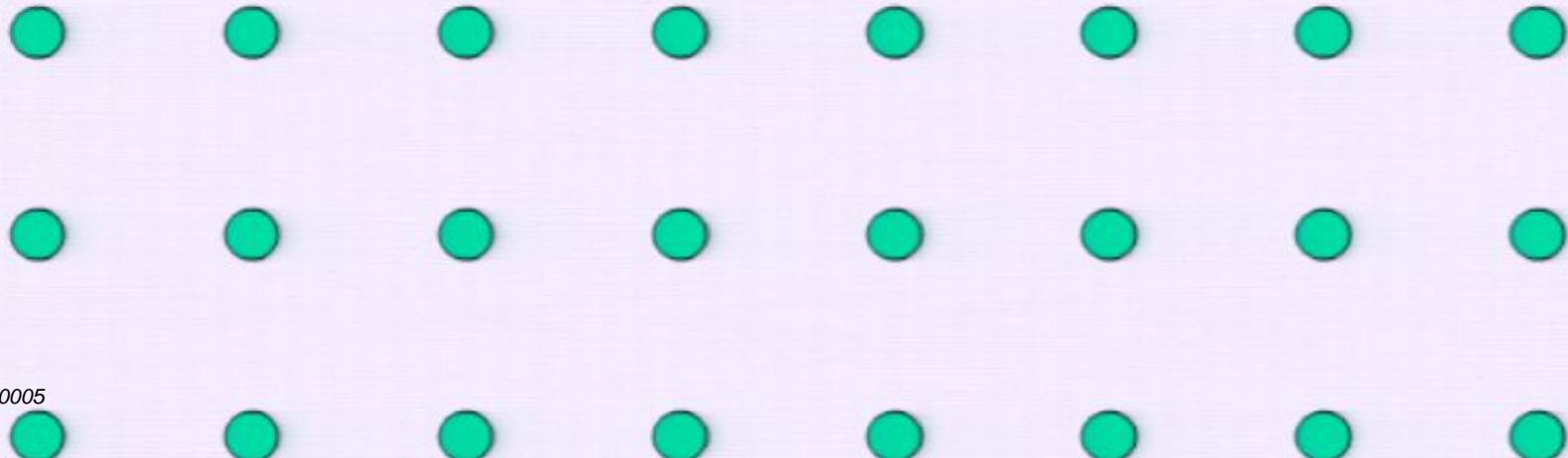
A. Why 2?

B. Experimental realization

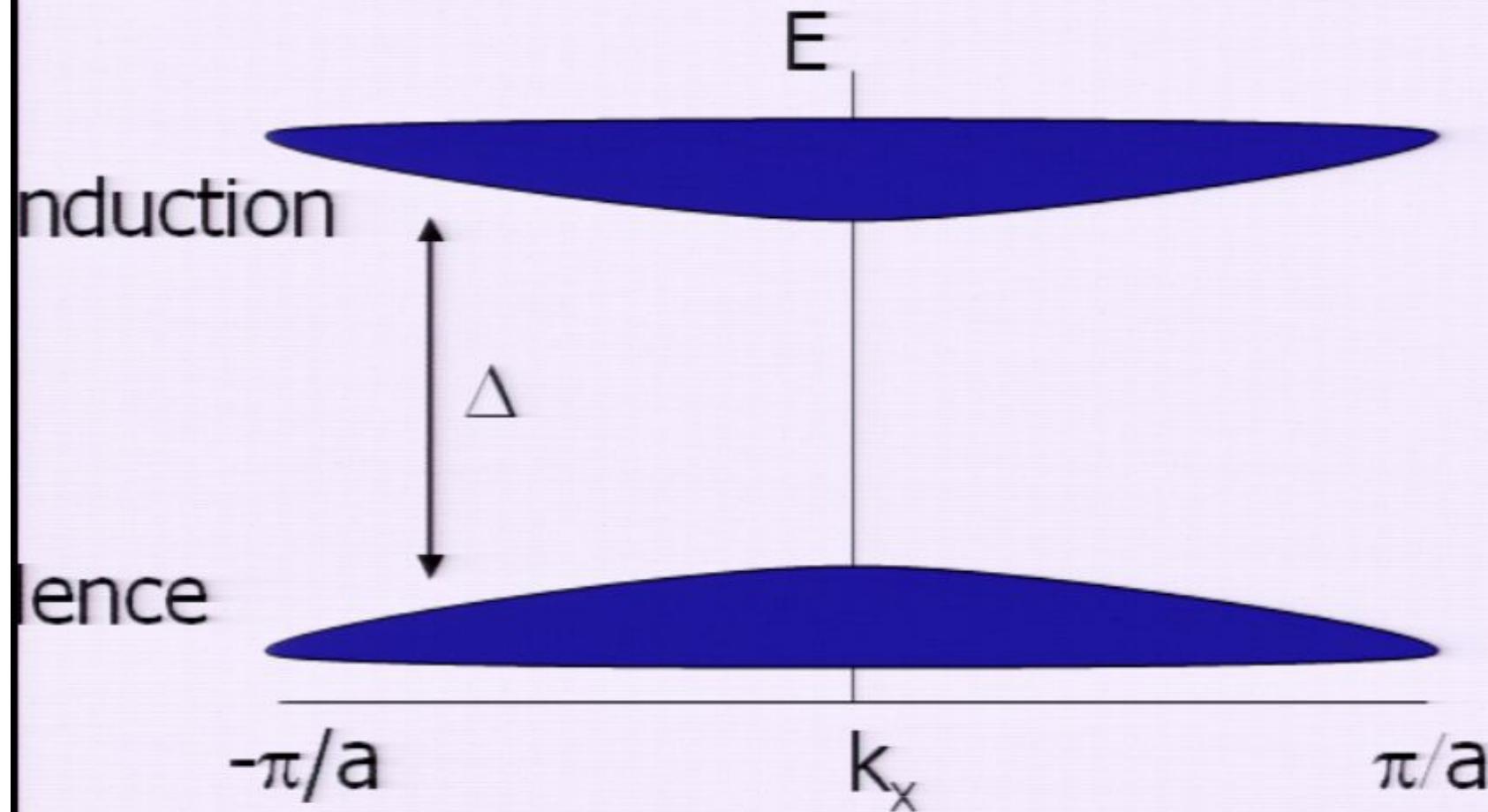
## II. Strongly correlated systems?

# Insulators with an edge

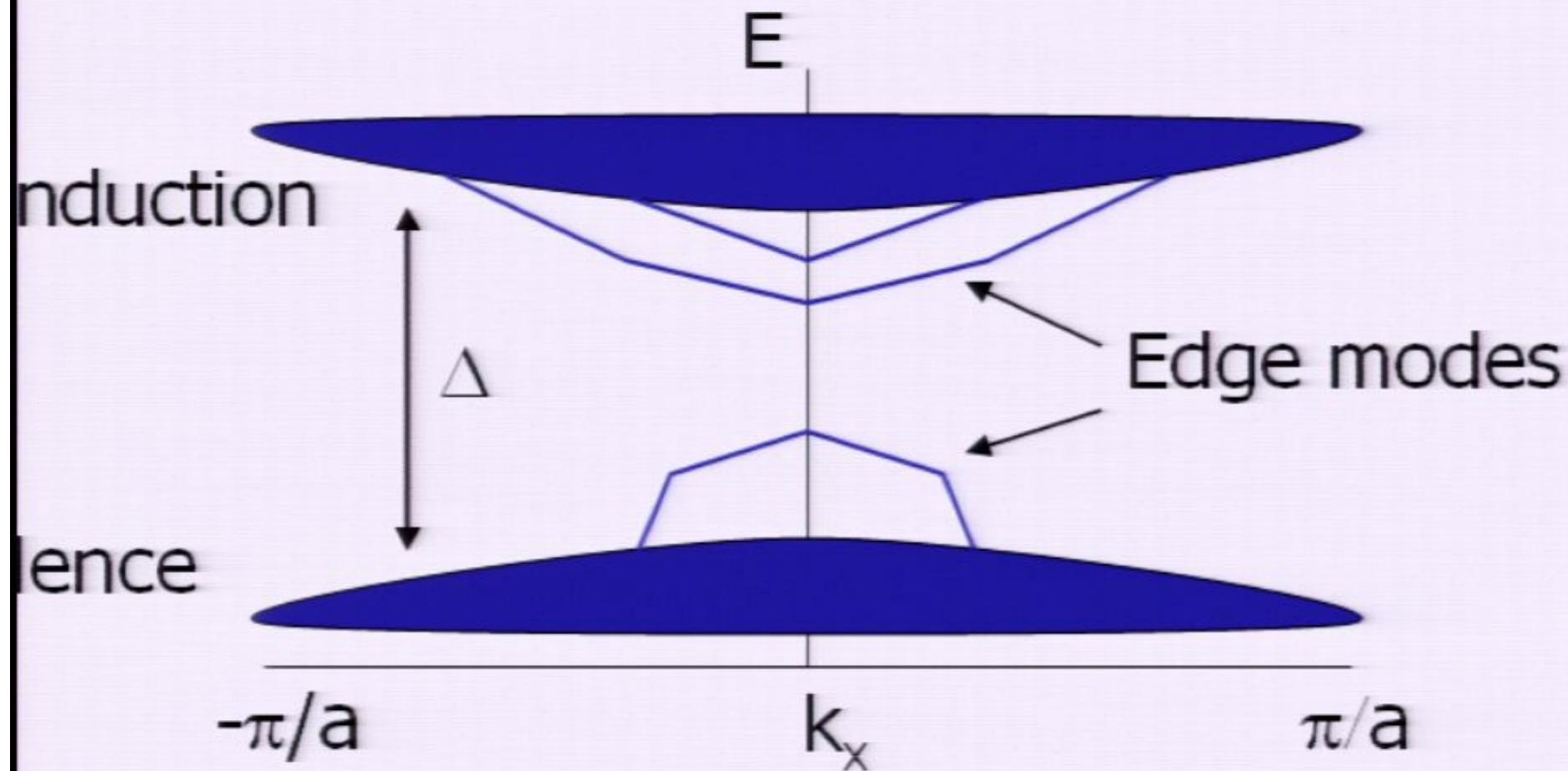
Vacuum



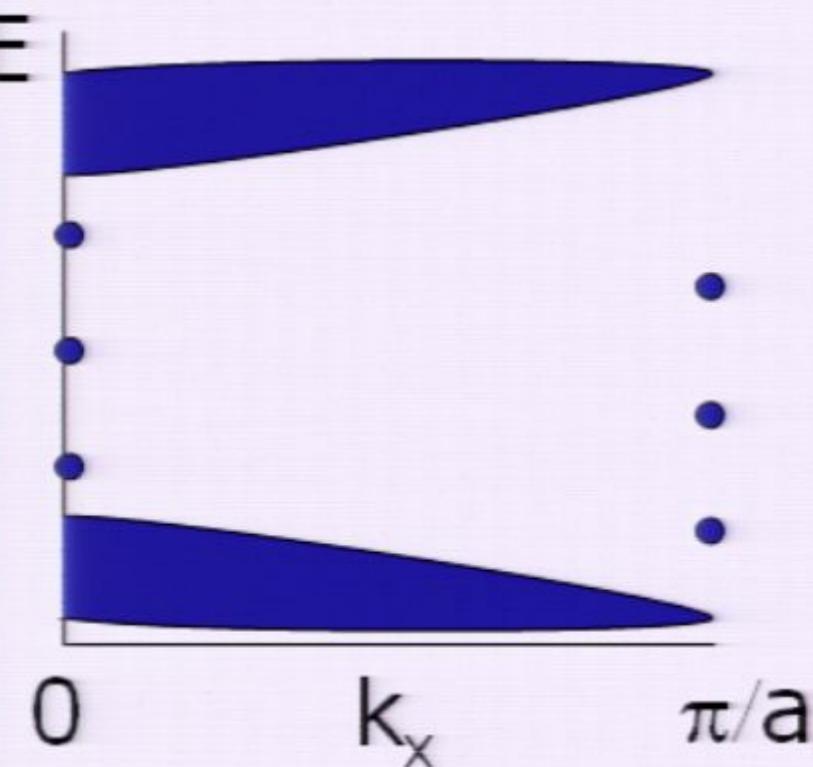
# Energy spectrum with an edge



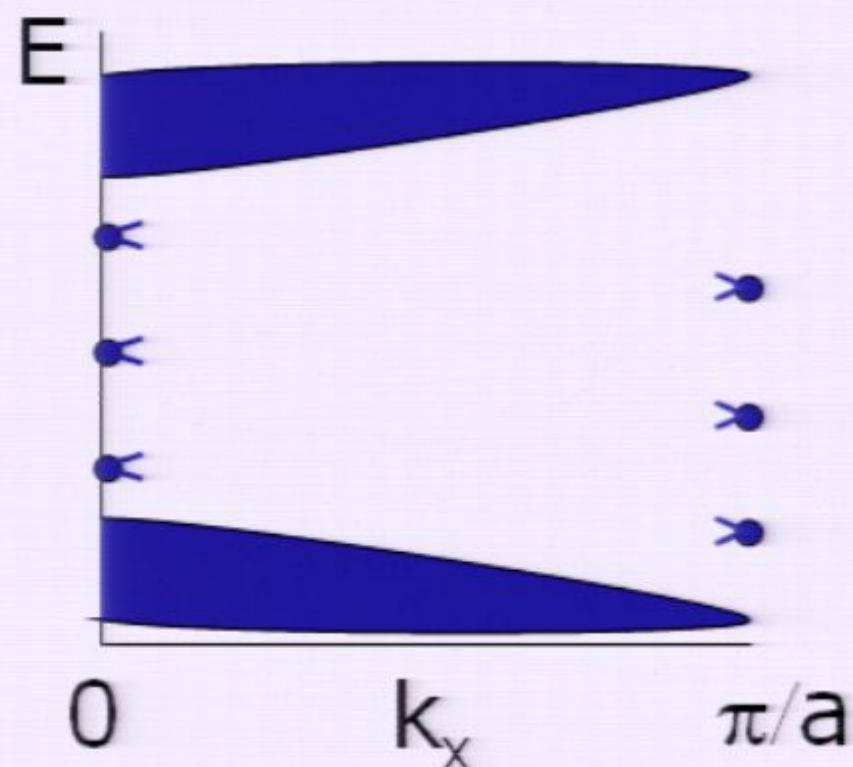
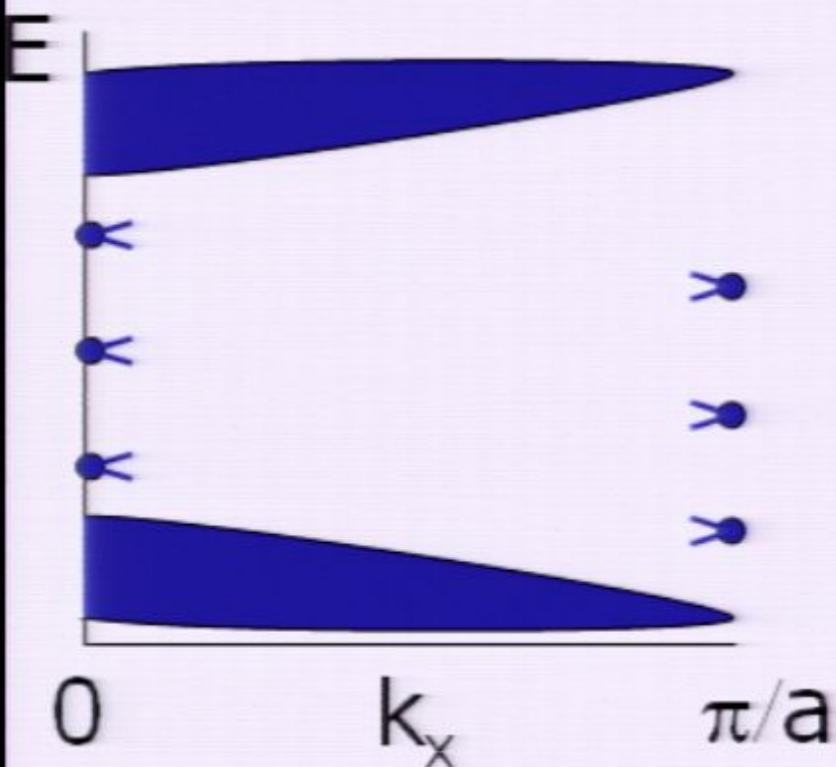
# Energy spectrum with an edge



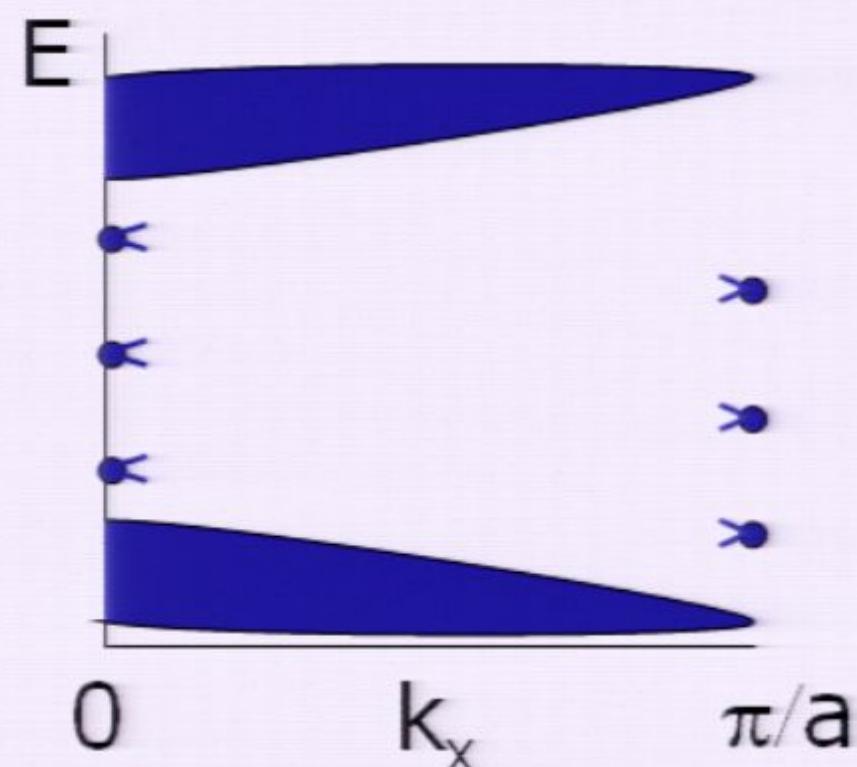
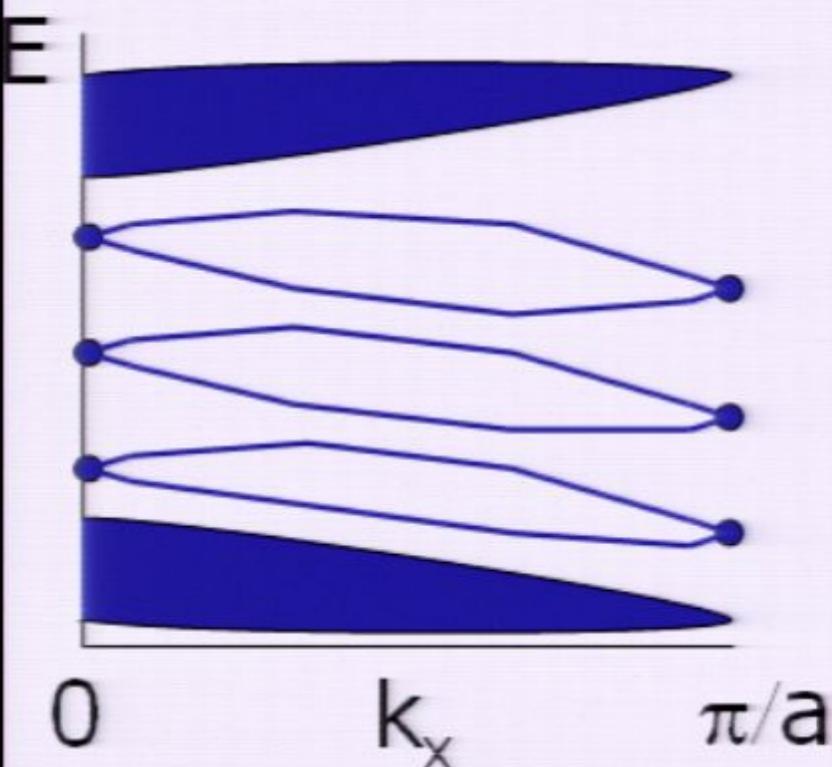
## Two types of edge spectra



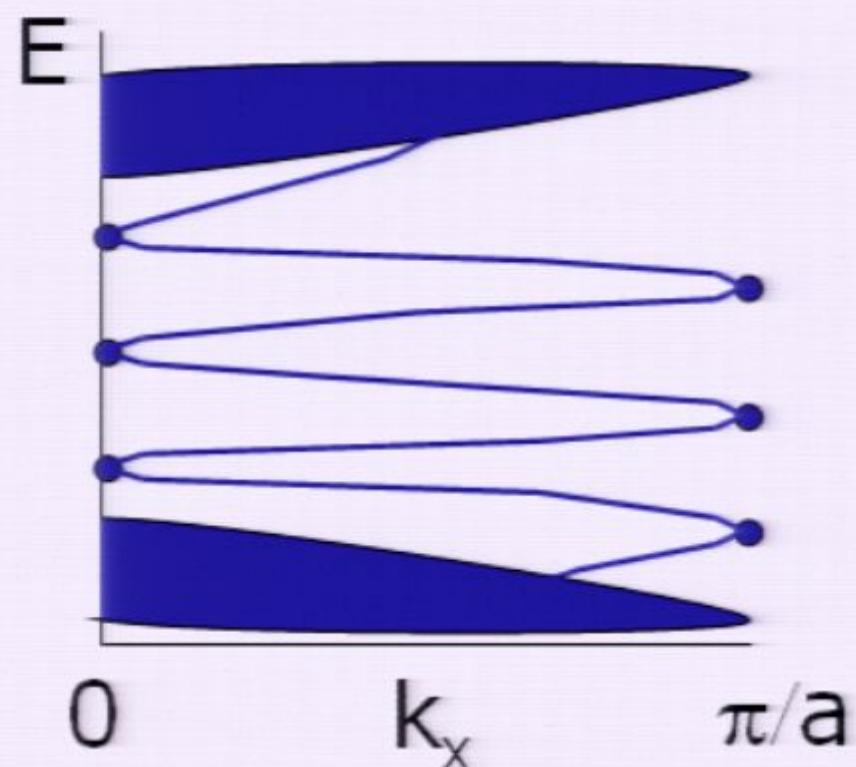
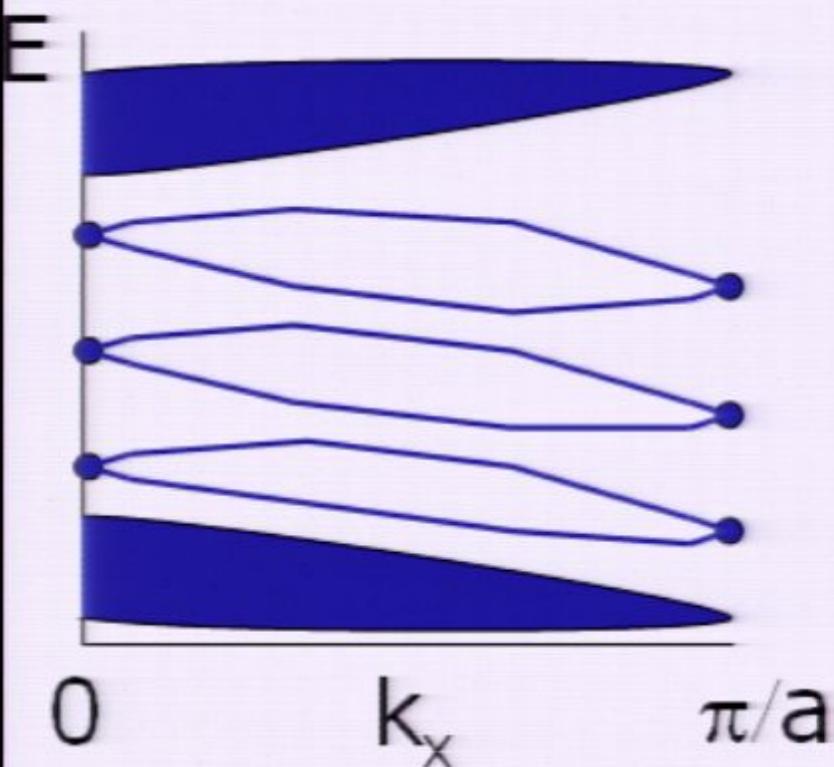
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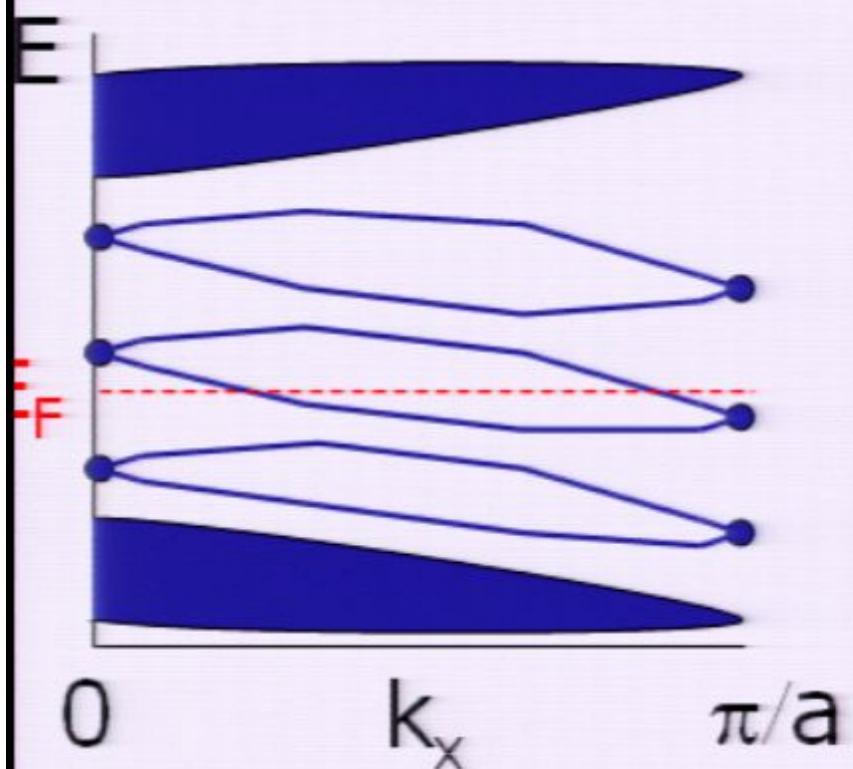
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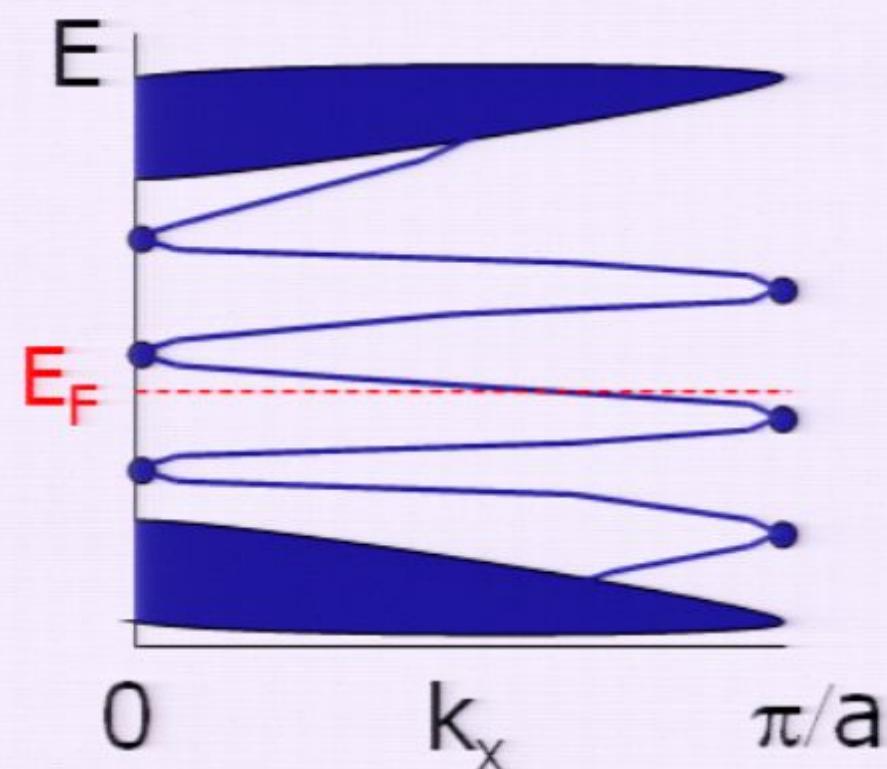
## Two types of edge spectra



## Two types of edge spectra



conventional insulator



"Topological insulator"

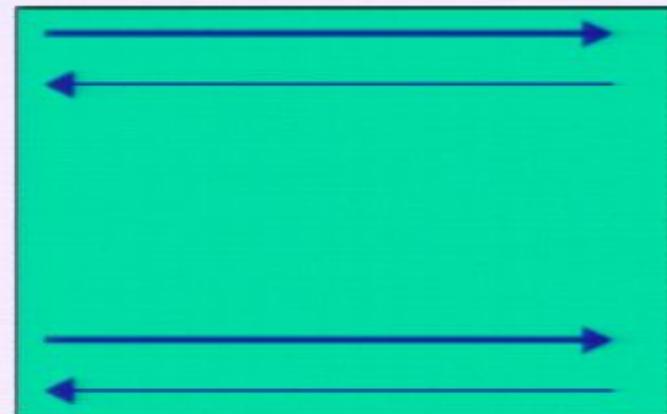
# Physical properties

conventional insulator      Topological insulator



No edge mode

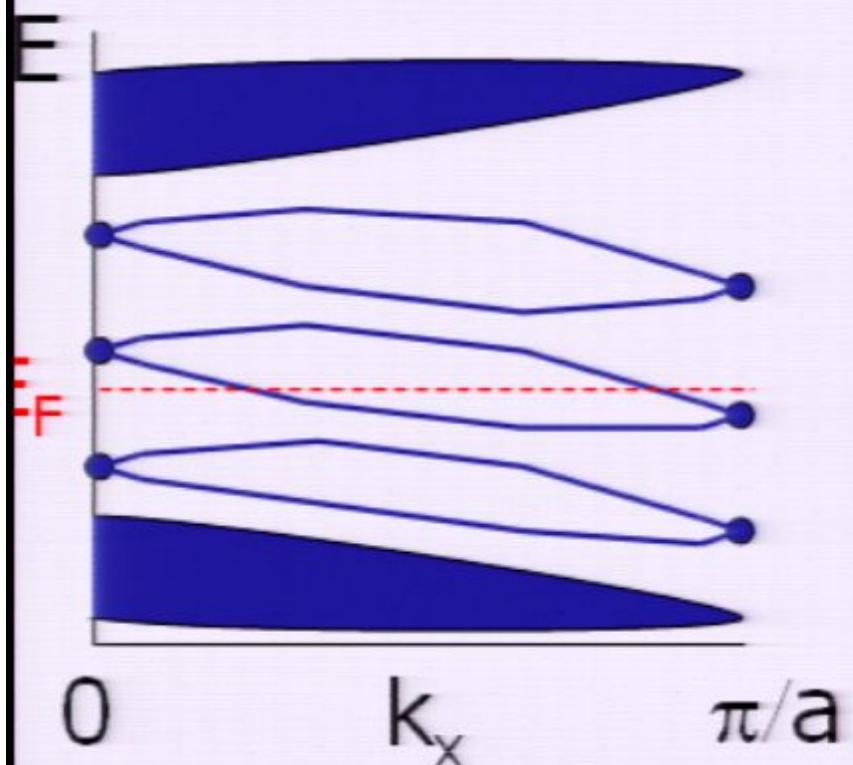
Conductance  $G = 0$



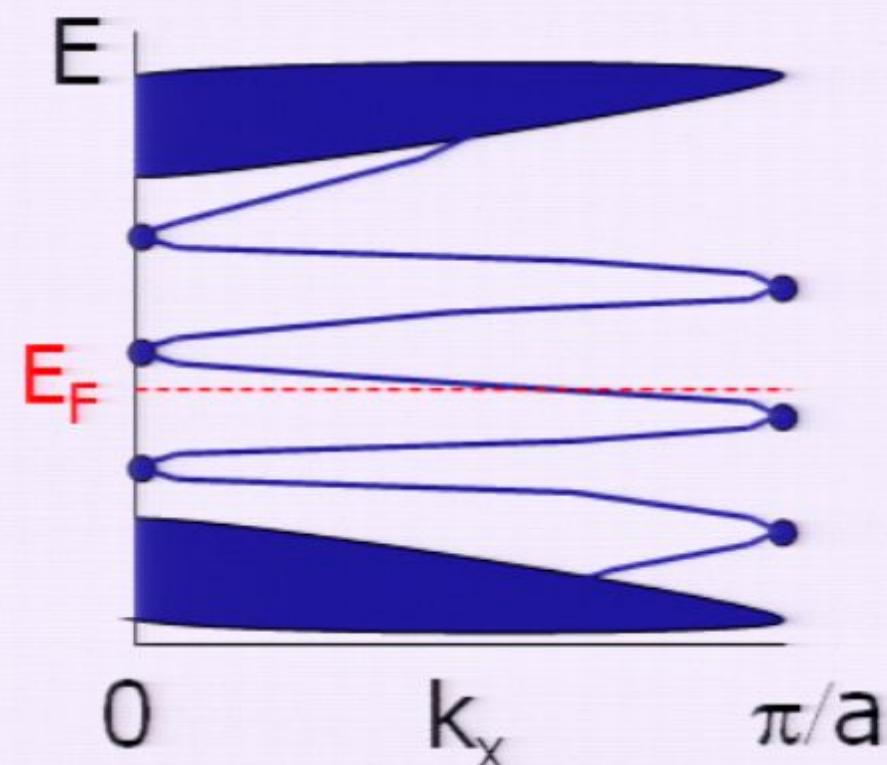
Pair of edge modes

Conductance  $G = 2e^2/h$

# Two types of edge spectra



conventional insulator



"Topological insulator"

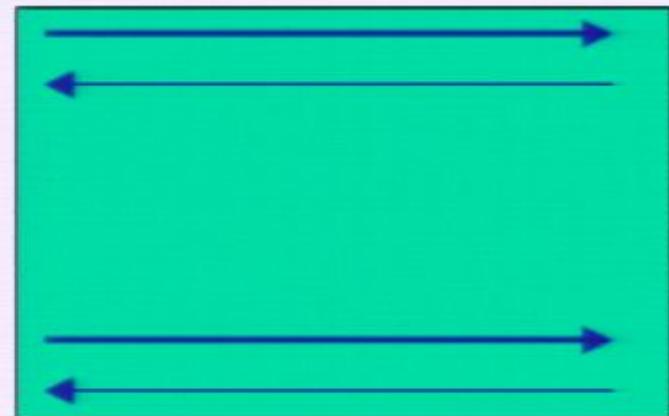
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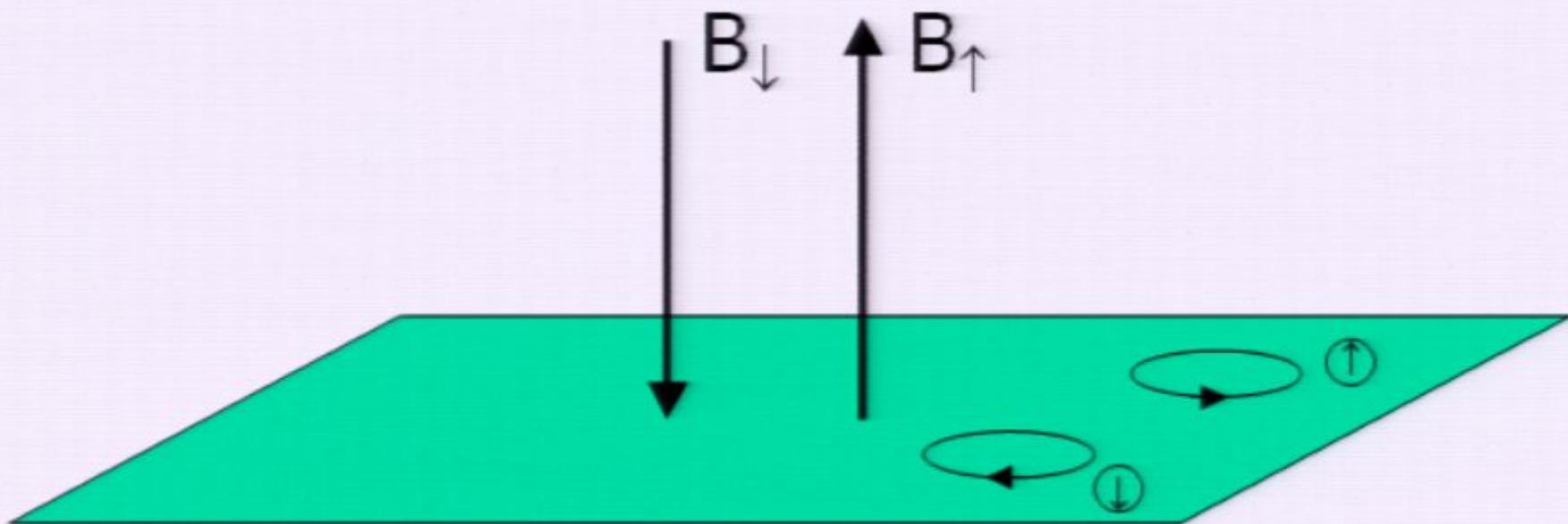


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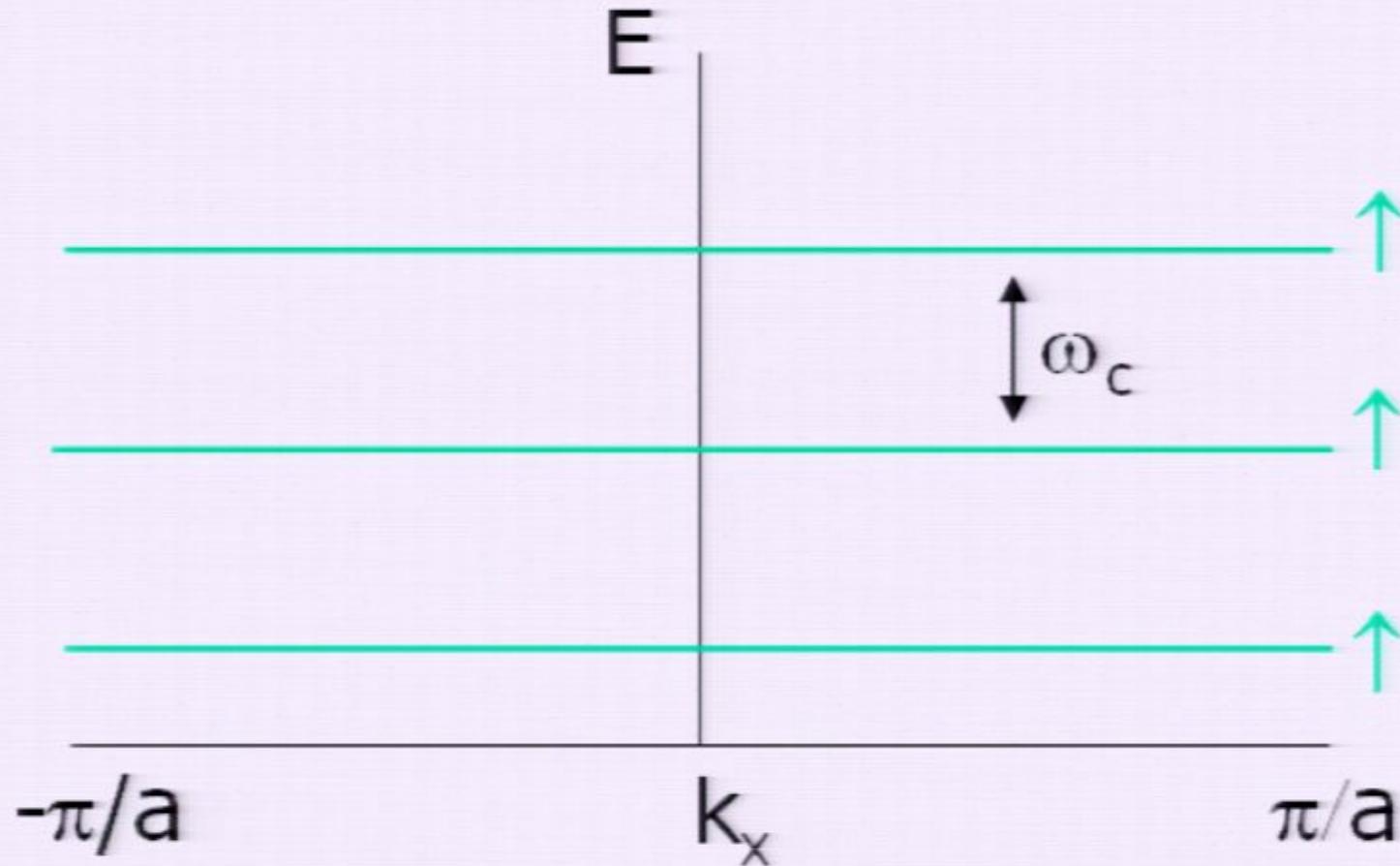
# Toy model

Electrons in a uniform, spin-dependent magnetic field

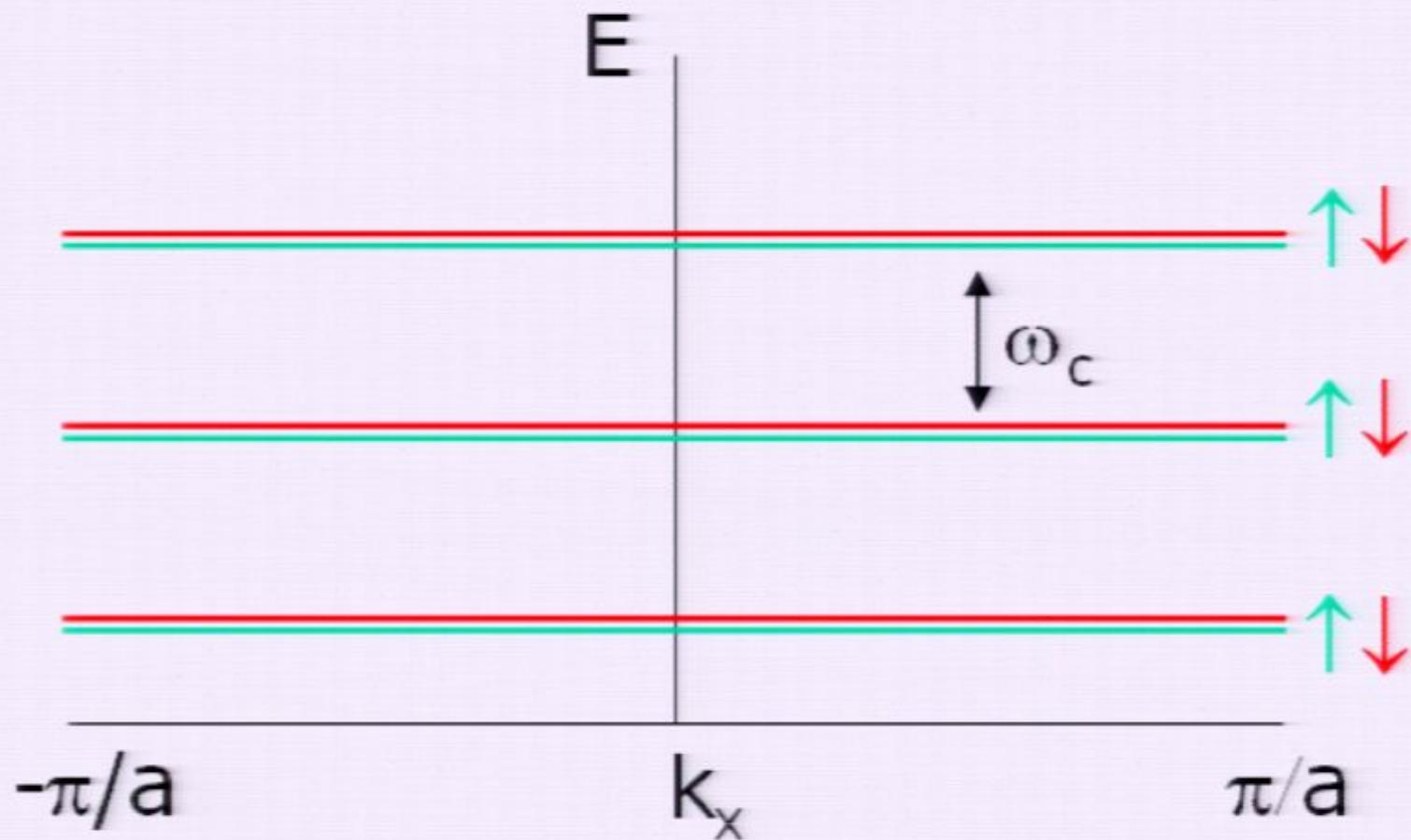


$$H = (p - e A \sigma^z)^2 / 2m$$

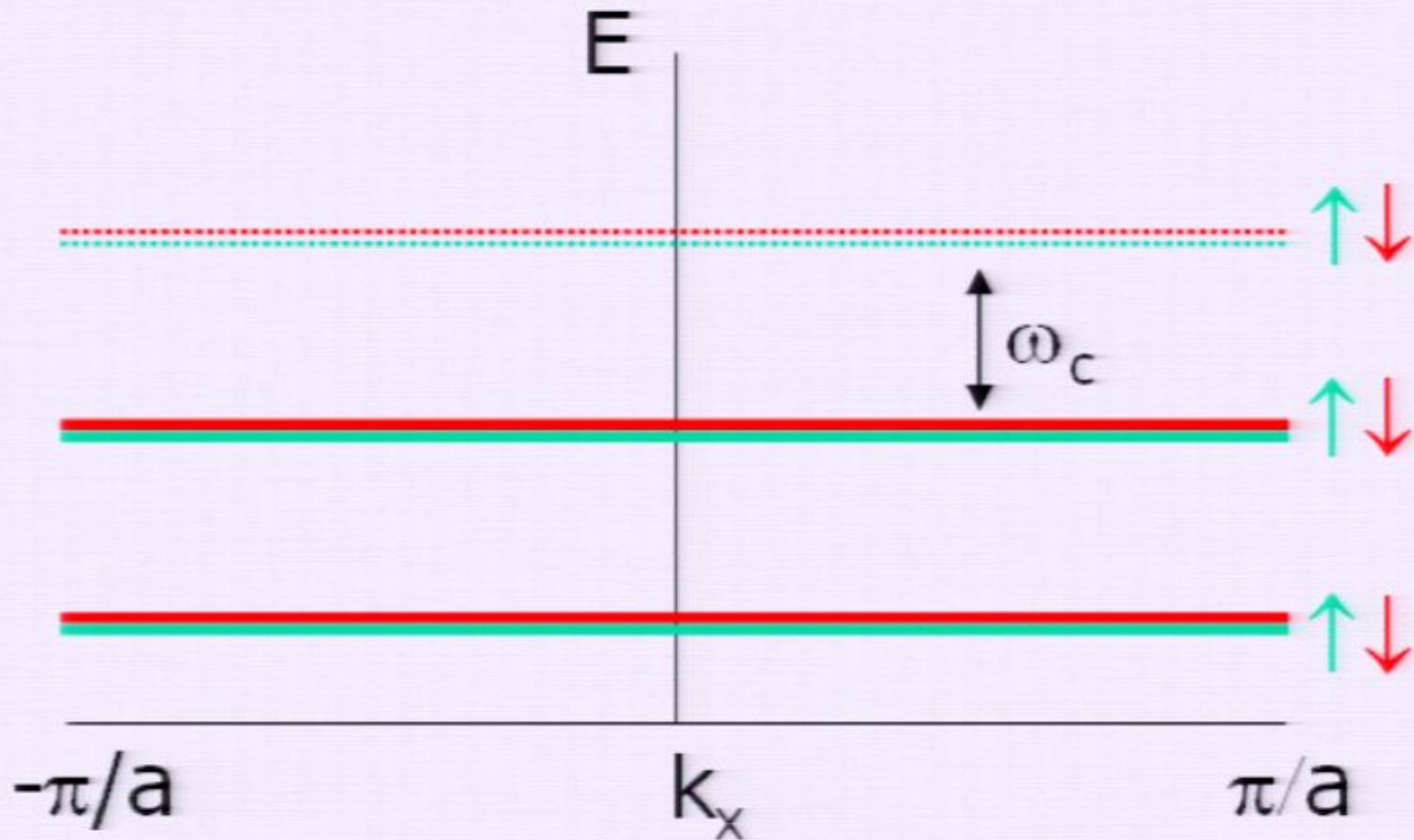
# Energy spectrum



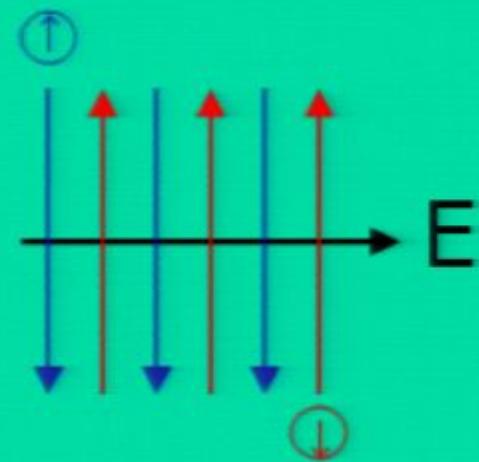
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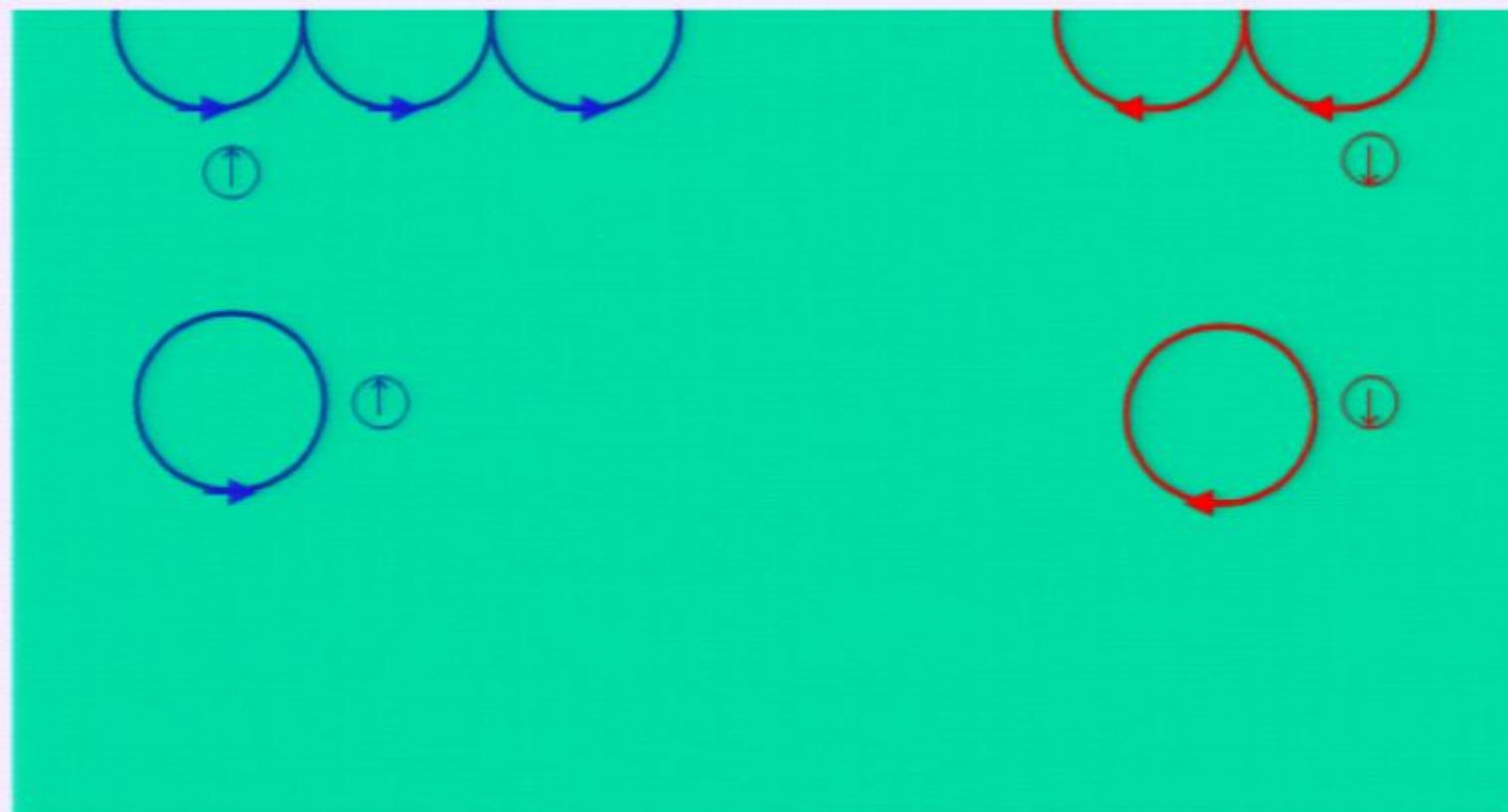
# Spin Hall effect



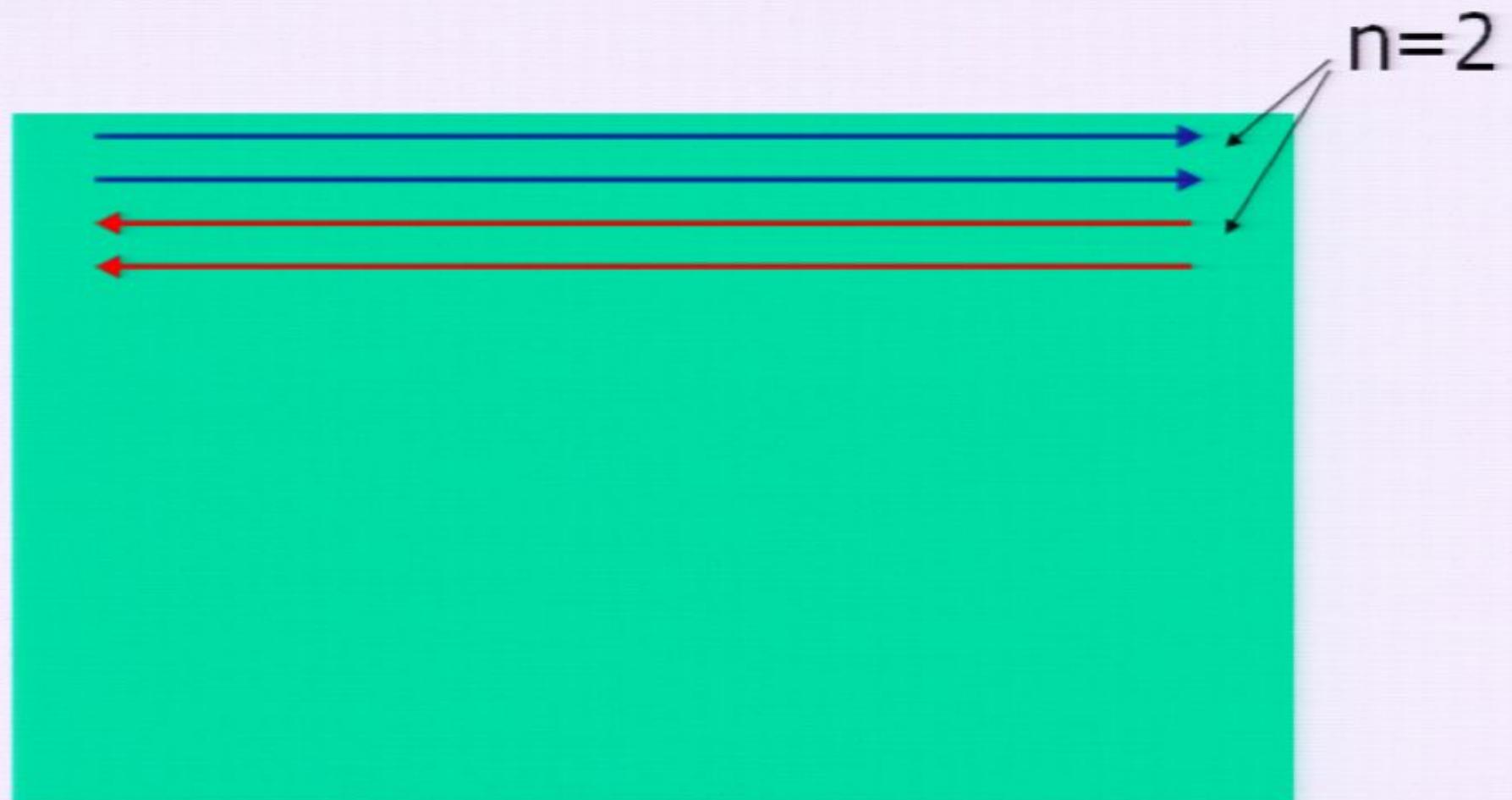
$$\mathbf{j}_s = \sigma_{\text{SH}} \cdot \mathbf{E}$$

$$\sigma_{\text{SH}} = n \cdot e / 2\pi$$

# Edge modes



# Edge modes



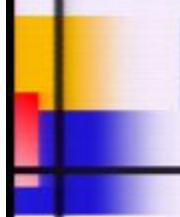


# Classification

$n$  even  $\Rightarrow$  conventional insulator  
 $n$  odd  $\Rightarrow$  topological insulator

$n = \#$ filled Landau levels per spin

= spin-Hall conductivity  $\sigma_{\text{SH}}$   
(in units of  $e/2\pi$ )

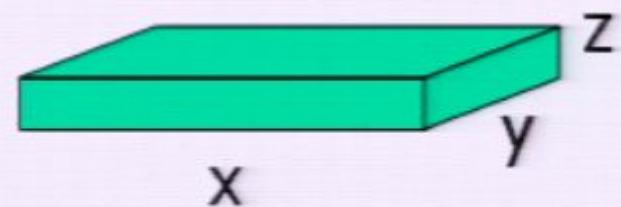


# Realizing topological insulators

$$H_{SO} \propto (\mathbf{p} \times \nabla V) \cdot \boldsymbol{\sigma}$$

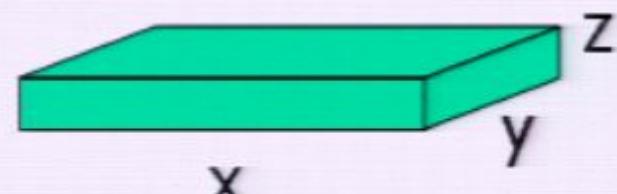
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If  $V$  symmetric about  $z=0$ ,

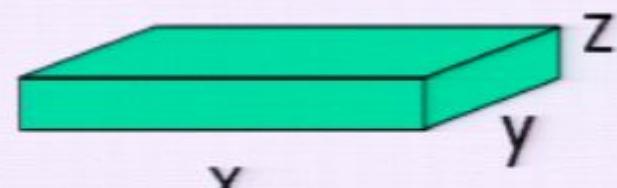
$$H_{SO} \propto \mathbf{p} \cdot \mathbf{A} \sigma^z$$

where

$$\mathbf{A} = (\partial_y V, -\partial_x V)$$

# Realizing topological insulators

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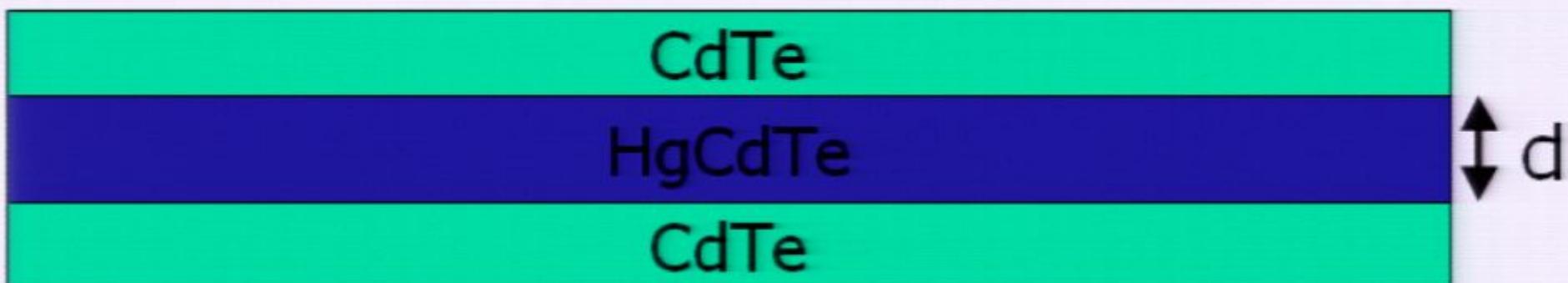
where

$$\mathbf{A} = (\partial_y V, -\partial_x V)$$

Effective spin-dependent magnetic field

# Experimental realization

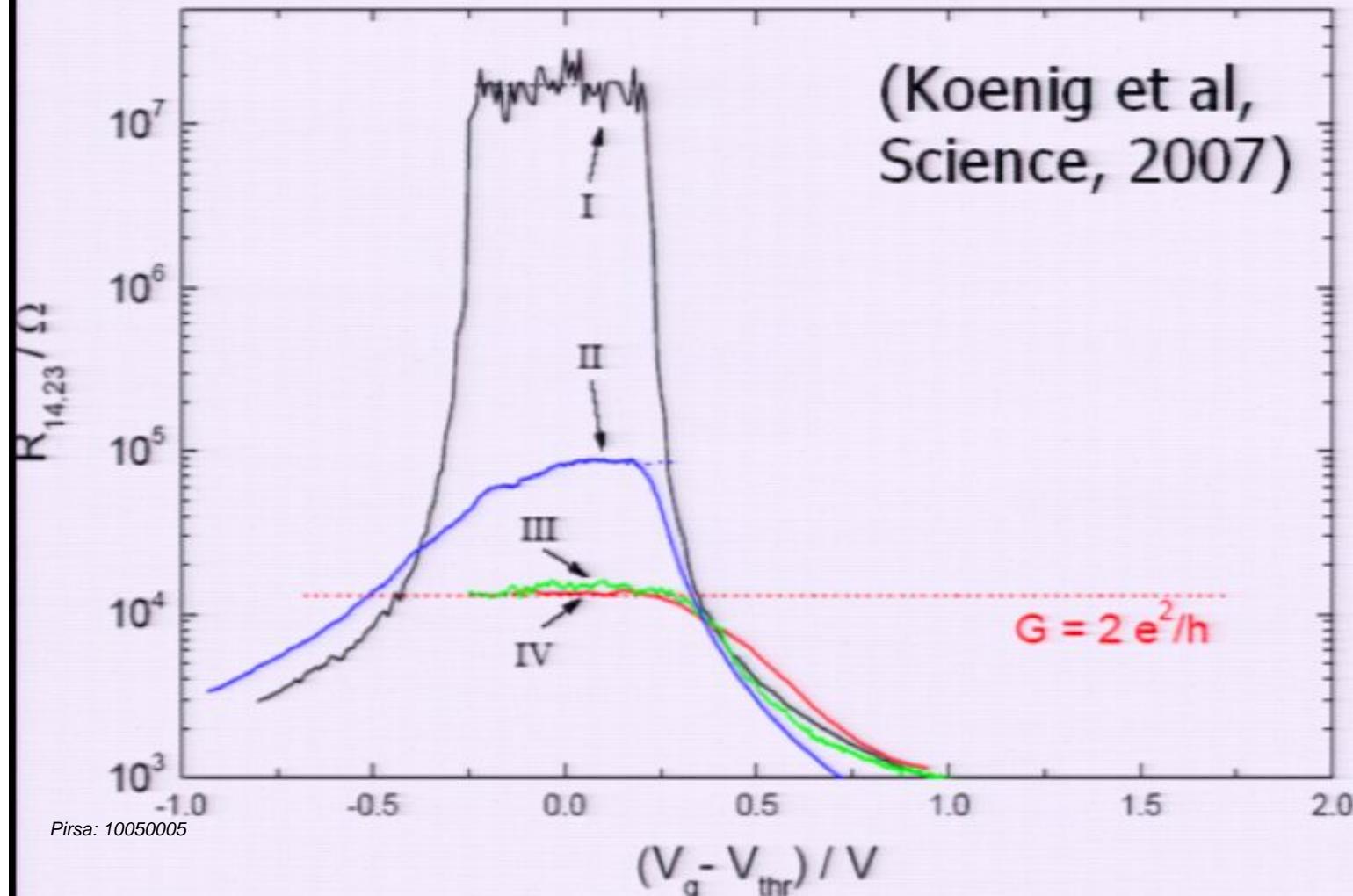
(Bernevig et al,  
Science, 2006)



$d < d_c \Rightarrow$  conventional insulator

$d > d_c \Rightarrow$  topological insulator

# Experiment



# Experimental realization

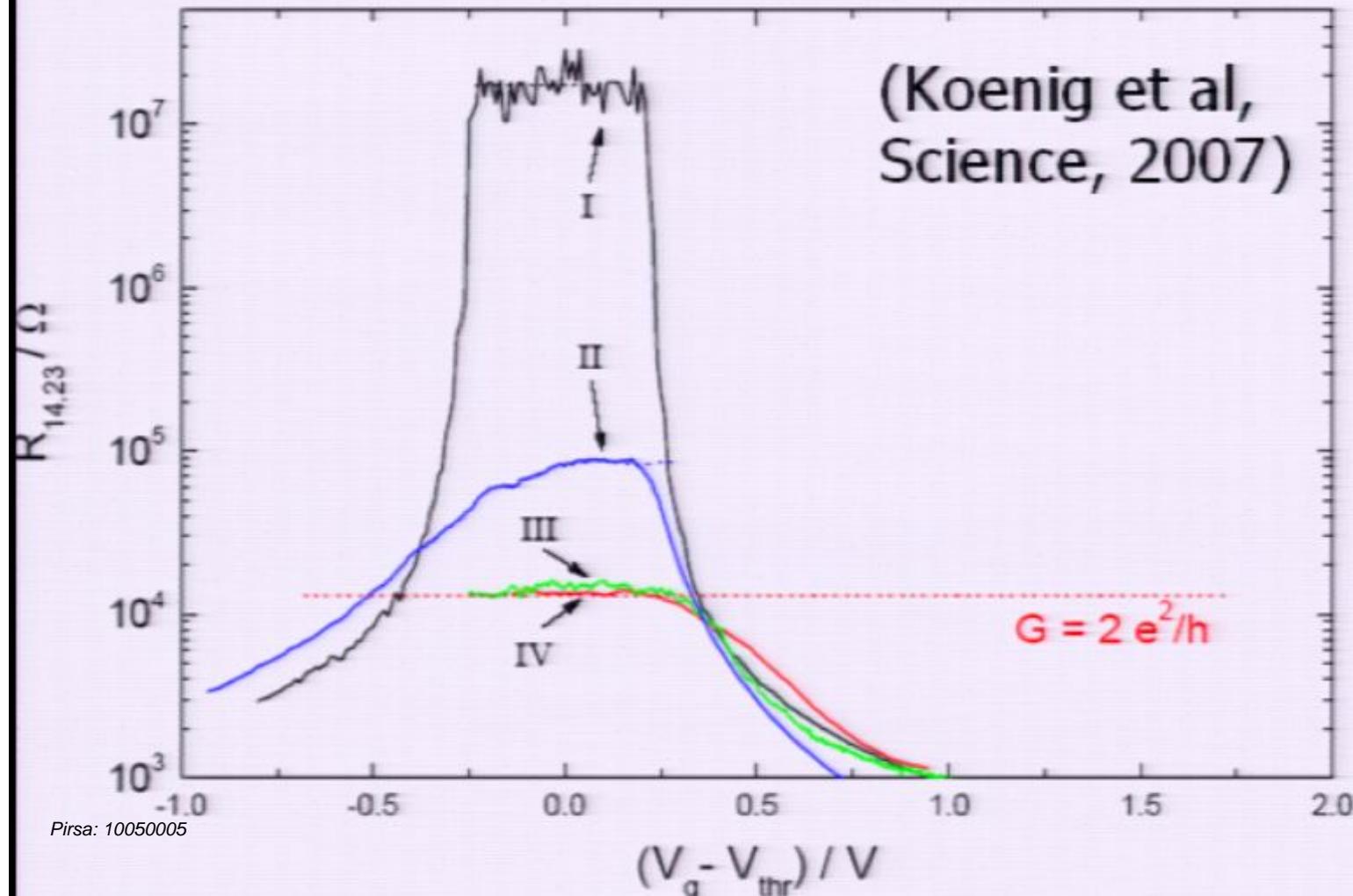
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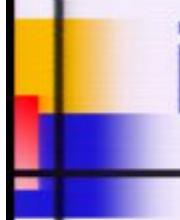
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# Experiment



- I.  $d < d_c$
- II.  $d < d_c$
- III.  $d > d_c$
- IV.  $d > d_c$

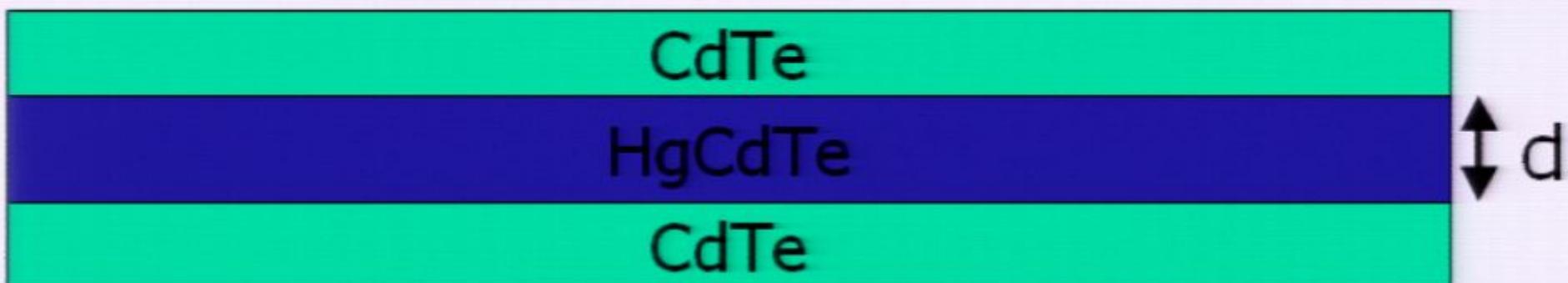


# Strongly interacting/correlated insulators?

- How can we make sense of these ideas in strongly correlated systems?
- Completely unexplored territory

# Experimental realization

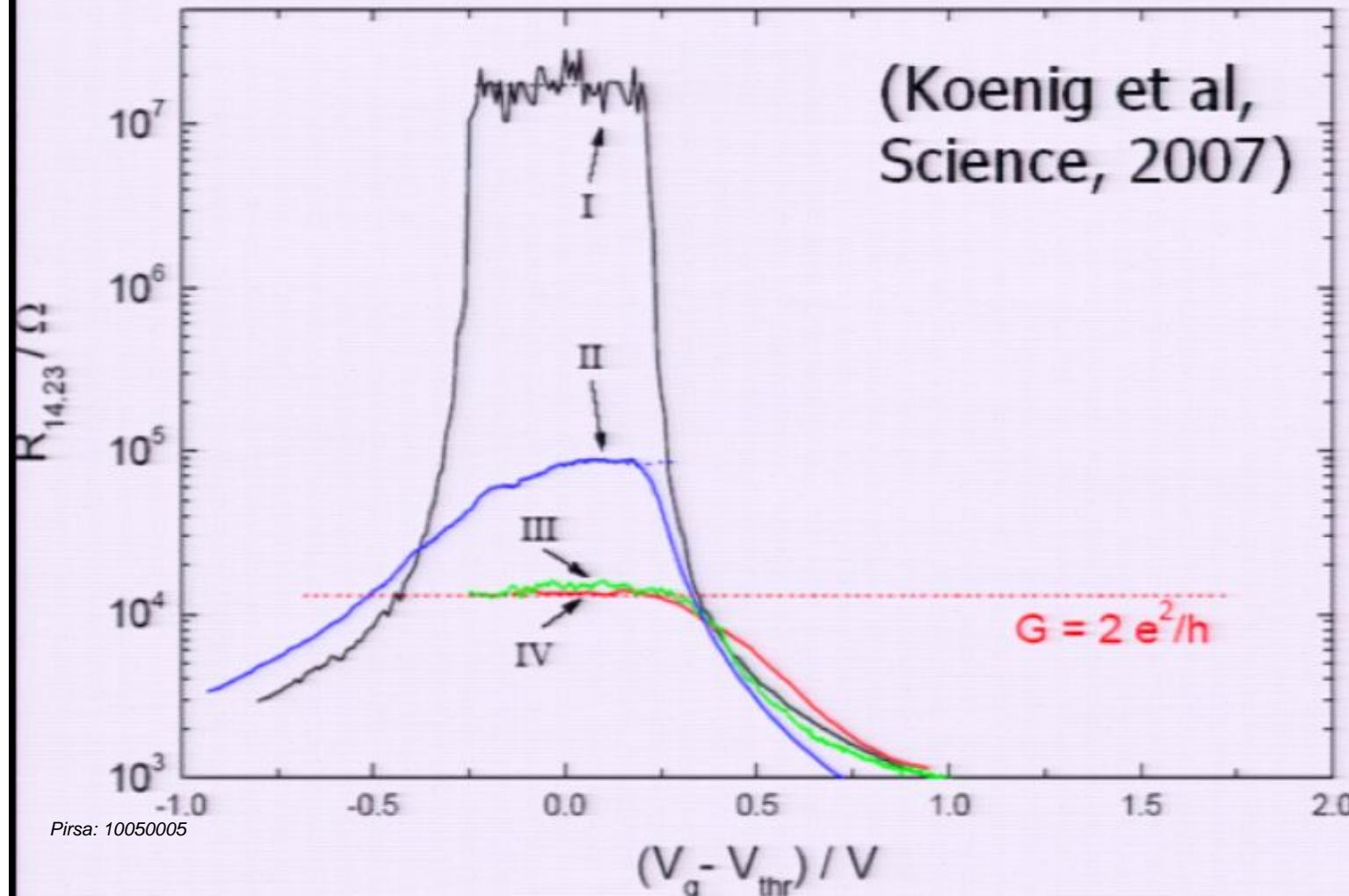
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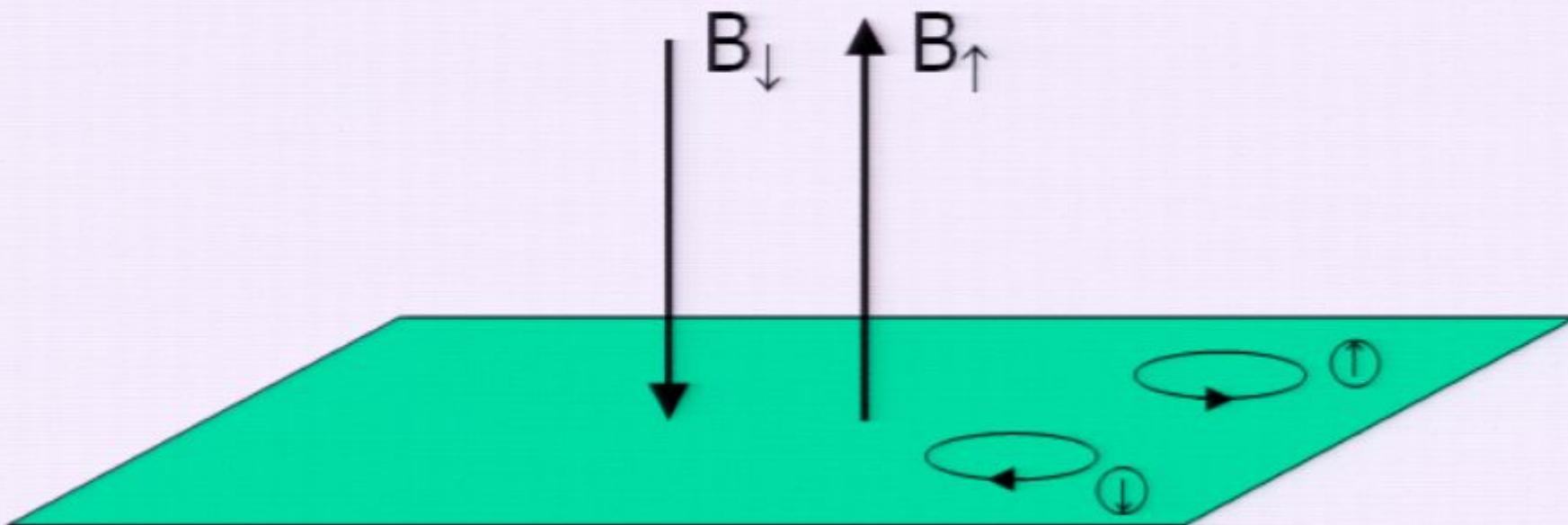


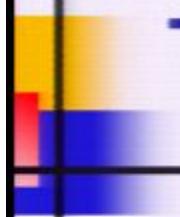
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# Toy model

Electrons in a spin-dependent magnetic field





## Toy model

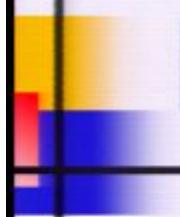
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- Previously considered integer filling  $\Rightarrow$  constructed band insulators
- Now generalize to get strongly correlated insulators

# Generalizing toy model construction

- Include interactions between electrons
  - Consider fractional filling
- ⇒ fractional quantum Hall states for each spin: e.g.

$$\Psi(z^\uparrow, z^\downarrow) = \prod (z_i^\uparrow - z_j^\uparrow)^3 \cdot \prod (z_i^*{}^\downarrow - z_j^*{}^\downarrow)^3$$

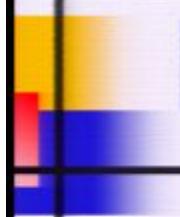


# Main result

(ML, Stern,  
PRL, 2009)

Edge mode if and only if  $\sigma_{\text{SH}}/e^*$  odd

$\sigma_{\text{SH}}$ : spin-Hall conductivity (in units of  $e/2\pi$ )  
 $e^*$ : elementary charge (in units of  $e$ )



# Main result

(ML, Stern,  
PRL, 2009)

Edge mode if and only if  $\sigma_{\text{SH}}/e^*$  odd

“Fractional topological insulator”

$\sigma_{\text{SH}}$ : spin-Hall conductivity (in units of  $e/2\pi$ )  
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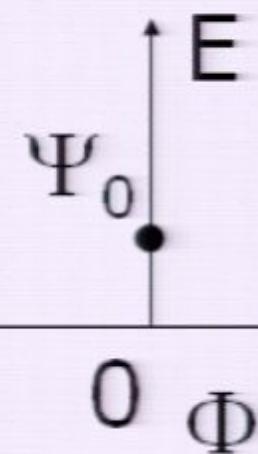
## Examples of criterion

	$\sigma_{\text{SH}}$	$e^*$	protected edge mode?
Laughlin $v = 1/3$	$1/3$	$1/3$	yes
Hierarchical $v = 2/5$	$2/5$	$1/5$	no
Strong pairing $v = 1/2$	$1/2$	$1/4$	no

# Flux insertion argument: integer case



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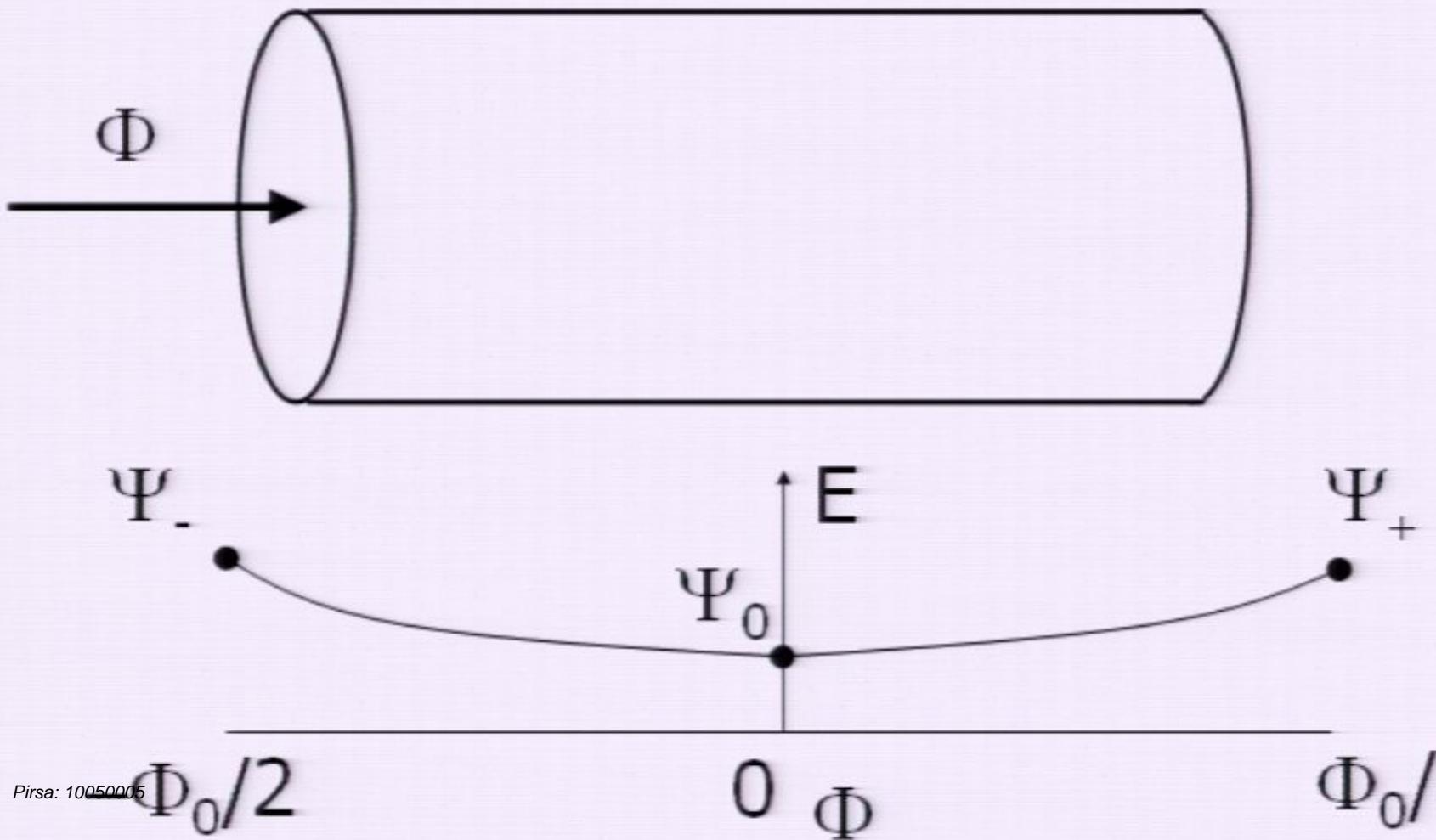


# Flux insertion argument: integer case

Notice that:

1.  $\Psi_{\pm}$  are eigenstates of same Hamiltonian
  2.  $\Psi_+ = T \Psi_- \Rightarrow E_+ = E_-$
- $\Rightarrow \Psi_{\pm}$  are degenerate eigenstates at  $\Phi_0/2$

# Flux insertion argument: integer case



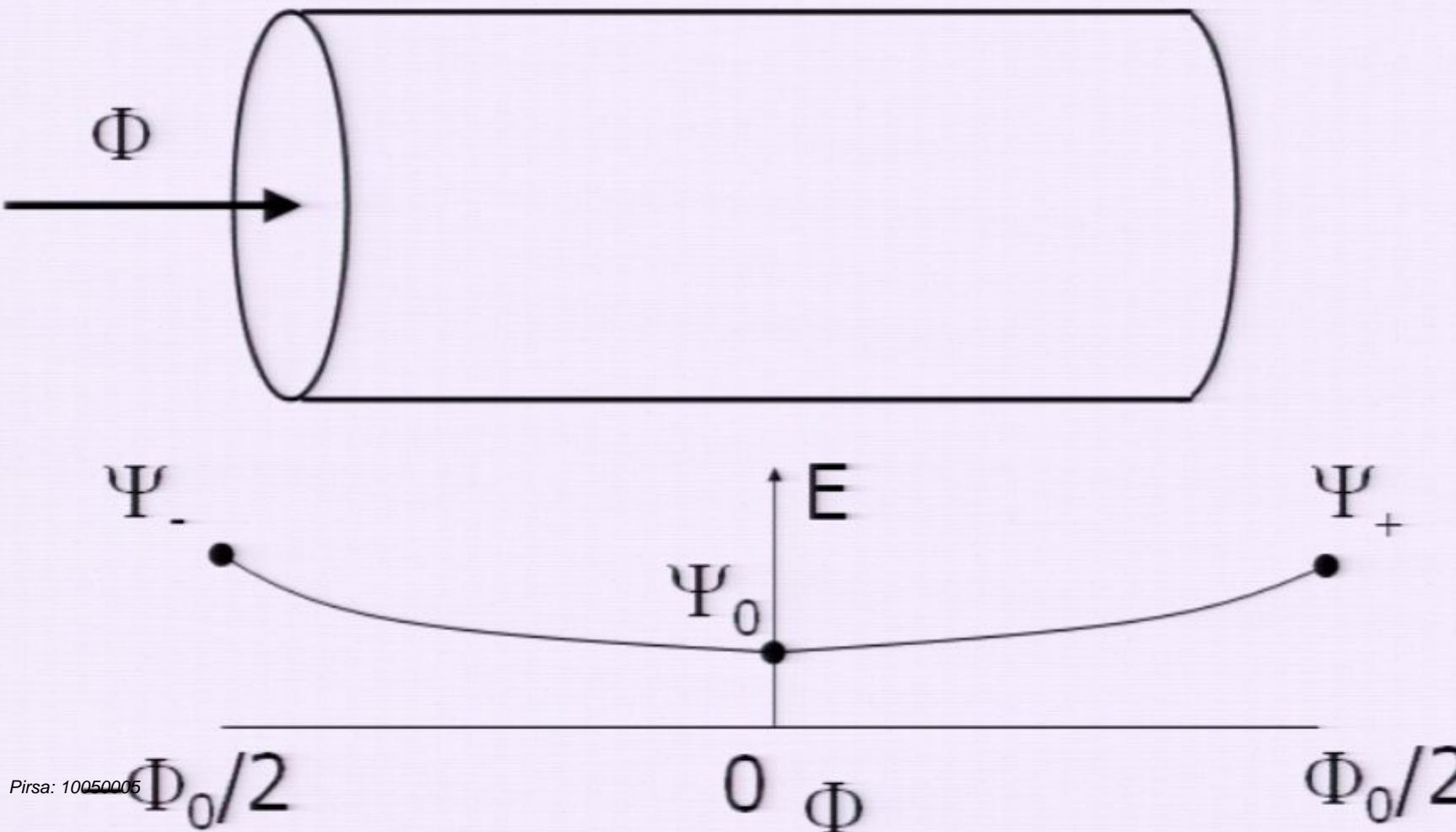


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$\Rightarrow \Psi_{\pm}$  are degenerate eigenstates at  $\Phi_0/2$

# Flux insertion argument: integer case

Now add an arbitrary TRI perturbation

What happens to the degeneracy  
between  $\Psi_{\pm}$  at  $\Phi_0/2$ ?

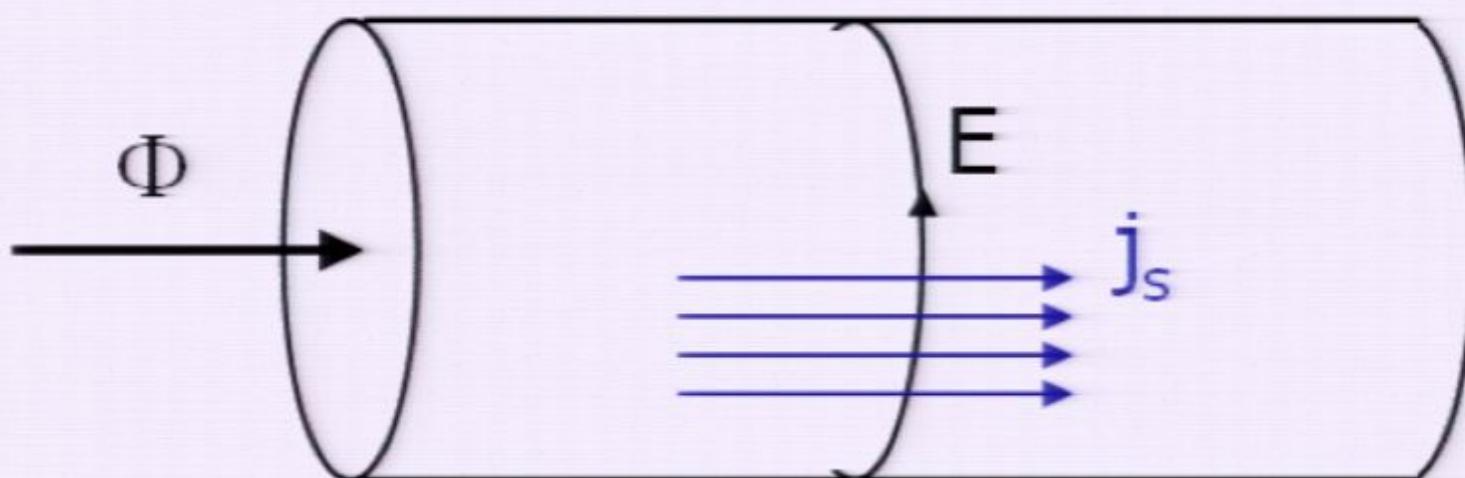
# Flux insertion argument: integer case

To go from  $\Psi_-$  to  $\Psi_+$ , insert  $\Phi_0$ :



# Flux insertion argument: integer case

To go from  $\Psi_-$  to  $\Psi_+$ , insert  $\Phi_0$ :



Transfers  $n$  spin-ups from left to right,  
 $n$  spin-downs from right to left

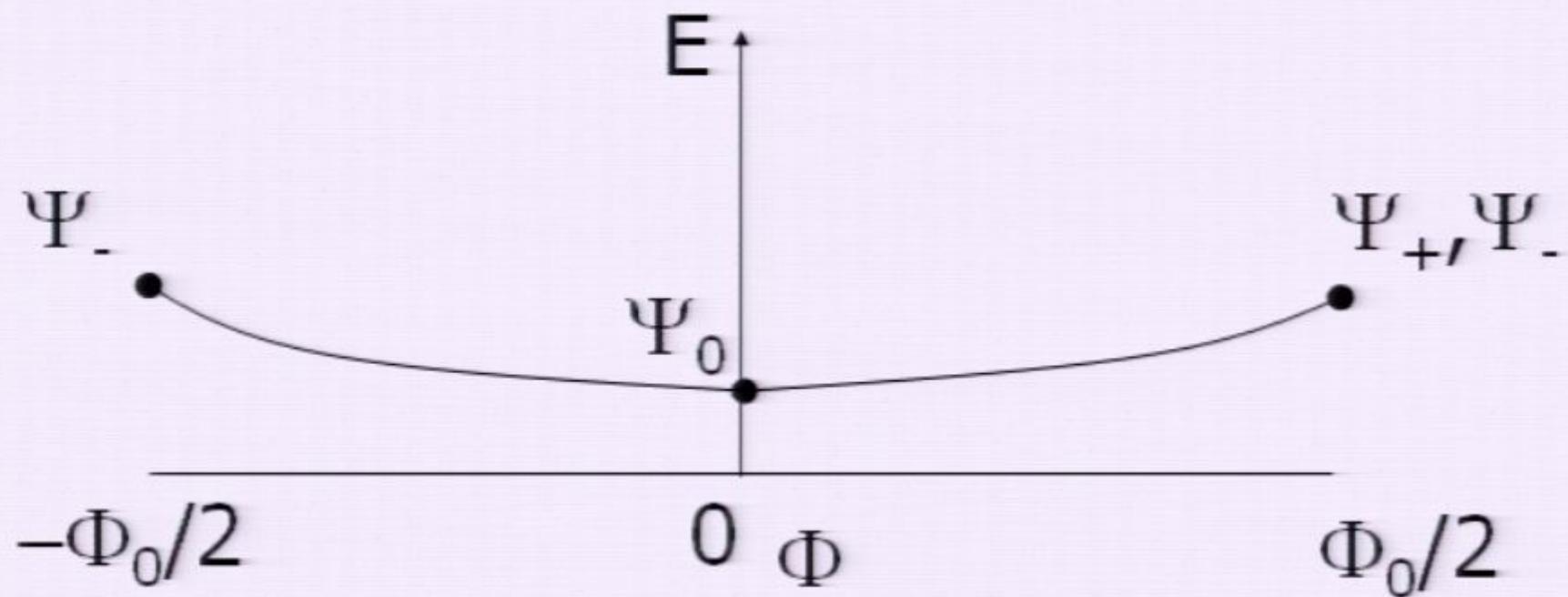
# Flux insertion argument: integer case

$\Psi_{\pm}$  differ by flipping n spins at each edge.

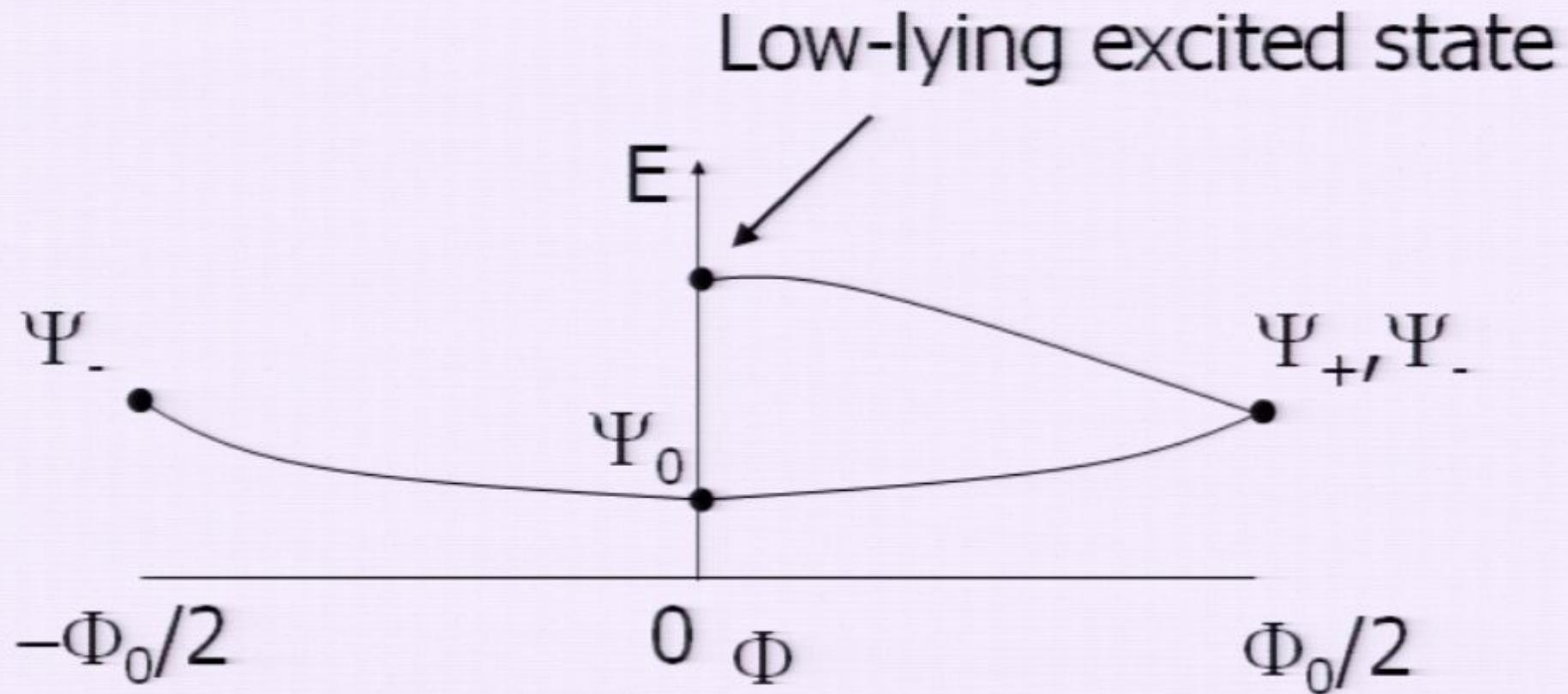
n odd  $\Rightarrow \Psi_{\pm}$  are Kramers pair

$\Rightarrow$  degeneracy at  $\Phi_0/2$  can't split

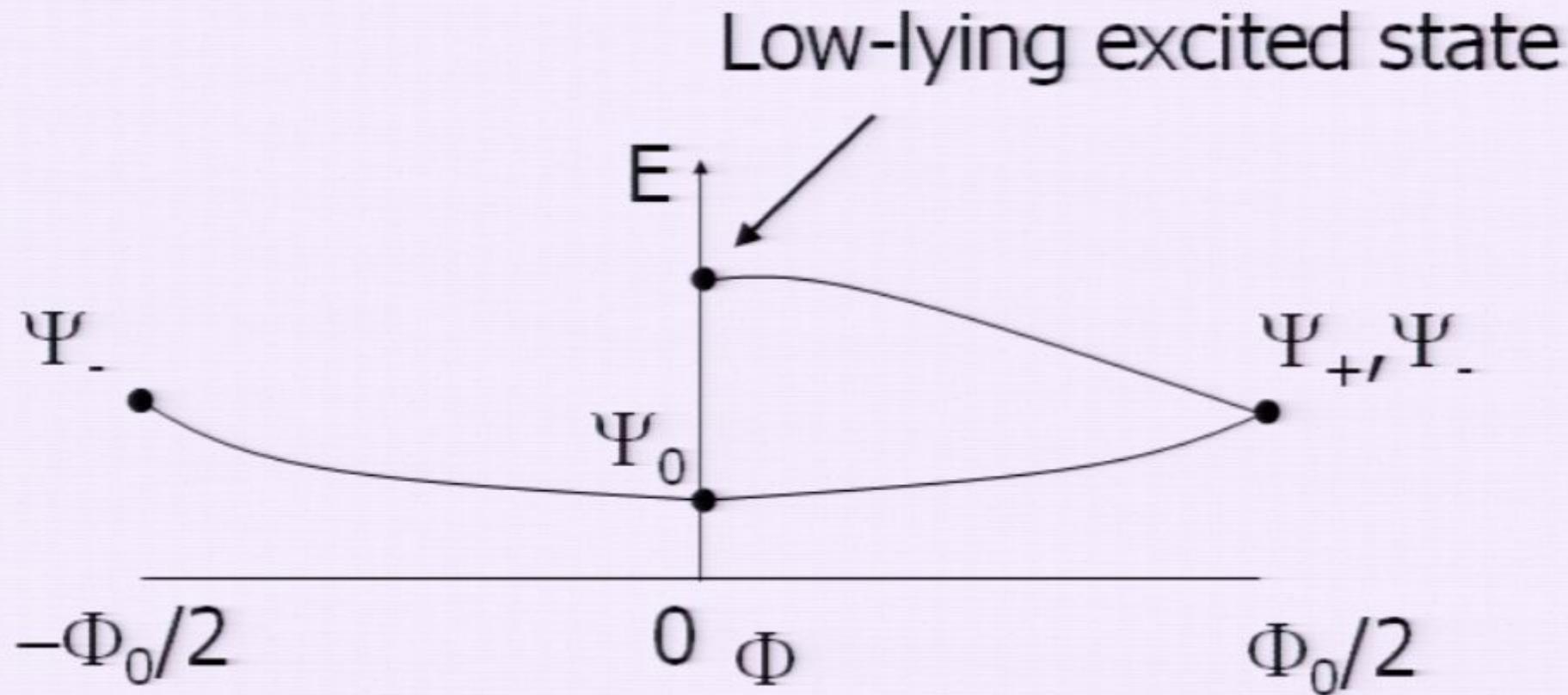
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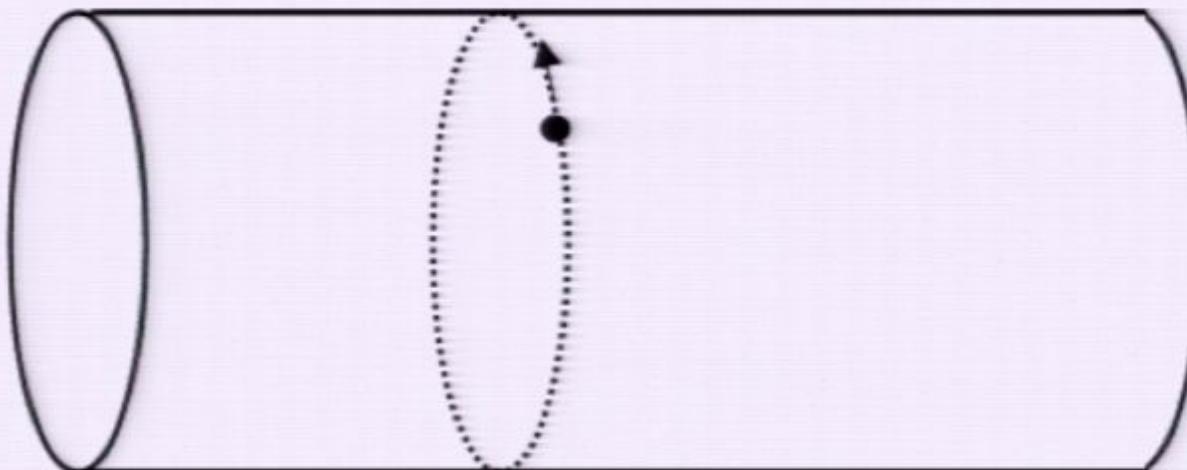
## Fractional case

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- Subtlety: always low-lying states on cylinder, even if edge is gapped
- Correspond to different “topological sectors”

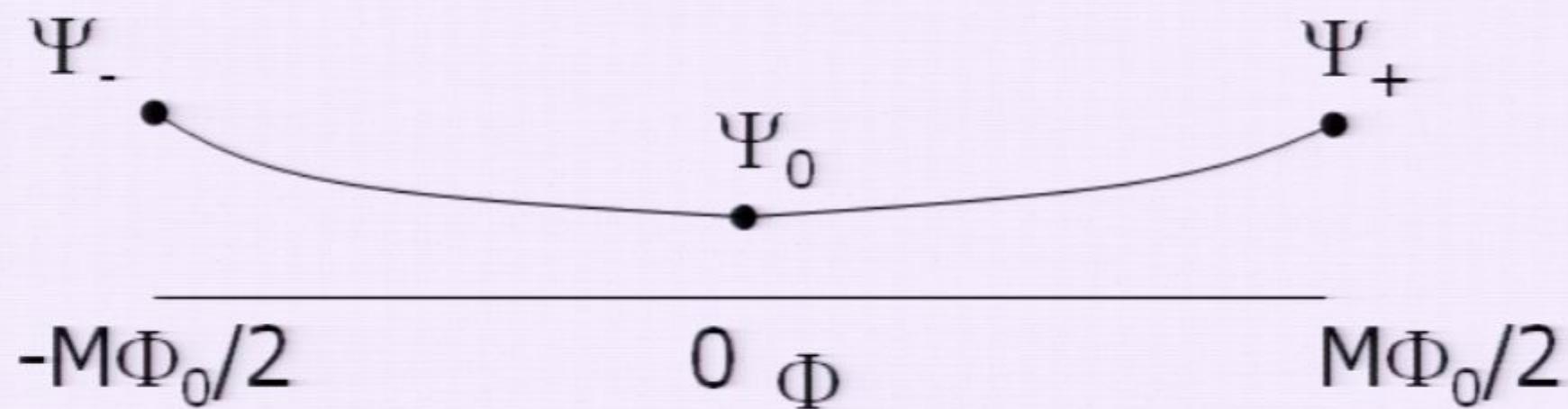
## Fractional case

Different topological sectors distinguished by Berry phases for quasiparticle transport:



# General flux insertion argument

Insert  $M\Phi_0/2$  flux:



Choose  $M$  large enough so that  $\Psi_{\pm}$  are in same sector

# General flux insertion argument

What is M?

Can show

$$M = 1/e^*$$

where  $e^*$  is elementary charge

# General flux insertion argument

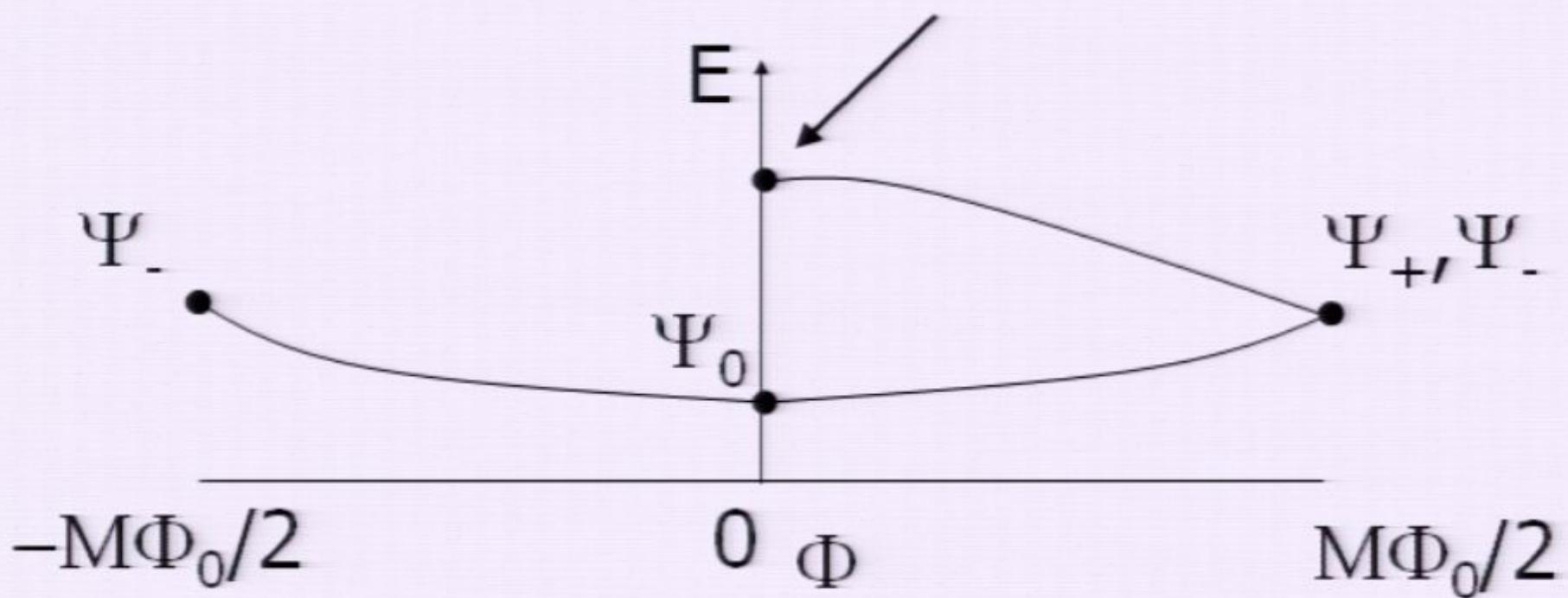
$\Psi_{\pm}$  differ by flipping  $\sigma_{\text{SH}} \cdot \mathbf{M} = \sigma_{\text{SH}}/e^*$  spins

$\sigma_{\text{SH}}/e^*$  odd  $\Rightarrow \Psi_{\pm}$  Kramers pair

$\Rightarrow$  degeneracy at  $M\Phi_0/2$   
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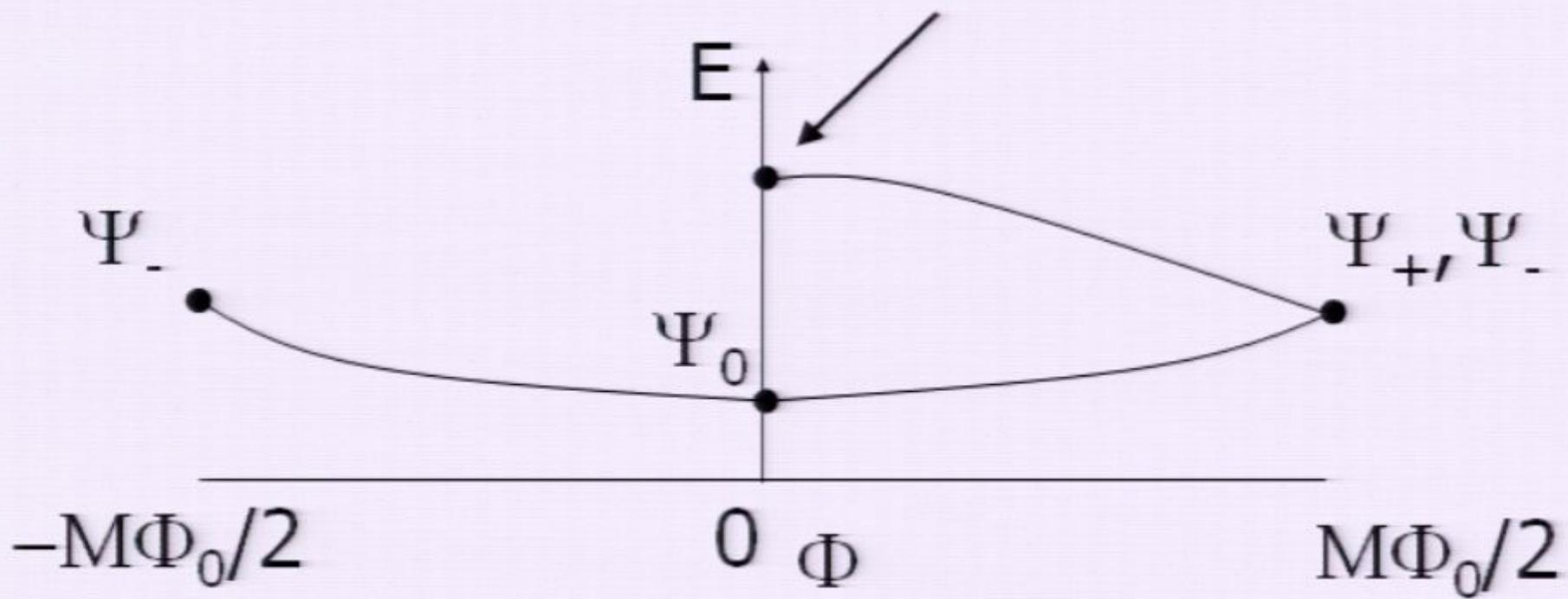
# Flux insertion argument: integer case

Low-lying excited state



# Flux insertion argument: integer case

Low-lying excited state





# Summary

- Generalized topological insulators to strongly interacting systems.
- Derived general criterion:

Fractional topological insulator iff  
 $\sigma_{\text{SH}}/e^*$  is odd