

Title: Deformations of General Relativity

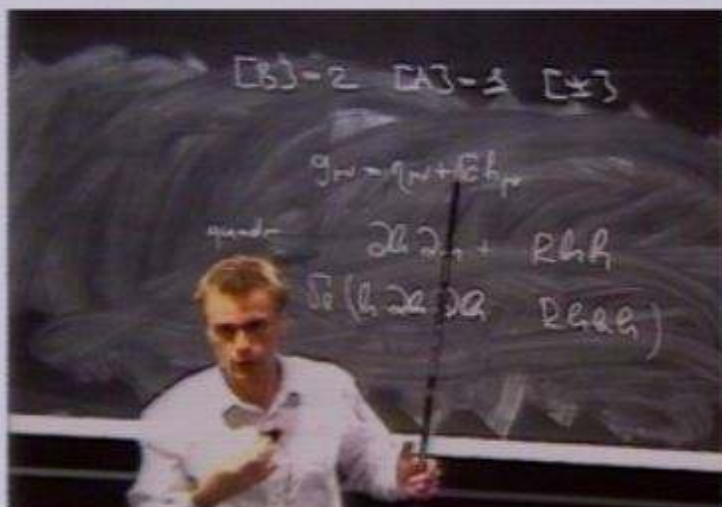
Date: May 05, 2010 04:00 PM

URL: <http://pirsa.org/10050002>

Abstract: I will describe a very special (infinite-parameter) family of gravity theories that all describe, exactly like General Relativity, just two propagating degrees of freedom. The theories are obtained by generalizing Plebanski's self-dual (chiral) formulation of GR. I will argue that this class of gravity theories provides a potentially powerful new framework for testing the asymptotic safety conjecture in quantum gravity.

Deformations of GR

Kirill Krasnov (Nottingham)
Perimeter Institute, May 5 2010



Perimeter talk on:
Renormalizable non-metric
quantum gravity?
November 30, 2006

<http://pirsa.org/06110041>

The present talk is about the same theory

Much better understood, many more reasons
to be interested in it!

Will only discuss aspects of relevance for quantum
gravity, ignoring other, e.g. modified gravity

Outline

- Introduction: Perturbative quantum gravity, non-renormalizability, asymptotic safety
- Deformations of GR - an infinite-parametric class of gravity theories with 2 propagating DOF
- On the UV fixed point

Part I: Asymptotic Safety

Effective Field Theory:

Standard Model plus GR Lagrangian - only the first few terms of an infinitely complicated Lagrangian comprising all terms compatible with symmetries

Explains why renormalizable is what is relevant at low energies

Parameters of the EFT Lagrangian - coupling constants - are energy scale dependent

Dimensionless couplings $\check{g}(\mu)$
that measure interaction
strength at given energy
scale μ

$$\check{g}(\mu) := g\mu^{-[g]}$$

g does not run $\Rightarrow \check{g}(\mu) \rightarrow 0$ as $\mu \rightarrow 0$
 $[g] < 0$

Non-renormalizable interactions are irrelevant at low energies

**Renormalizability is not fundamental,
but is a low-energy phenomenon**

Question of Quantum Gravity:

What provides a UV completion of the EFT Lagrangian of Standard Model + Gravity?

Standard expectation is that new DOF become relevant at the cutoff scale: pions \Rightarrow quarks and gluons

Possible alternative: Asymptotic Safety

$$\check{g}(\mu) \rightarrow \check{g}_* \quad \text{as} \quad \mu \rightarrow \infty$$

\check{g} does not run

plus finite number of attractive directions for predictive power

An AS theory is UV safe without need for new DOF

There is now more and more evidence that gravity (plus matter) may be an AS theory

Perimeter Conference on: Asymptotic Safety-30 Years After

<http://pirsa.org/C09025>

Drawbacks:

- In gravity the UV fixed point is necessarily a non-trivial one $\check{G}_* \neq 0$ (the trivial fixed point is the IR one) \Rightarrow UV physics is non-perturbative in nature
- By no means guaranteed that the UV theory is unitary (e.g. R^2 terms)
- Seems impossible to prove AS, as the RG flow in an infinite-dimensional space of couplings need to be considered

At least some of the difficulties would be removed if had a compact description of a “healthy” and sufficiently large closed under renormalization) class of Lagrangians.

The class of actions describing deformations of GR may be exactly such class, and has a potential to provide a powerful new perspective on asymptotic safety.

Deformations of GR are obtained by generalizing the so-called **Plebanski** self-dual (chiral) formulation.

Leibanski formulation of general relativity:

Consider a tetrad θ_{μ}^I for metric $g_{\mu\nu} = \theta_{\mu}^I \theta_{\nu}^J \eta_{IJ}$

where $\eta_{IJ} = \text{diag}(-1, 1, 1, 1)$

Take

$$\Sigma^a := i\theta^0 \wedge \theta^a - \frac{1}{2}\epsilon^{abc}\theta^b \wedge \theta^c. \quad \begin{array}{l} I = (0, a) \\ a = 1, 2, 3 \end{array}$$

Properties:

$$\Sigma^a \text{ are self-dual } \frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\Sigma_{\rho\sigma}^a = i\Sigma_{\mu\nu}^a$$

$$\Sigma^a \wedge \Sigma^b \sim \delta^{ab}$$

$$\Sigma^a \wedge (\Sigma^b)^* = 0$$

Consider γ^a such that

$$d\Sigma^a + \epsilon^{abc} \gamma^b \wedge \Sigma^c = 0$$

can solve for γ^a explicitly

$$\gamma_{\mu}^a = \frac{1}{2} \Sigma^{a\rho\nu} \Sigma_{\mu\nu}^b \nabla^{\alpha} \Sigma_{\alpha\rho}^b.$$

Here indices are raised and lowered with the metric and

$$\nabla_{\mu} : \nabla_{\mu} g_{\nu\rho} = 0$$

Not hard to check that

$$\gamma^a = i\Gamma^{0a} - \frac{1}{2}\epsilon^{abc}\Gamma^{bc}$$

where Γ^{IJ} is the Levi-Civita connection

$$d\theta^I + \Gamma^I_J \wedge \theta^J = 0$$

Define $F^a = d\gamma^a + \frac{1}{2}\epsilon^{abc}\gamma^b \wedge \gamma^c$

Not hard to show that $F^a = iF^{0a} - \frac{1}{2}\epsilon^{abc}F^{bc}$

where $F^{IJ} = d\Gamma^{IJ} + \Gamma^{IK} \wedge \Gamma_K^J$

Einstein equations (vacuum) can be stated as:

$F^{(IJ)}$ self-dual is self-dual as a two-form

Thus $F^a = \Phi^{ab} \Sigma^b$

where $\Psi^{ab} := \Phi^{ab} - \frac{1}{3} \delta^{ab} \text{Tr}(\Phi)$ is arbitrary

$$\text{Tr}(\Phi) \sim \Lambda$$

Gives a procedure for computing curvature components that is more efficient than the tetrad method!

E.g. Schwarzschild

$$ds^2 = -f^2(r)dt^2 + g^2(r)dr^2 + r^2 d\Omega^2$$

$$e^t = f(r)dt, \quad e^r = g(r)dr, \quad e^\theta = r d\theta, \quad e^\phi = r \sin(\theta) d\phi$$

$$\Sigma^1 = i f g dt \wedge dr - r^2 \sin(\theta) d\theta \wedge d\phi,$$

$$\Sigma^2 = i f r dt \wedge d\theta - g r \sin(\theta) d\phi \wedge dr,$$

$$\Sigma^3 = i f r \sin(\theta) dt \wedge d\phi - g r dr \wedge d\theta$$

$$A^1 = \frac{if'}{g} dt + \cos(\theta) d\phi, \quad A^2 = -\frac{\sin(\theta) d\phi}{g}, \quad A^3 = \frac{d\theta}{g}$$

$$F^1 = -\frac{1}{2fg} \left(\frac{f'}{g} \right)' (\bar{\Sigma}^1 + \Sigma^1) - \frac{1}{2r^2} \left(1 - \frac{1}{g^2} \right) (\bar{\Sigma}^1 - \Sigma^1),$$

$$F^2 = -\frac{1}{2g^2 r} \left(\frac{g'}{g} (\bar{\Sigma}^2 - \Sigma^2) + \frac{f'}{f} (\bar{\Sigma}^2 + \Sigma^2) \right),$$

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$$F^1 = -\frac{1}{2fg} \left(\frac{f'}{g} \right)' (\bar{\Sigma}^1 + \Sigma^1) - \frac{1}{2r^2} \left(1 - \frac{1}{g^2} \right) (\bar{\Sigma}^1 - \Sigma^1),$$

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Can reformulate everything in terms of two-forms -
do not need a metric!

Lemma: Given a triple Σ^a of
two-forms satisfying:

$$\begin{aligned}\Sigma^a \wedge \Sigma^b &\sim \delta^{ab}, \\ \Sigma^a \wedge (\Sigma^b)^* &= 0, \\ \text{Re}(\Sigma^a \wedge \Sigma_a) &= 0\end{aligned}$$

there exists a unique real Lorentzian
signature metric such that

$$\Sigma^a := i\theta^0 \wedge \theta^a - \frac{1}{2}\epsilon^{abc}\theta^b \wedge \theta^c.$$

Idea of the proof:

Declare Σ^a to span the space of self-dual two forms wrt some metric



fixes the conformal metric uniquely

$$\sqrt{-g}g_{\mu\nu} \sim \tilde{\epsilon}^{\alpha\beta\rho\sigma} \Sigma_{\mu\alpha}^a \Sigma_{\nu\beta}^b \Sigma_{\rho\sigma}^c \epsilon^{abc}$$

then use a multiple of $\frac{1}{i} \Sigma^a \wedge \Sigma_a$

as the volume form

Action principle (Plebanski)

$$S = i \int \Sigma^a \wedge F^a - \frac{1}{2} \Psi^{ab} \Sigma^a \wedge \Sigma^b$$

field equations:

$$\boxed{\Psi} \quad \Sigma^a \wedge \Sigma^b \sim \delta^{ab}$$

$$\boxed{\gamma} \quad D_\gamma \Sigma^a = 0$$

$$\boxed{\Sigma} \quad F^a = \Psi^{ab} \Sigma^b$$

$$\Lambda = 0$$

for simplicity and the
reality conditions

$$\Sigma^a \wedge (\Sigma^b)^* = 0,$$

$$\text{Re}(\Sigma^a \wedge \Sigma_a) = 0$$

to be imposed by hand

Constrained BF theory

$$S_{BF} = \int B^a \wedge F^a$$

The way propagating DOF are introduced is very interesting

$$S_{BF} = \int \tilde{E}^{ai} \dot{A}_i^a + A_0^a D_i \tilde{E}^{ai} + B_{0i}^a \tilde{\epsilon}^{ijk} F_{jk}^a$$

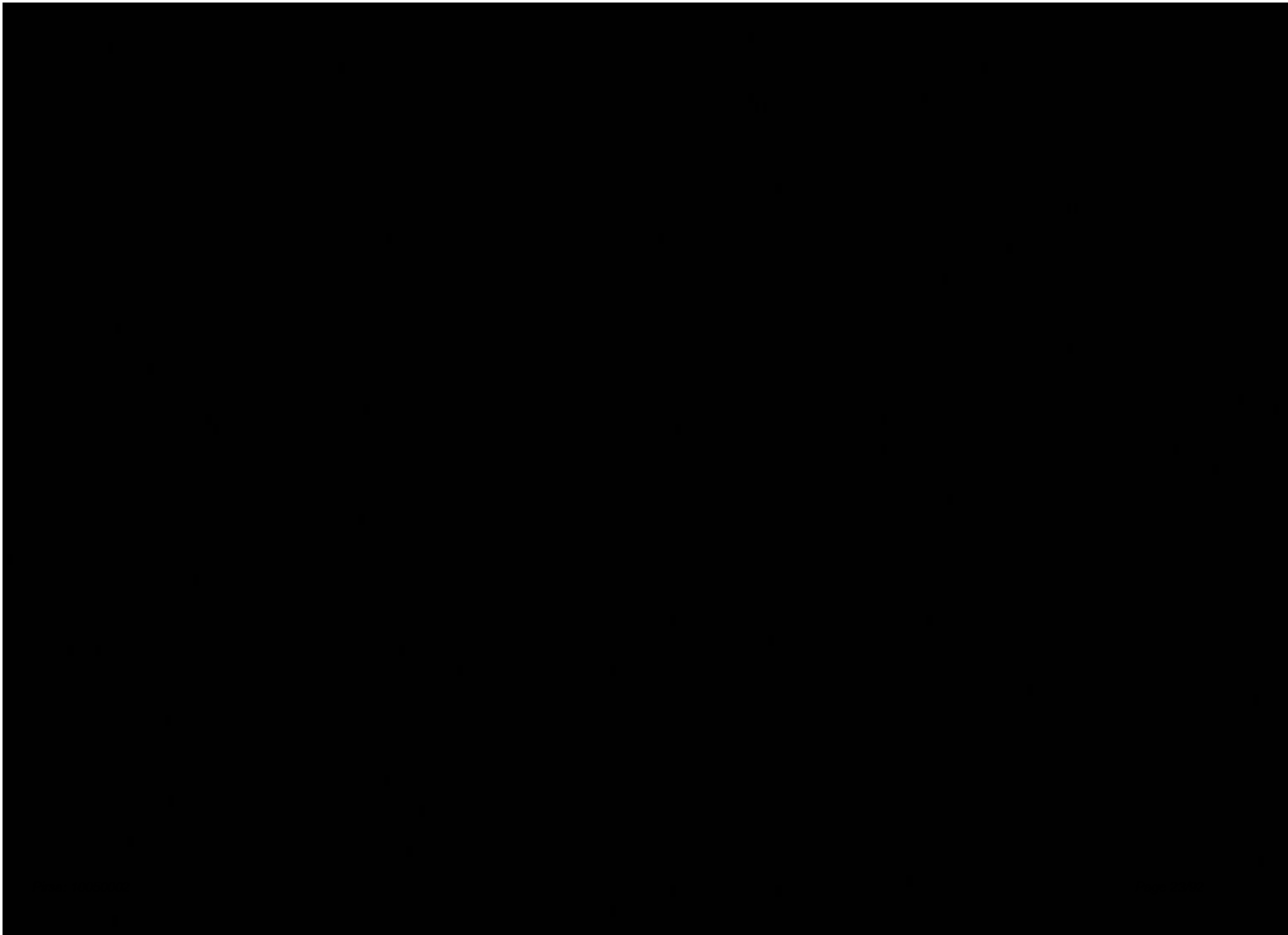
here $i = 1, 2, 3$ are spatial indices

B_{0i}^a are Lagrange multipliers for $F_{ij}^a = 0$

generators of

$$B^a \rightarrow B^a + d\eta^a$$

in Plebanski theory 5 out of 9 of these multipliers are set to zero by the Ψ^{ab} constraints, leaving only the diffeos (and $SO(3)$) as gauge



$$\exists^{abc} B_{ab}^i := \exists^{ci}$$

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Example:

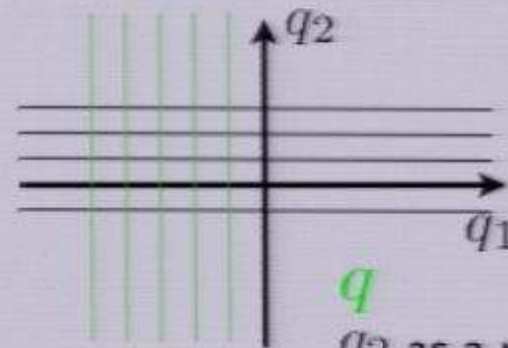
$$S = \int dt (p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2)$$

first class constraints: $p_1 \approx 0$ generate $q_1 \rightarrow q_1 + \lambda_1$
 $p_2 \approx 0$ $q_2 \rightarrow q_2 + \lambda_2$

can get a system with DOF via

$$S = \int dt (p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2 - \psi \lambda_2)$$

$$\boxed{\psi} \quad \begin{matrix} \lambda_1 \\ \lambda_2 = 0 \end{matrix}$$



to motivate a construction that follows consider
 a different way to get a system with DOF

$$S = \int dt (p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2 - V(\lambda_1, \lambda_2))$$

where $V(\alpha\lambda_1, \alpha\lambda_2) = \alpha V(\lambda_1, \lambda_2)$

introduce $r = \lambda_2/\lambda_1$

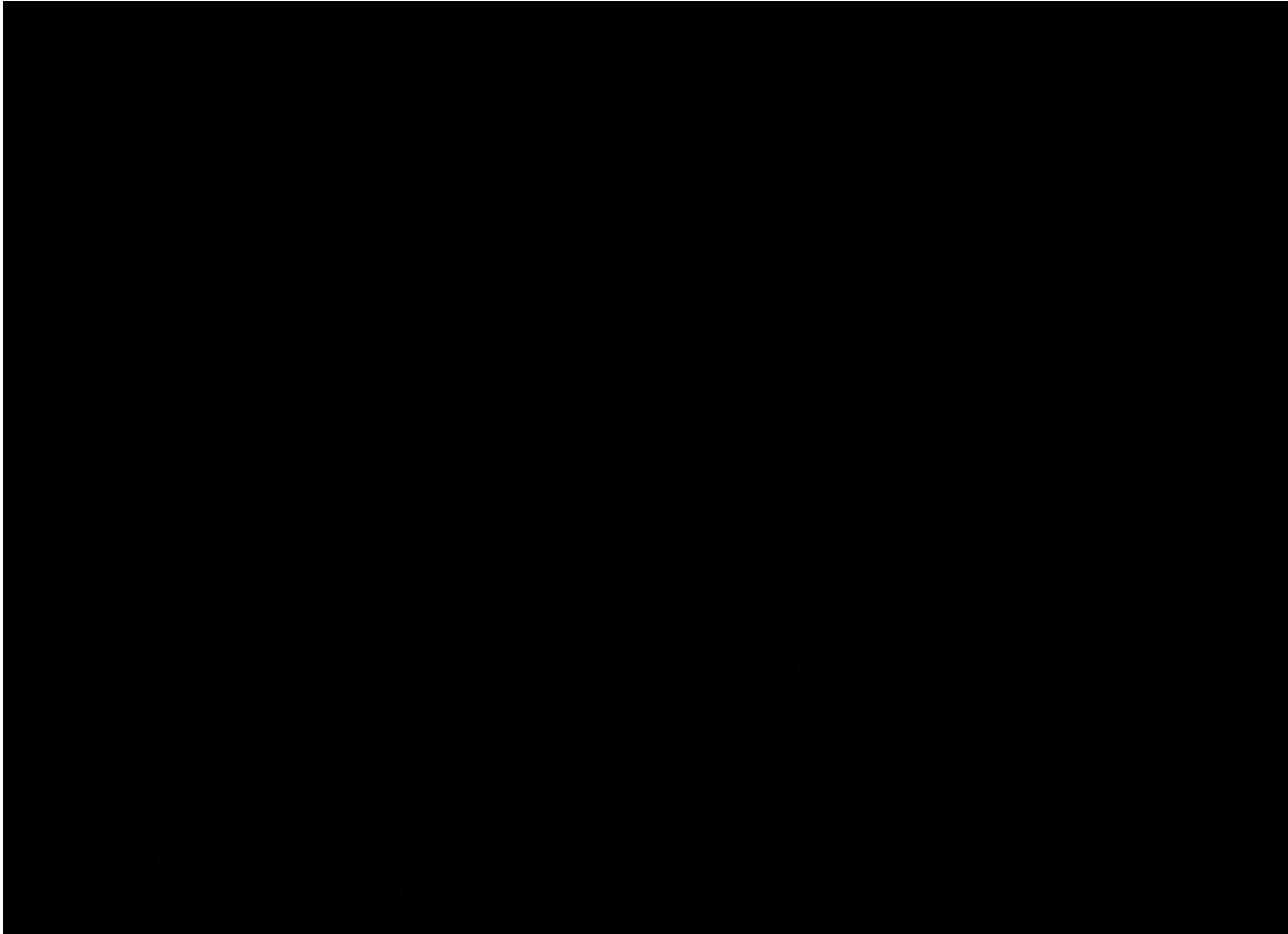
$$S = \int dt (p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1(p_1 + rp_2 + f(r)))$$

where $f(r) := V(1, r)$

r non-dynamical \Rightarrow

$$S = \int dt (p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1(p_1 + \tilde{f}(p_2)))$$

here $\tilde{f} = f - rf'$ Legendre transform



$$\overline{E}^{abc} B_{ab}^i := \overline{E}^{ci}$$

$$p_2 + f'(r) = 0$$

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Example:

$$f(r) = mr^2/2$$

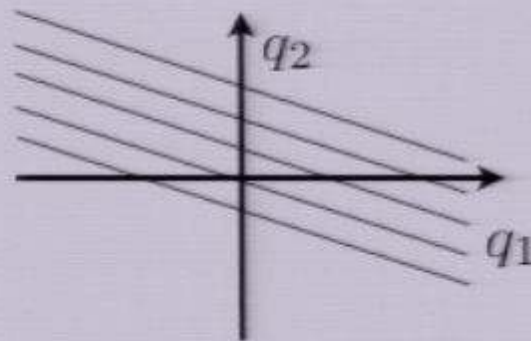
$$\Rightarrow \begin{aligned} r &= -p_2/m \\ \tilde{f}(p_2) &= -p_2^2/2m \end{aligned}$$

$$S = \int dt \left(p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1 \left(p_1 - \frac{p_2^2}{2m} \right) \right)$$

constraint generates

$$q_1 \rightarrow q_1 + \lambda_1$$

$$q_2 \rightarrow q_2 - \frac{p_2}{m} \lambda_1$$



$$p_2 \neq 0$$

$n \rightarrow \infty$ gives back
the original system

$n \rightarrow 0$ interchanges
the relevant vars

relevant coordinate

$$Q := q_2 + \frac{p_2}{m} q_1$$

$$\overline{E}^{abc} B_{ab}^i := \overline{F}^{ci}$$

$$p_z + f'(\Gamma) = 0$$

$$p_z + m\Gamma = 0$$

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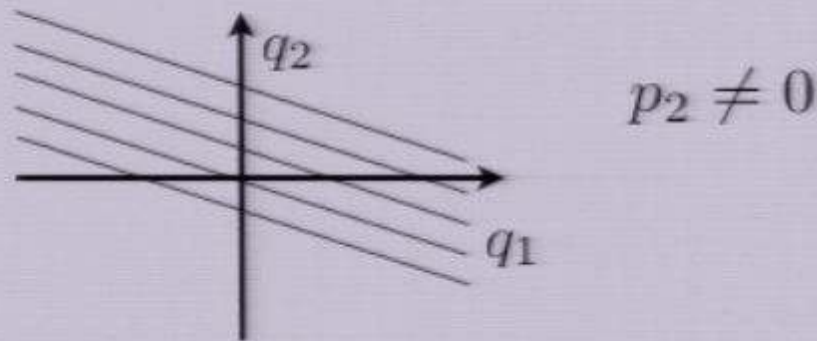
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Deformations of GR

$$S = \int B^i \wedge F^i + V(B^i \wedge B^j)$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$

$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

$$\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} = \tilde{X}^i$$

$$\epsilon^{abc} B_{ab}^i = F^ci$$

$$p_z + f'(r) = 0$$

$$p_z + m r = 0$$

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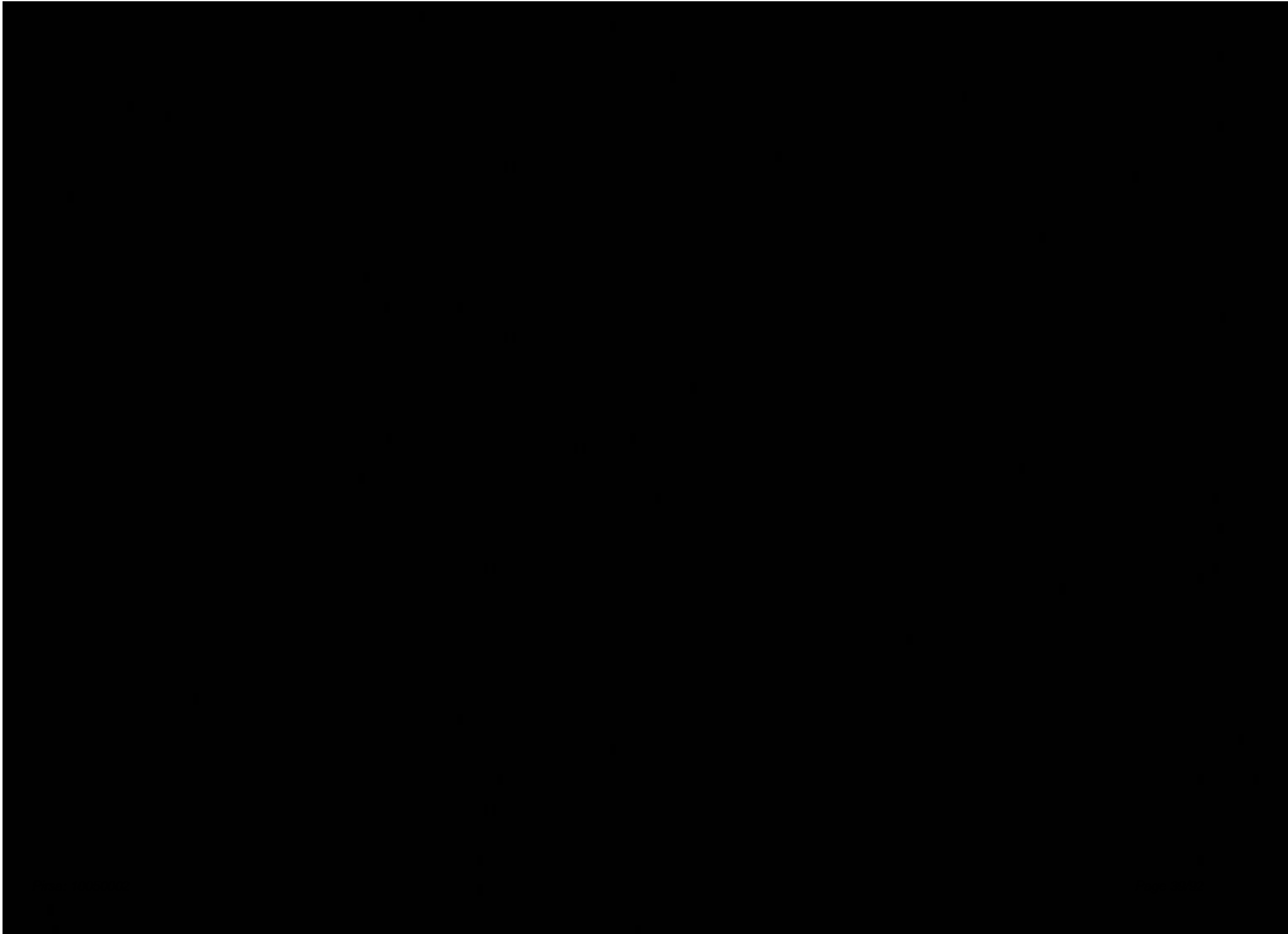
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Properties:

- Describes two propagating DOF for any $V : V'' \neq 0$ (complex for $SO(3, \mathbb{C})$)
- Deformations of GR, for any of the theories can be continuously deformed back to GR without changing dynamical content
- Can be shown to be about the conformal metric wrt which B^i are self-dual: matter moves along geodesics of one of the metric in this conformal class

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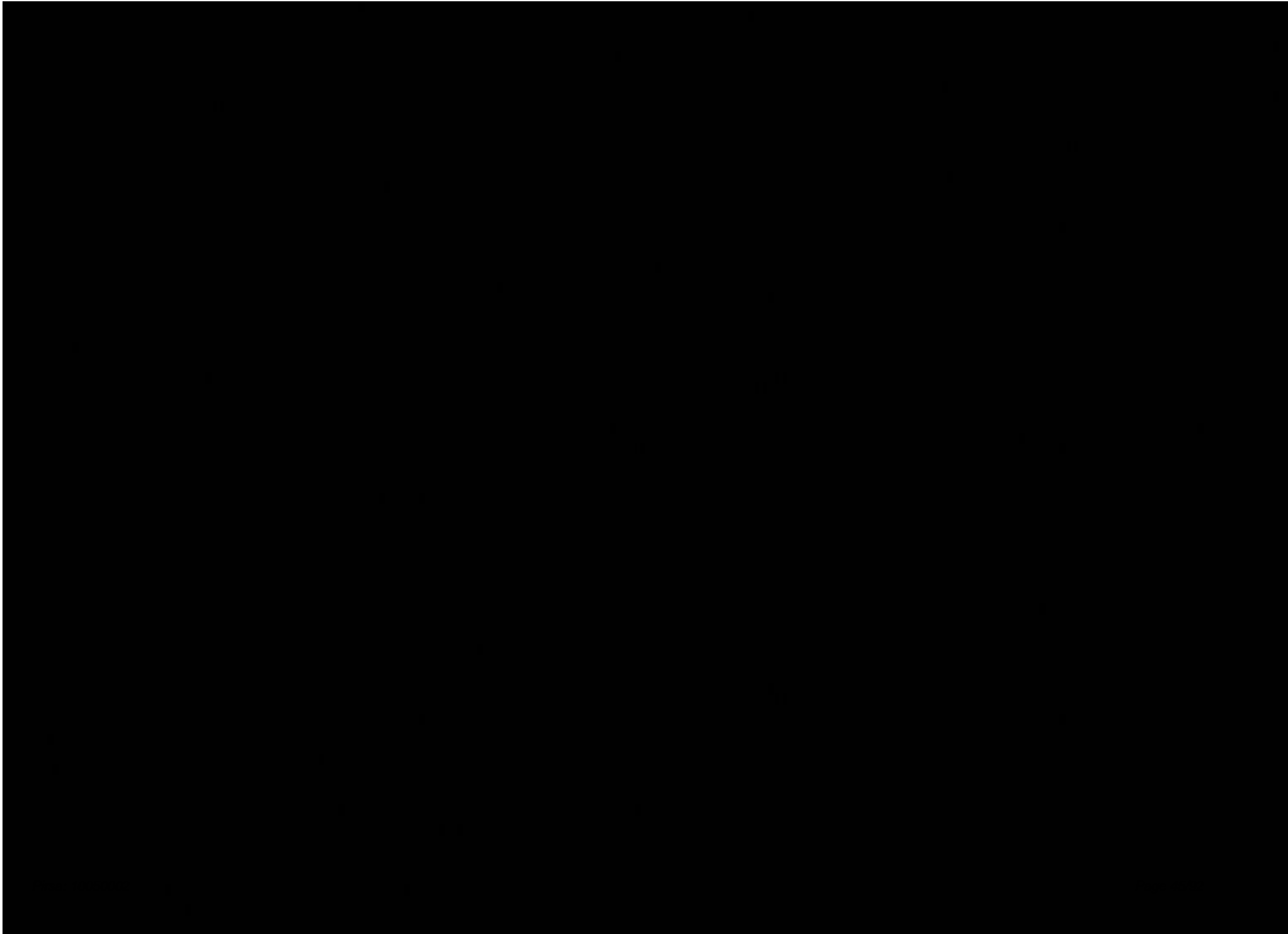
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Reality conditions:

$$B^i \wedge (B^j)^* = 0$$

plus a condition for
the volume element

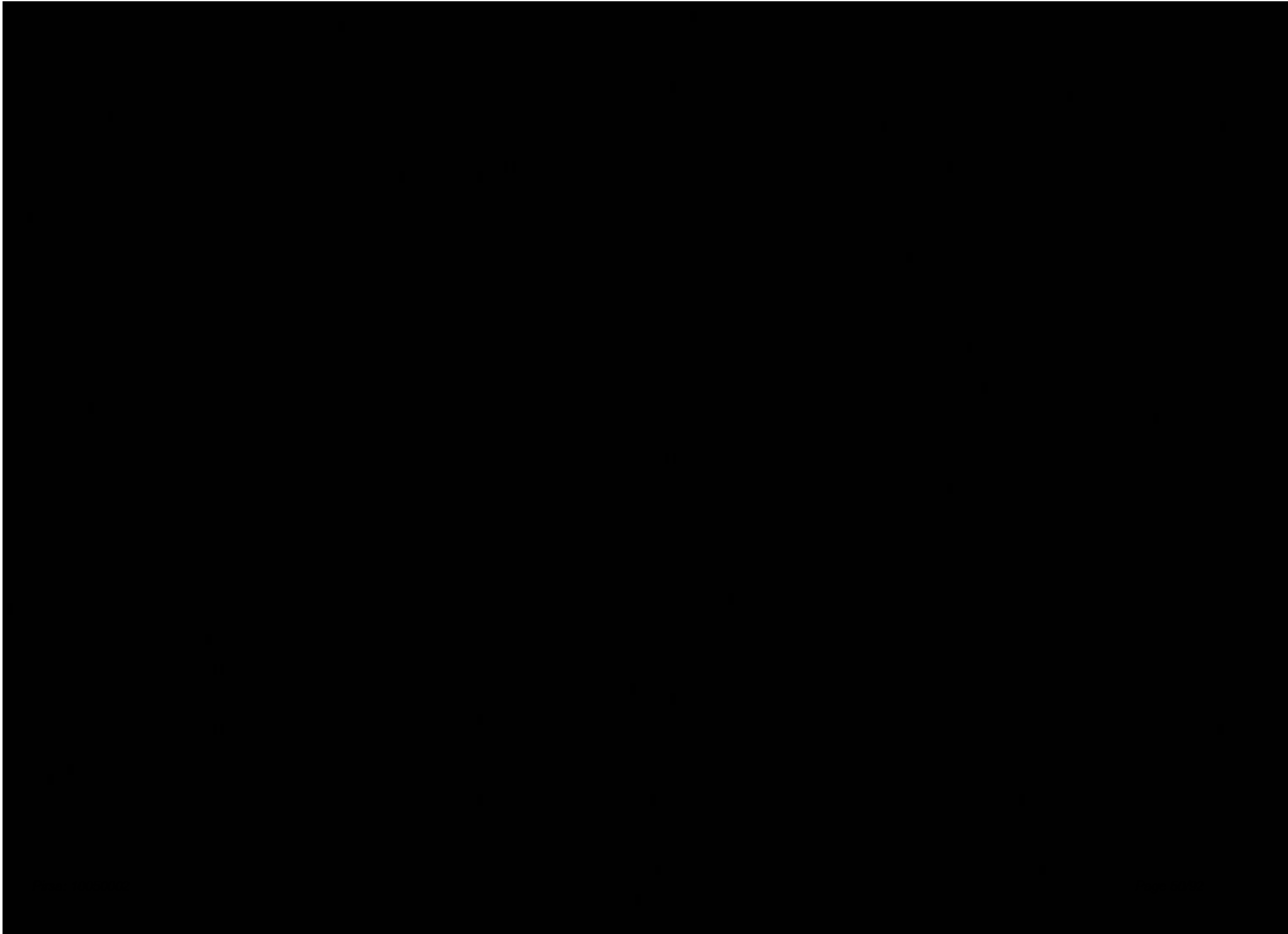
the physical metric is real

Unlike in the Plebanski case, this does not guarantee reality of the action.

Current prescription - take the real part of the arising complex action. Consistency?

A lot is known about the classical theory:

- spherically-symmetric problem can be solved exactly; spherical symmetry implies staticity; very interesting effects of singularity resolution inside BH's KK+Shtanov: 0705.2047, 0805.2668
- Newtonian limit - MOND-like (arbitrary function of second derivatives of the grav. potential)
- Linearized theory around Minkowski - gravitational waves unmodified Freidel: 0812.3200, KK: 0911.4903
- Friedman equations are unmodified KK+Shtanov: 1002.1210
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f (H)

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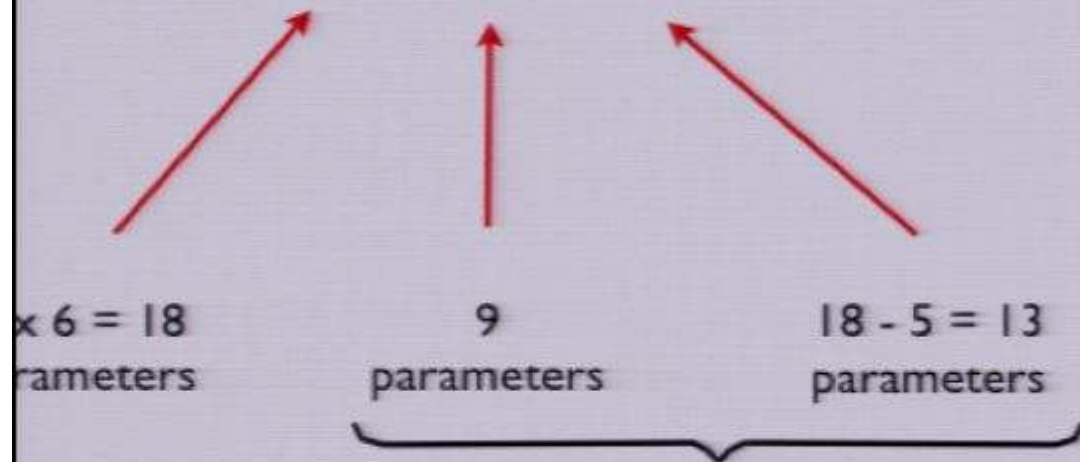
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Explicitly metric formulation is possible

B^i defines a (conformal) metric wrt which it is self-dual

construct Σ^a for this metric satisfying $\Sigma^a \wedge \Sigma^b \sim \delta^{ab}$

$$B^i = b_a^i \Sigma^a \quad \text{for some} \quad b_a^i \in \text{GL}(3, \mathbb{C})$$

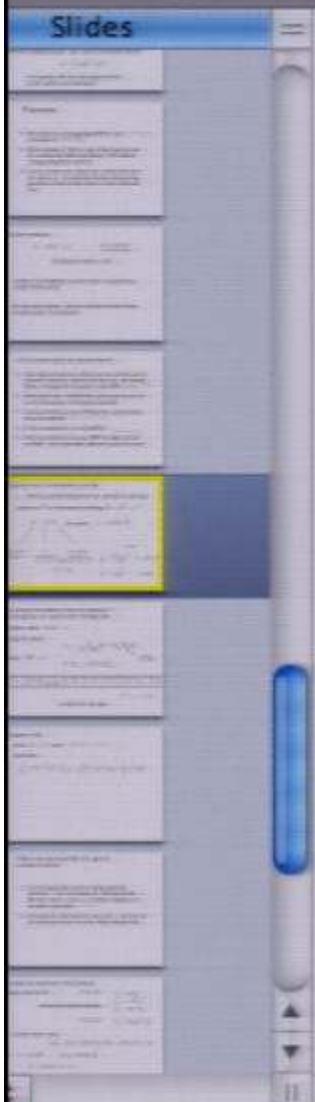


$$9 + 13 - 4 = 18$$

modulo

$$\begin{aligned}
 b_a^i &\rightarrow b_b^i (\Lambda^{-1})^b_a \\
 \Sigma^a &\rightarrow \Lambda^a_b \Sigma^b
 \end{aligned}
 \quad \Lambda \in \text{SO}(3, \mathbb{C})$$

$$\begin{aligned}
 b_a^i &\rightarrow \lambda^{-1} b_a^i \\
 \Sigma^a &\rightarrow \lambda \Sigma^a
 \end{aligned}
 \quad \lambda \in \mathbb{C}$$

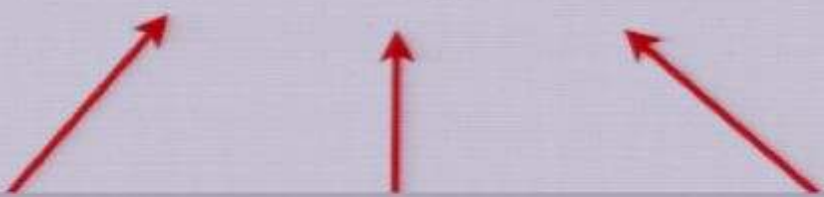


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Play the slideshow from the selected slide.

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$B^i = b_a^i \Sigma^a$ for some b



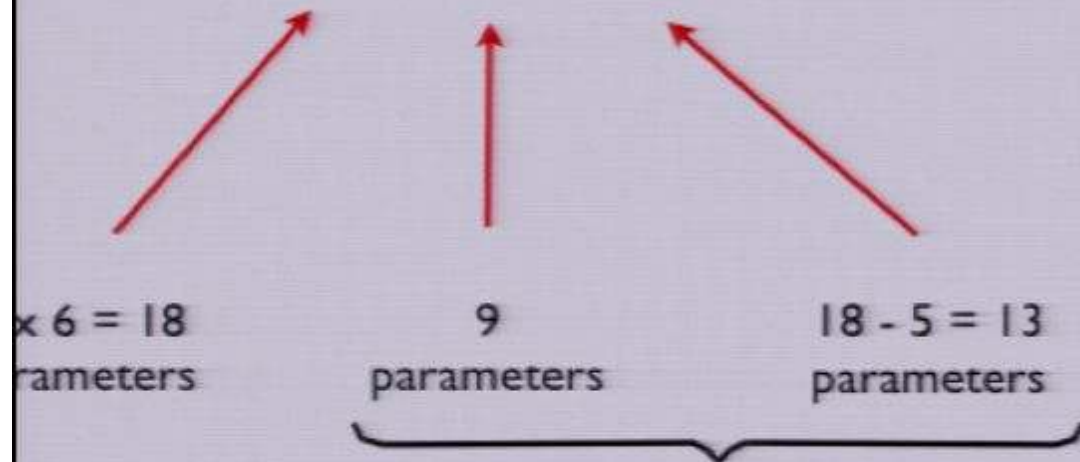
Slides

Explicitly metric formulation is possible

B^i defines a (conformal) metric wrt which it is self-dual

construct Σ^a for this metric satisfying $\Sigma^a \wedge \Sigma^b \sim \delta^{ab}$

$$B^i = b_a^i \Sigma^a \quad \text{for some} \quad b_a^i \in \text{GL}(3, \mathbb{C})$$



$$9 + 13 - 4 = 18$$

modulo

$$\begin{aligned} b_a^i &\rightarrow b_b^i (\Lambda^{-1})_a^b \\ \Sigma^a &\rightarrow \Lambda_b^a \Sigma^b \end{aligned} \quad \Lambda \in \text{SO}(3, \mathbb{C})$$

$$\begin{aligned} b_a^i &\rightarrow \lambda^{-1} b_a^i \\ \Sigma^a &\rightarrow \lambda \Sigma^a \end{aligned} \quad \lambda \in \mathbb{C}$$

can rewrite the theory as that of quantities b_a^i "propagating" on a given metric background

need to solve $D_A B^i = 0$

using the metric

$$A_\mu = \frac{1}{2 \det(B)} B^{i\rho\sigma} B_{\rho\mu}^j \nabla^\nu B_{\nu\sigma}^j$$

using $\mathcal{D}\Sigma^a = 0$

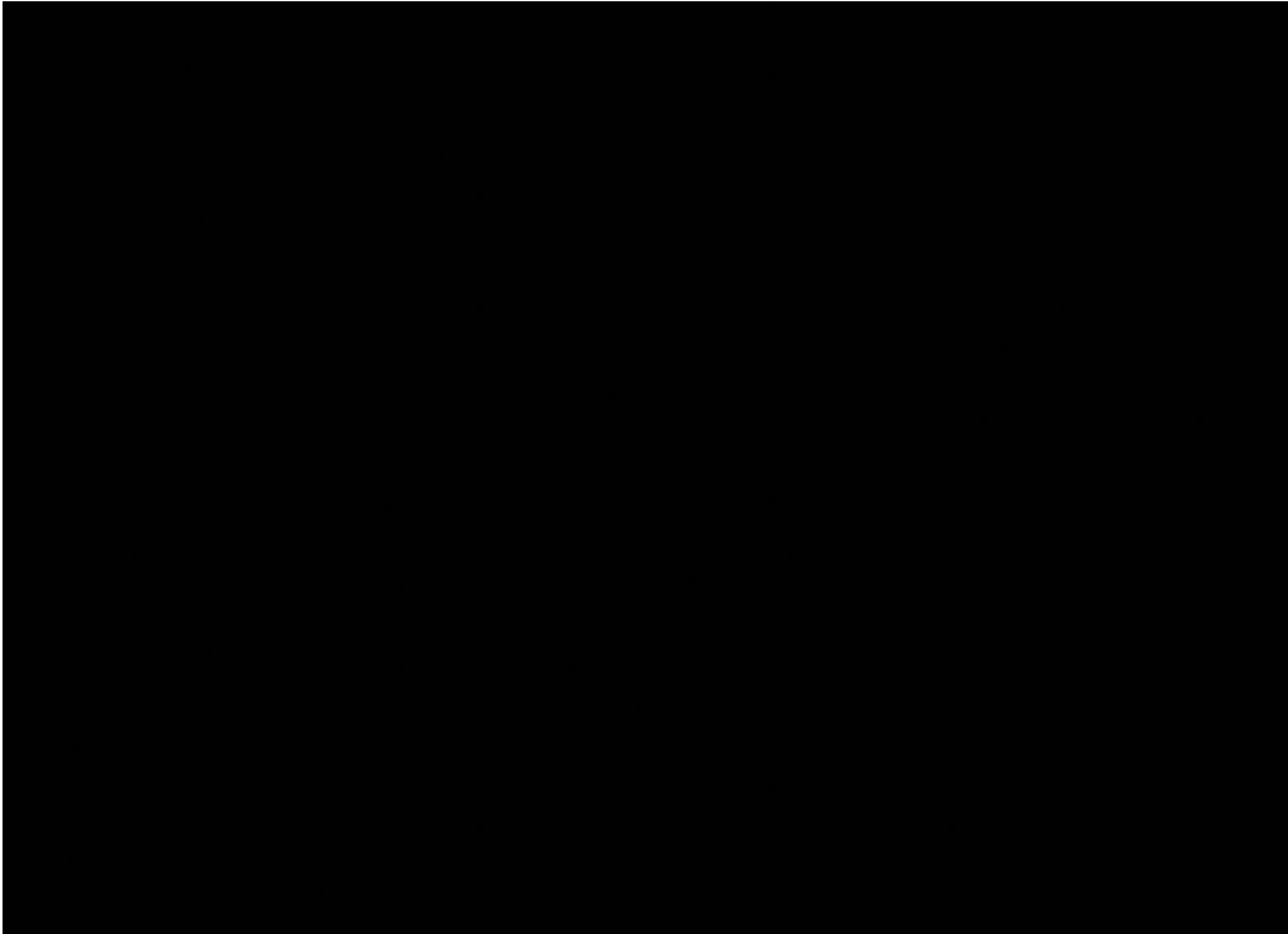
$$\nabla^\mu B_{\mu\nu}^i = (\mathcal{D}^\mu b_a^i) \Sigma_{\mu\nu}^a$$

metric compatible

$$[b] = \frac{i}{2} \int \sqrt{-g} \left(\frac{1}{4 \det(b)} (\Sigma_\alpha^a{}^\mu \Sigma_\mu^b{}^\nu \Sigma_\nu^c{}^\rho \Sigma_{\rho\beta}^d) (b_c^i \mathcal{D}^\alpha b_a^i) (b_b^j \mathcal{D}^\beta b_d^j) + 2V(m) \right)$$

here $m_{ab} = b_a^i b_b^j \delta_{ij}$

σ -model of a new type



Handwritten text in a cursive script, possibly a signature or a name, written in white on a dark background. The text is oriented vertically and appears to read "Ishere Ishere" or similar.

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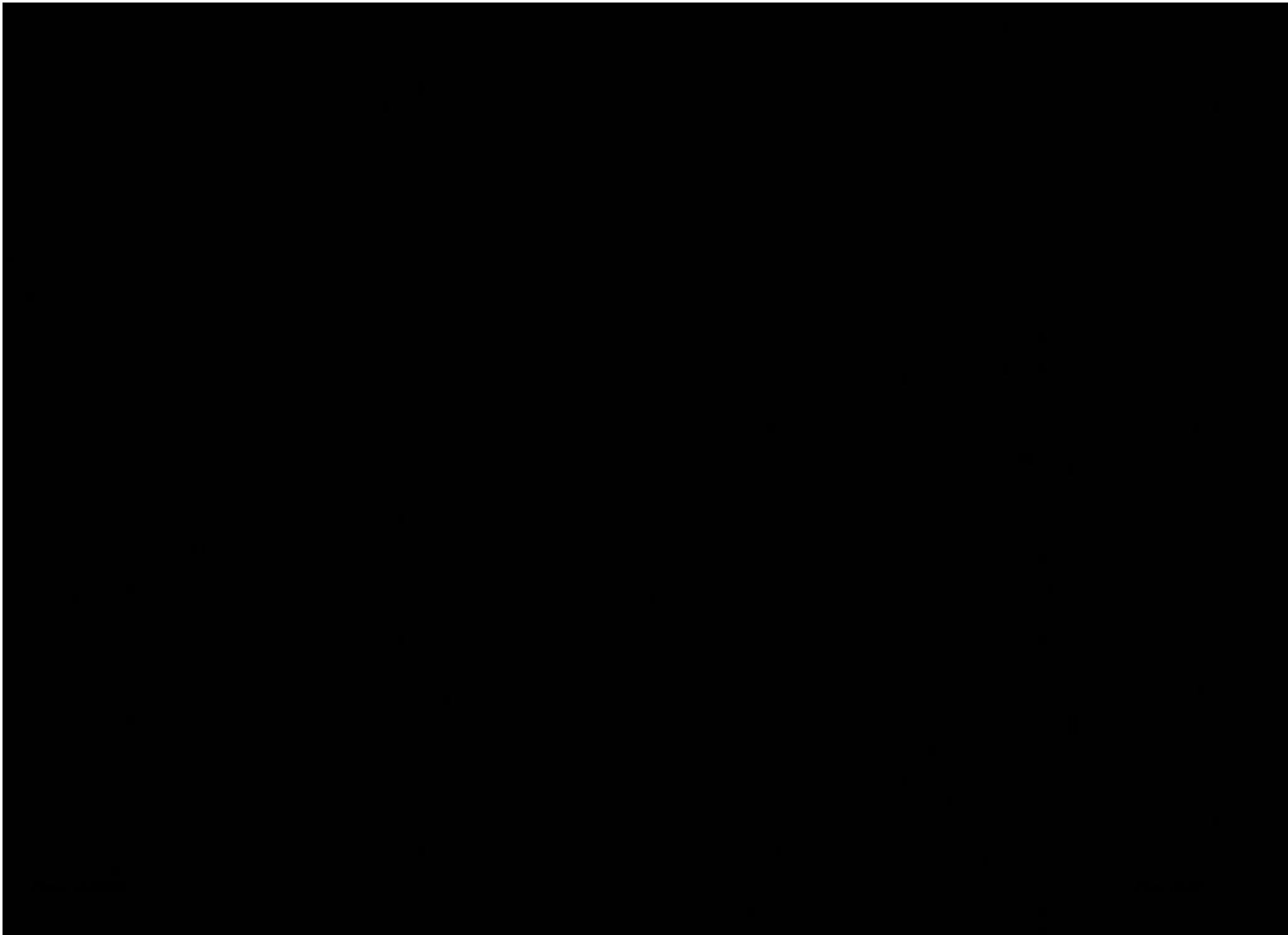
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What is the dynamical field is in general
a matter of choice

- Can interpret the metric as dynamical and eliminate b_a^i via constraints (or field equations) \Rightarrow Effective metric action as an infinite expansion in curvature invariants KK: 0911.4903
- Can solve for the metric in terms of b_a^i (at least in perturbation theory around a fixed background)



$$(\partial_\mu \varphi)^2 \rightarrow (\partial_\mu \varphi - \dot{\varphi})^2$$

$$\int \sqrt{-g} \, B_{\mu\nu}^i B_{\rho\sigma}^j = \tilde{X}^{ij}$$

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$$F^i$$

$$F^i \text{ "dual" }$$

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Consider the theory for a fixed metric

Gauge-symmetries:

$$\text{SO}(3, \mathbb{C})$$

$$b_a^i \rightarrow \Lambda_j^i b_a^j$$

conformal transformations

$$\Sigma^a \rightarrow \Omega^2 \Sigma^a$$

$$b_a^i \rightarrow \Omega^{-2} b_a^i$$

$$\text{SO}(3, \mathbb{C})$$

$$b_a^i \rightarrow b_b^i (\Lambda^{-1})_a^b$$

σ-model on the coset

$$\mathcal{M} = \text{GL}(3, \mathbb{C}) / \text{SO}(3, \mathbb{C}) \times \text{SO}(3, \mathbb{C}) \times \mathbb{C}$$

$$b = \Lambda A \tilde{\Lambda}^T, \quad \Lambda, \tilde{\Lambda} \in \text{SO}(3, \mathbb{C})$$

$$A = \text{diag}(a_1, a_2, a_3)$$

then \mathcal{M} coordinatized by ratios

$$a_2/a_1, a_3/a_1$$

Speculations on the nature of the UV fixed point

R

Infinitely steep potential V - General Relativity

(or potential of the order M_p^2)

relevant dynamical variables - metric

$b_a^i - \delta_a^i$ variables of the order $Weyl/M_p^2 \ll 1$

Torres-Gomez+KK: 0911.3793, KK: 0911.4903

effective metric Lagrangian -
expansion in curvature invariants



As energy increases the derivative terms become more important than the potential (alternatively, potential becomes flatter)

Approaching the topological BF theory corresponding to

$$V = \text{const}$$

fixed point!

even though the fixed point is not a dynamical theory, its neighbors are

Is this the fixed point controlling the UV behavior?

What are the relevant variables?

$$(b - \delta) \sim \frac{E^2}{M_p^2} \text{ (metric - Minkowski)}$$

for $E^2 \gg M_p^2$
the b -variables are more relevant

Topological symmetry and its gauge-fixings

$$B^i \rightarrow B^i + d\eta^i$$

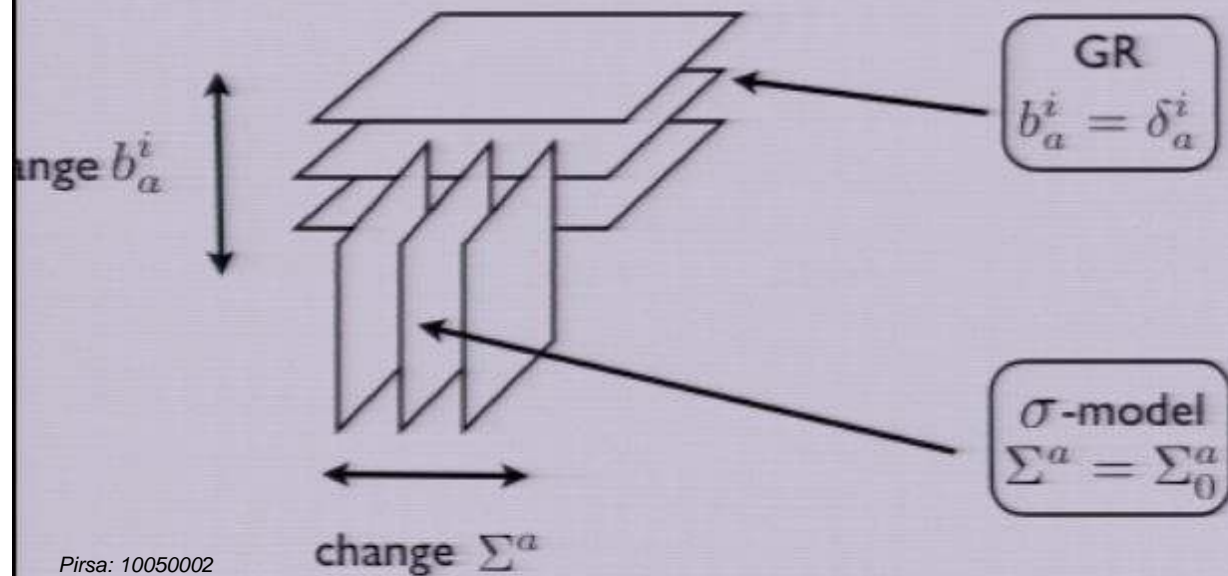
can use it to put any B^i to one of the two forms:

$$B^i = \delta_a^i \Sigma^a$$

$$B^i = b_a^i \Sigma_0^a$$

Σ_0^a -fixed metric two-forms

Diffeomorphisms + 5
extra transformations



σ -model as the relevant theory in the UV?

Summary:

- Infinite-parameter (parametrized by $V(X^{ij})$) class of gravity theories with 2 propagating DOF - effective metric Lagrangians
- Essentially $S = \int f(F^i \wedge F^j)$ so, naively at least, should be closed under renormalization
- IR - steep potential, $b_a^i = \delta_a^i$ - GR
- UV - flat potential, BF theory as the fixed point?
Coset b_a^i as relevant variables?

the idea of BF theory as the UV fixed point is not new - spin foams

Explicit description of the neighboring theories via σ -model? UV-valid perturbative description?

Slides

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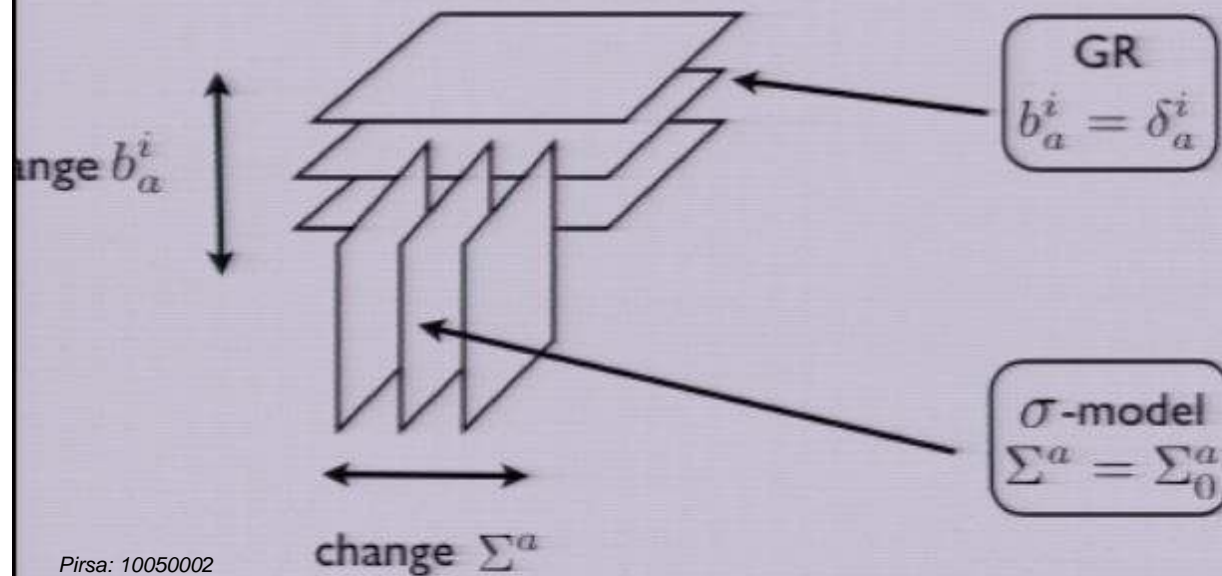
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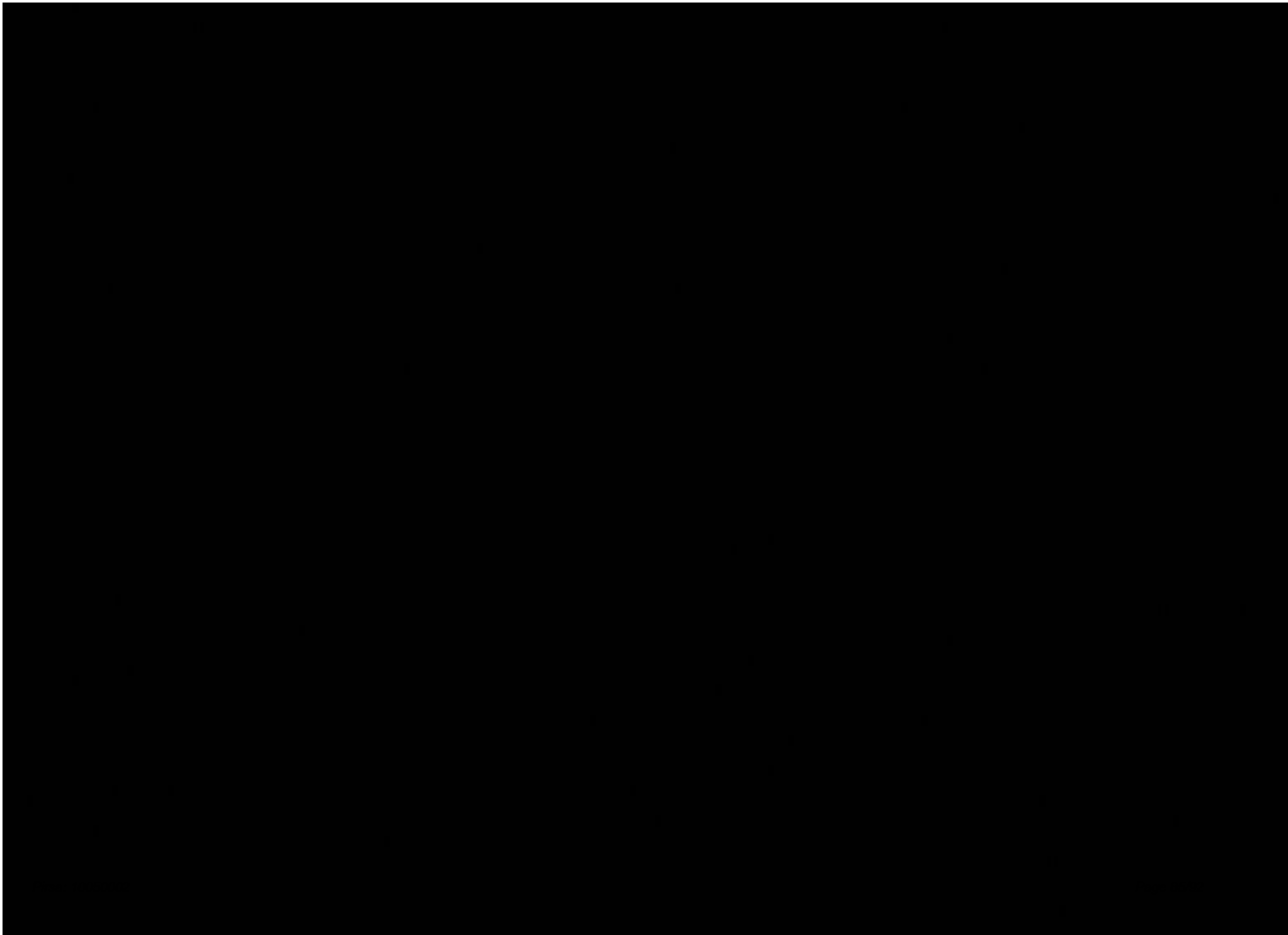
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$$g^{\mu\nu} \partial_\mu \Psi^\dagger \partial_\nu \Psi^\dagger$$

$$(\partial_\mu \Psi)^2 \rightarrow (\partial_\mu \Psi - \Psi)$$

$$\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}^a B_{\rho\sigma}^i = \tilde{X}^{ij}$$

$$\tilde{\epsilon}^{abc} B_{ab}^i = \tilde{F}^{ci}$$

$$p_z + f'(r) = 0$$

$$p_z + m\Gamma = 0$$

$$\frac{m\Gamma^2}{2} \cup$$

$$F^i + \frac{\partial V}{\partial B^i} = 0$$

B^i

$$f(\Psi_{,ij}^2)$$

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Example:

$$f(r) = mr^2/2$$

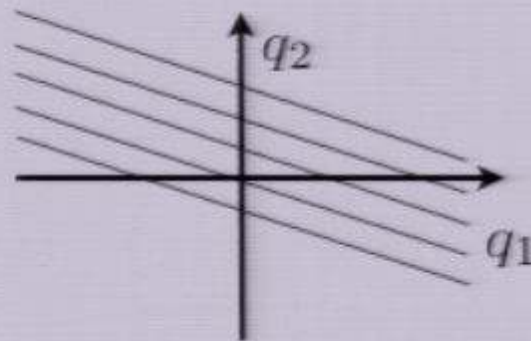
$$\Rightarrow \begin{aligned} r &= -p_2/m \\ \tilde{f}(p_2) &= -p_2^2/2m \end{aligned}$$

$$S = \int dt \left(p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1 \left(p_1 - \frac{p_2^2}{2m} \right) \right)$$

constraint generates

$$q_1 \rightarrow q_1 + \lambda_1$$

$$q_2 \rightarrow q_2 - \frac{p_2}{m} \lambda_1$$



$$p_2 \neq 0$$

$n \rightarrow \infty$ gives back
the original system

$n \rightarrow 0$ interchanges
the relevant vars

relevant coordinate

$$Q := q_2 + \frac{p_2}{m} q_1$$

Deformations of GR

$$S = \int B^i \wedge F^i + V(B^i \wedge B^j)$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$

$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

$$g^{\mu\nu} \partial_\mu \psi^I \partial_\nu \psi^I$$

$$B^i \rightarrow \underline{M_1^{ij} \dot{B}^j} + \underline{M_2^{ij} F^j}$$

$$\epsilon^{abc} B^i B^{ab} :=$$

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