Title: Deformations of General Relativity

Date: May 05, 2010 04:00 PM

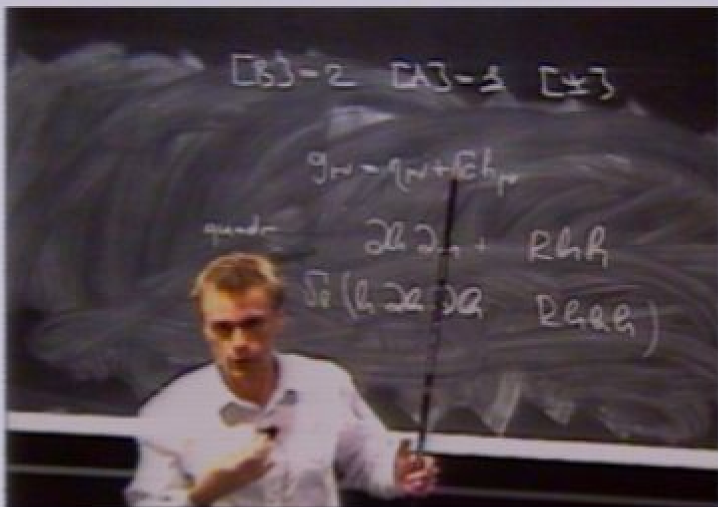
URL: http://pirsa.org/10050002

Abstract: I will describe a very special (infinite-parameter) family of gravity theories that all describe, exactly like General Relativity, just two propagating degrees of freedom. The theories are obtained by generalizing Plebanski's self-dual (chiral) formulation of GR. I will argue that this class of gravity theories provides a potentially powerful new framework for testing the asymptotic safety conjecture in quantum gravity.

Deformations of GR

Kirill Krasnov (Nottingham)

Perimeter Institute, May 5 2010



Perimeter talk on:
Renormalizable non-metric quantum gravity?
November 30, 2006

http://pirsa.org/06110041

The present talk is about the same theory

Much better understood, many more reasons to be interested in it!

Will only discuss aspects of relevance for quantum gravity, ignoring other, e.g. modified gravity

Outline

- Introduction: Perturbative quantum gravity, non-renormalizability, asymptotic safety
- Deformations of GR - an infinite-parametric class of gravity theories with 2 propagating DOF
- On the UV fixed point

Part I: Asymptotic Safety

Effective Field Theory:

Standard Model plus GR Lagrangian - only the first few terms of an infinitely complicated Lagrangian comprising all terms compatible with symmetries

Explains why renormalizable is what is relevant at low energies

Parameters of the EFT Lagrangian - coupling constants - are energy scale dependent

Dimensionless couplings $\breve{g}(\mu)$ that measure interaction strength at given energy scale μ

$$\breve{g}(\mu) := g\mu^{-[g]}$$

g does not run $\quad \Rightarrow \quad \breve{g}(\mu) \to 0 \quad$ as $\quad \mu \to 0$

$[g] < 0$

Non-renormalizable interactions are irrelevant at low energies

Renormalizability is not fundamental,
but is a low-energy phenomenon

Question of Quantum Gravity:

What provides a UV completion of the EFT Lagrangian of Standard Model + Gravity?

Standard expectation is that new DOF become relevant at the cutoff scale: pions $\Rightarrow$ quarks and gluons

Possible alternative: Asymptotic Safety

$$\breve{g}(\mu) \to \breve{g}_* \quad \text{as} \quad \mu \to \infty$$

$\breve{g}$ does not run

plus finite number of attractive directions for predictive power

An AS theory is UV safe without need for new DOF

There is now more and more evidence that gravity (plus matter) may be an AS theory

Perimeter Conference on: Asymptotic Safety-30 Years After

http://pirsa.org/C09025

Drawbacks:

- In gravity the UV fixed point is necessarily a non-trivial one $\breve{G}_* \neq 0$ (the trivial fixed point is the IR one) $\Rightarrow$ UV physics is non-perturbative in nature
- By no means guaranteed that the UV theory is unitary (e.g. R^2 terms)
- Seems impossible to prove AS, as the RG flow in an infinite-dimensional space of couplings need to be considered

At least some of the difficulties would be removed if had a compact description of a "healthy" and sufficiently large (closed under renormalization) class of Lagrangians.

The class of actions describing deformations of GR may be exactly such class, and has a potential to provide a powerful new perspective on asymptotic safety.

Deformations of GR are obtained by generalizing the so-called Plebanski self-dual (chiral) formulation.

Plebanski formulation of general relativity:

Consider a tetrad θ^I_μ for metric $g_{\mu\nu} = \theta^I_\mu \theta^J_\nu \eta_{IJ}$

where $\eta_{IJ} = \mathrm{diag}(-1,1,1,1)$

Take

$$\Sigma^a := \mathrm{i}\,\theta^0 \wedge \theta^a - \frac{1}{2}\epsilon^{abc}\theta^b \wedge \theta^c. \qquad I = (0,a) \quad a = 1,2,3$$

Properties:

Σ^a are self-dual $\frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\Sigma^a_{\rho\sigma} = i\Sigma^a_{\mu\nu}$

$$\Sigma^a \wedge \Sigma^b \sim \delta^{ab}$$

$$\Sigma^a \wedge (\Sigma^b)^* = 0$$

Consider γ^a such that

$$d\Sigma^a + \epsilon^{abc}\gamma^b \wedge \Sigma^c = 0$$

can solve for γ^a explicitly

$$\gamma^a_\mu = \frac{1}{2}\Sigma^{a\,\rho\nu}\Sigma^b_{\mu\nu}\nabla^\alpha\Sigma^b_{\alpha\rho}.$$

Here indices are raised and lowered with the metric and

$$\nabla_\mu : \nabla_\mu g_{\nu\rho} = 0$$

lot hard to check that

$$\gamma^a = i\Gamma^{0a} - \frac{1}{2}\epsilon^{abc}\Gamma^{bc}$$

where Γ^{IJ} is the Levi-Civita connection

$$d\theta^I + \Gamma^I{}_J \wedge \theta^J = 0$$

Define $F^a = d\gamma^a + \frac{1}{2}\epsilon^{abc}\gamma^b \wedge \gamma^c$

Not hard to show that $F^a = iF^{0a} - \frac{1}{2}\epsilon^{abc}F^{bc}$

where $F^{IJ} = d\Gamma^{IJ} + \Gamma^{IK} \wedge \Gamma_K{}^J$

instein equations (vacuum) can be stated as:

$$F^{(IJ)_{self-dual}}$$ is self-dual as a two-form

hus $\boxed{F^a = \Phi^{ab}\Sigma^b}$

where $\Psi^{ab} := \Phi^{ab} - \frac{1}{3}\delta^{ab}\mathrm{Tr}(\Phi)$ is arbitrary

$$\mathrm{Tr}(\Phi) \sim \Lambda$$

Gives a procedure for computing curvature components that is more efficient than the tetrad method!

E.g. Schwarzschild $ds^2 = -f^2(r)dt^2 + g^2(r)dr^2 + r^2 d\Omega^2$

$$e^t = f(r)dt, \qquad e^r = g(r)dr, \qquad e^\theta = r d\theta, \qquad e^\phi = r\sin(\theta)d\phi$$

$$\begin{aligned}
\Sigma^1 &= ifg dt \wedge dr - r^2 \sin(\theta) d\theta \wedge d\phi, \\
\Sigma^2 &= ifr dt \wedge d\theta - gr\sin(\theta) d\phi \wedge dr, \\
\Sigma^3 &= ifr\sin(\theta) dt \wedge d\phi - gr dr \wedge d\theta
\end{aligned}$$

$$A^1 = \frac{if'}{g}dt + \cos(\theta)d\phi, \qquad A^2 = -\frac{\sin(\theta)d\phi}{g}, \qquad A^3 = \frac{d\theta}{g}$$

$$\begin{aligned}
F^1 &= -\frac{1}{2fg}\left(\frac{f'}{g}\right)'(\bar{\Sigma}^1 + \Sigma^1) - \frac{1}{2r^2}\left(1 - \frac{1}{g^2}\right)(\bar{\Sigma}^1 - \Sigma^1), \\
F^2 &= -\frac{1}{2g^2 r}\left(\frac{g'}{g}(\bar{\Sigma}^2 - \Sigma^2) + \frac{f'}{f}(\bar{\Sigma}^2 + \Sigma^2)\right), \\
F^3 &= -\frac{1}{2g^2 r}\left(\frac{g'}{g}(\bar{\Sigma}^3 - \Sigma^3) + \frac{f'}{f}(\bar{\Sigma}^3 + \Sigma^3)\right).
\end{aligned}$$

lot hard to check that

$$\gamma^a = i\Gamma^{0a} - \frac{1}{2}\epsilon^{abc}\Gamma^{bc}$$

where Γ^{IJ} is the Levi-Civita connection

$$d\theta^I + \Gamma^I{}_J \wedge \theta^J = 0$$

Define $F^a = d\gamma^a + \frac{1}{2}\epsilon^{abc}\gamma^b \wedge \gamma^c$

Not hard to show that $F^a = iF^{0a} - \frac{1}{2}\epsilon^{abc}F^{bc}$

where $F^{IJ} = d\Gamma^{IJ} + \Gamma^{IK} \wedge \Gamma_K{}^J$

Consider γ^a such that

$$d\Sigma^a + \epsilon^{abc}\gamma^b \wedge \Sigma^c = 0$$

can solve for γ^a explicitly

$$\gamma^a_\mu = \frac{1}{2}\Sigma^{a\,\rho\nu}\Sigma^b_{\mu\nu}\nabla^\alpha\Sigma^b_{\alpha\rho}.$$

Here indices are raised and lowered with the metric and

$$\nabla_\mu : \nabla_\mu g_{\nu\rho} = 0$$

E.g. Schwarzschild $\quad ds^2 = -f^2(r)dt^2 + g^2(r)dr^2 + r^2 d\Omega^2$

$$e^t = f(r)dt, \qquad e^r = g(r)dr, \qquad e^\theta = r d\theta, \qquad e^\phi = r\sin(\theta)d\phi$$

$$\Sigma^1 = ifg dt\wedge dr - r^2\sin(\theta)d\theta\wedge d\phi,$$
$$\Sigma^2 = ifr dt\wedge d\theta - gr\sin(\theta)d\phi\wedge dr,$$
$$\Sigma^3 = ifr\sin(\theta)dt\wedge d\phi - gr dr\wedge d\theta$$

$$A^1 = \frac{if'}{g}dt + \cos(\theta)d\phi, \qquad A^2 = -\frac{\sin(\theta)d\phi}{g}, \qquad A^3 = \frac{d\theta}{g}$$

$$F^1 = -\frac{1}{2fg}\left(\frac{f'}{g}\right)'(\bar{\Sigma}^1 + \Sigma^1) - \frac{1}{2r^2}\left(1 - \frac{1}{g^2}\right)(\bar{\Sigma}^1 - \Sigma^1),$$
$$F^2 = -\frac{1}{2g^2 r}\left(\frac{g'}{g}(\bar{\Sigma}^2 - \Sigma^2) + \frac{f'}{f}(\bar{\Sigma}^2 + \Sigma^2)\right),$$
$$F^3 = -\frac{1}{2g^2 r}\left(\frac{g'}{g}(\bar{\Sigma}^3 - \Sigma^3) + \frac{f'}{f}(\bar{\Sigma}^3 + \Sigma^3)\right).$$

Can reformulate everything in terms of two-forms - do not need a metric!

Lemma: Given a triple Σ^a of two-forms satisfying:

$$\Sigma^a \wedge \Sigma^b \sim \delta^{ab},$$
$$\Sigma^a \wedge (\Sigma^b)^* = 0,$$
$$\mathrm{Re}(\Sigma^a \wedge \Sigma_a) = 0$$

there exists a unique real Lorentzian signature metric such that

$$\Sigma^a := \mathrm{i}\,\theta^0 \wedge \theta^a - \frac{1}{2}\epsilon^{abc}\theta^b \wedge \theta^c.$$

Idea of the proof:

Declare Σ^a to span the space of self-dual two forms wrt some metric

fixes the conformal metric uniquely

$$\sqrt{-g}\, g_{\mu\nu} \sim \tilde{\epsilon}^{\alpha\beta\rho\sigma} \Sigma^a_{\mu\alpha} \Sigma^b_{\nu\beta} \Sigma^c_{\rho\sigma} \epsilon^{abc}$$

then use a multiple of $\frac{1}{i} \Sigma^a \wedge \Sigma_a$

as the volume form

ction principle (Plebanski)

$$S = i \int \Sigma^a \wedge F^a - \frac{1}{2}\Psi^{ab}\Sigma^a \wedge \Sigma^b$$

field equations:

$\boxed{\Psi}$ $\quad \Sigma^a \wedge \Sigma^b \sim \delta^{ab}$

$\boxed{\gamma}$ $\quad D_\gamma \Sigma^a = 0$

$\boxed{\Sigma}$ $\quad F^a = \Psi^{ab}\Sigma^b$

$\Lambda = 0$

for simplicity and the reality conditions

$$\Sigma^a \wedge (\Sigma^b)^* = 0,$$
$$\mathrm{Re}(\Sigma^a \wedge \Sigma_a) = 0$$

to be imposed by hand

Constrained BF theory

$$S_{BF} = \int B^a \wedge F^a$$

The way propagating DOF are introduced is very interesting

$$S_{BF} = \int \tilde{E}^{ai} \dot{A}_i^a + A_0^a D_i \tilde{E}^{ai} + B_{0i}^a \tilde{\epsilon}^{ijk} F_{jk}^a$$

here $i = 1, 2, 3$ are spatial indices

B_{0i}^a are Lagrange multipliers for $F_{ij}^a = 0$

generators of

$B^a \to B^a + d\eta^a$

in Plebanski theory 5 out of 9 of these multipliers are set to zero by the Ψ^{ab} constraints, leaving only the diffeos (and SO(3)) as gauge

[redacted]

$$\tilde{\epsilon}^{abc} B^i_{ab} := \tilde{E}^{ci}$$

The way propagating DOF are introduced is very interesting

$$S_{BF} = \int \tilde{E}^{ai} \dot{A}_i^a + A_0^a D_i \tilde{E}^{ai} + B_{0i}^a \tilde{\epsilon}^{ijk} F_{jk}^a$$

here $i = 1, 2, 3$ are spatial indices

B_{0i}^a are Lagrange multipliers for $F_{ij}^a = 0$

generators of

$B^a \to B^a + d\eta^a$

in Plebanski theory 5 out of 9 of these multipliers are set to zero by the Ψ^{ab} constraints, leaving only the diffeos (and SO(3)) as gauge xample:

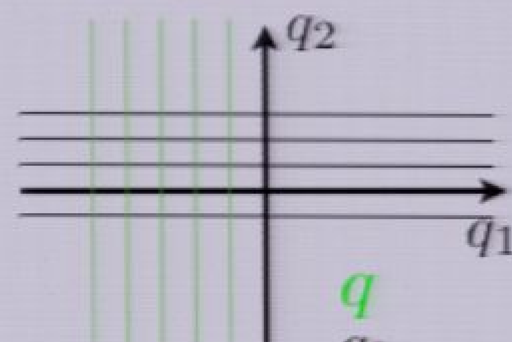
$$S = \int dt\,(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2)$$

first class constraints: $\begin{array}{l} p_1 \approx 0 \\ p_2 \approx 0 \end{array}$ generate $\begin{array}{l} q_1 \to q_1 + \lambda_1 \\ q_2 \to q_2 + \lambda_2 \end{array}$

can get a system with DOF via

$$S = \int dt\,(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2 \overset{-\psi\lambda_1}{- \psi\lambda_2})$$

$\boxed{\psi}\quad \overset{\lambda_1}{\lambda_2} = 0$



ompletely constrained system with a single DOF

q_2 as a relevant variable

o motivate a construction that follows consider
different way to get a system with DOF

$$S = \int dt\,(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2 - V(\lambda_1, \lambda_2))$$

where $V(\alpha\lambda_1, \alpha\lambda_2) = \alpha V(\lambda_1, \lambda_2)$

introduce $r = \lambda_2/\lambda_1$

$$S = \int dt\,(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1(p_1 + r p_2 + f(r)))$$

where $f(r) := V(1, r)$

r non-dynamical $\Rightarrow$

$$S = \int dt\,\left(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1(p_1 + \tilde{f}(p_2))\right)$$

here $\tilde{f} = f - r f'$ Legendre transform

[redacted]

$$\tilde{\epsilon}^{abc} B^{i}_{ab} := \tilde{E}^{ci}$$

$$p_2 + f'(r) = 0$$

To motivate a construction that follows consider a different way to get a system with DOF

$$S = \int dt\,(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1 p_1 - \lambda_2 p_2 - V(\lambda_1, \lambda_2))$$

where $V(\alpha\lambda_1, \alpha\lambda_2) = \alpha V(\lambda_1, \lambda_2)$

introduce $r = \lambda_2/\lambda_1$

$$S = \int dt\,(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1(p_1 + r p_2 + f(r)))$$

where $f(r) := V(1, r)$

r non-dynamical $\Rightarrow$

$$S = \int dt\left(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1(p_1 + \tilde{f}(p_2))\right)$$

o motivate a construction that follows consider
different way to get a system with DOF

$$S = \int dt\,(p_1\dot q_1 + p_2\dot q_2 - \lambda_1 p_1 - \lambda_2 p_2 - V(\lambda_1,\lambda_2))$$

where $V(\alpha\lambda_1,\alpha\lambda_2) = \alpha V(\lambda_1,\lambda_2)$

introduce $r = \lambda_2/\lambda_1$

$$S = \int dt\,(p_1\dot q_1 + p_2\dot q_2 - \lambda_1(p_1 + r p_2 + f(r)))$$

where $f(r) := V(1,r)$

r non-dynamical $\Rightarrow$

$$S = \int dt\left(p_1\dot q_1 + p_2\dot q_2 - \lambda_1(p_1 + \tilde f(p_2))\right)$$

here $\tilde f = f - r f'$ Legendre transform

Example: $f(r) = mr^2/2$

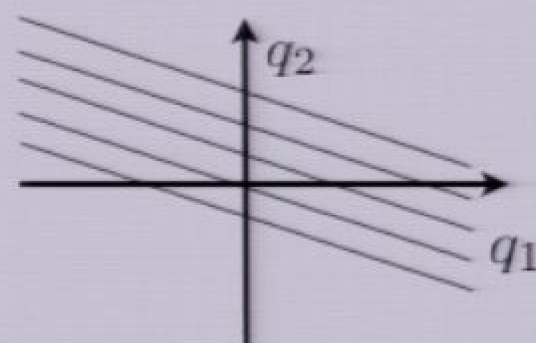
$$\Rightarrow \quad \begin{array}{l} r = -p_2/m \\ \tilde{f}(p_2) = -p_2^2/2m \end{array}$$

$$S = \int dt \left(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1 (p_1 - \frac{p_2^2}{2m}) \right)$$

constraint generates

$$q_1 \longrightarrow q_1 + \lambda_1$$

$$q_2 \longrightarrow q_2 - \frac{p_2}{m}\lambda_1$$



$p_2 \neq 0$

$m \to \infty$ gives back the original system

$m \to 0$ interchanges the relevant vars

relevant coordinate $Q := q_2 + \frac{p_2}{m} q_1$

$$\tilde{\epsilon}^{abc} B^i_{ab} := \tilde{E}^{ci}$$

$$p_2 + f'(r) = 0$$

$$p_2 + mr = 0$$

Example: $$f(r) = mr^2/2$$

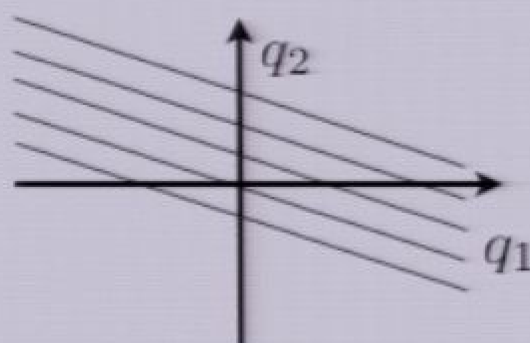
$$\Rightarrow \quad \begin{aligned} r &= -p_2/m \\ \tilde{f}(p_2) &= -p_2^2/2m \end{aligned}$$

$$S = \int dt \left(p_1 \dot{q}_1 + p_2 \dot{q}_2 - \lambda_1 (p_1 - \frac{p_2^2}{2m}) \right)$$

constraint generates

$$q_1 \longrightarrow q_1 + \lambda_1$$

$$q_2 \longrightarrow q_2 - \frac{p_2}{m} \lambda_1$$



$p_2 \neq 0$

$m \to \infty$ gives back the original system

$m \to 0$ interchanges the relevant vars

relevant coordinate $$Q := q_2 + \frac{p_2}{m} q_1$$

Deformations of GR

$$\boxed{S = \int B^i \wedge F^i + V(B^i \wedge B^j)}$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$
$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^{i}_{\mu\nu} B^{j}_{\rho\sigma} = \tilde{X}^{i}$$

$$\tilde{\epsilon}^{abc} B^{i}_{ab} := \tilde{E}^{ci}$$

$$p_z + f'(r) = 0$$

$$p_z + mr = 0$$

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^{i}_{\mu\nu} B^{j}_{\rho\sigma} = \tilde{X}^{ij}$$

$$\tilde{\epsilon}^{abc} B^{i}_{ab} := \tilde{E}^{ci}$$

$$p_z + f'(r) = 0$$

$$p_z + mr = 0$$

Deformations of GR

$$\boxed{S = \int B^i \wedge F^i + V(B^i \wedge B^j)}$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$
$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

[redacted]

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^{i}_{\mu\nu} B^{j}_{\rho\sigma} = \tilde{X}^{ij}$$

$$\tilde{\epsilon}^{abc} B^{i}_{ab} := \tilde{E}^{ci}$$

$$F^i + \text{"}\frac{\partial V}{\partial B^i}\text{"} = 0$$

$$p_2 + f'(r) = 0$$

$$p_2 + mr = 0$$

Deformations of GR

$$\boxed{S = \int B^i \wedge F^i + V(B^i \wedge B^j)}$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$
$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

Properties:

- Describes two propagating DOF for any $V : V'' \neq 0$ (complex for $SO(3,\mathbb{C})$)
- Deformations of GR, for any of the theories can be continuously deformed back to GR without changing dynamical content
- Can be shown to be about the conformal metric wrt which B^i are self-dual: matter moves along geodesics of one of the metric in this conformal class

Deformations of GR

$$\boxed{S = \int B^i \wedge F^i + V(B^i \wedge B^j)}$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$
$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

Properties:

- Describes two propagating DOF for any $V : V'' \neq 0$ (complex for $SO(3, \mathbb{C})$)
- Deformations of GR, for any of the theories can be continuously deformed back to GR without changing dynamical content
- Can be shown to be about the conformal metric wrt which B^i are self-dual: matter moves along geodesics of one of the metric in this conformal class

[redacted]

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^{i}_{\mu\nu} B^{j}_{\rho\sigma} = \tilde{X}^{ij}$$

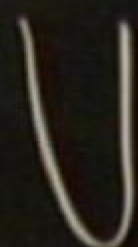
$$\tilde{\epsilon}^{abc} B^{i}_{ab} := \bar{E}^{ci}$$

$$F^{i} + \frac{\text{"}\partial V\text{"}}{\partial B^{i}} = 0$$

$$p_z + f'(r) = 0$$

$$p_z + m r = 0$$

$$\frac{m r^2}{2}$$



Properties:

- Describes two propagating DOF for any $V : V'' \neq 0$ (complex for $SO(3,\mathbb{C})$)
- Deformations of GR, for any of the theories can be continuously deformed back to GR without changing dynamical content
- Can be shown to be about the conformal metric wrt which B^i are self-dual: matter moves along geodesics of one of the metric in this conformal class

Reality conditions:

$$B^i \wedge (B^j)^* = 0$$

plus a condition for the volume element

the physical metric is real

Unlike in the Plebanski case, this does not guarantee reality of the action.

Current prescription - take the real part of the arising complex action. Consistency?

A lot is known about the classical theory:

- spherically-symmetric problem can be solved exactly; spherical symmetry implies staticity; very interesting effects of singularity resolution inside BH's KK+Shtanov: 0705.2047, 0805.2668
- Newtonian limit - MOND-like (arbitrary function of second derivatives of the grav. potential)
- Linearized theory around Minkowski - gravitational waves unmodified Freidel: 0812.3200, KK: 0911.4903
- Friedman equations are unmodified KK+Shtanov: 1002.1210
- Scalar perturbations around FRW background are modified - time-dependent effective speed of sound

[redacted]

B^i

$$+\left(\left|\mathcal{L}_i\right|^2\right)$$

B^i

$$\int \left(\left| \partial_i\, j \right| \right)^2$$

A lot is known about the classical theory:

- spherically-symmetric problem can be solved exactly; spherical symmetry implies staticity; very interesting effects of singularity resolution inside BH's KK+Shtanov: 0705.2047, 0805.2668
- Newtonian limit - MOND-like (arbitrary function of second derivatives of the grav. potential)
- Linearized theory around Minkowski - gravitational waves unmodified Freidel: 0812.3200, KK: 0911.4903
- Friedman equations are unmodified

KK+Shtanov: 1002.1210

- Scalar perturbations around FRW background are modified - time-dependent effective speed of sound

xplicitly metric formulation is possible

B^i defines a (conformal) metric wrt which it is self-dual

construct Σ^a for this metric satisfying $\Sigma^a \wedge \Sigma^b \sim \delta^{ab}$

$$B^i = b^i_a \Sigma^a \qquad \text{for some} \qquad b^i_a \in \mathrm{GL}(3, \mathbb{C})$$

x 6 = 18
rameters

9
parameters

18 - 5 = 13
parameters

9 + 13 - 4 =18

modulo

$$b^i_a \to b^i_b (\Lambda^{-1})^b_a \qquad \Lambda \in \mathrm{SO}(3, \mathbb{C})$$
$$\Sigma^a \to \Lambda^a_b \Sigma^b$$

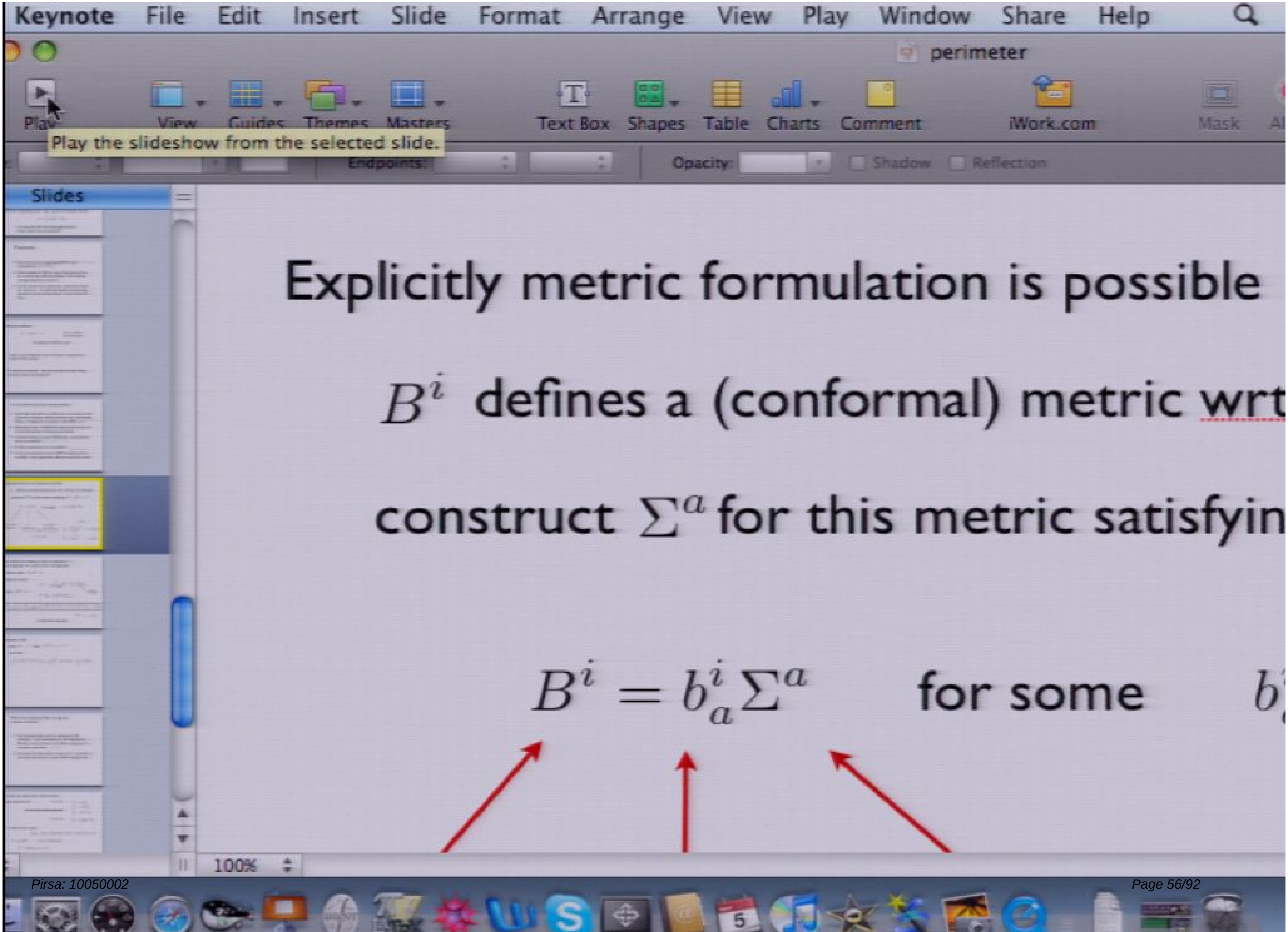
$$b^i_a \to \lambda^{-1} b^i_a \qquad \lambda \in \mathbb{C}$$
$$\Sigma^a \to \lambda \Sigma^a$$

Explicitly metric formulation is possible

B^i defines a (conformal) metric wrt

construct Σ^a for this metric satisfyin

$$B^i = b^i_a \Sigma^a \qquad \text{for some} \qquad b$$


Keynote File Edit Insert Slide Format Arrange View Play Window Share Help
perimeter
Play View Guides Themes Masters Text Box Shapes Table Charts Comment iWork.com Mask
Play the slideshow from the selected slide.
Endpoints: Opacity: Shadow Reflection
Slides
Explicitly metric formulation is possible
B^i defines a (conformal) metric wrt
construct Σ^a for this metric satisfyin
$B^i = b^i_a \Sigma^a$ for some
100%

Explicitly metric formulation is possible

B^i defines a (conformal) metric wrt which it is self-dual

construct Σ^a for this metric satisfying $\Sigma^a \wedge \Sigma^b \sim \delta^{ab}$

$$B^i = b^i_a \Sigma^a \qquad \text{for some} \qquad b^i_a \in \mathrm{GL}(3,\mathbb{C})$$

x 6 = 18
rameters

9
parameters

18 - 5 = 13
parameters

9 + 13 - 4 =18

modulo

$$b^i_a \to b^i_b (\Lambda^{-1})^b_a \qquad \Lambda \in \mathrm{SO}(3,\mathbb{C})$$
$$\Sigma^a \to \Lambda^a_b \Sigma^b$$

$$b^i_a \to \lambda^{-1} b^i_a \qquad \lambda \in \mathbb{C}$$
$$\Sigma^a \to \lambda \Sigma^a$$

an rewrite the theory as that of quantities b^i_a
propagating" on a given metric background

eed to solve $D_A B^i = 0$

sing the metric

$$A_\mu = \frac{1}{2\det(B)} B^{i\rho\sigma} B^j_{\rho\mu} \nabla^\nu B^j_{\nu\sigma}$$

metric
compatible

sing $\mathcal{D}\Sigma^a = 0$

$$\nabla^\mu B^i_{\mu\nu} = (\mathcal{D}^\mu b^i_a)\Sigma^a_{\mu\nu}$$

$$b] = \frac{i}{2}\int \sqrt{-g}\left(\frac{1}{4\det(b)}\left(\Sigma^a_\alpha{}^\mu \Sigma^b_\mu{}^\nu \Sigma^c_\nu{}^\rho \Sigma^d_{\rho\beta}\right)\left(b^i_c \mathcal{D}^\alpha b^i_a\right)\left(b^j_b \mathcal{D}^\beta b^j_d\right) + 2V(m)\right)$$

here $m_{ab} = b^i_a b^j_b \delta_{ij}$

σ-model of a new type

[redacted]

$$g^{\mu\nu}\,\partial_\mu \varphi^I\,\partial_\nu \varphi^I$$

an rewrite the theory as that of quantities b^i_a
propagating" on a given metric background

eed to solve $D_A B^i = 0$

sing the metric

$$A_\mu = \frac{1}{2\det(B)} B^{i\rho\sigma} B^j_{\rho\mu} \nabla^\nu B^j_{\nu\sigma}$$

metric compatible

sing $\mathcal{D}\Sigma^a = 0$

$$\nabla^\mu B^i_{\mu\nu} = (\mathcal{D}^\mu b^i_a)\Sigma^a_{\mu\nu}$$

$$b] = \frac{i}{2} \int \sqrt{-g} \left(\frac{1}{4\det(b)} \left(\Sigma^a_\alpha{}^\mu \Sigma^b_\mu{}^\nu \Sigma^c_\nu{}^\rho \Sigma^d_{\rho\beta}\right) \left(b^i_c \mathcal{D}^\alpha b^i_a\right) \left(b^j_b \mathcal{D}^\beta b^j_d\right) + 2V(m) \right)$$

here $m_{ab} = b^i_a b^j_b \delta_{ij}$

σ-model of a new type

elation to GR

when $b^i_a = \delta^i_a$ then $b^i_c \mathcal{D}^\alpha b^i_a = \epsilon^{cae}\gamma^{\alpha e}$

then have

$$\int \sqrt{-g}\epsilon^{abc}\Sigma^{a\mu\nu}\gamma^b_\mu \gamma^c_\nu \sim \int \Sigma^a \wedge F^a(\gamma) \sim \int \sqrt{-g}R$$

an rewrite the theory as that of quantities b^i_a propagating" on a given metric background

eed to solve $D_A B^i = 0$

sing the metric

$$A_\mu = \frac{1}{2\det(B)} B^{i\rho\sigma} B^j_{\rho\mu} \nabla^\nu B^j_{\nu\sigma}$$

metric compatible

sing $\mathcal{D}\Sigma^a = 0$

$$\nabla^\mu B^i_{\mu\nu} = (\mathcal{D}^\mu b^i_a)\Sigma^a_{\mu\nu}$$

$$\boxed{b] = \frac{i}{2}\int \sqrt{-g}\left(\frac{1}{4\det(b)}\left(\Sigma^a_\alpha{}^\mu \Sigma^b_\mu{}^\nu \Sigma^c_\nu{}^\rho \Sigma^d_{\rho\beta}\right)\left(b^i_c \mathcal{D}^\alpha b^i_a\right)\left(b^j_b \mathcal{D}^\beta b^j_d\right) + 2V(m)\right)}$$

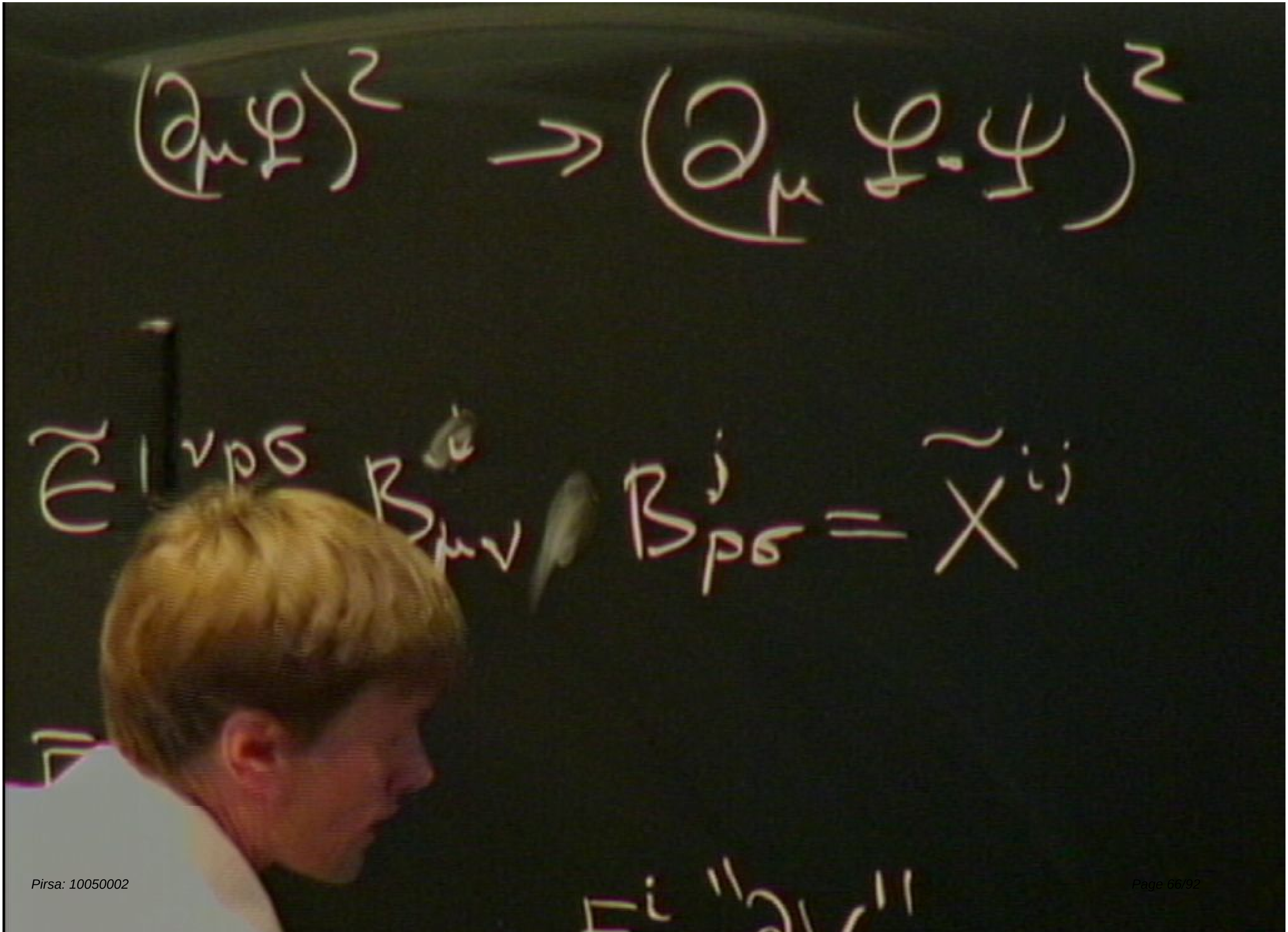
here $m_{ab} = b^i_a b^j_b \delta_{ij}$

σ-model of a new type

What is the dynamical field is in general a matter of choice

- Can interpret the metric as dynamical and eliminate b^i_a via constraints (or field equations) $\Rightarrow$ Effective metric action as an infinite expansion in curvature invariants KK: 0911.4903
- Can solve for the metric in terms of b^i_a (at least in perturbation theory around a fixed background)

[redacted]


$(\partial_\mu \varphi)^2 \rightarrow (\partial_\mu \varphi \cdot \varphi)^2$
$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^i_{\mu\nu} B^j_{\rho\sigma} = \tilde{X}^{ij}$

What is the dynamical field is in general a matter of choice

- Can interpret the metric as dynamical and eliminate b_a^i via constraints (or field equations) $\Rightarrow$ Effective metric action as an infinite expansion in curvature invariants KK: 0911.4903
- Can solve for the metric in terms of b_a^i (at least in perturbation theory around a fixed background)

$$(\partial_\mu \mathcal{L})^2 \to (\partial_\mu \mathcal{L} \cdot \Sigma)^2$$

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} \, B^i_{\mu\nu} \, B^j_{\rho\sigma} = \tilde{X}^{ij}$$

$$\bar{E}^{ci}$$

$$E^i \text{ "}\partial V\text{"}$$

$$(\partial_\mu \varphi)^2 \rightarrow (\partial_\mu \varphi \cdot \Psi)^2$$

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^i_{\mu\nu} B^j_{\rho\sigma} = \tilde{X}^{ij}$$

$$E^{ci}$$

E^i "∂V"

$$(\partial_\mu \varphi)^2 \rightarrow (\partial_\mu \varphi \cdot \psi)^2$$

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^i_{\mu\nu} B^j_{\rho\sigma} = \tilde{X}^{ij}$$

$$\bar{E}^{ci}$$

E^i "∂V"

What is the dynamical field is in general a matter of choice

- Can interpret the metric as dynamical and eliminate b^i_a via constraints (or field equations) $\Rightarrow$ Effective metric action as an infinite expansion in curvature invariants KK: 0911.4903
- Can solve for the metric in terms of b^i_a (at least in perturbation theory around a fixed background)

onsider the theory for a fixed metric

auge-symmetries: $\mathrm{SO}(3,\mathbb{C})$ $b^i_a \to \Lambda^i_j b^j_a$

conformal transformations $\Sigma^a \to \Omega^2 \Sigma^a$

$b^i_a \to \Omega^{-2} b^i_a$

$\mathrm{SO}(3,\mathbb{C})$ $b^i_a \to b^i_b (\Lambda^{-1})^b_a$

-model on the coset

$$\mathcal{M} = \mathrm{GL}(3,\mathbb{C})/\mathrm{SO}(3,\mathbb{C}) \times \mathrm{SO}(3,\mathbb{C}) \times \mathbb{C}$$

f $b = \Lambda A \tilde{\Lambda}^T, \qquad \Lambda, \tilde{\Lambda} \in \mathrm{SO}(3,\mathbb{C})$

$A = \mathrm{diag}(a_1, a_2, a_3)$

then $\mathcal{M}$ coordinatized by ratios

$a_2/a_1, a_3/a_1$

Speculations on the nature of the UV fixed point

IR

Infinitely steep potential V - General Relativity

(or potential of the order M_p^2)

relevant dynamical variables - metric

$b_a^i - \delta_a^i$ variables of the order $Weyl/M_p^2 \ll 1$

Torres-Gomez+KK: 0911.3793, KK: 0911.4903

effective metric Lagrangian -
expansion in curvature invariants

V

s energy increases the derivative terms become more important han the potential (alternatively, potential becomes flatter)

pproaching the topological BF theory corresponding to

$$V = const$$

fixed point!

even though the fixed point is not a dynamical theory, its neighbors are

this the fixed point controlling the UV behavior?

hat are the relevant variables?

$$(b - \delta) \sim \frac{E^2}{M_p^2}$$ (metric - Minkowski)

for $E^2 \gg M_p^2$ the b-variables are more relevant

opological symmetry and its gauge-fixings

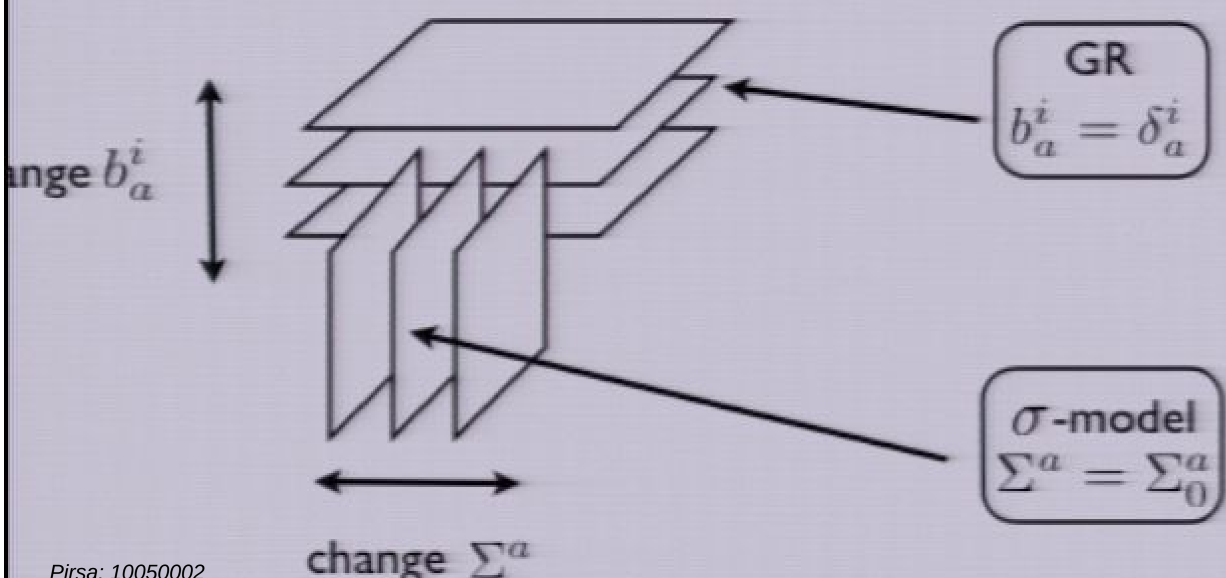
$$B^i \to B^i + d\eta^i$$

can use it to put any B^i to one of the two forms:

$$B^i = \delta^i_a \Sigma^a \qquad B^i = b^i_a \Sigma^a_0$$

Σ^a_0-fixed metric two-forms

ffeomorphisms + 5
tra transformations



σ-model as the relevant theory in the UV?

Summary:

- Infinite-parameter (parametrized by $V(X^{ij})$) class of gravity theories with 2 propagating DOF - effective metric Lagrangians
- Essentially $S = \int f(F^i \wedge F^j)$ so, naively at least, should be closed under renormalization
- IR - steep potential, $b^i_a = \delta^i_a$ - GR
- UV - flat potential, BF theory as the fixed point? Coset b^i_a as relevant variables?

the idea of BF theory as the UV fixed point is not new - spin foams

Explicit description of the neighboring theories via σ-model? UV-valid perturbative description?

Summary:

- Infinite-parameter (parametrize
gravity theories with 2 propagat
metric Lagrangians

- Essentially $S = \int f(F^i \wedge F^j)$ so, nai
be closed under renormalizatio

- IR - steep potential, $b^i_a = \delta^i_a$ - G

Keynote File Edit Insert Slide Format Arrange View Play Window Share Help

perimeter

Play the slideshow from the selected slide.

Summary:

- Infinite-parameter (parametrize
gravity theories with 2 propagat
metric Lagrangians

- Essentially $S = \int f(F^i \wedge F^j)$ so, naiv
be closed under renormalizatio

- IR - steep potential, $b^i_a = \delta^i_a$ - G

Summary:

- Infinite-parameter (parametrize
gravity theories with 2 propagat
metric Lagrangians

- Essentially $S = \int f(F^i \wedge F^j)$ so, naiv
be closed under renormalizatio

- IR - steep potential, $b^i_a = \delta^i_a$ - C

Summary:

- Infinite-parameter (parametrize
gravity theories with 2 propagat
metric Lagrangians

- Essentially $S = \int f(F^i \wedge F^j)$ so, naiv
be closed under renormalizatio

- IR - steep potential, $b^i_a = \delta^i_a$ - G

Summary:

- Infinite-parameter (parametrized by $V(X^{ij})$) class of gravity theories with 2 propagating DOF - effective metric Lagrangians
- Essentially $S = \int f(F^i \wedge F^j)$ so, naively at least, should be closed under renormalization
- IR - steep potential, $b^i_a = \delta^i_a$ - GR
- UV - flat potential, BF theory as the fixed point? Coset b^i_a as relevant variables?

the idea of BF theory as the UV fixed point is not new - spin foams

Explicit description of the neighboring theories via σ-model? UV-valid perturbative description?

...opological symmetry and its gauge-fixings

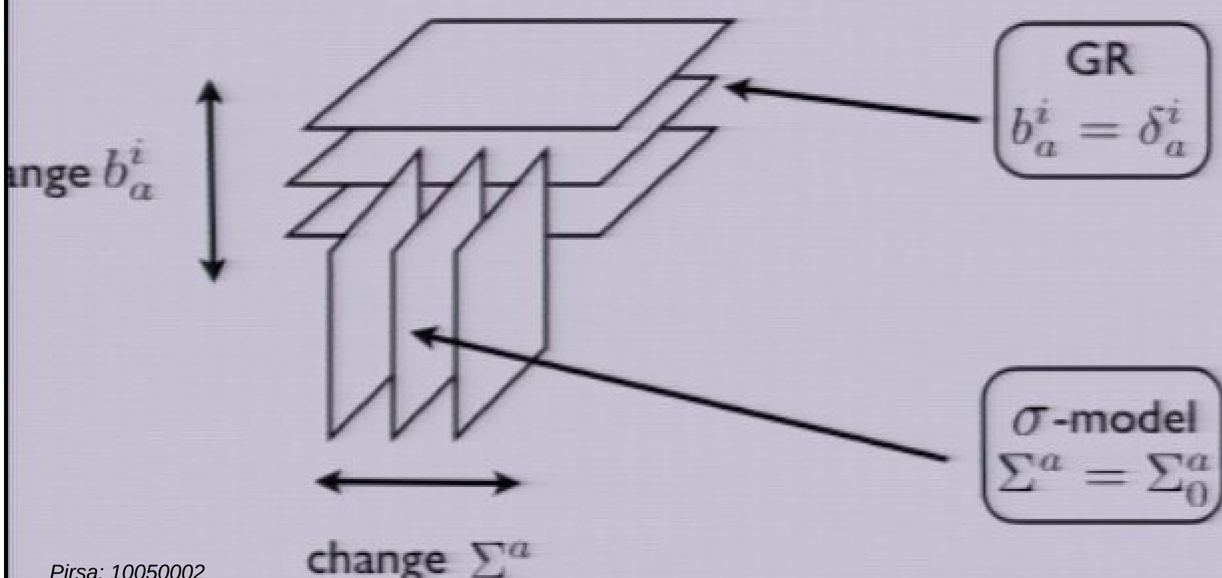
$$B^i \to B^i + d\eta^i$$

...ffeomorphisms + 5
...tra transformations

can use it to put any B^i to
one of the two forms:

$$B^i = \delta^i_a \Sigma^a \qquad\qquad B^i = b^i_a \Sigma^a_0$$

Σ^a_0-fixed metric two-forms



σ-model as the relevant theory in the UV?

elation to GR

when $b^i_a = \delta^i_a$ then $b^i_c \mathcal{D}^\alpha b^i_a = \epsilon^{cae}\gamma^{\alpha e}$

then have

$$\int \sqrt{-g}\epsilon^{abc}\Sigma^{a\mu\nu}\gamma^b_\mu\gamma^c_\nu \sim \int \Sigma^a \wedge F^a(\gamma) \sim \int \sqrt{-g}R$$

an rewrite the theory as that of quantities b^i_a
propagating" on a given metric background

eed to solve $D_A B^i = 0$

sing the metric

$$A_\mu = \frac{1}{2\det(B)} B^{i\rho\sigma} B^j_{\rho\mu} \nabla^\nu B^j_{\nu\sigma}$$

metric
compatible

sing $\mathcal{D}\Sigma^a = 0$

$$\nabla^\mu B^i_{\mu\nu} = (\mathcal{D}^\mu b^i_a)\Sigma^a_{\mu\nu}$$

$$b] = \frac{i}{2}\int \sqrt{-g}\left(\frac{1}{4\det(b)}\left(\Sigma^{a}_{\alpha}{}^{\mu}\Sigma^{b}_{\mu}{}^{\nu}\Sigma^{c}_{\nu}{}^{\rho}\Sigma^{d}_{\rho\beta}\right)\left(b^i_c\mathcal{D}^\alpha b^i_a\right)\left(b^j_b\mathcal{D}^\beta b^j_d\right) + 2V(m)\right)$$

here $m_{ab} = b^i_a b^j_b \delta_{ij}$

σ-model of a new type

[redacted]

$$(\partial_\mu \varphi)^2 \rightarrow (\partial_\mu \varphi - \psi)$$

$$g^{\mu\nu}\partial_\mu \varphi^I \partial_\nu \varphi^I$$

$$\tilde{\epsilon}^{\mu\nu\rho\sigma} B^i_{\mu\nu} B^j_{\rho\sigma} = \tilde{X}^{ij}$$

$$\tilde{\epsilon}^{abc} B^i_{ab} := \tilde{E}^{ci}$$

$$p_z + f'(r) = 0$$

$$F^i + \text{"}\frac{\partial V}{\partial B^i}\text{"} = 0$$

$$B^i$$

$$p_z + mr = 0$$

$$f\left((\varphi_i)^2\right)$$

$$\frac{mr^2}{2}$$

an rewrite the theory as that of quantities b^i_a
propagating" on a given metric background

eed to solve $D_A B^i = 0$

sing the metric

$$A_\mu = \frac{1}{2\det(B)} B^{i\rho\sigma} B^j_{\rho\mu} \nabla^\nu B^j_{\nu\sigma}$$

metric
compatible

sing $\mathcal{D}\Sigma^a = 0$

$$\nabla^\mu B^i_{\mu\nu} = (\mathcal{D}^\mu b^i_a)\Sigma^a_{\mu\nu}$$

$$\boxed{b] = \frac{i}{2}\int \sqrt{-g}\left(\frac{1}{4\det(b)}\left(\Sigma^a_\alpha{}^\mu \Sigma^b_\mu{}^\nu \Sigma^c_\nu{}^\rho \Sigma^d_{\rho\beta}\right)\left(b^i_c \mathcal{D}^\alpha b^i_a\right)\left(b^j_b \mathcal{D}^\beta b^j_d\right) + 2V(m)\right)}$$

here $m_{ab} = b^i_a b^j_b \delta_{ij}$

σ-model of a new type

Example: $f(r) = mr^2/2$

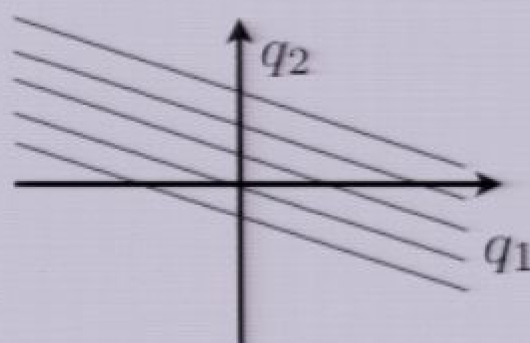
$$\Rightarrow \quad \begin{array}{l} r = -p_2/m \\ \tilde{f}(p_2) = -p_2^2/2m \end{array}$$

$$S = \int dt \left(p_1\dot{q}_1 + p_2\dot{q}_2 - \lambda_1(p_1 - \frac{p_2^2}{2m}) \right)$$

constraint generates

$$q_1 \longrightarrow q_1 + \lambda_1$$

$$q_2 \longrightarrow q_2 - \frac{p_2}{m}\lambda_1$$



$p_2 \neq 0$

$m \to \infty$ gives back the original system

$m \to 0$ interchanges the relevant vars

relevant coordinate $\quad Q := q_2 + \frac{p_2}{m} q_1$

Deformations of GR

$$\boxed{S = \int B^i \wedge F^i + V(B^i \wedge B^j)}$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$
$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;
closed under renormalization?

$$g^{\mu\nu}\,\partial_\mu \varphi^I\,\partial_\nu \varphi^I$$

$$B^i \to \underline{M_1^{ij} B^j} + \underline{M_2^{ij} F^j}$$

$$\epsilon^{abc}\, B^i_{ab} :=$$

$$g^{\mu\nu}\,\partial_\mu \varphi^I\,\partial_\nu \varphi^I$$

$$B^i \to \underline{M_1^{ij}\,B^j} + \underline{M_2^{ij}\,F^j}$$

$$\epsilon^{abc}\,B^i_{ab} :=$$

Deformations of GR

$$\boxed{S = \int B^i \wedge F^i + V(B^i \wedge B^j)}$$

where the potential satisfies

$$V(\alpha X^{ij}) = \alpha V(X^{ij})$$
$$V(\Lambda X \Lambda^T) = V(X) \quad \forall \Lambda \in SO(3)$$

Remark: integrating out B^i gets a pure connection theory

$$S = \int f(F^i \wedge F^j)$$

most general diff. invariant gauge theory;

closed under renormalization?