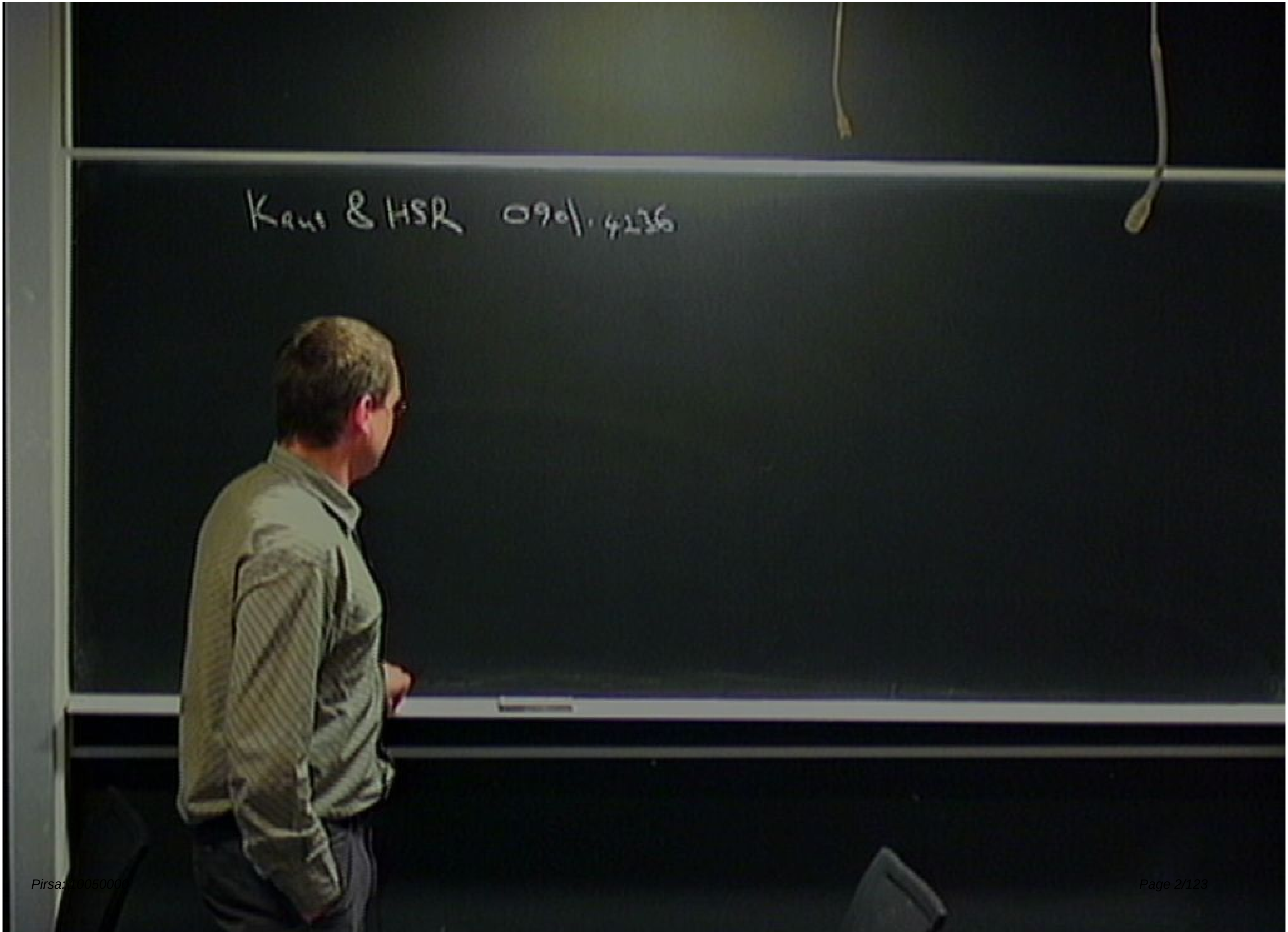
Title: Strong Gravity

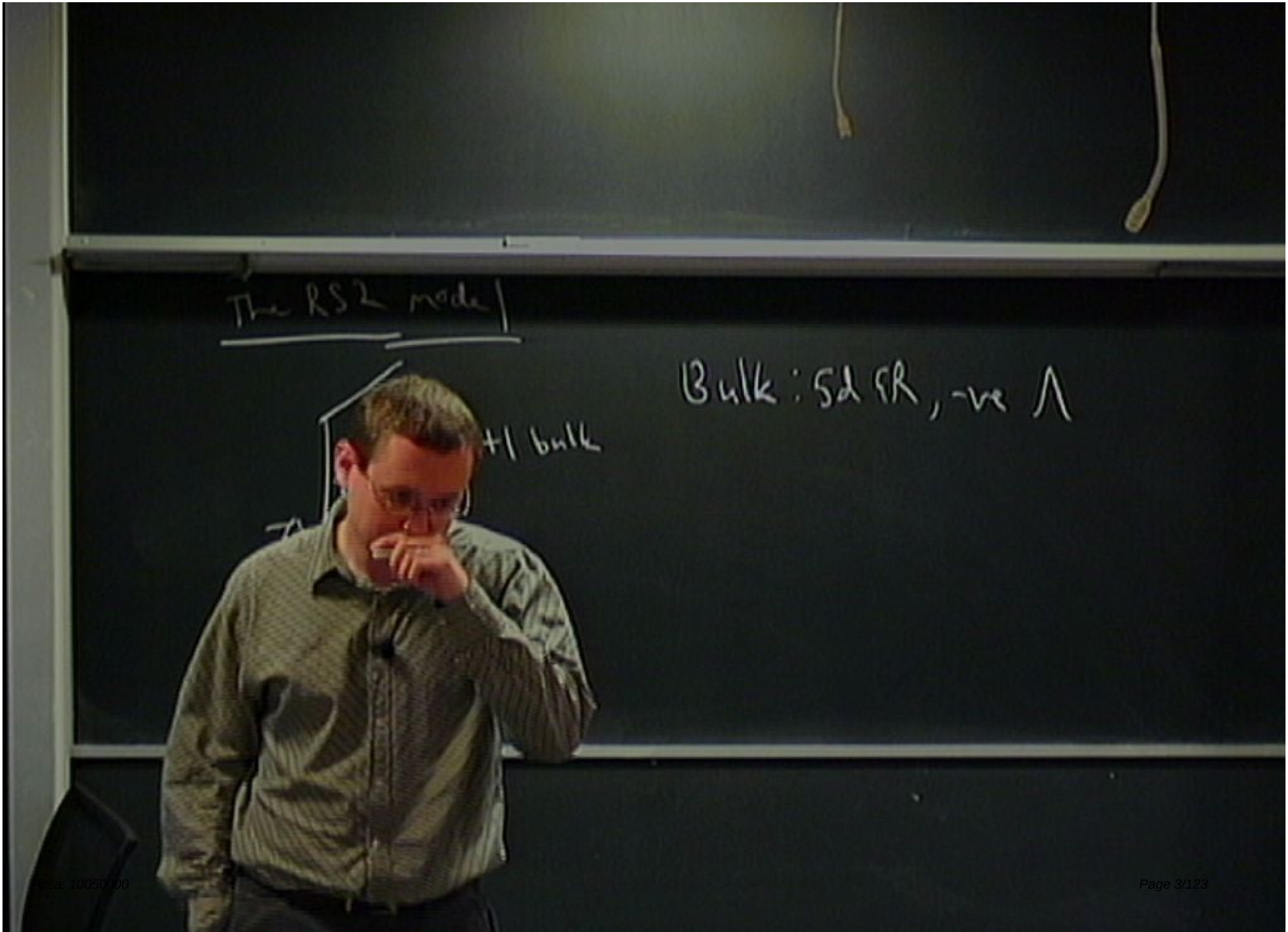
Date: May 06, 2010 01:00 PM

URL: http://pirsa.org/10050000

Abstract: TBA

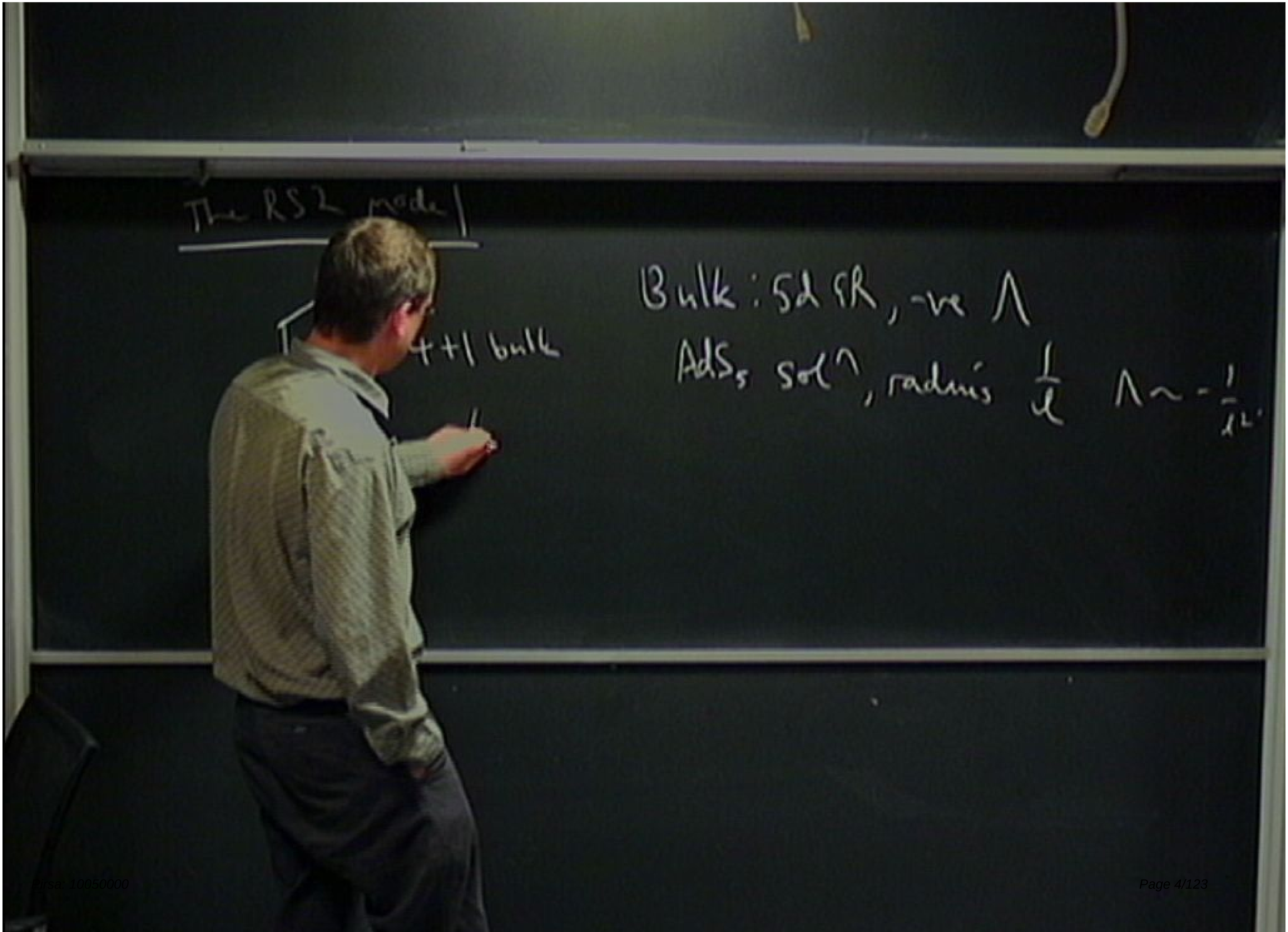


Kaus & HSR 0901.4236



The RS2 model

Bulk : 5d GR, -ve Λ

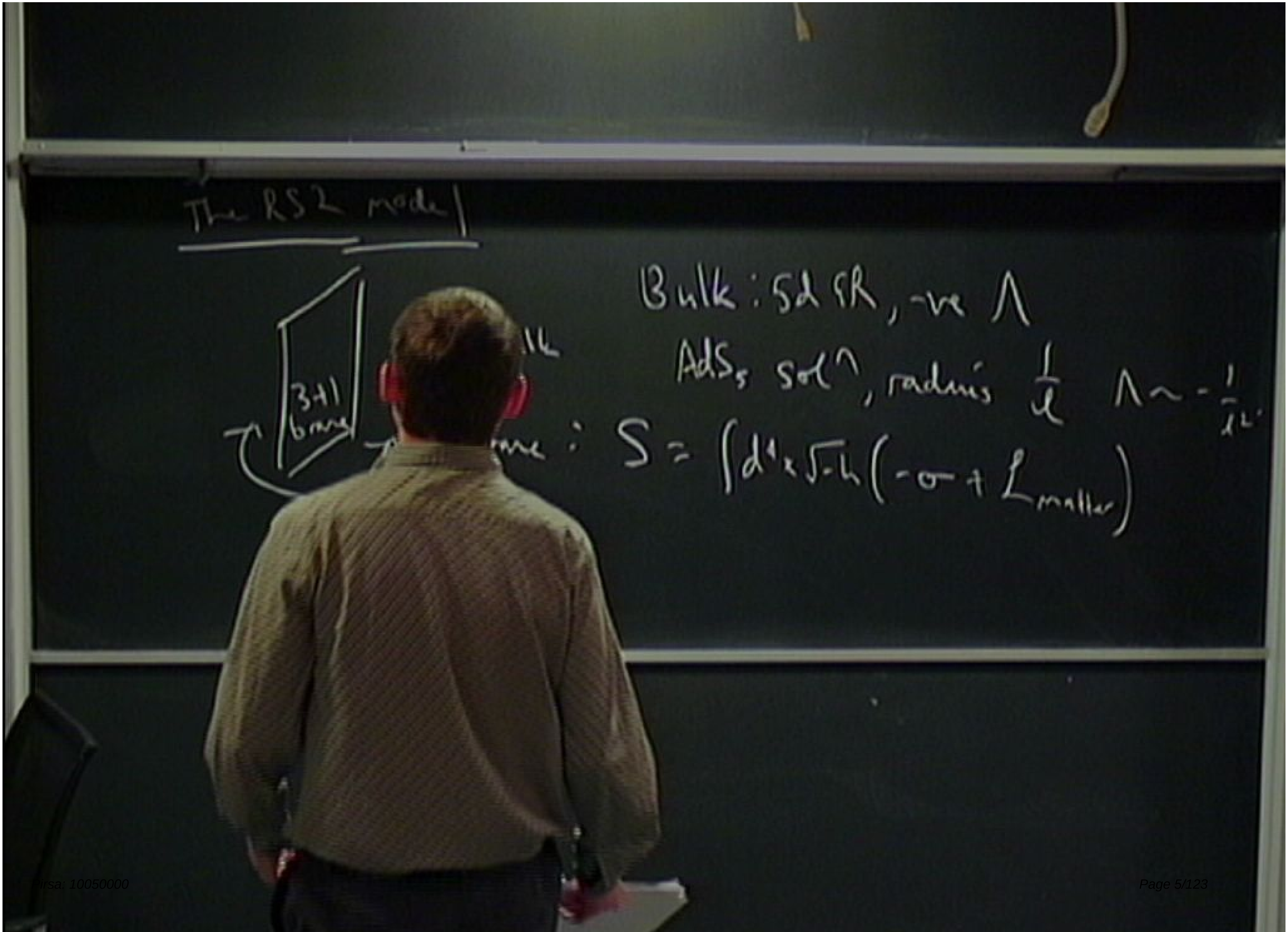


The RS2 model

4+1 bulk

Bulk: 5d GR, -ve Λ

AdS$_5$ sol^n, radius $\frac{1}{l}$ $\quad \Lambda \sim -\frac{1}{l^2}$

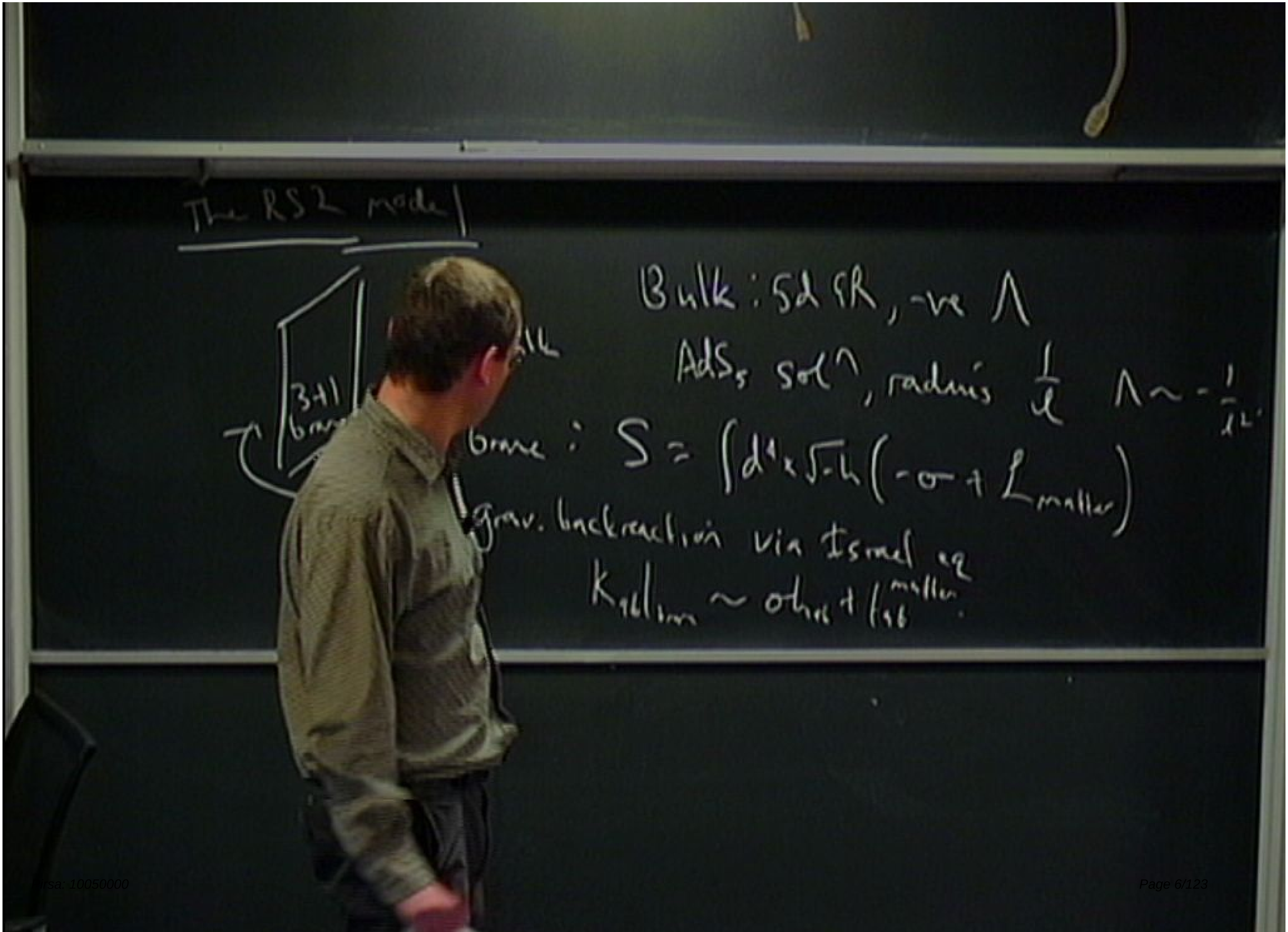


The RS2 model

Bulk : 5d GR, -ve Λ

AdS_5 sol^n, radius $\frac{1}{\ell}$ $\Lambda \sim -\frac{1}{\ell^2}$

me : $S = \int d^4x \sqrt{-h} \left(-\sigma + \mathcal{L}_{matter} \right)$



The RS2 model

Bulk: 5d GR, -ve Λ

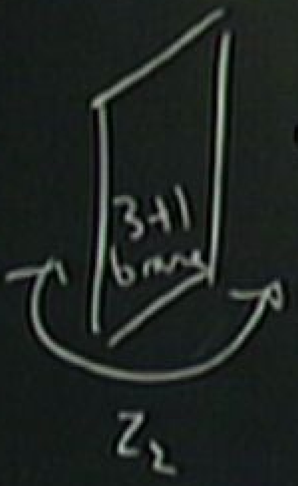
AdS_5 sol^n, radius $\frac{1}{\ell}$ $\Lambda \sim -\frac{1}{\ell^2}$

brane: $S = \int d^4x \sqrt{-h}\left(-\sigma + \mathcal{L}_{matter}\right)$

grav. backreaction via Israel eq

$K_{ab}|_{bran} \sim$ others $+ T_{ab}^{matter}$

The RS2 model



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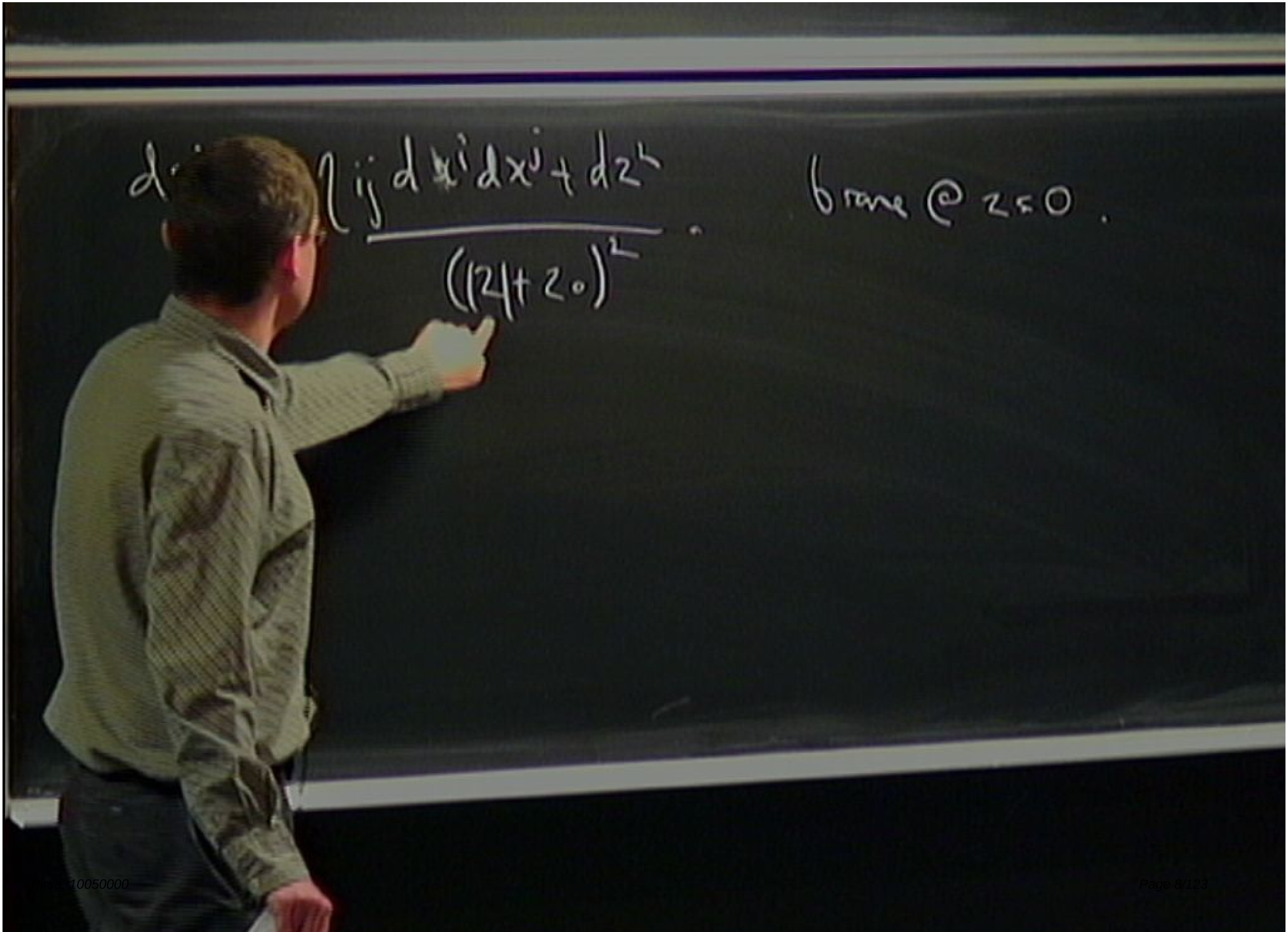
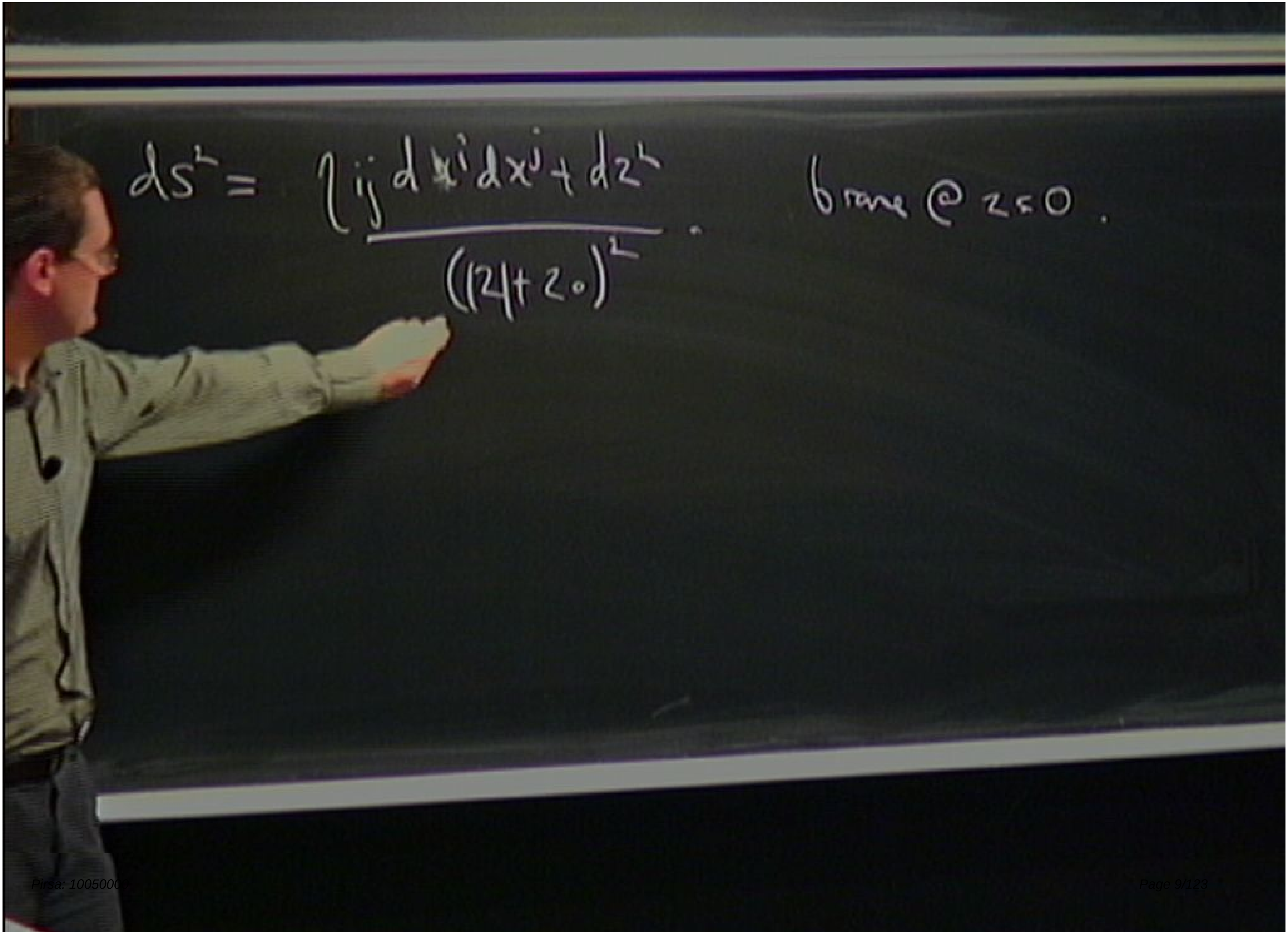
grav. backreaction via Israel eq

$K_{ab}|_{brane} \sim \sigma h_{ab} + T_{ab}^{matter}$



$$d\cdot \quad \eta_{ij}\frac{dx^i dx^j + dz^2}{(|z|+z_0)^2} .$$

brane @ z=0 .



$$ds^2 = \frac{\eta_{ij}\, dx^i dx^j + dz^2}{(|z|+z_0)^2}.$$

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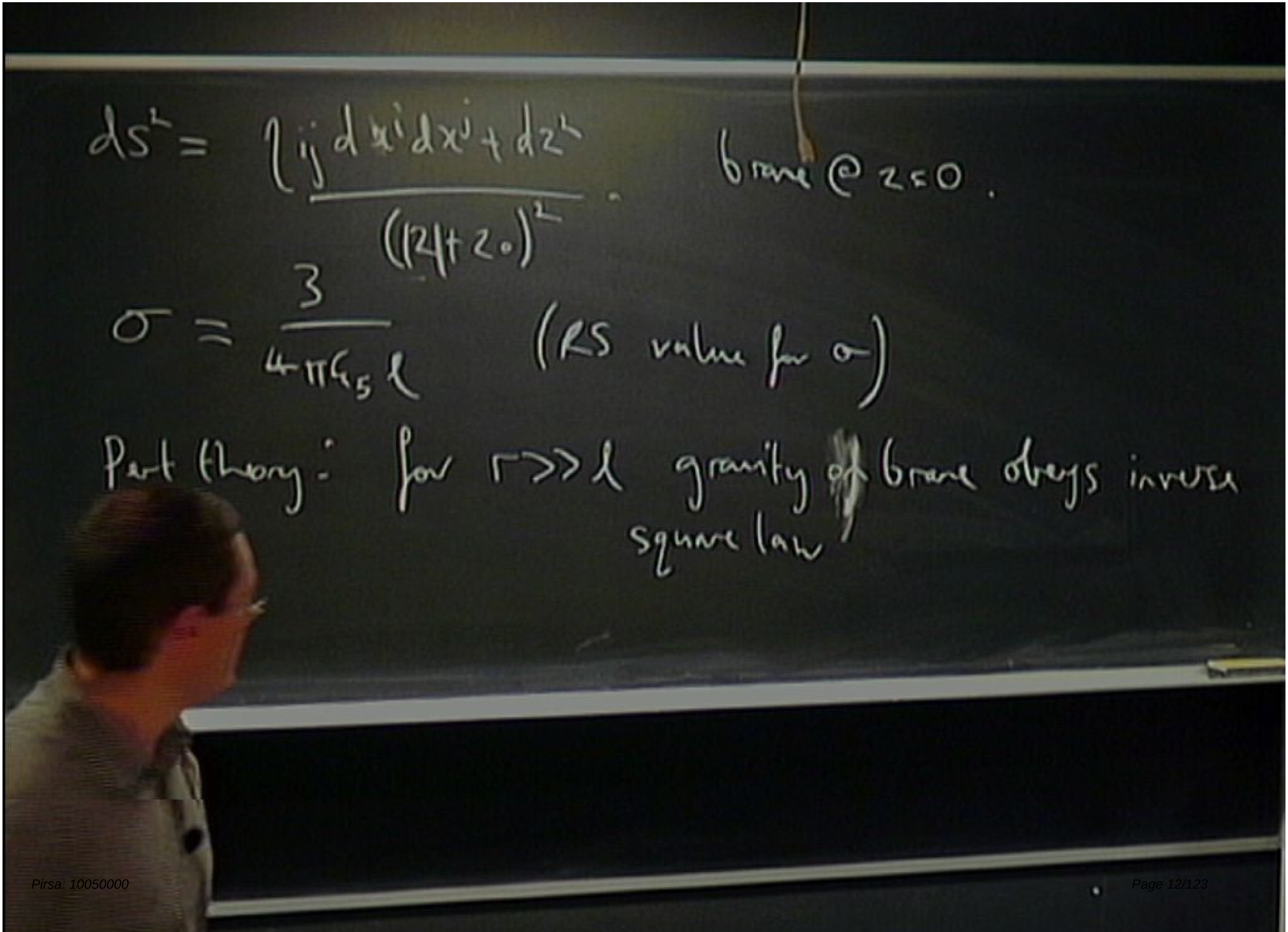
$$\sigma = \frac{3}{4\pi G_5 \ell}$$ (RS value for σ)



$$ds^2 = \frac{\eta_{ij}\,dx^i dx^j + dz^2}{(|z|+z_0)^2} .$$ brane @ $z=0$.

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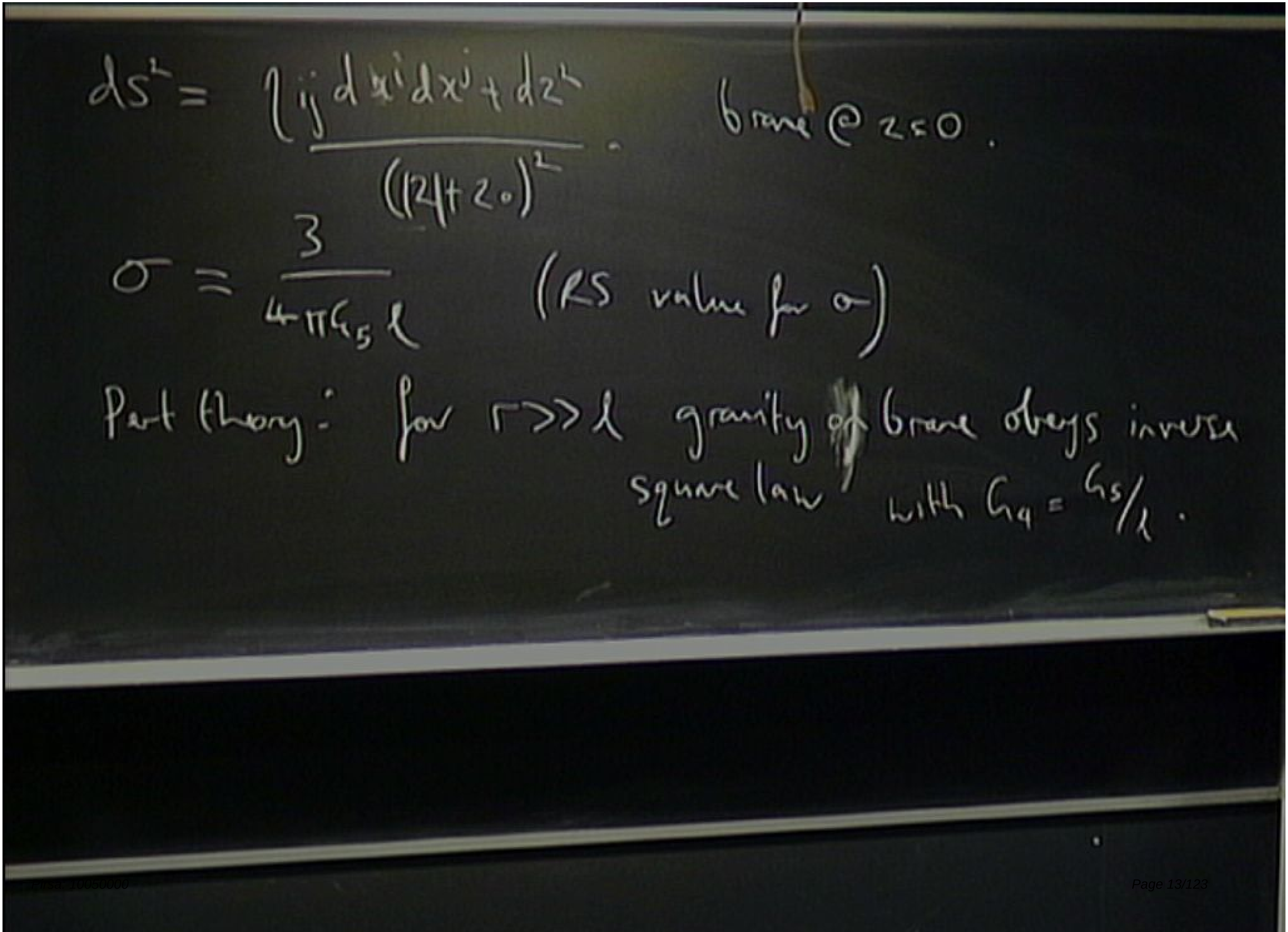
Pert theory: for $r \gg \ell$ gravity on brane obeys inverse square law



$$ds^2 = \frac{\eta_{ij}\,dx^i dx^j + dz^2}{(|z|+z_0)^2}. \qquad \text{brane @ } z=0.$$

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Pert theory: for $r \gg \ell$ gravity on brane obeys inverse square law with $G_4 = G_5/\ell$.

$$(|z|+z_0)^2$$

$$\sigma = \frac{3}{4\pi G_5 \ell} \quad (\text{RS value for } \sigma)$$

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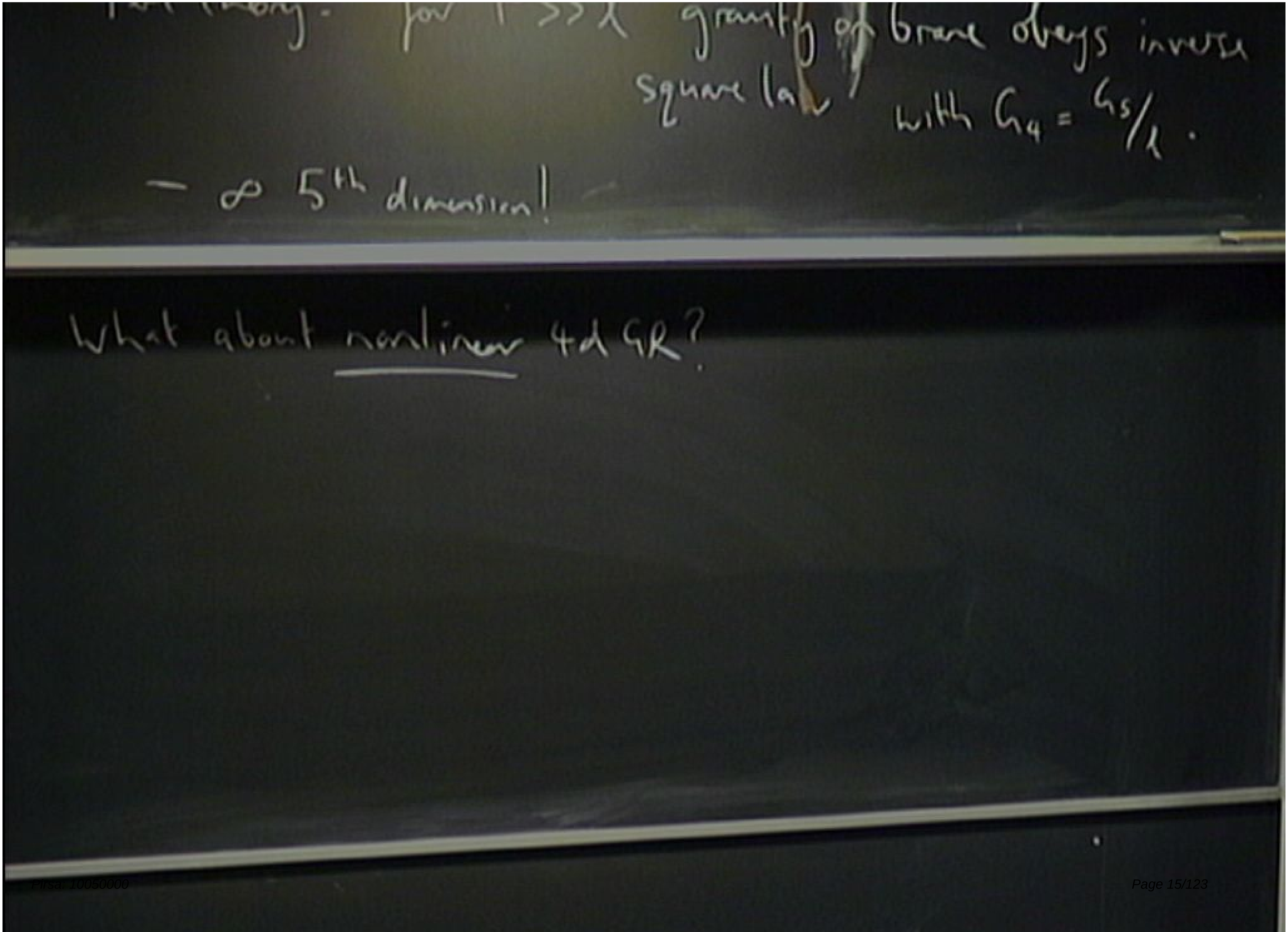
— ∞ 5th dimension!



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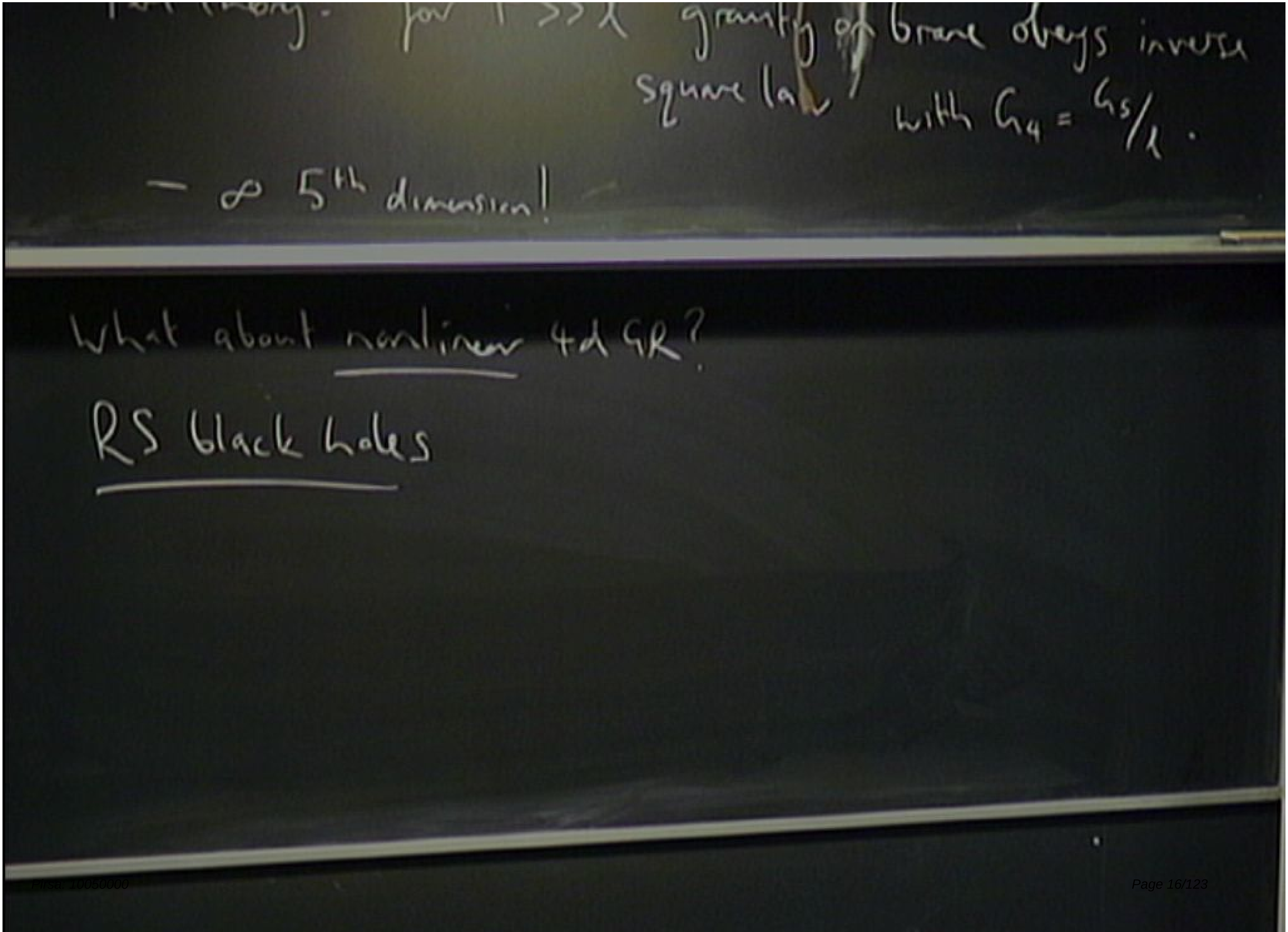
What about nonlinear 4d GR?



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RS black holes

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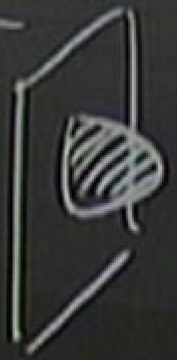
<u>RS black holes</u>



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RS black holes

Many attempts to find sol^n (exact or numerical) theory: for $r \gg \ell$ gravity of brane obeys inverse square law with $G_4 = G_5/\ell$.

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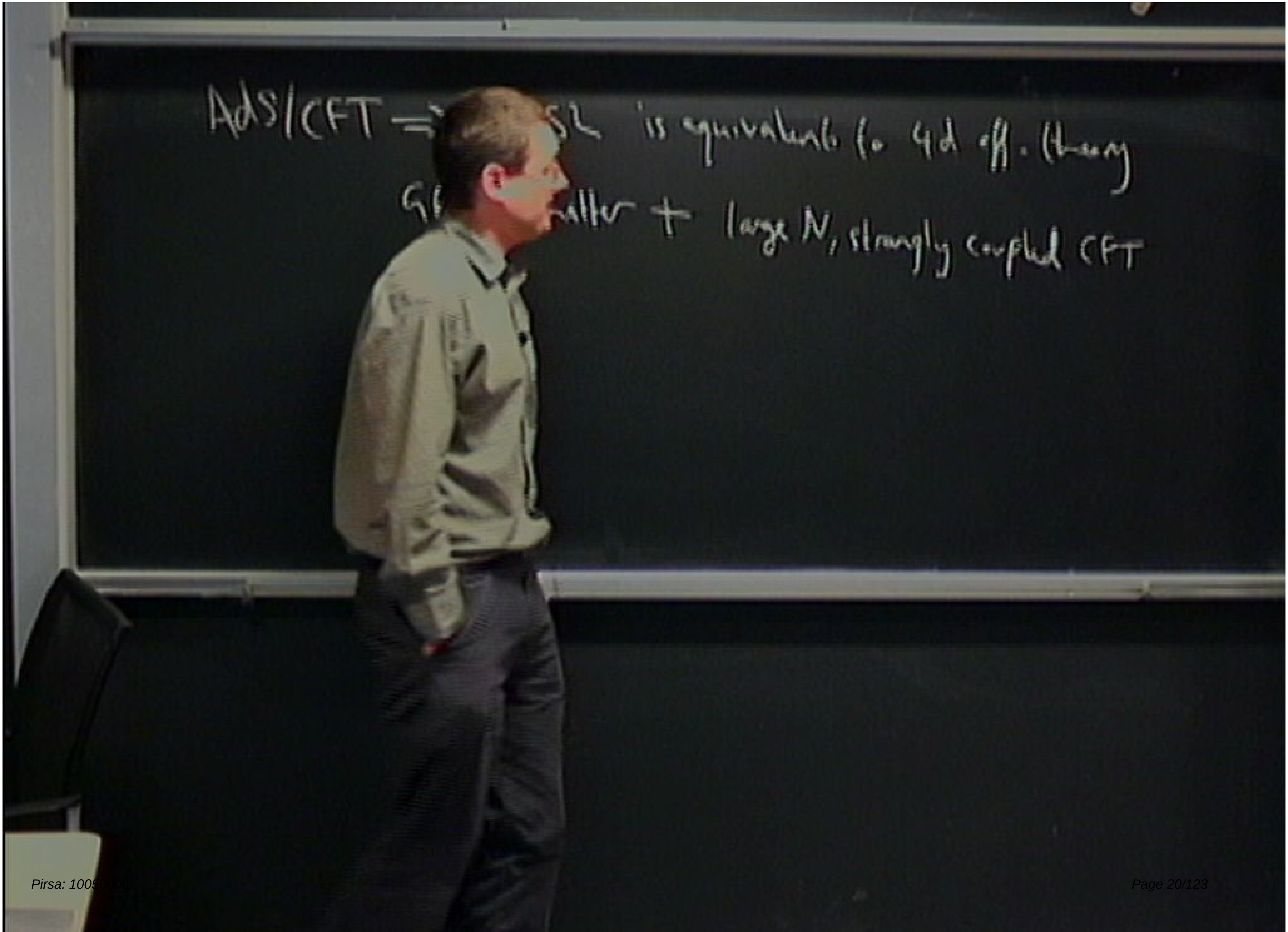
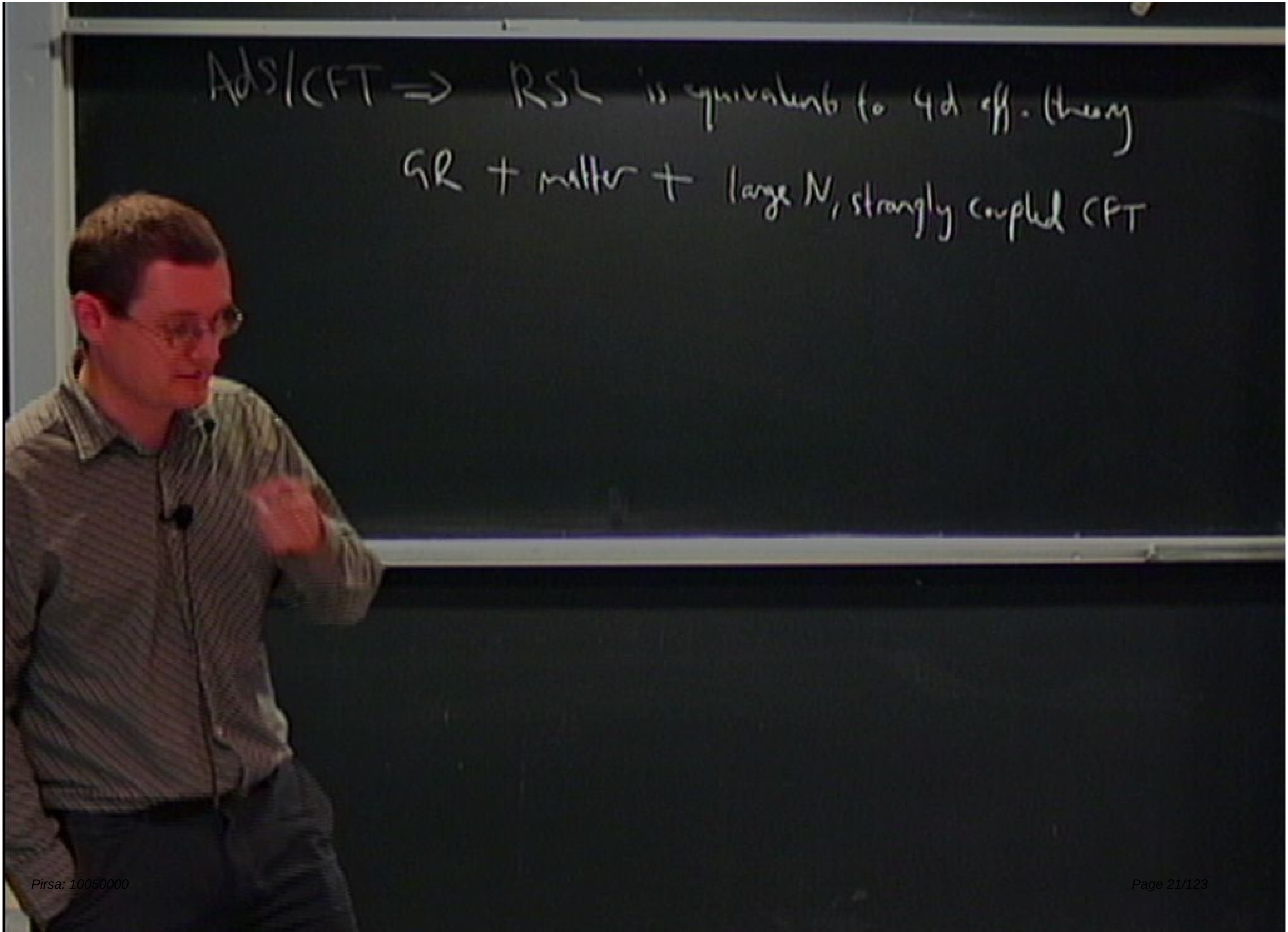
RS black holes

Many attempts to find sol^n (exact or numerical) describing RS2 BH with $G_4 M \gg \ell$.



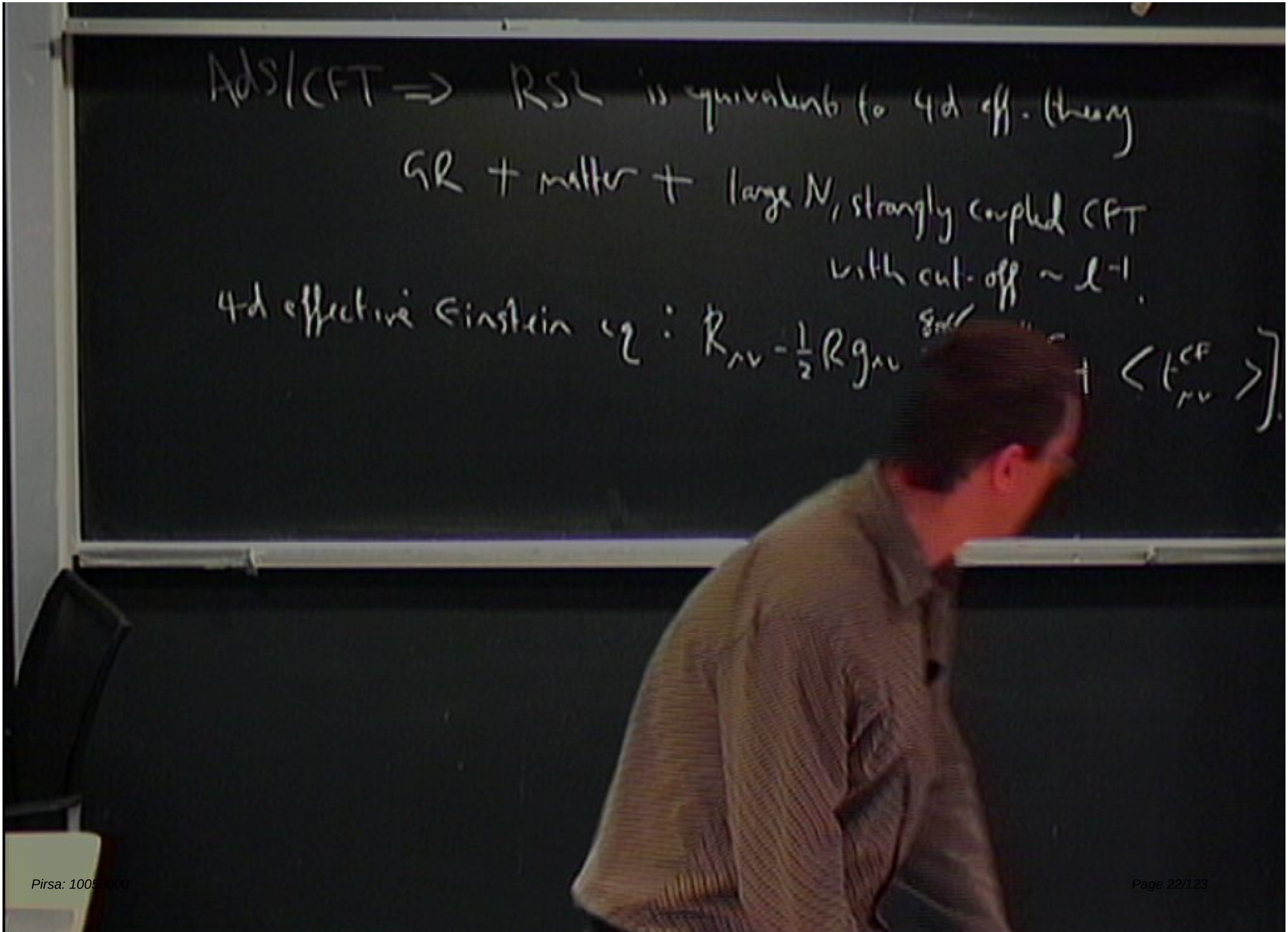
AdS/CFT ⇒ … S² is equivalent to 4d eff. theory

G… …atter + large N, strongly coupled CFT



AdS/CFT $\Rightarrow$ RS2 is equivalent to 4d eff. theory

GR + matter + large N, strongly coupled CFT

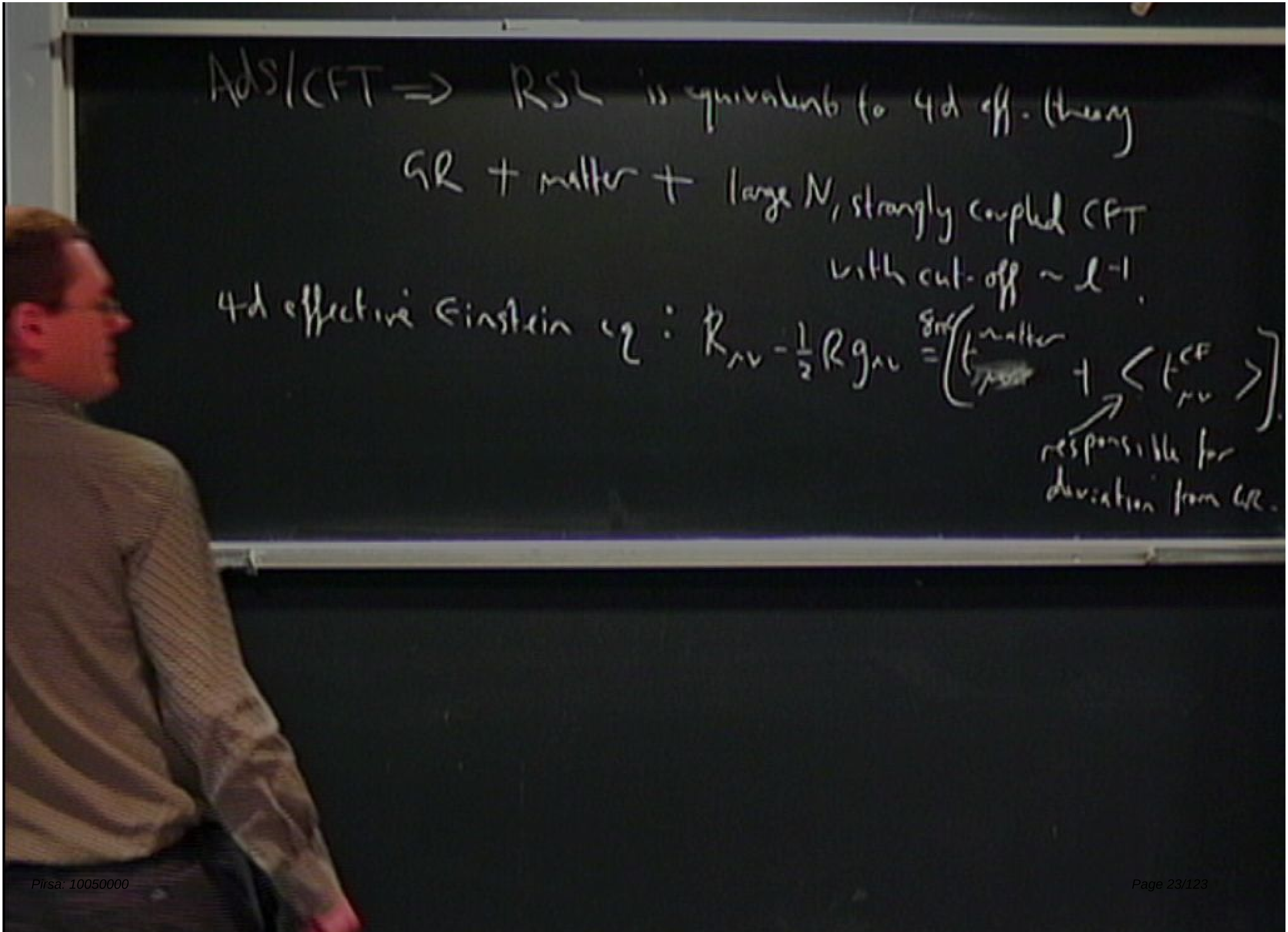


AdS/CFT ⇒ RS2 is equivalent to 4d eff. theory

GR + matter + large N, strongly coupled CFT

with cut-off ~ ℓ^{-1}.

4d effective Einstein eq : $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ $8\pi G$ [illegible] $+ \langle T^{CF}_{\mu\nu} \rangle]$.



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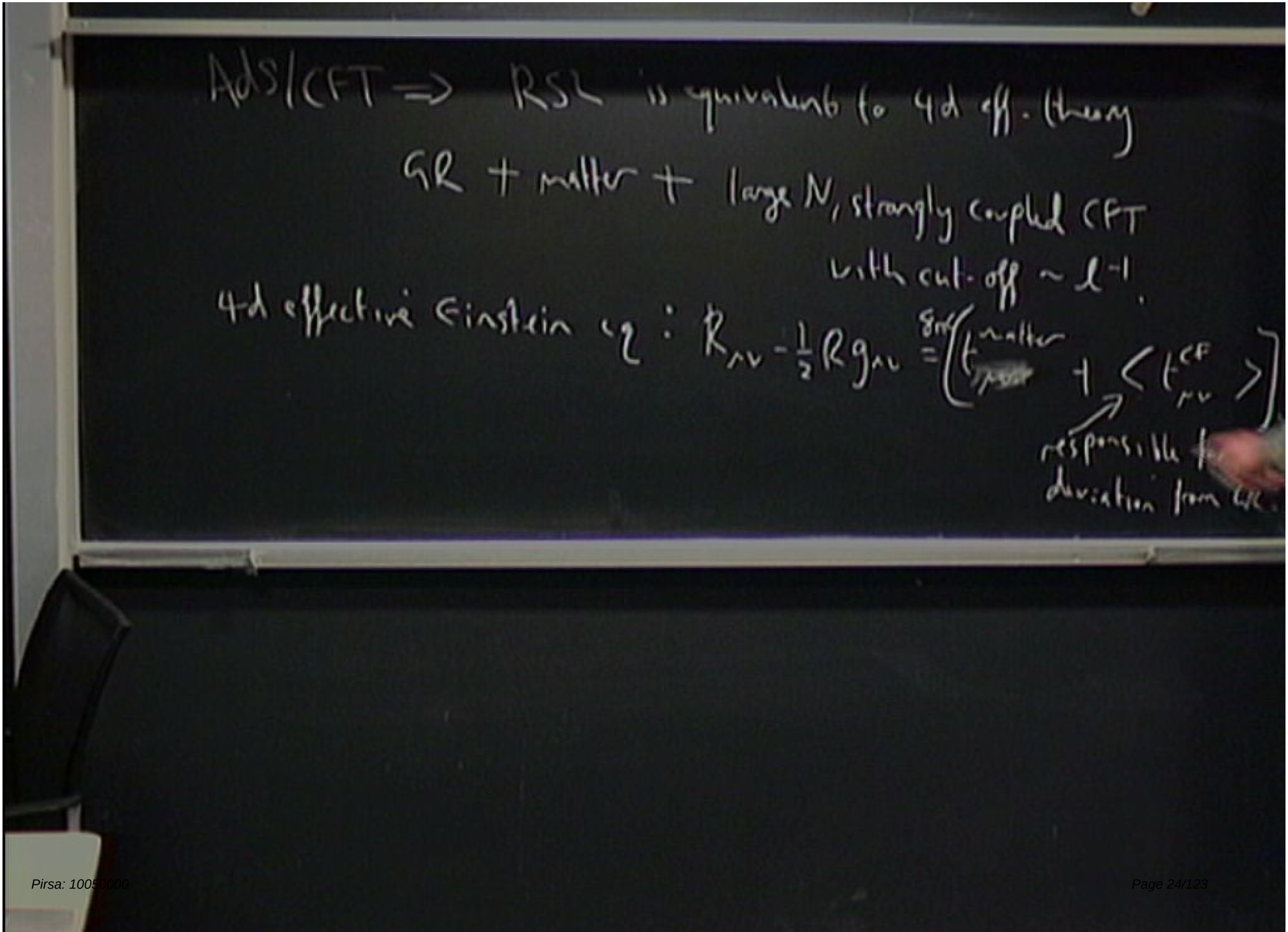
responsible for deviation from GR.



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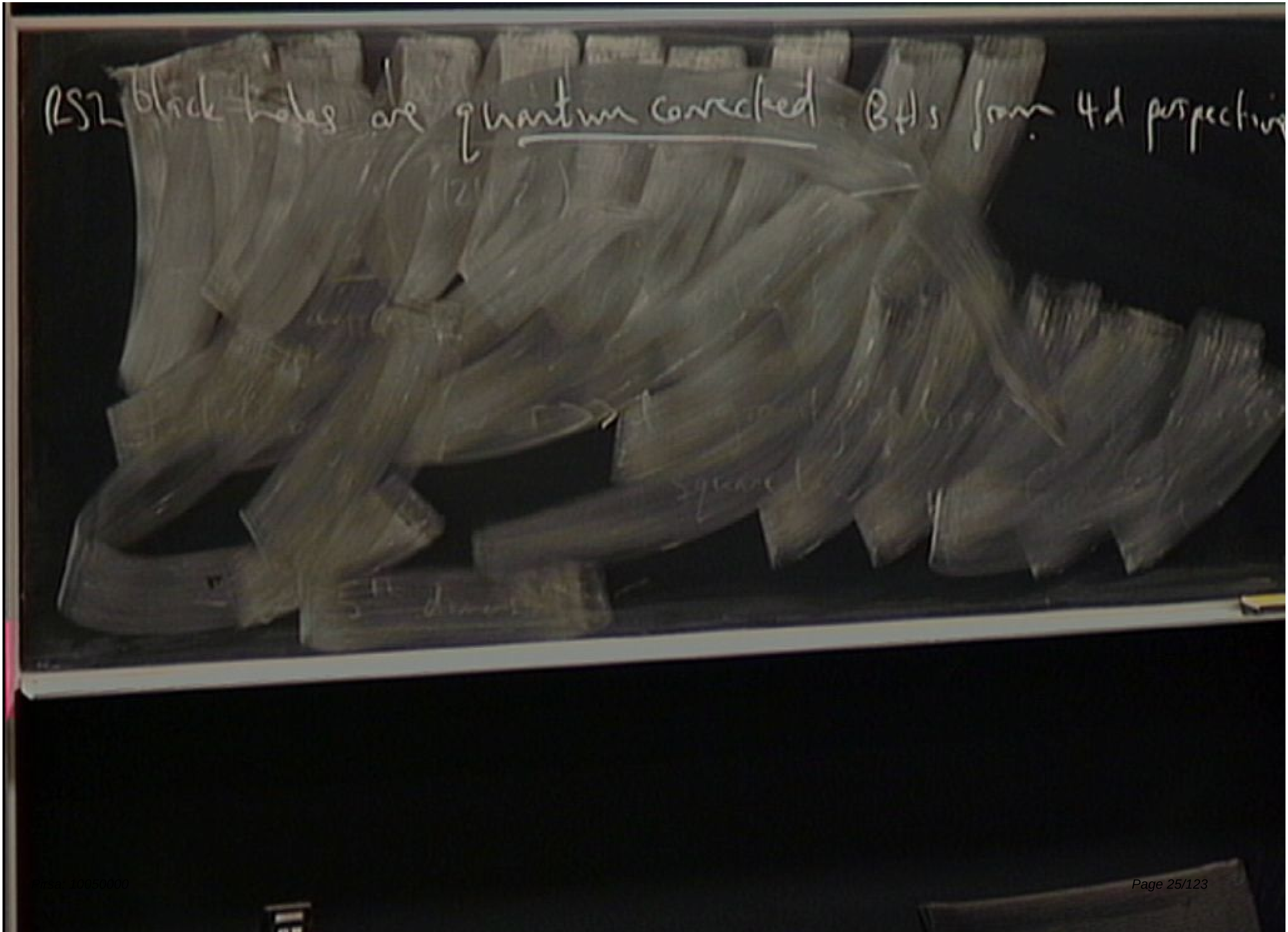
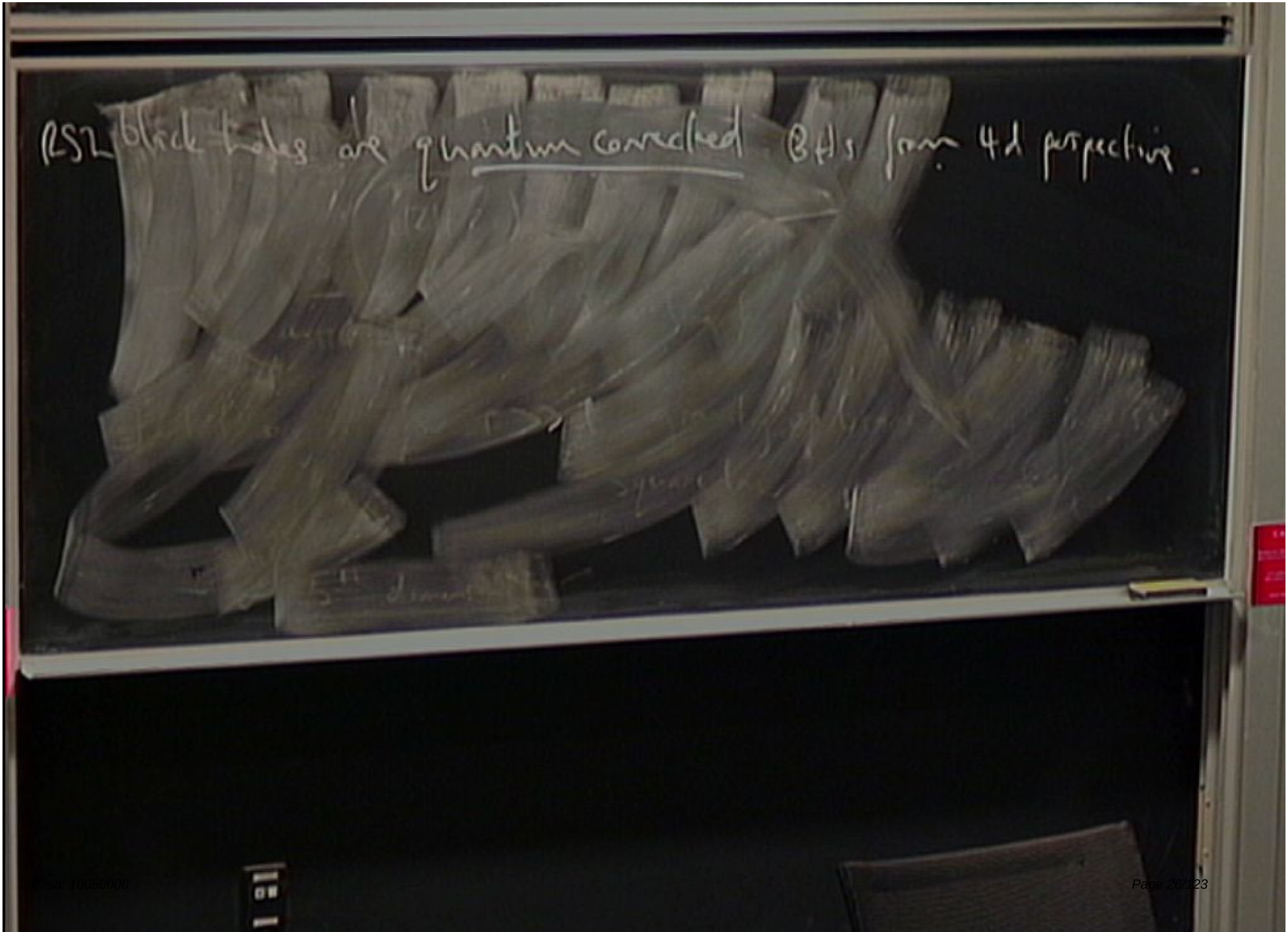


RS2 black holes are <u>quantum corrected</u> BHs from 4d perspectiv



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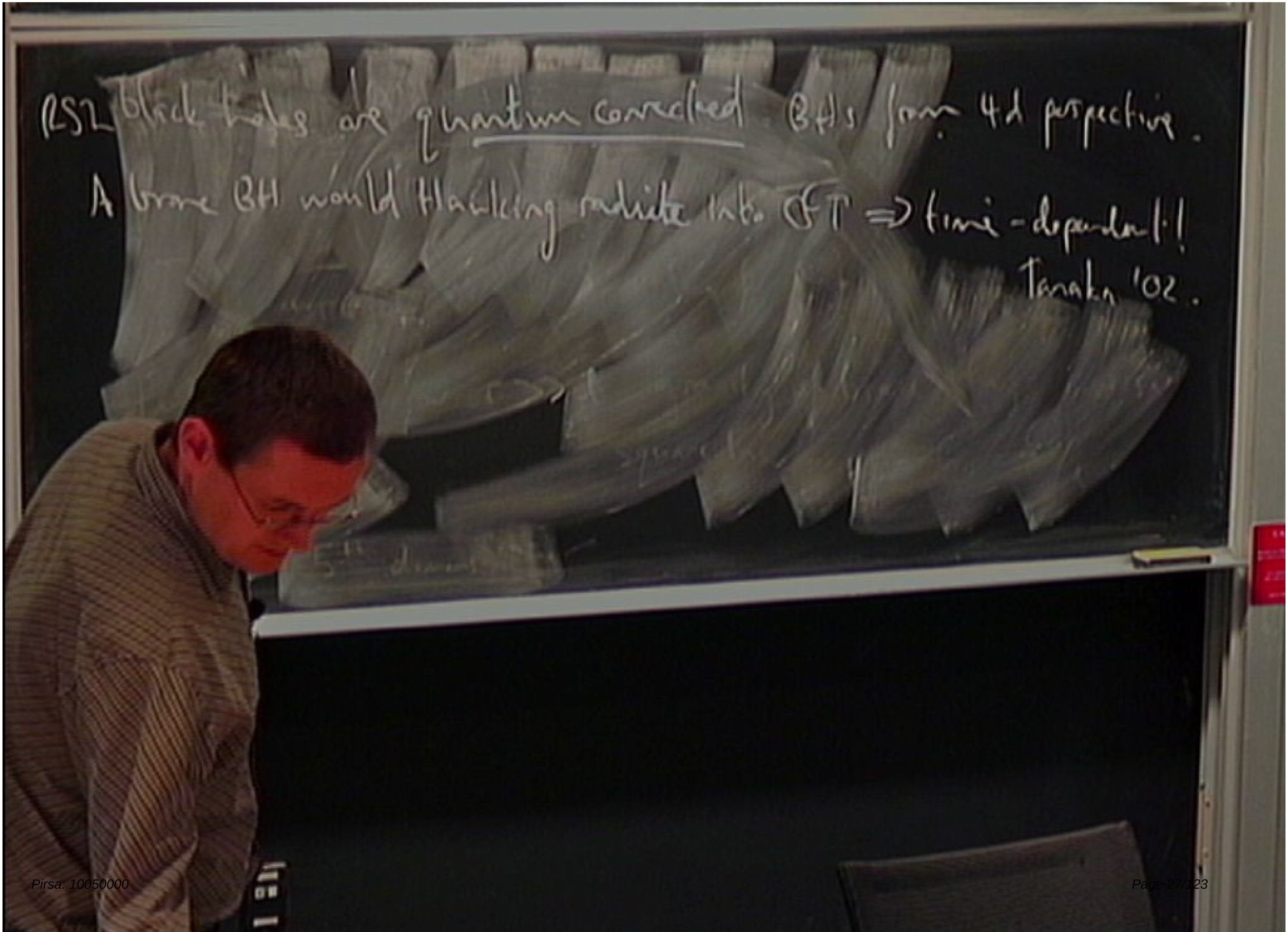
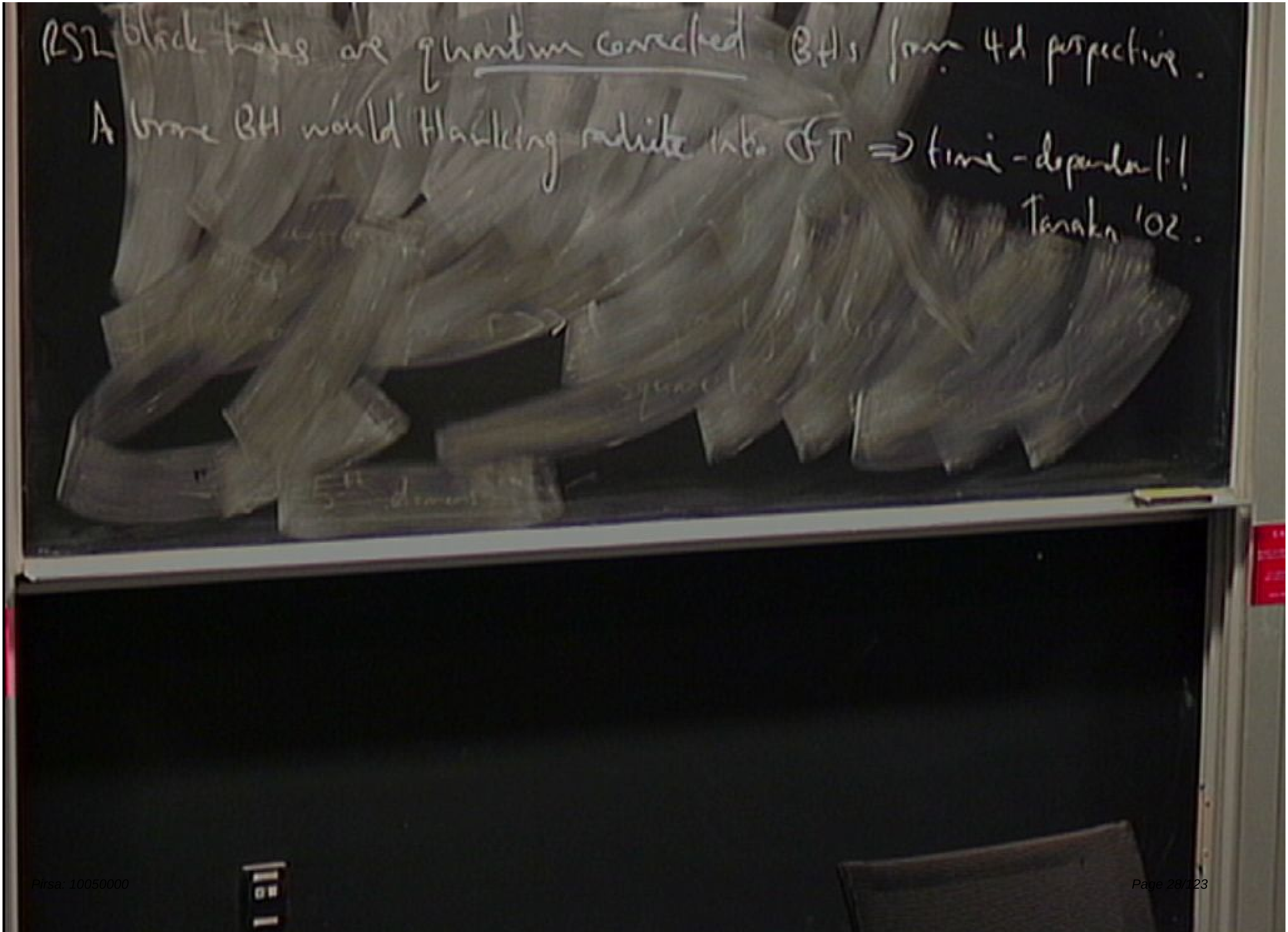


RS2 black holes are quantum corrected BHs from 4d perspective.

A brane BH would Hawking radiate into CFT ⇒ time-dependent!

Tanaka '02.



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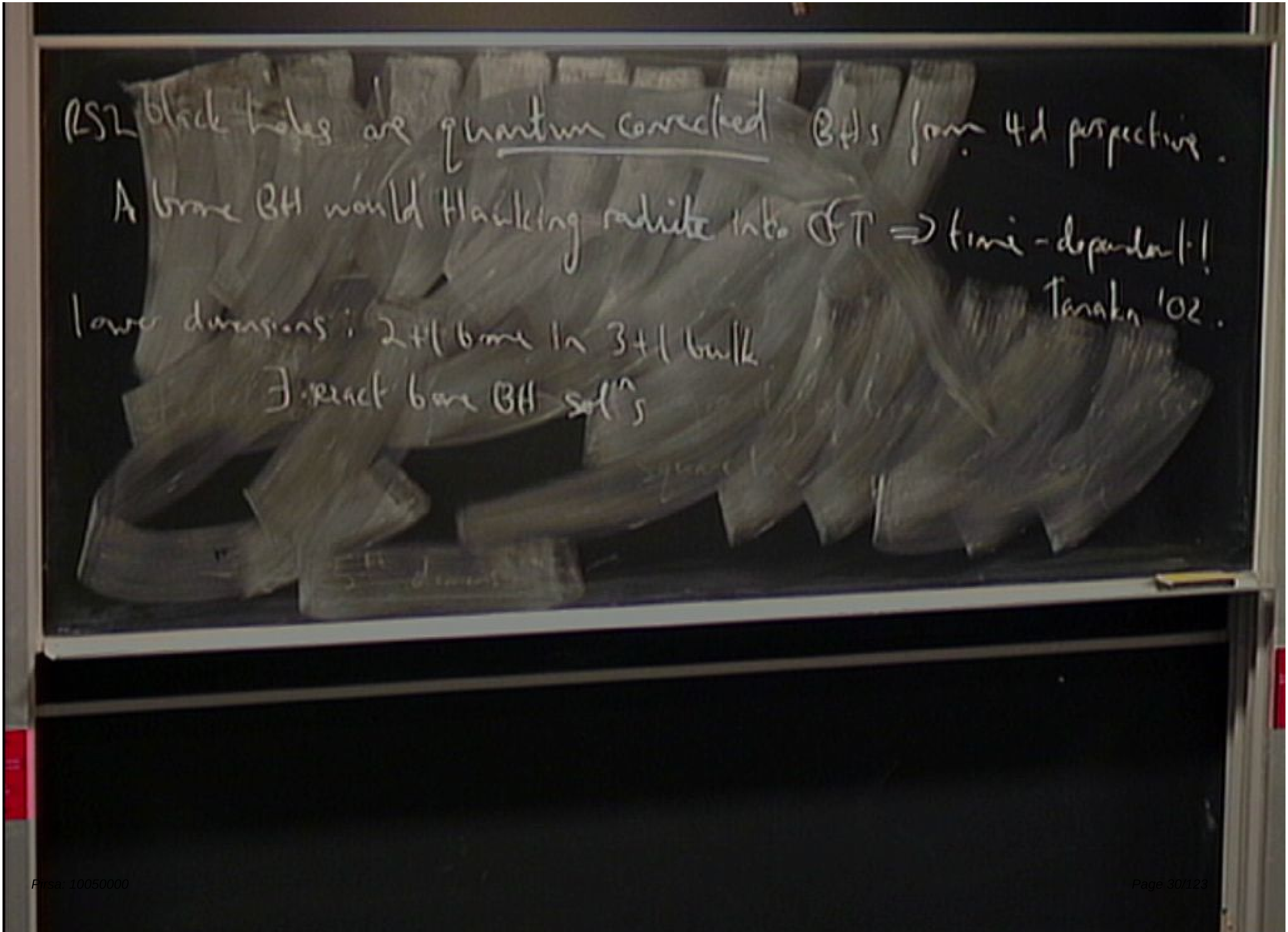

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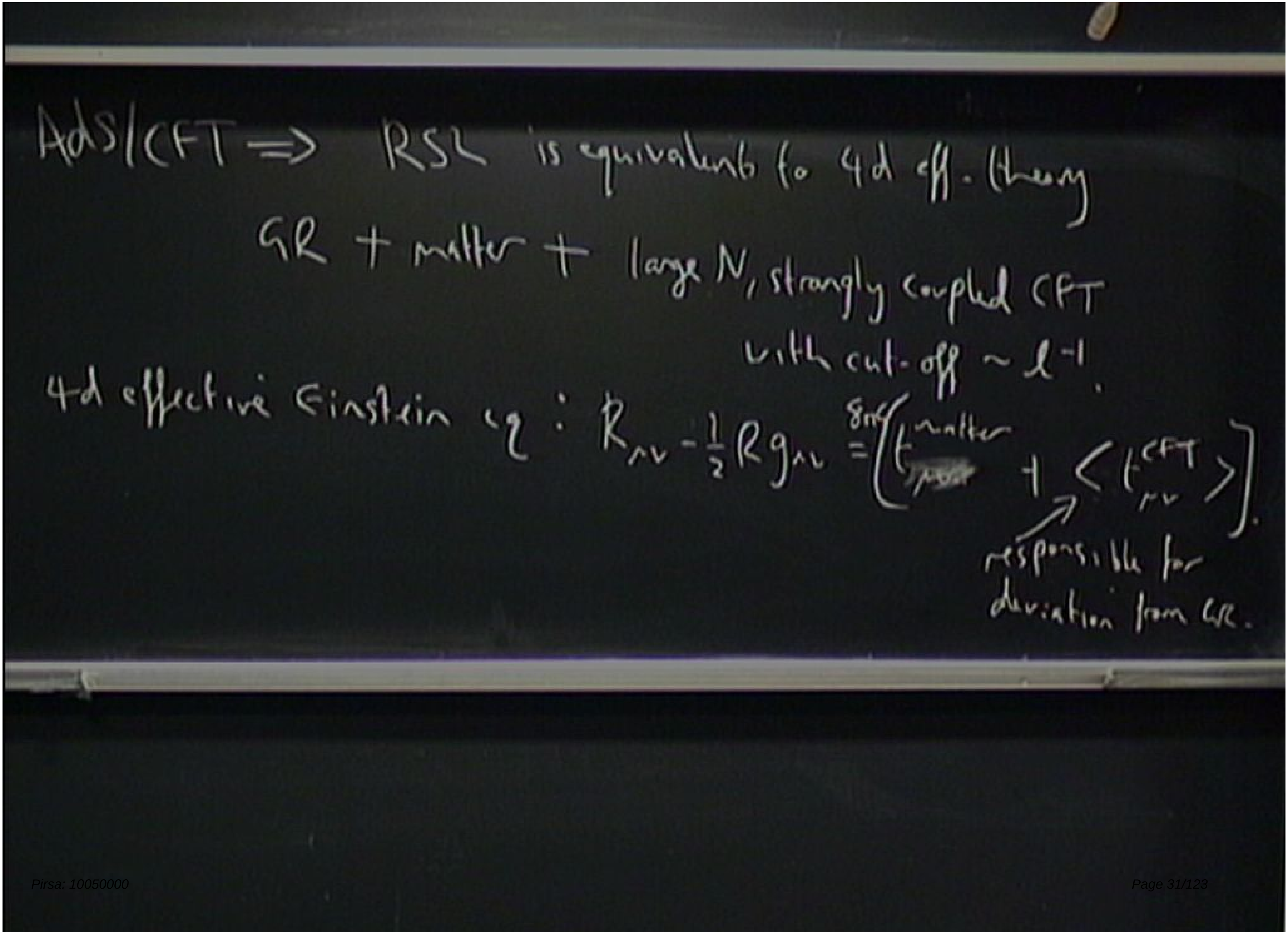
lower dimensions: 2+1 brane in 3+1 bulk

∃ exact brane BH sol^ns



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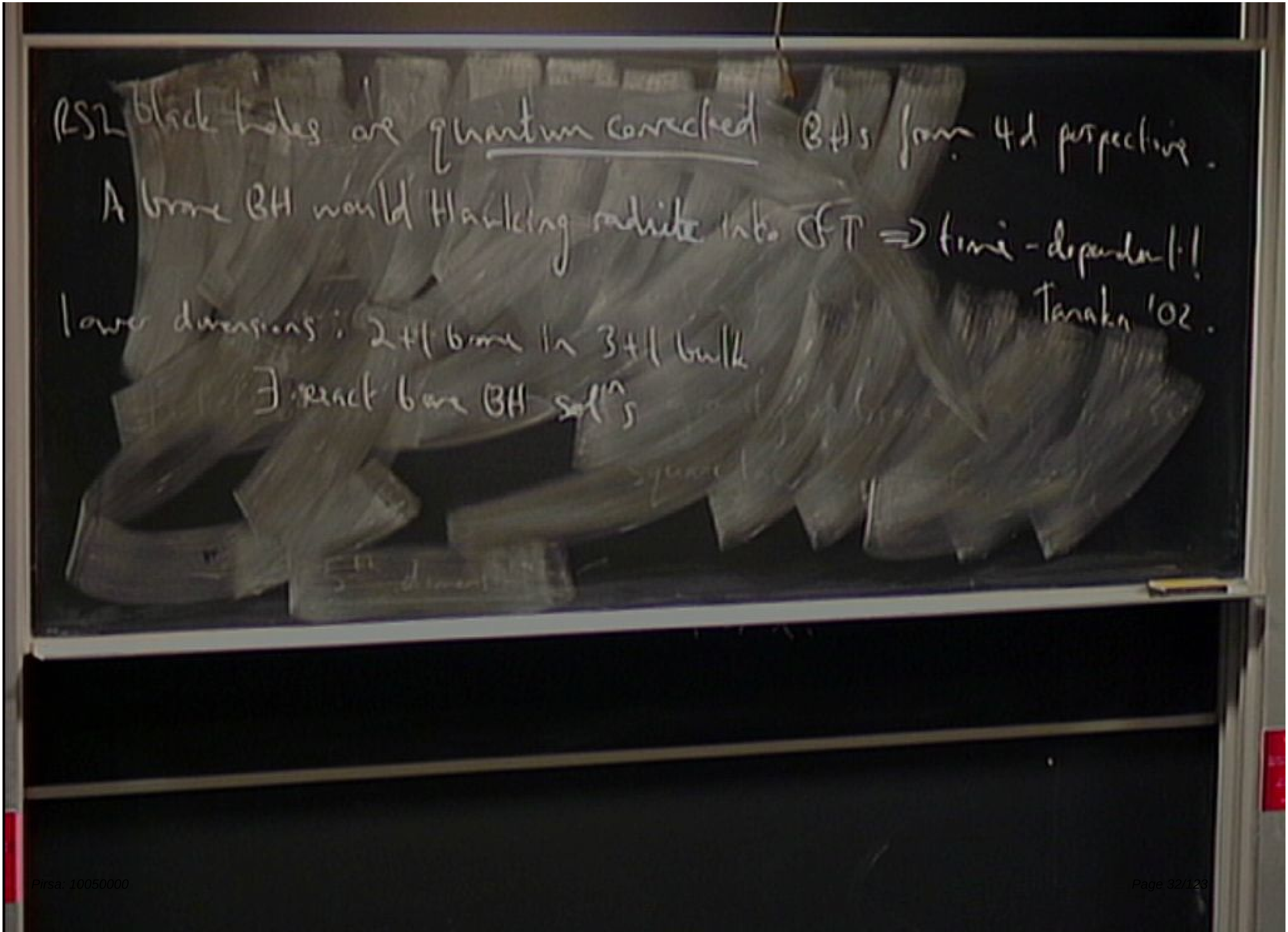

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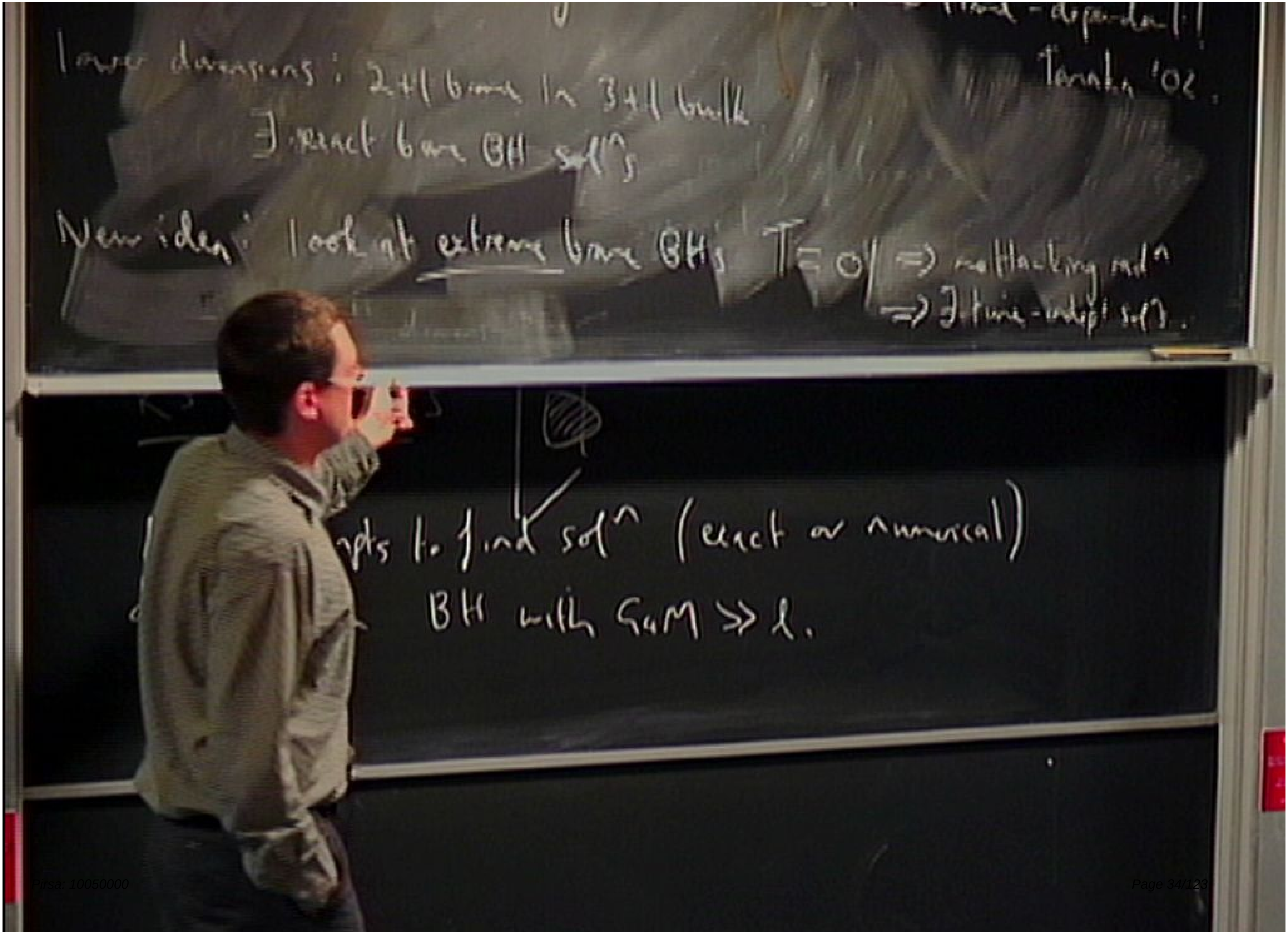
lower dimensions: 2+1 brane in 3+1 bulk Tanaka '02.

$\exists$ exact brane BH sol^{n}s

New idea:

Many attempts to find sol^{n} (exact or numerical) describing RS2 BH with $G_4 M \gg l$.



time-dependent!

lower dimensions: 2+1 brane in 3+1 bulk

$\exists$ exact brane BH sol^{n}s

Tanaka '02.

New idea: look at extreme brane BHs $T=0$ $\Rightarrow$ no Hawking rad^n

$\Rightarrow \exists$ time-indep^t sol^n.

pts to find sol^n (exact or numerical)

BH with $G_4 M \gg \ell$.

5d bulk

∃ exact brane BH sol^ns

New idea: look at extreme brane BHs (T = 0) ⇒ no Hawking rad^n
⇒ ∃ time-indep^t sol^n?

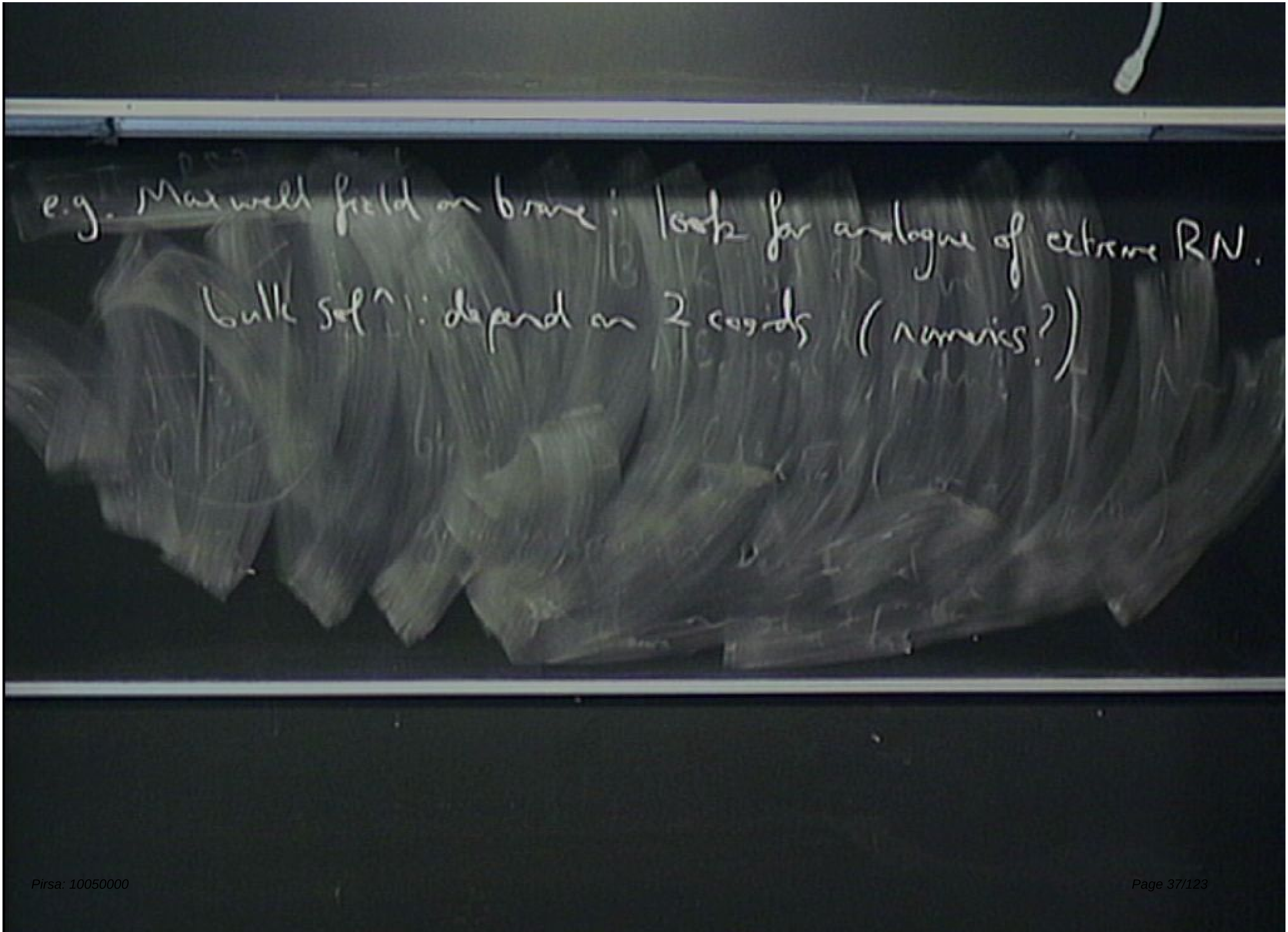
What about Kerr, New 4d GR?

RS black holes

Many attempts to find sol^n (exact or numerical)
describing RS2 BH with $G_4 M \gg \ell$.

e.g. Maxwell field on brane: look for analogue of extreme RN.

bulk [illegible] pend on 2 coords



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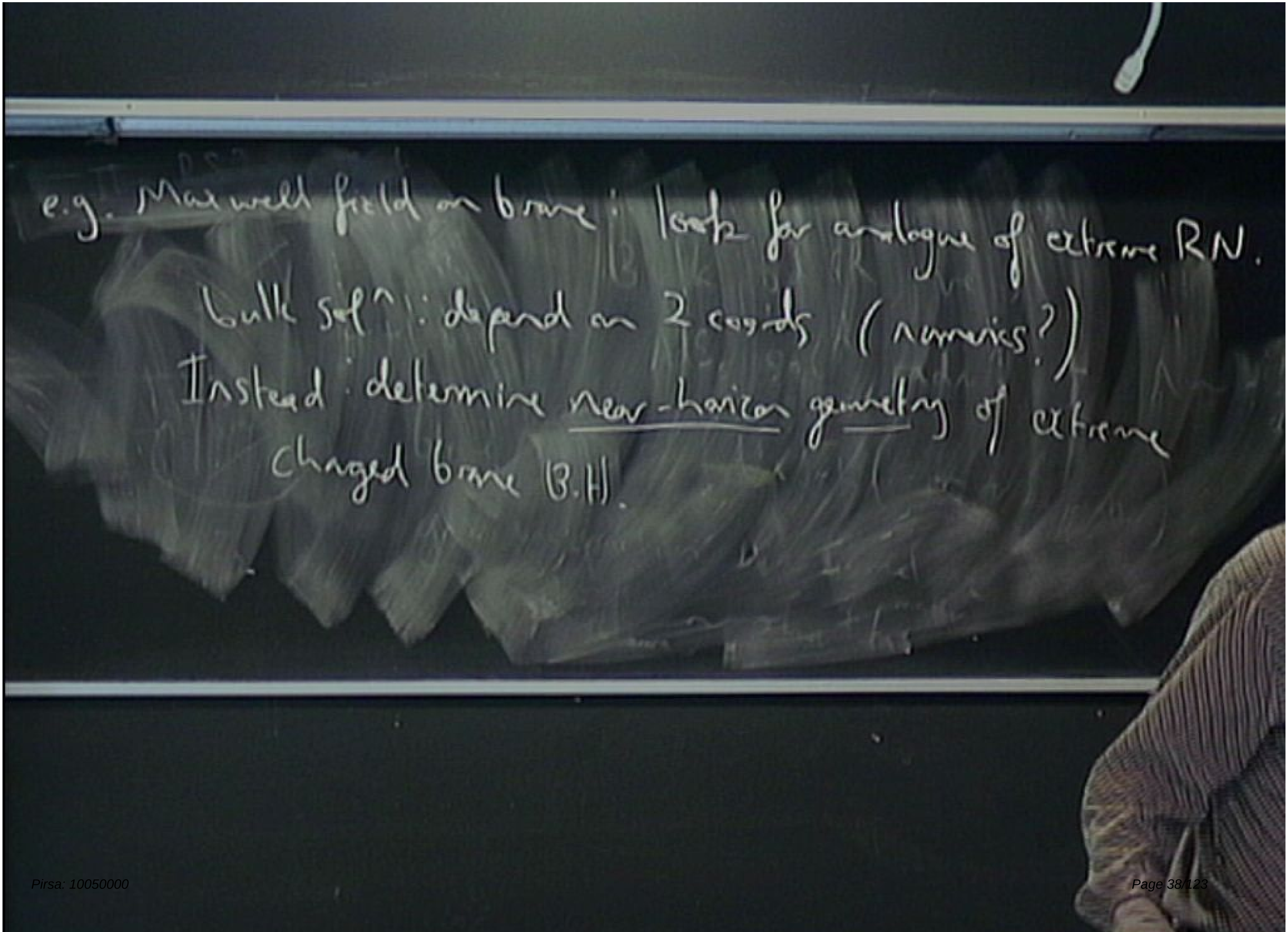

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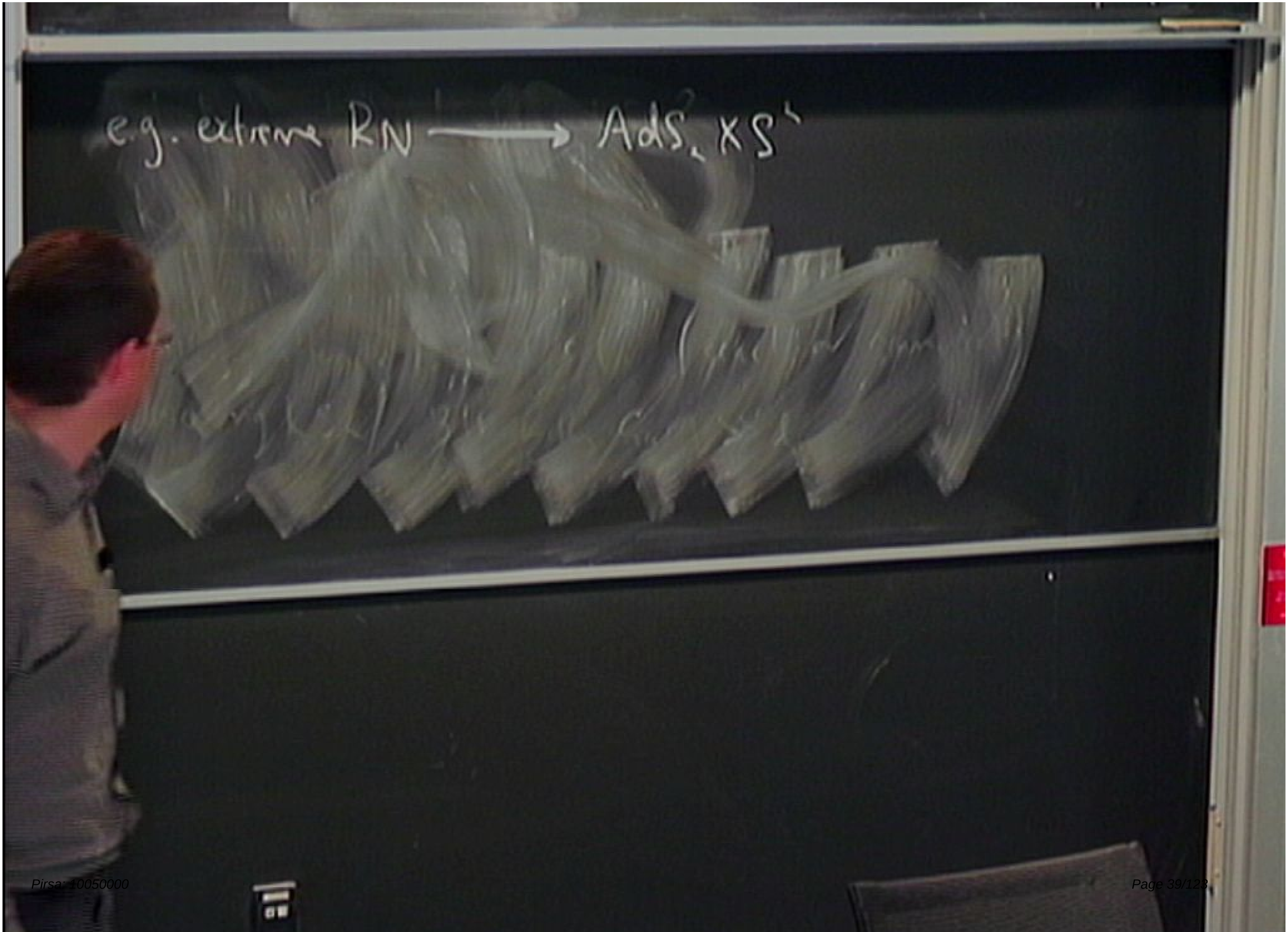
Instead: determine near-horizon geometry of extreme charged brane (B.H).



e.g. extreme RN $\longrightarrow AdS_2 \times S^2$

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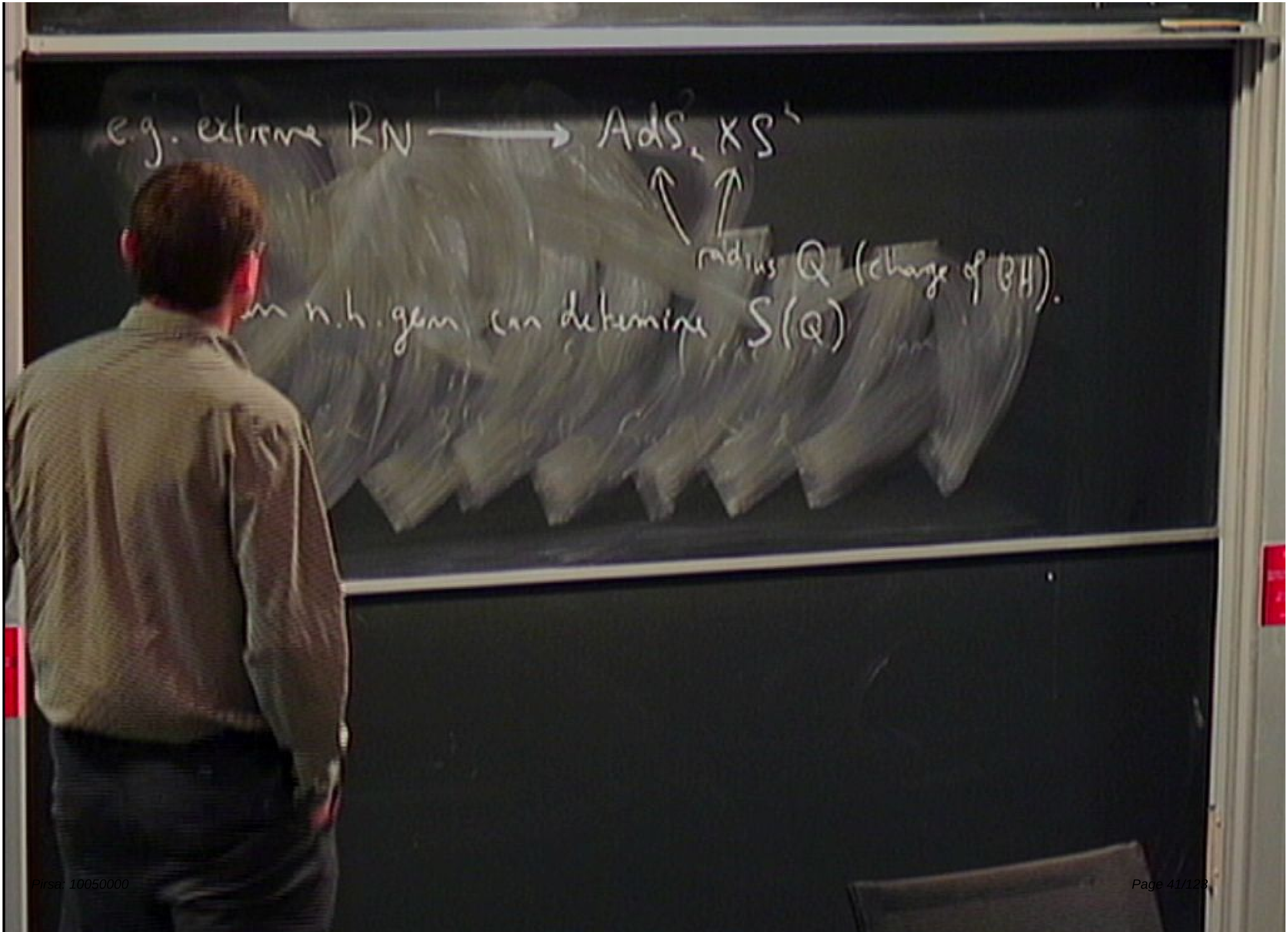
radius Q (charge of BH).



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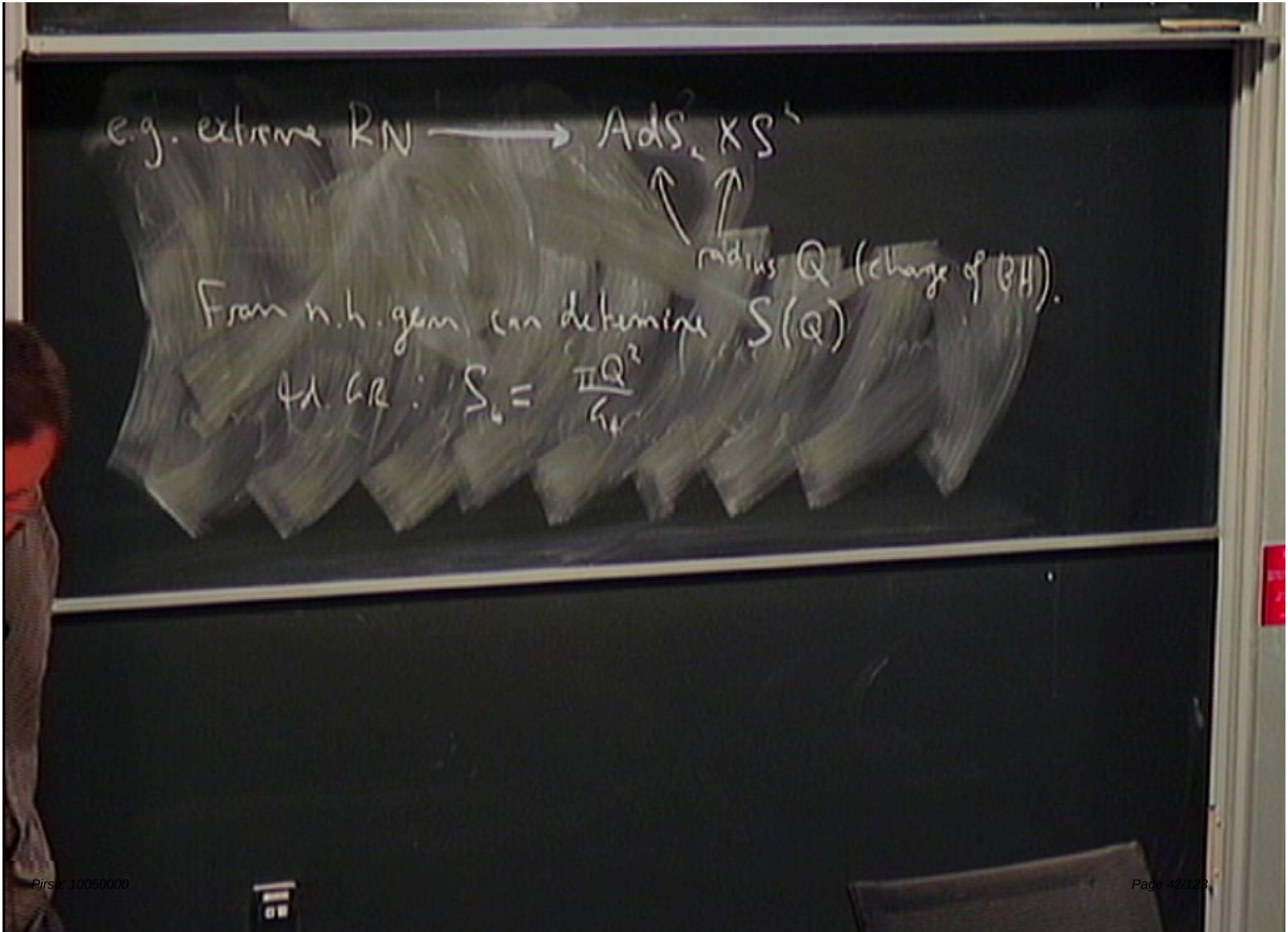
n.h. geom can determine $S(Q)$



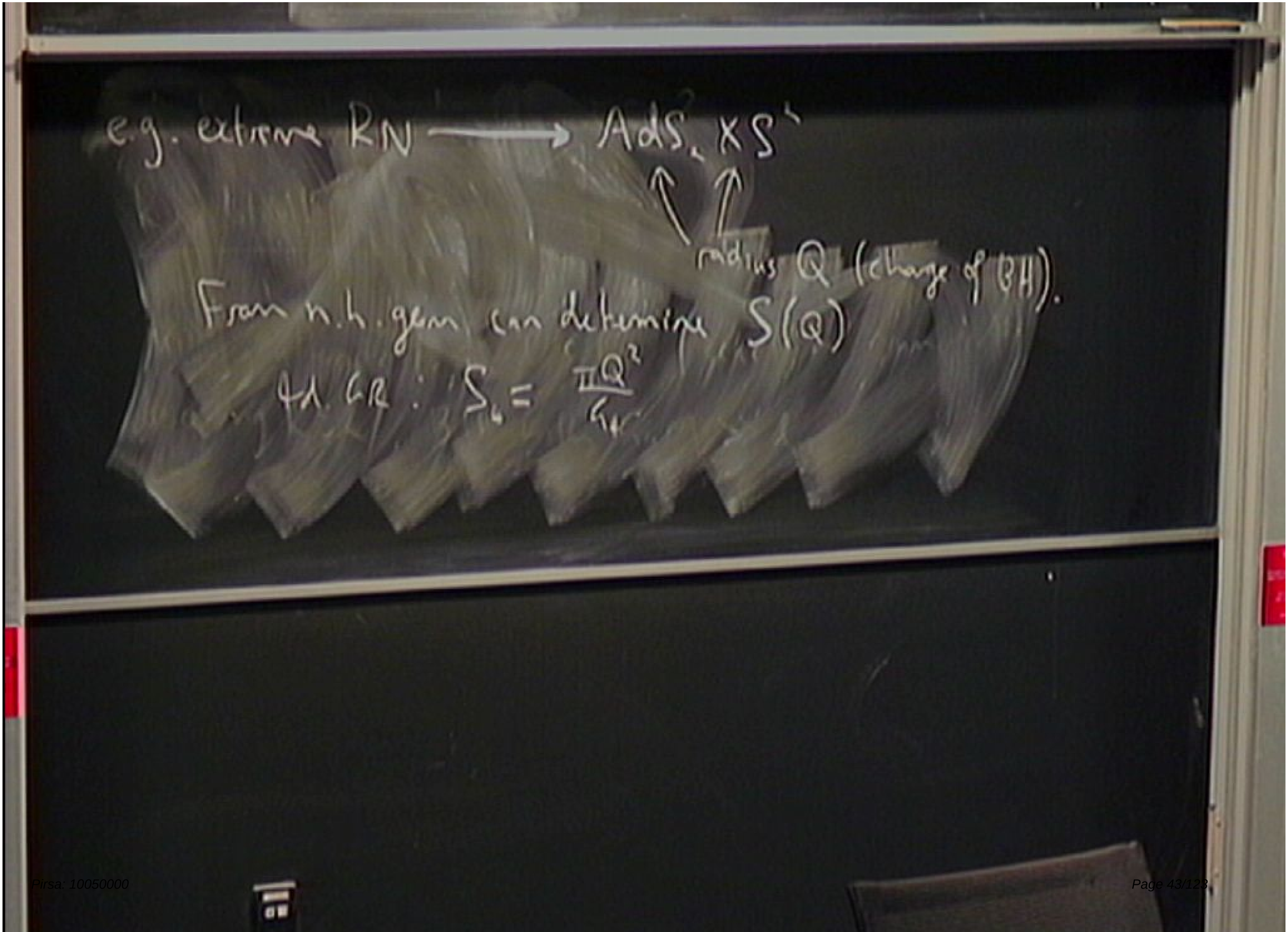
e.g. extreme RN $\longrightarrow AdS_2 \times S^2$

$\uparrow \quad \uparrow$

radius Q (charge of BH).

From n.h. geom can determine $S(Q)$

4d GR: $S_0 = \frac{\pi Q^2}{G_4}$



e.g. extreme RN $\longrightarrow AdS_2 \times S^2$

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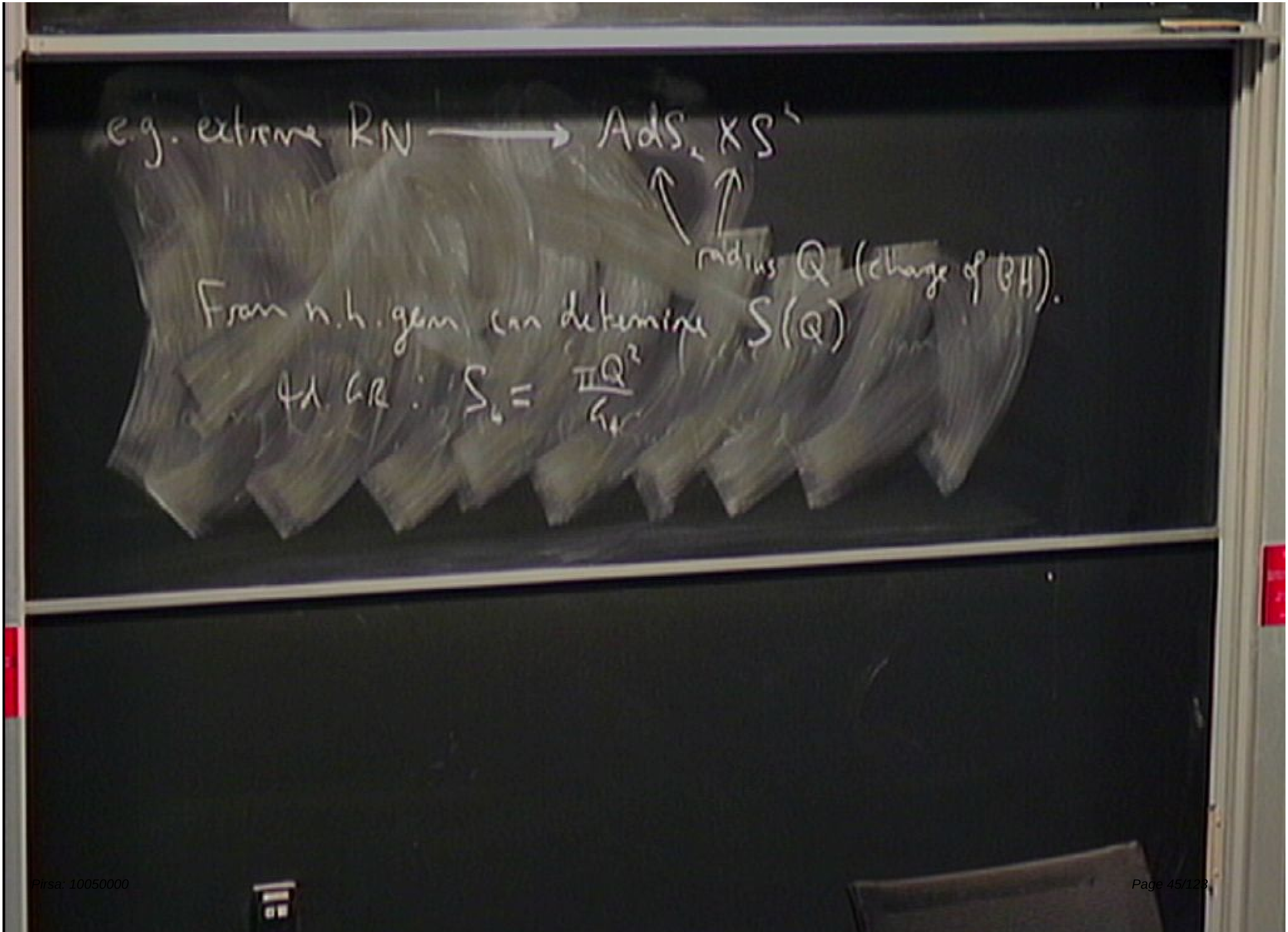

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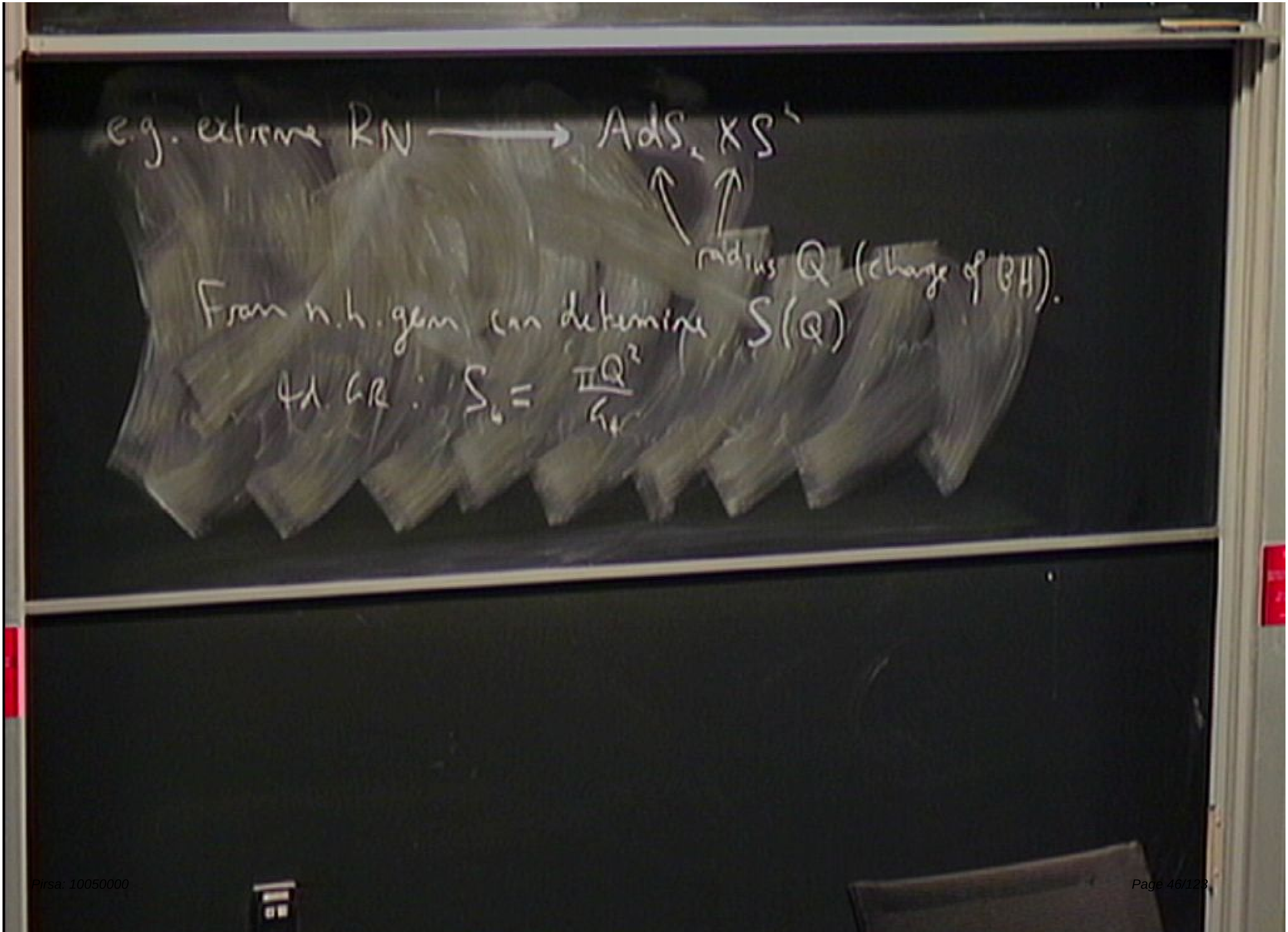

e.g. extreme RN $\longrightarrow$ $AdS_2 \times S^2$

$\uparrow \uparrow$ radius Q (charge of BH).

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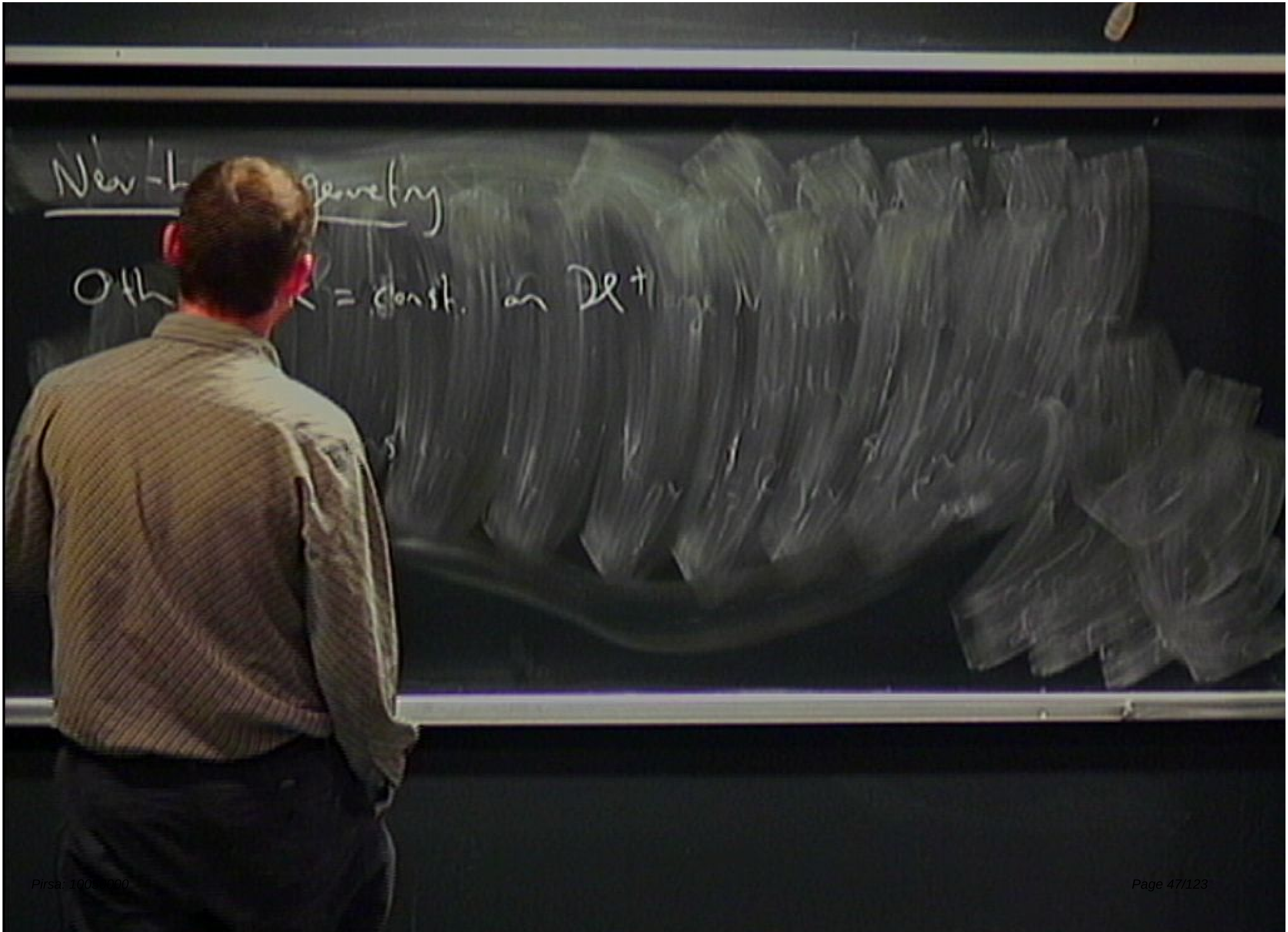
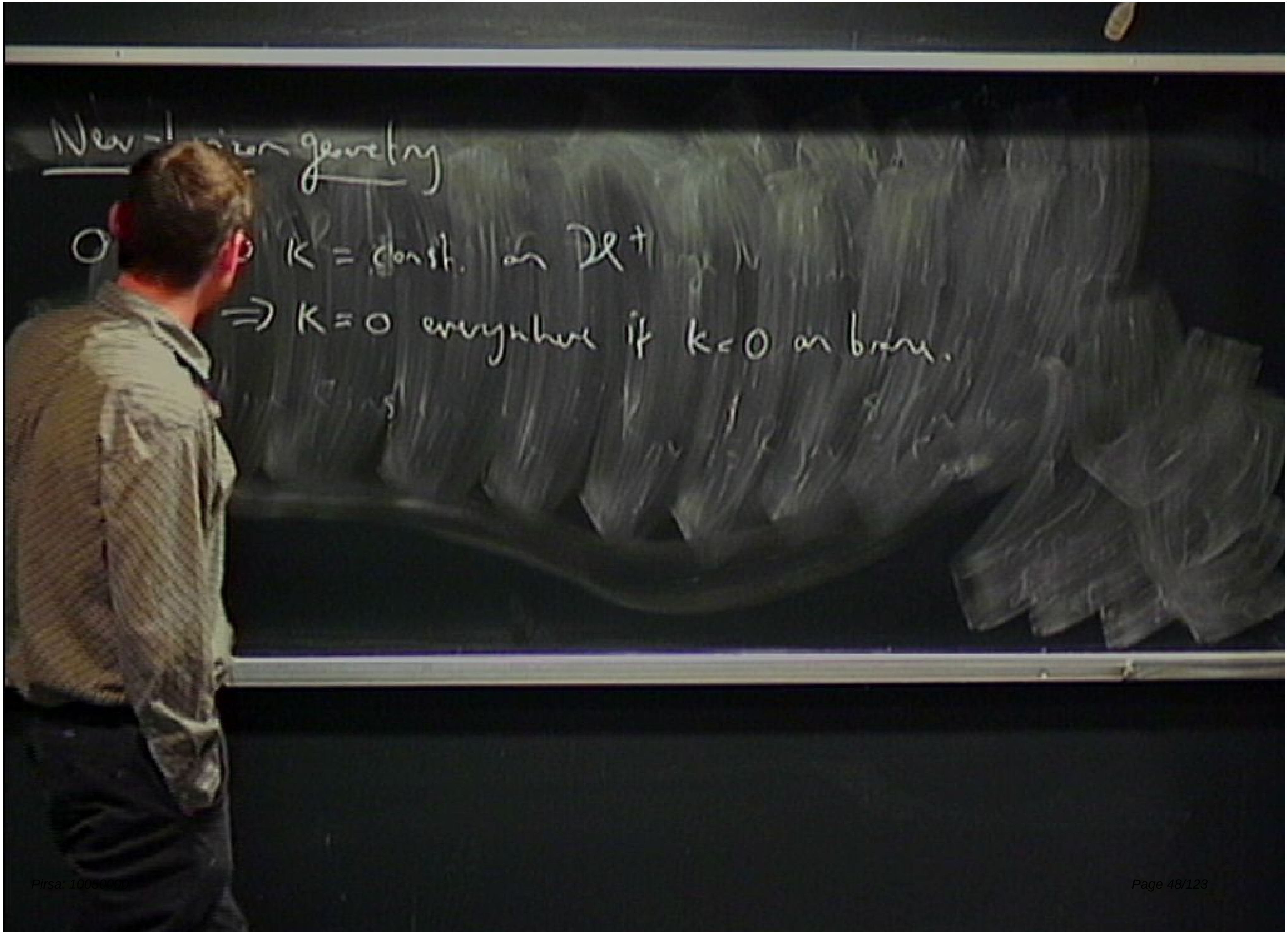
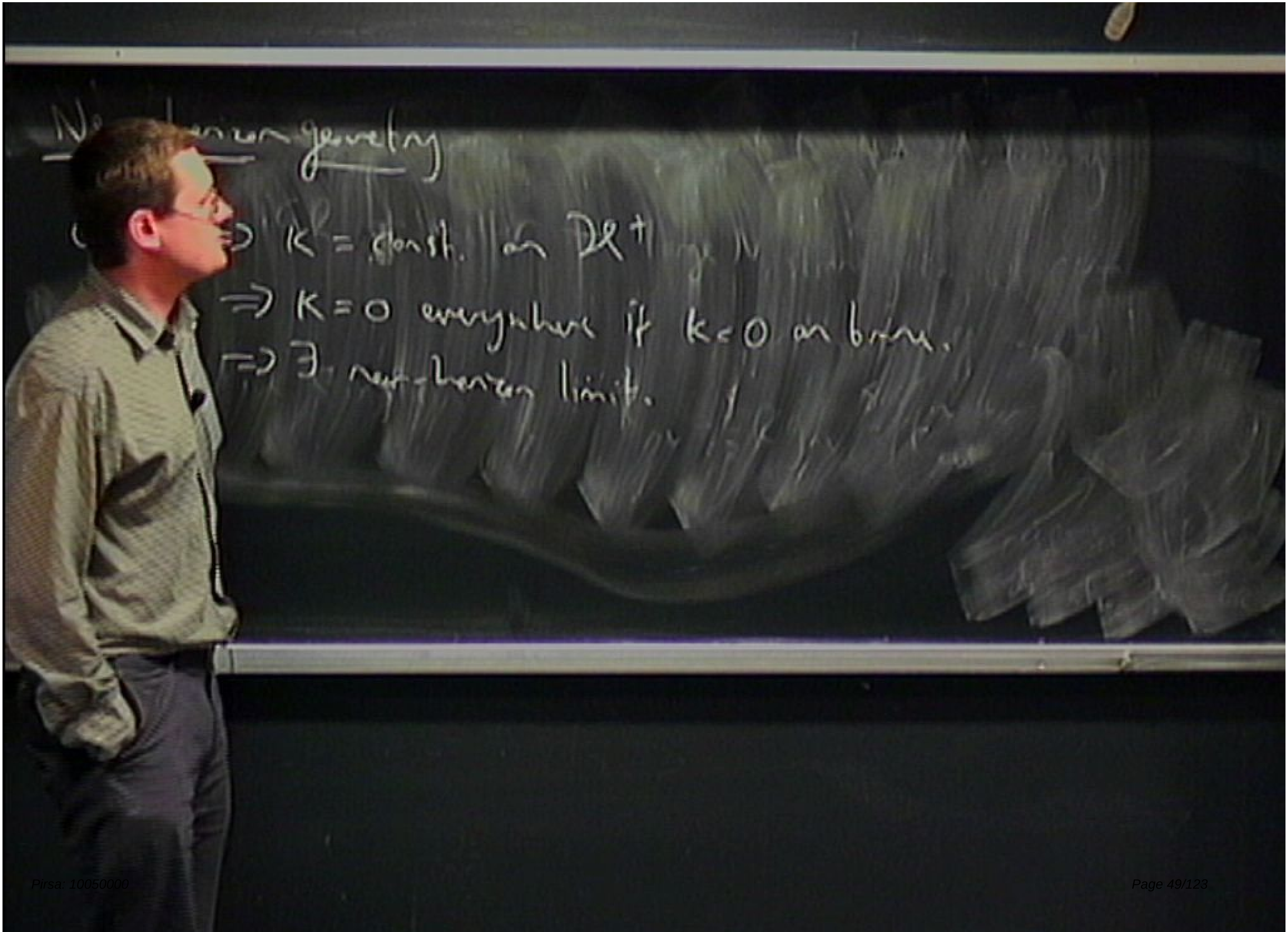

New-L geometry
0th = const. on De+



Near-h[illegible]izon geometry

0 [illegible] $K = \text{const.}$ on $\partial\mathcal{H}^+$

$\Rightarrow K = 0$ everywhere if $k < 0$ on brane.

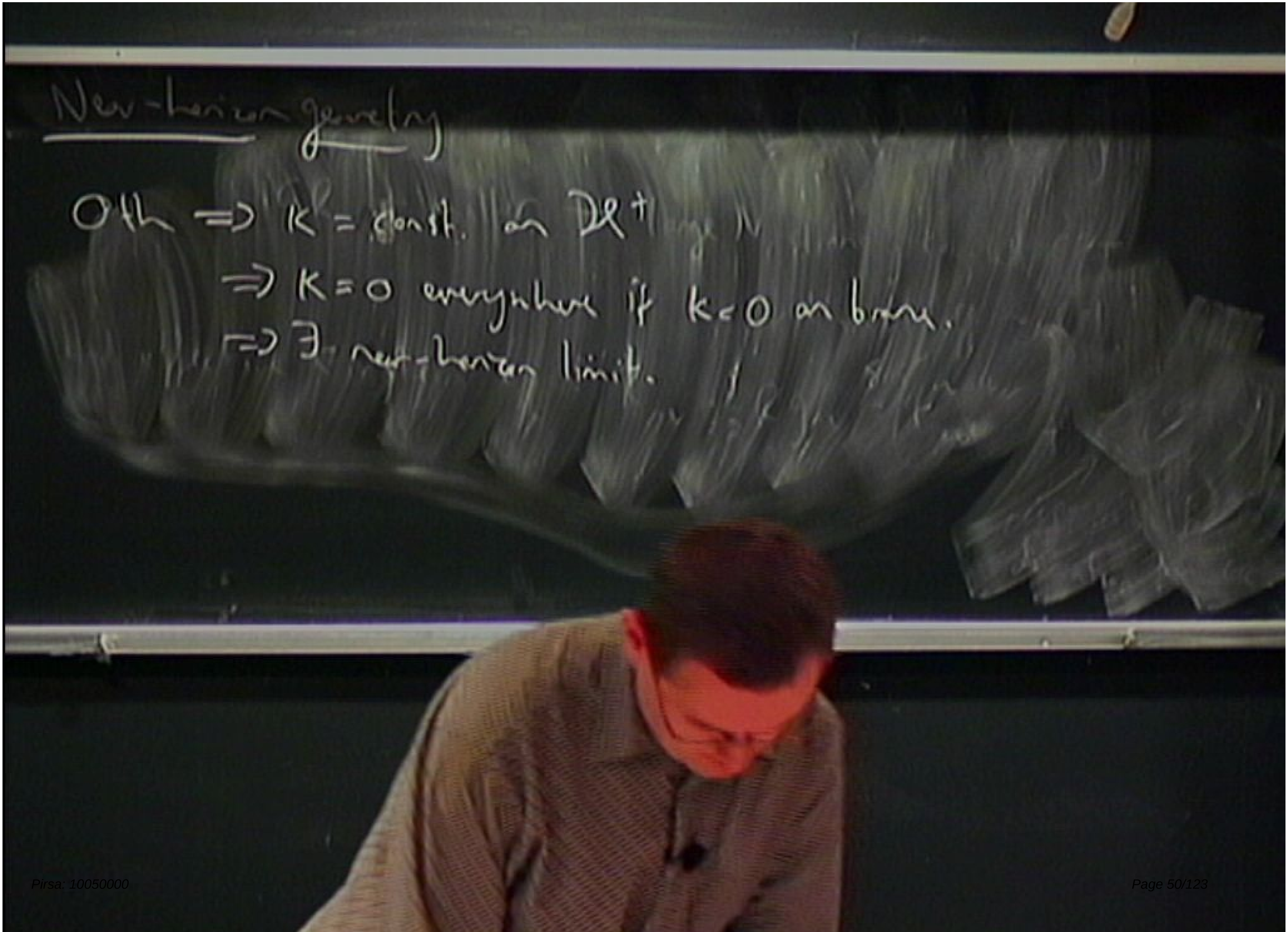


Ne[illegible]rizon geometry

(⇒ $K = \text{const.}$ on DR^+

⇒ $K=0$ everywhere if $k<0$ on brane.

⇒ ∃ near-horizon limit.

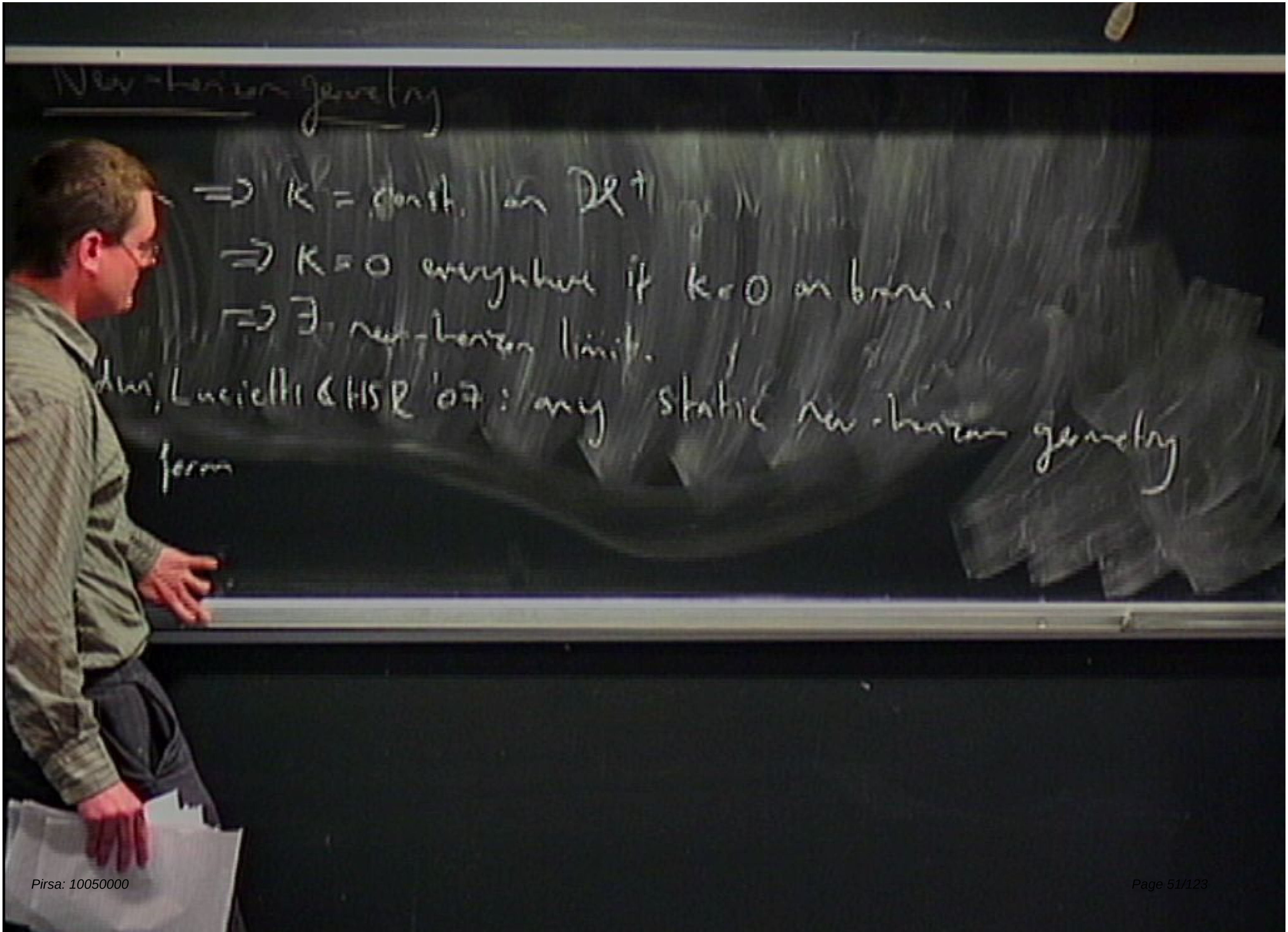


Near-horizon geometry

0th $\Rightarrow \kappa = \text{const.}$ on $D\ell^+$

$\Rightarrow \kappa = 0$ everywhere if $k<0$ on brane.

$\Rightarrow \exists$ near-horizon limit.



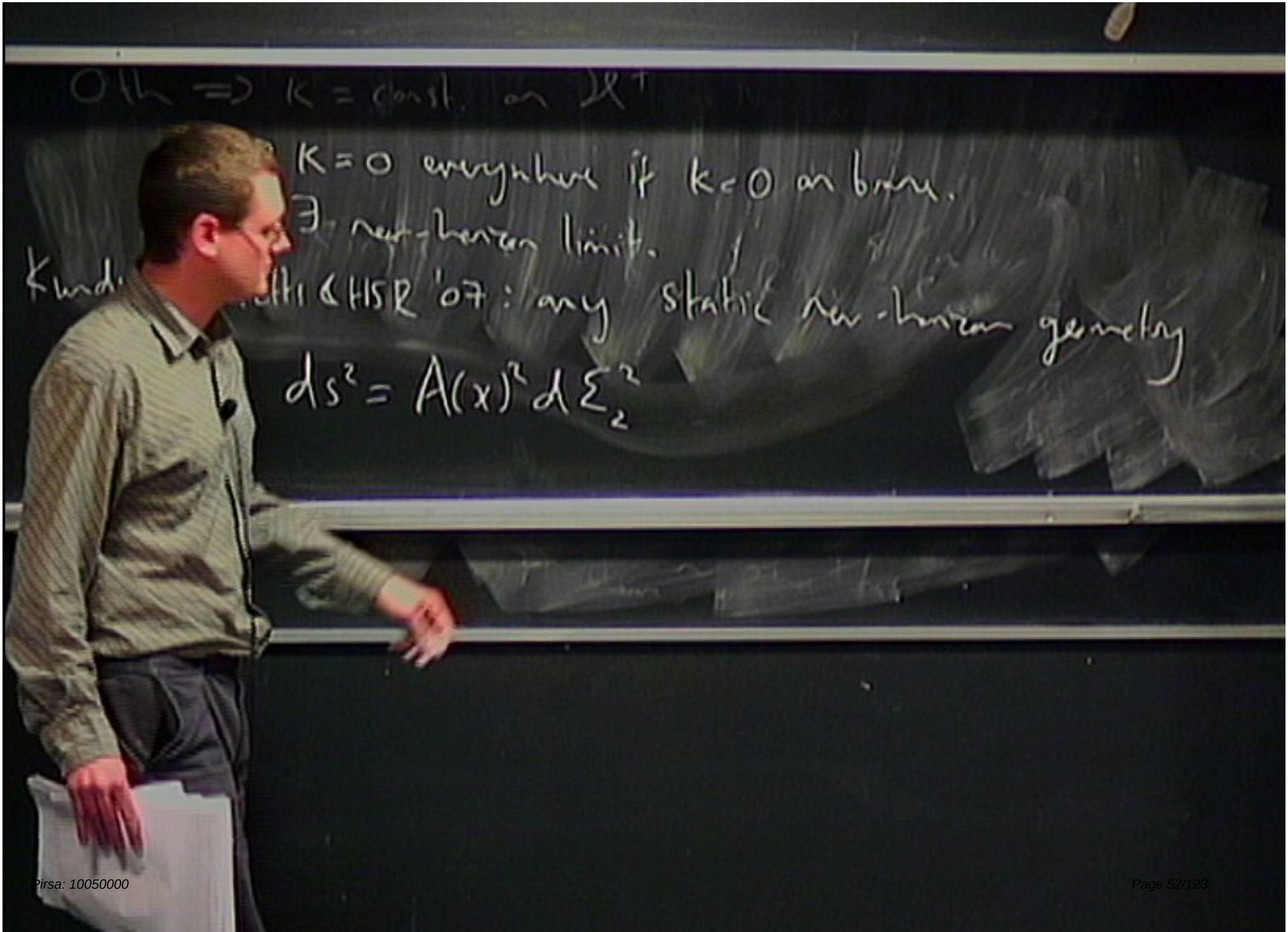
Near-horizon geometry

$\Rightarrow K = \text{const. on } \partial R^{+}$

$\Rightarrow K = 0$ everywhere if $K = 0$ on brane.

$\Rightarrow \exists$ near-horizon limit.

dri, Lucietti & HSR '07: any static near-horizon geometry form



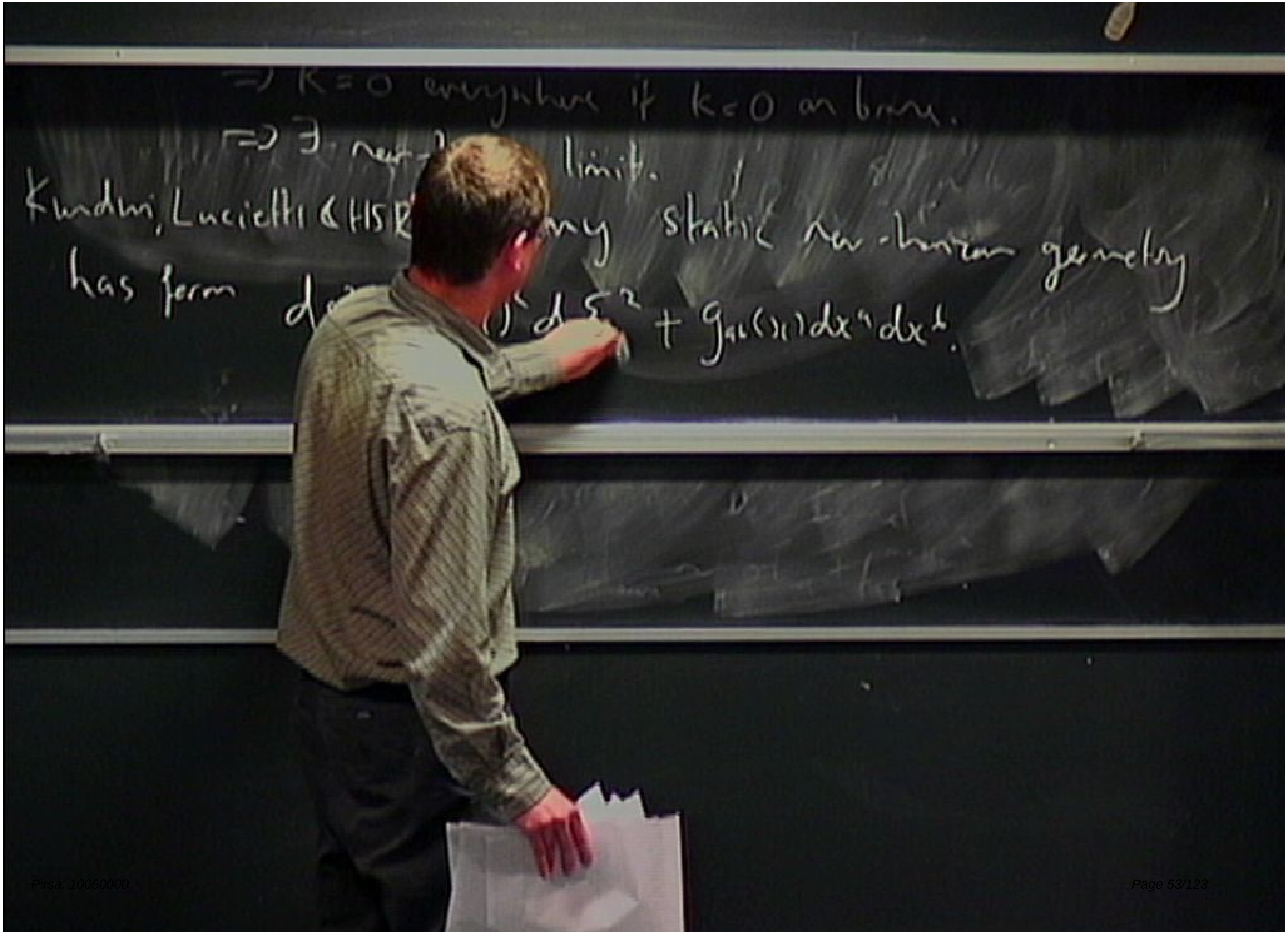
0th $\Rightarrow$ κ = const. on $\mathcal{H}^+$

$\kappa = 0$ everywhere if $\kappa = 0$ on bdry.

$\exists$ near-horizon limit.

Kund etti & HSR '07: any static near-horizon geometry

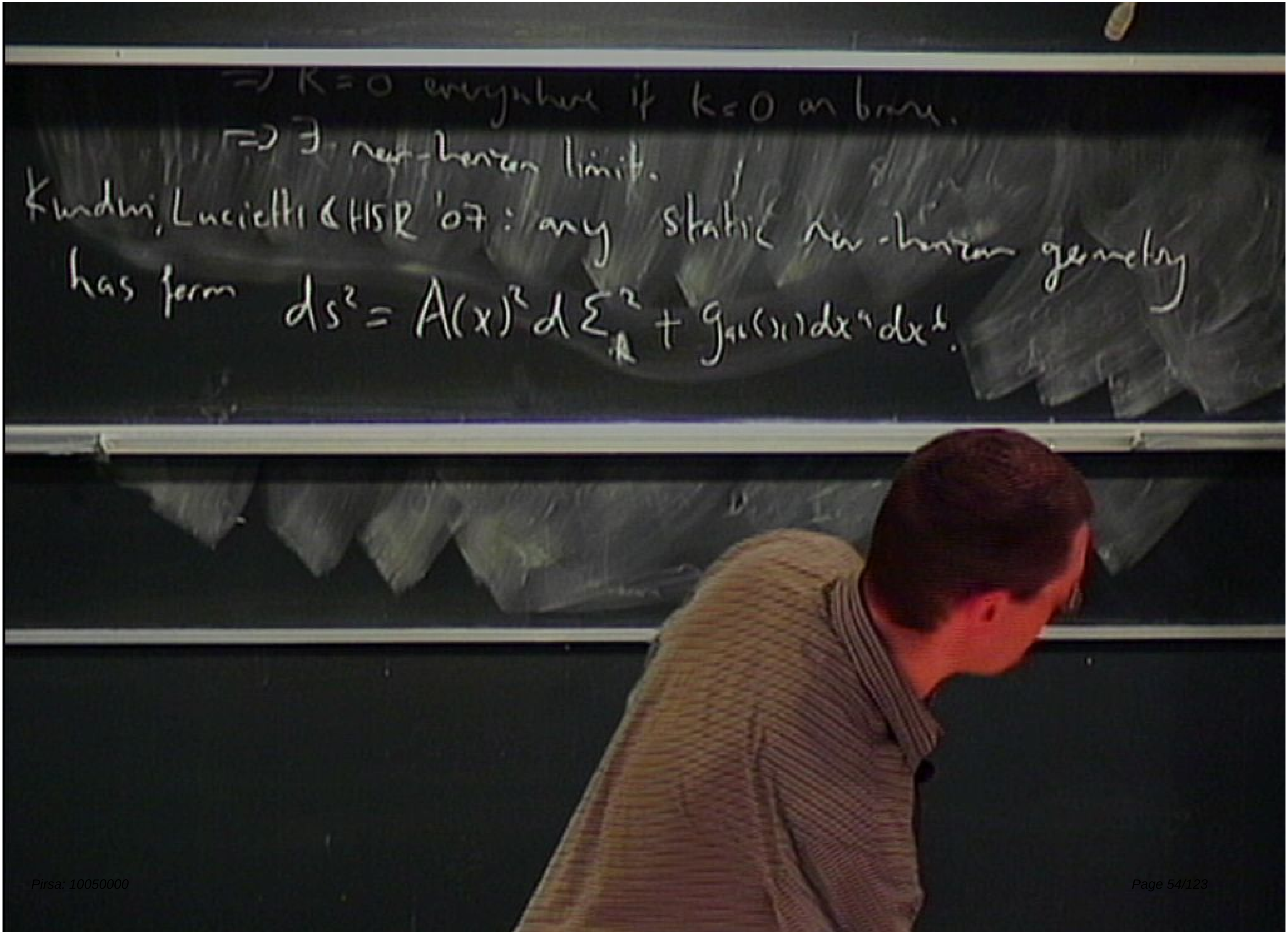
$$ds^2 = A(x)^2 d\Sigma_2^2$$



$\Rightarrow K = 0$ everywhere if $K = 0$ on brane.

$\Rightarrow \exists$ near- limit.

Kunduri, Lucietti & HSR any static near-horizon geometry has form d $)^2 d\xi^2 + g_{ab}(x)dx^a dx^b$.



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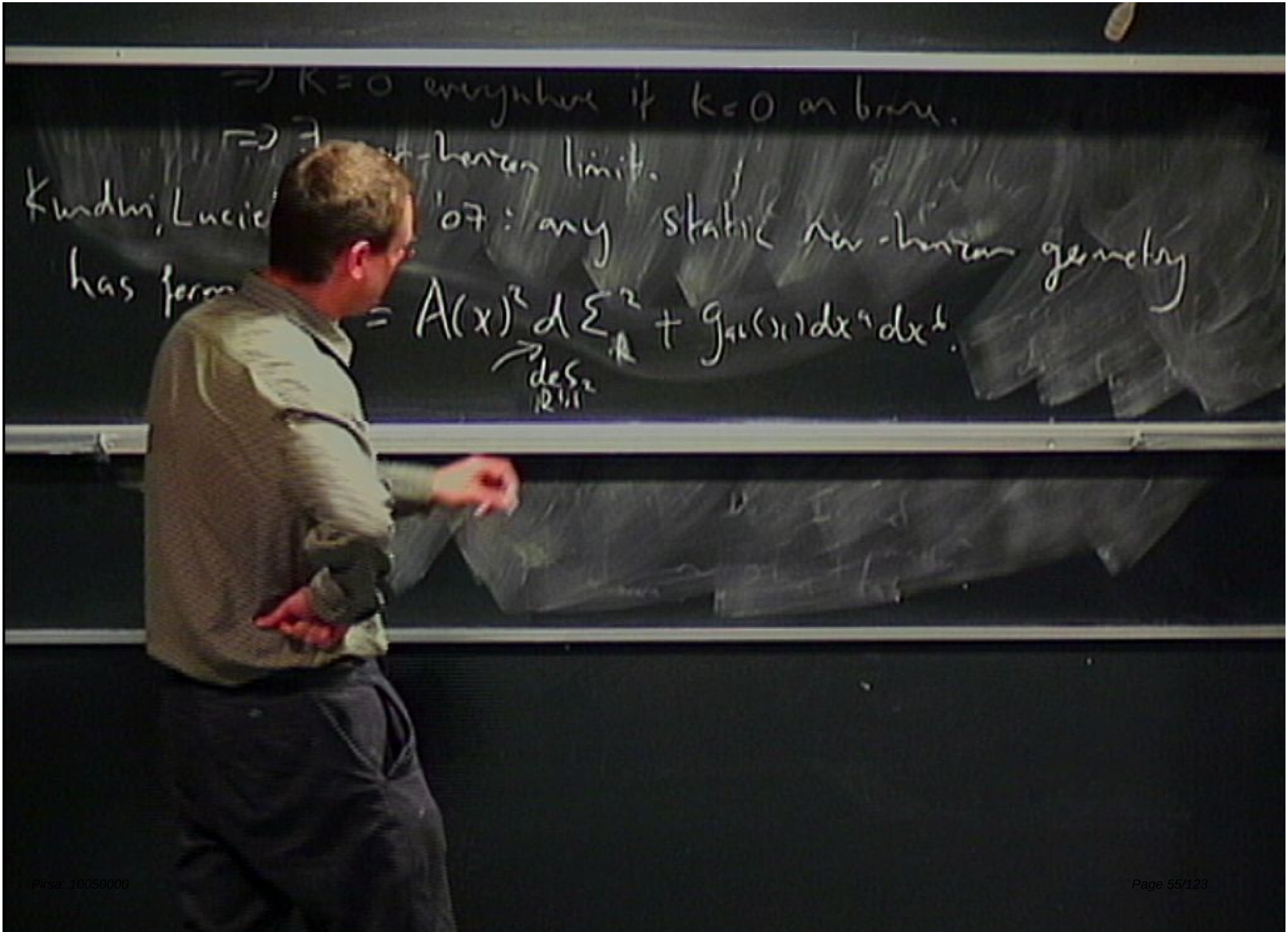
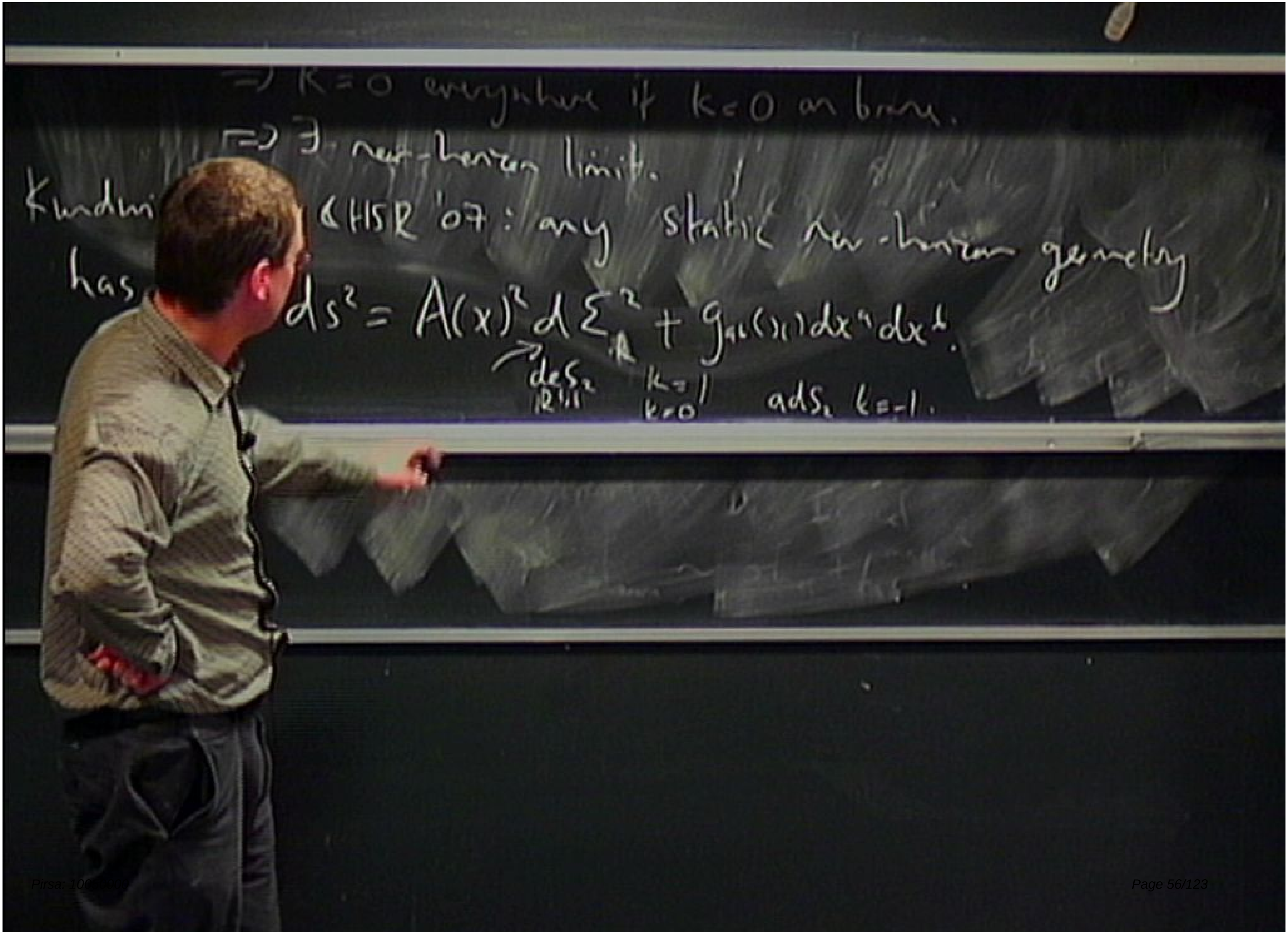
Kunduri, Lucietti & HSR '07: any static near-horizon geometry has form $ds^2 = A(x)^2 d\Sigma_k^2 + g_{ab}(x) dx^a dx^b$.





$\Rightarrow$ $k=0$ everywhere if $k=0$ on brane.

$\Rightarrow$ $\exists$ near-horizon limit.

Kundu[illegible] &HSR '07: any static near-horizon geometry has [illegible]

$$ds^2 = A(x)^2 d\Sigma_k^2 + g_{ab}(x) dx^a dx^b.$$

$\nearrow$ dS_2 $k=1$

$\mathbb{R}^{1,1}$ $k=0$

AdS_2 $k=-1$.

$\Rightarrow K = 0$ everywhere if $k = 0$ on brane.

$\Rightarrow \exists$ near-horizon limit.

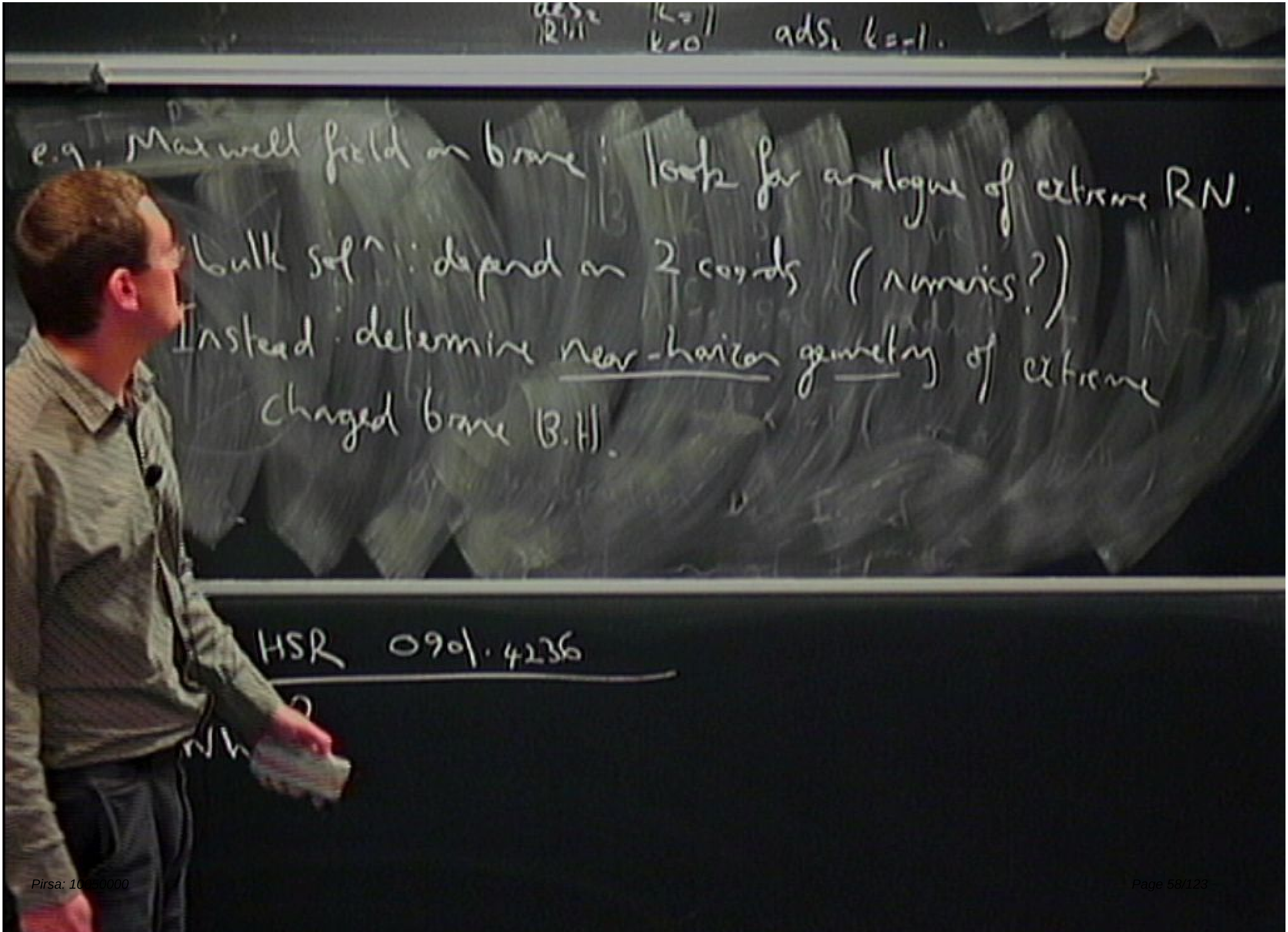
Kunduri, Lucietti & HSR '07: any static near-horizon geometry has form

$$ds^2 = A(x)^2 \, d\Sigma_k^2 + g_{ab}(x)\, dx^a dx^b .$$

dS_2 $k=1$

$\mathbb{R}^{1,1}$ $k=0$

adS_2 $k=-1$.



dS_2 $k=1$

$\mathbb{R}^{1,1}$ $k=0$

adS_2 $k=-1$.

e.g. Maxwell field on brane: look for analogue of extreme RN.

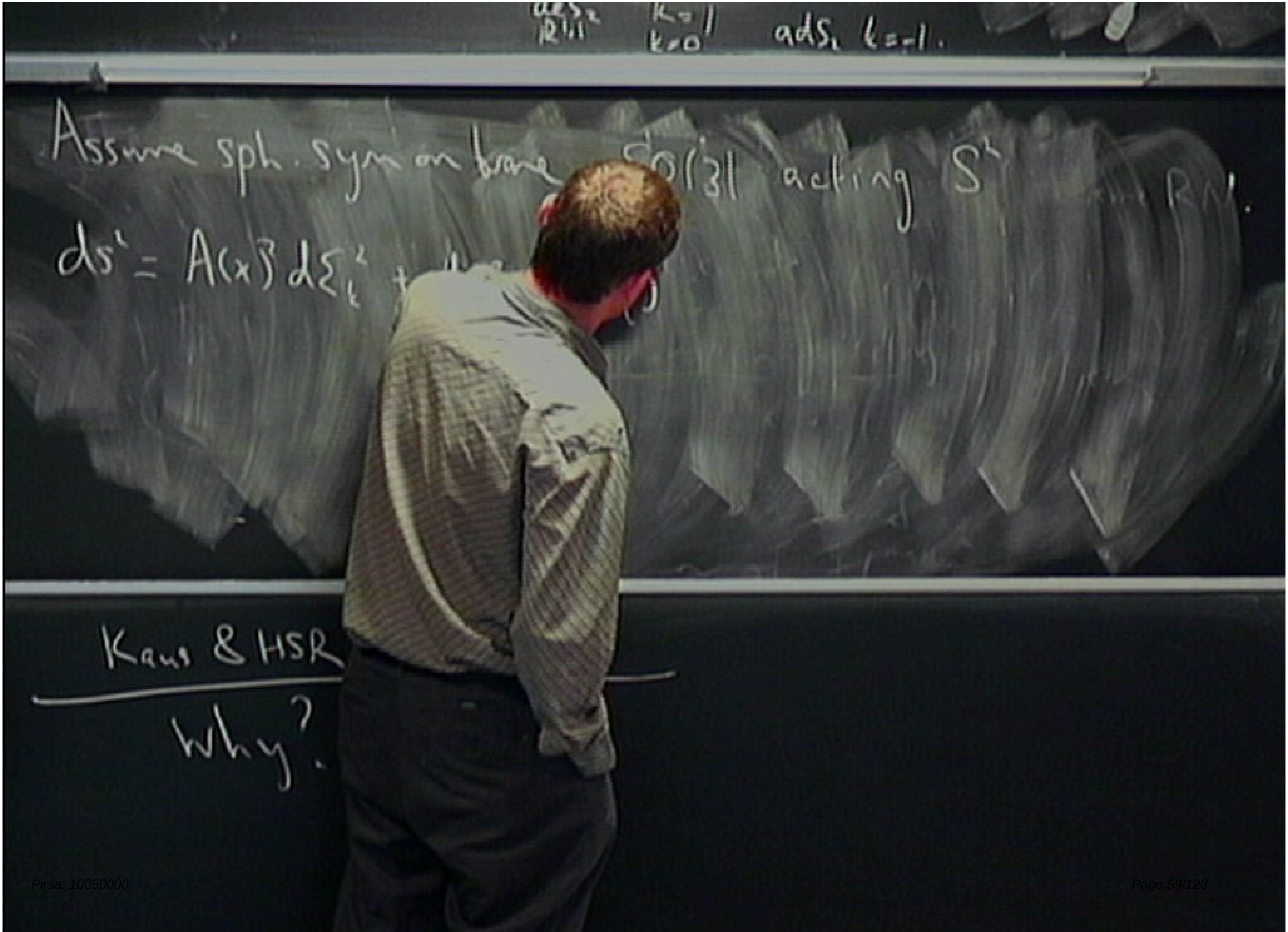
bulk sol^n: depend on 2 coords (numerics?)

Instead: determine near-horizon geometry of extreme charged brane B.H.

HSR 0901.4236

?

NH



adS_2 $k=1$ adS_2 $k=-1$.

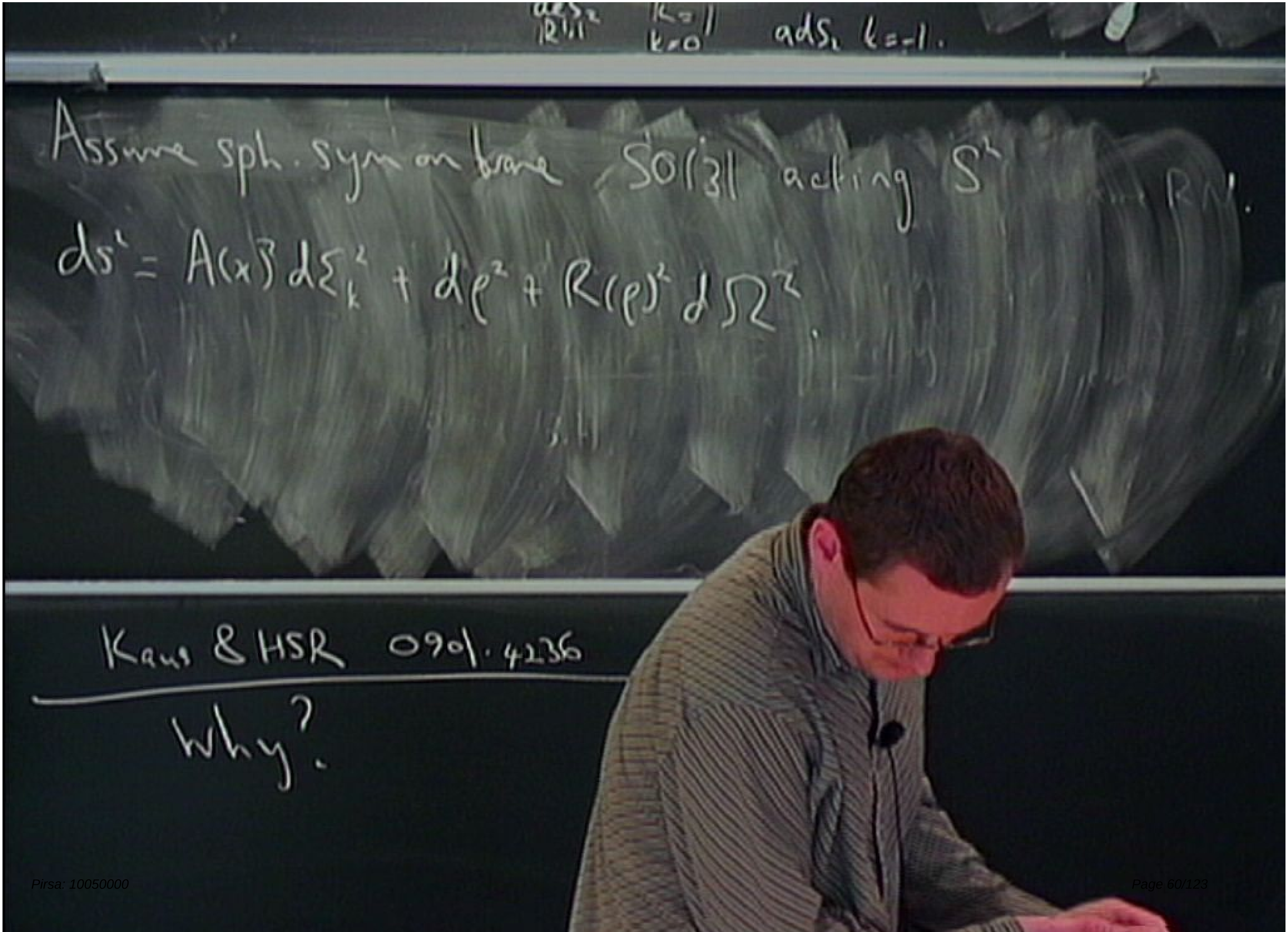
$\mathbb{R}^{1,1}$ $k=0$

Assume sph. sym on brane $SO(3)$ acting S^2 … RN.

$ds^2 = A(x)^3 \, d\Sigma_k^2 + \ldots$

Kaus & HSR

Why?.



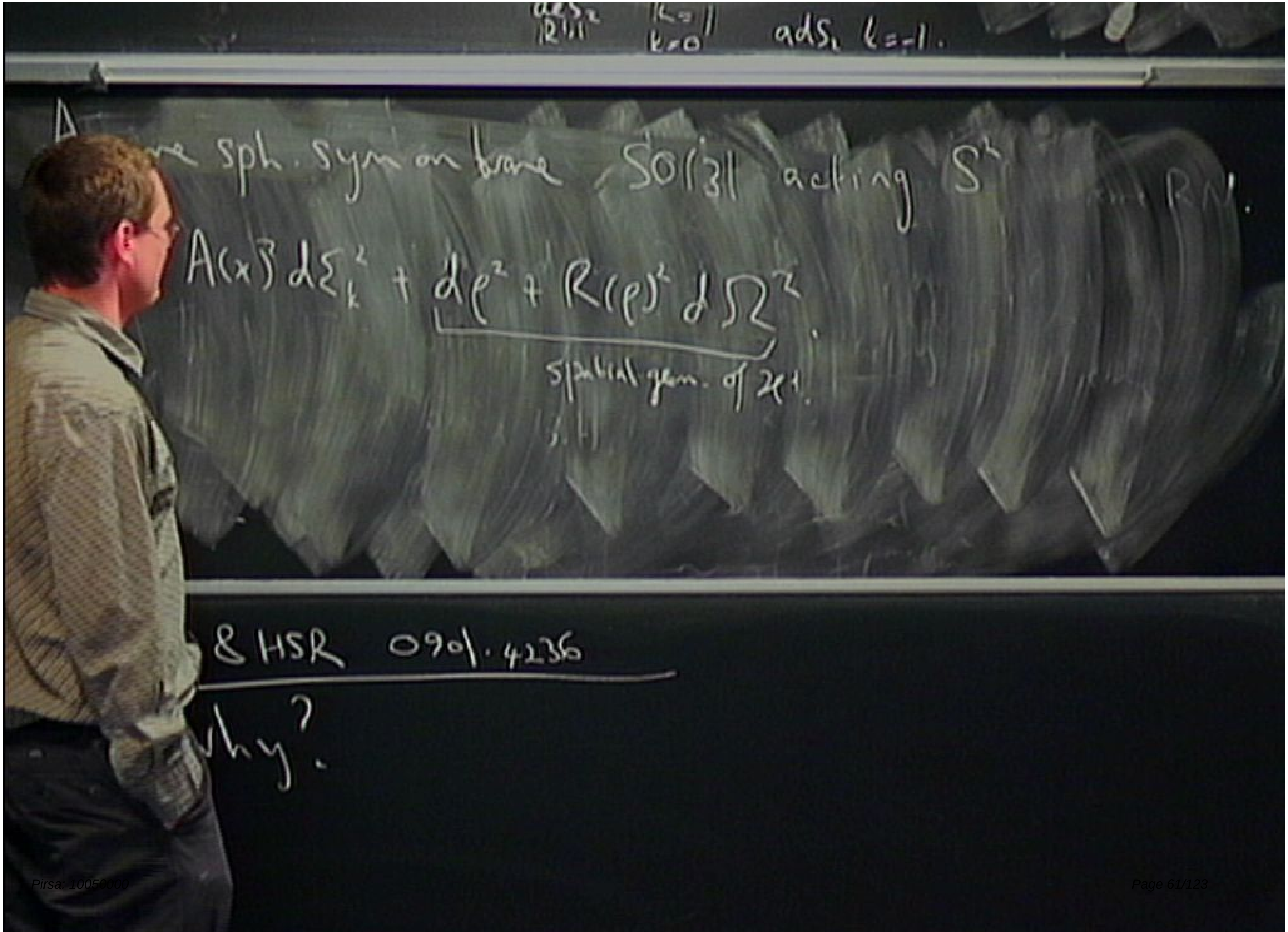
dS_2 $\mathbb{R}^{1,1}$ $k=1$ $k=0$ adS_2 $k=-1$.

Assume sph. sym on brane $SO(3)$ acting S^2 … RN.

$$ds^2 = A(x)^2 d\Sigma_k^2 + d\rho^2 + R(\rho)^2 d\Omega^2.$$

Karch & HSR 0901.4236

Why?.



dS_2 $k=1$

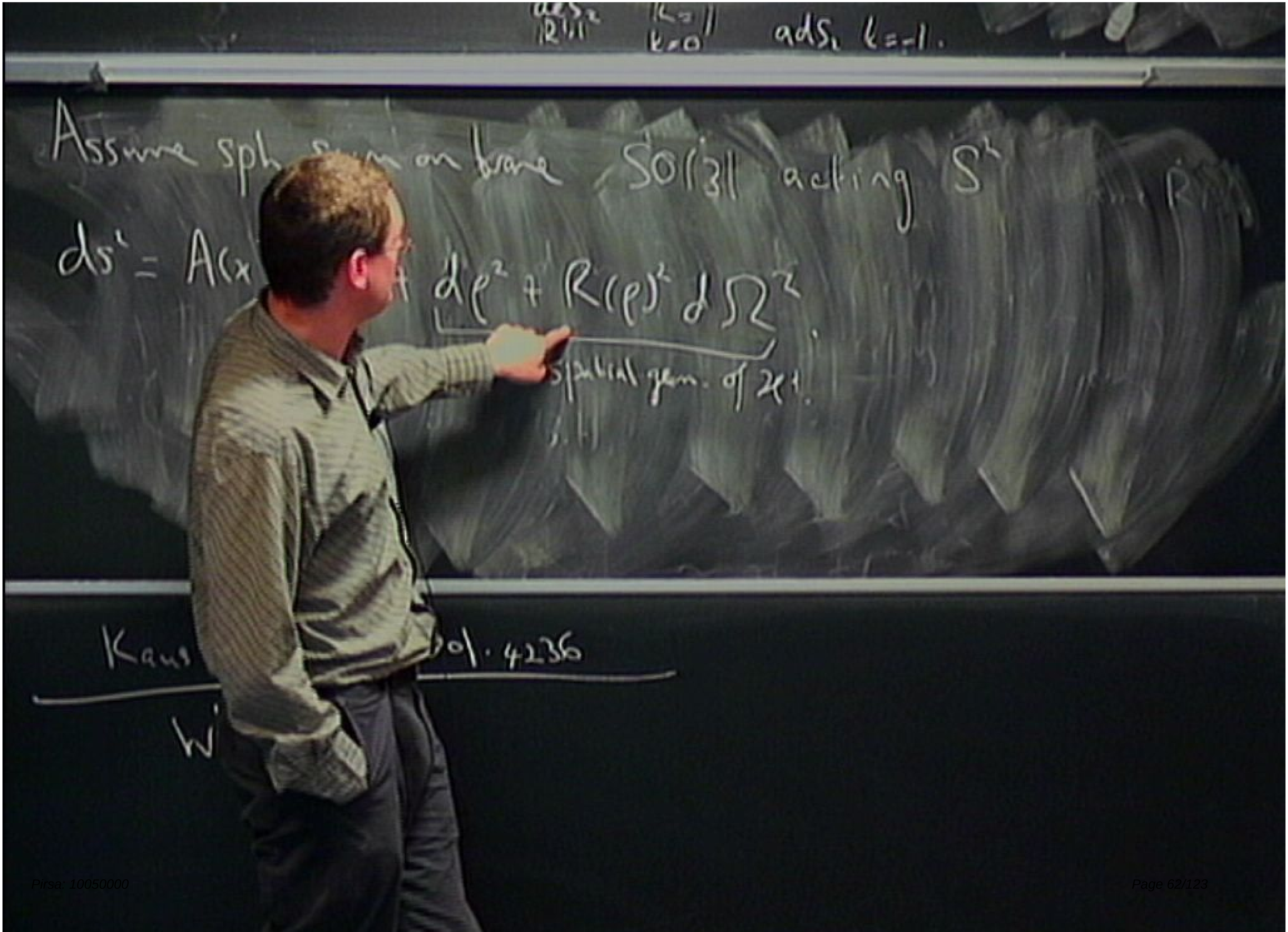
$\mathbb{R}^{1,1}$ $k=0$ adS_2 $k=-1$.

A…me sph. sym on brane $SO(3)$ acting S^2 … RN.

$$A(x)^2 d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

& HSR 0901.4236

…hy?



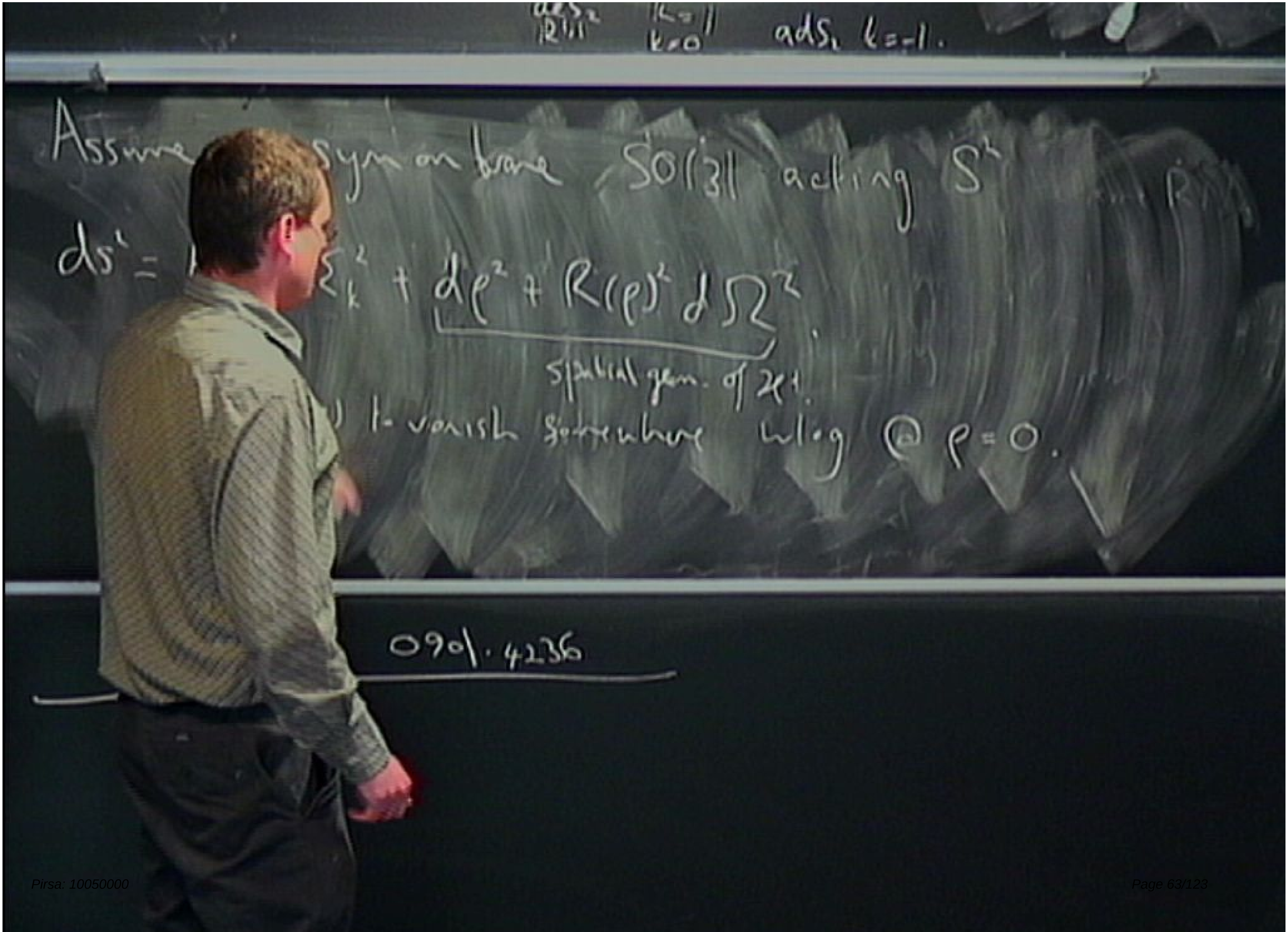
$R^{1,1}$ $k=0$ adS_2 $k=-1$.

Assume sph. s... on brane $SO(3)$ acting S^2 ... R

$$ds^2 = A(x \ldots + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

Kaus ... 01.4236

W



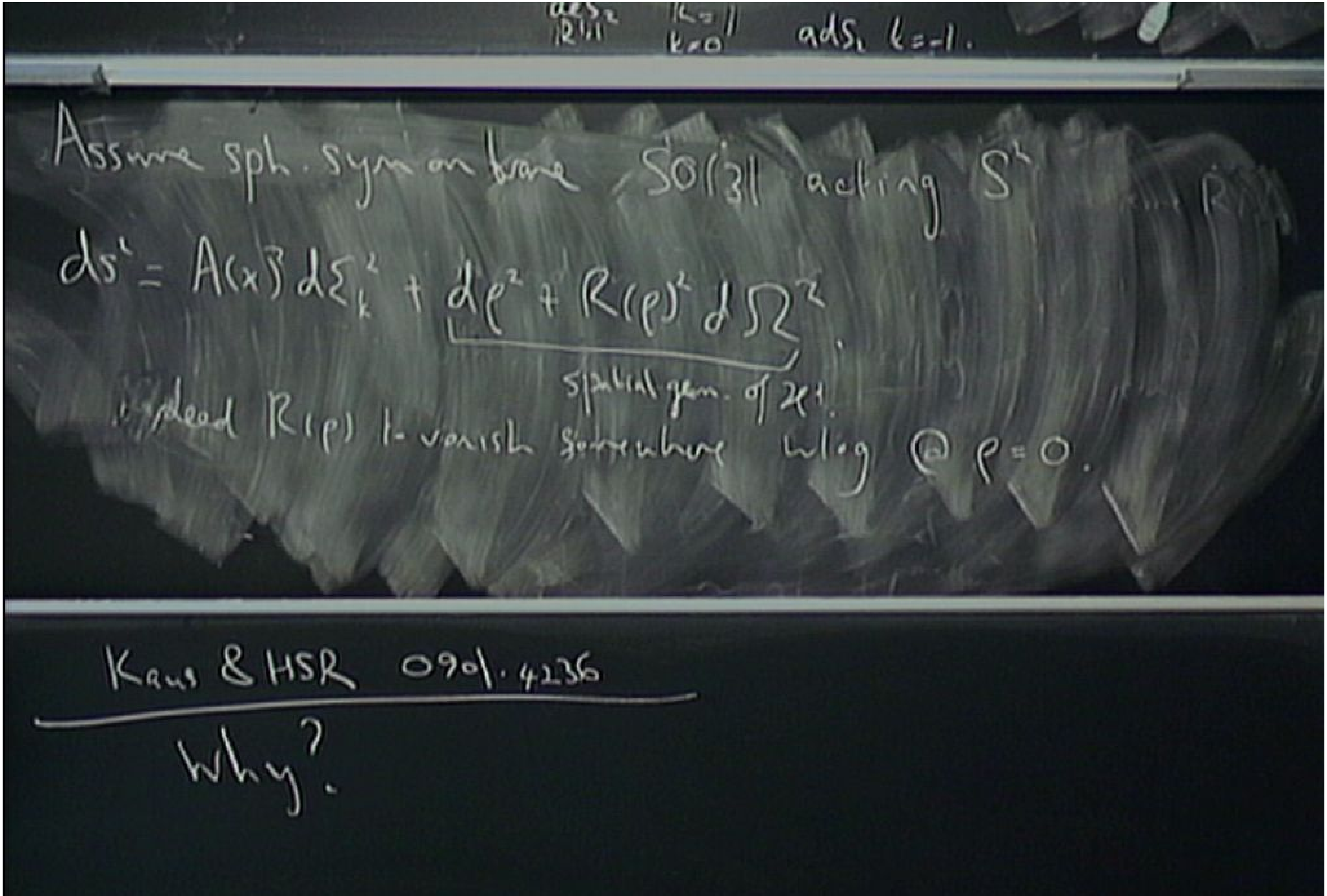
adS_2 $\mathbb{R}^{1,1}$ $k=1$ $k=0$ adS_2 $k=-1$.

Assume sym on brane $SO(3)$ acting S^2 ... $R(\rho)$

$$ds^2 = \ldots \Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

... to vanish somewhere wlog @ $\rho = 0$.

0901.4236



dS_2 $k=1$
$\mathbb{R}^{1,1}$ $k=0$ adS_2 $k=-1$.

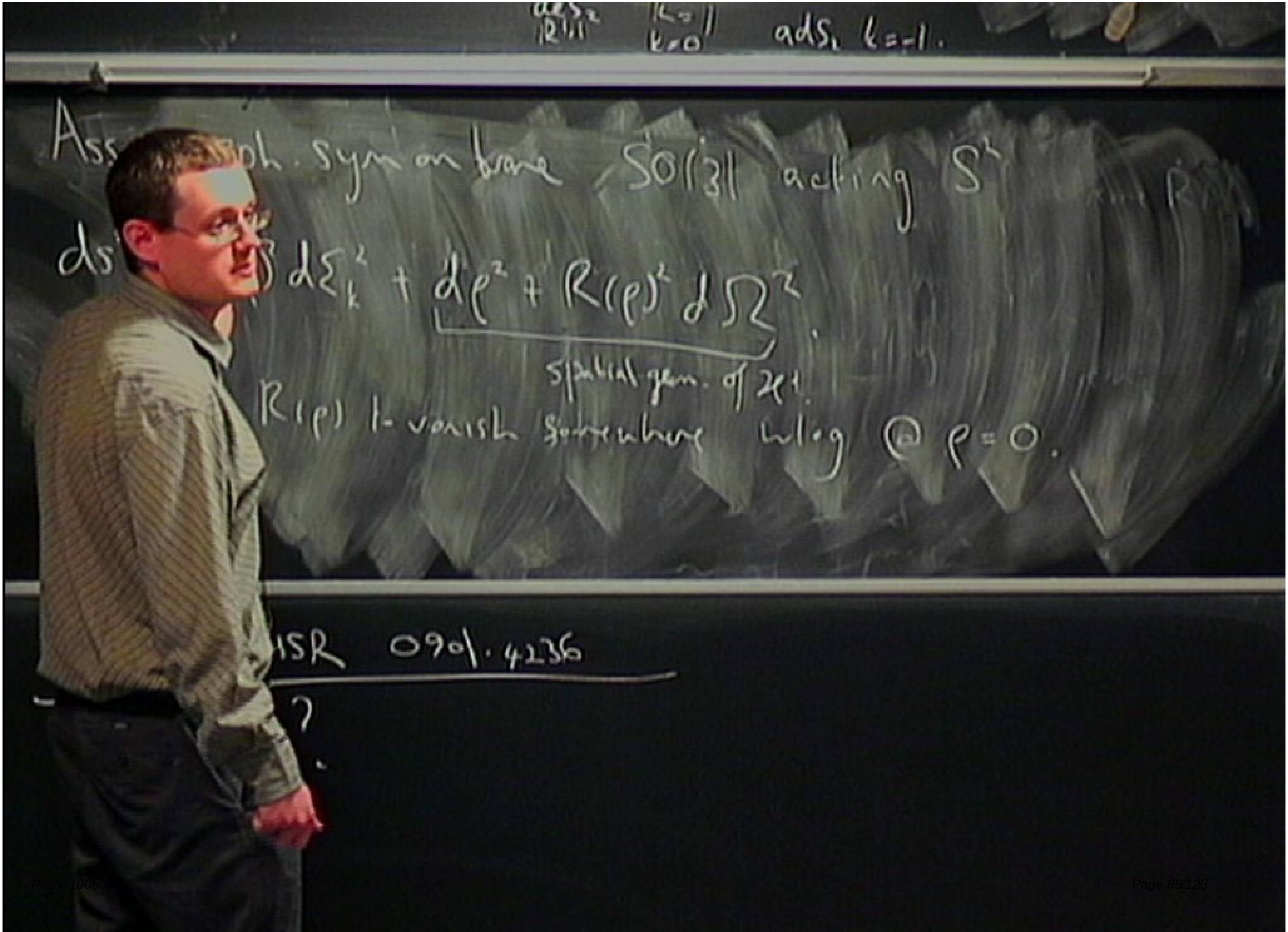
Assume sph. sym on brane $SO(3)$ acting S^2

$$ds^2 = A(x)^2 \, d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 \, d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

Indeed $R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

Kaus & HSR 0901.4236

Why?



adS_2 $R^{1,1}$ $k=1$ $k=0$ adS_2 $k=-1$.

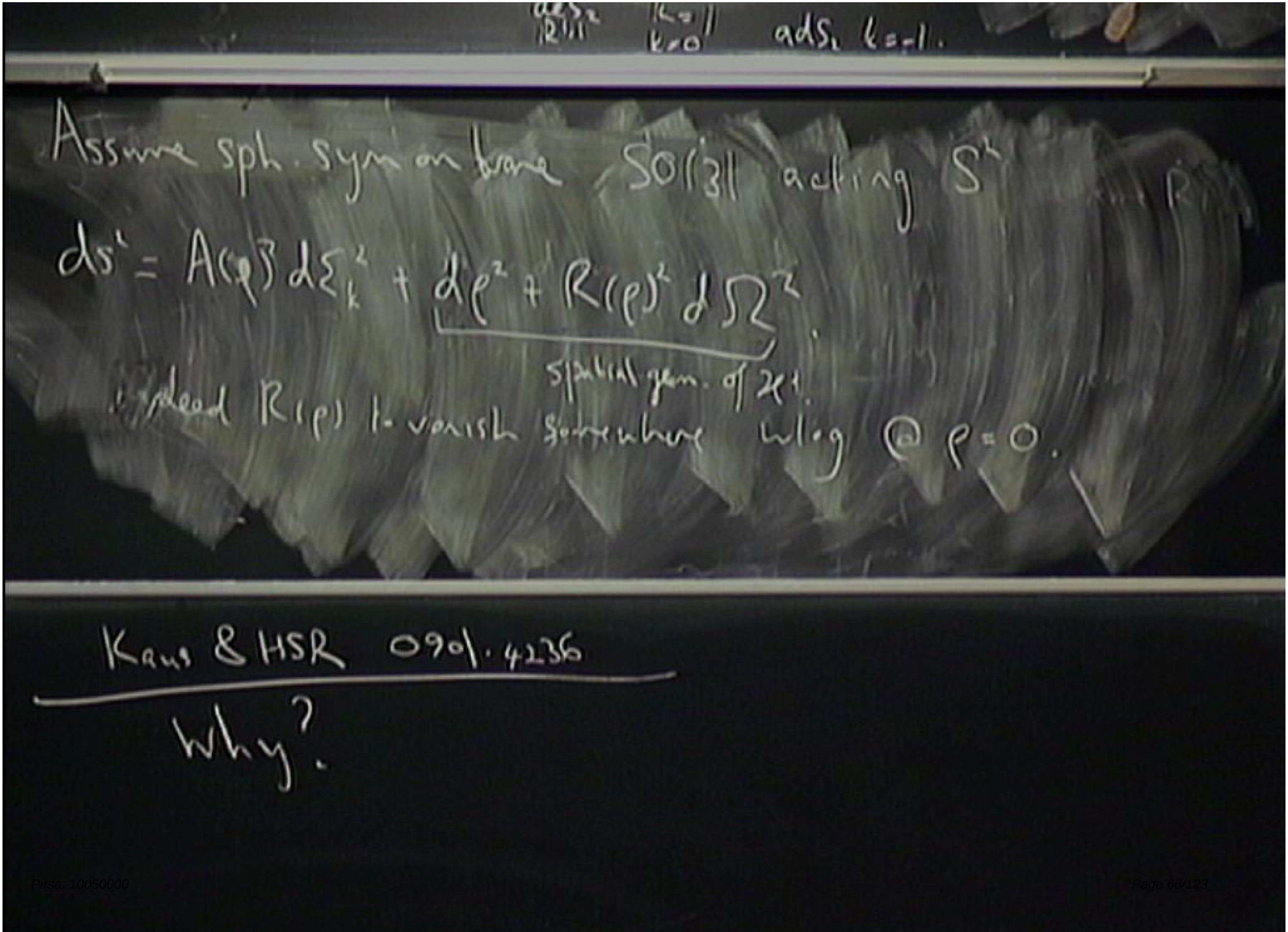
Ass ph. sym on brane $SO(3)$ acting S^2

$ds^2 = \; d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of 2+1}}$.

$R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

ISR 0901.4236

?



dS_2 $k=1$
$\mathbb{R}^{1,1}$ $k=0$ adS_2 $k=-1$.

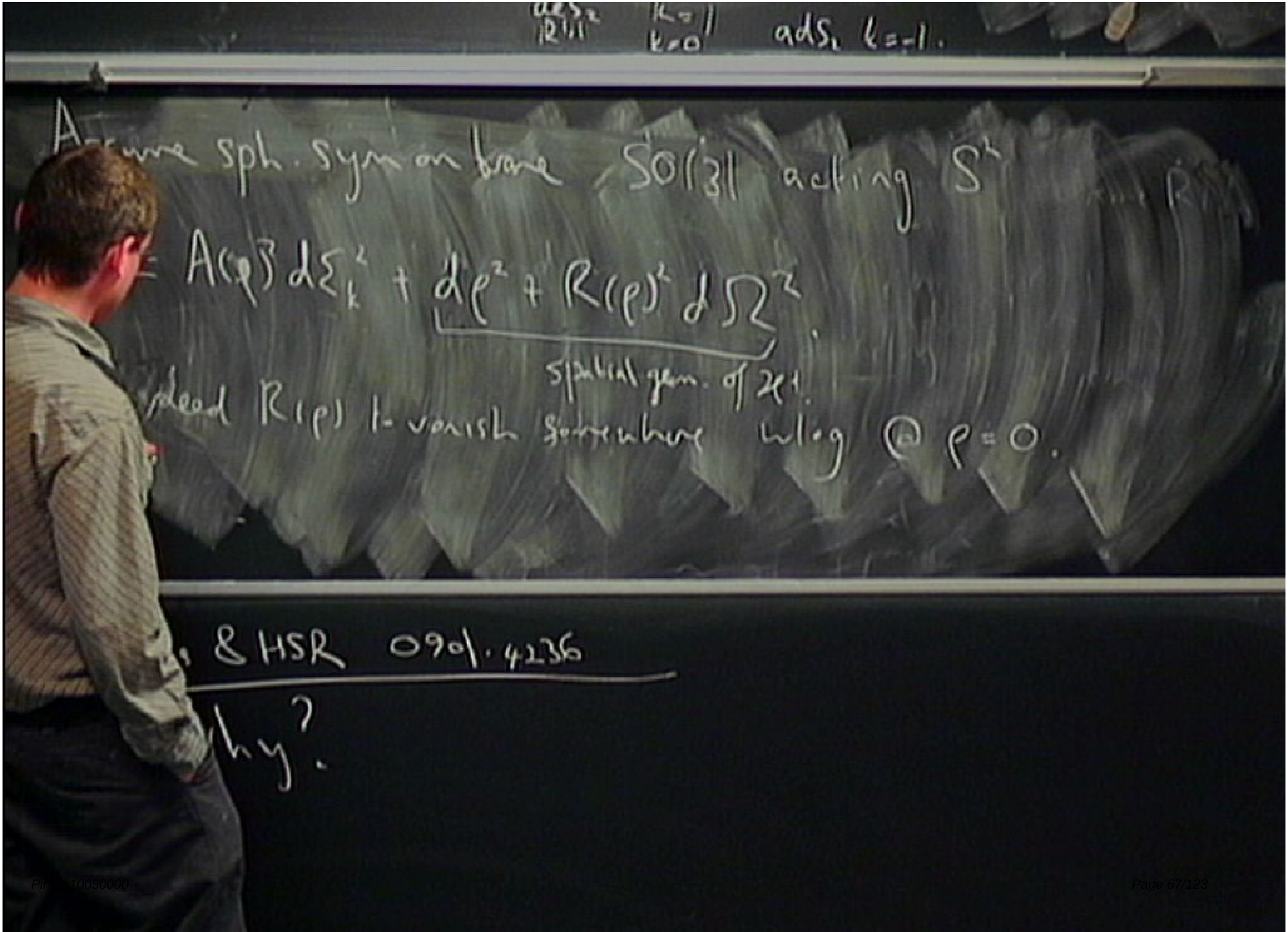
Assume sph. sym on brane SO(3) acting S^2 … R

$$ds^2 = A(\rho)^2 d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

Need $R(\rho)$ to vanish somewhere wlog @ $\rho=0$.

Kaus & HSR 0901.4236

Why?



dS_2 $k=1$ $\mathbb{R}^{1,1}$ $k=0$ adS_2 $k=-1$.

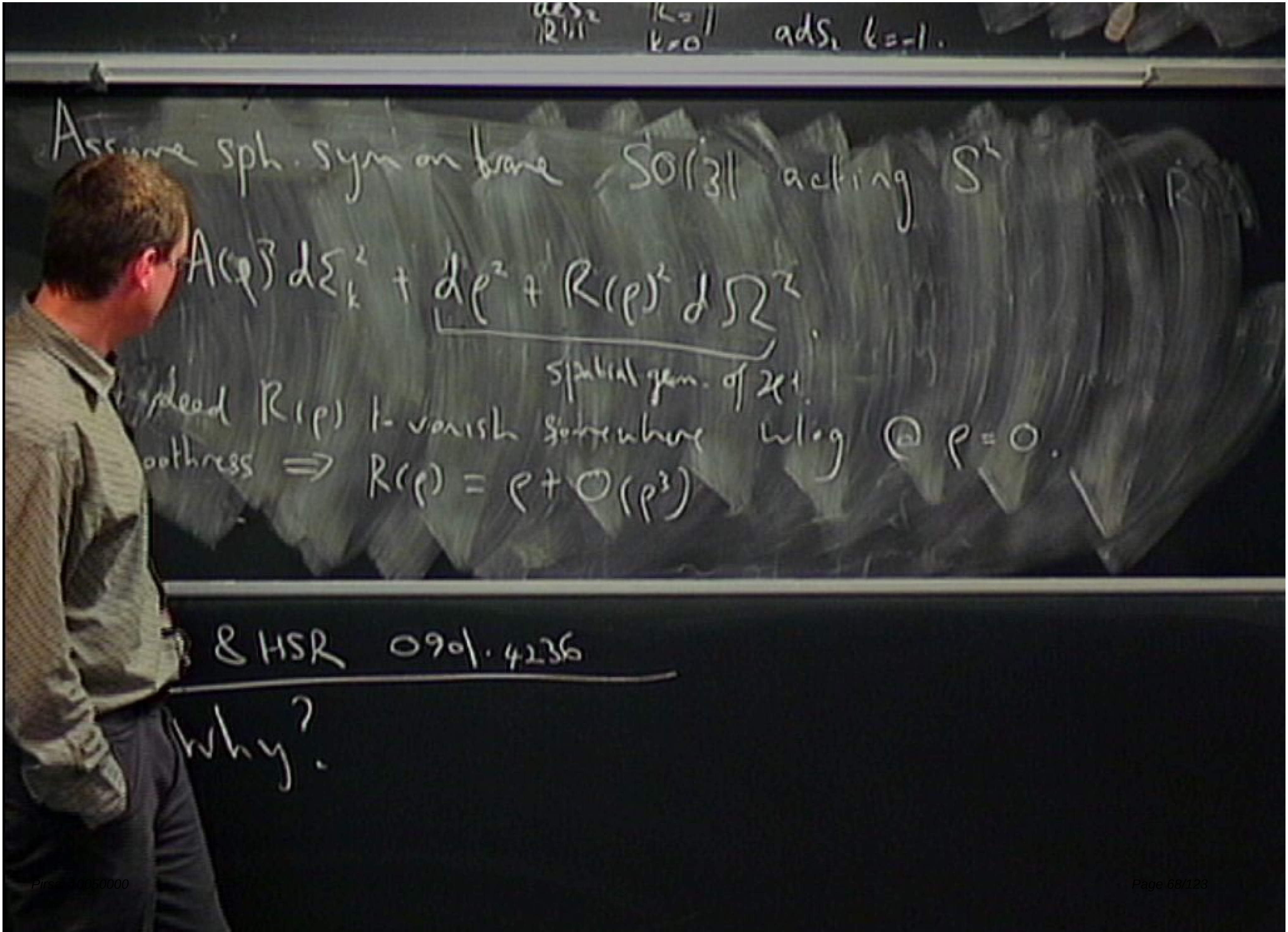
Assume sph. sym on brane $SO(3)$ acting S^2

$$= A(\rho)^2 d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

eed $R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

& HSR 0901.4236

hy?



dS_2 $\mathbb{R}^{1,1}$ $k=1$ $k=0$ adS_2 $k=-1$.

Assume sph. sym on brane SO(3) acting S^2 ... $\mathbb{R}^3$

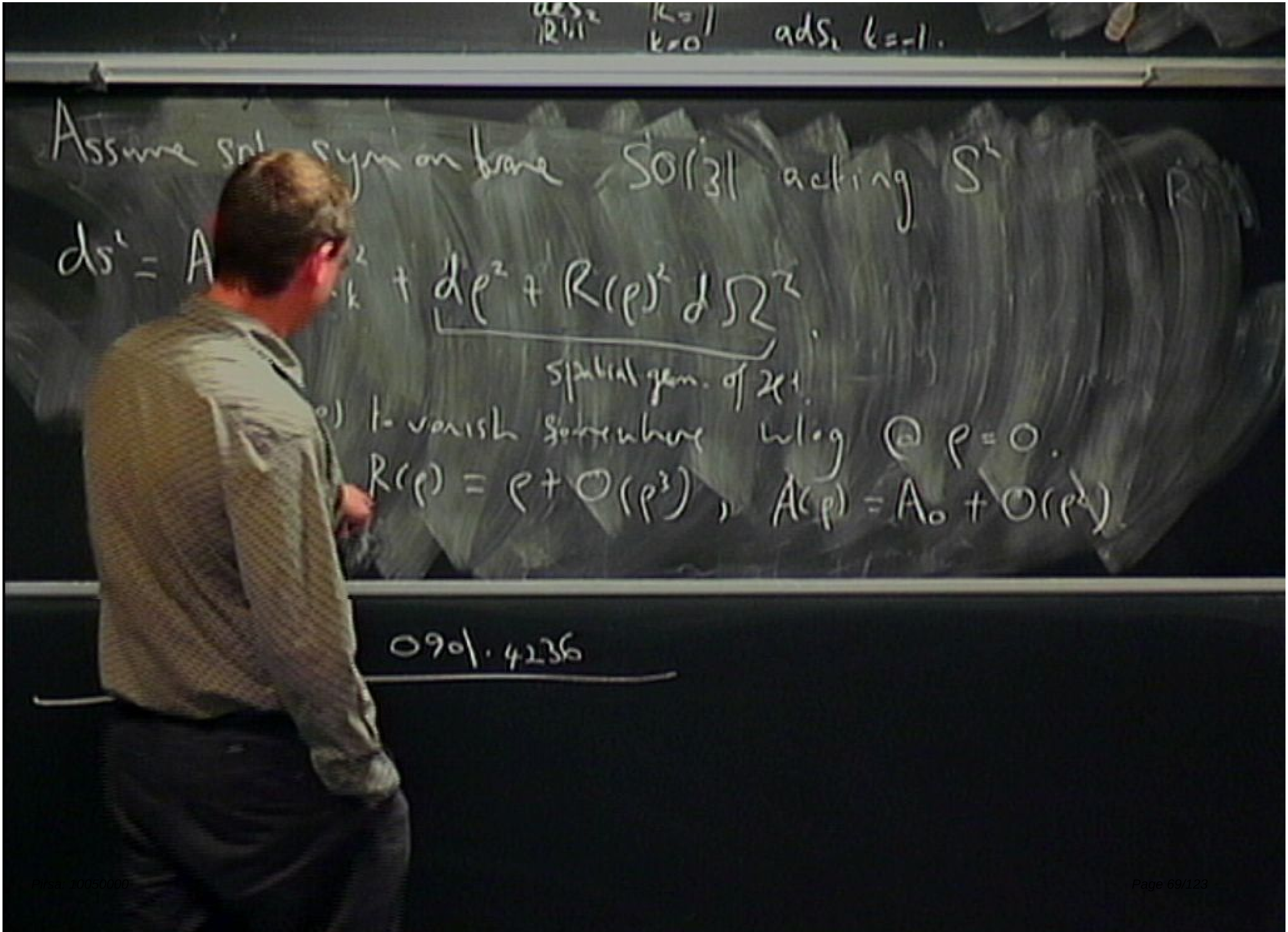
$$A(\rho)^2 d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}.$$

Need $R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

oothness $\Rightarrow$ $R(\rho) = \rho + O(\rho^3)$

& HSR 0901.4236

Why?.



dS_2 $\mathbb{R}^{1,1}$ $k=1$ $k=0$ adS_2 $k=-1$.

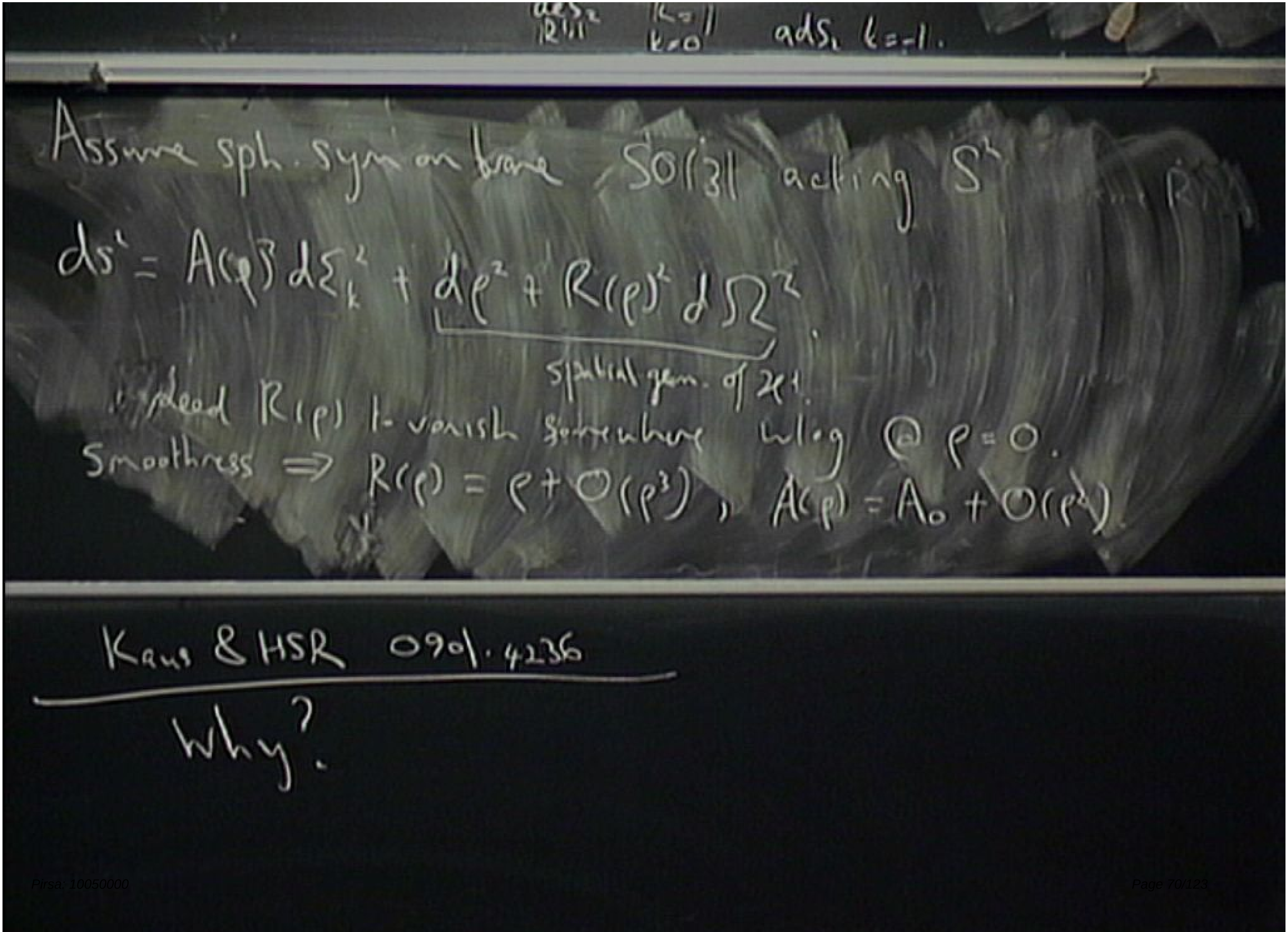
Assume sph. sym on brane SO(3) acting S^2

$ds^2 = A \quad {}_k + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of 2+1}}$.

) to vanish somewhere wlog @ $\rho = 0$.

$R(\rho) = \rho + O(\rho^3)$, $A(\rho) = A_0 + O(\rho^2)$

0901.4236



dS_2 $k=1$ $\mathbb{R}^{1,1}$ $k=0$ adS_2 $k=-1$.

Assume sph. sym on brane SO(3) acting S^2 ... $\mathbb{R}$

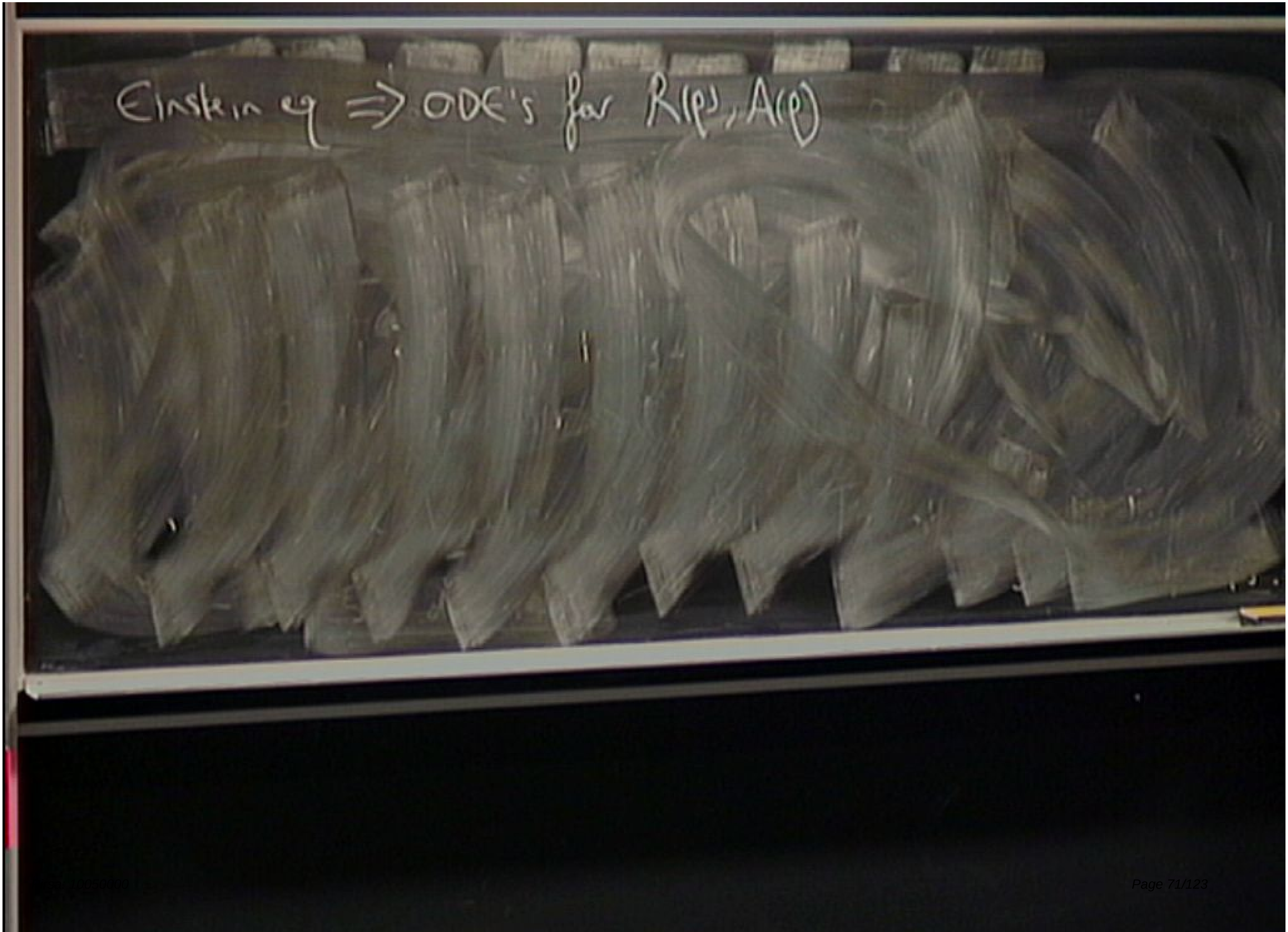
$$ds^2 = A(\rho)^2 d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of 2+1}}.$$

Need $R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

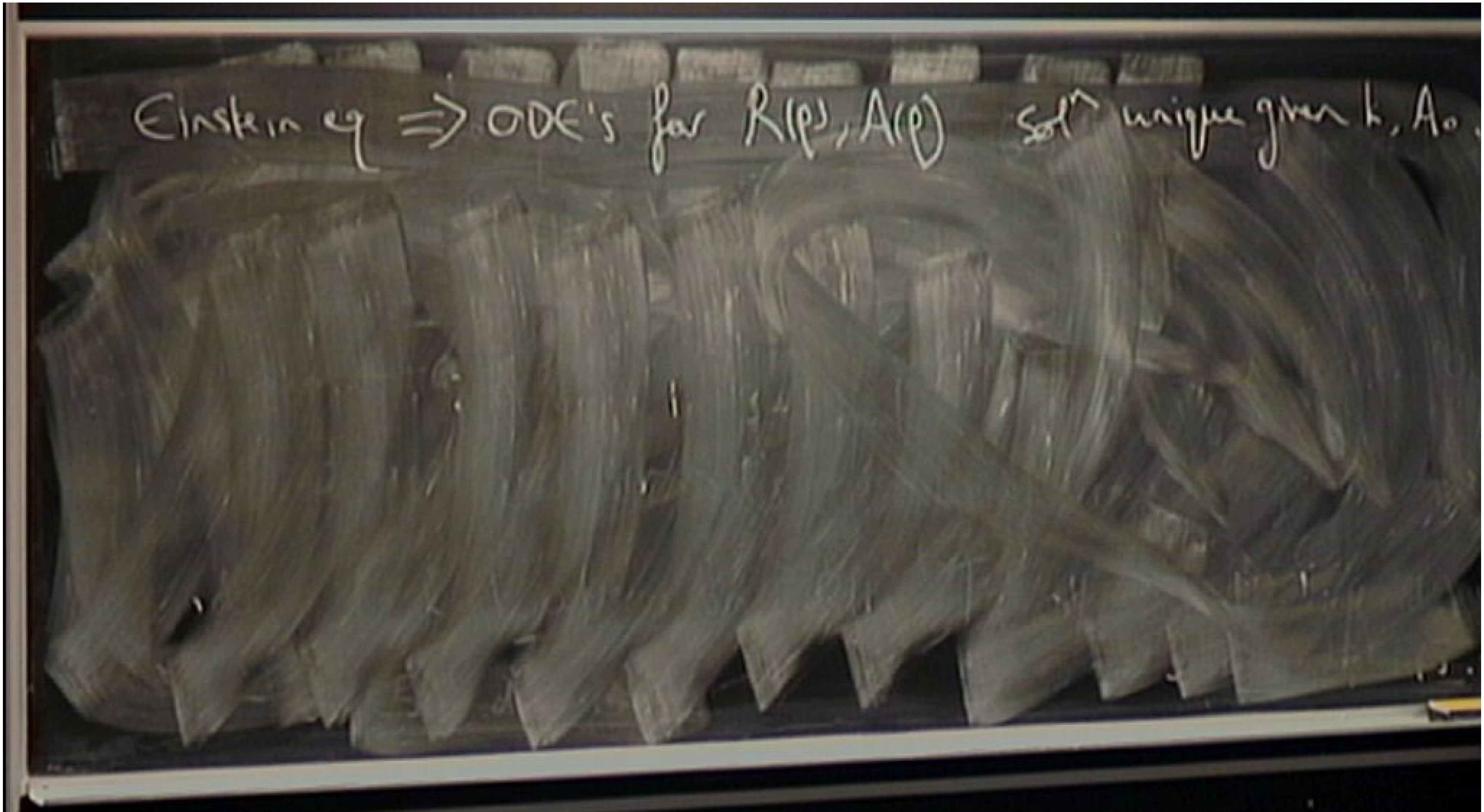
Smoothness $\Rightarrow$ $R(\rho) = \rho + O(\rho^3)$, $A(\rho) = A_0 + O(\rho^2)$.

Kaus & HSR 0901.4236

Why?



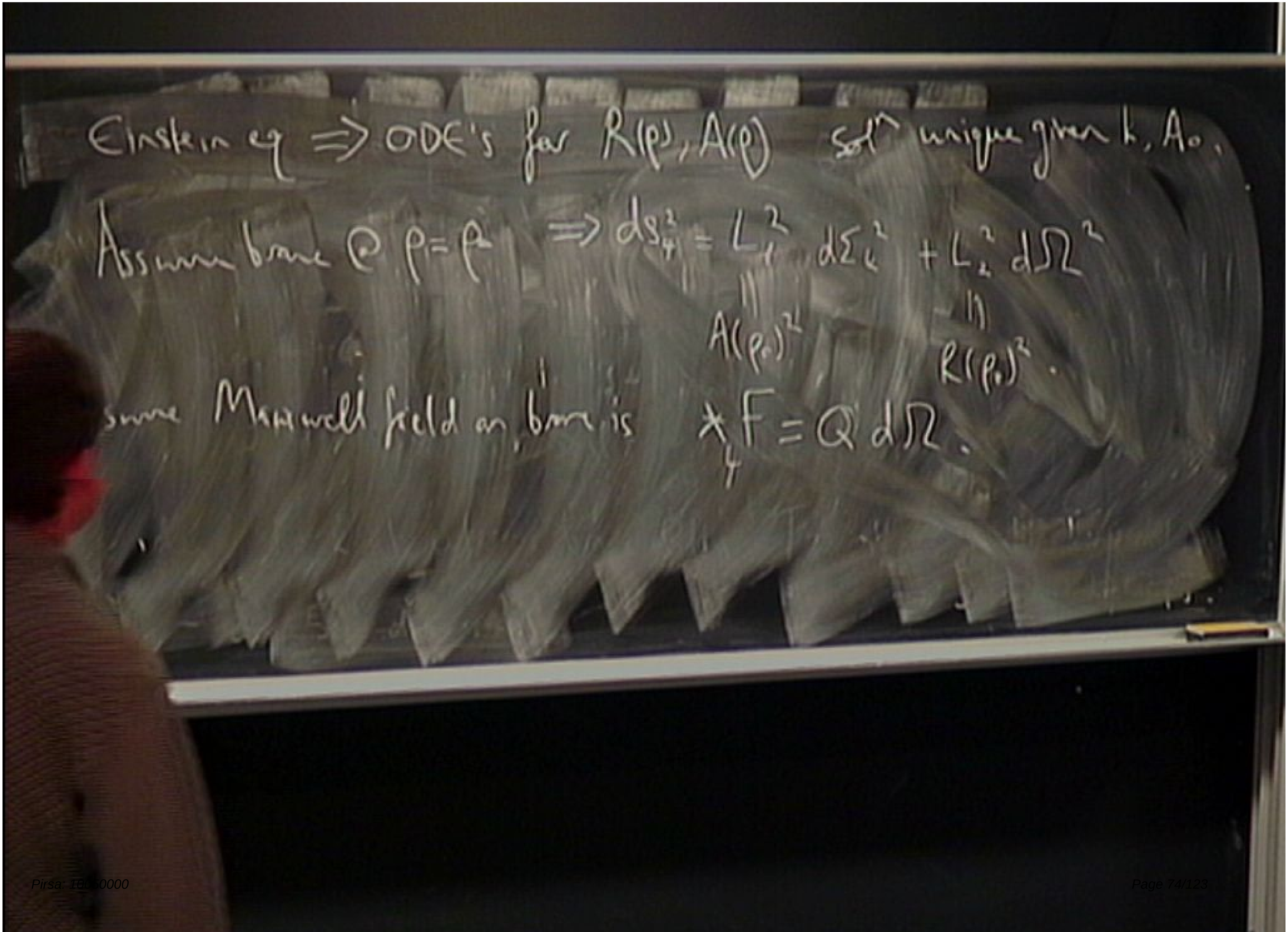
Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$



Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$ sol^n unique given h, A_0.

Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$ sol^{n} unique given b, A_0.

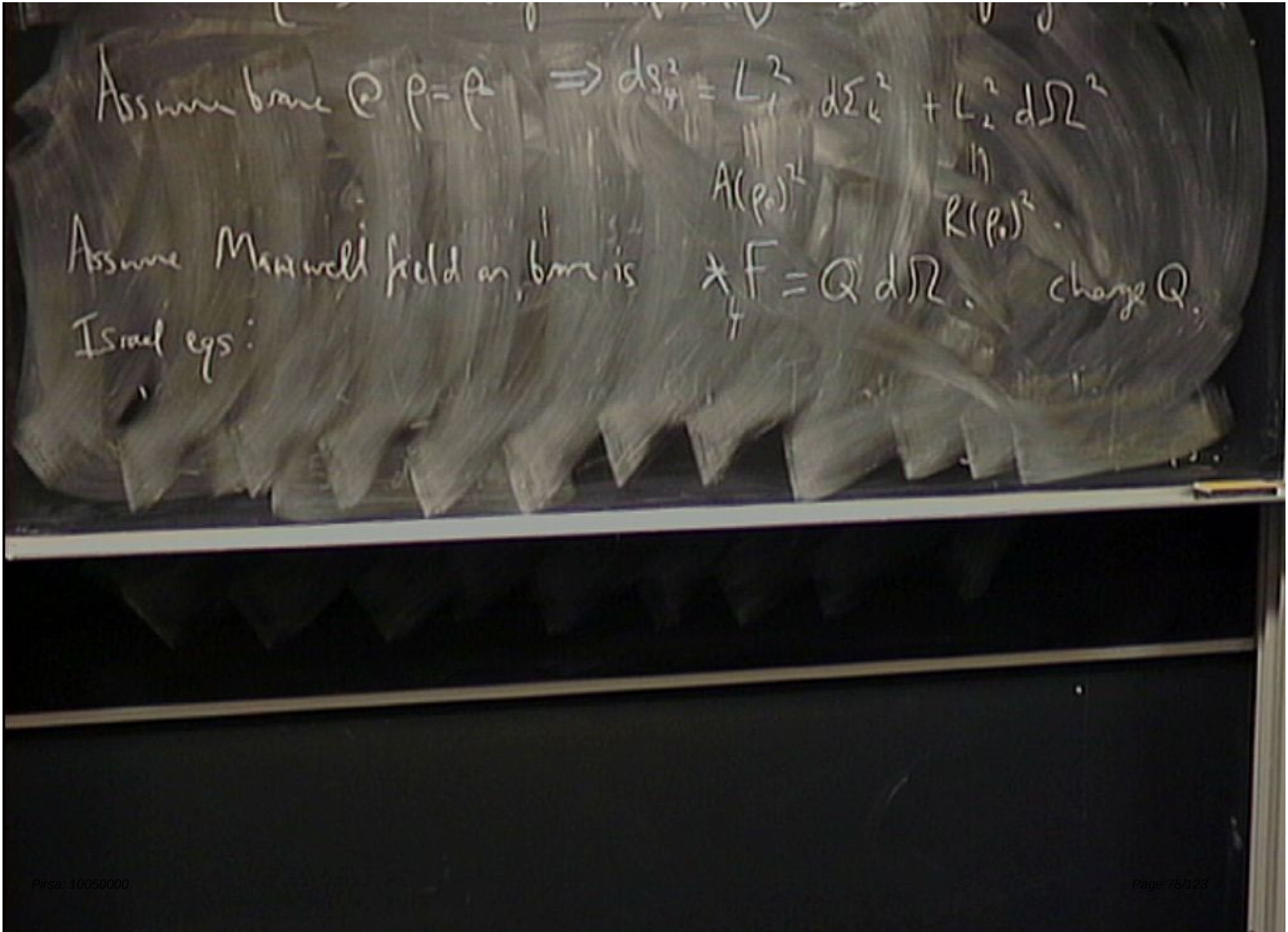
Assume brane @ $\rho = \rho_0$ $\Rightarrow$ $ds_4^2 = \underset{\substack{\| \\ A(\rho_0)^2}}{L_1^2}\, d\Sigma_4^2 + \underset{\substack{\| \\ R(\rho_0)^2}}{L_2^2}\, d\Omega^2$.



Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$ sol^n unique given k, A_0.

Assume brane @ $\rho = \rho_0$ $\Rightarrow$ $ds_4^2 = \underset{\overset{\|}{A(\rho_0)^2}}{L_1^2}\, d\Sigma_k^2 + \underset{\overset{\|}{R(\rho_0)^2}}{L_2^2}\, d\Omega^2$.

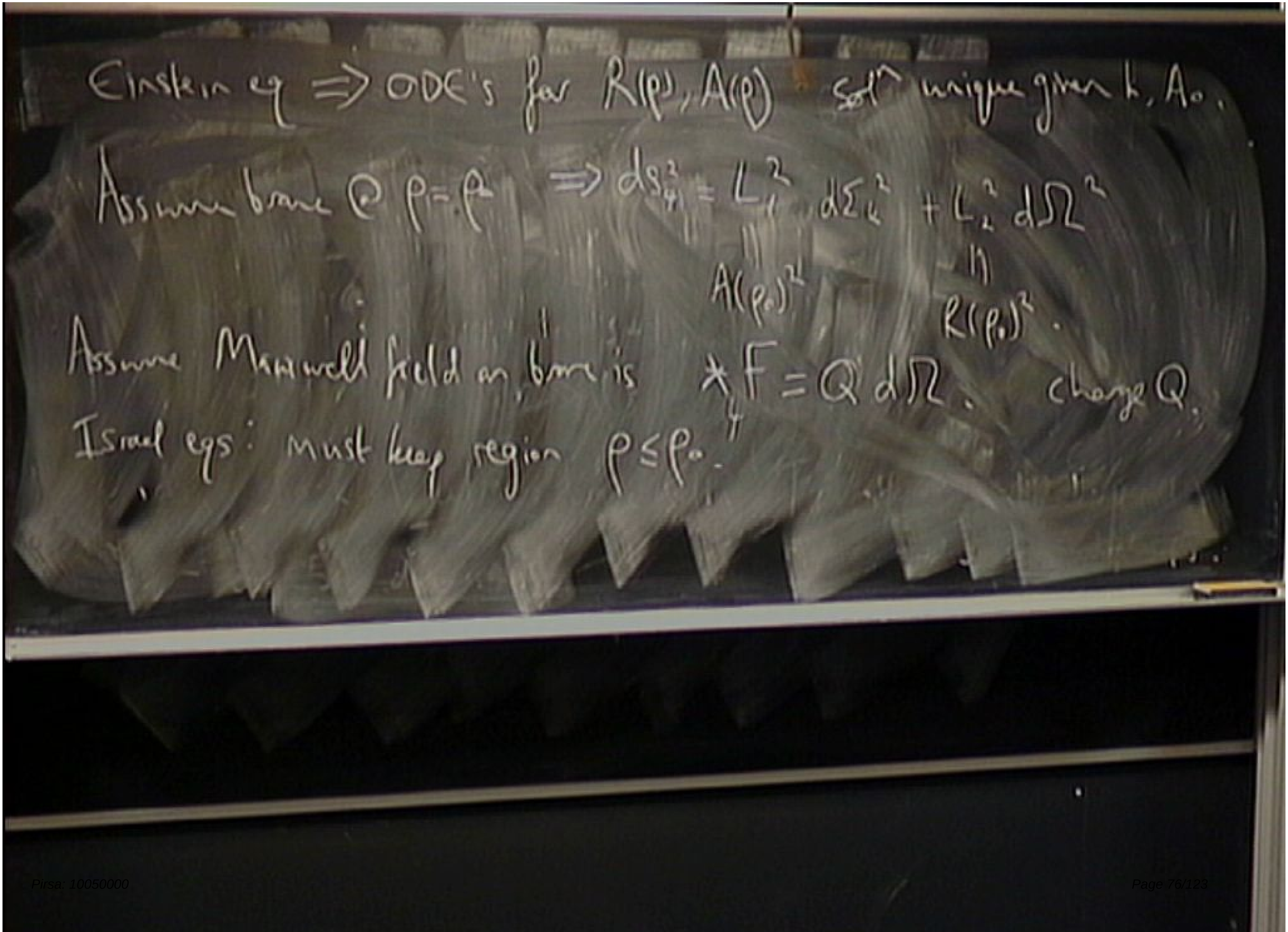
sume Maxwell field on brane is $*_4 F = Q\, d\Omega$.



Assume brane @ $\rho=\rho_0$ $\Rightarrow$ $ds_4^2 = \underset{A(\rho_0)^2}{\underset{\parallel}{L_1^2}}\, d\Sigma_k^2 + \underset{R(\rho_0)^2 .}{\underset{\parallel}{L_2^2}}\, d\Omega^2$

Assume Maxwell field on brane is $*_4 F = Q\, d\Omega$. charge Q.

Israel eqs:

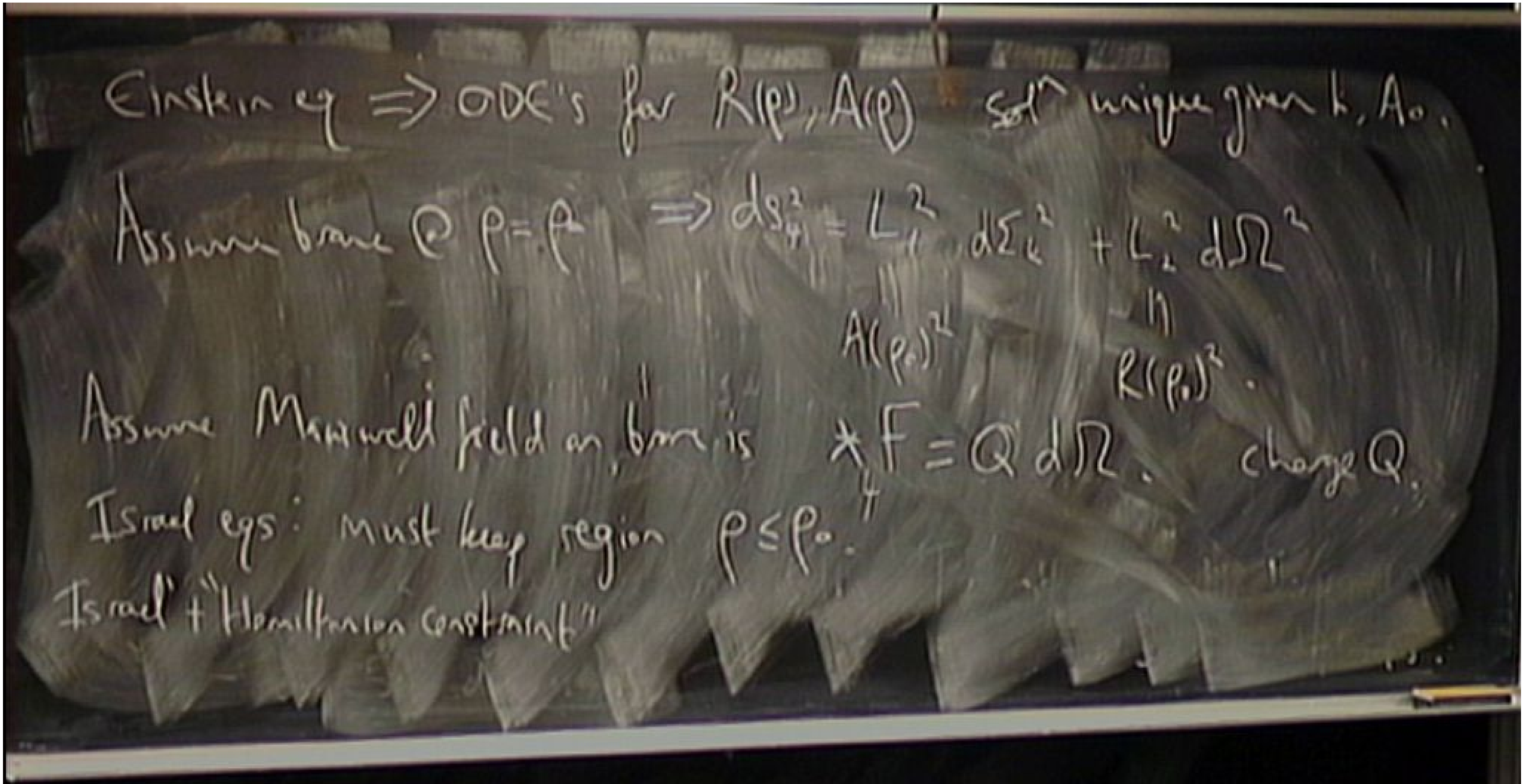


Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$. sol^{n} unique given L, A_0.

Assume brane @ $\rho=\rho_0$ $\Rightarrow$ $ds_4^2 = \underset{A(\rho_0)^2}{\underset{\parallel}{L_1^2}} \, d\Sigma_2^2 + \underset{R(\rho_0)^2}{\underset{\parallel}{L_2^2}} \, d\Omega^2$.

Assume Maxwell field on brane is $\star_4 F = Q \, d\Omega$. charge Q.

Israel eqs: must keep region $\rho \le \rho_0$.



Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$. sol^n unique given L, A_0.

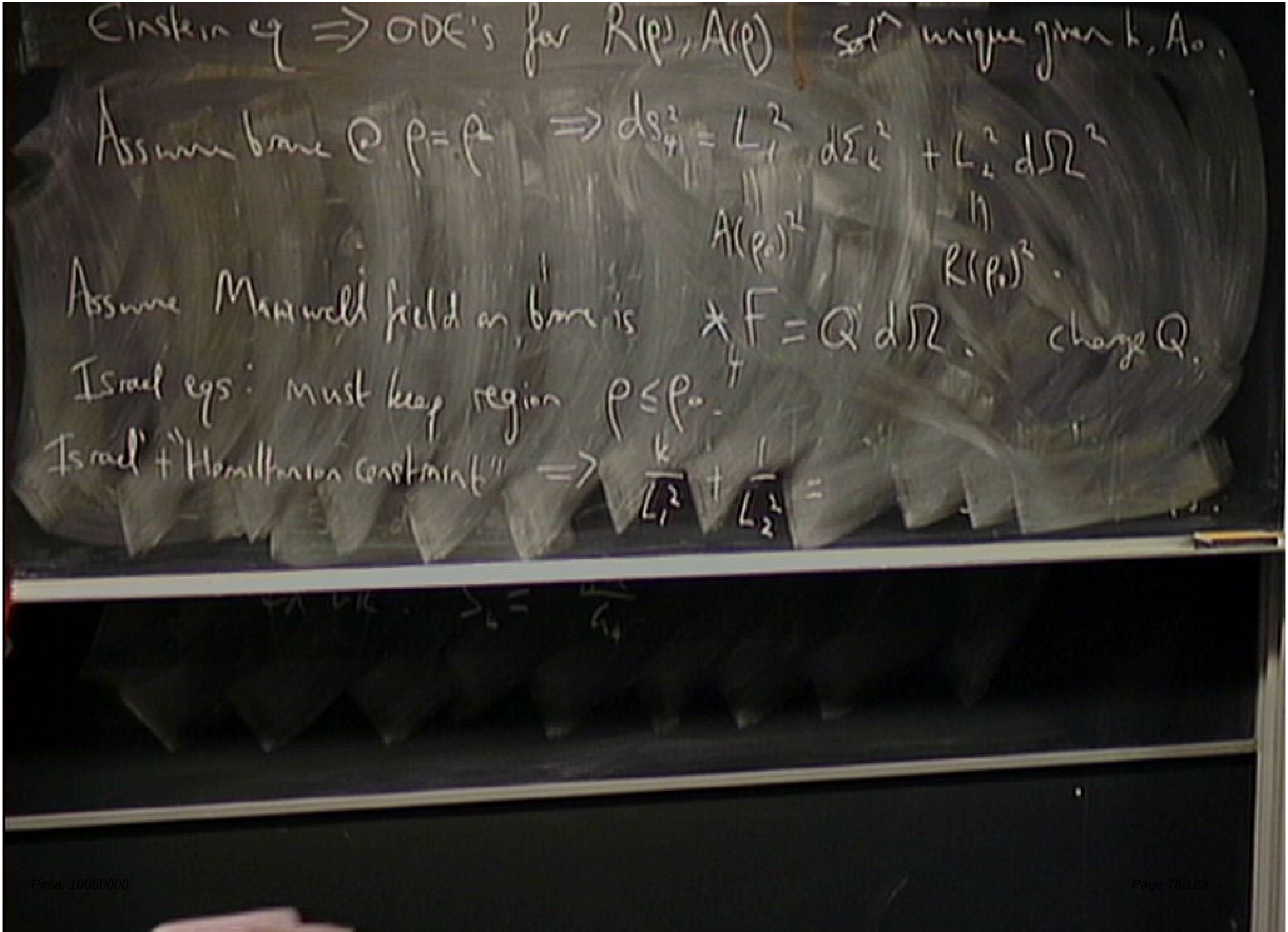
Assume brane @ $\rho = \rho_0$ $\Rightarrow$

$$ds_4^2 = \underset{A(\rho_0)^2}{\underset{\|}{L_1^2}}\, d\Sigma_2^2 + \underset{R(\rho_0)^2 .}{\underset{\|}{L_2^2}}\, d\Omega^2$$

Assume Maxwell field on brane is $*_4 F = Q\, d\Omega$. charge Q.

Israel eqs: must keep region $\rho \leq \rho_0$.

Israel + "Hamiltonian constraint"



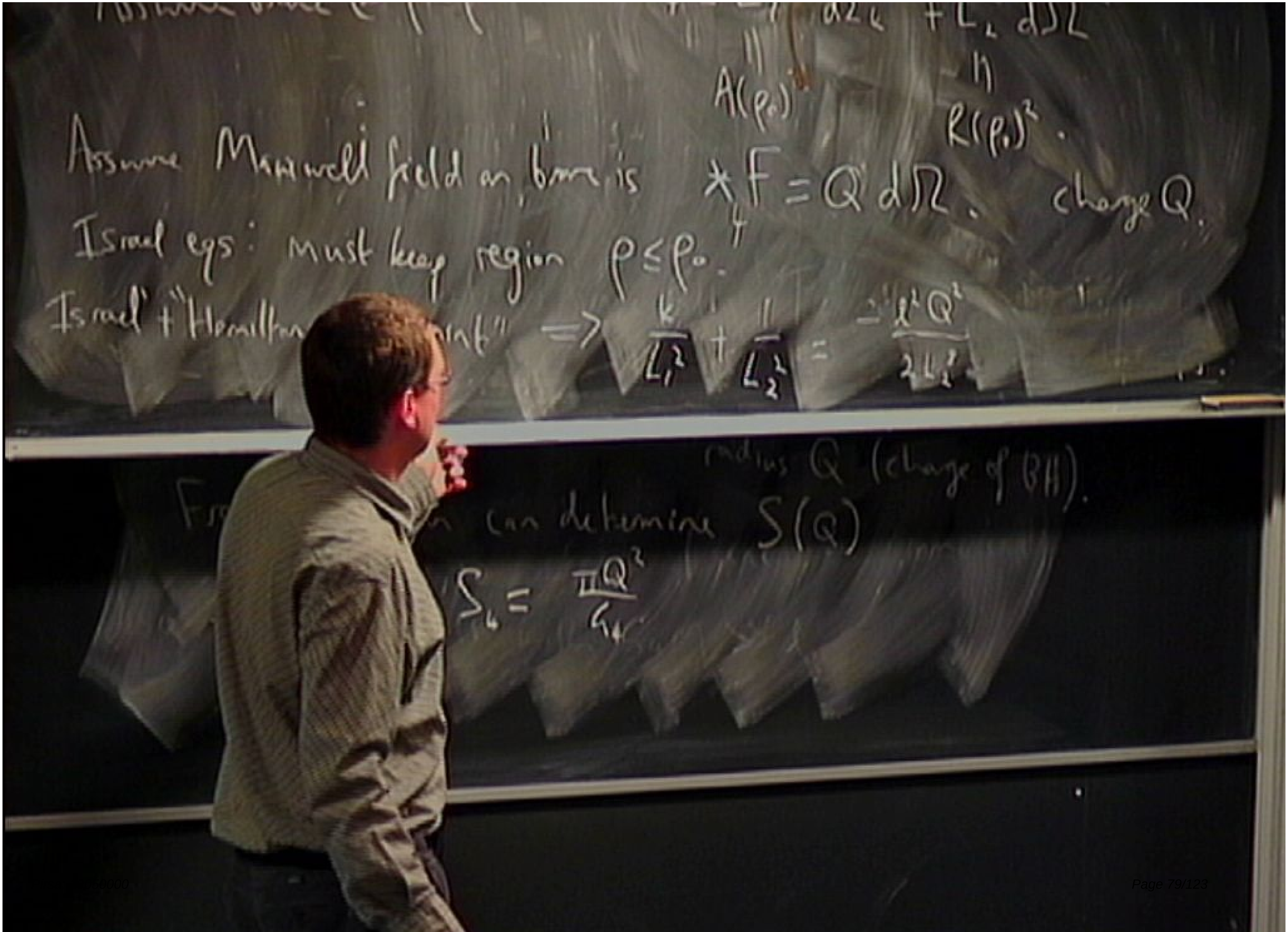
Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$ sol^n unique given k, A_0.

Assume brane @ $\rho = \rho_0$ $\Rightarrow$ $ds_4^2 = \underset{A(\rho_0)^2}{\underset{\parallel}{L_1^2}}\, d\Sigma_k^2 + \underset{R(\rho_0)^2}{\underset{\parallel}{L_2^2}}\, d\Omega^2$.

Assume Maxwell field on brane is $*_4 F = Q\, d\Omega$. charge Q.

Israel eqs: must keep region $\rho \leq \rho_0$.

Israel + "Hamiltonian constraint" $\Rightarrow$ $\frac{k}{L_1^2} + \frac{1}{L_2^2} =$



$A(\rho_0)$ $R(\rho_0)^2$.

Assume Maxwell field on brane is $*_4 F = Q\, d\Omega$. charge Q.

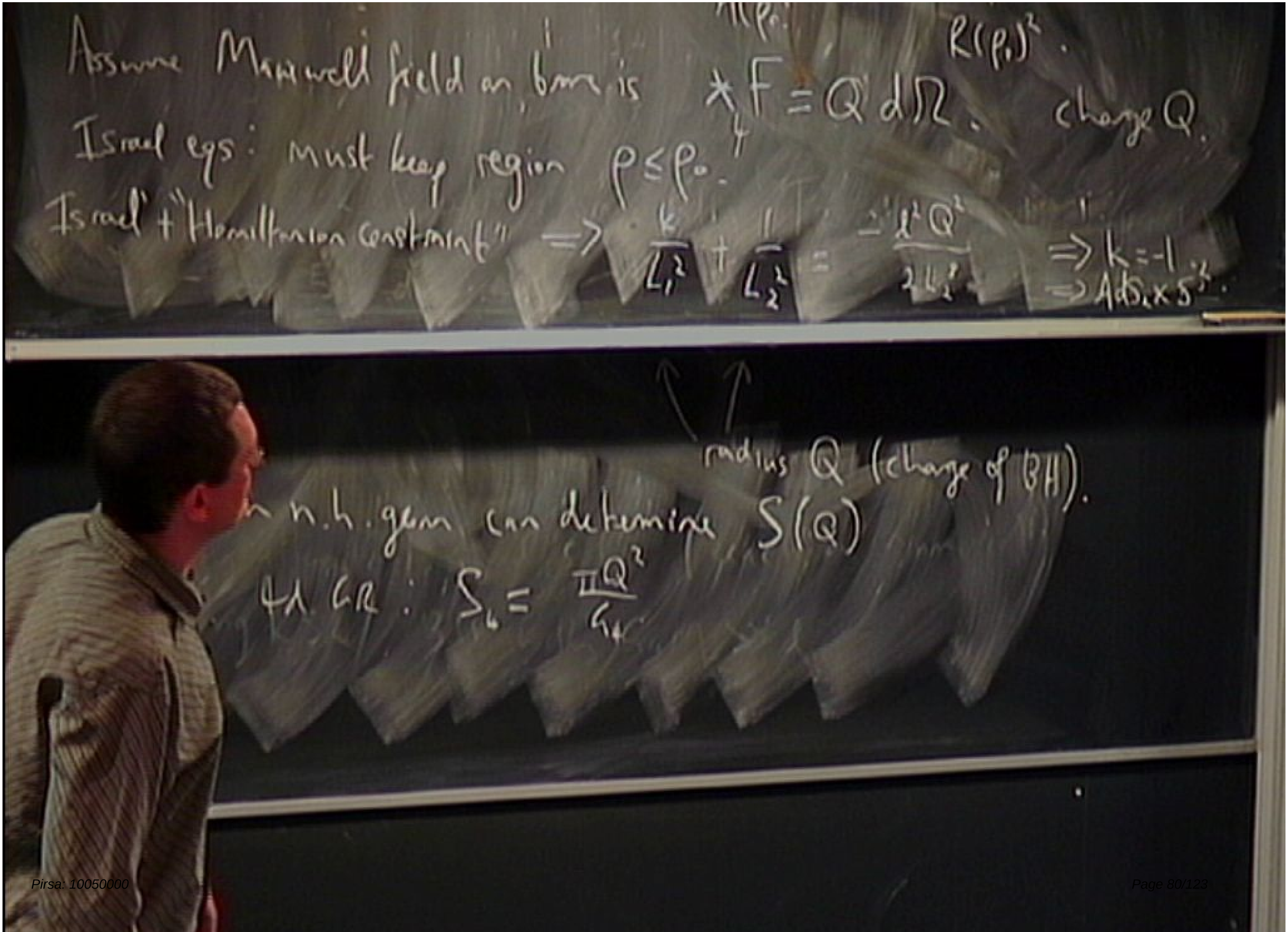
Israel eqs: must keep region $\rho \le \rho_0$.

Israel + "Hamilton… …int" $\Rightarrow \frac{k}{L_1^2} + \frac{1}{L_2^2} = -\frac{\ell^2 Q^2}{2 L_2^2}$

radius Q (charge of BH).

Fr… …n can determine $S(Q)$

$S_4 = \frac{\pi Q^2}{G_4}$



$R(\rho_0$ $\quad R(\rho_0)^2$.

Assume Maxwell field on brane is $\star_4 F = Q\, d\Omega$. charge Q.

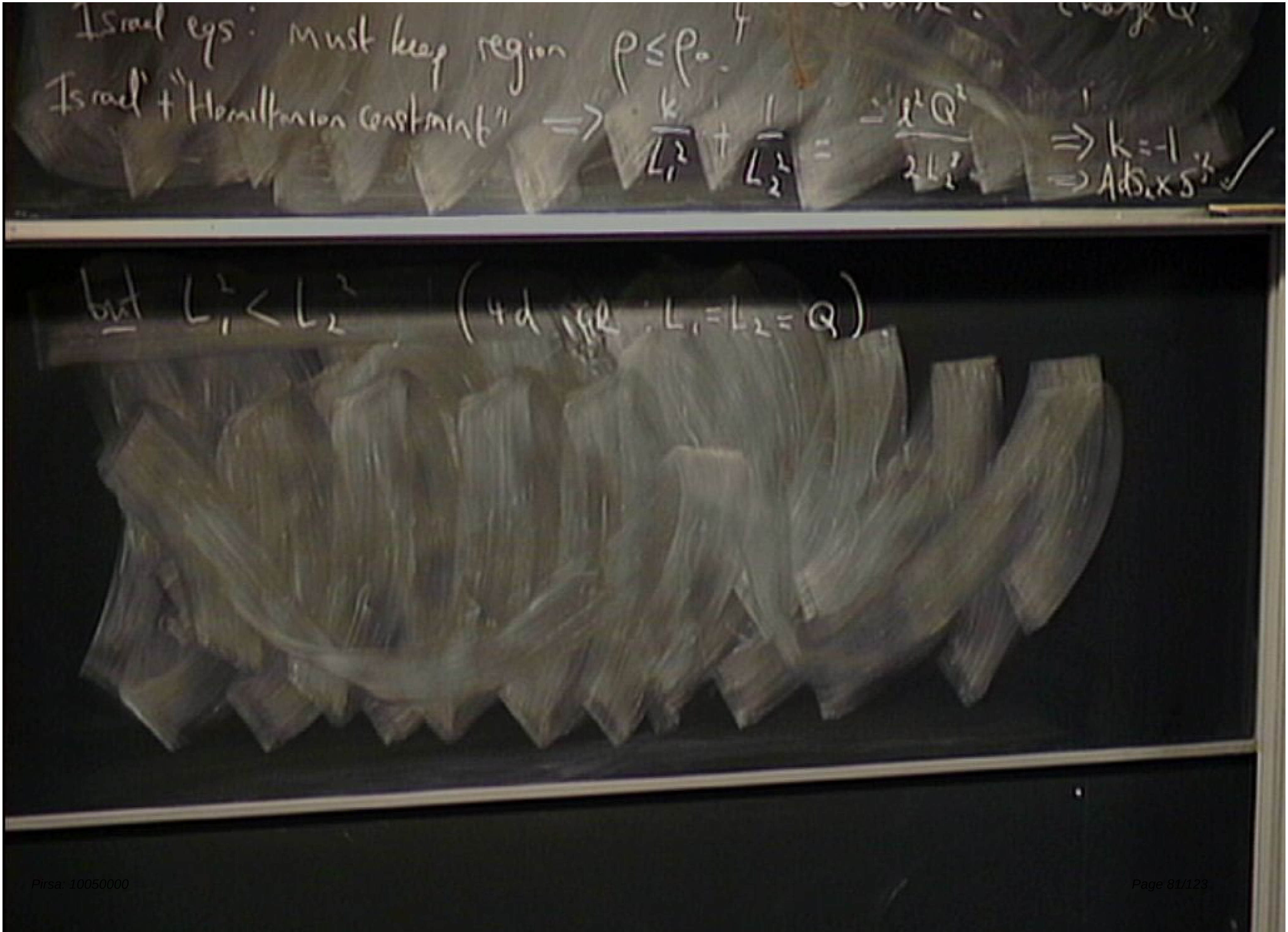
Israel eqs: must keep region $\rho \le \rho_0$.

Israel + "Hamiltonian constraint" $\Rightarrow \frac{k}{L_1^2} + \frac{1}{L_2^2} = -\frac{\ell^2 Q^2}{2 L_2^4}$ $\Rightarrow k = -1$, $\Rightarrow AdS_2 \times S^2$.

radius Q (charge of BH).

n.h. geom can determine $S(Q)$

4d GR: $S_b = \frac{\pi Q^2}{G_4}$



Israel eqs: Must keep region $\rho \le \rho_0$.

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but $L_1^2 < L_2^2$ (4d case: $L_1 = L_2 = Q$).

charge Q.

Israel eqs: Must keep region $\rho \leq \rho_0$.

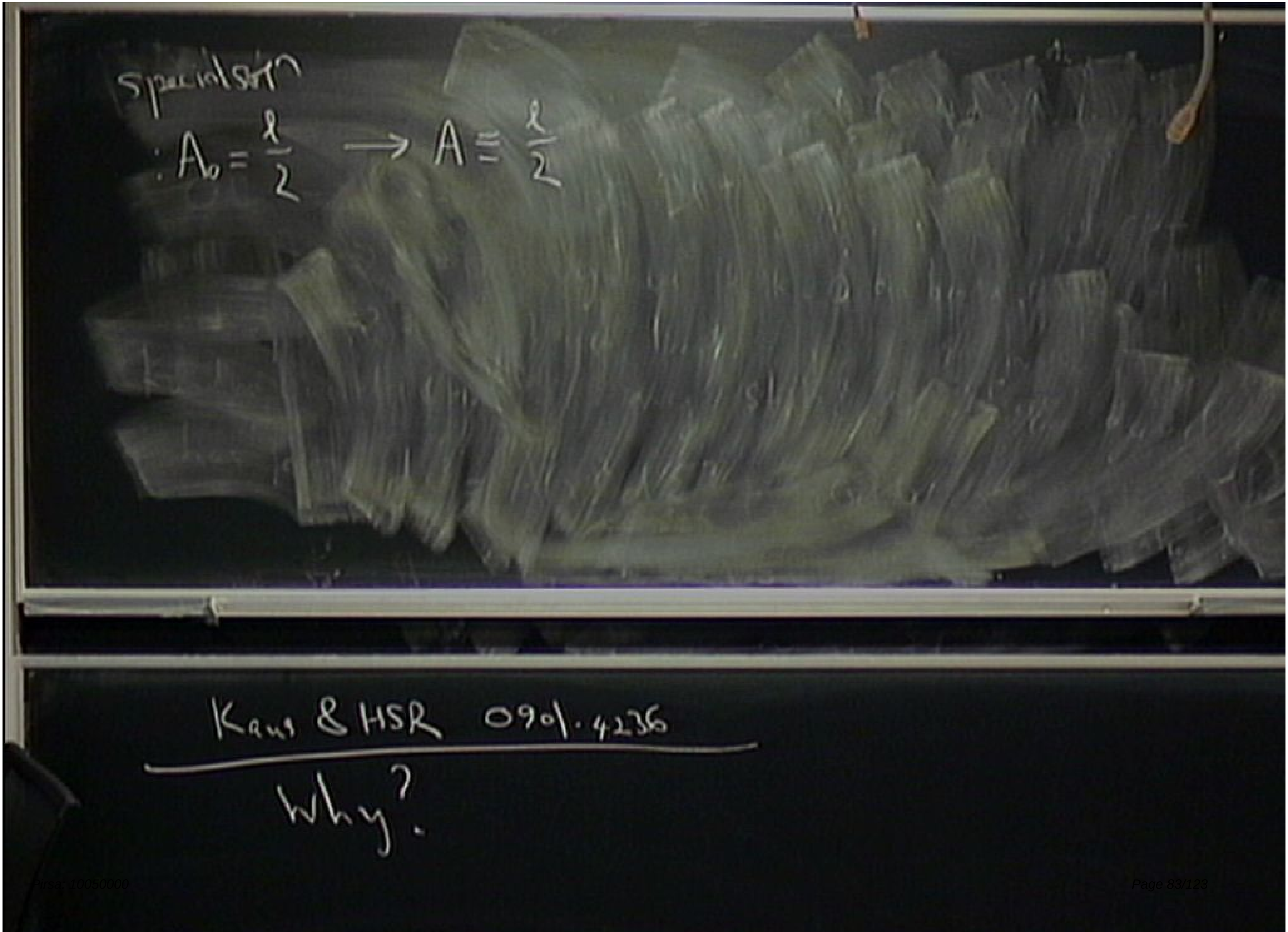
Israel + "Hamiltonian constraint" $\Rightarrow \frac{k}{L_1^2} + \frac{1}{L_2^2} = -\frac{\ell^2 Q^2}{2L_1^2}$ $\Rightarrow k = -1$, $\Rightarrow AdS_2 \times S^3$ ✓

but $L_1^2 < L_2^2$ (4d GR: $L_1 = L_2 = Q$).

Solutions

(i) determine bulk - 1 parameter A_0

(ii) Israel determine A_0, ρ_0 uniquely to Q.

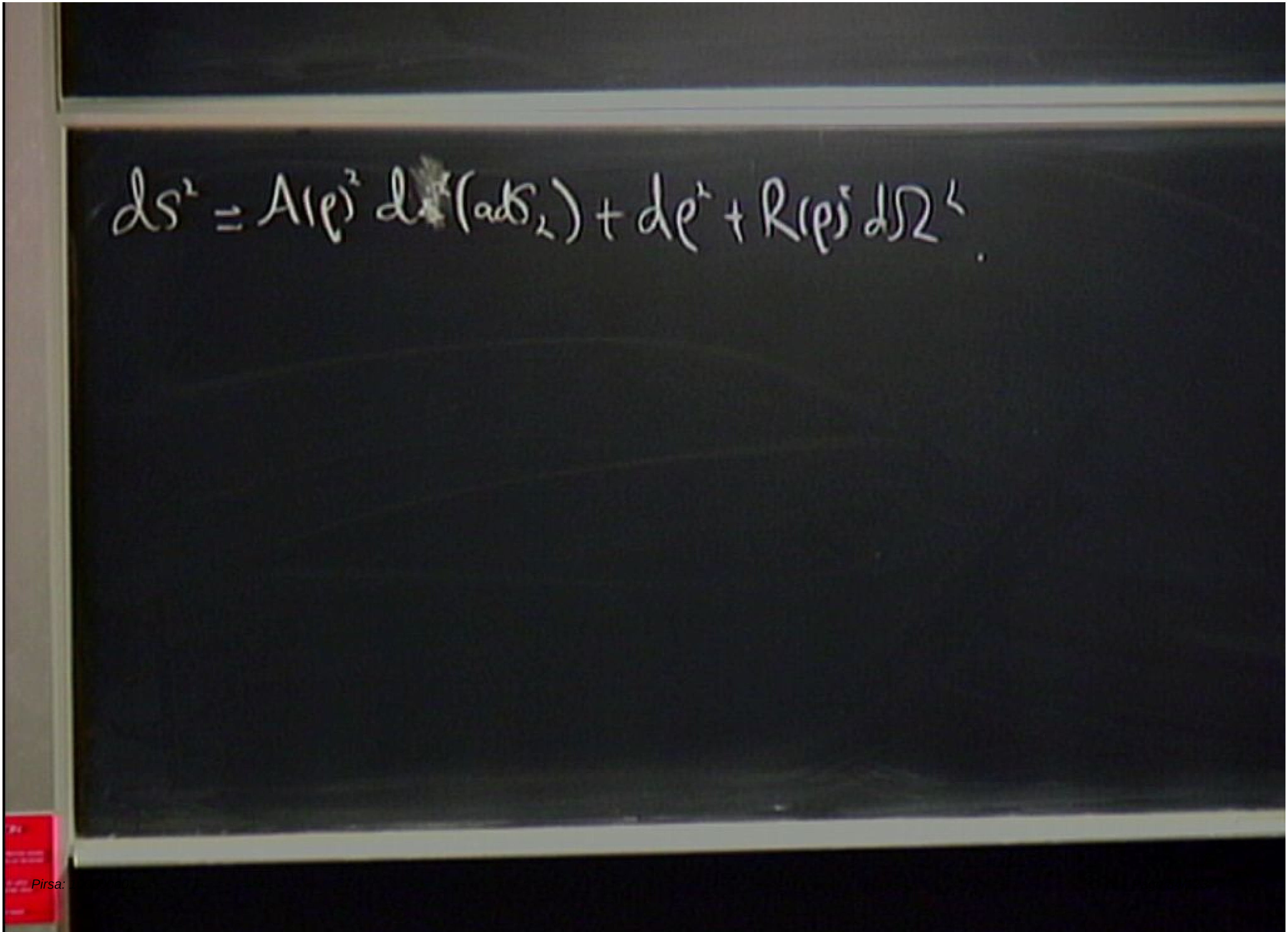


special soln

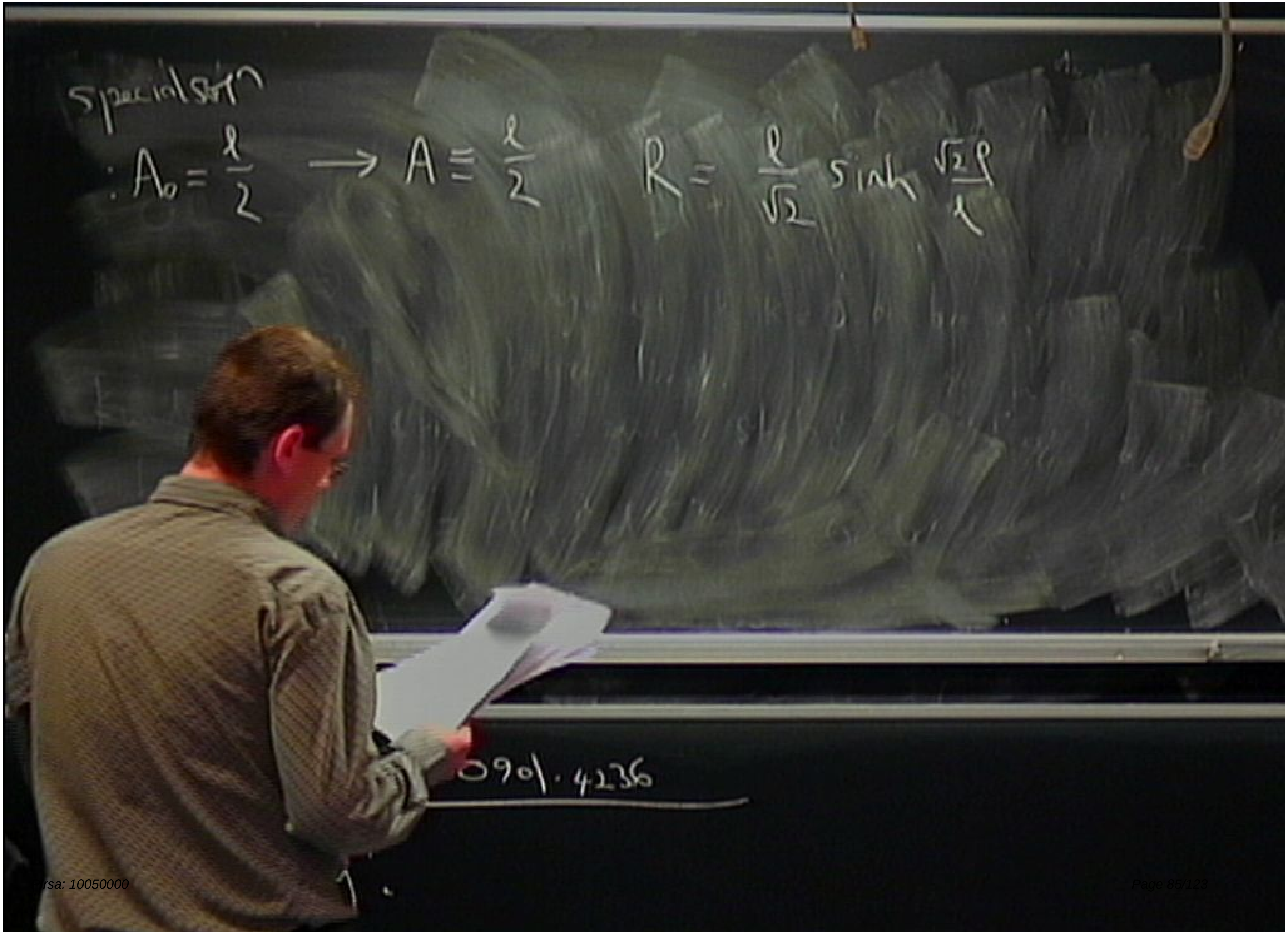
$$: A_0 = \frac{\ell}{2} \longrightarrow A \equiv \frac{\ell}{2}$$

Kaus & HSR 0901.4236

Why?.



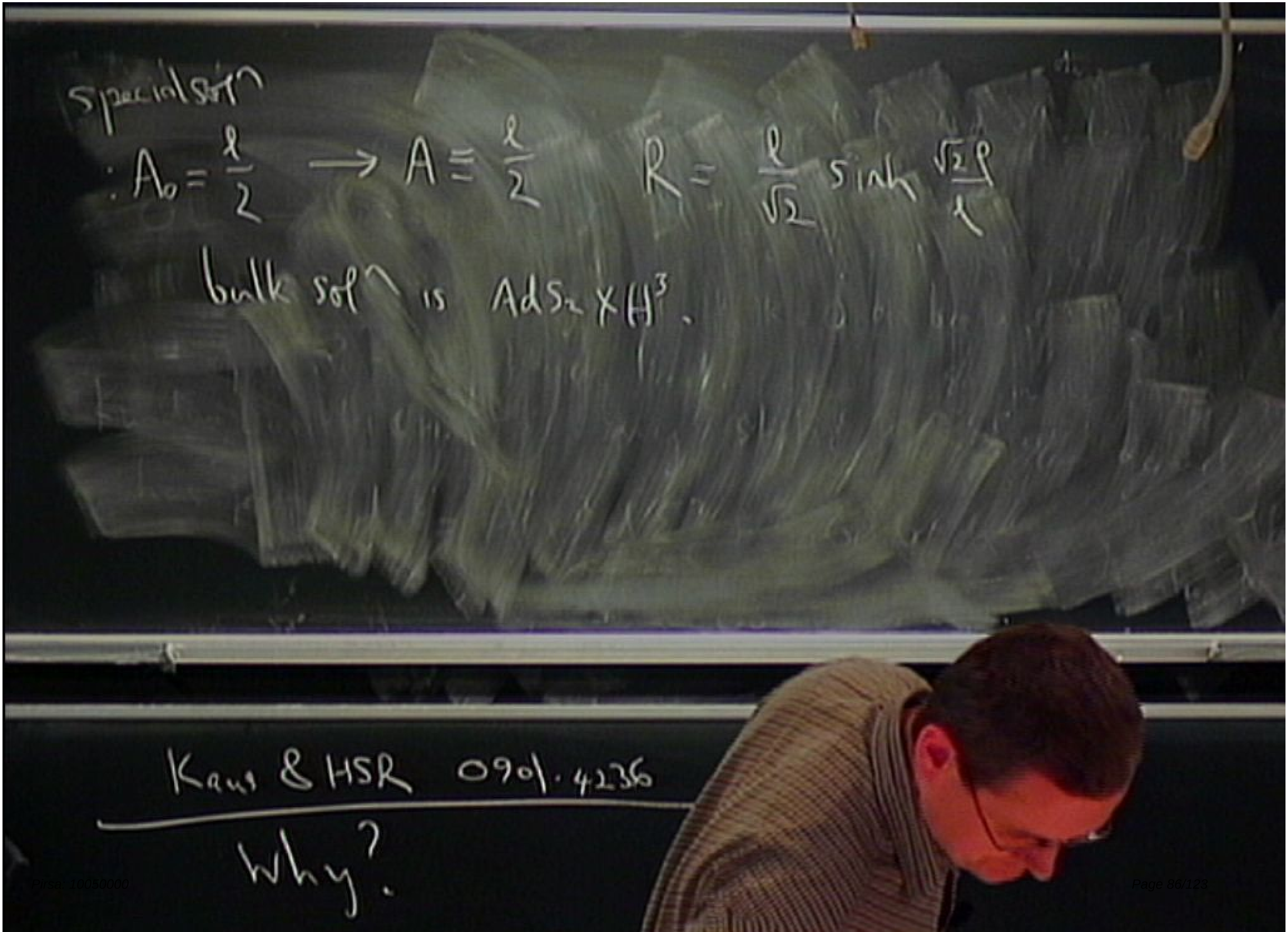
$$ds^2 = A(\rho)^2 \, ds^2(AdS_2) + d\rho^2 + R(\rho)^2 \, d\Omega^2 .$$



special soln

$$: A_0 = \frac{\ell}{2} \longrightarrow A \equiv \frac{\ell}{2} \qquad R = \frac{\ell}{\sqrt{2}} \sinh \frac{\sqrt{2}\rho}{\ell}$$

0901.4236



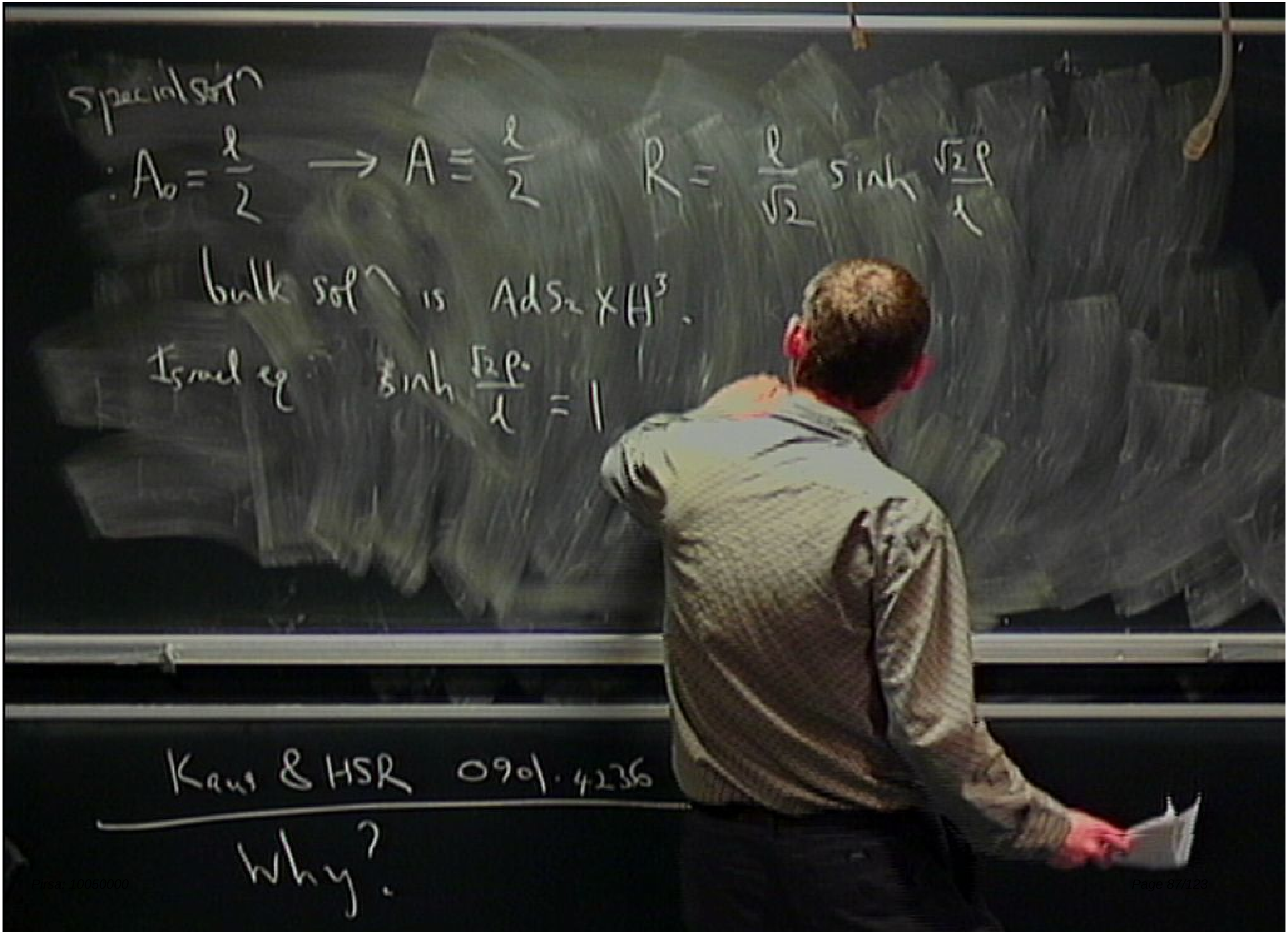
special soln

$$A_0 = \frac{\ell}{2} \longrightarrow A \equiv \frac{\ell}{2} \qquad R = \frac{\ell}{\sqrt{2}} \sinh \frac{\sqrt{2}\rho}{\ell}$$

bulk soln is $AdS_2 \times H^3$.

Kaus & HSR 0901.4236

Why?.



special sol^n

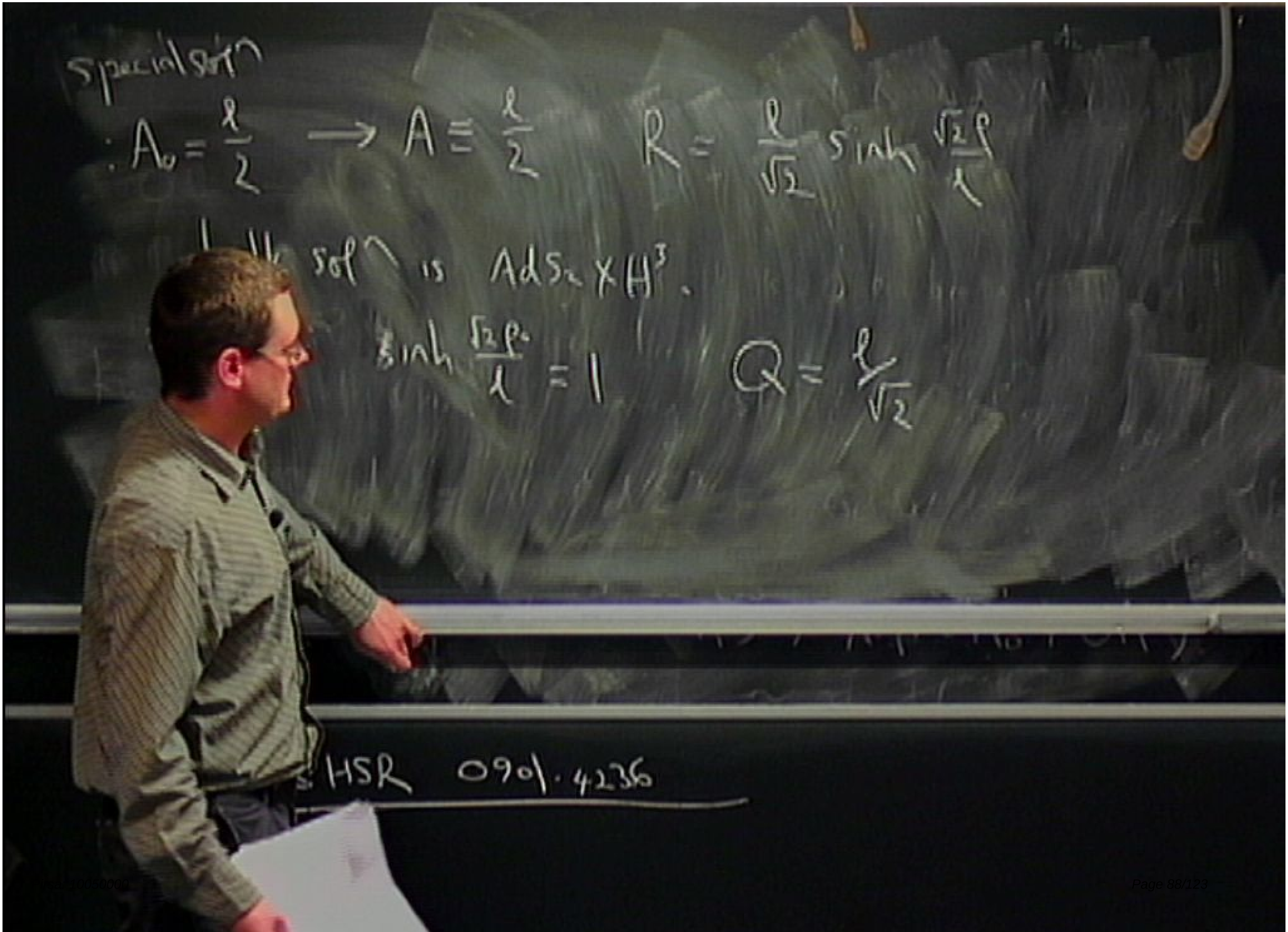
: $A_0 = \frac{\ell}{2} \longrightarrow A \equiv \frac{\ell}{2}$ $\quad R = \frac{\ell}{\sqrt{2}} \sinh \frac{\sqrt{2}\rho}{\ell}$

bulk sol^n is $AdS_2 \times H^3$.

Israel eq: $\sinh \frac{\sqrt{2}\rho_0}{\ell} = 1$

Kaus & HSR 0901.4236

Why?.



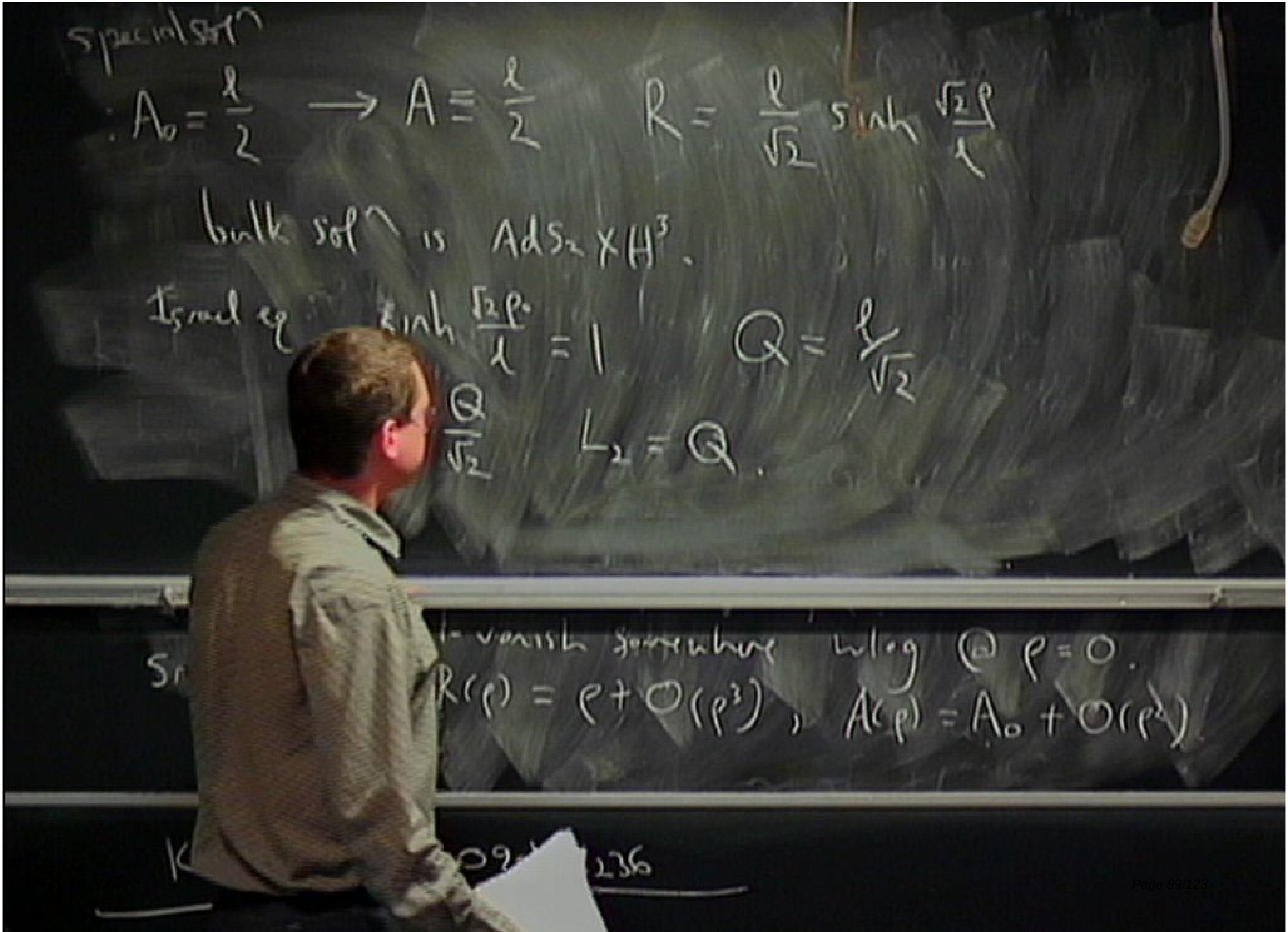
special soln

$$:A_0 = \frac{\ell}{2} \longrightarrow A \equiv \frac{\ell}{2} \qquad R = \frac{\ell}{\sqrt{2}} \sinh \frac{\sqrt{2}\rho}{\ell}$$

solⁿ is $AdS_2 \times H^3$.

$$\sinh \frac{\sqrt{2}\rho_0}{\ell} = 1 \qquad Q = \ell/\sqrt{2}$$

HSR 0901.4236



special soln

: $A_0 = \frac{\ell}{2} \longrightarrow A \equiv \frac{\ell}{2}$ $\quad R = \frac{\ell}{\sqrt{2}} \sinh \frac{\sqrt{2}\rho}{\ell}$

bulk soln is $AdS_2 \times H^3$.

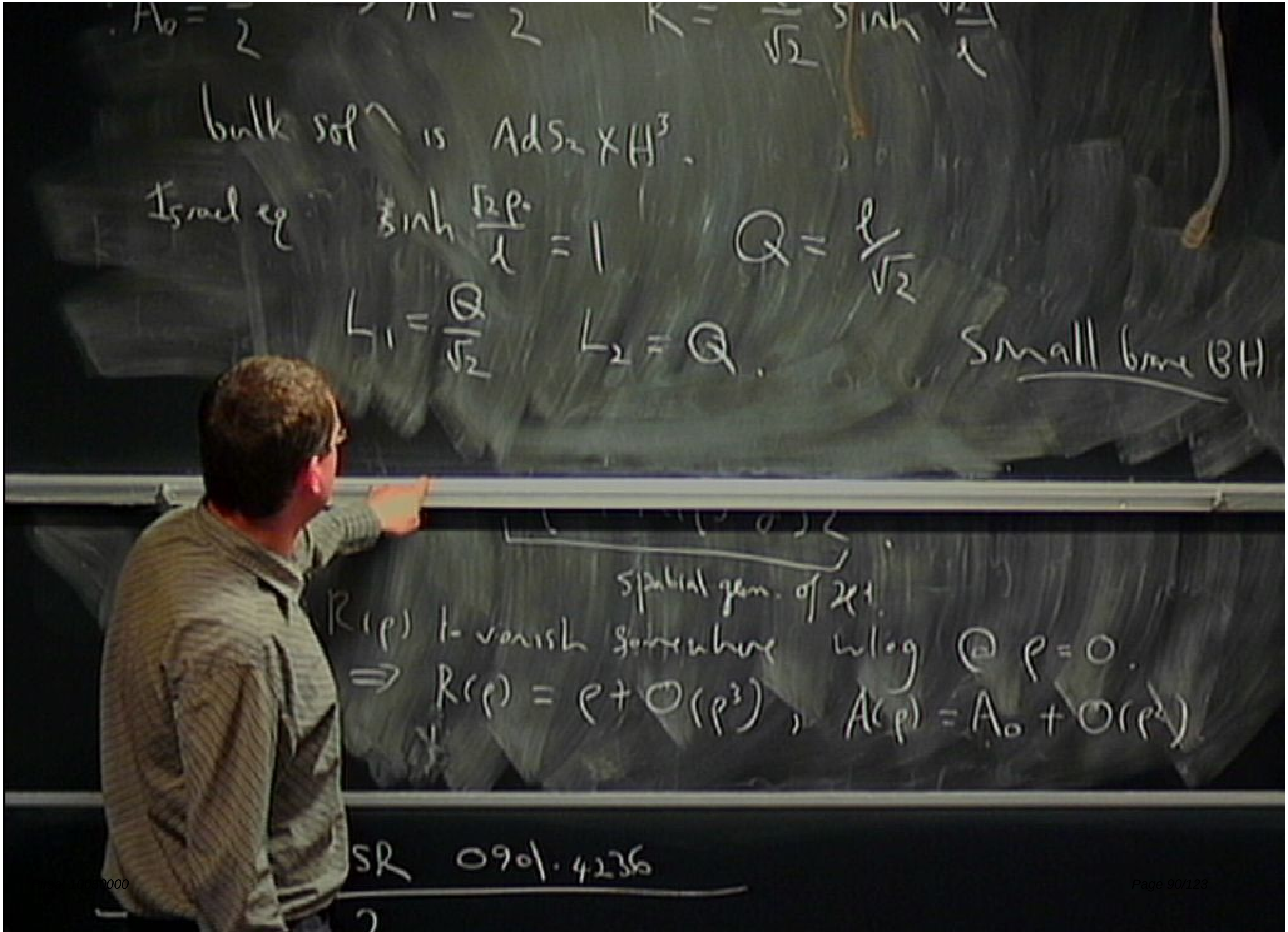
Israel eq: $\sinh \frac{\sqrt{2}\rho_0}{\ell} = 1$ $\quad Q = \ell/\sqrt{2}$

$\frac{Q}{\sqrt{2}}$ $\quad L_2 = Q$.

Sn -vanish somewhere wlog @ $\rho = 0$.

$R(\rho) = \rho + O(\rho^3)$, $\quad A(\rho) = A_0 + O(\rho^2)$

K 09 236



$\therefore A_0 = \frac{\;}{2} \Rightarrow A = \frac{\;}{2} \qquad R = \frac{\;}{\sqrt{2}} \sinh \frac{\sqrt{2}\;}{l}$

bulk sol$^{\text{n}}$ is $AdS_2 \times \mathbb{H}^3$.

Israel eq: $\quad \sinh \frac{\sqrt{2}\rho_0}{l} = 1 \qquad Q = l/\sqrt{2}$

$L_1 = \frac{Q}{\sqrt{2}} \qquad L_2 = Q.$

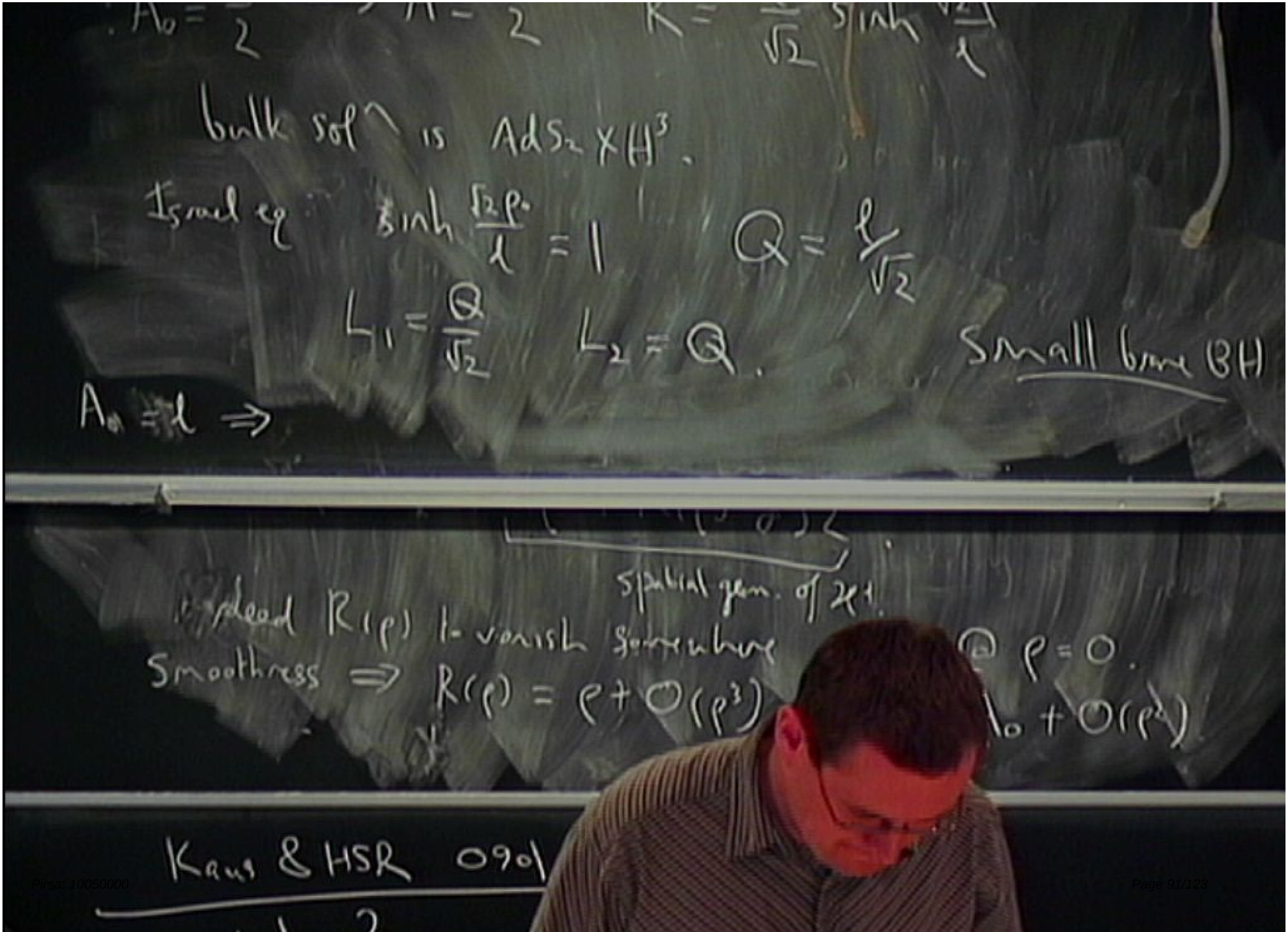
Small brane BH

spatial geom. of 2+1.

$R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

$\Rightarrow R(\rho) = \rho + O(\rho^3), \quad A(\rho) = A_0 + O(\rho^2)$

SR 0901.4236



$A_0 = \frac{\ }{2} \Rightarrow A = \frac{\ }{2} \qquad R = \frac{\ }{\sqrt{2}} \sinh \frac{\sqrt{2}\ }{\ell}$

bulk sol^n is $AdS_2 \times \mathbb{H}^3$.

Israel eq: $\sinh \frac{\sqrt{2}\rho_0}{\ell} = 1 \qquad Q = \ell/\sqrt{2}$

$L_1 = \frac{Q}{\sqrt{2}} \qquad L_2 = Q$.

Small brane BH

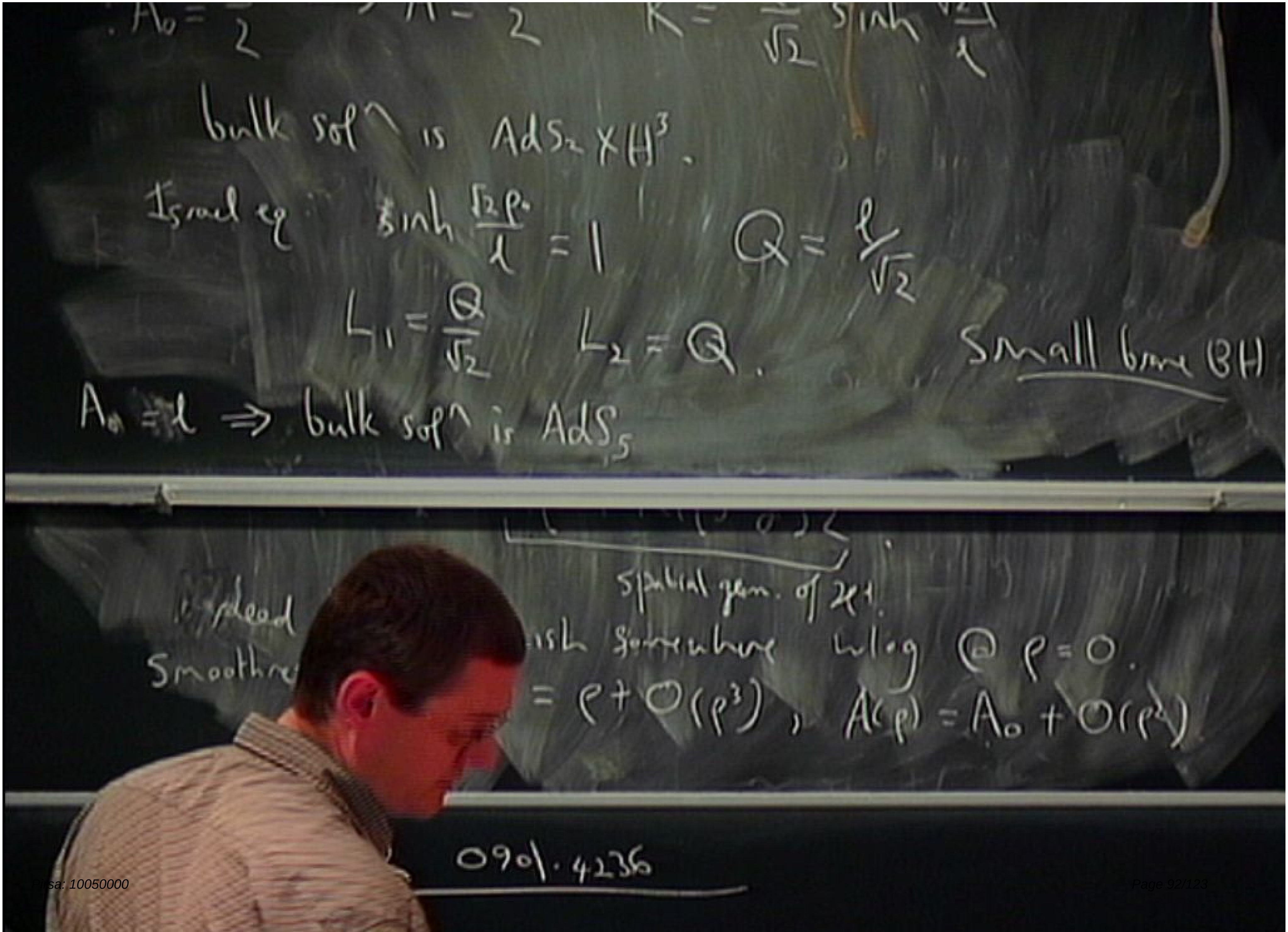
$A_0 = \ell \Rightarrow$

spatial geom. of $\mathcal{H}$+1.

Need $R(\rho)$ to vanish somewhere @ $\rho = 0$.

Smoothness $\Rightarrow R(\rho) = \rho + O(\rho^3)$ $A_0 + O(\rho^2)$

Kaus & HSR 0901



$\cdot A_0 = \frac{\quad}{2} \quad A = \frac{\quad}{2} \quad R = \frac{\quad}{\sqrt{2}} \sinh \frac{\sqrt{2}\quad}{l}$

bulk sol^n is $AdS_2 \times H^3$.

Israel eq: $\sinh \frac{\sqrt{2}\rho_0}{l} = 1 \qquad Q = l/\sqrt{2}$

$L_1 = \frac{Q}{\sqrt{2}} \qquad L_2 = Q.$

Small bare BH

$A_0 = l \Rightarrow$ bulk sol^n is AdS_5

spatial geom. of $\mathbb{R}^{1}$.

Indeed

Smoothne ish somewhere wlog @ $\rho = 0$.

$= \rho + O(\rho^3), \quad A(\rho) = A_0 + O(\rho^2)$

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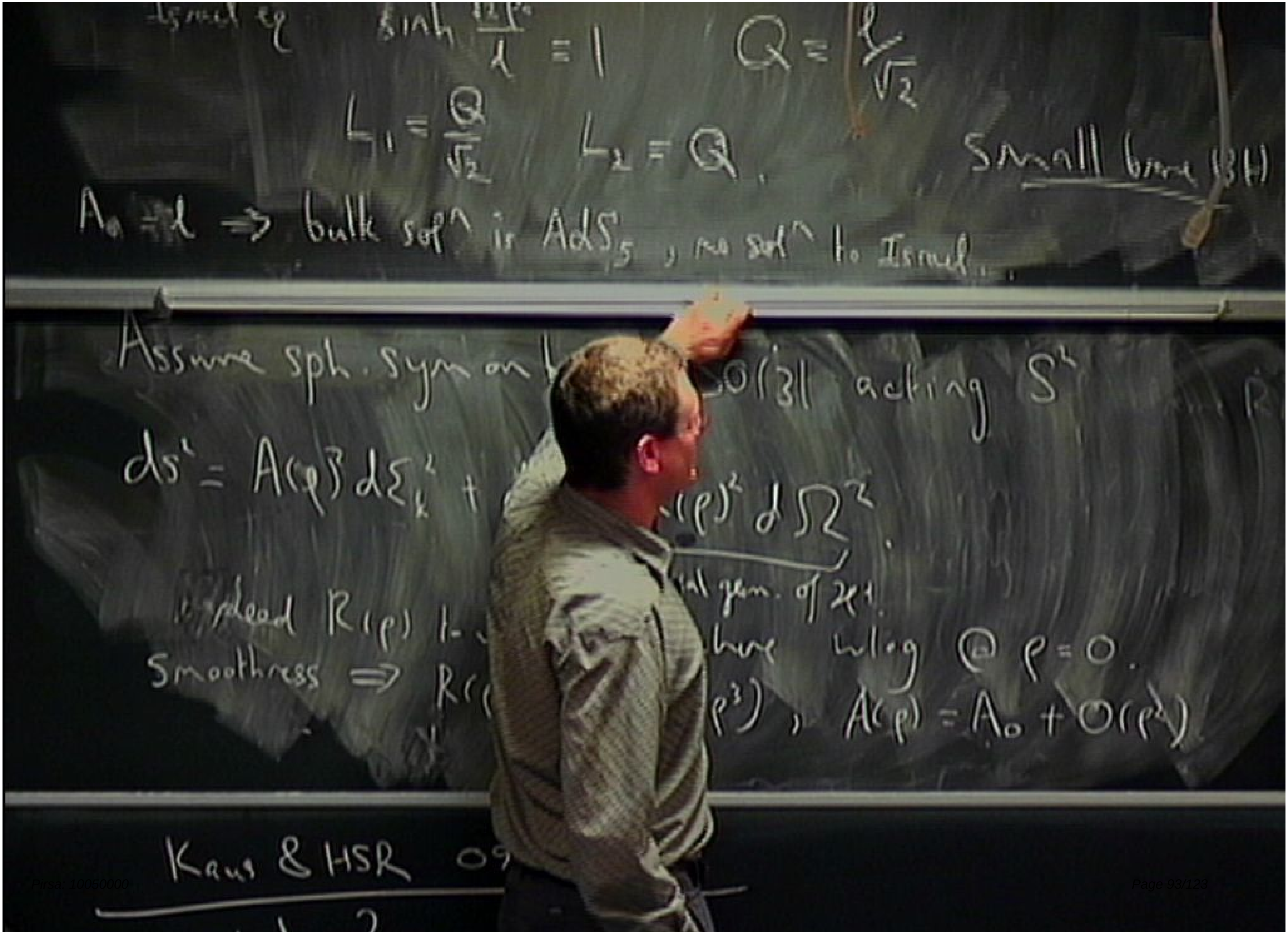

Israel eq $\sinh \frac{\sqrt{2}\rho_0}{\lambda} = 1$ $Q = \ell/\sqrt{2}$

$L_1 = \frac{Q}{\sqrt{2}}$ $L_2 = Q$.

Small brane BH

$A_0 \neq \ell \Rightarrow$ bulk sol^n is AdS_5, no sol^n to Israel.

Assume sph. sym on b… SO(3) acting S^2

$ds^2 = A(\rho)^2 d\Sigma_k^2 + \ldots (\rho)^2 d\Omega^2$.

…al geom. of k=1.

Indeed R(ρ) to … here wlog @ ρ = 0.

Smoothness ⇒ $R(\ldots \rho^3)$, $A(\rho) = A_0 + O(\rho^2)$

Kaus & HSR 09

[redacted]

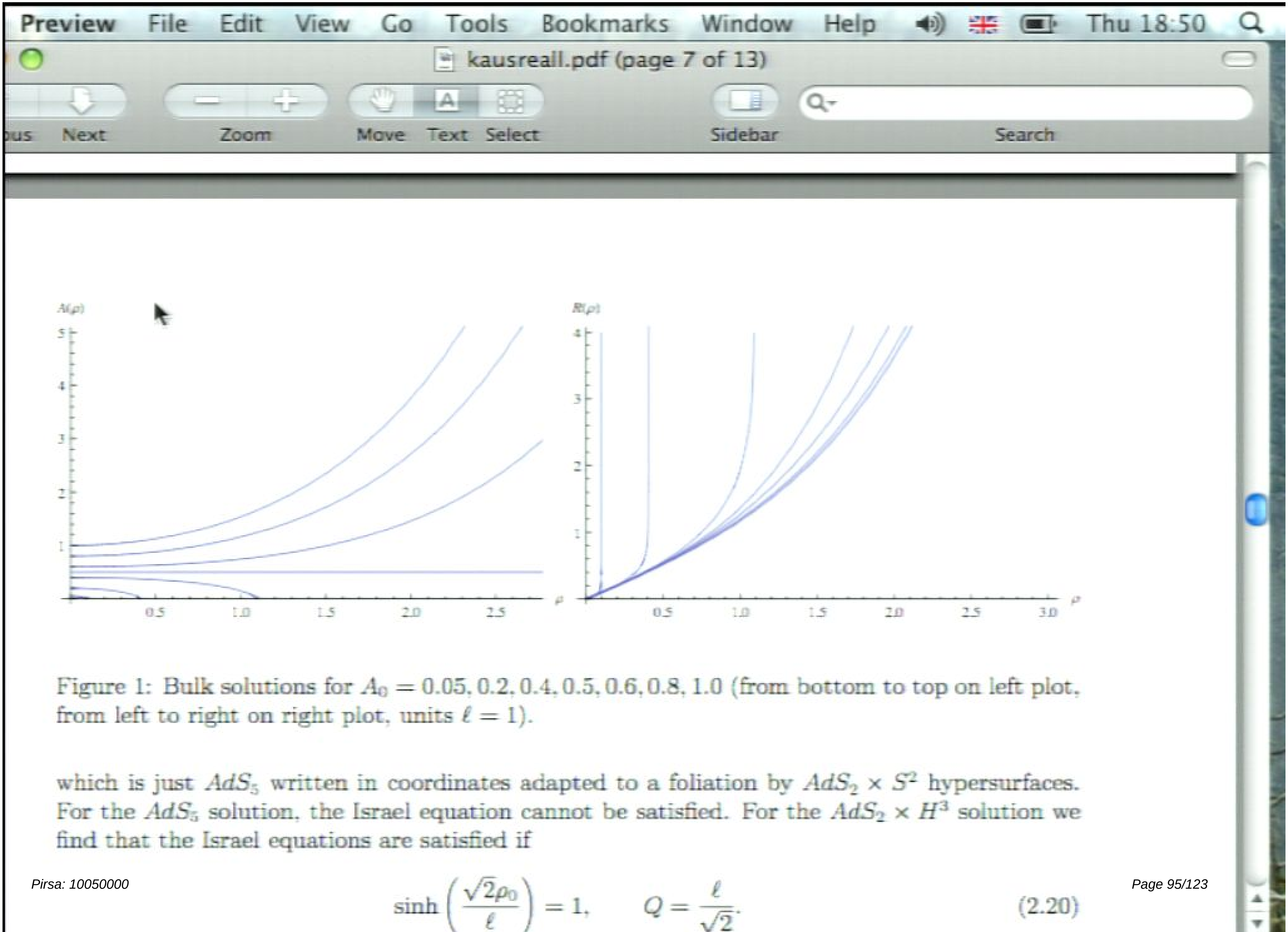

Figure 1: Bulk solutions for $A_0 = 0.05, 0.2, 0.4, 0.5, 0.6, 0.8, 1.0$ (from bottom to top on left plot, from left to right on right plot, units $\ell = 1$).

which is just AdS_5 written in coordinates adapted to a foliation by $AdS_2 \times S^2$ hypersurfaces. For the AdS_5 solution, the Israel equation cannot be satisfied. For the $AdS_2 \times H^3$ solution we find that the Israel equations are satisfied if

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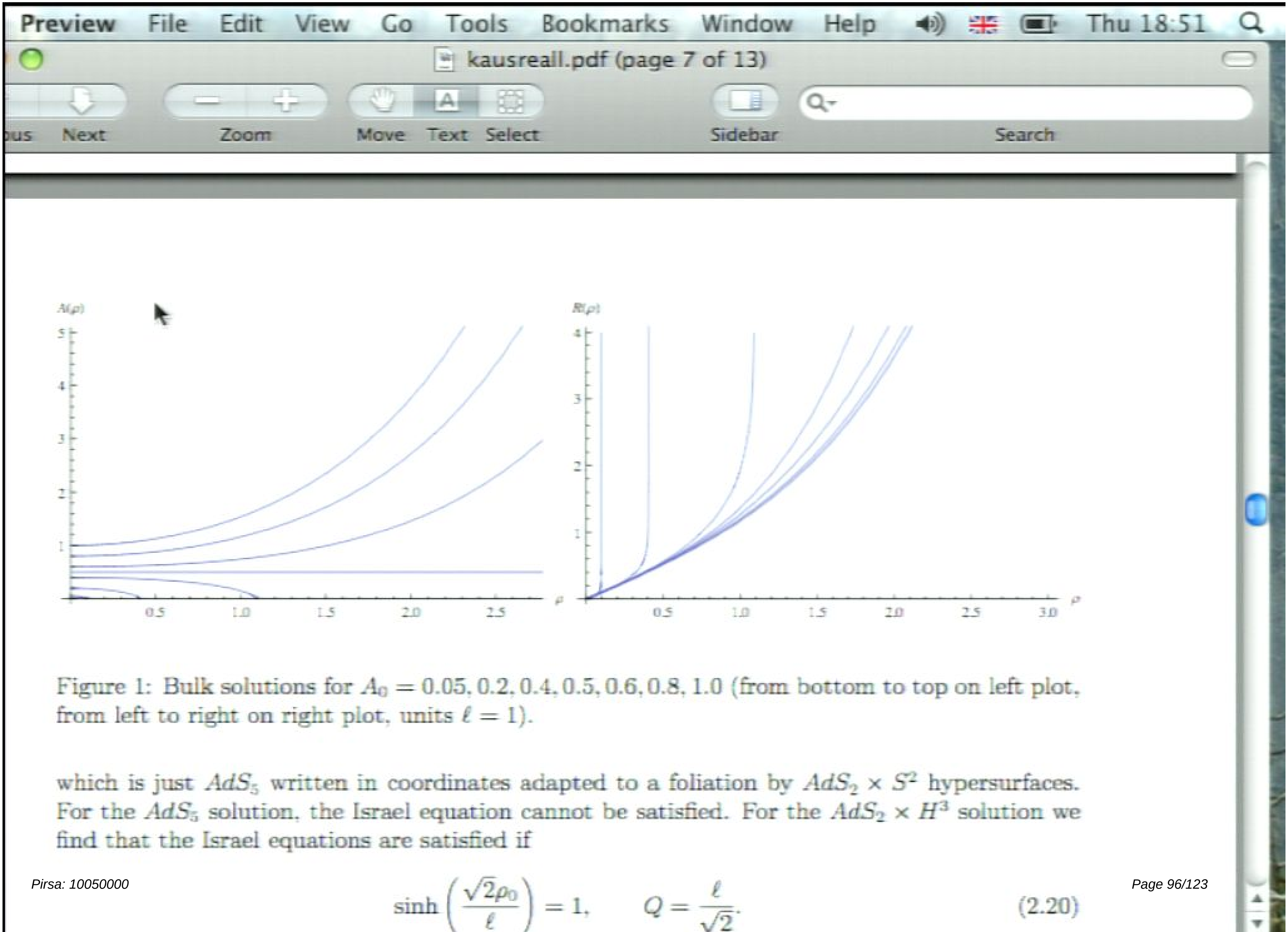

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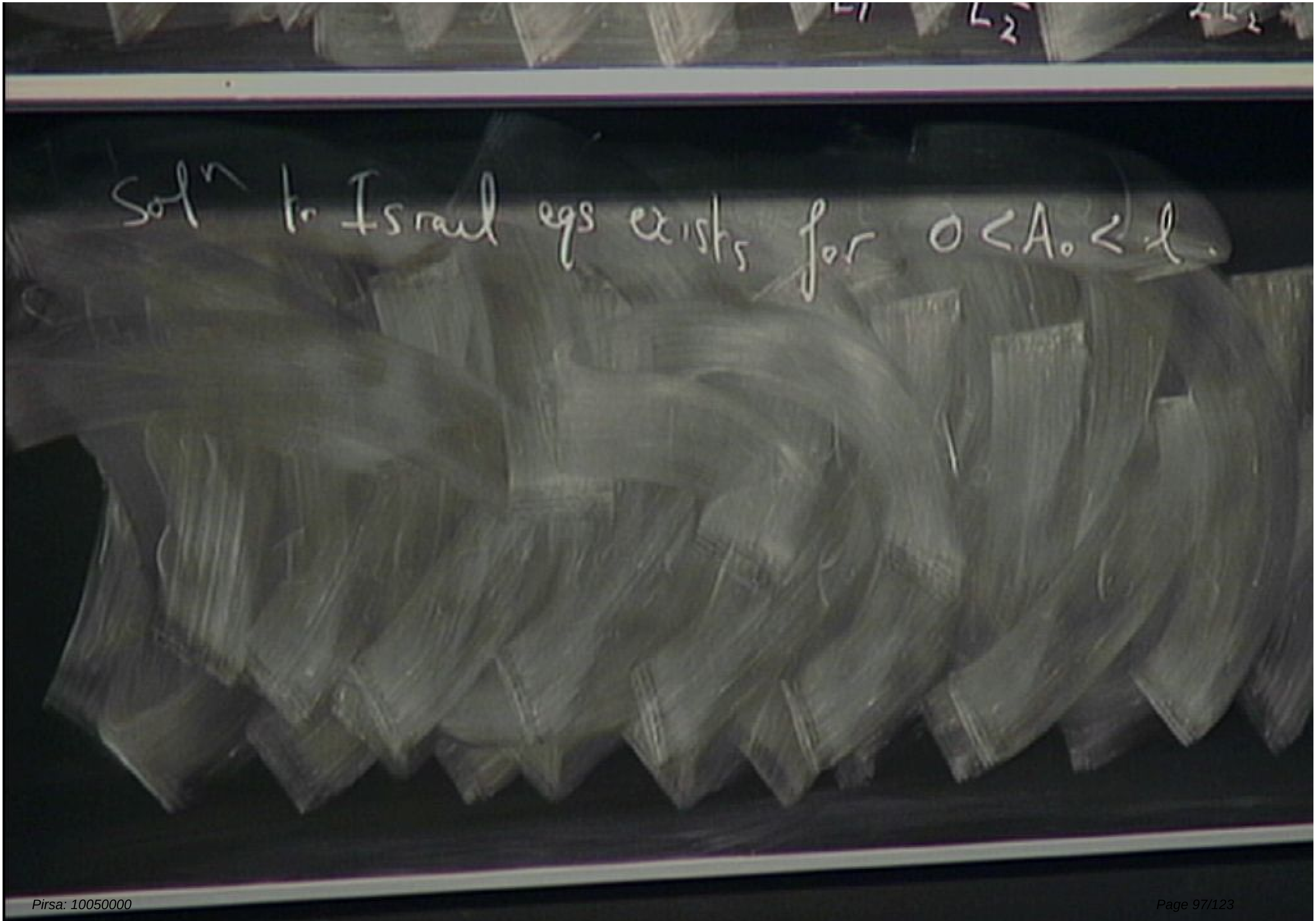
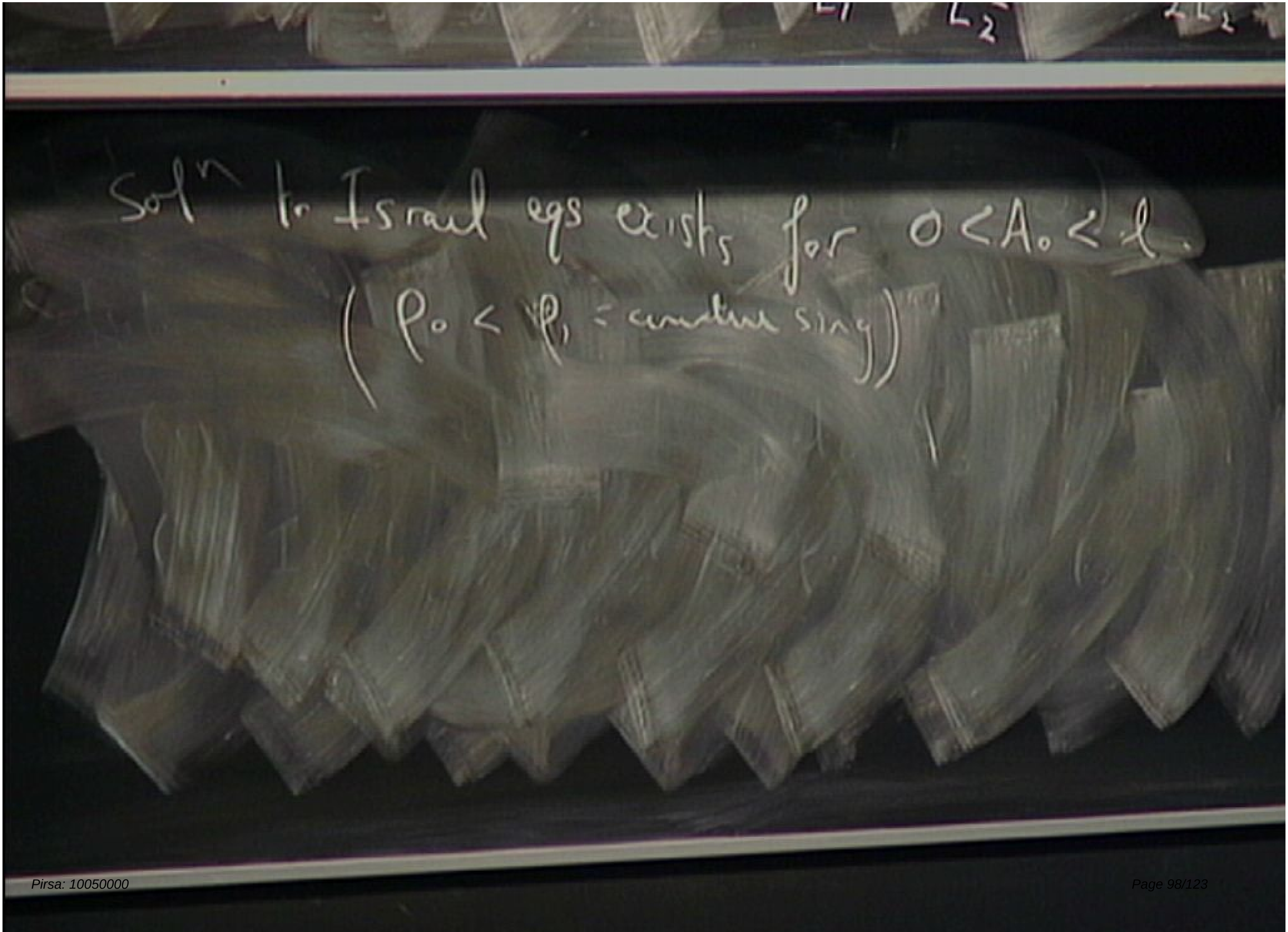


Sol^n to Israel eqs exists for $0 < A_0 < l$.



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($\rho_0 < \rho_1$ = curvature sing)

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treat Q as indept variable.

Einstein eq $\Rightarrow$ ODE's for $R(\rho)$, $A(\rho)$. Sol^n unique given k

Assume brane @ $\rho = \rho_0$ $\Rightarrow$ $$ds_4^2 = \underset{A(\rho_0)^2}{\underset{\parallel}{L_1^2}}\, d\Sigma_k^2 + \underset{R(\rho_0)^2}{\underset{\parallel}{L_2^2}}\, d\Omega^2$$

Assume Maxwell field on brane is $*_4 F = Q\, d\Omega$. charge

Israel eqs: must keep region $\rho \leq \rho_0$.

Israel + "Hamiltonian constraint" $\Rightarrow$ $$\frac{k}{L_1^2} + \frac{1}{L_2^2} = -\frac{\ell^2 Q^2}{2L_2^2}$$ $\Rightarrow k =$

$\Rightarrow AdS_2$

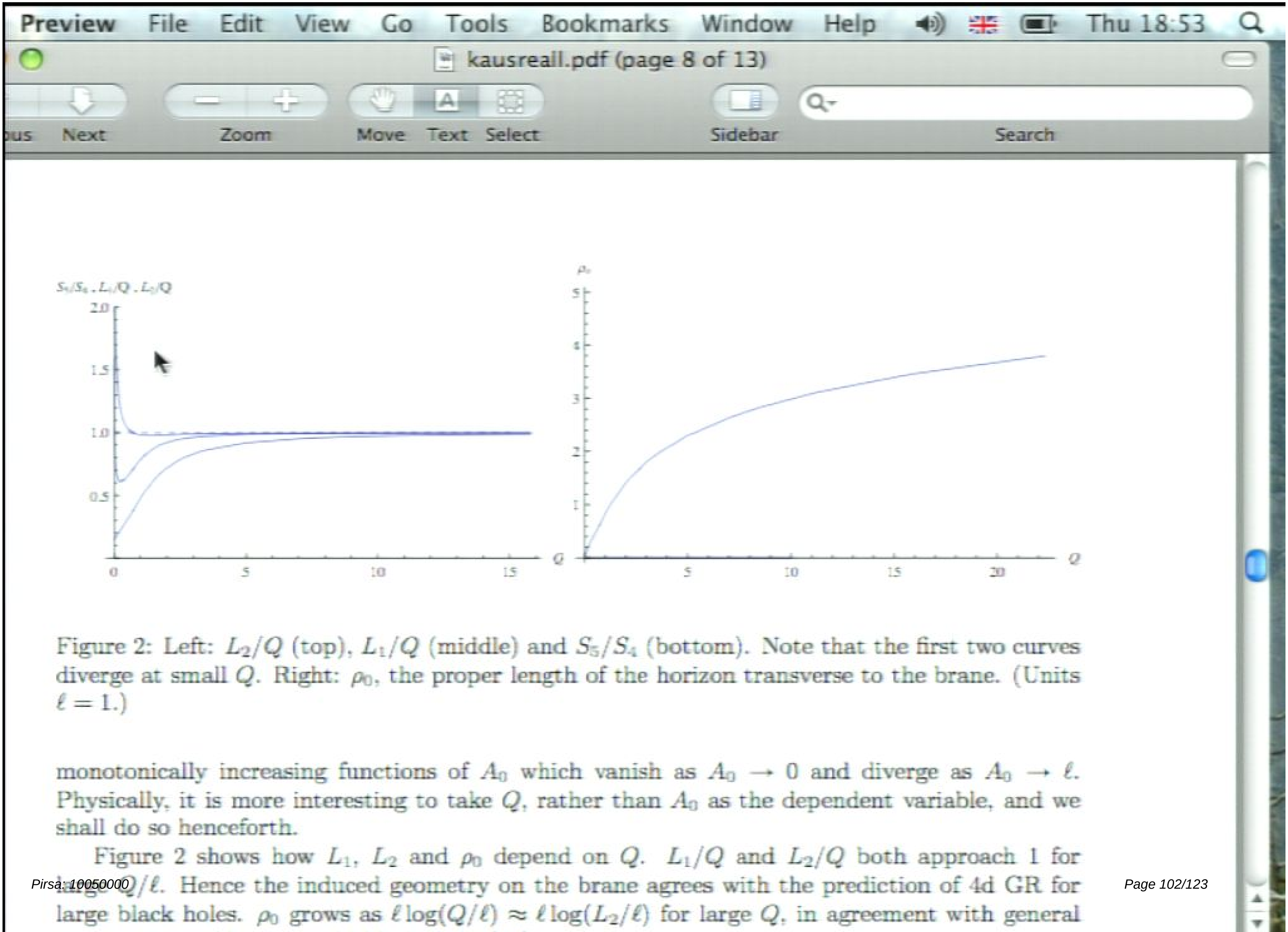
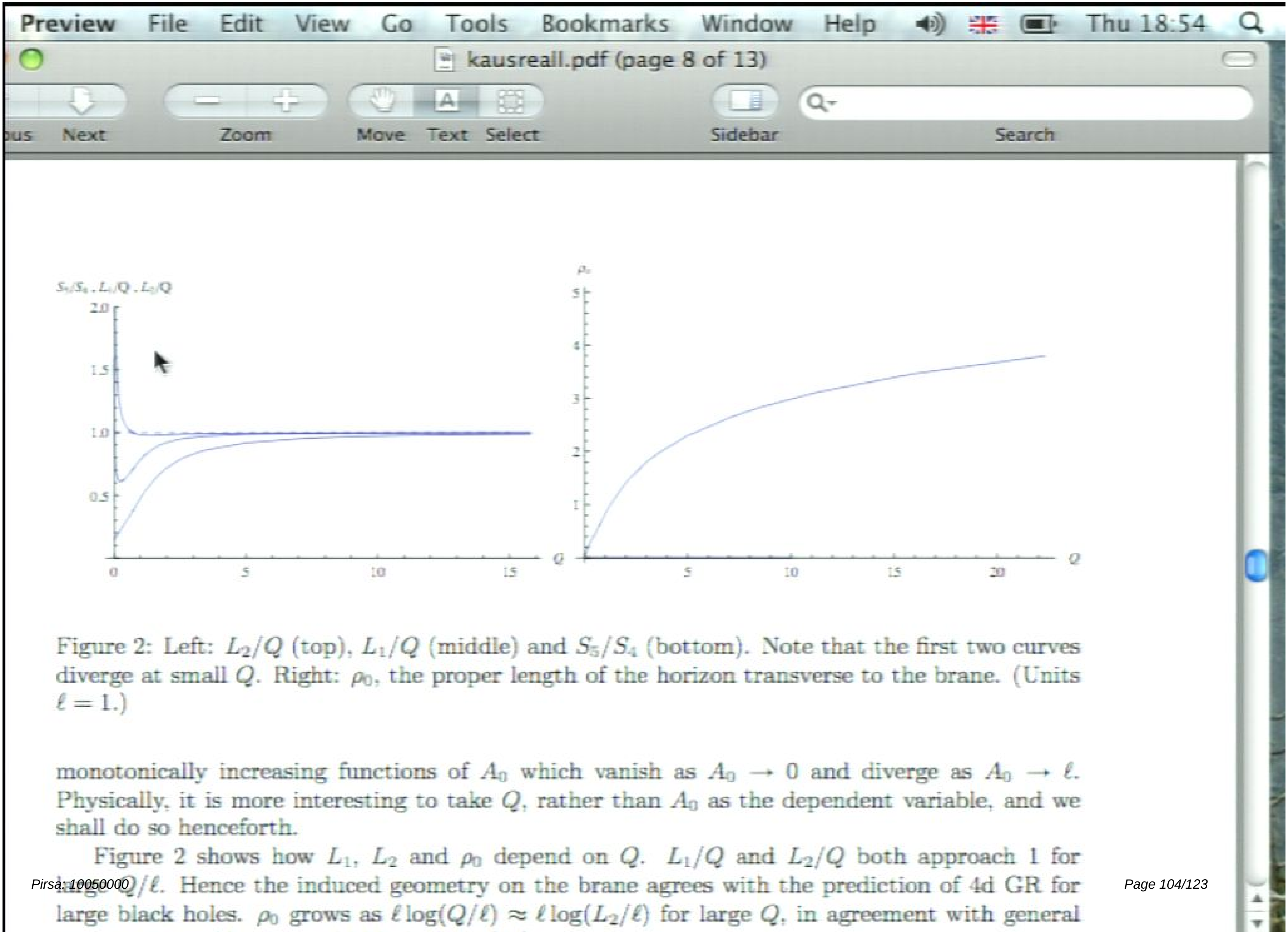


Figure 2: Left: L_2/Q (top), L_1/Q (middle) and S_5/S_4 (bottom). Note that the first two curves diverge at small Q. Right: ρ_0, the proper length of the horizon transverse to the brane. (Units $\ell = 1$.)

monotonically increasing functions of A_0 which vanish as $A_0 \to 0$ and diverge as $A_0 \to \ell$. Physically, it is more interesting to take Q, rather than A_0 as the dependent variable, and we shall do so henceforth.

Figure 2 shows how L_1, L_2 and ρ_0 depend on Q. L_1/Q and L_2/Q both approach 1 for large Q/ℓ. Hence the induced geometry on the brane agrees with the prediction of 4d GR for large black holes. ρ_0 grows as $\ell \log(Q/\ell) \approx \ell \log(L_2/\ell)$ for large Q, in agreement with general

$$ds^2 = A(\rho)^2\, d\,\text{[illegible]}^2(adS_2) + d\rho^2 + R(\rho)^2\, d\Omega^2 .$$



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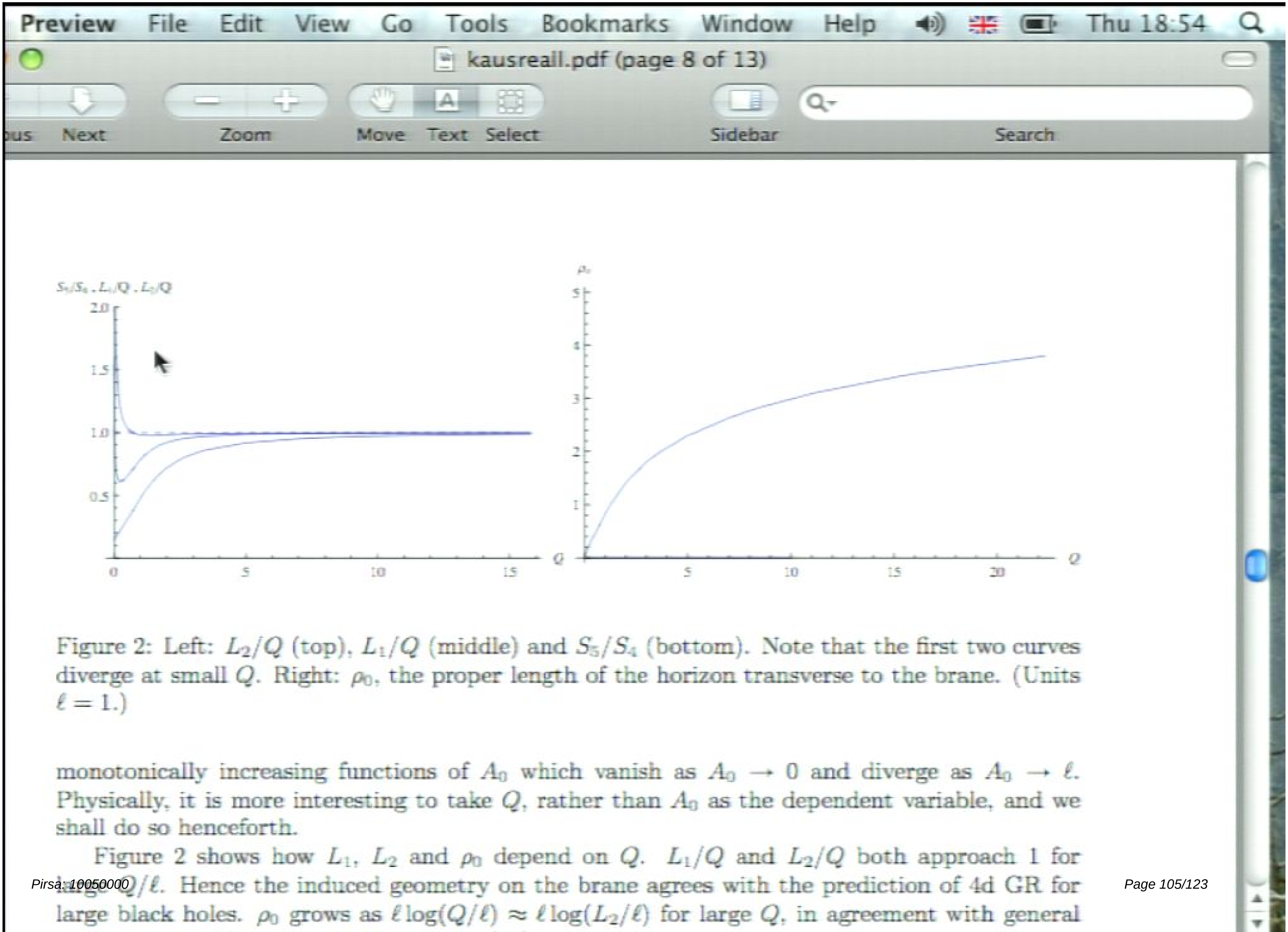

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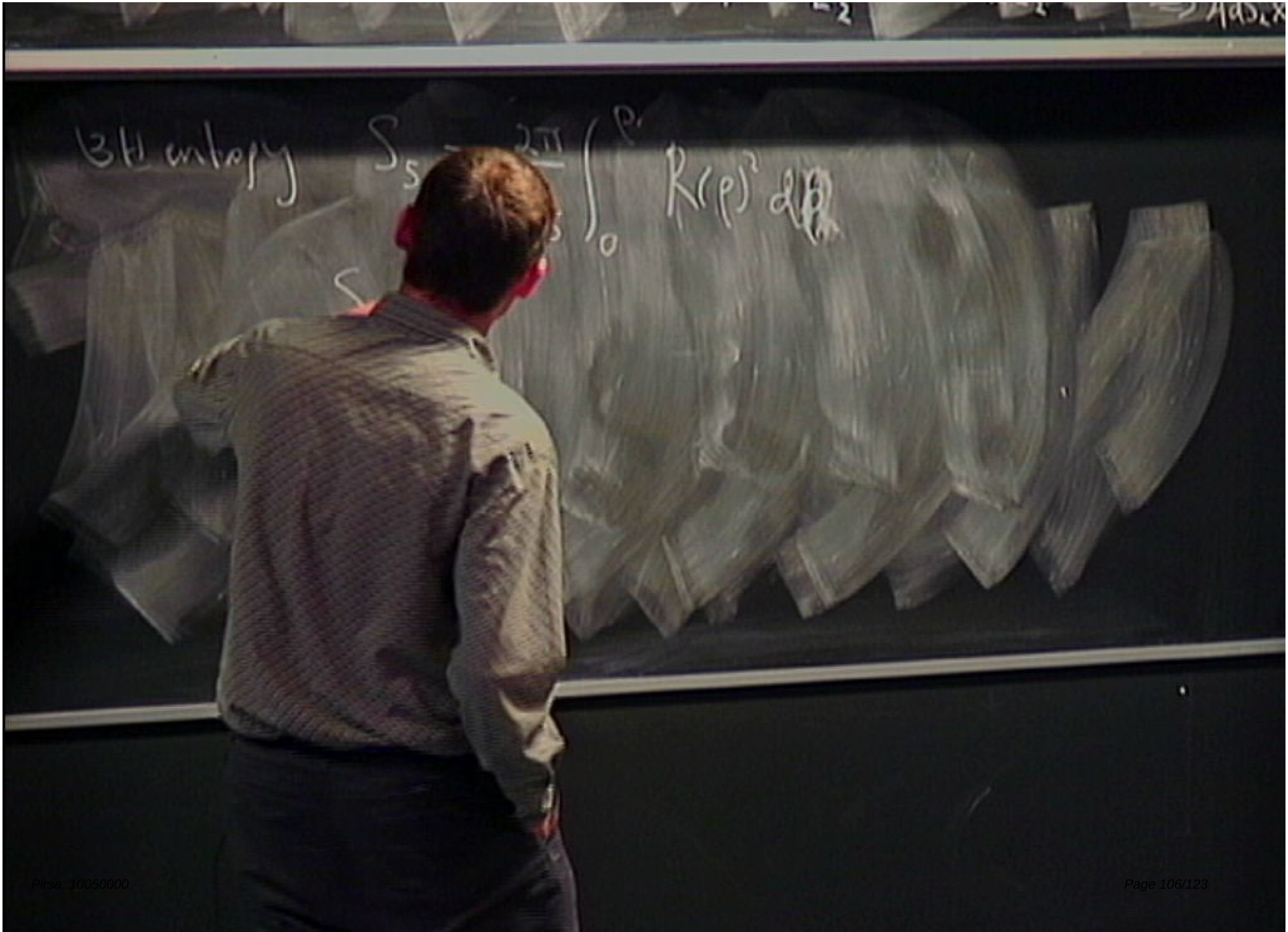
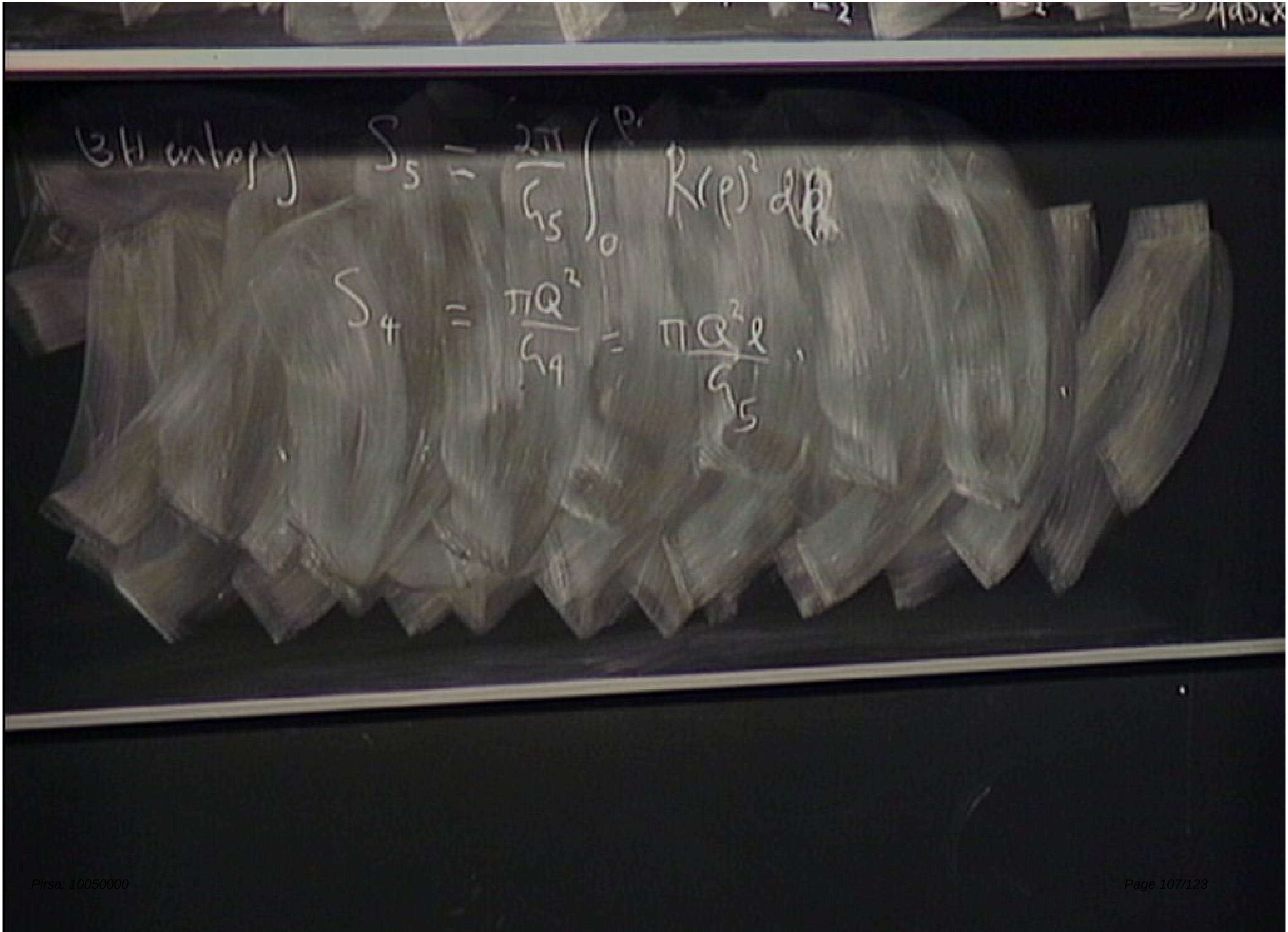

BH entropy $S_5 = \frac{2\pi}{G_5}\int_0^{\rho_+} R(\rho)^3 d\rho$
S



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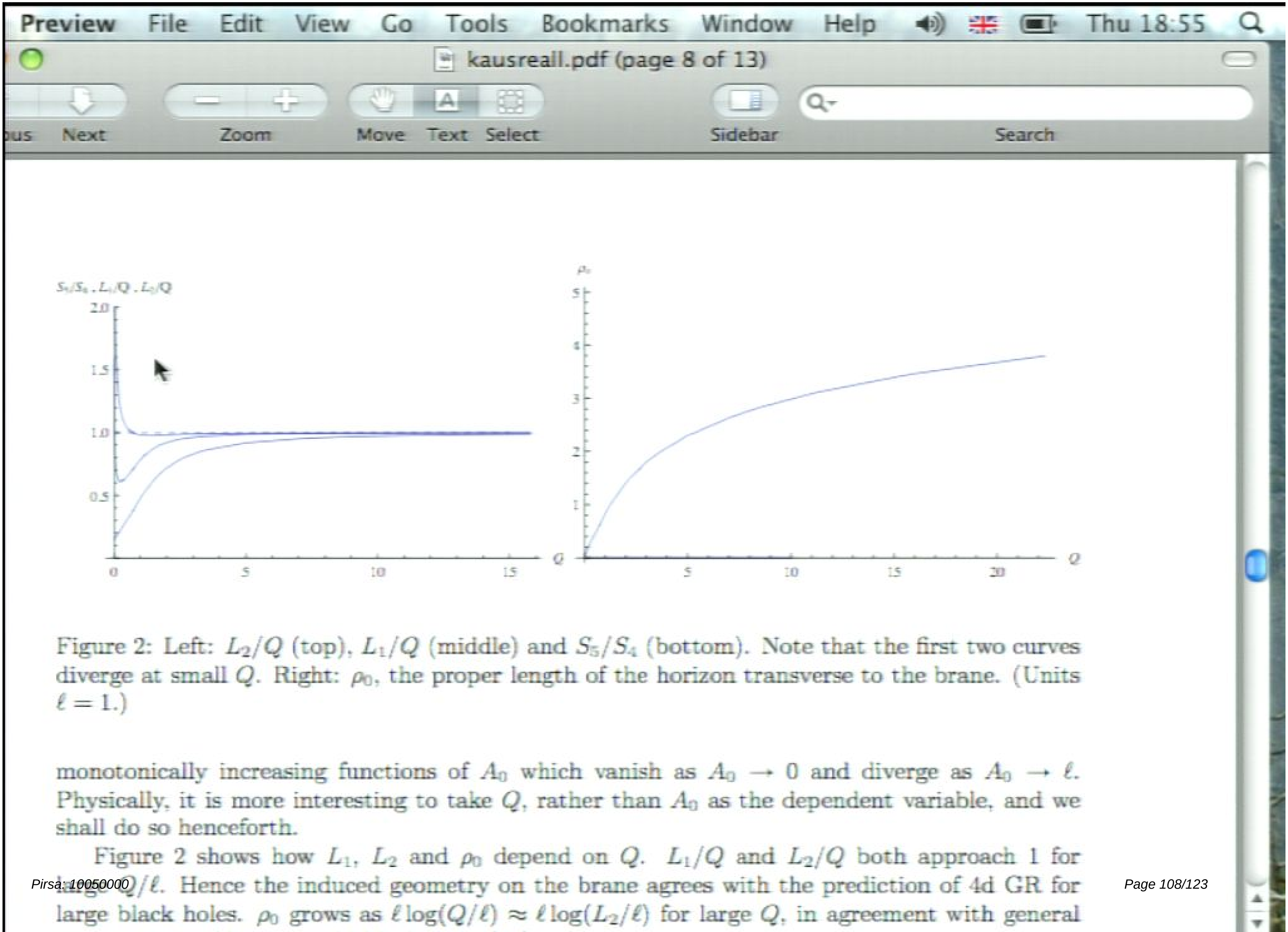

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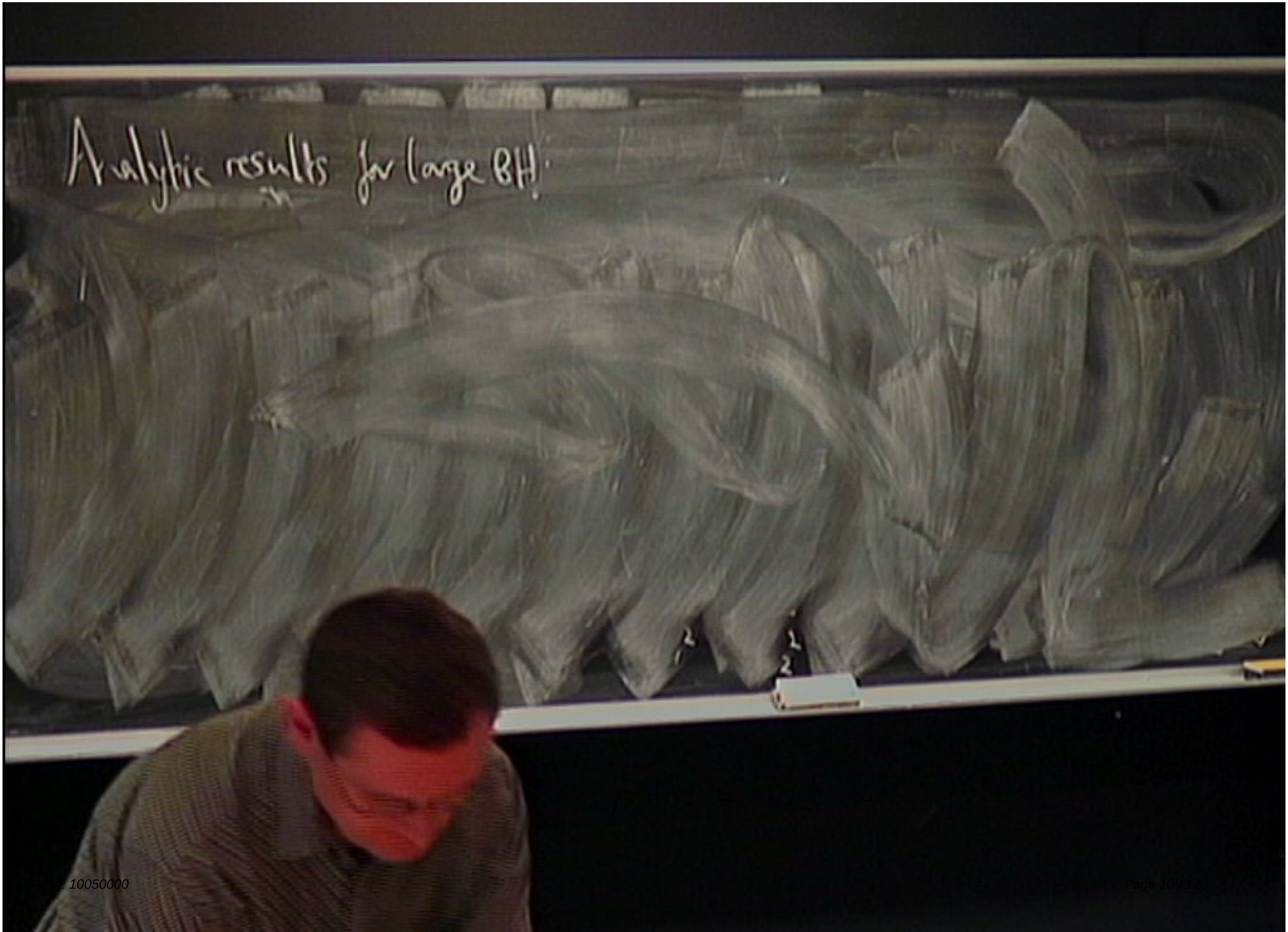
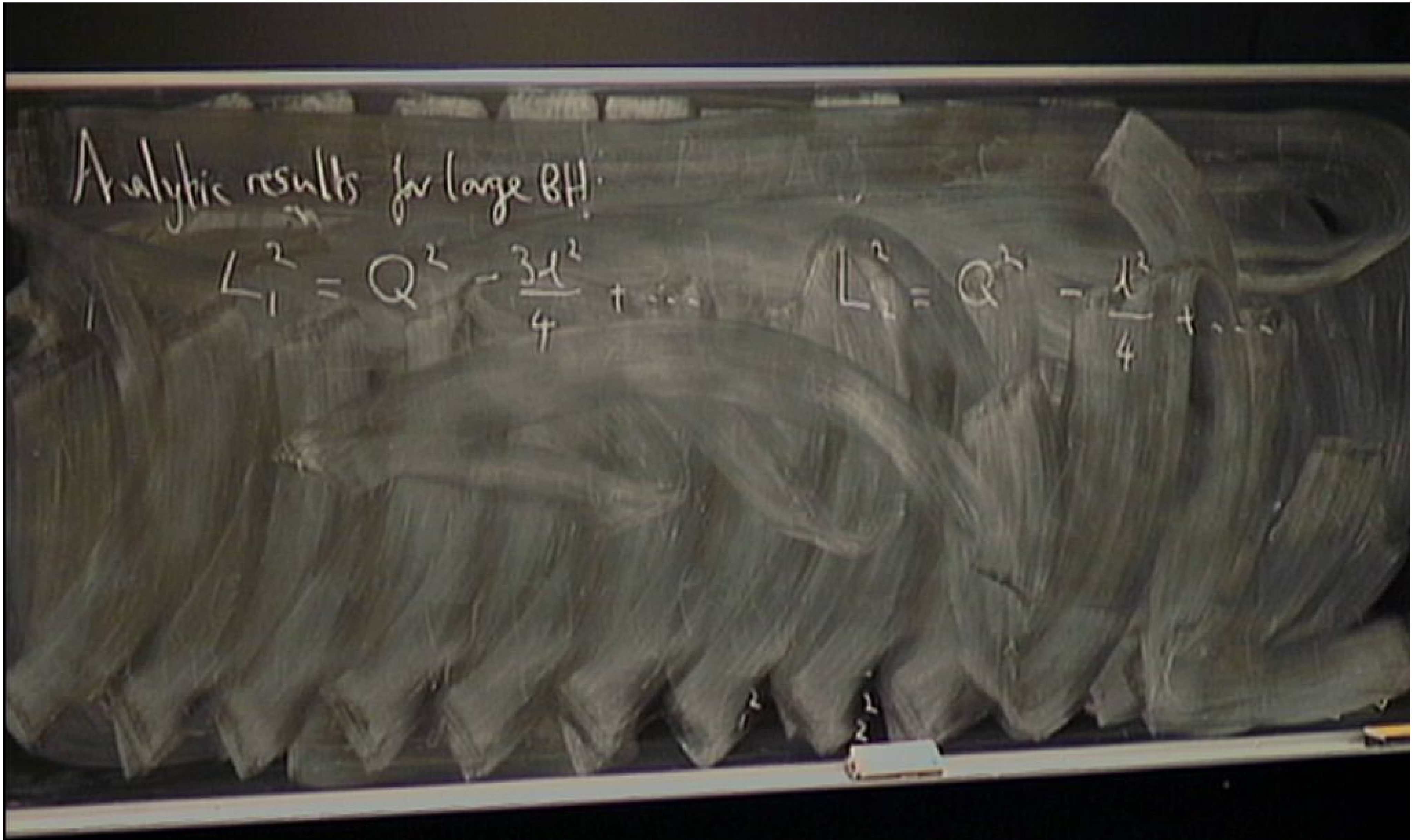


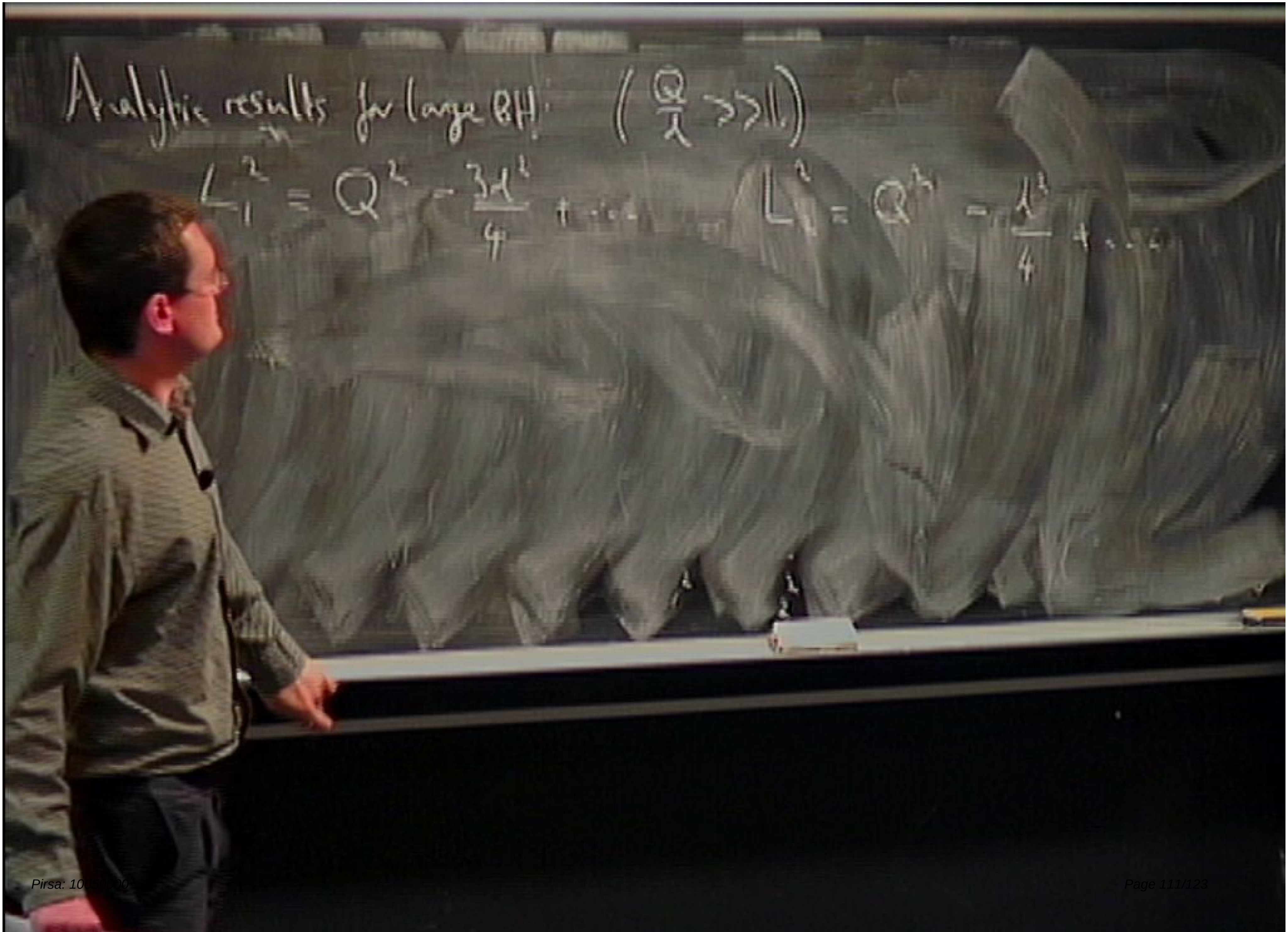
Analytic results for large BH:

in



Analytic results for large BH:

$$L_1^2 = Q^2 - \frac{3\lambda^2}{4} + \dots \qquad L_2^2 = Q^2 - \frac{\lambda^2}{4} + \dots$$



Analytic results for large BH: $\left(\frac{Q}{\lambda} \gg 1 \right)$

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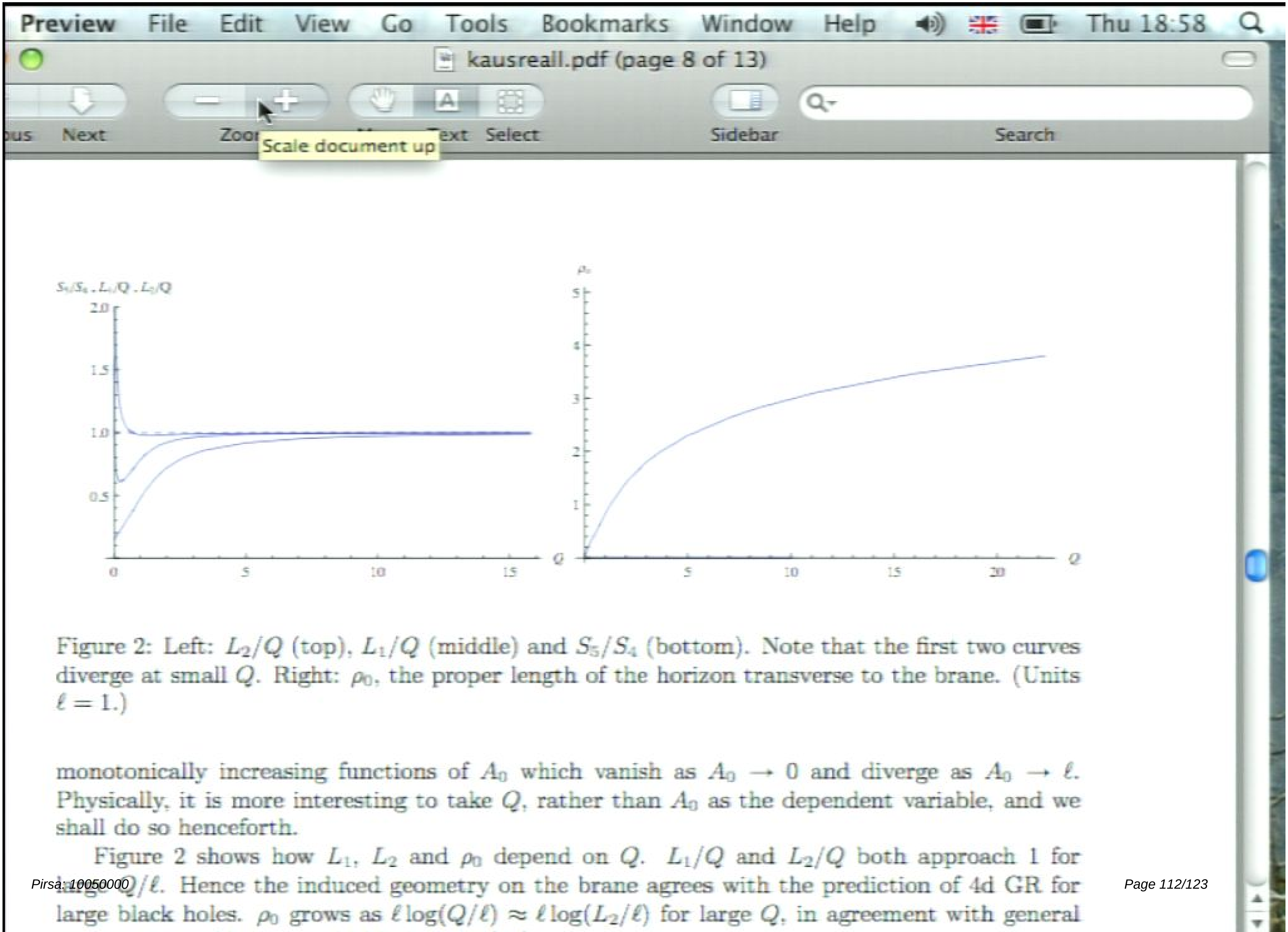

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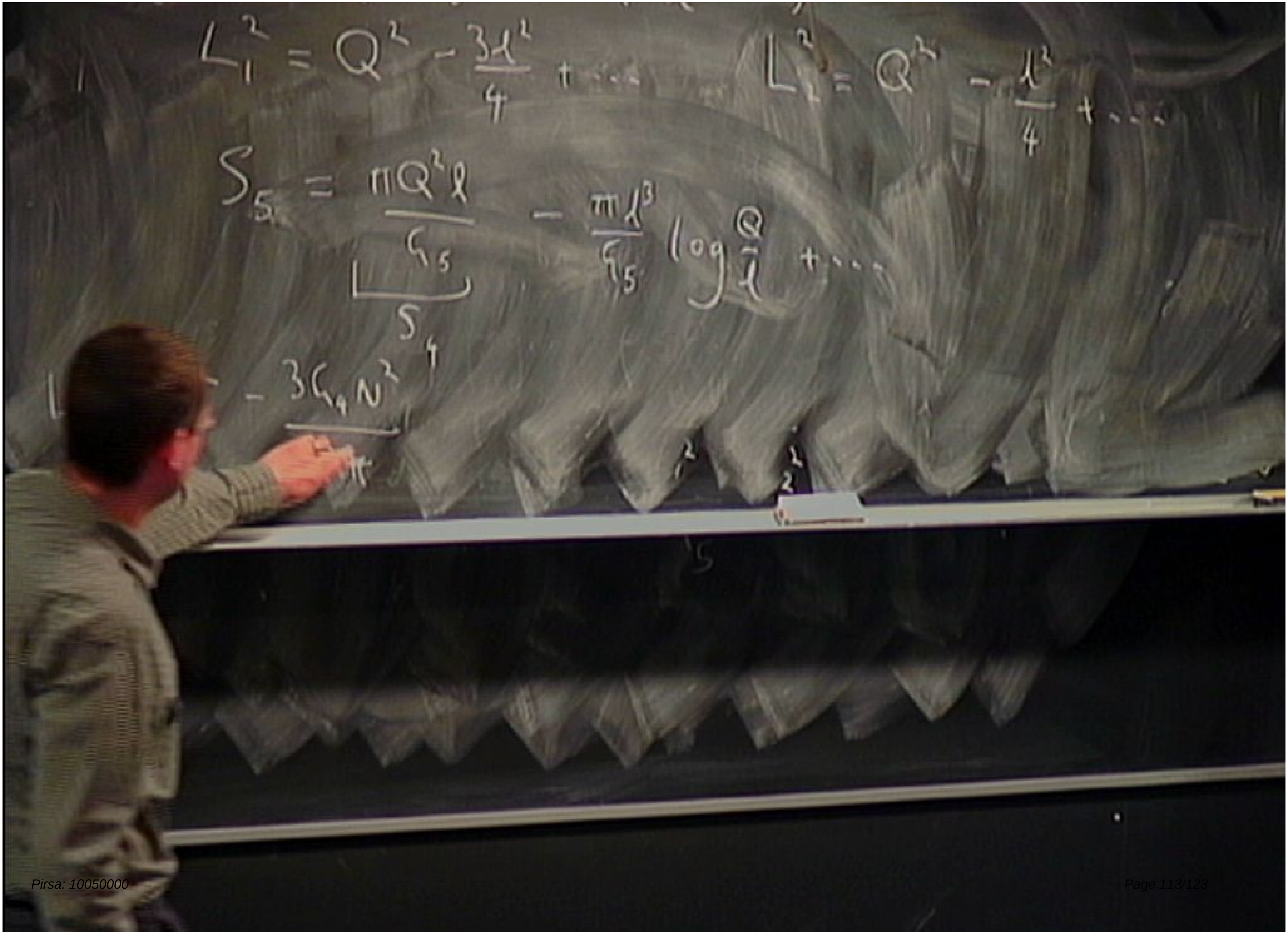
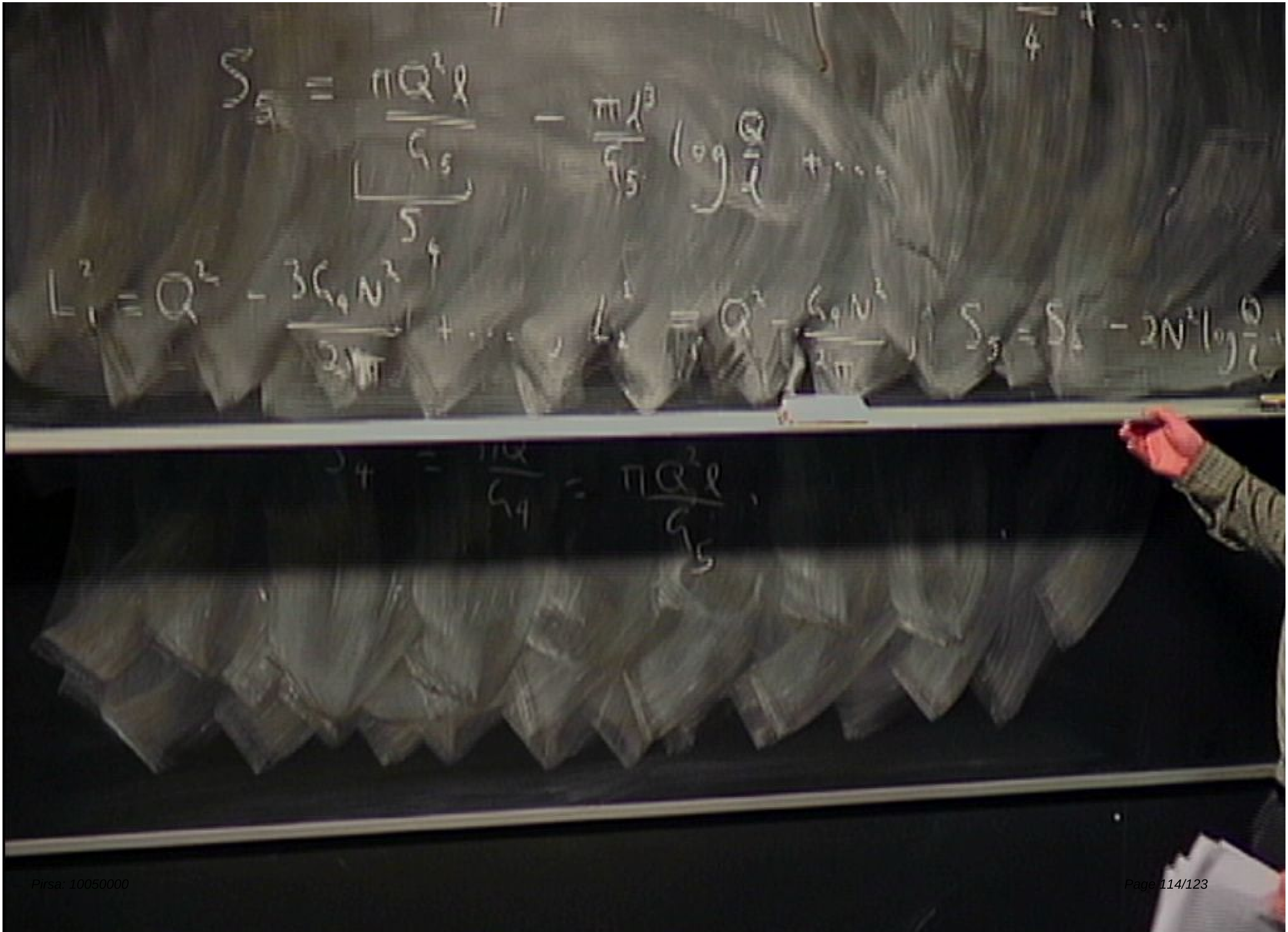


$$L_1^2 = Q^2 - \frac{3\ell^2}{4} + \dots \qquad L_2^2 = Q^2 - \frac{\ell^3}{4} + \dots$$

$$S_5 = \underbrace{\frac{\pi Q^2 \ell}{G_5}}_{S_4} - \frac{\pi \ell^3}{G_5} \log \frac{Q}{\ell} + \dots$$

$$L \quad - \frac{3 G_4 N^2}{\pi}$$



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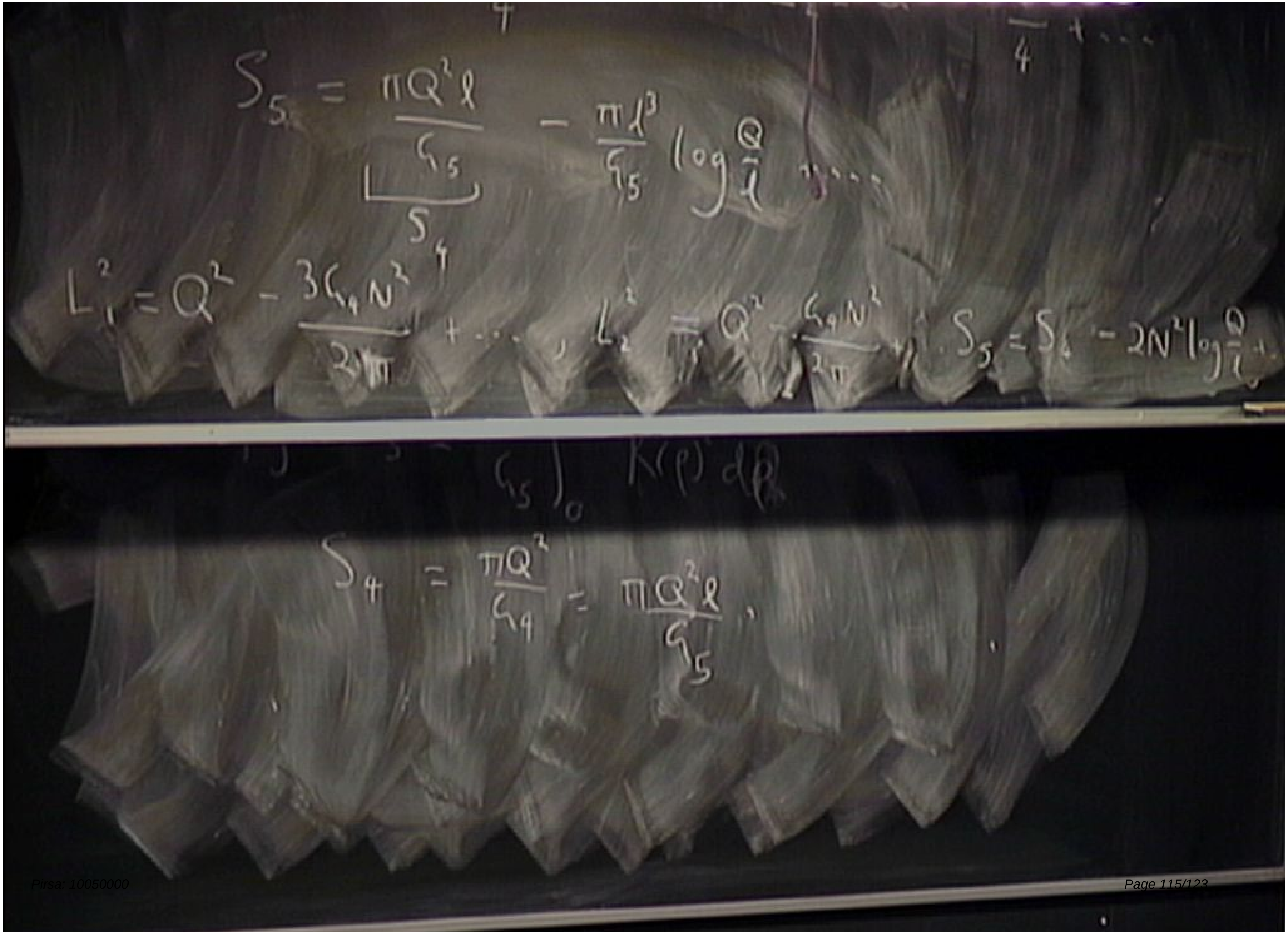
$$L_1^2 = Q^2 - \frac{3 G_4 N^2}{2\sqrt{\pi}} + \dots, \quad L_1^2 = Q^2 - \frac{G_4 N^2}{2\pi}, \quad S_5 = S_4 - 2N^2 \log\frac{Q}{\ell}$$

$$S_4 = \frac{\pi Q^2}{G_4} = \frac{\pi Q^2 \ell}{G_5},$$



4 $\qquad \frac{}{4} + \dots$

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$$G_5 \int_0 K(\rho)\, d\rho$$

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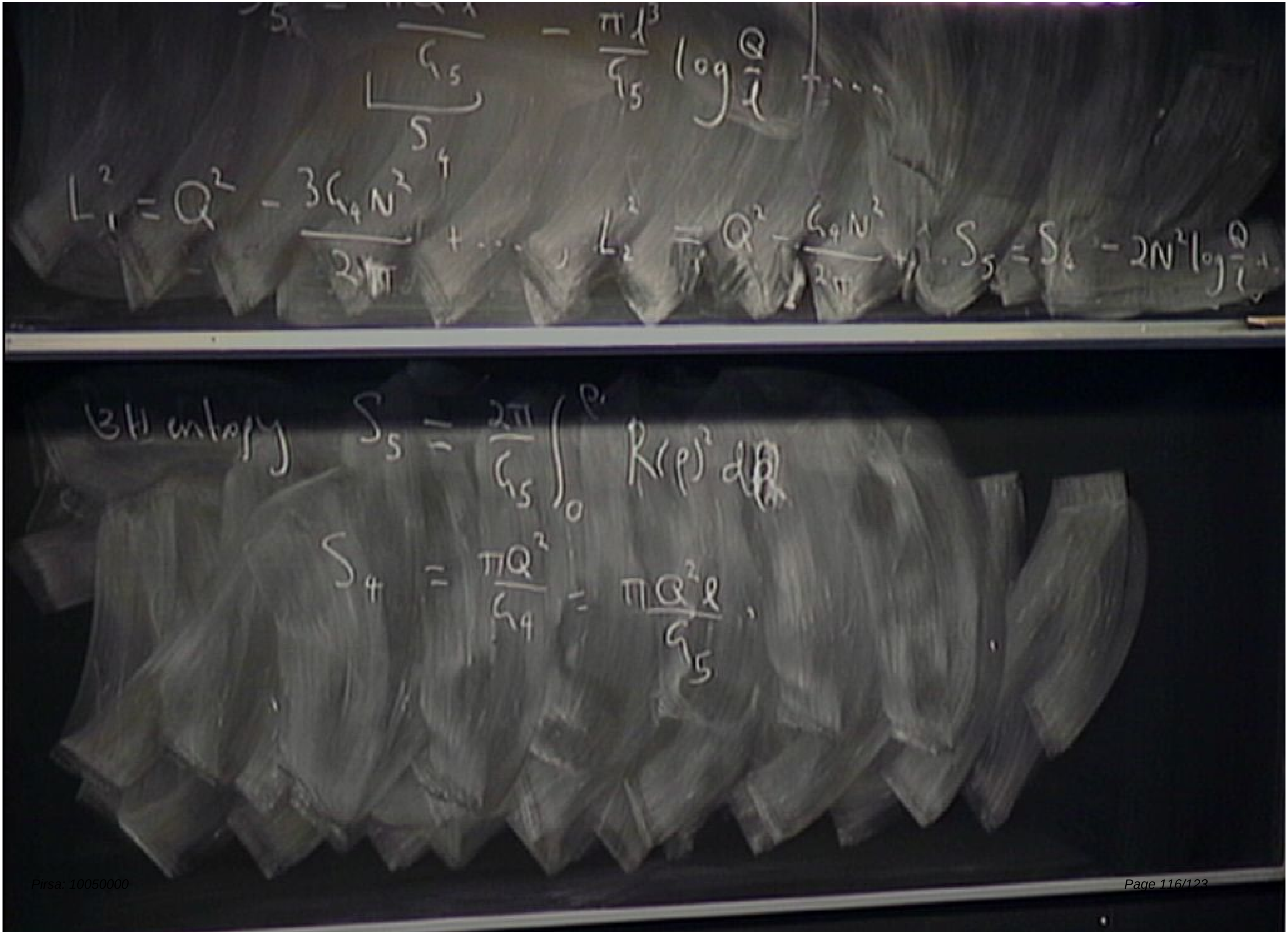

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BH entropy $S_5 = \frac{2\pi}{G_5}\int_0^{\rho'} R(\rho)^2 \, d\rho$

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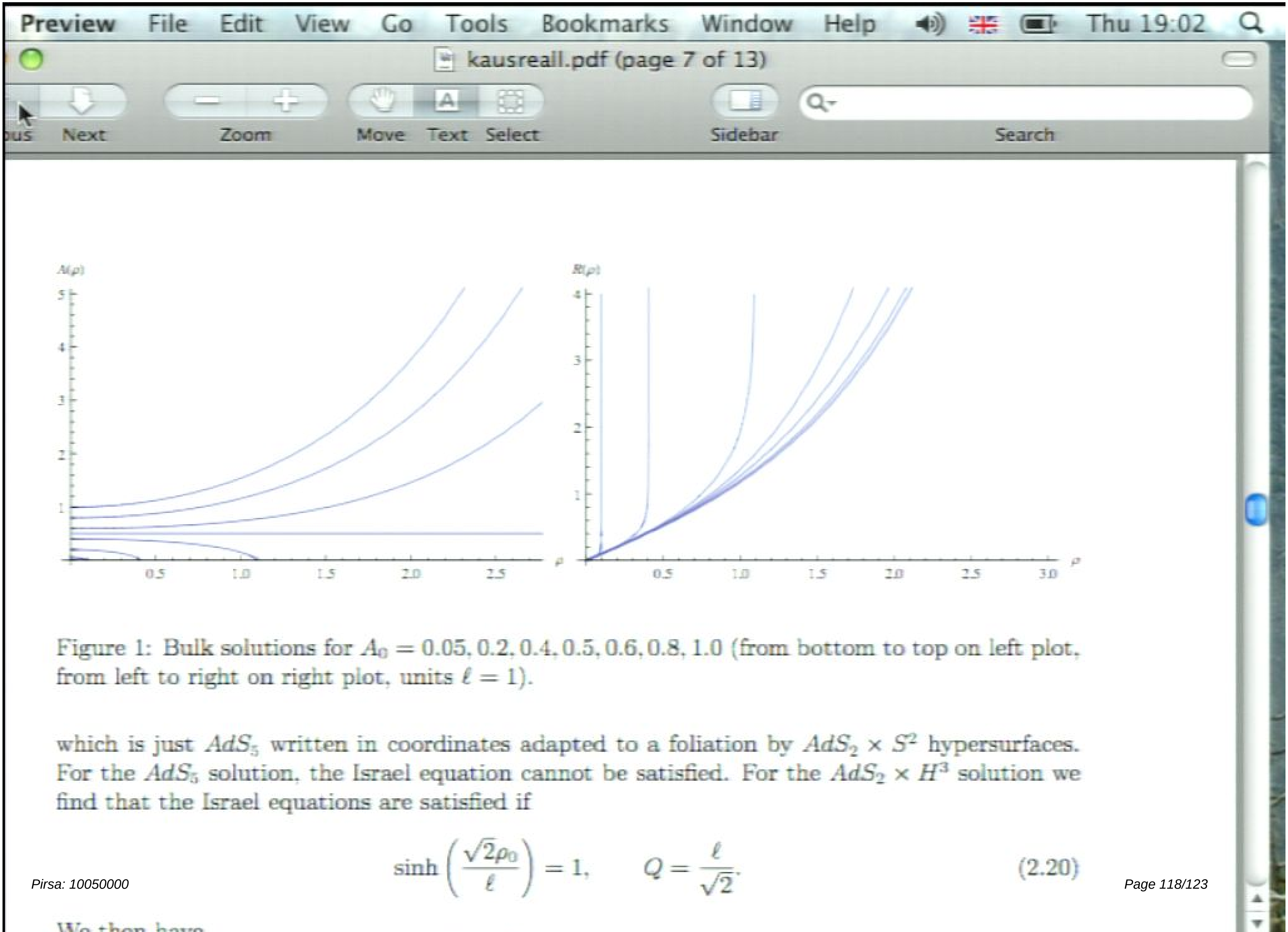
ignore brane

$A_0 > \frac{Q}{\ell}$



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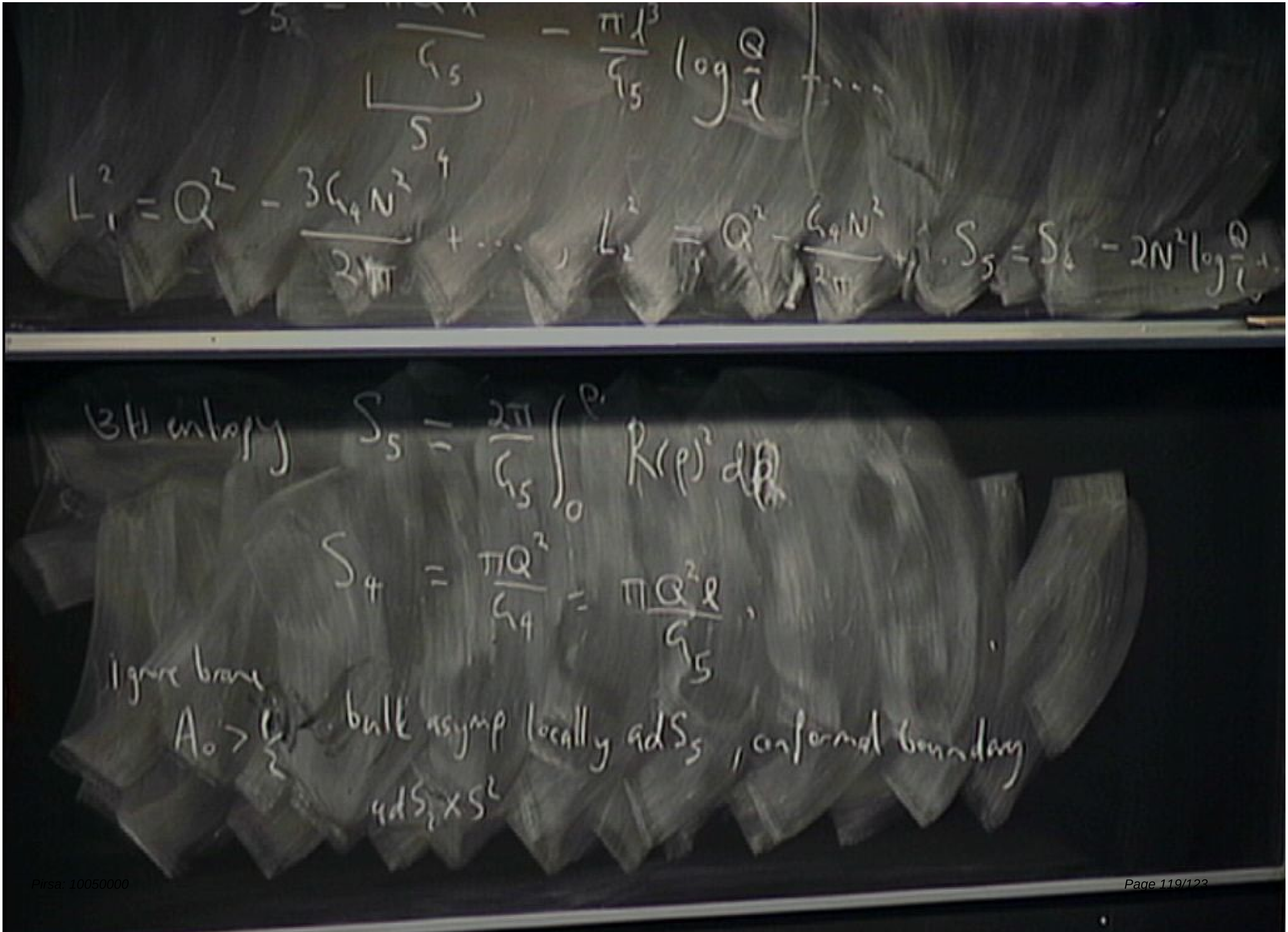
We then have



$$S_5 = \underbrace{\frac{\quad}{G_5}}_{S_4} - \frac{\pi \ell^3}{G_5}\log\frac{Q}{\ell} + \dots$$

$$L_1^2 = Q^2 - \frac{3G_4N^2}{2\pi} + \dots, \quad L_2^2 = Q^2 - \frac{G_4N^2}{2\pi} + \dots \quad S_5 = S_4 - 2N^2\log\frac{Q}{\ell} + \dots$$

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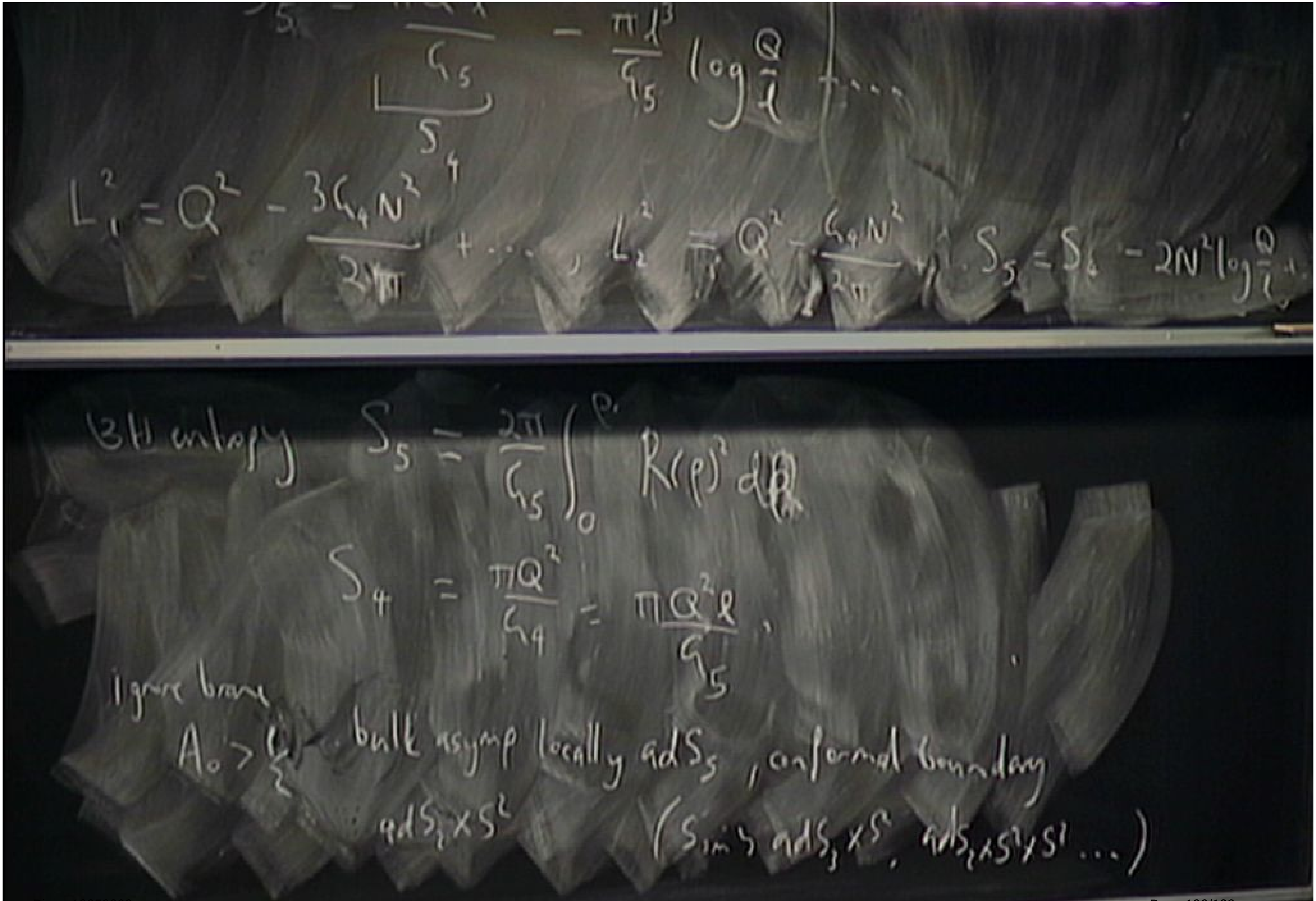
ignore brane

$A_0 > \ell$ – bulk asymp locally adS_5, conformal boundary $adS_2 \times S^2$



$$\underbrace{\frac{\quad}{G_5}}_{S_4} - \frac{\pi \ell^3}{G_5} \log\frac{Q}{\ell} + \dots$$

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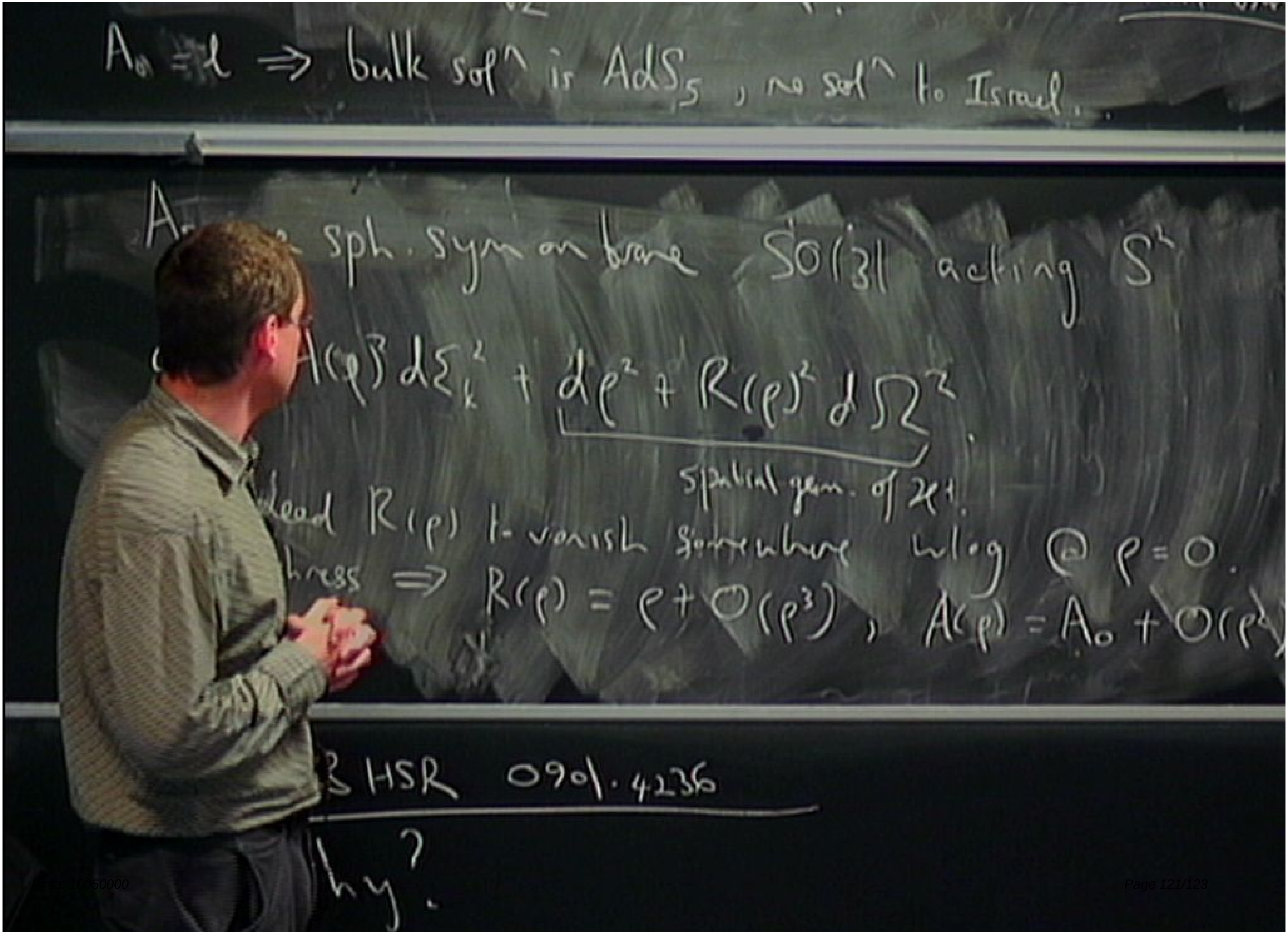
ignore brane

$A_0 > \{$ bulk asymp locally adS_5, conformal boundary

$adS_2 \times S^2$ (S_1's $adS_3 \times S^1$, $adS_2 \times S^1 \times S^1$ ...)



$A_0 = \ell \Rightarrow$ bulk sol^ is AdS_5, no sol^ to Israel.

A[illegible] sph. sym on brane $SO(3)$ acting S^2

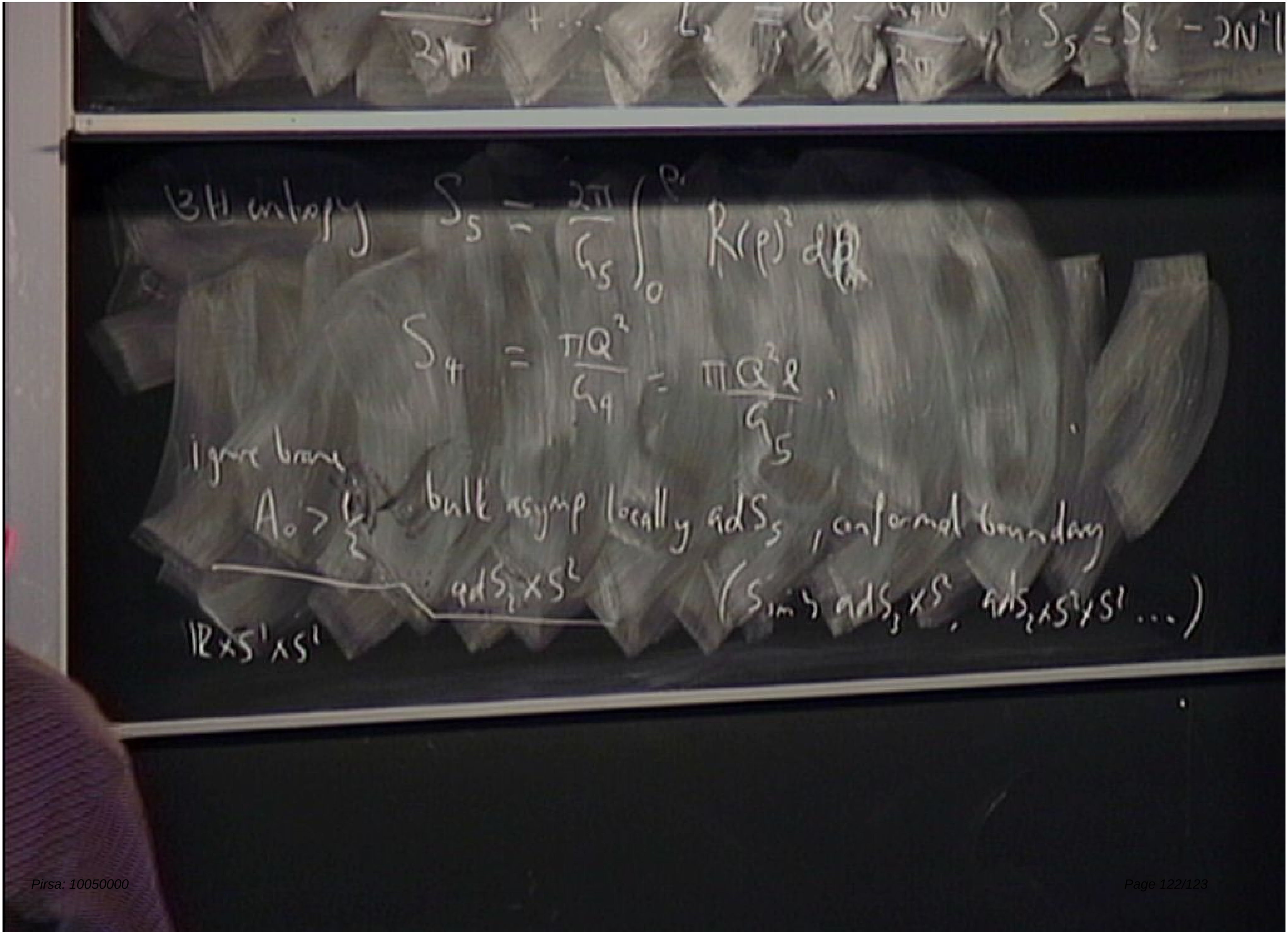
[illegible] $A(\rho)^2 d\Sigma_k^2 + \underbrace{d\rho^2 + R(\rho)^2 d\Omega^2}_{\text{spatial geom. of } 2+1}$.

[illegible]ead $R(\rho)$ to vanish somewhere wlog @ $\rho = 0$.

[illegible]hness $\Rightarrow$ $R(\rho) = \rho + O(\rho^3)$, $A(\rho) = A_0 + O(\rho^2)$

[illegible] HSR 0901.4236

[illegible]hy?



$\frac{}{2\sqrt{\pi}} + \dots$, L_2 … Q … $S_5 = S_6 - 2N^2$

BH entropy $S_5 = \frac{2\pi}{G_5}\int_0^{\rho_1} R(\rho)^2\, d\rho$

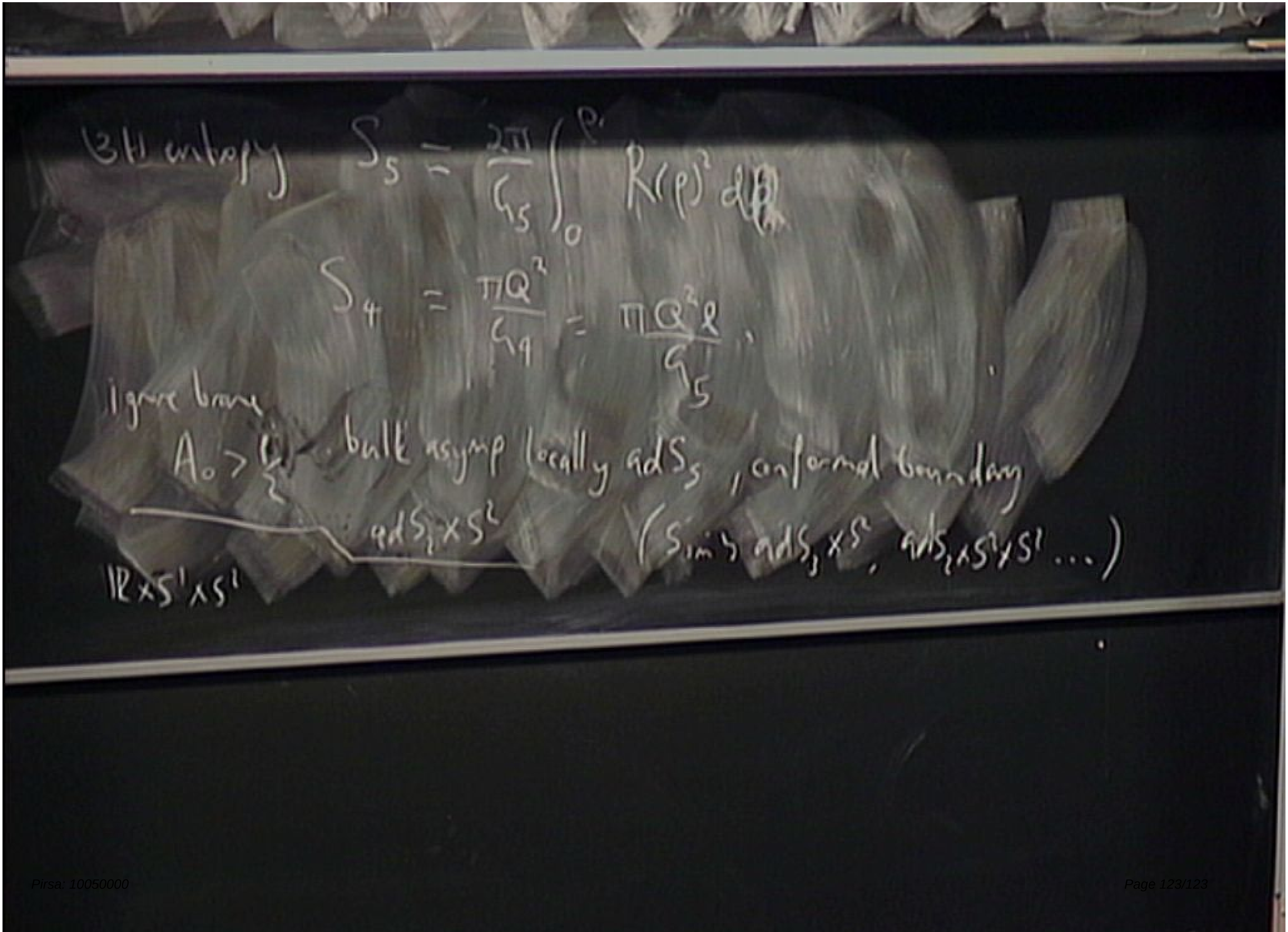
$$S_4 = \frac{\pi Q^2}{G_4} = \frac{\pi Q^2 \ell}{G_5} ,$$

ignore brane

$A_0 > \frac{1}{2}$ bulk asymp locally adS_5 , conformal boundary

$adS_2 \times S^2$ ($S_{\min}$'s $adS_3 \times S^2$, $adS_2 \times S^2 \times S^1$ ...)

$\mathbb{R} \times S^1 \times S^1$



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$A_0 > \ell$, bulk asymp locally adS_5 , conformal boundary

$adS_2 \times S^2$ (sim's $adS_3 \times S^2$, $adS_2 \times S^2 \times S^1$ ...)

$\mathbb{R} \times S^1 \times S^1$