

Title: Robustness and Violations of a Scalar Equivalence Principle

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Abstract: Modified gravity theories under consideration typically reduce to a scalar-tensor form in the appropriate limits.

I will discuss in what sense a universal scalar coupling is stable against quantum corrections, when the scalar equivalence principle is violated, how to look for such violations, and the connection with cosmic acceleration.

Robustness and Violations of a Scalar Equivalence Principle

Work with Alberto Nicolis, Chris Stubbs

Ubiquitousness of scalar-tensor theories:

e.g. $f(R)$, DGP, massive gravity, degravitation, ghost condensate, galileon, extrinsic curvature with auxillary dim., cucustom, TeVeS, STV, etc.

Probably due to Weinberg's theorem: at low energy, a Lorentz invariant theory of massless spin-2 particle must be GR.

Let's therefore consider (Einstein frame):

$$S = \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) + \alpha\varphi T_m \right] + \dots + h_{\mu\nu}T_m^{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

φ dimensionless, $M_P = 1$

α = scalar-matter coupling = $\mathcal{O}(1)$

Plan:

1. Is a universally coupled scalar stable against classical and quantum corrections? Answer: partial yes, for corrections in the matter sector, i.e. a scalar equivalence principle.
2. First caveat: equivalence principle can be violated if graviton self interactions are important, e.g. black holes (Nordvedt effect).
3. Second caveat: equivalence principle can be violated if the scalar self interactions are important (e.g. chameleon) unless protected by galileon/shift symmetry (e.g. DGP).
4. Observational consequences ($\mathcal{O}(1)$ violations) and some random comments.

Universally coupled scalar:

$$S = \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) + \alpha\varphi T_m \right] + \dots + h_{\mu\nu} T_m^{\mu\nu}$$

- Consider an object in the presence of a long wavelength external φ (i.e. ignoring tides).



$$S_{\text{int}} \sim -\alpha Q \int d\tau \varphi \quad \text{with scalar charge} \quad Q \sim - \int d^3x T_m \quad .$$

(ignore grav./scalar self-int.)

- Scalar charge to mass ratio = 1:

$$\begin{aligned} Q &= - \int d^3x T_m^0{}_0 - \int d^3x T_m^i{}_i \\ &= M - \int d^3x \partial_j (x^i T_m^j{}_i) \\ &= M \end{aligned}$$

↑
“grav.” mass ↗ inertial mass

use conservation & stationarity

- (Graviton + scalar) equivalence principle:

$$M \ddot{\vec{X}} = -M \vec{\nabla} \Phi_{\text{ext.}} - \alpha M \vec{\nabla} \varphi_{\text{ext.}}$$

Different objects fall at the same rate,
no matter what internal structure i.e.
geodesic motion in Jordan frame

- The above derivation works quantum mechanically as well:

$$Q = -\langle \Psi | \int d^3x T_m{}^\mu{}_\mu | \Psi \rangle = -\langle \Psi | \int d^3x T_m{}^0{}_0 | \Psi \rangle = M \quad \text{e.g. proton}$$

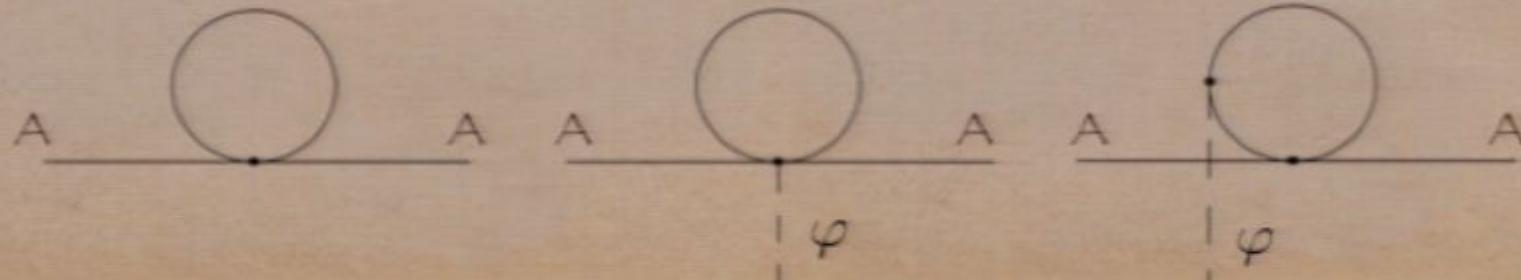
- This makes clear why the universal scalar coupling appears robust against quantum corrections in explicit calculations (c.f. Fujii).

e.g. $S = \int d^4x -\frac{1}{2}(\partial\psi_A)^2 - \frac{1}{2}m_A^2\psi_A^2 - \frac{1}{2}(\partial\psi_B)^2 - \frac{1}{2}m_B^2\psi_B^2 - \sum_{i,j} \lambda_{ij}\psi_i^2\psi_j^2$

$$+ \alpha\varphi \left[-(\partial\psi_A)^2 - 2m_A^2\psi_A^2 - (\partial\psi_B)^2 - 2m_B^2\psi_B^2 - 4\sum_{i,j} \lambda_{ij}\psi_i^2\psi_j^2 \right]$$

renormalize this m_A

renormalize this m_A



Caveat 1: graviton self-interactions

- True inertial mass is $M = - \int d^3x \tau^0_0 \neq - \int d^3x T_m{}^0_0 = Q$.
 τ^{μ}_{ν} from matter + graviton + scalar

Best example is a black hole, whose mass comes entirely from gravitational binding energy.

Its $Q/M = 0$ (no hair theorem).

- Therefore, a black hole and a regular star will fall at rates that are $O(1)$ different (Nordvedt effect).
More generally: $Q/M \sim 1 - v^2/c^2$.

Caveat 2: scalar self-interactions

$$S = \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) + \alpha\varphi T_m \right] + \dots + h_{\mu\nu}T_m^{\mu\nu}$$

- One might think objects simply move on geodesics in Jordan frame, where matter is minimally coupled to metric i.e.

$$(\alpha\varphi\eta_{\mu\nu} + h_{\mu\nu})T_m^{\mu\nu} \rightarrow \tilde{h}_{\mu\nu}T_m^{\mu\nu}$$

↑ ↑
Einstein Jordan

- The issue is that the scalar charge is renormalized by the scalar self-interaction:

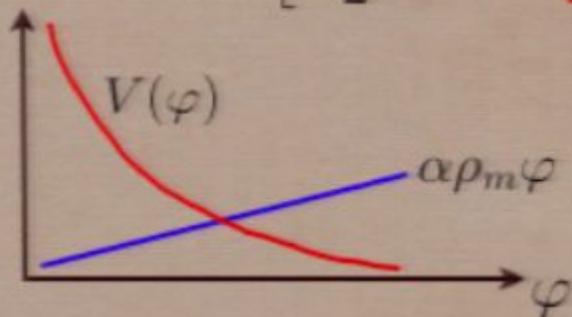
$$Q = - \int d^3x \left[T_m^0{}_0 + \frac{1}{\alpha} \frac{\partial \mathcal{L}_{\text{int}}}{\partial \varphi} \right]$$

- Scalar self-interaction is useful for screening out the scalar on small scales, which is mandatory for solar system tests.

Chameleon screening - environment dependent mass

Khoury & Weltman

$$S_{\text{scalar}} \sim \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 - \underline{V(\varphi)} + \alpha\varphi T_m{}^\mu{}_\mu \right] \quad (\text{Einstein frame})$$



e.o.m.:

$$\square\varphi \sim [V + \alpha\rho_m\varphi]_{,\varphi} \quad (T_m{}^\mu{}_\mu \sim -\rho_m)$$

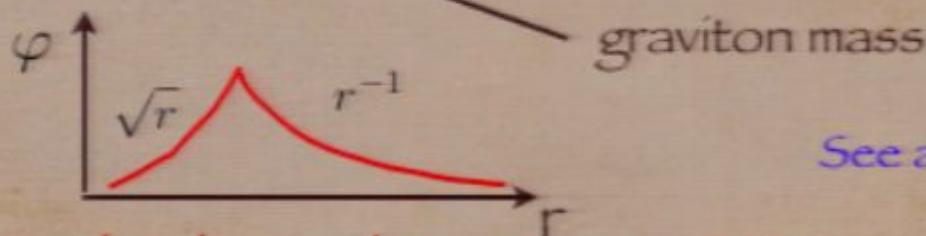
(φ dimensionless, $M_P = 1$)

Vainshtein screening - scale dependent interactions

e.g. DGP

$$S_{\text{scalar}} \sim \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 - \underline{\frac{1}{m^2}(\partial\varphi)^2\square\varphi} + \alpha\varphi T_m{}^\mu{}_\mu \right] \quad (\text{Einstein frame})$$

e.o.m.: $\square\varphi + \frac{1}{m^2} [(\square\varphi)^2 - \partial^\mu\partial^\nu\varphi\partial_\mu\partial_\nu\varphi] \sim \alpha\rho_m$



$\varphi \propto \frac{1}{r}$ larger r
 $\varphi \propto \sqrt{r}$ small r

point mass solution

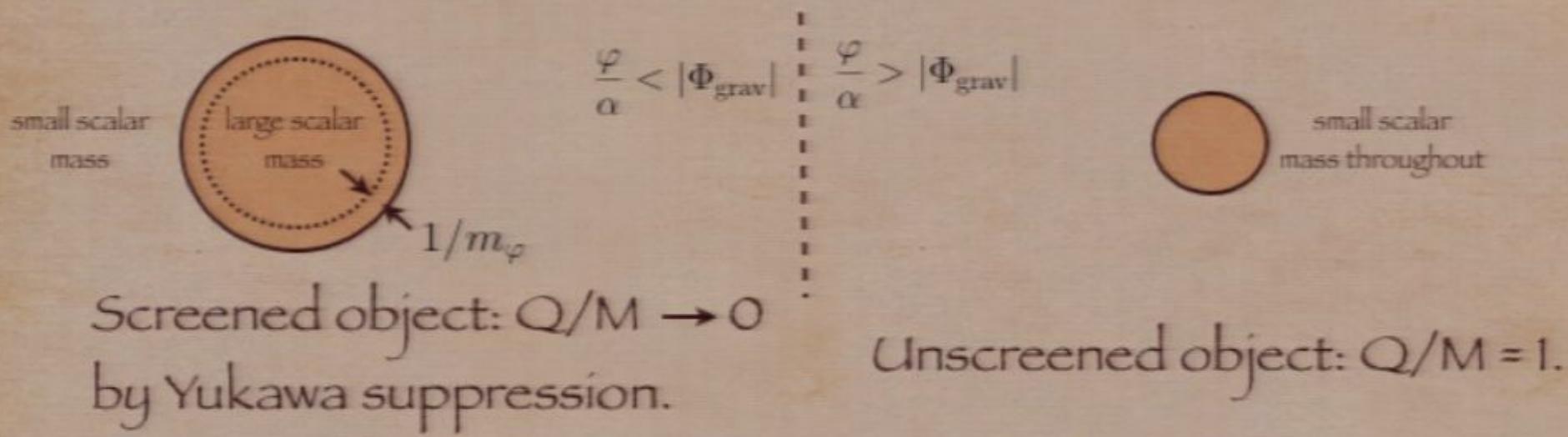
See also: galileon (Nicolis, Rattazzi, Trincherini)

Key in both: nonlinear interaction

α = scalar-matter coupling = $\mathcal{O}(1)$ generally

Chameleon case:

Screened and unscreened objects have $O(1)$ difference in Q/M , and therefore $O(1)$ equivalence principle violation. i.e. screened objects do not move on Jordan frame geodesics. This is a non-relativistic effect.



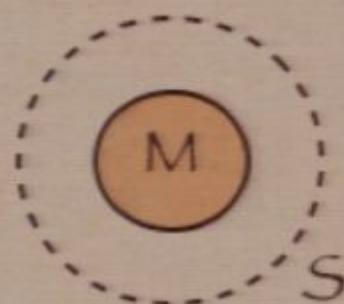
Note: similar arguments apply to symmetron screening too (Hinterbichler & Khoury).

Vainshtein case:

$$\text{Eqt for } \varphi : \partial_\mu J^\mu \sim \frac{\rho_m}{M_P^2} \quad \text{where} \quad J^\mu \sim \partial^\mu \varphi + \frac{1}{m^2} \partial^\mu \varphi \partial^2 \varphi$$

The scalar charge is conserved, by shift/galilean symmetry.
Therefore, no non-relativistic violation of equivalence principle, though Nordvedt effect is still present.

Motion of an extended object - a more precise argument:



$$\text{momentum } P_i = \int d^3x t_i^0$$

momentum flux
↓

$$\dot{P}_i = \int d^3x \partial_0 t_i^0 = - \int d^3x \partial_j t_i^j = - \oint dS \hat{x}_j t_i^j$$

where t_{μ}^{ν} = pseudo energy-momentum

Trick: choose S so that $h_{\mu\nu}$ is small at S , but not necessarily at object. Advantage: perturbative at S (even if perturbations are large at object), and bypass consideration of self-forces.

Works in both Einstein AND Jordan frame.

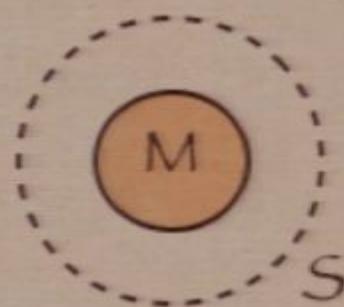
Einstein, Hofmann, Infeld; Damour

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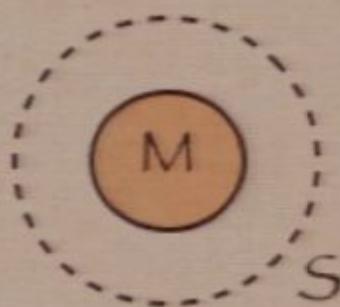
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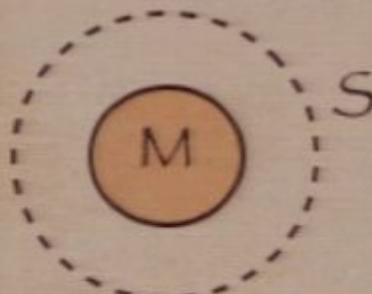
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The fact that objects don't fall at the same rate means they don't all follow the Jordan frame geodesic.

This might seem surprising given that matter is minimally coupled to the Jordan frame metric.



Jordan frame:

$$\text{momentum } P_i = \int d^3x t_i^0$$

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↓
momentum flux

Jordan frame extra

$$G_\mu^\nu + \Delta_\mu^\nu = 8\pi G T_\mu^\nu$$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ($h_{\mu\nu}$ not small)

$$G^{(1)}_\mu^\nu = 8\pi G \left[T_\mu^\nu - \frac{G^{(2)}_\mu^\nu}{8\pi G} - \frac{\Delta_\mu^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu^\nu$$

Geodesic motion requires cancellation between them.

Jordan frame summary for chameleon:

$$M \ddot{X}_i = -M \left[\frac{1 + 2\epsilon\alpha^2}{1 + 2\alpha^2} \right] \partial_i \Phi_{\text{ext}}$$

Milky way & Sun has $|\Phi_{\text{object}}| \sim 10^{-6}$
 $\rightarrow \varphi/\alpha \lesssim 10^{-6}$

(modulo local group)

$\epsilon \sim 1$ for unscreened objects and $\epsilon \sim 0$ for screened objects
 $(\varphi/\alpha > |\Phi_{\text{object}}|)$ $(\varphi/\alpha < |\Phi_{\text{object}}|)$

grav. mass = inertial mass grav. mass \neq inertial mass

Generically $\alpha \sim 1$, so expect O(1) violation of equivalence principle between screened and unscreened objects.

Only unscreened objects move on Jordan frame geodesics.

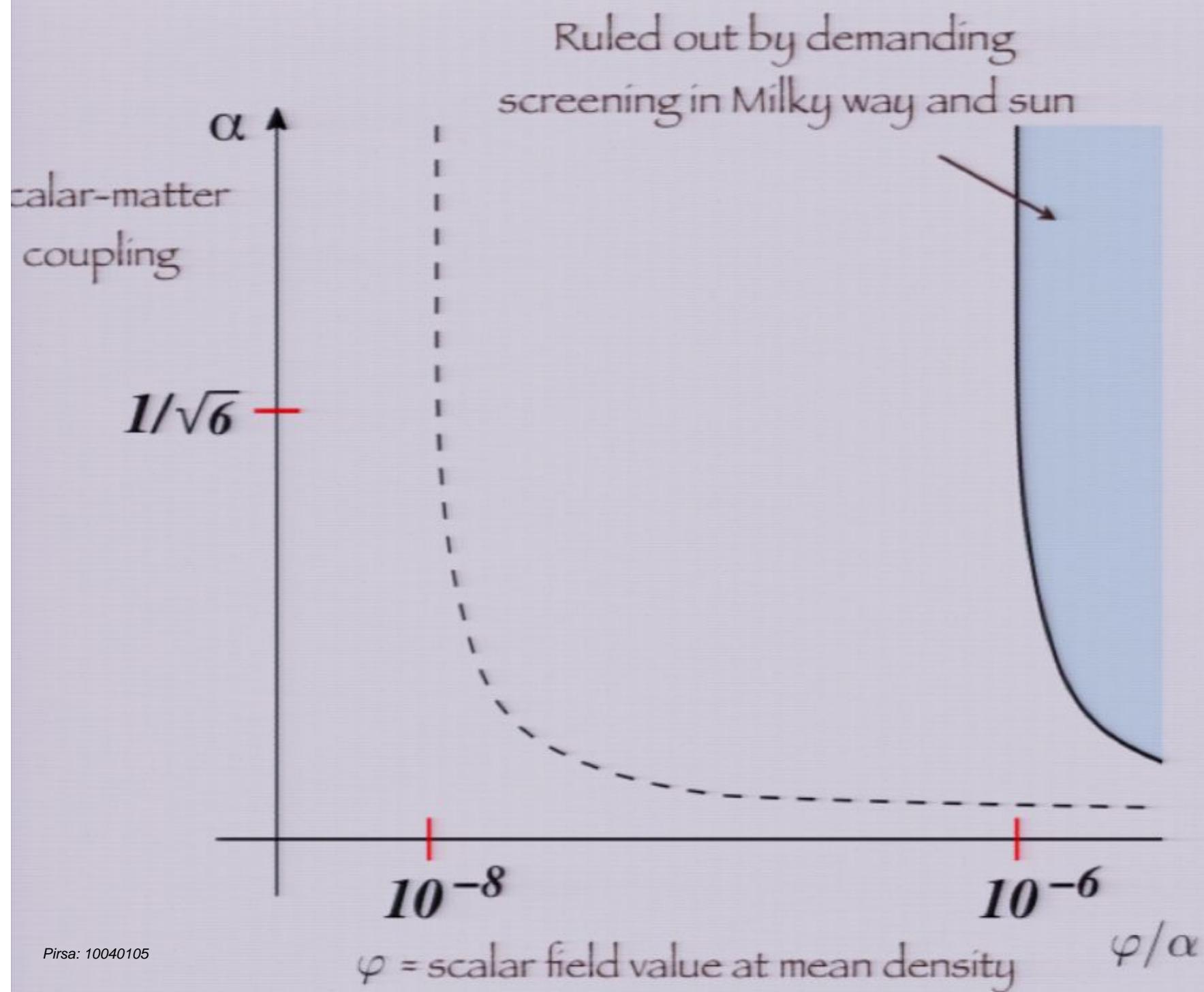
E.g. $f(R)$: $\alpha = 1/\sqrt{6}$, unscreened/screened grav. mass = $4/3$.

Note: $f(R)$'s special α is not protected against quantum corrections.

Important parameters: α & $\frac{\varphi}{\alpha}$

scalar-matter coupling;
controls e.p. violation level

controls screening



Bulk motion tests:

Idea - unscreened small galaxies, screened large galaxies.

1. Small galaxies should move faster than large galaxies (i.e. an effective velocity bias - redshift distortion needs to be reworked) in unscreened environments. Beware: Yukawa suppression.
2. Small galaxies should stream out of voids faster than large galaxies creating larger than expected voids defined by small galaxies (see Peebles; note: effect cares about sign of grav. pot.).

Internal motion tests:

Idea - unscreened HI gas clouds, screened stars.

3. Diffuse gas (e.g. HI) should move faster than stars in small galaxies even if they are on the same orbit. Beware: asymmetric drift.
4. Gravitational lensing mass should agree with dynamical mass from stars, but disagree with that from HI in small galaxies.



Key: avoid blanket screening.

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Random comment 1: compton wavelength (at mean density) in $f(R)$ is fairly short.

$$m_\varphi^2 = \frac{\partial^2 V}{\partial \varphi^2} \sim \frac{1}{\varphi} \frac{\partial V}{\partial \varphi} \sim \frac{H^2}{\varphi}$$

Therefore : $m_\varphi^{-1} \sim 10^{-3} H^{-1} \left(\frac{\varphi}{10^{-6}} \right)^{1/2}$

Yukawa suppression of the scalar force therefore makes it hard to see equivalence principle violation in bulk motions of galaxies. Internal motion is more promising.

- However, the above scaling can be violated to give longer compton wavelength (also possible for symmetron, Hinterbichler & Khoury).

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Random comment 2: lensing mass will agree with dynamical mass from screened objects, but will disagree with dynamical mass from unscreened objects.



Because neither photons nor screened objects see the scalar force, while unscreened objects do.

- Therefore, Pengjie Zhang et al.'s test would find no deviation from GR for screened galaxies, but will find deviation for unscreened galaxies.
- Comparison between stellar dynamical mass of a galaxy and lensing mass should not yield deviation, because stars like the sun is screened ($M/R \sim \text{const.}$ for m.s.).

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