Title: Non-linear structure formation in modified gravity

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Abstract: Instead of adding another dark component to the energy budget of the Universe, one can ask whether the observed accelerated expansion might in fact be due to the behavior of gravity itself on the largest scales.
In this talk I will focus on two popular modified gravity theories which realize this scenario: f(R) gravity and the DGP model. While these models yield an accelerated expansion, they also affect the formation of structure on much smaller scales. We have studied this with cosmological N-body simulations which consistently solve for the modified gravitational force. I will discuss the effects of modified gravity on dark matter halo properties as well as cosmological observables. For
f(R) gravity, our first simulation-calibrated constraints from the observed abundance of massive clusters improve on previous constraints from the CMB and ISW by a factor of ~1000. This exemplifies the sensitivity of cosmological observables in the non-linear regime as probes of gravity.

Non-linear Structure Formation in Modified Gravity

Fabian Schmidt
Caltech - TAPIR

with **Wayne Hu, Marcos Lima, Alexey Vikhlinin, Hiroaki Oyaizu**

Perimeter Institute 4/28

Modified Gravity: Challenges

Theoretical Challenge:

- **Gravity constrained on wide range of scales:**
  - Early Universe: BBN, CMB
  - Growth of structure
  - **Solar System**

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Theoretical Challenge:

- **Gravity constrained on wide range of scales:**
  - Early Universe: BBN, CMB
  - Growth of structure
  - **Solar System**
- **Idea: reduce to GR in high-curvature regime**
  - Applies to Early Universe as well as high-density regions today

Modified Gravity: Challenges

Observational Challenge:

- **How can we distinguish Modified Gravity from GR + Dark Energy ?**
  - (Almost) any expansion possible with Dark Energy

Modified Gravity: Challenges

Observational Challenge:

- **How can we distinguish Modified Gravity from GR + Dark Energy ?**
  - (Almost) any expansion possible with Dark Energy
- **Beyond background: growth of structure**
  - Predictions straightforward in *linear regime*
  - *Non-linear regime* less so...

Probing gravity: linear vs nonlinear regime

Linear regime: CMB, SN, ISW, *BAO*

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- Specialized N-body simulations necessary
- Wealth of observables available
- Lots of statistics and S/N

Modified Gravity Models

Two known and fully worked models achieving acceleration:

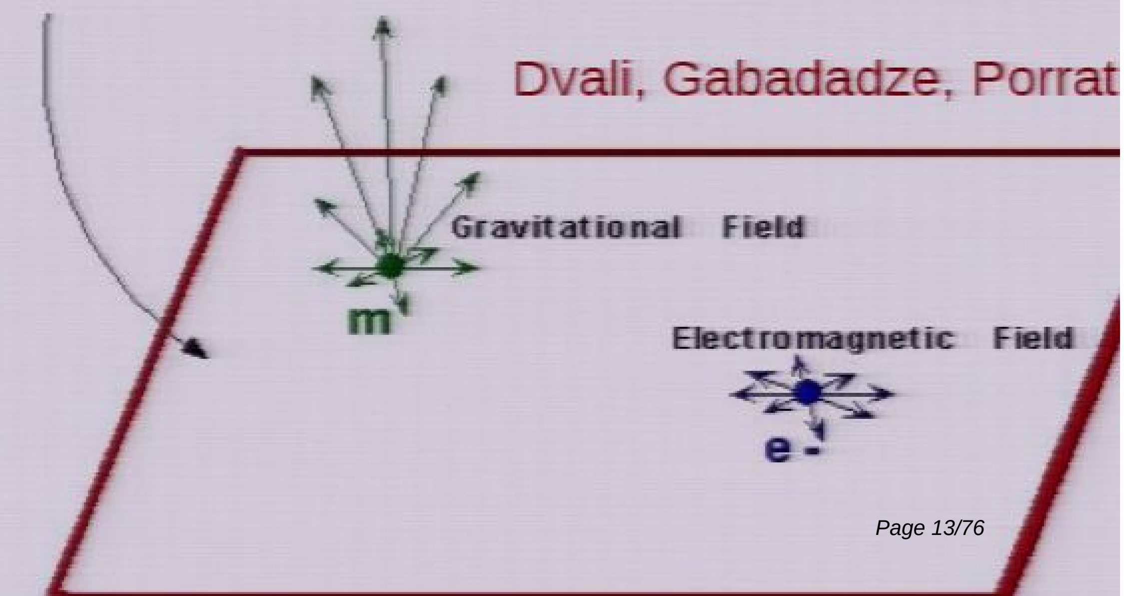
cf. Wayne's ta

- **DGP braneworld model**
  - Gravity "leaks" into large extra dimension
- **f(R) model**
  - Phenomenological extension of GR
  - Equivalent to scalar-tensor theory

- **Both use *non-linear mechanism* to restore GR locally**

DGP Braneworld cosmology

- **Dvali-Gabadadze-Porrati model:**
  - Matter / radiation confined to 4D brane in 5D Minkowski space
  - Action constructed to reduce to GR on small scales



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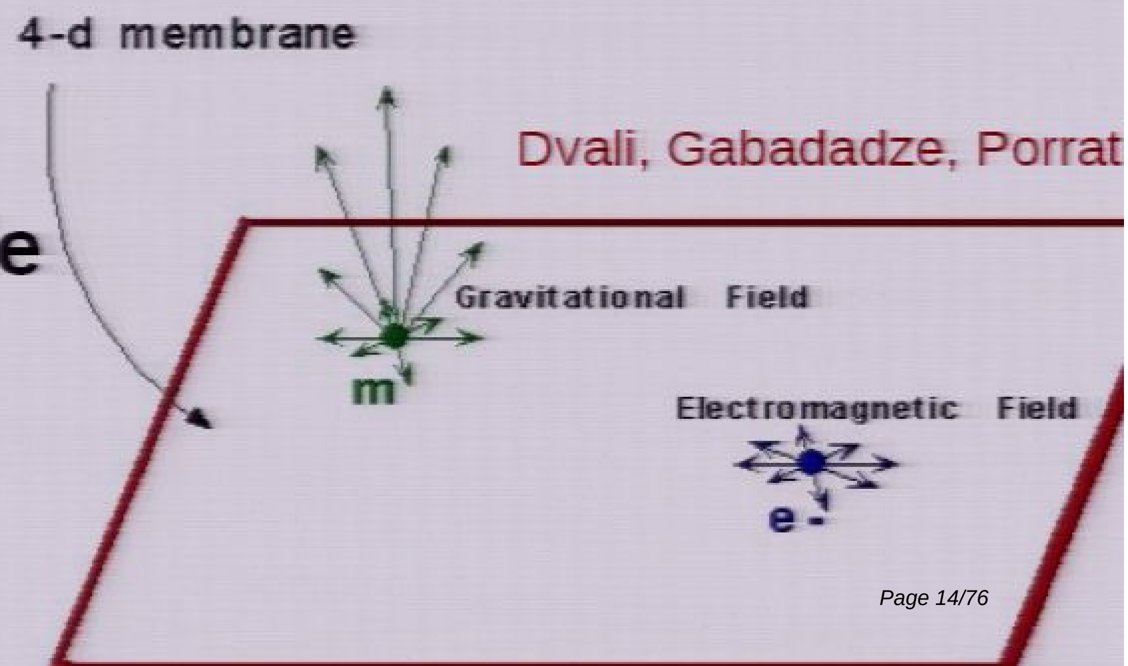

- **Cross-over scale**

$$r_c \equiv \frac{G^{(5)}}{2G^{(4)}}$$

DGP Branches

- **Self-accelerating branch**
  - Accelerating today if $r_c \sim H_0^{-1} \sim 3000\,\mathrm{Mpc/h}$
  - w_{eff} ~ -0.5 ... -0.8
  - *~4σ conflict* with CMB+Supernovae Fang et al. 08

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Fang et al. 08

- **Normal branch**
  - Have to add Λ or *dark energy* on brane

Lombriser et al.
FS 09b

Growth of structure in DGP

- **Sub-horizon scales: effective scalar-tensor theory**
  - Massless field φ - *brane-bending mode*
  - φ contributes to dynamical potential:

$$\Psi = \Psi_N + \frac{1}{2}$$

Newtonian p

Koyama & Maartens 06,
Nicolis & Rattazzi 04

Growth of structure in DGP

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  - Massless field φ - *brane-bending mode*
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$$\Psi = \Psi_N + \frac{1}{2}\varphi$$

Newtonian p

  - *Normal branch:* φ attractive
  - *Self-acc. branch:* φ *repulsive*

Koyama & Maartens 06,
Nicolis & Rattazzi 04

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Brane-bending mode

- **On linear scales :** $\varphi = \frac{2}{3\beta}\Psi_N, \quad \beta(a) \propto H\, r_c$

  – Effective grav. constant $$\boxed{G_{\rm eff} = G_N\left(1 + \frac{1}{3\beta(a)}\right)}$$

Brane-bending mode

- When $\delta\rho/\bar{\rho} \gtrsim 1$, non-linear interactions of φ important:

$$\nabla^2\varphi + \frac{r_c^2}{3\beta\, a^2}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)(\nabla^i\nabla^j\varphi)\right] = \frac{8\pi\, G\, a^2}{3\beta}\delta$$

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  - Time derivatives neglected: *sub-horizon scales*
  - Type of interactions **generic** to braneworld models *(Gauss-Codazzi eq.)*

Non-linear interactions

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- **Hard: non-linear in derivatives of φ**
  - No superposition principle
- **Only numerical solution in general**
  - As part of N-body simulation

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- **Two analytically solvable cases:**

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- **Two analytically solvable cases:**

1. Plane wave: $\varphi \sim e^{i\mathbf{k}\cdot\mathbf{x}} \Rightarrow -k^2\varphi = \frac{8\pi G a^2}{3\beta}\delta\rho$

Non-linearity cancels !

Non-linear interactions

$$\nabla^2\varphi + \frac{r_c^2}{3\beta\, a^2}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)(\nabla^i\nabla^j\varphi)\right] = \frac{8\pi\, G\, a^2}{3\beta}\delta$$

- **Two analytically solvable cases:**

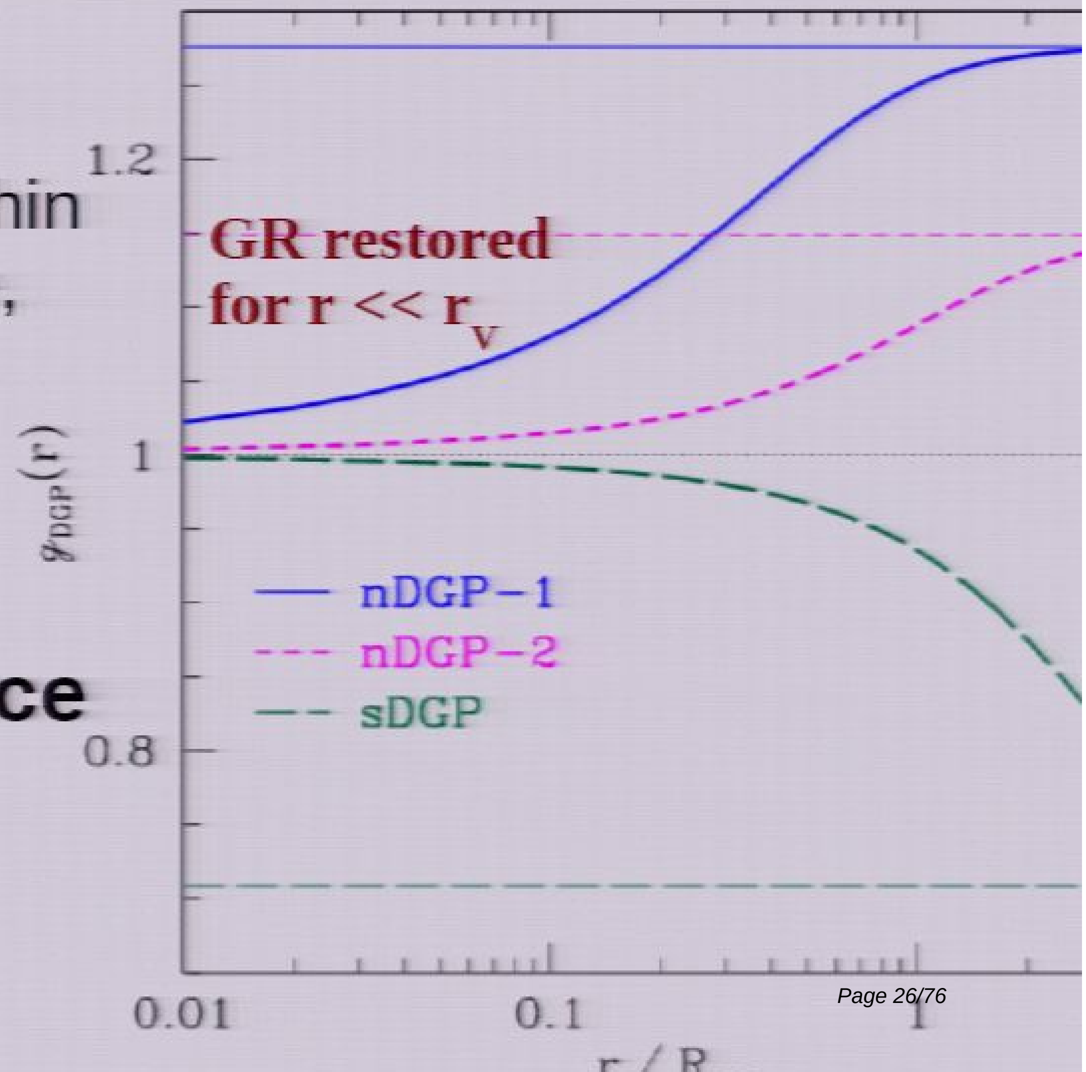
2. Spherically symmetric mass

- *φ saturates within Vainshtein radius* $R_* = \left(\frac{8 r_s r_c^2}{9\beta^2}\right)^{1/}$

$$R_{*,\odot} \sim 100\,\mathrm{pc} \;\; \text{for} \;\; r_c \sim \mathrm{Gpc}$$

Vainshtein mechanism

- **Spherical mass:**
  - Field suppressed within characteristic scale, *Vainshtein radius:*
    $r_V \propto (r_c^2 \, r_s)^{1/3}$



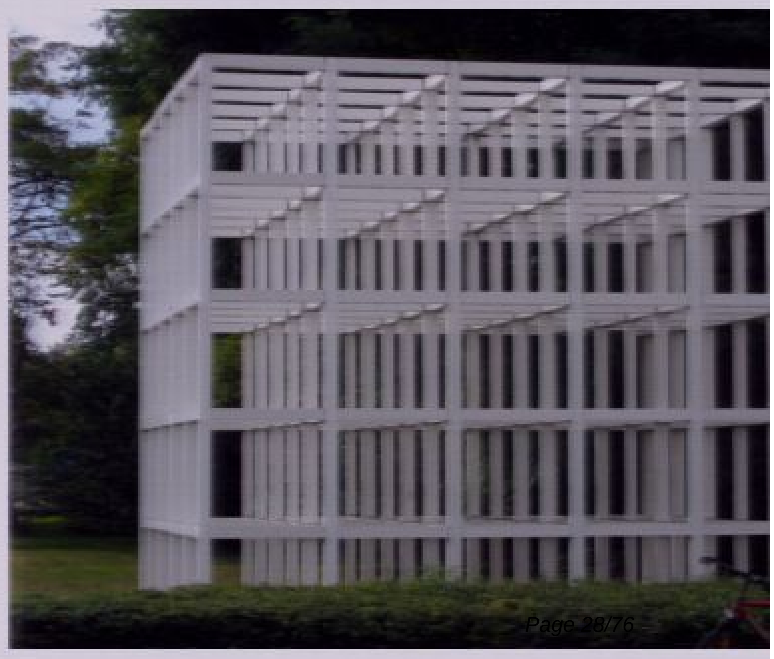
- **Modified grav. force**
  - within halos -->

Simulating DGP

- Need self-consistent solution of nonlinear φ field and dark matter

Simulating DGP

- **Need self-consistent solution of nonlinear φ field and dark matter**
- ***Particle-mesh code:***
  - Density and potential are evaluated on cubic grid
  - Given modified potential, propagation of particles unchanged



Main task: solve for potential

- **Newtonian potential Ψ_N:**
  - Obtained via Fourier transform of density

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  - Non-linear relaxation scheme (Newton-Raphson)
    - Parallelized with multi-grid acceleration

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- **Finally:** $\Psi = \Psi_N + \frac{1}{2}\varphi$
- **Non-linear relaxation *time-consuming:***
  - CPU time ~20x that of ordinary GR simulations

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Simulated Models

sDGP: Best-fit flat self-accelerating DGP model

Fang et al.

- No Λ or dark energy

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sDGP: Best-fit flat self-accelerating DGP model Fang et al.

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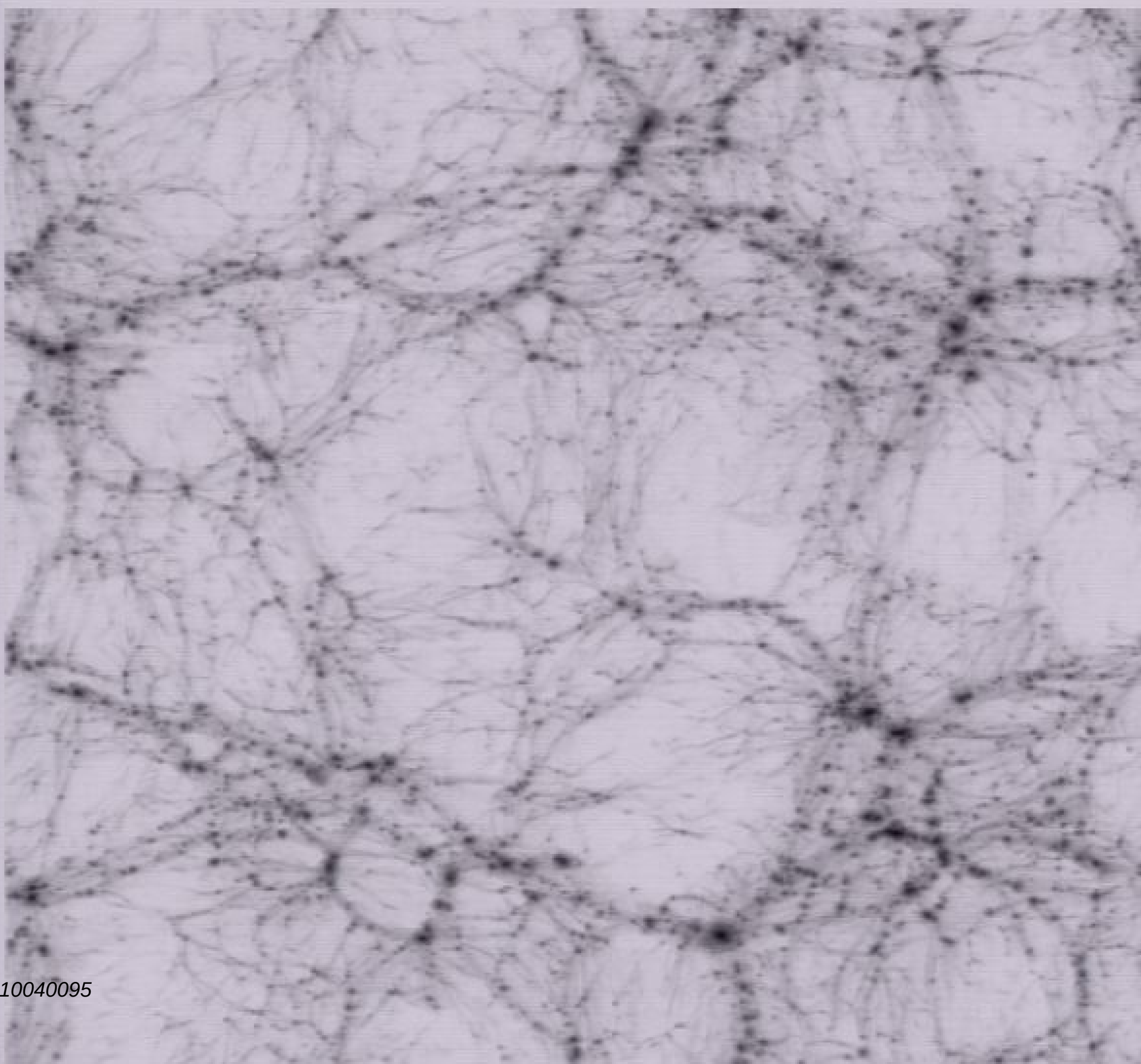
nDGP: normal-branch with dark energy

- Exact ΛCDM expansion history: r_c unconstrained FS (
- Contrived model... but fully understood
- Effective model for generalized braneworlds

Simulated Models

- **Simulate three models in each case:**
  - GR with matched expansion history:
    "QCDM" / ΛCDM
  - ***Full DGP***
  - ***Linearized DGP***

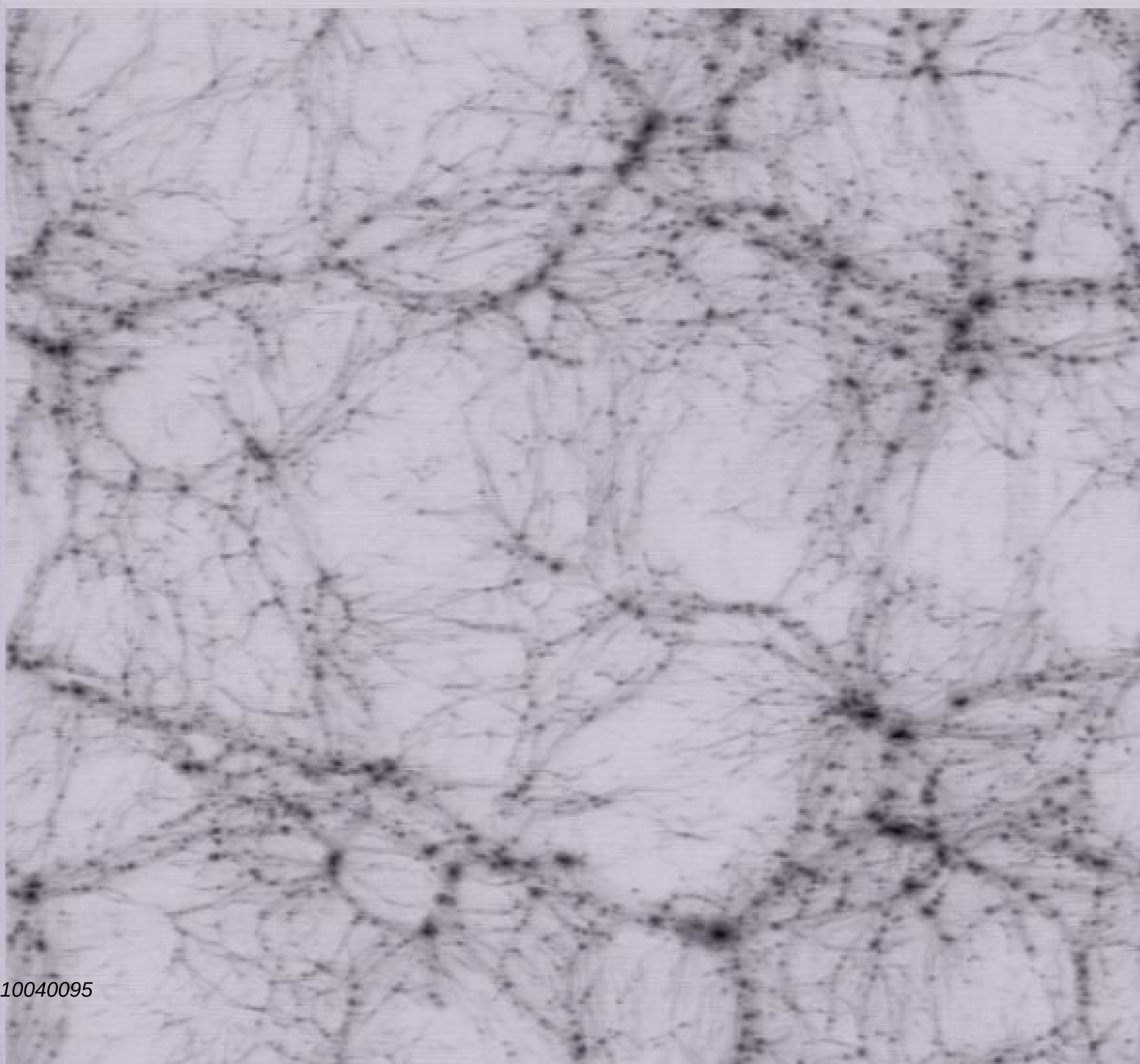
Results: Structure Formation



Density field
Slice through simulation at z=0, size: 64 Mpc/h

GR – ΛCDM

Results: Structure Formation



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DGP normal branch + D
r_c = 3000 Mpc

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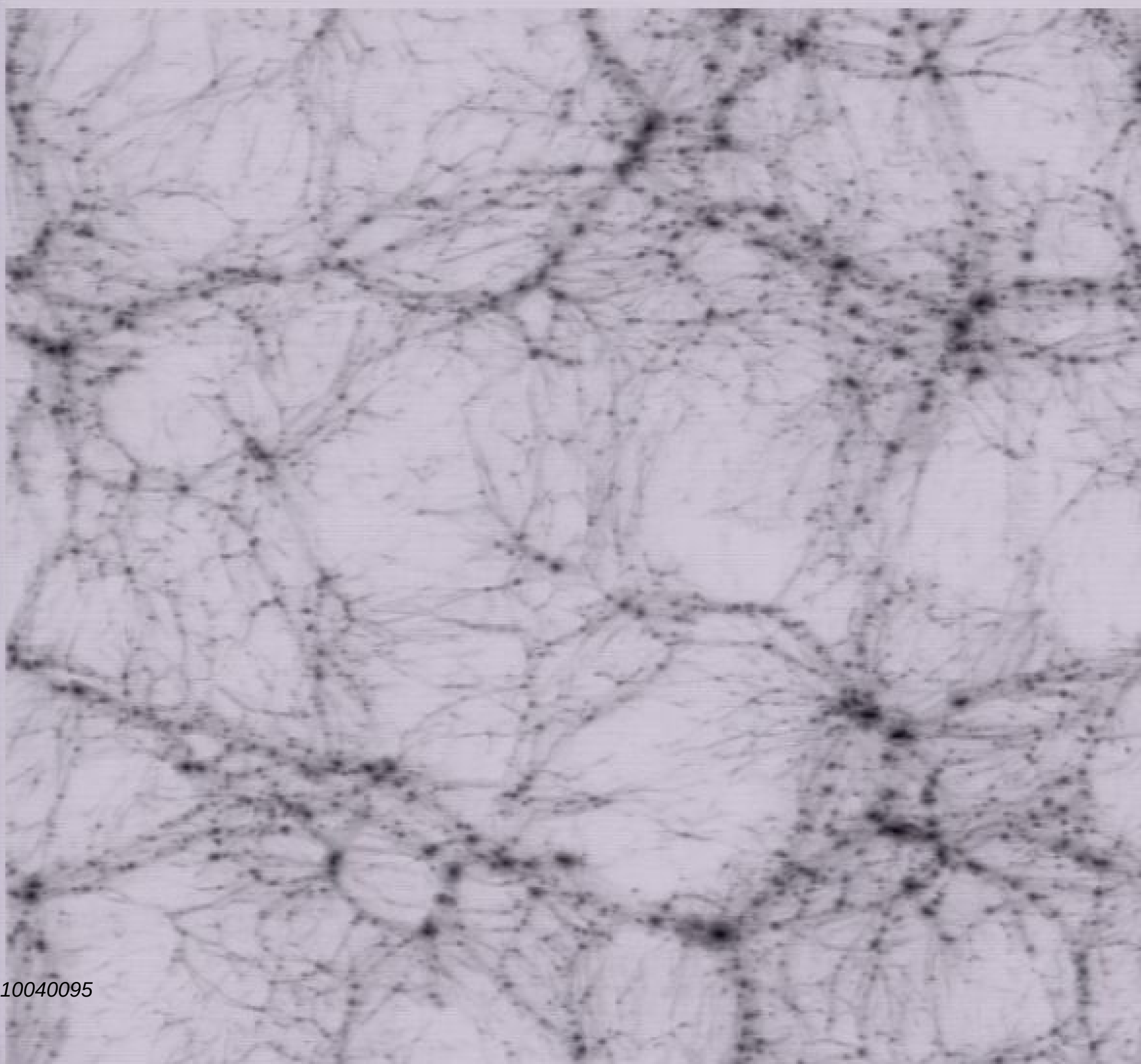

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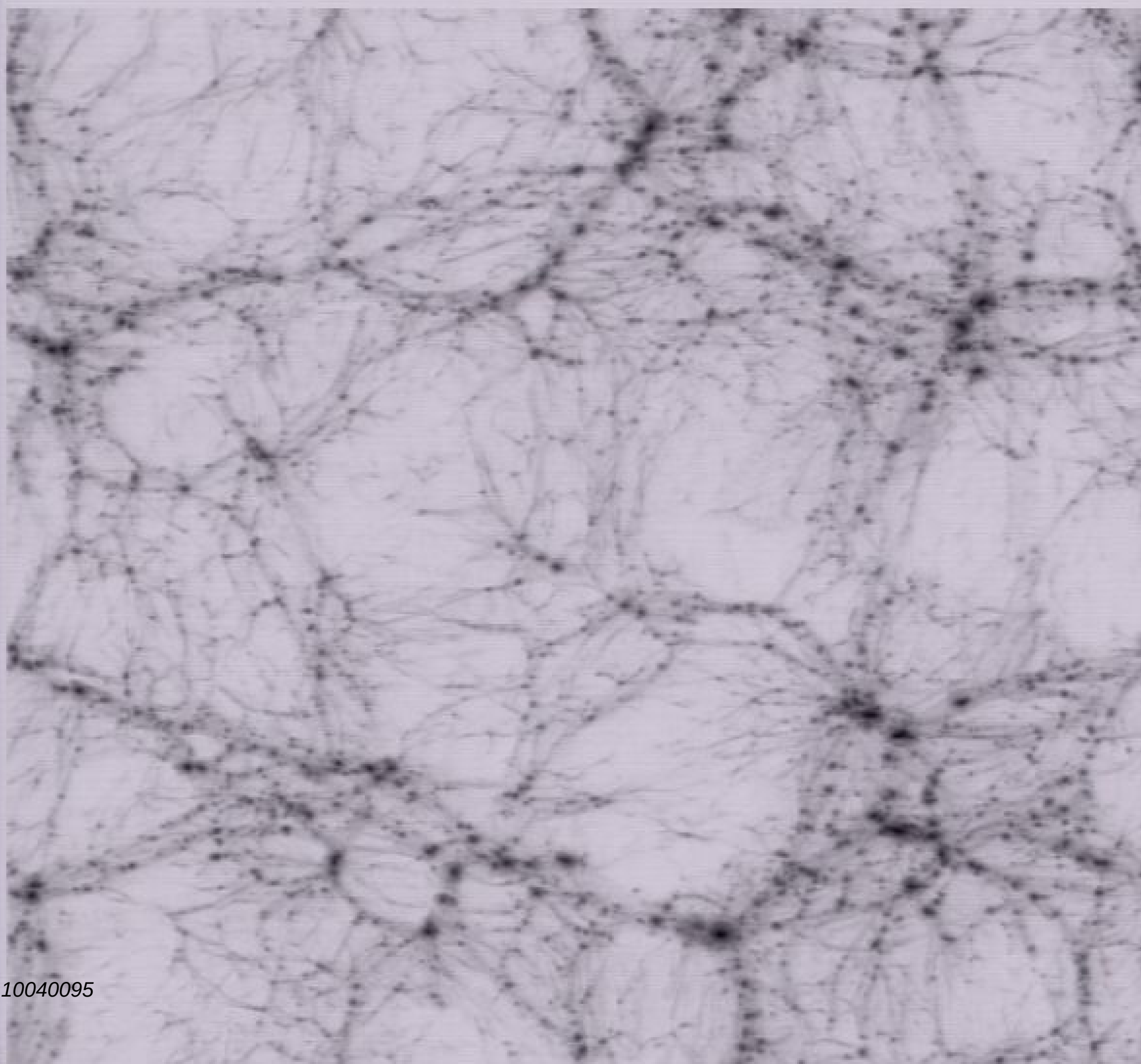

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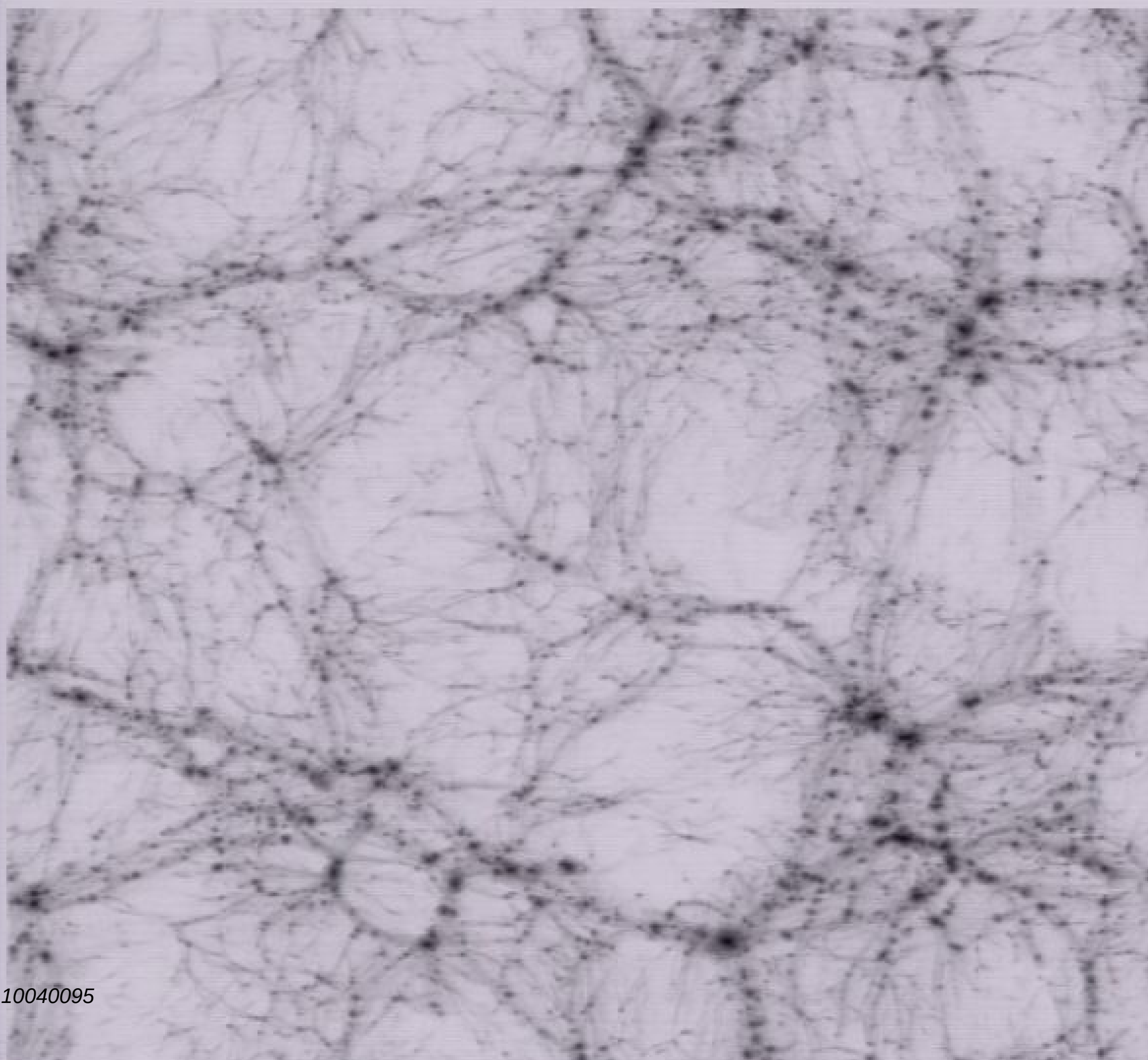

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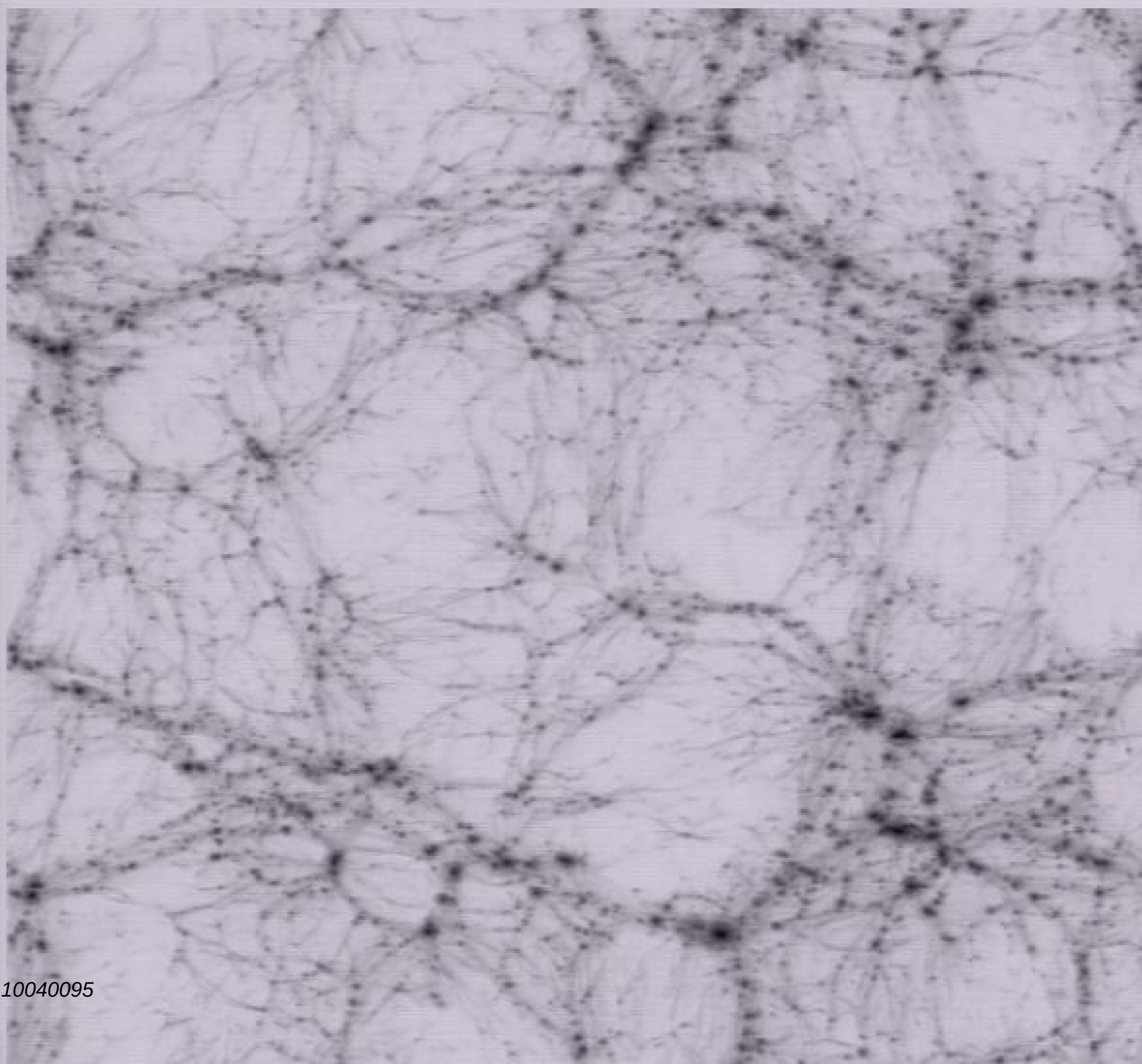

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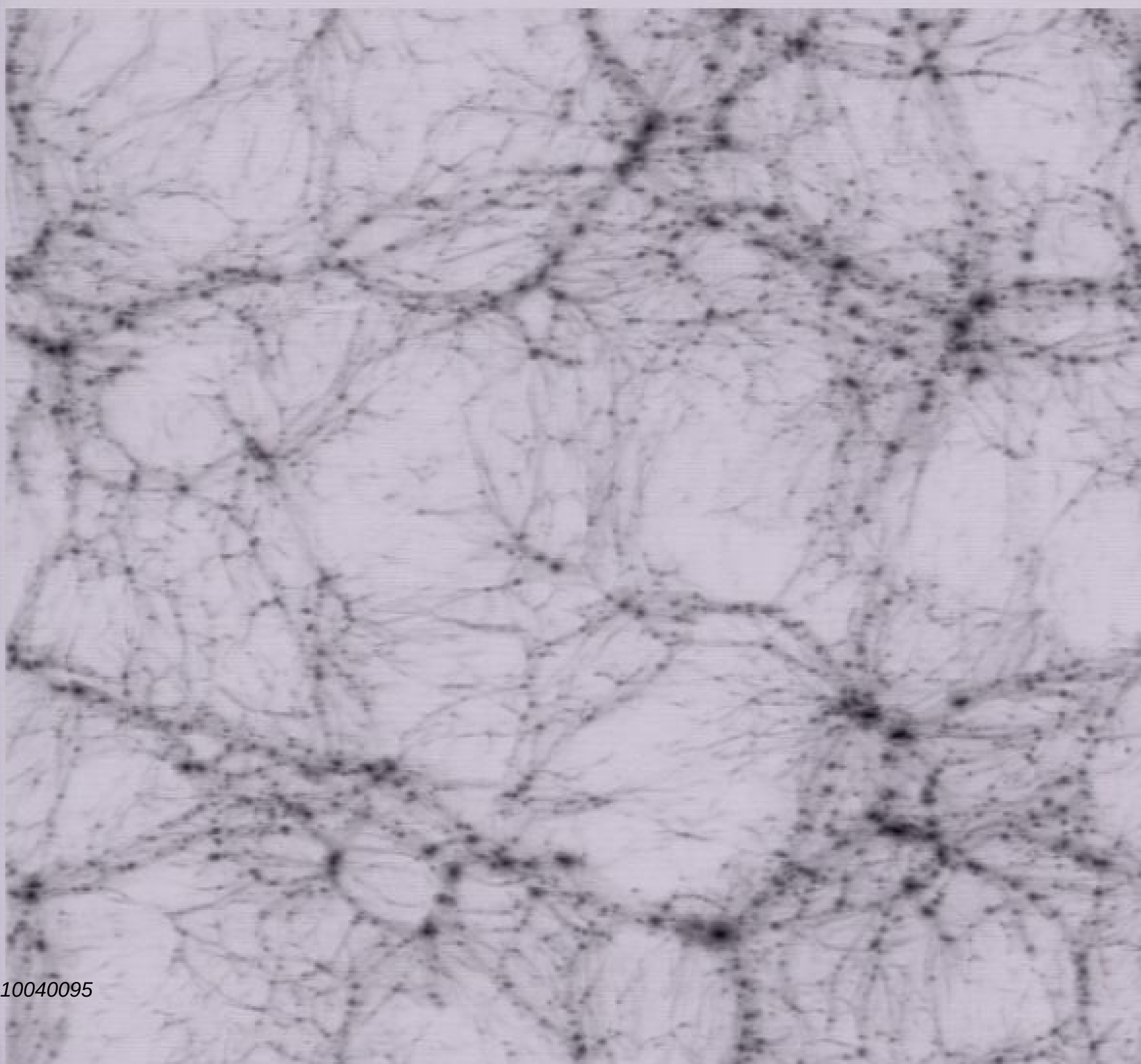

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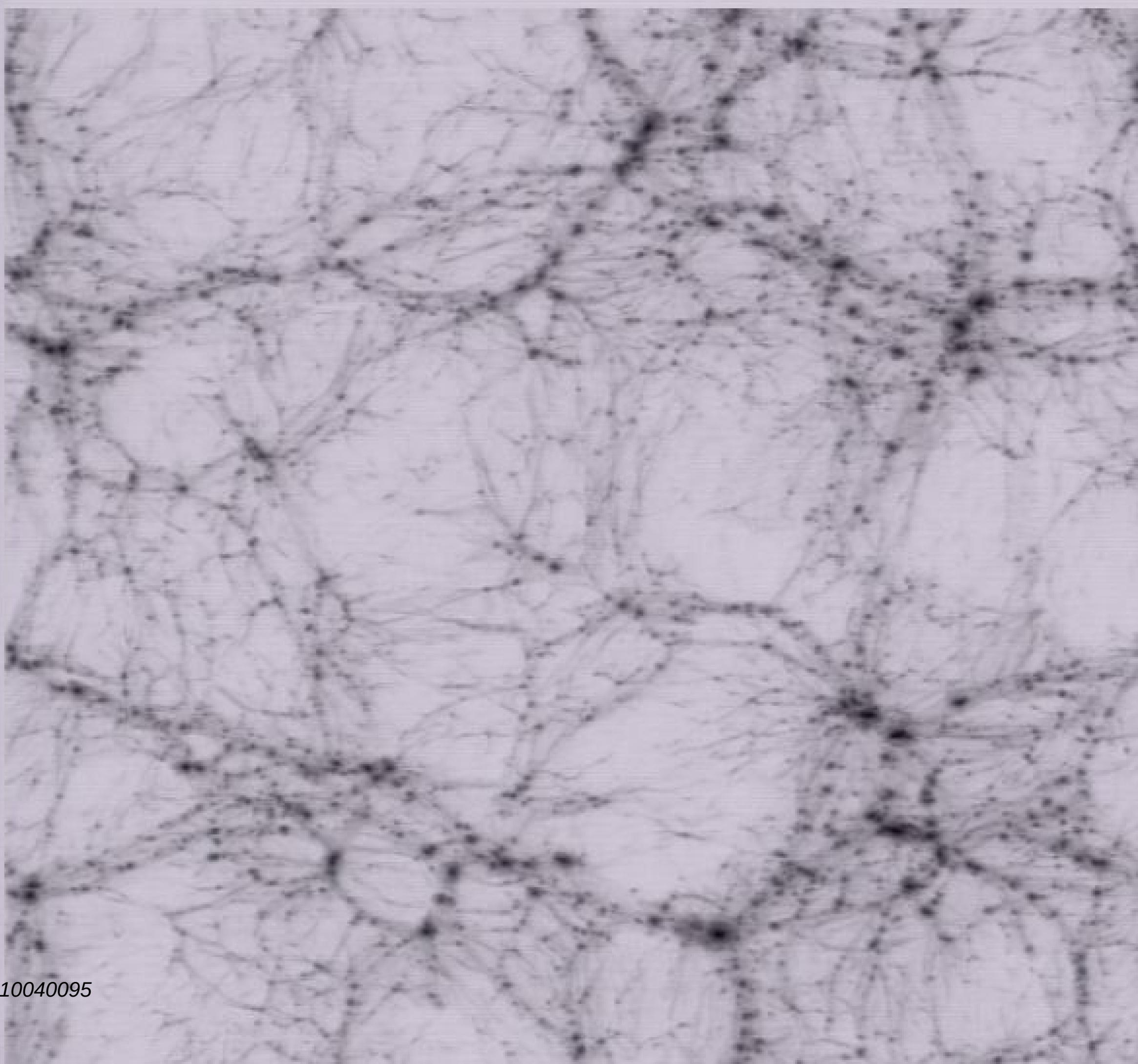

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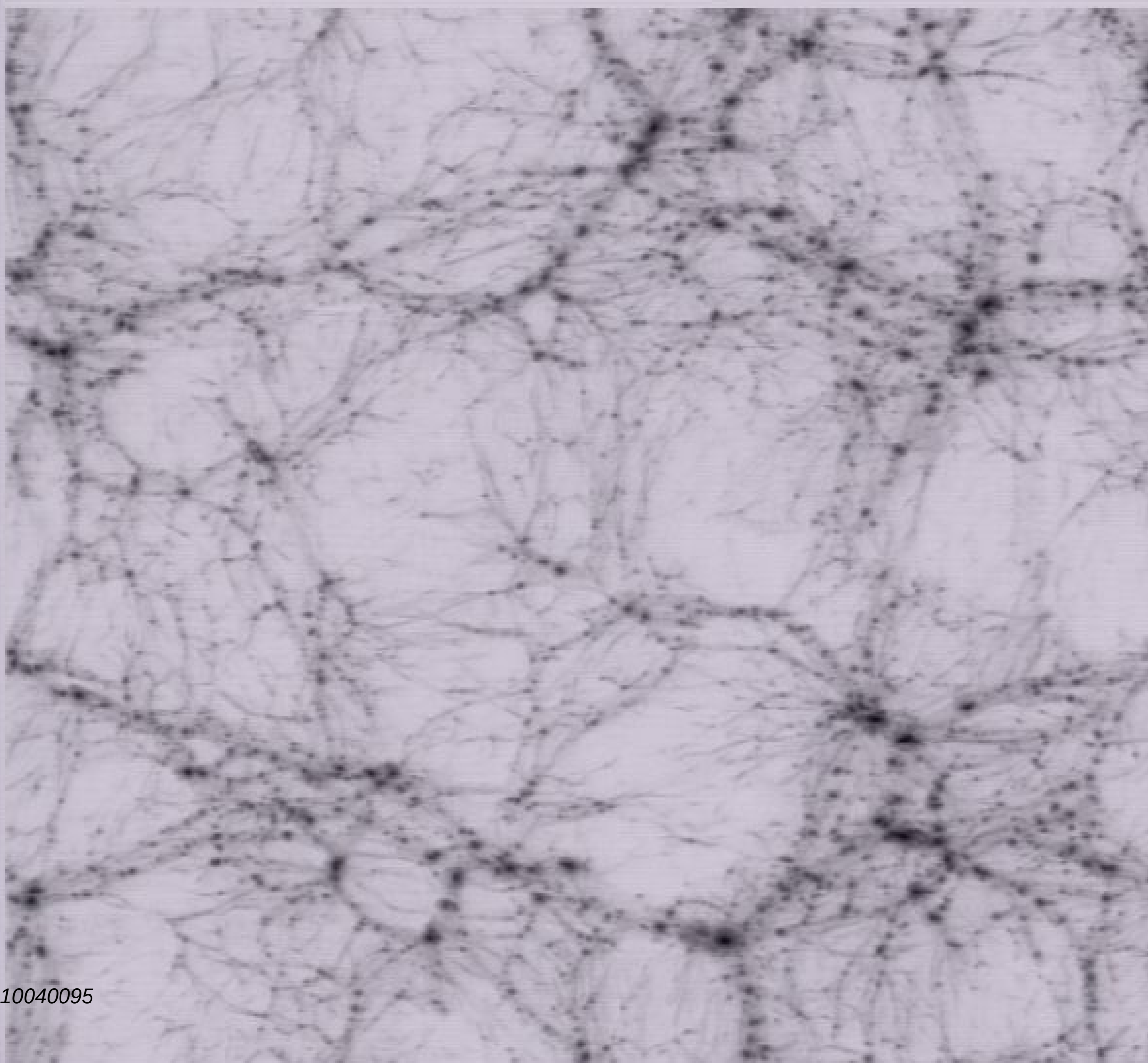

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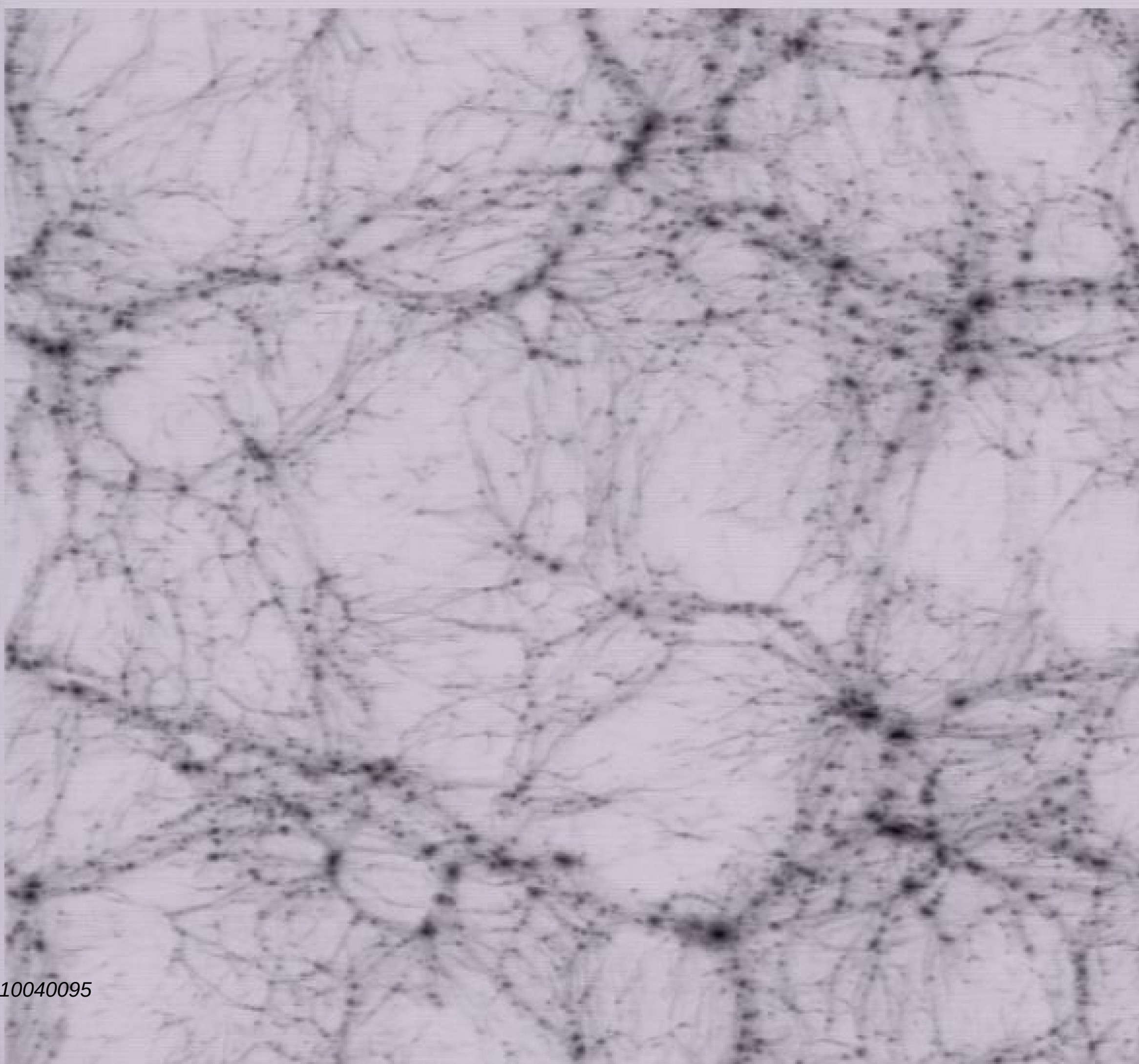

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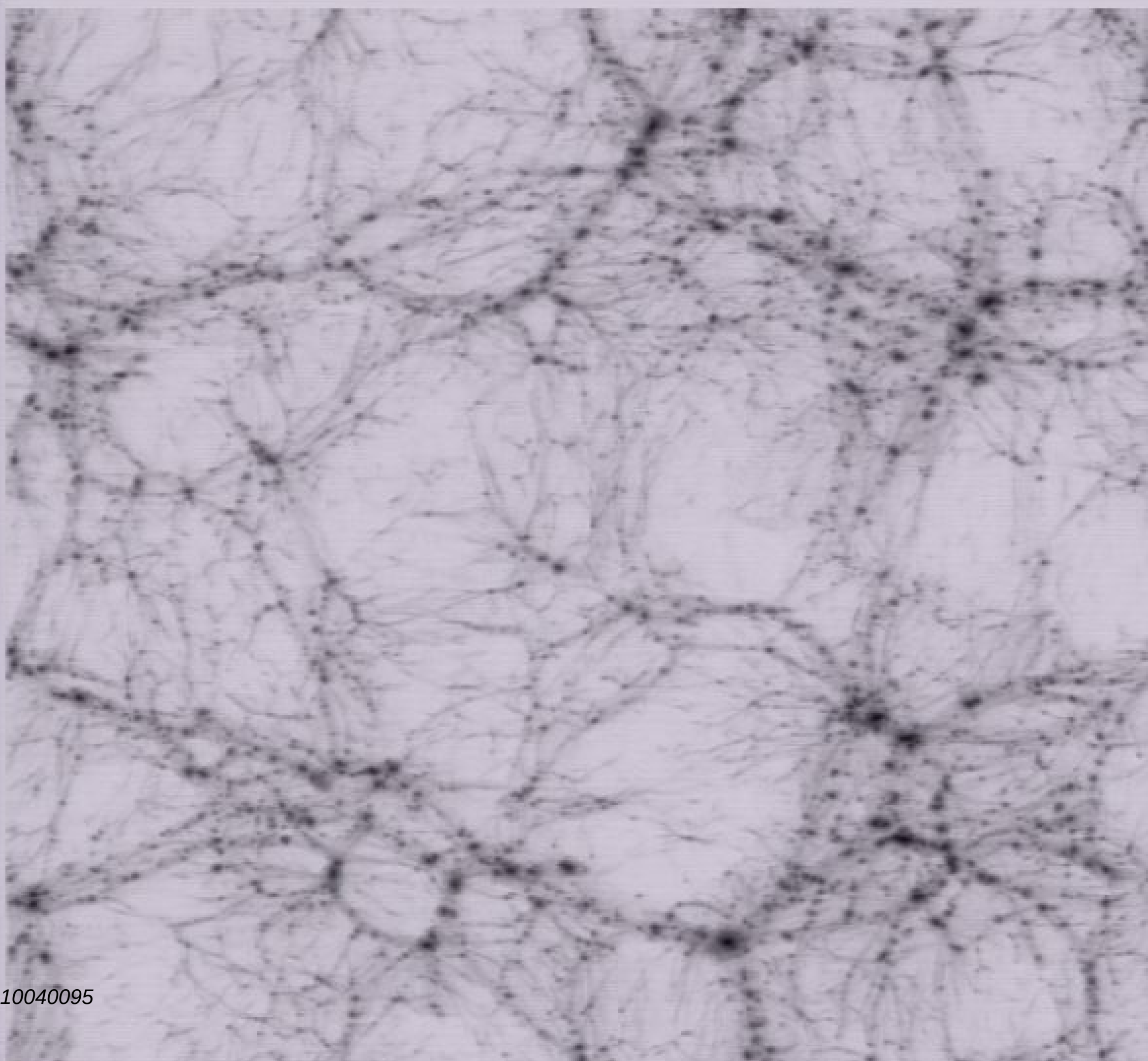

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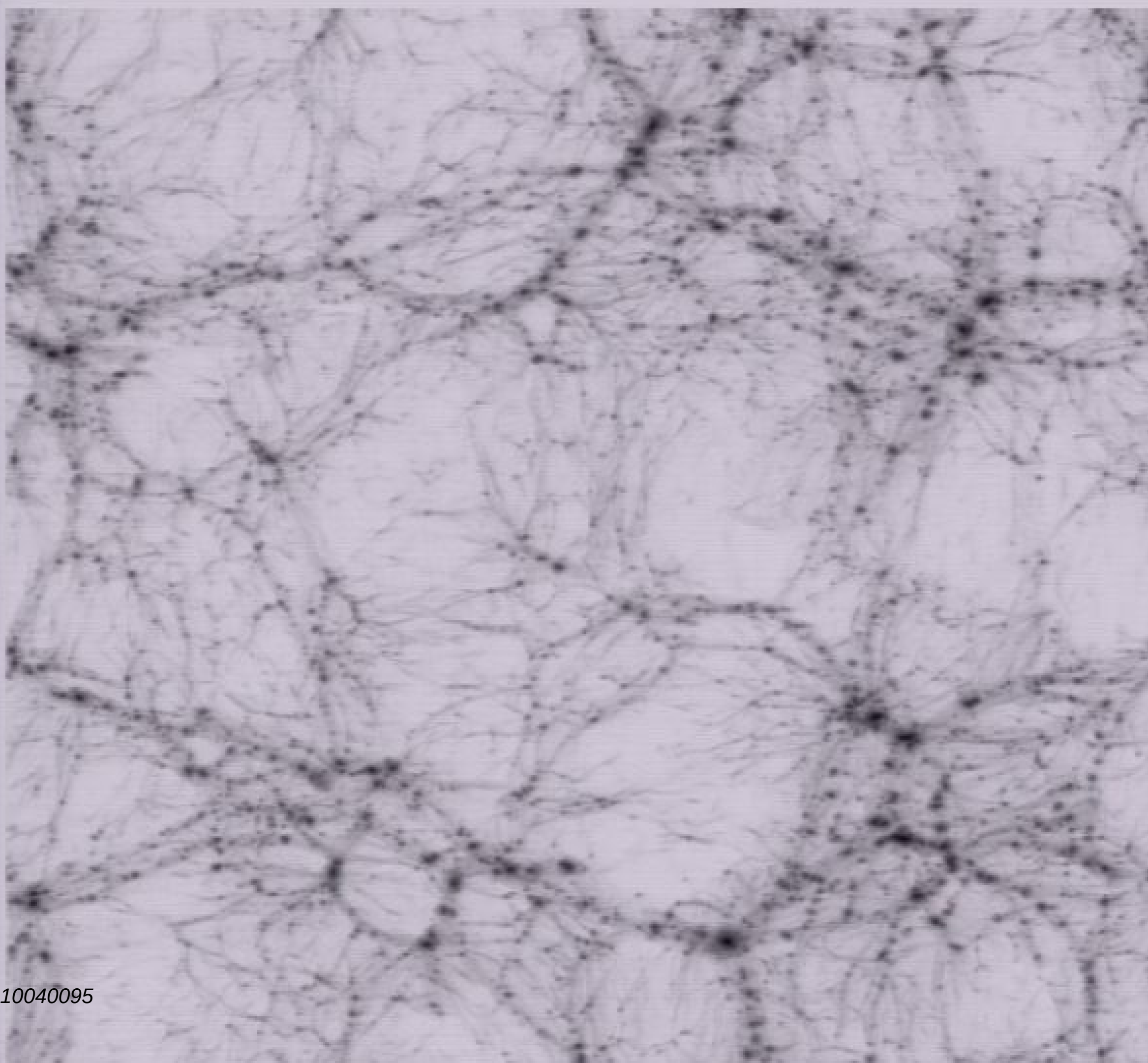

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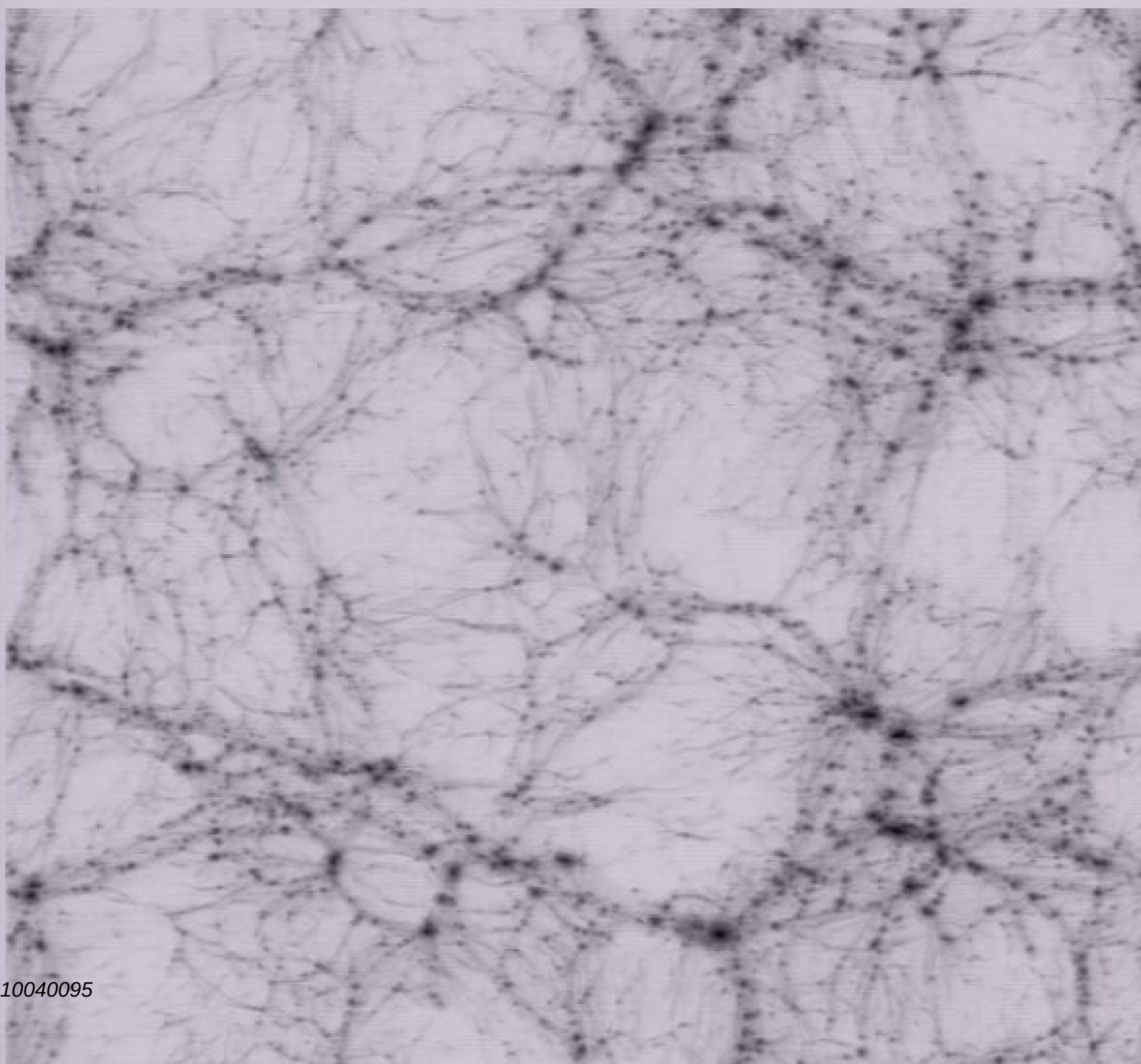

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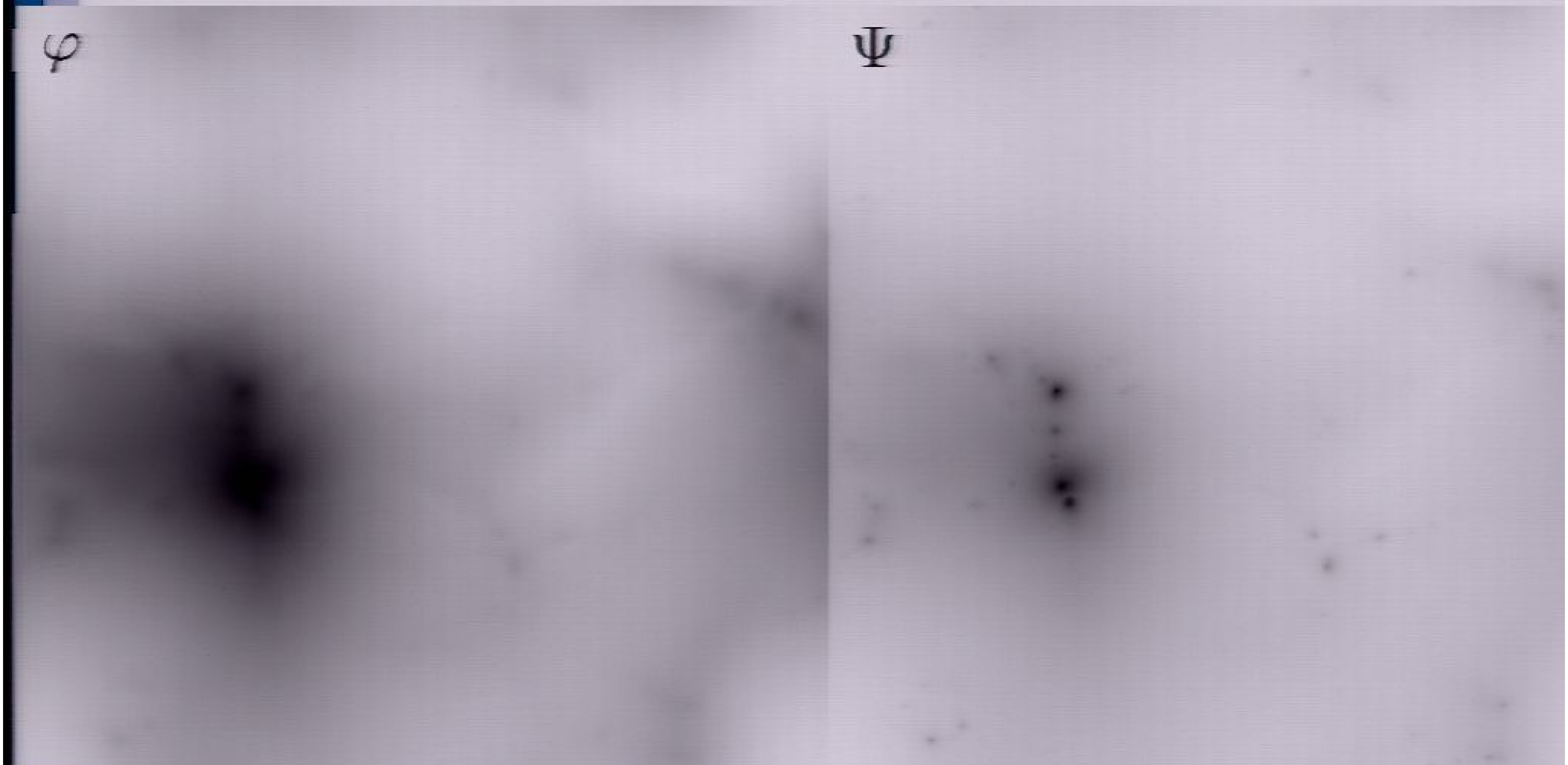
Brane-bending mode & Potential



sDGP simulation, 64 Mpc/h box,

Results: Matter Power Spectrum

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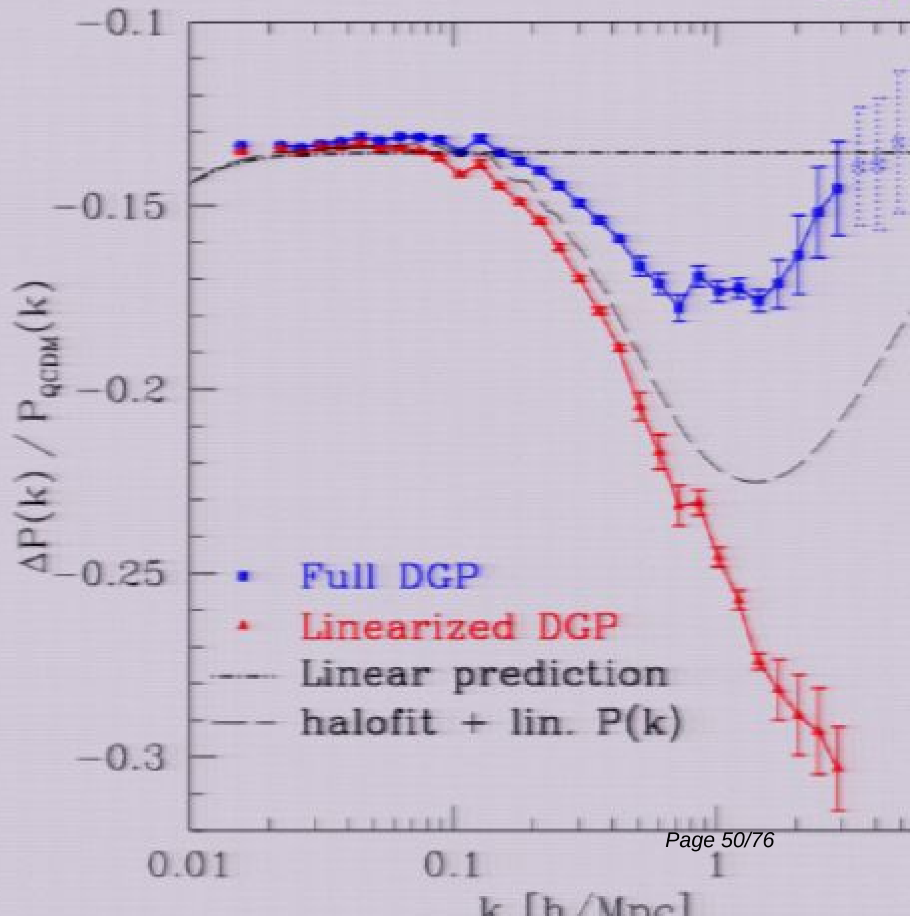
- **Full** and **linearized** DGP vs GR (z=0)

sDG



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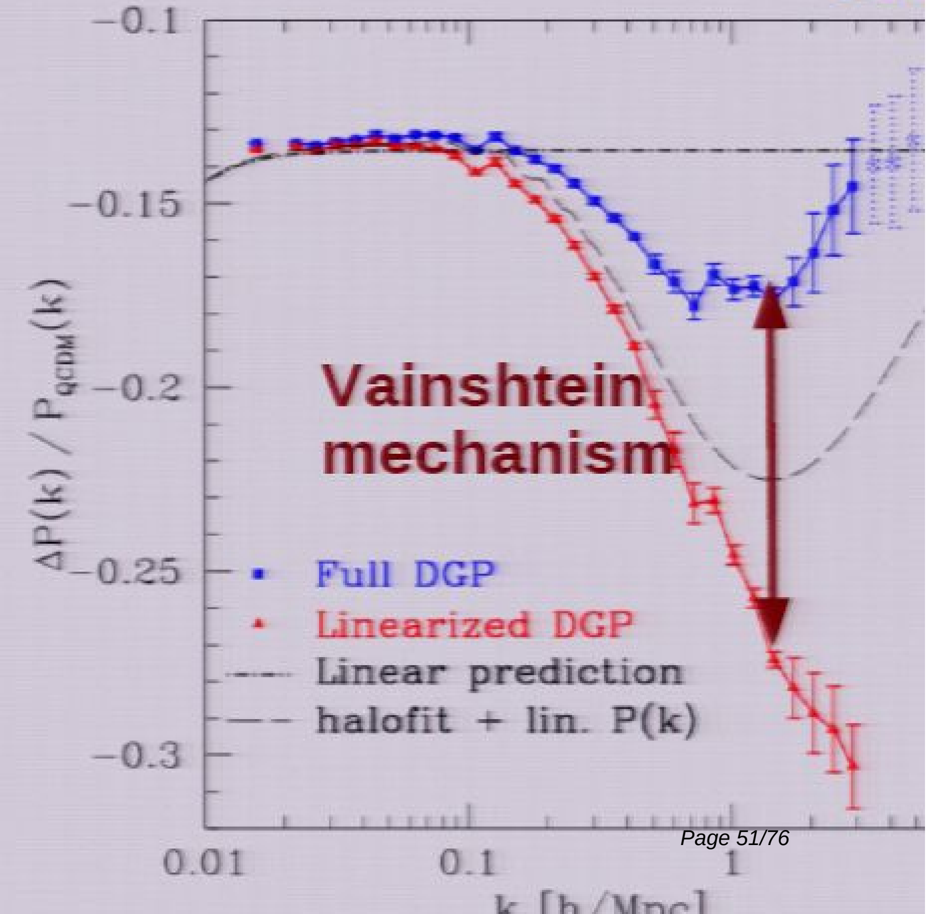

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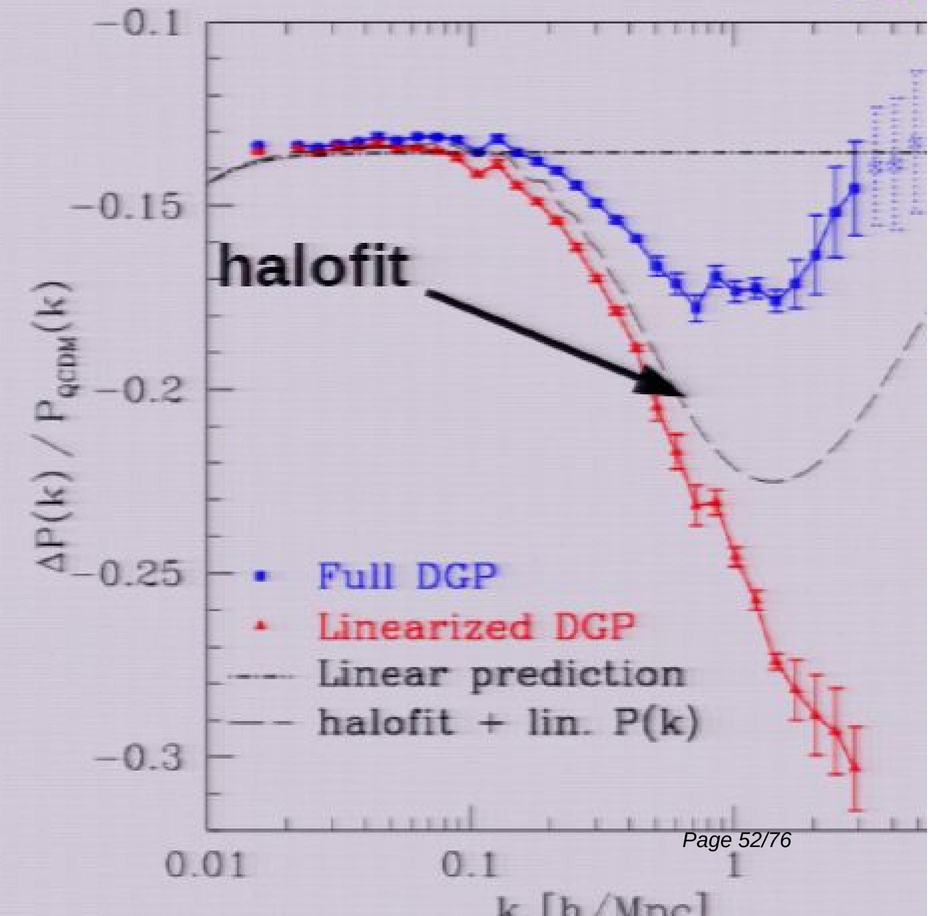

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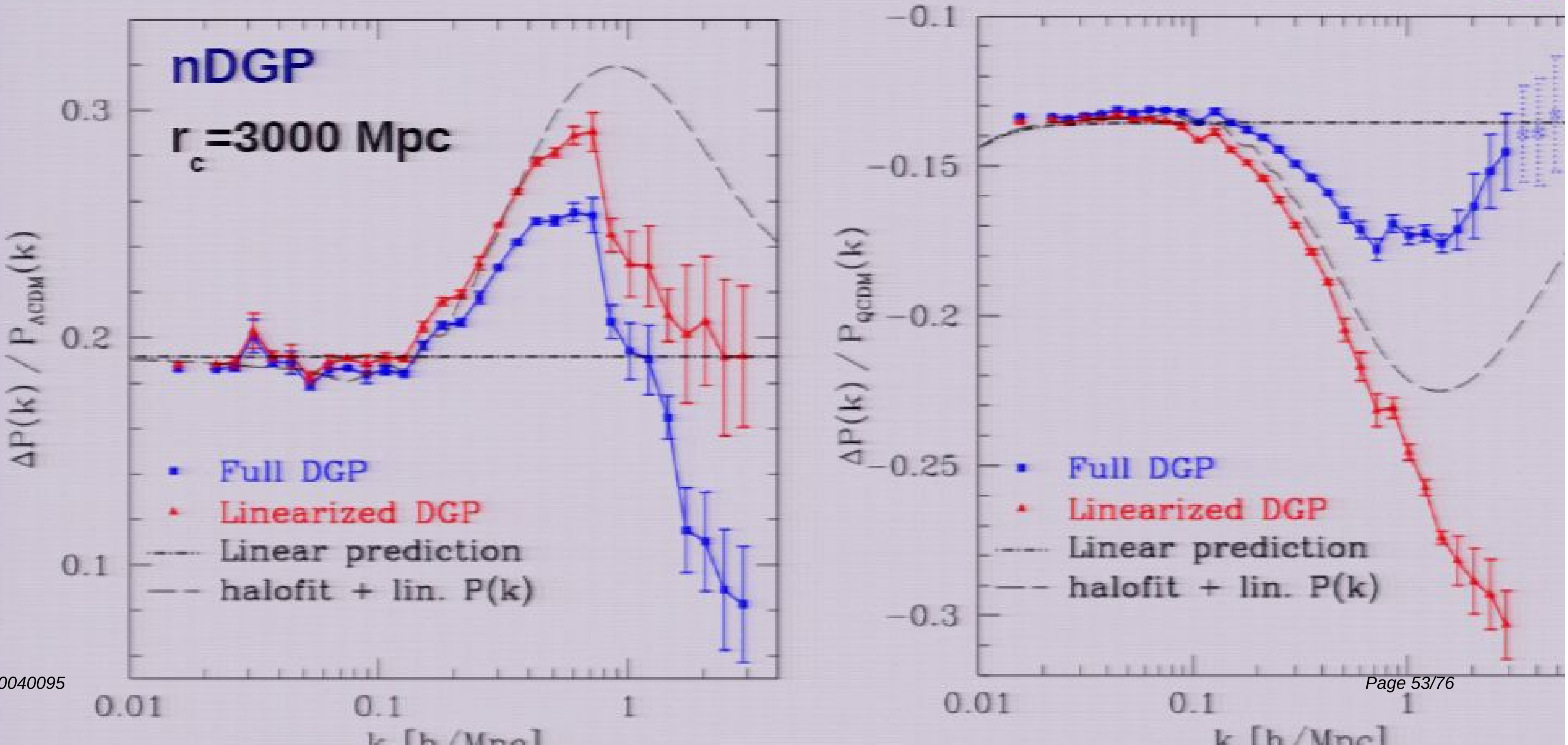
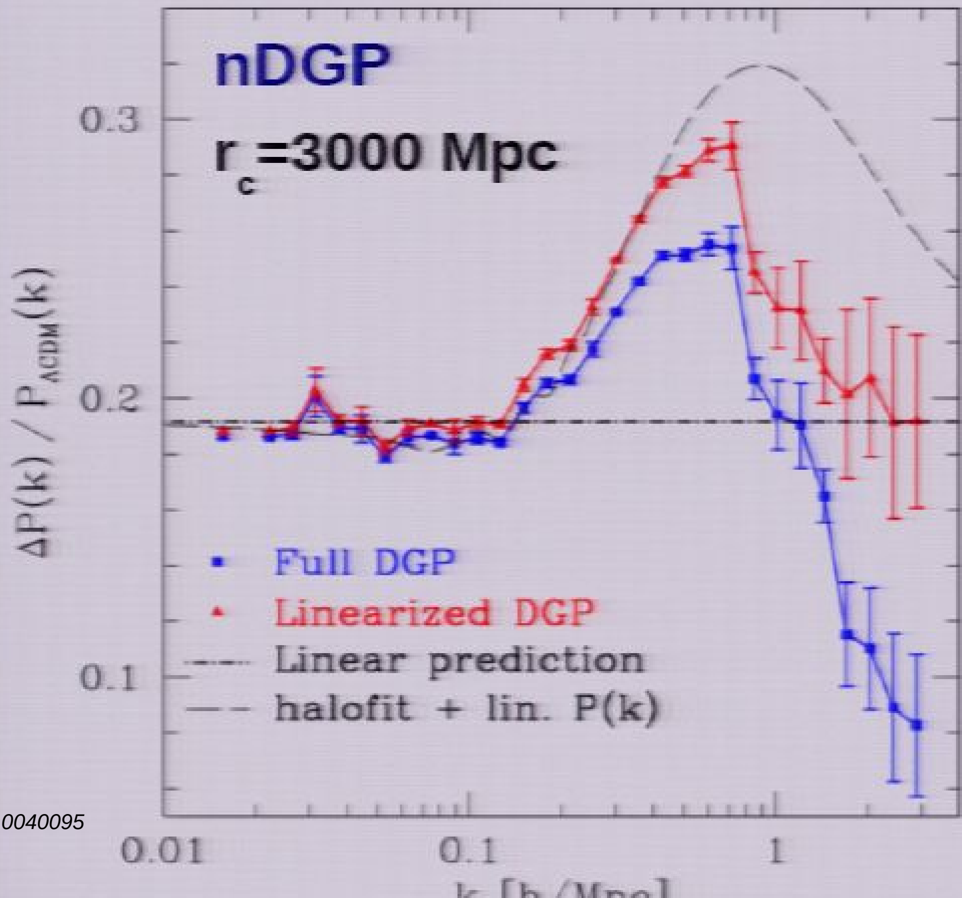


Results: Matter Power Spectrum

- **Can we model DGP effects without running 300 hr simulations ?**



- Understand physics behind DGP effects
- Extend predictions to different cosmological parameter sets

Halo Model of Large-Scale Structure

Goal: map *linear initial density* field to *non-linear large scale structure* today

Ansatz: all matter in bound ***dark matter halos***

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Basic halo properties:

1. **Mass function:** abundance
2. **Halo bias:** clustering
3. **Halo density profiles:** interior matter distribution

Halo Model of Large-Scale Structure

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Ansatz: all matter in bound ***dark matter halos***

Basic halo properties:

1. Mass function

2. Halo bias

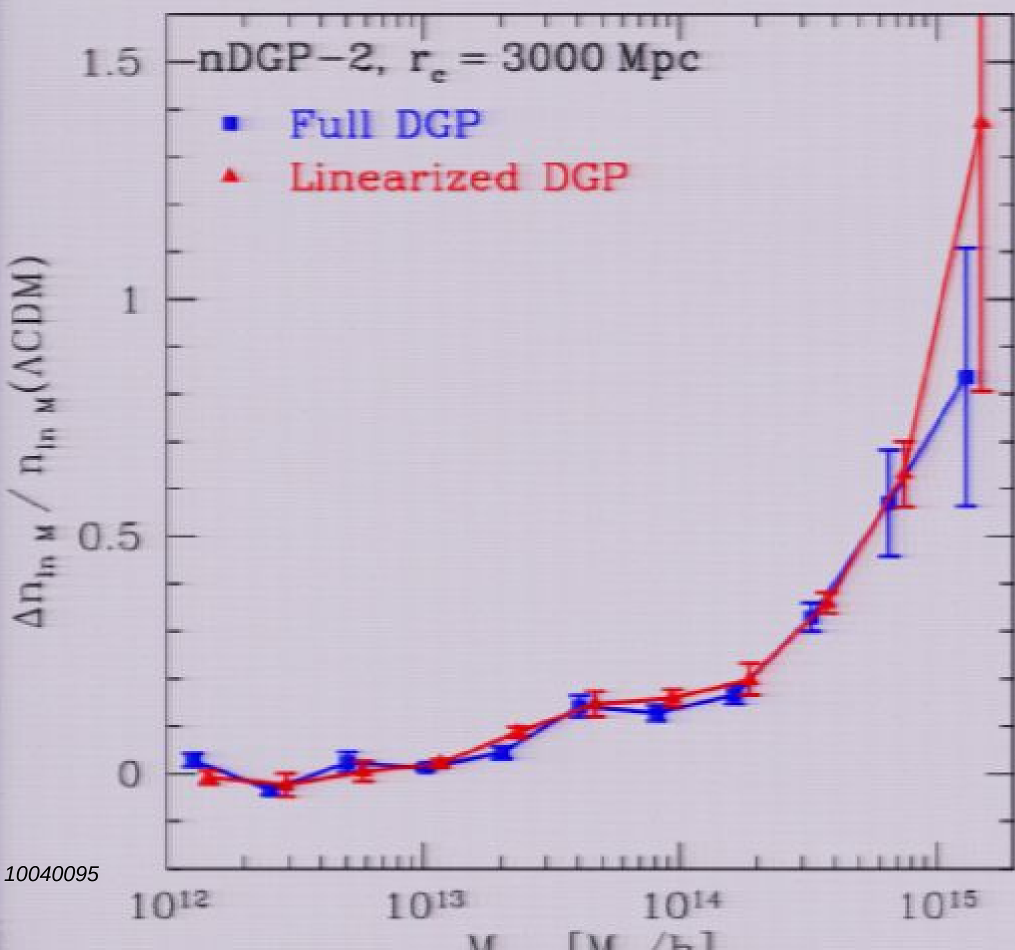
} Spherical Collapse & Press-Schechter Theory (Sheth-Tormen)

3. Halo density profiles

← NFW profile + concentration relatio

Halo mass function in DGP

Sensitive probe of growth of structure



nDGP: relative deviation of dn/dln M from ΛCDM

Order unity enhancement at cluster masses

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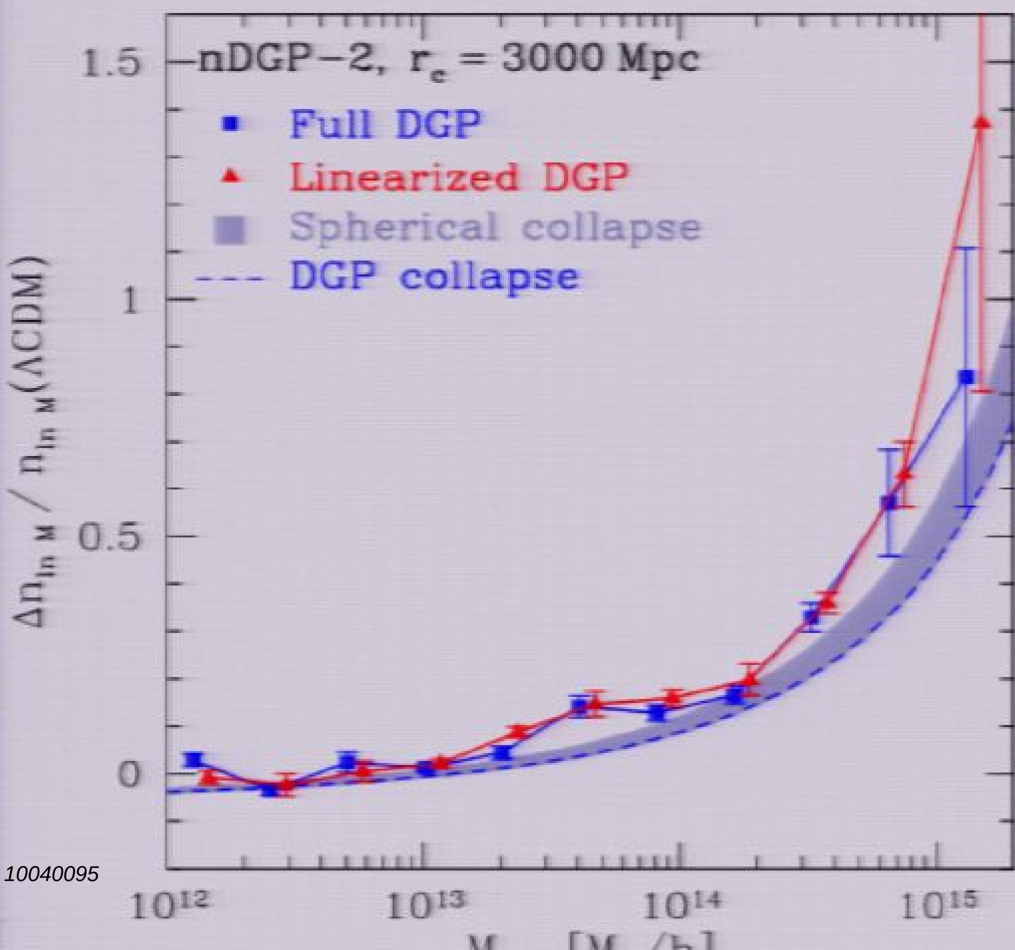
Halo mass function in DGP

Spherical collapse + Sheth-Tormen mass fct.



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Order unity enhancement at cluster masses ~ 10^{14} M_0

Spherical collapse
- range between "full" and no Vainshtein mechanism

FS, et al. 08,
FS, Hu, Lima

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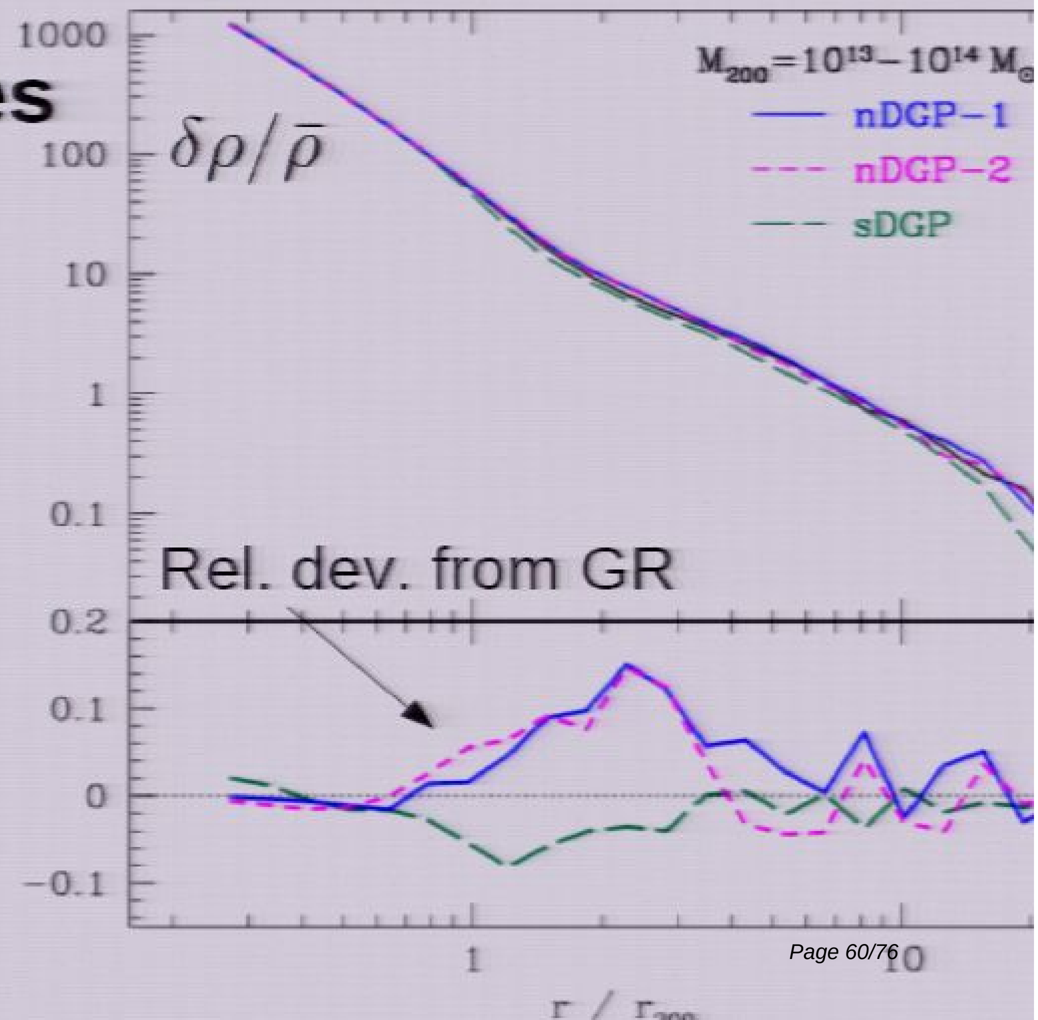
Halo Density Profiles

- **No strong DGP effects in inner cores**
  - Scale radius r_s unchanged
  - Cores formed early
- **Some effects in infall region**
  - at few R_{200}



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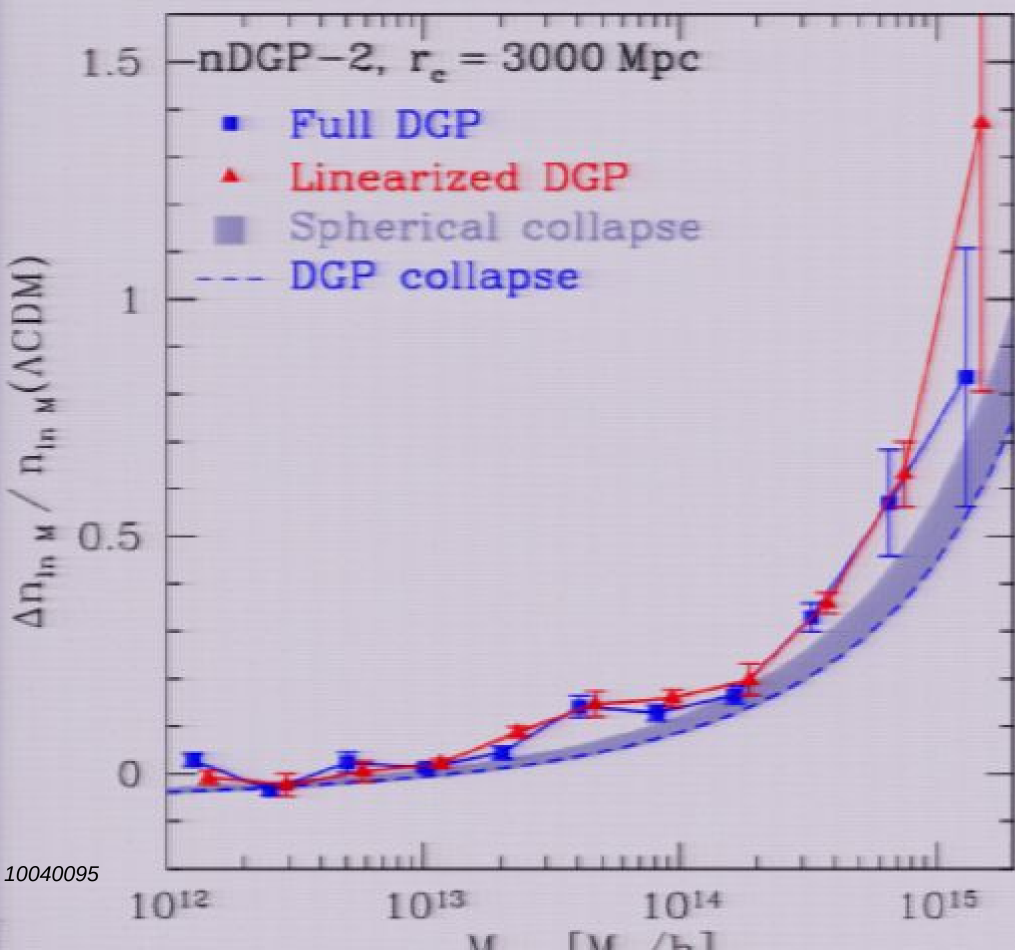

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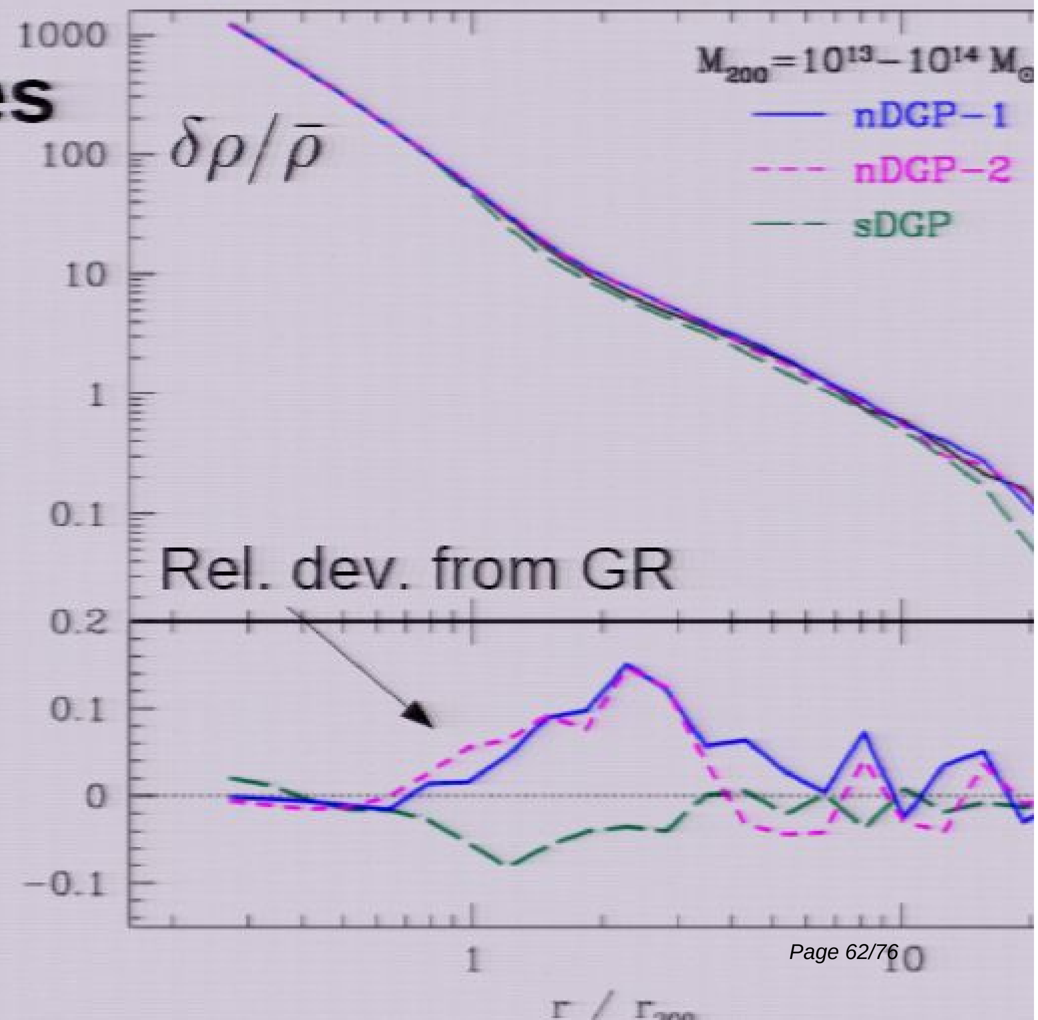


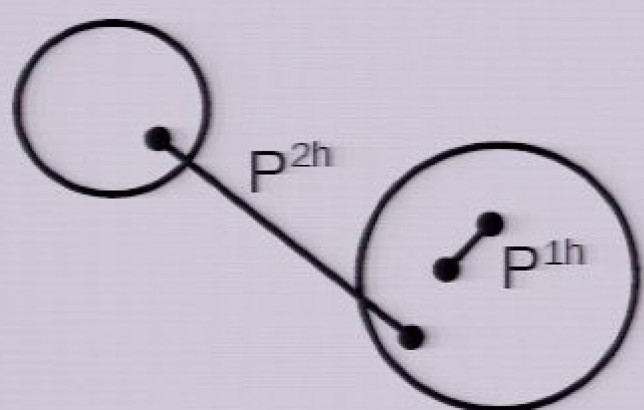
Halo model power spectrum

Halo mass function, bias, + profiles --> P(k)

- $P(k) = P^{2h}(k) + P^{1h}(k)$

2-halo, large scales

1-halo, small scales



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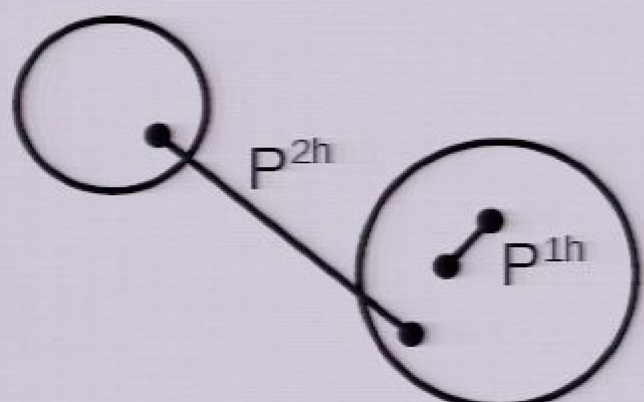

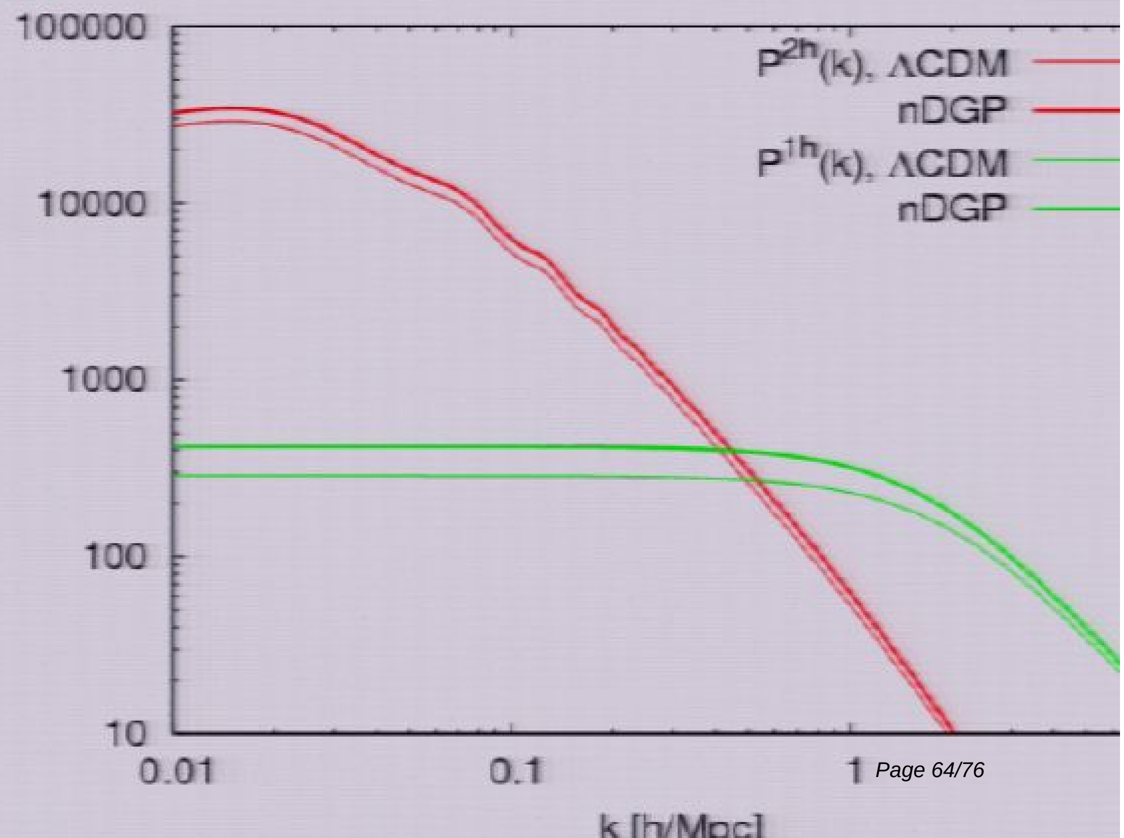

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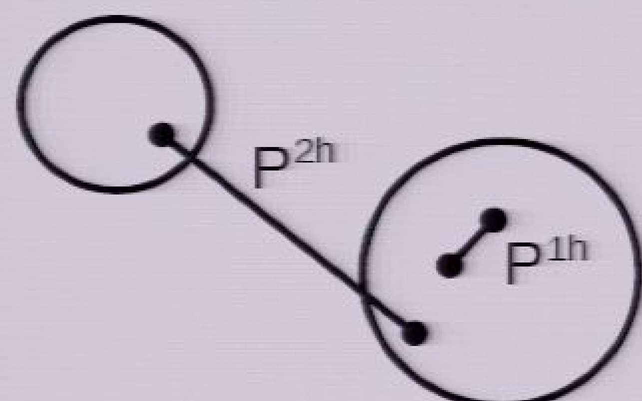
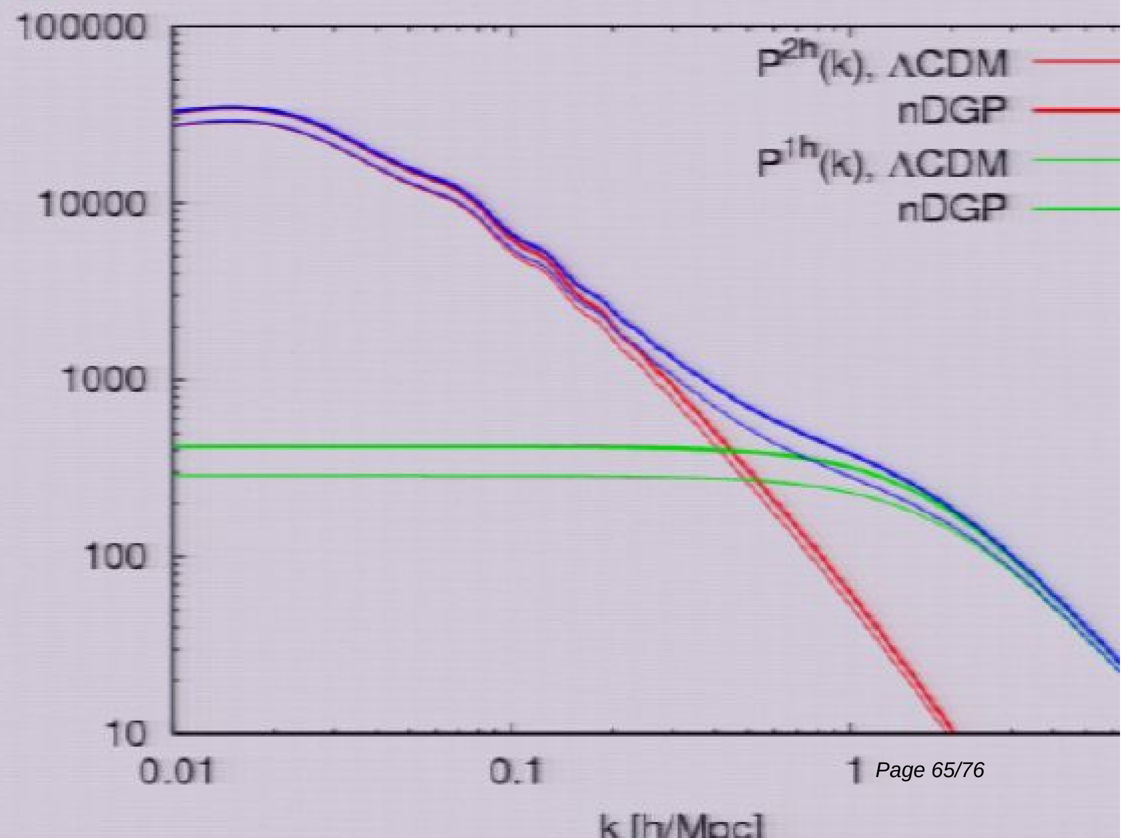




Halo model power spectrum

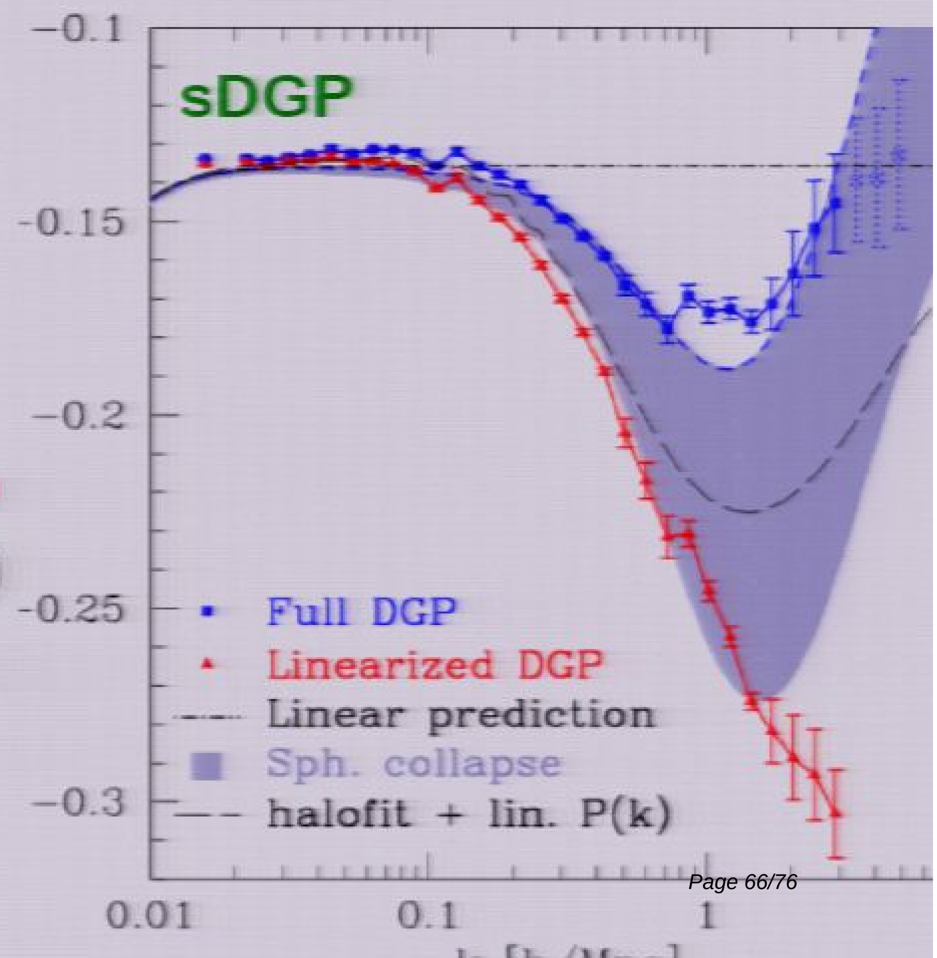
Halo mass function, bias, + profiles --> P(k)

- $P(k) = P^{2h}(k) + P^{1h}(k)$

  2-halo, large scales

  1-halo, small scales

- Assume *unmodified halo profiles* (cf simulations)
- **Excellent match to full sDGP simulations**



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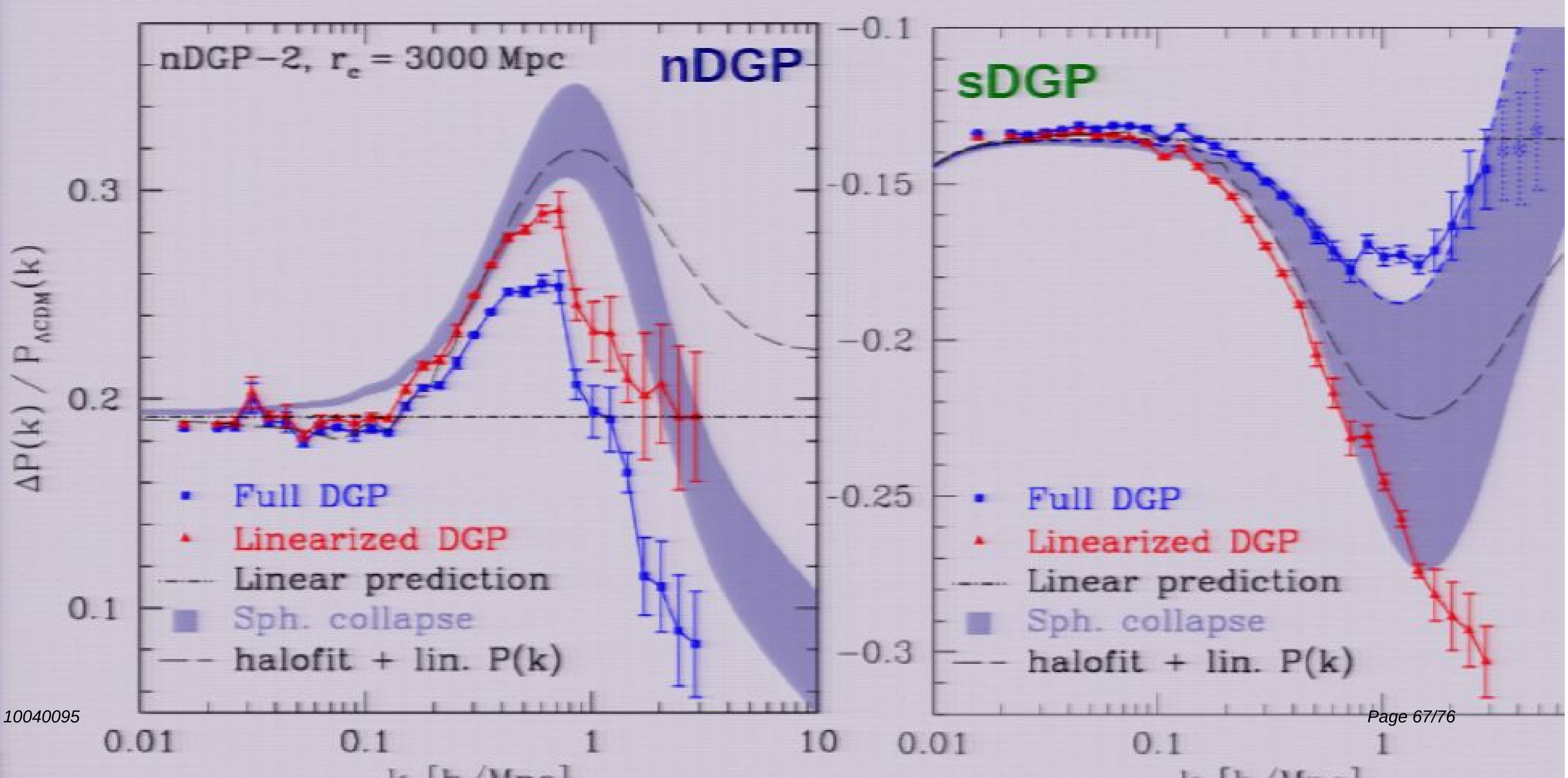

Cosmic Test of Gravity: Dynamics vs Lensing

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  - Propagation of non-rel. objects vs light

Cosmic Test of Gravity: Dynamics vs Lensing

- **Scalar-tensor theories affect *dynamics* differently than *lensing***
  - Propagation of non-rel. objects vs light
- **Can be probed on cosmological scales:**
  - *Large scales:* Redshift distortions and galaxy-galaxy lensing

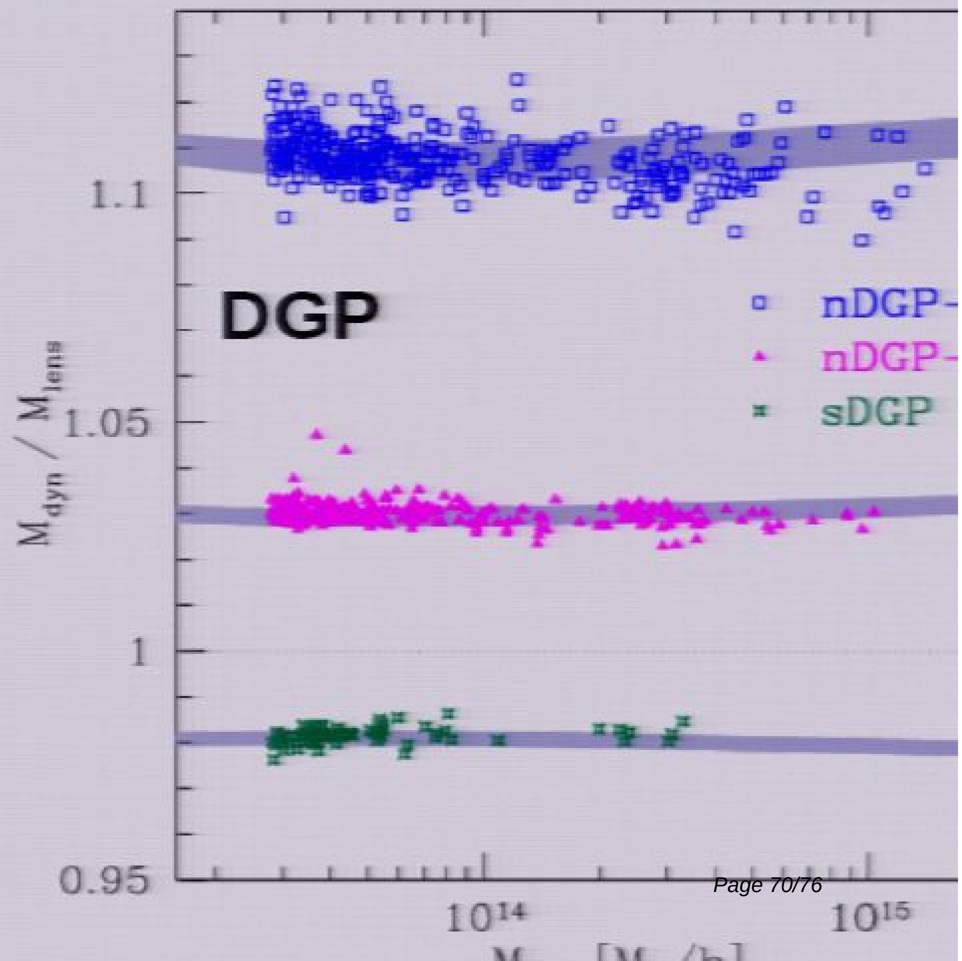
    Zhang et al
    Reyes et al
  - *Small scales:* Lensing mass vs Dynamical mass of clusters,

    FS, '10
    - X-rays, SZ, galaxy velocity dispersions

Dynamical vs lensing mass

arXiv:1003.0

- M_{dyn}/M_{lens}: Analytical models & simulations



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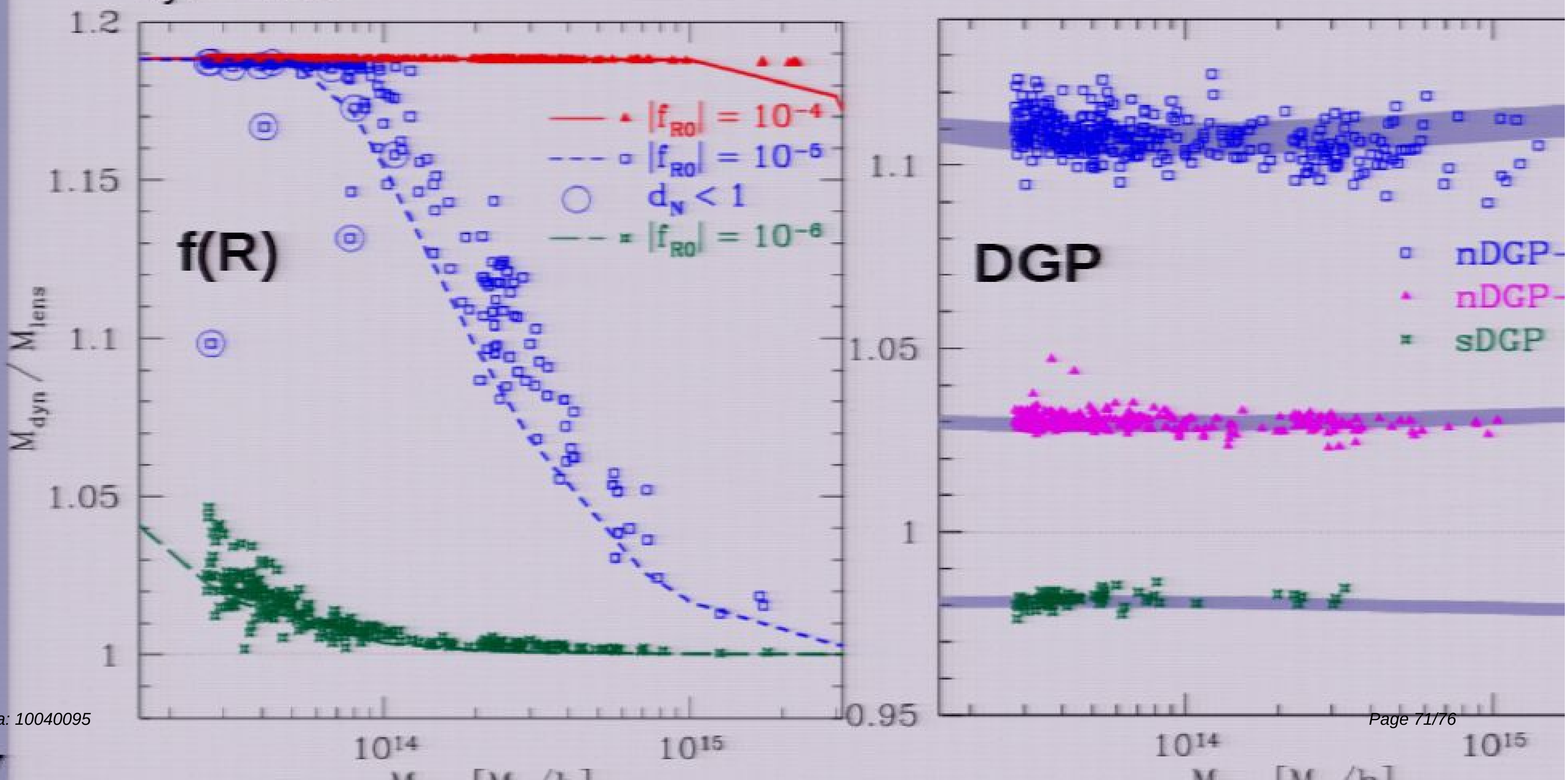
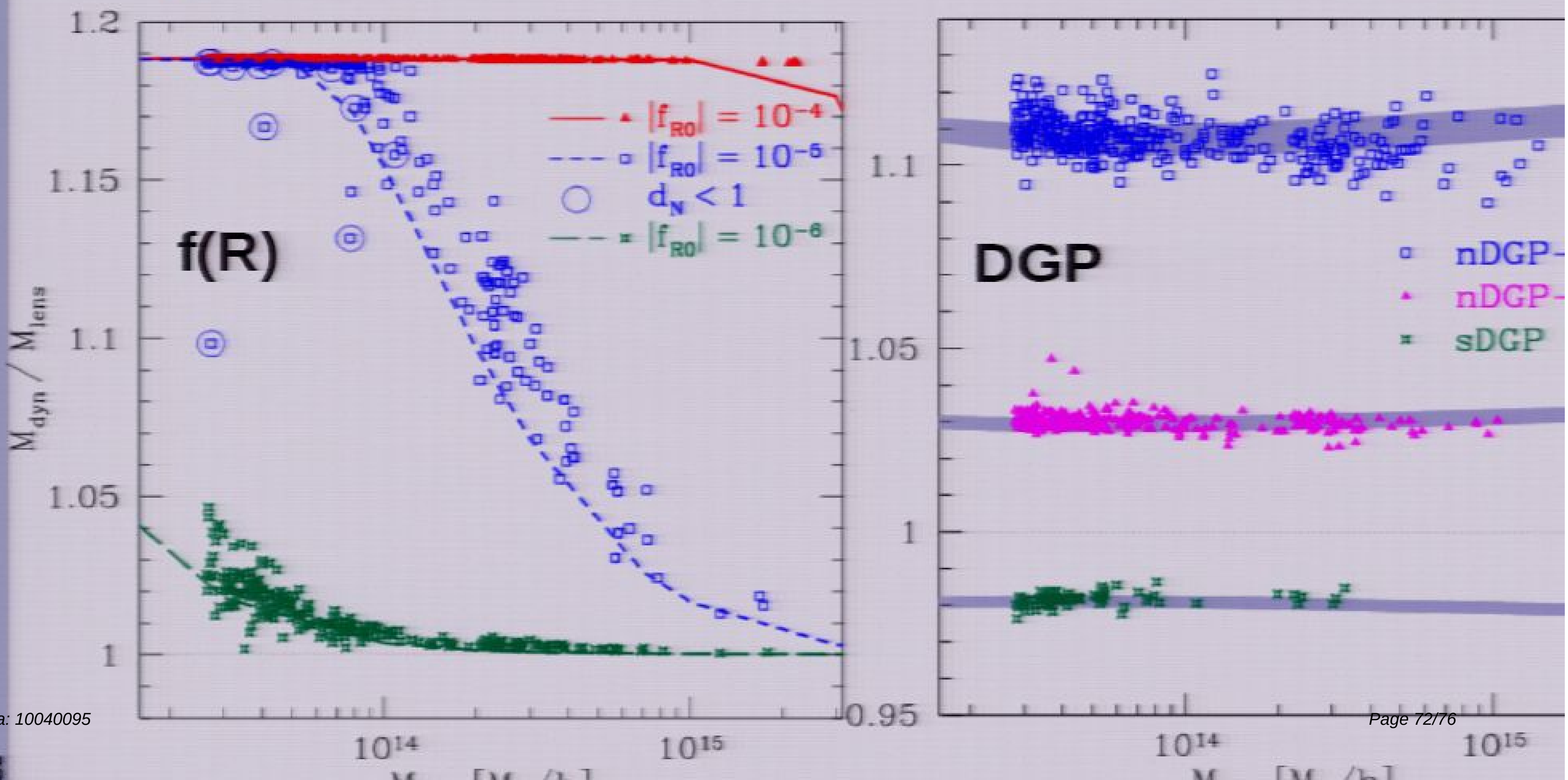


Dynamical vs lensing mass

arXiv:1003.0

- Important for cluster abundance constraints!



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Conclusions

- **Modified Gravity:** fundamental alternative to *Dark Energy*
- **Understanding structure formation in non-linear regime crucial**
  - prove viability of models and place constraints
  - With X-ray clusters, constraints on f(R) already improved by ~1000 over linear regime
- **Simulations of modified gravity are opening the door to probing gravity on Mpc scales**
  - Model-independent parametrization possible ?