Title: TBA

Date: Apr 28, 2010 04:00 PM

URL: http://pirsa.org/10040094

Abstract: TBA

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There are basically three derivations of this effect:

Peak Background Split (PBS): objects correspond to $\,\delta_{
m lin} > \delta_c$

$$\Delta b_1(k, f_{\rm NL}) = \frac{2f_{\rm NL}}{M(k)}(b_{10} - 1)\delta_c$$

Gaussian Field Peaks in high-threshold limit $(\nu \gg 1)$

$$\Delta b_1(k, f_{\rm NL}) = \frac{2f_{\rm NL}}{M(k)} \nu^2$$

Local Eulerian bias model

$$(\delta_g = b_1 \delta + \frac{b_2}{2} \delta^2 + \ldots)$$

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here M relates the density to the Bardeen potential through the Poisson eqn

$$M(k) = \frac{2c^2k^2T(k)D(z)}{3\Omega_m H_0^2} \sim k^2 \qquad (k \to 0)$$

n local Eulerian models and peaks there is a generic formula (for any type primordial non-Gaussianity) for the low-k power change

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$$\phi = \phi_{\ell} + \phi_s$$

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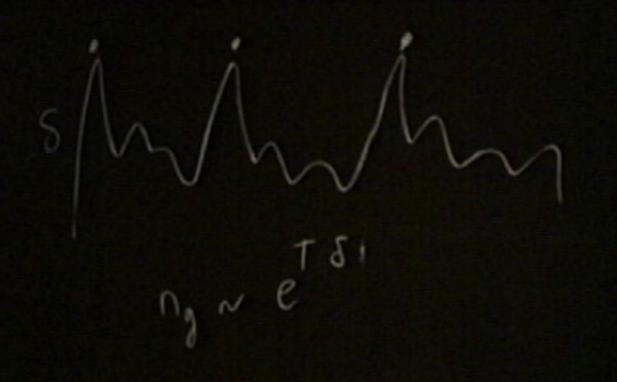
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all three formulae agree in the high-peak limit!

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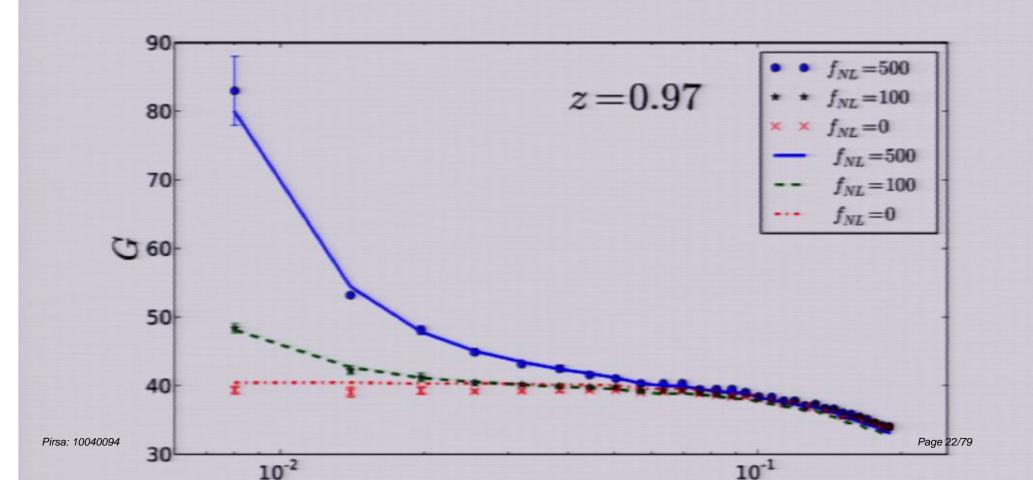
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Halo propagator in N-body simulations:

learly, local models are wrong...



Large-Scale Bias in non-local PNG

In single-field inflationary models, we are instead interested in models that correspond to non-local PNG. For example, the equilateral model has a Bardeen potential bispectrum,

$$(6f_{\rm NL})^{-1}B_{\rm equil} = -P_1P_2 - 2(P_1P_2P_3)^{2/3} + P_1^{1/3}P_2^{2/3}P_3$$

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$$(6f_{\rm NL})^{-1}B_{\rm ortho} = -3P_1P_2 - 8(P_1P_2P_3)^{2/3} + 3P_1^{1/3}P_2^{2/3}P_3$$

We are interested in generating such bispectra from quadratic (non-local) nodels, i.e.

$$\Phi = \phi + f_{\rm NL} K[\phi, \phi]$$

where K is the appropriate non-local quadratic kernel that generates the desired bispectrum. For simplicity we assume scale-invariance.

Introduce some handy non-local operators

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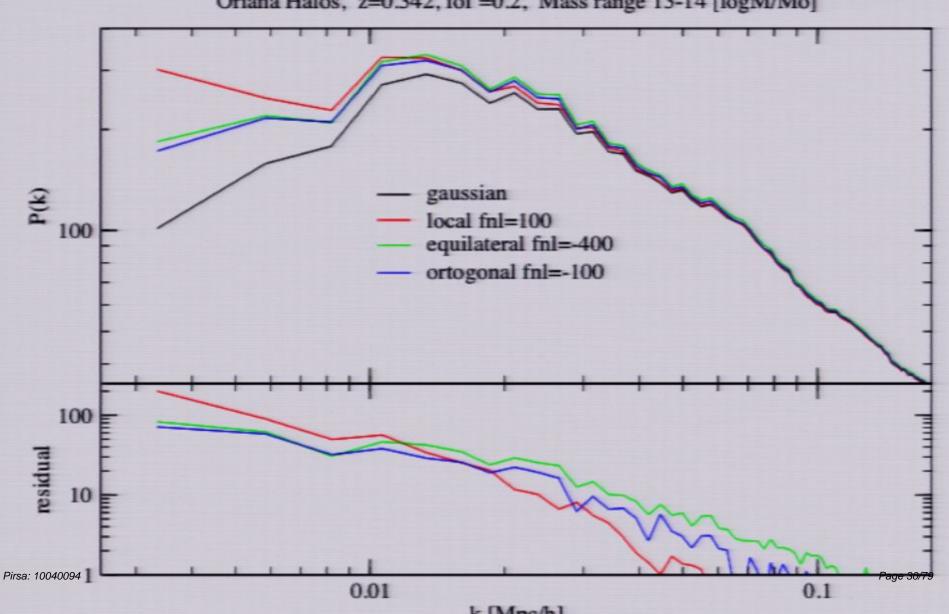
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Power Spectrum for non-gaussian models

Oriana Halos, z=0.342, fof =0.2, Mass range 13-14 [logM/Mo]



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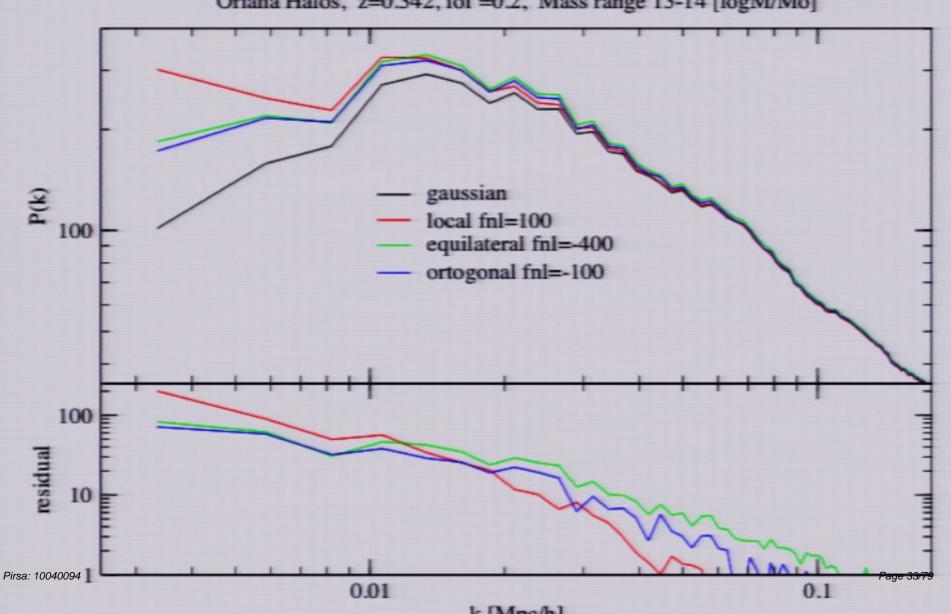
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Page 38/79

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Large-Scale Bias in non-local PNG

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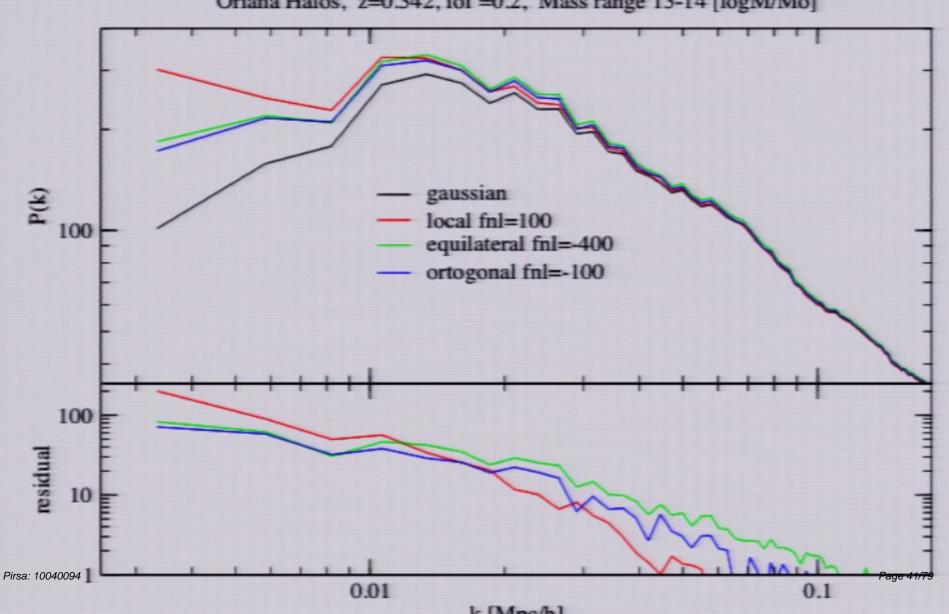
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Power Spectrum for non-gaussian models

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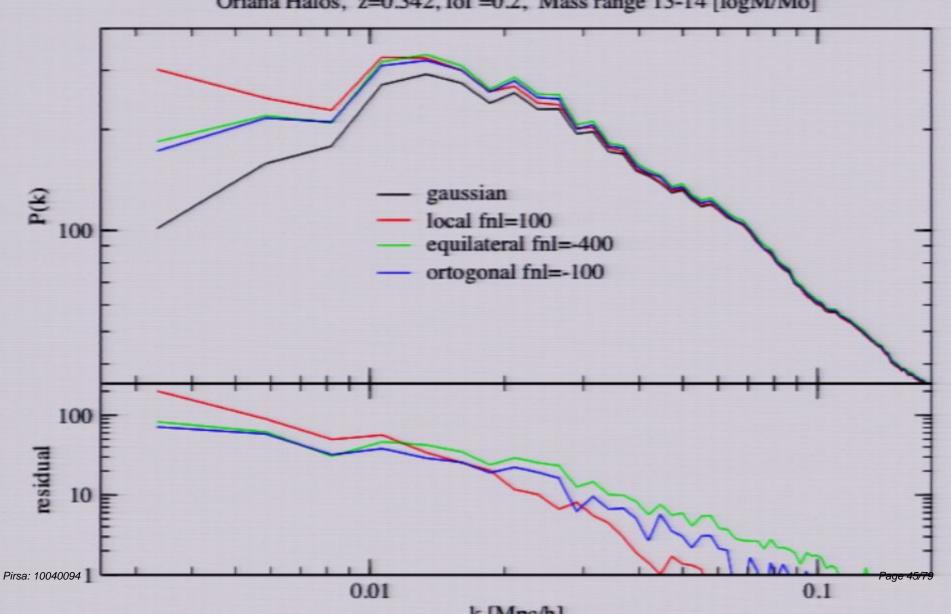
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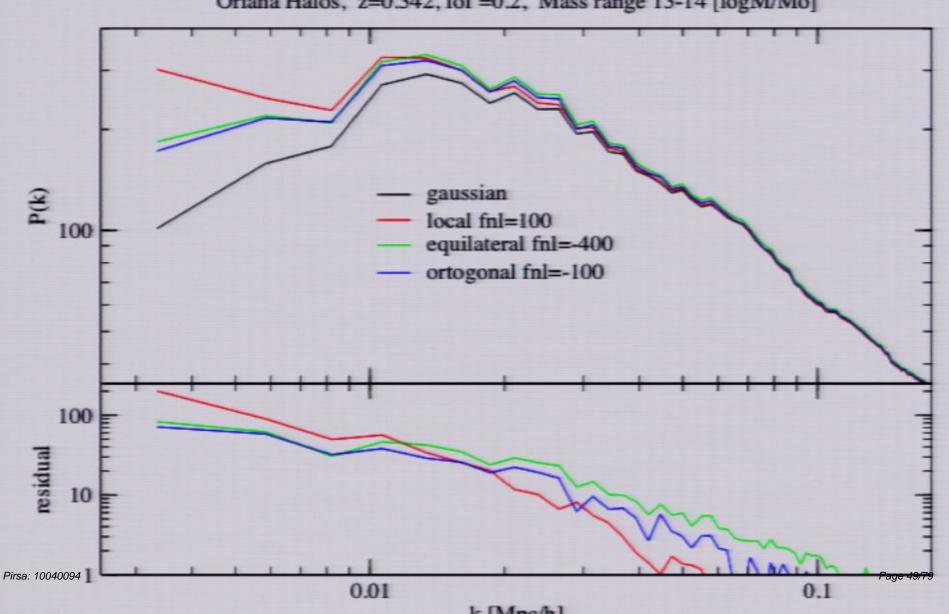
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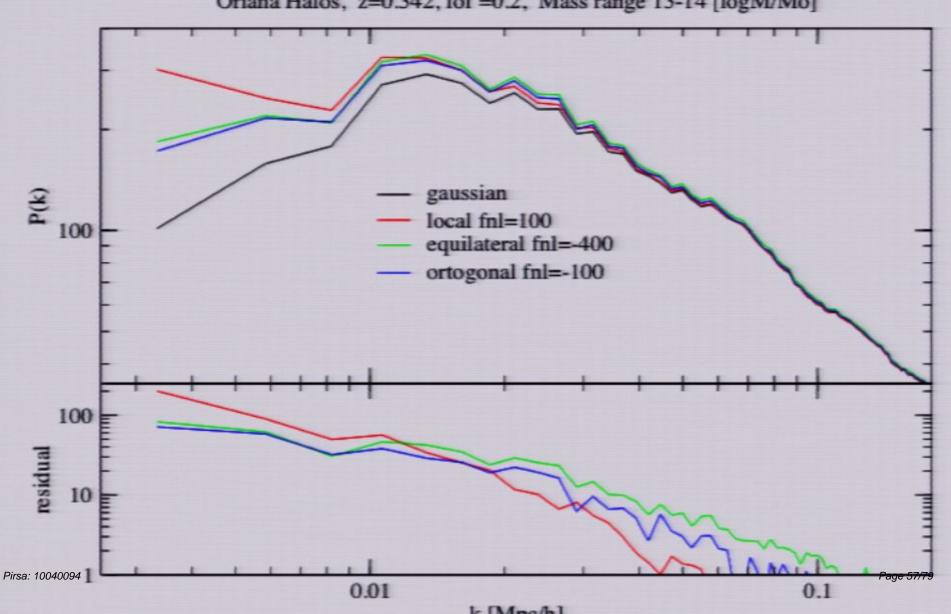
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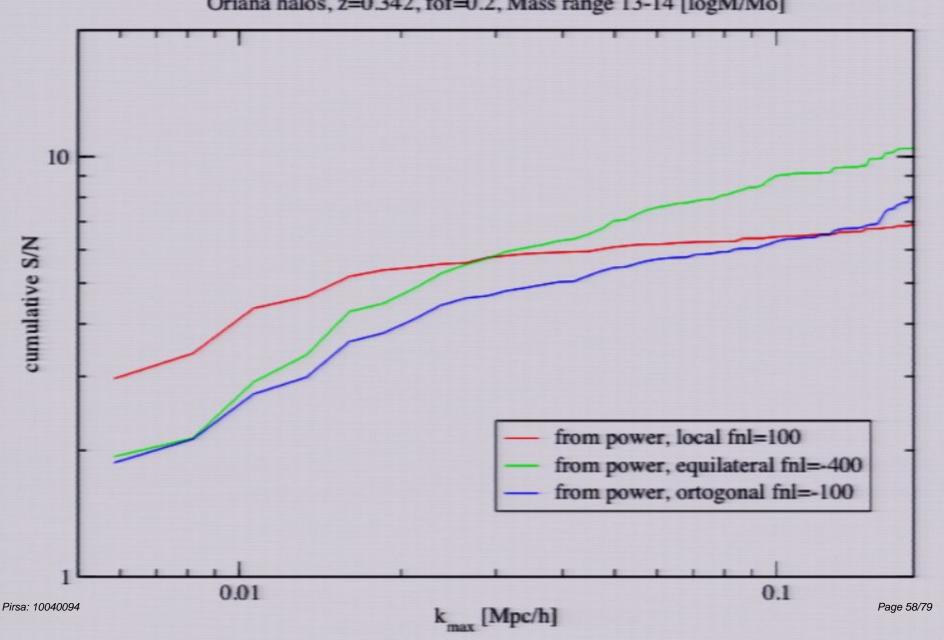
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Oriana Halos, z=0.342, fof =0.2, Mass range 13-14 [logM/Mo]



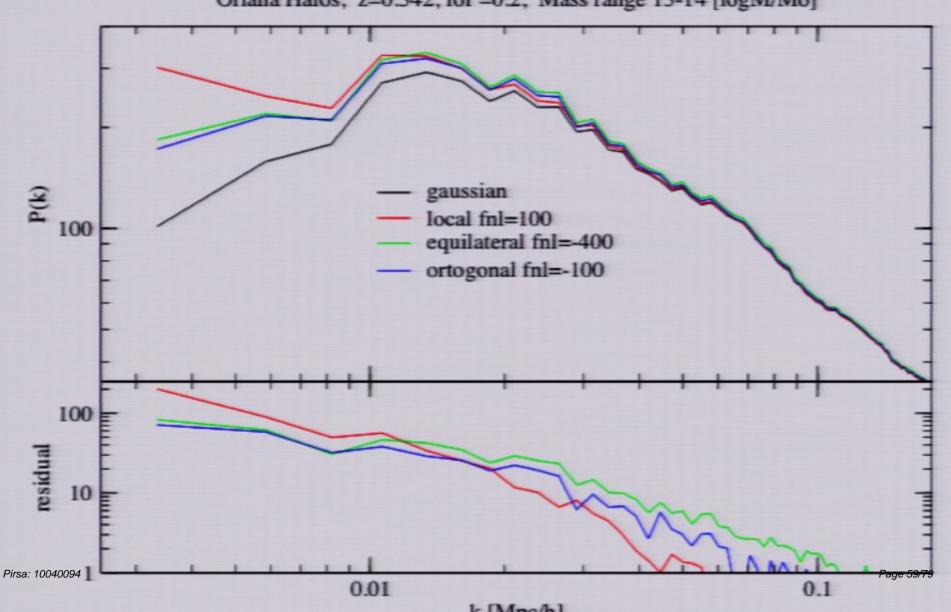
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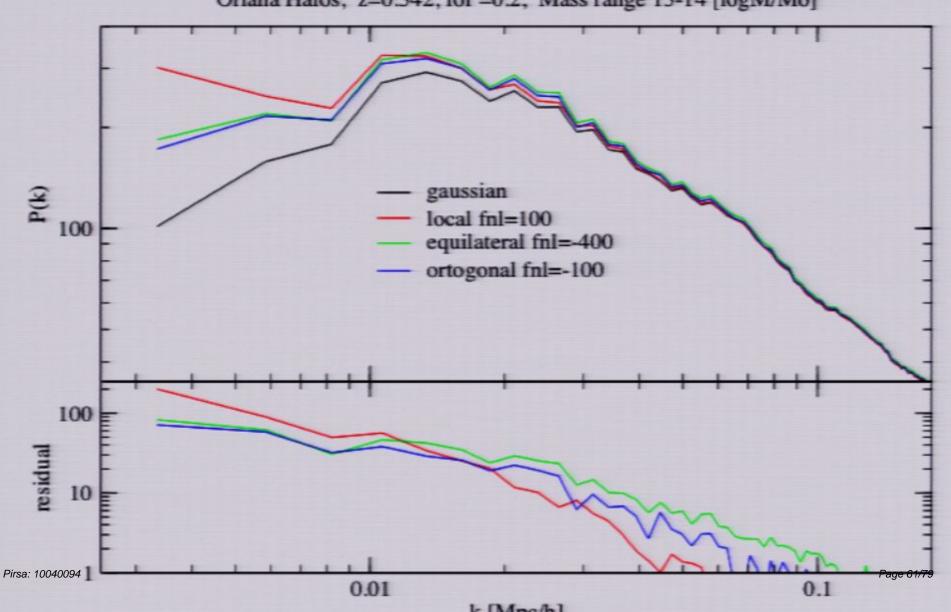
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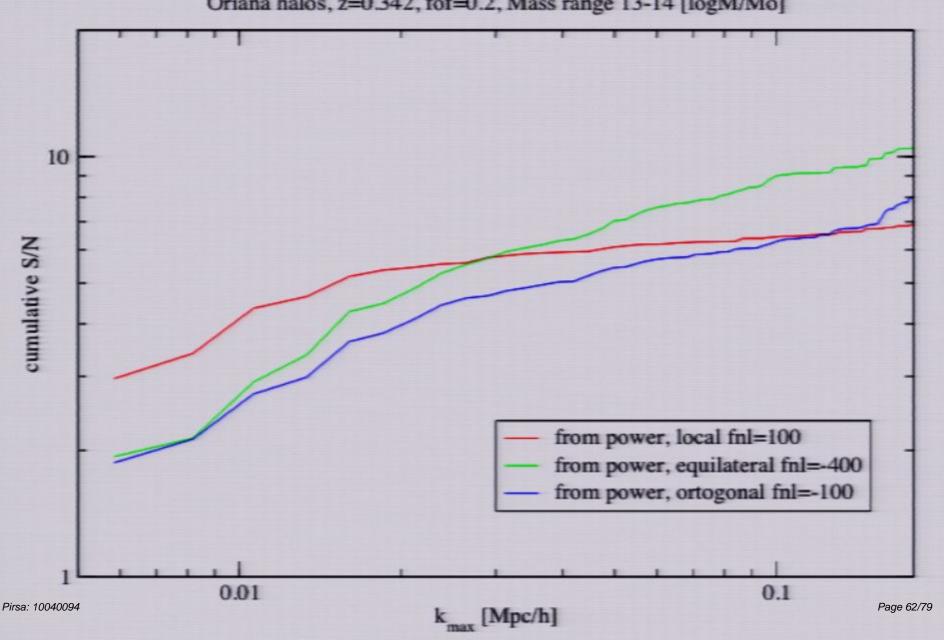
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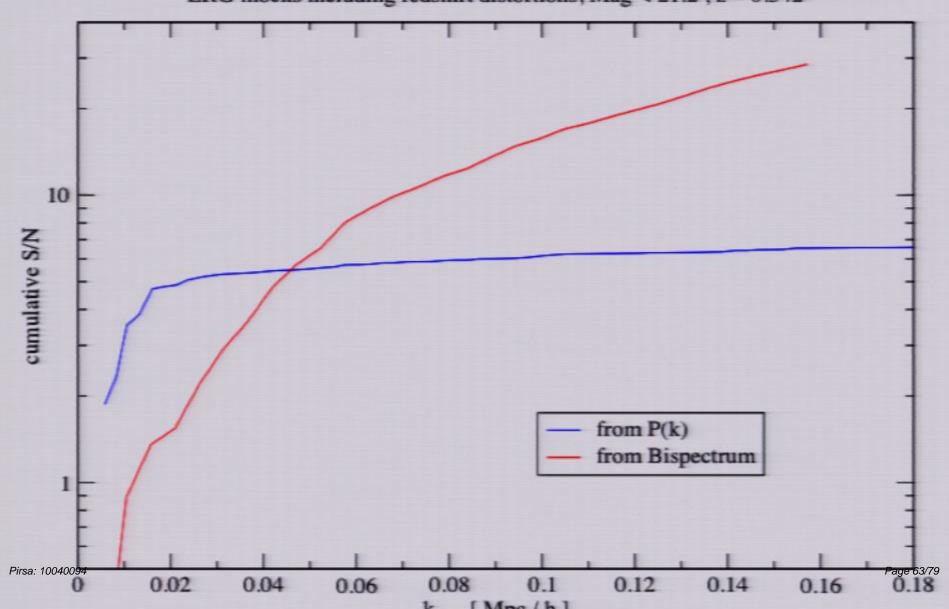


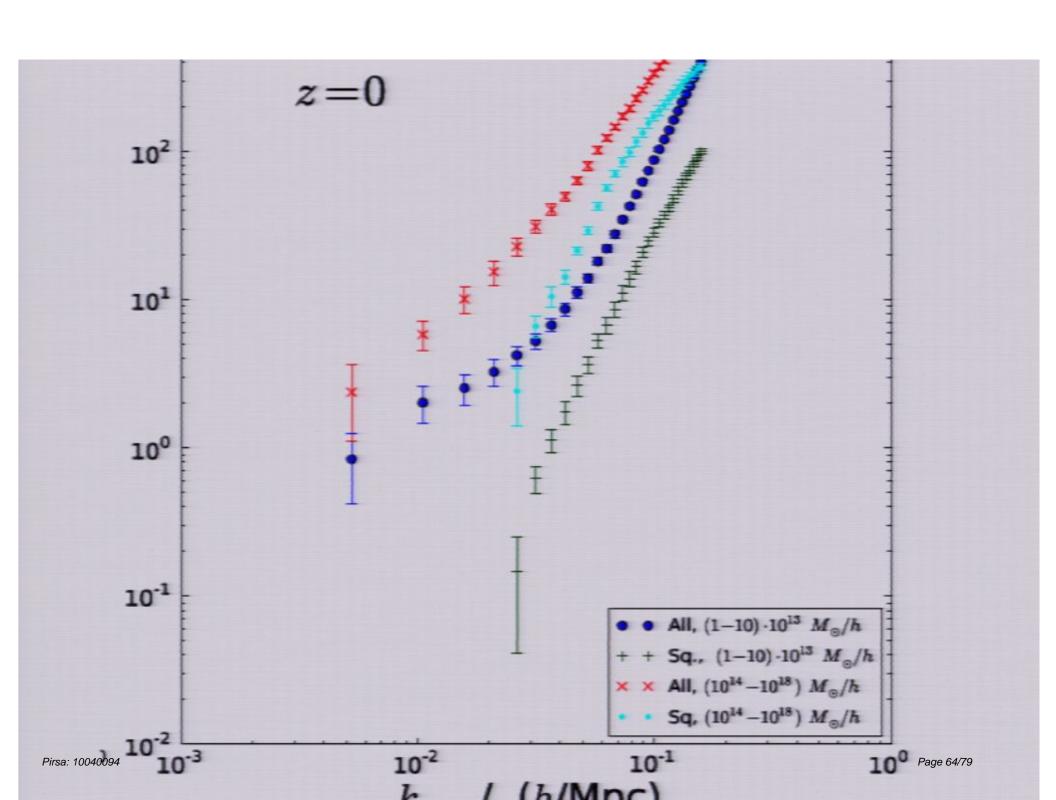
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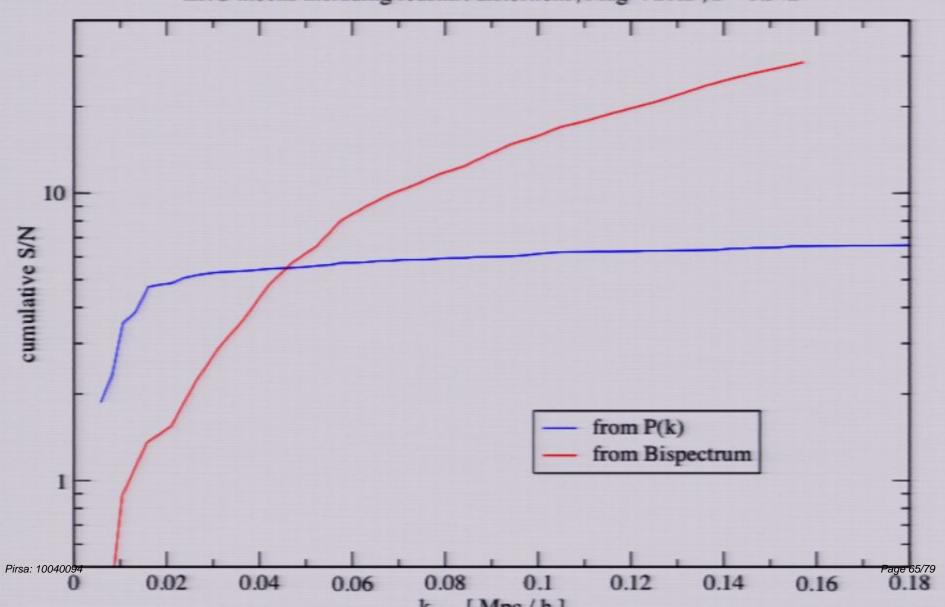


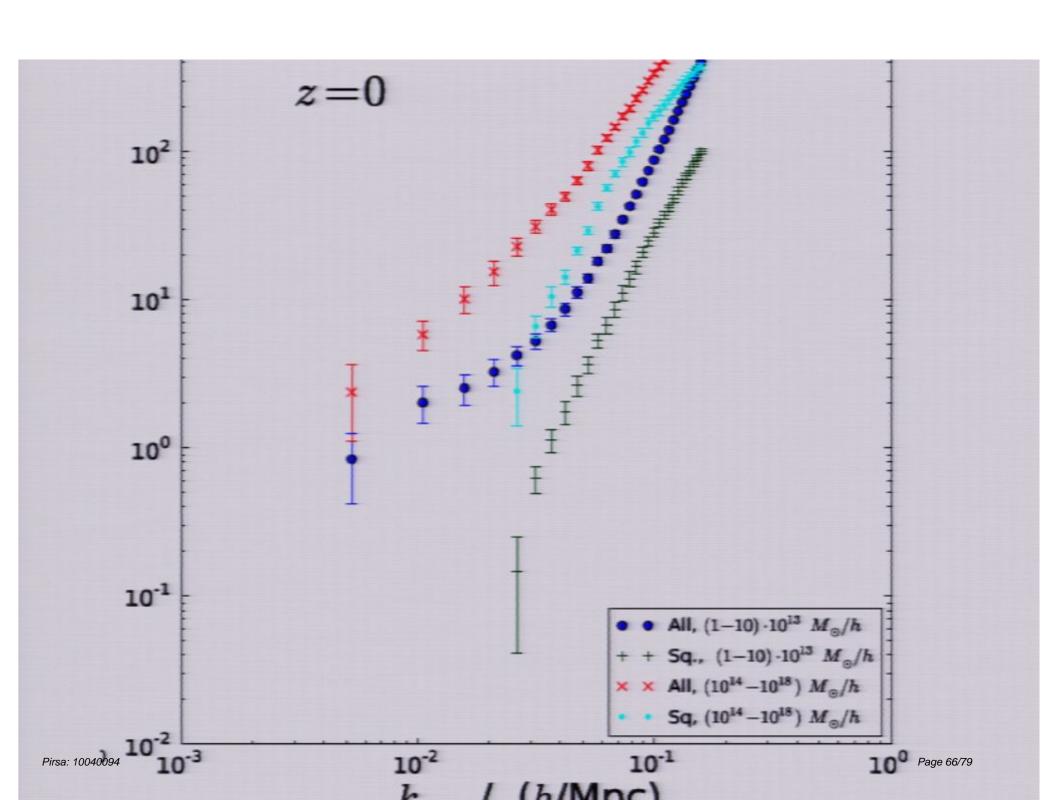
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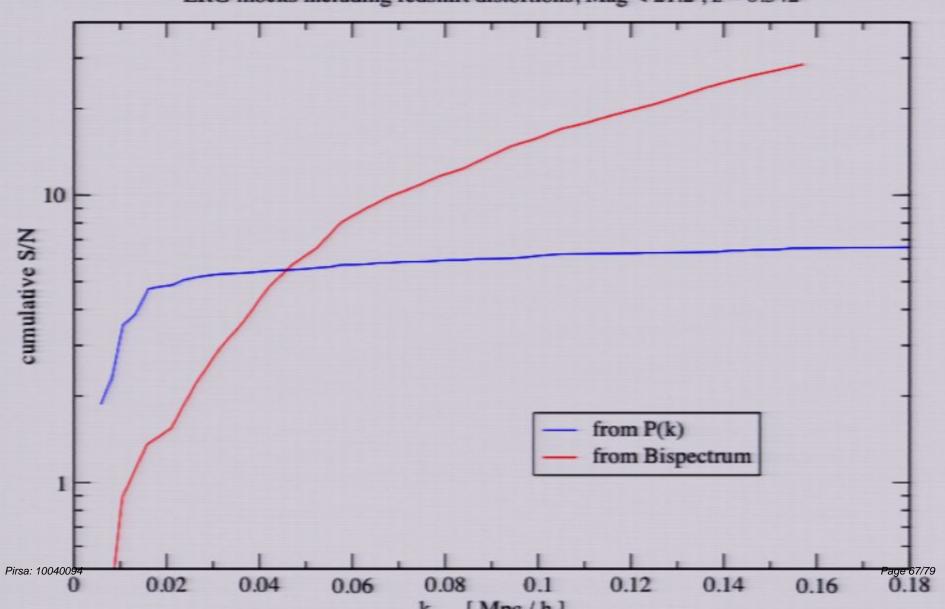


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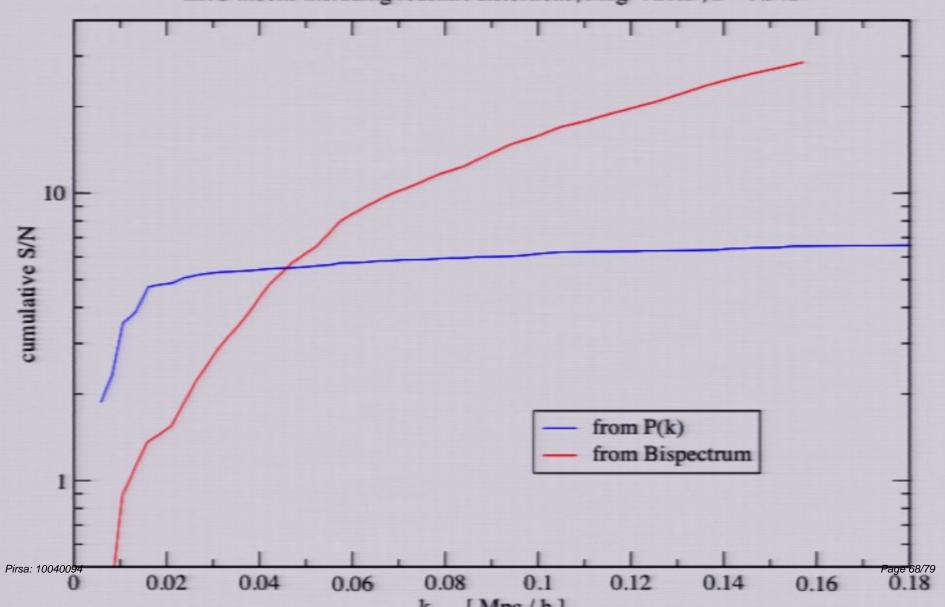


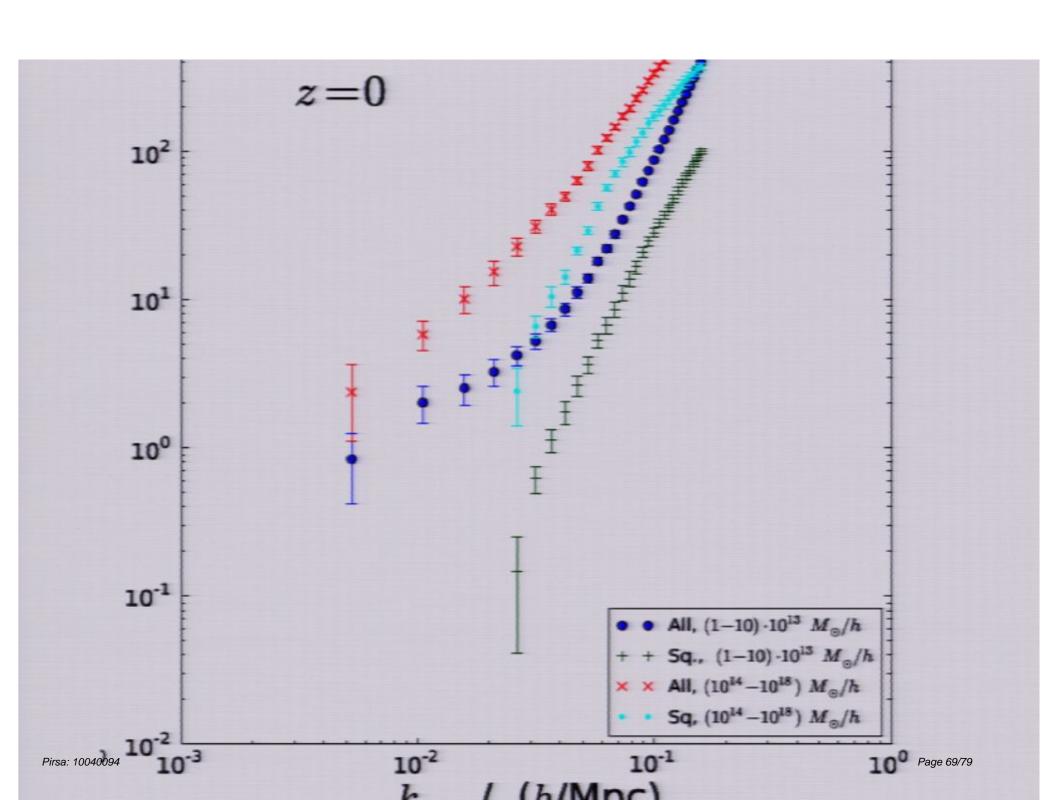


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Conclusions

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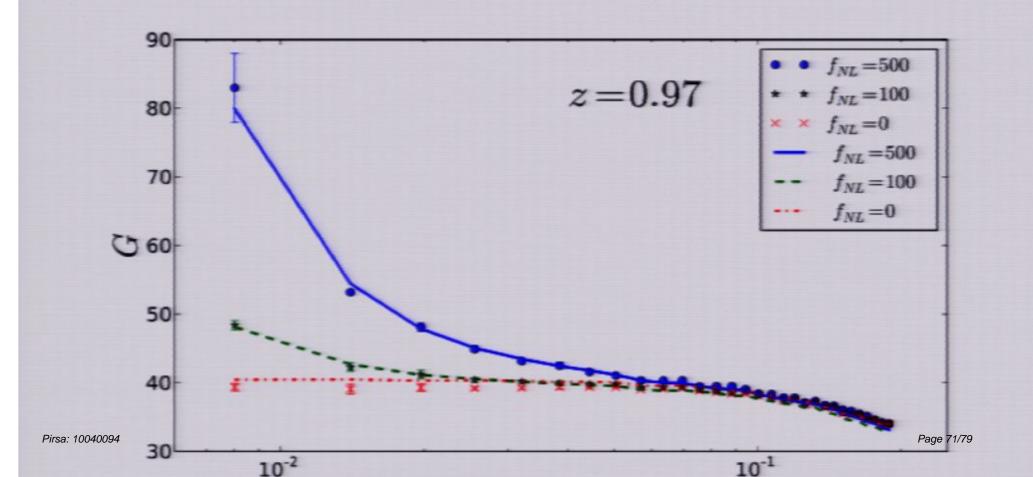
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Halo propagator in N-body simulations:

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Large-Scale Bias in local PNG

In local models of primordial non-Gaussianity (PNG) we have for the Bardeen potential,

$$\Phi = \phi + f_{\rm NL}\phi^2$$

which implies for it a bispectrum,

$$B = 2f_{\rm NL}P_1P_2 + {\rm cyc.}$$

For biased tracers (galaxies, halos), this model leads to a scale-dependent bia it large scales (Dalal et al 2008),

$$b_1(k) = b_{10} + \Delta b_1(k, f_{\rm NL})$$

where b~I/k^2 at low-k.

There are basically three derivations of this effect:

Peak Background Split (PBS): objects correspond to $\,\delta_{
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