Title: Holography for Cosmology

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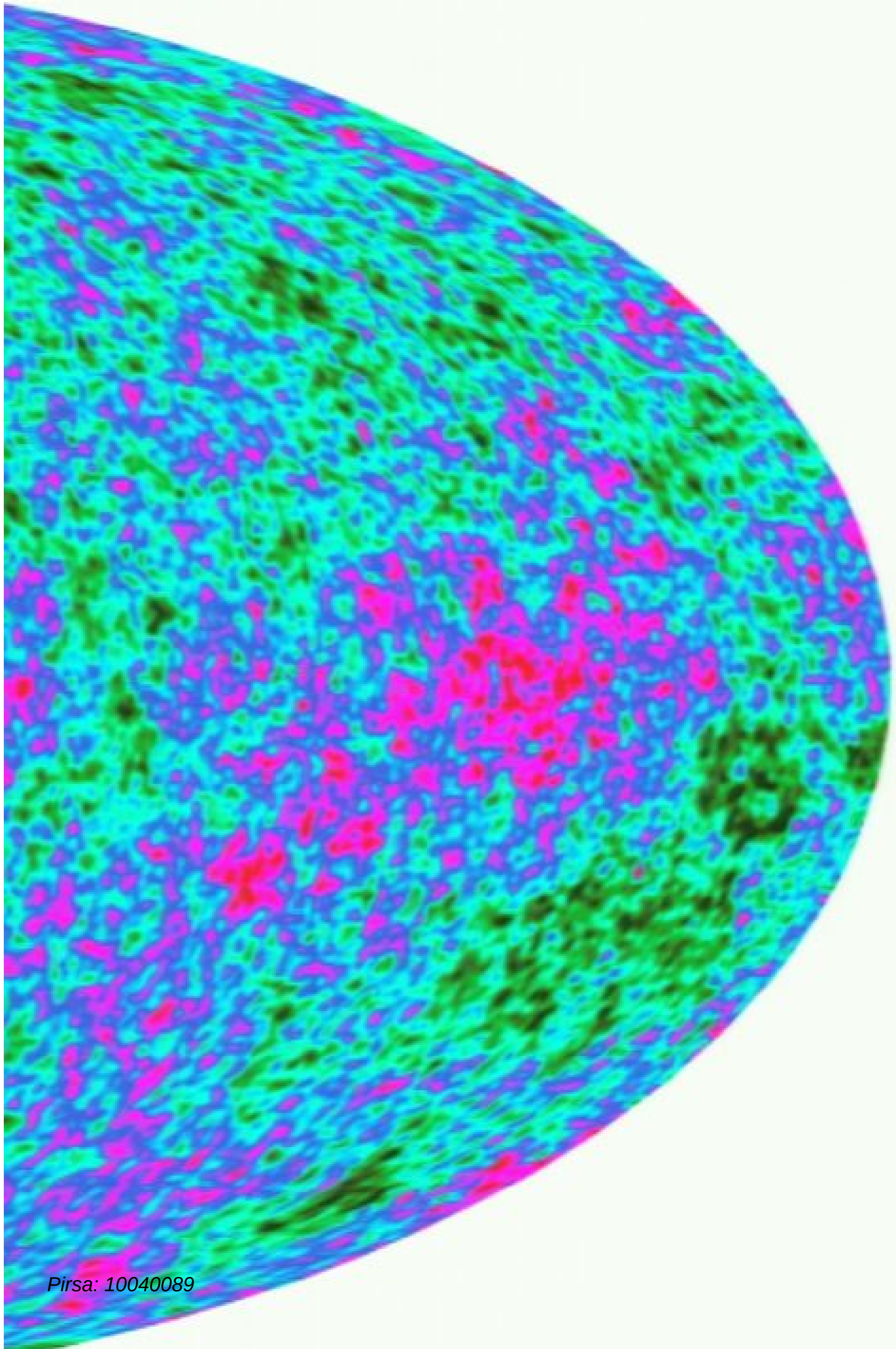
Abstract: TBA



Holography for Cosmology

Paul McFadden
Universiteit van Amsterdam

work with Kostas Skenderis
arXiv:0907.5542 & 1001.2007

Holography

Holography states that any quantum theory of gravity should have a dual description in terms of a QFT (*without* gravity) in one dimension less.

Examples found in string theory involve spacetimes with a negative cosmological constant (e.g. AdS/CFT).

- Here we propose a holographic framework for inflationary cosmology.

Specifically, we seek a dual description of four-dimensional inflationary cosmology in terms of a three-dimensional QFT (without gravity).

Holography *for Cosmology*

Any proposed holographic framework for cosmology should specify:

1. The precise nature of the dual QFT.
2. How to compute cosmological observables (e.g. the primordial power spectrum) from the correlation functions of the dual QFT.

Having defined such a duality,

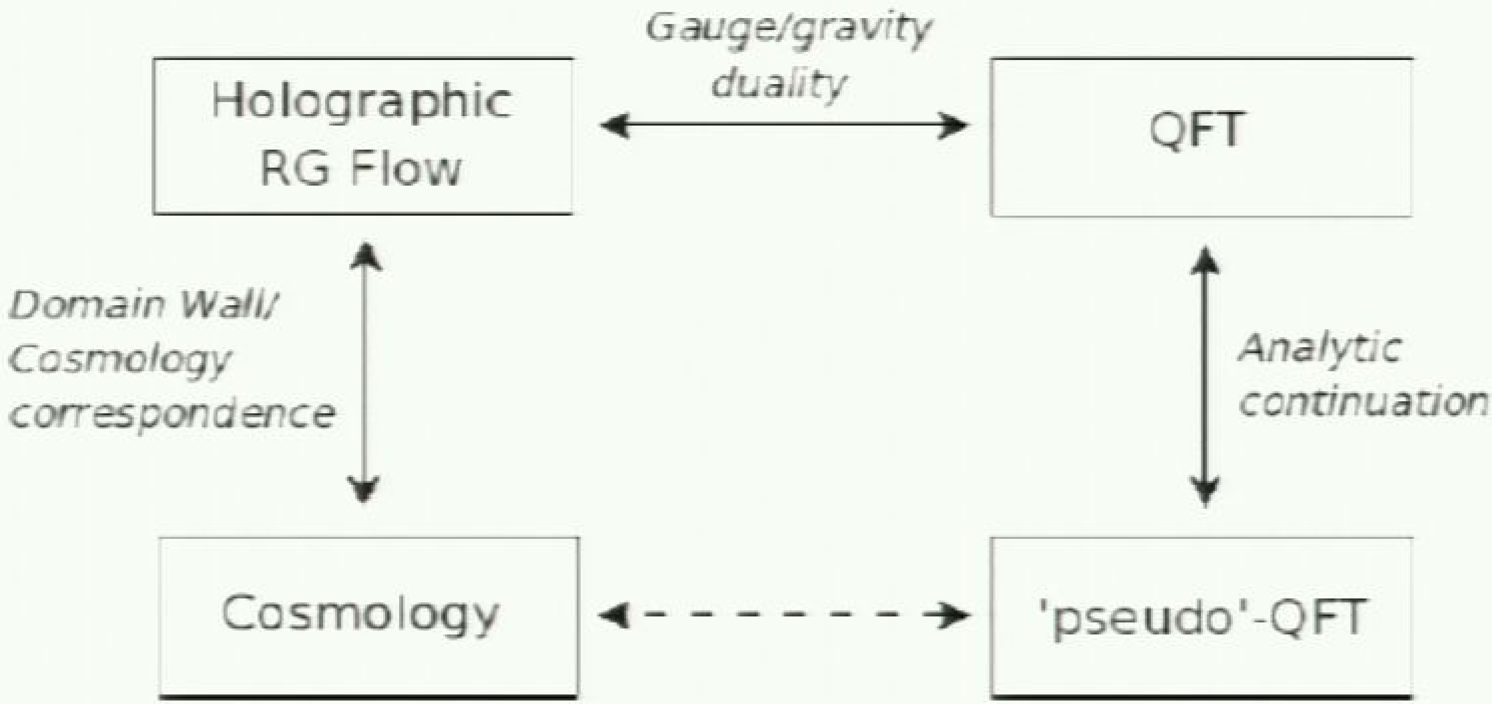
3. Must recover standard inflationary predictions in their regime of validity (namely, when a perturbative quantisation of fluctuations is possible, i.e. weakly coupled gravity = strongly coupled QFT).

Strong gravity

New results then follow by applying the holographic framework in the opposite regime where gravity is *strongly coupled* and a perturbative quantisation of fluctuations breaks down. The dual QFT is then *weakly coupled*.

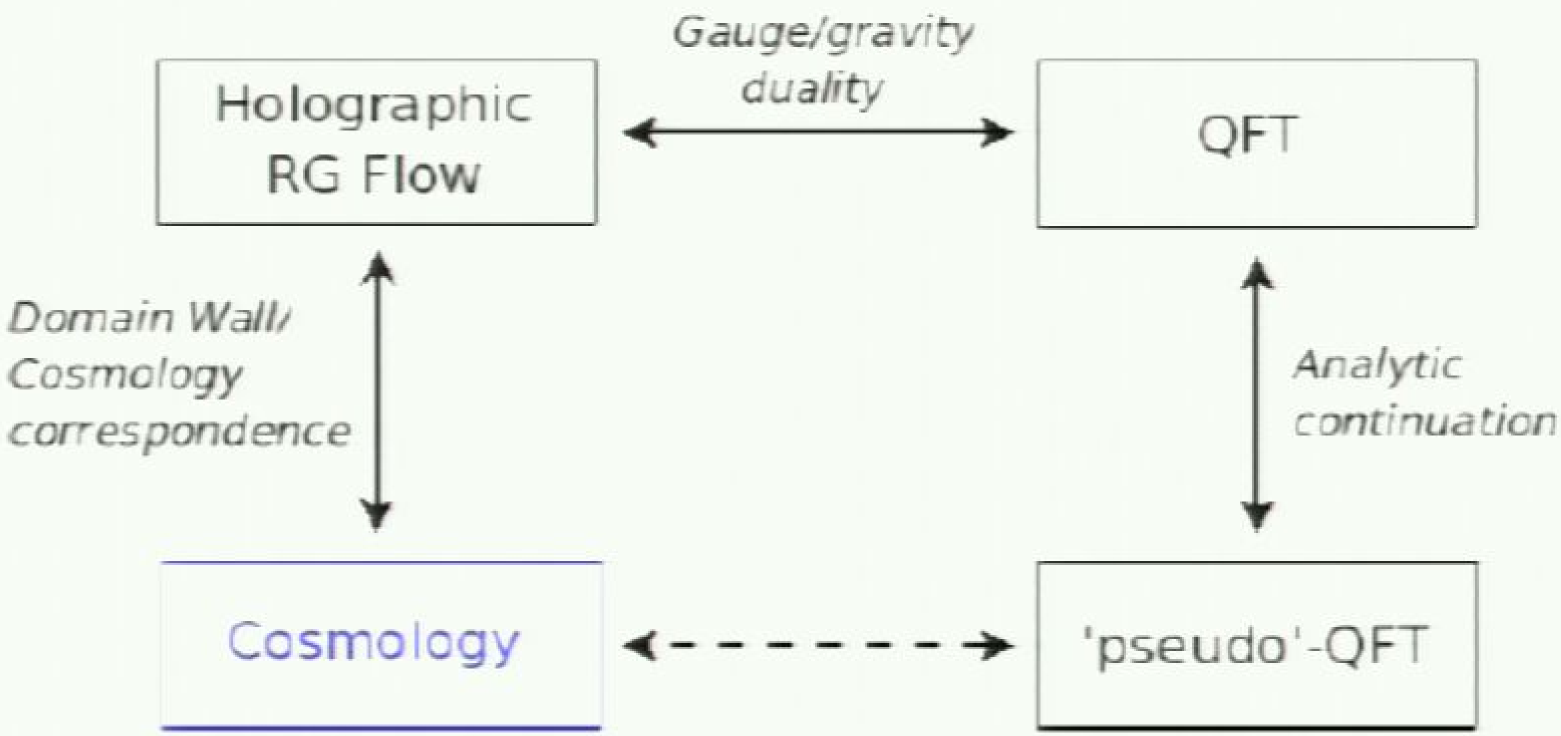
- Compute cosmological observables *holographically* using only perturbative QFT.
- Qualitatively different predictions from standard inflation.
- Simple to find holographic models consistent with observation.

Plan of talk



- Part I: Holography for cosmology
- Part II: Strong gravity regime: overview of cosmological results.

Ref: arXiv:1001.2007 & 0907.5542.


Gauge/gravity duality
Holographic RG Flow
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

Cosmological perturbations

We start by reviewing standard inflationary cosmology and the cosmological observables we would like to compute holographically.

- For simplicity, we discuss single-field 4d inflationary models:

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi)].$$

- We assume a spatially flat background and perturb

$$\begin{aligned} \mathrm{d}s^2 &= -\mathrm{d}t^2 + a^2(t)[\delta_{ij} + h_{ij}(t,\vec{x})]\mathrm{d}x^i\mathrm{d}x^j, \\ \Phi &= \varphi(t) + \delta\varphi(t,\vec{x}). \end{aligned}$$

where $h_{ij} = -2\psi(z,\vec{x})\delta_{ij} + 2\partial_i\partial_j\chi(z,\vec{x}) + \gamma_{ij}(z,\vec{x})$.

- γ_{ij} is transverse traceless and we form the gauge-invariant combination $\zeta = \psi + (H/\dot{\varphi})\delta\varphi$.

Cosmological perturbations

- The equations of motion for the perturbations are:

$$
\begin{aligned}
0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} + a^{-2}q^2\zeta. \\
0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + a^{-2}q^2\gamma_{ij}.
\end{aligned}
$$

where $H = \dot{a}/a$ is the Hubble rate and $\epsilon = -\dot{H}/H^2$ is the 'slow-roll' parameter. We are not assuming that ϵ is small.

Power spectra

In the inflationary paradigm, cosmological perturbations are assumed to originate on sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\begin{aligned} \langle \zeta(t, \vec{q}) \zeta(t, -\vec{q}) \rangle &= |\zeta_q(t)|^2, \\ \langle \gamma_{ij}(t, \vec{q}) \gamma_{kl}(t, -\vec{q}) \rangle &= 2 |\gamma_q(t)|^2 \Pi_{ijkl}, \end{aligned}$$

  where Π_{ijkl} is the transverse traceless projection operator while $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are then given by

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2} |\zeta_{q(0)}|^2, \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2} |\gamma_{q(0)}|^2,$$

  where $\gamma_{q(0)}$ and $\zeta_{q(0)}$ are the constant late-time values of the mode functions, with initial conditions set by the Bunch-Davies vacuum.

Power spectra via response functions

In preparation for our holographic discussion, we rewrite the power spectrum as follows.

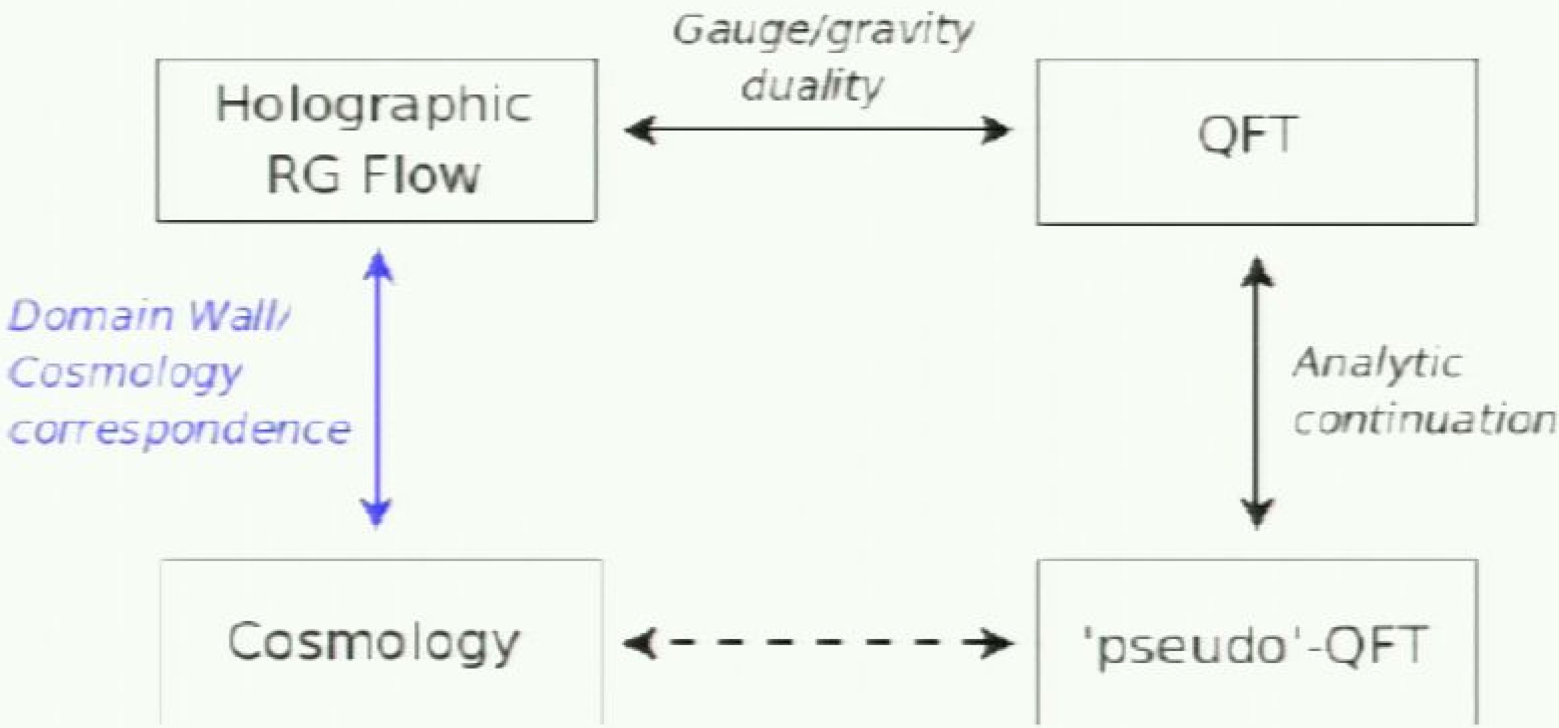
- We define the *response functions* as

$$\Pi^{(\zeta)} = \Omega\,\zeta, \quad \Pi^{(\gamma)}_{ij} = E\,\gamma_{ij}.$$

  where $\Pi^{(\zeta)}$ and $\Pi^{(\gamma)}_{ij}$ are the canonical momenta.
- One can show that

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E(q)].$$

  hence the power spectra may be expressed in terms of the late-time behaviour of the response functions.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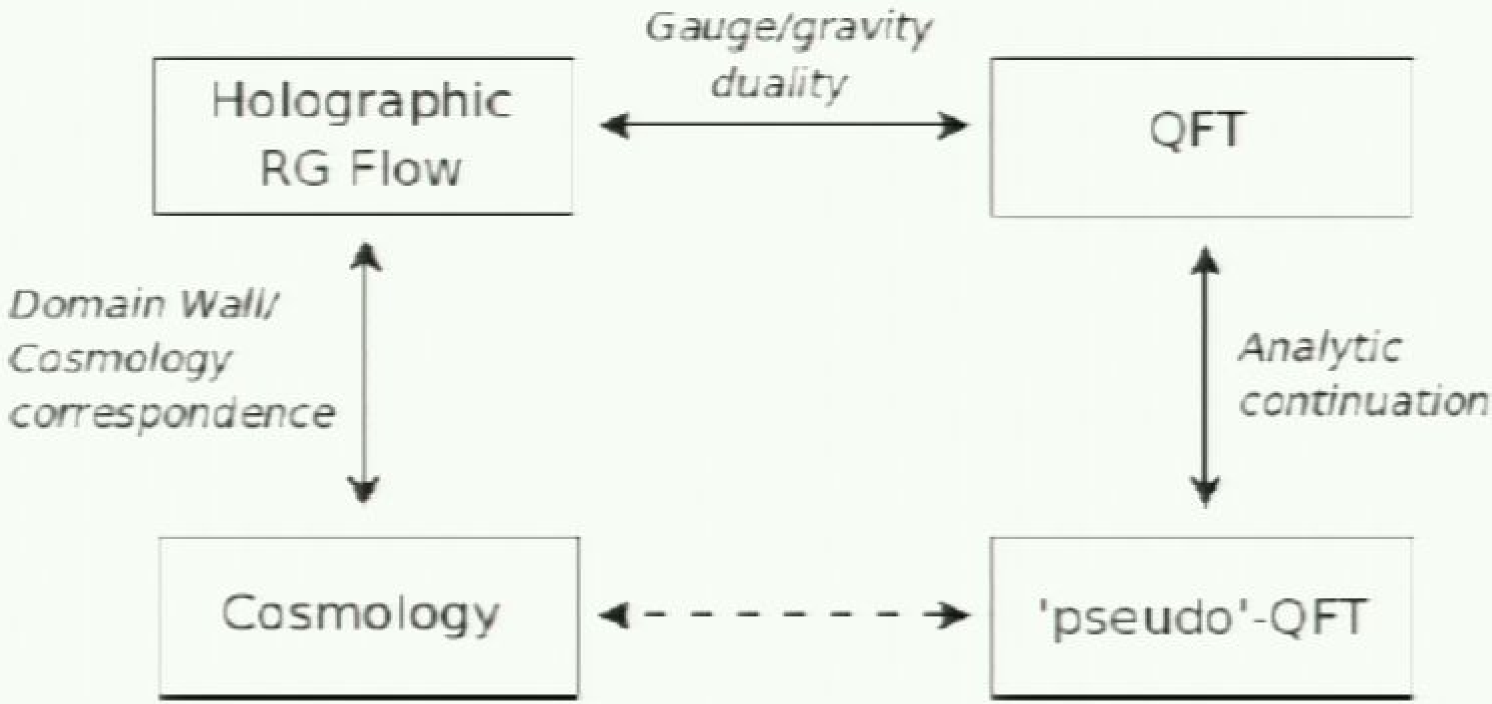

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Domain-wall/cosmology correspondence

- 'Domain-wall' spacetimes are closely related to cosmological spacetimes:

$$ds^2 = \eta\, dz^2 + a^2(z) d\vec{x}^2. \qquad \Phi = \varphi(z).$$

  where $\eta = +1$ for a (Euclidean) DW and $\eta = -1$ for cosmology.
- They play a prominent role in holography where they describe holographic RG flows (*i.e.* radial evolution of DW $\leftrightarrow$ RG flow of dual QFT).
- The DW action is

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{g}\, [-R + (\partial\Phi)^2 + 2\kappa^2 V(\Phi)].$$

Domain-wall/cosmology correspondence

- Including perturbations, the equations of motion for DW/C read:

$$\begin{aligned} H &= -(1/2)W(\varphi), \quad \dot{\varphi} = W_{,\varphi}, \quad 2\eta\kappa^2 V = (W_{,\varphi})^2 - (3/2)W^2, \\ 0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \eta a^{-2} q^2 \zeta, \quad 0 = \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \eta a^{-2} q^2 \gamma_{ij}. \end{aligned}$$

- Defining the analytically continued variables

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq,$$

we see that a cosmological solution written in terms of (κ, q) continues to a DW solution expressed in terms of $(\bar{\kappa}, \bar{q})$.

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Domain-wall/cosmology correspondence

- This particular bulk continuation was chosen as it has a clear interpretation in terms of dual QFT variables.
- Our choice of sign in the continuation of q ensures that the Bunch-Davies vacuum on the cosmology side maps to a solution that is *regular* in the interior of the domain-wall:

$$\zeta, \gamma \sim \exp(-iq\tau) \quad \rightarrow \quad \zeta, \gamma \sim \exp(\bar{q}\tau)$$

  where $\tau = \int \mathrm{d}z/a$ and the DW interior is $\tau \rightarrow -\infty$.
- One can define response functions $\bar{\Omega}$ and $\bar{E}$ for the DW spacetime. They are related to their cosmological counterparts by the analytic continuations $\bar{\Omega}(-iq) = \Omega(q)$ and $\bar{E}(-iq) = E(q)$.

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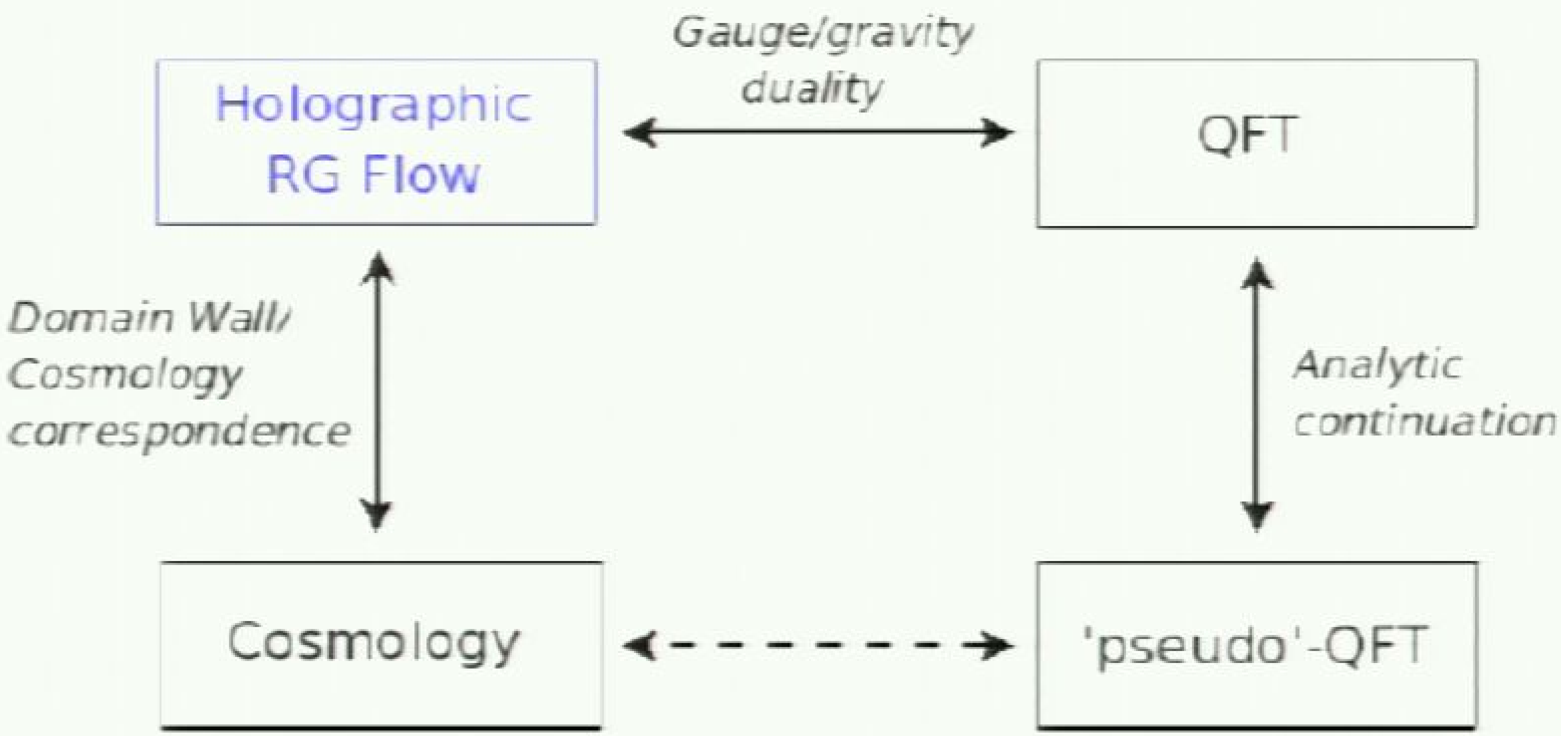
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Holographic RG flows

There are two classes of domain-wall spacetimes whose holographic interpretation is well understood:

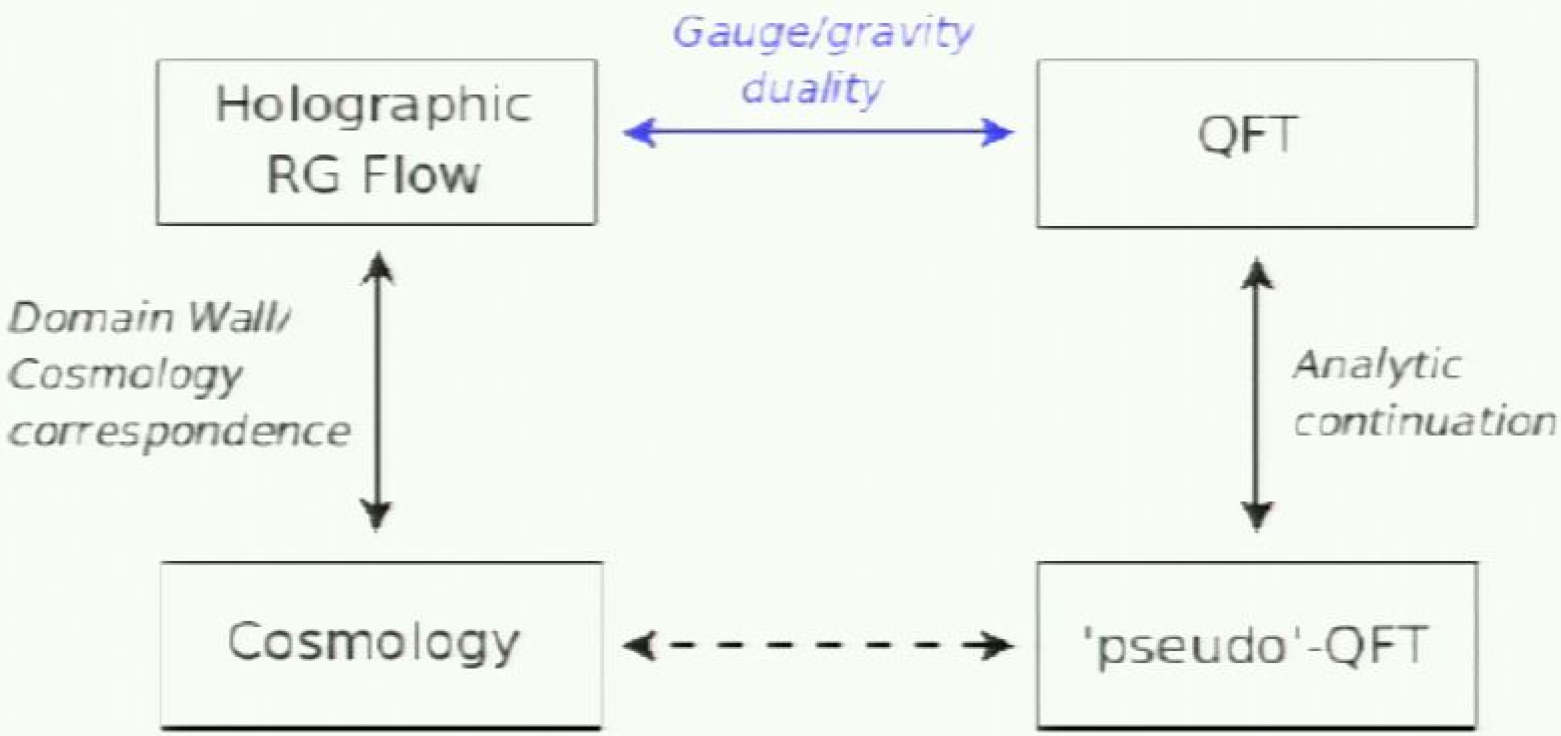
1. Asymptotically AdS solutions: $a \sim e^z$, $\varphi \sim 0$ as $z \to \infty$.

- These solutions describe a QFT that approaches a fixed point in the UV. The fixed point is the CFT dual to the asymptotic AdS spacetime.
- Under the DW/C correspondence, these solutions are mapped to cosmologies that are asymptotically de Sitter at late times.

Holographic RG flows

2. Asymptotically power-law solutions: $a \sim (z/z_0)^n$, $\varphi \sim \sqrt{2n}\ln(z/z_0)$ as $z \to \infty$.

- Specific cases of such spacetimes may be obtained by taking the near-horizon limit of non-conformal branes (e.g. D2 brane $\leftrightarrow n = 7$).
- These solutions describe QFTs with a dimensionful coupling constant in the regime where the dimensionality of the coupling constant drives the dynamics[1].
- Under the DW/C correspondence, they are mapped to cosmologies that undergo asymptotic power-law inflation.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Holography: a primer

Our holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

- There is a 1-to-1 correspondence between local gauge-invariant operators of the boundary QFT and bulk supergravity modes:
  - $\Rightarrow$ The bulk metric corresponds to the stress-energy tensor T_{ij} of the boundary theory.
  - $\Rightarrow$ Bulk scalar fields correspond to boundary scalar operators, e.g. $\mathrm{tr}F_{ij}F^{ij}$.
- Correlation functions of the dual QFT may be read off from the asymptotics of the bulk solution. Conversely, given appropriate QFT data, one can reconstruct the bulk asymptotics.

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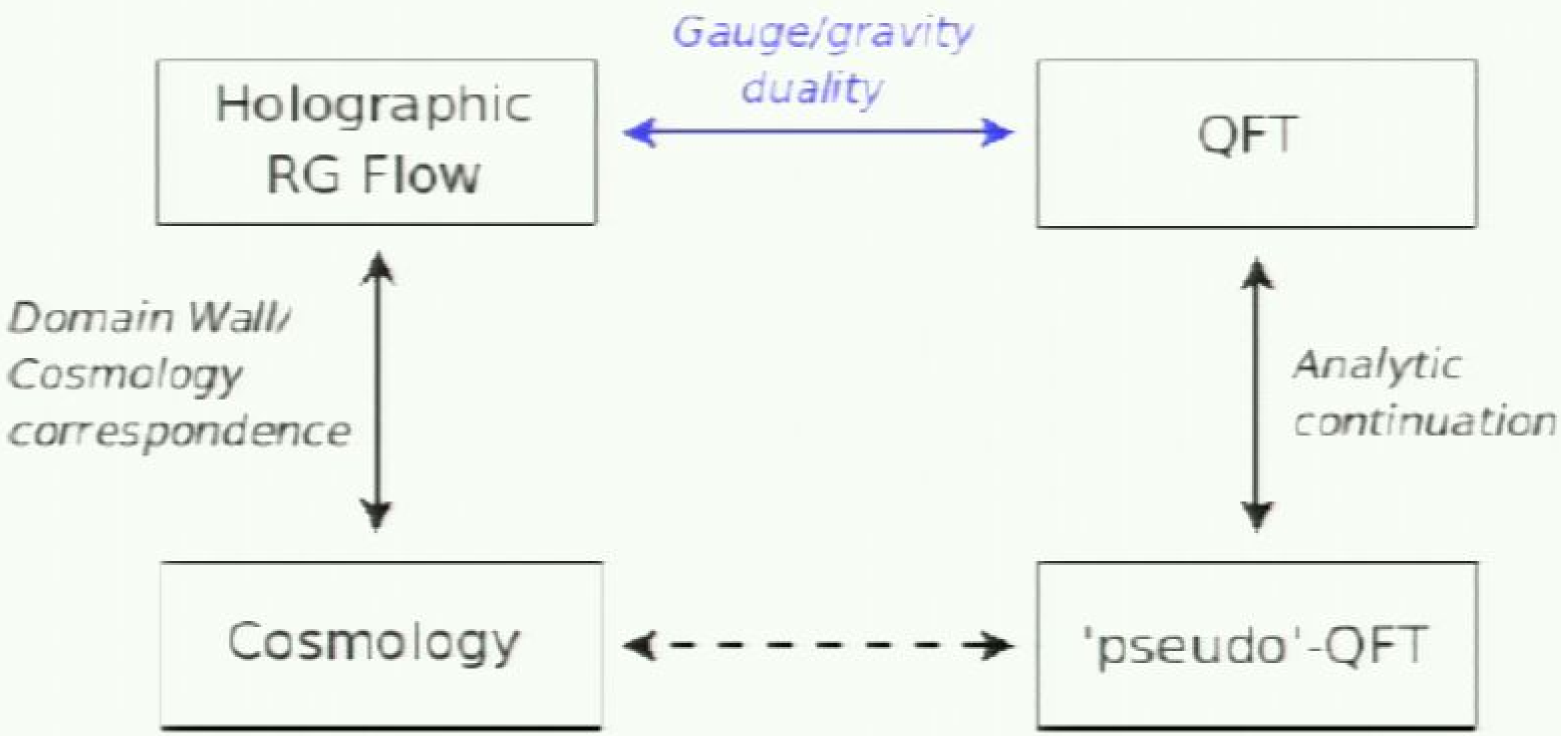
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- Correlation functions of the dual QFT may be read off from the asymptotics of the bulk solution. Conversely, given appropriate QFT data, one can reconstruct the bulk asymptotics.

Bulk asymptotics

The general asymptotic solution for the 4d bulk metric reads:

$$\mathrm{d}s^2 = \mathrm{d}r^2 + e^{2r} g_{ij}(r,x)\mathrm{d}x^i\mathrm{d}x^j,$$
$$g_{ij}(r,x) = g_{(0)ij}(x) + e^{-2r} g_{(2)ij}(x) + \ldots + e^{-2\sigma r} g_{(2\sigma)ij}(x) + \ldots$$

- $g_{(0)ij}(x)$ is the metric seen by the dual QFT, and hence acts as the *source* for the dual stress tensor T_{ij}.
- The $g_{(2k)ij}(x)$ with $k < \sigma$ are locally determined in terms of $g_{(0)ij}(x)$ via the asymptotic analysis of the field equations.
- $g_{(2\sigma)ij}(x)$ is only partially constrained by the asymptotic analysis of the field equations, and is related to the dual 1-pt function:

$$\langle T_{ij} \rangle = \frac{1}{2\bar{\kappa}^2}(2\sigma g_{(2\sigma)ij}).$$

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- From the bulk asymptotics, we can read off $\langle T_{ij} \rangle$. Equivalently, given $\langle T_{ij} \rangle$, we can reconstruct the bulk asymptotics.
- This remains true even in the regime where gravity is *strongly coupled* and the description in terms of low-energy fields (such as the metric) breaks down deep in the interior.
- The metric description is still valid *asymptotically*, however, and takes the same form as before. Gauge/gravity duality requires the value of $g_{(2\sigma)ij}$ deriving from stringy dynamics to match that derived from the dual weakly coupled QFT.

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$$g_{ij}(r,x) = g_{(0)ij}(x) + e^{-2r} g_{(2)ij}(x) + \ldots + e^{-2\sigma r} g_{(2\sigma)ij}(x) + \ldots$$

- $g_{(0)ij}(x)$ is the metric seen by the dual QFT, and hence acts as the *source* for the dual stress tensor T_{ij}.
- The $g_{(2k)ij}(x)$ with $k < \sigma$ are locally determined in terms of $g_{(0)ij}(x)$ via the asymptotic analysis of the field equations.
- $g_{(2\sigma)ij}(x)$ is only partially constrained by the asymptotic analysis of the field equations, and is related to the dual 1-pt function:

$$\langle T_{ij} \rangle = \frac{1}{2\bar{\kappa}^2}(2\sigma g_{(2\sigma)ij}).$$

Bulk asymptotics

- From the bulk asymptotics, we can read off $\langle T_{ij} \rangle$. Equivalently, given $\langle T_{ij} \rangle$, we can reconstruct the bulk asymptotics.
- This remains true even in the regime where gravity is *strongly coupled* and the description in terms of low-energy fields (such as the metric) breaks down deep in the interior.
- The metric description is still valid *asymptotically*, however, and takes the same form as before. Gauge/gravity duality requires the value of $g_{(2\sigma)ij}$ deriving from stringy dynamics to match that derived from the dual weakly coupled QFT.

Two-point functions

- Higher-point functions may be obtained by differentiating the 1-pt function w.r.t. the sources and then setting the sources to their background values,

$$\text{e.g.} \qquad \langle T_{ij}(x) T_{kl}(y) \rangle \sim \frac{\delta g_{(2\sigma)ij}(x)}{\delta g_{(0)kl}(y)}\Big|_{g_{(0)}=\delta}$$

- To compute 2-pt functions one only needs to solve for the fluctuations to *linear* order.
- On general grounds, the 2-pt function for the stress tensor admits the decomposition

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

where the transverse and transverse traceless projection operators are

$$\pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2. \quad \Pi_{ijkl} = \pi_{i(k} \pi_{l)j} - (1/2) \pi_{ij} \pi_{kl}.$$

Holographic analysis

- Employing the radial Hamiltonian formulation of holographic renormalisation[2], we showed that for both asymptotically AdS and asymptotically power-law DW spacetimes,

$$A(\bar{q}) = 4\bar{E}_{(0)}(\bar{q}), \quad B(\bar{q}) = (1/4)\bar{\Omega}_{(0)}(\bar{q}).$$

- Thus, the 2-pt function $\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle$ may be extracted from the DW response functions. The subscript indicates taking the term with appropriate scaling in the asymptotic expansion.

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(Some details)

- In the Hamiltonian formalism, the asymptotic radial expansion of the metric is replaced by a *covariant* expansion in eigenfunctions of the dilatation operator:

$$\delta_D = \partial_r + O(e^{-2r}), \qquad \delta_D A_{(m)} = -m A_{(m)}.$$

- The dual 1-pt function is then

$$\langle T^i_j \rangle = \left(\frac{-2}{\sqrt{g}} \Pi^i_j \right)_{(3)}$$

where Π^i_j is the radial canonical momentum.

- For asymptotically power-law spacetimes, the holographic analysis takes place in the *dual frame*, where the metric is asymptotically AdS.

(Some details)

Under a linear variation of the QFT sources $g_{(0)ij}$ and $\varphi_{(0)}$,

$$\delta\langle T_{ij}\rangle = -\frac{1}{2}\sqrt{g_{(0)}}\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle\delta g_{(0)}^{kl} - \sqrt{g_{(0)}}\langle T_{ij}(\bar{q})\mathcal{O}(-\bar{q})\rangle\delta\varphi_{(0)}$$
$$= \frac{1}{2}A(\bar{q})\gamma^{i}_{j(0)} - 2B(\bar{q})\psi_{(0)}\pi^{i}_{j} - \langle T^{i}_{j}(\bar{q})\mathcal{O}(-\bar{q})\rangle\delta\varphi_{(0)}.$$

In comparison, perturbing the bulk radial canonical momentum to linear order, we find (*e.g.* in the asymptotically AdS case)

$$\delta\langle T^{i}_{j}\rangle = \left[\frac{2\bar{E}(\bar{q})}{a^3}\gamma^{i}_{j} - \left(\frac{\bar{q}^2}{\bar{\kappa}^2 a^2 H} + \frac{\bar{\Omega}(\bar{q})}{2a^3}\right)\psi\pi^{i}_{j} - (\ldots)\delta\varphi\right]_{(3)}$$

Extracting the terms with appropriate dilatation weight then yields:

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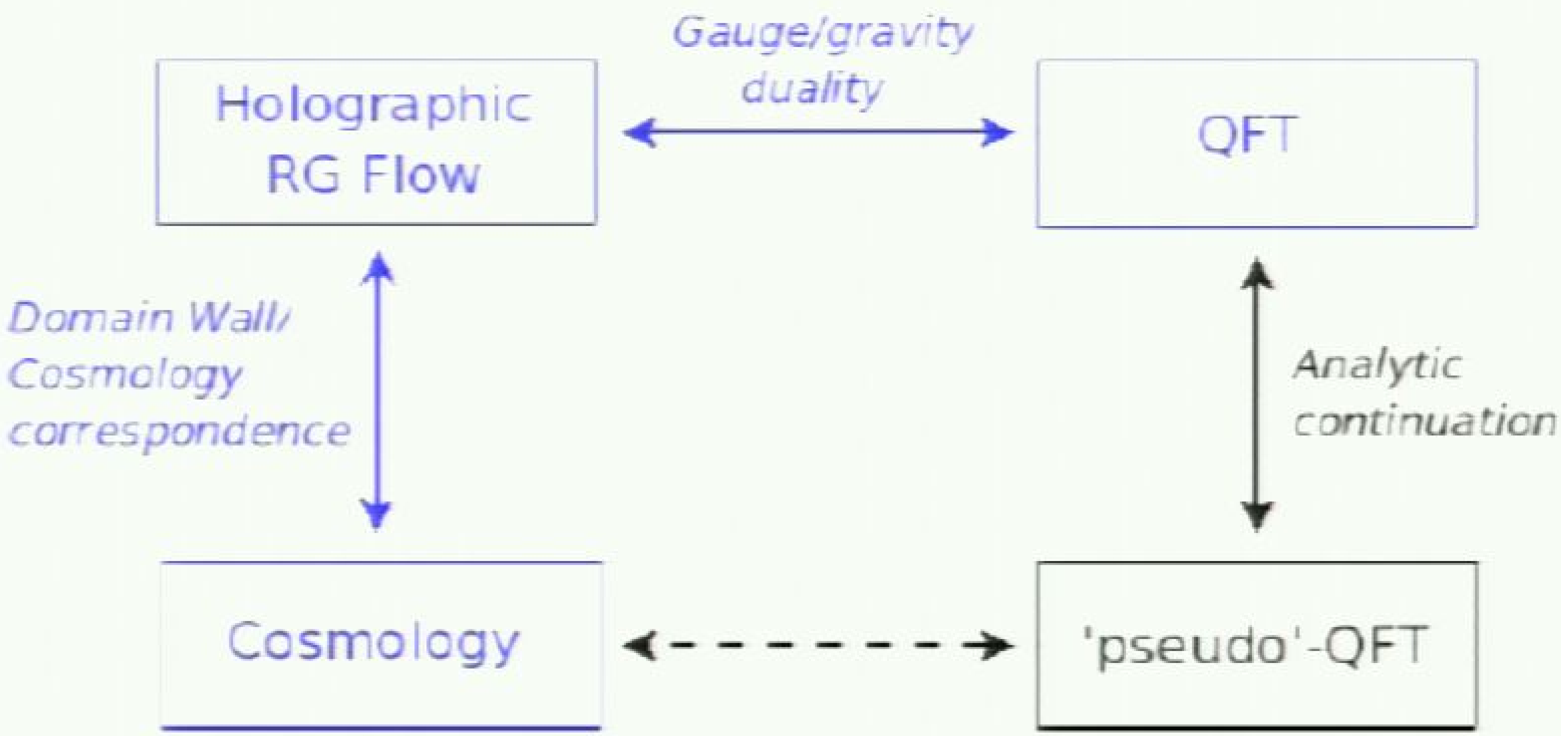
Gauge/gravity
duality
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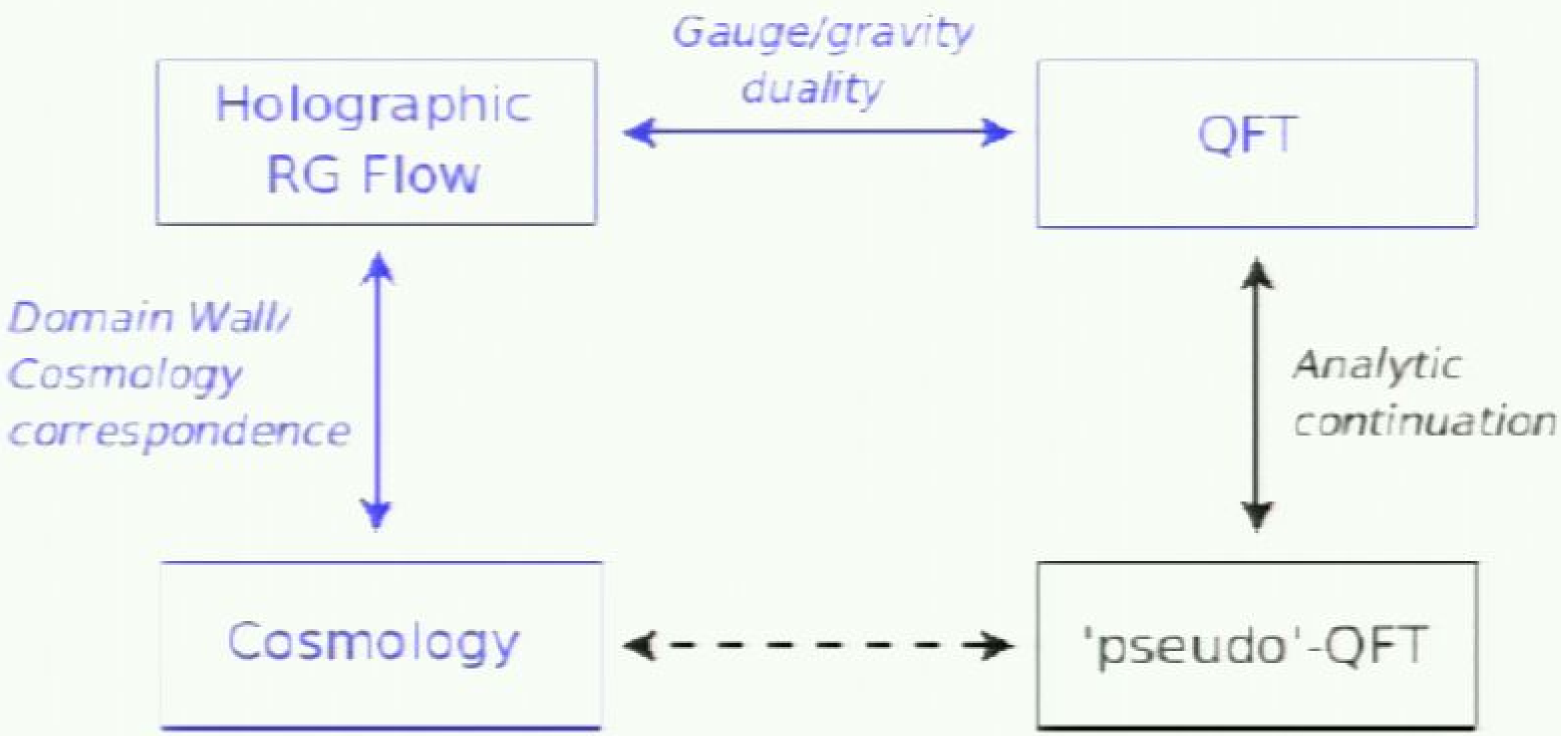
Holographic RG Flow
Gauge/gravity duality
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From cosmology to QFT

Applying the analytic continuations $\bar{\kappa}^2 = -\kappa^2$, $\bar{q} = -iq$, we find a direct relation between the cosmological power spectra and the 2-pt functions of the dual QFT:

$$\Delta_S^2(q) = \frac{-q^3}{16\pi^2 \mathrm{Im} B(-iq)}, \quad \Delta_T^2(q) = \frac{-2q^3}{\pi^2 \mathrm{Im} A(-iq)}.$$

where

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}.$$