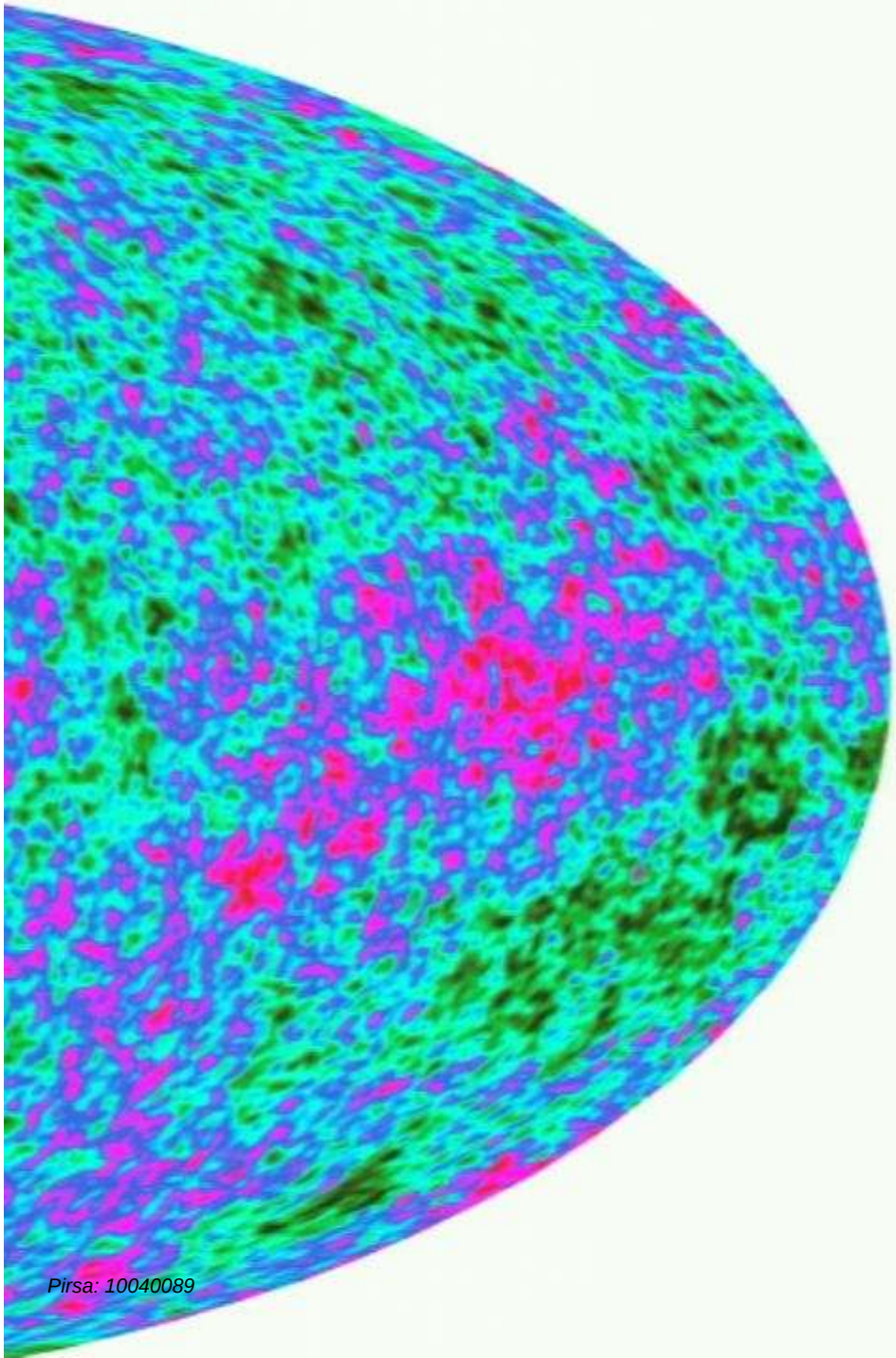
Title: Holography for Cosmology

Date: Apr 15, 2010 01:00 PM

URL: http://pirsa.org/10040089

Abstract: TBA



Holography for Cosmology

Paul McFadden
Universiteit van Amsterdam

work with Kostas Skenderis
arXiv:0907.5542 & 1001.2007

Holography

Holography states that any quantum theory of gravity should have a dual description in terms of a QFT (*without* gravity) in one dimension less.

Examples found in string theory involve spacetimes with a negative cosmological constant (e.g. AdS/CFT).

- Here we propose a holographic framework for inflationary cosmology.

Specifically, we seek a dual description of four-dimensional inflationary cosmology in terms of a three-dimensional QFT (without gravity).

Holography *for Cosmology*

Any proposed holographic framework for cosmology should specify:

1. The precise nature of the dual QFT.
2. How to compute cosmological observables (e.g. the primordial power spectrum) from the correlation functions of the dual QFT.

Having defined such a duality,

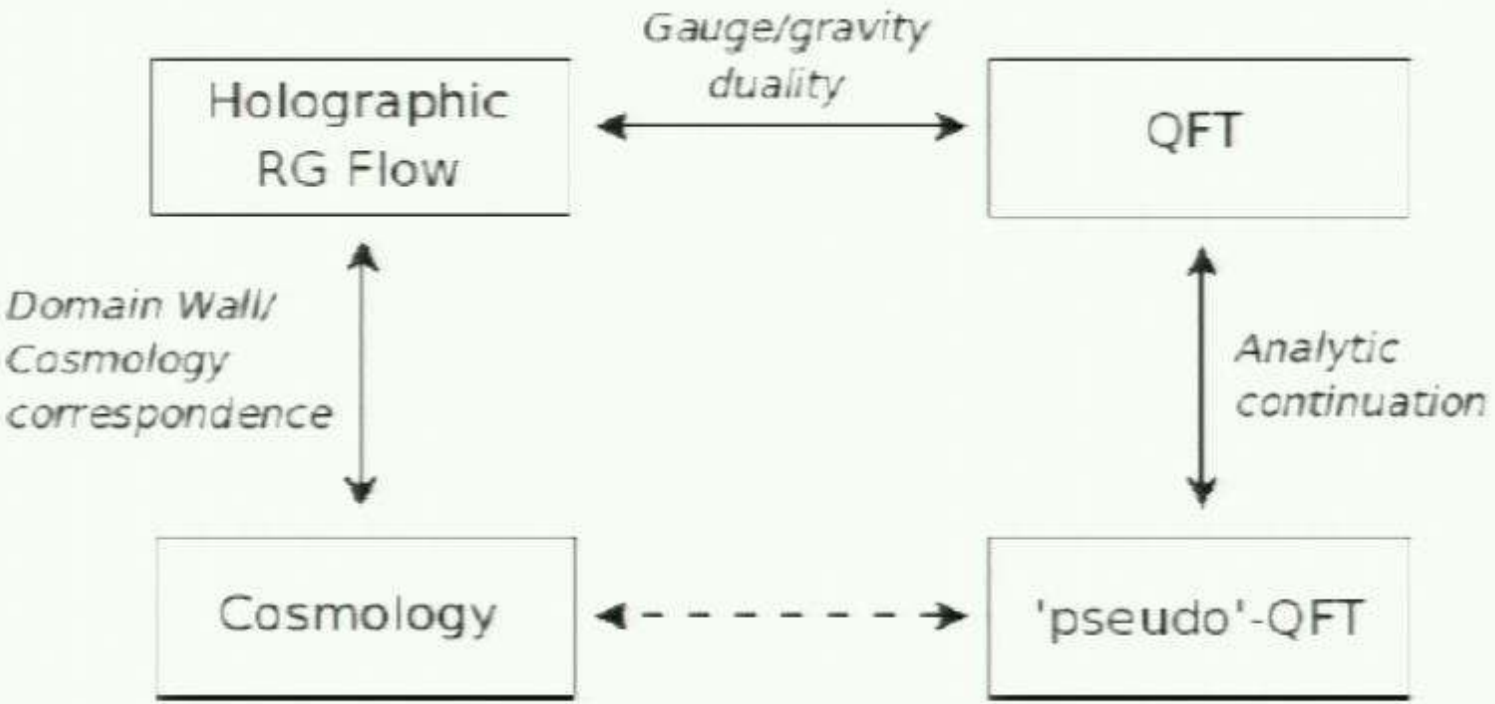
3. Must recover standard inflationary predictions in their regime of validity (namely, when a perturbative quantisation of fluctuations is possible, i.e. weakly coupled gravity = strongly coupled QFT).

Strong gravity

New results then follow by applying the holographic framework in the opposite regime where gravity is *strongly coupled* and a perturbative quantisation of fluctuations breaks down. The dual QFT is then *weakly coupled*.

- Compute cosmological observables *holographically* using only perturbative QFT.
- Qualitatively different predictions from standard inflation.
- Simple to find holographic models consistent with observation.

Plan of talk



- Part I: Holography for cosmology
- Part II: Strong gravity regime: overview of cosmological results.

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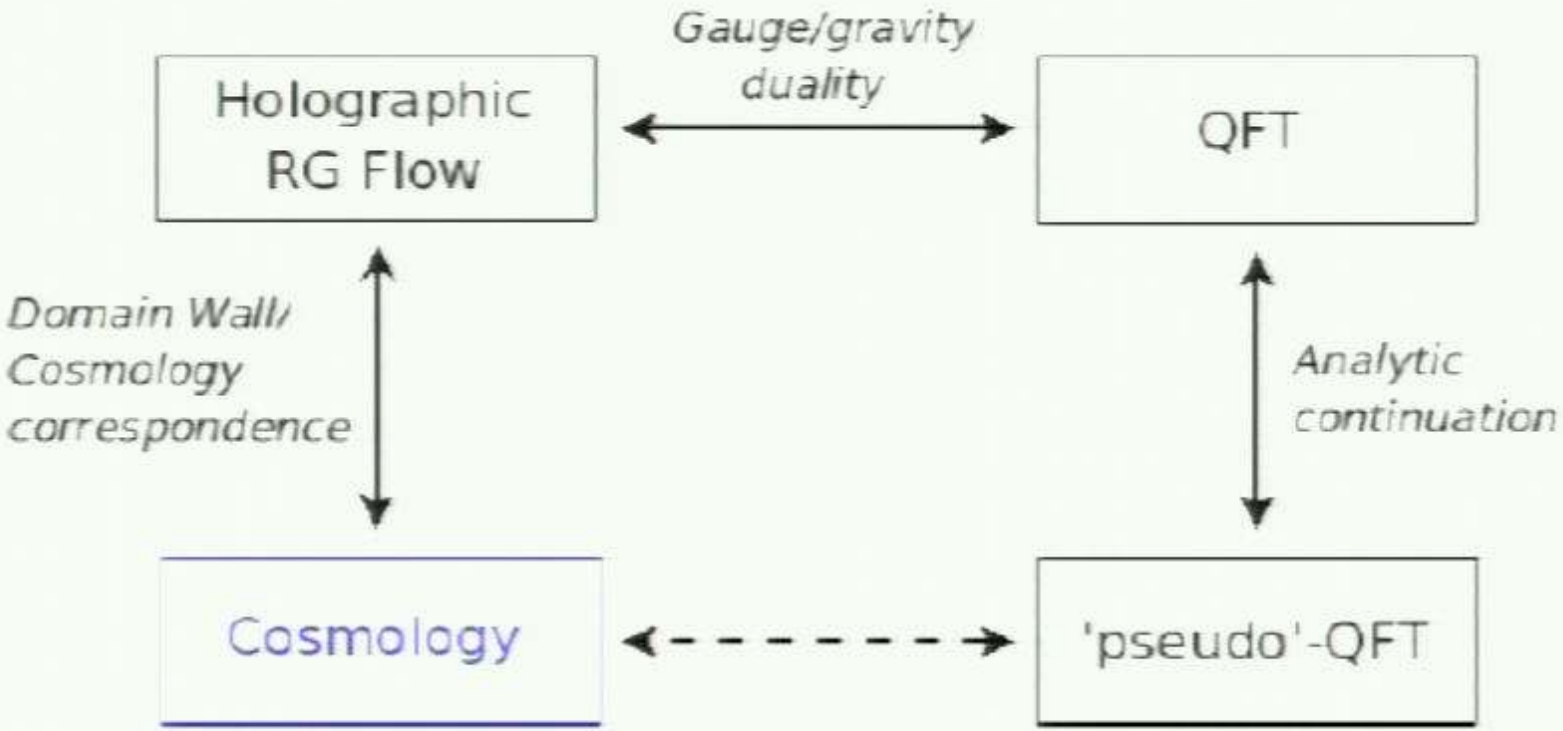
Holographic
RG Flow
Gauge/gravity
duality
QFT
Domain Wall/
Cosmology
correspondence
Analytic
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Cosmological perturbations

We start by reviewing standard inflationary cosmology and the cosmological observables we would like to compute holographically.

- For simplicity, we discuss single-field 4d inflationary models:

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi)].$$

- We assume a spatially flat background and perturb

$$\begin{aligned} \mathrm{d}s^2 &= -\mathrm{d}t^2 + a^2(t)[\delta_{ij} + h_{ij}(t.\vec{x})]\mathrm{d}x^i\mathrm{d}x^j. \\ \Phi &= \varphi(t) + \delta\varphi(t.\vec{x}). \end{aligned}$$

  where $h_{ij} = -2\psi(z.\vec{x})\delta_{ij} + 2\partial_i\partial_j\chi(z.\vec{x}) + \gamma_{ij}(z.\vec{x})$.

- γ_{ij} is transverse traceless and we form the gauge-invariant combination $\zeta = \psi + (H/\dot{\varphi})\delta\varphi$.

Cosmological perturbations

- The equations of motion for the perturbations are:

$$\begin{aligned} 0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} + a^{-2}q^2\zeta. \\ 0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + a^{-2}q^2\gamma_{ij}. \end{aligned}$$

where $H = \dot{a}/a$ is the Hubble rate and $\epsilon = -\dot{H}/H^2$ is the 'slow-roll' parameter. We are not assuming that ϵ is small.

Power spectra

In the inflationary paradigm, cosmological perturbations are assumed to originate on sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\begin{aligned} \langle \zeta(t,\vec{q})\zeta(t,-\vec{q})\rangle &= |\zeta_q(t)|^2, \\ \langle \gamma_{ij}(t,\vec{q})\gamma_{kl}(t,-\vec{q})\rangle &= 2|\gamma_q(t)|^2 \Pi_{ijkl}. \end{aligned}$$

where Π_{ijkl} is the transverse traceless projection operator while $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are then given by

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}|\zeta_{q(0)}|^2, \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}|\gamma_{q(0)}|^2.$$

where $\gamma_{q(0)}$ and $\zeta_{q(0)}$ are the constant late-time values of the mode functions, with initial conditions set by the Bunch-Davies vacuum.

Power spectra via response functions

In preparation for our holographic discussion, we rewrite the power spectrum as follows.

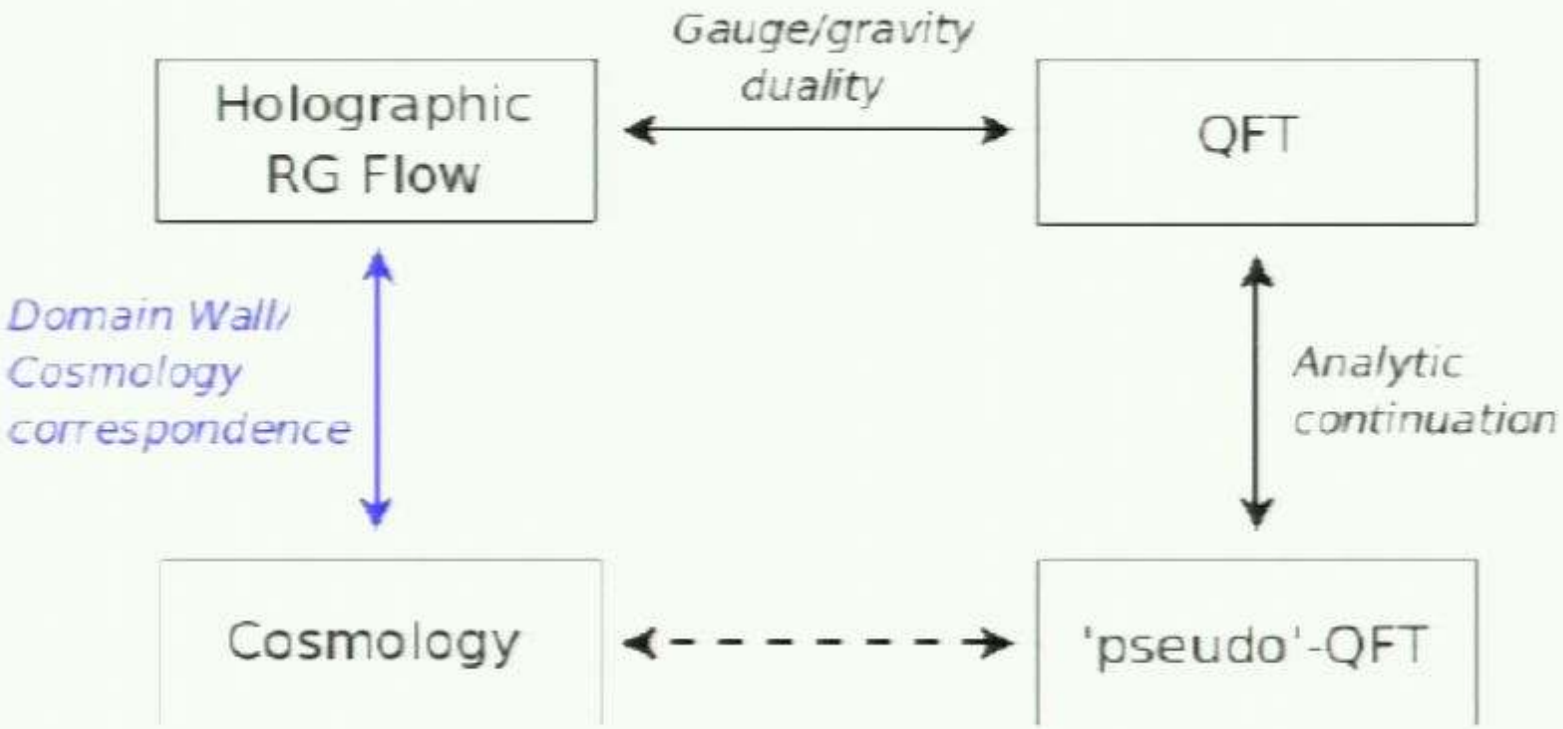
- We define the *response functions* as

$$\Pi^{(\zeta)} = \Omega\,\zeta, \quad \Pi^{(\gamma)}_{ij} = E\,\gamma_{ij},$$

  where $\Pi^{(\zeta)}$ and $\Pi^{(\gamma)}_{ij}$ are the canonical momenta.
- One can show that

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E(q)].$$

  hence the power spectra may be expressed in terms of the late-time behaviour of the response functions.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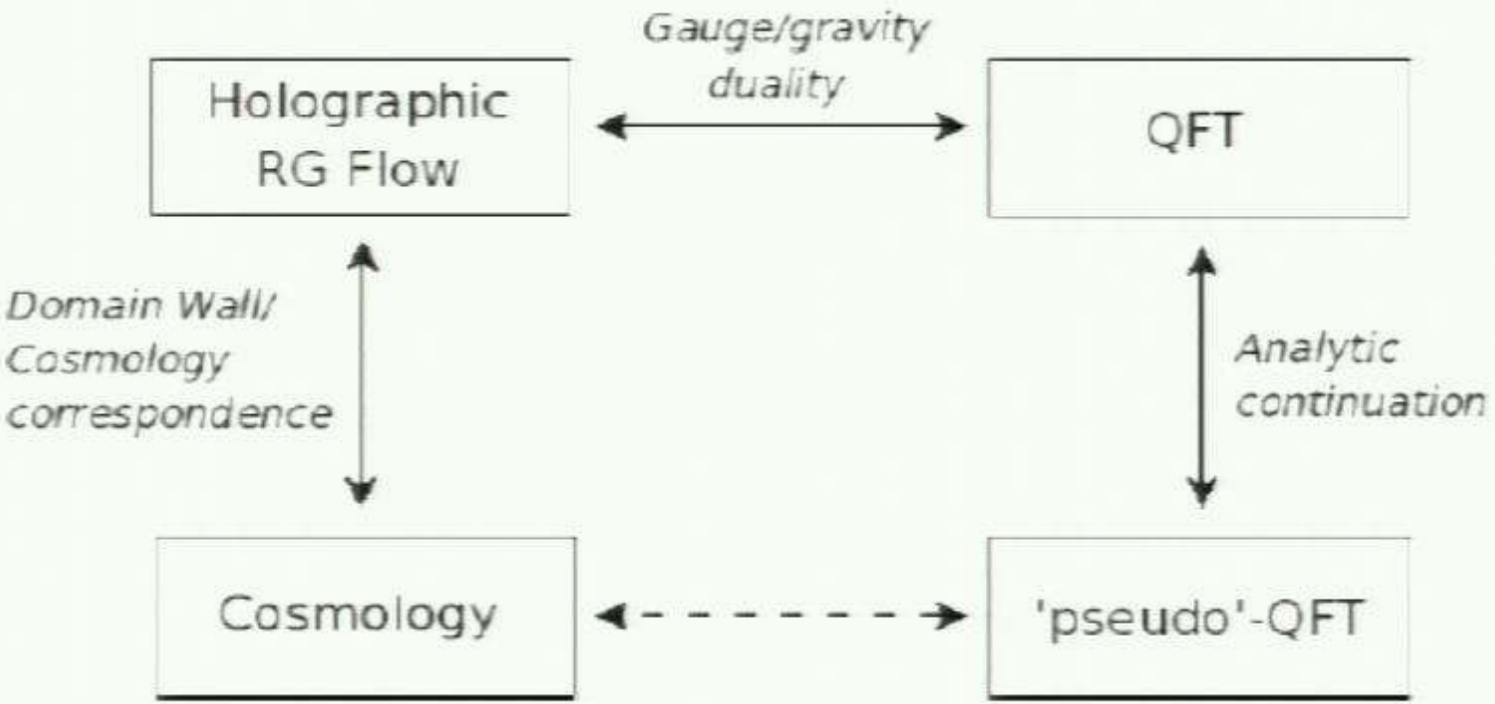

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Domain-wall/cosmology correspondence

- 'Domain-wall' spacetimes are closely related to cosmological spacetimes:

$$\mathrm{d}s^2 = \eta\,\mathrm{d}z^2 + a^2(z)\mathrm{d}\vec{x}^2, \qquad \Phi = \varphi(z),$$

where $\eta = +1$ for a (Euclidean) DW and $\eta = -1$ for cosmology.

- They play a prominent role in holography where they describe holographic RG flows (*i.e.* radial evolution of DW $\leftrightarrow$ RG flow of dual QFT).
- The DW action is

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{g}\,[-R + (\partial\Phi)^2 + 2\kappa^2 V(\Phi)].$$

Domain-wall/cosmology correspondence

- Including perturbations, the equations of motion for DW/C read:

$$H = -(1/2)W(\varphi), \quad \dot{\varphi} = W_{,\varphi}, \quad 2\eta\kappa^2 V = (W_{,\varphi})^2 - (3/2)W^2,$$
$$0 = \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \eta a^{-2} q^2 \zeta, \quad 0 = \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \eta a^{-2} q^2 \gamma_{ij},$$

- Defining the analytically continued variables

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq,$$

we see that a cosmological solution written in terms of (κ, q) continues to a DW solution expressed in terms of $(\bar{\kappa}, \bar{q})$.

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Domain-wall/cosmology correspondence

- This particular bulk continuation was chosen as it has a clear interpretation in terms of dual QFT variables.
- Our choice of sign in the continuation of q ensures that the Bunch-Davies vacuum on the cosmology side maps to a solution that is *regular* in the interior of the domain-wall:

$$\zeta, \gamma \sim \exp(-iq\tau) \quad \rightarrow \quad \zeta, \gamma \sim \exp(\bar{q}\tau)$$

  where $\tau = \int \mathrm{d}z/a$ and the DW interior is $\tau \to -\infty$.

- One can define response functions $\bar{\Omega}$ and $\bar{E}$ for the DW spacetime. They are related to their cosmological counterparts by the analytic continuations $\bar{\Omega}(-iq) = \Omega(q)$ and $\bar{E}(-iq) = E(q)$.

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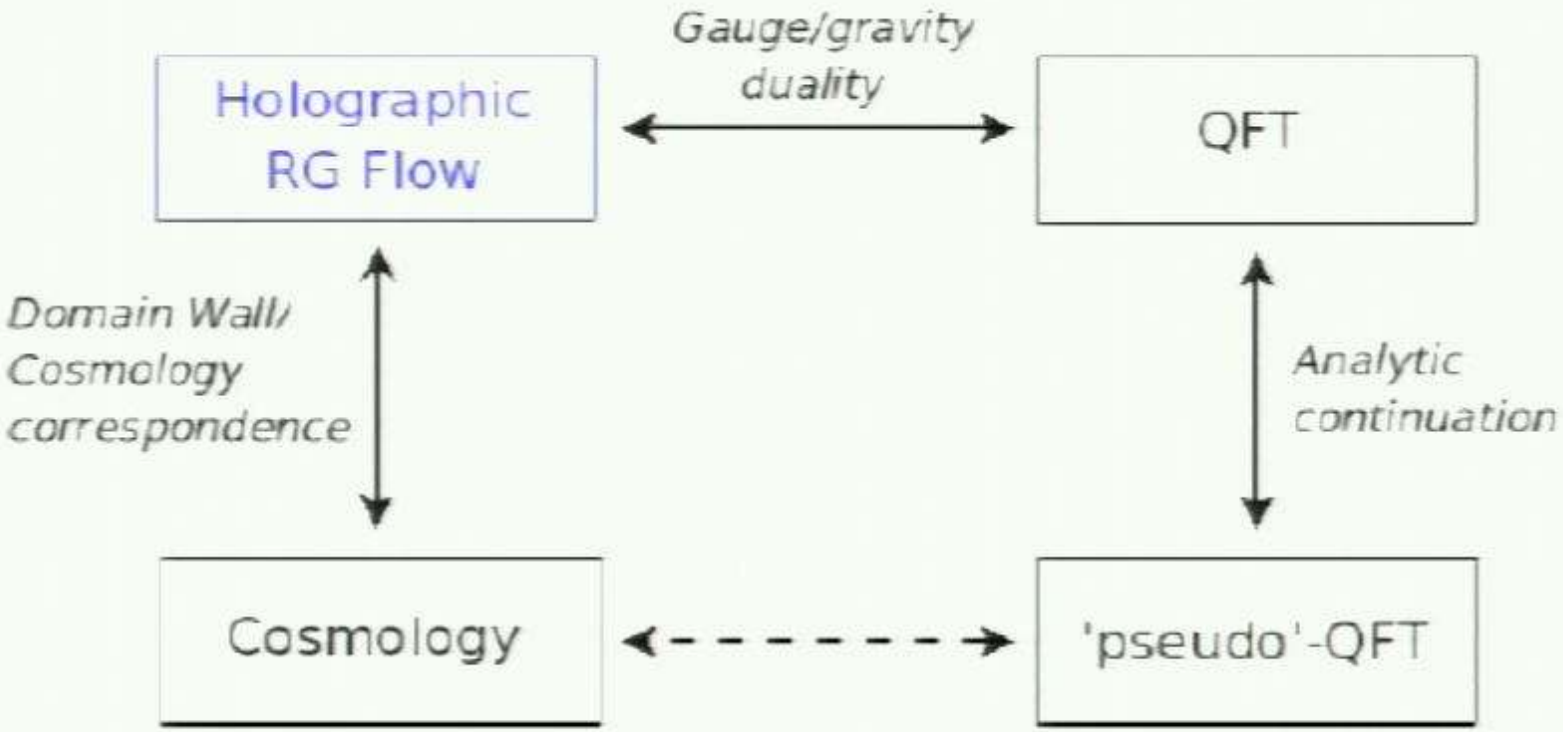
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Holographic RG flows

There are two classes of domain-wall spacetimes whose holographic interpretation is well understood:

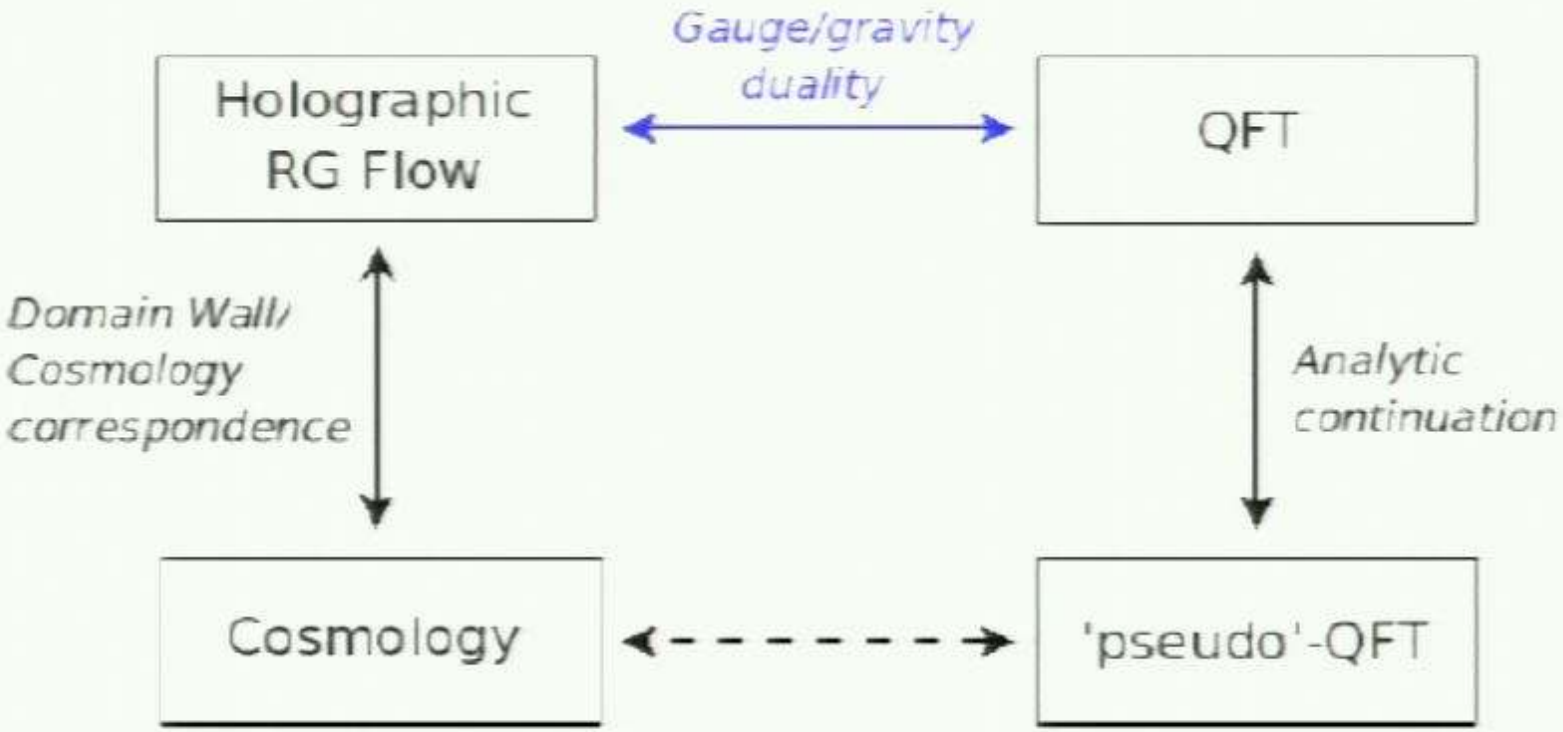
1. Asymptotically AdS solutions: $a \sim e^z$, $\varphi \sim 0$ as $z \to \infty$.

- These solutions describe a QFT that approaches a fixed point in the UV. The fixed point is the CFT dual to the asymptotic AdS spacetime.
- Under the DW/C correspondence, these solutions are mapped to cosmologies that are asymptotically de Sitter at late times.

Holographic RG flows

2. Asymptotically power-law solutions: $a \sim (z/z_0)^n$, $\varphi \sim \sqrt{2n}\ln(z/z_0)$ as $z \to \infty$.

- Specific cases of such spacetimes may be obtained by taking the near-horizon limit of non-conformal branes (e.g. D2 brane $\leftrightarrow n = 7$).
- These solutions describe QFTs with a dimensionful coupling constant in the regime where the dimensionality of the coupling constant drives the dynamics[1].
- Under the DW/C correspondence, they are mapped to cosmologies that undergo asymptotic power-law inflation.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Holography: a primer

Our holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

- There is a 1-to-1 correspondence between local gauge-invariant operators of the boundary QFT and bulk supergravity modes:
  - $\Rightarrow$ The bulk metric corresponds to the stress-energy tensor T_{ij} of the boundary theory.
  - $\Rightarrow$ Bulk scalar fields correspond to boundary scalar operators, e.g. $\mathrm{tr} F_{ij} F^{ij}$.
- Correlation functions of the dual QFT may be read off from the asymptotics of the bulk solution. Conversely, given appropriate QFT data, one can reconstruct the bulk asymptotics.

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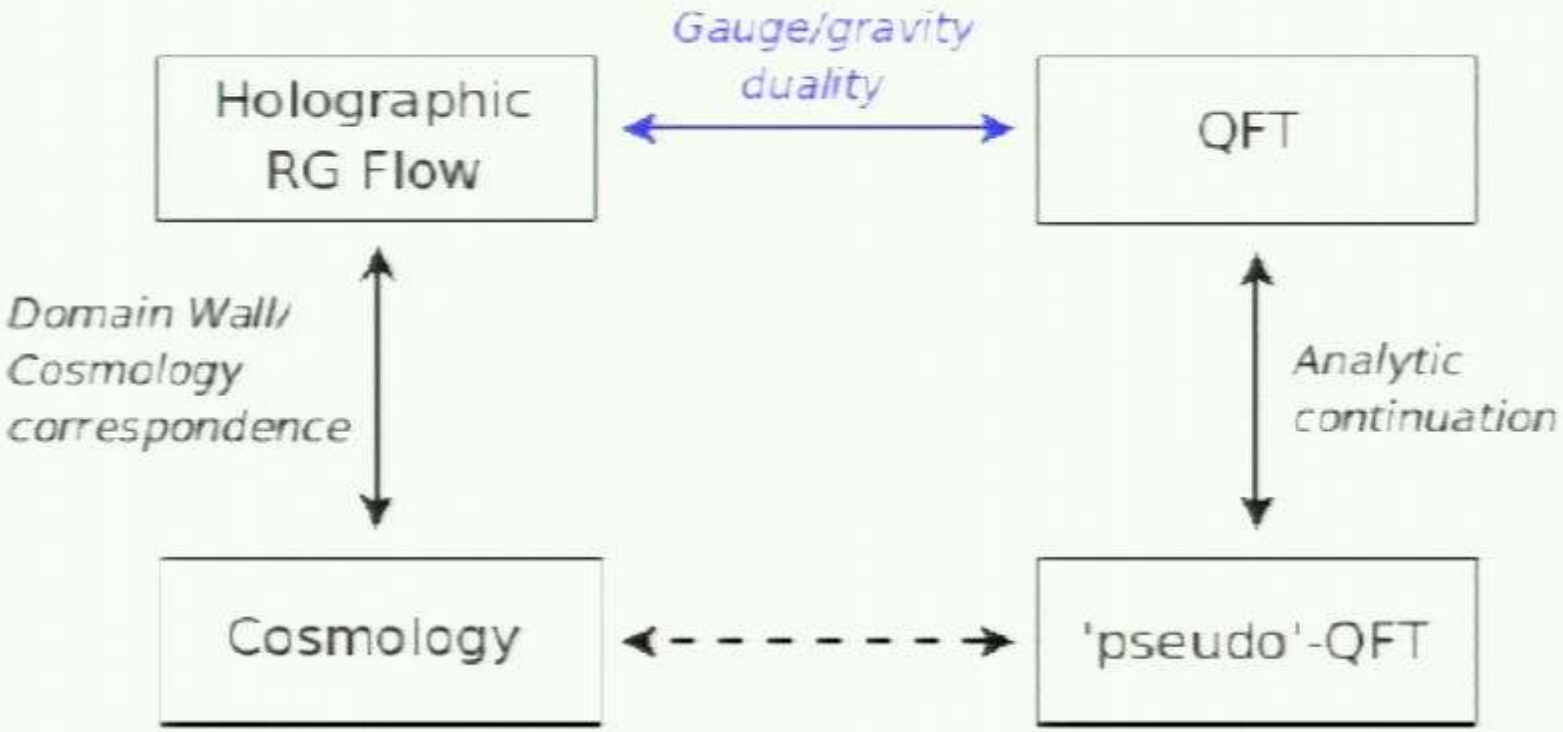
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Bulk asymptotics

The general asymptotic solution for the 4d bulk metric reads:

$$ds^2 = dr^2 + e^{2r} g_{ij}(r, x) dx^i dx^j,$$
$$g_{ij}(r, x) = g_{(0)ij}(x) + e^{-2r} g_{(2)ij}(x) + \ldots + e^{-2\sigma r} g_{(2\sigma)ij}(x) + \ldots$$

- $g_{(0)ij}(x)$ is the metric seen by the dual QFT, and hence acts as the *source* for the dual stress tensor T_{ij}.
- The $g_{(2k)ij}(x)$ with $k < \sigma$ are locally determined in terms of $g_{(0)ij}(x)$ via the asymptotic analysis of the field equations.
- $g_{(2\sigma)ij}(x)$ is only partially constrained by the asymptotic analysis of the field equations, and is related to the dual 1-pt function:

$$\langle T_{ij} \rangle = \frac{1}{2\bar{\kappa}^2} (2\sigma g_{(2\sigma)ij}).$$

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Bulk asymptotics

- From the bulk asymptotics, we can read off $\langle T_{ij} \rangle$. Equivalently, given $\langle T_{ij} \rangle$, we can reconstruct the bulk asymptotics.
- This remains true even in the regime where gravity is *strongly coupled* and the description in terms of low-energy fields (such as the metric) breaks down deep in the interior.
- The metric description is still valid *asymptotically*, however, and takes the same form as before. Gauge/gravity duality requires the value of $g_{(2\sigma)ij}$ deriving from stringy dynamics to match that derived from the dual weakly coupled QFT.

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- This remains true even in the regime where gravity is *strongly coupled* and the description in terms of low-energy fields (such as the metric) breaks down deep in the interior.
- The metric description is still valid *asymptotically*, however, and takes the same form as before. Gauge/gravity duality requires the value of $g_{(2\sigma)ij}$ deriving from stringy dynamics to match that derived from the dual weakly coupled QFT.

Bulk asymptotics

The general asymptotic solution for the 4d bulk metric reads:

$$ds^2 = dr^2 + e^{2r} g_{ij}(r, x) dx^i dx^j,$$
$$g_{ij}(r, x) = g_{(0)ij}(x) + e^{-2r} g_{(2)ij}(x) + \ldots + e^{-2\sigma r} g_{(2\sigma)ij}(x) + \ldots$$

- $g_{(0)ij}(x)$ is the metric seen by the dual QFT, and hence acts as the *source* for the dual stress tensor T_{ij}.
- The $g_{(2k)ij}(x)$ with $k < \sigma$ are locally determined in terms of $g_{(0)ij}(x)$ via the asymptotic analysis of the field equations.
- $g_{(2\sigma)ij}(x)$ is only partially constrained by the asymptotic analysis of the field equations, and is related to the dual 1-pt function:

$$\langle T_{ij} \rangle = \frac{1}{2\bar{\kappa}^2} (2\sigma g_{(2\sigma)ij}).$$

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Two-point functions

- Higher-point functions may be obtained by differentiating the 1-pt function w.r.t. the sources and then setting the sources to their background values,

$$\text{e.g.} \qquad \langle T_{ij}(x) T_{kl}(y) \rangle \sim \frac{\delta g_{(2\sigma)ij}(x)}{\delta g_{(0)kl}(y)}\Big|_{g_{(0)}=\delta}$$

- To compute 2-pt functions one only needs to solve for the fluctuations to *linear* order.
- On general grounds, the 2-pt function for the stress tensor admits the decomposition

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}.$$

where the transverse and transverse traceless projection operators are

$$\pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2. \quad \Pi_{ijkl} = \pi_{i(k} \pi_{l)j} - (1/2) \pi_{ij} \pi_{kl}.$$

Holographic analysis

- Employing the radial Hamiltonian formulation of holographic renormalisation[2], we showed that for both asymptotically AdS and asymptotically power-law DW spacetimes,

$$A(\bar{q}) = 4\bar{E}_{(0)}(\bar{q}), \quad B(\bar{q}) = (1/4)\bar{\Omega}_{(0)}(\bar{q}).$$

- Thus, the 2-pt function $\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle$ may be extracted from the DW response functions. The subscript indicates taking the term with appropriate scaling in the asymptotic expansion.

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(Some details)

- In the Hamiltonian formalism, the asymptotic radial expansion of the metric is replaced by a *covariant* expansion in eigenfunctions of the dilatation operator:

$$\delta_D = \partial_r + O(e^{-2r}). \qquad \delta_D A_{(m)} = -m A_{(m)}.$$

- The dual 1-pt function is then

$$\langle T^i_j \rangle = \left(\frac{-2}{\sqrt{g}} \bar{\Pi}^i_j \right)_{(3)}$$

where $\bar{\Pi}^i_j$ is the radial canonical momentum.

- For asymptotically power-law spacetimes, the holographic analysis takes place in the *dual frame*, where the metric is asymptotically AdS.

(Some details)

Under a linear variation of the QFT sources $g_{(0)ij}$ and $\varphi_{(0)}$,

$$\delta\langle T_{ij}\rangle = -\frac{1}{2}\sqrt{g_{(0)}}\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle\delta g^{kl}_{(0)} - \sqrt{g_{(0)}}\langle T_{ij}(\bar{q})\mathcal{O}(-\bar{q})\rangle\delta\varphi_{(0)}$$
$$= \frac{1}{2}A(\bar{q})\gamma^{i}_{j(0)} - 2B(\bar{q})\psi_{(0)}\pi^{i}_{j} - \langle T^{i}_{j}(\bar{q})\mathcal{O}(-\bar{q})\rangle\delta\varphi_{(0)}.$$

In comparison, perturbing the bulk radial canonical momentum to linear order, we find (*e.g.* in the asymptotically AdS case)

$$\delta\langle T^{i}_{j}\rangle = \left[\frac{2\bar{E}(\bar{q})}{a^3}\gamma^{i}_{j} - \left(\frac{\bar{q}^2}{\bar{\kappa}^2 a^2 H} + \frac{\bar{\Omega}(\bar{q})}{2a^3}\right)\psi\pi^{i}_{j} - (\ldots)\delta\varphi\right]_{(3)}$$

Extracting the terms with appropriate dilatation weight then yields:

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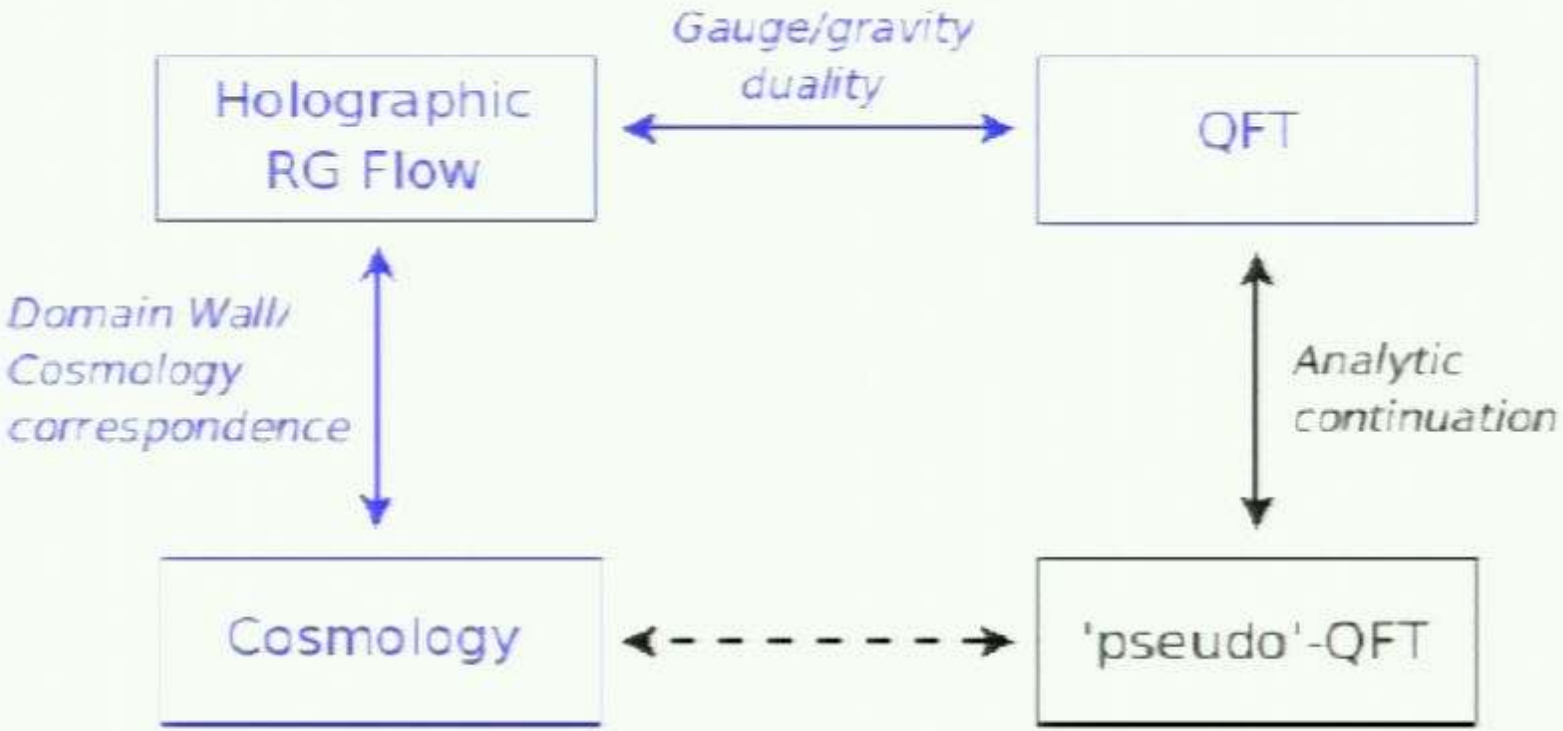
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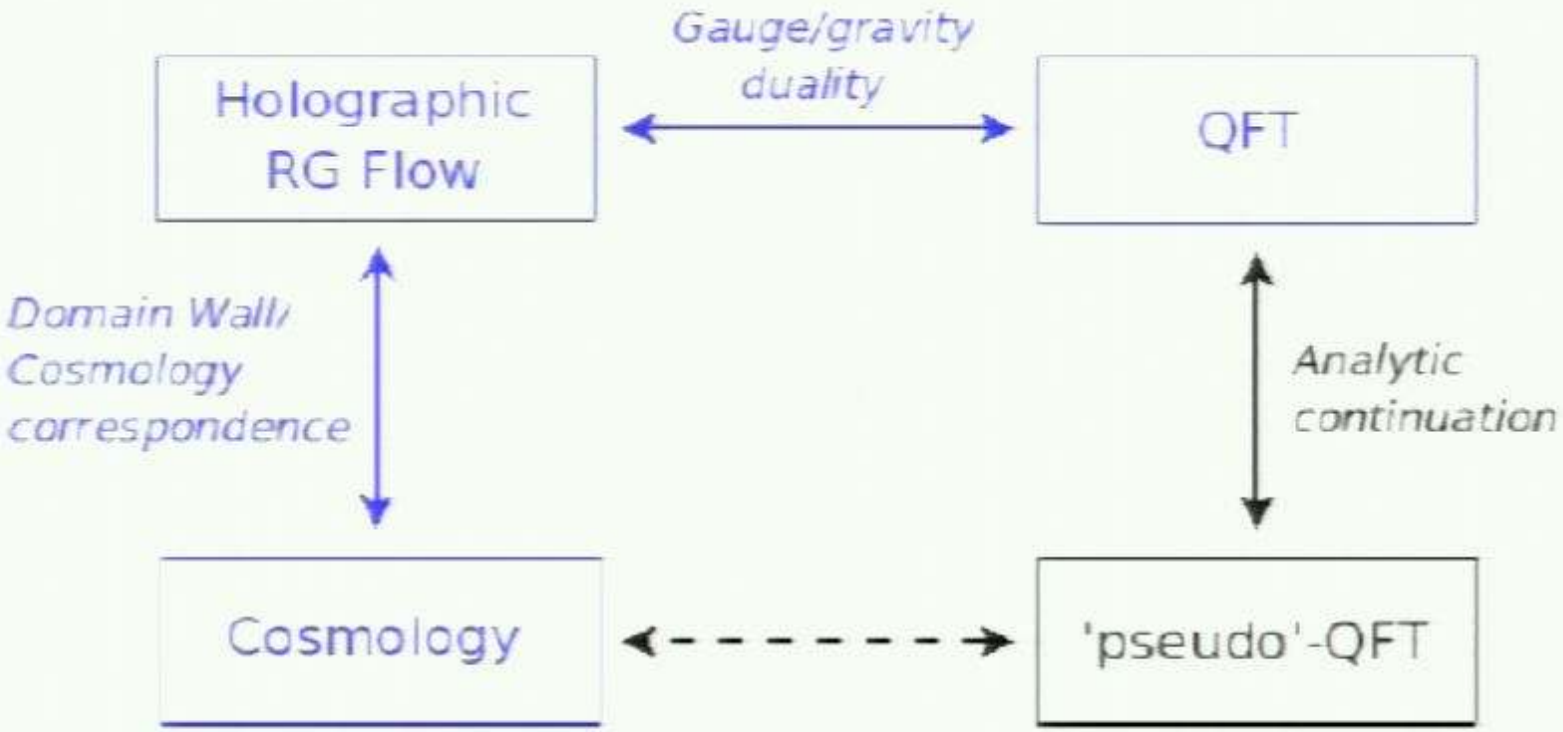
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From cosmology to QFT

Applying the analytic continuations $\bar{\kappa}^2 = -\kappa^2$, $\bar{q} = -iq$. we find a direct relation between the cosmological power spectra and the 2-pt functions of the dual QFT:

$$\Delta_S^2(q) = \frac{-q^3}{16\pi^2 \mathrm{Im} B(-iq)}, \quad \Delta_T^2(q) = \frac{-2q^3}{\pi^2 \mathrm{Im} A(-iq)},$$

where

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}.$$