Title: Fractional Quantum Hall Effect and Featureless Mott Insulators

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Abstract: We point out and explicitly demonstrate a close connection that exists between featureless Mott insulators and fractional quantum Hall liquids. Using magnetic Wannier states as the single-particle basis in the lowest Landau level (LLL), we demonstrate that the Hamiltonian of interacting bosons in the LLL maps onto a Hamiltonian of a featureless Mott insulator on triangular lattice, formed by the magnetic Wannier states. The Hamiltonian is remarkably simple and consists only of short-range repulsion and ring-exchange terms.

Fractional Quantum Hall Effect and Featureless Mott Insulators

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Outline

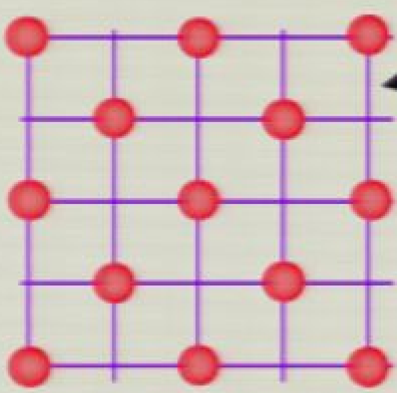
- Introduction: what's the connection and why it's interesting.
- FQHL in magnetic Wannier basis: Mott insulator on triangular lattice.

FQHL vs Mott insulator

- Lowest Landau level: $N_\phi = \dfrac{L_x L_y}{2\pi\ell^2}$ degenerate single-particle states.
- Fill with repulsively interacting particles at filling factor $\nu = N/N_\phi$.
- At some $\nu = \dfrac{p}{q}$ ground states are incompressible liquids with topological order.

FQHL vs Mott insulator

- Mott insulator: fill degenerate Wannier orbitals in a crystal lattice with repulsively interacting particles.
- At filling factors $\nu = \frac{p}{q}$ get incompressible states.



Mott insulator at $\nu = 1/2$ on square lattice

Important difference from FQHE: Mott insulators at fractional filling typically break lattice symmetries.

FQHL is related to featureless Mott insulator

Related work

- Kalmeyer-Laughlin spin liquid.
- FQHL on a thin torus: Seidel, Fu, Lee, Leinaas, Moore; Bergholtz & Karlhede

Why is it not trivial to make the connection explicit?

- Standard choices for LLL orbital eigenstates are not localized.
- This makes the energy penalty for doubly occupying a particular orbital vanish in the thermodynamic limit.
- Then it's hard to make an analogy to Mott insulator.

Magnetic Wannier basis

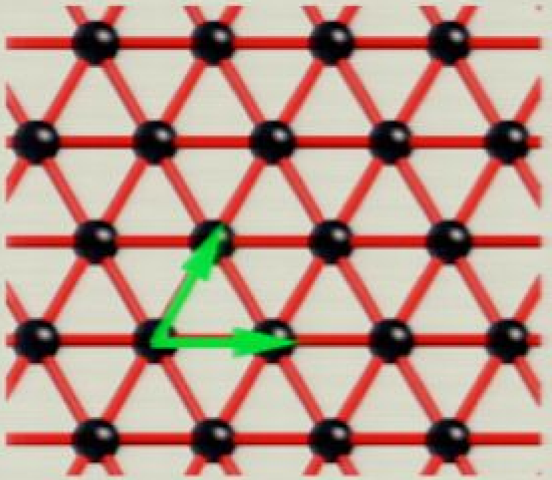
- To make the 2D Mott insulator connection explicit need to choose a LLL basis, consisting of wavefunctions, localized in all directions in the 2D plane, analogous to Wannier functions in insulators.
- Thouless: exponentially localized Wannier functions are incompatible with nonzero Chern number, thus the above seems impossible.
- But turns out to be possible to construct "quasilocalized" Wannier functions: normalizable but with $1/r^2$ tail.

Rashba, Zhukov, Efros, PRB 55, 5306 (1997)

Magnetic Wannier basis

- Symmetric-gauge LLL wavefunction with zero angular momentum:

$$c_0(\mathbf{r}) = \frac{1}{\sqrt{2\pi\ell^2}} e^{-r^2/4\ell^2}$$



- Translate to sites of any 2D Bravais lattice with one flux quantum per unit cell:

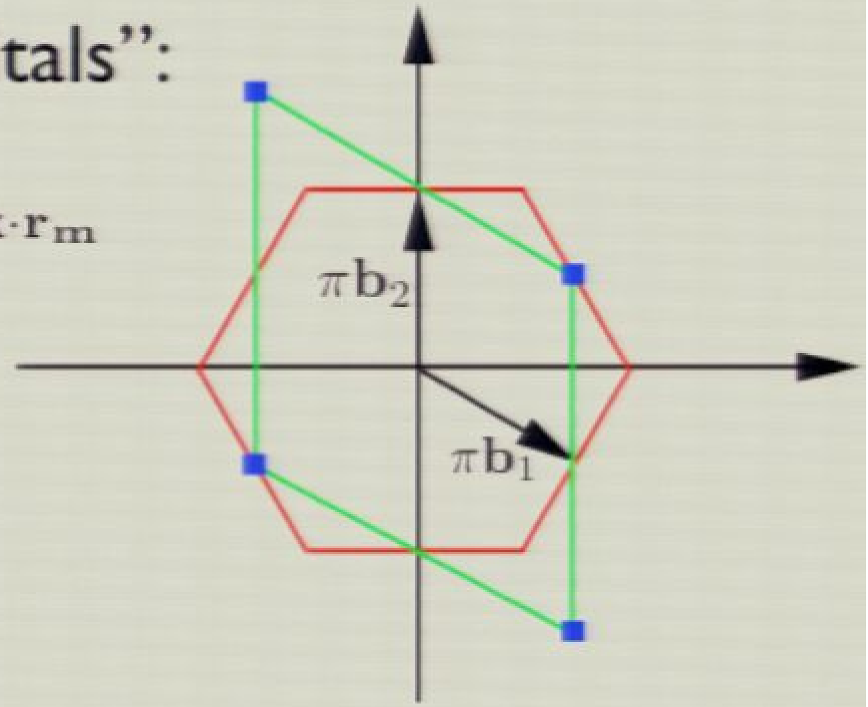
$$c_{\mathbf{m}}(\mathbf{r}) = T_{m_1\mathbf{a}_1} T_{m_2\mathbf{a}_2} c_0(\mathbf{r}) = \frac{(-1)^{m_1 m_2}}{\sqrt{2\pi\ell^2}} e^{-(\mathbf{r}-\mathbf{r}_{\mathbf{m}})^2/4\ell^2 + (i/2\ell^2)\hat{z}\cdot(\mathbf{r}\times\mathbf{r}_{\mathbf{m}})}$$

- Perelomov overcompleteness identity: $\sum_{\mathbf{m}} (-1)^{m_1+m_2} c_{\mathbf{m}}(\mathbf{r}) = 0$

This is the origin of the nontrivial topological properties of Bloch states, i.e. nonvanishing Chern number

Magnetic Bloch and Wannier states

- Construct Bloch states out of linear combinations of "atomic orbitals":



$$\Psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N_\phi \nu(\mathbf{k})}} \sum_{\mathbf{m}} c_{\mathbf{m}}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}_{\mathbf{m}}}$$

$$\mathbf{k} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2$$

$\nu(\mathbf{k})$ vanishes at $(k_1, k_2) = (\pi, \pi)$

- Wannier functions are obtained by inverse Fourier transform:

$$\phi_{\mathbf{m}}(\mathbf{r}) = \frac{1}{\sqrt{N_\phi}} \sum_{\mathbf{k}} \Psi_{\mathbf{k}}(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}_{\mathbf{m}}}$$

Hamiltonian in Wannier basis

$$H = \sum_{\mathbf{m}_1,\mathbf{m}_2,\mathbf{m}_3,\mathbf{m}_4} \langle \mathbf{m}_1 \mathbf{m}_2 | V | \mathbf{m}_3 \mathbf{m}_4 \rangle b^\dagger_{\mathbf{m}_1} b^\dagger_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3}$$

- Matrix elements are finite in thermodynamic limit and short-range $\sim 1/r^6$
- However, no obvious symmetries apart from the symmetry of triangular lattice, thus a lot of different nonzero matrix elements.
- In this form, the Hamiltonian is useless.
- But there is a hidden symmetry.

Magnetic Bloch states and the Abrikosov vortex lattice

$$\sum_{\mathbf{m}} (-1)^{m_1+m_2} c_{\mathbf{m}}(\mathbf{r}) = 0$$

- It follows from the Perelomov identity that the zeros of $\Psi_{\mathbf{k}}(\mathbf{r})$ form a triangular lattice with one flux quantum per unit cell.
- Zeros are located at: $\mathbf{r}_{\mathbf{mk}} = \mathbf{r}_{\mathbf{m}} + \frac{1}{2}(\mathbf{a}_1 + \mathbf{a}_2) + \ell^2 \hat{z} \times \mathbf{k}$
- Thus magnetic Bloch states correspond to Abrikosov vortex lattices, with the first BZ momentum labeling different vortex lattice positions.
- Ground states of interacting bosons in LLL at large filling factors thus correspond to condensation into Bloch states with a particular momentum.

Hamiltonian in Wannier basis

$$H = \sum_{\mathbf{m}_1,\mathbf{m}_2,\mathbf{m}_3,\mathbf{m}_4} \langle \mathbf{m}_1 \mathbf{m}_2 | V | \mathbf{m}_3 \mathbf{m}_4 \rangle b^\dagger_{\mathbf{m}_1} b^\dagger_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3}$$

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Abrikosov lattice in Wannier basis

$$H = \sum_{\mathbf{m}_1,\mathbf{m}_2,\mathbf{m}_3,\mathbf{m}_4} \langle \mathbf{m}_1 \mathbf{m}_2 | V | \mathbf{m}_3 \mathbf{m}_4 \rangle b^\dagger_{\mathbf{m}_1} b^\dagger_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3}$$

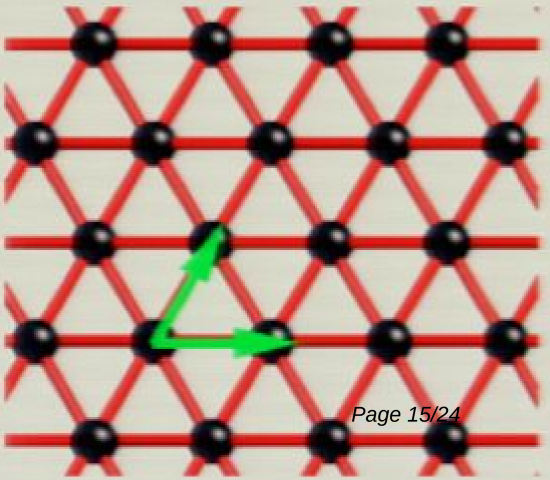
$$b^\dagger_{\mathbf{m}} = \frac{1}{\sqrt{N_\phi}} \sum_{\mathbf{k}} b^\dagger_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_{\mathbf{m}}}$$

Abrikosov lattice = condensate at a single momentum = uniform phase winding in Wannier representation

$$\mathbf{k} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2$$

Periodic boundary conditions: $k_{1,2} = \frac{2\pi n_{1,2}}{\sqrt{N_\phi}}$

$n_{1,2}$ are phase winding numbers



Abrikosov lattice states in Wannier rep. are labelled by phase winding numbers with respect to triangular lattice basis directions

Phase fluctuations

Since all states with uniform phase gradients are degenerate, the long-wavelength phase action can't depend on gradient of the phase:

$$S \sim \int d\tau d\mathbf{r} \left[(\partial_\tau \theta)^2 + (\nabla^2 \theta)^2 \right]$$

This means that the dispersion of phase fluctuations is quadratic, instead of linear:

$$\omega_{\mathbf{q}} \sim \mathbf{q}^2$$

Sinova, Hanna, MacDonald

Doesn't necessarily mean that system is not superfluid: vortices may still be localized and superfluid stiffness finite

Center-of-mass conservation

- Absence of the $(\nabla\theta)^2$ term in the phase action is a consequence of an emergent conservation law: conservation of the center of mass of the bosons

$$H = \sum_{\mathbf{k},\mathbf{q},\mathbf{q}'} \langle \mathbf{k}+\mathbf{q}, \mathbf{k}+\mathbf{q}' | V | \mathbf{k}, \mathbf{k}+\mathbf{q}+\mathbf{q}' \rangle b^\dagger_{\mathbf{k}+\mathbf{q}} b^\dagger_{\mathbf{k}+\mathbf{q}'} b_{\mathbf{k}+\mathbf{q}+\mathbf{q}'} b_{\mathbf{k}}$$

- All Bloch functions are related to each other by magnetic translations:

$$\Psi_{\mathbf{k}}(\mathbf{r}) = e^{i\gamma_{\mathbf{k}}} e^{\frac{i}{2}\mathbf{k}\cdot\mathbf{r}} \Psi_0(\mathbf{r} - \ell^2 \hat{z} \times \mathbf{k})$$

- It follows that the interaction matrix element is independent of $\mathbf{k}$, as long as Umklapp processes can be neglected, i.e. when q and q' are small.

- Can describe vortex liquids with long correlation length:

$$\xi \gg \ell$$

Center-of-mass conservation

$$H = \sum_{\mathbf{k},\mathbf{q},\mathbf{q}'} \langle \mathbf{k}+\mathbf{q}, \mathbf{k}+\mathbf{q}' | V | \mathbf{k}, \mathbf{k}+\mathbf{q}+\mathbf{q}' \rangle b^\dagger_{\mathbf{k}+\mathbf{q}} b^\dagger_{\mathbf{k}+\mathbf{q}'} b_{\mathbf{k}+\mathbf{q}+\mathbf{q}'} b_{\mathbf{k}}$$

↓

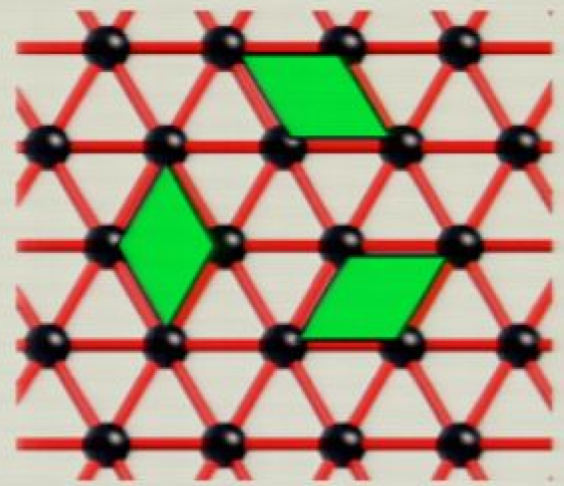
$$H = \sum_{\mathbf{m}_1,\mathbf{m}_2,\mathbf{m}_3,\mathbf{m}_4} \langle \mathbf{m}_1 \mathbf{m}_2 | V | \mathbf{m}_3 \mathbf{m}_4 \rangle b^\dagger_{\mathbf{m}_1} b^\dagger_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3}$$

$$\langle \mathbf{m}_1 \mathbf{m}_2 | V | \mathbf{m}_3 \mathbf{m}_4 \rangle = f(\mathbf{m}_1 - \mathbf{m}_4, \mathbf{m}_2 - \mathbf{m}_4) \delta_{\mathbf{m}_1+\mathbf{m}_2, \mathbf{m}_3+\mathbf{m}_4}$$

Wannier-basis Hamiltonian has emergent center-of-mass conservation

Ring-exchange model on triangular lattice

$$H = -K \sum_{P} b^{\dagger}_{\mathbf{m}_1} b^{\dagger}_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3} + U \sum_{\mathbf{m}} n^2_{\mathbf{m}} + \sum_{\mathbf{m}\mathbf{m}'} V_{\mathbf{m}\mathbf{m}'} n_{\mathbf{m}} n_{\mathbf{m}'}$$



Shortest-range ring-exchange + repulsive interactions on triangular lattice = bosons in the LLL

Sanity check

$$H = -K \sum_P b^\dagger_{\mathbf{m}_1} b^\dagger_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3} + U \sum_{\mathbf{m}} n^2_{\mathbf{m}} + \sum_{\mathbf{m}\mathbf{m}'} V_{\mathbf{m}\mathbf{m}'} n_{\mathbf{m}} n_{\mathbf{m}'}$$

$$\Downarrow \quad \nu \gg 1$$

$$H = -K \cos(\theta_{\mathbf{m}_1} + \theta_{\mathbf{m}_2} - \theta_{\mathbf{m}_3} - \theta_{\mathbf{m}_4})$$

Classical ground state at positive K: uniform phase gradients along basis directions, magnitude of gradient not fixed = Abrikosov lattice

Balents & Paramekanti, 2003

Half-filled Landau level

$\nu = 1/2$ Laughlin liquid $\Psi(z_1, \ldots z_N) = \prod_{i<j} (z_i - z_j)^2 e^{-\frac{1}{4}\sum_i |z_i|^2}$

$$H = -K \sum_P b^\dagger_{\mathbf{m}_1} b^\dagger_{\mathbf{m}_2} b_{\mathbf{m}_4} b_{\mathbf{m}_3} + U \sum_{\mathbf{m}} n^2_{\mathbf{m}} + \sum_{\mathbf{m}\mathbf{m}'} V_{\mathbf{m}\mathbf{m}'} n_{\mathbf{m}} n_{\mathbf{m}'}$$

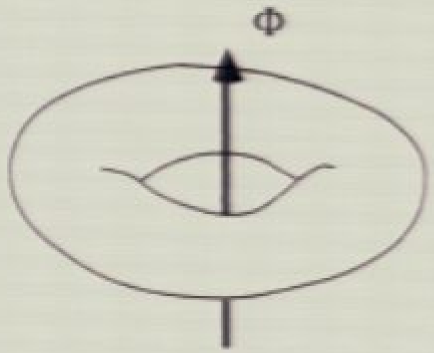
Large U

$$H = -K \sum_P S^+_{\mathbf{m}_1} S^+_{\mathbf{m}_2} S^-_{\mathbf{m}_3} S^-_{\mathbf{m}_4} + V \sum_{\langle \mathbf{m}\mathbf{m}' \rangle} S^z_{\mathbf{m}} S^z_{\mathbf{m}'}$$

K = 0: classical Ising model on triangular lattice,
ground state has extensive degeneracy

Ground state for K < V is a gapped spin liquid with 4-fold degeneracy on a torus

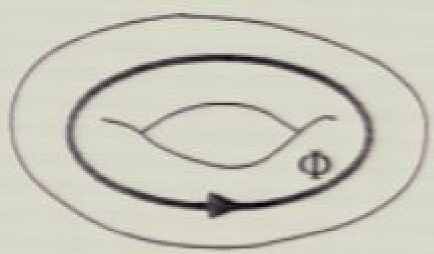
Kalmeyer-Laughlin liquid



(a)

$$H = -K \sum_{P} S^+_{\mathbf{m}_1} S^+_{\mathbf{m}_2} S^-_{\mathbf{m}_3} S^-_{\mathbf{m}_4} + V \sum_{\langle \mathbf{m}\mathbf{m}' \rangle} S^z_{\mathbf{m}} S^z_{\mathbf{m}'}$$

H is time-reversal invariant, thus 4-fold degeneracy on torus



(b)

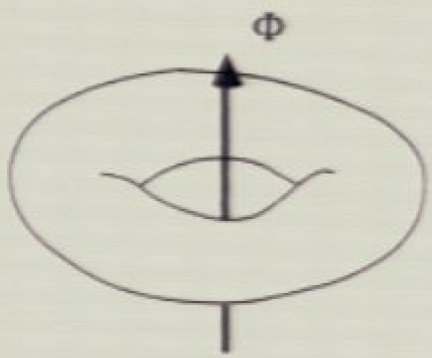
But Laughlin liquid is only 2-fold degenerate

Resolution: spontaneous time-reversal breaking, degeneracy between the two pairs of states lifted by otherwise irrelevant COM-nonconserving terms in H

Conclusions

- Interacting bosons in the LLL map onto a time-reversal invariant model of bosons on triangular lattice with ring-exchange and repulsive interactions.
- At filling factors at which the bosons in the LLL exhibit FQHE, the ground state of the equivalent model on triangular lattice is a featureless Mott insulator with topological order.
- Bose metal states at odd-denominator fractions?
- All assertions can be checked by QMC on large system sizes, since there is no sign problem (unlike if one directly simulates charged bosons in magnetic field).

Kalmeyer-Laughlin liquid



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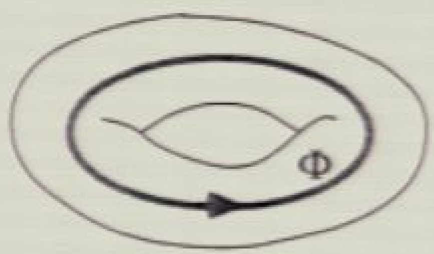

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Resolution: spontaneous time-reversal breaking, degeneracy between the two pairs of states lifted by otherwise irrelevant COM-nonconserving terms in H