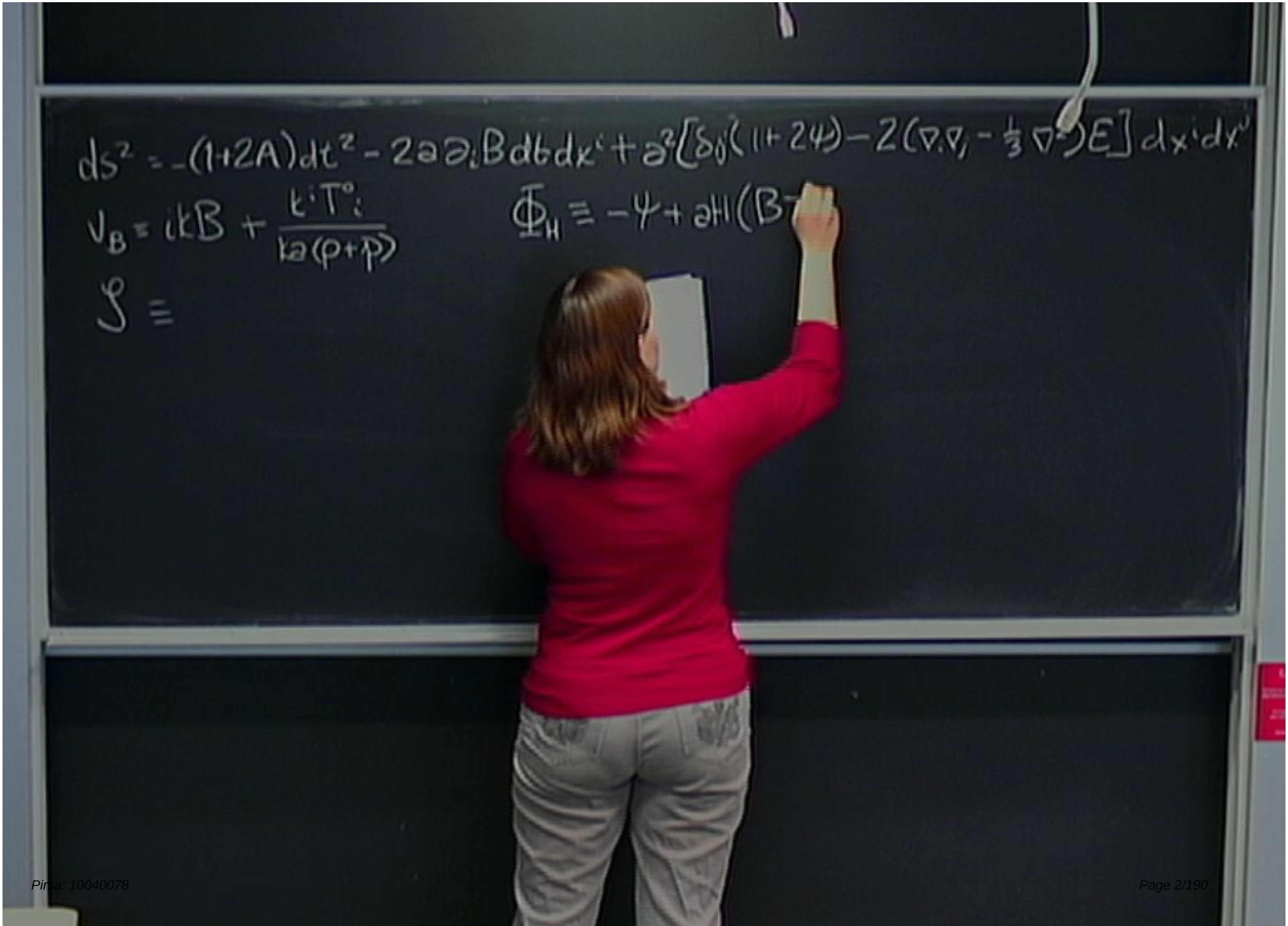
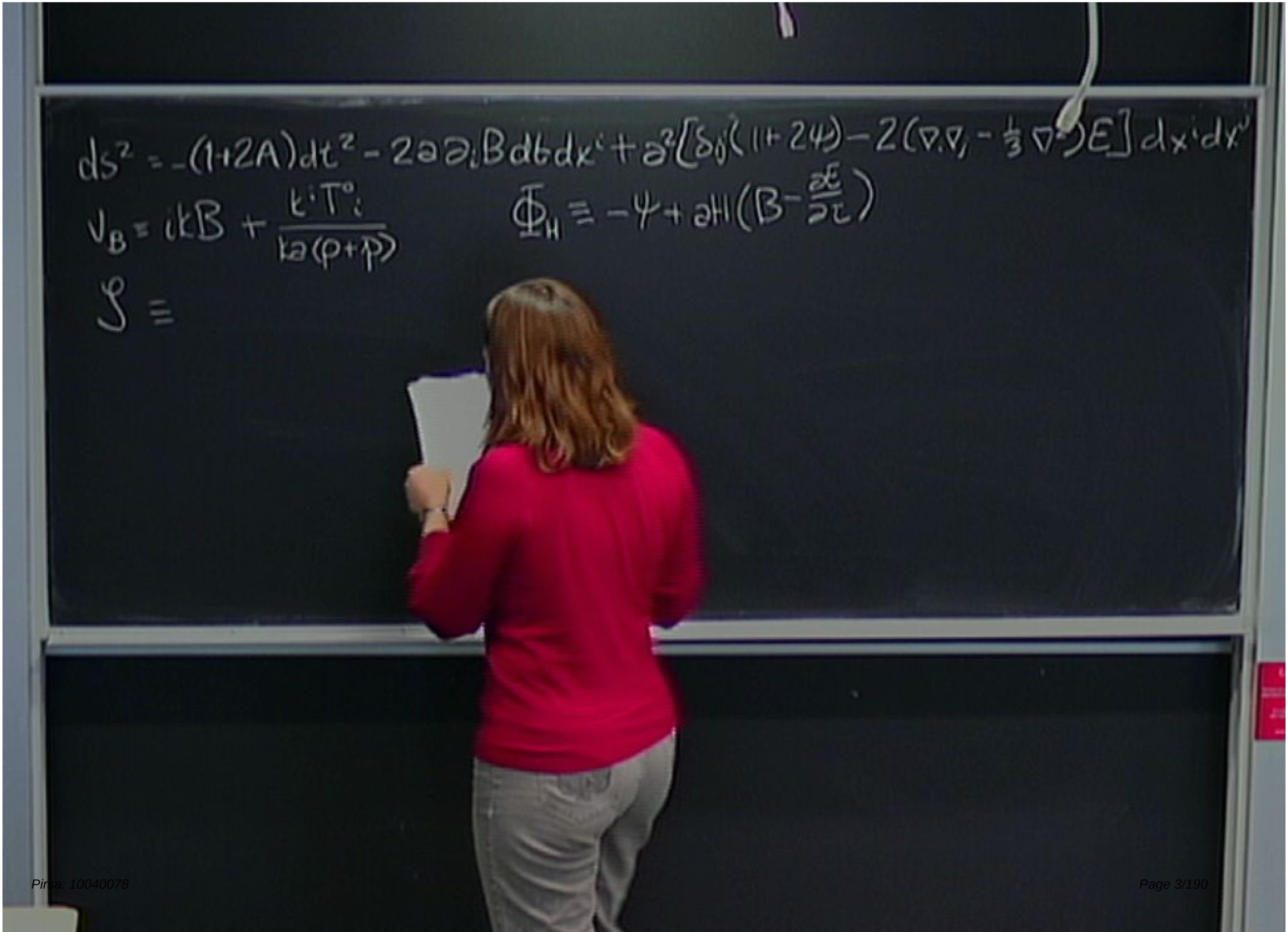
Title: Generation of Perturbations During Inflation

Date: Apr 20, 2010 12:00 PM

URL: http://pirsa.org/10040078

Abstract: Lecture 4


$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2[\delta_{ij}(1+2\psi) - 2(\nabla_i\nabla_j - \frac{1}{3}\nabla^2)E]\, dx^i dx^j$
$v_B \equiv ikB + \frac{k^i T^0_i}{ka(\rho+p)}$
$\Phi_H \equiv -\psi + aH(B -$
$\zeta \equiv$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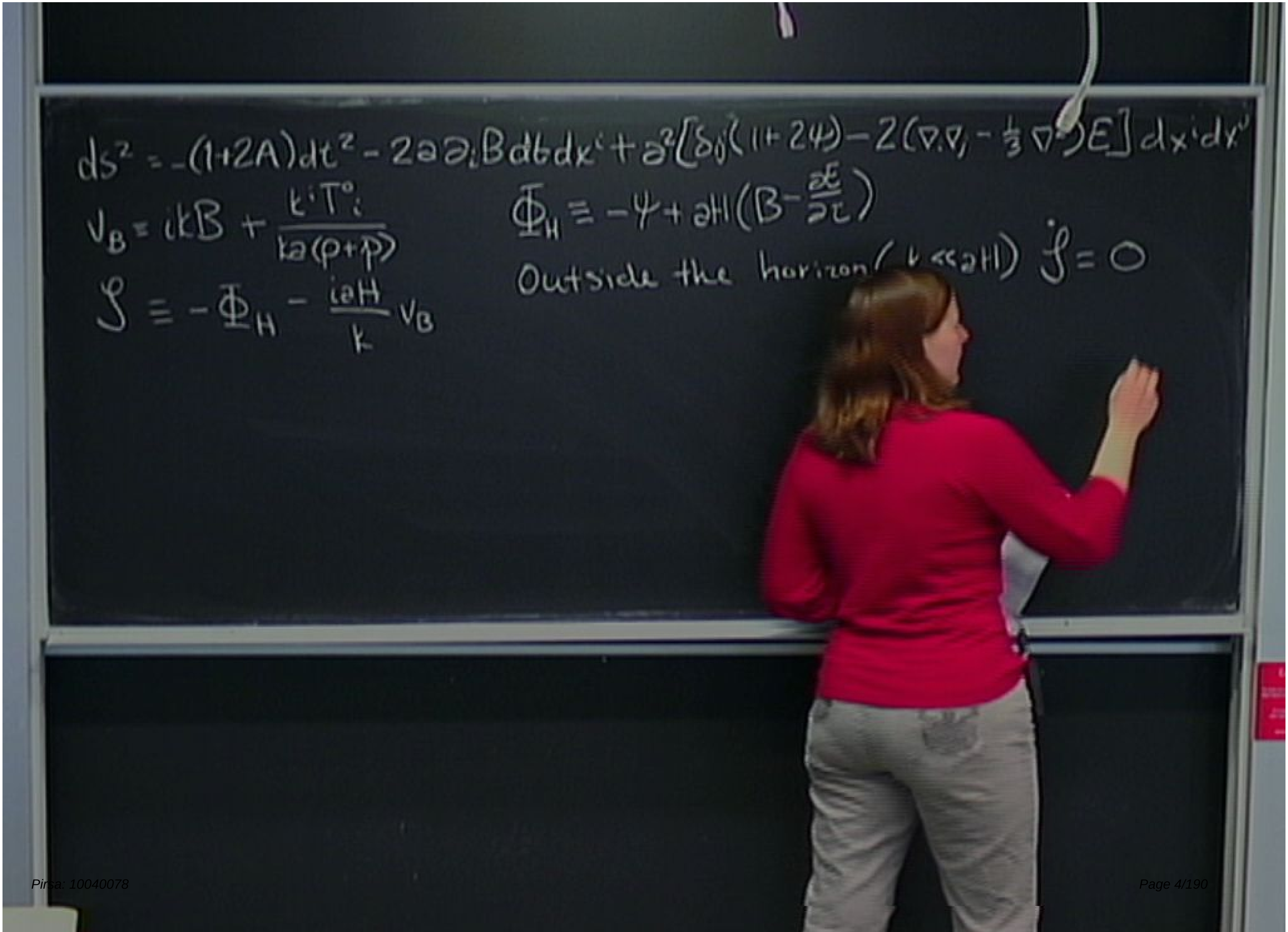
$\zeta \equiv$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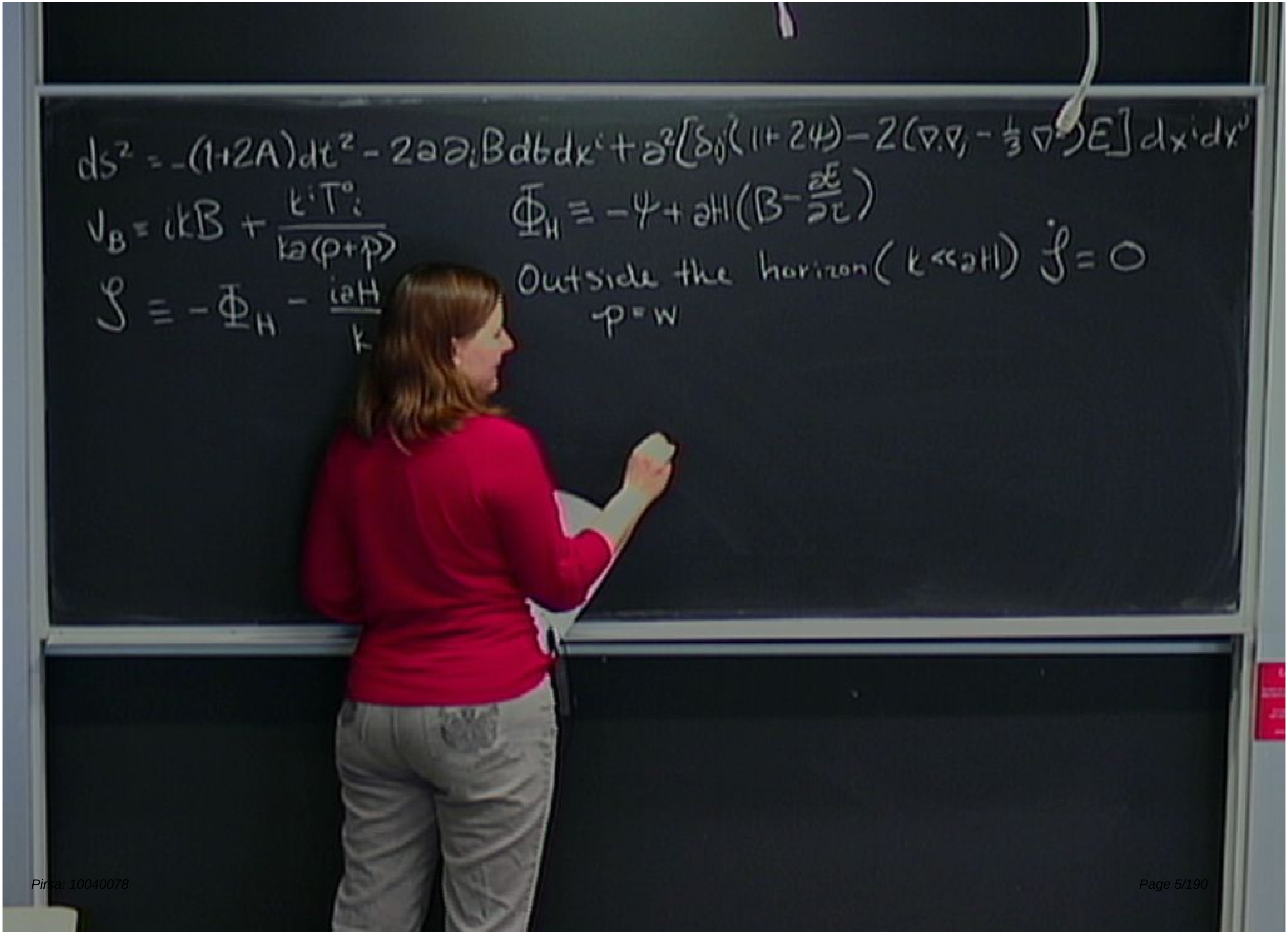
$\Phi_H \equiv -\psi + aH(B - \frac{\partial E}{\partial \tau})$
$\zeta \equiv -\Phi_H - \frac{iaH}{k} v_B$
Outside the horizon ($k \ll aH$) $\dot{\zeta} = 0$


$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\,dt\,dx^i + a^2[\delta_{ij}(1+2\psi) - 2(\nabla_i\nabla_j - \frac{1}{3}\nabla^2)E]\,dx^i dx^j$
$v_B \equiv ikB + \frac{k^i T^0_i}{ka(\rho+p)}$
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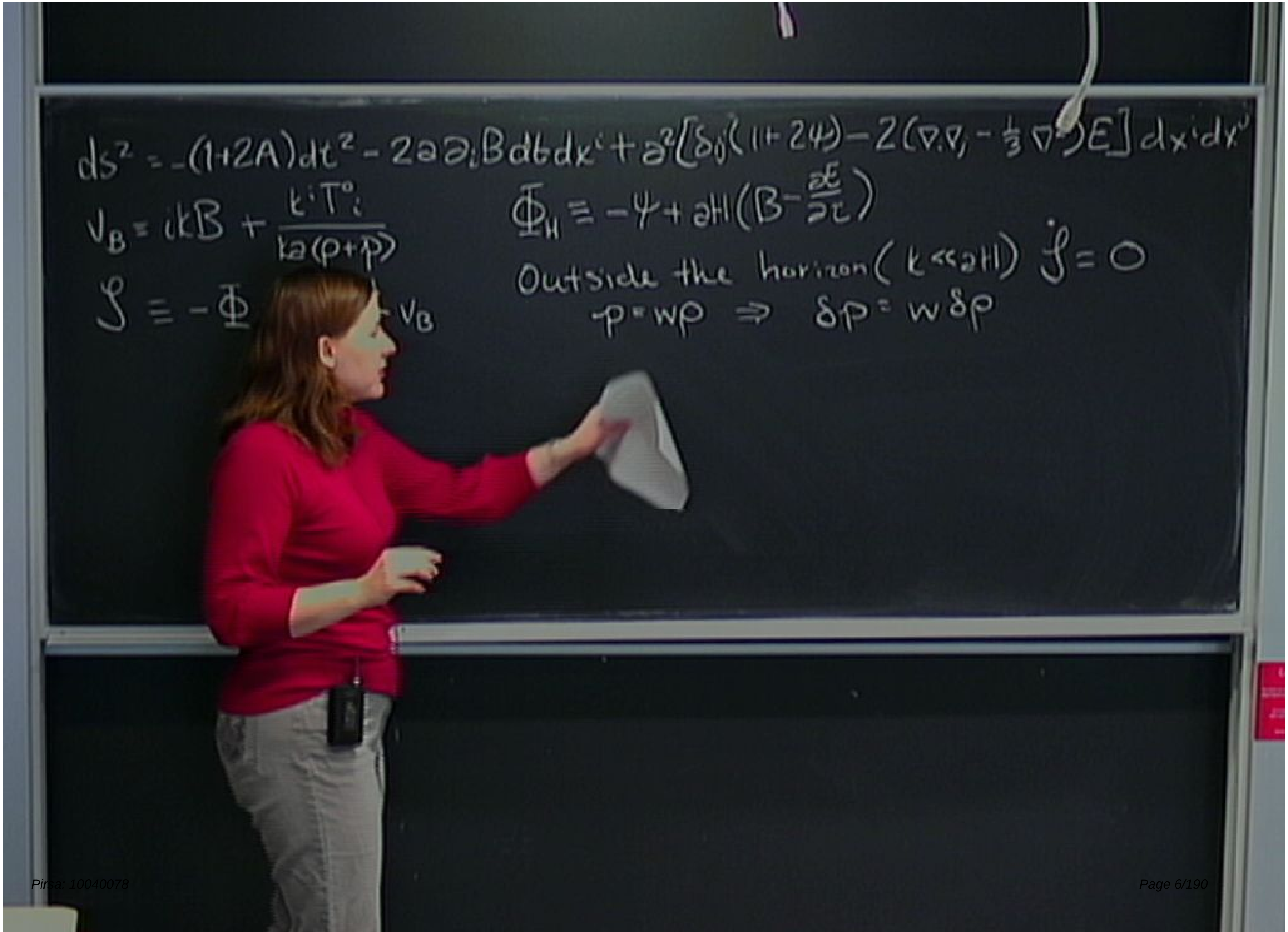
$\zeta \equiv -\Phi_H - \frac{iaH}{k}$
Outside the horizon ($k \ll aH$) $\dot{\zeta} = 0$
$p = w$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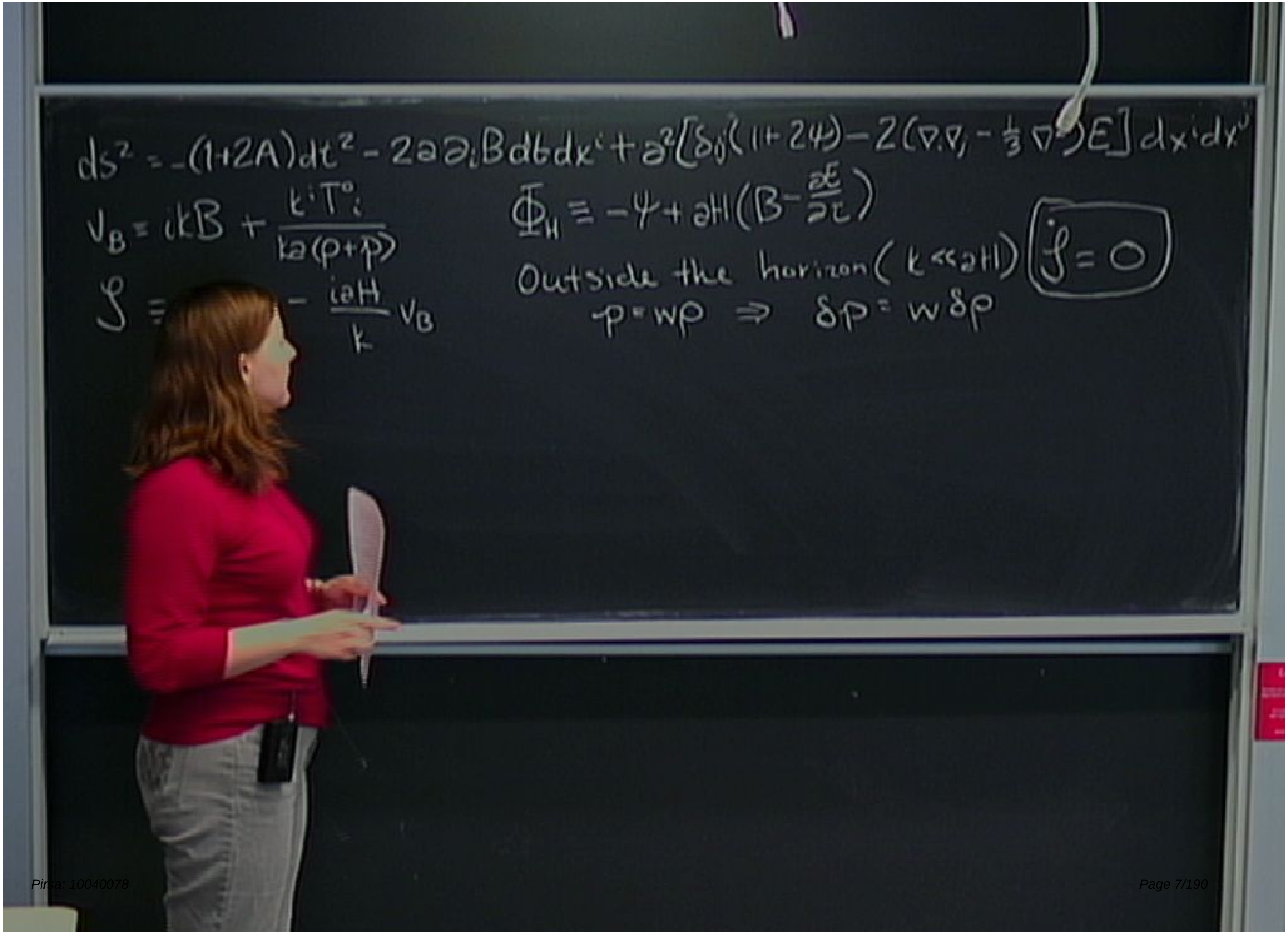
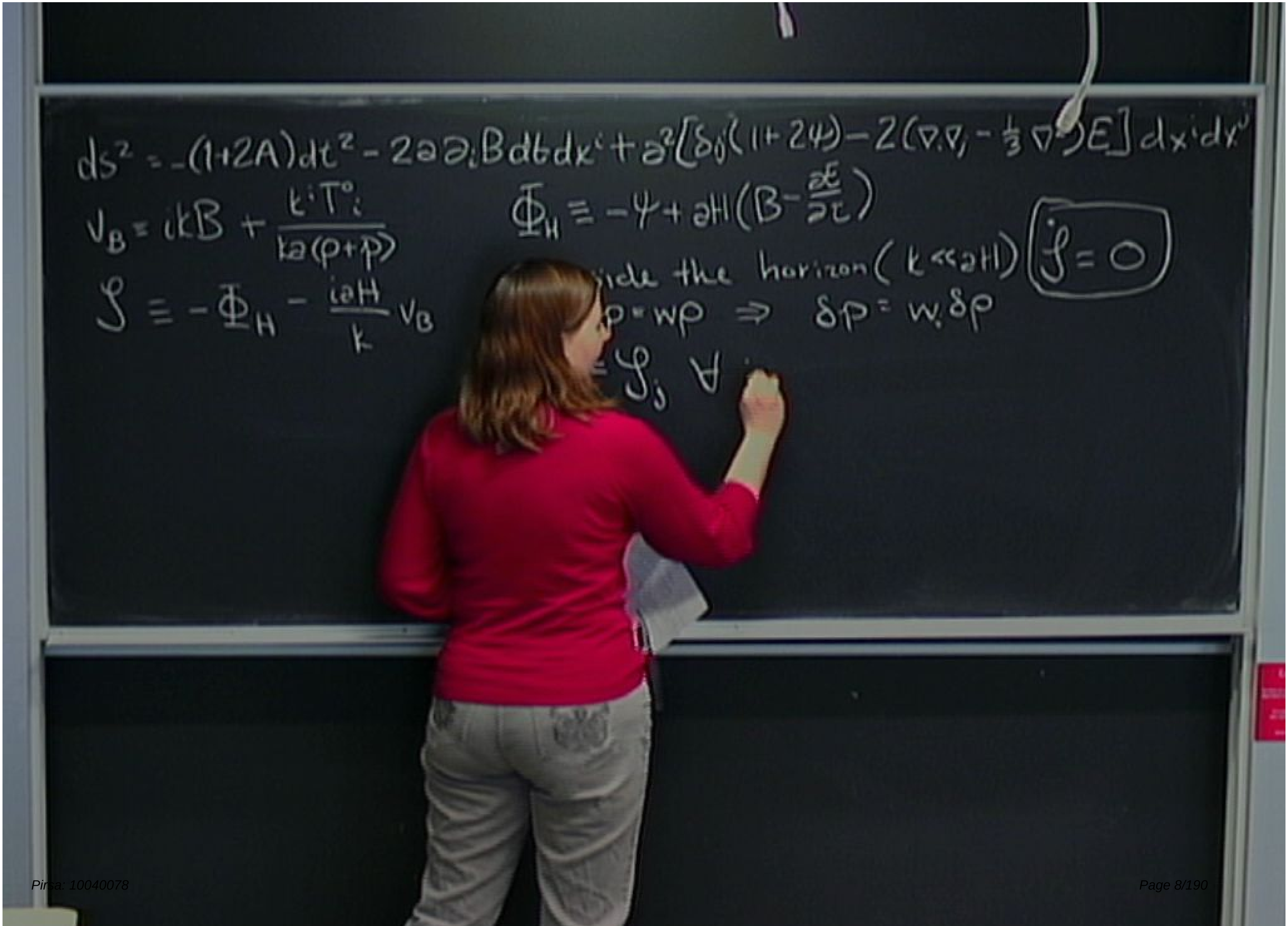
$v_B \equiv ikB + \frac{k^i T^0_i}{ka(\rho+p)}$
$\Phi_H \equiv -\psi + aH(B - \frac{\partial E}{\partial \tau})$
Outside the horizon ($k \ll aH$) $\dot{\zeta} = 0$
$p = w\rho \Rightarrow \delta p = w\delta\rho$


ds² = −(1+2A)dt² − 2a∂ᵢB dt dxⁱ + a²[δᵢⱼ(1+2ψ) − 2(∇ᵢ∇ⱼ − ⅓∇²)E] dxⁱdxʲ
v_B = ikB + kⁱT⁰ᵢ / ka(ρ+p)
ζ = … − (iaH / k) v_B
Φ_H = −ψ + aH(B − ∂E/∂t)
Outside the horizon (k ≪ aH)
ζ̇ = 0
p = wρ ⇒ δp = wδρ



$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\,dt\,dx^i + a^2[\delta_{ij}(1+2\psi) - 2(\nabla_i\nabla_j - \frac{1}{3}\nabla^2)E]dx^i dx^j$
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ide the horizon ($k \ll aH$)
$\dot{\zeta} = 0$
$p = w\rho \Rightarrow \delta p = w_i \delta\rho$
$= \zeta_j\ \forall$

$$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2\left[\delta_{ij}(1+2\psi) - 2\left(\nabla_i\nabla_j - \tfrac{1}{3}\nabla^2\right)E\right]dx^i dx^j$$

$$v_B \equiv ikB + \frac{k^i T^0_i}{ka(\rho+p)}$$

$$\zeta \equiv -\Phi_H - \frac{iaH}{k} v_B$$

$$\zeta$$

$$\Phi_H \equiv -\psi + aH\left(B - \frac{\partial E}{\partial t}\right)$$

Outside the horizon ($k \ll aH$) $\dot{\zeta} = 0$

$$p = w\rho \Rightarrow \delta p = w_i \delta\rho$$

$\zeta_i = \zeta_j \;\forall\, i,j \Rightarrow$ adiabatic

$$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2\left[\delta_{ij}(1+2\psi) - 2\left(\nabla_i\nabla_j - \tfrac{1}{3}\nabla^2\delta_{ij}\right)E\right]dx^i dx^j$$

$$v_B \equiv ikB + \frac{k^i T^0{}_i}{ka(\rho+p)}$$

$$\Phi_H \equiv -\psi + aH\left(B - \frac{\partial E}{\partial t}\right)$$

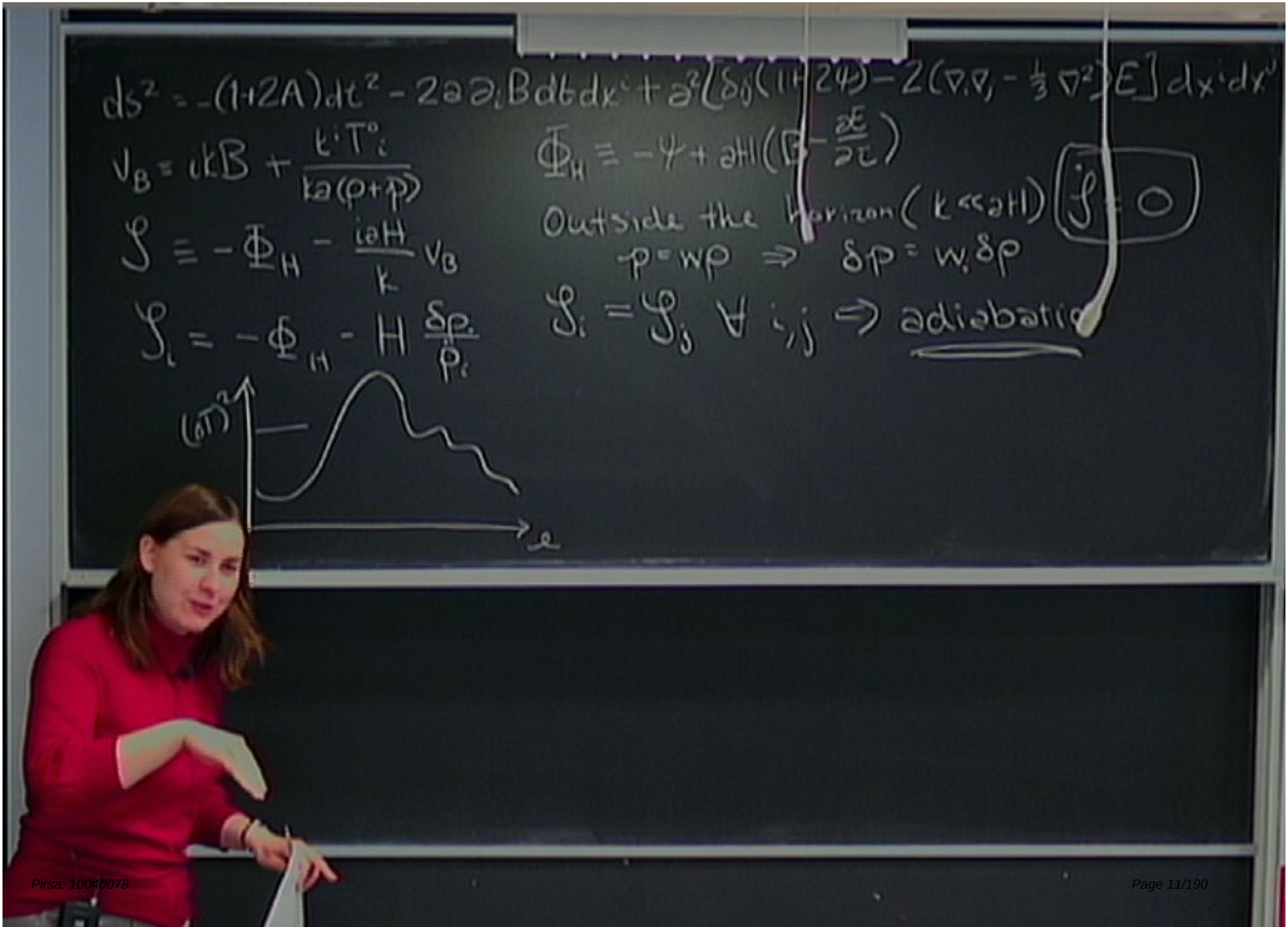
$$\zeta \equiv -\Phi_H - \frac{iaH}{k} v_B$$

Outside the horizon ($k \ll aH$) $\boxed{\dot{\zeta} = 0}$

$$p = w\rho \Rightarrow \delta p = w_i\,\delta\rho$$

$$\zeta_i = -\Phi_H - H\frac{\delta\rho_i}{\dot{\rho}_i}$$

$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiabatic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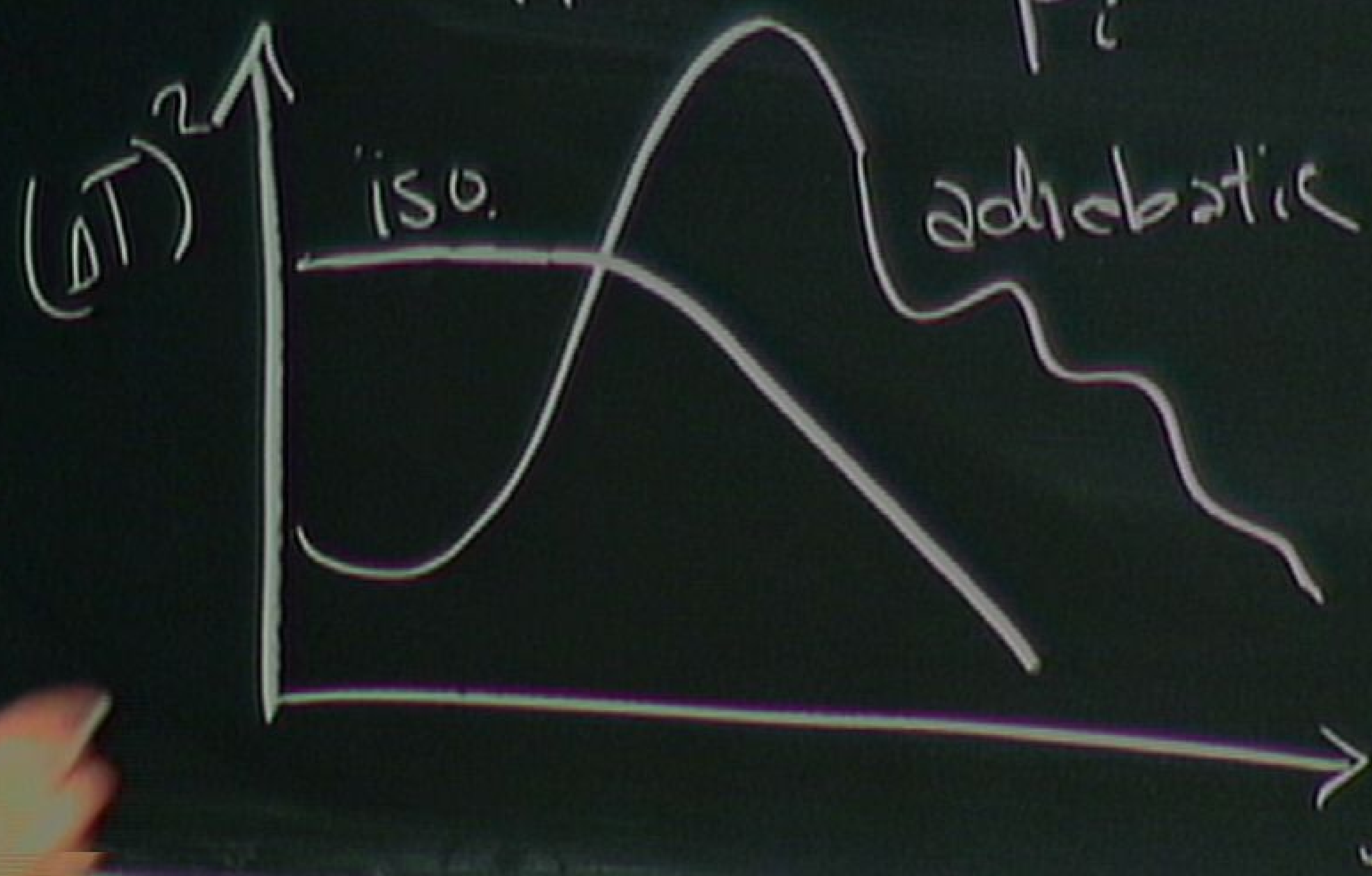
$v_B = ikB + \frac{k^i T^0_i}{ka(\rho+p)}$
$\Phi_H = -\psi + aH(B - \frac{\partial E}{\partial \tau})$
$\zeta = -\Phi_H - \frac{iaH}{k} v_B$
Outside the horizon $(k \ll aH)$ $\dot\zeta = 0$
$p = w\rho \Rightarrow \delta p = w_i \delta\rho$
$\zeta_i = -\Phi_H - H\frac{\delta\rho_i}{\dot\rho_i}$
$\zeta_i = \zeta_j\ \forall\ i,j \Rightarrow$ adiabatic
$(\delta T)^2$
l

$$\zeta_i = -\Phi_H - H \frac{\delta\rho_i}{\dot{\rho}_i}$$

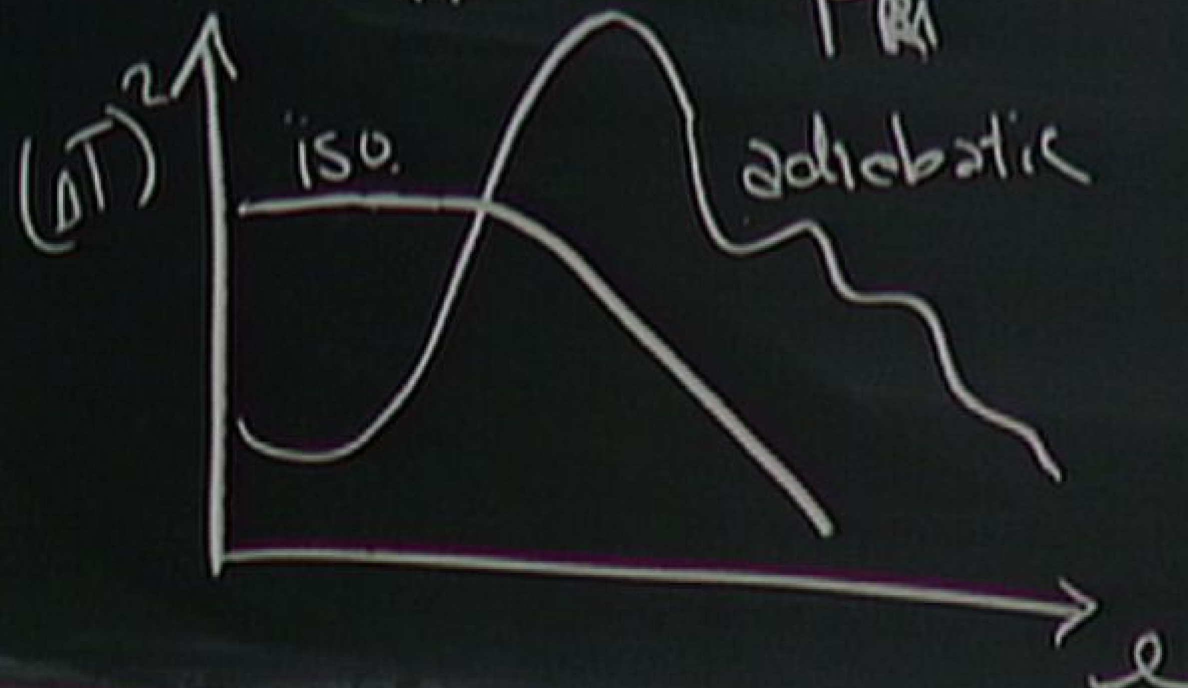


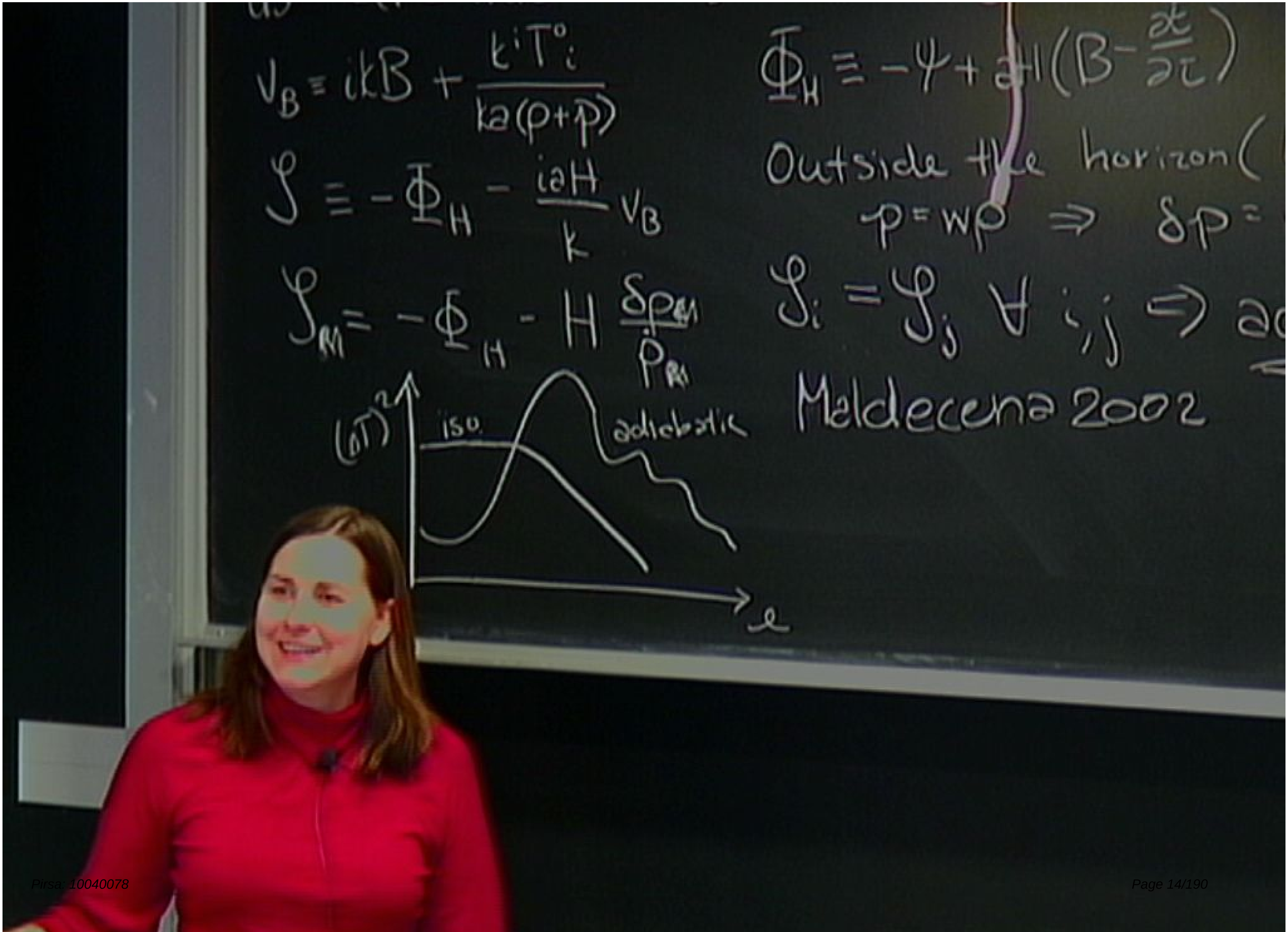
$$\zeta \equiv -\Phi_H - \frac{iaH}{k} v_B$$

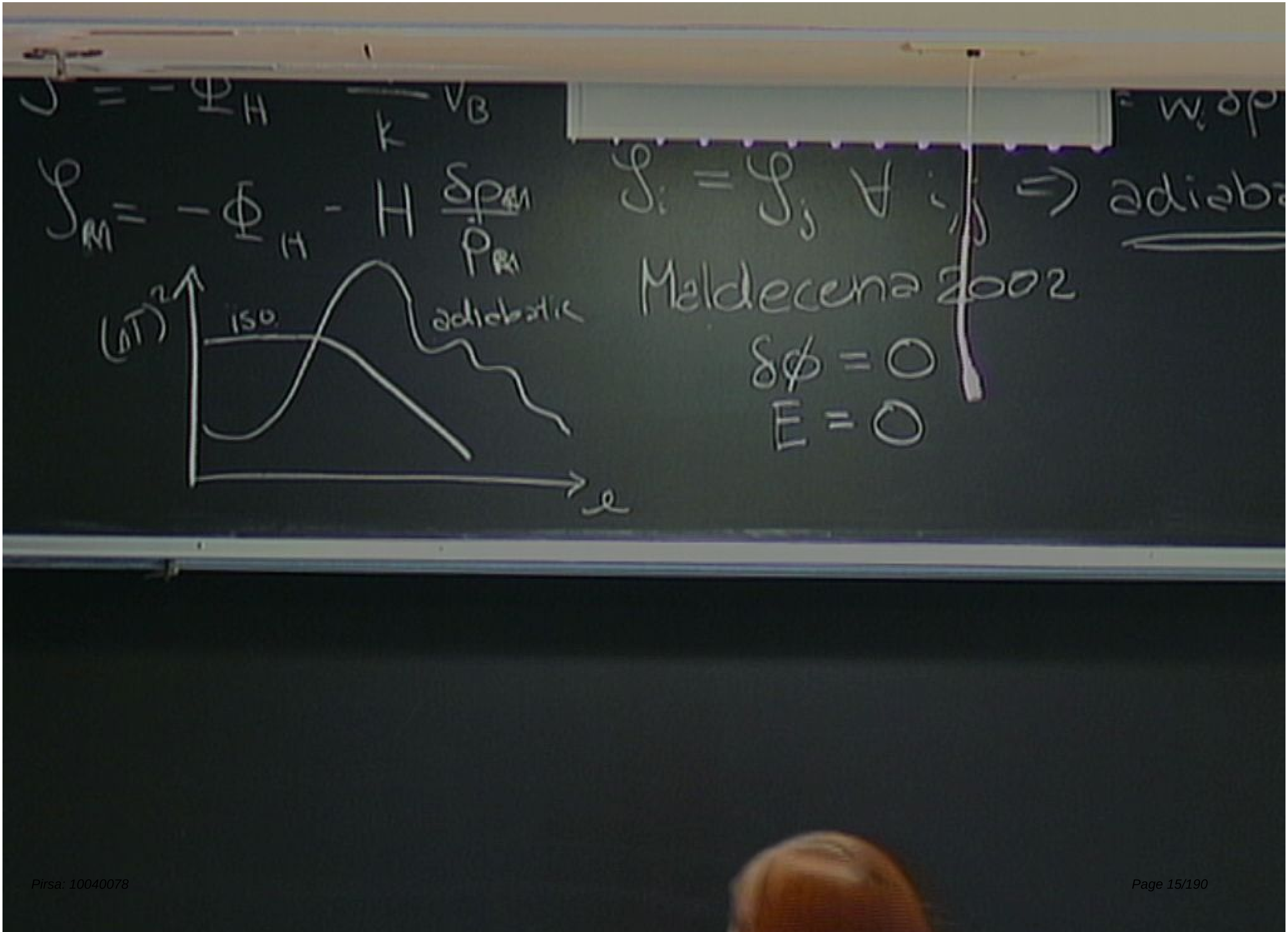
Outsi

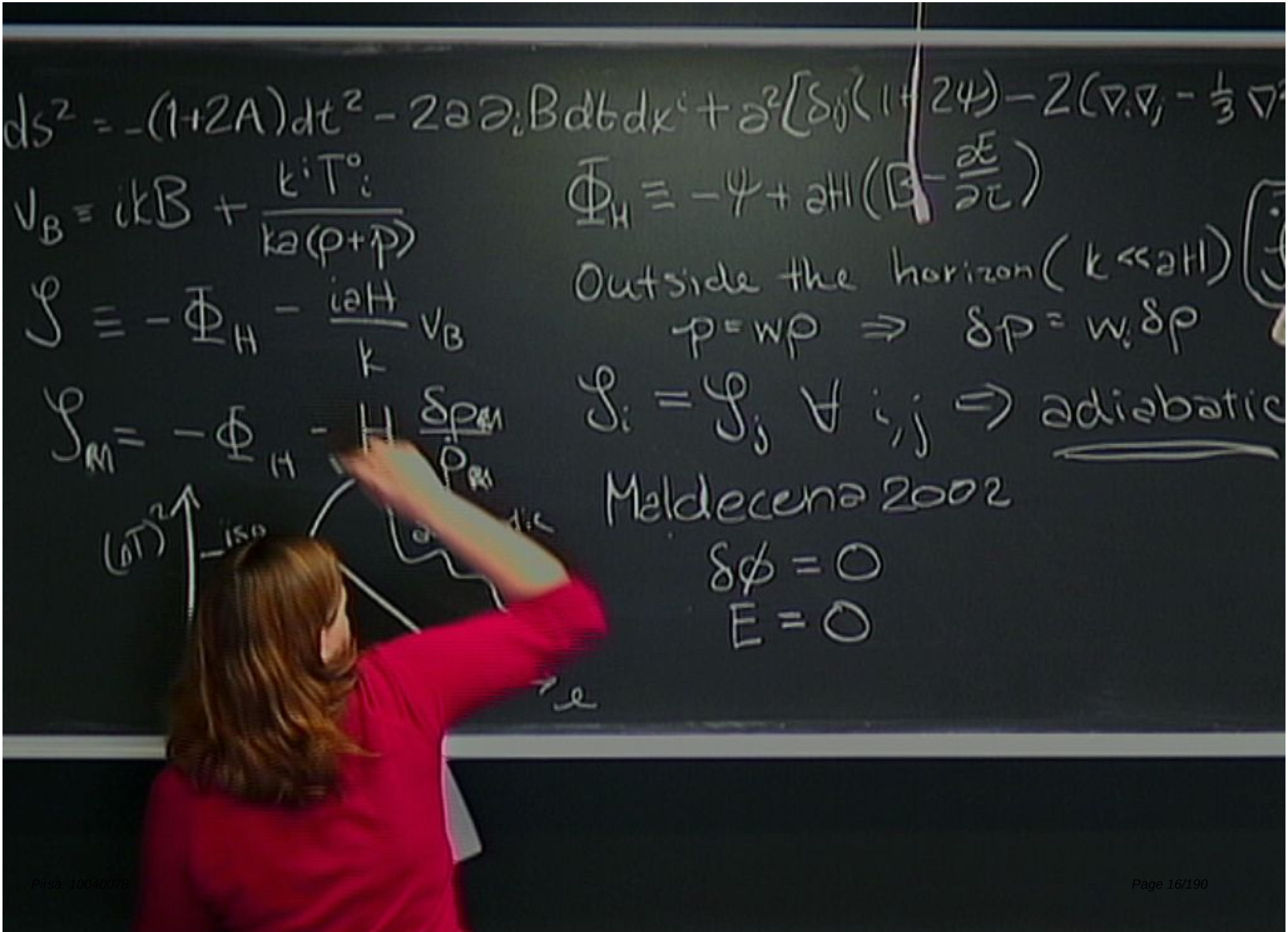
$$\zeta_{rm} = -\Phi_H - H \frac{\delta\rho_{rm}}{\dot{\rho}_{rm}}$$

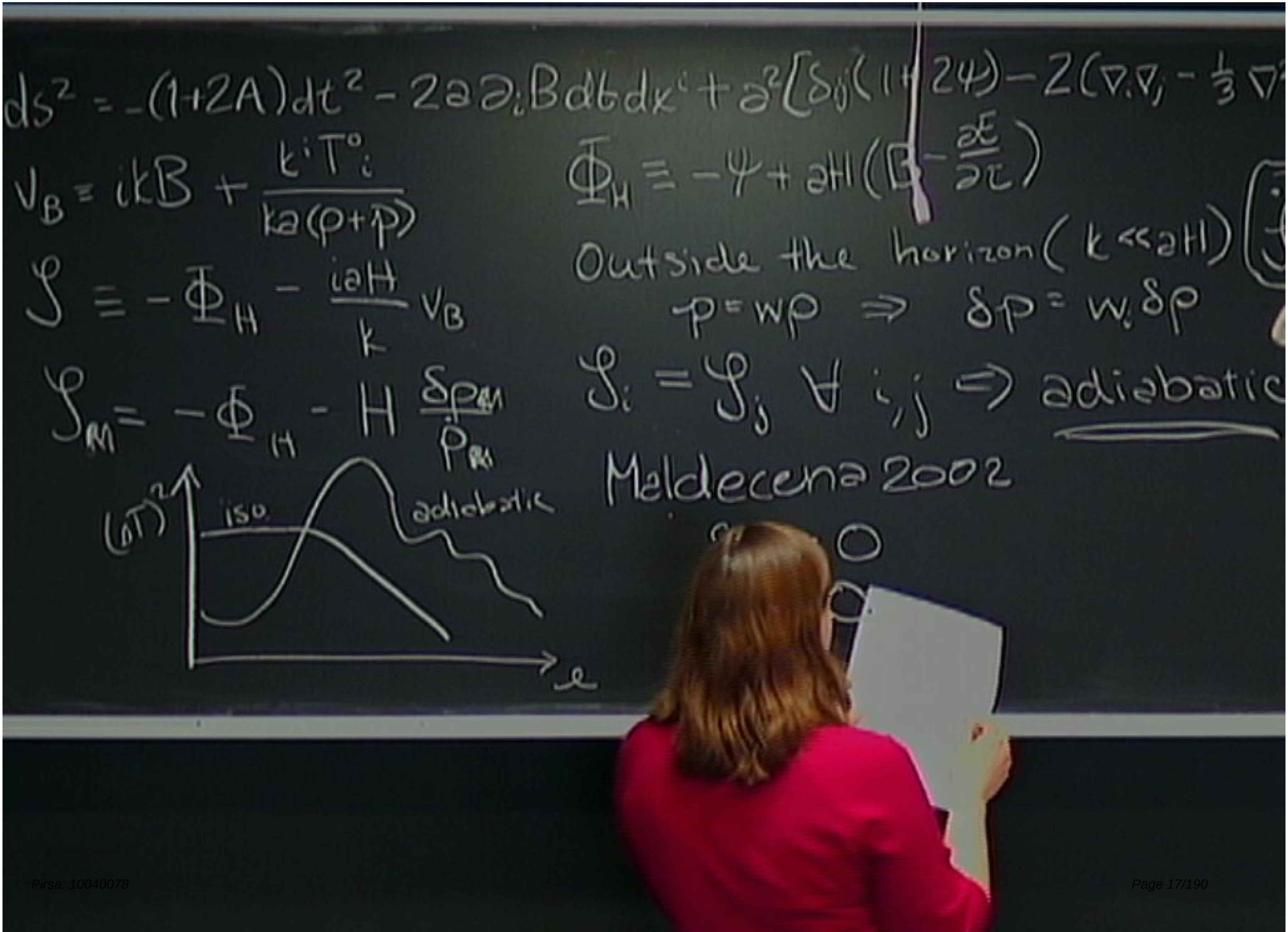
$\zeta_i =$

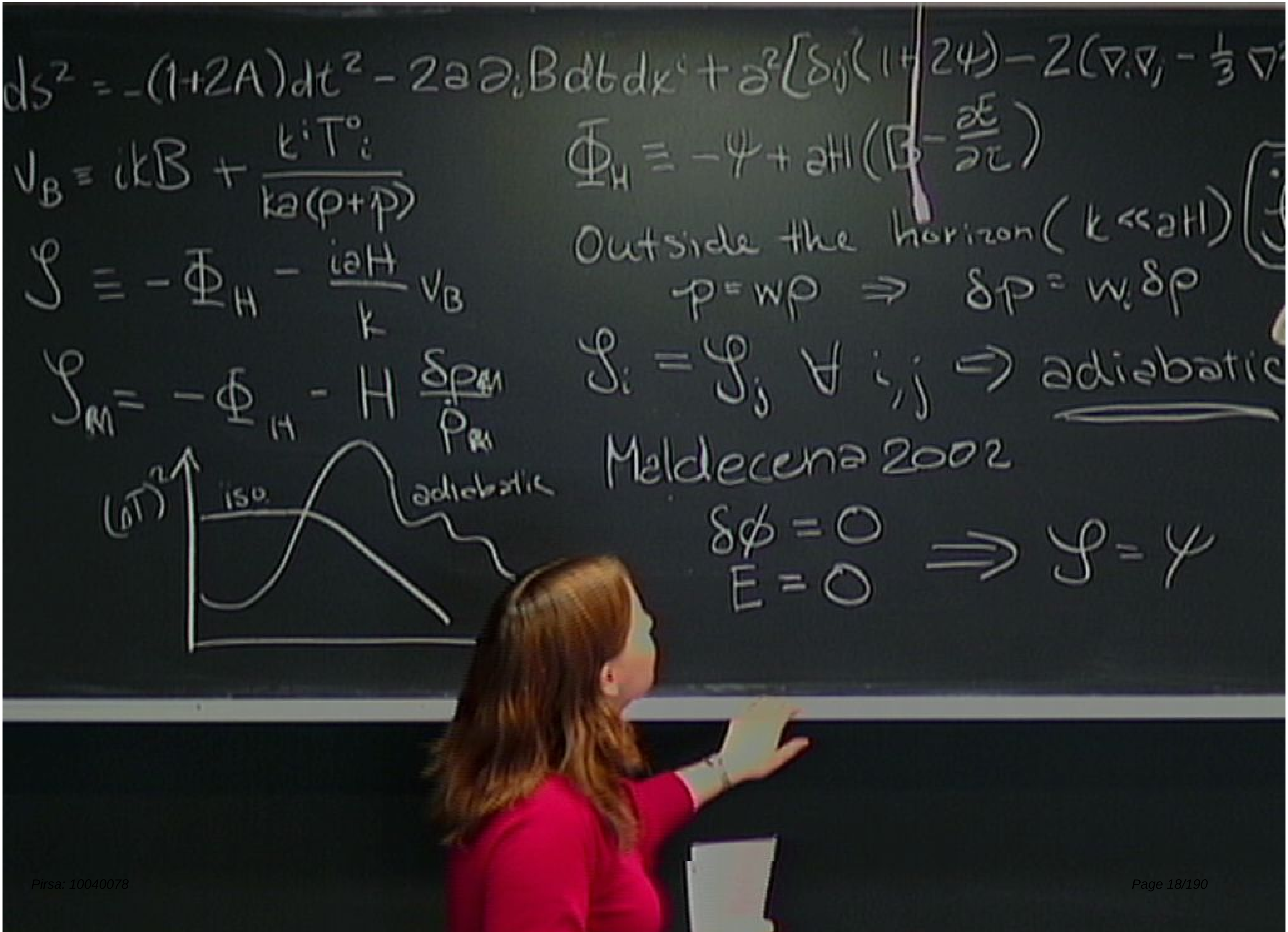



Outside the horizon
Maldecena 2002
iso.
adiabatic


$\zeta_{RM} = -\Phi_H - H \frac{\delta\rho_{RM}}{\dot{\rho}_{RM}}$
$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiaba
Maldecena 2002
$\delta\phi = 0$
$E = 0$
$(\Delta T)^2$
iso
adiabatic
ℓ


ds² = −(1+2A)dt² − 2a∂ᵢB dt dxⁱ + a²[δᵢⱼ(1+2ψ) − 2(∇ᵢ∇ⱼ − ⅓∇²
V_B ≡ ikB + kⁱT⁰ᵢ / ka(ρ+p)
ζ ≡ −Φ_H − iaH/k V_B
ζ_M = −Φ_H − H δρ_M / ρ̇_M
Φ_H ≡ −ψ + aH(B − ∂E/∂τ)
Outside the horizon (k ≪ aH)
p = wρ ⇒ δp = wᵢ δρ
ζᵢ = ζⱼ ∀ i, j ⇒ adiabatic
Maldecena 2002
δφ = 0
E = 0


Outside the horizon
adiabatic
Maldecena 2002
iso.
adiabatic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Φ_H ≡ -ψ + aH(B - ∂E/∂τ)
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Outside the horizon (k ≪ aH)
p = wρ ⇒ δp = wᵢδρ
ζ_M = -Φ_H - H δρ_M/ρ̇_M
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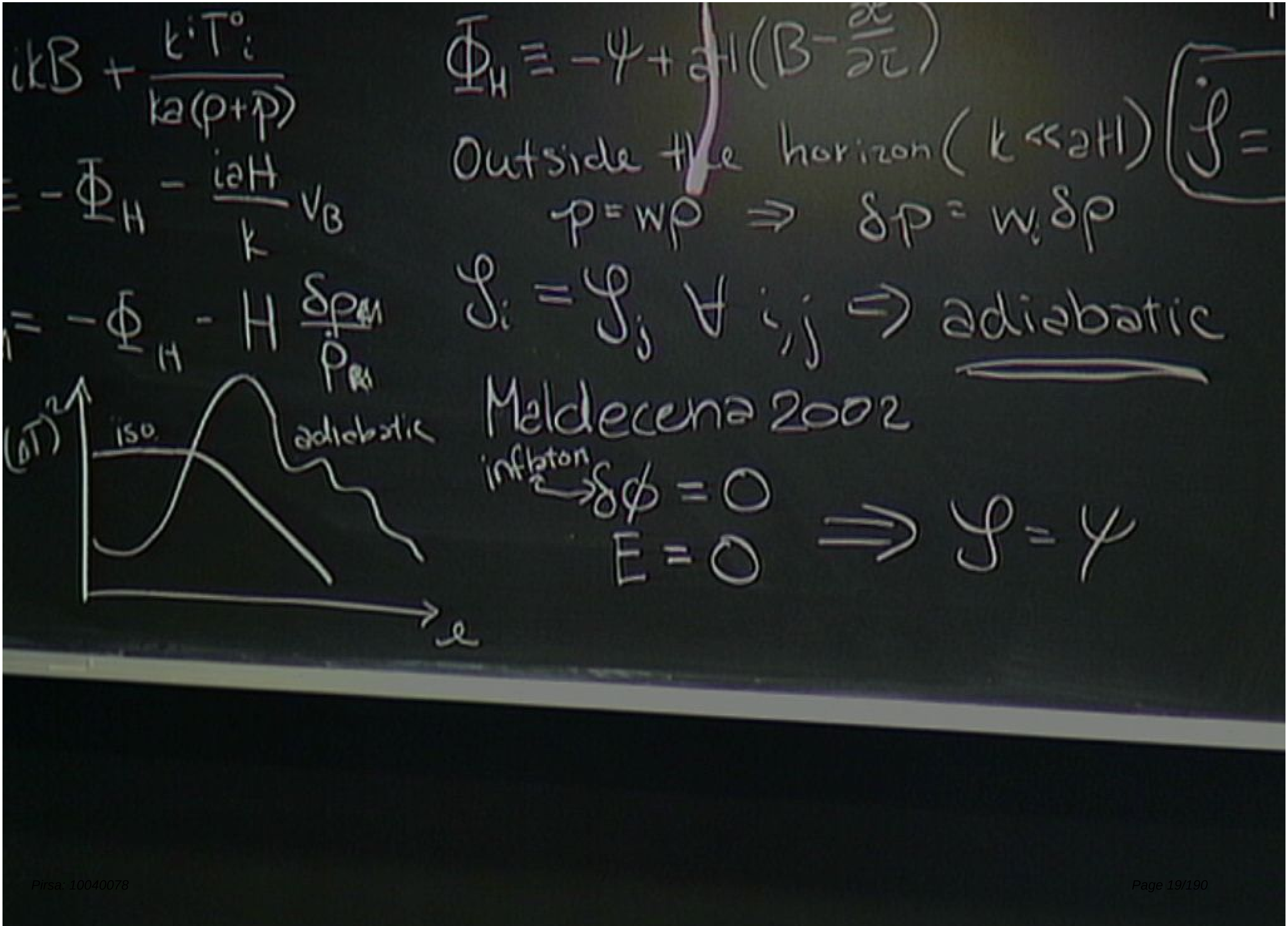
(ΔT)²
iso.
adiabatic
Maldecena 2002
δφ = 0
E = 0
⇒ ζ = ψ


ikB + k^i T^0_i / ka(ρ+p)
= −Φ_H − iaH/k v_B
= −Φ_H − H δρ/ρ̇
Φ_H ≡ −ψ + aH(B − ∂E/∂τ)
Outside the horizon (k ≪ aH)
p = wρ ⇒ δp = w_i δρ
ζ_i = ζ_j ∀ i,j ⇒ adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0 ⇒ ζ = ψ
(ΔT)²
iso
adiabatic
ℓ

$-(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2[\delta_{ij}(1+2\psi) - 2(\nabla_i\nabla_j - \frac{1}{3}\nabla^2)E]\, dx$

$kB + \frac{k^i T^0{}_i}{ka(\rho+p)}$

$\Phi_H \equiv -\psi + aH(B - \frac{\partial E}{\partial \tau})$

$-\Phi_H - \frac{iaH}{k} v_B$

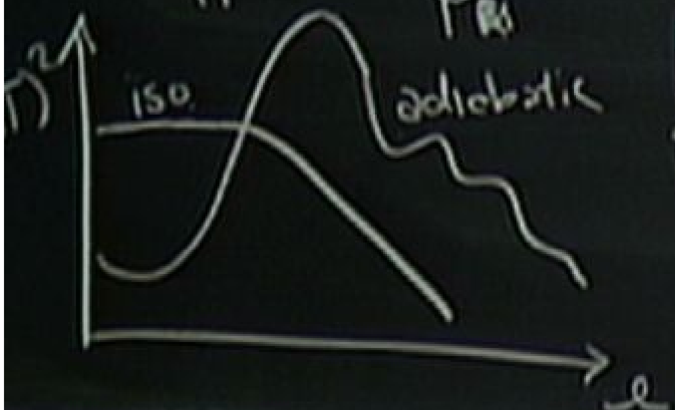
$-\Phi_H - H\frac{\delta\rho_{\alpha}}{\dot{\rho}_{\alpha}}$

Outside the horizon ($k \ll aH$)

$\boxed{\dot{\zeta} = 0}$

$p = w\rho \Rightarrow \delta p = w_i \delta\rho$

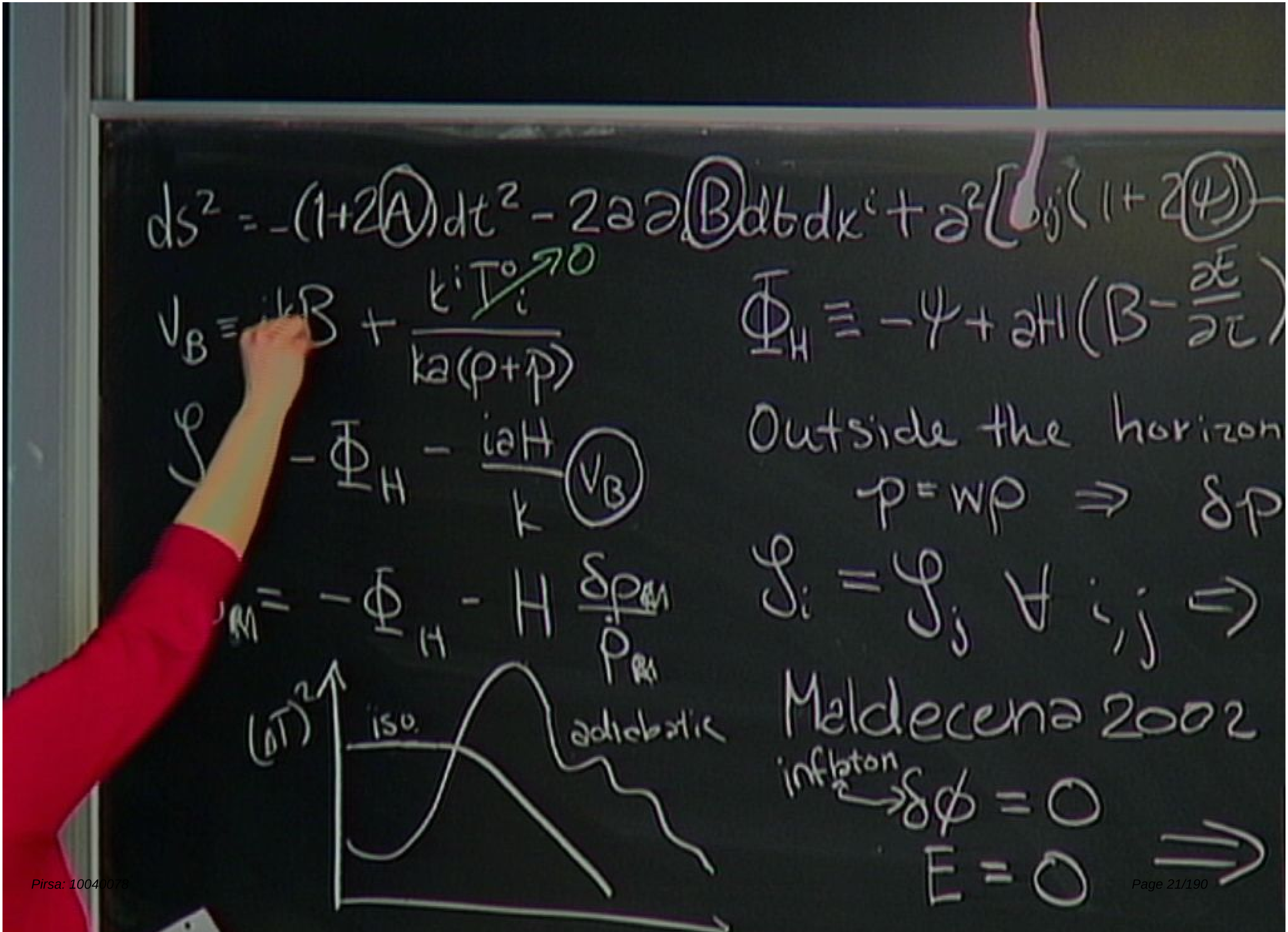
$\zeta_i = \zeta_j \;\forall\, i,j \Rightarrow$ adiabatic



Maldecena 2002

inflaton $\to \delta\phi = 0$

$E = 0 \Rightarrow \boxed{\zeta = \psi}$



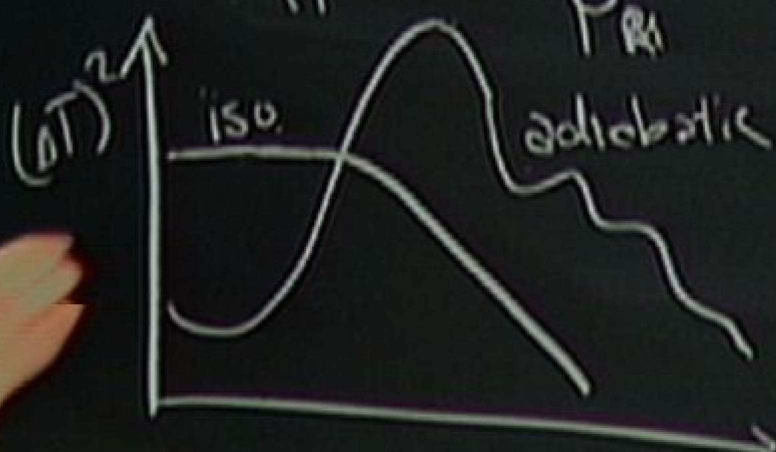
$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2[\delta_{ij}(1+2\psi)$
$V_B \equiv ikB + \frac{k^i I^0_i}{ka(\rho+p)}$ → 0
$\Phi_H \equiv -\psi + aH\left(B - \frac{\partial E}{\partial \tau}\right)$
$\zeta = -\Phi_H - \frac{iaH}{k} V_B$
Outside the horizon
$p = w\rho \Rightarrow \delta p$
$\zeta_{RM} = -\Phi_H - H\frac{\delta\rho_{RM}}{\dot\rho_{RM}}$
$\zeta_i = \zeta_j \;\forall\, i,j \Rightarrow$
$(\Delta T)^2$
iso.
adiabatic
Maldecena 2002
inflaton
$\delta\phi = 0$
$E = 0 \Rightarrow$

$$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2[\delta_{ij}(1+2\psi) -$$

$$v_B \equiv ikB + \frac{k^i T^0_i}{ka(\rho+p)} \quad (k^i T^0_i \to 0)$$

$$\zeta \equiv -\Phi_H - \frac{iaH}{k} v_B$$

$$\zeta_m = -\Phi_H - H\frac{\delta\rho_m}{\dot{\rho}_m}$$



$$\Phi_H \equiv -\psi + aH\left(B - \frac{\partial E}{\partial \tau}\right)$$

Outside the horizon

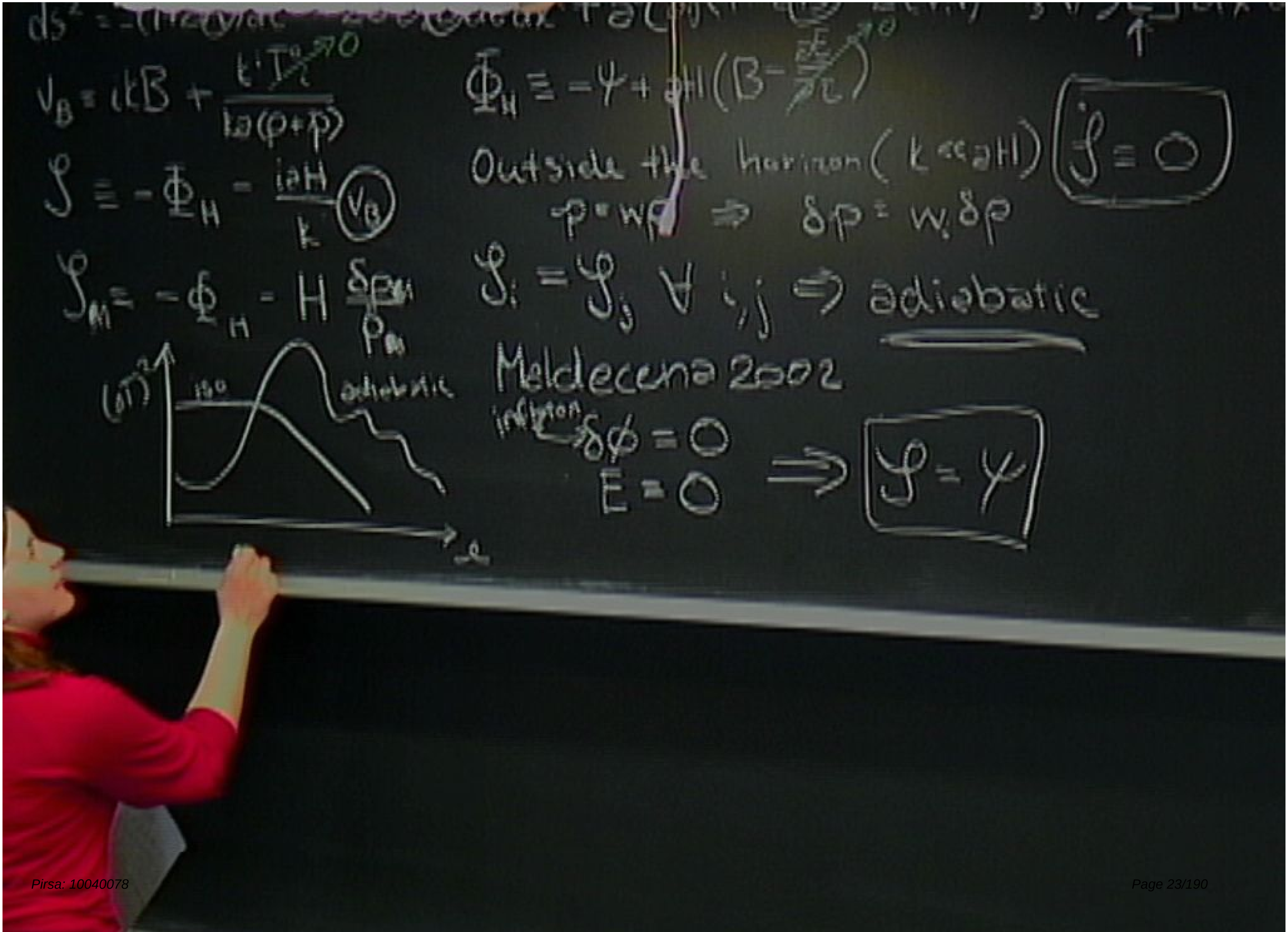
$p = w\rho \Rightarrow \delta p$

$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$

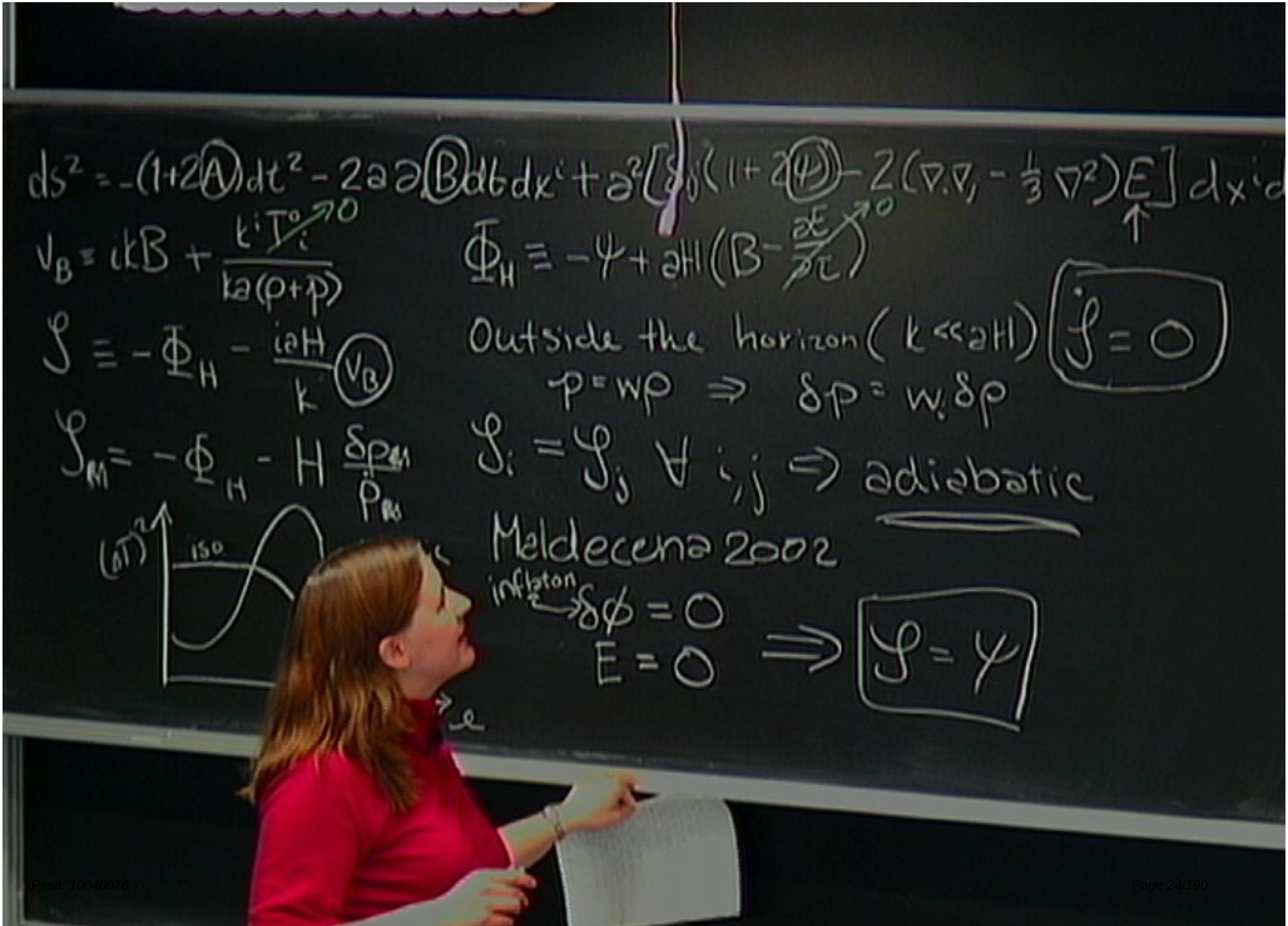
Maldecena 2002

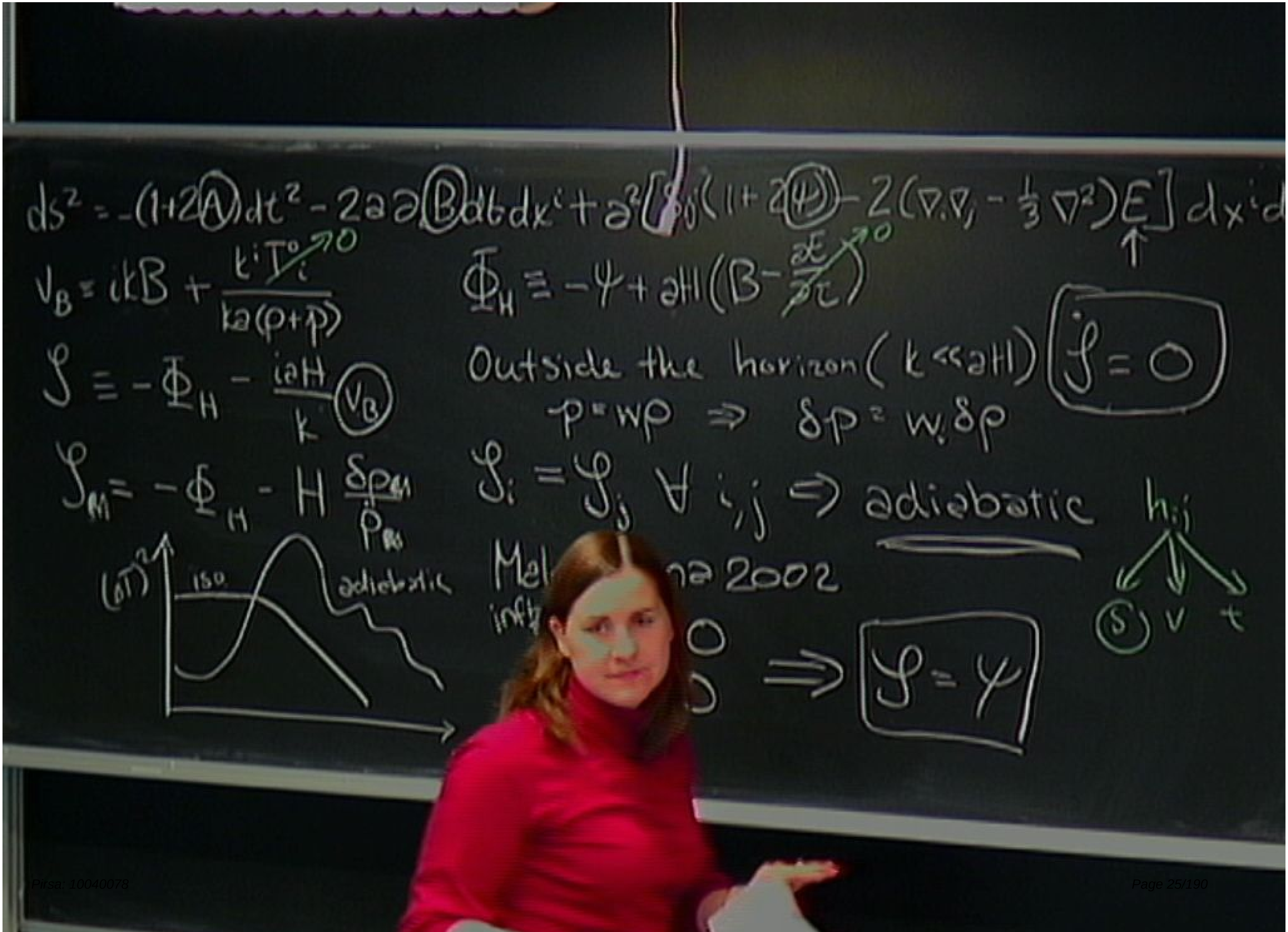
inflaton $\to \delta\phi = 0$

$E = 0 \Rightarrow$



Outside the horizon (k << aH)
ζ = 0
p = wρ ⇒ δp = w δρ
ζ_i = ζ_j ∀ i,j ⇒ adiebatic
Maldecena 2002
inflation δφ = 0
E = 0 ⇒ ζ = ψ
iso
adiabatic


ds² = -(1+2A)dt² - 2a∂ᵢB dt dxⁱ + a²[δᵢⱼ(1+2ψ) - 2(∇ᵢ∇ⱼ - ⅓∇²)E] dxⁱdxʲ
v_B = ikB + kⁱT⁰ᵢ / ka(ρ+p) → 0
Φ_H ≡ -ψ + aH(B - ∂E/∂τ) → 0
ζ ≡ -Φ_H - iaH/k v_B
Outside the horizon (k << aH)
ζ̇ = 0
p = wρ ⇒ δp = wᵢδρ
ζ_M = -Φ_H - H δρ_M/ρ̇_M
ζᵢ = ζⱼ ∀ i,j ⇒ adiabatic
Maldecena 2002
(ΔT)²
iso
inflaton δφ = 0
E = 0 ⇒ ζ = ψ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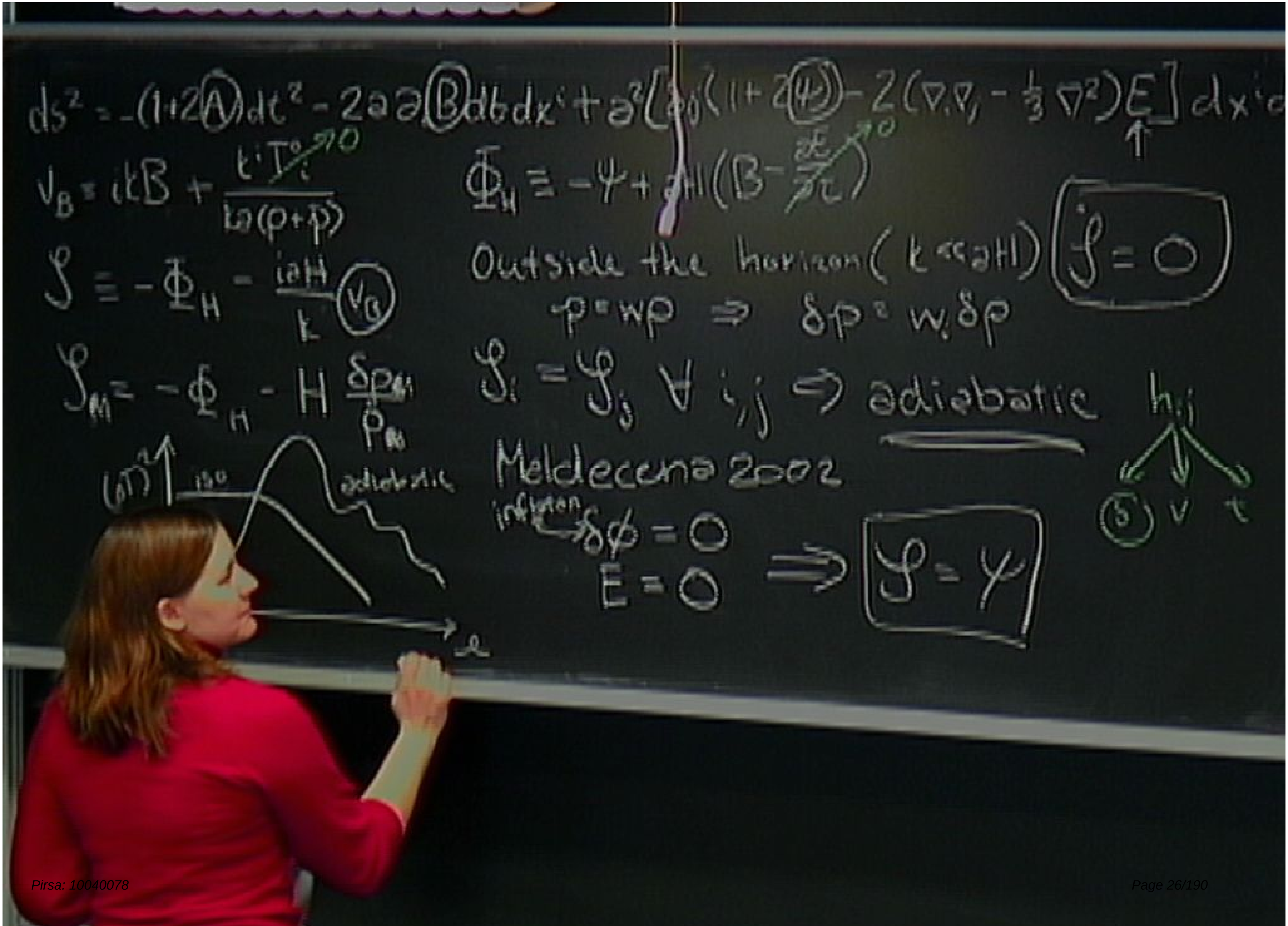
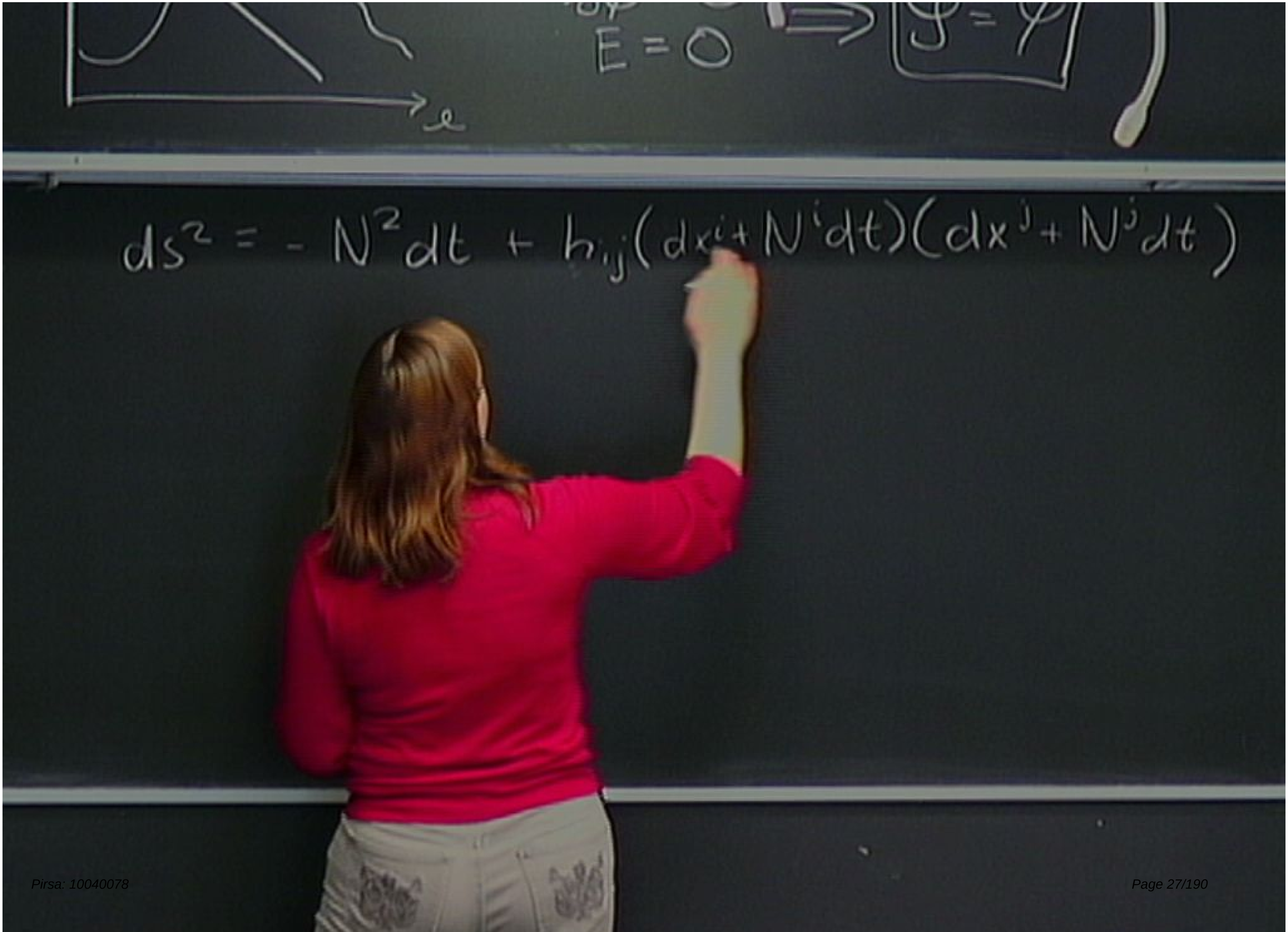
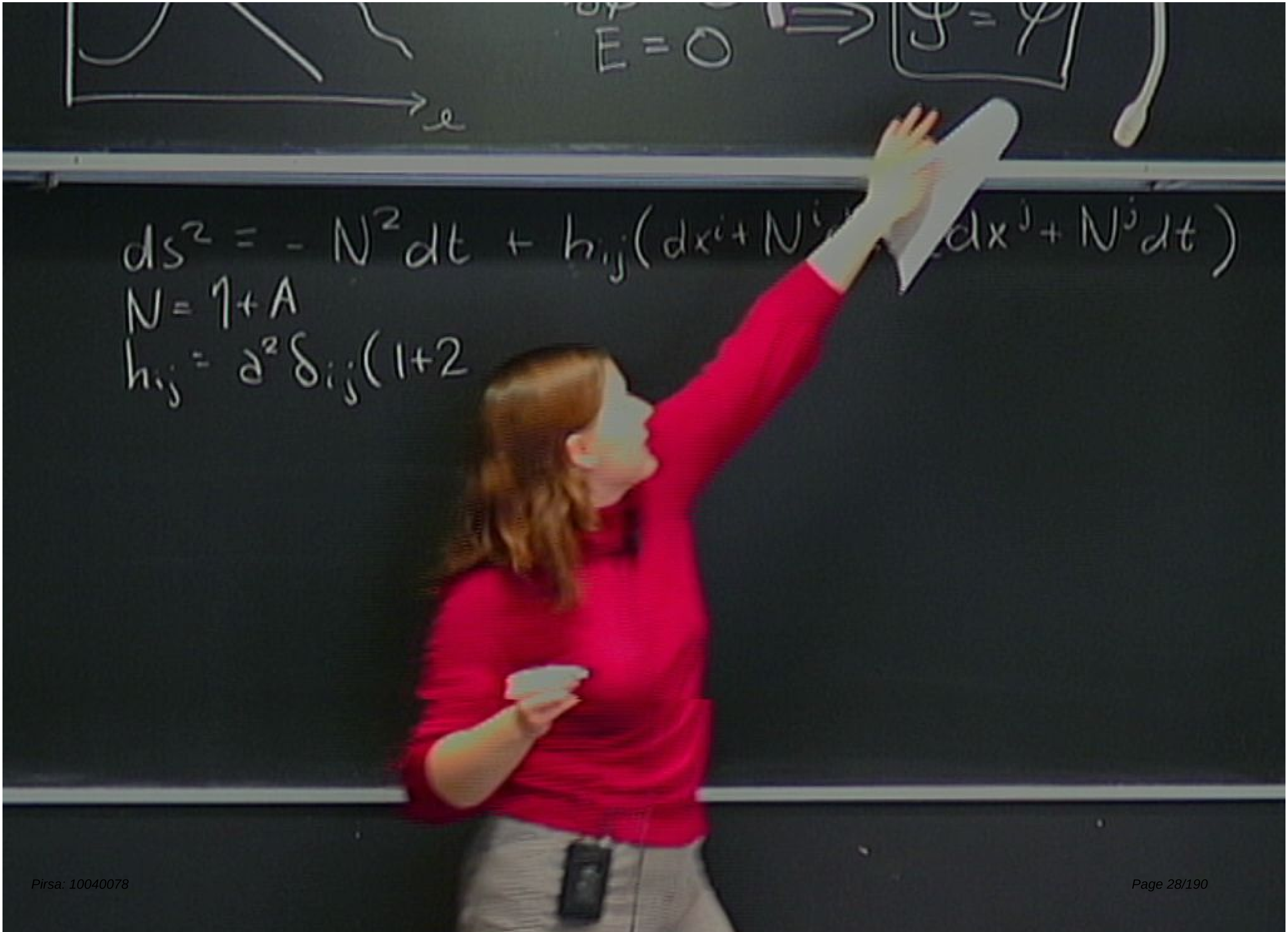
$v_B = ikB + \frac{k^i T^0_i}{ka(\rho+p)}$
$\Phi_H \equiv -\psi + aH(B - \frac{\partial E}{\partial t})$
$\zeta \equiv -\Phi_H - \frac{iaH}{k} v_B$
$\zeta_M = -\Phi_H - H \frac{\delta\rho_M}{\dot\rho_M}$
Outside the horizon ($k \ll aH$)
$\dot\zeta = 0$
$p = w\rho \Rightarrow \delta p = w_i \delta\rho$
$\zeta_i = \zeta_j \;\forall\, i,j \Rightarrow$ adiabatic
h_{ij}
S
v
t
iso
adiabatic
$(\delta T)^2$
2002
$\Rightarrow$ $\zeta = \psi$


Outside the horizon (k << aH)
Maldacena 2002
adiabatic
inflation
δφ = 0
E = 0


E = 0
ds² = - N²dt + h_ij(dx^i + N^i dt)(dx^j + N^j dt)



E = 0
ds² = - N²dt + h_ij(dx^i + N^i
dx^j + N^j dt)
N = 1 + A
h_ij = a² δ_ij(1+2

$$E = 0 \quad \Longrightarrow \quad \boxed{\psi = \varphi}$$

$$ds^2 = -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$N = 1 + A$$

$$h_{ij} = a^2 \delta_{ij}(1 + 2\psi)$$

$$N^i = -\frac{1}{a}\partial^i B$$

inflaton $\rightarrow \delta\phi = 0$

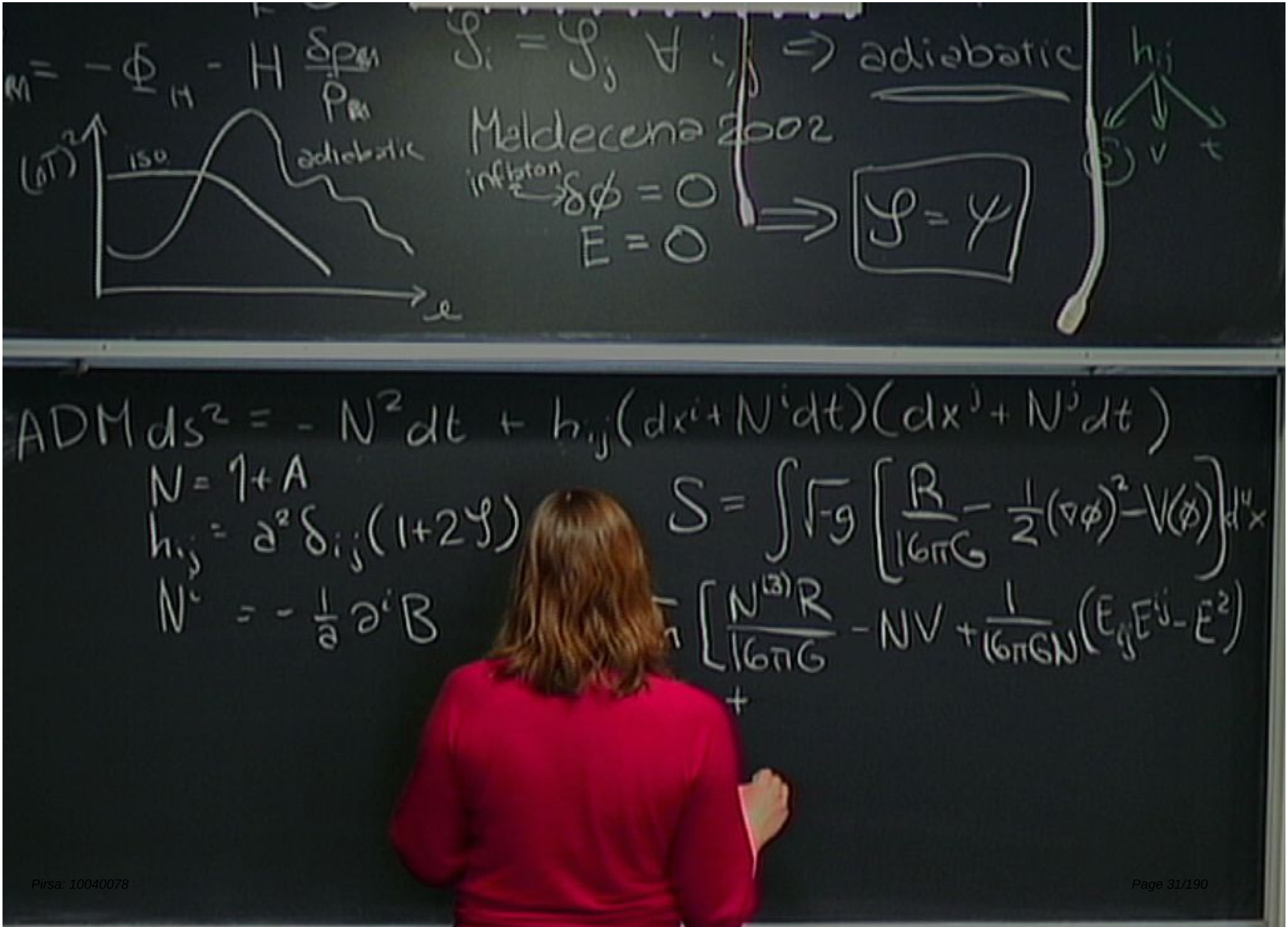
$E = 0$

$\Longrightarrow \boxed{\zeta = \psi}$

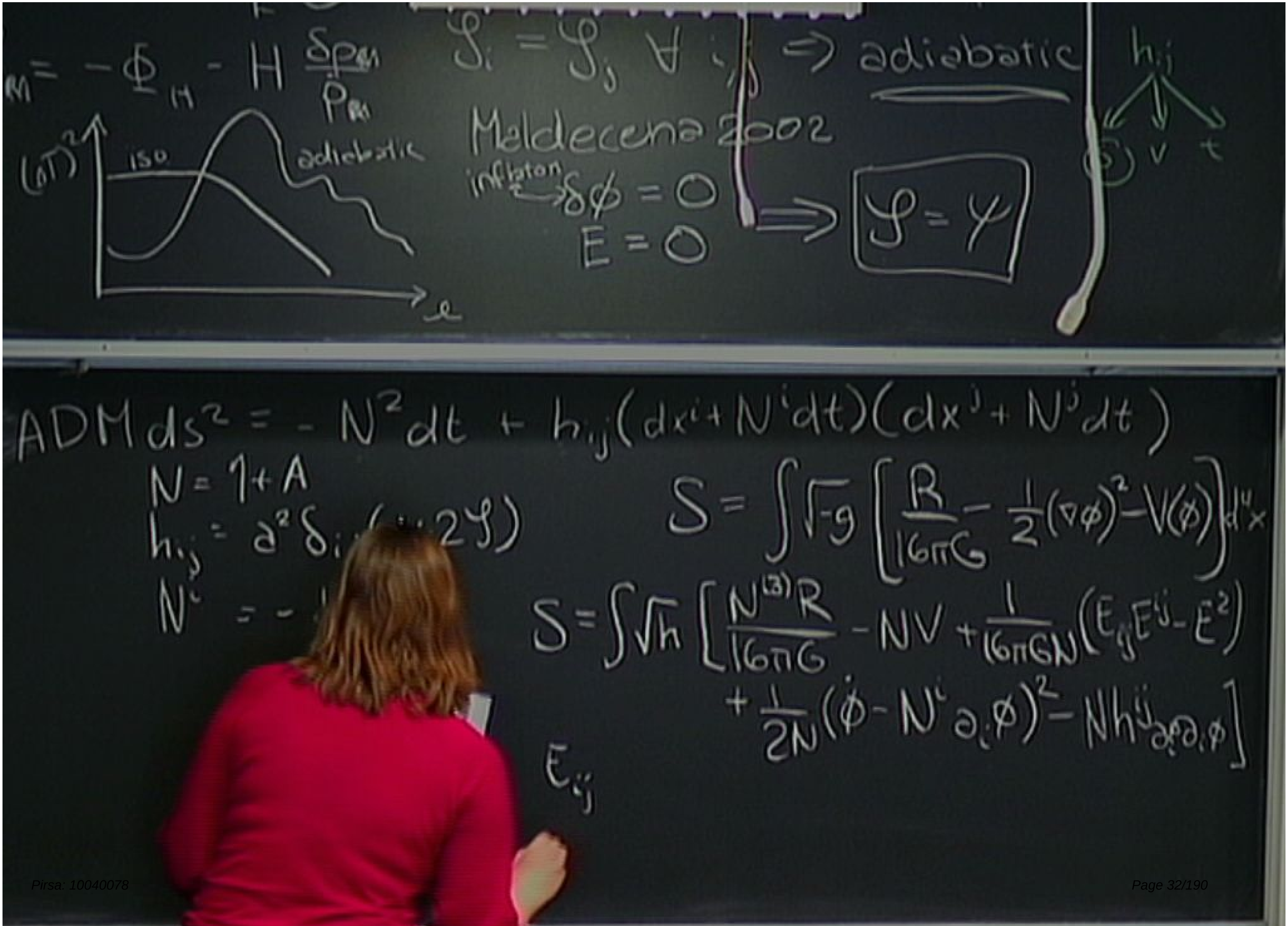
$$\ldots t + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

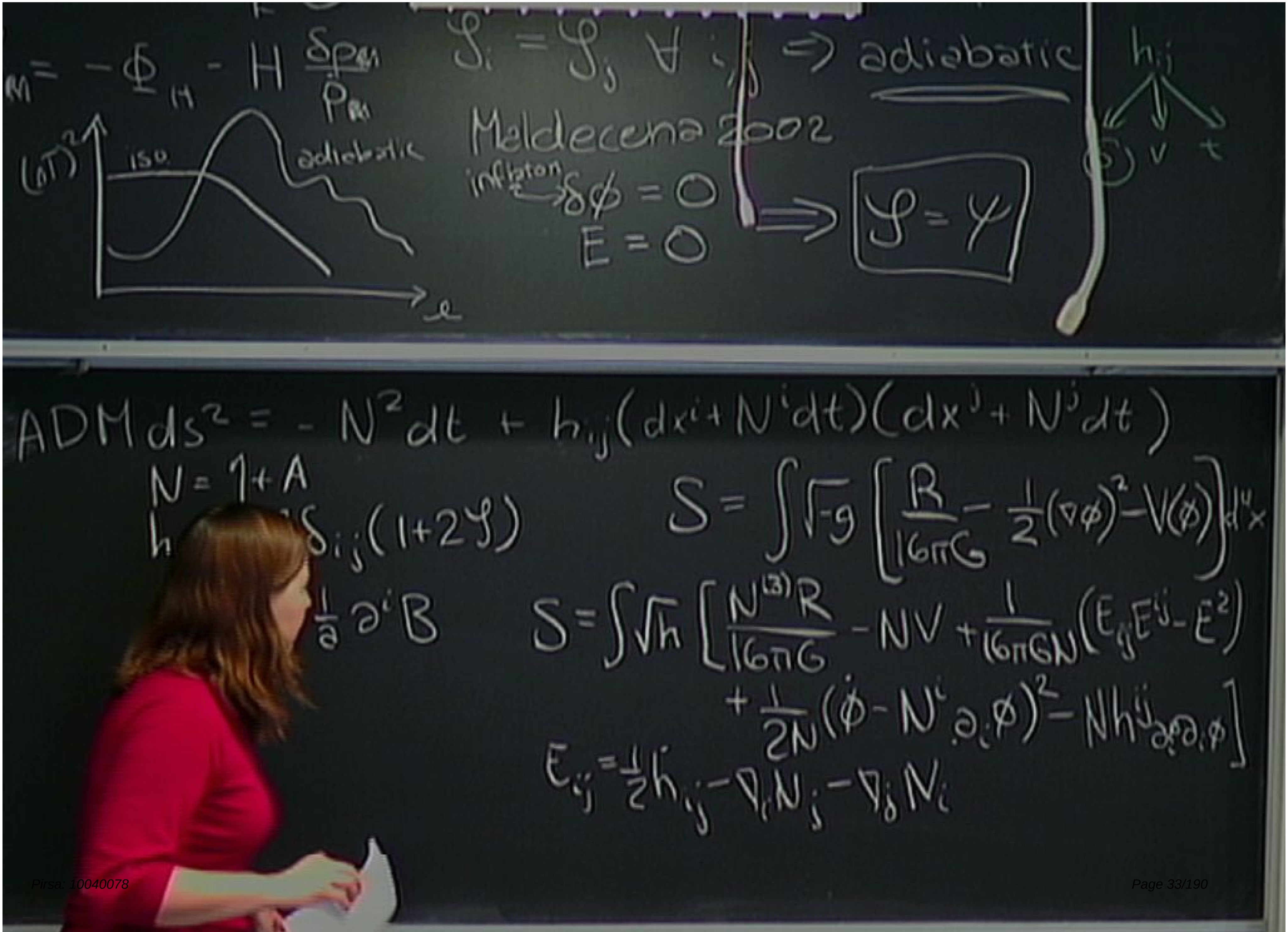
$$\ldots 2\zeta)$$

$$S = \int \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] d^4x$$



$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiabatic
h_{ij}
iso
adiabatic
Maldacena 2002
inflaton $\rightarrow \delta\phi = 0$
$E = 0$
$\Rightarrow \zeta = \psi$
ADM $ds^2 = -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N = 1 + A$
$h_{ij} = a^2 \delta_{ij}(1 + 2\zeta)$
$N^i = -\frac{1}{a}\partial^i B$
$S = \int \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] d^4x$
$\left[\frac{N^{(3)}R}{16\pi G} - NV + \frac{1}{16\pi G N}\left(E_{ij}E^{ij} - E^2\right)\right.$

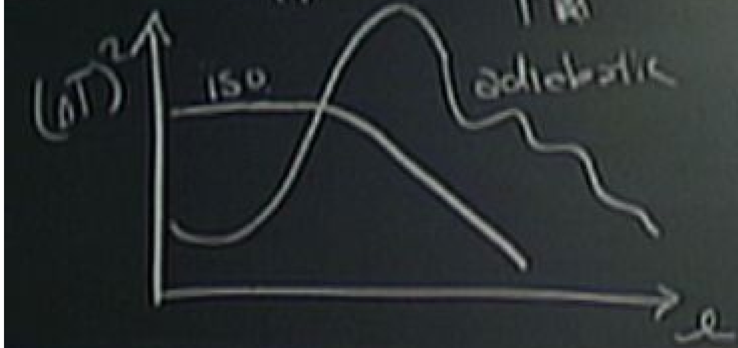

adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic
ADM ds² = − N²dt + h_ij (dx^i + N^i dt)(dx^j + N^j dt)
N = 1 + A
S = ∫ √−g [R/16πG − ½(∇φ)² − V(φ)] d⁴x
S = ∫ √h [N ⁽³⁾R/16πG − NV + 1/16πGN (E_ij E^ij − E²) + 1/2N (φ̇ − N^i ∂_i φ)² − N h^ij ∂_i φ ∂_j φ]
E_ij


ζ_i = ζ_j ∀ i,j ⇒ adiabatic
Maldacena 2002
inflaton δφ = 0
E = 0
ζ = ψ
iso
adiabatic
ADM ds² = −N²dt + h_ij(dx^i + N^i dt)(dx^j + N^j dt)
N = 1 + A
S = ∫√−g [R/16πG − ½(∇φ)² − V(φ)] d⁴x
S = ∫√h [N ⁽³⁾R/16πG − NV + 1/16πGN (E_ij E^ij − E²) + 1/2N (φ̇ − N^i ∂_i φ)² − N h^ij ∂_i φ ∂_j φ]
E_ij = ½ ḣ_ij − ∇_i N_j − ∇_j N_i

$\mathcal{R}_M = -\Phi_H - H \frac{\delta\rho_M}{\dot{\rho}_M}$

$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiabatic

h_{ij} → S, V, T



Maldacena 2002

inflaton $\to \delta\phi = 0$

$E = 0$ $\Rightarrow$ $\boxed{\zeta = \psi}$

ADM $ds^2 = -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

$N = 1 + A$

$h_{ij} = a^2 \delta_{ij}(1 + 2\zeta)$

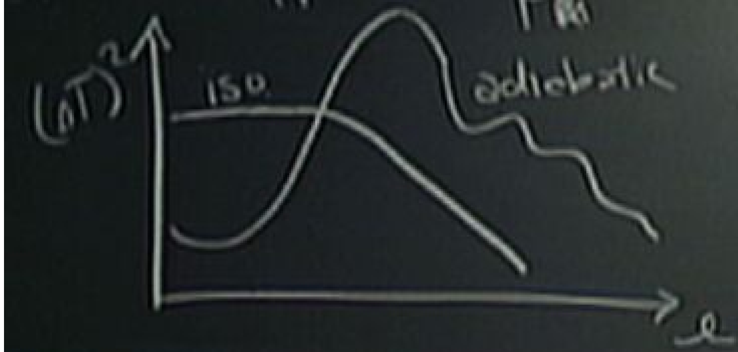
$N^i = -\frac{1}{a}\partial^i B$

$$S = \int \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] d^4x$$

$$S = \int \sqrt{h}\left[\frac{N\,{}^{(3)}R}{16\pi G} - NV + \frac{1}{16\pi G N}\left(E_{ij}E^{ij} - E^2\right) + \frac{1}{2N}\left(\dot{\phi} - N^i\partial_i\phi\right)^2 - N h^{ij}\partial_i\phi\,\partial_j\phi\right]$$

$$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i$$

$\zeta_M = -\Phi_H - H \frac{\delta\rho_M}{\dot{\rho}_M}$



$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiabatic

Maldecena 2002

inflaton $\rightarrow \delta\phi = 0$

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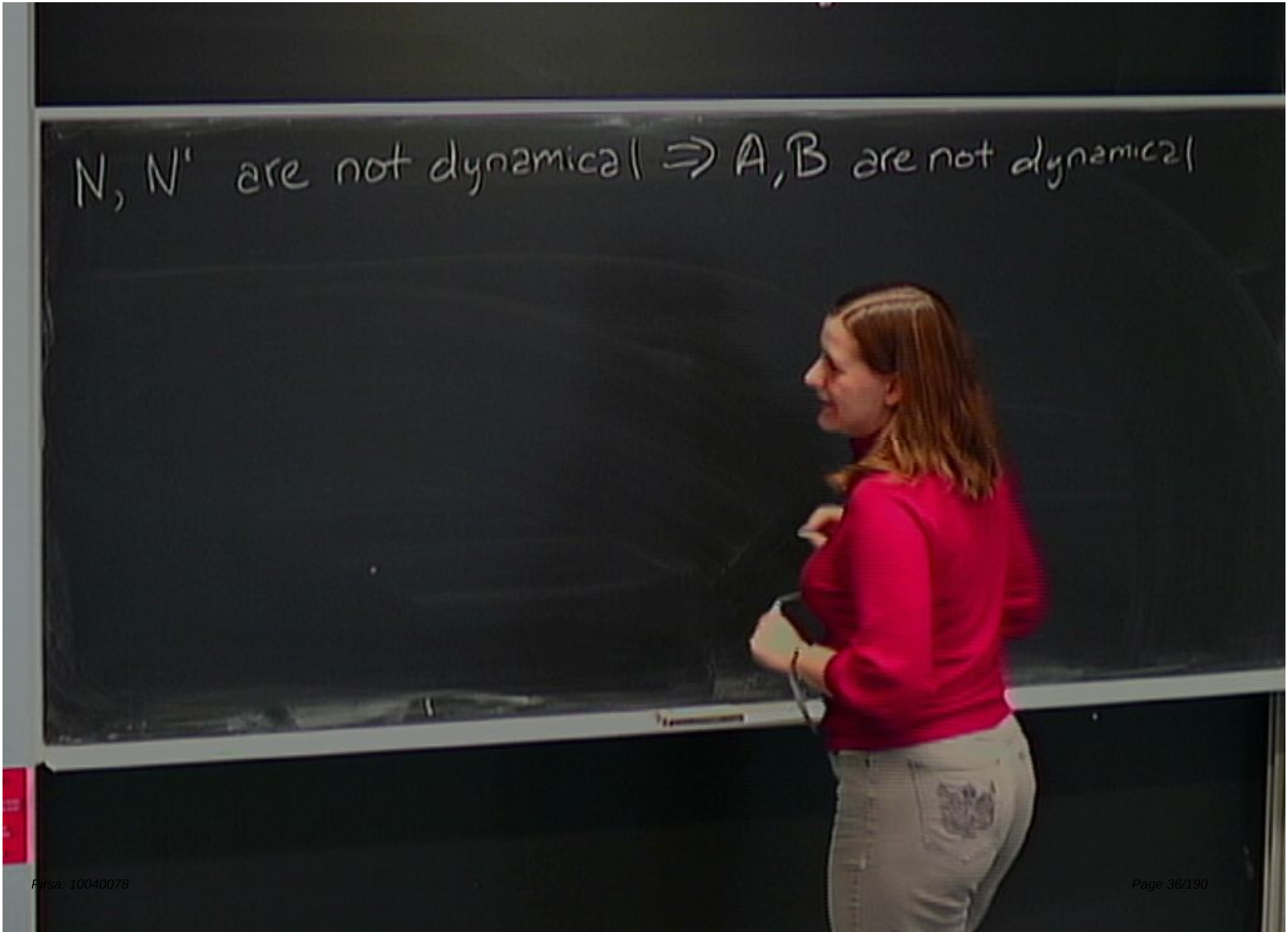
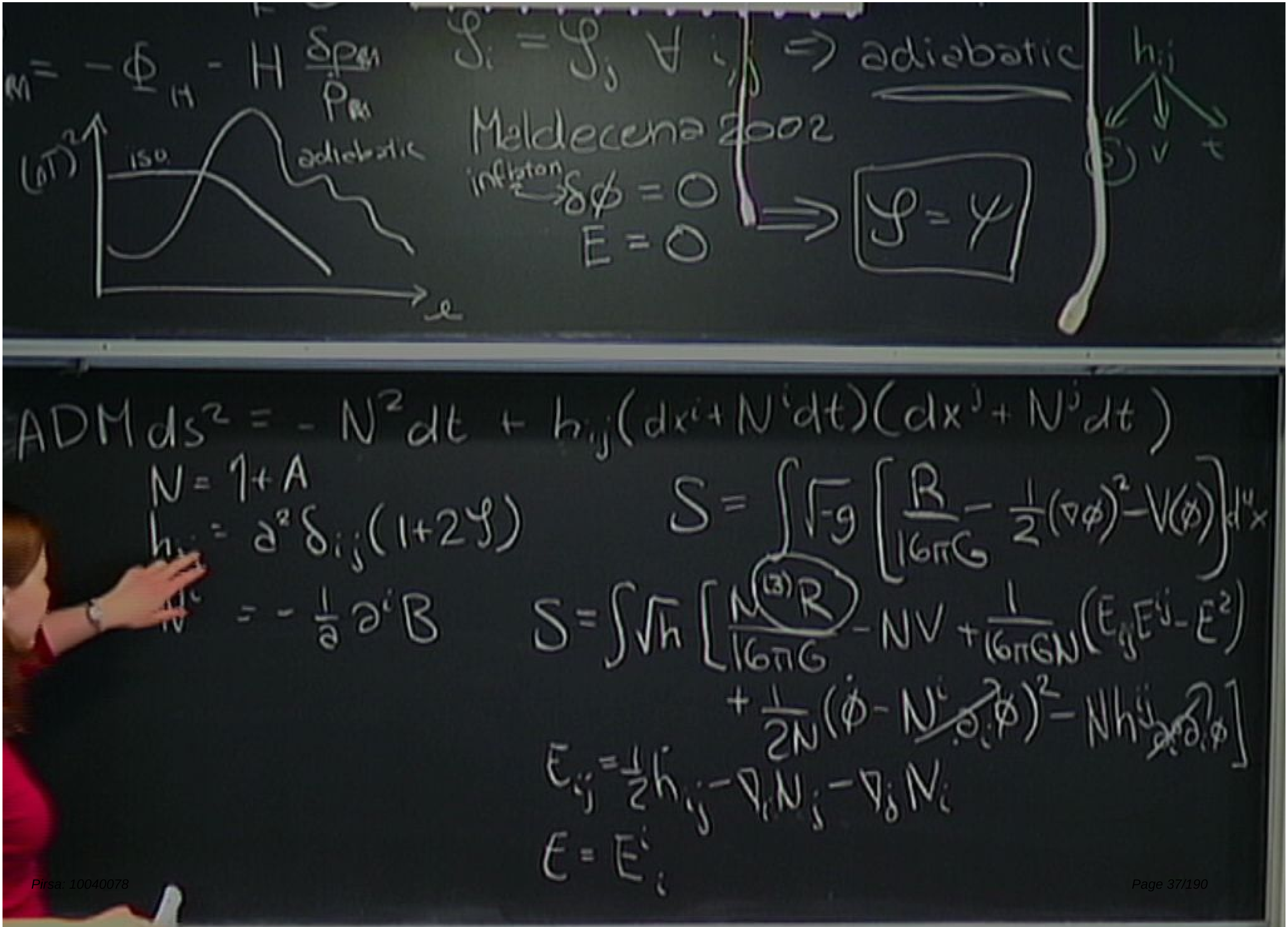
$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i$

$E = E^i_i$


N, N' are not dynamical ⇒ A, B are not dynamical



$\Phi_H - H \frac{\delta\rho}{\dot{\rho}}$
$\zeta_i = \zeta_j \;\forall\, i,j \Rightarrow$ adiabatic
h_{ij}
S V T
$(\delta T)^2$
iso.
adiabatic
Maldacena 2002
inflaton $\to \delta\phi = 0$
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$S = \int \sqrt{h}\left[\frac{N\,{}^{(3)}R}{16\pi G} - NV + \frac{1}{16\pi G N}(E_{ij}E^{ij} - E^2) + \frac{1}{2N}(\dot\phi - N^i \partial_i \phi)^2 - N h^{ij}\partial_i\phi\partial_j\phi\right]$
$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i$
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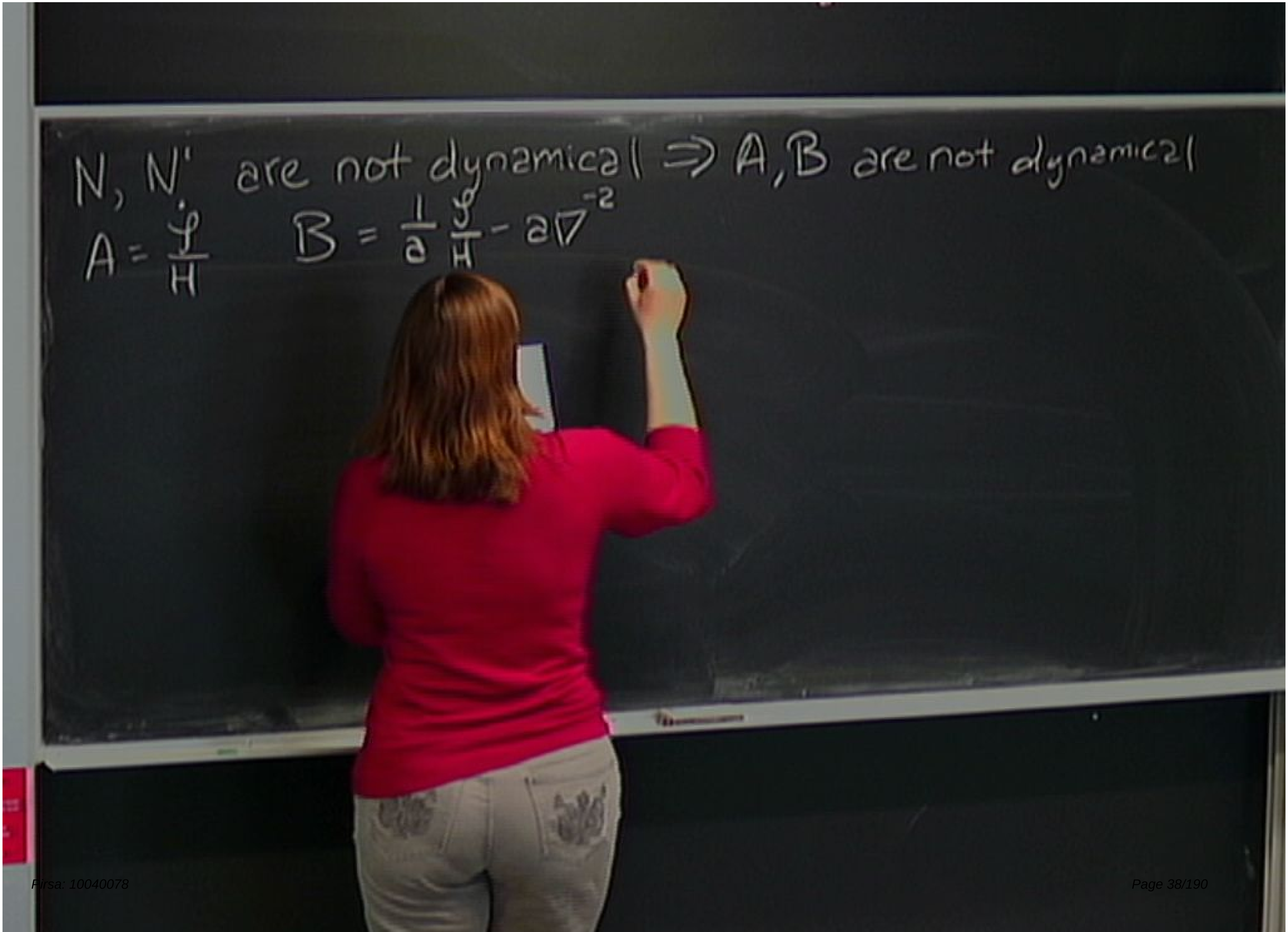
N, N' are not dynamical ⇒ A, B are not dynamical
A = φ̇ / H
B = 1/a φ/H − a∇^-2

N, N' are not dynamical $\Rightarrow$ A, B are not dynamical

$$A = \frac{\dot{\zeta}}{H} \qquad B = \frac{1}{a}\frac{\zeta}{H} - a\nabla^{-2}\left(\frac{\dot{\phi}^2}{2H^2}\dot{\zeta}\right)(8\pi G)^3$$

N, N' are not dynamical $\Rightarrow$ A, B are not dynamical

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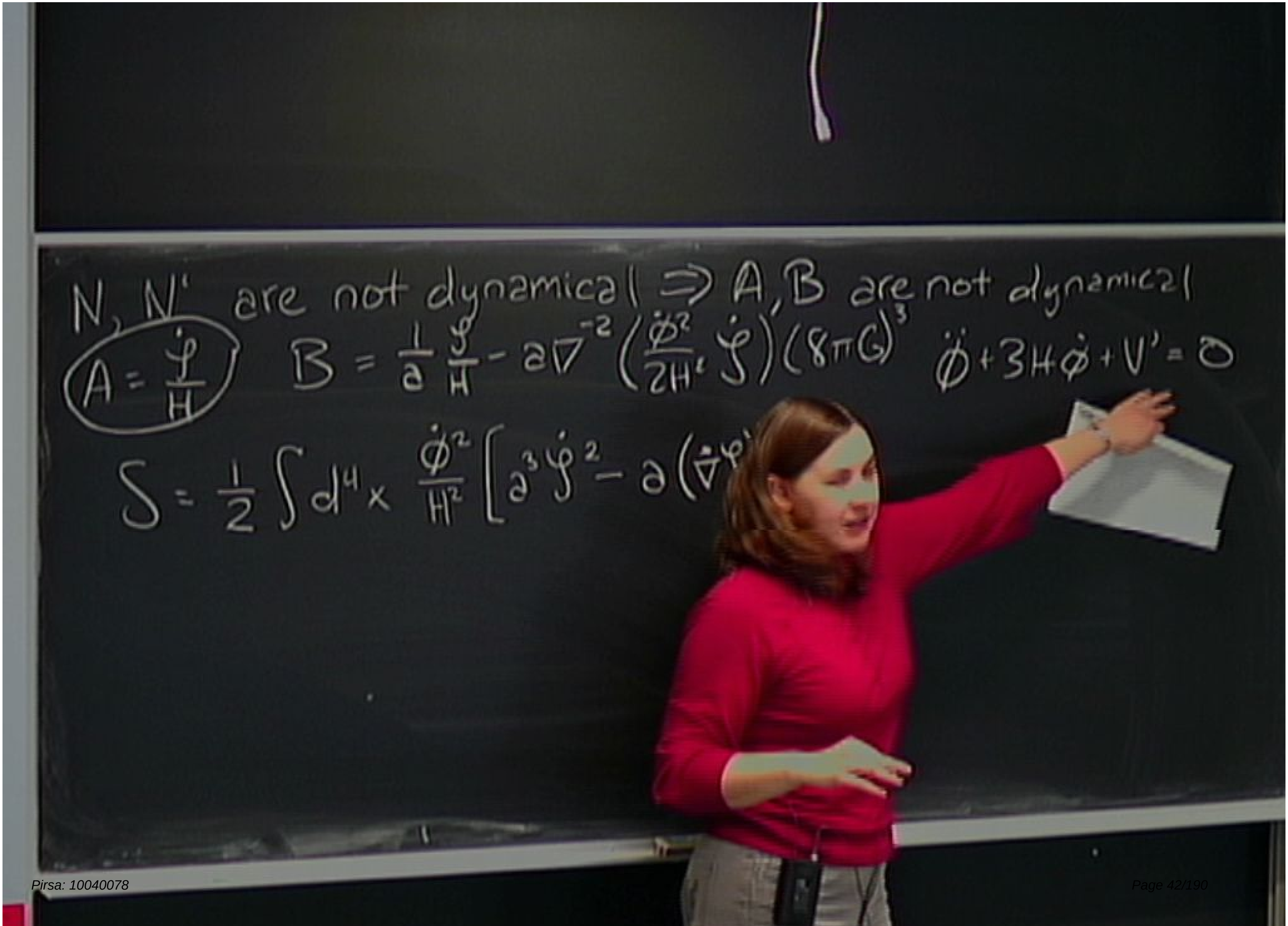
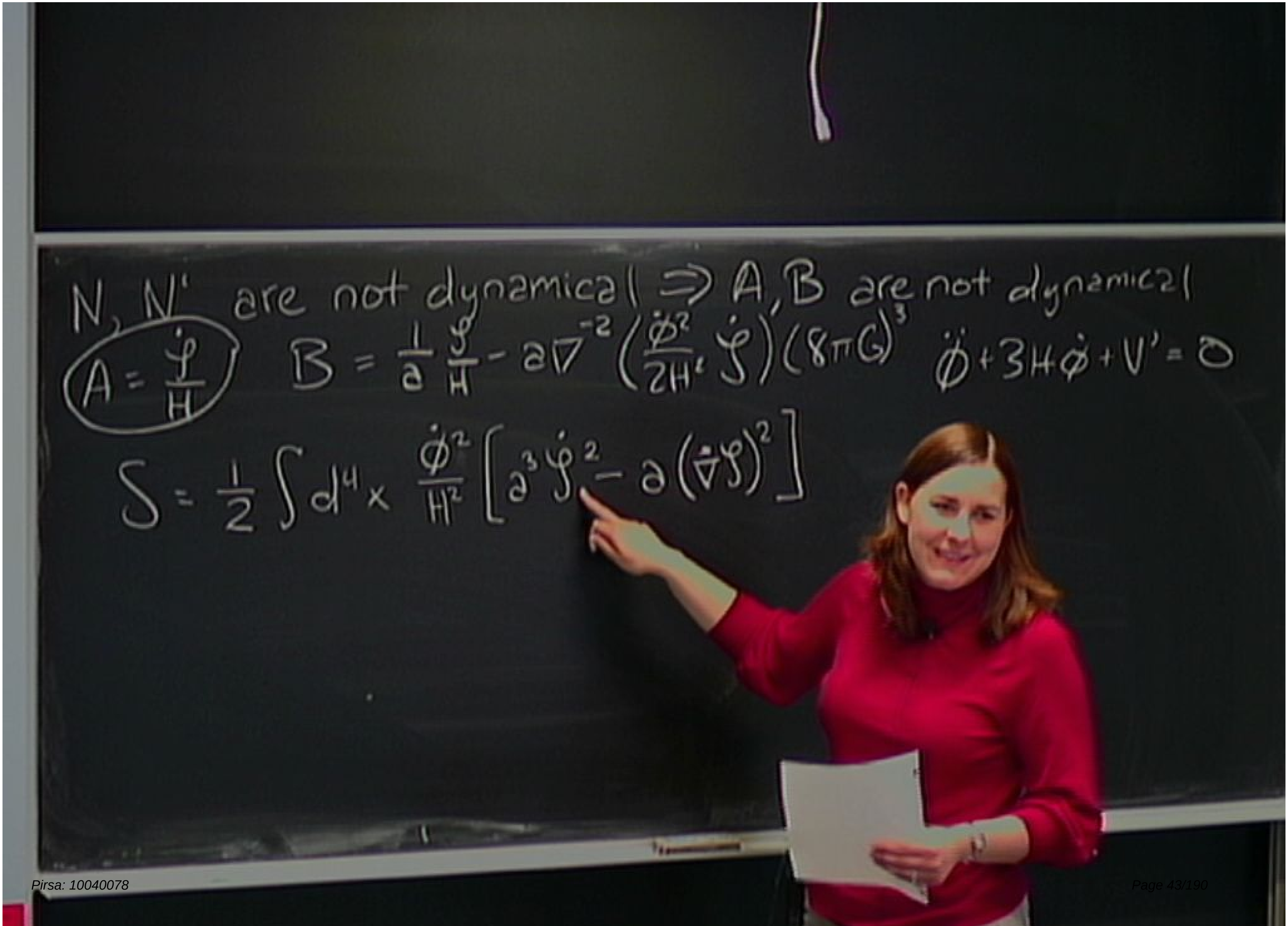

N, N' are not dynamical ⇒ A, B are not dynamical
$A = \frac{\dot{\varphi}}{H}$
$B = \frac{1}{a}\frac{\varphi}{H} - a\nabla^{-2}\left(\frac{\dot{\phi}^2}{2H^2}\dot{\varphi}\right)(8\pi G)^3$
$\ddot{\phi} + 3H\dot{\phi} + V' = 0$
$S = \frac{1}{2}\int d^4x \, \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\varphi}^2 - a(\vec{\nabla}\varphi\right.$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$$-\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2}a^3\dot{\varphi}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\varphi\right) = 0$$

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$$\varepsilon \ \ldots \ \frac{\dot{\phi}^2}{H^2}$$

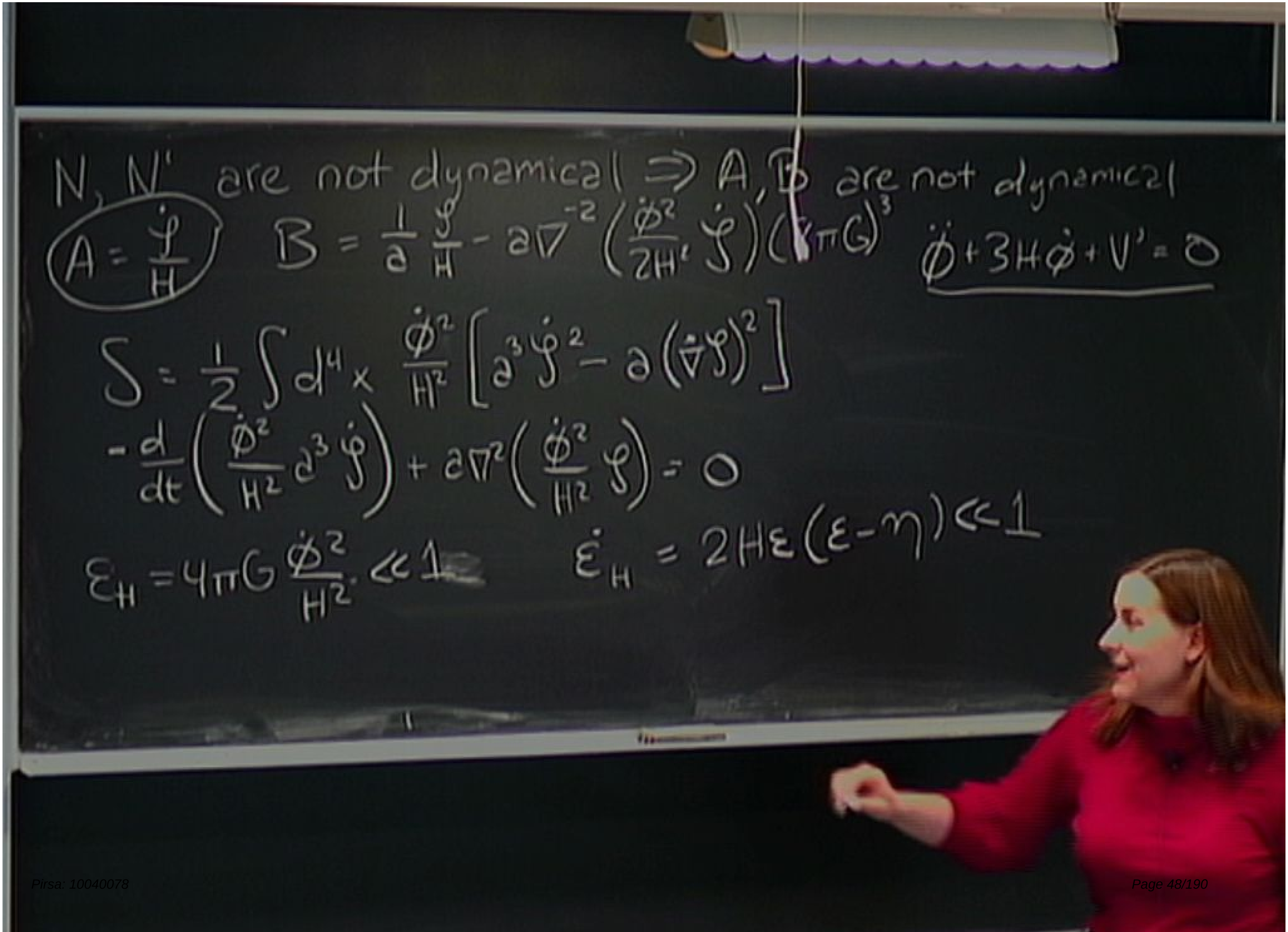
N, N' are not dynamical ⇒ A, B are not dynamical

$$A = \frac{\dot{\varphi}}{H} \qquad B = \frac{1}{a}\frac{\varphi}{H} - a\nabla^{-2}\left(\frac{\dot{\phi}^2}{2H^2}\dot{\varphi}\right)(8\pi G)^3 \qquad \ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$S = \frac{1}{2}\int d^4x\, \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\varphi}^2 - a(\vec{\nabla}\varphi)^2\right]$$

$$-\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2}a^3\dot{\varphi}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\varphi\right) = 0$$

$$\varepsilon_H = 4\pi G\frac{\dot{\phi}^2}{H^2}$$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$\ddot\phi + 3H\dot\phi + V' = 0$
$S = \frac{1}{2}\int d^4x \, \frac{\dot\phi^2}{H^2}\left[a^3\dot\varphi^2 - a(\vec\nabla\varphi)^2\right]$
$-\frac{d}{dt}\left(\frac{\dot\phi^2}{H^2}a^3\dot\varphi\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\varphi\right) = 0$
$\varepsilon_H = 4\pi G\frac{\dot\phi^2}{H^2} \ll 1$
$\dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$

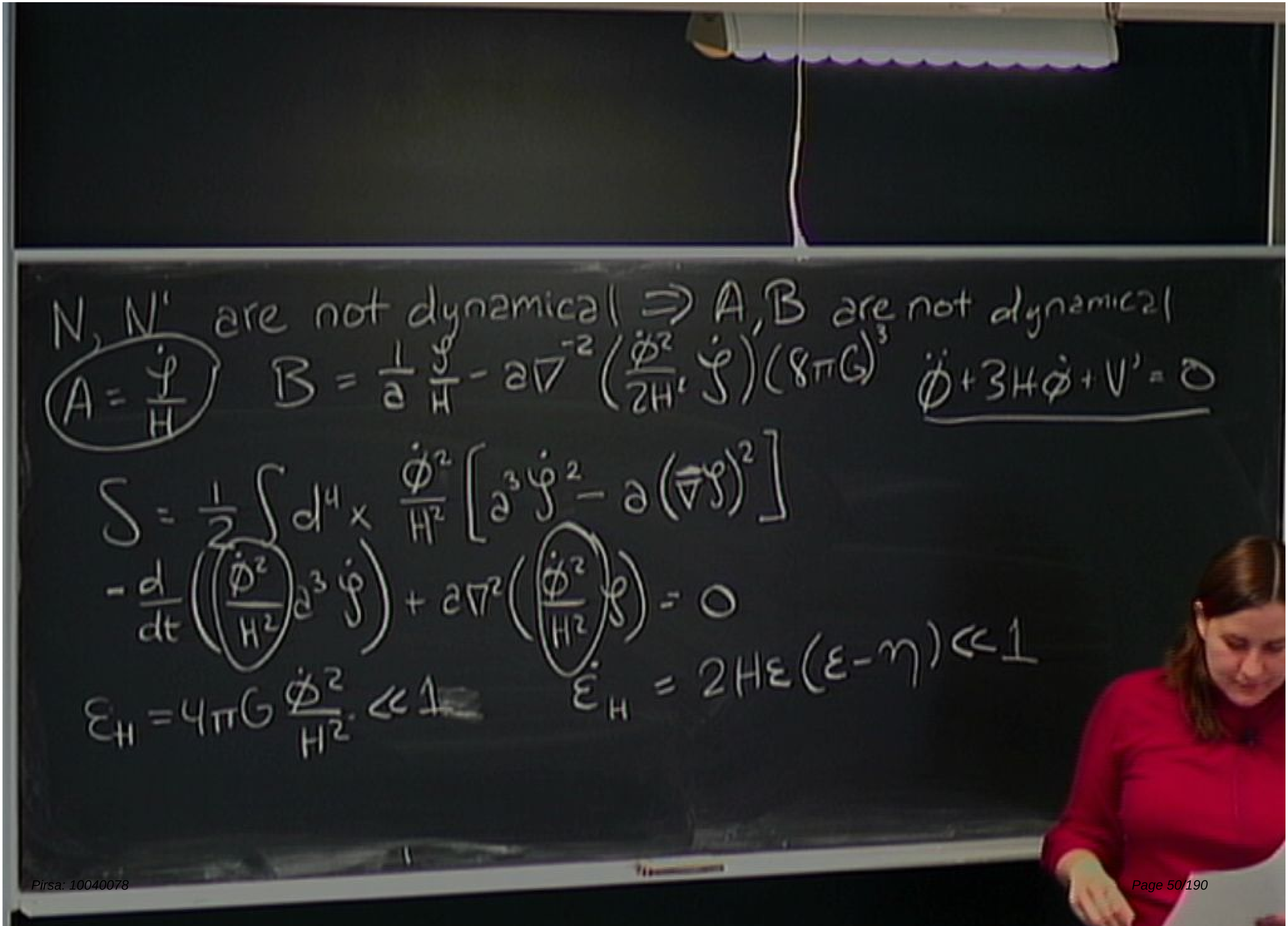
N, N' are not dynamical $\Rightarrow A, B$ are not dynamical

$$A = \frac{\dot{\zeta}}{H} \qquad B = \frac{1}{a}\frac{\zeta}{H} - a\nabla^{-2}\left(\frac{\dot{\phi}^2}{2H^2}\dot{\zeta}\right)(4\pi G)^3 \qquad \ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$S = \frac{1}{2}\int d^4x \, \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\zeta}^2 - a(\vec{\nabla}\zeta)^2\right]$$

$$-\frac{d}{dt}\left(\left(\frac{\dot{\phi}^2}{H^2}\right)a^3\dot{\zeta}\right) + a\nabla^2\left(\left(\frac{\dot{\phi}^2}{H^2}\right)\zeta\right) = 0$$

$$\varepsilon_H = 4\pi G\frac{\dot{\phi}^2}{H^2} \ll 1 \qquad \dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$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$-\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2}a^3\dot{\zeta}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\zeta\right) = 0$
$\varepsilon_H = 4\pi G\frac{\dot{\phi}^2}{H^2} \ll 1$
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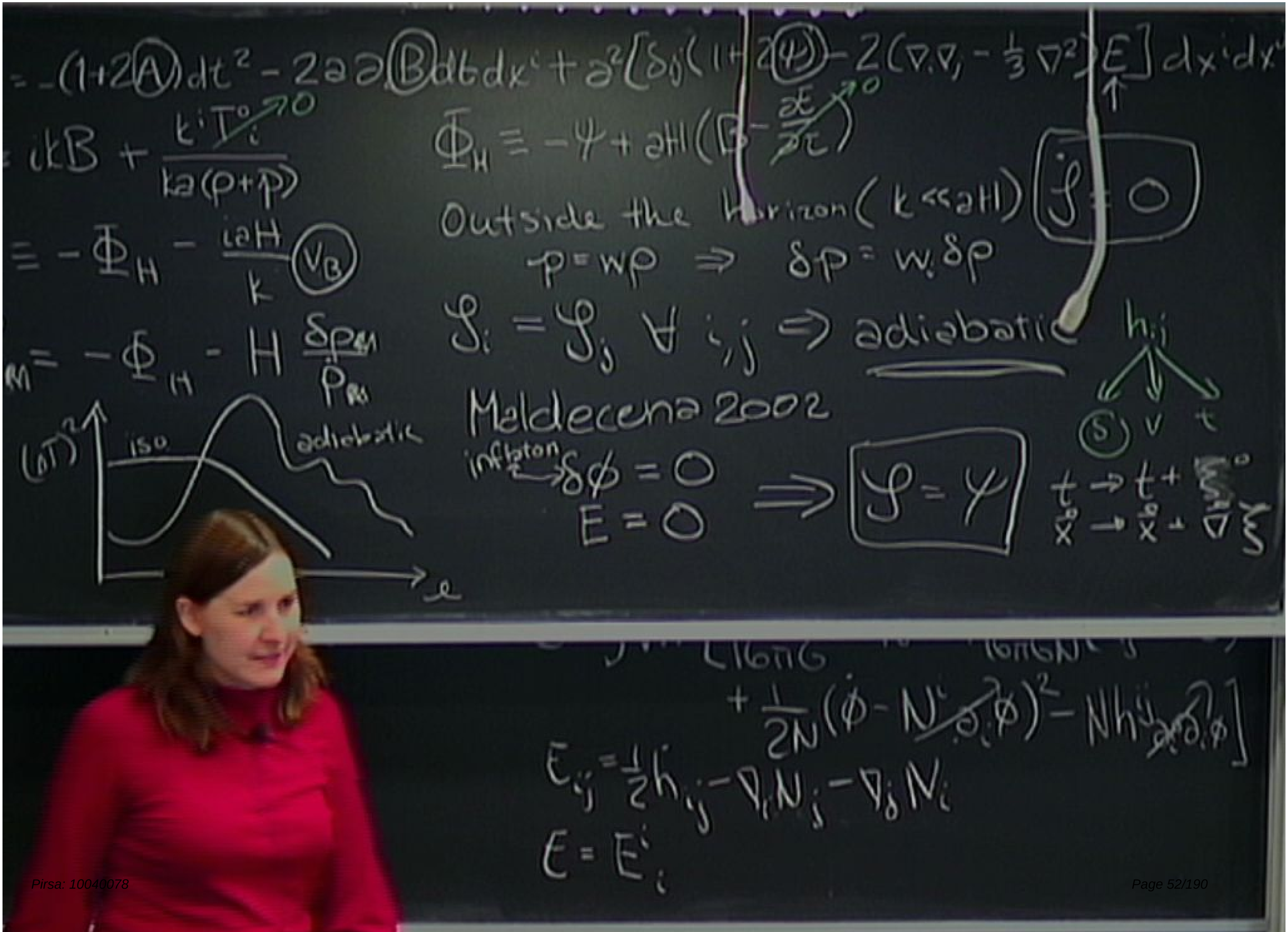
N, N' are not dynamical $\Rightarrow A, B$ are not dynamical

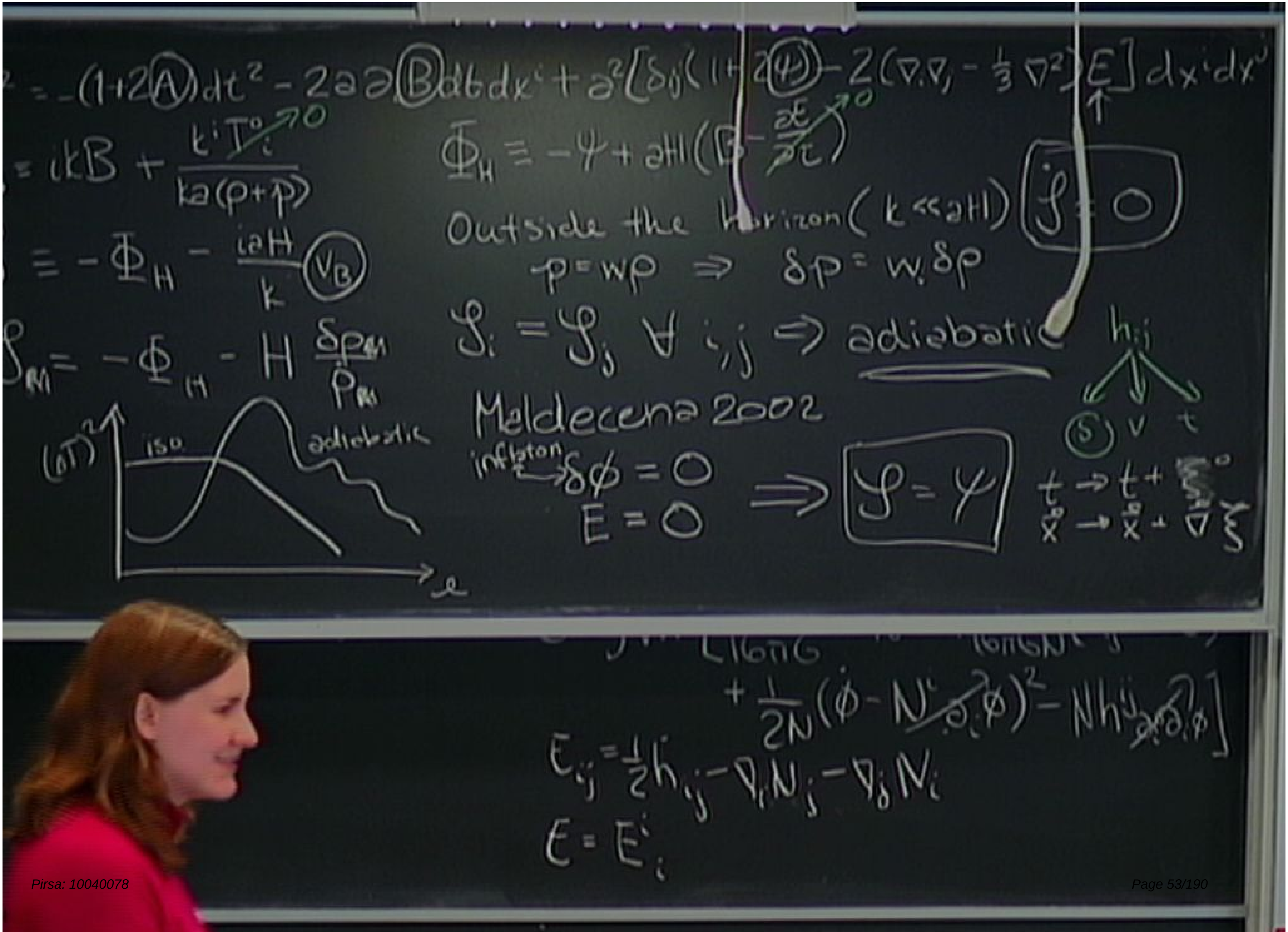
$$A = \frac{\dot{\zeta}}{H} \qquad B = \frac{1}{a}\frac{\zeta}{H} - a\nabla^{-2}\left(\frac{\dot{\phi}^2}{2H^2}\dot{\zeta}\right)(8\pi G)^3 \qquad \ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$S = \frac{1}{2}\int d^4x \, \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\zeta}^2 - a(\nabla\zeta)^2\right] \qquad f \equiv \frac{\dot{\phi}}{H}\zeta \propto \sqrt{\varepsilon_H}\,\zeta$$

$$-\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2}a^3\dot{\zeta}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\zeta\right) = 0$$

$$\varepsilon_H = 4\pi G\frac{\dot{\phi}^2}{H^2} \ll 1 \qquad \dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$


Outside the horizon (k << aH)
p = wρ ⇒ δp = w δρ
adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic


Outside the horizon (k << aH)
p = wρ ⇒ δp = w δρ
adiabatic
Maldecena 2002
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δφ = 0
E = 0
iso.
adiabatic
h_ij
t → t + ξ
x → x + ∇ξ
E = E^i_i

$$ds^2 = -(1+2A)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2\left[\delta_{ij}(1+2\psi) - 2\left(\nabla_i\nabla_j - \tfrac{1}{3}\nabla^2\right)E\right]dx^i dx^j$$

$$v = ikB + \frac{k^i T^0_i}{ka(\rho+p)}$$

$$\Phi_H \equiv -\psi + aH\left(B - \frac{\partial E}{\partial t}\right)$$

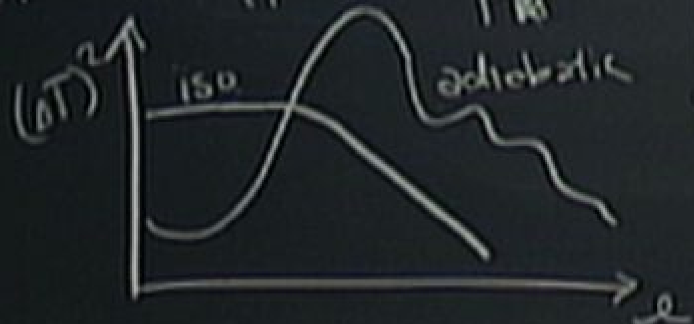
$$\mathcal{R} \equiv -\Phi_H - \frac{iaH}{k} v_B$$

$$\zeta_M = -\Phi_H - H\frac{\delta\rho_M}{\dot\rho_M}$$

Outside the horizon ($k \ll aH$) $\boxed{\dot\zeta = 0}$

$p = w\rho \Rightarrow \delta p = w_i\,\delta\rho$

$\zeta_i = \zeta_j \;\forall\, i,j \Rightarrow$ adiabatic



Meldecena 2002

inflaton $\delta\phi = 0$

$E = 0 \Rightarrow \boxed{\zeta = \psi}$

$h_{ij} \to \zeta,\ v,\ t$

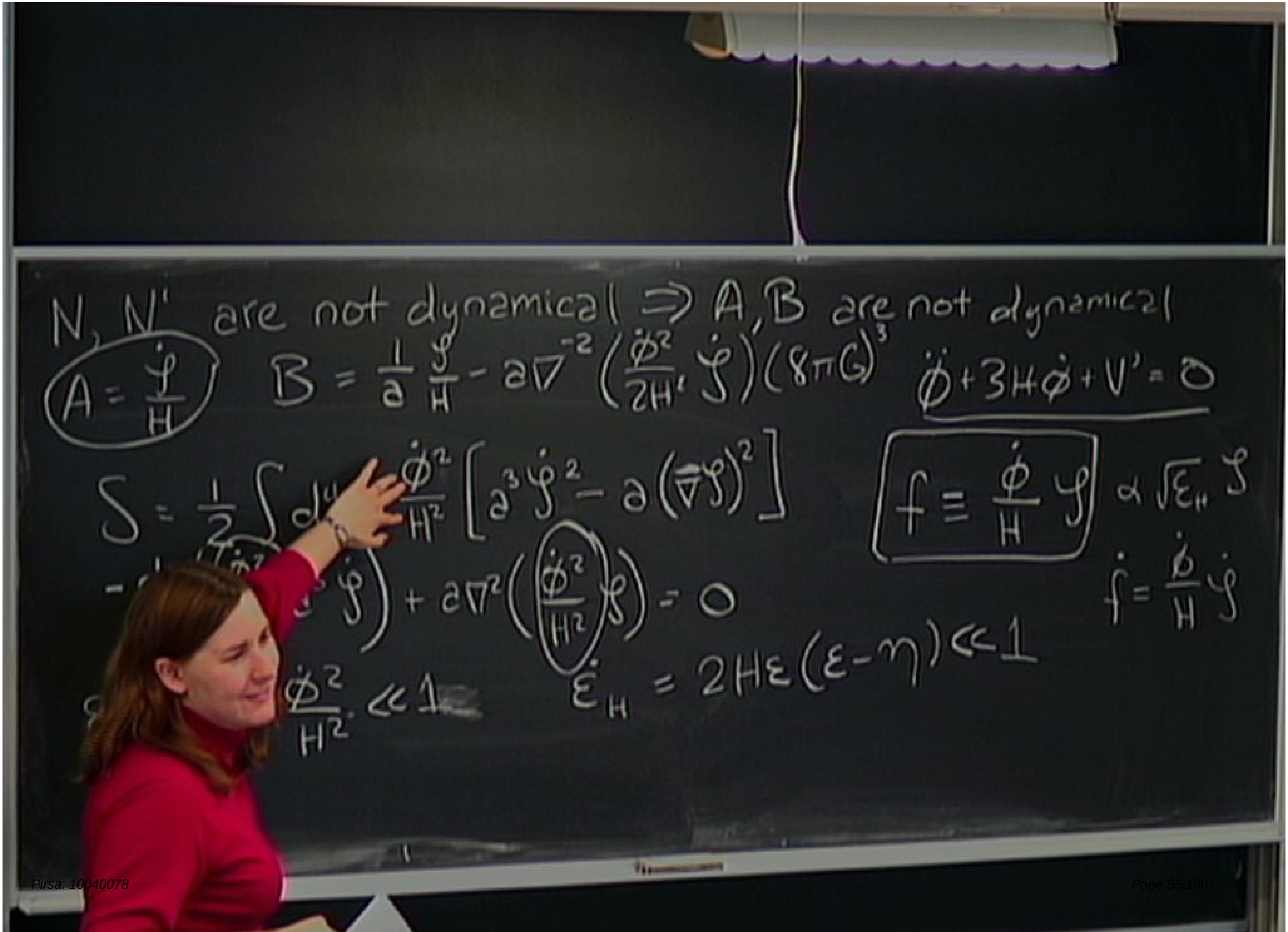
$t \to t + \xi^0$

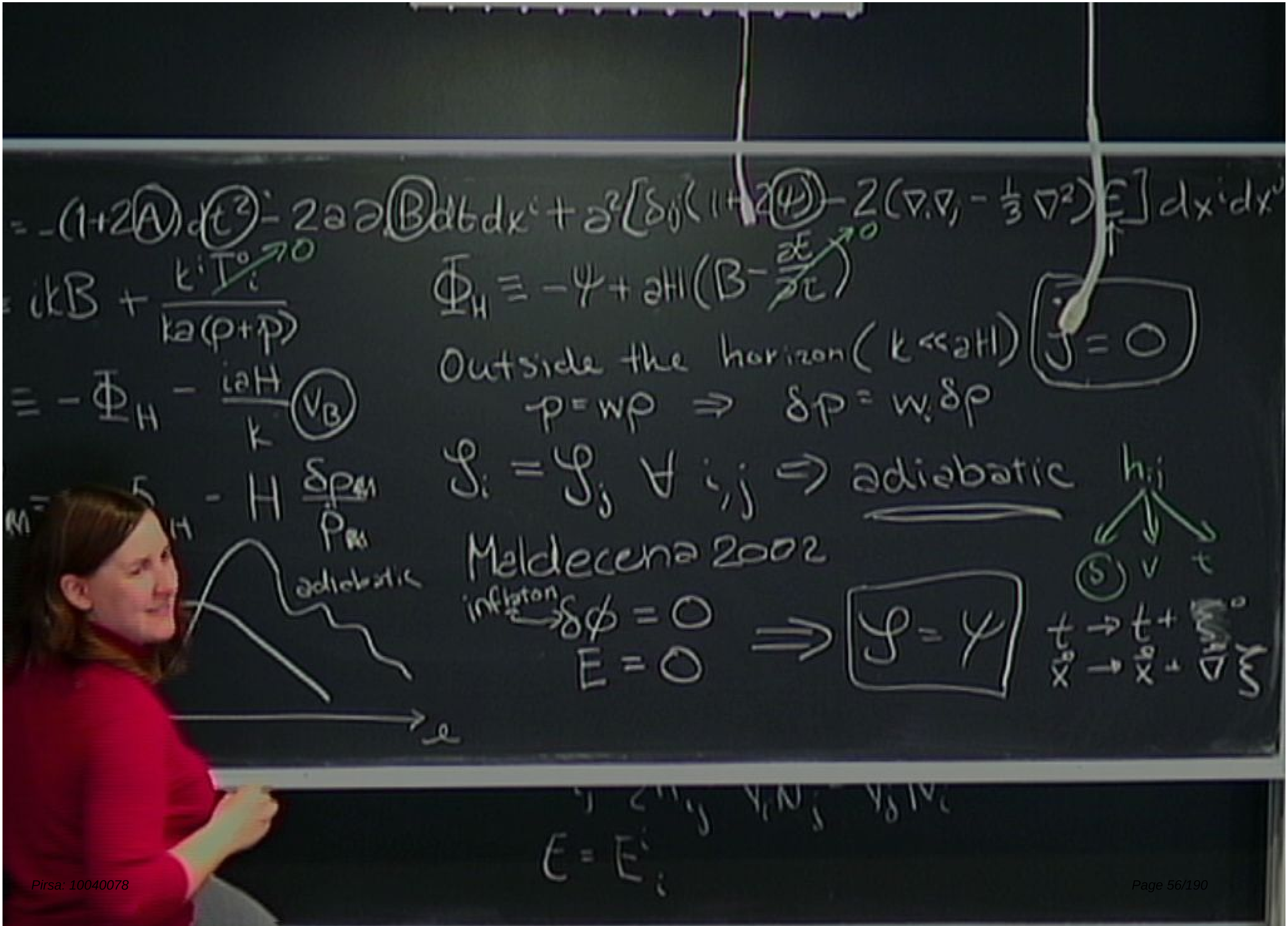
$\vec{x} \to \vec{x} + \vec{\nabla}\xi$

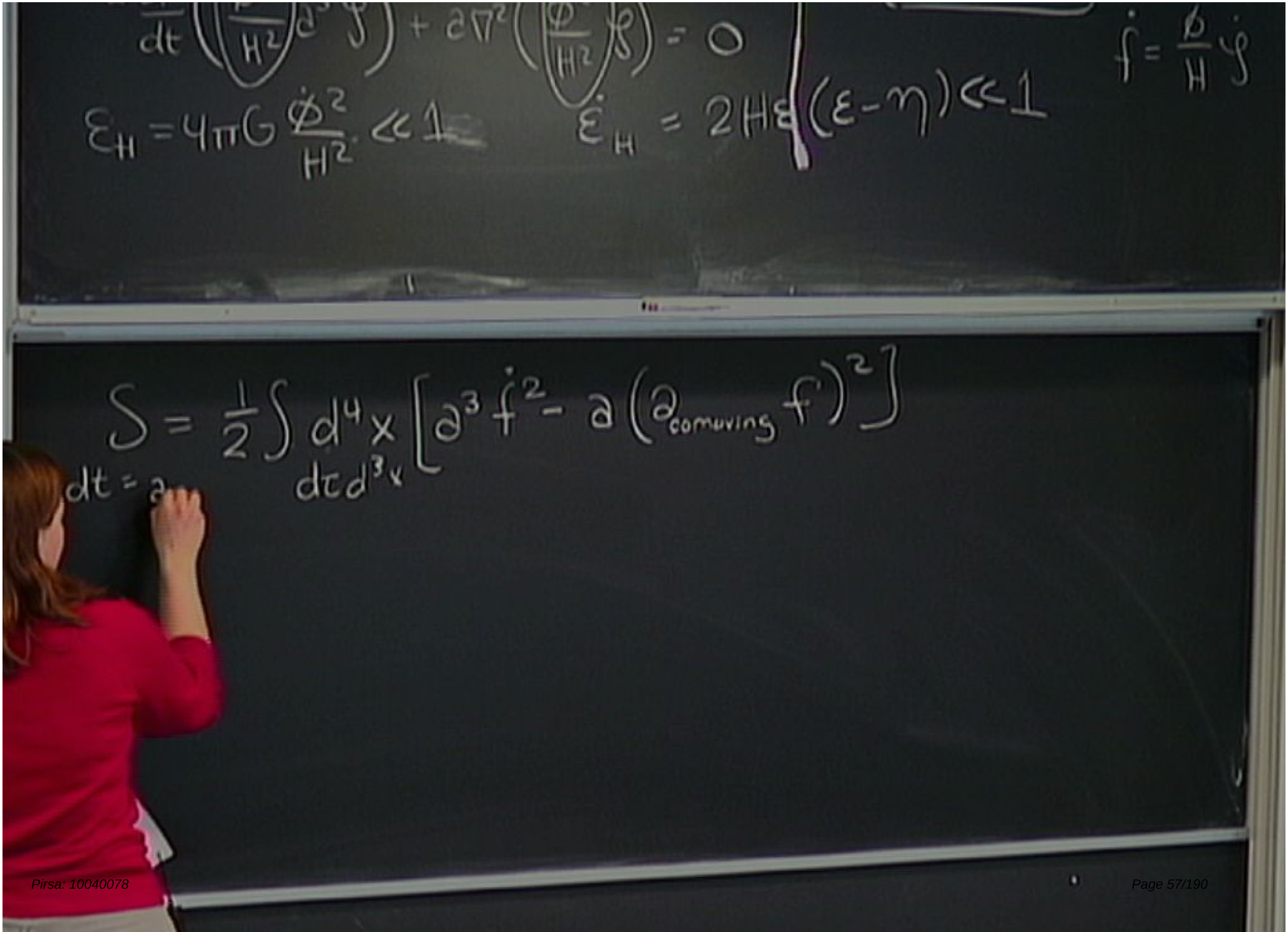
$$+\frac{1}{2N}\left(\dot\phi - N^i\partial_i\phi\right)^2 - Nh^{ij}\partial_i\phi\,\partial_j\phi\Big]$$

$$E_{ij} = \frac{1}{2}\dot h_{ij} - \nabla_i N_j - \nabla_j N_i$$

$$E = E^i_i$$


N, N' are not dynamical ⇒ A, B are not dynamical
$A = \frac{\dot{\varphi}}{H}$
$B = \frac{1}{a}\frac{\varphi}{H} - a\nabla^{-2}\left(\frac{\dot{\phi}^2}{2H^2}\dot{\varphi}\right)(8\pi G)^3$
$\ddot{\phi} + 3H\dot{\phi} + V' = 0$
$S = \frac{1}{2}\int d^4x \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\varphi}^2 - a(\nabla\varphi)^2\right]$
$f \equiv \frac{\dot{\phi}}{H}\varphi \propto \sqrt{\varepsilon_H}\,\varphi$
$+ a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\varphi\right) = 0$
$\dot{f} = \frac{\dot{\phi}}{H}\dot{\varphi}$
$\dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$
$\frac{\dot{\phi}^2}{H^2} \ll 1$


Outside the horizon (k << aH)
ζ = 0
p = wρ ⇒ δp = w δρ
ζ_i = ζ_j ∀ i,j ⇒ adiabatic
h_ij
S V t
adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
ζ = ψ
E = E_i



$\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2} a^3 \dot{\zeta}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\zeta\right) = 0$
$f = \frac{\dot{\phi}}{H}\zeta$
$\varepsilon_H = 4\pi G \frac{\dot{\phi}^2}{H^2} \ll 1$
$\dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$
$S = \frac{1}{2}\int d^4x \left[a^3 \dot{f}^2 - a\left(\partial_{comoving} f\right)^2\right]$
$dt = a$
$d\tau d^3x$

$$\frac{d}{dt}\left(\left(\frac{\dot{\phi}^2}{H^2}\right)a^3\dot{\varphi}\right) + a\nabla^2\left(\left(\frac{\dot{\phi}^2}{H^2}\right)\varphi\right) = 0$$

$$\dot{f} = \frac{\dot{\phi}}{H}\dot{\varphi}$$

$$\varepsilon_H = 4\pi G\frac{\dot{\phi}^2}{H^2} \ll 1 \qquad \dot{\varepsilon}_H = 2H\varepsilon(\varepsilon-\eta) \ll 1$$

$$S = \frac{1}{2}\int d^4x\left[a^3\dot{f}^2 - a\left(\partial_{comoving} f\right)^2\right]$$

$dt = a\,d\tau$ $\qquad d\tau\, d^3x$

$$\frac{d}{dt}\left(\left(\frac{\dot{\phi}^2}{H^2}\right) a^3 \dot{\varphi}\right) + a\nabla^2\left(\left(\frac{\dot{\phi}^2}{H^2}\right)\varphi\right) = 0$$

$$f = \frac{\dot{\phi}}{H}\varphi$$

$$\varepsilon_H = 4\pi G \frac{\dot{\Phi}^2}{H^2} \ll 1 \qquad \dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$

$$S = \frac{1}{2}\int \underset{dt\,d^3x}{d^4x}\left[a^3 \dot{f}^2 - a\left(\partial_{\text{comoving}} f\right)^2\right]$$

$$\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2} a^3 \dot{\varphi}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\varphi\right) = 0$$

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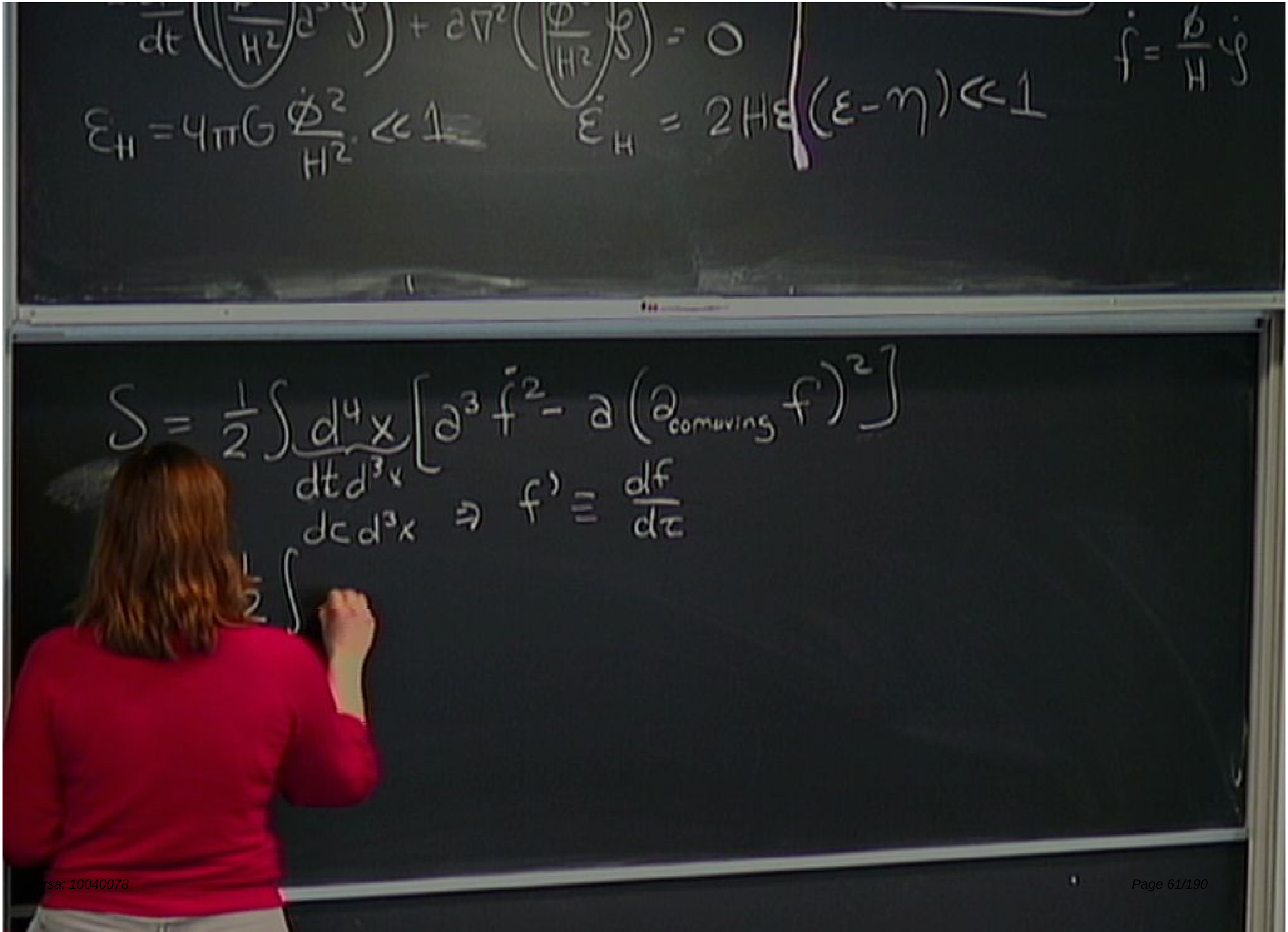
$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3 \dot{f}^2 - a\left(\partial_{\text{comoving}} f\right)^2\right]$$



d/dt (φ̇²/H² a³ ḟ) + a∇²(φ̇²/H² f) = 0
ḟ = φ̇/H φ̇
ε_H = 4πG φ̇²/H² ≪ 1
ε̇_H = 2Hε(ε − η) ≪ 1
S = 1/2 ∫ d⁴x [a³ ḟ² − a (∂_comoving f)²]
dt d³x
dτ d³x ⇒ f' ≡ df/dτ

$$\frac{d}{dt}\left(\left(\frac{\dot\phi^2}{H^2}\right)a^3\dot{\varphi}\right) + a\nabla^2\left(\left(\frac{\dot\phi^2}{H^2}\right)\varphi\right) = 0$$

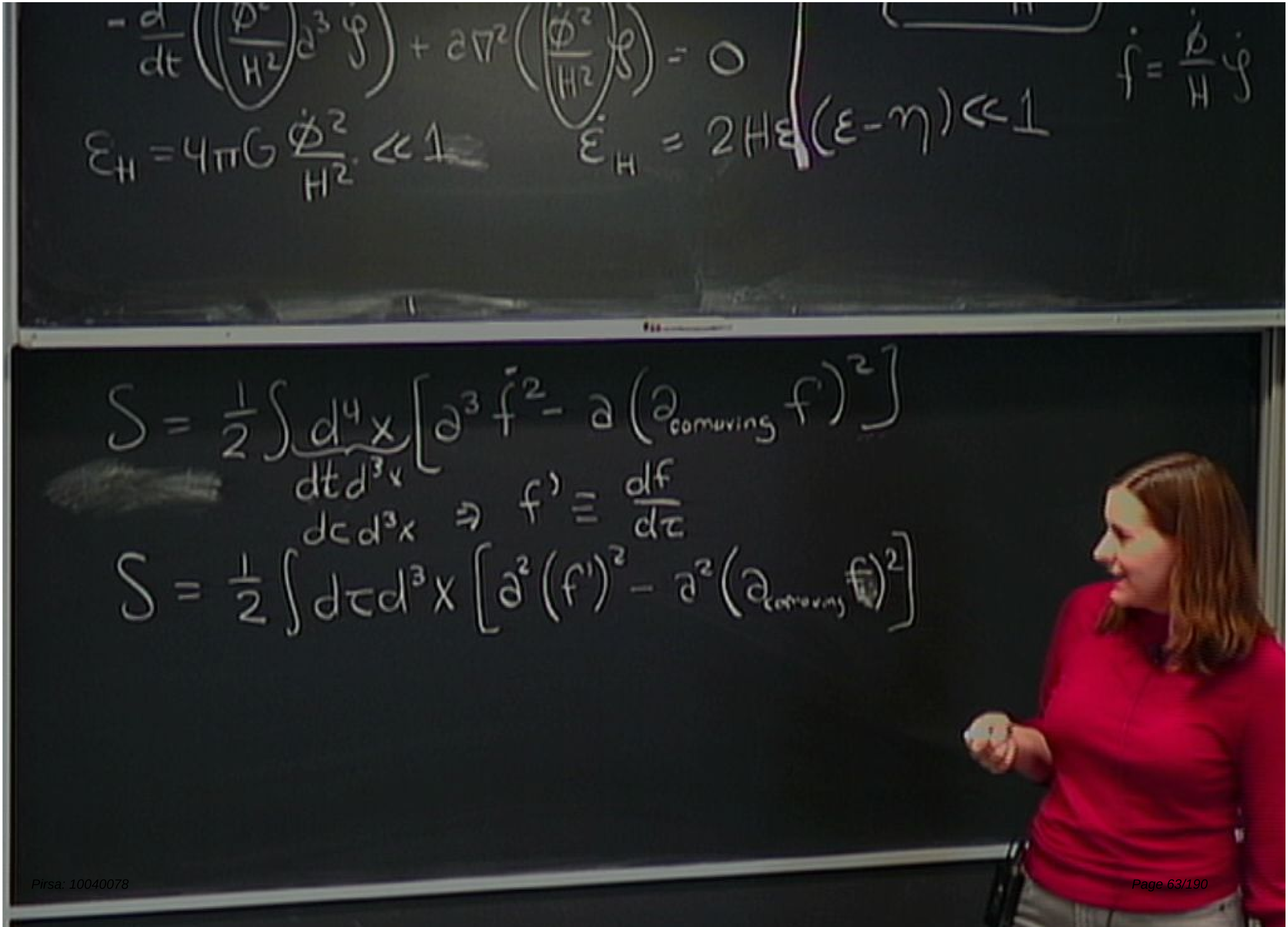
$$\dot f = \frac{\dot\phi}{H}\varphi$$

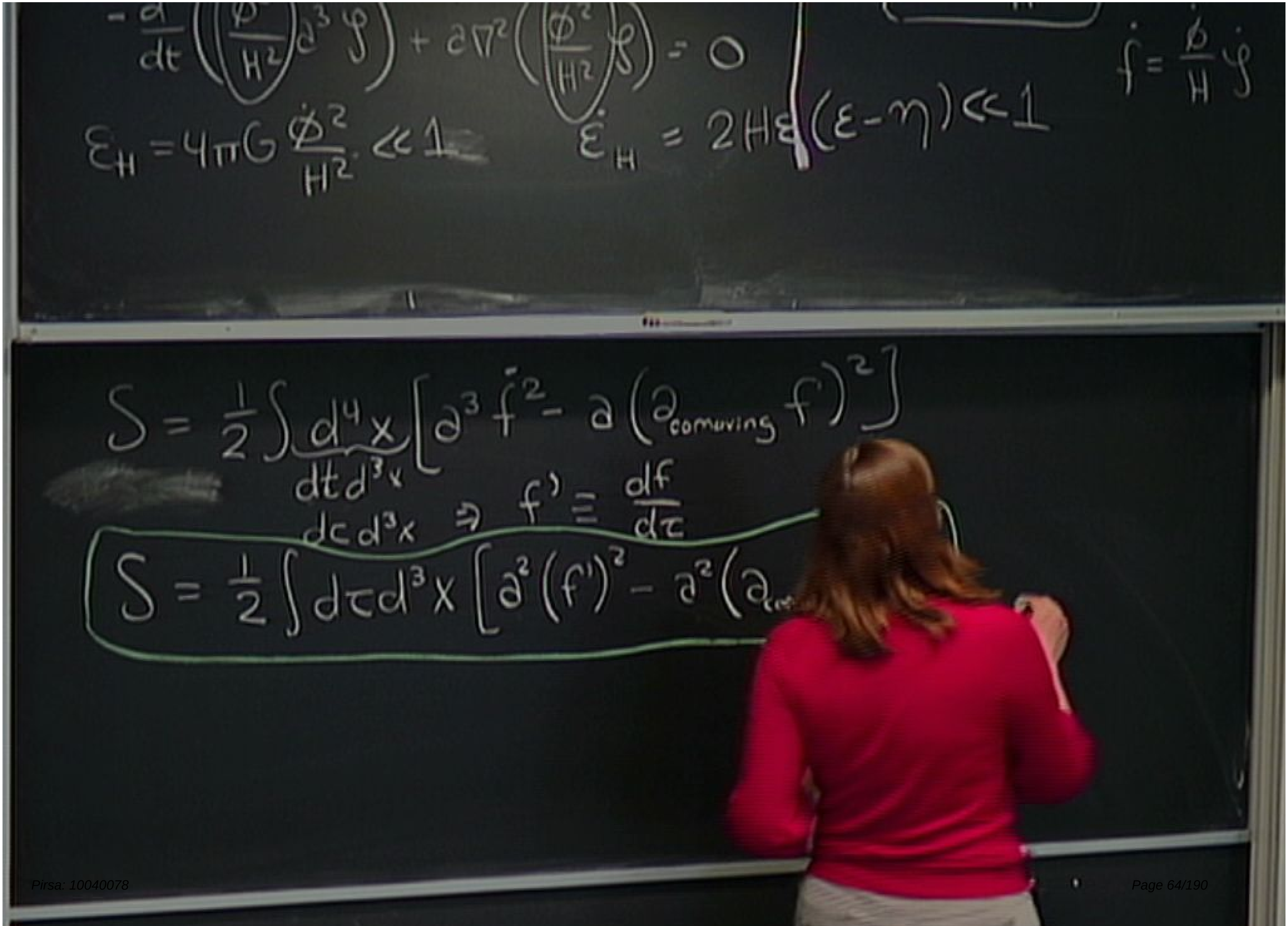
$$\varepsilon_H = 4\pi G\frac{\dot\phi^2}{H^2} \ll 1 \qquad \dot\varepsilon_H = 2H\varepsilon(\varepsilon-\eta) \ll 1$$

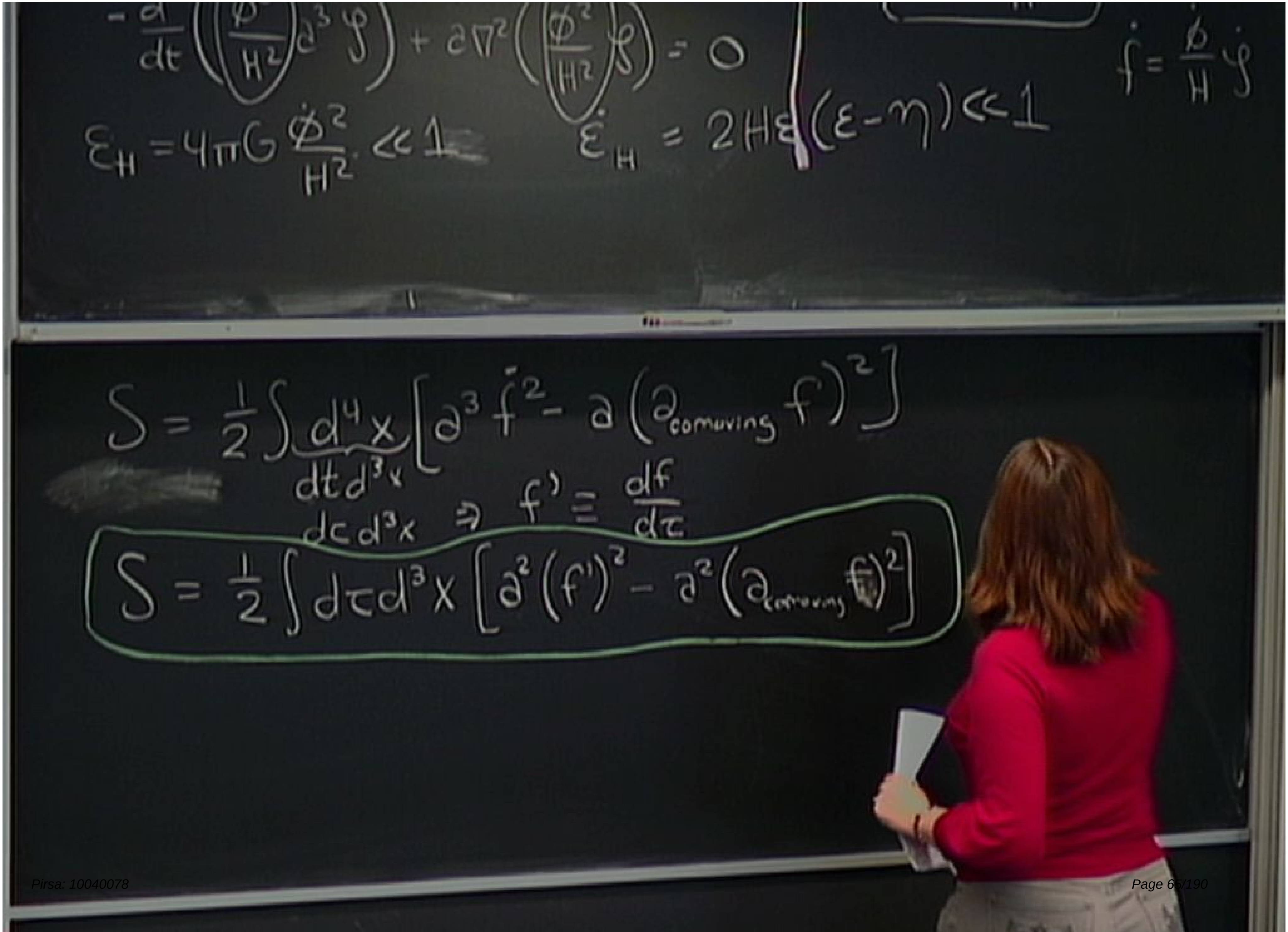
$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3\dot f^2 - a\left(\partial_{comoving} f\right)^2\right]$$

$$dt\,d^3x \qquad d\tau\,d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

$$S = \frac{1}{2}\int d\tau\,d^3x\left[a^2(f')^2 - a^2\left(\partial_{comoving} f\right)^2\right.$$


$-\frac{d}{dt}\left(\frac{\dot\phi^2}{H^2} a^3 \dot\varphi\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\varphi\right) = 0$
$\epsilon_H = 4\pi G \frac{\dot\phi^2}{H^2} \ll 1$
$\dot\epsilon_H = 2H\epsilon(\epsilon-\eta) \ll 1$
$f = \frac{\dot\phi}{H}\varphi$
$S = \frac{1}{2}\int d^4x \left[a^3 \dot f^2 - a(\partial_{comoving} f)^2\right]$
$dt\, d^3x$
$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$
$S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2(\partial_{comoving} f)^2\right]$


$-\frac{d}{dt}\left(\left(\frac{\dot\phi^2}{H^2}\right)a^3\dot\varphi\right) + a\nabla^2\left(\left(\frac{\dot\phi^2}{H^2}\right)\varphi\right) = 0$
$f = \frac{\dot\phi}{H}\varphi$
$\varepsilon_H = 4\pi G\frac{\dot\phi^2}{H^2} \ll 1$
$\dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$
$S = \frac{1}{2}\int d^4x\left[a^3\dot f^2 - a(\partial_{comoving} f)^2\right]$
$dt\,d^3x$
$d\tau\,d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$
$S = \frac{1}{2}\int d\tau\,d^3x\left[a^2(f')^2 - a^2(\partial_{co}$


$\varepsilon_H = 4\pi G \frac{\dot{\phi}^2}{H^2} \ll 1$
$\dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$
$f = \frac{\dot{\phi}}{H} \varphi$
$S = \frac{1}{2}\int d^4x \left[a^3 \dot{f}^2 - a(\partial_{comoving} f)^2\right]$
$dt\, d^3x$
$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$
$S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2 (\partial_{comoving} f)^2\right]$

$$-\frac{d}{dt}\left(\frac{\dot\phi^2}{H^2} a^3 \dot\varphi\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\varphi\right) = 0$$

$$f = \frac{\dot\phi}{H}\varphi$$

$$\varepsilon_H = 4\pi G \frac{\dot\phi^2}{H^2} \ll 1 \qquad \dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3 \dot f^2 - a\left(\partial_{\text{comoving}} f\right)^2\right]$$

$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2\left(\partial_{\text{comoving}} f\right)^2\right]}$$

$$f = \frac{\dot\phi}{H}\varphi$$

$$-\frac{d}{dt}\left(\frac{\dot\phi^2}{H^2} a^3 \dot{\varphi}\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\varphi\right) = 0$$

$$f = \frac{\dot\phi}{H}\varphi$$

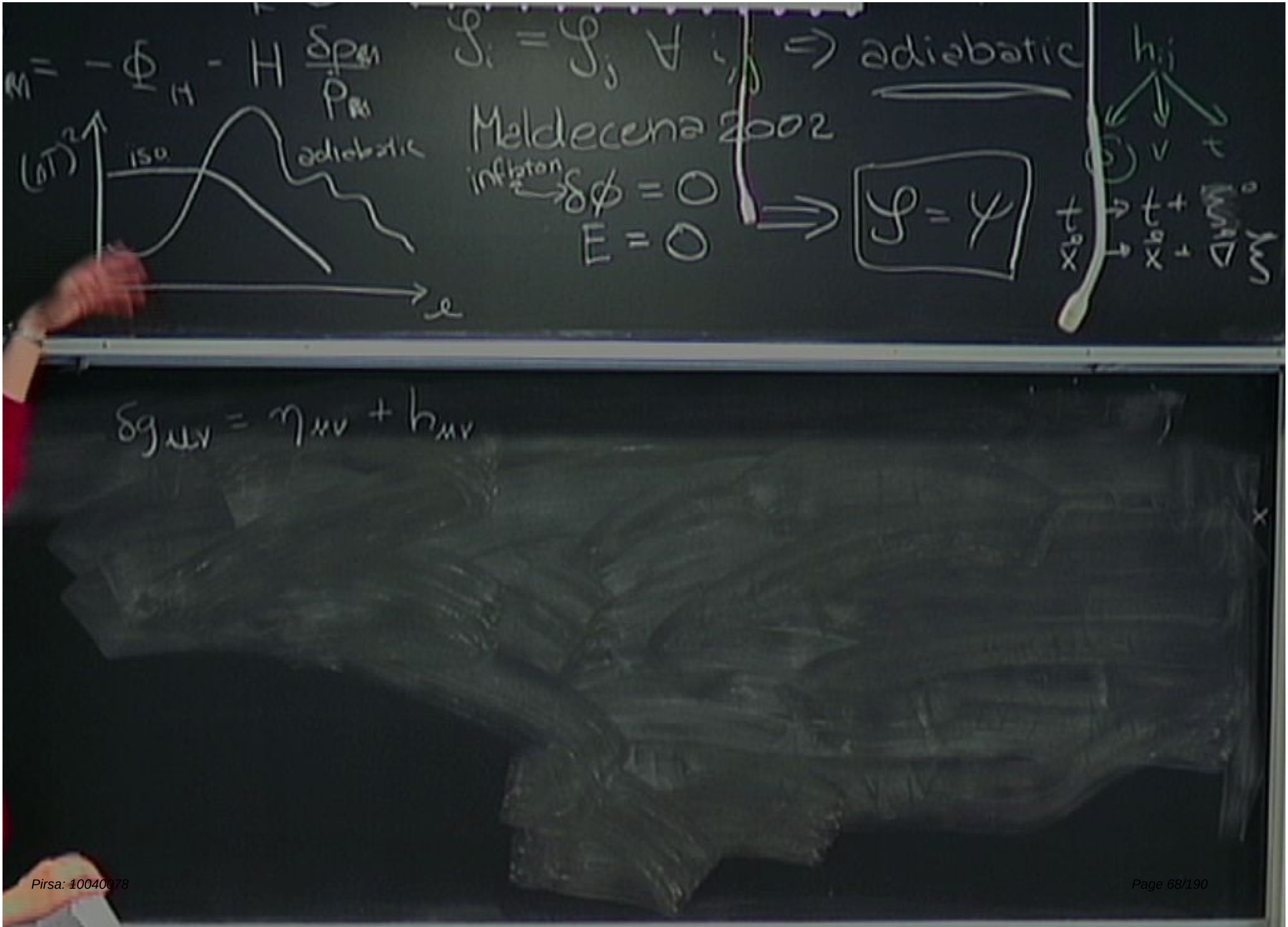
$$\varepsilon_H = 4\pi G \frac{\dot\phi^2}{H^2} \ll 1 \qquad \dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x} \left[a^3 \dot f^2 - a\left(\partial_{comoving} f\right)^2\right]$$

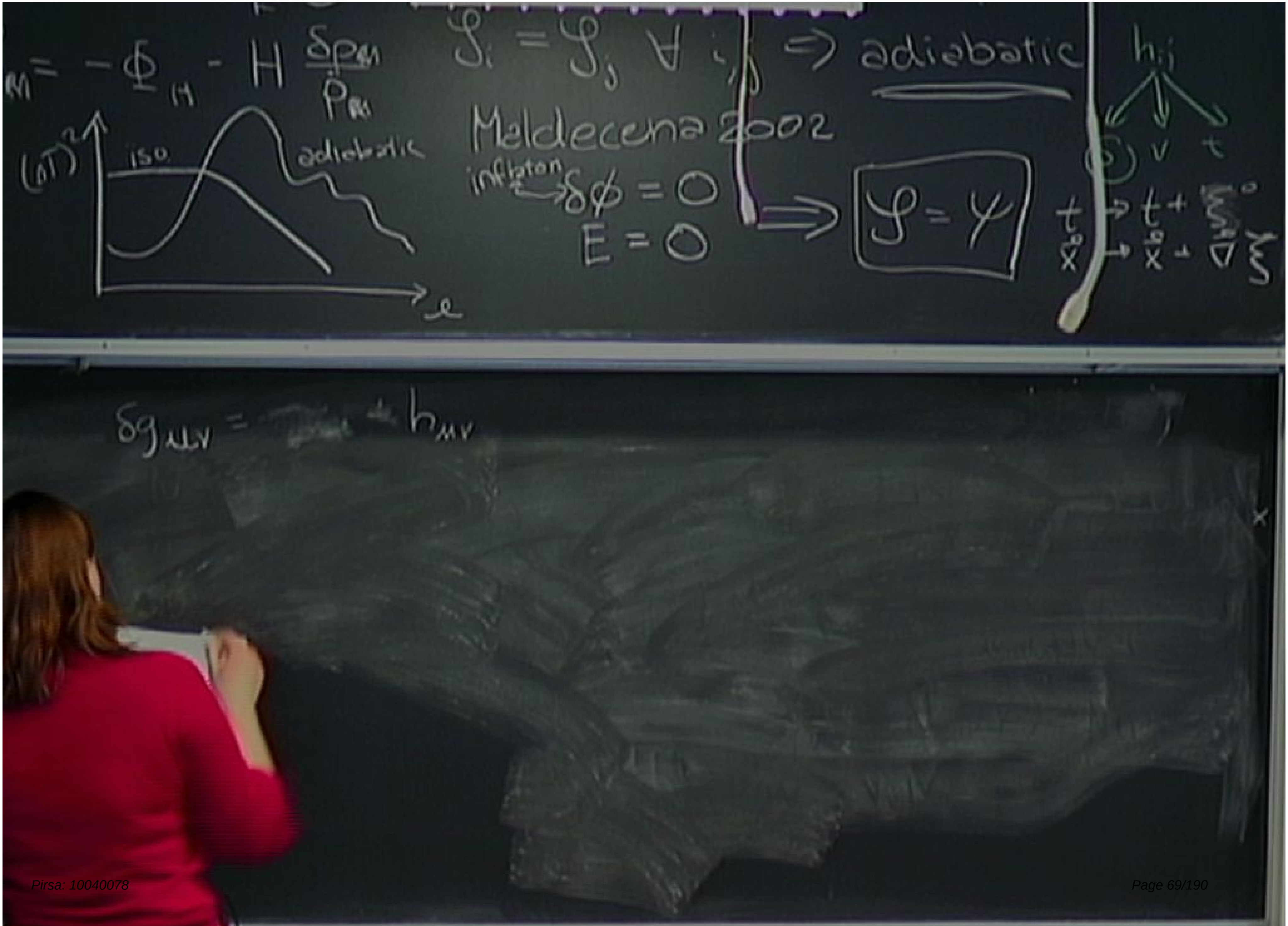
$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

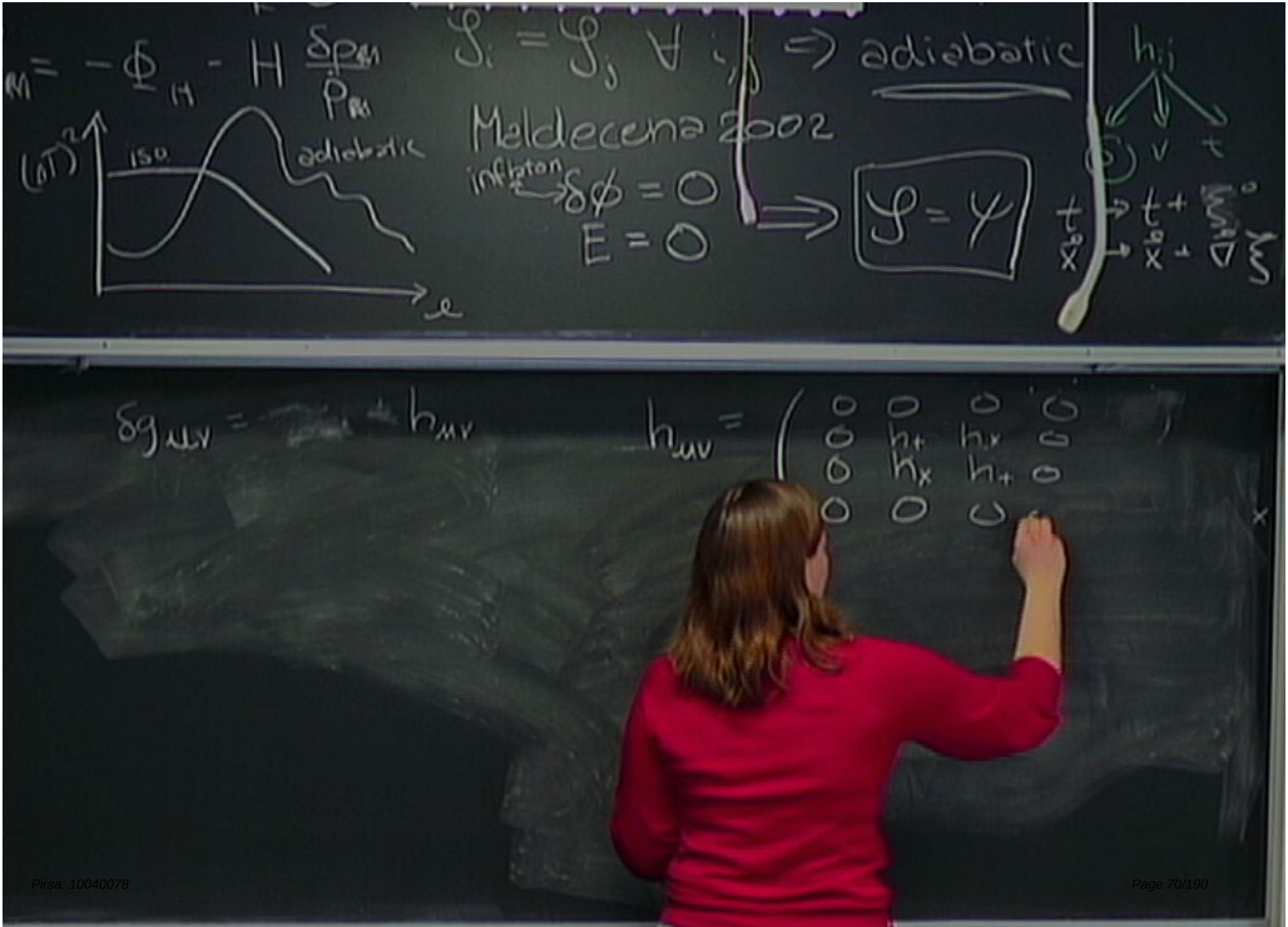
$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2\left(\partial_{comoving} f\right)^2\right]}$$

$$f = \frac{\dot\phi}{H}\varphi$$

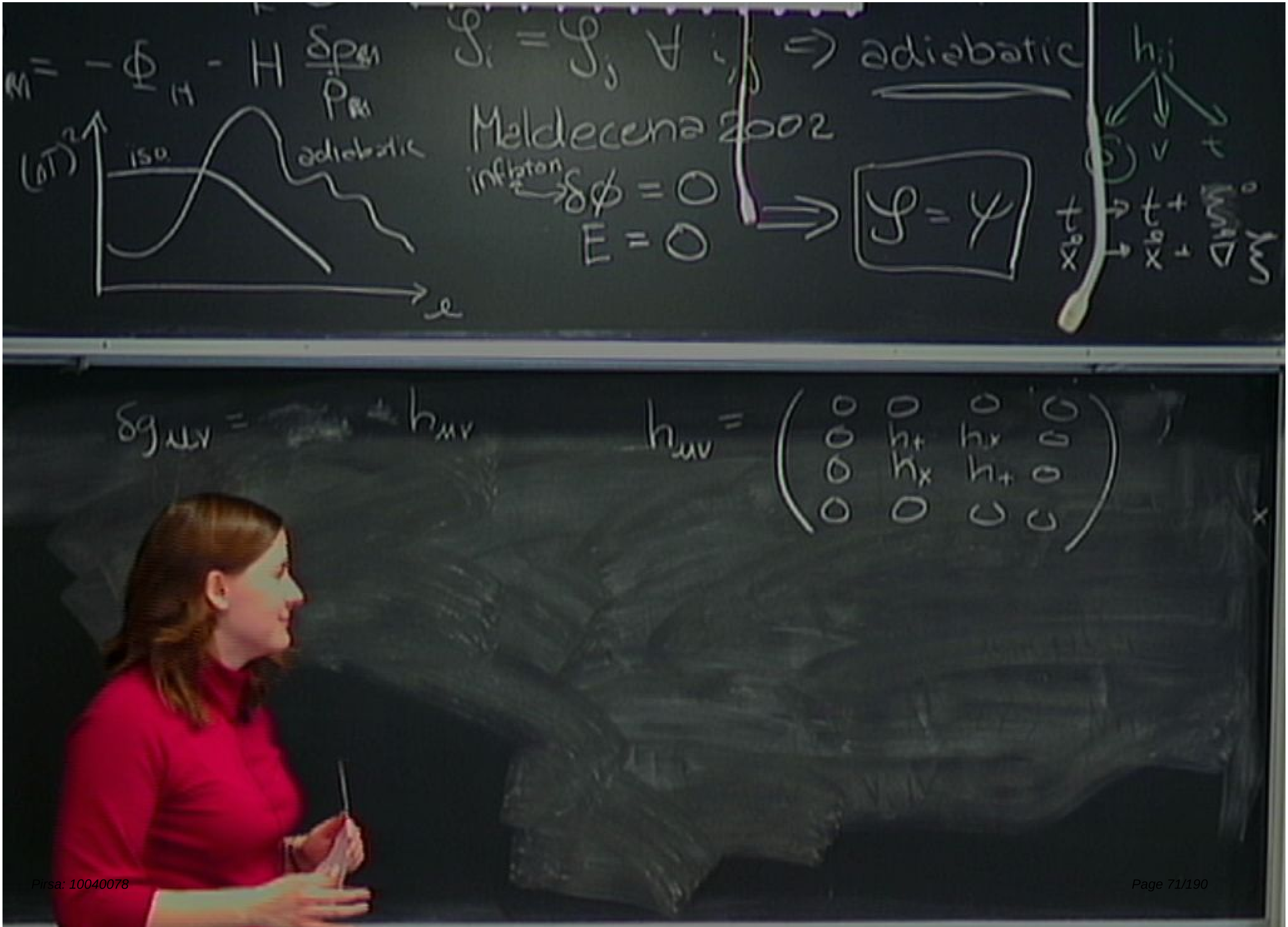


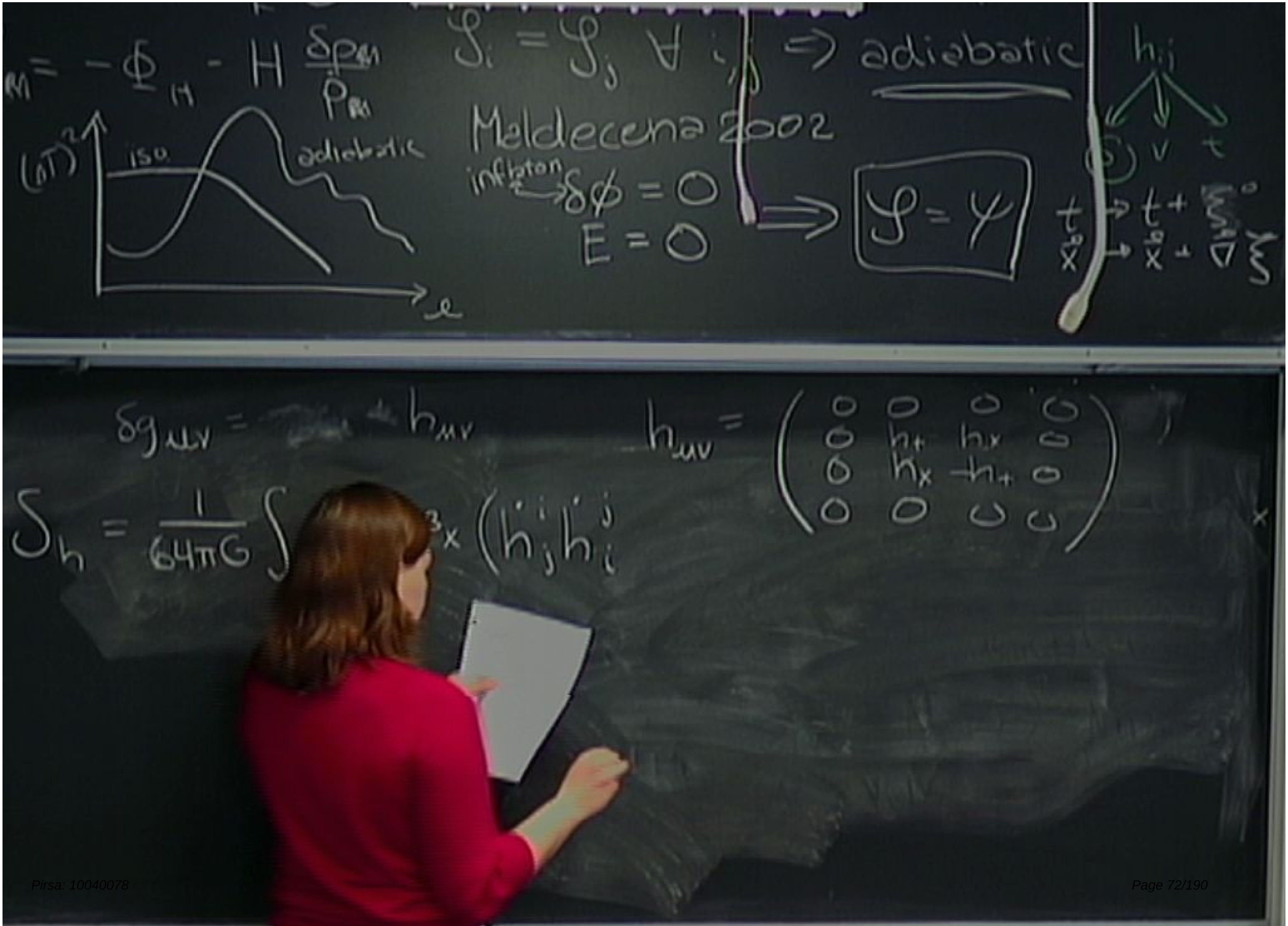
= − Φ_H − H δρ/ρ̇
ς_i = ς_j ∀ i,j ⇒ adiabatic
h_ij
(ΔT)²
iso
adiabatic
ℓ
Meldecena 2002
inflaton
δφ = 0
E = 0
ς = ψ
t → t +
x → x + ∇
δg_μν = η_μν + h_μν


adiabatic
h_{ij}
Maldecena 2002
iso
adiabatic
inflaton
$\delta\phi = 0$
$E = 0$
$\delta g_{\mu\nu} =$
$h_{\mu\nu}$

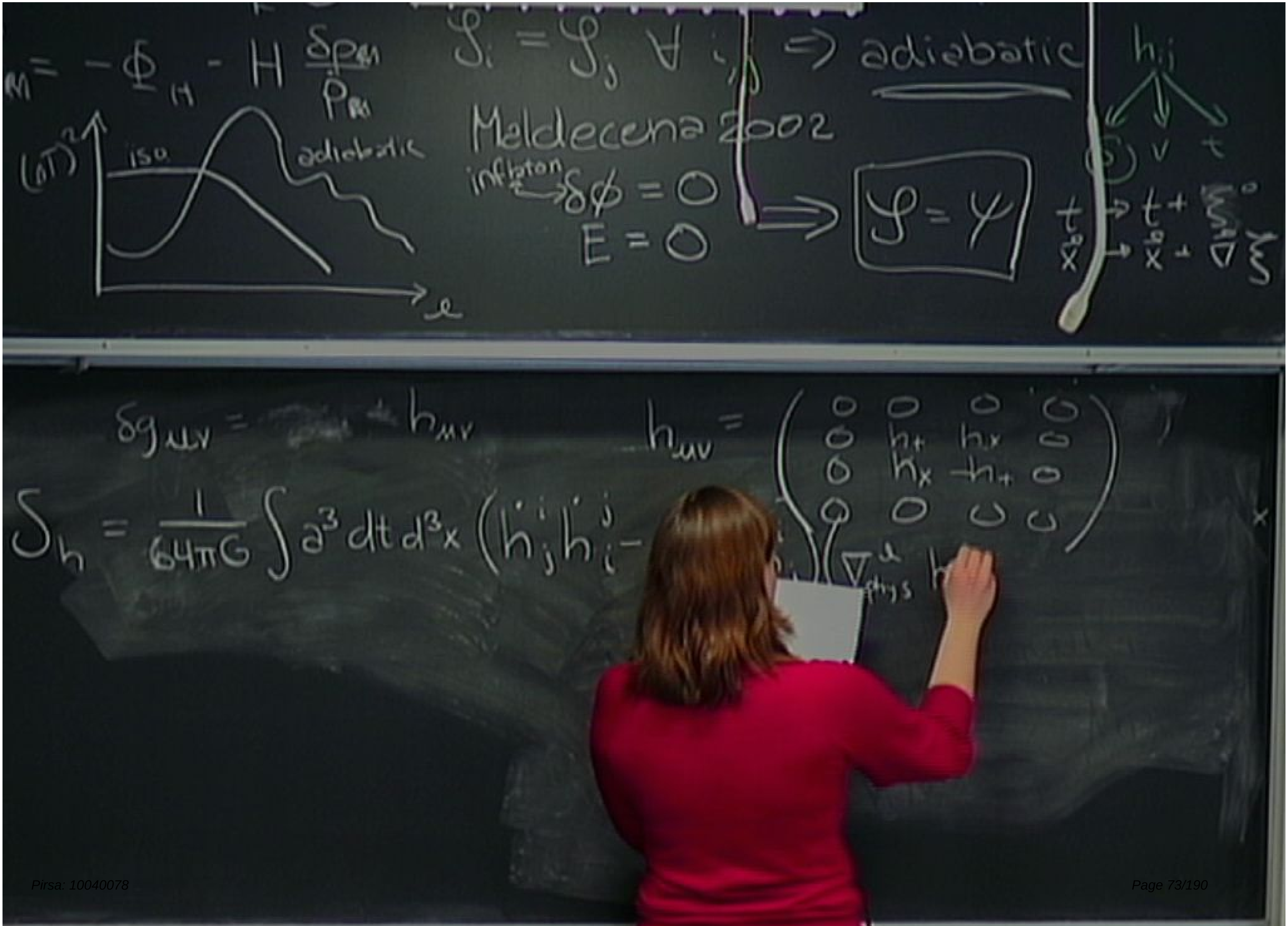


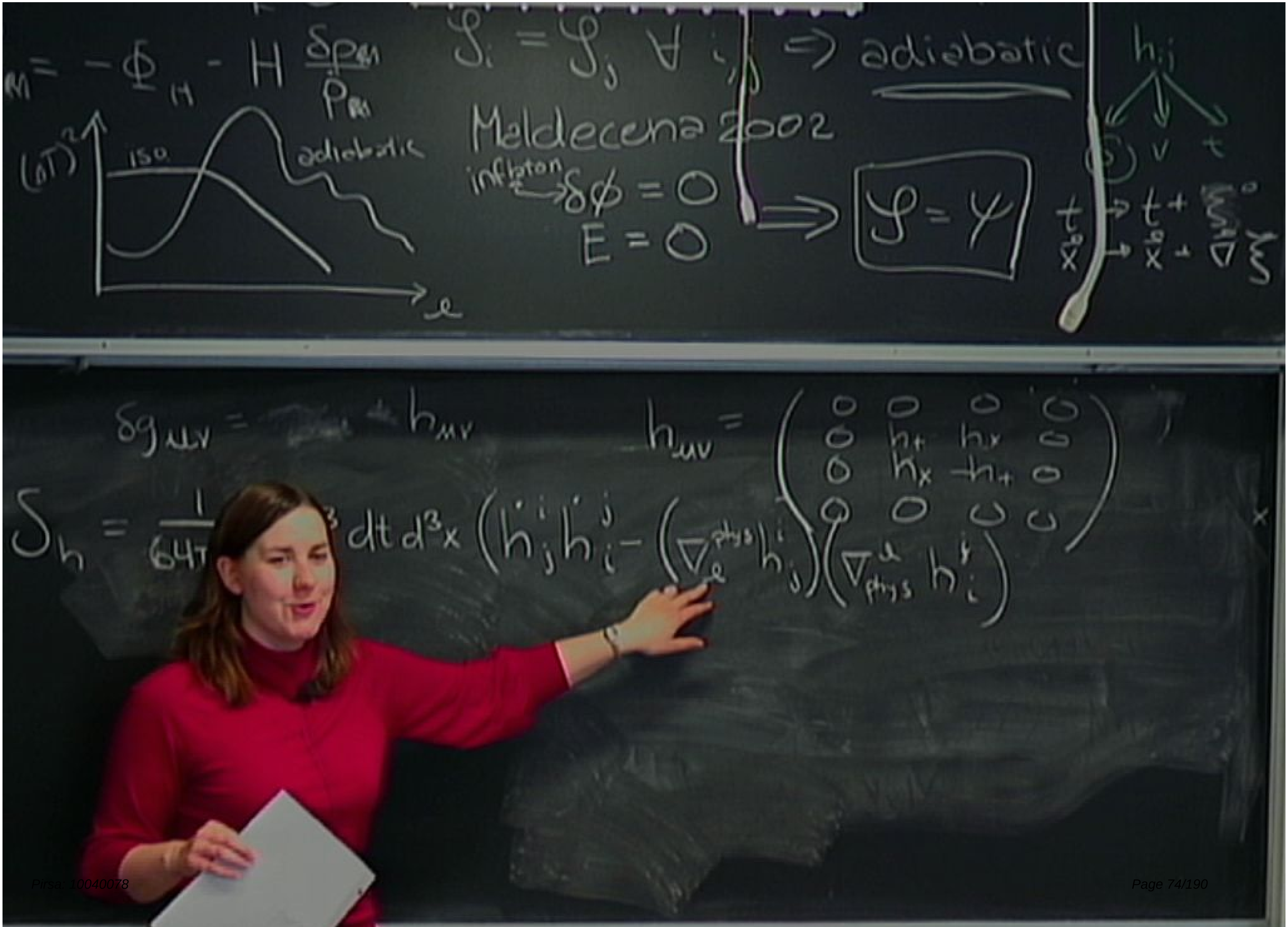
⇒ adiabatic
Maldecena 2002
iso
adiabatic
inflaton
δφ = 0
E = 0
δg_μν =
h_μν
h_μν =

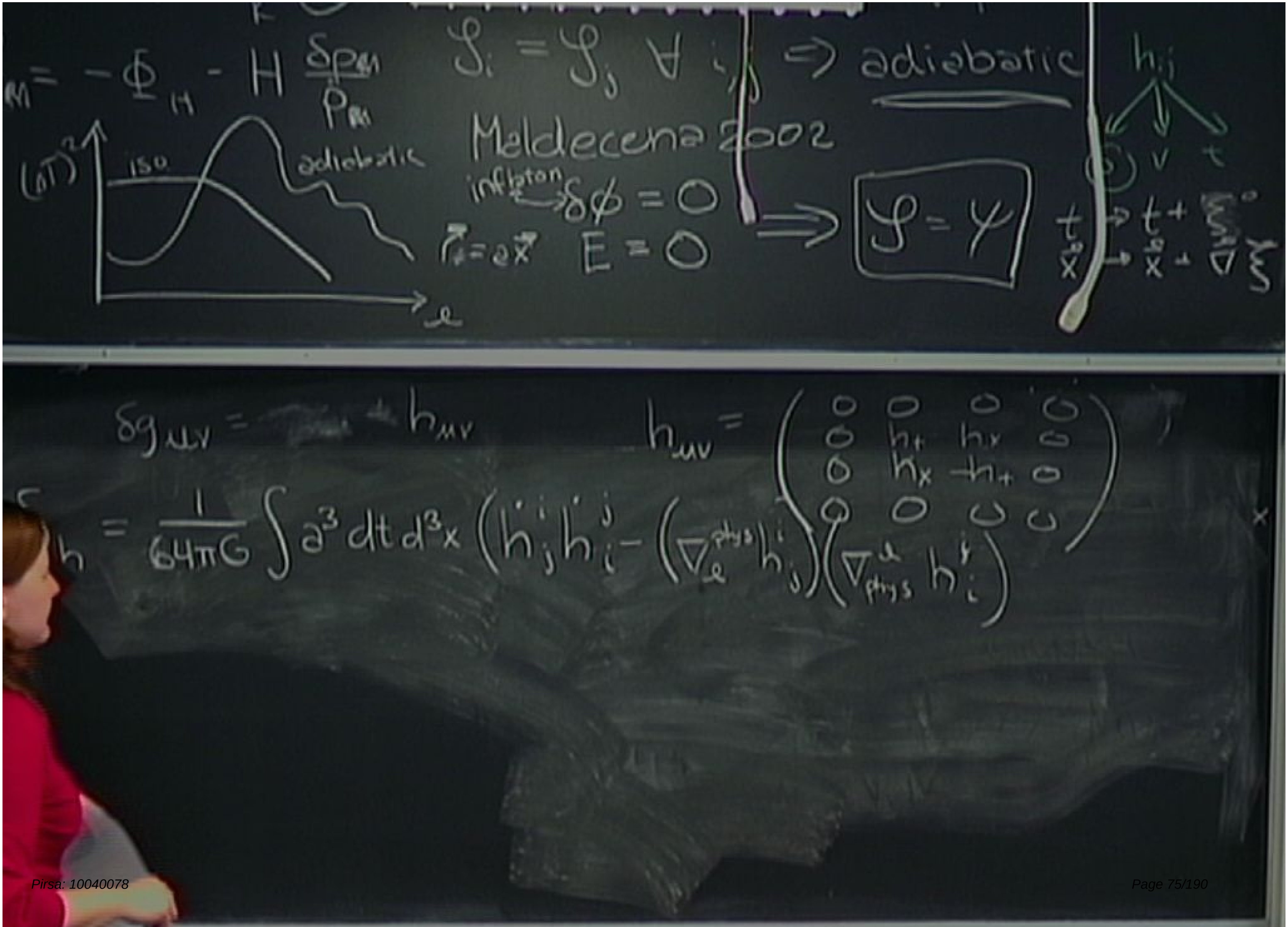

adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso.
adiabatic
h_ij
δg_μν = h_μν
h_μν =
0 0 0 0
0 h+ hx 0
0 hx h+ 0
0 0 0 0

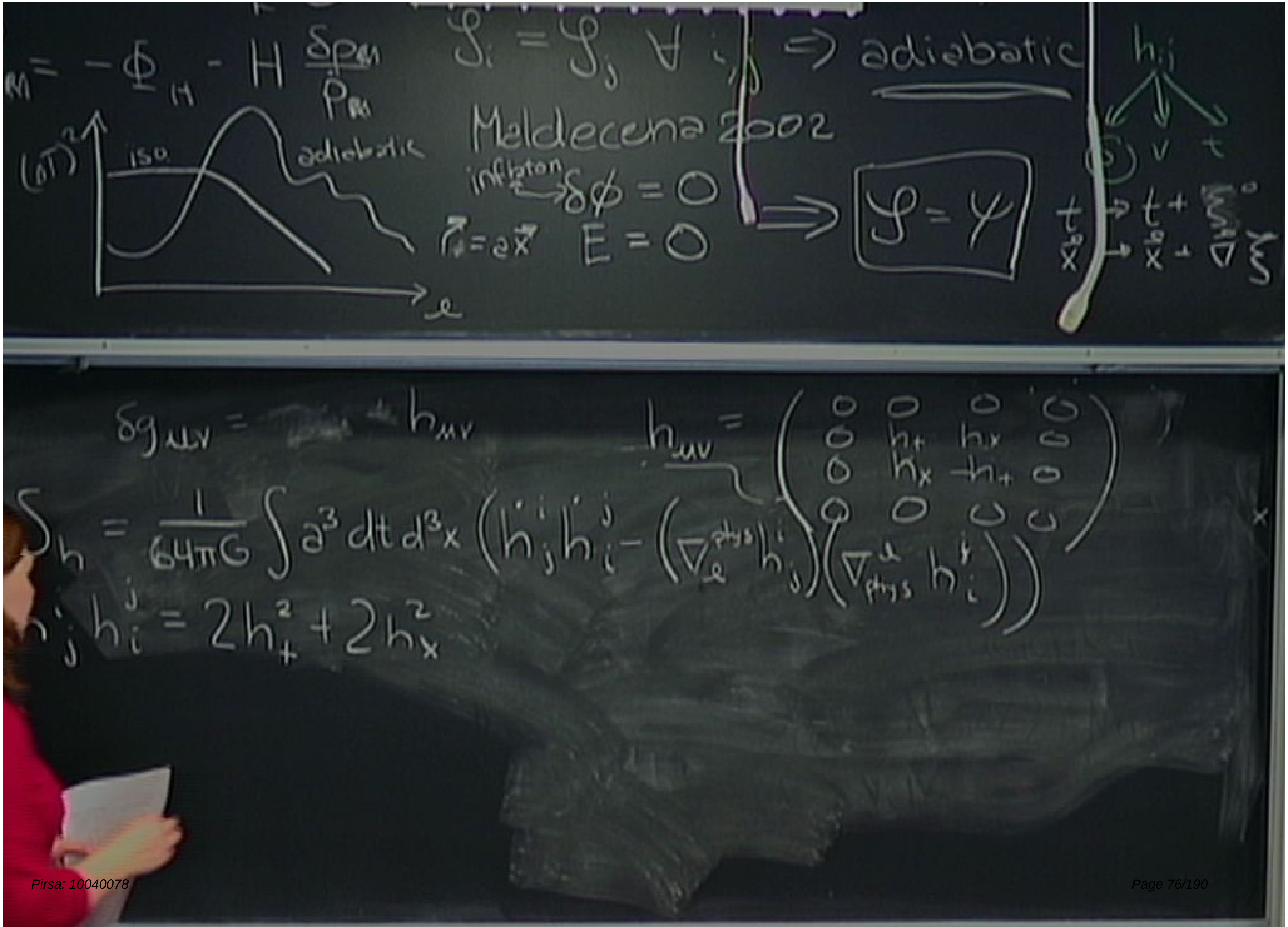


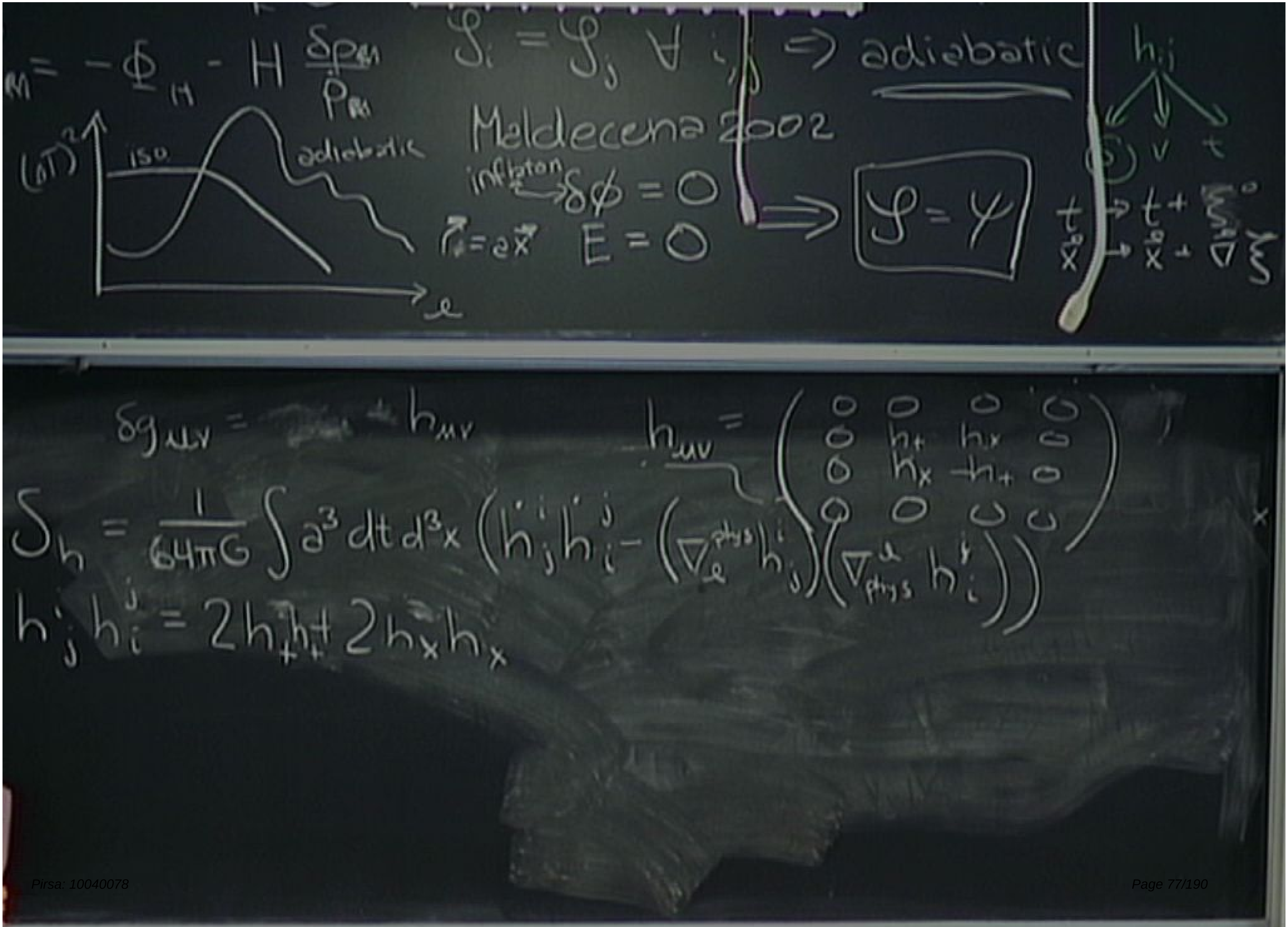
adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic
δg_μν =
h_μν
h_μν =
S_h = 1/64πG ∫

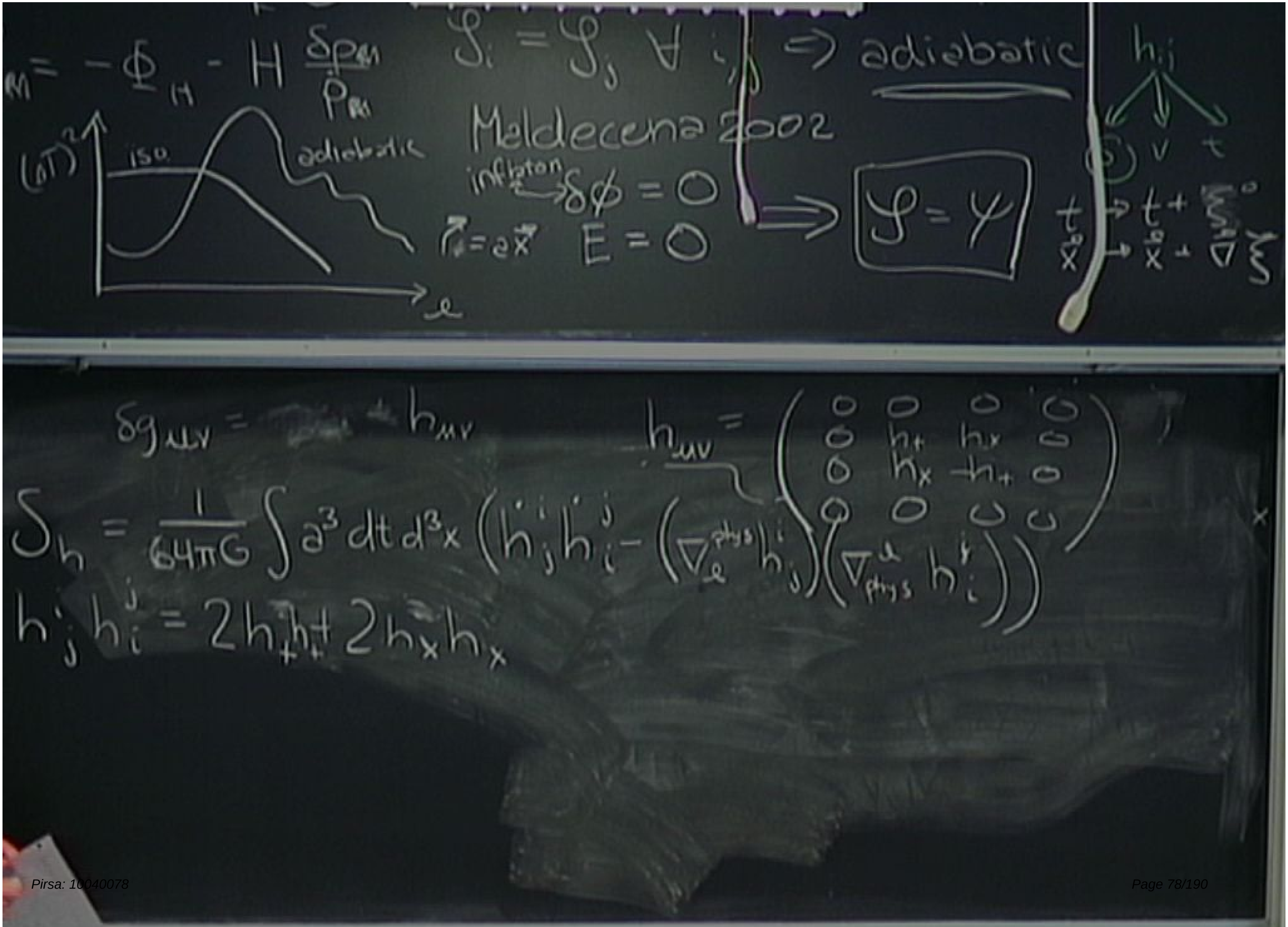

adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic
h_ij
δg_μν = h_μν
h_μν =
h+ h×
h× -h+
S_h = 1/64πG ∫ a³ dt d³x
phys

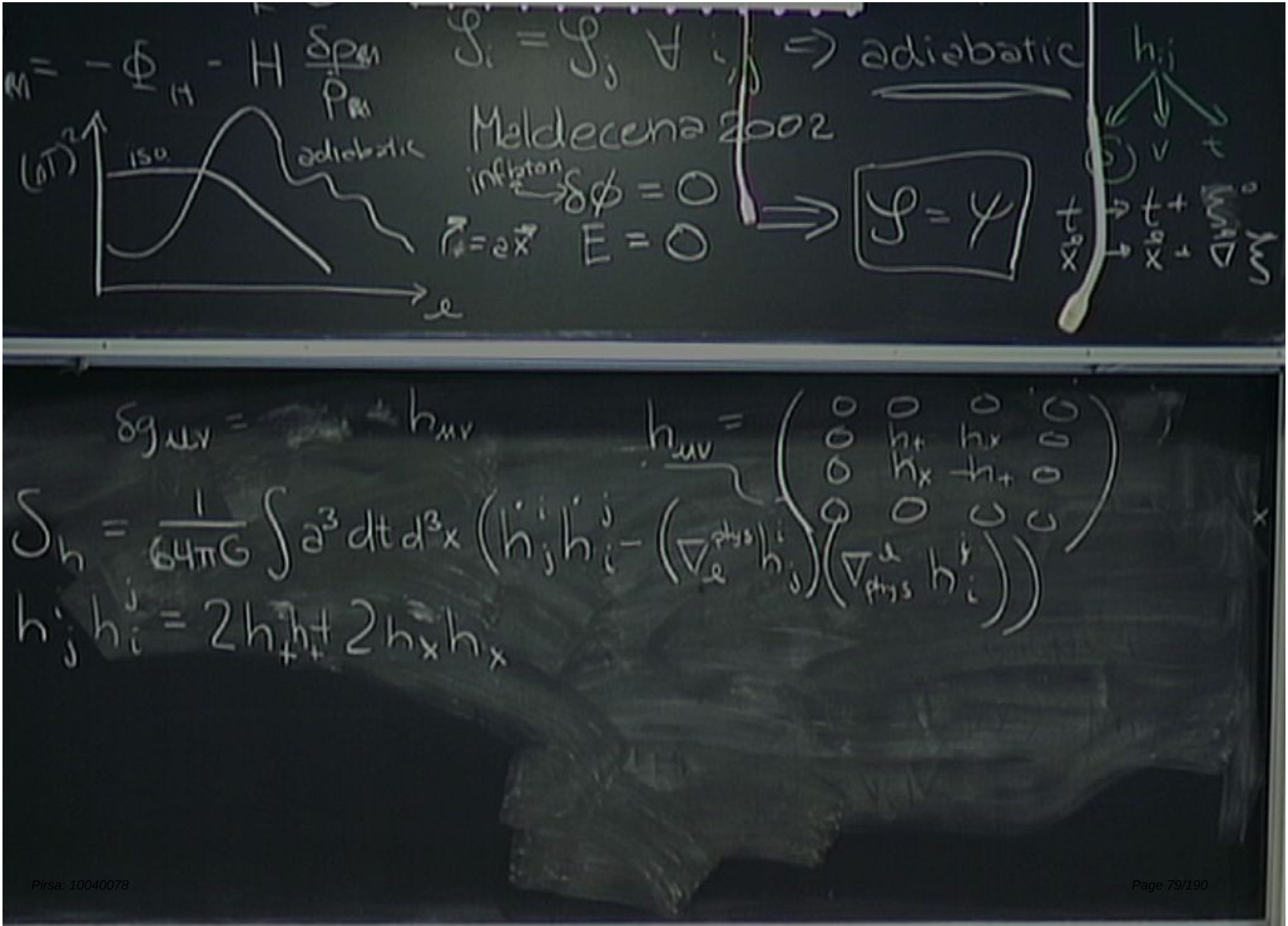

adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
150
adiabatic
δg_μν =
h_μν
h_μν =
S_h =
dt d³x
phys


adiabatic
Maldecena 2002
inflaton
E = 0
150
adiabatic
h_ij
t

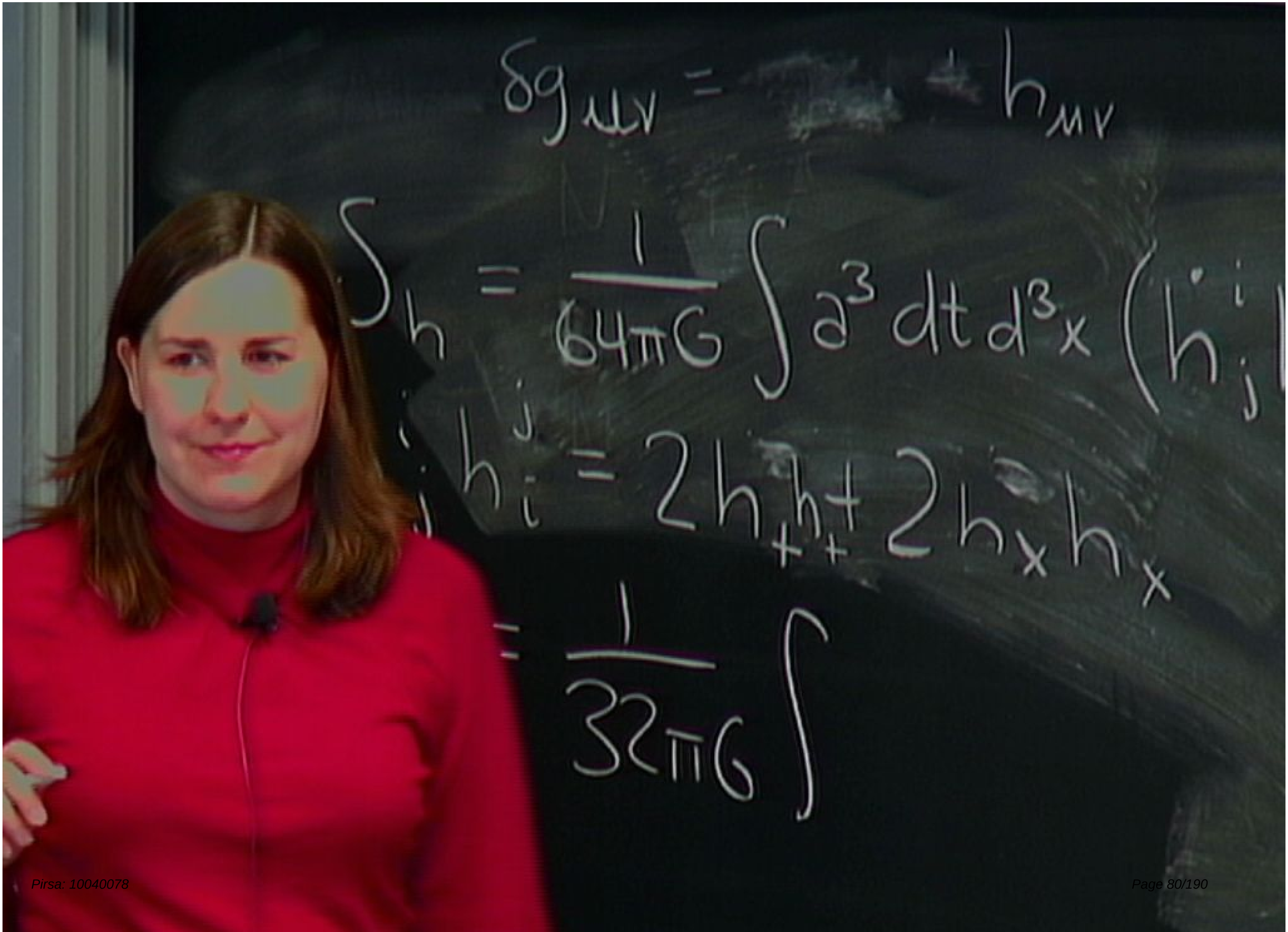

$\mathcal{S}_i = \mathcal{S}_j \ \forall\, i,j \Rightarrow$ adiabatic
Maldecena 2002
inflaton $\delta\phi = 0$
$E = 0$
$\mathcal{S} = \psi$
iso
adiabatic
$\delta g_{\mu\nu} = h_{\mu\nu}$
$S_h = \frac{1}{64\pi G}\int a^3\, dt\, d^3x \left(\dot h^i_j \dot h^j_i - (\nabla^{phys}_\ell h^i_j)(\nabla^\ell_{phys} h^j_i)\right)$
$\dot h^i_j \dot h^j_i = 2\dot h_+^2 + 2\dot h_\times^2$

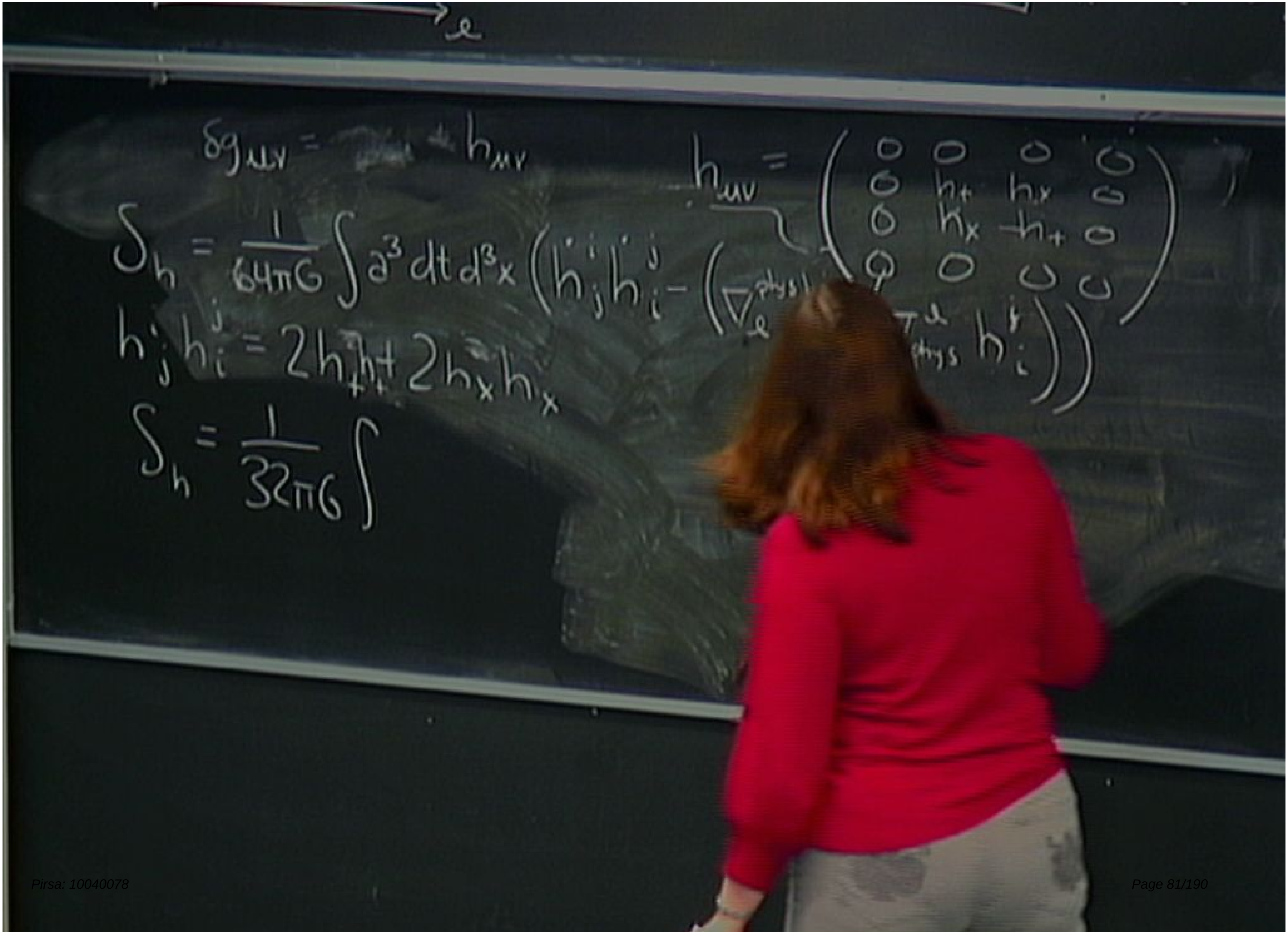

adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic
h_{ij}
$\delta g_{\mu\nu} = h_{\mu\nu}$
$S_h = \frac{1}{64\pi G}\int a^3\,dt\,d^3x\,(\dot h^i_j \dot h^j_i - (\nabla^{phys}_l h^i_j)(\nabla^l_{phys} h^j_i))$
$h^i_j h^j_i = 2h_+h_+ + 2h_\times h_\times$

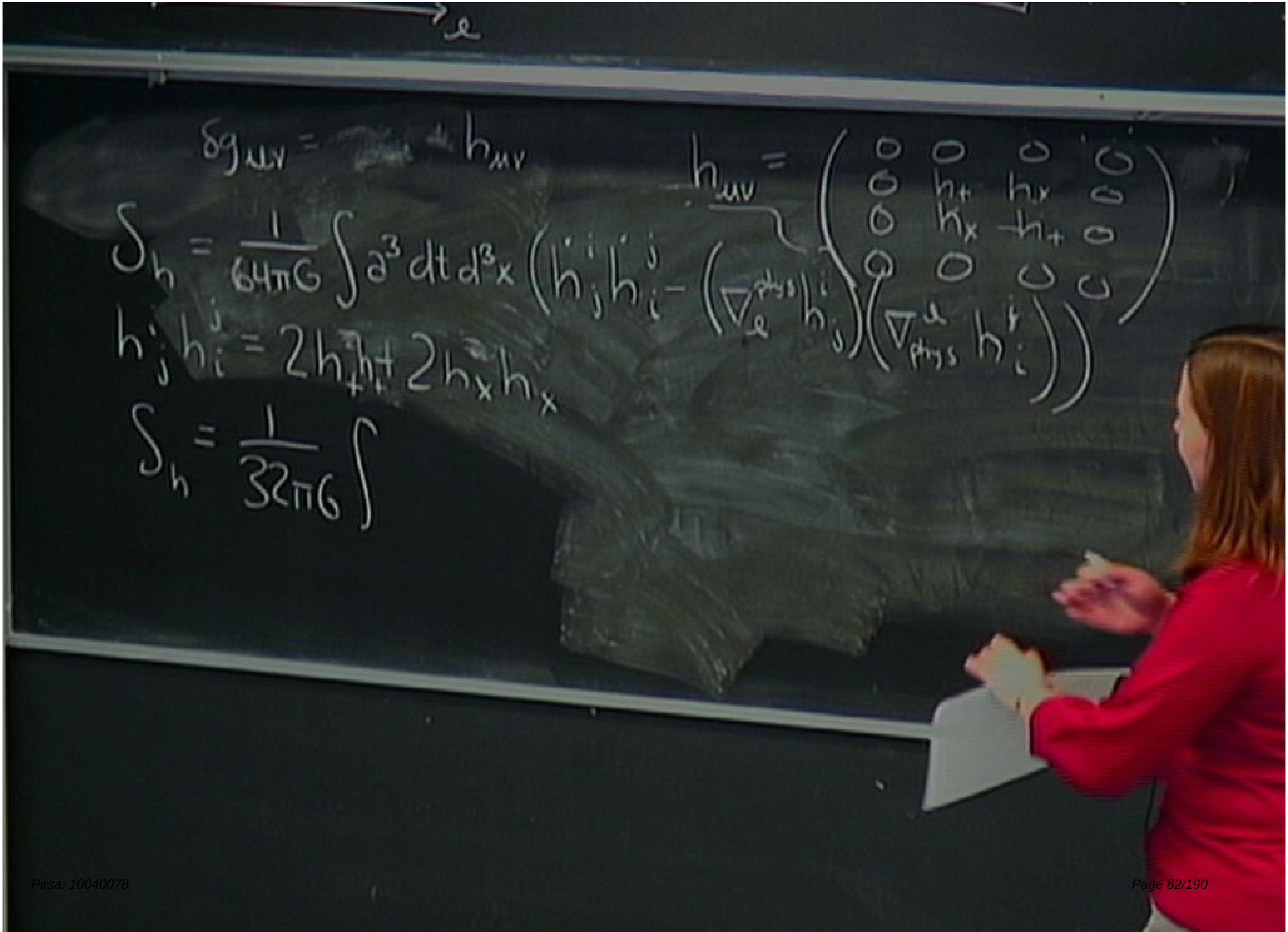

= − Φ_H − H δρ/ρ̇
δ_i = δ_j ∀ i,j ⇒ adiabatic
h_ij
Maldacena 2002
iso
adiabatic
inflaton → δφ = 0
r = a x
E = 0
δ = ψ
t → t + ξ
x → x + ∇ξ
δg_μν = h_μν
h_μν = (0 0 0 0 / 0 h+ h× 0 / 0 h× −h+ 0 / 0 0 0 0)
S_h = 1/(64πG) ∫ a³ dt d³x (ḣ^i_j ḣ^j_i − (∇^phys_ℓ h^i_j)(∇^ℓ_phys h^j_i))
h^i_j h^j_i = 2h+h+ + 2h×h×

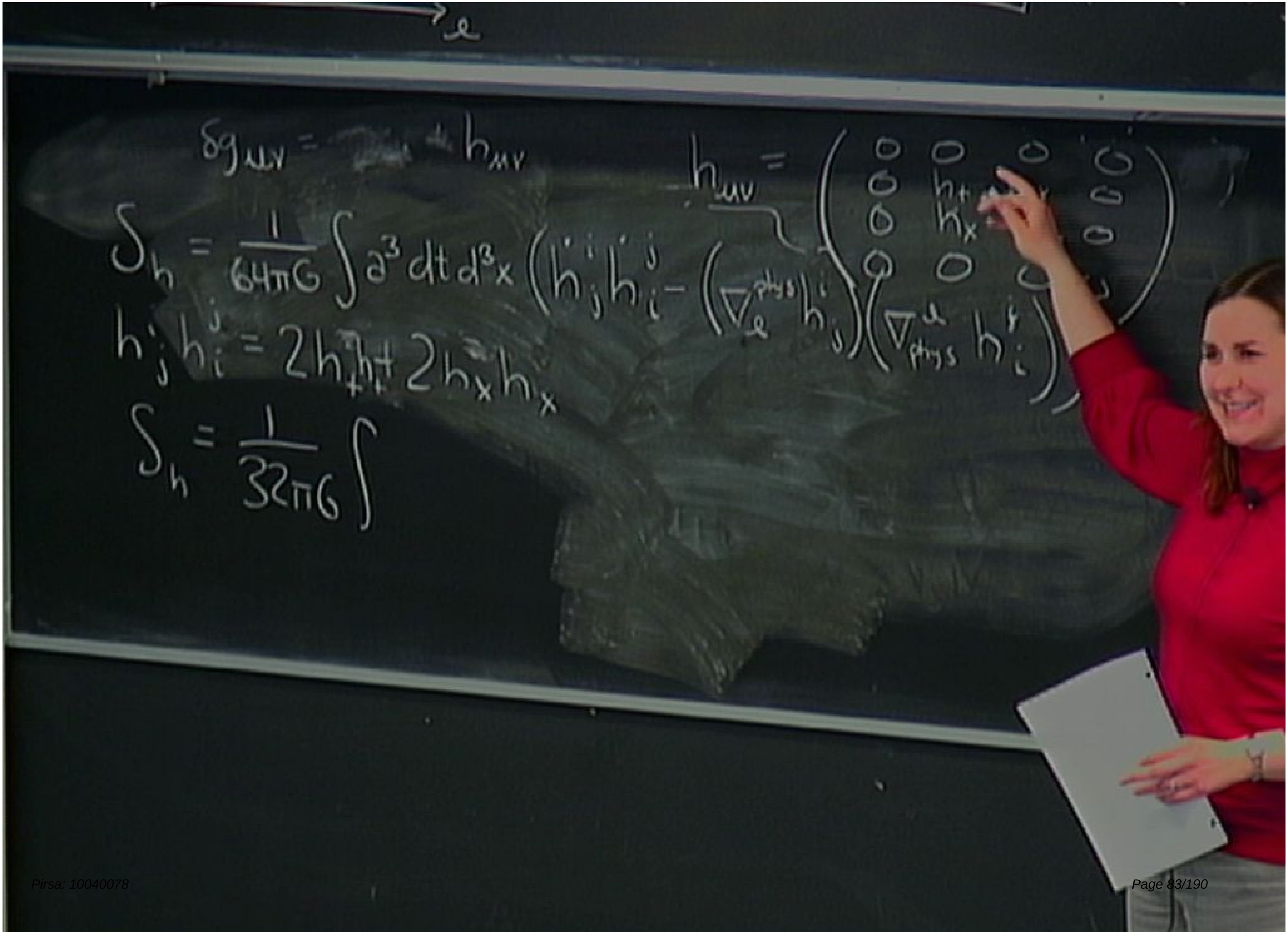


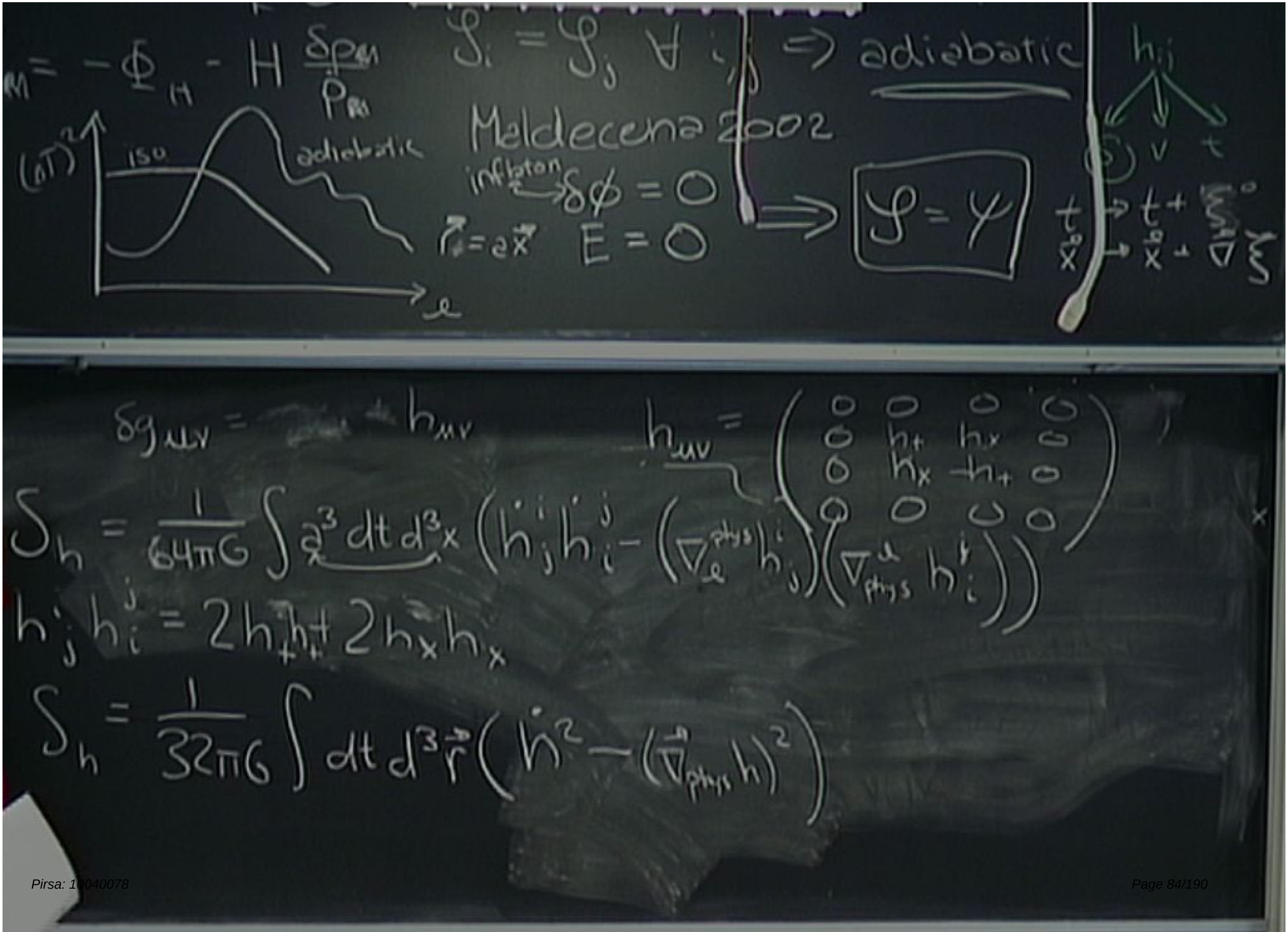
$-\Phi_H - H \frac{\delta\rho_M}{\dot\rho_M}$
$\zeta_i = \zeta_j \ \forall\ i,j \Rightarrow$ adiabatic
h_{ij}
S V T
$t \to t + \xi^0$
$\vec x \to \vec x + \nabla\xi$
$(\delta T)^2$
iso
adiabatic
Maldecena 2002
inflaton $\to \delta\phi = 0$
$\vec r = a\vec x$ $E = 0$
$\Rightarrow \zeta = \psi$
$\delta g_{\mu\nu} = h_{\mu\nu}$
$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$S_h = \frac{1}{64\pi G}\int a^3\, dt\, d^3x \left(\dot h^i_j \dot h^j_i - (\nabla^{phys}_l h^i_j)(\nabla^l_{phys} h^j_i)\right)$
$h^i_j h^j_i = 2h_+h_+ + 2h_\times h_\times$

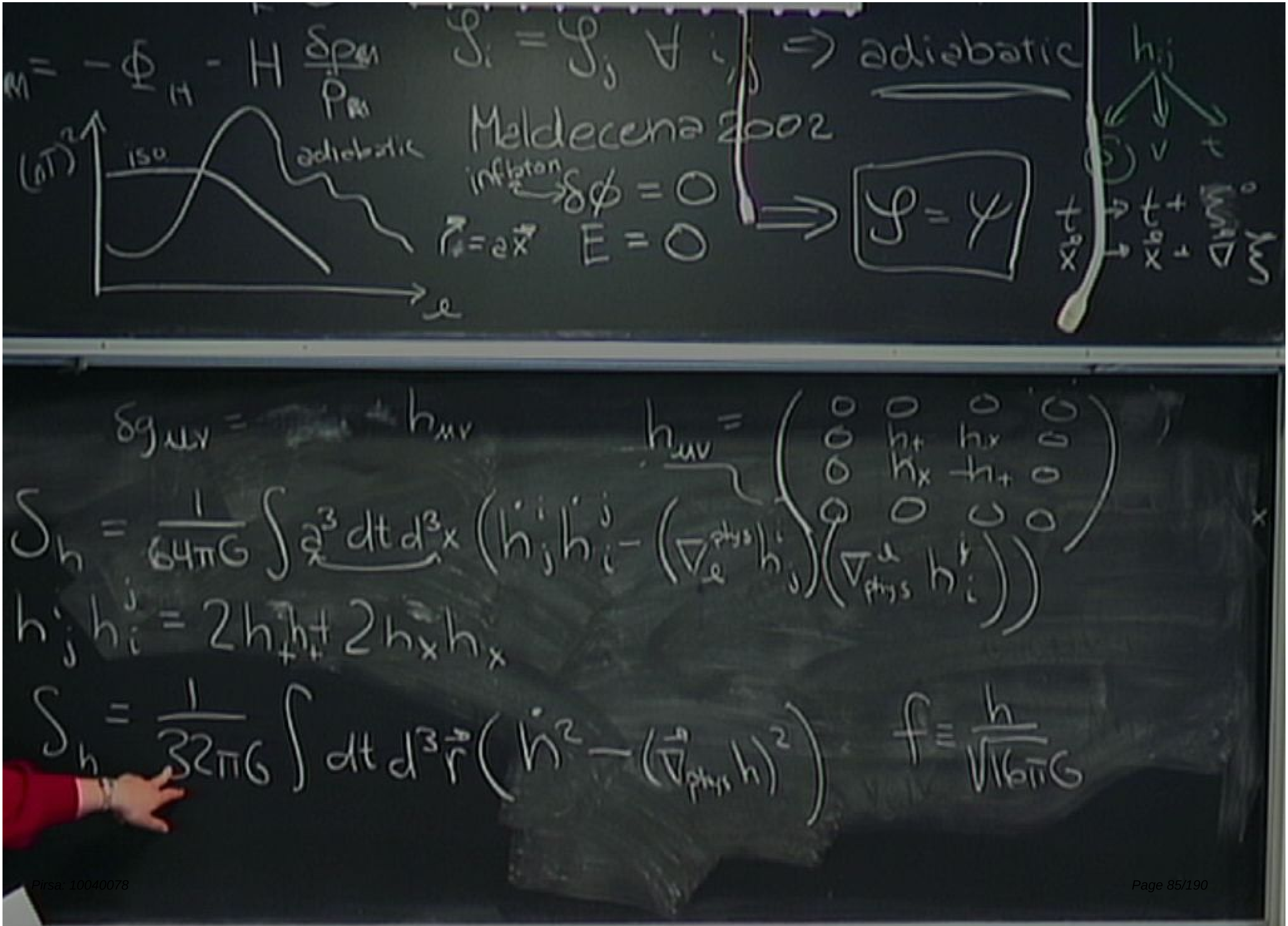

$\delta g_{\mu\nu} = h_{\mu\nu}$
$S_h = \frac{1}{64\pi G} \int a^3 \, dt \, d^3x \, (\dot{h}^i_j$
$h^j_i = 2h_+h_+ + 2h_\times h_\times$
$= \frac{1}{32\pi G} \int$

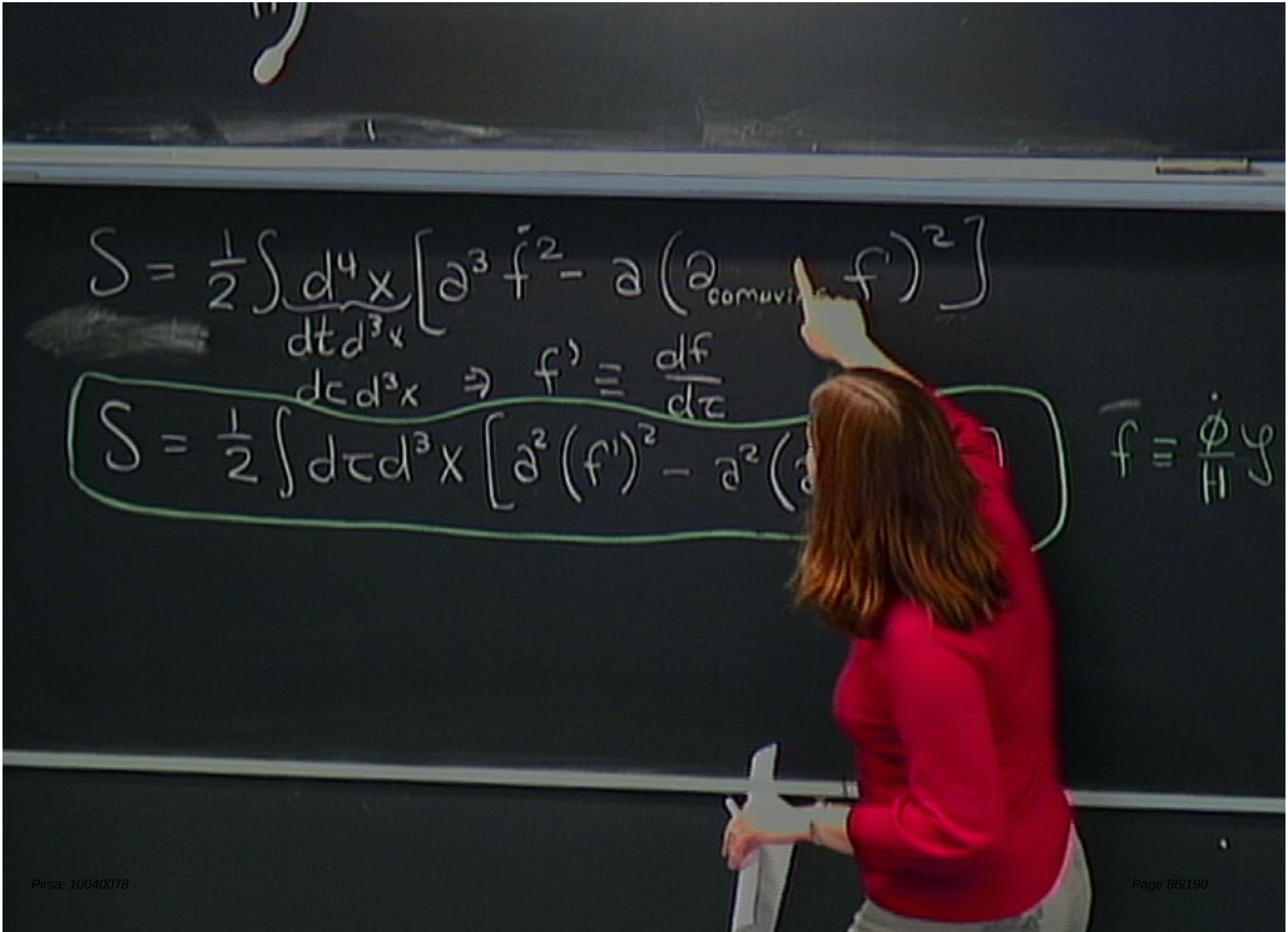

δg_μν = h_μν
h_μν = (0 0 0 0 / 0 h+ hx 0 / 0 hx −h+ 0 / 0 0 0 0)
S_h = 1/(64πG) ∫ a³ dt d³x (ḣ^i_j ḣ^j_i − (∇_l^phys
∇^l_phys h^j_i))
ḣ^i_j ḣ^j_i = 2ḣ+ḣ+ + 2ḣxḣx
S_h = 1/(32πG) ∫


$\delta g_{\mu\nu} = \quad h_{\mu\nu}$
$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$S_h = \frac{1}{64\pi G} \int a^3 \, dt \, d^3x \left(\dot{h}^i_j \dot{h}^j_i - \left(\nabla_e^{phys} h^i_j\right)\left(\nabla^e_{phys} h^j_i\right) \right)$
$\dot{h}^i_j \dot{h}^j_i = 2\dot{h}_+\dot{h}_+ + 2\dot{h}_\times \dot{h}_\times$
$S_h = \frac{1}{32\pi G} \int$


δg_μν = h_μν
h_μν = (0 0 0 0 / 0 h+ h× 0 / 0 h× −h+ 0 / 0 0 0 0)
S_h = 1/(64πG) ∫ a³ dt d³x (h'^i_j h'^j_i − (∇^phys_ℓ h^i_j)(∇^ℓ_phys h^j_i))
h'^i_j h'^j_i = 2h'+h'+ + 2h'×h'×
S_h = 1/(32πG) ∫


adiabatic
Maldecena 2002
iso
adiabatic
inflaton
δφ = 0
E = 0
h_{ij}
$t \to t + \xi^0$
$\delta g_{\mu\nu} = h_{\mu\nu}$
$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$S_h = \frac{1}{64\pi G}\int a^3\, dt\, d^3x \left(\dot h^i_j \dot h^j_i - (\nabla^{phys}_\ell h^i_j)(\nabla^\ell_{phys} h^j_i)\right)$
$h^i_j h^j_i = 2h_+h_+ + 2h_\times h_\times$
$S_h = \frac{1}{32\pi G}\int dt\, d^3\vec r \left(\dot h^2 - (\vec\nabla_{phys} h)^2\right)$

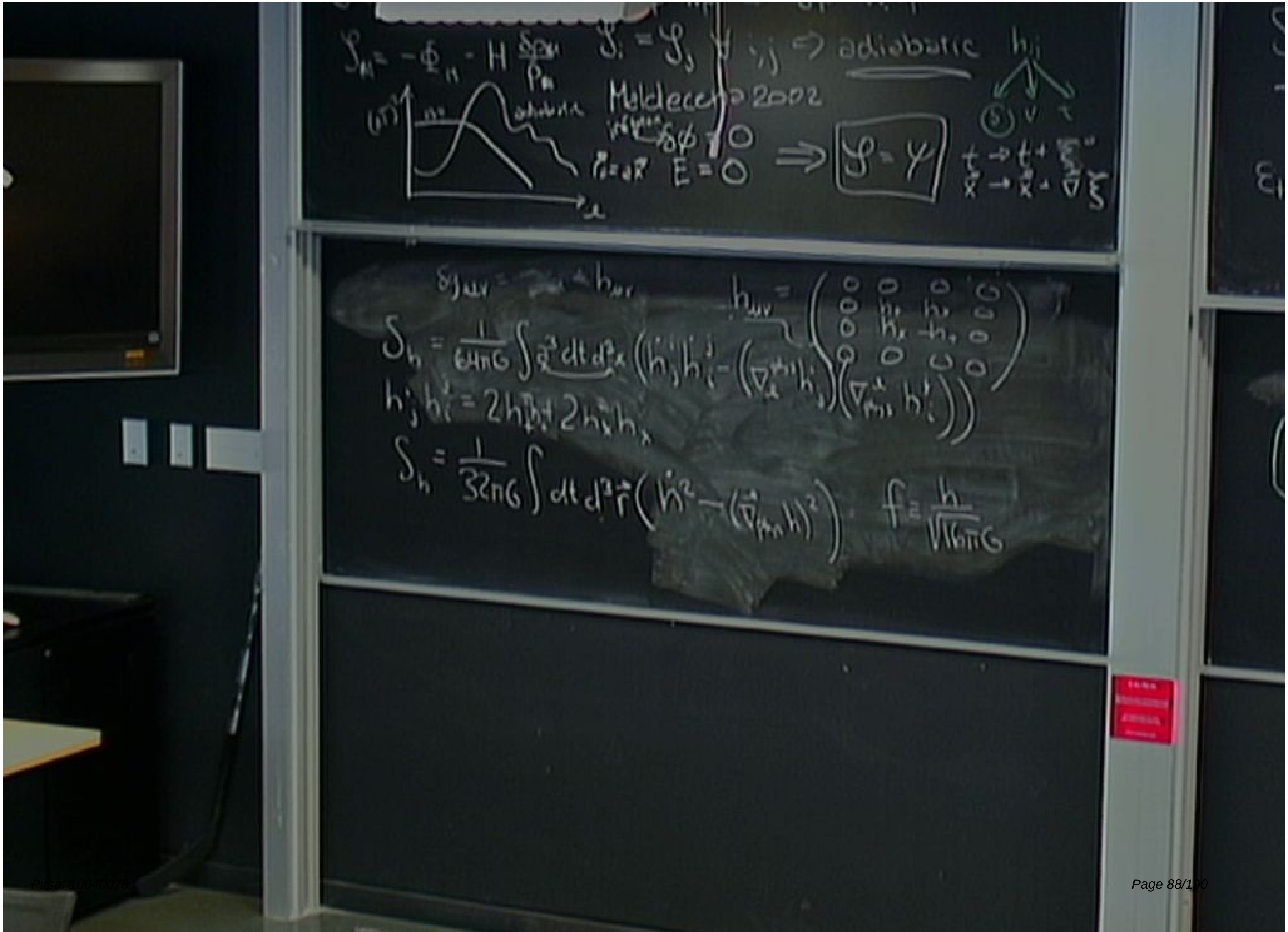

Maldecena 2002
adiabatic
iso
adiabatic
inflaton
$\delta\phi = 0$
$E = 0$
$\varsigma = \psi$
$\delta g_{\mu\nu} = h_{\mu\nu}$
$S_h = \frac{1}{64\pi G}\int a^3\, dt\, d^3x \left(\dot h^i_j \dot h^j_i - (\nabla^{phys}_l h^i_j)(\nabla^l_{phys} h^j_i)\right)$
$h^i_j h^j_i = 2h_+h_+ + 2h_\times h_\times$
$S_h = \frac{1}{32\pi G}\int dt\, d^3\vec r \left(\dot h^2 - (\vec\nabla_{phys} h)^2\right)$
$f = \frac{h}{\sqrt{16\pi G}}$

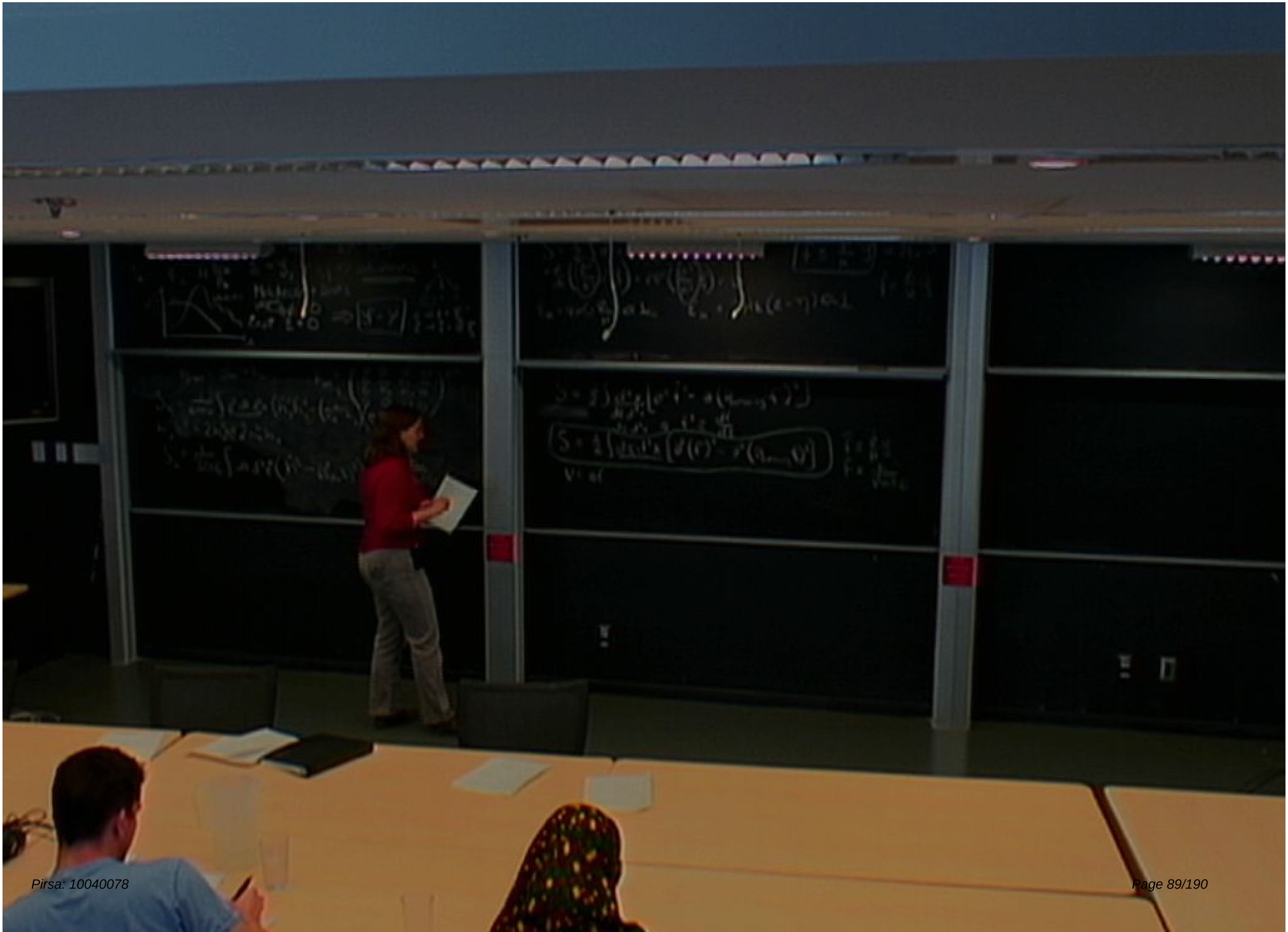


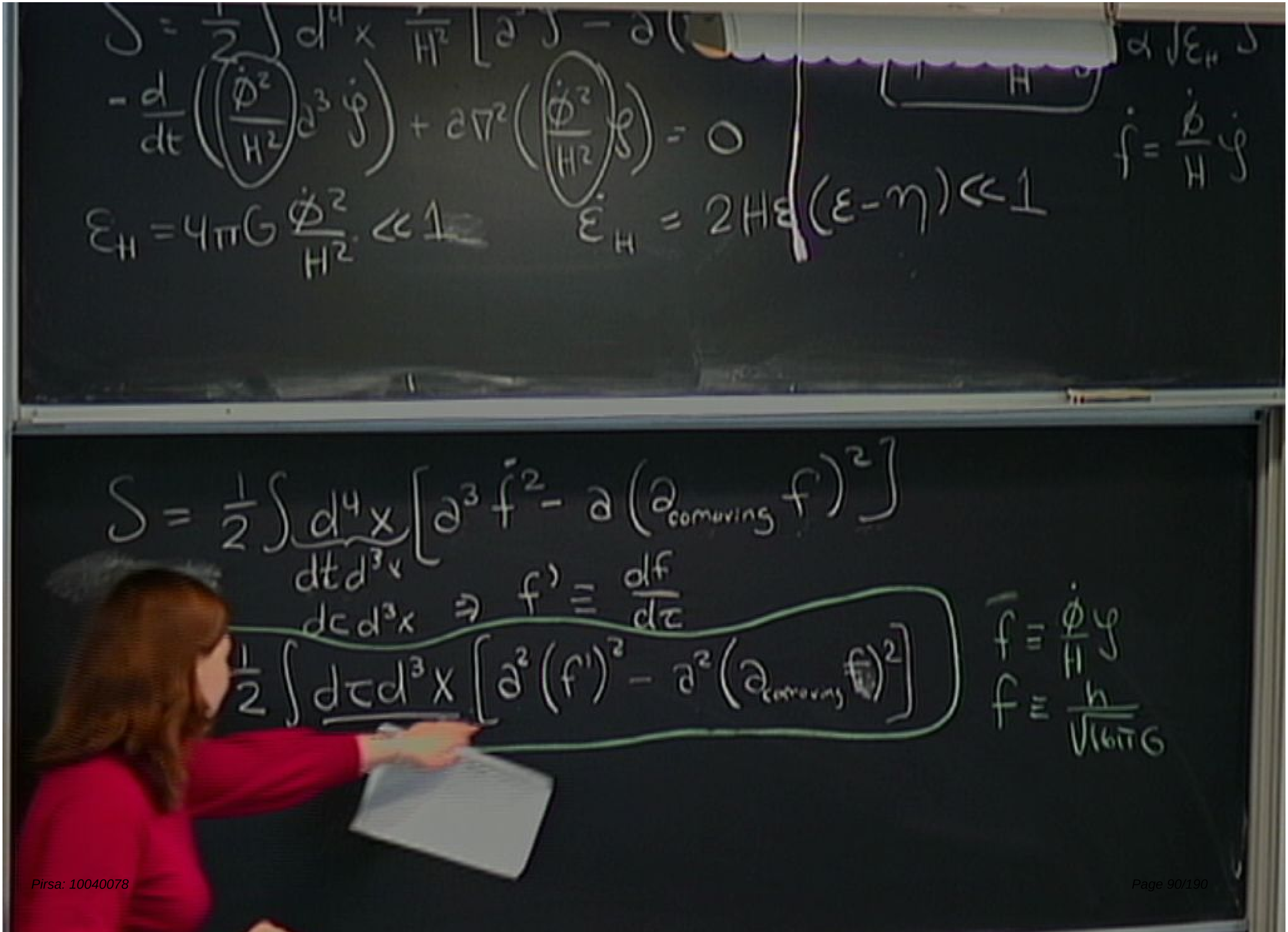
S = 1/2 ∫ d⁴x [a³ ḟ² − a (∂comoving f)²]
dt d³x
dτ d³x ⇒ f' = df/dτ
S = 1/2 ∫ dτ d³x [a² (f')² − a² (
f = φ̇/H

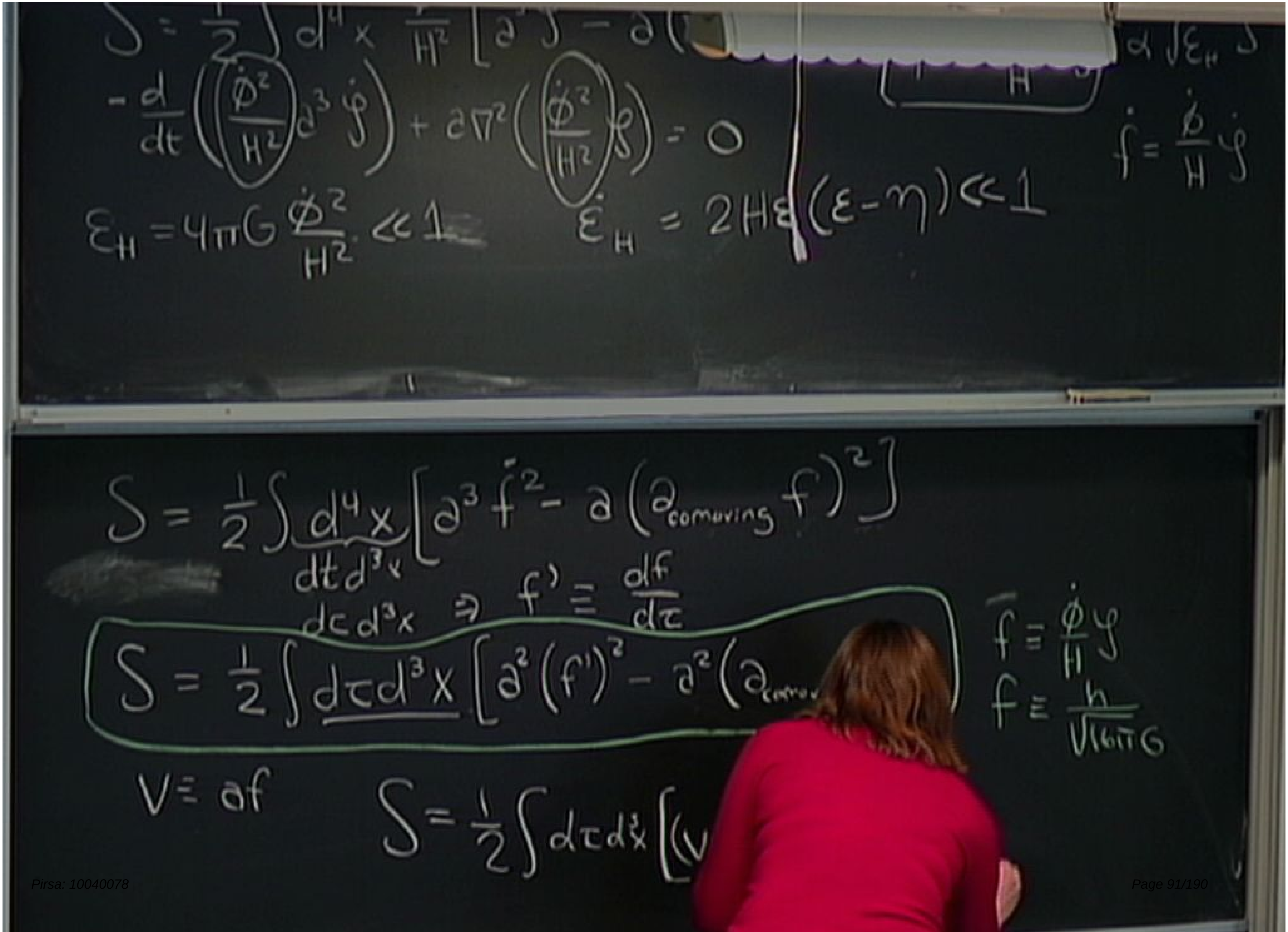
$$f \equiv \frac{\dot{\phi}}{H}\varphi$$

$$f \equiv \frac{h}{\sqrt{16\pi G}}$$


adiabatic
Maldacena 2002
E = 0




- d/dt ((φ̇²/H²) a³ ϱ̇) + a∇²((φ̇²/H²) ϱ) = 0
ε_H = 4πG φ̇²/H² ≪ 1
ε̇_H = 2Hε(ε - η) ≪ 1
f = φ̇/H ϱ
S = 1/2 ∫ d⁴x [a³ ḟ² - a (∂_comoving f)²]
dt d³x
dτ d³x ⇒ f' ≡ df/dτ
1/2 ∫ dτ d³x [a² (f')² - a² (∂_comoving f)²]
f = φ̇/H ϱ
f ≡ h/√16πG


$-\frac{d}{dt}\left(\left(\frac{\dot\phi^2}{H^2}\right)a^3\dot{\mathcal{R}}\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\mathcal{R}\right) = 0$
$\varepsilon_H = 4\pi G\frac{\dot\phi^2}{H^2} \ll 1$
$\dot\varepsilon_H = 2H\varepsilon(\varepsilon-\eta) \ll 1$
$f = \frac{\dot\phi}{H}\mathcal{R}$
$S = \frac{1}{2}\int d^4x\left[a^3\dot f^2 - a(\partial_{comoving} f)^2\right]$
$dt\,d^3x$
$d\tau\,d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$
$S = \frac{1}{2}\int d\tau d^3x\left[a^2(f')^2 - a^2(\partial_{comov}$
$f = \frac{\dot\phi}{H}\mathcal{R}$
$f = \frac{h}{\sqrt{16\pi G}}$
$V \equiv af$
$S = \frac{1}{2}\int d\tau d^3x\left[(V\right.$

$$S = \frac{1}{2}\int d^4x \, \frac{\dot{\phi}^2}{H^2}\left[a^3 \dot{\varrho}^2 - a(\partial\varrho)^2\right]$$

$$-\frac{d}{dt}\left(\frac{\dot{\phi}^2}{H^2} a^3 \dot{\varrho}\right) + a\nabla^2\left(\frac{\dot{\phi}^2}{H^2}\varrho\right) = 0$$

$$\propto \sqrt{\varepsilon_H}\,\varrho \qquad f = \frac{\dot{\phi}}{H}\varrho$$

$$\varepsilon_H = 4\pi G \frac{\dot{\phi}^2}{H^2} \ll 1 \qquad \dot{\varepsilon}_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3 \dot{f}^2 - a\left(\partial_{\text{comoving}} f\right)^2\right]$$

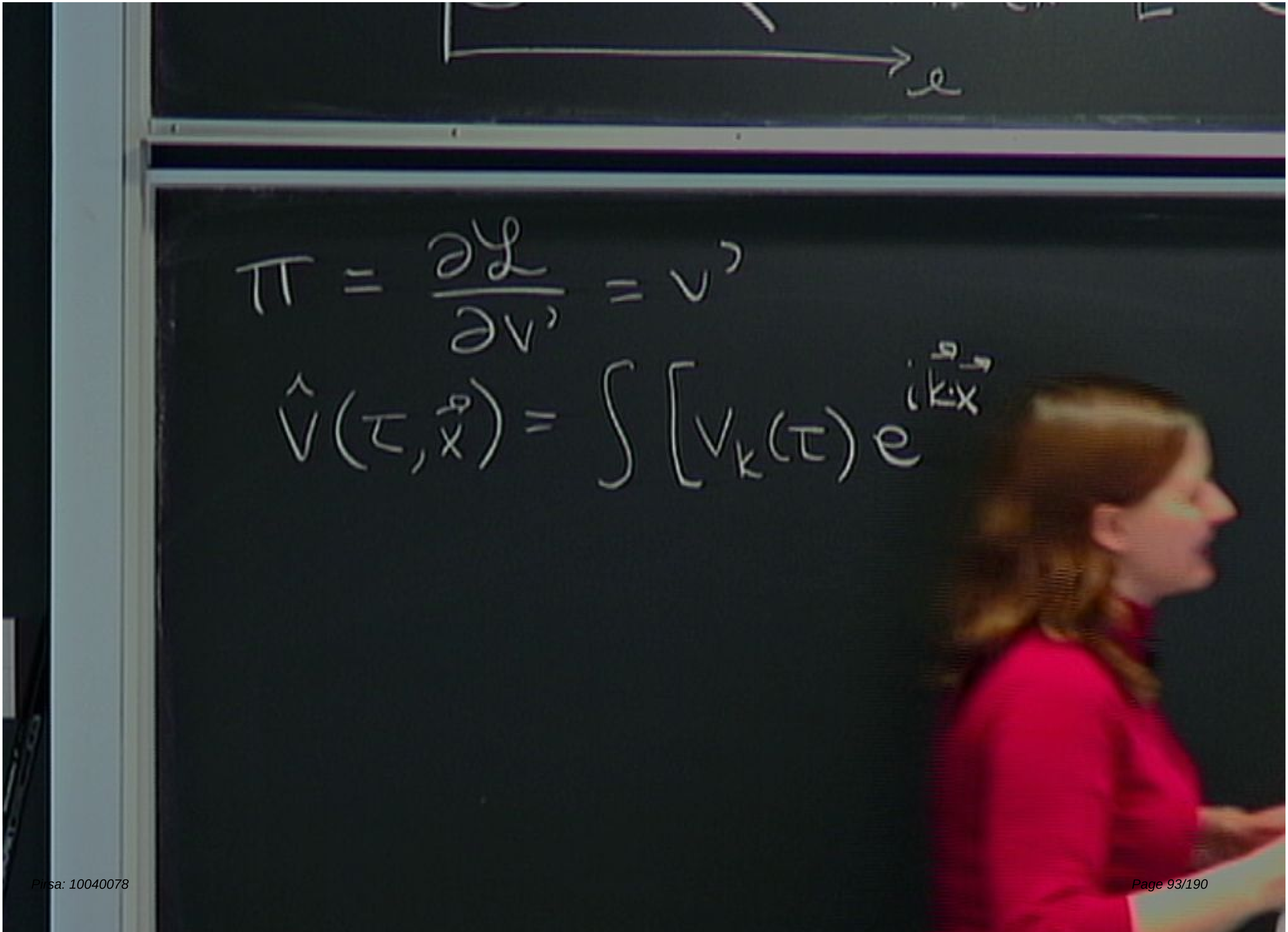
$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2\left(\partial_{\text{comoving}} f\right)^2\right]}$$

$$f = \frac{\dot{\phi}}{H}\varrho \qquad f \equiv \frac{h}{\sqrt{16\pi G}}$$

$$V \equiv af$$

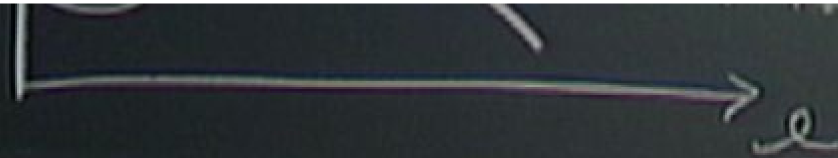
$$S = \frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2\right]$$



$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$
$\hat{v}(\tau, \vec{x}) = \int [v_k(\tau) e^{i \vec{k} \cdot \vec{x}}$

$$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$$

$$\hat{v}(\tau, \vec{x}) = \int \Big[v_k(\tau) e^{i\vec{k}\cdot\vec{x}}$$



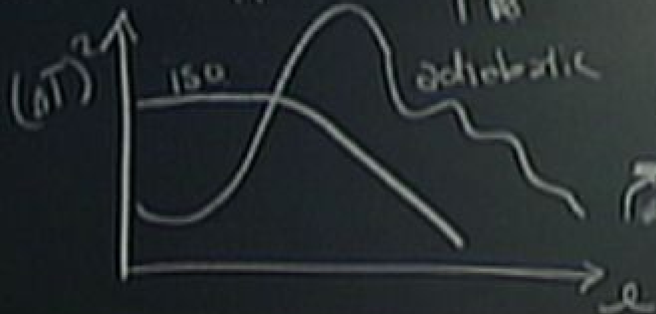
$$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$$

$$\hat{v}(\tau, \vec{x}) = \int \left[v_k(\tau) e^{i\vec{k}\cdot\vec{x}} a_k + \right.$$

$\zeta = -\Phi_H$ $\quad k \; (v_B)$ $\quad w_i \delta p$

$$\zeta_M = -\Phi_H - H \frac{\delta \rho_M}{\dot{\rho}_M}$$

$$\zeta_i = \zeta_j \;\; \forall\, i,j \Rightarrow \text{adiabatic}$$



Maldacena 2002

inflaton $\delta\phi = 0$

$\vec{r} = a\vec{x}$ $\quad E = 0$ $\Longrightarrow$ $\boxed{\zeta = \psi}$

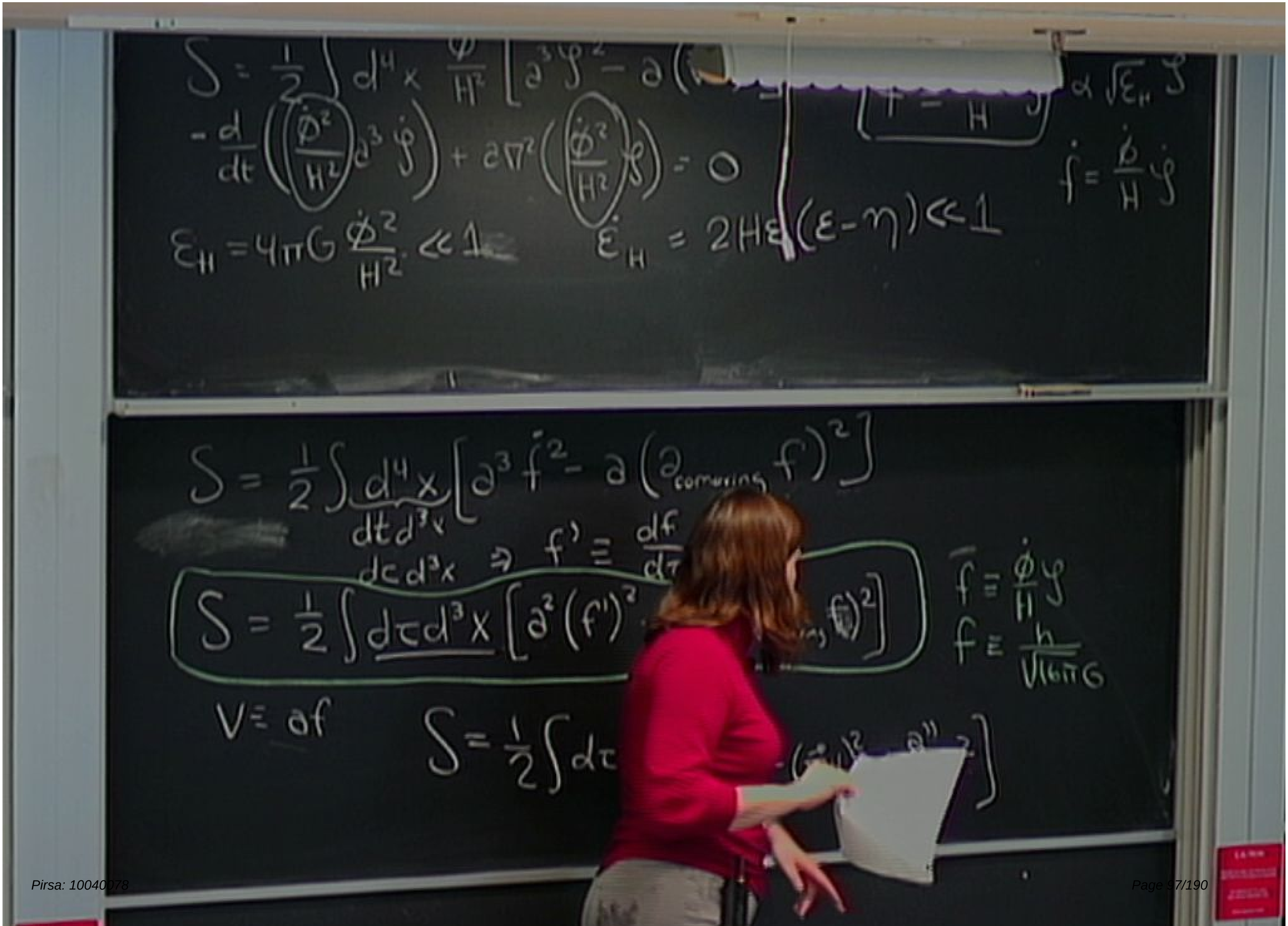
$h_{ij} \to$ S, V, T

$t \to t + \xi^0$

$\vec{x} \to \vec{x} + \nabla \xi$

$$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$$

$$\hat{v}(\tau, \vec{x}) = \int \left[v_k(\tau) e^{i\vec{k}\cdot\vec{x}} a_k + v_k^* e^{-i\vec{k}\cdot\vec{x}} a_k^\dagger \right] \frac{d^3k}{(2\pi)^3}$$


$S = \frac{1}{2}\int d^4x \frac{\dot\phi^2}{H^2}\left[a^3\dot{\zeta}^2 - a(\ldots\right.$
$\propto \sqrt{\varepsilon_H}\,\zeta$
$-\frac{d}{dt}\left(\left(\frac{\dot\phi^2}{H^2}\right)a^3\dot\zeta\right) + a\nabla^2\left(\left(\frac{\dot\phi^2}{H^2}\right)\zeta\right) = 0$
$\dot f = \frac{\dot\phi}{H}\dot\zeta$
$\varepsilon_H = 4\pi G\frac{\dot\phi^2}{H^2} \ll 1$
$\dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$
$S = \frac{1}{2}\int \frac{d^4x}{dt\,d^3x}\left[a^3\dot f^2 - a(\partial_{comoving} f)^2\right]$
$d\tau\, d^3x \Rightarrow f' = \frac{df}{d\tau}$
$S = \frac{1}{2}\int d\tau\, d^3x\left[a^2(f')^2 \ldots\right]$
$f = \frac{\dot\phi}{H}\zeta$
$f = \frac{h}{\sqrt{16\pi G}}$
$V = af$
$S = \frac{1}{2}\int d\tau \ldots$

$$S = \frac{1}{2}\int d^4x \, \frac{\dot\phi^2}{H^2}\left[a^3\dot\zeta^2 - a(\ldots\right.$$

$$-\frac{d}{dt}\left(\frac{\dot\phi^2}{H^2} a^3 \dot\zeta\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\zeta\right) = 0$$

$$\varepsilon_H = 4\pi G \frac{\dot\phi^2}{H^2} \ll 1 \qquad \dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$

$$\boxed{f = \frac{\dot\phi}{H}\zeta} \propto \sqrt{\varepsilon_H}\,\zeta$$

$$\dot f = \frac{\dot\phi}{H}\dot\zeta$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3 \dot f^2 - a\left(\partial_{\text{comoving}} f\right)^2\right] \qquad \frac{a''}{a} = \frac{2}{\tau^2}$$

$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2\left(\partial_{\text{comoving}} f\right)^2\right]}$$

$$f = \frac{\dot\phi}{H}\zeta \qquad f = \frac{h}{\sqrt{16\pi G}}$$

$$v = af \qquad S = \frac{1}{2}\int d\tau\, d^3x\left[(v')^2 - (\vec\nabla v)^2 + \frac{a''}{a}v^2\right]$$

$$S = \frac{1}{2}\int d^4x \, \frac{\dot\phi}{H^2}\left[a^3 \dot{\varphi}^2 - a(\ldots\right.$$

$$-\frac{d}{dt}\left(\left(\frac{\dot\phi^2}{H^2}\right)a^3\dot{\varphi}\right) + a\nabla^2\left(\left(\frac{\dot\phi^2}{H^2}\right)\varphi\right) = 0$$

$$\alpha \sqrt{\varepsilon_H}\, \varphi$$

$$f = \frac{\dot\phi}{H}\varphi$$

$$\varepsilon_H = 4\pi G \frac{\dot\phi^2}{H^2} \ll 1 \qquad \dot\varepsilon_H = 2H\varepsilon(\varepsilon - \eta) \ll 1$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3 \dot f^2 - a\left(\partial_{\text{comoving}} f\right)^2\right] \qquad \frac{a''}{a} = \frac{2}{\tau^2}$$

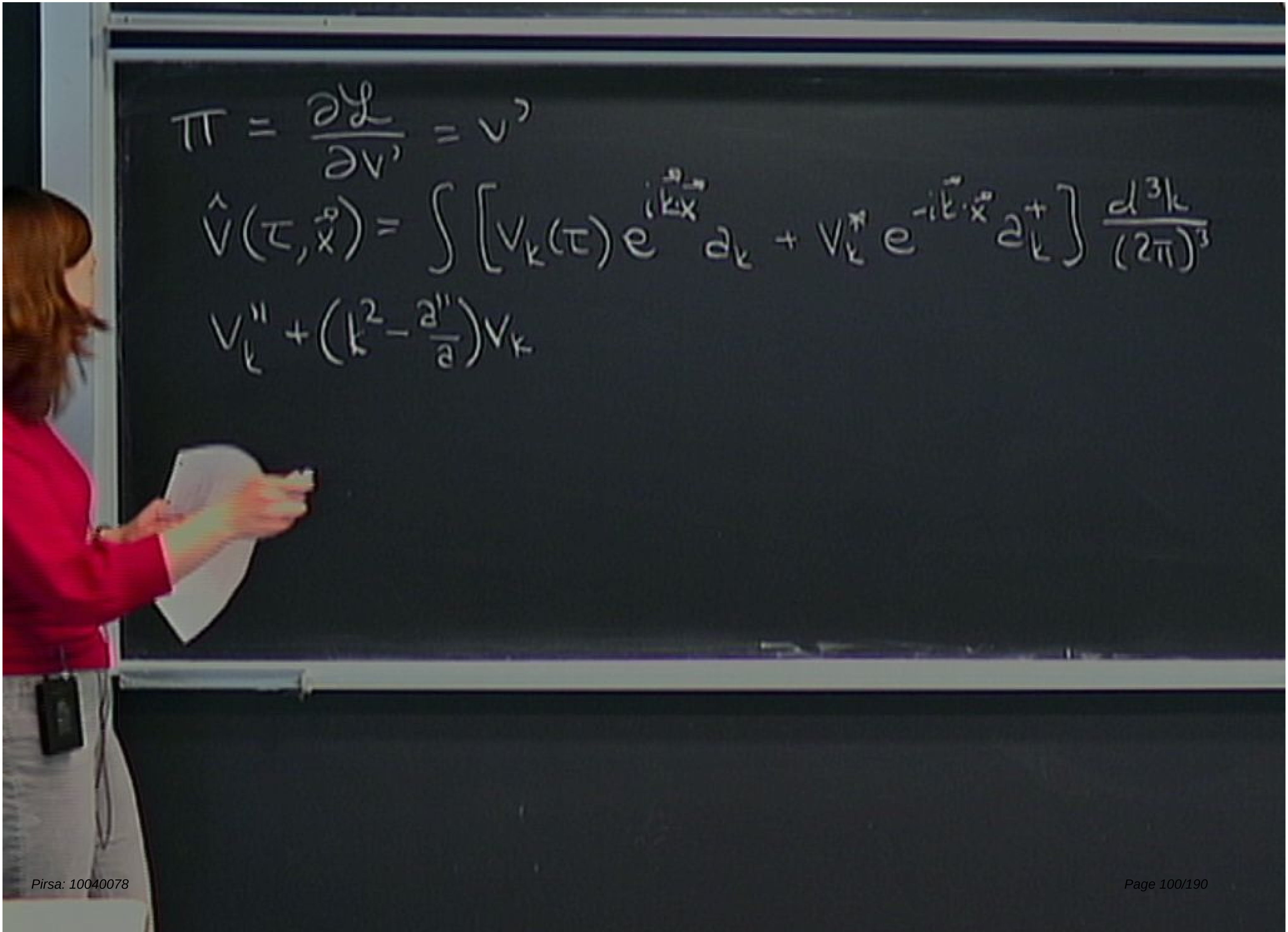
$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x\left[a^2 (f')^2 - a^2\left(\partial_{\text{comoving}} f\right)^2\right]}$$

$$f = \frac{\dot\phi}{H}\varphi \qquad f = \frac{h}{\sqrt{16\pi G}}$$

$$V \equiv af$$

$$S = \frac{1}{2}\int d\tau\, d^3x\left[(v')^2 - (\vec\nabla v)^2 + \frac{a''}{a}v^2\right]$$


$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$
$\hat{v}(\tau, \vec{x}) = \int \left[v_k(\tau) e^{i\vec{k}\cdot\vec{x}} a_k + v_k^* e^{-i\vec{k}\cdot\vec{x}} a_k^+ \right] \frac{d^3k}{(2\pi)^3}$
$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k$

$$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$$

$$\hat{v}(\tau, \vec{x}) = \int \left[v_k(\tau) e^{i\vec{k}\cdot\vec{x}} a_k + v_k^* e^{-i\vec{k}\cdot\vec{x}} a_k^\dagger \right] \frac{d^3k}{(2\pi)^3}$$

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$S = \frac{1}{2}\int d^4x\, \frac{\dot\phi^2}{H^2}\left[a^3\dot\varphi^2 - a(\vec\nabla\varphi)^2\right]$$

$$f \equiv \frac{\dot\phi}{H}\varphi \propto \sqrt{\varepsilon_H}\,\zeta$$

$$-\frac{d}{dt}\left(\frac{\dot\phi^2}{H^2}a^3\dot\varphi\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\varphi\right) = 0$$

$$\dot f = \frac{\dot\phi}{H}\dot\varphi$$

$$\varepsilon_H = 4\pi G\frac{\dot\phi^2}{H^2} \ll 1 \qquad \dot\varepsilon_H = 2H\varepsilon(\varepsilon-\eta) \ll 1$$

$$k\tau \gg 1$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x}\left[a^3 f^2 - a(\partial_{comoving} f)^2\right] \qquad \frac{a''}{a} = \frac{2}{\tau^2}$$

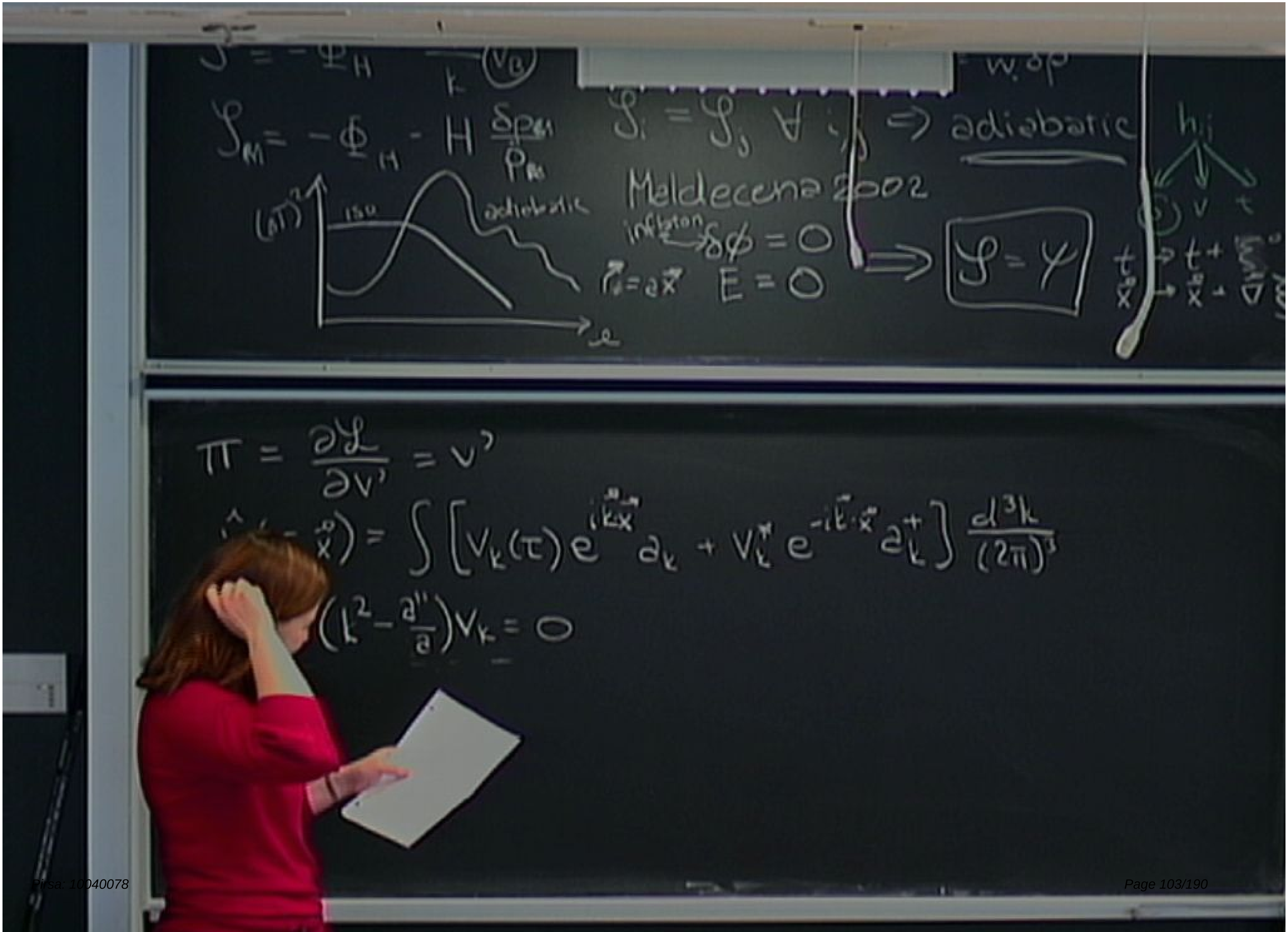
$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

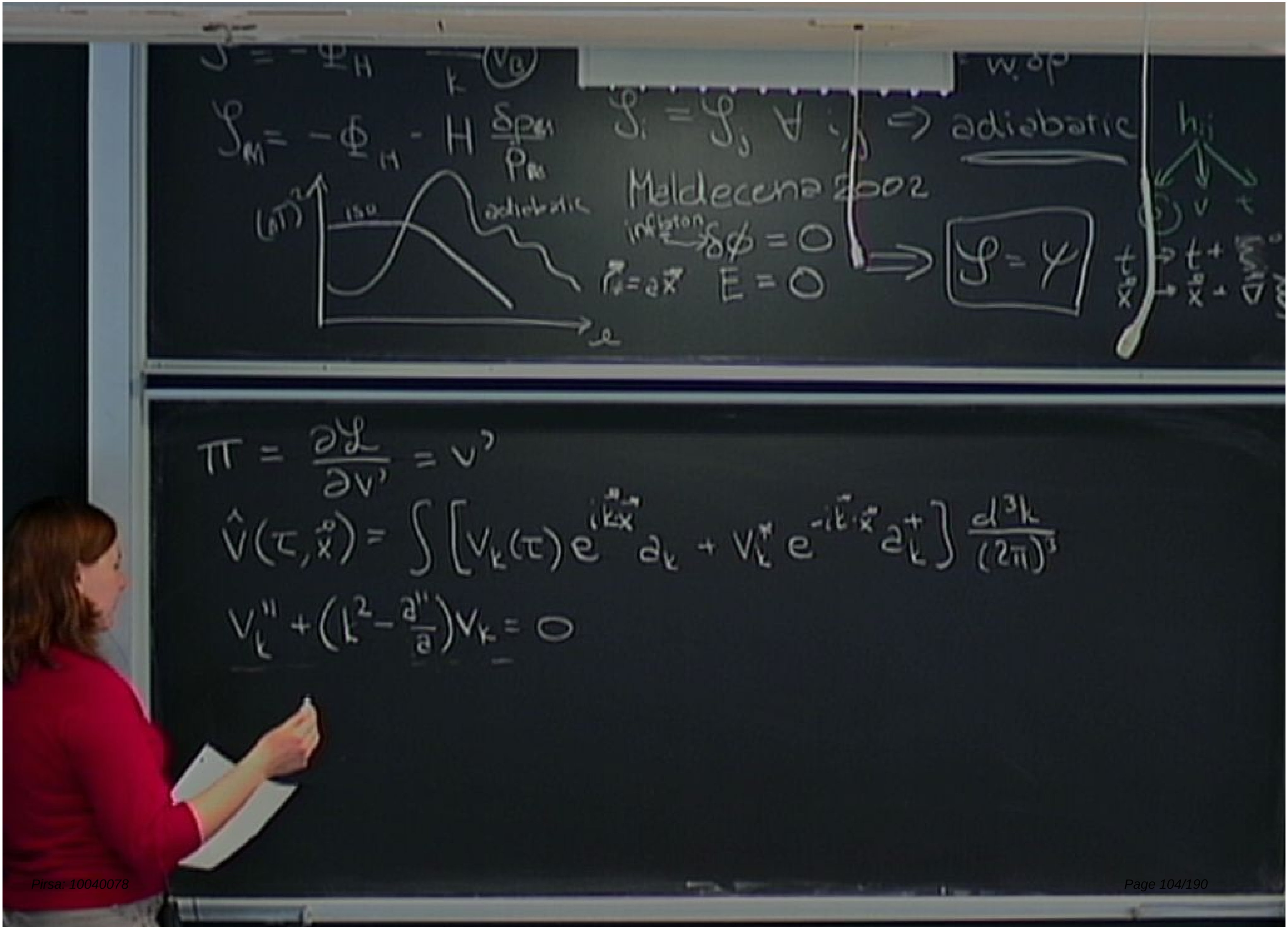
$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x\left[a^2(f')^2 - a^2(\partial_{comoving} f)^2\right]}$$

$$f = \frac{\dot\phi}{H}\varphi \qquad f \equiv \frac{h}{\sqrt{16\pi G}}$$

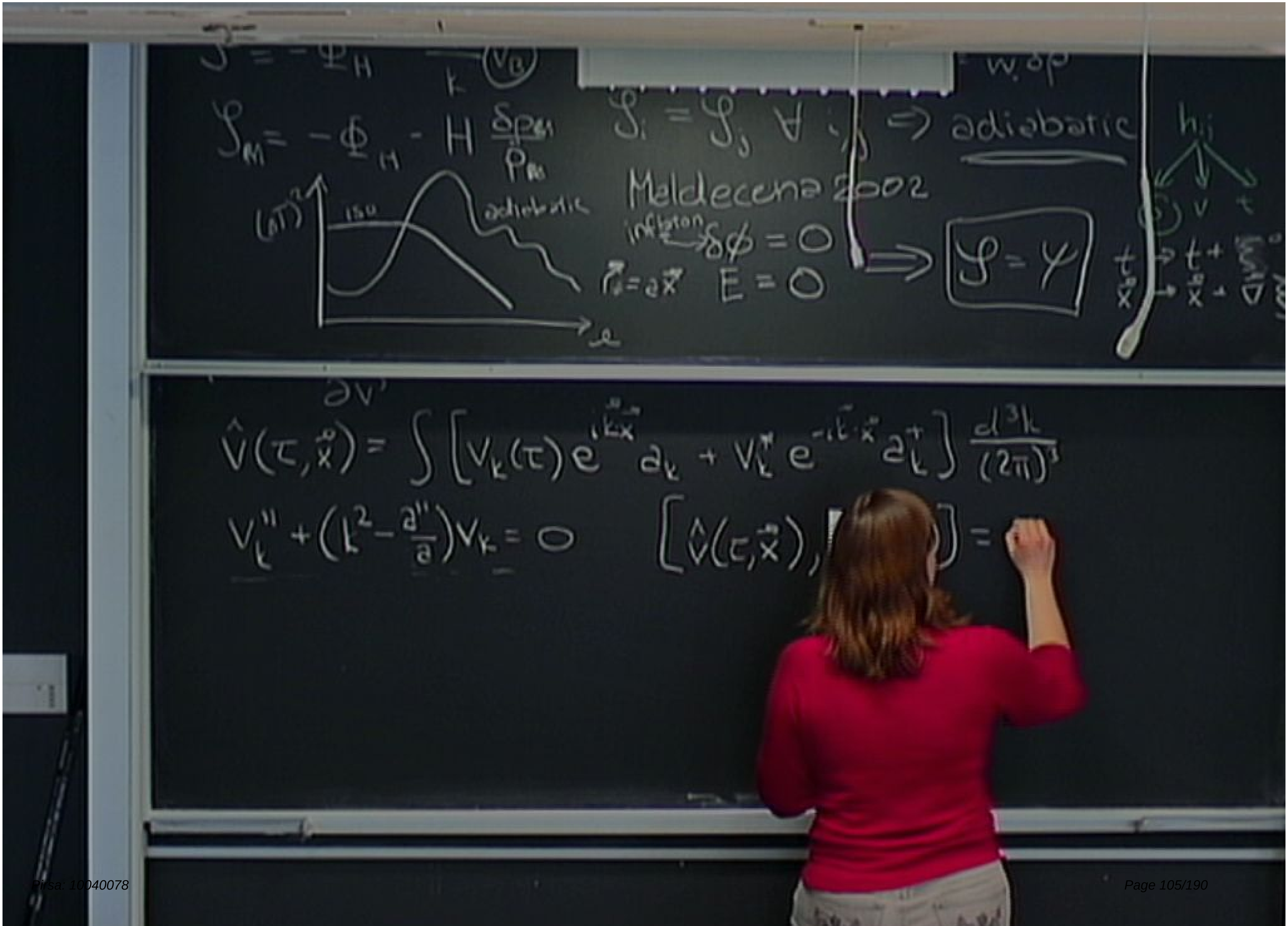
$$v = af$$

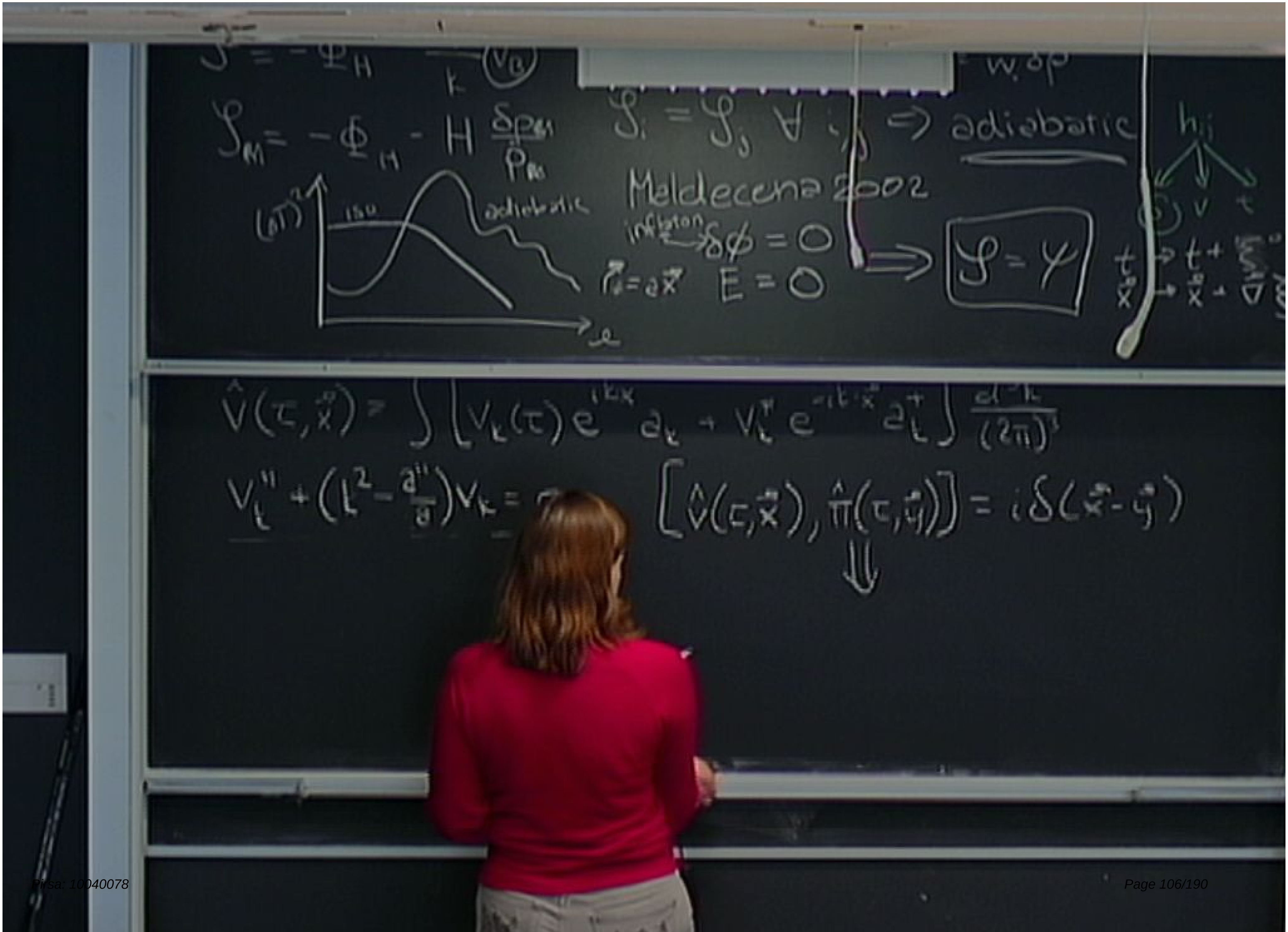
$$S = \frac{1}{2}\int d\tau\, d^3x\left[(v')^2 - (\vec\nabla v)^2 + \frac{a''}{a}v^2\right]$$

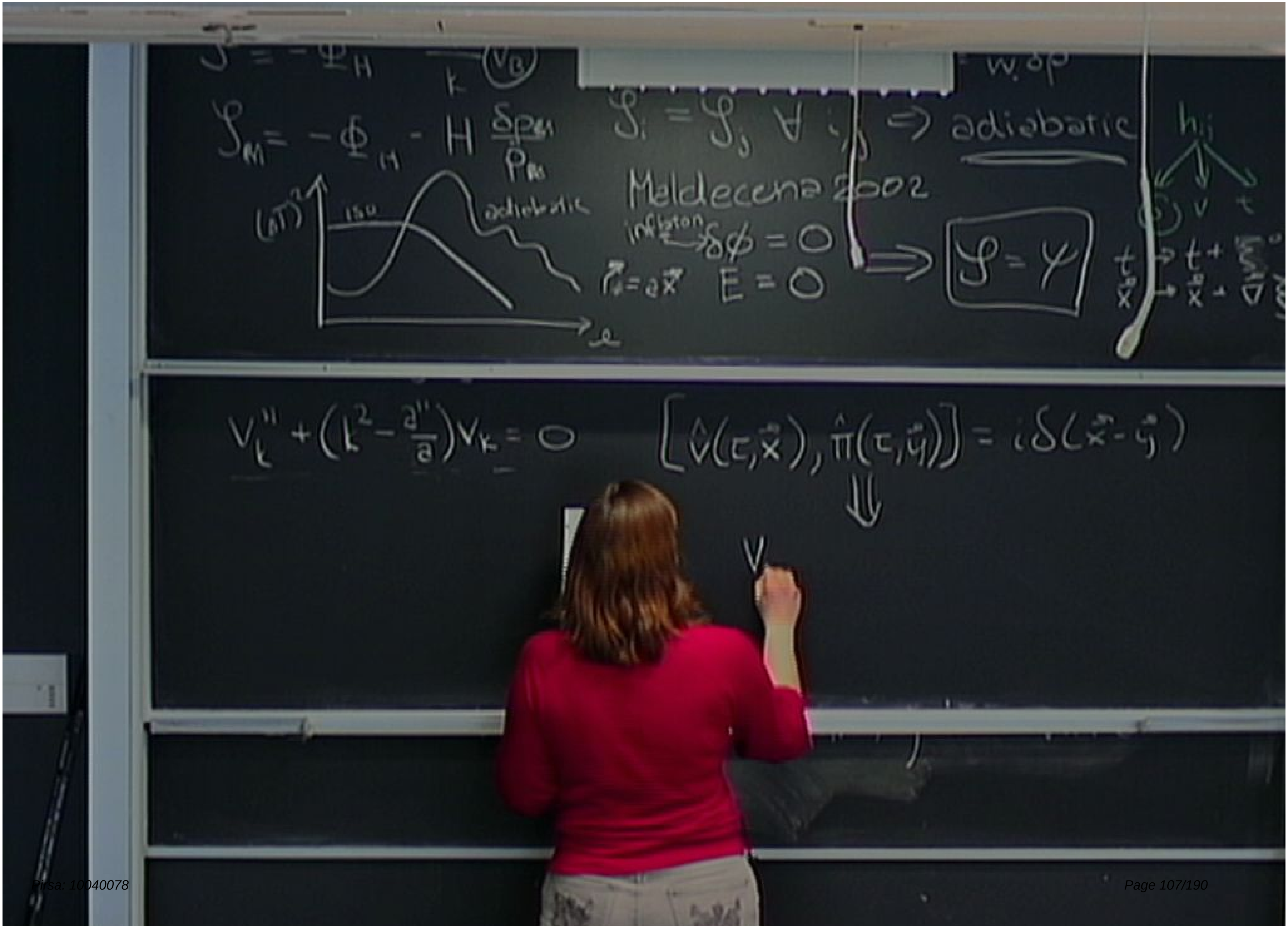

adiabatic
Maldecena 2002
iso
adiabatic
inflaton
δφ = 0
E = 0
π = ∂L/∂v' = v'

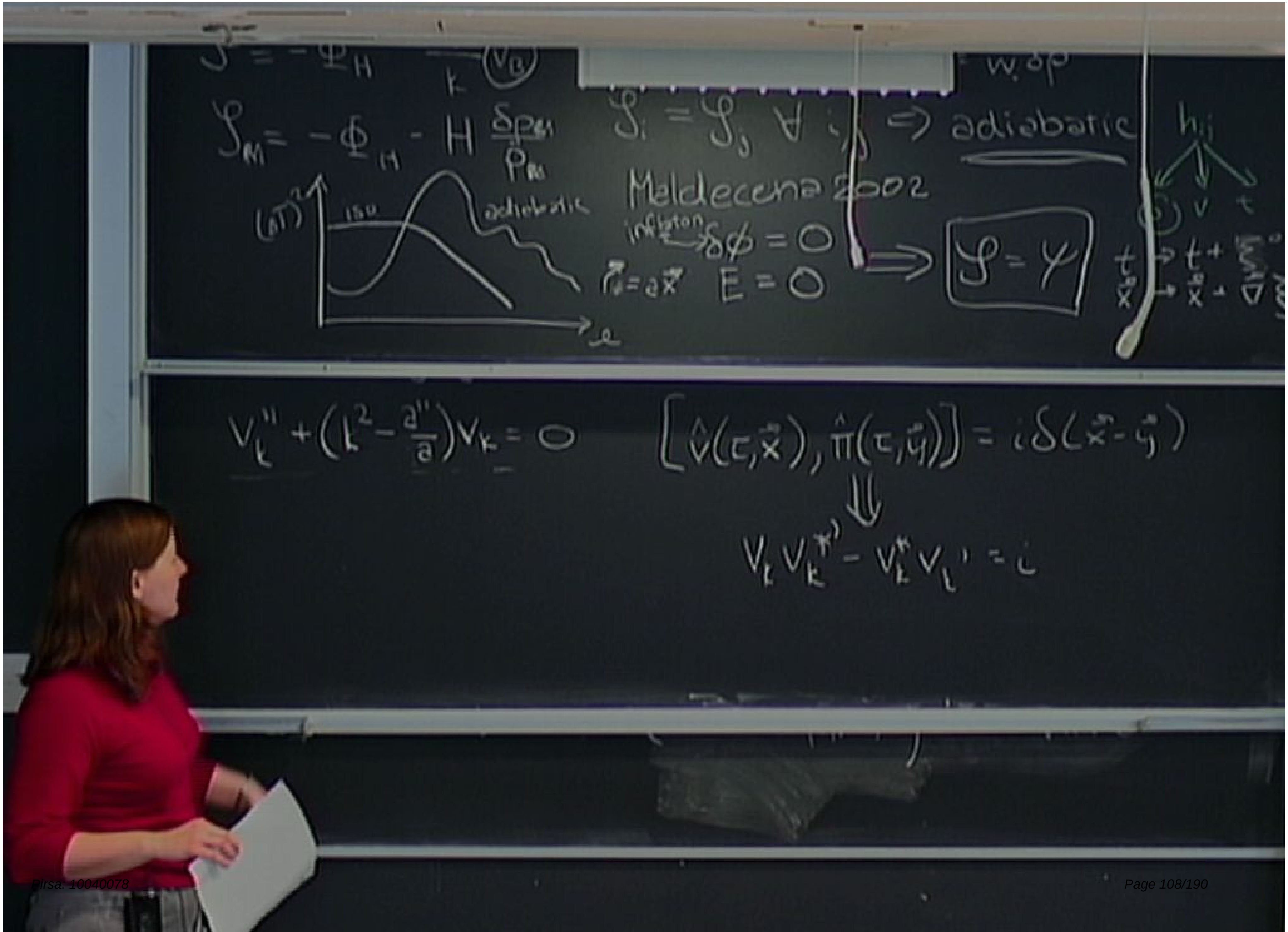


Maldecena 2002
adiabatic
inflaton
δφ = 0
E = 0
ζ = ψ
iso
adiabatic
π = ∂ℒ/∂v′ = v′
v̂(τ, x) = ∫ [v_k(τ) e^{ik·x} a_k + v_k* e^{−ik·x} a_k†] d³k/(2π)³
v_k″ + (k² − a″/a) v_k = 0


adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
ISO
adiabatic
V̂(τ, x⃗) = ∫ [V_k(τ) e^{i k⃗·x⃗} a_k + V_k^* e^{-i k⃗·x⃗} a_k^†] d³k/(2π)³
V_k'' + (k² − a''/a) V_k = 0
[V̂(τ, x⃗),

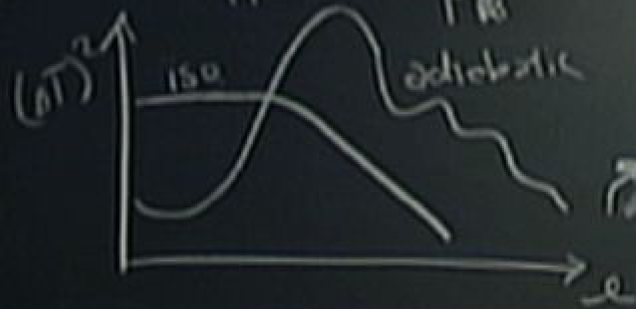

adiabatic
Maldacena 2002
inflaton
δφ = 0
E = 0
ζ = ψ
adiabatic
isu


adiabatic
Maldecena 2002
inflaton
E = 0
adiabatic


adiabatic
Maldacena 2002
inflaton
$\delta\phi = 0$
$E = 0$
adiabatic
$V_k'' + \left(k^2 - \frac{a''}{a}\right) V_k = 0$
$\left[\hat{v}(\tau, \vec{x}), \hat{\pi}(\tau, \vec{y})\right] = i\delta(\vec{x} - \vec{y})$

$\zeta = -\Phi_H$ ... k (v_B) ... $= w_i \delta\rho$

$$\zeta_M = -\Phi_H - H \frac{\delta\rho_M}{\dot\rho_M}$$



$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiabatic

Maldacena 2002

inflaton $\delta\phi = 0 \Rightarrow \boxed{\zeta = \psi}$

$\vec r = a\vec x \quad E = 0$

$h_{ij} \to$ (s) v t

$t \to t + \xi^0$

$\vec x \to \vec x + \nabla \xi$

$$\hat v(\tau, \vec x) = \int \left[v_k(\tau) e^{i\vec k\cdot\vec x} a_k + v_k^* e^{-i\vec k\cdot\vec x} a_k^\dagger \right] \frac{d^3k}{(2\pi)^3}$$

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\left[\hat v(\tau, \vec x), \hat\pi(\tau, \vec y)\right] = i\delta(\vec x - \vec y)$$

$$\Downarrow$$

$$v_k v_k^{*\prime} - v_k^* v_k' = i$$

N, N' are not dynamical $\Rightarrow$ A, B are not dynamical

$$A = \frac{\dot\zeta}{H} \qquad B = \frac{1}{a}\frac{\zeta}{H} - a\nabla^{-2}\left(\frac{\dot\phi^2}{2H^2}\dot\zeta\right)(8\pi G)^3 \qquad \ddot\phi + 3H\dot\phi + V' = 0$$

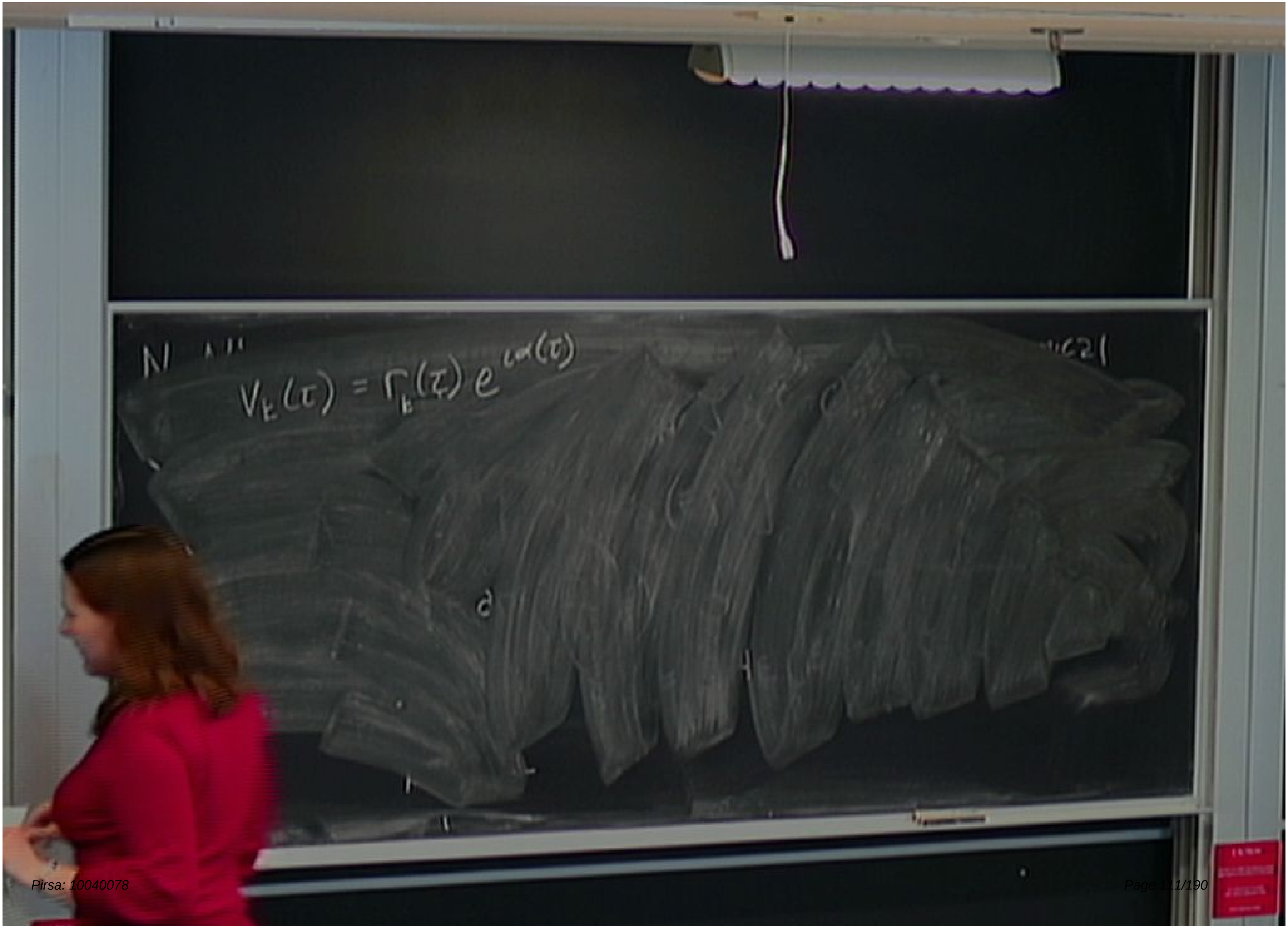
$$S = \frac{1}{2}\int d^4x\, \frac{\dot\phi^2}{H^2}\left[a^3\dot\zeta^2 - a(\nabla\zeta)^2\right] \qquad f \equiv \frac{\dot\phi}{H}\zeta \propto \sqrt{\epsilon_H}\,\zeta$$

$$\left(\frac{\dot\phi^2}{H^2}a^3\dot\zeta\right) + a\nabla^2\left(\frac{\dot\phi^2}{H^2}\zeta\right) = 0 \qquad \dot f = \frac{\dot\phi}{H}\dot\zeta$$

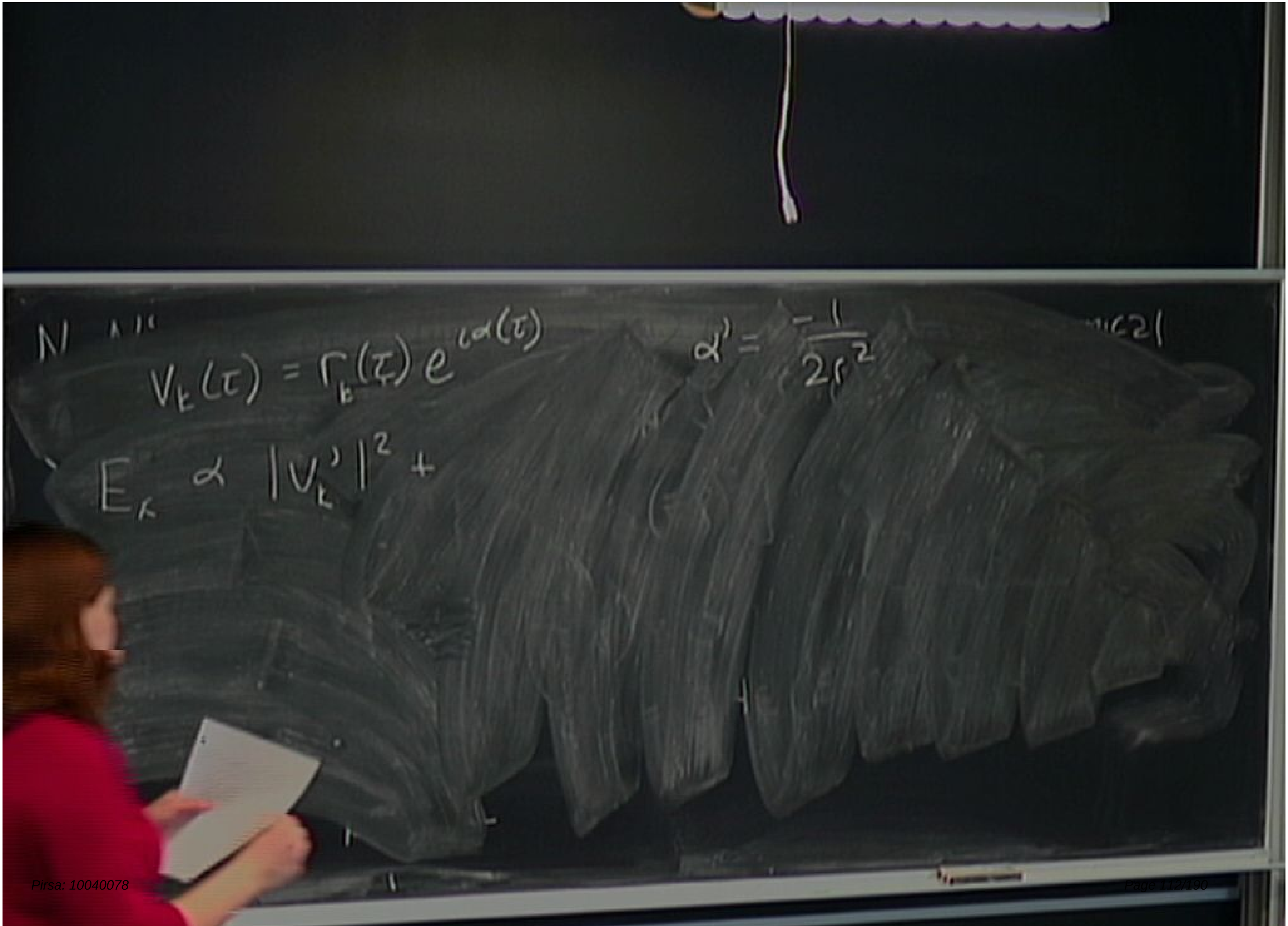
$$\dot\epsilon_H = 2H\epsilon(\epsilon - \eta) \ll 1$$

$$G\frac{\dot\phi^2}{H^2} \ll 1$$

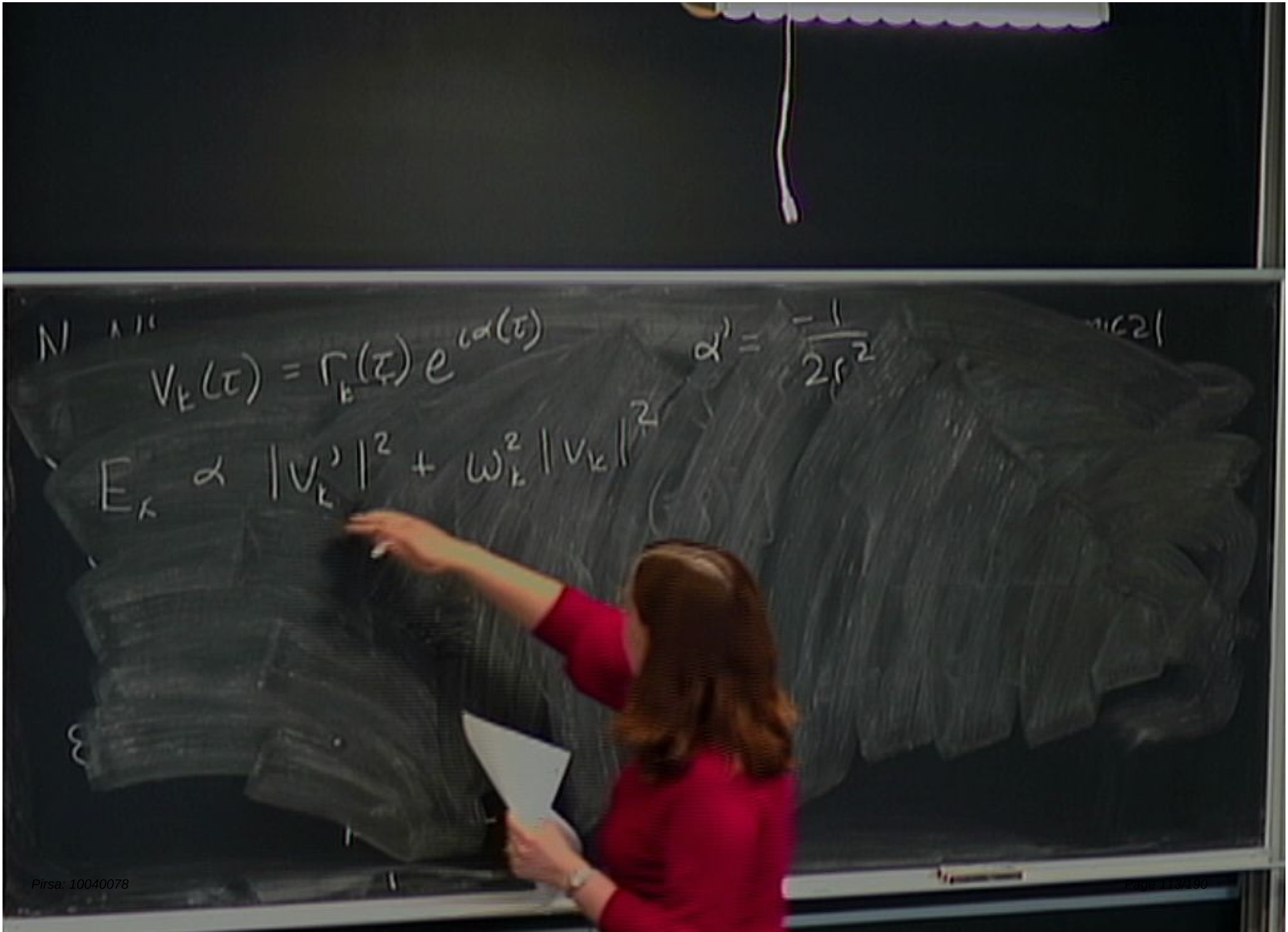
$$k\tau \gg 1$$



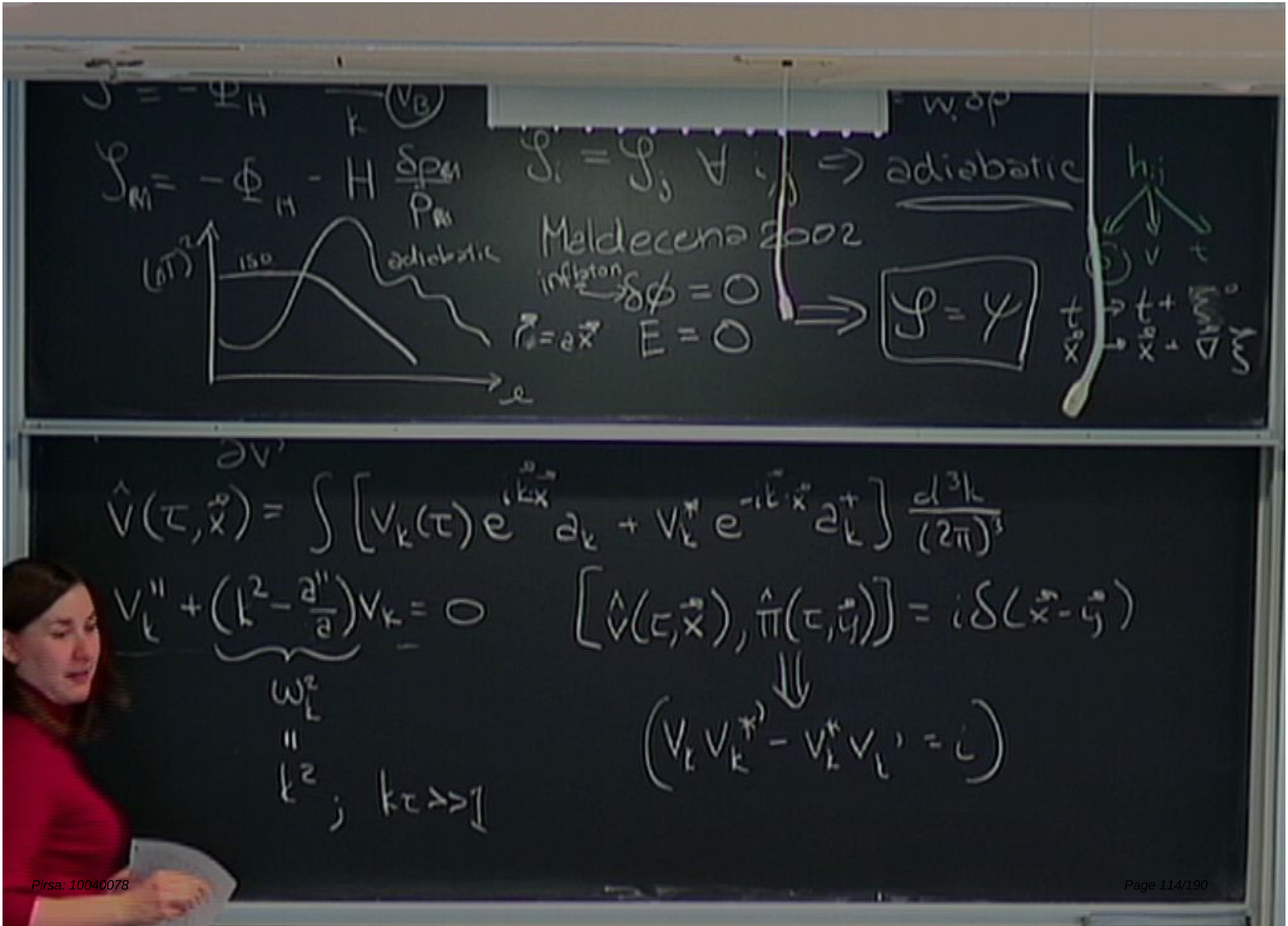
$V_t(\tau) = \Gamma_t(\tau)\, e^{i\alpha(\tau)}$

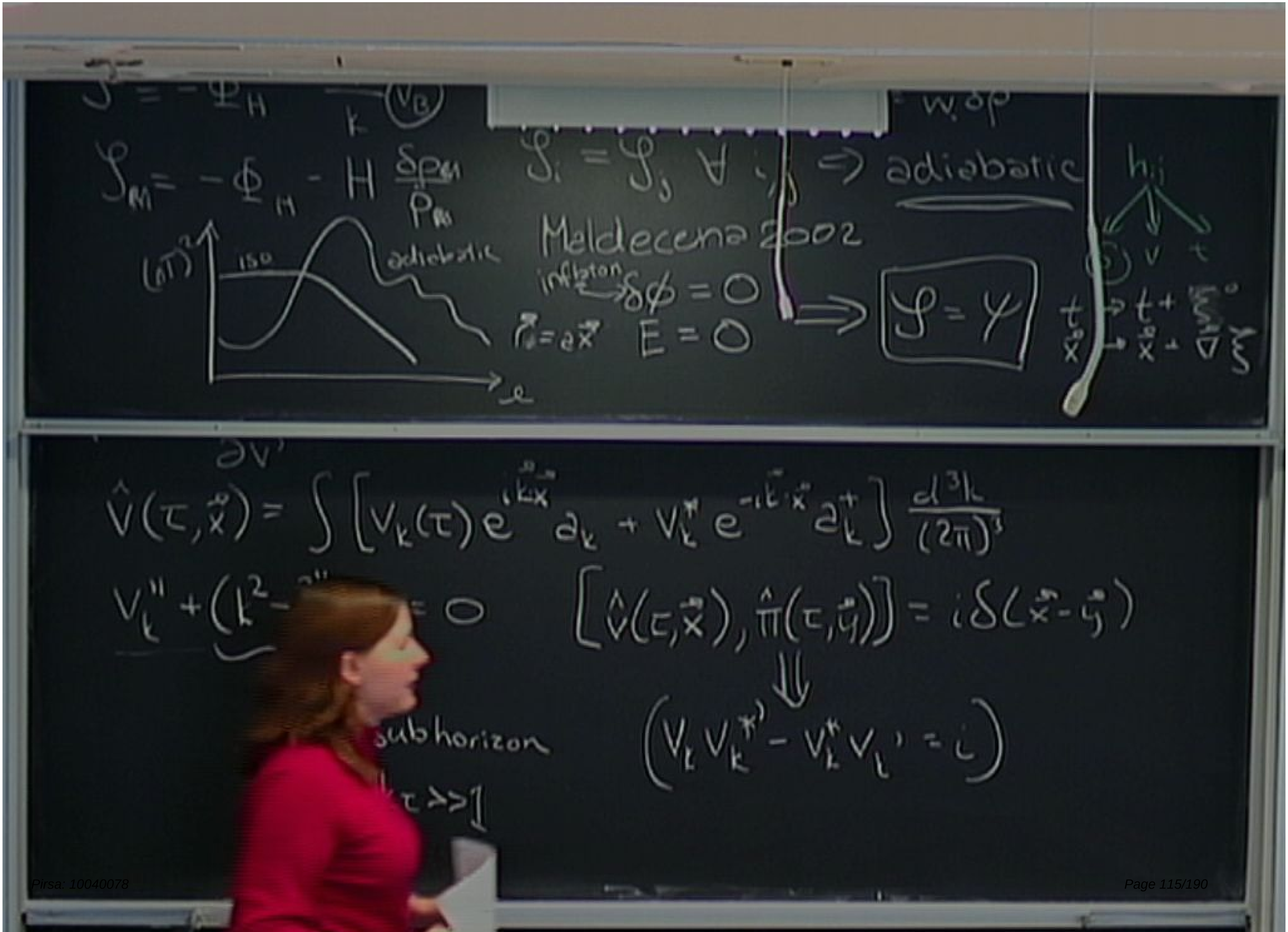


$V_k(\tau) = r_k(\tau) e^{i\alpha(\tau)}$
$\alpha' = \frac{-1}{2r^2}$
$E_k \propto |v_k'|^2 +$

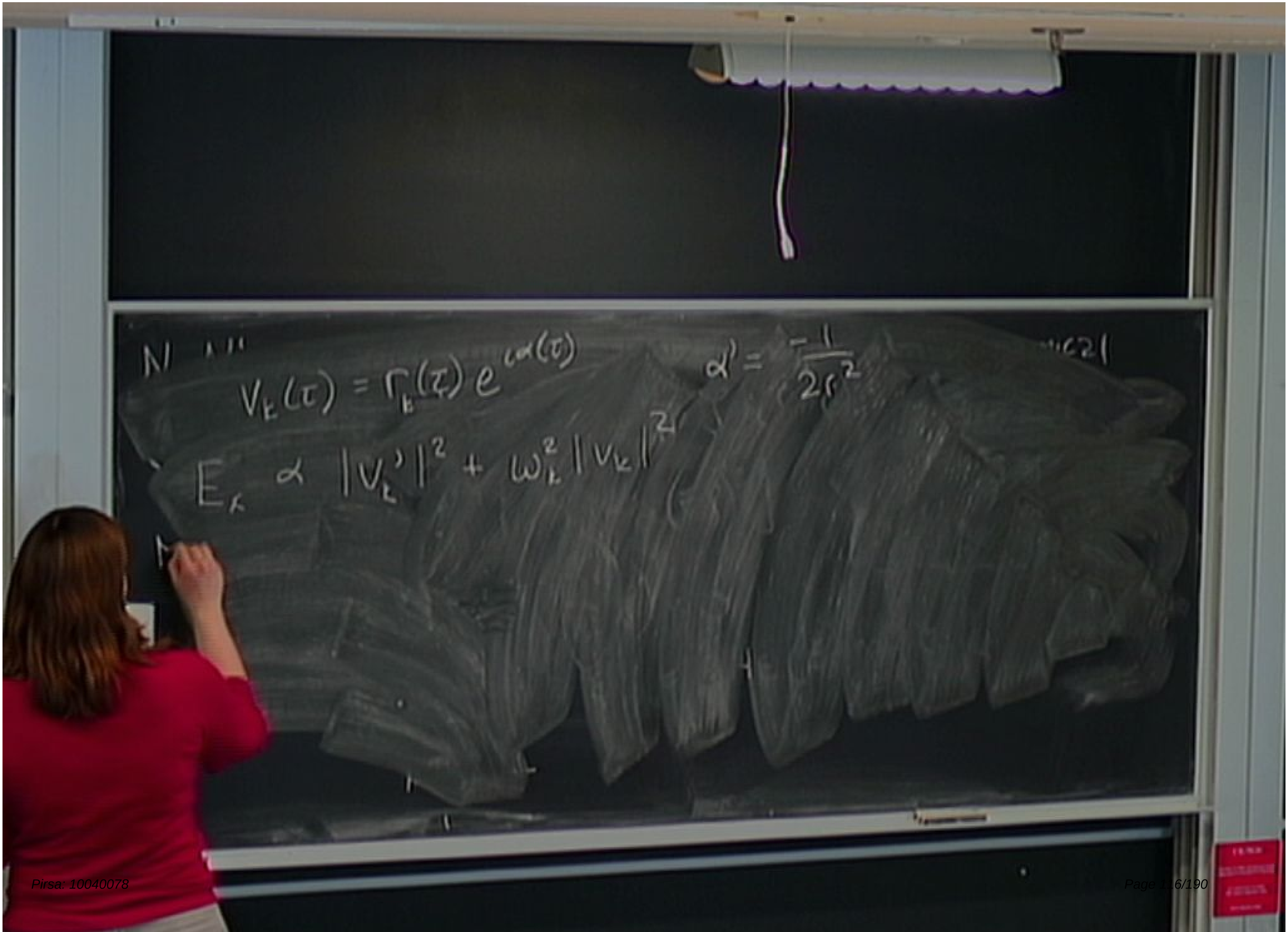


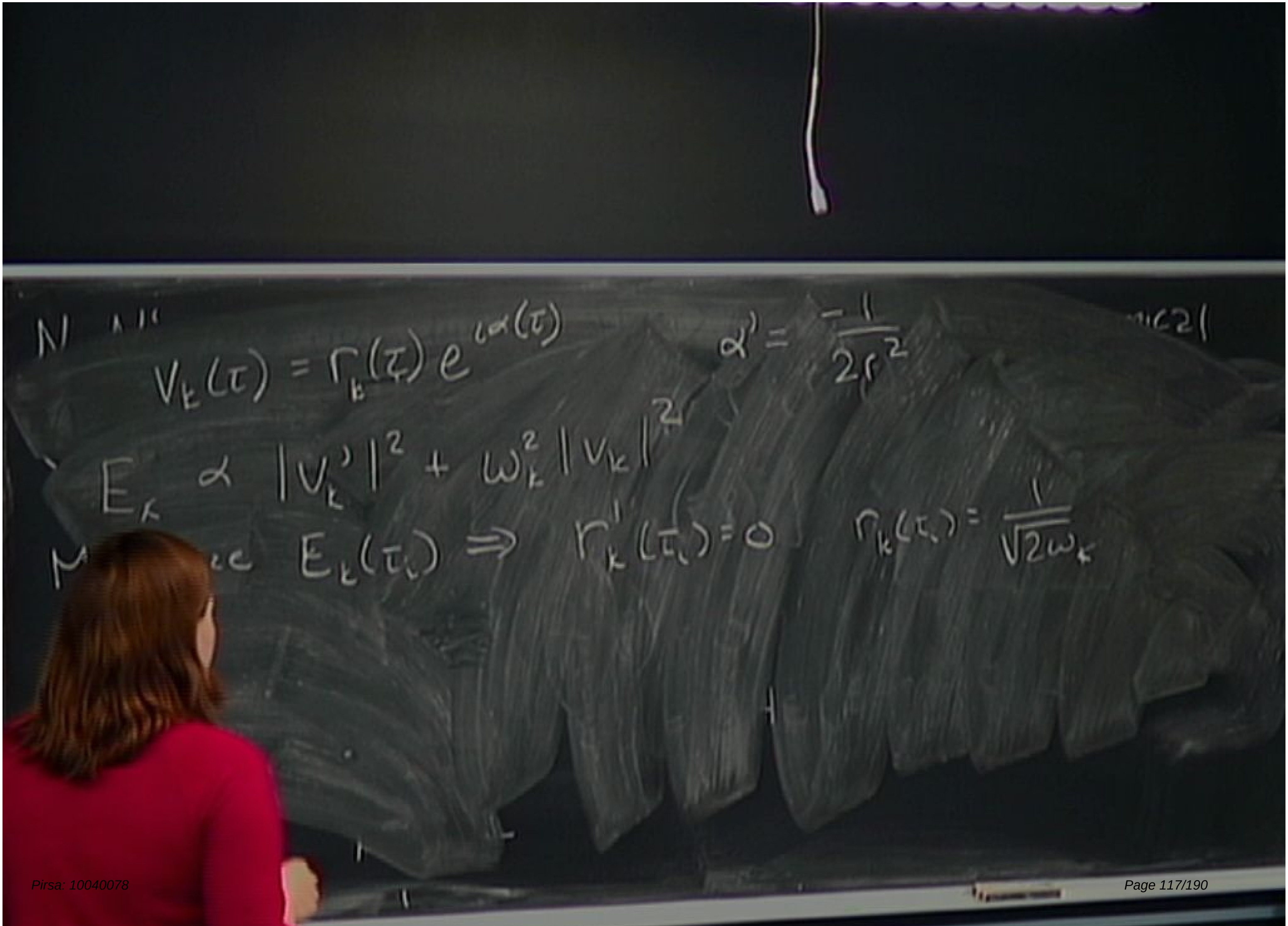
$V_k(\tau) = r_k(\tau) e^{i\alpha(\tau)}$
$\alpha' = \frac{-1}{2r^2}$
$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$


Maldacena 2002
adiabatic
inflaton
δφ = 0
E = 0
ISO
adiabatic
hij
v
t
ωk²
kτ >> 1



adiabatic
Maldacena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic
subhorizon

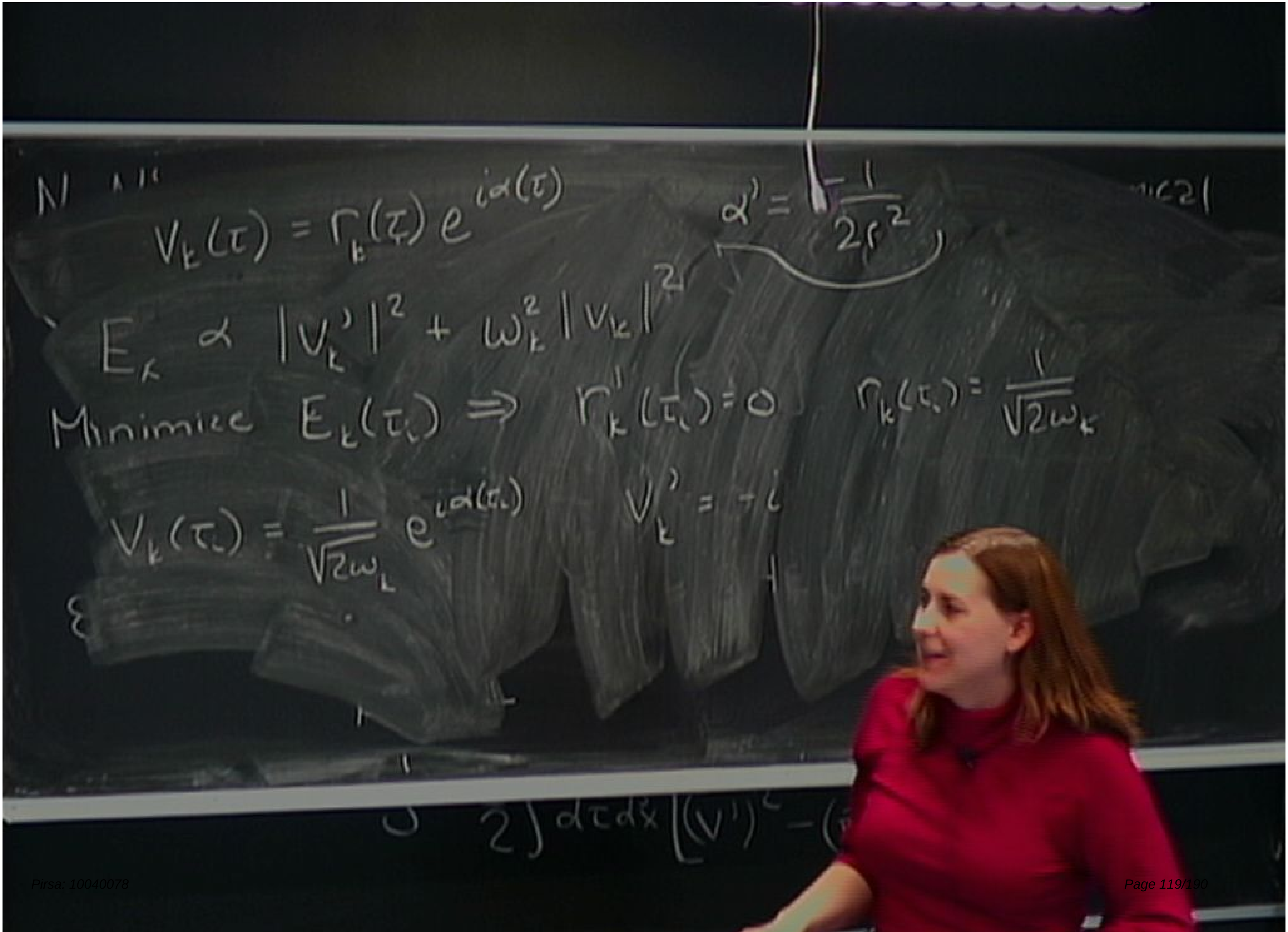

$V_k(\tau) = r_k(\tau) e^{i\alpha(\tau)}$
$\alpha' = \frac{-1}{2r^2}$
$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$


V_k(τ) = r_k(τ) e^{iα(τ)}
α' = −1/(2r²)
E_k ∝ |v_k'|² + ω_k² |v_k|²
E_k(τ_i) ⇒ r_k'(τ_i) = 0
r_k(τ_i) = 1/√(2ω_k)

$$V_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)} \qquad \alpha' = \frac{-1}{2r^2}$$

$$E_k \propto |V_k'|^2 + \omega_k^2 |V_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$


$V_k(\tau) = r_k(\tau) e^{i\alpha(\tau)}$
$\alpha' = \frac{1}{2r^2}$
$E_k \propto |V_k'|^2 + \omega_k^2 |V_k|^2$
Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0$
$r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$
$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)}$
$V_k' = -i$

$$V_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)} \qquad \alpha' = \frac{-1}{2r^2}$$

$$E_k \propto |V_k'|^2 + \omega_k^2 |V_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}\, e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$$

$$\frac{}{2}\int d\tau\, d\vec{x} \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2 \right]$$

$$V_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)} \qquad \alpha' = -\frac{1}{2r^2}$$

$$E_k \propto |V_k'|^2 + \omega_k^2 |V_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$$

$$\frac{}{2}\int d\tau\, d^3x \left[(V')^2 - (\vec{\nabla}V)^2 + \frac{a''}{a}V^2\right]$$

$$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$$

$\omega \simeq k$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2 (\partial_{\text{comoving}} f)^2\right]$$

$f = \frac{\phi}{H}\varphi$

$f = \frac{h}{\sqrt{16\pi G}}$

$V = af$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2\right]$$

$$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0$ $\quad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$v_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad v_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$$

$\omega \simeq k$

$$v_k(\tau) = \frac{1}{\sqrt{2k}} e^{ik\tau}$$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2 (\partial_{\text{comoving}} f)^2\right]$$

$f = \frac{\phi}{H}\varphi$

$f = \frac{h}{\sqrt{16\pi G}}$

$v = af$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2\right]$$

$$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0$ $\quad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$v_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad v_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$$

& $\omega \simeq k$ $\quad$ $v_k(\tau) = \frac{1}{\sqrt{2k}} e^{ik\tau}$ $\quad$ $k\tau \gg 1$

$$d\tau\, d^3x \Rightarrow \quad ' = \frac{d}{d\tau}$$

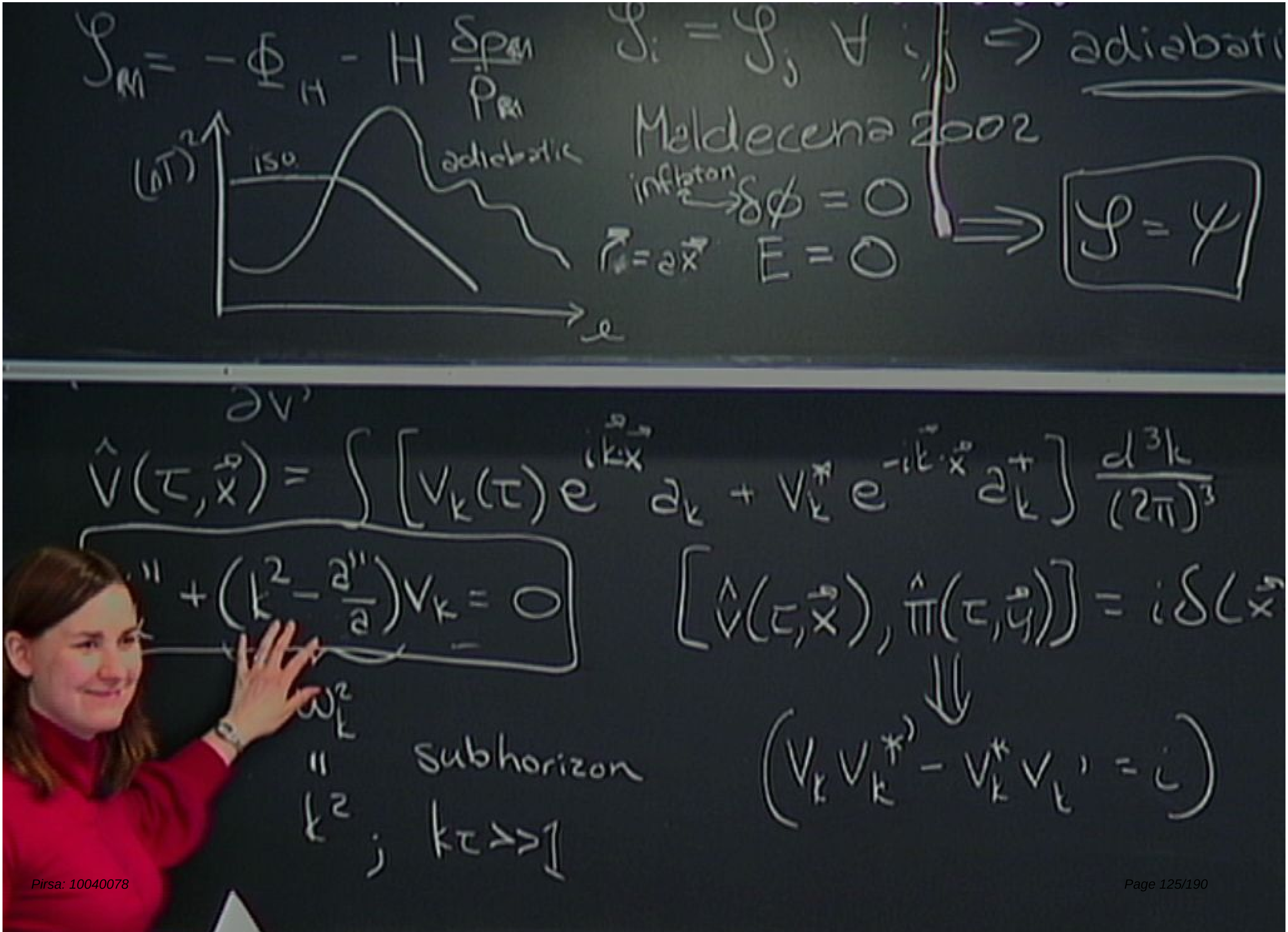
$$S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2 (\partial_{\text{comoving}} f)^2\right]$$

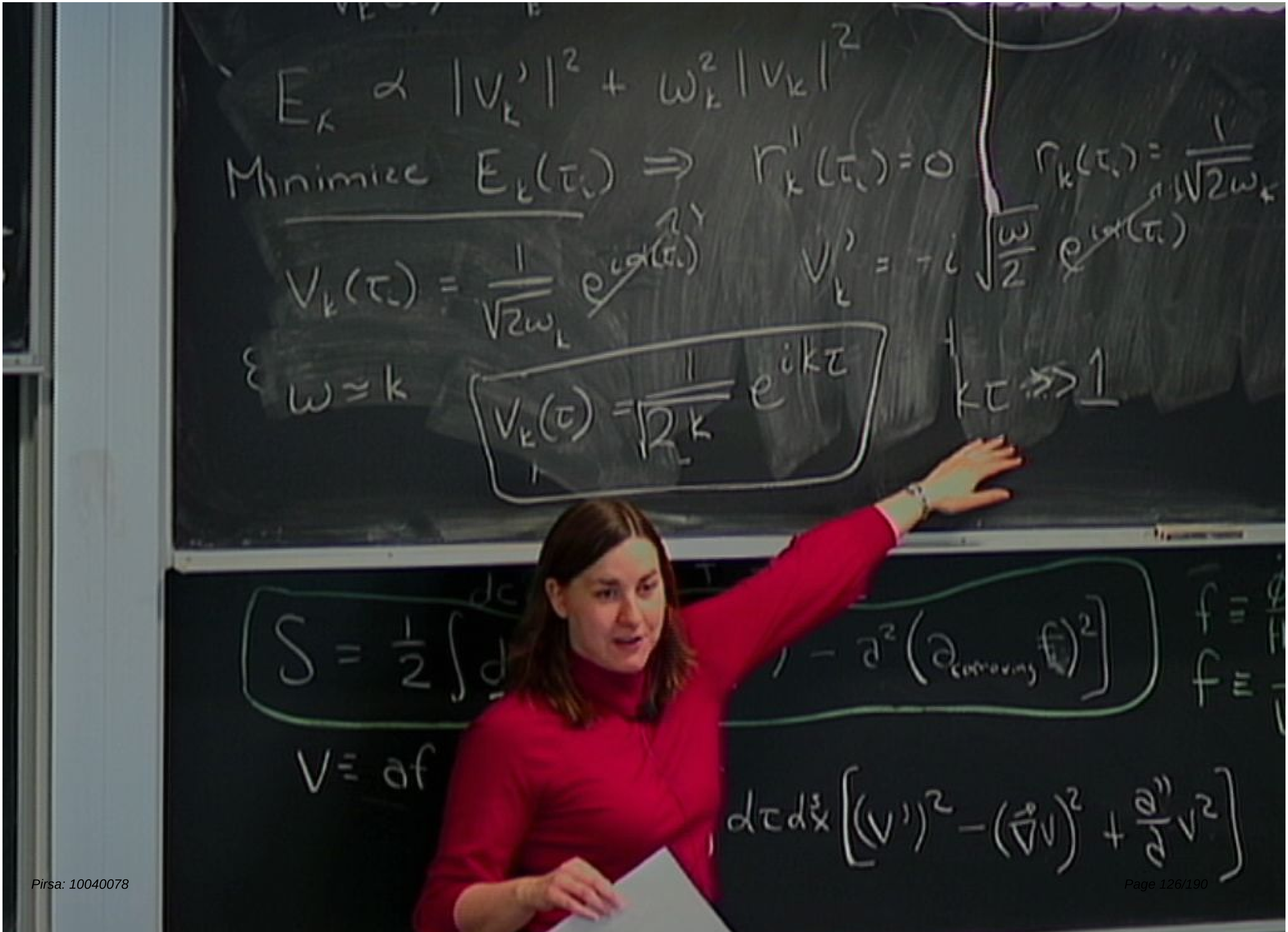
$f = \frac{\dot\phi}{H}\varphi$

$f = \frac{h}{\sqrt{16\pi G}}$

$v = af$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec\nabla v)^2 + \frac{a''}{a} v^2\right]$$


adiabatic
Maldecena 2002
inflaton
$\delta\phi = 0$
$E = 0$
iso
adiabatic
subhorizon
$k\tau \gg 1$


Minimize

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i)$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$$

$$\omega \simeq k \qquad \boxed{V_k(\tau) = \frac{1}{\sqrt{2k}} e^{ik\tau}} \qquad k\tau \gg 1$$

$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x} \left[a^3 \dot{f}^2 - a\left(\partial_{comoving} f\right)^2\right] \qquad \boxed{\frac{a''}{a} = \frac{2}{\tau^2}}$$

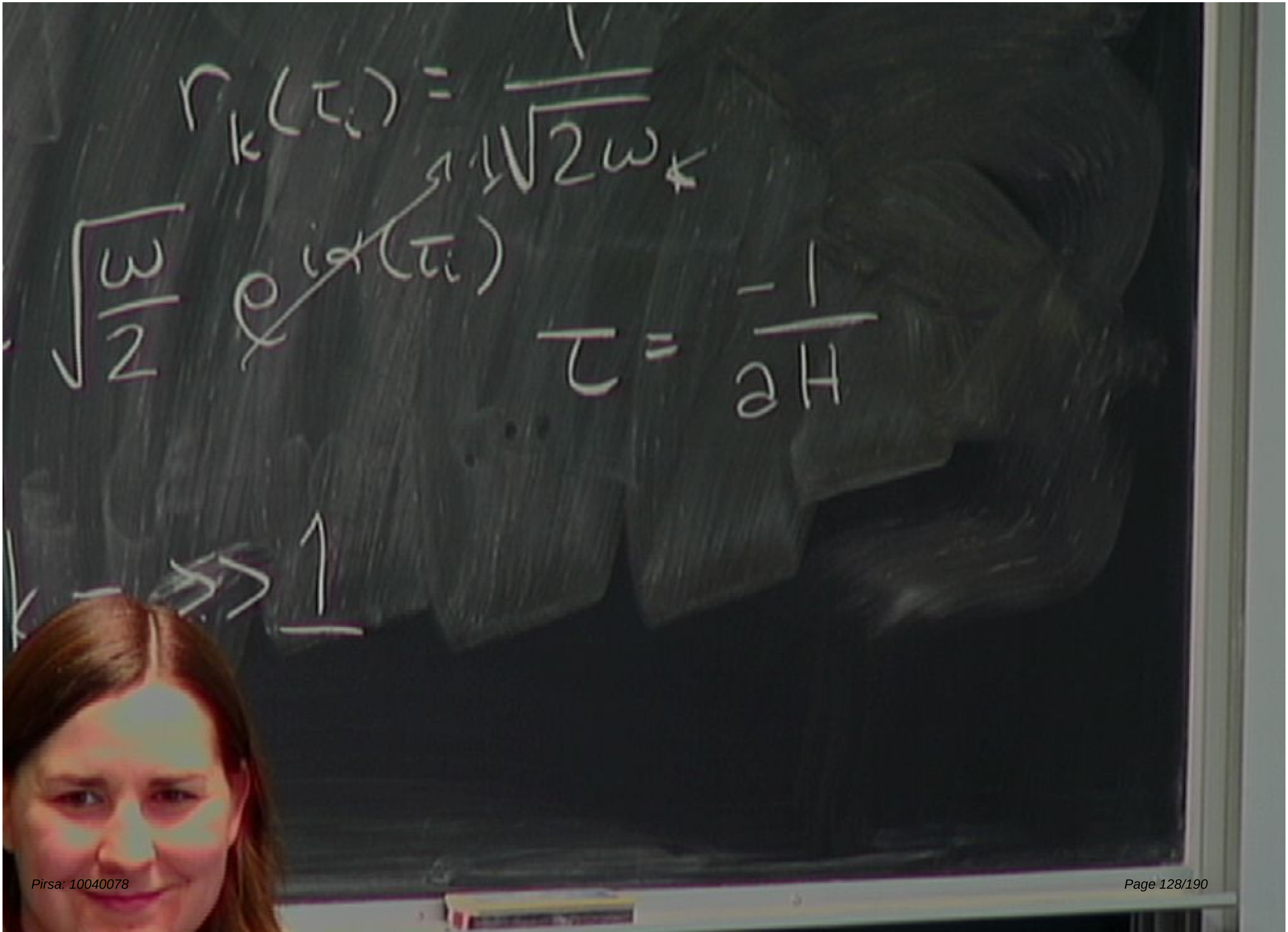
$$d\tau\, d^3x \Rightarrow f' = \frac{df}{d\tau}$$

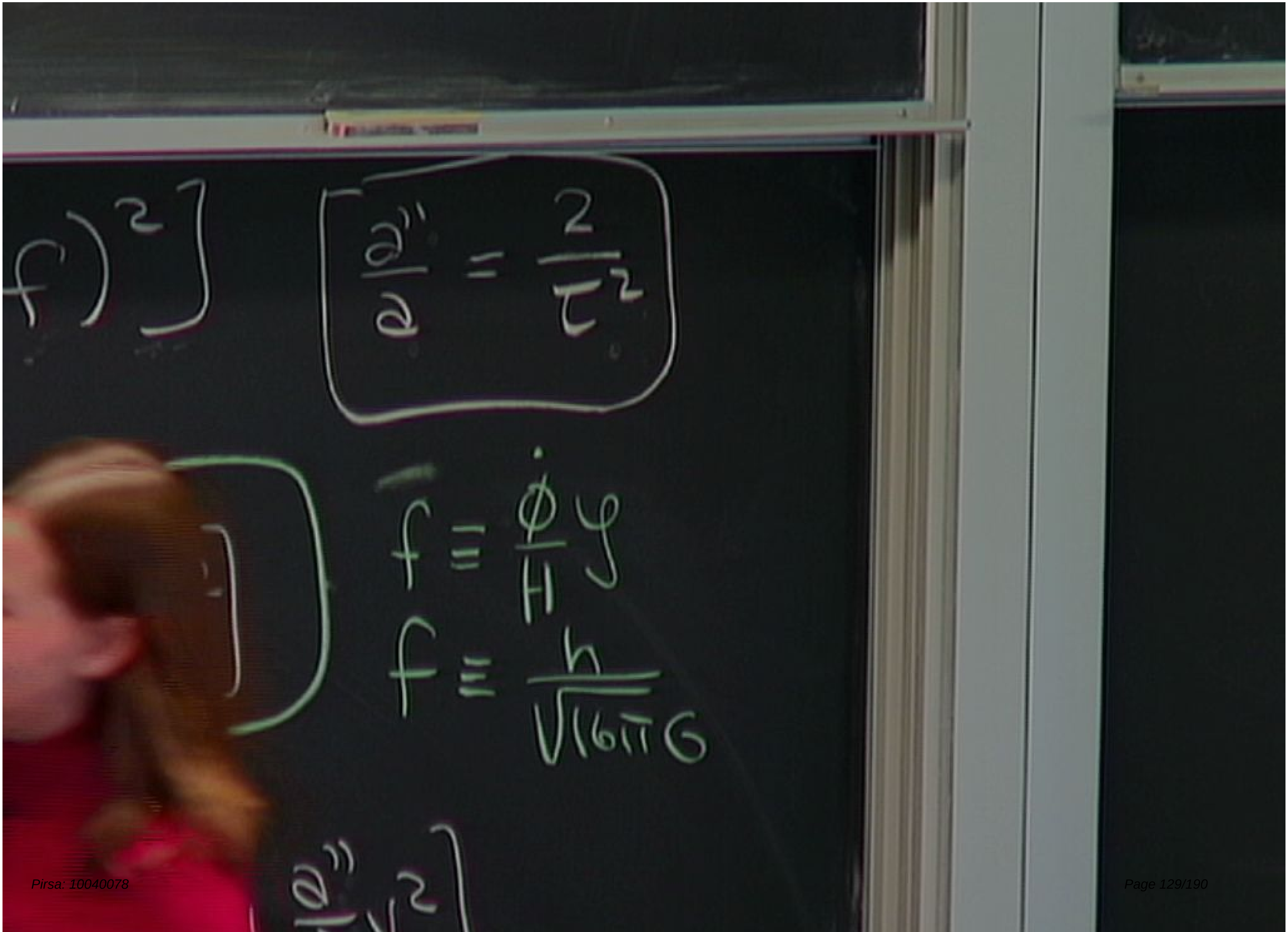
$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2\left(\partial_{comoving} f\right)^2\right]}$$

$$f = \frac{\dot{\phi}}{H}\varphi \qquad f = \frac{h}{\sqrt{16\pi G}}$$

$$V = af$$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[(V')^2 - (\vec{\nabla} V)^2 + \frac{a''}{a} V^2\right]$$


$r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$
$\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)}$
$\tau = \frac{-1}{aH}$
$\gg 1$


f)²]
a''/a = 2/τ²
f ≡ φ̇/H φ
f ≡ h/√(16πG)

$$v_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)} \qquad \alpha' = \frac{-1}{2r^2}$$

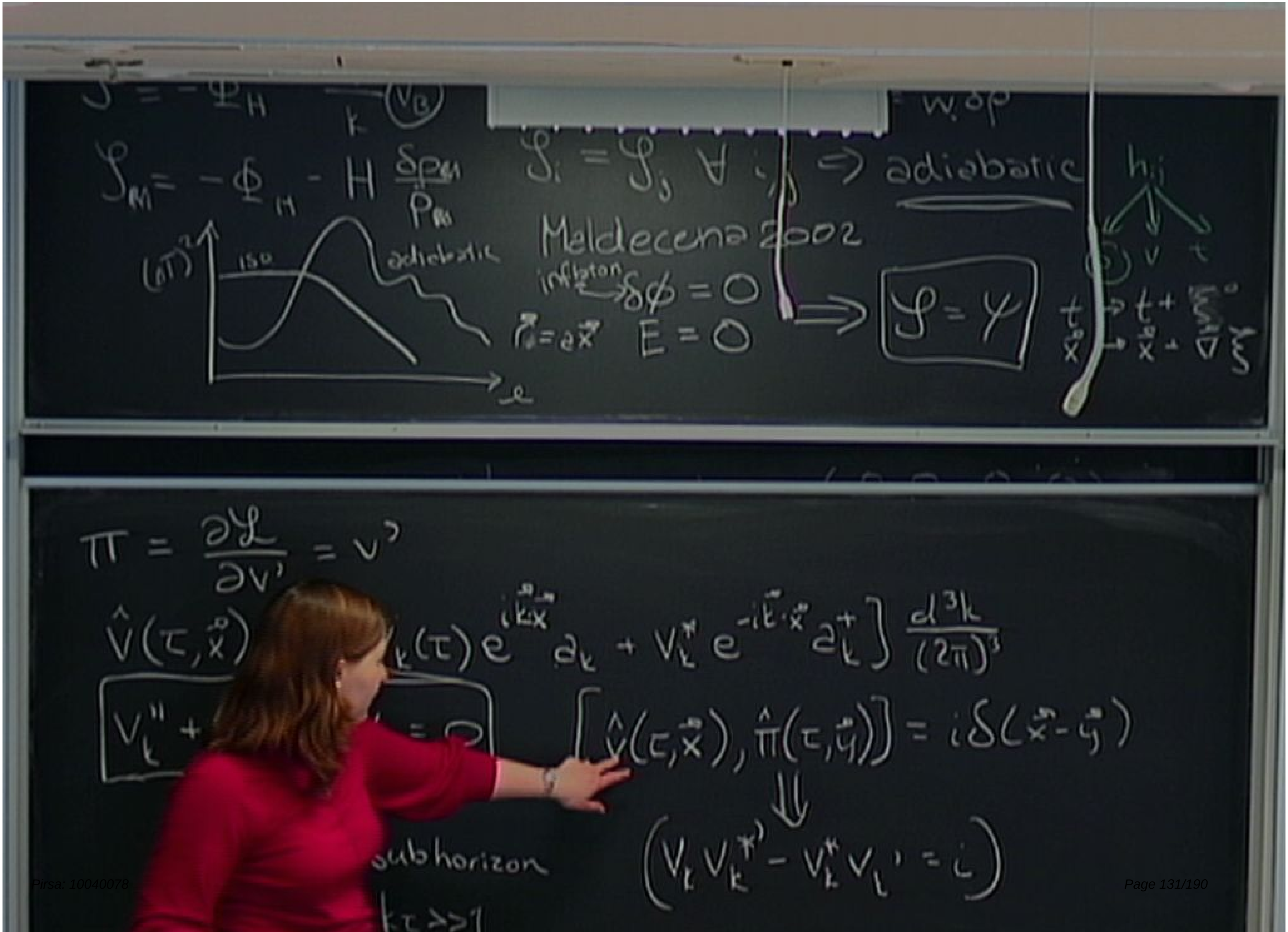
$$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$v_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}\, e^{i\alpha(\tau_i)} \qquad v_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)} \qquad \tau = \frac{-1}{aH}$$

$$\omega \simeq k \qquad v_k(\tau) = \frac{1}{\sqrt{2k}}\, e^{ik\tau} \qquad k\tau \gg 1$$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2\right]$$


adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
adiabatic
iso
subhorizon
$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$
$[\hat{v}(\tau,\vec{x}), \hat{\pi}(\tau,\vec{y})] = i\delta(\vec{x}-\vec{y})$
$(v_k v_k^{*\prime} - v_k^* v_k' = i)$

$$V_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)} \qquad \alpha' = \frac{-1}{2r^2}$$

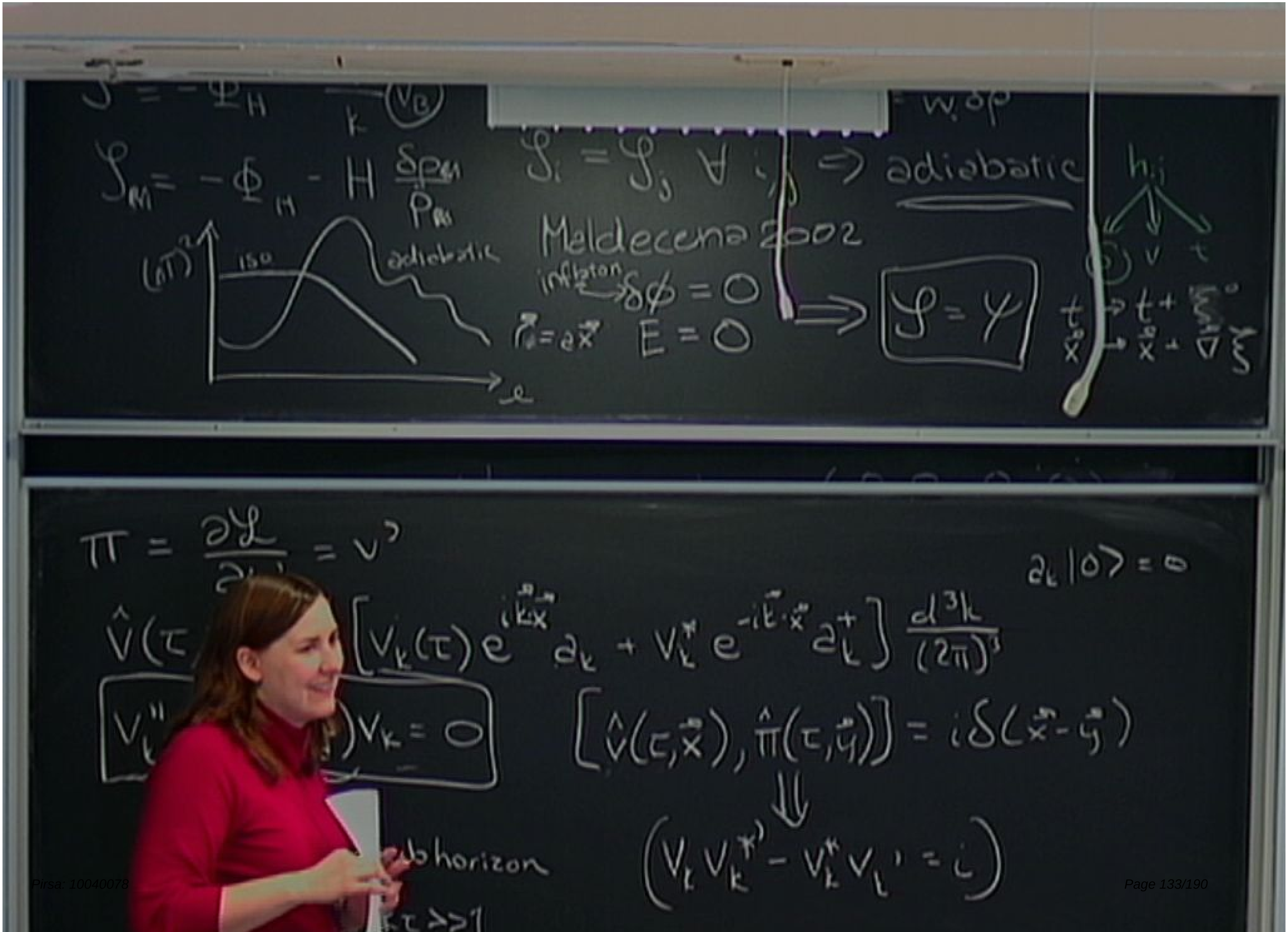
$$E_k \propto |v_k'|^2 + \omega_k^2 |v_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)} \qquad \tau = \frac{-1}{aH}$$

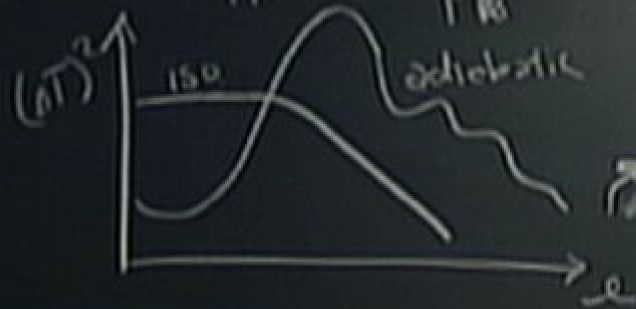
$$\omega \simeq k \qquad V_k(\tau) = \frac{1}{\sqrt{2k}} e^{ik\tau} \qquad k\tau \gg 1$$

$$S = \frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2 \right]$$



adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
ζ = ψ
π = ∂L/∂v' = v'
a_k |0> = 0
[v̂(τ, x), π̂(τ, y)] = i δ(x − y)
subhorizon
V_k V_k^{*'} − V_k^* V_k' = i

$\zeta_M = -\Phi_H - H \frac{\delta\rho_M}{\dot\rho_M}$



$\zeta_i = \zeta_j \ \forall\, i,j \Rightarrow$ adiabatic

Maldecena 2002

inflaton $\longrightarrow \delta\phi = 0$

$\vec r = a\vec x \quad E = 0$

$\Longrightarrow \boxed{\zeta = \psi}$

h_{ij} → s, v, t

$t \to t + \xi^0$

$\vec x \to \vec x + \nabla \xi$

$\pi = \frac{\partial \mathcal{L}}{\partial v'} = v'$

$a_k|0\rangle = 0$

$\hat v(\tau, \vec x) = \int \left[v_k(\tau) e^{i\vec k\cdot\vec x} a_k + v_k^* e^{-i\vec k\cdot\vec x} a_k^\dagger \right] \frac{d^3k}{(2\pi)^3}$

$\boxed{v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0}$

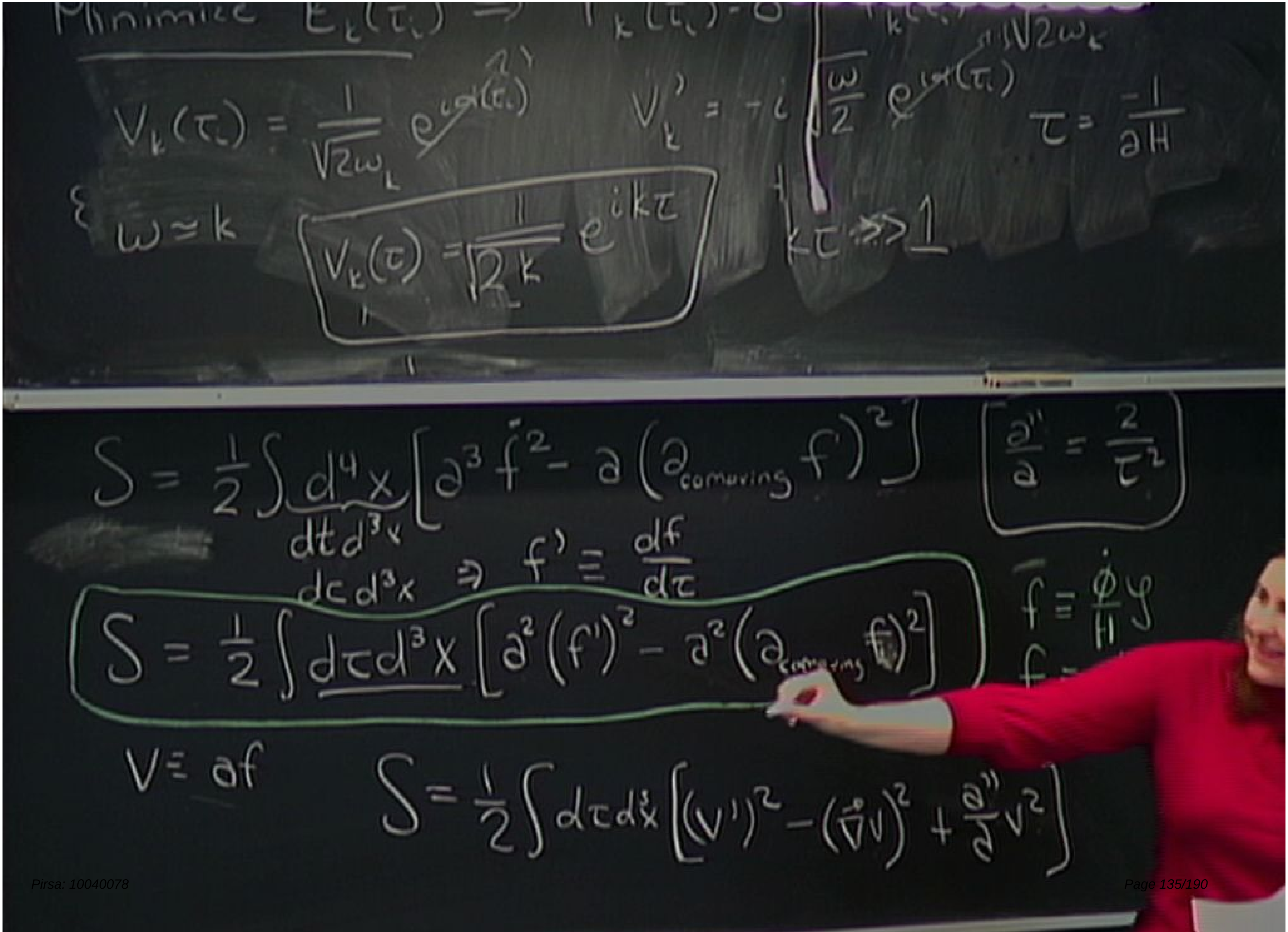
$\left[\hat v(\tau, \vec x), \hat\pi(\tau, \vec y)\right] = i\delta(\vec x - \vec y)$

$\Downarrow$

$\left(v_k v_k^{*\prime} - v_k^* v_k' = i\right)$

$\omega_k^2 = k^2$ subhorizon

$k\tau \gg 1$



Minimize
ω ≃ k
V_k(τ) = 1/√(2k) e^{ikτ}
kτ >> 1
τ = -1/(aH)
S = 1/2 ∫ d^4x [a^3 f^2 - a (∂_comoving f)^2]
dt d^3x
dτ d^3x ⇒ f' ≡ df/dτ
a''/a = 2/τ^2
S = 1/2 ∫ dτ d^3x [a^2 (f')^2 - a^2 (∂_comoving f)^2]
V ≡ af
S = 1/2 ∫ dτ d^3x [(V')^2 - (∇V)^2 + a''/a V^2]

Minimize $E_k(\tau_i) \Rightarrow$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}} e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)} \qquad \tau = \frac{-1}{aH}$$

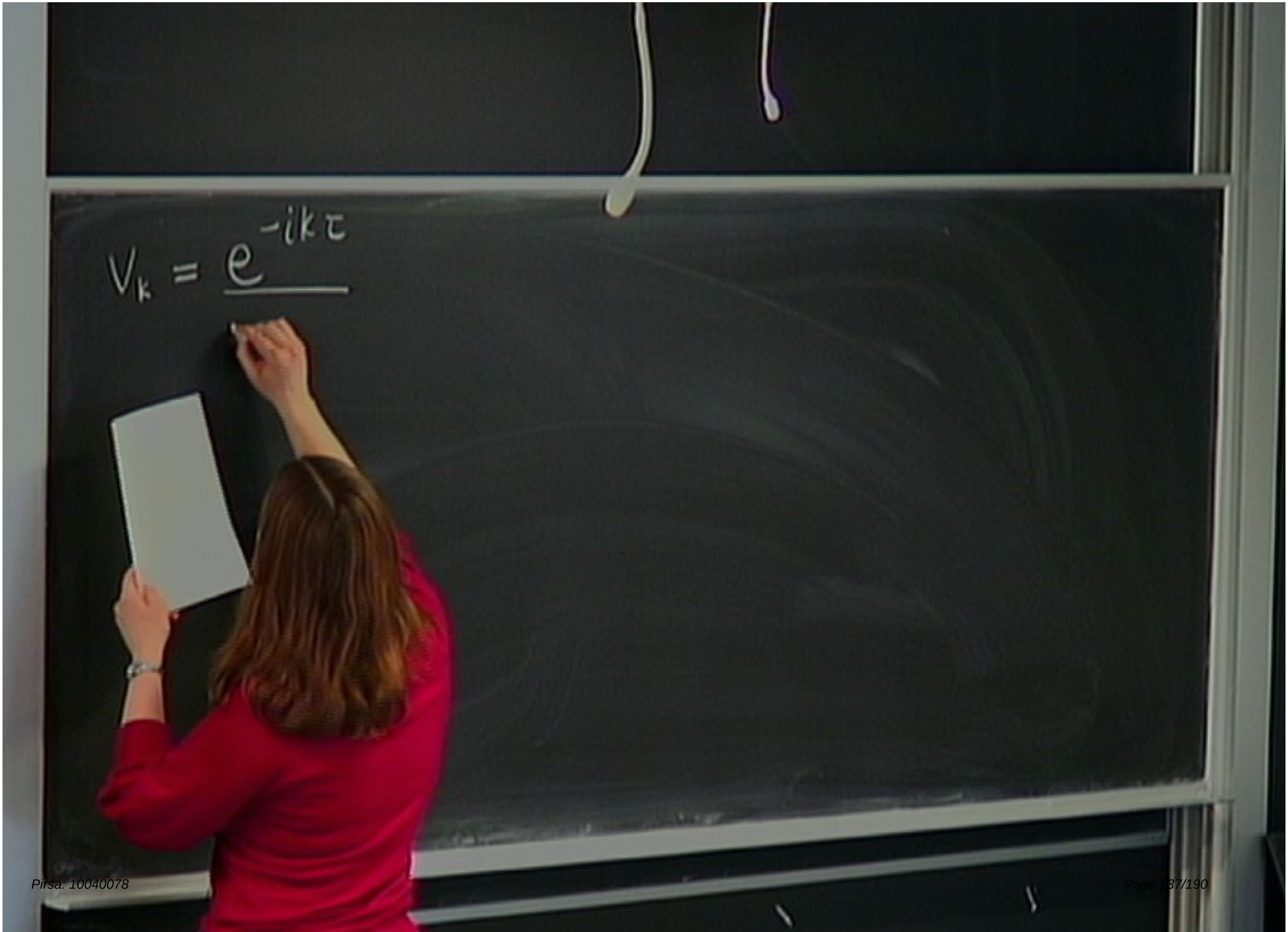
$$\omega \simeq k \qquad \boxed{V_k(\tau) = \frac{1}{\sqrt{2k}} e^{ik\tau}} \qquad k\tau \gg 1$$

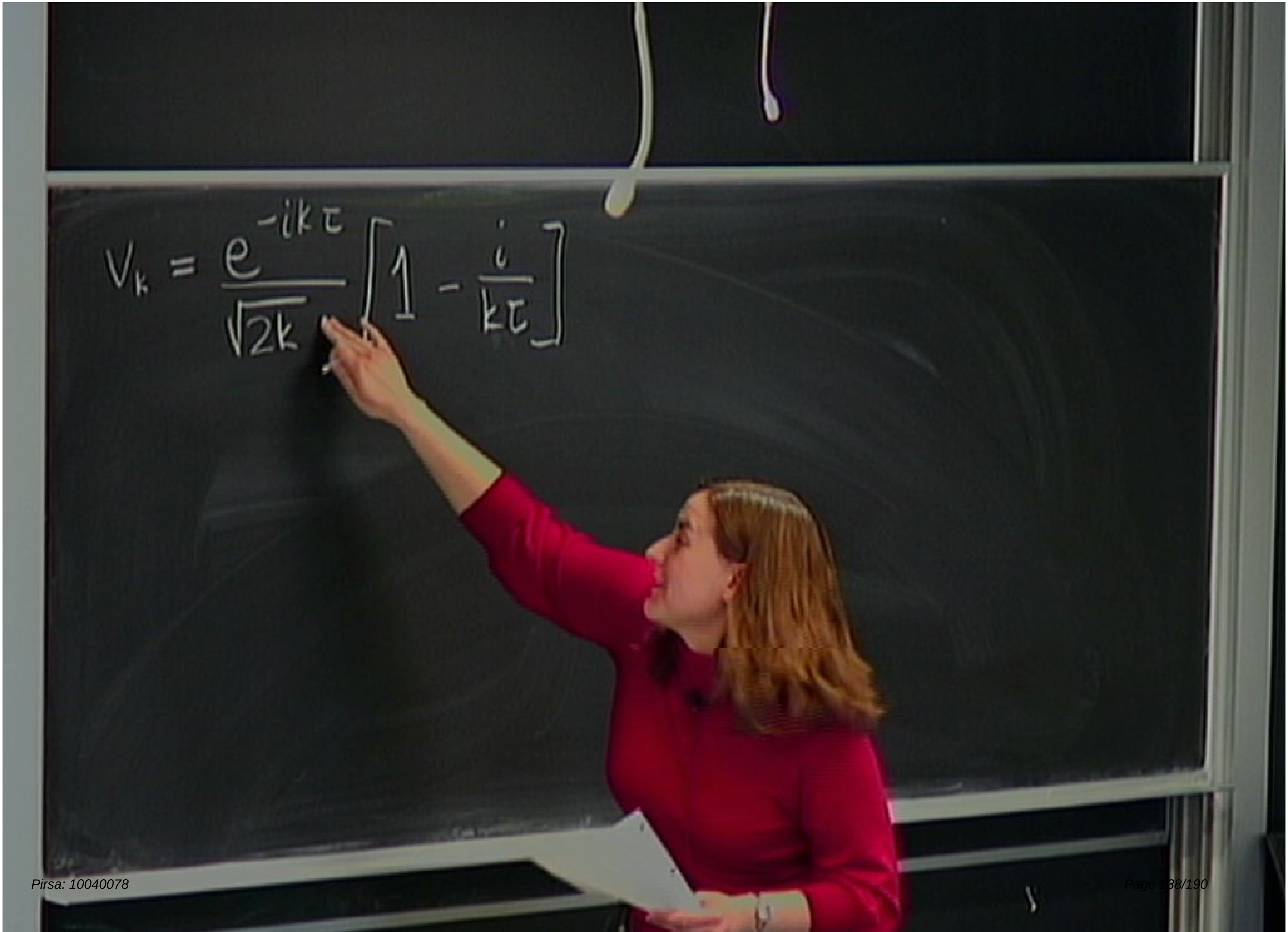
$$S = \frac{1}{2}\int \underbrace{d^4x}_{dt\,d^3x} \left[a^3 \dot{f}^2 - a\left(\partial_{\text{comoving}} f\right)^2\right] \qquad \boxed{\frac{a''}{a} = \frac{2}{\tau^2}}$$

$$d\tau\, d^3x \Rightarrow f' \equiv \frac{df}{d\tau}$$

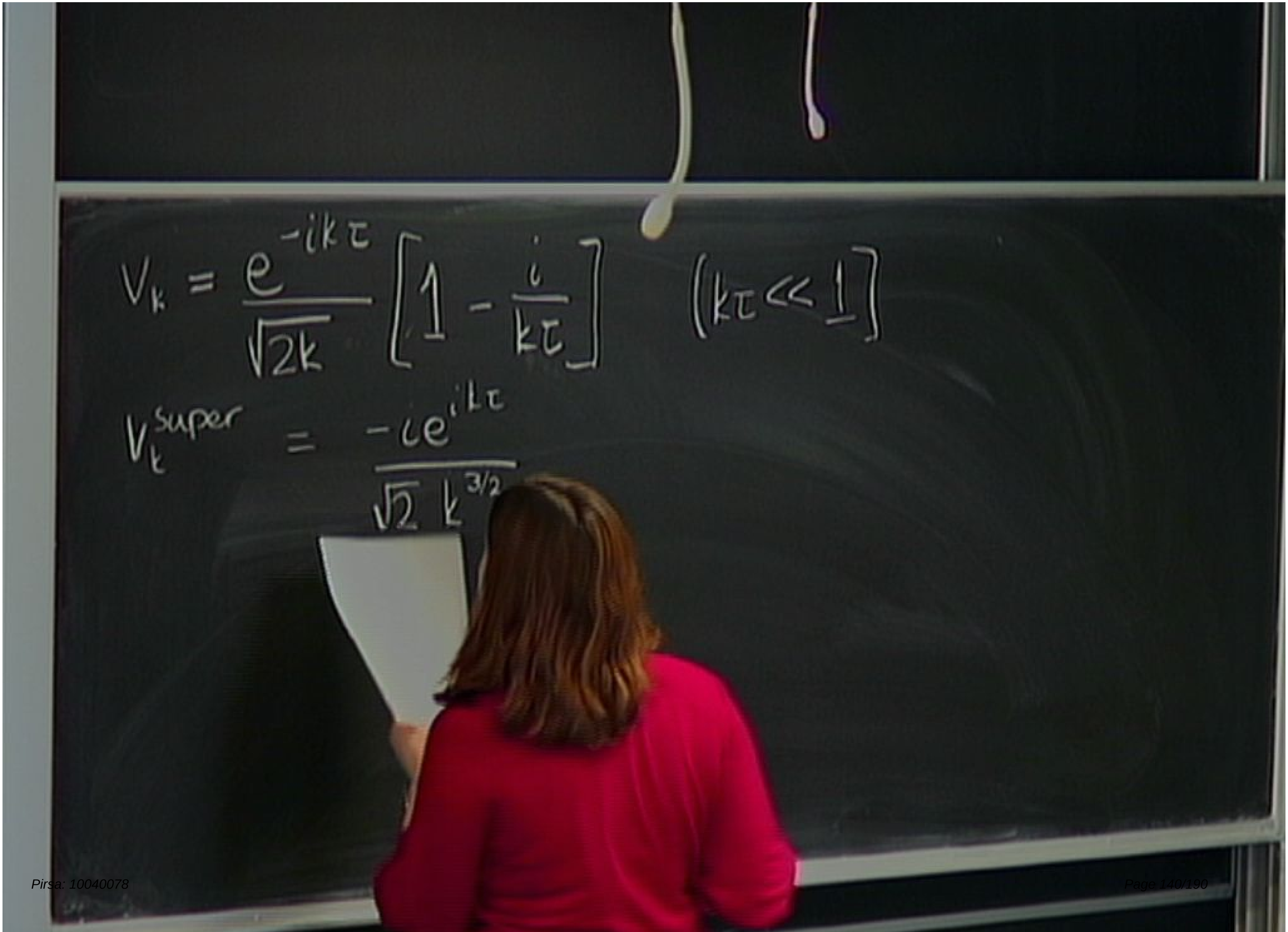
$$\boxed{S = \frac{1}{2}\int d\tau\, d^3x \left[a^2 (f')^2 - a^2\left(\partial_{\text{comoving}} f\right)^2\right]} \qquad f = \frac{\dot{\phi}}{H}\varphi \qquad f = \frac{h}{\sqrt{16\pi G}}$$

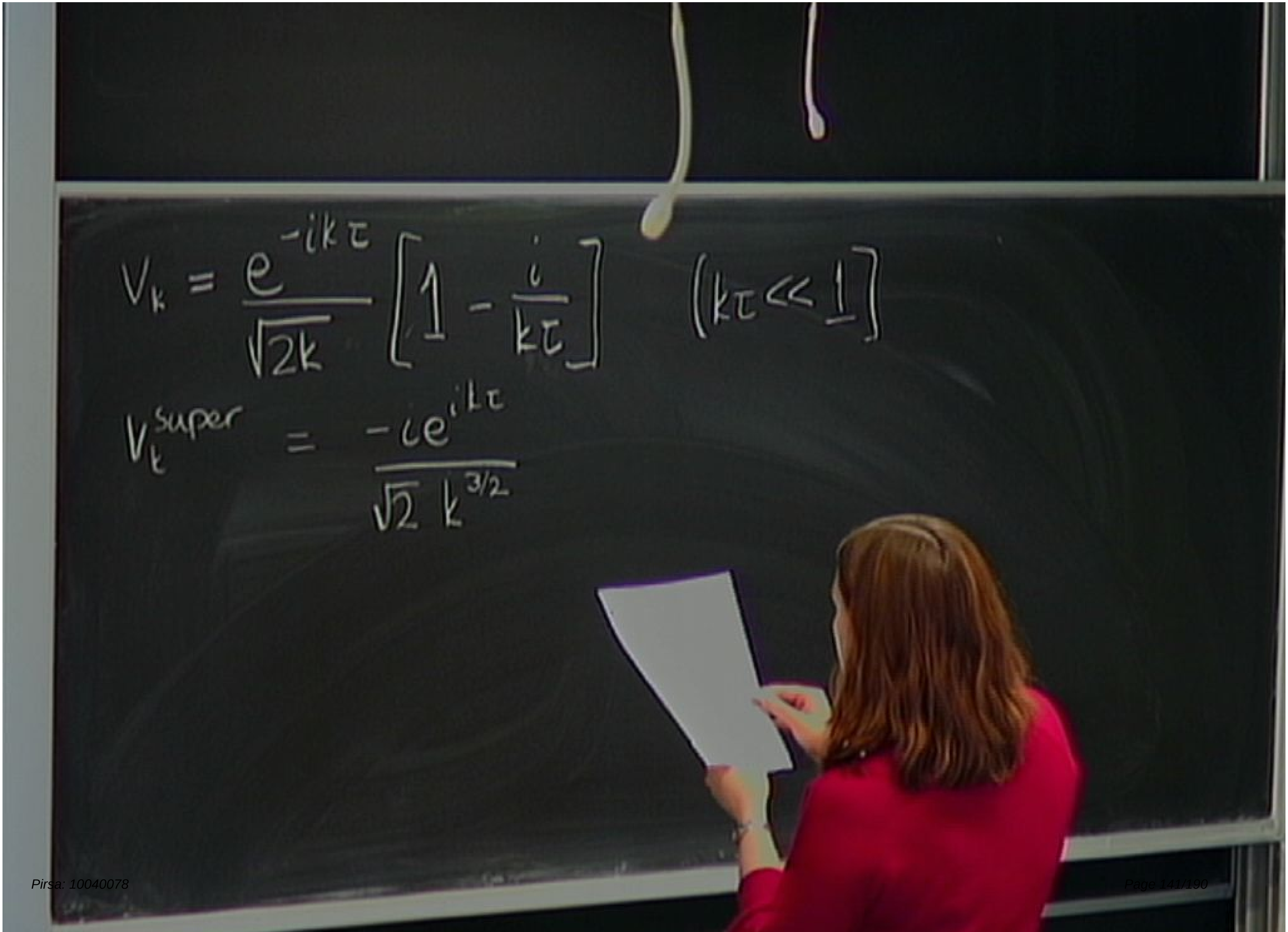
$$V \equiv af \qquad S = \frac{1}{2}\int d\tau\, d^3x \left[(V')^2 - (\vec{\nabla} V)^2 + \frac{a''}{a} V^2\right]$$

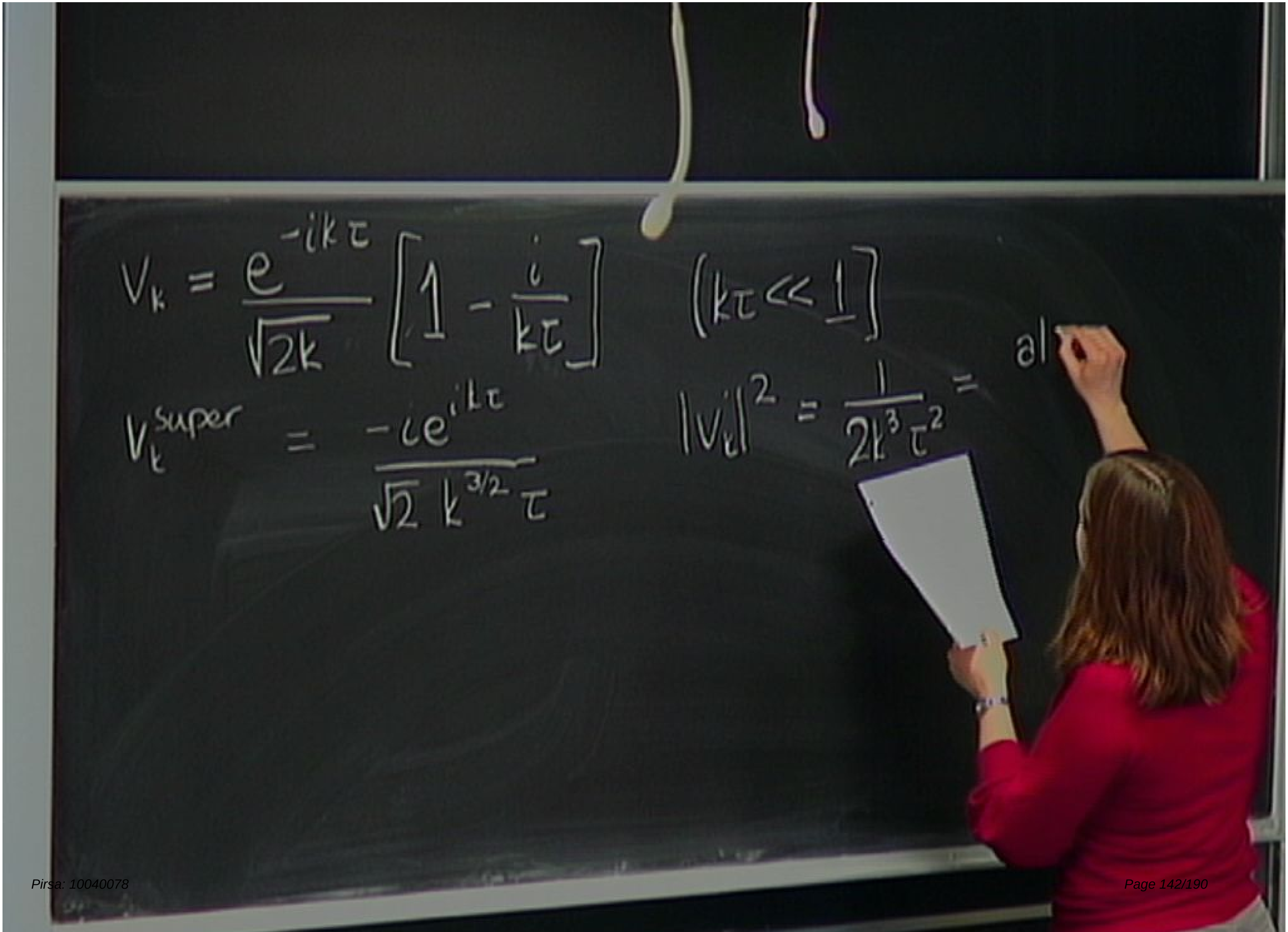

$V_k = \frac{e^{-ik\tau}}{}$


$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$

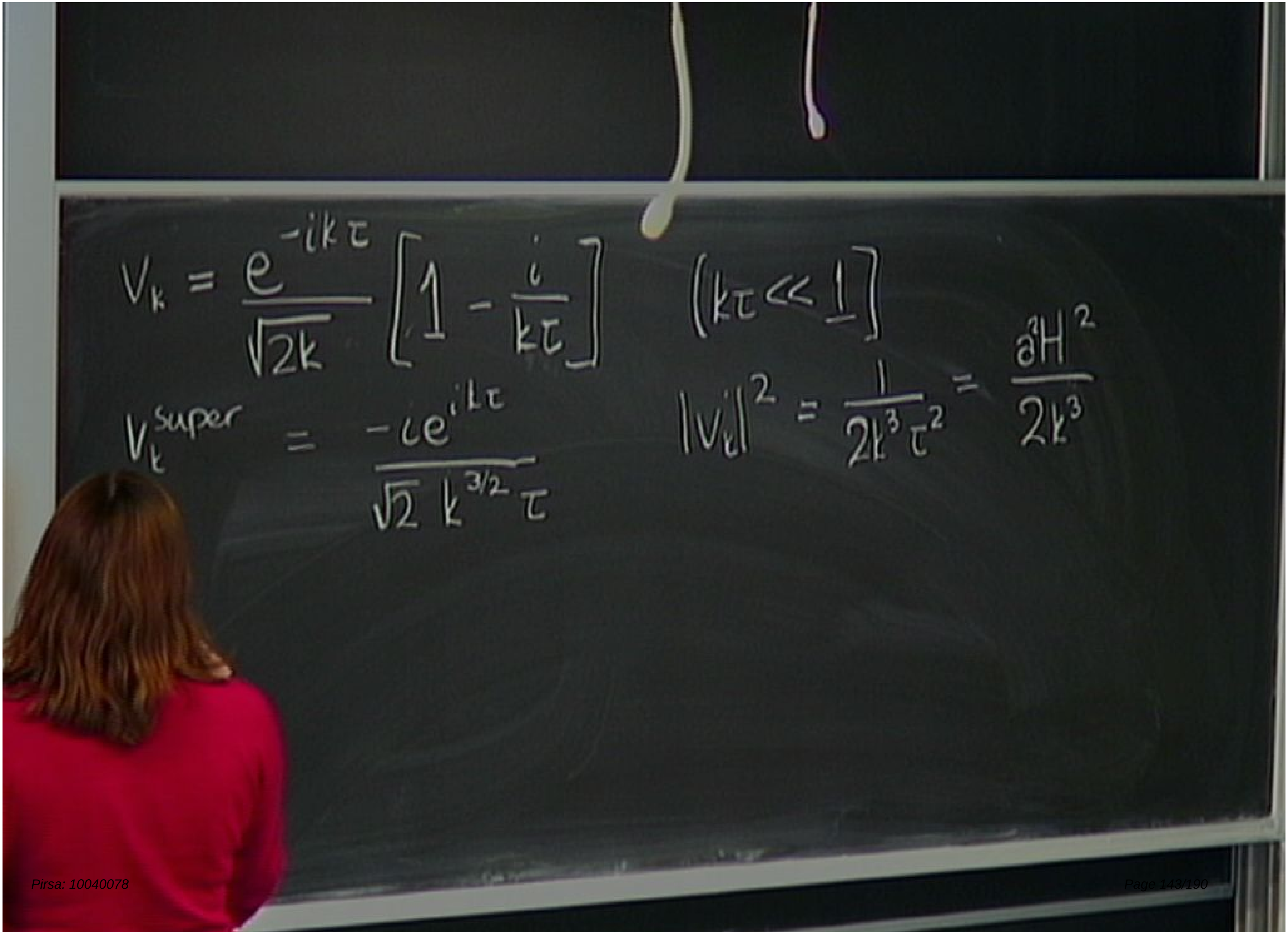
$$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$$

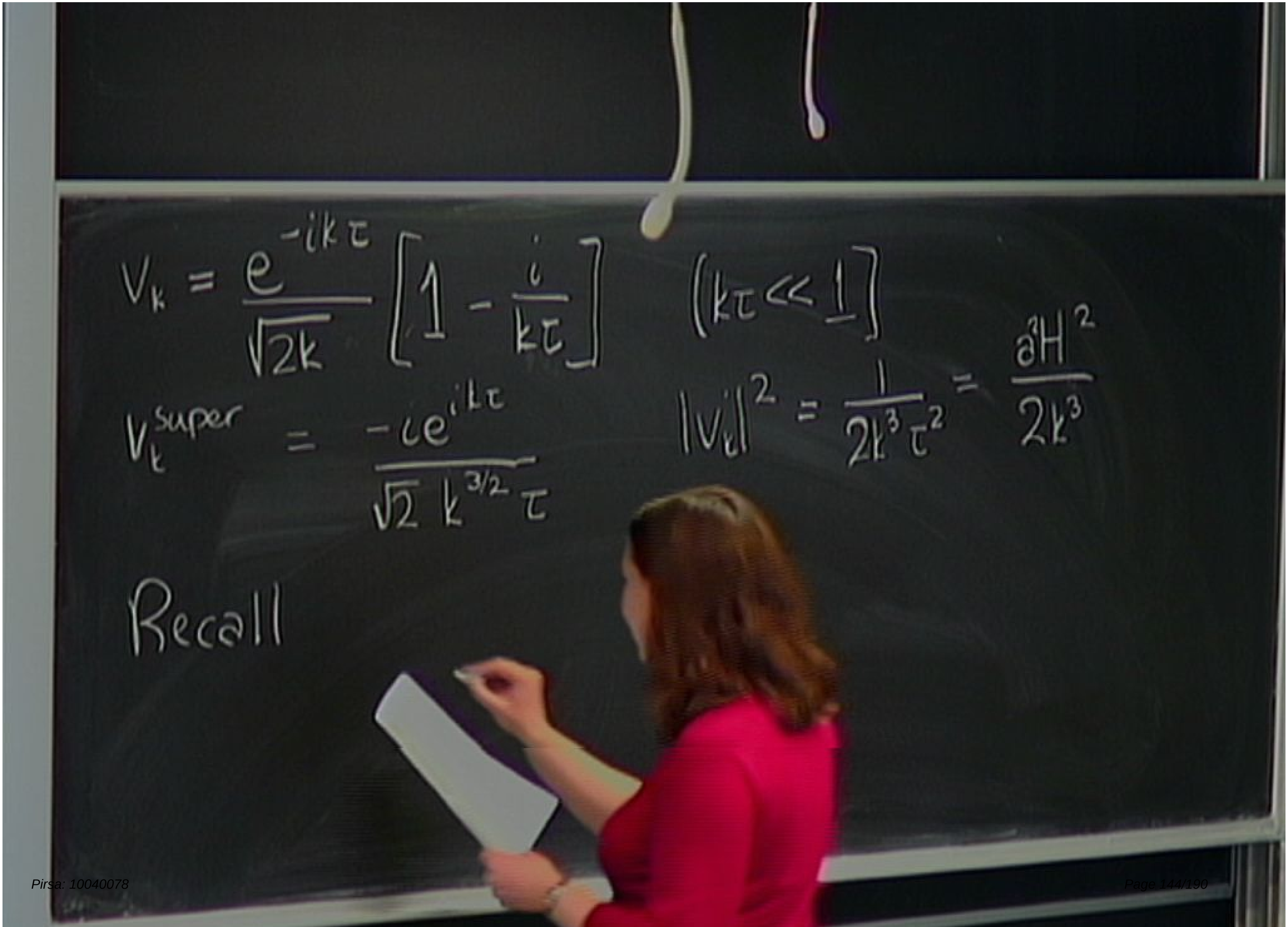

$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
$(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}}$

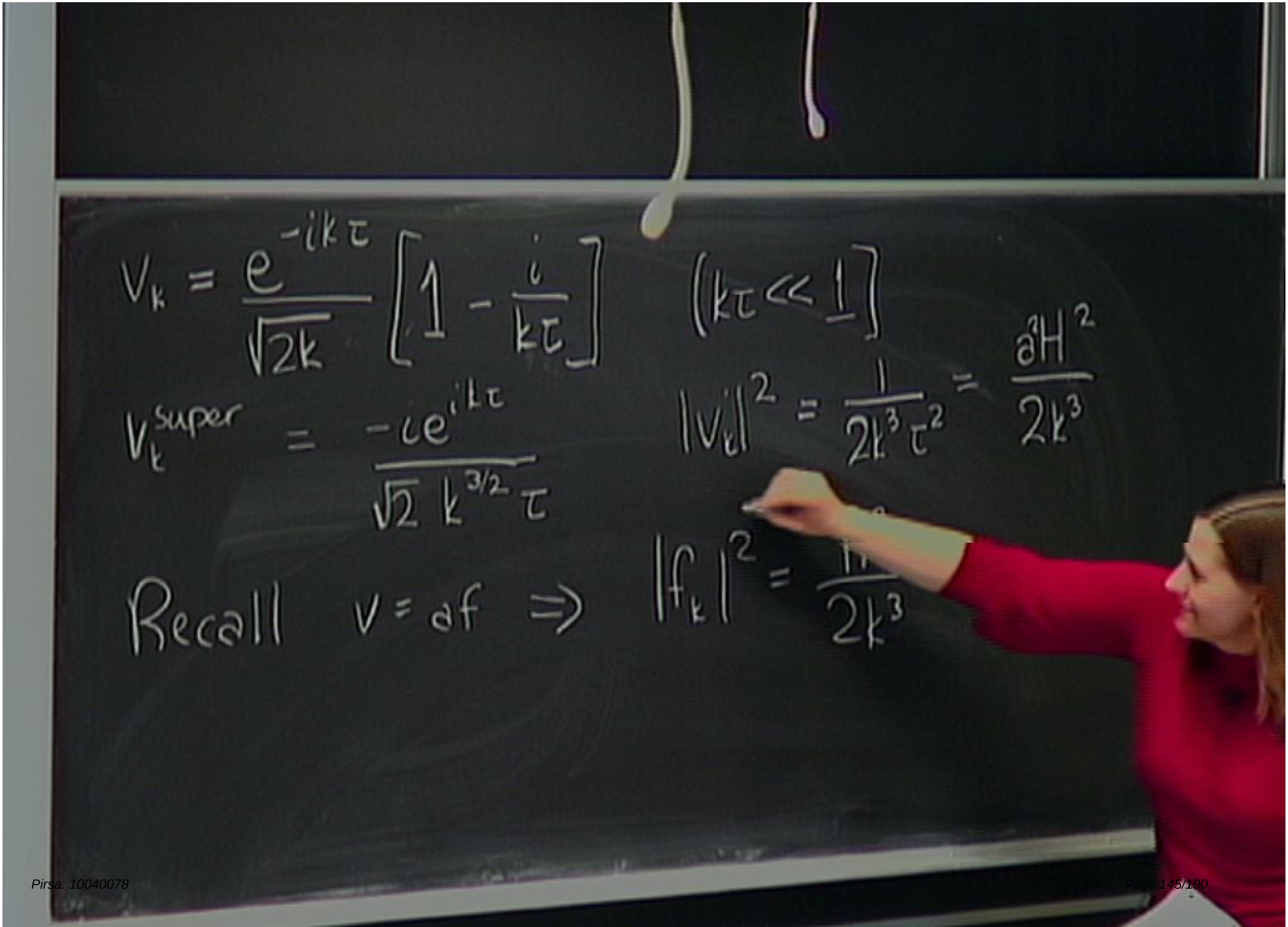

$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
$(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}}$



$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
$(k\tau << 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}\tau}$
$|v_k|^2 = \frac{1}{2k^3\tau^2} = a|$


$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
$(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}\tau}$
$|V_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$


$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
$(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}\tau}$
$|v_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$
Recall


$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
$(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\, k^{3/2}\tau}$
$|v_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$
Recall $v = af \Rightarrow$ $|f_k|^2 =$ $\frac{\ }{2k^3}$

$$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right] \qquad (k\tau \ll 1)$$

$$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}\,\tau} \qquad |V_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$$

Recall $v = af \Rightarrow$ $\boxed{|f_k|^2 = \frac{H^2}{2k^3}}$

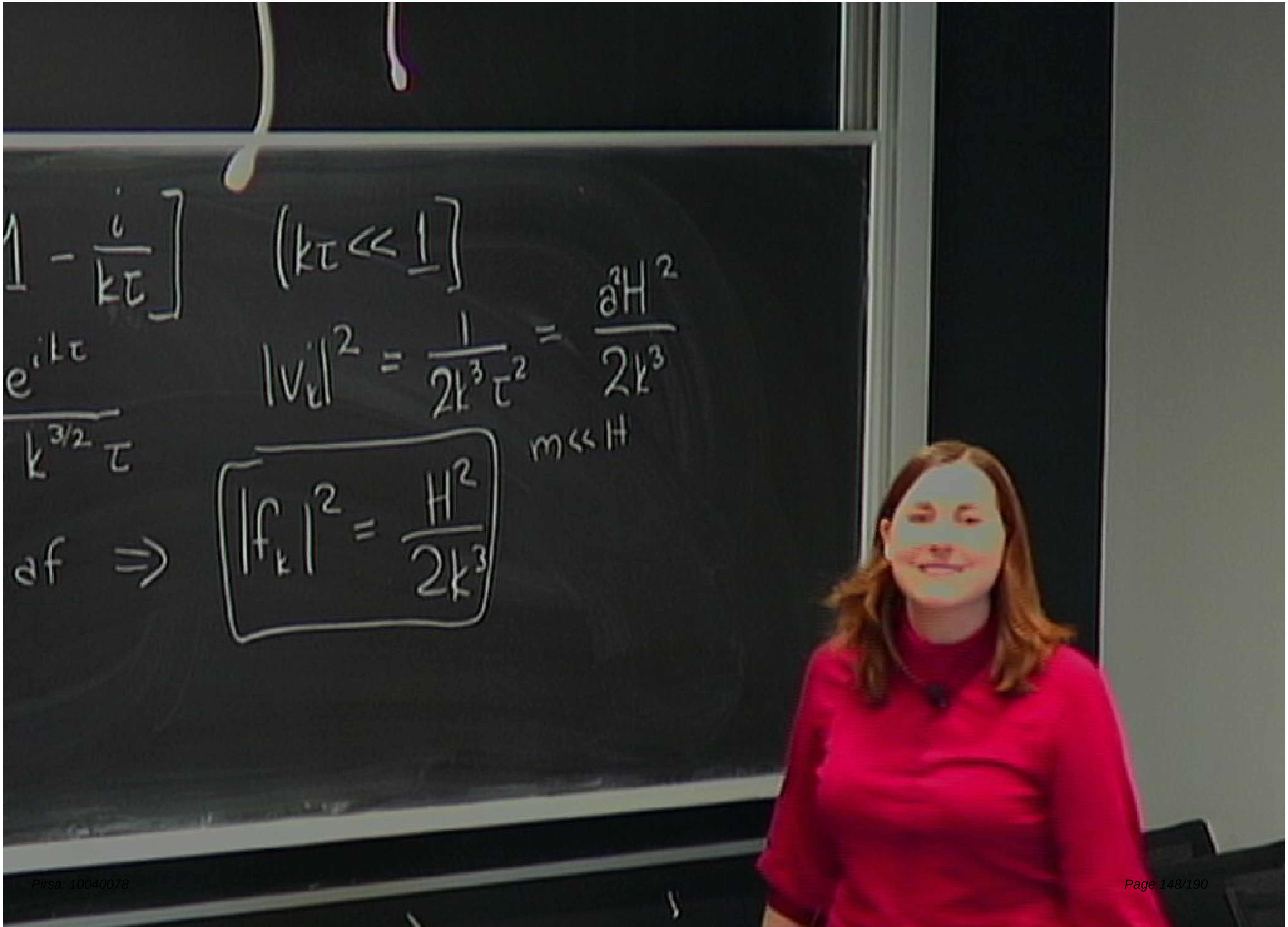
$$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right] \quad (k\tau \ll 1)$$

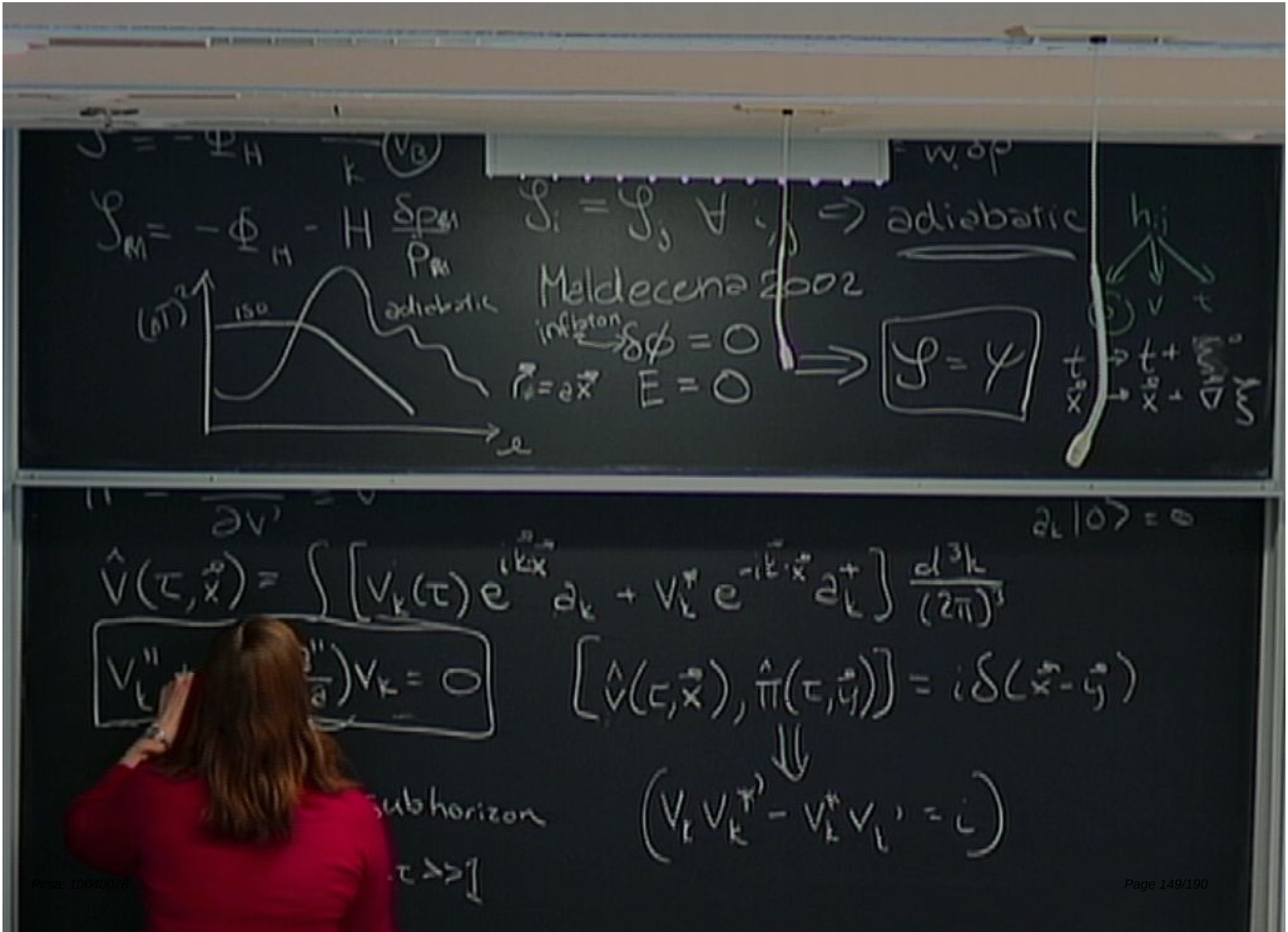
$$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\, k^{3/2}\,\tau}$$

$$|v_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$$

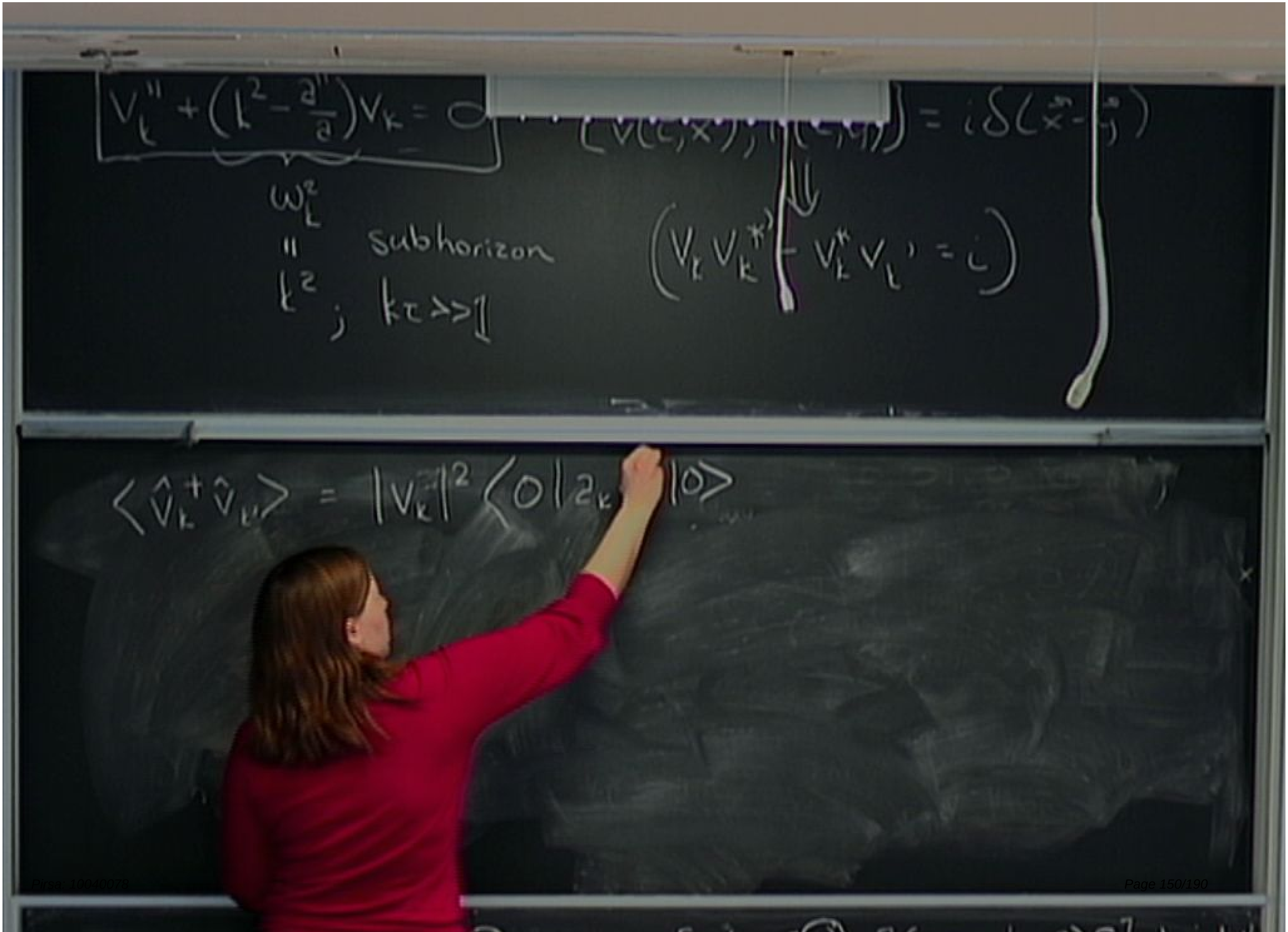
$m \ll H$

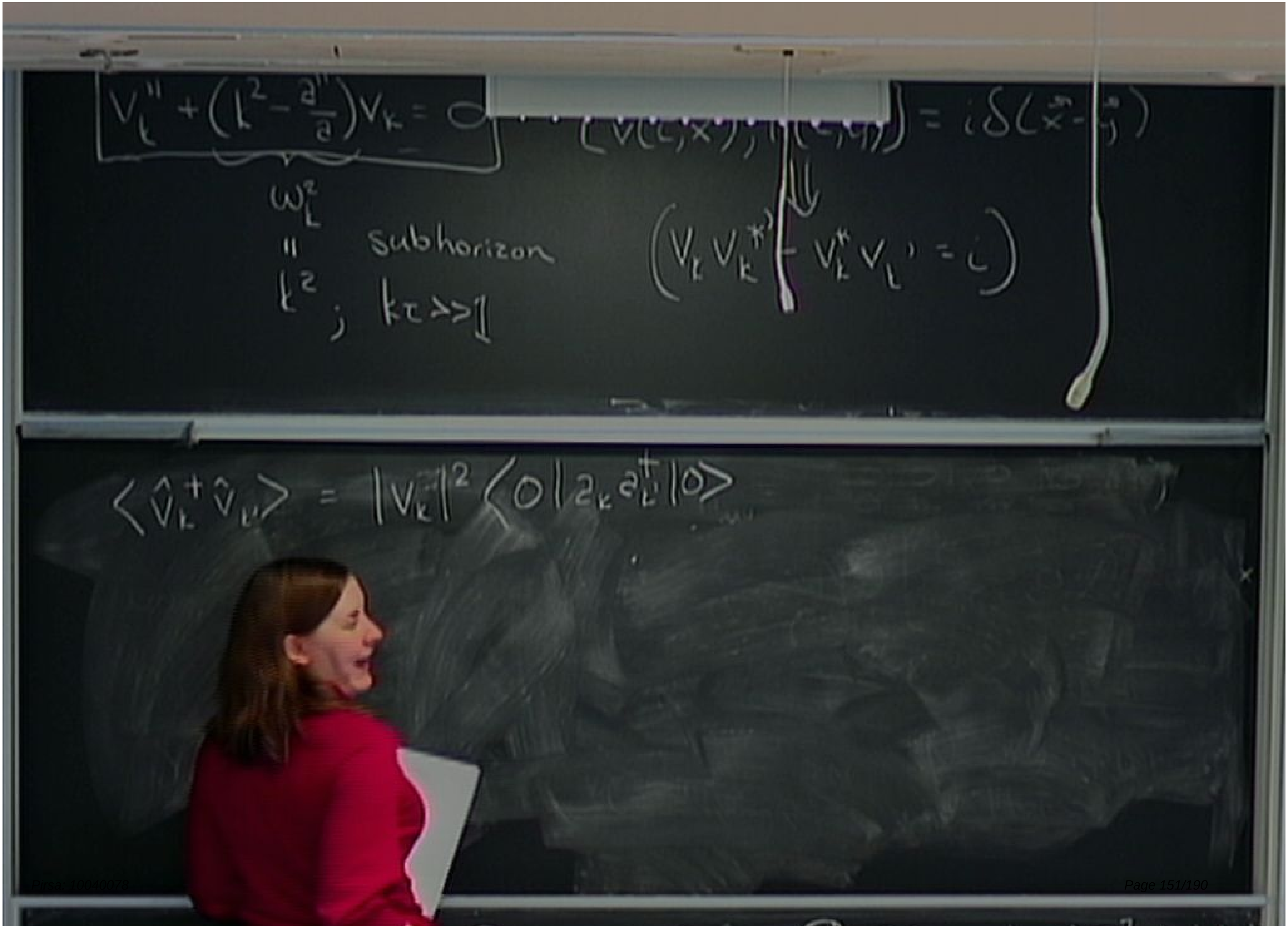
Recall $v = af \Rightarrow$ $$\boxed{|f_k|^2 = \frac{H^2}{2k^3}}$$

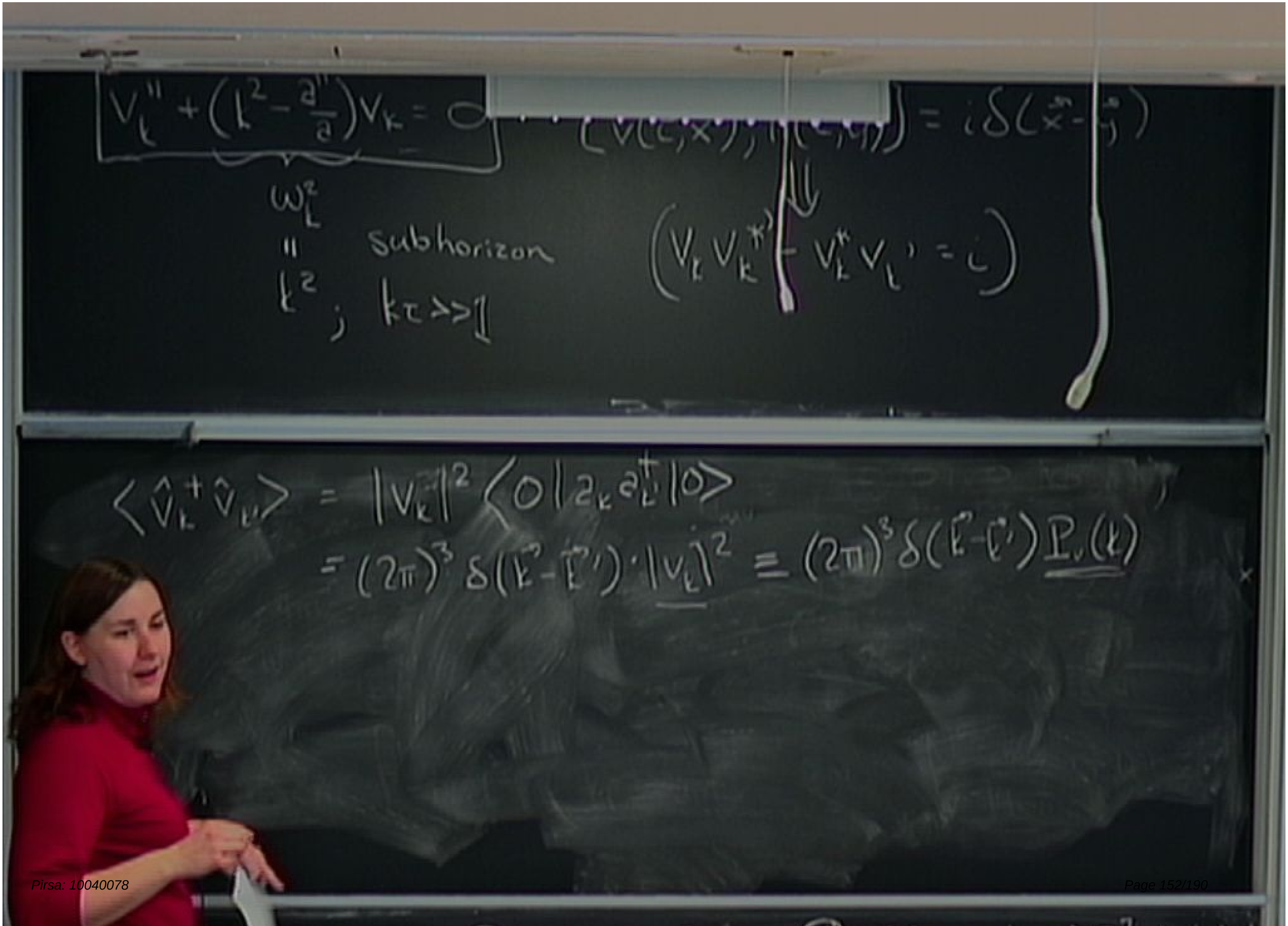

(kτ << 1)
m << H



adiabatic
Maldecena 2002
inflaton
δφ = 0
E = 0
iso
adiabatic
subhorizon


V_k'' + (k^2 - a''/a) V_k = 0
= iδ(x - y)
ω_k^2
subhorizon
k^2 ; kτ >> 1
(V_k V_k^*' - V_k^* V_k' = i)
⟨v̂_k^† v̂_k⟩ = |V_k|^2 ⟨0| a_k |0⟩


v_k'' + (k^2 - a''/a) v_k = 0
ω_k^2
subhorizon
k^2 ; kτ >> 1
= iδ(x - y)
(v_k v_k*' - v_k* v_k' = i)
⟨v̂_k^† v̂_k'⟩ = |v_k|^2 ⟨0| a_k a_k'^† |0⟩



v_k'' + (k^2 - a''/a) v_k = 0
ω_k^2
= subhorizon
k^2 ; kτ >> 1
[v(τ, x), π(τ, y)] = i δ(x - y)
(v_k v_k*' - v_k* v_k' = i)
⟨v̂_k^† v̂_k'⟩ = |v_k|^2 ⟨0| a_k a_k'^† |0⟩
= (2π)^3 δ(k - k') · |v_k|^2 ≡ (2π)^3 δ(k - k') P_v(k)

$$\boxed{v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0}$$

ω_k^2 " k^2 ; $k\tau \gg 1$ subhorizon

$[v(\tau,\vec{x}), \pi(\tau,\vec{y})] = i\delta(\vec{x}-\vec{y})$

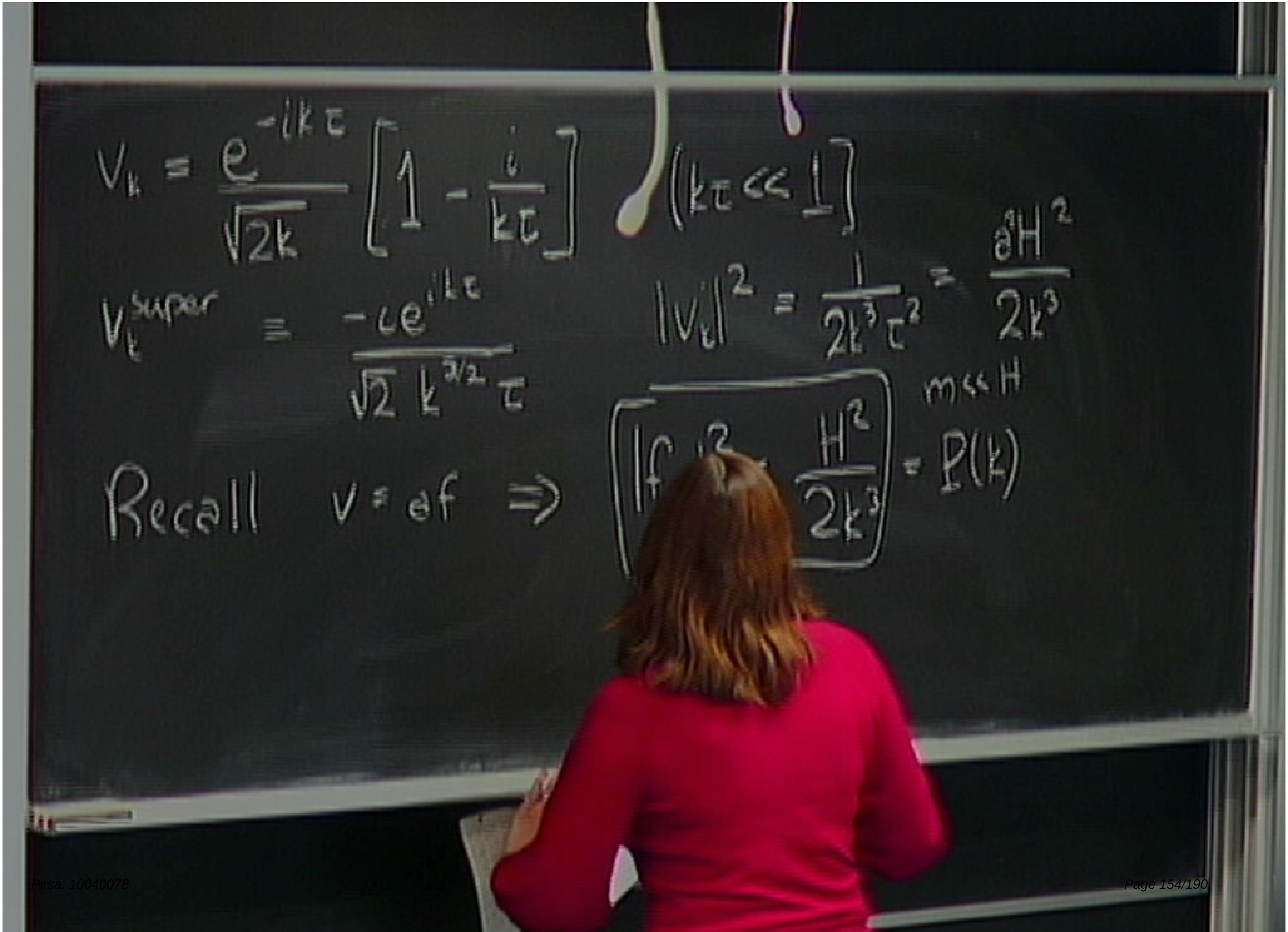
$\Downarrow$

$\left(v_k v_k^{*\prime} - v_k^* v_k' = i \right)$

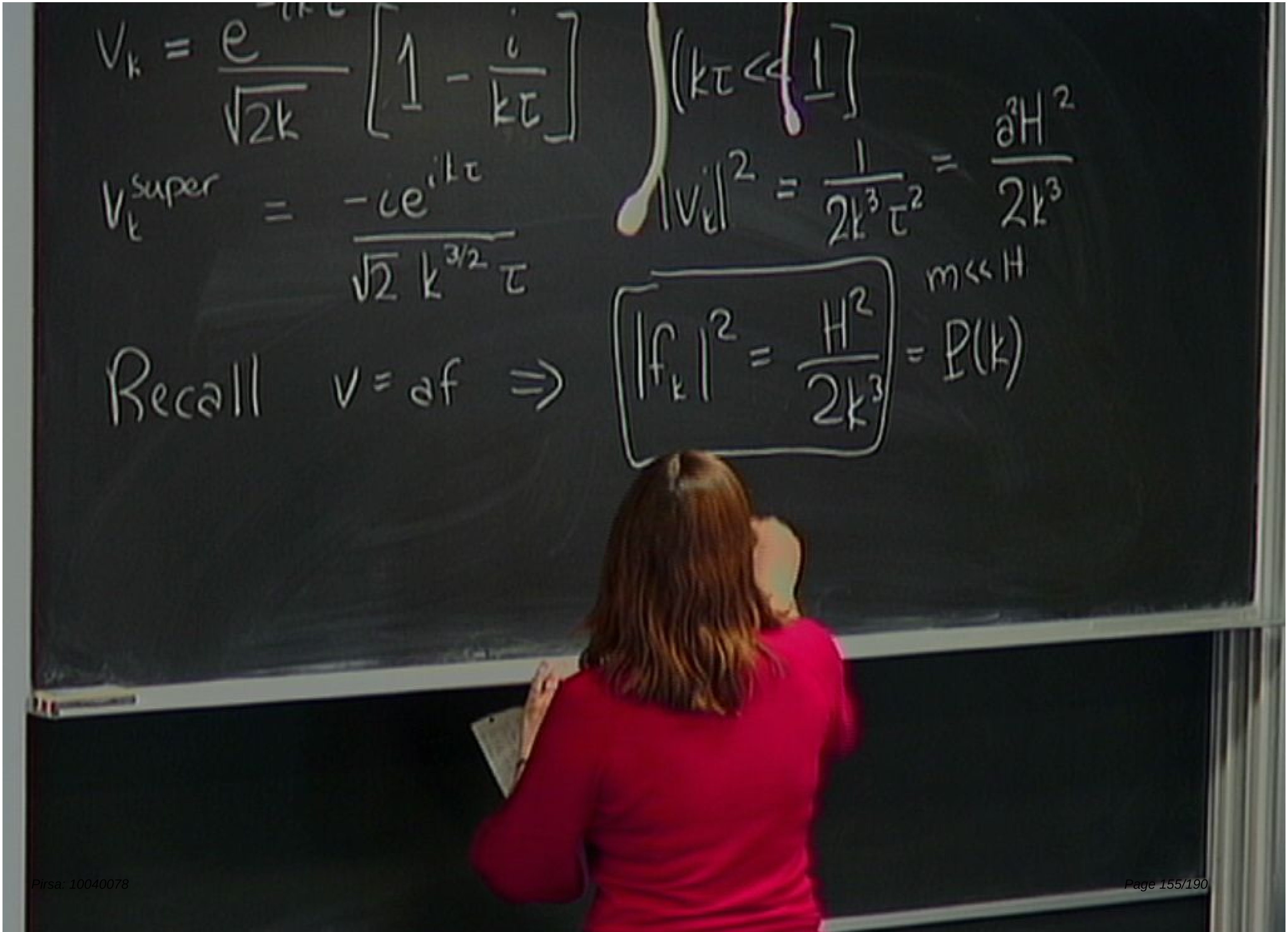
$$\langle \hat{v}_k^\dagger \hat{v}_{k'} \rangle = |v_k|^2 \langle 0 | a_k a_{k'}^\dagger | 0 \rangle$$

$$= (2\pi)^3 \delta(\vec{k}-\vec{k}') \cdot |v_k|^2 \equiv (2\pi)^3 \delta(\vec{k}-\vec{k}') \underline{P_v(k)}$$

$$\langle \hat{v}(\vec{x}) \hat{v}(\vec{y}) \rangle = \int \underbrace{\frac{P_v(k) k^3}{2\pi^2}}_{\mathcal{P}(k)} \frac{\sin(kr)}{kr} \frac{dk}{k}$$



$V_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$ $(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}\,\tau}$
$|V_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$
Recall $v = af \Rightarrow$ $|f_k|^2 = \frac{H^2}{2k^3} = P(k)$
$m \ll H$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$(k\tau \ll 1)$
$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\,k^{3/2}\tau}$
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$$V_k^{super} = \frac{-ie^{ik\tau}}{\sqrt{2}\, k^{3/2}\, \tau}$$

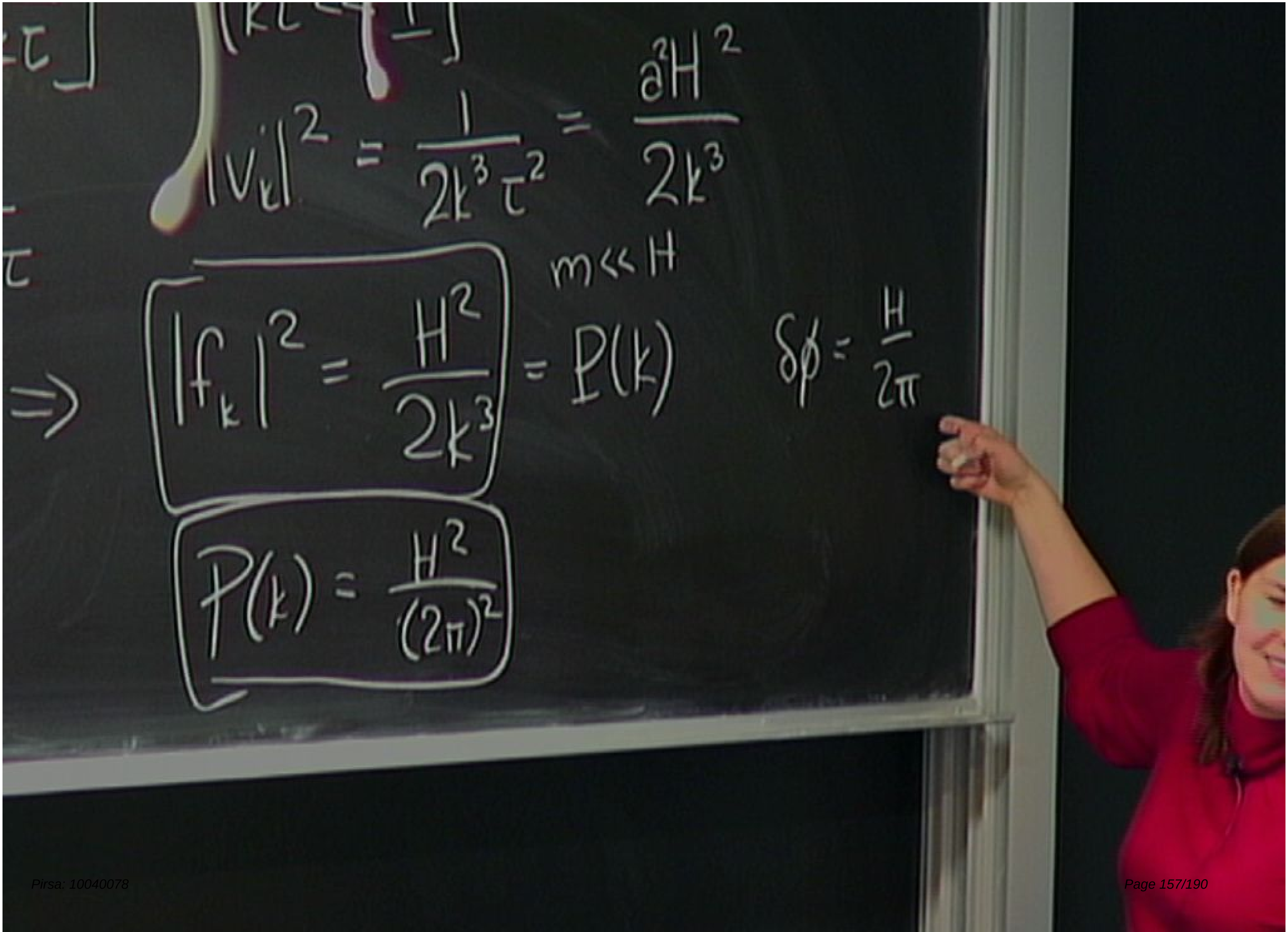
$$|V_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$$

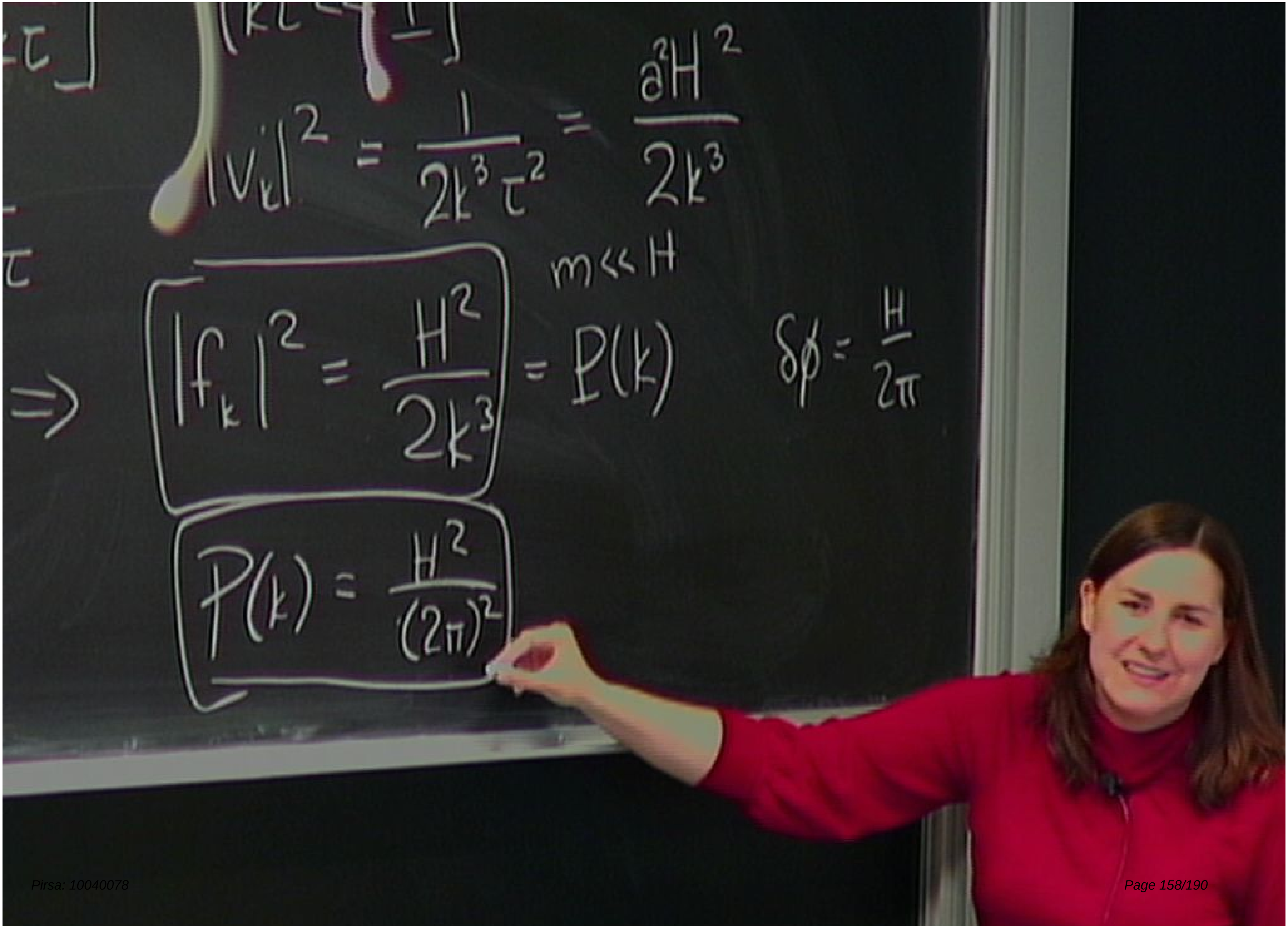
$m \ll H$

Recall $v = af \Rightarrow$

$$\boxed{|f_k|^2 = \frac{H^2}{2k^3}} = P(k)$$

$$\mathcal{P}(k) = \frac{H^2}{(2\pi)^2}$$


$|v_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$
$m \ll H$
$|f_k|^2 = \frac{H^2}{2k^3} = P(k)$
$\delta\phi = \frac{H}{2\pi}$
$P(k) = \frac{H^2}{(2\pi)^2}$


$|v_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$
$m \ll H$
$\Rightarrow |f_k|^2 = \frac{H^2}{2k^3} = P(k)$
$\delta\phi = \frac{H}{2\pi}$
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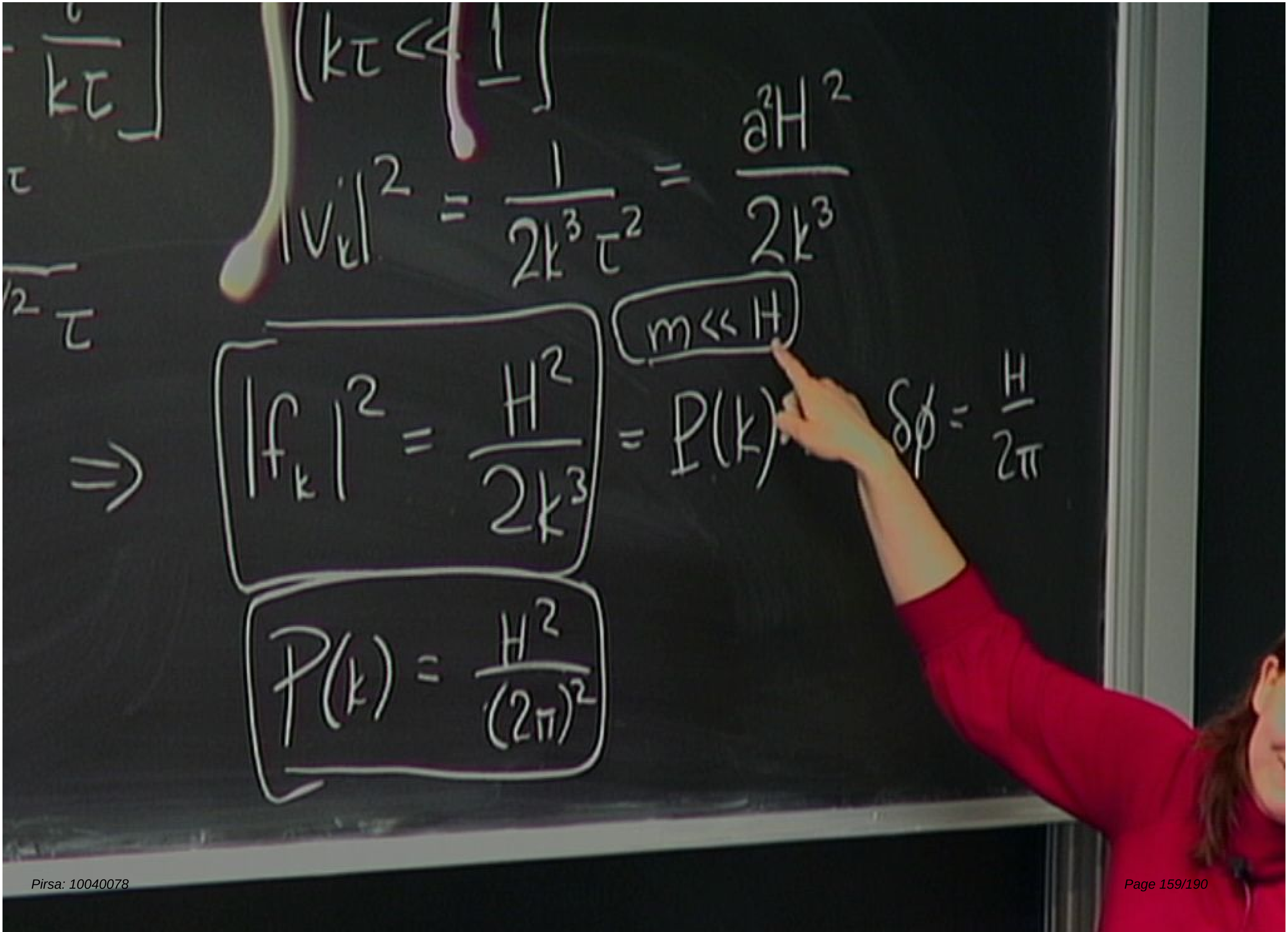
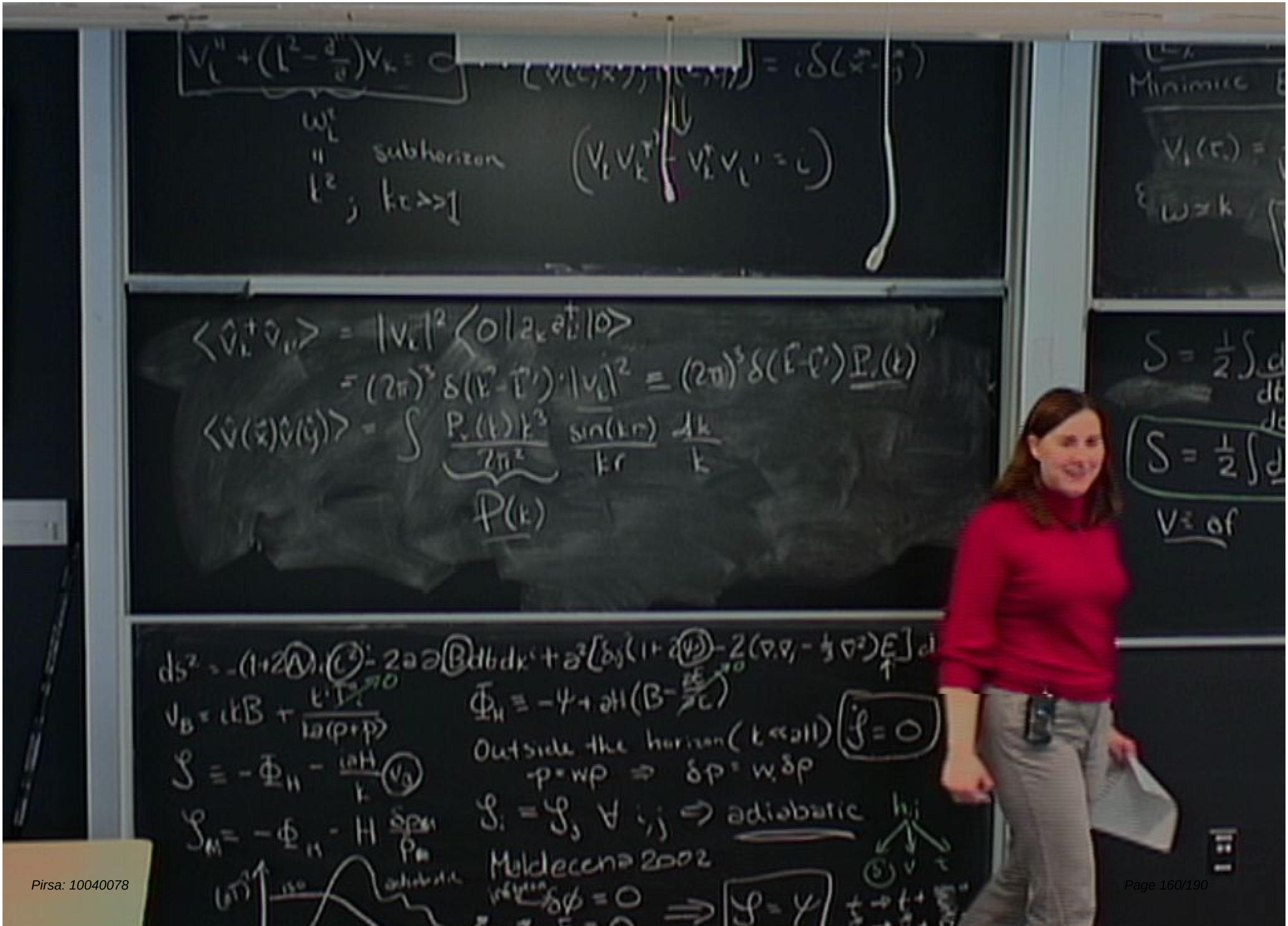
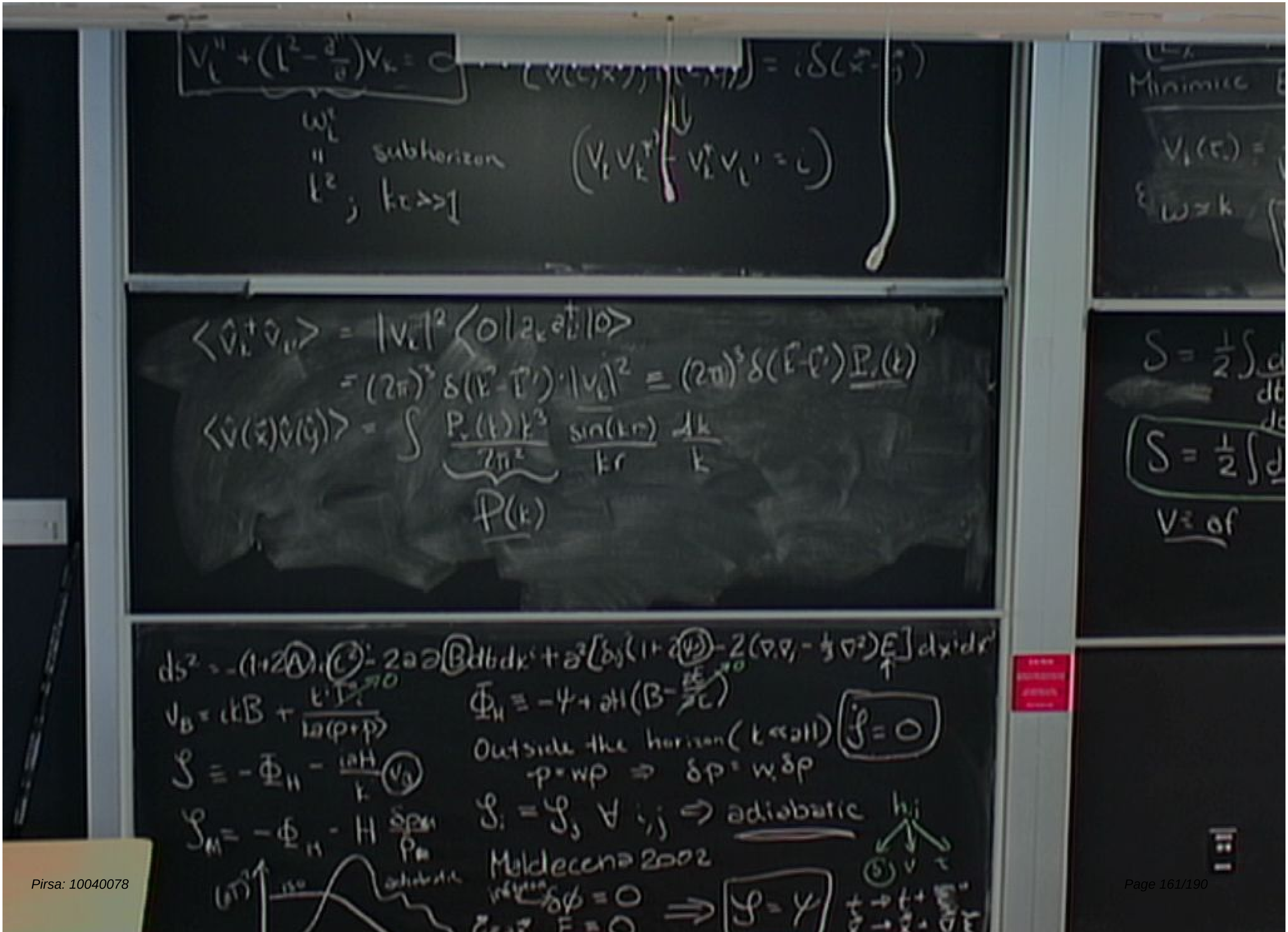
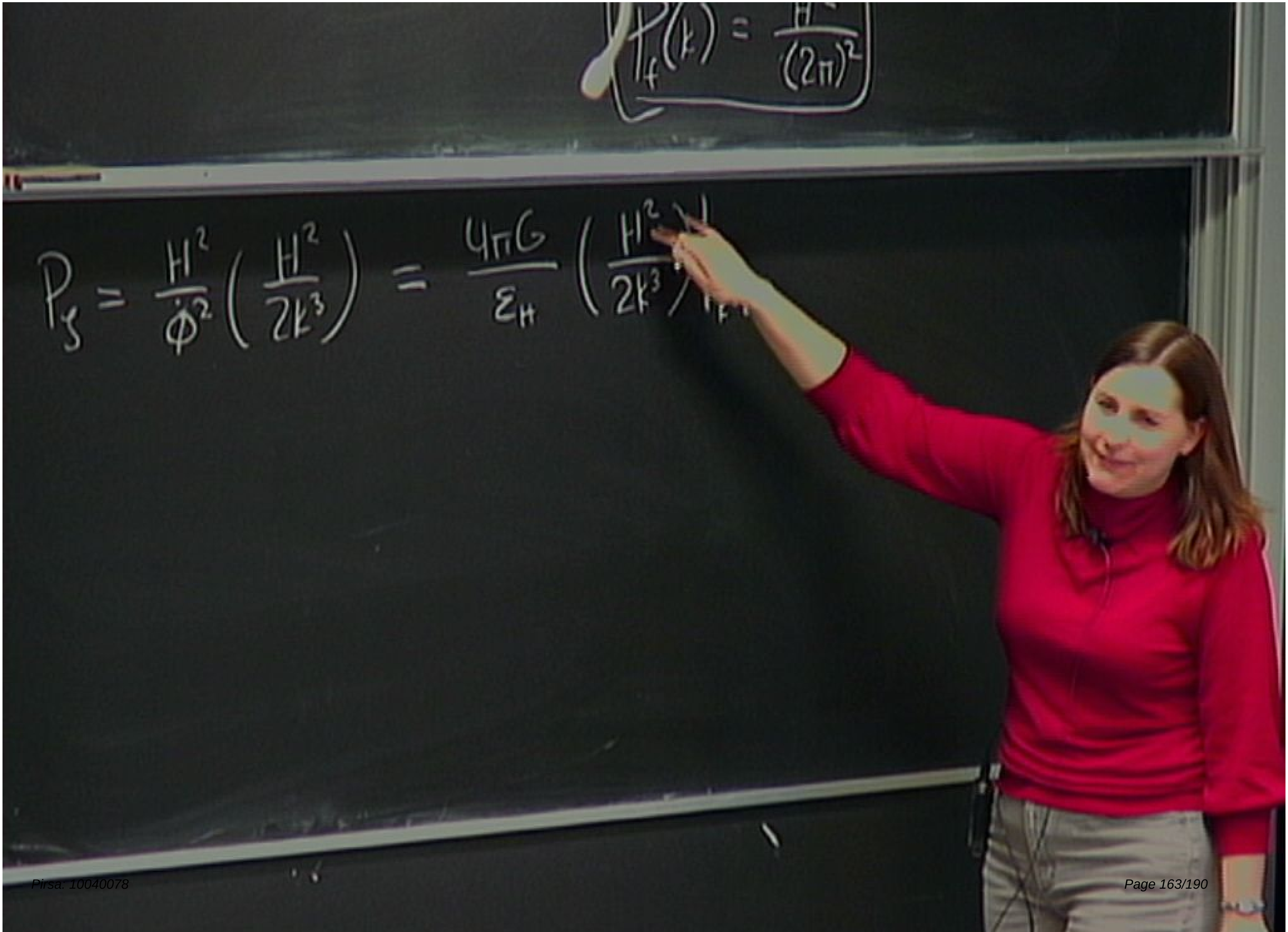
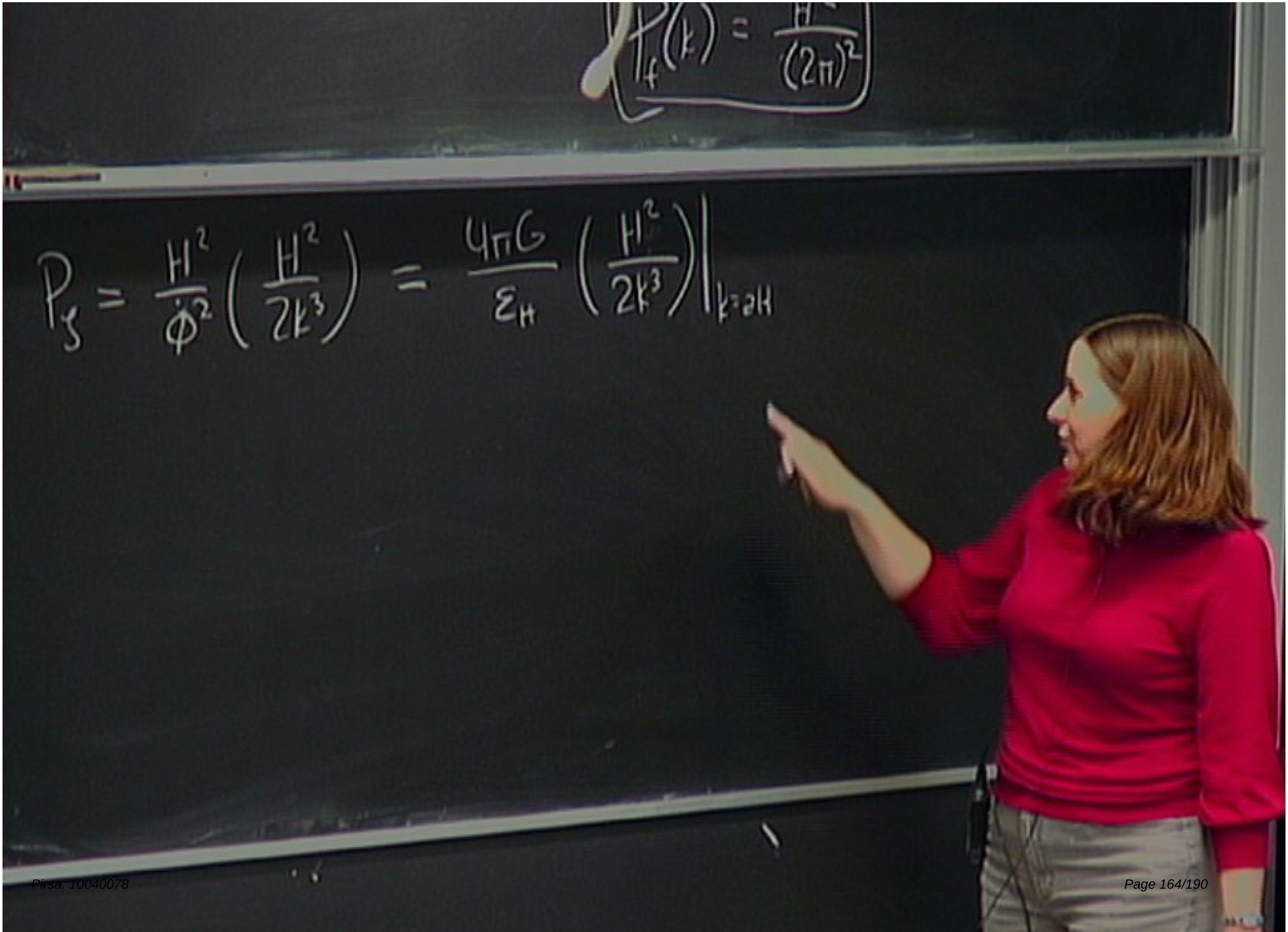

$(k\tau \ll 1)$
$|v_k|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3}$
$m \ll H$
$\Rightarrow |f_k|^2 = \frac{H^2}{2k^3} = P(k)$
$\delta\phi = \frac{H}{2\pi}$
$P(k) = \frac{H^2}{(2\pi)^2}$

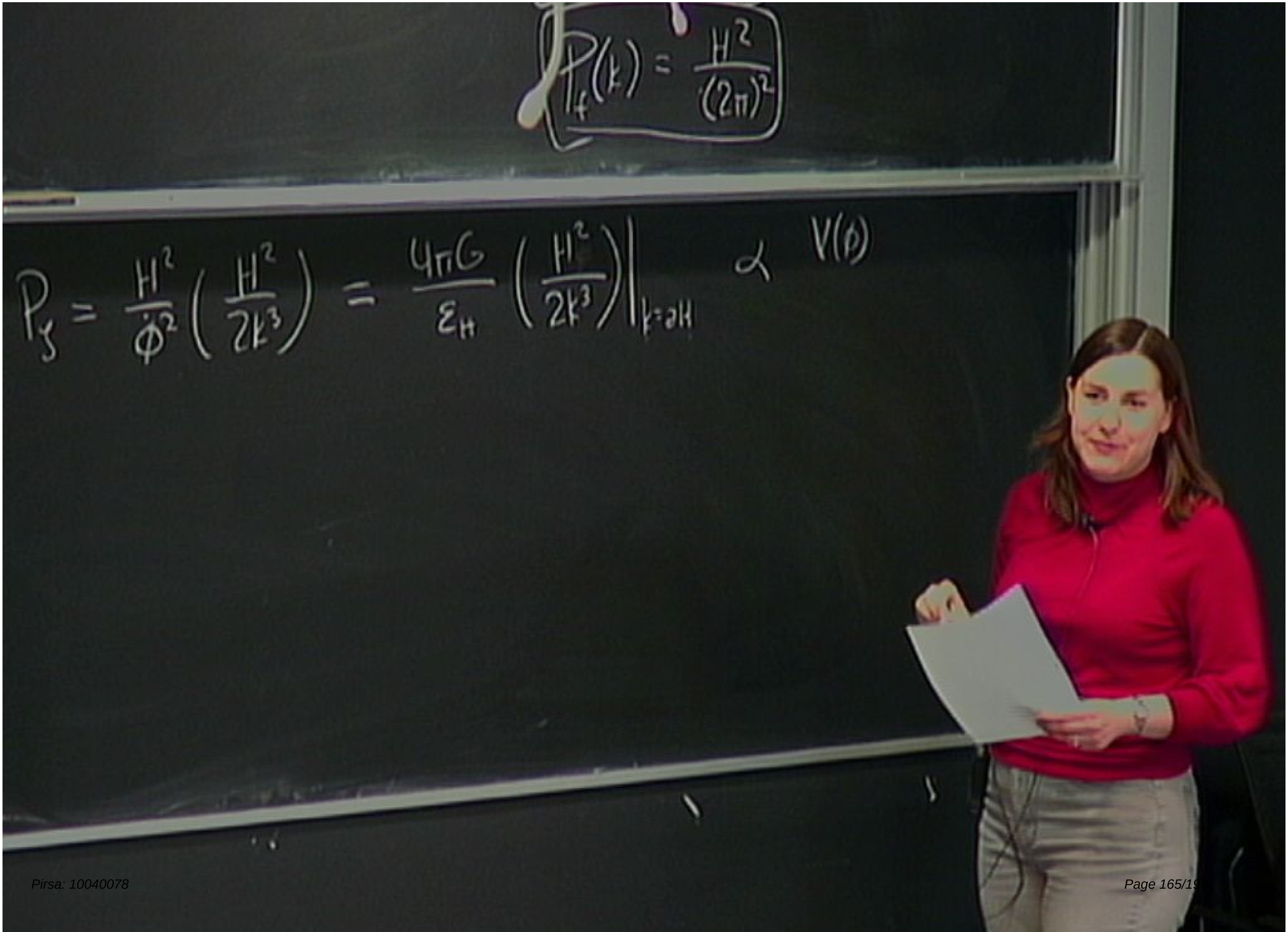

subhorizon
Outside the horizon
adiabatic
Maldecena 2002

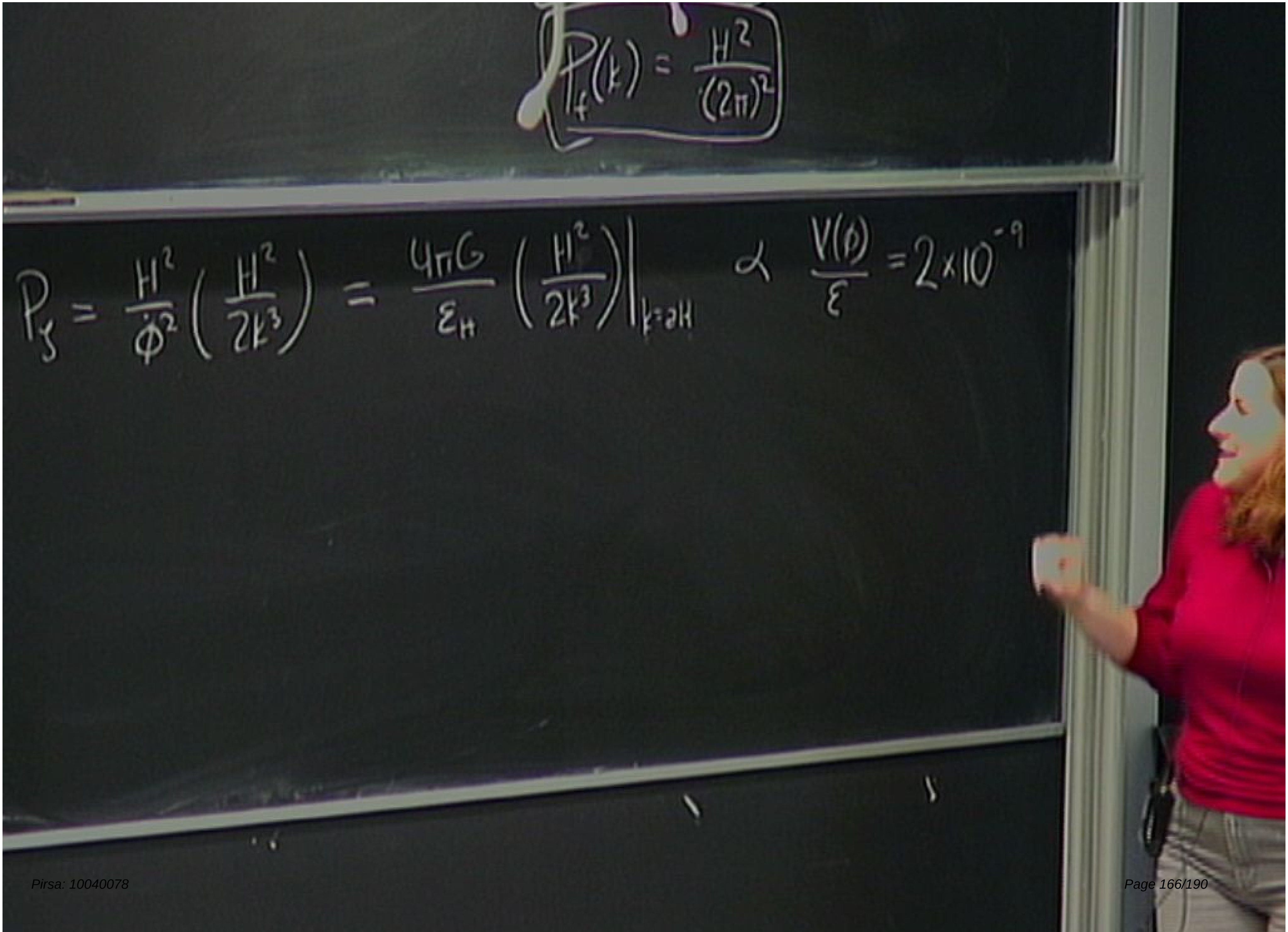

subhorizon
Minimize
Outside the horizon
adiabatic
Maldecena 2002

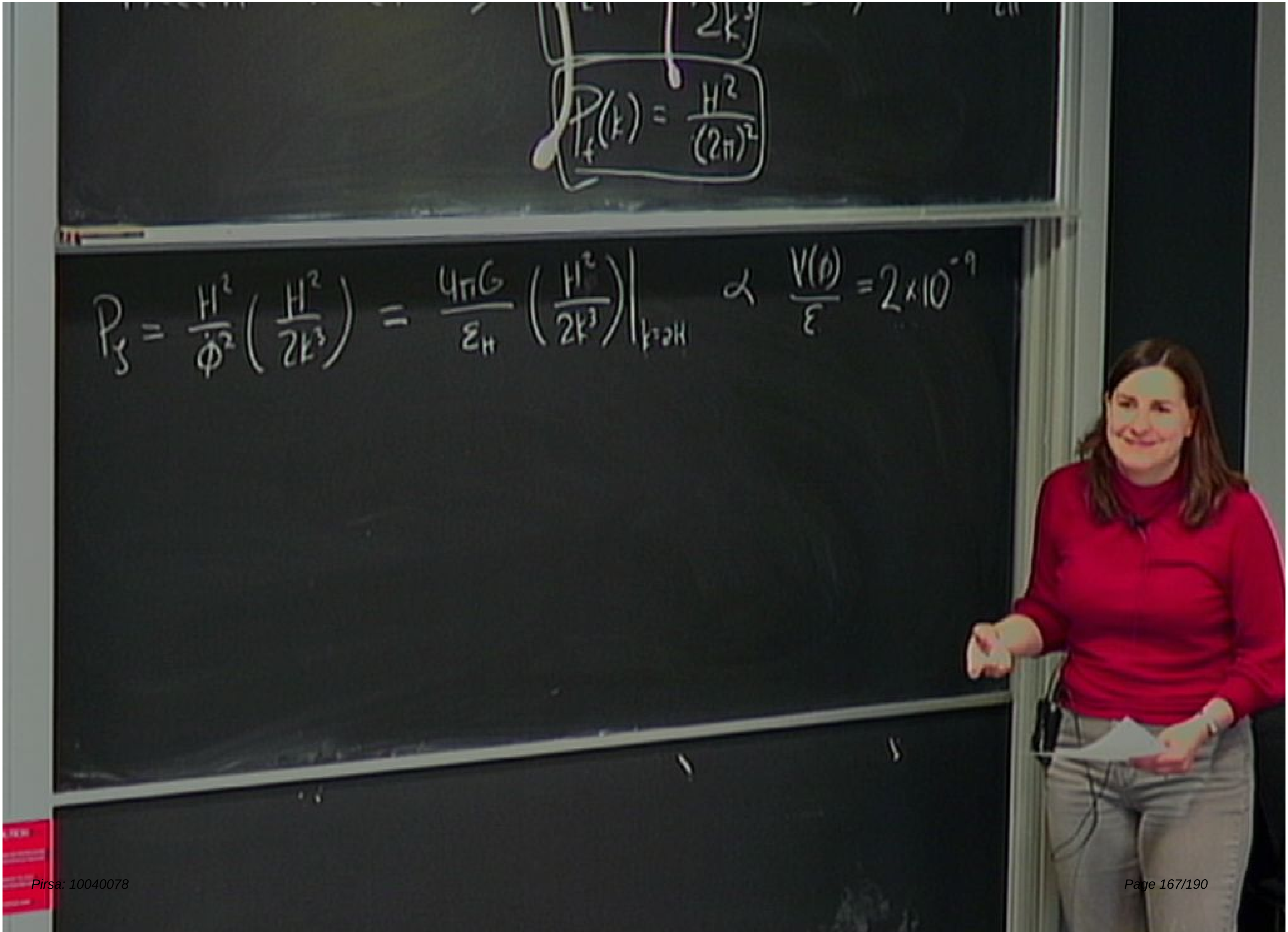
$$P_\zeta = \frac{H^2}{\dot{\phi}^2}\left(\frac{H^2}{2k^3}\right)$$


$P_t(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)$


$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta = \frac{H^2}{\dot{\phi}^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH}$


$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \propto V(\phi)$

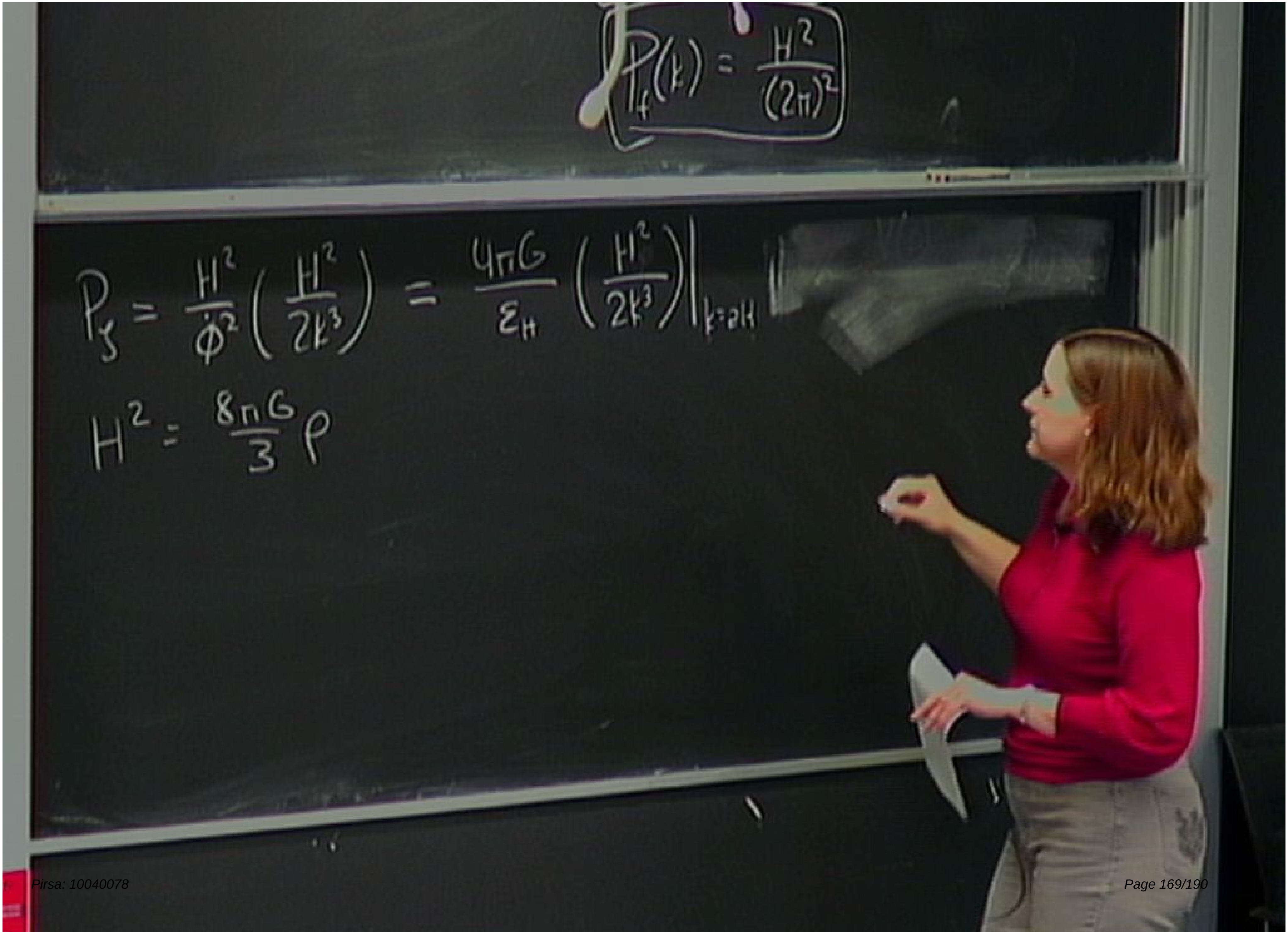

$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta = \frac{H^2}{\dot{\phi}^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \propto \frac{V(\phi)}{\varepsilon} = 2\times 10^{-9}$


2k³
P_φ(k) = H²/(2π)²
P_ζ = H²/φ̇² (H²/2k³) = 4πG/ε_H (H²/2k³)|_{k=aH} ∝ V(φ)/ε = 2×10⁻⁹

$$\boxed{P_\phi(k) = \frac{H^2}{(2\pi)^2}}$$

$$P_\zeta = \frac{H^2}{\dot{\phi}^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Bigg|_{k=aH} \propto \frac{V(\phi)}{\varepsilon} = 2\times 10^{-9}$$

$$H^2 = \frac{8\pi G}{3}\rho$$


$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH}$
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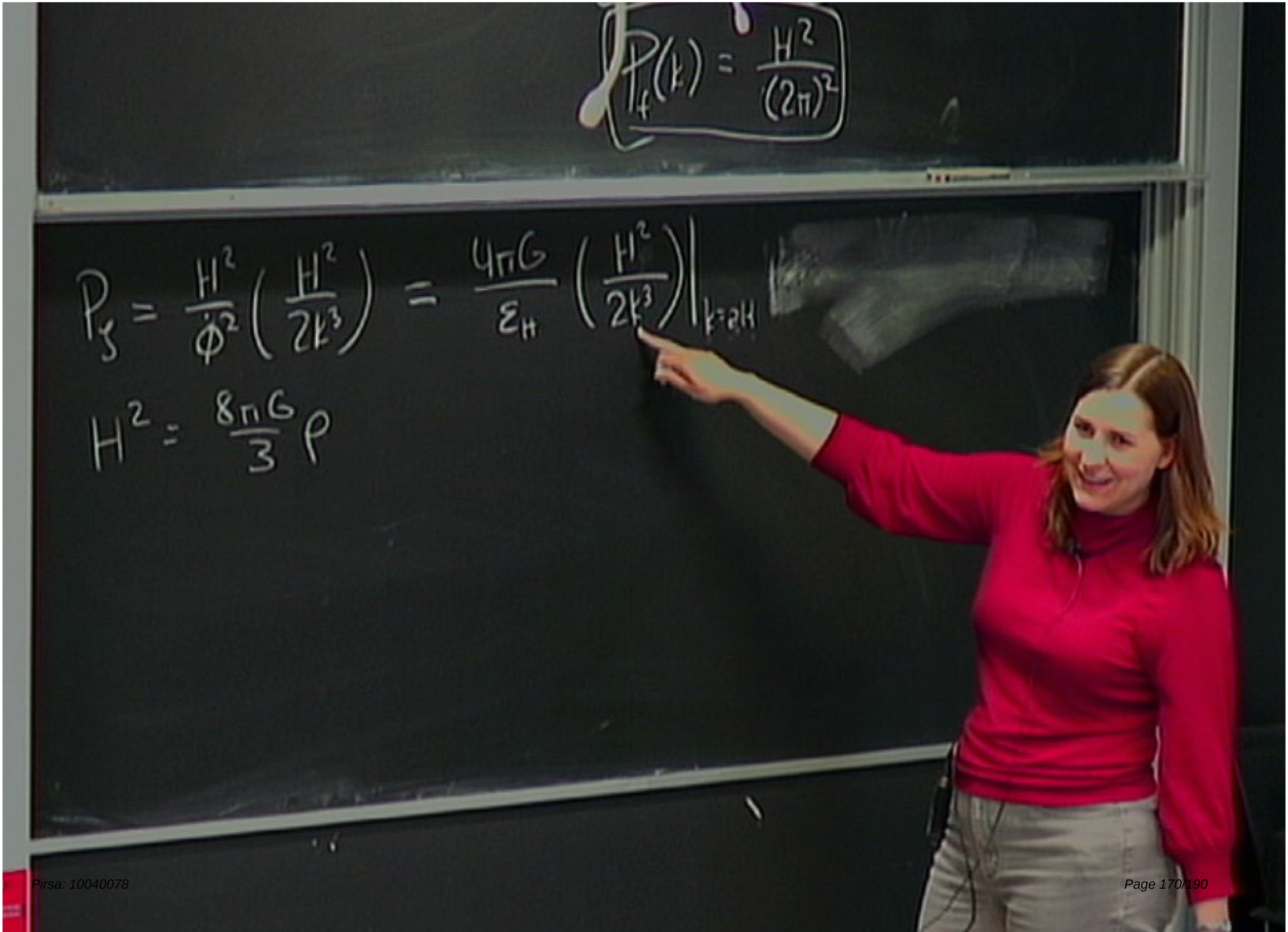
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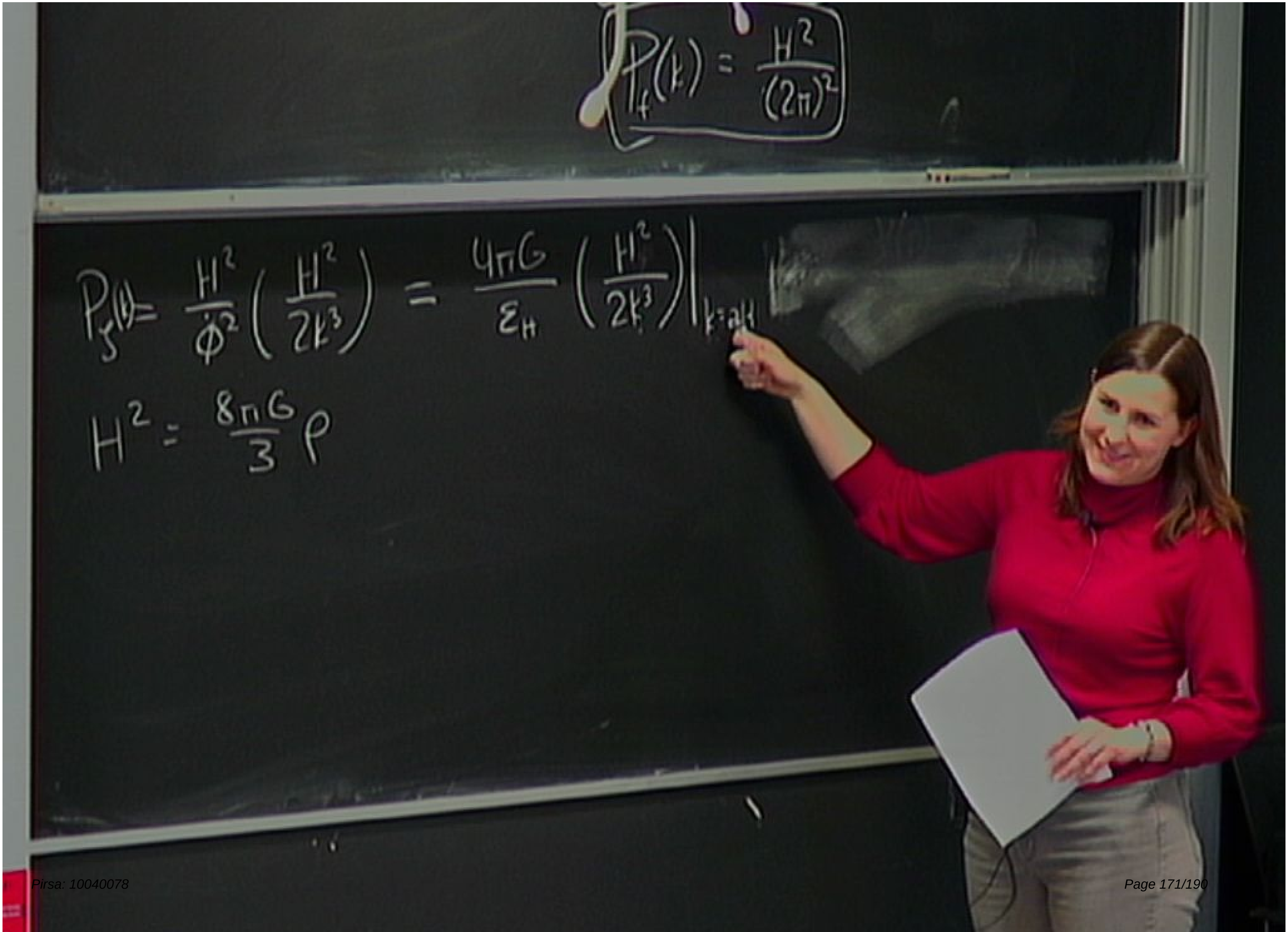
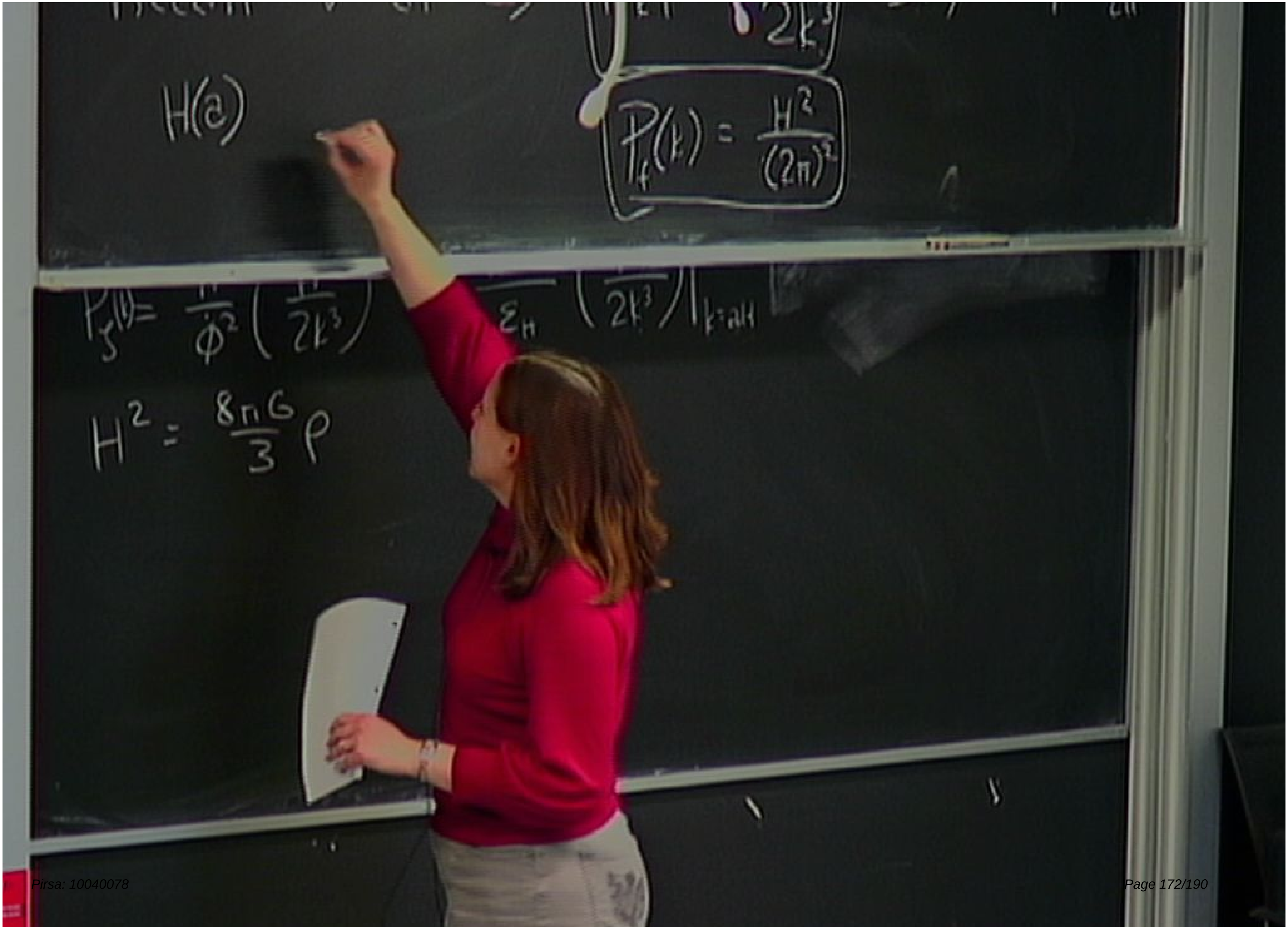
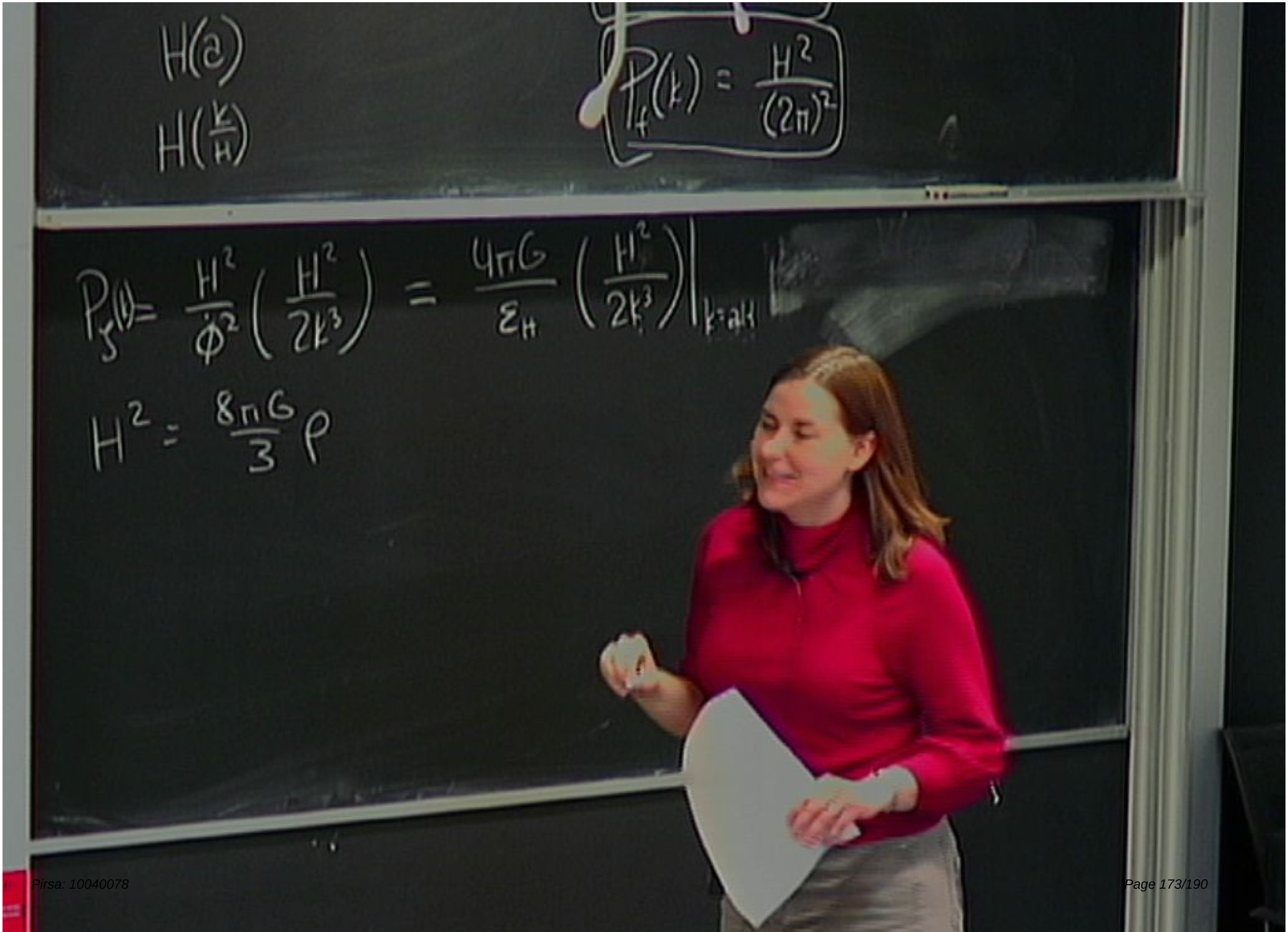
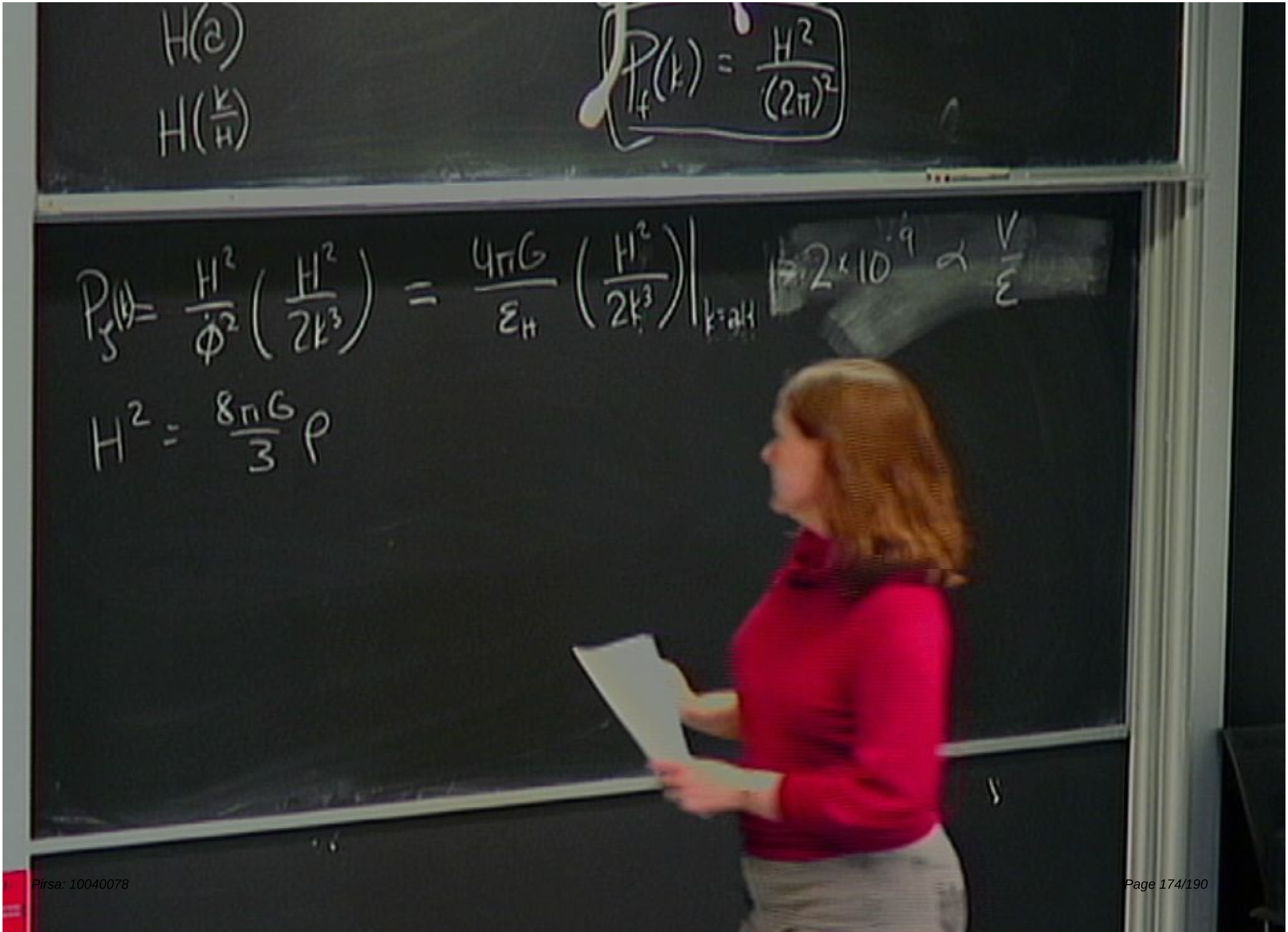
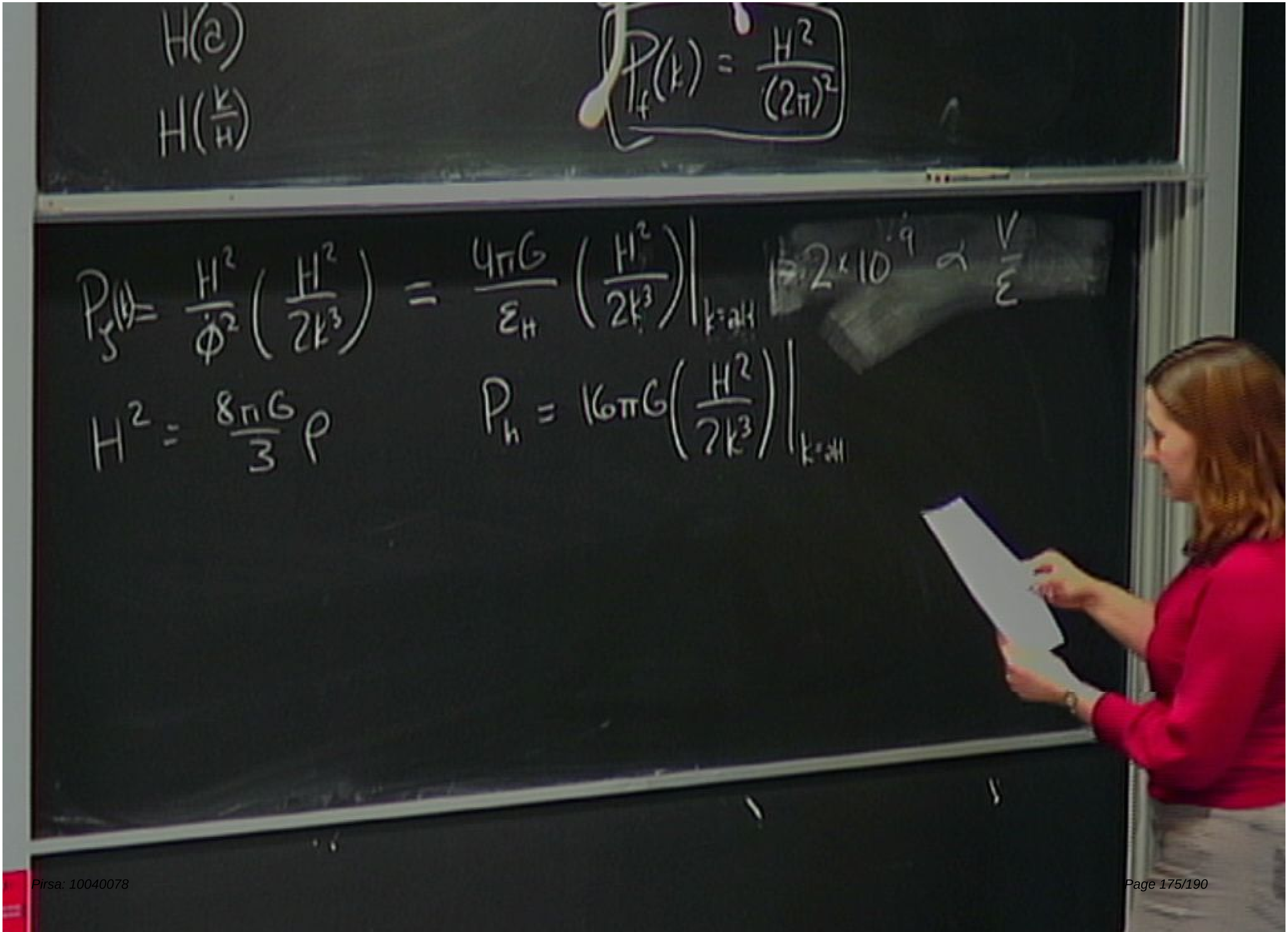
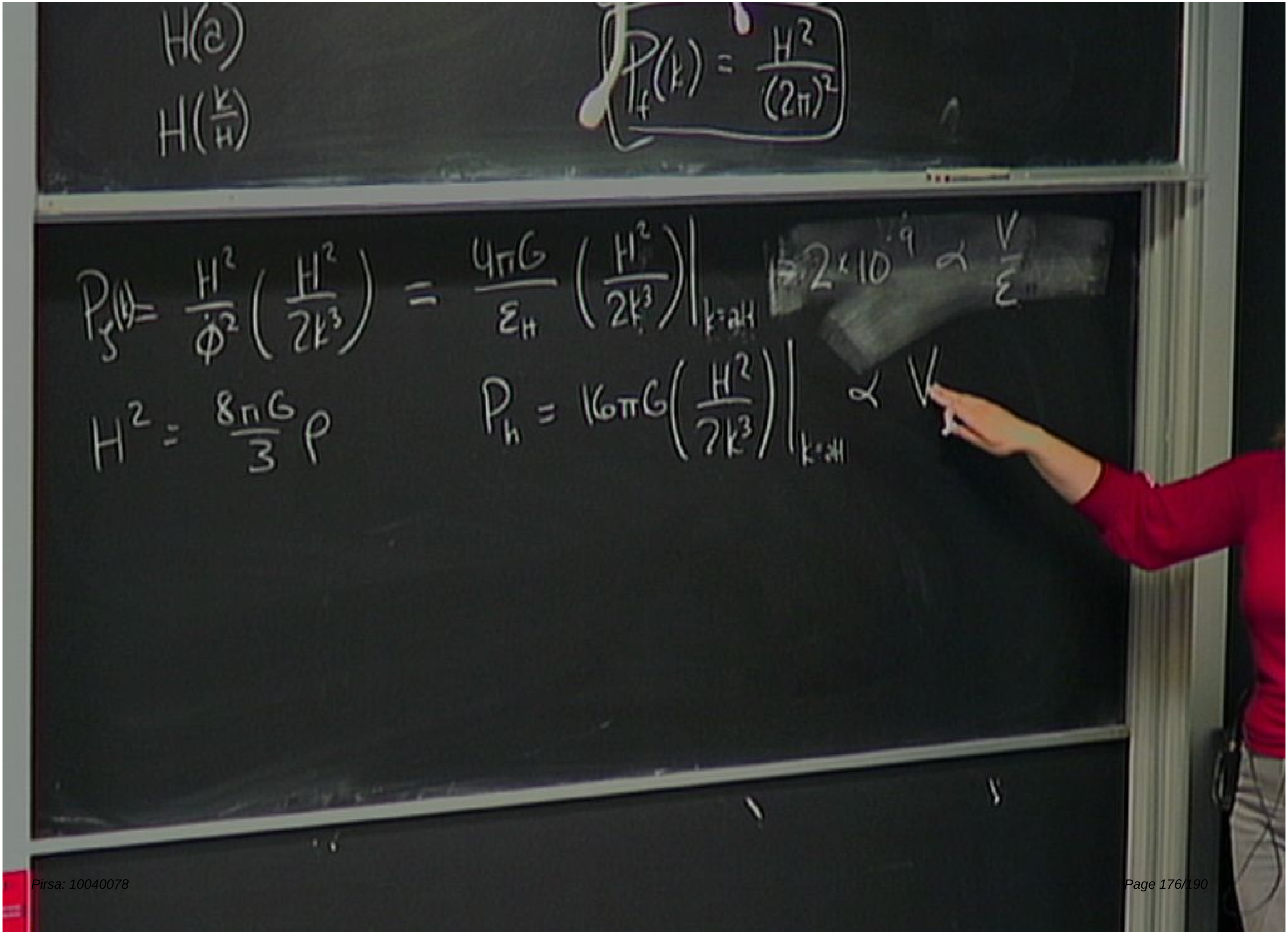

$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta(k) = \frac{H^2}{\dot{\phi}^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH}$
$H^2 = \frac{8\pi G}{3}\rho$


H(a)
$P_\phi(k) = \frac{H^2}{(2\pi)^2}$
$H^2 = \frac{8\pi G}{3} \rho$

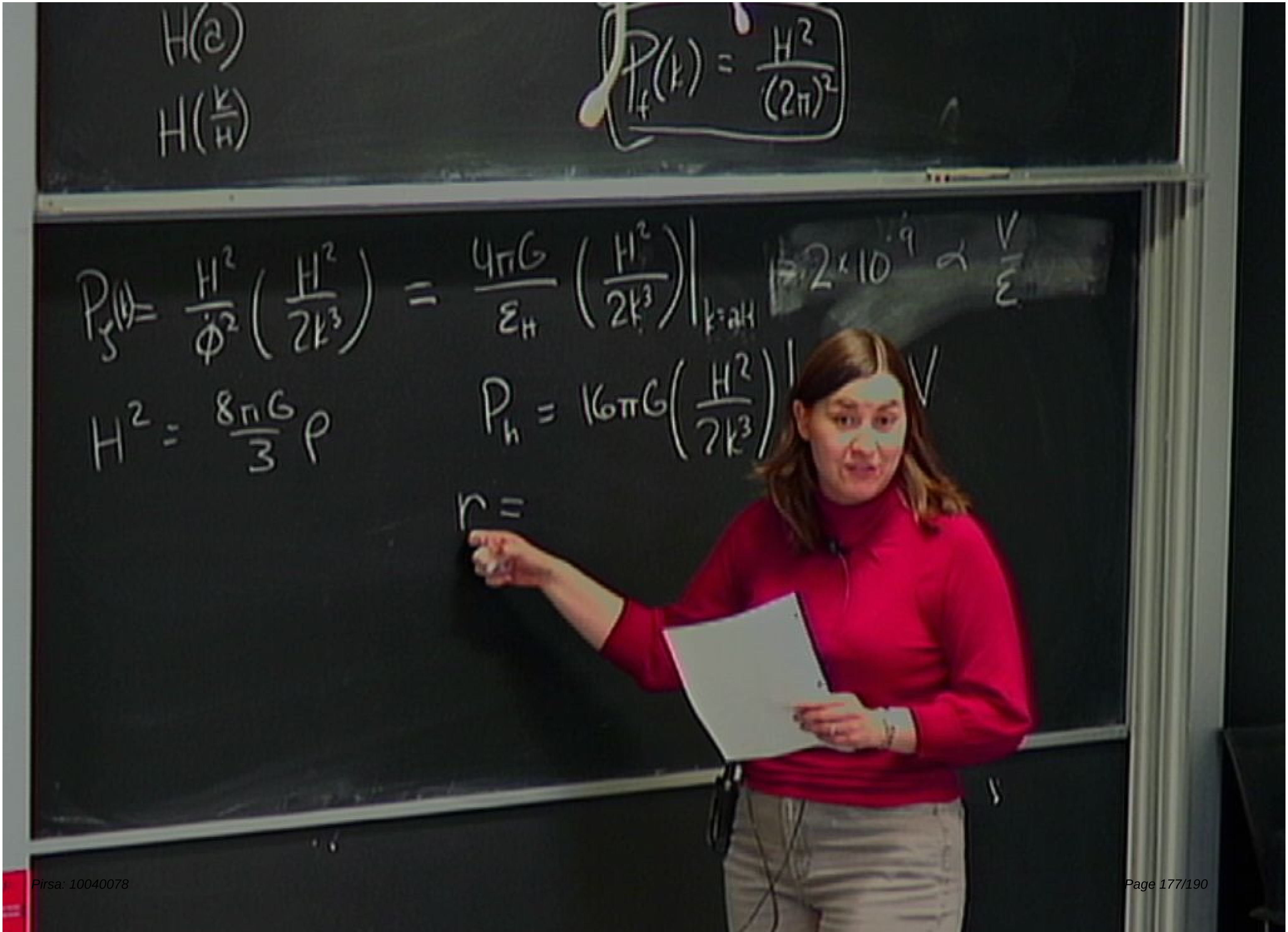

H(a)
H(k/H)
P_ψ(k) = H^2/(2π)^2
P_ζ(k) = H^2/φ̇^2 (H^2/2k^3) = 4πG/ε_H (H^2/2k^3)|_{k=aH}
H^2 = 8πG/3 ρ

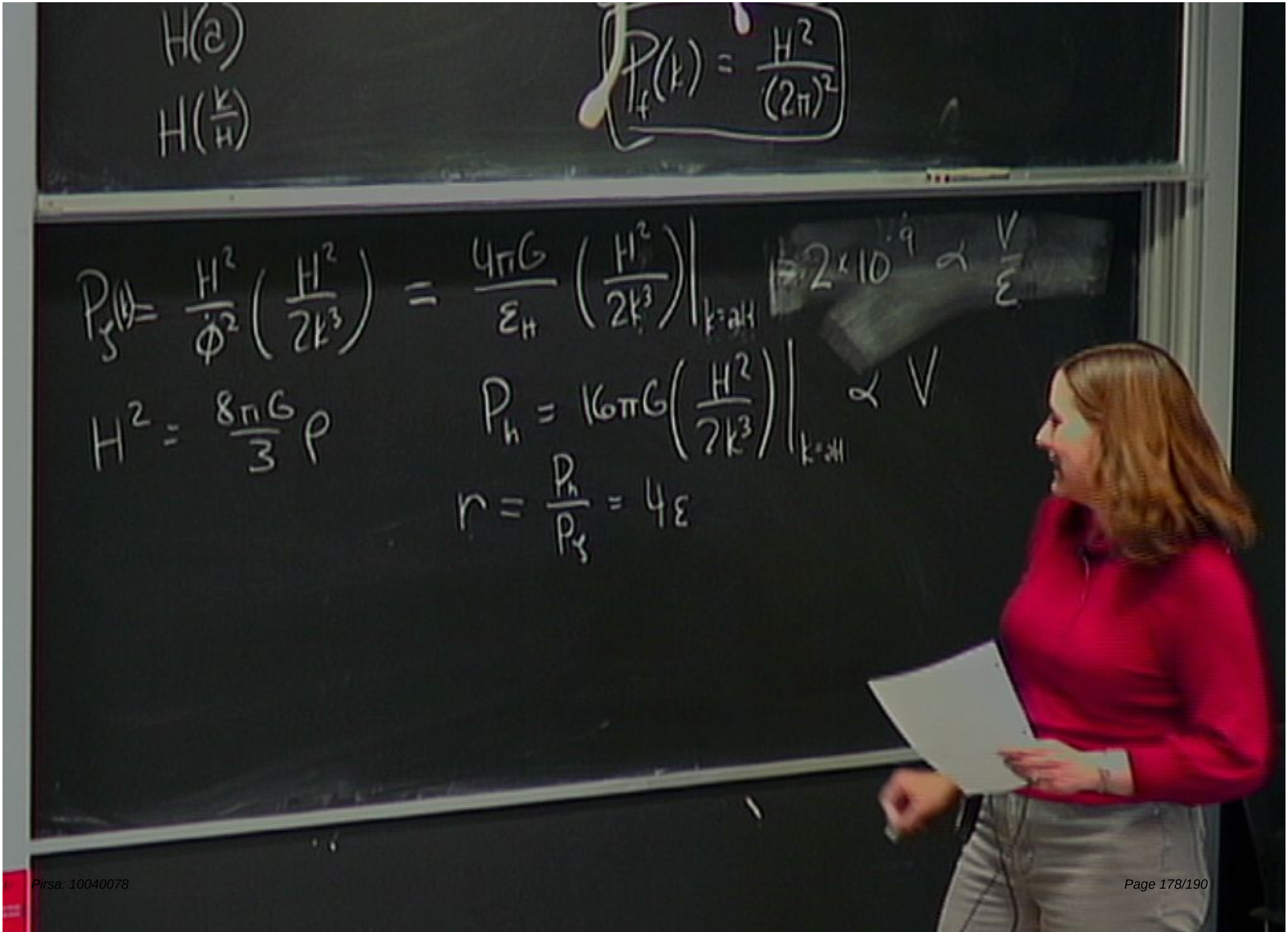

H(a)
H(k/H)
P_t(k) = H^2/(2π)^2
P_ζ(k) = H^2/φ̇^2 (H^2/2k^3) = 4πG/ε_H (H^2/2k^3)|_{k=aH} ≈ 2×10^-9 ∝ V/ε
H^2 = 8πG/3 ρ


H(a)
H(k/H)
P_f(k) = H^2/(2π)^2
P_ζ(k) = H^2/φ̇^2 (H^2/2k^3) = 4πG/ε_H (H^2/2k^3)|_{k=aH} ≈ 2×10^-9 ∝ V/ε
H^2 = 8πG/3 ρ
P_h = 16πG (H^2/2k^3)|_{k=aH}



$H(a)$
$H(\frac{k}{H})$
$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta(k) = \frac{H^2}{\dot{\phi}^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \approx 2\times 10^{-9} \propto \frac{V}{\varepsilon}$
$H^2 = \frac{8\pi G}{3}\rho$
$P_h = 16\pi G\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \propto V$


H(a)
H(k/H)
P_t(k) = H^2/(2π)^2
P_ζ(k) = H^2/φ̇^2 (H^2/2k^3) = 4πG/ε_H (H^2/2k^3)|_{k=aH} ≈ 2×10^-9 ∝ V/ε
H^2 = 8πG/3 ρ
P_h = 16πG(H^2/2k^3)
r =


H(a)
$H(\frac{k}{H})$
$P_f(k) = \frac{H^2}{(2\pi)^2}$
$P_\zeta(k) = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} = 2\times10^{-9} \propto \frac{V}{\varepsilon}$
$H^2 = \frac{8\pi G}{3}\rho$
$P_h = 16\pi G\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \propto V$
$r = \frac{P_h}{P_\zeta} = 4\varepsilon$

$H(a)$

$H(\frac{k}{H})$

$$P_f(k) = \frac{H^2}{(2\pi)^2}$$

$$P_\zeta(k) = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Bigg|_{k=aH} = 2\times10^{-9} \propto \frac{V}{\varepsilon}$$

$$H^2 = \frac{8\pi G}{3}\rho$$

$$P_h = 16\pi G\left(\frac{H^2}{2k^3}\right)\Bigg|_{k=aH} \propto V$$

$$r = \frac{P_h}{P_\zeta} = 4\varepsilon < 0.25$$

$H(a)$

$H\left(\frac{k}{H}\right)$

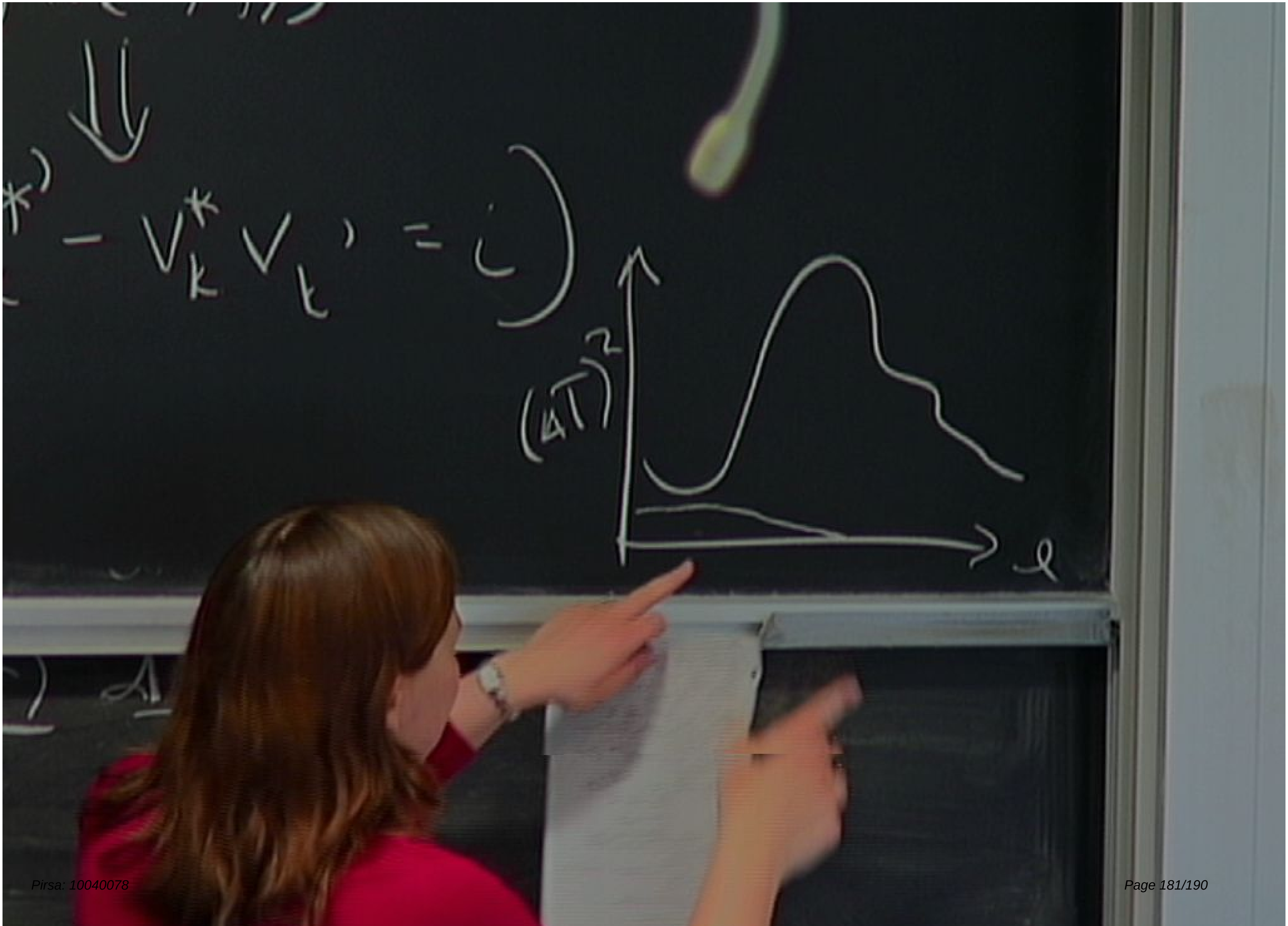
$$P_\phi(k) = \frac{H^2}{(2\pi)^2}$$

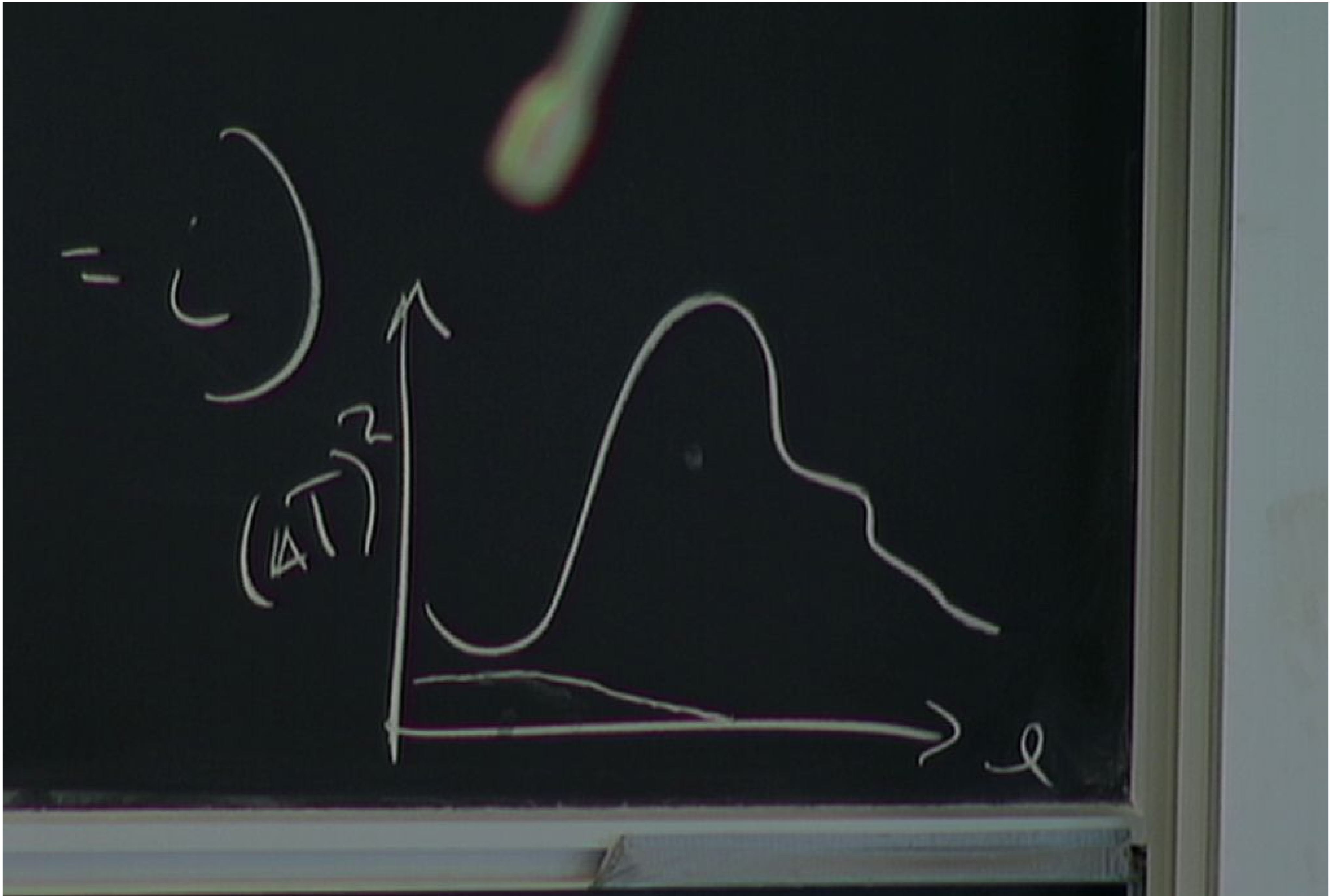
$$P_\zeta(k) = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \approx 2\times10^{-9} \propto \frac{V}{\varepsilon}$$

$$H^2 = \frac{8\pi G}{3}\rho$$

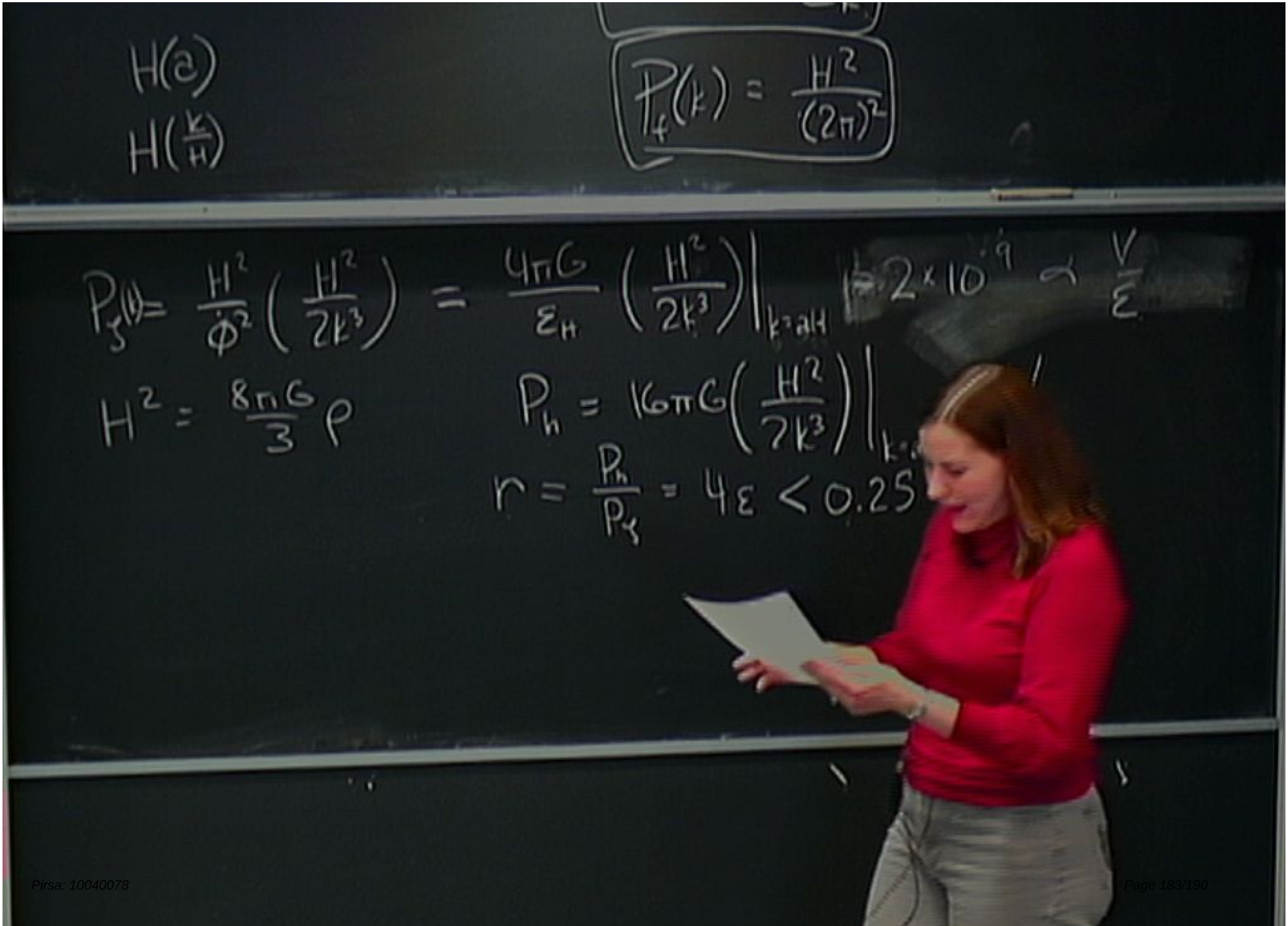
$$P_h = 16\pi G\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \propto V$$

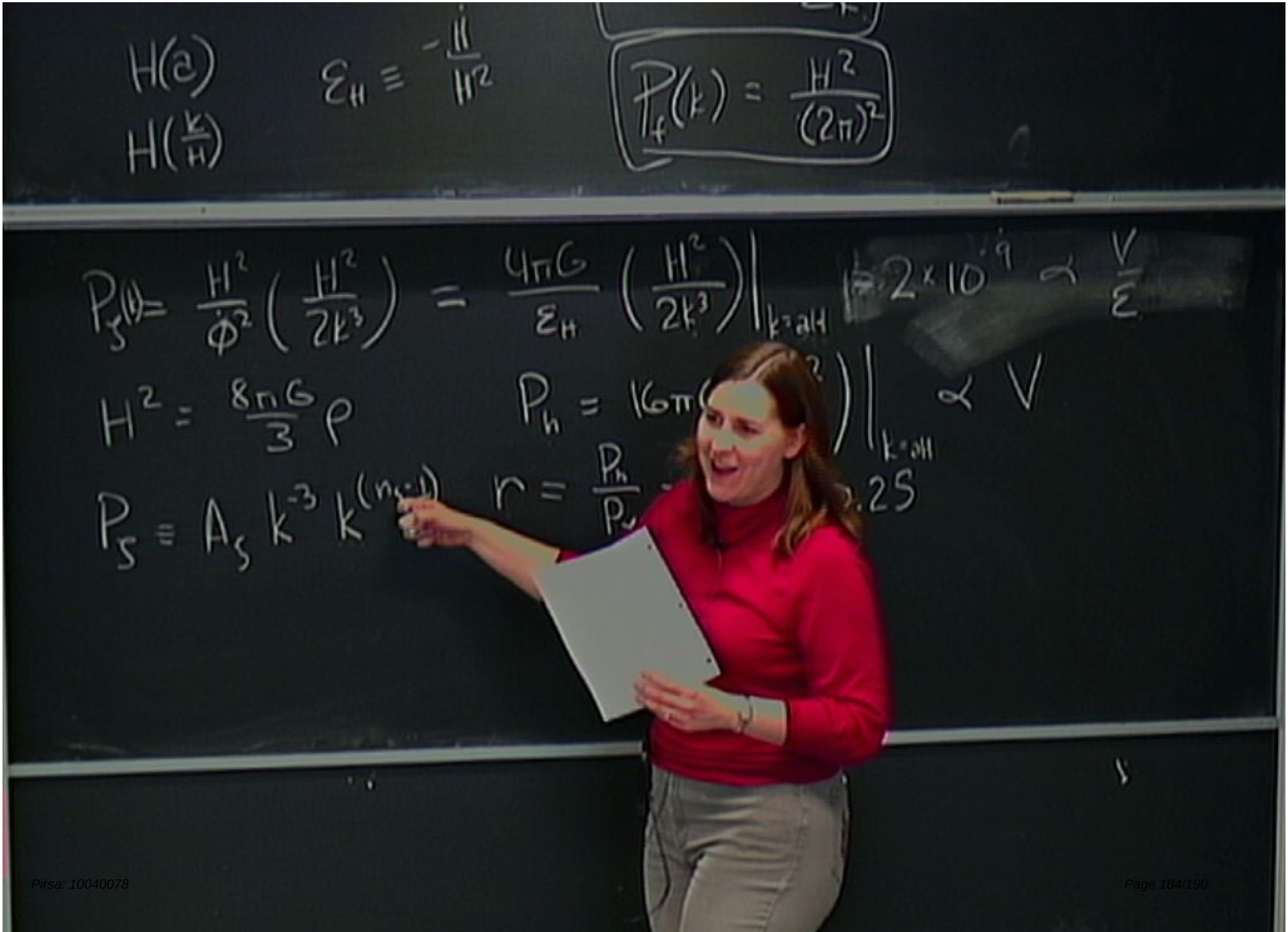
$$r = \frac{P_h}{P_\zeta} = 4\varepsilon < 0.25$$

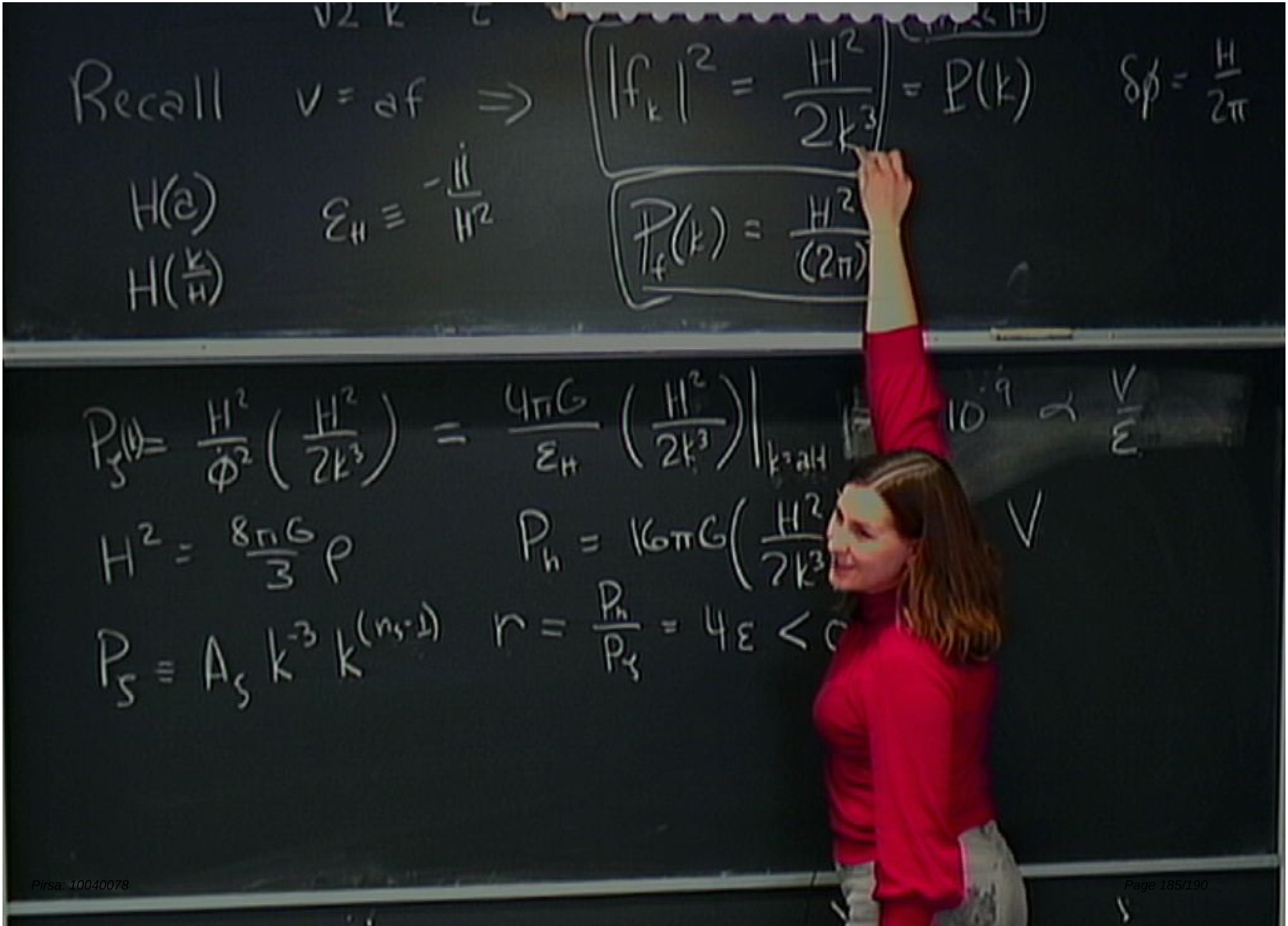

$-V_k^* V_t' = i$
$(4T)^2$
ℓ



$(\Delta T)^2$
ℓ


H(a)
H(k/H)
P_f(k) = H^2/(2π)^2
P_ζ(k) = H^2/φ̇^2 (H^2/2k^3) = 4πG/ε_H (H^2/2k^3)|_{k=aH} ≈ 2×10^-9 ∝ V/ε
H^2 = 8πG/3 ρ
P_h = 16πG (H^2/2k^3)|
r = P_h/P_ζ = 4ε < 0.25


H(a)
H(k/H)
ε_H ≡ −Ḣ/H²
P_φ(k) = H²/(2π)²
P_ζ(k) = H²/φ̇² (H²/2k³) = 4πG/ε_H (H²/2k³)|_{k=aH} ≈ 2×10⁻⁹ ∝ V/ε
H² = 8πG/3 ρ
P_h = 16π
∝ V
P_ζ ≡ A_s k⁻³ k^(n_s−1)
r = P_h/P_ζ
.25


Recall v = af ⇒ |f_k|^2 = H^2 / 2k^3 = P(k)
δφ = H / 2π
H(a)
H(k/H)
ε_H ≡ −Ḣ / H^2
P_f(k) = H^2 / (2π)
P_ζ(k) = H^2/φ̇^2 (H^2 / 2k^3) = 4πG/ε_H (H^2 / 2k^3)|_{k=aH}
10^-9 ∝ V/ε
H^2 = 8πG/3 ρ
P_h = 16πG (H^2 / 2k^3)
V
P_ζ ≡ A_ζ k^-3 k^(n_s-1)
r = P_h / P_ζ = 4ε <

$$V_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)}$$

$$\alpha' = \frac{-1}{2r^2}$$

$$P_{\delta\rho} \propto k$$

$$E_k \propto |V_k'|^2 + \omega_k^2 |V_k|^2$$

Minimice $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}\, e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)} \qquad \tau = \frac{-1}{aH}$$

$$\omega \simeq k \qquad V_k(\tau) = \frac{1}{\sqrt{2k}}\, e^{ik\tau} \qquad k\tau \gg 1$$

$$V_k(\tau) = r_k(\tau)\, e^{i\alpha(\tau)} \qquad \alpha' = \frac{-1}{2r^2} \qquad P_{\delta\rho} \propto k$$

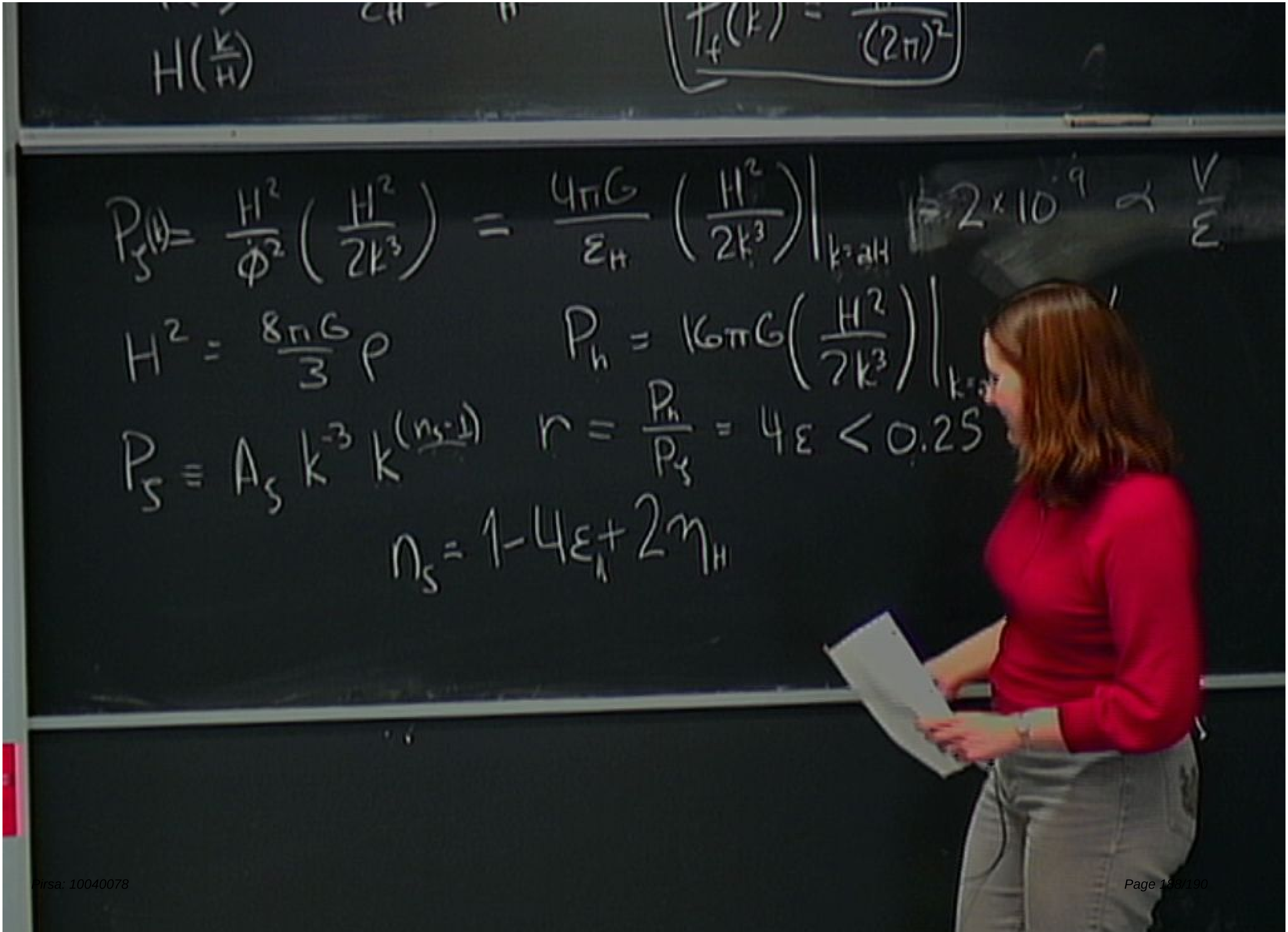
$$E_k \propto |V_k'|^2 + \omega_k^2 |V_k|^2$$

Minimize $E_k(\tau_i) \Rightarrow r_k'(\tau_i) = 0 \qquad r_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}$

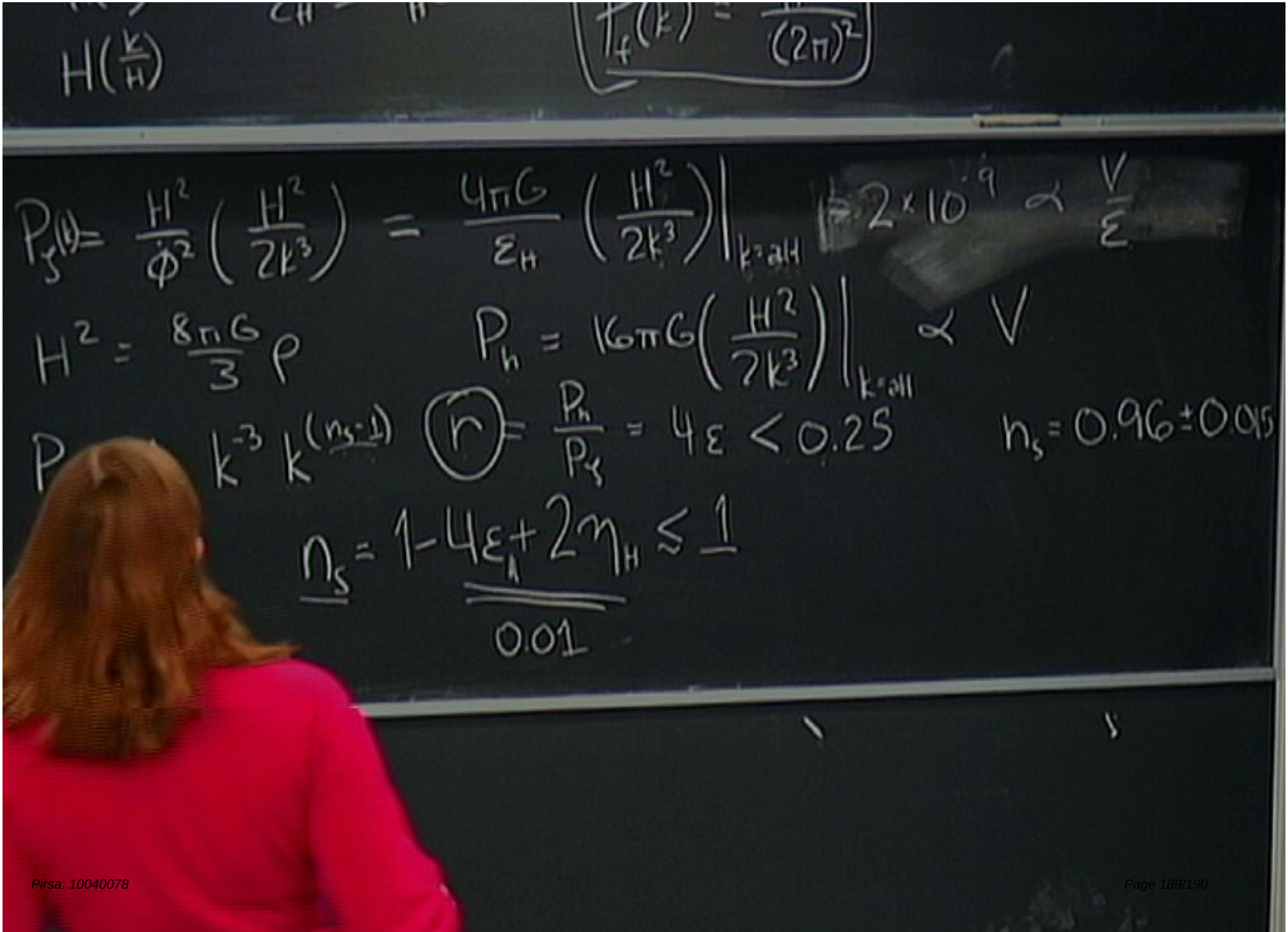
$$V_k(\tau_i) = \frac{1}{\sqrt{2\omega_k}}\, e^{i\alpha(\tau_i)} \qquad V_k' = -i\sqrt{\frac{\omega}{2}}\, e^{i\alpha(\tau_i)} \qquad \tau = \frac{-1}{aH}$$

$$\omega \simeq k \qquad V_k(\tau) = \frac{1}{\sqrt{2k}}\, e^{ik\tau} \qquad k\tau \gg 1$$

$$\frac{1}{2}\int d\tau\, d^3x \left[(v')^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2\right]$$



$H(\frac{k}{H})$
$P_\zeta(k) = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\varepsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \approx 2\times10^{-9} \propto \frac{V}{\varepsilon}$
$H^2 = \frac{8\pi G}{3}\rho$
$P_h = 16\pi G\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH}$
$P_\zeta \equiv A_s k^{-3} k^{(n_s-1)}$
$r = \frac{P_h}{P_\zeta} = 4\varepsilon < 0.25$
$n_s = 1-4\varepsilon_H+2\eta_H$


H(k/H)
P_ζ(k) = H^2/φ̇^2 (H^2/2k^3) = 4πG/ε_H (H^2/2k^3)|_{k=aH} ≈ 2×10^-9 ∝ V/ε
H^2 = 8πG/3 ρ
P_h = 16πG (H^2/2k^3)|_{k=aH} ∝ V
k^-3 k^(n_s-1)
r = P_h/P_ζ = 4ε < 0.25
n_s = 0.96 ± 0.05
n_s = 1 - 4ε_H + 2η_H ≲ 1
0.01

$H(\frac{k}{H})$ $\quad \epsilon_H \quad$ $P_t(k) = \frac{\ldots}{(2\pi)^2}$

$$P_\zeta(k) = \frac{H^2}{\dot\phi^2}\left(\frac{H^2}{2k^3}\right) = \frac{4\pi G}{\epsilon_H}\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \approx 2\times 10^{-9} \propto \frac{V}{\epsilon}$$

$$H^2 = \frac{8\pi G}{3}\rho \qquad P_h = 16\pi G\left(\frac{H^2}{2k^3}\right)\Big|_{k=aH} \propto V$$

$$P_\zeta = A_s k^{-3} k^{(n_s-1)} \qquad r = \frac{P_h}{P_\zeta} = 4\epsilon < 0.25 \qquad n_s = 0.96 \pm 0.015$$

$$P_h = A_h k^{-3} k^{n_T} \qquad n_s = 1 - 4\epsilon_H + 2\eta_H \lesssim 1 \qquad n_T = -2\epsilon$$

$$0.01$$