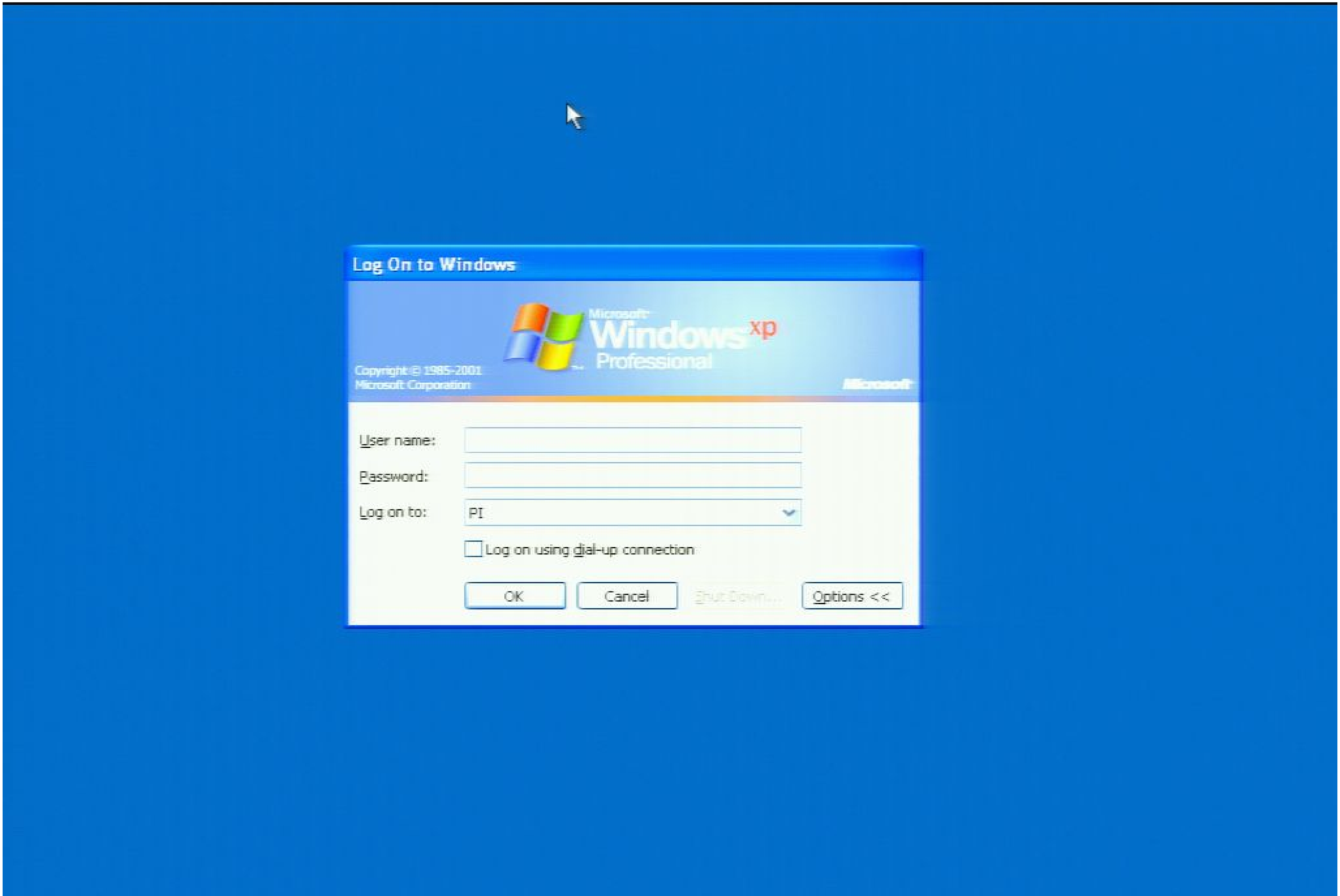
Title: Quantum Field Theory for Cosmology - Lecture 23

Date: Apr 06, 2010 04:00 PM

URL: http://pirsa.org/10040074

Abstract:


Log On to Windows
Microsoft
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Microsoft Corporation
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Password:
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Log on using dial-up connection
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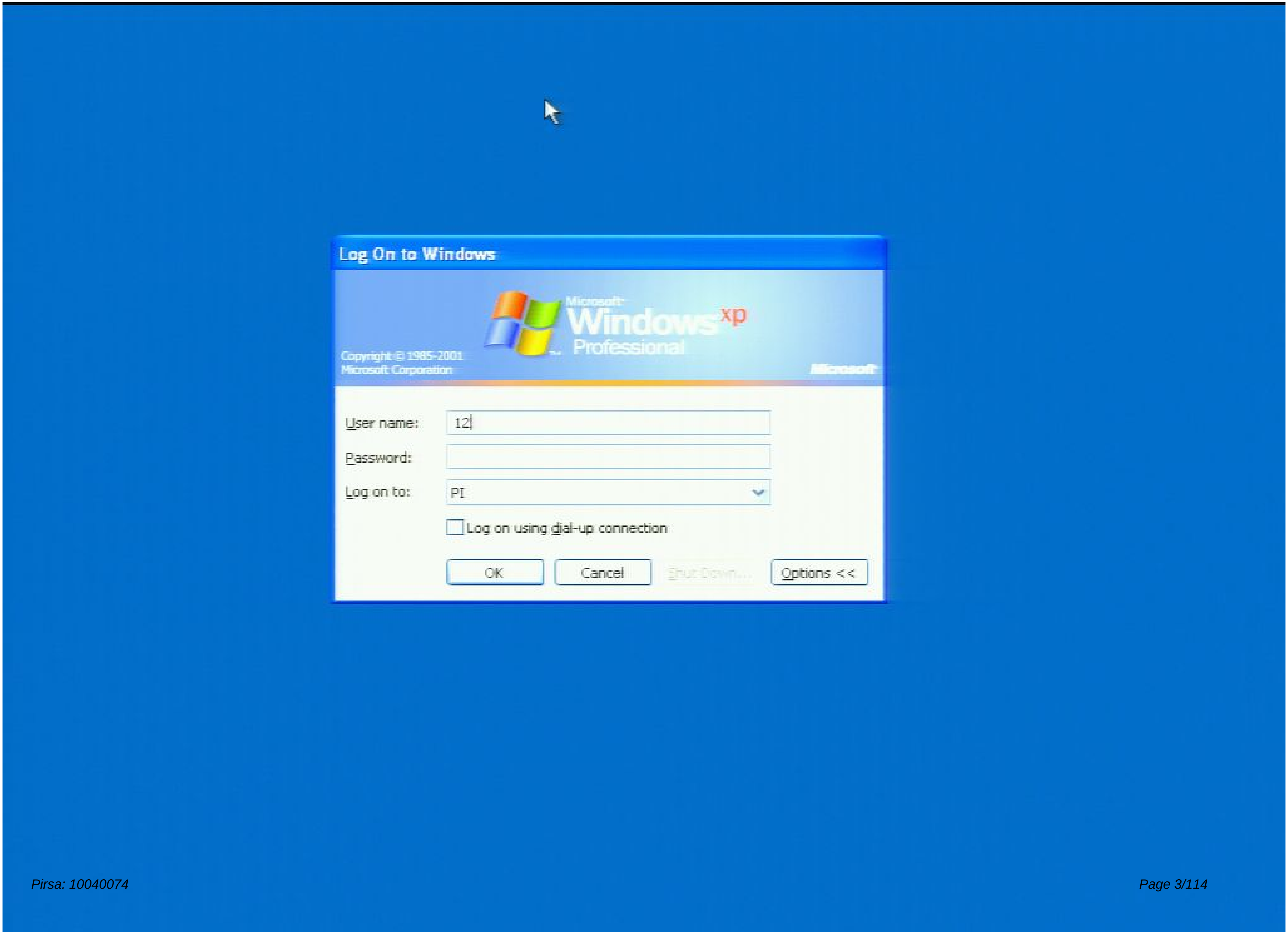
Log On to Windows
Microsoft
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Microsoft Corporation
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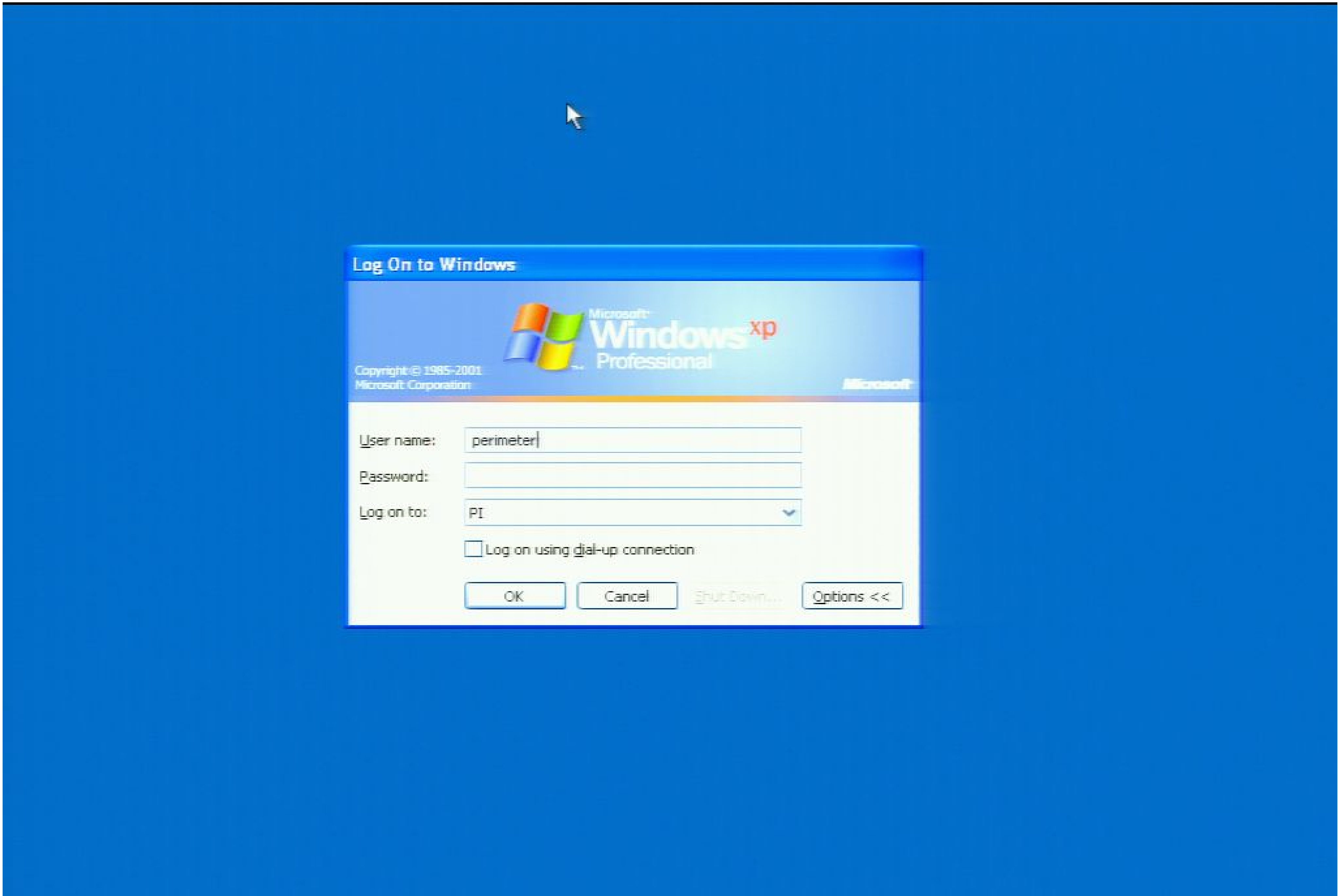
Log On to Windows
Microsoft
Windows xp
Professional
Copyright © 1985-2001
Microsoft Corporation
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User name:
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Password:
Log on to:
PI
Log on using dial-up connection
OK
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Options <<

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Recall strategy:

we continue here

SR, 1^{st}Q
Hamiltonian
formalism

$\xrightarrow[\text{(equivalence)}]{\text{step 1: Legendre transform}}$

SR, 1^{st}Q.
Lagrangian
formalism

step 2 $\downarrow$ allow curvature

GR, 1^{st}Q
Hamiltonian
formalism

$\xleftarrow[\text{(equivalence)}]{\text{step 3: Legendre transform}}$

GR, 1^{st}Q
Lagrangian
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step 4 $\downarrow$

GR, 2^{nd}Q
Hamiltonian
formalism

$\xleftrightarrow[\text{(equivalence)}]{\text{Dyson Schwinger eqns are same}}$

GR, 2^{nd} Q
Lagrangian formalism
(Path integral of QFT)

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OK, $\mathcal{L}$ & Hamiltonian formalism

Dyson Schwinger eqns are same (equivalence)

OK, $\mathcal{L}$ & Lagrangian formalism (Path integral of QFT)

2nd quantization using Feynman's path integral

- Assume a fixed spacetime is chosen and we are given its metric $g_{\mu\nu}(x)$ in some arbitrary coordinate system
- Then, for each field $\phi(\vec{x},t)$ we can calculate its action $S[\phi,g]$, e.g., for the Klein Gordon fields:

$$S_{KG}[\phi,g] = \frac{1}{2}\int_{\mathbb{R}^4}\left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - m^2\phi^2 - \lambda\phi^4\right)\sqrt{|g|}\, d^4x$$

- Following Feynman, we obtain a "probability amplitude"

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$$\text{prob. ampl.}[\phi] := \frac{1}{c}\, e^{\frac{i}{\hbar} S_{KG}[\phi,g]}$$

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for any particular field evolution $\phi(x,t)$ to occur.

- Note: $w[\phi] = e^{iS[\phi]}$ is an un-normalized prob. amplitude

Therefore, c is the normalization constant.

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- **Recall:**

  - Assume given unnormalized probabilities $w(A_i)$ for some values A_i to be found.

  - Then, the expectation value $\bar{A}$ is given by:

$$\bar{A} = \frac{1}{c} \sum_i A_i \, w(A_i) \quad \text{where } c = \sum_i w(A_i)$$

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Example 1: Average field amplitude

Note:
The path integral is ill-defined analytically. But, algebraically it yields a consistent algorithm for calculating expectation values.
One expects that there are UV and IR cutoffs in nature which render the path integrals of QFT also analytically well-defined.

"Path Integral",
integral over all
variables $\phi(x)$ $\forall$
for $\phi(x) \in (-\infty, \infty$

$$\bar{\phi}(x) = \frac{\int \underbrace{\phi(x)}_{\text{odd in } \phi} e^{iS[\phi]} \, D[\phi]}{\int e^{iS[\phi]} \, D[\phi]}$$

$\leftarrow$ Normalization

Assume e.g.:

$$\underbrace{S[\phi]}_{\text{is then even in } \phi} = \int \frac{1}{2} \phi(x)(\Box - m^2)\phi(x) + \frac{\lambda}{4!} \phi^4(x) \, d^4x$$

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Assume e.g.:

$$\int e^{iS[\phi]} D[\phi] = \lim_{N \to \infty} \iiint e^{iS_{\text{discrete}}[\phi]} \, d\phi(x_1)\, d\phi(x_2) \ldots d\phi(x_N)$$

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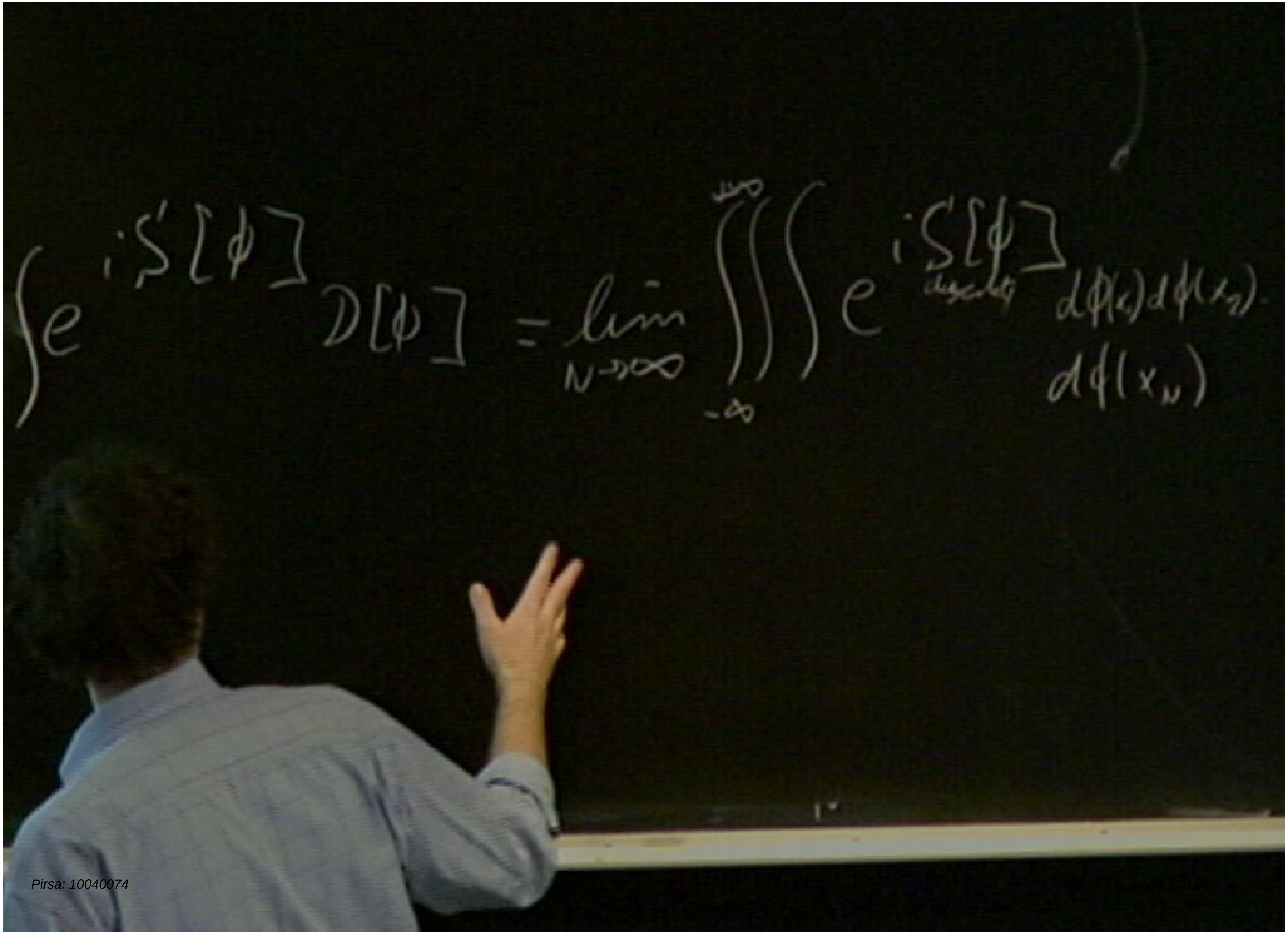

$$\int e^{iS[\phi]} D[\phi] = \lim_{N\to\infty} \int_{-\infty}^{\infty}\int\int e^{iS_{\text{discrete}}[\phi]} \, d\phi(x_1)\, d\phi(x_2) \cdots d\phi(x_N)$$

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Steiner

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$$\Longrightarrow \bar{\phi}(x) = 0$$

We obtained this very quickly!

Compare: How did we calculate $\bar{\phi}(x)$ previously?

$$\bar{\phi}(x) = \langle 0 | \hat{\phi}(x) | 0 \rangle$$

Notice that we can

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Notice that we can say more when using the path integral approach, and more quickly.

Hard to evaluate except if $\lambda = 0$. Then:

$$\hat{\phi}(x) = \int e^{ikx} \left(u_k(t)\, a_k + u_k^*(t)\, a_k^+ \right) d^3k$$

Thus: $\bar{\phi}(x) = \langle 0 | \cancel{x}\, a + \cancel{x}\, a^+ | 0 \rangle = 0$

<u>Example 2:</u> 2-point correlation function

$$G^{(2)}(x,x') := \frac{\int \phi(x)\,\phi(x')\, e^{iS[\phi]}\, D[\phi]}{\int e^{iS[\phi]}\, D[\phi]}$$

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* Meaning of $G^{(2)}(x,x')$?

It shows how much the field amplitudes at events x and x' are correlated over some spatial and temporal distance.

* How to calculate $G^{(2)}(x,x')$ with old methods?

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$$G^{(2)}(x,x') = \langle 0 |\, \hat{\phi}(x)\, \hat{\phi}(x') \,| 0 \rangle \qquad (A)$$

Here, we assume for now that: $x_0 \geq x_0'$.

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* In space times where we have an explicit mode decomposition, e.g., FRW,

$$\hat{\phi}(x) = \int e^{ikx}\left(u_k(t)\,a_k + u_k^*(t)\,a_k^+\right) d^3k \qquad \text{(B)}$$

Recall: This means that the result depends on the identification of the vacuum state

we obtain $G^{(2)}(x,x')$ in terms of $u_k(t)$.

* Why $x_0 \geq x_0'$?

We have $[\hat{\phi}(x), \hat{\phi}(x')] = 0$ if $x_0 = x_0'$

but, in general, they don't commute.

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we obtain $G^{(1)}(x,x')$ in terms of $u_k(t)$.

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we obtain $G^{(t)}(x,x')$ in terms of $u_k(t)$.

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Opposite choice could be made but -1 would have to be absorbed somewhere.

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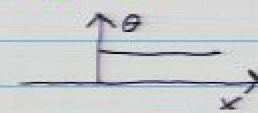
* To automatize the bookkeeping, define T:

$$G^{(2)}(x,x') := \langle 0 | T\, \hat{\phi}(x)\, \hat{\phi}(x') | 0 \rangle$$

"Time ordering operator"

$$= \langle 0 | \theta(x_0 - x_0')\, \hat{\phi}(x)\, \hat{\phi}(x') + \theta(x_0' - x_0)\, \hat{\phi}(x')\, \hat{\phi}(x) | 0 \rangle$$

Heaviside step function



Remarks:

* From Eqs. **A**, **B** follows $G^{(2)}(x,x')$.

* $G^{(2)}(x,x')$ is nonzero almost everywhere!

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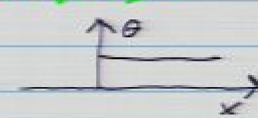

Remarks:

* From Eqs. A, B follows $G^{(2)}(x,x')$.

* $G^{(2)}(x,x')$ is nonzero almost everywhere!

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some spatial and temporal distance.

* How to calculate $G^{(2)}(x,x')$ with old methods?

$$G^{(2)}(x,x') = \langle 0 | \hat{\phi}(x)\, \hat{\phi}(x') | 0 \rangle \quad (A)$$

Here, we assume for now that: $x_0 \geq x_0'$.

* In space times where we have an explicit mode decomposition, e.g., FRW,

$$\hat{\phi}(x) = \int e^{ikx}\left(u_k(t)\, a_k + u_k^*(t)\, a_k^+\right) d^3k \quad (B)$$

Recall: This means that the result depends on the identification of the vacuum state

we obtain $G^{(2)}(x,x')$ in terms of $u_k(t)$.

* Why $x_0 \geq x_0'$?

We have $[\hat{\phi}(x), \hat{\phi}(x')] = 0$ if $x_0 = x_0'$

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* But $G^{(2)}(x,x') \neq 0$ even if x, x' are spacelike separated, e.g., if $x_0 = x_0'$!

This cannot be due to communication, i.e., propagation. Indeed: $|0\rangle$ is entangled!

Note also:

One can calculate the fluctuation spectrum from averaging $G^{(2)}(x,x)$:

$$(\Delta\phi)^2_{\text{region } B} = \langle 0|\int \hat{\phi}(x)\hat{\phi}(x)\, W(x)\, d^3x\,|0\rangle$$

some window fctn for region B.

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made but - I would have
to be absorbed somewhere.

* To automatize the bookkeeping, define T:

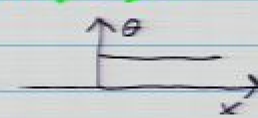
$$G^{(2)}(x,x') := \langle 0 | T\, \hat{\phi}(x)\, \hat{\phi}(x') | 0 \rangle$$

"Time ordering operator"

$$= \langle 0 | \theta(x_0 - x_0')\, \hat{\phi}(x)\, \hat{\phi}(x')$$

$$+ \theta(x_0' - x_0)\, \hat{\phi}(x')\, \hat{\phi}(x) | 0 \rangle$$

Heaviside step function



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$$G^{(2)}(x,x') := \frac{\int \phi(x)\phi(x') e^{iS[\phi]} D[\phi]}{\int e^{iS[\phi]} D[\phi]}$$

Consider integral of a total derivative:

$$\int \frac{\delta}{\delta\phi(x)} \left(\phi(x') e^{iS[\phi]}\right) D[\phi]$$

$$= \text{boundary term}$$

$$= 0$$

This equation is ill defined, as is the path integral. But, it holds in well-defined regularized versions. In particular, notice that after analytic extension to imaginary time, get $e^{-S[\phi]}$

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Thus:

$$0 = \int \frac{\delta}{\delta\phi(x)} \left(\phi(x')\, e^{iS[\phi]}\right) D[\phi]$$

$$= \int \Big(\delta^4(x-x') + \phi(x')\, i \underbrace{\frac{\delta S}{\delta\phi(x)}}_{\shortparallel}\Big) e^{iS[\phi]} D[\phi]$$

(assume now $\lambda = 0$) $\rightarrow$

$$= \int \Big(\delta^4(x-x') + i\phi(x') \overbrace{(\Box_x - m^2)\phi(x)}\Big) e^{iS[\phi]} D[\phi]$$

Nontrivial on curved space. Includes $\sqrt{g}$.

$$\Rightarrow\quad 0 = \delta^4(x-x')\int e^{iS[\phi]} D[\phi] + i(\Box_x - m^2)\int \phi(x)\,\phi(x')\, e^{iS[\phi]} D[\phi]$$

next:

$$\cdot \frac{1}{\int e^{iS[\phi]} \ldots}$$

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Thus, $G^{(2)}$ is called a Green's function for the Klein Gordon equation.

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Note: $\Box$ For any given metric g this becomes an explicit partial differential equation.

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- But is the solution unique?

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- If we can solve it, we know all about the propagation and also all about the correlations caused by the entanglement of the vacuum state.

- But is the solution unique?

  Does the operator $(\Box_x - m^2)$ have a unique right inverse?

- **No!** Because $\exists$ solutions to $(\Box_x - m^2)\, G^{(2)}_{hom}(x,x') = 0$!

- Example Minkowski space:

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next:

$$\Rightarrow \quad 0 = \delta^4(x-x')\int e^{iS[\phi]} D[\phi] + i(\Box_x - m^2)\int \phi(x)\,\phi(x')\, e^{iS[\phi]} D[\phi] \qquad \cdot \frac{1}{\int e^{iS[\phi]} D[\phi]}$$

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▫ Example Minkowski space:

Fourier transform spatial coordinates, to obtain:

$$\left(\partial_t^2 + \vec{k}^2 + m^2\right) G^{(2)}(t,t',k) = i\,\delta(t-t')$$

Thus, $G^{(2)}(t,t',k)$ is unique only up to the choice of a solution to

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These are the mode solutions:

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The central quantities in elementary particle physics:

- Of central significance in quantum field theory are the fields' n-point correlation functions:

$$G(x_1, x_2, \ldots, x_n) := N \int_{\text{all } \phi} \phi(x_1)\,\phi(x_2)\ldots\phi(x_n)\, e^{iS[\phi]}\, D[\phi]$$

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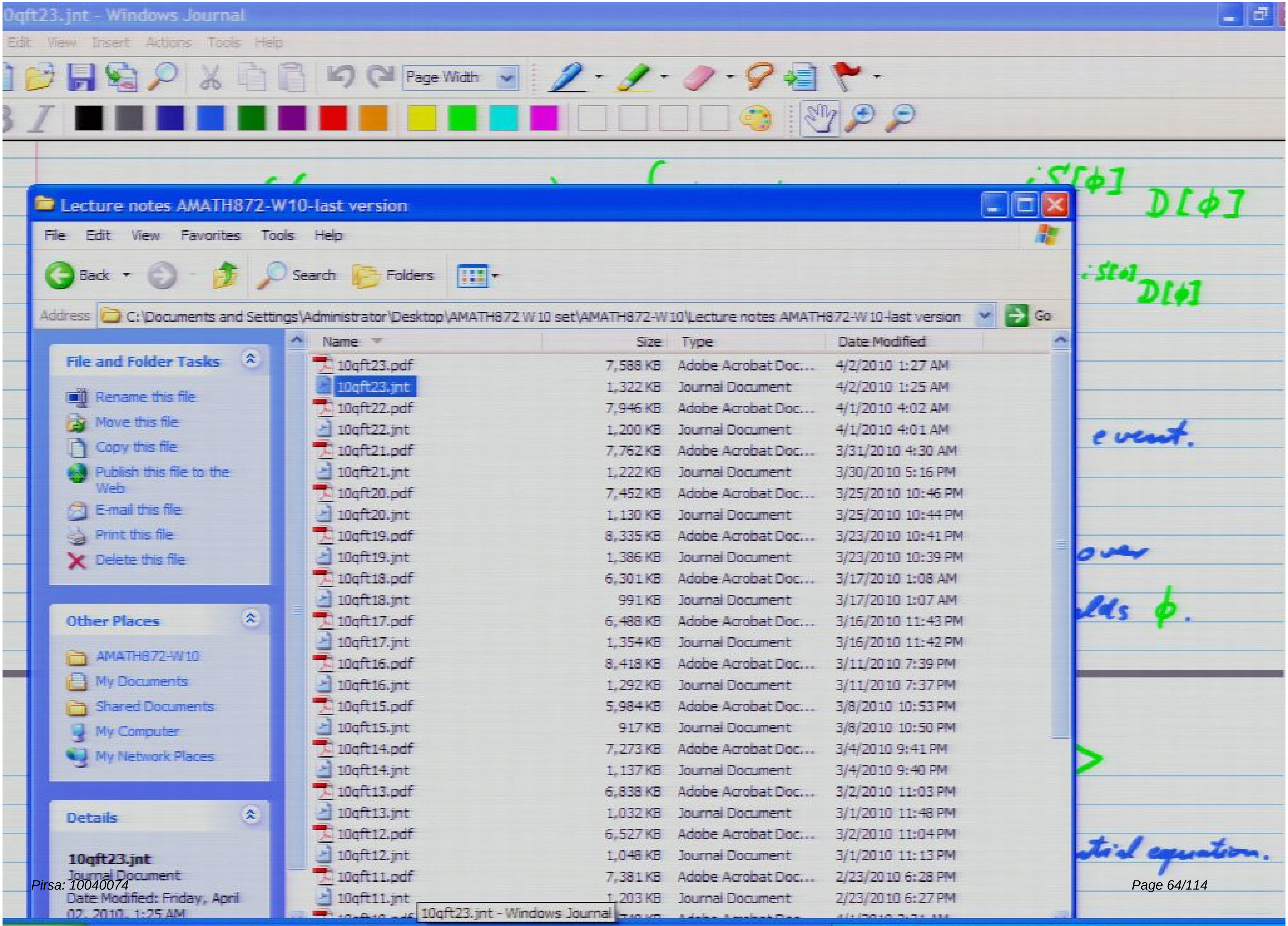
Proposition: $G^{(n)}(x_1, \ldots, x_n) = \langle 0 | T\, \hat{\phi}(x_1) \cdot \ldots \cdot \hat{\phi}(x_n) | 0 \rangle$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Proposition: $G^{(n)}(x_1,\dots,x_n) = \langle 0| T\hat\phi(x_1)\cdot\ldots\cdot\hat\phi(x_n)|0\rangle$

Proof strategy: Show that LHS and RHS obey same differential equation.

□ Case $n=2$:

$$(\Box_x + m^2)\, G^{(2)}(x,x') = i\,\delta^4(x-x') \quad \text{(we showed)}$$

$$(\Box_x + m^2)\langle 0|T\hat\phi(x)\hat\phi(x')|0\rangle = i\,\delta^4(x-x') \quad \text{(given as exercise)}$$

□ For general n: "Schwinger-Dyson equations"

From path integral, easy to derive, generalizing above ansatz which worked for $n=2$:

$$0 = \int \frac{\delta}{\delta\phi(x)}\left(\phi(x_2)\cdot\ldots\cdot\phi(x_n)\, e^{iS[\phi]}\right) D[\phi]$$

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$$(\Box_x + m^2)\,\langle 0|T\hat\phi(x)\hat\phi(x')|0\rangle = i\,\delta^4(x-x') \quad \text{(given as exercise)}$$

- For general n: "Schwinger-Dyson equations"

From path integral, easy to derive, generalizing above ansatz which worked for $n=2$:

$$0 = \int \frac{\delta}{\delta\phi(x)}\left(\phi(x_2)\cdot\ldots\cdot\phi(x_n)\, e^{iS[\phi]}\right) D[\phi]$$

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Proposition: $G^{(n)}(x_1, \dots, x_n) = \langle 0 | T \hat{\phi}(x_1) \cdot \dots \cdot \hat{\phi}(x_n) | 0 \rangle$

Proof strategy: Show that LHS and RHS obey same differential equation.

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For general n: "Schwinger-Dyson equations"

From path integral, easy to derive, generalizing above ansatz which worked for $n=1$:

$$0 = \int \frac{\delta}{\delta \phi(x_1)} \left(\phi(x_2) \cdot \ldots \cdot \phi(x_n) e^{iS[\phi]} \right) D[\phi]$$

... then work out the derivatives.

It is much more work in the operator formalism to derive that the $\langle 0 | T \hat{\phi}(x_1) \cdot \ldots \cdot \hat{\phi}(x_n) | 0 \rangle$ obey the same equations. But it can be done.

Why cumbersome in operator framework?

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Note:
In the "imaginary time" formalism, the KG eqn is elliptic, thus has no homogeneous solutions $\Rightarrow G^{(2)}$ unique in P.I. approach. Then, analytic continuation to real time yields a unique choice – and generally the right choice. Problem: Imaginary time formalism not generally available on curved spacetimes.

Thus, QFT in operator and PI formalism yield same predictions for the correlation functions (and everything else), provided the Dyson Schwinger equations are solved with the same initial conditions. (i.e., same vacuum)

Notice: PI approach does not fix the initial conditions (and thus the vacuum state)

What PI approach is best for: Perturbation theory for interactions

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What PI approach is best for: Perturbation theory for interactions

* We introduce an auxiliary field, J, called a "source field":

$$G(x_1, x_2, \dots, x_n) := N \int_{\text{all } \phi} \phi(x_1)\phi(x_2)\dots\phi(x_n)\, e^{iS[\phi]}\, D[\phi]$$

Source field

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$$= (-i)^n \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)}\, N \int_{\text{all }\phi} e^{iS[\phi] + i\int \overbrace{J(x)}^{\text{Source field}}\phi(x)\,d^4x}\, D[\phi] \Bigg|_{J=0}$$

* Let us define an action, $\tilde{S}[\phi, J]$, that includes the sources:

$$\tilde{S}[\phi, J] := \int_{\mathbb{R}^4} \overbrace{\tfrac{1}{2}\phi(x)\left(\frac{\partial^2}{\partial x_0^2} - \Delta + m^2\right)\phi(x) + \frac{\lambda}{8}\phi(x)^4}^{\text{Recall: } = S[\phi]} + J(x)\,\phi(x)\, d^4x$$

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The "partition function", $Z[J]$, is defined as:

$$Z[J] = N \int_{\text{all } \phi} e^{i\tilde{S}[\phi, J]} D[\phi]$$

* We can now write the correlation functions in this form:

$$G(x_1, \dots, x_n) = (-i)^n \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} Z[J] \Big|_{J=0}$$

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How can we now calculate the n-point functions?

* To this end, it obviously suffices to calculate $Z[J]$, since $Z[J]$ is "generating functional" for the $G(x_1, \ldots, x_n)$.

* Explicitly, e.g., for $\lambda\phi^4$-theory:

$$Z[J] = N \int_{\text{all } \phi} e^{i\tilde{S}[\phi, J]} D[\phi]$$

$$= N \int_{\text{all } \phi} e^{i\int_{\mathbb{R}^4} \frac{1}{2}\phi(x)\left(\frac{\partial^2}{\partial x_0^2} - \Delta + m^2\right)\phi(x) + \frac{\lambda}{8}\phi(x)^4 + J(x)\phi(x)\, d^4x} D[\phi]$$

Note: this step is also analytically ill defined. $\longrightarrow$

$$= e^{i\int \frac{\lambda}{8}\left(\frac{\delta}{\delta J(\tilde{x})}\right)^4 d^4\tilde{x}}\, N \int_{\text{all } \phi} e^{i\int_{\mathbb{R}^4} \frac{1}{2}\phi(x)\left(\frac{\partial^2}{\partial x_0^2} - \Delta + m^2\right)\phi(x) + J(x)\phi(x)\, d^4x} D[\phi]$$

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* Introduce a shorthand matrix & vector notation:

$$\int_{\mathbb{R}^4} \frac{1}{2}\phi(x)\left(\frac{\partial^2}{\partial x^{0\,2}} - \Delta + m^2\right)\phi(x)\, d^4x + \int_{\mathbb{R}^4} J(x)\phi(x)\, d^4x$$

$$Z[j] = \int e^{-f^2 - \lambda f^4 + j f} \, df$$

$$\int e^{iS[\phi]} D[\phi] = \lim_{N \to \infty} \int_{-\infty}^{+\infty}$$

$$Z[j] = \int e^{-f^2 - \lambda f^4 + jf}\, df \overset{?}{=} e^{-\lambda\left(\frac{\partial}{\partial j}\right)^4} \int e^{-f^2 + jf}\, df$$

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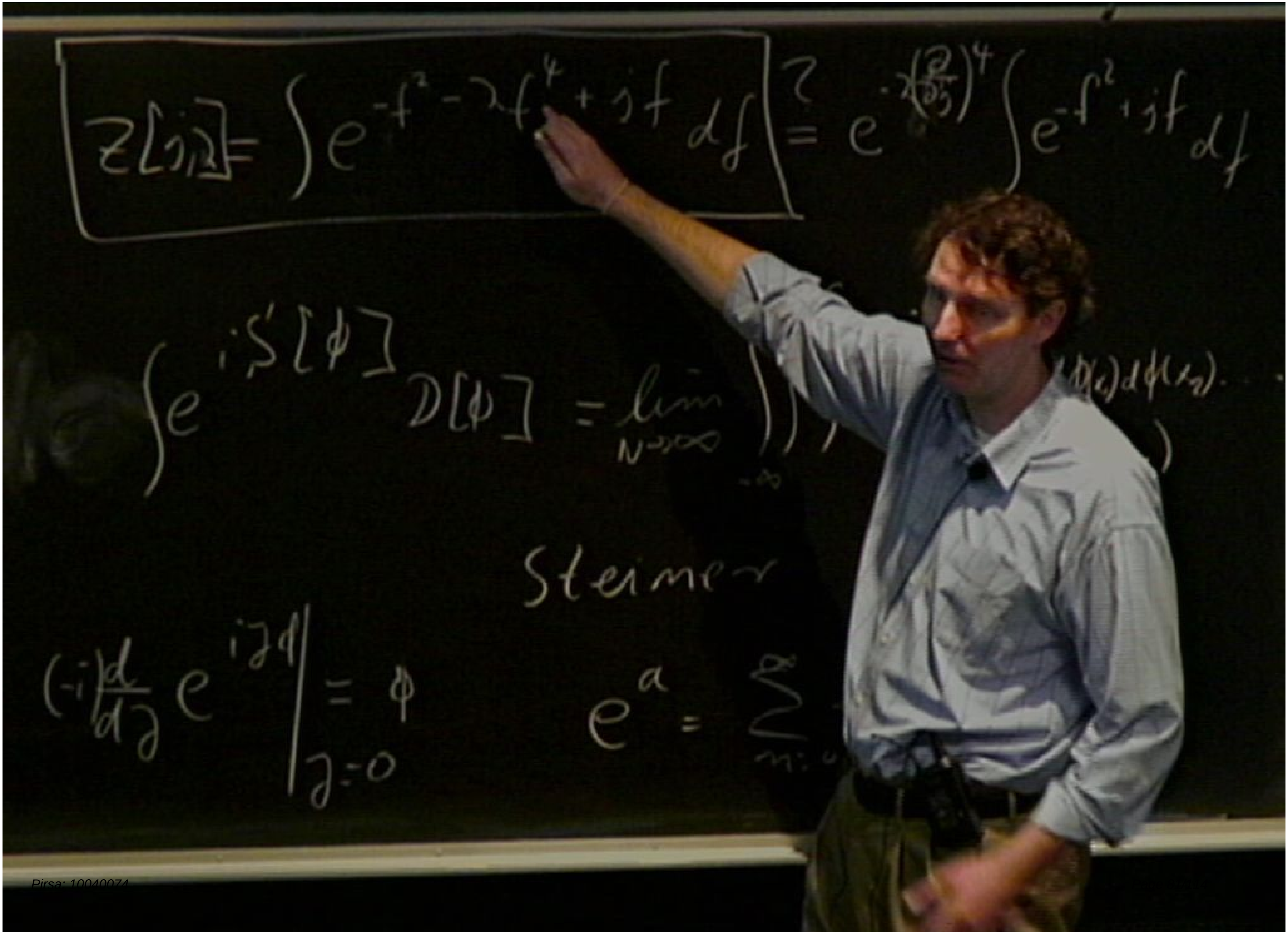
Steiner

$$(-i)\frac{d}{dj} e^{ij\phi}\Big|_{j=0} = \phi \qquad e^a = \sum_{n=0}^{\infty} \frac{a^n}{n!}$$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With suitable choice of a countable basis in the function space of the fields.

$$= \sum_{i,j} \frac{1}{2} f_i M_{ij} f_j + \sum_k j_k f_k$$

$$= \frac{1}{2} f M f + j f$$

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$$Z[J] = e^{i\int \frac{\lambda}{8}\left(\frac{\delta}{\delta J(\tilde{x})}\right)^4 d^4\tilde{x}}\, N\int_{\text{all }\phi} e^{i\int_{\mathbb{R}^4} \frac{1}{2}\phi(x)\left(\frac{\partial^2}{\partial x^{0\,2}} - \Delta + m^2\right)\phi(x) + J(x)\phi(x)\, d^4x}\, D[\phi]$$

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$$\int_{\text{all } f}$$

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Normalization constant

* Back in original notation:

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$$M = \partial_t^2 - \Delta + m^2$$

* Thus:

$K = M^{-1}$ is a Green's function:

* I.e., K obeys:

$$(\partial_t^2 - \Delta + m^2) K(x,x') = \delta^4(x-x')$$

* Here, $K(x,x')$ is called the propagator.

* $K(x,x')$ will be represented by an edge $\overline{x \quad\quad x'}$:

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Wireless Network Connection is now connected
Connected to: pi-wireless(unsecured)
Signal Strength: Low

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$$Z[J] = \tilde{N}\, e^{\frac{i\lambda}{8}\int_{\mathbb{R}^4}\left(\frac{\delta}{\delta J(\tilde{x})}\right)^4 d^4\tilde{x}}\quad e^{-\frac{i}{2}\int_{\mathbb{R}^4} J(x)K(x,x')J(x')\,d^4x\,d^4x'}$$

* Recall that:

$$M = \partial_t^2 - \Delta + m^2$$

* Thus:

$K = M^{-1}$ is a Green's function:

* I.e., K obeys:

$$(\partial_t^2 - \Delta + m^2)\,K(x,x') = \delta^4(x-x')$$

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Where graphs come in:

* The generating function

$K(x,x')$ is known explicitly (e.g., Hankel fctns)

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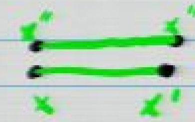
may now be viewed as a sum of graphs: (all x, x' etc integrated)

$= \tilde{N}\Big[\, 1 - \frac{i}{2}$  $+ \frac{1}{2!}\left(\frac{-i}{2}\right)^2$  $+ \frac{1}{3!}\left(\frac{-i}{2}\right)^3$  $+ \dots$

$\int d^4\tilde{x}\; K(\tilde{x},\tilde{x})^2$

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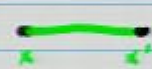
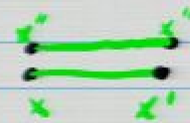
$= \tilde{N}\Big[\,1 - \frac{i}{2}$  $+ \frac{1}{2!}\left(\frac{-i}{2}\right)^2$  $+ \frac{1}{3!}\left(\frac{-i}{2}\right)^3$  $+ \dots$

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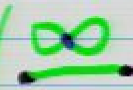
$+ \frac{i\lambda}{8}\Big(\sigma + \frac{c_1}{2!}\left(\frac{i}{2}\right)^2$  $+ \frac{c_2}{3!}\left(\frac{-i}{2}\right)^3\Big($  $+$  $+ \dots\Big) + \dots\Big]$

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* The generating function

explicitly (e.g., Hankel fctns)

$$Z[J] = \tilde N \, e^{\frac{i\lambda}{8}\int_{\mathbb{R}^4}\left(\frac{\delta}{\delta J(\tilde x)}\right)^4 d^4\tilde x} \; e^{-\frac{i}{2}\int_{\mathbb{R}^4} J(x)\,K(x,x')\,J(x')\,d^4x\,d^4x'}$$

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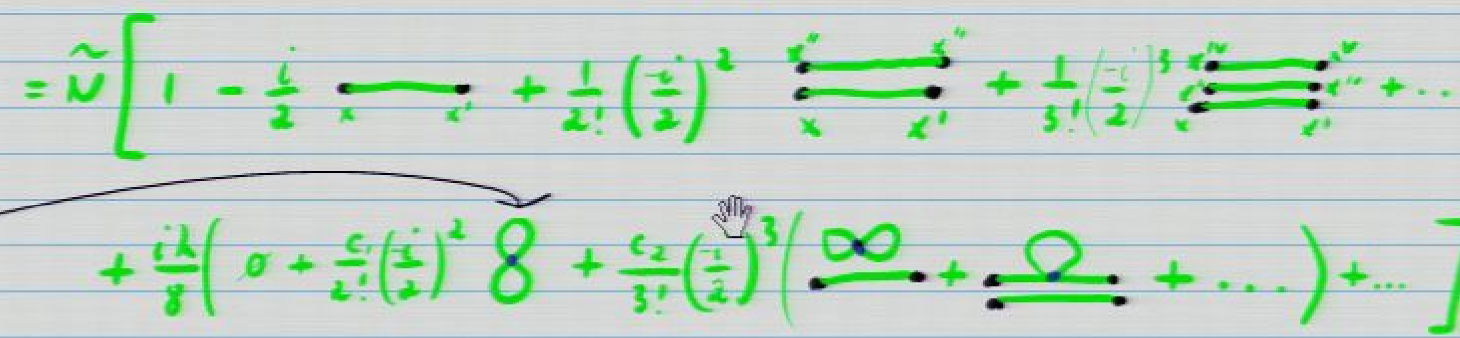

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$$\int d^4\tilde{x}\; K(\tilde{x},\tilde{x})^2$$

$$+ \frac{i\lambda}{8}\left(0 + \frac{c_1}{2!}\left(\frac{-i}{2}\right)^2 8 + \frac{c_2}{3!}\left(\frac{-i}{2}\right)^3\left(\dots + \dots + \dots\right) + \dots\right]$$

* We have...

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$$\int d^4\tilde{x}\; K(\tilde{x},\tilde{x})^2$$

$$= \tilde{N}\Big[\, 1 - \frac{i}{2}\;\underset{x \qquad\quad x'}{\rule{3em}{0.4pt}} + \frac{1}{2!}\left(\frac{-i}{2}\right)^2 \;\ldots\; + \frac{1}{3!}\left(\frac{-i}{2}\right)^3 \;\ldots\; + \dots$$

$$+ \frac{i\lambda}{8}\left(0 + \frac{c_1}{2!}\left(\frac{-i}{2}\right)^2 \; 8 \; + \frac{c_2}{3!}\left(\frac{-i}{2}\right)^3 \left(\ldots + \ldots + \dots \right) + \dots \right) \Big]$$

* We have:

- One kind of edge:

  $\underset{x \qquad\quad x'}{\rule{3em}{0.4pt}} = K(x,x')$ "propagator"

- Two kinds of vertices:

  $\underset{x}{\bullet} = \frac{i}{2} J(x)$ A free end: an ingoing or outgoing particle

  $\underset{x}{\bullet} = \frac{i\lambda}{8}\, \delta^4(x-x_1)\,\delta^4(x-x_2)\,\delta^4(x-x_3)\,\delta^4(x-x_4)$

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This vertex describes the collision of particles

* Note: $Z[J]$ contains connected and disconnected graphs.

$$\left(\frac{\delta J(x)}{\delta J(x')} = \delta^4(x-x')\right)$$

$$= \tilde{N}\left(1 + \frac{i\lambda}{8}\int_{\mathbb{R}^4}\left(\frac{\delta}{\delta J(x)}\right)^4 d^4x + \dots\right)\left(1 - \frac{i}{2}\int_{\mathbb{R}^4} J(x)K(x,x')J(x')\,d^4x\,d^4x' + \dots\right)$$

may now be viewed as a sum of graphs: (all x, x' etc integrated)

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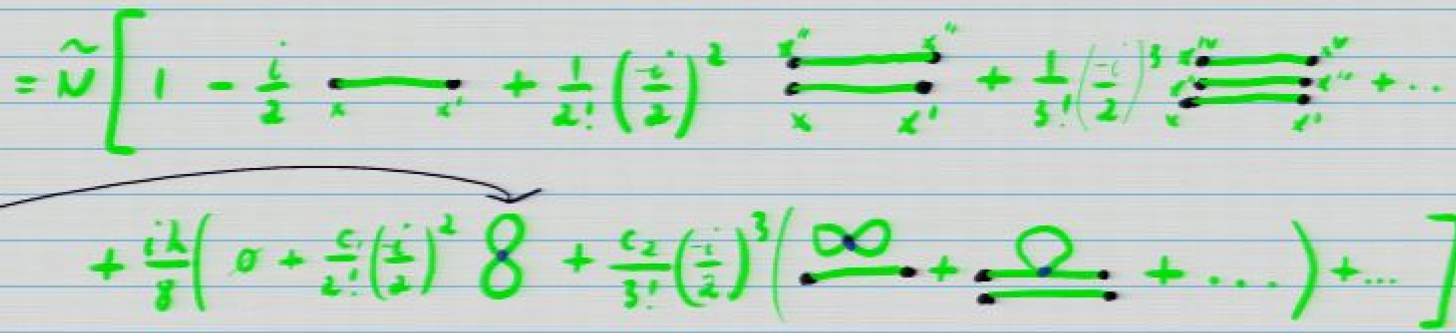

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- One kind of edge:

$= K(x,x')$ "propagator"

- Two kinds of vertices:

$= i\lambda(x)$ A new vertex: an ...

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* Definition: Let $iW[J]$ be the sum of only all connected graphs.

* Proposition: We have:

$$Z[J] = \sum_{N=0}^{\infty} \frac{1}{N!}\left(iW[J]\right)^N = e^{iW[J]}$$

This is clear because:

- disconnected graphs are products of connected graphs
- the factor $1/N!$ avoids overcounting because in $Z[J]$ the order of the connected subgraphs does not matter (since the vertices can be re-labeled).

* Thus: $W[J] = -i \ln Z[J]$ is the generating functional for the connected

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Outlook:

Why work with $W[J]$, instead of $Z[J] = e^{iW[J]}$?

Recall: $Z[J] = N \int_{all\,\phi} e^{iS[\phi,J]} D[\phi]$ i.e.: $Z[0] = 1$ means

N^{-1} = sum over all graphs without end vertices, i.e., it is the sum over all disconnected graphs.

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Thus:

$$(i)^n \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} Z[J]\Big|_{J=0}$$

inherits N and the disconnected graphs but in

$$(i)^n \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J]\Big|_{J=0}$$

they cancel!

Exercise: Show this, using that $(\ln f)' = \frac{f'}{f}$

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