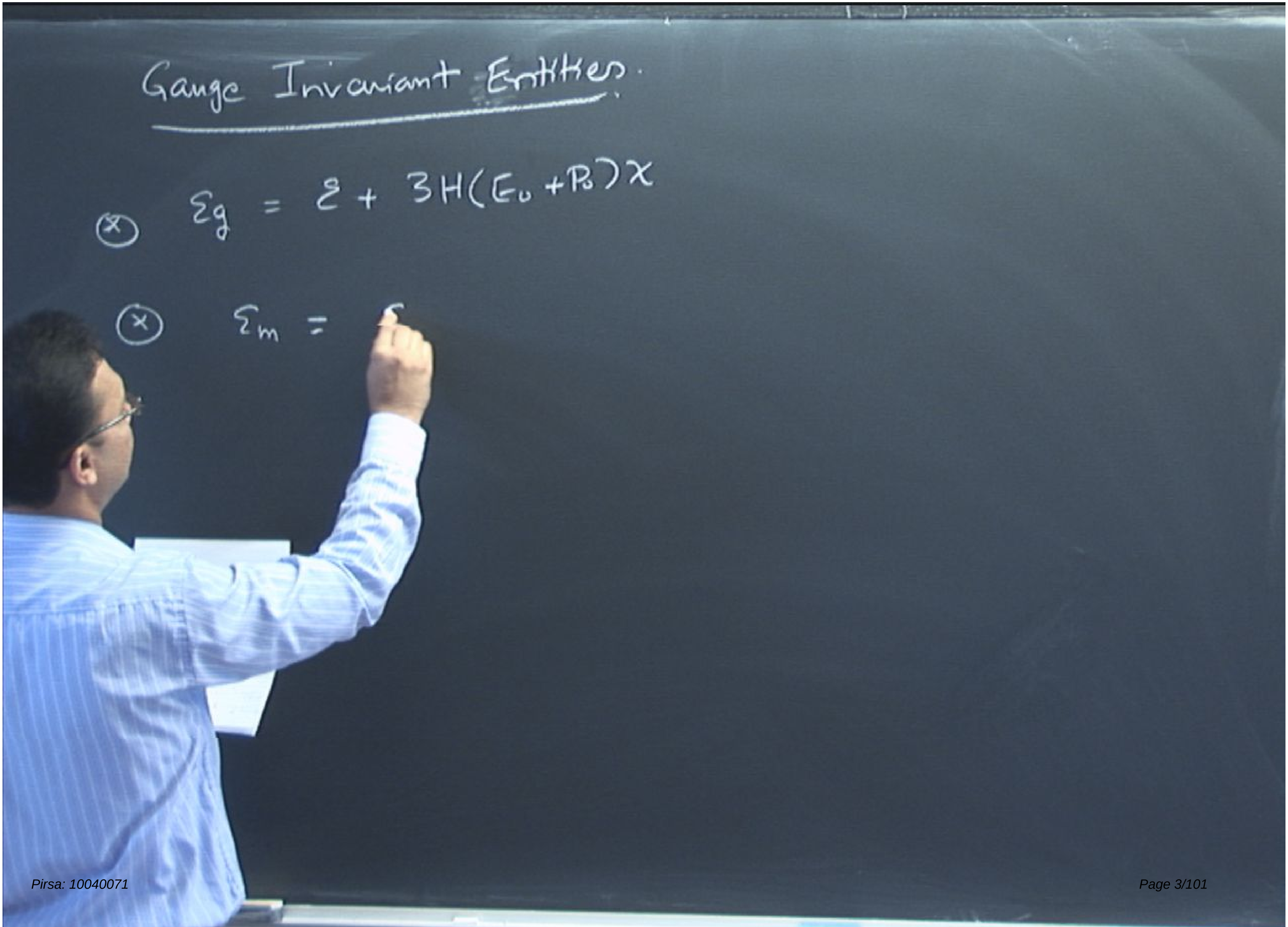
Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 14

Date: Apr 22, 2010 09:30 AM

URL: http://pirsa.org/10040071

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.


Gauge Invariant Entities.
$\varepsilon_g = \varepsilon + 3H(E_0 + P_0)x$
$\varepsilon_m =$

Gauge Invariant Entities.

- $\varepsilon_g = \varepsilon + 3H(\varepsilon_0 + P_0)\chi$; $\varepsilon_g = \varepsilon$ in zero shear gauge

- $\varepsilon_m = \varepsilon - 3H\psi$; $\varepsilon_m = \varepsilon$ in comoving gauge.

Gauge Invariant Entities.

⊗ $\mathcal{E}_g = \mathcal{E} + 3H(E_0 + P_0)\chi$; $\mathcal{E}_g = \mathcal{E}$ in zero shear gauge

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⊗ $\psi_g = \psi + (E_0 + P_0)\chi$

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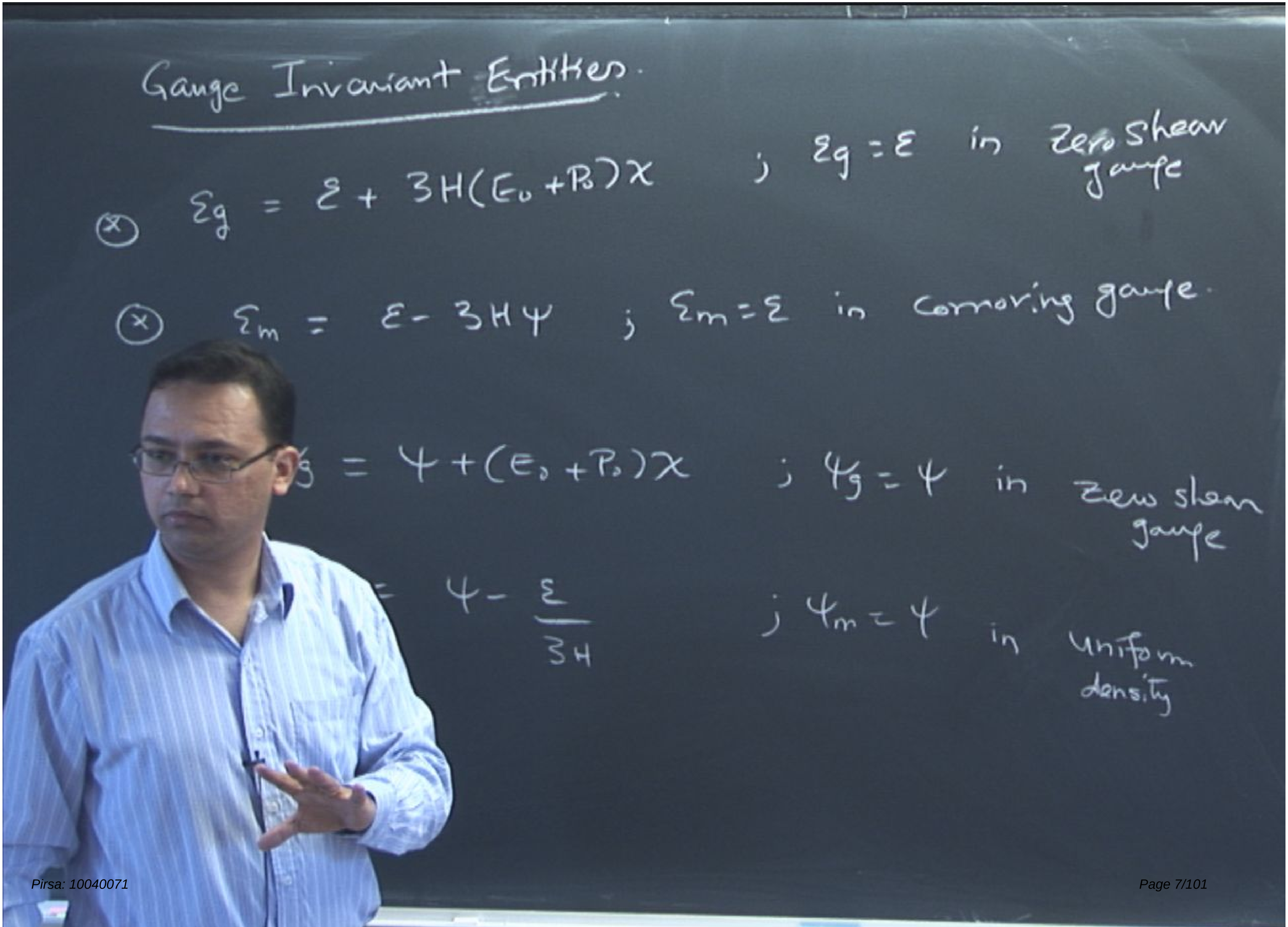
⊛


Gauge Invariant Entities.
$\varepsilon_g = \varepsilon + 3H(E_0 + P_0)\chi$; $\varepsilon_g = \varepsilon$ in zero shear gauge
$\varepsilon_m = \varepsilon - 3H\psi$; $\varepsilon_m = \varepsilon$ in comoving gauge.
$= \psi + (E_0 + P_0)\chi$; $\psi_g = \psi$ in zero shear gauge
$\psi - \frac{\varepsilon}{3H}$; $\psi_m = \psi$ in uniform density

Gauge Invariant Entities.

$\otimes$ $\mathcal{E}_g = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)\chi$; $\mathcal{E}_g = \mathcal{E}$ in zero shear gauge

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$\otimes$ $\psi_g = \psi + (\mathcal{E}_0 + P_0)\chi$; $\psi_g = \psi$ in zero shear gauge

$\otimes$ $\psi_m = \psi - \dfrac{\mathcal{E}}{3H}$; $\psi_m = \psi$ in uniform density gauge

$$\zeta_B = \varphi + \frac{\varepsilon}{3(E_0 + P_0)} \quad \text{is. G.I.}$$

$$\zeta_L \equiv \mathcal{R} = \varphi + \frac{H\psi}{(E_0 + P_0)}$$

$$\psi = a\,(E_0 + P_0)\,(v - B)$$

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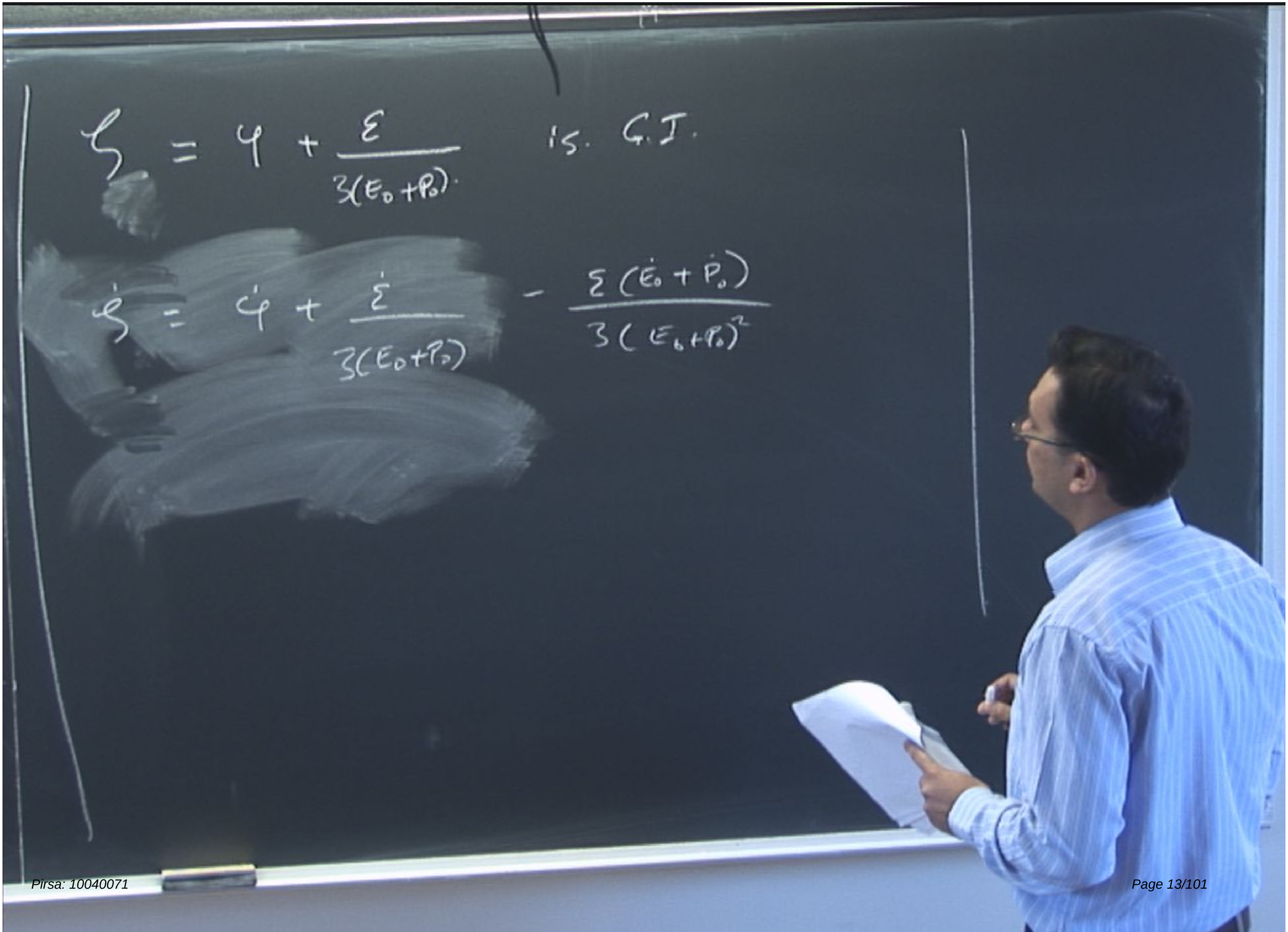
$\zeta = \varphi + \frac{\varepsilon}{3(E_0+P_0)}$ is G.I.
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$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\pi}{\varepsilon}$$

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(41) $\dot{\varphi} = H\alpha - \frac{k}{3} - \frac{\Delta\chi}{3}$

(46)

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$$\dot{\mathcal{E}} = -3H(\mathcal{E}+\Pi) + (E_0+P_0)(k-3H\alpha) - \Delta\psi$$

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$$\dot{\zeta} = \ldots + \frac{(k - 3H\alpha)}{3} - \frac{\Delta\psi}{3} - \varepsilon$$

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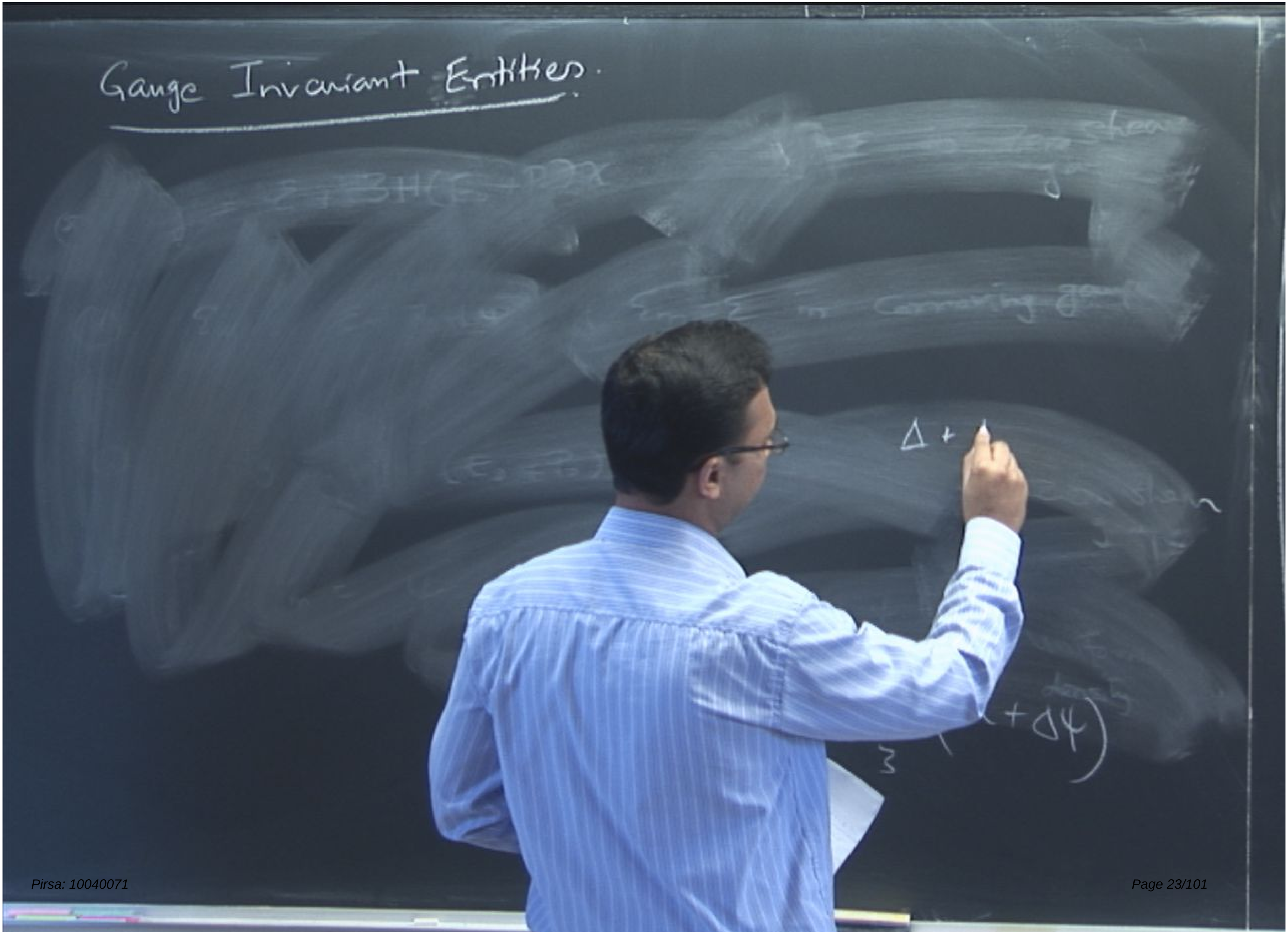
Gauge Invariant Entities.
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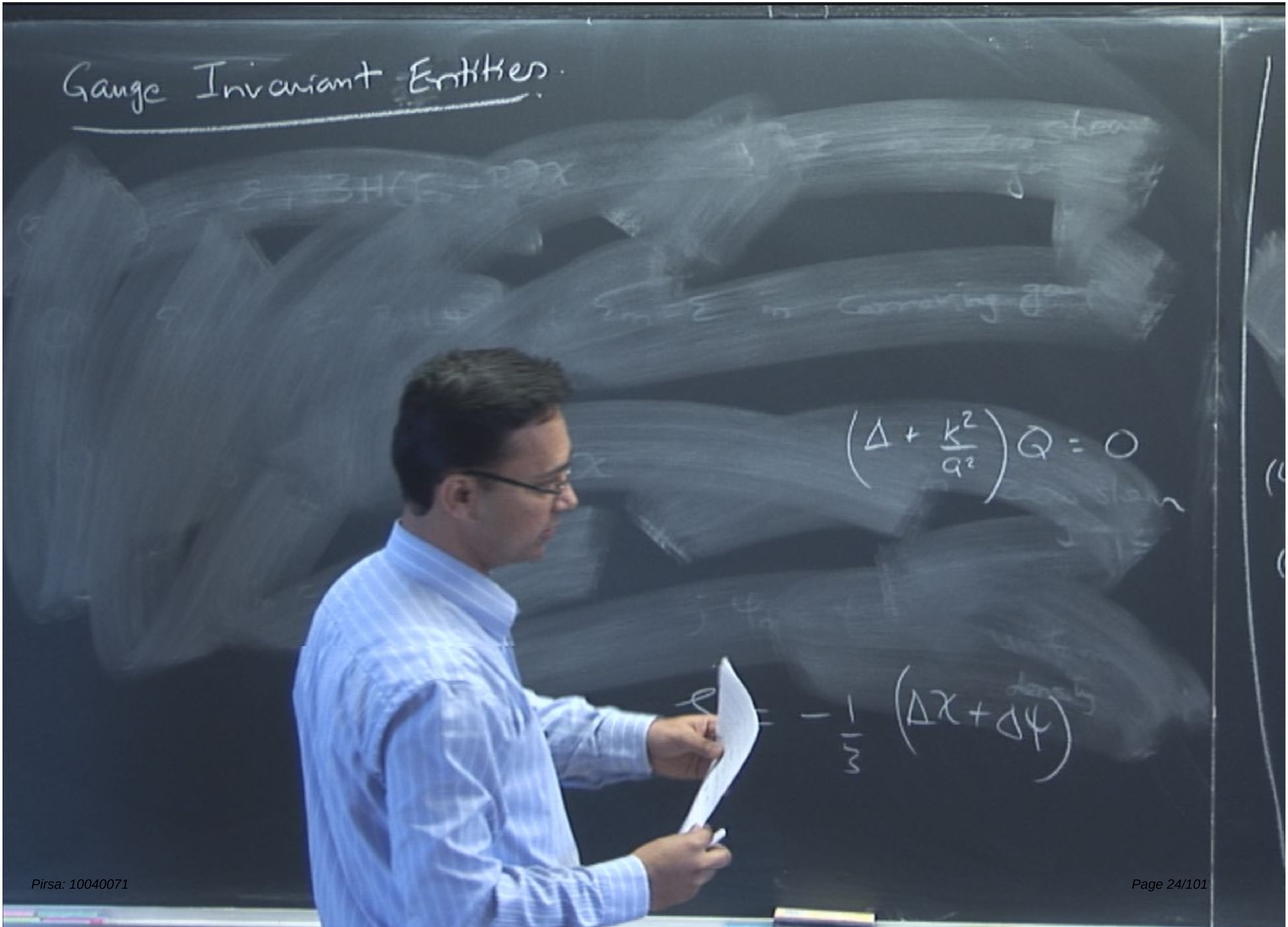
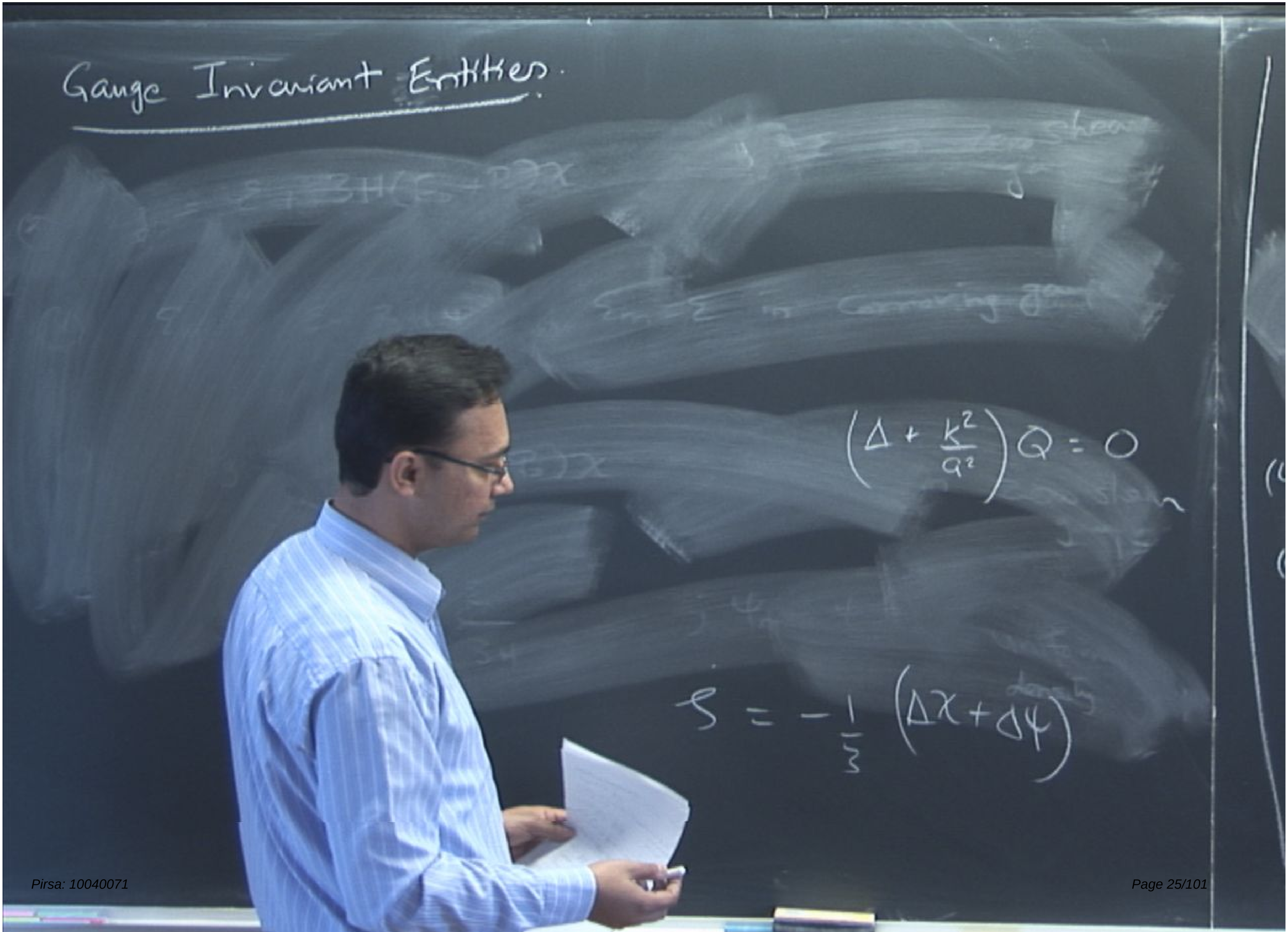
+ Δψ)


Gauge Invariant Entities
$\left(\Delta + \frac{k^2}{a^2}\right) Q = 0$
$-\frac{1}{3}\left(\Delta\chi + \Delta\psi\right)$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Gauge Invariant Entities.

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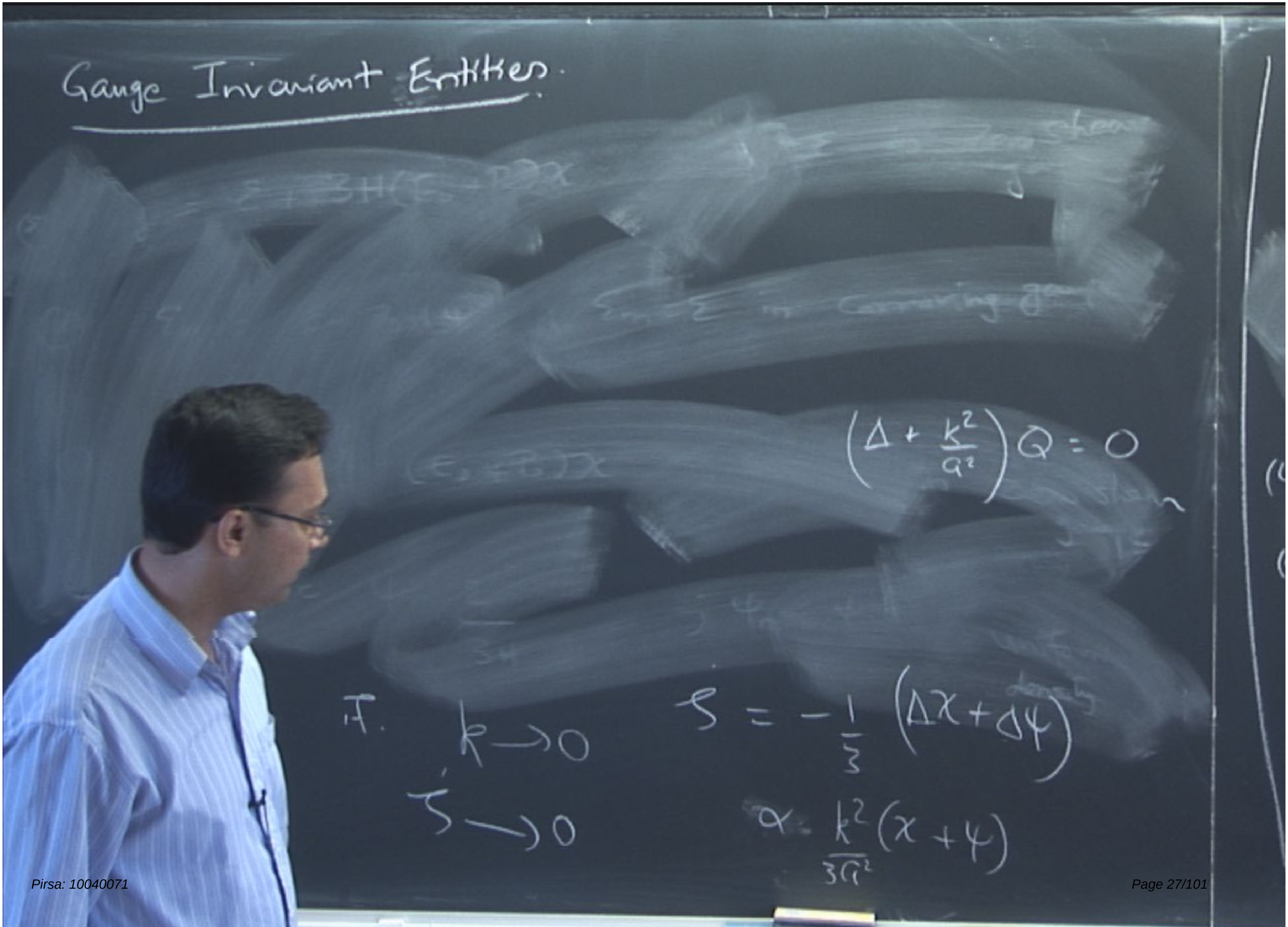
$$k \to 0 \qquad \zeta = -\frac{1}{3}\left(\Delta\chi + \Delta\psi\right)$$

$$\propto \frac{k^2}{3a^2}(\chi + \psi)$$


Gauge Invariant Entities.
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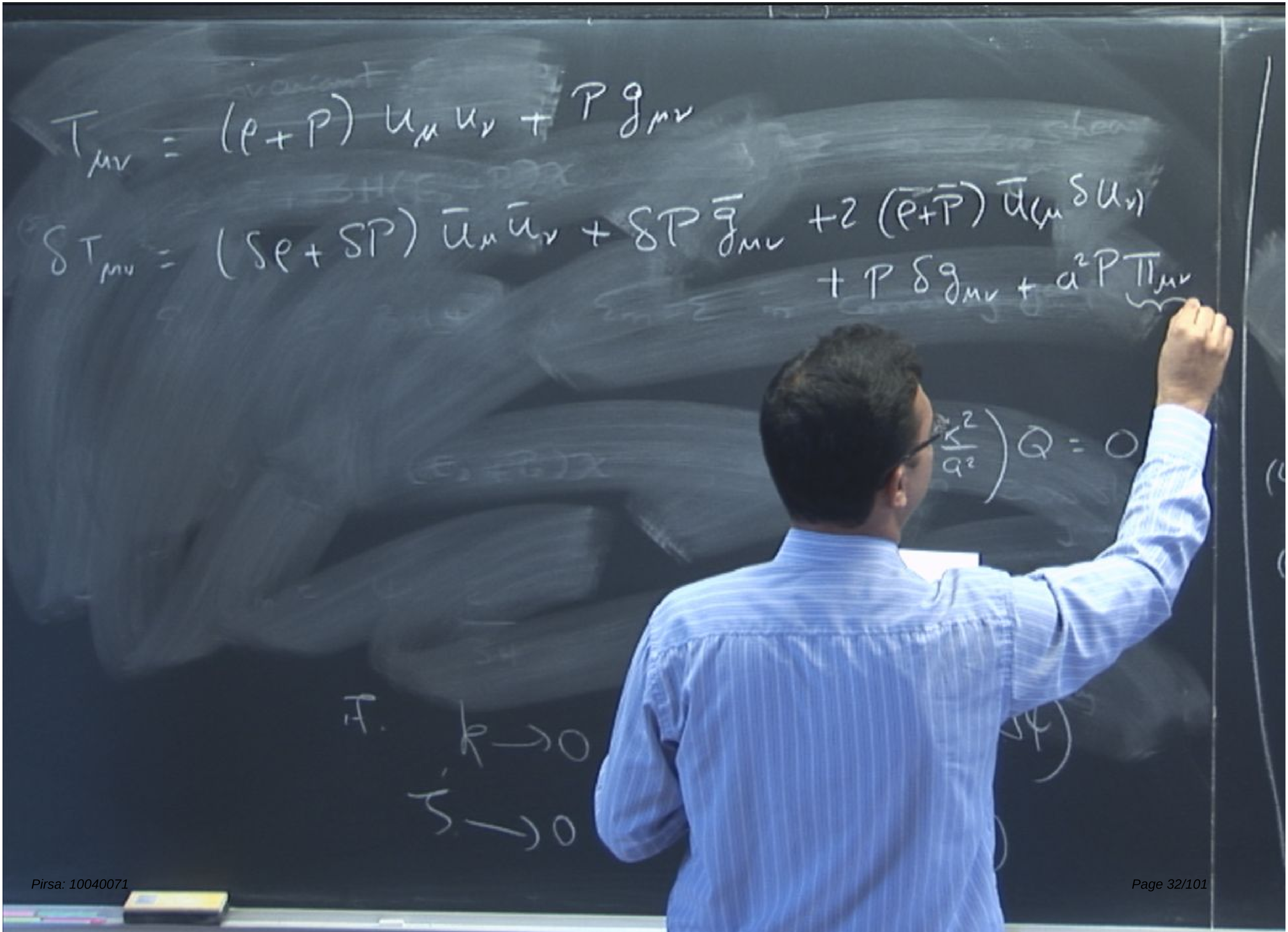

$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$
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$+ P \delta g_{\mu\nu} + a^2 P \Pi_{\mu\nu}$
$\frac{k^2}{a^2}\Big) Q = 0$
$k \to 0$
$\zeta' \to 0$

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$\bar{u}$

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$$\bar{u}_\mu = -a\, \delta^0_\mu$$

F. $k \to 0$

$\zeta' \to 0$

$$\zeta = -\frac{1}{3}\left(\Delta\chi + \Delta\psi\right) \propto \frac{k^2}{3a^2}(\chi + \psi)$$

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$$\bar{u}_\mu = -a\,\delta^0_\mu \;;\quad \bar{u}^\mu = \frac{1}{a}\,\delta^\mu_0 \;;$$

$$\delta u^\mu = \frac{1}{a}(-\alpha, v^i)$$

perturbation

if. $k \to 0$ $\quad \zeta = -\frac{1}{3}(\Delta\chi + \Delta\psi)$

$\zeta \to 0$ $\quad \propto \frac{k^2}{3a^2}(\chi + \psi)$

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$$\delta u^\mu = \frac{1}{a}(-\alpha, \underbrace{v^i}_{\text{perturbation in spatial velocity}}) \;;\quad \delta u_\mu = a(-\alpha,\, v_i - \beta_{,i})$$

$$\text{If. } \quad k \to 0 \qquad \zeta = -\frac{1}{3}\left(\Delta\chi + \Delta\psi\right)$$

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$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P\, g_{\mu\nu}$$

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v^i ↓ perturbation in spatial velocity

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P\, g_{\mu\nu}$$

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$$\delta T^0{}_i =$$

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v^i ↓ perturbation in spatial velocity

q. 9 & 35b)

$$\delta T^0{}_i = \frac{1}{a} D_i \psi$$

$$\delta T_{0i} = -a D_i \psi$$

nal)
me

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$$

$$\delta T_{\mu\nu} = (\delta\rho + \delta P)\, \bar{u}_\mu \bar{u}_\nu + \delta P\, \bar{g}_{\mu\nu} + 2(\overline{\rho + P})\, \bar{u}_{(\mu} \delta u_{\nu)} + P\, \delta g_{\mu\nu} + a^2 P\, \underbrace{\Pi_{\mu\nu}}_{\text{anisotropic stress}}$$

$$\bar{u}_\mu = -a\,\delta^0_\mu \;;\; \bar{u}^\mu = \frac{1}{a}\,\delta^\mu_0 \;;$$

$$\delta u^\mu = \frac{1}{a}(-\alpha \ldots \;;\; \delta u_\mu = a(-\alpha, v_i - \beta_{,i})$$

(Eq. 9 & 35b)

in ...

$$\delta T^0{}_i = \frac{1}{a} D_i \psi \ldots$$

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$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$$

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$$\delta u^\mu = \frac{1}{a}(-\alpha, v^i) \ ; \quad \delta u_\mu = a(\ldots, v_i)$$

perturbation in spatial velocity

(Eq. 9 & 35b)

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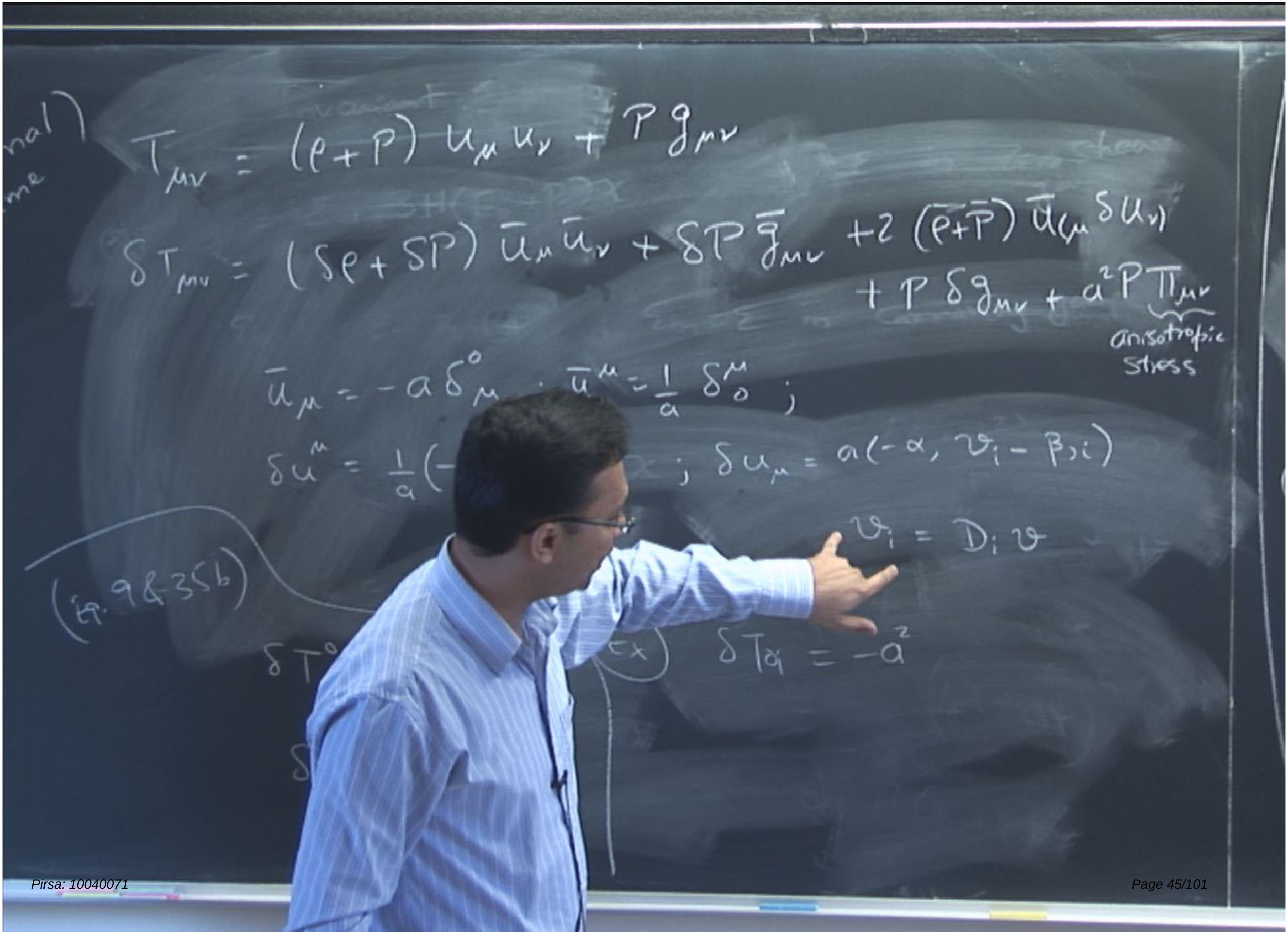
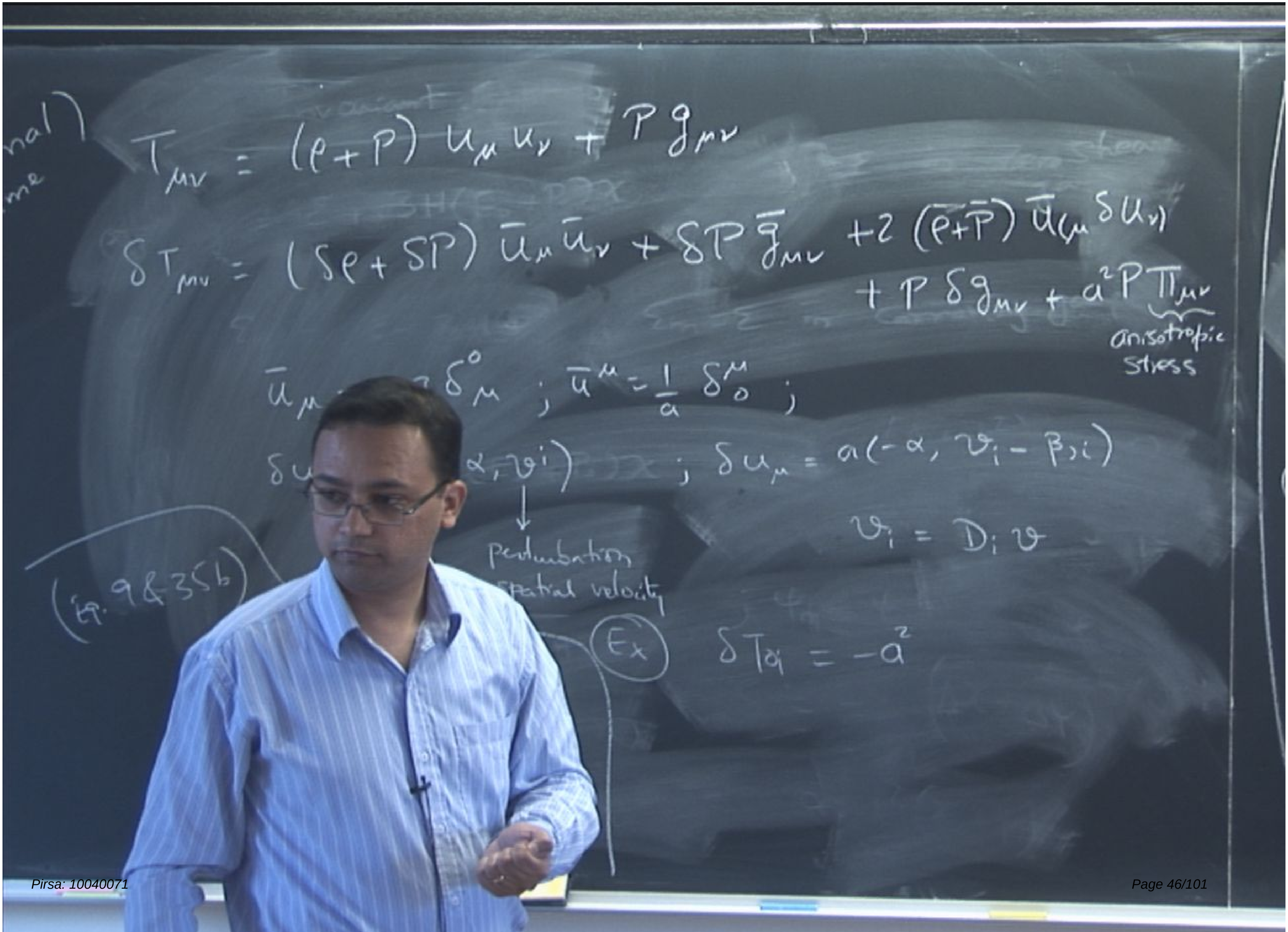
perturbation in spatial velocity

(eq. 9 & 35b)

Ex



$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$
$\delta T_{\mu\nu} = (\delta\rho + \delta P)\, \bar{u}_\mu \bar{u}_\nu + \delta P \bar{g}_{\mu\nu} + 2(\overline{\rho + P})\, \bar{u}_{(\mu} \delta u_{\nu)}$
$+ P \delta g_{\mu\nu} + a^2 P \underbrace{\Pi_{\mu\nu}}_{\text{anisotropic stress}}$
$\bar{u}_\mu = -a \delta^0_\mu$; $\bar{u}^\mu = \frac{1}{a} \delta^\mu_0$;
$\delta u^\mu = \frac{1}{a}(-$; $\delta u_\mu = a(-\alpha, v_i - \beta_{,i})$
$v_i = D_i v$
(Eq. 9 & 35b)
δT^0
$\delta T^0_i = -a^2$



$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$
$\delta T_{\mu\nu} = (\delta\rho + \delta P)\bar{u}_\mu \bar{u}_\nu + \delta P \bar{g}_{\mu\nu} + 2(\overline{\rho + P})\bar{u}_{(\mu}\delta u_{\nu)}$
$+ P \delta g_{\mu\nu} + a^2 P \Pi_{\mu\nu}$
anisotropic stress
$\bar{u}^\mu = \frac{1}{a}\delta^\mu_0$;
$\delta u_\mu = a(-\alpha, v_i - \beta_{,i})$
perturbation
spatial velocity
$v_i = D_i v$
(Eq. 9 & 35b)
Ex
$\delta T_{0i} = -a^2$

nal)
me

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$$

$$\delta T_{\mu\nu} = (\delta\rho + \delta P)\, \bar{u}_\mu \bar{u}_\nu + \delta P\, \bar{g}_{\mu\nu} + 2(\overline{\rho + P})\, \bar{u}_{(\mu} \delta u_{\nu)} + P\, \delta g_{\mu\nu} + a^2 P\, \underbrace{\Pi_{\mu\nu}}_{\text{anisotropic stress}}$$

$$\bar{u}_\mu = -a\,\delta^0_\mu \;;\; \bar{u}^\mu = \tfrac{1}{a}\,\delta^\mu_0 \;;$$

$$\delta u^\mu = \frac{1}{a}(-\alpha, v^i) \qquad \delta u_\mu = a(-\alpha, v_i - \beta_{,i})$$

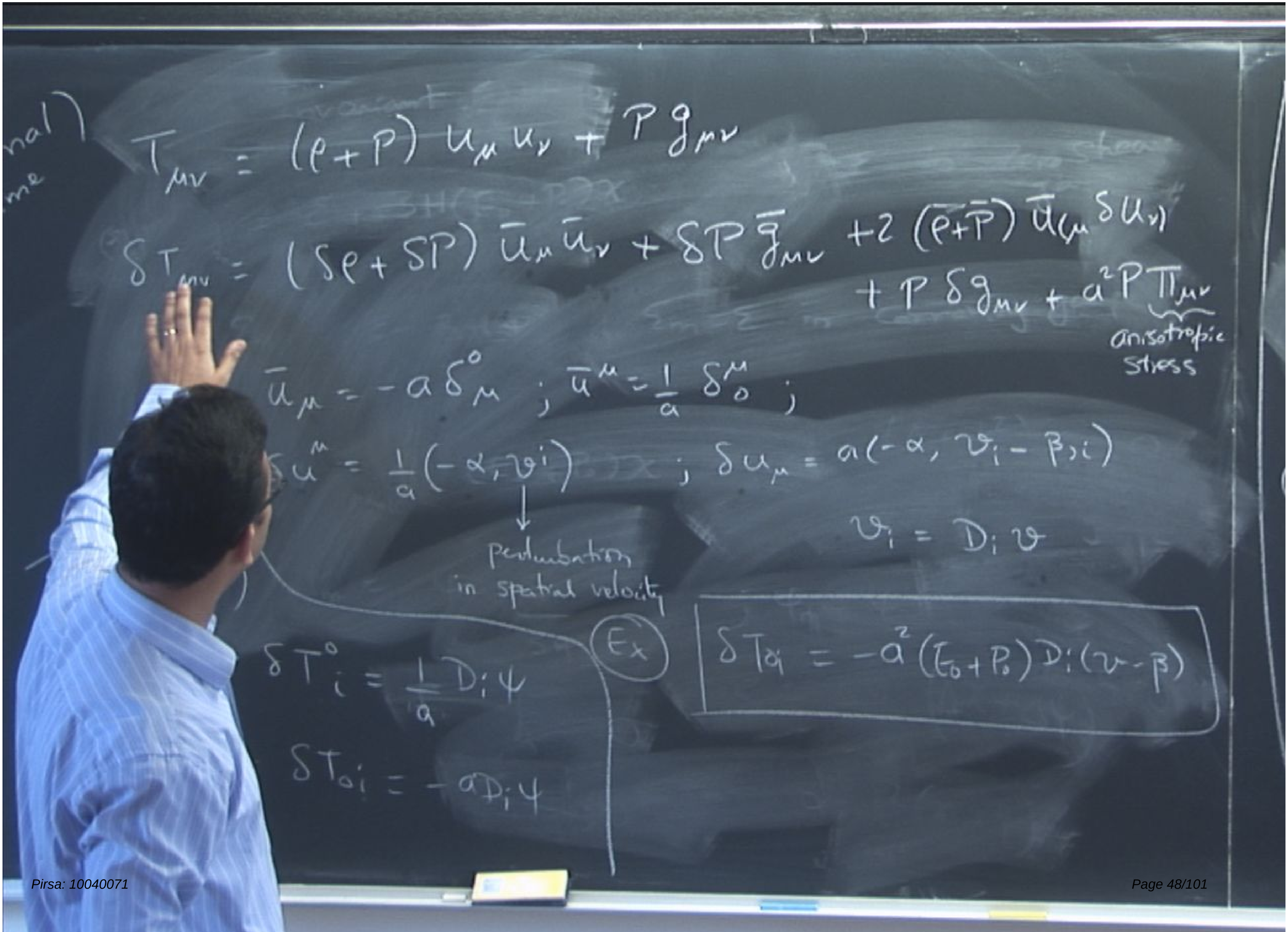
Pecul...

$$v_i = D_i v$$

(Eq. 9 & 35b)

$$\delta T^0 \ldots$$

$$\boxed{\delta T^0_i = - \ldots (\bar\rho_0 + P_0)\, D_i (v - \beta)}$$


$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$
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$\bar{u}_\mu = -a\,\delta^0_\mu \; ; \; \bar{u}^\mu = \frac{1}{a}\delta^\mu_0 \; ;$
$\delta u^\mu = \frac{1}{a}(-\alpha, v^i) \; ; \; \delta u_\mu = a(-\alpha, v_i - \beta_{,i})$
perturbation in spatial velocity
$v_i = D_i v$
$\delta T^0{}_i = \frac{1}{a} D_i \psi$
$\delta T_{0i} = -a D_i \psi$
Ex
$\boxed{\delta T_{0i} = -a^2(\rho_0 + P_0)\, D_i (v - \beta)}$

nal)
me

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$$

$$\delta T_{\mu\nu} = (\delta\rho + \delta P)\, \bar{u}_\mu \bar{u}_\nu + \delta P\, \bar{g}_{\mu\nu} + 2(\bar{\rho} + \bar{P})\, \bar{u}_{(\mu} \delta u_{\nu)} + P\, \delta g_{\mu\nu} + a^2 P \underbrace{\Pi_{\mu\nu}}_{\text{anisotropic stress}}$$

$$\bar{u}_\mu = -a\, \delta^0_\mu \;;\; \bar{u}^\mu = \frac{1}{a}\, \delta^\mu_0 \;;$$

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perturbation in spatial velocity

$$v_i = D_i v$$

Ex

$$\boxed{\delta T_{0i} = -a^2(\bar{\rho}_0 + \bar{P}_0)\, D_i(v - \beta)}$$

$$\delta T^0{}_i = \frac{1}{a} D_i \psi$$

$$\delta T_{0i} = -a D_i \psi$$

nal)
me

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$$

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$\bar{u}_\mu = -a \delta^0_\mu$; $\bar{u}^\mu = \frac{1}{a} \delta^\mu_0$;

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perturbation in spatial velocity

$v_i = D_i v$

(Eq. 9 & 35b)

$\delta T^0{}_i = \frac{1}{a} D_i \psi$

$\delta T_{0i} = -a D_i \psi$

Ex

$$\boxed{\delta T_{0i} = -a^2 (E_0 + P_0) D_i (v - \beta)}$$

$$\boxed{\psi = a (E_0 + P_0)(v - \beta)}$$

nal)
me

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}$$

$$\delta T_{\mu\nu} = (\delta\rho + \delta P)\, \bar{u}_\mu \bar{u}_\nu + \delta P \bar{g}_{\mu\nu} + 2(\bar{\rho} + \bar{P})\, \bar{u}_{(\mu} \delta u_{\nu)} + P \delta g_{\mu\nu} + a^2 P \underbrace{\Pi_{\mu\nu}}_{\text{anisotropic stress}}$$

$$\bar{u}_\mu = -a \delta^0_\mu \; ; \; \bar{u}^\mu = \frac{1}{a} \delta^\mu_0 \; ;$$

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perturbation in spatial velocity

$$v_i = D_i v$$

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$$\delta T^0{}_i = \frac{1}{a} D_i \psi$$

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Ex

$$\boxed{\delta T^0_i = -a^2 (E_0 + P_0) D_i (v - \beta)}$$

$$\boxed{\psi = a(E_0 + P_0)(v - \beta)}$$

$$\zeta = \psi + \frac{\mathcal{E}}{3(E_0+P_0)} \quad \text{is G.I.}$$

$$\dot{\zeta} = \dot{\psi} + \frac{\dot{\mathcal{E}}}{3(E_0+P_0)} - \frac{\mathcal{E}(\dot{E}_0+\dot{P}_0)}{3(E_0+P_0)^2}$$

$$V = v + \gamma'$$

$$-\Delta\psi$$

$$-\frac{\mathcal{E}\dot{E}_0\left(1+\frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0+P_0)(E_0+P_0)}$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

$$= \frac{\Pi}{\mathcal{E}}$$

$$\zeta = \varphi + \frac{\mathcal{E}}{3(E_0+P_0)} \quad \text{is. G.I.}$$

$$\dot{\zeta} = \dot{\varphi} + \frac{\dot{\mathcal{E}}}{3(E_0+P_0)} - \frac{\mathcal{E}(\dot{E}_0+\dot{P}_0)}{3(E_0+P_0)^2}$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\mathcal{E}}$$

$$V = v + \gamma'$$

$$v \to v + L'$$

$$\gamma \to \gamma - L$$

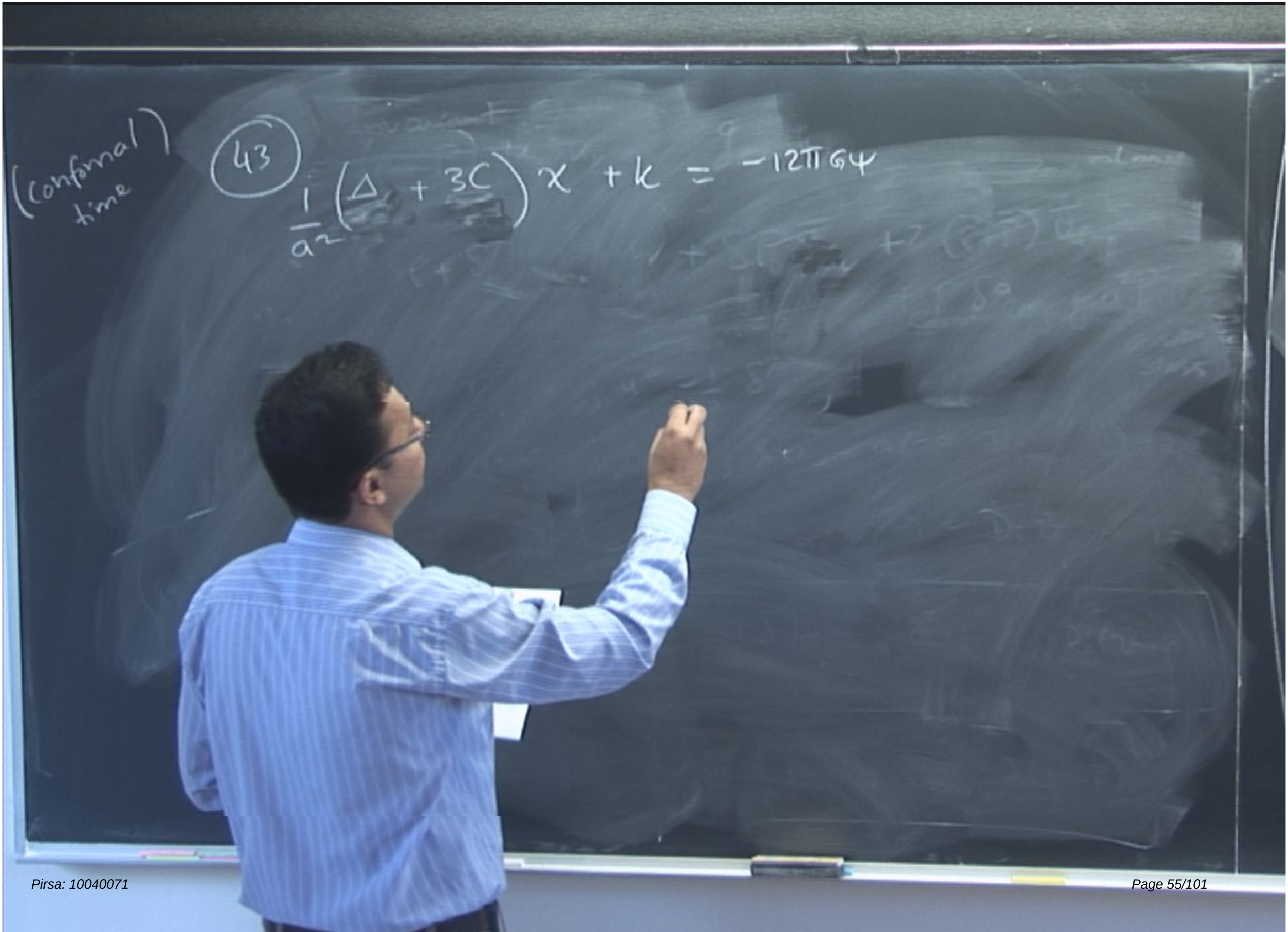
$$\Delta\psi$$

$$-\frac{\mathcal{E}\dot{E}_0\left(1+\frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0+P_0)(E_0+P_0)}$$

(conformal time)

43

$$\left(\frac{\Delta}{a^4} + \frac{3C}{a^2}\right)\chi + k = -12\pi G\psi$$


(conformal) time
43
1/a² (Δ + 3C) x + k = −12πGψ

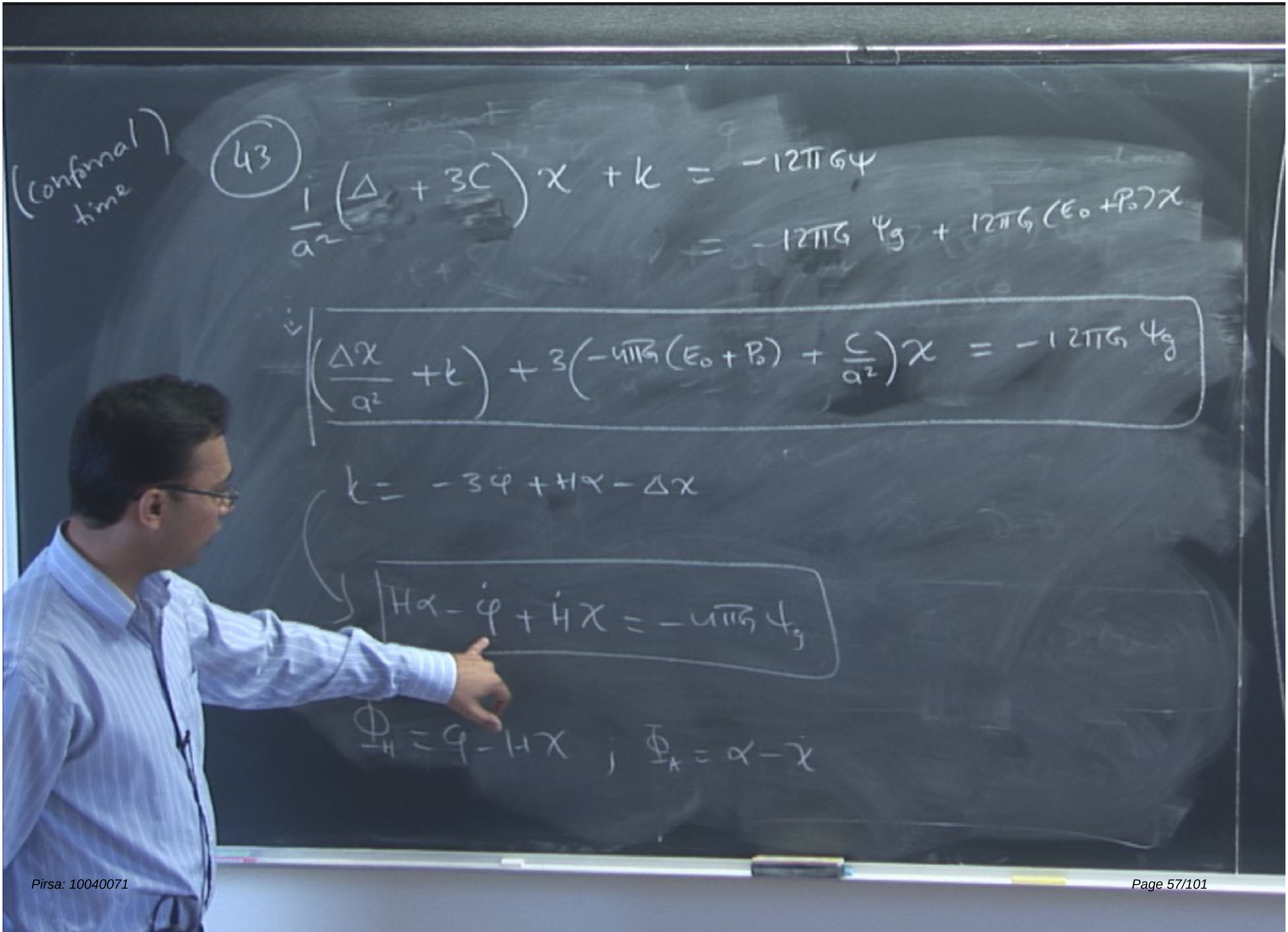
(conformal time)

43

$$\frac{1}{a^2}\left(\Delta + 3C\right)\chi + k = -12\pi G \psi$$

$$= -12\pi G\, \psi_g + 12\pi G\,(\varepsilon_0 + P_0)\chi$$

$$\Rightarrow \boxed{\left(\frac{\Delta\chi}{a^2} + k\right) + 3\left(-4\pi G(\varepsilon_0 + P_0) + \frac{C}{a^2}\right)\chi = -12\pi G\, \psi_g}$$


(conformal time)
43
$\frac{1}{a^2}(\Delta + 3C)\chi + k = -12\pi G\psi$
$= -12\pi G\,\psi_g + 12\pi G(\mathcal{E}_0 + P_0)\chi$
$\boxed{\left(\frac{\Delta\chi}{a^2} + k\right) + 3\left(-4\pi G(\mathcal{E}_0 + P_0) + \frac{C}{a^2}\right)\chi = -12\pi G\,\psi_g}$
$k = -3\dot{\varphi} + H\alpha - \Delta\chi$
$\boxed{H\alpha - \dot{\varphi} + \dot{H}\chi = -4\pi G\,\psi_g}$
$\Phi_H = \varphi - H\chi \;;\; \Phi_\alpha = \alpha - \dot{\chi}$

(conformal) time

43

$$\frac{1}{a^2}\left(\Delta + 3C\right)\chi + k = -12\pi G\psi$$

$$= -12\pi G\,\psi_g + 12\pi G(\varepsilon_0 + P_0)\chi$$

$$\boxed{\left(\frac{\Delta\chi}{a^2} + k\right) + 3\left(-4\pi G(\varepsilon_0 + P_0) + \frac{C}{a^2}\right)\chi = -12\pi G\,\psi_g}$$

$$k = -3\dot\varphi + H\alpha - \Delta\chi$$

$$\boxed{H\alpha - \dot\varphi + \dot H\chi = -4\pi G\,\psi_g}$$

$$\Phi_H = \varphi - H\chi \;;\; \Phi_A = \alpha - \dot\chi$$

$$\boxed{\dot\Phi_H - H\Phi_A = 4\pi G\,\psi_g}$$

(conformal time)

43

$$\frac{1}{a^2}(\Delta + 3C)\chi + k = -12\pi G \psi$$

$$= -12\pi G\, \psi_g + 12\pi G (\varepsilon_0 + P_0)\chi$$

$$\boxed{\left(\frac{\Delta\chi}{a^2} + k\right) + 3\left(-4\pi G(\varepsilon_0 + P_0) + \frac{C}{a^2}\right)\chi = -12\pi G\, \psi_g}$$

$$\psi = a(\varepsilon_0 + P_0)(v - \beta)$$

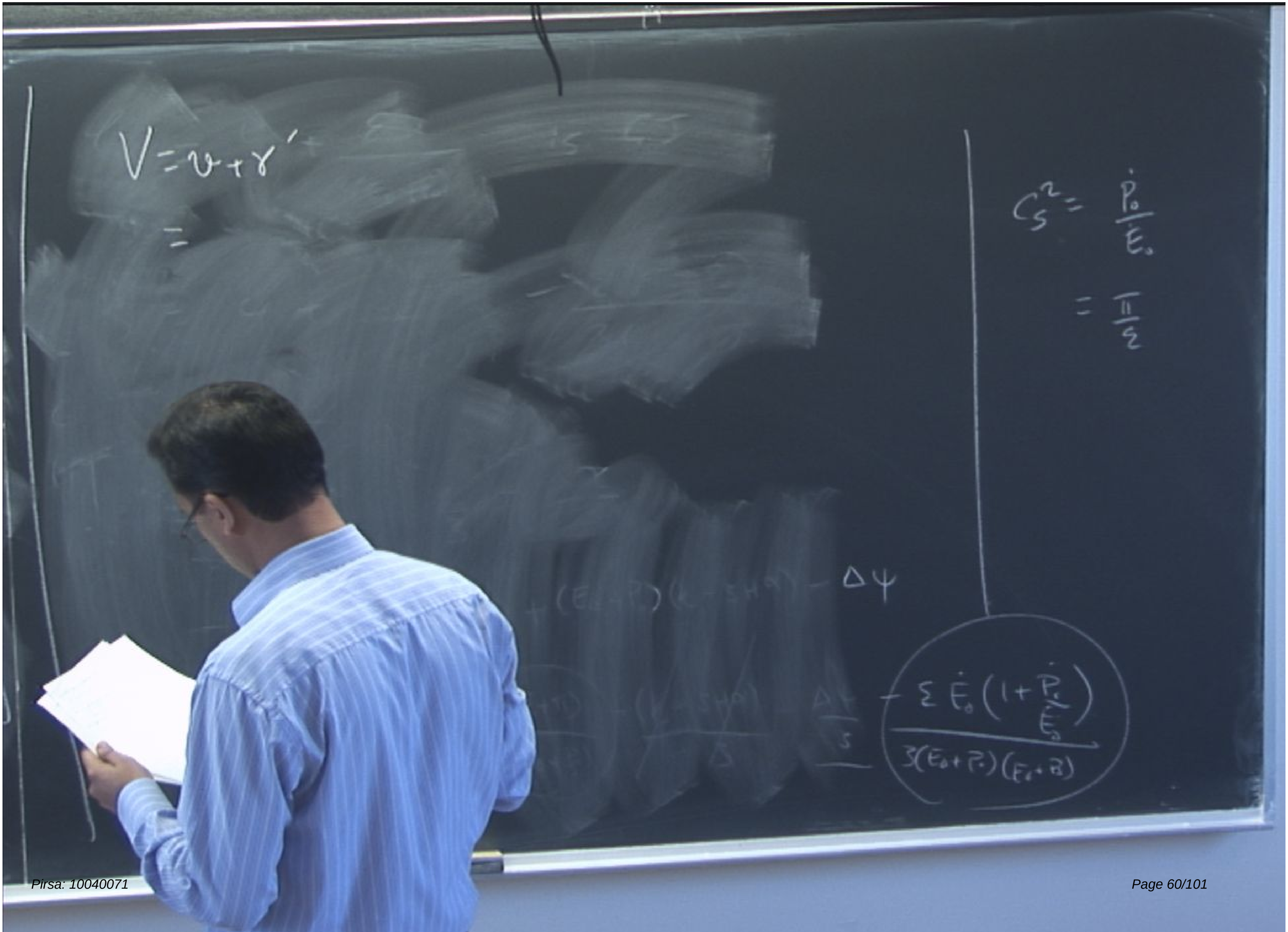
$$\psi_g = \psi + (\varepsilon_0 + P_0)\chi$$

$$k = -3\dot\varphi + H\alpha - \Delta\chi$$

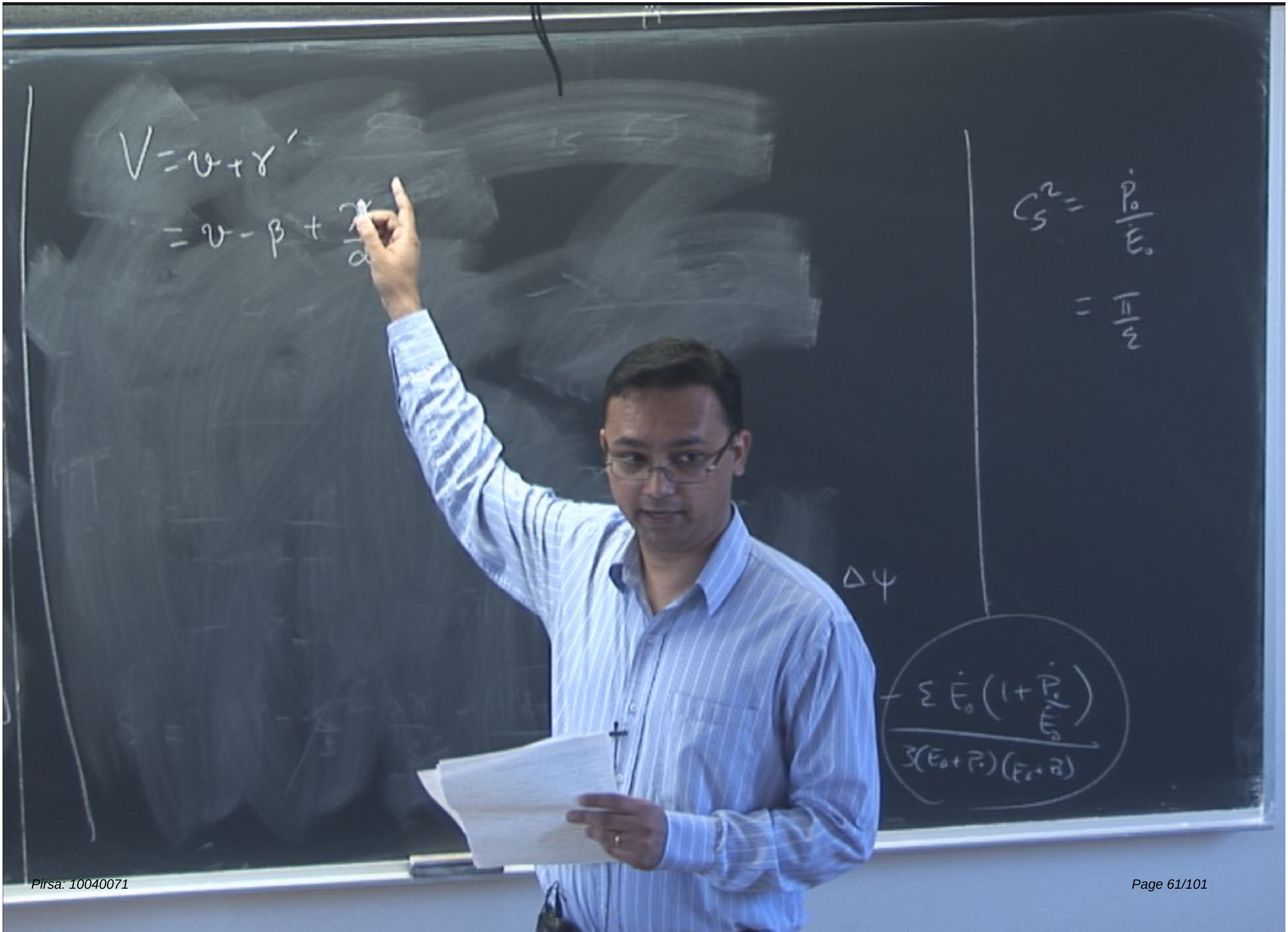
$$\boxed{H\alpha - \dot\varphi + \dot H\chi = -4\pi G\, \psi_g}$$

$$\Phi_H = \varphi - H\chi \;;\; \Phi_A = \alpha - \dot\chi$$

$$\boxed{\dot\Phi_H - H\Phi_A = 4\pi G\, \psi_g}$$



$V = v + \gamma'$
$c_s^2 = \dot{P}_0 / \dot{E}_0$
$= \Pi / \Sigma$
$\Delta\Psi$
$\dfrac{\Sigma \dot{E}_0 \left(1 + \dot{P}_0/\dot{E}_0\right)}{3(E_0+P_0)(E_0+P_0)}$


$V = v + \gamma'$
$= v - \beta + \frac{\gamma}{\alpha}$
$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$
$= \frac{\Pi}{\Sigma}$
$\Delta \Psi$
$\Sigma \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)$
$3(E_0 + P_0)(E_0 + P_0)$

$$V = v + \gamma'$$

$$= v - \beta + \frac{\chi}{\alpha}$$

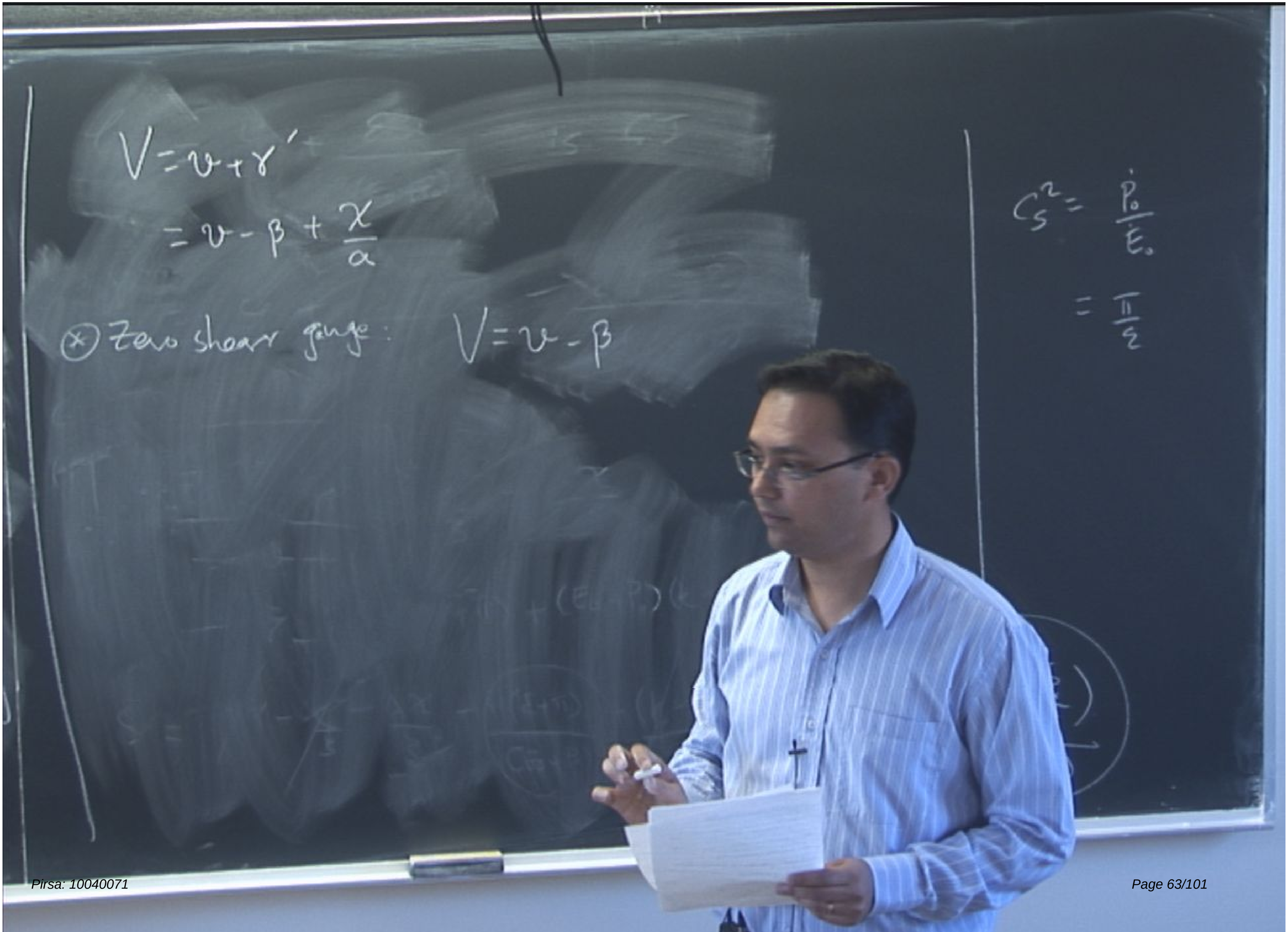
(*) Zero shear gauge:

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

$$= \frac{\Pi}{\Sigma}$$

$$- \Delta\Psi$$

$$\frac{\Sigma \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$


V = v + γ'
= v - β + χ/a
Zero shear gauge: V = v - β
c_s^2 = Ṗ_0 / Ė_0
= Π/Σ

$$V = v + \gamma'$$

$$= v - \beta + \frac{\chi}{a}$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

$$= \frac{\Pi}{\Sigma}$$

(*) Zero shear gauge: $V = v - \beta$

$$\Psi_g = \Psi = a(E_0 + P_0)\underbrace{(v - \beta)}_{V}$$

$$\Phi_H' - \mathcal{H}\Phi_A$$

$\Delta\Psi$

$$\frac{\Sigma \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$

$\pi G (E_0 + P_0) \chi$

$\boxed{- 12\pi G \, \Psi_g}$

$+ P_0)(v - \beta)$

$(E_0 + P_0) \chi$

$\boxed{H \Phi_A = 4\pi G \, \Psi_g}$

$$V = v + \gamma'$$

$$= v - \beta + \frac{\chi}{a}$$

⊛ Zero shear gauge: $V = v - \beta$

$$\Psi_g = \Psi = a (E_0 + P_1) \underbrace{(v - \beta)}_{V}$$

$$\longrightarrow \Phi'_H - \mathcal{H} \Phi_A$$

$\Delta\Psi$

$$\frac{-\Sigma \dot{E}_0 (1 +}{3(E_0 + P_0)(E_0}$$

$$= -12\pi G \Psi$$

$$= -12\pi G\, \Psi_g + 12\pi G(\epsilon_0 + P_0)\chi$$

$$\boxed{\epsilon_0 + P_0) + \frac{c}{a^2}\Big)\chi = -12\pi G\, \Psi_g}$$

$$\Psi = a(\epsilon_0 + P_0)(v - \beta)$$

$$\Psi_g = \Psi + (\epsilon_0 + P_0)\chi$$

$$\boxed{\Psi_g}$$

$$\boxed{\dot{\Phi}_H - H\Phi_A = 4\pi G\, \Psi_g}$$

$$\frac{\Phi_H'}{a}$$

$$V = v + \gamma'$$

$$= v - \beta + \frac{\chi}{a}$$

⊛ Zero shear gauge: $V = v - \beta$

$$\Psi_g = \Psi = a(\epsilon_0 + P_0)\underbrace{(v - \beta)}_{V}$$

$$\Phi_H' - \mathcal{H}\Phi_A$$

$$V = v + \gamma'$$

$$= v - \beta + \frac{\chi}{a}$$

(*) Zero shear gauge: $V = v - \beta$

$$\Psi_g = \Psi = a(E_0 + P_0)\underbrace{(v - \beta)}_{V}$$

$$\boxed{\begin{aligned} \Phi_H &= -\Psi \\ \Phi_A &= \Phi \end{aligned}}$$

$$\rightarrow \Phi_H' - \mathcal{H}\Phi_A = 4\pi G a^2 (E_0 + P_0) V$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\Sigma}$$

$$-\frac{\Sigma \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$

$$V = v + \gamma'$$
$$= v - \beta + \frac{\chi}{a}$$

(*) Zero shear gauge: $V = v - \beta$

$$\Psi_g = \Psi = a(E_0 + P_0)\underbrace{(v - \beta)}_{V}$$

$$\boxed{\begin{aligned}\Phi_H &= -\Psi \\ \Phi_A &= \Phi\end{aligned}}$$

$$\Phi_H' - \mathcal{H}\Phi_A = 4\pi G a^2 (E_0 + P_0) V$$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2 (\rho + P) V}$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\varepsilon}$$

$$-\frac{\varepsilon \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$

$$V = v + \gamma'$$

$$= v - \beta + \frac{\chi}{a}$$

(*) Zero shear gauge: $V = v - \beta$

$$\Psi_g = \Psi = a(E_0 + P_0)\underbrace{(v - \beta)}_{V}$$

$$\Phi_H' - \mathcal{H}\Phi_A = 4\pi G a^2 (E_0 + P_0) V$$

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$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

$$= \frac{\Pi}{\varepsilon}$$

$$-\frac{\varepsilon \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$

$= -12\pi G \Psi$

$= -12\pi G\, \Psi_g + 12\pi G (\mathcal{E}_0 + P_0)\chi$

$$\boxed{(\mathcal{E}_0 + P_0) + \frac{c}{a^2})\chi = -12\pi G\, \Psi_g}$$

$\Psi = a(\mathcal{E}_0 + P_0)(v - \beta)$

χ

$\Psi_g = \Psi + (\mathcal{E}_0 + P_0)\chi$

$\frac{\dot{a}}{a}$

$\boxed{4\pi G\, \Psi_g}$

$$\boxed{\dot{\Phi}_H - H\Phi_A = 4\pi G\, \Psi_g}$$

$\alpha - \dot{\chi}$

$\frac{\Phi_H'}{a}$

$V = v + \gamma'$

$= v - \beta + \frac{\chi}{a}$

⊛ Zero shear gauge: $V = v - \beta$

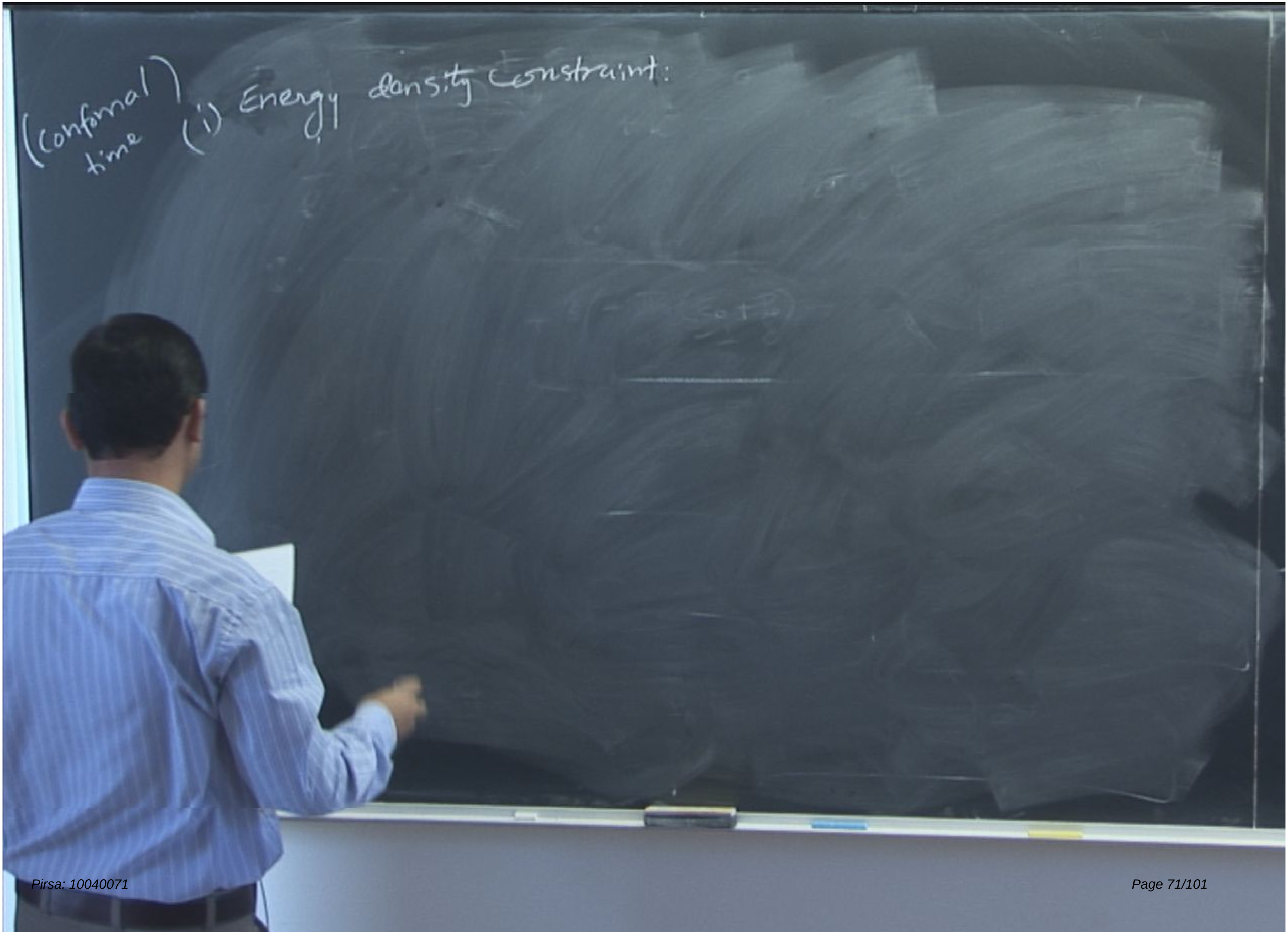
$\Psi_g = \Psi = a(\mathcal{E}_0 + P_0)\underbrace{(v - \beta)}_{V}$

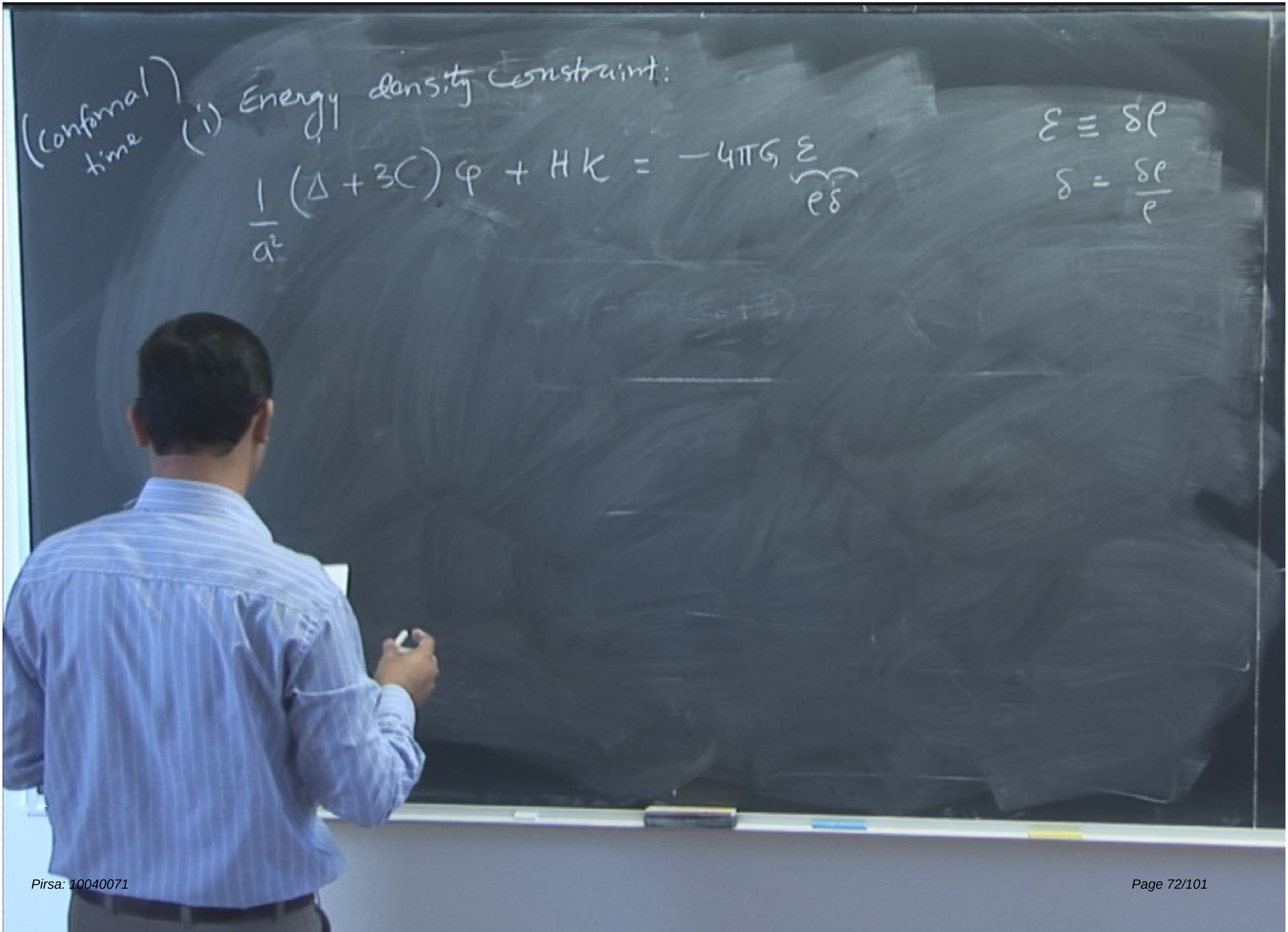
$\Phi_H' - \mathcal{H}\Phi_A = 4\pi G a^2 (\mathcal{E}_0 + P_0) V$

$\frac{a'}{a}$

$\boxed{\mathcal{H} = aH}$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2 (\rho + P) V}$$


(conformal) time
(i) Energy density constraint:


(conformal) time
(i) Energy density constraint:
$\frac{1}{a^2}(\Delta + 3C)\varphi + Hk = -4\pi G \underbrace{\varepsilon}_{\rho\delta}$
$\varepsilon \equiv \delta\rho$
$\delta = \frac{\delta\rho}{\rho}$

(conformal time)

(i) Energy density constraint:

$$\frac{1}{a^2}(\Delta + 3C)\varphi + Hk = -4\pi G \underbrace{\varepsilon}_{\rho\delta}$$

$$\varepsilon \equiv \delta\rho$$

$$\delta = \frac{\delta\rho}{\rho}$$

u.s.

$$\frac{1}{a^2}(\Delta + 3C)\chi + k = -12\pi G \psi$$

$$\varepsilon \longrightarrow \varepsilon_m = \varepsilon - 3H\psi$$

(conformal time) (i) Energy density constraint:

$$\varepsilon \equiv \delta\rho$$

$$\delta = \frac{\delta\rho}{\rho}$$

$$\frac{1}{a^2}(\Delta + 3C)\varphi + Hk = -4\pi G \underbrace{\varepsilon}_{\rho\delta}$$

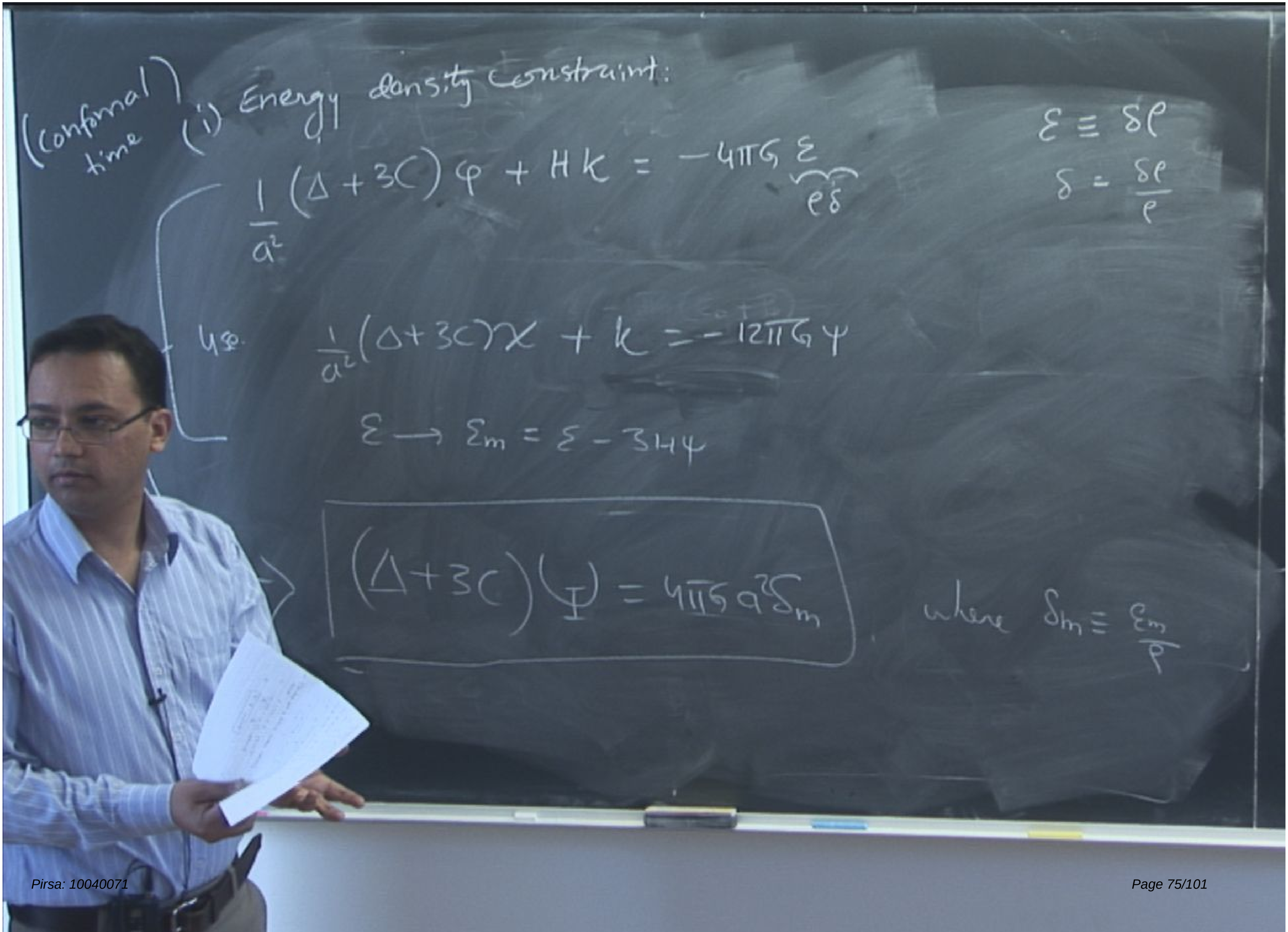
use

$$\frac{1}{a^2}(\Delta + 3C)\chi + k = -12\pi G \psi$$

$$\varepsilon \longrightarrow \varepsilon_m = \varepsilon - 3H\psi$$

$$\boxed{(\Delta + 3C)\Psi = 4\pi G a^2 \delta_m}$$

where $\delta_m \equiv \frac{\varepsilon_m}{\rho}$


(conformal time)
(i) Energy density constraint:
$\frac{1}{a^2}(\Delta + 3C)\varphi + Hk = -4\pi G \underbrace{\mathcal{E}}_{\rho\delta}$
$\mathcal{E} \equiv \delta\rho$
$\delta = \frac{\delta\rho}{\rho}$
use
$\frac{1}{a^2}(\Delta + 3C)\chi + k = -12\pi G \psi$
$\mathcal{E} \longrightarrow \mathcal{E}_m = \mathcal{E} - 3H\psi$
$\Rightarrow \boxed{(\Delta + 3C)\Psi = 4\pi G a^2 \delta_m}$
where $\delta_m \equiv \frac{\mathcal{E}_m}{\rho}$

(4.57) $$\left(D^i D_j - \frac{1}{3}\delta^i_j \Delta\right)\left(\dot{\chi} + H\chi - \alpha - \varphi - 8\pi G\sigma\right) = 0$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\varepsilon}$$

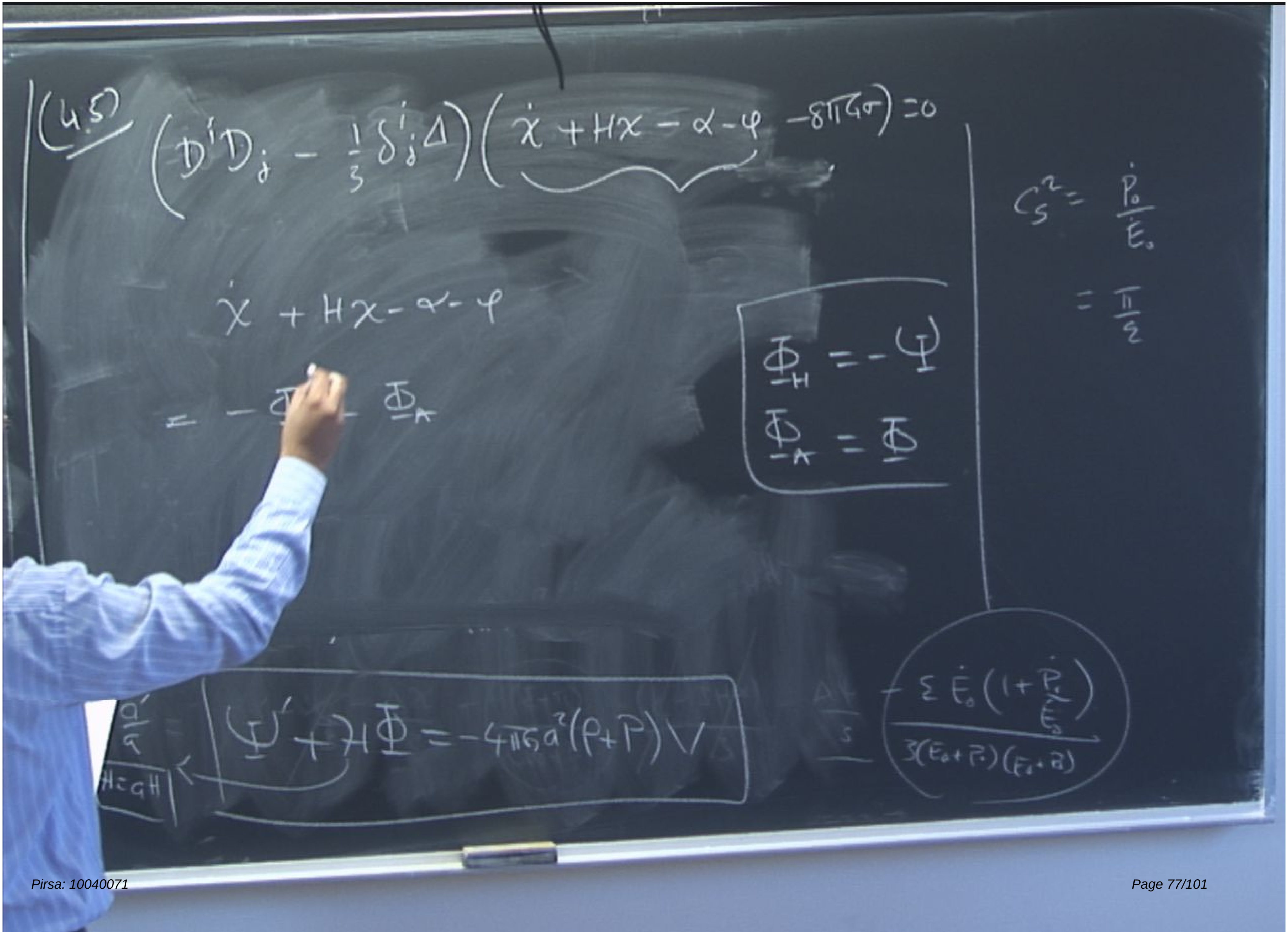
$$\boxed{\begin{aligned} \Phi_H &= -\Psi \\ \Phi_A &= \Phi \end{aligned}}$$

$\frac{a'}{a}$

$\boxed{\mathcal{H} = aH}$

$4\pi G a^2(\rho + P)V$

$$-\frac{\Sigma \dot{E}_0\left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$


(4.5)
$\left(D^i D_j - \frac{1}{3}\delta^i_j \Delta\right)\left(\dot{\chi} + H\chi - \alpha - \varphi - 8\pi G\sigma\right) = 0$
$\dot{\chi} + H\chi - \alpha - \varphi$
$= - \Phi_H - \Phi_A$
$\Phi_H = -\Psi$
$\Phi_A = \Phi$
$c_s^2 = \frac{\dot{p}_0}{\dot{E}_0}$
$= \frac{\Pi}{\varepsilon}$
$\Psi' + \mathcal{H}\Phi = -4\pi G a^2(\rho + P) V$
$\mathcal{H} = aH$
$\frac{\Sigma \dot{E}_0 \left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$

(4.5) $\left(D^i D_j - \frac{1}{3}\delta^i_j \Delta\right)\left(\underbrace{\dot{\chi} + H\chi - \alpha - \varphi}\ - 8\pi G\sigma\right) = 0$

$$\dot{\chi} + H\chi - \alpha - \varphi$$

$$= -\Phi_H - \Phi_A$$

$$\boxed{\begin{aligned}\Phi_H &= -\Psi \\ \Phi_A &= \Phi\end{aligned}}$$

$$(\ldots)(\Psi - \Phi) = (\ldots)\, 8\pi G\sigma$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\mathcal{E}}$$

$\frac{a'}{a}$ $\boxed{\mathcal{H} = aH}$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2 (\rho + P) V}$$

$$-\frac{\mathcal{E}\dot{E}_0\left(1 + \frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0 + P_0)(E_0 + P_0)}$$

(4.5) $\left(D^i D_j - \frac{1}{3}\delta^i_j \Delta\right)\left(\underbrace{\dot{\chi} + H\chi - \alpha - \varphi}\ -8\pi G\sigma\right) = 0$

$$\dot{\chi} + H\chi - \alpha - \varphi$$

$$= -\Phi_H - \Phi_A$$

$$\Phi_H = -\Psi$$

$$\Phi_A = \Phi$$

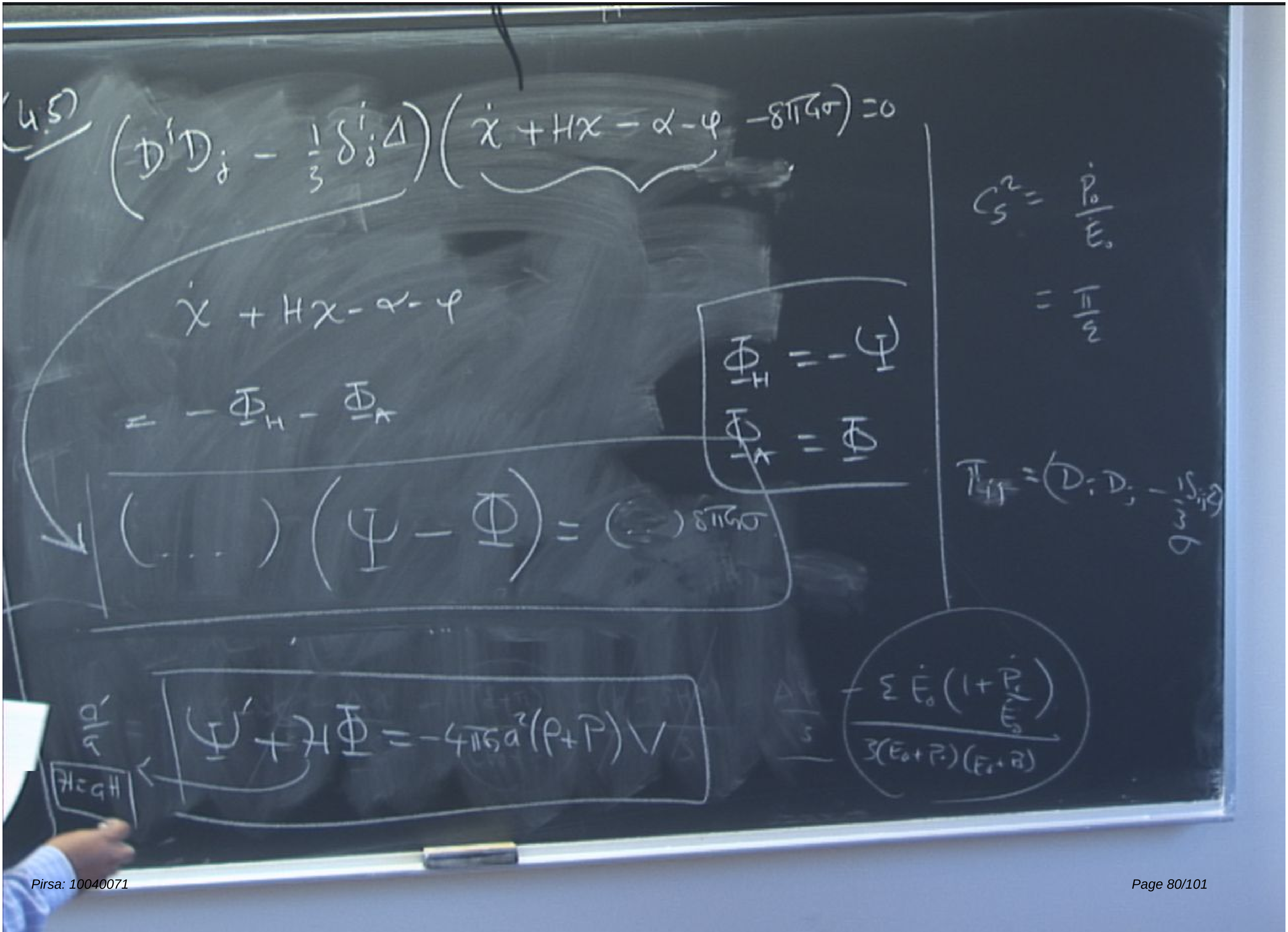
$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\varepsilon}$$

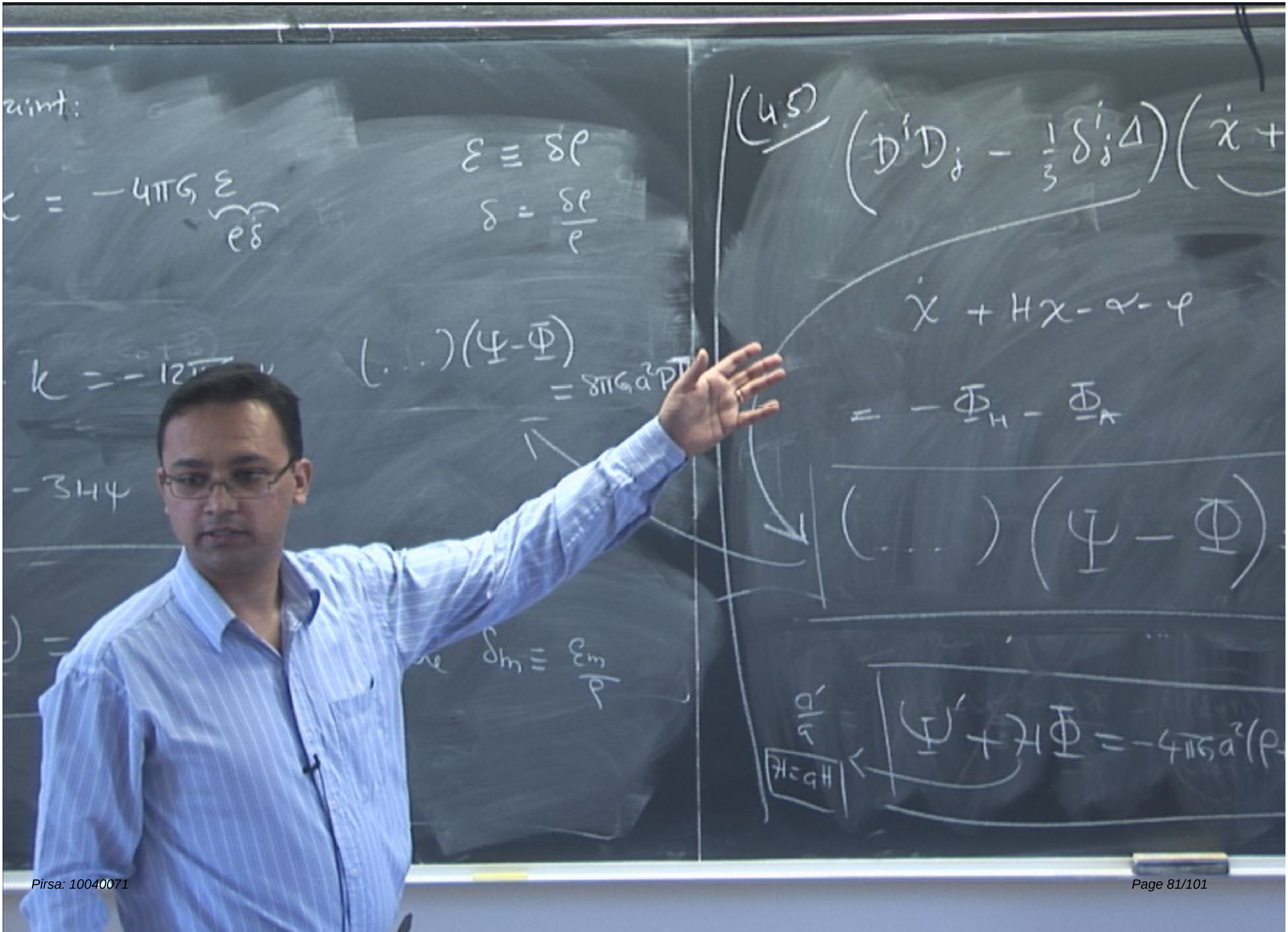
$$\boxed{(\ldots)\left(\Psi - \Phi\right) = (\ldots)\, 8\pi G\sigma}$$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2(\rho + P)V}$$

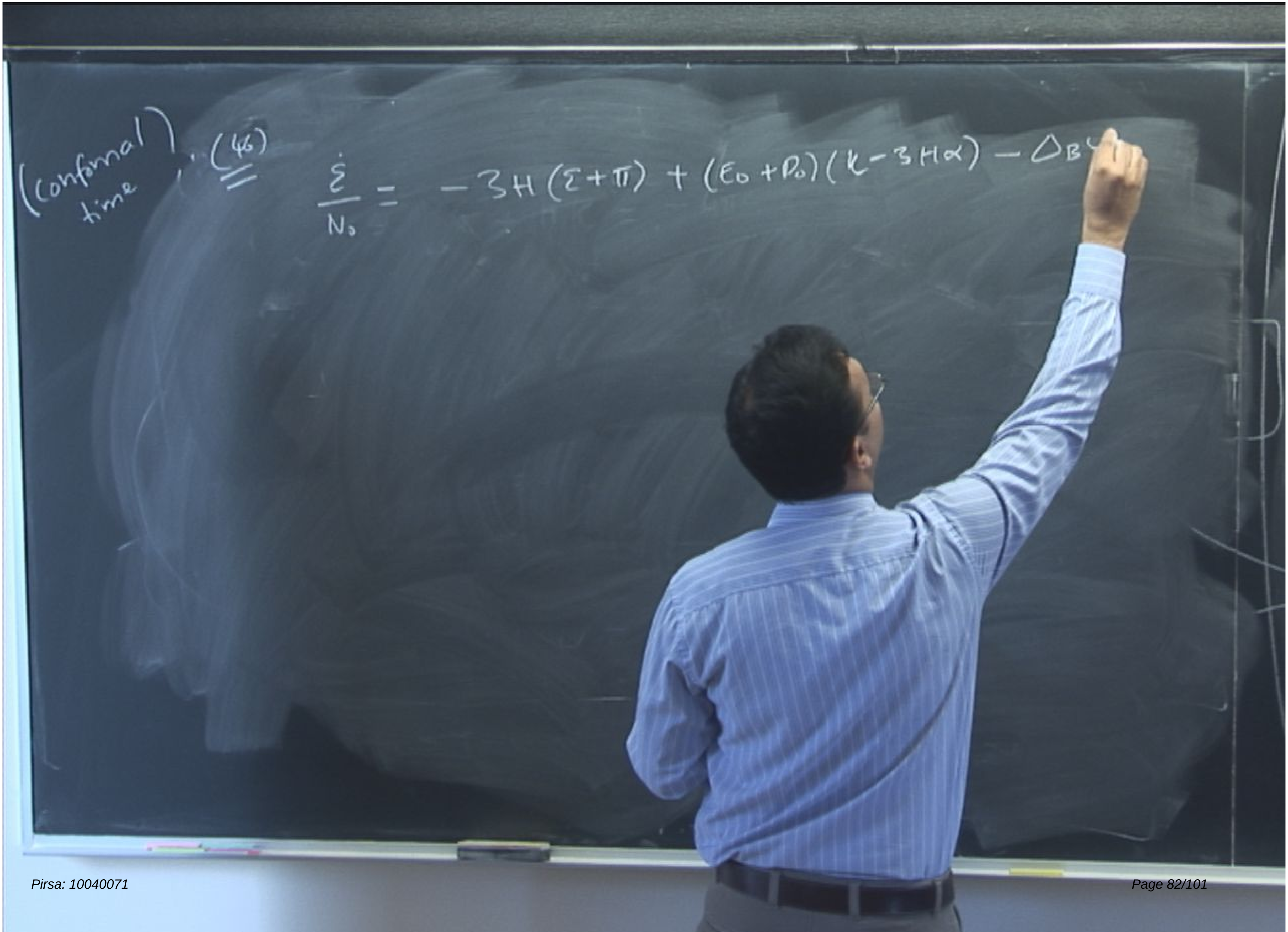
$\frac{a'}{a}$ $\boxed{\mathcal{H} = aH}$

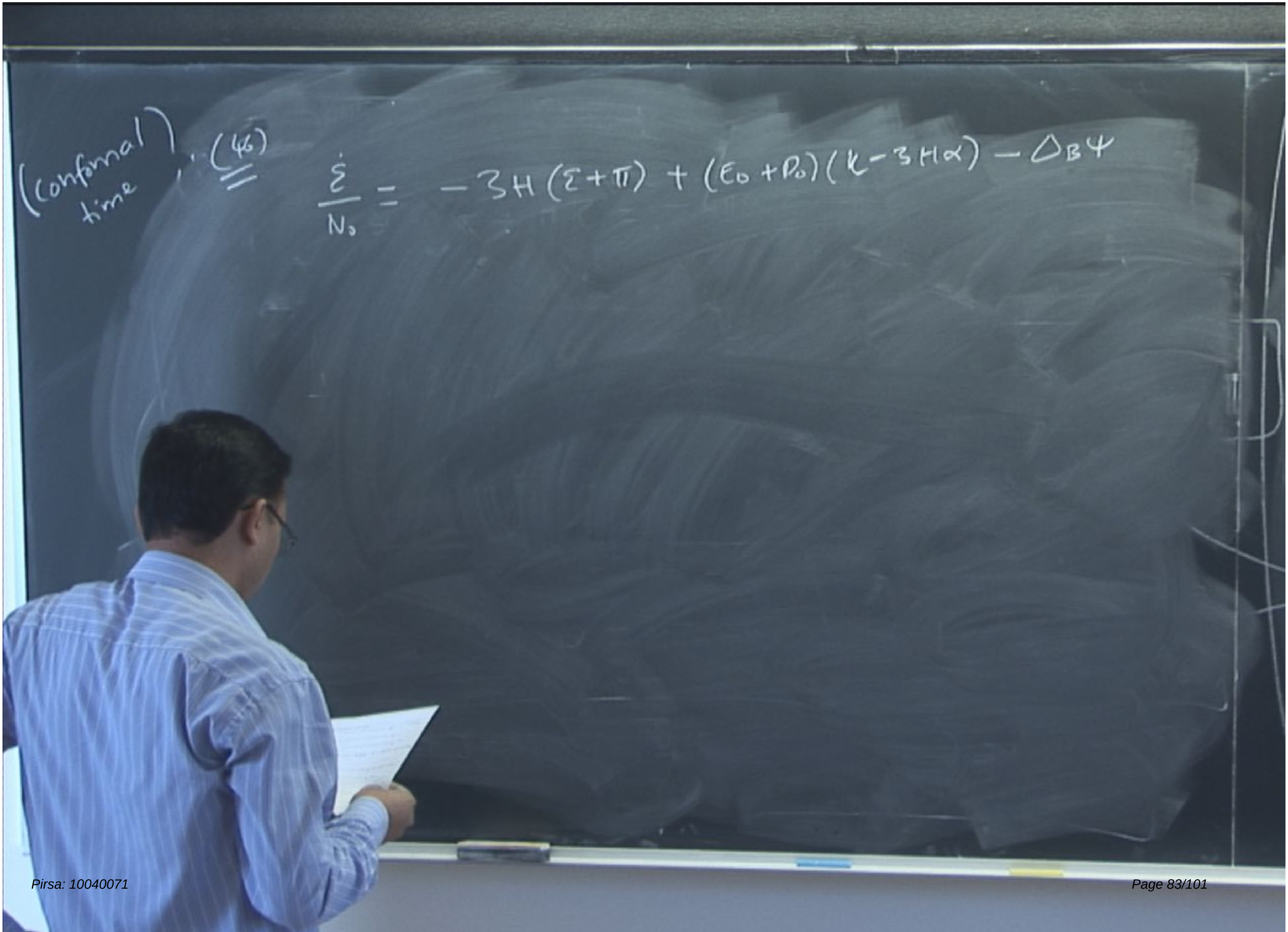
$$-\frac{\varepsilon \dot{E}_0\left(1+\frac{\dot{P}_0}{\dot{E}_0}\right)}{3(E_0+P_0)(E_0+P_0)}$$





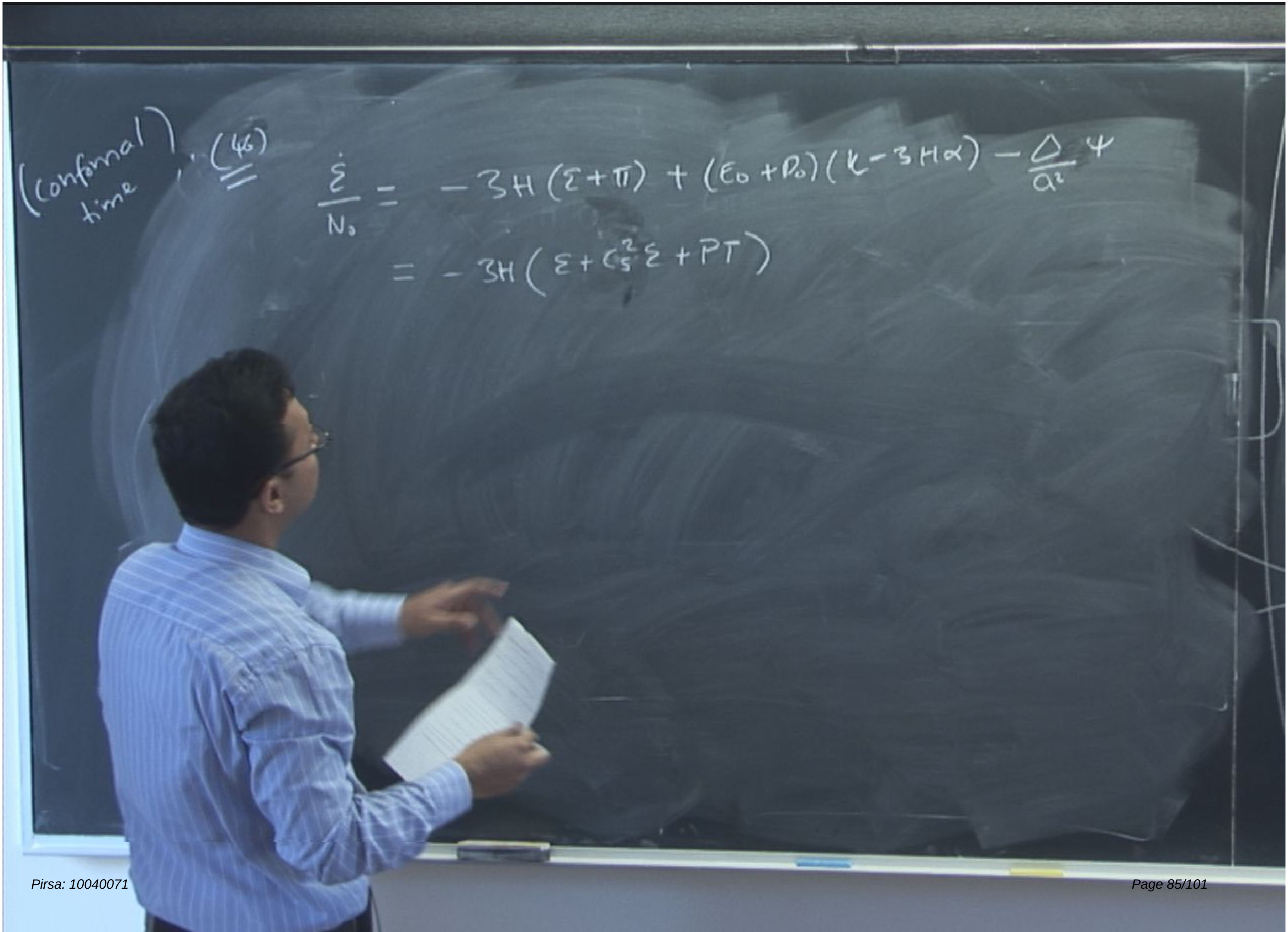
(4.5)
ε ≡ δρ
δ = δρ/ρ
−4πG ε
ρδ
(...)(Ψ − Φ)
= 8πGa²P
ẋ + Hχ − α − φ
= − Φ_H − Φ_A
(...)(Ψ − Φ)
δ_m ≡ ε_m/ρ
a′/a
Ψ′ + ℋΦ = −4πGa²(ρ
ℋ = aH

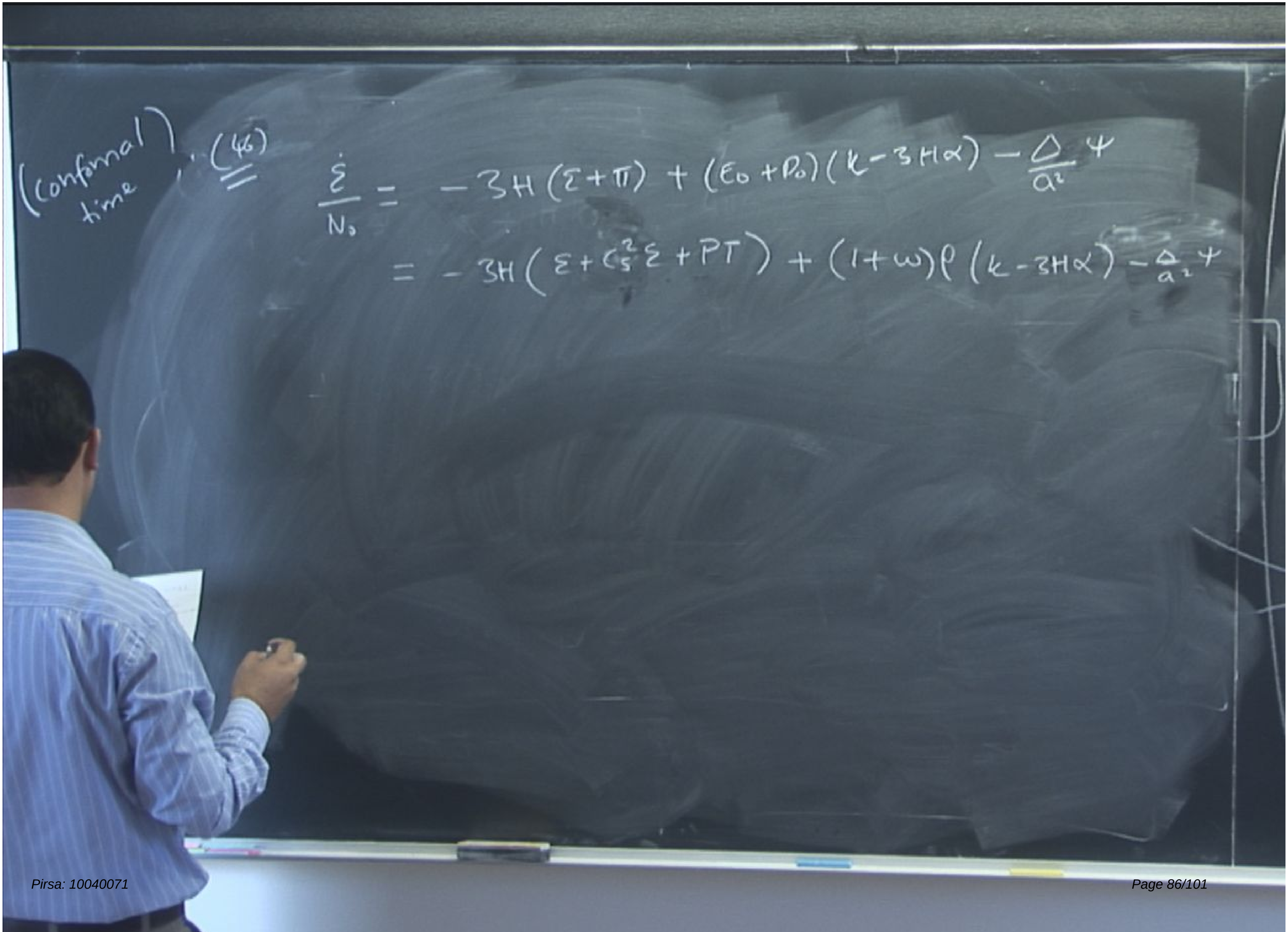

(conformal time)
(46)
$\frac{\dot{\varepsilon}}{N_0} = -3H(\varepsilon+\pi) + (E_0+P_0)(k-3H\alpha) - \Delta B$


(conformal time)
(46)
$\frac{\dot{\mathcal{E}}}{N_0} = -3H(\mathcal{E}+\Pi) + (\mathcal{E}_0+P_0)(\kappa-3H\alpha) - \Delta_B\Psi$

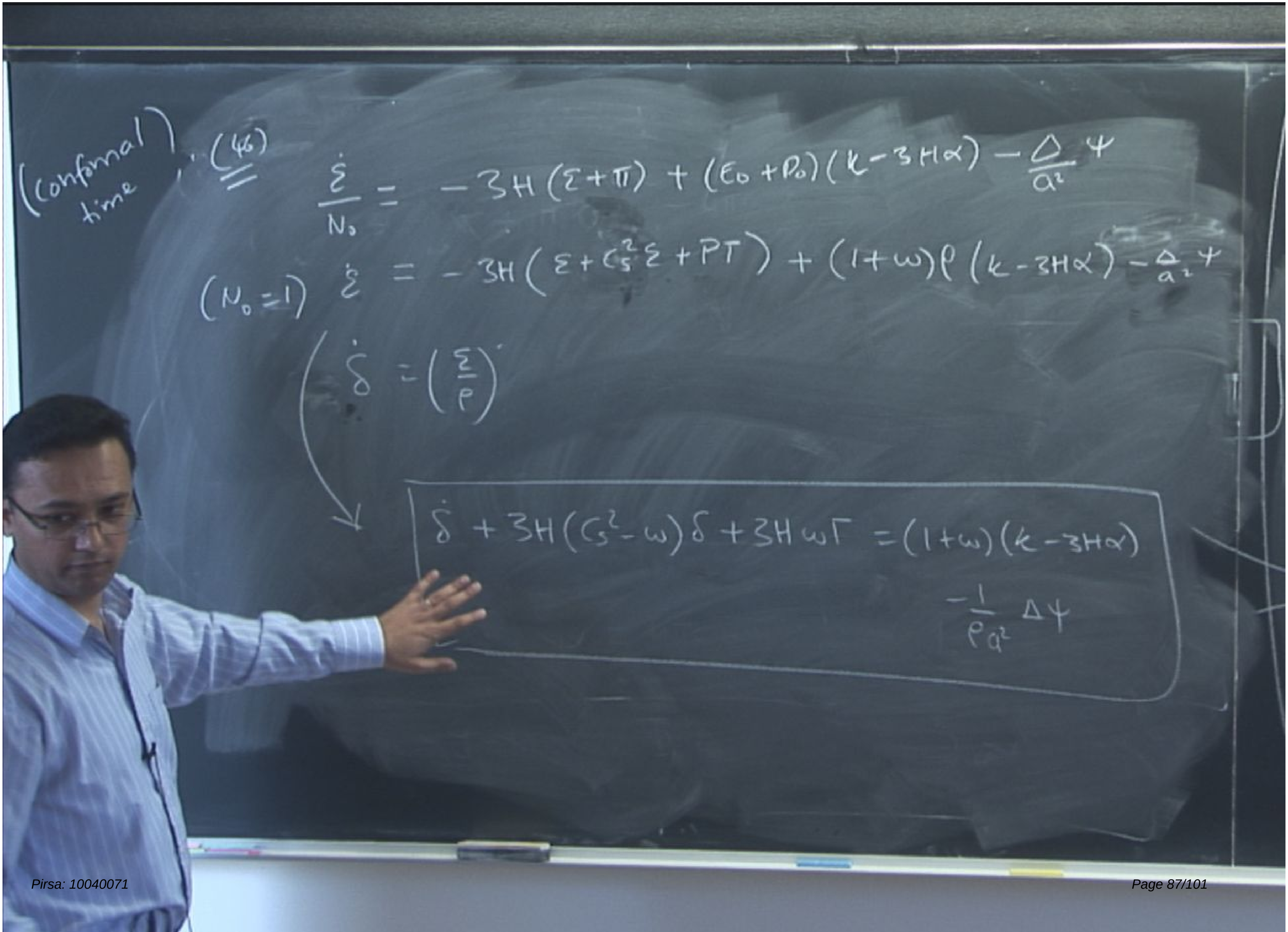
(conformal time)

(46)
$$\frac{\dot{\mathcal{E}}}{N_0} = -3H(\mathcal{E}+\Pi) + (\epsilon_0+p_0)(\kappa - 3H\alpha) - \frac{\Delta}{a^2}\Psi$$


(conformal time)
(46)
$\frac{\dot{\varepsilon}}{N_0} = -3H(\varepsilon + \pi) + (\varepsilon_0 + p_0)(\kappa - 3H\alpha) - \frac{\Delta}{a^2}\psi$
$= -3H(\varepsilon + c_s^2 \varepsilon + pT)$



(conformal time)
(46)
$\frac{\dot{\mathcal{E}}}{N_0} = -3H(\mathcal{E}+\Pi) + (\varepsilon_0+p_0)(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$= -3H(\mathcal{E} + c_s^2\mathcal{E} + pT) + (1+w)\rho(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$


(conformal time)
(46)
$\frac{\dot{\varepsilon}}{N_0} = -3H(\varepsilon+\pi) + (\varepsilon_0+P_0)(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$(N_0=1)$ $\dot{\varepsilon} = -3H(\varepsilon + c_s^2\varepsilon + P\Gamma) + (1+w)\rho(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$\dot{\delta} = \left(\frac{\varepsilon}{\rho}\right)^{\cdot}$
$\boxed{\dot{\delta} + 3H(c_s^2 - w)\delta + 3Hw\Gamma = (1+w)(k-3H\alpha) - \frac{1}{\rho a^2}\Delta\Psi}$

(conformal time) (46)

$$\frac{\dot{\mathcal{E}}}{N_0} = -3H(\mathcal{E}+\Pi) + (\varepsilon_0+P_0)(k-3H\alpha) - \frac{\Delta}{a^2}\psi$$

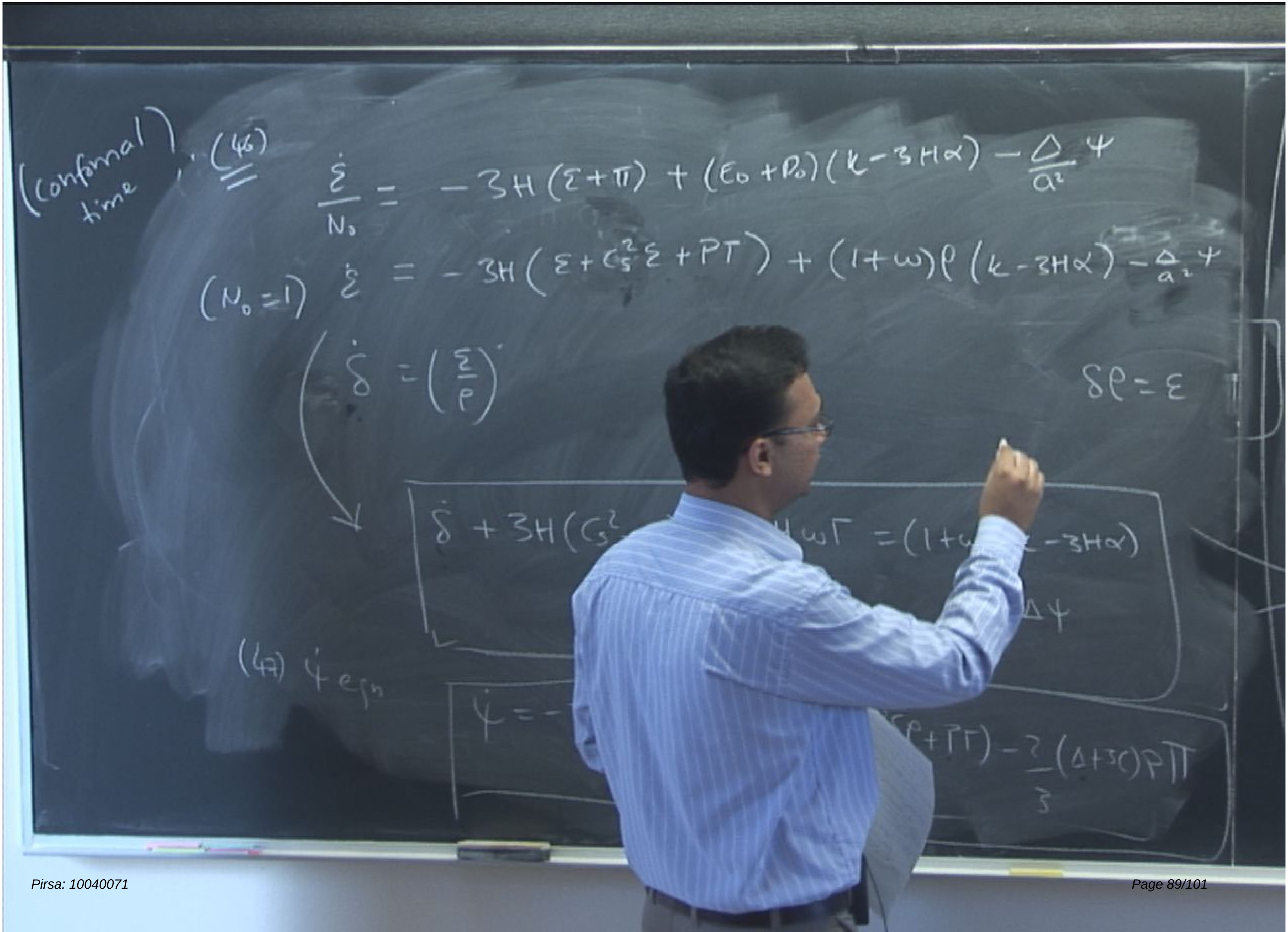
$(N_0=1)$ $$\dot{\mathcal{E}} = -3H(\mathcal{E}+c_s^2\mathcal{E}+P\Gamma) + (1+\omega)\rho(k-3H\alpha) - \frac{\Delta}{a^2}\psi$$

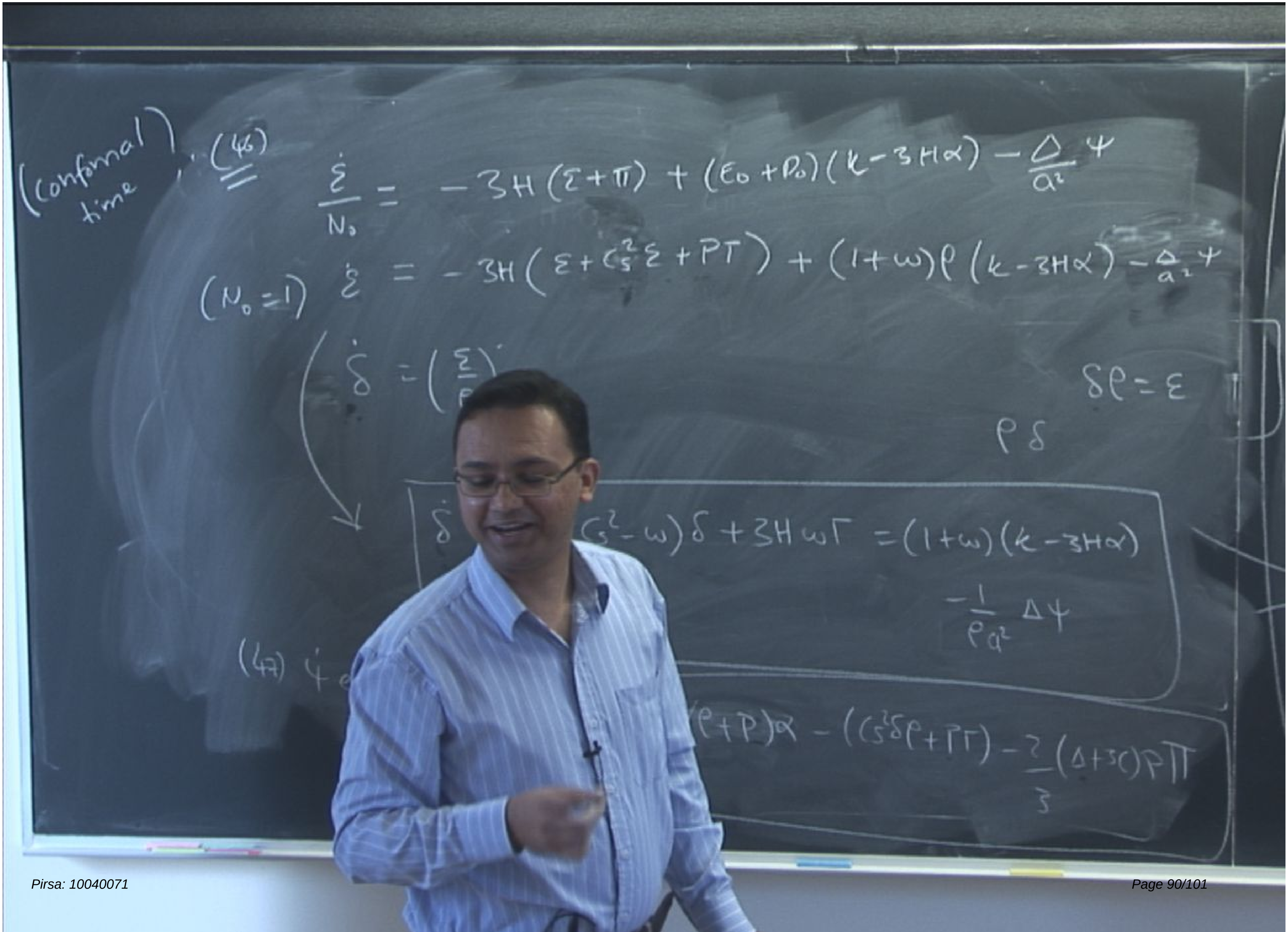
$$\dot{\delta} = \left(\frac{\mathcal{E}}{\rho}\right)^{\cdot}$$

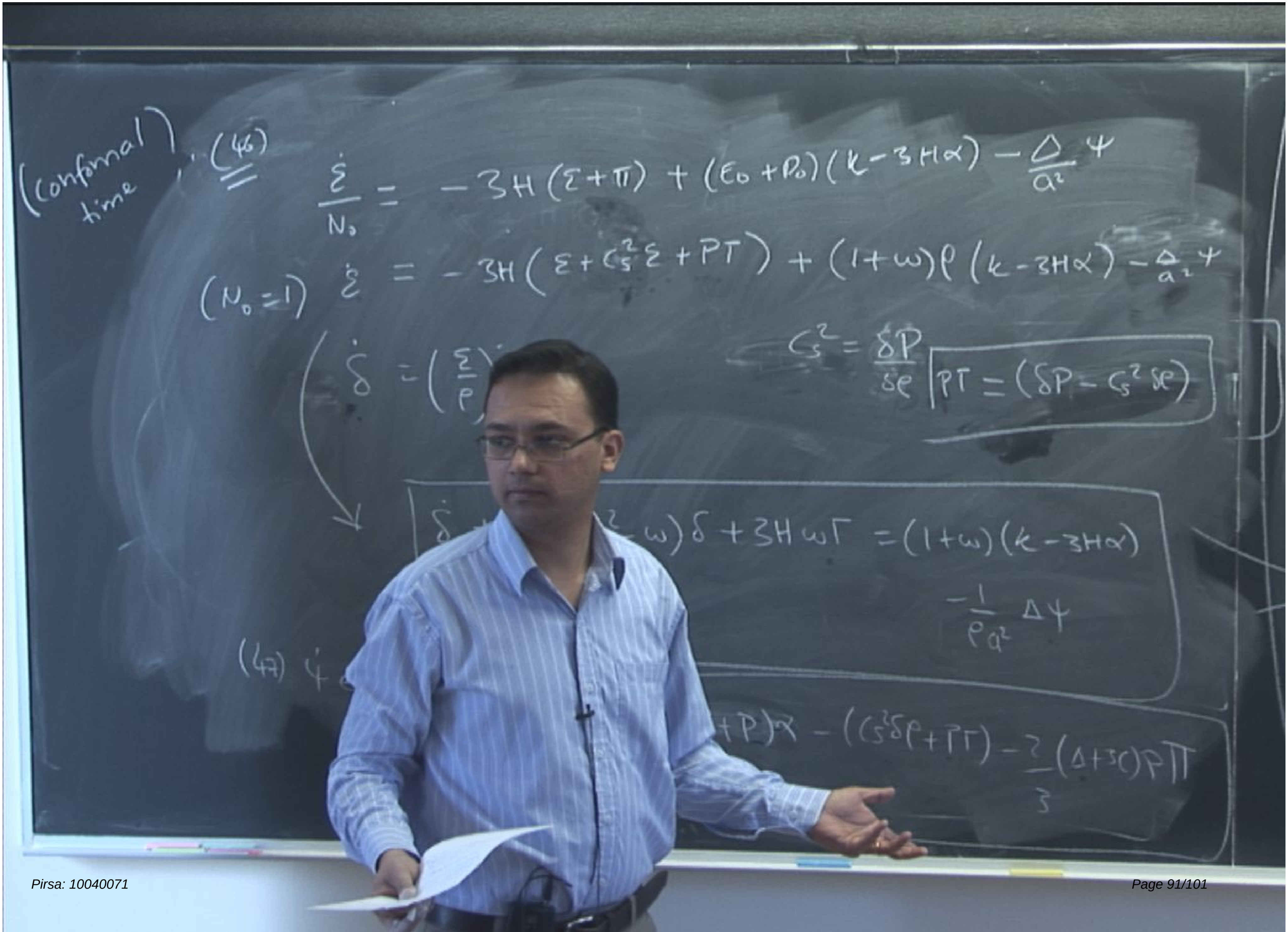
$$\boxed{\dot{\delta} + 3H(c_s^2-\omega)\delta + 3H\omega\Gamma = (1+\omega)(k-3H\alpha) - \frac{1}{\rho a^2}\Delta\psi}$$

(47) $\dot{\psi}$ eqn

$$\dot{\psi} = -3H\psi - (\rho+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta+3C)P\Pi$$


(conformal time)
(46)
$\frac{\dot{\varepsilon}}{N_0} = -3H(\varepsilon+\pi) + (\varepsilon_0+P_0)(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$(N_0=1)$ $\dot{\varepsilon} = -3H(\varepsilon + c_s^2\varepsilon + P\Gamma) + (1+w)\rho(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$\dot{\delta} = \left(\frac{\varepsilon}{\rho}\right)^{\cdot}$
$\delta\rho = \varepsilon$
(47) $\dot{\psi}$ eqn


(conformal time)
(46)
$\frac{\dot{\mathcal{E}}}{N_0} = -3H(\mathcal{E}+\Pi) + (\mathcal{E}_0+P_0)(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$(N_0=1)$ $\dot{\mathcal{E}} = -3H(\mathcal{E}+c_s^2\mathcal{E}+P\Gamma) + (1+\omega)\rho(k-3H\alpha) - \frac{\Delta}{a^2}\Psi$
$\delta\rho = \mathcal{E}$
$\rho\delta$
$(c_s^2-\omega)\delta + 3H\omega\Gamma = (1+\omega)(k-3H\alpha)$
$-\frac{1}{\rho a^2}\Delta\Psi$
(47)
$(\rho+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta+3\mathcal{K})P\Pi$


(conformal time)
(46)
$\frac{\dot{\varepsilon}}{N_0} = -3H(\varepsilon + \pi) + (\varepsilon_0 + P_0)(k - 3H\alpha) - \frac{\Delta}{a^2}\psi$
$(N_0 = 1)$ $\dot{\varepsilon} = -3H(\varepsilon + c_s^2 \varepsilon + P\Gamma) + (1+\omega)\rho(k - 3H\alpha) - \frac{\Delta}{a^2}\psi$
$\dot{\delta} = \left(\frac{\varepsilon}{\rho}\right)$
$c_s^2 = \frac{\delta P}{\delta \rho}$
$P\Gamma = (\delta P - c_s^2 \delta\rho)$
$\omega)\delta + 3H\omega\Gamma = (1+\omega)(k - 3H\alpha)$
$-\frac{1}{\rho a^2}\Delta\psi$
(47)
$+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta + 3K)P\Pi$

(conformal time) (46)

$$\frac{\dot{\mathcal{E}}}{N_0} = -3H(\mathcal{E}+\Pi) + (\mathcal{E}_0 + P_0)(k - 3H\alpha) - \frac{\Delta}{a^2}\psi$$

$(N_0 = 1)$

$$\dot{\mathcal{E}} = -3H(\mathcal{E} + c_s^2\mathcal{E} + P\Gamma) + (1+\omega)\rho(k - 3H\alpha) - \frac{\Delta}{a^2}\psi$$

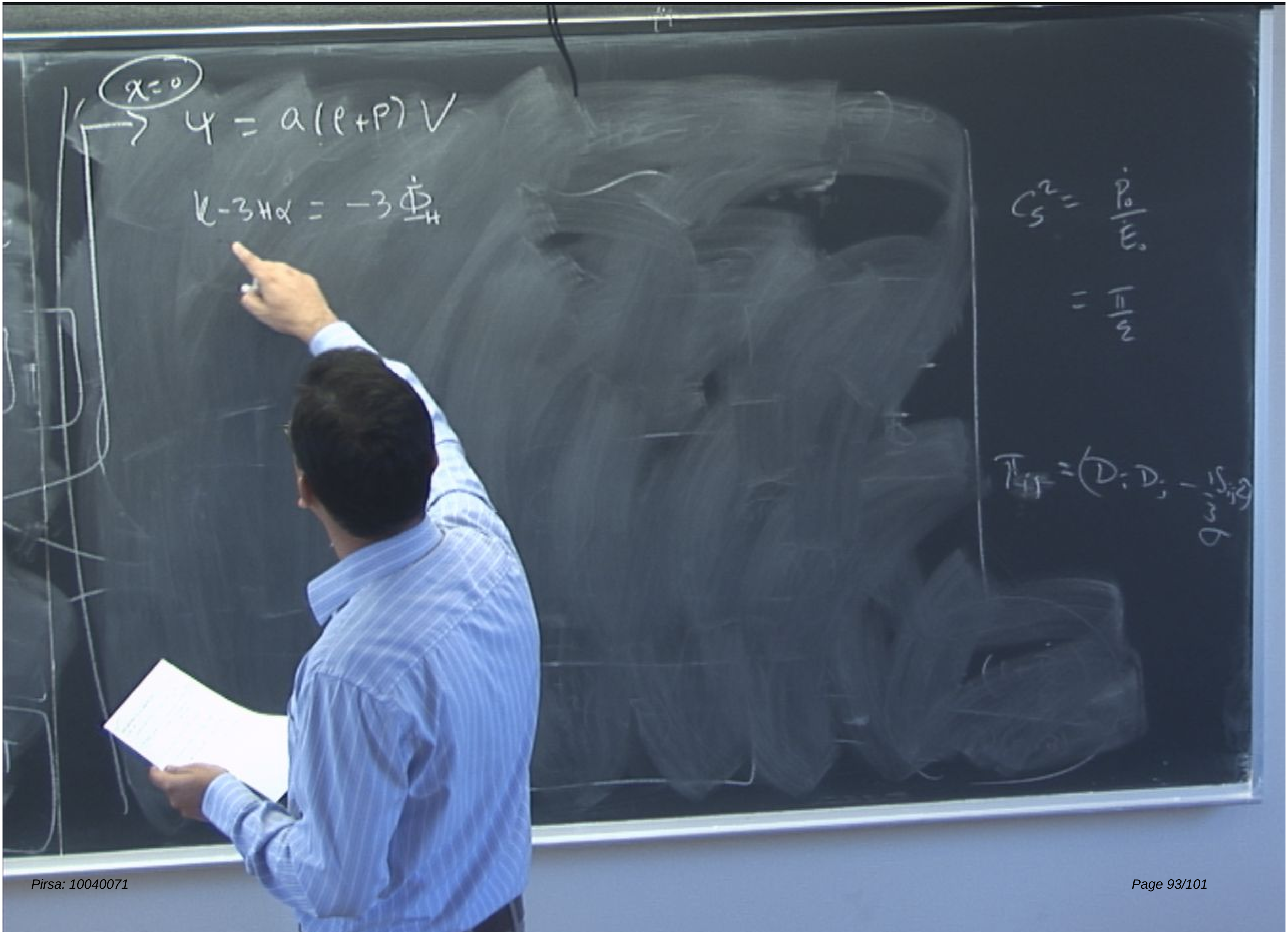
$$c_s^2 = \frac{\delta P}{\delta\rho} \qquad \boxed{P\Gamma = (\delta P - c_s^2\delta\rho)}$$

$$\dot{\delta} = \left(\frac{\mathcal{E}}{\rho}\right)^{\cdot}$$

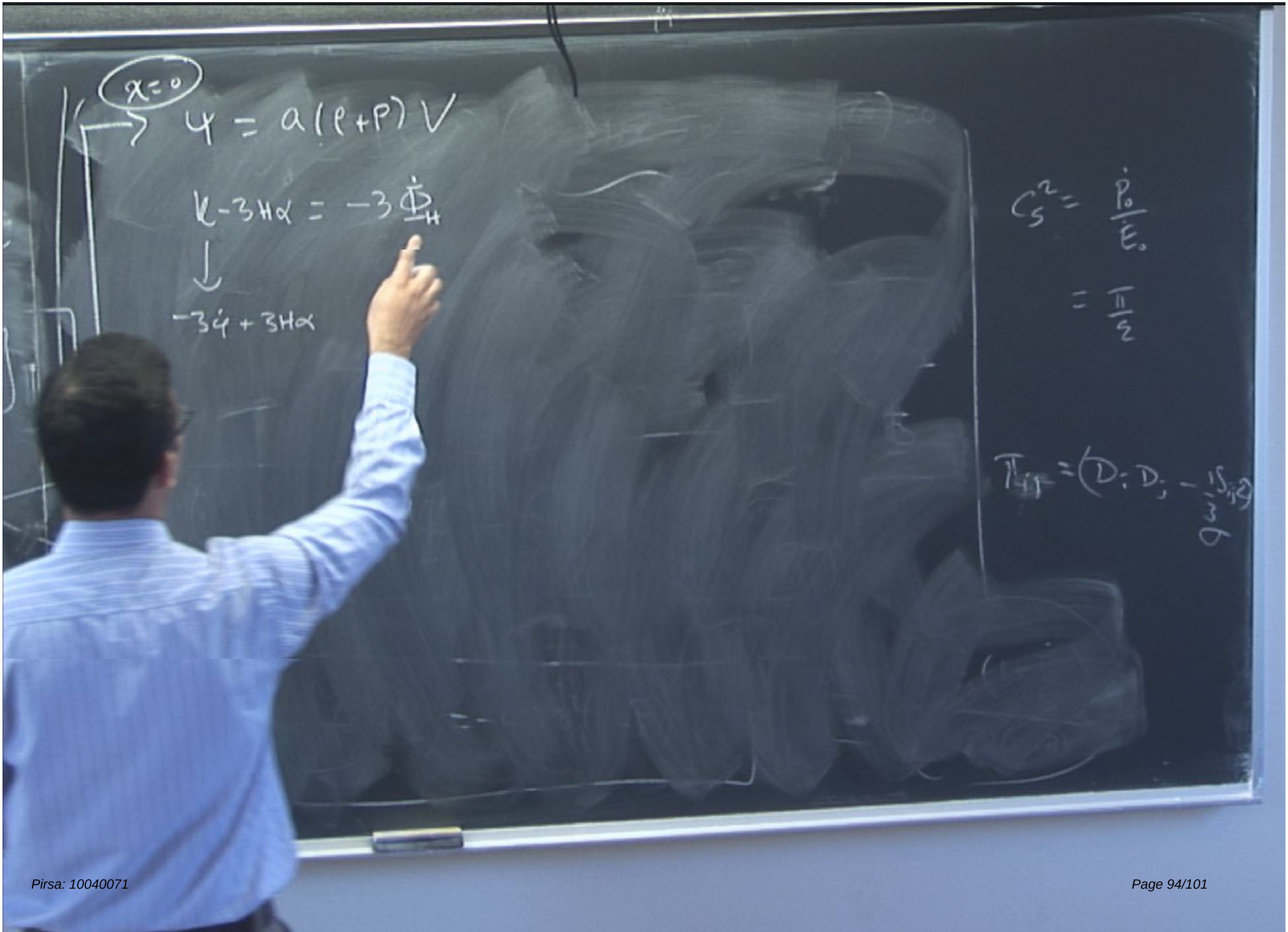
$$\boxed{\dot{\delta} + 3H(c_s^2 - \omega)\delta + 3H\omega\Gamma = (1+\omega)(k - 3H\alpha) - \frac{1}{\rho a^2}\Delta\psi}$$

(47) $\dot{\psi}$ eqn

$$\boxed{\dot{\psi} = -3H\psi - (\rho+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta + 3K)P\Pi}$$



$\alpha = 0$
$\Psi = a(\rho + P) V$
$\kappa - 3H\alpha = -3\dot{\Phi}_H$
$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$
$= \frac{\Pi}{E}$
$\Pi_{ij} = (D_i D_j - \frac{1}{3}\delta_{ij}\nabla^2)\sigma$



α=0
ψ = a(ρ+P)V
-3ψ̇ + 3Hα
c_s² = Ṗ₀/Ė₀
= Π/ε

$\alpha = 0$

$$\boxed{\begin{aligned} \psi &= a(\rho + P) V \\ \kappa - 3H\alpha &= -3\dot{\Phi}_H \end{aligned}}$$

$\downarrow$

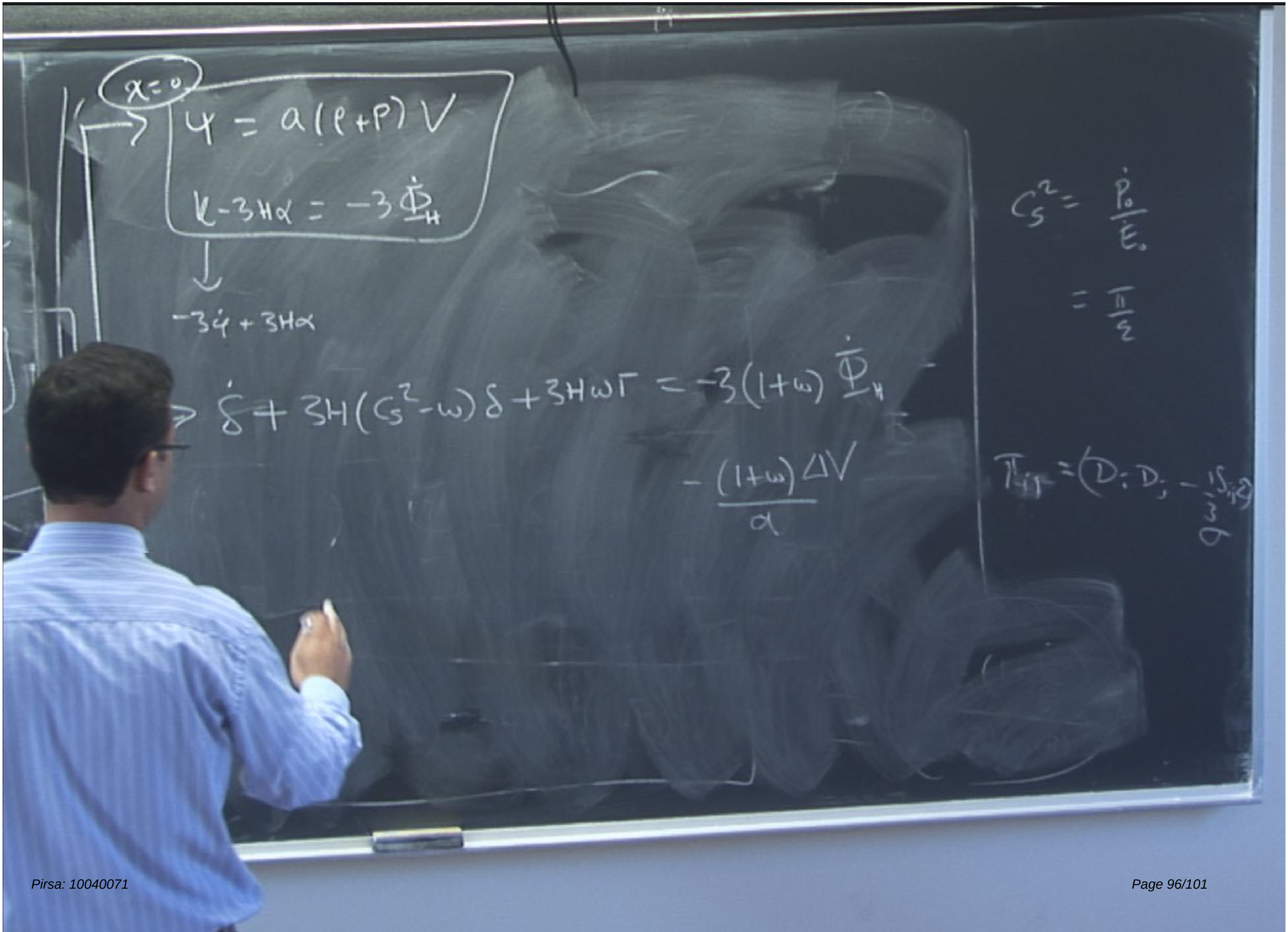
$-3\dot{\psi} + 3H\alpha$

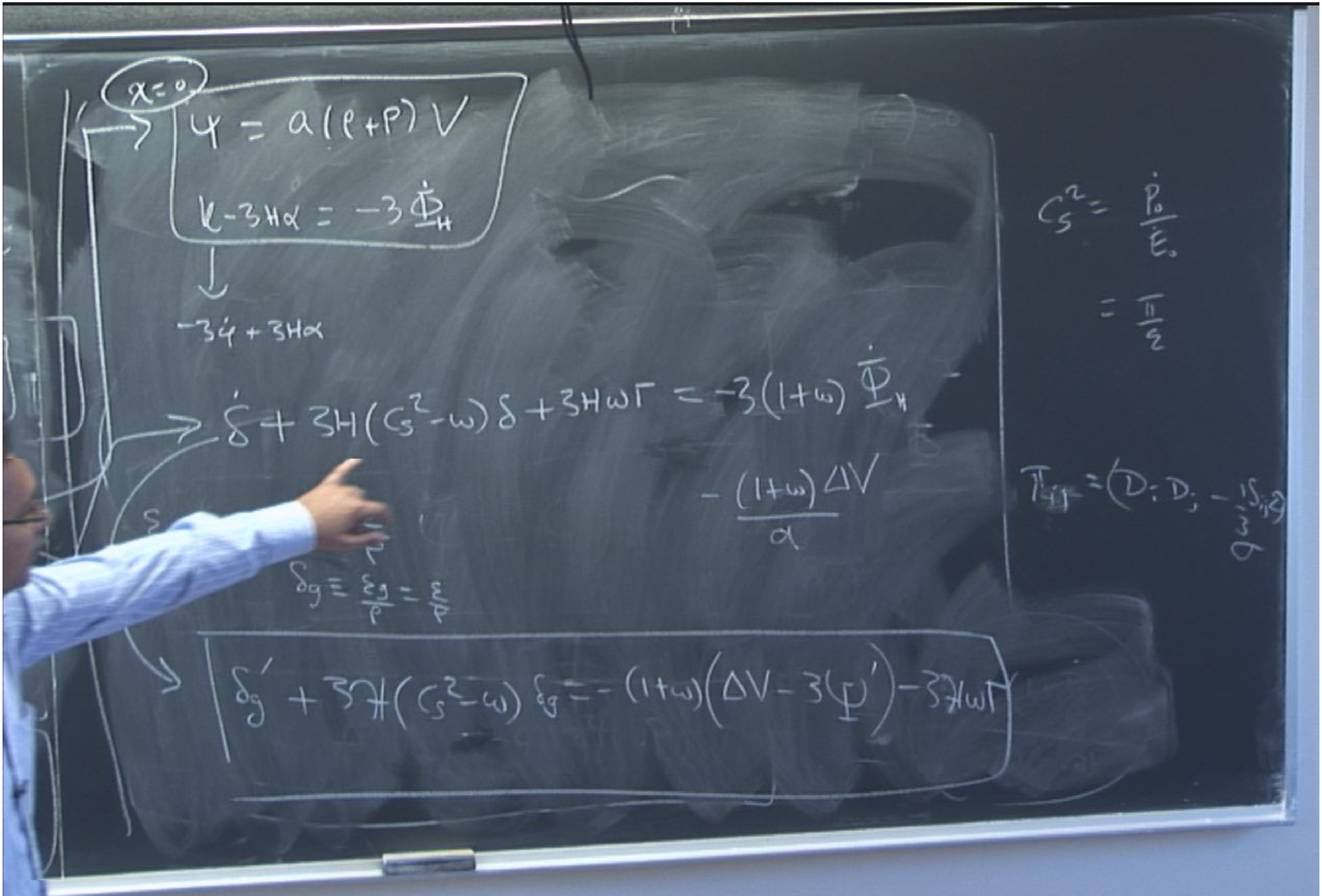
$$\dot{\delta} + 3H(c_s^2 - w)\delta$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

$$= \frac{\Pi}{\varepsilon}$$

$$\Pi_{ij} = \left(D_i D_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\sigma$$


$\alpha = 0$
$\psi = a(\rho + P)V$
$\kappa - 3H\alpha = -3\dot{\Phi}_H$
$-3\dot{\psi} + 3H\alpha$
$\Rightarrow \dot{\delta} + 3H(c_s^2 - w)\delta + 3Hw\Gamma = -3(1+w)\dot{\Phi}_H - \frac{(1+w)\Delta V}{a}$
$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$
$= \frac{\Pi}{\Sigma}$
$\Pi_{ij} = (D_i D_j - \frac{1}{3}\delta_{ij}\Delta)\sigma$


x = 0
ψ = a(ρ+P)V
κ − 3Hα = −3 Φ̇_H
−3ψ̇ + 3Hα
δ̇ + 3H(c_s² − ω)δ + 3HωΓ = −3(1+ω) Φ̇_H
− (1+ω)ΔV / a
δ_g ≡ ε_g/ρ = ε/ρ
δ_g′ + 3ℋ(c_s² − ω) δ_g = −(1+ω)(ΔV − 3Ψ′) − 3ℋωΓ
c_s² = Ṗ₀/Ė₀
= Π/ε

$\alpha = 0$

$$\boxed{\begin{aligned} \psi &= a(\rho + P)V \\ \kappa - 3H\alpha &= -3\dot{\Phi}_H \end{aligned}}$$

$$-3\dot{\psi} + 3H\alpha$$

$$\dot{\delta} + 3H(c_s^2 - \omega)\delta + 3H\omega\Gamma = -3(1+\omega)\dot{\Phi}_H - \frac{(1+\omega)\Delta V}{a}$$

$$\varepsilon_g = \varepsilon \qquad \delta \equiv \frac{\varepsilon}{\rho}$$

$$\delta_g \equiv \frac{\varepsilon_g}{\rho} = \frac{\varepsilon}{\rho}$$

$$\boxed{\delta_g' + 3\mathcal{H}(c_s^2 - \omega)\delta_g = -(1+\omega)\left(\Delta V - 3\Psi'\right) - 3\mathcal{H}\omega\Gamma}$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0} = \frac{\Pi}{\varepsilon}$$

$$\Pi_{ij} = \frac{\left(D_i D_j - \frac{1}{3}\delta_{ij}\Delta\right)}{\sigma}$$

(46)

$$V' + (1-3c_s^2)\mathcal{H}V + \Phi + \frac{c_s^2}{1+w}\delta_g + \frac{w}{1+w}\Gamma + \frac{2}{3}(\Delta+3K)\frac{w}{1+w}\Pi = 0$$

$$\dot{\delta} \; \ldots \; -w)\delta + 3Hw\Gamma = (1+w)(k-3H\alpha) - \frac{1}{\rho a^2}\Delta\psi$$

(47) $\dot{\psi}$ e …

$$\ldots\psi - (\rho+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta+3K)P\Pi$$

(46)

$$\boxed{V' + (1-3c_s^2)\mathcal{H}V + \Phi + \frac{c_s^2}{1+w}\delta_g + \frac{w}{1+w}\Gamma + \frac{2}{3}(\Delta+3C)\frac{w}{1+w}\Pi = 0}$$

$$\boxed{\dot{\delta} + 3H(c_s^2 - w)\delta + 3Hw\Gamma = (1+w)(k - 3H\alpha) - \frac{1}{\rho a^2}\Delta\psi}$$

(47) $\dot{\psi}$ eqn

$$\boxed{\dot{\psi} = -3H\psi - (\rho+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta+3C)P\Pi}$$

(46)

$$\boxed{V' + (1-3c_s^2)\mathcal{H}V + \Phi + \frac{c_s^2}{1+w}\delta_g + \frac{w}{1+w}\Gamma + \frac{2}{3}(\Delta+3C)\frac{w}{1+w}\Pi = 0}$$

$$\boxed{\dot\delta + 3H(c_s^2-w)\delta + 3Hw\Gamma = (1+w)(k-3H\alpha) - \frac{1}{\rho a^2}\Delta\psi}$$

(47) $\dot\psi$ eqn.

$$\boxed{\dot\psi = -3H\psi - (\rho+P)\alpha - (c_s^2\delta\rho + P\Gamma) - \frac{2}{3}(\Delta+3C)P\Pi}$$