Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 13

Date: Apr 21, 2010 09:30 AM

URL: http://pirsa.org/10040070

Abstract:

Derivation of adiabatic g and d modes
from Friedmann.

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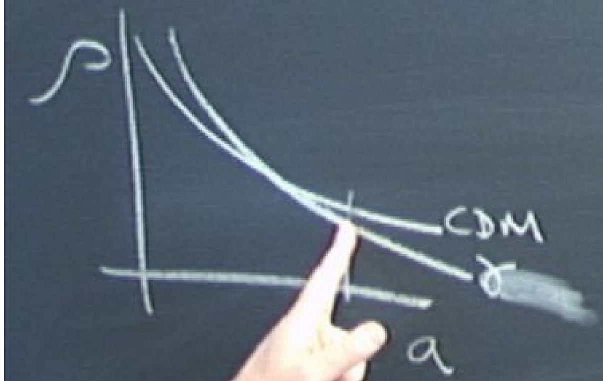
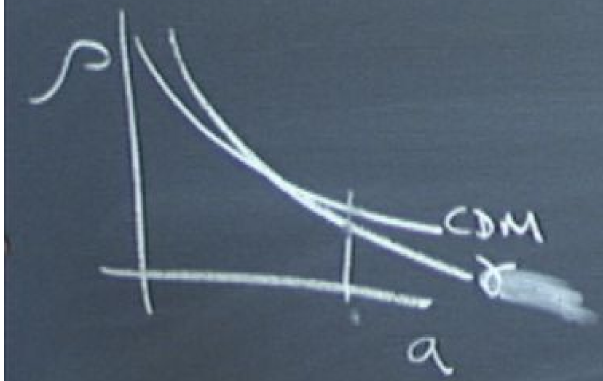


Derivation of adiabatic g and d modes from Friedmann.



PROPER TIME

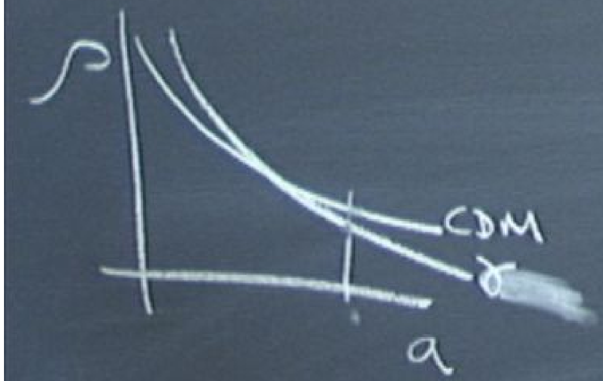
($\Omega_0 = 1$)

$a_{rad} \propto t^{1/2}$

$a_{mat} \propto t^{2/3}$

Derivation of adiabatic g and d modes

from Friedmann.



PROPER

TIME

$(\Omega_0 = 1)$

$a_{rad} \propto t^{1/2}$

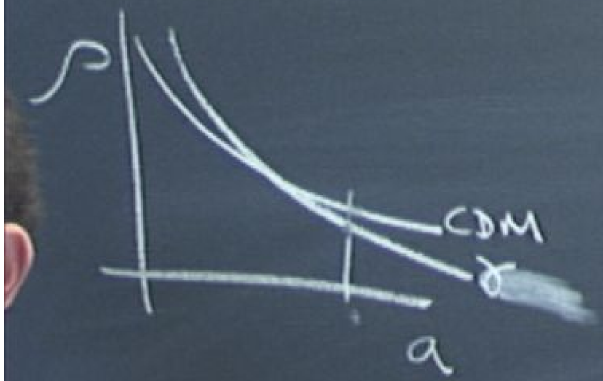
$a_{mat} \propto t^{2/3}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

rdu $\dot{a} \propto a^{-1} \rightarrow a \propto t^{1/2}$

Mdu $\dot{a} \propto a^{-1/2} \rightarrow a \propto t^{2/3}$.

Derivation of adiabatic g and d modes from Friedmann.



PROPER TIME

$(\Omega_0 = 1)$

$a_{rad} \propto t^{1/2}$

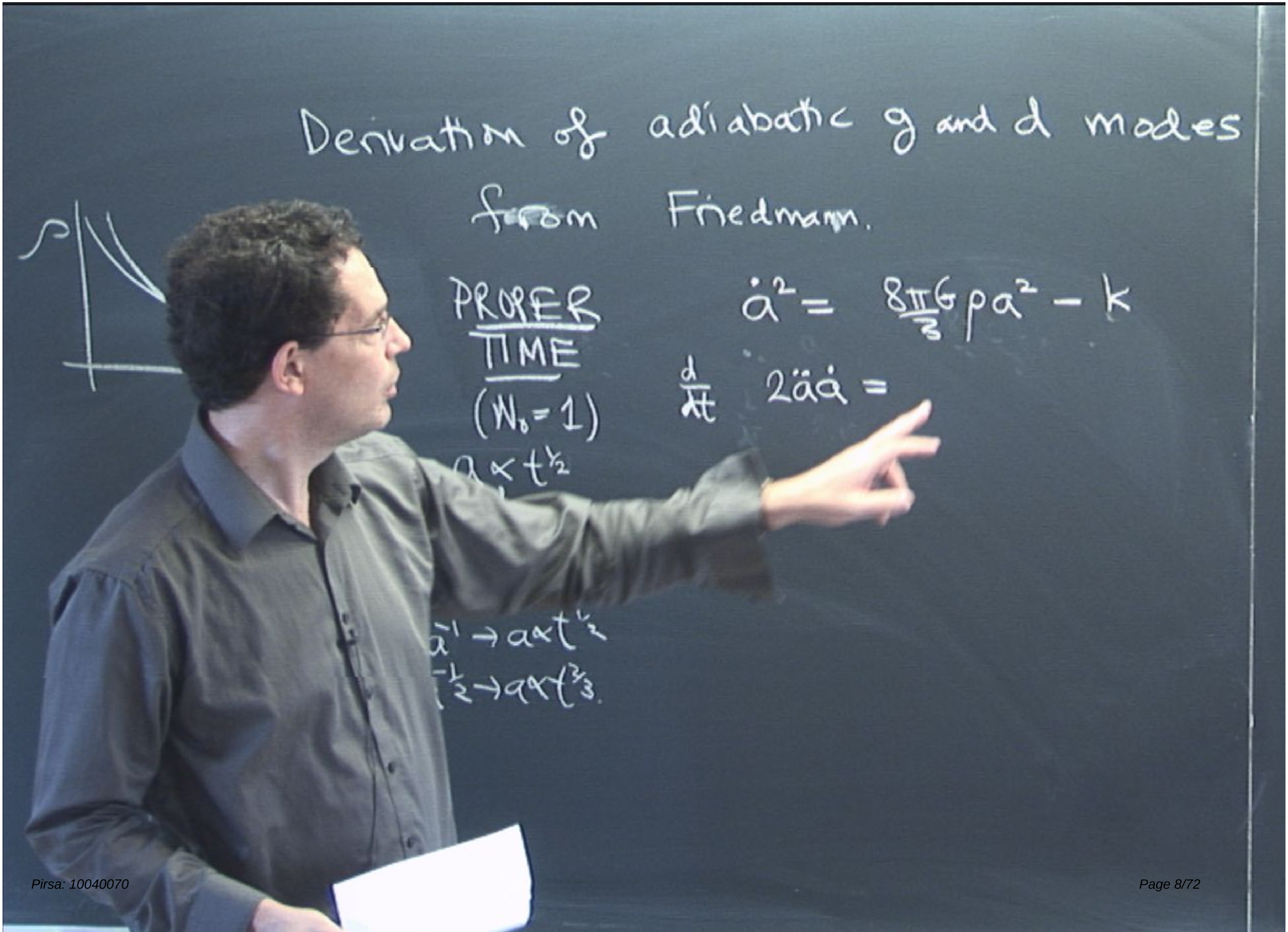
$a_{mat} \propto t^{2/3}$

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 - k$$

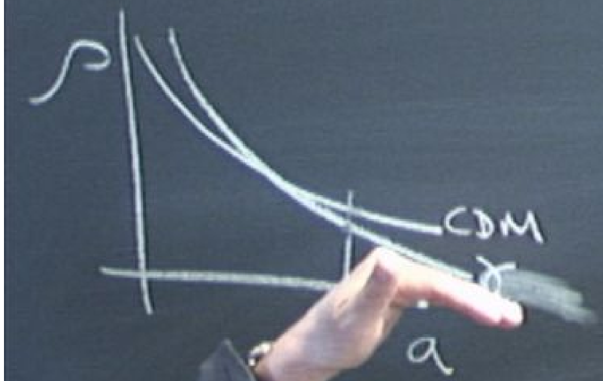
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho$$

rdu $\dot{a} \propto a^{-1} \rightarrow a \propto t^{1/2}$

Mdu $\dot{a} \propto a^{-1/2} \rightarrow a \propto t^{2/3}$.


Derivation of adiabatic g and d modes
from Friedmann.
PROPER TIME
$(\Omega_0 = 1)$
$a \propto t^{1/2}$
$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$
$\frac{d}{dt}\quad 2\ddot{a}\dot{a} =$
$a^{-1} \to a \propto t^{1/2}$
$^{-1/2} \to a \propto t^{2/3}$

Derivation of adiabatic g and d modes from Friedmann.



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($\Omega_0 = 1$)

$a_{rad} \propto t^{1/2}$

$a_{mat} \propto t^{2/3}$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

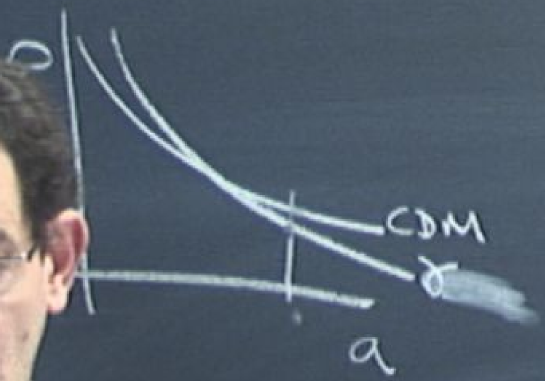
rdn $\dot a \propto a^{-1} \rightarrow a \propto t^{1/2}$

Mdn $\dot a \propto a^{-1/2} \rightarrow a \propto t^{2/3}$.

$$\dot a^2 = \frac{8\pi G}{3}\rho_{(a)} a^2 - k$$

$$\frac{d}{dt} \quad 2\ddot a \dot a =$$

Derivation of adiabatic g and d modes from Friedmann.



PROPER TIME

$(\Omega_0 = 1)$

$a_{rad} \propto t^{1/2}$

$a_{mat} \propto t^{2/3}$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho(a)\, a^2 - k$$

$$\frac{d}{dt} \quad 2\ddot{a}\dot{a} = \frac{d}{da}(\quad)\,\dot{a}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

rdn $\dot{a} \propto a^{-1} \rightarrow a \propto t^{1/2}$

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Derivation of adiabatic g and d modes

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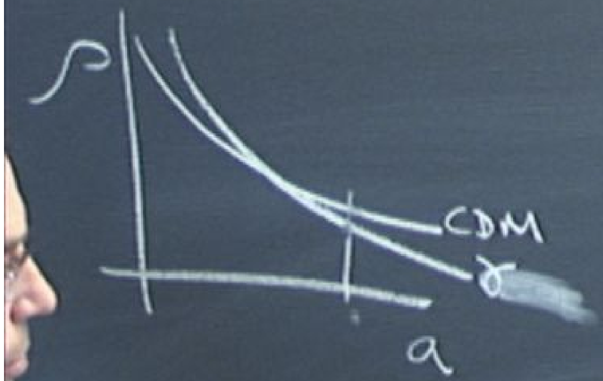

PROPER TIME

$(\Omega_0 = 1)$

$a_{rad} \propto t^{1/2}$

$a_{mat} \propto t^{2/3}$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{(a)} a^2 - k$$

$$\frac{d}{dt} \quad 2\ddot{a}\cancel{\dot{a}} = \frac{d}{da}(\quad)\cancel{\dot{a}}$$

$$\to a = a_0(1+\alpha)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

rdu $\dot{a} \propto a^{-1} \to a \propto t^{1/2}$

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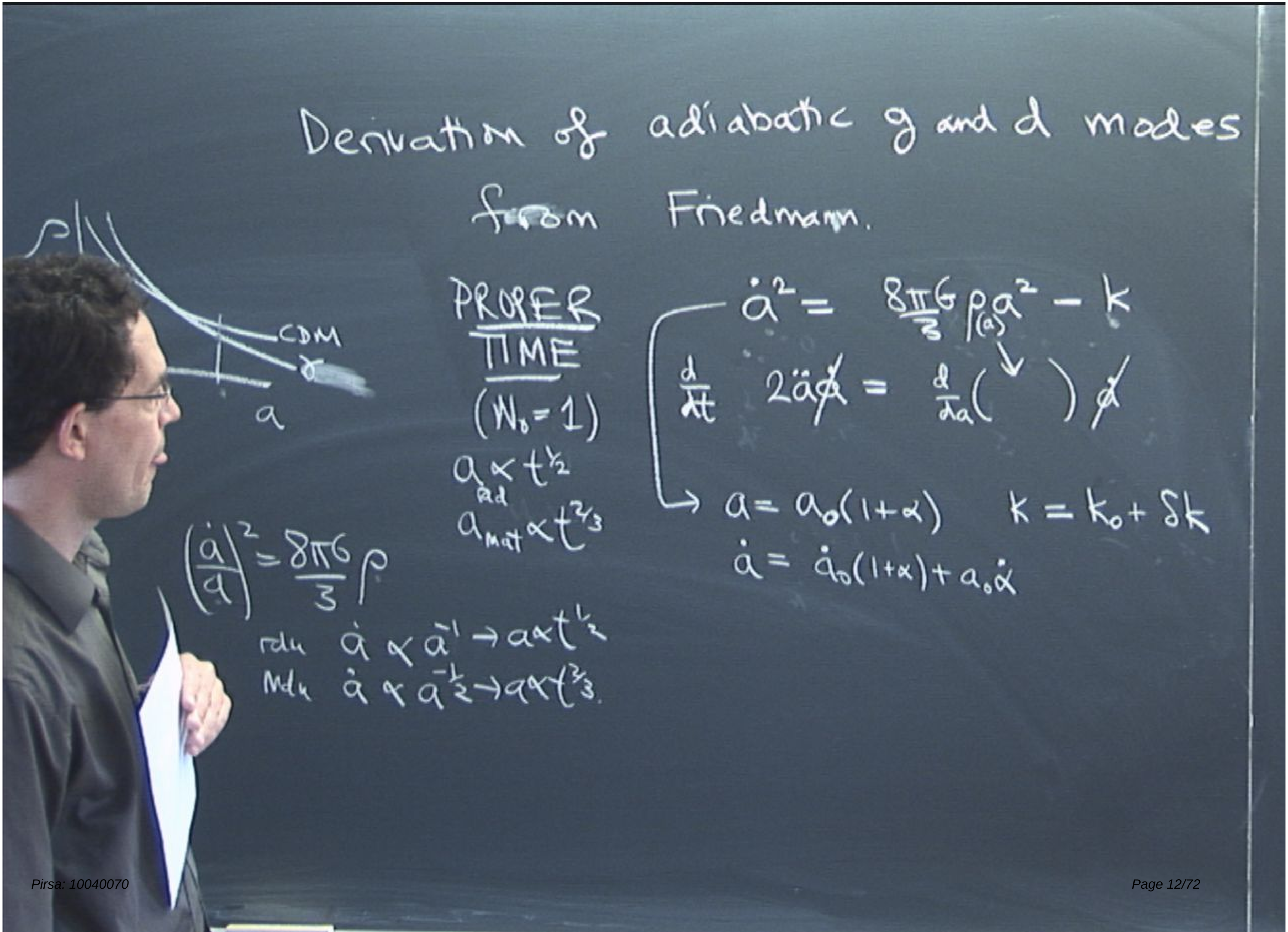
Derivation of adiabatic g and d modes
from Friedmann.
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$a = a_0(1+\alpha) \qquad k = k_0 + \delta k$
$\dot{a} = \dot{a}_0(1+\alpha) + a_0\dot{\alpha}$
ρ
CDM
γ
a
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
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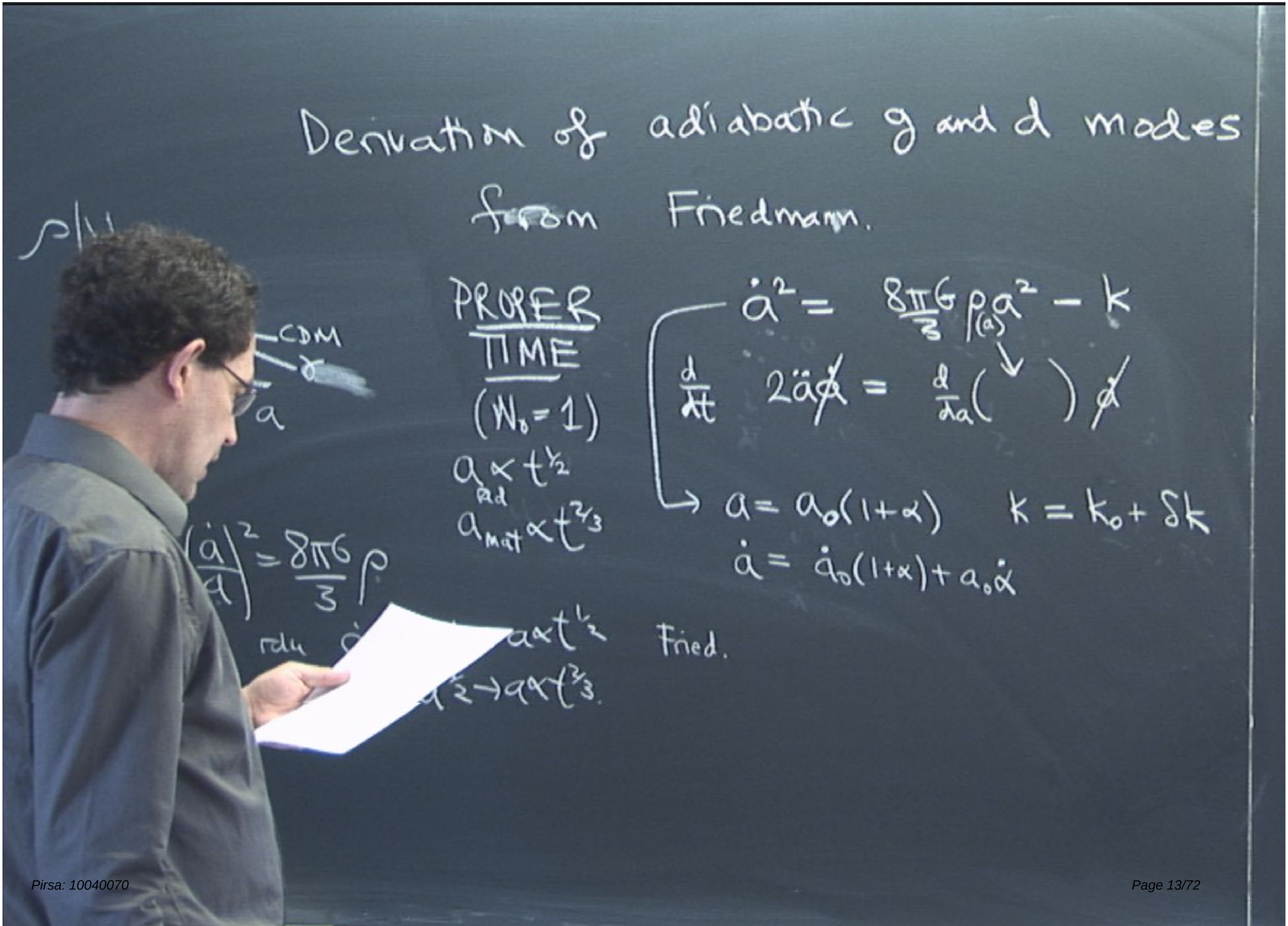
Derivation of adiabatic g and d modes
from Friedmann.
PROPER
TIME
$(\Omega_0 = 1)$
$a_{rad} \propto t^{1/2}$
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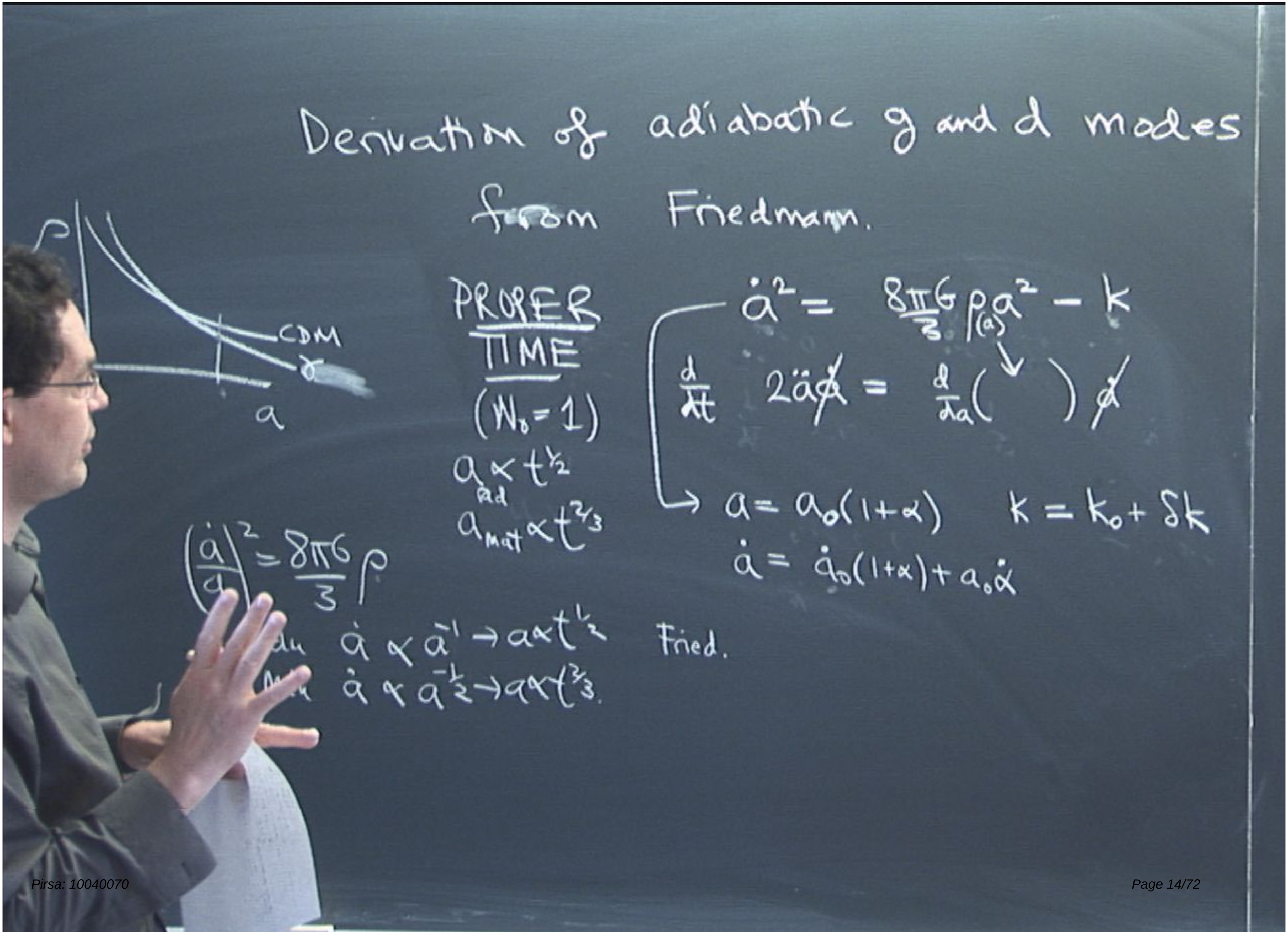
$\dot{a} = \dot{a}_0(1+\alpha) + a_0\dot{\alpha}$
Fried.
CDM
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
$a \propto t^{1/2}$
$\rightarrow a \propto t^{2/3}$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PROPER TIME
$(\Omega_0 = 1)$
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CDM
γ
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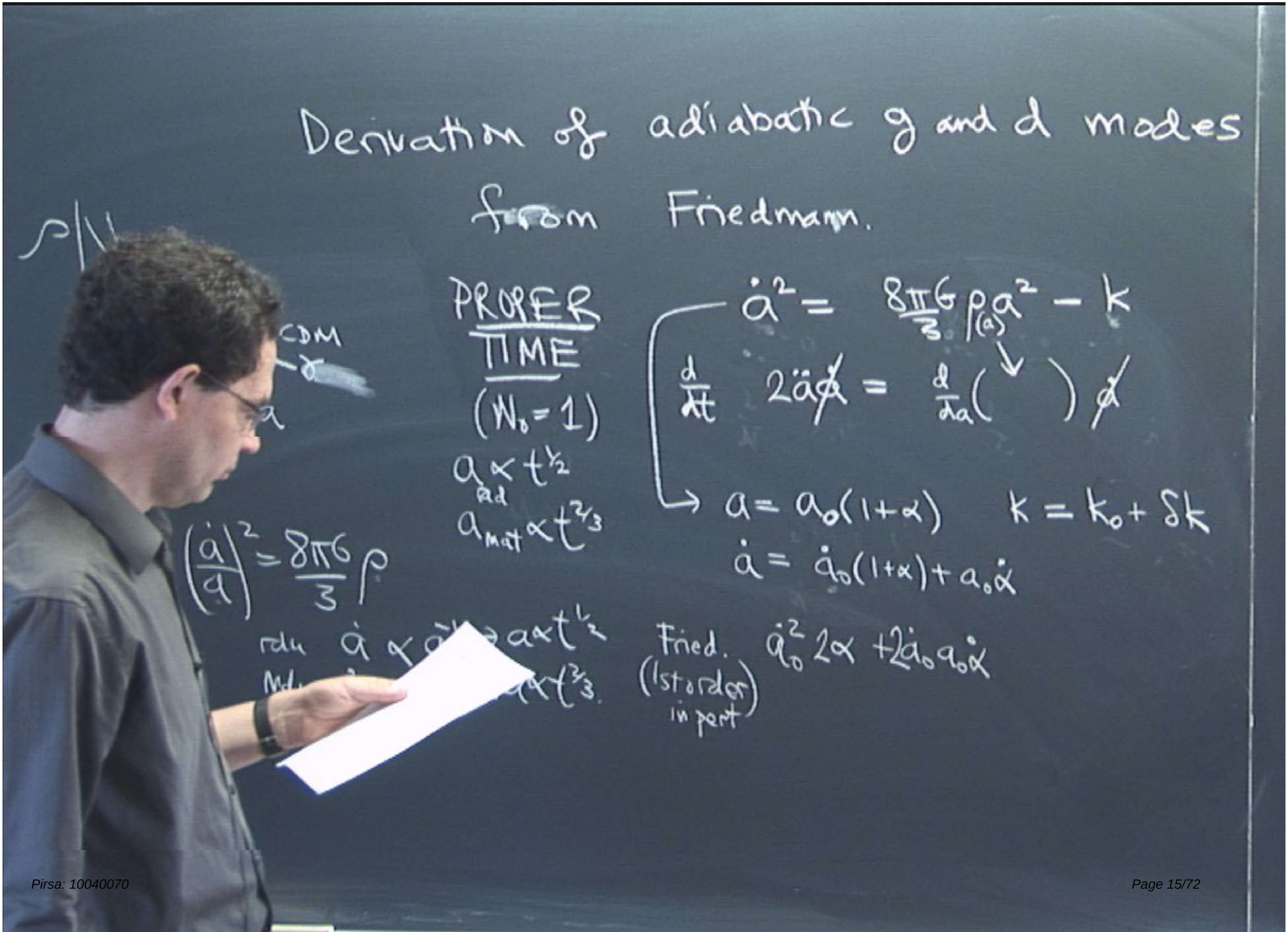
Fried.


Derivation of adiabatic g and d modes
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PROPER TIME
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$\dot{a}^2 = \frac{8\pi G}{3} \rho(a) a^2 - k$
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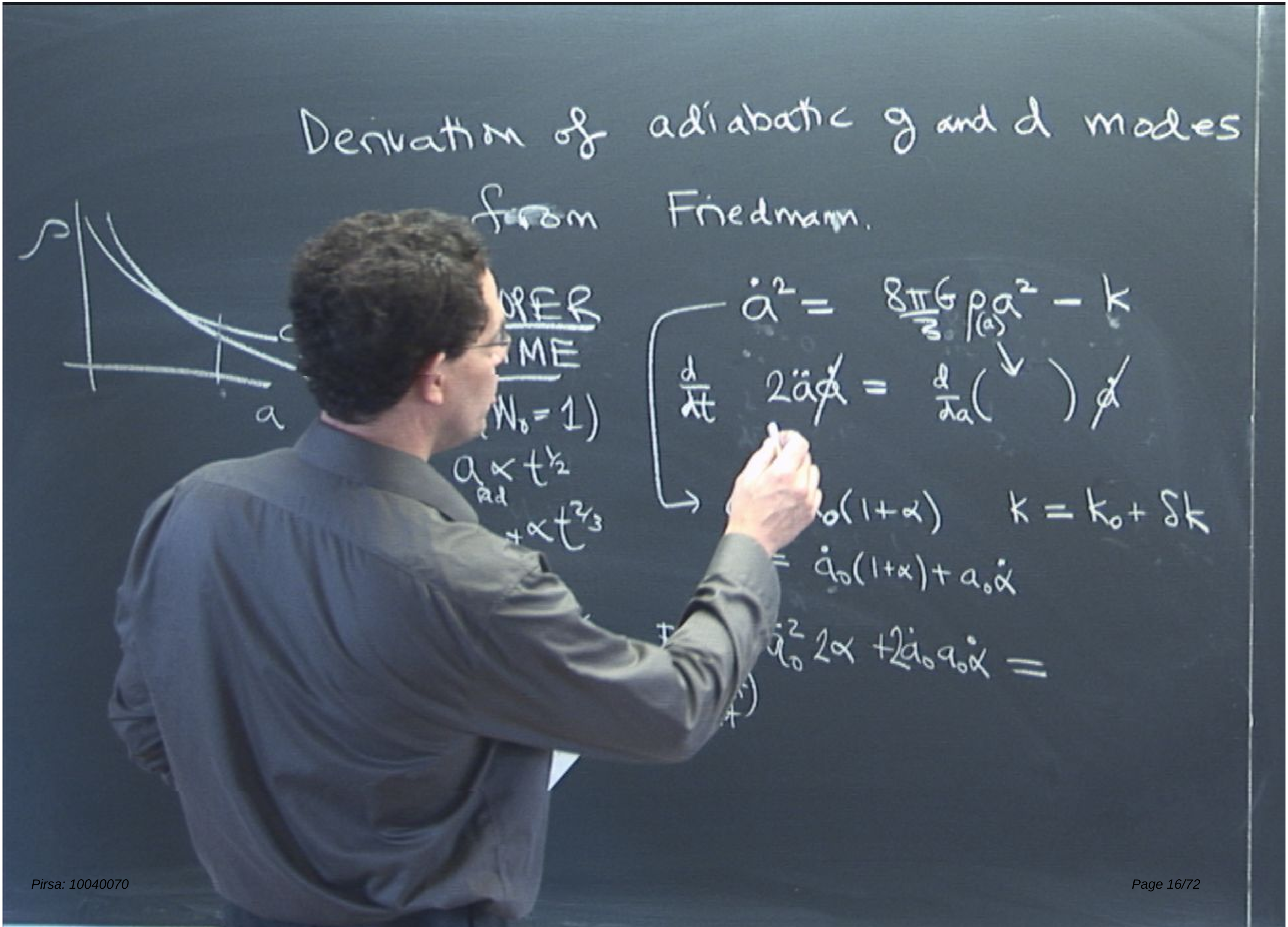
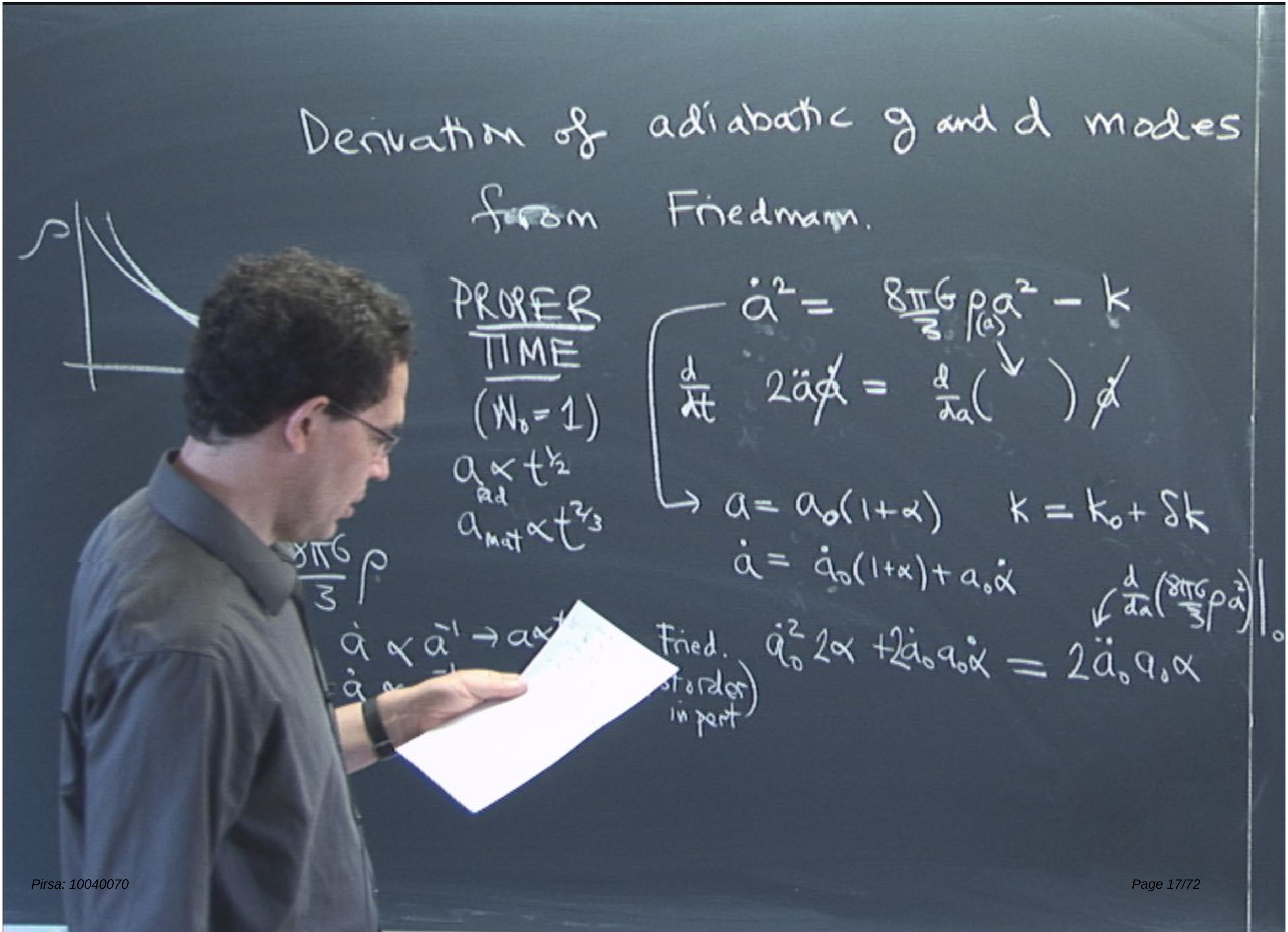
$a = a_0(1+\alpha) \quad k = k_0 + \delta k$
$\dot{a} = \dot{a}_0(1+\alpha) + a_0\dot{\alpha}$
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
rdn $\dot{a} \propto$ $a \propto t^{1/2}$
Md $a \propto t^{2/3}$
Fried. (1st order in pert) $\dot{a}_0^2 2\alpha + 2\dot{a}_0 a_0 \dot{\alpha}$
CDM


Derivation of adiabatic g and d modes
from Friedmann.
ρ
a
$\Omega_0 = 1)$
$a_{rad} \propto t^{1/2}$
$\propto t^{2/3}$
$\dot{a}^2 = \frac{8\pi G}{3} \rho(a) a^2 - k$
$\frac{d}{dt} \quad 2\ddot{a}\cancel{\dot{a}} = \frac{d}{da}(\quad) \cancel{\dot{a}}$
$a_0(1+\alpha) \quad k = k_0 + \delta k$
$= \dot{a}_0(1+\alpha) + a_0\dot{\alpha}$
$\dot{a}_0^2 2\alpha + 2\dot{a}_0 a_0 \dot{\alpha} =$


Derivation of adiabatic g and d modes
from Friedmann.
PROPER
TIME
$(\Omega_0 = 1)$
$a_{rad} \propto t^{1/2}$
$a_{mat} \propto t^{2/3}$
$\frac{8\pi G}{3}\rho$
$\dot{a}^2 = \frac{8\pi G}{3}\rho_{(a)} a^2 - k$
$\frac{d}{dt} \quad 2\ddot{a}\cancel{\dot{a}} = \frac{d}{da}(\quad)\cancel{\dot{a}}$
$a = a_0(1+\alpha) \quad k = k_0 + \delta k$
$\dot{a} = \dot{a}_0(1+\alpha) + a_0\dot{\alpha}$
$\frac{d}{da}\left(\frac{8\pi G}{3}\rho a^2\right)\Big|_0$
Fried. (1st order in pert)
$\dot{a}_0^2 2\alpha + 2\dot{a}_0 a_0 \dot{\alpha} = 2\ddot{a}_0 a_0 \alpha$

g adiabatic g and d modes

n Friedmann.

$$\dot a^2 = \frac{8\pi G}{3}\rho_{(a)} a^2 - k$$

$$\frac{d}{dt} \quad 2\ddot a \cancel{\dot a} = \frac{d}{da}(\quad)\cancel{\dot a}$$

$$a = a_0(1+\alpha) \qquad k = k_0 + \delta k$$

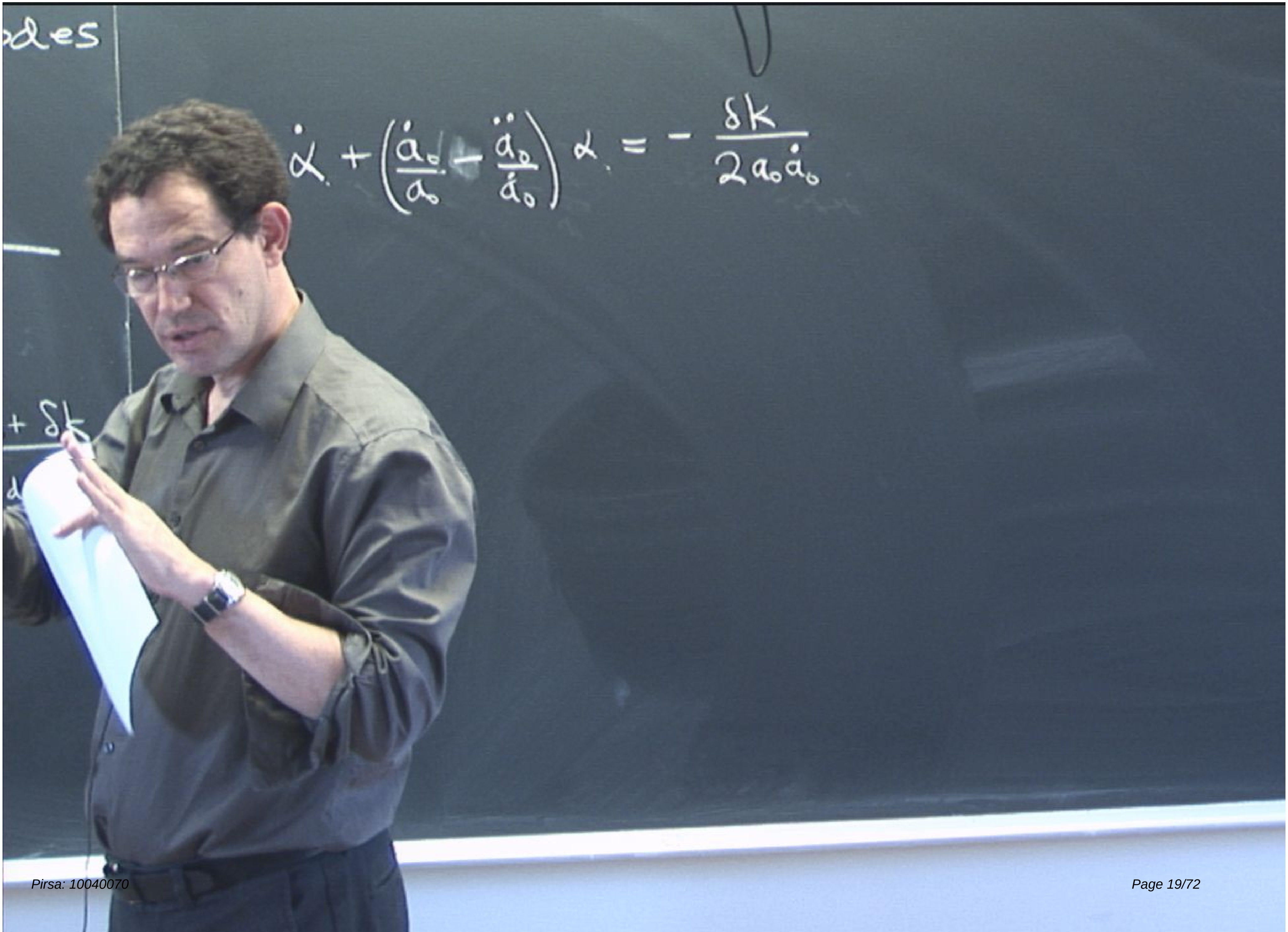
$$\dot a = \dot a_0(1+\alpha) + a_0\dot\alpha$$

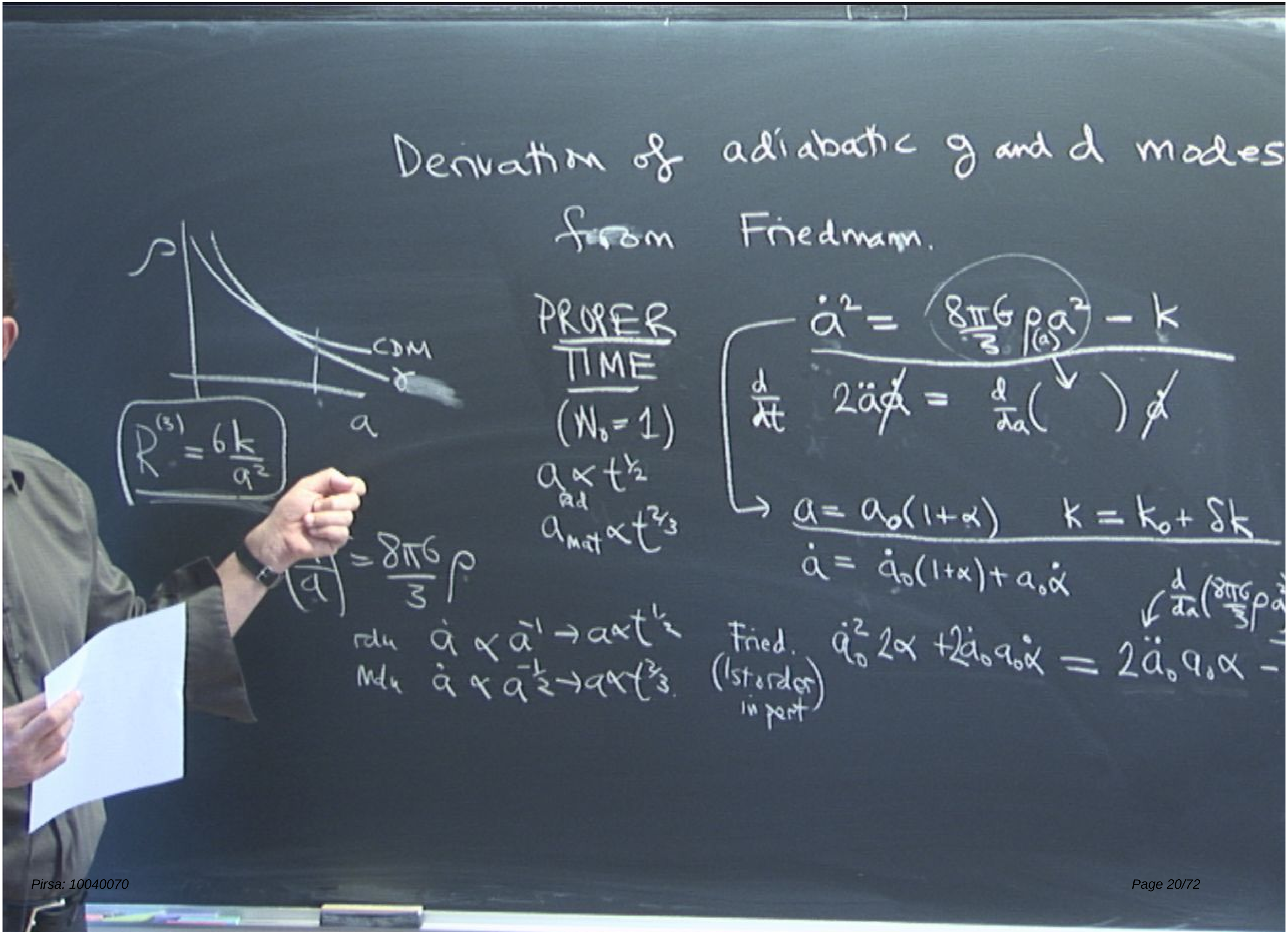
$$\frac{d}{da}\left(\frac{8\pi G}{3}\rho a^2\right)\Big|_0$$

Fried. (1st order in pert) $\dot a_0^2\, 2\alpha + 2\dot a_0 a_0\dot\alpha = 2\ddot a_0 a_0\alpha - \delta k$

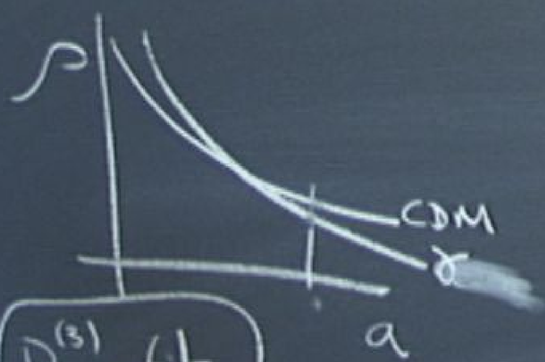
$t^{2/3}$ $t^{1/2}$ $t^{2/3}$.

$$\dot\alpha + \left(\frac{\dot a_0}{a_0} - \frac{\ddot a_0}{\dot a_0}\right)\alpha$$


$\dot{\alpha} + \left(\frac{\dot{a}_0}{a_0} - \frac{\ddot{a}_0}{\dot{a}_0}\right)\alpha = -\frac{\delta k}{2 a_0 \dot{a}_0}$


Derivation of adiabatic g and d modes
from Friedmann.
PROPER
TIME
($W_0 = 1$)
$a_{rad} \propto t^{1/2}$
$a_{mat} \propto t^{2/3}$
$\dot{a}^2 = \frac{8\pi G}{3} \rho_{(a)} a^2 - k$
$\frac{d}{dt}$ $2\ddot{a}\cancel{\dot{a}} = \frac{d}{da}(\ \ )\cancel{\dot{a}}$
$a = a_0(1+\alpha)$ $k = k_0 + \delta k$
$\dot{a} = \dot{a}_0(1+\alpha) + a_0\dot{\alpha}$
$\frac{d}{da}(\frac{8\pi G}{3}\rho a^2$
Fried. (1st order in pert)
$\dot{a}_0^2\, 2\alpha + 2\dot{a}_0 a_0 \dot{\alpha} = 2\ddot{a}_0 a_0 \alpha -$
CDM
γ
a
$R^{(3)} = 6\frac{k}{a^2}$
$\left(\frac{\dot{a}}{a}\right) = \frac{8\pi G}{3}\rho$
rdn $\dot{a} \propto a^{-1} \rightarrow a \propto t^{1/2}$
Mdn $\dot{a} \propto a^{-1/2} \rightarrow a \propto t^{2/3}$

Derivation of adiabatic g and d modes from Friedmann.



$$\boxed{R^{(3)} = 6\frac{k}{a^2}}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

rdn $\dot a \propto a^{-1} \to a \propto t^{1/2}$

Mdn $\dot a \propto a^{-1/2} \to a \propto t^{2/3}$

PROPER TIME

$(\Omega_0 = 1)$

$a_{rad} \propto t^{1/2}$

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$$\dot a^2 = \frac{8\pi G}{3}\rho_{(a)} a^2 - k$$

$$\frac{d}{dt} \quad 2\ddot a \cancel{\dot a} = \frac{d}{da}(\quad)\cancel{\dot a}$$

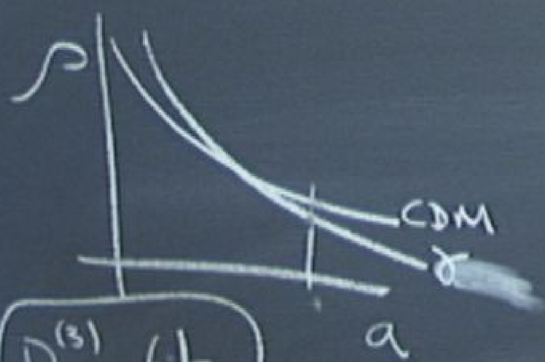
$$a = a_0(1+\alpha) \qquad k = k_0 + \delta k$$

$$\dot a = \dot a_0(1+\alpha) + a_0\dot\alpha$$

Fried. (1st order in pert)

$$\dot a_0^2\, 2\alpha + 2\dot a_0 a_0 \dot\alpha = 2\ddot a_0 a_0 \alpha - \ldots \qquad \frac{d}{da}\left(\frac{8\pi G}{3}\rho a^2\right)$$

Derivation of adiabatic g and d modes from Friedmann.



$$\boxed{R^{(3)} = 6\frac{k}{a^2}} = \frac{\#}{r_c^2}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\#}{r_c}$$

PROPER TIME

$(\Omega_0 = 1)$

$a_{rad} \propto t^{1/2}$

$a_{mat} \propto t^{2/3}$

rdn $\dot a \propto a^{-1} \rightarrow a \propto t^{1/2}$

Mdn $\dot a \propto a^{-\frac{1}{2}} \rightarrow a \propto t^{2/3}$.

$$\dot a^2 = \frac{8\pi G}{3}\rho_{(a)} a^2 - k$$

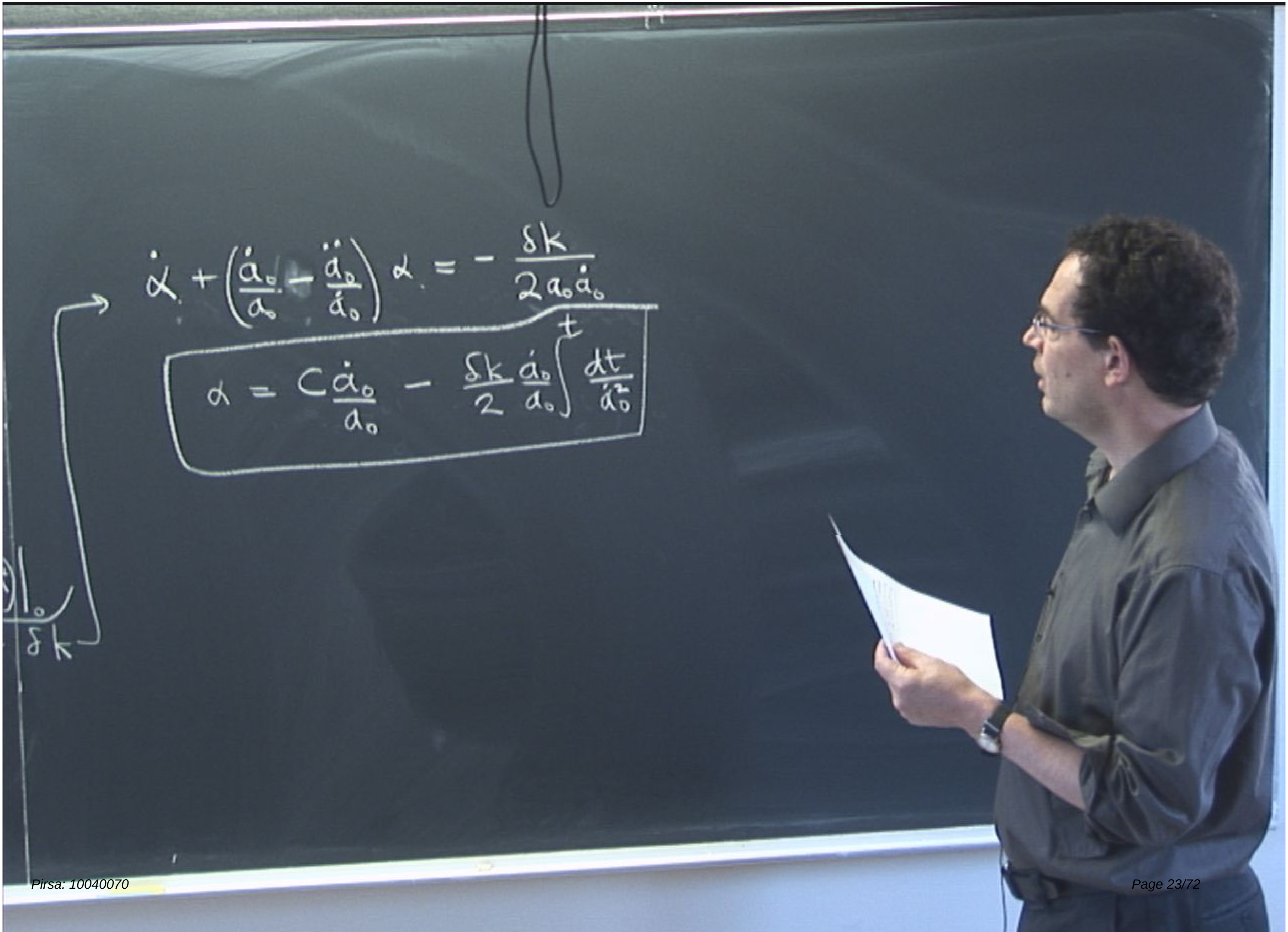
$$\frac{d}{dt} \quad 2\ddot a \cancel{\dot a} = \frac{d}{da}(\quad) \cancel{\dot a}$$

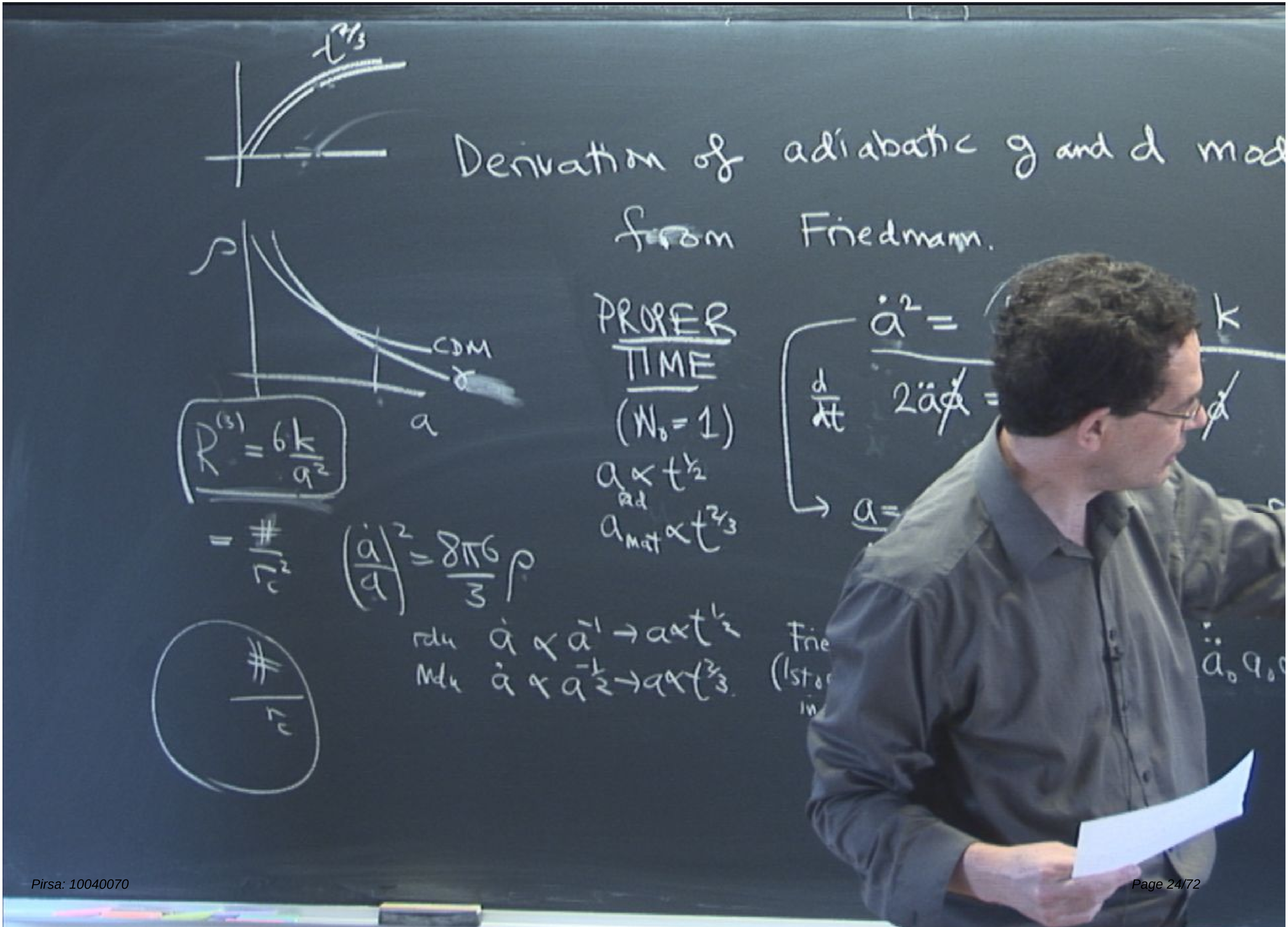
$$a = a_0(1+\alpha) \qquad k = k_0 + \delta k$$

$$\dot a = \dot a_0(1+\alpha) + a_0\dot\alpha$$

Fried. (1st order in pert)

$$\dot a_0^2\, 2\alpha + 2\dot a_0 a_0 \dot\alpha = 2\ddot a_0 a_0 \alpha - \quad \frac{d}{da}\left(\frac{8\pi G}{3}\rho a^2\right.$$


$\dot{\alpha} + \left(\frac{\dot{a}_0}{a_0} - \frac{\ddot{a}_0}{\dot{a}_0}\right)\alpha = -\frac{\delta k}{2 a_0 \dot{a}_0}$
$\alpha = C\frac{\dot{a}_0}{a_0} - \frac{\delta k}{2}\frac{\dot{a}_0}{a_0}\int^t \frac{dt}{\dot{a}_0^2}$


Derivation of adiabatic g and d mod
from Friedmann.
$t^{2/3}$
ρ
CDM
γ
a
$R^{(3)} = 6\frac{k}{a^2}$
PROPER TIME
$(\Omega_0 = 1)$
$a_{rad} \propto t^{1/2}$
$a_{mat} \propto t^{2/3}$
$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$
rdn $\dot a \propto a^{-1} \rightarrow a \propto t^{1/2}$
Mdn $\dot a \propto a^{-1/2} \rightarrow a \propto t^{2/3}$
$\dot a^2 =$
k
$\frac{d}{dt}$
$2\ddot a \dot a =$

$$\dot{\alpha} + \left(\frac{\dot{a}_0}{a_0} - \frac{\ddot{a}_0}{\dot{a}_0}\right)\alpha = -\frac{\delta k}{2 a_0 \dot{a}_0}$$

$$\boxed{\alpha = C\frac{\dot{a}_0}{a_0} - \frac{\delta k}{2}\frac{\dot{a}_0}{a_0}\int^{t}\frac{dt}{\dot{a}_0^2}}$$

$$a_0 \propto t^{2/3}$$
$$\dot{a}_0 \propto t^{-1/3}$$

$$dt = a(t)\,dt_c$$
$$t_c = \int\frac{dt}{a(t)} \propto t^{1/3}$$

$$\delta k\,\frac{1}{t}\,t^{5/3} = \delta k\,t^{2/3} = \delta k\,\tau^2$$

e.g. cdm U. $a_0 \propto t^{2/3} \propto t_c^{2}$

$$\alpha = C\frac{2}{3t}$$
$$\downarrow$$
$$\propto t_c^{-3}$$

δk

$$\dot{\alpha} + \left(\frac{\dot{a}_0}{a_0} - \frac{\ddot{a}_0}{\dot{a}_0}\right)\alpha = -\frac{\delta k}{2 a_0 \dot{a}_0}$$

$$\boxed{\alpha = C\frac{\dot{a}_0}{a_0} - \frac{\delta k}{2}\frac{\dot{a}_0}{a_0}\int^t \frac{dt}{\dot{a}_0^2}}$$

$dt = a(t)\, dt_c$

$t_c = \int \frac{dt}{a(t)} \propto t^{1/3}$

$a_0 \propto t^{2/3}$

$\dot{a}_0 \propto t^{-1/3}$

$\delta k \frac{1}{t} t^{5/3}$

$= \delta k\, t^{2/3}$

$= \delta k\, \tau^2$

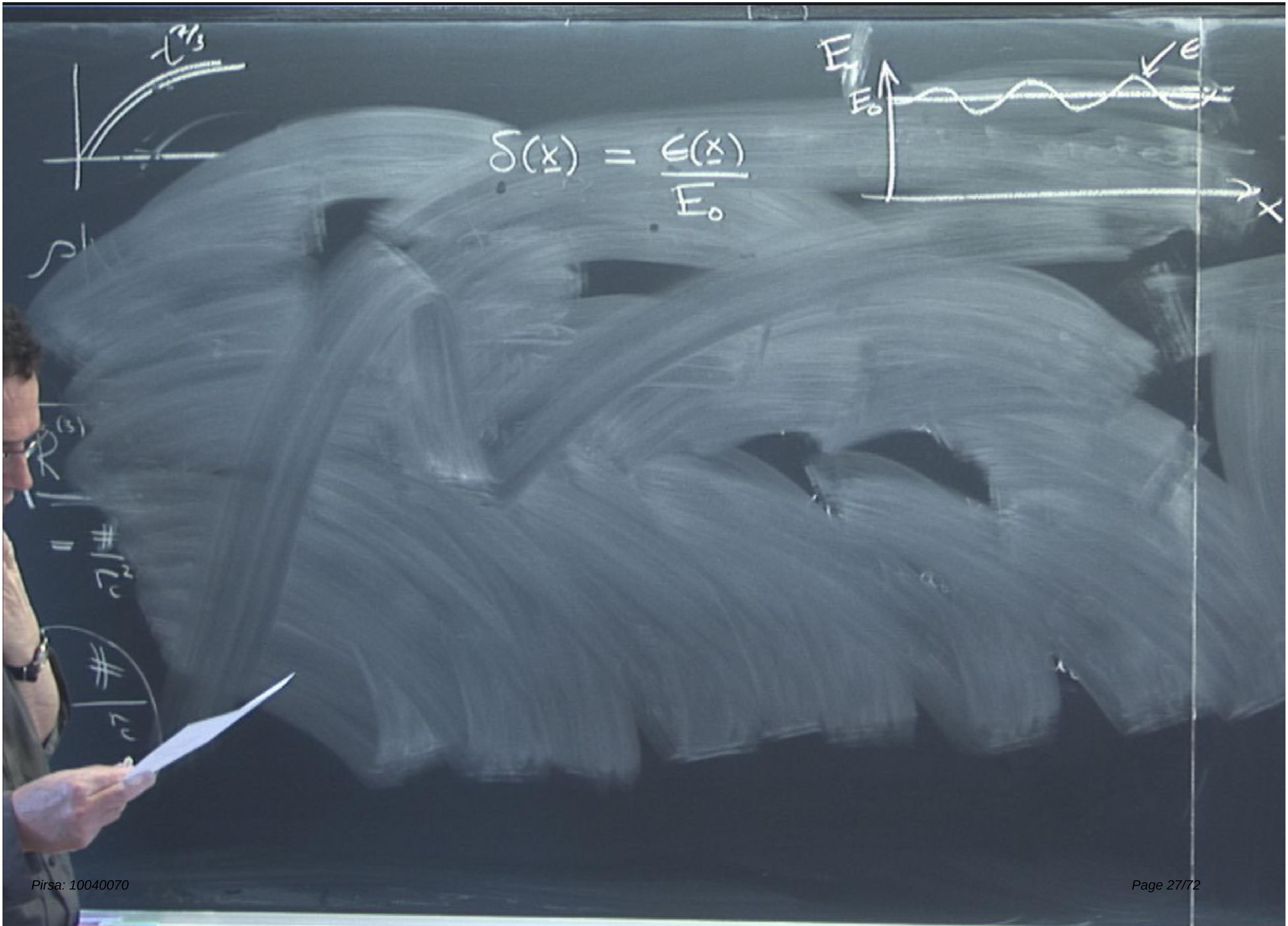
e.g. cdm U. $a_0 \propto t^{2/3} \propto t_c^2$

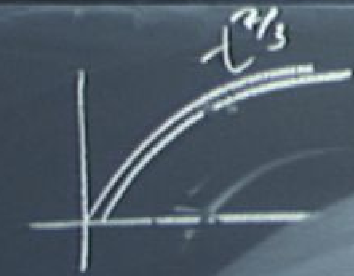
$\alpha = C\frac{2}{3t}$

$\propto t_c^{-3}$

rdu $\to$ g mode $\propto t_c^2 \propto t$

δk

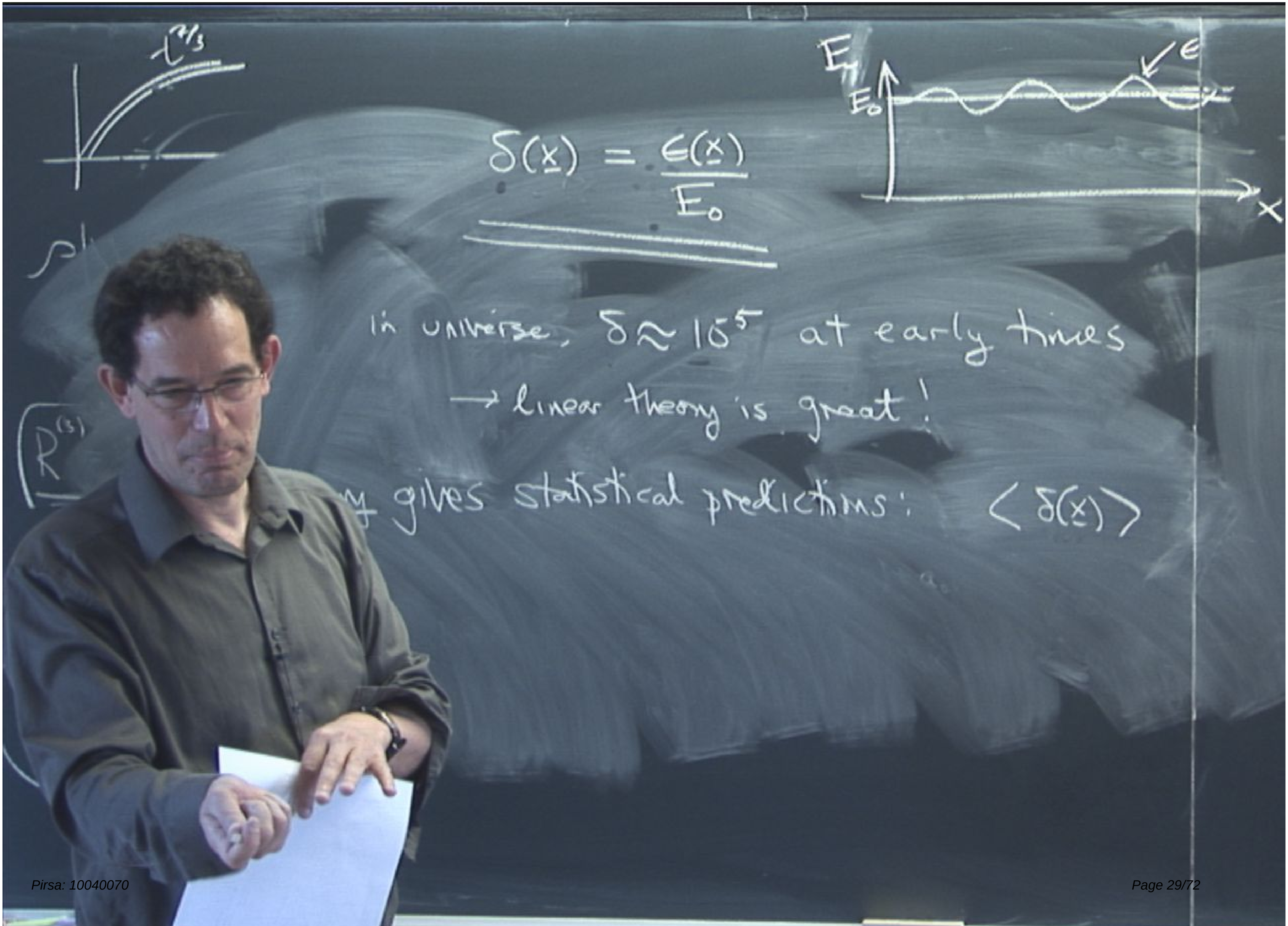

δ(x) = ∈(x) / E₀
E
E₀
x
∈

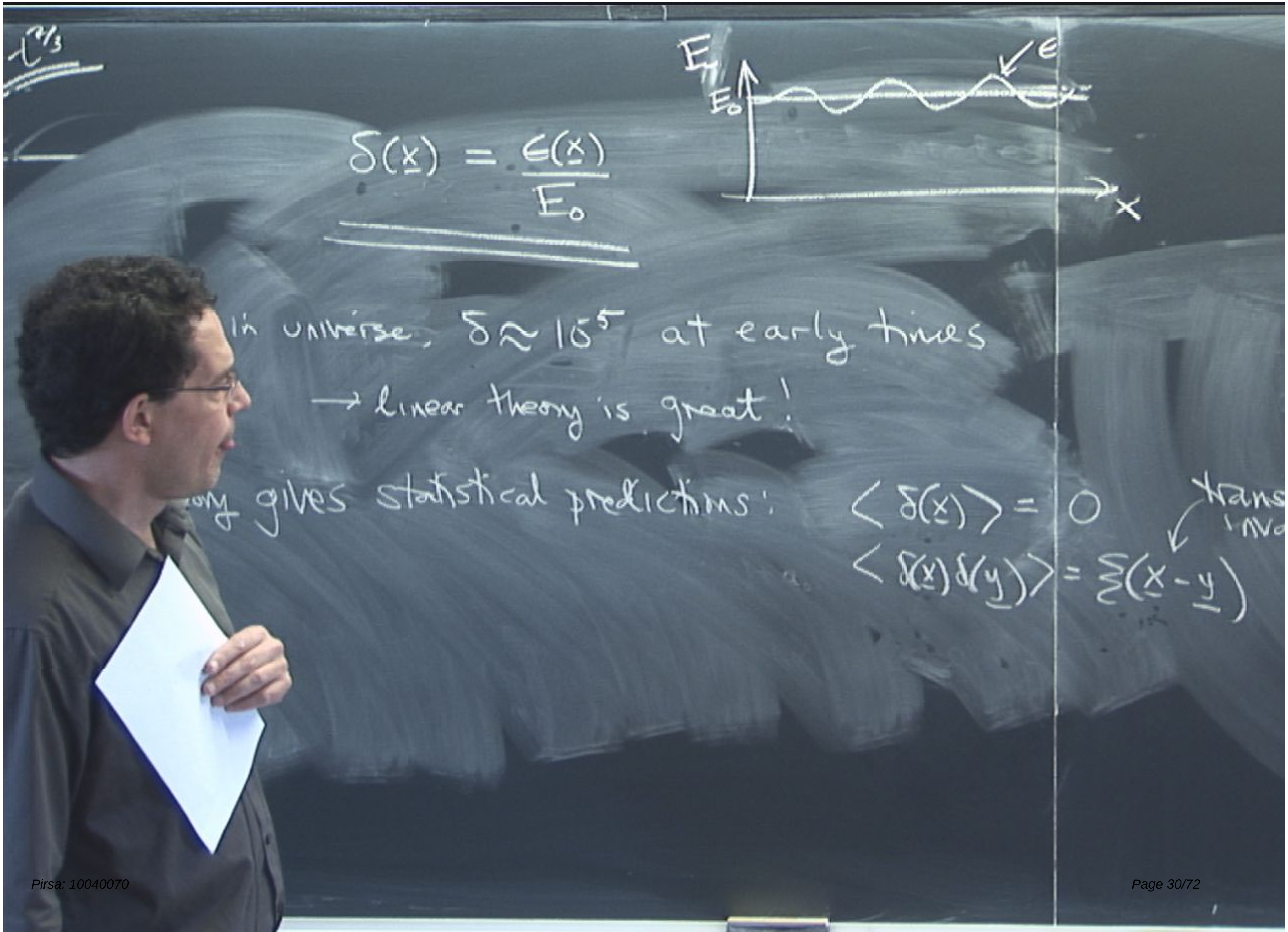


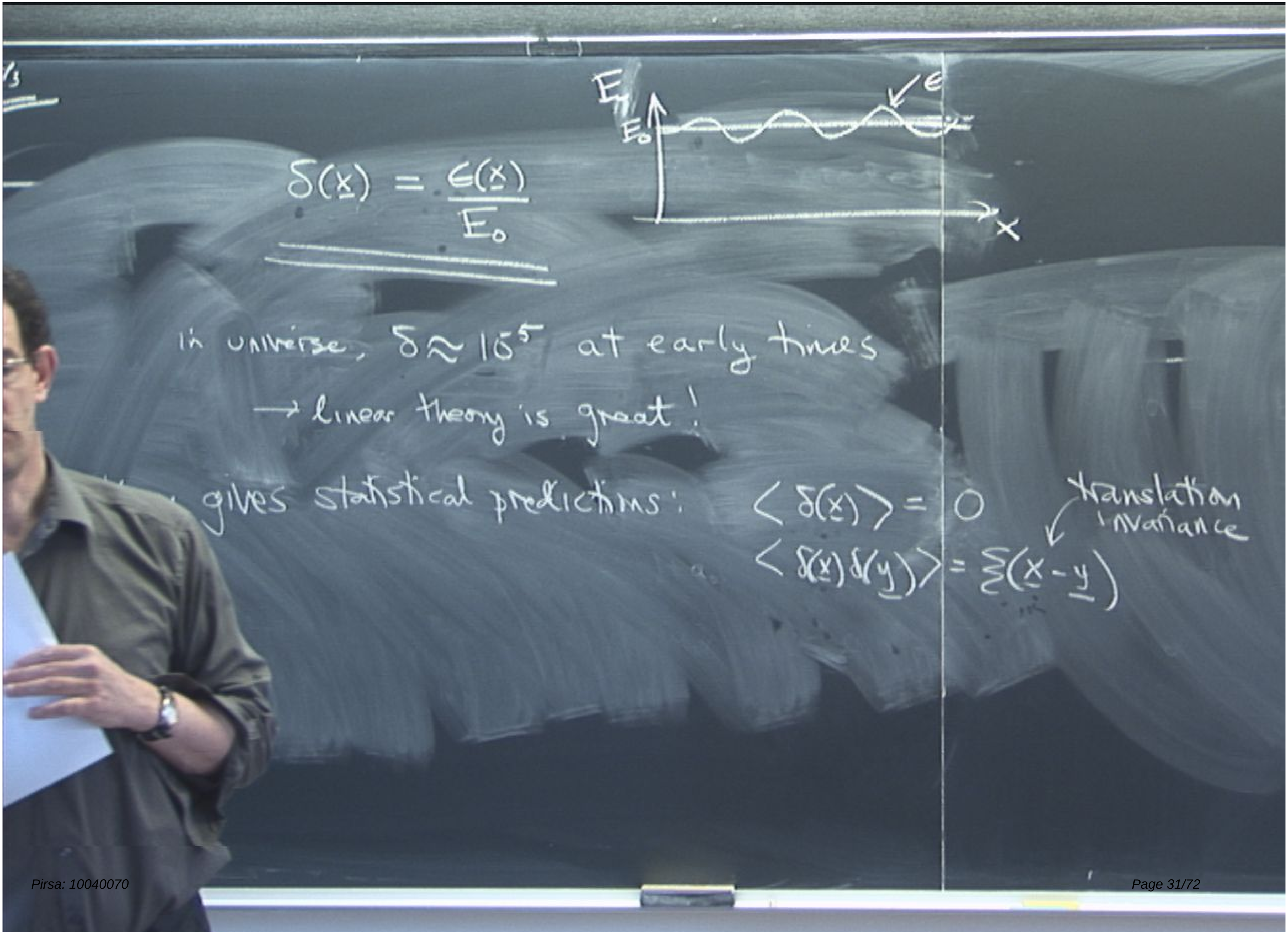
$$\delta(\underline{x}) = \frac{\epsilon(\underline{x})}{E_0}$$

in universe, $\delta \approx 10^{-5}$ at early times

$\rightarrow$ linear theory is great!


E
E₀
ε
x
δ(x) = ε(x) / E₀
in universe, δ ≈ 10⁻⁵ at early times
→ linear theory is great!
R^(3)
gives statistical predictions: ⟨δ(x)⟩


$t^{2/3}$
$\delta(\underline{x}) = \frac{\epsilon(\underline{x})}{E_0}$
E
E_0
ϵ
x
in universe, $\delta \approx 10^{-5}$ at early times
→ linear theory is great!
ory gives statistical predictions: $\langle \delta(\underline{x}) \rangle = 0$
$\langle \delta(\underline{x})\delta(\underline{y}) \rangle = \xi(\underline{x}-\underline{y})$
trans
inva


$\delta(\underline{x}) = \frac{\epsilon(\underline{x})}{E_0}$
E
E_0
ϵ
x
in universe, $\delta \approx 10^{-5}$ at early times
→ linear theory is great!
gives statistical predictions: $\langle \delta(\underline{x}) \rangle = 0$
$\langle \delta(\underline{x}) \delta(\underline{y}) \rangle = \xi(\underline{x} - \underline{y})$
translation invariance

convenient to F.T.

$$\delta(\underline{x}) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}}\,\delta_{\underline{k}}$$

$\rightarrow x$

translation invariance

$= \xi(\underline{x}-\underline{y})$

convenient to F.T.

$$\delta(\underline{x}) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}}\,\delta_{\underline{k}}$$

V

pbcs.

$$\sum_{\underline{k}} \approx V\int\frac{d^3k}{(2\pi)^3}$$

translation invariance

$$= \xi(\underline{x}-\underline{y})$$

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V pbcs

$$\sum_{\underline{k}} \simeq V\int \frac{d^3\underline{k}}{(2\pi)^3}$$

translation invariance

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$\boxed{V}$ pbcs.

$$\sum_{\underline{k}} \approx V \int \frac{d^3 k}{(2\pi)^3}$$

$$\delta_{\underline{k}} = \int \frac{d^3 x}{V} e^{-i\underline{k}\cdot\underline{x}} \delta(\underline{x})$$

$\rightarrow x$

0

translation invariance

$= \xi(\underline{x} - \underline{y})$

convenient to F.T.

$$\delta(\underline{x}) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}}\, \delta_{\underline{k}}$$

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$\to x$

0

translation invariance

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$$\delta_{\underline{k}} = \int \frac{d^3x}{V} e^{-i\underline{k}\cdot\underline{x}}\,\delta(\underline{x})$$

$$\langle \delta_{\underline{k}}\,\delta_{\underline{k}'} \rangle = \int_x \int_y e^{-i\underline{k}\cdot x - i\underline{k}'\cdot y} \langle \delta(\underline{x})\,\delta(\underline{y}) \rangle$$

$\rightarrow x$

0

translation invariance

$\rangle = \xi(\underline{x}-\underline{y})$

convenient to F.T.

$$\delta(\underline{x}) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}}\, \delta_{\underline{k}}$$

$\boxed{V}$ pbcs

$$\sum_{\underline{k}} \approx V \int \frac{d^3k}{(2\pi)^3}$$

$$\delta_{\underline{k}} = \int \frac{d^3x}{V} e^{-i\underline{k}\cdot\underline{x}}\, \delta(\underline{x})$$

$$\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle = \int_x \int_y e^{-i\underline{k}\cdot\underline{x} - i\underline{k}'\cdot\underline{y}} \langle \underbrace{\delta(\underline{x})\, \delta(\underline{y})}_{\xi(\underline{x}-\underline{y})} \rangle$$

translation invariance

$$\rangle = \xi(\underline{x}-\underline{y})$$

$$\underline{x} = \underline{y} + \underline{\Delta}$$

$$= \int \frac{d^3\Delta}{V} e^{-i\underline{k}\cdot\underline{\Delta}}\, \xi(\underline{\Delta}) \underbrace{\int \frac{d^3y}{V} e^{-i(\underline{k}+\underline{k}')\cdot\underline{y}}}_{\delta_{\underline{k}+\underline{k}',\underline{0}}}$$

convenient to F.T.

$$\delta(\underline{x}) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}}\,\delta_{\underline{k}}$$

$\boxed{V}$ pbcs.

$$\sum_{\underline{k}} \approx V\int \frac{d^3k}{(2\pi)^3}$$

$$\delta_{\underline{k}} = \int \frac{d^3x}{V}\, e^{-i\underline{k}\cdot\underline{x}}\,\delta(\underline{x})$$

$$\langle \delta_{\underline{k}}\,\delta_{\underline{k}'} \rangle = \int_x \int_y e^{-i\underline{k}\cdot\underline{x} - i\underline{k}'\cdot\underline{y}} \langle \underbrace{\delta(\underline{x})\,\delta(\underline{y})}_{\xi(\underline{x}-\underline{y})} \rangle$$

O

translation invariance

$\rangle = \xi(\underline{x}-\underline{y})$

$$\underline{x} = \underline{y} + \underline{\Delta}$$

$$= \underbrace{\int \frac{d^3\Delta}{V}\, e^{-i\underline{k}\cdot\underline{\Delta}}\, \underbrace{\xi(\underline{\Delta})}_{\xi(|\underline{\Delta}|)}}\; \underbrace{\int \frac{d^3y}{V}\, e^{-i(\underline{k}+\underline{k}')\cdot\underline{y}}}_{\delta_{\underline{k}+\underline{k}',0}}$$

$$\equiv P(|\underline{k}|)\;\delta_{\underline{k}+\underline{k}',\underline{0}}$$

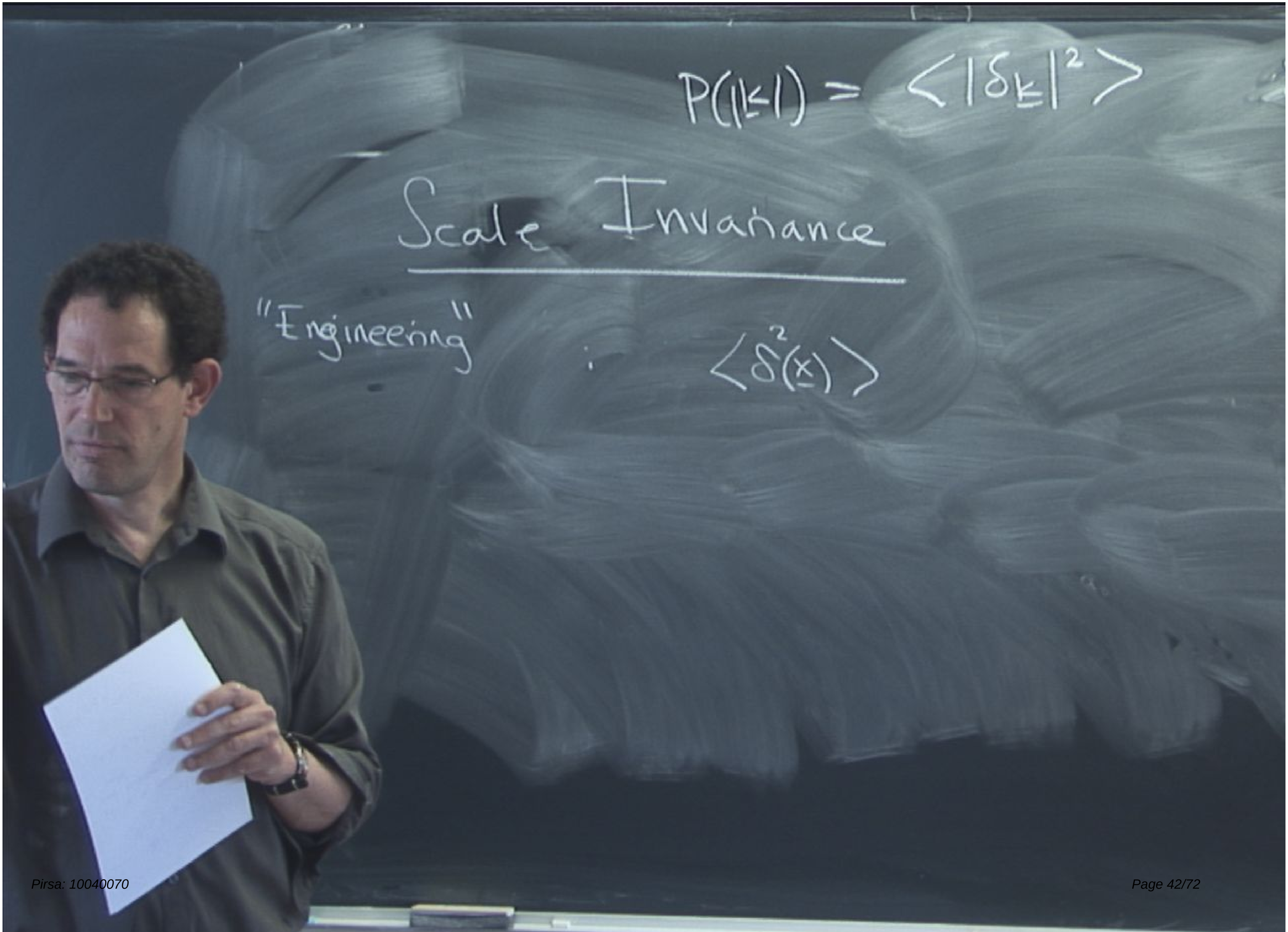
Power Spectrum.

$$P(|\underline{k}|) = \langle |\delta_{\underline{k}}|^2 \rangle$$

$$P(|\underline{k}|) = \langle |\delta_{\underline{k}}|^2 \rangle$$

Scale Invariance

"Engineering"


$P(|\underline{k}|) = \langle |\delta_{\underline{k}}|^2 \rangle$
Scale Invariance
"Engineering" :
$\langle \delta^2(\underline{x}) \rangle$

$\delta_{\underline{k}}^*$ reality of $\delta(\underline{x})$

convenient to F.T.

$\boxed{V}$ pbcs

$$\delta(\underline{x}) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}} \delta_{\underline{k}}$$

$$\delta_{\underline{k}} = \int \frac{d^3x}{V} e^{-i\underline{k}\cdot\underline{x}} \delta(\underline{x})$$

$$\sum_{\underline{k}} \simeq V\int \frac{d^3k}{(2\pi)^3}$$

$$\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle = \int_x \int_y e^{-i\underline{k}\cdot\underline{x} - i\underline{k}'\cdot\underline{y}} \langle \underbrace{\delta(\underline{x})\,\delta(\underline{y})}_{\xi(\underline{x}-\underline{y})} \rangle$$

$$\underline{x} = \underline{y} + \underline{\Delta}$$

$$= \underbrace{\int \frac{d^3\Delta}{V} e^{-i\underline{k}\cdot\underline{\Delta}} \underbrace{\xi(\underline{\Delta})}_{\xi(|\underline{\Delta}|)}} \underbrace{\int \frac{d^3y}{V} e^{-i(\underline{k}+\underline{k}')\cdot\underline{y}}}_{\delta_{\underline{k}+\underline{k}',\underline{0}}}$$

$$\xi(\underline{\Delta}) = V\int d^3k\, e^{i\underline{k}\cdot\underline{\Delta}} P(|\underline{k}|)$$

$$\equiv P(|\underline{k}|)\ \delta_{\underline{k}+\underline{k}',\underline{0}}$$

Power Spectrum.

$$P(|\underline{k}|) = \langle |\delta_{\underline{k}}|^2 \rangle \qquad \delta_{-\underline{k}} = \delta_{\underline{k}}^*$$

Scale Invariance

"Engineering" : $\langle \delta^2(\underline{x}) \rangle = V \int d^3k \, \underbrace{P(|\underline{k}|)}_{\frac{1}{V}\frac{1}{|\underline{k}|^3}}$

if δ dimensionless $= \int \frac{d^3k}{|\underline{k}|^3}$

$\sim \int d(\ln k) \sim \int \frac{dk}{k}$ log dgt in UV and IR

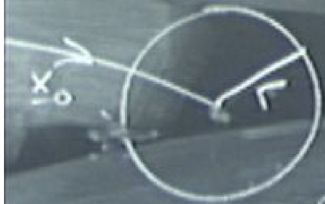
equal contrn's to variance from each logarithmic interval in k.

Mean of δ in some volume

Mean of δ in some volume

$$\bar{\delta} = \frac{\int_0^r d^3\underline{x}' \sum_{\underline{k}} \delta_{\underline{k}} \, e^{-i\underline{k}\cdot\underline{x}'}}{\frac{4\pi}{3} r^3}$$

Mean of δ in some volume



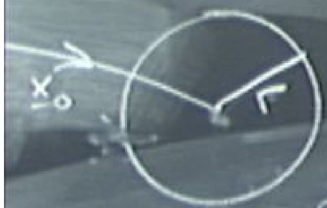
Ex

$$\bar{\delta} = \frac{\int_0^r d^3\underline{x}' \sum_{\underline{k}} \delta_{\underline{k}} e^{-i\underline{k}\cdot(\underline{x}_0+\underline{x}')}}{\frac{4\pi}{3}r^3} \quad ; \quad \langle \bar{\delta}^2 \rangle = V\int d^3k\, P(|\underline{k}|)\, W(kr)$$

$$W(kr) = \left(\frac{3(\sin kr - kr\cos kr)}{(kr)^3}\right)^2$$

Mean of δ in some volume

Ex



$$\bar{\delta} = \frac{\int_0^r d^3\underline{x}' \sum_{\underline{k}} \delta_{\underline{k}} e^{-i\underline{k}\cdot(\underline{x}_0+\underline{x}')}}{\frac{4\pi}{3} r^3}$$

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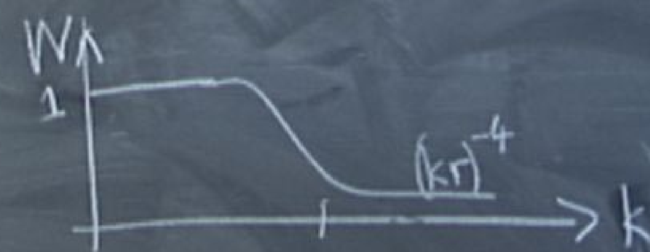
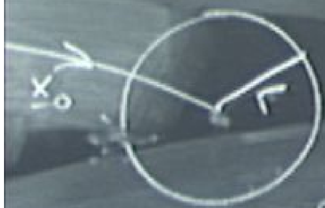


$\bar{\delta}^2 \approx$

Mean of δ in some volume

$\int k^2 dk$



Ex

$$\bar{\delta} = \frac{\int_0^r d^3\underline{x}' \sum_{\underline{k}} \delta_{\underline{k}} e^{-i\underline{k}\cdot(\underline{x}_0+\underline{x}')}}{\frac{4\pi}{3} r^3}$$

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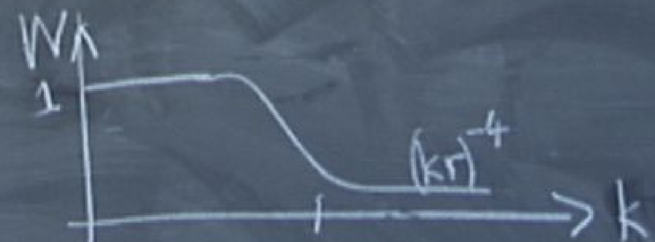
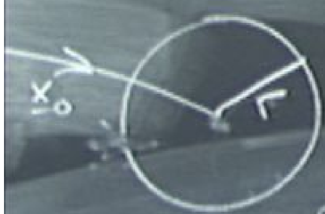


$$\langle \bar{\delta}^2 \rangle \approx k^3 P(|\underline{k}|) \Big|_{k=1/r}$$

Mean of δ in some volume



$$\bar{\delta} = \frac{\int_0^r d^3\underline{x}' \sum_{\underline{k}} \delta_{\underline{k}} e^{-i\underline{k}\cdot(\underline{x}_0+\underline{x}')}}{\frac{4\pi}{3} r^3}$$

$$\langle \bar{\delta}^2 \rangle \sim r^{-(3+n)}$$

if $P(|\underline{k}|) \propto k^n$

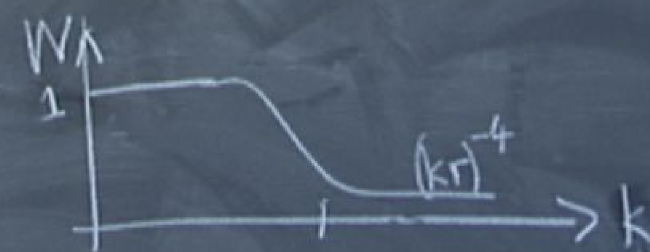
$-3 < n < 1$

$\int k^2 dk$

Ex

$$\langle \bar{\delta}^2 \rangle = V \int d^3k \, P(|\underline{k}|) W(kr)$$

$$W(kr) = \left(\frac{3(\sin kr - kr\cos kr)}{(kr)^3} \right)^2$$

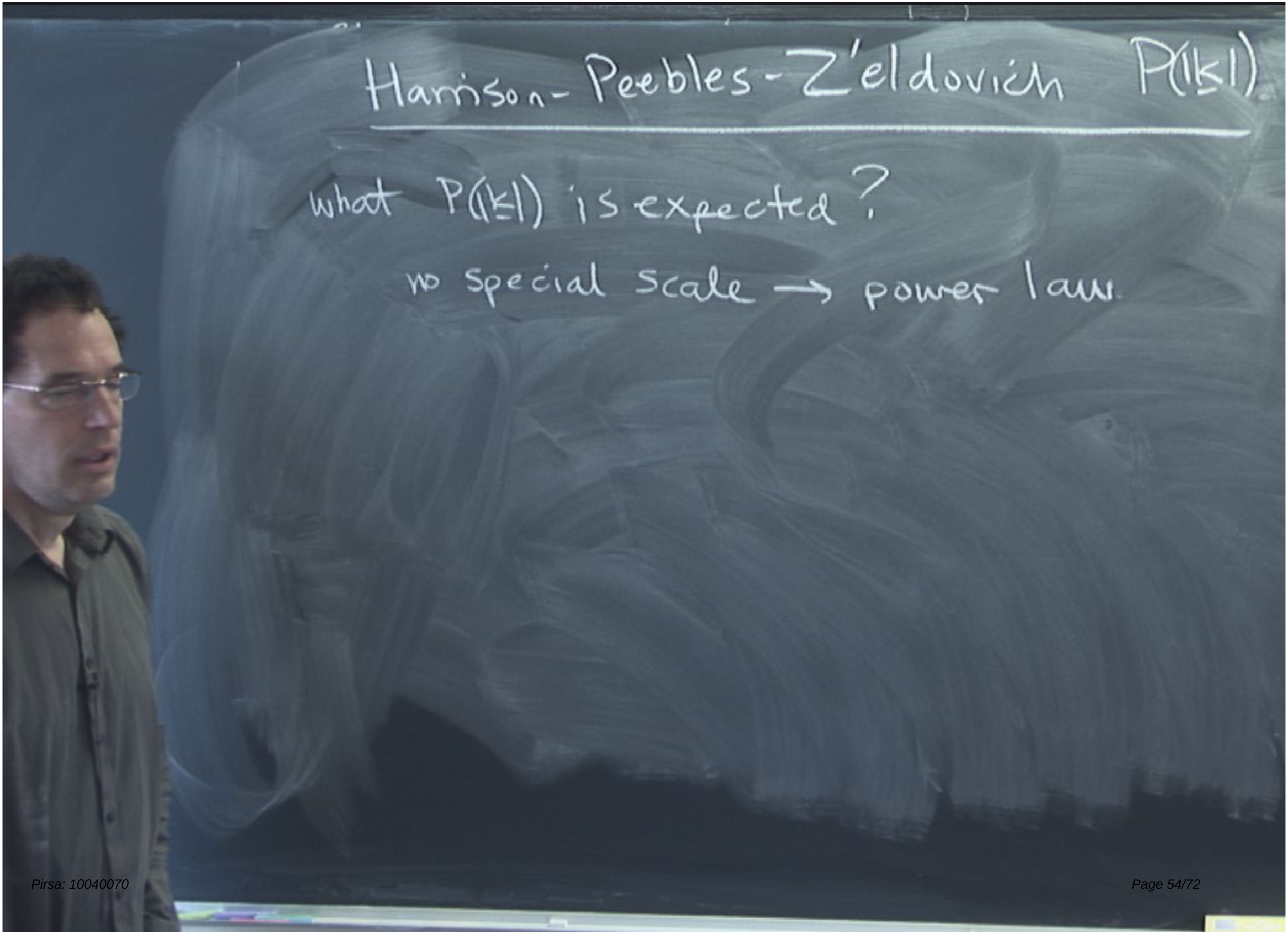


$$\langle \bar{\delta}^2 \rangle \approx k^3 P(|\underline{k}|) \Big|_{k=1/r}$$

Harrison - Peebles - Z'eldovich $P(|\underline{k}|)$

Harrison - Peebles - Z'eldovich $P(|\underline{k}|)$

what $P(|\underline{k}|)$ is expected?


Harrison-Peebles-Z'eldovich P(|k|)
what P(|k|) is expected?
no special scale → power law

Harrison - Peebles - Z'eldovich $P(|\underline{k}|)$

what $P(|\underline{k}|)$ is expected?

no special scale $\longrightarrow$ power law

small scales : Φ ? $\quad \frac{GM}{c^2 r} = \frac{v^2}{c^2} \sim 10^{-5}$

Harrison-Peebles-Z'eldovich $P(|\underline{k}|)$

what $P(|\underline{k}|)$ is expected?

no special scale $\rightarrow$ power law.

small scales : Φ? $\frac{GM}{c^2 r} = \frac{v^2}{c^2} \sim 10^{-5}$

large scales : $\frac{\delta T}{T}$ small. $\rightarrow$ Φ small large scales.

$$\rightarrow \langle |\Phi_k|^2 \rangle = \frac{\#}{V k^3} \qquad \# \sim 10^{-10}$$

Harrison-Peebles-Z'eldovich $P(|\underline{k}|)$

what $P(|\underline{k}|)$ is expected?

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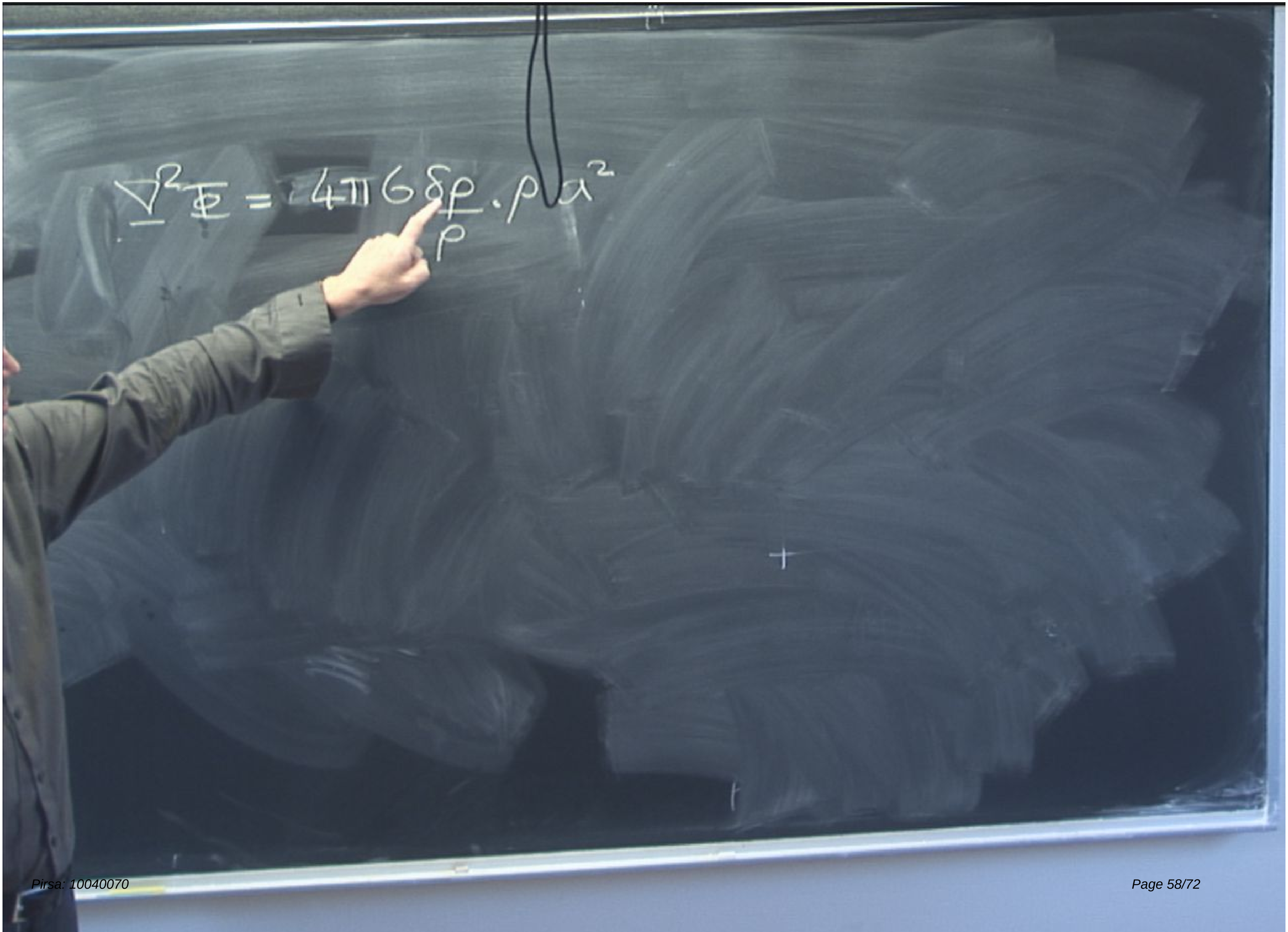
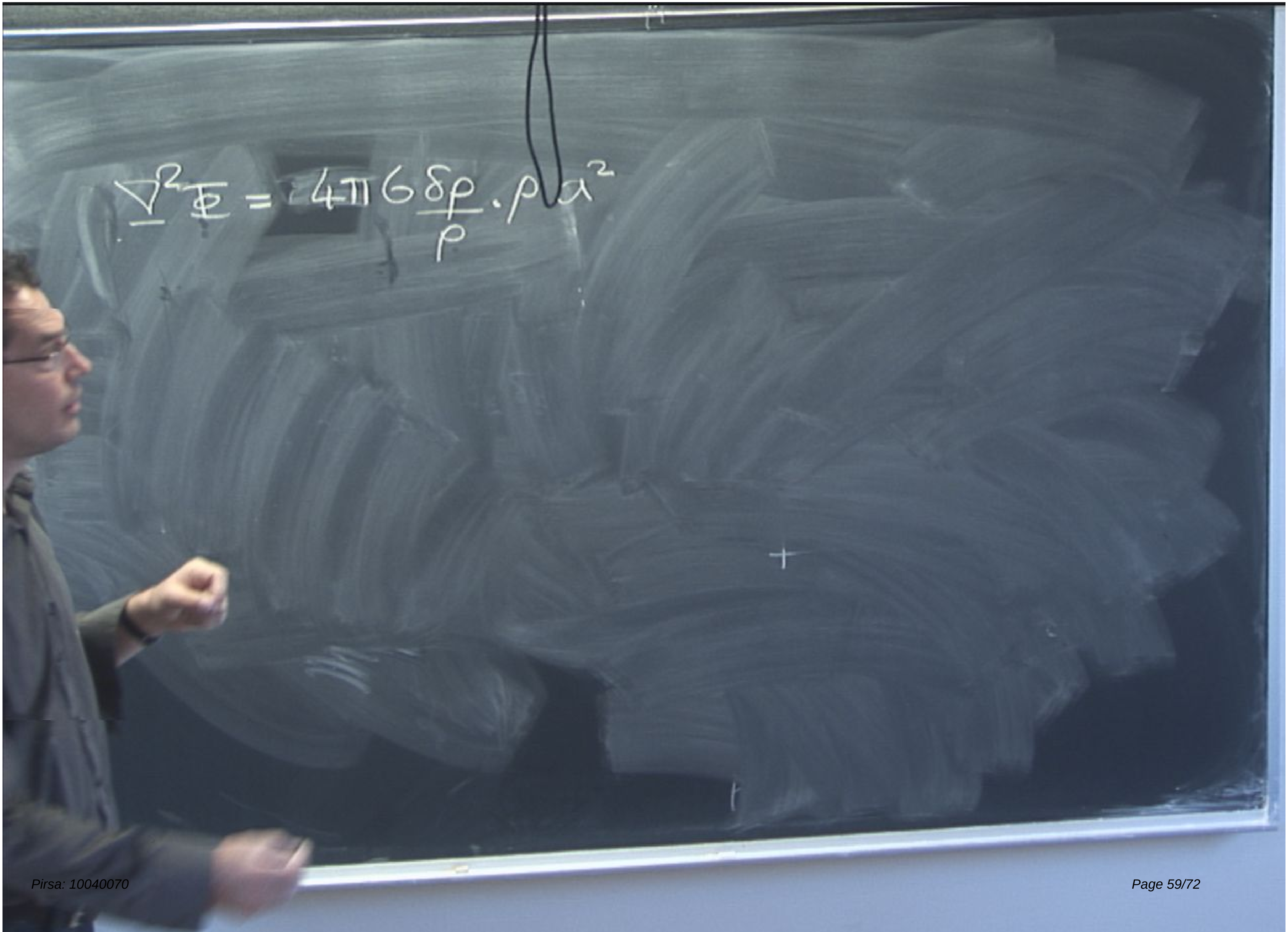
$\rightarrow \langle |\Phi_k|^2 \rangle = \frac{\#}{V k^3}$ $\qquad \underline{\# \sim 10^{-10}}$

$\langle \Phi^2 \rangle = \#$

0

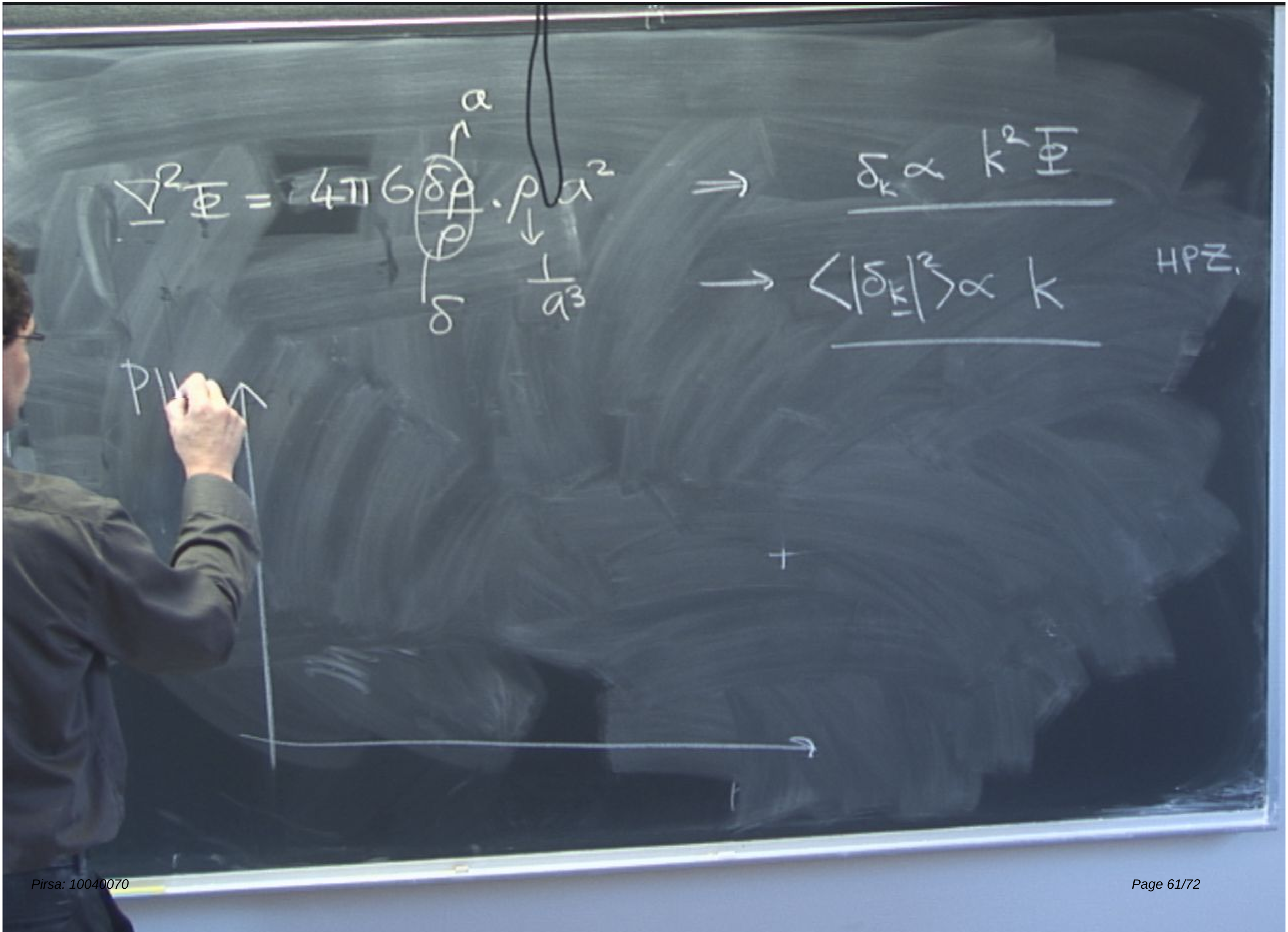
$\underline{x}_0$


$\underline{\nabla}^2 \underline{\Phi} = 4\pi G \frac{\delta\rho}{\rho} \cdot \rho a^2$


$\nabla^2 \Phi = 4\pi G \frac{\delta\rho}{\rho} \cdot \rho a^2$

$$\nabla^2 \Phi = 4\pi G \frac{\delta\rho}{\rho} \cdot \rho\, a^2 \quad \Rightarrow \quad \underline{\delta \propto k^2 \Phi}$$

$$\frac{\delta\rho}{\rho} \to \delta, \quad a \leftarrow \frac{\delta\rho}{\rho}, \quad \rho \to \frac{1}{a^3}$$


$\underline{\nabla}^2 \Phi = 4\pi G \frac{\delta\rho}{\rho} \cdot \rho a^2$
a
δ
$\frac{1}{a^3}$
$\Rightarrow \delta_k \propto k^2 \Phi$
$\longrightarrow \langle |\delta_{\underline{k}}|^2 \rangle \propto k$
HPZ.

$$\underline{\nabla}^2 \Phi = 4\pi G \frac{\delta\rho}{\rho} \cdot \rho a^2$$

$$\frac{\delta\rho}{\rho} \to \delta \to a, \qquad \rho \to \frac{1}{a^3}$$

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$$\to \quad \underline{\langle |\delta_{\underline{k}}|^2 \rangle \propto k} \qquad \text{HPZ.}$$

$$\underline{\nabla}^2 \Phi = 4\pi G \underbrace{\frac{\delta\rho}{\rho}}_{\delta} \cdot \underbrace{\rho}_{\frac{1}{a^3}} a^2 \implies \underline{\delta_k \propto k^2 \Phi}$$

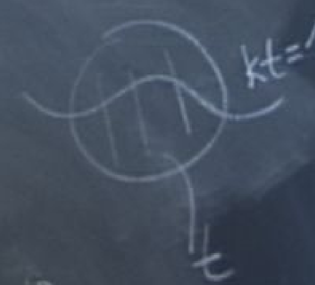
$$\longrightarrow \underline{\langle |\delta_{\underline{k}}|^2 \rangle \propto k} \qquad \text{HPZ.}$$

$$\underline{\nabla}^2 \Phi = 4\pi G \underbrace{\frac{\delta\rho}{\rho}}_{\delta} \cdot \underbrace{\rho}_{\frac{1}{a^3}} a^2 \Longrightarrow \underline{\delta_k \propto k^2 \Phi}$$

$$\longrightarrow \underline{\langle|\delta_{\underline{k}}|^2\rangle \propto k t^4} \quad \text{HPZ.}$$

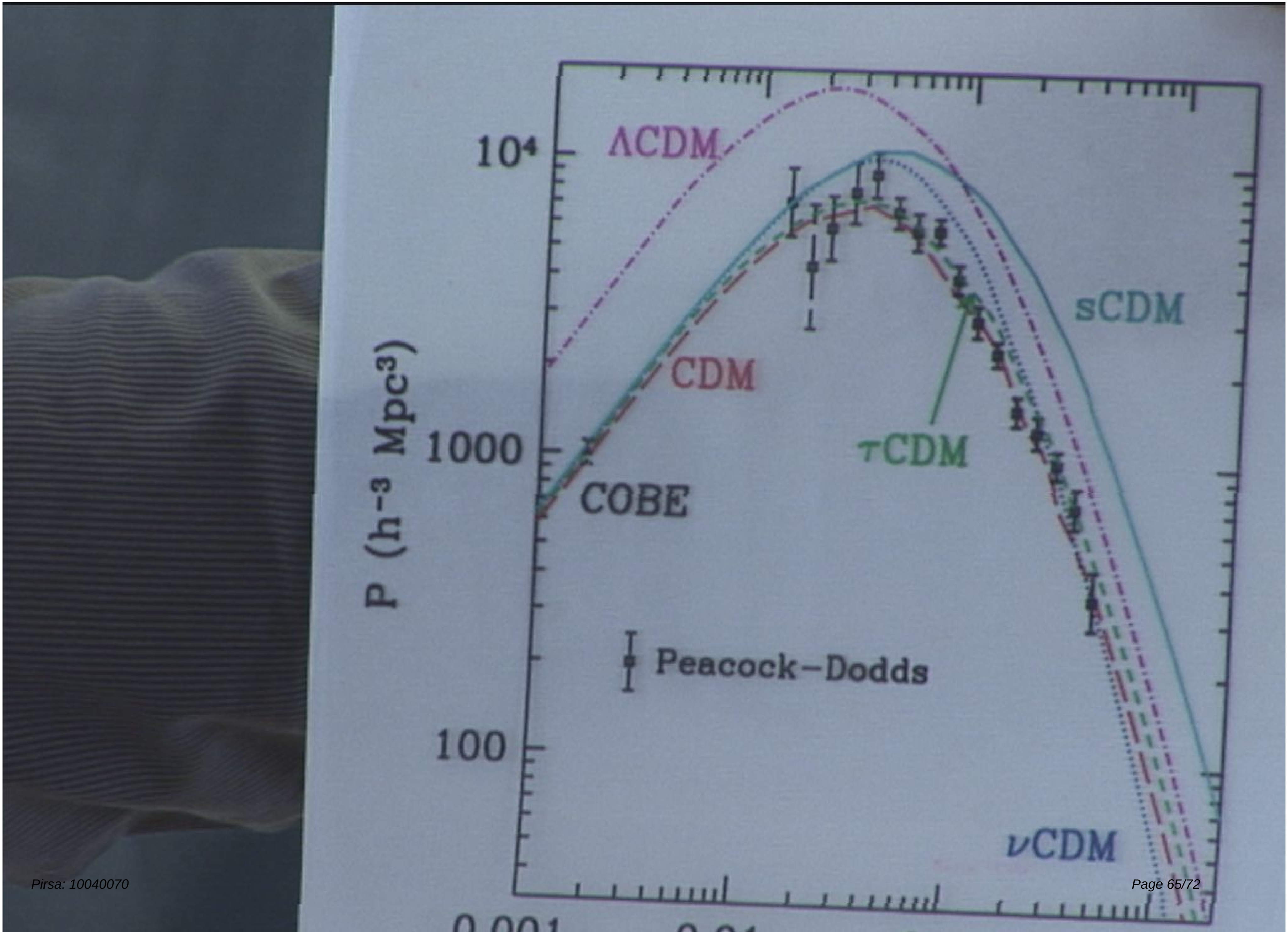
conformal time

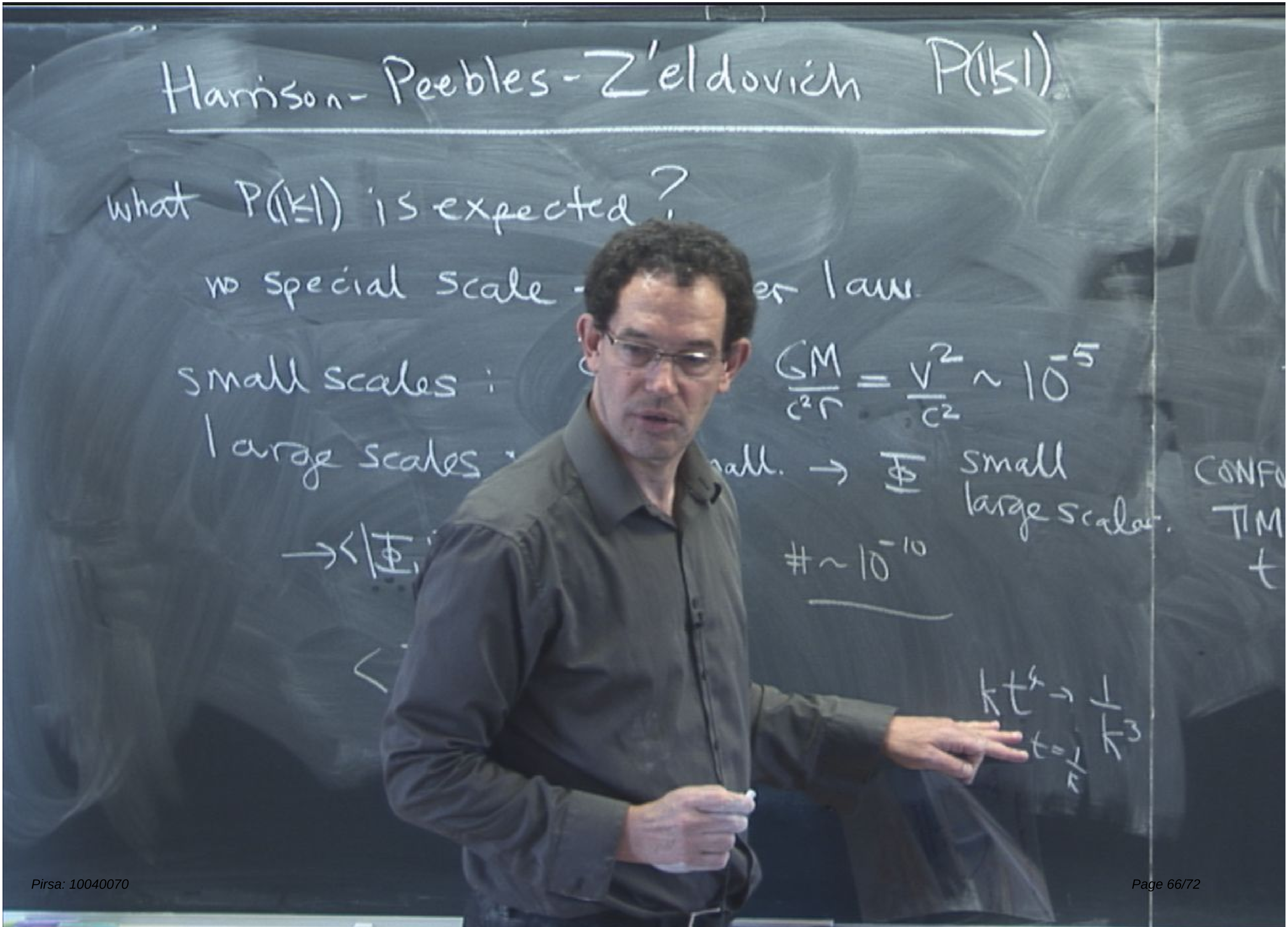
CONFORMAL TIME t

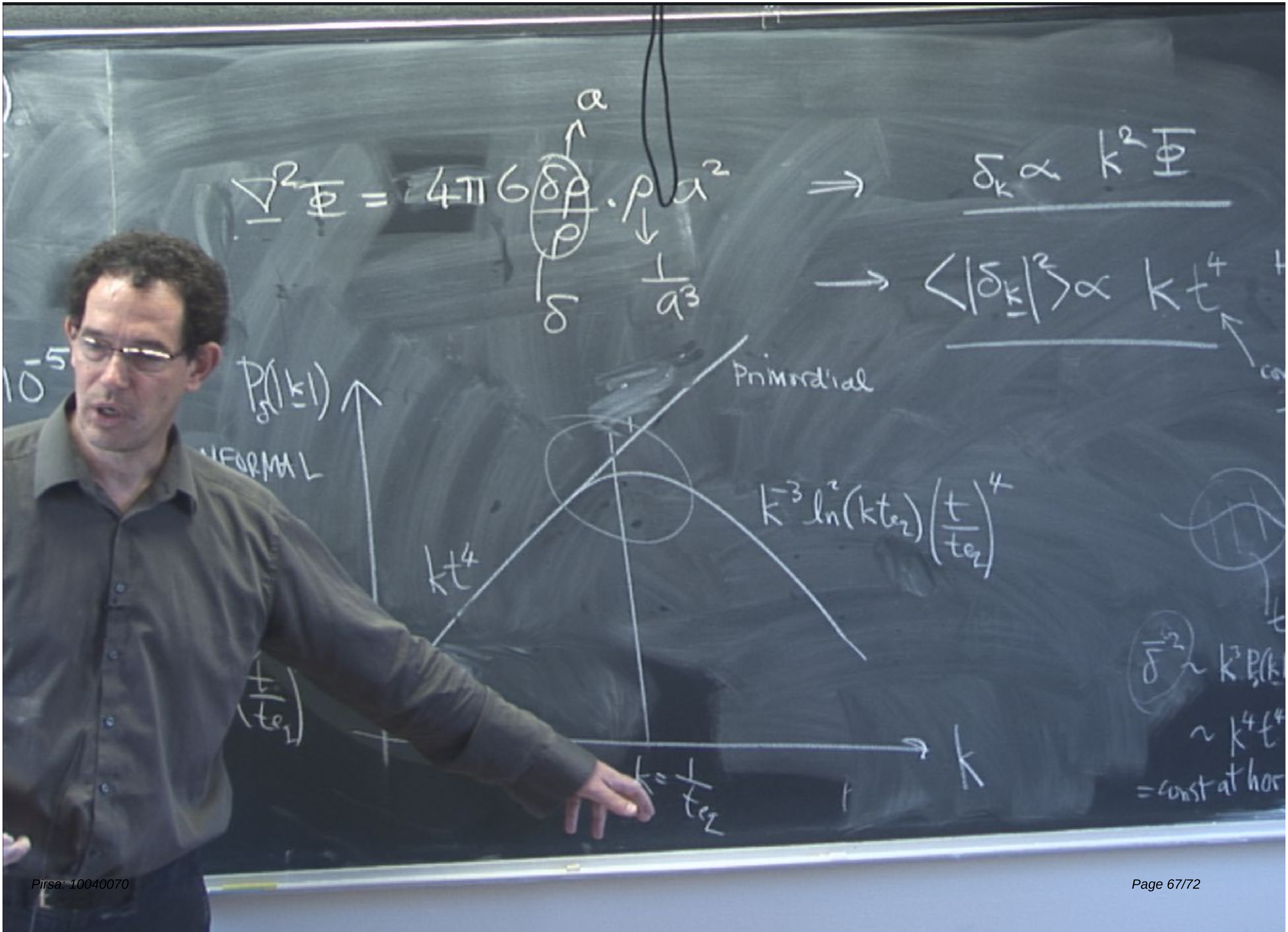


$$\bar{\delta}^2 \sim k^3 P_\delta(|\underline{k}|)$$

$$\sim k^4 t^4$$


ΛCDM
10^4
1000
100
P (h^{-3} Mpc^3)
CDM
COBE
τCDM
sCDM
Peacock–Dodds
νCDM


Harrison-Peebles-Z'eldovich P(|k|)
what P(|k|) is expected?
no special scale
law.
small scales :
GM/c²r = v²/c² ~ 10⁻⁵
large scales
small.
→ Φ small
large scales.
~ 10⁻¹⁰
kt⁴ → 1/k³
t = 1/k
CONF
TIM
t



$\nabla^2 \Phi = 4\pi G \frac{\delta\rho}{\rho} \cdot \rho a^2$
$\Rightarrow \delta_k \propto k^2 \Phi$
$\rightarrow \langle |\delta_{\underline{k}}|^2 \rangle \propto k t^4$
$P_\delta(|\underline{k}|)$
Primordial
$k^{-3} \ln^2(k t_{eq}) \left(\frac{t}{t_{eq}}\right)^4$
$k t^4$
$k = \frac{1}{t_{eq}}$
k
$\bar{\delta}^2 \sim k^3 P_\delta(k)$
$\sim k^4 t^4$
= const at hor

$$\nabla^2 \Phi = 4\pi G \frac{\delta\rho}{\rho}\cdot \rho\, a^2 \Longrightarrow \delta_k \propto k^2 \Phi$$

$a \qquad \delta \qquad \frac{1}{a^3}$

$$\longrightarrow \langle|\delta_{\underline{k}}|^2\rangle \propto k t^4$$

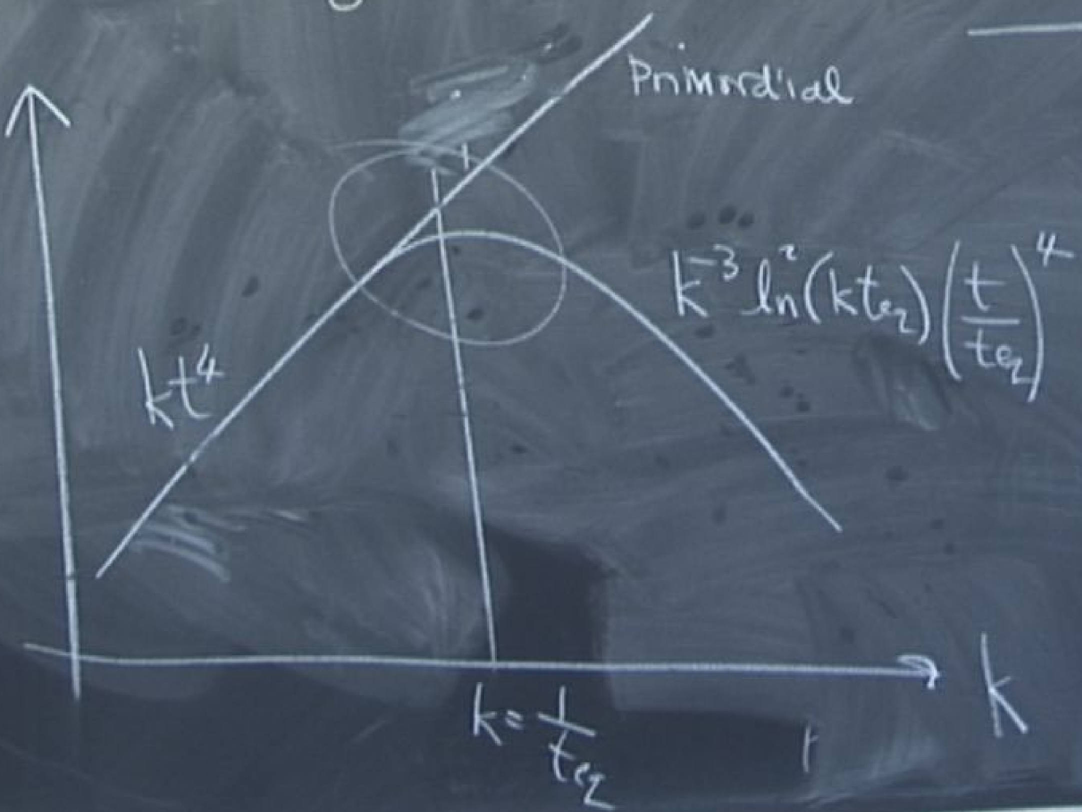
$\bar{\delta}^2$

10^{-5}

ll

ge scalar.

CONFORMAL TIME t



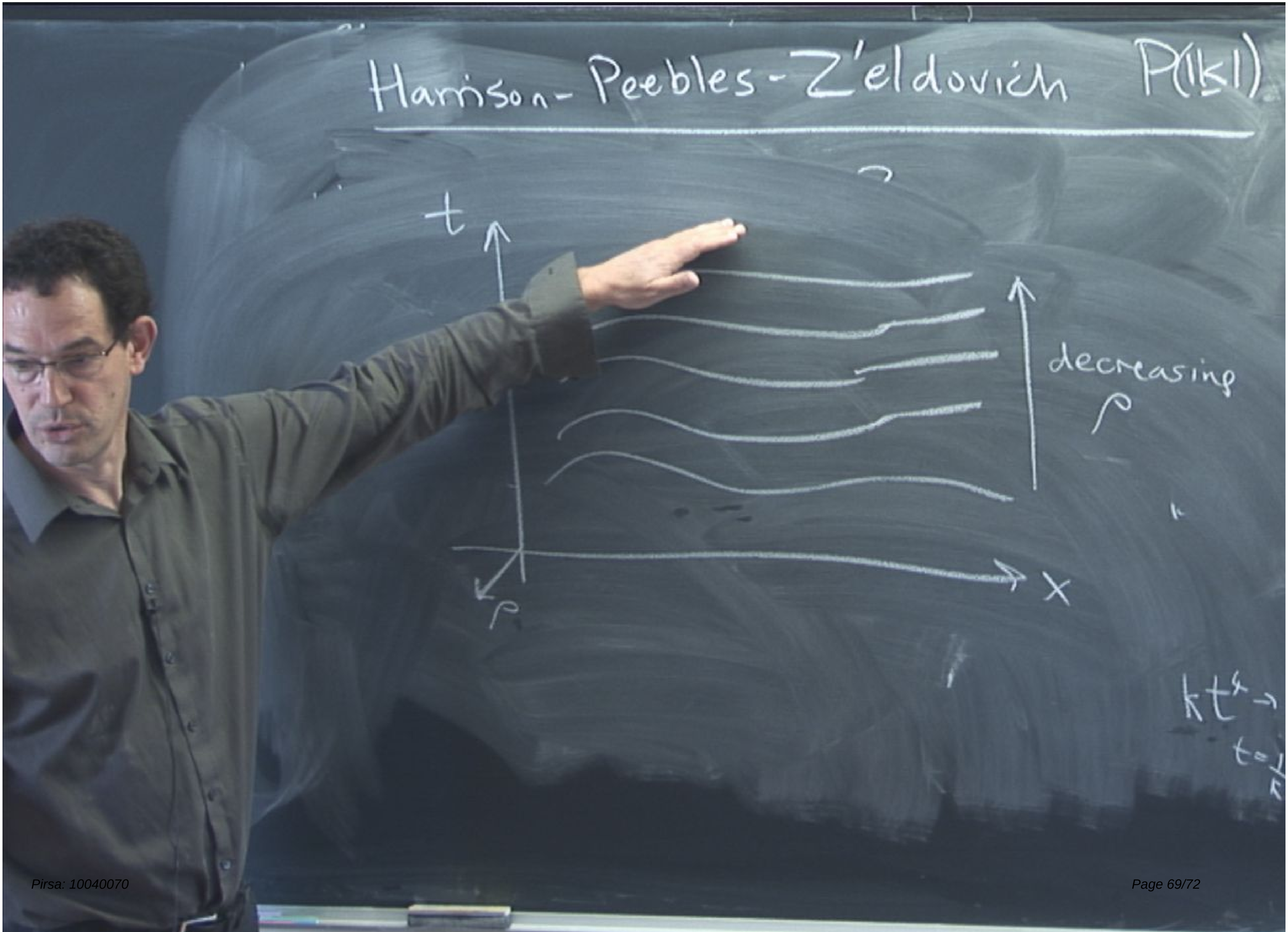
$t^4 \to \frac{1}{k^3}\ln^2\left(\frac{t}{t_{eq}}k\right)\left(\frac{t}{t_{eq}}\right)^4$

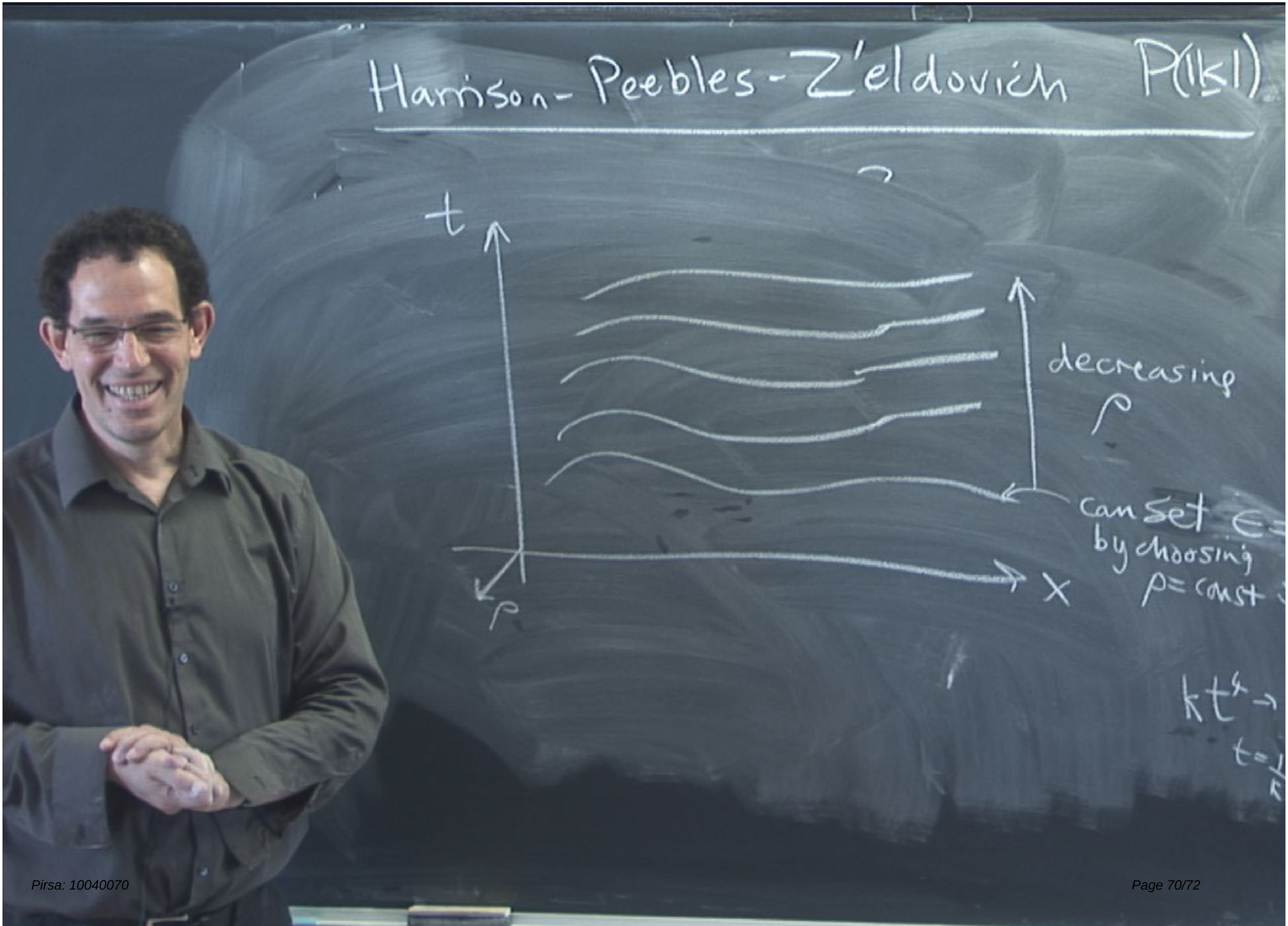
$t = \frac{1}{k}$

$\bar{\delta}^2 \sim k^3 P_\delta(k)$

$\sim k^4 t^4$

$=$ const at hor


Harrison - Peebles - Z'eldovich P(|k|)
t
decreasing
ρ
x

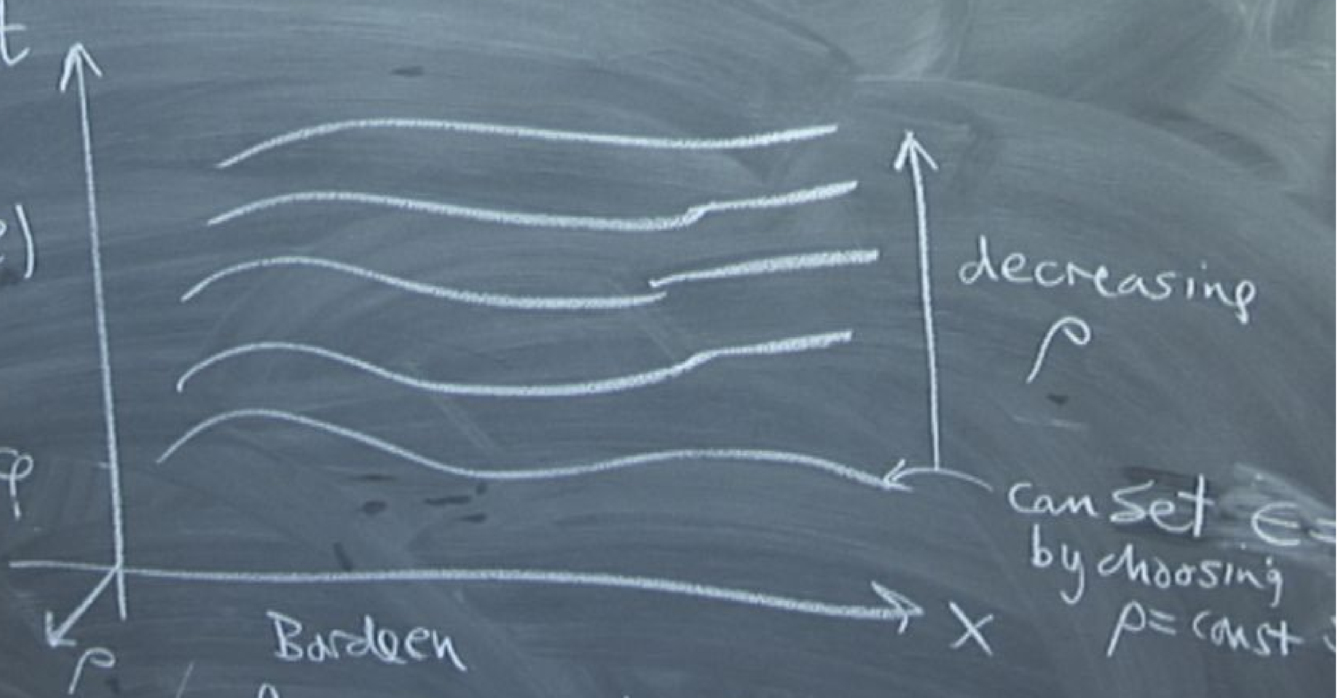

Harrison-Peebles-Z'eldovich P(|k|)
t
decreasing
ρ
can set ε=
by choosing
ρ= const
x
ρ

Harrison - Peebles - Z'eldovich $P(|\underline{k}|)$

$$h_{ij} = a^2 f_{ij}(1+2\varphi) + D_i D_j \gamma$$

conf

$$\delta R^{(3)} = \left(\Delta + \frac{3C}{a^3}\right)\varphi$$



Bardeen

$$\zeta = \varphi + \frac{1}{3}\frac{\epsilon}{P_0 + E_0}$$

$k t^{\alpha} \rightarrow$

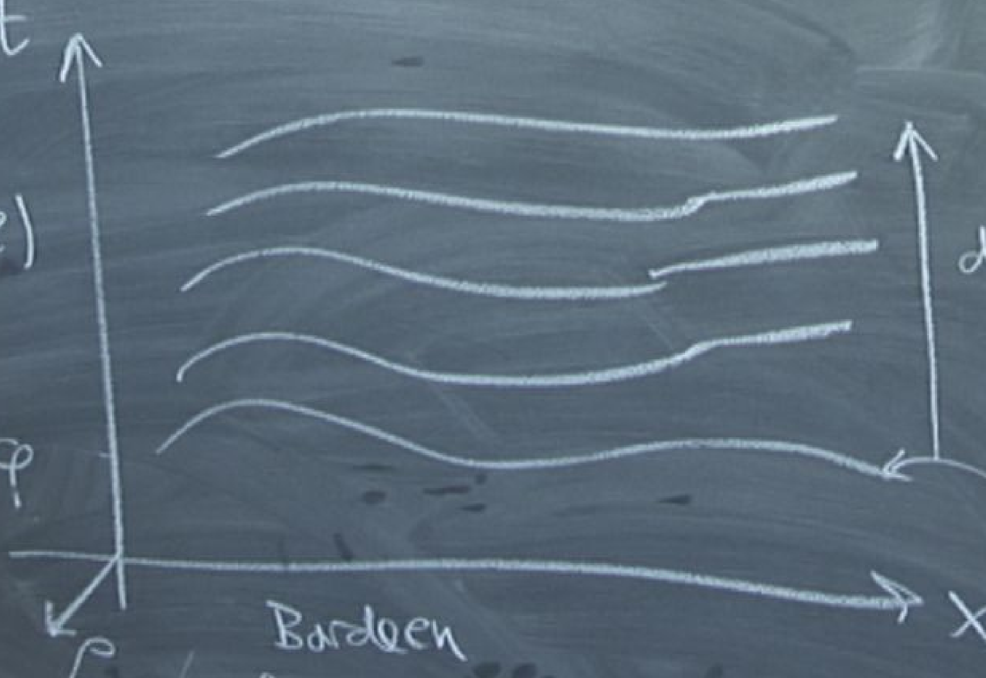
$t = \frac{1}{k}$

Harrison-Peebles-Z'eldovich $P(|\underline{k}|)$

conf

$$h_{ij} = a^2 f_{ij}(1+2\varphi) + D_i D_j \gamma$$

$$\delta R^{(3)} = \left(\Delta + \frac{3C}{a^3}\right)\varphi$$



Bardeen

$$\zeta = \varphi + \frac{1}{3}\frac{\epsilon}{P_0 + E_0}$$

gauge invariant

$k t^\alpha \to$

$t = \frac{1}{k}$

$\to$ HPZ spectrum.