

Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 12

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URL: <http://pirsa.org/10040069>

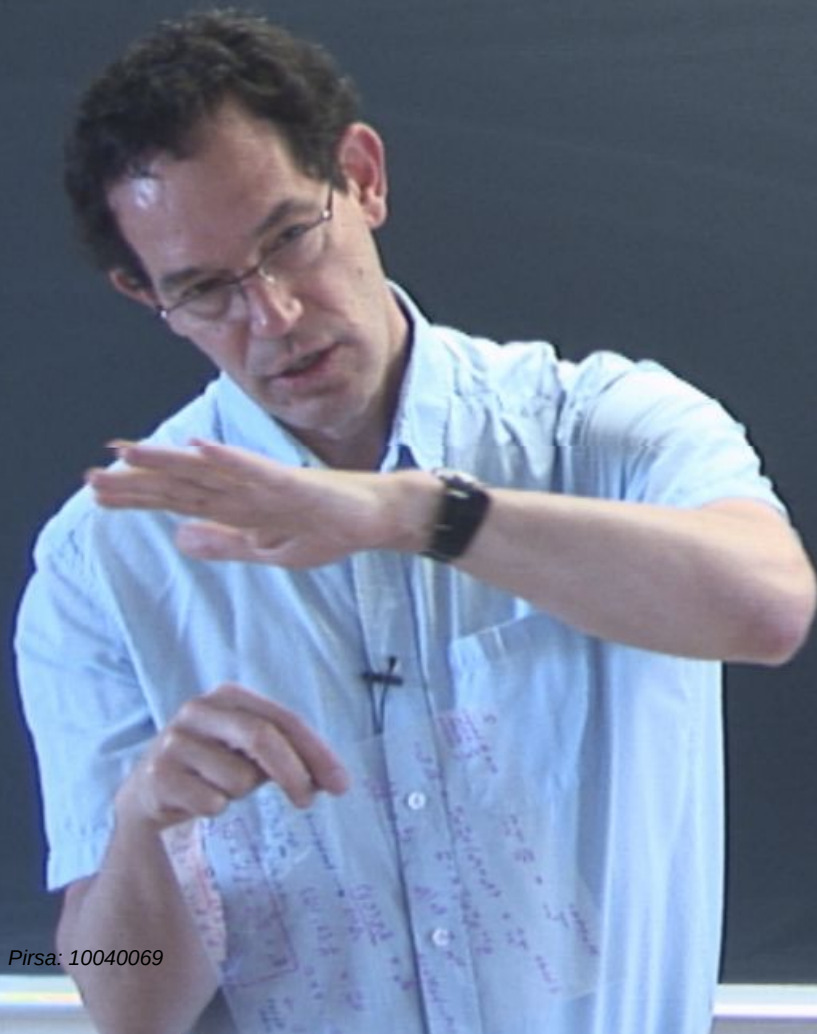
Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

$$\dot{\delta} = -3\mathcal{H}(w - c_s^2)\delta \dots$$

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Perfect fluid : special cases

① CDM



Perfect fluid : special cases

① CDM $P=0 \rightarrow \dot{\delta}+$

Perfect fluid : special cases

① CDM $P=0 \rightarrow \dot{\delta} = -\theta - \frac{\delta h}{2}$

Perfect fluid : special cases

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CDM

$$P=0$$

$$\dot{\delta} = -\theta - \frac{\delta \dot{h}}{2}$$

conformal pert.

$$h_{ij} = a^2(f_{ij}(1+z\varphi) + D_i D_j \chi)$$

$$\delta h =$$

Perfect fluid : special cases

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CDM

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conformal pert.

$$h_{ij} = a^2 \left(f_{ij} \underbrace{(1+2\varphi)}_{\text{conformal pert.}} + D_i D_j \chi \right)$$

$$\delta h = 6\varphi + \Delta_c \chi$$

Perfect fluid : special cases

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$$\dot{\theta} = -\frac{\dot{a}}{a}\theta$$

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$$N_0 = a$$

$$ds^2 = a^2[-dt^2 + h_{ij}dx^i dx^j]$$

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$$\dot{\theta} = -\frac{\dot{a}}{a} \theta$$

$$\nabla \cdot v \quad \theta = \frac{C(x)}{a(t)}$$

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② Radiation $P = \frac{1}{3} \rho$

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$$\theta = \frac{C(x)}{a(t)}$$

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(valid on $\lambda \gg \lambda_{\text{mfp}}$)

$$\dot{\delta} = -\frac{4}{3} \theta - \frac{2}{3} \delta h$$

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$$\dot{\theta} = -\frac{1}{4}\Delta_c \delta$$

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 conformal rest v.

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$$\dot{\delta} = -\theta - \frac{\delta h}{2}$$

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$$\theta = \frac{C(x)}{a(t)}$$

"ultralocal"
 no gradients.

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$$\dot{\delta} = -\frac{4}{3} \theta - \frac{2}{3} \delta h$$

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Need one more eqⁿ.

(44)

Need one more eqⁿ ($\alpha=0$)

$$(44) \quad \frac{\dot{\kappa}}{N_0} + 2H\kappa = 4\pi G(\epsilon + 3p)$$

Need one more eqⁿ. ($\alpha = 0$)

$$(44) \quad \frac{\dot{\kappa}}{N_0} + 2 + H\kappa = 4\pi G(\epsilon + 3\pi)$$

Need one more eqⁿ ($\alpha = 0$)

$$(44) \quad \frac{\dot{\kappa}}{N_0} + 2 + \kappa = 4\pi G(\epsilon + 3\pi)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

Need one more eqⁿ. ($\alpha=0$)

$$(44) \quad \frac{\dot{K}}{N_0} + 2H K = 4\pi G(\epsilon + 3P)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(P + \rho)$$

Need one more eqⁿ. ($\alpha = 0$)

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$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$\rightarrow \ddot{a} = -4\pi G a(\rho + 3p)$$

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$$\ddot{r} = -\frac{GM}{r^2}$$

Need one more eqⁿ. ($\alpha = 0$)

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$$\ddot{r} = -\frac{G}{r^2} \frac{4\pi}{3} \rho \cdot r$$

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$$\ddot{r} = -G \frac{4\pi}{3} \rho_{\text{matter}} r^3$$

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$$\ddot{r} = -G \frac{4\pi}{3} \rho \cdot r^3$$

$$\ddot{a}^2 = \frac{8\pi G}{3} \rho a^2 - k$$

$$2\ddot{a}a =$$

Need one more eqⁿ ($\alpha = 0$)

$$(44) \quad \frac{\dot{K}}{N_0} + 2 + 1K = 4\pi G(\epsilon + 3\pi)$$

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$$\ddot{r} = -G \frac{\frac{4\pi}{3}\rho \cdot r^3}{r^2} \text{ matter}$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$$

$$2\ddot{a}a = \frac{8\pi G}{3}\left(-3\frac{\dot{a}}{a}(\rho + p)\right)$$

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Need one more eqⁿ ($\alpha=0$)

$$(44) \quad \frac{\dot{K}}{N_0} + 2H K = 4\pi G(\epsilon + 3P)$$

Ex: try to derive just
by considering FRW.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(P + \rho)$$

$$\rightarrow \ddot{a} = -\frac{4\pi G}{3} a(\rho + 3P)$$



$$\ddot{r} = -G \frac{4\pi}{3} \rho \frac{r^3}{r^2} \text{ / mitte}$$

$$\ddot{a} = \frac{8\pi G}{3} a(\rho + 3P)$$

$$2\ddot{a} = \frac{8\pi G}{3} a(\rho + 3P)$$

Need one more eqⁿ ($\alpha=0$)

$$(44) \quad \frac{\dot{\kappa}}{N_0} + 2 + 1\kappa = 4\pi G(\epsilon + 3\pi)$$

Ex: try to derive just
by considering FRW.

$$\begin{aligned} E &= E_0 + \epsilon \\ P &= P_0 + \pi \end{aligned}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + P)$$

$$\rightarrow \ddot{a} = -\frac{4\pi G}{3}a(\rho + 3P)$$



$$\ddot{r} = -\frac{G}{r^2} \frac{4\pi}{3} \rho r^3$$

$$\ddot{a}^2 = \frac{8\pi G}{3} \rho a^2 - k$$

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$$(44) \quad \frac{\dot{K}}{N_0} + 2H\dot{K} = 4\pi G(\epsilon + 3p)$$

Ex: try to derive just
by considering FRW.

$$K = -\frac{\dot{h}}{2a}$$

$$\begin{aligned} E &= E_0 + \epsilon \\ p &= p_0 + \pi \end{aligned}$$

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$$\ddot{r} = -\frac{G}{r^2} \frac{4\pi}{3}\rho \cdot r^3$$

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Ex: try to derive just by considering FRW.

$$K = -\frac{\dot{h}}{2a}$$

$$\ddot{h} + \frac{\dot{a}}{a}\dot{h} = -8\pi G(1+3c_s^2)\epsilon_0 \delta a^2$$

$$\delta = \frac{\epsilon}{\epsilon_0}$$

$$\begin{aligned} E &= E_0 + \epsilon \\ p &= p_0 + \pi \end{aligned}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + p)$$

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$$\ddot{r} = -G \frac{4\pi}{3} \rho \frac{r^3}{r^2}$$

$$\ddot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$$

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Need one more eqn. ($\alpha=0$)

$$(44) \quad \frac{\dot{K}}{N_0} + 2H K = 4\pi G (\epsilon + 3p)$$

Ex: try to derive just by considering FRW.

$$K = -\frac{\dot{h}}{2a}$$

$$\Rightarrow \ddot{h} + \frac{\dot{a}}{a} \dot{h} = -8\pi G (1 + 3c_s^2) E_0 \delta a^2$$

$$\delta \equiv \frac{\epsilon}{E_0}$$

$$\begin{aligned} E &= E_0 + \epsilon \\ P &= P_0 + \pi \end{aligned}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + P)$$

$$\rightarrow \ddot{a} = -\frac{4\pi G}{3} a(\rho + 3P)$$



$$\ddot{r} = -G \frac{4\pi}{3} \frac{\rho r^3}{r^2}$$

$$\ddot{a} = \frac{8\pi G}{3} \rho a^2 - \frac{k}{a}$$

$$2\frac{\ddot{a}}{a} = \frac{8\pi G}{3} \left(-3\frac{\dot{a}}{a}(\rho + P) + 2\frac{\dot{a}}{a}\rho a^2 - \frac{k}{a^2} \right)$$

Need one more eqⁿ. ($\alpha = 0$)

$$(44) \quad \frac{\dot{K}}{N_0} + 2H K = 4\pi G(\epsilon + 3p)$$

Ex: try to derive just by considering FRW.

$$K = -\frac{\dot{h}}{2a}$$

$$\delta \equiv \frac{\epsilon}{E_0}$$

$$\Rightarrow \dot{h}'' + \frac{\dot{a}}{a} \dot{h}' = -8\pi G(1 + 3c_s^2) E_0 \delta a^2$$

$$\begin{aligned} E &= E_0 + \epsilon \\ P &= P_0 + \pi \end{aligned} \quad \text{proper time}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + P)$$

$$\rightarrow \ddot{a} = -\frac{4\pi G}{3} a(\rho + 3P)$$



$$\ddot{r} = -G \frac{4\pi}{3} \frac{\rho r^3}{r^2}$$

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$$2\ddot{a} = \frac{8\pi G}{3} \left(-3\frac{\dot{a}}{a}(\rho + P)a^2 + 2\frac{\dot{a}}{a}\rho a^2 \right) - (P + 3P)$$

CDM only, with $\Omega = 0$, flat universe

background $H^2 = ?$

CDM only, with $\theta = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3}$$

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3} : \quad \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3} : \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

$$\rightarrow \dot{a}^2 \propto a$$

$$\dot{a} \propto a^{1/2}$$

$$\rightarrow a \propto t^2$$

$\uparrow$
conformal time

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3} : \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

$$dt_p = a dt$$

$$t_p = \int a(t) dt$$

=

$$\rightarrow \dot{a}^2 \propto a$$

$$\dot{a} \propto a^{1/2}$$

$$\rightarrow a \propto t^2$$

$\uparrow$
conformal time

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3}; \quad \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

$$dt_p = a dt$$

$$t_p = \int a(t) dt$$

$$\propto t^3$$

$$\Rightarrow a \propto t^2 \propto t_p^{2/3}$$

$$\rightarrow \ddot{a}^2 \propto a$$

$$\ddot{a} \propto a^{1/2}$$

$$\rightarrow a \propto t^2$$

$\uparrow$
conformal time

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3} : \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

$$dt_p = a dt$$

$$t_p = \int a(t) dt$$

$$\propto t^3$$

$$\Rightarrow a \propto t^2 \propto t_p^{2/3}$$

$$\rightarrow \dot{a}^2 \propto a$$

$$\dot{a} \propto a^{1/2}$$

$$\rightarrow a \propto t^2$$

conformal time

CDM only, with $\Omega = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$P = 0$

$$E_0 \propto \frac{1}{a^3} : \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

$$dt_p = a dt$$

$$t_p = \int a dt$$

$$\rightarrow \dot{a}^2 \propto a$$

$$\dot{a} \propto a^{\frac{1}{2}}$$

$$\rightarrow a \propto \underline{\underline{t^2}}$$

$\uparrow$
conformal time

$$\Rightarrow \frac{\dot{a}}{a} = \frac{2}{t}$$

$$\Rightarrow \frac{8\pi G}{3} E_0 a^2 = \frac{4}{t^2}$$

CDM only, with $\theta = 0$, flat universe

background

$$H^2 = \frac{8\pi G}{3} E_0$$

$$P_0 = 0$$

$$E_0 \propto \frac{1}{a^3} : \frac{1}{a^2} \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_0 \propto \frac{1}{a^3}$$

$\uparrow$
 N_0^2

$$dt_p = a dt$$

$$t_p = \int a(t) dt$$

$$\propto t^3$$

$$\Rightarrow a \propto t^2 \propto t_p^{2/3}$$

$$\rightarrow \dot{a}^2 \propto a$$

$$\Rightarrow \frac{\dot{a}}{a} = \frac{2}{t}$$

$$\dot{a} \propto a^{1/2}$$

$$\rightarrow a \propto t^2$$

$\uparrow$
conformal time

$$\Rightarrow \frac{8\pi G}{3} E_0 a^2 = \frac{4}{t^2}$$

$$\theta=0 \Rightarrow \dot{h} = -2\dot{a}$$

$$\Rightarrow \delta \ddot{h} + \frac{\dot{a}}{a} \delta \dot{h} = -8\pi G (1+3c_s^2) E_0 \delta a^2$$

(conformal time)

$$\propto \frac{1}{a^3}$$

al time

$$\frac{a}{(p+3p)}$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +8\pi G$$

$$\Rightarrow \delta \ddot{h} + \frac{\dot{a}}{a} \delta \dot{h} = -8\pi G (1 + 3c_s^2) E_0 \delta a^2$$

(conformal time)

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G (1+3c_s^2) E_0 a^2 \delta$$

CDM
=

$$\Rightarrow \delta \ddot{h} + \frac{\dot{a}}{a} \delta \dot{h} = -8\pi G (1+3c_s^2) E_0 \delta a^2$$

(conformal time)

$$\propto \frac{1}{a^3}$$

al time

$$\frac{a}{(p+3p)}$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G (1+3c_s^2) E_0 a^2 \delta$$

$$\text{for CDM: } \underline{\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta}$$

$$\Rightarrow \underline{\delta \ddot{h} + \frac{\dot{a}}{a} \delta \dot{h} = -8\pi G (1+3c_s^2) E_0 \delta a^2}$$

(conformal time)

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta \quad \delta \propto t^n \quad n=2 \text{ or } -3.$

$$4\pi G(1+3c_s^2) E_0 \delta a^2$$

$$\frac{a}{(p+3p)}$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta \quad \delta \propto t^n \quad n=2 \text{ or } -3.$

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode decaying mode

$$\frac{a}{(p+3p)}$$

$$\theta = 0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta$ $\delta \propto t^n$ $n = 2 \text{ or } -3$

self-gravity

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode decaying mode

estimate growth factor during matter domination

$$\frac{a}{(p+3p)}$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta$ $\delta \propto t^n$ $n=2 \text{ or } -3$

self-gravity.

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode

decaying mode

estimate growth factor during matter domination

$$\frac{a_{eq}}{a_0}$$

$$\frac{a}{(p+3p)}$$

$$\theta=0 \Rightarrow \dot{\delta} = -2\dot{a}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta$ $\delta \propto t^n$ $n=2 \text{ or } -3$

self-gravity.

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode

decaying mode

estimate growth factor during matter domination

$$\frac{a_0}{a_{eq}} = 2.5 \times 10^4 \Omega_M h^2$$

$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$

(p+3p)

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta$ $\delta \propto t^n$ $n=2 \text{ or } -3$

$$\propto \frac{1}{a^3}$$

self-gravity.

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode decaying mode

estimate growth factor during matter domination

$$\frac{a_0}{a_{eq}} = 2.5 \times 10^4 \Omega_M h^2$$

~ few thousand.

$$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$$

(p+3p)

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

$$\text{CDM: } \ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta \quad \delta \propto t^n \quad n=2 \text{ or } -3.$$

$$\propto \frac{1}{a^3}$$

self-gravity.

$$\rightarrow \delta(t, x) = \underbrace{A(x) t^2}_{\text{growing mode}} + \underbrace{B(x) t^{-3}}_{\text{decaying mode}}$$

estimate growth factor during matter domination

$$a_{\text{now}} = 2.5 \times 10^4 \Omega_M h^2$$

(recall $t^2 \propto a$) $\sim$ few thousand.

$$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$$

(p13p)

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta$ $\delta \propto t^n$ $n=2 \text{ or } -3$

self-gravity.

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode decaying mode

estimate growth factor during matter domination

$$\frac{a_0}{a_{eq}} = 2.5 \times 10^4 \Omega_M h^2$$

(recall $t^2 \propto a$) $\sim$ few thousand.

$$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$$

$\frac{a}{(p+3p)}$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}$$

$$\rightarrow \ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = +4\pi G(1+3c_s^2) E_0 a^2 \delta$$

for CDM: $\ddot{\delta} + \frac{2}{t} \dot{\delta} = \frac{6}{t^2} \delta$ $\delta \propto t^n$ $n=2 \text{ or } -3$

self-gravity.

$$\rightarrow \delta(t, x) = A(x) t^2 + B(x) t^{-3}$$

growing mode

decaying mode

estimate growth factor during matter domination

$$\frac{a_0}{a_{eq}} = 2.5 \times 10^4 \Omega_M h^2$$

(recall $t \propto a$)

a few thousand.

$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$

CDM in radiation era.
+ radiation



CDM in radiation era.
 radiation



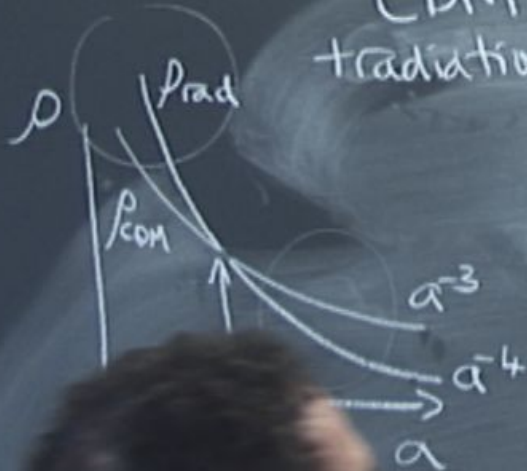
cross here,

$$\text{where } \frac{a_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{cdm,0}}$$

CDM in radiation era.
radiation

↓ worry about v_c^2

Fourier transform $\delta(t, x) = \sum_{\mathbf{k}} \delta_{\mathbf{k}}$



$$\frac{\rho_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{CDM,0}}$$

CDM in radiation era.
radiation

↓ worry about v_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

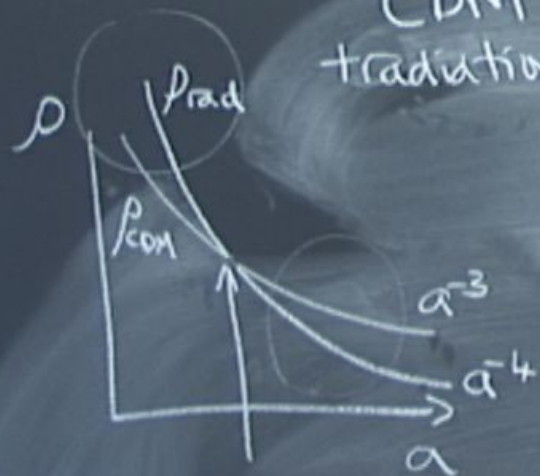


cross here,
 where $\frac{a_{\text{eq}}}{a} = 1$

CDM in radiation era.
radiation

↓ worry about ∇_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$



cross here,

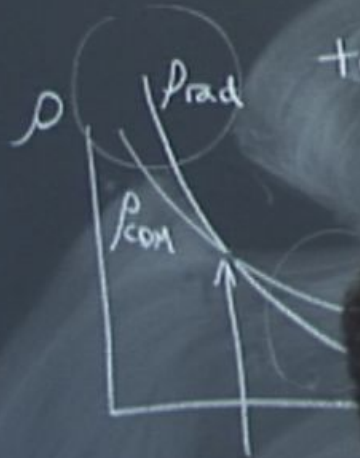
where $\frac{a_{\text{eq}}}{a_0} = \frac{\rho_{\text{rad},0}}{\rho_{\text{CDM},0}}$

CDM in radiation era.
radiation

↓ worry about v_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" perturbation



cross here
 where $\rho_{\text{rad}} = \rho_{\text{m}}$

CDM in radiation era.

↓ worry about v_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert's



cross over,

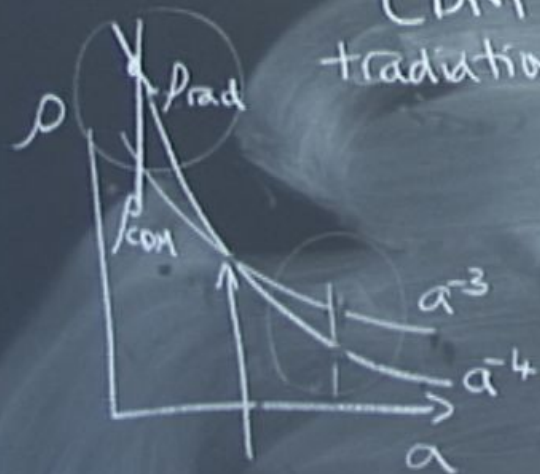
$$\frac{a_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{CDM,0}}$$

CDM in radiation era.
radiation

↓ worry about V_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert's



cross here,

where $\frac{a_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{CDM,0}}$

CDM in radiation era.
radiation

↓ worry about ∇_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert's

$\dot{\delta}_c =$



cross here,

where $\frac{a_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{CDM,0}}$

CDM in radiation era.
 radiation

↓ worry about ∇_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert'n's



cross here,
 where $\frac{a_{\text{eq}}}{a_0} = \frac{\rho_{\text{rad},0}}{\rho_{\text{CDM},0}}$

$$\dot{\delta}_c = -\frac{\delta h}{2} - \theta$$

$$\dot{\theta} = -\frac{\dot{a}}{a} \theta$$

CDM in radiation era.



cross here,
where $\frac{a_{\text{eq}}}{a_0} = \frac{\rho_{\text{rad},0}}{\rho_{\text{CDM},0}}$

Worry about ∇_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert'n's

$$\dot{\delta}_c = -\frac{\dot{h}}{2} - \dot{\theta}_c$$

$$\dot{\theta} = -\frac{\dot{a}}{a} \theta_c$$

$$\dot{\delta}_r = -\frac{2}{3} \dot{h} - \frac{4}{3} \dot{\theta}_r$$

$$\dot{\theta} = -\frac{1}{4} \dot{\theta}_r$$

$$\Rightarrow \dot{\delta}_c - \frac{3}{4} \dot{\delta}_r = 0 \quad \text{if } \dot{\theta} \text{ can be ignored.}$$

$$\Rightarrow \boxed{\delta_c = \frac{3}{4} \delta_r} + \text{const}$$

CDM in radiation era.
 radiation

↓ worry about ∇_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert's

$$\dot{\delta}_c = -\frac{\dot{h}}{2} - \cancel{\dot{\theta}_c}$$

$$\dot{\theta} = -\frac{\dot{a}}{a} \theta_c$$

$$\dot{\delta}_r = -\frac{2}{3} \dot{h} - \frac{4}{3} \dot{\theta}_r$$

$$\dot{\theta} = -\frac{1}{4} \cancel{\dot{\theta}_r}$$

$$\dot{\delta}_c - \frac{3}{4} \dot{\delta}_r = 0 \quad \text{if } \dot{\theta} \text{ can be ignored.}$$

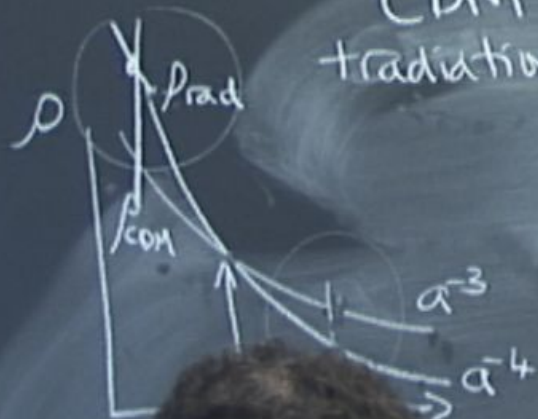
$$\Rightarrow \boxed{\delta_c = \frac{3}{4} \delta_r} + \text{const}$$

Adiabatic pert's
 have

$$\boxed{\delta_c = \frac{3}{4} \delta_r}$$

$$\rho_c \propto a^{-3} \Rightarrow \frac{\dot{\rho}_c}{\rho_c} = -3 \frac{\dot{a}}{a}$$

$$\rho_r \propto a^{-4} \Rightarrow \frac{\dot{\rho}_r}{\rho_r} = -4 \frac{\dot{a}}{a}$$



CDM in radiation era.
radiation



cross here,
 where $\frac{a_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{CDM,0}}$

↓ worry about ∇_c^2

Fourier transform $\delta(t, \underline{x}) = \sum_{\underline{k}} \delta_{\underline{k}}(t) e^{i \underline{k} \cdot \underline{x}}$

"Adiabatic" pert's

$$\dot{\delta}_c = -\frac{\dot{h}}{2} - \cancel{\theta_c}$$

$$\dot{\theta} = -\frac{\dot{a}}{a} \theta_c$$

$$\dot{\delta}_r = -\frac{2}{3} \dot{h} - \frac{4}{3} \theta_r$$

$$\dot{\theta} = -\frac{1}{4} \cancel{\theta_r}$$

Adiabatic pert's
 have

$$\boxed{\delta_c = \frac{3}{4} \delta_r}$$

$\Rightarrow$

$$\dot{\delta}_c - \frac{3}{4} \dot{\delta}_r = 0 \quad \text{if } \theta \dot{s} \text{ can be ignored.}$$

$$\begin{aligned} \rho_c \propto a^{-3} & \quad \delta_c = -3 \frac{\delta a}{a} \\ \rho_r \propto a^{-4} & \quad \delta_r = -4 \frac{\delta a}{a} \end{aligned}$$

$$\Rightarrow \boxed{\delta_c = \frac{3}{4} \delta_r} + \text{const}$$

CDM in radiation era.
radiation

↓ worry about ∇_c^2

Fourier transform $\delta(t, x) = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i \vec{k} \cdot \vec{x}}$

"Adiabatic" pert's



cross here,
 where $\frac{a_{eq}}{a_0} = \frac{\rho_{rad,0}}{\rho_{cdm,0}}$

Adiabatic pert's
 have

$$\boxed{\delta_c = \frac{3}{4} \delta_r}$$

$$\begin{aligned} \rho_c \propto a^{-3} & \quad \delta_c = -3 \frac{\delta a}{a} \\ \rho_r \propto a^{-4} & \quad \delta_r = -4 \frac{\delta a}{a} \end{aligned}$$

$$\dot{\delta}_c = -\frac{\dot{a}}{a} \delta_c - \dot{\theta}_c$$

$$\dot{\theta} = -\frac{\dot{a}}{a} \theta_c$$

$$\dot{\delta}_r = -\frac{2}{3} \dot{a} \delta_r - \frac{4}{3} \dot{\theta}_r$$

$$\dot{\theta} = -\frac{1}{4} \dot{\theta}_r$$

$$\dot{\delta}_c - \frac{3}{4} \dot{\delta}_r = 0 \quad \text{if } \dot{\theta} \text{ can be ignored.}$$

$$\Rightarrow \boxed{\delta_c = \frac{3}{4} \delta_r} + \text{const}$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_{or} a^2 \delta_r + 4\pi G E_{oc} a^2 \delta_c$$

can
read.

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_{or} a^2 \delta_r + 4\pi G E_{oc} a^2 \delta_c$$

in radⁿ. era, $E_{oc} \ll E_{or}$

can
read.

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_{or} a^2 \delta_r + 4\pi G E_{oc} a^2 \delta_c$$

in radⁿ. era, $E_{oc} \ll E_{or}$

negligible

$$\text{rad}^n \text{ era } \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} E_o a^2 \propto a^{-2}$$

$$\Rightarrow a \propto t$$

can

read.

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\ddot{a}}{a} \delta_c = +4\pi G(1+3c_s^2) E_{or} a^2 \delta_r + 4\pi G E_{oc} a^2 \delta_c \quad (*)$$

in radⁿ. era, $E_{oc} \ll E_{or}$

$$\text{rad}^n \text{ era} \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} E_o a^2 \propto a^{-2}$$

$\Rightarrow a \propto t$ in radⁿ. era.

$$\Rightarrow \frac{8\pi G}{3} E_o a^2 = \frac{1}{t^2}$$

$$(*) \rightarrow \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = \underbrace{8\pi G E_{or} a^2}_{\frac{3}{t^2}} \left(\frac{4}{3} \delta_c\right)$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_{or} a^2 \delta_r + 4\pi G E_{oc} a^2 \delta_c \quad (*)$$

in radⁿ. era, $E_{oc} \ll E_{or}$

negligible

$$\text{rad}^n \text{ era } \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} E_o a^2 \propto a^{-2}$$

$\Rightarrow a \propto t$ in radⁿ. era.

$$\Rightarrow \frac{8\pi G}{3} E_o a^2 = \frac{1}{t^2}$$

$$\ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = \frac{4}{t^2} \delta_c$$

$$\Rightarrow \boxed{\delta_c = A(x) t^2 + B(x) t^{-2}}$$

g-mode

$$(*) \rightarrow \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = \underbrace{8\pi G E_{or} a^2}_{\frac{3}{t^2}} \left(\frac{4}{3} \delta_c\right)$$

can
read.

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_{or} a^2 \delta_r + 4\pi G E_{oc} a^2 \delta_c \quad (*)$$

in radⁿ. era, $E_{oc} \ll E_{or}$

negligible

$$\text{rad}^n \text{ era } \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} E_o a^2 \propto a^{-2}$$

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$$\ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = \frac{4}{t^2} \delta_c$$

$$\Rightarrow \boxed{\delta_c = A(x) t^2 + B(x) t^{-2}}$$

g-mode

valid for long wavelengths

$\lambda \gg$ Hubble radius,

$$(*) \rightarrow \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = \underbrace{\frac{8\pi G E_{or} a^2}{\frac{3}{t^2}}}_{\frac{4}{t^2}} \left(\frac{4}{3} \delta_c\right)$$

Short wavelengths :



Short wavelengths :



δy oscillates

Short wavelengths :



δx oscillates (neglect δh term)

$$\begin{aligned}\ddot{\delta x} &= \frac{1}{3} \Delta_c \delta x \\ &= -\frac{k^2}{3} \delta x\end{aligned}$$

Short wavelengths:



δx oscillates (neglect $\delta \dot{h}$ term)
→ averages to zero

$$\ddot{\delta x} = \frac{1}{3} \Delta_c \delta x$$
$$= -\frac{k^2}{3} \delta x$$

period $\sim \frac{1}{k}$

Short wavelengths:

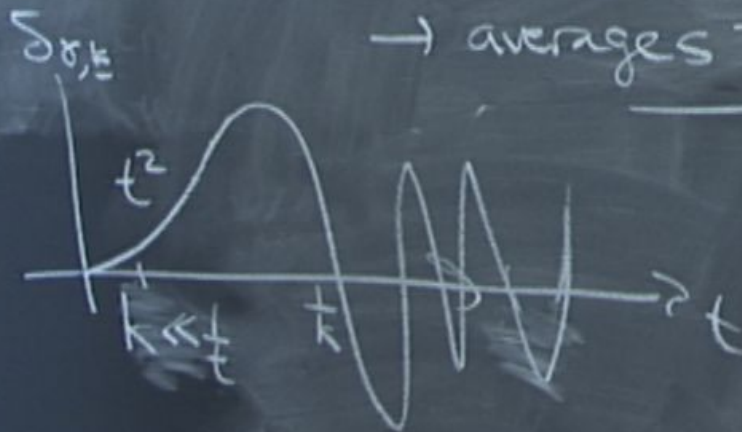


δ_x oscillates (neglect $\delta \dot{h}$ term)
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$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_0 a^2 \delta_r + 4\pi G \rho \delta_c \quad (*)$$

in radⁿ. era, $E_{0c} \ll E_{0r}$

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$$\rightarrow \boxed{\delta_c = A t^{-2}}$$

$$\theta=0 \Rightarrow \dot{\delta h} = -2\dot{\delta}_c$$

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$$\ddot{\delta}_c + \dots \delta_c$$

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g-mode

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Short wavelengths:

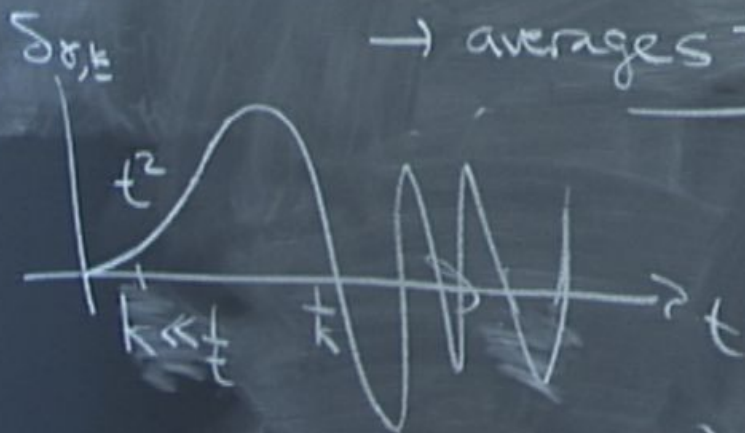


δ_x oscillates (neglect $\delta \dot{h}$ term)
 $\rightarrow$ averages to zero

$$\ddot{\delta}_x = \frac{1}{3} \Delta_c \delta_x$$

$$= -\frac{k^2}{3} \delta_x$$

period $\sim \frac{1}{k}$

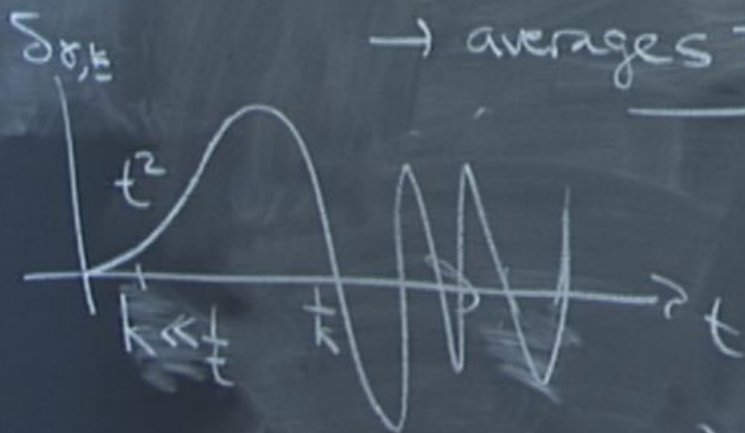


$$\rightarrow \ddot{\delta}_c + \frac{1}{t} \delta_c = 0$$

Short wavelengths:



δ_x oscillates (neglect δh term)
 $\rightarrow$ averages to zero



$$\ddot{\delta}_x = \frac{1}{3} \Delta_c \delta_x$$

$$= -\frac{k^2}{\omega} \delta_x$$

period $\sim \frac{1}{k}$

$$\rightarrow \ddot{\delta}_c + \frac{1}{t} \delta_c = 0$$

$$\Rightarrow \delta_c$$

Short wavelengths:

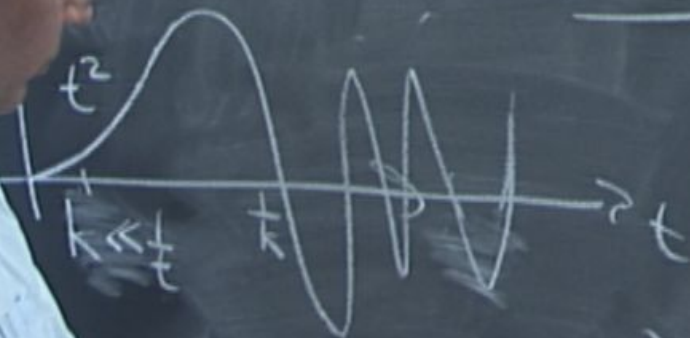


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$$= -\frac{k^2}{3} \delta_x$$

period $\sim \frac{1}{k}$



$\rightarrow \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = 0$ "Mészáros" effect

$$\text{Ex} \Rightarrow \delta_c = A(x) \ln t + B(x)$$

Short wavelengths:

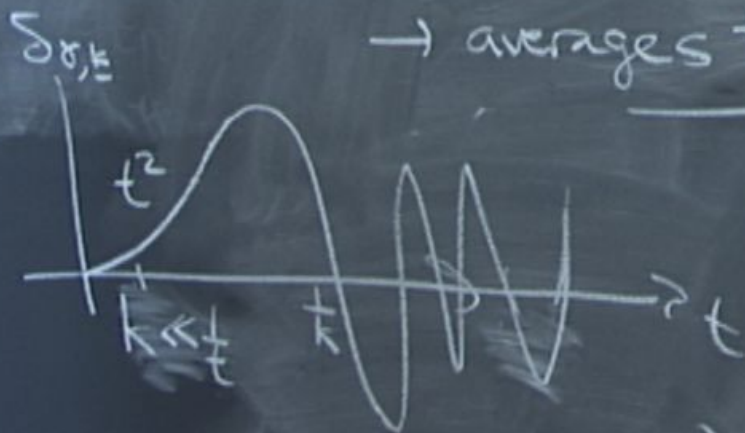


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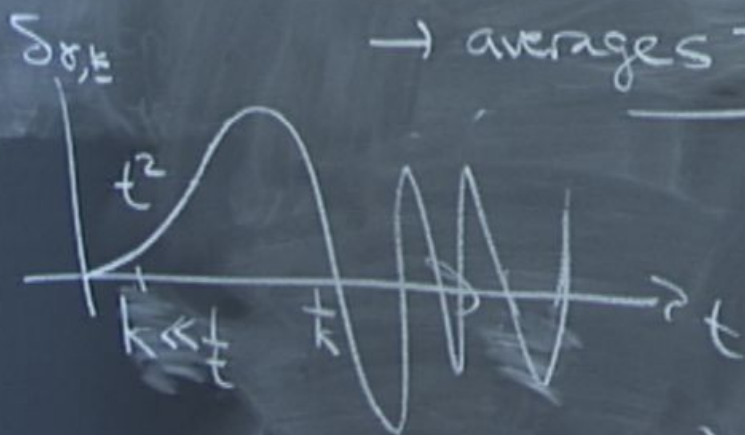
Short wavelengths:



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period $\sim \frac{1}{k}$



$\rightarrow \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = 0$ "Meszaros" effect

$$I_x \Rightarrow \delta_c = A(x) \ln t + B(x)$$

$$\theta=0 \Rightarrow \delta \dot{h} = -2\dot{\delta}_c$$

$$\rightarrow \ddot{\delta}_c + \frac{\dot{a}}{a} \dot{\delta}_c = +4\pi G(1+3c_s^2) E_{0r} a^2 \delta_r \quad (*)$$

$$+4\pi G E_{0c} a^2 \delta_c \quad \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c$$

in radⁿ. era, $E_{0c} \ll E_{0r}$

$$\text{rad}^n \text{ era} \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} E_0 a^2 \propto a^{-2}$$

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$$(*) \rightarrow \ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = \underbrace{8\pi G E_{0r} a^2}_{\frac{3}{t^2}} \left(\frac{4}{3} \delta_c\right)$$

$$\theta=0 \Rightarrow \delta h = -2\delta_c$$

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$$+ 4\pi G E_{oc} a^2 \delta_c$$

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g-mode

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radⁿ. era $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G E_o a^2}{3} \propto a^{-2}$

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g-mode

valid for long wavelengths

$\lambda \gg$ Hubble radius,

Short wavelengths :



$\delta x, t$

δx oscillates (neglect δh term)
 $\rightarrow$ averages to zero

$$\ddot{\delta x} = \frac{1}{3} \Delta_c \delta x$$

$$= -\frac{k^2}{3} \delta x$$

period $\sim \frac{1}{k}$

$$\ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = 0 \quad \text{"Meszaros" effect}$$

$$\Rightarrow \delta_c = A(x) \ln t + B(x)$$

Short wavelengths:



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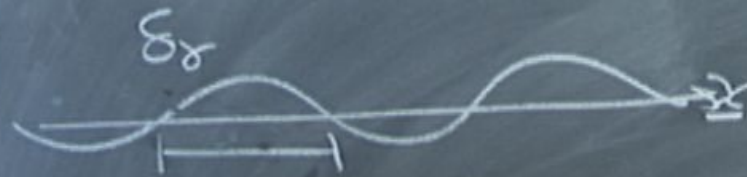


$$\ddot{\delta}_c + \frac{1}{t} \dot{\delta}_c = 0$$

"Meszaros" effect

$$\delta_c = A(x) \ln t + B(x)$$

Short wavelengths:



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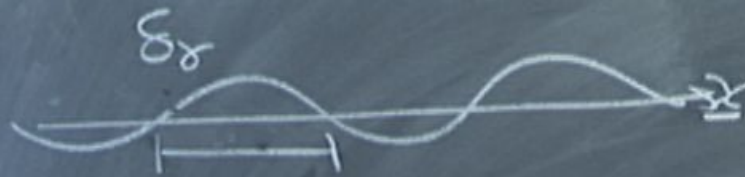


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rad here

Short wavelengths:

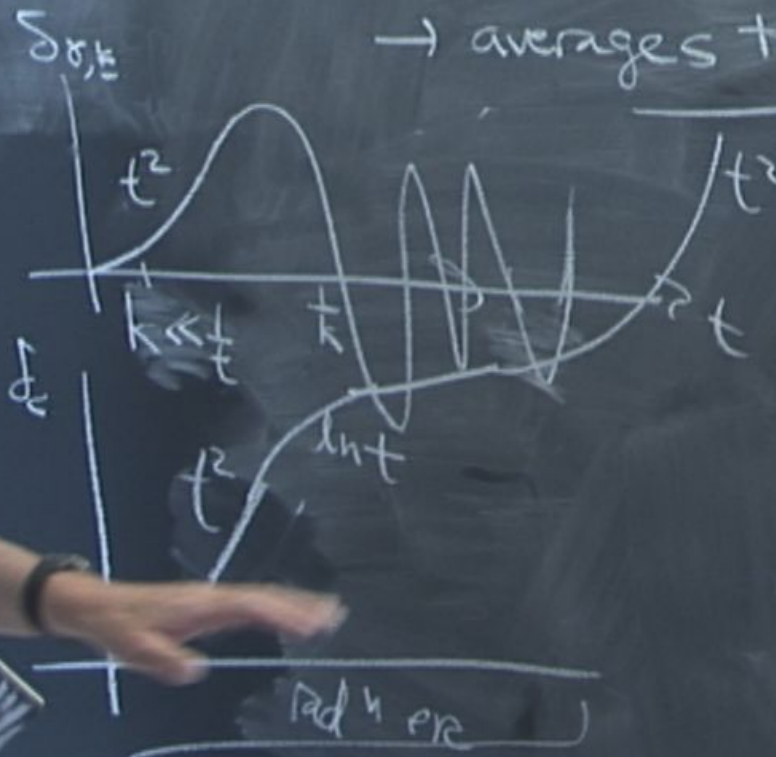


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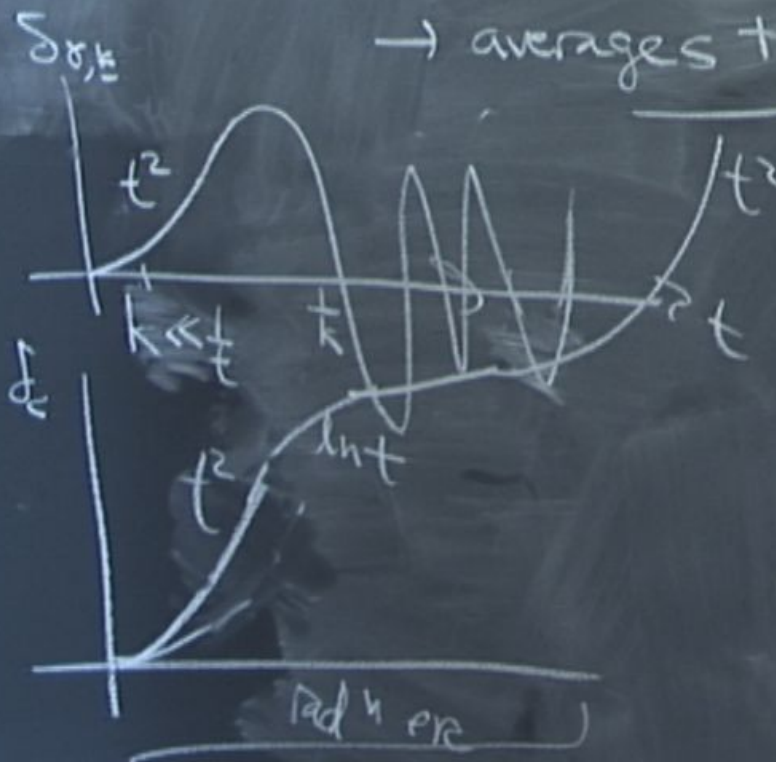


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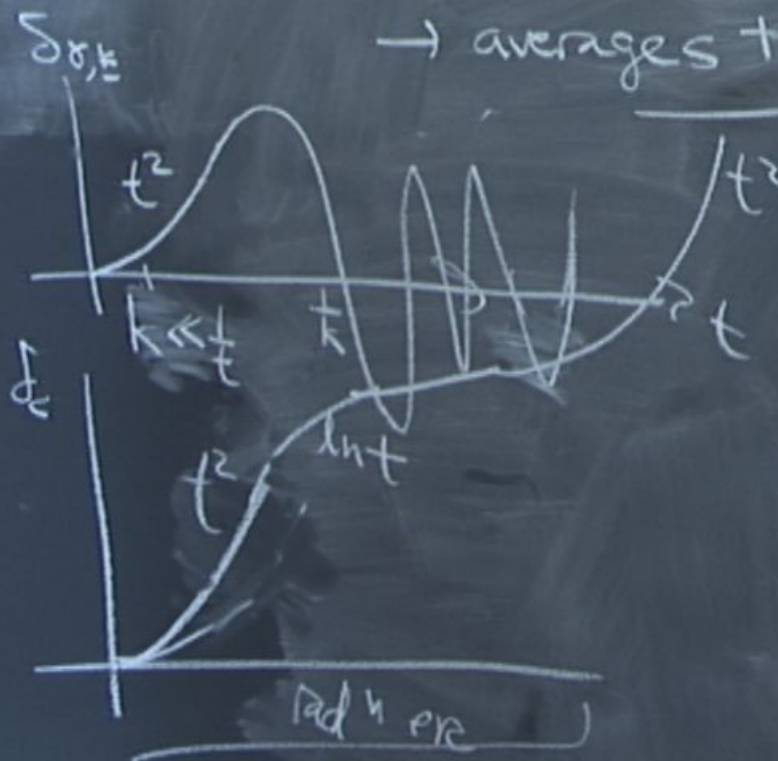


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Power spectrum of Perturbations

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$\delta(x)$

Power spectrum of Perturbations

$$\delta(\underline{x}) = \sum_{\underline{k}} \delta_{\underline{k}} e^{i\underline{k} \cdot \underline{x}}$$

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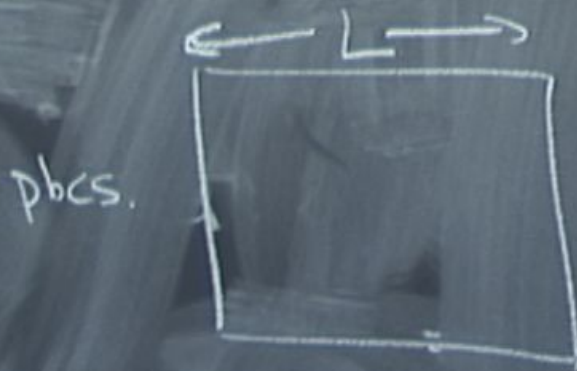


Volume L^3

$$\text{pbcs.} \rightarrow \underline{k} = \frac{2\pi}{L} (n_1, n_2, n_3)$$

Power spectrum of Perturbations

$$\delta(\underline{x}) = \sum_{\underline{k}} \delta_{\underline{k}} e^{i \underline{k} \cdot \underline{x}}$$



Volume L^3

$$\text{pbcs.} \rightarrow \underline{k} = \frac{2\pi}{L} (n_1, n_2, n_3)$$

Statistical ensemble of $\delta_{\underline{k}}$'s

Power spectrum of Perturbations

$$\delta(\underline{x}) = \sum_{\underline{k}} \delta_{\underline{k}} e^{i \underline{k} \cdot \underline{x}}$$

$\leftarrow L$

volume L^3

p.b.c.s.

$$\text{p.b.c.s.} \rightarrow \underline{k} = \frac{2\pi}{L} (n_1, n_2, n_3)$$

Sta

$$\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle$$

Power spectrum of Perturbations

$$\delta(\underline{x}) = \sum_{\underline{k}} \delta_{\underline{k}} e^{i \underline{k} \cdot \underline{x}}$$



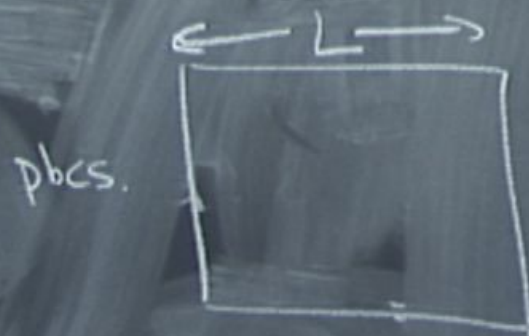
Volume L^3

$$\text{pbcs.} \rightarrow \underline{k} = \frac{2\pi}{L} (n_1, n_2, n_3)$$

Statistical ensemble of $\delta_{\underline{k}}$'s : $\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle = \delta_{\underline{k} + \underline{k}', 0}$

Power spectrum of Perturbations

$$\delta(x) = \sum_{\underline{k}} \delta_{\underline{k}} e^{i\underline{k} \cdot \underline{x}}$$



Volume L^3

p.b.c.s. $\rightarrow \underline{k} = \frac{2\pi}{L} (n_1, n_2, n_3)$ translation invariance

Statistical ensemble of $\delta_{\underline{k}}$'s : $\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle = \delta_{\underline{k} + \underline{k}', 0}$

Power spectrum of Perturbations

$$\delta(x) = \sum_{\underline{k}} \delta_{\underline{k}} e^{i\underline{k} \cdot \underline{x}}$$



Volume L^3

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Statistical ensemble of $\delta_{\underline{k}}$'s : $\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle = \delta_{\underline{k}+\underline{k}',0} P_{\underline{k}}$

Power spectrum of Perturbations

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Volume L^3

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Power spectrum of Perturbations

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Statistical ensemble of $\delta_{\underline{k}}$'s : $\langle \delta_{\underline{k}} \delta_{\underline{k}'} \rangle = \delta_{\underline{k} + \underline{k}', 0} P(|\underline{k}|)$

$\uparrow$ homogeneity $\uparrow$ isotropy

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now that

$$\langle \delta(x) \delta(y) \rangle = \sum_{\underline{k}} e^{i\underline{k} \cdot (\underline{x} - \underline{y})} P(|\underline{k}|)$$

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x: show that

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$$\sum_{\underline{k}} \approx V \int \frac{d^3 \underline{k}}{(2\pi)^3}$$

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Ex: show that

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in
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