Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 11

Date: Apr 19, 2010 09:30 AM

URL: http://pirsa.org/10040068

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

Getting rid of N_i

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad t = t'$$

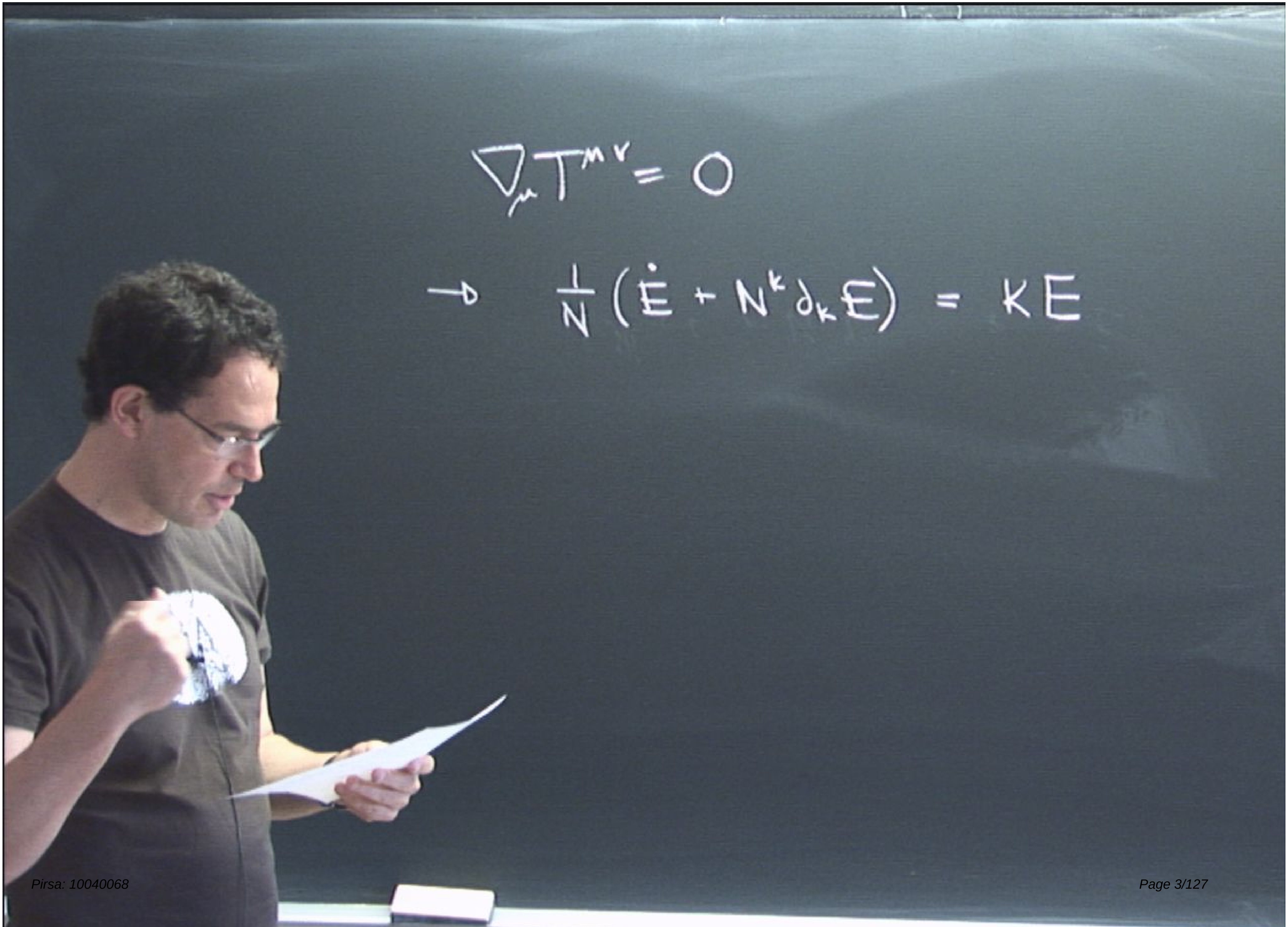
$$x^i = x'^i + \int^{t'} dt' \, N^i(t', x')$$

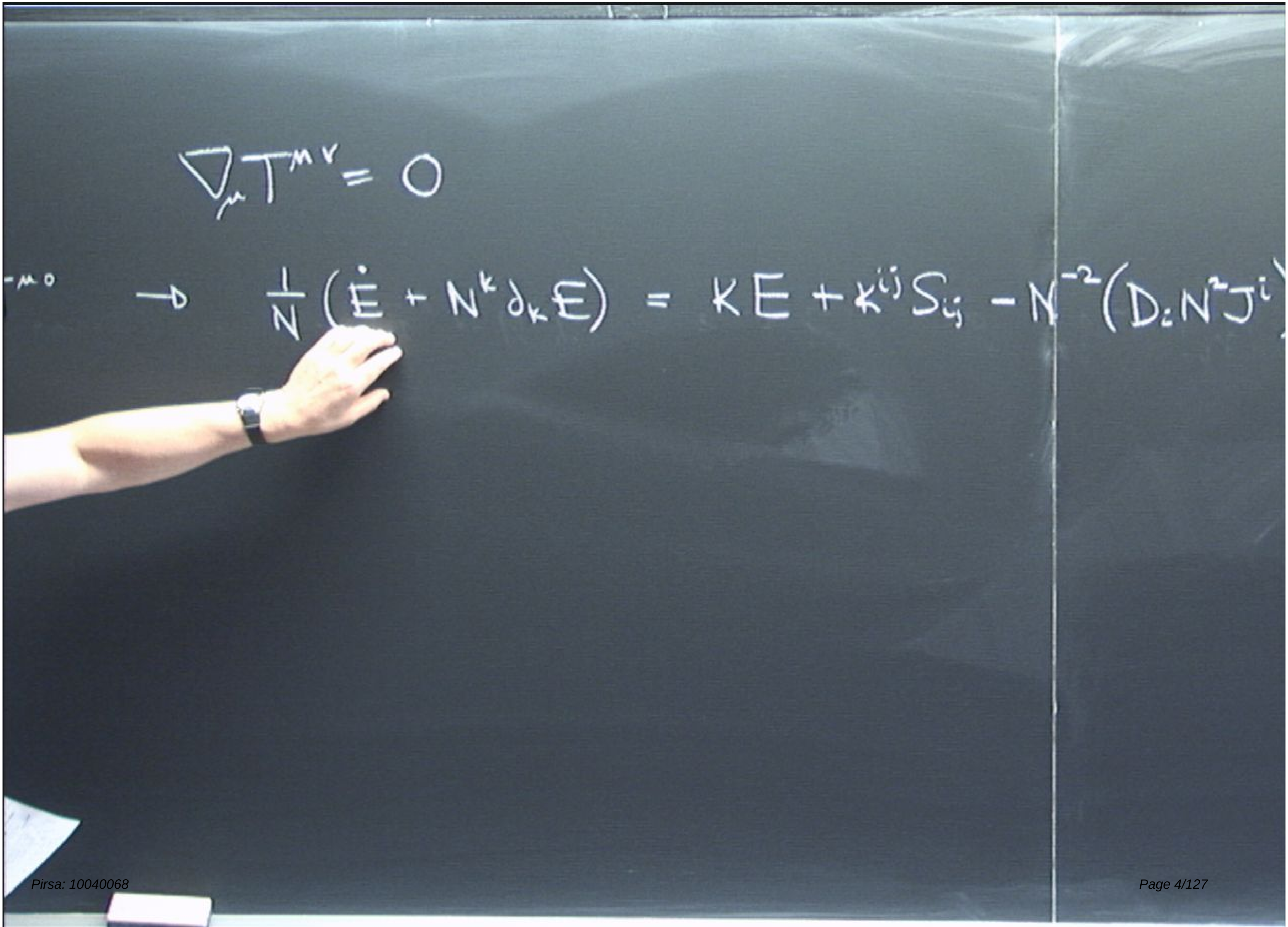
$$\text{Ex:} \quad g'_{00}(x') = -N^2$$

$$g'_{0i}(x') = 0$$

$$g'_{ij}(x') = \frac{\partial x^l}{\partial x'^i} \frac{\partial x^m}{\partial x'^j} h_{lm}$$

repar of Σ


$\nabla_\mu T^{\mu\nu} = 0$
$\to \frac{1}{N}(\dot{E} + N^k \partial_k E) = K E$


$\nabla_\mu T^{\mu\nu} = 0$
$-\mu 0$
$\to \frac{1}{N}\left(\dot{E} + N^k \partial_k E\right) = KE + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$-\mu 0 \quad \rightarrow \quad \frac{1}{N}\left(\dot{E} + N^k \partial_k E\right) = K E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu T^{\mu 0} \longrightarrow \frac{1}{N}\left(\dot{E} + N^k \partial_k E\right) = K E + K^{ij} S_{ij} - N$$

$$\nabla_\mu T^{\mu i} \longrightarrow \dot{J}_i + N^k J_k + N^k_{,i} J_k = N K J_i - ($$

$= 0$

Bardeen

$$\dot{E} + N^k \partial_k E) = K E + K^{ij} S_{ij} - N^{-2} (D_i N^2 J^i) \qquad (11)$$

$$+ N^k J_k + N^k_{,i} J_k = N K J_i - (E \delta_i^k + S_i^k) N_{,k} - N D_k S_i^k \qquad (12)$$

Getting rid of N_i

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad t = t'$$

$$x^i = x'^i + \int^{t'} dt' \, N^i(t', x')$$

Ex: $g'_{00}(x') = -N^2$

$g'_{0i}(x') = 0$

$g'_{ij}(x') = \dfrac{\partial x^l}{\partial x'^i} \dfrac{\partial x^m}{\partial x'^j} h_{lm}$ repar of Σ

Getting rid of N_i:

$$g'_{\mu\nu}(x') = \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\, g$$

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad t = t'$$

$$x^i = x'^i + \int^{t'} dt'\, N^i(t', x')$$

Ex:

$$g'_{00}(x') = -N^2$$

$$g'_{0i}(x') = 0$$

repar of Σ

$$g'_{ij}(x') = \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^j}\, h_{lm}$$

Getting rid of N_i:

$$g'_{\mu\nu}(x') = \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\, g$$

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad t = t'$$

$$x^i = x'^i + \int^{t'} dt'\, N^i(t';x')$$

Ex: $g'_{00}(x') = -N^2$

$$g'_{0i}(x') = 0$$

repar of Σ

$$g'_{ij}(x') = \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^j}\, h_{lm}$$

Getting rid of N_i

$$g'_{\mu\nu}(x') = \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\, g$$

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad t = t'$$

$$x^i = x'^i + \int^{t'} dt'\, N^i(t', x') + f^i$$

Ex: $g'_{00}(x') = -N^2$

$g'_{0i}(x') = 0$

$g'_{ij}(x') = \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^j} h_{lm}$ ← repar of Σ

Getting rid of N_i

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}\, g_{\alpha\beta}(x)$$

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2+N^iN_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad t = t'$$

$$x^i = x'^i + \int^{t'} dt'\, N^i(t', x')$$

Ex: $g'_{00}(x') = -N^2$

$g'_{0i}(x') = 0$

repar of Σ

$g'_{ij}(x') = \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^j}\, h_{lm}$

Getting rid of N_i

$$g'_{\mu\nu}(x') = \frac{\partial x^\sigma}{\partial x'^\nu}\frac{\partial x^\rho}{\partial x'^\mu}\, g_{\sigma\rho}(x)$$

$$g_{\mu\nu}(x) = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix} \quad ; \quad \text{set} \quad \begin{aligned} t &= t' \\ x^i &= x'^i + \int^{t'} dt'\, N^i(t';x') \end{aligned}$$

Ex: $g'_{00}(x') = -N^2$

$g'_{0i}(x') = 0$

repar of Σ

$$g'_{ij}(x') = \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^j}\, h_{lm}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\rightarrow \quad \frac{1}{N}\big(\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}\big) = K E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

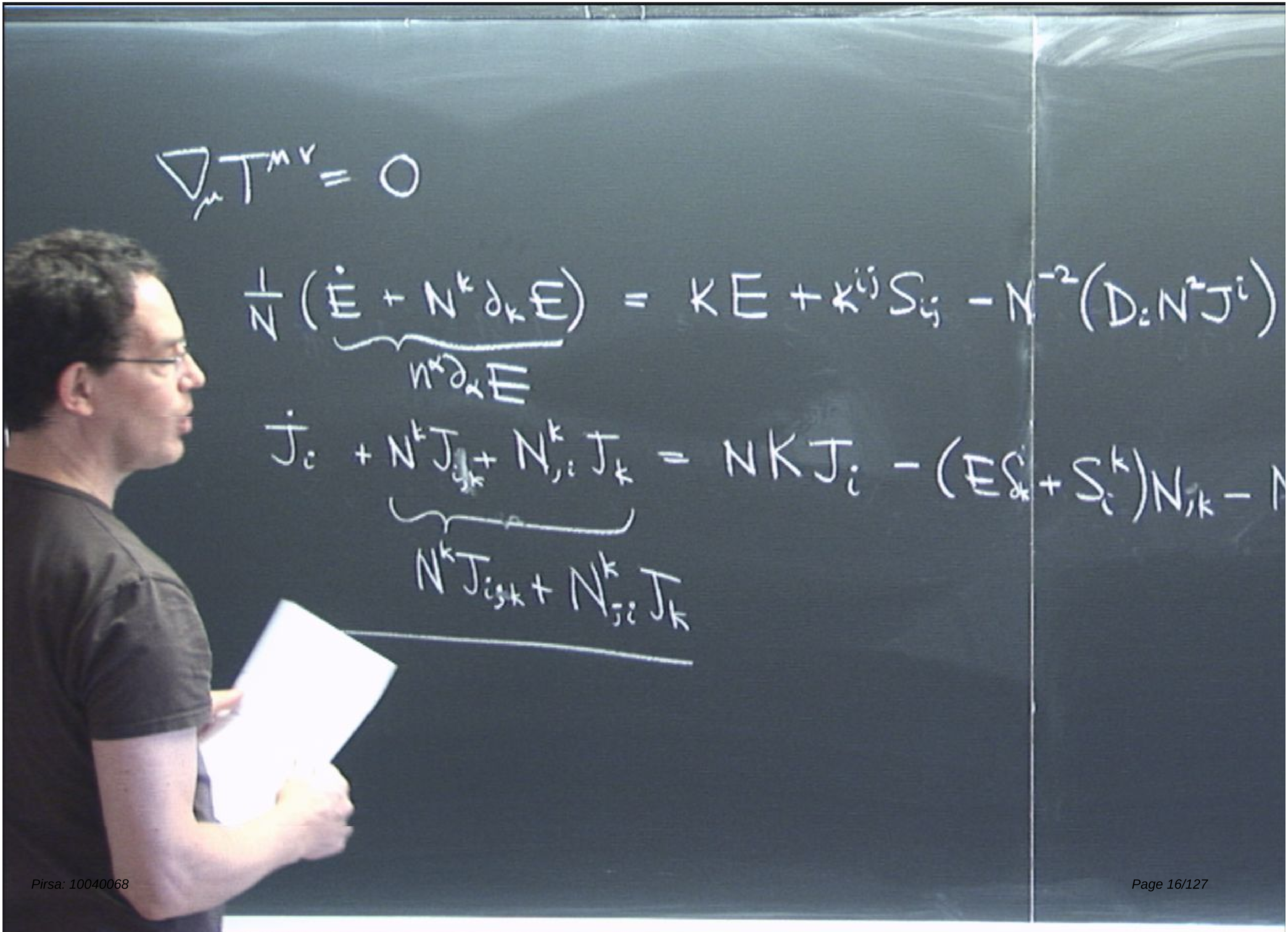
$T^{\mu i}$

$$\ldots + N^k J_k + N^k_{,i} J_k = N K J_i - \left(E\delta^k_i + S_i^{\ k}\right) N_{,k} - \ldots$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\to \quad \frac{1}{N}\big(\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}\big) = KE + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$$T^{\mu i} \to \quad \dot{J}_i + \underbrace{N^k J_{i,k} + N^k_{,i} J_k}_{N^k J_{i;k} + N^k_{;i} J_k} = N K J_i - (E\delta^k_i + S^k_i) N_{,k} - N$$


$\nabla_\mu T^{\mu\nu} = 0$
$\frac{1}{N}(\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}) = KE + K^{ij} S_{ij} - N^{-2}(D_i N^2 J^i)$
$\dot{J}_i + \underbrace{N^k J_{i,k} + N^k_{,i} J_k}_{N^k J_{i;k} + N^k_{;i} J_k} = NKJ_i - (E\delta^k_i + S_i^k) N_{,k} - N$

$$\nabla_\mu T^{\mu\nu} = 0$$

$\mu 0$ $\rightarrow$ $$\frac{1}{N}\Big(\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}\Big) = K E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$T^{\mu i}$ $\rightarrow$ $$\dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = N K J_i - (E\delta_i^k + S_i^k) N_{,k} - N$$

Cosmological perturbations

$$E = E_0 + \epsilon$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$\nu = 0 \quad \rightarrow$

$$\frac{1}{N}\underbrace{\left(\dot{E} + N^k \partial_k E\right)}_{n^\alpha \partial_\alpha E} = K E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$T^{\mu i} \quad \rightarrow$

$$\dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = N K J_i - (E \delta^k_i + S_i^k) N_{,k} - \ldots$$

Cosmological perturbations

$$E = E_0 + \epsilon$$

background: $\quad H^2 = \frac{8\pi G}{3} E_0$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\rightarrow \quad \frac{1}{N}\underbrace{\left(\dot{E} + N^k \partial_k E\right)}_{n^\alpha \partial_\alpha E} = K E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$$\dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = N K J_i - (E\delta_i^k + S_i^k) N_{,k} - N$$

Cosmological perturbations $\quad E = E_0 + \epsilon$

background: $\quad H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

$$\dot{E}_0 =$$

$$\nabla_\mu T^{\mu\nu} = 0$$

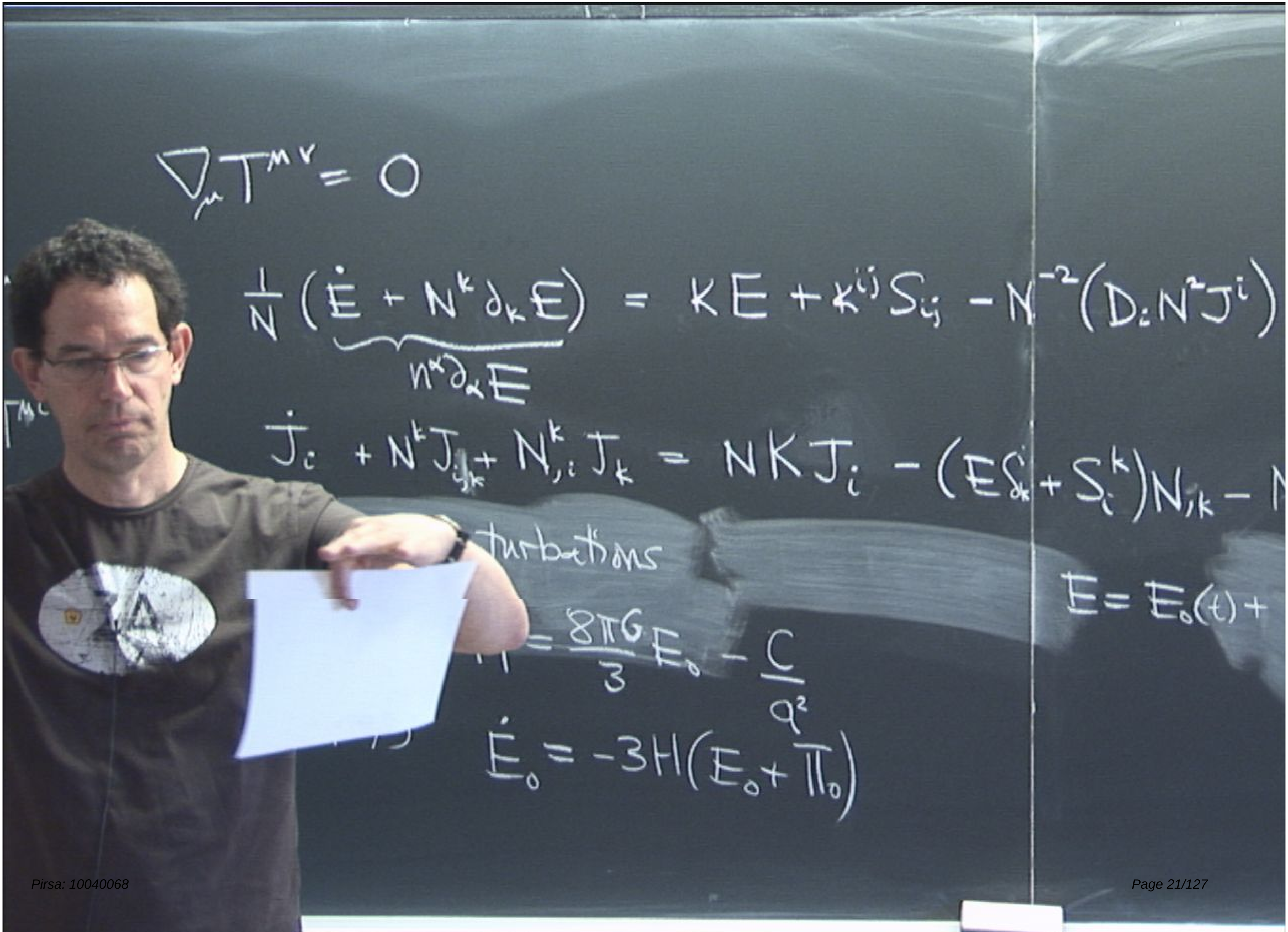
$$\overset{\mu 0}{} \rightarrow \quad \frac{1}{N}\big(\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}\big) = KE + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$$T^{\mu i} \rightarrow \quad \dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = NKJ_i - \left(E\delta^k_i + S_i^k\right) N_{,k} - N$$

Cosmological perturbations $\quad E = E_0 + \epsilon$

background: $\quad H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

$\dot{\rho} = -3H(p+\rho) \qquad \dot{E}_0 = -3H\left(E_0 + \Pi_0\right)$


$\nabla_\mu T^{\mu\nu} = 0$
$\frac{1}{N}(\dot{E} + N^k \partial_k E) = K E + K^{ij} S_{ij} - N^{-2}(D_i N^2 J^i)$
$n^\alpha \partial_\alpha E$
$\dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = N K J_i - (E \delta_i^k + S_i^k) N_{,k} - N$
turbations
$E = E_0(t) +$
$= \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$
$\dot{E}_0 = -3H(E_0 + \Pi_0)$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\to \quad \frac{1}{N}\left(\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}\right) = K E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right)$$

$$T^{\mu i} \longrightarrow \quad \dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = N K J_i - (E\delta^k_i + S_i^k) N_{,k} - \ldots$$

Cosmological perturbations

$E = E_0(t) + \ldots$

background: $H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

$\dot{\rho} = -3H(p+\rho)$ $\quad \dot{E}_0 = -3H(E_0 + \Pi_0)$

Bardeen.

$$^{-2}\left(D_i N^2 J^i\right) \qquad (11)$$

$$+S_i^{\,k})N_{,k} - N D_k S_i^{\,k} \qquad (12)$$

perturbation

$$E = E_0(t) + \epsilon(t, x)$$

$= 0$

Bardeen.

$$\underbrace{\dot{E} + N^k \partial_k E)}_{n^\alpha \partial_\alpha E} = K E + K^{ij} S_{ij} - N^{-2}(D_i N^2 J^i) \qquad (11)$$

$$+ N^k J_{i,k} + N^k_{,i} J_k = N K J_i - (E \delta^k_i + S_i^k) N_{,k} - N D_k S_i^k \qquad (12)$$

-turbations

$$E = E_0(t) + \epsilon(t, x)$$ perturbation

J_i

$$^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$$

$$\dot{E}_0 = -3H(E_0 + \Pi_0)$$

$= 0$

Bardeen.

$$\dot{E} + N^k \partial_k E) = KE + K^{ij} S_{ij} - N^{-2}(D_i N^2 J^i) \quad (11)$$

$$\underbrace{\dot{E} + N^k \partial_k E}_{n^\alpha \partial_\alpha E}$$

$$+ N^k J_{i;k} + N^k_{,i} J_k = NKJ_i - (E S^k_i + S_i^k) N_{,k} - N D_k S_i^k \quad (12)$$

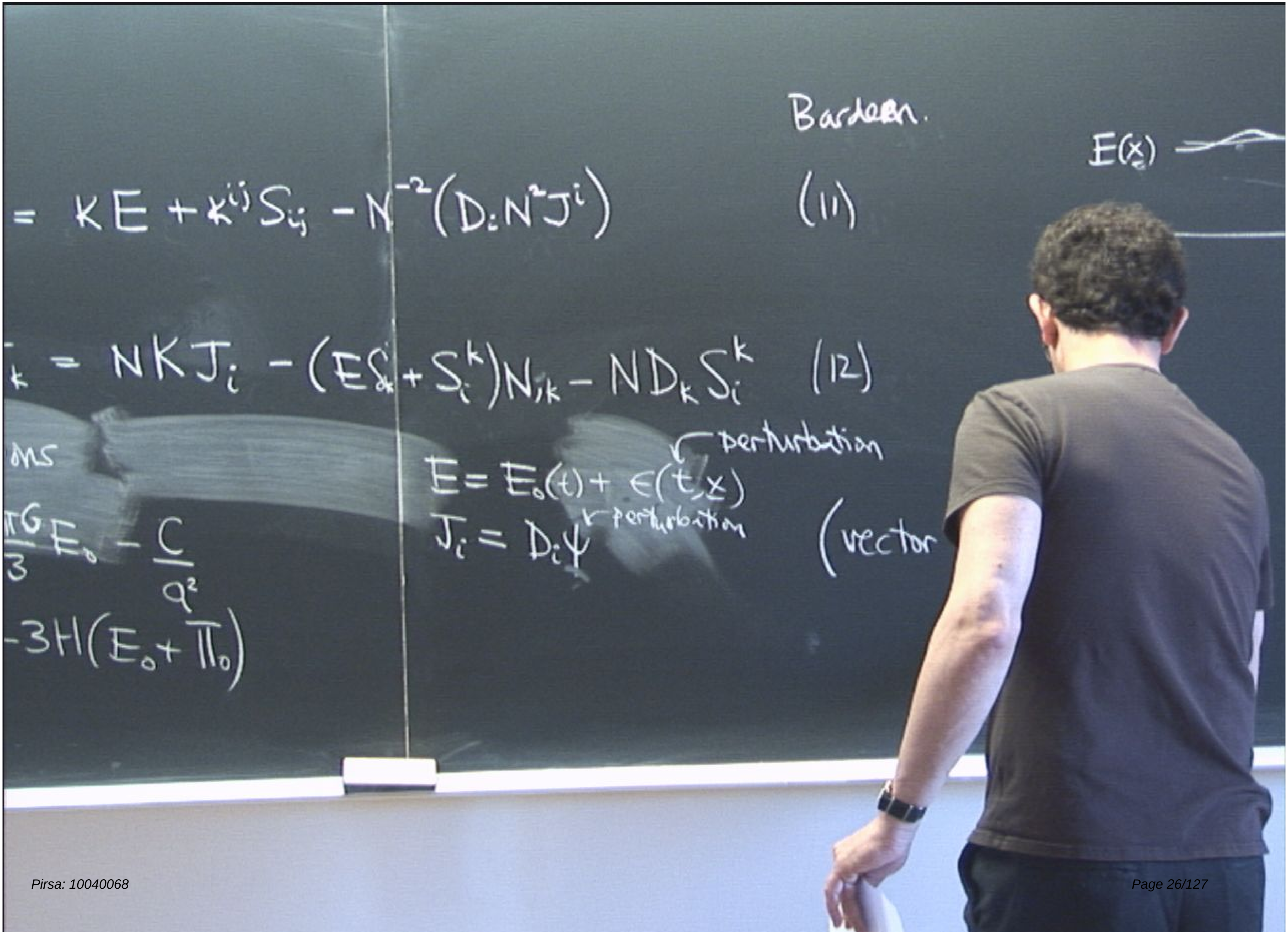
gical perturbations

ound: $H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

$(p+\rho)$ $\dot{E}_0 = -3H(E_0 + \Pi_0)$

$E = E_0(t) + \epsilon(t, x)$ perturbation

$J_i = D_i \psi$ perturbation


Bardeen.
$E(x)$
$= KE + K^{ij}S_{ij} - N^{-2}(D_i N^2 J^i)$ (11)
$= NKJ_i - (E\delta_{ik} + S_i^k)N_{,k} - N D_k S_i^k$ (12)
ons
perturbation
$E = E_0(t) + \epsilon(t, x)$
$J_i = D_i \psi$
perturbation
(vector
$\frac{\pi G}{3} E_0 - \frac{C}{a^2}$
$-3H(E_0 + \Pi_0)$

Bardeen.

$E(\underset{\sim}{x})$

$$\ldots E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right) \qquad (11)$$

$$N K J_i - \left(E \delta_i^k + S_i^k\right) N_{,k} - N D_k S_i^k \qquad (12)$$

perturbation

$$E = E_0(t) + \epsilon(t, \underline{x})$$

perturbation

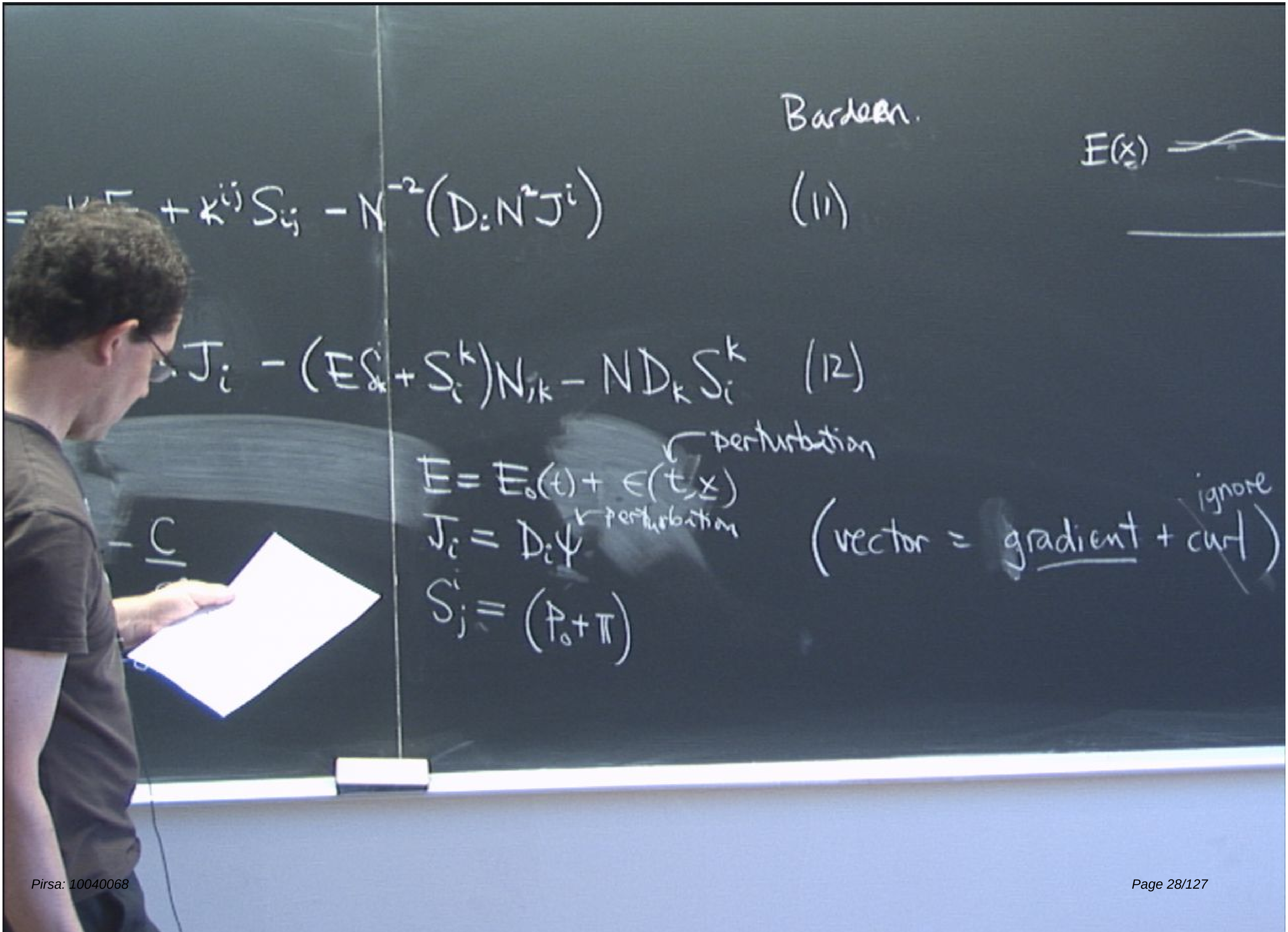
$$J_i = D_i \psi$$

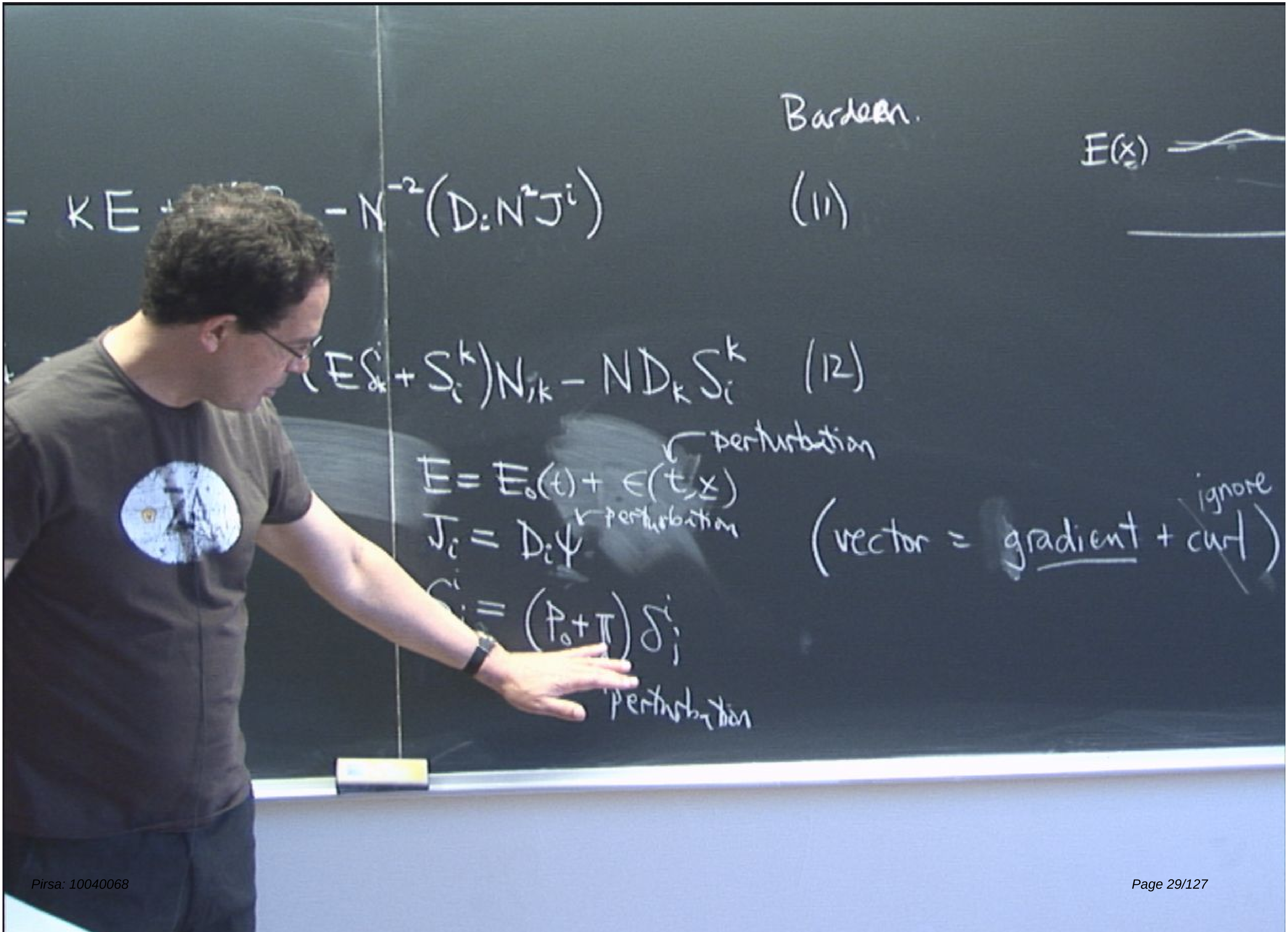
ignore

(vector = gradient + ~~curl~~)

$$- \frac{C}{a^2}$$

$$\left(E_0 + \Pi_0\right)$$


Bardeen.
$E(\underline{x})$
$+ K^{ij} S_{ij} - N^{-2}(D_i N^2 J^i)$ (11)
$J_i - (E\delta_i^k + S_i^k)N_{,k} - N D_k S_i^k$ (12)
perturbation
$E = E_0(t) + \epsilon(t,\underline{x})$
perturbation
$J_i = D_i \psi$
$S^i_j = (P_0 + \Pi)$
ignore
(vector = gradient + curl)
C


Bardeen.
E(x)
= KE + ... − N^-2 (D_i N^2 J^i)   (11)
(E δ^i_k + S_i^k) N_,k − N D_k S_i^k   (12)
perturbation
E = E_0(t) + ε(t, x)
J_i = D_i ψ
perturbation
S^i_j = (P_0 + π) δ^i_j
perturbation
(vector = gradient + curl)
ignore

Bardeen.

$E(\underset{\sim}{x})$

$= KE + K^{ij} S_{ij} - N^{-2}(D_i N^2 J^i)$ (11)

$= NKJ_i - (E\delta_i^k + S_i^k) N_{,k} - N D_k S_i^k$ (12)

ons

$\frac{G}{3} E_0 - \frac{C}{a^2}$

$3H(E_0 + P_0)$

perturbation

$E = E_0(t) + \epsilon(t, \underline{x})$

perturbation

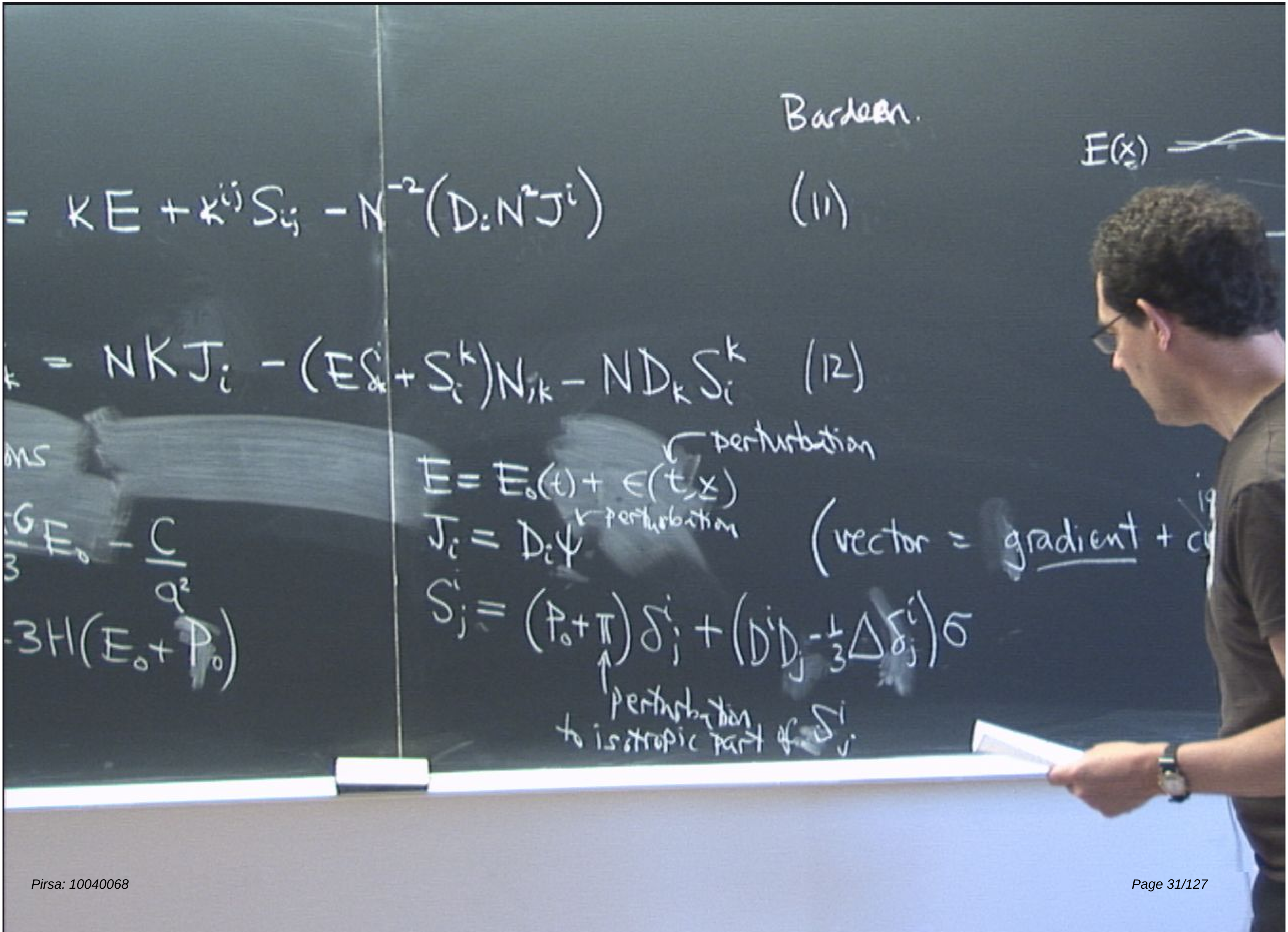
$J_i = D_i \psi$

$S^i_j = (P_0 + \Pi)\delta^i_j$

perturbation

(vector = $\underline{\text{gradient}}$ + ~~curl~~)

ignore


Bardeen.
E(x)
= K E + K^ij S_ij − N^−2 (D_i N^2 J^i)
(11)
= N K J_i − (E δ_i^k + S_i^k) N,_k − N D_k S_i^k
(12)
ons
perturbation
E = E_0(t) + ε(t, x)
perturbation
J_i = D_i ψ
(vector = gradient + c
S^i_j = (P_0 + π) δ^i_j + (D^i D_j − 1/3 Δ δ^i_j) σ
−3H(E_0 + P_0)
perturbation
to isotropic part of S^i_j

Bardeen.

$E(\underset{\sim}{x})$

$= KE + K^{ij}S_{ij} - N^{-2}(D_i N^2 J^i)$ (11)

$_k = NKJ_i - (E\delta_i^k + S_i^k)N_{,k} - ND_k S_i^k$ (12)

ons

$\frac{G}{3}E_0 - \frac{C}{a^2}$

$-3H(E_0 + P_0)$

perturbation

$E = E_0(t) + \epsilon(t, \underset{\sim}{x})$

perturbation

$J_i = D_i \psi$

(vector = gradient + ~~curl~~) ignore

$S^i_j = (P_0 + \Pi)\delta^i_j + \underbrace{\left(D^i D_j - \frac{1}{3}\Delta \delta^i_j\right)\sigma}$ → anisotropic str

perturbation to isotropic part of S^i_j

Bardeen.

$E(\underline{x})$

$$E + K^{ij} S_{ij} - N^{-2}\left(D_i N^2 J^i\right) \qquad (11)$$

$$N K J_i - \left(E \delta_i^k + S_i^k\right) N_{,k} - N D_k S_i^k \qquad (12)$$

$- \dfrac{C}{a^2}$

$(E_0 + P_0)$

perturbation

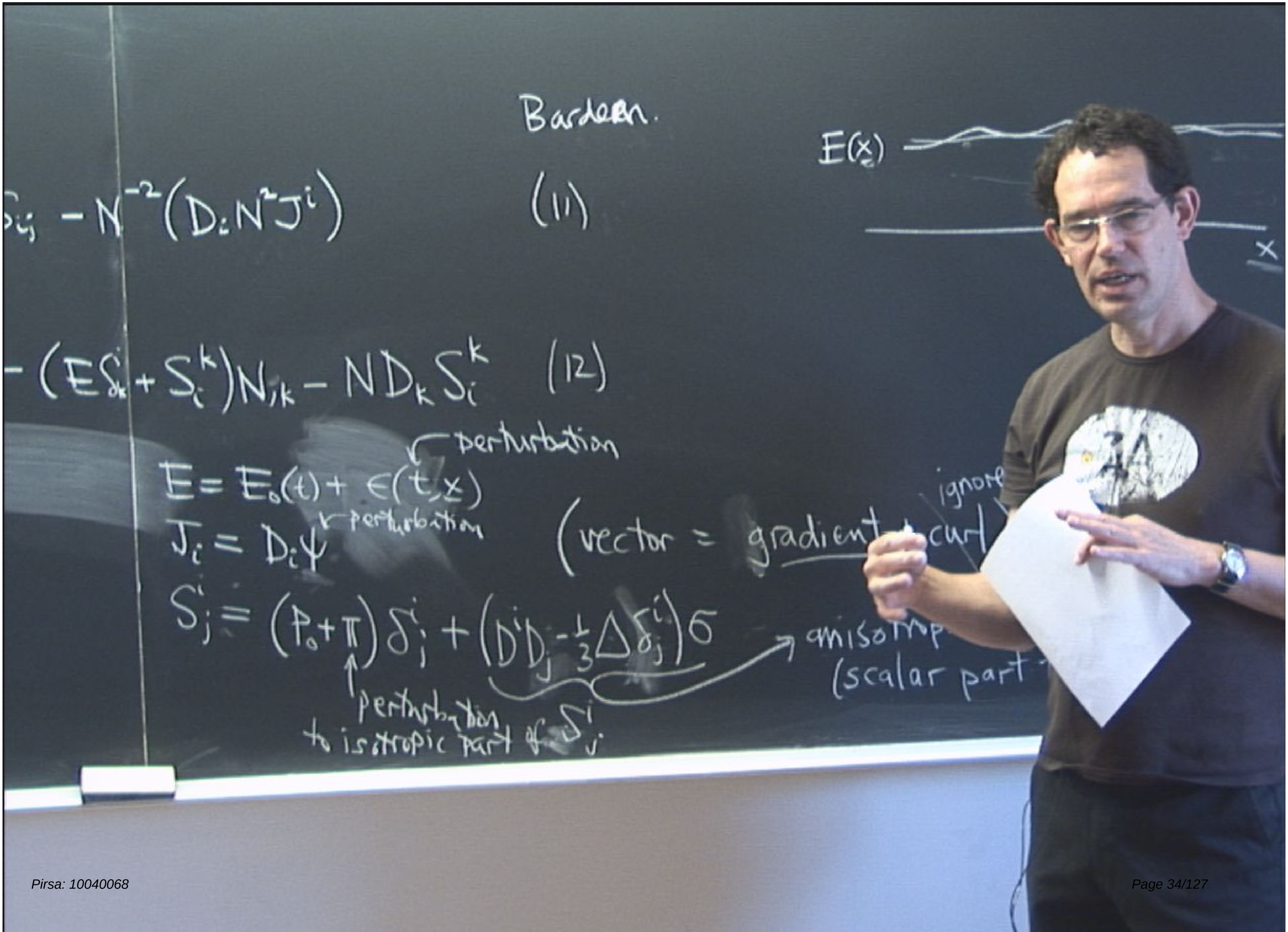
$$E = E_0(t) + \epsilon(t, \underline{x})$$

$$J_i = D_i \psi \quad \text{perturbation}$$

$$\left(\text{vector} = \underline{\text{gradient}} + \cancel{\text{curl}}\right) \quad \text{ignore}$$

$$S^i_j = (P_0 + \Pi)\,\delta^i_j + \underbrace{\left(D^i D_j - \tfrac{1}{3}\Delta \delta^i_j\right)\sigma} \rightarrow \text{anisotropic stress}$$

perturbation to isotropic part of S^i_j



Bardeen.
$E(x)$
x
$S_{ij} - N^{-2}(D_i N^2 J^i)$ (11)
$-(E\delta^i_k + S^k_i)N_{,k} - N D_k S^k_i$ (12)
perturbation
$E = E_0(t) + \epsilon(t,x)$
perturbation
$J_i = D_i\psi$
ignore
(vector = gradient + curl)
$S^i_j = (P_0 + \Pi)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta\delta^i_j)\sigma$
anisotrop
(scalar part
perturbation to isotropic part of S^i_j

Bardeen.

$$S_{ij} - N^{-2}\left(D_i N^2 J^i\right) \qquad (11)$$

$$-\left(E\delta^i_k + S_i^k\right)N_{,k} - N D_k S_i^k \qquad (12)$$

$$E = E_0(t) + \underbrace{\epsilon(t,x)}_{}$$ perturbation

$$J_i = D_i \psi$$ perturbation

(vector = $\underline{\text{gradient}}$ + $\cancel{\text{curl}}$) ignore

$$S^i_j = (P_0 + \Pi)\,\delta^i_j + \underbrace{\left(D^i D_j - \tfrac{1}{3}\Delta\delta^i_j\right)\sigma}_{} \rightarrow$$ anisotropic stress (scalar part thereof)

Π: perturbation to isotropic part of δ^i_j

Bardeen.

$$\ldots\delta_{ij} - N^{-2}\left(D_i N^2 J^i\right) \qquad (11)$$

$$-\left(E\delta^i_k + S^k_i\right)N_{,k} - N D_k S^k_i \qquad (12)$$

$$E = E_0(t) + \underbrace{\epsilon(t,\underline{x})}_{\text{perturbation}}$$

$$J_i = D_i \psi \quad \leftarrow \text{perturbation}$$

(vector = $\underline{\text{gradient}}$ + ~~curl~~) ignore

$$S^i_j = \underbrace{(P_0 + \Pi)}_{\text{perturbation to isotropic part of } S^i_j}\delta^i_j + \left(D^i D_j - \tfrac{1}{3}\Delta\delta^i_j\right)\sigma \rightarrow \text{anisotropic stress (scalar part thereof)}$$

$^{-2}(D_i N^2 J^i)$

$+ S_i^k) N_{,k} - N D_k S_i^k \quad (12)$

perturbation

$E = E_0(t) + \epsilon(t, \underline{x})$

perturbation

$J_i = D_i \psi$

(vector = $\underline{\text{gradient}}$ + $\cancel{\text{curl}}$) ignore

$S^i_j = (P_0 + \Pi)\delta^i_j + \underbrace{\left(D^i D_j - \frac{1}{3}\Delta \delta^i_j\right)\sigma}$

perturbation to isotropic part of S^i_j

anisotropic stress (scalar part thereof)

zero for perfect fluid

$$^{-2}\left(D_i N^2 J^i\right)$$

$$+ S_i^k) N_{,k} - N D_k S_i^k \quad (12)$$

perturbation

$$E = E_0(t) + \epsilon(t, \underline{x})$$

perturbation

$$J_i = D_i \psi$$

$$S^i_j = (P_0 + \pi)\,\delta^i_j + \left(D^i D_j - \tfrac{1}{3}\Delta \delta^i_j\right)\sigma$$

perturbation to isotropic part of S^i_j

anisotropic stress (scalar part thereof)

(vector = gradient + ~~curl~~) ignore

zero f… perfe… fluid

1st order pert^n equations

$N^{-2}(D_i N^2 J^i)$

$(E\delta_i^k + S_i^k)N_{,k} - N D_k S_i^k$ (12)

perturbation

$E = E_0(t) + \underline{\epsilon(t,x)}$

perturbation

$J_i = D_i \underline{\Psi}$

(vector = $\underline{\text{gradient}}$ + $\cancel{\text{curl}}$)

ignore

$S^i_j = (P_0 + \underline{\Pi})\delta^i_j + \underbrace{(D^i D_j - \frac{1}{3}\Delta\delta^i_j)\underline{\sigma}}$

perturbation to isotropic part of S^i_j

anisotropic stress (scalar part thereof)

zero for perfect $\underline{\text{fluid}}$

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon+\pi) + (E_0+P_0)(k-3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$^{2}(D_i N^2 J^i)$

$+ S_i^k) N_{,k} - N D_k S_i^k \quad (12)$

perturbation

$E = E_0(t) + \underline{\epsilon(t,x)}$

perturbation

$J_i = D_i \underline{\psi}$

$S^i_j = (P_0 + \underline{\pi})\delta^i_j + \underbrace{\left(D^i D_j - \frac{1}{3}\Delta\delta^i_j\right)\underline{\sigma}}$ → anisotropic stress (scalar part thereof) → zero for perfect fluid

perturbation to isotropic part of S^i_j

(vector = gradient + $\overset{\text{ignore}}{\cancel{\text{curl}}}$)

1st order pert^n equations

$$\frac{\mathring{\epsilon}}{N_0} = -3H(\epsilon + \pi) + (E_0 + P_0)(\kappa - 3H\alpha) - \Delta\psi$$

$$\frac{\mathring{\psi}}{N_0} = -3H\psi - (E_0 + P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$$+ S_i^k)N_{,k} - N D_k S_i^k \quad (12)$$

perturbation

$$E = E_0(t) + \epsilon(t, x)$$

perturbation

$$J_i = D_i \psi$$

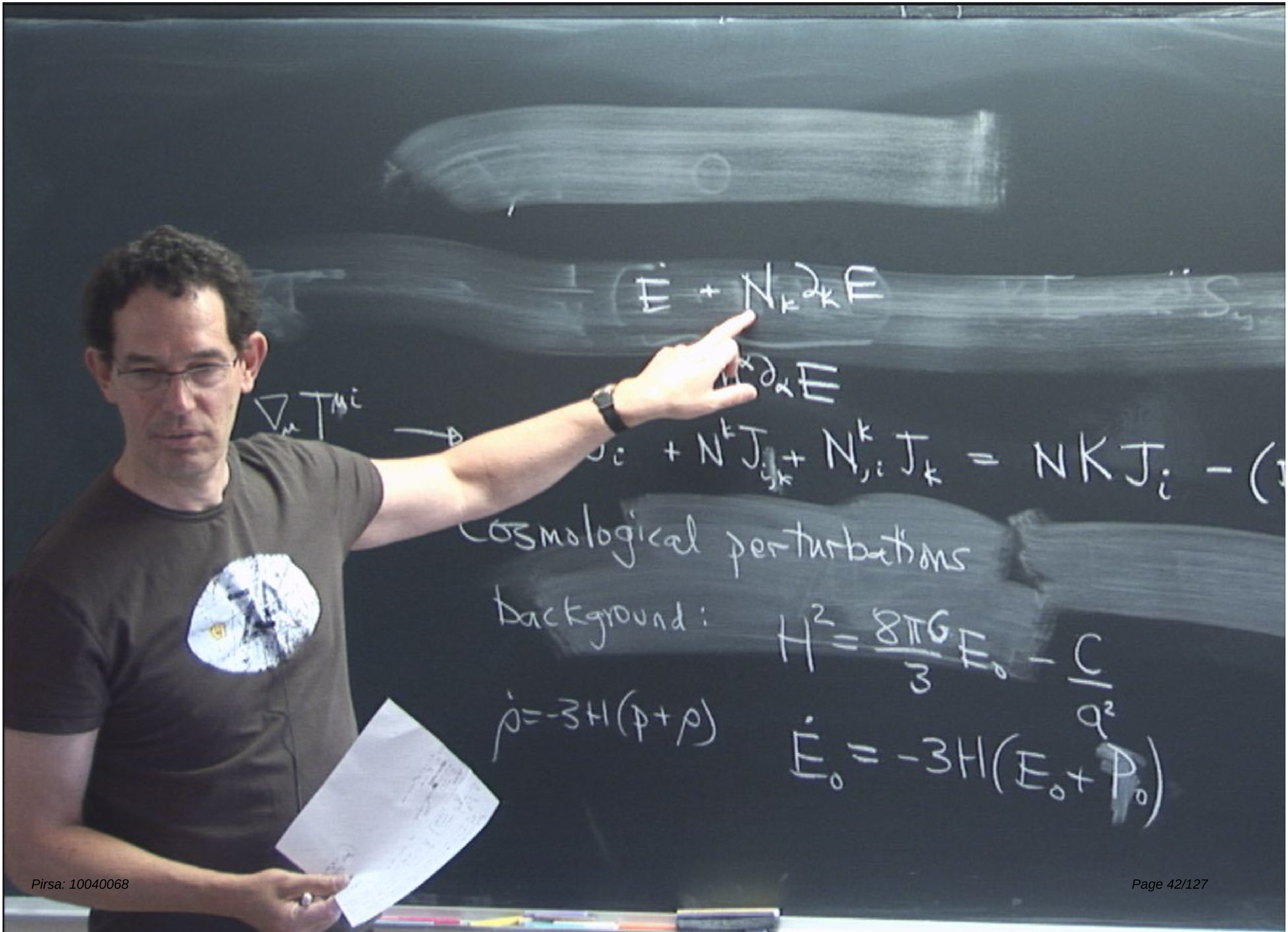
(vector = gradient + curl) ignore

$$S^i_j = (P_0 + \Pi)\delta^i_j + \underbrace{\left(D^i D_j - \tfrac{1}{3}\Delta \delta^i_j\right)}\sigma$$

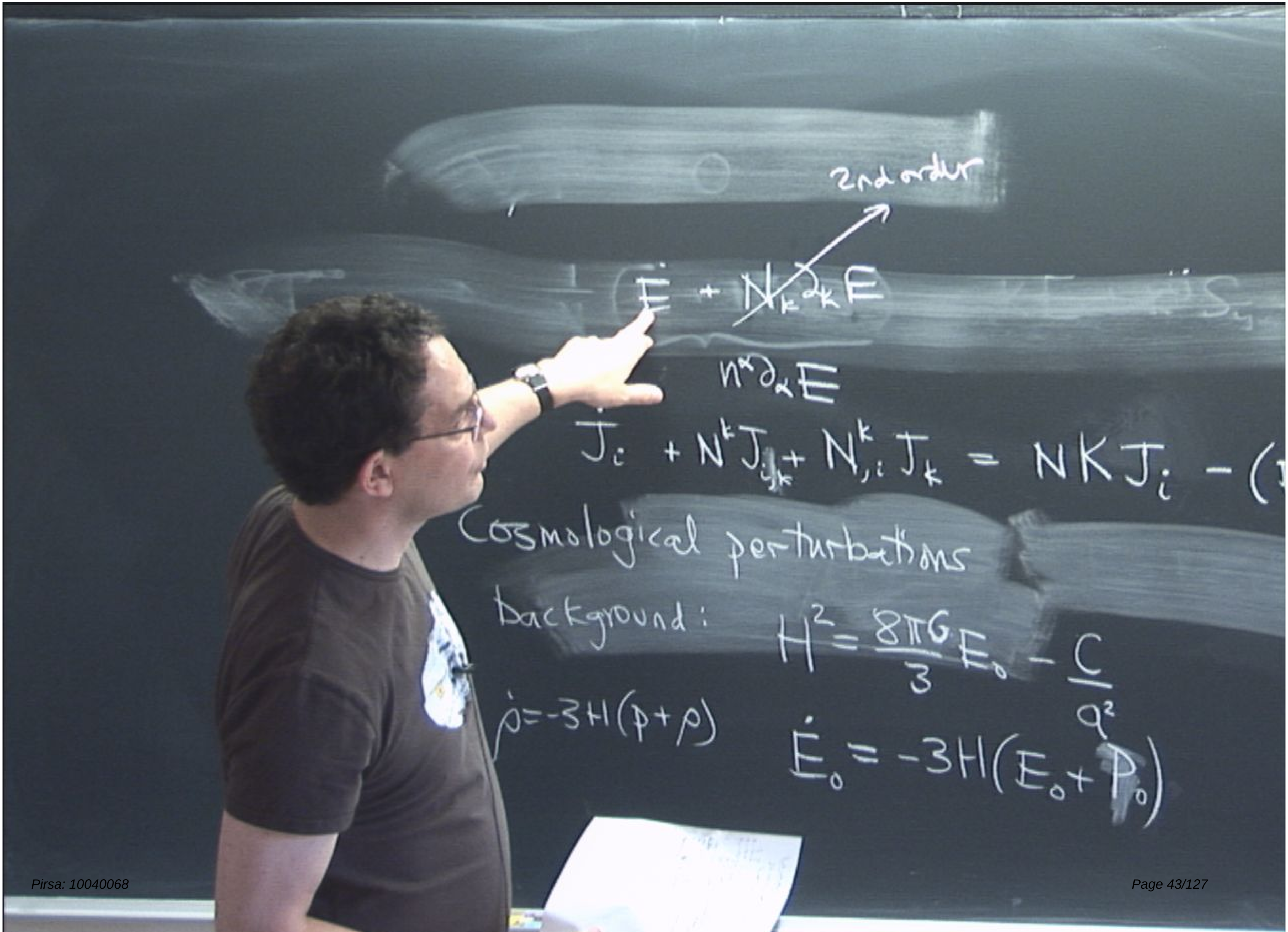
anisotropic stress (scalar part thereof)

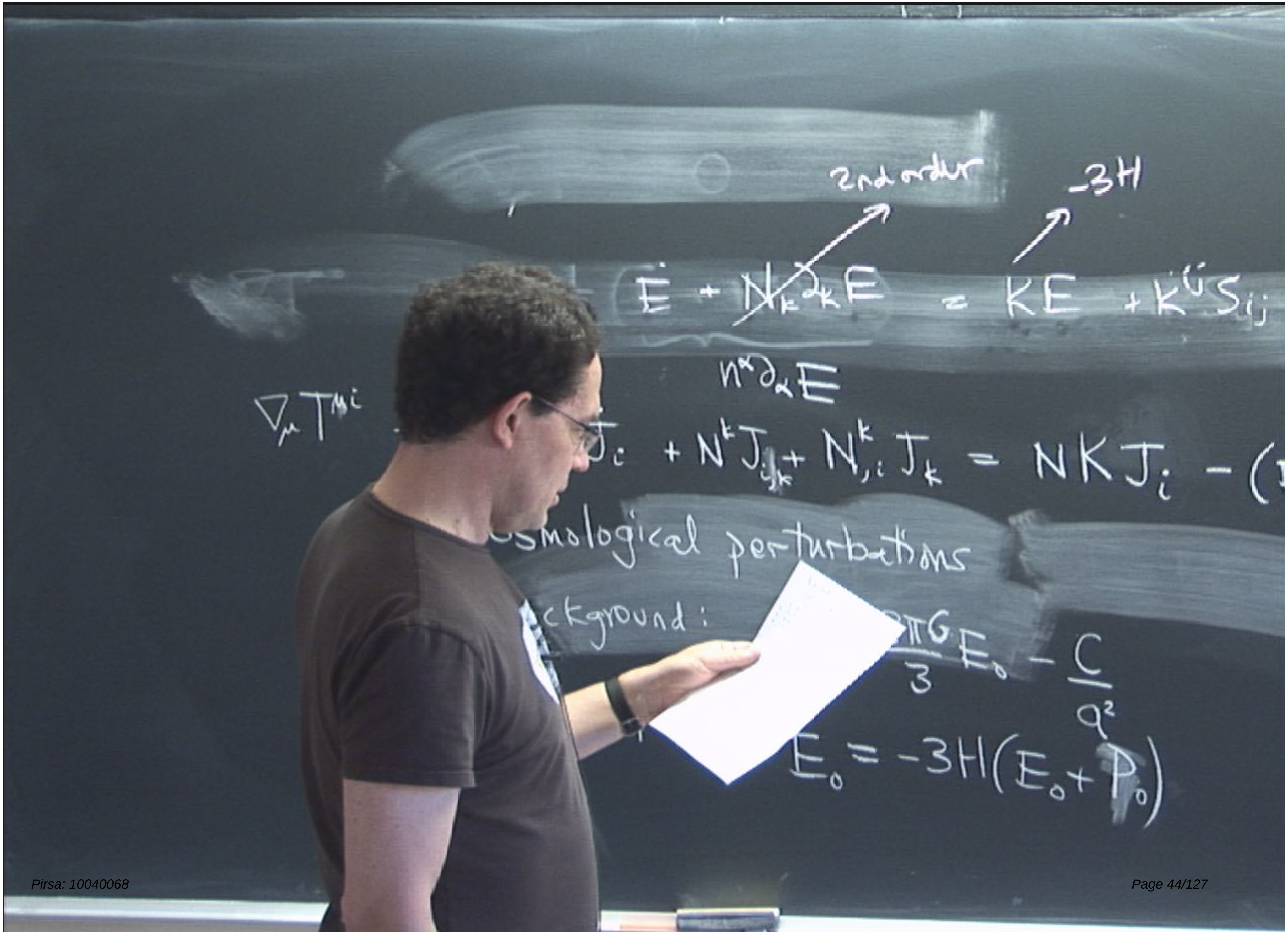
zero for perfect fluid

perturbation to isotropic part of S^i_i



$\dot{E} + N_k \partial_k E$
$\partial_\alpha E$
$\nabla_\mu T^{\mu i}$
$+ N^k J_{i,k} + N^k_{,i} J_k = N K J_i - ($
Cosmological perturbations
background:
$H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$
$\dot{\rho} = -3H(p+\rho)$
$\dot{E}_0 = -3H(E_0 + P_0)$


2nd order
$\dot{E} + \cancel{N_k} \partial_k E$
$n^\alpha \partial_\alpha E$
$\dot{J}_i + N^k J_{i,k} + N^k_{,i} J_k = NKJ_i - ($
Cosmological perturbations
background:
$H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$
$\dot{\rho} = -3H(p+\rho)$
$\dot{E}_0 = -3H(E_0 + P_0)$


2nd order
-3H
E + N^k ∂_k E = KE + K^ij S_ij
n^α ∂_α E
∇_μ T^μi
J_i + N^k J_i;k + N^k_,i J_k = NKJ_i − (
smological perturbations
ckground:
πG/3 E_0 − C/a^2
E_0 = −3H(E_0 + P_0)

1st order pert

$\frac{\dot{\epsilon}}{N_0} = -3$

$\frac{\dot{\psi}}{N_0} = -$

2nd order

$-3H$

$KE = KE + K^{ij}S_{ij} - (D_i N J^i)$

$NKJ_i - (E\delta^k_i + S^k_i)N_{,k} - ND_k S^k_i$ (12)

rturbations

$= \frac{8\pi G}{3} E_0 - \frac{C}{}$

$+ P_0)$

perturbation

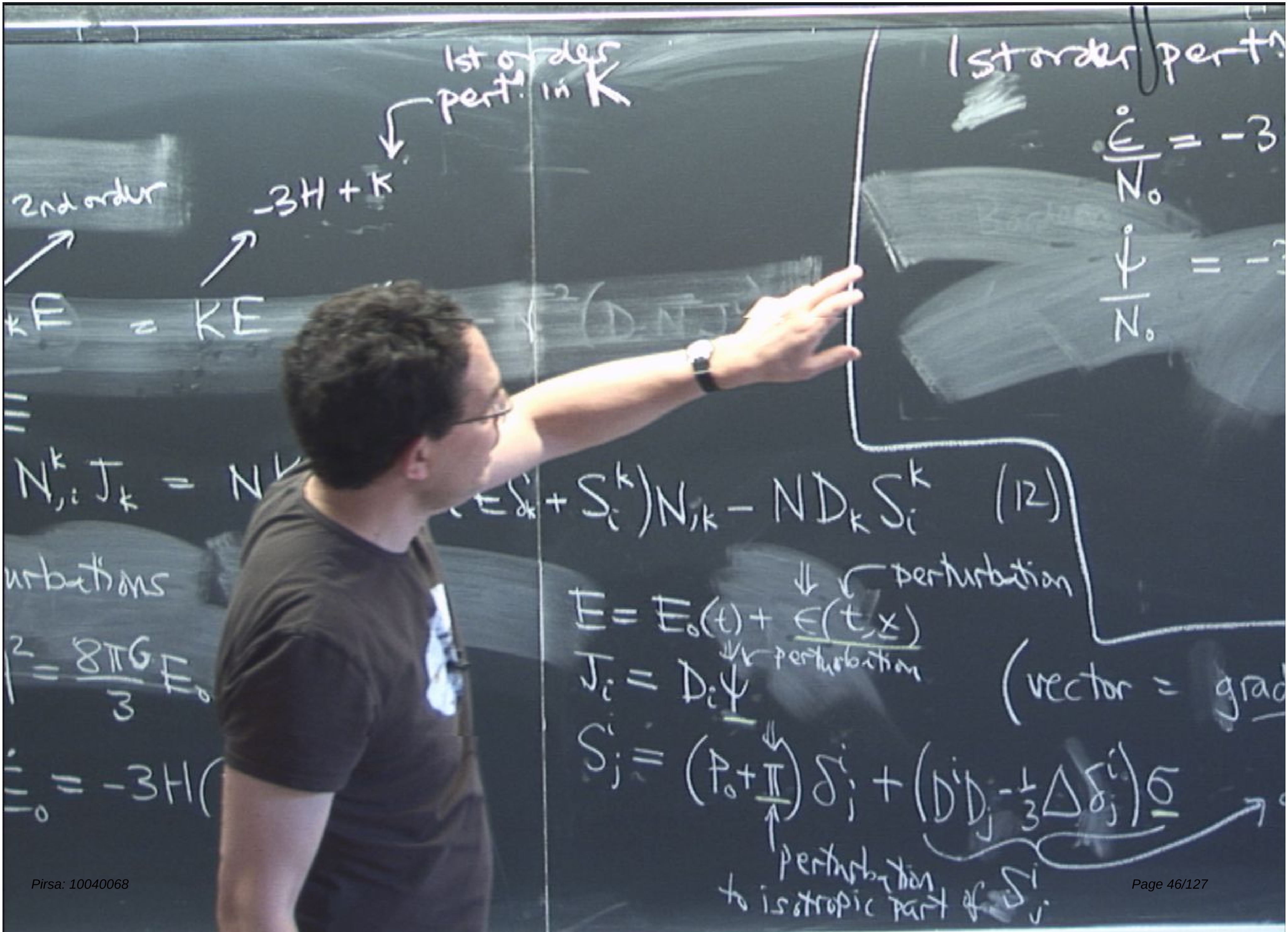
$E = E_0(t) + \epsilon(t, x)$

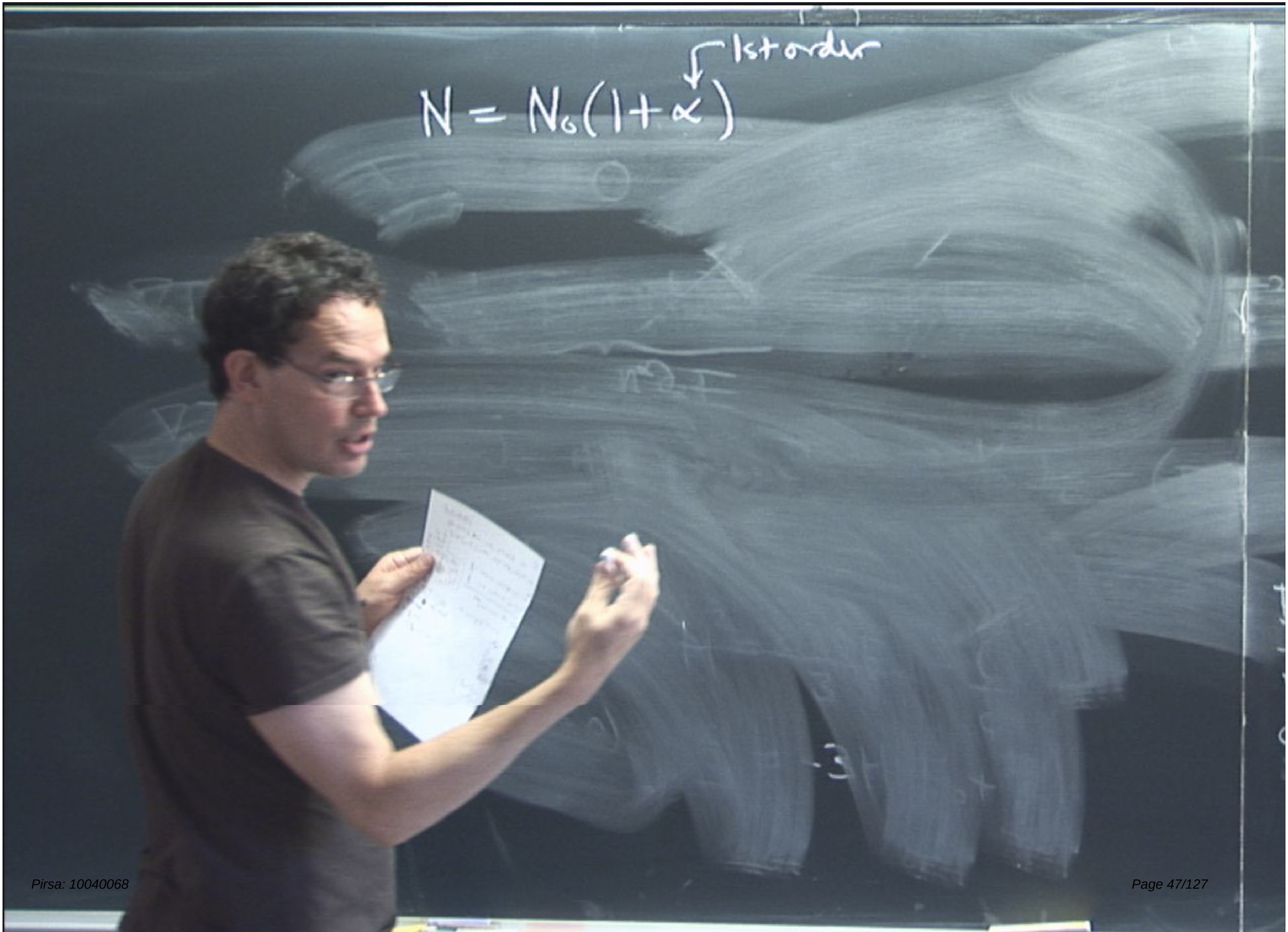
perturbation

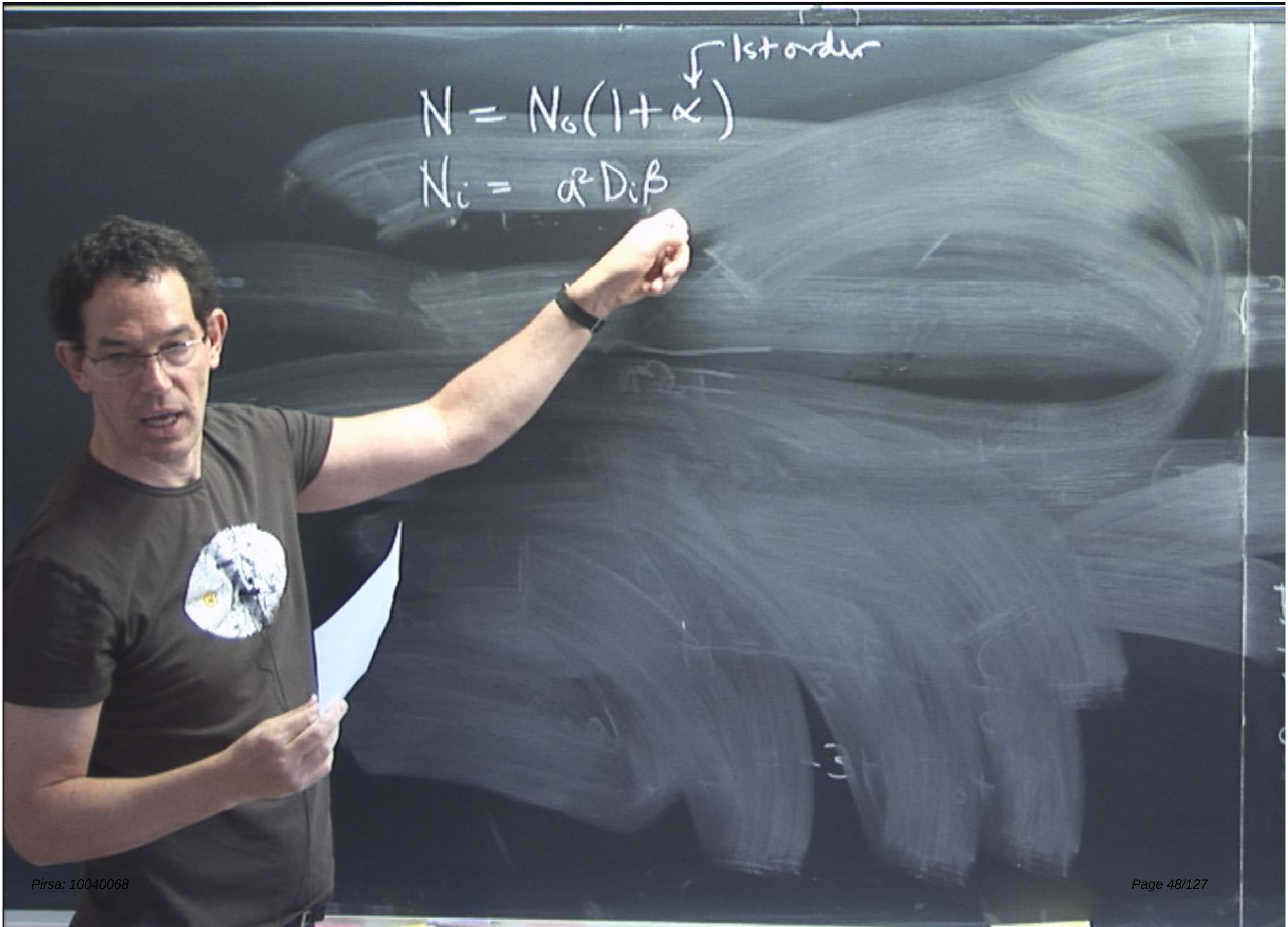
$J_i = D_i \psi$ (vector = grad

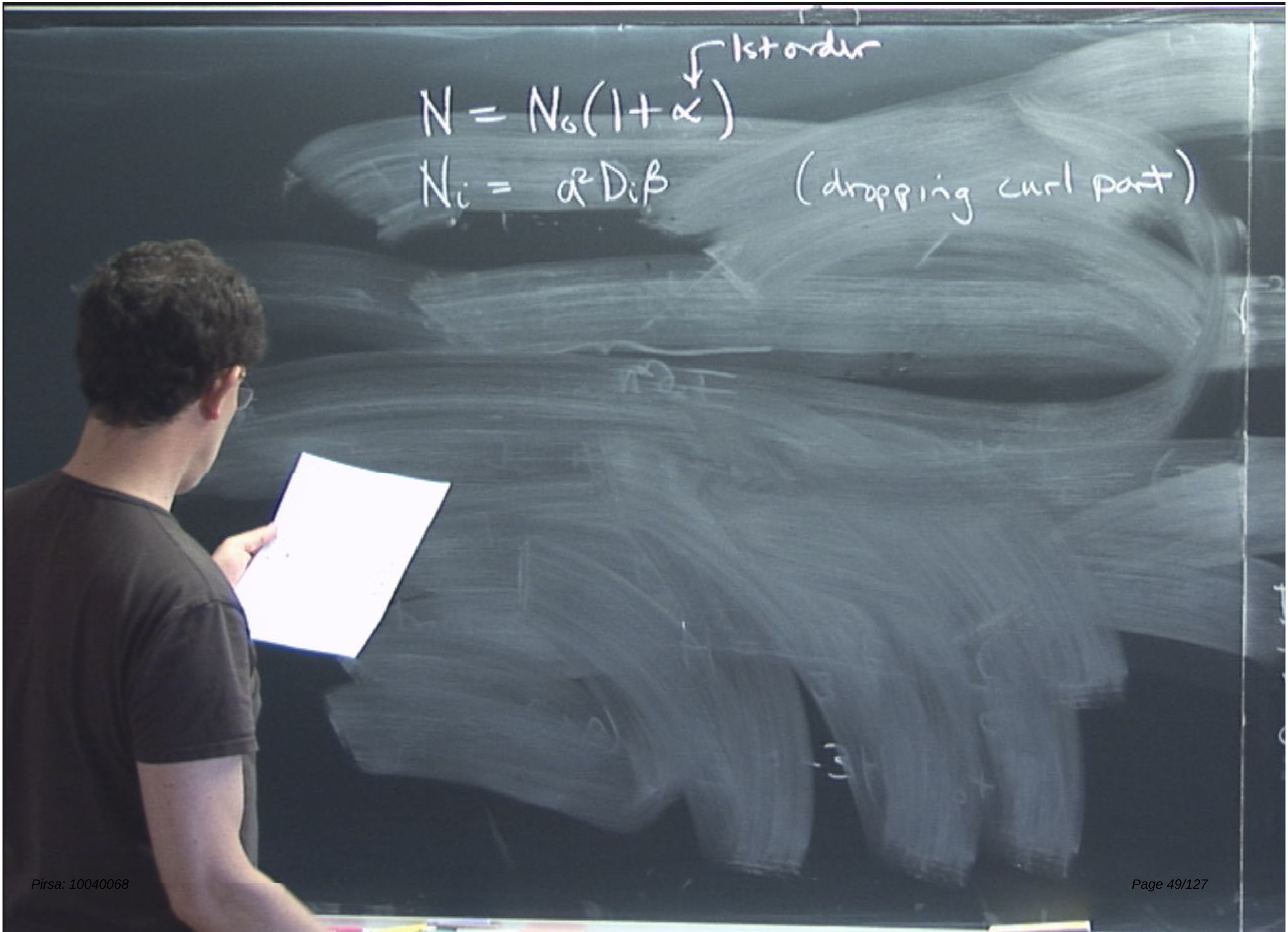
$S^i_j = (P_0 + \Pi)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta\delta^i_j)\sigma$

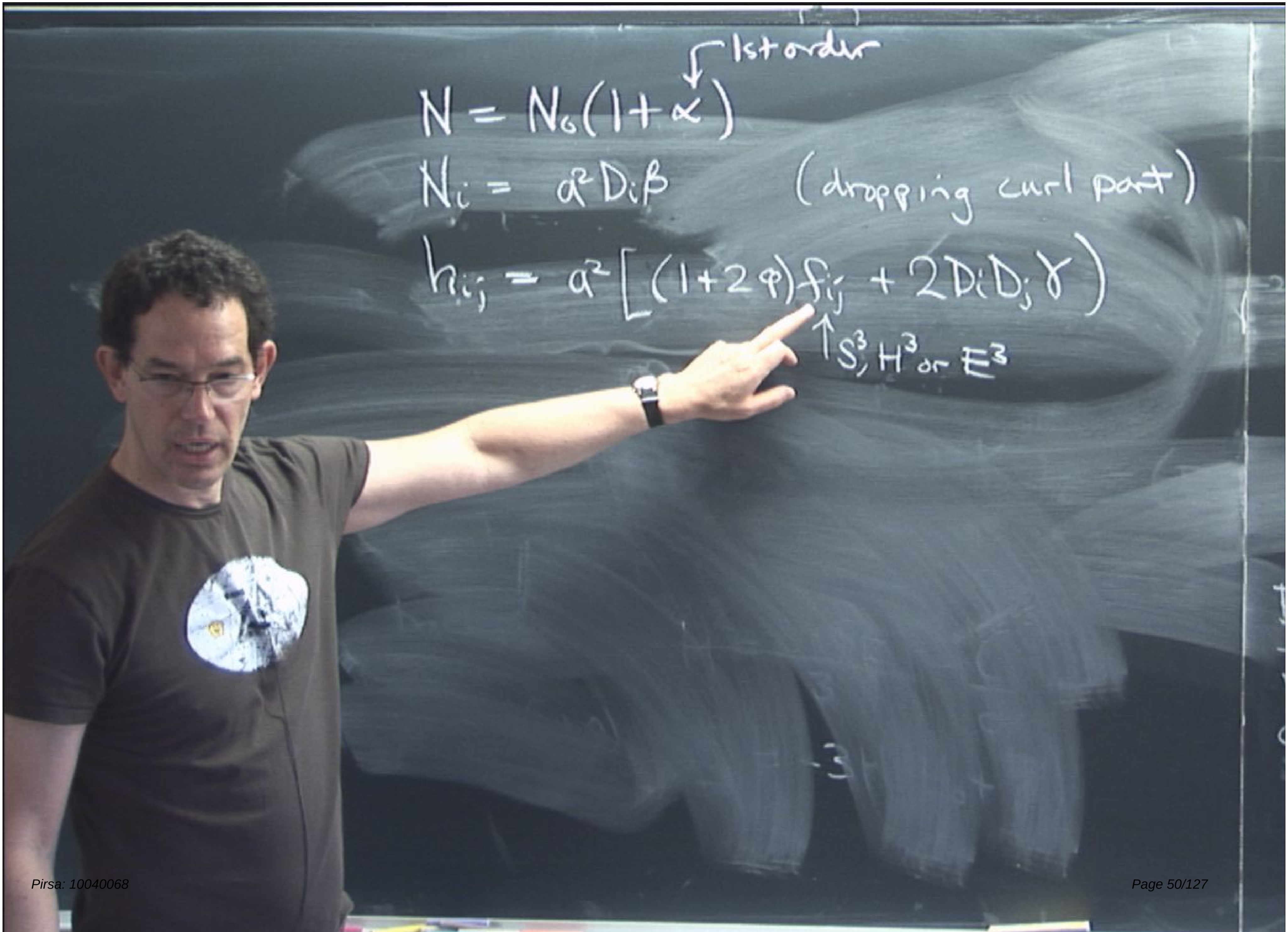
perturbation to isotropic part of S^i_j


1st order pert.ⁿ in K
-3H + K
2nd order
KE = KE
1st order pert.ⁿ
$\frac{\dot{\epsilon}}{N_0} = -3$
$\frac{\dot{\psi}}{N_0} = -$
$N^k_{,i} J_k = N$
$+ S_i^k) N_{,k} - N D_k S_i^k$ (12)
urbations
perturbation
$E = E_0(t) + \underline{\epsilon(t,x)}$
perturbation
$J_i = D_i \underline{\psi}$
(vector = grad
$= \frac{8\pi G}{3} E_0$
$S^i_j = (P_0 + \underline{\Pi})\delta^i_j + (D^i D_j - \frac{1}{3}\Delta\delta^i_j)\underline{\sigma}$
$\dot{E}_0 = -3H($
perturbation
to isotropic part of S^i_j


1st order
$N = N_0(1+\alpha)$


1st order
$N = N_0(1+\alpha)$
$N_i = \alpha^2 D_i \beta$


1st order
$N = N_0(1+\alpha)$
$N_i = a^2 D_i \beta$
(dropping curl part)


1st order
$N = N_0(1+\alpha)$
$N_i = a^2 D_i \beta$
(dropping curl part)
$h_{ij} = a^2\left[(1+2\varphi)f_{ij} + 2D_iD_j\gamma\right)$
S^3, H^3 or E^3

1st order

$$N = N_0(1+\alpha)$$

$$N_i = a^2 D_i \beta \qquad \text{(dropping curl part)}$$

$$h_{ij} = a^2\left[(1+2\underline{\varphi}) f_{ij} + 2 D_i D_j \underline{\gamma}\right]$$

$\uparrow S^3, H^3$ or E^3

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon+\pi) + (E_0+P_0)(k-3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{2}\right)\sigma$$

(12)

perturbation

$$E = E_0(t) + \epsilon(t,x)$$

perturbation

$$J_i = D_i\psi$$

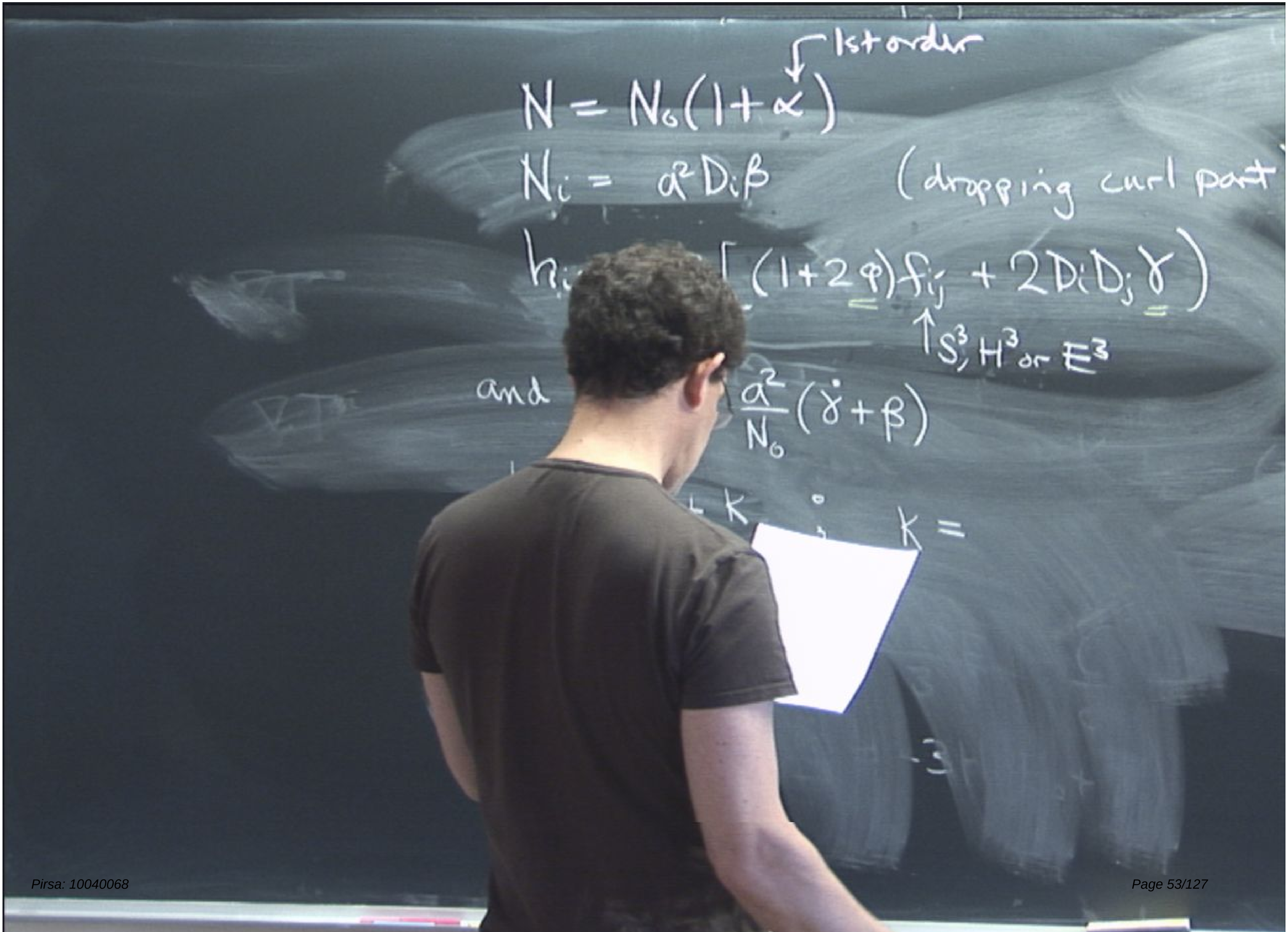
$$S^i_j = (P_0 + \Pi)\,\delta^i_j + \left(D^iD_j - \tfrac{1}{3}\Delta\delta^i_j\right)\sigma$$

perturbation to isotropic part of S^i_j

anisotropic stress (scalar part there...

(vector = gradient + ~~curl~~) ignore

ze... → p...


1st order
N = N₀(1+α)
Nᵢ = a²Dᵢβ (dropping curl part
(1+2φ)fᵢⱼ + 2DᵢDⱼγ)
S³, H³ or E³
and
a²/N₀ (γ̇+β)
k =

$$N = N_0(1+\overset{\text{1st order}}{\alpha})$$

$$N_i = a^2 D_i \beta \qquad \text{(dropping curl part)}$$

$$h_{ij} = a^2\left[(1+2\underline{\varphi})\underset{\uparrow\, S^3,\, H^3 \text{ or } E^3}{f_{ij}} + 2D_iD_j\underline{\gamma}\right)$$

and $\chi = \dfrac{a^2}{N_0}(\dot{\gamma}+\beta)$

$$K = -3H + \kappa \; ; \quad \kappa = -3\left(\frac{\dot{\varphi}}{N_0} - H\alpha\right) - \Delta\chi$$

-3

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon + \pi) + (E_0+P_0)(k - 3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$D_k D_i D^k \sigma$

(12)

perturbation

$E = E_0(t) + \underline{\epsilon(t, x)}$

perturbation

$J_i = D_i \underline{\psi}$

(vector = gradient + ~~curl~~)

ignore

$S^i_j = (P_0 + \underline{\pi})\,\delta^i_j + \underbrace{\left(D^i D_j - \frac{1}{3}\Delta\delta^i_j\right)\underline{\sigma}}$

anisotropic stress
(scalar part thereof)

perturbation
to isotropic part of S^i_j

zer[illegible]
perf[illegible]
fl[illegible]

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon + \pi) + (E_0 + P_0)(k - 3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0 + P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$$D_k D_i D^k \sigma = D_i D_k D^k \sigma + R_{ik} D^k \sigma$$

(12)

$$E = E_0(t) + \underbrace{\epsilon(t, \underline{x})}_{\text{perturbation}}$$

$$J_i = D_i \underbrace{\psi}_{\text{perturbation}}$$

$$S^i_j = (P_0 + \underline{\Pi})\,\delta^i_j + \left(D^i D_j - \frac{1}{3}\Delta \delta^i_j\right)\underline{\sigma}$$

perturbation to isotropic part of S^i_j

anisotropic stress (scalar part thereof)

(vector = gradient + ~~curl~~) ignore

zero for perfect fluid

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon + \pi) + (E_0+P_0)(k - 3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$$D_k D_i D^k \sigma \quad \frac{2c}{a^2}$$

$$= D_i D_k D^k \sigma + R_{ik} D^k \sigma$$

(12)

perturbation

$$E = E_0(t) + \underline{\epsilon(t, \underline{x})}$$

perturbation

$$J_i = D_i \underline{\psi}$$

ignore

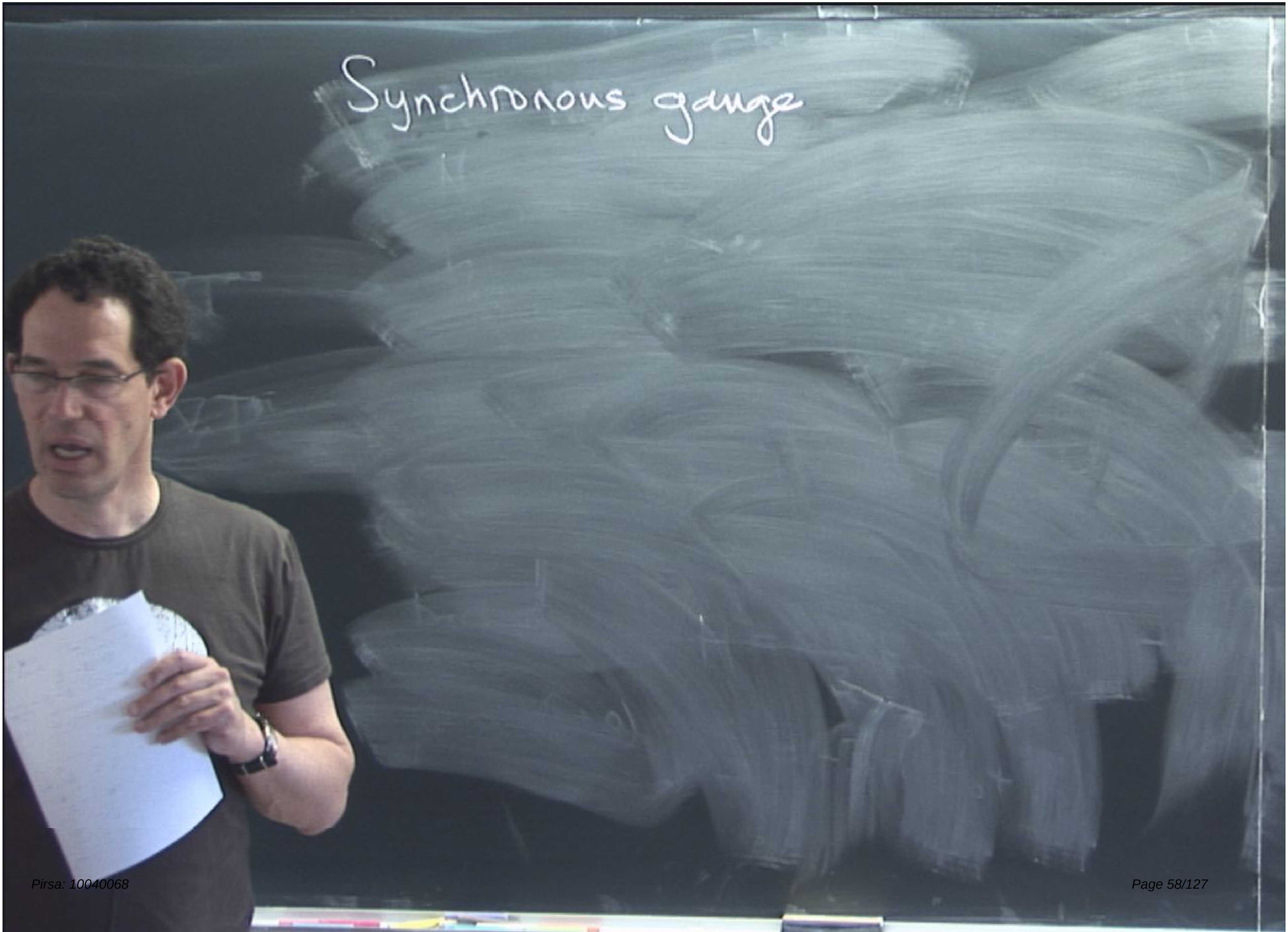
$$(\text{vector} = \underline{\text{gradient}} + \cancel{\text{curl}})$$

$$S^i_j = (P_0 + \underline{\pi})\,\delta^i_j + \left(D^i D_j - \tfrac{1}{3}\Delta \delta^i_j\right)\underline{\sigma}$$

perturbation to isotropic part of δ^i_j

anisotropic stress (scalar part thereof)

zero for perfect <u>fluid</u>


Synchronous gauge

Synchronous gauge (simplest for studying matter evolut

Synchronous gauge (simplest for studying matter evolution)

Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}dx^i dx^j$

Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

(Ex : show SG exists)

$ds^2 = -dt^2 + h_{ij} dx^i dx^j$

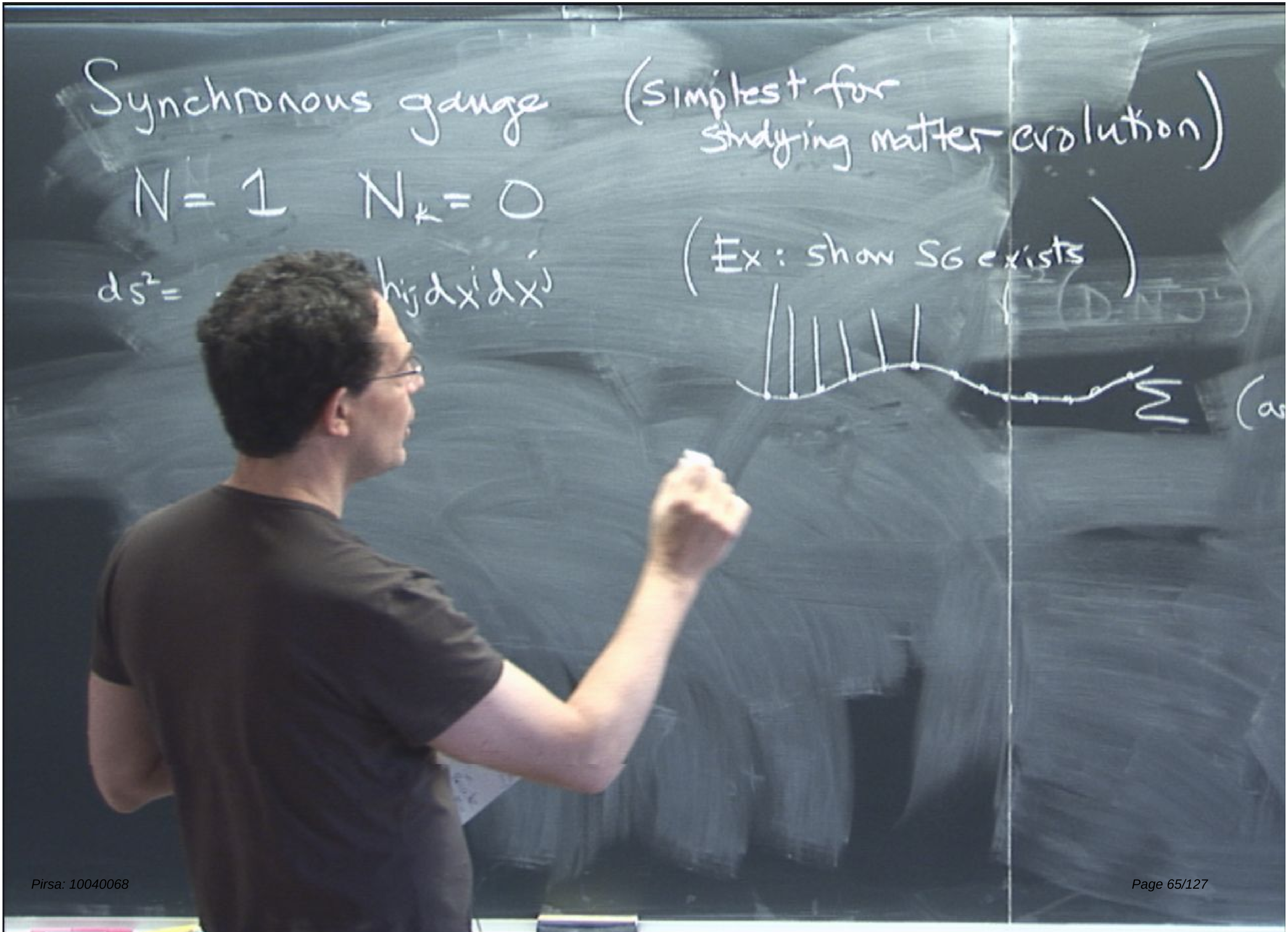
Synchronous gauge (simplest for studying matter evolution)

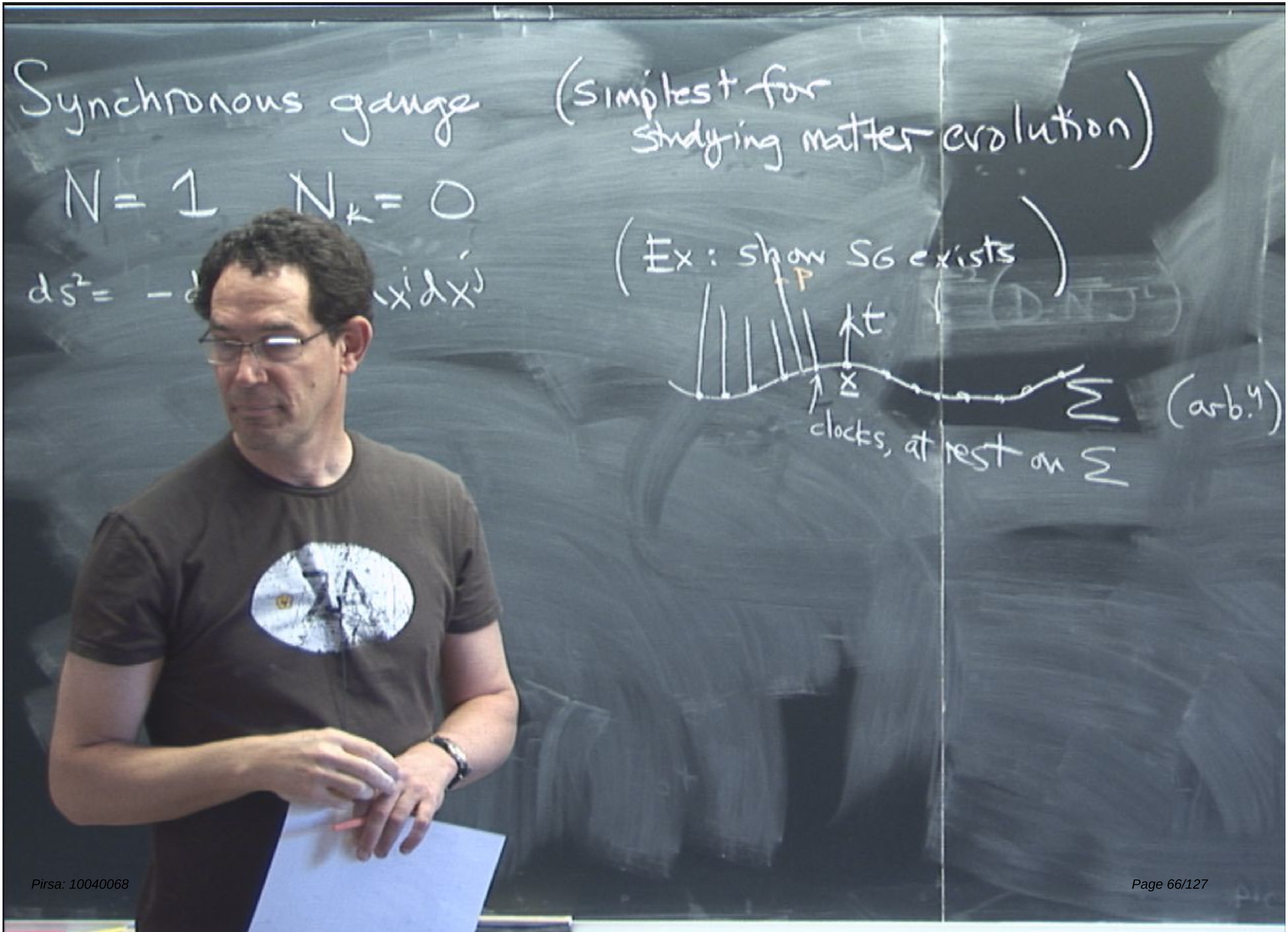
$N = 1 \quad N_k = 0$

(Ex: show SG exists)

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

(a


Synchronous gauge (simplest for studying matter evolution)
N = 1   N_k = 0
ds² =
h_ij dx^i dx^j
(Ex: show SG exists)
Σ

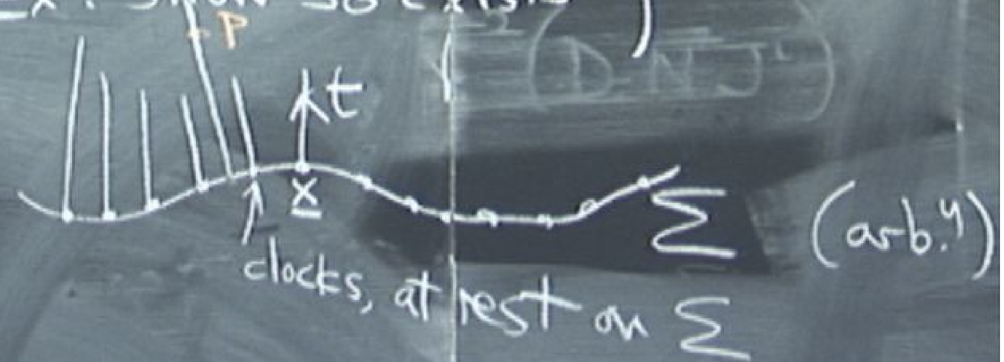

Synchronous gauge (simplest for studying matter evolution)
N = 1 N_k = 0
ds² = −d
x^i dx^j
(Ex: show SG exists)
P
t
x
Σ (arb.y)
clocks, at rest on Σ

Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

(Ex: show SG exists)

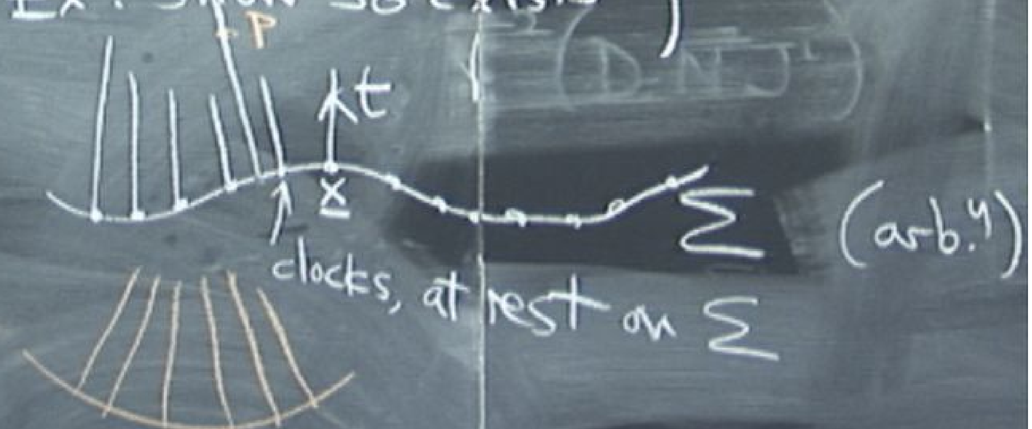


Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

(Ex: show SG exists)

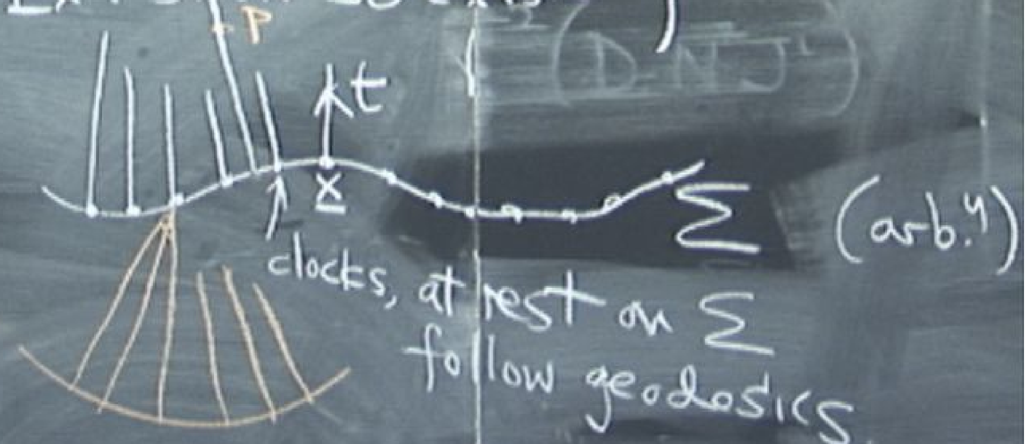


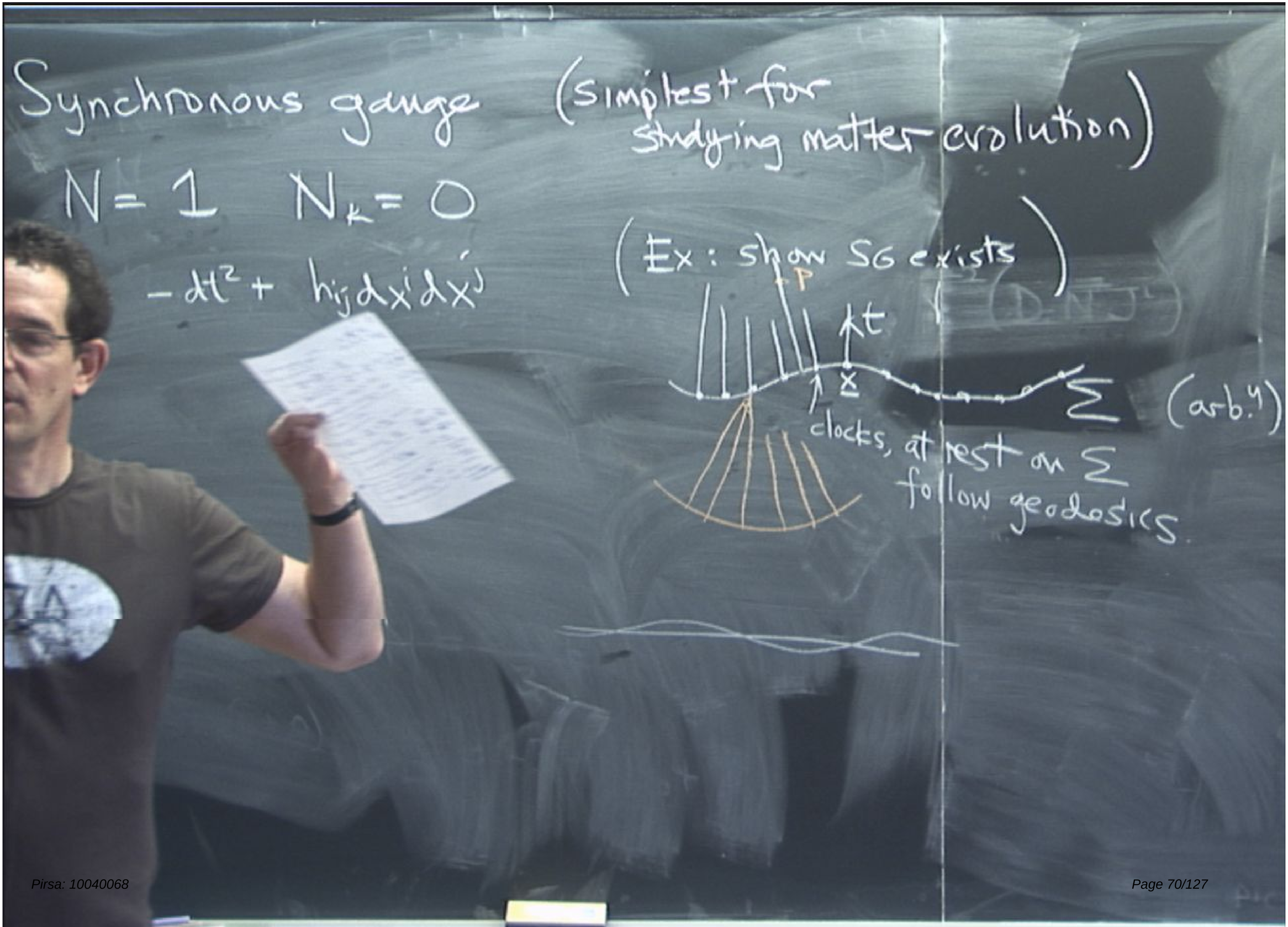
Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

(Ex: show SG exists)



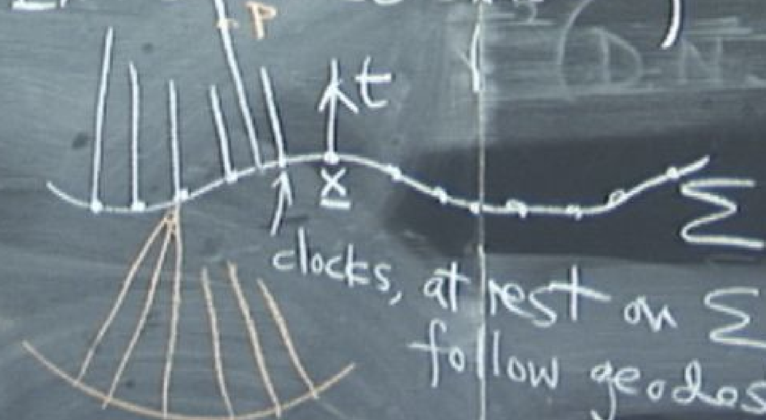

Synchronous gauge (simplest for studying matter evolution)
$N = 1$ $N_k = 0$
$-dt^2 + h_{ij}dx^i dx^j$
(Ex: show SG exists)
P
t
x
Σ (arb.y)
clocks, at rest on Σ
follow geodesics.

Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

d ... $h_{ij}\, dx^i dx^j$

(Ex: show SG exists)



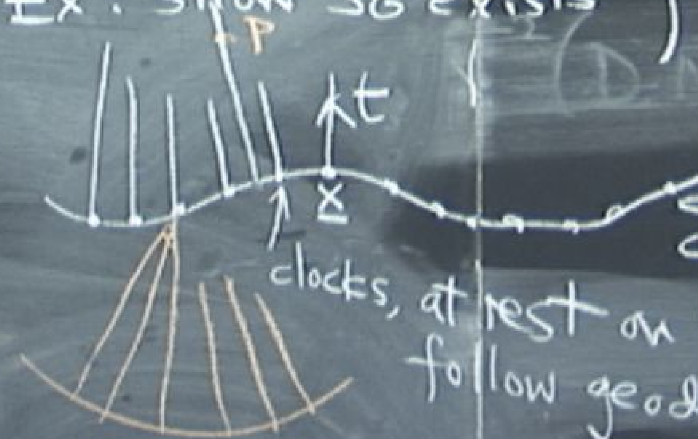
Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

$N = N_0(t)$

(Ex: show SG exists)



Σ (arb

clocks, at rest on Σ
follow geodesics

Synchronous gauge (simplest for studying matter evolution)

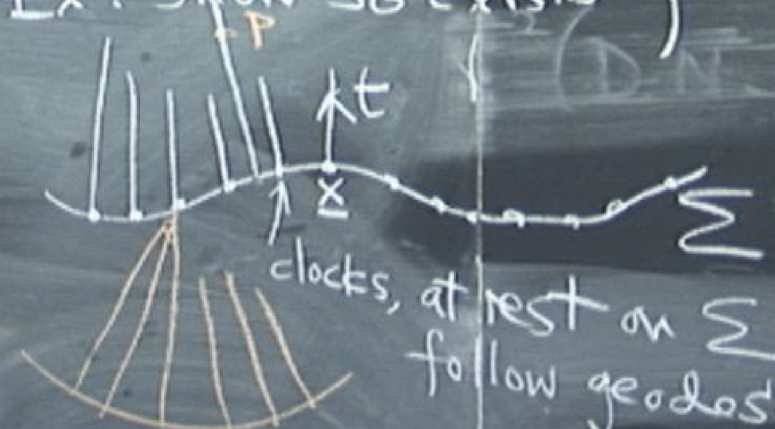
$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

(Ex: show SG exists)

$N = N_0(t)$

$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$



(arb

clocks, at rest on Σ follow geodesics

Synchronous gauge (simplest for studying matter evolution)

$$N = 1 \quad N_k = 0$$

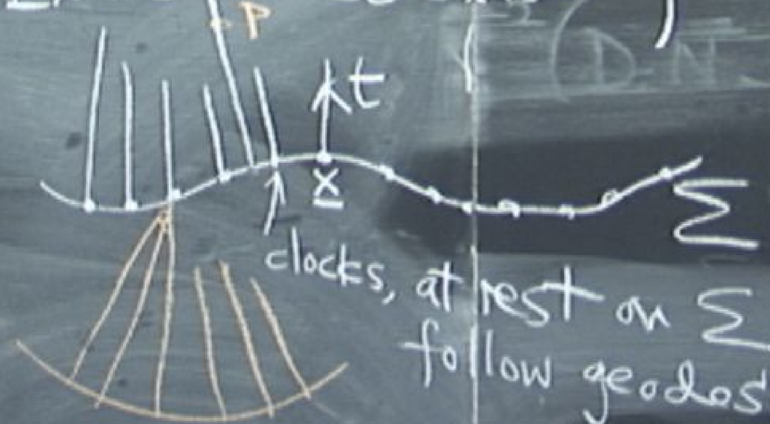
$$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$$

$$N = N_0(t)$$

$$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$$

$$h_{ij} = a^2\left(f_{ij} + \delta h_{ij}\right)$$

(Ex: show SG exists)



Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}dx^i dx^j$

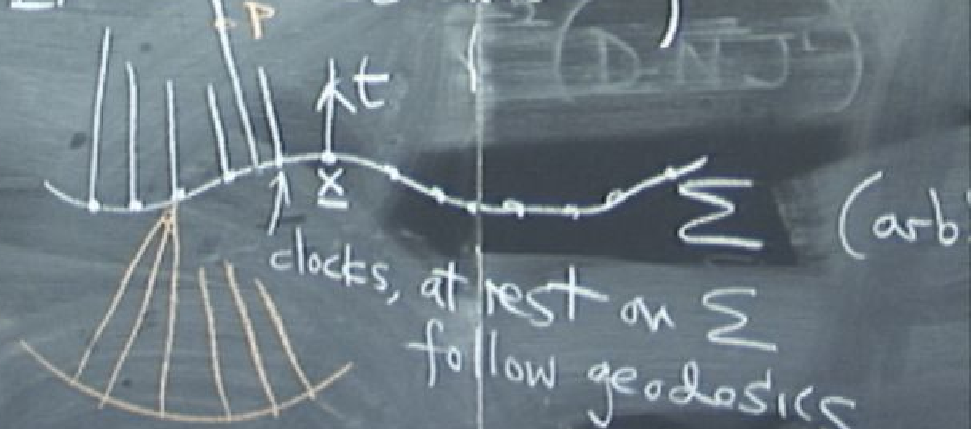
$N = N_0(t)$

$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}dx^i dx^j$

$h_{ij} = a^2(f_{ij} + \delta h_{ij})$

$\delta h = \delta h^i{}_i$

(Ex: show SG exists)



Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

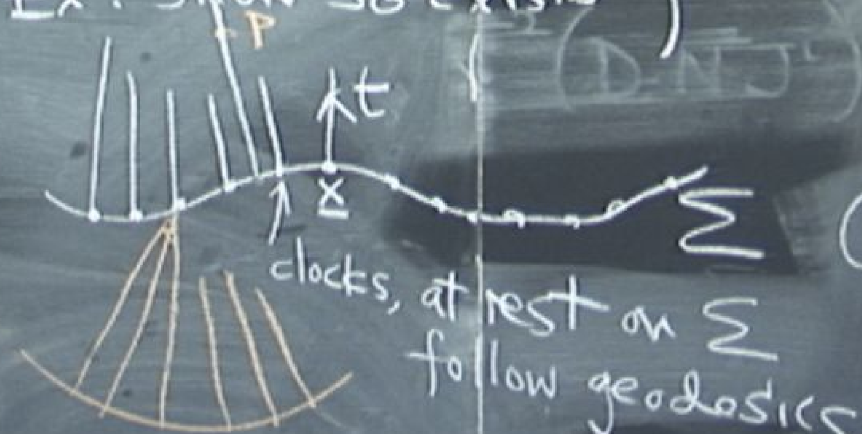
(Ex: show SG exists)

$N = N_0(t)$

$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$

$$h_{ij} = a^2\big(f_{ij} + \underbrace{\delta h_{ij}}_{\text{1st order}}\big)$$

$\delta h = \delta h^i{}_i$



(arb

Synchronous gauge (simplest for studying matter evolution)

$N = 1 \quad N_k = 0$

(Ex: show SG exists)

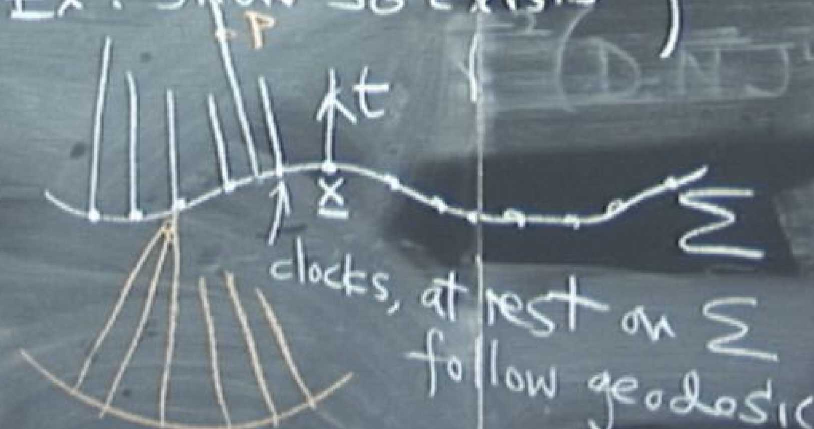
$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

$N = N_0(t)$

$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$

$h_{ij} = a^2(f_{ij} + \underbrace{\delta h_{ij}}_{\text{1st order}})$

$\delta h = \delta h^i{}_i$ (index raised with f^{ij})



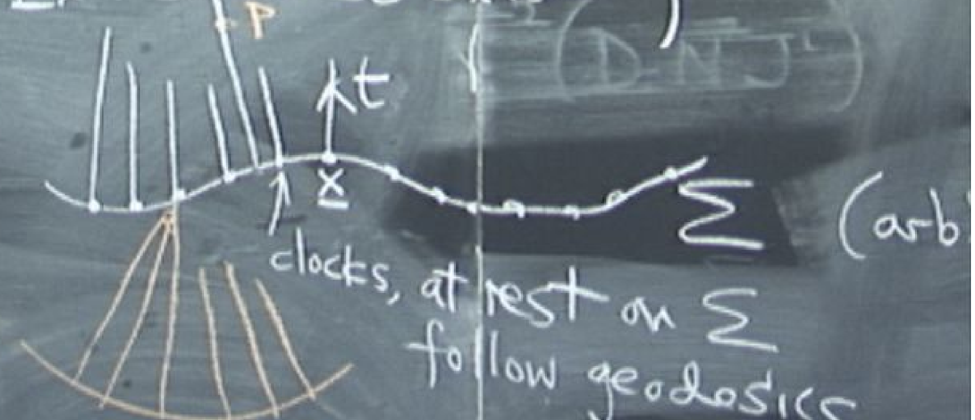
(arb

Synchronous gauge (simplest for studying matter evolution)

$$N = 1 \qquad N_k = 0$$

(Ex: show SG exists)

$$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$$



$$N = N_0(t)$$

$$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$$

clocks, at rest on Σ follow geodesics

$$h_{ij} = a^2\left(f_{ij} + \underbrace{\delta h_{ij}}_{\text{1st order}}\right)$$

$$\delta h = \delta h^i{}_i \quad (\text{index raised with } f^{ij})$$

Synchronous gauge (simplest for studying matter evolution)

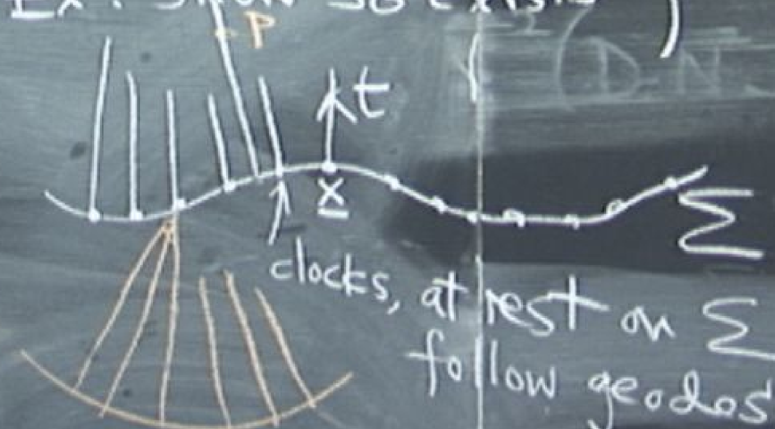
$N = 1 \quad N_k = 0$

(Ex: show SG exists)

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

$N = N_0(t)$

$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$



clocks, at rest on Σ follow geodesics

(arb

$h_{ij} = a^2(f_{ij} + \underbrace{\delta h_{ij}}_{\text{1st order}})$

$\delta h = \delta h^i{}_i$ (index raised with f^{ij})

Synchronous gauge (simplest for studying matter evolution)

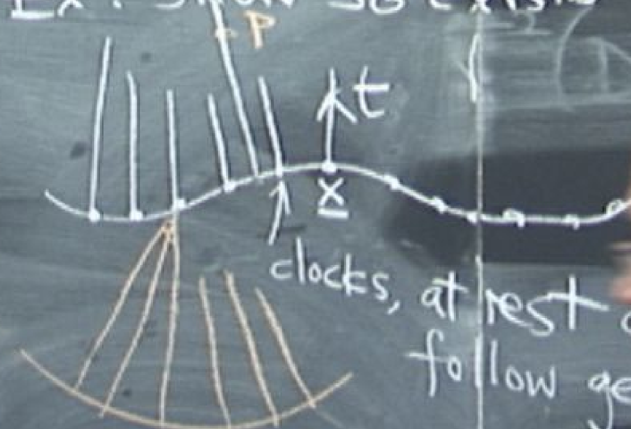
$N = 1 \quad N_k = 0$

$ds^2 = -dt^2 + h_{ij} dx^i dx^j$

(Ex: show SG exists)

$N = N_0(t)$

$ds^2 = -N_0^2(t)\, dt^2 + h_{ij} dx^i dx^j$



clocks, at rest a…
follow geo…

$h_{ij} = a^2 (f_{ij} + \delta h_{ij})$ ← 1st order

$\delta h = \delta h^i{}_i$ (index raised with f^{ij})

Synchronous gauge (simplest for studying matter evolution)

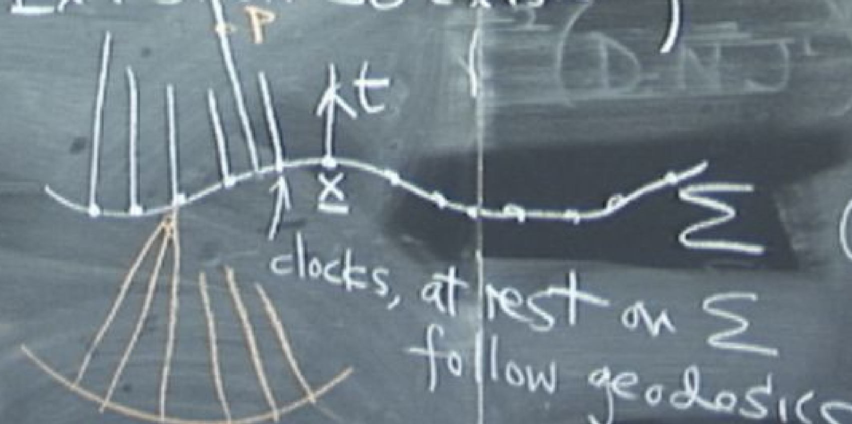
$N = 1 \quad N_k = 0$

(Ex: show SG exists)

$ds^2 = -dt^2 + h_{ij}\,dx^i dx^j$

$N = N_0(t)$

$ds^2 = -N_0^2(t)\,dt^2 + h_{ij}\,dx^i dx^j$



$$h_{ij} = a^2\big(f_{ij} + \underbrace{\delta h_{ij}}_{\text{1st order}}\big)$$

$\delta h = \delta h^i{}_i$ (index raised with f^{ij})

(in literature δh_{ij} is called h_{ij})

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon + \pi) + (E_0 + P_0)(k - 3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0 + P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

evolution)

ists)

Σ (arb.y)

rest on Σ

llow geodesics

evolution)

ists)

Σ (arb.y)

rest on Σ
llow geodesics

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon+\pi) + (E_0+P_0)(k-3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$$\delta h = 2(3\varphi + \Delta_c \gamma)$$

$$\Delta = a^{-2} f^{ij} D_i D_j = a^{-2}\Delta_c$$

evolution)

ists)

Σ (arb.y)

rest on Σ
llow geodesics

1st order pert^{n} equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon + \pi) + (E_0 + P_0)(k - 3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0 + P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$$\delta h = 2(3\varphi + \Delta_c \gamma)$$

$$\Delta = a^{-2} f^{ij} D_i D \ldots$$
$$= a^{-2}\Delta_c$$

$$K = -3\frac{\dot{\varphi}}{N_0} - \Delta_c \frac{\dot{\gamma}}{N} = -\frac{\delta \dot{h}}{\ldots}$$

evolution)

ists)

Σ (arb.y)

rest on Σ

llow geodesics

1st order pert^n equations

$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon+\pi) + (E_0+P_0)(k-3H\alpha) - \Delta\psi$$

$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma$$

$$\delta h = 2(3\varphi + \Delta_c \gamma) \qquad \Delta = a^{-2} f^{ij} D_i D_j = a^{-2}\Delta_c$$

$$K = -3\frac{\dot{\varphi}}{N_0} - \Delta_c \frac{\dot{\gamma}}{N_0} = -\frac{\delta \dot{h}}{2N_0}$$

volution)

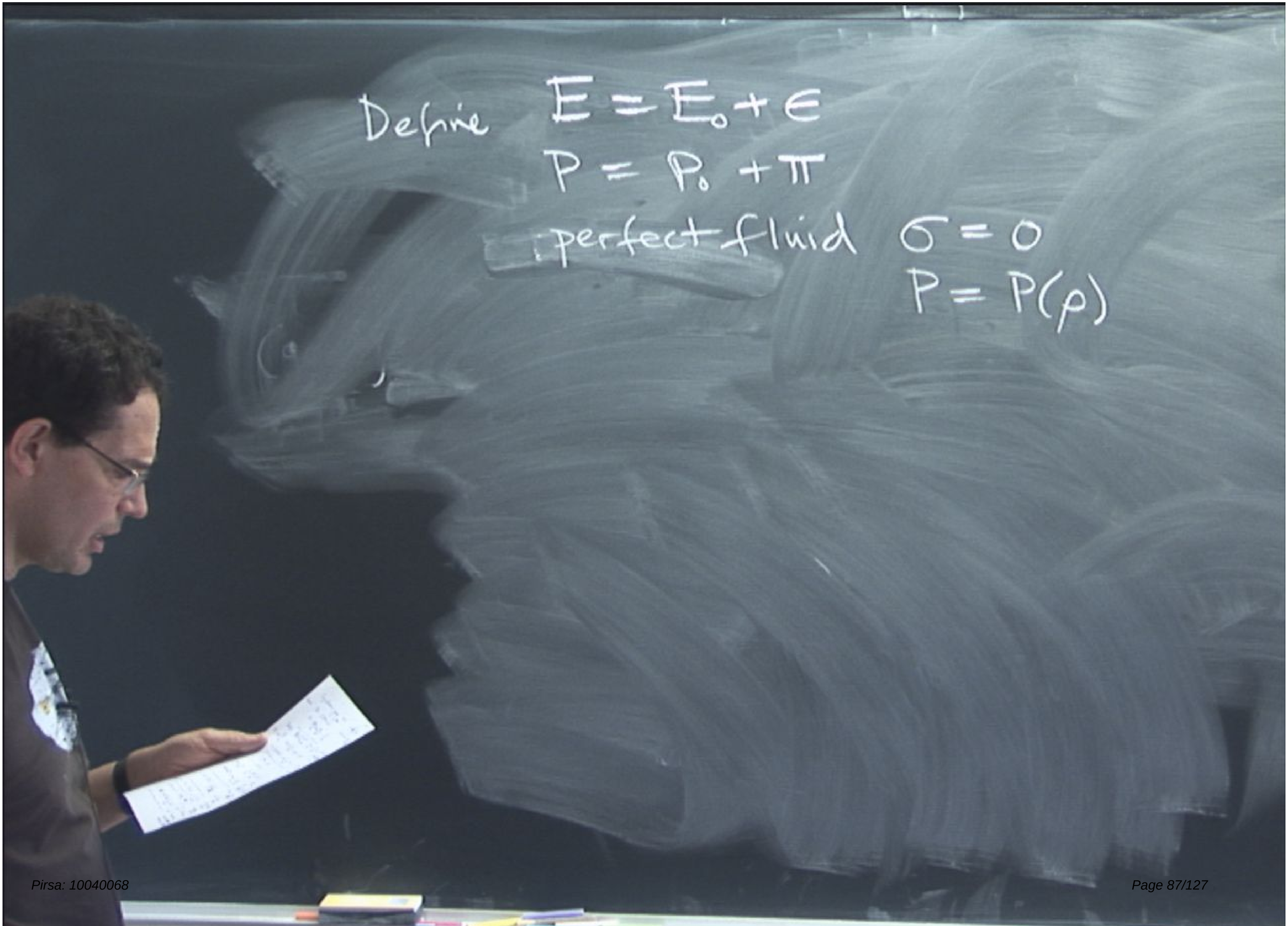
1st order pert^{n} equations

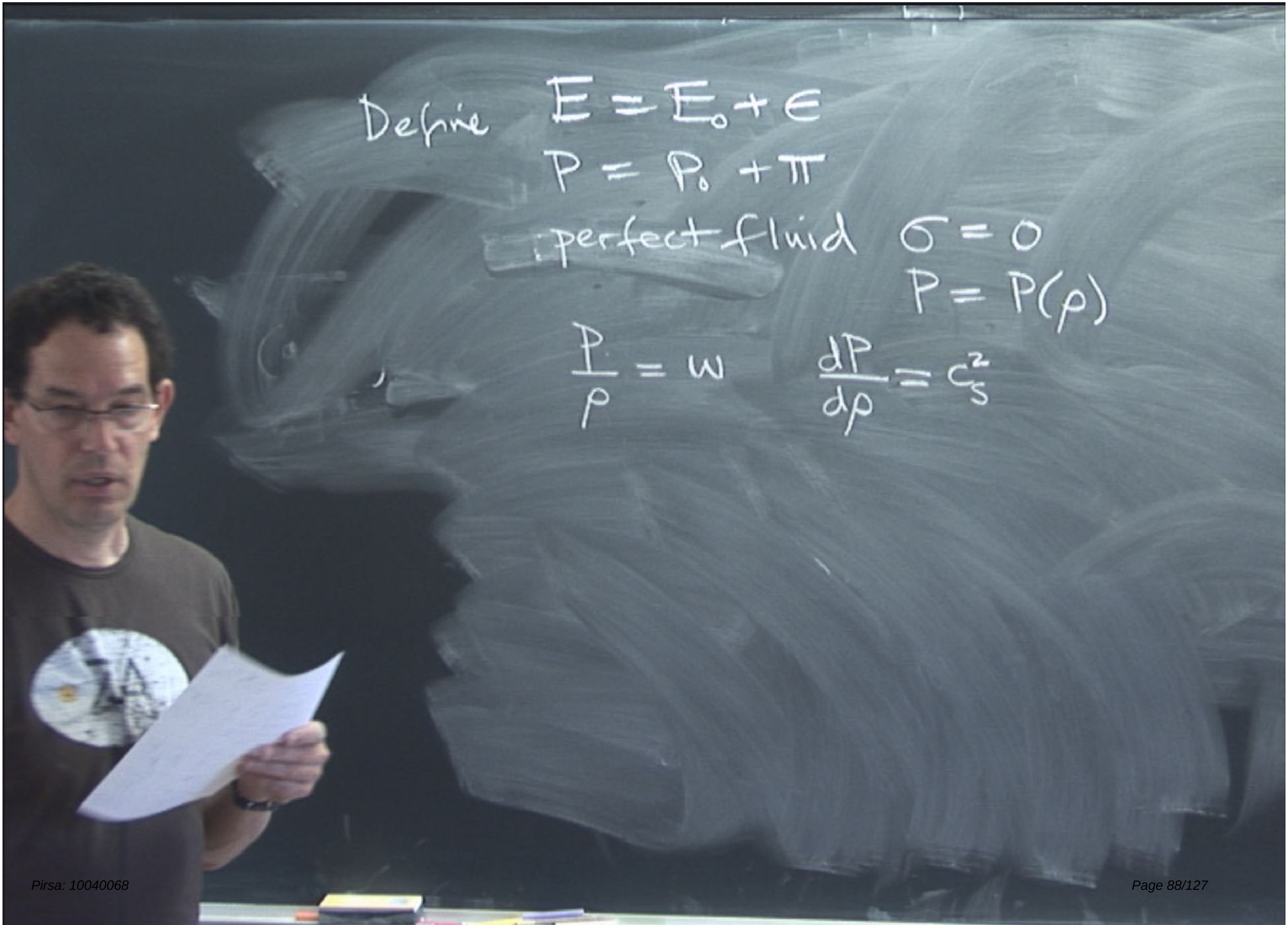
$$\frac{\dot{\epsilon}}{N_0} = -3H(\epsilon+\pi) + (E_0+P_0)(K-3H\alpha) - \Delta\psi \qquad (P1)$$

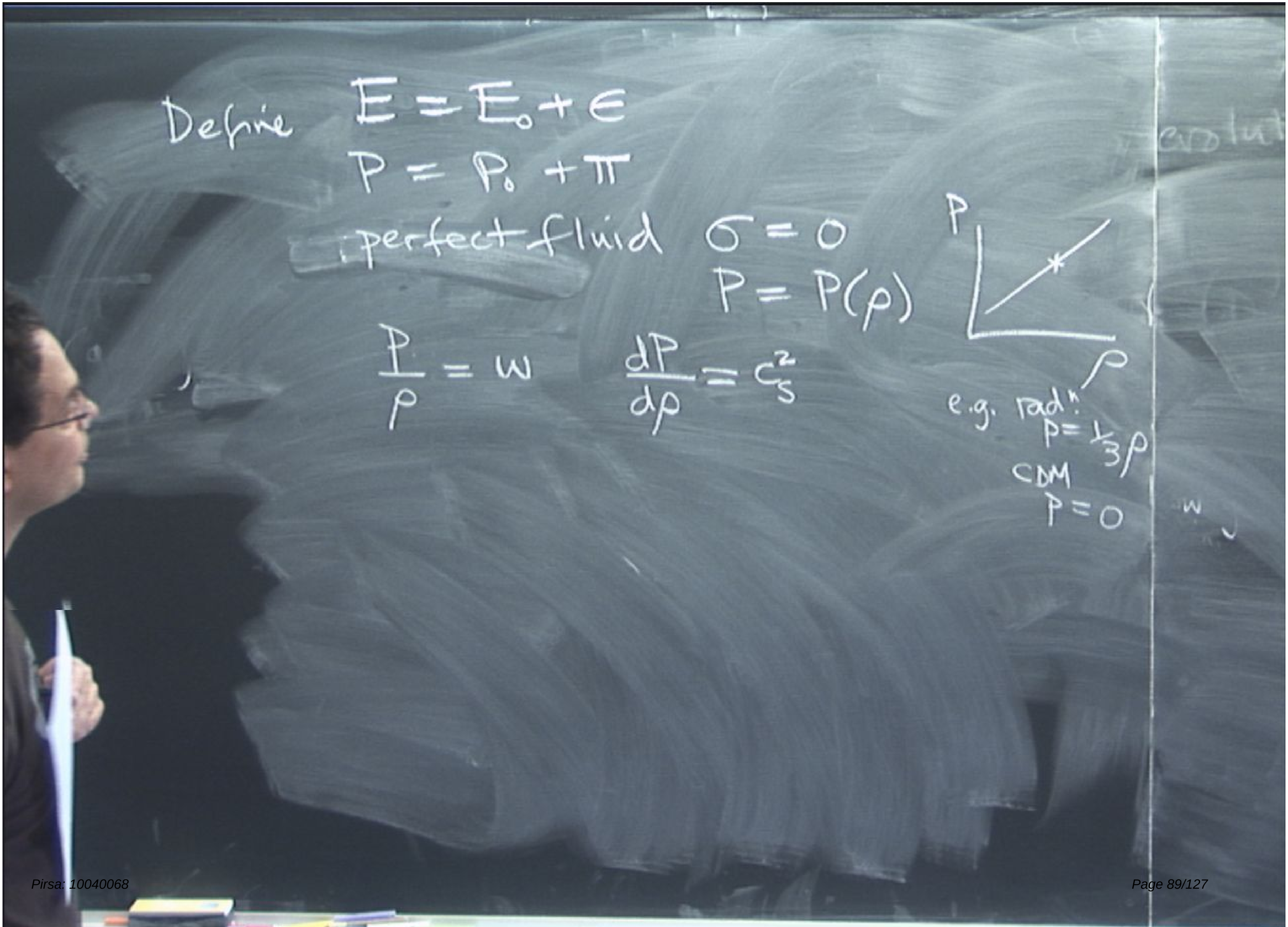
$$\frac{\dot{\psi}}{N_0} = -3H\psi - (E_0+P_0)\alpha - \pi - \frac{2}{3}\left(\Delta + \frac{3c}{a^2}\right)\sigma \qquad (P2)$$

$$\delta h = 2(3\varphi + \Delta_c \gamma) \qquad \Delta = a^{-2} f^{ij} D_i D_j$$

$$K = -3\frac{\dot{\varphi}}{N_0} - \Delta_c \frac{\dot{\gamma}}{N_0} = -\frac{\delta \dot{h}}{2N_0} \qquad = a^{-2}\Delta_c$$


Define $E = E_0 + \epsilon$
$P = P_0 + \pi$
perfect fluid $\sigma = 0$
$P = P(\rho)$


Define $E = E_0 + \epsilon$
$P = P_0 + \pi$
perfect fluid $\sigma = 0$
$P = P(\rho)$
$\frac{P}{\rho} = w$
$\frac{dP}{d\rho} = c_s^2$


Define $E = E_0 + \epsilon$
$P = P_0 + \pi$
perfect fluid $\sigma = 0$
$P = P(\rho)$
$\frac{P}{\rho} = w$
$\frac{dP}{d\rho} = c_s^2$
P
ρ
e.g. rad^n: $P = \frac{1}{3}\rho$
CDM $P = 0$

Define $E = E_0 + \epsilon$

$P = P_0 + \Pi$

perfect fluid $\sigma = 0$

$P = P(\rho)$

$$\frac{P}{\rho} = w \qquad \frac{dP}{d\rho} = c_s^2$$

e.g. rad^n: $P = \frac{1}{3}\rho$

CDM $P = 0$

$$\Rightarrow \Pi = c_s^2 \epsilon$$

Define $E = E_0 + \epsilon$

$P = P_0 + \pi$

perfect fluid $\sigma = 0$

$P = P(\rho)$

$$\frac{P}{\rho} = w \qquad \frac{dP}{d\rho} = c_s^2$$

e.g. rad^n: $P = \frac{1}{3}\rho$

CDM $P = 0$

$$\Rightarrow \pi = c_s^2 \epsilon$$

$$P1 \Longrightarrow \frac{\dot{\epsilon}}{N_0} = -3H(1+c_s^2)\epsilon + (P_0+\rho_0)K - \Delta\psi$$

Define $E = E_0 + \epsilon$

$P = P_0 + \Pi$

perfect fluid $\sigma = 0$

$P = P(\rho)$

$\frac{P}{\rho} = w \qquad \frac{dP}{d\rho} = c_s^2$

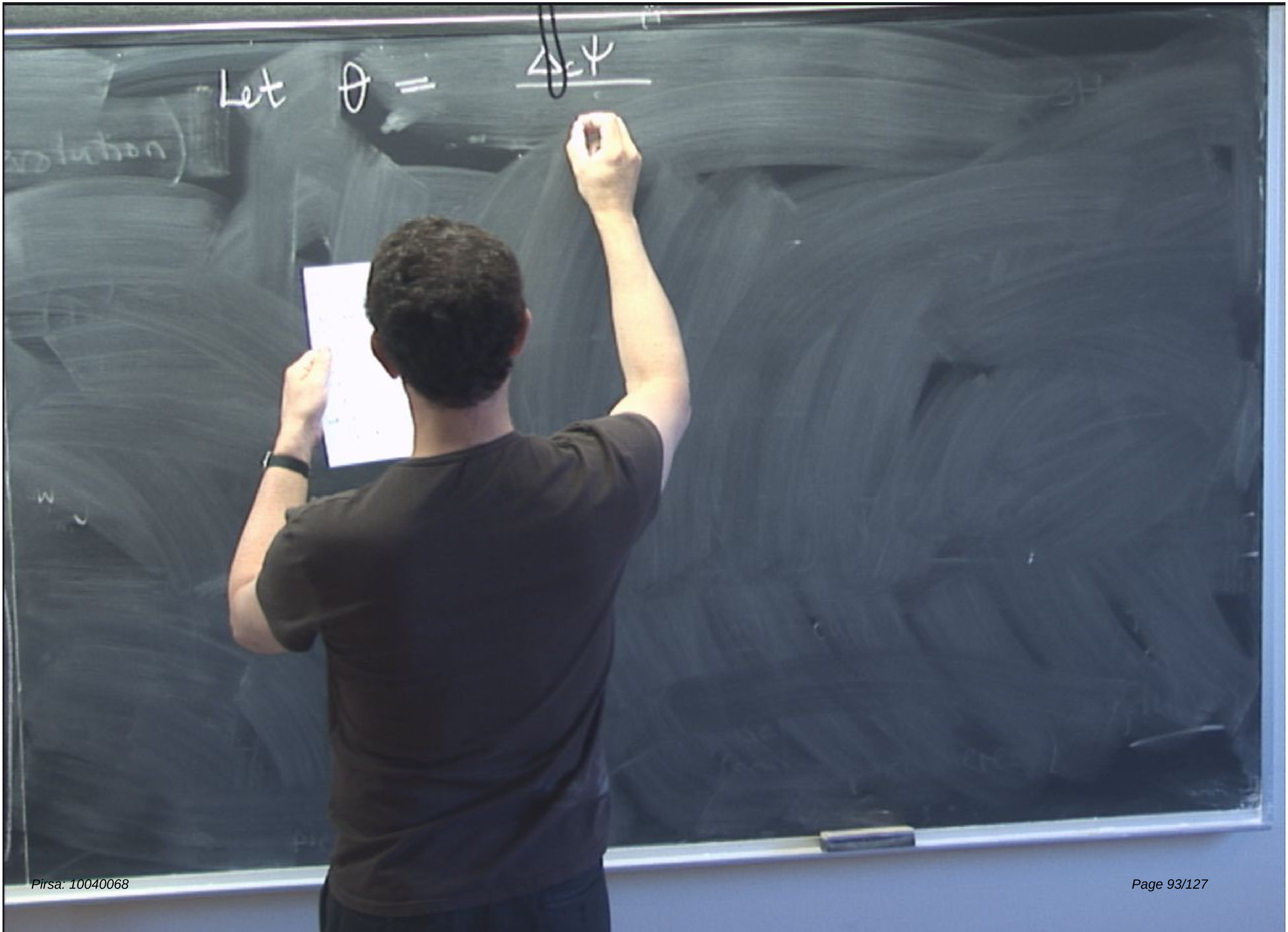
e.g. rad^n: $P = \frac{1}{3}\rho$

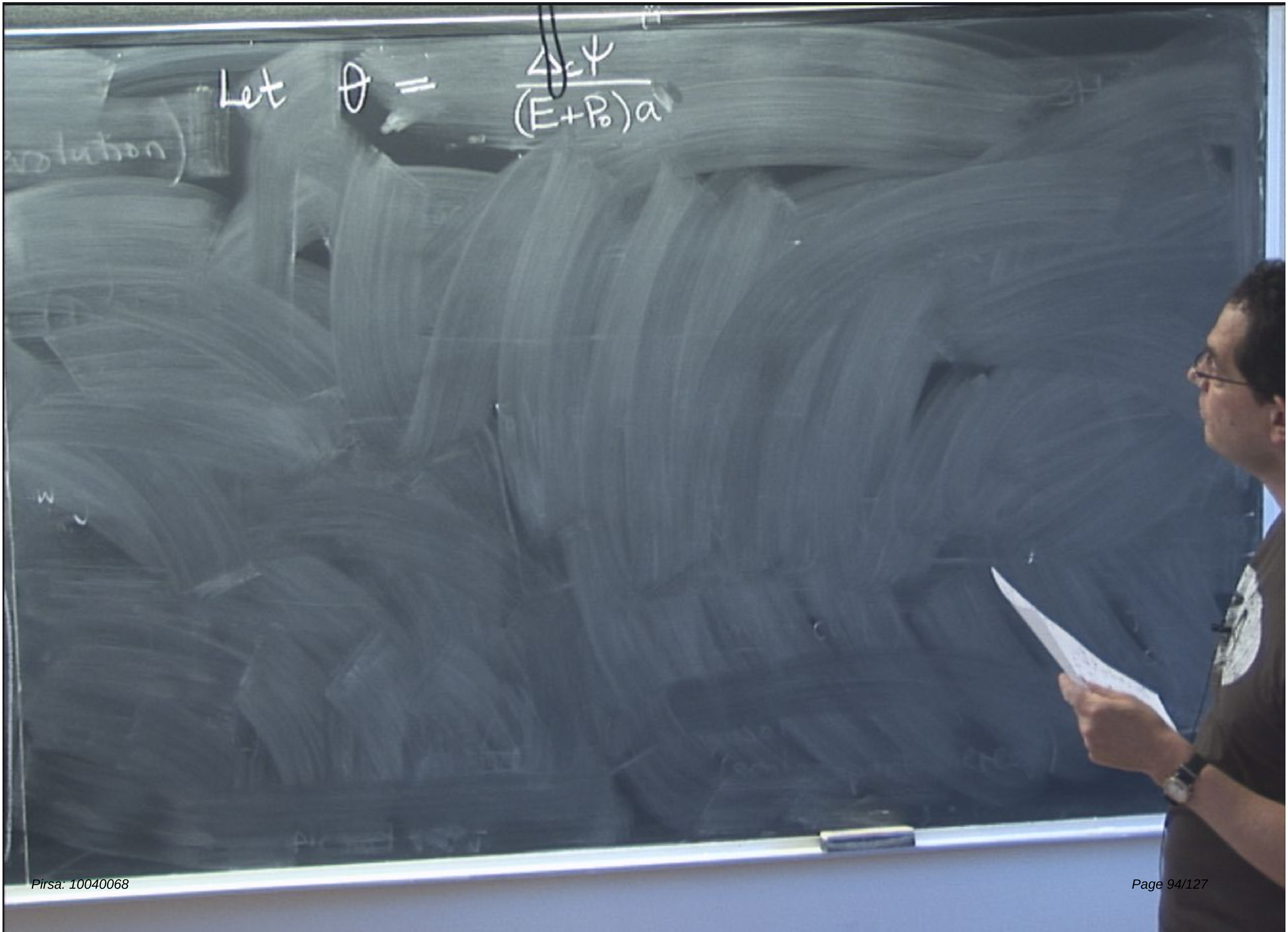
CDM $P = 0$

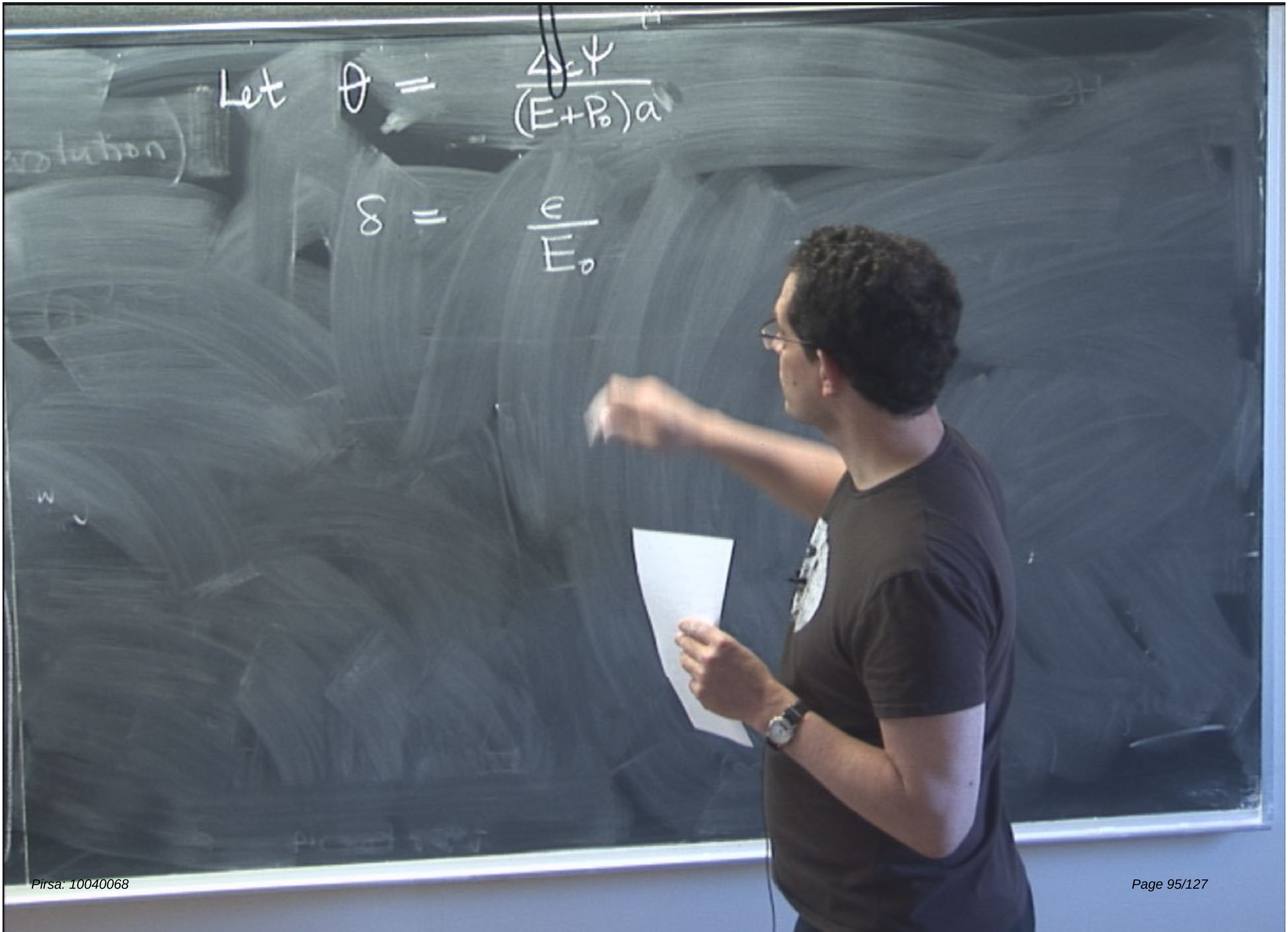
$\Rightarrow \Pi = c_s^2 \epsilon$

P1 $\Longrightarrow$

$$\boxed{\begin{aligned} \frac{\dot{\epsilon}}{N_0} &= -3H(1+c_s^2)\epsilon + (P_0+\rho_0)K - \Delta\psi \\ \frac{\dot{\psi}}{N_0} &= -3H\psi - c_s^2\epsilon \end{aligned}}$$


Let θ = Δcψ

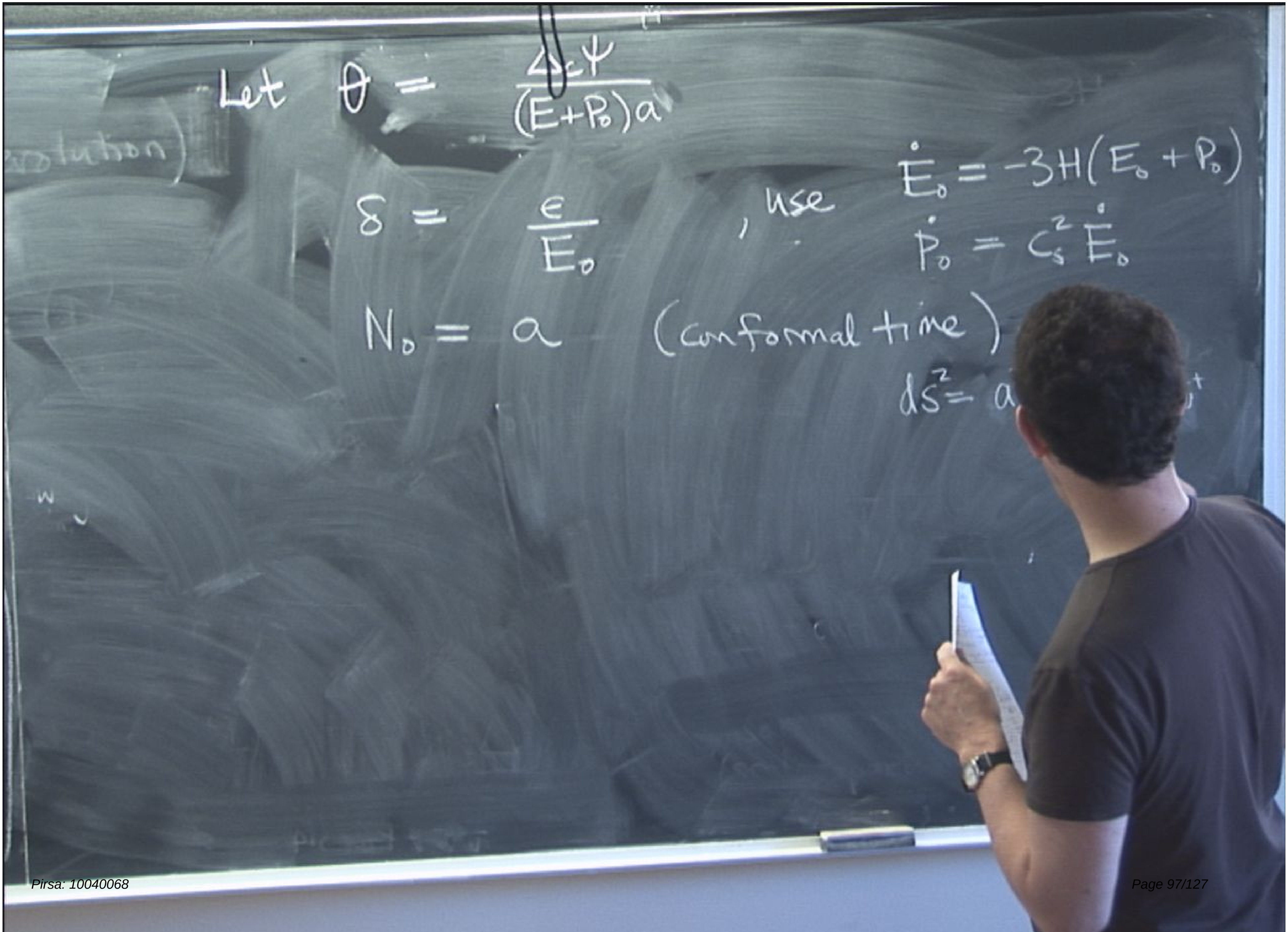

Let $\theta = \frac{\Delta_c \psi}{(E+P_0)a}$


Let $\theta = \frac{\Delta_c \psi}{(E+P_0)a}$
$\delta = \frac{\epsilon}{E_0}$

Let $\theta = \dfrac{\Delta_c \psi}{(E+P_0)a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$


Let θ = Δ_c ψ / (E+P₀)a
(evolution)
δ = ε / E₀ , use Ė₀ = −3H(E₀ + P₀)
Ṗ₀ = c_s² Ė₀
N₀ = a (conformal time)
ds² = a

Let $\theta = \dfrac{\Delta_c \Psi}{(E+P_0)a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$

$N_0 = a$ (conformal time)

$$ds^2 = a^2\left[-dt^2 + (f_{ij} + \delta h_{ij})\,dx^i dx^j\right]$$

Define $E = E_0 + \epsilon$

$P = P_0 + \Pi$

perfect fluid $\sigma = 0$

$P = P(\rho)$

$\frac{P}{\rho} = w \qquad \frac{dP}{d\rho} = c_s^2$

e.g. rad^n: $P = \frac{1}{3}\rho$

CDM $P = 0$

$\Rightarrow \Pi = c_s^2 \epsilon$

P1 $\Longrightarrow$

$$\boxed{\begin{aligned} \frac{\dot{\epsilon}}{N_0} &= -3H(1+c_s^2)\epsilon + (P_0+E_0)K - \Delta\psi \\ \frac{\dot{\psi}}{N_0} &= -3H\psi - c_s^2\epsilon \end{aligned}}$$

...olution)

Let $\theta = \dfrac{\Delta_c \psi}{(E+P_0)a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$

$N_0 = a$ (conformal time)

$$ds^2 = a^2\left[-dt^2 + (f_{ij} + \delta h_{ij})dx^i dx^j\right]$$

Ex:

(…olution)

Let $\theta = \frac{\Delta_c \psi}{(E+P_0)a}$

$\delta = \frac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$

$N_0 = a$ (conformal time)

$H = \frac{\dot{a}}{N_0 a} = \frac{\dot{a}}{a^2}$ $ds^2 = a^2\left[-dt^2 + (f_{ij} + \delta h_{ij})dx^i dx^j\right]$

Ex:

Let $\theta = \dfrac{\Delta_c \psi}{(E+P_0)a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$

$N_0 = a$ (conformal time)

$H = \dfrac{\dot{a}}{N_0 a} = \dfrac{\dot{a}}{a^2}$ $ds^2 = a^2\left[-dt^2 + (f_{ij} + \delta h_{ij})dx^i dx^j\right]$

$= \frac{1}{a}\mathcal{H}$ $\mathcal{H} = \frac{\dot{a}}{a}$

Ex:

Let $\theta = \dfrac{\Delta_c \psi}{(E+P_0)\,a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$

$N_0 = a$ (conformal time)

$H = \dfrac{\dot a}{N_0 a} = \dfrac{\dot a}{a^2}$ $\quad ds^2 = a^2\left[-dt^2 + (f_{ij} + \delta h_{ij})\,dx^i dx^j\right]$

$= \dfrac{1}{a}\mathcal{H} \qquad \mathcal{H} = \dfrac{\dot a}{a}$

Ex:

$$\dot\delta = 3\mathcal{H}(c_s^2 - w)\delta - \frac{\dot h}{2}(1+w) - (1+w)\theta$$

$$\dot\theta = -\mathcal{H}(1-3c_s^2)\theta - \frac{c_s^2}{(1+w)}\Delta_c \delta$$

Let $\theta = \dfrac{\Delta_c \Psi}{(E+P_0)a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 = -3H(E_0 + P_0)$

$\dot{P}_0 = c_s^2 \dot{E}_0$

$N_0 = a$ (conformal time)

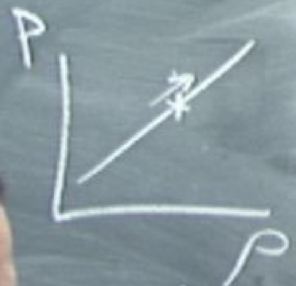
$H = \dfrac{\dot{a}}{N_0 a} = \dfrac{\dot{a}}{a^2}$ $\quad ds^2 = a^2\left[-dt^2 + (f_{ij} + \delta h_{ij})dx^i dx^j\right]$

$= \dfrac{1}{a}\mathcal{H}$ $\quad \mathcal{H} = \dfrac{\dot{a}}{a}$

Ex:

$$\boxed{\begin{aligned} \dot{\delta} &= 3\mathcal{H}(c_s^2 - w)\delta - \frac{\dot{h}}{2}(1+w) - (1+w)\theta \\ \dot{\theta} &= -\mathcal{H}(1-3c_s^2)\theta - \frac{c_s^2}{(1+w)}\Delta_c \delta \end{aligned}}$$

evolution)



e.g. rad^n: $p = \frac{1}{3}\rho$

NM $= 0$

Ex:

$$\delta = \frac{\delta\rho}{\rho}$$

$$\theta = \nabla \cdot v$$

Let $\theta = \dfrac{\Delta_c \Psi}{(E+P_0)a}$

$\delta = \dfrac{\epsilon}{E_0}$, use $\dot{E}_0 =$

$\dot{P}_0 =$

$N_0 = a$ (conformal time)

$H = \dfrac{\dot{a}}{N_0 a} = \dfrac{\dot{a}}{a^2}$ $\quad ds^2 = a^2\left[-dt^2 + (f_{i}\right.$

$= \dfrac{1}{a}\mathcal{H} \qquad \mathcal{H} = \dfrac{\dot{a}}{a}$

$$\boxed{\begin{aligned} \dot{\delta} &= 3\mathcal{H}(c_s^2 - w)\delta - \frac{\dot{h}}{2}(1+w) - (1+w)\theta \\ \dot{\theta} &= -\mathcal{H}(1-3c_s^2)\theta - \frac{c_s^2}{(1+w)}\Delta_c \delta \end{aligned}}$$

(solution)

e.g. rad^n: $p = \frac{1}{3}\rho$

NM $= 0$

Let $\theta = \frac{\Delta_c \psi}{(E+P_0)a}$

$\delta = \frac{\epsilon}{E_0}$, use $\dot{E}_0 =$ ... $\dot{P}_0 =$ ...

$N_0 = a$ (conformal time)

$H = \frac{\dot{a}}{N_0 a} = \frac{\dot{a}}{a^2}$ $\quad ds^2 = a^2\left[-dt^2 + (f_{ij} ...\right.$

$= \frac{1}{a}\mathcal{H} \qquad \mathcal{H} = \frac{\dot{a}}{a}$

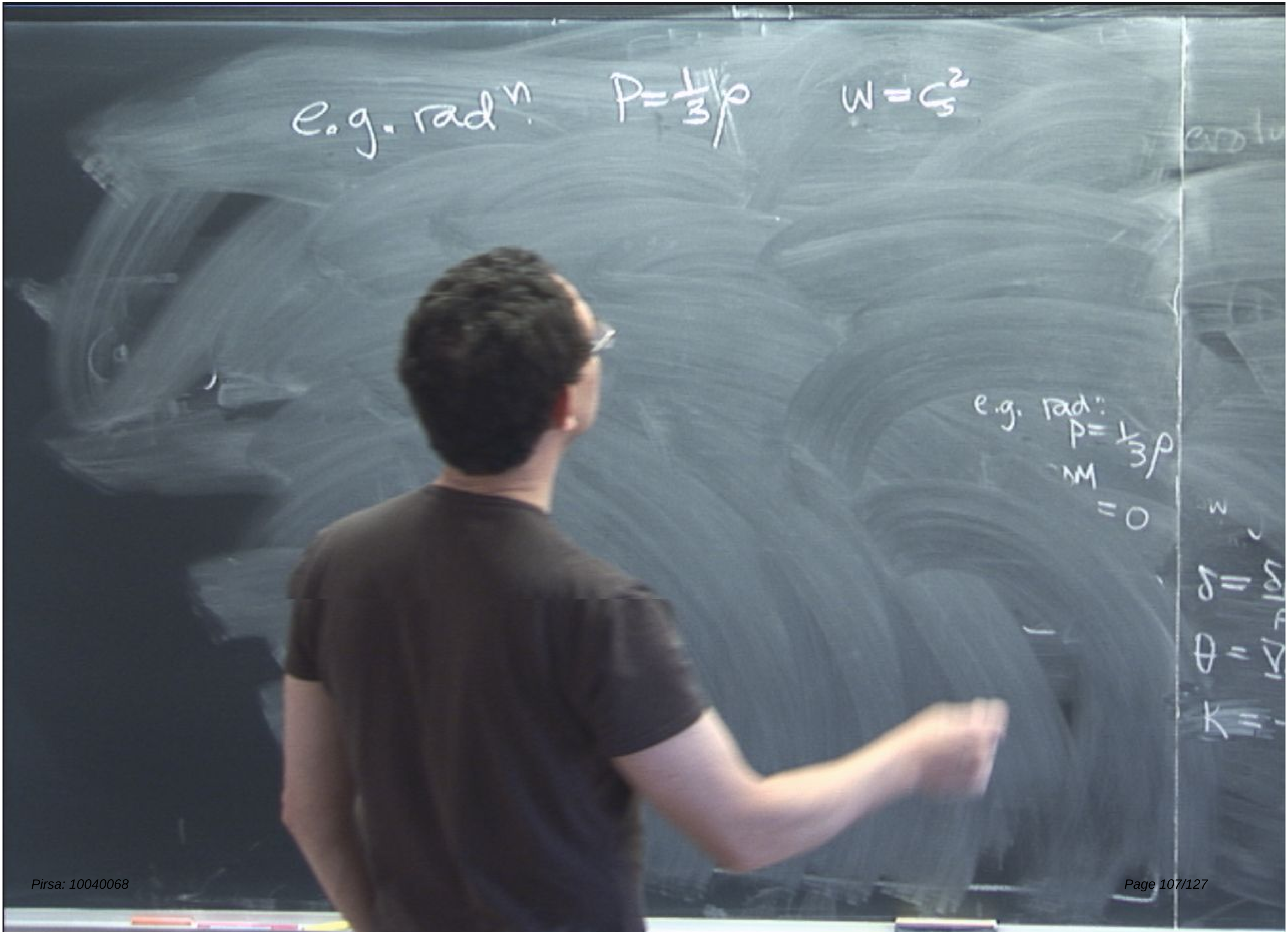
Ex:

$\delta = \frac{\delta\rho}{\rho}$

$\theta = \underline{\nabla}\cdot\underline{v}$

$K = -\frac{\delta\dot{h}}{2N_0}$

$$\boxed{\begin{aligned} \dot{\delta} &= 3\mathcal{H}(c_s^2 - w)\delta - \frac{\delta\dot{h}}{2}(1+w) - (1+w)\theta \\ \dot{\theta} &= -\mathcal{H}(1-3c_s^2)\theta - \frac{c_s^2}{(1+w)}\Delta_c\delta \end{aligned}}$$


e.g. radn: P = 1/3 ρ
w = c_s^2
e.g. radn: P = 1/3 ρ
=0
δ =
θ =
K =

e.g. rad^n: $P = \frac{1}{3}\rho$ $w = c_s^2 = \frac{1}{3}$

$$\dot{\delta} = -\frac{2}{3}\delta\dot{h} - \frac{4}{3}\theta$$

$$\dot{\theta} = -\frac{1}{4}\Delta_c \delta$$

e.g. rad^n: $P = \frac{1}{3}\rho$

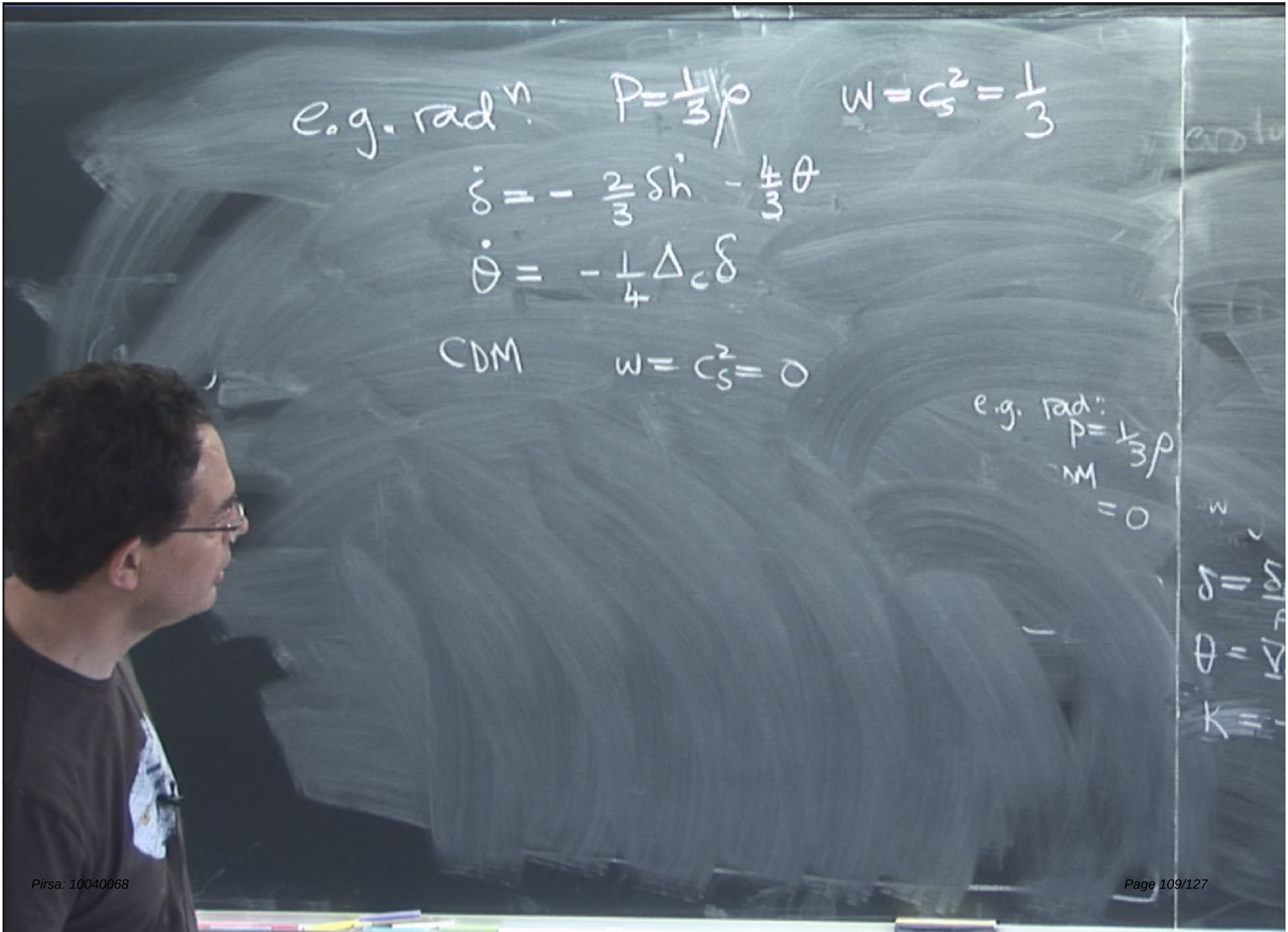
...M

$= 0$

$\delta = \frac{\delta}{\rho}$...

$\theta = \nabla$...

$K = -$...



e.g. rad^n: $P = \frac{1}{3}\rho$ $w = c_s^2 = \frac{1}{3}$
$\dot{\delta} = -\frac{2}{3}\delta\dot{h} - \frac{4}{3}\theta$
$\dot{\theta} = -\frac{1}{4}\Delta_c \delta$
CDM $w = c_s^2 = 0$
e.g. rad^n:
$P = \frac{1}{3}\rho$
$= 0$

e.g. rad^n: $P = \frac{1}{3}\rho$ $w = c_s^2 = \frac{1}{3}$

$$\dot{\delta} = -\frac{2}{3}\dot{\delta h} - \frac{4}{3}\theta$$

$$\dot{\theta} = -\frac{1}{4}\Delta_c \delta$$

CDM $w = c_s^2 = 0$

$$\dot{\delta} = -\frac{\dot{\delta h}}{2}$$

e.g. rad^n: $P = \frac{1}{3}\rho$

DM

$= 0$

w

$\delta = \frac{\delta}{\rho}$

$\theta = \nabla$

$K = -$

e.g. rad^n: $P = \frac{1}{3}\rho$ $\quad w = c_s^2 = \frac{1}{3}$

$$\boxed{\begin{aligned} \dot{\delta} &= -\frac{2}{3}\dot{\delta h} - \frac{4}{3}\theta \\ \dot{\theta} &= -\frac{1}{4}\Delta_c \delta \end{aligned}}$$

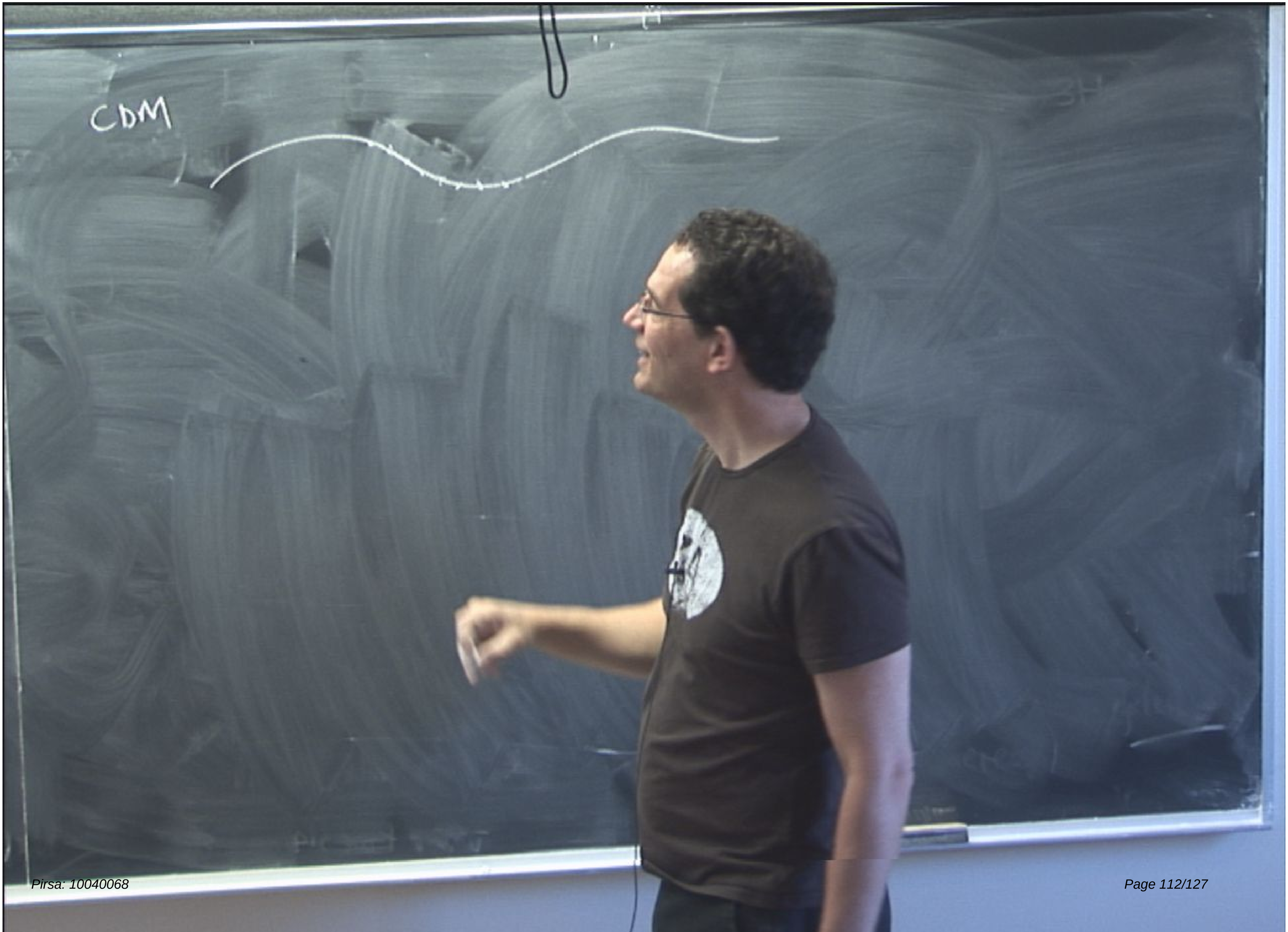
CDM $\quad w = c_s^2 = 0$

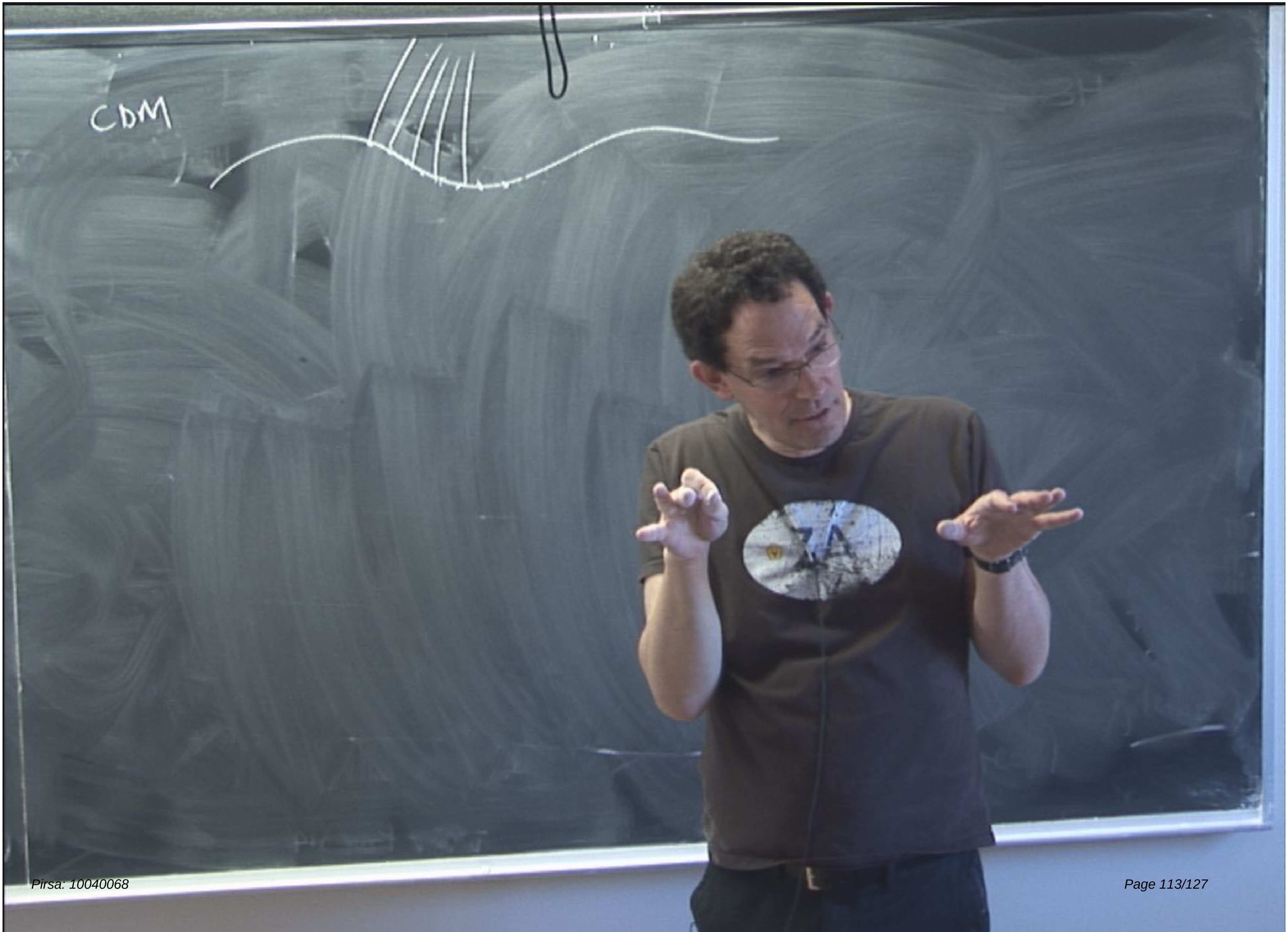
$$\boxed{\begin{aligned} \dot{\delta} &= -\frac{\dot{\delta h}}{2} - \theta \\ \dot{\theta} &= -\mathcal{H}\theta \end{aligned}}$$

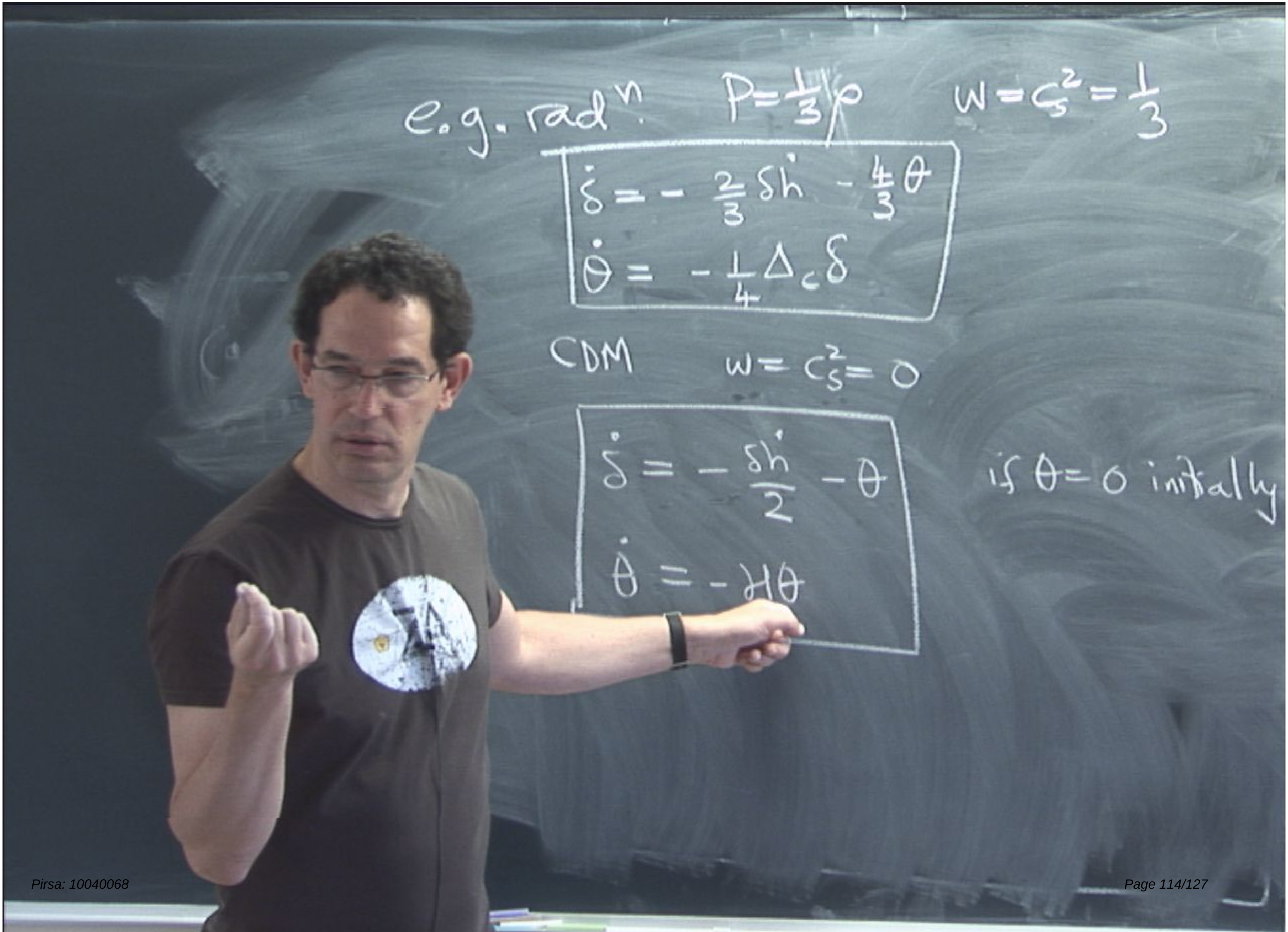
e.g. rad^n: $P = \frac{1}{3}\rho$

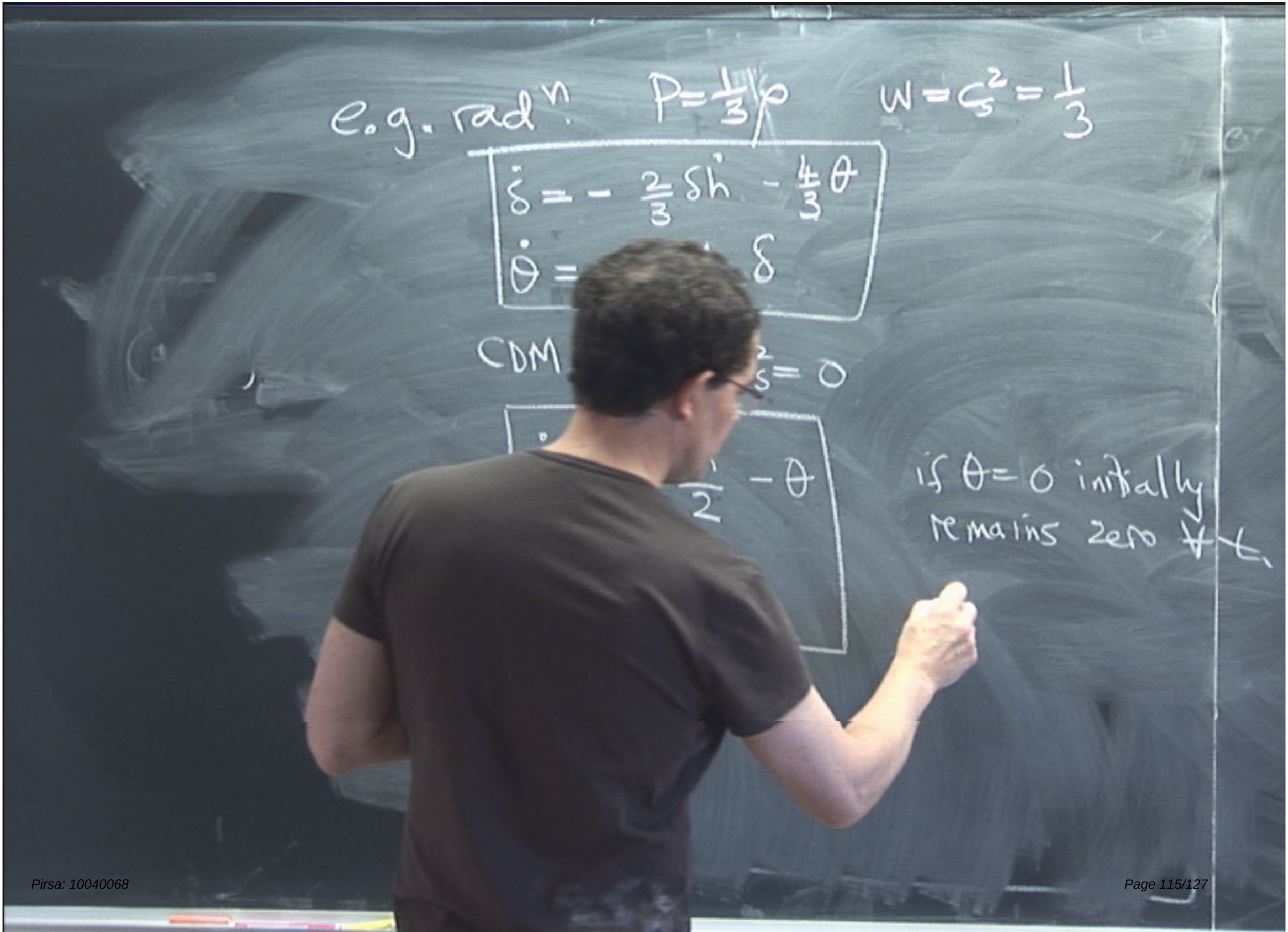
DM

$= 0$


CDM


CDM


e.g. rad^n: $P=\frac{1}{3}\rho$ $w=c_s^2=\frac{1}{3}$
$\dot{\delta} = -\frac{2}{3}\delta\dot{h} - \frac{4}{3}\theta$
$\dot{\theta} = -\frac{1}{4}\Delta_c\delta$
CDM $w=c_s^2=0$
$\dot{\delta} = -\frac{\delta\dot{h}}{2} - \theta$
$\dot{\theta} = -\mathcal{H}\theta$
if $\theta=0$ initially



e.g. rad^n: $P = \frac{1}{3}\rho$ $w = c_s^2 = \frac{1}{3}$
$\dot{\delta} = -\frac{2}{3}\dot{h} - \frac{4}{3}\theta$
$\dot{\theta} =$ δ
CDM
$= 0$
$- \theta$
if $\theta = 0$ initially
remains zero $\forall t$

e.g. rad^n $\quad P = \frac{1}{3}\rho \quad w = c_s^2 = \frac{1}{3}$

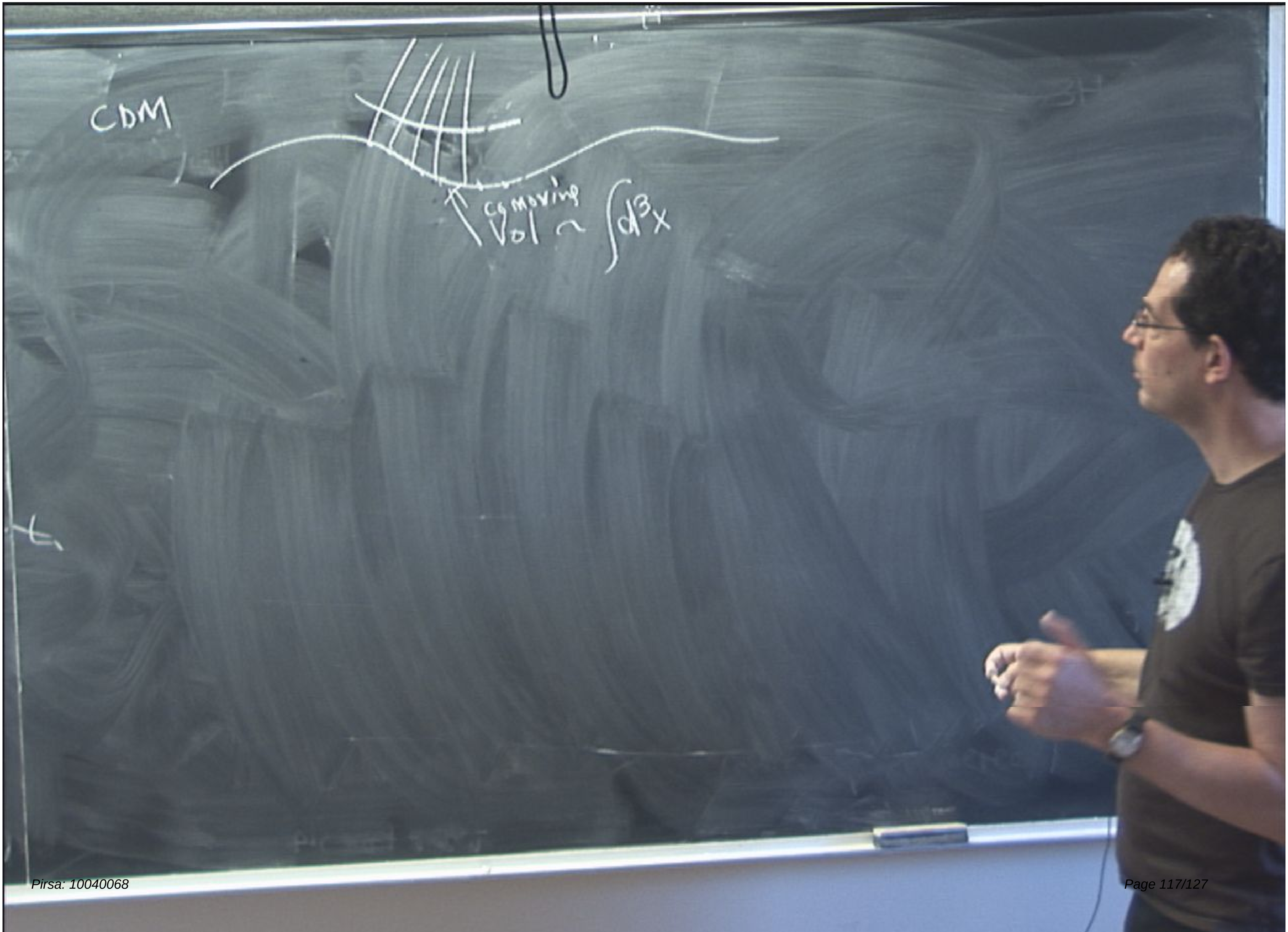
$$\boxed{\begin{aligned} \dot{\delta} &= -\frac{2}{3}\delta\dot{h} - \frac{4}{3}\theta \\ \dot{\theta} &= -\frac{1}{4}\Delta_c \delta \end{aligned}}$$

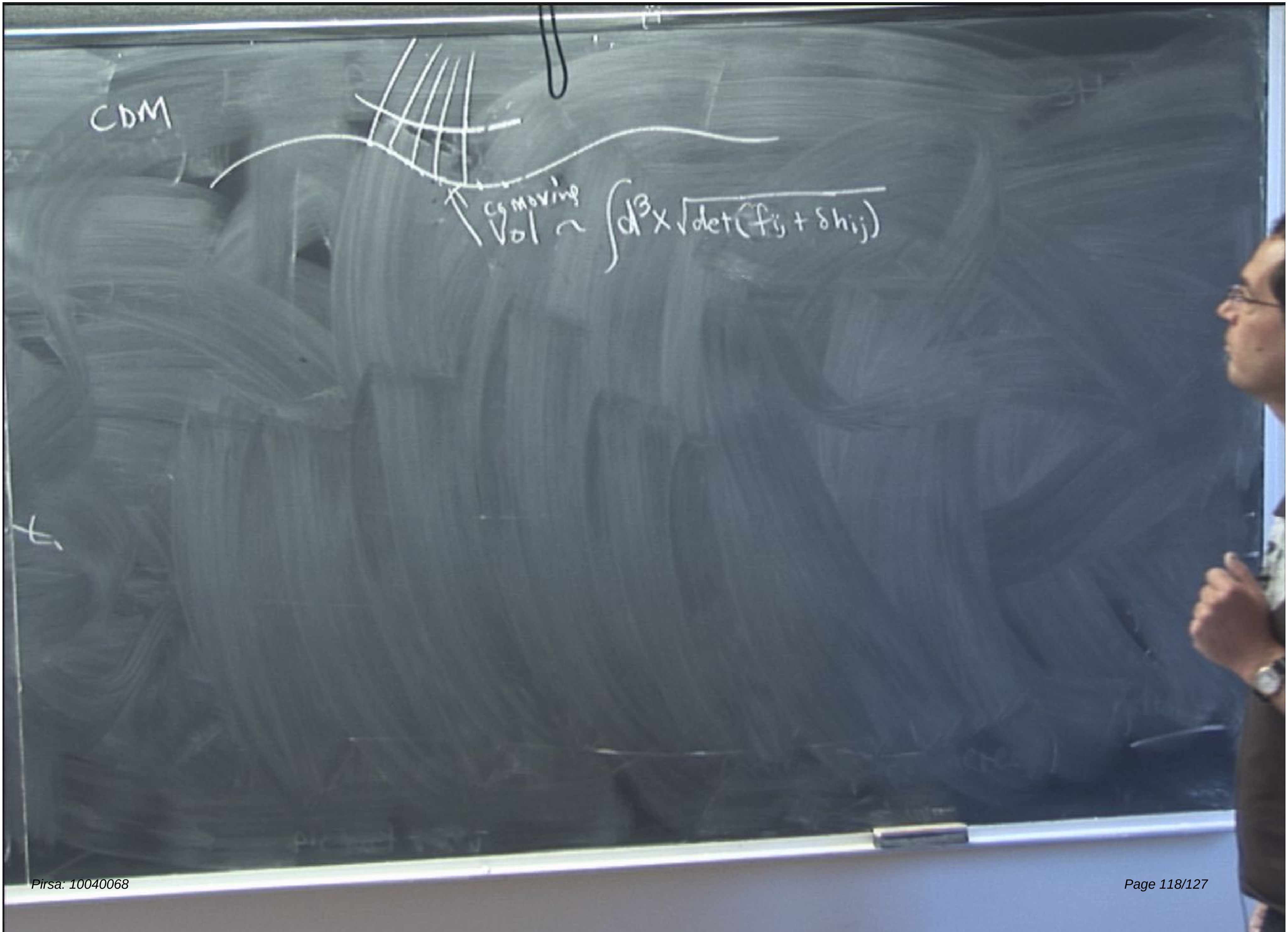
CDM $\quad w = c_s^2 = 0$

$$\boxed{\begin{aligned} \dot{\delta} &= -\frac{\delta\dot{h}}{2} - \theta \\ \dot{\theta} &= -\mathcal{H}\theta \end{aligned}}$$

if $\theta = 0$ initially remains zero $\forall\, t$.

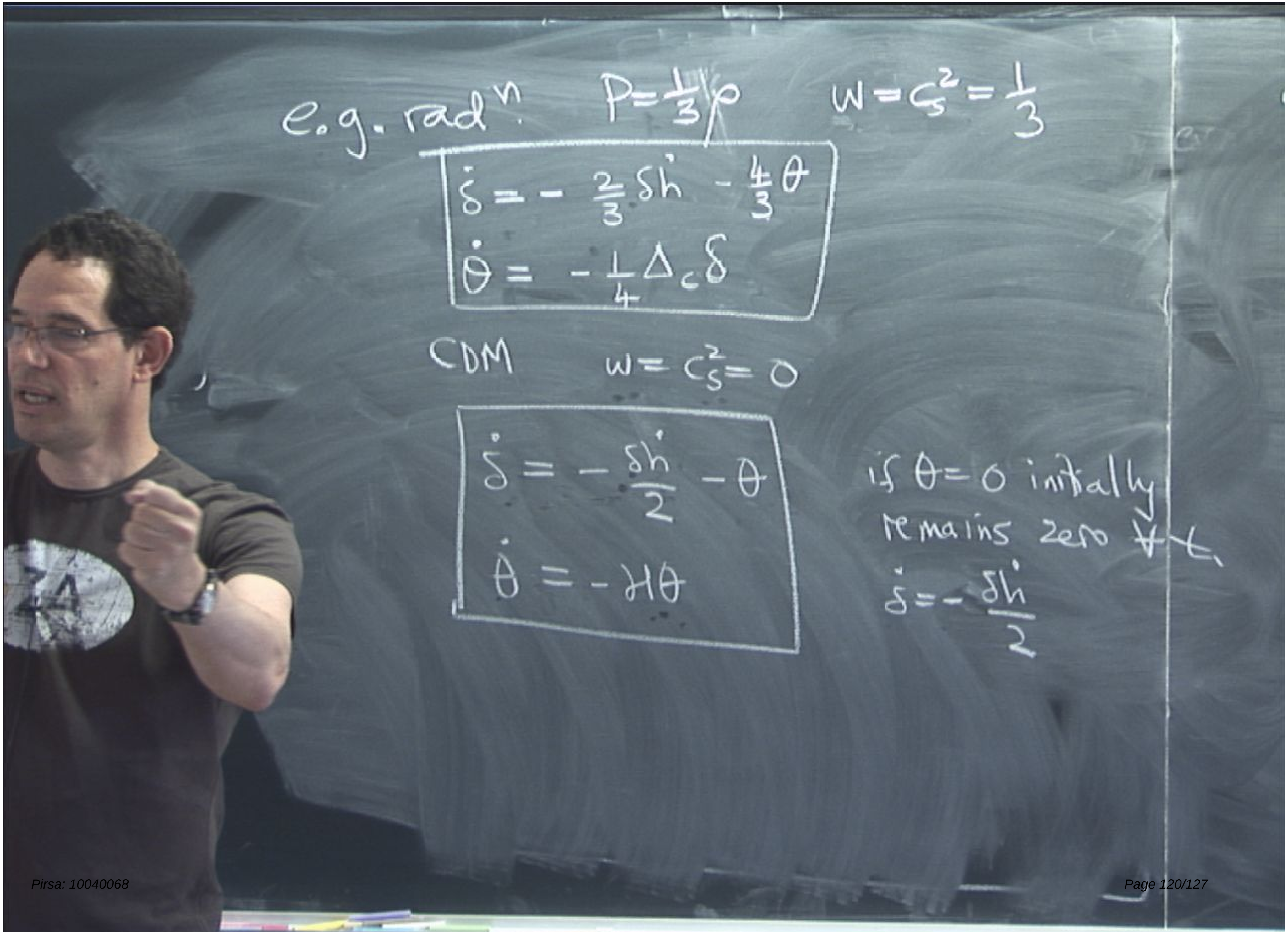
$$\dot{\delta} = -\frac{\delta\dot{h}}{2}$$


CDM
comoving
Vol ~ ∫d³x

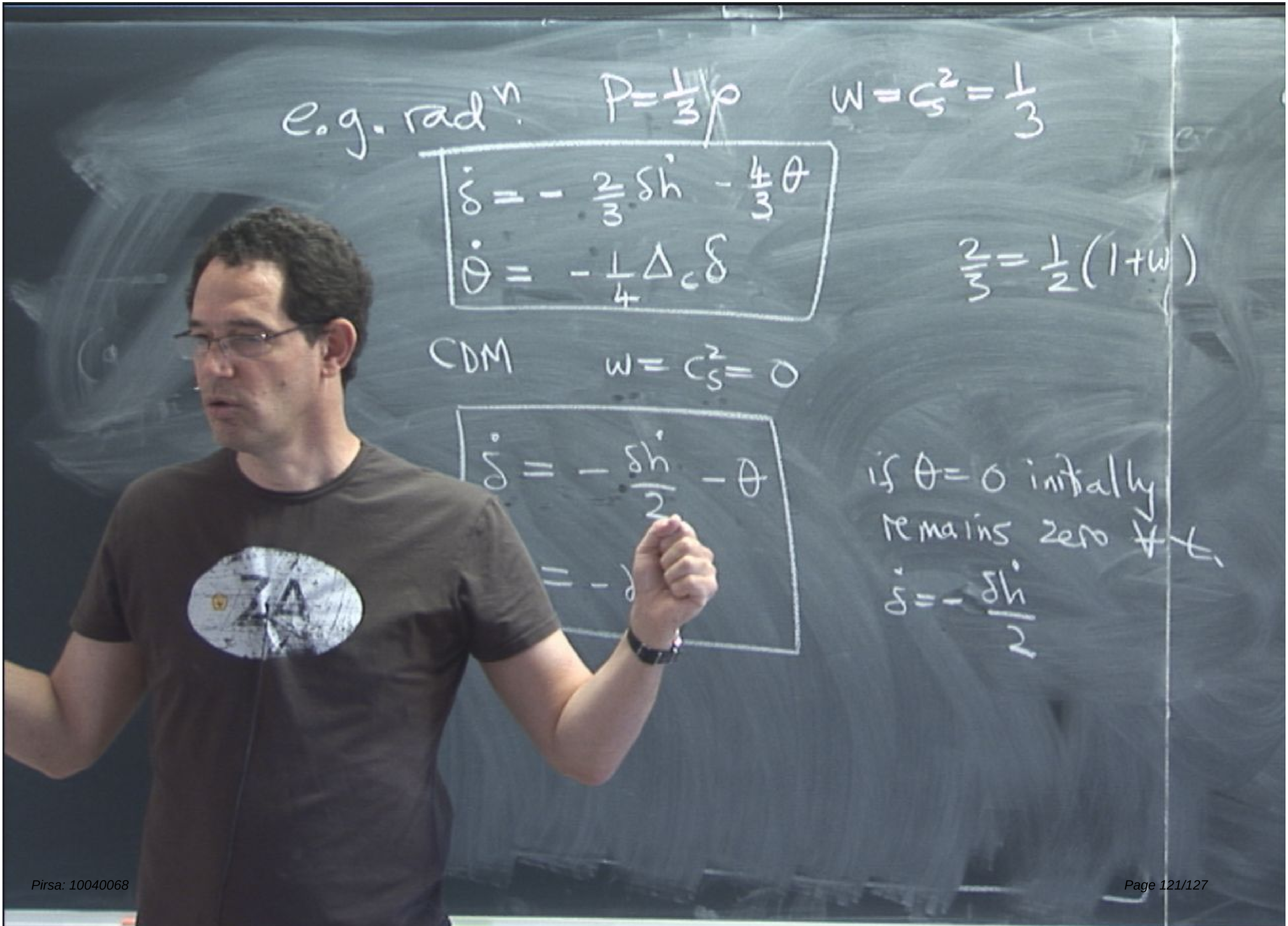

CDM
comoving
Vol ~ $\int d^3x \sqrt{\det(\gamma_{ij} + \delta h_{ij})}$

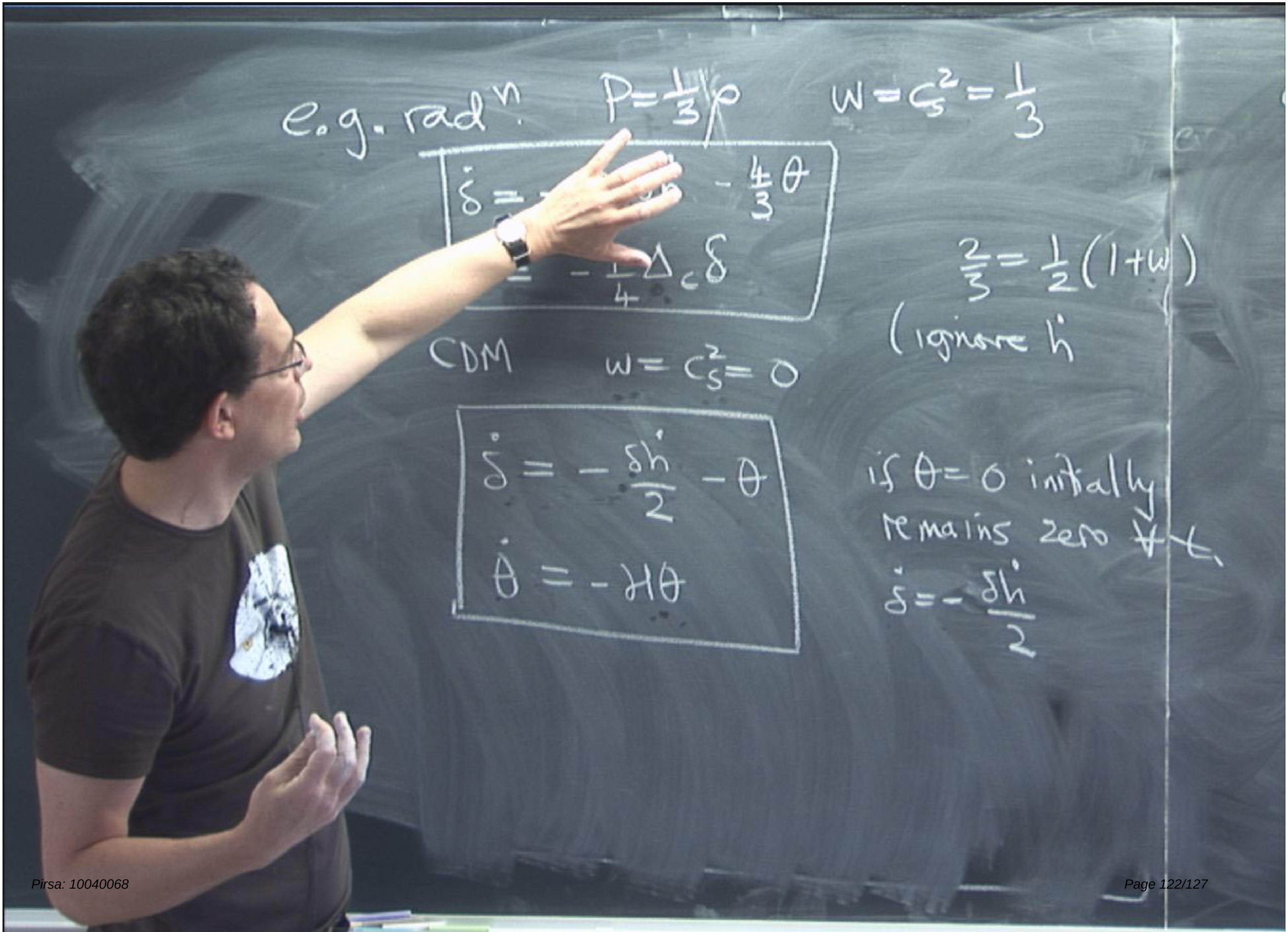
CDM

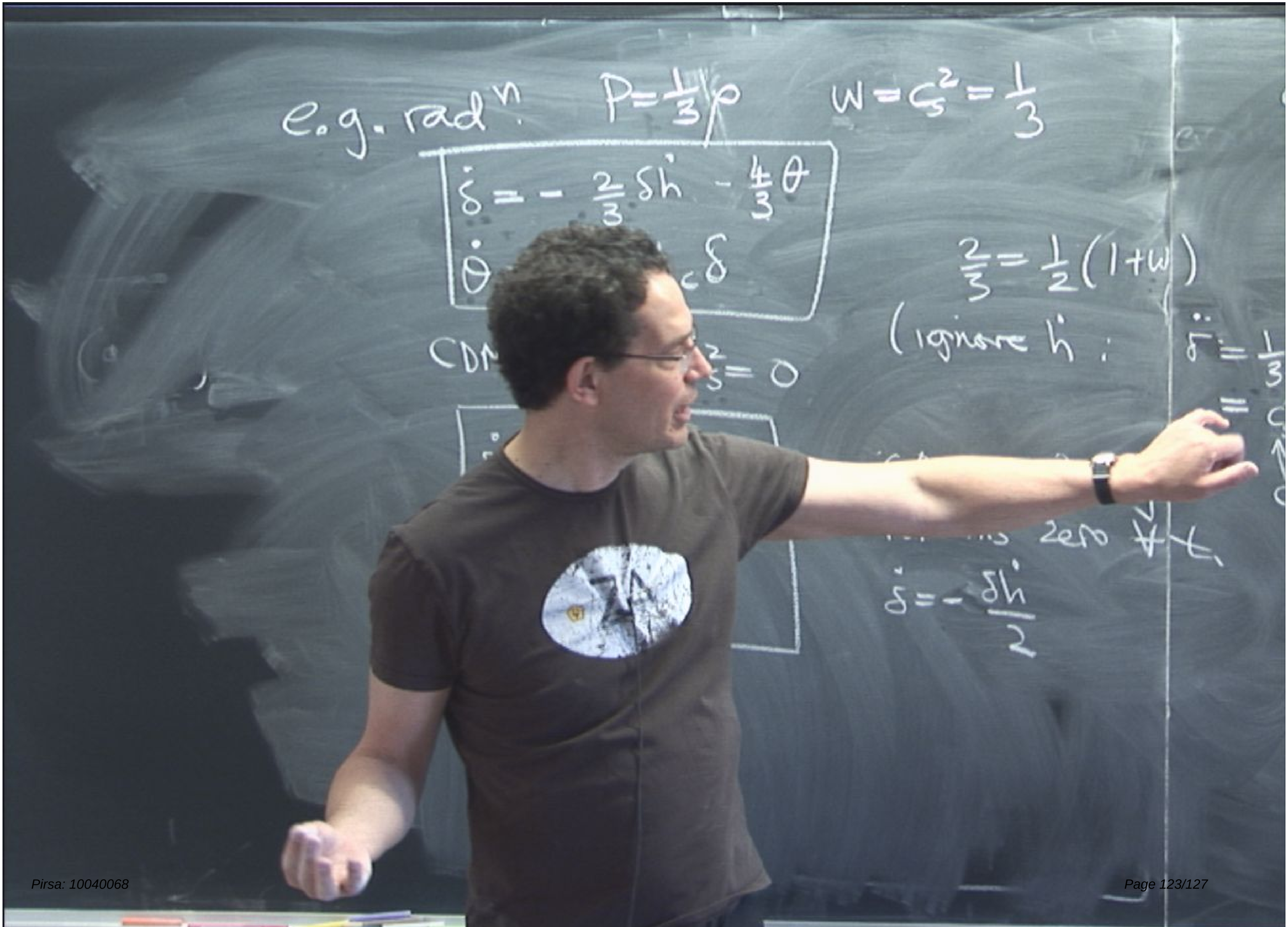
$$\text{comoving Vol} \sim \int d^3x \, \underbrace{\sqrt{\det(f_{ij} + \delta h_{ij})}}_{\det f \, (1 + \frac{1}{2}\delta h)}$$



e.g. rad^{n}: $P = \frac{1}{3}\rho$ $w = c_s^2 = \frac{1}{3}$
$\dot{\delta} = -\frac{2}{3}\delta\dot{h} - \frac{4}{3}\theta$
$\dot{\theta} = -\frac{1}{4}\Delta_c\delta$
CDM $w = c_s^2 = 0$
$\dot{\delta} = -\frac{\delta\dot{h}}{2} - \theta$
$\dot{\theta} = -\mathcal{H}\theta$
if $\theta = 0$ initially
remains zero $\forall t$.
$\dot{\delta} = -\frac{\delta\dot{h}}{2}$


e.g. radn: P = 1/3 ρ
w = cs2 = 1/3
δ̇ = − 2/3 δḣ − 4/3 θ
θ̇ = − 1/4 Δc δ
2/3 = 1/2 (1+w)
CDM w = cs2 = 0
δ̇ = − δḣ/2 − θ
if θ = 0 initially
remains zero ∀ t.
δ̇ = − δḣ/2


e.g. rad^n: $P = \frac{1}{3}\rho$
$w = c_s^2 = \frac{1}{3}$
$\dot{\delta} = - \frac{4}{3}\theta$
$- \frac{1}{4}\Delta_c \delta$
$\frac{2}{3} = \frac{1}{2}(1+w)$
(ignore $\dot{h}$
CDM
$w = c_s^2 = 0$
$\dot{\delta} = -\frac{\delta\dot{h}}{2} - \theta$
$\dot{\theta} = -\mathcal{H}\theta$
if $\theta = 0$ initially
remains zero $\forall t$.
$\dot{\delta} = -\frac{\delta\dot{h}}{2}$



e.g. rad^n: P = 1/3 ρ
w = c_s^2 = 1/3
δ̇ = − 2/3 δḣ − 4/3 θ
2/3 = 1/2 (1+w)
(ignore ḣ :
CDM
zero ∀ t.
δ̇ = − δḣ/2

e.g. rad^n: $P=\frac{1}{3}\rho$ $\quad w=c_s^2=\frac{1}{3}$

CDM

$$\boxed{\begin{aligned}\dot\delta &= -\frac{2}{3}\delta\dot h - \frac{4}{3}\theta\\ \dot\theta &= -\frac{1}{4}\Delta_c\delta\end{aligned}}$$

$$\frac{2}{3} = \frac{1}{2}(1+w)$$

(ignore $\dot h$: $\ddot\delta = \frac{1}{3}\Delta_c\delta$)

$$= c_s^2\Delta_c\delta$$

$\uparrow$

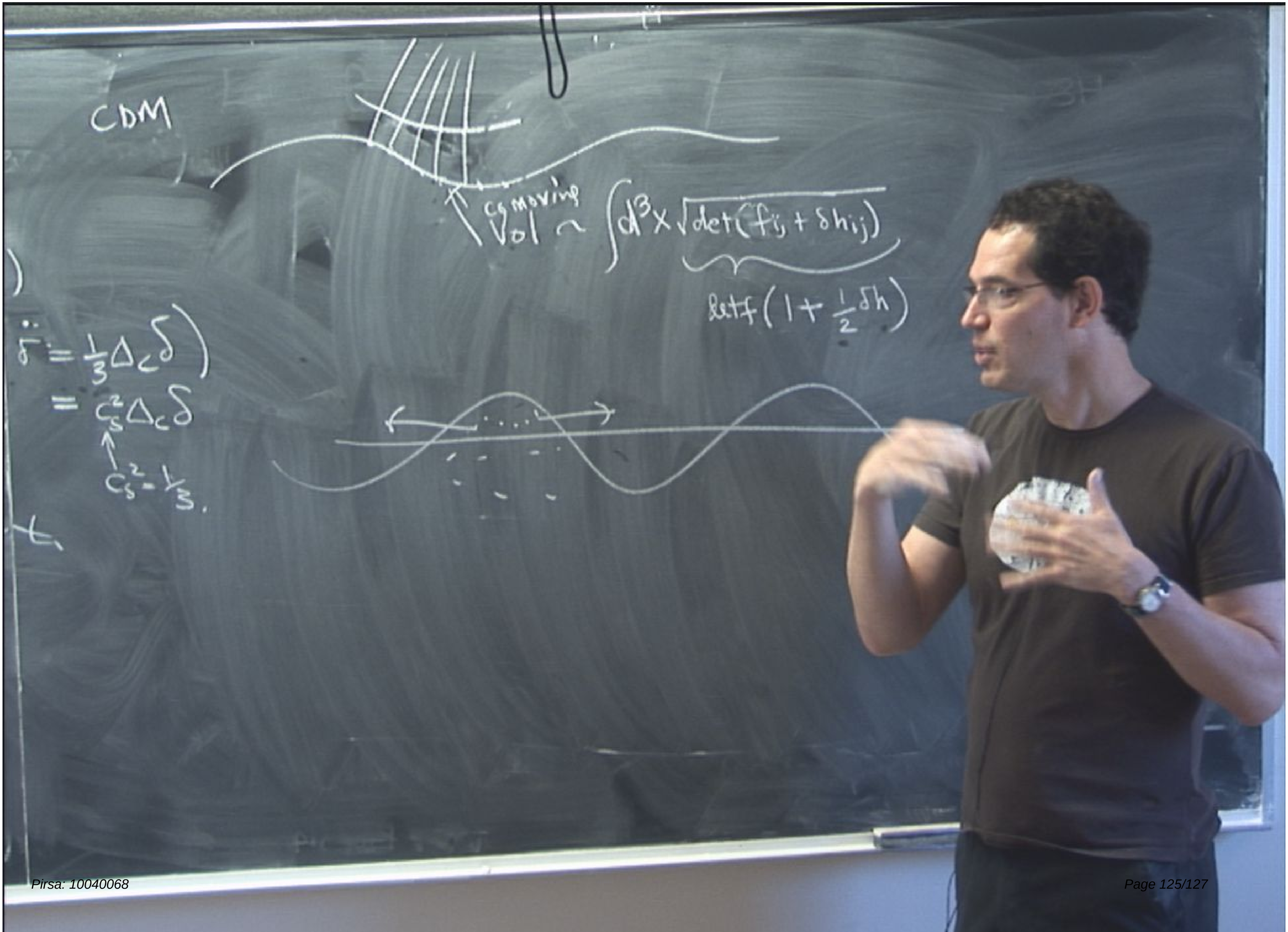
$c_s^2 = 1/3$.

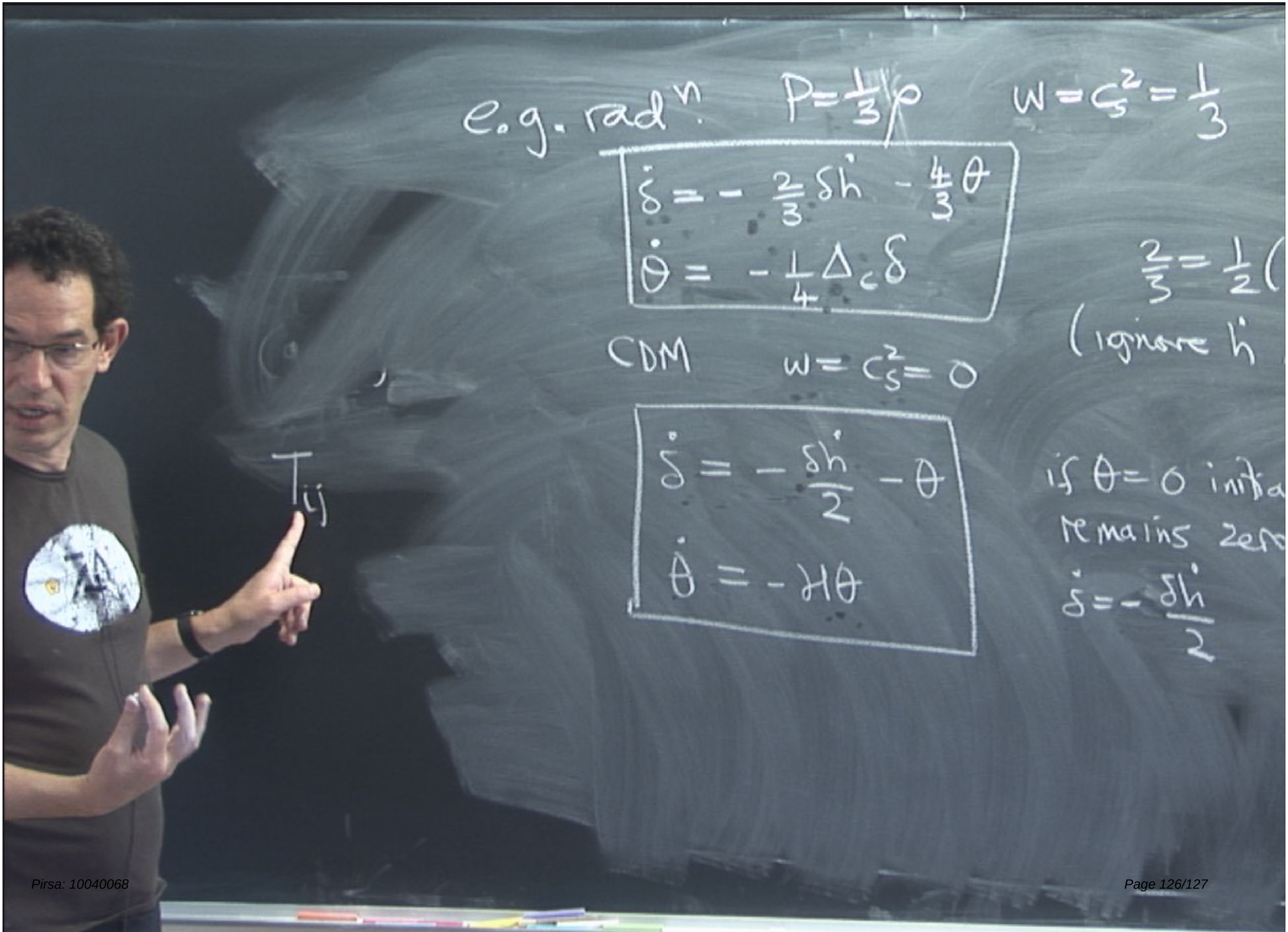
CDM $\quad w = c_s^2 = 0$

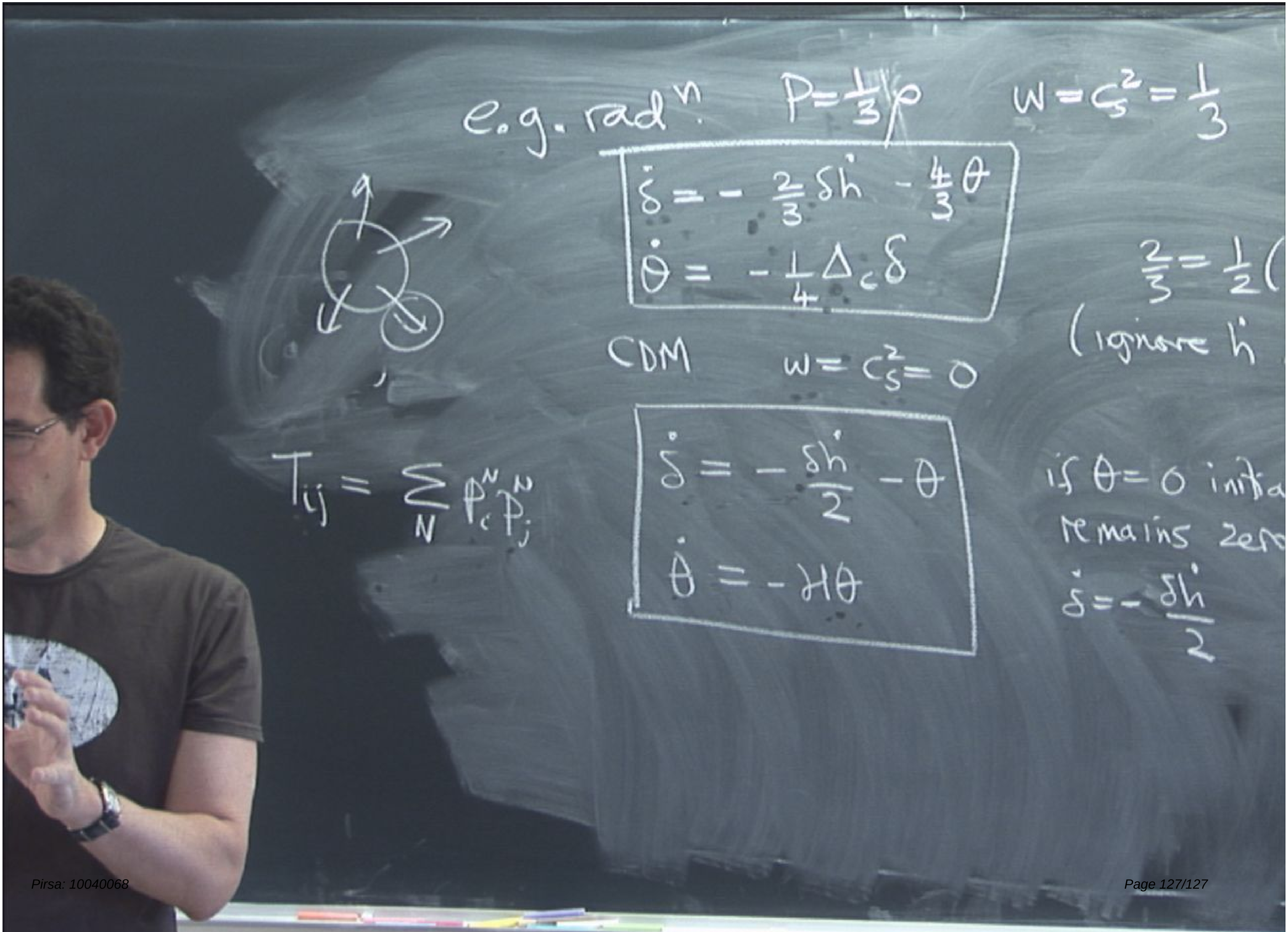
$$\boxed{\begin{aligned}\dot\delta &= -\frac{\delta\dot h}{2} - \theta\\ \dot\theta &= -\mathcal{H}\theta\end{aligned}}$$

if $\theta = 0$ initially remains zero $\forall t$.

$$\dot\delta = -\frac{\delta\dot h}{2}$$


CDM
comoving Vol $\sim \int d^3x \sqrt{\det(f_{ij} + \delta h_{ij})}$
$\det f \left(1 + \frac{1}{2}\delta h\right)$
$= \frac{1}{3}\Delta_c \delta$
$= c_s^2 \Delta_c \delta$
$c_s^2 = 1/3.$


e.g. radn. $P=\frac{1}{3}\rho$ $w=c_s^2=\frac{1}{3}$
$\dot{\delta}=-\frac{2}{3}\delta\dot{h}-\frac{4}{3}\theta$
$\dot{\theta}=-\frac{1}{4}\Delta_c\delta$
$\frac{2}{3}=\frac{1}{2}($
(ignore $\dot{h}$
CDM $w=c_s^2=0$
T_{ij}
$\dot{\delta}=-\frac{\delta\dot{h}}{2}-\theta$
$\dot{\theta}=-\mathcal{H}\theta$
if $\theta=0$ initia
remains zer
$\dot{\delta}=-\frac{\delta\dot{h}}{2}$


e.g. radn. $P = \frac{1}{3}\rho$ $w = c_s^2 = \frac{1}{3}$
$\dot{\delta} = -\frac{2}{3}\delta\dot{h} - \frac{4}{3}\theta$
$\dot{\theta} = -\frac{1}{4}\Delta_c\delta$
$\frac{2}{3} = \frac{1}{2}($
(ignore $\dot{h}$
CDM $w = c_s^2 = 0$
$T_{ij} = \sum_N p_i^N p_j^N$
$\dot{\delta} = -\frac{\delta\dot{h}}{2} - \theta$
$\dot{\theta} = -\mathcal{H}\theta$
if $\theta = 0$ initia
remains zer
$\dot{\delta} = -\frac{\delta\dot{h}}{2}$