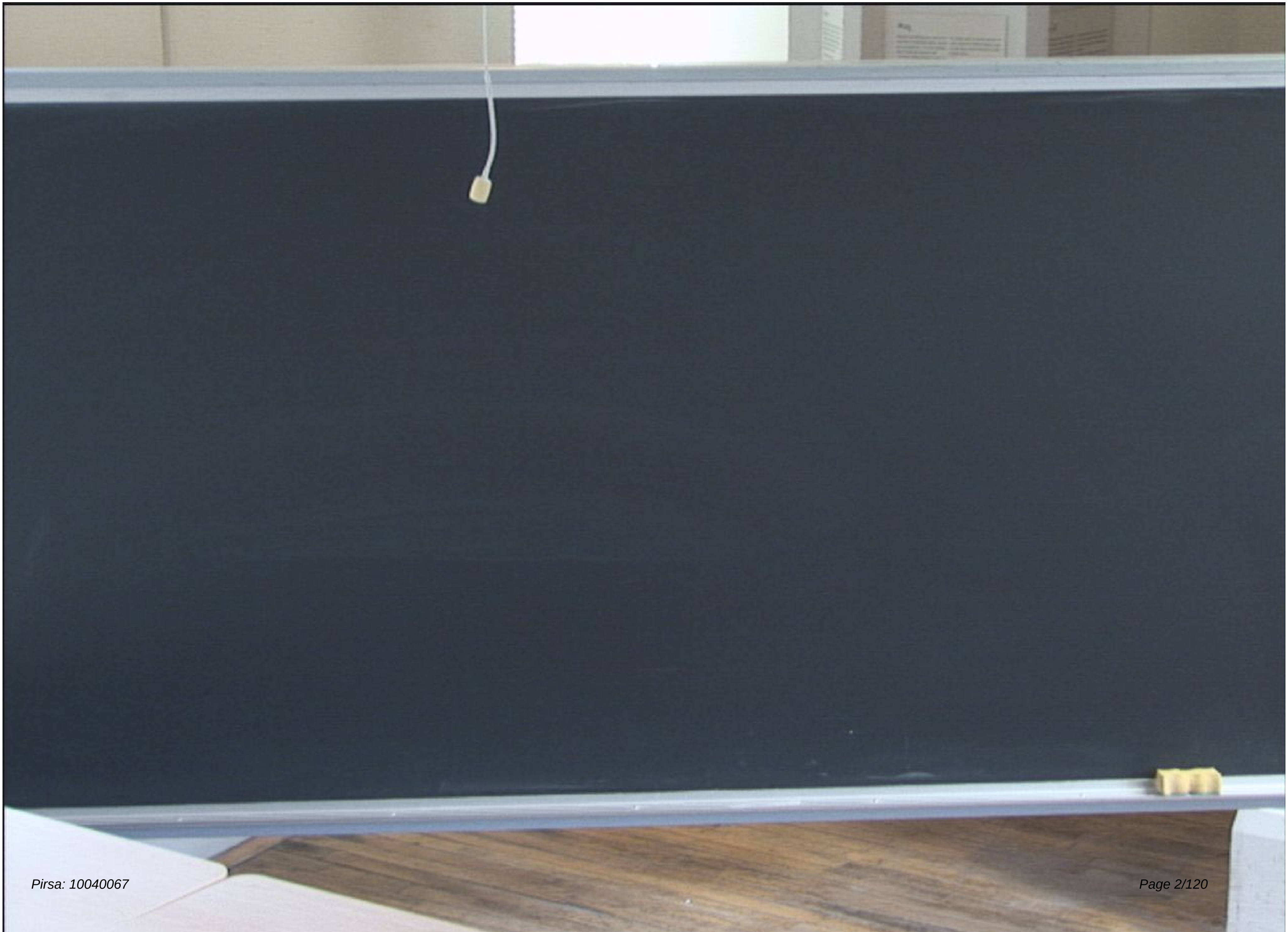
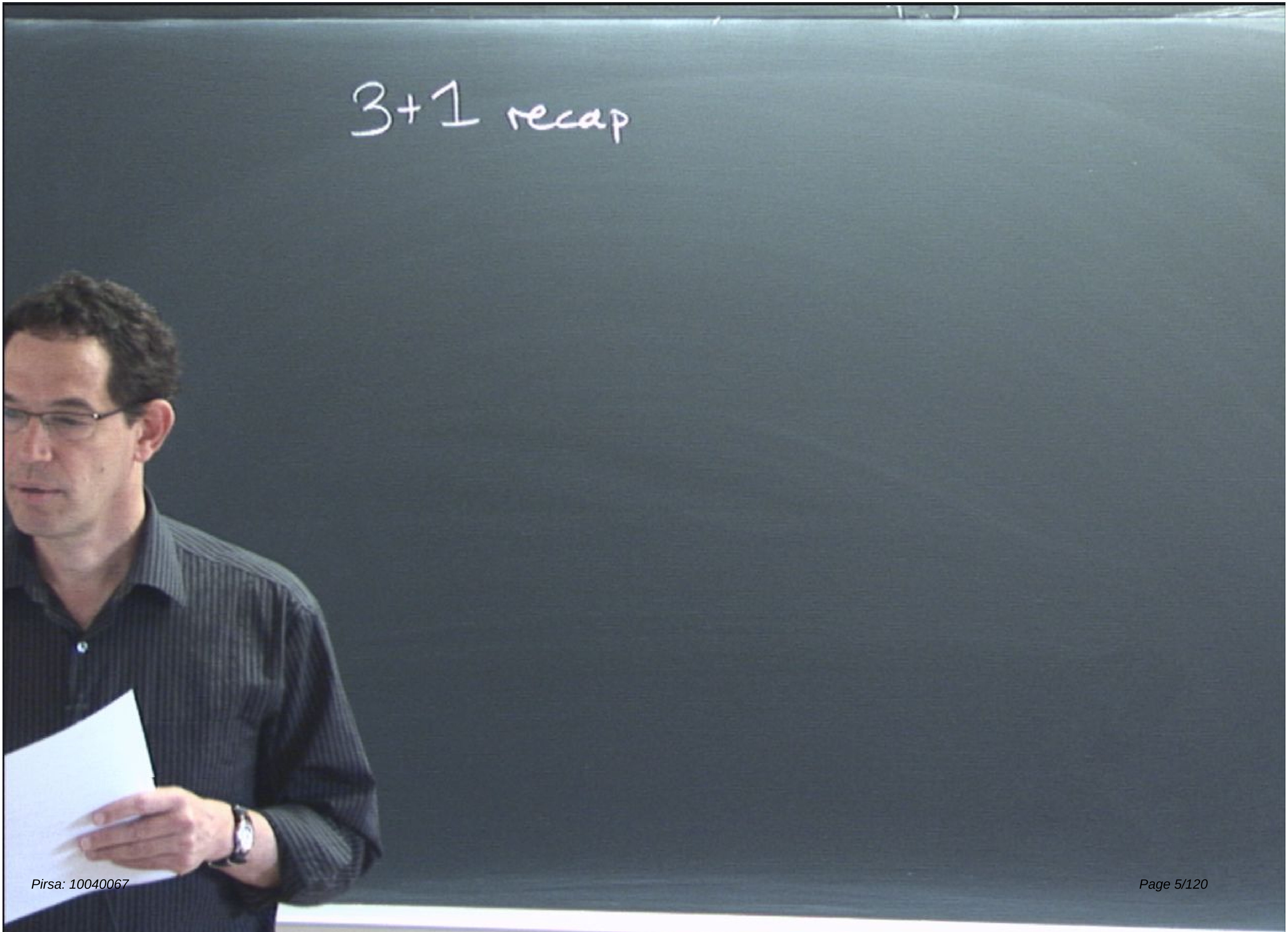
Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 10

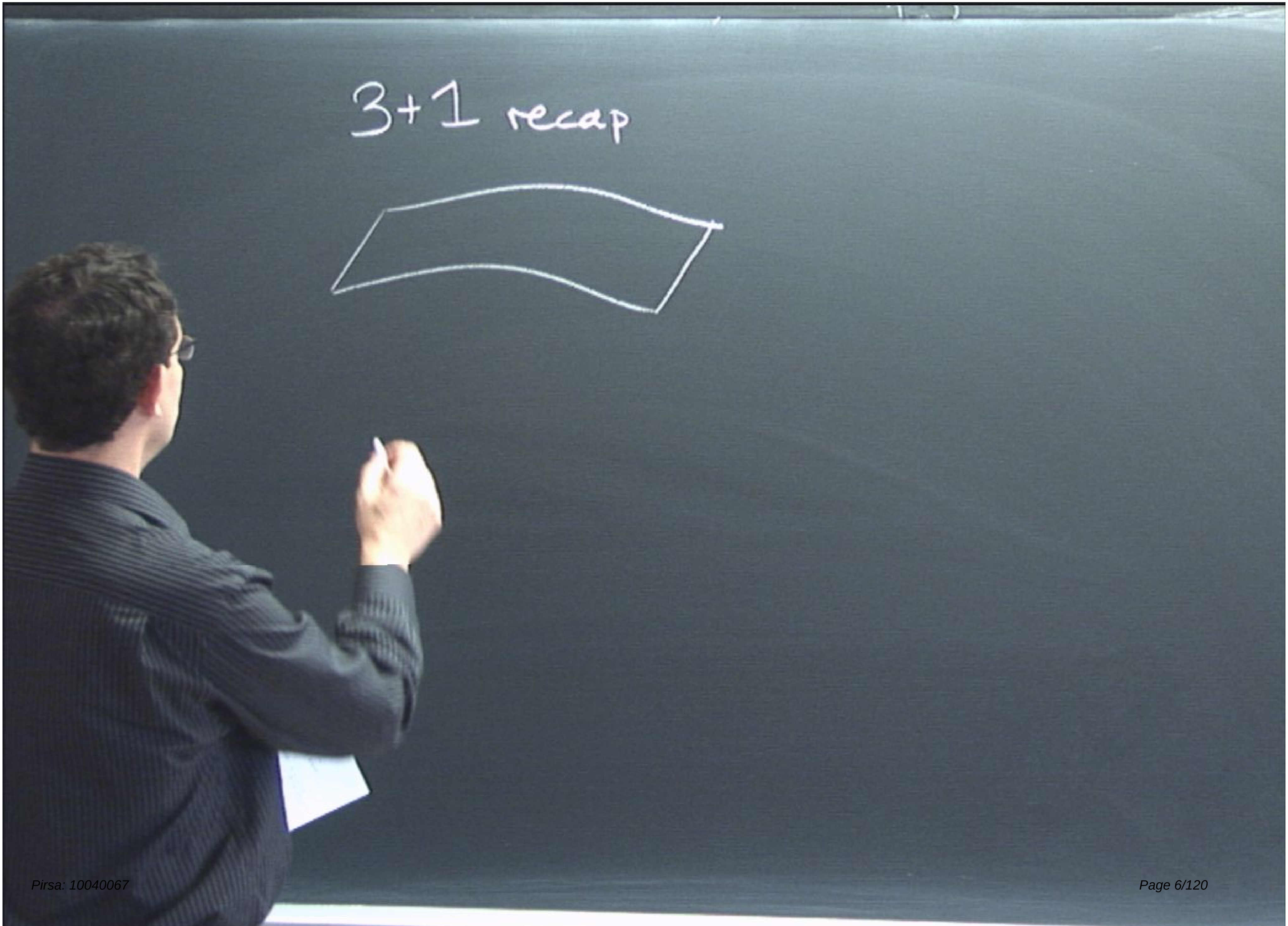
Date: Apr 16, 2010 09:30 AM

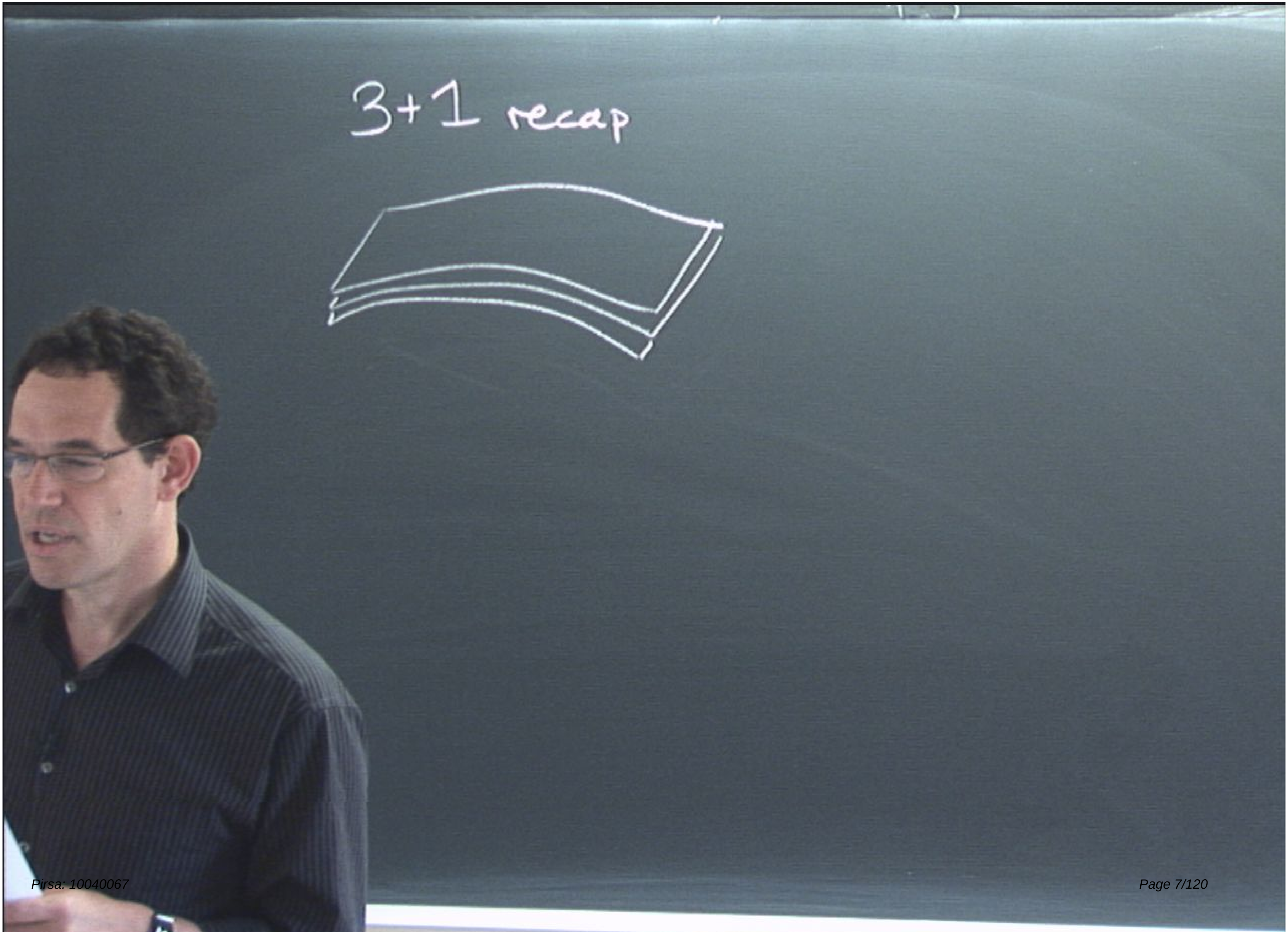
URL: http://pirsa.org/10040067

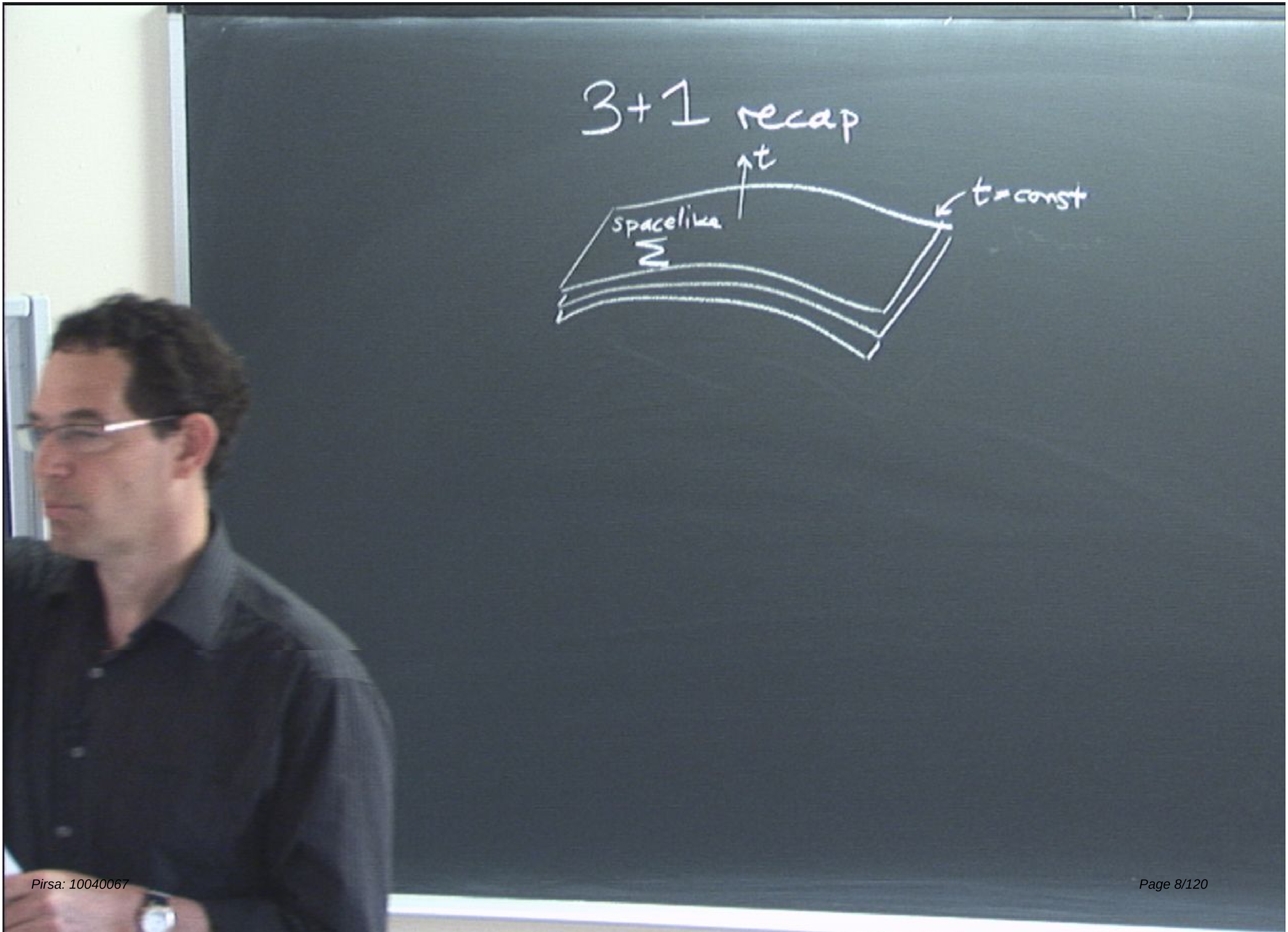
Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

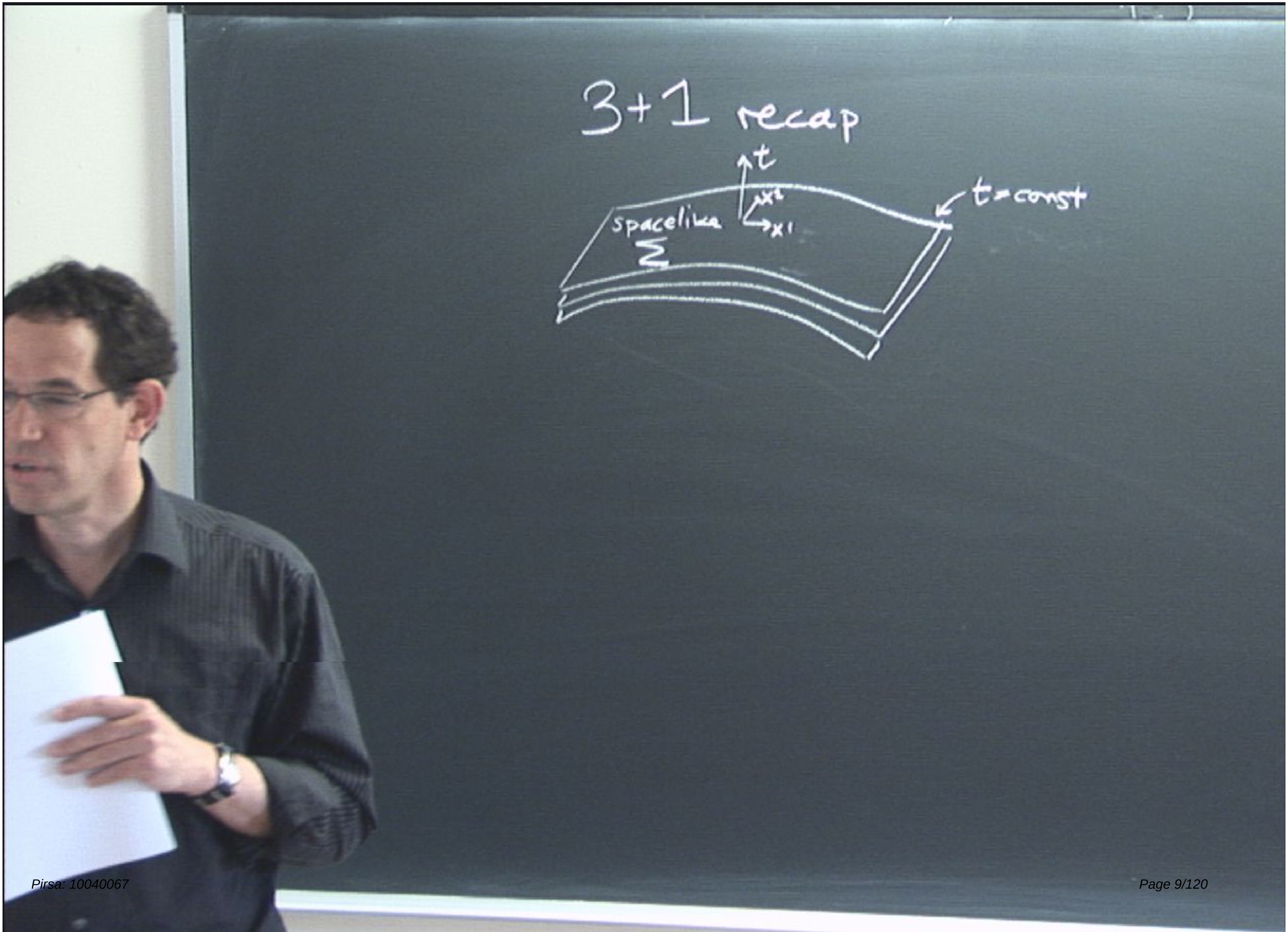



3+1 recap

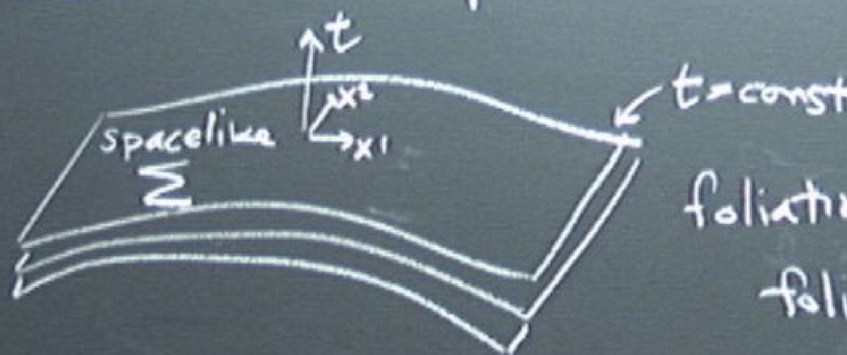

3+1 recap


3+1 recap


3+1 recap
t
t=const
spacelike
Σ

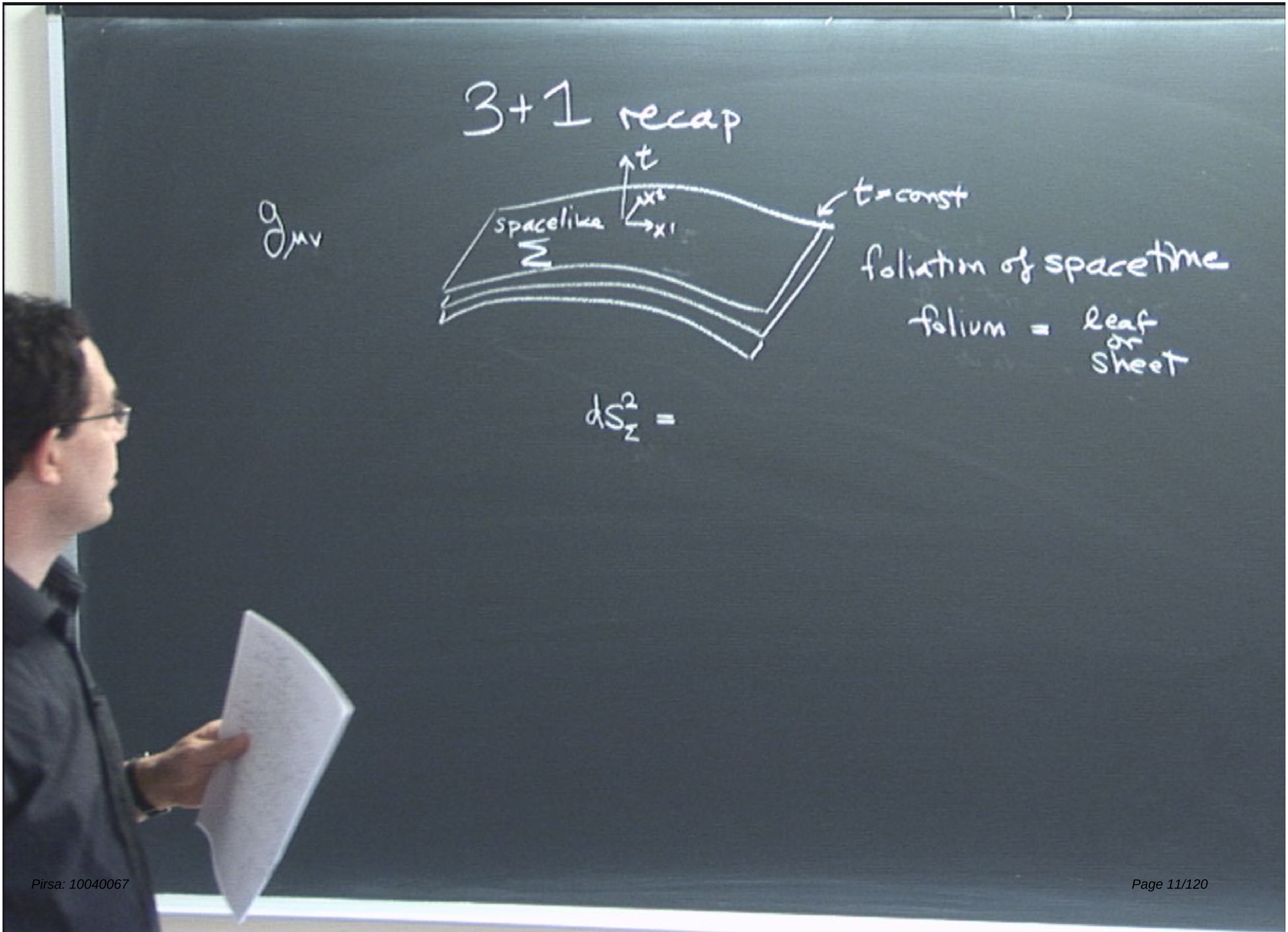

3+1 recap
t
x^2
x^1
spacelike
Σ
t=const

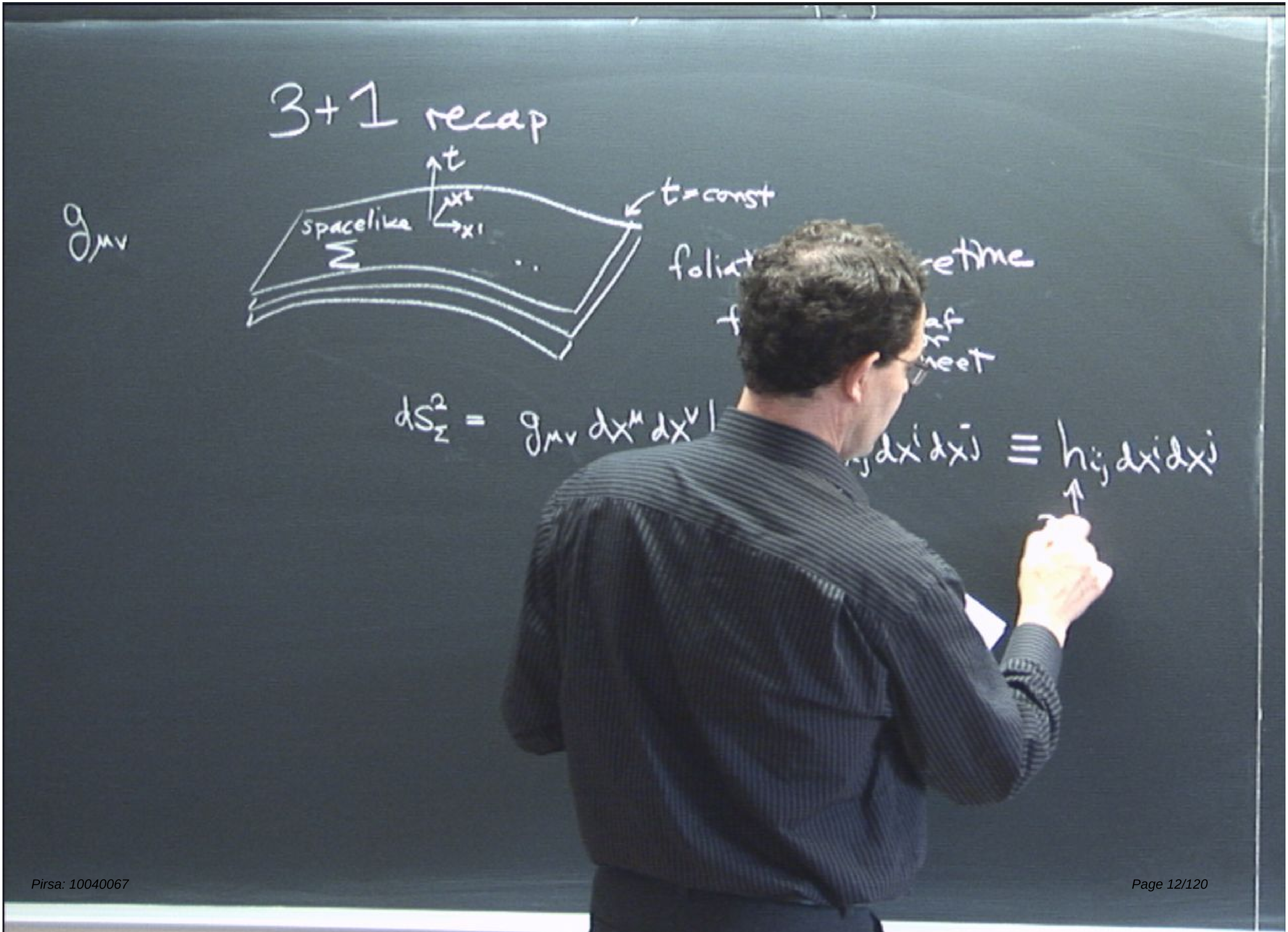
3+1 recap



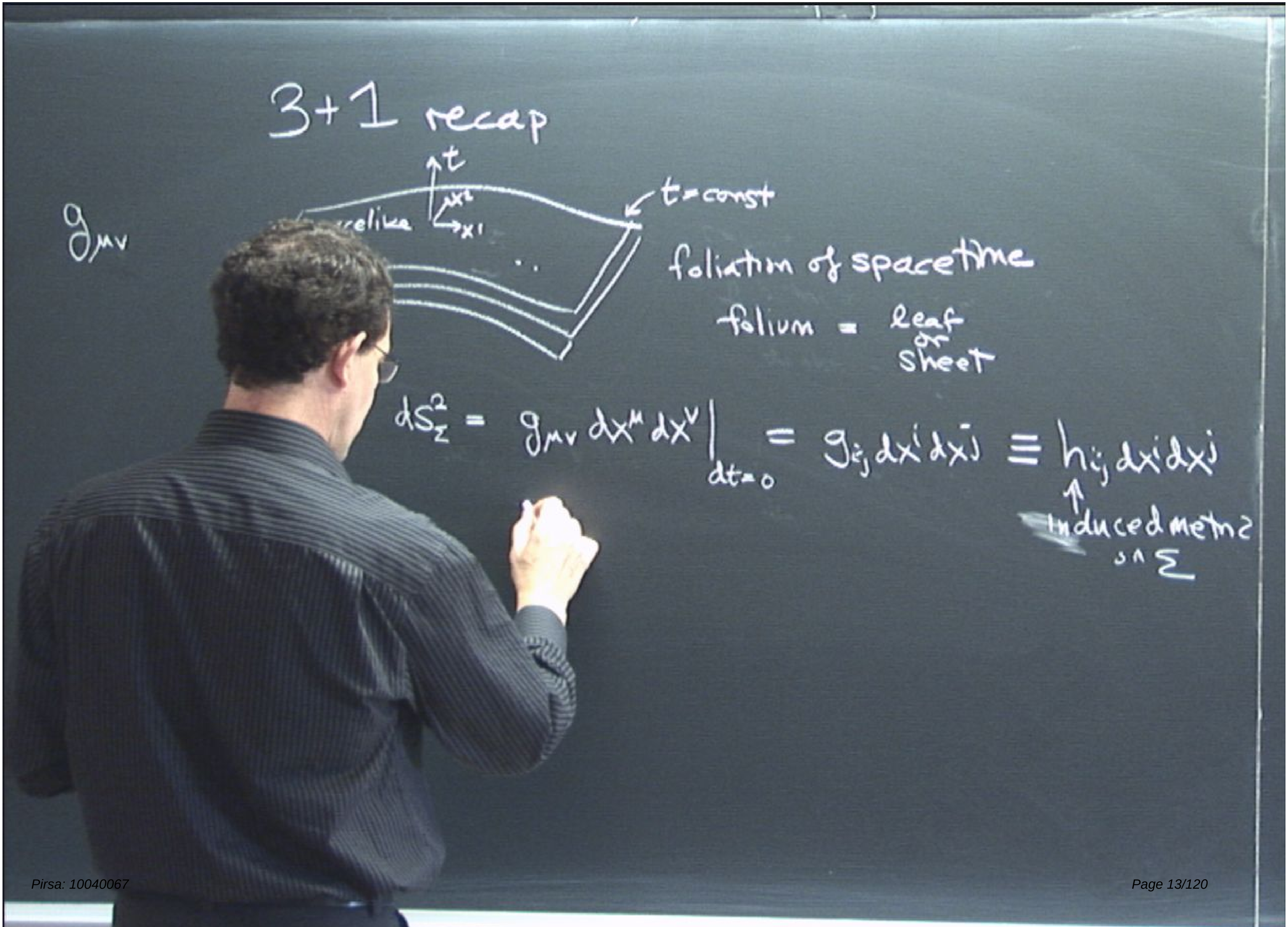
foliation of spacetime

folium = leaf or sheet


3+1 recap
$g_{\mu\nu}$
t
x^2
x^1
spacelike
Σ
t=const
foliation of spacetime
folium = leaf or sheet
$dS^2_\Sigma =$



3+1 recap
$g_{\mu\nu}$
t
x^2
x^1
spacelike
Σ
t=const
$ds_\Sigma^2 = g_{\mu\nu}\, dx^\mu dx^\nu |$
$dx^i dx^j \equiv h_{ij}\, dx^i dx^j$


3+1 recap
t
x^2
x^1
t=const
$g_{\mu\nu}$
foliation of spacetime
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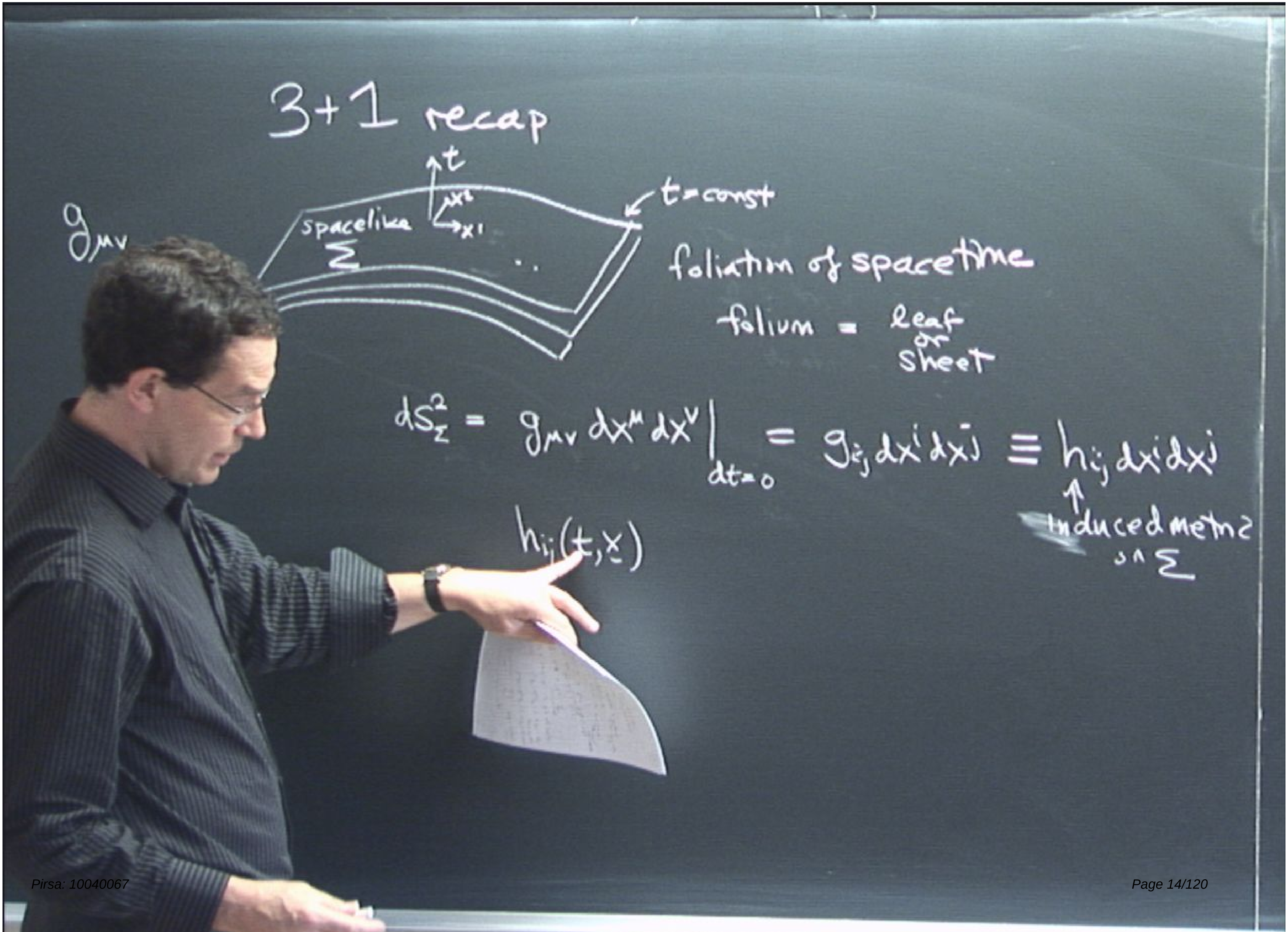
$ds_\Sigma^2 = g_{\mu\nu}\, dx^\mu dx^\nu \Big|_{dt=0} = g_{ij}\, dx^i dx^j \equiv h_{ij}\, dx^i dx^j$
induced metric on Σ


3+1 recap
$g_{\mu\nu}$
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foliation of spacetime
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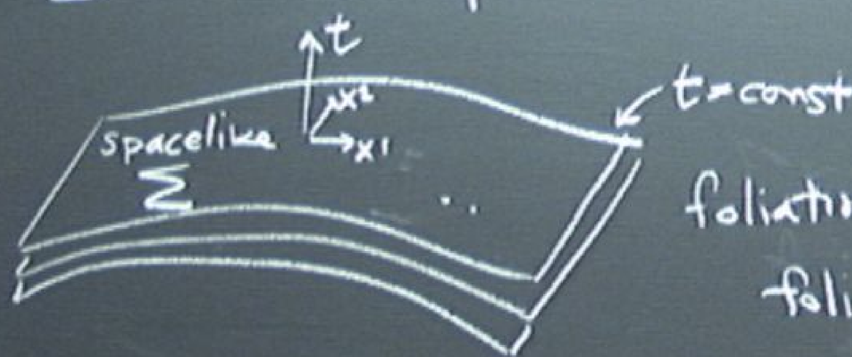
$ds^2_\Sigma = g_{\mu\nu}\,dx^\mu dx^\nu \big|_{dt=0} = g_{ij}\,dx^i dx^j \equiv h_{ij}\,dx^i dx^j$
induced metric on Σ
$h_{ij}(t, \underline{x})$

3+1 recap

$g_{\mu\nu}$



foliation of spacetime

folium = leaf or sheet

$$ds_\Sigma^2 = g_{\mu\nu}\,dx^\mu dx^\nu\Big|_{dt=0} = g_{ij}\,dx^i dx^j \equiv h_{ij}\,dx^i dx^j$$

induced metric on Σ

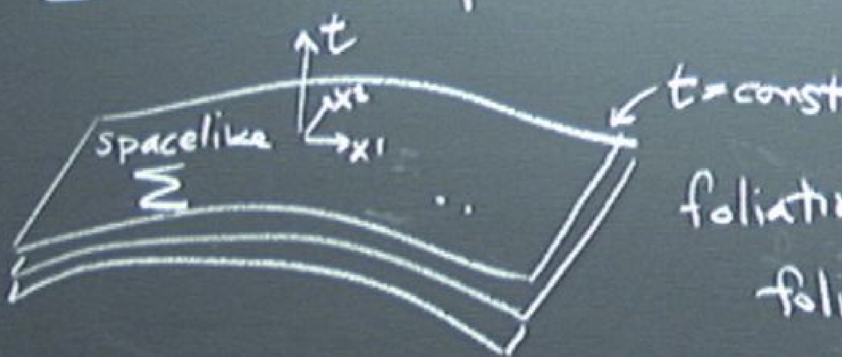
$$\underline{h_{ij}(t,\underline{x}) = g_{ij}(t,\underline{x})} \quad (1)$$

3+1 recap

$g_{\mu\nu}$

$\mu = 0,1,2,3$

$i = 1,2,3$



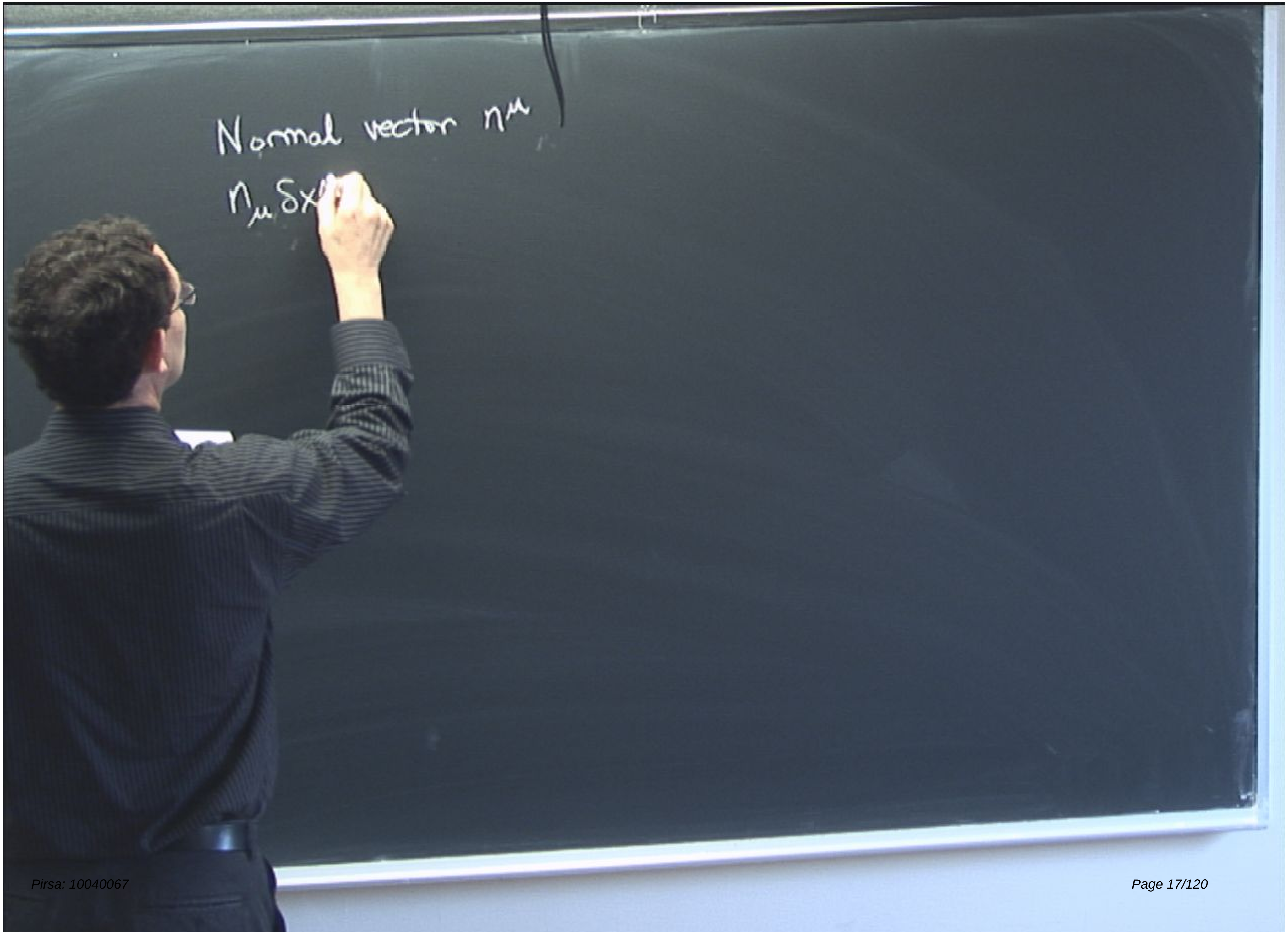
foliation of spacetime

folium = leaf or sheet

$$ds^2_\Sigma = g_{\mu\nu}\,dx^\mu dx^\nu\Big|_{dt=0} = g_{ij}\,dx^i dx^j \equiv h_{ij}\,dx^i dx^j$$

↑ induced metric on Σ

$$\underline{h_{ij}(t,\underline{x}) = g_{ij}(t,\underline{x})} \qquad (1)$$


Normal vector n^μ
$n_\mu \delta x$

Normal vector n^μ

$n_\mu \delta x^\mu = 0 \quad \forall\ \delta x^\mu$ in Σ

eg $\delta x^1 \neq 0$

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eg $\delta x^1 \neq 0$

want $n^0 > 0$

i.e. n^μ to be future-pointing

$\rightarrow \quad n_i = 0 \quad \longrightarrow \quad n_\mu = (-N, \underline{0})$

$(g^{00} < 0)$

Normal vector n^μ

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want $n^0 > 0$

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$(g^{00} < 0)$

Normalise n^μ: $g_{\mu\nu} n^\mu n^\nu = -1$

$= g^{\mu\nu} n_\mu n_\nu = N^2 g^{00} \rightarrow \underline{g^{00} = -N^{-2}}$ (2)

Normal vector n^μ

$n_\mu \delta x^\mu = 0 \quad \forall \; \delta x^\mu$ in Σ

eg $\delta x^1 \neq 0$

$\rightarrow \; n_i = 0 \longrightarrow n_\mu = (-N, \underline{0})$ ← want $n^0 > 0$

i.e. n^μ to be future-pointing $(g^{00} < 0)$

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Let $n^i = \frac{N^i}{N}$ so $n^\mu = \frac{1}{N}(1, N^i)$

Normal vector n^μ

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$\rightarrow \dfrac{N^i}{N}$

Normal vector n^μ

$n_\mu \delta x^\mu = 0 \quad \forall\ \delta x^\mu$ in Σ

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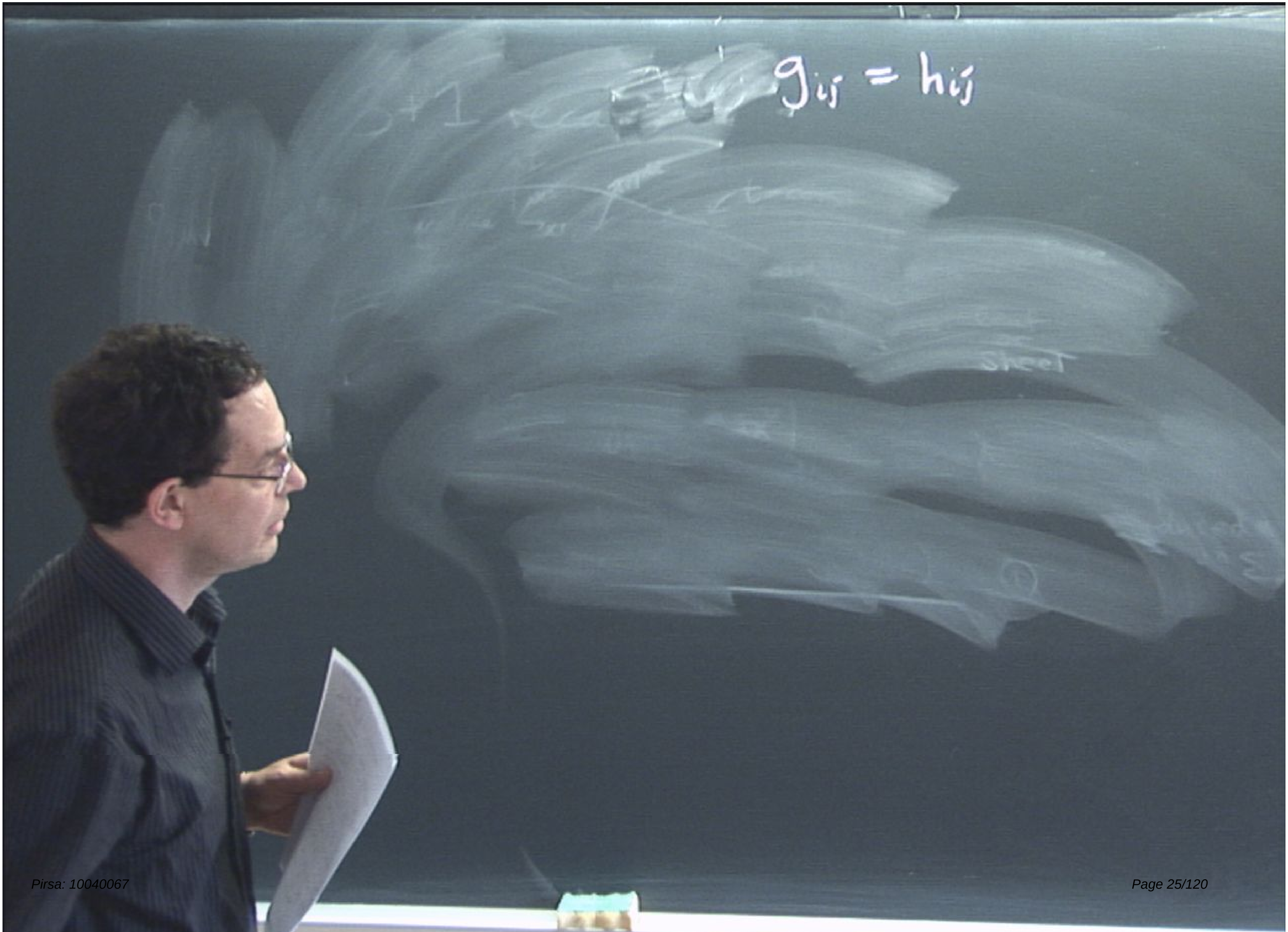
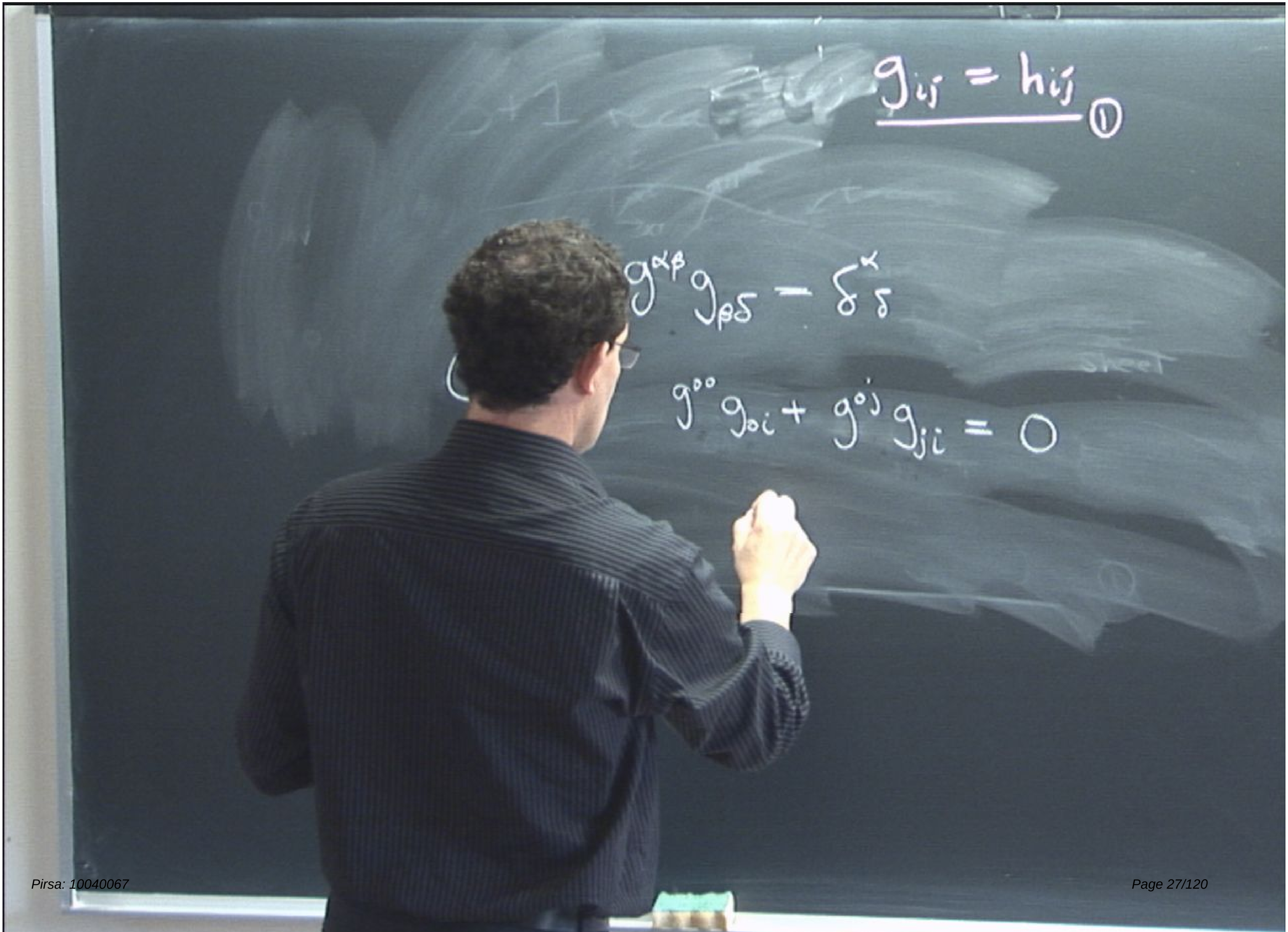
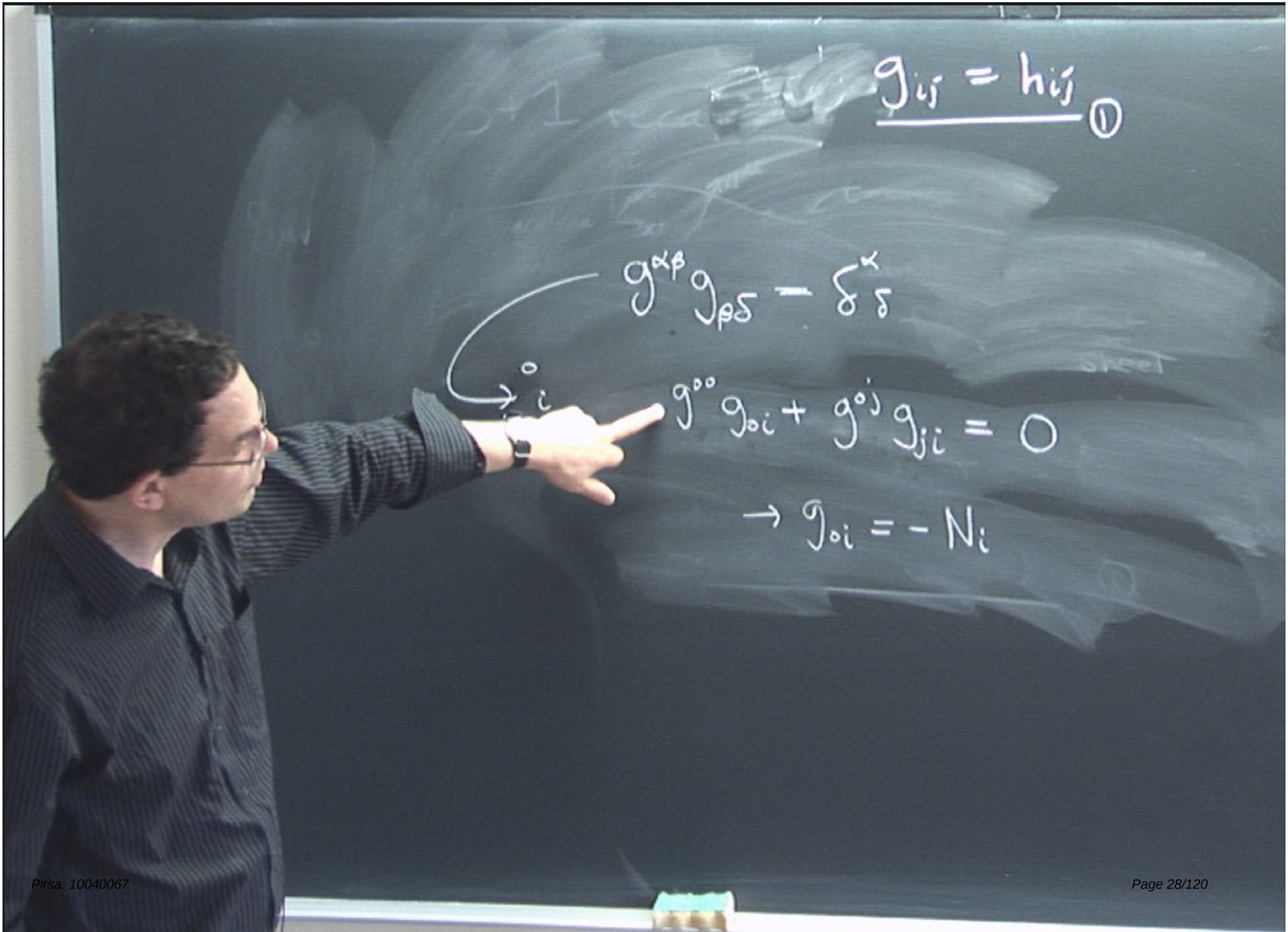
Let $n^i = \frac{N^i}{N}$ so $n^\mu = \frac{1}{N}(1, N^i) = g^{\mu\nu} n_\nu = g^{\mu 0} n_0$

$\rightarrow \frac{N^i}{N} = g^{i0}(-N) \Rightarrow \underline{g^{i0} = \frac{-N^i}{N^2}}$ (3)


$g_{ij} = h_{ij}$

$$g_{ij} = h_{ij} \quad (1)$$


$g_{ij} = h_{ij}$ ①
$g^{\alpha\beta} g_{\beta\delta} = \delta^{\alpha}_{\delta}$
$g^{00} g_{0i} + g^{0j} g_{ji} = 0$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${}^0{}_i$
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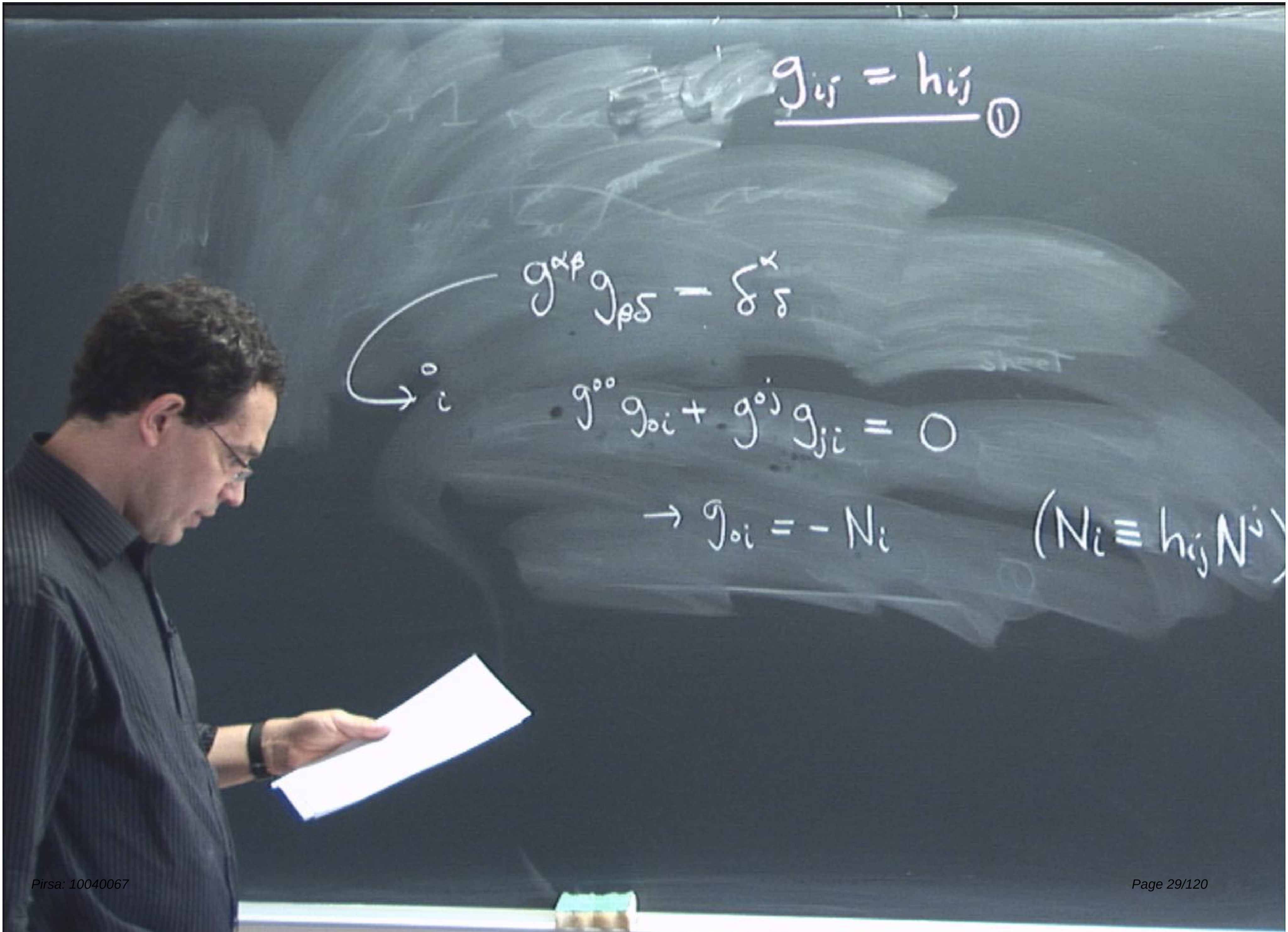
$\rightarrow g_{0i} = -N_i$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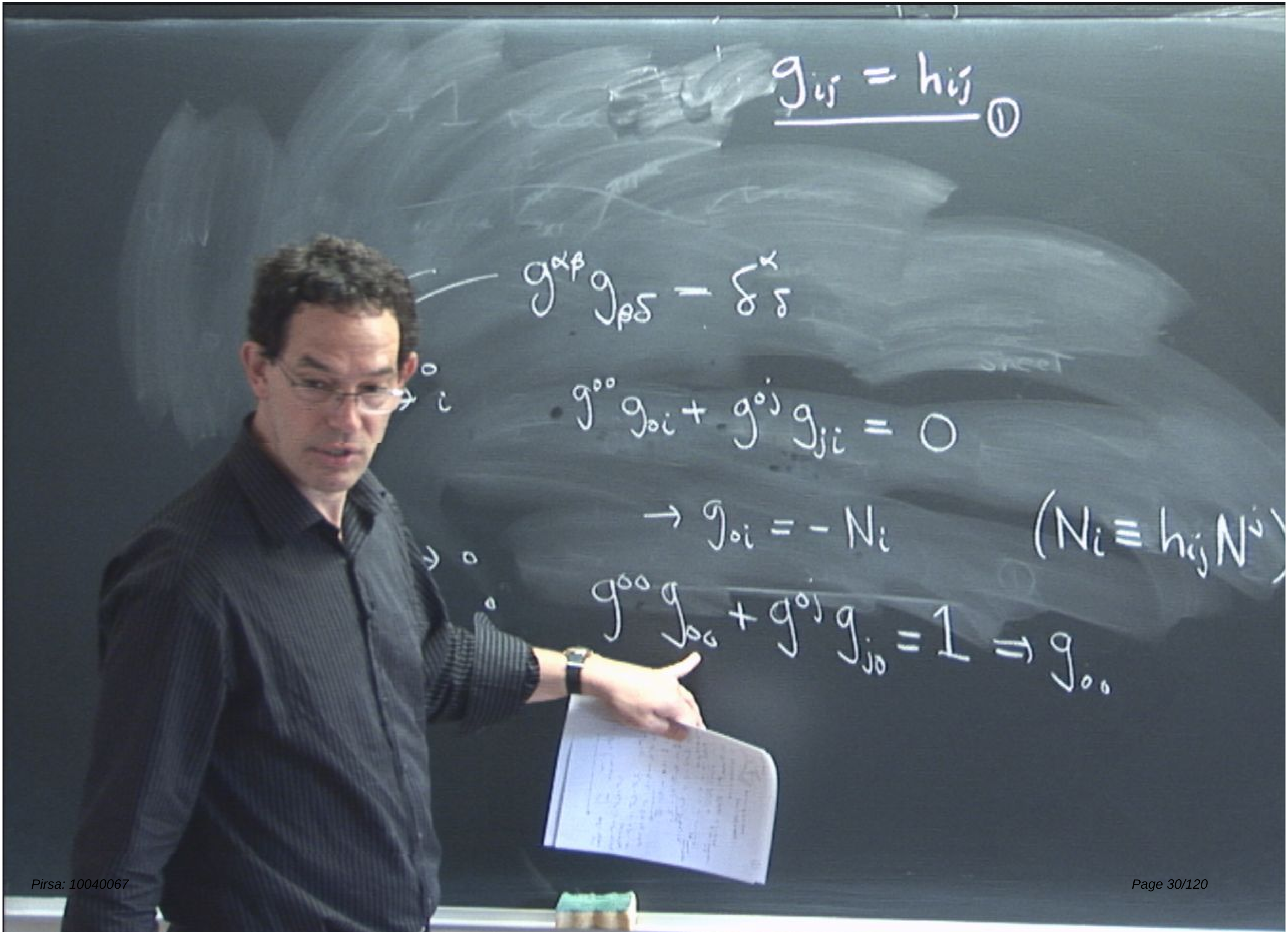
$(N_i \equiv h_{ij} N^j)$


$g_{ij} = h_{ij}$ ①
$g^{\alpha\beta} g_{\beta\delta} = \delta^{\alpha}_{\delta}$
$g^{00} g_{0i} + g^{0j} g_{ji} = 0$
$\to g_{0i} = -N_i$
$(N_i \equiv h_{ij} N^j)$
$g^{00} g_{00} + g^{0j} g_{j0} = 1 \Rightarrow g_{00}$

$$\underline{g_{ij} = h_{ij}} \quad ①$$

$$g^{\alpha\beta} g_{\beta\delta} = \delta^{\alpha}_{\delta}$$

${}^{0}{}_{i}$

$$g^{00} g_{0i} + g^{0j} g_{ji} = 0$$

$$\rightarrow \underline{g_{0i} = -N_i} \qquad (N_i \equiv h_{ij} N^j)$$

${}^{0}{}_{0}$

$$g^{00} g_{00} + g^{0j} g_{j0} = 1 \Rightarrow \underline{g_{00} = -N^2 + N^i N_i}$$

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix}$$

$$g_{ij} = h_{ij} \quad ①$$

$$g^{\alpha\beta} g_{\beta\delta} = \delta^{\alpha}_{\delta}$$

${}^{0}{}_{i}$ $$g^{00} g_{0i} + g^{0j} g_{ji} = 0$$

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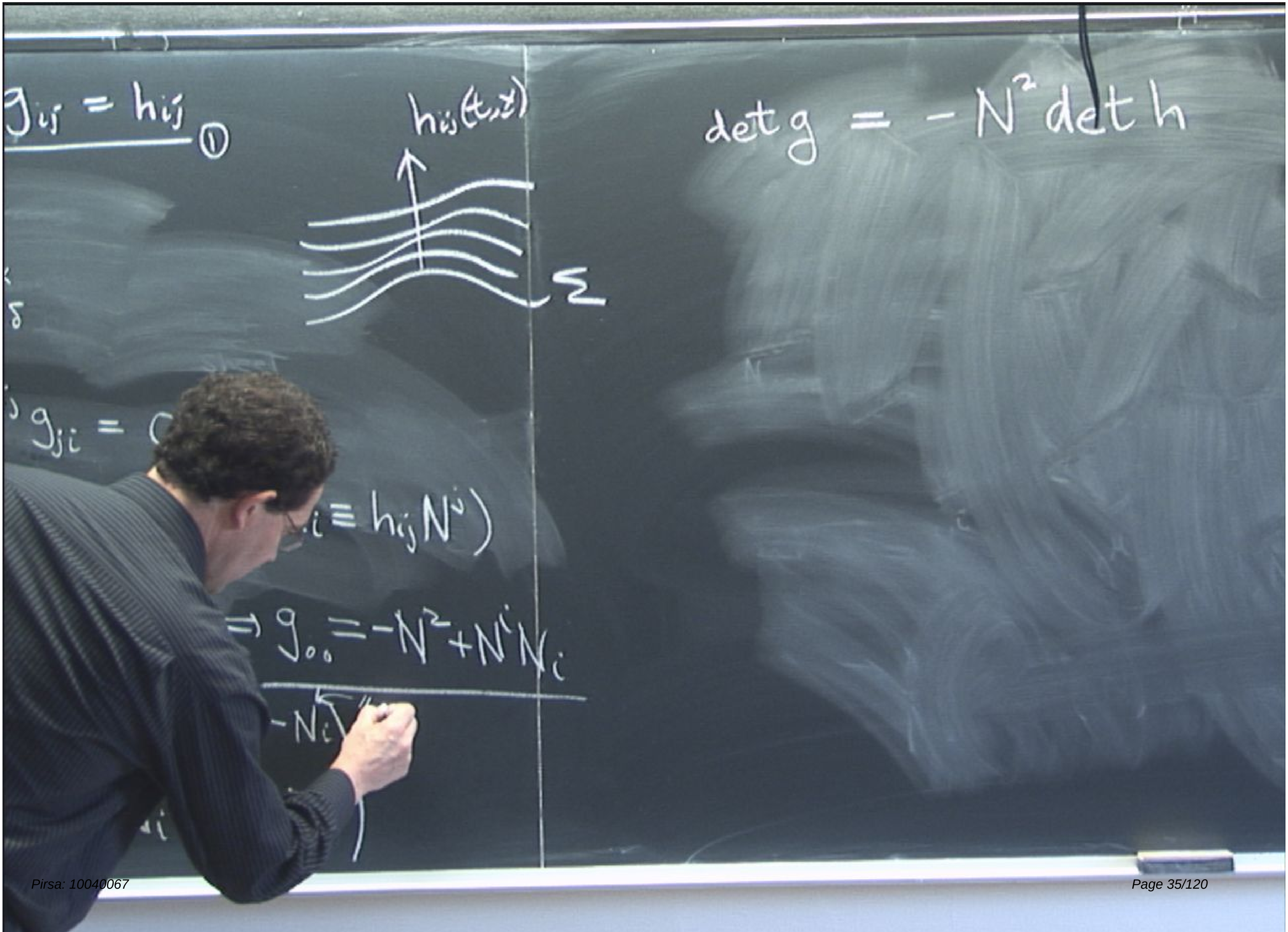
$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix}$$

$$\det g = -N^2 \det h \qquad (Ex)$$

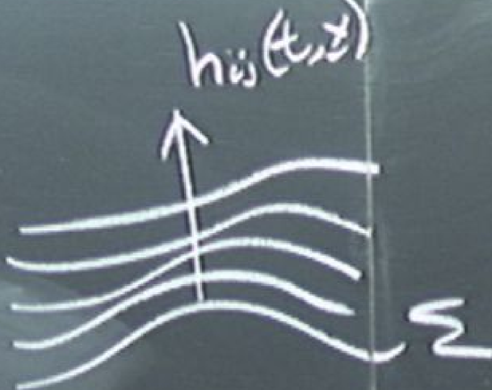
N_i

$$\det g = -N^2 \det h \qquad (Ex)$$

N_i


$g_{ij} = h_{ij}$ ①
$h_{ij}(t,x)$
Σ
$\det g = -N^2 \det h$
$g_{ji} =$
$i \equiv h_{ij} N^j)$
$\Rightarrow g_{00} = -N^2 + N^i N_i$
$-N_i$

$g_{ij} = h_{ij}$ ①



δ

$g_{ji} = 0$

$= -N_i \qquad (N_i \equiv h_{ij} N^j)$

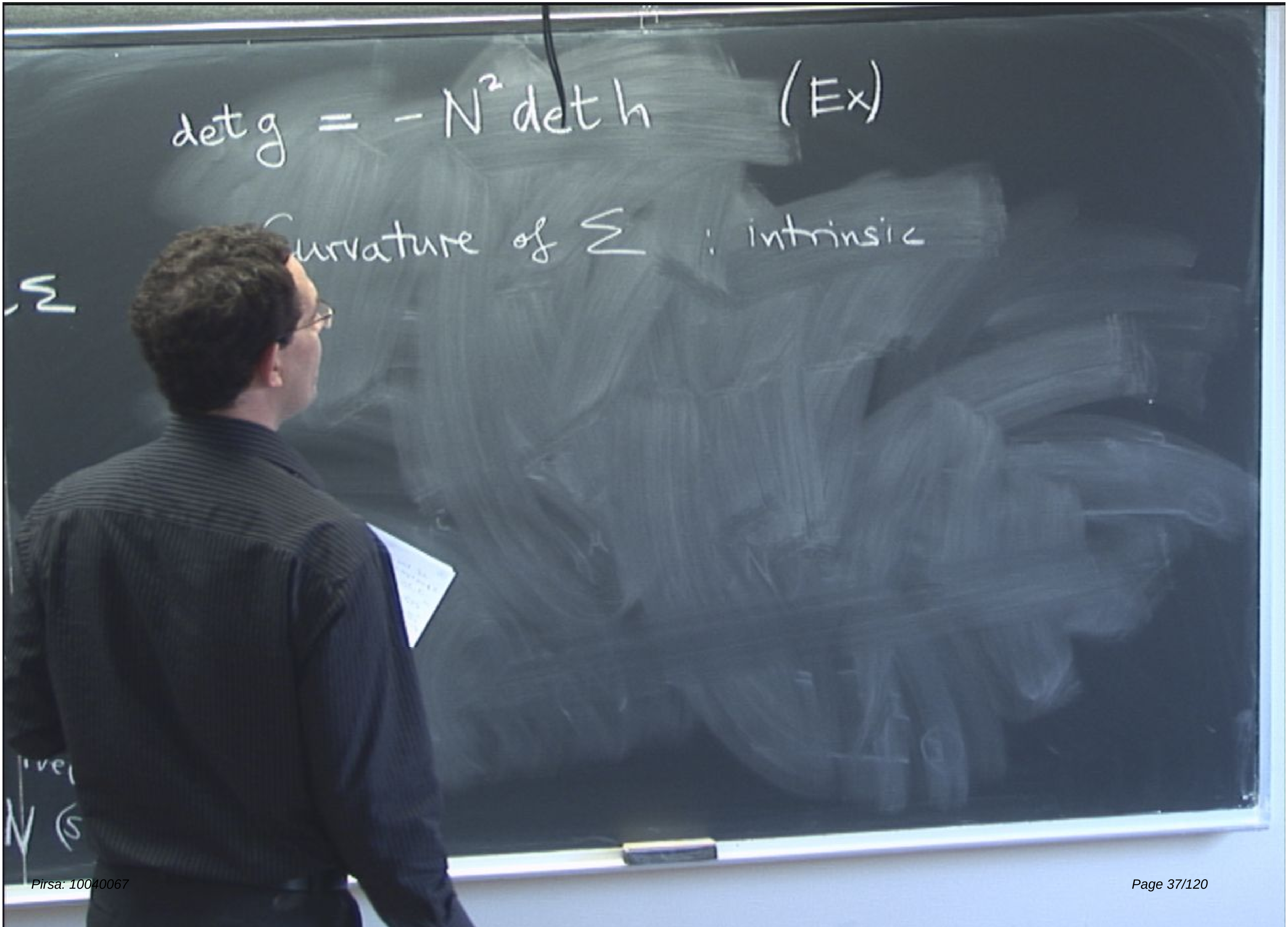
$g^{0j} g_{j0} = 1 \Rightarrow g_{00} = -N^2 + N^i N_i$

$$\begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & h_{ij} \end{pmatrix}$$

"shift" N^i

"lapse" N

$\det g = -N^2 \det h$

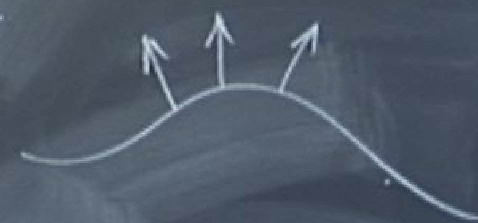


$\det g = -N^2 \det h$ (Ex)
Curvature of Σ : intrinsic

$$\det g = -N^2 \det h \qquad (Ex)$$

Curvature of Σ : intrinsic
+ extrinsic

Extrinsic curvature : from $\nabla_\mu n_\nu$



Σ

N_i (vector on Σ)

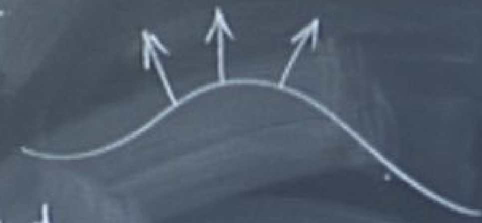
N (scalar on Σ)

$$\det g = -N^2 \det h \qquad (\mathrm{Ex})$$

Curvature of Σ : intrinsic
+ extrinsic

Extrinsic curvature : from $\nabla_\mu n_\nu$

tensor in 4d.



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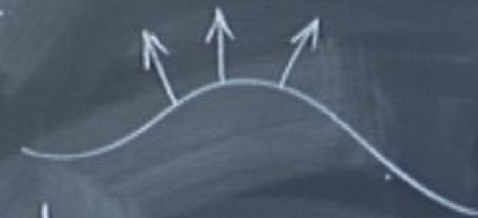
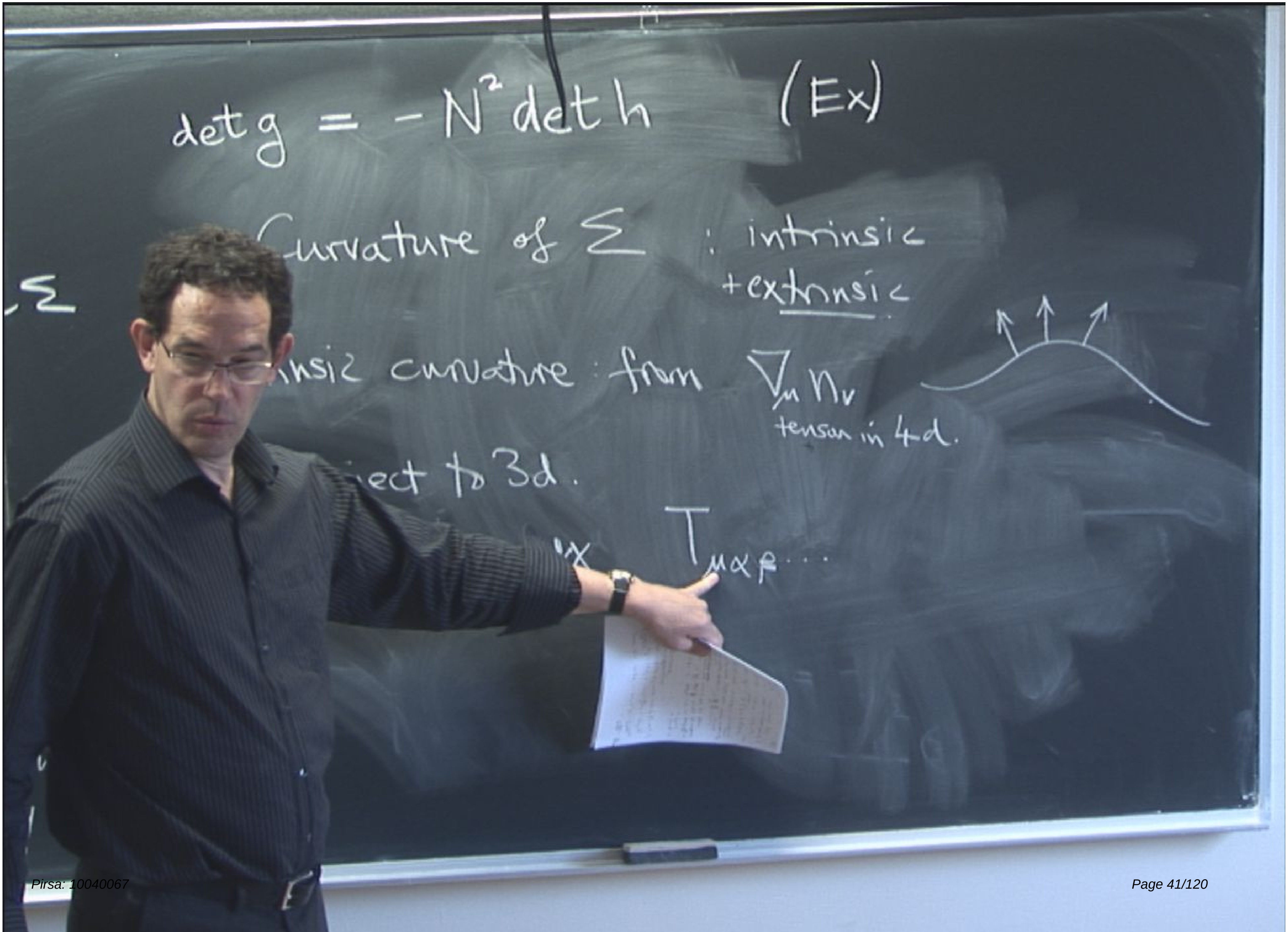


need to project to 3d.

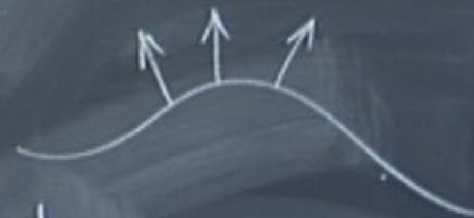
(Y∂r on Σ)
Scalar on Σ)


det g = − N² det h (Ex)
Curvature of Σ : intrinsic
+ extrinsic
insic curvature : from ∇μ nν
tensor in 4d.
ject to 3d.
Tμαβ ...

$$\det g = -N^2 \det h \qquad (\text{Ex})$$

Curvature of Σ : intrinsic
+ extrinsic

Extrinsic curvature : from $\nabla_\mu n_\nu$
tensor in 4d.



need to project to 3d.

Any tensor index μ, $T_{\mu\alpha\beta\cdots}$ can be projected into a piece along n^μ, and a piece $\perp$r n^μ.

Σ

N_i (vector on Σ)

N (Scalar on Σ)

$\det g = -N^2 \det h$ (Ex)

Curvature of Σ : intrinsic
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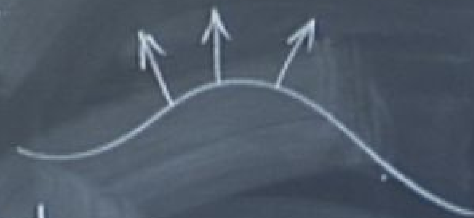
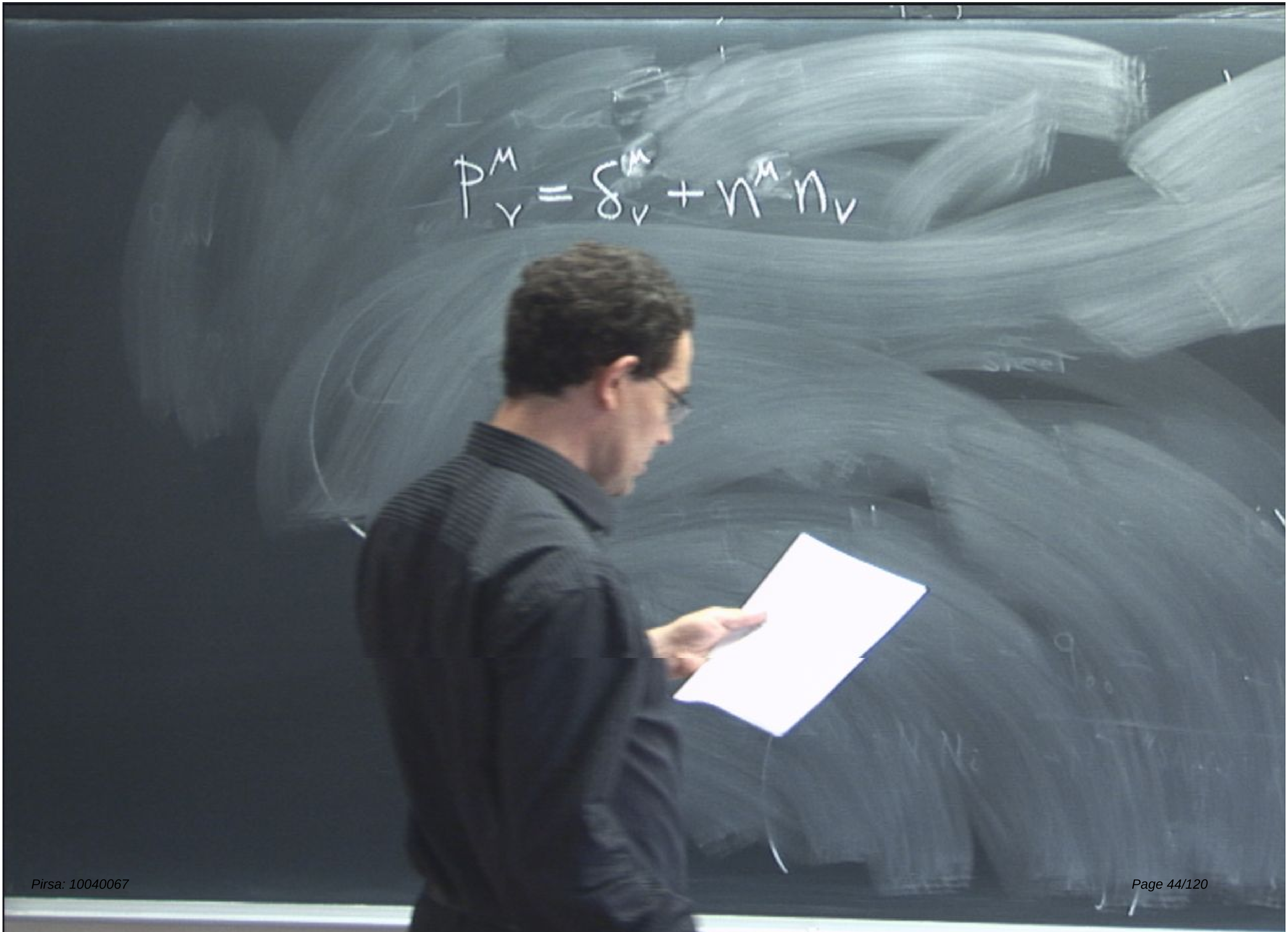
Extrinsic curvature : from $\nabla_\mu n_\nu$
tensor in 4d.

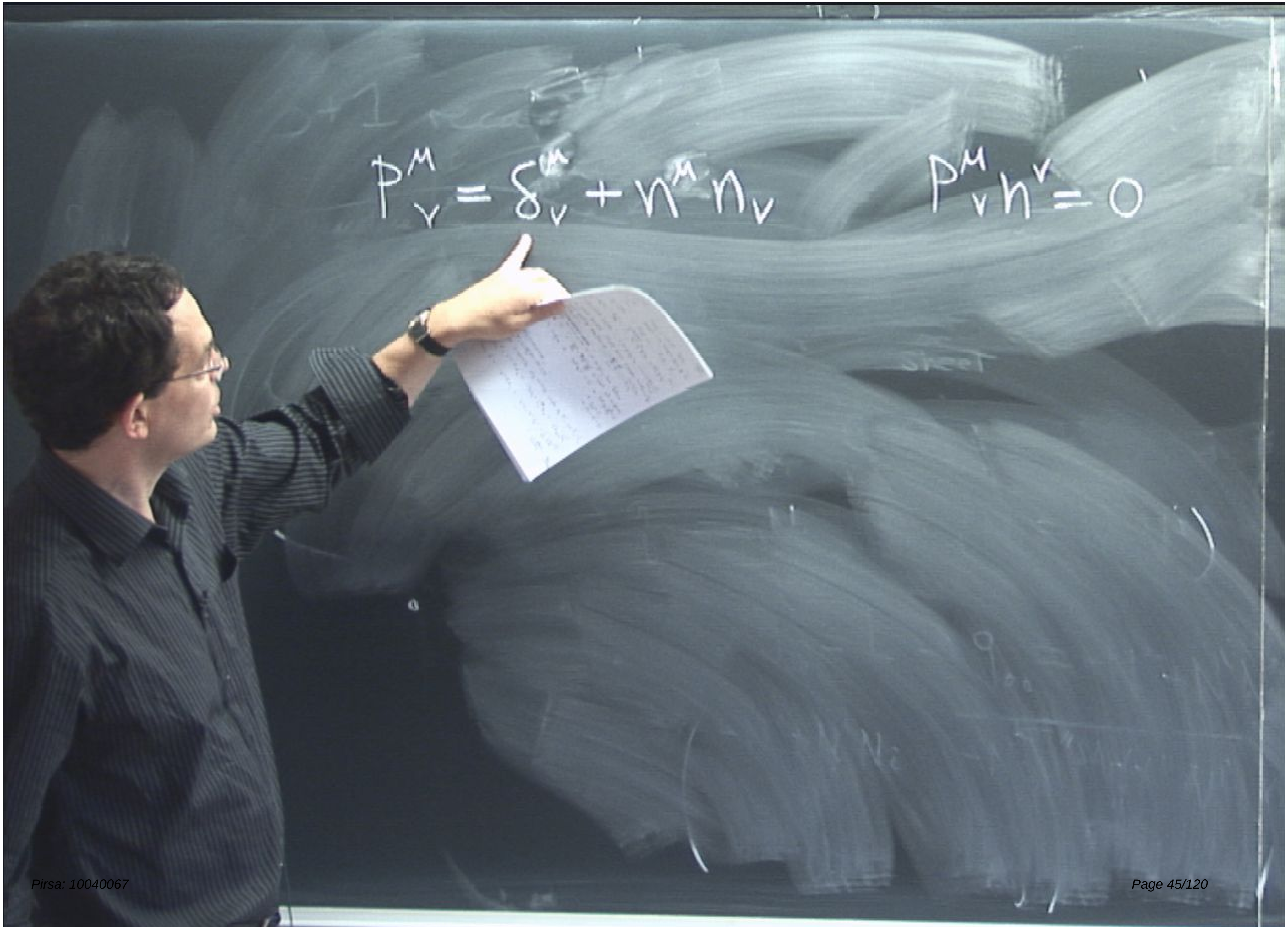


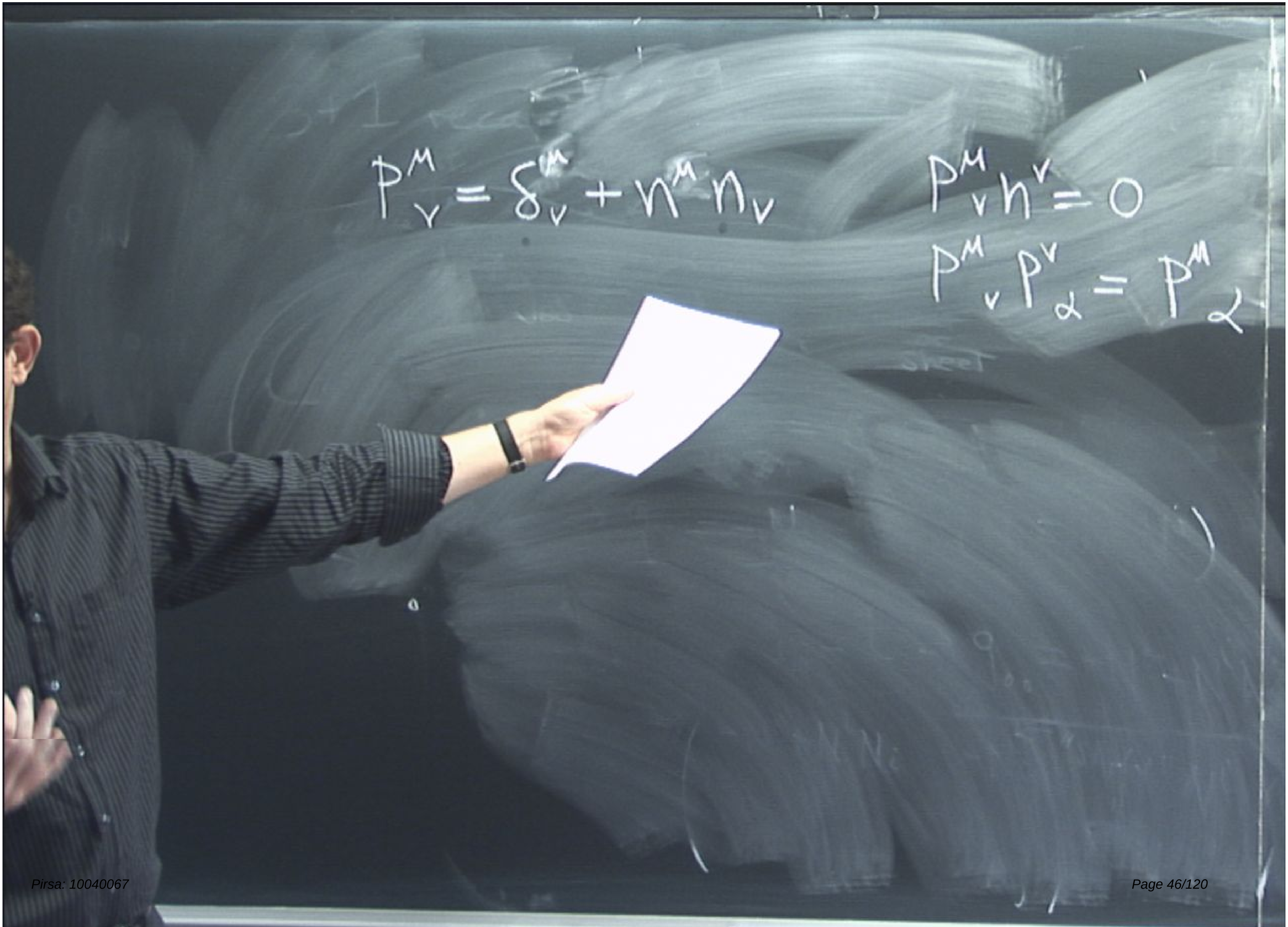
need to project to 3d.

Any tensor index μ, $T_{\mu\alpha\beta\ldots}$ can be projected into a piece along n^μ, and a piece $\perp$r n^μ.

$n^\mu T_{\mu\alpha\beta\ldots}$


$P^{\mu}_{\nu} = \delta^{\mu}_{\nu} + n^{\mu} n_{\nu}$


$P^\mu_\nu = \delta^\mu_\nu + n^\mu n_\nu$
$P^\mu_\nu n^\nu = 0$


$P^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} + n^{\mu} n_{\nu}$
$P^{\mu}{}_{\nu} n^{\nu} = 0$
$P^{\mu}{}_{\nu} P^{\nu}{}_{\alpha} = P^{\mu}{}_{\alpha}$

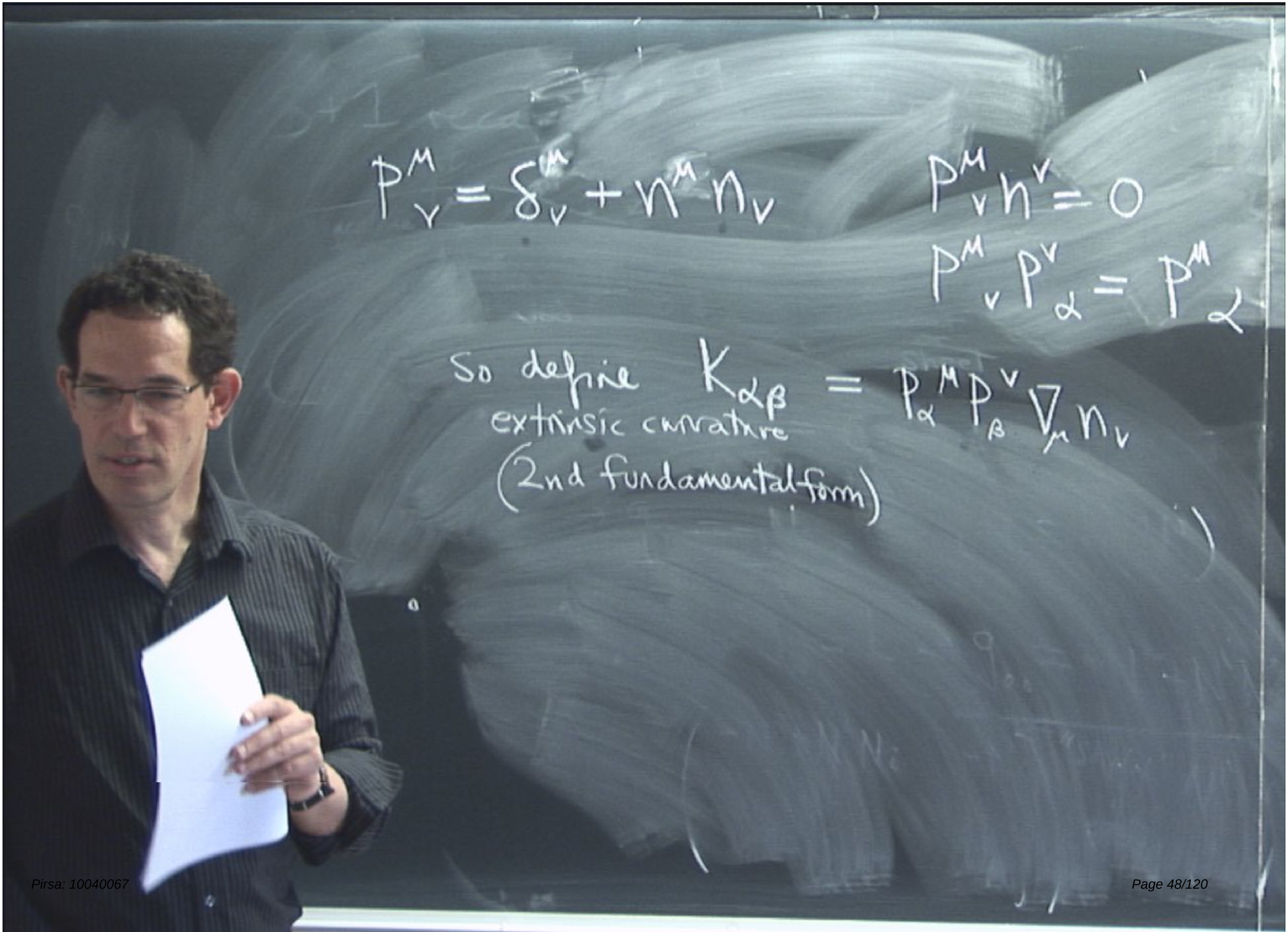
$$P^\mu{}_\nu = \delta^\mu_\nu + n^\mu n_\nu$$

$$P^\mu{}_\nu n^\nu = 0$$

$$P^\mu{}_\nu P^\nu{}_\alpha = P^\mu{}_\alpha$$

So define $K_{\alpha\beta}$

ex


$P^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} + n^{\mu} n_{\nu}$
$P^{\mu}{}_{\nu} n^{\nu} = 0$
$P^{\mu}{}_{\nu} P^{\nu}{}_{\alpha} = P^{\mu}{}_{\alpha}$
So define $K_{\alpha\beta} = P_{\alpha}{}^{\mu} P_{\beta}{}^{\nu} \nabla_{\mu} n_{\nu}$
extrinsic curvature
(2nd fundamental form)

$$P^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} + n^{\mu} n_{\nu}$$

$$P^{\mu}{}_{\nu} n^{\nu} = 0$$

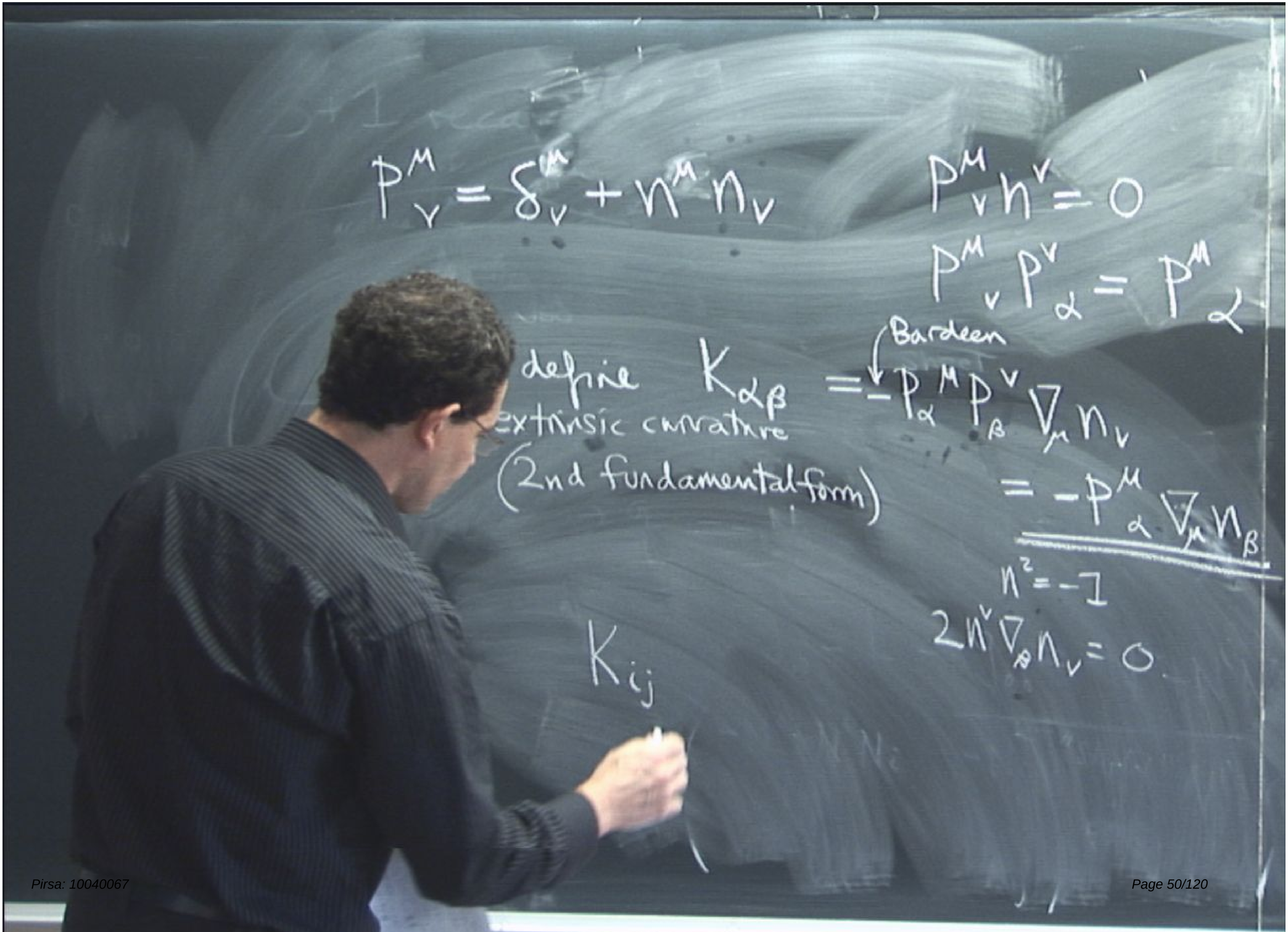
$$P^{\mu}{}_{\nu} P^{\nu}{}_{\alpha} = P^{\mu}{}_{\alpha}$$

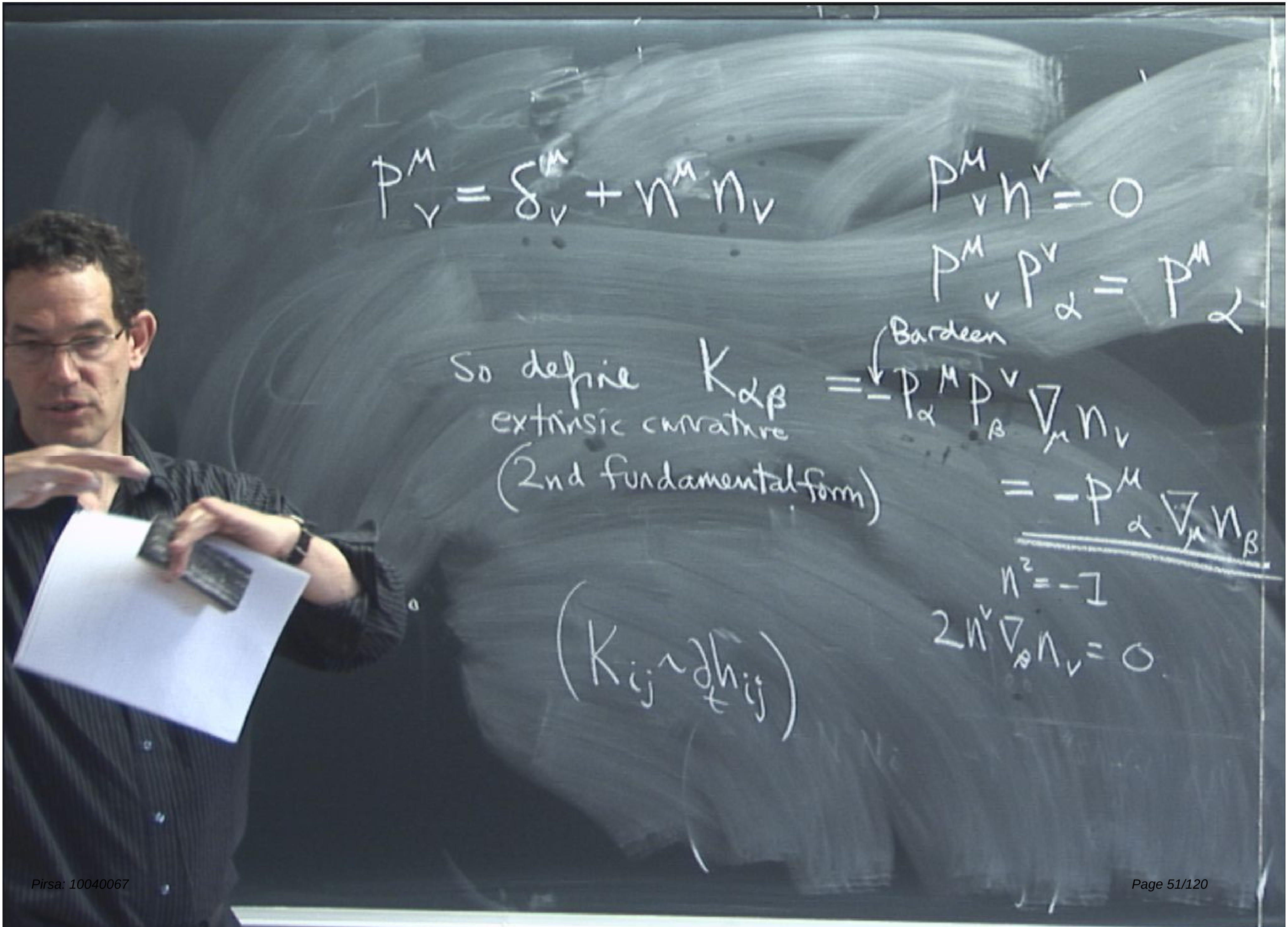
Bardeen

So define $K_{\alpha\beta} = -P_{\alpha}{}^{\mu} P_{\beta}{}^{\nu} \nabla_{\mu} n_{\nu}$

extrinsic curvature

(2nd fundamental form)


$P^\mu{}_\nu = \delta^\mu_\nu + n^\mu n_\nu$
$P^\mu{}_\nu n^\nu = 0$
$P^\mu{}_\nu P^\nu{}_\alpha = P^\mu{}_\alpha$
Bardeen
define $K_{\alpha\beta} = -P_\alpha{}^\mu P_\beta{}^\nu \nabla_\mu n_\nu$
extrinsic curvature
(2nd fundamental form)
$= -P^\mu{}_\alpha \nabla_\mu n_\beta$
$n^2 = -1$
$2n^\nu \nabla_\alpha n_\nu = 0$
K_{ij}


$P^\mu{}_\nu = \delta^\mu_\nu + n^\mu n_\nu$
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Bardeen
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extrinsic curvature
(2nd fundamental form)
$= -P^\mu{}_\alpha \nabla_\mu n_\beta$
$n^2 = -1$
$2n^\nu \nabla_\rho n_\nu = 0$
$(K_{ij} \sim \partial_t h_{ij})$

$$P^\mu{}_\nu = \delta^\mu_\nu + n^\mu n_\nu$$

$$P^\mu{}_\nu n^\nu = 0$$

$$P^\mu{}_\nu P^\nu{}_\alpha = P^\mu{}_\alpha$$

Bardeen

So define $K_{\alpha\beta} = -P_\alpha{}^\mu P_\beta{}^\nu \nabla_\mu n_\nu$

extrinsic curvature

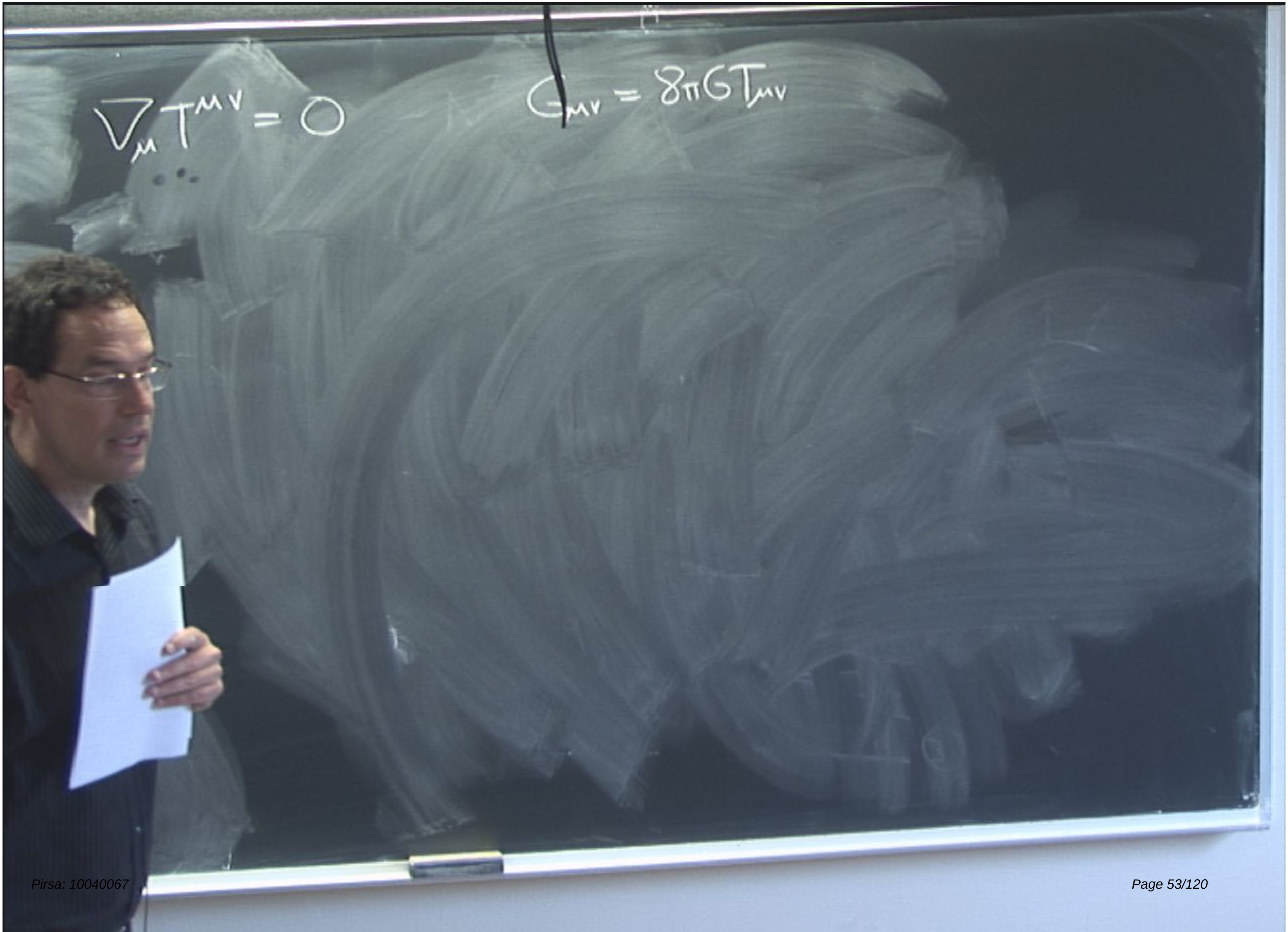
(2nd fundamental form)

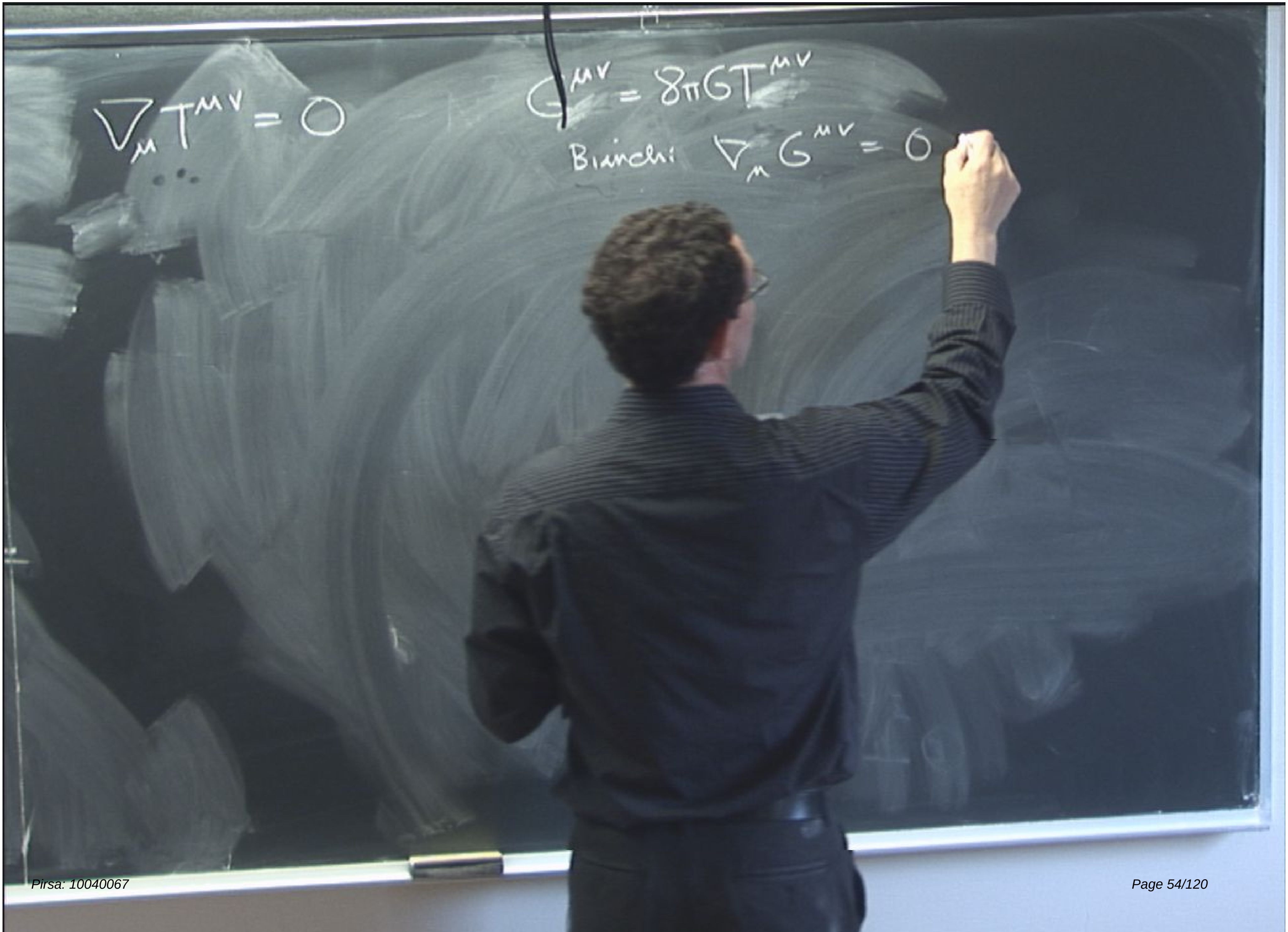
$$= -P^\mu{}_\alpha \nabla_\mu n_\beta$$

$$n^2 = -1$$

$$2n^\nu \nabla_\rho n_\nu = 0$$

$$\left(K_{ij} \sim \partial_t h_{ij} \right)$$


$\nabla_\mu T^{\mu\nu} = 0$
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$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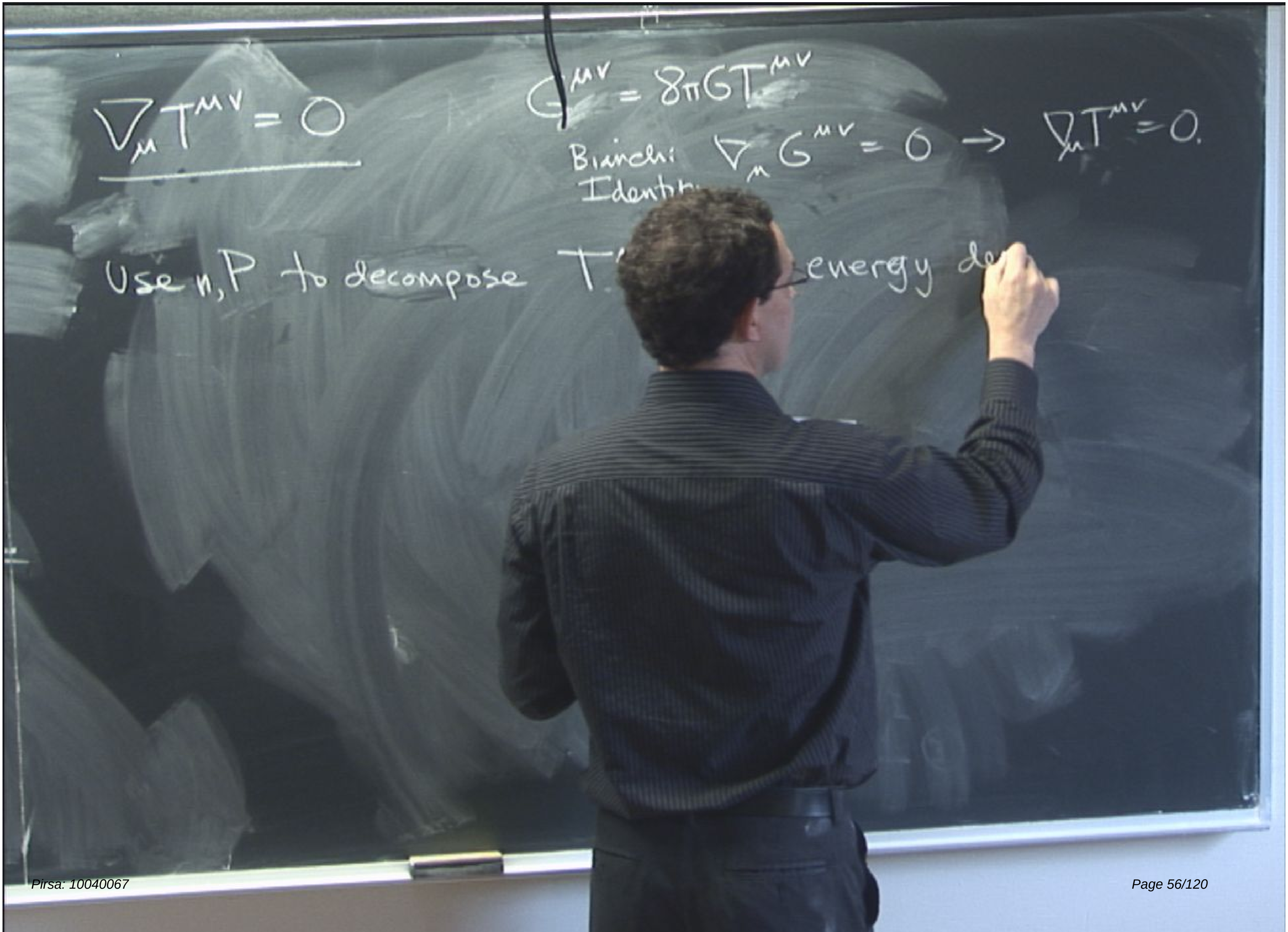
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Use n,P to decompose T
energy

$\nabla_\mu T^{\mu\nu} = 0$

$G^{\mu\nu} = 8\pi G T^{\mu\nu}$

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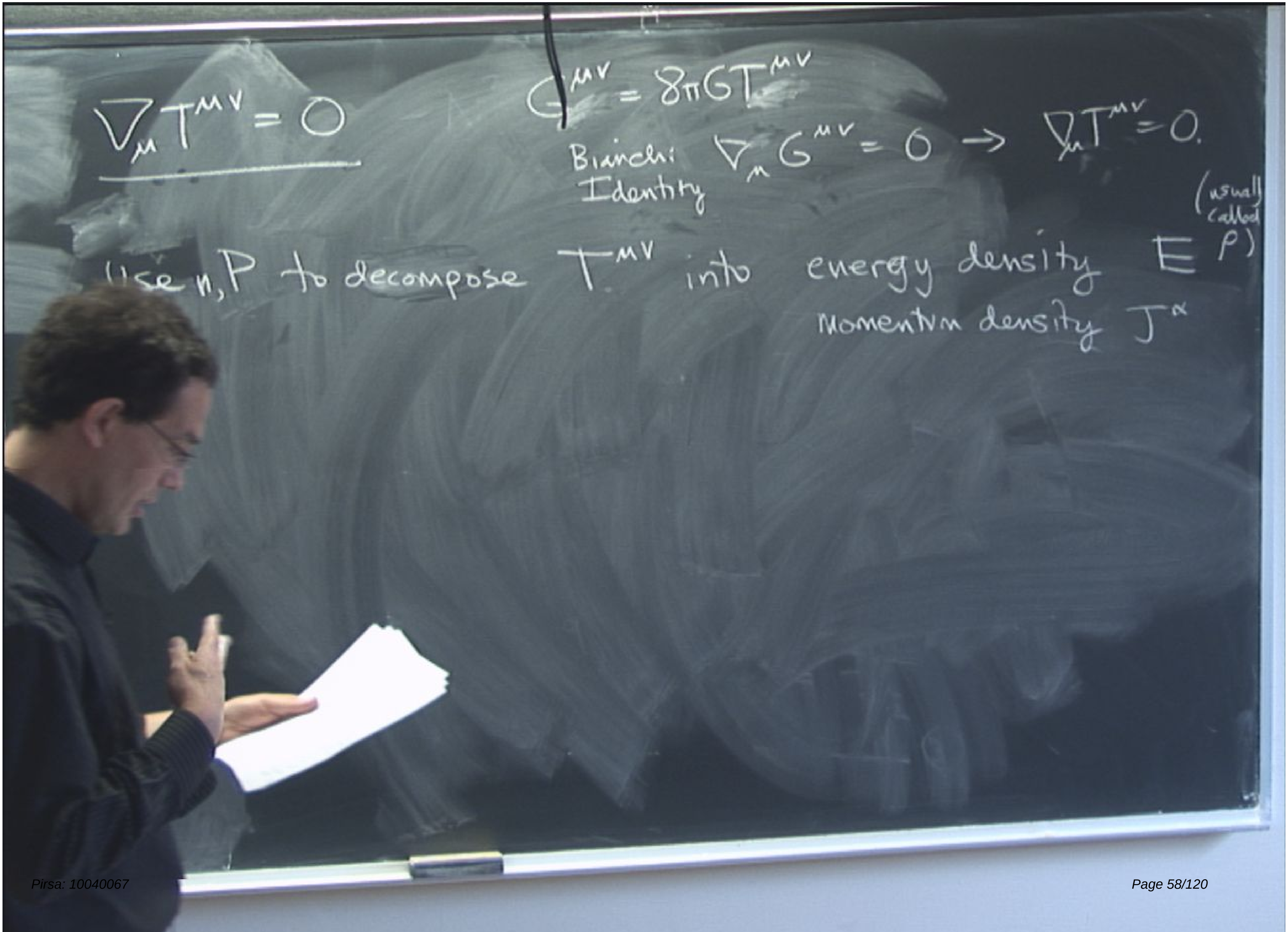
Use n, P to decompose $T^{\mu\nu}$ into energy density E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Use n, P to decompose $T^{\mu\nu}$ into energy density E (usually called ρ)
momentum density J^α

$\nabla_\mu T^{\mu\nu} = 0$

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Use n, P to decompose $T^{\mu\nu}$ into energy density E (usually called ρ)

momentum density J^α

spatial stress $S^{\alpha\beta}$

$\underline{\nabla_\mu T^{\mu\nu} = 0}$

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Use n, P to decompose $T^{\mu\nu}$ into energy density E (usually called ρ)

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$$E = n_\mu n_\nu T^{\mu\nu}$$

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Use n, P to decompose $T^{\mu\nu}$ into energy density E (usually called ρ)

spatial Momentum density J^α

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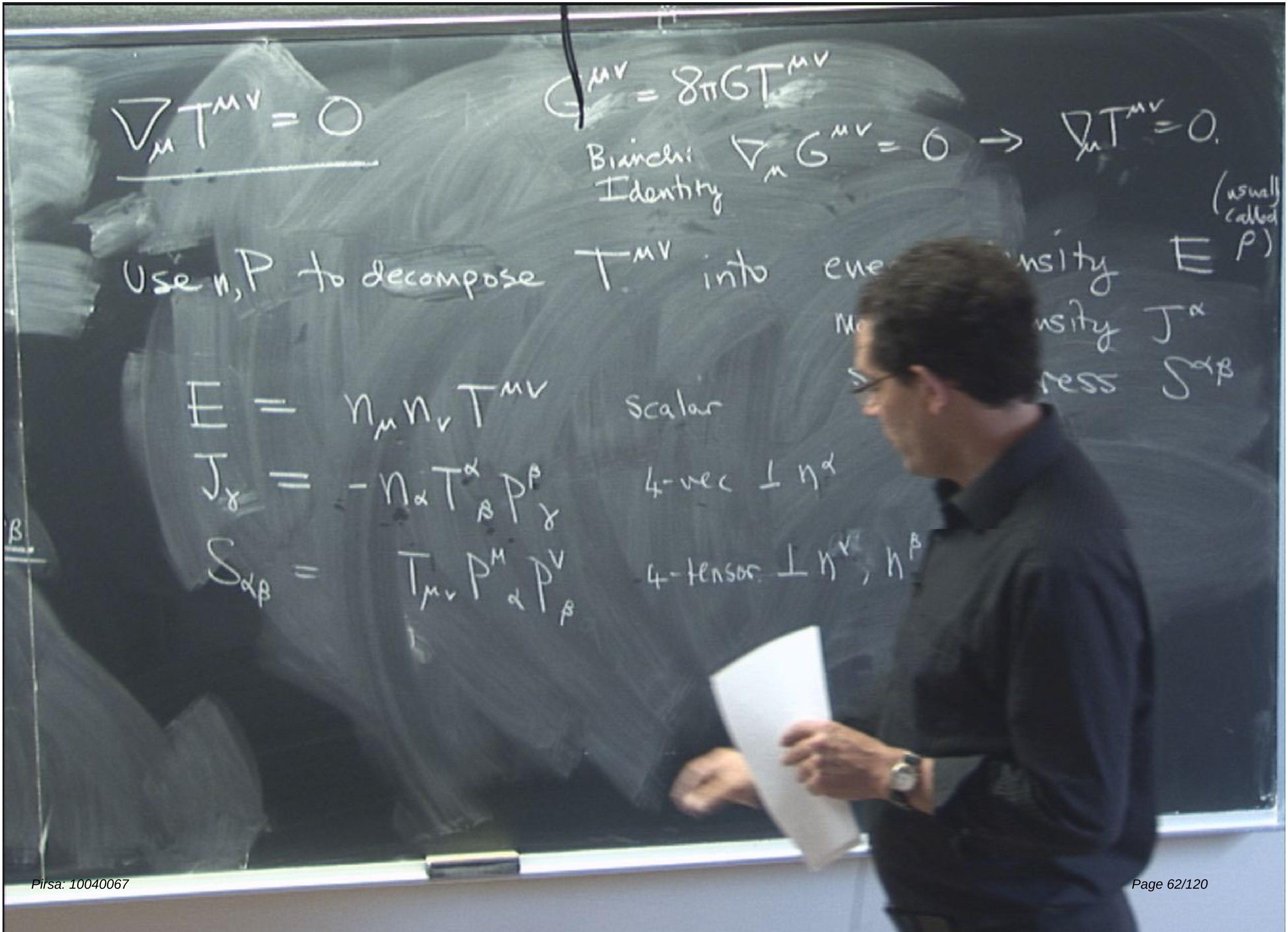
$$E = n_\mu n_\nu T^{\mu\nu}$$

$$J_\gamma = -n_\alpha T^\alpha{}_\beta P^\beta{}_\gamma$$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Use n,P to decompose $T^{\mu\nu}$ into ene nsity E
Mo nsity J^α
ress $S^{\alpha\beta}$
$E = n_\mu n_\nu T^{\mu\nu}$ scalar
$J_\gamma = -n_\alpha T^\alpha{}_\beta P^\beta{}_\gamma$ 4-vec $\perp n^\alpha$
$S_{\alpha\beta} = T_{\mu\nu} P^\mu{}_\alpha P^\nu{}_\beta$ 4-tensor $\perp n^\nu, n^\beta$

$\nabla_\mu T^{\mu\nu} = 0$

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Bianchi Identity: $\nabla_\mu G^{\mu\nu} = 0 \rightarrow \nabla_\mu T^{\mu\nu} = 0.$

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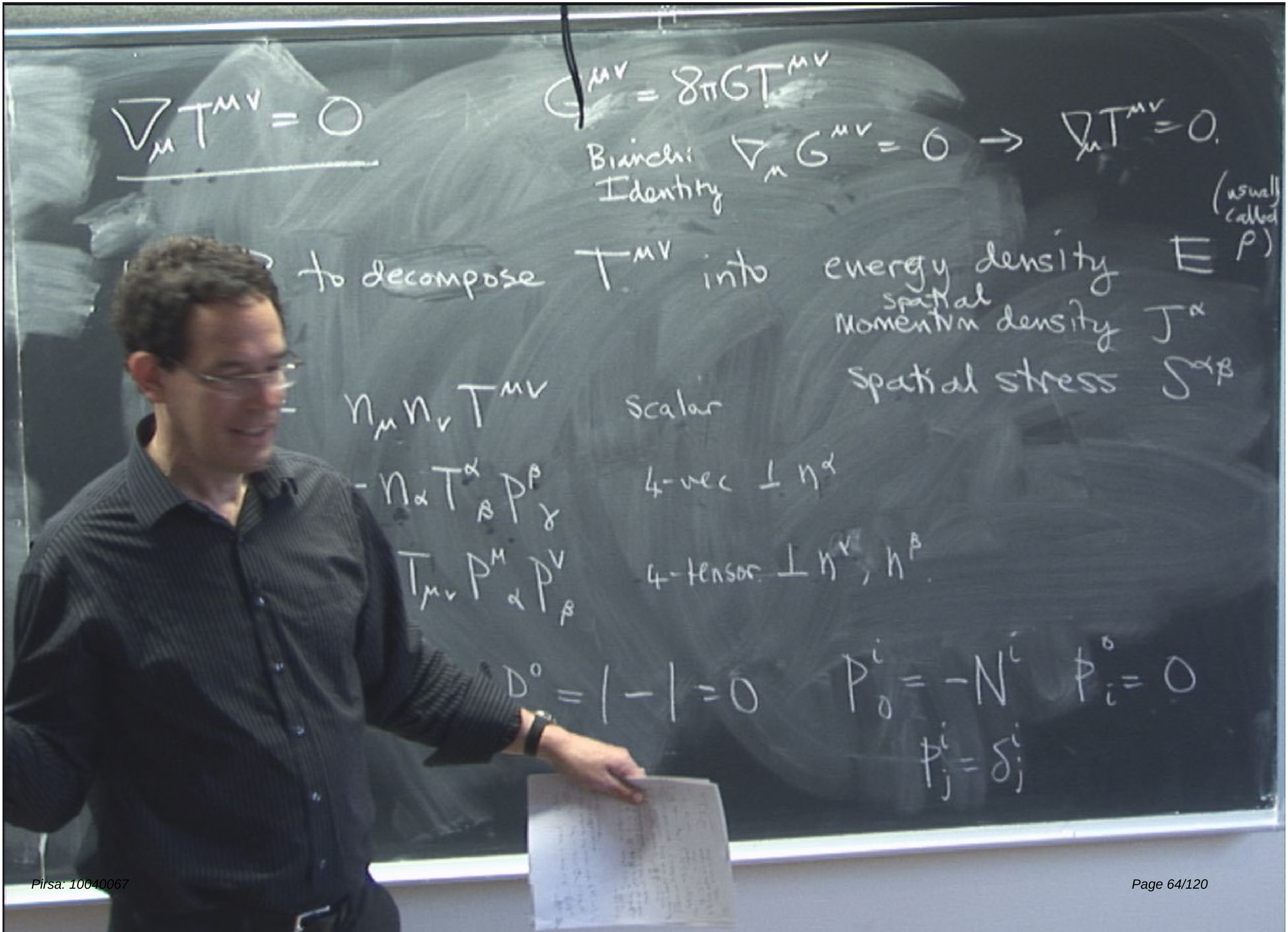
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spatial momentum density J^α
spatial stress $S^{\alpha\beta}$
$n_\mu n_\nu T^{\mu\nu}$ scalar
$-n_\alpha T^\alpha{}_\beta P^\beta{}_\gamma$ 4-vec $\perp n^\alpha$
$T_{\mu\nu} P^\mu{}_\alpha P^\nu{}_\beta$ 4-tensor $\perp n^\alpha, n^\beta$
$P^0{}_0 = 1 - 1 = 0$ $P^i{}_0 = -N^i$ $P^0{}_i = 0$
$P^i{}_j = \delta^i_j$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$G^{\mu\nu} = 8\pi G T^{\mu\nu}$$

Bianchi Identity $\nabla_\mu G^{\mu\nu} = 0 \rightarrow \nabla_\mu T^{\mu\nu} = 0.$

Use n, P to decompose $T^{\mu\nu}$ into energy density E (usually called ρ)

spatial Momentum density J^α

spatial stress $S^{\alpha\beta}$

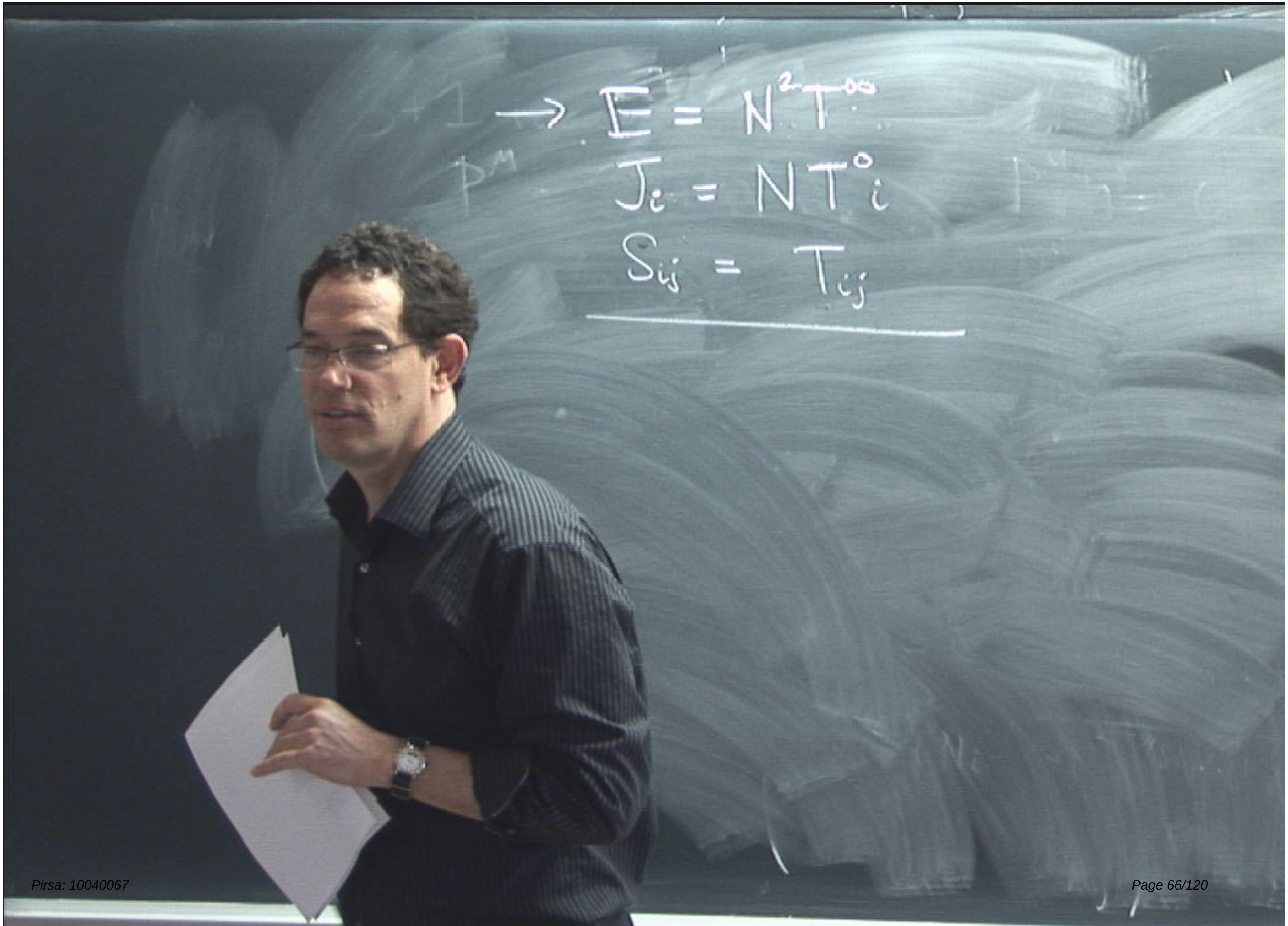
$$E = n_\mu n_\nu T^{\mu\nu} \quad \text{scalar}$$

$$J_\gamma = -n_\alpha T^\alpha{}_\beta P^\beta{}_\gamma \quad \text{4-vec} \perp n^\alpha$$

$$S_{\alpha\beta} = T_{\mu\nu} P^\mu{}_\alpha P^\nu{}_\beta \quad \text{4-tensor} \perp n^\alpha, n^\beta$$

$$P^\mu{}_\nu = \delta^\mu_\nu + n^\mu n_\nu \qquad P^0{}_0 = 1 - 1 = 0 \qquad P^i{}_0 = -N^i \qquad P^0{}_i = 0$$

$$P^i{}_j = \delta^i_j$$

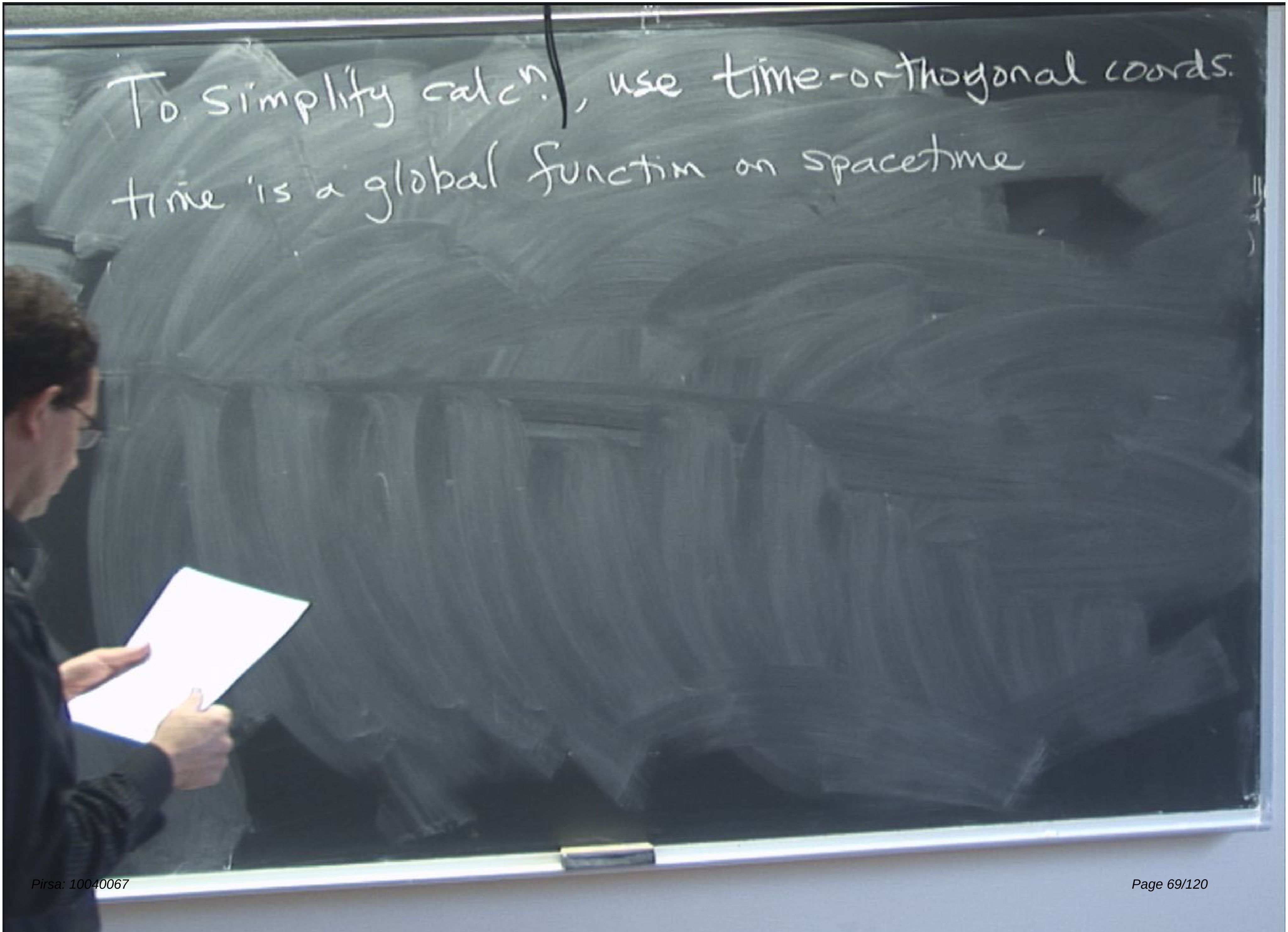

$\rightarrow E = N^2 T^{00}$
$J_i = N T^0{}_i$
$S_{ij} = T_{ij}$

$$\longrightarrow E = N^2 T^{00}$$

$$J_i = N T^0{}_i$$

$$S_{ij} = T_{ij}$$

To simplify calc^{n}, use time-orthogonal coords.


To simplify calc^{n}, use time-orthogonal coords.
time is a global function on spacetime

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time is any global function on spacetime which is always increasing in all future-pointing directions

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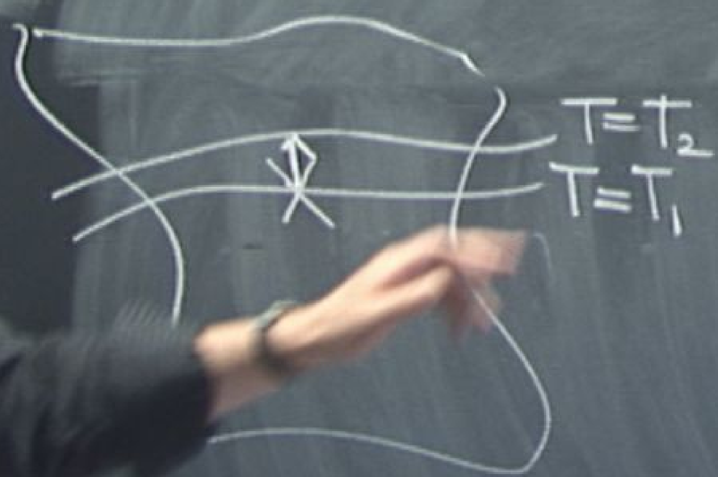

To simplify calc^{n}., use time-orthogonal coords.

time is any global function on spacetime which is always increasing in all future-pointing directions

$T(x^{\mu})$

To simplify calcⁿ., use time-orthogonal coords.

time is any global function on spacetime which is always increasing in all future-pointing directions

$T(x^\mu)$ $\quad V^\mu \frac{\partial}{\partial x^\mu} T > 0 \quad V^\mu$ inside future l.c.

To simplify calc^n, use time-orthogonal coords.

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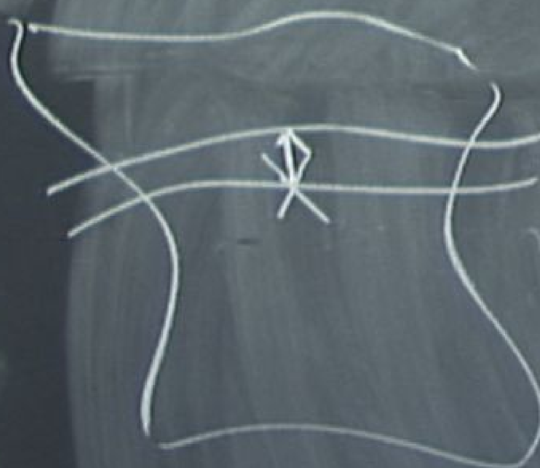
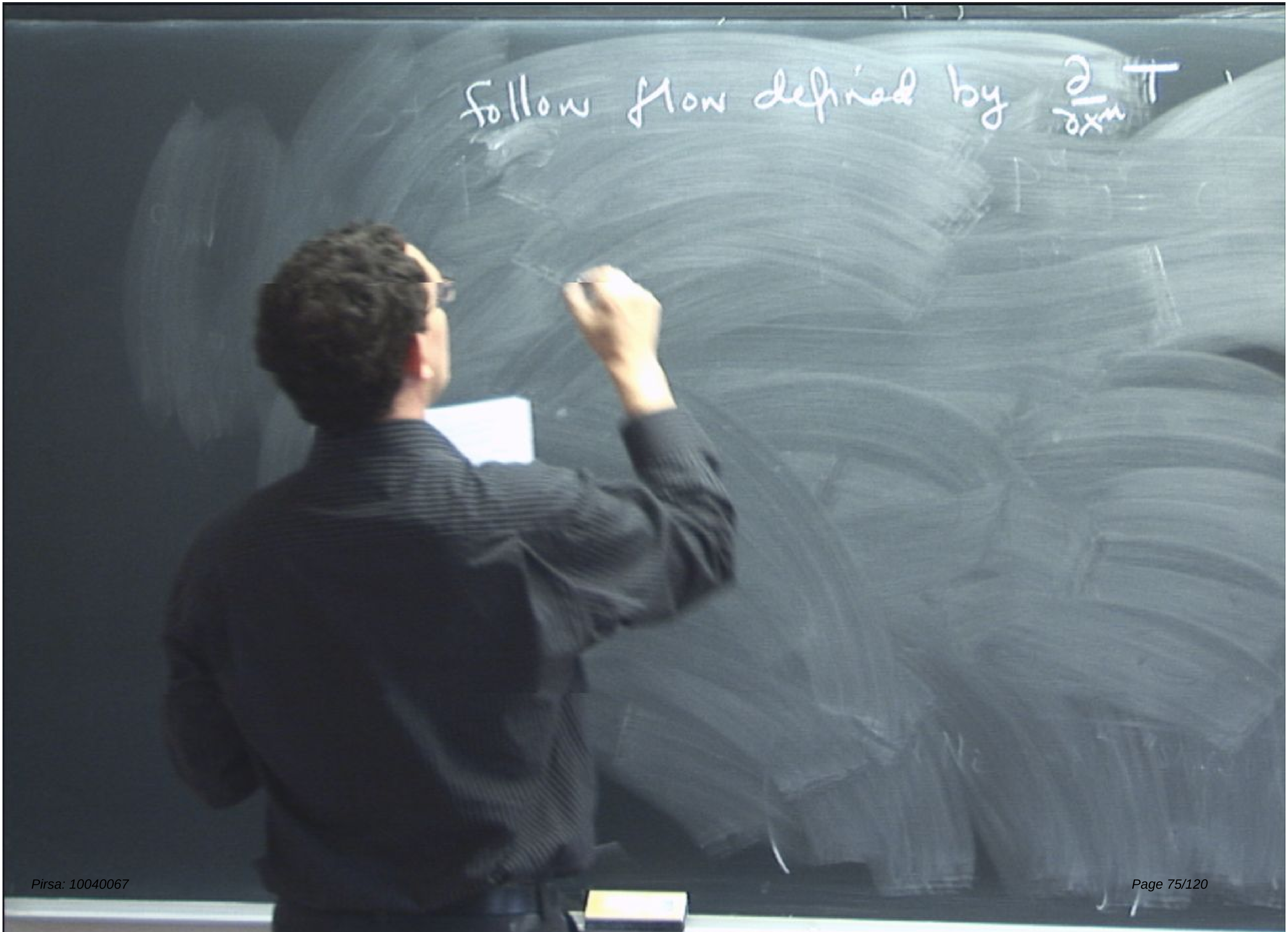
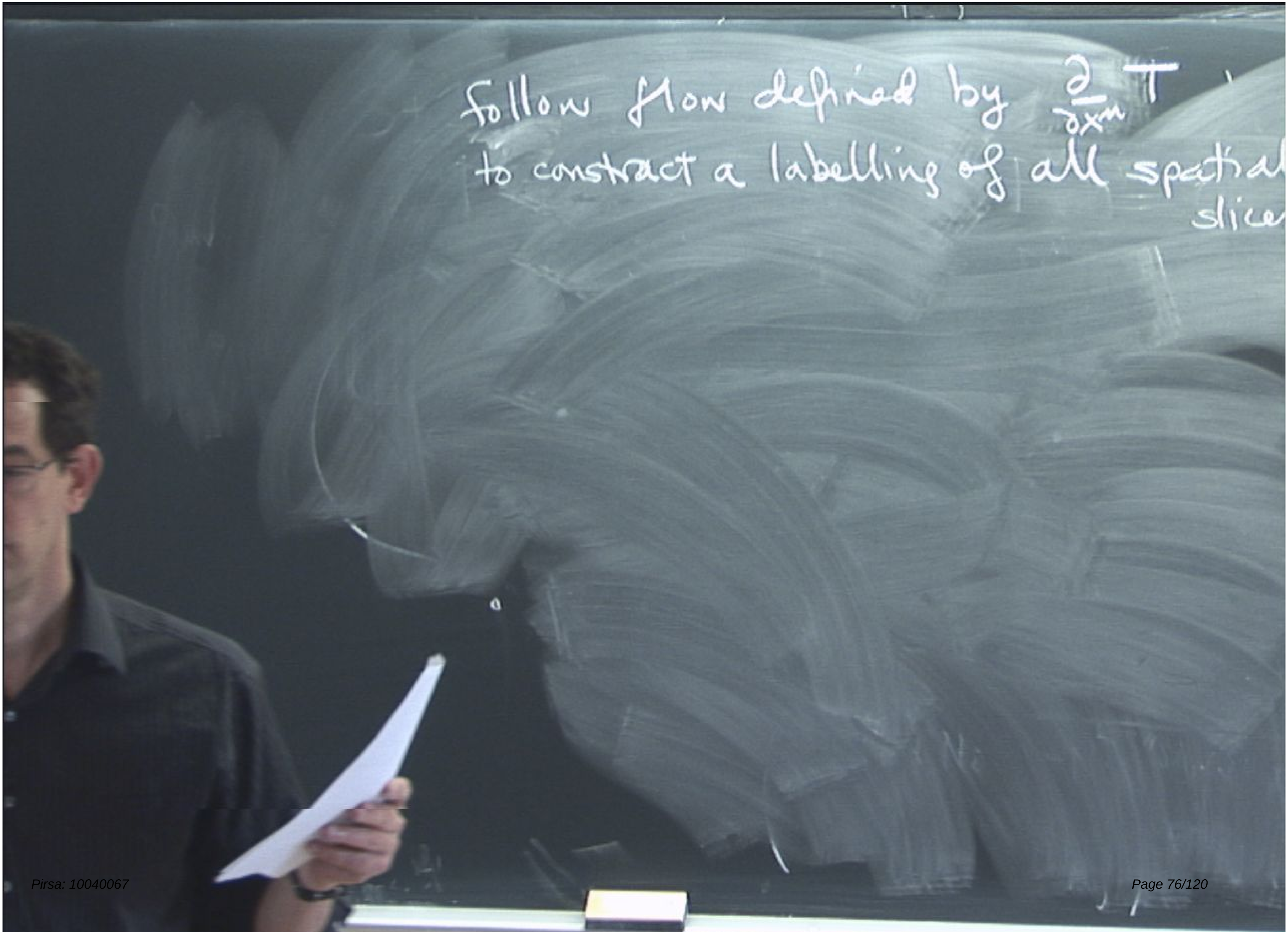
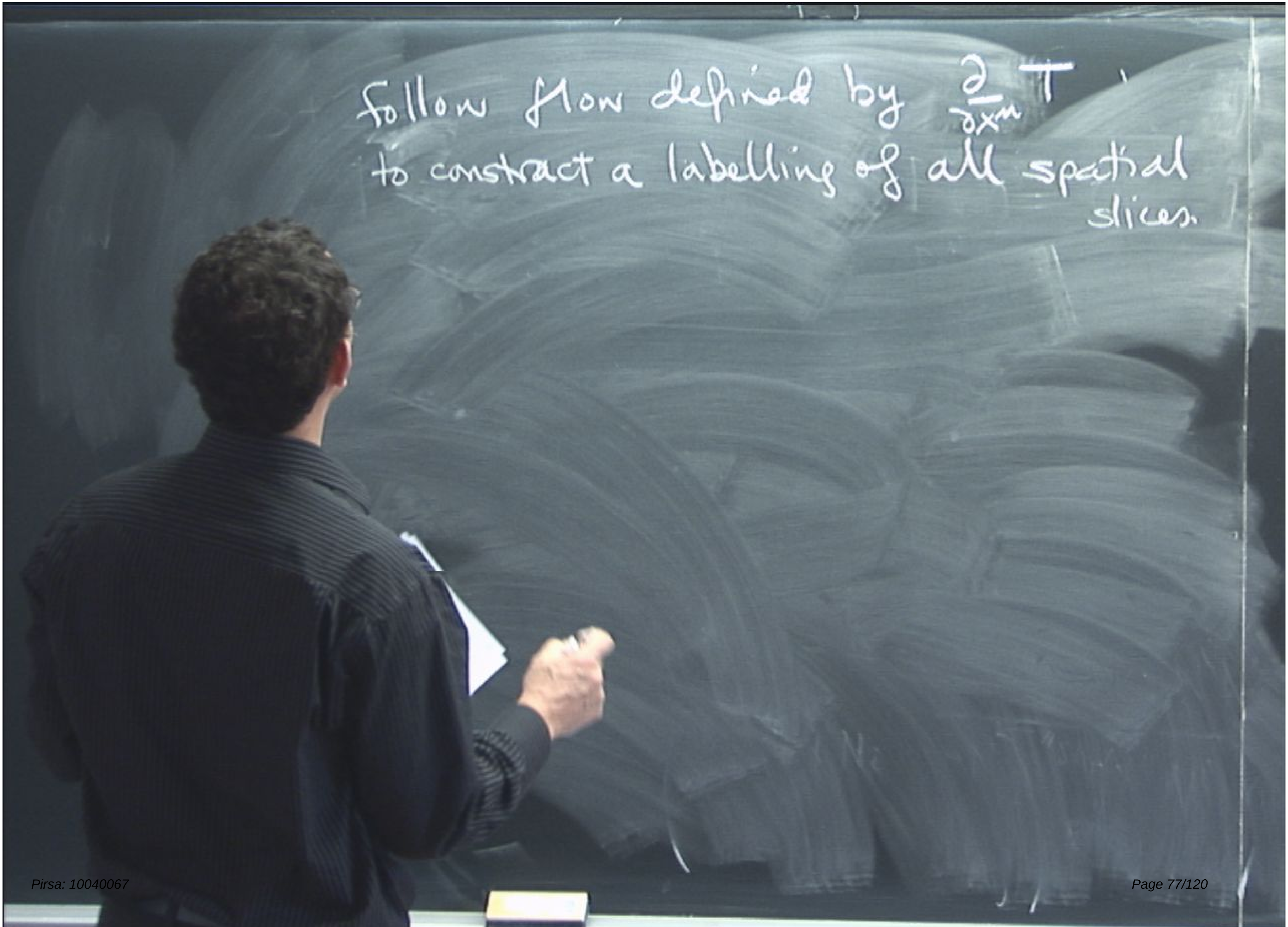
$T(x^\mu)$ $\quad V^\mu \frac{\partial}{\partial x^\mu} T > 0 \quad V^\mu$ inside future l.c.



level sets of T = spatial hypersurfaces Σ

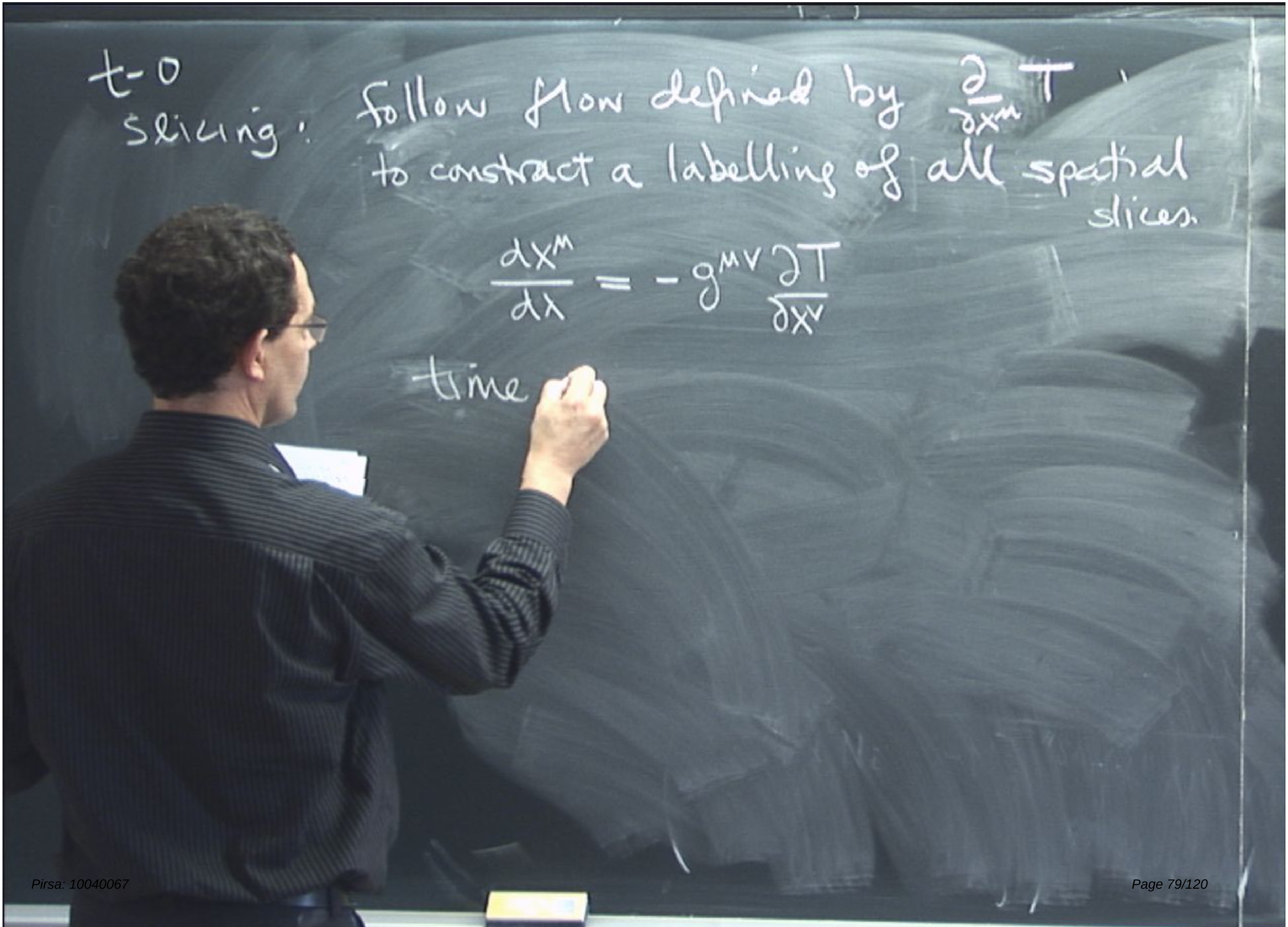

follow flow defined by $\frac{\partial}{\partial x^\mu} T$


follow flow defined by $\frac{\partial}{\partial x^m} T$
to construct a labelling of all spatial
slice


follow flow defined by $\frac{\partial}{\partial x^\mu} T$
to construct a labelling of all spatial
slices.

t-0 slicing: follow flow defined by $\frac{\partial}{\partial x^\mu} T$ to construct a labelling of all spatial slices.

$$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu}\frac{\partial T}{\partial x^\nu}$$


t-0
slicing: follow flow defined by $\frac{\partial}{\partial x^\mu} T$
to construct a labelling of all spatial slices.
$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu} \frac{\partial T}{\partial x^\nu}$
time

t-0

slicing: follow flow defined by $\frac{\partial}{\partial x^\mu} T$
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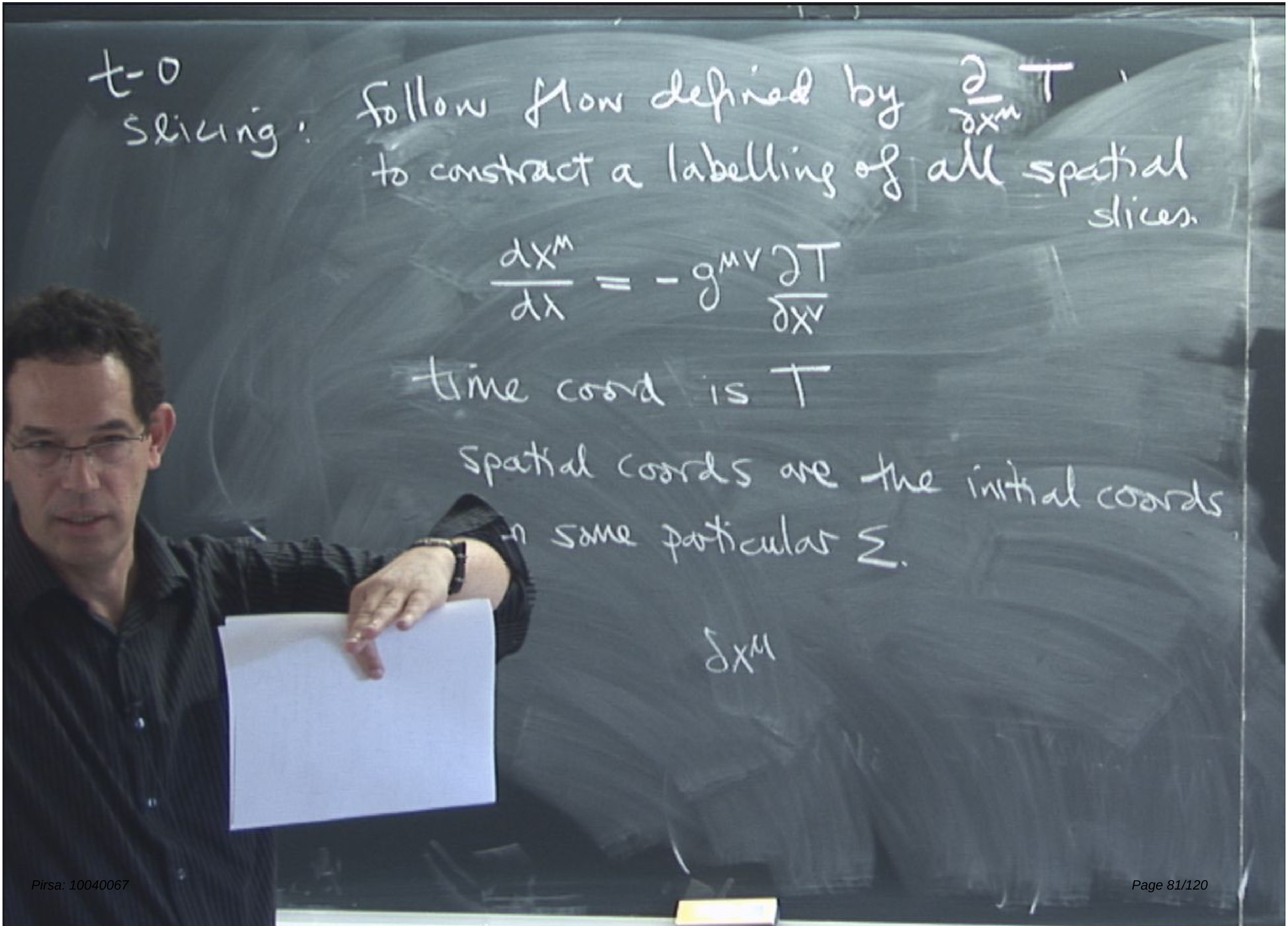
$$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu}\frac{\partial T}{\partial x^\nu}$$

time coord is λ or any monotonic f^n of it

spatial coords are the initial coords on some particular Σ.



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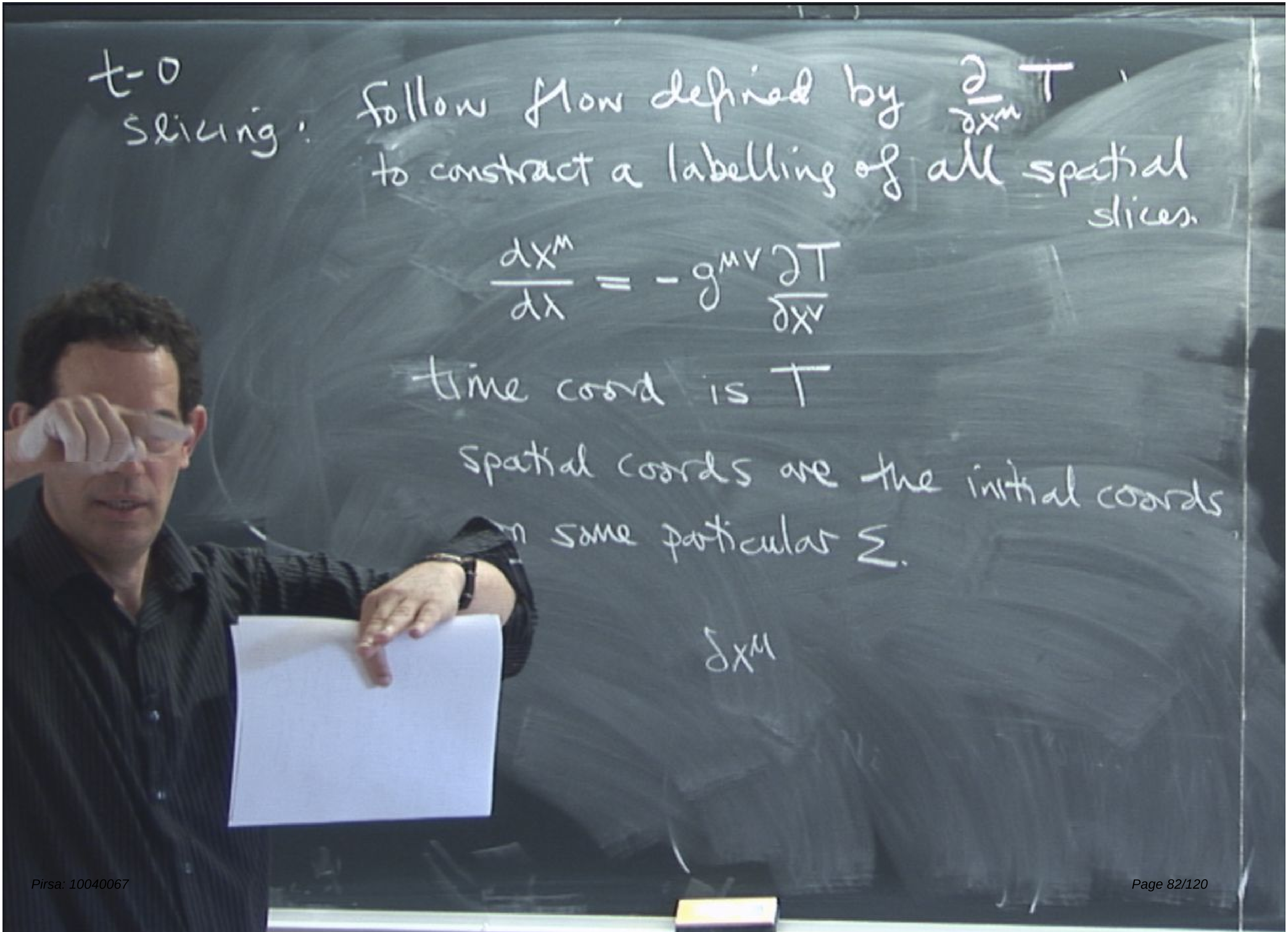
$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu} \frac{\partial T}{\partial x^\nu}$
time coord is T
spatial coords are the initial coords
on some particular Σ.
δx^μ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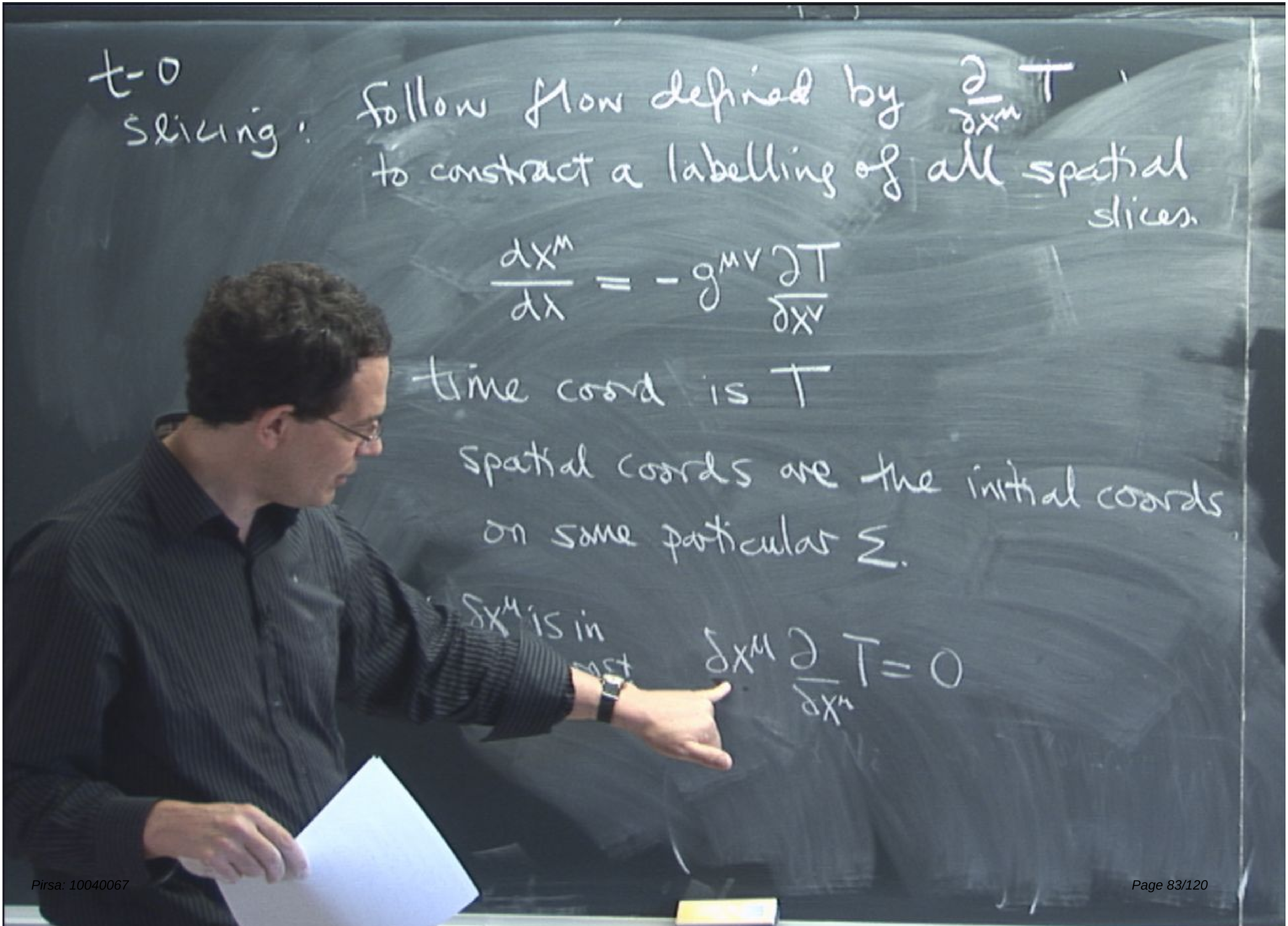
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$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu} \frac{\partial T}{\partial x^\nu}$
time coord is T
spatial coords are the initial coords on some particular Σ.
δx^μ is in
$\delta x^\mu \frac{\partial}{\partial x^\mu} T = 0$

$=0$
slicing: follow flow defined by $\frac{\partial}{\partial x^\mu} T$
to construct a labelling of all spatial slices.

$$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu}\frac{\partial T}{\partial x^\nu}$$

time coord is T

spatial coords are the initial coords on some particular Σ.

- if δx^μ is in a $T=$ const surface. $\delta x^\mu \frac{\partial}{\partial x^\mu} T = 0 \Rightarrow \delta x^\mu \frac{dx^\nu}{d\lambda} g_{\mu\nu} = 0$

$= 0$

slicing: follow flow defined by $\frac{\partial}{\partial x^\mu} T$
to construct a labelling of all spatial slices.

$$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu} \frac{\partial T}{\partial x^\nu}$$

time coord is T

spatial coords are the initial coords on some particular Σ.

- if δx^μ is in a $T =$ const surface $\quad \delta x^\mu \frac{\partial}{\partial x^\mu} T = 0 \Rightarrow \delta x^\mu \frac{dx^\nu}{d\lambda} g_{\mu\nu} = 0$

$$\Rightarrow g_{i0} = 0$$

$\to N_i = 0$

$g_{\mu\nu} =$

$\rightarrow N_i = 0$

int-o coords, only nonzero components of P^μ_ν are $P^i_j = \delta^i_j$

" " $K_{\mu\nu}$

$g_{\mu\nu} =$

$\rightarrow N_i = 0$

int-o coords, only nonzero components of $P^{\mu}{}_{\nu}$ are $P^i{}_j = \delta^i_j$

" $K_{\mu\nu}$

are K_{ij}

$g_{\mu\nu} =$

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int-o coords, only nonzero components of $P^\mu{}_\nu$ are $P^i{}_j = \delta^i{}_j$

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are $K_{ij} = -\nabla_i n_j$

$g_{\mu\nu} =$

$\rightarrow N_i = 0$

int-o coords, only nonzero components of $P^{\mu}{}_{\nu}$ are $P^i{}_j = \delta^i_j$

" $K_{\mu\nu}$

are $K_{ij} = -\nabla_i n_j = -\partial_i n_j + \Gamma^{\alpha}_{ji} n_\alpha$

$g_{\mu\nu} =$

$\rightarrow N_i = 0$

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are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji} n_0 = -\frac{1}{2} h_{ij,0}$

$g_{\mu\nu} =$

$\rightarrow N_i = 0 \rightarrow n_i = 0$

int-o coords, only nonzero components of P^μ_ν are $P^i_j = \delta^i_j$

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are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji} n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

$g_{\mu\nu} =$

$\rightarrow N_i = 0 \rightarrow n_i = 0$

int-o coords, only nonzero components of P^μ_ν are $P^i_j = \delta^i_j$

" " $K_{\mu\nu}$

are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji} n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

trace $h^{ij} K_{ij} = -h^{ij} \nabla_i n_j = -\frac{1}{2} h_{ij,0} h^{ij} N^{-1}$

$= -\frac{1}{\sqrt{h}} \frac{1}{N} \frac{\partial}{\partial t} \sqrt{h}$

$g_{\mu\nu} =$

$\rightarrow N_i = 0 \rightarrow n_i = 0$

int-0 coords, only nonzero components of $P^{\mu}_{\ \nu}$ are $P^i_{\ j} = \delta^i_j$

" $K_{\mu\nu}$

are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji} n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

trace $h^{ij} K_{ij} = -h^{ij} \nabla_i n_j = -\frac{1}{2} h_{ij,0} h^{ij} N^{-1}$

$= -\frac{1}{\sqrt{h}} \frac{1}{N} \frac{\partial}{\partial t} \sqrt{h} = \quad - d\,\mathrm{Vol}$

$g_{\mu\nu} =$

$\mathrm{Vol} = \int d^3x \sqrt{h}$

$\to N_i = 0 \to n_i = 0$

int-o coords, only nonzero components of $P^\mu{}_\nu$ are $P^i{}_j = \delta^i{}_j$

" $K_{\mu\nu}$

are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji}\, n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

trace $h^{ij} K_{ij} = -h^{ij}\nabla_i n_j = -\frac{1}{2} h_{ij,0}\, h^{ij} N^{-1}$

$= -\frac{1}{\sqrt{h}} \frac{1}{N} \frac{\partial}{\partial t}\sqrt{h} = - d\, V$

$g_{\mu\nu} =$

$Vol = \int d^3x \sqrt{h}$

$\to N_i = 0 \to n_i = 0$

int-o coords, only nonzero components of $P^\mu{}_\nu$ are $P^i{}_j = \delta^i{}_j$

" $K_{\mu\nu}$

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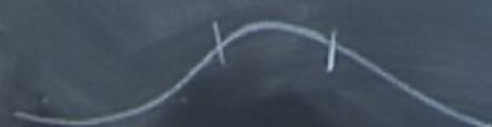
trace $h^{ij} K_{ij} = -h^{ij} \nabla_i n_j = -\frac{1}{2} h_{ij,0} h^{ij} N^{-1}$

$$= -\frac{1}{\sqrt{h}} \frac{1}{N} \frac{\partial}{\partial t} \sqrt{h} = -\frac{d\, Vol}{d t_p}$$

$g_{\mu\nu} =$



$Vol = \int d^3x \sqrt{h}$

$\rightarrow N_i = 0 \rightarrow n_i = 0$

int-o coords, only nonzero components of $P^\mu{}_\nu$ are $P^i{}_j = \delta^i{}_j$

" $K_{\mu\nu}$

are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji}\, n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

trace $K \equiv h^{ij} K_{ij} = -h^{ij}\nabla_i n_j = -\frac{1}{2} h_{ij,0} h^{ij} N^{-1}$

$K = -\frac{1}{Vol}\frac{d\,Vol}{dt_p}$

$= -\frac{1}{\sqrt{h}}\frac{1}{N}\frac{\partial}{\partial t}\sqrt{h} = -\frac{1}{Vol}\frac{d\,Vol}{dt_p}$

$g_{\mu\nu} =$

$Vol = \int d^3x \sqrt{h}$

$\rightarrow N_i = 0 \rightarrow n_i = 0$

t-0 coords, only nonzero components of $P^\mu_{\ \nu}$ are $P^i_{\ j} = \delta^i_j$

" " $K_{\mu\nu}$

are $K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^{0}_{ji}\, n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

trace $K \equiv h^{ij} K_{ij} = -h^{ij}\nabla_i n_j = -\frac{1}{2} h_{ij,0}\, h^{ij} N^{-1}$

$K = -\frac{1}{Vol}\frac{d\,Vol}{dt_p}$ $\qquad = -\frac{1}{\sqrt{h}}\frac{1}{N}\frac{\partial}{\partial t}\sqrt{h} = -\frac{1}{Vol}\frac{d\,Vol}{dt_p}$

$g_{\mu\nu} =$

FRW $= -3H$

$Vol = \int d^3x \sqrt{h}$

$\rightarrow N_i = 0 \rightarrow n_i = 0$ $\qquad g_{\mu\nu} = \begin{pmatrix} -N^2 & 0 \\ 0 & h_{ij} \end{pmatrix}$

t-o coords, only nonzero components of $P^\mu{}_\nu$ are $P^i{}_j = \delta^i{}_j$

" $\qquad$ " $\qquad K_{\mu\nu}$

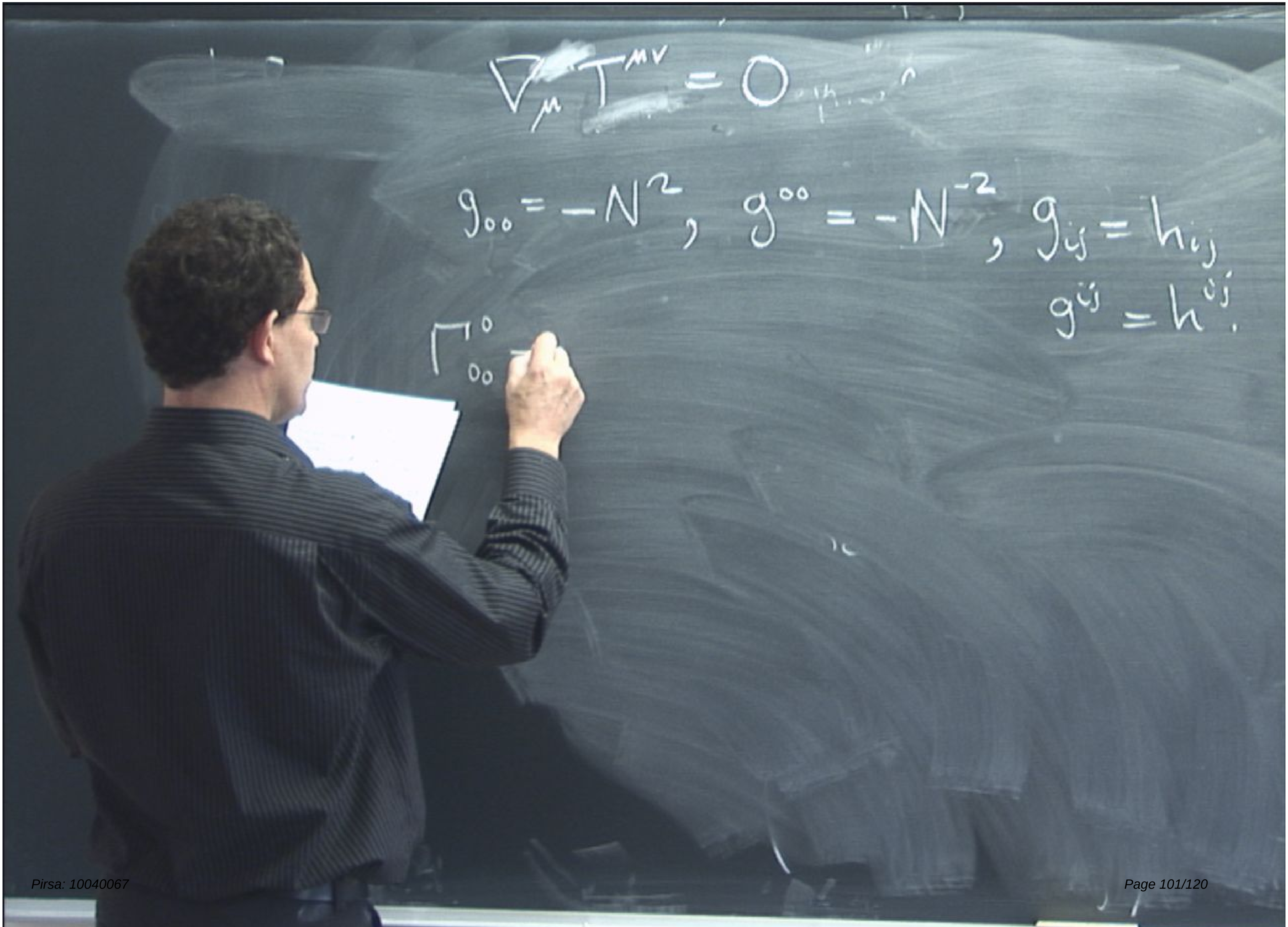
are $\quad K_{ij} = -\nabla_i n_j = -\cancel{\partial_i n_j} + \Gamma^0_{ji}\, n_0 = -\frac{1}{2} h_{ij,0} N^{-1}$

trace $K \equiv h^{ij} K_{ij} = -h^{ij} \nabla_i n_j = -\frac{1}{2} h_{ij,0} h^{ij} N^{-1}$

$$K = -\frac{1}{Vol}\frac{d\,Vol}{dt_p} \qquad = -\frac{1}{\sqrt{h}}\frac{1}{N}\frac{\partial}{\partial t}\sqrt{h} = -\frac{1}{Vol}\frac{d\,Vol}{dt_p}$$

FRW $\quad = -3H$

$$Vol = \int d^3x \sqrt{h}$$


$\nabla_\mu T^{\mu\nu} = 0$
$g_{00} = -N^2, \; g^{00} = -N^{-2}, \; g_{ij} = h_{ij}$
$g^{ij} = h^{ij}$
$\Gamma^0_{00} =$

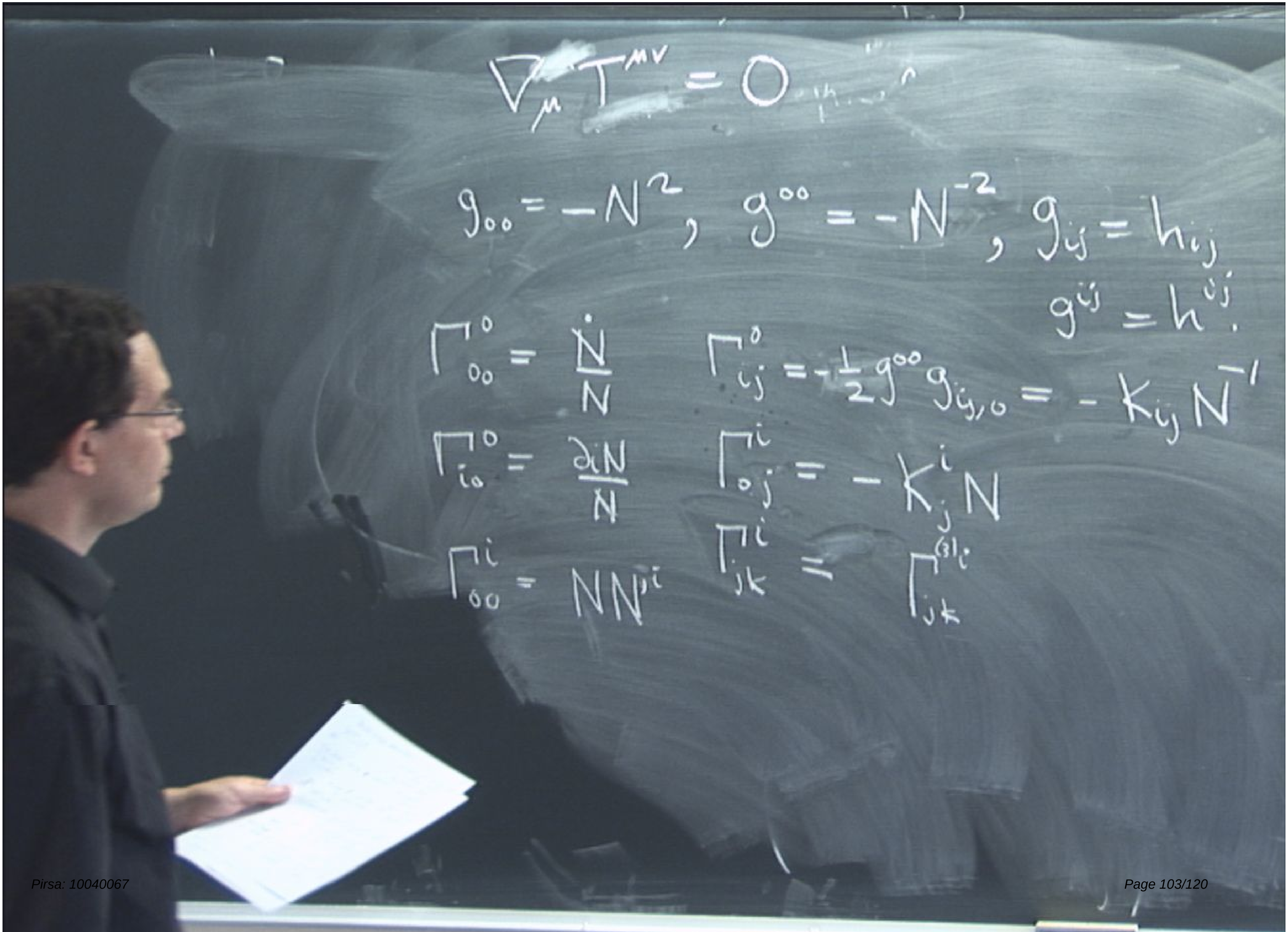
$$\nabla_\mu T^{\mu\nu} = 0$$

$$g_{00} = -N^2, \quad g^{00} = -N^{-2}, \quad g_{ij} = h_{ij}$$

$$g^{ij} = h^{ij}.$$

$$\Gamma^0_{00} = \frac{\dot N}{N} \qquad \Gamma^0_{ij} = -\frac{1}{2} g^{00} g_{ij,0} =$$

$$\Gamma^0_{i0} = \frac{\partial_i N}{N}$$



$\nabla_\mu T^{\mu\nu} = 0$
$g_{00} = -N^2, \quad g^{00} = -N^{-2}, \quad g_{ij} = h_{ij}$
$g^{ij} = h^{ij}$
$\Gamma^0_{00} = \frac{\dot{N}}{N} \quad \Gamma^0_{ij} = -\frac{1}{2} g^{00} g_{ij,0} = -K_{ij} N^{-1}$
$\Gamma^0_{i0} = \frac{\partial_i N}{N} \quad \Gamma^i_{0j} = -K^i_j N$
$\Gamma^i_{00} = N N^{,i} \quad \Gamma^i_{jk} = {}^{(3)}\Gamma^i_{jk}$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$g_{00} = -N^2, \quad g^{00} = -N^{-2}, \quad g_{ij} = h_{ij}$$

$$g^{ij} = h^{ij}.$$

$$\Gamma^0_{00} = \frac{\dot{N}}{N} \qquad \Gamma^0_{ij} = -\frac{1}{2} g^{00} g_{ij,0} = -K_{ij} N^{-1}$$

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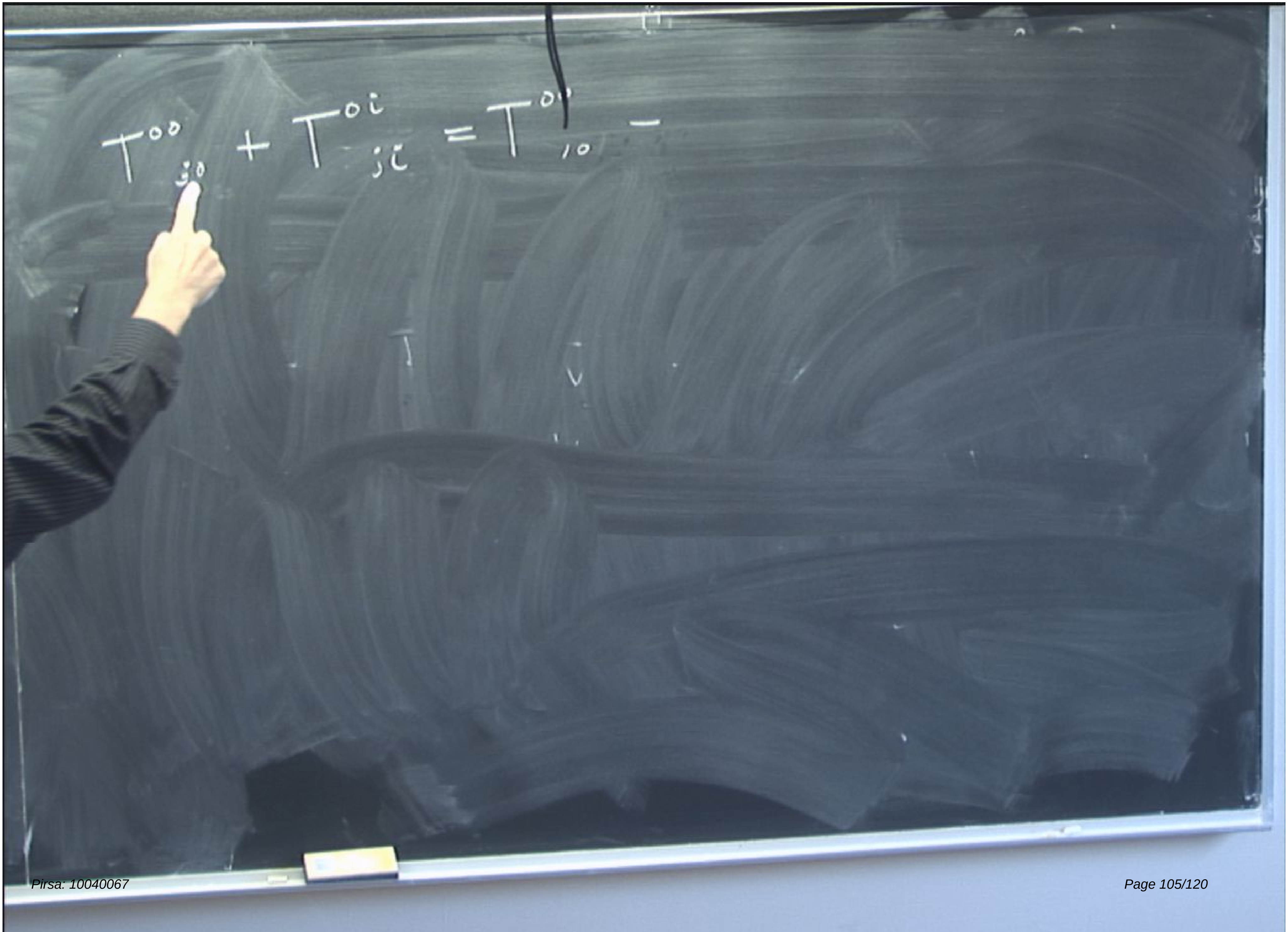
$T^{00}{}_{,0} + T^{0i}{}_{,i} = T^{0}{}_{,0} -$

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^0_{\alpha 0} T^{\alpha 0} + \Gamma^0_{\alpha 0} T^{0\alpha}$$
$$+ T^{0i}{}_{,i} + \Gamma^0_{\alpha i} T^{\alpha i} + \Gamma^i_{\alpha i} T^{0\alpha}$$

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^{0}_{\alpha 0} T^{\alpha 0} + \Gamma^{0}_{\alpha 0} T^{0\alpha}$$

$$+ T^{0i}{}_{,i} + \Gamma^{0}_{\alpha i} T^{\alpha i} + \Gamma^{i}_{\alpha i} T^{0\alpha}$$

$$= T^{00}{}_{,0} + 2\Gamma^{0}_{00} T^{00} + 3\Gamma^{0}_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^{0}_{ij} T^{ij} +$$

$$\Gamma^{i}_{0i} T^{00} + \Gamma^{i}_{ji} T^{0j}$$

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^{0}_{\alpha 0} T^{\alpha 0} + \Gamma^{0}_{\alpha 0} T^{0\alpha}$$

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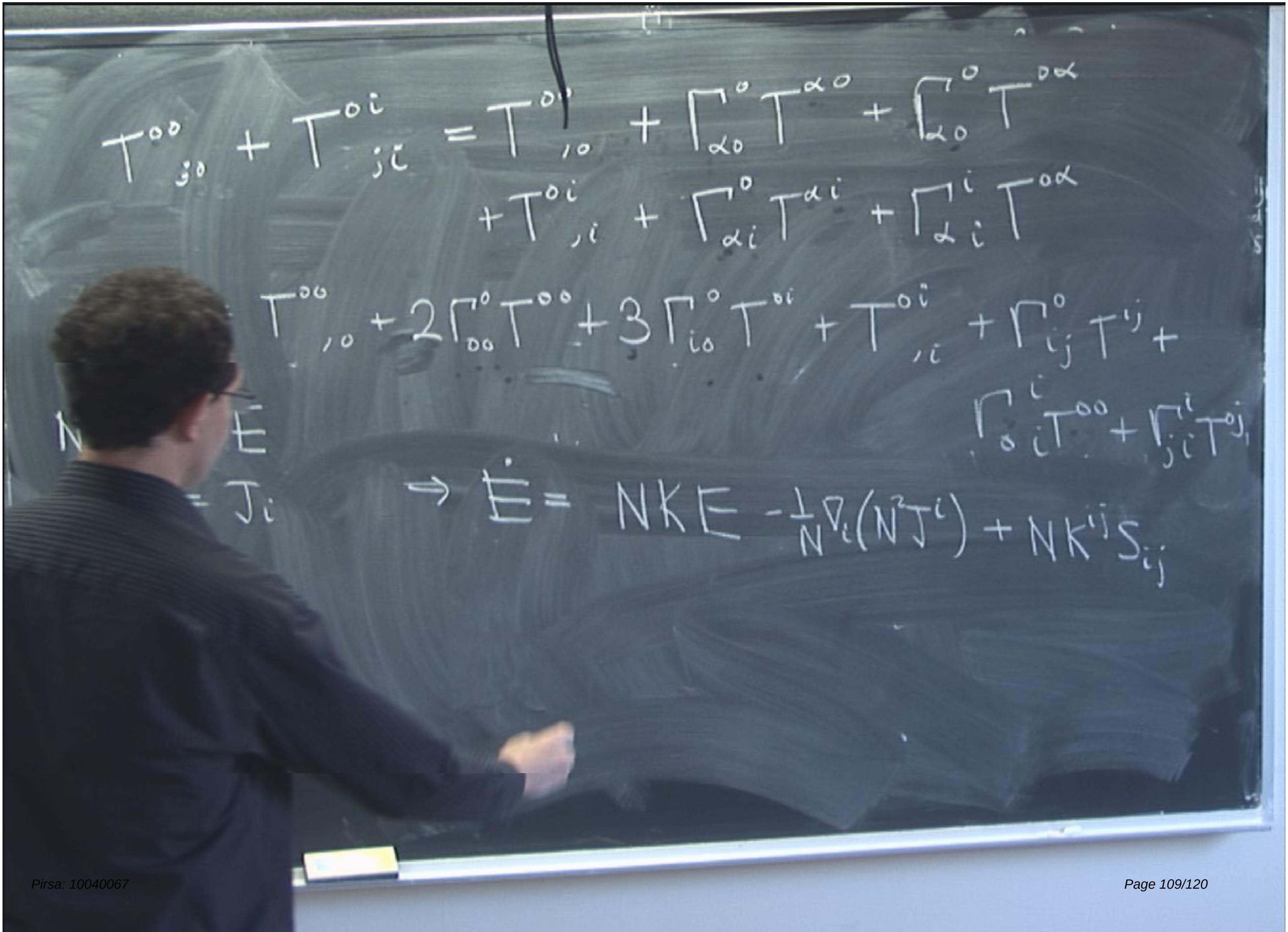
$$= T^{00}{}_{,0} + 2\Gamma^{0}_{00} T^{00} + 3\Gamma^{0}_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^{0}_{ij} T^{ij} +$$

$$\Gamma^{i}_{0i} T^{00} + \Gamma^{i}_{ji} T^{0j}$$


$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^{0}_{\alpha 0} T^{\alpha 0} + \Gamma^{0}_{\alpha 0} T^{0\alpha}$
$+ T^{0i}{}_{,i} + \Gamma^{0}_{\alpha i} T^{\alpha i} + \Gamma^{i}_{\alpha i} T^{0\alpha}$
$T^{00}{}_{,0} + 2\Gamma^{0}_{00} T^{00} + 3\Gamma^{0}_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^{0}_{ij} T^{ij} +$
$\Gamma^{i}_{0i} T^{00} + \Gamma^{i}_{ji} T^{0j}$
$\Rightarrow \dot{E} = NKE - \frac{1}{N}\nabla_i (N^2 J^i) + NK^{ij} S_{ij}$

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^0_{\alpha 0} T^{\alpha 0} + \Gamma^0_{\alpha 0} T^{0\alpha}$$

$$+ T^{0i}{}_{,i} + \Gamma^0_{\alpha i} T^{\alpha i} + \Gamma^i_{\alpha i} T^{0\alpha}$$

$$= T^{00}{}_{,0} + 2\Gamma^0_{00} T^{00} + 3\Gamma^0_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^0_{ij} T^{ij} + \Gamma^i_{0i} T^{00} + \Gamma^i_{ji} T^{0j}$$

$$N^2 T^{00} = E$$

$$N T^0{}_i = J_i$$

$$T_{ij} = S_{ij}$$

$$\Rightarrow \dot{E} = NKE - \frac{1}{N}\nabla_i (N^2 J^i) + N K^{ij} S_{ij}$$

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^{0}_{\alpha 0} T^{\alpha 0} + \Gamma^{0}_{\alpha 0} T^{0\alpha}$$
$$+ T^{0i}{}_{,i} + \Gamma^{0}_{\alpha i} T^{\alpha i} + \Gamma^{i}_{\alpha i} T^{0\alpha}$$

$$= T^{00}{}_{,0} + 2\Gamma^{0}_{00} T^{00} + 3\Gamma^{0}_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^{0}_{ij} T^{ij} +$$
$$\Gamma^{i}_{0i} T^{00} + \Gamma^{i}_{ji} T^{0j}$$

$N^2 T^{00} = E$

$N T^{0}{}_{i} = J_i$

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$$\Rightarrow \dot{E} = NKE - \frac{1}{N}\nabla_i (N^2 J^i) + N K^{ij} S_{ij}$$

(in coord syst with $N^i = 0$).

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^{0}_{\alpha 0} T^{\alpha 0} + \Gamma^{0}_{\alpha 0} T^{0\alpha}$$
$$+ T^{0i}{}_{,i} + \Gamma^{0}_{\alpha i} T^{\alpha i} + \Gamma^{i}_{\alpha i} T^{0\alpha}$$

$$= T^{00}{}_{,0} + 2\Gamma^{0}_{00} T^{00} + 3\Gamma^{0}_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^{0}_{ij} T^{ij} +$$
$$\Gamma^{i}_{0i} T^{00} + \Gamma^{i}_{ji} T^{0j}$$

$N^2 T^{00} = E$

$N T^{0}{}_{i} = J_i$ $\Rightarrow$ $\dot{E} = NKE - \frac{1}{N}\nabla_i(N^2 J^i) + NK^{ij} S_{ij}$

$T_{ij} = S_{ij}$ (in coord syst with $N^i = 0$).

$$T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{00}{}_{,0} + \Gamma^0_{\alpha 0} T^{\alpha 0} + \Gamma^0_{\alpha 0} T^{0\alpha}$$
$$+ T^{0i}{}_{,i} + \Gamma^0_{\alpha i} T^{\alpha i} + \Gamma^i_{\alpha i} T^{0\alpha}$$

$$= T^{00}{}_{,0} + 2\Gamma^0_{00} T^{00} + 3\Gamma^0_{i0} T^{0i} + T^{0i}{}_{,i} + \Gamma^0_{ij} T^{ij} +$$
$$\Gamma^i_{0i} T^{00} + \Gamma^i_{ji} T^{0j}$$

$N^2 T^{00} = E$

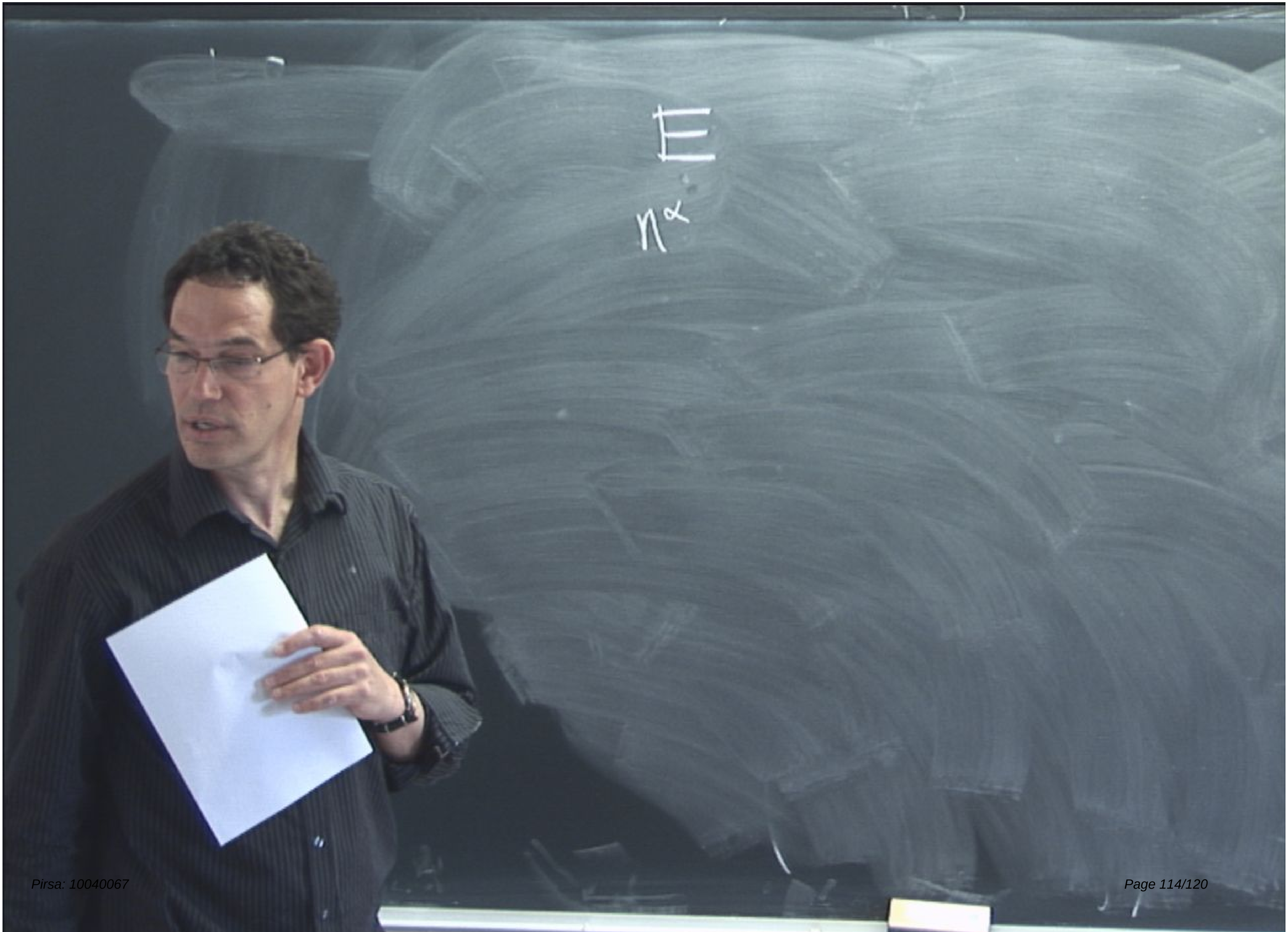
$N T^0{}_i = J_i$

$T_{ij} = S_{ij}$

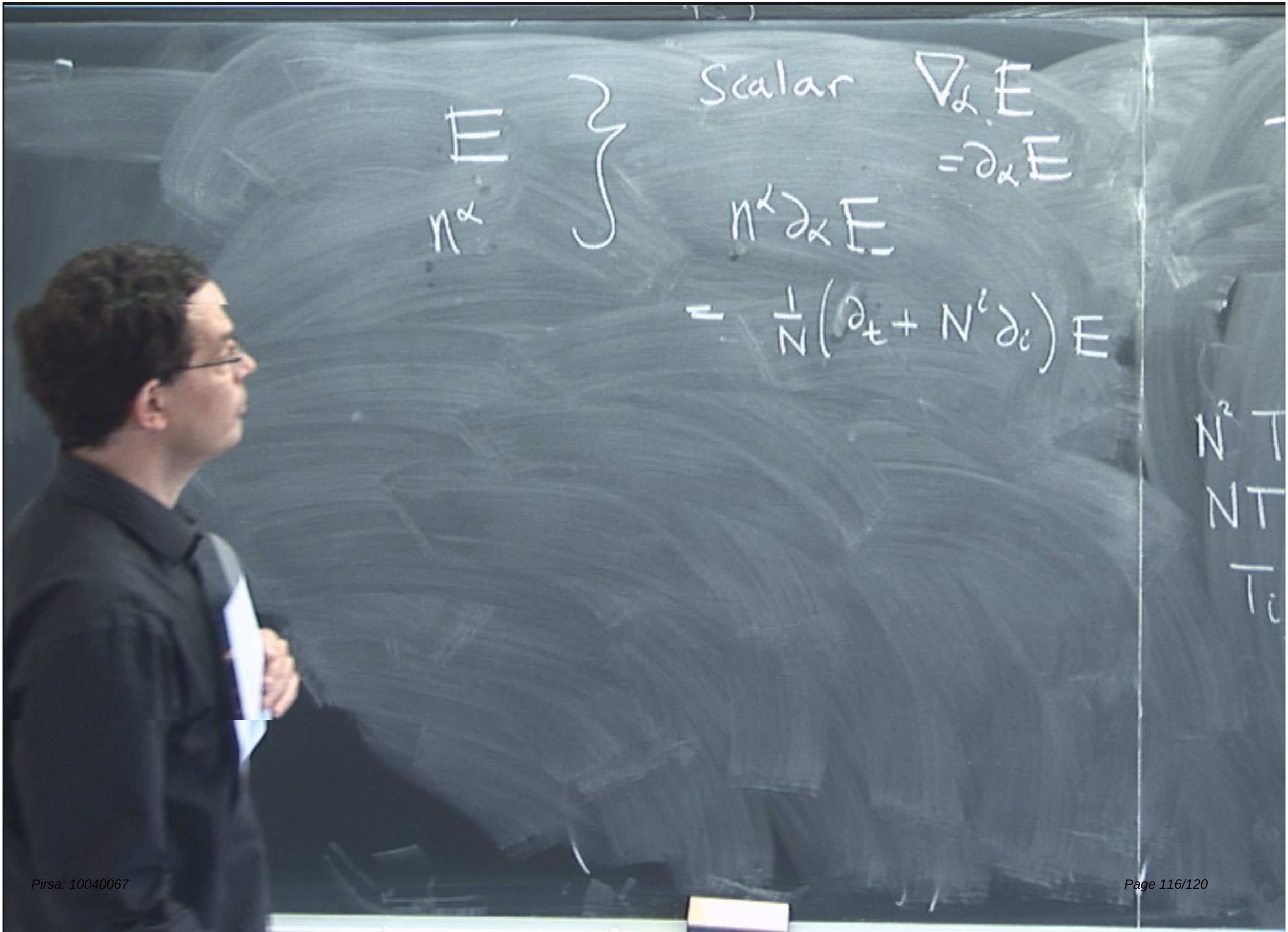
$$\Rightarrow \dot{E} = NKE - \frac{1}{N}\nabla_i (N^2 J^i) + NK^{ij} S_{ij}$$

(in coord syst with $N^i = 0$).

$\dot{E}$ is only quantity in this eq^n which isn't a coord scalar


E
n^α


Scalar
$\nabla_\alpha E$
E
n^α



Scalar $\nabla_\alpha E$
$= \partial_\alpha E$
E
n^α
$n^\alpha \partial_\alpha E$
$= \frac{1}{N}(\partial_t + N^i \partial_i) E$
$N^2 T$
NT
T_i

$$E \quad n^\alpha \Bigg\} \quad \text{Scalar} \quad \nabla_\alpha E = \partial_\alpha E$$

$$n^\alpha \partial_\alpha E = \frac{1}{N}\left(\partial_t + N^i \partial_i\right) E$$

i.e. a scalar eqn which reduces to (*) in t-σ coords is:

$$n^\alpha \partial_\alpha E = \frac{1}{N}\left(\partial_t + N^i \partial_i\right) E$$

$$= KE - \frac{1}{N^2}\nabla_i\left(N^2 J^i\right) + N K^{ij} S_{ij}$$

$T^{\cdot}$

$N^2 T^{00}$

$N T^0{}_i$

T_{ij}

$$\left.\begin{matrix} E \\ n^\alpha \end{matrix}\right\} \quad \text{Scalar} \quad \nabla_\alpha E = \partial_\alpha E$$

$$n^\alpha \partial_\alpha E = \frac{1}{N}\left(\partial_t + N^i \partial_i\right) E$$

i.e. a scalar eqn which reduces to (*) in t-o coords is:

$$n^\alpha \partial_\alpha E = \frac{1}{N}\left(\partial_t + N^i \partial_i\right) E$$

$$= KE - \frac{1}{N^2}\nabla_i\left(N^2 J^i\right) + NK^{ij}S_{ij}$$

T

$N^2 T^{00}$

$NT^0{}_i$

T_{ij}

$$E, \; n^\alpha \Big\} \quad \text{Scalar} \quad \nabla_\alpha E = \partial_\alpha E$$

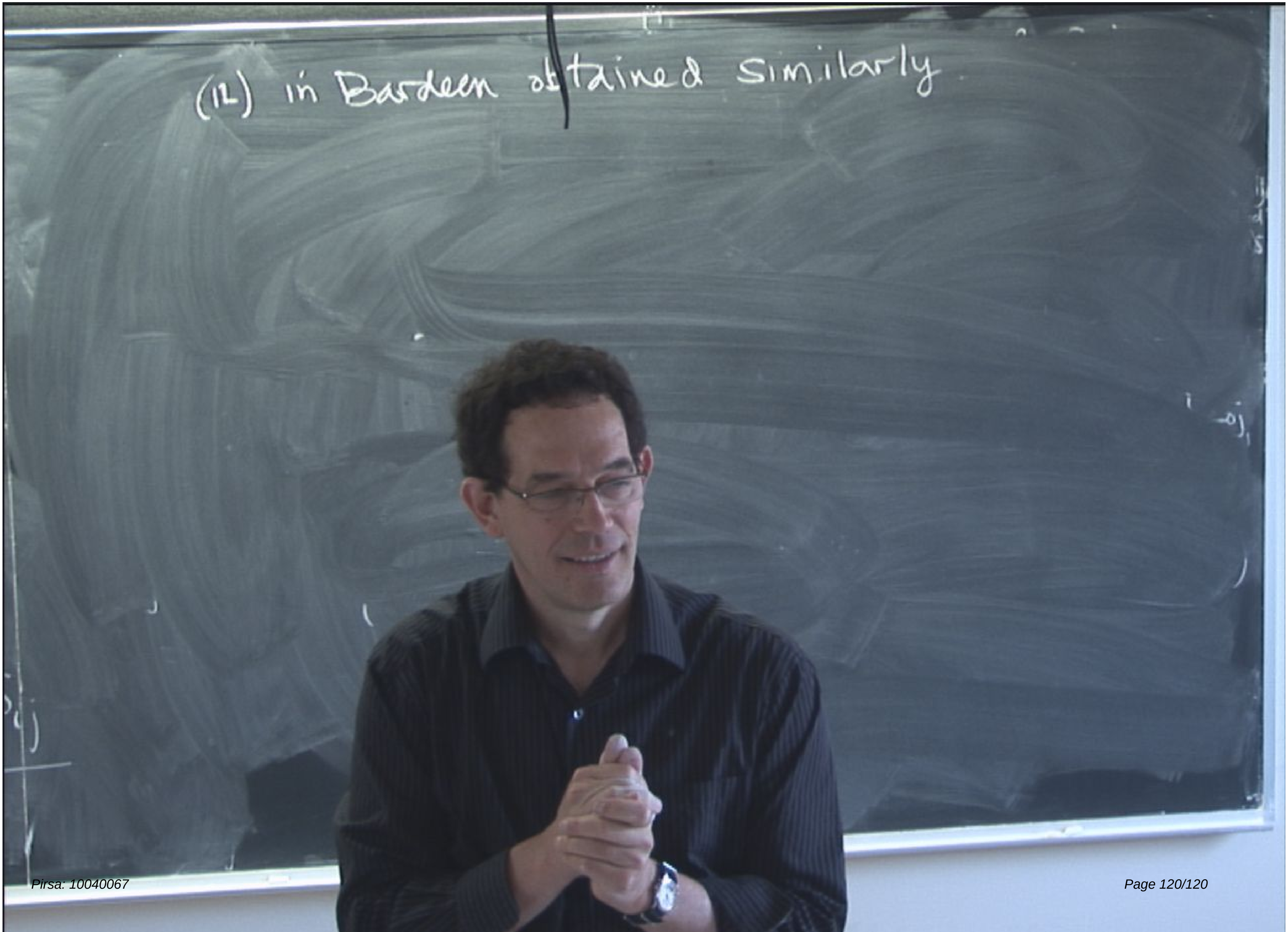
$$n^\alpha \partial_\alpha E = \frac{1}{N}\left(\partial_t + N^i \partial_i\right) E$$

i.e. a scalar eqn which reduces to (*) in t-o coords is:

$$n^\alpha \partial_\alpha E = \frac{1}{N}\left(\partial_t + N^i \partial_i\right) E$$

$$= KE - \frac{1}{N^2} \nabla_i \left(N^2 J^i\right) + N K^{ij} S_{ij}$$

(11) in Bardeen.


(12) in Bardeen obtained similarly