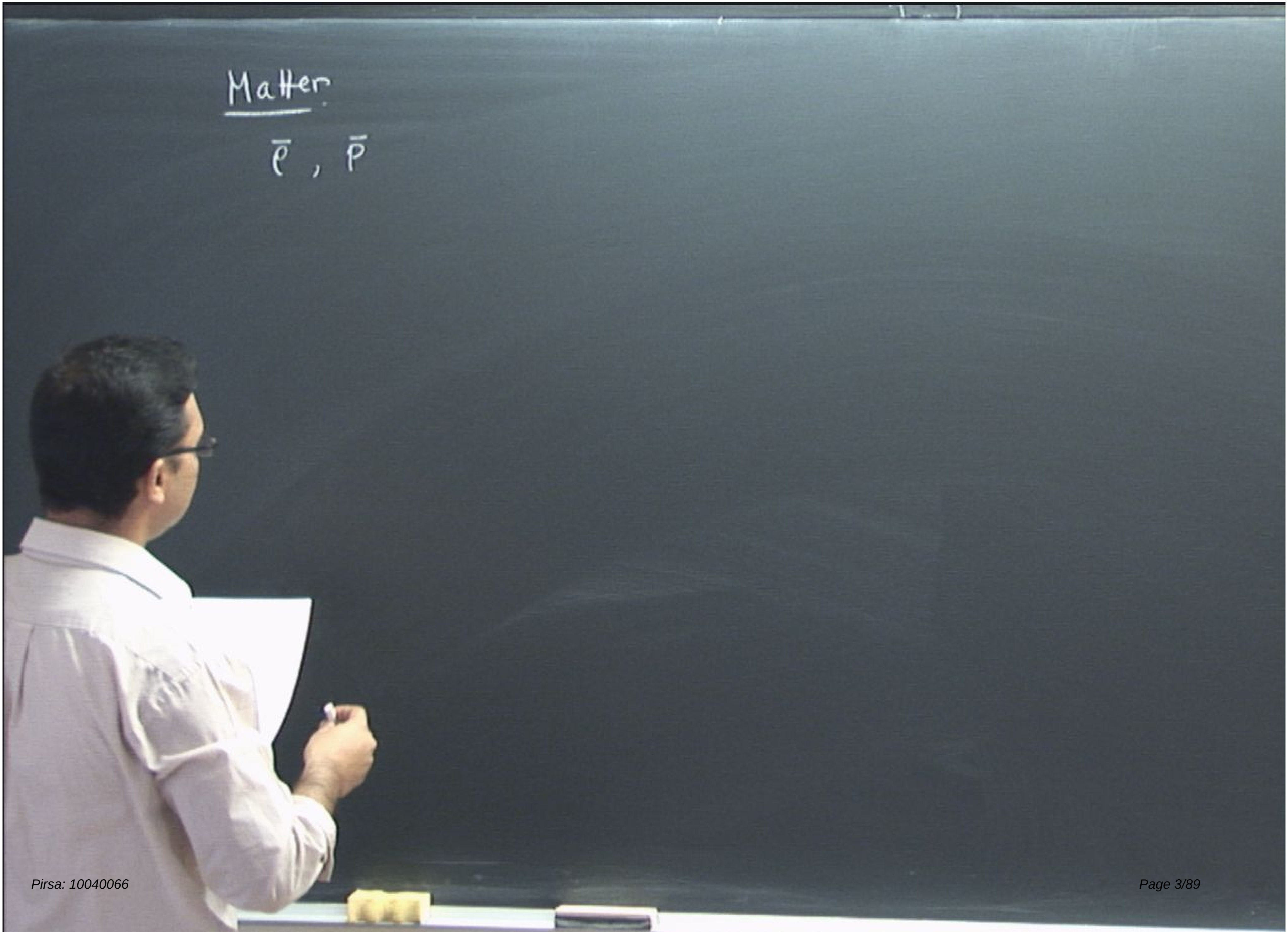
Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 9

Date: Apr 15, 2010 09:30 AM

URL: http://pirsa.org/10040066

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

Matter


Matter
$\bar{e}$, $\bar{p}$

Matter

$\bar{\rho}\ ,\ \bar{P}$

$$x^\mu \longrightarrow x^\mu + \epsilon^\mu$$

Ex $\quad \delta\tilde{q} = \delta q - q_{,\alpha}\,\epsilon^\alpha$

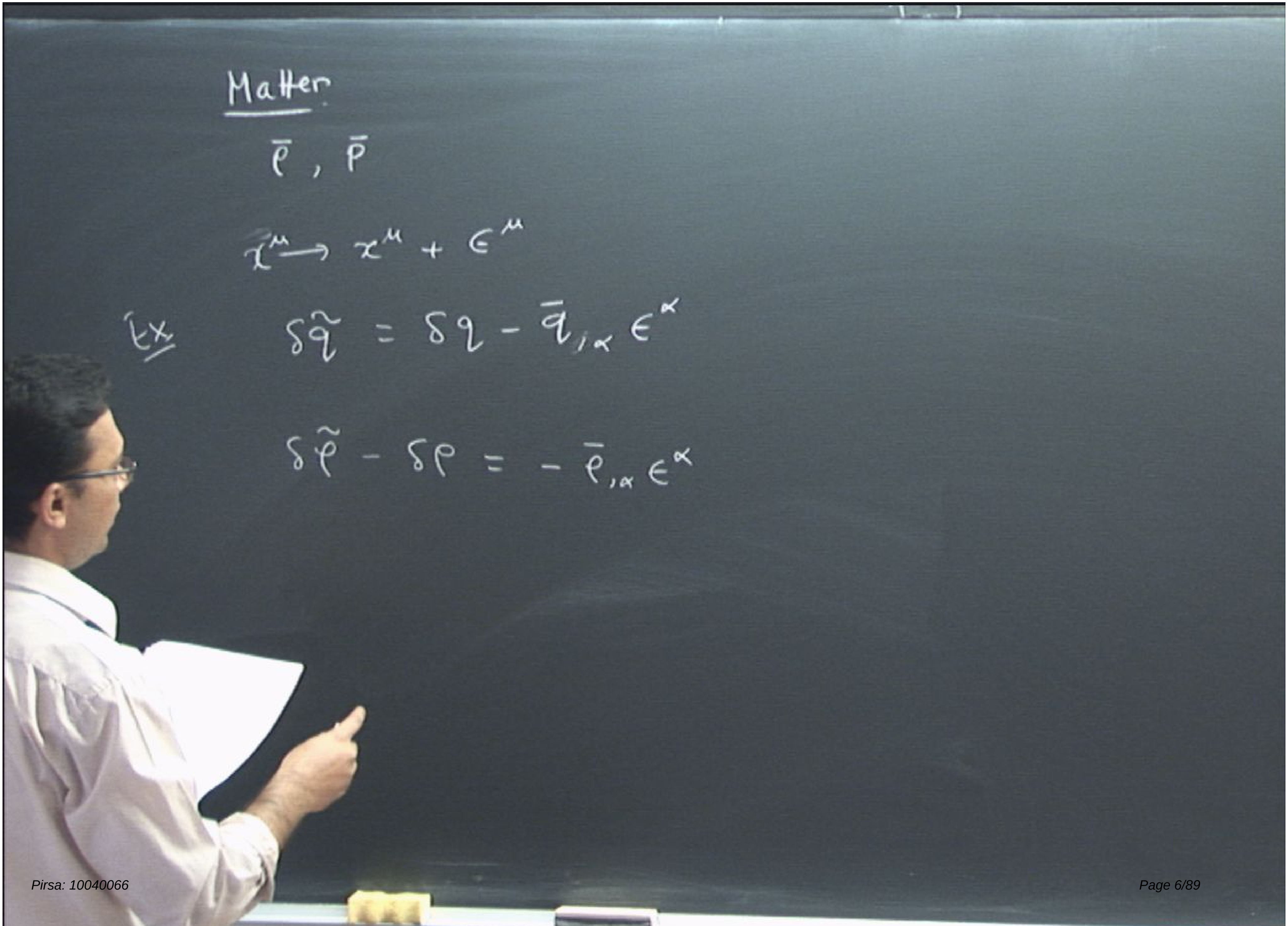
Matter

$\bar{\rho}\ ,\ \bar{p}$

$x^{\mu} \longrightarrow x^{\mu} + \epsilon^{\mu}$

Ex. $\delta\tilde{q} = \delta q - \bar{q}_{,\alpha}\epsilon^{\alpha}$

$\delta\tilde{\rho} - \delta\rho = -\bar{\rho}_{,\alpha}\epsilon^{\alpha}$


Matter
$\bar{\rho}$, $\bar{P}$
$x^\mu \longrightarrow x^\mu + \epsilon^\mu$
Ex.
$\delta\tilde{q} = \delta q - \bar{q}_{,\alpha}\,\epsilon^\alpha$
$\delta\tilde{\rho} - \delta\rho = -\bar{\rho}_{,\alpha}\,\epsilon^\alpha$

Matter

$\bar{\rho}\,,\ \bar{P}$

$x^{\mu} \longrightarrow x^{\mu} + \epsilon^{\mu}$

Ex. $\delta\tilde{q} = \delta q - \bar{q}_{,\alpha}\,\epsilon^{\alpha}$

$\delta\tilde{\rho} - \delta\rho = -\bar{\rho}_{,\alpha}\,\epsilon^{\alpha} = -\bar{\rho}_{,0}\,\epsilon^{0}$

$\delta\tilde{\rho} \downarrow \tilde{\mathcal{E}}$, $\delta\rho \downarrow \mathcal{E}$

$\tilde{\mathcal{E}} = \mathcal{E} - \dot{E}_{0}\,T$

$\mathcal{E} = \mathcal{E} + 3H(E_{0} + P_{0})\,T$

Matter

$\bar{\rho}\,,\ \bar{P}$

$x^\mu \longrightarrow x^\mu + \epsilon^\mu$

Ex.

$$\delta\tilde{q} = \delta q - \bar{q}_{,\alpha}\,\epsilon^\alpha$$

$$\delta\tilde{\rho} - \delta\rho = -\bar{\rho}_{,\alpha}\,\epsilon^\alpha = -\bar{\rho}_{,0}\,\epsilon^0$$

$\delta\tilde{\rho} \to \tilde{\varepsilon}$, $\delta\rho \to \varepsilon$

$$\tilde{\varepsilon} = \varepsilon - \dot{E}_0\,T$$

$$\boxed{\tilde{\varepsilon} = \varepsilon + 3H(E_0 + P_0)\,T}$$

Matter

$\bar{\rho}\,,\ \bar{P}$

$x^\mu \longrightarrow x^\mu + \epsilon^\mu$

Ex

$$\delta\tilde{q} = \delta q - \bar{q}_{,\alpha}\,\epsilon^\alpha$$

$$\delta\tilde{\rho} - \delta\rho = -\bar{\rho}_{,\alpha}\,\epsilon^\alpha = -\bar{\rho}_{,0}\,\epsilon^0$$

$\delta\tilde{\rho} \to \tilde{\mathcal{E}}$, $\delta\rho \to \mathcal{E}$

$$\tilde{\mathcal{E}} = \mathcal{E} - \dot{E}_0 T$$

$$\boxed{\tilde{\mathcal{E}} = \mathcal{E} + 3H(E_0 + P_0)\,T}$$

$$\epsilon^\mu + \epsilon^\mu$$

$$= \delta q - \bar{q}_{,\alpha} \epsilon^\alpha$$

$$- \delta\rho = -\bar{\rho}_{,\alpha} \epsilon^\alpha = -\bar{\rho}_{,0} \epsilon^0$$

$$\downarrow$$

$$\varepsilon$$

$$\tilde{\varepsilon} = \varepsilon - \dot{E}_0 T$$

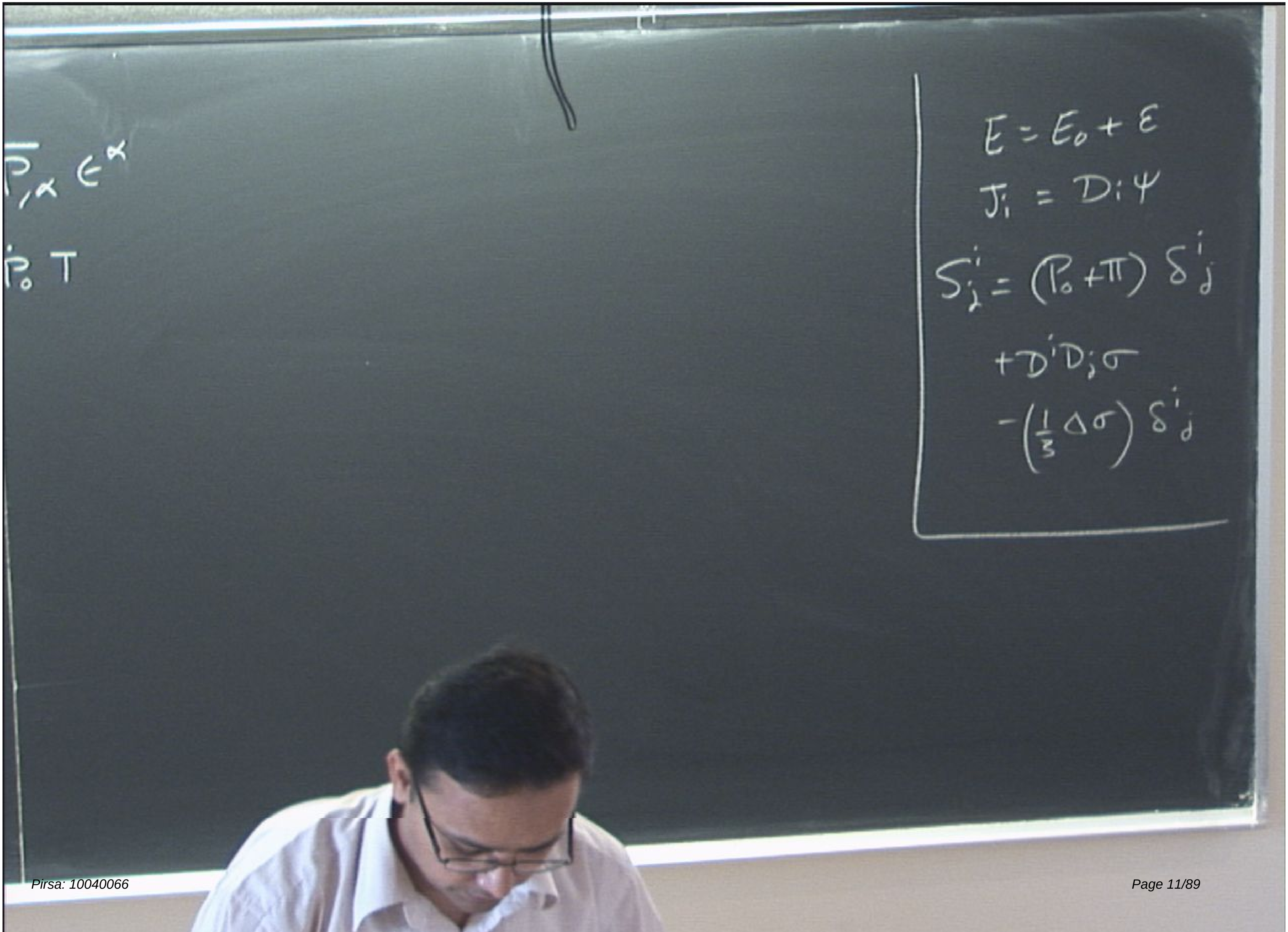
$$\boxed{\tilde{\varepsilon} = \varepsilon + 3H(E_0 + P_0) T}$$

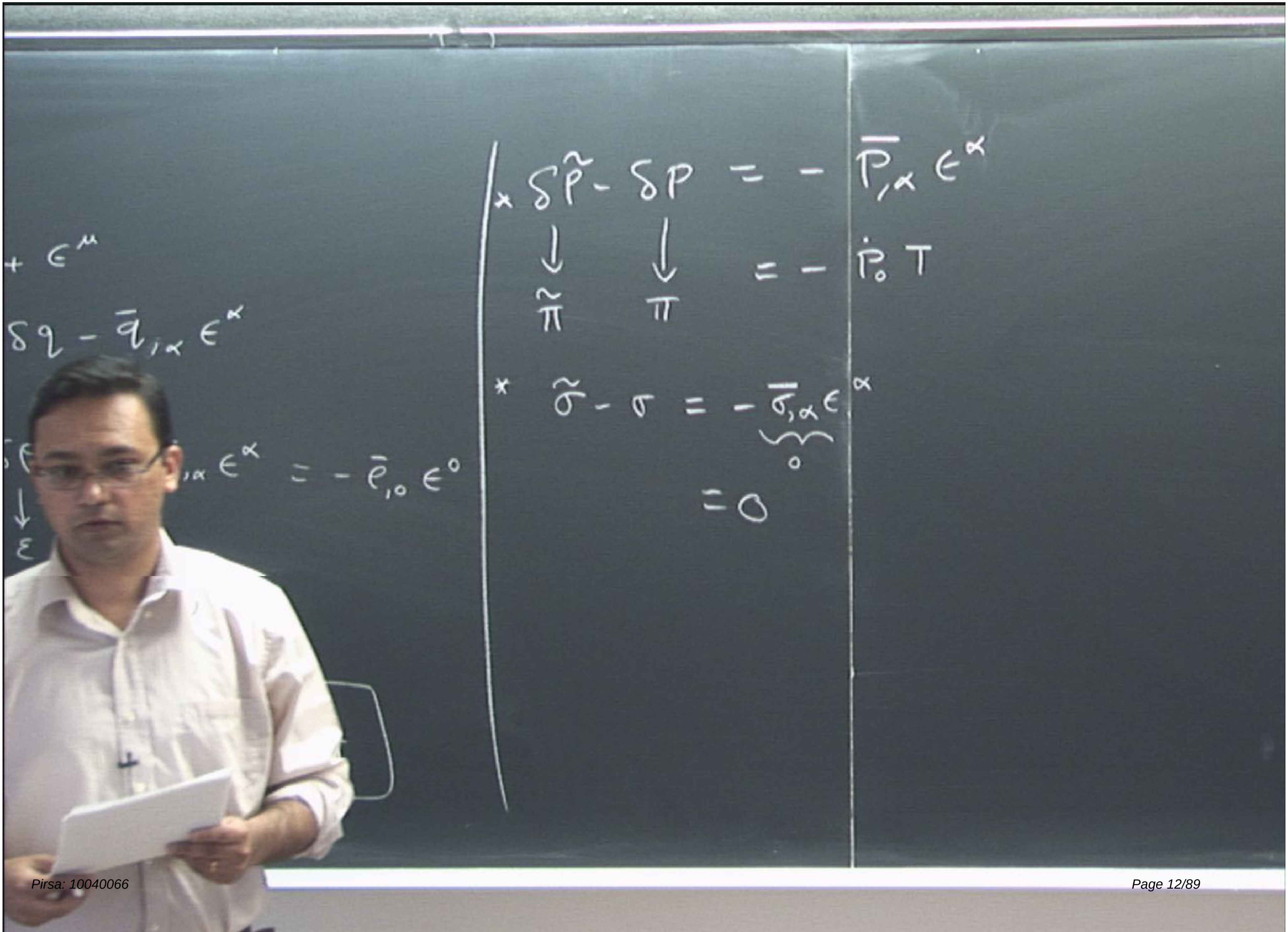
$$* \quad \delta\tilde{P} - \delta P = -\bar{P}_{,\alpha} \epsilon^\alpha$$

$$\downarrow \qquad \downarrow$$

$$\tilde{\pi} \qquad \pi \qquad = -\dot{P}_0 T$$

$$*$$


$\bar{P}_{,\alpha} \epsilon^{\alpha}$
$\dot{P}_0 T$
$E = E_0 + \varepsilon$
$J_i = D_i \psi$
$S^i_j = (P_0 + \Pi) \delta^i_j$
$+ D^i D_j \sigma$
$- \left(\frac{1}{3} \Delta \sigma\right) \delta^i_j$


+ ε^μ
δq - q̄,α ε^α
,α ε^α = - ē,0 ε^0
* δp̃ - δp = - p̄,α ε^α
= - ṗ0 T
π̃ π
* σ̃ - σ = - σ̄,α ε^α
= 0

$+ \epsilon^{\mu}$

$\delta q - \bar{q}_{,\alpha} \epsilon^{\alpha}$

$\delta\rho = -\bar{\rho}_{,\alpha}\epsilon^{\alpha} = -\bar{\rho}_{,0}\epsilon^{0}$

$\downarrow$

ε

$= \mathcal{E} - \dot{E}_0 T$

$\mathcal{E} + 3H(E_0 + P_0)T$

$* \quad \delta\tilde{p} - \delta p = -\bar{P}_{,\alpha}\epsilon^{\alpha}$

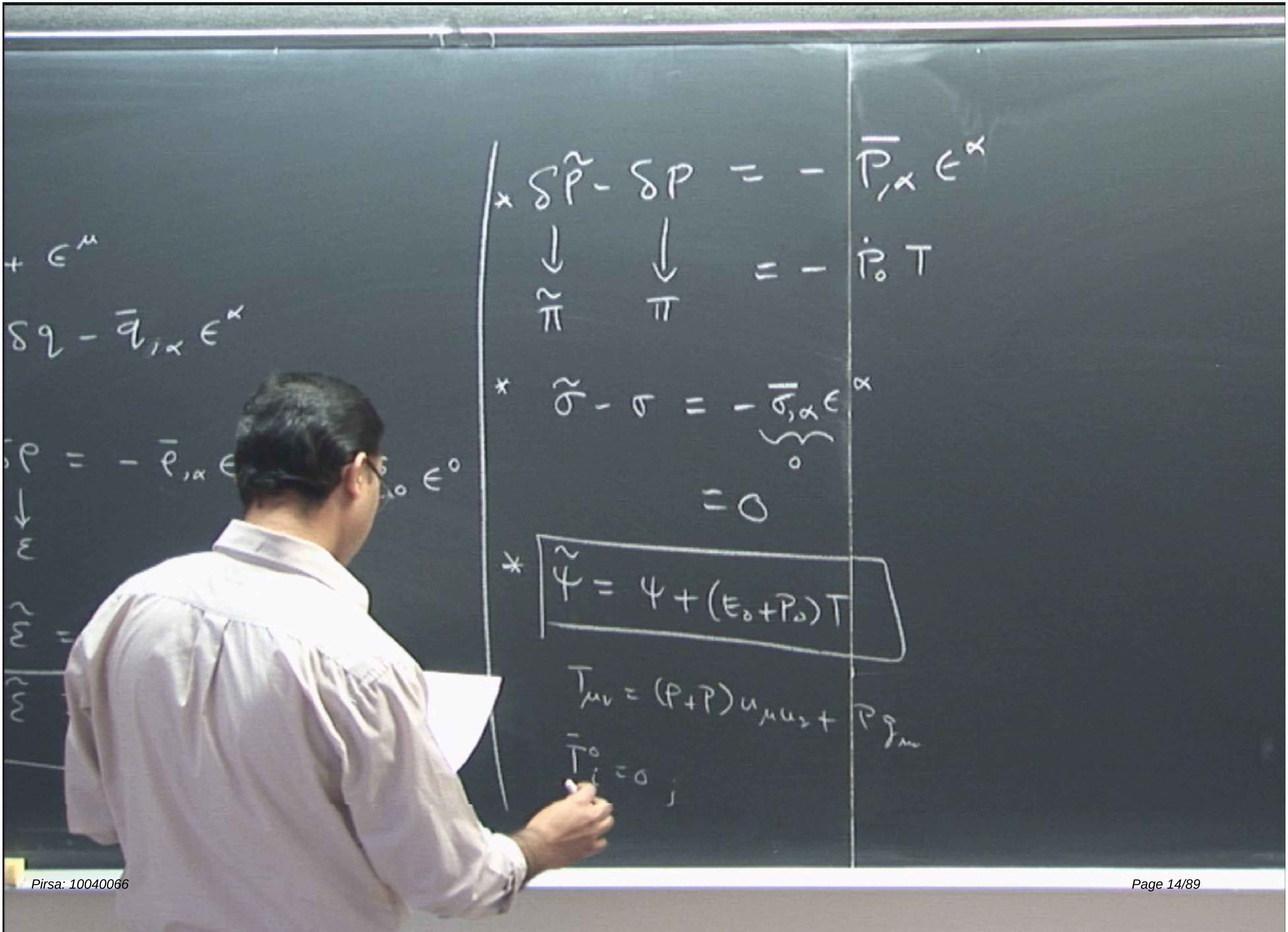
$\downarrow \qquad \downarrow \qquad = -\dot{P}_0 T$

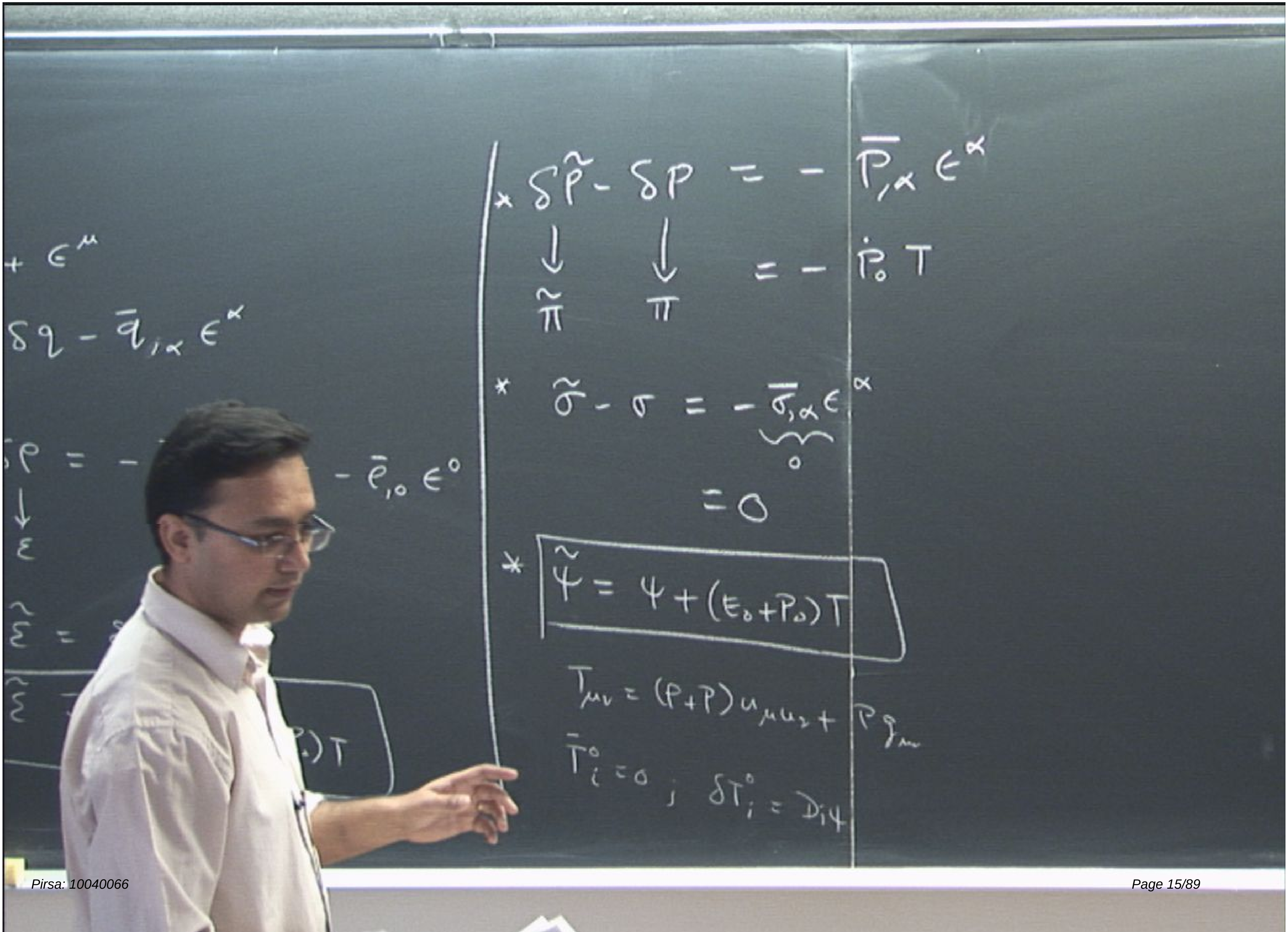
$\tilde{\pi} \qquad \pi$

$* \quad \tilde{\sigma} - \sigma = -\underbrace{\bar{\sigma}_{,\alpha}\epsilon^{\alpha}}_{0}$

$= 0$

$* \quad \boxed{\tilde{\Psi} = \Psi + (E_0 + P_0)T}$


$+ \epsilon^\mu$
$\delta q - \bar{q}_{,\alpha} \epsilon^\alpha$
$\delta\rho = - \bar{\rho}_{,\alpha} \epsilon$
$_{,0} \epsilon^0$
ε
$\tilde{\varepsilon} =$
$\tilde{\varepsilon}$
$* \ \delta\tilde{P} - \delta P = - \bar{P}_{,\alpha} \epsilon^\alpha$
$\tilde{\pi} \quad \pi \quad = - \dot{P}_0 T$
$* \ \tilde{\sigma} - \sigma = - \bar{\sigma}_{,\alpha} \epsilon^\alpha$
0
$= 0$
$* \ \tilde{\Psi} = \Psi + (\varepsilon_0 + P_0) T$
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$
$\bar{T}^0_i = 0 \ ;$



δp̃ - δp = - p̄,α ε^α
= - ṗ₀ T
π̃   π
σ̃ - σ = - σ̄,α ε^α
= 0
ψ̃ = ψ + (ε₀ + P₀) T
T_μν = (ρ + P) u_μ u_ν + P g_μν
T̄⁰_i = 0 ; δT⁰_i = D_i ψ
δq - q̄,α ε^α
- ρ̄,₀ ε⁰

$$+ \epsilon^{\mu}$$

$$\delta q - \bar{q}_{,\alpha}\,\epsilon^{\alpha}$$

$$\delta\rho = -\bar{\rho}_{,\alpha}\,\epsilon^{\alpha} = -\bar{\rho}_{,0}\,\epsilon^{0}$$

$$\downarrow \quad \mathcal{E}$$

$$\tilde{\mathcal{E}} = \mathcal{E} - \dot{E}_0\, T$$

$$\boxed{\tilde{\mathcal{E}} = \mathcal{E} + 3H(E_0 + P_0)\, T}$$

$$* \quad \delta\tilde{p} - \delta p = -\bar{p}_{,\alpha}\,\epsilon^{\alpha} = -\dot{p}_0\, T$$

$$\downarrow \tilde{\pi} \qquad \downarrow \pi$$

$$* \quad \tilde{\sigma} - \sigma = -\underbrace{\bar{\sigma}_{,\alpha}}_{0}\,\epsilon^{\alpha} = 0$$

$$* \quad \boxed{\tilde{\psi} = \psi + (E_0 + P_0)\, T}$$

$$T_{\mu\nu} = (\rho + P)\, u_{\mu} u_{\nu} + P\, g_{\mu\nu}$$

$$\bar{T}^{0}_{i} = 0 \;;\quad \delta T^{0}_{i} = D_i \psi$$

$$\delta\tilde{T}^{0}_{i} = \ldots$$

$+ \epsilon^\mu$

$\delta q - \bar{q}_{,\alpha} \epsilon^\alpha$

$\delta\rho = -\bar{\rho}_{,\alpha}\epsilon^\alpha = -\bar{\rho}_{,0}\epsilon^0$

$\downarrow$

$\mathcal{E}$

$\tilde{\mathcal{E}} = \mathcal{E} - \dot{E}_0 T$

$$\tilde{\mathcal{E}} = \mathcal{E} + 3H(E_0 + P_0)T$$

$* \; \delta\tilde{p} - \delta p = -\bar{P}_{,\alpha}\epsilon^\alpha$

$\downarrow \quad \downarrow \qquad = -\dot{P}_0 T$

$\tilde{\pi} \quad \pi$

$* \; \tilde{\sigma} - \sigma = -\underbrace{\bar{\sigma}_{,\alpha}}_{0}\epsilon^\alpha$

HW. $\qquad = 0$

$$* \; \tilde{\Psi} = \Psi + (E_0 + P_0)T$$

$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$

$\bar{T}^0_i = 0 \; ; \; \delta T^0_i = D_i \Psi$

$\delta\tilde{T}^0_i - \delta T^0_i$

$$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)T$$

$$\tilde{\pi} = \pi - \dot{P}_0 T$$

$$\tilde{\sigma} = \sigma$$

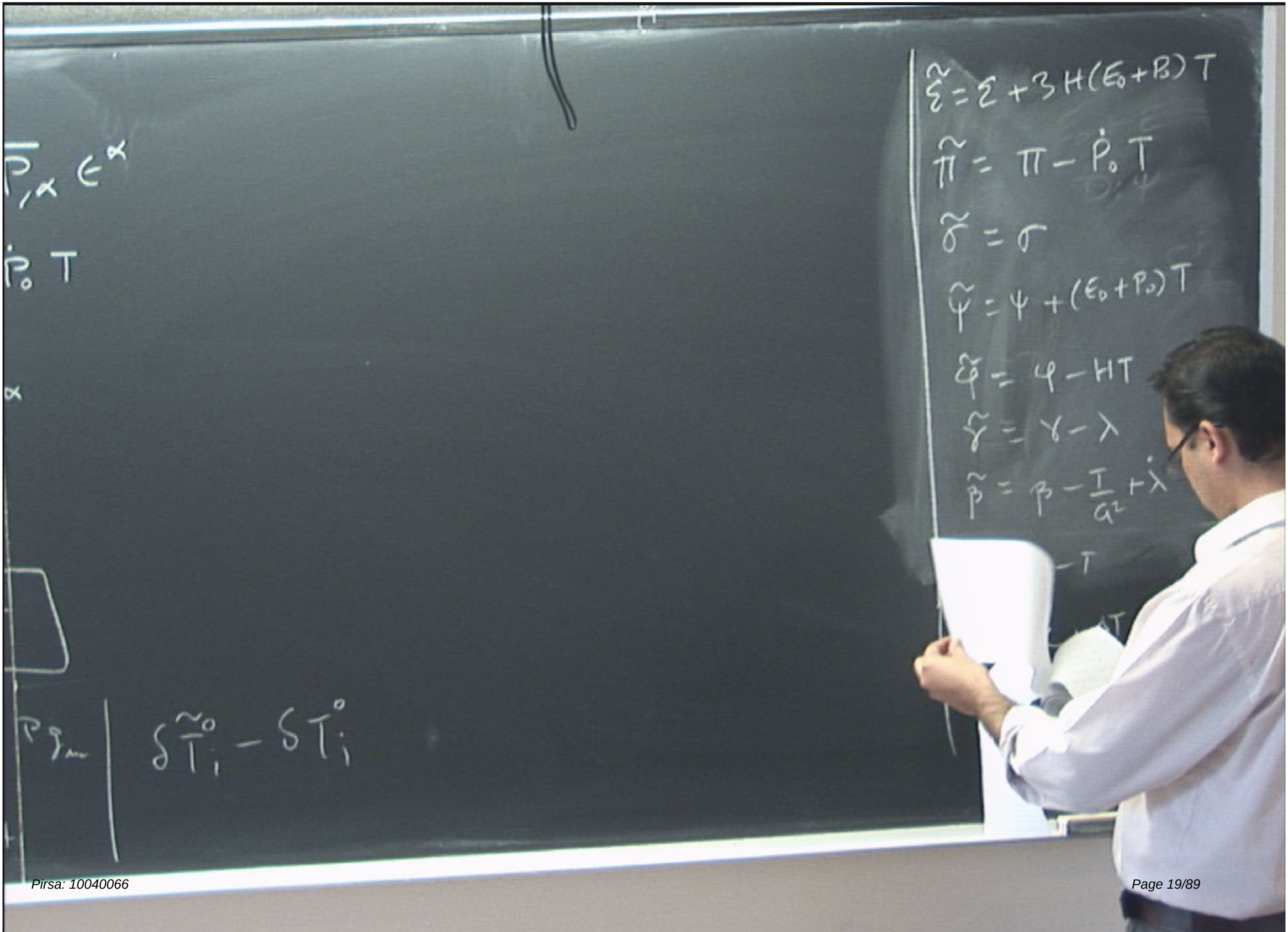
$$\tilde{\psi} = \psi + (\mathcal{E}_0 + P_0)T$$

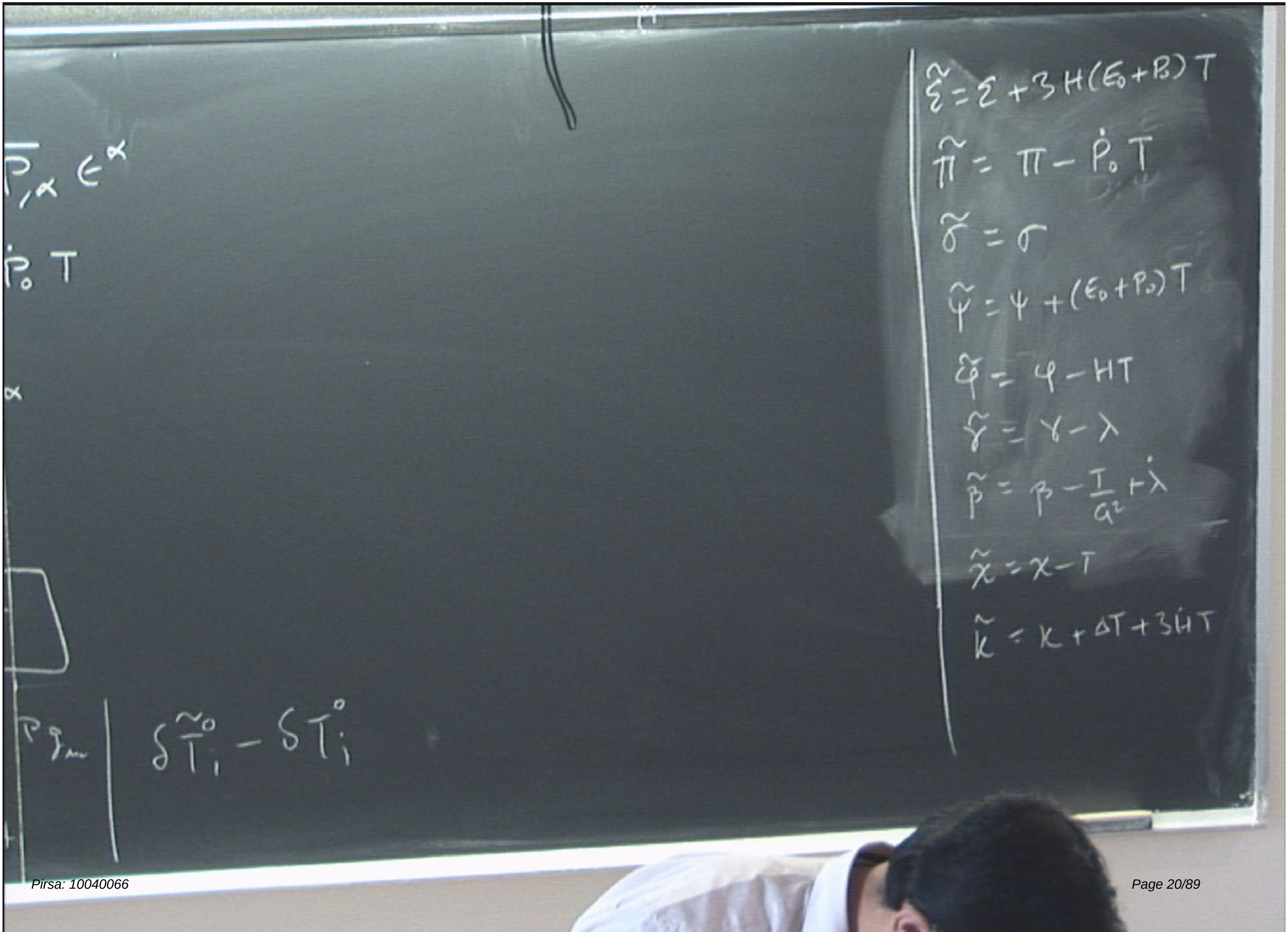
$P_{,\alpha}\,\epsilon^{\alpha}$

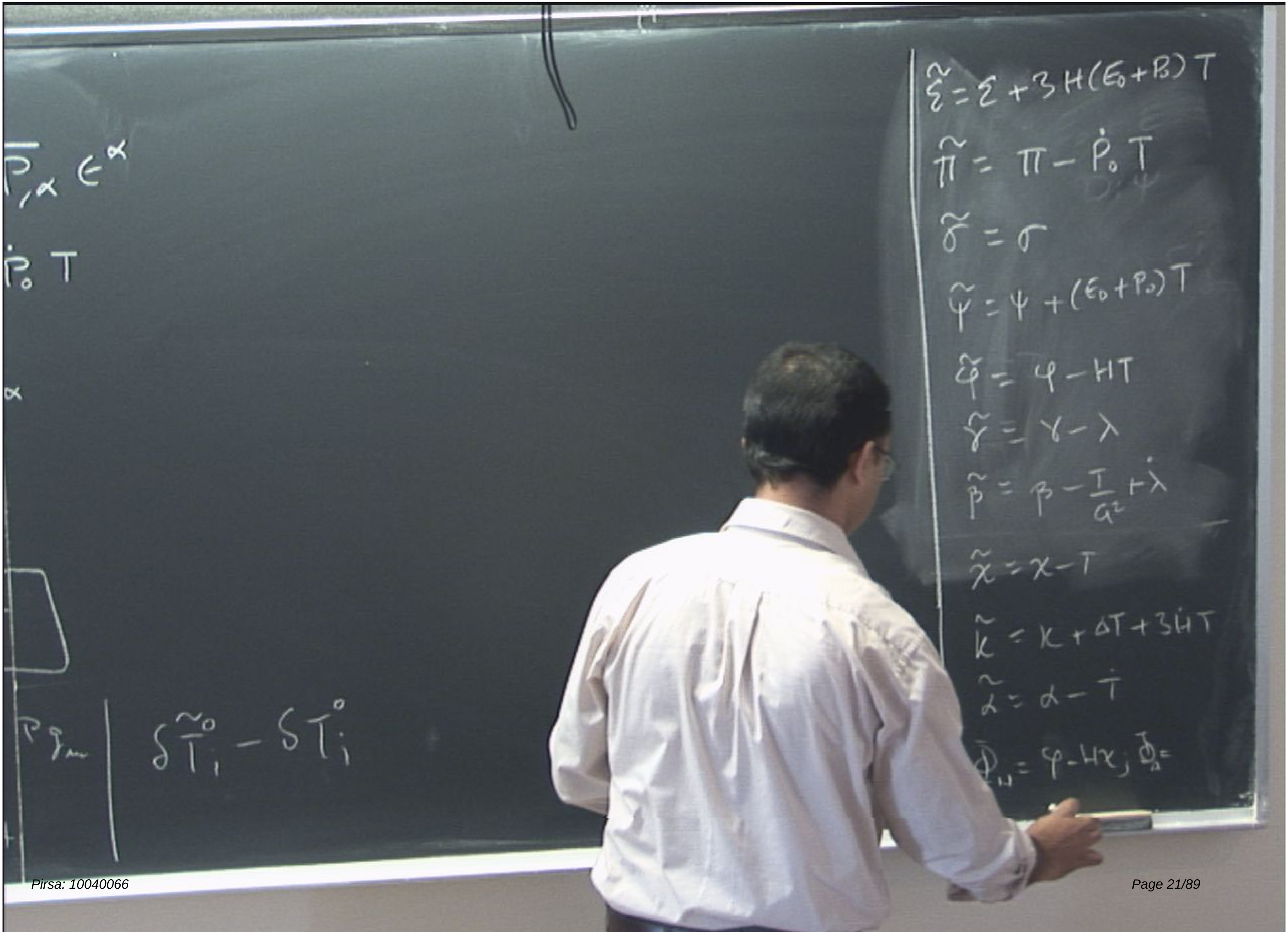
$\dot{P}_0\, T$

α

$P g_{\mu\nu}$ | $\delta\tilde{T}^0_i - \delta T^0_i$


$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)T$
$\tilde{\pi} = \pi - \dot{P}_0 T$
$\tilde{\sigma} = \sigma$
$\tilde{\psi} = \psi + (\mathcal{E}_0 + P_0)T$
$\tilde{\varphi} = \varphi - HT$
$\tilde{\gamma} = \gamma - \lambda$
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$\delta \tilde{T}^0_i - \delta T^0_i$


$\tilde{\varepsilon} = \varepsilon + 3H(\varepsilon_0 + P_0)T$
$\tilde{\pi} = \pi - \dot{P}_0 T$
$\tilde{\sigma} = \sigma$
$\tilde{\psi} = \psi + (\varepsilon_0 + P_0)T$
$\tilde{\varphi} = \varphi - HT$
$\tilde{\gamma} = \gamma - \lambda$
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$\tilde{\chi} = \chi - T$
$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$
$\delta \tilde{T}^0_i - \delta T^0_i$



ε̃ = ε + 3H(E₀ + P₀) T
π̃ = π − Ṗ₀ T
σ̃ = σ
ψ̃ = ψ + (E₀ + P₀) T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
Φ_H = φ − Hχ ; Φ_A =
P̄,α ε^α
Ṗ₀ T
δT̃⁰ᵢ − δT⁰ᵢ

$\bar{P}_{,\alpha}\,\epsilon^{\alpha}$

$\dot{P}_0\, T$

α

$\delta \tilde{T}^0_i - \delta T^0_i$

$$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)T$$

$$\tilde{\pi} = \pi - \dot{P}_0 T$$

$$\tilde{\sigma} = \sigma$$

$$\tilde{\psi} = \psi + (\mathcal{E}_0 + P_0)T$$

$$\tilde{\varphi} = \varphi - HT$$

$$\tilde{\gamma} = \gamma - \lambda$$

$$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$$

$$\tilde{\chi} = \chi - T$$

$$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$$

$$\tilde{\alpha} = \alpha - \dot{T}$$

$$\Phi_H = \varphi - H\chi; \quad \Phi_A = \alpha - \chi$$

$\bar{P}_{,\alpha}\, \epsilon^{\alpha}$

$\dot{P}_0\, T$

α

$P\, g_{\mu\nu}$ | $\delta \tilde{T}^0_i - \delta T^0_i$

$$\tilde{\varepsilon} = \varepsilon + 3H(\varepsilon_0 + P_0)T$$

$$\tilde{\pi} = \pi - \dot{P}_0 T$$

$$\tilde{\sigma} = \sigma$$

$$\tilde{\psi} = \psi + (\varepsilon_0 + P_0)T$$

$$\tilde{\varphi} = \varphi - HT$$

$$\tilde{\gamma} = \gamma - \lambda$$

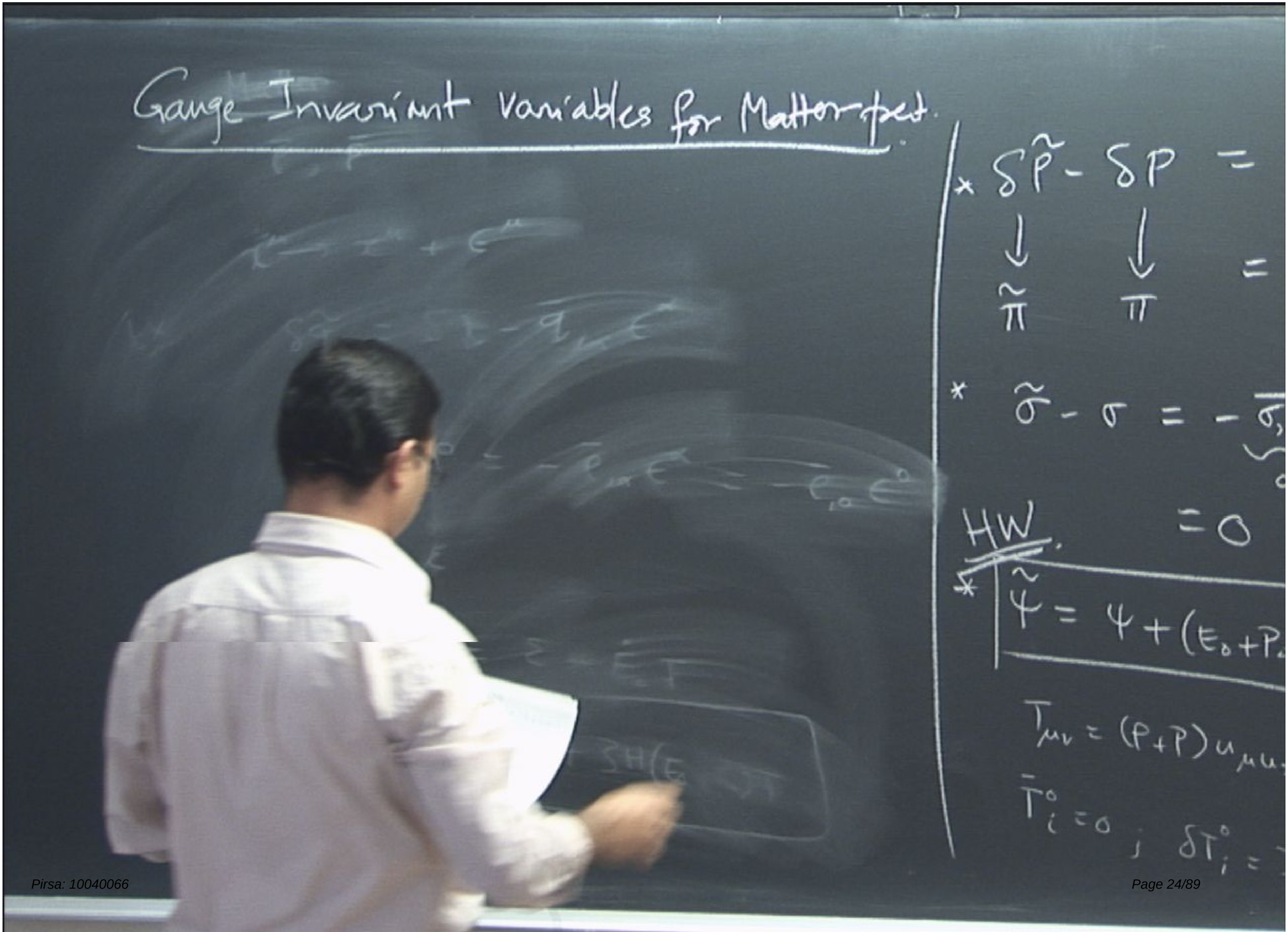
$$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$$

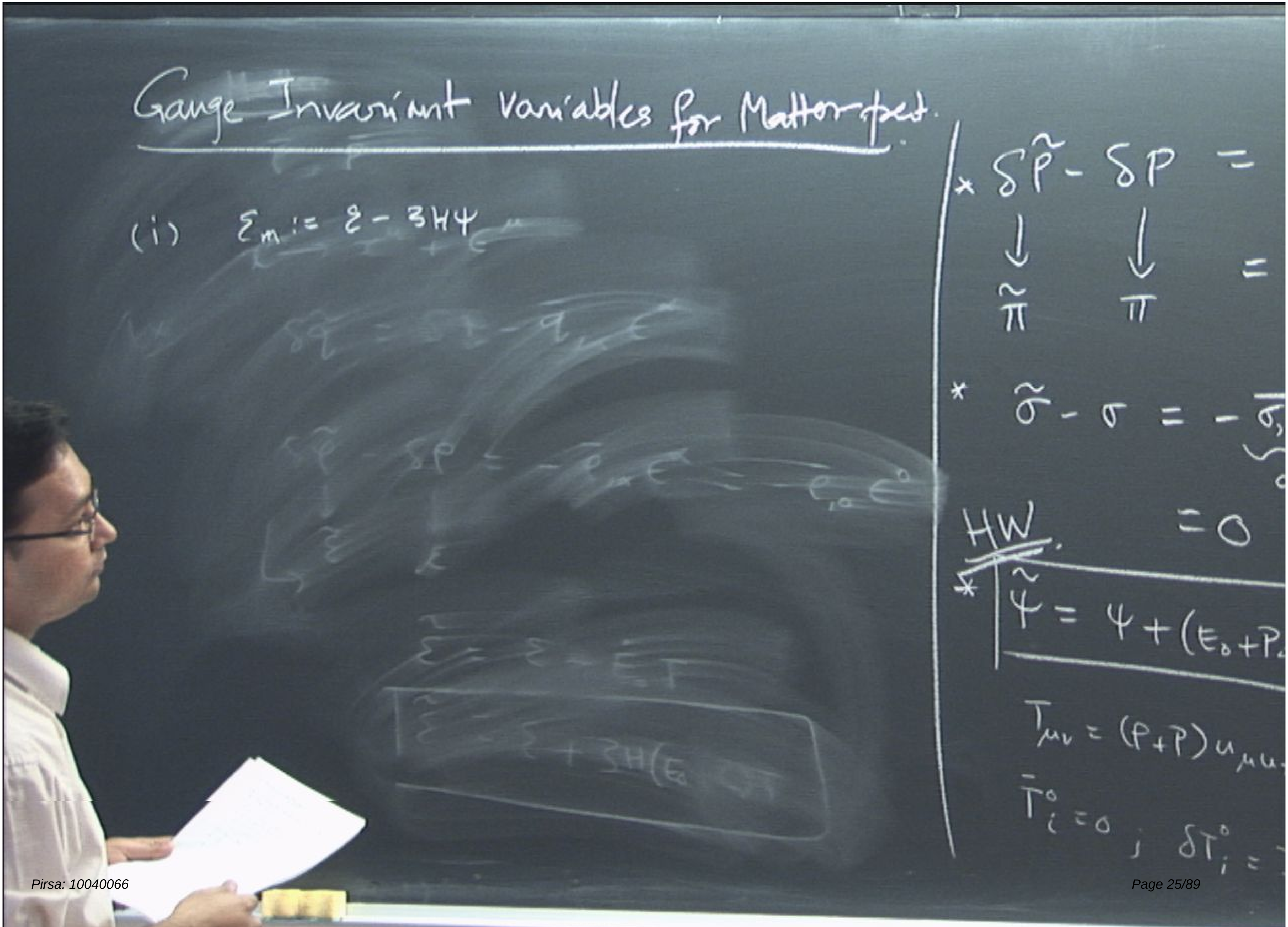
$$\tilde{\chi} = \chi - T$$

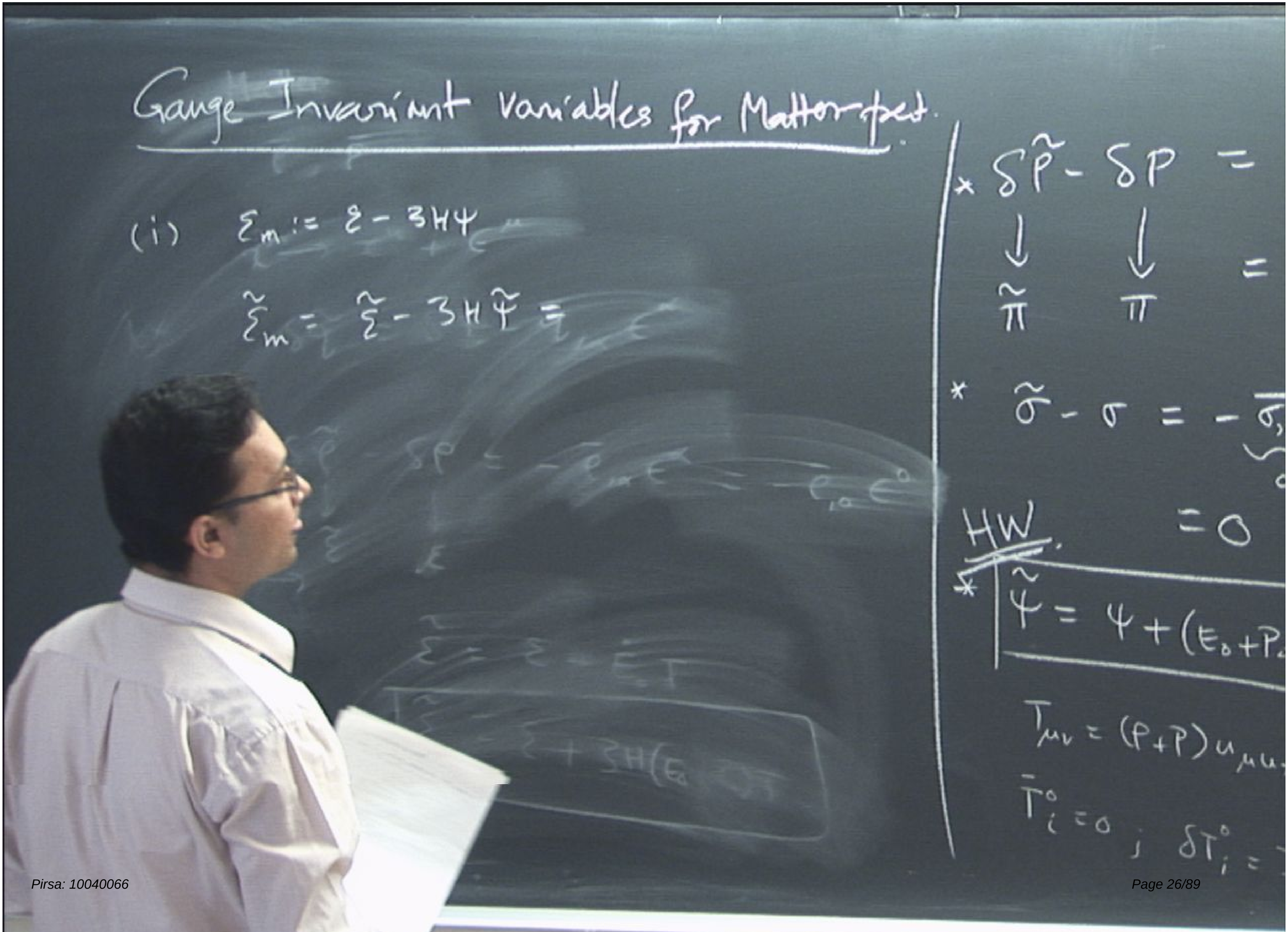
$$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$$

$$\tilde{\alpha} = \alpha - \dot{T}$$

$$\Phi_H = \varphi - H\chi;\quad \Phi_A = \alpha - \chi$$


Gauge Invariant variables for Matter part.
* δρ̃ − δP =
π̃ π =
* σ̃ − σ = −
HW =0
Ψ̃ = Ψ + (E₀+P
Tμν = (ρ+P) u_μ u
T̄°ᵢ = 0 ; δT°ᵢ =


Gauge Invariant variables for Matter part.
(i) ε_m := ε − 3Hψ
* δp̃ − δp =
π̃ π =
* σ̃ − σ = − σ̄,
HW =0
* ψ̃ = ψ + (ε₀+P
T_μν = (ρ+P) u_μ u
T̄⁰_i = 0 ; δT⁰_i =


Gauge Invariant variables for Matter pert.
(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$
$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} =$
$* \; \delta\tilde{P} - \delta P =$
$\tilde{\pi} \quad \pi$
$* \; \tilde{\sigma} - \sigma = -$
HW
$= 0$
$* \; \tilde{\Psi} = \Psi + (\mathcal{E}_0 + P$
$T_{\mu\nu} = (\rho + P) u_\mu u$
$\bar{T}^0_i = 0 \; ; \; \delta T^0_i =$

Gauge Invariant variables for Matter part.

(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$

$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$

* $\delta\tilde{P} - \delta P =$

$\tilde{\pi}$ π $=$

* $\tilde{\sigma} - \sigma = -\bar{\sigma},$

HW. $= 0$

* $\tilde{\Psi} = \Psi + (E_0 + P_0$

$T_{\mu\nu} = (\rho + P) u_\mu u$

$\bar{T}^0_i = 0$; $\delta T^0_i =$

Gauge Invariant variables for Matter-part.

(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$

$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$

→ in the gauge $\Psi = 0$

$\mathcal{E}_m = \mathcal{E}$

* $\delta\tilde{P} - \delta P =$

$\downarrow \quad \downarrow \quad =$

$\tilde{\pi} \quad \pi$

* $\tilde{\sigma} - \sigma = -\bar{\sigma}$

HW. $= 0$

* $\tilde{\Psi} = \Psi + (\mathcal{E}_0 + P_0$

$T_{\mu\nu} = (\rho + P) u_{\mu} u_{\nu}$

$\bar{T}^0_i = 0$; $\delta T^0_i =$

Gauge Invariant variables for Matter part.

(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$

$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$

in the gauge $\Psi = 0$ (comoving gauge)

$\mathcal{E}_m = \mathcal{E}$

* $\delta\tilde{P} - \delta P =$

$\downarrow \quad \downarrow \quad =$

$\tilde{\pi} \quad \pi$

* $\tilde{\sigma} - \sigma = -\bar{\sigma}$

HW. $= 0$

* $\tilde{\Psi} = \Psi + (E_0 + P_0$

$T_{\mu\nu} = (\rho + P) u_\mu u_\nu$

$\bar{T}^0_i = 0 \; ; \; \delta T^0_i =$

Gauge Invariant variables for Matter pert.

(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$

$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$

→ in the gauge $\Psi = 0$ (comoving gauge)

$\mathcal{E}_m = \mathcal{E}$

(ii) $\mathcal{E}_g$

* $\delta\tilde{P} - \delta P =$

$\downarrow \quad \downarrow \quad =$

$\tilde{\pi} \quad \pi$

* $\tilde{\sigma} - \sigma = -\sigma_{,}$

HW. $= 0$

* $\tilde{\Psi} = \Psi + (\mathcal{E}_0 + P_0$

$T_{\mu\nu} = (\rho + P) u_\mu u_\nu$

$\bar{T}^0_i = 0 \; ; \; \delta T^0_i =$

Gauge Invariant variables for Matter pert.

(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$

$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$

→ in the gauge $\Psi = 0$ (comoving gauge)

$\mathcal{E}_m = \mathcal{E}$

(ii) $\mathcal{E}_g = \mathcal{E} + 3H(E_0+P_0)\chi$

$\tilde{\mathcal{E}}_g = \mathcal{E} + 3H(E_0+P_0)T + 3H(E_0+P_0)\chi$

$= \mathcal{E}_g \qquad - 3H(E_0+P_0)T$

$* \ \delta\tilde{P} - \delta P =$

$\downarrow \qquad \downarrow \qquad =$

$\tilde{\pi} \qquad \pi$

$* \ \tilde{\sigma} - \sigma = -\bar{\sigma}_{,}$

HW. $\qquad = 0$

$* \ \tilde{\Psi} = \Psi + (E_0 + P_0$

$T_{\mu\nu} = (\rho + P) u_{\mu} u_{\nu}$

$\bar{T}^0_i = 0 \ ; \ \delta T^0_i =$

Gauge Invariant variables for Matter part.

(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$

$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$

in the gauge $\Psi = 0$ (comoving gauge)

$\mathcal{E}_m = \mathcal{E}$

(ii) $\mathcal{E}_g = \mathcal{E} + 3H(E_0+P_0)\chi$

$\tilde{\mathcal{E}}_g = \tilde{\mathcal{E}} + 3H(E_0+P_0)T + 3H(E_0+P_0)\chi$

$= \mathcal{E}_g \qquad - 3H(E_0+P_0)T$

* $\delta\tilde{P} - \delta P =$

$\downarrow \quad \downarrow \quad =$

$\tilde{\pi} \quad \pi$

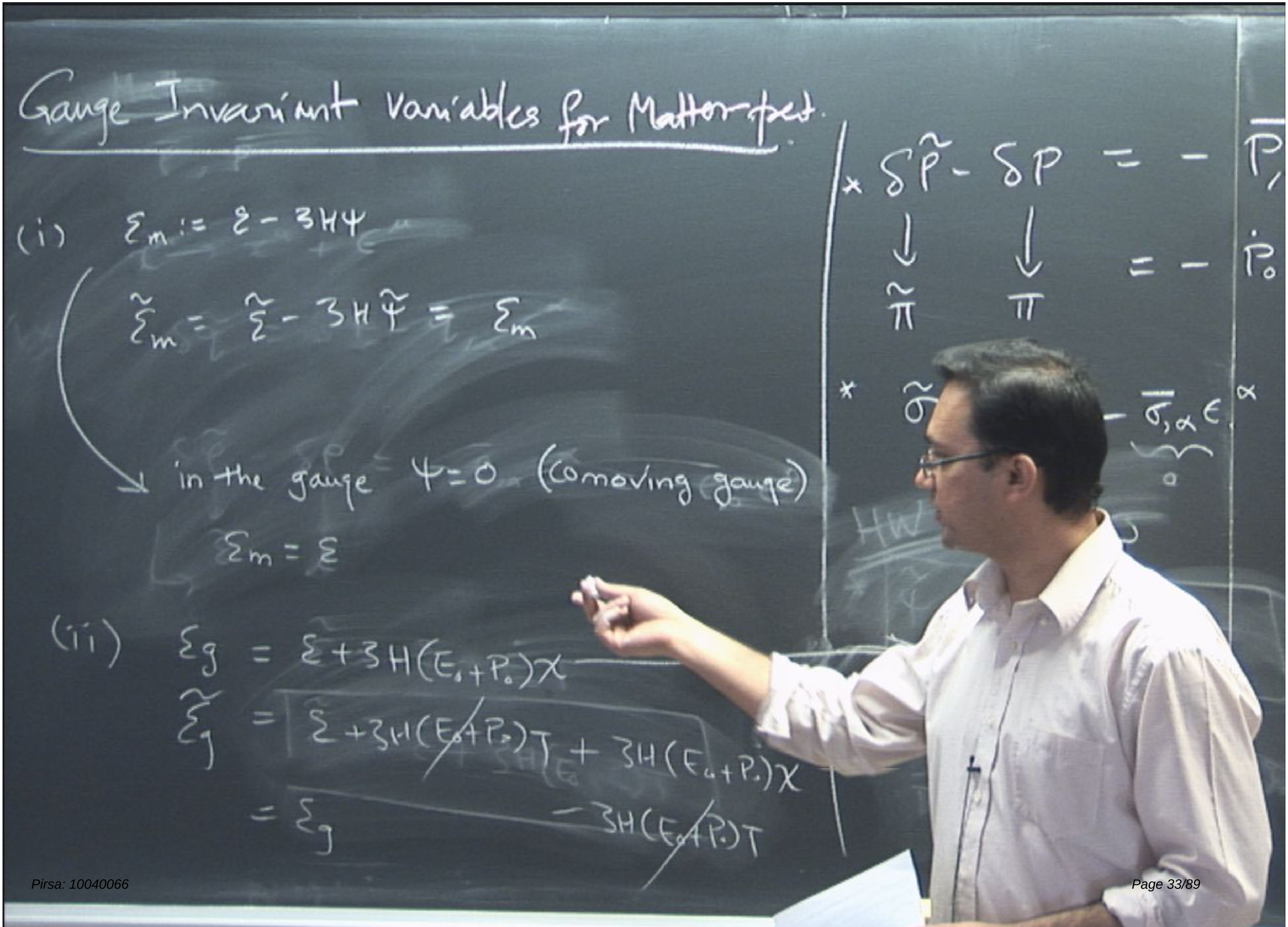
* $\tilde{\sigma} - \sigma = -\bar{\sigma}$

HW $= 0$

* $\tilde{\Psi} = \Psi + (E_0 + P_0$

$T_{\mu\nu} = (\rho + P) u_\mu u$

$\bar{T}^0_i = 0 \; ; \; \delta T^0_i =$


Gauge Invariant variables for Matter pert.
(i) $\mathcal{E}_m := \mathcal{E} - 3H\Psi$
$\tilde{\mathcal{E}}_m = \tilde{\mathcal{E}} - 3H\tilde{\Psi} = \mathcal{E}_m$
in the gauge $\Psi = 0$ (comoving gauge)
$\mathcal{E}_m = \mathcal{E}$
(ii) $\mathcal{E}_g = \mathcal{E} + 3H(E_0 + P_0)\chi$
$\tilde{\mathcal{E}}_g = \mathcal{E} + 3H(E_0 + P_0)T + 3H(E_0 + P_0)\chi$
$= \mathcal{E}_g - 3H(E_0 + P_0)T$
* $\delta\tilde{P} - \delta P = - \bar{P}_{,}$
$\tilde{\pi}$ π $= - \dot{P}_0$
* $\tilde{\sigma}$ $- \bar{\sigma}_{,\alpha}\epsilon^{\alpha}$
HW

Gauge Invariant variables for Matter pert.

(i) $\varepsilon_m := \varepsilon - 3H\psi$

$\tilde{\varepsilon}_m = \tilde{\varepsilon} - 3H\tilde{\psi} = \varepsilon_m$

→ in the gauge $\psi = 0$ (comoving gauge)

$\varepsilon_m = \varepsilon$

(ii) $\varepsilon_g = \varepsilon + 3H(E_0 + P_0)\chi$

$\tilde{\varepsilon}_g = \varepsilon + 3H(E_0 + P_0)T + 3H(E_0 + P_0)\chi$

$= \varepsilon_g \quad - 3H(E_0 + P_0)T$

→ $\varepsilon_g = \varepsilon$ in $\chi = 0$ gauge.

i.e. Newtonian gauge, zero shear

$* \ \delta\tilde{P} - \delta P = - \bar{P}_{,}$

$\downarrow \quad \downarrow \qquad = - \dot{P}_0$

$\tilde{\pi} \quad \pi$

$* \ \tilde{\sigma} - \sigma = - \bar{\sigma}_{,\alpha}\epsilon^{\alpha}$

0

0

HW

Gauge Invariant variables for Matter sect.

(i) $\varepsilon_m := \varepsilon - 3H\psi$

$\tilde{\varepsilon}_m = \tilde{\varepsilon} - 3H\tilde{\psi} = \varepsilon_m$

→ in the gauge $\psi = 0$ (comoving gauge)

$\varepsilon_m = \varepsilon$

(ii) $\varepsilon_g = \varepsilon + 3H(E_0 + P_0)\chi$ → $\varepsilon_g = \varepsilon$ in $\chi = 0$ gauge. i.e. Newtonian gauge. "zero shear"

$\tilde{\varepsilon}_g = \varepsilon + 3H(E_0 + P_0)T + 3H(E_0 + P_0)\chi$

$= \varepsilon_g \qquad - 3H(E_0 + P_0)T$

* $\delta\tilde{P} - \delta P = - \bar{P}_{,}$

$\downarrow \quad \downarrow \qquad = - \dot{P}_0$

$\tilde{\pi} \quad \pi$

* $\tilde{\sigma} - \sigma = - \bar{\sigma}_{,\alpha}\epsilon^{\alpha}$

0

HW

$$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)\,T$$

$$\tilde{\pi} = \pi - \dot{P}_0\,T$$

$$\tilde{\sigma} = \sigma$$

$$\tilde{\psi} = \psi + (\mathcal{E}_0 + P_0)\,T$$

$$\tilde{\varphi} = \varphi \; [illegible]$$

$$\tilde{\gamma} = \gamma - \lambda$$

$$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$$

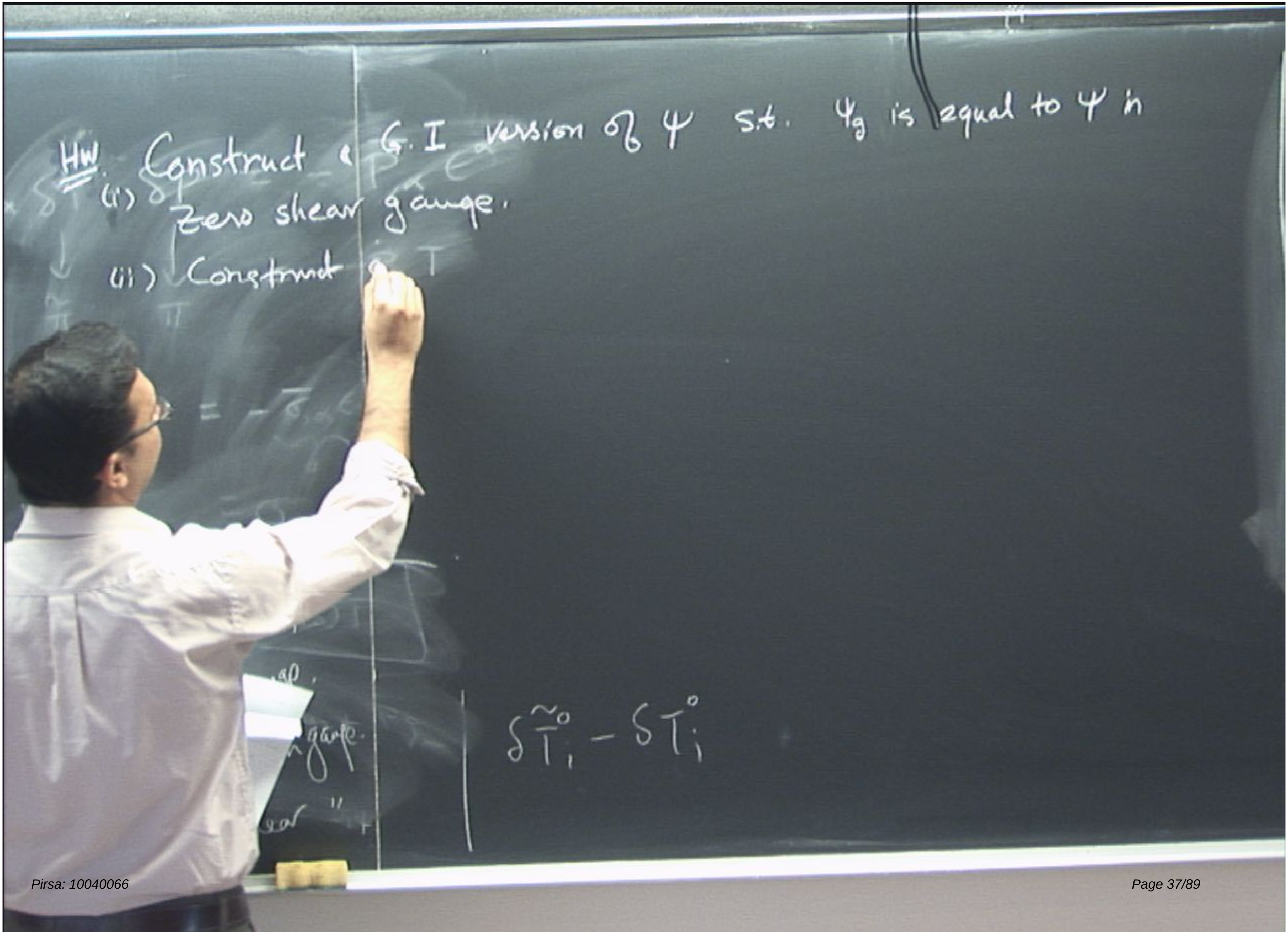
$$\tilde{\chi} = \chi - T$$

$$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}\,T$$

$$\tilde{\alpha} = \alpha - \dot{T}$$

$$\Phi_H = \varphi - H\chi, \; [illegible]$$

$$\delta \tilde{T}^0_i - \delta T^0_i$$


HW. Construct a G.I version of ψ s.t. ψ_g is equal to ψ in
(i) Zero shear gauge.
(ii) Construct
δT̃^0_i − δT^0_i

HW. Construct a G.I version of Ψ s.t. Ψ_g is equal to Ψ in

(i) zero shear gauge.

(ii) Construct another GI version of Ψ s.t. Ψ_m is equal to Ψ when $\varepsilon = 0$ (uniform density gauge).

HW

$\Rightarrow \varepsilon_g = \varepsilon$ in

$\chi = 0$ gauge.

ie. Newtonian gauge.

zero shear "

$\delta \tilde{T}^0_i - \delta T^0_i$

HW. Construct a G.I version of Ψ s.t. Ψ_g is equal to Ψ in

(i) Zero shear gauge.

(ii) Construct another GI version of Ψ s.t. Ψ_m is equal Ψ when $\varepsilon = 0$ (uniform density gauge).

(*) $P = \omega \rho$

HW Construct a G.I version of Ψ s.t. Ψ_g is equal to Ψ in

(i) Zero shear gauge.

(ii) Construct another GI version of Ψ s.t. Ψ_m is equal Ψ when $\varepsilon = 0$ (uniform density gauge).

(*) $P = \omega \rho$

Adiabatic : $c_s^2 = \frac{\delta P}{\delta \rho} = \frac{\dot{P}}{\dot{\rho}}$

HW. Construct a G.I version of Ψ s.t. Ψ_g is equal to Ψ in

(i) zero shear gauge.

(ii) Construct another GI version of Ψ s.t. Ψ_m is equal to Ψ when $\varepsilon = 0$ (uniform density gauge).

(*) $P = \omega \rho$

Adiabatic : $c_s^2 = \frac{\delta P}{\delta \rho} = \frac{\dot{P}}{\dot{\rho}}$

More generally we have departures from adiabaticity measured.

HW. Construct a G.I version of ψ s.t. ψ_g is equal to ψ in

(i) zero shear gauge.

(ii) Construct another GI version of ψ s.t. ψ_m is equal to ψ when $\varepsilon = 0$ (uniform density gauge).

(*) $P = \omega \rho$

Adiabatic : $c_s^2 = \dfrac{\delta P}{\delta \rho} = \dfrac{\dot{P}}{\dot{\rho}}$

More generally we have departures from adiabaticity measured entropy perturbations 'Γ'

$$\Gamma = \frac{1}{\omega \rho}\left(\delta P - c_s^2 \delta \rho\right) \longrightarrow P\Gamma =$$

HW. Construct a G.I version of Ψ s.t. Ψ_g is equal to Ψ in

(i) zero shear gauge.

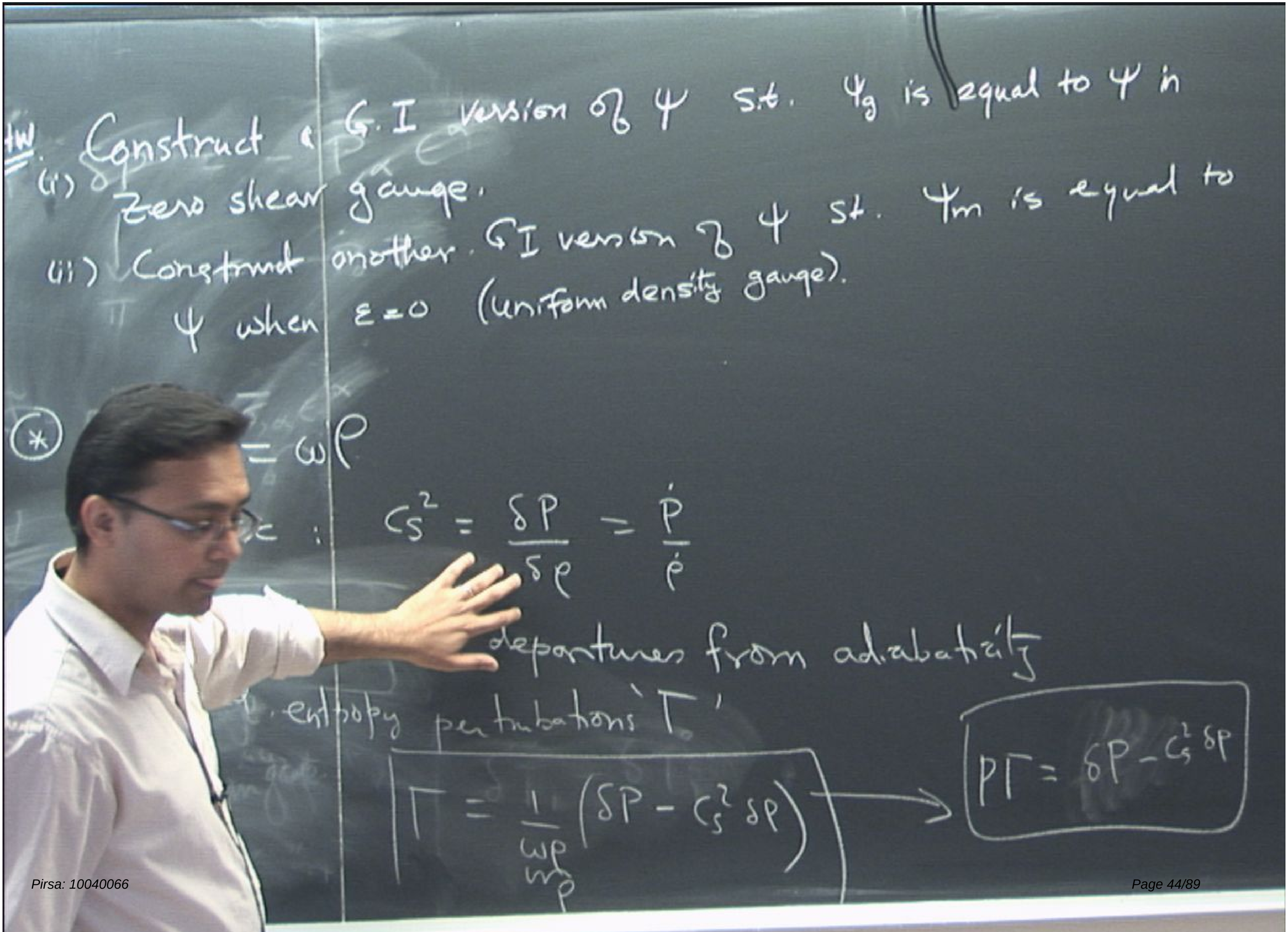
(ii) Construct another G.I version of Ψ s.t. Ψ_m is equal to Ψ when $\varepsilon = 0$ (uniform density gauge).

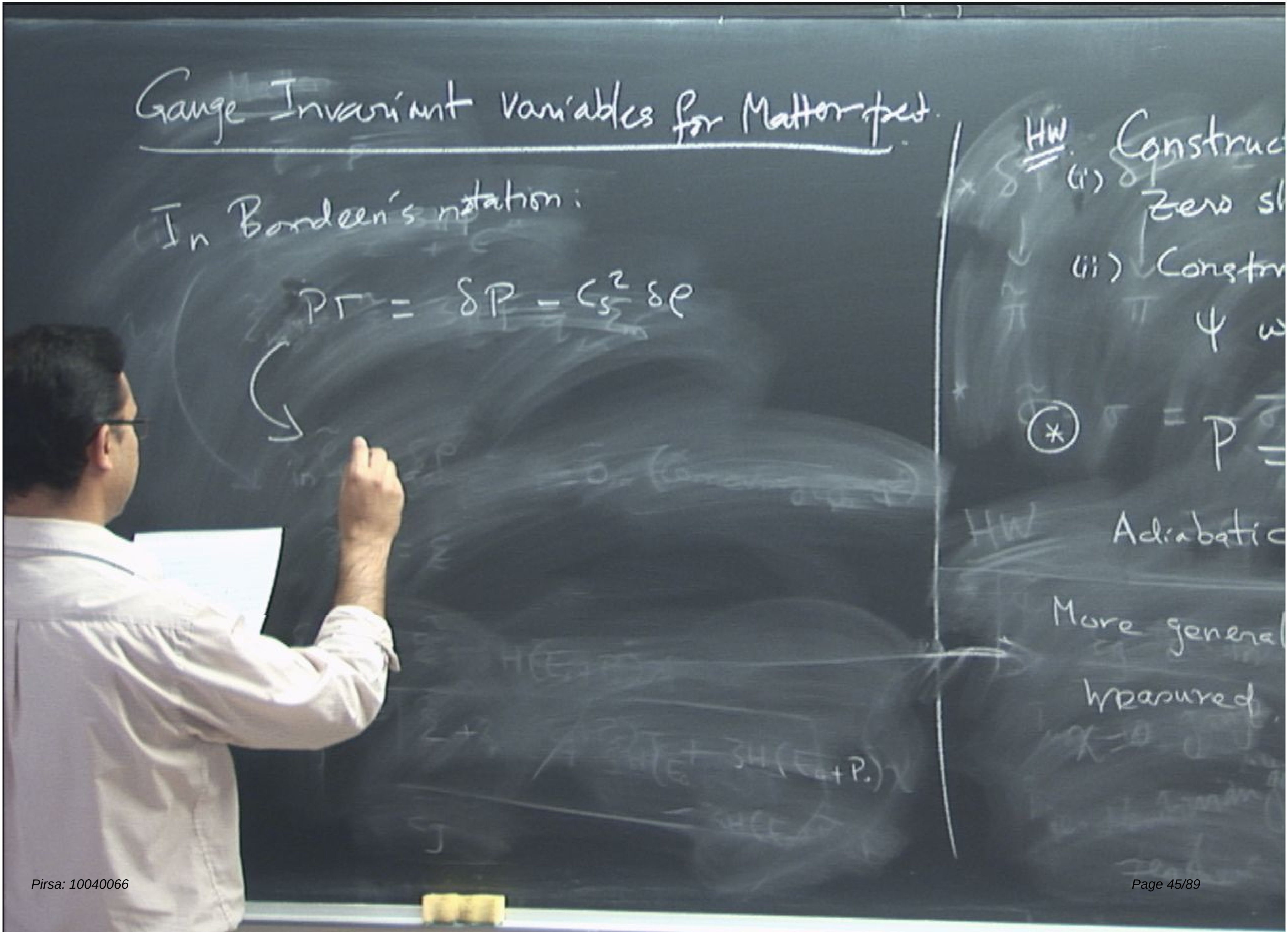
(*) $P = \omega \rho$

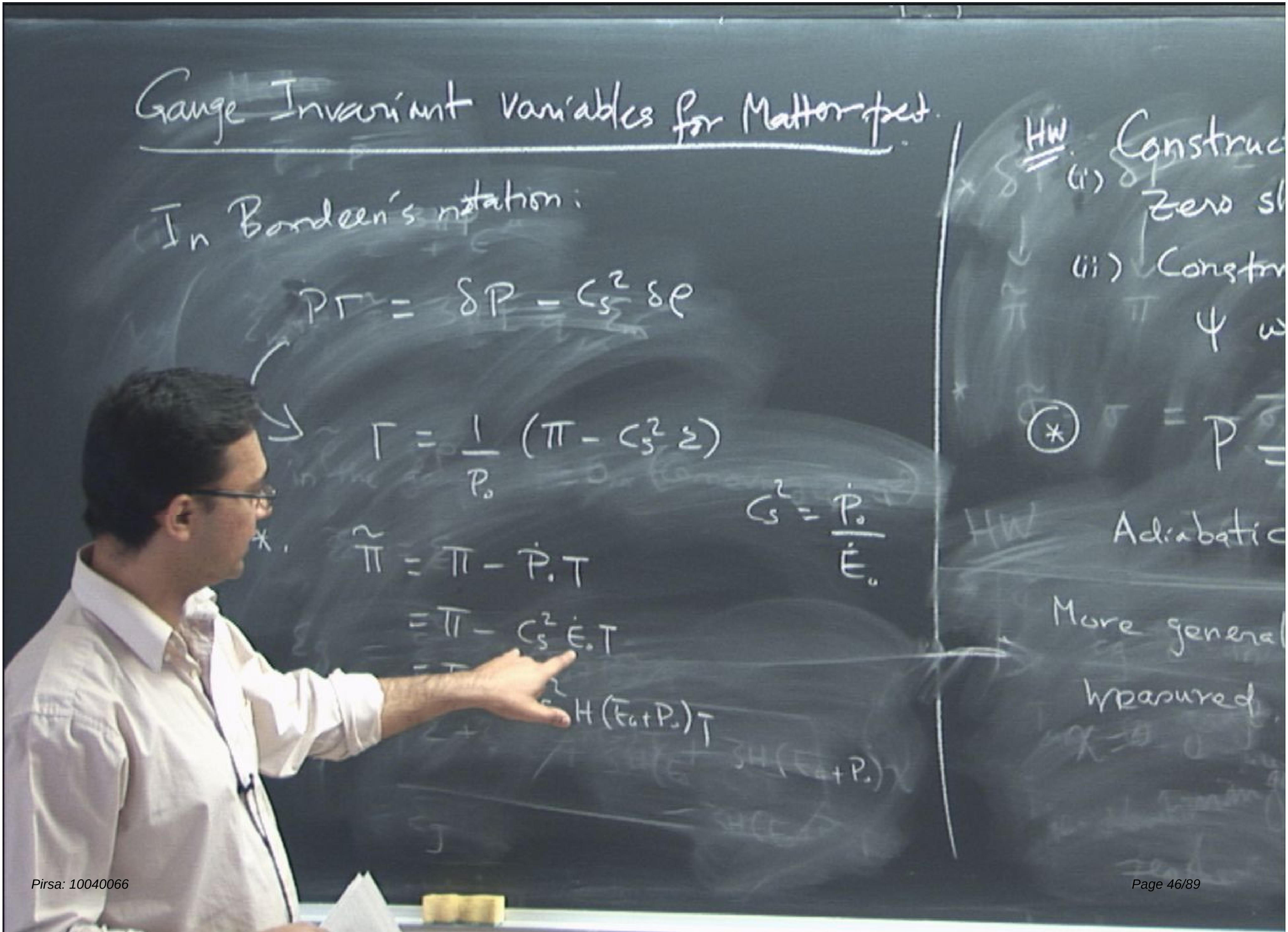
Adiabatic : $c_s^2 = \frac{\delta P}{\delta \rho} = \frac{\dot{P}}{\dot{\rho}}$

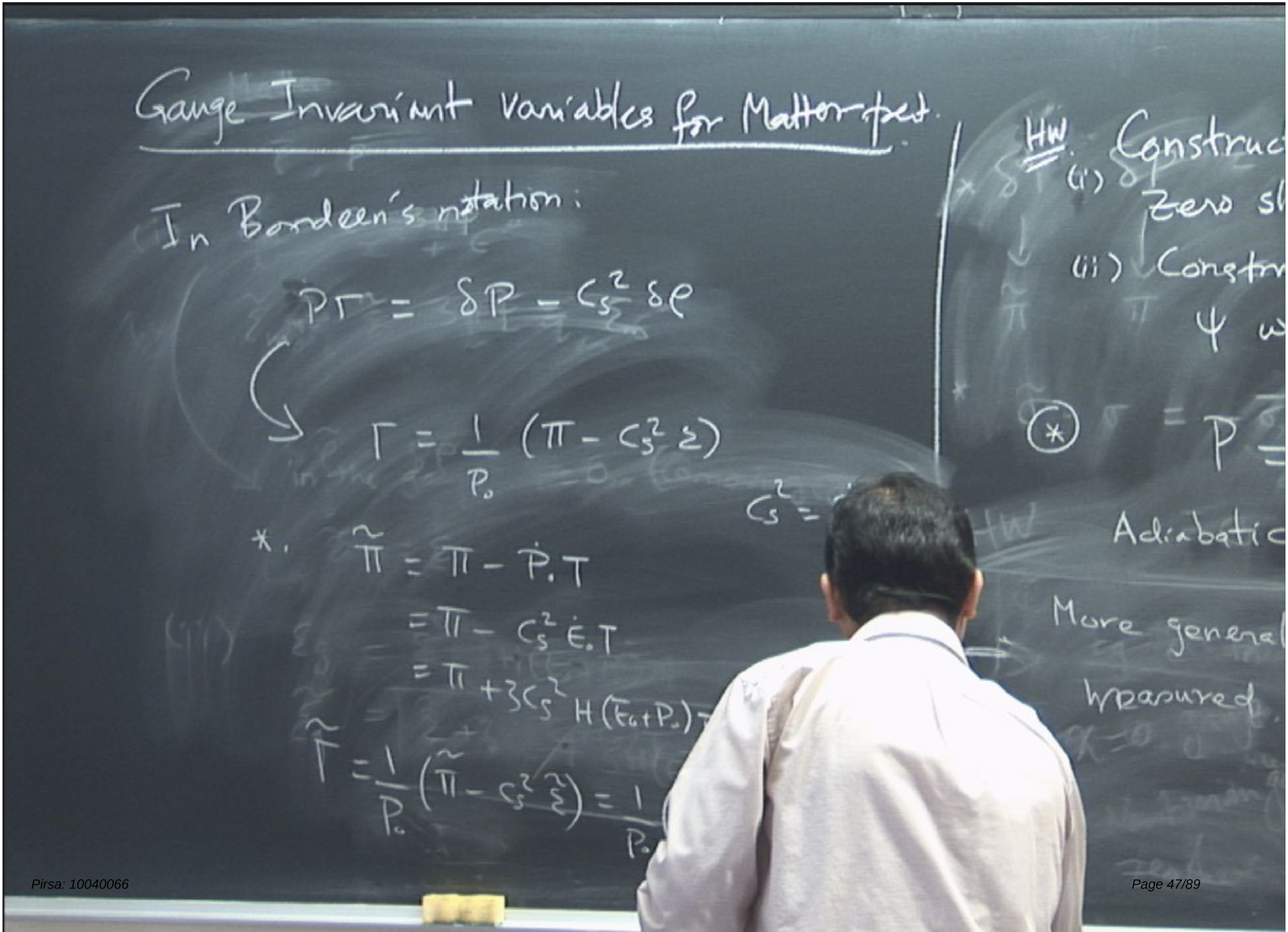
More generally we have departures from adiabaticity measured entropy perturbations 'Γ'

$$\Gamma = \frac{1}{\omega \rho}\left(\delta P - c_s^2 \delta \rho\right) \rightarrow P\Gamma = \delta P - c_s^2 \delta \rho$$


HW. Construct a G.I version of ψ s.t. ψg is equal to ψ in
(i) zero shear gauge.
(ii) Construct another G.I version of ψ st. ψm is equal to
ψ when ε=0 (uniform density gauge).
= ωρ
cs² = δP/δρ = Ṗ/ρ̇
departures from adiabaticity
entropy perturbations 'Γ'
Γ = 1/ωρ (δP − cs² δρ)
pΓ = δP − cs² δρ


Gauge Invariant variables for Matter part.
In Bardeen's notation:
$p\Gamma = \delta P - c_s^2 \delta\rho$
HW. Construct
(i)
Zero sh
(ii) Constr
Adiabatic
More general
measured


Gauge Invariant variables for Matter part.
In Bardeen's notation:
pΓ = δP − cs² δρ
Γ = 1/P₀ (π − cs² ε)
cs² = Ṗ₀/Ė₀
π̃ = π − Ṗ₀ T
= π − cs² Ė₀ T
H (E₀+P₀) T
HW. Construc
(i) δ
Zero sh
(ii) Constr
ψ w
P
Adiabatic
More general
measured


Gauge Invariant variables for Matter part.
In Bardeen's notation:
$P\Gamma = \delta P - c_s^2 \delta\rho$
$\Gamma = \frac{1}{P_0}(\pi - c_s^2 \varepsilon)$
$c_s^2 =$
$\tilde{\pi} = \pi - \dot{P}_0 T$
$= \pi - c_s^2 \dot{\varepsilon}_0 T$
$= \pi + 3c_s^2 H(\varepsilon_0 + P_0)$
$\tilde{\Gamma} = \frac{1}{P_0}(\tilde{\pi} - c_s^2 \tilde{\varepsilon}) = \frac{1}{P_0}$
HW. Construc
(i) δ
Zero sh
(ii) Constr
ψ w
$P =$
Adiabatic
More general
Measured

Gauge Invariant variables for Matter part.

In Bardeen's notation:

$$P\Gamma = \delta P - c_s^2 \delta\rho$$

$$\Gamma = \frac{1}{P_0}(\Pi - c_s^2 \Sigma)$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

$$*.\quad \tilde{\Pi} = \Pi - \dot{P}_0 T$$

$$= \Pi - c_s^2 \dot{E}_0 T$$

$$= \Pi + 3c_s^2 H(E_0 + P_0) T$$

$$\tilde{\Gamma} = \frac{1}{P_0}(\tilde{\Pi} - c_s^2 \tilde{\Sigma}) = \frac{1}{P_0}\left(\Pi + 3c_s^2 H(E_0+P_0)T - c_s^2\Sigma - 3c_s^2 H(E_0+P_0)T\right) = \ldots$$

HW. Construc...

(i) δ ... Zero sh...

(ii) Constr...

Ψ w...

(*) P ...

Adiabatic...

More general...

measured...

Gauge Invariant variables for Matter sect.

In Bardeen's notation:

$$P\Gamma = \delta P - c_s^2 \delta\rho$$

$$\Gamma = \frac{1}{P_0}\left(\Pi - c_s^2 \mathcal{E}\right)$$

$$c_s^2 = \frac{\dot{P}_0}{\dot{E}_0}$$

*. $\tilde{\Pi} = \Pi - \dot{P}_0 T$

$= \Pi - c_s^2 \dot{E}_0 T$

$= \Pi + 3c_s^2 H (E_0 + P_0) T$

$$\tilde{\Gamma} = \frac{1}{P_0}\left(\tilde{\Pi} - c_s^2 \tilde{\mathcal{E}}\right) = \frac{1}{P_0}\left(\Pi + 3c_s^2 H (E_0+P_0)T - c_s^2 \mathcal{E} - 3c_s^2 H (E_0+P_0)T\right) = \Gamma$$

HW. Construct a G.I...

(i) Zero shear ga...

(ii) Construct anoth...

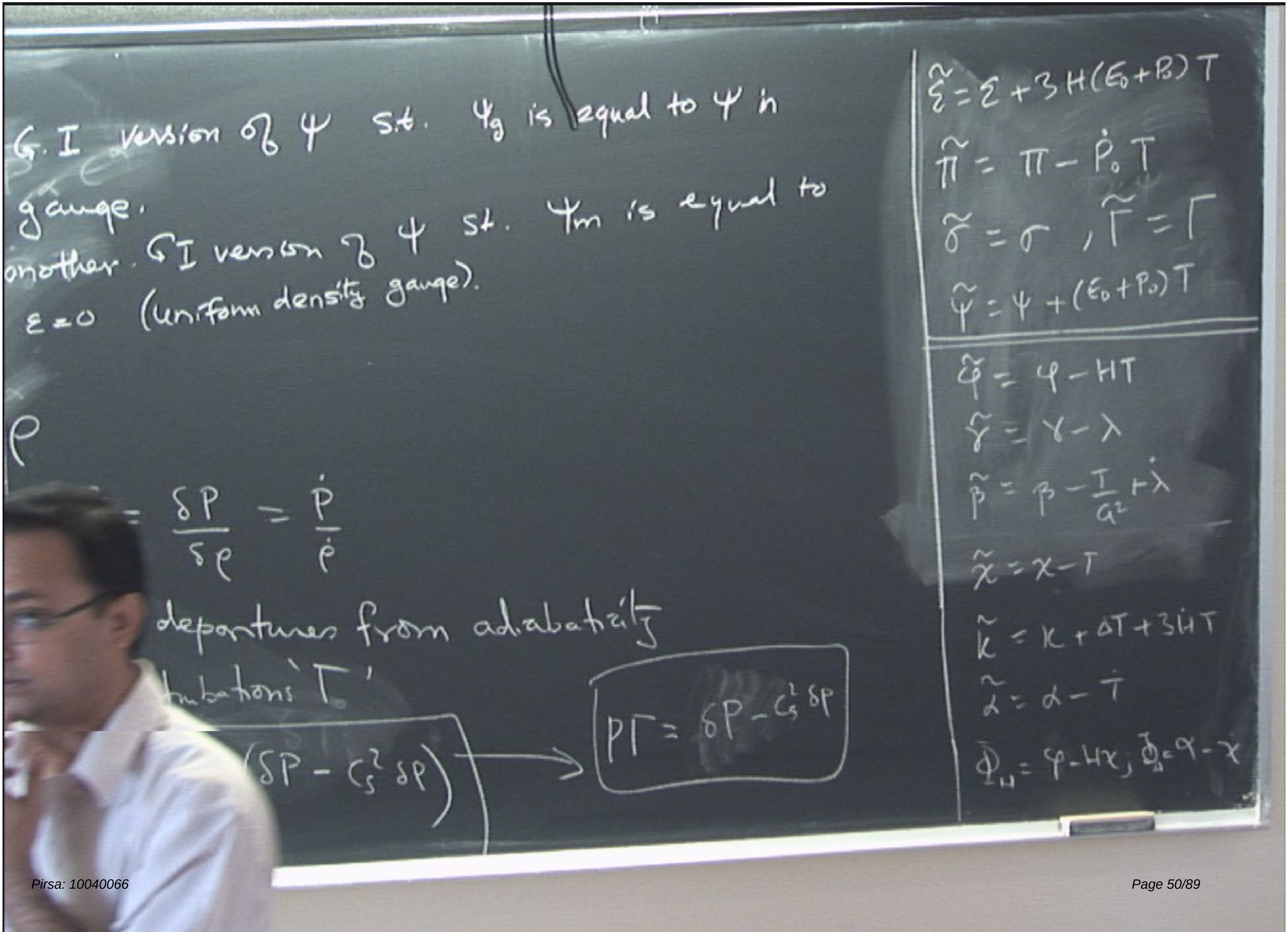
ψ when $\mathcal{E} =$ ...

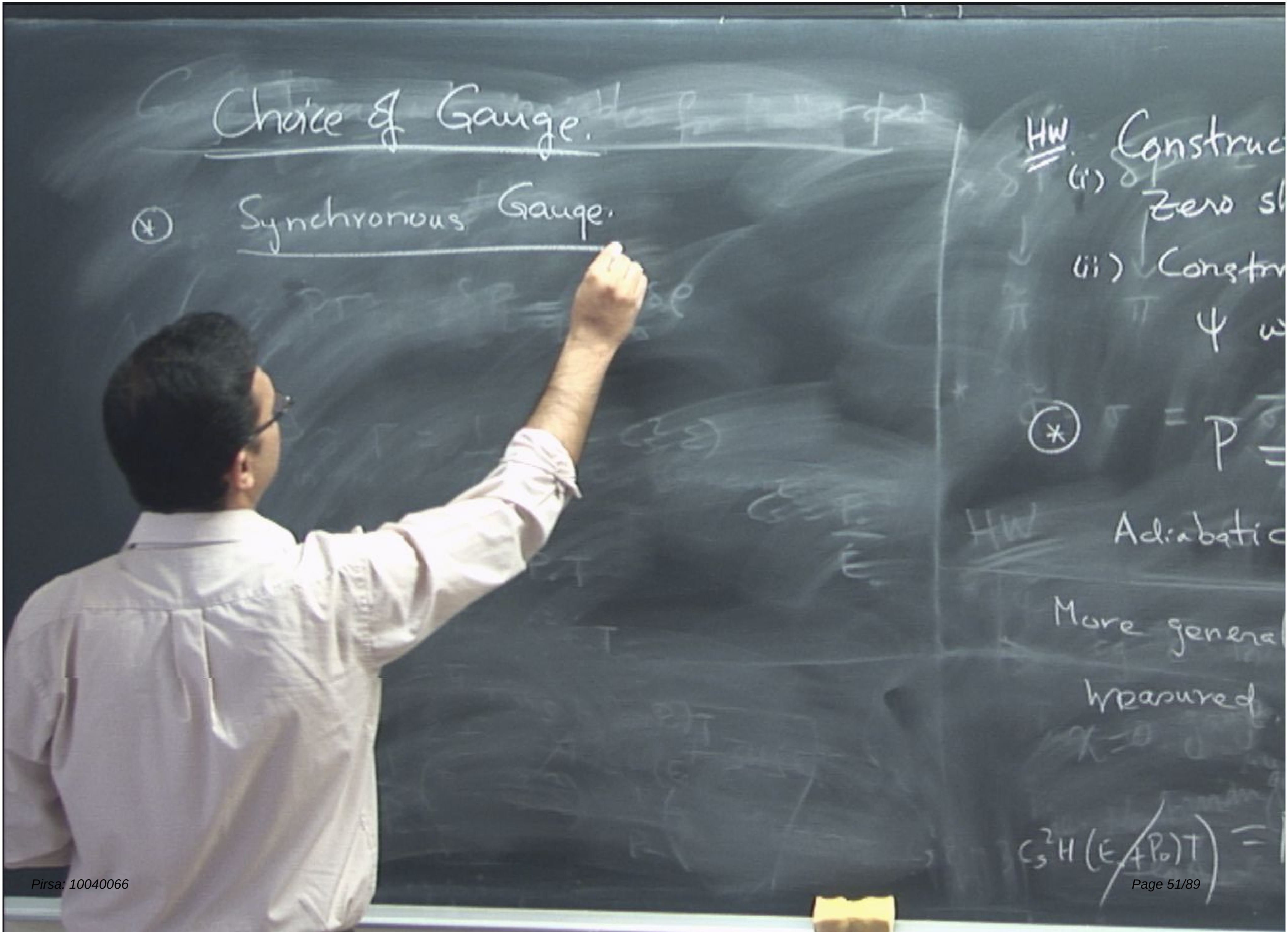
(*) $P = \omega \rho$

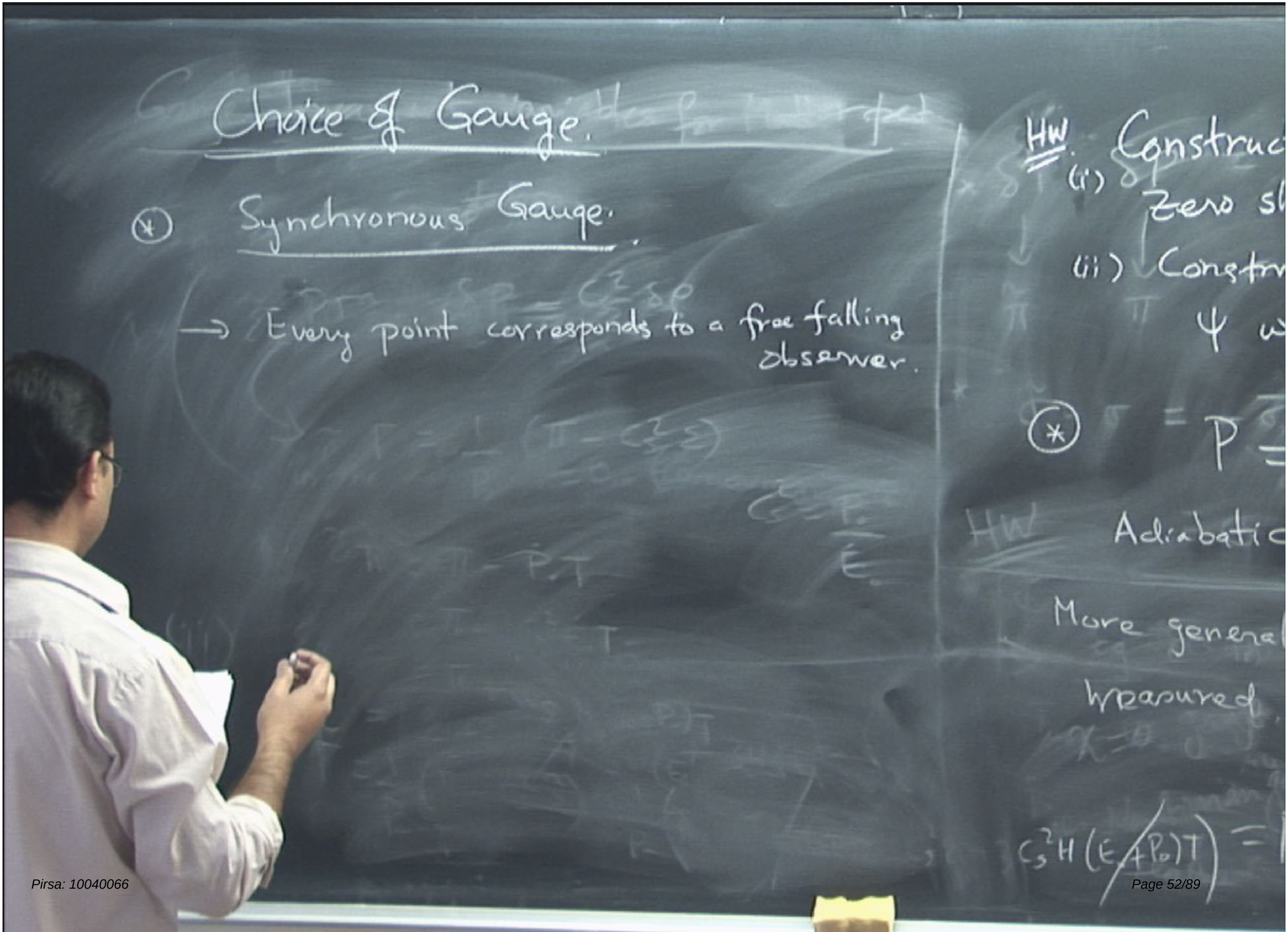
Adiabatic : c_s ...

More generally we ha...

measured entropy


G.I version of Ψ s.t. Ψ_g is equal to Ψ in
gauge.
another GI version of Ψ s.t. Ψ_m is equal to
$\varepsilon = 0$ (uniform density gauge).
ρ
$= \frac{\delta P}{\delta \rho} = \frac{\dot{P}}{\dot{\rho}}$
departures from adiabaticity
perturbations 'Γ'
$(\delta P - c_s^2 \delta \rho)$
$P\Gamma = \delta P - c_s^2 \delta \rho$
$\tilde{\varepsilon} = \varepsilon + 3H(\varepsilon_0 + P_0) T$
$\tilde{\pi} = \pi - \dot{P}_0 T$
$\tilde{\sigma} = \sigma \ , \ \tilde{\Gamma} = \Gamma$
$\tilde{\Psi} = \Psi + (\varepsilon_0 + P_0) T$
$\tilde{\varphi} = \varphi - HT$
$\tilde{\gamma} = \gamma - \lambda$
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$\tilde{\chi} = \chi - T$
$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H} T$
$\tilde{\alpha} = \alpha - \dot{T}$
$\Phi_H = \varphi - H\chi, \ \Phi_A = \alpha - \dot{\chi}$


Choice of Gauge.
Synchronous Gauge.
HW. Construc
(i)
Zero sl
(ii) Constr
Ψ w
P =
Adiabatic
More general
measured


Choice of Gauge.
Synchronous Gauge.
→ Every point corresponds to a free falling observer.
HW. Construc
(i) Zero sl
(ii) Constr
Adiabatic
More genera
measured

Lapse $\rightarrow \alpha$

Shift $\rightarrow \beta$

$h_{ij} \rightarrow \varphi, \psi$

…nds to a free falling observer.

Adiabatic

More generally … from ad…

measured entropy perturbations 'Γ'

$$c_s^2 H (E + P_0) T = \Gamma$$

$$\Gamma = \frac{1}{w\rho}\left(\delta P - c_s^2 \delta\rho\right)$$

Choice of Gauge.

(*) Synchronous Gauge.

→ Every point corresponds to a free falling observer.

→ $\alpha = 0, \beta = 0$

Lapse → α

Shift → β

h_{ij} → φ

(*)

Adiabatic

More general

Measured

$c_s^2 H (E + P_0) T$ =

Choice of Gauge.

(*) Synchronous Gauge.

→ Every point corresponds to a free falling observer.

→ $\alpha = 0, \beta = 0$

$$g_{00} = -1, \quad g_{0i} = 0$$

Lapse → α

Shift → β

h_{ij} → φ,

Adiabatic

More general

Measured

$c_s^2 H (E + P_0) T$ =

Choice of Gauge.

(*) Synchronous Gauge.

→ Every point corresponds to a free falling observer.

→ $\alpha = 0, \beta = 0$

$g_{00} = -1, \; g_{0i} = 0$

→ We foliated the manifold at $t = $ const and put spatial coordinates such that all clocks are synchronized

Lapse → α

Shift → β

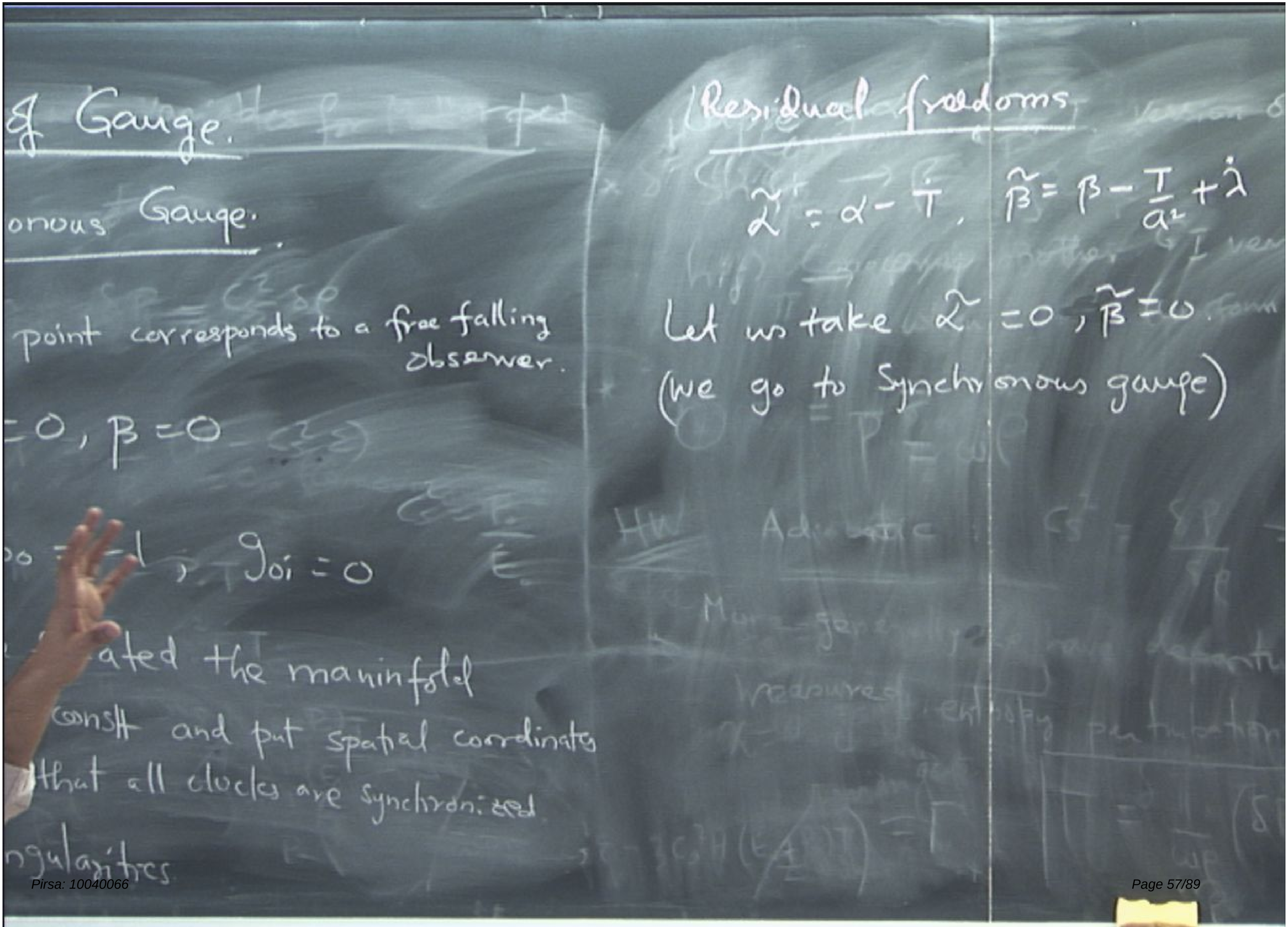
h_{ij} → φ,

Adiabati

More genera

measured

$c_s^2 H (E + P_0) T) =$


g Gauge.
onous Gauge.
point corresponds to a free falling observer.
=0, β=0
goi = 0
ated the maninfold
const and put spatial coordinates
that all clocks are synchronized
ngularities
Residual freedoms
$\tilde{\alpha} = \alpha - \dot{T}$, $\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
Let us take $\tilde{\alpha} = 0$, $\tilde{\beta} = 0$.
(we go to Synchronous gauge)

...g Gauge.

...onous Gauge.

...point corresponds to a free falling observer.

$...= 0, \ \beta = 0$

$...{}_{00} = -1, \ g_{0i} = 0$

...foliated the maninfold

$t = \text{const}$ and put spatial coordinates

...that all clocks are synchronized

...ngularities

Residual freedoms

$$\tilde{\alpha} = \alpha - \dot{T}, \quad \tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$$

Let us take $\tilde{\alpha} = 0, \ \tilde{\beta} = 0$.

(we go to Synchronous gauge)

$$\tilde{\alpha} = 0 \Rightarrow \dot{T} = \alpha$$

$$\tilde{\beta} = 0 \Rightarrow \dot{\lambda} = \frac{T}{a^2} - \beta$$

$$T = \int \alpha \, dt + C_1$$

$$\lambda = -\int \beta \, dt + \int \frac{T}{a^2} dt + C_2$$

of Gauge.

...onous Gauge.

... point corresponds to a free falling observer.

$= 0, \beta = 0$

$g_{00} = -1, \; g_{0i} = 0$

... foliated the maninfold
t = const and put spatial coordinates
... that all clocks are synchronized

...ngularities

Residual freedoms

$$\tilde{\alpha} = \alpha - \dot{T}, \quad \tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$$

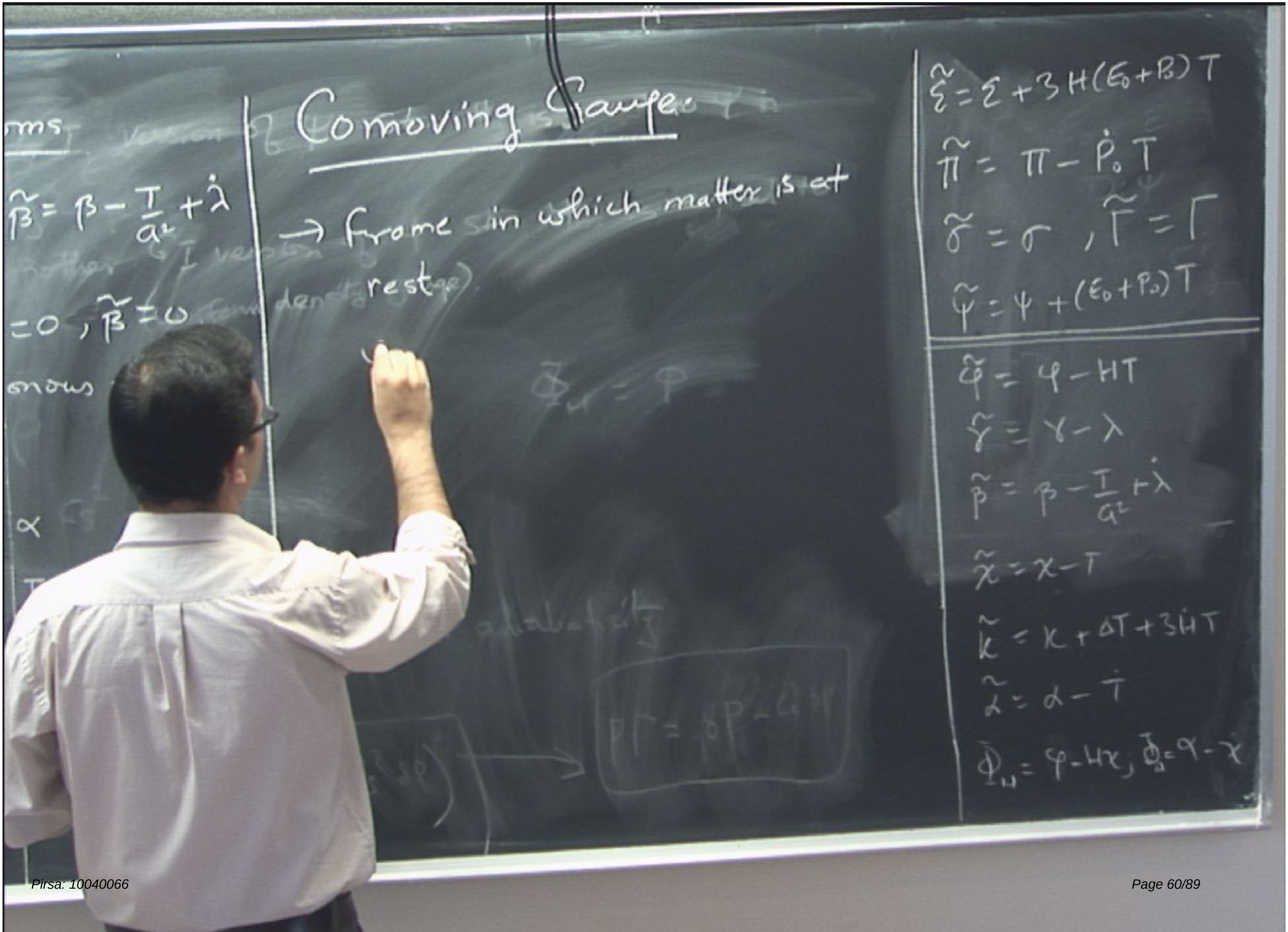
Let us take $\tilde{\alpha} = 0, \tilde{\beta} = 0$.
(we go to Synchronous gauge)

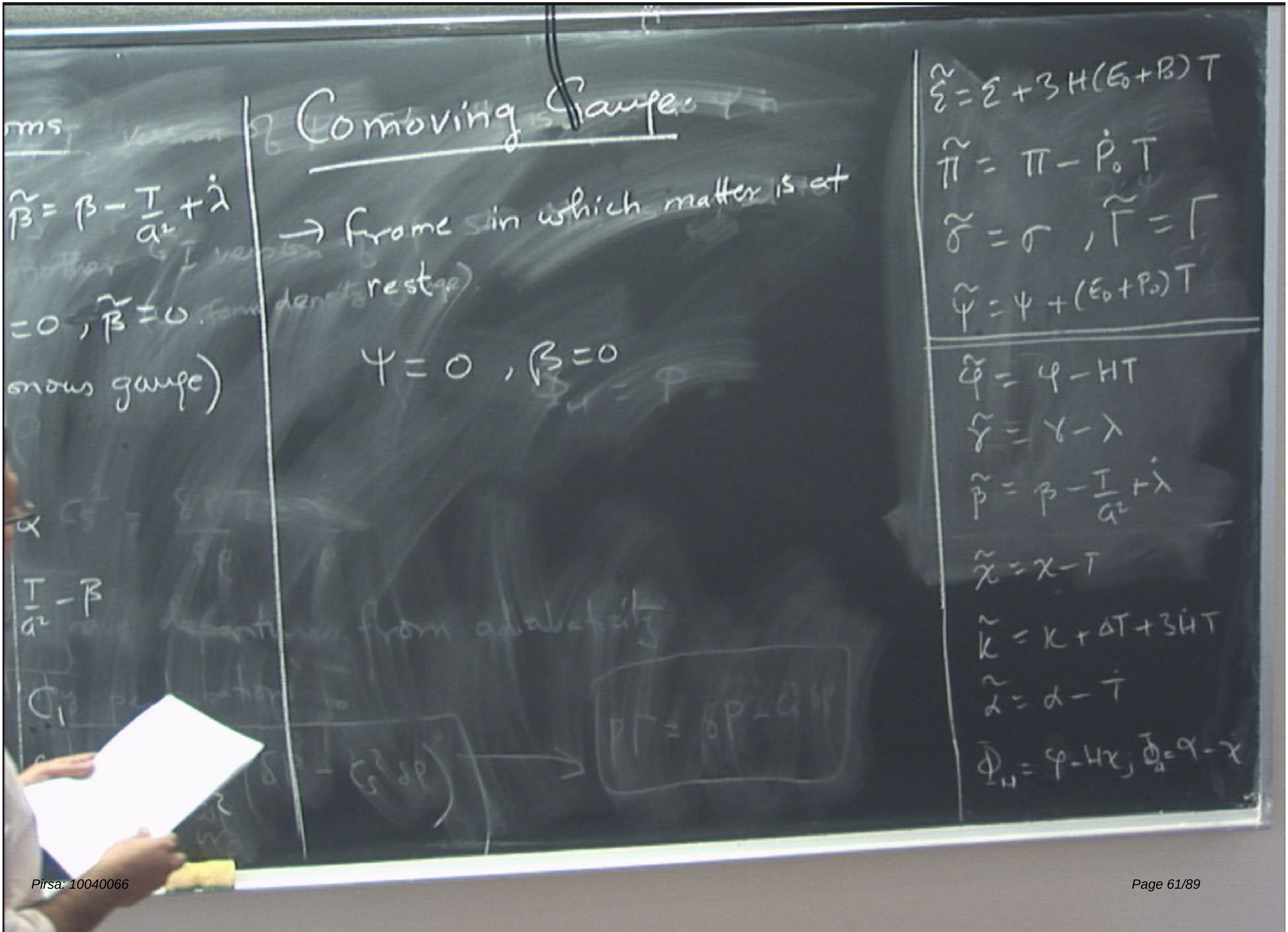
$$\tilde{\alpha} = 0 \Rightarrow \dot{T} = \alpha$$

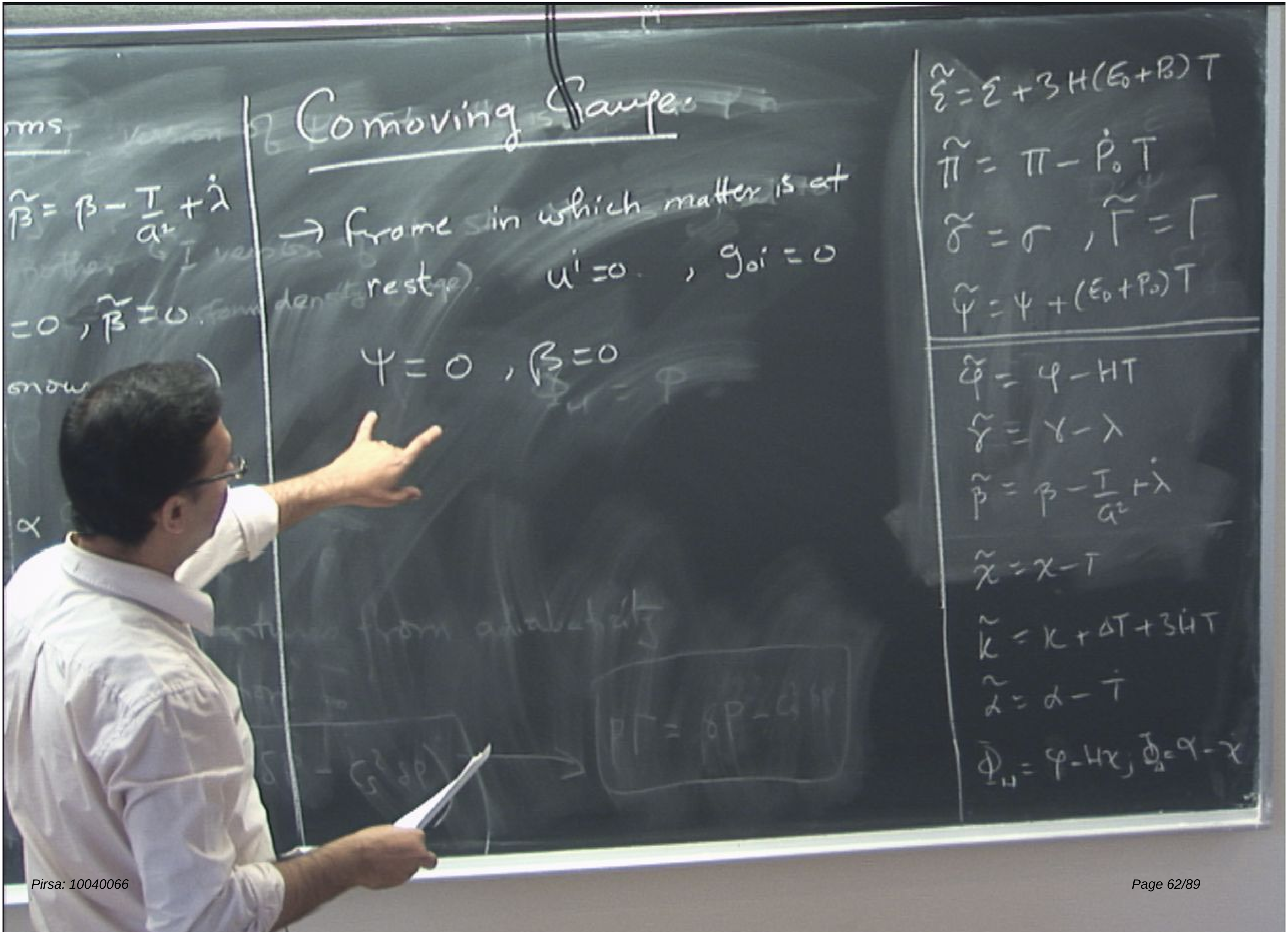
$$\tilde{\beta} = 0 \Rightarrow \dot{\lambda} = \frac{T}{a^2} - \beta$$

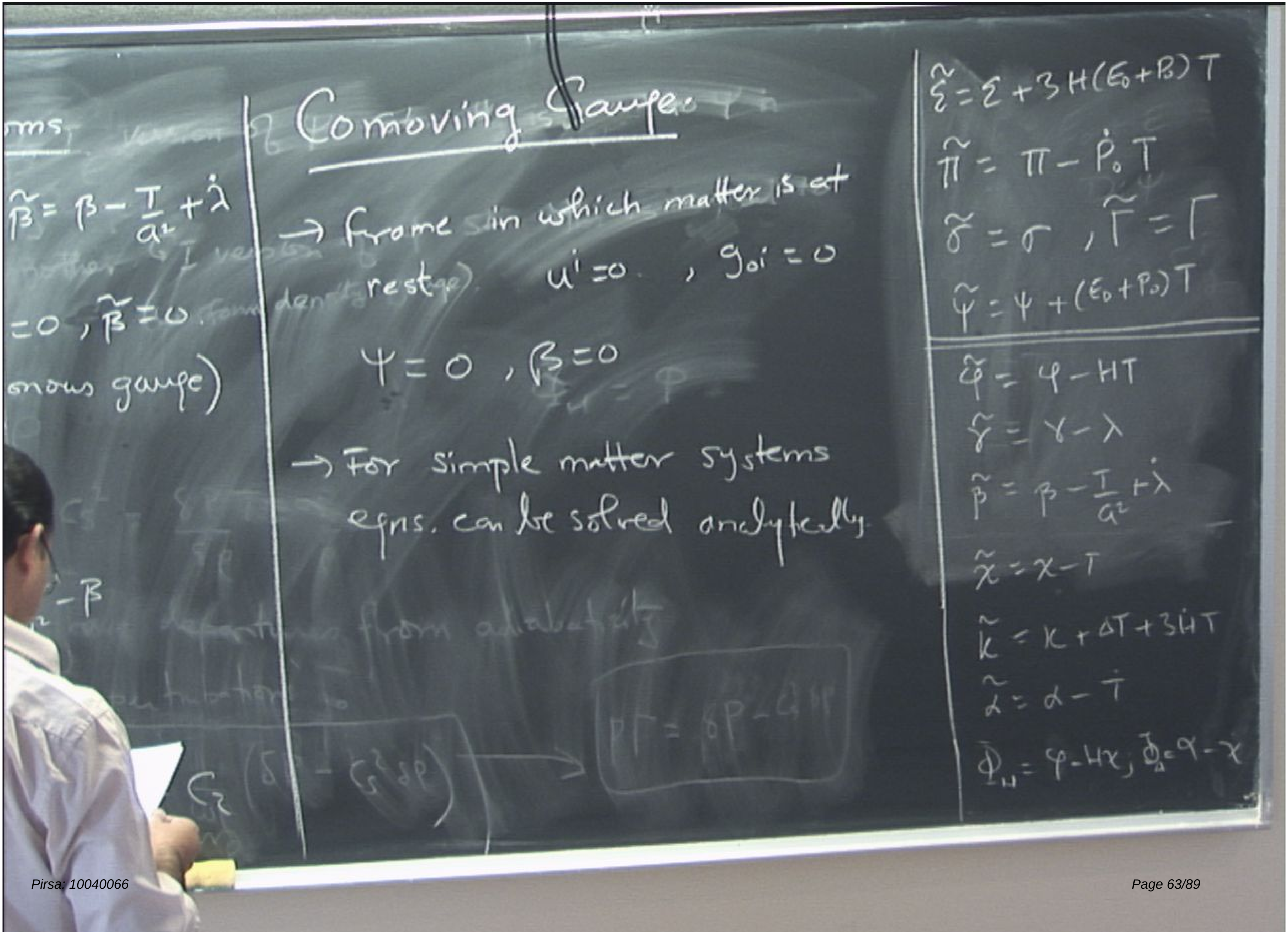
$$T = \int \alpha \, dt + C_1$$

$$\lambda = -\int \beta \, dt + \int \frac{T}{a^2} dt + C_2$$


Comoving Gauge:
→ frame in which matter is at rest
ε̃ = ε + 3H(ε0 + P0) T
π̃ = π − Ṗ0 T
σ̃ = σ , Γ̃ = Γ
ψ̃ = ψ + (ε0 + P0) T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
Φ_H = φ − Hχ ; Φ_A = α − χ̇


Comoving Gauge
→ frame in which matter is at rest
$\Psi = 0 \;,\; \beta = 0$
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$= 0 \;,\; \tilde{\beta} = 0$
nous gauge)
$\frac{T}{a^2} - \beta$
$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)\,T$
$\tilde{\pi} = \pi - \dot{P}_0\,T$
$\tilde{\sigma} = \sigma \;,\; \tilde{\Gamma} = \Gamma$
$\tilde{\Psi} = \Psi + (\mathcal{E}_0 + P_0)\,T$
$\tilde{\varphi} = \varphi - HT$
$\tilde{\gamma} = \gamma - \lambda$
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$\tilde{\chi} = \chi - T$
$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$
$\tilde{\alpha} = \alpha - \dot{T}$
$\Phi_H = \varphi - H\chi \;;\; \Phi_A = \alpha - \dot{\chi}$


Comoving Gauge:
→ frame in which matter is at rest
u^i = 0 , g_0i = 0
ψ = 0 , β = 0
β̃ = β − T/a² + λ̇
= 0 , β̃ = 0
ξ̃ = ξ + 3H(ε₀ + P₀) T
π̃ = π − Ṗ₀ T
σ̃ = σ , Γ̃ = Γ
ψ̃ = ψ + (ε₀ + P₀) T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
Φ_H = φ − Hχ; Φ_A = α − χ̇


Comoving Gauge
→ frame in which matter is at rest
$u^i = 0$, $g_{0i} = 0$
$\psi = 0$, $\beta = 0$
→ For simple matter systems eqns. can be solved analytically
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$= 0$, $\tilde{\beta} = 0$
...nous gauge)
$\tilde{\varepsilon} = \varepsilon + 3H(\varepsilon_0 + P_0)T$
$\tilde{\pi} = \pi - \dot{P}_0 T$
$\tilde{\sigma} = \sigma$, $\tilde{\Gamma} = \Gamma$
$\tilde{\psi} = \psi + (\varepsilon_0 + P_0)T$
$\tilde{\varphi} = \varphi - HT$
$\tilde{\gamma} = \gamma - \lambda$
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$
$\tilde{\chi} = \chi - T$
$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$
$\tilde{\alpha} = \alpha - \dot{T}$
$\Phi_H = \varphi - H\chi$; $\Phi_A = \alpha - \dot{\chi}$

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$= 0\ ,\ \tilde{\beta} = 0$

(…nous gauge)

$\frac{T}{a^2} - \beta$

C_1

$\int \frac{T}{a^2} d\tau + C_2$

Comoving Gauge:

→ frame in which matter is at rest. $u^i = 0$, $g_{0i} = 0$

$$\boxed{\Psi = 0\ ,\ \beta = 0}$$

→ For simple matter systems eqns. can be solved analytically.

→ there is still residual freedom left.

$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)T$

$\tilde{\pi} = \pi - \dot{P}_0 T$

$\tilde{\sigma} = \sigma\ ,\ \tilde{\Gamma} = \Gamma$

$\tilde{\Psi} = \Psi + (\mathcal{E}_0 + P_0)T$

$\tilde{\varphi} = \varphi - HT$

$\tilde{\gamma} = \gamma - \lambda$

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$\tilde{\chi} = \chi - T$

$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$

$\tilde{\alpha} = \alpha - \dot{T}$

$\Phi_H = \varphi - H\chi\ ;\ \Phi_A = \alpha - \dot{\chi}$

rms

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$= 0$, $\tilde{\beta} = 0$

onous gauge)

α

$\frac{T}{a^2} - \beta$

C_1

$\int \frac{T}{a^2} d\tau + C_2$

Comoving Gauge:

→ frame in which matter is at rest) $u^i = 0$, $g_{0i} = 0$

$$\boxed{\Psi = 0 \ , \ \beta = 0}$$

→ For simple matter systems eqns. can be solved analytically.

→ there is still a λ residual freedom left.

$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)T$

$\tilde{\pi} = \pi - \dot{P}_0 T$

$\tilde{\sigma} = \sigma$, $\tilde{\Gamma} = \Gamma$

$\tilde{\Psi} = \Psi + (\mathcal{E}_0 + P_0)T$

$\tilde{\varphi} = \varphi - HT$

$\tilde{\gamma} = \gamma - \lambda$

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$\tilde{\chi} = \chi - T$

$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$

$\tilde{\alpha} = \alpha - \dot{T}$

$\Phi_H = \varphi - H\chi$; $\Phi_A = \alpha - \dot{\chi}$

Comoving Gauge

→ frame in which matter is at rest. $u^i = 0$, $g_{0i} = 0$

$$\Psi = 0 \ , \ \beta = 0$$

→ For simple matter systems eqns. can be solved analytically.

→ there is still a residual freedom left.

→ $T^i_0 = 0$

$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0)T$

$\tilde{\pi} = \pi - \dot{P}_0 T$

$\tilde{\sigma} = \sigma \ , \ \tilde{\Gamma} = \Gamma$

$\tilde{\Psi} = \Psi + (\mathcal{E}_0 + P_0)T$

$\tilde{\varphi} = \varphi - HT$

$\tilde{\gamma} = \gamma - \lambda$

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$\tilde{\chi} = \chi - T$

$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$

$\tilde{\alpha} = \alpha - \dot{T}$

$\Phi_H = \varphi - H\chi, \ \Phi_A = \alpha - \dot{\chi}$

ms

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$= 0, \ \tilde{\beta} = 0$

onous gauge)

α

$\frac{T}{a^2} - \beta$

C_1

$\int \frac{T}{a^2} d\tau + C_2$

...doms

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$\tilde{\lambda} = 0 , \tilde{\beta} = 0$

(...chronous gauge)

$= \alpha$

$+ C_1$

$+ \int \frac{T}{a^2} d\tau + C_2$

Comoving Gauge

→ frame in which matter is at rest. $u^i = 0$, $g_{0i} = 0$

$$\boxed{\psi = 0 , \beta = 0}$$

→ For simple matter systems eqns. can be solved analytically

→ there is still a residual freedom left.

→ $T^i_0 = 0$

$\tilde{\mathcal{E}} = \mathcal{E} + 3H(\mathcal{E}_0 + P_0) T$

$\tilde{\pi} = \pi - \dot{P}_0 T$

$\tilde{\sigma} = \sigma , \tilde{\Gamma} = \Gamma$

$\tilde{\psi} = \psi + (\mathcal{E}_0 + P_0) T$

$\tilde{\varphi} = \varphi - HT$

$\tilde{\gamma} = \gamma - \lambda$

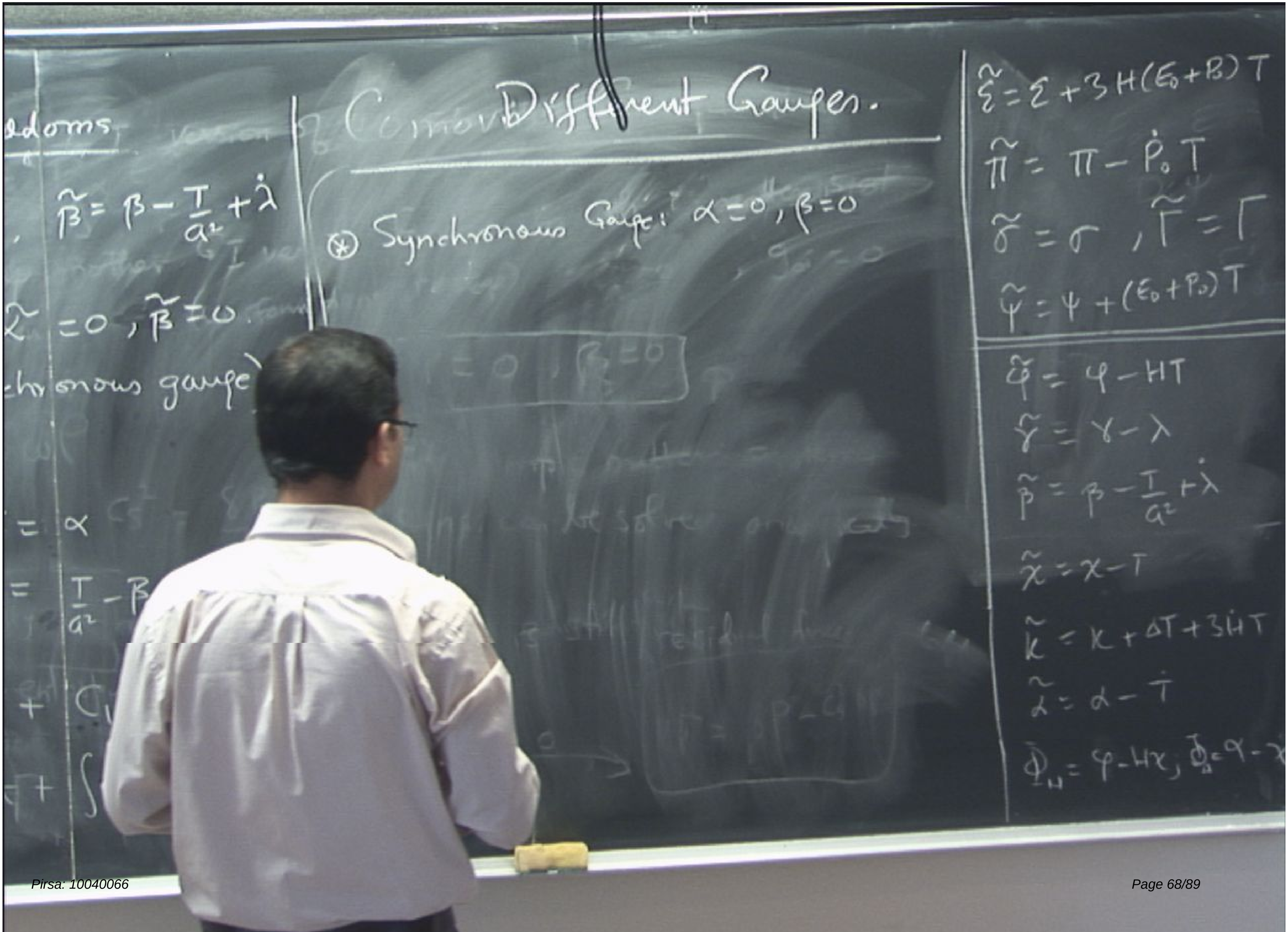
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

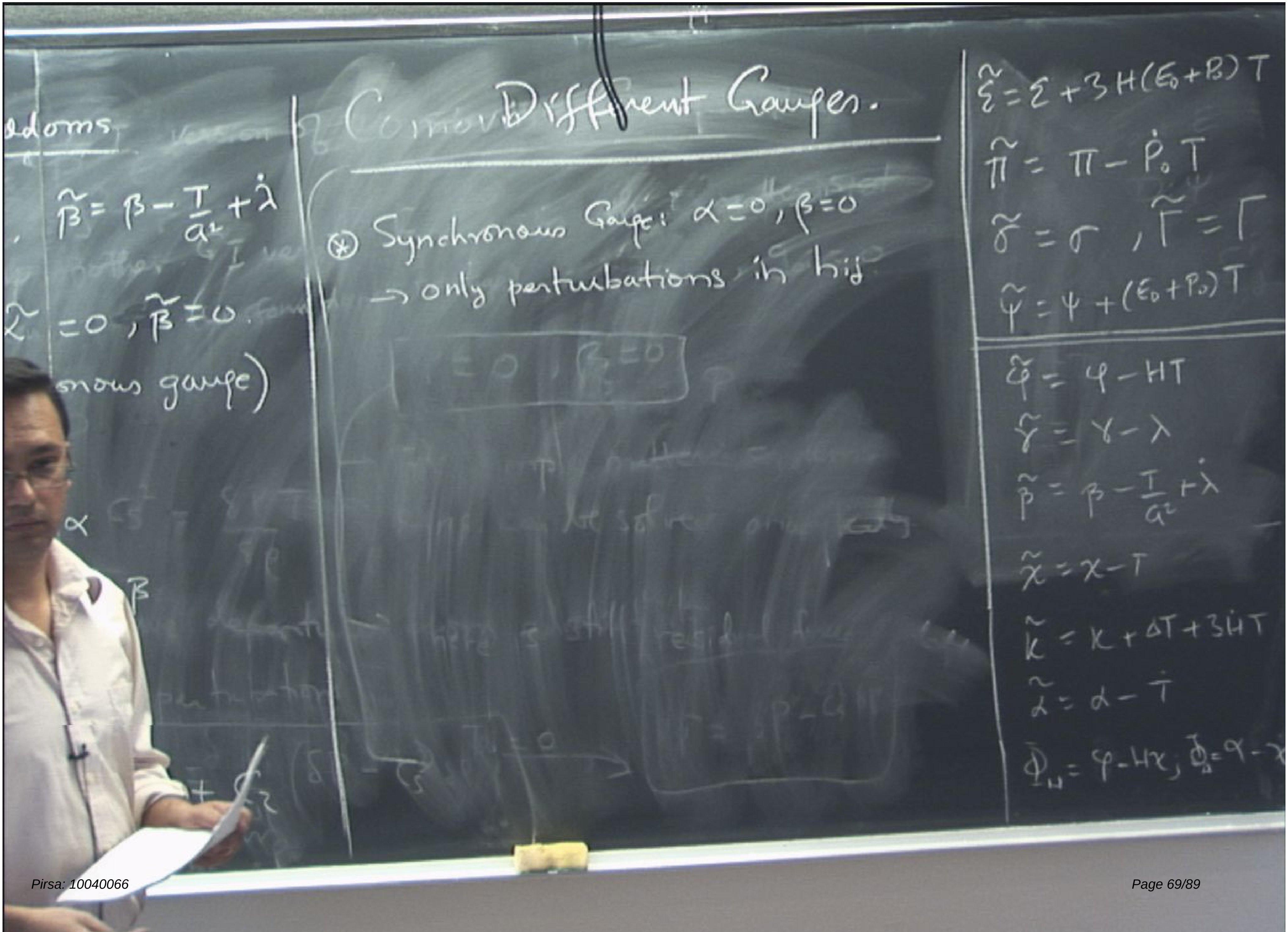
$\tilde{\chi} = \chi - T$

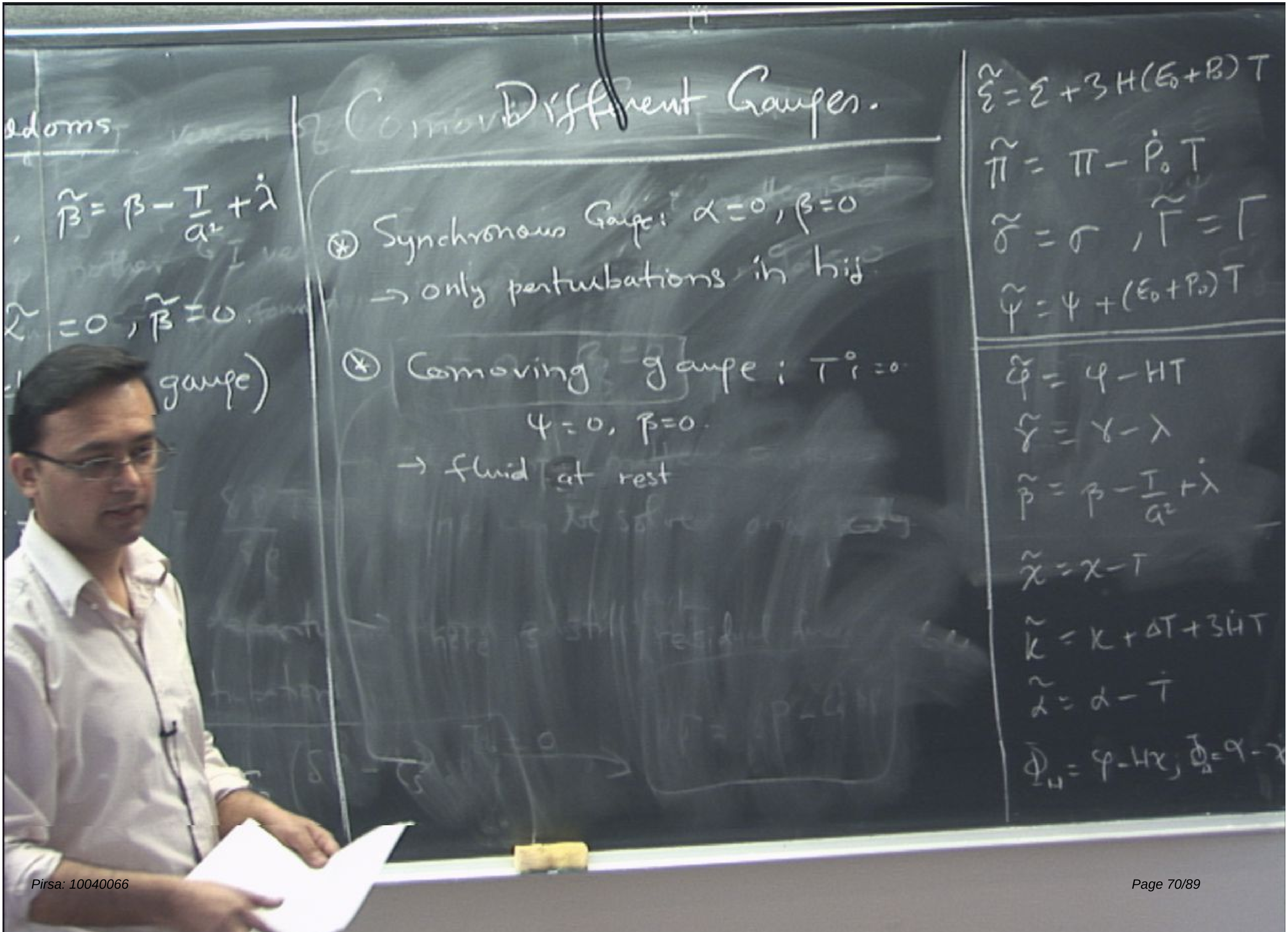
$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H} T$

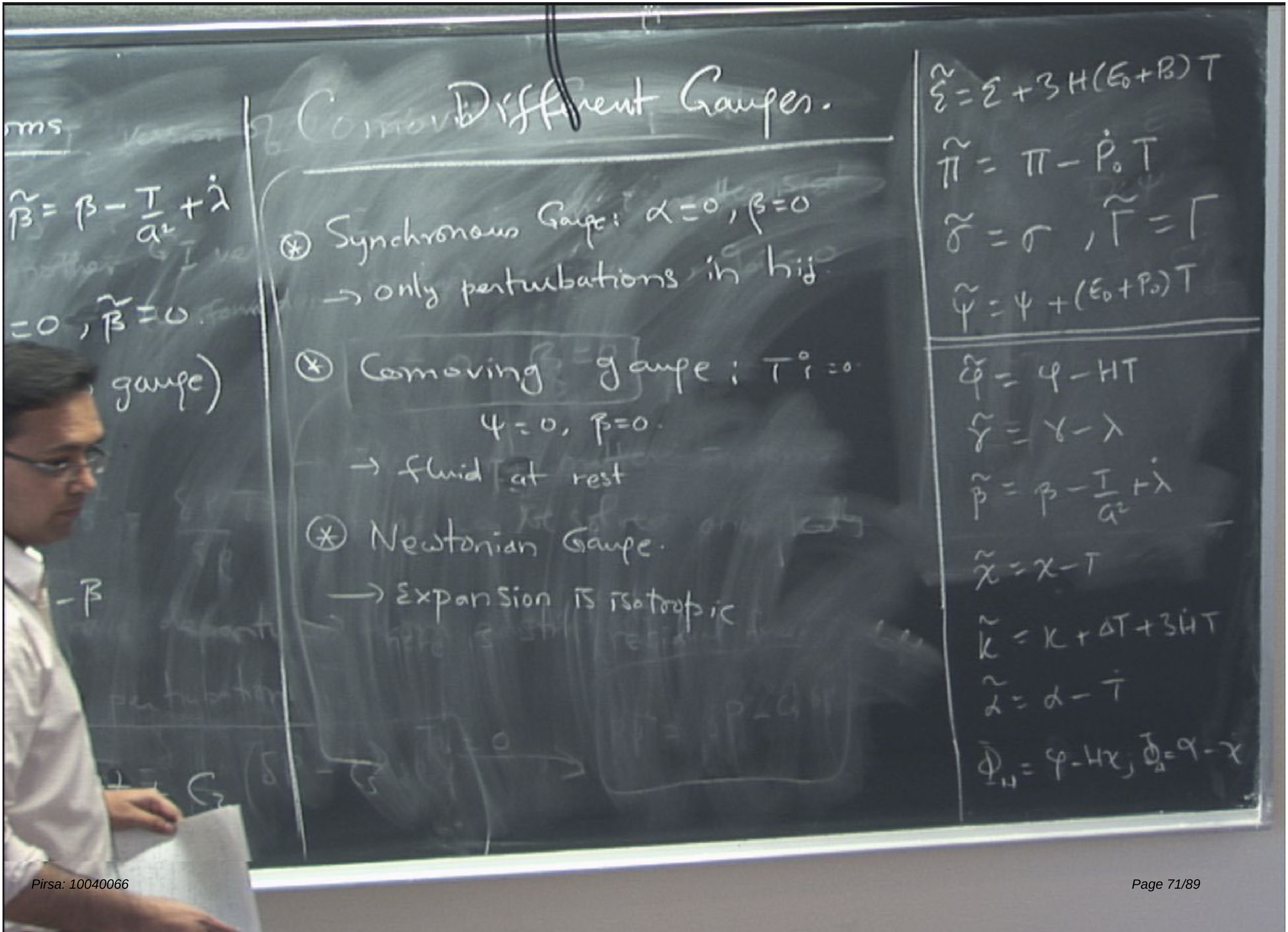
$\tilde{\alpha} = \alpha - \dot{T}$

$\Phi_H = \varphi - H\chi , \Phi_A = \alpha - \dot{\chi}$


Different Gauges.
⊛ Synchronous Gauge: α = 0, β = 0
β̃ = β − T/a² + λ̇
α̃ = 0, β̃ = 0
chronous gauge
ε̃ = ε + 3H(ε₀ + P₀)T
π̃ = π − Ṗ₀T
σ̃ = σ, Γ̃ = Γ
ψ̃ = ψ + (ε₀ + P₀)T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
Φ_H = φ − Hχ, Φ_A = α − χ̇


Different Gauges.
⊛ Synchronous Gauge: α = 0, β = 0
→ only perturbations in hij
β̃ = β − T/a² + λ̇
= 0 , β̃ = 0
synchronous gauge)
ε̃ = ε + 3H(ε0 + P0) T
π̃ = π − Ṗ0 T
σ̃ = σ , Γ̃ = Γ
ψ̃ = ψ + (ε0 + P0) T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
ΦH = φ − Hχ, ΦA = α − χ̇


Different Gauges.
Synchronous Gauge: α = 0, β = 0
→ only perturbations in hij
Comoving gauge: Tⁱ₀ = 0
ψ = 0, β = 0.
→ fluid at rest
Σ̃ = Σ + 3H(E₀ + P₀)T
π̃ = π − Ṗ₀ T
σ̃ = σ , Γ̃ = Γ
ψ̃ = ψ + (E₀ + P₀)T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
Φ_H = φ − Hχ, Φ_A = α − χ̇
β̃ = β − T/a² + λ̇
α̃ = 0 , β̃ = 0
gauge)
edoms


Different Gauges.
⊛ Synchronous Gauge: α=0, β=0
→ only perturbations in hij
⊛ Comoving gauge: T^0_i = 0
ψ=0, β=0
→ fluid at rest
⊛ Newtonian Gauge.
→ Expansion is isotropic
ε̃ = ε + 3H(ε0+P0)T
π̃ = π − Ṗ0 T
σ̃ = σ, Γ̃ = Γ
ψ̃ = ψ + (ε0+P0)T
φ̃ = φ − HT
γ̃ = γ − λ
β̃ = β − T/a² + λ̇
χ̃ = χ − T
κ̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
ΦH = φ − Hχ; ΦA = α − χ̇
β̃ = β − T/a² + λ̇
=0, β̃ = 0
gauge)
−β

Choice of Gauge.

Longitudinal / Zero Shear / Newtonian gauge

$\chi = 0 \Rightarrow \frac{a^2}{N_0}(\dot{\gamma} + \beta) = 0$

$\dot{\gamma} = 0, \quad \beta = 0$

(Residual f

$\tilde{\alpha} = \alpha -$

Let us take

(we go to Syn

$\tilde{\alpha} = 0 \Rightarrow \dot{T}$

$\tilde{\beta} = 0 \Rightarrow \dot{\lambda}$

$T = \int \alpha \, dt$

$\lambda = -\int \beta \, dt$

Choice of Gauge.

Longitudinal / Zero Shear / Newtonian gauge

$\chi = 0 \Rightarrow \frac{a^2}{N_0}(\dot{\gamma} + \beta) = 0$

$\dot{\gamma} = 0, \quad \beta = 0$

→ There is no residual freedom.

e.g. $T = \chi$

Residual f

$\tilde{\alpha} = \alpha -$

Let us take

(we go to Syn

$\tilde{\alpha} = 0 \Rightarrow \dot{T}$

$\tilde{\beta} = 0 \Rightarrow \dot{\lambda}$

$T = \int \alpha \, dt$

$\lambda = -\int \beta \, dt$

Choice of Gauge.

Longitudinal / Zero Shear / Newtonian gauge

$\chi = 0 \Rightarrow \frac{a^2}{N_0}(\dot{\gamma} + \beta) = 0$

Residual f

$\tilde{\alpha} = \alpha -$

Let us take

(we go to Syn

$\tilde{\alpha} = 0 \Rightarrow$

$\tilde{\beta} = 0 \Rightarrow \dot{\lambda}$

$T = \int \alpha \, d$

$\lambda = -\int \beta \, d$

...rms

$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$= 0$, $\tilde{\beta} = 0$

...onous gauge)

α

$\frac{T}{a^2} - \beta$

C_1

$\int \frac{T}{a^2} d\tau + C_2$

Different Gauges.

⊛ Synchronous Gauge: $\alpha = 0$, $\beta = 0$

→ only perturbations in h_{ij}

⊛ Comoving gauge: $T^0_i = 0$

$\psi = 0$, $\beta = 0$

→ fluid at rest

⊛ Newtonian Gauge.

⟶ Expansion is isotropic

⟶ No residual d.o.f

$\tilde{\varepsilon} = \varepsilon + 3H(\varepsilon_0 + P_0)T$

$\tilde{\pi} = \pi - \dot{P}_0 T$

$\tilde{\sigma} = \sigma$, $\tilde{\Gamma} = \Gamma$

$\tilde{\psi} = \psi + (\varepsilon_0 + P_0)T$

$\tilde{\varphi} = \varphi - HT$

$\tilde{\gamma} = \gamma - \lambda$

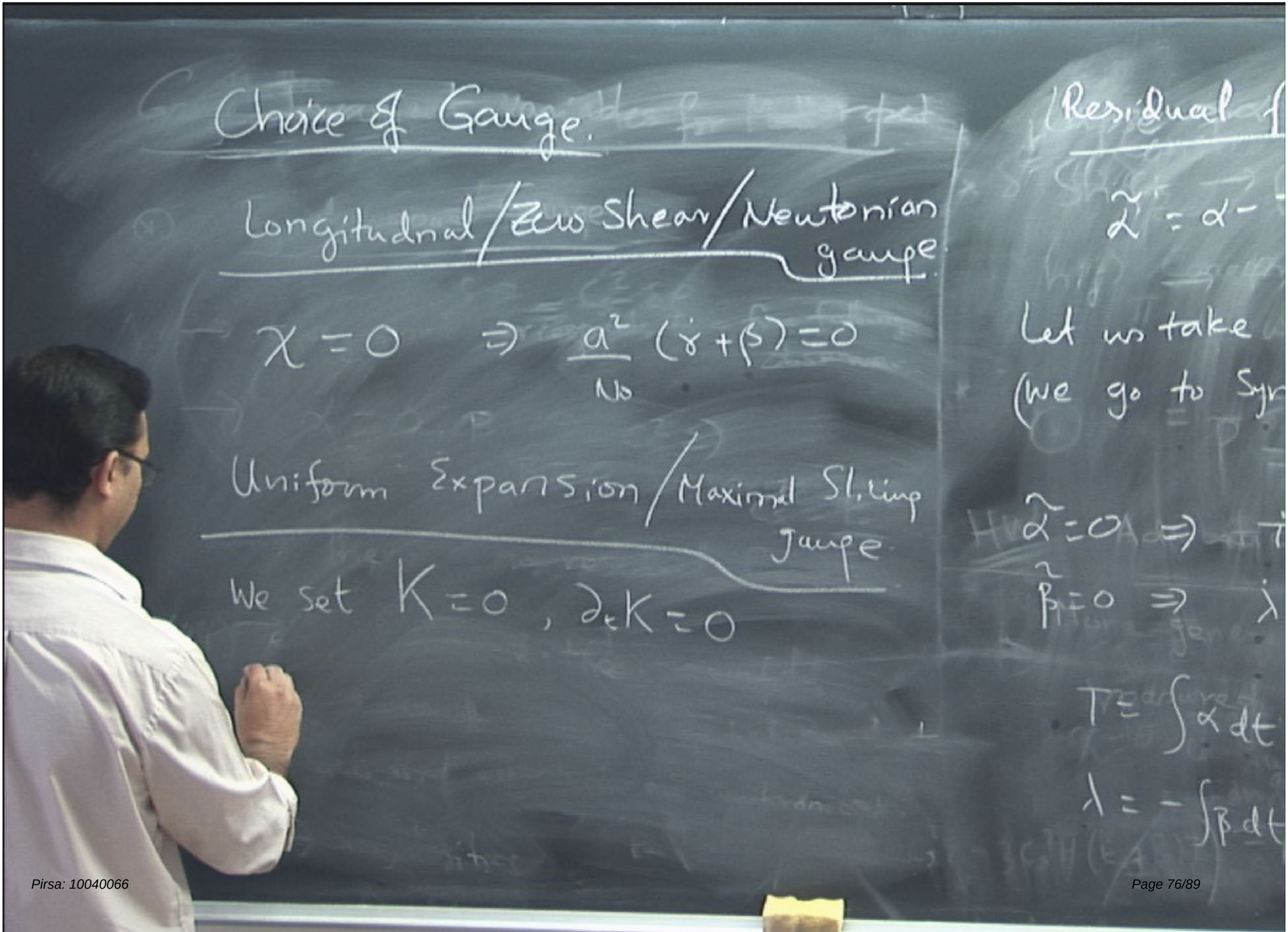
$\tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$

$\tilde{\chi} = \chi - T$

$\tilde{\kappa} = \kappa + \Delta T + 3\dot{H}T$

$\tilde{\alpha} = \alpha - \dot{T}$

$\Phi_H = \varphi - H\chi$; $\Phi_A = \alpha - \dot{\chi}$


Choice of Gauge.
Longitudinal / Zero Shear / Newtonian gauge
$\chi = 0 \Rightarrow \frac{a^2}{N_0}(\dot{\gamma} + \beta) = 0$
Uniform Expansion / Maximal Slicing gauge
We set $K = 0$, $\partial_t K = 0$
Residual
$\tilde{\alpha} = \alpha -$
Let us take
(we go to Syn
$\tilde{\alpha} = 0 \Rightarrow$
$\tilde{\beta} = 0 \Rightarrow \dot{\lambda}$
$T = \int \alpha \, dt$
$\lambda = -\int \beta \, dt$

Choice of Gauge.

Longitudinal / Zero Shear / Newtonian gauge

$$\chi = 0 \Rightarrow \frac{a^2}{N_0}(\dot{\gamma} + \beta) = 0$$

Uniform Expansion / Maximal Slicing gauge

We set $K = 0$, $\partial_t K = 0$

In cosmology: $K = \text{const}$, $\partial_t K = 0$

(Residual f…

$\tilde{\alpha} = \alpha -$ …

Let us take …

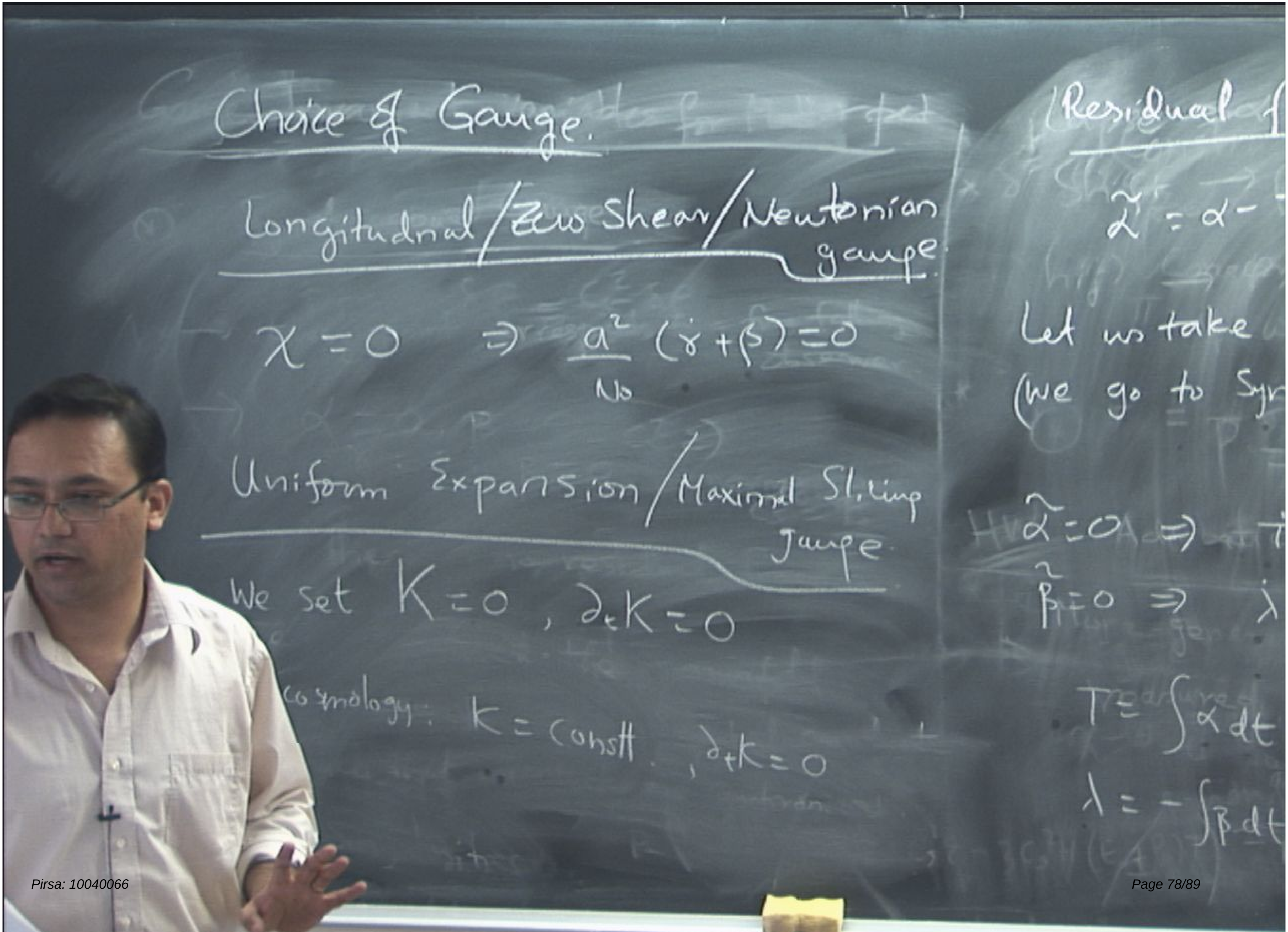
(we go to Syn…

$\tilde{\alpha} = 0 \Rightarrow$ …

$\tilde{\beta} = 0 \Rightarrow \lambda$ …

$T = \int \alpha \, dt$

$\lambda = -\int \beta \, dt$



Choice of Gauge.
Longitudinal / Zero Shear / Newtonian gauge
χ = 0 ⇒ a^2/N0 (γ̇ + β) = 0
Uniform Expansion / Maximal Slicing gauge
We set K = 0 , ∂t K = 0
cosmology: K = const , ∂t K = 0
Residual
Let us take
(we go to
T = ∫ α dt
λ = − ∫ β dt

/Newtonian gauge

$+\beta) = 0$

axial Slicing gauge

0

$\partial_t K = 0$

Residual freedoms

$$\tilde{\alpha} = \alpha - \dot{T}, \quad \tilde{\beta} = \beta - \frac{T}{a^2} + \dot{\lambda}$$

Let us take $\tilde{\alpha} = 0, \tilde{\beta} = 0$.

(we go to Synchronous gauge)

Uniform density gauge.

$\varepsilon = 0$

ψ_m is ψ

Comov. Diff

⊛ Synchronous Ga

→ only perturba

⊛ Comoving

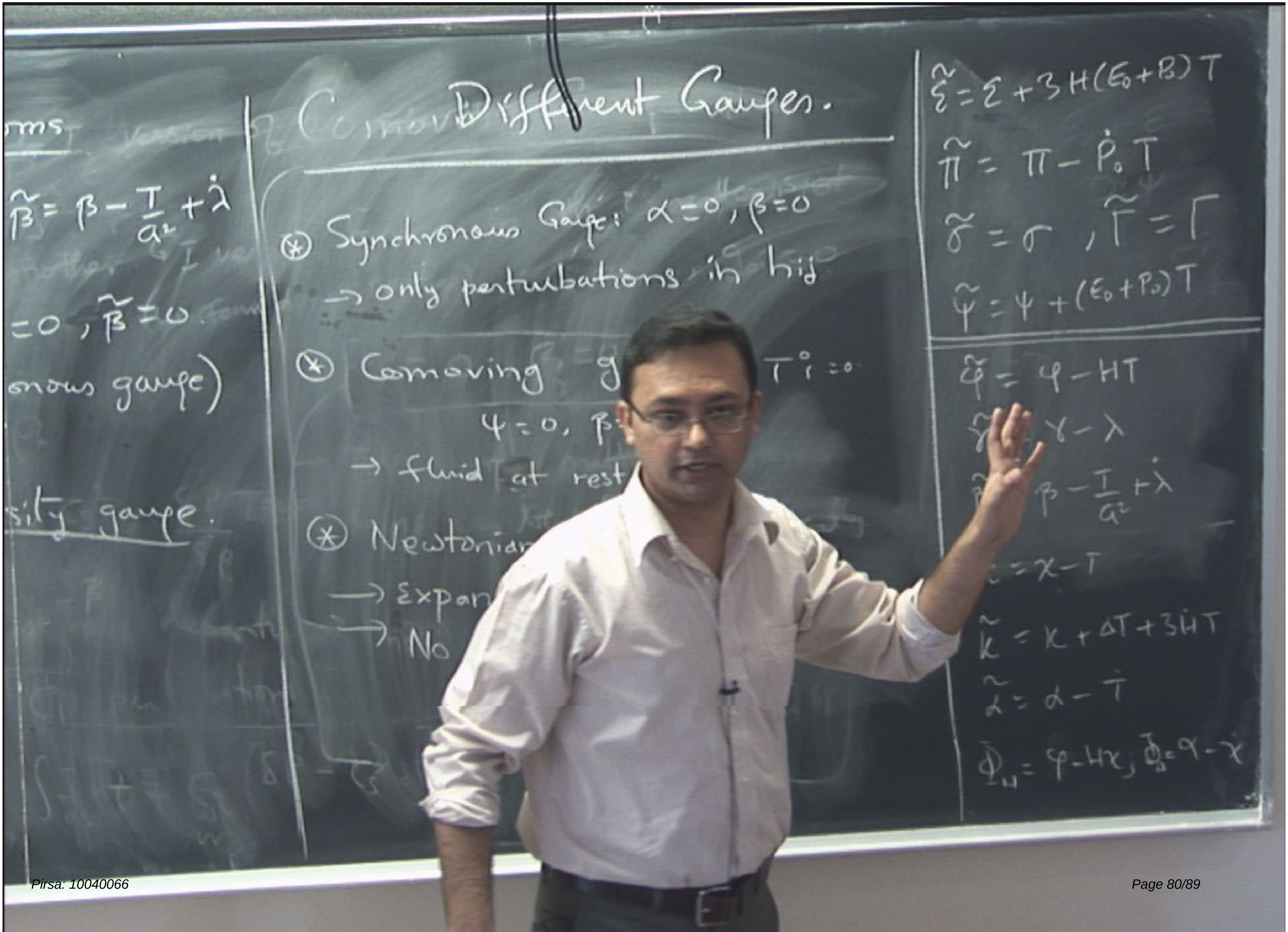
$\psi = 0,$

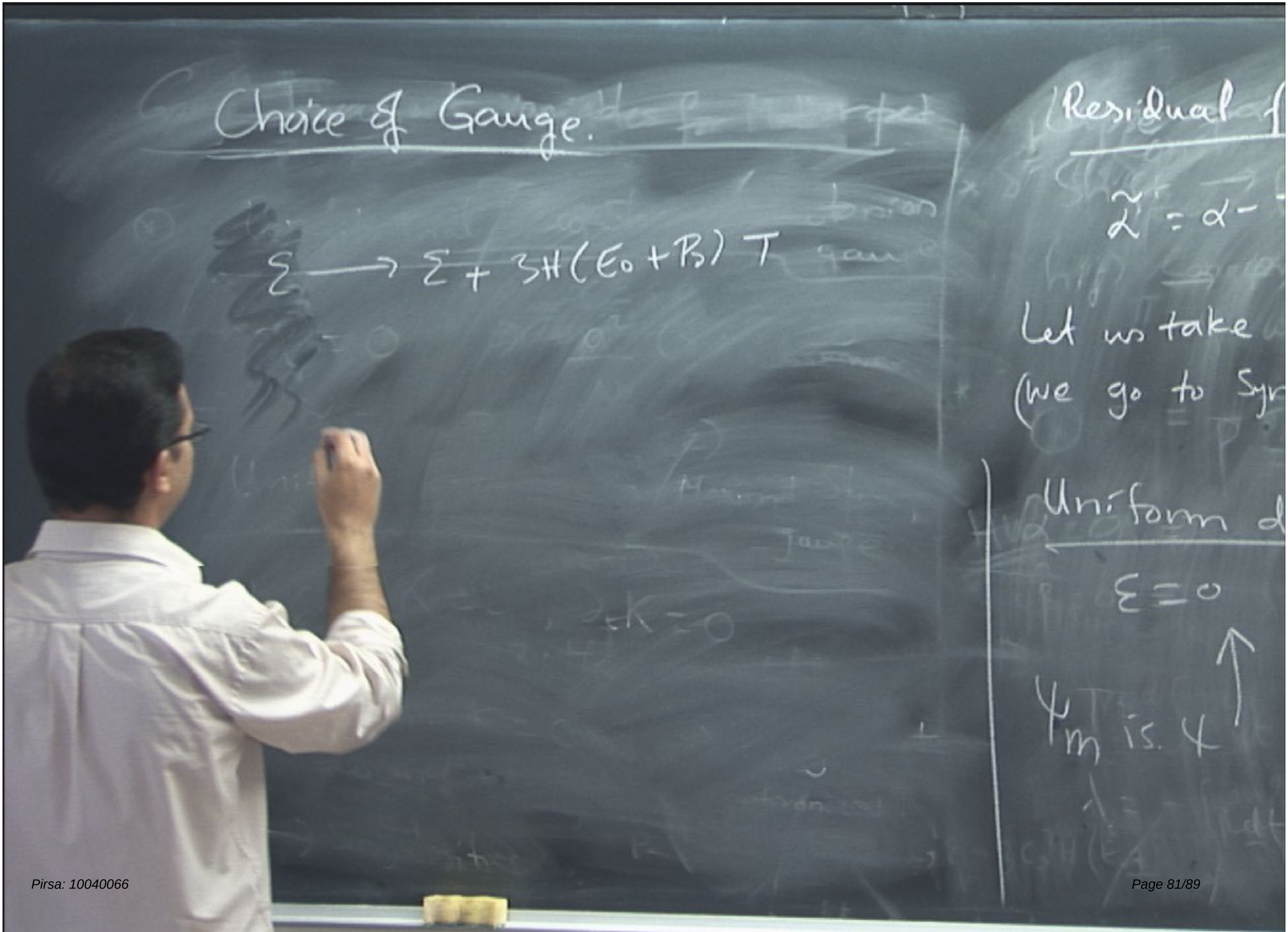
→ fluid at re

⊛ Newtonian Ga

→ Expansion is

→ No residu


Different Gauges.
Synchronous Gauge: α = 0, β = 0
→ only perturbations in hij
Comoving
ψ = 0,
→ fluid at rest
Newtonian
→ Expan
→ No
β̃ = β − T/a² + λ̇
= 0 , β̃ = 0
onous gauge)
sity gauge.
ε̃ = ε + 3H(ε0 + P0) T
π̃ = π − Ṗ0 T
σ̃ = σ , Γ̃ = Γ
ψ̃ = ψ + (ε0 + P0) T
φ̃ = φ − HT
γ − λ
β − T/a² + λ̇
= χ − T
k̃ = κ + ΔT + 3ḢT
α̃ = α − Ṫ
ΦH = φ − Hχ; ΦA = α − χ̇


Choice of Gauge.
Σ → Σ + 3H(E₀ + P₀) T
Residual
Let us take
(we go to
Uniform
ε = 0
Ψ_m is Ψ

Choice of Gauge.

$$\varepsilon \longrightarrow \varepsilon + 3H(E_0 + P_0)\, T$$

Define $\zeta = \varphi + \dfrac{\varepsilon}{3(E_0 + P_0)}$

Residual f

$\tilde{\alpha} = \alpha -$

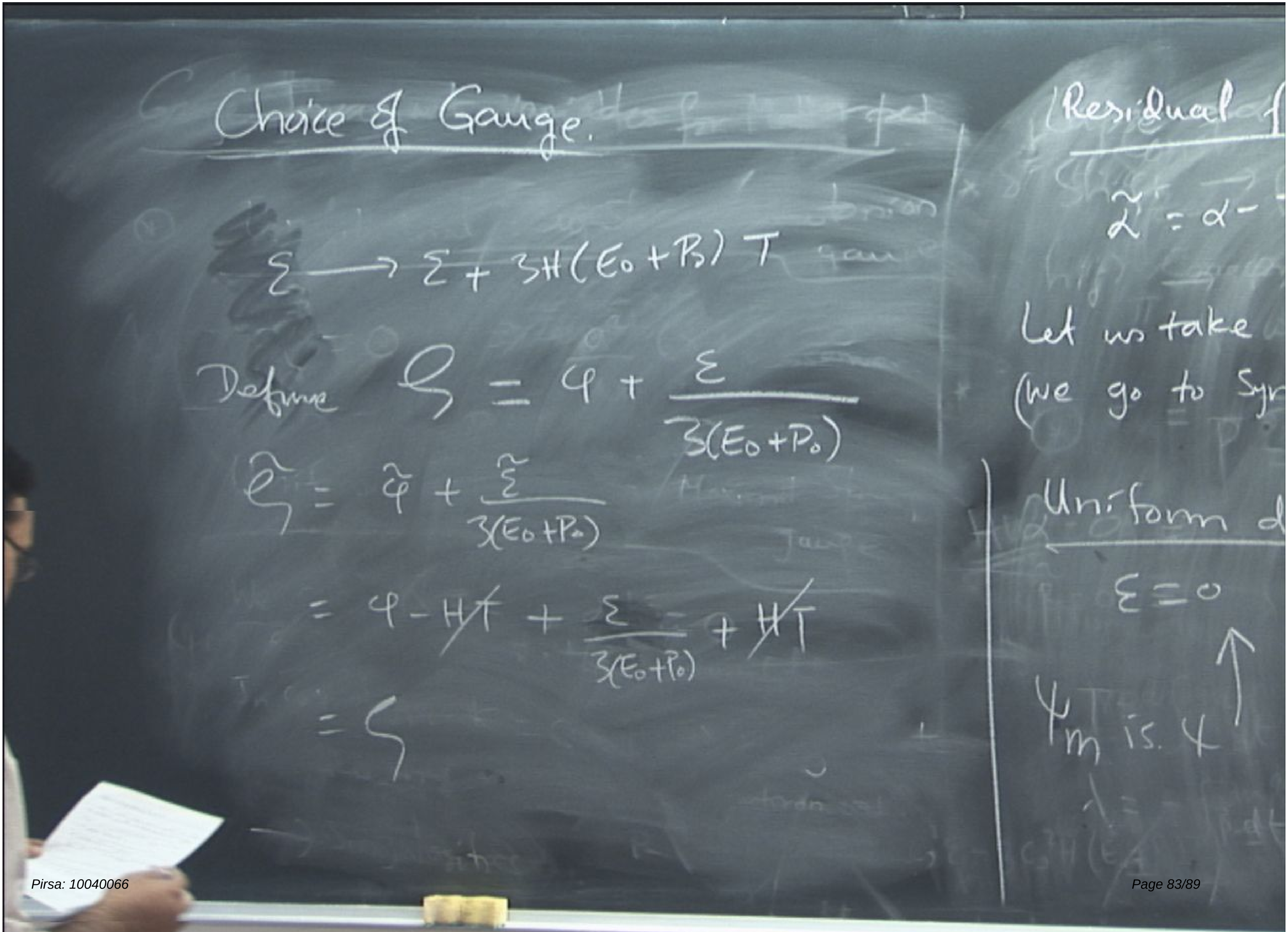
Let us take

(we go to Syn

Uniform d

$\varepsilon = 0$

ψ_m is ψ


Choice of Gauge.
$\mathcal{E} \to \mathcal{E} + 3H(E_0 + P_0)\,T$
Define $\zeta = \varphi + \frac{\mathcal{E}}{3(E_0+P_0)}$
$\tilde{\zeta} = \tilde{\varphi} + \frac{\tilde{\mathcal{E}}}{3(E_0+P_0)}$
$= \varphi - HT + \frac{\mathcal{E}}{3(E_0+P_0)} + HT$
$= \zeta$
Residual
$\tilde{\alpha} = \alpha -$
Let us take
(we go to Syn
Uniform d
$\mathcal{E} = 0$
ψ_m is ψ

$(E_0 + P_0)\,T$

$\varphi + \dfrac{\varepsilon}{3(E_0 + P_0)}$

$\dfrac{\varepsilon}{3(E_0 + P_0)} + \cancel{H}T$

(*) ζ is conserved at large scales

Comov

(*) Synchron

→ only

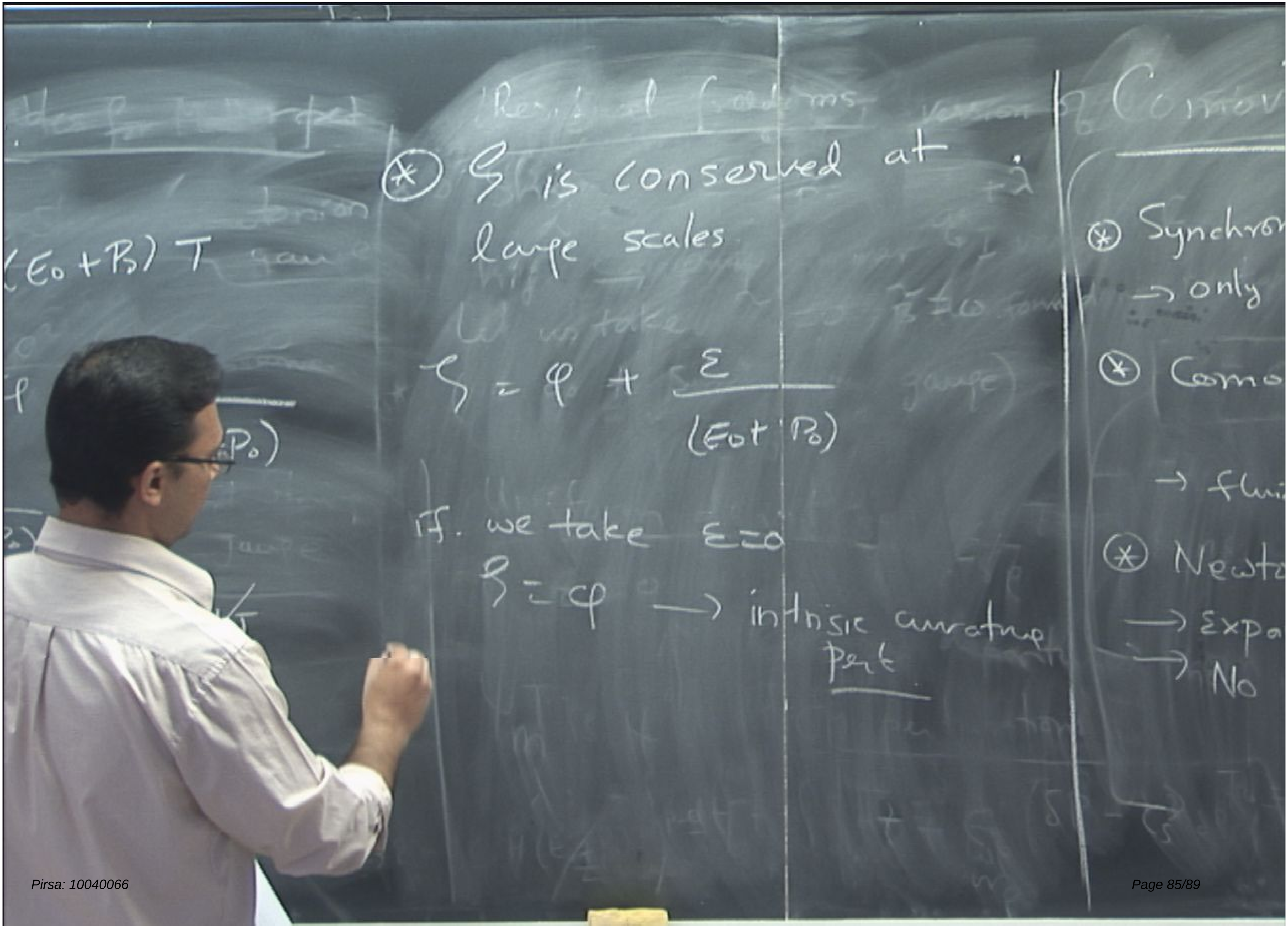
(*) Como

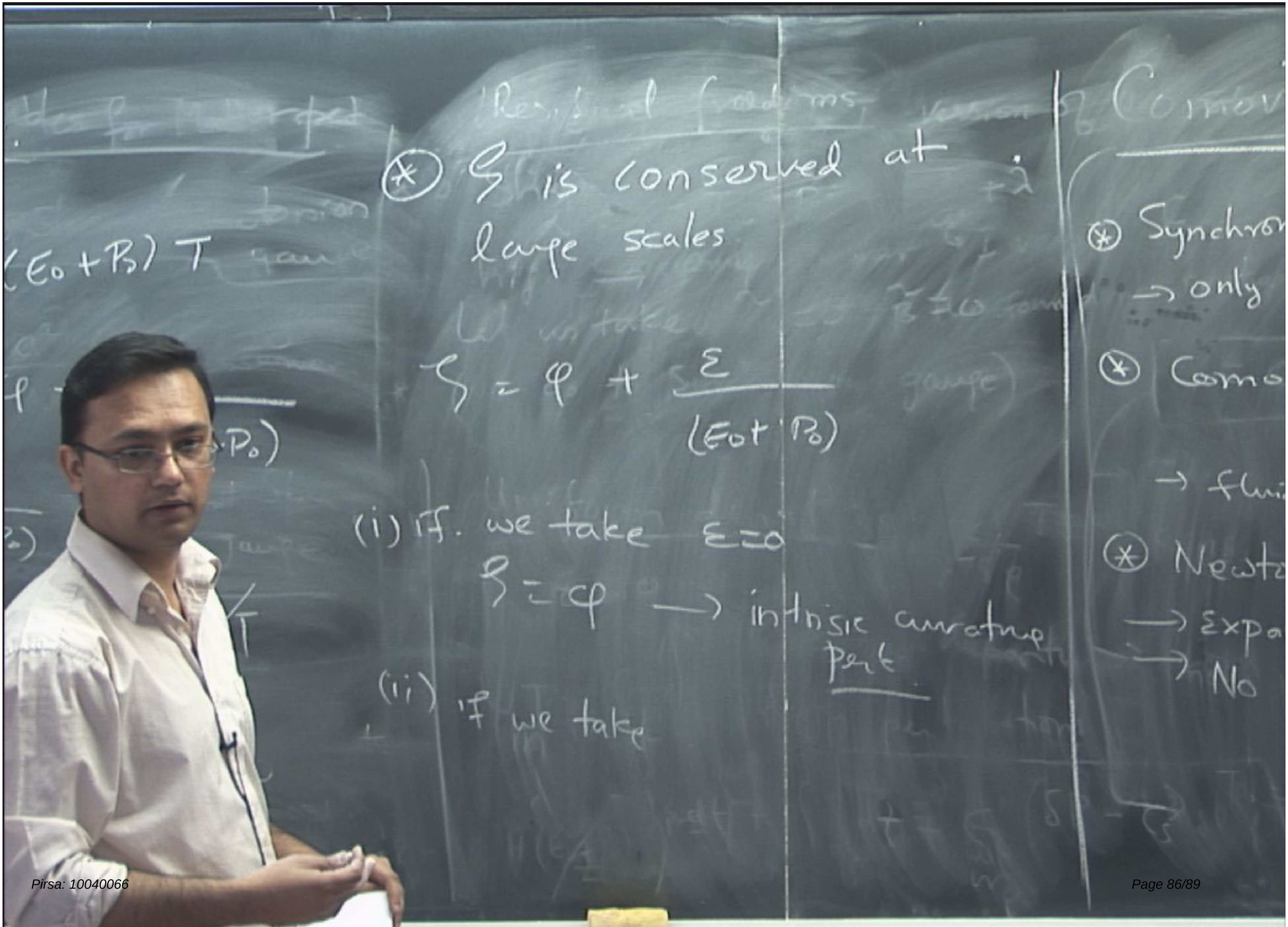
→ flui

(*) Newto

→ Expa

→ No


ζ is conserved at
large scales
ζ = φ + ε / (ε₀ + P₀)
if we take ε = 0
ζ = φ → intrinsic curvature
part
Synchron
→ only
Comov
→ flui
Newto
→ Expa
→ No
(ε₀ + P₀) T


ζ is conserved at
large scales
ζ = φ + ε / (ε₀ + P₀)
(i) if we take ε = 0
ζ = φ → intrinsic curvature part
(ii) if we take
(ε₀ + P₀) T
Synchron
→ only
Comov
→ flui
Newto
→ Expa
→ No

$(E_0 + P_0)\,T$

$\varphi + \dfrac{\mathcal{E}}{3(E_0+P_0)}$

$\dfrac{\mathcal{E}}{3(E_0+P_0)} + \not{H}T$

(*) ζ is conserved at large scales

$$\zeta = \varphi + \frac{\mathcal{E}}{(E_0 + P_0)}$$

(i) if. we take $\mathcal{E} = 0$

$\zeta = \varphi$ → intrinsic curvature part

(ii) if we take $\varphi = 0$ gauge. ie gauge in which intrinsic curvature part vanishes

then $\zeta = \dfrac{\mathcal{E}}{(E_0 + P_0)}$

Comov…

(*) Synchron…
→ only

(*) Comov…

→ flui…

(*) Newto…
→ Expa…
→ No

$(E_0+P_0)\,T$

$\varphi + \frac{\varepsilon}{3(E_0+P_0)}$

$\frac{\varepsilon}{3(E_0+P_0)} + \not{H}\,T$

⊛ ζ is conserved at large scales

$$\zeta = \varphi + \frac{\varepsilon}{(E_0+P_0)}$$

(i) if we take $\varepsilon = 0$

$\zeta = \varphi \longrightarrow$ intrinsic curvature part

(ii) if we take $\varphi = 0$ gauge i.e. gauge in which intrinsic curvature part vanishes

then $\zeta = \frac{\varepsilon}{(E_0+P_0)}$

⊛ Synchron…

→ only

⊛ Como…

→ flu…

⊛ Newto…

→ Expa…

→ No

⊛ ζ is conserved at large scales

$$\zeta = \varphi + \frac{\varepsilon}{(\varepsilon_0 + P_0)}$$

(i) if we take $\varepsilon = 0$

$\zeta = \varphi$ → intrinsic curvature pert

(ii) if we take $\varphi = 0$ gauge, i.e. gauge in which intrinsic curvature pert vanishes

then $\zeta = \frac{\varepsilon}{(\varepsilon_0 + P_0)}$

Comov Differen

⊛ Synchronous Gauge:

→ only perturbatio

⊛ Comoving ga

$\psi = 0$, $\beta = 0$

→ fluid at rest

⊛ Newtonian Gauge

→ Expansion is iso

→ No residual