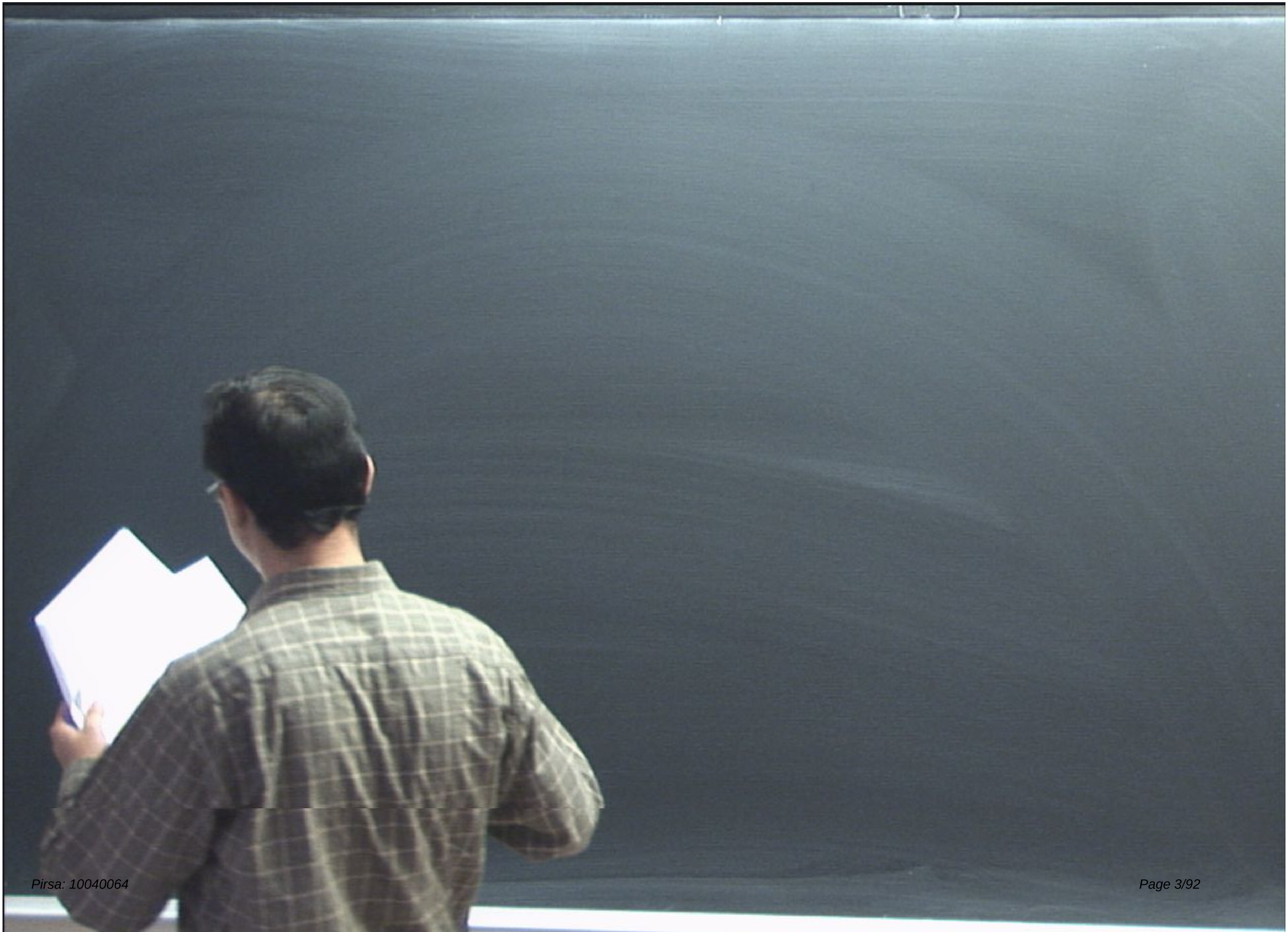
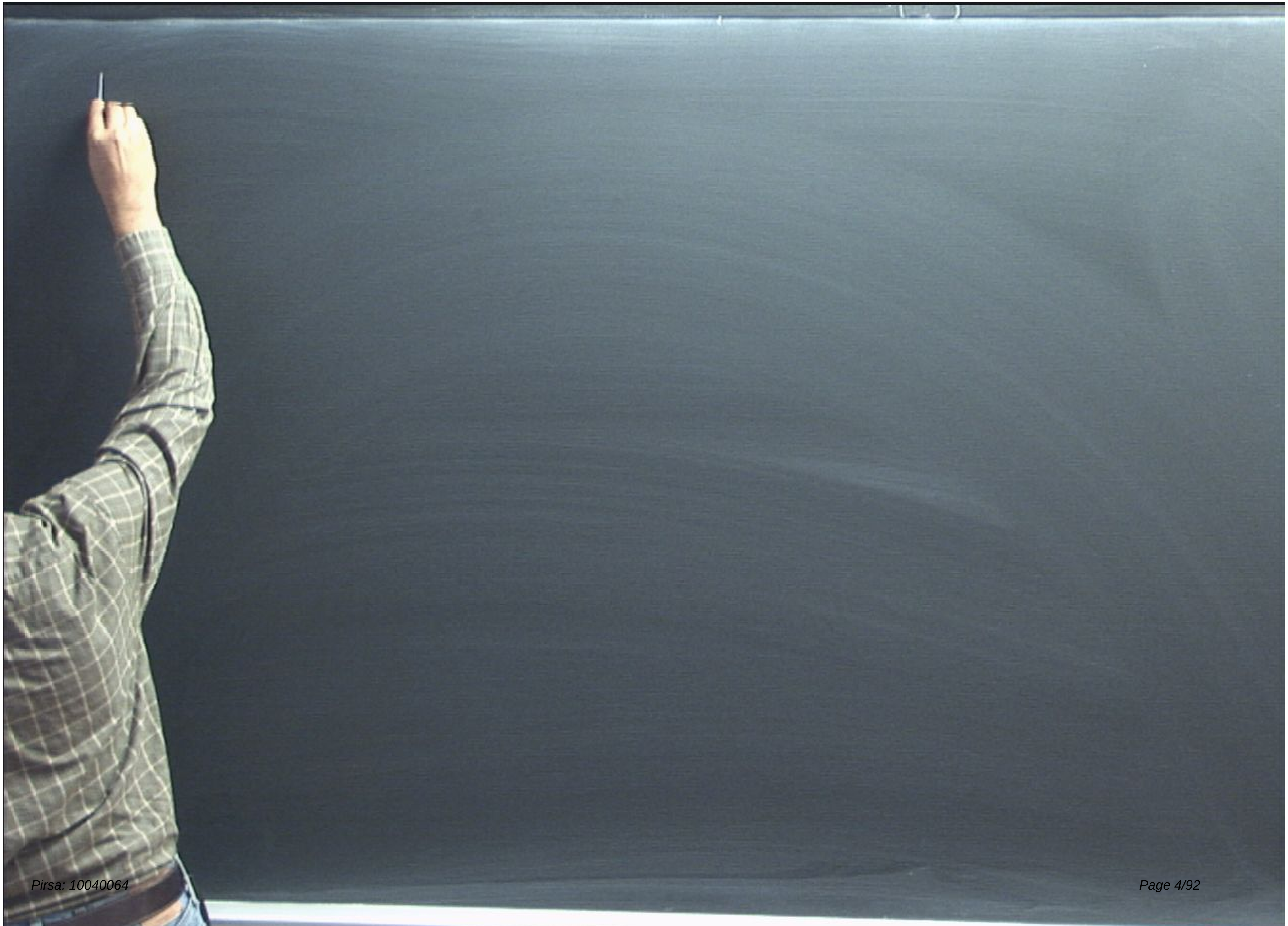
Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 7

Date: Apr 13, 2010 09:30 AM

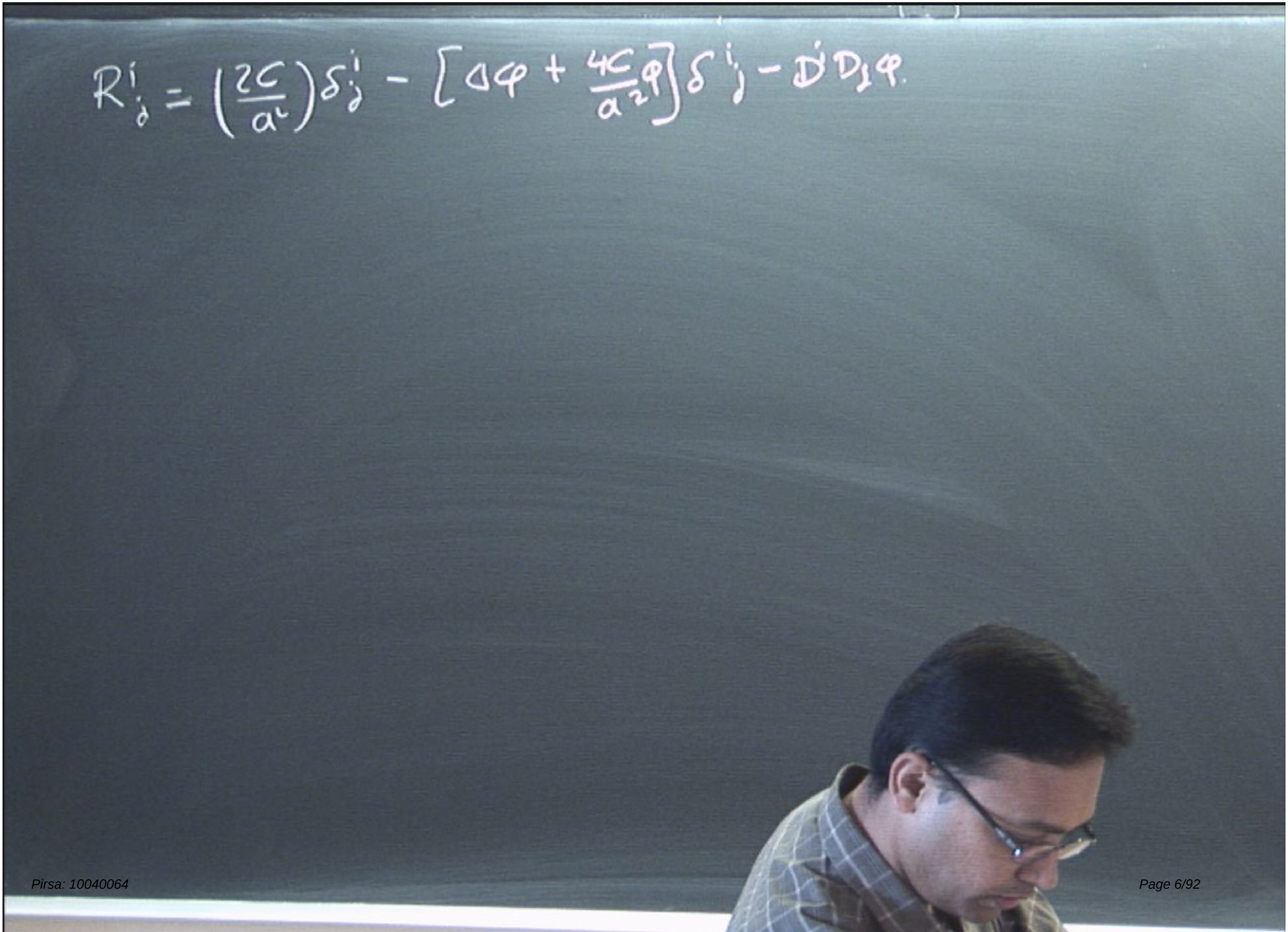
URL: http://pirsa.org/10040064

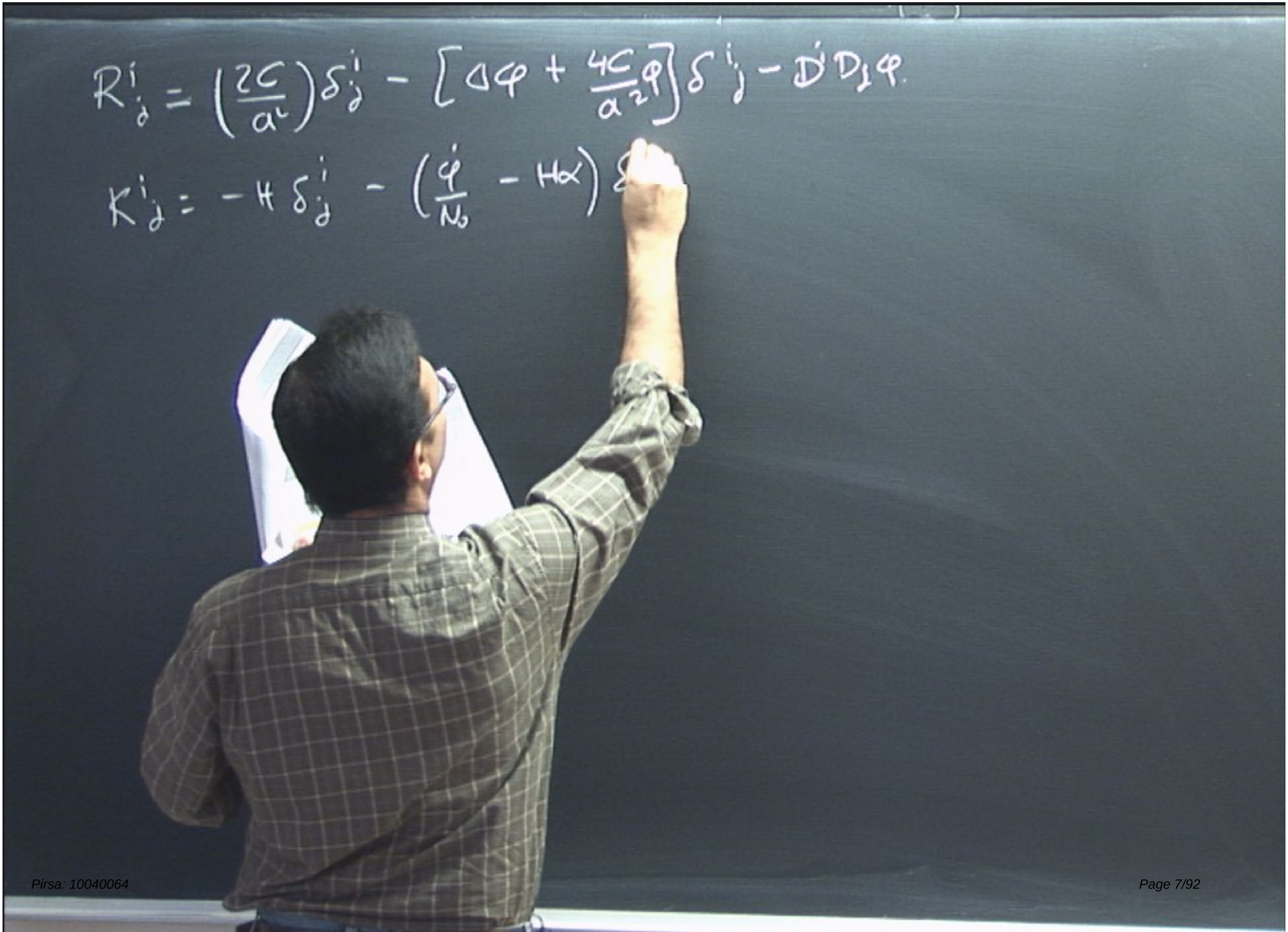
Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.





$$R^i{}_j = \left(\frac{2c}{a^2}\right)\delta^i{}_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i{}_j -$$


$R^i_j = \left(\frac{2c}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$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$K^i_j = -H\,\delta^i_j - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta$

$$R^i_{\ j} = \left(\frac{2C}{a^2}\right)\delta^i_{\ j} - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_{\ j} - D^i D_j \varphi.$$

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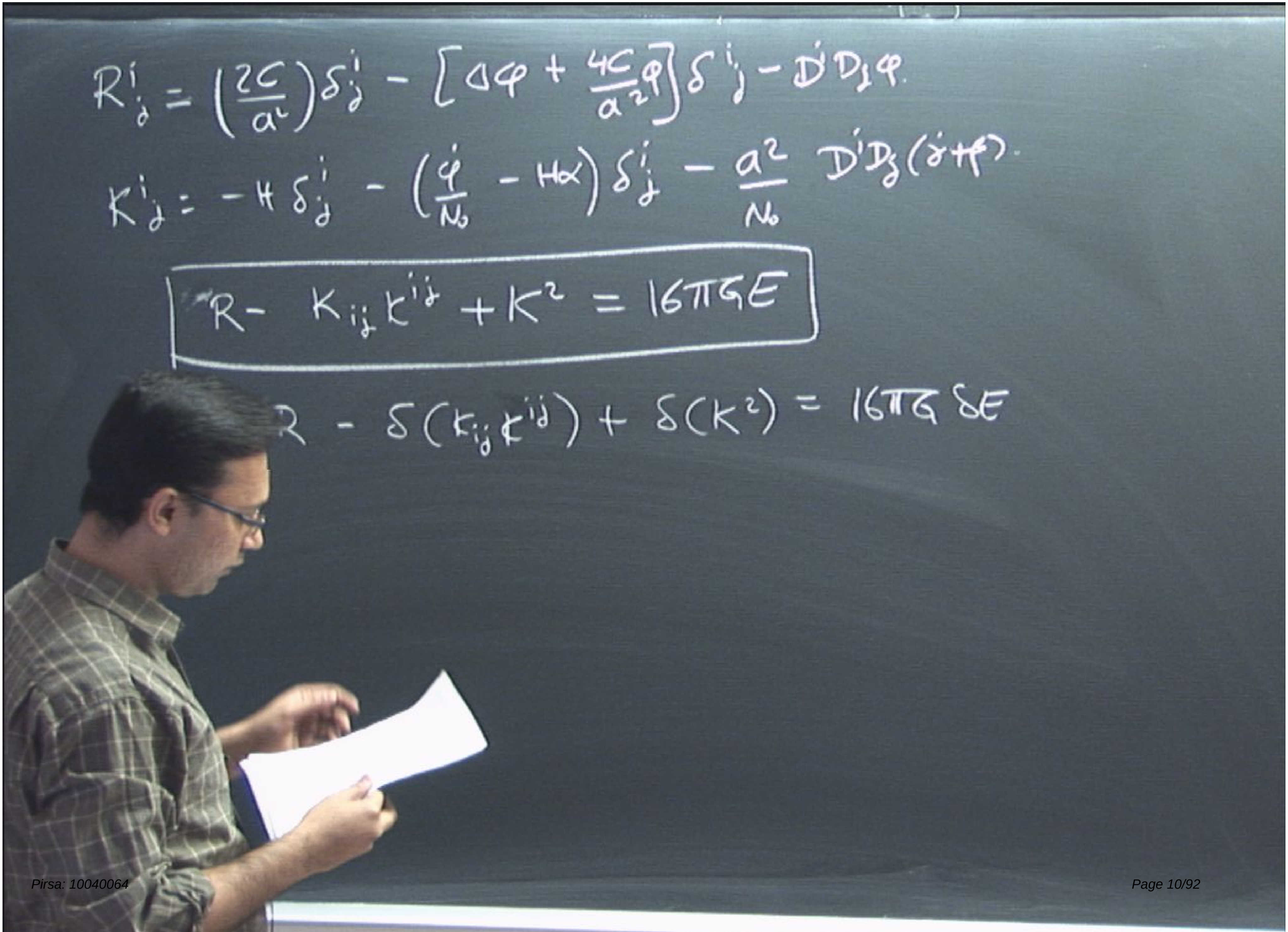
$$\boxed{{}^{(3)}R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$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$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$
$R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\, \delta E$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi .$$

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$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E$$

Trace

$$R^i_{\ j} = \left(\frac{2C}{a^2}\right)\delta^i_{\ j} - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_{\ j} - D^i D_j \varphi.$$

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Trace $\quad R = \dfrac{6C}{a^2} - 3\left(\Delta\varphi + \dfrac{4C}{a^2}\varphi\right) - \Delta\varphi$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i{}_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi.$$

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Trace

$$R = \underbrace{\frac{6C}{a^2}}_{\text{background}} \underbrace{- 3\left(\Delta\varphi + \frac{4C}{a^2}\varphi\right) - \Delta\varphi}_{\delta R}$$

$$R^i{}_j = \left(\frac{2c}{a^2}\right)\delta^i{}_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i{}_j - D^iD_j\varphi$$

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$$\boxed{\delta R = -4\,\Delta\varphi - \frac{12c}{a^2}\varphi}$$

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Trace

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$\chi = \frac{a^2}{N_0}(\dot{\gamma} + \beta)$
$k = -3($

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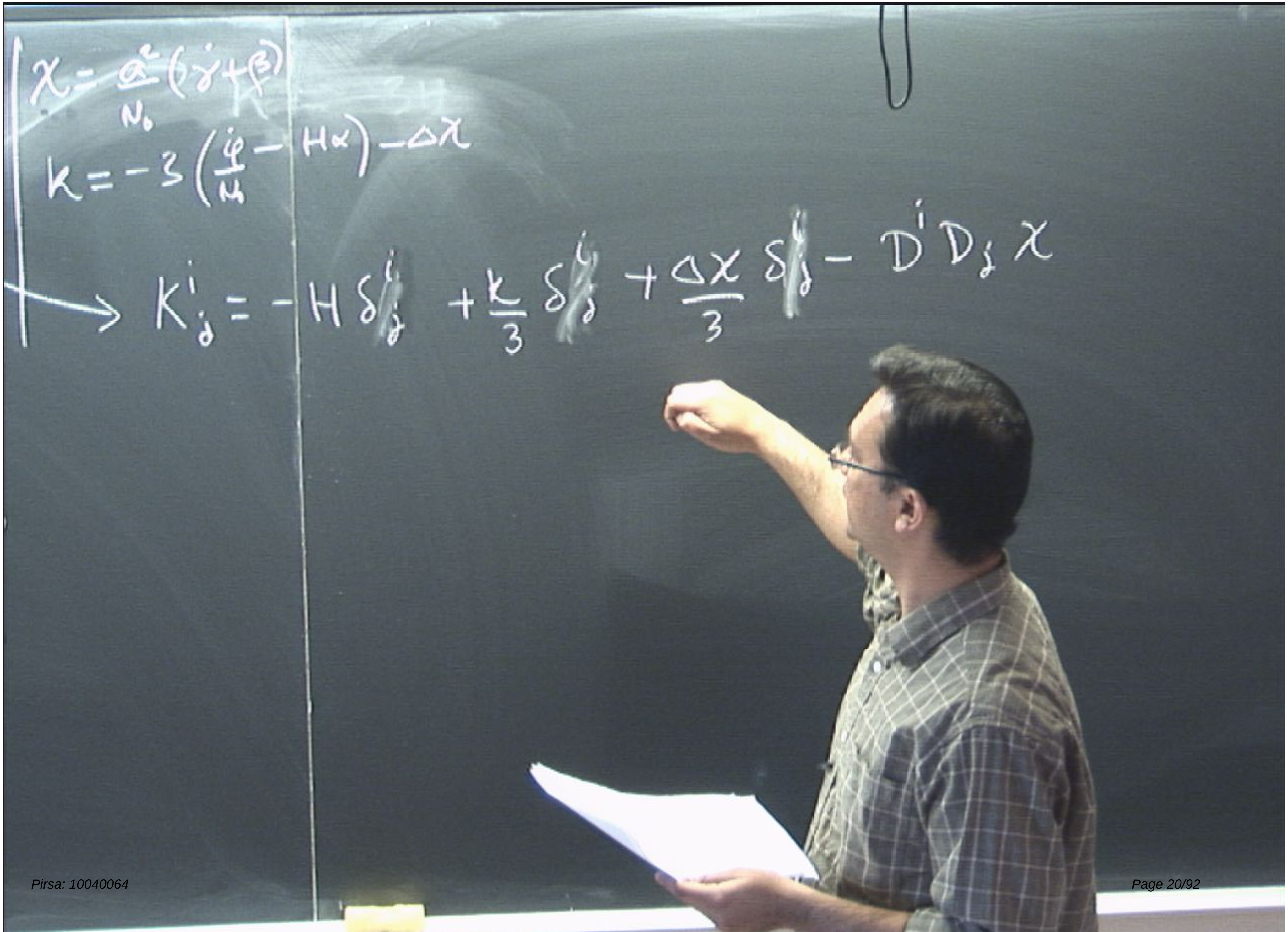
$$\rightarrow K^i_{\ j} = -H\delta_i^{\ j} + \frac{k}{3}\delta_i^{\ j}$$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$$K = -3H + k + \underbrace{\Delta\chi - \Delta\chi}_{0}$$

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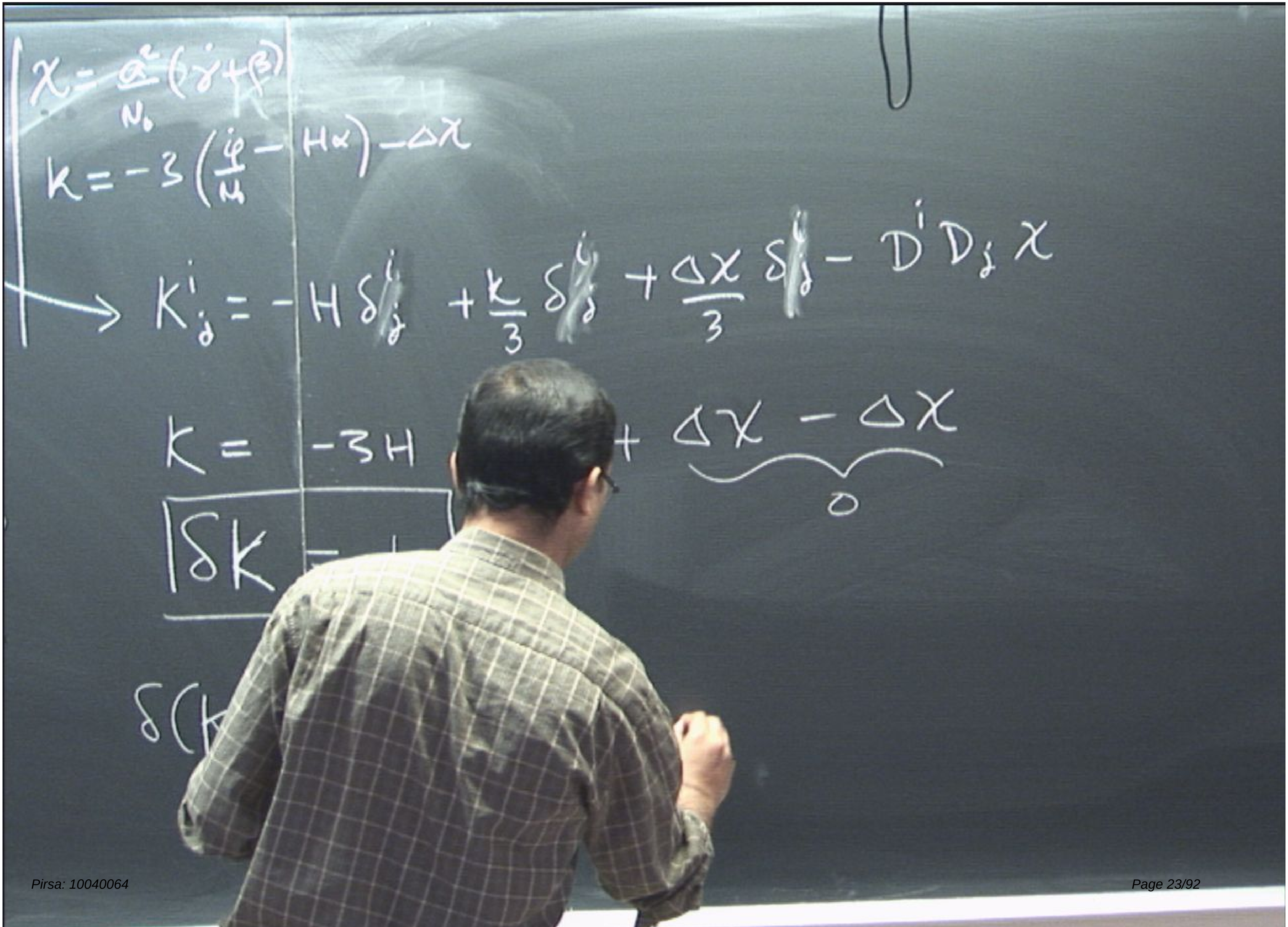
$$\boxed{\delta K = k}$$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$K = -3H$
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$\boxed{\delta K =}$
$\delta(K$

$$\chi = \frac{a^2}{N_0}(\dot{\gamma} + \beta)$$

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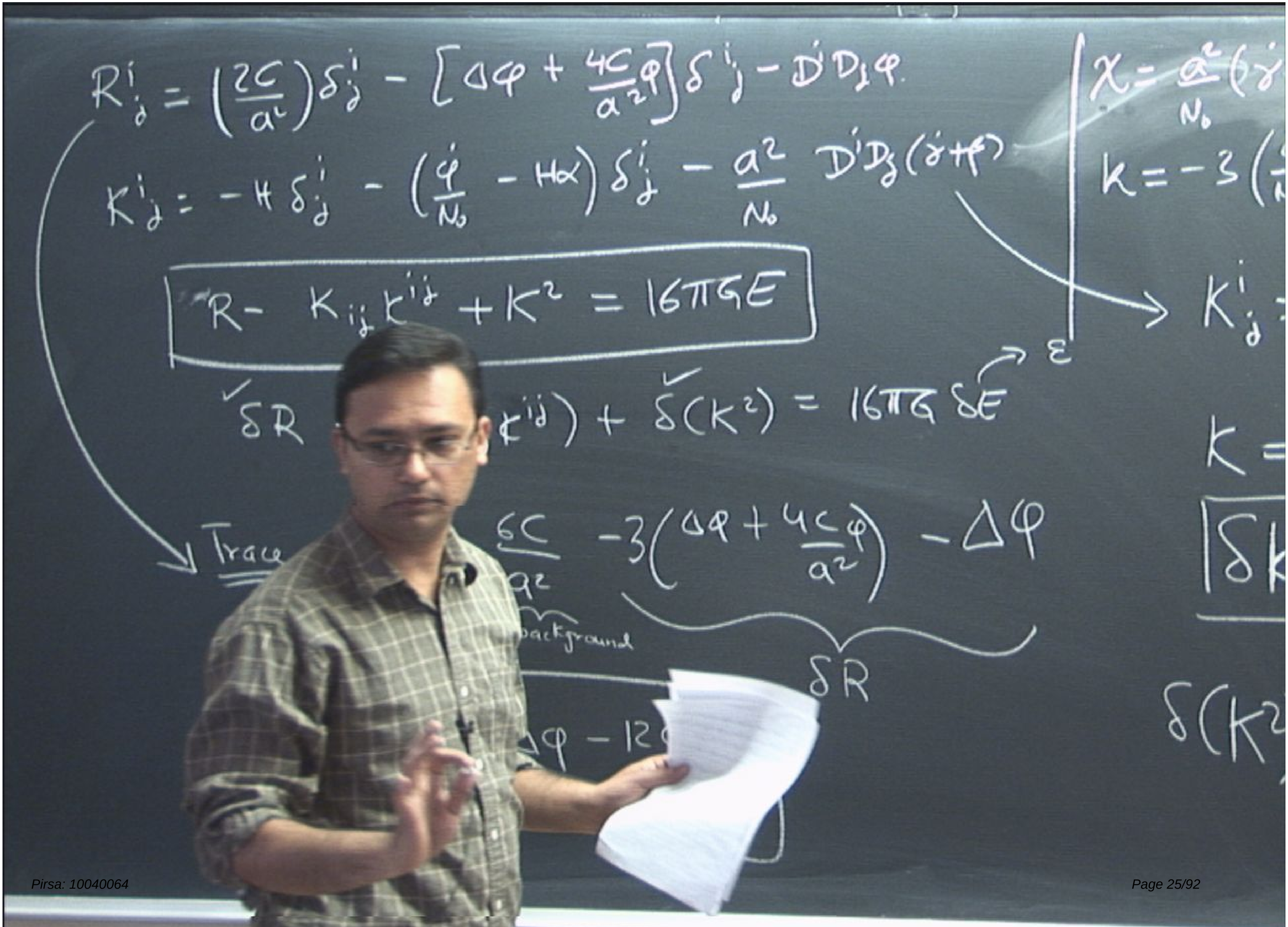
$$\boxed{\delta K = k}$$

$$\delta(K^2) \overset{(b)}{=} 2K\delta K = -6Hk$$



$R^i_j = \left(\frac{2c}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$
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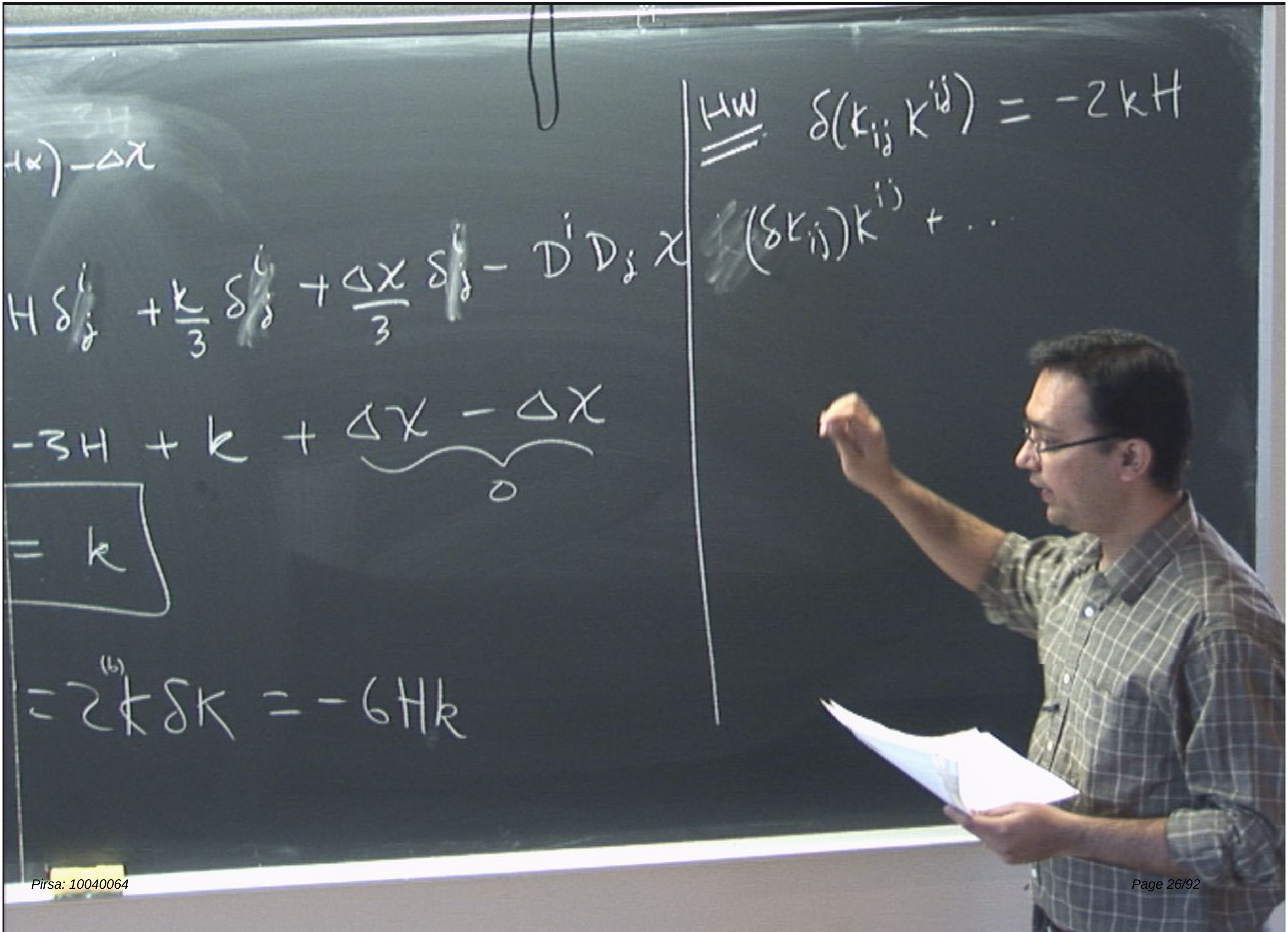
$R - K_{ij}K^{ij} + K^2 = 16\pi G E$
$\delta R \quad K^{ij}) + \delta(K^2) = 16\pi G\,\delta E$
Trace
$\frac{6c}{a^2} - 3\left(\Delta\varphi + \frac{4c}{a^2}\varphi\right) - \Delta\varphi$
background
δR
$\Delta\varphi - 12$
$\chi = \frac{a^2}{N_0}(\dot\gamma$
$K = -3($
K^i_j
$K =$
δK
$\delta(K^2$


Hα) − Δχ
H δ^i_j + k/3 δ^i_j + Δχ/3 δ^i_j − D^i D_j χ
−3H + k + Δχ − Δχ
0
= k
(b)
= 2k δK = −6Hk
HW δ(K_ij K^ij) = −2kH
(δK_ij)K^ij + . . .

$\alpha) - \Delta\chi$

$H\delta^i_j + \frac{k}{3}\delta^i_j + \frac{\Delta\chi}{3}\delta^i_j - D^i D_j \chi$

$-3H + k + \underbrace{\Delta\chi - \Delta\chi}_{0}$

$\boxed{= k}$

$= 2\overset{(b)}{k}\,\delta K = -6Hk$

$\underline{\underline{HW}}$ $\delta(K_{ij}K^{ij}) = -2kH$

$(\delta K_{ij})K^{ij} + \ldots$

$K_{ij} = (\quad)$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

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$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\boxed{\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E}$$

Trace

$$R = \underbrace{\frac{6C}{a^2}}_{\text{background}} \underbrace{- 3\left(\Delta\varphi + \frac{4C}{a^2}\varphi\right) - \Delta\varphi}_{\delta R}$$

$$\boxed{\delta R = -4\Delta\varphi - \frac{12C}{a^2}\varphi}$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma$$

$$K = -3\,($$

$$K^i{}_j =$$

$$K =$$

$$\delta K$$

$$\delta(K^2$$

$$R^i_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$$

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$$\boxed{\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E}$$

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$$\chi = \frac{a^2}{N_0}(\dot\gamma \ldots$$

$$K = -3\left(\ldots\right.$$

$$K^i_j = \ldots$$

$$K = \ldots$$

$$\delta K \ldots$$

$$\delta(K^2 \ldots$$

$$H\alpha) - \Delta\chi$$

$$H\delta^i_j + \frac{k}{3}\delta^i_j + \frac{\Delta\chi}{3}\delta^i_j - D^i D_j \chi$$

$$-3H + k + \underbrace{\Delta\chi - \ldots}_{0}$$

$$\boxed{= k}$$

$$= 2\overset{(b)}{k}\,\delta K = -($$

$$\underline{\underline{HW}}\quad \delta(K_{ij}K^{ij}) = -2kH$$

$$E = E_0 + \mathcal{E} \rightarrow \delta\rho$$

$$\rho = \rho_0 + \delta\rho$$

$H\alpha) - \Delta\chi$

$H\delta^i_j + \frac{k}{3}\delta^i_j + \frac{\Delta\chi}{3}\delta^i_j - D^i D_j \chi$

$-3H + k + \ldots - \Delta\chi$

$= k$

$= 2\overset{(b)}{k}\delta K$

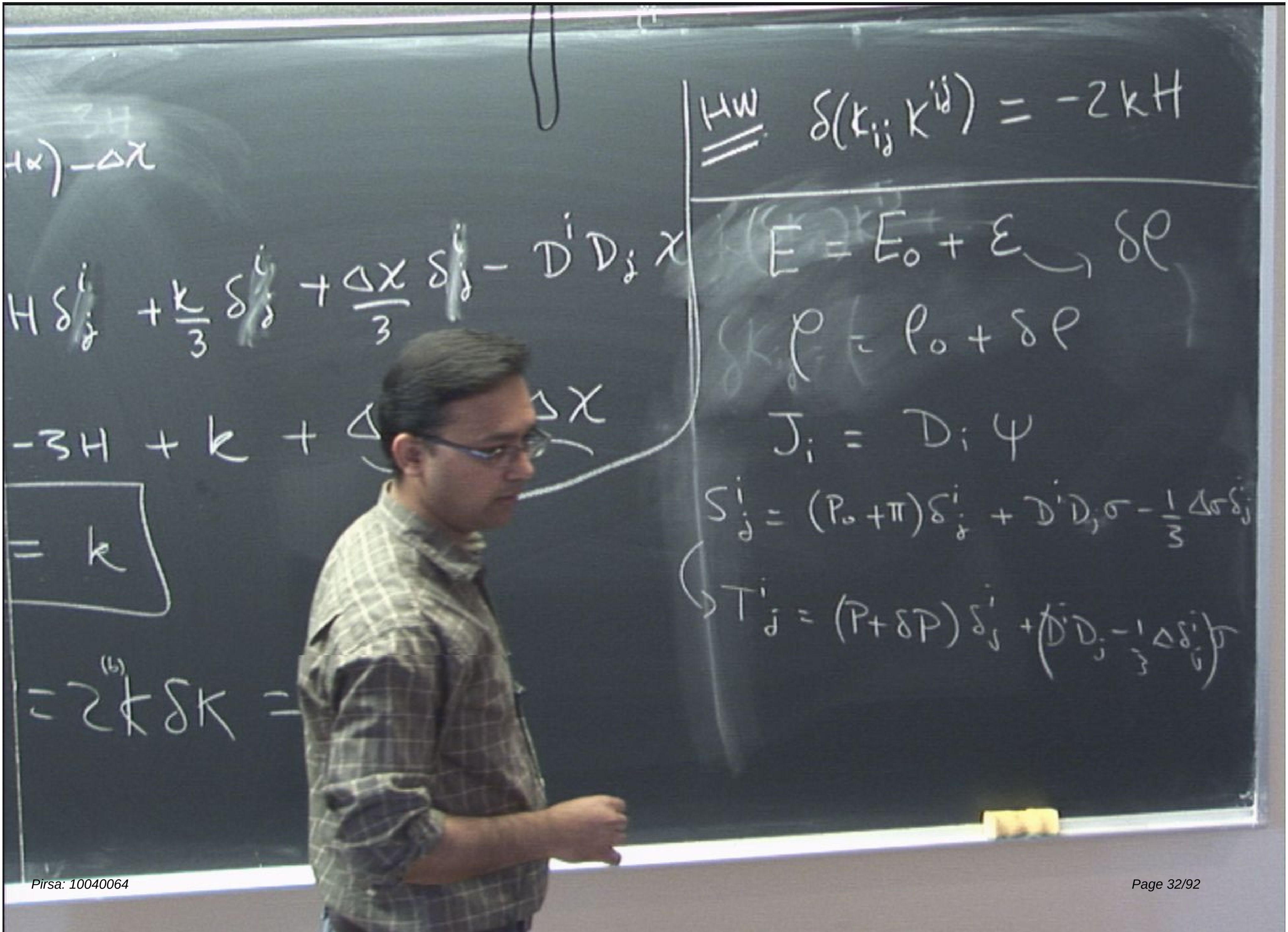
HW $\delta(K_{ij}K^{ij}) = -2kH$

$E = E_0 + \mathcal{E} \rightarrow \delta\rho$

$\rho = \rho_0 + \delta\rho$

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$



HW $\delta(K_{ij}K^{ij}) = -2kH$
$E = E_0 + \mathcal{E} \to \delta\rho$
$\rho = \rho_0 + \delta\rho$
$J_i = D_i \psi$
$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\delta^i_j$
$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta\delta^i_j)\sigma$
$H\delta^i_j + \frac{k}{3}\delta^i_j + \frac{\Delta\chi}{3}\delta^i_j - D^i D_j \chi$
$-3H + k +$
$= k$
$= 2k\delta K =$

$H\alpha) - \Delta\chi$

$H\delta^i_j + \frac{k}{3}\delta^i_j + \frac{\Delta\chi}{3}\delta^i_j - D^i D_j \chi$

$-3H + k + \underbrace{\Delta\chi - \Delta\chi}_{0}$

$\boxed{= k}$

$= 2\overset{(b)}{k}\,\delta K = -6Hk$

HW $\delta(K_{ij}K^{ij}) = -2kH$

$E = E_0 + \mathcal{E} \rightarrow \delta\rho$

$\rho = \rho_0 + \delta\rho$

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$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta\delta^i_j)\sigma$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi.$$

$$K^i{}_j = -H\,\delta^i_j - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^i_j - \frac{a^2}{N_0} D^i D_j(\dot\gamma + \varphi)$$

$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\boxed{\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E}$$

$$-4\Delta\varphi - \frac{12C}{a^2}\varphi + 2kH - 6kH = 16\pi G\,\varepsilon$$

$$\boxed{\delta R = -4\Delta\varphi - \frac{12C}{a^2}\varphi}$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma \ldots$$

$$K = -3(\ldots$$

$$K^i{}_j = \ldots$$

$$K = \ldots$$

$$\boxed{\delta K \ldots}$$

$$\delta(K^2 \ldots$$

$$R^i_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$$

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$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

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$$-4\Delta\varphi - \frac{12C}{a^2}\varphi + 2kH - 6kH = 16\pi G\, \varepsilon$$

$$\boxed{\left(\Delta\varphi + \frac{3C}{a^2}\varphi\right) + kH = -4\pi G\, \varepsilon}$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma$$

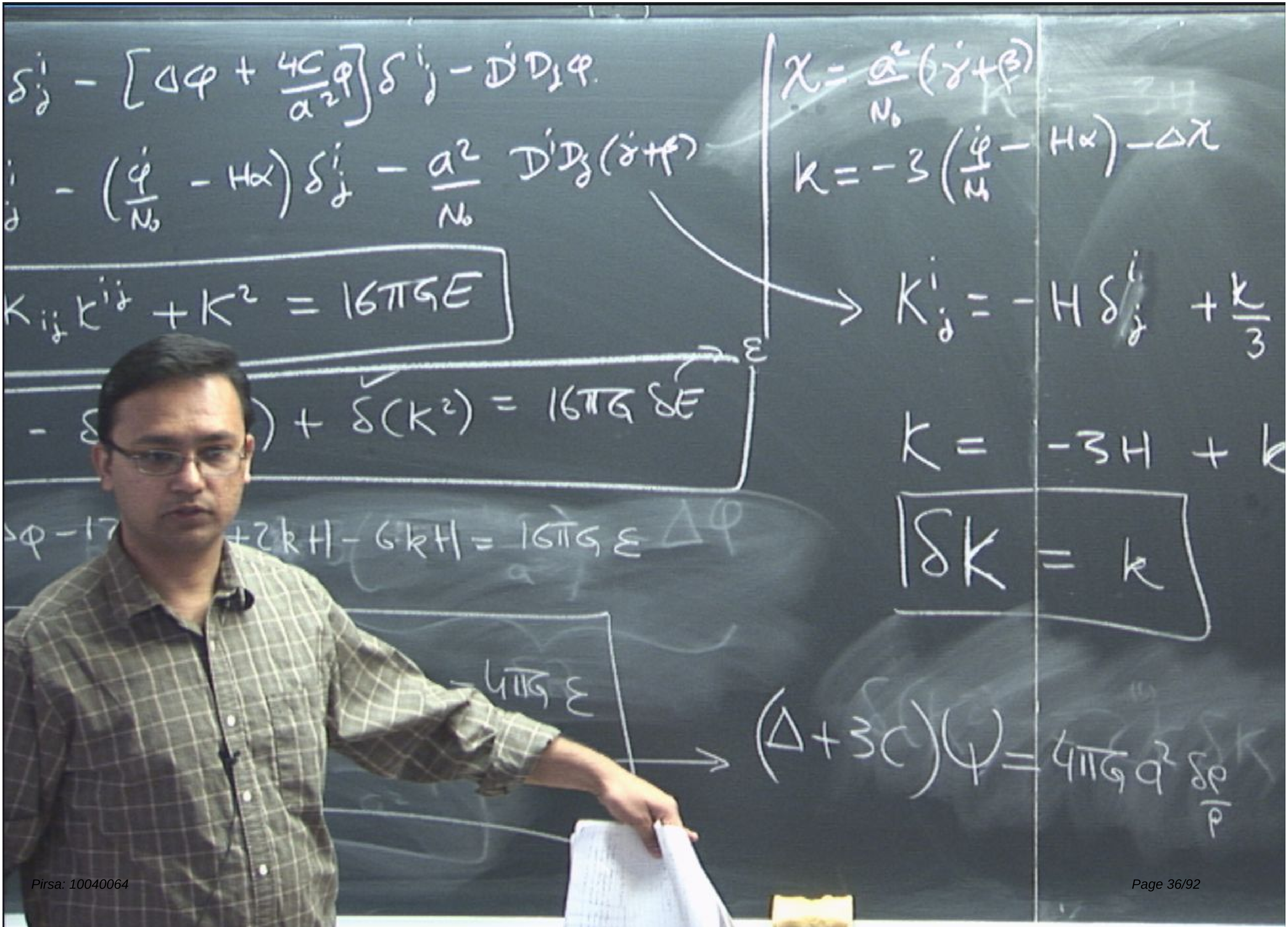
$$k = -3($$

$$K^i_j$$

$$k =$$

$$\boxed{\delta K}$$

$$\delta(K^2$$


$\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^iD_j\varphi$
$- \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^i_j - \frac{a^2}{N_0} D^iD_j(\dot\gamma+\beta)$
$K_{ij}K^{ij} + K^2 = 16\pi G E$
$\delta(K^2) = 16\pi G\,\delta E$
$+2kH - 6kH = 16\pi G\,\varepsilon$
$4\pi G\,\varepsilon$
$\chi = \frac{a^2}{N_0}(\dot\gamma+\beta)$
$k = -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi$
$K^i_j = -H\delta^i_j + \frac{k}{3}$
$K = -3H + k$
$\boxed{\delta K = k}$
$(\Delta+3C)\varphi = 4\pi G a^2 \frac{\delta\rho}{\rho}$

$$\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j\varphi$$

$$^i_j - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^i_j - \frac{a^2}{N_0} D^i D_j(\dot\gamma + \beta)$$

$$\boxed{K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\boxed{-\delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E} \quad \varepsilon$$

$$\Delta\varphi - \frac{12C}{a^2}\varphi + 2kH - 6kH = 16\pi G\,\varepsilon$$

$$\boxed{\left(\Delta\varphi + \frac{3C}{a^2}\varphi\right) + kH = -4\pi G\,\varepsilon}$$

$$\longrightarrow (\Delta + 3C)\psi = 4\pi G a^2 \frac{\delta\rho}{\rho}$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma + \beta)$$

$$k = -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi$$

$$K^i_j = -H\delta^i_j + \frac{k}{3}$$

$$K = -3H + k$$

$$\boxed{\delta K = k}$$

$$R^i_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$$

$$K^i_j = -H\,\delta^i_j - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^i_j - \frac{a^2}{N_0} D^i D_j(\dot\gamma + \beta)$$

$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\boxed{\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E} \quad \delta E \to \varepsilon$$

$$-4\Delta\varphi - \frac{12C}{a^2}\varphi + 2kH - 6kH = 16\pi G\,\varepsilon$$

$$\boxed{\left(\Delta\varphi + \frac{3C}{a^2}\varphi\right) + kH = -4\pi G\,\varepsilon} \longrightarrow (\tilde\Delta + 3C)\varphi = \ldots$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma + \beta)$$

$$k = -3\left(\frac{\dot\varphi}{N_0} - \ldots\right.$$

$$K^i_j = - \ldots$$

$$k = \ldots$$

$$\boxed{\delta K = \ldots}$$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$K^i{}_j = -H\,\delta^i_j - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^i_j - \frac{a^2}{N_0} D^i D_j(\dot\gamma + \beta)$$

$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\boxed{\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E}$$

$$-4\Delta\varphi - \frac{12C}{a^2}\varphi + 2kH - 6kH = 16\pi G\,\varepsilon$$

$$\boxed{\left(\Delta\varphi + \frac{3C}{a^2}\varphi\right) + kH = -4\pi G\,\varepsilon}$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma + \beta)$$

$$k = -3\left(\frac{\dot\varphi}{N_0} - \right.$$

$$K^i{}_j = -$$

$$k =$$

$$\boxed{\delta K}$$

$$(\tilde\Delta + 3C)\varphi =$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma + \beta)$$

$$k = -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi$$

$$\rightarrow K^i_j = -H\delta^i_j + \frac{k}{3}\delta^i_j + \frac{\Delta\chi}{3}\delta^i_j - D^i D_j \chi$$

$$K = -3H + k + \underbrace{\Delta\chi - \Delta\chi}_{0}$$

$$\boxed{\delta K = k}$$

$$G^0_0 = T^0_0$$

$$(\tilde\Delta + 3C)\psi = 4\pi G a^2 S$$

HW $\delta(K_{ij}$

$E = E$

$J_i =$

$S^i_j = (P_0 + \pi$

$T^i_j = (P + \delta$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$K^i{}_j = -H\,\delta^i_j - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^i_j - \frac{a^2}{N_0} D^i D_j(\dot\gamma + \beta)$$

$$\boxed{R - K_{ij}K^{ij} + K^2 = 16\pi G E}$$

$$\boxed{\delta R - \delta(K_{ij}K^{ij}) + \delta(K^2) = 16\pi G\,\delta E}$$

$$-4\Delta\varphi - \frac{12C}{a^2}\varphi + 2kH - 6kH = 16\pi G\,\varepsilon$$

$$\boxed{\left(\Delta\varphi + \frac{3C}{a^2}\varphi\right) + kH = -4\pi G\,\varepsilon} \rightarrow (\tilde\Delta + 3C)(\varphi + \ldots$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma \ldots$$

$$k = -3(\ldots$$

$$K^i{}_j \ldots$$

$$k = \ldots$$

$$\delta k \ldots$$

$$R^i_{\ j} = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_{\ j} - D^i D_j \varphi$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma$$

$$k = -3\,($$

$$K^i_{\ j} =$$

$$k =$$

$$\boxed{\delta k}$$

$$(\tilde\Delta + 3C)($$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$\chi = \frac{a^2}{N_0}(\dot{\gamma}$$

$$k = -3($$

$$K^i_j$$

$$K =$$

$$\boxed{\delta K}$$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$\rightarrow \quad K^j_i = -H\delta^j_i + \frac{k}{3}\delta^j_i + \frac{\Delta X}{3}\delta^j_i - D_i D^j X$$

$$\chi = \frac{a^2}{N_0}(\dot{\gamma}$$

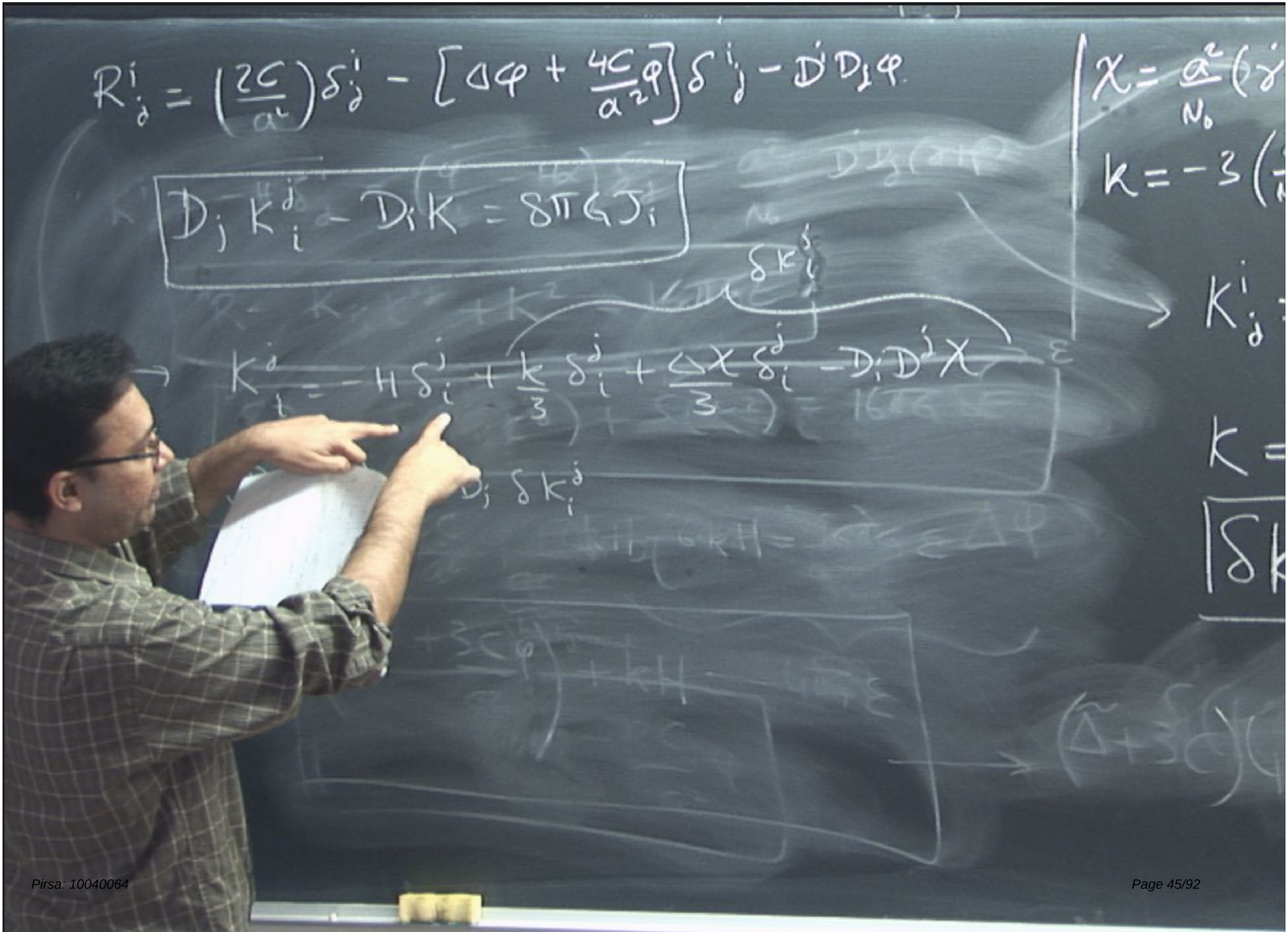
$$k = -3($$

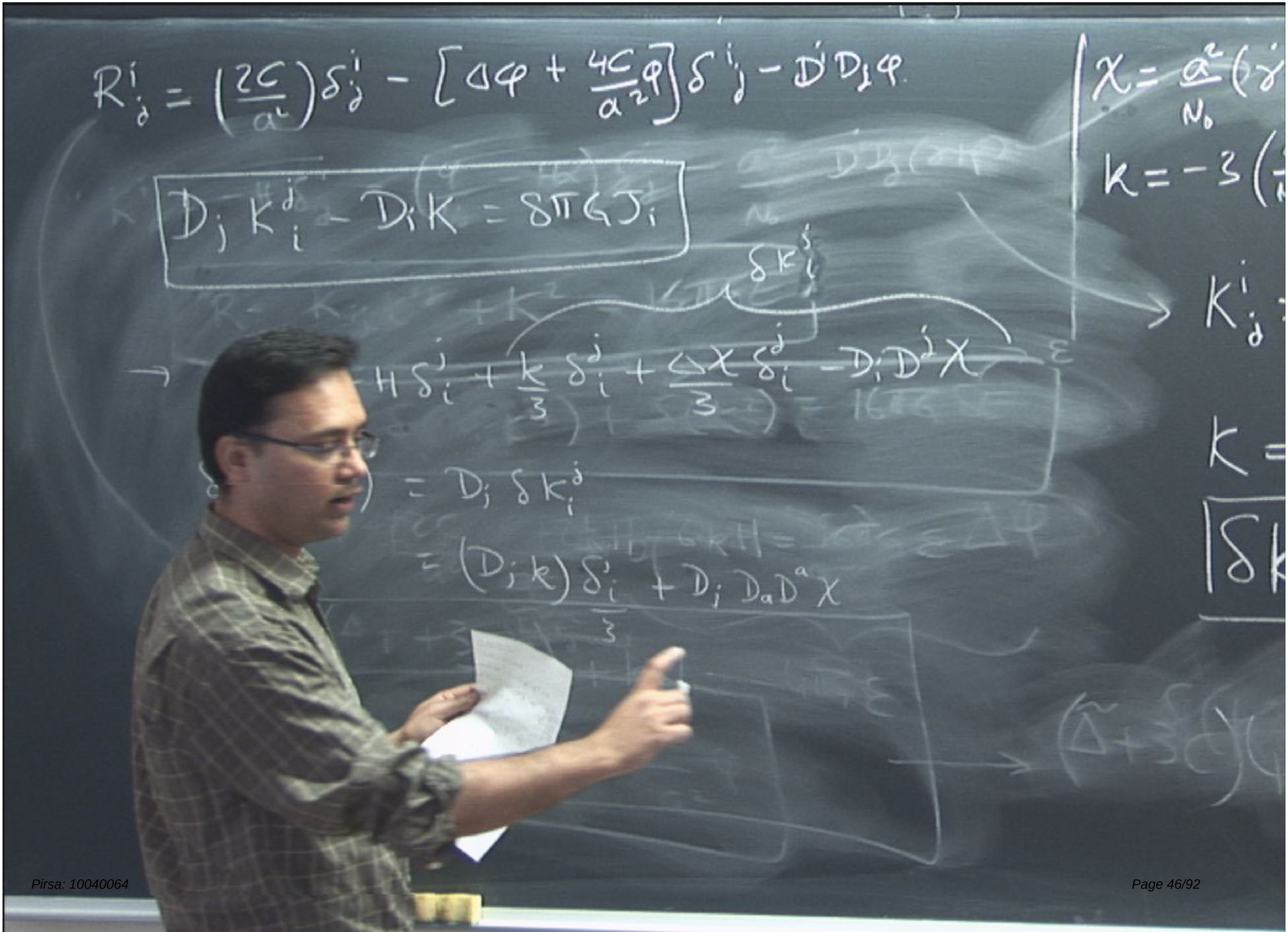
$$K^i_j$$

$$K =$$

$$\delta K$$

$$(\tilde{\Delta} + 3C)($$


$R^i_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$
$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$
δK^j_i
$K^j_i = -H\delta^j_i + \frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi$
$D_j \delta K^j_i$
$\chi = \frac{a^2}{N_0}$
$k = -3$



$R^i_j = \left(\frac{2c}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$
$D_j K^j_i - D_i K = 8\pi G J_i$
δK^j_i
$\to -H\delta^j_i + \frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi$
$= D_j \delta K^j_i$
$= (D_j k)\frac{\delta^j_i}{3} + D_j D_a D^a \chi$
$\chi = \frac{a^2}{N_0}($
$k = -3($
K^i_j
$K =$
δK

$$R^i_{\ j} = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_{\ i} - D_i K = 8\pi G J_i}$$

$$\rightarrow \quad K^j_{\ i} = -H\delta^j_i + \underbrace{\frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$\delta(D_j K^j_{\ i}) = D_j \delta K^j_i$$

$$= \frac{(D_j k)\delta^j_i}{3} + \frac{D_j D_a D^a \chi}{3}\delta^j_i - D_j D_i D^j \chi$$

$$\chi = \frac{a^2}{N_0}(\dots$$

$$k = -3(\dots$$

$$K^i_{\ j} \dots$$

$$K = \dots$$

$$\boxed{\delta k \dots}$$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$\rightarrow K^j_i = -H\delta^j_i + \frac{k}{3}\delta^j_i + \underbrace{\frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$\delta(D_j K^j_i) = D_j \delta K^j_i$$

$$= \frac{(D_j k)\delta^j_i}{3} + \frac{D_j D_a D^a \chi \,\delta^j_i}{3} - D_j D_i D^j \chi$$

$$\chi = \frac{a^2}{N_0}(\dot{\gamma}$$

$$k = -3($$

$$K^i_j$$

$$K =$$

$$\delta k$$

$$R^i_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma$$

$$k = -3\,($$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$K^j_i = -H\delta^j_i + \underbrace{\frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$K^i_j$$

$$K =$$

$$\boxed{\delta K}$$

$$\delta(D_j K^j_i) = D_j \delta K^j_i$$

(i) $$= (D_j k)\frac{\delta^j_i}{3} + D_j \frac{D_a D^a \chi}{3}\delta^j_i - D_j D_i D^j \chi$$

(ii) $$\delta(D_i K) = D_i \delta K = D_i k$$

$$R^i{}_j = \left(\frac{2c}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$\rightarrow \quad K^j_i = -H\delta^j_i + \underbrace{\frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$\delta(D_j K^j_i) = D_j \delta K^j_i$$

$$= \frac{(D_j k)\delta^j_i}{3} + \frac{D_j D_a D^a \chi\, \delta^j_i}{3} - D_j D_i D^j \chi$$

(i)

$$\delta(D_i K) = D_i \delta K = D_i k$$

$$\chi = \frac{a^2}{N_0}(\dot{\gamma}$$

$$k = -3($$

$$K^i_j$$

$$K =$$

$$\delta k$$

$$R^i_{\ j} = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_{\ i} - D_i K = 8\pi G J_i}$$

$$\rightarrow \quad K^j_{\ i} = -H\delta^j_i + \underbrace{\frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$\delta(D_j K^j_{\ i}) = D_j \delta K^j_i$$

(i)
$$= \frac{(D_j k)\delta^j_i}{3} + \frac{D_j D_a D^a \chi\, \delta^j_i}{3} - \underbrace{D_j D_i D^j \chi}$$

(ii) $$\delta(D_i K) = D_i \delta K = D_i k$$

$$\delta(D_j K^j_{\ i} - D_i k) = -\frac{2}{3}(D_i k + D_i \Delta\chi) - \frac{2C}{a^2} D_i \chi$$

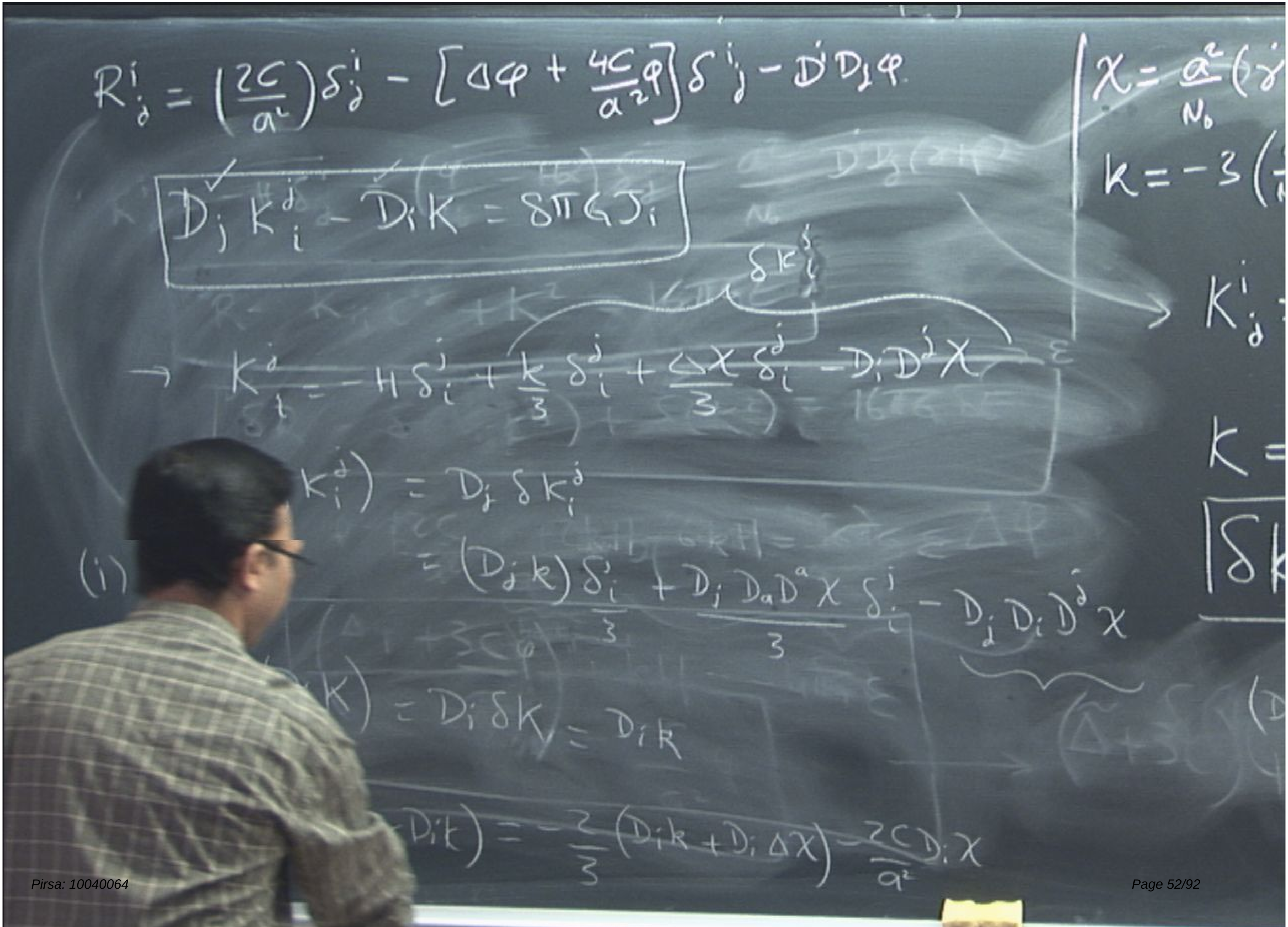
$$\chi = \frac{a^2}{N_0}(\dot\gamma \ldots$$

$$k = -3(\ldots$$

$$K^i_{\ j} = \ldots$$

$$K = \ldots$$

$$\delta k \ldots$$


$R^i_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i_j - D^i D_j \varphi$
$D_j K^j_i - D_i K = 8\pi G J_i$
δK^j_i
$K^j_i = -H\delta^j_i + \frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi$
$K^j_i) = D_j \delta K^j_i$
(i)
$= (D_j k)\frac{\delta^j_i}{3} + \frac{D_j D_a D^a \chi}{3}\delta^j_i - D_j D_i D^j \chi$
$K) = D_i \delta K = D_i k$
$D_i k) = -\frac{2}{3}(D_i k + D_i \Delta\chi) - \frac{2C}{a^2} D_i \chi$
$\chi = \frac{a^2}{N_0}$
$k = -3$
K^i_j
$K =$
δK

$$R^i{}_j = \left(\frac{2c}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4c}{a^2}\varphi\right]\delta^i{}_j - D^i D_j \varphi$$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$\rightarrow \quad K^j_i = -H\delta^j_i + \underbrace{\frac{k}{3}\delta^j_i + \frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$\delta(D_j K^j_i) = D_j \delta K^j_i$$

(i)

$$= \frac{(D_j k)\delta^j_i}{3} + \frac{D_j D_a D^a \chi}{3}\delta^j_i - \underbrace{D_j D_i D^j \chi}$$

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$$k = -3($$

$$K^i_j$$

$$K =$$

$$R^i{}_j = \left(\frac{2C}{a^2}\right)\delta^i_j - \left[\Delta\varphi + \frac{4C}{a^2}\varphi\right]\delta^i{}_j - D^j D_j \varphi$$

$$\chi = \frac{a^2}{N_0}(\dot\gamma$$

$$k = -3\Big($$

$$\boxed{D_j K^j_i - D_i K = 8\pi G J_i}$$

$$K^i_j$$

$$\rightarrow \quad K^j_i = -H\delta^j_i + \frac{k}{3}\delta^j_i + \underbrace{\frac{\Delta\chi}{3}\delta^j_i - D_i D^j \chi}_{\delta K^j_i}$$

$$\delta(D_j K^j_i) = D_j \delta K^j_i$$

(i)

$$= (D_j k)\frac{\delta^j_i}{3} + \frac{D_j D_a D^a \chi}{3}\delta^j_i - \underbrace{D_j D_i D^j \chi}$$

(ii) $\delta(D_i K) = D_i \delta K = D_i k$

$$\boxed{\delta(D_j K^j_i - D_i K) = -\frac{2}{3}(D_i k + D_i \Delta\chi) - \frac{2C}{a^2} D_i \chi}$$

$$8\pi G J_i \rightarrow 8\pi G D_i \psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi\right) - \frac{2c}{a^2} D_i \chi = 8\pi G D_i \psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi + \frac{3c}{a^2} D_i \chi\right) = 8\pi G D_i \psi$$

$$\boxed{\delta K = k}$$

$$G^0{}_0 = T^0{}_0$$

HW $\delta(K_{ij} K^{ij}) = -2$

$$E = E_0 + \mathcal{E}$$

$$\rho = \rho_0 + \delta\rho$$

$$J_i = D_i \psi$$

$$S^i{}_j = (P_0 + \pi)\delta^i{}_j + D^i D_j \sigma$$

$$T^i{}_j = (P + \delta P)\delta^i{}_j + D^i D_j$$

$$8\pi G J_i \rightarrow 8\pi G D_i \Psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi\right) - \frac{2C}{a^2} D_i \chi = 8\pi G D_i \Psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi + \frac{3C}{a^2} D_i \chi\right) = 8\pi G D_i \Psi$$

$$\boxed{D_i\left(\Delta \chi + k + \frac{3C}{a^2} + 12\pi G \Psi\right) = 0}$$

δk

$D^j \chi$

HW $\delta(K_{ij} K^{ij}) =$

$E = E_0 + \mathcal{E}$

$\rho = \rho_0 +$

$J_i = D_i \Psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$\delta \pi G J_i \longrightarrow 8\pi G D_i \psi$

$$\rightarrow -\frac{2}{3}\left(D_i k + D_i \Delta X\right) - \frac{2C}{a^2} D_i X = 8\pi G D_i \psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta X + \frac{3C}{a^2} D_i X\right) = 8\pi G D_i \psi$$

δk

$$\boxed{D_i\left(\Delta X + k + \frac{3C}{a^2} + 12\pi G \psi\right) = 0}$$

$e^{i k_i x^i}$

$\underline{\underline{HW}}$ $\delta(K_{ij} K^{ij}) =$

$E = E_0 + \mathcal{E}$

$\rho = \rho_0 +$

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$8\pi G J_i \to 8\pi G D_i \psi$

$\to -\frac{2}{3}(D_i k + D_i \Delta \chi) - \frac{2C}{a^2} D_i \chi = 8\pi G D_i \psi$

$-\frac{2}{3}\left(D_i k + D_i \Delta \chi + \frac{3C}{a^2} D_i \chi\right) = 8\pi G D_i \psi$

$$\boxed{D_i\left(\Delta\chi + k + \frac{3C}{a^2} + 12\pi G \psi\right) = 0}$$

$Q \sim e^{i k_i x^i}$

$D^j \chi$

HW $\delta(K_{ij} K^{ij}) =$

$E = E_0 + \mathcal{E}$

$\rho = \rho_0 + $

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$8\pi G J_i \rightarrow 8\pi G D_i \Psi$

$$\rightarrow -\frac{2}{3}\left(D_i k + D_i \Delta X\right) - \frac{2C}{a^2} D_i X = 8\pi G D_i \Psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta X + \frac{3C}{a^2} D_i X\right) = 8\pi G D_i \Psi$$

$$\boxed{D_i\left(\Delta X + k + \frac{3C}{a^2}X + 12\pi G \Psi\right) = 0}$$

$D^j X$

$Q \sim e^{i k_i x^i}$

HW $\delta(K_{ij}K^{ij}) =$

$E = E_0 + \mathcal{E}$

$\rho = \rho_0 +$

$J_i = D_i \Psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$8\pi G J_i \rightarrow 8\pi G D_i \Psi$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi\right) - \frac{2C}{a^2} D_i \chi = 8\pi G D_i \Psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi + \frac{3C}{a^2} D_i \chi\right) = 8\pi G D_i \Psi$$

$$\boxed{D_i\left(\Delta \chi + k + \frac{3C}{a^2}\chi + 12\pi G \Psi\right) = 0}$$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2 (1+w)\rho V}$$

HW $\delta(K_{ij}K^{ij}) =$

$E = E_0 + \mathcal{E}$

$\rho = \rho_0 +$

$J_i = D_i \Psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$8\pi G J_i \to 8\pi G D_i \psi$

HW $\delta(K_{ij} K^{ij}) =$

$-\frac{2}{3}(D_i k + D_i \Delta \chi) - \frac{2C}{a^2} D_i \chi = 8\pi G D_i \psi$

$-\frac{2}{3}\left(D_i k + D_i \Delta \chi + \frac{3C}{a^2} D_i \chi\right) = 8\pi G D_i \psi$

$$\boxed{D_i\left(\Delta \chi + k + \frac{3C}{a^2}\chi + 12\pi G \psi\right) = 0}$$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2 (1+w)\rho V}$$

$E = E_0 + \varepsilon$

$\rho = \rho_0 +$

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$D^j \chi$

$$8\pi G J_i \longrightarrow 8\pi G D_i \psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi\right) - \frac{2C}{a^2} D_i \chi = 8\pi G D_i \psi$$

$$-\frac{2}{3}\left(D_i k + D_i \Delta \chi + \frac{3C}{a^2} D_i \chi\right) = 8\pi G D_i \psi$$

$$\boxed{D_i\left(\Delta \chi + k + \frac{3C}{a^2}\chi + 12\pi G \psi\right) = 0}$$

$$\boxed{\Psi' + \mathcal{H}\Phi = -4\pi G a^2 (1+w)\rho V}$$

HW $\delta(K_{ij} K^{ij}) =$

$E = E_0 + \mathcal{E}$

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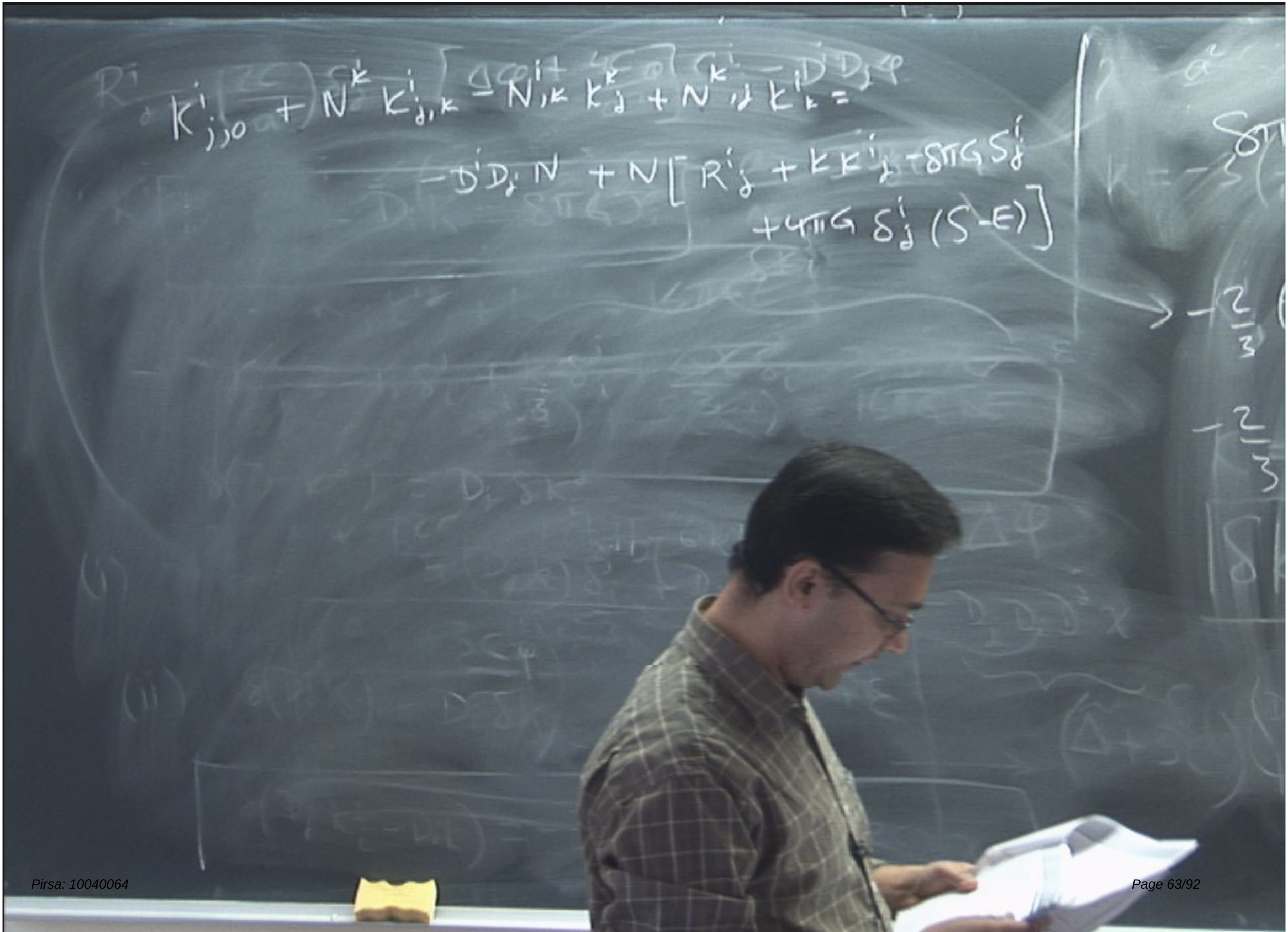
$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j +$

$T^i_j = (P + \delta P)\delta^i_j$

$D^j \chi$


$K^i_{j,0} + N^k K^i_{j,k} - N^i_{,k} K^k_j + N^k_{,j} K^i_k =$
$-D^i D_j N + N[R^i_j + K K^i_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S - E)]$

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$

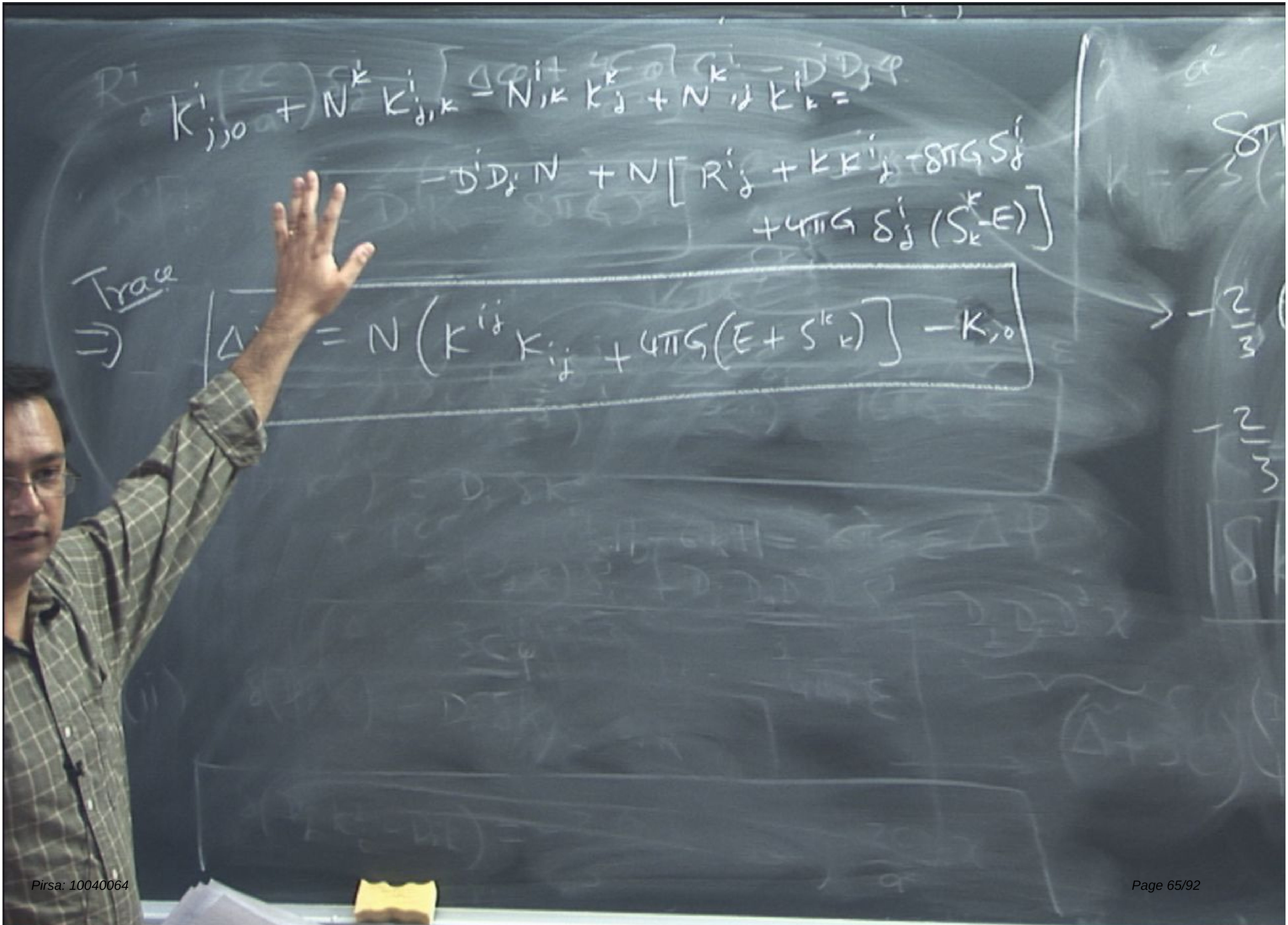
$$-D^i D_j N + N\left[R^i{}_j + K K^i{}_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E)\right]$$

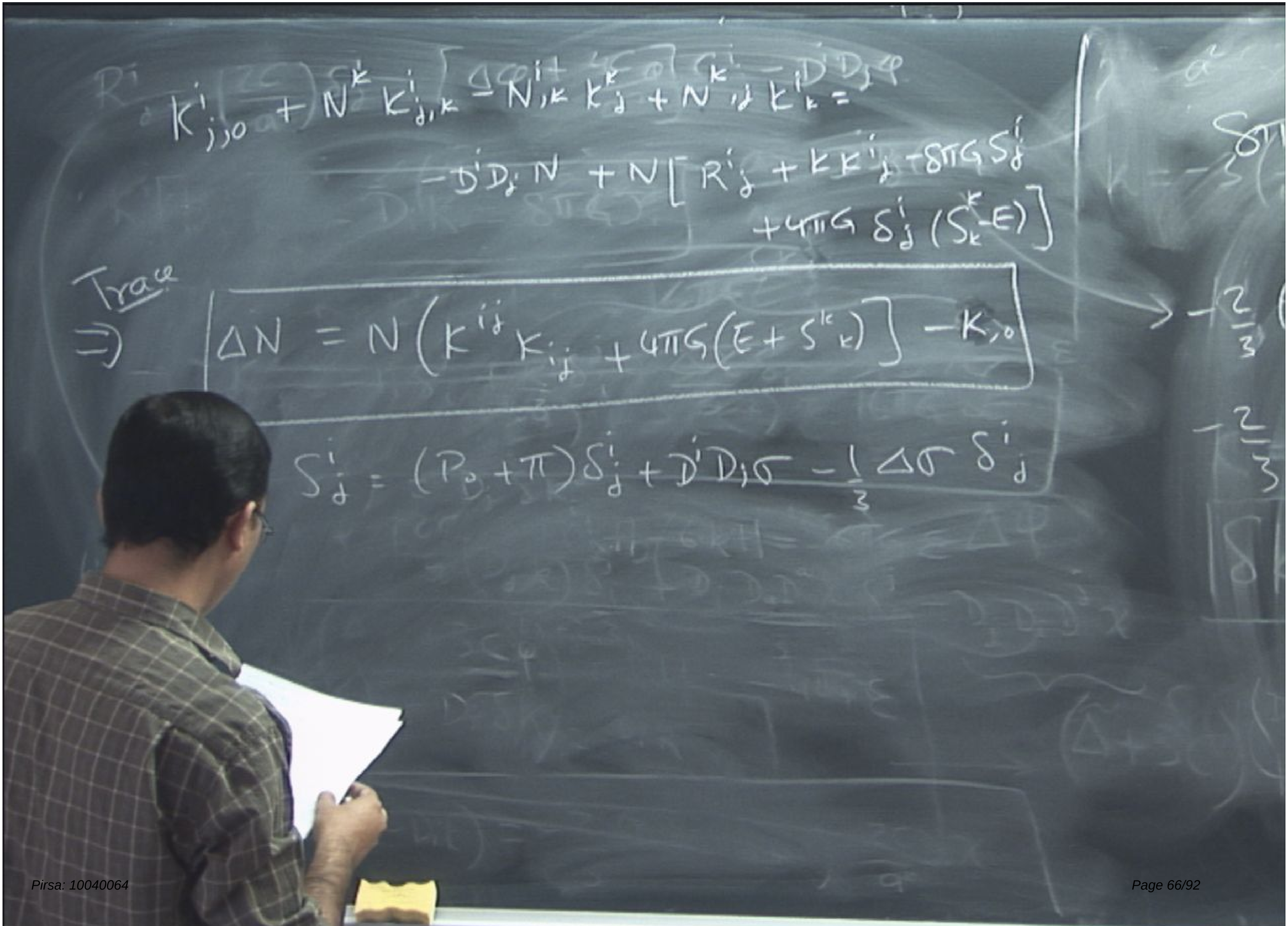
Trace

$$\Rightarrow \quad \Delta N = N\left(K^{ij} K_{ij} + 4\pi G\left(E + S^k{}_k\right)\right] - K_{,0}$$

$$-\frac{2}{3}$$

$$-\frac{2}{3}$$


$K^i_{j,0} + N^k K^i_{j,k} - N^i_{,k} K^k_j + N^k_{,j} K^i_k =$
$-D^i D_j N + N[R^i_j + K K^i_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E)]$
Trace
$\Rightarrow \Delta N = N(K^{ij} K_{ij} + 4\pi G(E + S^k_k)] - K_{,0}$


$K^i_{j,0} + N^k K^i_{j,k} - N^i_{,k} K^k_j + N^k_{,j} K^i_k =$
$-D^i D_j N + N[R^i_j + K K^i_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E)]$
Trace
$\Rightarrow \boxed{\Delta N = N(K^{ij} K_{ij} + 4\pi G(E + S^k_k)] - K_{,0}}$
$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\,\delta^i_j$

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$
$$- D^i D_j N + N\left[R^i{}_j + K K^i{}_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E)\right]$$

Trace

$$\Rightarrow \boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G (E + S^k{}_k)\right] - K_{,0}}$$

$$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\,\delta^i_j$$

$$S^i_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$-\frac{2}{3}$

$-\frac{2}{3}$

$$K^i_{j,0} + N^k K^i_{j,k} - N^i_{,k} K^k_j + N^k_{,j} K^i_k =$$

$$- D^i D_j N + N\left[R^i_j + K K^i_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E) \right]$$

Trace

$$\Rightarrow \quad \boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G(E + S^k_k)\right] - K_{,0}}$$

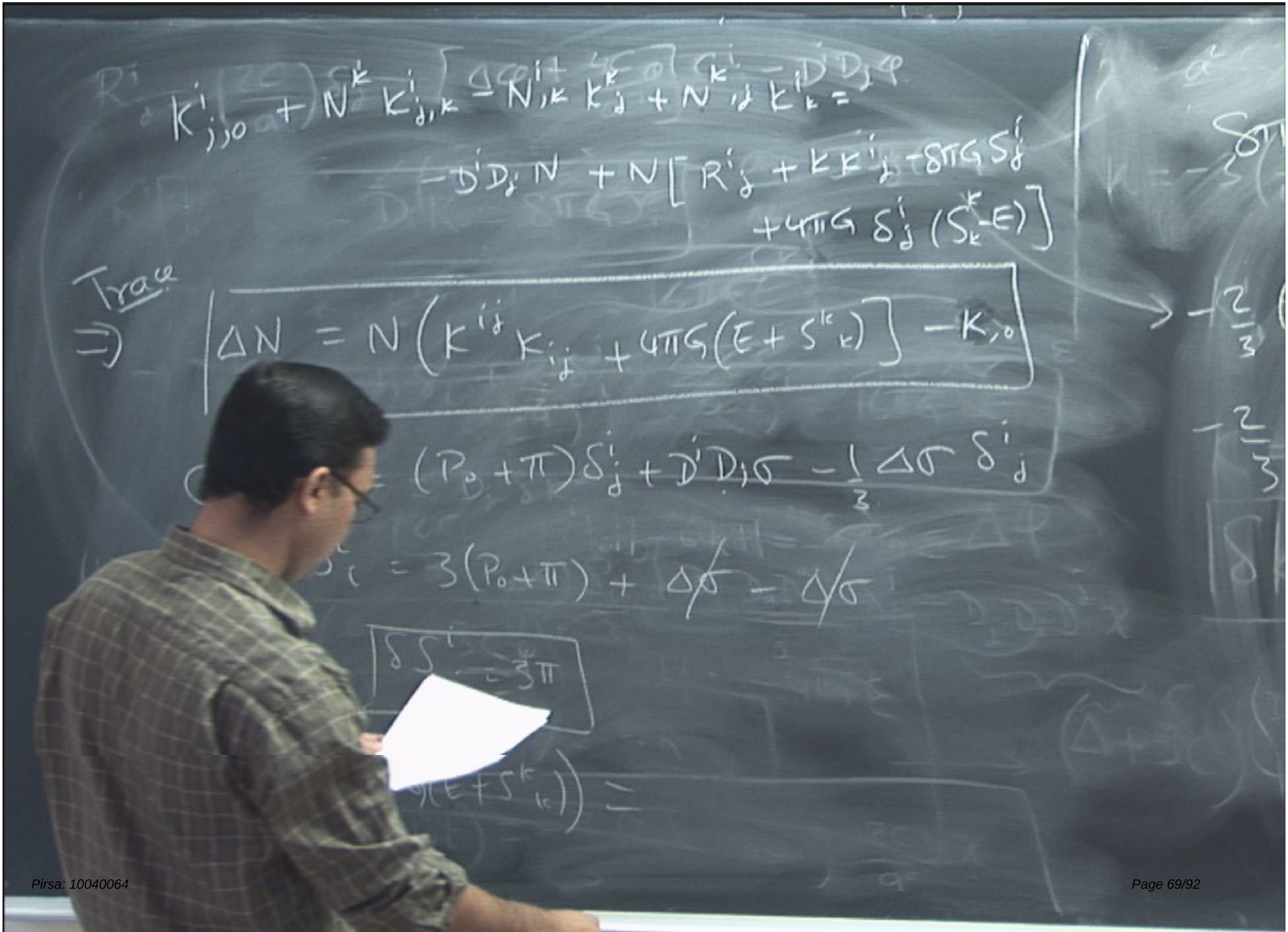
$$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\,\delta^i_j$$

$$S^i_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$$\boxed{\delta S^i_i = 3\pi}$$

$-\frac{2}{3}$

$-\frac{2}{3}$


$K^i_{j,0} + N^k K^i_{j,k} - N^i_{,k} K^k_j + N^k_{,j} K^i_k =$
$-D^i D_j N + N[R^i_j + K K^i_j - 8\pi G S^i_j$
$+ 4\pi G \delta^i_j (S^k_k - E)]$
Trace
$\Rightarrow \Delta N = N(K^{ij} K_{ij} + 4\pi G(E + S^k_k)] - K_{,0}$
$(P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma \delta^i_j$
$= 3(P_0 + \pi) + \Delta\sigma - \Delta\sigma$
$\delta S^i_i = 3\pi$

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$

$$- D^i D_j N + N \Big[R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j$$

$$+ 4\pi G \, \delta^i_j (S^k{}_k - E) \Big]$$

Trace

$$\Rightarrow \quad \boxed{\Delta N = N \left(K^{ij} K_{ij} + 4\pi G (E + S^k{}_k) \right] - K_{,0}}$$

$$(*) \quad S^i{}_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3} \Delta\sigma \, \delta^i_j$$

$$S^i{}_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$$\boxed{\delta S^i{}_i = 3\pi}$$

$$(*) \quad \delta\left(4\pi G (E + S^k{}_k)\right) = 4\pi G (\mathcal{E} + 3\pi)$$

$$-\frac{2}{3}$$

$$-\frac{2}{3}$$

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$

$$- D^i D_j N + N\left[R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i_j (S^k{}_k - E)\right]$$

Trace

$$\Rightarrow \boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G(E + S^k{}_k)\right] - K_{,0}}$$

$$(*) \quad S^i{}_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\, \delta^i$$

$$S^i{}_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$$\boxed{\delta S^i{}_i = 3\pi}$$

$$(*) \quad \delta\left(4\pi G(E + S^k{}_k)\right) = 4\pi G(\varepsilon + 3\pi)$$

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$

$$- D^i D_j N + N\left[R^i{}_j + K K^i{}_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E) \right]$$

$$8\pi G S_i$$

$$\boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G (E + S^k{}_k)\right] - K_{,0}}$$

$$\rightarrow -\frac{2}{3}(D_i k + $$

$$-\frac{2}{3}(D_i k$$

$$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\,\delta^i_j$$

$$S^i_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$$\boxed{\delta S^i_i = 3\pi}$$

$$\delta\left(4\pi G(E + S^k{}_k)\right) = 4\pi G(\varepsilon + 3\pi)$$

$$K_{,0} \rightarrow (\delta K)_{,0}$$

$$\delta K = k$$

$$K_{,0} \xrightarrow{\text{Perturbation}} k_{,0}$$

D_i

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$

$$- D^i D_j N + N\left[R^i{}_j + K K^i{}_j - 8\pi G S^i_j + 4\pi G\, \delta^i_j (S^k_k - E)\right]$$

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$$\boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G (E + S^k{}_k)\right] - K_{,0}}$$

$$\to -\frac{2}{3}(D_i k + $$

$$-\frac{2}{3}(D_i k$$

(*) $$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\, \delta^i_j$$

$$S^i_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$$\boxed{\delta S^i_i = 3\pi}$$

(*) $$\delta\left(4\pi G (E + S^k{}_k)\right) = 4\pi G (\varepsilon + 3\pi)$$

$$K_{,0} \longrightarrow (\delta K)_{,0}$$

$$\delta K = k$$

$$K_{,0} \xrightarrow{\text{Perturbation}} k_{,0}$$

$$\delta(K_{ij}K^{ij}) = 2kH$$

D_i

$$K^i_{j,0} + N^k K^i_{j,k} - N_{,k} K^k_j + N^i_{,j} K^j_k = -D^i D_j N + N\left[R^i_j + K K^i_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E) \right]$$

$$\boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G (E + S^k_k) \right] - K_{,0}}$$

$$\circledast \quad S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}\Delta\sigma\, \delta^i_j$$

$$S^i_i = 3(P_0 + \pi) + \cancel{\Delta\sigma} - \cancel{\Delta\sigma}$$

$$\boxed{\delta S^i_i = 3\pi}$$

$$\circledast \quad \delta\left(4\pi G (E + S^k_k)\right) = 4\pi G (\mathcal{E} + 3\pi)$$

$$K_{,0} \longrightarrow (\delta K)_{,0}$$

$$\delta K = k$$

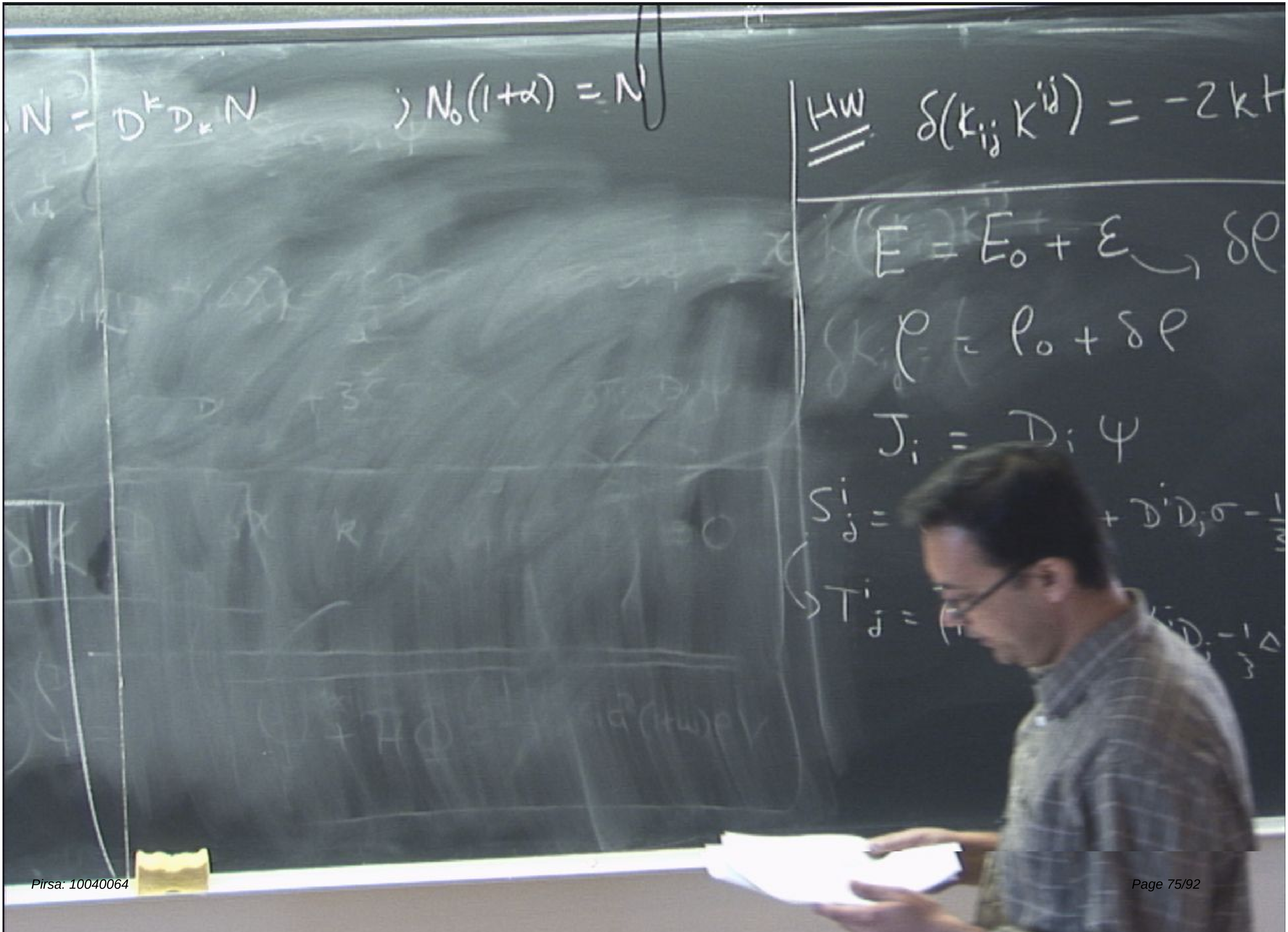
$$K_{,0} \xrightarrow{\text{Perturbation}} k_{,0}$$

$$\delta(K_{ij} K^{ij}) = -2kH$$

$$8\pi G S_i$$

$$-\frac{2}{3}(D_i k + \ldots$$

$$-\frac{2}{3}(D_i \ldots$$


N = D^k D_k N
; N_0(1+α) = N
HW
δ(K_ij K^ij) = −2kH
E = E_0 + ε
δρ
ρ = ρ_0 + δρ
J_i = D_i Ψ
S^i_j =
+ D^i D_j σ −
T^i_j =

$\Delta N = D^k D_k N \quad ; N_0(1+\alpha) = N$

$= D^k D_k N_0 (1+\alpha)$

$\delta(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$

HW $\delta(K_{ij} K^{ij}) = -2kH$

$E = E_0 + \mathcal{E} \to \delta\rho$

$\rho = \rho_0 + \delta\rho$

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}$

$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta$

$\Delta N = D^k D_k N \qquad ; N_0(1+\alpha) = N$

$= D^k D_k N_0 (1+\alpha)$

$\delta(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$

$\Delta N = N\left[K^{ij} K_{ij} + 4\pi G (E + S^k_k)\right] - K_{,0}$

HW $\delta(K_{ij} K^{ij}) = -2kH$

$E = E_0 + \varepsilon \to \delta\rho$

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$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta$

$$\Delta N = D^k D_k N \qquad ; N_0(1+\alpha) = N$$

$$= D^k D_k N_0 (1+\alpha)$$

$$\delta(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$$

$$\Delta N = \underbrace{N_0(1+\alpha)}_{N} \left[K^{ij} K_{ij} + 4\pi G (E + S^k_k) \right] - K_{,0}$$

HW $\delta(K_{ij} K^{ij}) = -2kH$

$$E = E_0 + \mathcal{E} \rightarrow \delta\rho$$

$$\rho = \rho_0 + \delta\rho$$

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$$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta$$

$\Delta N = D^k D_k N \qquad ; N_0(1+\alpha) = N$

$= D^k D_k N_0(1+\alpha)$

$\delta(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$

$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\left[K^{ij}K_{ij} + 4\pi G(E+S^k_k)\right] - K_{,0}$

background

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pert

pert

backg

HW $\delta(K_{ij}K^{ij}) = -2kH$

$E = E_0 + \mathcal{E} \rightarrow \delta\rho$

$\rho = \rho_0 + \delta\rho$

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}$

$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta$

$\Delta N = D^k D_k N \qquad ; N_0(1+\alpha) = N$

$= D^k D_k N_0(1+\alpha)$

$\delta(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$

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background + pert

pert

pert

backg

HW $\delta(K_{ij} K^{ij}) = -2kH$

$E = E_0 + \mathcal{E} \rightarrow \delta\rho$

$\rho = \rho_0 + \delta\rho$

$J_i = D_i \psi$

$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}$

$T^i_j = (P + \delta P)\delta^i_j + (D^i D_j - \frac{1}{3}\Delta$

$$\Delta N = D^k D_k N \qquad ; N_0(1+\alpha) = N$$

$$= D^k D_k N_0(1+\alpha)$$

$$\delta(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$$

$$\Delta N = \underbrace{N_0(1+\alpha)}_{N} \underbrace{\left[K^{ij} K_{ij} + 4\pi G (E + S^k_k) \right]}_{\text{pert}} - K_{,0}$$

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$${}^{(0)}K_{ij} {}^{(0)}K^{ij} = 3H^2$$

$$4\pi G (E_0$$

HW $\delta(K_{ij} K^{ij}) = -2KH$

$$E = E_0 + \mathcal{E} \rightarrow \delta\rho$$

$$\rho = \rho_0 + \delta\rho$$

$$J_i = D_i \psi$$

$$S^i_j = (P_0 + \pi)\delta^i_j + D^i D_j \sigma - \frac{1}{3}$$

$$T^i_j = (P + \delta P)\delta^i_j + D^i D_j - \frac{1}{3}\Delta$$

$= D^k D_k N$; $N_0(1+\alpha) = N$

$= D^k D_k N_0(1+\alpha)$

$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$

$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\left[K^{ij}K_{ij} + 4\pi G(E + S^k_k)\right] - K_{,0}$

background

+

pert

pert

backg

$K^{(0)}_{ij}K^{(0)ij} = 3H^2$

$4\pi G(E_0 + 3P_0)$

background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$N_0 \Delta\alpha = N_0(1+\alpha)\left[3H^2 - 2kH\right.$

$$= D^k D_k N \quad ; N_0(1+\alpha) = N$$

$$= D^k D_k N_0(1+\alpha)$$

$$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta\alpha$$

$$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\Big[K^{ij} K_{ij} + 4\pi G(E + S^k_k$$

background + pert

pert

pert

backg

$$K^{(0)}_{ij} K^{ij} = 3H^2$$

$$4\pi G(E_0 + 3P_0)$$

background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$$N_0\Delta\alpha = N_0(1+\alpha)\Big[3H^2 - 2kH + 4\pi G(E_0 + 3P_0) + 4\pi G(\varepsilon + 3\pi)\Big] - k_{,0}$$

$$\Delta\alpha = -2kN_0H + 4\pi G N_0(\varepsilon + 3\pi)$$

$= D^k D_k N \quad ; N_0(1+\alpha) = N$

$= D^k D_k N_0(1+\alpha)$

$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$

$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\underbrace{\left[K^{ij}K_{ij} + 4\pi G(E + S^k_k)\right]}_{\text{pert}} - K_{,0}$

background

+

pert

backg

$^{(0)}K^{(0)}_{ij}K^{ij} = 3H^2$

$4\pi G(E_0 + 3P_0)$

background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$$N_0\Delta\alpha = N_0(1+\alpha)\left[3H^2 - 2kH + 4\pi G(E_0 + 3P_0) + 4\pi G(\mathcal{E} + 3\Pi)\right] - k_{,0}$$

$$N_0\Delta\alpha = -2kN_0H + 4\pi G N_0(\mathcal{E} + 3\Pi) - k_{,0}$$

$$= D^k D_k N \qquad ; N_0(1+\alpha) = N$$

$$= D^k D_k N_0(1+\alpha)$$

$$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$$

$$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\left[K^{ij}K_{ij} + 4\pi G(E + S^k{}_k)\right] - K_{,0}$$

background

+

pert

pert

backg

$$K^{(0)}_{ij}K^{(0)ij} = 3H^2$$

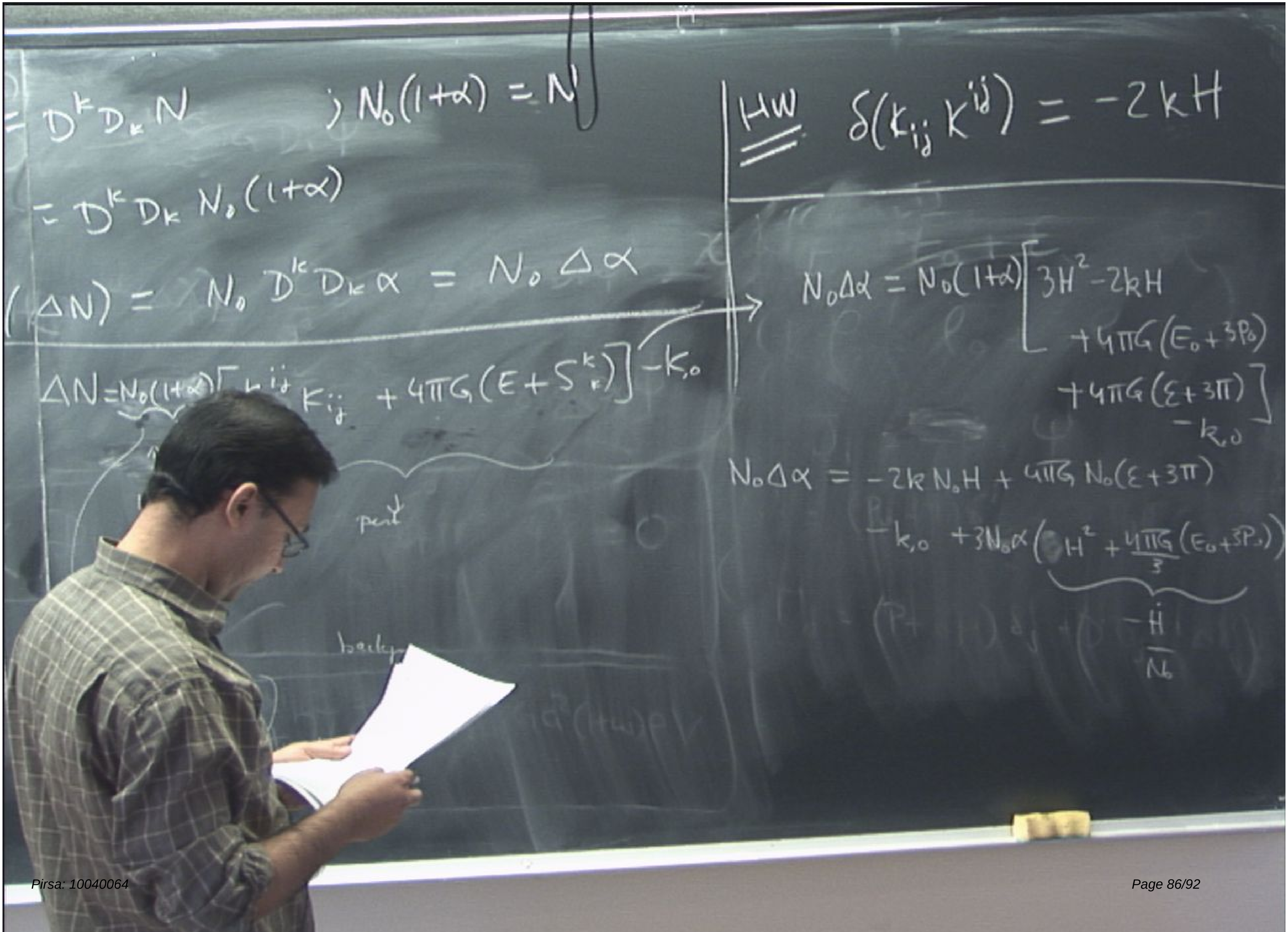
$$4\pi G(E_0 + 3P_0)$$

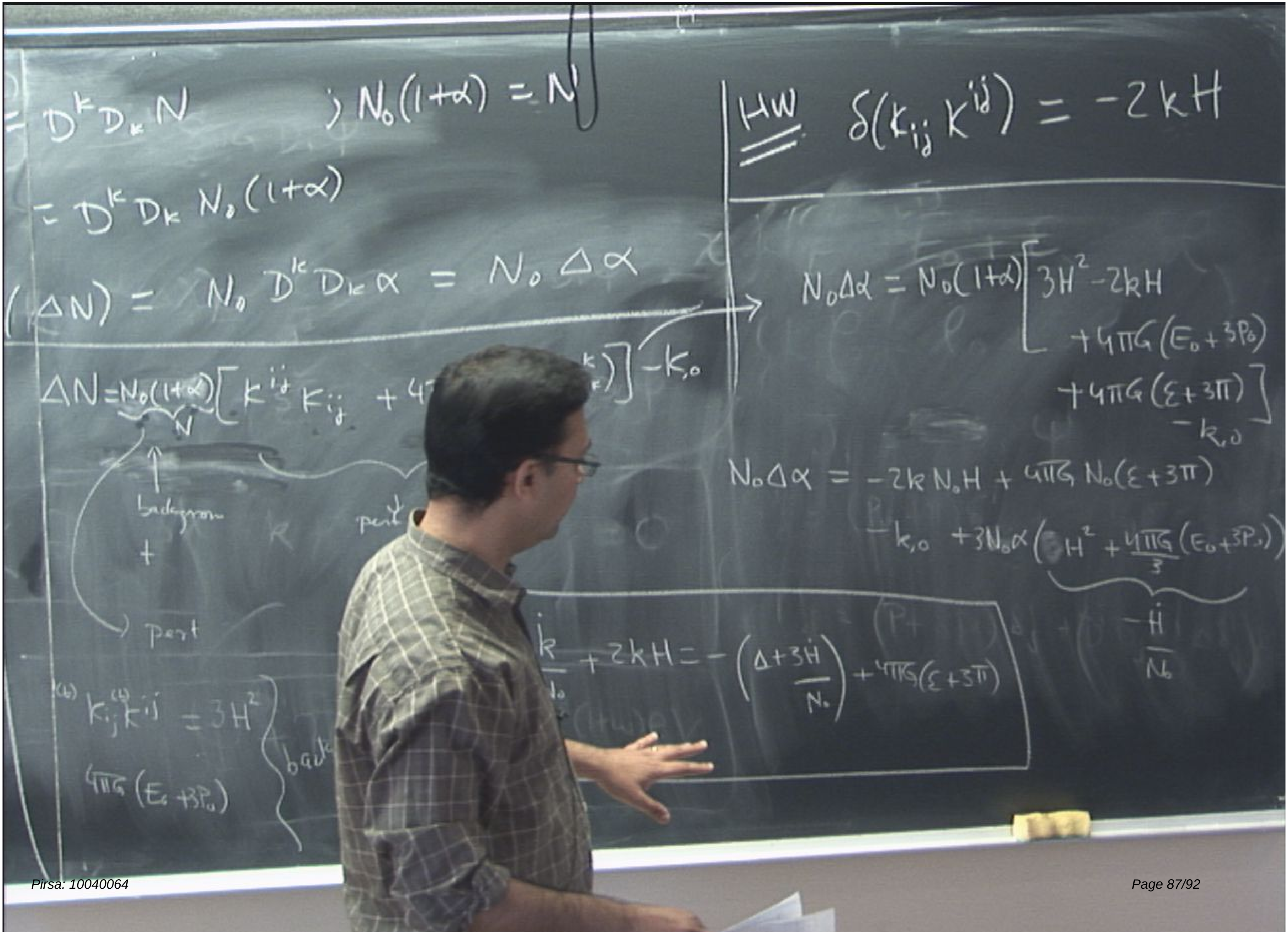
background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$$N_0\Delta\alpha = N_0(1+\alpha)\left[3H^2 - 2kH + 4\pi G(E_0 + 3P_0) + 4\pi G(\varepsilon + 3\pi)\right] - k_{,0}$$

$$N_0\Delta\alpha = -2kN_0H + 4\pi G N_0(\varepsilon + 3\pi) - k_{,0} + 3N_0\alpha\left(H^2 + \frac{4\pi G}{3}(E_0 + 3P_0)\right)$$


= DᵏDₖN ; N₀(1+α) = N
= DᵏDₖN₀(1+α)
(ΔN) = N₀DᵏDₖα = N₀Δα
ΔN = N₀(1+α)[Kⁱʲ Kᵢⱼ + 4πG(E + Sᵏₖ)] − k,₀
pert
HW δ(Kᵢⱼ Kⁱʲ) = −2kH
N₀Δα = N₀(1+α)[3H² − 2kH + 4πG(E₀ + 3P₀) + 4πG(ε + 3π)] − k,₀
N₀Δα = −2kN₀H + 4πG N₀(ε + 3π) − k,₀ + 3N₀α(H² + 4πG/3 (E₀ + 3P₀))
−Ḣ/N₀


D^k D_k N ; N_0(1+α) = N
= D^k D_k N_0(1+α)
(ΔN) = N_0 D^k D_k α = N_0 Δα
ΔN = N_0(1+α)[K^ij K_ij + 4 ...)] - K,0
background
+
pert
pert
HW δ(K_ij K^ij) = -2kH
N_0Δα = N_0(1+α)[3H^2 - 2kH + 4πG(E_0 + 3P_0) + 4πG(ε + 3π)] - k,0
N_0Δα = -2kN_0H + 4πG N_0(ε + 3π) - k,0 + 3N_0α(H^2 + 4πG/3 (E_0 + 3P_0))
-Ḣ/N_0
k̇/N_0 + 2kH = -(Δ + 3Ḣ/N_0) + 4πG(ε + 3π)
K_ij K^ij = 3H^2
4πG(E_0 + 3P_0)
back

$= D^k D_k N \qquad ; N_0(1+\alpha) = N$

$= D^k D_k N_0(1+\alpha)$

$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta\alpha$

$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\left[K^{ij}K_{ij} + 4\pi G(E + S^k_k)\right]$

background + pert

pert

backg

$K^{(0)}_{ij}K^{ij} = 3H^2$

$4\pi G(E_0 + 3P_0)$

background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$N_0\Delta\alpha = N_0(1+\alpha)\left[3H^2 - 2kH + 4\pi G(E_0 + 3P_0) + 4\pi G(\mathcal{E} + 3\Pi)\right] - k_{,0}$

$\alpha = -2kN_0H + 4\pi G N_0(\mathcal{E} + 3\Pi) - k_{,0} + 3N_0\alpha\underbrace{\left(H^2 + \frac{4\pi G}{3}(E_0 + 3P_0)\right)}_{-\frac{\dot H}{N_0}}$

$\left(\Delta + \frac{3\dot H}{N_0}\right) + 4\pi G(\mathcal{E} + 3\Pi)$

$$= D^k D_k N \qquad ; N_0(1+\alpha) = N$$

$$= D^k D_k N_0(1+\alpha)$$

$$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$$

$$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\left[K^{ij}K_{ij} + 4\pi G(E + S^k_k)\right] - K_{,0}$$

background + pert — pert, backg

$$^{(0)}K_{ij}\,^{(0)}K^{ij} = 3H^2$$

$$4\pi G(E_0 + 3P_0)$$

background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$$N_0\Delta\alpha = N_0(1+\alpha)\left[3H^2 - 2kH + 4\pi G(E_0 + 3P_0) + 4\pi G(\mathcal{E} + 3\Pi)\right] - K_{,0}$$

$$N_0\Delta\alpha = -2kN_0H + 4\pi G N_0(\mathcal{E}+3\Pi) - K_{,0} + 3N_0\alpha\underbrace{\left(H^2 + \frac{4\pi G}{3}(E_0+3P_0)\right)}_{-\frac{\dot H}{N_0}}$$

$$\boxed{\frac{\dot k}{N_0} + 2kH = -\left(\Delta + \frac{3\dot H}{N_0}\right)\alpha + 4\pi G(\mathcal{E}+3\Pi)}$$

$$= D^k D_k N \qquad ; N_0(1+\alpha) = N$$

$$= D^k D_k N_0(1+\alpha)$$

$$(\Delta N) = N_0 D^k D_k \alpha = N_0 \Delta \alpha$$

$$\Delta N = \underbrace{N_0(1+\alpha)}_{N}\Big[\underbrace{K^{ij}K_{ij} + 4\pi G(E + S^k_k)}_{\text{pert}}\Big] - K_{,0}$$

background

+

pert

backg

$$K^{(0)}_{ij}K^{(0)ij} = 3H^2$$

$$4\pi G(E_0 + 3P_0)$$

background

HW $\delta(K_{ij}K^{ij}) = -2kH$

$$N_0\Delta\alpha = N_0(1+\alpha)\Big[3H^2 - 2kH + 4\pi G(E_0 + 3P_0) + 4\pi G(\mathcal{E} + 3\Pi)\Big] - k_{,0}$$

$$N_0\Delta\alpha = -2kN_0H + 4\pi G N_0(\mathcal{E} + 3\Pi) - k_{,0} + 3N_0\alpha\Big(\underbrace{H^2 + \frac{4\pi G}{3}(E_0 + 3P_0)}_{-\frac{\dot H}{N_0}}\Big)$$

$$\boxed{\frac{\dot k}{N_0} + 2kH = -\left(\Delta + \frac{3\dot H}{N_0}\right)\alpha + 4\pi G(\mathcal{E} + 3\Pi)}$$

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k =$$

$$- D^i D_j N + N\left[R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i_j (S^k_k - E)\right]$$

Trace

$$\Rightarrow \boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G (E + S^k{}_k)\right] - K_{,0}}$$

Eq(45) $\longrightarrow$

$$\boxed{\delta S^i{}_i = 3\pi}$$

$$\circledast \quad \delta\left(4\pi G (E + S^k{}_k)\right) = 4\pi G (\mathcal{E} + 3\pi)$$

$K_{,0} \longrightarrow (\delta K)_{,0}$

$\delta K = k$

$K_{,0} \xrightarrow{\text{Perturbation}} k_{,0}$

$\delta(K_{ij} K^{ij}) = -2kH$

$$K^i_{j,0} + N^k K^i_{j,k} - N^i_{,k} K^k_j + N^k_{,j} K^i_k =$$

$$- D^i D_j N + N\left[R^i_j + K K^i_j - 8\pi G S^i_j + 4\pi G \delta^i_j (S^k_k - E) \right]$$

Trace

$$\Rightarrow \quad \boxed{\Delta N = N\left(K^{ij} K_{ij} + 4\pi G (E + S^k_k) \right] - K_{,0}}$$

$$\text{Eq}(45) \longrightarrow \boxed{\Psi - \Phi = 8\pi G \sigma}$$

$$K_{,0} \rightarrow (\delta K)_{,0}$$

$$\delta K = k$$

$$K_{,0} \xrightarrow{\text{Perturbation}} k_{,0}$$

$$\delta(K_{ij} K^{ij}) = -2kH$$

(i)

(ii)

$$\boxed{\delta S^i_i = 3\pi}$$

$$\circledast \quad \delta\left(4\pi G (E + S^k_k)\right) = 4\pi G (\mathcal{E} + 3\pi)$$