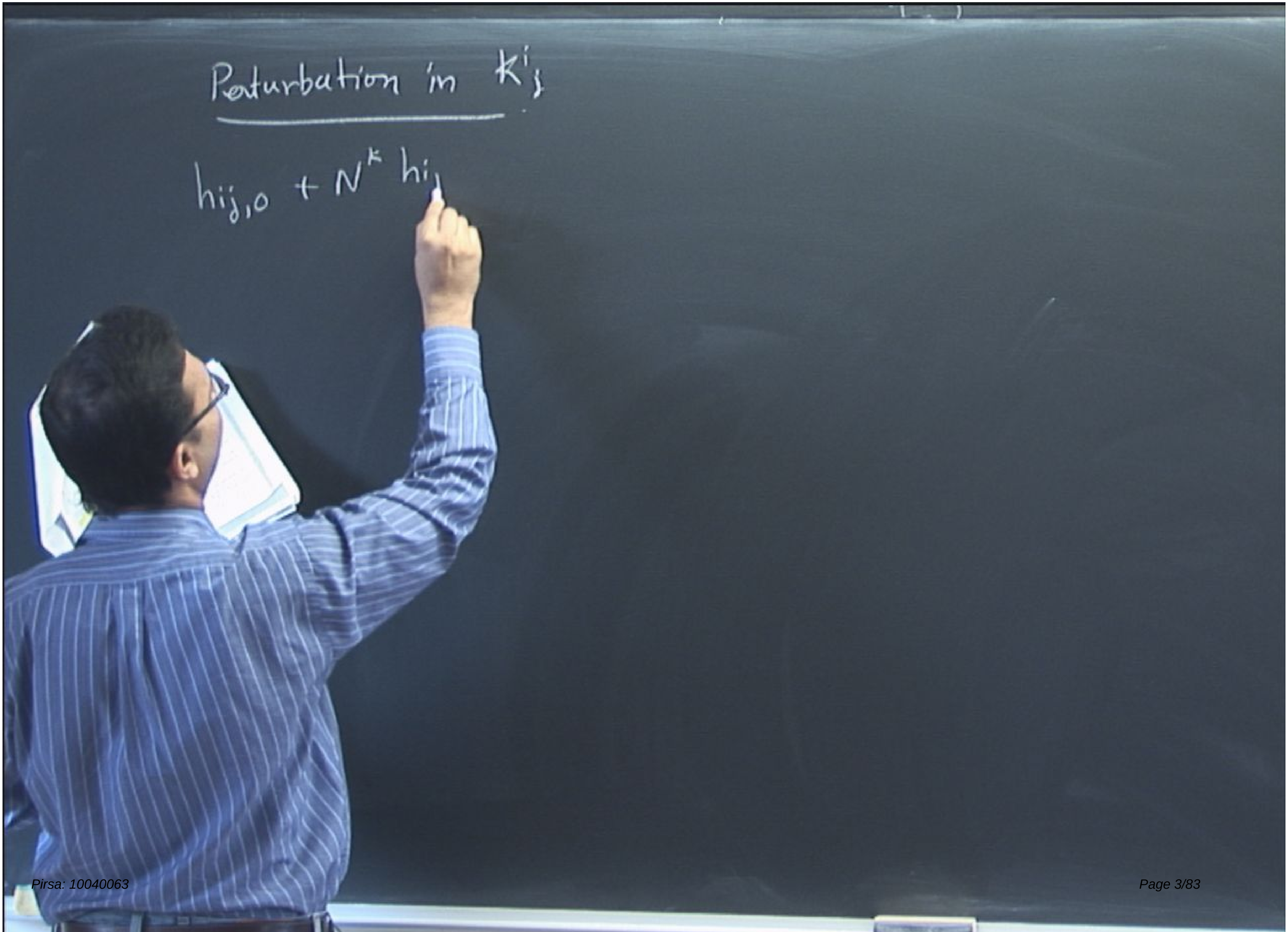
Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 6

Date: Apr 12, 2010 09:30 AM

URL: http://pirsa.org/10040063

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

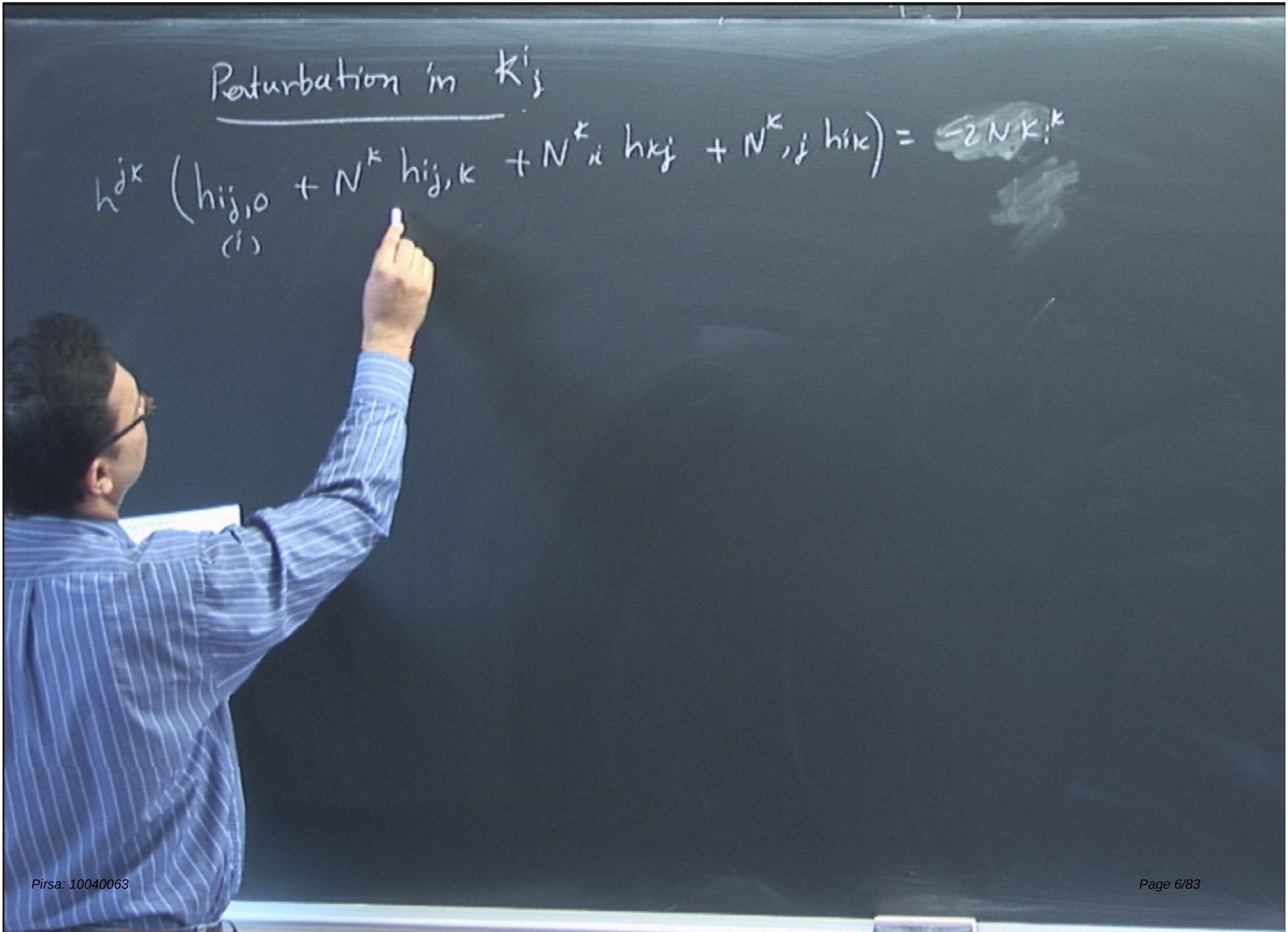

Perturbation in K^i_j
$h_{ij,0} + N^k h_{i}$

Perturbation in $K^i{}_j$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$$

Perturbation in $K^i{}_j$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$$



Perturbation in $K^i{}_j$
$h^{jk}\left(h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik}\right) = -2NK_i{}^k$
(1)

Perturbation in K^i_j

$$h^{jk}\Big(\underset{(i)}{h_{ij,0}} + \underset{(ii)}{N^k h_{ij,k}} + \underset{(iii)}{N^k{}_{,i} h_{kj}} + \underset{(iv)}{N^k{}_{,j} h_{ik}}\Big) = -2NK_i{}^k$$

$$(i) \quad h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^j D^k \gamma\right)$$

Perturbation in $K^i{}_j$

$$h^{jk}\left(h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i}\, h_{kj} + N^k{}_{,j}\, h_{ik}\right) = -2NK_i{}^k$$

(i) (ii) (iii) (iv)

(i) $h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD\right.\right.$

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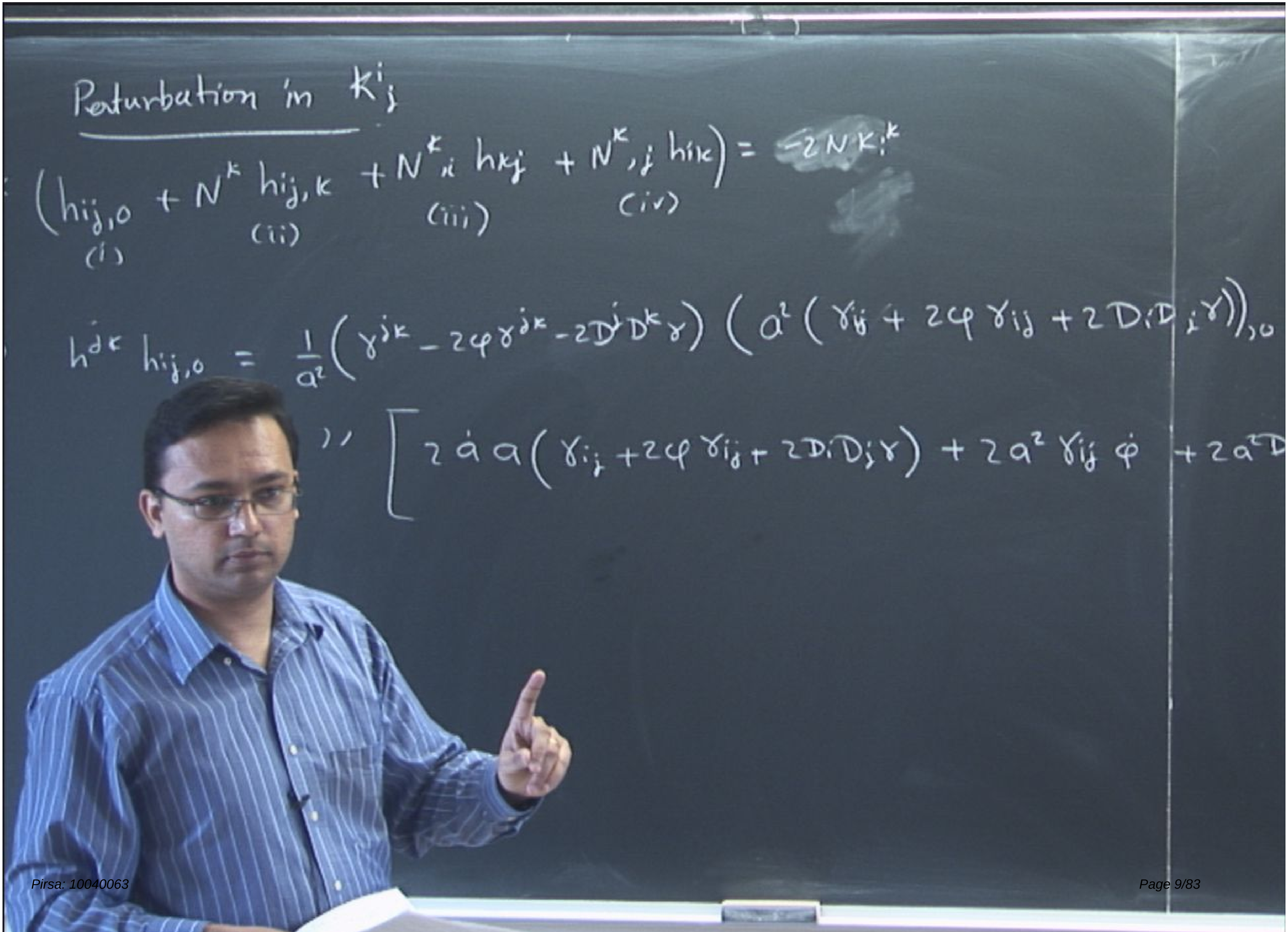

Perturbation in K^i_j
$(h_{ij,0} + N^k h_{ij,k} + N^k_{,i} h_{kj} + N^k_{,j} h_{ik}) = -2NK_i^k$
(i) (ii) (iii) (iv)
$h^{jk} h_{ij,0} = \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma)(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma))_{,0}$
" $[2\dot{a}a(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D$

Perturbation in K^i_j

$$\left(h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik}\right) = -2NK_i{}^k$$

$$\text{(i)} \quad \text{(ii)} \quad \text{(iii)} \quad \text{(iv)}$$

$$h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^j D^k \gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i D_j \gamma\right)\right)_{,0}$$

$$\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^j D^k\gamma\Big)\Big[2\dot{a}a\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i D_j\gamma\right) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D$$

Perturbation in $K^i{}_j$

$$h^{jk}\Big(\underset{(i)}{h_{ij,0}} + \underset{(ii)}{N^k h_{ij,k}} + \underset{(iii)}{N^k{}_{,i}\, h_{kj}} + \underset{(iv)}{N^k{}_{,j}\, h_{ik}}\Big) = -2NK_i{}^k$$

$$\text{(i)}\quad h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\Big(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right)\Big)_{,0}$$

$$\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\Big)\Big[2\dot a a\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot\varphi + 2a^2 D_iD_j\dot\gamma\Big]$$

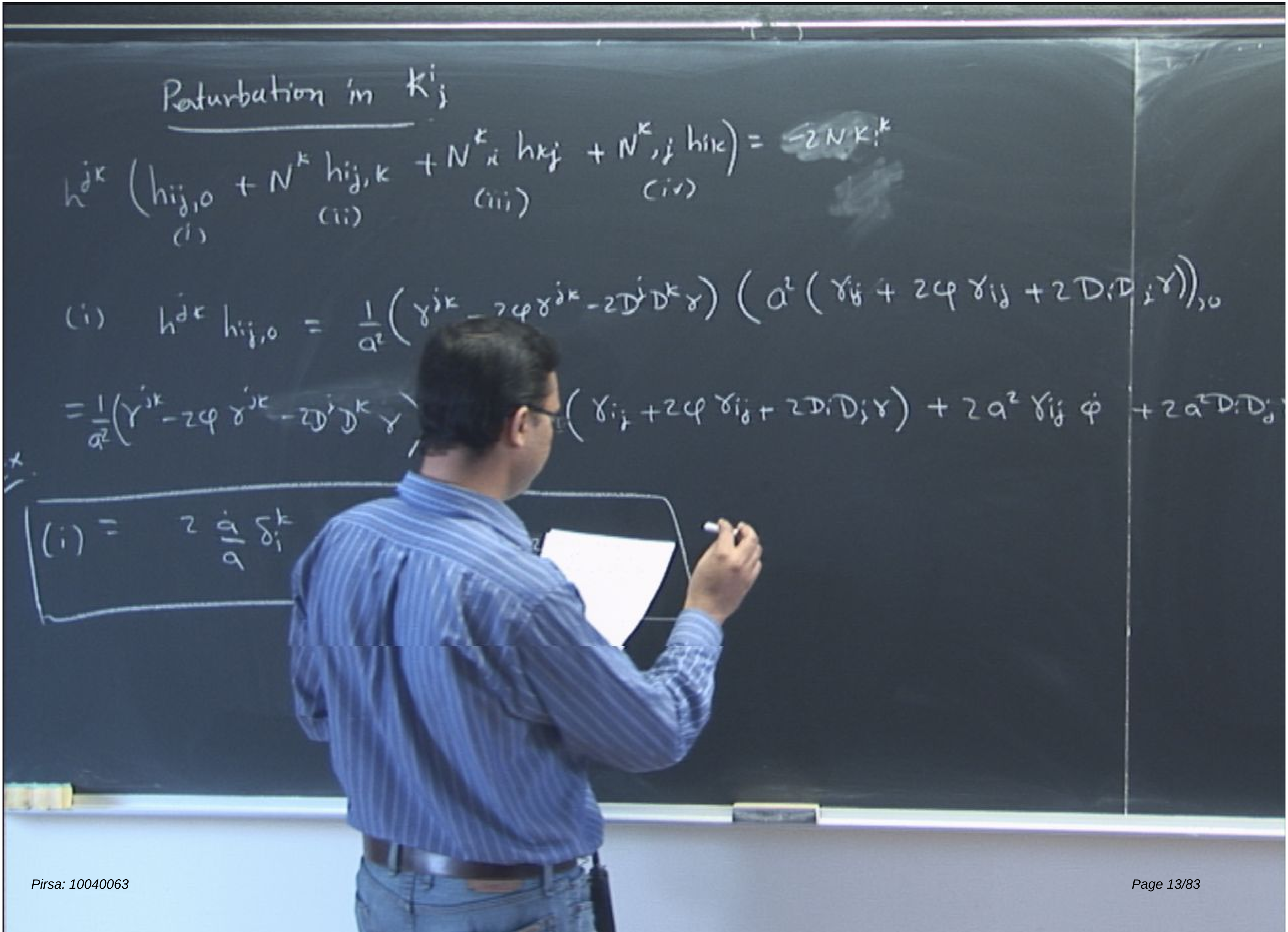
Perturbation in $K^i{}_j$

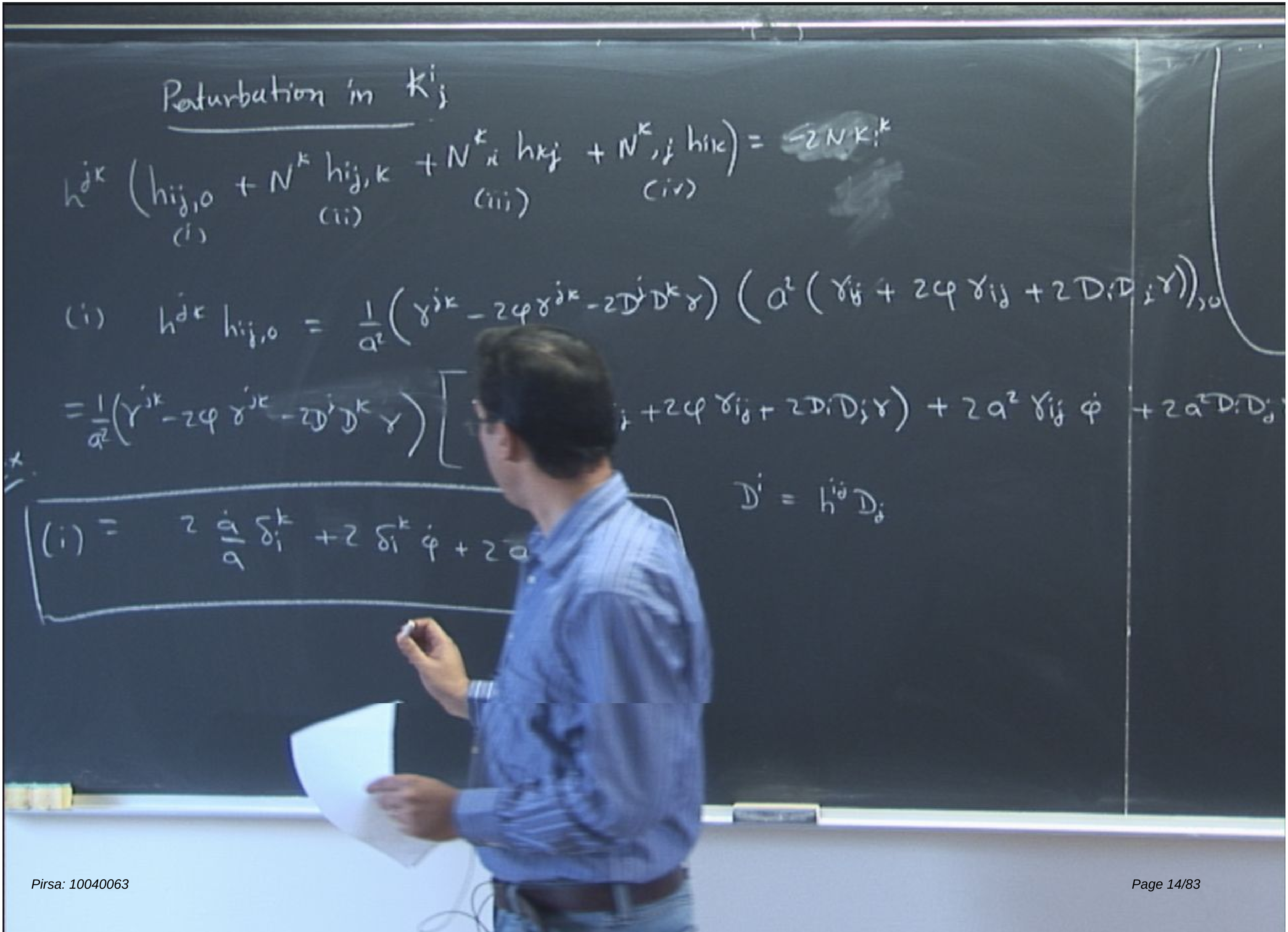
$$h^{jk}\Big(\underset{(i)}{h_{ij,0}} + \underset{(ii)}{N^k h_{ij,k}} + \underset{(iii)}{N^k{}_{,i} h_{kj}} + \underset{(iv)}{N^k{}_{,j} h_{ik}}\Big) = -2NK_i{}^k$$

$$(i)\quad h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right)\right)_{,0}$$

$$= \frac{1}{a^2}\left(\gamma^{jk} \ldots 2D^jD^k\gamma\right)\left[2\dot a a\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot\varphi + 2a^2 D_iD_j\dot\gamma\right]$$

$$= \ldots + 2\delta_i{}^k\dot\varphi + 2a^2 D_iD^k\dot\gamma$$


Perturbation in $K^i{}_j$
$h^{jk}\left(h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik}\right) = -2NK_i{}^k$
(i) (ii) (iii) (iv)
(i) $h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right)\right)_{,0}$
$= \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)$ $\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot\varphi + 2a^2D_iD_j$
(i) = $2\frac{\dot a}{a}\delta_i^k$


Perturbation in K^i_j
$h^{jk}\left(h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik}\right) = -2NK_i^k$
(i) (ii) (iii) (iv)
(i) $h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right)\right)_{,0}$
$= \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left[\ \ldots + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot\varphi + 2a^2D_iD_j$
$D^i = h^{ij}D_j$
(i) = $2\frac{\dot a}{a}\delta_i^k + 2\delta_i^k\dot\varphi + 2a$

Perturbation in K^i_j

$$h^{jk}\Big(\underset{(i)}{h_{ij,0}} + \underset{(ii)}{N^k h_{ij,k}} + \underset{(iii)}{N^k{}_{,i} h_{kj}} + \underset{(iv)}{N^k{}_{,j} h_{ik}}\Big) = -2NK_i{}^k$$

$$(i)\quad h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^j D^k \gamma\right)\Big(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i D_j \gamma\right)\Big)_{,0}$$

$$= \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^j D^k\gamma\right)\Big[2\dot{a}a\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_i D_j \dot{\gamma}$$

$$D^i = h^{ij} D_j$$

$$\boxed{(i) = 2\underbrace{\frac{\dot{a}}{a}}\delta_i^k + 2\delta_i^k\dot{\varphi} + 2a^2 D_i D^k \dot{\gamma}}$$

(ii) $N^k h_{ij,k} =$

$\ldots_i \gamma)\big)\big)_{,0}$

$+ 2a^2 D_i D_j \gamma \Big]$

$N_i = a^2 D_i \beta$

$N^k = h^{ki} N_i$

$= \frac{1}{a^2} \gamma^{ki} a^2 D_i \beta$

$= \gamma^{ki} D_i \beta$

$$(ii)\quad N^k h_{ij,k} = \left(a^2 D^k\beta\right)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma),_k\right)$$

$$\ldots_i\gamma)\Big)\Big),_0$$

$$+2a^2D_iD_j\gamma\Big]$$

$$N_i = a^2 D_i\beta$$

$$N^k = h^{ki} N_i$$

$$= \frac{1}{a^2}\gamma^{ki} a^2 D_i\beta$$

$$= \gamma^{ki} D_i\beta$$

$$= a^2 D^k\beta$$

$$(ii)\quad N^k h_{ij,k} = (a^2 D^k\beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)_{,k}\right)$$

$$\simeq a^2 D^k\beta\,\underbrace{(a^2\gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$i\gamma)\big)\big)_{,0}$$

$$+2a^2 D_iD_j\gamma\,\Big]$$

$$N_i = a^2 D_i\beta$$

$$N^k = h^{ki}N_i$$

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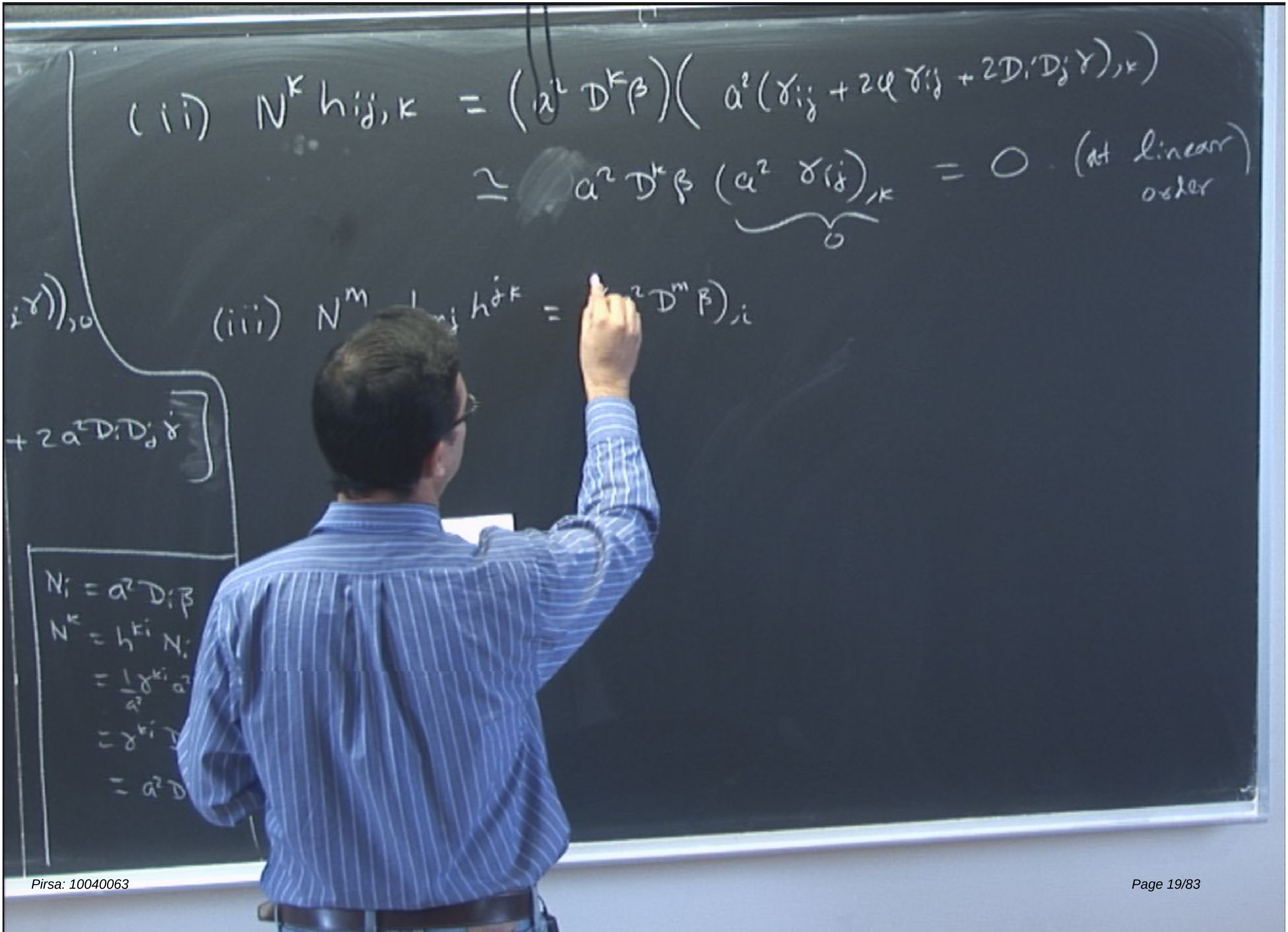
$$= a^2 D^k\beta$$


(ii) N^k h_ij,k = (a^2 D^k β)(a^2(γ_ij + 2ψ γ_ij + 2D_i D_j γ),k)
≃ a^2 D^k β (a^2 γ_ij),k = 0 (at linear order)
0
(iii) N^m
2 D^m β),i
+ 2a^2 D_i D_j γ
N_i = a^2 D_i β
N^k = h^ki N_i

$$(ii)\quad N^k h_{ij,k} = (a^2 D^k\beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)_{,k}\right)$$

$$\simeq a^2 D^k\beta\,\underbrace{(a^2\,\gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$(iii)\quad N^m{}_{,i}\, h_{mj}\, h^{jk} = \left[(a^2 D^m\beta)_{,i}\right] a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_mD_j\gamma)$$

$$\frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - D^jD^k\gamma)$$

$$\gamma))_{,0}$$

$$\boxed{+2\ \ ^2D_iD_j\gamma}$$

$$(ii)\quad N^k h_{ij,k} = \left(a^2 D^k \beta\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right)_{,k}\right)$$

$$\simeq a^2 D^k \beta \underbrace{\left(a^2 \gamma_{ij}\right)_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$(iii)\quad N^m{}_{,i}\, h_{mj}\, h^{jk} = \left[\left(a^2 D^m \beta\right)_{,i}\right] a^2\left(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_mD_j\gamma\right)$$

$$\frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - D^jD^k\gamma\right)$$

$\gamma)\Big)_{,0}$

$+2a^2 D_i D_j \gamma$

(ii) $N^k h_{ij,k} = (a^2 D^k \beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)_{,k}\right)$

$\simeq a^2 D^k \beta \, \underbrace{(a^2 \gamma_{ij})_{,k}}_{0} = 0$ (at linear order)

(iii) $N^m{}_{,i} h_{mj} h^{jk} = \left[(a^2 D^m \beta)_{,i}\right]\left[a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_mD_j\gamma) \; \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - D^jD^k\gamma)\right]$

$a^2(D^m \beta)$

$N_i = a^2 D_i$

$N^k = h^{ki}$

$= \frac{1}{a^2}\gamma$

$=$

$=$

$$(ii)\quad N^k h_{ij,k} = (a^2 D^k\beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)_{,k}\right)$$

$$\simeq a^2 D^k\beta\,\underbrace{(a^2\,\gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$\gamma))_{,0}$$

$$D_iD_j\gamma$$

$$(iii)\quad N^m{}_{,i}\, h_{mj}\, h^{jk} = \left[(a^2 D^m\beta)_{,i}\right]\begin{bmatrix} a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_mD_j\gamma) \\ \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - D^jD^k\gamma) \end{bmatrix}$$

$$\simeq a^2(D_iD^m\beta)\,\delta^k{}_m$$

$$(iv)\quad N^k{}_{,j}\, h_{ik}$$

$$\gamma)\big)_{,0}$$

$$+ 2a^2 D_i D_j \gamma \Big]$$

$$N_i = a^2 D_i \beta$$
$$N^k = h^{ki} N_i$$
$$= \frac{1}{a^2}\gamma^{ki} a^2 D_i \beta$$
$$= \gamma^{ki} D_i \beta$$
$$= a^2 D^k \beta$$

(ii) $$N^k h_{ij,k} = (a^2 D^k \beta)\big(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2 D_i D_j \gamma)_{,k}\big)$$

$$\simeq a^2 D^k \beta\, \underbrace{(a^2\, \gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

(iii) $$N^m{}_{,i}\, h_{mj} h^{jk} = \left[(a^2 D^m \beta)_{,i}\right] \begin{bmatrix} a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2 D_m D_j \gamma) \\ \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - D^j D^k \gamma) \end{bmatrix}$$

$$\simeq a^2 (D_i D^m \beta)\, \delta^k{}_m$$

(iv) $$N^k{}_{,j}\, h_{im} h^{jm} =$$

Perturbation in $K^i{}_j$

$$h^{jk}\Big(h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{i\cdot}\Big) = -2NK_i{}^k$$

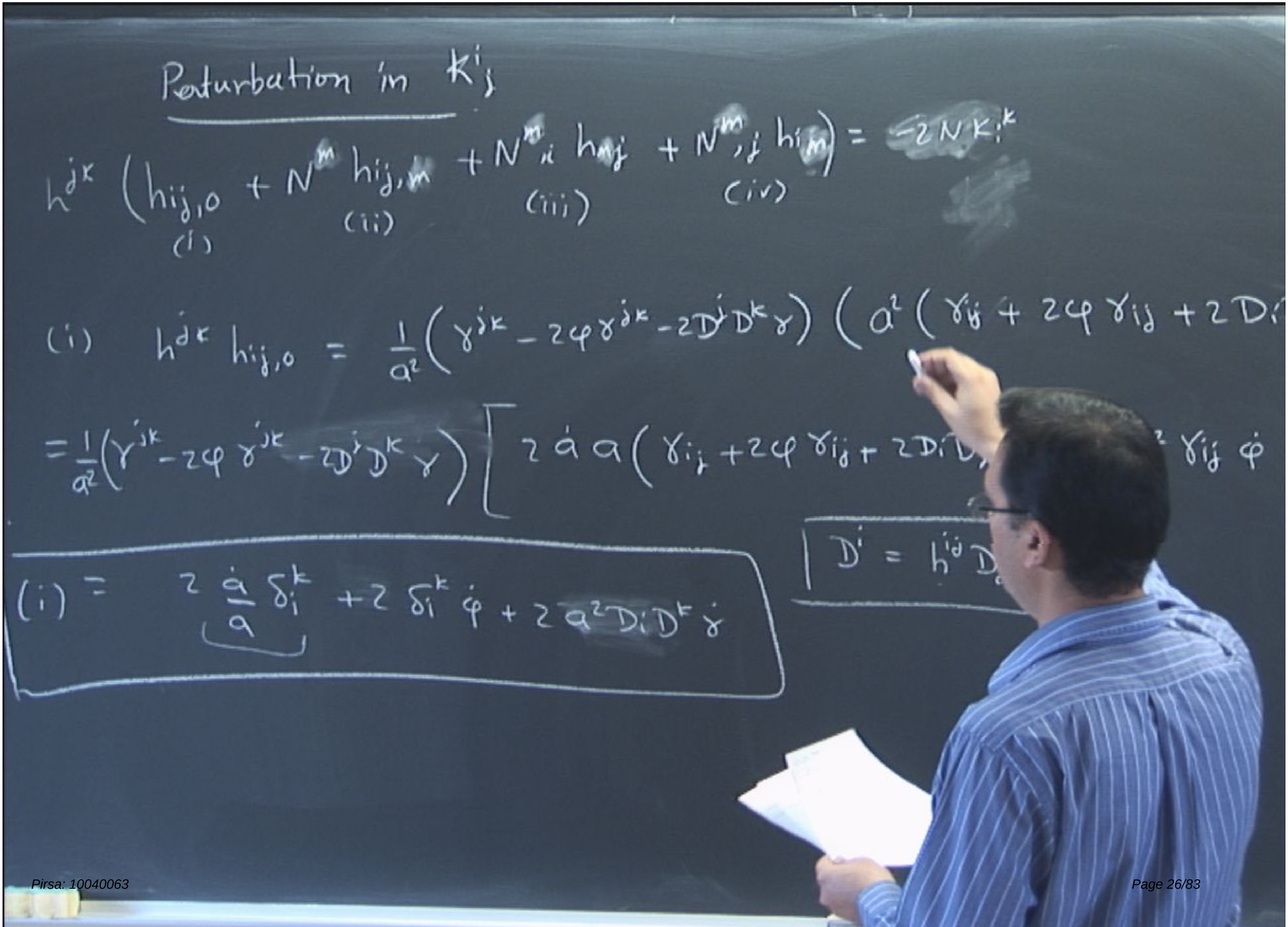
(i) (ii) (iii) (iv)

$$\text{(i)} \quad h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i\right.\right.$$

$$= \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\Big[\ \ldots \gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot\varphi$$

$$\boxed{D^i = h^{ij}D_j}$$

$$\text{(i)} = 2\underbrace{\frac{\dot a}{a}}\delta_i^k + 2\delta_i^k\dot\varphi + 2\ldots$$


Perturbation in $K^i{}_j$
$h^{jk}\big(\underset{(i)}{h_{ij,0}} + \underset{(ii)}{N^m h_{ij,m}} + \underset{(iii)}{N^m{}_{,i} h_{mj}} + \underset{(iv)}{N^m{}_{,j} h_{im}}\big) = -2NK_i{}^k$
(i) $h^{jk} h_{ij,0} = \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma)\,(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i$
$= \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma)\,[2\dot a a(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD$ … $\gamma_{ij}\dot\varphi$
$D^i = h^{ij}D_j$
$\boxed{(i) = 2\underbrace{\frac{\dot a}{a}}\delta_i^k + 2\delta_i^k\dot\varphi + 2a^2D_iD^k\dot\gamma}$

Perturbation in $K^i{}_j$

$$h^{jk}\Big(\underset{(i)}{h_{ij,0}} + \underset{(ii)}{N^m h_{ij,m}} + \underset{(iii)}{N^m{}_{,i}\, h_{mj}} + \underset{(iv)}{N^m{}_{,j}\, h_{im}}\Big) = -2NK_i{}^k$$

(i) $$h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i\right.\right.$$

$$= \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\Big[2\dot a a\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot\varphi$$

$$\boxed{D^i = h^{ij}D_j}$$

$$\boxed{(i) = 2\underbrace{\frac{\dot a}{a}}\delta_i^k + 2\delta_i^k\dot\varphi + 2a^2D_iD^k\dot\gamma}$$

$$(ii) \quad N^k h_{ij,k} = \left(a^2 D^k\beta\right)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)_{,k}\right)$$

$$\simeq a^2 D^k\beta\, \underbrace{(a^2\gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$(iii) \quad N^m{}_{,i}\, h_{mj}\, h^{jk} = \left[(a^2 D^m\beta)_{,i}\right]\left[a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_mD_j\gamma)\ \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - D^jD^k\gamma)\right]$$

$$\simeq a^2(D_iD^m\beta)\,\delta^k{}_m$$

$$(iii) + (iv) = 2a^2 D_i D^k\beta$$

$$-2N\, k_i{}^k = -2N_0(1+\alpha)k_i{}^k = -2N_0 k_i{}^k$$

$$N_i = a^2 D_i\beta$$
$$N^k = h^{ki}N_i = \frac{1}{a^2}\gamma^{ki}a^2 D_i\beta = \gamma^{ki}D_i\beta = a^2 D^k\beta$$

$$\ldots \gamma))_{,0} \qquad + 2a^2 D_iD_j\gamma \,]$$

$$(ii)\quad N^k h_{ij,k} = (a^2 D^k\beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)_{,k}\right)$$

$$\simeq a^2 D^k\beta\, \underbrace{(a^2\,\gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$(iii)\quad N^m{}_{,i}\, h_{mj}\, h^{jk} = \left[(a^2 D^m\beta)_{,i}\right]\left[a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_mD_j\gamma)\quad \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - \ldots \right]$$

$$\simeq a^2 (D_i D^m\beta)\,\delta^k{}_m$$

$$(iii) + (iv) = 2\, a^2 D_i D^k\beta$$

$$-2N K_i{}^k = -2N_o(1+\alpha)K_i{}^k = -2N_o K_i{}^k + 2N_o\alpha H\delta_i{}^k$$

$$\ldots\gamma))_{,0}$$

$$+ 2a^2 D_iD_j\gamma$$

$$N_i = a^2 D_i\beta$$
$$N^k = h^{ki}N_i$$
$$= \frac{1}{a^2}\gamma^{ki}a^2 D_i\beta$$
$$= \gamma^{ki}D_i\beta$$
$$= a^2 D^k\beta$$

$_i\gamma))_{,0}$

$+2a^2 D_i D_j \gamma$

(ii) $N^k h_{ij,k} = (a^2 D^k \beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i D_j \gamma)\right)_{,k}$

$\simeq a^2 D^k \beta \underbrace{(a^2 \gamma_{ij})_{,k}}_{0} = 0$ (at linear order)

(iii) $N^m{}_{,i} h_{mj} h^{jk} = \left[(a^2 D^m \beta)_{,i}\right]\left[a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_m D_j \gamma) \; \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - \ldots\right]$

$\simeq a^2 (D_i D^m \beta)\delta^k{}_m$

$(iii) + (iv) = 2a^2 D_i D^k \beta$

$N_i = a^2 D_i \beta$

$N^k = h^{ki} N_i$

$= \frac{1}{a^2}\gamma^{ki} a^2 D_i \beta$

$= \gamma^{ki} D_i \beta$

$= a^2 D^k \beta$

$-2N K_i{}^k = -2N_0(1+\alpha)K_i{}^k = -2N_0 K_i{}^k + 2N_0 \alpha H \delta_i{}^k$

$$(ii)\quad N^k h_{ij,k} = (a^2 D^k\beta)\left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_i D_j\gamma)_{,k}\right)$$

$$\simeq a^2 D^k \beta\, \underbrace{(a^2\, \gamma_{ij})_{,k}}_{0} = 0 \quad \text{(at linear order)}$$

$$(iii)\quad N^m{}_{,i}\, h_{mj}\, h^{jk} = \left[(a^2 D^m\beta)_{,i}\right] \begin{bmatrix} a^2(\gamma_{mj} + 2\varphi\gamma_{mj} + 2D_m D_j\gamma) \\ \frac{1}{a^2}(\gamma^{jk} - 2\varphi\gamma^{jk} - D^j D^k\gamma) \end{bmatrix}$$

$$\simeq a^2(D_i D^m \beta)\delta^k{}_m$$

$$(iii) + (iv) = 2a^2 D_i D^k \beta$$

$$-2N K_i{}^k = -2N_0(1+\alpha)K_i{}^k = -2N_0 K_i{}^k + 2N_0\alpha H \delta_i{}^k$$

$$N_i = a^2 D_i\beta$$
$$N^k = h^{ki} N_i = \frac{1}{a^2}\gamma^{ki} a^2 D_i\beta = \gamma^{ki} D_i\beta = a^2 D^k\beta$$

$$\ldots{}_i\gamma))_{,0} \qquad + 2a^2 D_i D_j \gamma$$

Perturbation in K^i_j

$$h^{jk}\left(h_{ij,0} + N^m h_{ij,m} + N^m{}_{,i} h_{mj} + N^m{}_{,j} h_{im}\right) = -2NK_i{}^k$$
$$\quad\;\; \text{(i)} \qquad\quad \text{(ii)} \qquad\quad \text{(iii)} \qquad\quad \text{(iv)}$$

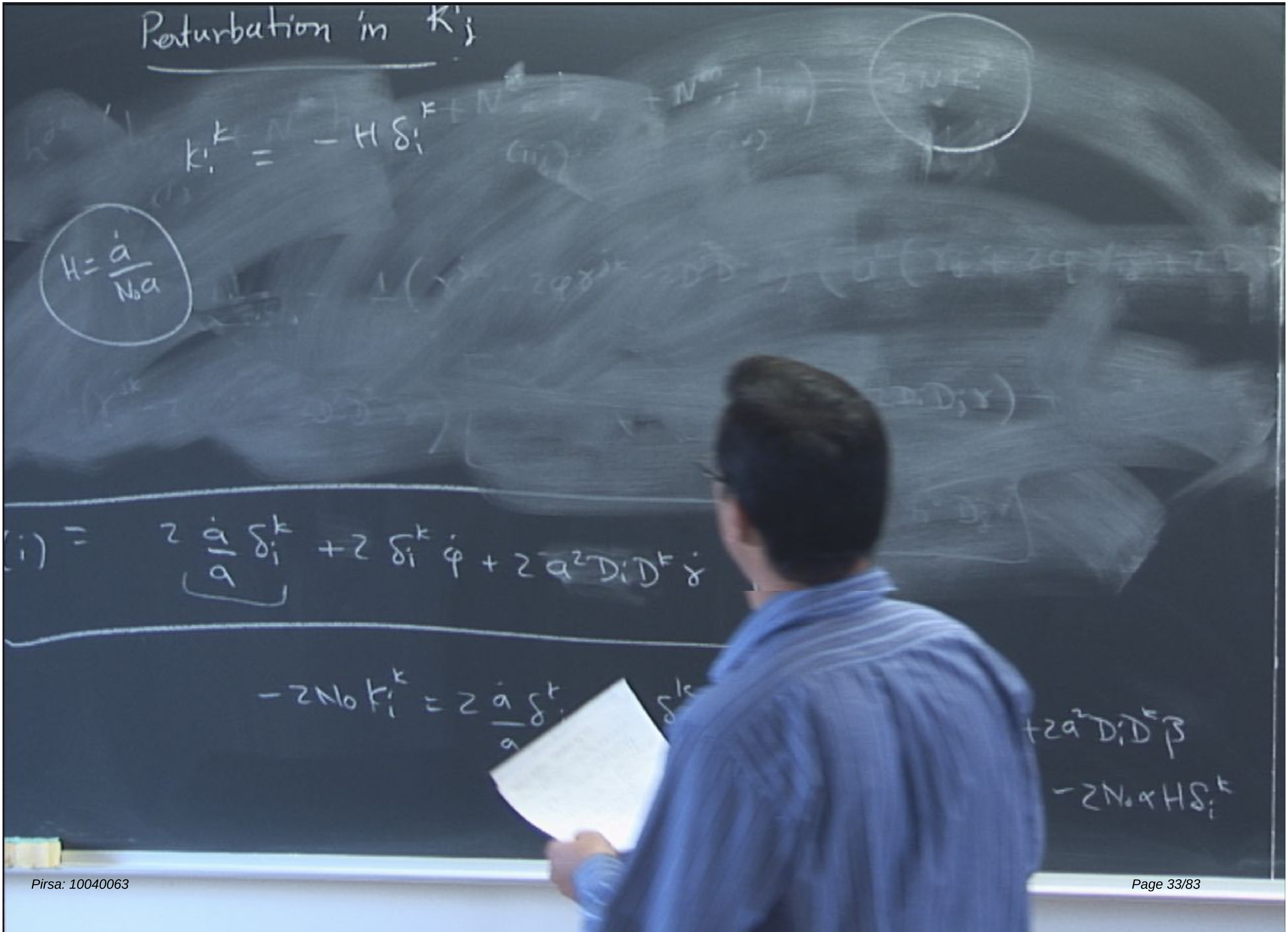
(i) $$h^{jk} h_{ij,0} = \frac{1}{a^2}\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left(a^2\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD\right.\right.$$

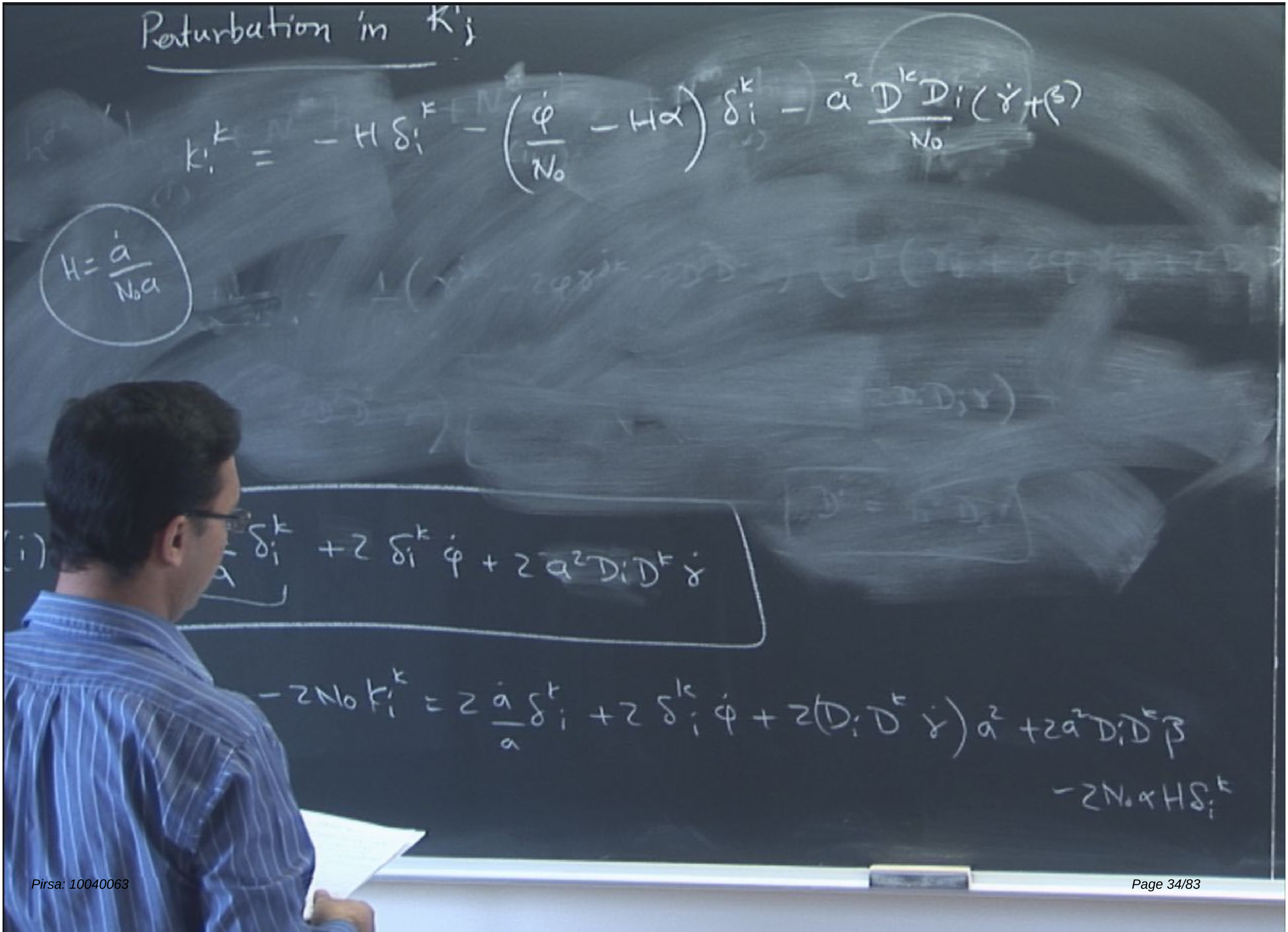
$$\left(\gamma^{jk} - 2\varphi\gamma^{jk} - 2D^jD^k\gamma\right)\left[2\dot{a}a\left(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma\right) + 2a^2\gamma_{ij}\dot{\varphi}\right.$$

$$\boxed{D^i = h^{ij}D_j}$$

$$\boxed{2\underbrace{\frac{\dot{a}}{a}}\delta_i^k + 2\delta_i^k\dot{\varphi} + 2a^2D_iD^k\dot{\gamma}}$$

$$-2N_0K_i{}^k = 2\frac{\dot{a}}{a}\delta^k{}_i + 2\delta^k{}_i\dot{\varphi} + 2\left(D_iD^k\dot{\gamma}\right)a^2 + 2a^2D_iD^k\beta$$
$$-2N_0\alpha H\delta_i^k$$


Perturbation in K^i_j
H = ȧ / N₀a
K_i^k = − H δ_i^k
(i) = 2 (ȧ/a) δ_i^k + 2 δ_i^k φ̇ + 2 a² D_i D^k γ̇
−2N₀ K_i^k = 2 (ȧ/a) δ^k_i
+2a² D_i D^k β
−2N₀ α H δ_i^k



Perturbation in K^i_j
$K_i^k = -H\delta_i^k - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta_i^k - a^2\frac{D^k D_i}{N_0}(\dot\gamma + \beta)$
$H = \frac{\dot a}{N_0 a}$
(i) $\frac{\dot a}{a}\delta_i^k + 2\delta_i^k\dot\varphi + 2a^2 D_i D^k\dot\gamma$
$-2N_0 K_i^k = 2\frac{\dot a}{a}\delta^k_i + 2\delta^k_i\dot\varphi + 2(D_i D^k\dot\gamma)a^2 + 2a^2 D_i D^k\beta$
$-2N_0\alpha H\delta_i^k$

Perturbation in K^i_j

$$\boxed{K_i^{\,k} = -H\delta_i^k - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta_i^k - a^2\frac{D^k D_i}{N_0}(\dot\gamma + \beta)}$$

$$H = \frac{\dot a}{N_0 a}$$

$$\boxed{\ldots) = 2\underbrace{\frac{\dot a}{a}}\delta_i^k + 2\delta_i^k\dot\varphi + 2a^2 D_i D^k\dot\gamma}$$

$$-2N_0 K_i^{\,k} = 2\frac{\dot a}{a}\delta^k_i + 2\delta^k_i\dot\varphi + 2(D_i D^k\dot\gamma)a^2 + 2a^2 D_i D^k\beta$$
$$-2N_0\alpha H\delta_i^k$$

Perturbation in K^i_j

$$\boxed{K_i^{\ k} = -H\delta_i^{\ k} - \underbrace{\left(\frac{\dot{\varphi}}{N_0} - H\alpha\right)\delta_i^k - a^2\frac{D^k D_i(\dot{\gamma}+\beta)}{N_0}}_{\text{perturbation}}}$$

background

$$H = \frac{\dot{a}}{N_0 a}$$

$$\boxed{(i) = 2\frac{\dot{a}}{a}\delta_i^k + 2\delta_i^k\dot{\varphi} + 2a^2 D_i D^k\dot{\gamma}}$$

$$-2N_0 K_i^{\ k} = 2\frac{\dot{a}}{a}\delta^k_{\ i} + 2\delta^k_{\ i}\dot{\varphi} + 2(D_i D^k\dot{\gamma})a^2 + 2a^2 D_i D^k\beta$$

$$-2N_0\alpha H\delta_i^{\ k}$$

Perturbation in K^i_j

$$\boxed{K_i^k = -H\delta_i^k \underbrace{-\left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta_i^k - a^2\frac{D^kD_i}{N_0}(\dot\gamma + \beta)}_{\text{perturbation}}}$$

background

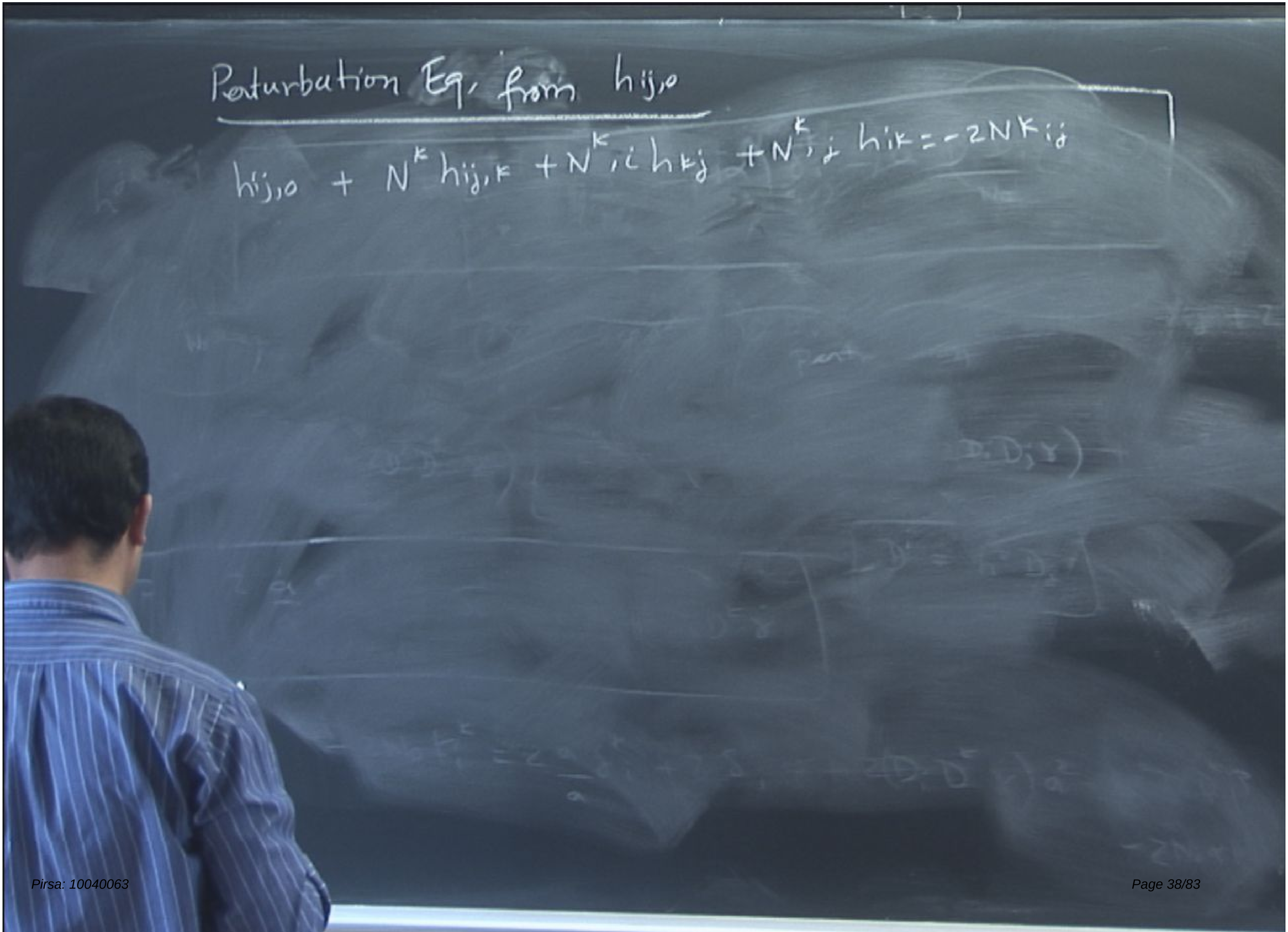
$= \frac{\dot a}{N_0 a}$

$$\boxed{= 2\frac{\dot a}{a}\delta_i^k + 2\delta_i^k\dot\varphi + 2a^2 D_i D^k\dot\gamma}$$

$$-2N_0 K_i^k = 2\frac{\dot a}{a}\delta_i^k + 2\delta_i^k\dot\varphi + 2(D_iD^k\dot\gamma)a^2 + 2a^2D_iD^k\beta$$
$$-2N_0\alpha H\delta_i^k$$

N_i

N^k


Perturbation Eq. from $h_{ij,0}$
$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$

Perturbation Eq. from $h_{ij,0}$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$$

$$\circledast. \quad h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$= \cdots \dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)$$

Perturbation Eq. from $h_{ij,0}$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$$

$$\circledast.\quad h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \Big[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j}$$

Perturbation Eq. from $\dot{h}_{ij,0}$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}$$

$$h_{ij,0} = (a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma))_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

Ex

$$N^k h_{ij,k} \simeq 0$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

(a). $$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

Ex (b) $N^k h_{ij,k} \simeq 0$

$\rightarrow N^k{}_{,i} h_{kj} \simeq a^4(D_iD^k\beta)\gamma_{kj}$

Perturbation Eq. from $h_{ij,0}$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i}\, h_{kj} + N^k{}_{,j}\, h_{ik} = -2N K_{ij}$$

① $$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

② $$N^k h_{ij,k} \simeq 0$$

$$N^k{}_{,i}\, h_{kj} \simeq a^4(D_iD^k\tilde{\beta})\gamma_{kj}$$

$$N^k{}_{,j}\, h_{ik}$$

Perturbation Eq. from $h_{ij,0}$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

$\rightarrow$ $N^k h_{ij,k} \simeq 0$

$\rightarrow$ $N^k{}_{,i} h_{kj} \simeq a^4(D_iD^k\beta)\gamma_{kj}$

$\rightarrow$ $N^k{}_{,j} h_{ik} \simeq a^4(D_iD^k\beta)\gamma_{k}$

Perturbation Eq. from $h_{ij,0}$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

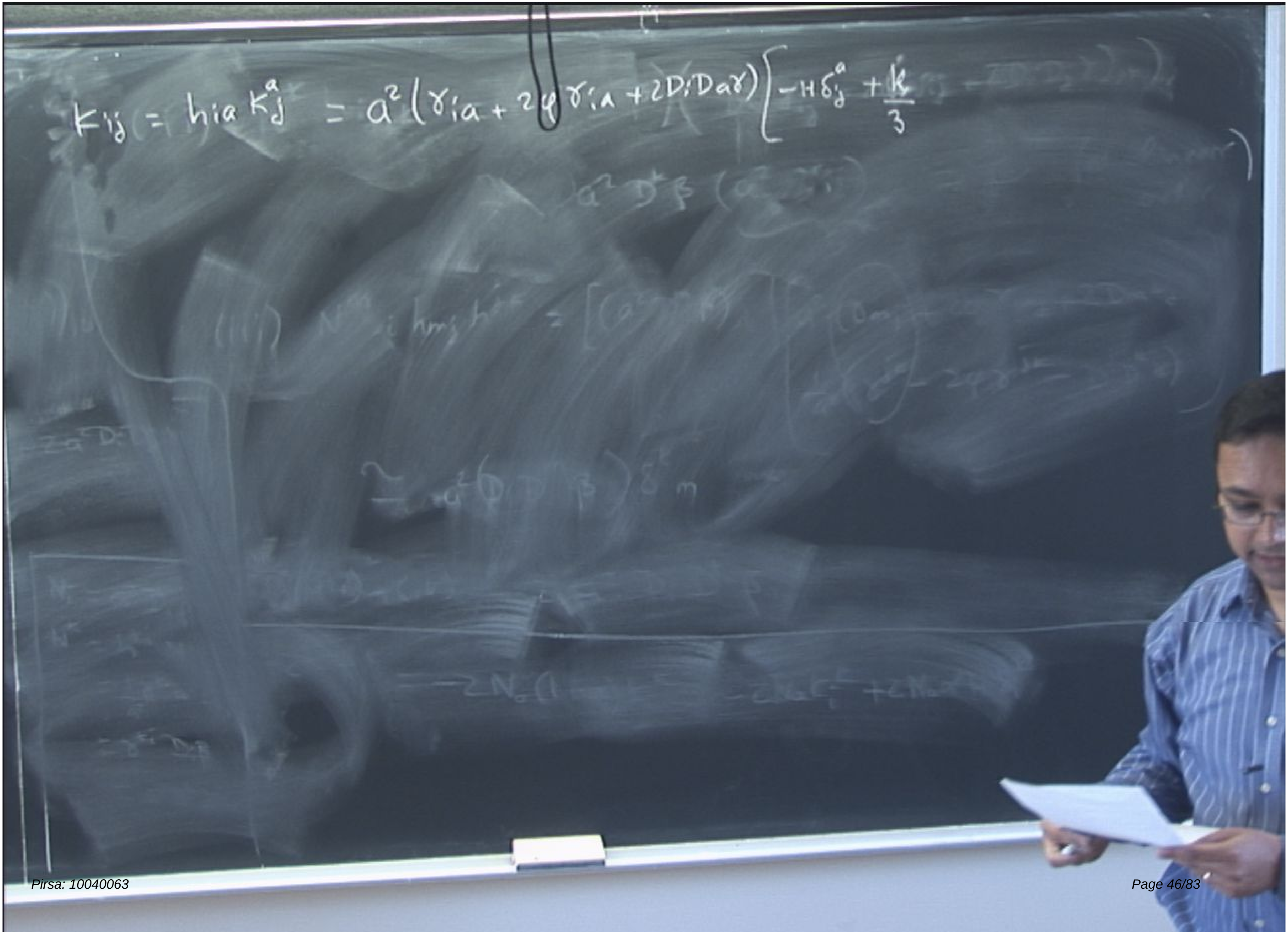
$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

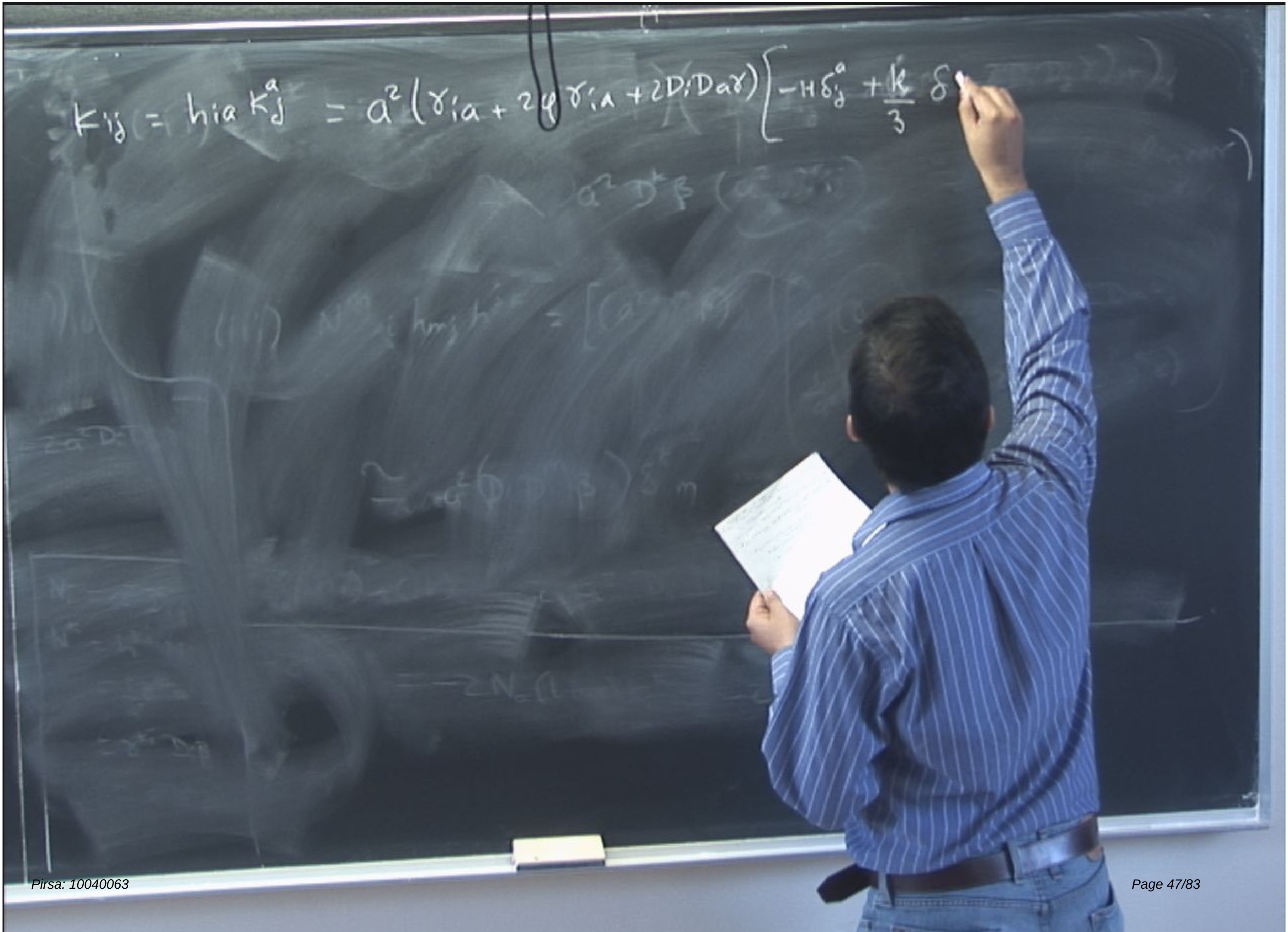
Ex

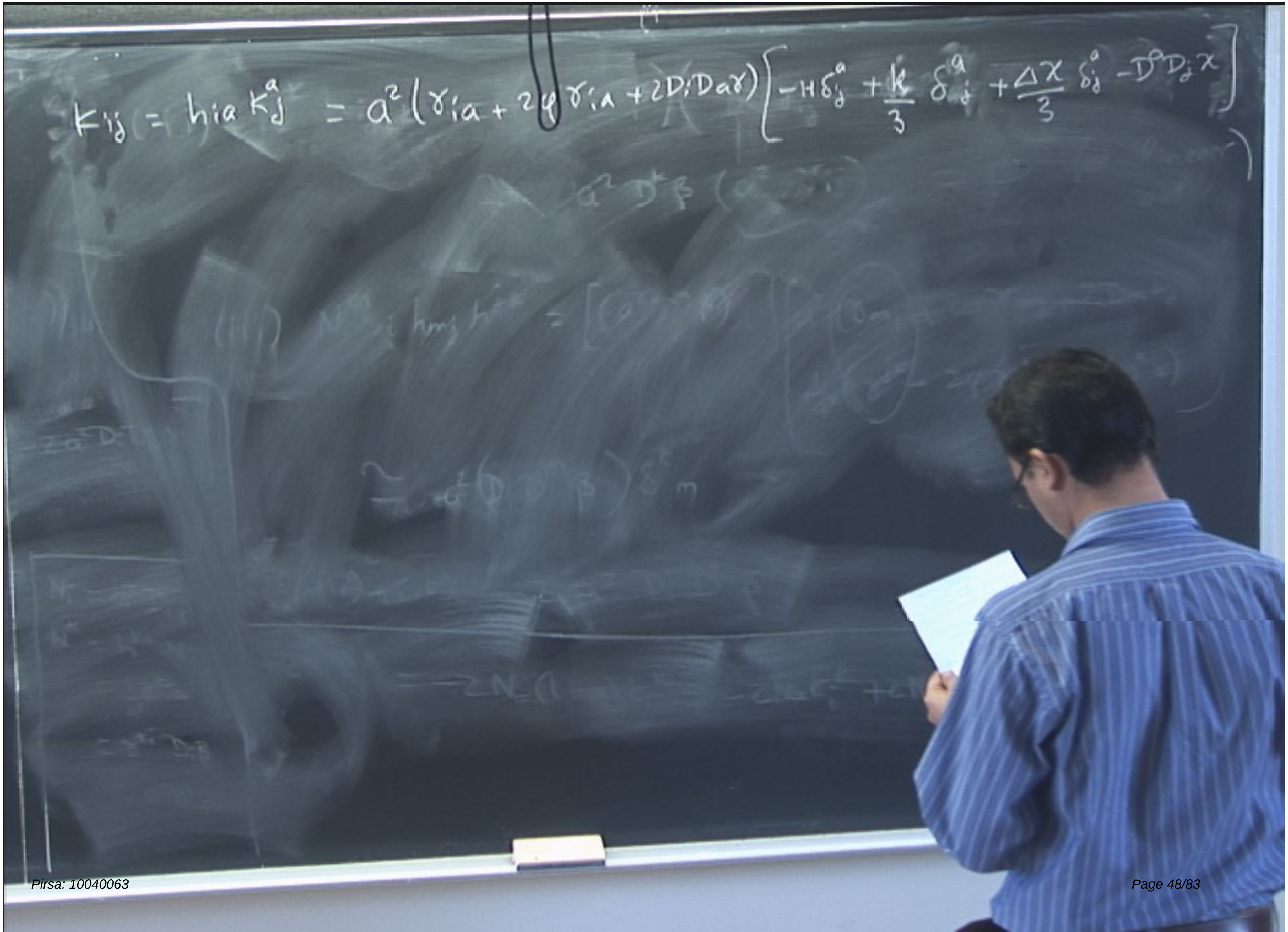
$$N^k h_{ij,k} \simeq 0$$

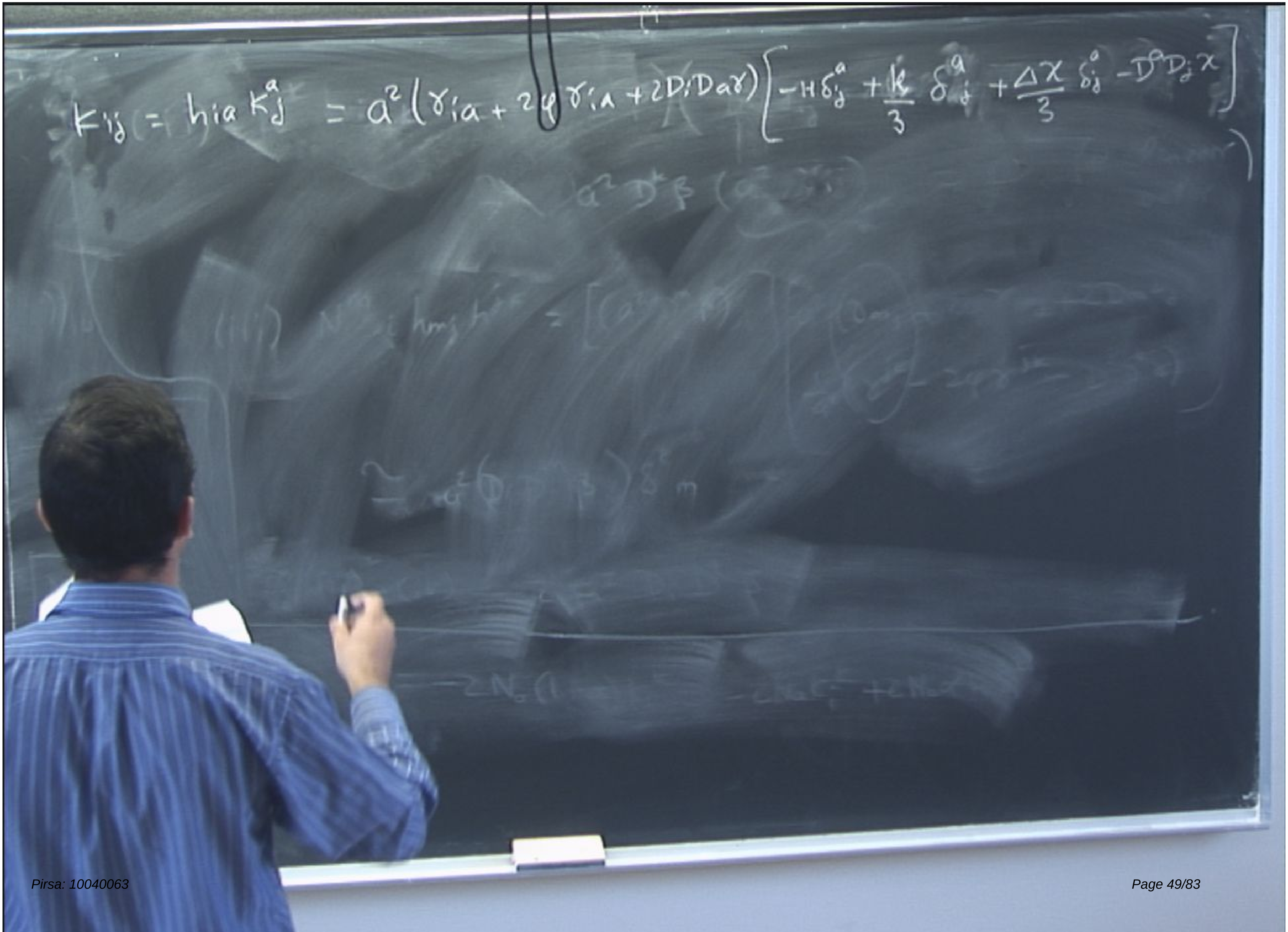
$$\to N^k{}_{,i} h_{kj} \simeq a^4(D_iD^k\beta)\gamma_{kj}$$

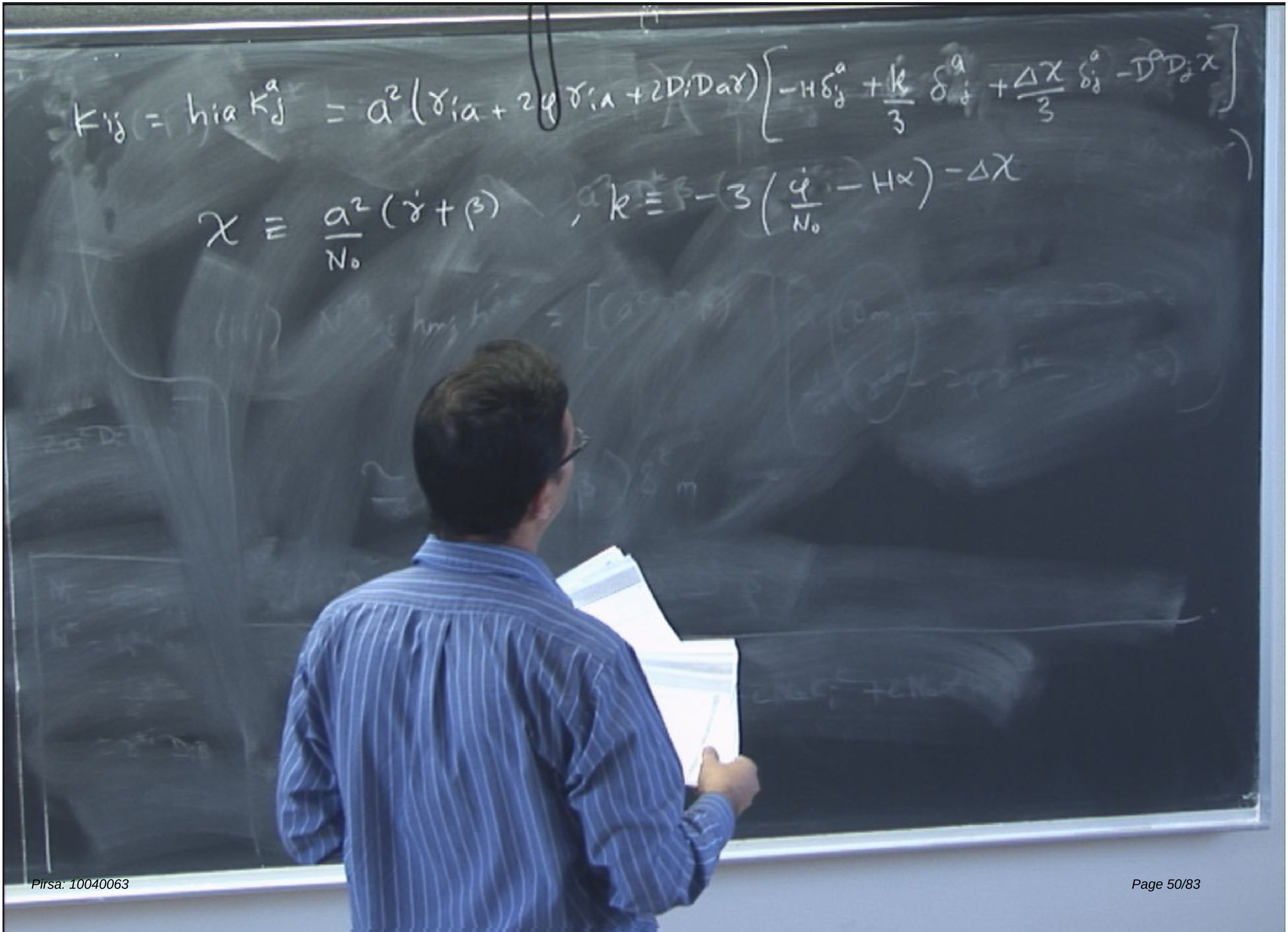
$$\to N^k{}_{,j} h_{ik} \simeq a^4(D_jD^k\beta)\gamma_{ik}$$


$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{\dot{k}}{3}\right.$


Kij = hia K^a_j = a²(γia + 2ψ γia + 2DiDaγ)[−H δ^a_j + k/3 δ


$K_{ij} = h_{ia} K^a_j = a^2(\gamma'_{ia} + 2\varphi \gamma'_{ia} + 2D_i D_a \gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^a D_j \chi\right]$


$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\psi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$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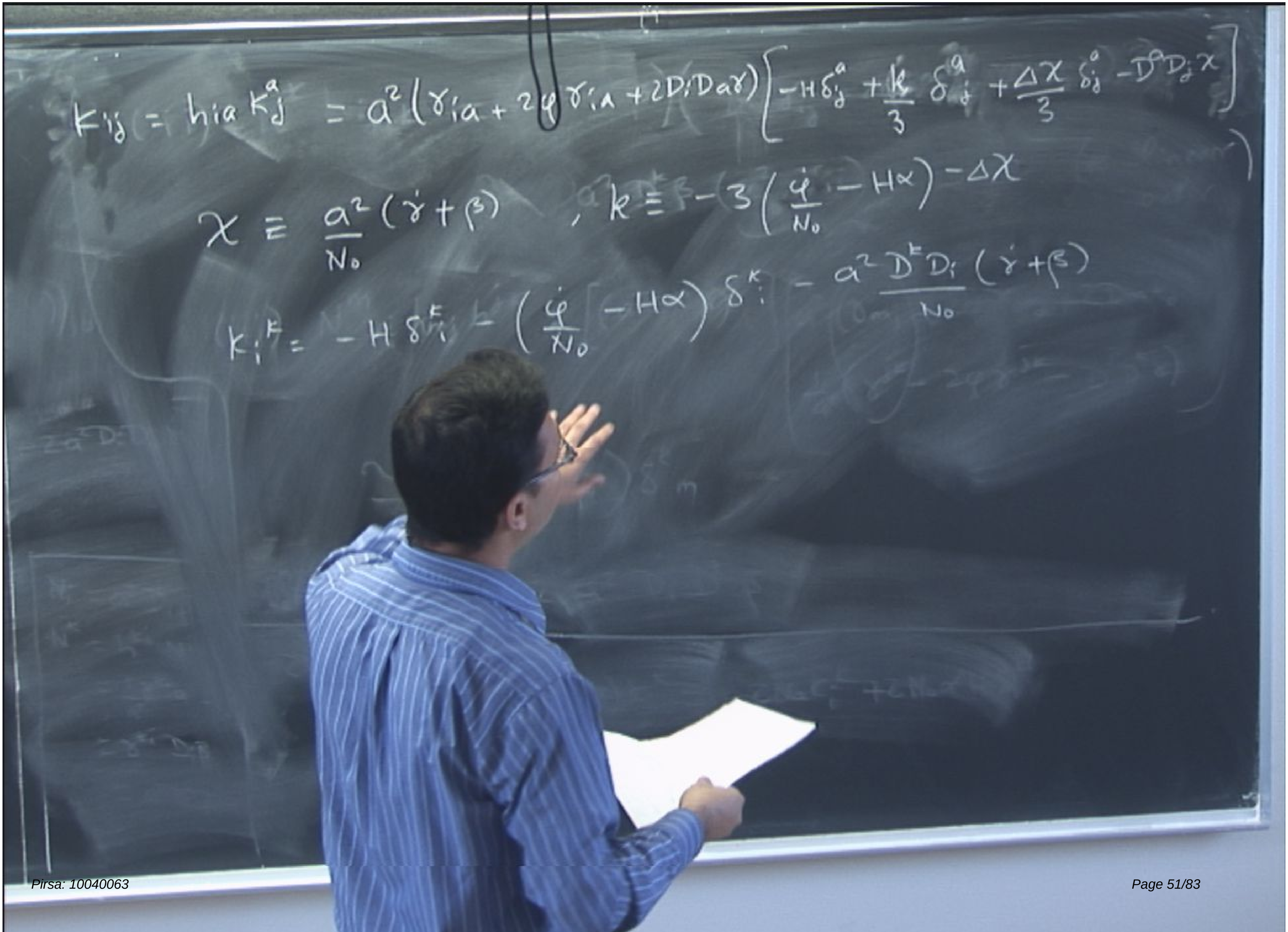
$\chi \equiv \frac{a^2}{N_0}(\dot{\gamma} + \beta)$; $k \equiv -3\left(\frac{\dot{\varphi}}{N_0} - H\alpha\right) - \Delta\chi$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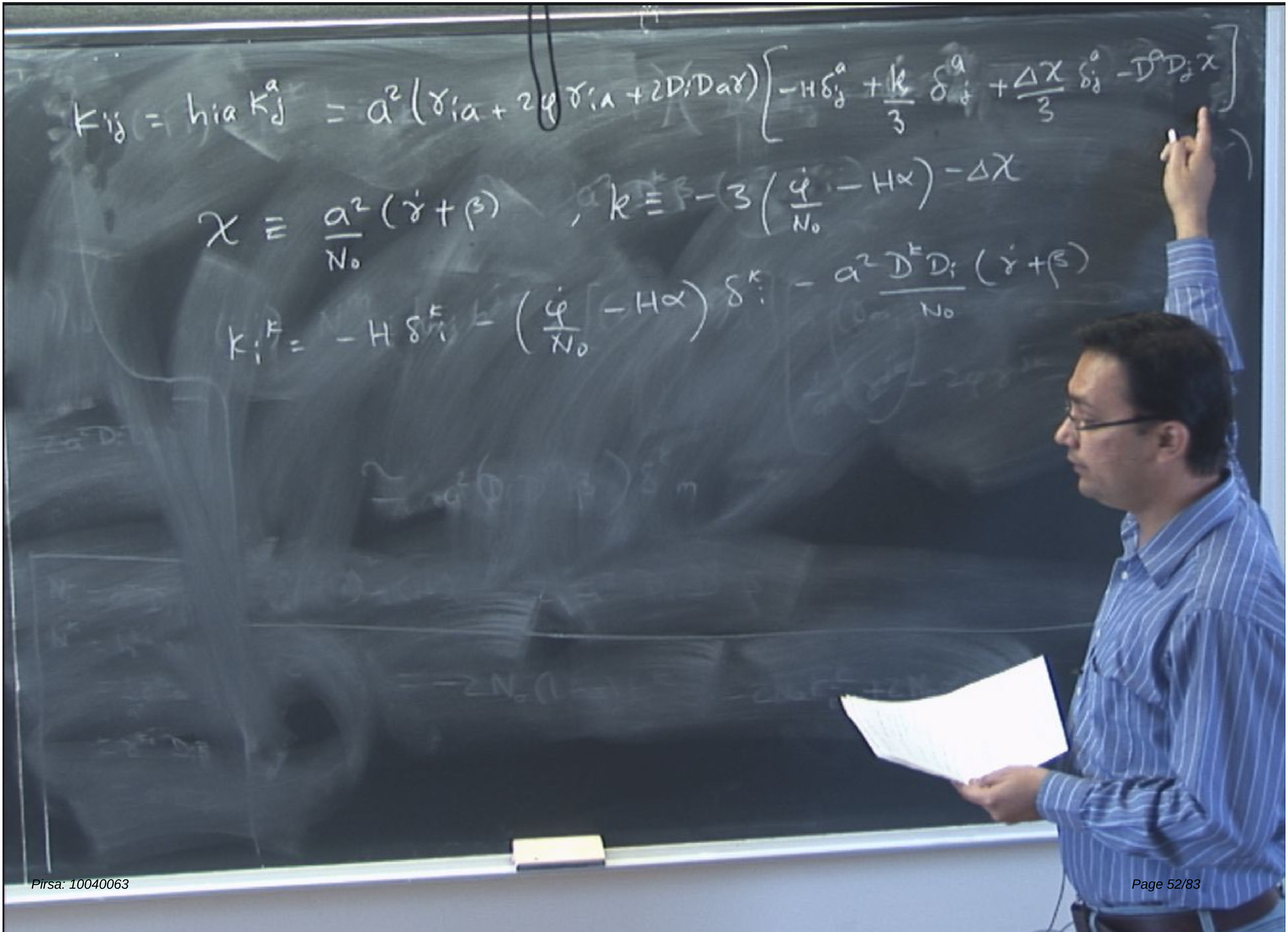
$K_i^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma+\beta)$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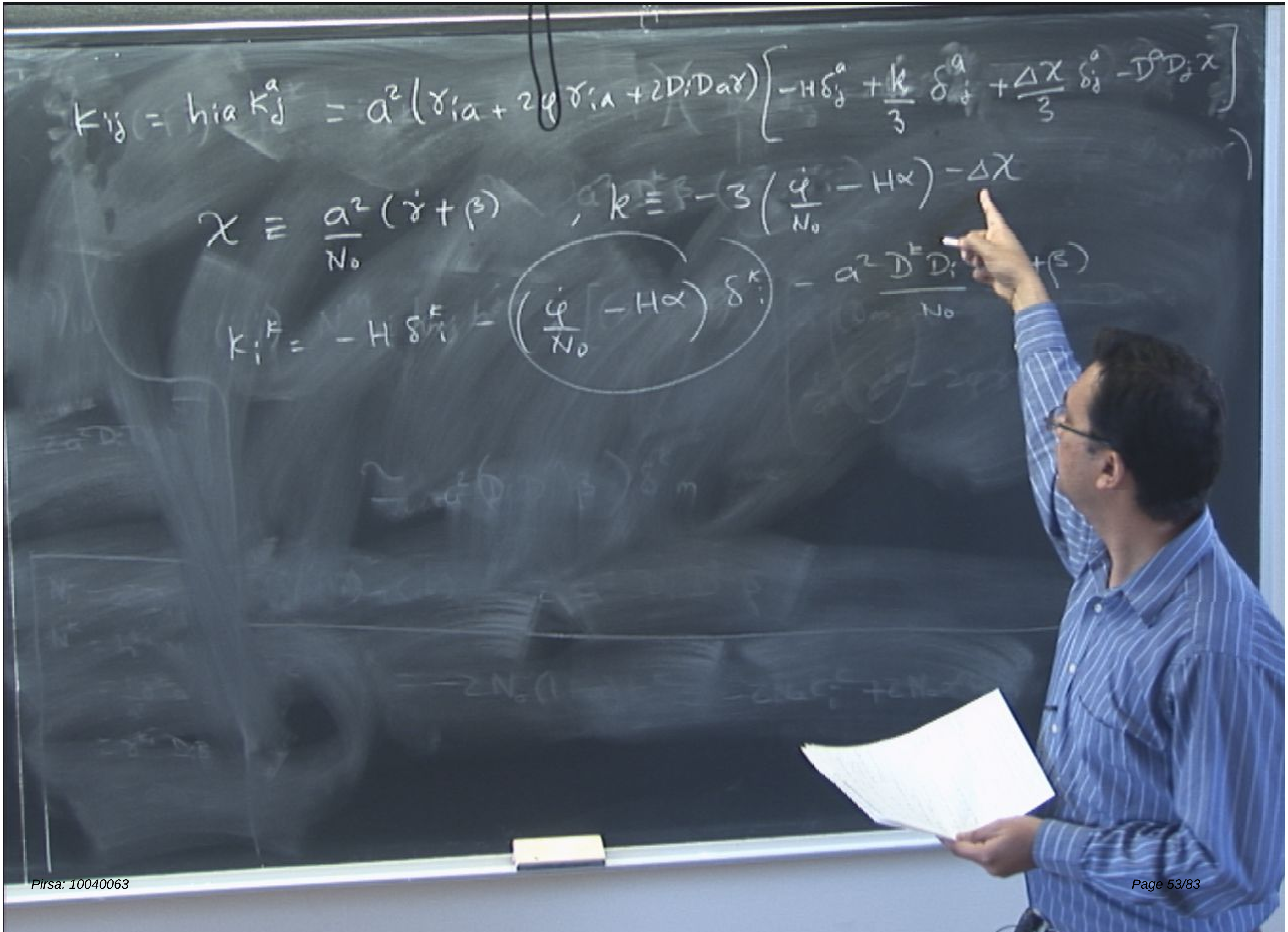
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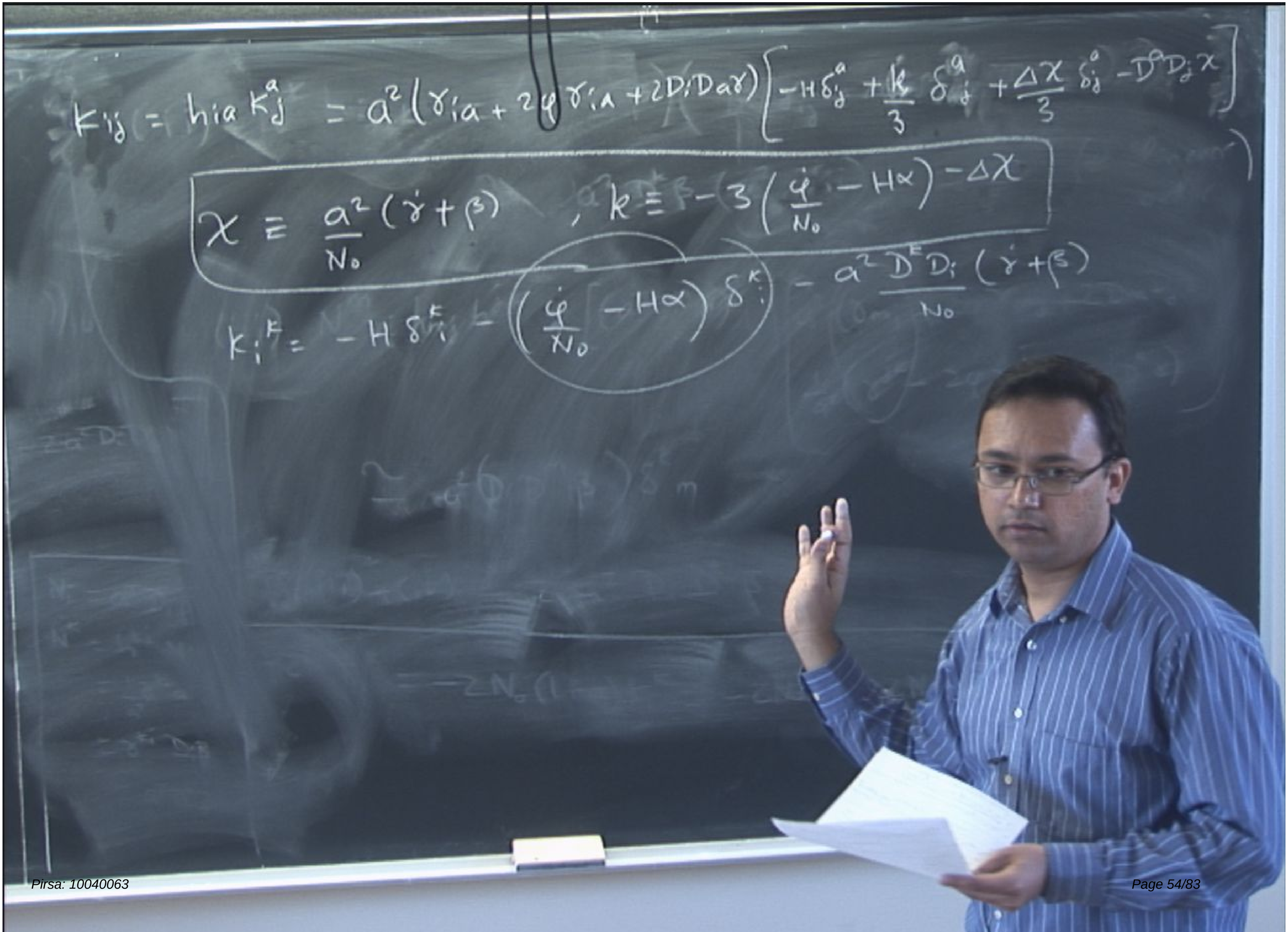
$\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta)$, $k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi$
$K_i^{\;k} = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - \frac{a^2 D^kD_i}{N_0}$ $+\beta)$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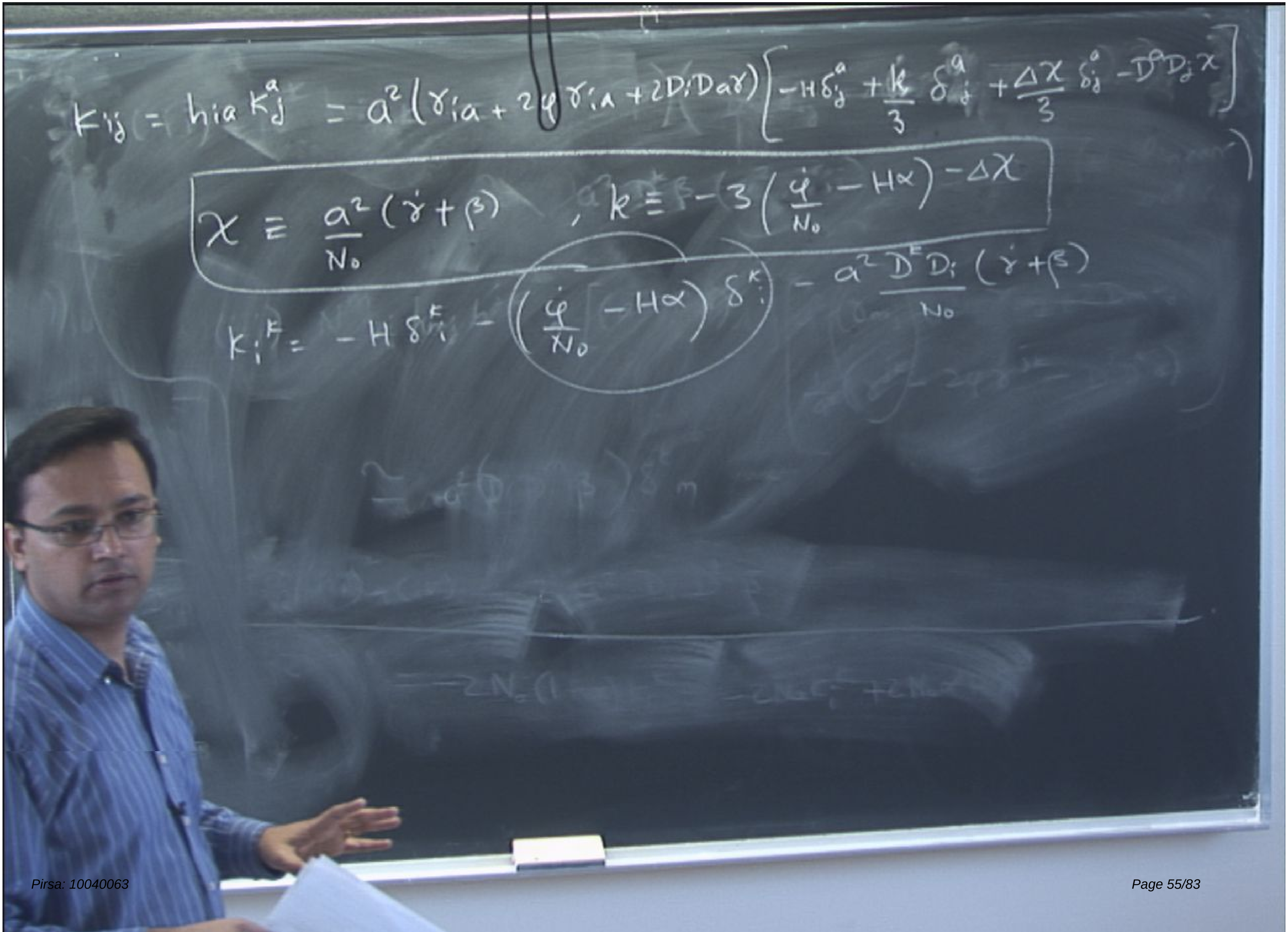
$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma+\beta) \quad , k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$
$K_i^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i(\dot\gamma+\beta)}{N_0}$



$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$
$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot{\gamma}+\beta) \quad , k \equiv -3\left(\frac{\dot{\varphi}}{N_0} - H\alpha\right) - \Delta\chi}$
$K_i^k = -H\delta^k_i - \left(\frac{\dot{\varphi}}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot{\gamma}+\beta)$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma+\beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma+\beta)$$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

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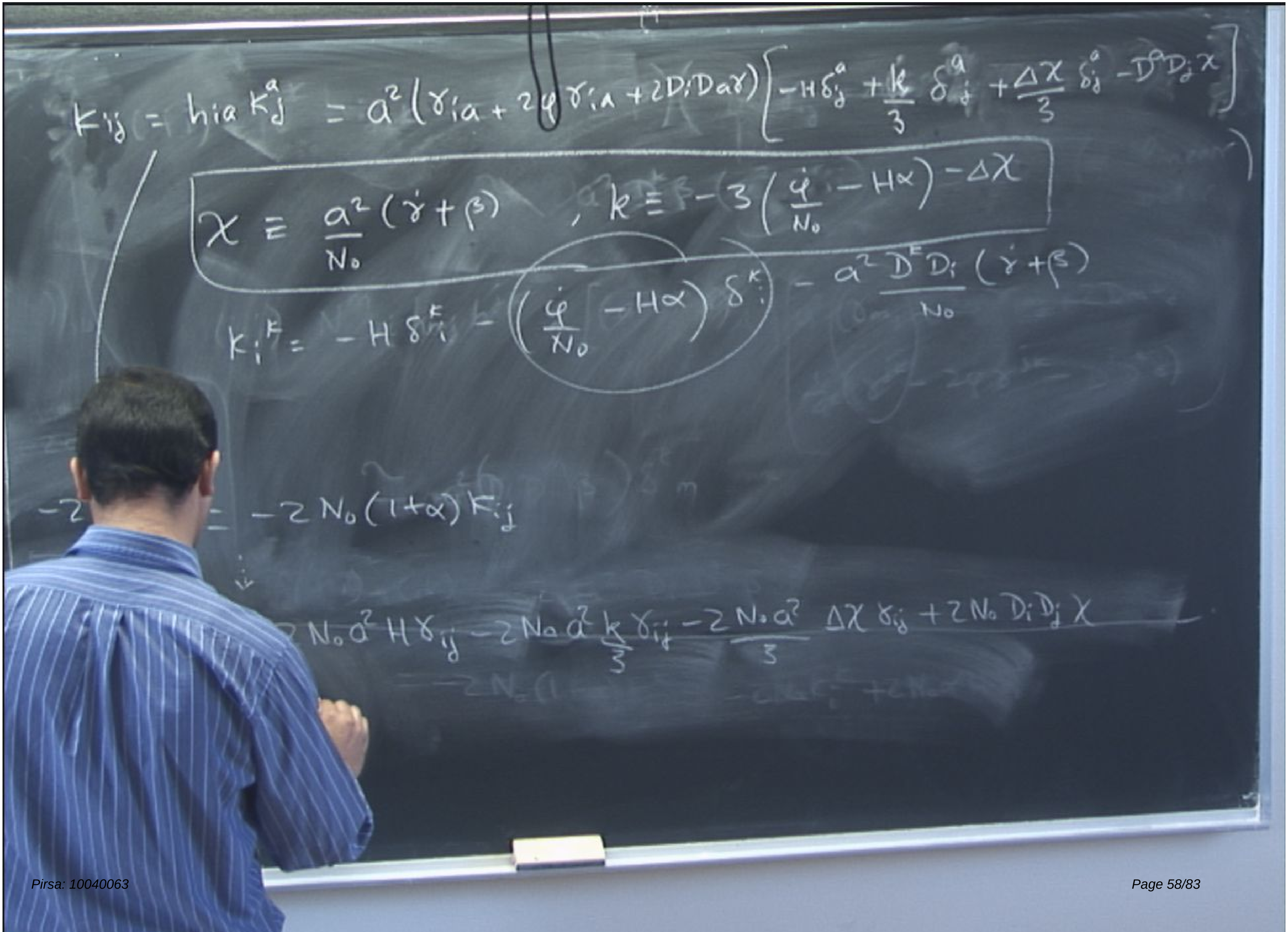
$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - \frac{a^2 D^k D_i(\dot\gamma+\beta)}{N_0}$$

$$-2 \ldots = -2N_0(1+\alpha)K_{ij}$$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$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma+\beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$
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$= -2N_0(1+\alpha)K_{ij}$
$2N_0a^2H\gamma_{ij} - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0a^2}{3}\Delta\chi\gamma_{ij} + 2N_0D_iD_j\chi$

$$K_{ij} = h_{ia} K^a_j = a^2\left(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma\right)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma + \beta)$$

$$-2NK_{ij} = -2N_0(1+\alpha)K_{ij}$$

$$= 2N_0a^2H\gamma_{ij} - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0a^2}{3}\Delta\chi\gamma_{ij} + 2N_0D_iD_j\chi$$

$$+ 4\varphi N_0a^2\gamma_{ij}H + 4N_0a^2(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - \frac{a^2 D^k D_i}{N_0}(\dot\gamma + \beta)$$

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$$+ 4\varphi N_0a^2\gamma_{ij}H + 4N_0a^2(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - \frac{a^2 D^k D_i}{N_0}(\dot\gamma + \beta)$$

$$-2NK_{ij} = -2N_0(1+\alpha)K_{ij}$$

$$= 2N_0 a^2 H\gamma_{ij} - 2N_0 a^2 \frac{k}{3}\gamma_{ij} - 2\frac{N_0 a^2}{3}\Delta\chi\gamma_{ij} + 2N_0 D_iD_j\chi$$

$$+ 4\varphi N_0 a^2 \gamma_{ij} H + 4N_0 a^2 (D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

Ex

$$N^k h_{ij,k} \simeq 0$$

$$\to N^k{}_{,i} h_{kj} \simeq a^4(D_iD^k\beta)\gamma_{kj}$$

$$\to N^k{}_{,j} h_{ik} \simeq a^4(D_jD^k\beta)\gamma_{ik}$$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - \frac{a^2 D^k D_i}{N_0}(\dot\gamma + \beta)$$

$$2\cancel{N_0} a^2 \frac{\dot a}{\cancel{N_0 a}}\gamma_{ij} = 2a\dot a\gamma_{ij}$$

$$-2NK_{ij} = -2N_0(1+\alpha)K_{ij}$$

$$= 2N_0a^2H\gamma_{ij} - 2N_0a^2\frac{k}{3}\gamma_{ij} - \frac{2N_0a^2}{3}\Delta\chi\gamma_{ij} + 2N_0D_iD_j\chi$$

$$+ 4\varphi N_0a^2\gamma_{ij}H + 4N_0a^2(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\ \ \gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$⊗. \quad h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma} + 2a^2D_iD_j\beta}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

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$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\gamma + 2a^2D_iD_j\beta}_{2N_0D_iD_j\chi}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma} + 2a^2D_iD_j\beta}_{2N_0D_iD_j\chi}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$\circledast.\quad h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2 D_iD_j\dot{\gamma} + 2a^2 D_iD_j\beta}_{2N_0 D_iD_j\chi}$$

$$= 2a\dot{a}($$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$\boxed{\chi \equiv \frac{a^2(\dot\gamma + \beta)}{N_0} \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - \frac{a^2 D^k D_i(\dot\gamma + \beta)}{N_0}$$

$$2N_0 a^2 \frac{\dot a}{N_0 a}\gamma_{ij} = 2a\dot a\gamma_{ij}$$

$$-2NK_{ij} = -2N_0(1+\alpha)K_{ij}$$

$$= 2N_0a^2H\gamma_{ij} - 2N_0a^2\frac{k}{3}\gamma_{ij} - \frac{2N_0a^2}{3}\Delta\chi\gamma_{ij} + 2N_0D_iD_j\chi$$

$$+ 4\varphi N_0a^2\gamma_{ij}H + 4N_0a^2(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

② $h_{ij,0} = (a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma))_{,0}$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2 D_iD_j\dot{\gamma} + 2a^2 D_iD_j\beta}_{2N_0 D_iD_j\chi}$$

$$= 2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD \quad D_iD_j\beta$$

$$\chi$$

$$= 2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) - 2N_0a^2\frac{k}{3}\gamma_{i} \quad \gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma} + 2a^2D_iD_j\beta}_{2N_0D_iD_j\chi}$$

$$= 2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0}{3}a^2\Delta\chi\gamma_{ij}$$

$$+ 2\,|$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma}}_{2N_0D_i} + 2a^2 \ldots$$

$$= 2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0}{3}a^2\Delta\chi \ldots$$
$$+ 2N_0D_iD_j\chi + 2N_0aH\gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$\cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} + 2a^2\gamma_{ij}\dot{\varphi} + 2 \ldots + 2a^2 D_iD_j\beta$$

$$\cancel{N_0 D_iD_j}\chi$$

$$= \cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} - 2 \ldots a^2\Delta\chi\gamma_{ij}$$

$$+ \cancel{2N_0 D_iD_j}\chi + 2N_0 a \ldots$$

$$\Rightarrow 2a^2\gamma_{ij}\dot{\varphi} = \ldots 2N_0 a \ldots$$

Perturbation Eq. from $\dot{h}_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

① $$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

$$\cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2 D_iD_j\dot{\gamma} + 2a^2 D_iD_j\beta}_{\cancel{2N_0 D_iD_j\chi}}$$

$$= \cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0}{3}a^2\Delta\chi\gamma_{ij}$$
$$+ \cancel{2N_0D_iD_j\chi} + 2N_0 a H\gamma_{ij}$$

$$\Rightarrow 2a^2\gamma_{ij}\dot{\varphi} = -2N_0a^2\frac{k}{3}\gamma_{ij} - \frac{2N_0}{3}a^2\Delta\chi\gamma_{ij} + 2N_0 a H\gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$\cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma} + 2a^2D_iD_j\beta}_{\cancel{2N_0D_iD_j\chi}}$$

$$= \cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0}{3}a^2\Delta\chi\gamma_{ij}$$
$$+ \cancel{2N_0D_iD_j\chi} + 2N_0aH\gamma_{ij}$$

$$\Rightarrow 2a^2\gamma_{ij}\dot{\varphi} = -2N_0a^2\frac{k}{3}\gamma_{ij} - \frac{2N_0}{3}a^2\Delta\chi\gamma_{ij} + 2N_0aH\gamma_{ij}$$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$$N_0(1+\alpha)$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma+\beta) \quad , \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma+\beta)$$

$$2\cancel{N_0}a^{\cancel{2}}\frac{\dot a}{\cancel{N_0 a}}\gamma_{ij} = 2a\dot a\gamma_{ij}$$

$$-2NK_{ij} = -2N_0(1+\alpha)K_{ij}$$

$$+4\varphi N_0 a^2\gamma_{ij}H + 4N_0a^2(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

$$K_{ij} = h_{ia} K^a_j = a^2(\dot\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$$

$N_0(1+\alpha)$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta) \quad ; \; k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma + \beta)$$

$$2\cancel{N_0}\cancel{a}^2 \frac{\dot a}{\cancel{N_0 a}}\gamma_{ij} = 2a\dot a\gamma_{ij}$$

$-2NK_{ij} = -2N_0(1+\alpha)K_{ij}$

$$\boxed{\frac{\dot\varphi}{N_0} \quad \ldots H\alpha - \frac{\Delta\chi}{3}}$$

$+2N_0 a(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$

$$K_{ij} = h_{ia} K^a_j = a^2\left(\gamma_{ia} + 2\varphi\,\gamma_{ia} + 2D_iD_a\gamma\right)\left[-H\delta^a_j + \frac{k}{3}\,\delta^a_j + \frac{\Delta\chi}{3}\,\delta^a_j - D^aD_j\chi\right]$$

$$\to N_0(1+\alpha)$$

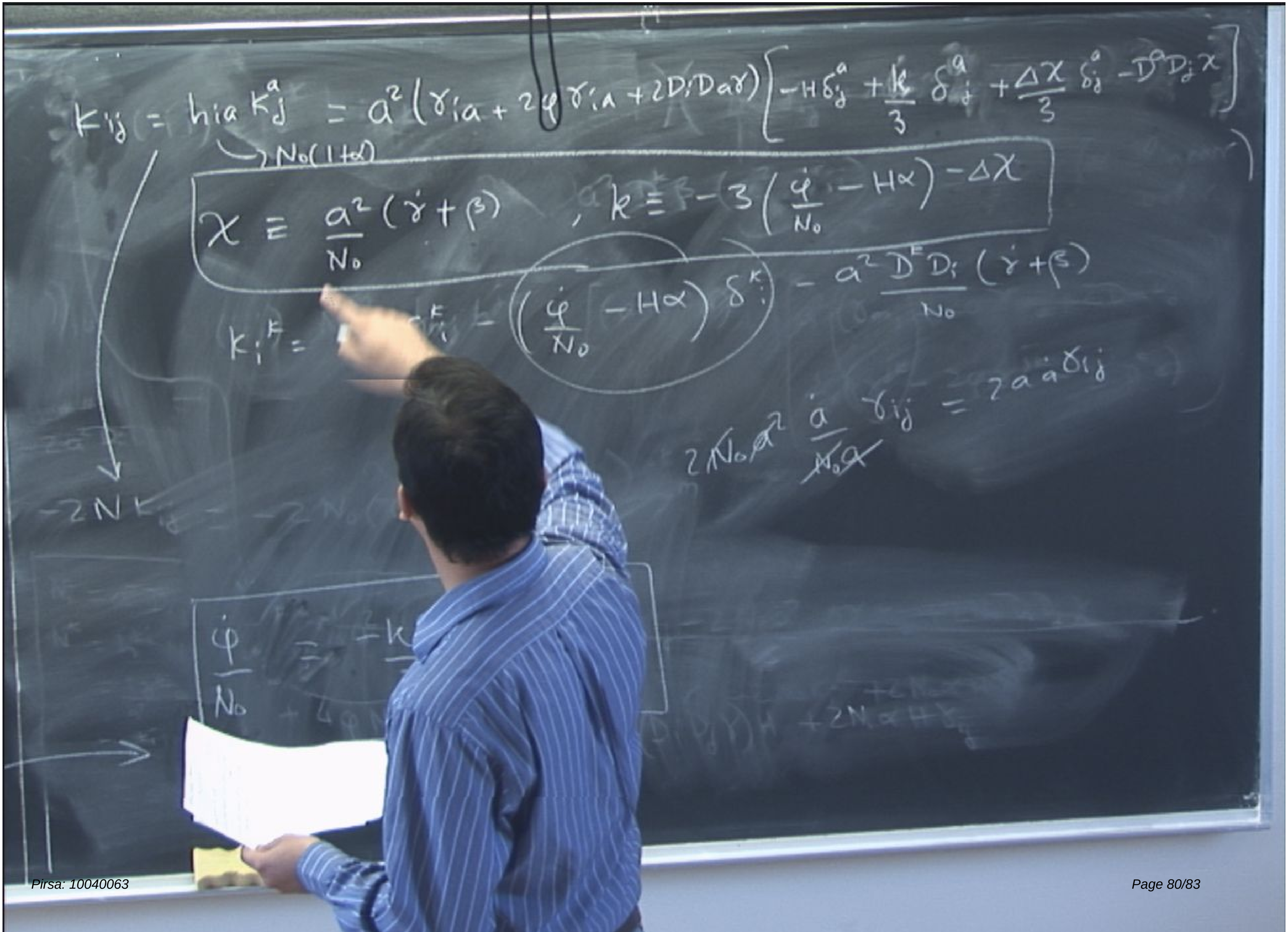
$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma+\beta)\,, \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i{}^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma+\beta)$$

$$2\cancel{N_0}a^{\cancel{2}}\frac{\dot a}{\cancel{N_0 a}}\gamma_{ij} = 2a\dot a\gamma_{ij}$$

$$-2NK$$

$$\frac{\dot\varphi}{N_0} \quad \frac{\Delta\chi}{3}$$



$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^aD_j\chi\right]$
$N_0(1+\alpha)$
$\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta)$, $k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi$
$K_i^k =$
$\left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^kD_i}{N_0}(\dot\gamma + \beta)$
$= 2a\dot a\gamma_{ij}$
$2N_0a^2\frac{\dot a}{N_0 a}\gamma_{ij}$
$-2NK$
$\frac{\dot\varphi}{N_0}$

$$K_{ij} = h_{ia} K^a_j = a^2(\gamma_{ia} + 2\varphi\gamma_{ia} + 2D_iD_a\gamma)\left[-H\delta^a_j + \frac{k}{3}\delta^a_j + \frac{\Delta\chi}{3}\delta^a_j - D^a D_j \chi\right]$$

$$N_0(1+\alpha)$$

$$\boxed{\chi \equiv \frac{a^2}{N_0}(\dot\gamma + \beta) \quad ; \quad k \equiv -3\left(\frac{\dot\varphi}{N_0} - H\alpha\right) - \Delta\chi}$$

$$K_i^k = -H\delta^k_i - \left(\frac{\dot\varphi}{N_0} - H\alpha\right)\delta^k_i - a^2\frac{D^k D_i}{N_0}(\dot\gamma + \beta)$$

$$2\cancel{N_0}\cancel{a^2}\frac{\dot a}{\cancel{N_0 a}}\gamma_{ij} = 2a\dot a\gamma_{ij}$$

$$-2NK = -2N_0(1+\alpha)K_{ij}$$

$$\boxed{\frac{\dot\varphi}{N_0} = -\frac{k}{3} + H\alpha - \frac{\Delta\chi}{3}}$$

$$+4\varphi N_0 a^2\gamma_{ij} + 4N_0 a^2(D_iD_j\gamma)H + 2N_0\alpha H\gamma_{ij}$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2D_iD_j\dot{\gamma}\right]}$$

$$\cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma} + 2a^2D_iD_j\beta}_{\cancel{2N_0D_iD_j\chi}}$$

$$= \cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} - 2N_0a^2\frac{k}{3}\gamma_{ij} - 2\frac{N_0}{3}a^2\Delta\chi\gamma_{ij}$$

$$+ \cancel{2N_0D_iD_j\chi} + 2N_0\alpha H\gamma_{ij}a^2$$

$$\Rightarrow\ 2a^2\gamma_{ij}\dot{\varphi} = -2N_0a^2\frac{k}{3}\gamma_{ij} - \frac{2N_0}{3}a^2\Delta\chi\gamma_{ij} + 2N_0\alpha H\gamma_{ij}a^2$$

Perturbation Eq. from $h_{ij,0}$

$$\boxed{h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij}}$$

$$h_{ij,0} = \left(a^2(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)\right)_{,0}$$

$$\boxed{h_{ij,0} = \left[2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma) + 2a^2\gamma_{ij}\dot{\varphi} + 2a^2 D_iD_j\dot{\gamma}\right]}$$

$$\cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} + 2a^2\gamma_{ij}\dot{\varphi} + \underbrace{2a^2D_iD_j\dot{\gamma} + 2a^2D_iD_j\beta}_{\cancel{2N_0 D_iD_j\chi}}$$

$$= \cancel{2a\dot{a}(\gamma_{ij} + 2\varphi\gamma_{ij} + 2D_iD_j\gamma)} - 2N_0 a^2 \frac{k}{3}\gamma_{ij} - 2\frac{N_0}{3}a^2\Delta\chi\gamma_{ij}$$
$$+ \cancel{2N_0 D_iD_j\chi} + 2N_0 a H \gamma_{ij} a^2$$

$$\Rightarrow 2a^2\gamma_{ij}\dot{\varphi} = -2N_0 a^2\frac{k}{3}\gamma_{ij} - \frac{2N_0}{3}a^2\Delta\chi\gamma_{ij} + 2N_0 a H\gamma_{ij}a^2$$