

Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 5

Date: Apr 09, 2010 09:30 AM

URL: <http://pirsa.org/10040062>

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

$$ds^2 = a^2(\eta) \left[-(1+2A) d\eta^2 - 2B_i dx^i d\eta + (\gamma_{ij} + h_{ij}) dx^i dx^j \right]$$

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SVT decomposition:

Any vector field $= D^i B + B$

$$ds^2 = a^2(\eta) \left[-(1+2A) d\eta^2 - 2B_i dx^i d\eta + (\gamma_{ij} + h_{ij}) dx^i dx^j \right]$$

$$ds^2 = a^2(\eta) \left[-d\eta^2 + \gamma_{ij} dx^i dx^j \right]$$

SVT decomposition :

Any vector field $B^i = D^i B + \bar{B}^i$ st. $D^i \bar{B}_i = 0$

$$ds^2 = a^2(\eta) \left[-(1+2A) d\eta^2 - 2B_i dx^i d\eta + (\gamma_{ij} + h_{ij}) dx^i dx^j \right]$$

$$ds^2 = a^2(\eta) \left[-d\eta^2 + \gamma_{ij} dx^i dx^j \right]$$

SVT decomp.

Any vector field $= D^i B + \bar{B}^i$ st. $D^i \bar{B}_i = 0$

A scalar $h_{ij} = 2\bar{C}$

$$ds^2 = a^2(\eta) \left[-(1+2A) d\eta^2 - 2B_i dx^i d\eta + (\gamma_{ij} + h_{ij}) dx^i dx^j \right]$$

$$ds^2 = a^2(\eta) \left[-d\eta^2 + \gamma_{ij} dx^i dx^j \right]$$

SVT decomposition:

Any vector field $B_i = D^i B + \bar{B}_i$ st. $D^i \bar{B}_i = 0$

A rank-2 tensor $h_{ij} = 2C\gamma_{ij} + 2D_i D_j E + 2D_{(i} \bar{E}_{j)} + 2$

$$ds^2 = a^2(\eta) \left[-(1+2A) d\eta^2 - 2B_i dx^i d\eta + (\gamma_{ij} + h_{ij}) dx^i dx^j \right]$$

$$ds^2 = a^2(\eta) \left[-d\eta^2 + \gamma_{ij} dx^i dx^j \right]$$

SVT decomposition :

Any vector field : $B^i = D^i B + \bar{B}^i$ st. $D^i \bar{B}_i = 0$

A rank 2 symmetric tensor : $h_{ij} = 2C\gamma_{ij} + 2D_i D_j E + 2D_{(i} \bar{E}_{j)}$

with $D_i \bar{E}^{ij} = 0$, $\bar{E}^i_i = 0$

$$ds^2 = a^2(\eta) \left[-(1+2A) d\eta^2 - 2B_i dx^i d\eta + (\gamma_{ij} + h_{ij}) dx^i dx^j \right]$$

$$ds^2 = -a^2(\eta) \left[-d\eta^2 + \gamma_{ij} dx^i dx^j \right]$$

SVT decomposition :

Any vector field : $B^i = D^i B + \bar{B}^i$ st. $D^i \bar{B}_i = 0$

A rank 2 symmetric tensor : $h_{ij} = 2C\gamma_{ij} + 2D_i D_j E + 2D_{(i} \bar{E}_{j)} + 2\bar{E}_{ij}$
 with $D_i \bar{E}^{ij} = 0$, $\bar{E}^i_i = 0$

} 4 scalar d.o.f : A, B, C, E

+2 $\bar{E}_{ij}$

4 scalar perturbations: $A, B, C, E \rightarrow 4$
 2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 4$
 1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

$$+ 2 \bar{E}_{ij}$$

4 scalar perturbations: $A, B, C, E \rightarrow 4$

2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 4$

1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

Scalar perturbations of the metric:

$$ds^2 = a^2(\eta) \left[-(1+zA)d\eta^2 - 2(D_i B) dx^i d\eta + ((1+zC)\gamma_{ij} + 2D_i D_j E) \frac{dx^i dx^j}{dx^i} \right]$$

$+ 2\bar{E}_{ij}$

4 scalar perturbations: $A, B, C, E \rightarrow 4$
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Scalar perturbations of the metric:

$$ds^2 = a^2(\eta) \left[-(1+zA)d\eta^2 - 2(D_i B) dx^i d\eta + ((1+zC)\gamma_{ij} + 2D_i D_j E) \frac{dx^i dx^j}{2} \right]$$

$+ 2\bar{E}_{ij}$
 $\bar{E}_i = 0$

Mukhanov
(+ ---)

$$\left[-(1+\epsilon) dy^0 - 2B_i dx^i dy^0 + (X_i - h_{ij} x^j) dx^i \right]$$

$$12 = a^2(\eta) \left[-a\eta^2 + \frac{1}{2} \int dx^i dx^j \right]$$

composition

$$x^i = \int B^i \quad B^i \quad \text{st. } D^i B_i = 0$$

$$2D^i \quad 2D^j$$

$$D_i E^i = 0, \quad E_i =$$

hanov
---)

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1-2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right].$$

$$ds^2 = a^2(\eta) \left[-d\eta^2 + \gamma_{ij} dx^i dx^j \right]$$

decomposition

$$A = \frac{1}{3} B^i B_i \quad B^i \quad \text{st. } D^i B_i = 0$$

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1+2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right].$$

$$ds^2 = -(1+2A) dt^2 - 2a^2 (D_i B) dx^i dt + a^2 (\delta_{ij} (1+2\psi))$$

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1-2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right].$$

$$ds^2 = -(1+2A) dt^2 - 2a^2 (D_i B) dx^i dt + a^2 (\delta_{ij} (1+2\psi))$$

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1-2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right]$$

$$ds^2 = -(1+2A) dt^2 - 2a^2 (D_i B) dx^i dt + a^2 (\delta_{ij} (1+2\psi) - 2D_i D_j E) dx^i dx^j$$

decomposition

$$B^i = D^i B \quad B^i \text{ st. } D_i B_i = 0$$

Mukhanov
(+ ---)

Dodelson
(- + + +)

Ratra

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1-2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right]$$

$$ds^2 = -dt^2 - 2a^2(D_i B) dx^i dt + a^2(\gamma_{ij}(1+2\psi) - 2D_i D_j \chi) dx^i dx^j$$

$$ds^2 = -N^2(1+2\alpha) dt^2 - 2a^2(D_i \beta) dx^i dt + a^2(\gamma_{ij}(1+2\phi) + 2D_i D_j \chi) dx^i dx^j$$

Mukhanov
(+ ---)

Dodelson
(- + + +)

Bordean
(- + + +)

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1-2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right]$$

$$ds^2 = -(1+2A) dt^2 - 2a^2(D_i B) dx^i dt + a^2(\delta_{ij}(1+2\psi) - 2D_i D_j E) dx^i dx^j$$

$$ds^2 = -N^2(1+2\alpha) dt^2 - 2a^2(D_i \beta) dx^i dt$$

$$+ a^2(\delta_{ij}(1+2\phi) + 2D_i D_j \chi) dx^i dx^j$$

Mukhanov
(+ ---)

Dodelson
(- +++)

Bardeen
(- +++)

$$ds^2 = a^2 \left[(1+2\phi) d\eta^2 + 2(D_i B) dx^i d\eta - ((1-2\psi)\delta_{ij} - 2D_i D_j E) dx^i dx^j \right]$$

$$ds^2 = -(1+2A) dt^2 - 2a^2(D_i B) dx^i dt + a^2(\delta_{ij}(1+2\psi) - 2D_i D_j E) dx^i dx^j$$

$$ds^2 = -N^2(1+2\alpha) dt^2 - 2a^2(D_i \beta) dx^i dt$$

$$+ a^2(\delta_{ij}(1+2\phi) + 2D_i D_j \gamma) dx^i dx^j$$

Bardeen	Mukhanov	Dodelson
α	ϕ	A
β	B	B
ϕ	$-\psi$	ψ
γ	$-E$	$-E$

4 scalar perturbations: $A, B, C, E \rightarrow 4$

2 vector perturbations $\bar{B}^i, \bar{E}^i \rightarrow 4$

1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

Jeans' Potential $\Phi_H = \phi - H\chi$; $\bar{\Phi}_A = \alpha - \frac{\dot{\chi}}{N_0}$

$$ds^2 = a^2(\eta) \left[-(1+zA)d\eta^2 - z(D_i B)dx^i + (1+z\phi)\delta_{ij}dx^i dx^j + 2z(D_i E)dx^i \right]$$

$dx^i dx^j$ | 4 scalar perturbations: $A, B, C, E \rightarrow 4$
 $\rightarrow 4$
 2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 2$
 1 Tensor perturbation: $\bar{E}^{ij}$

Bardeen's $\Phi_H = \phi - H\chi$; $\bar{\Phi}_A = \alpha - \frac{\dot{\chi}}{N_0}$

Mukhanov | Dodelson
 $-\Psi$

7]

$dx^i dx^j$

4 scalar perturbations: $A, B, C, E \rightarrow 4$

2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 4$

1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

Bardeen's Potential $\Phi_H = \phi - H\chi$; $\bar{\Phi}_A = \alpha - \frac{\chi}{N_0}$

Bardeen	Mukhanov	Dodelson
$\bar{\Phi}_H$	$-\Psi$	$-\bar{\Phi}$
$\bar{\Phi}_A$	$\bar{\Phi}$	Ψ

$$\delta R^i_{j\mu\nu} = \delta(\alpha) \delta^i_j + 2(D_\mu \beta) \delta^i_\nu - (1+2\alpha) \delta^i_\mu \delta^j_\nu - 2D_\mu D_\nu \alpha$$

$$ds^2 = -(1+2\alpha) dt^2 - 2a^2(D_\mu \beta) dx^\mu dt + a^2(\gamma_{ij} + 2D_i D_j \alpha) dx^i dx^j$$

$$a^2(D_\mu \beta) dx^\mu dt + a^2(\gamma_{ij} + 2D_i D_j \alpha) dx^i dx^j$$

$$(1+2\alpha) dt^2 - 2a^2(D_\mu \beta) dx^\mu dt$$

$$(\delta_{ij}(1+2\alpha) + 2D_i D_j \alpha)$$

A

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) dx^i + [(1+2\phi)\delta_{ij} - 2D_i D_j E] dx^i dx^j$$

$$ds^2 = - (1+2A) dt^2 - 2a(D_i B) dx^i dt + a^2(\delta_{ij}(1+2\phi) - 2D_i D_j E) dx^i dx^j$$

$$ds^2 = -a^2(1+2\alpha) dt^2 - 2a^2(D_i \beta) dx^i dt + a^2(\delta_{ij}(1+2\phi) - 2D_i D_j \gamma) dx^i dx^j$$

$$(\delta_{ij}(1+2\phi) + 2D_i D_j \gamma)$$

Metric

ϕ

β

γ

$-E$

A

B

C

D

E

Bar

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i \beta) dx^i dx^j - ((1+2\phi)\delta_{ij} - 2D_i D_j E) dx^i dx^j$$

$$h_{ij}^2 = a^2 (\gamma_{ij} + 2\phi \gamma_{ij} + 2D_i D_j \gamma) + a^2 (\gamma_{ij} (1+2\phi) - 2D_i D_j E) dx^i dx^j$$

$$ds^2 = a^2 (1+2\phi) dt^2 - 2a^2 (D_i \beta) dx^i dt - a^2 (\gamma_{ij} (1+2\phi) + 2D_i D_j \gamma) dx^i dx^j$$

$$Metric = (1+2\phi) dt^2 - 2a^2 (D_i \beta) dx^i dt - a^2 (\gamma_{ij} (1+2\phi) + 2D_i D_j \gamma) dx^i dx^j$$

$$A = 1+2\phi$$

$$B = -2a^2 D_i \beta$$

$$C = a^2 \gamma_{ij} (1+2\phi) + 2a^2 D_i D_j \gamma$$

$$D = 0$$

$$E = 0$$

$$F = 0$$

$$G = 0$$

$$H = 0$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i E) dx^i \dots (1+2\phi)\gamma_{ij} - 2D_i D_j E) dx^i dx^j$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\phi \gamma_{ij} + 2D_i D_j \gamma)$$

$$= (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) \delta x^i - (1 + 2\phi) \delta x^i - 2D_i D_j E \delta x^j$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\phi \gamma_{ij} + 2D_i D_j \gamma) + \bar{h}_{ij} = a^2 \gamma_{ij}$$

$$= (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) dx^i \quad (1.24) \delta_{ij} = 2D_i D_j E$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\phi \gamma_{ij} + 2D_i D_j \gamma) \quad + \quad \bar{h}_{ij} = a^2 \gamma_{ij}$$

$$\rightarrow \delta h^{ik} R_{kj} + h^{ik} (\delta R_{kj})$$

$$= (\delta h^{ik}) \bar{R}_{kj} + \bar{h}^{ik} (\delta R_{kj})$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) \delta x^i - (L_4) \delta_{ij} - 2D_i D_j E$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\varphi \gamma_{ij} + 2D_i D_j \gamma) + \bar{h}_{ij} = a^2 \gamma_{ij}$$

$$\rightarrow = (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

4 scalar perturbations: $A, B, C, E \rightarrow 4$

2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 4$

1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

Bardeen's Potential $\Phi_H = \phi - H\chi$; $\bar{\Phi}_A = \alpha - \frac{\dot{\chi}}{N_0}$

Bardeen	Mukhanov	Dodelson	Straumann
Φ_H	$-\Psi$	$-\Phi$	
$\bar{\Phi}_A$	Φ	Ψ	

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) d\psi + (1 - 2\psi)\delta_{ij} - 2D_i D_j E$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\psi\gamma_{ij} + 2D_i D_j \gamma) + \bar{h}_{ij} = a^2 \gamma_{ij}$$

$$= (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

Palatini Identity :

$$\delta R_{\mu\nu} = (\delta \Gamma^\alpha_{\mu\nu})_{;\alpha} - (\delta \Gamma^\alpha_{\mu\alpha})_{;\nu}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) \delta x^i - (2\psi \delta_{ij} - 2D_i D_j E)$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\psi \gamma_{ij} + 2D_i D_j \gamma) + \bar{h}_{ij} = a^2 \delta_{ij}$$

$$= (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

Palatini Identity :

$$\delta R_{\mu\nu} = (\delta \Gamma^\alpha_{\mu\nu})_{;\alpha} - (\delta \Gamma^\alpha_{\mu\alpha})_{;\nu}$$

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\nu\mu} - \partial_\nu \Gamma^\alpha_{\alpha\mu} + \Gamma^\alpha_{\alpha\lambda} \Gamma^\lambda_{\nu\mu} - \Gamma^\alpha_{\nu\lambda} \Gamma^\lambda_{\alpha\mu}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i E) \delta x^i_j - (D_i \psi) \delta x^i_j - 2D_i D_j E$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\psi \gamma_{ij} + 2D_i D_j \psi) + \bar{h}_{ij} = a^2 \gamma_{ij}$$

$$= (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

Identity:

$$\delta R_{\mu\nu} = (\delta \Gamma^\alpha_{\mu\nu})_{;\alpha} - (\delta \Gamma^\alpha_{\mu\alpha})_{;\nu}$$

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\nu\mu} - \partial_\nu \Gamma^\alpha_{\alpha\mu} + \Gamma^\alpha_{\alpha\lambda} \Gamma^\lambda_{\nu\mu} - \Gamma^\alpha_{\nu\lambda} \Gamma^\lambda_{\alpha\mu}$$

$$\delta R_{\mu\nu} = \partial_\alpha \delta \Gamma^\alpha_{\nu\mu} - \partial_\nu \delta \Gamma^\alpha_{\alpha\mu} + (\delta \Gamma^\alpha_{\alpha\lambda}) \Gamma^\lambda_{\nu\mu} + \Gamma^\alpha_{\alpha\lambda} \delta \Gamma^\lambda_{\nu\mu} - \dots$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) \delta x^i - (D_i \psi) \delta x^i - 2D_i D_j E$$

$$h_{ij} = a^2 (\gamma_{ij} + 2\phi \gamma_{ij} + 2D_i D_j \gamma) + \bar{h}_{ij} = a^2 \gamma_{ij}$$

$$= (\delta h^{ik}) R_{kj} + h^{ik} (\delta R_{kj})$$

Palatini Identity:

$$\delta R_{\mu\nu} = (\delta \Gamma^\alpha_{\mu\nu})_{;\alpha} - (\delta \Gamma^\alpha_{\mu\alpha})_{;\nu}$$

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\nu\mu} - \partial_\nu \Gamma^\alpha_{\alpha\mu} + \Gamma^\alpha_{\alpha\lambda} \Gamma^\lambda_{\nu\mu} - \Gamma^\alpha_{\nu\lambda} \Gamma^\lambda_{\alpha\mu}$$

$$\delta R_{\mu\nu} = \partial_\alpha \delta \Gamma^\alpha_{\nu\mu} - \partial_\nu \delta \Gamma^\alpha_{\alpha\mu} + (\delta \Gamma^\alpha_{\alpha\lambda}) \Gamma^\lambda_{\nu\mu} + \Gamma^\alpha_{\alpha\lambda} \delta \Gamma^\lambda_{\nu\mu} -$$

$$\nabla_\alpha \delta \Gamma^\alpha_{\nu\mu} = \partial_\alpha \delta \Gamma^\alpha_{\nu\mu} - (\delta \Gamma^\alpha_{\lambda\mu}) \Gamma^\lambda_{\nu\alpha} + \Gamma^\alpha_{\nu\lambda} \delta \Gamma^\lambda_{\alpha\mu} - \delta \Gamma^\alpha_{\alpha\lambda} \Gamma^\lambda_{\nu\mu}$$

4 scalar perturbations: $A, B, C, E \rightarrow 4$

2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 4$

1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

$$\boxed{\delta R_{\mu\nu} = \nabla_\alpha \delta \Gamma^\alpha_{\nu\mu} - \nabla_\nu \delta \Gamma^\alpha_{\alpha\mu}}$$

$$\delta \Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left[(\delta g_{\nu\alpha})_{;\beta} + (\delta g_{\nu\beta})_{;\alpha} - (\delta g_{\alpha\beta})_{;\nu} \right]$$

$$\delta R^i_j = \delta(h^{ik} R_{kj})$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\delta h_{ab} R_{ij} + \dots \right]$$

Palatini Identity:

$$\delta R_{\mu\nu} = (\delta \Gamma^\alpha_{\mu\nu})_{;\alpha} - \Gamma^\alpha_{\mu\nu} (\delta \Gamma^\alpha_{\alpha\lambda})_{;\lambda}$$

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\nu\mu} - \partial_\nu \Gamma^\alpha_{\alpha\mu} + \Gamma^\alpha_{\alpha\lambda} \Gamma^\lambda_{\nu\mu} - \Gamma^\alpha_{\nu\lambda} \Gamma^\lambda_{\alpha\mu}$$

$$\delta R_{\mu\nu} = \partial_\alpha \delta \Gamma^\alpha_{\nu\mu} - \partial_\nu \delta \Gamma^\alpha_{\alpha\mu} + (\delta \Gamma^\alpha_{\alpha\lambda}) \Gamma^\lambda_{\nu\mu} - \Gamma^\alpha_{\nu\lambda} \delta \Gamma^\lambda_{\alpha\mu}$$

$$\nabla_\alpha \delta \Gamma^\alpha_{\nu\mu} = \partial_\alpha \delta \Gamma^\alpha_{\nu\mu} - (\delta \Gamma^\alpha_{\alpha\lambda}) \Gamma^\lambda_{\nu\mu}$$

4 scalar perturbations: $A, B, C, E \rightarrow 4$

2 vector perturbations: $\bar{B}^i, \bar{E}^i \rightarrow 4$

1 Tensor perturbation: $\bar{E}^{ij} \rightarrow 2$

$$\boxed{\delta R_{\mu\nu} = \nabla_\alpha \delta \Gamma^\alpha_{\nu\mu} - \nabla_\nu \delta \Gamma^\alpha_{\alpha\mu}}$$

$$\delta \Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left[(\delta g_{\nu\alpha})_{;\beta} + (\delta g_{\nu\beta})_{;\alpha} - (\delta g_{\alpha\beta})_{;\nu} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \chi$$

$$R^i_j = \delta(h^{ik} R_{kj}) + 2(D_i B) \delta_{ij} - (1/2) \delta_{ij} - 2D_i D_j E$$

$$R_{ij} = \frac{1}{2} h^{ab} \left[\delta h_{b|j|a} - \delta h_{ab|i} + \delta h_{b|a|i} - \delta h_{ij|ab} \right]$$

identity:

$$\delta R_{\mu\nu} = (\delta \Gamma_{\mu\nu}^\alpha)_{;\alpha} - (\delta \Gamma_{\mu\alpha}^\alpha)_{;\nu}$$

$$= \partial_\alpha \Gamma_{\nu\mu}^\alpha - \partial_\nu \Gamma_{\alpha\mu}^\alpha + \Gamma_{\alpha\lambda}^\alpha \Gamma_{\nu\mu}^\lambda - \Gamma_{\nu\lambda}^\alpha \Gamma_{\alpha\mu}^\lambda$$

$$- \partial_\nu \delta \Gamma_{\alpha\mu}^\alpha + (\delta \Gamma_{\alpha\lambda}^\alpha) \Gamma_{\nu\mu}^\lambda + \Gamma_{\alpha\lambda}^\alpha \delta \Gamma_{\nu\mu}^\lambda -$$

$$- \partial_\alpha \delta \Gamma_{\nu\mu}^\alpha - (\delta \Gamma_{\lambda\mu}^\alpha) \Gamma_{\nu\alpha}^\lambda + \Gamma_{\nu\lambda}^\alpha \delta \Gamma_{\alpha\mu}^\lambda - \delta \Gamma_{\alpha\lambda}^\alpha \Gamma_{\nu\mu}^\lambda$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad -2D_i D_j E$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{b|l|j|a}} - \underset{(ii)}{\delta h_{ab|ij}} + \underset{(iii)}{\delta h_{b|l|ia}} - \underset{(iv)}{\delta h_{ij|ab}} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \delta_{ij} + 2a^2 D_i D_j \varphi \quad -2D_i D_j E$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{b|l|j} a} - \underset{(ii)}{\delta h_{ab|ij}} + \underset{(iii)}{\delta h_{b|l|ia}} - \underset{(iv)}{\delta h_{ij|ab}} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \delta_{ij} + 2a^2 D_i D_j \varphi$$

$$\frac{1}{2} h^{ab} \delta h_{b|l|j} a =$$

$$R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad -2D; D; E$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi|ja}} - \underset{(ii)}{\delta h_{ab|i}j} + \underset{(iii)}{\delta h_{b|j}ia} - \underset{(iv)}{\delta h_{i|a}jb} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \delta_{ij} + 2a^2 D_i D_j \varphi$$

$$\frac{1}{2} h^{ab} \delta h_{bi|ja} =$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu} \\ = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad -2D; D; E$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi|ja}} - \underset{(ii)}{\delta h_{ab|i j}} + \underset{(iii)}{\delta h_{b|j|ia}} - \underset{(iv)}{\delta h_{i|j|ab}} \right]$$

$$\delta h_{ij} = 2\phi a^2 \delta_{ij} + 2a^2 D_i D_j \phi$$

$$\frac{1}{2} h^{ab} \delta h_{bi|ja} =$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad \text{--- 2D, 2E}$$

$$h^{ab} \left[\underset{(i)}{\delta h_{bi} l_{ja}} - \underset{(ii)}{\delta h_{ab} l_{ij}} + \underset{(iii)}{\delta h_{bj} l_{ia}} - \underset{(iv)}{\delta h_{ij} l_{ab}} \right]$$

$$2\varphi a^2 \delta_{ij} + 2a^2 D_i D_j \delta$$

$$l_{ja} = \frac{1}{2}$$

$$\delta g^{mv} = g^{mv} - \bar{g}^{mv}$$

$$\delta g_{mv} = g_{mv} - \bar{g}_{mv}$$

4 scalar

2 vector

1 Tensor

$$\boxed{\delta R_{\mu\nu} = \nabla_\alpha \dots}$$

$$\delta \Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\lambda} \dots$$

$$h_{ij} = 2\varphi a^2$$

$$(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad -2D_i D_j E$$

$$h^{ab} \left[\underset{(i)}{\delta h_{b|l|j|a}} - \underset{(ii)}{\delta h_{ab|ij}} + \underset{(iii)}{\delta h_{b|l|ia}} - \underset{(iv)}{\delta h_{ij|ab}} \right]$$

$$2\varphi a^2 \delta_{ij} + 2a^2 D_i D_j \delta$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$= \frac{1}{2}$$

4 scalar

2 vector

1 Tensor

$$\delta R_{\mu\nu} = \nabla_\alpha$$

$$\delta \Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^\mu$$

$$h_{ij} = a^2 \delta_{ij} + 2\varphi a^2$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad \text{--- } 2D: D_i E$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi|ja}} - \underset{(ii)}{\delta h_{ab|i}j} + \underset{(iii)}{\delta h_{b|j}ia} - \underset{(iv)}{\delta h_{i|j}ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \gamma$$

$$(i) \quad \frac{1}{2} h^{ab} \delta h_{bi|ja} = \frac{2}{2} \frac{\gamma^{ab}}{a^2} \left(a^2 \varphi \gamma_{bi} + a^2 D_b D_i \gamma \right) \Big|_{ja}$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi}|ja} - \underset{(ii)}{\delta h_{ab}|ij} + \underset{(iii)}{\delta h_{bi}|ia} - \underset{(iv)}{\delta h_{ij}|ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \varphi$$

$$(i) \frac{1}{2} h^{ab} \delta h_{bi}|ja = \frac{1}{2} \left(\varphi \gamma_{bi} + a^2 D_b D_i \varphi \right) |ja$$

$$= \gamma^{ab} \gamma_{bi} D_a D_j \varphi$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi}|ja} - \underset{(ii)}{\delta h_{ab}|ij} + \underset{(iii)}{\delta h_{bj}|ia} - \underset{(iv)}{\delta h_{ij}|ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \varphi$$

$$\begin{aligned} (i) \quad \frac{1}{2} h^{ab} \delta h_{bi}|ja &= \frac{1}{2} \frac{\gamma^{ab}}{a^2} \left(a^2 \varphi \gamma_{bi} + a^2 D_b D_i \varphi \right) |ja \\ &= \gamma^{ab} \gamma_{bi} D_a D_j \varphi + \gamma^{ab} D_a D_j D_b D_i \varphi \end{aligned}$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi}|ja} - \underset{(ii)}{\delta h_{ab}|ij} + \underset{(iii)}{\delta h_{bi}|ia} - \underset{(iv)}{\delta h_{ij}|ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \varphi$$

$$\begin{aligned} (i) \quad \frac{1}{2} h^{ab} \delta h_{bi}|ja &= \frac{1}{2} \frac{\gamma^{ab}}{a^2} \left(a^2 \varphi \gamma_{bi} + a^2 D_b D_i \varphi \right) |ja \\ &= \gamma^{ab} \gamma_{bi} D_a D_j \varphi + \gamma^{ab} D_a D_j D_b D_i \varphi \end{aligned}$$

$$(ii) \quad -\frac{1}{2} h^{ab} \delta h_{ab}|ij = -3 D_i D_j \varphi - D_j D_i \Delta \varphi \quad ; \quad \Delta \equiv D^i D_i$$

$$\begin{aligned} \delta g^{\mu\nu} &= g^{\mu\nu} - \bar{g}^{\mu\nu} \\ \delta g_{\mu\nu} &= g_{\mu\nu} - \bar{g}_{\mu\nu} \end{aligned}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi}|ja} - \underset{(ii)}{\delta h_{ab}|ij} + \underset{(iii)}{\delta h_{bj}|ia} - \underset{(iv)}{\delta h_{ij}|ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \gamma$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$(i) \quad \frac{1}{2} h^{ab} \delta h_{bi}|ja = \frac{2}{2} \frac{\gamma^{ab}}{a^2} \left(a^2 \varphi \gamma_{bi} + a^2 D_b D_i \gamma \right) |ja$$

$$= \gamma^{ab} \gamma_{bi} D_a D_j \varphi + \gamma^{ab} D_a D_j D_b D_i \gamma$$

$$(ii) \quad \frac{1}{2} h^{ab} \delta h_{ab}|ij = -3 D_i D_j \varphi - D_j D_i \Delta \gamma \quad ; \quad \Delta \equiv D^i D_i$$

$$(iii) \quad \frac{1}{2} h^{ab} \delta h_{bj}|ia = \gamma^{ab} \gamma_{bj} D_a D_i \varphi + D_a D_i D^b D_b \gamma$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi}|ja} - \underset{(ii)}{\delta h_{ab}|ij} + \underset{(iii)}{\delta h_{bj}|ia} - \underset{(iv)}{\delta h_{ij}|ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \gamma$$

$$(i) \quad \frac{1}{2} h^{ab} \delta h_{bi}|ja = \frac{2}{2} \frac{\gamma^{ab}}{a^2} \left(a^2 \varphi \gamma_{bi} + a^2 D_b D_i \gamma \right) |ja$$

$$= \gamma^{ab} \gamma_{bi} D_a D_j \varphi + \gamma^{ab} D_a D_j D_b D_i \gamma$$

$$(ii) \quad -\frac{1}{2} h^{ab} \delta h_{ab}|ij = -3 D_i D_j \varphi - D_j D_i \Delta \gamma \quad ; \quad \Delta \equiv D^i D_i$$

$$(iii) \quad \frac{1}{2} h^{ab} \delta h_{bj}|ia = \gamma^{ab} \gamma_{bj} D_a D_i \varphi + D_a D_i D^a D_j \gamma$$

$$(iv) \quad -\frac{1}{2} \gamma^{ab} \delta g_{ij}|ab = -a^2 \Delta \varphi \gamma_{ij} - \Delta D_i D_j \gamma$$

$$\delta g^{\mu\nu} = g^{\mu\nu} - \bar{g}^{\mu\nu}$$

$$\delta g_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi}|ja} - \underset{(ii)}{\delta h_{ab}|ij} + \underset{(iii)}{\delta h_{bj}|ia} - \underset{(iv)}{\delta h_{ij}|ab} \right]$$

$$\delta h_{ij} = 2\varphi a^2 \gamma_{ij} + 2a^2 D_i D_j \gamma$$

$$(i) \quad \frac{1}{2} h^{ab} \delta h_{bi}|ja = \frac{\gamma}{2} \frac{\gamma^{ab}}{a^2} \left(a^2 \varphi \gamma_{bi} + a^2 D_b D_i \gamma \right) |ja$$

$$= \gamma^{ab} \gamma_{bi} D_a D_j \varphi + \gamma^{ab} D_a D_j D_b D_i \gamma$$

$$(ii) \quad -\frac{1}{2} h^{ab} \delta h_{ab}|ij = -3 D_i D_j \varphi - D_j D_i \Delta \gamma \quad ; \quad \Delta \equiv D^i D_i$$

$$(iii) \quad \frac{1}{2} h^{ab} \delta h_{bj}|ia = \gamma^{ab} \gamma_{bj} D_a D_i \varphi + D_a D_i D^a D_j \gamma$$

$$(iv) \quad -\frac{1}{2} g^{ab} \delta g_{ij}|ab = -a^2 \Delta \varphi \gamma_{ij} - \Delta D_i D_j \gamma$$

$$\delta h_{ij} = -\frac{2\phi}{a^2} \gamma_{ij} - \frac{2}{a^2} D_i D_j \chi$$

$$(\delta h_{ij}) R_{ij} = \left(-\frac{2\phi}{a^2} \gamma_{ij} \right)$$

$$\delta h_{ij} = -2\phi \gamma_{ij} - \frac{2}{a^2} D^i D^j \chi$$

$$(\delta h_{ik}) \bar{R}_{kj} = \left(-\frac{2\phi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \chi \right) \left(\frac{2c}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$\delta h_{ij} = -2\varphi \gamma_{ij} - \frac{2}{a^2} D^i D^j \gamma$$

$$(\delta h_{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$= -\frac{4C}{a^2} \varphi \delta^i_j$$

$$\delta h^{ij} = -\frac{2\phi}{a^2} \gamma^{ij} - \frac{2}{a^2} D^i D^j \gamma$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\phi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \phi \delta^i_j - \frac{4C}{a^2} D^i D_j \gamma$$

$$\delta h_{ij} = -2\varphi \gamma_{ij} - \frac{2}{a^2} D^i D^j \chi$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \chi \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$\rightarrow \delta R_{ij} = -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + D_a D_i D^a D_j \chi - D_j$$

$$\delta h^{ij} = -2\varphi \gamma^{ij} - \frac{2}{a^2} D^i D^j \chi$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \chi \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$\delta R_{ij} = -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + D_a D_i D^a D_j \chi - D_j D_i \Delta \chi + D_a D_i D^a D_j \chi - \Delta$$

$$\delta h^{ij} = -\frac{2\varphi}{a^2} \gamma^{ij} - \frac{2}{a^2} D^i D^j \chi$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \chi \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$\rightarrow \delta R_{ij} = -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + D_a D_i D^a D_j \chi - D_j D_i \Delta \chi + D_a D_i D^a D_j \chi - \Delta D_i D_j \chi$$

$$\delta h_{ij} = -\frac{2\phi}{a^2} \gamma_{ij} - \frac{2}{a^2} D^i D^j \chi$$

$$(\delta h_{ik}) \bar{R}_{kj} = \left(-\frac{2\phi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \chi \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$\boxed{(\delta h^i_i) = -\frac{4C}{a^2} \phi \delta^i_i - \frac{4C}{a^2} D^i D_i \chi}$$

$$\delta \gamma_{ij} = D_i D_j \phi - a^2 \Delta \phi \gamma_{ij} + D_a D_i D^a D_j \chi - D_j D_i \Delta \chi$$

$$+ D_a D_i D^a D_j \chi - \Delta D_i D_j \chi$$

$$D_j \chi = \chi_j$$

$$\delta h^{ij} = -2\varphi \gamma^{ij} - \frac{2}{a^2} D^i D^j \chi$$

along a perturbation

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \chi \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \delta_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$\delta R_{ij} = -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + D_a D_i D^a D_j \chi - D_j D_i \Delta \chi$$

$$+ D_a D_i D^a D_j \chi - \underbrace{\Delta D_i D_j \chi}_{D_a D^a D_i D_j \chi}$$

$$D_j \chi = \chi_j$$

$$D_a (D_i D^a - D^a D_i) \chi_j$$

$$\delta h_{ij} = -\frac{2\phi}{a^2} \gamma_{ij} - \frac{2}{a^2} D^i D^j \gamma$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\phi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \gamma_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} - \frac{4C}{a^2} D^i D_j \gamma$$

$$\rightarrow \delta R_{ij} = -D_i D_j \phi \gamma_{ij} + D_a D_i D^a D_j \gamma - D_j D_i \Delta \gamma$$

$$D_i D^a D_j \gamma - \underbrace{\Delta D_i D_j \gamma}_{D_a D^a}$$

$$D_j \gamma = \chi_j$$

$$D_a (D_i D^a - D^a D_i) \chi_j$$

$$\delta h^{ij} = -\frac{2\varphi}{a^2} \gamma^{ij} - \frac{2}{a^2} D^i D^j \gamma$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \gamma_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \gamma$$

$$\begin{aligned} \delta R_{ij} &= -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + \left[D_a D_i D^a D_j \gamma - D_j D_i \Delta \gamma \right. \\ &\quad \left. + D_a D_i D^a D_j \gamma - \Delta D_i D_j \gamma \right] \\ &= -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + \frac{4C}{a^2} D_i D_j \gamma \end{aligned}$$

$$\begin{aligned} D_j \gamma &= \chi_j \\ D_a (D_i D^a - D^a D_i) \chi_j \end{aligned}$$

$$\delta h^{ij} = -\frac{2\varphi}{a^2} \gamma^{ij} - \frac{2}{a^2} D^i D^j \gamma$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \gamma_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \gamma$$

$$\begin{aligned} \delta R_{ij} &= -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + \left[D_a D_i D^a D_j \gamma - D_j D_i \Delta \gamma \right. \\ &\quad \left. + D_a D_i D^a D_j \gamma - \underbrace{\Delta D_i D_j \gamma}_{D_a D^a} \right] \\ &= -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + \frac{4C}{a^2} D_i D_j \gamma \end{aligned}$$

$$\begin{aligned} D_j \gamma &= \chi_j \\ D_a (D_i D^a - D^a D_i) \chi_j \end{aligned}$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi} | a} - \underset{(ii)}{\delta h_{ab} | i} + \underset{(iii)}{\delta h_{bi} | a} - \underset{(iv)}{\delta h_{ij} | ab} \right]$$

$$(\delta R^i_j) = h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj}$$

$$h^{ab} \delta h_{ab} | i j = -3 D_i D_j \varphi - D_j D_i \Delta \chi \quad ; \quad \Delta \equiv D^i D_i$$

$$h^{ab} \delta h_{bi} | a = \gamma^{ab} \gamma_{bi} D_a D_i \varphi + D_a D_i D^a D_j \chi$$

$$\frac{1}{2} \gamma^{ab} \delta g_{ij} | ab = -g^2 \Delta \varphi \gamma_{ij} - \Delta D_i D_j \chi$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi} | j a} - \underset{(ii)}{\delta h_{ab} | i j} + \underset{(iii)}{\delta h_{bj} | i a} - \underset{(iv)}{\delta h_{ij} | a b} \right]$$

$$(\delta R^i_j) = h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj}$$

$$\varphi - \Delta \varphi \delta^i_j + \frac{4C}{q^2} D^i D_j \chi$$

$$(ii) - \frac{1}{2} \delta^{ab} = -3 D_i D_j \varphi - D_j D_i \Delta \chi \quad ; \quad \Delta \equiv D^i D_i$$

$$(iii) \delta^{ab} \gamma_{bj} D_a D_i \varphi + D_a D_i D^a D_j \chi$$

$$(iv) - q^2 \Delta \varphi \chi - \Delta D_i D_j \chi$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad \text{--- 2D, D.E.}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underbrace{\delta h_{bi} |_{ja}}_{(i)} - \underbrace{\delta h_{ab} |_{ij}}_{(ii)} + \underbrace{\delta h_{bj} |_{ia}}_{(iii)} - \underbrace{\delta h_{ij} |_{ab}}_{(iv)} \right]$$

$$\begin{aligned} (\delta R^i_j) &= h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj} \\ &= -D^i D_j \varphi - \Delta \varphi \delta^i_j + \frac{4C}{a^2} D^i D_j \chi \end{aligned}$$

$$(ii) \quad -\frac{1}{2} h^{ab} \delta h_{ab} |_{ij} = -3 D^i D_j \varphi - D_j D_i \Delta \chi \quad ; \quad \Delta \equiv D^i D_i$$

$$(iii) \quad \frac{1}{2} h^{ab} \delta h_{bj} |_{ia} = \gamma^{ab} \gamma_{bj} D_a D_i \varphi + D_a D_i D^a D_j \chi$$

$$(iv) \quad -\frac{1}{2} g^{ab} \delta g_{ij} |_{ab} = -a^2 \Delta \varphi \gamma_{ij} - \Delta D_i D_j \chi$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi} | j a} - \underset{(ii)}{\delta h_{ab} | i j} + \underset{(iii)}{\delta h_{bj} | i a} - \underset{(iv)}{\delta h_{ij} | a b} \right]$$

$$(\delta R^i_j) = h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj}$$

$$= -D^i D_j \varphi - \Delta \varphi \delta^i_j + \frac{4C}{a^2} D^i D_j \chi$$

$$= -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$\delta R^i_j = -D^i D_j \varphi - \frac{4C}{a^2} \varphi \delta^i_j - \Delta \varphi \delta^i_j$$

$$(iv) -\frac{1}{2} g^{ab} \delta g_{ij} |_{ab} = -a^2 \Delta \varphi \chi_{ij} - \Delta D_i D_j \chi$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad \text{--- 2D, 2E}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi} | j a} - \underset{(ii)}{\delta h_{ab} | i j} + \underset{(iii)}{\delta h_{bj} | i a} - \underset{(iv)}{\delta h_{ij} | a b} \right]$$

$$(\delta R^i_j) = h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj}$$

$$= -D^i D_j \varphi - \Delta \varphi \delta^i_j + \frac{4C}{a^2} D^i D_j \chi$$

$$\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$-D^i D_j \varphi - \frac{4C}{a^2} \varphi \delta^i_j - \Delta \varphi \delta^i_j$$

$$R^i_j + \delta R$$

$$\delta R^i_j = \delta(h^{ik} R_{kj}) = (\delta h^{ik}) R_{kj} + h^{ik} \delta R_{kj} \quad \text{--- 2D, R, E}$$

$$\delta R_{ij} = \frac{1}{2} h^{ab} \left[\underset{(i)}{\delta h_{bi} | j a} - \underset{(ii)}{\delta h_{ab} | i j} + \underset{(iii)}{\delta h_{bj} | i a} - \underset{(iv)}{\delta h_{ij} | a b} \right]$$

$$(\delta R^i_j) = h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj}$$

$$= -D^i D_j \varphi - \Delta \varphi \delta^i_j + \frac{4C}{a^2} D^i D_j \chi$$

$$= -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \chi$$

$$\delta R^i_j = -D^i D_j \varphi - \frac{4C}{a^2} \varphi \delta^i_j - \Delta \varphi \delta^i_j$$

$$R^i_j = \bar{R}^i_j + \delta R^i_j = \frac{2C}{a^2} h^i_j$$

$$\delta h^{ik} = -\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma$$

$$(\delta h^{ik}) \bar{R}_{kj} = \left(-\frac{2\varphi}{a^2} \gamma^{ik} - \frac{2}{a^2} D^i D^k \gamma \right) \left(\frac{2C}{a^2} \right) \underbrace{h_{kj}}_{a^2 \gamma_{kj}}$$

$$(\delta h^{ik}) \bar{R}_{kj} = -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \gamma$$

$$\begin{aligned} \delta R_{ij} &= -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + \left[D_a D_i D^a D_j \gamma - D_j D_i \Delta \gamma \right. \\ &\quad \left. + D_a D_i D^a D_j \gamma - \underbrace{\Delta D_i D_j \gamma}_{D_a D^a} \right] \\ &= -D_i D_j \varphi - a^2 \Delta \varphi \gamma_{ij} + \frac{4C}{a^2} D_i D_j \gamma \end{aligned}$$

$$h_{ij} = a^2 \gamma_{ij} + 2a^2 \varphi \gamma_{ij} + 2a^2 D_i D_j \gamma$$

$$\begin{aligned} D_j \gamma &= \chi_j \\ D_a (D_i D^a - D^a D_i) \chi_j \end{aligned}$$

$$\delta R^i_j = h^{ik} \delta R_{kj} + (\delta h^{ik}) R_{kj}$$

$$= -D^i D_j \varphi - \Delta \varphi \delta^i_j + \frac{4C}{a^2} D^i D_j \varphi$$

$$= -\frac{4C}{a^2} \varphi \delta^i_j - \frac{4C}{a^2} D^i D_j \varphi$$

$$\boxed{\delta R^i_j = -D^i D_j \varphi - \frac{4C}{a^2} \varphi \delta^i_j - \Delta \varphi \delta^i_j}$$

$$R^i_j = \bar{R}^i_j + \delta R^i_j = \frac{2C}{a^2} \delta^i_j - D^i D_j \varphi - \frac{4C}{a^2} \varphi \delta^i_j -$$