Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 4

Date: Apr 08, 2010 09:30 AM

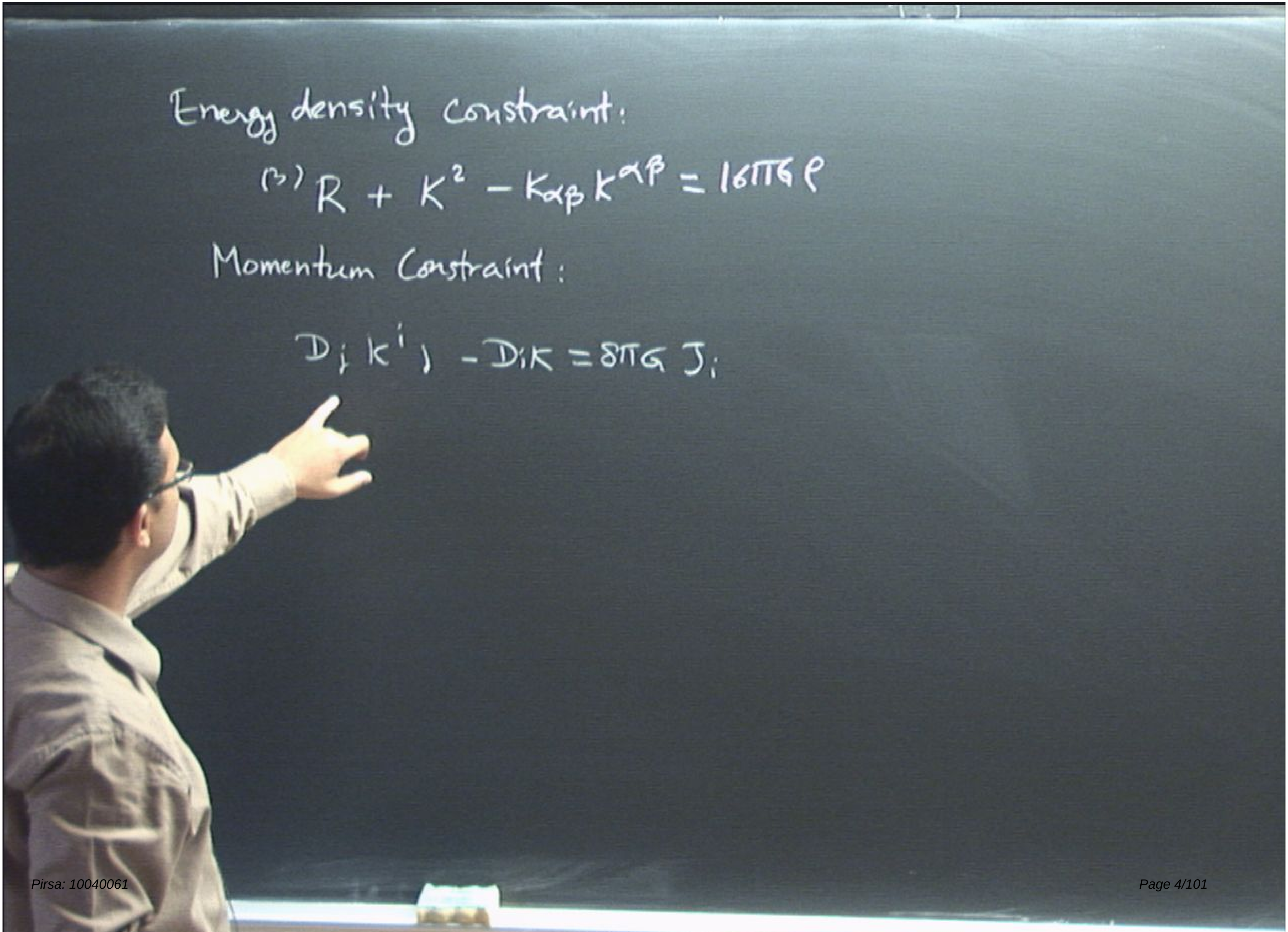
URL: http://pirsa.org/10040061

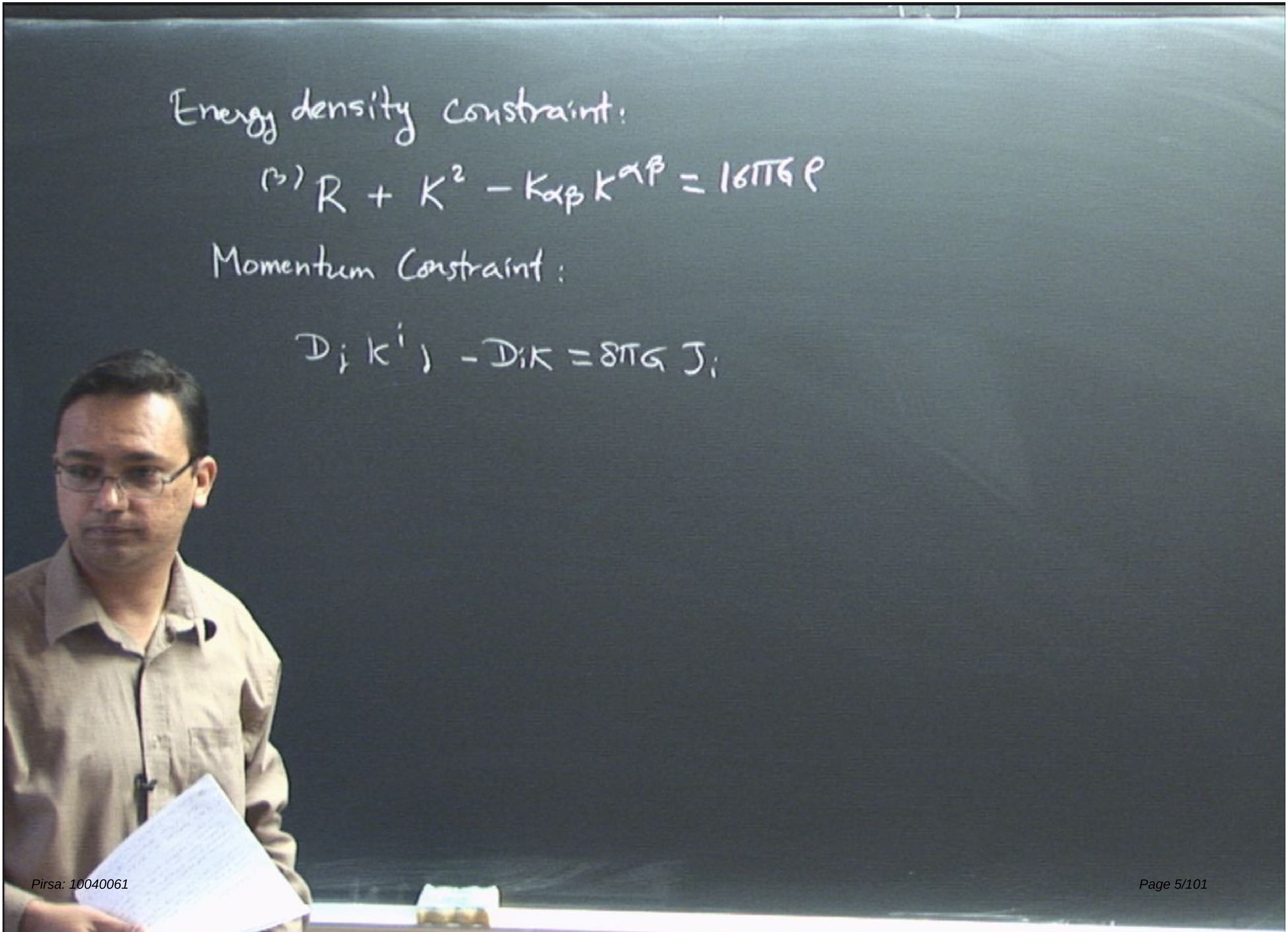
Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum (


Energy density constraint:
$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$
Momentum Constraint:
$D_j K^i{}_j - D_i K = 8\pi G J_i$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Gauss Eqn: $$P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} P^{\sigma}{}_{\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\alpha\mu}K_{\beta\nu} - K_{\alpha\nu}K_{\beta\mu}$$

Energy density constraint:

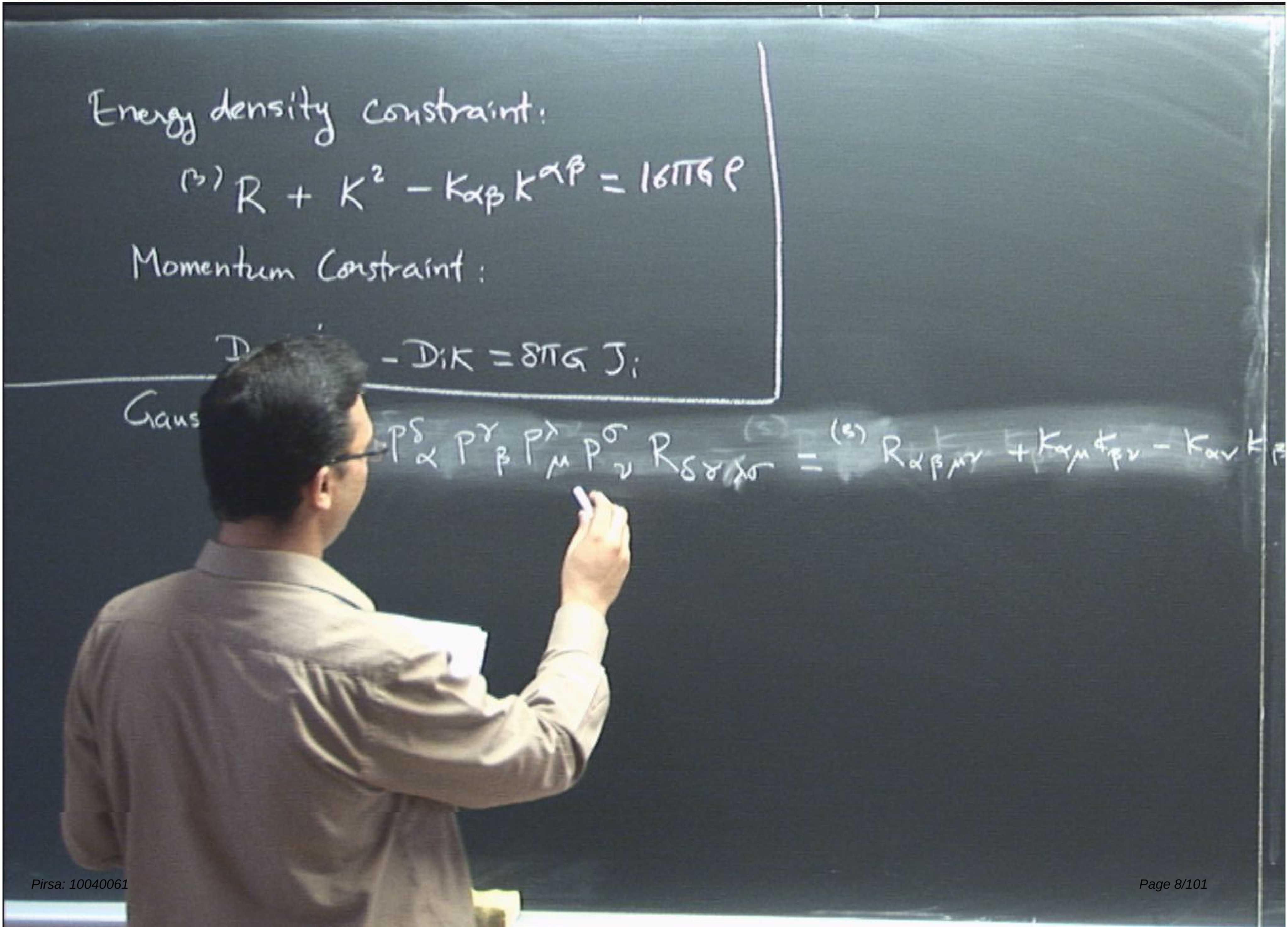
$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_j K^i{}_j - D_i K = 8\pi G J_i $$

Gauss Eqn:

$$ P^\delta{}_\alpha P^\gamma{}_\beta P^\lambda{}_\mu P^{\sigma}{}_\nu R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\alpha\mu}K_{\beta\nu} - K_{\alpha\nu}K_{\beta\mu} $$


Energy density constraint:
$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$
Momentum Constraint:
$- D_i K = 8\pi G J_i$
Gaus
$P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} P^{\sigma}{}_{\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\alpha\mu}K_{\beta\nu} - K_{\alpha\nu}K$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_i \;[\text{illegible}]\; D_i K = 8\pi G J_i$$

Gauss [illegible]

$$[\text{illegible}]\, P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} P^{\sigma}{}_{\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\alpha\mu}K_{\beta\nu} - K_{\alpha\nu}K_{\beta\mu}$$

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Momentum Constraint:

$$ D_j K^i{}_j - D_i K = 8\pi G J_i $$

Gauss Eqn:

$$ P^\delta_\alpha P^\gamma_\nu P^\lambda_\mu P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\gamma\mu} + K_{\alpha\mu}K - K_\alpha^\beta K_{\beta\mu} $$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

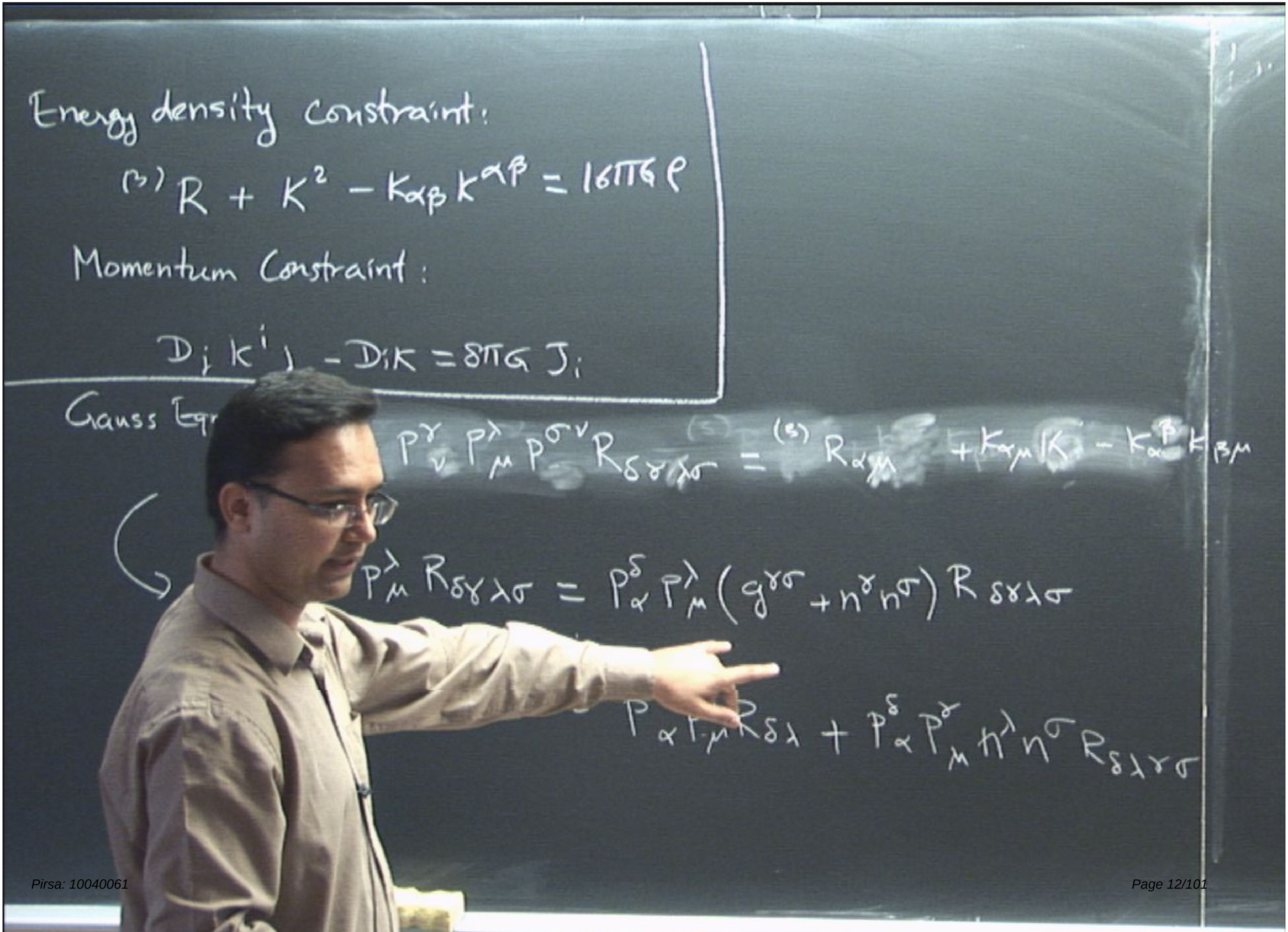
$$D_j K^i{}_j \ldots D_i K = 8\pi G J_i$$

Gauss Eqn:

$$\ldots P^\lambda_\mu P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\gamma\mu} + K_{\alpha\mu}K - K_\alpha^\beta K_{\beta\mu}$$

$$P^\delta \ldots R_{\delta\gamma\lambda\sigma} = P^\delta_\alpha P^\lambda_\mu \left(g^{\gamma\sigma} + n^\gamma n^\sigma\right) R_{\delta\gamma\lambda\sigma}$$

$$=$$



Energy density constraint:
$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$
Momentum Constraint:
$D_j K^i{}_j - D_i K = 8\pi G J_i$
Gauss Eq
$P^\gamma_\nu P^\lambda_\mu P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu}K - K_\alpha^\beta K_{\beta\mu}$
$P^\lambda_\mu R_{\delta\gamma\lambda\sigma} = P^\delta_\alpha P^\lambda_\mu (g^{\gamma\sigma} + n^\gamma n^\sigma) R_{\delta\gamma\lambda\sigma}$
$P_\alpha P_\mu R_{\delta\lambda} + P^\delta_\alpha P^\gamma_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma}$

...rgy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_j K^i{}_j - D_i K = 8\pi G J_i$$

Gauss Eqn:

$$P^\delta_\alpha P^\gamma_\nu P^\lambda_\mu P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} \ldots + K_{\alpha\mu}K - K_\alpha{}^\beta K_{\beta\mu}$$

$$P^\delta_\alpha P^{\gamma\sigma} P^\lambda_\mu R_{\delta\gamma\lambda\sigma} = P^\delta_\alpha P^\lambda_\mu (\ldots) R_{\delta\gamma\lambda\sigma}$$

$$= P^\delta_\alpha P^\lambda_\mu R_{\delta} \ldots n^\sigma R_{\delta\lambda\gamma\sigma}$$

$$\boxed{P^\delta_\mu P^\lambda_\nu R_{\delta\lambda} + P^\delta_\alpha \ldots + K_{\nu\mu}K - K_\alpha{}^\nu K_{\nu\mu}}$$

...rgy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

...Momentum Constraint:

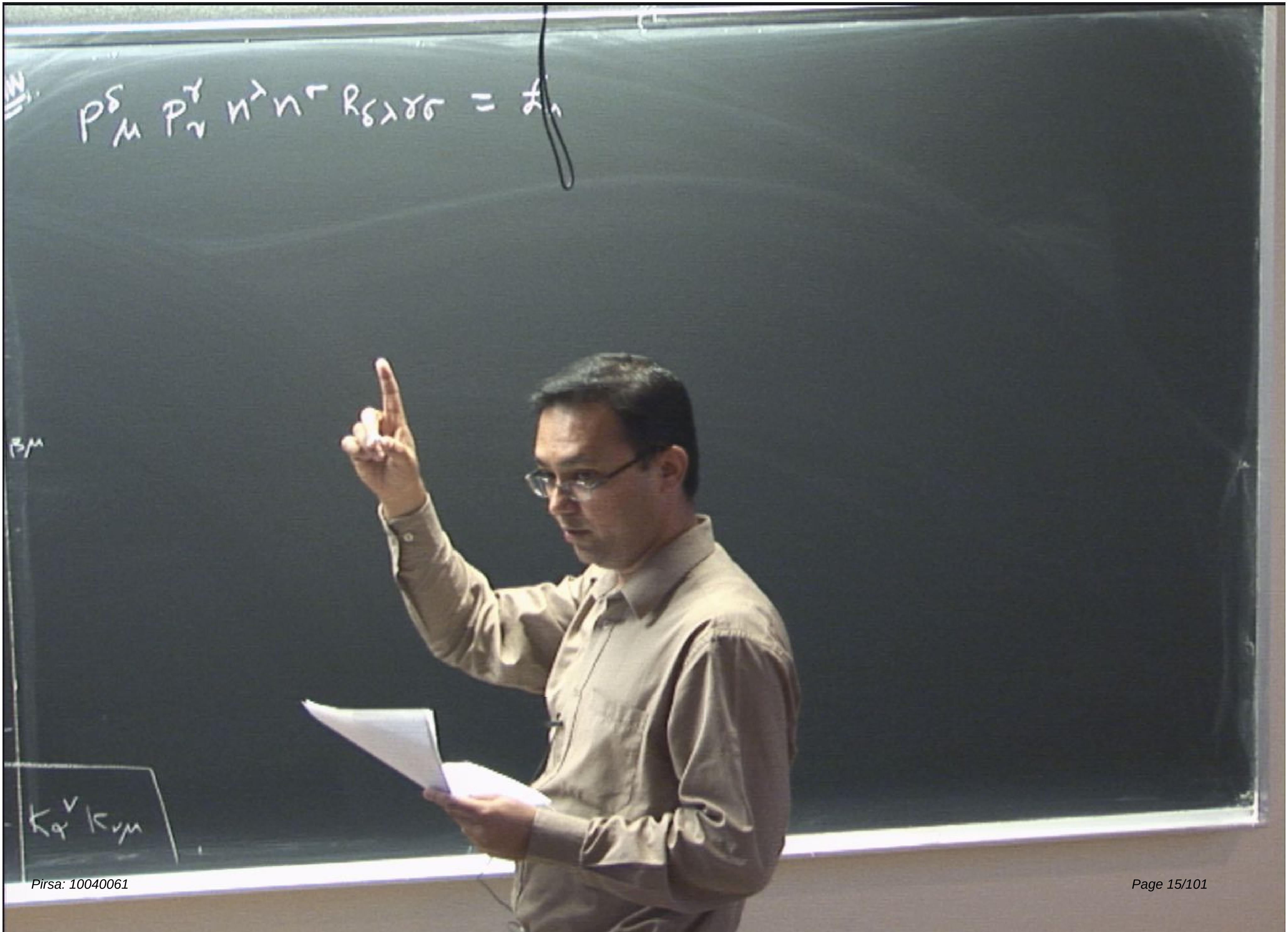
$$D_j K^i{}_j - D_i K = 8\pi G J_i$$

...Gauss Eqn: $P^\delta_\alpha P^\gamma_\nu P^\lambda_\mu P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\gamma\mu} + K_{\alpha\mu}K - K_\alpha{}^\beta K_{\beta\mu}$

$$P^\delta_\alpha P^{\gamma\sigma} P^\lambda_\mu R_{\delta\gamma\lambda\sigma} = P^\delta_\alpha P^\lambda_\mu (g^{\gamma\sigma} + n^\gamma n^\sigma) R_{\delta\gamma\lambda\sigma}$$

$$= P^\delta_\alpha P^\lambda_\mu R_{\delta\lambda} + P^\delta_\alpha P^\gamma_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma}$$

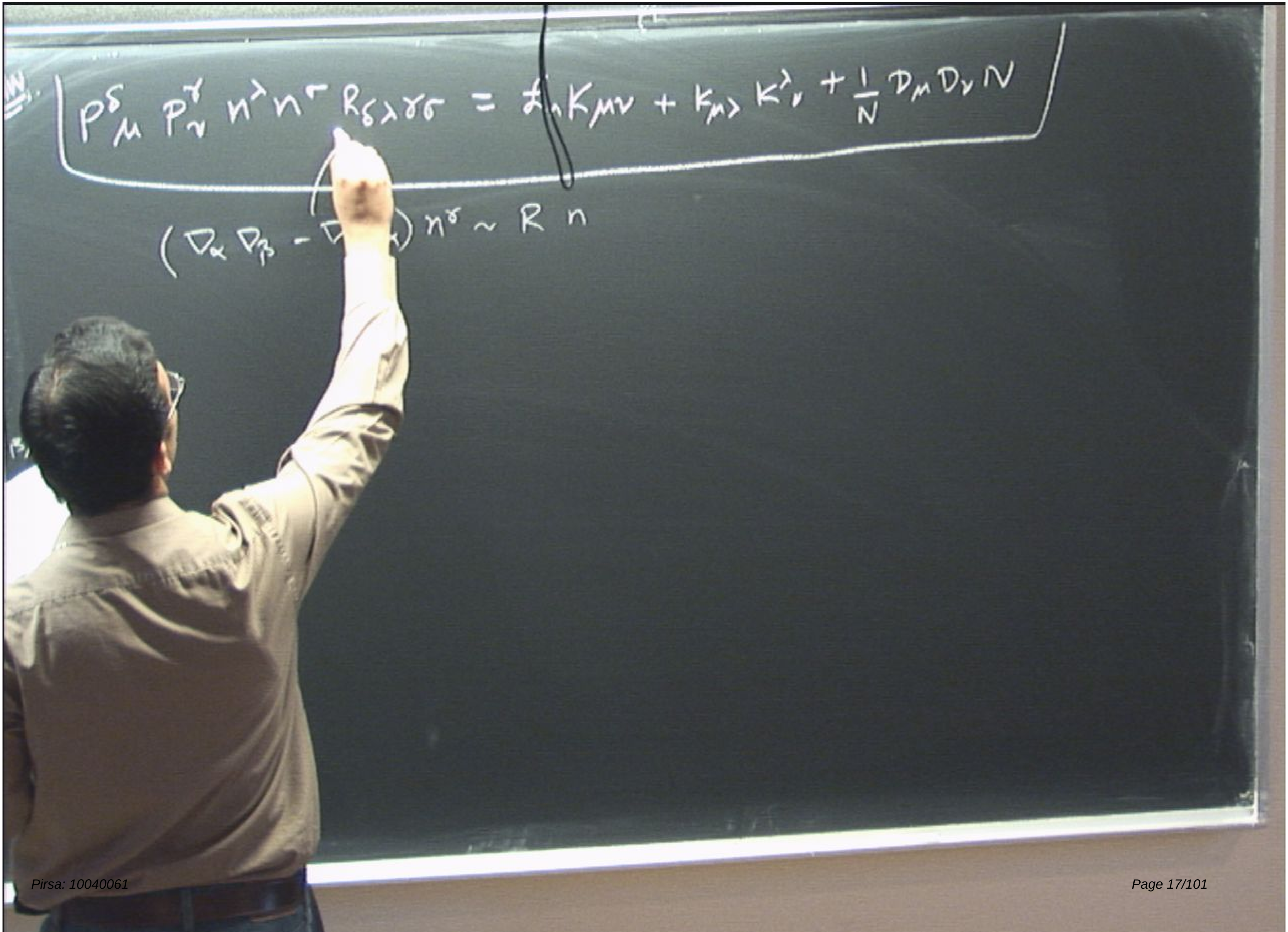
$$\boxed{P^\delta_\mu P^\lambda_\nu R_{\delta\lambda} + P^\delta_\alpha P^\gamma_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu}K - K_\alpha{}^\nu K_{\nu\mu}}$$


$P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} =$
$K_{\alpha}^{\ \nu} K_{\nu\mu}$

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \mathcal{L}_{n} K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_{\mu} D_{\nu} N}$$

β^{μ}

$K_{\alpha}{}^{\nu} K_{\nu\mu}$


$P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N$
$(D_\alpha D_\beta - \ldots) n^\gamma \sim R\, n$

HW.

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_{\mu} D_{\nu} N}$$

use (i) $(D_\alpha D_\beta - D_\beta D_\alpha) n^\sigma \sim R \; n$

(ii) $K_{\mu\nu}$

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_{\mu} D_{\nu} N}$$

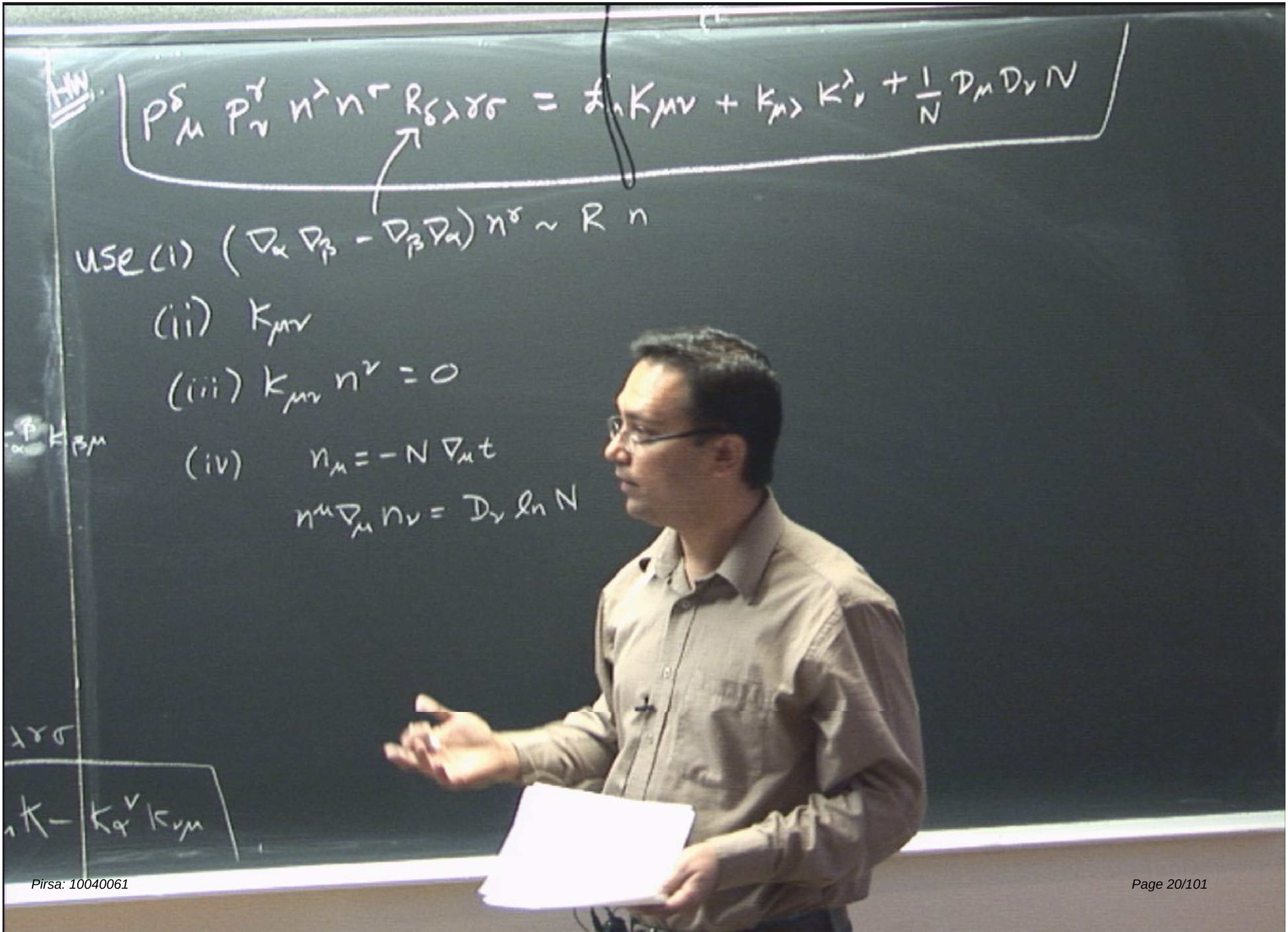
use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\ n$

(ii) $K_{\mu\nu}$

(iii) $K_{\mu\nu} n^{\nu} = 0$

(iv) $n_\mu = -N \nabla_\mu t$

$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$


HW
$P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N$
use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$
(ii) $K_{\mu\nu}$
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HW.

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N}$$

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$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_\nu{}^{\lambda} + \frac{1}{N} D_\mu D_\nu N$$

$$=$$

HW.

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N}$$

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$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_\nu{}^\lambda + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma{}_\nu$$

$$- P^\delta_\mu P^\lambda_\nu R_{\delta\lambda\ldots}$$

$-{}^\beta_\alpha K_{\beta\mu}$

$\lambda\gamma\sigma$

$K - K_\alpha{}^\nu K_{\nu\mu}$

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N}$$

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$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$

$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_\nu{}^{\lambda} + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^{\gamma}{}_{\nu} - P^{\delta}_{\mu} P^{\lambda}_{\nu} R_{\delta\lambda}$$

$$\pounds_t K_{\mu\nu} - \pounds_{N^i} K_{\mu\nu} = -D_\mu D_\nu N + N(\quad)$$

$K_\alpha{}^\nu K_{\nu\mu}$

$$\boxed{P^\delta_\mu P^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda{}_\nu + \frac{1}{N} D_\mu D_\nu N}$$

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$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_\nu{}^\lambda + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma{}_\nu - P^\delta_\mu P^\lambda_\nu R_{\delta\lambda}$$

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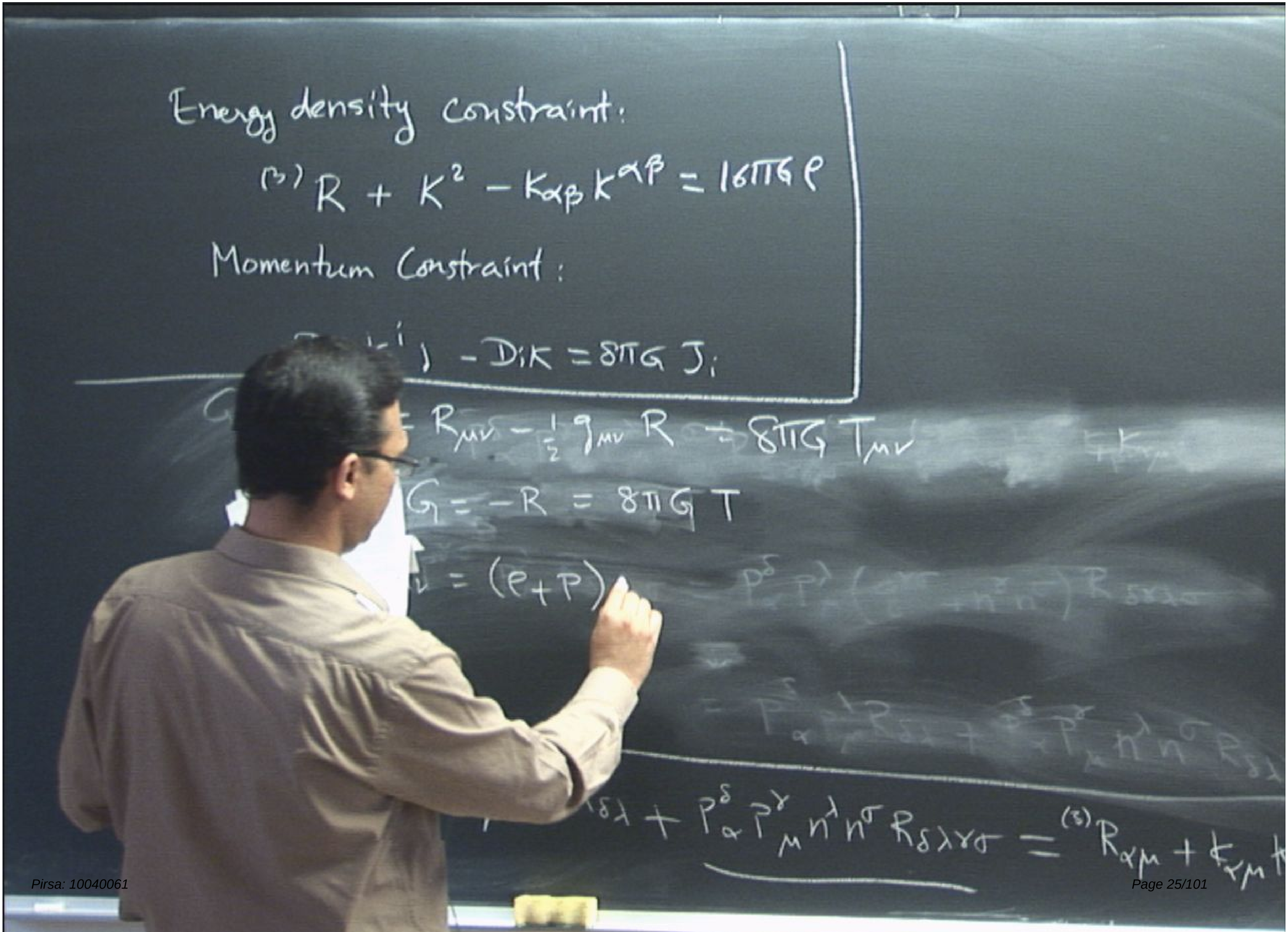
$K_\alpha{}^\nu K_{\nu\mu}$


Energy density constraint:
(3)R + K2 − Kαβ Kαβ = 16πG ρ
Momentum Constraint:
− DiK = 8πG Ji
= Rμν − ½ gμν R = 8πG Tμν
G = −R = 8πG T
= (ρ+P)
+ Pδα Pγμ nλ nσ Rδλγσ = (3)Rαμ + K

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

Momentum Constraint:

$$D_j K^i{}_j - D_i K = 8\pi G J_i$$

Gauss Eqn

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$G = -R = 8\pi G T$$

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

$$\boxed{P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} \ldots}$$

Energy density constraint:

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Momentum Constraint:

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Energy density constraint:

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Gauss Eqn.

$$ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} $$

$$ G = -R = 8\pi G T = 8\pi G (S - \rho) $$

$$ T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu} $$

$$ T_{ij} = S_{ij} $$

$$ P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu} K $$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_j K^i{}_j - D_i K = 8\pi G J_i $$

Gauss Eqn

$$ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} $$

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$$ P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K \ldots $$

density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

entum Constraint:

$$D_j K^i{}_j - D_i K = 8\pi G J_i$$

HW. $P^\delta{}_\mu P$

use (i) (

(ii)

(iii)

(iv)

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$G = -R = 8\pi G T = 8\pi G (S - \rho)$$

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

$$T_{ij} = S_{ij}$$

$$\Longrightarrow \boxed{R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (S - \rho\right.}$$

$$\boxed{P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - K_\alpha{}^\nu K_{\nu\mu}}$$

density constraint:

$${}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

entum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$G = -R = 8\pi G T = 8\pi G (S - \rho)$$

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

$$T_{ij} = S_{ij} \Longrightarrow R_{\mu\nu} = 8\pi G (T_{\mu\nu}$$

$$P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} =$$

HW.

$P^\delta{}_\mu P$

use (i) (

(ii)

(iii)

(iv)

H.W.:

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + }$$

use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$

(ii) $K_{\mu\nu}$

(iii) $K_{\mu\nu} n^{\nu} = 0$

(iv) $\nabla_\mu t$

$= D_\nu \ln N$

$\pounds_n$

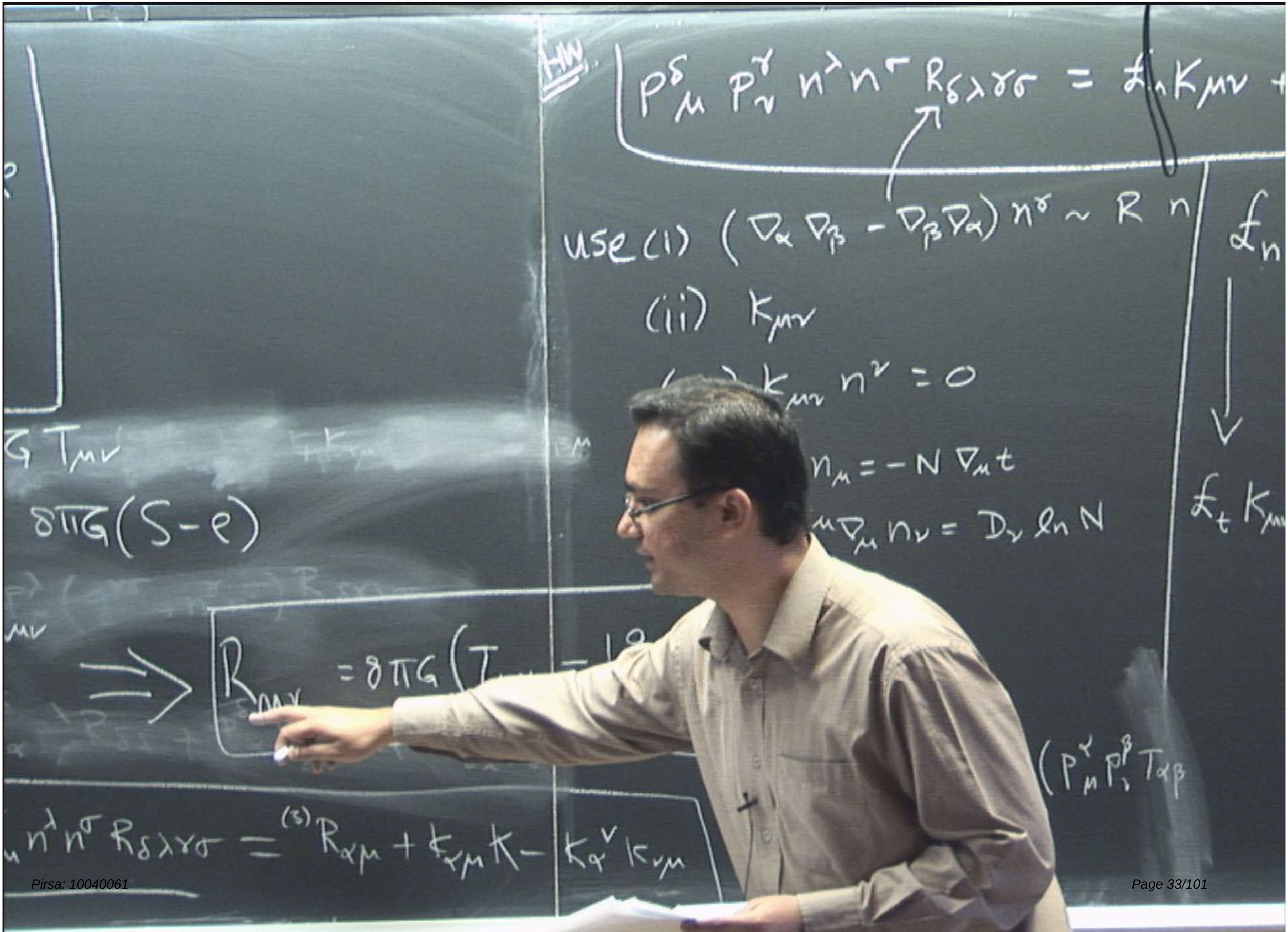
$\pounds_t K_{\mu\nu}$

$G\, T_{\mu\nu}$

$8\pi G (S - \rho)$

$$\Longrightarrow \boxed{R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \right.}$$

$n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - K$


HW.
$P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} +$
use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$
(ii) $K_{\mu\nu}$
$K_{\mu\nu} n^\nu = 0$
$n_\mu = -N \nabla_\mu t$
$\nabla_\mu n_\nu = D_\nu \ln N$
$\pounds_n$
$\pounds_t K_{\mu\nu}$
$G T_{\mu\nu}$
$8\pi G(S-\rho)$
$R_{\mu\nu} = 8\pi G($
$(P^\alpha_\mu P^\beta_\nu T_{\alpha\beta}$
$n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu} K - K_\alpha{}^\nu K_{\nu\mu}$

HW: $$\boxed{P^\delta_\mu P^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} +}$$

use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$

(ii) $K_{\mu\nu}$

(iii) $K_{\mu\nu} n^\nu = 0$

(iv) $n_\mu = -N \nabla_\mu t$

$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$

$\mathcal{L}_n$

$\mathcal{L}_t K_{\mu\nu}$

$G\, T_{\mu\nu}$

$8\pi G (S - \rho)$

$$\Longrightarrow \boxed{R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (S - \rho)\right)} \Longrightarrow$$

$P^\alpha_\mu P^\beta_\nu R_{\alpha\beta} = 8\pi G \left(P^\alpha_\mu P^\beta_\nu T_{\alpha\beta} \right.$

$n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu} K - K_\alpha{}^\nu K_{\nu\mu}$

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}_{\nu} + \frac{1}{N} D_\mu D_\nu N}$$

use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$

(ii) $K_{\mu\nu}$

(iii) $K_{\mu\nu} n^\nu = 0$

(iv) $n_\mu = -N \nabla_\mu t$

$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$

$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_\nu^{\lambda} + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^{\gamma}_{\nu} - P^{\delta}_{\mu} P^{\lambda}_{\nu} R_{\delta\lambda}$$

$$\pounds_t K_{\mu\nu} - \pounds \ldots D_\mu D_\nu N$$

$$\ldots {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^{\gamma}_{\nu})$$

$$- \frac{1}{2} g_{\mu\nu} (S - \rho))$$

$$P^{\alpha}_{\mu} P^{\beta}_{\nu} R_{\alpha\beta} = 8\pi G \left(P^{\gamma}_{\mu} P^{\beta}_{\nu} T_{\alpha\beta} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2} (S - \rho \ldots \right.$$

$$K_\alpha{}^{\nu} K_{\nu\mu}$$

$$\boxed{P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N}$$

use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^{\gamma} \sim R\, n$

(ii) $K_{\mu\nu}$

(iii) $K_{\mu\nu} n^{\nu} = 0$

(iv) $n_\mu = -N \nabla_\mu t$

$n^{\mu} \nabla_\mu n_\nu = D_\nu \ln N$

$$\mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K_{\nu}{}^{\lambda} + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^{\gamma}{}_{\nu} - P^{\delta}_{\mu} P^{\lambda}_{\nu} R_{\delta\lambda}$$

$$\mathcal{L}_t K_{\mu\nu} - \mathcal{L}_{N^i} K_{\mu\nu} = -D_\mu D_\nu N + N\left({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^{\gamma}{}_{\nu}\right) + 4\pi G\left(\gamma_{\mu\nu}(S-\rho) - 2 S_{\mu\nu}\right)$$

$$-\frac{1}{2} g_{\mu\nu}(S-\rho))$$

$$P^{\alpha}_{\mu} P^{\beta}_{\nu} R_{\alpha\beta} = 8\pi G \Big(\underbrace{P^{\gamma}_{\mu} P^{\beta}_{\nu} T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2}(S-\rho) \Big)$$

$K_{\alpha}{}^{\nu} K_{\nu\mu}$

...gy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

...omentum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

HW.

$$P^\delta{}_\mu$$

use (i)

(ii)

(iii)

(iv)

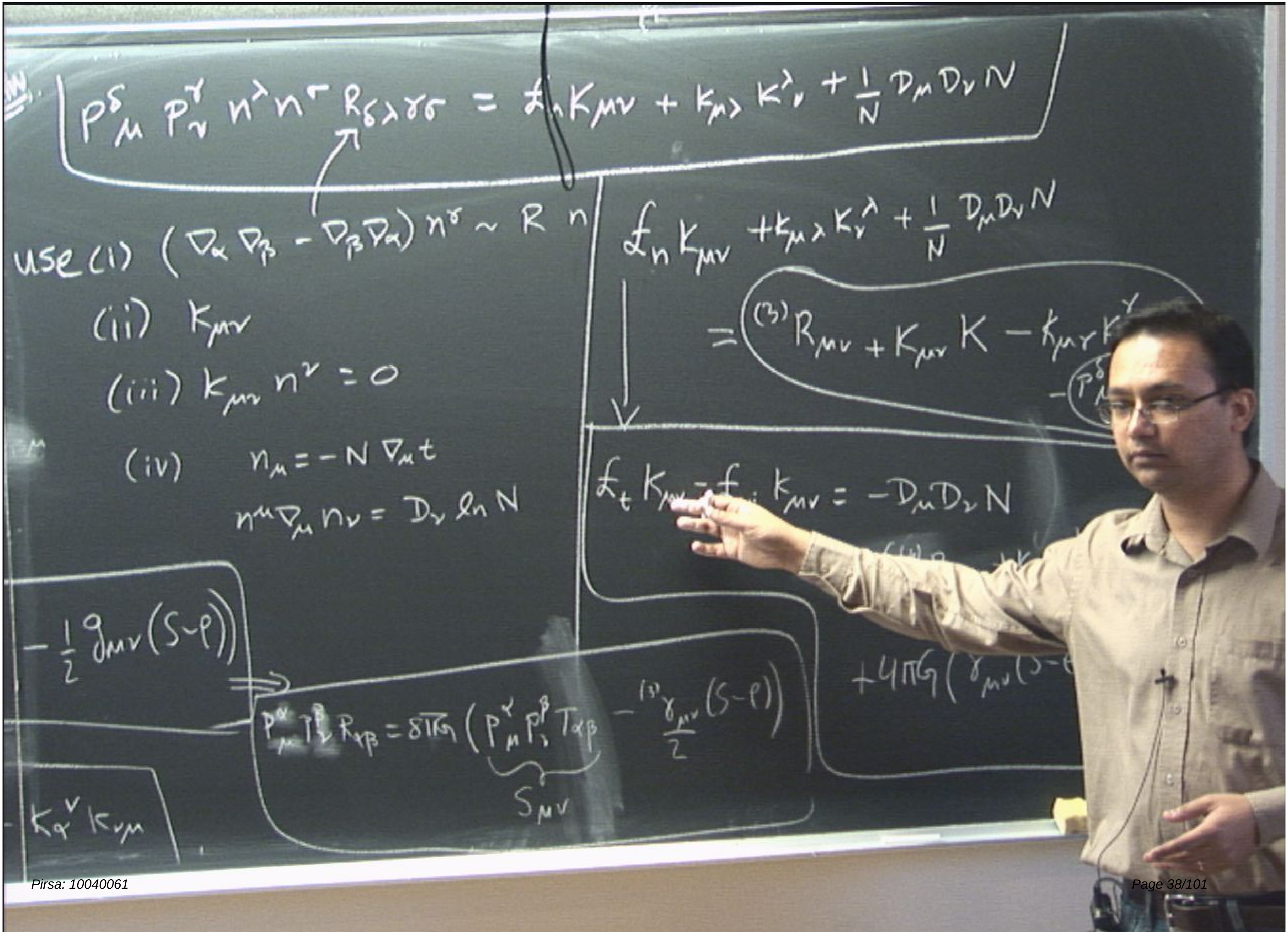
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$G = -R = 8\pi G T = 8\pi G (S - \rho)$$

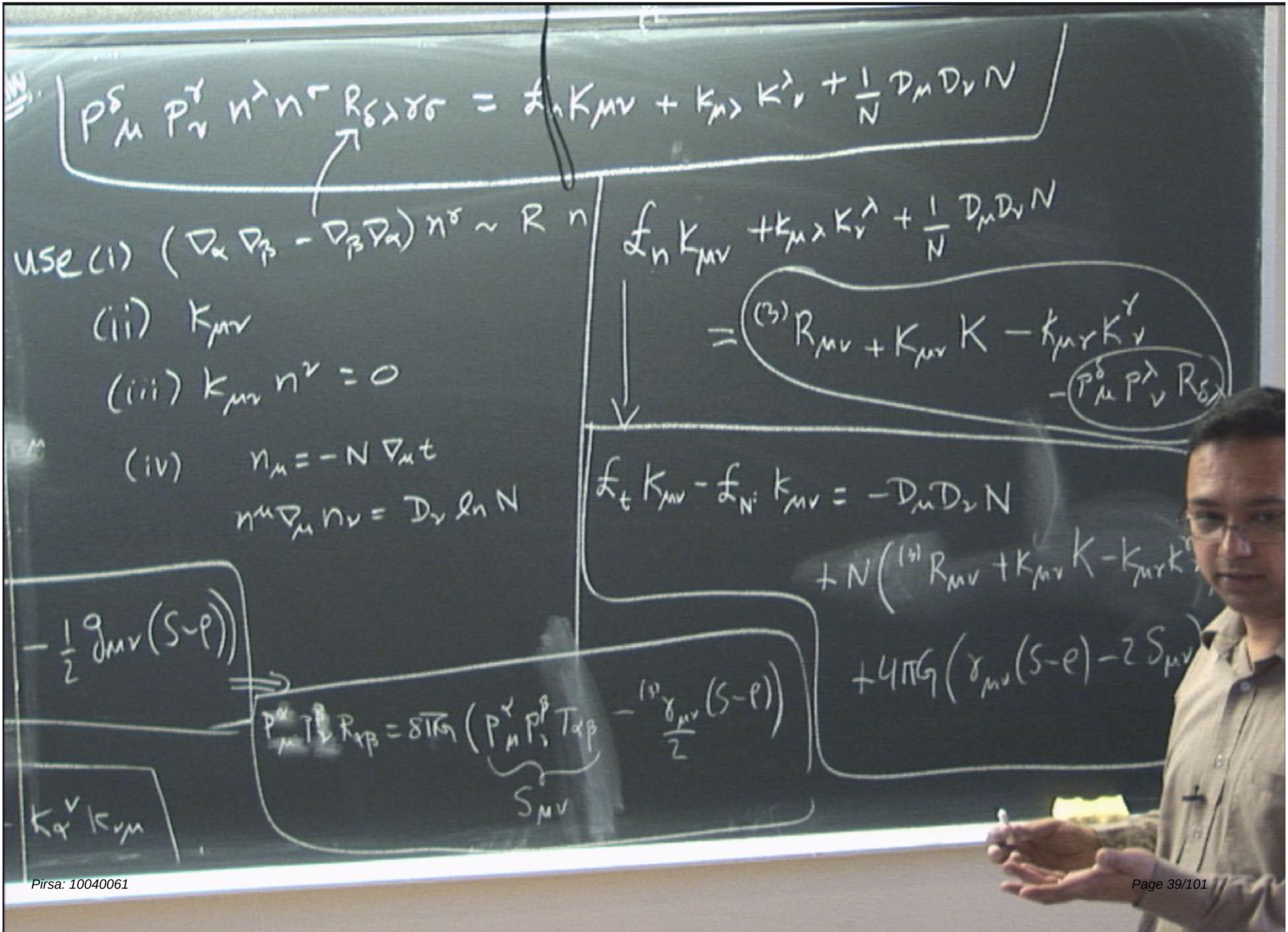
$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

$$T_{ij} = S_{ij} \implies \boxed{R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(S\right.}$$

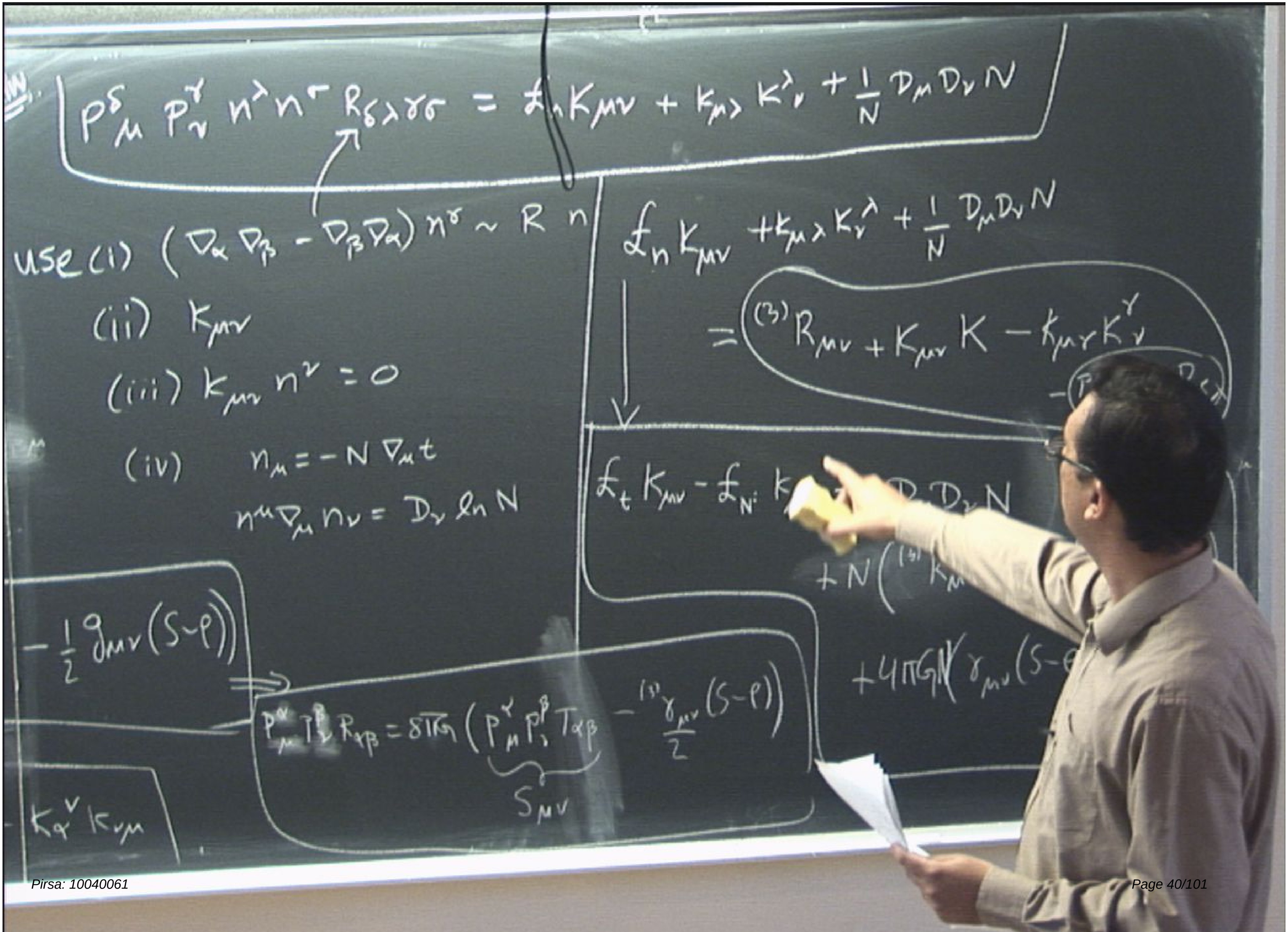
$$\boxed{P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - K_\alpha{}^\nu K_{\nu\mu}}$$



$P^\delta_\mu P^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda{}_\nu + \frac{1}{N} D_\mu D_\nu N$
use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$
(ii) $K_{\mu\nu}$
(iii) $K_{\mu\nu} n^\nu = 0$
(iv) $n_\mu = -N \nabla_\mu t$
$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$
$\mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K_\nu^\lambda + \frac{1}{N} D_\mu D_\nu N$
$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma$
$\mathcal{L}_t K_{\mu\nu} - \mathcal{L}_\beta K_{\mu\nu} = -D_\mu D_\nu N$
$+4\pi G \left(\gamma_{\mu\nu}(S-\rho\right.$
$-\frac{1}{2} g_{\mu\nu}(S-\rho))$
$P^\alpha_\mu P^\beta_\nu R_{\alpha\beta} = 8\pi G \left(\underbrace{P^\alpha_\mu P^\beta_\nu T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2}(S-\rho) \right)$
$K_\alpha{}^\nu K_{\nu\mu}$



$P^\delta_\mu P^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N$
use (i) $(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) n^\gamma \sim R\, n$
(ii) $K_{\mu\nu}$
(iii) $K_{\mu\nu} n^\nu = 0$
(iv) $n_\mu = -N \nabla_\mu t$
$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$
$\mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K_\nu^\lambda + \frac{1}{N} D_\mu D_\nu N$
$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma_\nu - P^\delta_\mu P^\lambda_\nu R_{\delta\lambda}$
$\mathcal{L}_t K_{\mu\nu} - \mathcal{L}_{N^i} K_{\mu\nu} = -D_\mu D_\nu N$
$+ N ({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma$
$+ 4\pi G (\gamma_{\mu\nu} (S - \rho) - 2 S_{\mu\nu}$
$- \frac{1}{2} g_{\mu\nu} (S - \rho))$
$P^\alpha_\mu P^\beta_\nu R_{\alpha\beta} = 8\pi G \Big(\underbrace{P^\alpha_\mu P^\beta_\nu T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2} (S - \rho) \Big)$
$K_\alpha{}^\nu K_{\nu\mu}$



P^δ_μ P^γ_ν n^λ n^σ R_δλγσ = £_n K_μν + K_μλ K^λ_ν + 1/N D_μ D_ν N
use (i) (∇_α ∇_β − ∇_β ∇_α) n^γ ~ R n
(ii) K_μν
(iii) K_μν n^ν = 0
(iv) n_μ = −N ∇_μ t
n^μ ∇_μ n_ν = D_ν ln N
£_n K_μν + K_μλ K_ν^λ + 1/N D_μ D_ν N
= ⁽³⁾R_μν + K_μν K − K_μγ K^γ_ν
£_t K_μν − £_N K
+ 4πGN (γ_μν (S −
− 1/2 g_μν (S − ρ))
P^α_μ P^β_ν R_αβ = 8πG (P^γ_μ P^β_ν T_αβ − ⁽³⁾γ_μν/2 (S − ρ))
S_μν
K_α^ν K_νμ

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

Momentum Constraint:

$$D_j K^i{}_j - D_i K = 8\pi G J_i$$

$$\boxed{\gamma^{\mu\alpha} \pounds_{Nn} K_{\alpha\nu} = \pounds_{Nn} K^\mu{}_\nu + 2N K^{\mu\alpha} K_{\alpha\nu}}$$

$$\boxed{R_{\mu\nu} = 8\pi G (T_{\mu\nu}}$$

$$P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu}K - K$$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_j K^j{}_i - D_i K = 8\pi G J_i $$

$$ \boxed{\gamma^{\mu\alpha} \mathcal{L}_{Nn} K_{\alpha\nu} = \mathcal{L}_{Nn} K^{\mu}{}_{\nu} + 2N K^{\mu\alpha} K_{\alpha\nu}} $$

$$ \mathcal{L}_{Nn} K^{\mu}{}_{\nu} = \mathcal{L}_{Nn}(\gamma^{\mu\alpha} K_{\alpha\nu}) $$

$$ R_{\mu\nu} = 8\pi G (T_{\mu\nu} - $$

$$ P^{\delta}{}_{\mu} P^{\lambda}{}_{\nu} R_{\delta\lambda} + \underline{P^{\delta}{}_{\alpha} P^{\gamma}{}_{\mu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma}} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - K $$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_j K^i{}_j - D_i K = 8\pi G J_i $$

$$ \boxed{\gamma^{\mu\alpha}\mathcal{L}_{Nn} K_{\alpha\nu} = \mathcal{L}_{Nn} K^\mu{}_\nu + 2N K^{\mu\alpha} K_{\alpha\nu}} $$

$$ \mathcal{L}_{Nn} K^\mu{}_\nu = \mathcal{L}_{Nn}(\gamma^{\mu\alpha} K_{\alpha\nu}) $$
$$ = (\mathcal{L}_{Nn}\gamma^{\mu\alpha}) K_{\alpha\nu} + \gamma^{\mu\alpha}\mathcal{L}_{Nn} K_{\alpha\nu} $$

$$ R_{\mu\nu} = 8\pi G (T_{\mu\nu} $$

$$ P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - $$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

Momentum Constraint:

$$D_i K^i{}_j - D_i K = 8\pi G J_i$$

$$\ldots_{Nn} K_{\alpha\nu} = \mathcal{L}_{Nn} K^{\mu}{}_{\nu} + 2N K^{\mu\alpha} K_{\alpha\nu}$$

$$\ldots_{Nn} K^{\mu}{}_{\nu} = \mathcal{L}_{Nn}\left(\gamma^{\mu\alpha} K_{\alpha\nu}\right)$$

$$= \left(\mathcal{L}_{Nn}\gamma^{\mu\alpha}\right) K_{\alpha\nu} + \gamma^{\mu\alpha}\mathcal{L}_{Nn} K_{\alpha\nu}$$

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} \ldots\right.$$

$$\ldots{}_{\nu} R_{\delta\lambda} + P^{\delta}_{\alpha} P^{\gamma}_{\mu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - \ldots$$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_j K^j{}_i - D_i K = 8\pi G J_i $$

$$ \gamma^{\mu\alpha} \pounds_{Nn} K_{\alpha\nu} = \pounds_{Nn} K^\mu{}_\nu + 2N K^{\mu\alpha} K_{\alpha\nu} $$

$$ \pounds_{Nn} K^\mu{}_\nu = \pounds_{Nn}(\gamma^{\mu\alpha} K_{\alpha\nu}) $$
$$ = (\pounds_{Nn}\gamma^{\mu\alpha}) K_{\alpha\nu} + \gamma^{\mu\alpha}\pounds_{Nn} K_{\alpha\nu} $$
$$ = -2N K^{\mu\alpha} K_{\alpha\nu} + \gamma^{\mu\alpha}\pounds_{Nn} K_{\alpha\nu} $$

$$ \boxed{R_{\mu\nu} = 8\pi G (T_{\mu\nu} - } $$

$$ P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - K $$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

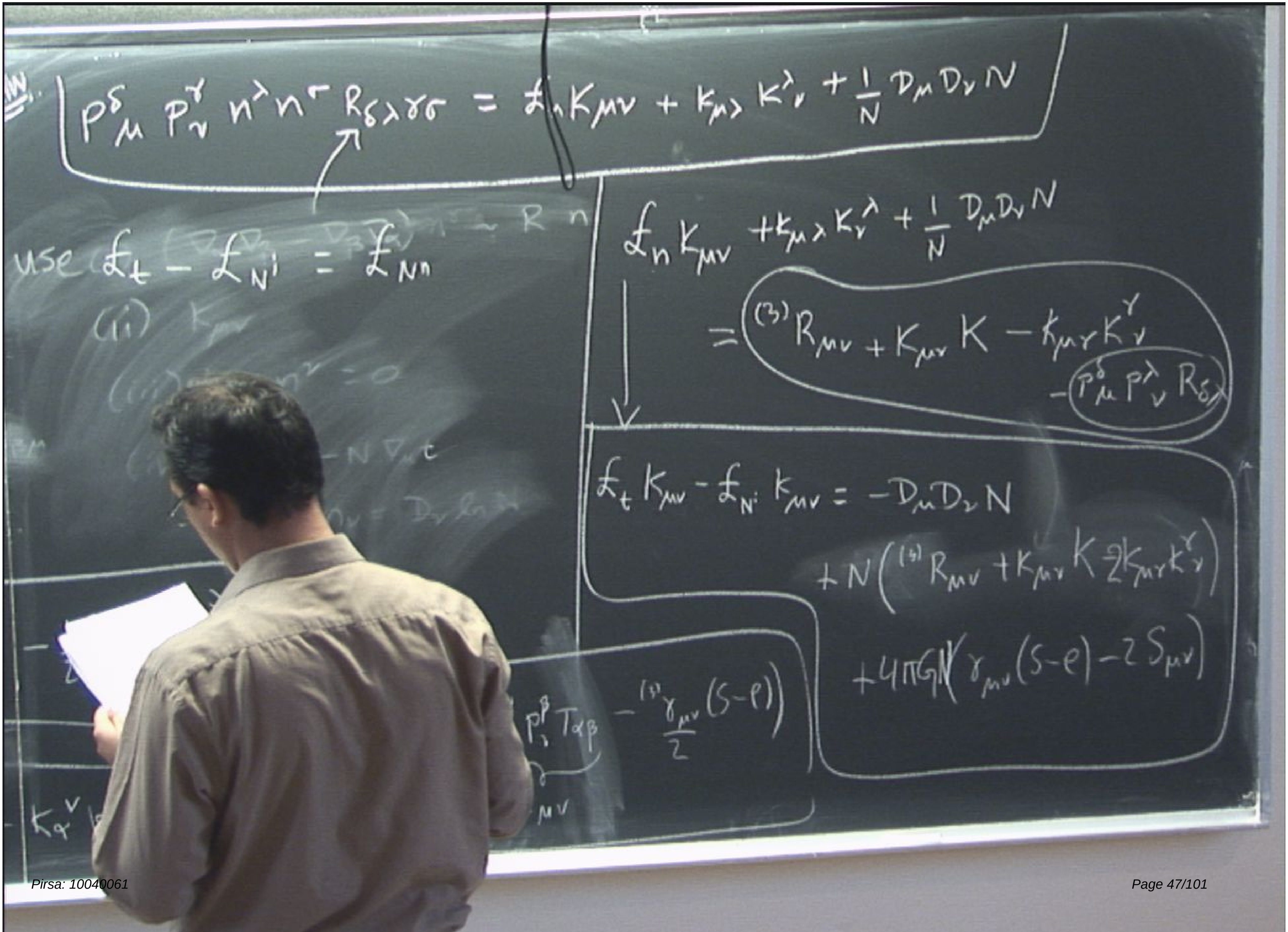
$$ D_j K^i{}_j - D_i K = 8\pi G J_i $$

$$ \boxed{\gamma^{\mu\alpha} \mathcal{L}_{Nn} K_{\alpha\nu} = \mathcal{L}_{Nn} K^\mu{}_\nu + 2N K^{\mu\alpha} K_{\alpha\nu}} $$

$$ \mathcal{L}_{Nn} K^\mu{}_\nu = \mathcal{L}_{Nn}(\gamma^{\mu\alpha} K_{\alpha\nu}) $$
$$ = (\mathcal{L}_{Nn}\gamma^{\mu\alpha}) K_{\alpha\nu} + \gamma^{\mu\alpha}\mathcal{L}_{Nn} K_{\alpha\nu} $$
$$ = -2N K^{\mu\alpha} K_{\alpha\nu} + \gamma^{\mu\alpha} \mathcal{L}_{Nn} K_{\alpha\nu} $$

$$ R_{\mu\nu} = 8\pi G (T $$

$$ P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R $$


$p^\delta_\mu p^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N$
use $\pounds_t - \pounds_{N^i} = \pounds_{Nn}$
$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N$
$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma_\nu - p^\delta_\mu p^\lambda_\nu R_{\delta\lambda}$
$\pounds_t K_{\mu\nu} - \pounds_{N^i} K_{\mu\nu} = -D_\mu D_\nu N$
$+ N\left({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - 2K_{\mu\gamma} K^\gamma_\nu\right)$
$+ 4\pi G N\left(\gamma_{\mu\nu}(S-\rho) - 2 S_{\mu\nu}\right)$

HW. $$P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}_{\nu} + \frac{1}{N} D_\mu D_\nu N$$

use $\pounds_t - \pounds_{N^i} = \pounds_{Nn}$

(ii) $K_{\mu\nu}$

$$\pounds_t K^{\mu}{}_{\nu} - \pounds_{N^i} K^{\mu}{}_{\nu} = -D^{\mu} D_\nu N + N\left[{}^{(3)}R^{\mu}{}_{\nu} + K K^{\mu}{}_{\nu}\right.$$

$$n^\mu \nabla_\mu n_\nu = D_\nu \ln N$$

$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_\nu^\lambda + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\nu}$$

$$\pounds_t K_{\mu\nu} - \pounds_{N^i} K_{\mu\nu} = -D_\mu D_\nu N + N\left({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K\right.$$

$$+ 4\pi G N\left(\gamma_{\mu\nu}(S-\rho)\right.$$

$$= 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(S-\rho)\right)$$

$$\Longrightarrow$$

$$P^{\alpha}_{\mu} P^{\beta}_{\nu} R_{\alpha\beta} = 8\pi G \left(\underbrace{P^{\gamma}_{\mu} P^{\beta}_{\nu} T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2}(S-\rho)\right)$$

$$R_{\alpha\mu} + K_{\alpha\mu} K - K_\alpha{}^\nu K_{\nu\mu}$$

HW. $\boxed{P^\delta_\mu P^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N}$

use $\mathcal{L}_t - \mathcal{L}_{N^i} = \mathcal{L}_{Nn}$

$$\mathcal{L}_t K^\mu_\nu - \mathcal{L}_{N^i} K^\mu_\nu = -D^\mu D_\nu N + N\left[{}^{(3)}R^\mu_\nu + K K^\mu_\nu - 8\pi G S^\mu_\nu + 4\pi G \delta^i_j\right.$$

$$= 8\pi G\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(S-\rho)\right)$$

$$R_{\alpha\mu} + K_{\gamma\mu} K - K_\alpha{}^\nu K_{\nu\mu}$$

$$\Rightarrow P^\alpha_\mu P^\beta_\nu R_{\alpha\beta} = 8\pi G\Big(\underbrace{P^\gamma_\mu P^\beta_\nu T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2}(S-\rho)\Big)$$

$$\mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N = {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\lambda} \ldots$$

$$\mathcal{L}_t K_{\mu\nu} - \mathcal{L}_{N^i} K_{\mu\nu} = -D_\mu D_\nu N + N\left({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K \ldots\right. + 4\pi G N\left(\gamma_{\mu\nu}(S-\rho)\ldots\right.$$

HW. $$P^{\delta}_{\mu} P^{\gamma}_{\nu} n^{\lambda} n^{\sigma} R_{\delta\lambda\gamma\sigma} = \pounds_n K_{\mu\nu} + K_{\mu\lambda} K^{\lambda}{}_{\nu} + \frac{1}{N} D_\mu D_\nu N$$

use $\pounds_t - \pounds_{N^i} = \pounds_{Nn}$

$$\pounds_t K^{\mu}{}_{\nu} - \pounds_{N^i} K^{\mu}{}_{\nu} = -D^{\mu} D_\nu N + N\left({}^{(3)}R^{\mu}{}_{\nu} + K K^{\mu}{}_{\nu} - 8\pi G S^{\mu}{}_{\nu} + 4\pi G \delta^{\mu}_{\nu}(S-\rho)\right)$$

$$\pounds_n K_{\mu\nu} + K_{\mu\lambda} K_{\nu}{}^{\lambda} + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\lambda} \ldots$$

$$\pounds_t K_{\mu\nu} - \pounds_{N^i} K_{\mu\nu} = -D_\mu D_\nu N + N\left({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K \ldots + 4\pi G N \gamma_{\mu\nu}(S-\rho)\right)$$

$$= 8\pi \ldots \frac{1}{2} g_{\mu\nu}(S-\rho))$$

$$P^{\alpha}_{\mu} P^{\beta}_{\nu} R_{\alpha\beta} = 8\pi G\left(\underbrace{P^{\gamma}_{\mu} P^{\beta}_{\nu} T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2}(S-\rho)\right)$$

$K_{\nu\mu}$

nergy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta} = 16\pi G \rho$$

Momentum Constraint:

$$D_j K^i{}_j - D_i K = 8\pi G J_i$$

H.W.:

$$\mathcal{L}_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha}$$

use

$$\mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K^\mu$$

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\right.$$

$$P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu} K - K_\alpha{}^\nu K_{\nu\mu}$$

ergy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

$$\pounds_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha$$

HW: P^{δ}

use $\pounds$

$$\pounds_t K^\mu{}_\nu - \pounds_{N^i} K^\mu{}_\nu$$

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\right.$$

$$P^\delta{}_\alpha P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\alpha\mu}K - K_\alpha{}^\nu K_{\nu\mu}$$

gy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

omentum Constraint:

$$ D_i K^i{}_j - D_j K = 8\pi G J_i $$

$$ \mathcal{L}_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha $$

$$ - N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha $$

HW.

$$ P^\delta{}_\mu $$

use $\mathcal{L}_t$

$$ \mathcal{L}_t K^\mu{}_\nu - \mathcal{L}_{N^i} K^\mu{}_\nu $$

$$ R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (S \right. $$

$$ P^\delta{}_\mu P^\lambda{}_\nu R_{\delta\lambda} + P^\delta{}_\alpha P^\gamma{}_\mu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = {}^{(3)}R_{\alpha\mu} + K_{\gamma\mu} K - K_\alpha{}^\nu K_{\nu\mu} $$

$$P^\delta_\mu P^\gamma_\nu n^\lambda n^\sigma R_{\delta\lambda\gamma\sigma} = \mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N$$

use $\mathcal{L}_t - \mathcal{L}_{N^i} = \mathcal{L}_{Nn}$

$$\mathcal{L}_{N^i} K^\mu_\nu = -D^\mu D_\nu N + N\left({}^{(3)}R^\mu_\nu + K K^\mu_\nu - 8\pi G S^\mu_\nu + 4\pi G \delta^\mu_\nu (S-\rho)\right)$$

$$-\frac{1}{2} g_{\mu\nu}(S-\rho))$$

$$P^\alpha_\mu P^\beta_\nu R_{\alpha\beta} = 8\pi G\Big(\underbrace{P^\alpha_\mu P^\beta_\nu T_{\alpha\beta}}_{S_{\mu\nu}} - \frac{{}^{(3)}\gamma_{\mu\nu}}{2}(S-\rho)\Big)$$

$$K_\alpha{}^\nu K_{\nu\mu}$$

$$\mathcal{L}_n K_{\mu\nu} + K_{\mu\lambda} K^\lambda_\nu + \frac{1}{N} D_\mu D_\nu N$$

$$= {}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - K_{\mu\gamma} K^\gamma_\nu - P^\delta_\mu P^\lambda_\nu R_{\delta\lambda}$$

$$\mathcal{L}_t K_{\mu\nu} - \mathcal{L}_{N^i} K_{\mu\nu} = -D_\mu D_\nu N + N\left({}^{(3)}R_{\mu\nu} + K_{\mu\nu} K - 2K_{\mu\gamma} K^\gamma_\nu\right) + 4\pi G N\left(\gamma_{\mu\nu}(S-\rho) - 2 S_{\mu\nu}\right)$$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_i K^i{}_j - D_i K = 8\pi G J_i$$

$$\mathcal{L}_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha$$

$$\boxed{-N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha}$$

$N^\alpha \sim$ first order in perturbation

$N^k = -D^k\beta \qquad K^\alpha{}_\nu = -H\delta^\alpha{}_\nu$

$$\mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K$$

HW.

use

$(T_{\mu\nu}$

$-\frac{1}{2} g$

$K_\alpha{}^\nu K$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho $$

Momentum Constraint:

$$ D_j K^j{}_i - D_i K = 8\pi G J_i $$

$$ \boxed{\mathcal{L}_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha} $$

$$ \boxed{-N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha} $$

$N^\alpha \sim$ first order in perturbation

$N^k = -D^k\beta \qquad K^\alpha{}_\nu = -H\delta^\alpha{}_\nu$

HW:

use

$\mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K$

$(T_{\mu\nu}$

$-\frac{1}{2} g$

$K_\alpha{}^\nu K$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

$$\mathcal{L}_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha$$

$$(H\delta^\mu{}_\nu)_{,\alpha} = 0$$

$$\boxed{-N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha}$$

first order

$N^\alpha \sim$ first order in perturbation

$N^k = -D^k\beta \qquad K^\alpha{}_\nu = -H\delta^\alpha{}_\nu$

HW.

use

$\mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K$

$(T_{\mu\nu} - \frac{1}{2} g$

$K_\alpha{}^\nu K$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_j K^i{}_j - D_i K = 8\pi G J_i $$

$$ \mathcal{L}_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha $$

$$ (H\delta^\mu_\nu)_{,\alpha} = 0 $$

$$ \boxed{-N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha} $$

first order

$N^\alpha \sim$ first order in perturbation

$N^k = -D^k\beta \qquad K^\alpha{}_\nu = -H\delta^\alpha{}_\nu$

$$ \mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K $$

HW: use

$(T_{\mu\nu} \quad -\frac{1}{2} g$

$K_\alpha{}^\nu K$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

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$$(H\delta^\mu_\nu)_{,\alpha} = 0$$

$$\boxed{-N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha}$$

first order

$N^\alpha \sim$ first order in perturbation

$$N^k = -D^k\beta \qquad K^\alpha_\nu = -H\delta^\alpha_\nu$$

HW.

use

$$\mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K$$

$$K^\mu{}_{\nu,0} +$$

$$(T_{\mu\nu} \quad -\frac{1}{2} g$$

$$K_\alpha{}^\nu K$$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

$$T^{\mu\nu}{}_{;\mu} = 0$$

$$\pounds_t \rho$$

$$\pounds$$

$$\pounds_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha$$

$$(H\delta^\mu{}_\nu)_{,\alpha} = 0$$

$$-N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha$$

first order

$N^\alpha \sim$ first order in perturbation

$$N^k = -D^k\beta \qquad K^\alpha{}_\nu = -H\delta^\alpha{}_\nu$$

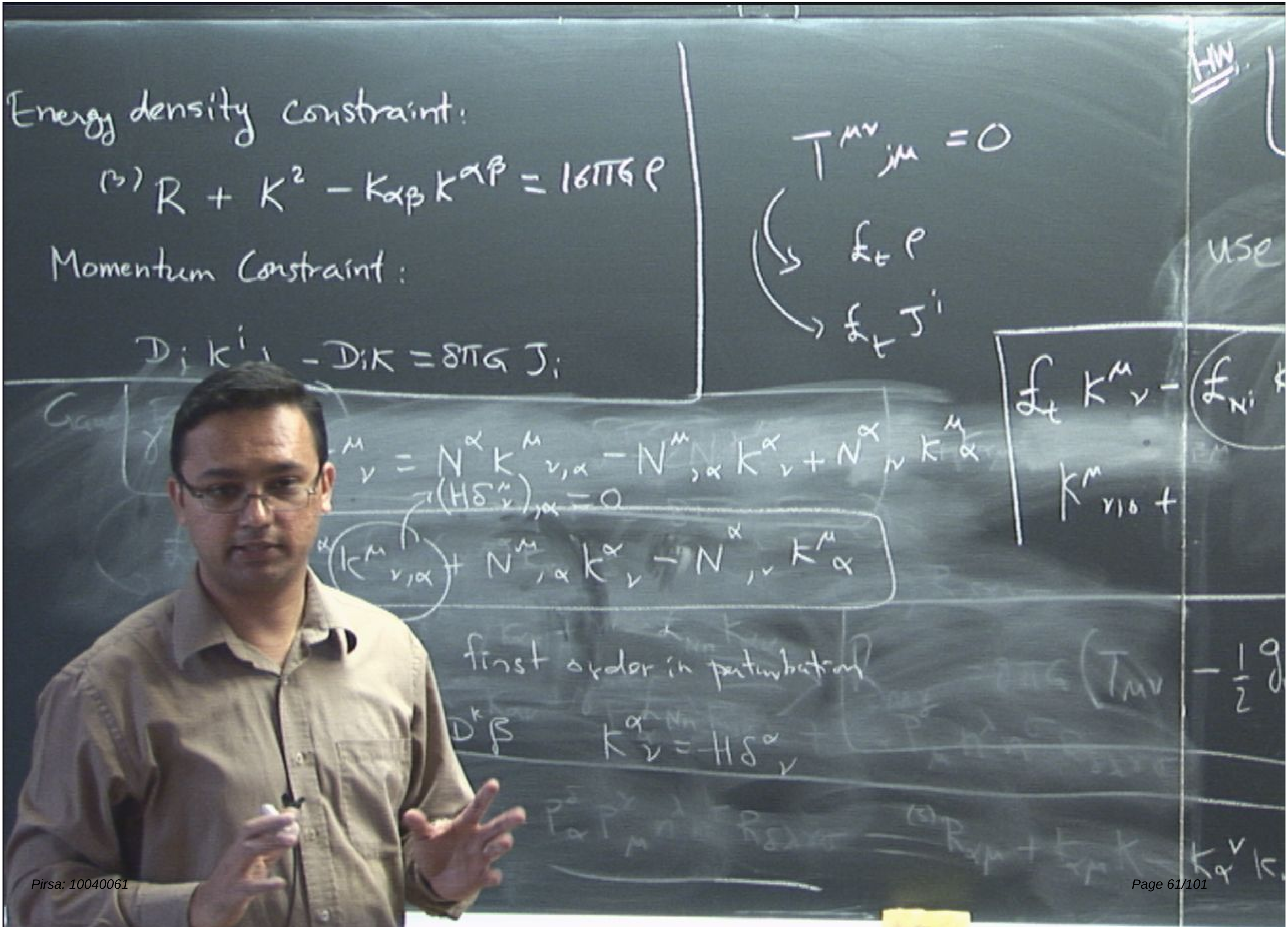
$$\pounds_t K^\mu{}_\nu - (\pounds_{N^i} K \ldots$$

$$K^\mu{}_{\nu,0} + \ldots$$

HW: use

$$(T_{\mu\nu} \quad -\frac{1}{2} g \ldots$$

$$K_\alpha{}^\nu K \ldots$$



Energy density constraint:
$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$
Momentum Constraint:
$D_j K^j{}_i - D_i K = 8\pi G J_i$
$T^{\mu\nu}{}_{;\mu} = 0$
$\pounds_t \rho$
$\pounds_t J^i$
HW
use
$\pounds_t K^\mu{}_\nu -$
$K^\mu{}_{\nu,0} +$
$= N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha$
$(H\delta^\mu_\nu)_{,\alpha} = 0$
$K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha$
first order in perturbation
$K^\alpha{}_\nu = -H\delta^\alpha{}_\nu$
$-\frac{1}{2}$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_i K^i{}_j - D_j K = 8\pi G J_j $$

$$ T^{\mu\nu}{}_{;\mu} = 0 $$

$$ \pounds_t \rho $$

$$ \pounds_t J^i $$

$$ \pounds_t K^\mu{}_\nu - (\pounds_{N^i} K $$

$$ K^\mu{}_{\nu,0} + $$

HW:

use

$$ \pounds_{N^i} K^\mu{}_\nu = N^\alpha K^\mu{}_{\nu,\alpha} - N^\mu{}_{,\alpha} K^\alpha{}_\nu + N^\alpha{}_{,\nu} K^\mu{}_\alpha $$

$$ (H\delta^\mu_\nu)_{,\alpha} = 0 $$

$$ - N^\alpha K^\mu{}_{\nu,\alpha} + N^\mu{}_{,\alpha} K^\alpha{}_\nu - N^\alpha{}_{,\nu} K^\mu{}_\alpha $$

first order

$N^\alpha \sim$ first order in perturbation

$$ N^k = -D^k\beta \qquad K^\alpha{}_\nu = -H\delta^\alpha{}_\nu $$

$$ (T_{\mu\nu} \qquad -\frac{1}{2} g $$

$$ K_\alpha{}^\nu K $$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$$

Momentum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

$T^{\mu\nu}{}_{;\mu} = 0$

$\mathcal{L}_t \rho$

$\mathcal{L}_t J^i$

HW:

use

$\mathcal{L}_t K^\mu{}_\nu - \mathcal{L}_{N^i} K$

$K^\mu{}_{\nu|0} +$

$(T_{\mu\nu} - \frac{1}{2} g$

$K_\alpha{}^\nu K$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_j K^j{}_i - D_i K = 8\pi G J_i$$

Dynamical Eqns.

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k$$

$$= -$$

$$T^{\mu\nu}{}_{;\mu} = 0$$

$$\mathcal{L}_t \rho$$

HW.

use

$$\mathcal{L}_t K^\mu{}_\nu - (\mathcal{L}_{N^i} K$$

$$K^\mu{}_{\nu,0} +$$

$$(T_{\mu\nu} \quad -\frac{1}{2} g$$

$$K_\alpha{}^\nu K$$

Energy density constraint:

$$ {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho $$

Momentum Constraint:

$$ D_i K^i{}_j - D_i K = 8\pi G J_i $$

<u>Dynamical Eqns.</u>

$$ K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k $$
$$ = -D^i D_j N + N\left(R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S-E)\right) $$

$$ T^{\mu\nu}{}_{;\mu} = 0 $$

$$ \mathcal{L}_t \rho $$

$$ \mathcal{L}_t J^i $$

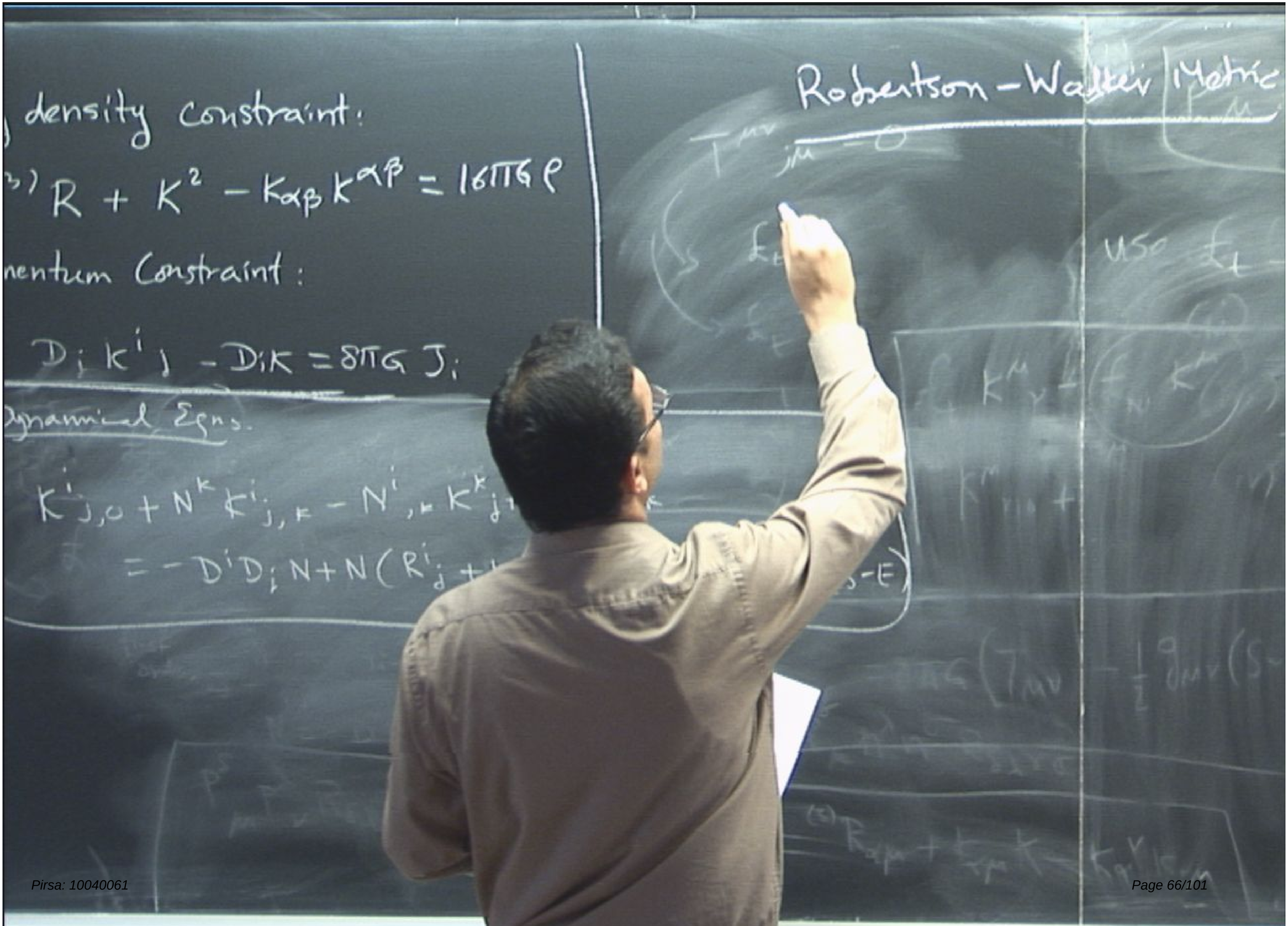
HW.

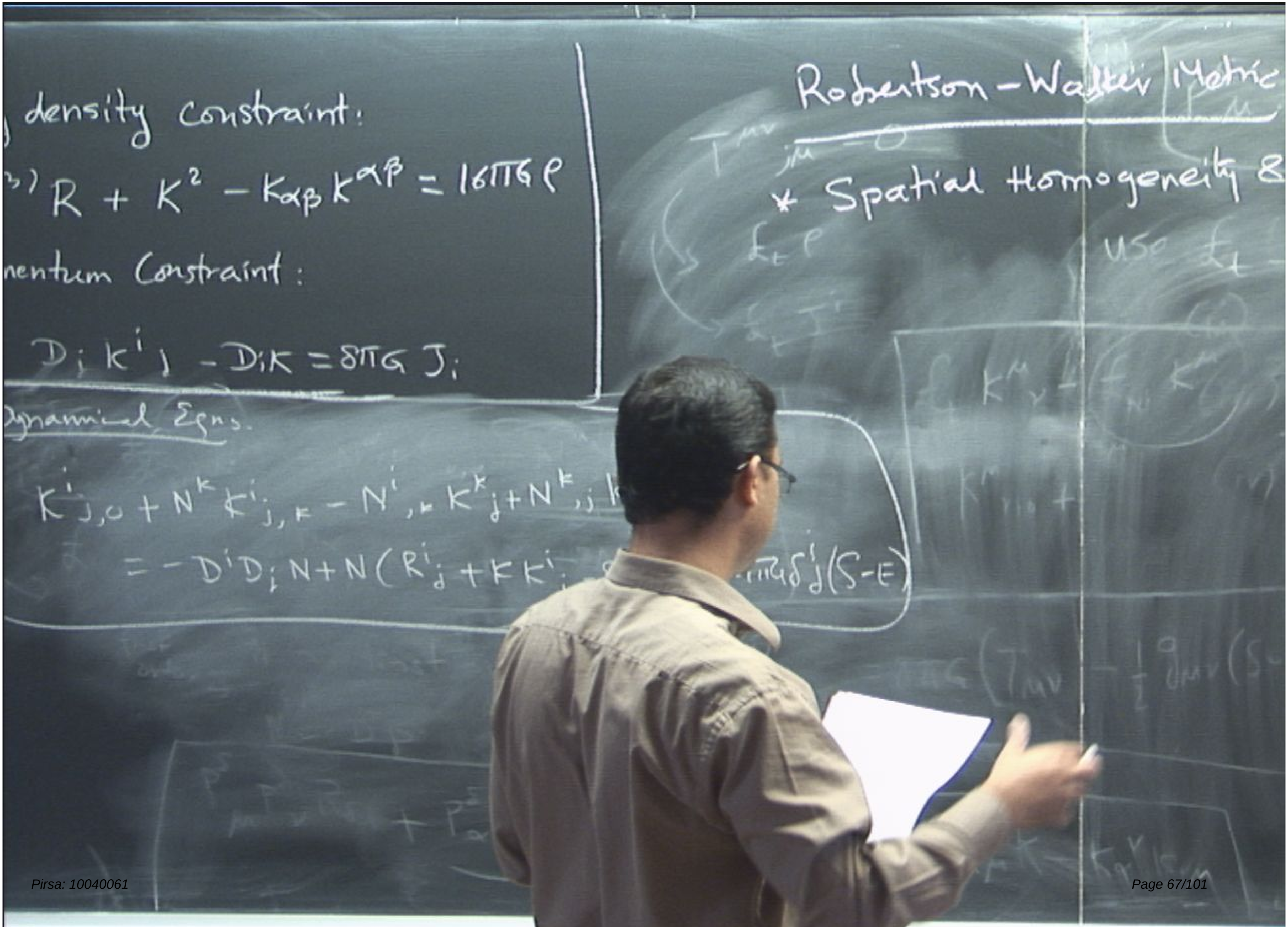
use

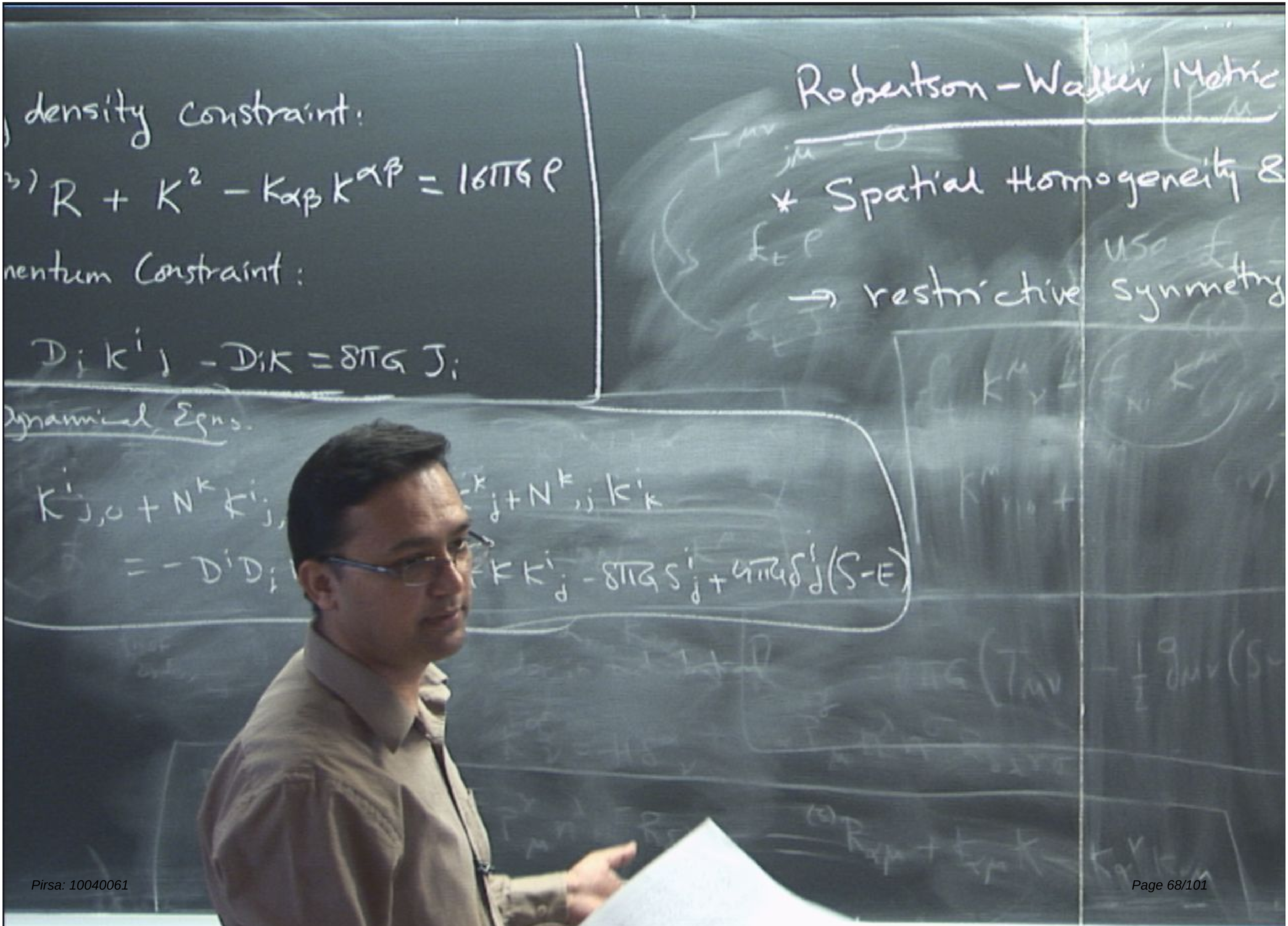
$$ \mathcal{L} K^\mu{}_\nu - (\mathcal{L}_N \ldots $$

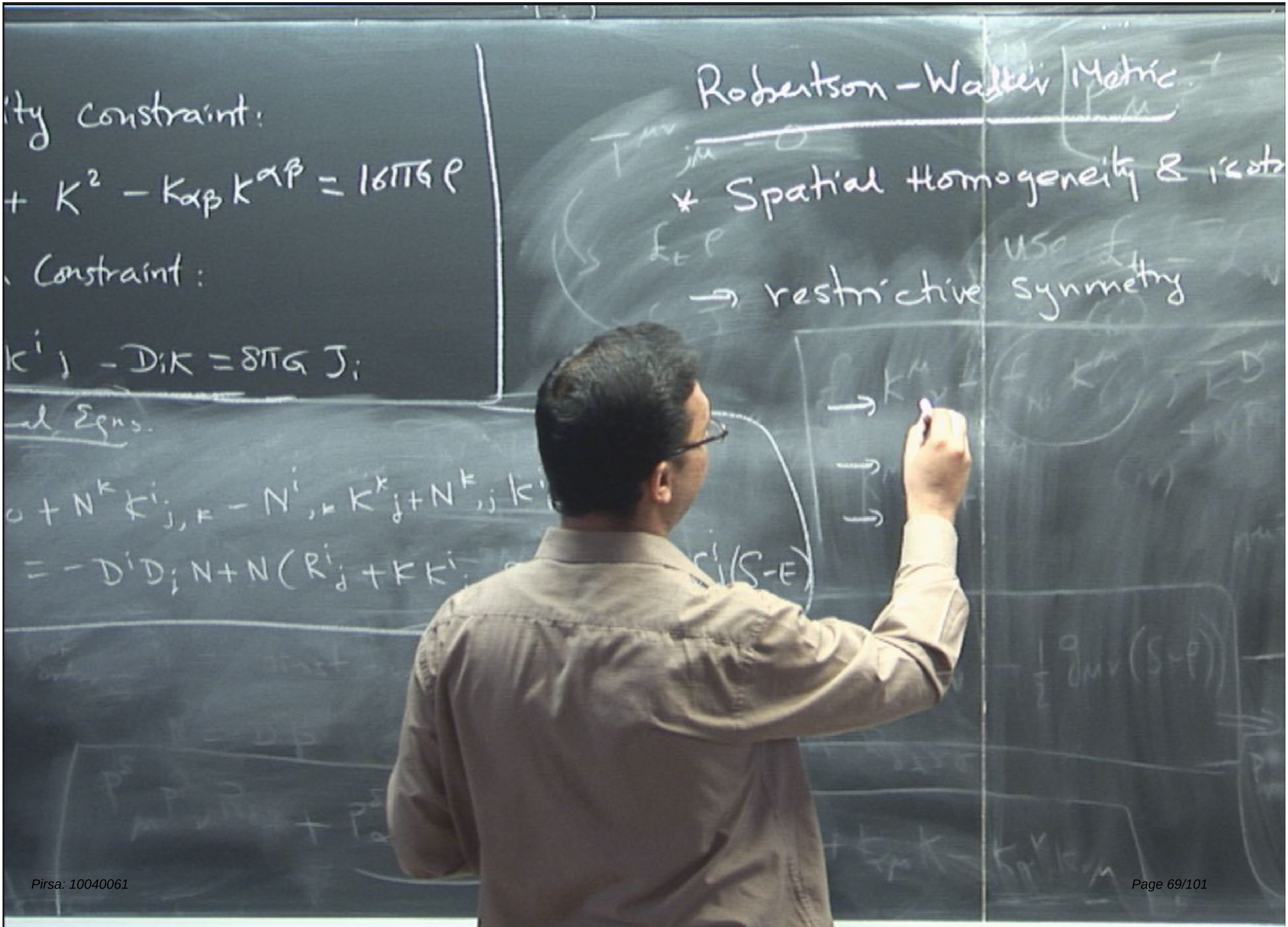
$$ K^\mu{}_{\nu,0} + $$

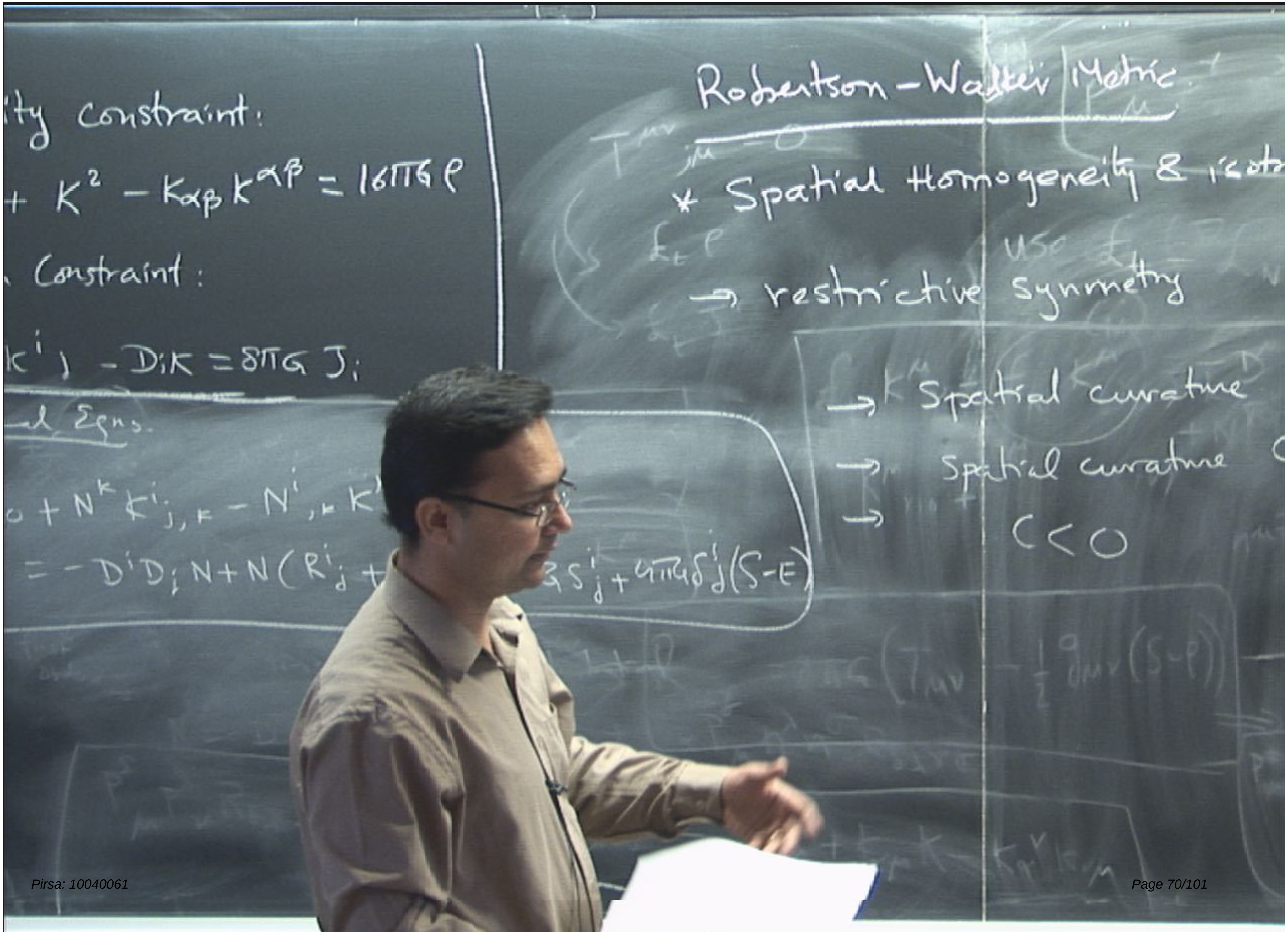
$$ \ldots\left(T_{\mu\nu} - \tfrac{1}{2} g \ldots\right. $$


density constraint:
$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$
mentum Constraint:
$D_j K^i{}_j - D_i K = 8\pi G J_i$
Dynamical Eqns.
Robertson-Walker Metric


density constraint:
$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$
mentum Constraint:
$D_j K^j{}_i - D_i K = 8\pi G J_i$
Dynamical Eqns.
$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j}$
$= - D^i D_j N + N(R^i{}_j + K K^i{}_j$
$\delta^i{}_j (S-E)$
Robertson-Walker Metric
* Spatial Homogeneity &


density constraint:
(3)R + K^2 - K_{αβ}K^{αβ} = 16πGρ
mentum Constraint:
D_i K^i_j - D_j K = 8πG J_j
Dynamical Eqns.
= -D^i D_j ... KK^i_j - 8πG S^i_j + 4πG δ^i_j (S-E)
Robertson-Walker Metric
* Spatial Homogeneity &
→ restrictive symmetry


ity constraint:
+ K^2 - Kαβ K^αβ = 16πGρ
Constraint:
K^i_j - D_iK = 8πG J_i
Eqns.
+ N^k K^i_j,k - N^i_,k K^k_j + N^k_,j K^i
= -D^iD_jN + N(R^i_j + KK^i_j
Robertson-Walker Metric
* Spatial Homogeneity & isotr
→ restrictive symmetry


Robertson-Walker Metric
* Spatial Homogeneity & isotr
→ restrictive symmetry
→ Spatial curvature
→ Spatial curvature
→ C<0
ity constraint:
+ K² - Kαβ K^αβ = 16πG ρ
Constraint:
- DiK = 8πG Ji

ity constraint:

$+ K^2 - K_{\alpha\beta} K^{\alpha\beta} = 16\pi G \rho$

Constraint:

$K^i{}_j - D_i K = 8\pi G J_i$

al Eqns.

$+ N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k$

$= -D^i D_j N + N(R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S-E))$

Robertson-Walker Metric

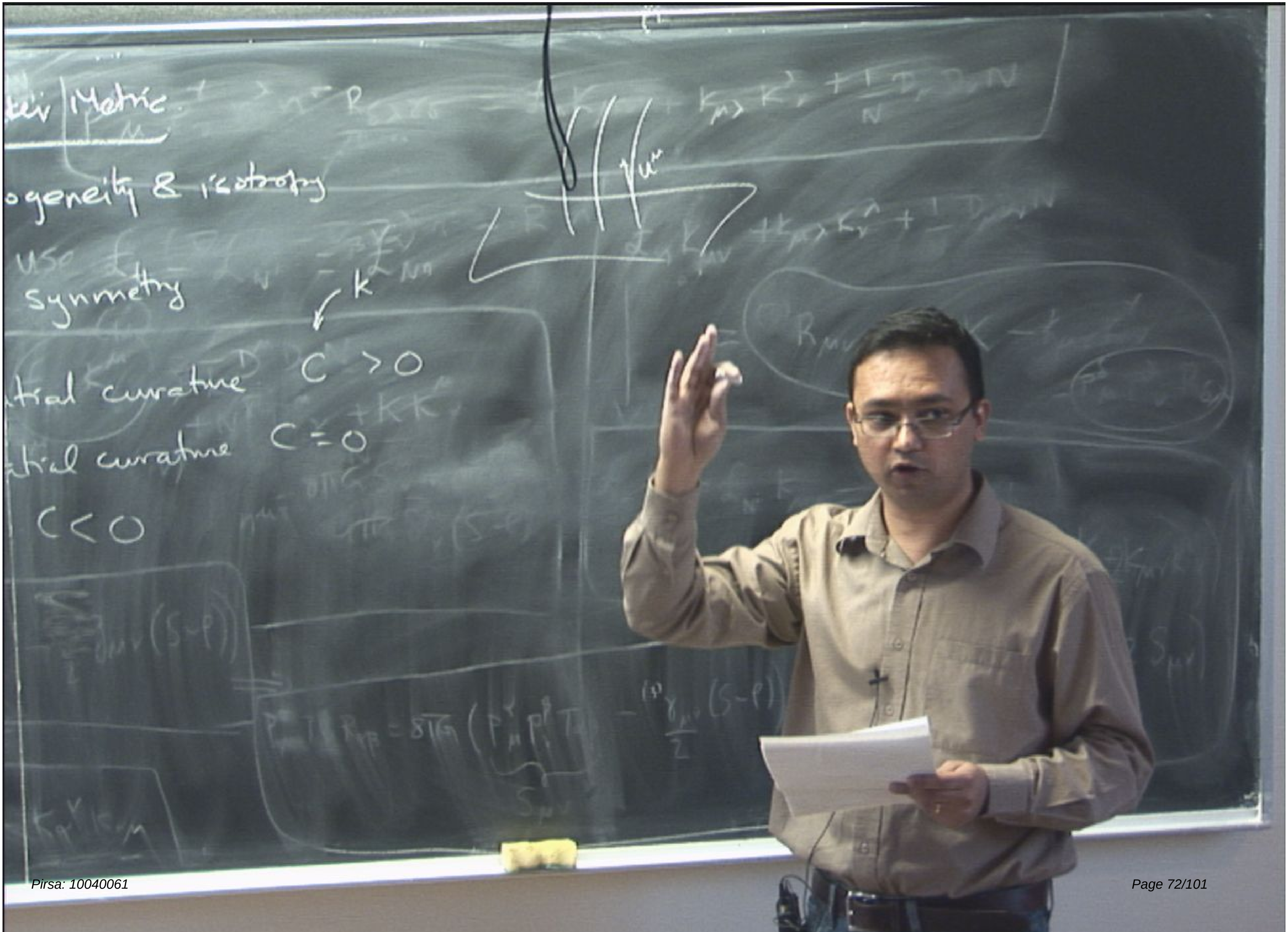
* Spatial Homogeneity & isotr

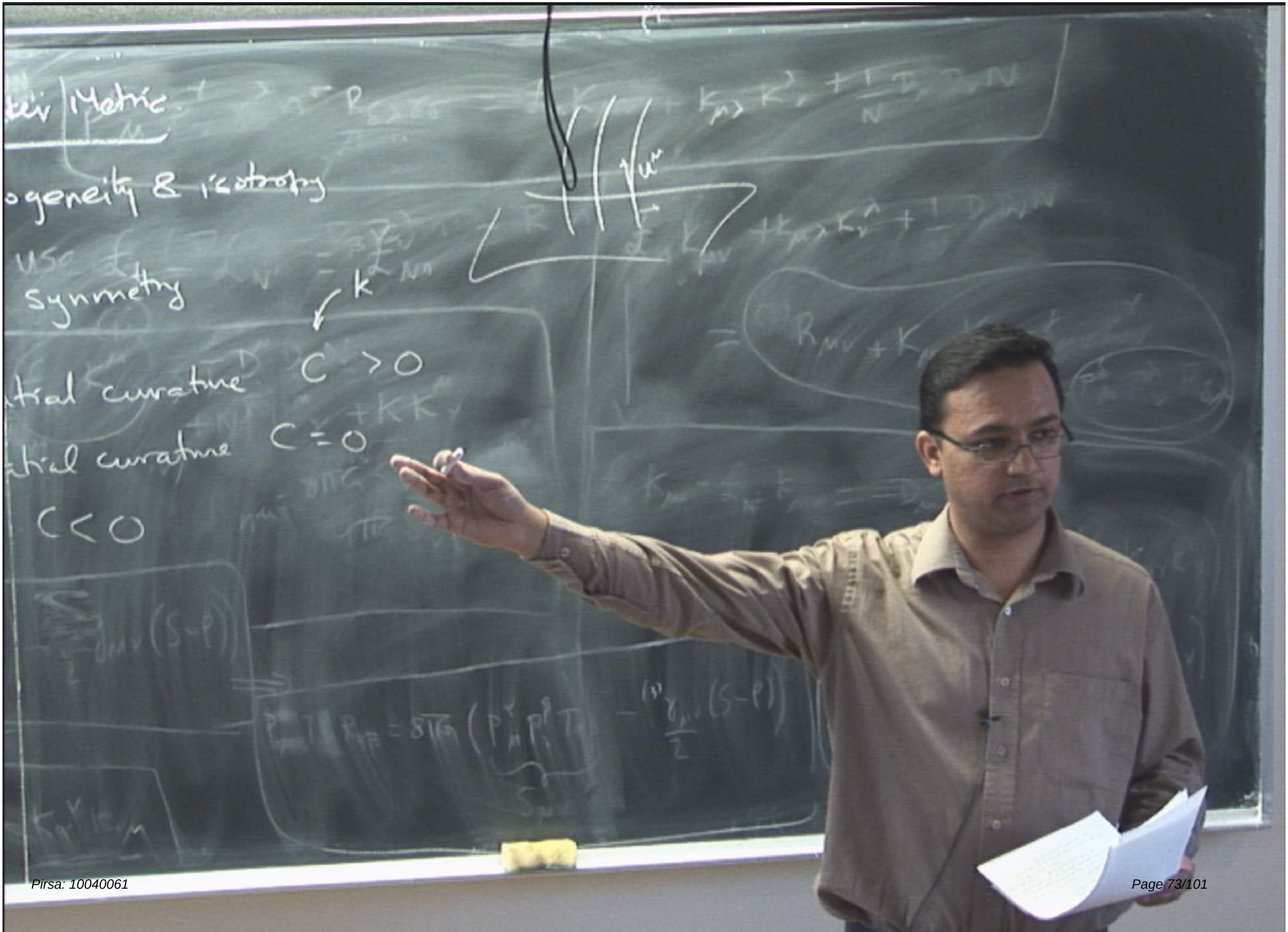
→ restrictive symmetry

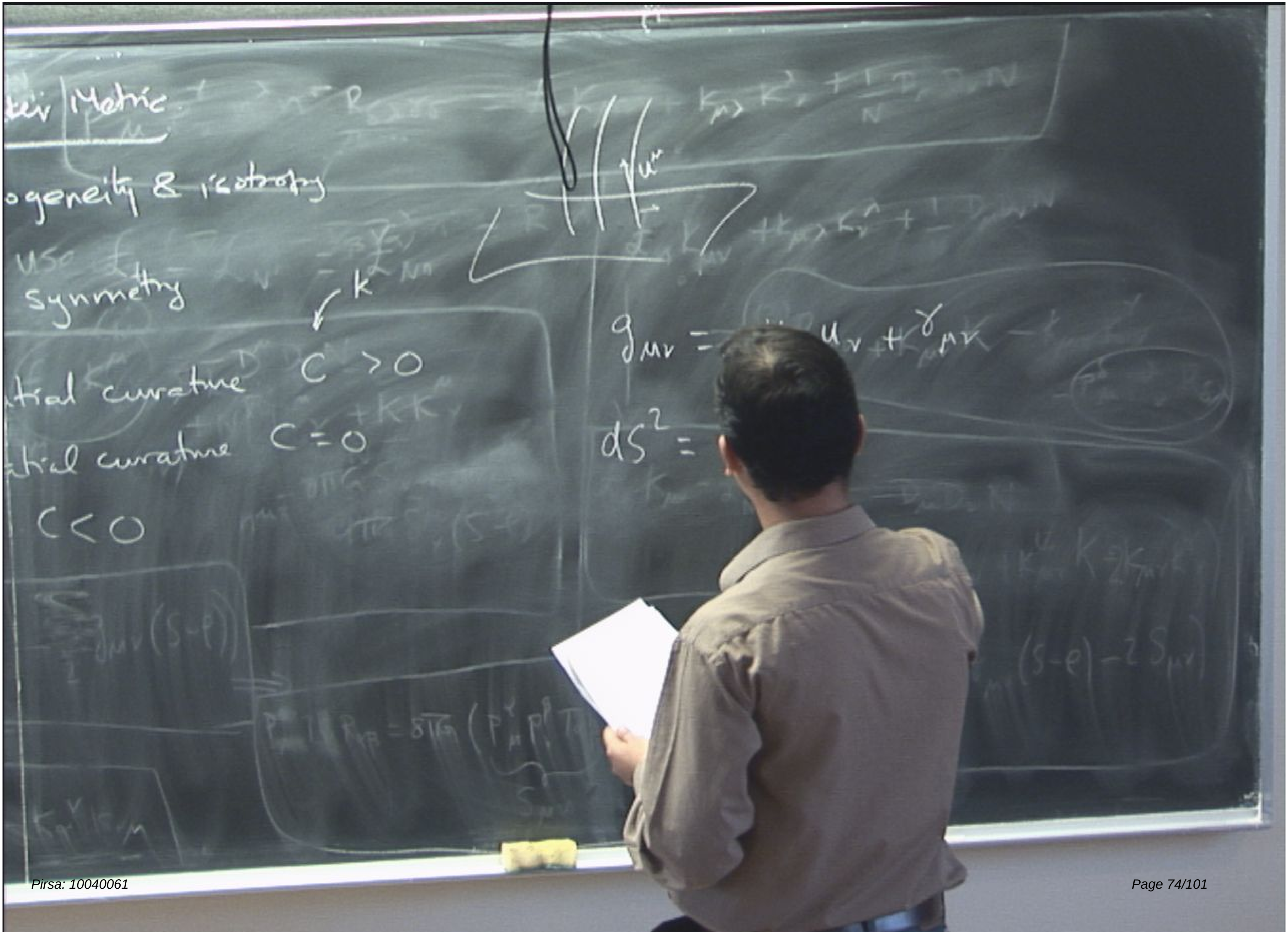
→ Spatial curvature

→ Spatial curvature

→ $C < 0$


Metric
ogeneity & isotropy
symmetry
k
curvature C > 0
curvature C = 0
C < 0


Metric
ogeneity & isotropy
symmetry
k
C > 0
curvature
C = 0
curvature
C < 0


Metric
ogeneity & isotropy
symmetry
k
C > 0
tial curvature
tial curvature
C = 0
C < 0
g_{μν} =
ds^2 =

...ker | Metric

...ogeneity & isotropy

... symmetry

k

...tial curvature $\quad C > 0$

...tial curvature $\quad C = 0$

$C < 0$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$$dt = u_\mu dx^\mu$$

…ker | Metric

…ogeneity & isotropy

symmetry

k

…tral curvature $C > 0$

…tial curvature $C = 0$

$C < 0$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$$dt = u_\mu dx^\mu$$

...ker | Metric

...ogeneity & isotropy

symmetry

k

...tial curvature $C > 0$

...tial curvature $C = 0$

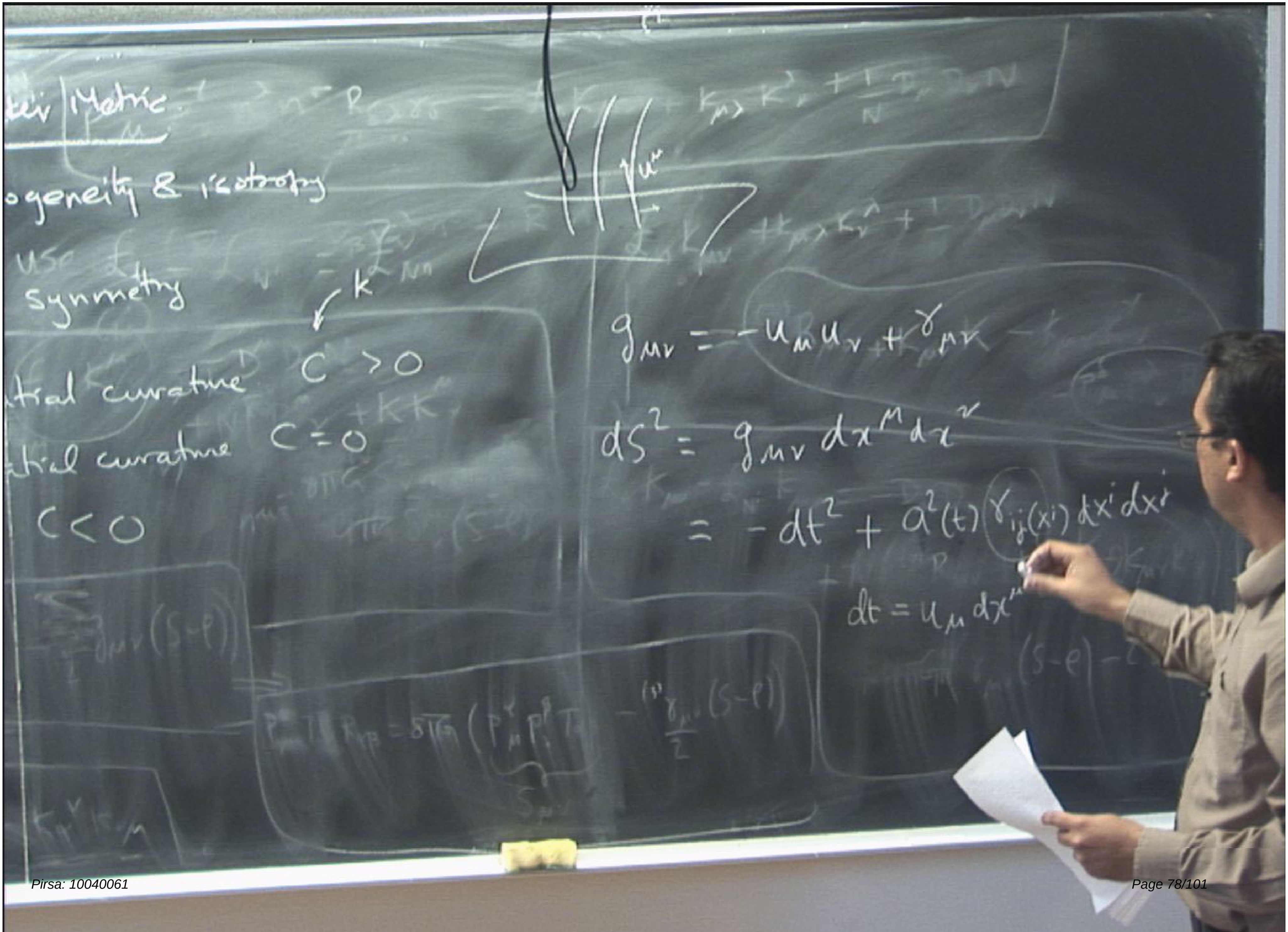
$C < 0$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$$dt = u_\mu dx^\mu$$


Metric
ogeneity & isotropy
symmetry
k
C > 0
tral curvature
tial curvature C = 0
C < 0
$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu}$
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$= -dt^2 + a^2(t)\gamma_{ij}(x^i) dx^i dx^j$
$dt = u_\mu dx^\mu$

...ker | Metric

...ogeneity & isotropy

symmetry

$\swarrow k$

...tial curvature $\quad C>0$

...tial curvature $\quad C=0$

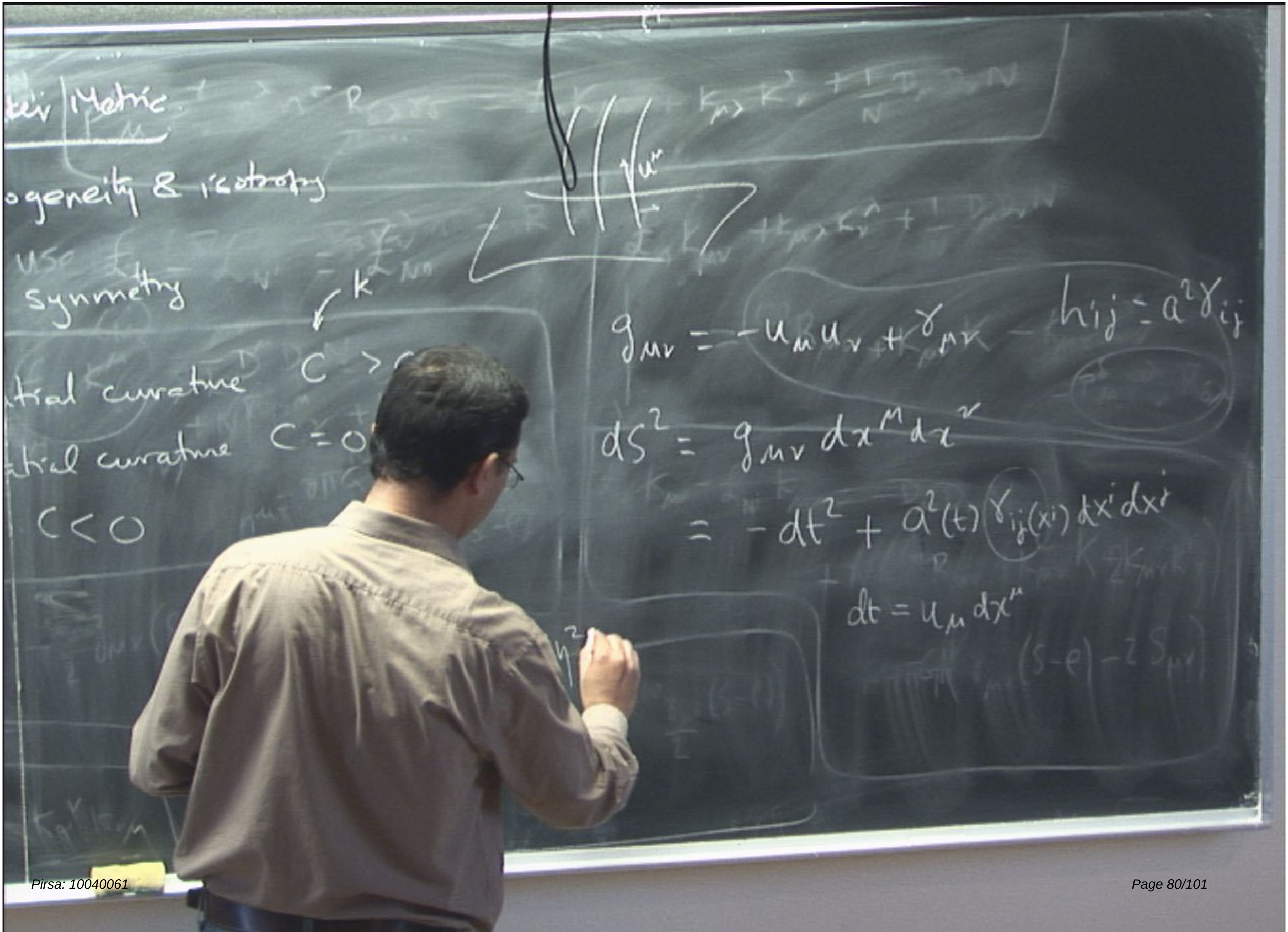
$C<0$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu} \qquad h_{ij} = a^2\gamma_{ij}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\,\gamma_{ij}(x^i)\, dx^i dx^j$$

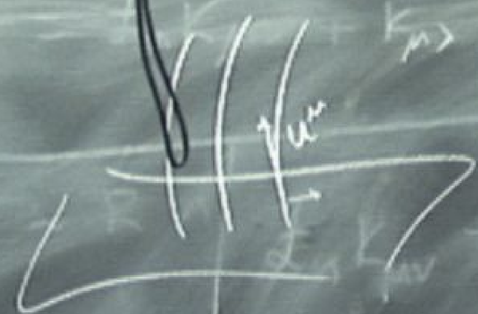
$$dt = u_\mu dx^\mu$$


Metric
ogeneity & isotropy
symmetry
k
$C > 0$
tial curvature
tial curvature $C = 0$
$C < 0$
$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu}$
$h_{ij} = a^2 \gamma_{ij}$
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$= -dt^2 + a^2(t) \gamma_{ij}(x^i) dx^i dx^j$
$dt = u_\mu dx^\mu$

...ker | Metric

...ogeneity & isotropy

symmetry



K

$C > 0$

...trial curvature $C = 0$

...tial curvature

$C < 0$

$dt = a(\eta)\, d\eta$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \gamma_{ij}\, dx^i dx^j\right)$$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu}$$

$$h_{ij} = a^2 \gamma_{ij}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$$dt = u_\mu\, dx^\mu$$

...ker | Metric

...ogeneity & isotropy

symmetry

k

...trial curvature $\quad C > 0$

...tial curvature $\quad C = 0$

$C < 0$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu} \qquad h_{ij} = a^2 \gamma_{ij}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$dt = a(\eta)\, d\eta \qquad = -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$$dt = u_\mu dx^\mu$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \gamma_{ij} dx^i dx^j\right)$$

...ker | Metric

...ogeneity & isotropy

symmetry

k

...tral curvature $C > 0$

...tial curvature $C = 0$

$C < 0$

$dt = a(\eta)\, d\eta$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu} \qquad h_{ij} = a^2 \gamma_{ij}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$$dt = u_\mu dx^\mu$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \gamma_{ij}\, dx^i dx^j\right)$$

$$+ f_C^2(r)\, d\Omega^2 \rightarrow d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

ker | Metric

...ogeneity & isotropy

symmetry

$\swarrow k$

...trial curvature $C > 0$

...tial curvature $C = 0$

$C < 0$

$dt = a(\eta)\, d\eta$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \gamma_{ij}\, dx^i dx^j\right)$$

$\ldots f_c^2(r)\, d\Omega^2$

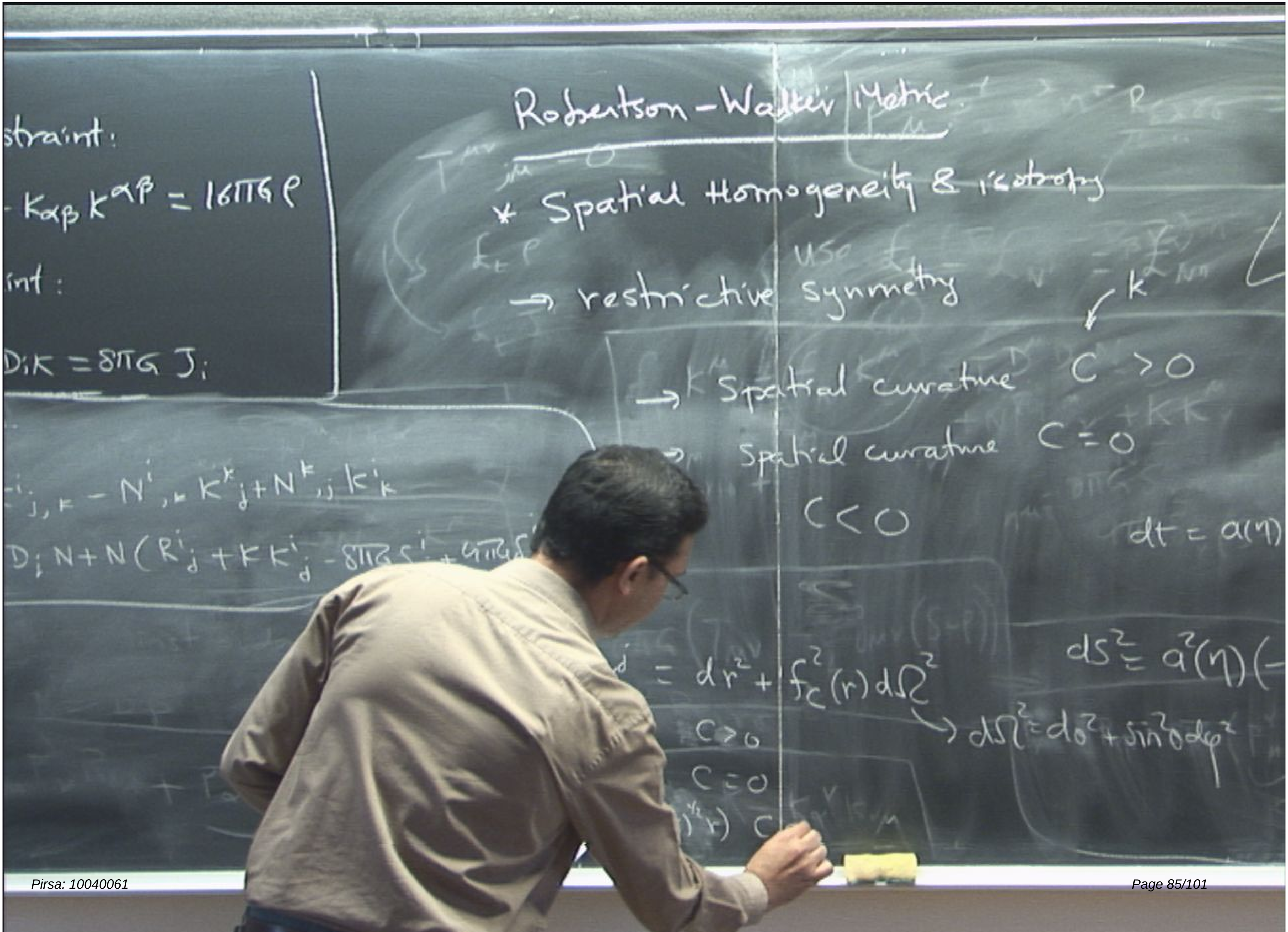
$\rightarrow d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$$g_{\mu\nu} = -u_\mu u_\nu + \gamma_{\mu\nu} \qquad h_{ij} = a^2 \gamma_{ij}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= -dt^2 + a^2(t)\, \gamma_{ij}(x^i)\, dx^i dx^j$$

$dt = u_\mu dx^\mu$



Robertson-Walker Metric
* Spatial Homogeneity & isotropy
→ restrictive symmetry
→ Spatial curvature C > 0
→ Spatial curvature C = 0
C < 0
dt = a(η)
ds² = a²(η)(−
= dr² + f_C²(r) dΩ²
dΩ² = dθ² + sin²θ dφ²
C > 0
C = 0
straint:
K_αβ K^αβ = 16πGρ
int:
D_i K = 8πG J_i

straint:

$- K_{\alpha\beta} K^{\alpha\beta} = 16\pi G \rho$

int:

$D_i K = 8\pi G J_i$

$\ldots^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k$

$D_j N + N (R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S - E))$

$S(r) = 4\pi f_c^2(r)$

Robertson-Walker Metric

* Spatial Homogeneity & isotropy

$\rightarrow$ restrictive symmetry

$\rightarrow$ Spatial curvature $C > 0$ (k)

$\rightarrow$ Spatial curvature $C = 0$

$\rightarrow$ $C < 0$

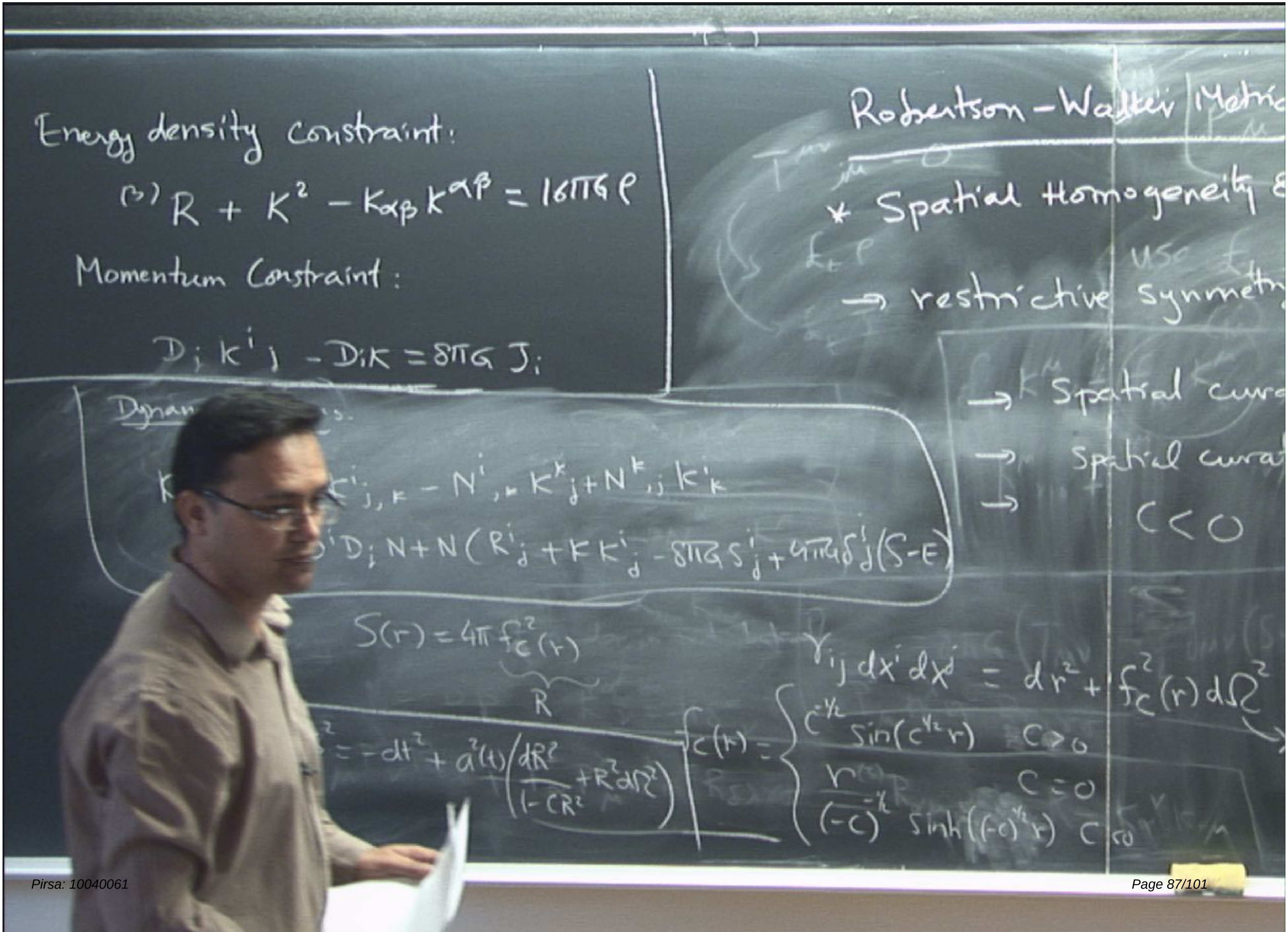
$dt = a(\eta)$

$ds^2 = a^2(\eta)(-$

$\gamma_{ij}\, dx^i dx^j = dr^2 + f_c^2(r)\, d\Omega^2$

$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$$f_c(r) = \begin{cases} C^{-1/2} \sin(C^{1/2} r) & C > 0 \\ r & C = 0 \\ (-C)^{-1/2} \sinh((-C)^{1/2} r) & C < 0 \end{cases}$$


Energy density constraint:
${}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$
Momentum Constraint:
$D_j K^i{}_j - D_i K = 8\pi G J_i$
Robertson-Walker Metric
* Spatial Homogeneity
→ restrictive symmetry
→ Spatial curv
→ Spatial curva
$C < 0$
$S(r) = 4\pi f_C^2(r)$
$\gamma_{ij}\,dx^i dx^j = dr^2 + f_C^2(r)\,d\Omega^2$
$= -dt^2 + a^2(t)\left(\frac{dR^2}{1-CR^2} + R^2 d\Omega^2\right)$
$C^{-1/2}\sin(C^{1/2} r)$ $C>0$
r $C=0$
$(-C)^{-1/2}\sinh((-C)^{1/2} r)$ $C<0$

Energy density constraint:

$$^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$$

Momentum Constraint:

$$D_i K^i{}_j - D_j K = 8\pi G J_j$$

Dynamical Eqns.

$$K^i{}_{j,0} + N^k K^i{}_{j,k} - N^i{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k$$
$$= -D^i D_j N + N\left(R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S-E)\right)$$

Robertson-Walker Metric

$T^{\mu\nu}{}_{;\mu} = 0$

* Spatial Homogeneity &

$\mathcal{L}_t \rho$

use

→ restrictive symmetry

→ Spatial curv…

→ Spatial curvat…

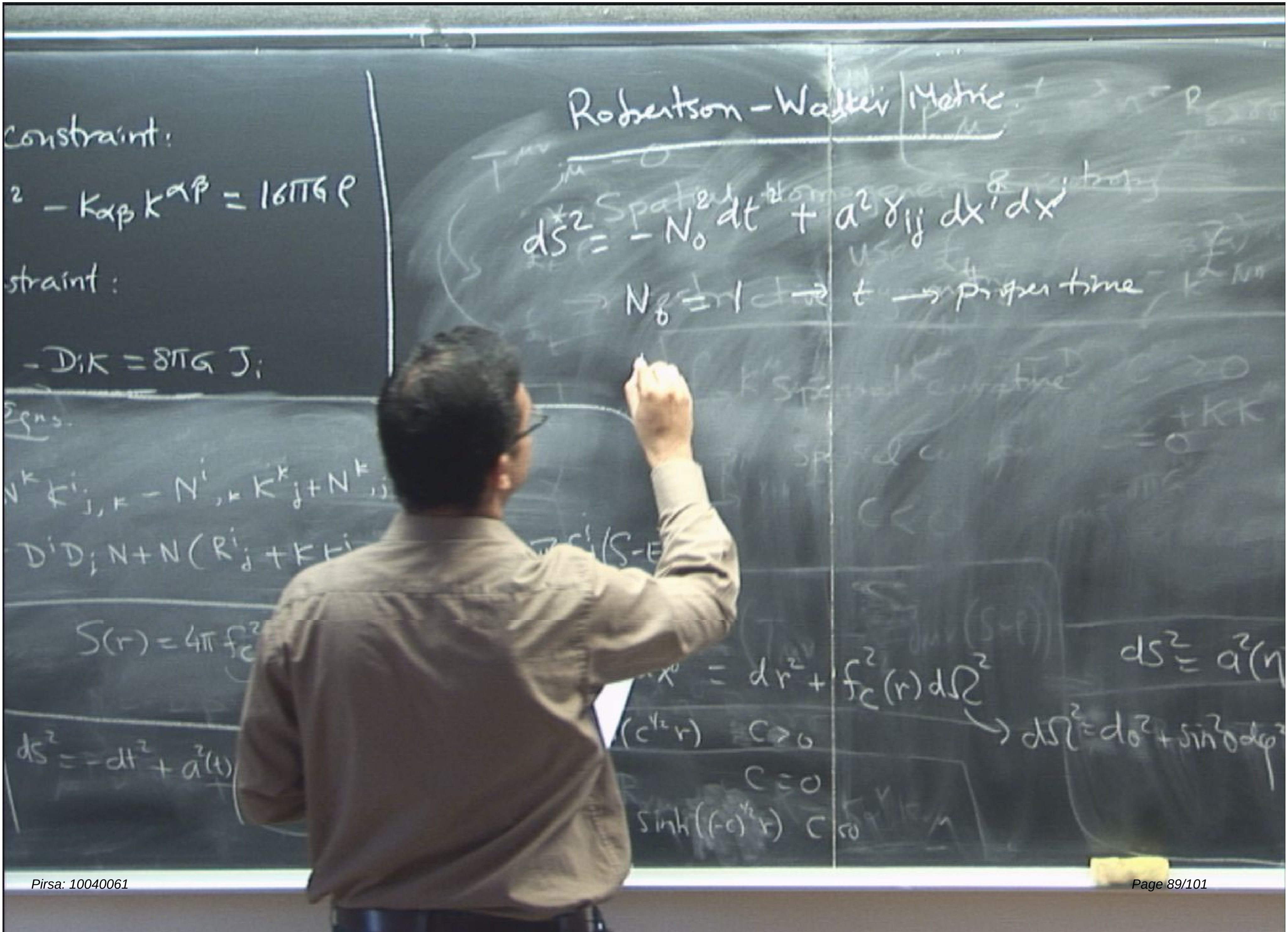
→ $C<0$

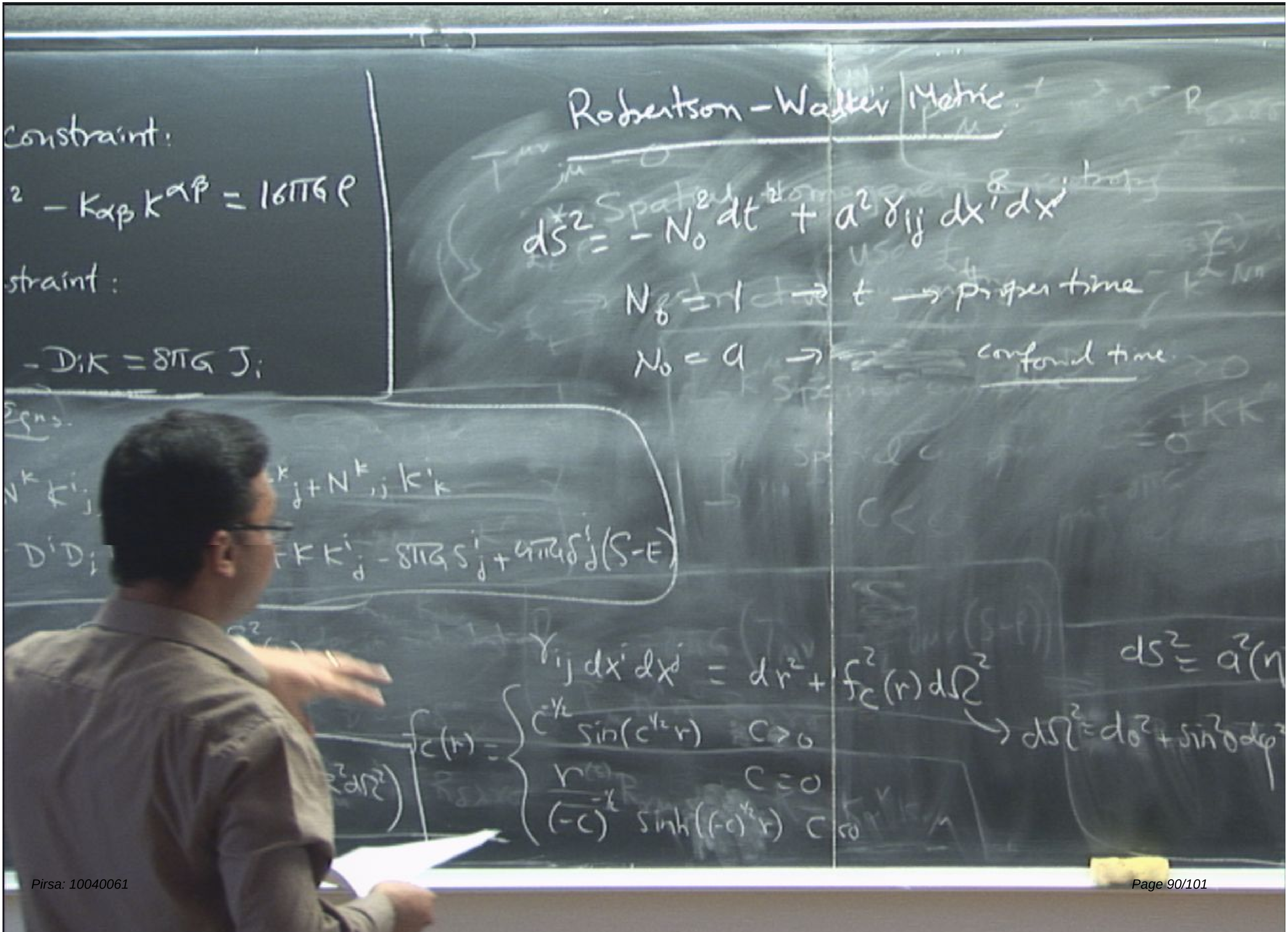
$$S(r) = 4\pi f_C^2(r)$$

$$\gamma_{ij}dx^i dx^j = dr^2 + f_C^2(r)\,d\Omega^2$$

$$\boxed{ds^2 = -dt^2 + a^2(t)\left(\frac{dR^2}{1-CR^2} + R^2 d\Omega^2\right)}$$

$$f_C(r) = \begin{cases} C^{-1/2}\sin(C^{1/2}r) & C>0 \\ r & C=0 \\ (-C)^{-1/2}\sinh((-C)^{1/2}r) & C<0 \end{cases}$$


Robertson-Walker Metric
constraint:
$-K_{\alpha\beta}K^{\alpha\beta} = 16\pi G \rho$
straint:
$-D_i K = 8\pi G J_i$
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
$S(r) = 4\pi f_c^2$
$= dr^2 + f_c^2(r) d\Omega^2$
$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$
$C > 0$
$C = 0$
$\sinh((-c)^{1/2} r) \quad C < 0$
$ds^2 = -dt^2 + a^2(t)$
$ds^2 = a^2(\eta$


Robertson-Walker Metric
Constraint:
$-K_{\alpha\beta}K^{\alpha\beta} = 16\pi G\rho$
straint:
$-D_iK = 8\pi G J_i$
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
$N_0 = a \rightarrow$ conformal time
$\gamma_{ij} dx^i dx^j = dr^2 + f_c^2(r) d\Omega^2$
$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$
$f_c(r) = C^{-1/2}\sin(C^{1/2}r)$ $C>0$
r $C=0$
$(-C)^{-1/2}\sinh((-C)^{1/2}r)$ $C<0$
$ds^2 = a^2(\eta$

Robertson-Walker Metric

$16\pi G\rho$

$$ds^2 = -N_0^2\,dt^2 + a^2\gamma_{ij}\,dx^i dx^j$$

$N_0 = 1 \rightarrow t \rightarrow$ proper time

$N_0 = a \rightarrow$ conformal time

$g_{\mu\nu}$

$ds^2 =$

${}^k{}_j + N^k{}_{,j} K^i{}_k$

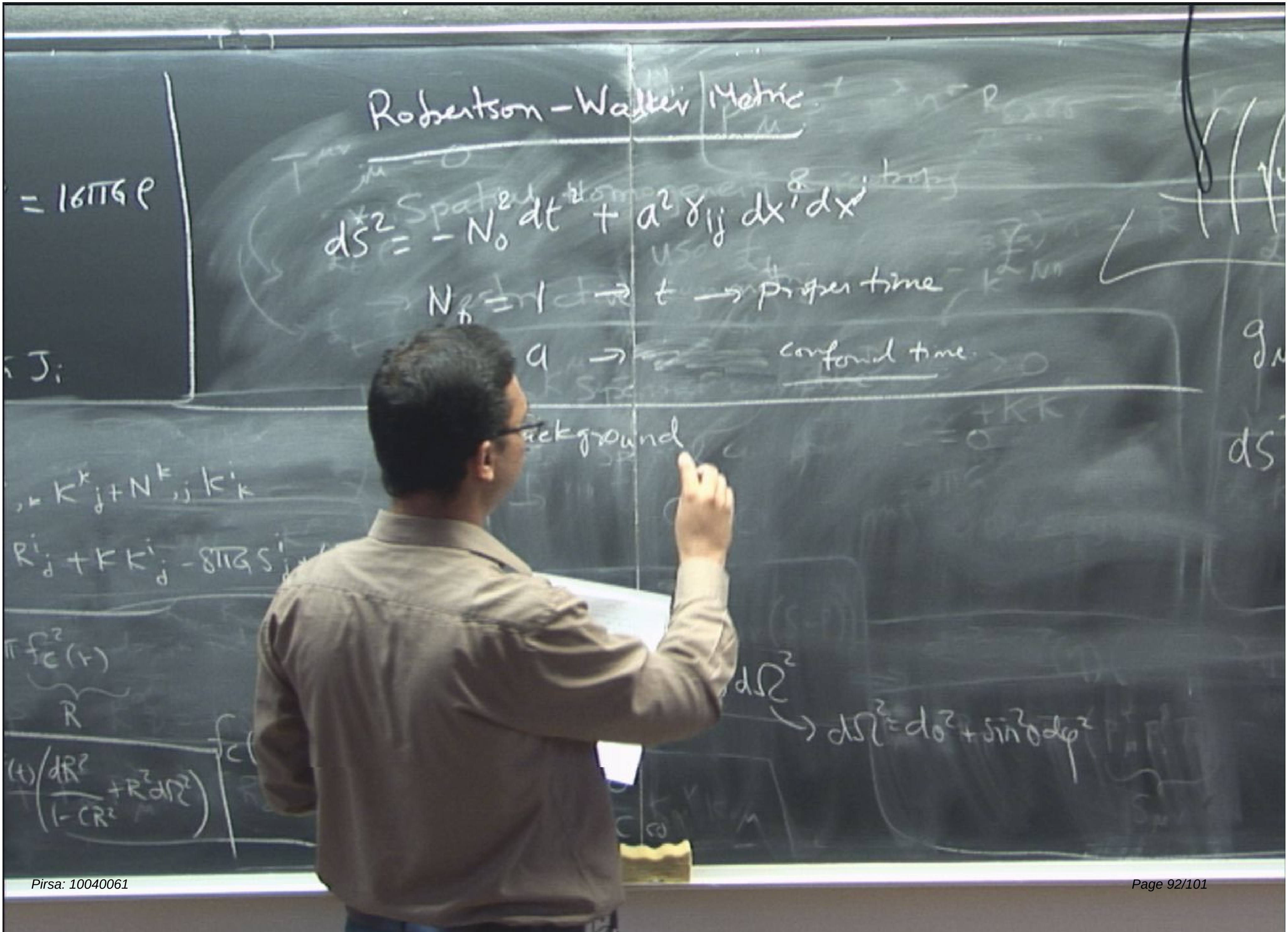
$K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S-E)$

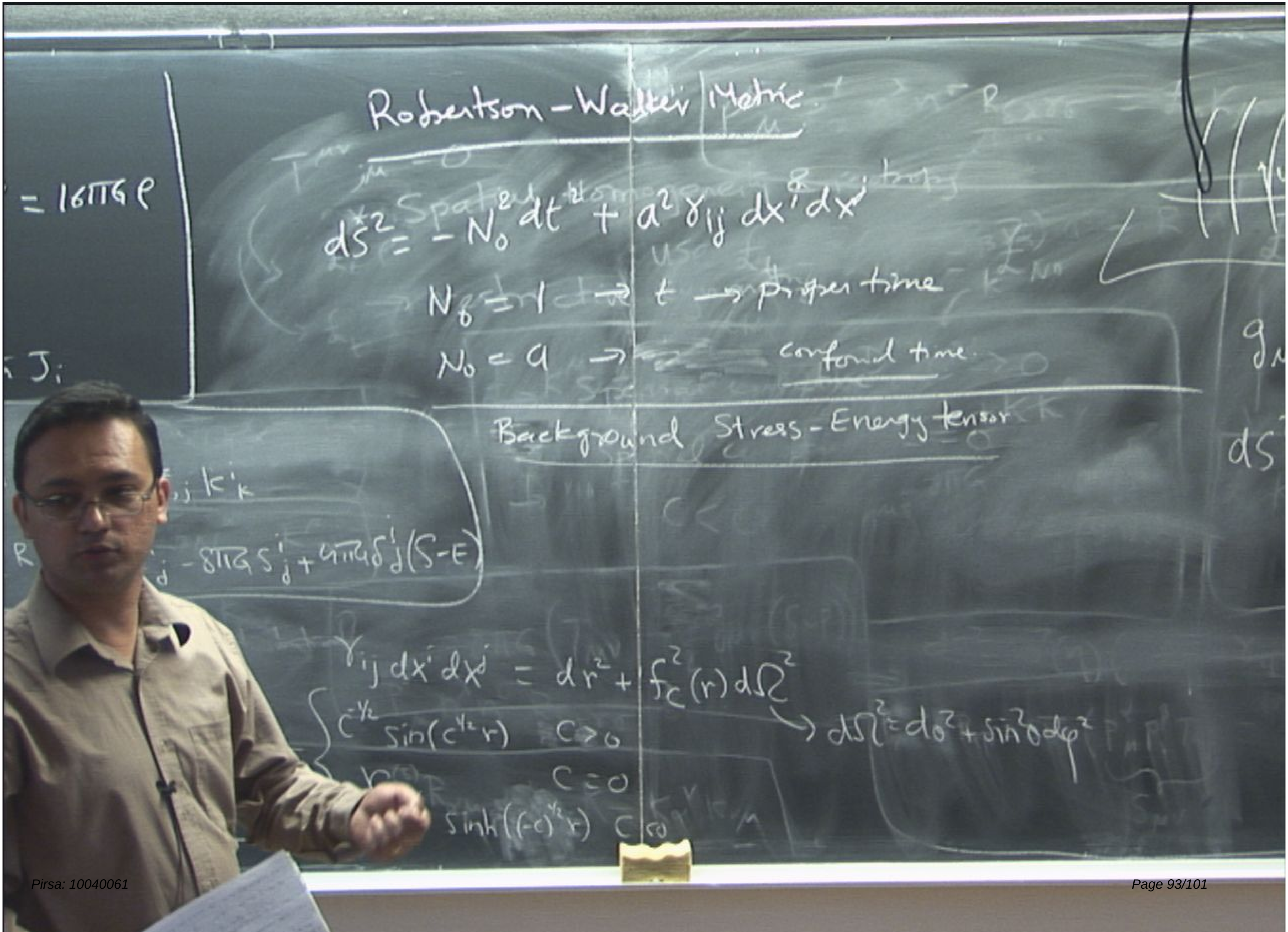
$$\gamma_{ij}\,dx^i dx^j = dr^2 + f_c^2(r)\,d\Omega^2$$

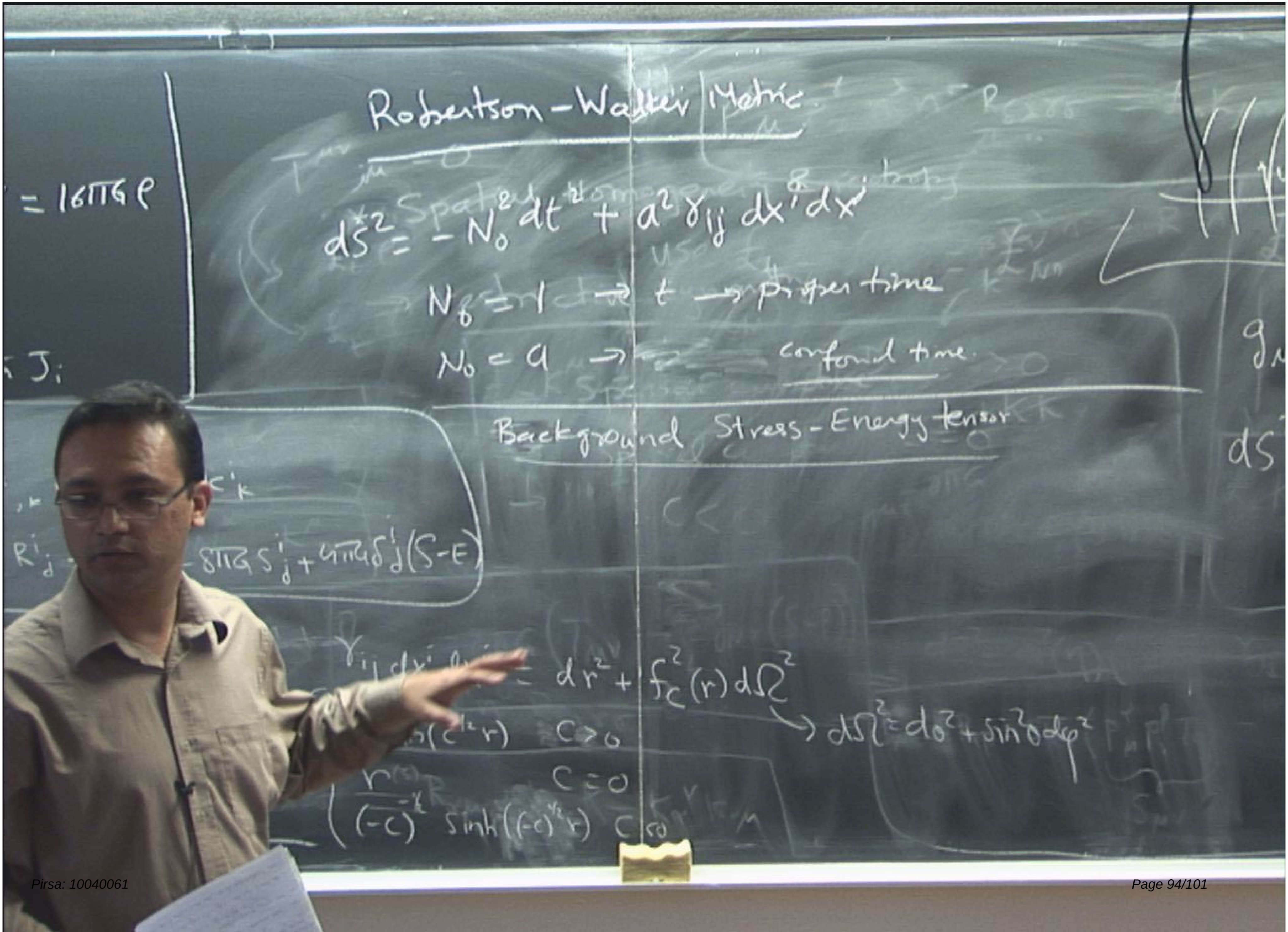
$$ds^2 = a^2(\eta)\left(-d\eta^2 + \gamma_{ij}\,dx \ldots\right.$$

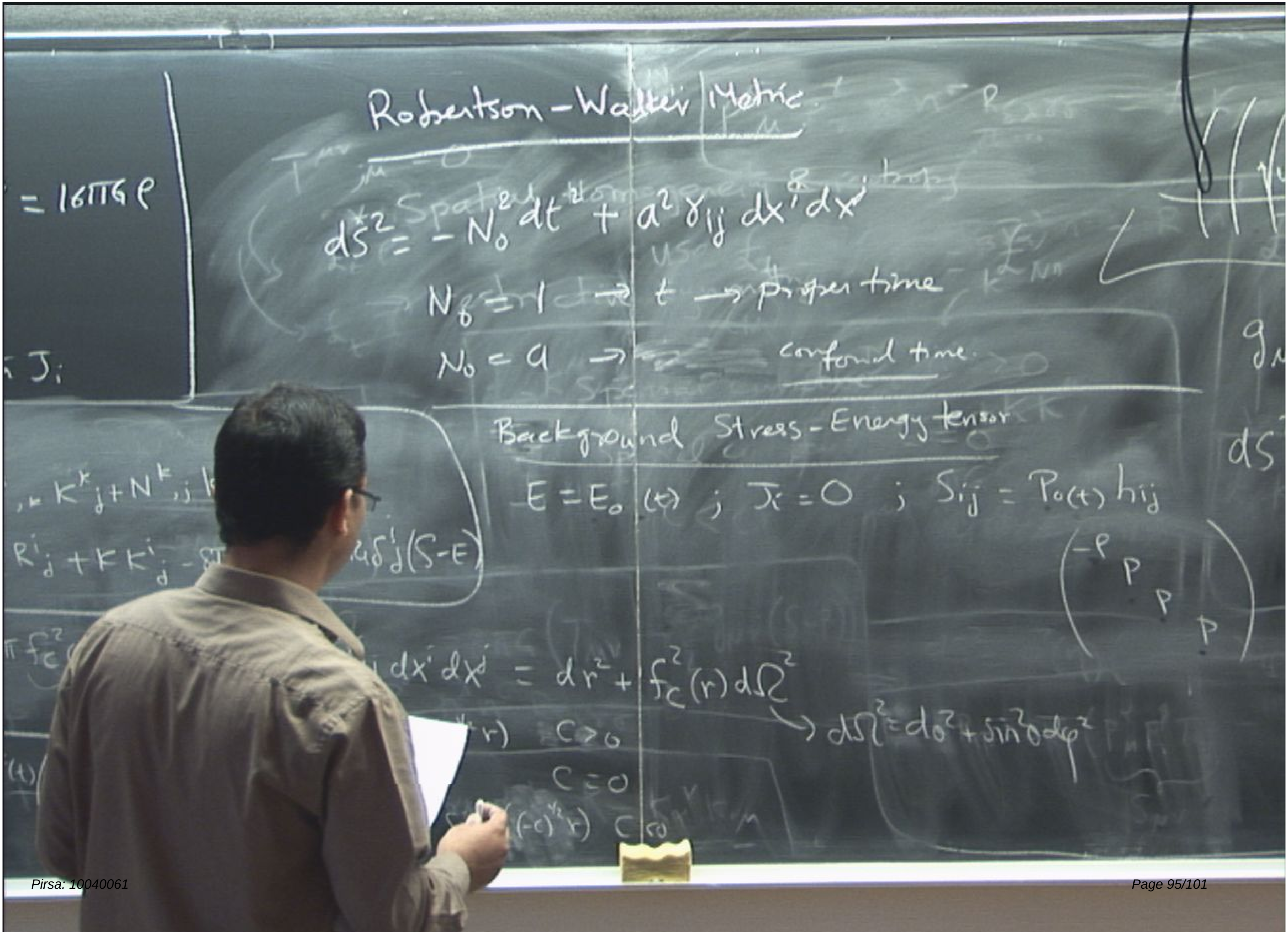
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

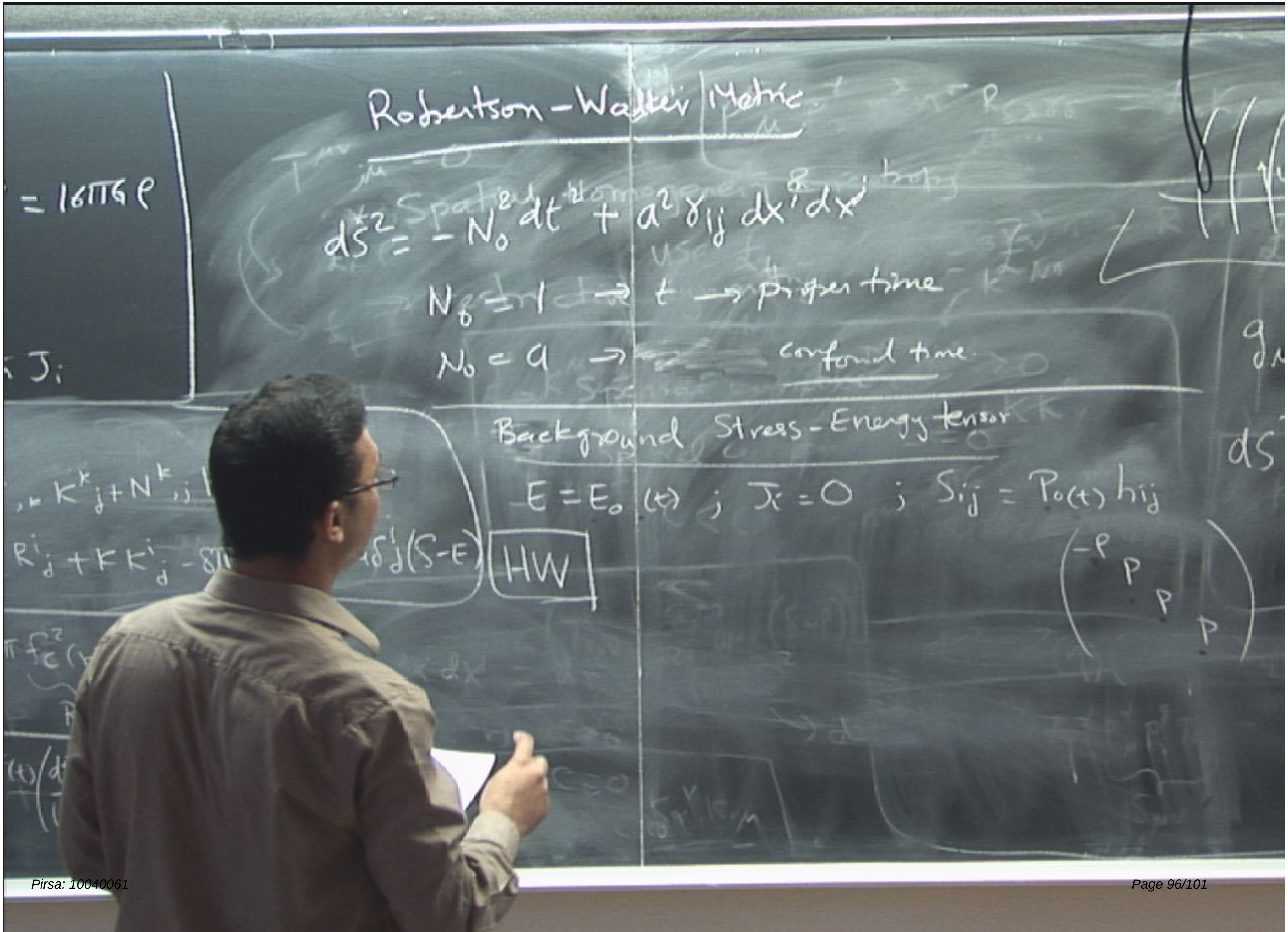
$$f_c(r) = \begin{cases} c^{-1/2} \ldots (c^{1/2} r) & c > 0 \\ \ldots & c = 0 \\ \ldots(-c)^{\ldots} \sinh\left((-c)^{1/2} r\right) & c < 0 \end{cases}$$


Robertson-Walker Metric
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
$a \rightarrow$ conformal time
$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$
$= 16\pi G \rho$


Robertson-Walker Metric
ds² = -N₀² dt² + a² γ_ij dx^i dx^j
N₀ = 1 → t → proper time
N₀ = a → conformal time
Background Stress-Energy tensor
γ_ij dx^i dx^j = dr² + f_c²(r) dΩ²
dΩ² = dθ² + sin²θ dφ²
c^{-1/2} sin(c^{1/2} r) c > 0
c = 0
sinh((-c)^{1/2} r) c < 0
= 16πGρ


= 16πG ρ
Robertson-Walker Metric
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
$N_0 = a \rightarrow$ conformal time
Background Stress-Energy tensor
$= dr^2 + f_C^2(r) d\Omega^2$
$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$
$C>0$
$C=0$
$(-C)^{-1/2} \sinh((-C)^{1/2} r)$ $C<0$


Robertson-Walker Metric
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
$N_0 = a \rightarrow$ conformal time.
Background Stress-Energy tensor
$E = E_0(t)$; $J_i = 0$; $S_{ij} = P_0(t) h_{ij}$
$\begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$
$dx^i dx^j = dr^2 + f_c^2(r) d\Omega^2$
$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$
$c > 0$
$c = 0$
$c < 0$
$= 16\pi G \rho$


Robertson-Walker Metric
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
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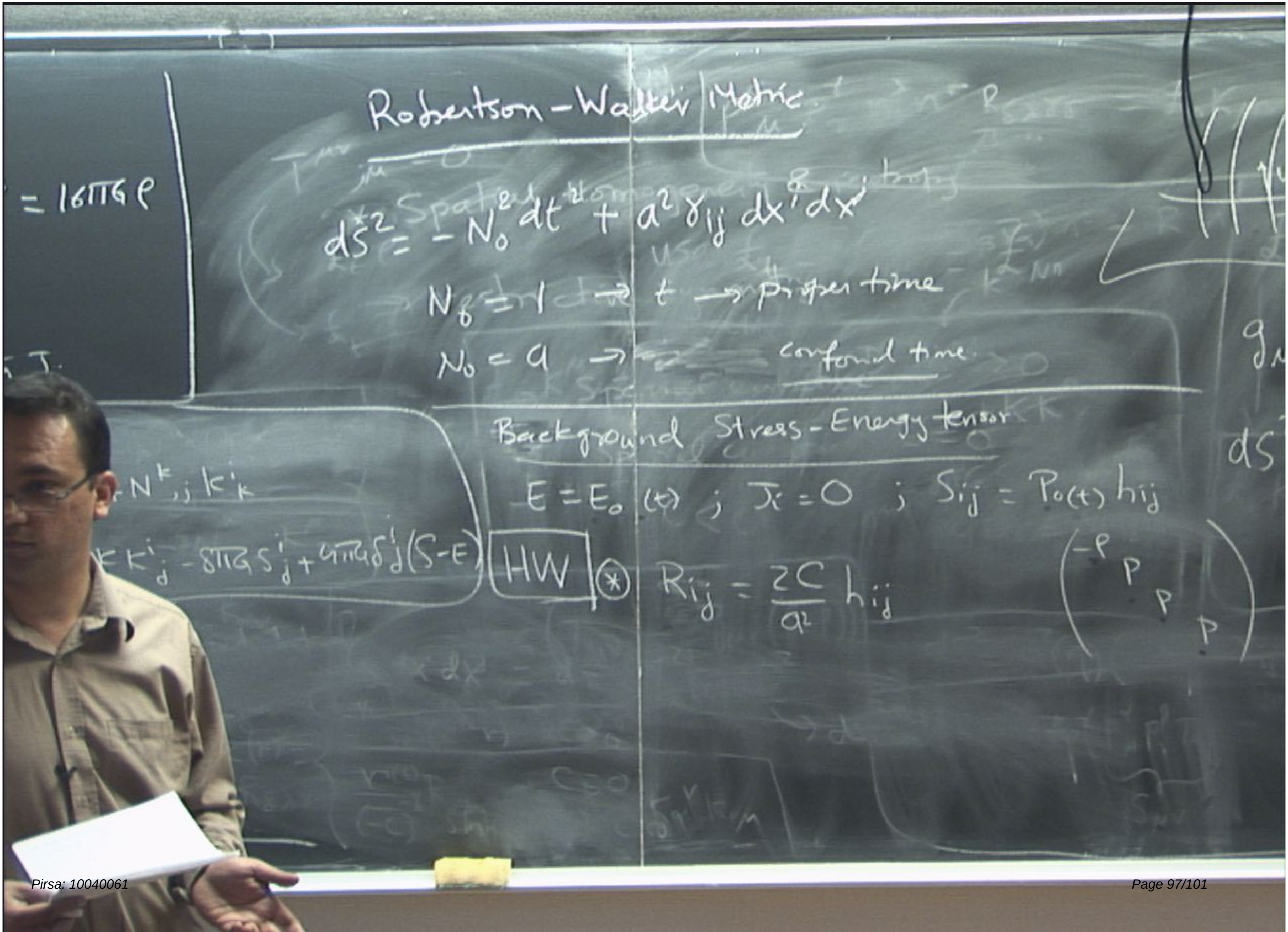
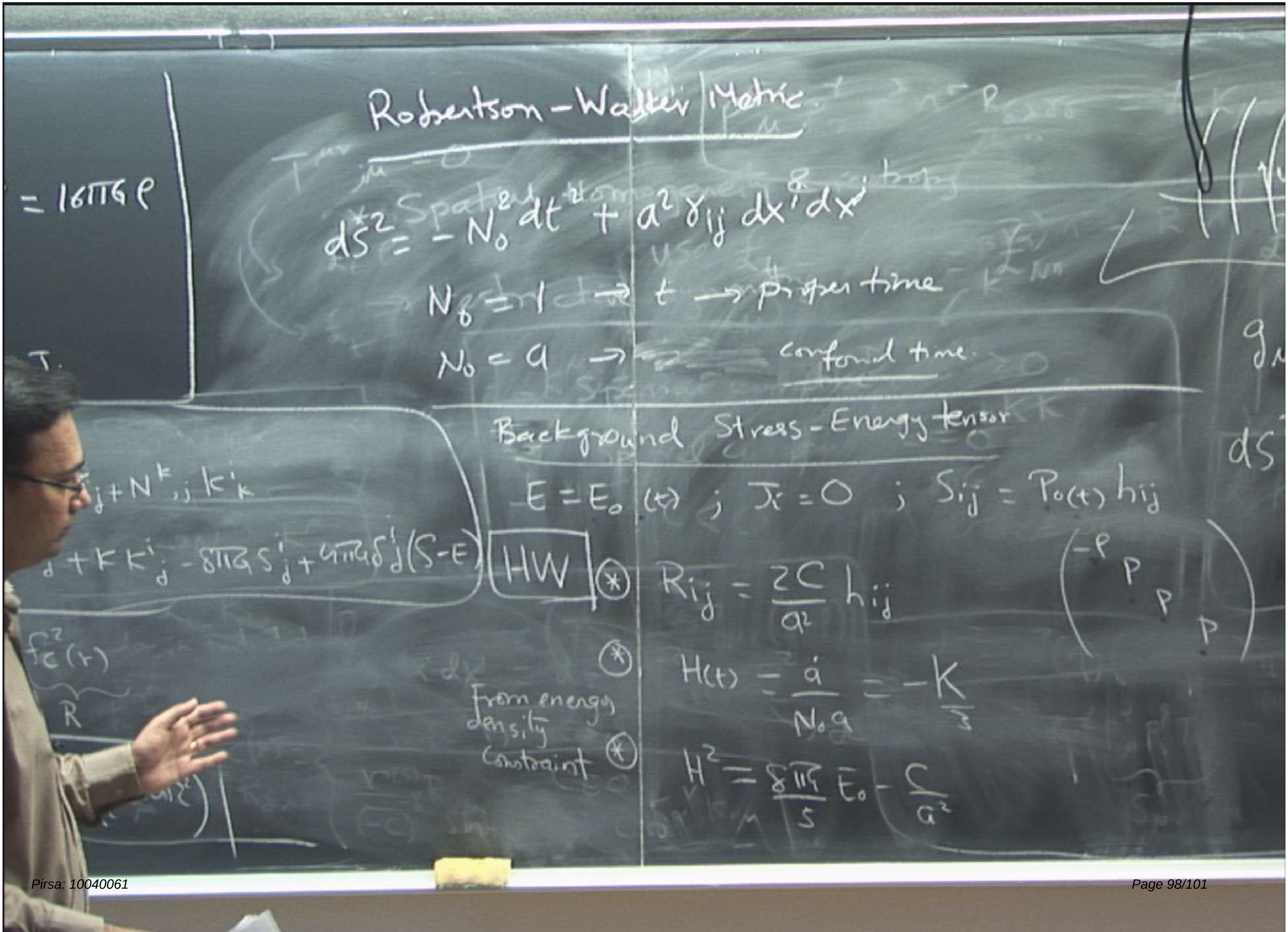
$E = E_0(t)$; $J_i = 0$; $S_{ij} = P_0(t) h_{ij}$
$\begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$
HW
$= 16\pi G \rho$



= 16πGρ
Robertson-Walker Metric
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \to t \to$ proper time
$N_0 = a \to$ conformal time
Background Stress-Energy tensor
$E = E_0(t)$; $J_i = 0$; $S_{ij} = P_0(t) h_{ij}$
HW
(*) $R_{ij} = \frac{2C}{a^2} h_{ij}$
$\begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$


Robertson-Walker Metric
$= 16\pi G \rho$
$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$
$N_0 = 1 \rightarrow t \rightarrow$ proper time
$N_0 = a \rightarrow$ conformal time.
Background Stress-Energy tensor
$E = E_0(t)$; $J_i = 0$; $S_{ij} = P_0(t) h_{ij}$
HW
$R_{ij} = \frac{2C}{a^2} h_{ij}$
$H(t) = \frac{\dot a}{N_0 a} = -\frac{K}{3}$
From energy density constraint
$H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

Robertson-Walker Metric

$$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$$

$N_0 = 1 \to t \to$ proper time

$N_0 = a \to$ conformal time.

Background Stress-Energy tensor

$$E = E_0(t) \; ; \; J_i = 0 \; ; \; S_{ij} = P_0(t) h_{ij}$$

$\boxed{\text{HW}}$ $\circledast$ $R_{ij} = \frac{2C}{a^2} h_{ij}$

$\circledast$ $H(t) = \frac{\dot a}{N_0 a} = -\frac{K}{3}$

From energy density constraint $\circledast$ $H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

K

$= 16\pi G \rho$

J_i

$K^k{}_j + N^k{}_{,i}$

$\delta^i{}_j (S-E)$

$f_c^2(r)$

R

$(t)\left(\frac{dR^2}{1-CR^2} + R^2 d\Omega^2\right)$

g_{μ}

ds

Robertson-Walker Metric

$$ds^2 = -N_0^2\,dt^2 + a^2 \gamma_{ij}\,dx^i dx^j$$

$N_0 = 1 \rightarrow t \rightarrow$ proper time

$N_0 = a \rightarrow$ conformal time

Background Stress-Energy tensor

$$E = E_0(t)\ ;\ J_i = 0\ ;\ S_{ij} = P_0(t)\, h_{ij}$$

$\boxed{\text{HW}}$ (*) $R_{ij} = \dfrac{2C}{a^2} h_{ij}$

(*) $H(t) = \dfrac{\dot a}{N_0 a} = -\dfrac{K}{3}$

From energy density constraint (*) $H^2 = \dfrac{8\pi G}{3} E_0 - \dfrac{C}{a^2}$

$K_{ij,0}$ Eqn

$$\frac{\dot H}{N_0} + H^2 = -\frac{4\pi G}{3}\ \ldots$$

$$\rightarrow \dot E_0 + 3H \ldots$$

$= 16\pi G \rho$

J_i

$\ldots{}_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k$

$R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S - E)$

$\pi \underbrace{f_C^2(r)}_{R}$

$(t)\left(\dfrac{dR^2}{1 - CR^2} + R^2 d\Omega^2\right)$

$g_{\mu\ldots}$

$ds^2 \ldots$

$= 16\pi G \rho$

J_i

$_{,k} K^k{}_j + N^k{}_{,j} K^i{}_k$

$R^i{}_j + K K^i{}_j - 8\pi G S^i{}_j + 4\pi G \delta^i{}_j (S - E)$

$\pi f_c^2(t)$ $\underbrace{\quad}_{R}$

$(t)\left(\frac{dR^2}{1 - CR^2} + R^2 d\Omega^2\right)$

Robertson–Walker Metric

$$ds^2 = -N_0^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j$$

$N_0 = 1 \rightarrow t \rightarrow$ proper time

$N_0 = a \rightarrow$ conformal time

Background Stress-Energy tensor

$$E = E_0(t) \; ; \; J_i = 0 \; ; \; S_{ij} = P_0(t) h_{ij}$$

$\boxed{\text{HW}}$

$(*)$ $R_{ij} = \frac{2C}{a^2} h_{ij}$

$(*)$ $H(t) = \frac{\dot a}{N_0 a} = -\frac{K}{3}$

From energy density constraint $(*)$ $H^2 = \frac{8\pi G}{3} E_0 - \frac{C}{a^2}$

$K_{ij,0}$ Eqn

$\frac{\dot H}{N_0} + H^2 = -\frac{4\pi G}{3}$

$\rightarrow \dot E_0 + 3H$

g_μ

ds