Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 3

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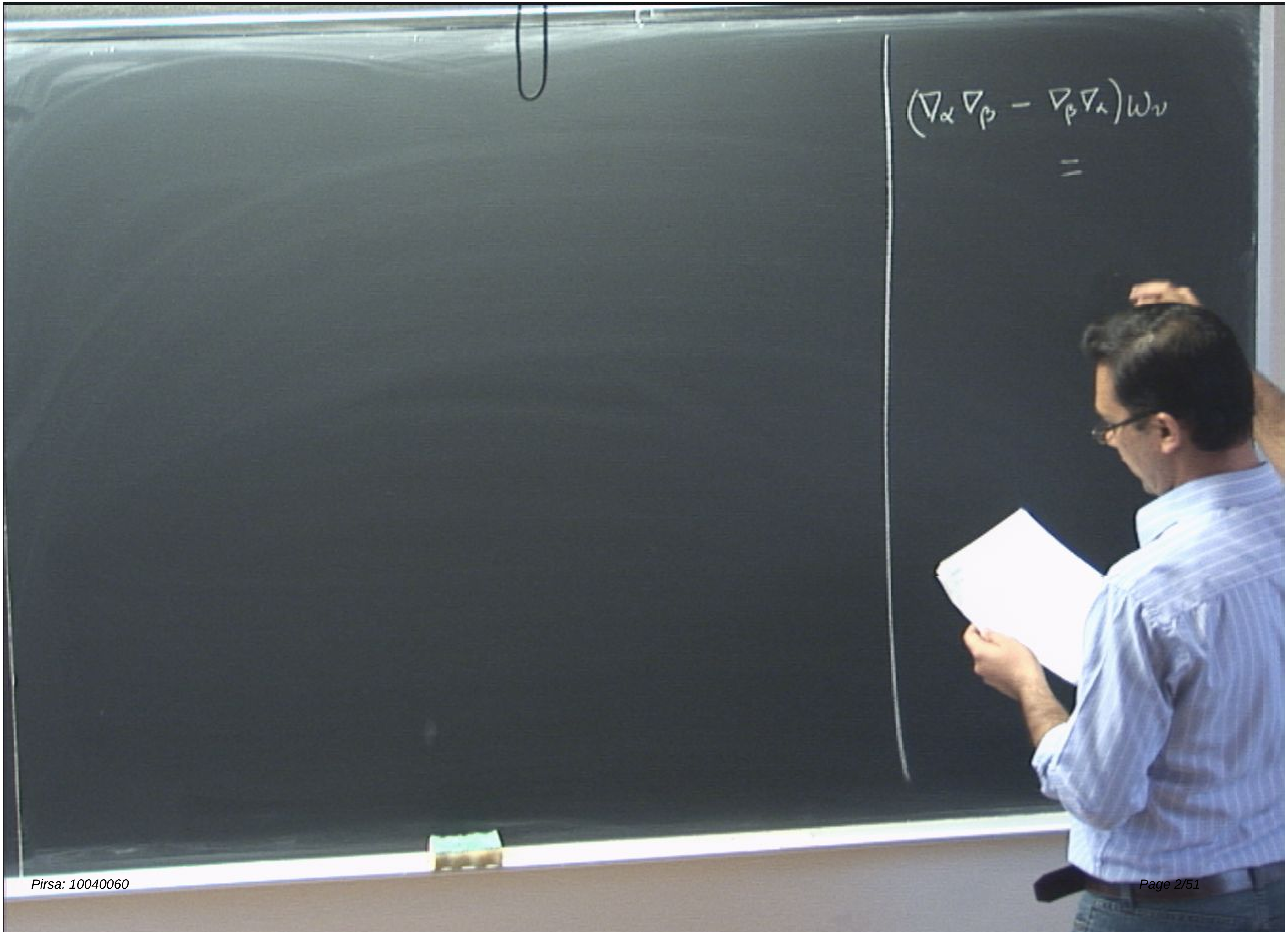
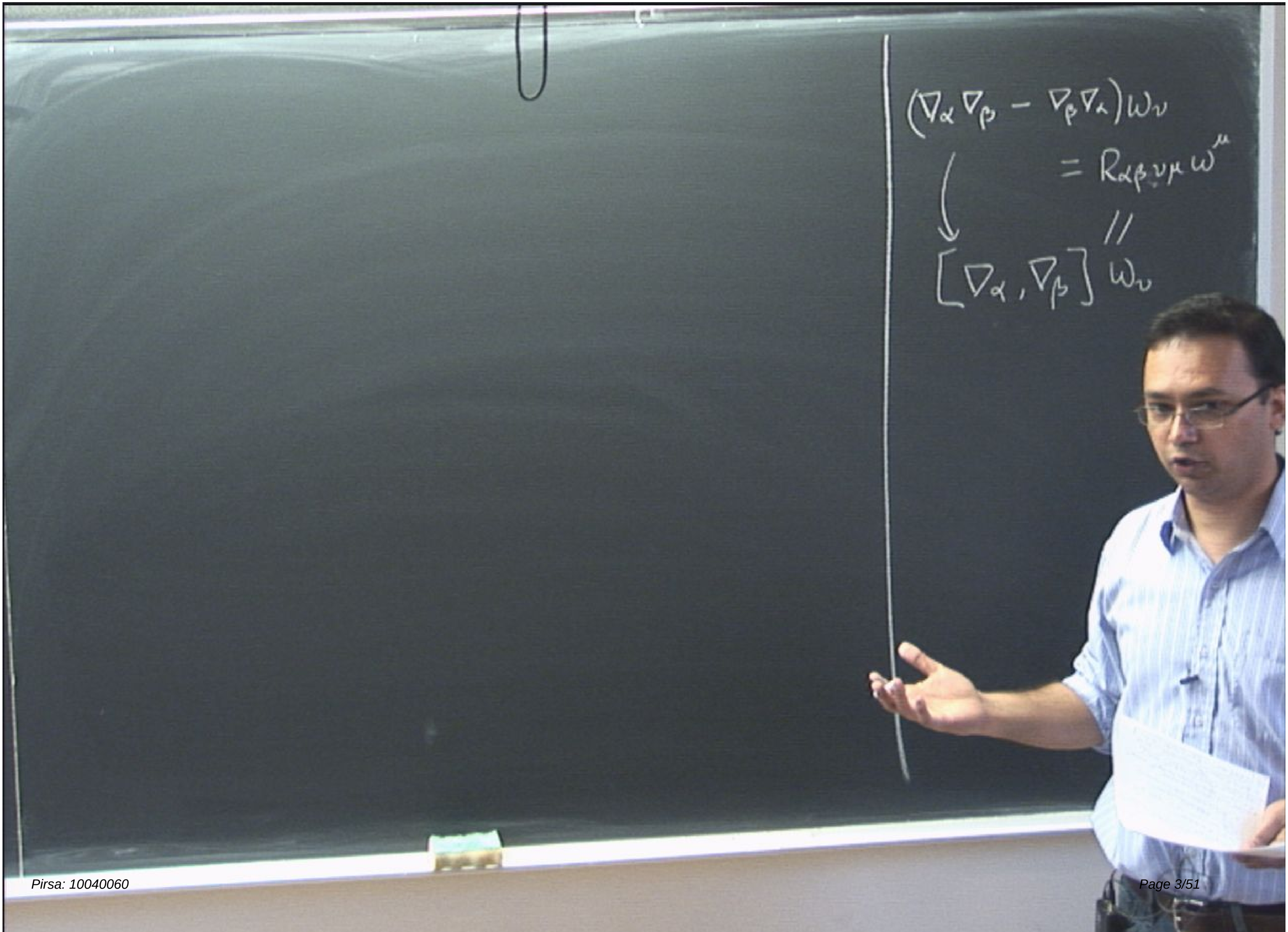
URL: http://pirsa.org/10040060

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

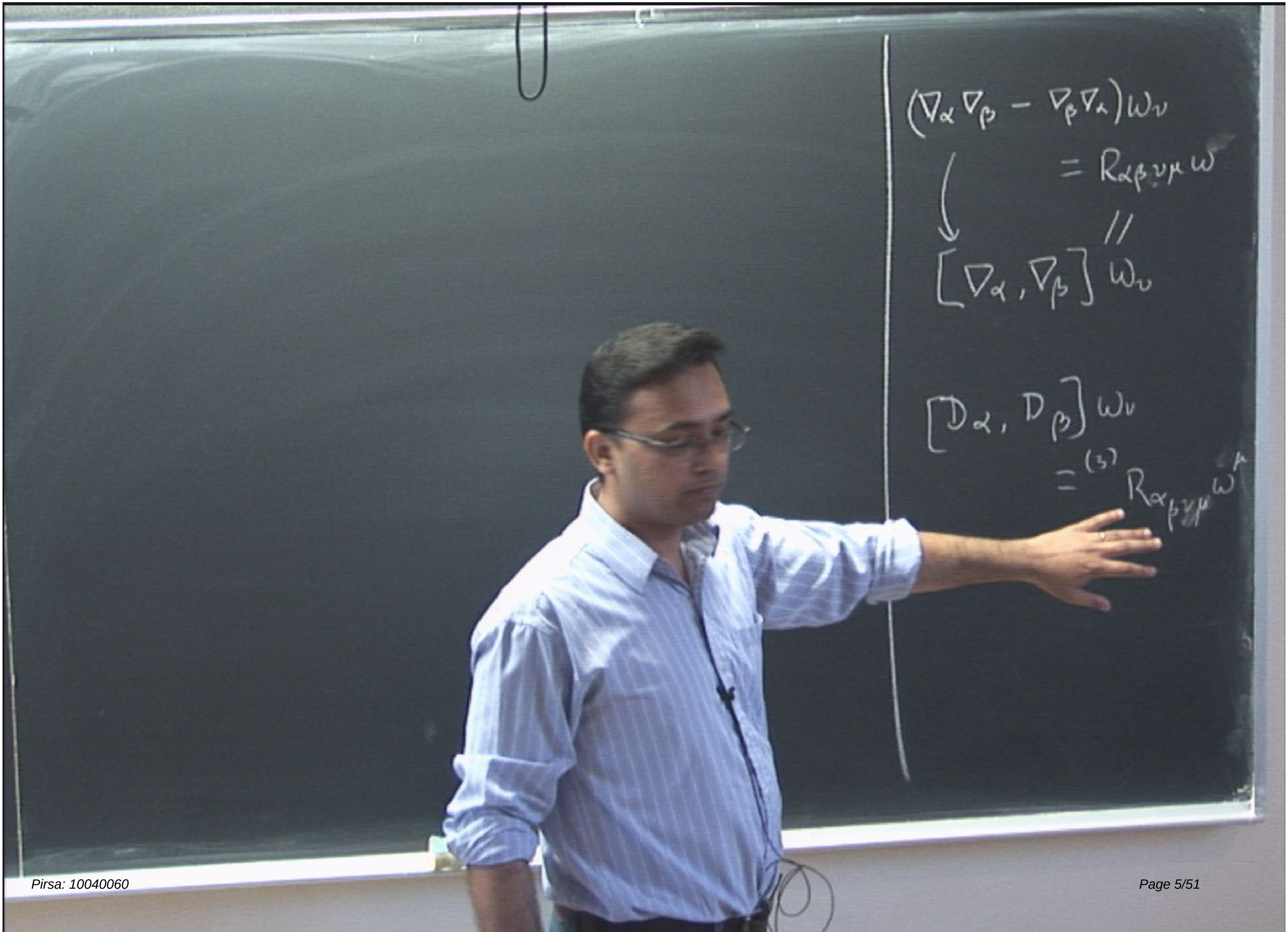

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$
$=$

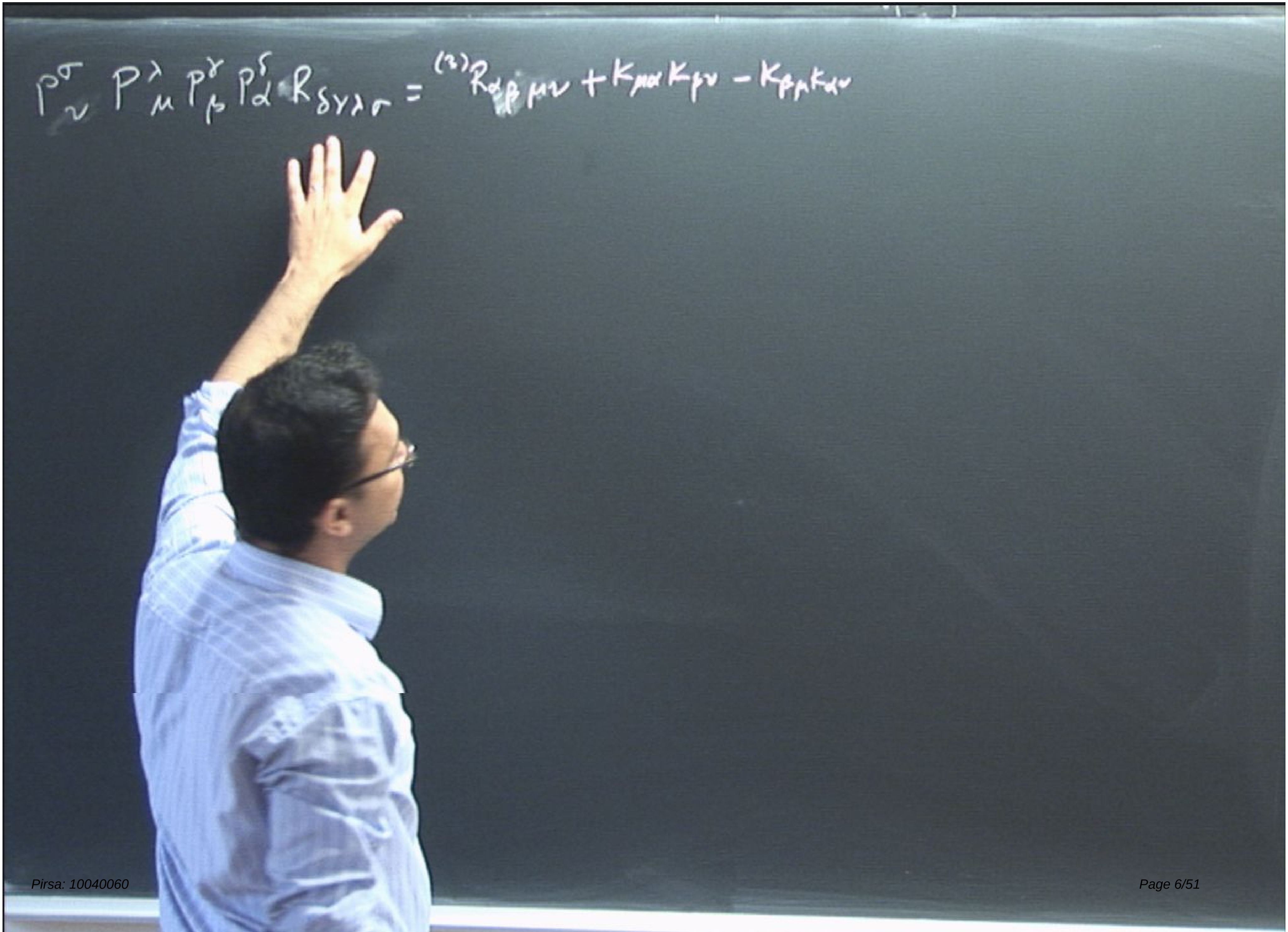

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$
$= R_{\alpha\beta\nu\mu}\,\omega^\mu$
$//$
$[\nabla_\alpha , \nabla_\beta]\,\omega_\nu$

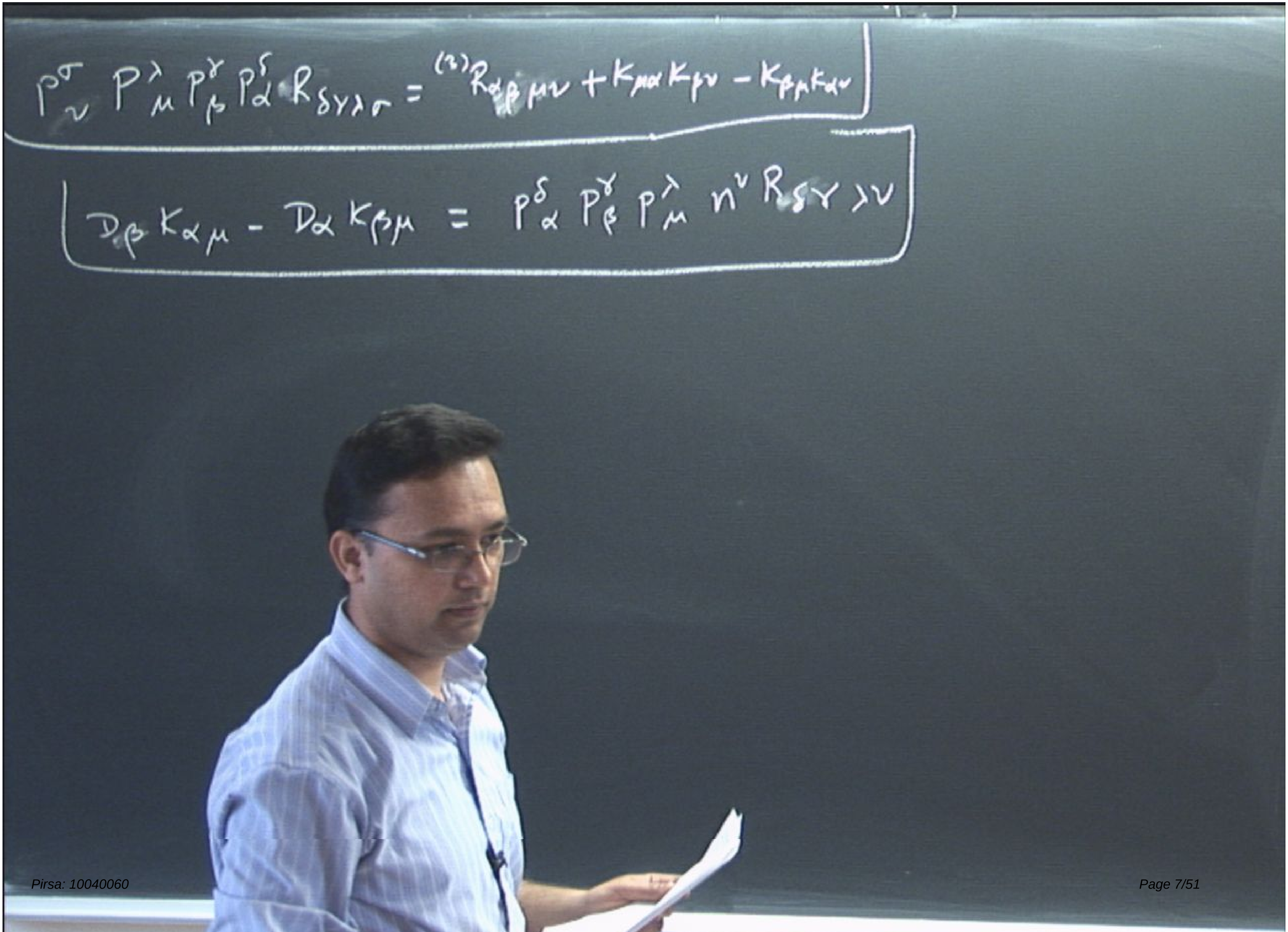
$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega^\mu$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

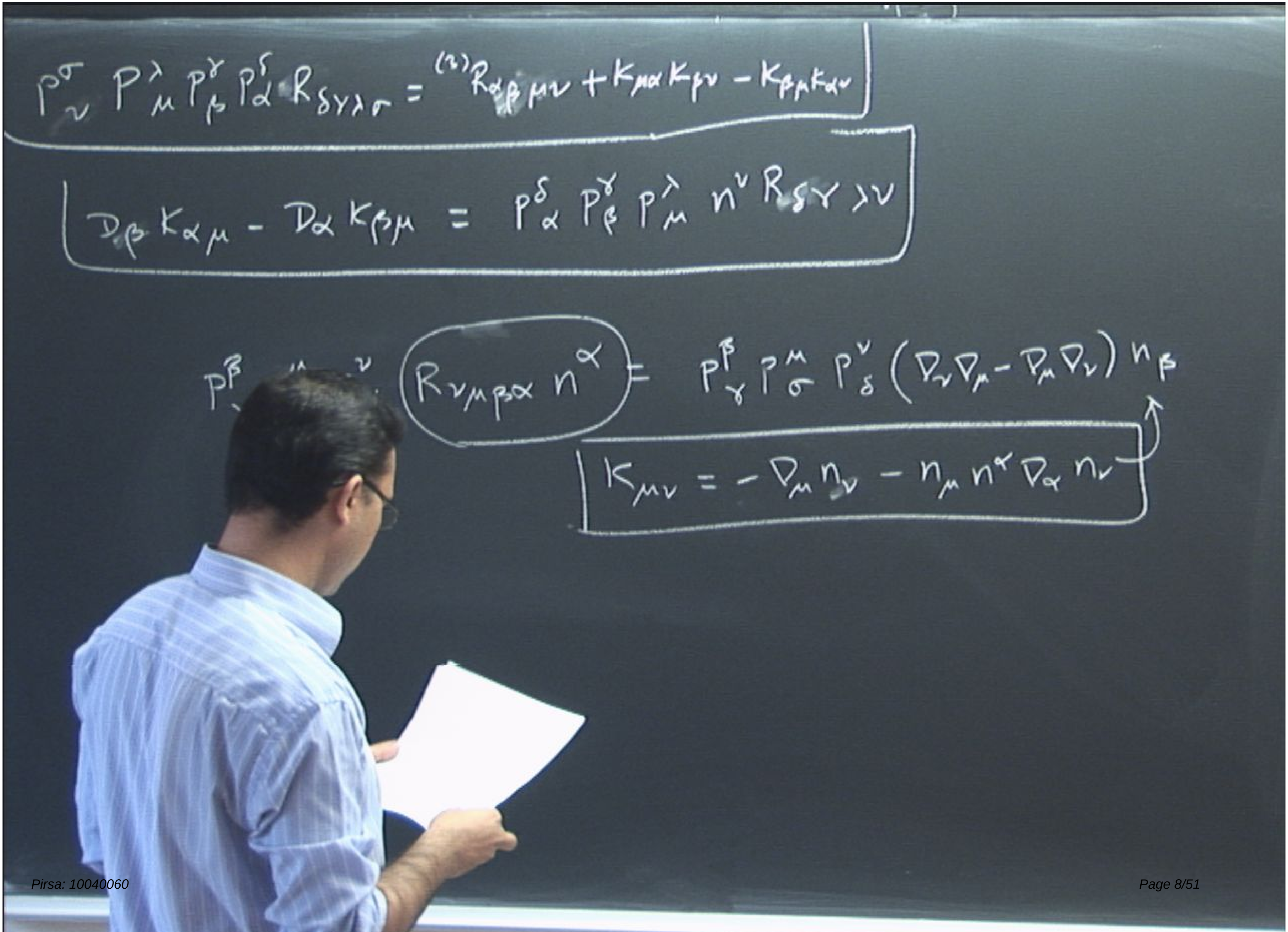
$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\mu\nu}\,\omega^\nu$$


$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$
$= R_{\alpha\beta\nu\mu}\omega$
$//$
$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$
$[D_\alpha, D_\beta]\,\omega_\nu$
$= {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^\mu$


$P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$



$\boxed{P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$
$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$


$\boxed{P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$
$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$
$P^{\beta} \;{}^{\mu}\; {}^{\nu}\; R_{\nu\mu\beta\alpha} n^{\alpha} = P^{\beta}_{\gamma} P^{\mu}_{\sigma} P^{\nu}_{\delta} \left(\nabla_{\nu}\nabla_{\mu} - \nabla_{\mu}\nabla_{\nu}\right) n_{\beta}$
$\boxed{K_{\mu\nu} = -\nabla_{\mu} n_{\nu} - n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu}}$

$$\boxed{P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$P^{\beta}_{\gamma} P^{\mu}_{\sigma} P^{\nu}_{\delta} R_{\nu\mu\beta\alpha} n^{\alpha} = P^{\beta}_{\gamma} P^{\mu}_{\sigma} P^{\nu}_{\delta} \left(\nabla_{\nu} \nabla_{\mu} - \nabla_{\mu} \nabla_{\nu} \right) n_{\beta}$$

$$\boxed{K_{\mu\nu} = - \nabla_{\mu} n_{\nu} - n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu}}$$

$$\boxed{P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$P^{\beta}_{\gamma} P^{\mu}_{\sigma} P^{\nu}_{\delta} R_{\nu\mu\beta\alpha} n^{\alpha} = P^{\beta}_{\gamma} P^{\mu}_{\sigma} P^{\nu}_{\delta} \left(\nabla_{\nu} \nabla_{\mu} - \nabla_{\mu} \nabla_{\nu} \right) n_{\beta}$$

$$\boxed{K_{\mu\nu} = -\nabla_{\mu} n_{\nu} - n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu}}$$

$$\boxed{P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$$

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}}$$

$$P^\beta \; {}^\mu \; P^\nu_\delta \left(R_{\nu\mu\beta\alpha} n^\alpha\right) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta \left(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu\right) n_\beta$$

$$\boxed{K_{\mu\nu} = -\nabla_\mu n_\nu - n_\mu n^\alpha \nabla_\alpha n_\nu}$$

$$\downarrow$$

$$D_\sigma K_{\gamma\delta} - D_\delta K_{\gamma\sigma}$$

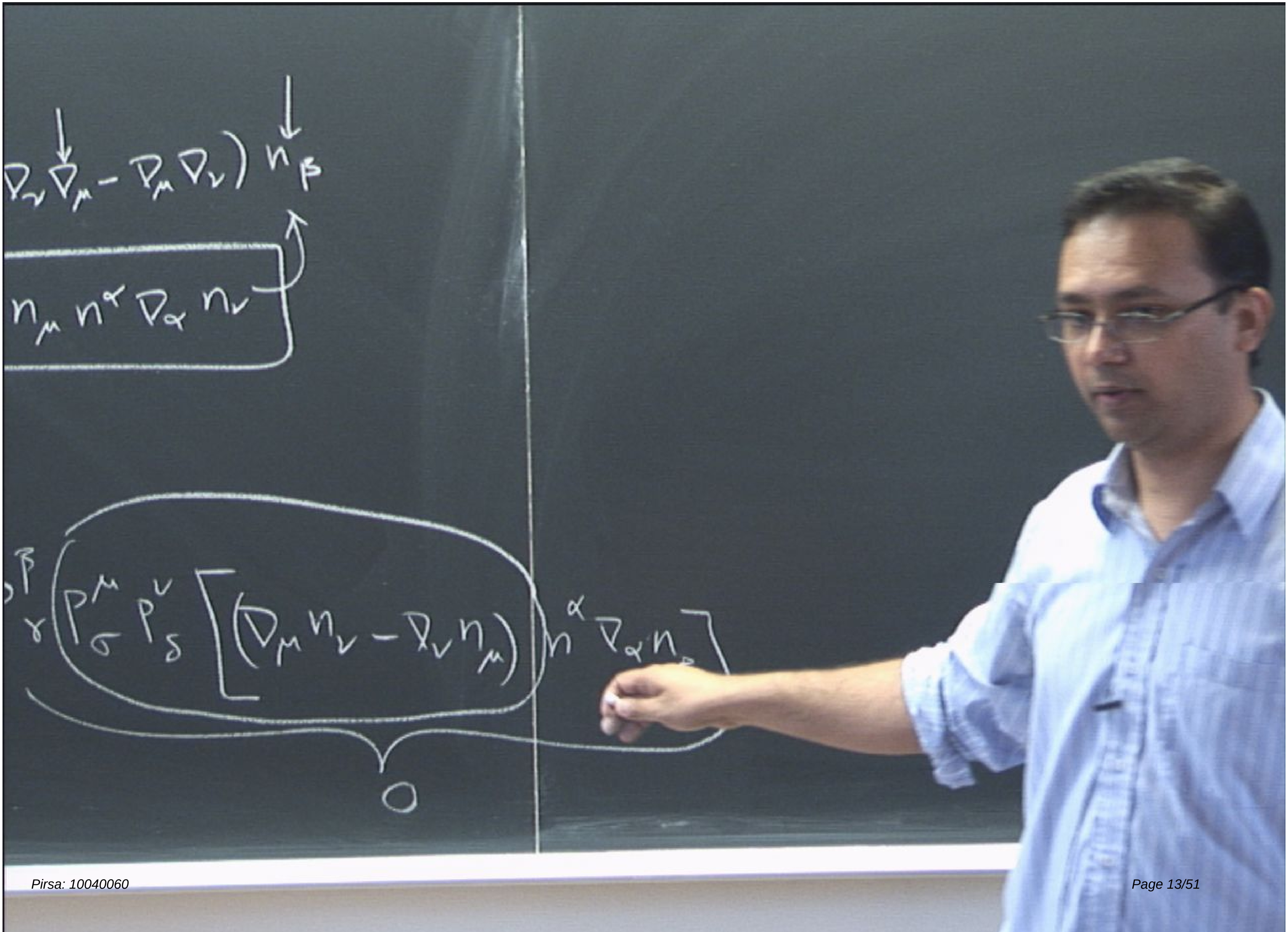
$$\boxed{P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$$

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}}$$

$$P^\mu_\sigma P^\nu_\delta \left(R_{\nu\mu\beta\alpha} n^\alpha\right) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta \left(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu\right) n_\beta$$

$$\boxed{K_{\mu\nu} = -\nabla_\mu n_\nu - n_\mu n^\alpha \nabla_\alpha n_\nu}$$

$$= D_\sigma K_{\gamma\delta} - D_\delta K_{\gamma\sigma} + \underbrace{P^\beta_\gamma P^\mu_\sigma P^\nu_\delta \left[\left(\nabla_\mu n_\nu - \nabla_\nu n_\mu\right) n^\alpha \nabla_\alpha n_\beta\right]}_{0}$$

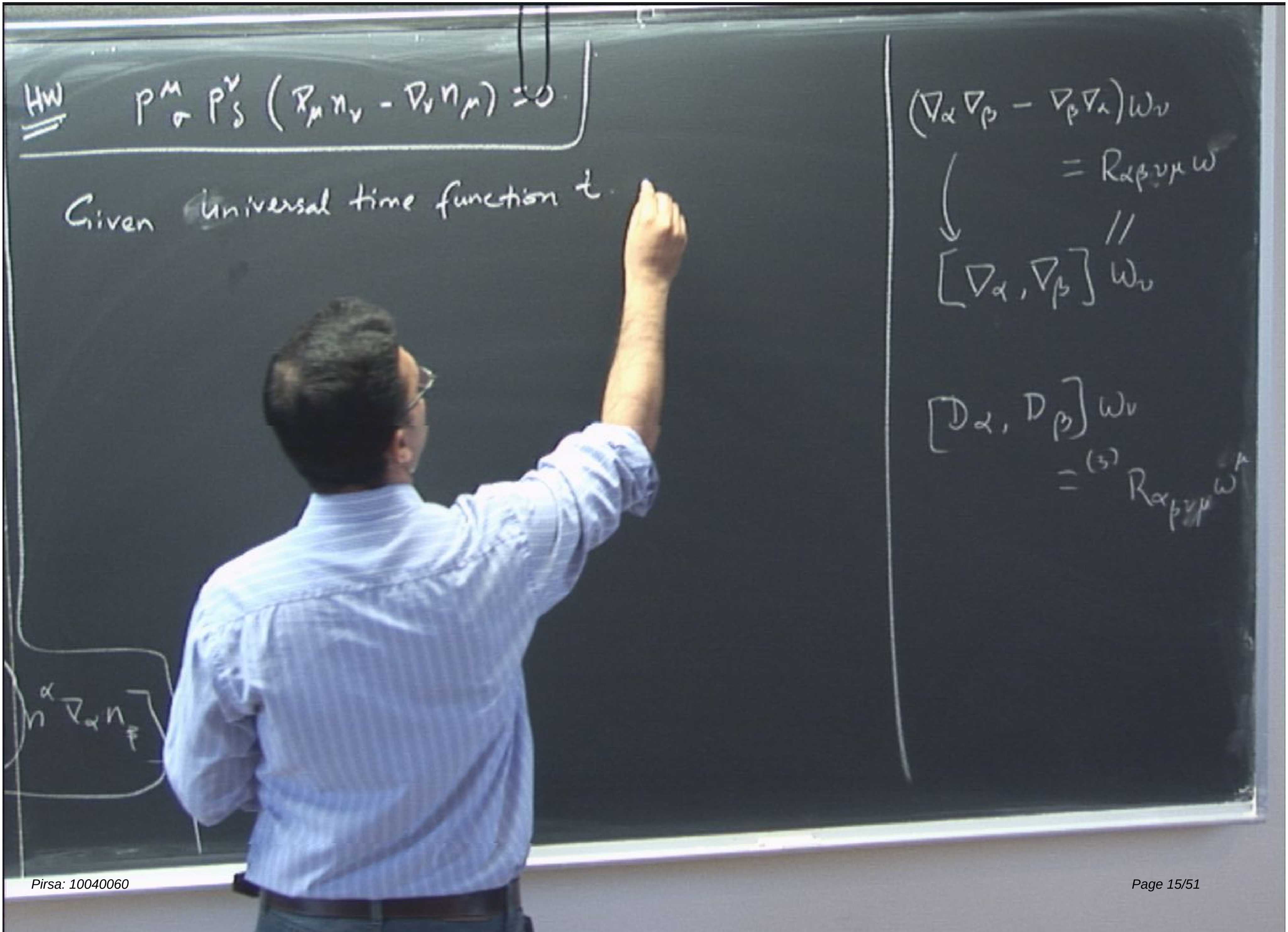

$\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) n_\beta$
$n_\mu n^\alpha \nabla_\alpha n_\nu$
$\gamma \left(P^\mu_\sigma P^\nu_\delta \left[(\nabla_\mu n_\nu - \nabla_\nu n_\mu) n^\alpha \nabla_\alpha n \right] \right)$
0

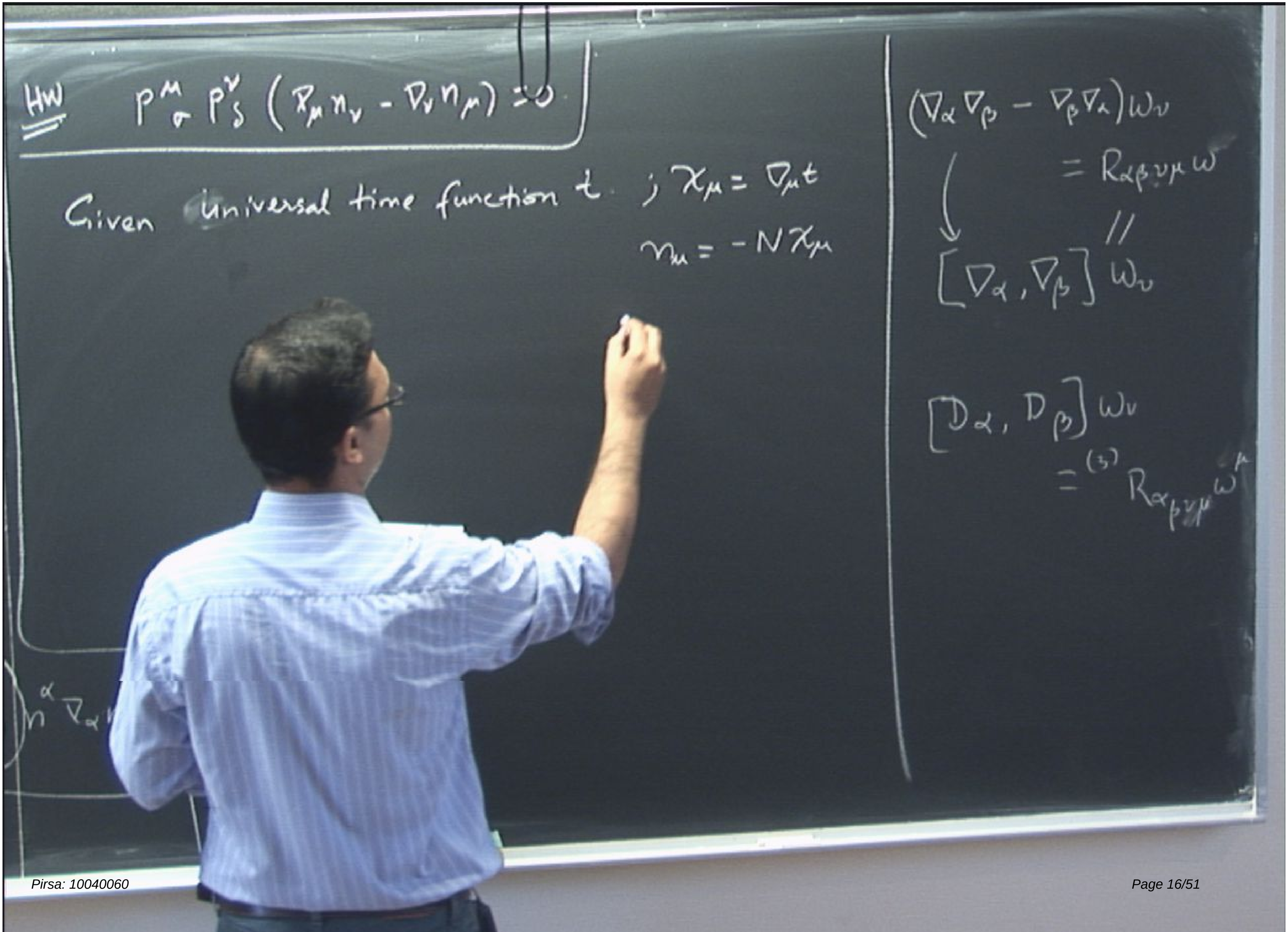
HW $\boxed{P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0}$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu}\, \omega^\mu$$

$$[\nabla_\alpha, \nabla_\beta]\, \omega_\nu$$

$$[D_\alpha, D_\beta]\, \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\, \omega^\mu$$


HW $P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$
Given universal time function t.
$n^\alpha \nabla_\alpha n_\beta$
$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$
$= R_{\alpha\beta\nu\mu}\omega^\mu$
$[\nabla_\alpha, \nabla_\beta]\omega_\nu$
$[D_\alpha, D_\beta]\omega_\nu$
$= {}^{(3)}R_{\alpha\beta\nu\mu}\omega^\mu$


HW $P^{\mu}_{\sigma} P^{\nu}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$
Given universal time function t ; $\chi_\mu = \nabla_\mu t$
$n_\mu = -N \chi_\mu$
$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$
$= R_{\alpha\beta\nu\mu}\omega$
//
$[\nabla_\alpha, \nabla_\beta]\omega_\nu$
$[D_\alpha, D_\beta]\omega_\nu$
$= {}^{(3)}R_{\alpha\beta\nu\mu}\omega^\mu$
$n^\alpha \nabla_\alpha$

HW $\quad P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0$

Given universal time function t. ; $\chi_\mu = \nabla_\mu t$

$$n_\mu = - N \chi_\mu$$

$$g^{\mu\nu} \chi_\mu \chi_\nu = - \frac{1}{N^2}$$

$$\left(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha \right) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$$[\nabla_\alpha, \nabla_\beta] \, \omega_\nu$$

$$[D_\alpha, D_\beta] \, \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \, \omega^\mu$$

$$\left. {}^{\alpha} \nabla_\alpha n_\beta \right]$$

HW $\quad P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0$

Given universal time function t ; $\chi_\mu = \nabla_\mu t$

$$n_\mu = -N \chi_\mu$$

$$g^{\mu\nu} \chi_\mu \chi_\nu = -\frac{1}{N^2}$$

$$\left(\nabla_\sigma n_\delta - \nabla_\delta n_\sigma \right) + \quad \ldots\ldots$$

$$\left(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha \right) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\left[\nabla_\alpha , \nabla_\beta \right] \omega_\nu$$

$$\left[D_\alpha , D_\beta \right] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $p^{\mu}_{\ \sigma}\, p^{\nu}_{\ \delta}\, (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

Given universal time function t ; $\chi_\mu = \nabla_\mu t$

$$n_\mu = -N \chi_\mu$$

$$g^{\mu\nu} \chi_\mu \chi_\nu = -\frac{1}{N^2}$$

$$\underbrace{(\nabla_\sigma n_\delta - \nabla_\delta n_\sigma)}_{\text{Some terms}} + \underbrace{\cdots\cdots}_{\text{Some term}}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega^{\mu}$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^{\mu}$$

$\underline{\underline{HW}}$ $\boxed{p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\,(\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0}$

Given universal time function t ; $\chi_\mu = \nabla_\mu t$

$$n_\mu = -N\chi_\mu$$

$$g^{\mu\nu}\chi_\mu \chi_\nu = -\frac{1}{N^2}$$

$$\underbrace{(\nabla_\sigma n_\delta - \nabla_\delta n_\sigma)}_{\text{Some terms}} + \underbrace{\cdots\cdots}_{\text{Some term}}$$

$$= 0$$

$$n^\alpha \nabla_\alpha n_\beta$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega$$

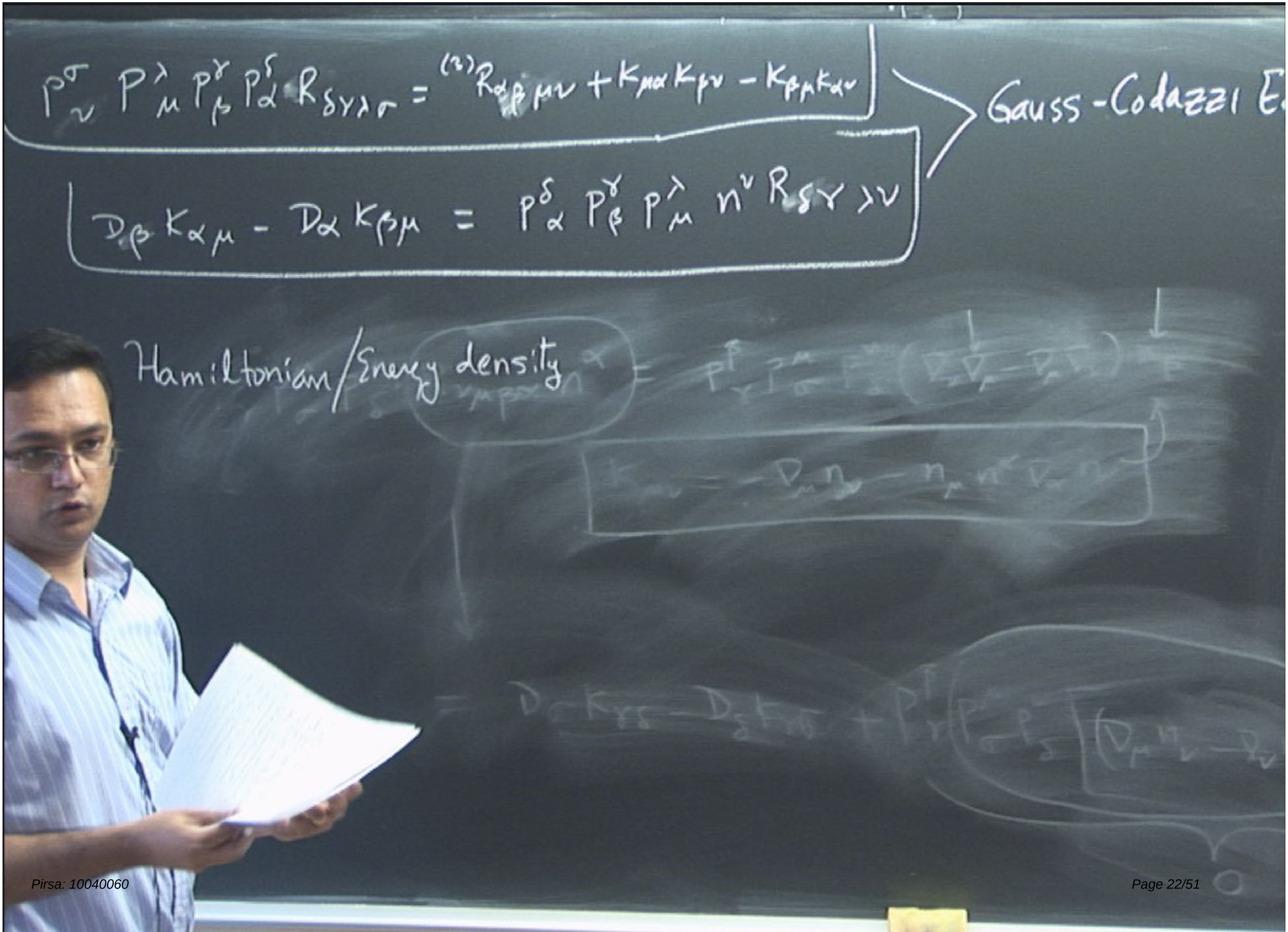
$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^\mu$$

$$\boxed{P^{\sigma}_{\ \nu} P^{\lambda}_{\ \mu} P^{\gamma}_{\ \beta} P^{\delta}_{\ \alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda}_{\ \mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

Gauss-Codazzi E


$P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$
$D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$
Gauss-Codazzi E
Hamiltonian/Energy density

$$P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

$$D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$$

Gauss-Codazzi E

Hamiltonian/Energy density Constraint:

$$\boxed{{}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}}$$

$$\boxed{P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

Gauss-Codazzi E

Hamiltonian/Energy density Constraint.

$$\boxed{{}^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta} = P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}}$$

$$\downarrow$$

$$\rho$$

$$P^{\delta\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R^{\alpha}{}_{\beta\mu}{}^{\nu} + K^{\alpha}_{\mu} K^{\nu}_{\beta} - K^{\alpha\nu} K_{\beta\mu}$$

$$\boxed{P^{\sigma}{}_{\nu} P^{\lambda}{}_{\mu} P^{\gamma}{}_{\beta} P^{\delta}{}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

Gauss-Codazzi E.

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

Hamiltonian/Energy density Constraint.

$K = K^i{}_i$

$$\boxed{{}^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta} = P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}}$$

$\downarrow$

ρ

$$P^{\delta\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R^{\alpha}{}_{\beta\mu}{}^{\nu} + K^{\alpha}{}_{\mu} K_{\beta}{}^{\nu} - K^{\alpha\nu} K_{\beta\mu}$$

Contract $\alpha \leftrightarrow \mu$, $\beta \leftrightarrow \nu$

$$\underline{\underline{HW}} \quad \boxed{P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0}$$

$$P^{\delta\alpha} P^{\gamma}{}_{\nu} P^{\lambda}{}_{\alpha} P^{\nu\sigma} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R + K^2$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

$\underline{\underline{HW}}$ $\boxed{p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0}$

$$\underbrace{p^{\delta\alpha}\, p^{\gamma}{}_{\nu}\, p^{\lambda}{}_{\alpha}\, p^{\nu\sigma}}_{p^{\delta\lambda}\, p^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^{\mu}$$

HW $P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\underbrace{P^{\delta\alpha} P^{\gamma}{}_{\nu} P^{\lambda}{}_{\alpha} P^{\nu\sigma} R_{\delta\gamma\lambda\sigma}}_{P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}} = {}^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta}$$

(x) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $2 G_{\mu\nu} n^\mu n^\nu =$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\underbrace{P^{\delta\alpha} P^{\gamma}{}_{\nu} P^{\lambda}{}_{\alpha} P^{\nu\sigma} R_{\delta\gamma\lambda\sigma}}_{P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}} = {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta}$$

(x) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $\boxed{2G_{\mu\nu} n^\mu n^\nu = R + 2R_{\mu\nu} n^\mu n^\nu}$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\omega$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\omega^\mu$$

HW $\boxed{P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0}$

$$\underbrace{P^{\delta\alpha} P^{\gamma}{}_{\nu} P^{\lambda}{}_{\alpha} P^{\nu\sigma} R_{\delta\gamma\lambda\sigma}}_{P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}} = {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta}$$

(*) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(**) $\boxed{2G_{\mu\nu} n^\mu n^\nu = R + 2R_{\mu\nu} n^\mu n^\nu}$

(*) $\boxed{P^{\alpha\mu} P^{\beta\nu} R_{\alpha\beta\mu\nu} = R + n^\alpha n^\mu R_{\alpha\mu} + n^\beta n^\nu R_{\beta\nu}}$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\omega^\mu$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\omega^\mu$$

HW $\quad P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\underbrace{P^{\delta\alpha} P^{\gamma}{}_{\nu} P^{\lambda}{}_{\alpha} P^{\nu\sigma}}_{P^{\delta\lambda} P^{\gamma\sigma}} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta}$$

$$\underline{P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}}$$

(*) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $\boxed{2 G_{\mu\nu} n^\mu n^\nu = R + 2 R_{\mu\nu} n^\mu n^\nu}$

(*) $\boxed{P^{\alpha\mu} P^{\beta\nu} R_{\alpha\beta\mu\nu} = R + n^\alpha n^\mu R_{\alpha\mu} + n^\beta n_\nu R_{\beta}{}^{\nu}}$

$$= R + 2 n^\alpha n^\beta R_{\alpha\beta} = 2 n^\alpha n^\beta G_{\alpha\beta}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$$

$$= R_{\alpha\beta\nu\mu} \omega^\mu$$

$$//$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu$$

$$= {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $\quad P^{\mu}{}_{\sigma} P^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0$

$$P^{\delta\alpha} P^{\gamma}{}_{\nu} P^{\lambda}{}_{\alpha} P^{\nu\sigma} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R + K^2 - K_{\alpha\beta} K^{\alpha\beta}$$

$$P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \ldots$$

Ex $\quad 2 G_{\mu\nu} n^\mu n^\nu = \ldots$

$$P^{\alpha\mu} P^{\beta\nu} R_{\alpha\beta\mu\nu} = R + \ldots n^\nu R_{\beta\nu}$$

$$= R + 2 n^\alpha \ldots G_{\alpha\beta}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\,(\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$p^{\delta\alpha}\, p^{\gamma}{}_{\nu}\, p^{\lambda}{}_{\alpha}\, p^{\nu\sigma} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta}$$

$$p^{\delta\lambda}\, p^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}$$

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

Ex $$\boxed{2G_{\mu\nu}\, n^\mu n^\nu = R + 2R_{\mu\nu}\, n^\mu n^\nu}$$

$$\boxed{p^{\alpha\mu} p^{\beta\nu} R_{\alpha\beta\mu\nu} = R + n^\alpha n^\mu R_{\alpha\mu} + n^\beta n^\nu R_{\beta\nu}}$$

$$= R + 2n^\alpha n^\beta R_{\alpha\beta} = 2n^\alpha n^\beta G_{\alpha\beta}$$

$$= 16\pi G\, n^\alpha n^\alpha T_{\alpha\alpha}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\,\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega$$

$$// \quad [\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$\boxed{P^{\sigma}_{\ \nu} P^{\lambda}_{\ \mu} P^{\gamma}_{\ \beta} P^{\delta}_{\ \alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$$

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda}_{\ \mu} n^\nu R_{\delta\gamma\lambda\nu}}$$

Gauss-Codazzi E

Hamiltonian/Energy density Constraint.

$K = K^i_{\ i}$

$$\boxed{{}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = P^{\delta\lambda} P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma} = 16\pi G\rho}$$

$\downarrow$

ρ

$$P^{\delta\alpha} P^{\gamma}_{\ \beta} P^{\lambda}_{\ \mu} P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R^{\alpha}_{\ \beta\mu}{}^{\nu} + K^{\alpha}_{\ \mu}K_{\beta}^{\ \nu} - K^{\alpha\nu}K_{\beta\mu}$$

Contract $\alpha \leftrightarrow \mu$, $\beta \leftrightarrow \nu$

$$\boxed{P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

Gauss-Codazzi E

$$\Rightarrow \quad D_{\beta} K_{\alpha}{}^{\mu} - D_{\alpha} K_{\beta}{}^{\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$$

$$P^{\sigma}{}_{\nu} P^{\lambda}{}_{\mu} P^{\gamma}{}_{\beta} P^{\delta}{}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$\Rightarrow D_\beta K_{\alpha}{}^{\mu} - D_\alpha K_{\beta}{}^{\mu} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$$

$\mu \longleftrightarrow \alpha$

$$\boxed{D_\beta K - D_\alpha K_{\beta}{}^{\alpha} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda\alpha} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$\boxed{P^{\sigma}{}_{\nu} P^{\lambda}{}_{\mu} P^{\gamma}{}_{\beta} P^{\delta}{}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}}$$

Gauss-Codazzi E

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda}{}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$\Rightarrow D_{\beta} K_{\alpha}{}^{\mu} - D_{\alpha} K_{\beta}{}^{\mu} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$$

$\mu \longleftrightarrow \alpha$

$$\boxed{D_{\beta} K - D_{\alpha} K_{\beta}{}^{\alpha} = P^{\delta}{}_{\alpha} P^{\gamma}{}_{\beta} P^{\lambda\alpha} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

two steps

$$D^{\alpha} K - D_{\beta} K^{\alpha\beta} = P^{\delta}{}_{\beta} P^{\gamma\alpha} P^{\lambda\beta} n^{\nu} R_{\delta\gamma\lambda\nu}$$

$$\boxed{P^{\sigma}_{\ \nu} P^{\lambda}_{\ \mu} P^{\gamma}_{\ \beta} P^{\delta}_{\ \alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$$

Gauss-Codazzi E

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda}_{\ \mu} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$D_\beta K_\alpha^{\ \mu} - D_\alpha K_\beta^{\ \mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$$

$\mu \to \alpha$

$$\boxed{D_\beta K - D_\alpha K_\beta^{\ \alpha} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda\alpha} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

two steps

$$D^\alpha K - D_\beta K^{\alpha\beta} = \underbrace{P^{\delta}_{\ \beta} P^{\gamma\alpha} P^{\lambda\beta} n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$\boxed{P^{\sigma}_{\nu}\, P^{\lambda}_{\mu}\, P^{\gamma}_{\beta}\, P^{\delta}_{\alpha}\, R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$$

Gauss-Codazzi E

$$\boxed{D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha}\, P^{\gamma}_{\beta}\, P^{\lambda}_{\mu}\, n^{\nu} R_{\delta\gamma\lambda\nu}}$$

$$\Rightarrow D_{\beta} K_{\alpha}{}^{\mu} - D_{\alpha} K_{\beta}{}^{\mu} = P^{\delta}_{\alpha}\, P^{\gamma}_{\beta}\, P^{\lambda\mu}\, n^{\nu} R_{\delta\gamma\lambda\nu}$$

$\mu \leftrightarrow \alpha$

$$\boxed{D_{\beta} K - D_{\alpha} K_{\beta}{}^{\alpha} = P^{\delta}_{\alpha}\, P^{\gamma}_{\beta}\, P^{\lambda\alpha}\, n^{\nu} R_{\delta\gamma\lambda\nu}}$$

two steps

$$\boxed{D^{\alpha} K - D_{\beta} K^{\alpha\beta} = \underbrace{P^{\delta}_{\beta}\, P^{\gamma\alpha}\, P^{\lambda\beta}\, n^{\nu} R_{\delta\gamma\lambda\nu}}}$$

$$\boxed{P^{\sigma}_{\ \nu} P^{\lambda}_{\ \mu} P^{\gamma}_{\ \beta} P^{\delta}_{\ \alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}}$$

Gauss-Codazzi E

$$\boxed{D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda}_{\ \mu} n^\nu R_{\delta\gamma\lambda\nu}}$$

$$\Rightarrow \quad D_\beta K_\alpha^{\ \mu} - D_\alpha K_\beta^{\ \mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

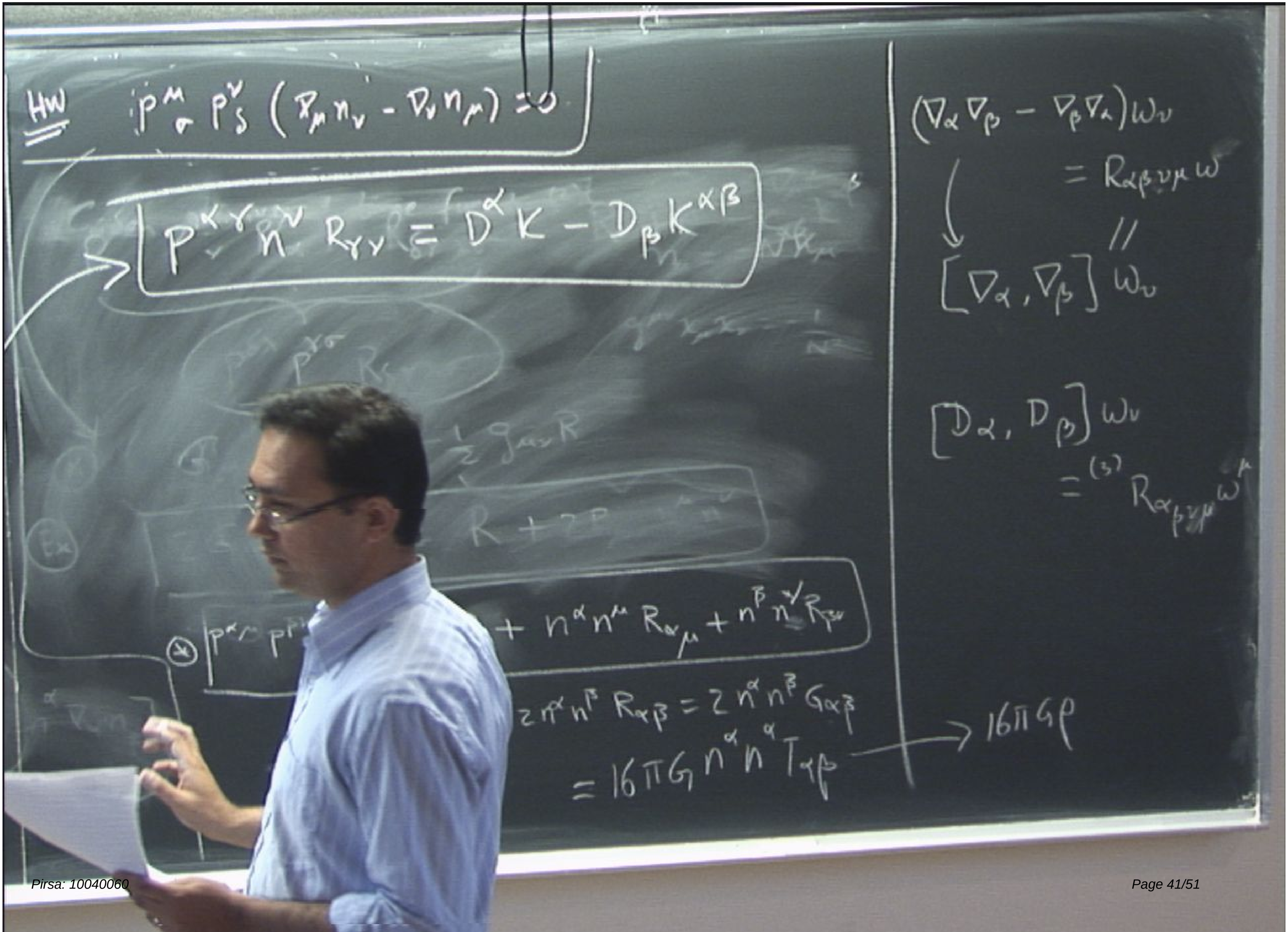
$\mu \longleftrightarrow \alpha$

$$\boxed{D_\beta K - D_\alpha K_\beta^{\ \alpha} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda\alpha} n^\nu R_{\delta\gamma\lambda\nu}}$$

two steps

$$\boxed{D^\alpha K - D_\beta K^{\alpha\beta} = \underbrace{P^{\delta}_{\ \beta} P^{\gamma\alpha} P^{\lambda\beta} n^\nu R_{\delta\gamma\lambda\nu}}}$$

$$\boxed{P^{\delta}_{\ \beta} R_{\delta\gamma}^{\ \ \ \beta}{}_{\nu} n^\nu = R_{\gamma\nu} n^\nu}$$


HW $p^{\mu}{}_{\alpha} p^{\nu}{}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$
$p^{\alpha\gamma} n^{\nu} R_{\gamma\nu} = D^{\alpha} K - D_{\beta} K^{\alpha\beta}$
$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$
$= R_{\alpha\beta\nu\mu}\,\omega^{\mu}$
$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$
$[D_\alpha, D_\beta]\,\omega_\nu$
$= {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^{\mu}$
$+ n^{\alpha} n^{\mu} R_{\alpha\mu} + n^{\beta} n^{\nu} R_{\beta\nu}$
$2 n^{\alpha} n^{\beta} R_{\alpha\beta} = 2 n^{\alpha} n^{\beta} G_{\alpha\beta}$
$= 16\pi G\, n^{\alpha} n^{\beta} T_{\alpha\beta} \to 16\pi G \rho$

HW $\quad P^{\mu}{}_{\alpha} P^{\nu}{}_{\delta} \left(\nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = 0$

$$\boxed{P^{\alpha\gamma} n^{\nu} R_{\gamma\nu} = D^{\alpha} K - D_{\beta} K^{\alpha\beta}}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$\boxed{P^{\alpha\mu} P^{\beta\nu} R_{\alpha\beta\mu\nu} = R + n^{\alpha} n^{\mu} R_{\alpha\mu} + n^{\beta} n^{\nu} R_{\beta\nu}}$$

$$= R + 2 n^{\alpha} n^{\beta} R_{\alpha\beta} = 2 n^{\alpha} n^{\beta} G_{\alpha\beta}$$

$$= 16\pi G\, n^{\alpha} n^{\alpha} T_{\alpha\beta} \longrightarrow 16\pi G \rho$$

$$\left(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha \right) \omega_\nu = R_{\alpha\beta\nu\mu} \omega$$

$$[\nabla_\alpha, \nabla_\beta]\, \omega_\nu$$

$$[D_\alpha, D_\beta]\, \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^{\mu}$$

HW $p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\, (\nabla_{\mu} n_{\nu} - \nabla_{\nu} n_{\mu}) = 0$

$$\boxed{P^{\alpha\gamma} n^{\nu} R_{\gamma\nu} = D^{\alpha} K - D_{\beta} K^{\alpha\beta}}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$\boxed{P^{\alpha\mu} n^{\nu} G_{\mu\nu} = P^{\alpha\mu} n^{\nu} R_{\mu\nu}}$$

$(\nabla_{\alpha}\nabla_{\beta} - \nabla_{\beta}\nabla_{\alpha})\omega_{\nu}$

$= R_{\alpha\beta\nu\mu}\,\omega$

$[\nabla_{\alpha}, \nabla_{\beta}]\,\omega_{\nu}$

$[D_{\alpha}, D_{\beta}]\,\omega_{\nu}$

$= {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^{\mu}$

$\rightarrow 16\pi G\rho$

HW $\quad p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\, (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\boxed{p^{\alpha\gamma} n^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}}$$

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

$$\boxed{p^{\alpha\mu} n^\nu G_{\mu\nu} = p^{\alpha\mu} n^\nu R_{\mu\nu}}$$

$$8\pi G\, p^{\alpha\mu} n^\nu T_{\mu\nu}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^\mu$$

$$\rightarrow 16\pi G \rho$$

HW $\quad p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\,(\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\boxed{P^{\alpha\gamma} n^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$\boxed{P^{\alpha\mu} n^\nu G_{\mu\nu} = P^{\alpha\mu} n^\nu R_{\mu\nu}}$$

$$8\pi G\, P^{\alpha\mu} n^\nu T_{\mu\nu}$$

$$= -8\pi G\, J^\alpha$$

$$\boxed{D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$$

$$= R_{\alpha\beta\nu\mu}\,\omega$$

$$//$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu$$

$$= {}^{(3)}R$$

HW $\quad p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\,(\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\boxed{p^{\alpha\gamma} n^{\nu} R_{\gamma\nu} = D^{\alpha} K - D_{\beta} K^{\alpha\beta}}$$

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

$$\boxed{p^{\alpha\mu} n^{\nu} G_{\mu\nu} = p^{\alpha\mu} n^{\nu} R_{\mu\nu}}$$

$$8\pi G\, p^{\alpha\mu} n^{\nu} T_{\mu\nu}$$

$$= -8\pi G\, J^{\alpha}$$

Momentum constraint.

$$\boxed{D^{\beta} K_{\alpha\beta} - D_{\alpha} K = 8\pi G\, J_{\alpha}}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\,\omega_\nu = R_{\alpha\beta\nu\mu}\,\omega$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^{\mu}$$

$$\rightarrow 16\pi G \rho$$

HW $\quad p^{\mu}{}_{\sigma}\, p^{\nu}{}_{\delta}\left(\nabla_\mu n_\nu - \nabla_\nu n_\mu\right) = 0$

$$\boxed{P^{\alpha\gamma} n^{\nu} R_{\gamma\nu} = D^{\alpha} K - D_{\beta} K^{\alpha\beta}}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$\boxed{p^{\alpha\mu} n^{\nu} G_{\mu\nu} = P^{\alpha\mu} n^{\nu} R_{\mu\nu}}$$

$$8\pi G\, p^{\alpha\mu} n^{\nu} T_{\mu\nu}$$

$$= -8\pi G\, J^{\alpha}$$

Momentum constraint.

$$\boxed{D^{\beta} K_{\alpha\beta} - D_{\alpha} K = 8\pi G\, J_{\alpha}}$$

$$\boxed{D^{j} K^{i}{}_{j} - D_{i} K = 8\pi G\, J_{i}}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$$

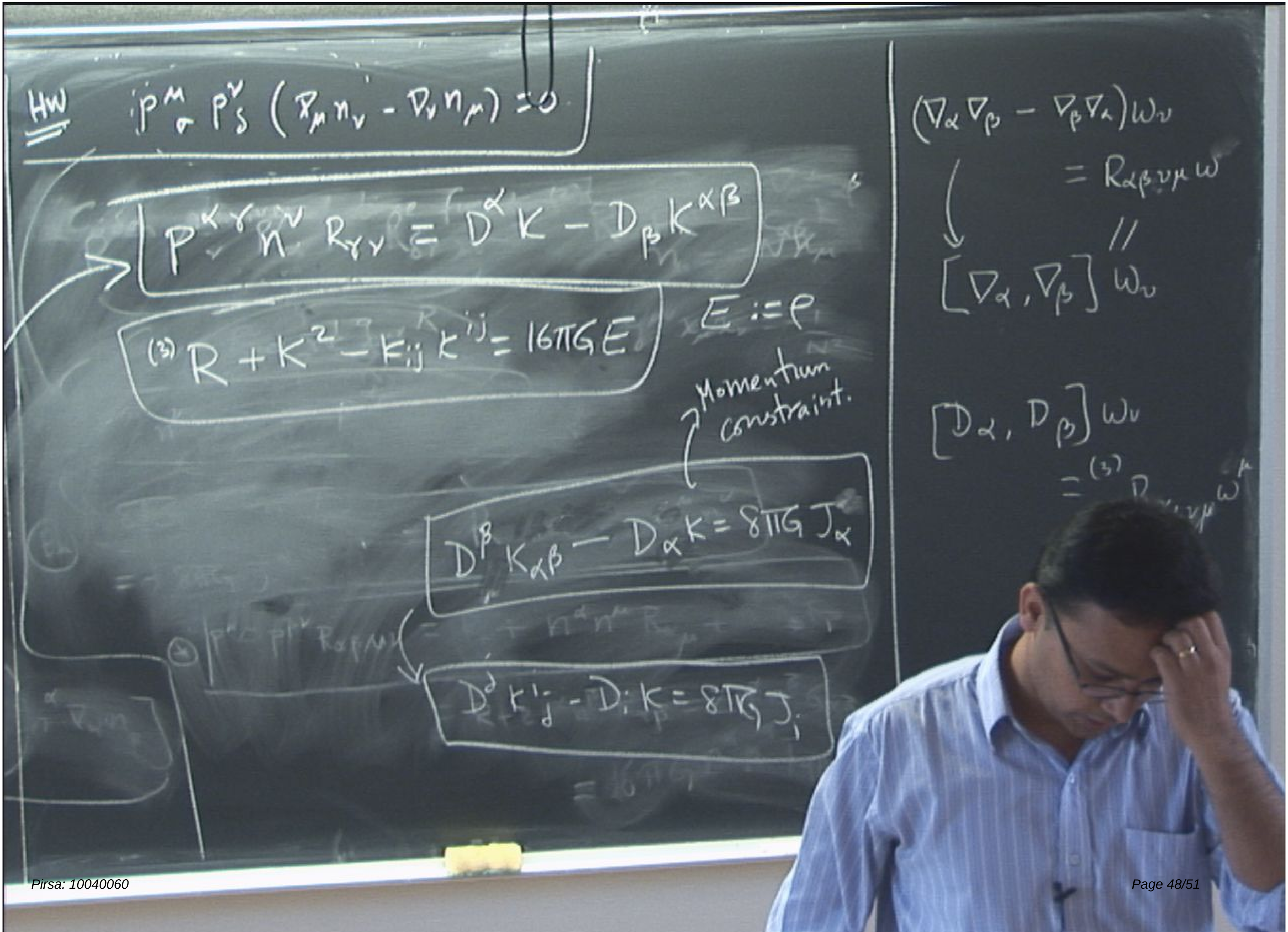
$$= R_{\alpha\beta\nu\mu}\,\omega$$

$$//$$

$$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$$

$$[D_\alpha, D_\beta]\,\omega_\nu$$

$$= {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^{\mu}$$


HW $P^{\mu}_{\sigma} P^{\nu}_{\delta} (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$
$P^{\alpha\gamma} n^{\nu} R_{\gamma\nu} = D^{\alpha} K - D_{\beta} K^{\alpha\beta}$
$^{(3)}R + K^2 - K_{ij} K^{ij} = 16\pi G E$
$E := \rho$
Momentum constraint.
$D^{\beta} K_{\alpha\beta} - D_{\alpha} K = 8\pi G J_{\alpha}$
$D^{j} K^{i}{}_{j} - D_{i} K = 8\pi G J_{i}$
$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu = R_{\alpha\beta\nu\mu}\omega$
$[\nabla_\alpha, \nabla_\beta]\omega_\nu$
$[D_\alpha, D_\beta]\omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu}\omega^{\mu}$

HW $P^\mu{}_\alpha P^\nu{}_\delta (\nabla_\mu n_\nu - \nabla_\nu n_\mu) = 0$

$$\boxed{P^{\alpha\gamma} n^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}}$$

$$\boxed{{}^{(3)}R + K^2 - K_{ij} K^{ij} = 16\pi G E}$$

$E := \rho$

Momentum constraint.

4 constraints

$$\boxed{D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha}$$

$$\boxed{D^j K^i{}_j - D_i K = 8\pi G J_i}$$

$\rightarrow 16\pi G\rho$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\omega_\nu$

$= R_{\alpha\beta\nu\mu}\omega$

$//$

$[\nabla_\alpha, \nabla_\beta]\,\omega_\nu$

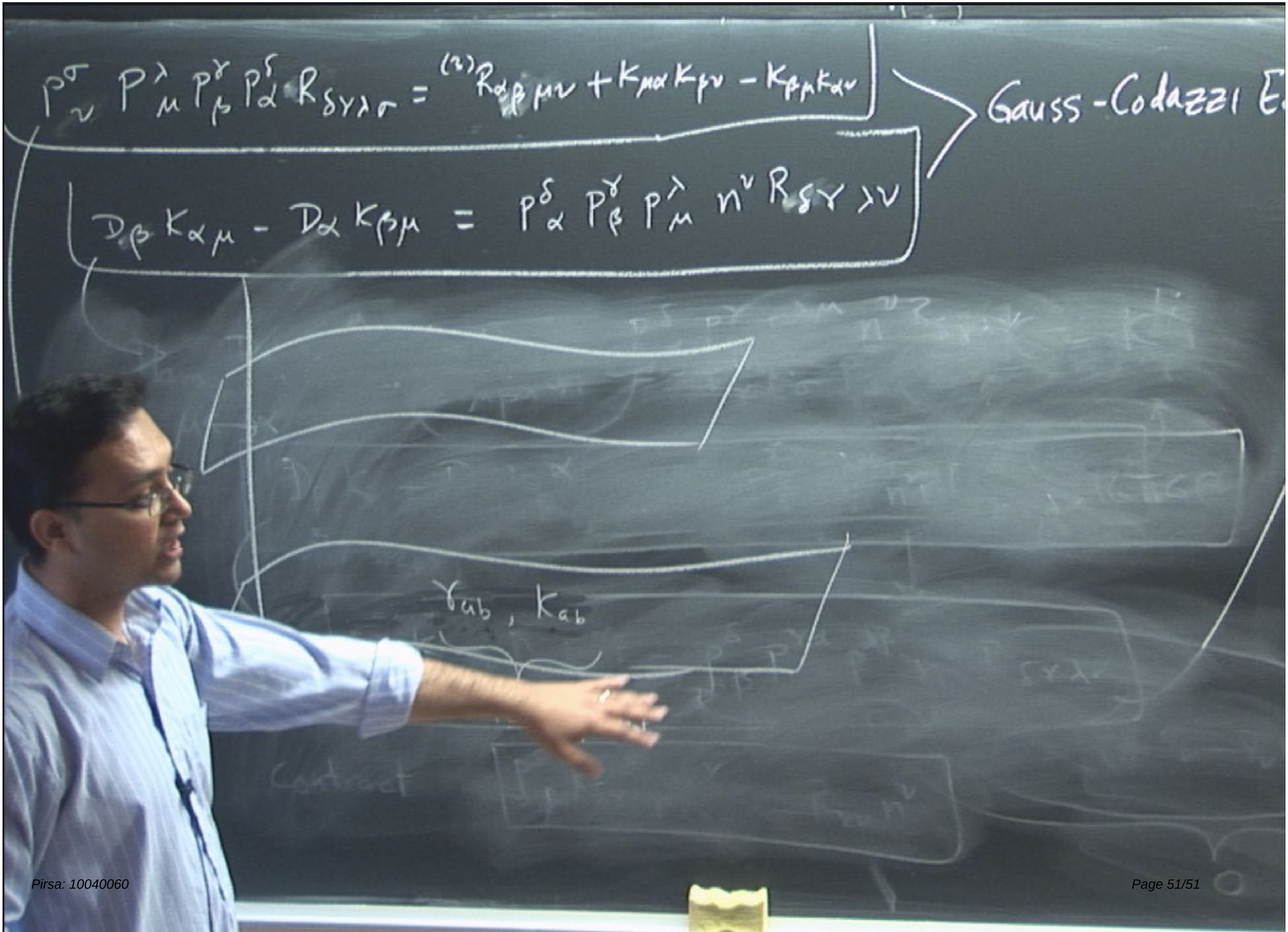
$[D_\alpha, D_\beta]\,\omega_\nu$

$= {}^{(3)}R_{\alpha\beta\nu\mu}\,\omega^\mu$

$$P^{\sigma}_{\ \nu} P^{\lambda}_{\ \mu} P^{\gamma}_{\ \beta} P^{\delta}_{\ \alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

$$D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\ \alpha} P^{\gamma}_{\ \beta} P^{\lambda}_{\ \mu} n^{\nu} R_{\delta\gamma\lambda\nu}$$

Gauss-Codazzi E


$P^{\sigma}_{\nu} P^{\lambda}_{\mu} P^{\gamma}_{\beta} P^{\delta}_{\alpha} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$
Gauss-Codazzi E
$D_{\beta} K_{\alpha\mu} - D_{\alpha} K_{\beta\mu} = P^{\delta}_{\alpha} P^{\gamma}_{\beta} P^{\lambda}_{\mu} n^{\nu} R_{\delta\gamma\lambda\nu}$
γ_{ab}, K_{ab}