

Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 3

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Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu =$$

$$\begin{aligned}
 (\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu &= R_{\alpha\beta\nu\mu} \omega^\mu \\
 \downarrow &\quad \parallel \\
 [\nabla_\alpha, \nabla_\beta] \omega_\nu &
 \end{aligned}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$$

$$= R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

//

$$[D_\alpha, D_\beta] \omega_\nu$$

$$= {}^{(3)}R_{\alpha\beta\mu\nu} \omega^\mu$$

$$\begin{aligned}
 (\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu &= R_{\alpha\beta\nu\mu} \omega^\mu \\
 \downarrow &\quad // \\
 [\nabla_\alpha, \nabla_\beta] \omega_\nu &
 \end{aligned}$$

$$\begin{aligned}
 [D_\alpha, D_\beta] \omega_\nu &= {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu
 \end{aligned}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\rho P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\rho\nu} - K_{\rho\mu}K_{\alpha\nu}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$



$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (R_{\nu\mu\beta\alpha} n^\alpha) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (D_\nu D_\mu - D_\mu D_\nu) n_\beta$$

$$K_{\mu\nu} = -D_\mu n_\nu - n_\mu n^\alpha D_\alpha n_\nu$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (R_{\nu\mu\beta\alpha} n^\alpha) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (D_\nu D_\mu - D_\mu D_\nu) n_\beta$$

$$K_{\mu\nu} = -D_\mu n_\nu - n_\mu n^\alpha D_\alpha n_\nu$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (R_{\nu\mu\beta\alpha} n^\alpha) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (D_\nu D_\mu - D_\mu D_\nu) n_\beta$$

$$K_{\mu\nu} = -D_\mu n_\nu - n_\mu n^\alpha D_\alpha n_\nu$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (R_{\nu\mu\beta\sigma} n^\alpha) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (\overset{\downarrow}{D_\nu} \overset{\downarrow}{D_\mu} - \overset{\downarrow}{D_\mu} \overset{\downarrow}{D_\nu}) \overset{\downarrow}{n^\beta}$$

$$K_{\mu\nu} = -D_\mu n_\nu - n_\mu n^\alpha D_\alpha n_\nu$$



$$D_\sigma K_{\gamma\delta} - D_\delta K_{\gamma\sigma}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\rho P^\delta_\alpha R_{\delta\gamma\lambda\rho} = {}^{(2)}R_{\rho\mu\nu} + K_{\mu\alpha} K_{\rho\nu} - K_{\rho\mu} K_{\alpha\nu}$$

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\beta_\sigma P^\mu_\gamma P^\nu_\delta (R_{\nu\mu\rho\sigma} n^\rho) = P^\beta_\gamma P^\mu_\sigma P^\nu_\delta (\overset{\downarrow}{D_\nu} \overset{\downarrow}{D_\mu} - \overset{\downarrow}{D_\mu} \overset{\downarrow}{D_\nu}) \overset{\downarrow}{n^\rho}_\rho$$

$$K_{\mu\nu} = -D_\mu n_\nu - n_\mu n^\alpha D_\alpha n_\nu$$

$$= \nu_\sigma K_{\gamma\delta} - D_\delta K_{\gamma\sigma} + \underbrace{P^\beta_\gamma P^\mu_\sigma P^\nu_\delta \left[(D_\mu n_\nu - D_\nu n_\mu) n^\alpha D_\alpha n_\rho \right]}_0$$

$$\downarrow \quad \downarrow$$

$$(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) n_\beta$$

$$\boxed{n_\mu n^\alpha \nabla_\alpha n_\nu}$$

$$\delta \mathcal{L} = \frac{1}{2} \left(P_\sigma^\mu P_\delta^\nu \left[(\nabla_\mu n_\nu - \nabla_\nu n_\mu) n^\alpha \nabla_\alpha n_\beta \right] \right)$$

HW

$$P^\mu_\alpha P^\nu_\beta (\nabla_\mu \eta_\nu - \nabla_\nu \eta_\mu) = 0$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$$
$$= R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow //$$
$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu$$
$$= {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $P^\mu_\alpha P^\nu_\beta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

Given universal time function t .

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow \quad \quad \quad //$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

$$n^\alpha \partial_\alpha n_\beta$$

HW $P^\mu_\alpha P^\nu_\beta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

Given universal time function t ; $\chi_\mu = \partial_\mu t$
 $\eta_\mu = -N \chi_\mu$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

↓

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $P^\mu_\alpha P^\nu_\beta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

Given universal time function t ; $x_\mu = \partial_\mu t$
 $\eta_\mu = -N x_\mu$
 $g^{\mu\nu} x_\mu x_\nu = -\frac{1}{N^2}$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$$

$$= R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow //$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu$$

$$= {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\partial^\alpha \partial_\alpha \eta_\beta$$

HW $(P^\mu_\sigma P^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

Given universal time function t ; $\chi_\mu = \partial_\mu t$
 $\eta_\mu = -N \chi_\mu$
 $g^{\mu\nu} \chi_\mu \chi_\nu = -\frac{1}{N^2}$

$(\partial_\sigma \eta_\delta - \partial_\delta \eta_\sigma) + \dots$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$$

$$= R_{\alpha\beta\nu\mu} \omega^\mu$$

$\downarrow$ // $[\nabla_\alpha, \nabla_\beta] \omega_\nu$

$$[D_\alpha, D_\beta] \omega_\nu$$

$$= {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $(P^\mu_\sigma P^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0)$

Given universal time function t ; $\chi_\mu = \partial_\mu t$
 $\eta_\mu = -N \chi_\mu$
 $g^{\mu\nu} \chi_\mu \chi_\nu = -\frac{1}{N^2}$

$$\underbrace{(\partial_\sigma \eta_\delta - \partial_\delta \eta_\sigma)}_{\text{Some term}} + \underbrace{\dots}_{\text{Some term}}$$

$$\begin{aligned} (\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu \\ \downarrow \\ [\nabla_\alpha, \nabla_\beta] \omega_\nu \end{aligned} \quad \begin{aligned} &= R_{\alpha\beta\nu\mu} \omega^\mu \\ &\parallel \\ & \end{aligned}$$

$$\begin{aligned} [D_\alpha, D_\beta] \omega_\nu \\ = {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu \end{aligned}$$

HW $(P^\mu_\sigma P^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

Given universal time function t ; $\chi_\mu = \partial_\mu t$
 $\eta_\mu = -N \chi_\mu$
 $g^{\mu\nu} \chi_\mu \chi_\nu = -\frac{1}{N^2}$

$$\underbrace{(\partial_\sigma \eta_\delta - \partial_\delta \eta_\sigma)}_{\text{Some term}} + \underbrace{\dots}_{\text{Some term}} = 0$$

$$\left[\eta^\alpha \partial_\alpha \eta_\beta \right]$$

$$\begin{aligned} (\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu \\ = R_{\alpha\beta\nu\mu} \omega^\mu \\ \downarrow // \\ [\nabla_\alpha, \nabla_\beta] \omega_\nu \end{aligned}$$

$$\begin{aligned} [D_\alpha, D_\beta] \omega_\nu \\ = {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu \end{aligned}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\mu\beta}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\beta_\gamma P^\mu_\alpha P^\nu_\delta R_{\nu\mu\beta\alpha} n^\alpha$$

$$P^\beta_\gamma P^\mu_\alpha P^\nu_\delta (D_\nu D_\mu - D_\mu D_\nu) n^\alpha$$

$$K_{\mu\alpha} = -D_\mu n_\alpha - n_\mu n^\nu D_\nu \alpha$$

$$= D_\alpha K_{\gamma\delta} - D_\delta K_{\gamma\alpha} + P^\beta_\gamma P^\mu_\alpha P^\nu_\delta (D_\mu n_\nu - D_\nu n_\mu)$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

Hamiltonian/Energy density

$$K_{\mu\nu} = -D_\mu n_\nu - n_\mu n^\alpha D_\alpha n_\nu$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

Hamiltonian/Energy density Constraint.

$${}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = P^\delta_\lambda P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

Hamiltonian/Energy density Constraint.

$${}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = P^\delta_\lambda P^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}$$

$$P^{\delta\alpha} P^\gamma_\beta P^\lambda_\mu P^{\sigma\nu} R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\beta\mu}{}^\alpha{}_\nu + K^\alpha_\mu K_{\beta\nu} - K^{\alpha\nu} K_{\beta\mu} n_\nu - \dots$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

Hamiltonian/Energy density Constraint.

$$K = K^i_i$$

$${}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = P^{\delta\lambda}P^{\gamma\sigma}R_{\delta\gamma\lambda\sigma}$$

$$P^{\delta\alpha}P^{\gamma\beta}P^\lambda_\mu P^{\sigma\nu}R_{\delta\gamma\lambda\sigma} = {}^{(3)}R^\alpha_{\beta\mu}{}^\nu + K^\alpha_\mu K^\nu_\beta - K^{\alpha\nu}K_{\beta\mu}$$

Contract $\alpha \leftrightarrow \mu, \beta \leftrightarrow \nu$

HW $p^\mu_\sigma p^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$p^\delta_\alpha p^\gamma_\nu p^\lambda_\alpha p^\sigma_\beta R_{\delta\gamma\lambda\sigma} = R + k^2$

universal wave function (3)

$\eta_\mu = -N \chi_\mu$

$g_{\mu\nu} \chi_\mu \chi_\nu = -\frac{1}{N^2}$

$(\partial_\sigma \eta_\delta - \partial_\delta \eta_\sigma) +$

Some terms

$= 0$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu$
 $= R_{\alpha\beta\nu\mu} w^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] w_\nu$

$[D_\alpha, D_\beta] w_\nu$
 $= {}^{(3)}R_{\alpha\beta\nu\mu} w^\mu$

HW $p^\mu_\sigma p^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$p^{\delta\alpha} p^{\gamma\lambda} p^{\nu\sigma} p^{\alpha\beta} R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$
 (universal wave function)
 $p^{\delta\lambda} p^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}$

$(\partial_\sigma \eta_\delta - \partial_\delta \eta_\sigma) +$

Some term

$= 0$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu$
 $= R_{\alpha\beta\nu\mu} w^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] w_\nu$

$[D_\alpha, D_\beta] w_\nu$
 $= {}^{(3)} R_{\alpha\beta\nu\mu} w^\mu$

HW $p^\mu_\sigma p^\nu_\delta (\delta^\mu_\nu - \nabla_\nu \eta_\mu) = 0$

universal wave function (3)
 $p^\delta_\alpha p^\gamma_\nu p^\lambda_\alpha p^\sigma R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$
 $p^\delta_\lambda p^\gamma_\sigma R_{\delta\gamma\lambda\sigma}$

(x) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(x) $2 G_{\mu\nu} \eta^\mu \eta^\nu =$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu = R_{\alpha\beta\nu\mu} w^\mu$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] w_\nu$$

$$[D_\alpha, D_\beta] w_\nu = {}^{(3)} R_{\alpha\beta\nu\mu} w^\mu$$

HW $p^\mu_\sigma p^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

universal wave function (3)

$$p^{\delta\alpha} p^{\gamma\lambda} p^{\nu\sigma} p^{\rho\mu} R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$$

$$p^{\delta\lambda} p^{\gamma\sigma} R_{\delta\gamma\lambda\sigma}$$

(*) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $2G_{\mu\nu} \eta^\mu \eta^\nu = R + 2R_{\mu\nu} \eta^\mu \eta^\nu$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$\downarrow$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $p^\mu_\sigma p^\nu_\delta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$p^{\delta\alpha} p^\nu_\nu p^\lambda_\alpha p^\sigma R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$
 $p^{\delta\lambda} p^\gamma_\sigma R_{\delta\gamma\lambda\sigma}$

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

$2G_{\mu\nu} \eta^\mu \eta^\nu = R + 2R_{\mu\nu} \eta^\mu \eta^\nu$

$p^{\alpha\mu} p^{\beta\nu} R_{\alpha\beta\mu\nu} = R + \eta^\alpha \eta^\mu R_{\alpha\mu} + \eta^\beta \eta_\nu R_{\beta\nu}$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$
 $= R_{\alpha\beta\nu\mu} \omega^\mu$

$[\nabla_\alpha, \nabla_\beta] \omega_\nu$

$[D_\alpha, D_\beta] \omega_\nu$
 $= {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

universal solution function (3)
 $p^\delta p^\nu p^\alpha p^\sigma R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$
 $p^\delta p^\sigma R_{\delta\gamma\lambda\sigma}$

(*) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $2G_{\mu\nu} \eta^\mu \eta^\nu = R + 2R_{\mu\nu} \eta^\mu \eta^\nu$

(*) $p^\alpha p^\beta R_{\alpha\beta\mu\nu} = R + \eta^\alpha \eta^\mu R_{\alpha\mu} + \eta^\beta \eta_\nu R_{\beta\nu}$
 $= R + 2\eta^\alpha \eta^\beta R_{\alpha\beta} = 2\eta^\alpha \eta^\beta G_{\alpha\beta}$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$
 $= R_{\alpha\beta\nu\mu} \omega^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] \omega_\nu$

$[D_\alpha, D_\beta] \omega_\nu$
 $= {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$p^\delta p^\gamma p^\alpha p^\beta R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$

$p^\delta p^\gamma R_{\delta\gamma\lambda\sigma}$

(*) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $2 G_{\mu\nu} \eta^\mu \eta^\nu = 2 R + \dots$

(*) $p^\alpha p^\beta R_{\alpha\beta\mu\nu} = R + \dots$

$= R + 2 R^{\alpha\alpha}$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] \omega_\nu$

$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

universal wave function (p)
 $p^\delta p^\nu p^\alpha p^\lambda R_{\delta\gamma\lambda\sigma} = R + k^2 - k_{\alpha\beta} k^{\alpha\beta}$

$p^\delta p^\nu R_{\delta\gamma\lambda\sigma}$

(*) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

(Ex) $2G_{\mu\nu} \eta^\mu \eta^\nu = R + 2R_{\mu\nu} \eta^\mu \eta^\nu$

(*) $p^\alpha p^\beta R_{\alpha\beta\mu\nu} = R + \eta^\alpha \eta^\mu R_{\alpha\mu} + \eta^\beta \eta^\nu R_{\beta\nu}$

$= R + 2\eta^\alpha \eta^\beta R_{\alpha\beta} = 2\eta^\alpha \eta^\beta G_{\alpha\beta}$
 $= 16\pi G \eta^\alpha \eta^\alpha T_{\alpha\alpha}$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$
 $= R_{\alpha\beta\nu\mu} \omega^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] \omega_\nu$

$[D_\alpha, D_\beta]$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\mu\beta}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

Hamiltonian/Energy density Constraint.

$$K = K^i_i$$

$${}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta} = P^{\delta\lambda}P^{\gamma\sigma}R_{\delta\gamma\lambda\sigma} = 16\pi G\rho$$

$$P^{\delta\alpha}P^{\gamma\beta}P^{\lambda\mu}P^{\sigma\nu}R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\mu\beta}K_{\alpha\nu}$$

Contract $\alpha \leftrightarrow \mu, \beta \leftrightarrow \nu$

$$p^\sigma_\nu p^\lambda_\mu p^\gamma_\beta p^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = p^\delta_\alpha p^\gamma_\beta p^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$\Rightarrow D_\beta K^\mu_{\alpha} - D_\alpha K^\mu_{\beta} = p^\delta_\alpha p^\gamma_\beta p^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\mu\beta}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$\Rightarrow D_\beta K^\mu_\alpha - D_\alpha K^\mu_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

$\mu \leftrightarrow \alpha$

$$D_\beta K - D_\alpha K^\alpha_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\alpha} n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$\Rightarrow D_\beta K^\mu_\alpha - D_\alpha K^\mu_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

$\mu \leftrightarrow \alpha$

$$D_\beta K - D_\alpha K^\alpha_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\alpha} n^\nu R_{\delta\gamma\lambda\nu}$$

two steps

$$\Rightarrow D^\alpha K - D_\beta K^{\alpha\beta} = P^\delta_\beta P^{\gamma\alpha} P^{\lambda\beta} n^\nu R_{\delta\gamma\lambda\nu}$$

Contract $\alpha \rightarrow \mu$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\mu\beta}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$D_\beta K^\mu_\alpha - D_\alpha K^\mu_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

$$D_\beta K^\alpha - D_\alpha K^\alpha_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\alpha} n^\nu R_{\delta\gamma\lambda\nu}$$

two steps

$$D^\alpha K - D_\beta K^{\alpha\beta} = P^\delta_\beta P^{\gamma\alpha} P^{\lambda\beta} n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$\Rightarrow D_\beta K^\mu_\alpha - D_\alpha K^\mu_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

$\mu \leftrightarrow \alpha$

$$D_\beta K - D_\alpha K^\alpha_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\alpha} n^\nu R_{\delta\gamma\lambda\nu}$$

two steps

$$D^\alpha K - D_\beta K^{\alpha\beta} = P^\delta_\beta P^{\gamma\alpha} P^{\lambda\beta} n^\nu R_{\delta\gamma\lambda\nu}$$

Contract $\alpha \leftrightarrow \beta$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\mu\beta}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$\Rightarrow D_\beta K^\mu_\alpha - D_\alpha K^\mu_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\mu} n^\nu R_{\delta\gamma\lambda\nu}$$

$\mu \leftrightarrow \alpha$

$$D_\beta K - D_\alpha K^\alpha_\beta = P^\delta_\alpha P^\gamma_\beta P^{\lambda\alpha} n^\nu R_{\delta\gamma\lambda\nu}$$

two steps

$$D^\alpha K - D_\beta K^{\alpha\beta} = P^\delta_\beta P^{\gamma\alpha} P^{\lambda\beta} n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\delta_\beta R_{\delta\gamma}{}^\beta{}_\nu n^\nu = R_{\gamma\nu} n^\nu$$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$\rightarrow p^\alpha \eta^\nu R_{\alpha\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$

$\circ p^\alpha p^\beta + n^\alpha n^\mu R_{\alpha\mu} + n^\beta n^\nu R_{\beta\nu}$

$2n^\alpha n^\beta R_{\alpha\beta} = 2n^\alpha n^\beta G_{\alpha\beta}$
 $= 16\pi G n^\alpha n^\alpha T_{\alpha\beta} \rightarrow 16\pi G \rho$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu$
 $= R_{\alpha\beta\nu\mu} \omega^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] \omega_\nu$

$[D_\alpha, D_\beta] \omega_\nu$
 $= {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$p^\alpha \gamma^\mu R_{\mu\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$1 = R + 2D_\mu \eta^\mu$$

$$\textcircled{*} p^\alpha p^\beta R_{\alpha\beta\mu\nu} = R + n^\alpha n^\mu R_{\alpha\mu} + n^\beta n^\nu R_{\beta\nu}$$

$$= R + 2n^\alpha n^\beta R_{\alpha\beta} = 2n^\alpha n^\beta G_{\alpha\beta}$$

$$= 16\pi G n^\alpha n^\alpha T_{\alpha\beta}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu = R_{\alpha\beta\nu\mu} w^\mu$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] w_\nu$$

$$[D_\alpha, D_\beta] w_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} w^\mu$$

$$\rightarrow 16\pi G \rho$$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$p^\alpha \eta^\nu R_{\nu\alpha} = D^\alpha K - D_\beta K^{\alpha\beta}$

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

$p^\mu \eta^\nu G_{\mu\nu} = p^\mu \eta^\nu R_{\mu\nu}$

$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu$
 $= R_{\alpha\beta\nu\mu} w^\mu$

$\downarrow$
 $[\nabla_\alpha, \nabla_\beta] w_\nu$

$[D_\alpha, D_\beta] w_\nu$
 $= {}^{(3)} R_{\alpha\beta\nu\mu} w^\mu$

$\rightarrow 16\pi G\rho$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$p^\alpha \eta^\nu R_{\alpha\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$p^\alpha \eta^\nu G_{\alpha\nu} = p^\alpha \eta^\nu R_{\alpha\nu}$$

$$\rightarrow 8\pi G, p^\alpha \eta^\nu T_{\alpha\nu}$$

$$p^\alpha p^\nu R_{\alpha\mu\nu} = R + \eta^\alpha \eta^\mu R_{\alpha\mu} + \dots$$

$$= R + 2\eta^\mu \eta^\mu R_{\mu\mu} = 2\eta^\mu \eta^\mu G_{\mu\mu}$$

$$= 16\pi G \eta^\mu \eta^\mu T_{\mu\mu}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)} R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\rightarrow 16\pi G \rho$$

HW $p^\mu p^\nu (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$p^\alpha \eta^\nu R_{\nu\alpha} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$p^\alpha \eta^\nu G_{\mu\nu} = p^\alpha \eta^\nu R_{\mu\nu}$$

$$\rightarrow 8\pi G p^\alpha \eta^\nu T_{\mu\nu} = -8\pi G J^\alpha$$

$$D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha$$

$$\textcircled{*} p^\alpha p^\beta R_{\alpha\beta\mu\nu} = R + \eta^\alpha \eta^\beta R_{\alpha\beta} + \dots$$

$$= R + 2\eta^\alpha \eta^\beta R_{\alpha\beta} = 2\eta^\alpha \eta^\beta G_{\alpha\beta}$$

$$= 16\pi G \eta^\alpha \eta^\beta T_{\alpha\beta}$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow \quad \quad \quad //$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu} \omega^\nu$$

HW $p^\mu_\alpha p^\nu_\beta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$p^\alpha_\gamma n^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$p^\alpha_\mu n^\nu G_{\mu\nu} = p^\alpha_\mu n^\nu R_{\mu\nu}$$

Momentum constraint.

$$\begin{aligned} &\rightarrow 8\pi G p^\alpha_\mu n^\nu T_{\mu\nu} \\ &= -8\pi G J^\alpha \end{aligned}$$

$$D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha$$

$$p^\alpha_\mu p^\nu_\beta R_{\alpha\mu\nu} = \dots + n^\alpha n^\mu R_{\alpha\mu} + \dots$$

$$\begin{aligned} &= R + 2 n^\mu n^\nu R_{\mu\nu} = 2 n^\mu n^\nu G_{\mu\nu} \\ &= 16\pi G n^\mu n^\nu T_{\mu\nu} \end{aligned}$$

$$\rightarrow 16\pi G \rho$$

$$\begin{aligned} (\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu \\ = R_{\alpha\beta\nu\mu} w^\mu \end{aligned}$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] w_\nu$$

$$\begin{aligned} [D_\alpha, D_\beta] w_\nu \\ = {}^{(3)}R_{\alpha\beta\nu\mu} w^\mu \end{aligned}$$

HW $p^\mu_\alpha p^\nu_\beta (\delta_{\mu\nu} - \partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$p^\alpha_\gamma n^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$p^\alpha_\mu n^\nu G_{\mu\nu} = p^\alpha_\mu n^\nu R_{\mu\nu}$$

Momentum constraint.

$$\begin{aligned} &\rightarrow 8\pi G, p^\alpha_\mu n^\nu T_{\mu\nu} \\ &= -8\pi G J^\alpha \end{aligned}$$

$$D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha$$

$$D^j K^i_j - D_i K = 8\pi G J_i$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu = R_{\alpha\beta\nu\mu} w^\mu$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] w_\nu$$

$$[D_\alpha, D_\beta] w_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} w^\mu$$

HW $p^\mu_\alpha p^\nu_\beta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$p^\alpha_\gamma \eta^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$(3) R + K^2 - K_{ij} K^{ij} = 16\pi G E$$

$$E := \rho$$

Momentum constraint.

$$D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha$$

$$D^j K^i_j - D_i K = 8\pi G J_i$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) \omega_\nu = R_{\alpha\beta\nu\mu} \omega^\mu$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] \omega_\nu$$

$$[D_\alpha, D_\beta] \omega_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} \omega^\mu$$

HW $p^\mu_\alpha p^\nu_\beta (\partial_\mu \eta_\nu - \partial_\nu \eta_\mu) = 0$

$$P^{\alpha\gamma} \eta^\nu R_{\gamma\nu} = D^\alpha K - D_\beta K^{\alpha\beta}$$

$$(3) R + K^2 - K_{ij} K^{ij} = 16\pi G E$$

$$E := \rho$$

Momentum constraint.

4 constraints

$$D^\beta K_{\alpha\beta} - D_\alpha K = 8\pi G J_\alpha$$

$$D^j K^i_j - D_i K = 8\pi G J_i$$

$$\rightarrow 16\pi G \rho$$

$$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha) w_\nu = R_{\alpha\beta\nu\mu} w^\mu$$

$$\downarrow \quad \parallel$$

$$[\nabla_\alpha, \nabla_\beta] w_\nu$$

$$[D_\alpha, D_\beta] w_\nu = {}^{(3)}R_{\alpha\beta\nu\mu} w^\mu$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(3)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

$$P^\sigma_\nu P^\lambda_\mu P^\gamma_\beta P^\delta_\alpha R_{\delta\gamma\lambda\sigma} = {}^{(2)}R_{\alpha\beta\mu\nu} + K_{\mu\alpha}K_{\beta\nu} - K_{\beta\mu}K_{\alpha\nu}$$

Gauss-Codazzi E.

$$D_\beta K_{\alpha\mu} - D_\alpha K_{\beta\mu} = P^\delta_\alpha P^\gamma_\beta P^\lambda_\mu n^\nu R_{\delta\gamma\lambda\nu}$$

γ_{ab}, K_{ab}

Contract