

Title: Explorations in Theoretical Astrophysics (PHYS 7890) - Lecture 2

Date: Apr 06, 2010 09:30 AM

URL: <http://pirsa.org/10040059>

Abstract: Gauge Invariant Cosmological Perturbation theory from 3+1 formulation of General Relativity. This course will aim to study in detail the 3+1 decomposition in General Relativity and use the formalism to derive Gauge invariant perturbation theory at the linear order. Some applications will be studied.







$$P^\alpha_\beta = \delta^\alpha_\beta + n^\alpha n_\beta$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta$$

↑

P

$$P^\alpha_\beta = \delta^\alpha_\beta + n^\alpha n_\beta$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

$$(b) g_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta$$

$$\uparrow$$

$$P^\mu_\alpha P^\nu_\beta g_{\mu\nu}$$

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$$\uparrow$$

$$P^\mu_\alpha P^\nu_\beta g_{\mu\nu}$$

$$K_{\mu\nu} = -(\underbrace{P^\alpha_\mu \nabla_\alpha}_{\text{circled}} n_\nu$$

$$= -D_\mu n_\nu$$

$$g_{\mu\nu} = 0$$

$$P^\alpha_\beta = \delta^\alpha_\beta + n^\alpha n_\beta$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

$$g_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta$$

$$\uparrow$$

$$P^\mu_\alpha P^\nu_\beta g_{\mu\nu}$$

$$K_{\mu\nu} = - \underbrace{P^\alpha_\mu \nabla_\alpha n_\nu}_{\text{circled}}$$

$$= -D_\mu n_\nu$$

$$g_{\mu\nu ; \mu} = 0$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_\eta \gamma_{\mu\nu} = -\frac{1}{2} \mathcal{L}_\eta g_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_\eta \gamma_{\mu\nu} = -\frac{1}{2} \mathcal{L}_\eta g_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon X^\mu$

$$T_x \rightarrow T_{x'}$$

$$\mathcal{L}_\eta$$

$$K_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} \gamma_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} g_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon X^\mu$

$$T_x \rightarrow T_{x'}$$

$$\lim_{\epsilon \rightarrow 0} T = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon}$$

$$K_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} \gamma_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} g_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon X^\mu$

$$T_x \rightarrow T_{x'}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (T_{x'} - T_x)$$

$$K_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} \gamma_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} g_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon X^\mu$

$$T_x \rightarrow T_{x'}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (T_{x'} - T_x)$$

$$\left(\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (T_{x'} - T_x) \right)_{\mu\nu} = T_{\mu\nu;\alpha} X^\alpha + T_{\alpha\nu} X^\alpha_{;\mu} + T_{\mu\alpha} X^\alpha_{;\nu}$$

$$K_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} \gamma_{\mu\nu} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} g_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon X^\mu$

$$T_x \rightarrow T_{x'}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (T_{x'} - T_x)$$

$$\left(\lim_{\epsilon \rightarrow 0} T \right)_{\mu\nu} = T_{\mu\nu;\alpha} X^\alpha + T_{\alpha\nu} X^\alpha_{;\mu} + T_{\mu\alpha} X^\alpha_{;\nu}$$

$$X^\mu = (t, x^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_\eta \gamma_{\mu\nu} = -\frac{1}{2} \mathcal{L}_\eta g_{\mu\nu}$$

$$\mathcal{L}_\eta \gamma_{\mu\nu}$$

Given $X^\mu \rightarrow X'^\mu = X^\mu + \epsilon \xi^\mu$

$$T_x \rightarrow T_{x'}$$

$$\mathcal{L}_x T = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (T_{x'} - T_x)$$

$$\boxed{(\mathcal{L}_x T)_{\mu\nu} = T_{\mu\nu;\alpha} X^\alpha + T_{\alpha\nu} X^\alpha_{;\mu} + T_{\mu\alpha} X^\alpha_{;\nu}}$$

$$X^\mu = (t, x^i)$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

=

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha$$

$$\begin{aligned} \mathcal{L}_n \gamma_{\mu\nu} &= n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha \\ &= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha \end{aligned}$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (\eta_\mu \eta_\nu)$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (\eta_\mu \eta_\nu) + (g_{\mu\alpha} + \eta_\mu \eta_\alpha) \nabla_\nu n^\alpha + (g_{\nu\alpha} + \eta_\nu \eta_\alpha) \nabla_\mu n^\alpha$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (\eta_\mu \eta_\nu) + (g_{\mu\alpha} + \eta_\mu \eta_\alpha) \nabla_\nu n^\alpha + (g_{\nu\alpha} + \eta_\nu \eta_\alpha) \nabla_\mu n^\alpha$$

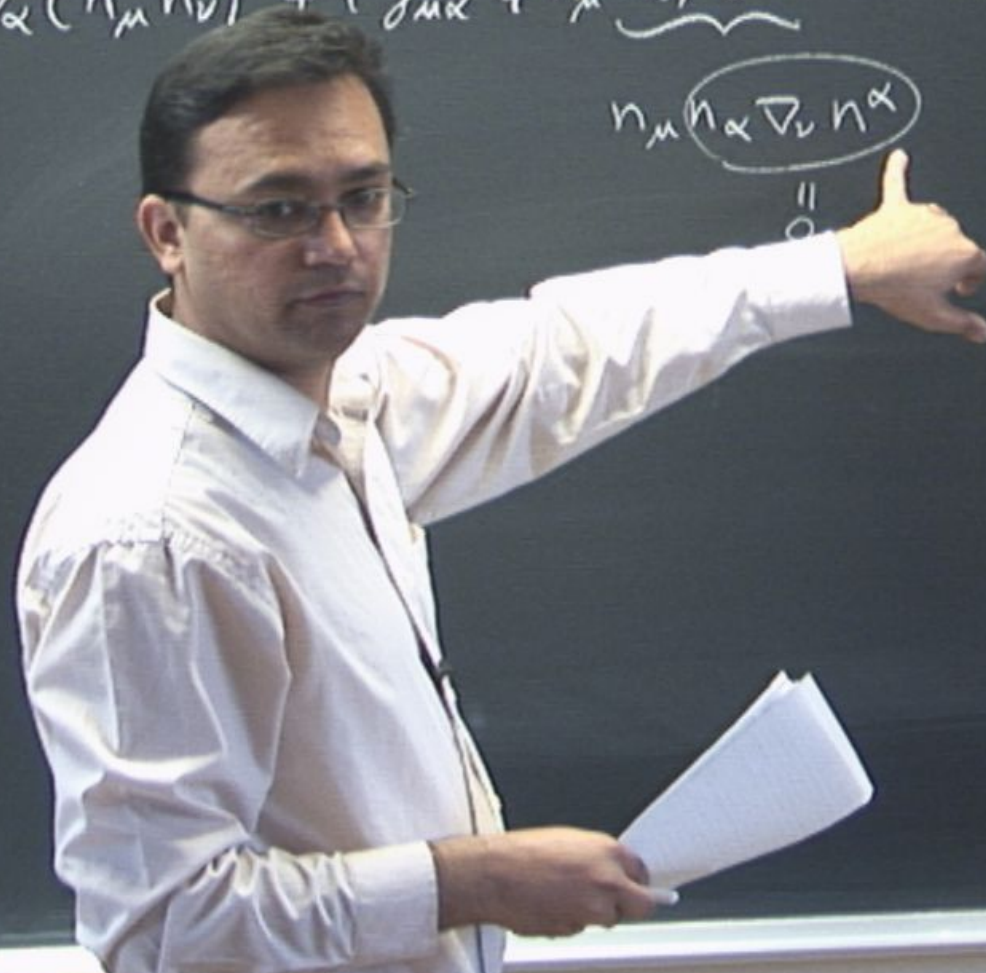


$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + \underbrace{(g_{\mu\alpha} + n_\mu n_\alpha)}_{\gamma_{\mu\alpha}} \nabla_\nu n^\alpha + (g_{\nu\alpha} + n_\nu n_\alpha) \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + \underbrace{n_\mu n_\alpha \nabla_\nu n^\alpha}_{=0} + (g_{\nu\alpha} + n_\nu n_\alpha) \nabla_\mu n^\alpha$$



$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (\eta_\mu \eta_\nu) + \underbrace{(g_{\mu\alpha} + \eta_\mu \eta_\alpha)}_{\substack{n_\mu \eta_\alpha \nabla_\nu n^\alpha \\ = 0}} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + \eta_\nu \eta_\alpha)}_{= 0} \nabla_\mu n^\alpha$$

$$=$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + \underbrace{(g_{\mu\alpha} + n_\mu n_\alpha)}_{n_\mu n_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + n_\nu n_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$= \underbrace{n_\mu n_\alpha \nabla_\nu n^\alpha}_{=0}$$

$$\begin{aligned} n^\mu \nabla_\nu n_\mu &= n_\mu \nabla_\nu n^\mu \\ &= \frac{1}{2} \nabla_\nu (n_\mu n^\mu) \\ &= 0 \end{aligned}$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (\eta_\mu \eta_\nu) + \underbrace{(g_{\mu\alpha} + \eta_\mu \eta_\alpha)}_{\eta_\mu \eta_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + \eta_\nu \eta_\alpha)}_{\eta_\nu \eta_\alpha \nabla_\mu n^\alpha} \nabla_\mu n^\alpha$$

$$\eta_\mu \eta_\alpha \nabla_\nu n^\alpha$$

$$\eta^\mu \nabla_\nu \eta_\mu$$

$$= \eta_\mu \nabla_\nu \eta^\mu$$

$$= \frac{1}{2} \nabla_\nu (\eta_\mu \eta^\mu)$$

$$= 0$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + \underbrace{(g_{\mu\alpha} + n_\mu n_\alpha)}_{n_\mu n_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + n_\nu n_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$n_\mu n_\alpha \nabla_\nu n^\alpha$$

$$n^\mu \nabla_\nu n_\mu$$

$$= n_\mu \nabla_\nu n^\mu$$

$$= \frac{1}{2} \nabla_\nu (n_\mu n^\mu)$$

$$= 0$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + \eta_\mu \eta_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (\eta_\mu \eta_\nu) + \underbrace{(g_{\mu\alpha} + \eta_\mu \eta_\alpha)}_{\eta_\mu \eta_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + \eta_\nu \eta_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$\eta_\mu \eta_\alpha \nabla_\nu n^\alpha = 0$$

$$= n^\alpha \eta_\mu \nabla_\alpha \eta_\nu + n^\alpha \eta_\nu \nabla_\alpha \eta_\mu + \nabla_\nu \eta_\mu + \nabla_\mu \eta_\nu$$

$$\eta^\mu \nabla_\nu \eta_\mu$$

$$= \eta_\mu \nabla_\nu \eta^\mu$$

$$= \frac{1}{2} \nabla_\nu (\eta_\mu \eta^\mu)$$

$$= 0$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + h_\mu h_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (h_\mu h_\nu) + \underbrace{(g_{\mu\alpha} + h_\mu h_\alpha)}_{n_\mu h_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + h_\nu h_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$n_\mu h_\alpha \nabla_\nu n^\alpha$$

$$= n^\alpha h_\mu \nabla_\alpha h_\nu + n^\alpha h_\nu \nabla_\alpha h_\mu + \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

$$h^\mu \nabla_\nu h_\mu$$

$$= h_\mu \nabla_\nu h^\mu$$

$$= \frac{1}{2} \nabla_\nu (h_\mu h^\mu)$$

$$= 0$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + \underbrace{(g_{\mu\alpha} + n_\mu n_\alpha)}_{n_\mu n_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + n_\nu n_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$n_\mu n_\alpha \nabla_\nu n^\alpha$$

$$= n^\alpha n_\mu \nabla_\alpha n_\nu + n^\alpha n_\nu \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

==

$$n^\mu \nabla_\nu n_\mu$$

$$= n_\mu \nabla_\nu n^\mu$$

$$= \frac{1}{2} \nabla_\nu (n_\mu n^\mu)$$

$$= 0$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + h_\mu h_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (h_\mu h_\nu) + \underbrace{(g_{\mu\alpha} + h_\mu h_\alpha)}_{n_\mu h_\alpha \nabla_\nu n^\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + h_\nu h_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$n_\mu h_\alpha \nabla_\nu n^\alpha$$

$$= n^\alpha h_\mu \nabla_\alpha h_\nu + n^\alpha h_\nu \nabla_\alpha h_\mu + \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

||

$$h^\mu \nabla_\nu h_\mu$$

$$= h_\mu \nabla_\nu h^\mu$$

$$= \frac{1}{2} \nabla_\nu (h_\mu h^\mu)$$

$$= 0$$

$$\nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$+ (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$\nabla_\alpha (n_\mu n_\nu) + (g_{\mu\alpha} + n_\mu n_\alpha) \nabla_\nu n^\alpha + (g_{\nu\alpha} + n_\nu n_\alpha) \nabla_\mu n^\alpha$$

$$\quad \quad \quad \underbrace{n_\mu n_\alpha \nabla_\nu n^\alpha}_{=0} \quad \quad \quad \underbrace{(g_{\nu\alpha} + n_\nu n_\alpha) \nabla_\mu n^\alpha}_{=0}$$

$$n_\mu \nabla_\alpha n_\nu + n^\alpha n_\nu \nabla_\alpha n_\mu$$

$$\nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$n^\mu \nabla_\nu n_\mu$$

$$= n_\mu \nabla_\nu n^\mu$$

$$= \frac{1}{2} \nabla_\nu (n_\mu n^\mu)$$

$$= 0$$

$$P^\alpha_\beta = \delta^\alpha_\beta + n^\alpha n_\beta$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

(3) $g_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta$

$$\uparrow$$

$$P^\mu_\alpha P^\nu_\beta g_{\mu\nu}$$

$$K_{\mu\nu} = -P^\alpha_\mu \nabla_\alpha n_\nu$$

$$= -\nabla_\mu n_\nu$$

$$g_{\mu\nu}|_{m=0}$$

$$n_\mu n_\nu = \gamma_{\mu\nu} - g_{\mu\nu}$$

$$\gamma = h^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu h^\alpha + \gamma_{\nu\alpha} \nabla_\mu h^\alpha$$

$$= h^\alpha \nabla_\alpha (g_{\mu\nu} + h_\mu h_\nu) + \gamma_{\mu\alpha} \nabla_\nu h^\alpha + \gamma_{\nu\alpha} \nabla_\mu h^\alpha$$

$$= h^\alpha \nabla_\alpha (h_\mu h_\nu) + \underbrace{(g_{\mu\alpha} + h_\mu h_\alpha)}_{\substack{= \\ 0}} \nabla_\nu h^\alpha + \underbrace{(g_{\nu\alpha} + h_\nu h_\alpha)}_{\substack{= \\ 0}} \nabla_\mu h^\alpha$$

$$= h^\alpha h_\mu \nabla_\alpha h_\nu + h^\alpha h_\nu \nabla_\alpha h_\mu + \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

$$= (\gamma^\alpha_\mu - g^\alpha_\mu) \nabla_\alpha h_\nu + (\gamma^\alpha_\nu - g^\alpha_\nu) \nabla_\alpha h_\mu + \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

$$\begin{aligned} h^\mu \nabla_\nu h_\mu &= n_\mu \nabla_\nu h^\mu \\ &= \frac{1}{2} \nabla_\nu (h_\mu h^\mu) \\ &= 0 \end{aligned}$$

$$P^\alpha_\beta = \delta^\alpha_\beta$$

$$(3) g_{\alpha\beta}, \gamma$$

$$(b) g_{\alpha\beta} = g$$

$$\uparrow$$

$$P^\mu_\alpha P^\nu_\beta g_{\mu\nu}$$

$$K_{\mu\nu} = -P^\alpha_\mu$$

$$= -D$$

$$g_{\mu\nu} \text{ in } \mathbb{R}^4$$

$$h_\mu h_\nu = \gamma_{\mu\nu}$$

$\mathcal{L}_n \gamma$

$$\gamma^\alpha_\mu \nabla_\alpha h_\nu - \nabla_\mu h_\nu + \gamma^\alpha_\nu \nabla_\alpha h_\mu - \cancel{\nabla_\nu h_\mu} + \cancel{\nabla_\mu h_\nu} + \cancel{\nabla_\mu h_\nu}$$

$\mathcal{L}_n \gamma$

$$\gamma^\alpha_\mu \nabla_\alpha h_\nu - \nabla_\mu h_\nu + \gamma^\alpha_\nu \nabla_\alpha h_\mu - \nabla_\nu h_\mu + \nabla_\mu h_\nu + \nabla_\nu h_\mu$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + (g_{\mu\alpha} + n_\mu n_\alpha) \nabla_\nu n^\alpha + (g_{\nu\alpha} + n_\nu n_\alpha) \nabla_\mu n^\alpha$$

$$n_\mu n_\alpha \nabla_\nu n^\alpha \quad \parallel \quad 0$$

$$= n^\alpha n_\mu \nabla_\alpha n_\nu + n^\alpha n_\nu \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$= (\gamma^\alpha_\mu - g^\alpha_\mu) \nabla_\alpha n_\nu + (\gamma^\alpha_\nu - g^\alpha_\nu) \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$= \gamma^\alpha_\mu \nabla_\alpha n_\nu + \gamma^\alpha_\nu \nabla_\alpha n_\mu =$$

$$\begin{aligned} n^\mu \nabla_\nu n_\mu &= n_\mu \nabla_\nu n^\mu \\ &= \frac{1}{2} \nabla_\nu (n_\mu n^\mu) \\ &= 0 \end{aligned}$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + h_\mu h_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (h_\mu h_\nu) + \underbrace{(g_{\mu\alpha} + h_\mu h_\alpha)}_{h_\mu h_\alpha} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + h_\nu h_\alpha)}_{h_\nu h_\alpha} \nabla_\mu n^\alpha$$

$$h_\mu h_\alpha \nabla_\nu n^\alpha = 0$$

$$= n^\alpha h_\mu \nabla_\alpha h_\nu + n^\alpha h_\nu \nabla_\alpha h_\mu$$

$$+ \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

$$= (\gamma^\alpha_\mu - g^\alpha_\mu) \nabla_\alpha h_\nu + (\gamma^\alpha_\nu - g^\alpha_\nu) \nabla_\alpha h_\mu$$

$$+ \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

$$= \gamma^\alpha_\mu \nabla_\alpha h_\nu + \gamma^\alpha_\nu \nabla_\alpha h_\mu$$

$$\mathcal{L}_n \gamma_{\mu\nu} = n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + (g_{\mu\alpha} + n_\mu n_\alpha) \nabla_\nu n^\alpha + (g_{\nu\alpha} + n_\nu n_\alpha) \nabla_\mu n^\alpha$$

$$\underbrace{n_\mu n_\alpha \nabla_\nu n^\alpha}_{=0}$$

$$= n^\alpha n_\mu \nabla_\alpha n_\nu + n^\alpha n_\nu \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$= (\gamma^\alpha_\mu - g^\alpha_\mu) \nabla_\alpha n_\nu + (\gamma^\alpha_\nu - g^\alpha_\nu) \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$= \gamma^\alpha_\mu \nabla_\alpha n_\nu + \gamma^\alpha_\nu \nabla_\alpha n_\mu = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$\begin{aligned} n^\mu \nabla_\nu n_\mu &= n_\mu \nabla_\nu n^\mu \\ &= \frac{1}{2} \nabla_\nu (n_\mu n^\mu) \\ &= 0 \end{aligned}$$

$$\mathcal{L}_n \gamma_{\mu\nu}$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha h_\nu - \nabla_\mu h_\nu + \gamma^\alpha_\nu \nabla_\alpha h_\mu - \nabla_\nu h_\mu + \nabla_\mu h_\nu$$

$$= n^\alpha \nabla_\alpha \gamma_{\mu\nu} + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (g_{\mu\nu} + n_\mu n_\nu) + \gamma_{\mu\alpha} \nabla_\nu n^\alpha + \gamma_{\nu\alpha} \nabla_\mu n^\alpha$$

$$= n^\alpha \nabla_\alpha (n_\mu n_\nu) + \underbrace{(g_{\mu\alpha} + n_\mu n_\alpha)}_{n_\mu n_\alpha \nabla_\nu n^\alpha = 0} \nabla_\nu n^\alpha + \underbrace{(g_{\nu\alpha} + n_\nu n_\alpha)}_{=0} \nabla_\mu n^\alpha$$

$$= n^\alpha n_\mu \nabla_\alpha n_\nu + n^\alpha n_\nu \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$= (\gamma^\alpha_\mu - g^\alpha_\mu) \nabla_\alpha n_\nu + (\gamma^\alpha_\nu - g^\alpha_\nu) \nabla_\alpha n_\mu + \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$= \gamma^\alpha_\mu \nabla_\alpha n_\nu + \gamma^\alpha_\nu \nabla_\alpha n_\mu = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$\begin{aligned} n^\mu \nabla_\nu n_\mu &= n_\mu \nabla_\nu n^\mu \\ &= \frac{1}{2} \nabla_\nu (n_\mu n^\mu) \\ &= 0 \end{aligned}$$

$$P^\alpha_\beta = \delta^\alpha_\beta + n^\alpha n_\beta$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

$$b) g_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta$$

$$\uparrow$$

$$P^\mu_\alpha P^\nu_\beta g_{\mu\nu}$$

$$K_{\mu\nu} = -\underbrace{P^\alpha_\mu \nabla_\alpha n_\nu}_{= -\nabla_\mu n_\nu}$$

$$g_{\mu\nu} ; \mu=0$$

$$K_{\mu\nu} = -P^\alpha_\mu \nabla_\alpha n_\nu$$

$$= -(\nabla_\mu n_\nu)$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha h_\nu - \nabla_\mu h_\nu + \gamma^\alpha_\nu \nabla_\alpha h_\mu - \nabla_\nu h_\mu + \nabla_\mu h_\nu + \nabla_\nu h_\mu$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha h_\nu - \cancel{\nabla_\mu h_\nu} + \gamma^\alpha_\nu \nabla_\alpha h_\mu - \cancel{\nabla_\nu h_\mu} + \cancel{\nabla_\mu h_\mu} + \cancel{\nabla_\nu h_\nu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

(Ex) $t^i = 0, t^0 = 1$

$$\gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha n_\nu - \nabla_\mu n_\nu + \gamma^\alpha_\nu \nabla_\alpha n_\mu - \nabla_\nu n_\mu + \nabla_\mu n_\nu + \nabla_\nu n_\mu$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

(Ex) $t^i = 0, t^0 = 1$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha h_\nu - \cancel{\nabla_\mu h_\nu} + \gamma^\alpha_\nu \nabla_\alpha h_\mu - \cancel{\nabla_\nu h_\mu} + \cancel{\nabla_\mu h_\nu} + \cancel{\nabla_\nu h_\mu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

(Ex) $t^i = 0, t^0 = 1$

$$\mathcal{L}_{\phi n} \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha n_\nu - \cancel{\nabla_\mu n_\nu} + \gamma^\alpha_\nu \nabla_\alpha n_\mu - \cancel{\nabla_\nu n_\mu} + \cancel{\nabla_\mu n_\nu} + \cancel{\nabla_\nu n_\mu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

↓

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

Ex

$$t^i = 0, t^0 = 1$$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha n_\nu - \cancel{\nabla_\mu n_\nu} + \gamma^\alpha_\nu \nabla_\alpha n_\mu - \cancel{\nabla_\nu n_\mu} + \cancel{\nabla_\mu n_\nu} + \cancel{\nabla_\nu n_\mu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

(Ex) $t^i = 0, t^0 = 1$

$$\downarrow$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha n_\nu - \cancel{\nabla_\mu n_\nu} + \gamma^\alpha_\nu \nabla_\alpha n_\mu - \cancel{\nabla_\nu n_\mu} + \cancel{\nabla_\mu n_\nu} + \cancel{\nabla_\nu n_\mu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$t^i = 0, t^0 = 1$$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\downarrow$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha n_\nu - \nabla_\mu n_\nu + \gamma^\alpha_\nu \nabla_\alpha n_\mu - \nabla_\nu n_\mu + \nabla_\mu n_\nu$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

(Ex) $t^i = 0, t^0 = 1$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu}$$

$$\gamma_{\mu\nu} - \mathcal{L}_{N^i} \gamma_{\mu\nu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$t^i = 0, t^0 = 1$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

↓

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2N K_{\mu\nu}$$

$$\gamma^\alpha_\mu \nabla_\alpha n_\nu - \nabla_\mu n_\nu + \gamma^\alpha_\nu \nabla_\alpha n_\mu - \nabla_\nu n_\mu + \nabla_\mu n_\nu + \nabla_\nu n_\mu$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$t^i = 0, t^0 = 1$$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_{\phi n} \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

K_{ij}

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

↓

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2N K_{\mu\nu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

(Ex) $t^i = 0, t^0 = 1$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_{\phi n} \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$K_{ij} = -D_j n_i$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$



$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

(Ex) $t^i = 0, t^0 = 1$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\downarrow$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\gamma_{\mu\nu}; \mathcal{L}_\phi \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$\boxed{\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2NK_{\mu\nu}}$$

$$D_i n_j = -D_j n_i$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

(Ex) $t^i = 0, t^0 = 1$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

↓

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$-D_i n_i =$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

(Ex) $t^i = 0, t^0 = 1$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_{\phi n} \gamma_{\mu\nu} = \phi$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} =$$

$$K_{ij} = -D_j n_i =$$

$$= N \left[\frac{1}{2} \left(-\frac{1}{N} \right) \right]$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$



$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2N K_{\mu\nu}$$

$$t^\mu = N n^\mu + N^\mu$$

$$n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N} \right); N^\mu = (0, N^i)$$

$$t^i = 0, t^0 = 1$$

$$\mathcal{L}_n \gamma_{\mu\nu}; \mathcal{L}_{\phi n} \gamma_{\mu\nu} = \phi \mathcal{L}_n \gamma_{\mu\nu}$$

$$\mathcal{L}_{Nn} \gamma_{\mu\nu} = N \mathcal{L}_n \gamma_{\mu\nu}$$

$$K_{ij} = -D_j n_i = -\Gamma^0_{ij} n_0$$

$$= \left[\frac{1}{2} \left(-\frac{1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$



$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{n_i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{n_i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$K_{ij} = -\frac{1}{2N} (N_{ij,i} + N_{ij,j})$$

$$K_{mv} = -\frac{1}{2} f_n \gamma_{mv}$$

$$\downarrow$$

$$K_{mv} = -\frac{1}{2N} f_{Nn} \gamma_{mv}$$

$$= -\frac{1}{2N} (f_t - f_{n_i})$$

$$f_t \gamma_{mv} = f_{n_i} \gamma_{mv} - 2N K_{mv}$$

$$-D_j n_i = -\Gamma^0_{ij} n_0$$

$$N \left[\frac{1}{2} \left(-\frac{1}{Nz} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{Nz} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{ij} = -\frac{1}{2N} (N_{i,j} + N_{j,i} + h_{ij,0})$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

↓

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$= -D_j n_i = -\Gamma^0_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(-\frac{1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{ij} = -\frac{1}{2N} (N_{i,j} + N_{j,i} + h_{ij,0})$$

$$N^\alpha = h^{\alpha\beta}$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\downarrow$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2N K_{\mu\nu}$$

$$D_j n_i = -\Gamma^0_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(-\frac{1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{ij} = -\frac{1}{2N} (N_{i|j} + N_{j|i} + h_{ij,0})$$

$$N^\beta = h^{\alpha\beta} N_\alpha$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$



$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$K_{ij} = -D_j n_i = -\Gamma^0_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(-\frac{1}{N^z} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^z} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{ij} = -\frac{1}{2N} (N_{i,j} + N_{j,i} + h_{ij,0})$$

$$N^\beta = h^{\alpha\beta} N_\alpha$$

$$N^\beta = h^{\alpha\gamma} N^\gamma_{|\gamma}$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$



$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$\frac{1}{2} \left(-\frac{1}{N^e} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^e} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k})$$

$$K_{ij} = -\frac{1}{2N} (N_{i|j} + N_{j|i} + h_{ij,0})$$

$$N^\beta = h^{\alpha\beta} N_\alpha$$

$$N_{\alpha|\beta} = h_{\alpha\gamma} N^\gamma{}_{|\beta} + h_{\alpha\gamma|\beta} N^\gamma$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\downarrow$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{Nn} \gamma_{\mu\nu}$$

$$= -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N^i} \gamma_{\mu\nu} - 2N K_{\mu\nu}$$

$$K_{ij} = -D_j n_i = -\Gamma^0_{ij} n_0$$

$$N \left[\frac{1}{2} \left(-\frac{1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{ij} = -\frac{1}{2N} (N_{i|j} + N_{j|i} + h_{ij,0})$$

$$N^\beta = h^{\alpha\beta} N_\alpha$$

$$N_{\alpha|\beta} = h_{\alpha\gamma} N^\gamma{}_{|\beta} + \underbrace{h_{\alpha\gamma|\beta} N^\gamma}_0$$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2NK_{ij} = -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N_i})$$

$$K_{ij} = -D_j n_i = -\Gamma^0{}_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(-\frac{1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu}$$

$$\downarrow$$

$$K_{\mu\nu} = -\frac{1}{2N} \mathcal{L}_{N_n} \gamma_{\mu\nu}$$

$$\mathcal{L}_t \gamma_{\mu\nu} = \mathcal{L}_{N_i} \gamma_{\mu\nu} - 2NK_{\mu\nu}$$

$$K_{ij} = -\frac{1}{2N} (N_{i|j} + N_{j|i} + h_{ij,0})$$

$$N^\beta = h^{\alpha\beta} N_\alpha$$

$$N_{\alpha|\beta} = h_{\alpha\gamma} N^\gamma{}_{|\beta} + \underbrace{h_{\alpha\gamma|\beta} N^\gamma}_0$$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij} = -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N_i})$$

$$K_{ij} = -D_j n_i = -\Gamma^0_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(\frac{-1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(\frac{-N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{mv} = -\frac{1}{2} \mathcal{L}_n \gamma_{mv}$$

$$\downarrow$$

$$K_{mv} = -\frac{1}{2N} \mathcal{L}_{N_n} \gamma_{mv}$$

$$\mathcal{L}_t \gamma_{mv} = \mathcal{L}_{N_i} \gamma_{mv} - 2N K_{mv}$$

$$K_{ij} = -\frac{1}{2N} (N_{i|j} + N_{j|i} + h_{ij,0})$$

$$N^\beta = h^{\alpha\beta} N_\alpha$$

$$N_{\alpha|\beta} = h_{\alpha\gamma} N^\gamma{}_{|\beta} + \underbrace{h_{\alpha\gamma|\beta} N^\gamma}_0$$

$$h_{ij,0} + N^k h_{ij,k} + N^k{}_{,i} h_{kj} + N^k{}_{,j} h_{ik} = -2N K_{ij} = -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i})$$

$$K_{ij} = -D_j n_i = -\Gamma^0_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(-\frac{1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(-\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$K_{mv} = -\frac{1}{2} \mathcal{L}_n \gamma_{mv}$$

$$\downarrow$$

$$K_{mv} = -\frac{1}{2N} \mathcal{L}_{N^n} \gamma_{mv}$$

$$\mathcal{L}_t \gamma_{mv} = \mathcal{L}_{N^i} \gamma_{mv} - 2N K_{mv}$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\nu}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

$$g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -\nabla_{\mu} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$



Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$= n^{\alpha} n_{\nu} \nabla_{\alpha} n_{\nu} + n^{\alpha} n_{\nu} \nabla_{\alpha} n_{\mu}$$

$$+ \nabla_{\alpha} n_{\mu} + \nabla_{\alpha} n_{\nu}$$

$$= (n^{\alpha} n_{\mu}) \nabla_{\alpha} n_{\nu} + (\gamma^{\alpha}_{\nu} - g^{\alpha}_{\nu}) \nabla_{\alpha} n_{\mu}$$

$$+ \nabla_{\alpha} n_{\mu} + \nabla_{\alpha} n_{\nu}$$

$$= \gamma^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu} + \gamma^{\alpha}_{\nu} \nabla_{\alpha} n_{\mu} = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

(3) $g_{\alpha\beta}$, $\gamma_{\alpha\beta}$, has

(3) $g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -\nabla_{\mu} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$(\gamma^{\alpha}_{\nu} - g^{\alpha}_{\nu}) \nabla_{\alpha} n_{\mu} = 0$$

$$\nabla_{\alpha} n_{\mu} = -K_{\mu\alpha} - K_{\nu\mu} = -2K_{\mu\alpha}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(b) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(b) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -\nabla_{\mu} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$= (\nabla_{\mu} n_{\nu} + \nabla_{\nu} n_{\mu}) + (\gamma^{\alpha}_{\nu} - g^{\alpha}_{\nu}) \nabla_{\alpha} n_{\mu}$$

$$+ \nabla_{\nu} n_{\mu} + \nabla_{\mu} n_{\nu}$$

$$= \gamma^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu} + \gamma^{\alpha}_{\nu} \nabla_{\alpha} n_{\mu} = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -\nabla_{\mu} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega_{\mu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu})$$

$$P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\gamma}^{\sigma} (\nabla_{\delta} \nabla_{\sigma} - \nabla_{\sigma} \nabla_{\delta}) \omega_{\mu}$$

$$+ \nabla_{\mu} n_{\alpha} + \nabla_{\alpha} n_{\mu}$$

$$= (\nabla_{\mu} n_{\alpha} + \nabla_{\alpha} n_{\mu}) + (\delta^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu})$$

$$+ \nabla_{\mu} n_{\alpha} + \nabla_{\alpha} n_{\mu}$$

$$= \delta^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$\delta^{\alpha}_{\beta} P_{\gamma}^{\delta} P_{\delta}^{\sigma} R_{\gamma\sigma\alpha}^{\mu} \omega_{\mu}$$

$$= 0$$

$$-k_{\nu\mu} = -2K_{\mu\nu}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$+ \nabla_{\nu} \omega_{\mu} + \nabla_{\mu} \omega_{\nu}$$

$$= (\nabla_{\alpha} n_{\nu} + \nabla_{\nu} n_{\alpha}) + (\gamma^{\alpha}_{\nu} - g^{\alpha}_{\nu}) \nabla_{\alpha} n_{\mu}$$

$$+ \nabla_{\nu} n_{\mu} + \nabla_{\mu} n_{\nu}$$

$$= \gamma^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu} + \gamma^{\alpha}_{\nu} \nabla_{\alpha} n_{\mu} = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$\gamma^{\sigma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} (\nabla_{\delta} \nabla_{\sigma} - \nabla_{\sigma} \nabla_{\delta}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\gamma} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$n_{\nu} \nabla_{\mu} n_{\nu} + (\gamma^{\alpha}_{\nu} - g^{\alpha}_{\nu}) \nabla_{\alpha} n_{\mu} = 0$$

$$+ \nabla_{\nu} n_{\mu} + \nabla_{\mu} n_{\nu}$$

$$\gamma^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu} + \gamma^{\alpha}_{\nu} \nabla_{\alpha} n_{\mu} = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu}(\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$\gamma^{\alpha}{}_{\beta} P^{\delta}{}_{\gamma} P^{\sigma}{}_{\nu} (\nabla_{\delta} \nabla_{\sigma} - \nabla_{\sigma} \nabla_{\delta}) \omega^{\nu} = P^{\gamma}{}_{\alpha} P^{\delta}{}_{\beta} P^{\sigma}{}_{\gamma} R_{\gamma\sigma\delta}{}^{\mu} \omega_{\mu}$$

$$P^{\mu}{}_{\alpha} P^{\nu}{}_{\beta} \nabla_{\alpha} \omega_{\beta} =$$

$$= \gamma^{\alpha}{}_{\mu} \nabla_{\alpha} n_{\nu} + \gamma^{\alpha}{}_{\nu} \nabla_{\alpha} n_{\mu} = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$P^{\alpha}{}_{\beta} = \delta^{\alpha}{}_{\beta} + n^{\alpha} n_{\beta}$$

$$(b) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(b) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}{}_{\alpha} P^{\nu}{}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -\nabla_{\mu} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}{}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} \nabla_{\alpha} \omega_{\beta} =$$

$$= \gamma^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu} + \gamma^{\alpha}_{\nu} \nabla_{\alpha} n_{\mu} = -K_{\mu\nu} - K_{\nu\mu} = -2K_{\mu\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, h_{\alpha\beta}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$(\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$\nabla_{\gamma} (\nabla_{\delta} \nabla_{\epsilon} - \nabla_{\epsilon} \nabla_{\delta}) \omega_{\sigma} = P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\gamma}^{\sigma} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$\omega_{\beta} = (\delta_{\alpha}^{\mu} + n^{\mu} n_{\alpha}) (\delta_{\beta}^{\nu} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\nu}$$

$$\omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, h_{\alpha\beta}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} \nabla_{\alpha} \omega_{\beta} = (\delta_{\alpha}^{\mu} + n^{\mu} n_{\alpha}) (\delta_{\beta}^{\nu} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow D_{\alpha} P_{\beta} \omega_{\mu} = P_{\alpha}^{\sigma} \nabla_{\sigma} P_{\beta}^{\gamma} P_{\mu}^{\nu} \nabla_{\gamma} \omega_{\nu}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, h_{\alpha\beta}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -D_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} \nabla_{\alpha} \omega_{\beta} = (\delta_{\alpha}^{\mu} + n^{\mu} n_{\alpha}) (\delta_{\beta}^{\nu} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow D_{\alpha} P_{\beta} \omega_{\mu} = P_{\alpha}^{\sigma} \nabla_{\sigma} P_{\beta}^{\gamma} P_{\mu}^{\nu} \nabla_{\gamma} \omega_{\nu}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, h_{\alpha\beta}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -D_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -(D_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

$$D_\alpha D_\beta w_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma w_\nu - K_{\alpha\mu} K_{\beta\nu} w^\nu$$

$$h_{ij,0} + N^k h_{ij,k} - N_{,i} h_{jk} = -2NK_{ij} = -\frac{1}{2N} (\mathcal{L}_t - \mathcal{L}_{N^i}) g_{ij}$$

$$K_{ij} = -D_j n_i$$

$$= N \left[\frac{1}{2} (\mathcal{L}_t g_{ij} - h_{ij,0}) + \frac{1}{2} \left(\frac{N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} R_{\gamma\delta\sigma\mu} \omega_{\mu}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} \nabla_{\alpha} \omega_{\beta} = (\delta^{\mu}_{\alpha} + n^{\mu} n_{\alpha}) (\delta^{\nu}_{\beta} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow D_{\alpha} P_{\beta} \omega_{\mu} = P^{\sigma}_{\alpha} \nabla_{\sigma} P^{\gamma}_{\beta} P^{\nu}_{\mu} \nabla_{\gamma} \omega_{\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -\nabla_{\mu} n_{\nu}$$

$$= -D_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

$$D_\alpha D_\beta w_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma w_\nu - K_{\alpha\mu} K_{\beta\nu} w^\nu$$

$$h_{ij,0} + N^k h_{ij,k} + N^k_{,i} h_{kj} + N^k_{,j} h_{ik} = -2NK_{ij} = -\frac{1}{2N} (\dot{f}_t - \dot{f}_{ni})$$

$$K_{ij} = -D_j n_i = -\Gamma^0_{ij} n_0$$

$$= N \left[\frac{1}{2} \left(\frac{-1}{N^2} \right) (-N_{i,j} - N_{j,i} - h_{ij,0}) + \frac{1}{2} \left(\frac{-N^k}{N^2} \right) (h_{ik,j} + h_{jk,i} - h_{ij,k}) \right]$$

$$\dot{f}_t \gamma_{mv} = \dot{f}_{ni} \gamma_{mv} - 2NK_{ij}$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$(P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu}) (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} R_{\gamma\delta\sigma\mu} \omega_{\mu}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} \nabla_{\alpha} \omega_{\beta} = (\delta^{\mu}_{\alpha} + n^{\mu} n_{\alpha}) (\delta^{\nu}_{\beta} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow \nabla_{\alpha} P_{\beta} \omega_{\mu} = P^{\sigma}_{\alpha} \nabla_{\sigma} P^{\gamma}_{\beta} P^{\nu}_{\mu} \nabla_{\gamma} \omega_{\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$(P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu}) (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} \nabla_{\alpha} \omega_{\beta} = (\delta^{\mu}_{\alpha} + n^{\mu} n_{\alpha}) (\delta^{\nu}_{\beta} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow D_{\alpha} P_{\beta} \omega_{\mu} = P^{\sigma}_{\alpha} \nabla_{\sigma} P^{\gamma}_{\beta} P^{\nu}_{\mu} \nabla_{\gamma} \omega_{\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -D_{\mu} n_{\nu}$$

$$g_{\mu\nu, m} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\gamma^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_\gamma^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\gamma^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_\gamma^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\gamma^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_\gamma^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= (D_\delta D_\sigma - D_\sigma D_\delta) \omega_\gamma + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\gamma^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_\gamma^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= (D_\delta D_\sigma - D_\sigma D_\delta) \omega_\gamma + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\gamma^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_\gamma^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= (D_\delta D_\sigma - D_\sigma D_\delta) \omega_\gamma + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

=

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P_{\alpha}^{\gamma} P_{\beta}^{\delta} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} \nabla_{\alpha} \omega_{\beta} = (\delta_{\alpha}^{\mu} + n^{\mu} n_{\alpha}) (\nabla_{\alpha} \omega_{\beta})$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} - n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow D_{\alpha} P_{\beta} \omega_{\mu} = P_{\alpha}^{\sigma} \nabla_{\sigma} P_{\beta}^{\gamma} \omega_{\gamma}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -D_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P_{\mu}^{\alpha} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$= {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$\boxed{P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha}$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^{\sigma\nu} P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{Y\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma Y} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma Y\alpha} \omega^\alpha + K_{Y\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma Y} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

$$P_\nu^\sigma P_\mu^\chi P_\beta^\gamma P_\alpha^\delta R_{\delta\chi\gamma\nu} = {}^{(3)}R_{\chi\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^{\sigma\nu} P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \left[(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta \right] + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

→ Gauss Eq.

$$P_\nu^\sigma P_\mu^\chi P_\beta^\gamma P_\alpha^\delta R_{\delta\chi\gamma\nu} = {}^{(3)}R_{\gamma\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^{\sigma\nu} P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

→ Gauss Eq.

$$P_\nu^\sigma P_\mu^\chi P_\beta^\gamma P_\alpha^\delta R_{\delta\chi\gamma\nu} = {}^{(3)}R_{\chi\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P_{\alpha}^{\gamma} P_{\beta}^{\delta} P_{\nu}^{\sigma} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} \nabla_{\alpha} \omega_{\beta} = (\delta_{\alpha}^{\mu} + n^{\mu} n_{\alpha}) (\delta_{\beta}^{\nu} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow D_{\alpha} P_{\beta} \omega_{\mu} = P_{\alpha}^{\sigma} \nabla_{\sigma} P_{\beta}^{\gamma} P_{\mu}^{\nu} \nabla_{\gamma} \omega_{\nu}$$

$$P_{\beta}^{\alpha} = \delta_{\beta}^{\alpha} + n^{\alpha} n_{\beta}$$

$$(b) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(b) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P_{\alpha}^{\mu} P_{\beta}^{\nu} g_{\mu\nu}$$

$$K_{\mu\nu} = -$$

$$= -$$

$$g$$

Gauss Equation

$$R_{\alpha\beta\mu}{}^{\nu} \omega_{\nu} = (\nabla_{\alpha} \nabla_{\beta} - \nabla_{\beta} \nabla_{\alpha}) \omega_{\mu}$$

$$\rightarrow \omega^{\nu} n_{\nu} = 0$$

$$\nabla_{\mu} (\omega^{\nu} n_{\nu}) = 0 \rightarrow n_{\nu} \nabla_{\mu} \omega^{\nu} = -\omega^{\nu} \nabla_{\mu} n_{\nu}$$

$$P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} (\nabla_{\gamma} \nabla_{\delta} - \nabla_{\delta} \nabla_{\gamma}) \omega_{\sigma} = P^{\gamma}_{\alpha} P^{\delta}_{\beta} P^{\sigma}_{\nu} R_{\gamma\delta\sigma\mu} \omega^{\mu}$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} \nabla_{\alpha} \omega_{\beta} = (\delta^{\mu}_{\alpha} + n^{\mu} n_{\alpha}) (\delta^{\nu}_{\beta} + n^{\nu} n_{\beta}) \nabla_{\alpha} \omega_{\beta}$$

$$= \nabla_{\mu} \omega_{\nu} + n^{\mu} n_{\alpha} \nabla_{\alpha} \omega_{\nu} + n^{\nu} n_{\beta} \nabla_{\mu} \omega_{\beta} + n^{\mu} n_{\alpha} n^{\nu} n_{\beta} \nabla_{\alpha} \omega_{\beta}$$

$$\Rightarrow \nabla_{\alpha} P_{\beta} \omega_{\mu} = P^{\sigma}_{\alpha} \nabla_{\sigma} P^{\gamma}_{\beta} P^{\nu}_{\mu} \nabla_{\gamma} \omega_{\nu}$$

$$P^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$(3) g_{\alpha\beta}, \gamma_{\alpha\beta}, \text{ has}$$

$$(3) g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}$$

$$\uparrow$$

$$P^{\mu}_{\alpha} P^{\nu}_{\beta} g_{\mu\nu}$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -\nabla_{\mu} n_{\nu}$$

$$g_{\mu\nu} = 0$$

$$K_{\mu\nu} = -P^{\alpha}_{\mu} \nabla_{\alpha} n_{\nu}$$

$$= -(\nabla_{\mu} n_{\nu} + n_{\mu} n^{\alpha} \nabla_{\alpha} n_{\nu})$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \left[(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta \right] + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

→ Gauss Eq.

$$P_\nu^\sigma P_\mu^\chi P_\beta^\gamma P_\alpha^\delta R_{\delta\chi\gamma\nu} = {}^{(3)}R_{\chi\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\gamma^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_\gamma^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_\gamma^\beta R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

→ Gauss Eq.

$$P_\mu^\alpha P_\beta^\gamma P_\delta^\sigma R_{\alpha\beta\gamma\sigma} = {}^{(3)}R_{\gamma\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$

$$D_\alpha D_\beta \omega_\mu = P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu - K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_\alpha^\sigma P_\beta^\gamma P_\mu^\nu \nabla_\sigma \nabla_\gamma \omega_\nu = D_\alpha D_\beta \omega_\mu + K_{\alpha\mu} K_{\beta\nu} \omega^\nu$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = P_Y^\beta P_\sigma^\mu P_\delta^\nu (\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) \omega_\beta$$

$$= \boxed{(D_\delta D_\sigma - D_\sigma D_\delta) \omega_\beta} + K_{\gamma\delta} K_{\sigma\beta} \omega^\beta - K_{\sigma\gamma} K_{\delta\beta} \omega^\beta$$

$$P_Y^\beta P_\sigma^\mu P_\delta^\nu R_{\alpha\beta\mu\nu} \omega^\alpha = {}^{(3)}R_{\delta\sigma\gamma\alpha} \omega^\alpha + K_{\gamma\delta} K_{\sigma\alpha} \omega^\alpha - K_{\sigma\gamma} K_{\delta\alpha} \omega^\alpha$$

$$\omega^\alpha = P_\alpha^\epsilon \omega_\epsilon = \delta_\alpha^\epsilon \omega_\epsilon$$

→ Gauss Eq.

$$P_\nu^\sigma P_\mu^\chi P_\beta^\gamma P_\alpha^\delta R_{\delta\gamma\chi\nu} = {}^{(3)}R_{\chi\beta\mu\nu} + K_{\mu\alpha} K_{\beta\nu} - K_{\beta\mu} K_{\alpha\nu}$$