Title: Quantum Spin Simulations (PHYS 7380) - Lecture 1

Date: Apr 05, 2010 09:30 AM

URL: http://pirsa.org/10040058

Abstract:

perimeter scholars
international

General Relativistic Gauge invariant Cosmological Perturbations

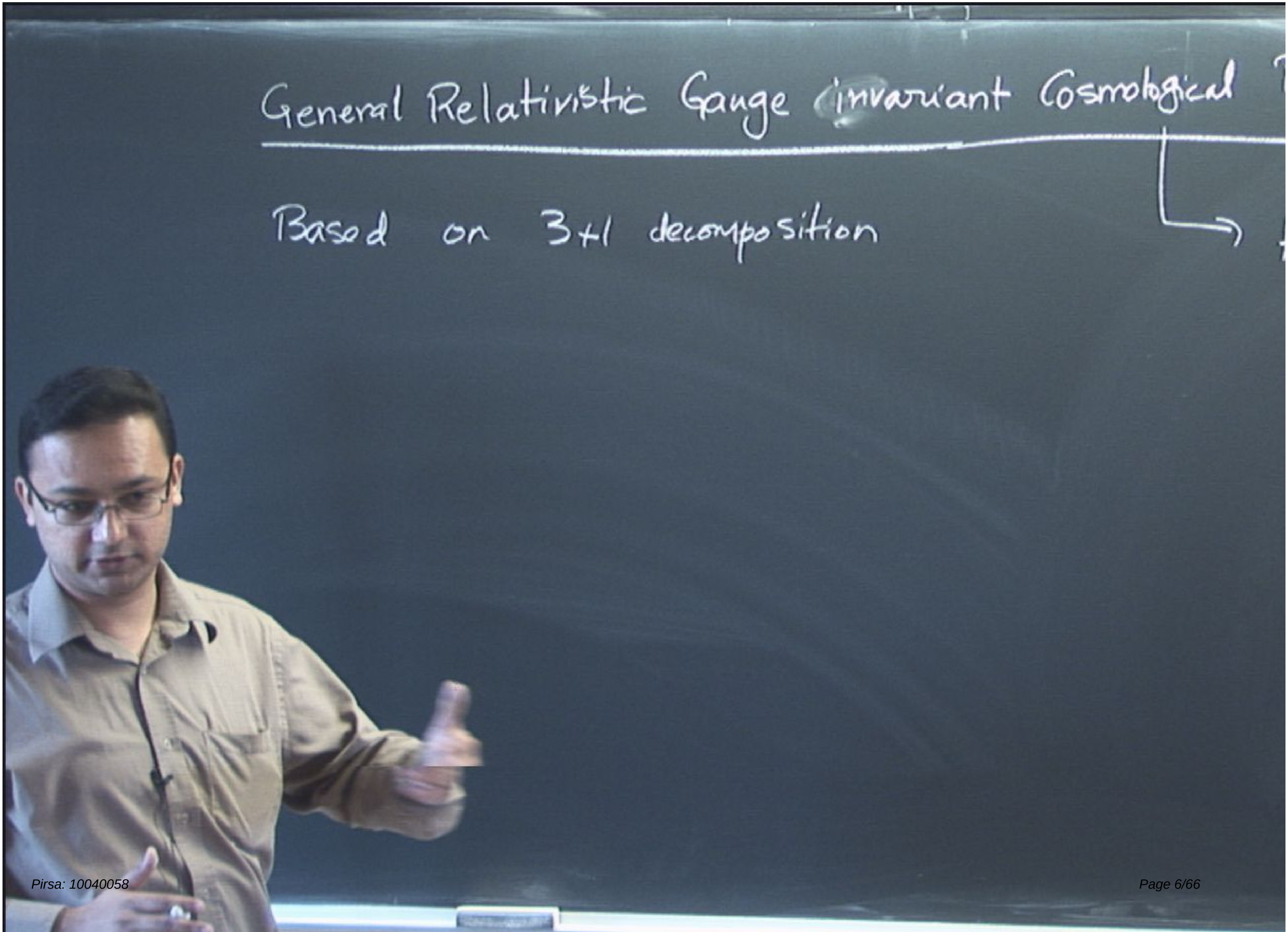
J. Bardeen 1988

On Cosmological Perturbations

elativistic Gauge invariant Cosmological Perturbations

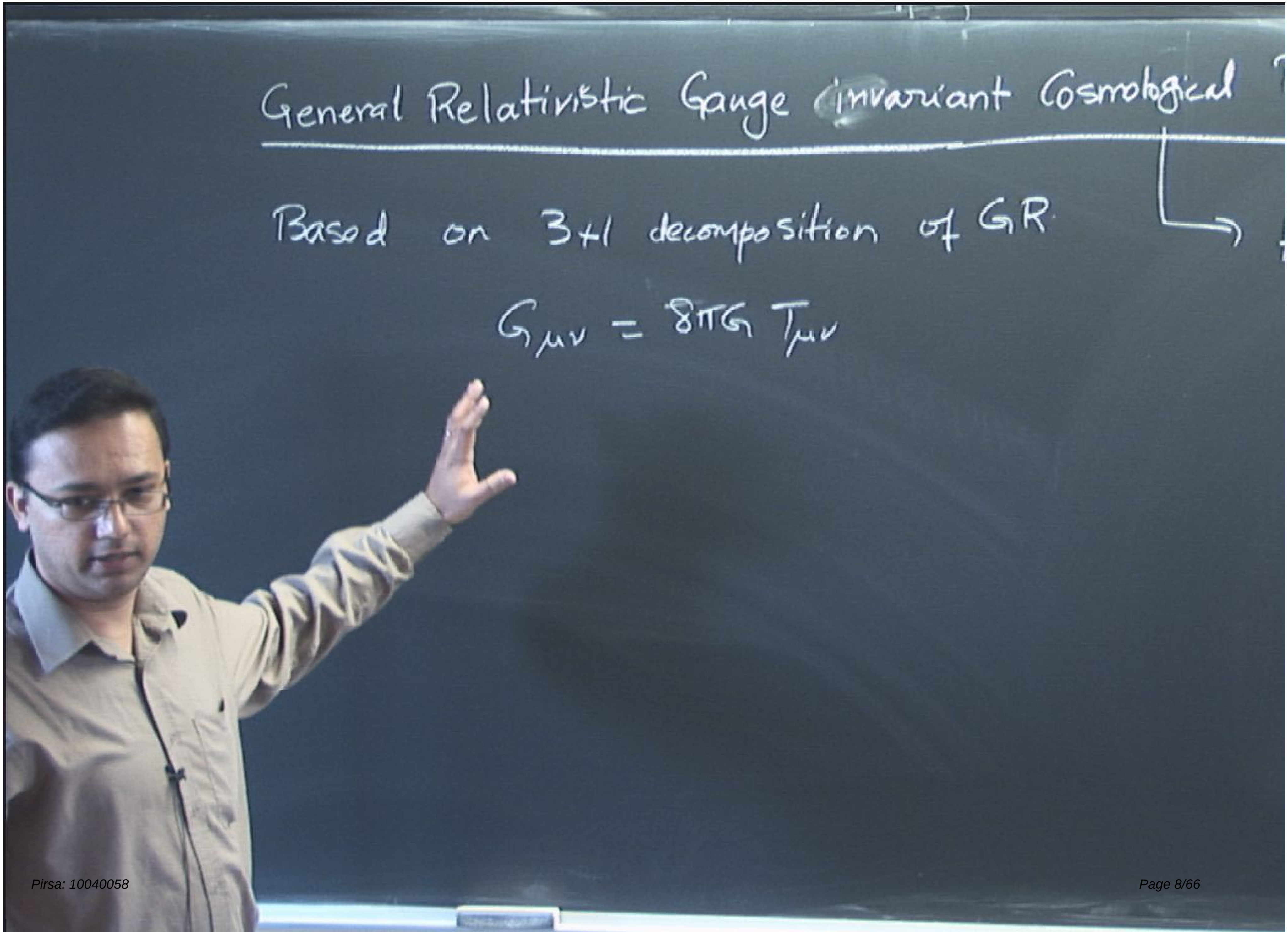
first order

scalar


General Relativistic Gauge invariant Cosmological
Based on 3+1 decomposition

General Relativistic Gauge invariant Cosmological

Based on 3+1 decomposition of GR

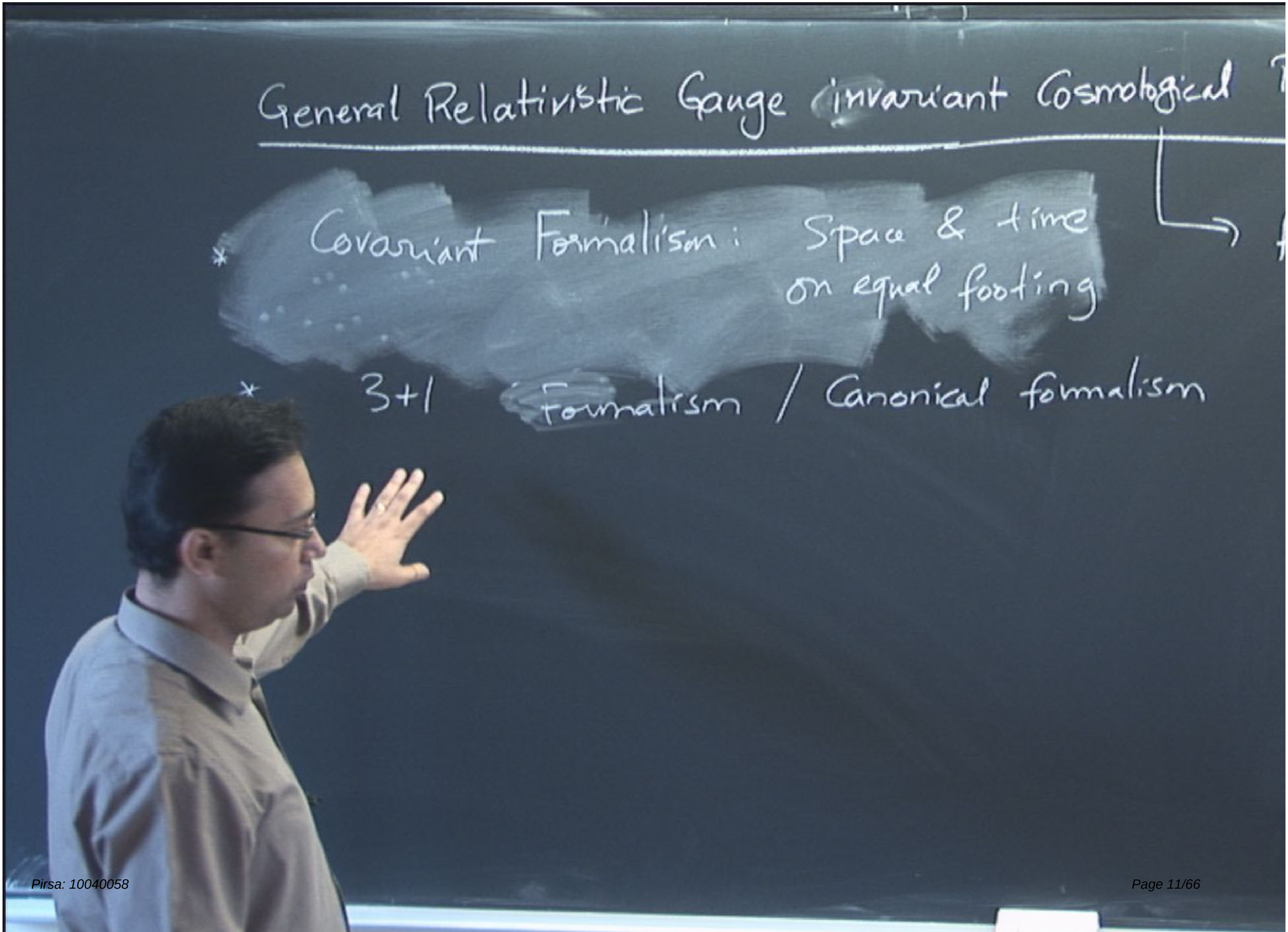

General Relativistic Gauge invariant Cosmological
Based on 3+1 decomposition of GR
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$

General Relativistic Gauge invariant Cosmological P

Covariant Formalism :

General Relativistic Gauge invariant Cosmological P

Covariant Formalism: Space & time on equal footing


General Relativistic Gauge invariant Cosmological
* Covariant Formalism: Space & time on equal footing
* 3+1 Formalism / Canonical formalism

General Relativistic Gauge invariant Cosmological

* Covariant Formalism: Space & time on equal footing

* 3+1 Formalism / Canonical formalism

Very useful if we wish to study evolution of systems

General Relativistic Gauge invariant Cosmological

* Covariant Formalism: Space & time on equal footing

* 3+1 Formalism / Canonical formalism

Very useful if we wish to study evolution of systems

→ Existence of a Cauchy surface.

If such a surface exists then we have globally hyperbolic

urbations

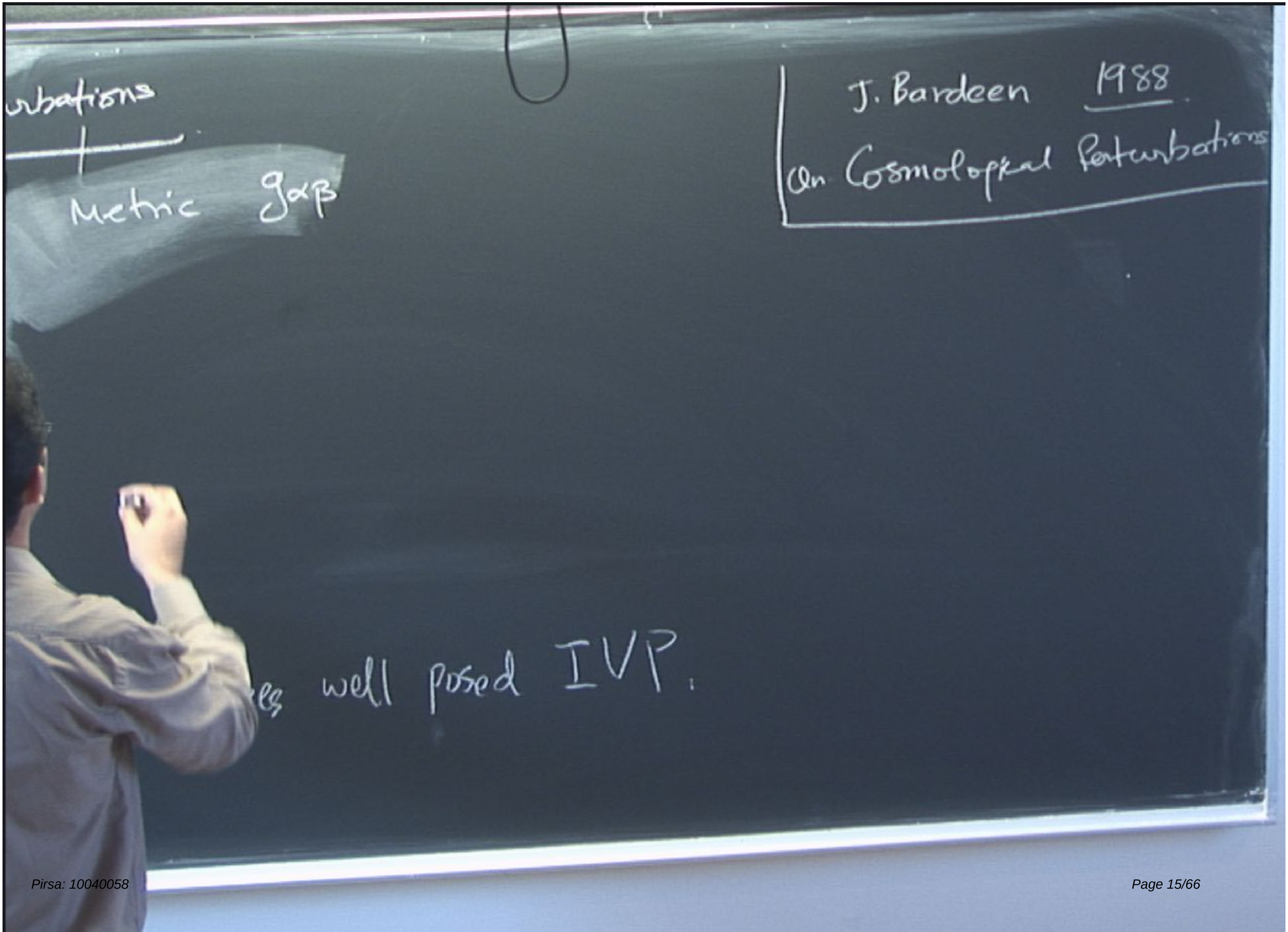
scalar

order

J. Bardeen $\underline{1988}$

On Cosmological Perturbations

$\rightarrow$ guarantees well posed IVP.


urbations
Metric $g_{\alpha\beta}$
J. Bardeen 1988
On Cosmological Perturbations
well posed IVP.

urbations

Metric $g_{\alpha\beta}$

→ spacetime can be foliated by spacelike hypersurfaces

J. Bardeen 1988

On Cosmological Perturbations

→ guarantees well posed IVP.

urbations

Metric $g_{\alpha\beta}$

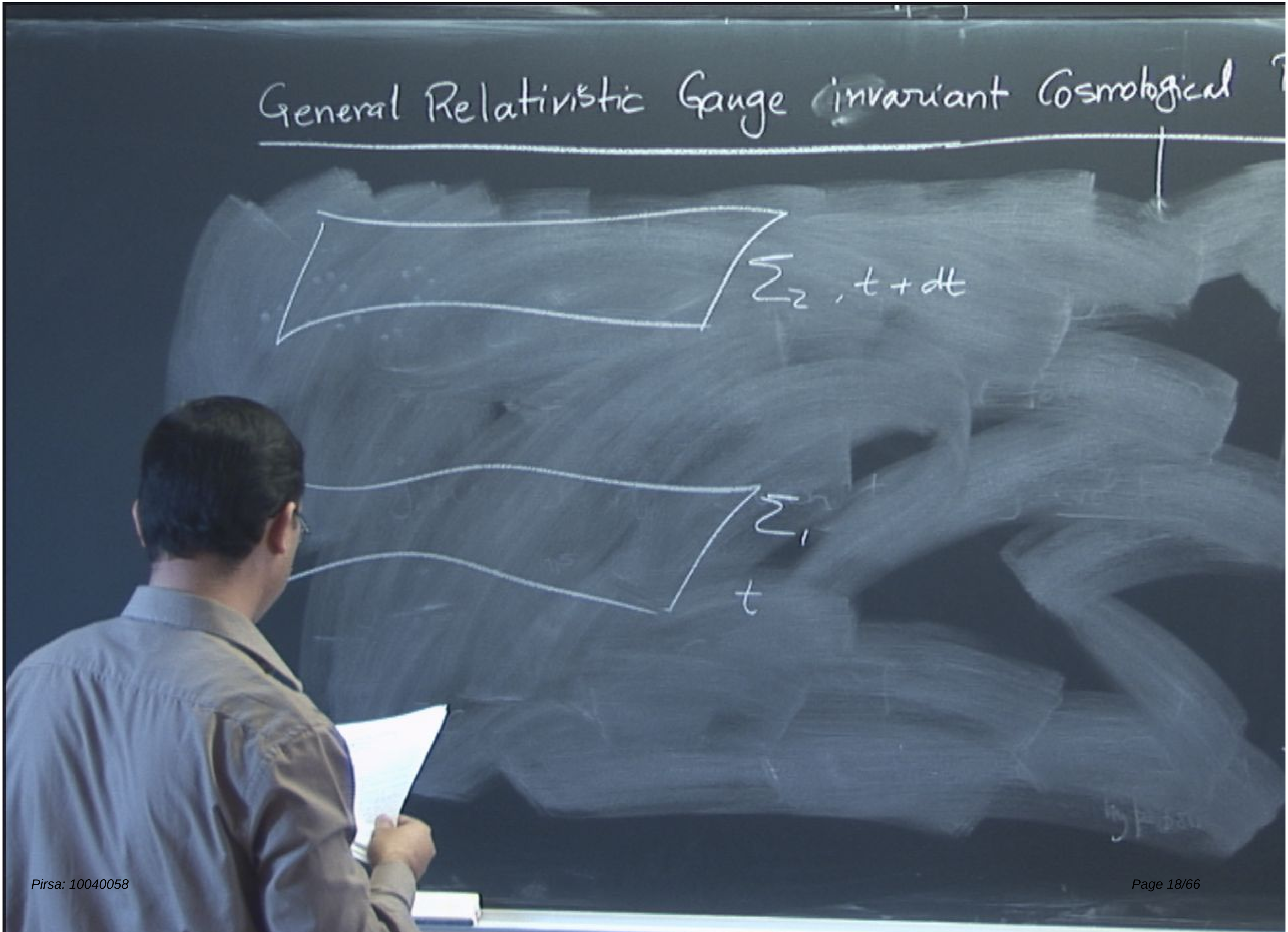
J. Bardeen 1988

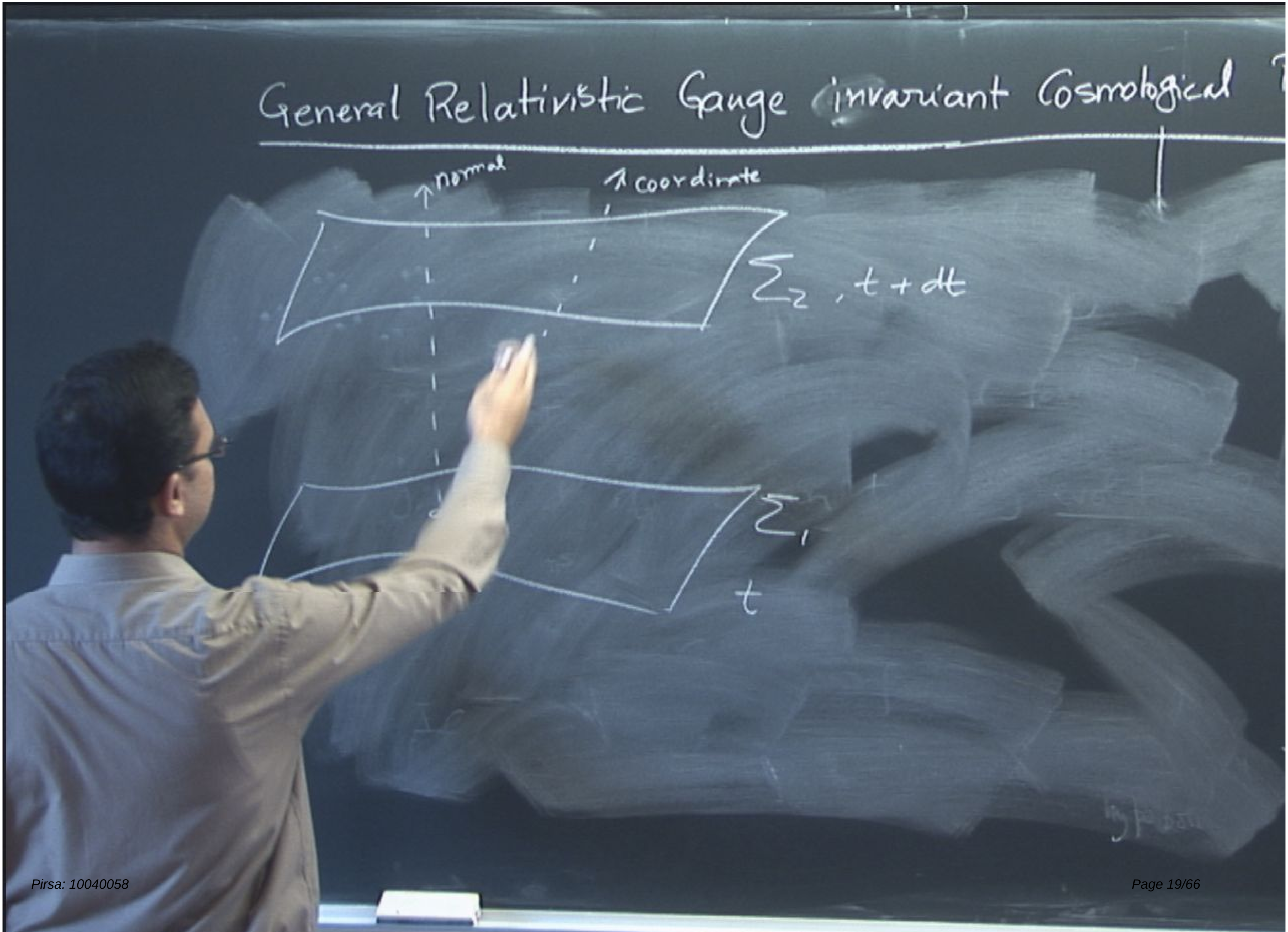
On Cosmological Perturbations

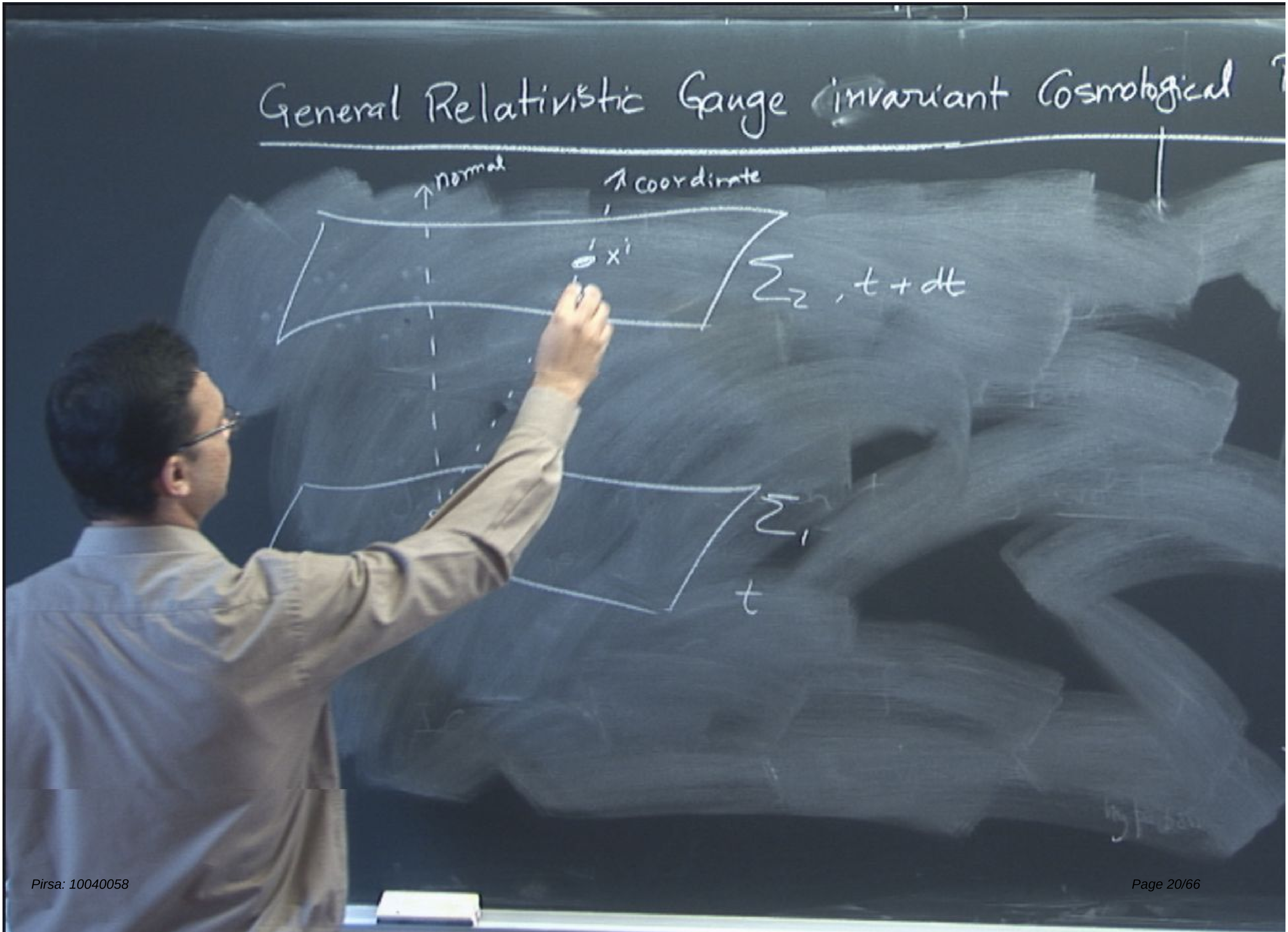
→ spacetime can be foliated by spacelike hypersurfaces

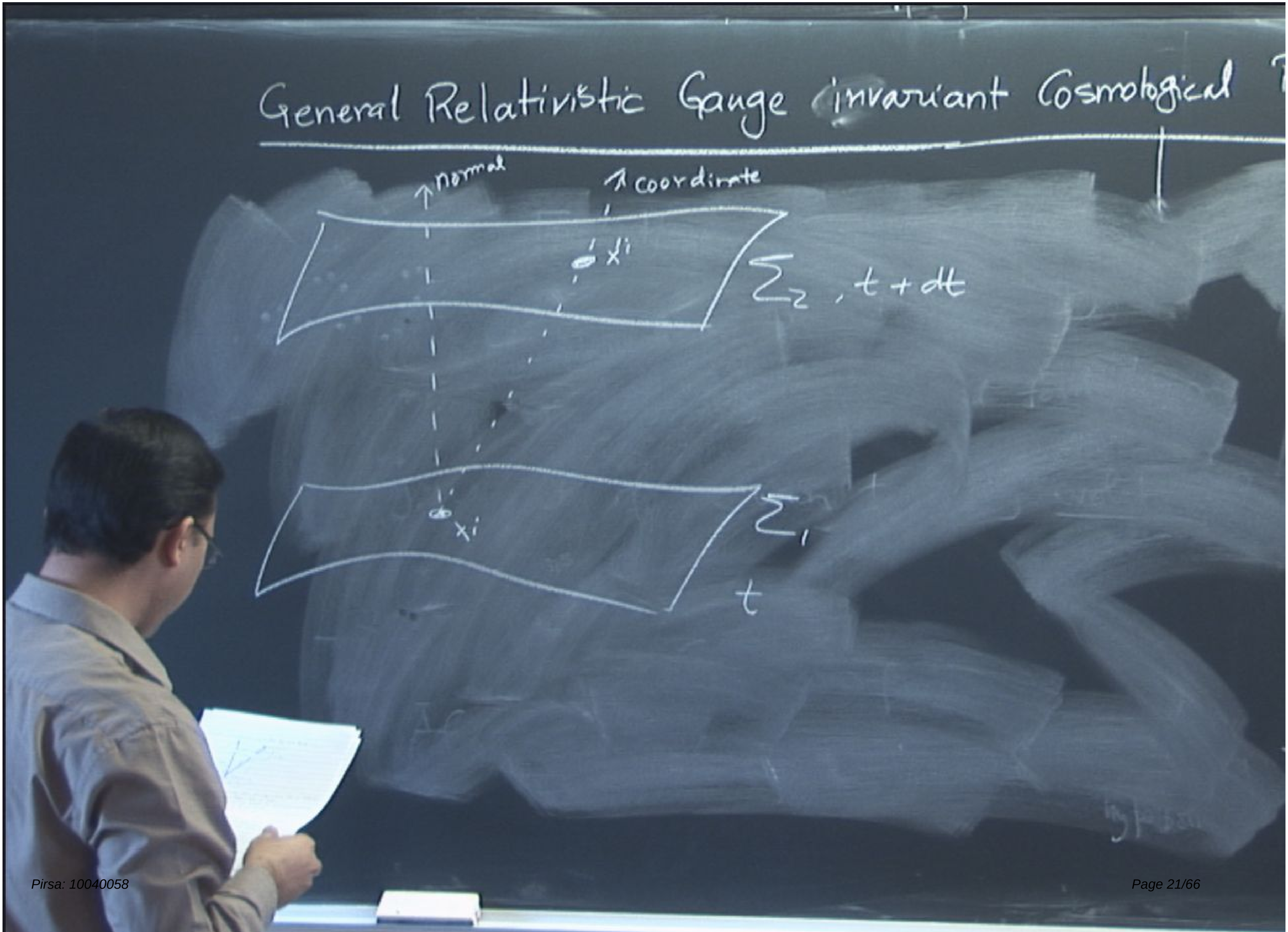
→ identify universal parameter 't' as a measure of time

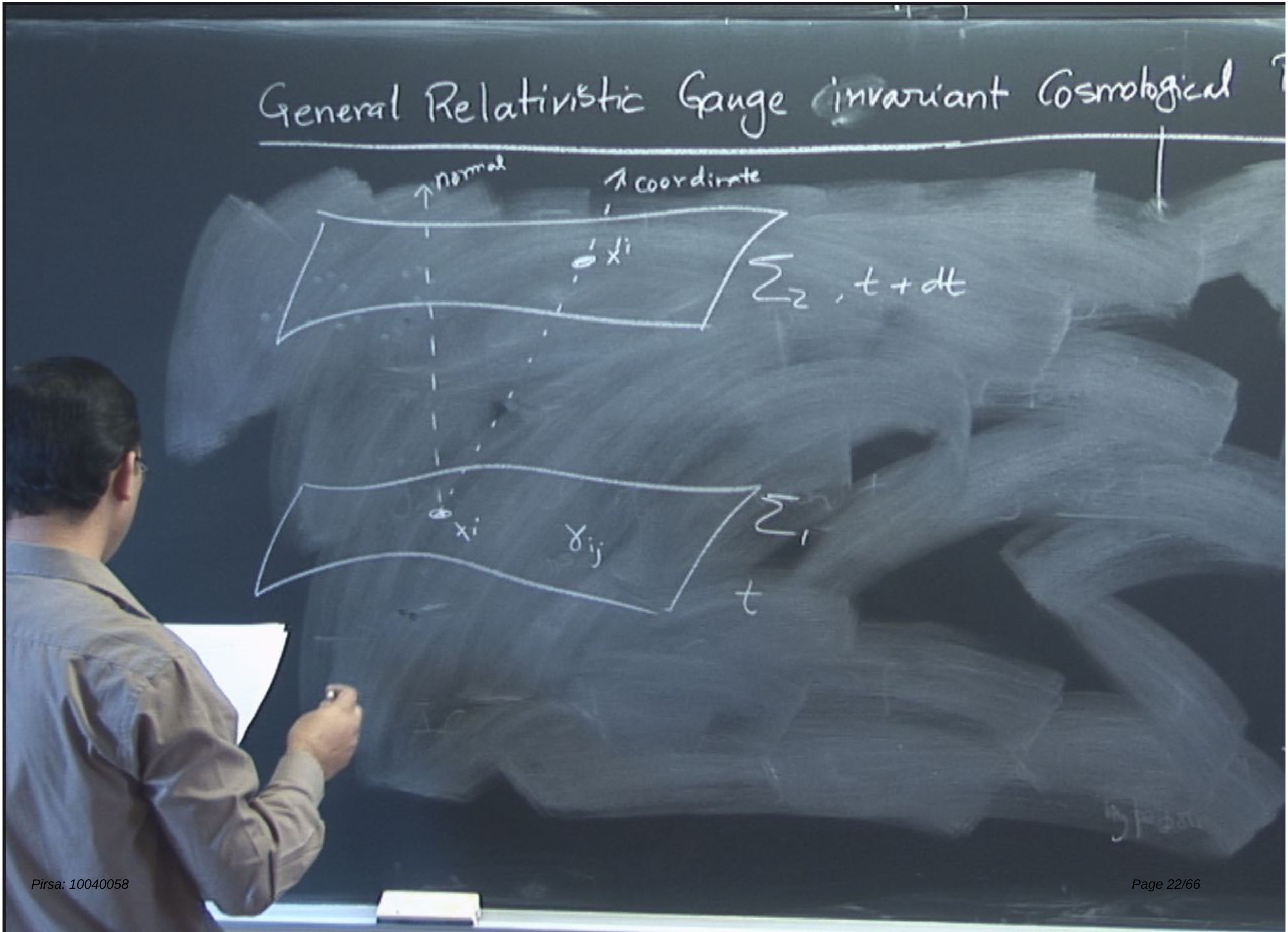
→ guarantees well posed IVP.

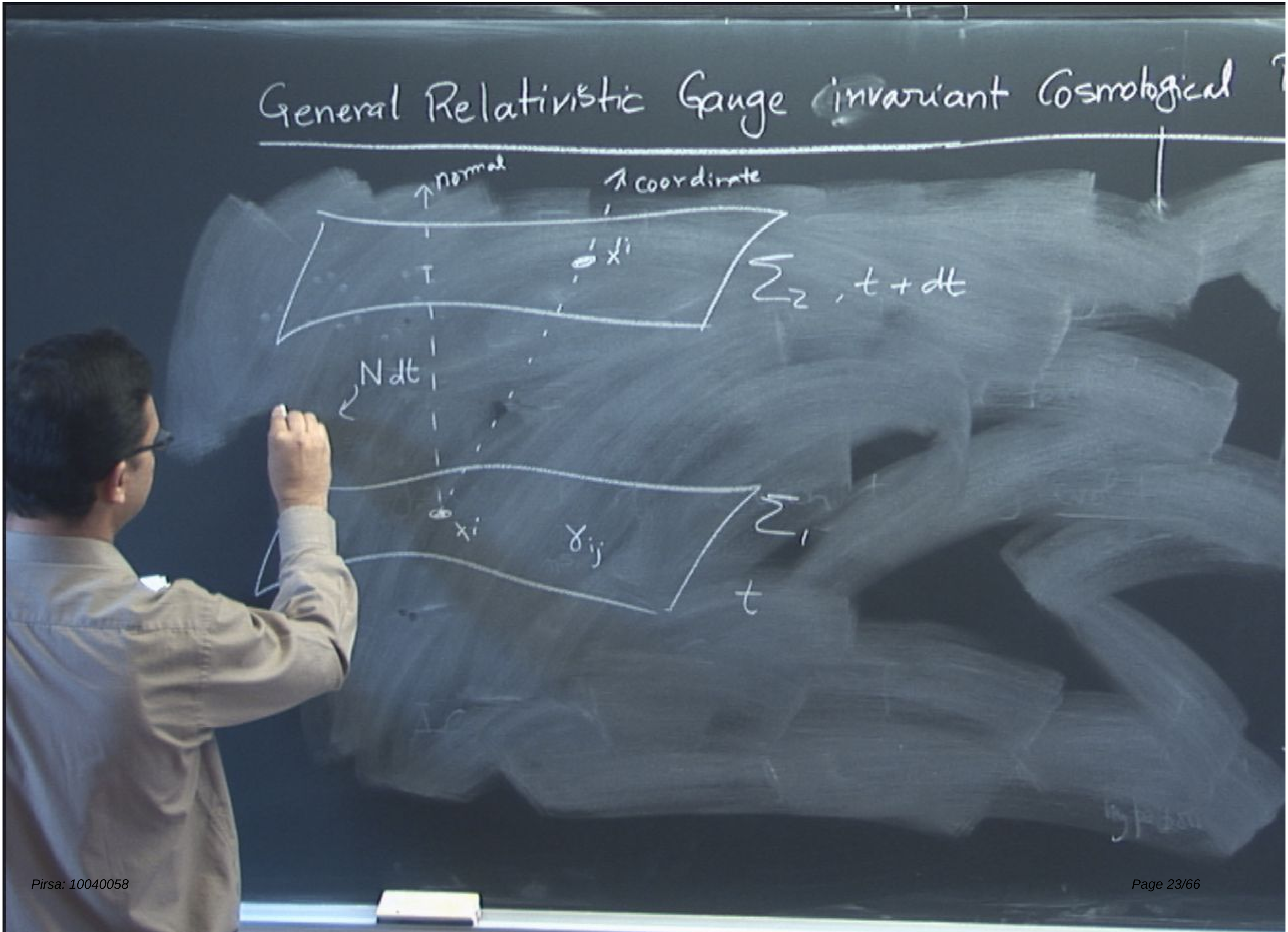

General Relativistic Gauge invariant Cosmological
Σ_2 , $t + dt$
Σ_1
t

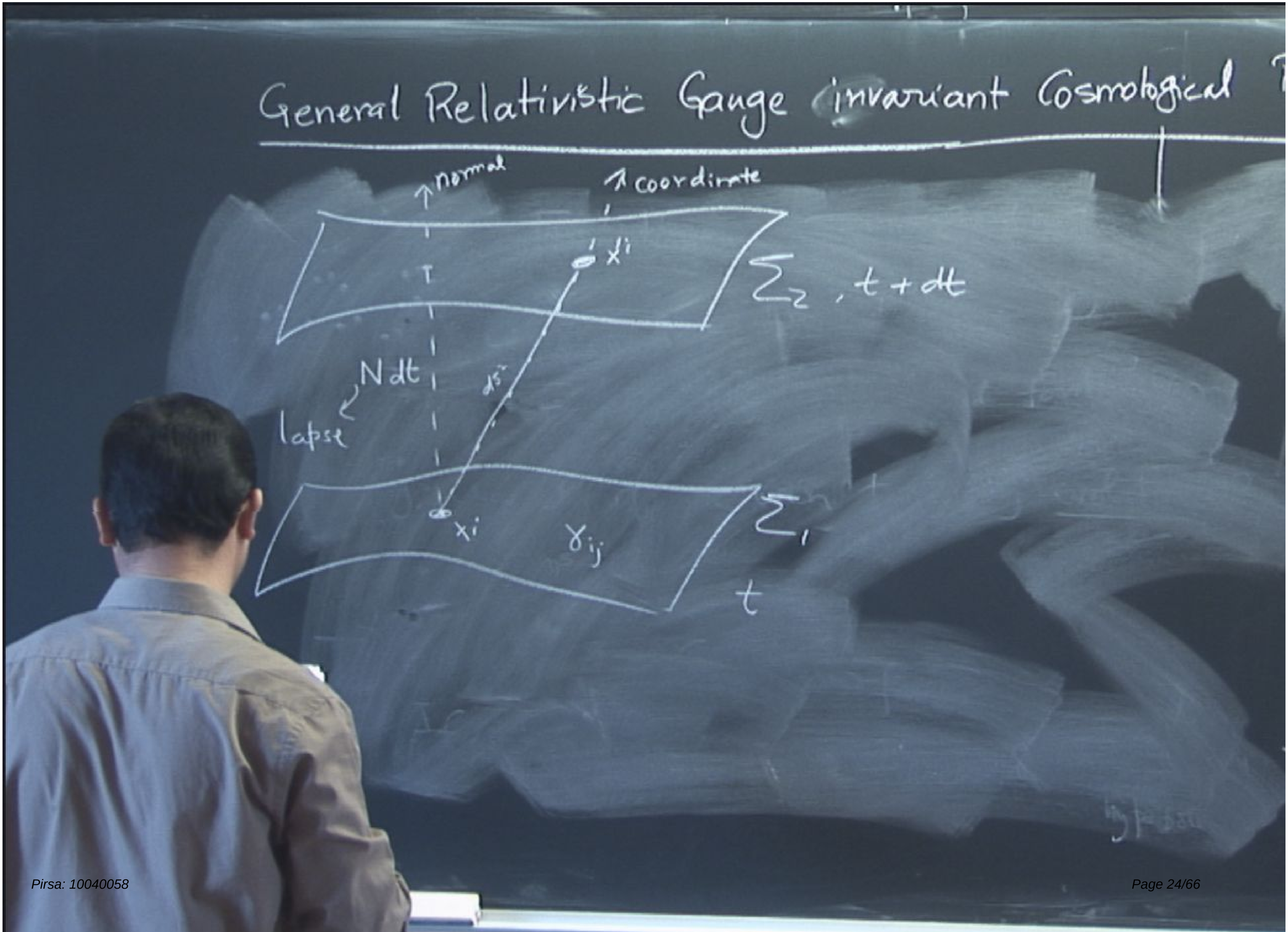

General Relativistic Gauge invariant Cosmological
normal
coordinate
Σ_2 , $t + dt$
Σ_1
t

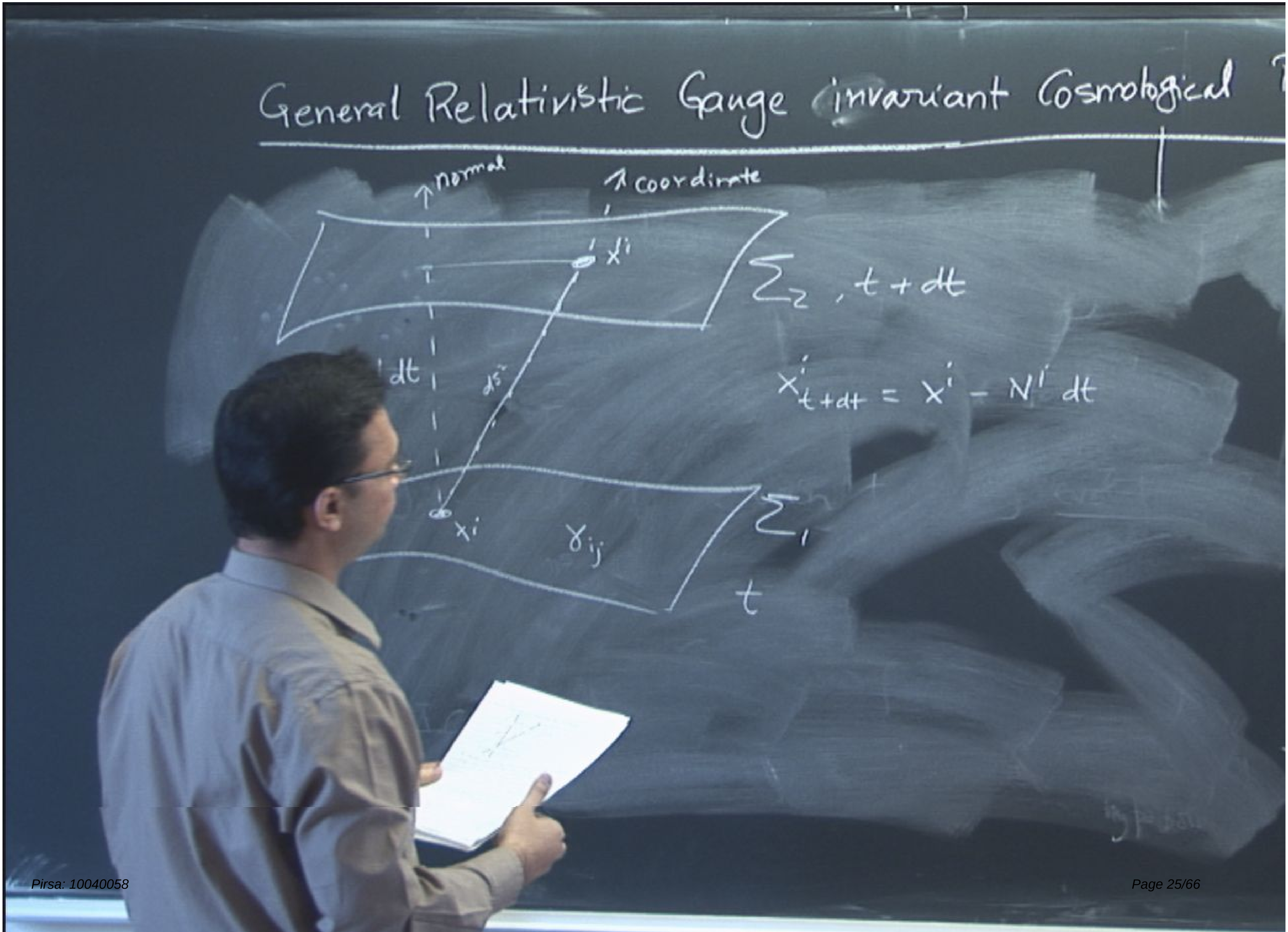

General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2 , $t + dt$
Σ_1
t


General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2 , $t + dt$
x^i
Σ_1
t

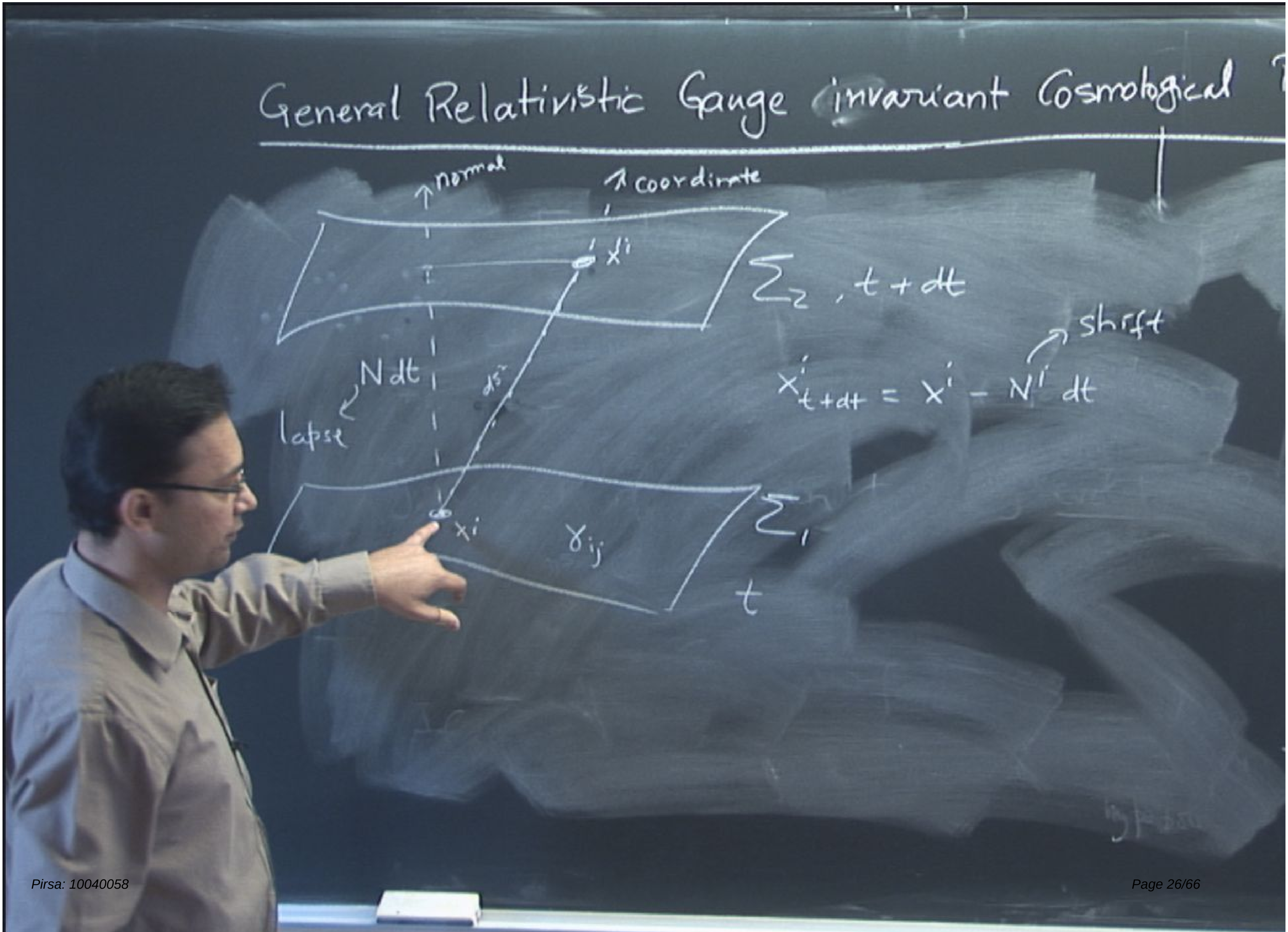

General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2, $t+dt$
Σ_1
x^i
γ_{ij}
t

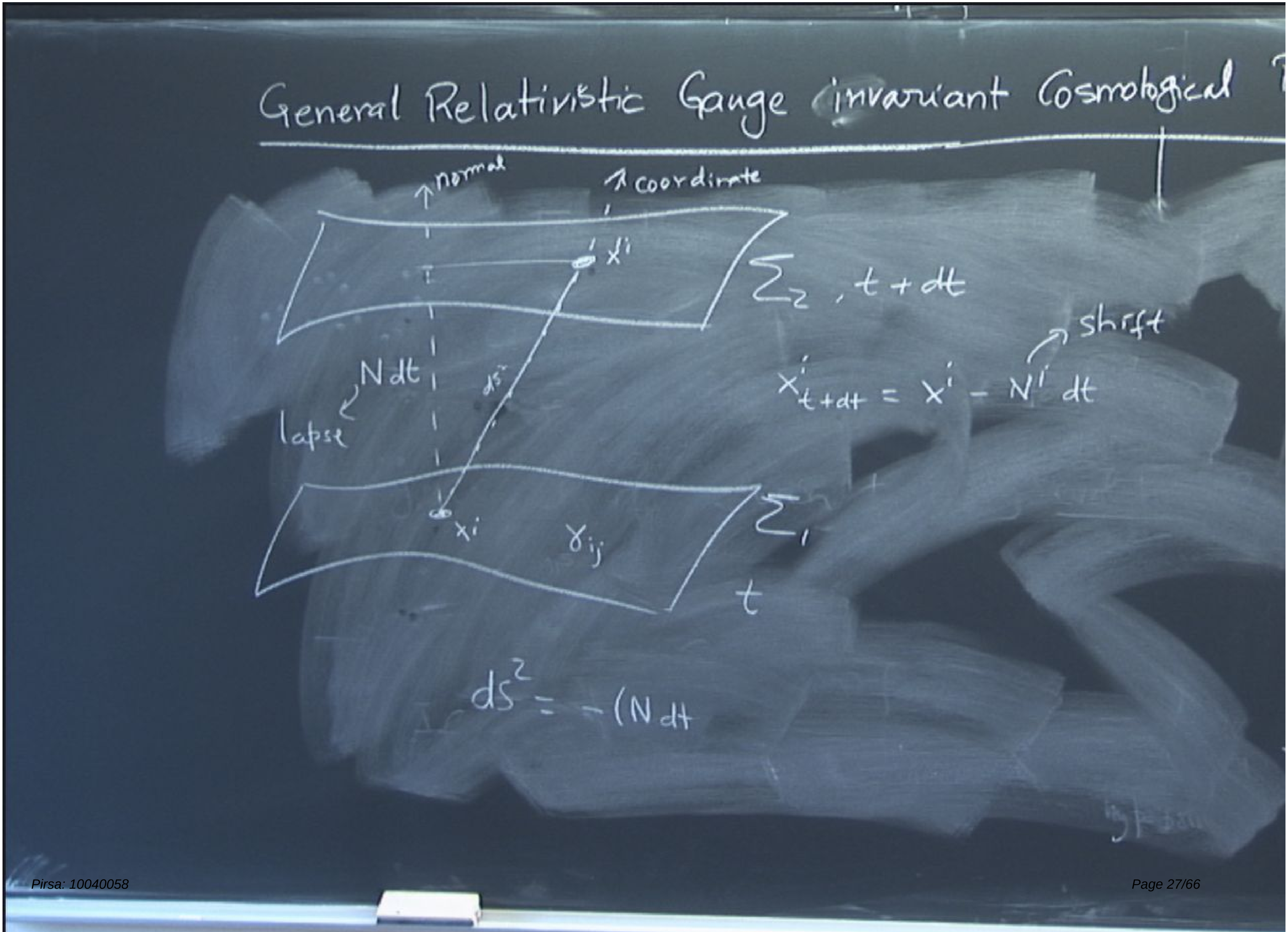

General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2 , t + dt
N dt
x^i
γ_ij
Σ_1
t

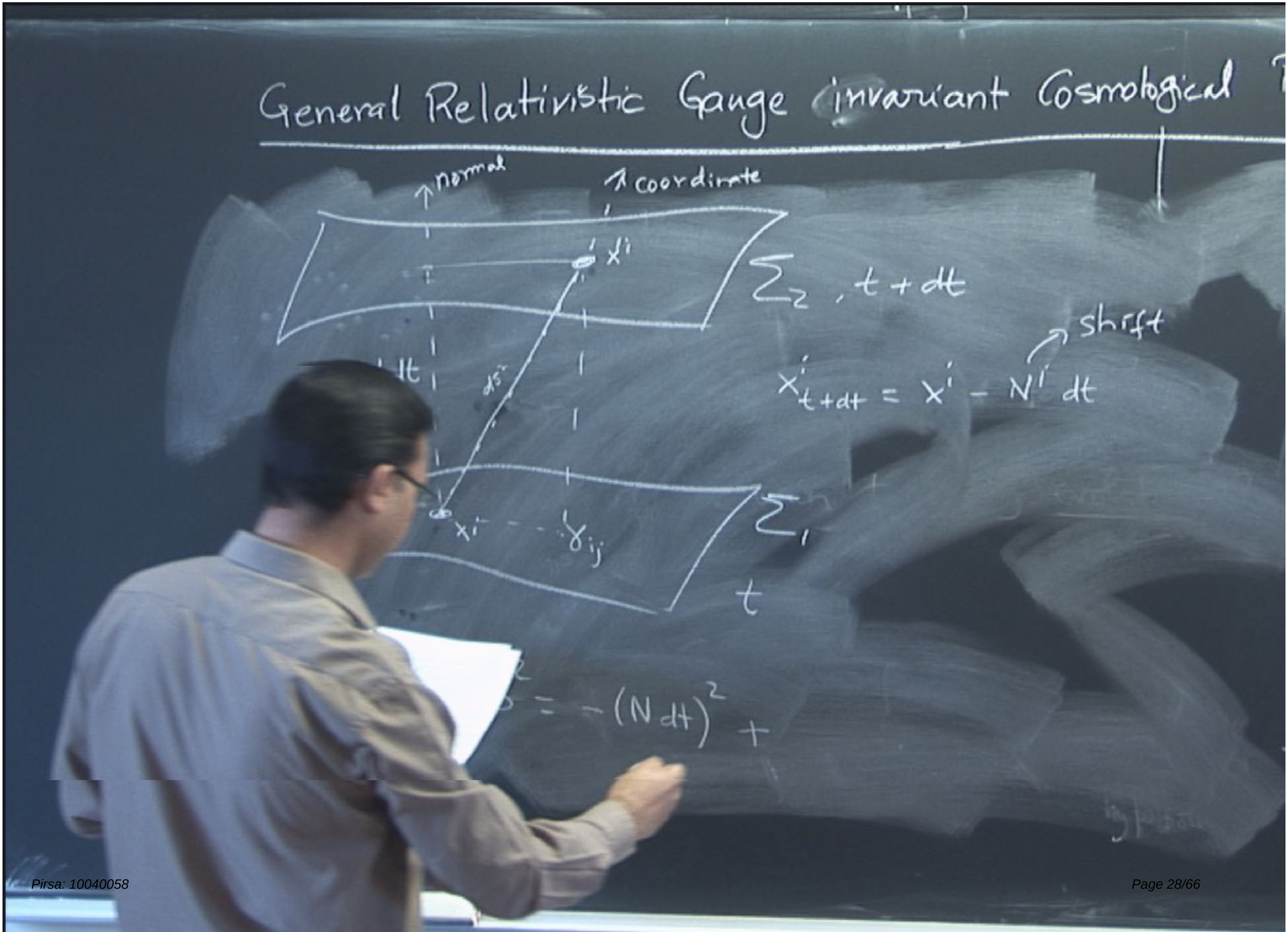

General Relativistic Gauge invariant Cosmological
normal
coordinate
x'^i
Σ_2 , t + dt
N dt
lapse
x^i
γ_ij
Σ_1
t

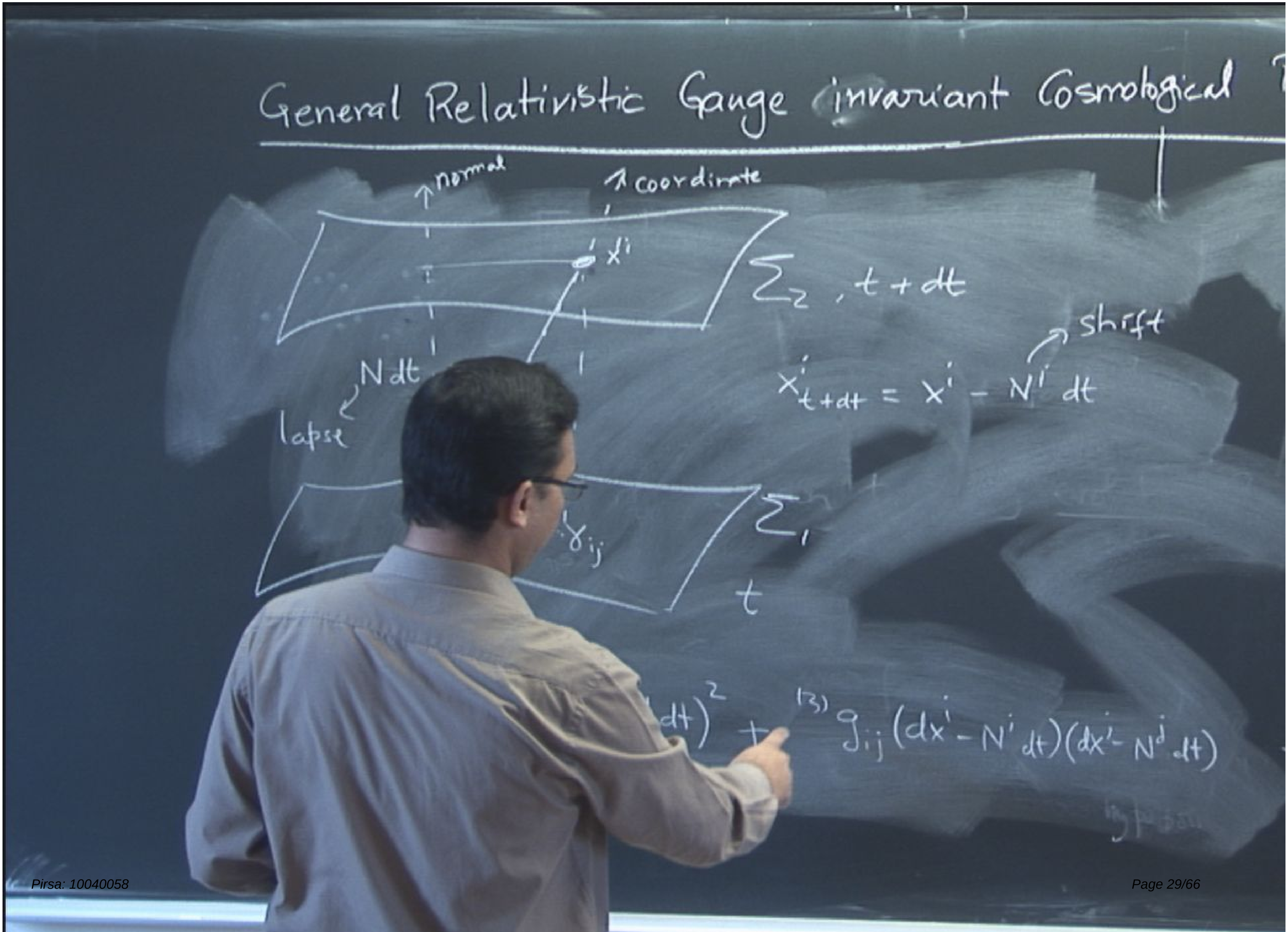


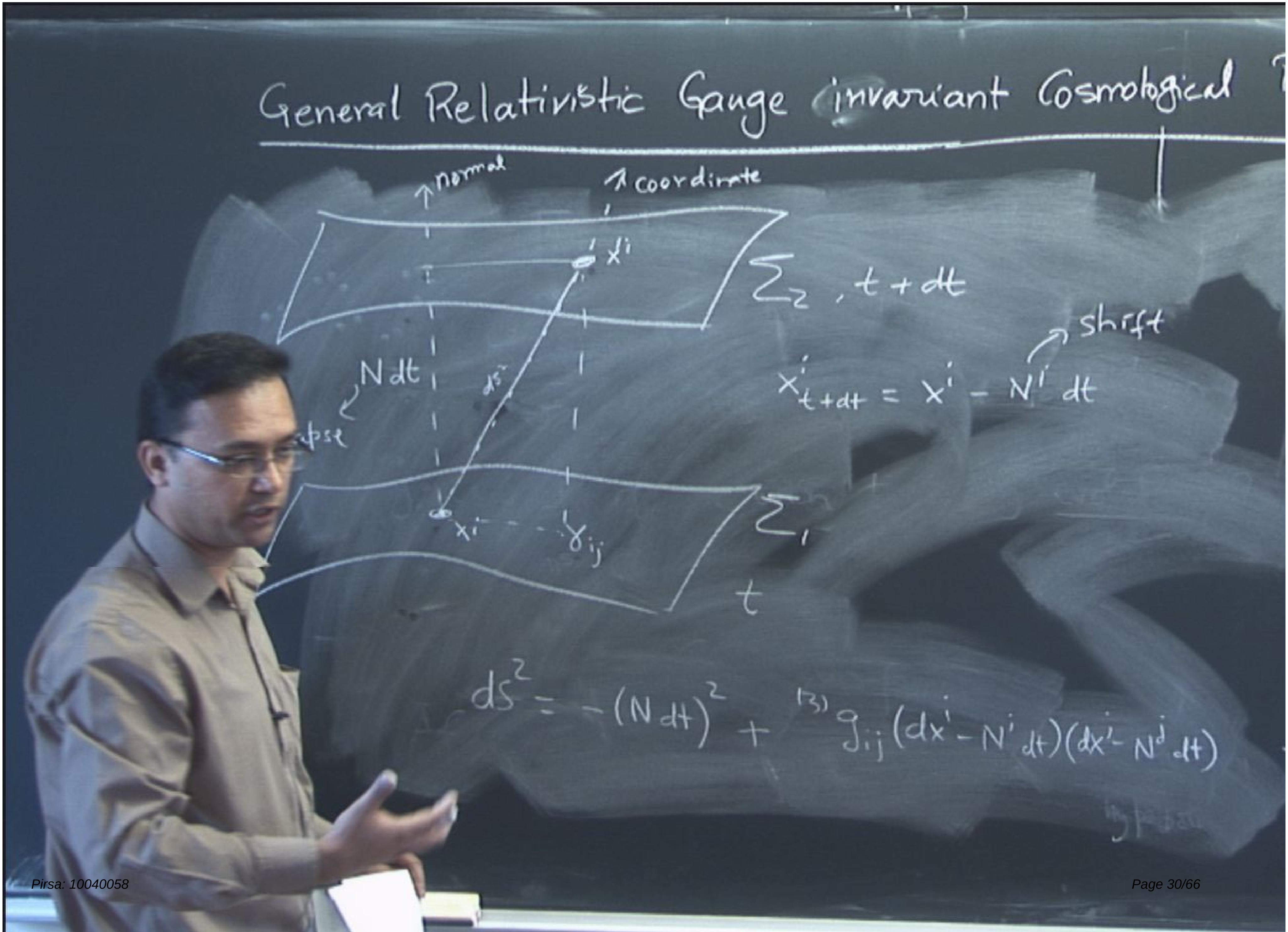
General Relativistic Gauge invariant Cosmological
normal
coordinate
x'^i
Σ_2 , $t + dt$
dt
ds^i
$x^i_{t+dt} = x^i - N^i\, dt$
x^i
γ_{ij}
Σ_1
t


General Relativistic Gauge invariant Cosmological
normal
coordinate
x'^i
Σ_2 , $t + dt$
shift
$N\,dt$
lapse
$x^i_{t+dt} = x^i - N^i\,dt$
x^i
γ_{ij}
Σ_1
t


General Relativistic Gauge invariant Cosmological
normal
coordinate
x'^i
Σ_2 , $t+dt$
shift
$N\,dt$
lapse
ds
$x^i_{t+dt} = x^i - N^i\,dt$
Σ_1
x^i
γ_{ij}
t
$ds^2 = -(N\,dt$


General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2 , $t + dt$
shift
$x^i_{t+dt} = x^i - N^i dt$
Σ_1
t
x^i
γ_{ij}
$= -(N dt)^2 +$

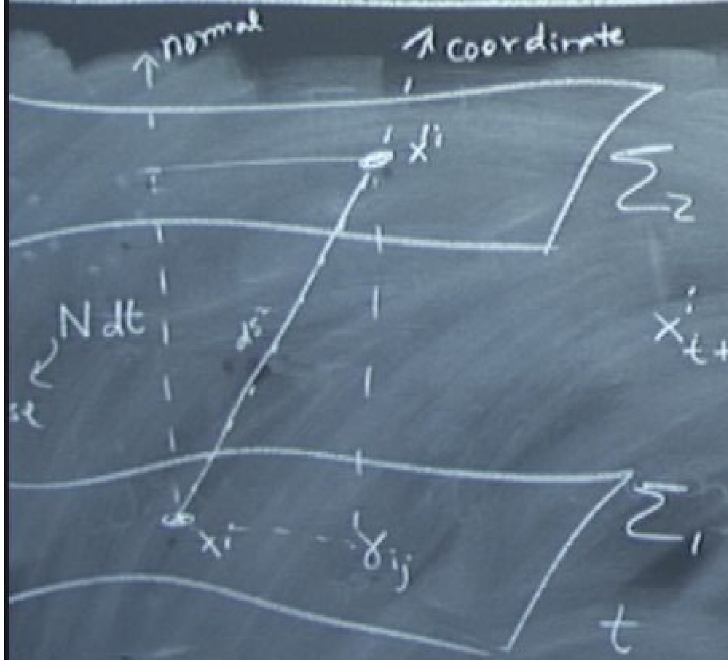

General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2 , t + dt
shift
N dt
lapse
x^i_{t+dt} = x^i − N^i dt
Σ_1
γ_ij
t
dt)^2 + ^(3)g_ij (dx^i − N^i dt)(dx^j − N^j dt)


General Relativistic Gauge invariant Cosmological
normal
coordinate
x^i
Σ_2 , $t + dt$
shift
$N dt$
ds
$x^i_{t+dt} = x^i - N^i dt$
Σ_1
x^i
γ_{ij}
t
$ds^2 = -(N dt)^2 + {}^{(3)}g_{ij}(dx^i - N^i dt)(dx^j - N^j dt)$

measure of time

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

...eral Relativistic Gauge invariant Cosmological Perturbations



shift

$$x^i_{t+dt} = x^i - N^i\,dt$$

$$ds^2 = -(N\,dt)^2 + {}^{(3)}g_{ij}(dx^i - N^i\,dt)(dx^j - N^j\,dt) \longrightarrow \text{guarantees}$$

$$= (-N^2 + N_i N^i)\,dt^2 - 2N_i\,dt\,dx^i + {}^{(3)}g_{ij}\,dx^i\,dx^j \qquad N_i = {}^{(3)}g_{ij} N^j$$

...rbations

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & \gamma_{ij} \end{pmatrix}$$

J. Bardeen 1988

On Cosmological Perturbations

$(- + + +)$

a measure of time

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

guarantees well posed IVP.

$N_i = {}^{(3)}g_{ij} N^j$

urbations

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & \gamma_{ij} \end{pmatrix}$$

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & -N^i/N^2 \\ -N^i/N^2 & \gamma^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix}$$

ranteeѕ well posed IVP.

N^j

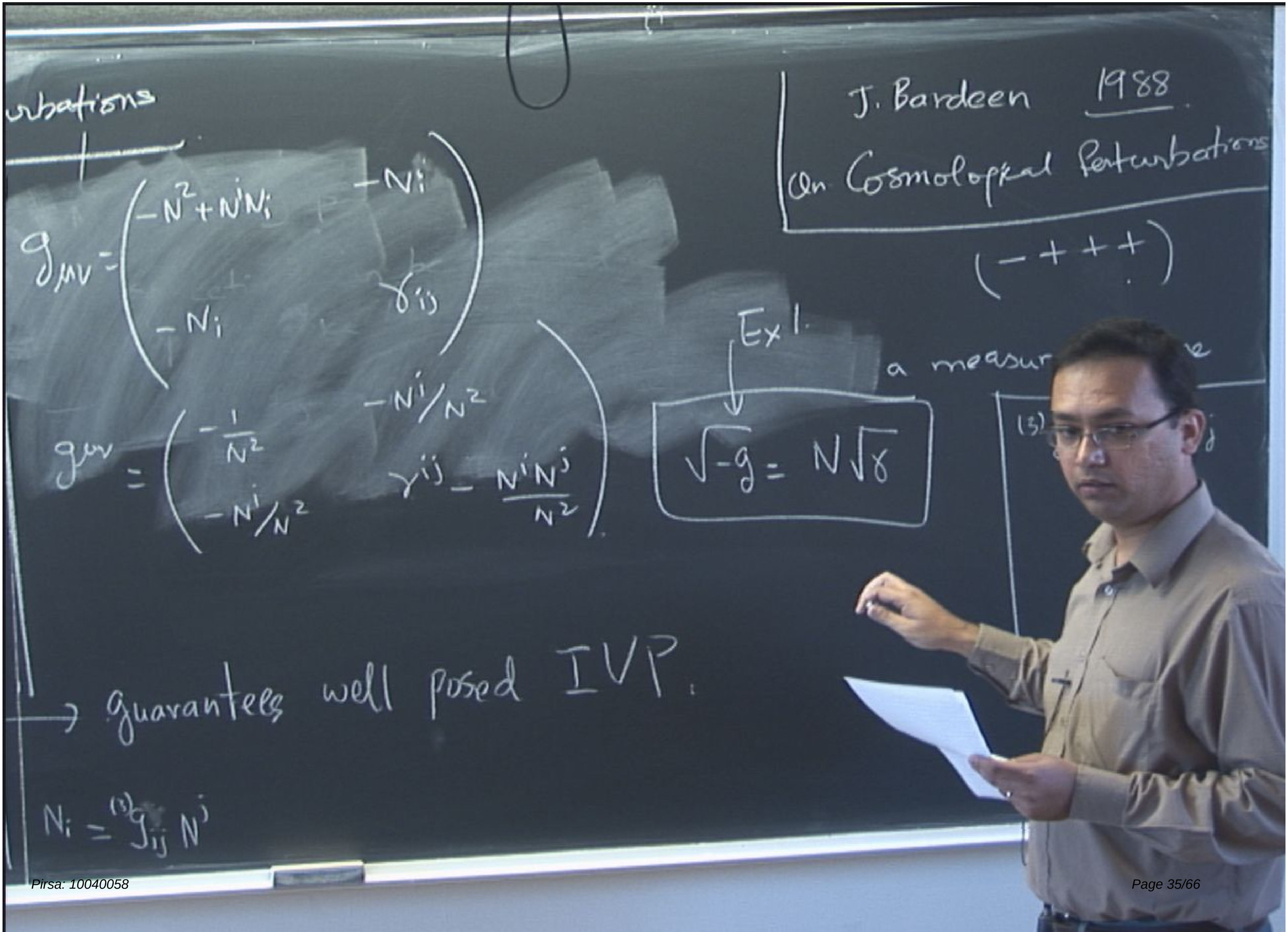
J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

a measure of time

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$



rbations
J. Bardeen 1988
On Cosmological Perturbations
(- + + +)
$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N_i & -N_i \\ -N_i & \gamma_{ij} \end{pmatrix}$
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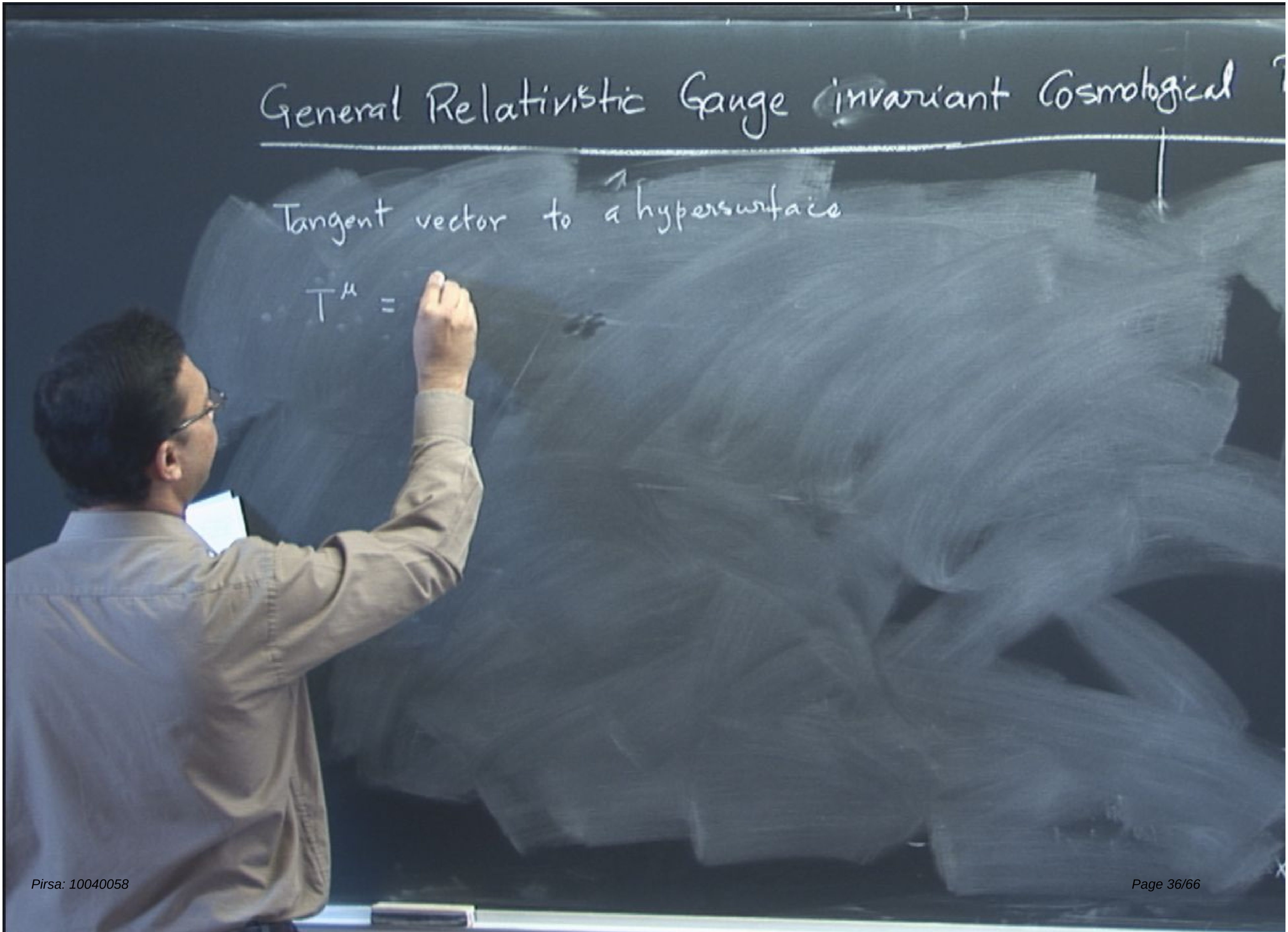
Ex 1.
$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$
a measur
→ guarantees well posed IVP.
$N_i = {}^{(3)}g_{ij} N^j$


General Relativistic Gauge invariant Cosmological
Tangent vector to a hypersurface
$T^\mu =$

General Relativistic Gauge invariant Cosmological

Tangent vector to a hypersurface

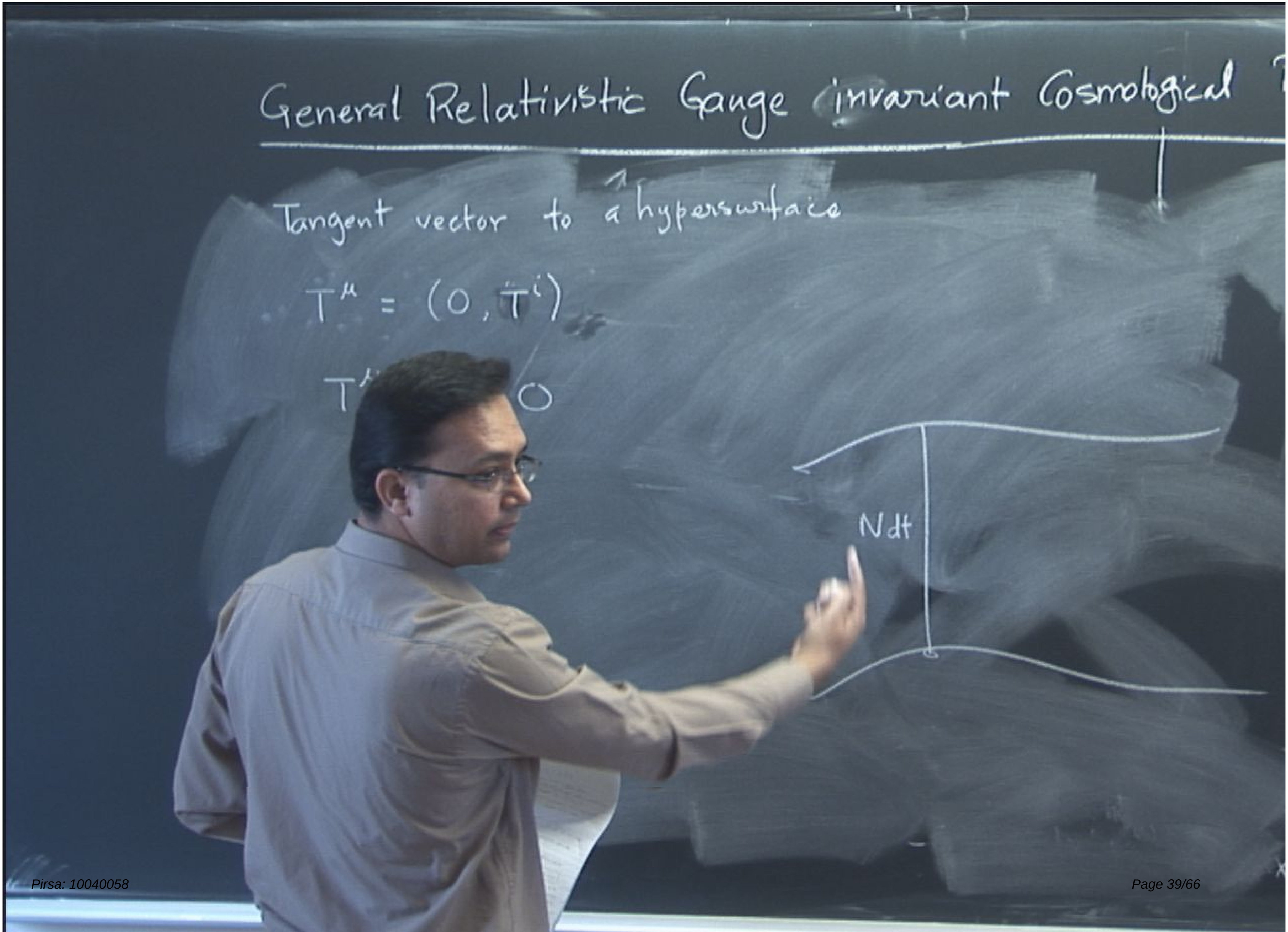
$$T^\mu = (0, T^i)$$

General Relativistic Gauge invariant Cosmological

Tangent vector to a hypersurface

$$T^\mu = (0, T^i)$$

$$T^\mu n_\mu = 0$$


General Relativistic Gauge invariant Cosmological
Tangent vector to a hypersurface
$T^\mu = (0, T^i)$
O
Ndt

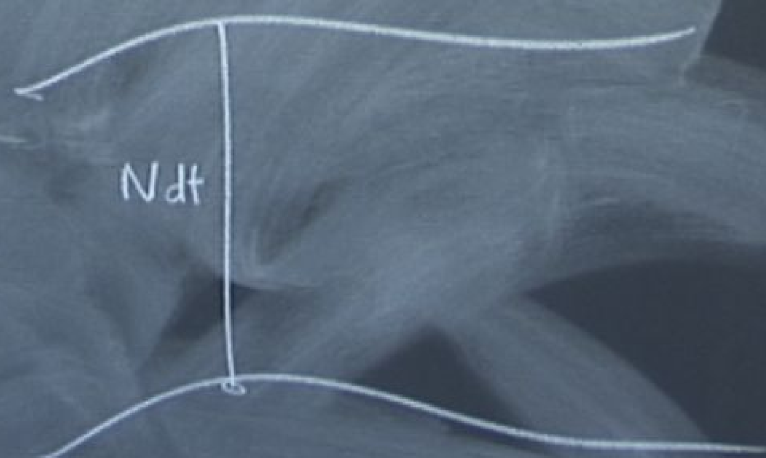
General Relativistic Gauge invariant Cosmological

Tangent vector to a hypersurface

$$T^\mu = (0, T^i)$$

$$T^\mu n_\mu = 0$$

$$n_\mu = (-N, 0, 0, 0)$$



Ex. 2

$$n^\mu = \frac{1}{N}(-1, -N^i)$$

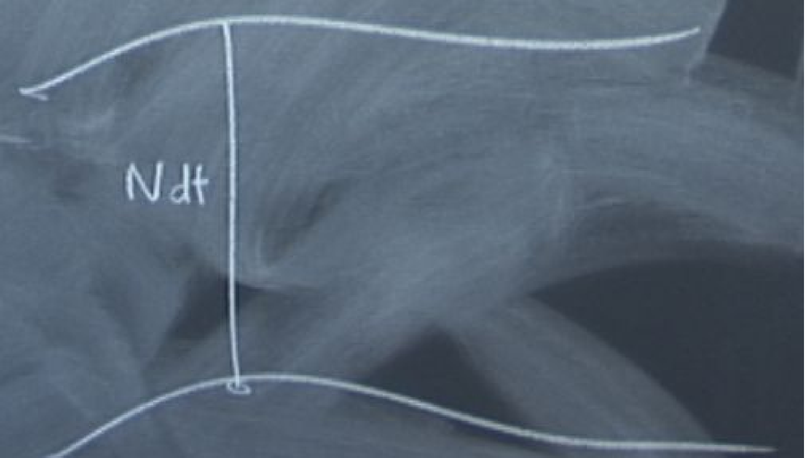
General Relativistic Gauge invariant Cosmological

Tangent vector to a hypersurface

$$T^\mu = (0, T^i)$$

$$T^\mu n_\mu = 0$$

$$n_\mu = (-N, 0, 0, 0)$$



Ex. 2

$$n^\mu = \frac{1}{N}\left(-1, -N^i\right)$$

$$n^\mu n_\mu = -1$$

urbations

$$P^\alpha{}_\beta := \delta^\alpha_\beta + n^\alpha n_\beta$$

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

Ex 1.

a measure of time

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$N_i = {}^{(3)}g_{ij} N^j$

...rbations

$$P^{\alpha}{}_{\beta} := \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$$^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$$

J. Bardeen 1988

On Cosmological Perturbations

$(- + + +)$

Ex 1.

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

a measure of time

$^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

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$${}^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$$

$$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$$

$N_i = {}^{(3)}g_{ij} N^j$

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

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J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

$${}^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$$

$$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$$

Ex 1.

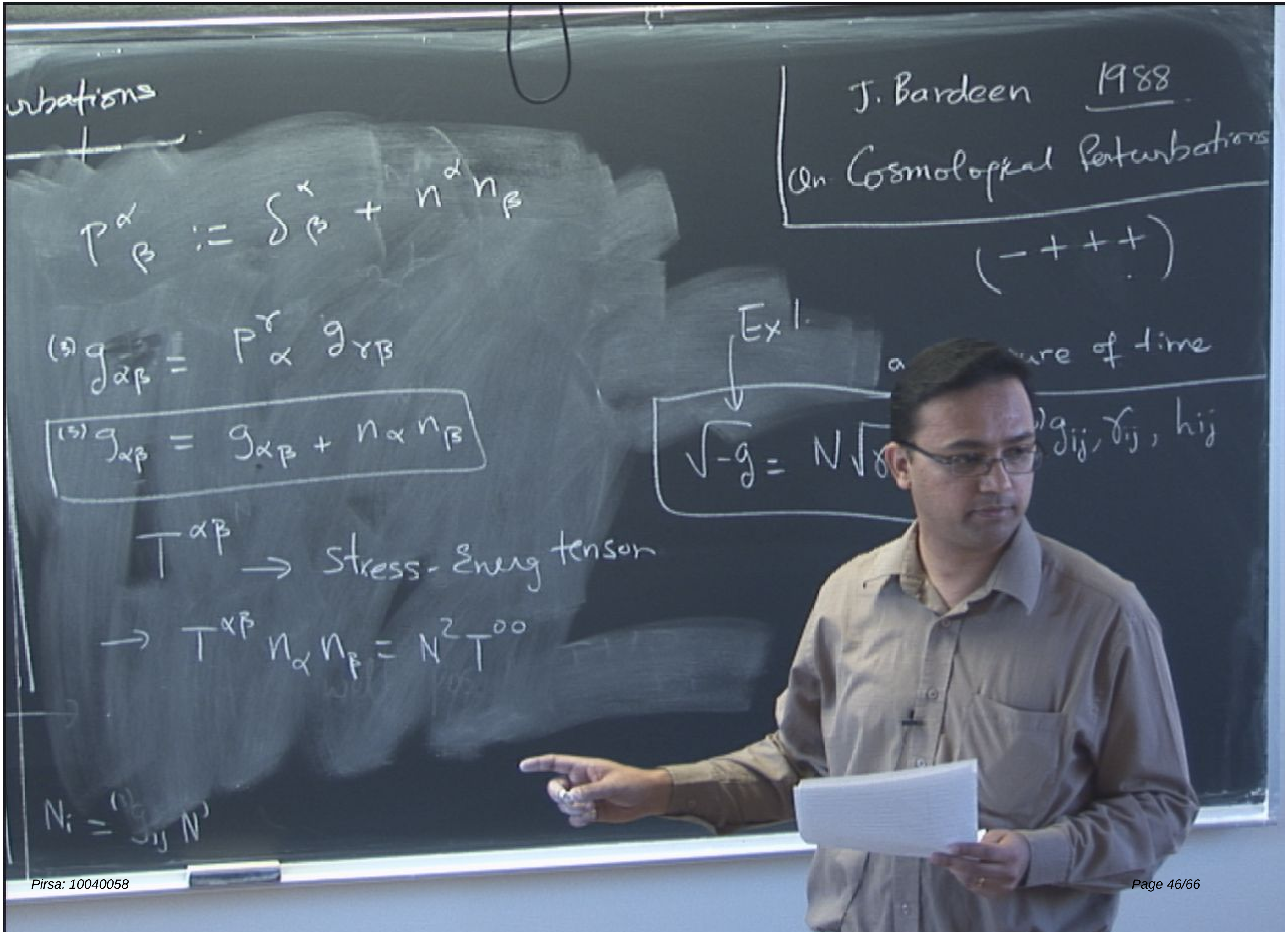
a measure of time

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$T^{\alpha\beta} \rightarrow$ Stress-Energy tensor

$N_i = {}^{(3)}g_{ij} N^j$


urbations
J. Bardeen 1988
On Cosmological Perturbations
(− + + +)
$P^{\alpha}{}_{\beta} := \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$
${}^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$
$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$
$T^{\alpha\beta} \rightarrow$ Stress-Energy tensor
$\rightarrow T^{\alpha\beta} n_{\alpha} n_{\beta} = N^2 T^{00}$
$N_i = {}^{(3)}g_{ij} N^j$
Ex 1.
ure of time
$\sqrt{-g} = N\sqrt{\gamma}$
$g_{ij}, \gamma_{ij}, h_{ij}$

urbations

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

$$P^{\alpha}{}_{\beta} := \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$${}^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$$

$$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$$

Ex 1.

a measure of time

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$T^{\alpha\beta} \rightarrow$ Stress-Energy tensor

$$\rightarrow T^{\alpha\beta} n_{\alpha} n_{\beta} = N^2 T^{00} = \rho \longleftarrow \text{energy density}$$

...rbations

$$P^{\alpha}{}_{\beta} := \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$${}^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$$

$$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$$

$T^{\alpha\beta} \rightarrow$ Stress-Energ tensor

$\rightarrow T^{\alpha\beta} n_{\alpha} n_{\beta} = N^2 T^{00} = \rho \leftarrow$ energy density

$T^{\alpha\beta} P_{\mu\alpha} P_{\nu\beta} =$

$N_i = g_{ij} N^j$

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

Ex 1. a measure of time

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

rbations

$$P^{\alpha}{}_{\beta} := \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$${}^{(3)}g_{\alpha\beta} = P^{\gamma}{}_{\alpha}\, g_{\gamma\beta}$$

$$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$$

$T^{\alpha\beta} \rightarrow$ Stress-Energ tensor

$$\rightarrow T^{\alpha\beta} n_{\alpha} n_{\beta} = N^2 T^{00} = \rho \leftarrow \text{energy density}$$

$$T^{\alpha\beta} P_{\mu\alpha} P_{\nu\beta} = {}^{(3)}T_{\mu\nu} = S_{\mu\nu} \rightarrow S_{ij}$$

$$N_i = {}^{(3)}g_{ij} N^j$$

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

Ex 1.

a measure of time

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

urbations

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

$$P^{\alpha}_{\ \beta} := \delta^{\alpha}_{\beta} + n^{\alpha} n_{\beta}$$

$${}^{(3)}g_{\alpha\beta} = P^{\gamma}_{\ \alpha}\, g_{\gamma\beta}$$

$$\boxed{{}^{(3)}g_{\alpha\beta} = g_{\alpha\beta} + n_{\alpha} n_{\beta}}$$

Ex 1.

a measure of time

$$\boxed{\sqrt{-g} = N\sqrt{\gamma}}$$

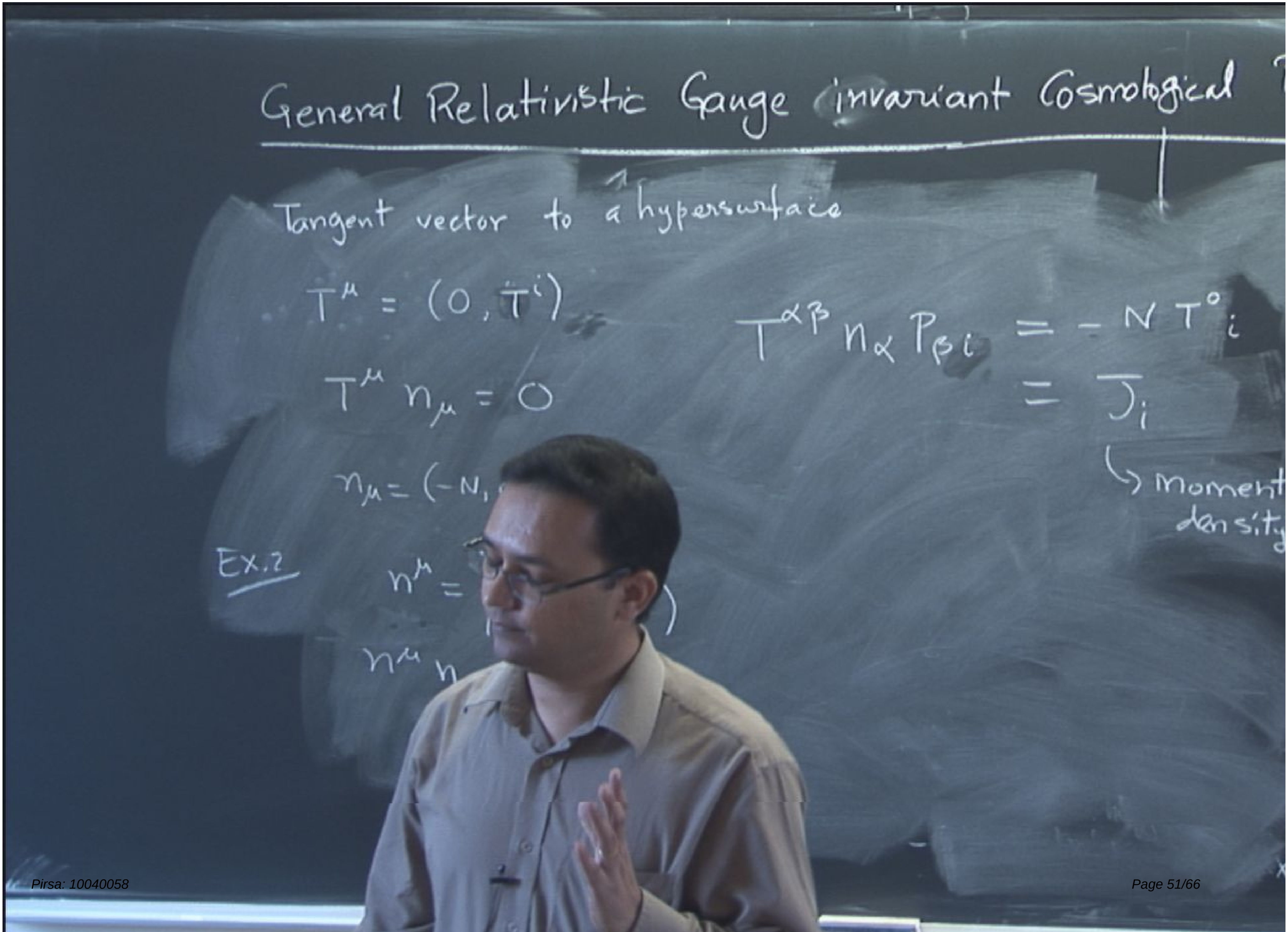
${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$T^{\alpha\beta} \rightarrow$ Stress-Energy tensor

$\rightarrow T^{\alpha\beta} n_{\alpha} n_{\beta} = N^2 T^{00} = \rho \leftarrow$ energy density

$T^{\alpha\beta} P_{\mu\alpha} P_{\nu\beta} = {}^{(3)}T_{\mu\nu} = S_{\mu\nu} \rightarrow S_{ij} \leftarrow$ Stress tensor

$N_i = {}^{(3)}g_{ij} N^j$



General Relativistic Gauge invariant Cosmological
Tangent vector to a hypersurface
$T^\mu = (0, T^i)$
$T^{\alpha\beta} n_\alpha P_{\beta i} = - N T^0{}_i$
$= J_i$
$T^\mu n_\mu = 0$
$n_\mu = (-N,$
moment
density
Ex.2
$n^\mu =$
$n^\mu n$

urbations

Extrinsic Curvature.

(i) Intrinsic curvature

J. Bardeen 1988

On Cosmological Perturbations

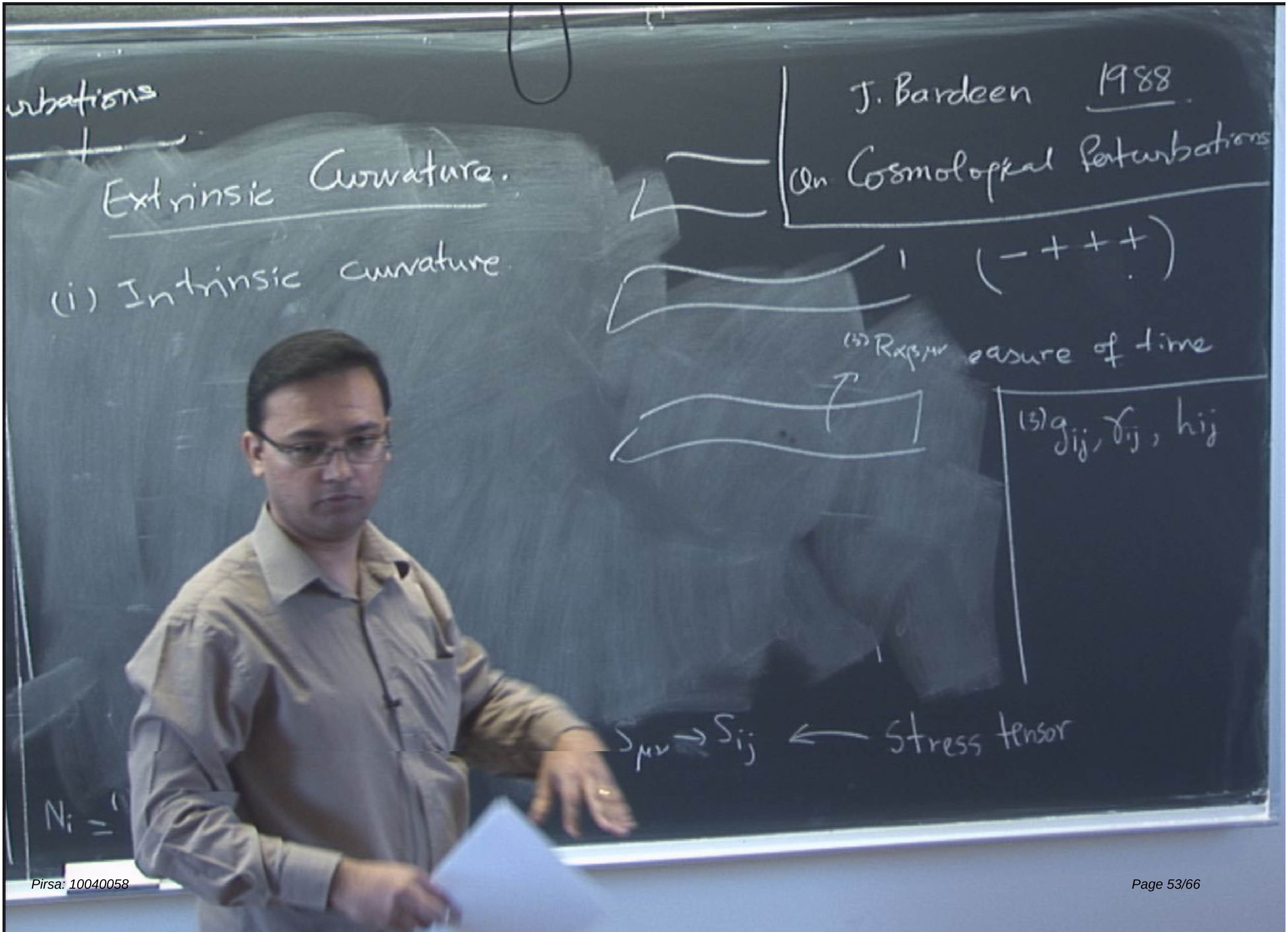
$(-+++)$

easure of time

$^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$S_{\mu\nu} \to S_{ij} \leftarrow$ Stress tensor

$N_i = {}^{(3)}g_{ij} N^j$


rbations
Extrinsic Curvature.
(i) Intrinsic curvature
J. Bardeen 1988
On Cosmological Perturbations
(- + + +)
(3) Rαβμν
easure of time
(3) gij, γij, hij
Sμν → Sij ← Stress tensor
Ni =

urbations

Extrinsic Curvature.

(i) Intrinsic curvature

↳ comes from ${}^{(3)}R_{\alpha\beta\mu\nu}$

(ii) the way hypersurfaces are embedded

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$

${}^{(4)}R_{\alpha\beta\mu\nu}$ easure of time

${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$S_{\mu\nu} \to S_{ij} \leftarrow$ Stress tensor

$N_i = {}^{(3)}g_{ij} N^j$

General Relativistic Gauge invariant Cosmological

Tangent vector to a hypersurface

$$T^\mu = (0, T^i)$$

$$T^\mu n_\mu = 0$$

$$n_\mu = (-N, 0, 0, 0)$$

Ex. 2

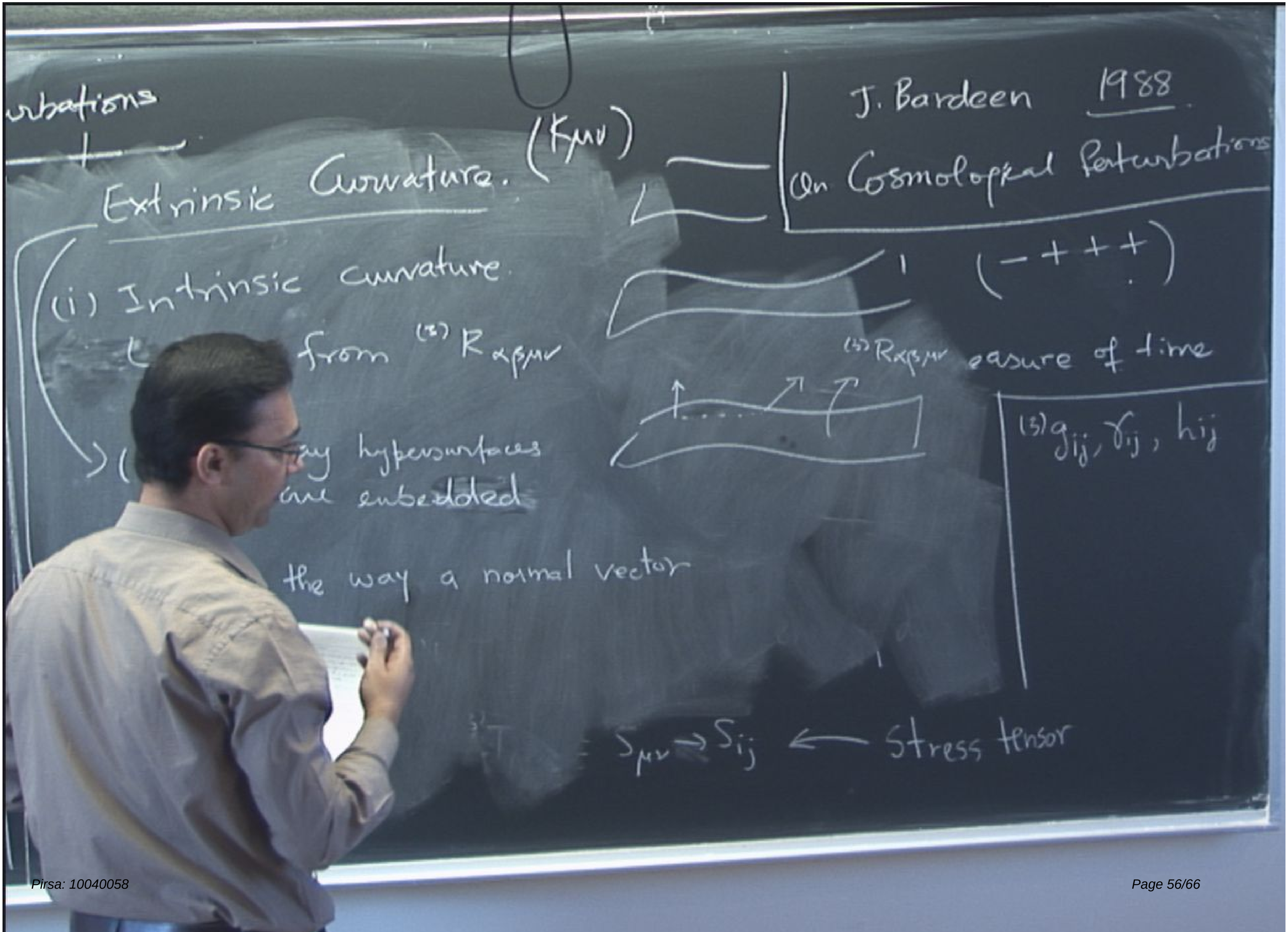
$$n^\mu = \frac{1}{N}(-1, -N^i)$$

$$n^\mu n_\mu = -1$$

$$T^{\alpha\beta} n_\alpha P_{\beta i} = -N T^0{}_i = \bar{J}_i$$

→ moment density

$N, N^i \rightarrow$ gauge variables



urbations
Extrinsic Curvature. ($K_{\mu\nu}$)
J. Bardeen 1988
On Cosmological Perturbations
(− + + +)
(i) Intrinsic curvature
from ${}^{(3)}R_{\alpha\beta\mu\nu}$
${}^{(3)}R_{\alpha\beta\mu\nu}$ easure of time
${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$
hypersurfaces
are embedded
the way a normal vector
$S_{\mu\nu} \rightarrow S_{ij}$ ← Stress tensor

urbations

Extrinsic Curvature. ($K_{\mu\nu}$)

(i) Intrinsic curvature

↳ comes from $^{(3)}R_{\alpha\beta\mu\nu}$

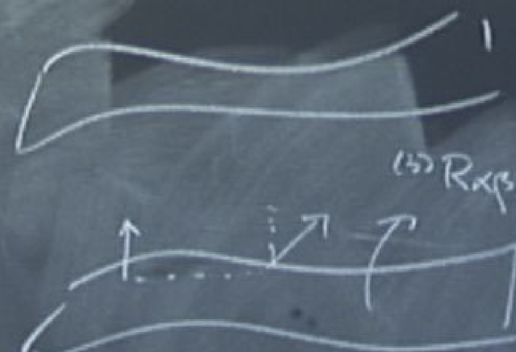
(ii) the way hypersurfaces are embedded

→ It measures the way a normal vector is parallel transport on Σ.

J. Bardeen 1988

On Cosmological Perturbations

$(-+++)$



$^{(3)}R_{\alpha\beta\mu\nu}$ easure of time

$^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$

$S_{\mu\nu} \to S_{ij}$ ← Stress tensor

$N_i = {}^{(3)}g_{ij} N^j$

(ii) the way hypersurfaces are embedded

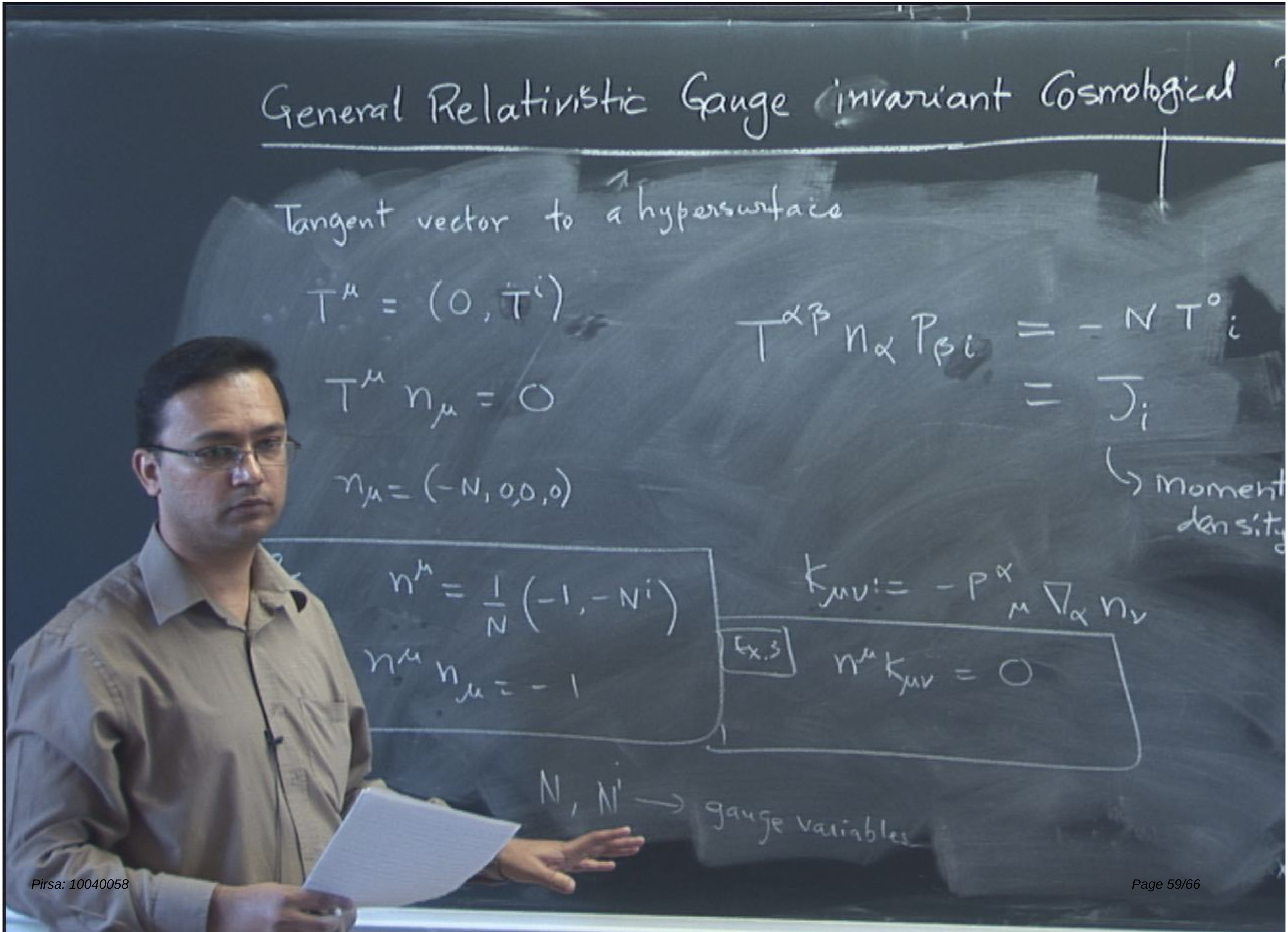
t measures the way a normal vector

s parallel transport on Σ.

$$\nabla_\nu A^\mu = A^\mu{}_{;\nu} = A^\mu{}_{,\nu} + A^\alpha \Gamma^\mu_{\alpha\nu} = S_{\mu\nu} \rightarrow$$

$^{(3)}g$

$g_{ij} N^j$


General Relativistic Gauge invariant Cosmological
Tangent vector to a hypersurface
$T^\mu = (0, T^i)$
$T^\mu n_\mu = 0$
$n_\mu = (-N, 0, 0, 0)$
$n^\mu = \frac{1}{N}(-1, -N^i)$
$n^\mu n_\mu = -1$
$T^{\alpha\beta} n_\alpha P_{\beta i} = -N T^0{}_i$
$= \bar{J}_i$
moment
density
$K_{\mu\nu} := -P^\alpha{}_\mu \nabla_\alpha n_\nu$
Ex.3
$n^\mu K_{\mu\nu} = 0$
$N, N^i \longrightarrow$ gauge variables

...rbations

Extrinsic Curvature. $(K_{\mu\nu})$

(i) Intrinsic curvature

↳ comes from ${}^{(3)}R_{\alpha\beta\mu\nu}$

(ii) the way hypersurfaces are embedded

→ It measures the way a normal vector is parallel transport on Σ.

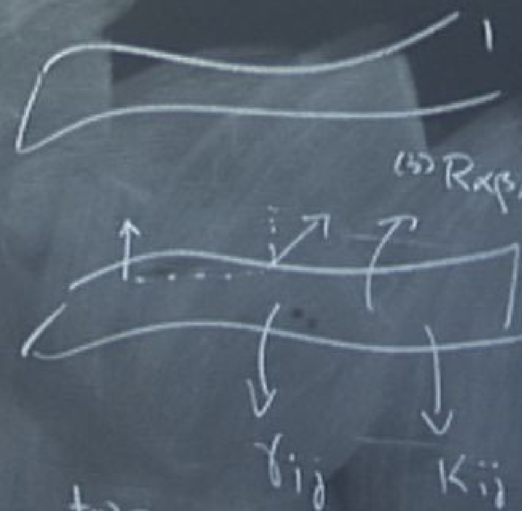
$$\nabla_\nu A^\mu = A^\mu{}_{;\nu} = A^\mu{}_{,\nu} + A^\alpha \Gamma^\mu_{\alpha\nu}$$

$S_{\mu\nu} \to S_{ij} \leftarrow$ Stress tensor

J. Bardeen 1988

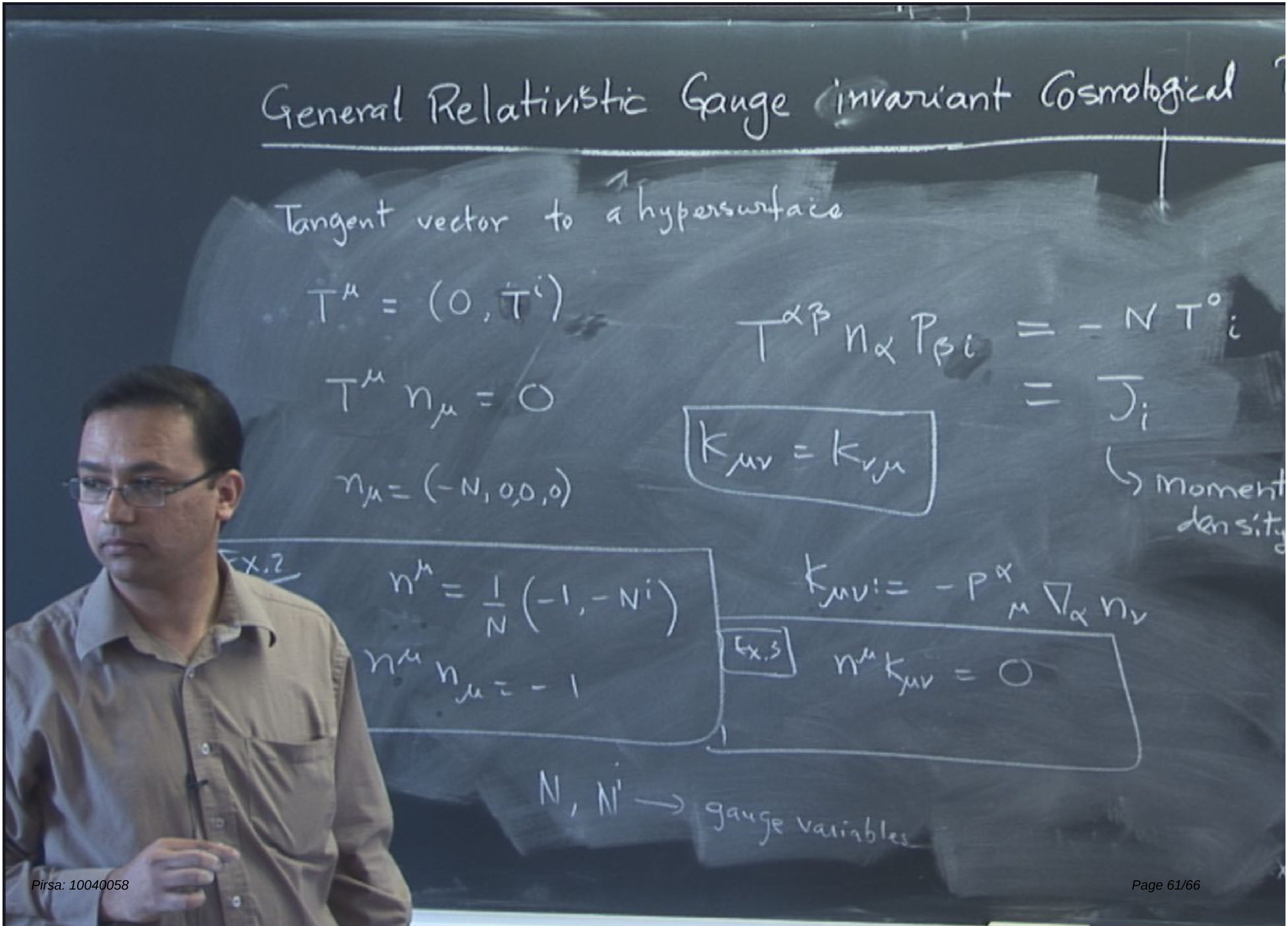
On Cosmological Perturbations

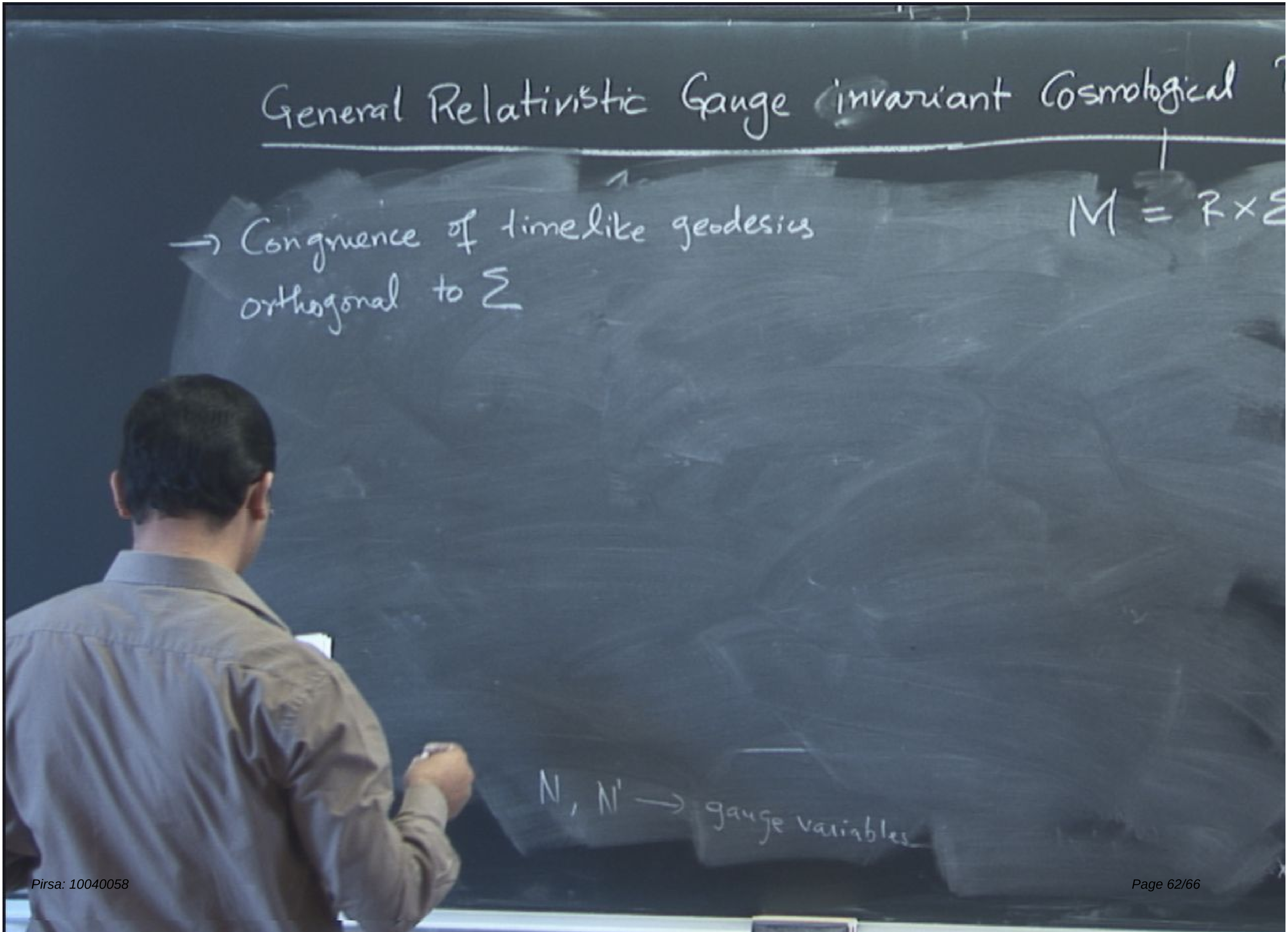
$(-+++)$

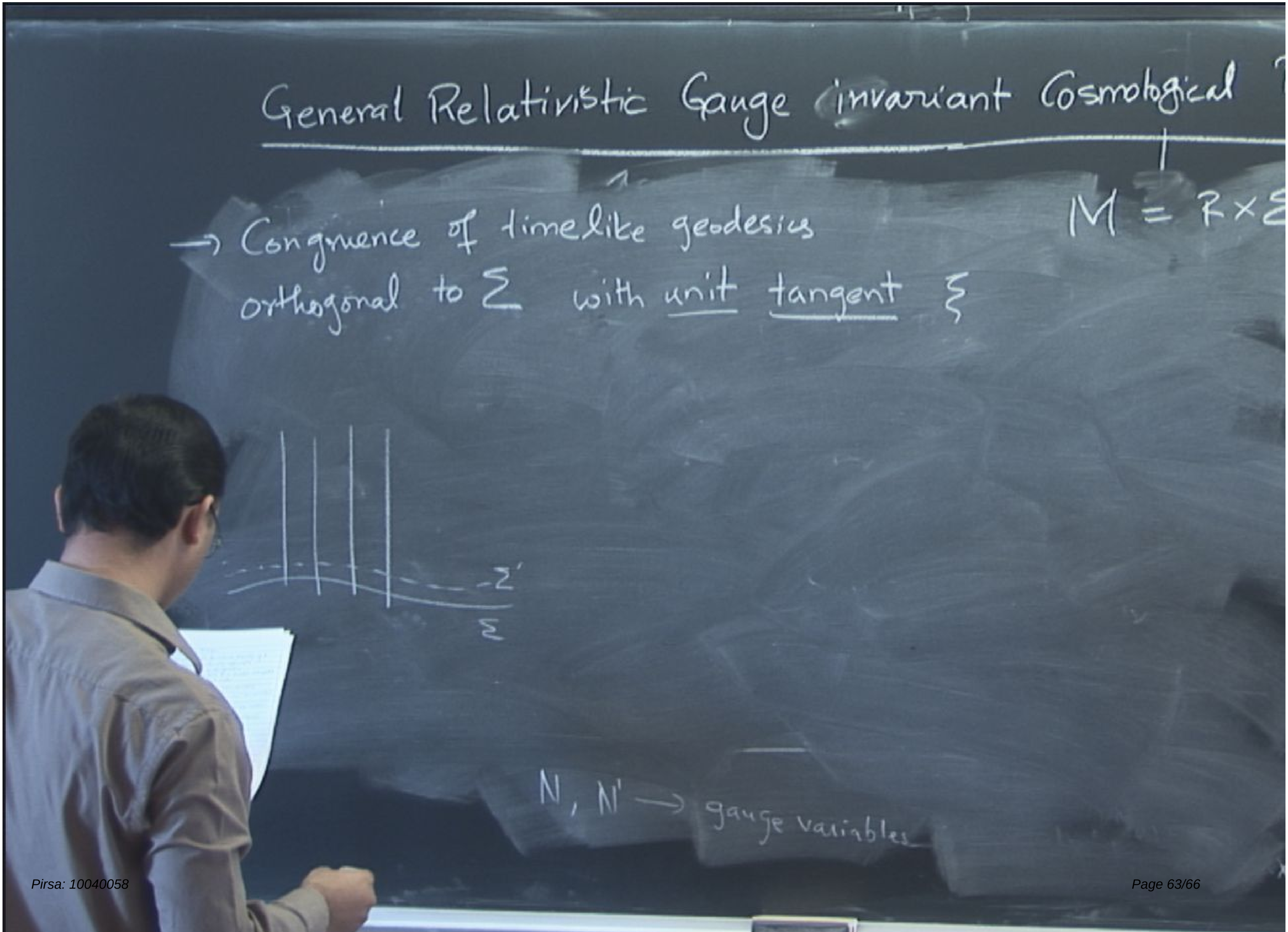


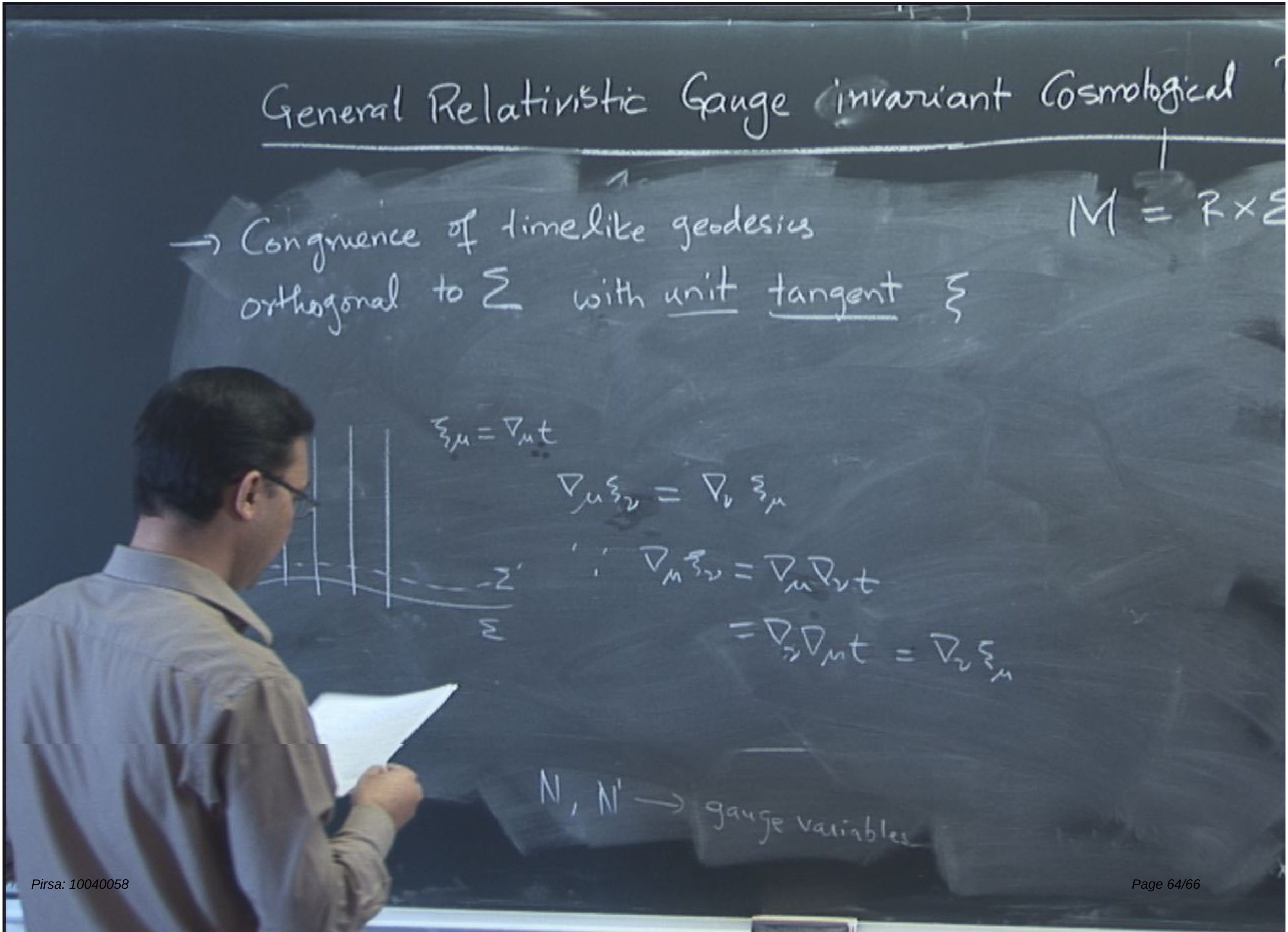
...easure of time

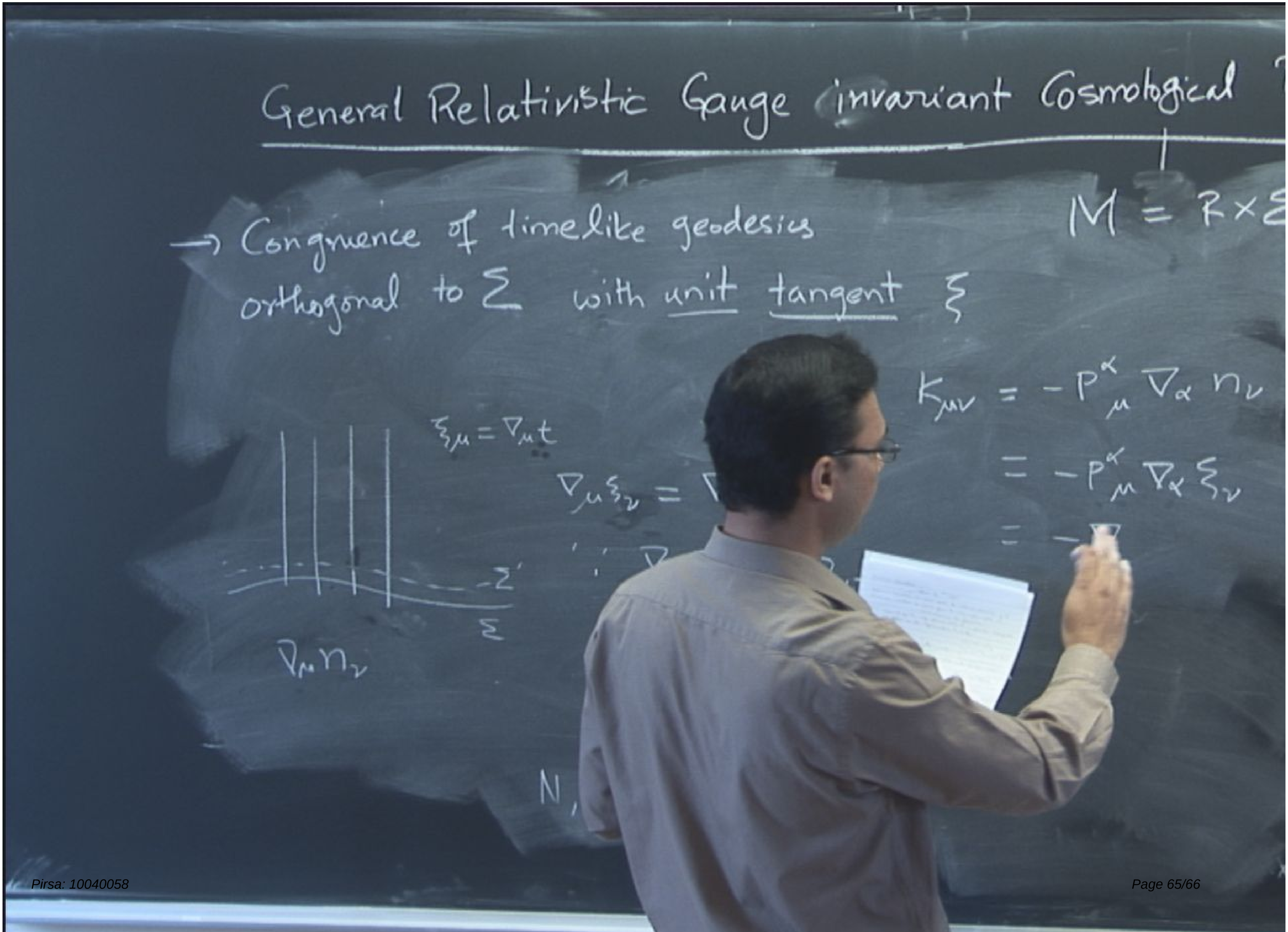
${}^{(3)}g_{ij}, \gamma_{ij}, h_{ij}$


General Relativistic Gauge invariant Cosmological
Tangent vector to a hypersurface
$T^\mu = (0, T^i)$
$T^\mu n_\mu = 0$
$n_\mu = (-N, 0, 0, 0)$
$T^{\alpha\beta} n_\alpha P_{\beta i} = -N T^0{}_i$
$= J_i$
momentum density
$\boxed{K_{\mu\nu} = K_{\nu\mu}}$
Ex. 2
$n^\mu = \frac{1}{N}(-1, -N^i)$
$n^\mu n_\mu = -1$
$K_{\mu\nu} := -P^\alpha{}_\mu \nabla_\alpha n_\nu$
Ex. 3
$n^\mu K_{\mu\nu} = 0$
$N, N^i \rightarrow$ gauge variables


General Relativistic Gauge invariant Cosmological
$M = R \times \Sigma$
→ Congruence of timelike geodesics
orthogonal to Σ
$N, N' \longrightarrow$ gauge variables


General Relativistic Gauge invariant Cosmological
M = R × Σ
→ Congruence of timelike geodesics
orthogonal to Σ with unit tangent ξ
Σ'
Σ
N, N' → gauge variables


General Relativistic Gauge invariant Cosmological
$M = R \times \Sigma$
→ Congruence of timelike geodesics
orthogonal to Σ with unit tangent ξ
$\xi_\mu = \nabla_\mu t$
$\nabla_\mu \xi_\nu = \nabla_\nu \xi_\mu$
$\because \nabla_\mu \xi_\nu = \nabla_\mu \nabla_\nu t$
$= \nabla_\nu \nabla_\mu t = \nabla_\nu \xi_\mu$
Σ'
Σ
$N, N' \rightarrow$ gauge variables

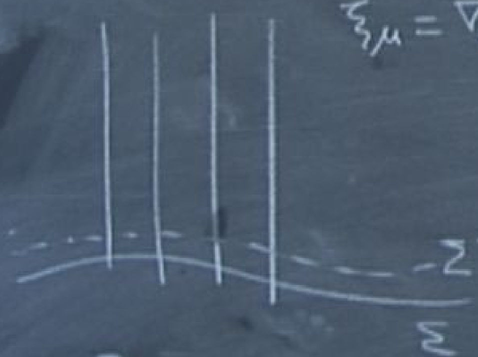

General Relativistic Gauge invariant Cosmological
$M = R \times \Sigma$
→ Congruence of timelike geodesics
orthogonal to Σ with unit tangent ξ
$K_{\mu\nu} = -P^{\alpha}_{\ \mu} \nabla_\alpha n_\nu$
$= -P^{\alpha}_{\ \mu} \nabla_\alpha \xi_\nu$
$\xi_\mu = \nabla_\mu t$
Σ'
Σ
$\nabla_\mu n_\nu$

General Relativistic Gauge invariant Cosmological

$M = R \times \Sigma$

$\rightarrow$ Congruence of timelike geodesics orthogonal to Σ with unit tangent ξ

$$\boxed{K_{\mu\nu} = K_{\nu\mu}}$$



$\xi_\mu = \nabla_\mu t$

$$\nabla_\mu \xi_\nu = \nabla_\nu \xi_\mu$$

$$\because \nabla_\mu \xi_\nu = \nabla_\mu \nabla_\nu t$$

$$= \nabla_\nu \nabla_\mu t = \nabla_\nu \xi_\mu$$

$\nabla_\mu n_\nu$

$$K_{\mu\nu} = -P^\alpha_{\ \mu} \nabla_\alpha n_\nu$$

$$= -P^\alpha_{\ \mu} \nabla_\alpha \xi_\nu$$

$$= -(\delta^\alpha_{\ \mu} + n^\alpha n_\mu)\nabla$$

$$= -\nabla_\mu \xi_\nu$$

$$= -\nabla_\nu \xi_\mu$$

$$= -P^\alpha_{\ \nu} \nabla_\alpha \xi_\mu$$

$$= -P^\alpha_{\ \nu} \nabla_\alpha n_\mu$$

$$= K_{\nu\mu}$$

$N, N' \longrightarrow$ gauge variables