Title: Quantum Spin Simulations (PHYS 7380) - Lecture 14

Date: Apr 22, 2010 11:00 AM

URL: http://pirsa.org/10040056

Abstract:

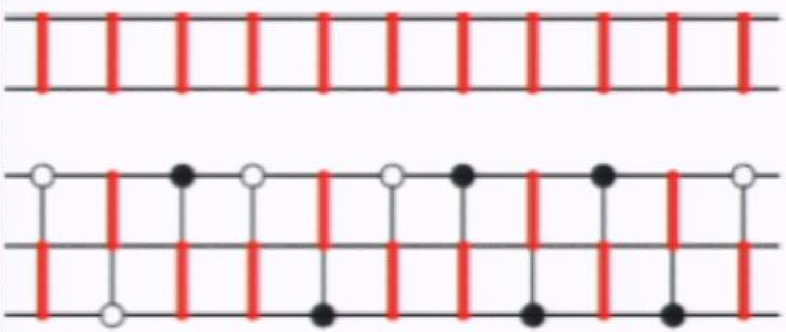
Ladder systems

E. Dagotto and T. M. Rice, Science 271, 618 (1996)

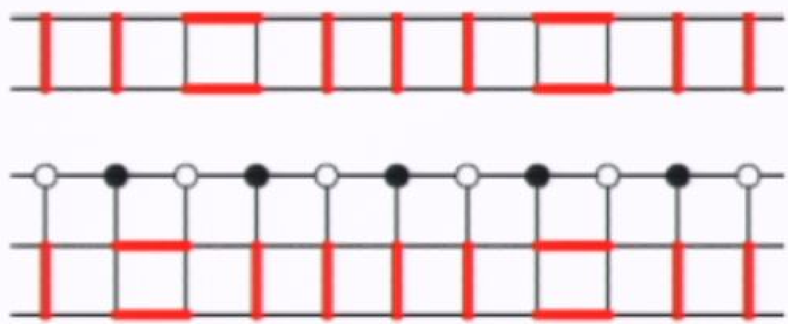
Coupled Heisenberg chains; $L_x \times L_y$ spins, $L_y \to \infty$, L_x finite
- systems with even and odd L_y have qualitatively different properties
  - spin gap $\Delta > 0$ for L_y even, $\Delta \to 0$ when $L_x \to \infty$
  - critical state, similar to single chain, for odd L_y
  - the 2D limit is approached in different ways

Consider anisotropic couplings; J_x and J_y
- the correct physics for all J_y/J_x can be understood based on large J_y/J_x
- short-range valence bond states

$J_y = 1, J_x = 0$



$0 < J_x/J_y \ll 1$



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$L_y = 2,4,\ldots$: $\Delta = J_y$ for $J_x = 0$
- gap persists for $J_x > 0$

$L_y = 3,5,\ldots$: $\Delta = 0$ for $J_x = 0$

Properties of Heisenberg ladders; large-scale SSE results

Magnetic susceptibility Low-T theoretical forms:

Odd L_y: from nonlinear -sigma model
Eggert, Affleck, Takahashi, PRL 73, 332 (1994)

$$\chi(T) = \frac{1}{2\pi c} + \frac{1}{4\pi c \ln(T_0/T)}$$

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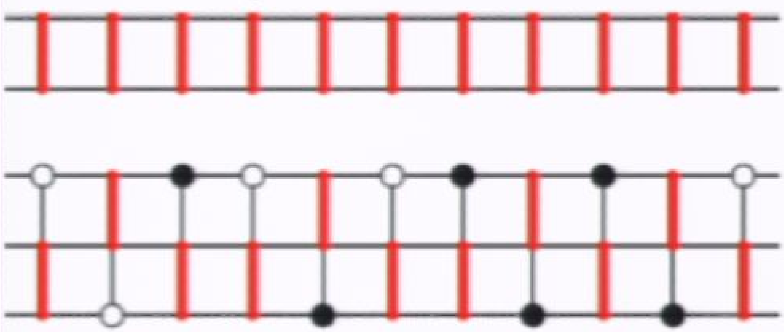

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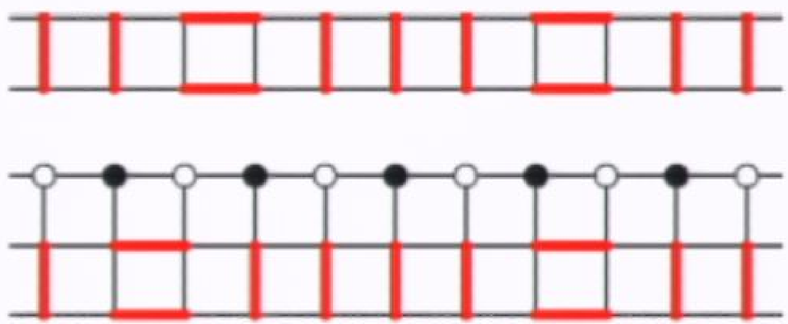

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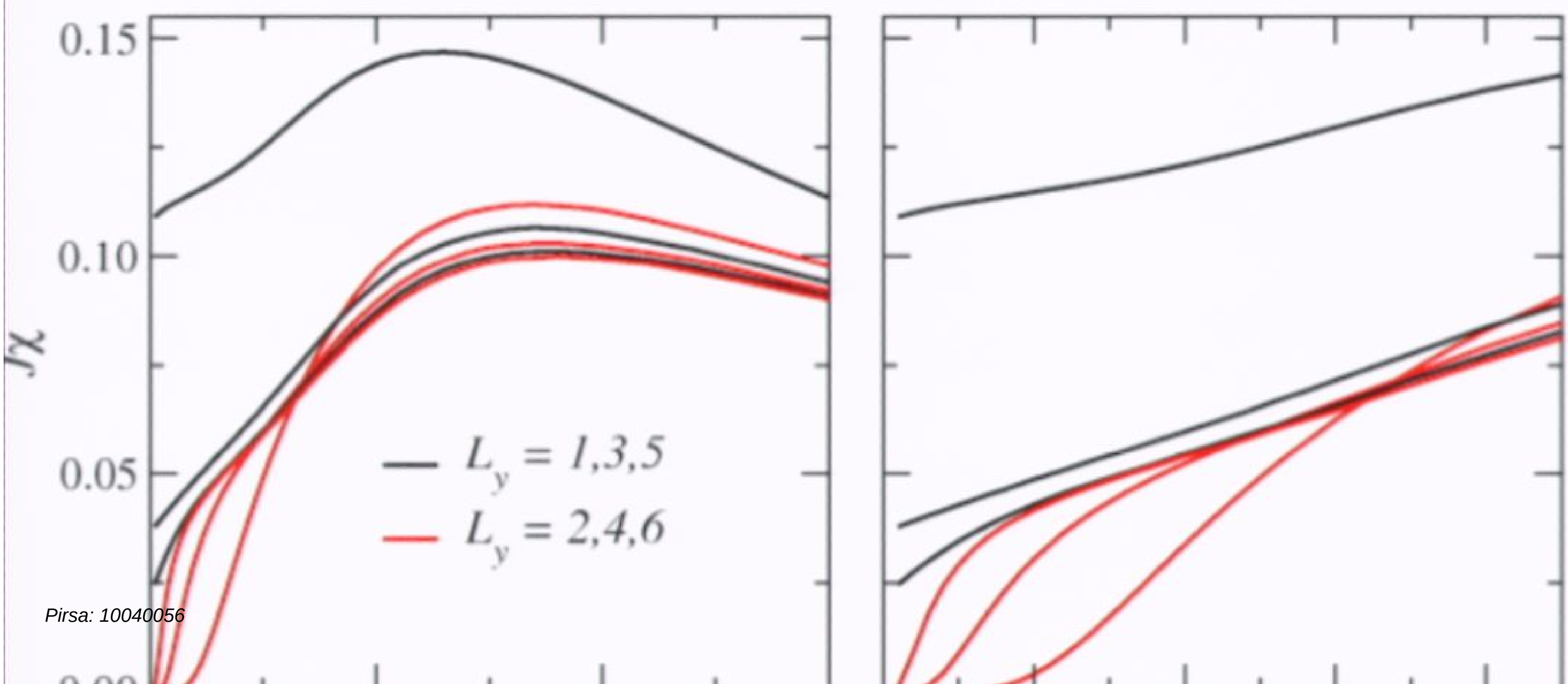

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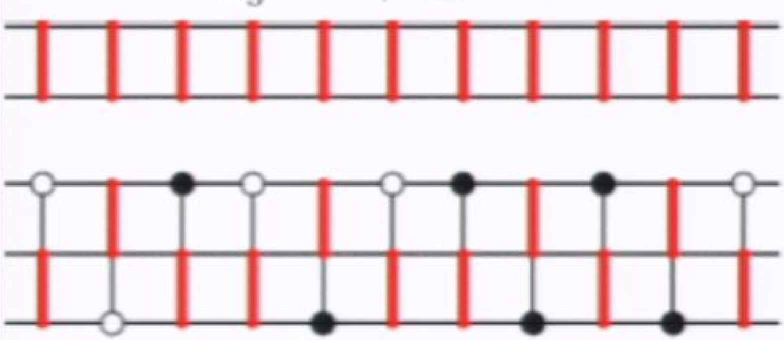

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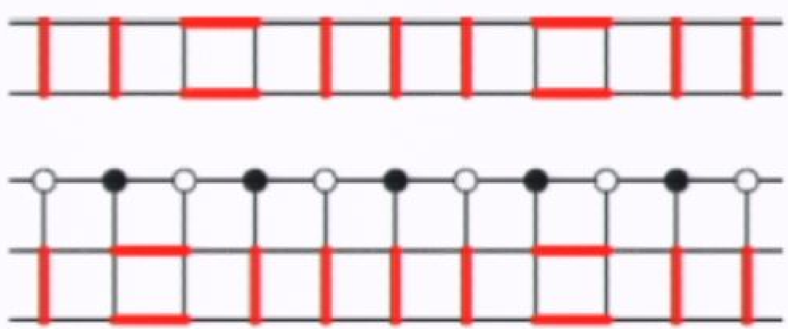

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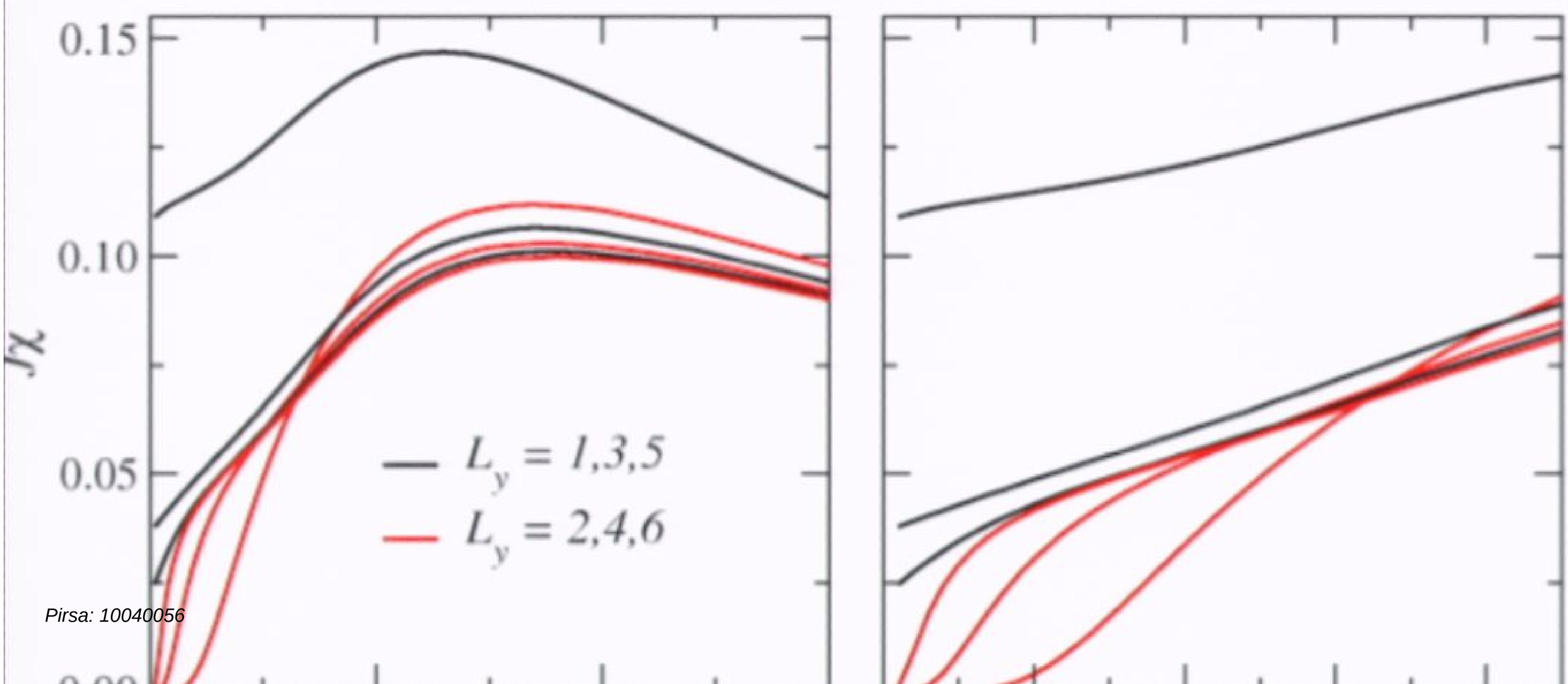
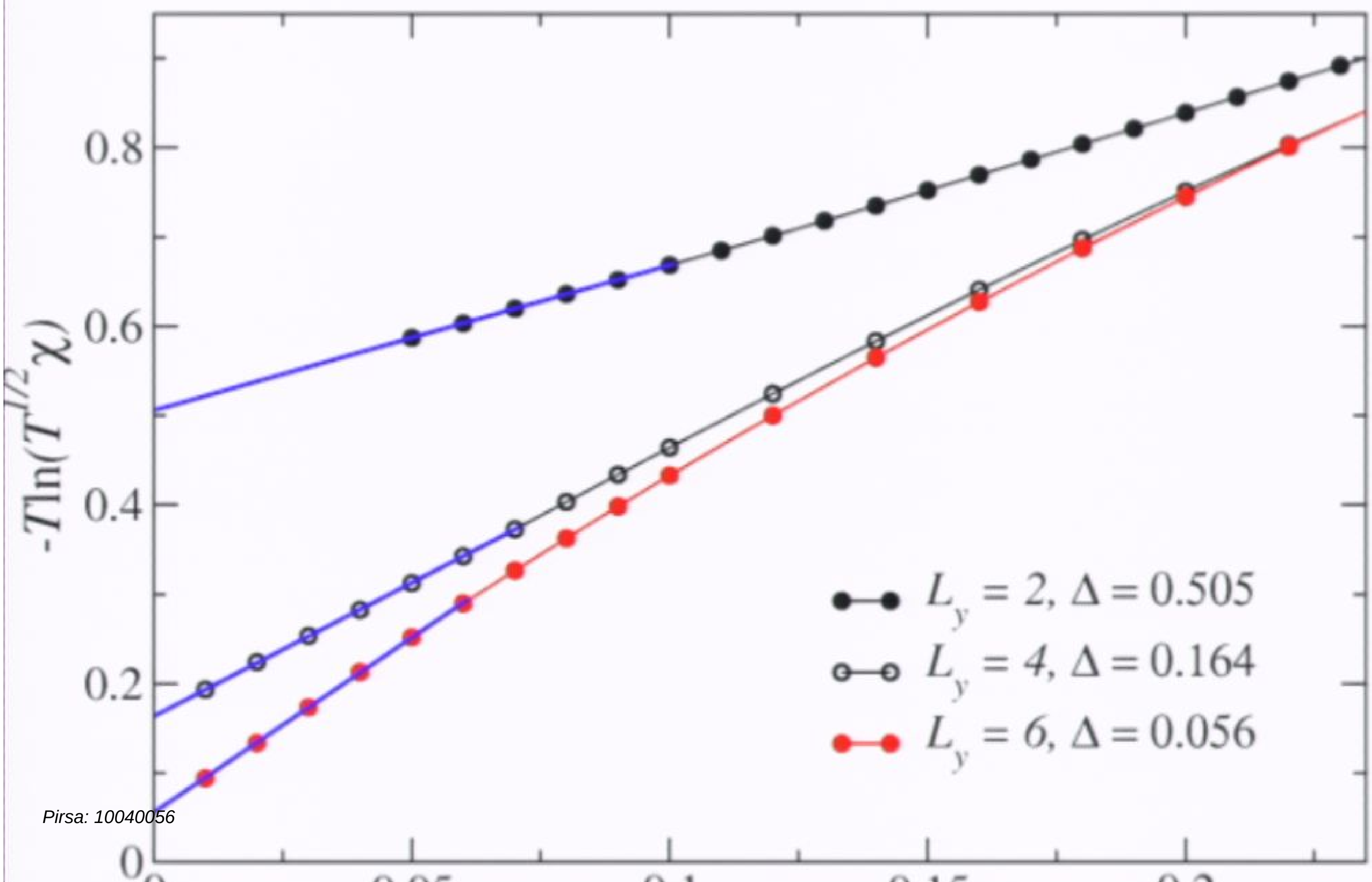


Extracting the gap for evel-L_y systems

From the low-T susceptibility form:

$$\chi(T) = \frac{a}{\sqrt{T}} e^{-\Delta/T} \;\Rightarrow\; -T\ln(\sqrt{T}\chi) = \Delta - T\ln(a)$$



T=0 spin correlations of ladders

Expected **asymptotic** behaviors

$$C(r) = A\frac{(-1)^r}{r}\ln\left(\frac{r}{r_0}\right)^{1/2}$$ (odd L_y)

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We also expect short-distance behavior reflecting 2D order for large L_y

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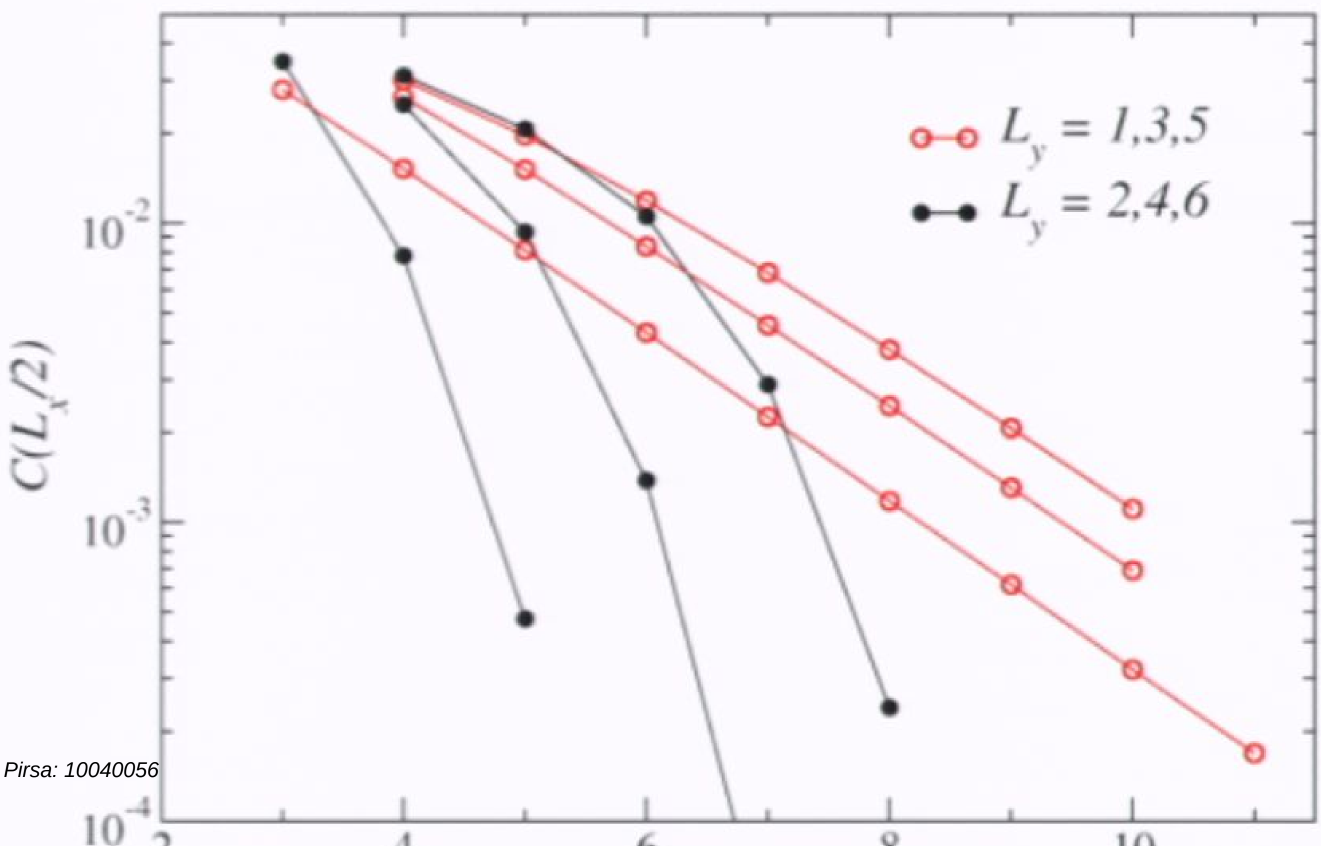

short-long distance cross-over behavior starts to become visible, but larger L_y needed to see signs of 2D order for $r<L_y$

- L×L lattices used to study 2D case

Correlation length for even-L_y

$$C(r) \propto \mathrm{e}^{-r/\xi}, \qquad \xi \propto \frac{1}{\Delta}$$

We need system lengths $L_x >> \xi$ to compute ξ reliably. Use:

$$\xi^2 = \frac{1}{q^2}\left(\frac{S(\pi,\pi)}{S(\pi-q,\pi)} - 1\right) \qquad S(\mathbf{q}) = \sum_{\mathbf{r}} \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{r}} C(\mathbf{r})$$

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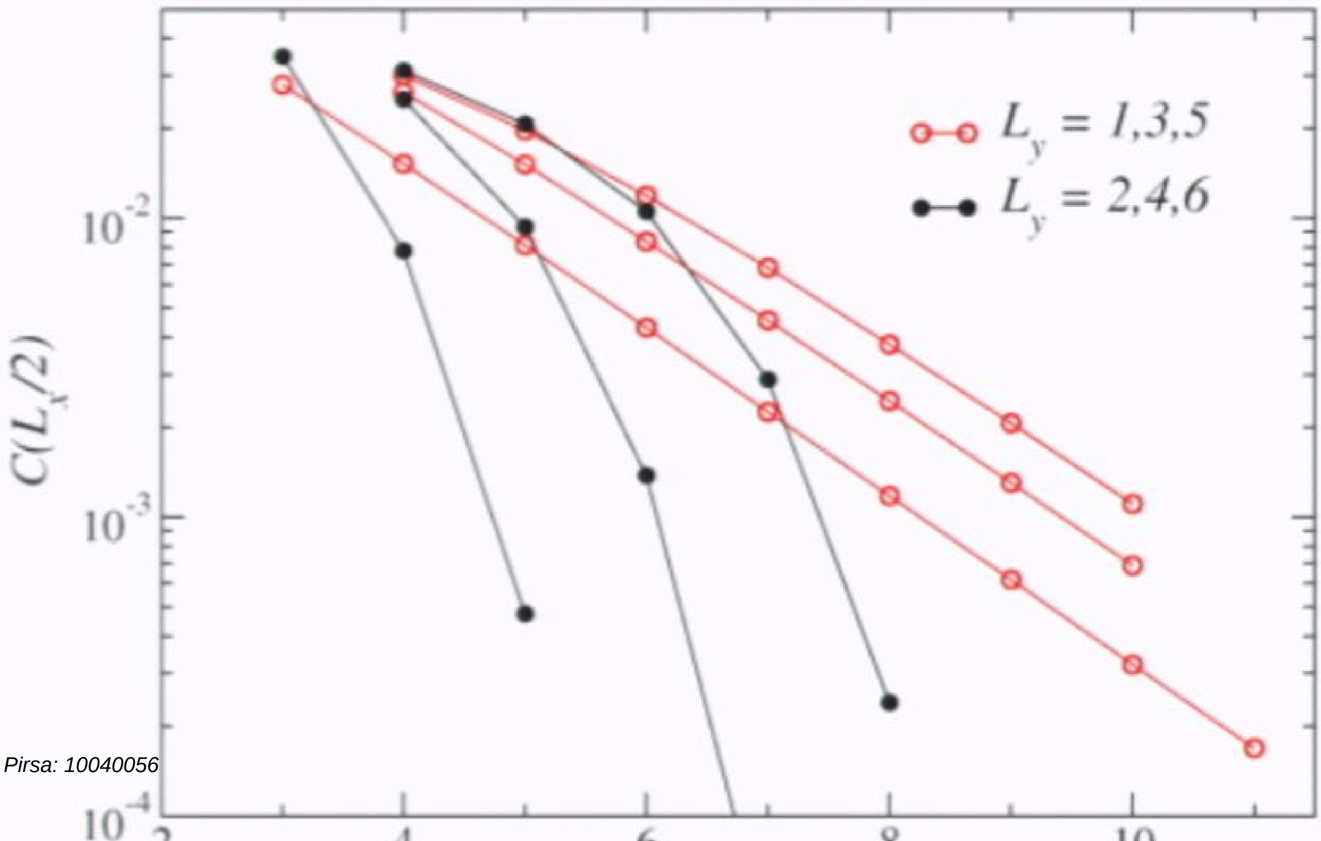

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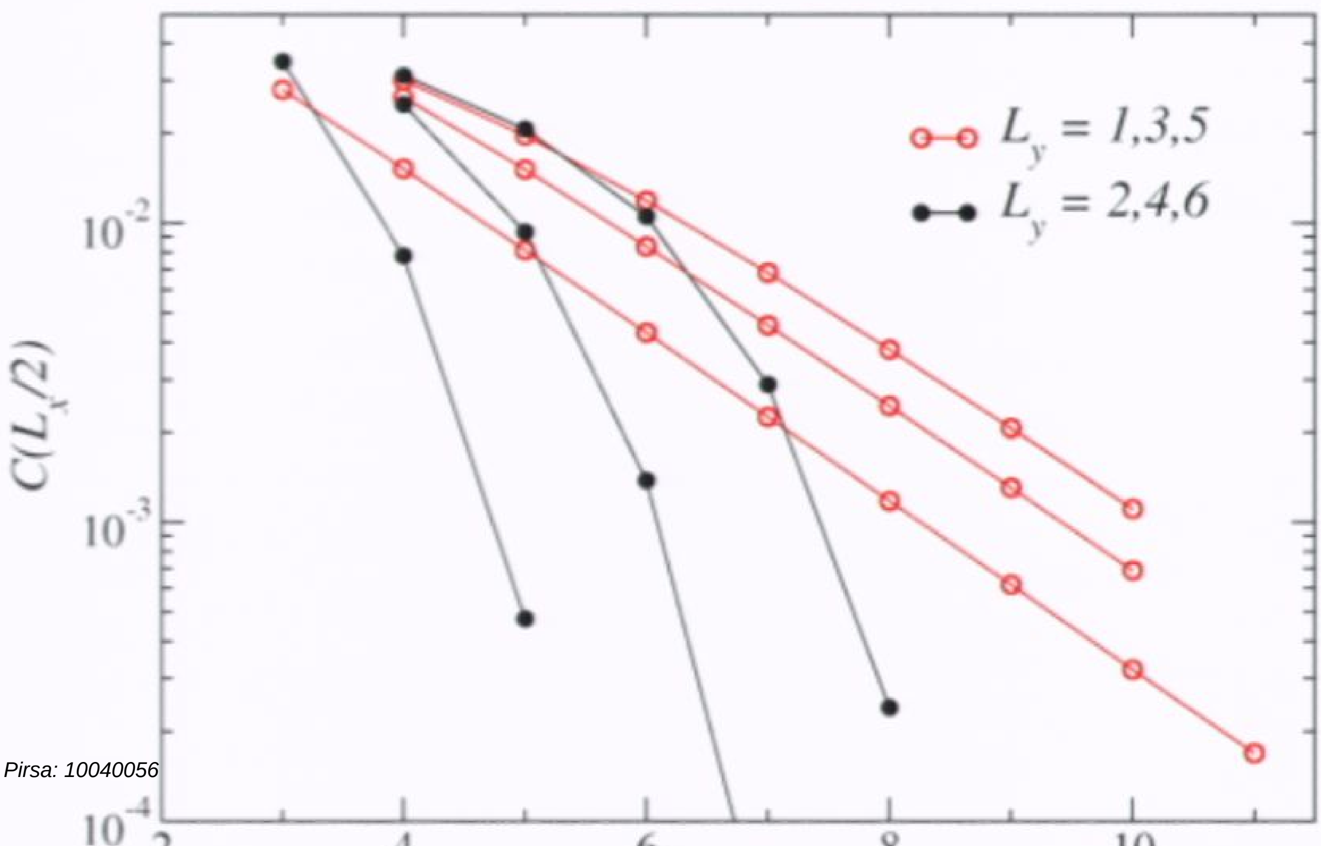

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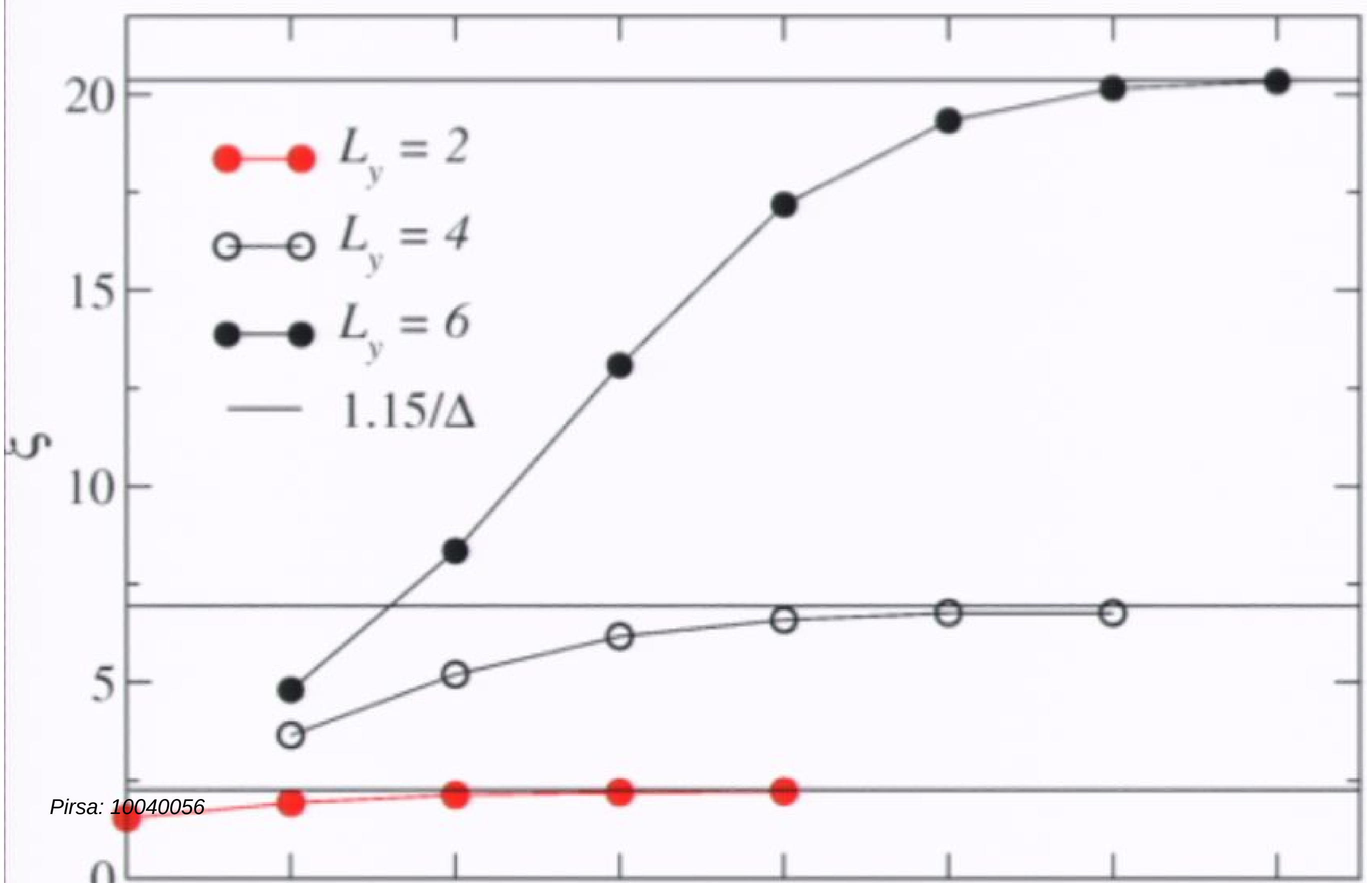
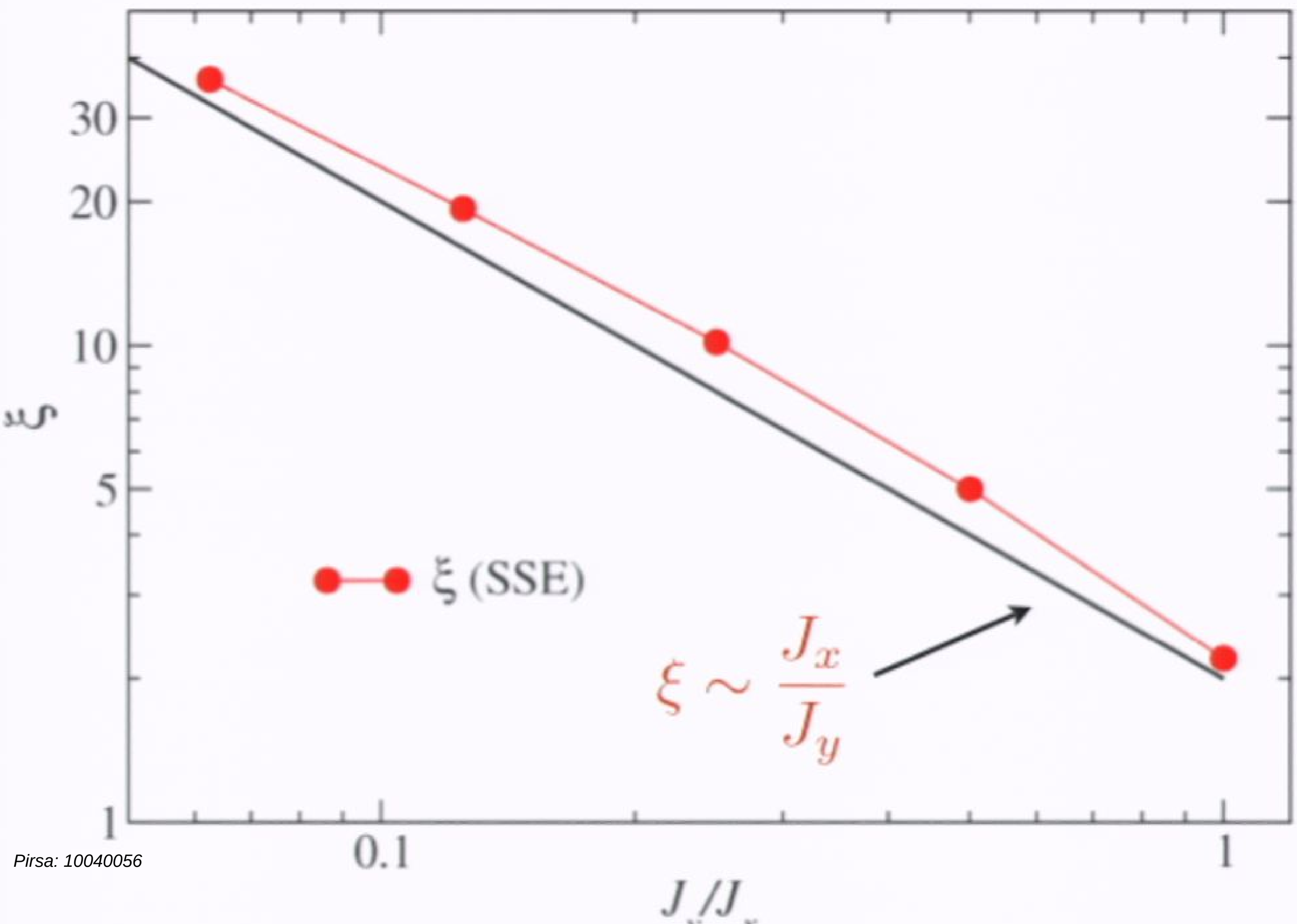


Correlation length versus J_y/J_x for L_y=2

the single chain is critical (1/r correlations) → ξ diverges as J_y/J_x → 0



2D Heisenberg model; long-range order at T=0

Spin-wave theory shows large sublattice magnetization; m_s=0.3034

- including up to $1/S^2$ corrections gives m_s=0.3070
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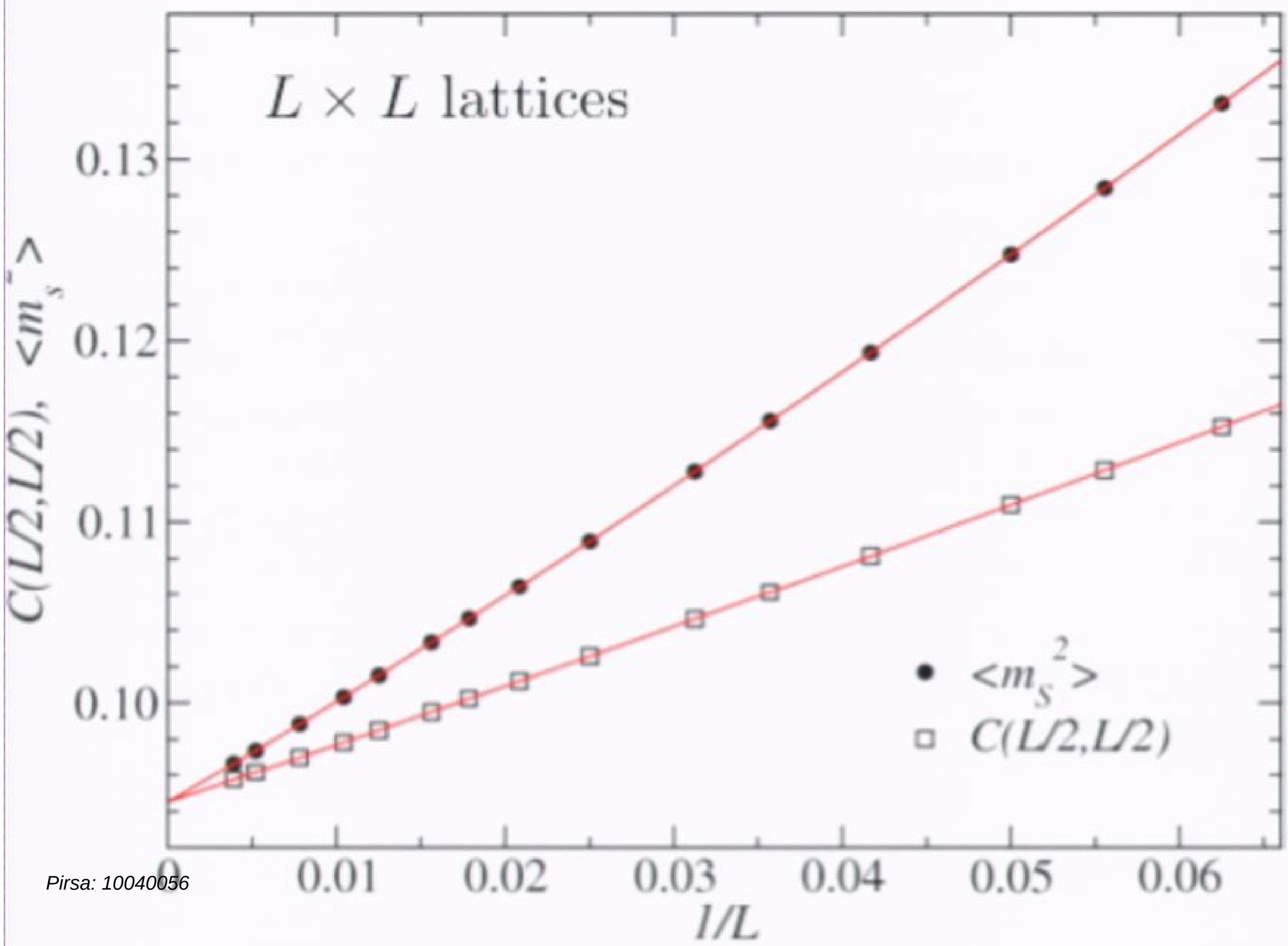

comparing results of

- m_s averaged over all sites (then squared)
- the spin correlation function C(L/2,L/2) at the longest distance

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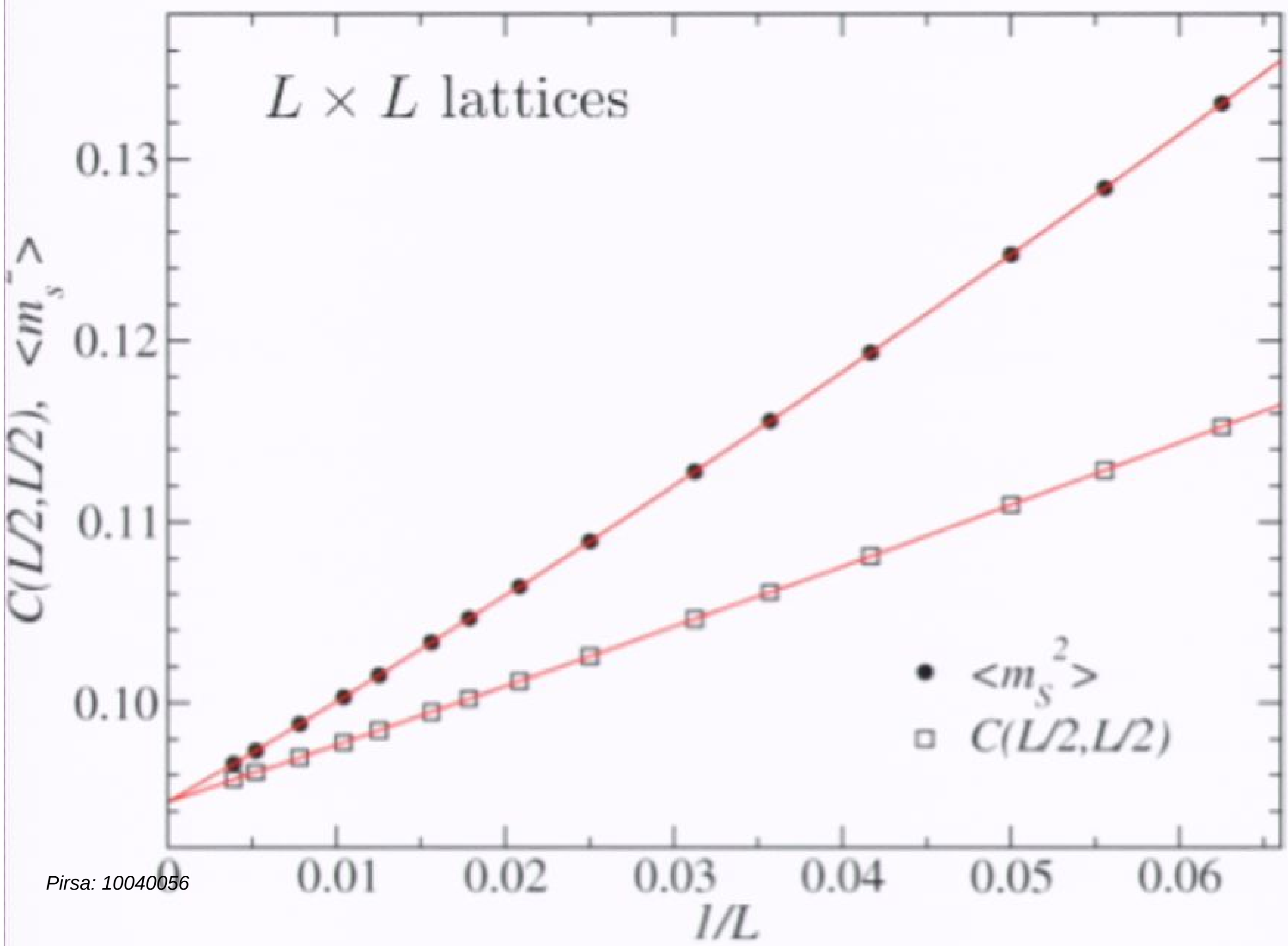

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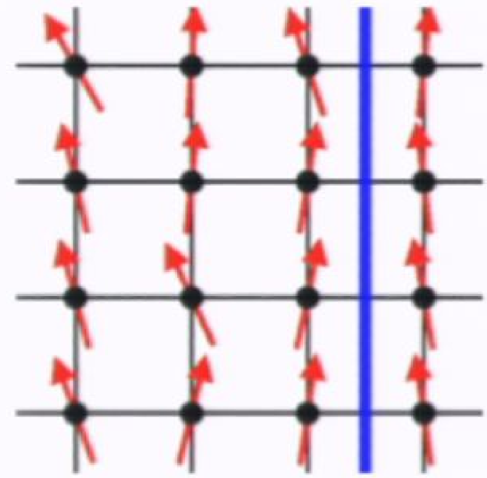
Linear size correction predicted from spin wave theory (and also more general symmetry arguments)

The spin stiffness (helicity modulus)

Corresponds to an Young's modulus of an elastic medium

- an important ground-state parameter of a spin system
- finite for an ordered state
- equivalent to the superfluid stiffness in boson language



Sensitivity of the ground-state energy (free energy at T>0) to "twisting" the spins along a boundary column

$$\rho_s^\gamma = \frac{1}{L}\frac{d^2\langle H(\phi)\rangle}{d\phi^2}, \quad \phi = \text{"twist" at boundary in } \gamma \text{ direction}$$

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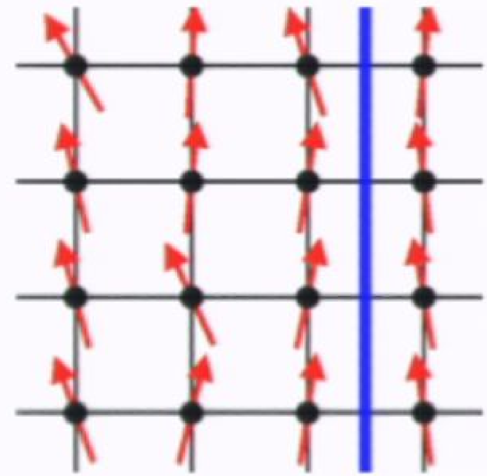

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Twist imposed by changing the Heisenberg interaction at the boundary

$$\mathbf{S}_i \cdot \mathbf{S}_j \to \mathbf{S}_i \cdot R\mathbf{S}_j, \qquad R = \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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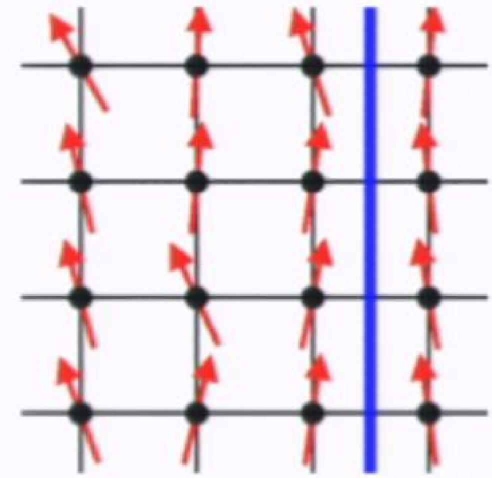

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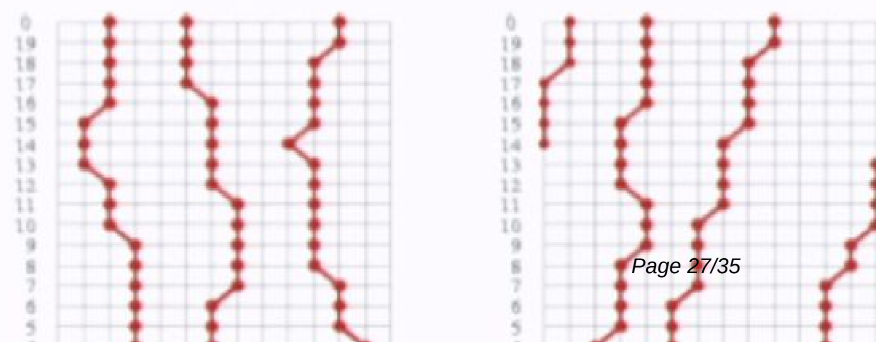

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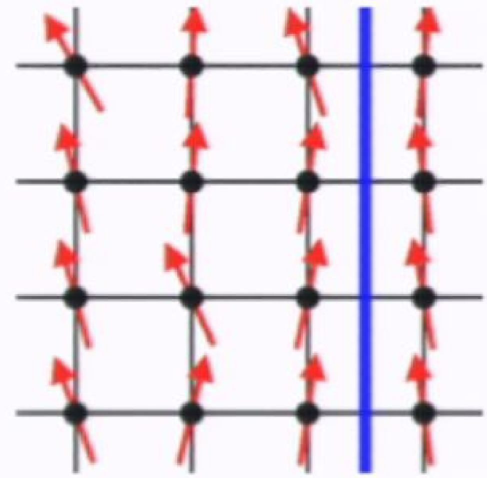

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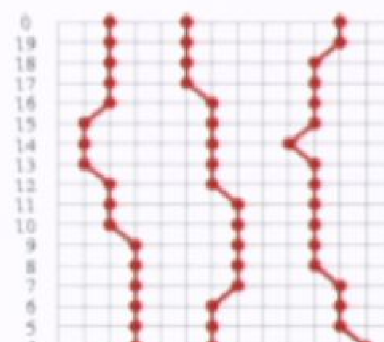
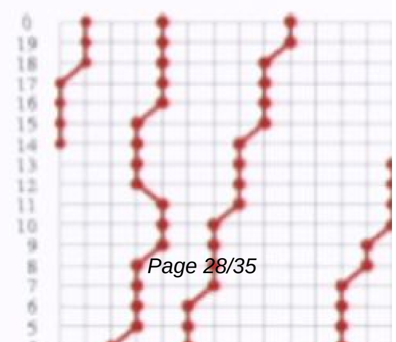


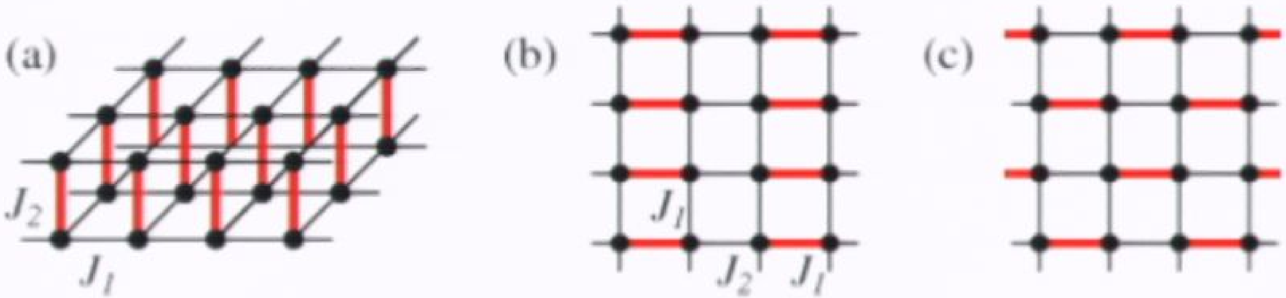


In SSE we have to count spin flip "events"

$$1 \sum^{n-1}$$

2D quantum-criticality (T=0 transition)

Examples: bilayer, dimerized single layer



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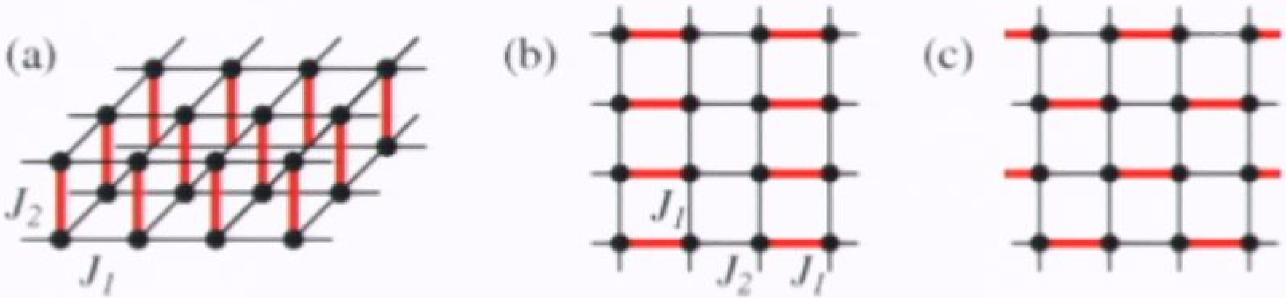
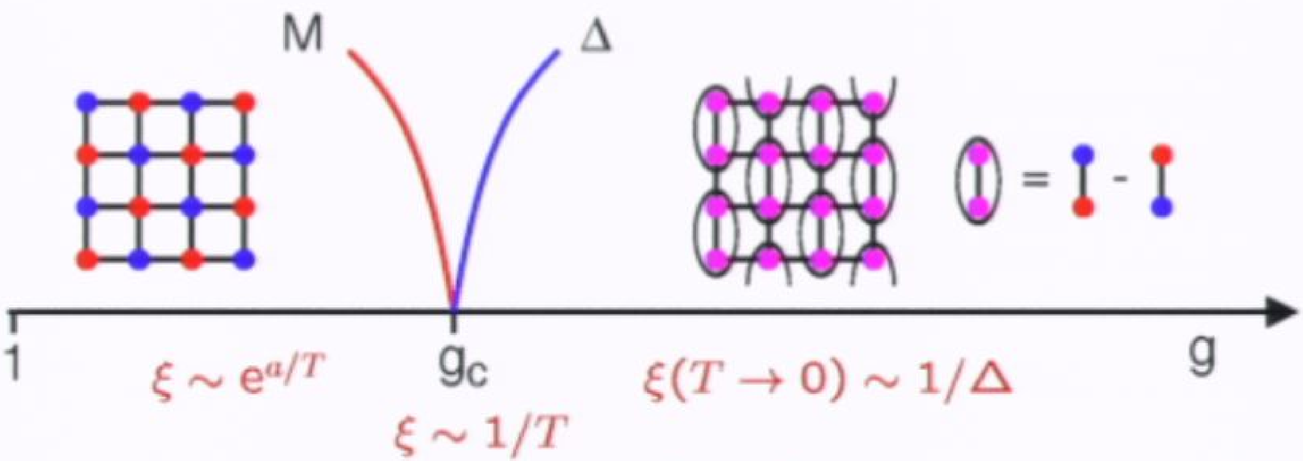


Singlet formation on strong bonds → Neel - disordered transition



2D quantum spins map onto (2+1)D classical spins (Haldane)

- Continuum field theory: nonlinear σ-model (Chakravarty, Halperin, Nelson)

⇒3D classical Heisenberg (O3) universality class expected

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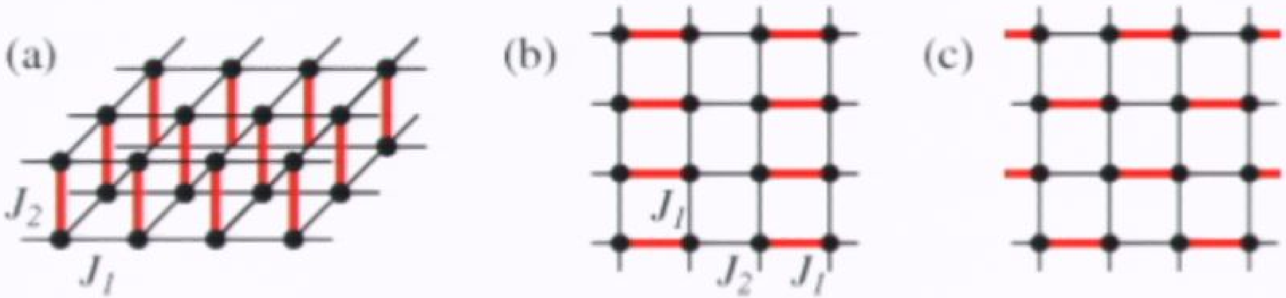

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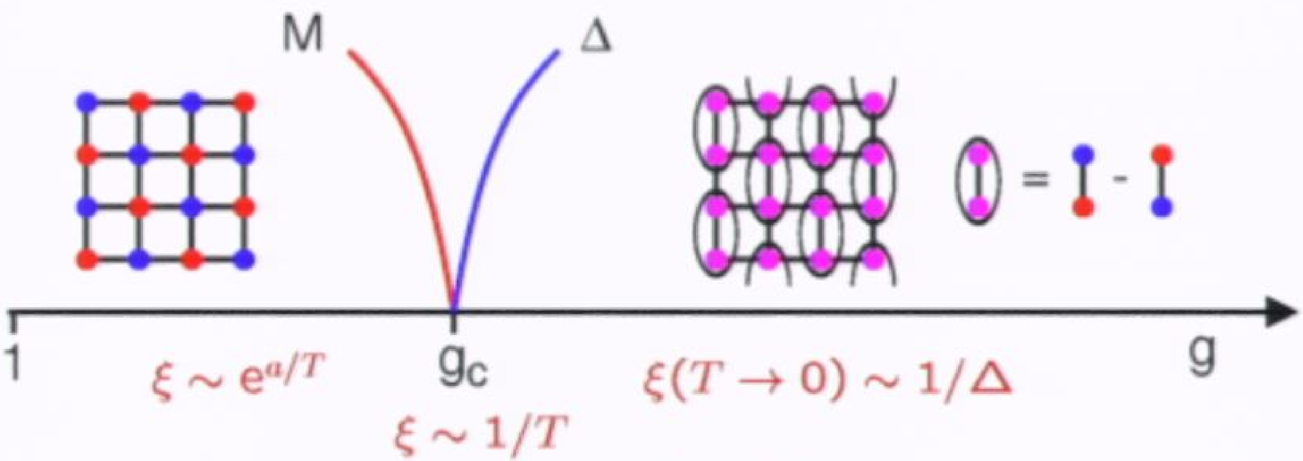

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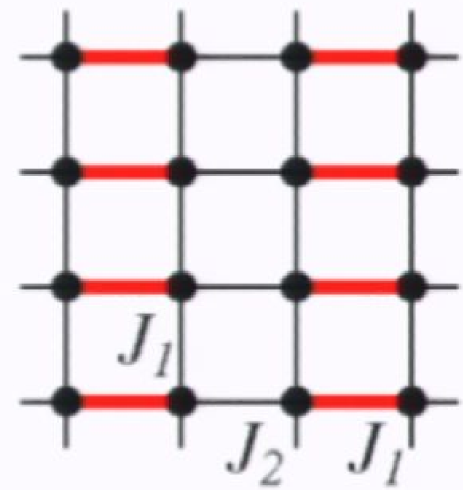
Dynamic Exponent z

- relates space and time directions $\xi_\tau \sim \xi_r^z, \quad \Delta \sim L^{-z}$

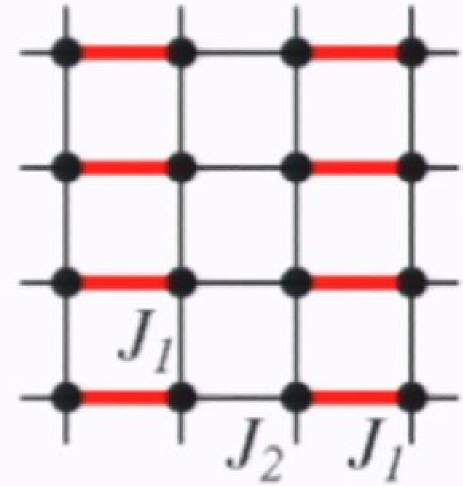
Analysis of the transition of dimerized (columnar) Heisenberg system



Analysis of the transition of dimerized (columnar) Heisenberg system

Two options of choosing the temperature in finite-lattice calculations

- get the ground state as $T \to 0$ limit
  - in practice **$T \ll \Delta$** (finite-size gap)
- use **$1/T = \beta = aL^z$** to analyze the transition
  - if z is known (or to test proposal)
  - the results should not depend on aspect ratio **a**



Analysis of the transition of dimerized (columnar) Heisenberg system

Two options of choosing the temperature in finite-lattice calculations

- get the ground state as $T \to 0$ limit
  - in practice **$T \ll \Delta$** (finite-size gap)
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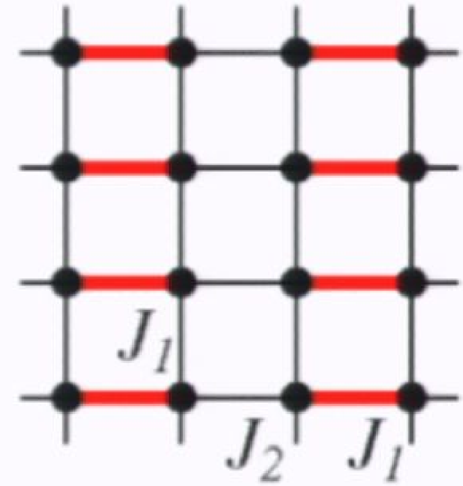

Use the Binder ratio

$$R_2 = \frac{\langle m_{sz}^4 \rangle}{\langle m_{sz}^2 \rangle^2}$$

to locate the critical coupling ratio g_c

Analysis of the transition of dimerized (columnar) Heisenberg system

Two options of choosing the temperature in finite-lattice calculations

- get the ground state as T→0 limit
  - in practice **T<<Δ** (finite-size gap)
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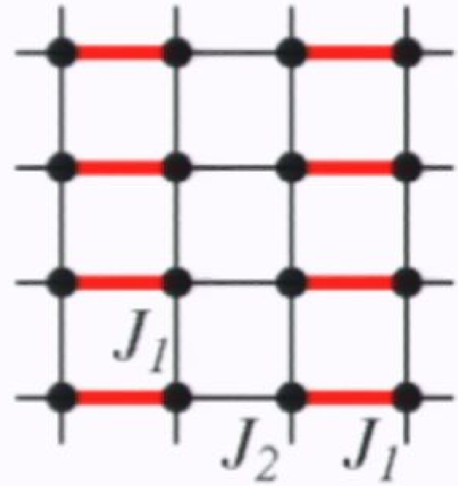

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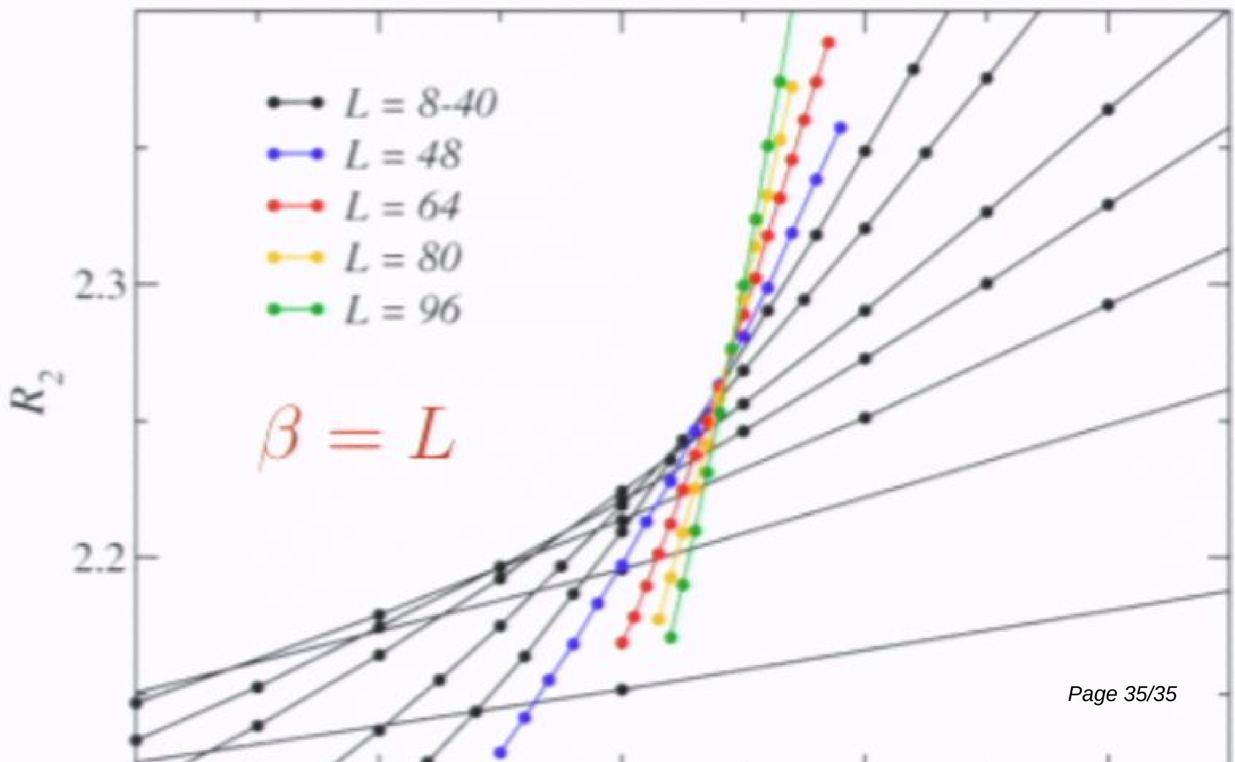
Significant drifts in the crossing points, large lattices needed



$g_c \approx 1.91$