Title: Predicting the Final Spin and Recoil for Black Hole Mergers

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Abstract: After prodigious work over several decades, binary black hole mergers can now be simulated in fully nonlinear numerical relativity. However, these simulations are still restricted to mass ratios q = m2/m1 > 1/10, initial spins a/M < 0.9, and initial separations r/M < 10. Fortunately, analytical techniques like black-hole perturbation theory and the post-Newtonian approximation allow us to study much of this region in parameter space that remains inaccessible to numerical relativity. I will use black-hole perturbation theory to establish a fundamental upper limit to the final spin that can be attained through binary mergers, and show how this limit can be used to improve predictions of final spins for finite mass ratios as well. I will also show that post-Newtonian inspirals between 1000 M < r < 10 M can align or anti-align black hole spins with each other, dramatically changing the distributions of final spins and recoil velocities that would be expected in astrophysical black hole mergers.

Predicting the Final Spin for Black Hole Mergers

Michael Kesden (Caltech)

Kesden, Lockhart, Phinney (in preparation)
Kesden, Sperhake, Berti arXiv:1002:2643
Kesden, Sperhake, Berti arXiv:1003:4993

Perimeter Institute – Waterloo, ON – April 6, 2010

Outline

- What are binary black hole mergers?
- Maximum spin from quasi-circular binary mergers
  - History of theoretical limits to BH spins
  - Calculation of new fundamental limit from BH mergers
  - Implications for predicting final spins
- Final spins from precessing binary black-hole mergers
  - Spin-orbit resonances (Cassini states?)
  - Alignment of BBH spins
  - Implications for final spins and recoil velocities

Spinning Black Holes

- Kerr 1963 discovered the unique stationary, axisymmetric vacuum solution to Einstein's equations

$$ds^2 = (r^2 + a^2\cos^2\theta)(d\theta^2 + \sin^2\theta d\phi^2) + 2(du + a\sin^2\theta d\phi)$$
$$\times (dr + a\sin^2\theta d\phi) - \left(1 - \frac{2mr}{r^2 + a^2\cos^2\theta}\right)$$
$$\times (du + a\sin^2\theta d\phi)^2,$$

- This solution ceases to be a BH for spins above the Kerr limit $a_* \equiv a/m = 1$, where its event horizon vanishes.
- Can BHs with spins at or beyond this Kerr limit be realized in nature?

Yes they can!!!

- Bardeen 1970 showed that if BHs accrete a rest mass ΔM_0 from the marginally stable (ms) orbit

$$\Delta M = E^{\dagger}{}_{\rm ms} \Delta M_0 \,, \qquad \Delta J = L^{\dagger}{}_{\rm ms} \Delta M_o$$

$$\frac{da_*}{d \ln M} \equiv \frac{d(J/M^2)}{d \ln M} = \frac{1}{M} \frac{L^{\dagger}{}_{\rm ms}(a_*, M)}{E^{\dagger}{}_{\rm ms}(a_*)} - 2a_*$$

$$dM/dM_o = E^{\dagger}{}_{\rm ms}(a_*) \,.$$

- A non-spinning BH of mass M_i reaches the Kerr limit $a_* = 1$ after growing to a mass $M_f = \sqrt{6}\, M_i$

No they can't!!!

- Thorne 1974 showed that if this rest mass is accreted from a geometrically thin, optical thick accretion disk:
  - Energy must be radiated for the mass to reach the ISCO
  - Some of these radiated photons will be captured by the BH
  - Photons with *retrograde* orbital angular momentum will be preferentially captured
- These retrograde photons spin down the BH, establishing an upper limit to the BH spin $a_{lim} < 1$.
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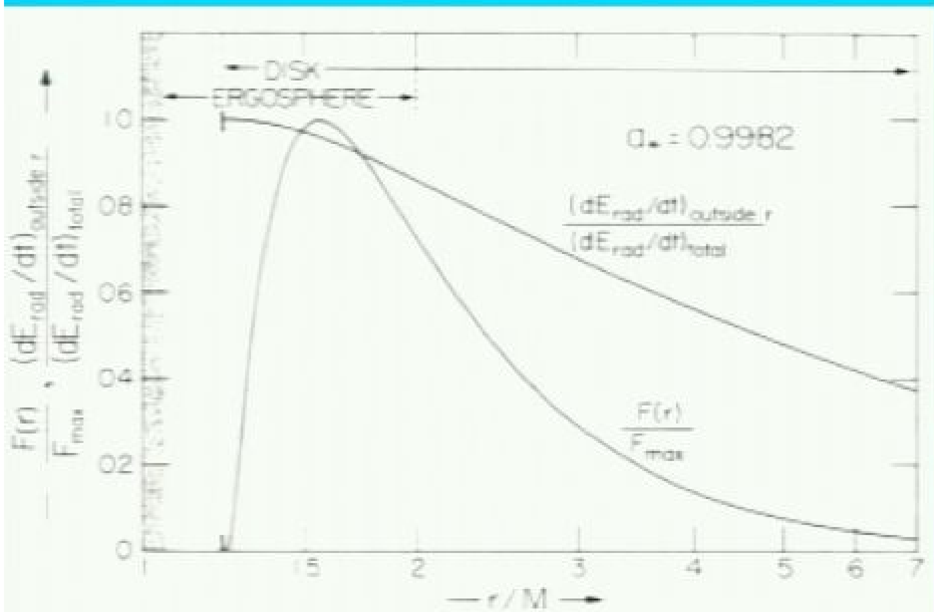
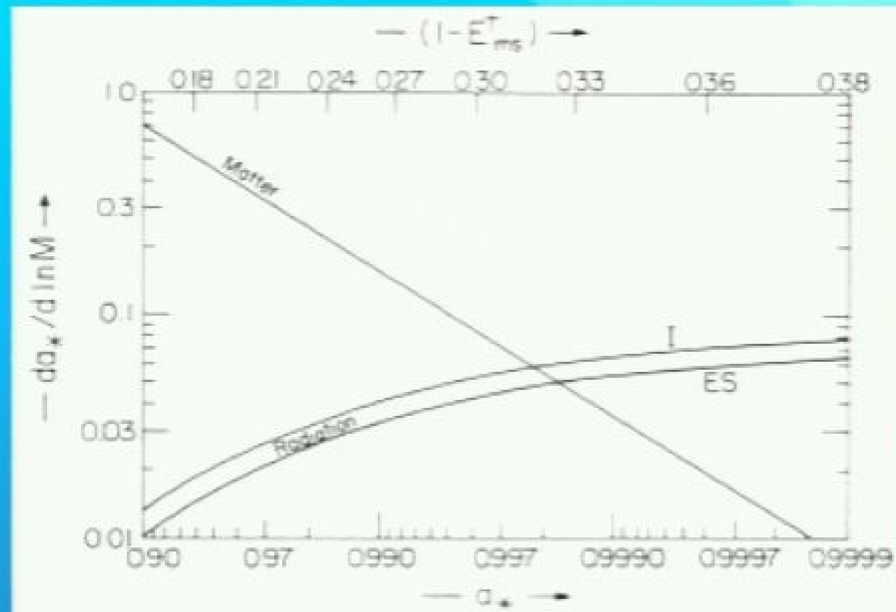
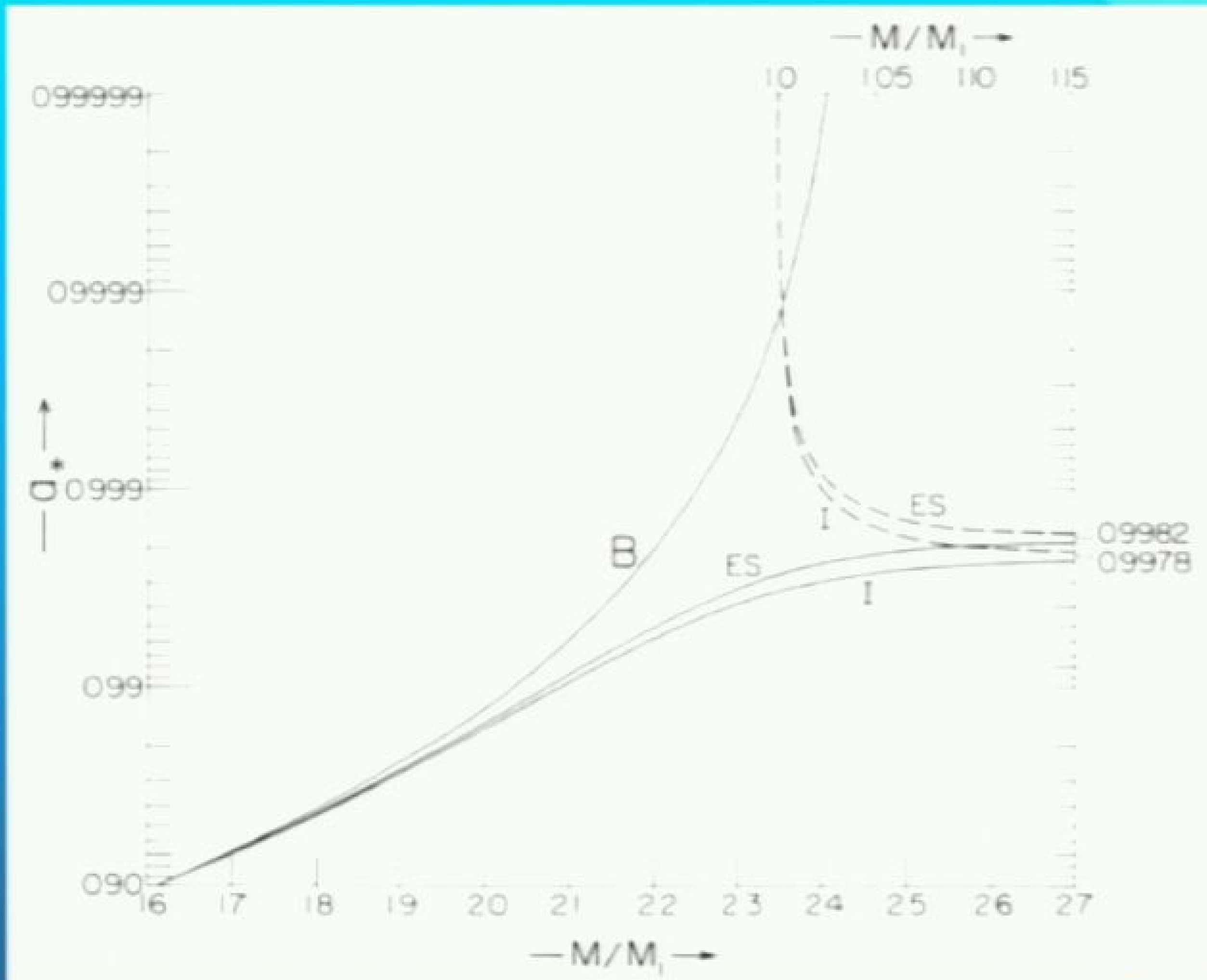
Radiatively efficient disks





- **I:** $\langle I(\Theta, \Phi)\rangle$ independent of Θ and Φ
- **ES:** $\langle I(\Theta, \Phi)\rangle \propto 1 + 2\cos\Theta$

A limit to BH spins



How fundamental is this limit?

- This limiting spin depends on the intensity $<I(\Theta, \Phi)>$ of radiated photons.
- What if no photons are emitted as all, as during binary black hole inspirals?
- Photons (γs)
  - $\lambda \ll R_S = 2GM/c^2$
  - Incoherently emitted γs act like particles
  - Captured when null geodesics cross horizon
- Gravitational waves (GWs)
  - $\lambda \sim \Omega_{orb}^{-1} \gg R_S$
  - Coherently emitted GWs act like classical waves
  - When are GWs captured?

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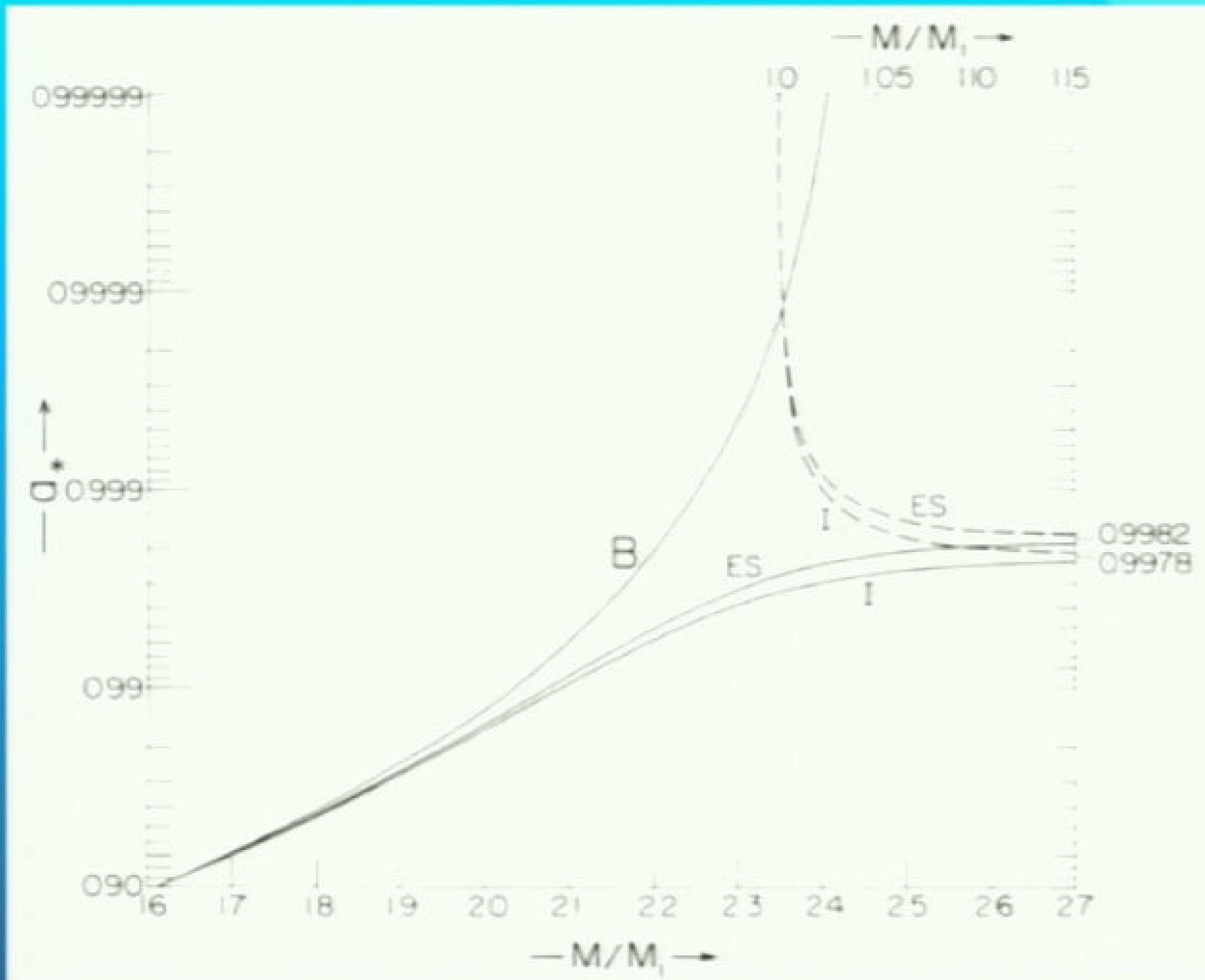

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Black Hole Perturbation Theory

- Einstein's equation is *nonlinear* in the metric

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}.$$

- In the test-particle limit $q \equiv m/M \ll 1$
  - Small metric perturbations about Kerr background
  - Vacuum perturbations fully described by Weyl scalar

$$\psi_4 \equiv -C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$$

  - *Linearize* Einstein's eq. in these small perturbations

Teukolsky equation

- *Nonlinear* Einstein eq. ⇒ *linear* Teukolsky eq.
- Teukolsky eq. is *separable*
  - decompose Ψ_4 into harmonics:

$$\psi_4 = \frac{1}{(r - ia\cos\theta)^4} \int_{-\infty}^{\infty} d\omega \sum_{lm} R_{lm\omega}(r) \, {}_{-2}S_{lm}^{a\omega}(\theta) e^{im\phi} e^{-i\omega t}$$

  - radial function satisfies wave equation:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \Rightarrow \quad \Delta^2 \frac{d}{dr}\left(\frac{1}{\Delta}\frac{dR_{lm\omega}}{dr}\right) - V(r) R_{lm\omega}(r) = -T_{lm\omega}(r)$$

- Calculate GW fluxes to ∞ and down the horizon at r_+
- Implemented with GREMLIN code (Hughes 2000)

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Superradiant scattering

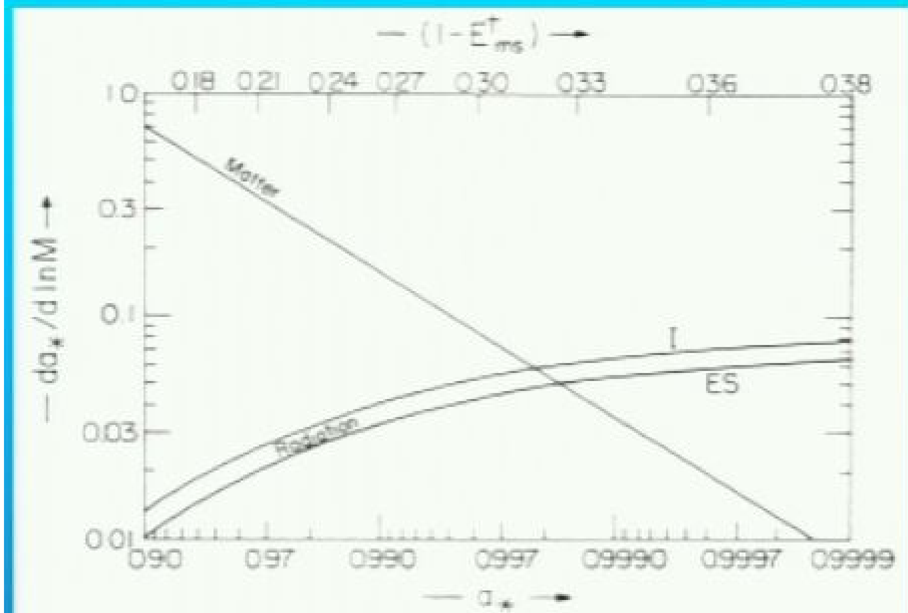
- For highly spinning BHs, GWs are scattered to ∞ with *larger* amplitudes than they were emitted with
- Energy and angular momentum are extracted from BH

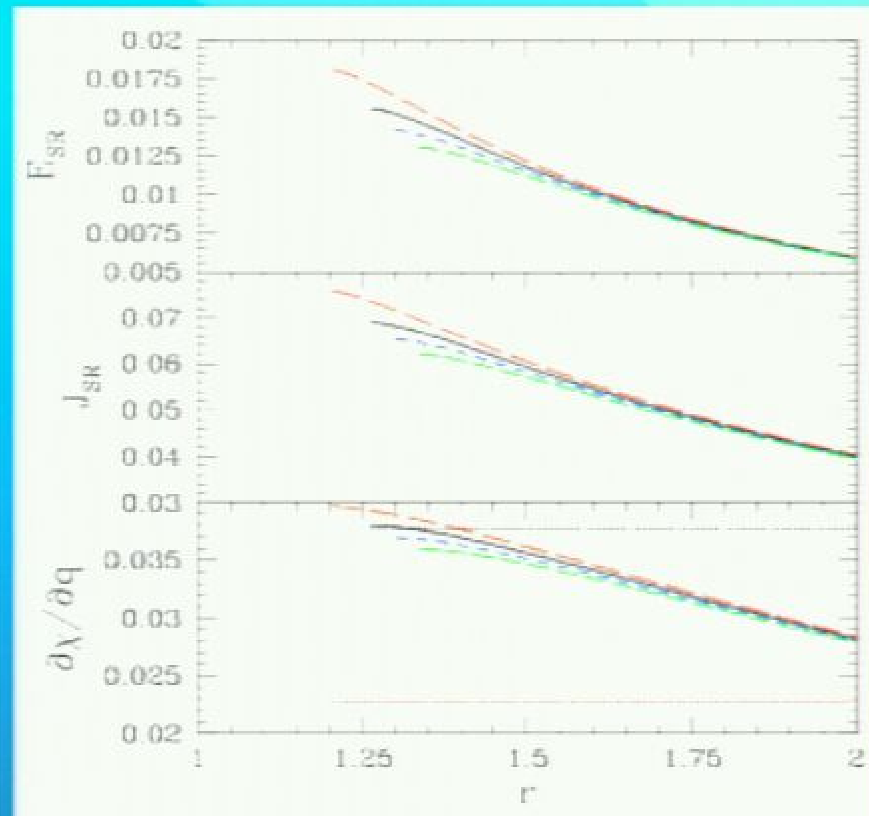
$$E_{\rm SR}(\chi, r) = \int_r^\infty \left(\frac{dE}{dt}\right)^{\rm rad}_{r\to r_+} \frac{dr'}{\dot{r}}$$

$$J_{\rm SR}(\chi, r) = \int_r^\infty \left(\frac{dL_z}{dt}\right)^{\rm rad}_{r\to r_+} \frac{dr'}{\dot{r}}$$

- This sets a new GR limit to BH final spins, where the positive torque from the accreted material balances the negative torque from the scattered GWs.

Torque balance





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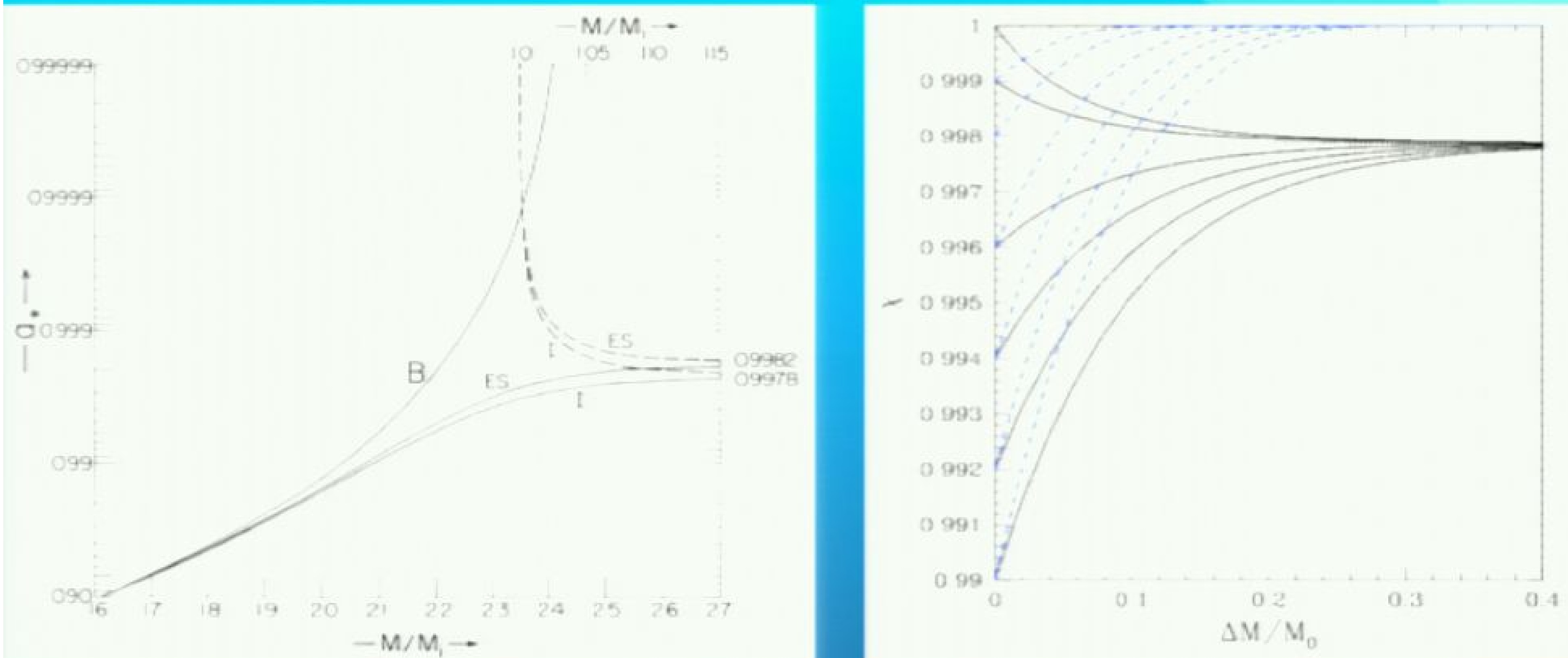
$$\frac{\partial \chi_f}{\partial q}(\chi_1, r) = \frac{\partial \chi_{\rm ISCO}}{\partial q}(\chi_1) - \frac{\partial \chi_{\rm SR}}{\partial q}(\chi_1, r)$$

$$\frac{\partial \chi_{\rm ISCO}}{\partial q}(\chi_1) \to L_{\rm ISCO}(\chi_1) - 2\chi_1 E_{\rm ISCO}(\chi_1)$$

$$\frac{\partial \chi_{\rm SR}}{\partial q}(\chi_1, r) \to J_{\rm SR}(\chi_1, r) - 2\chi_1 E_{\rm SR}(\chi_1, r)$$

Limiting spin



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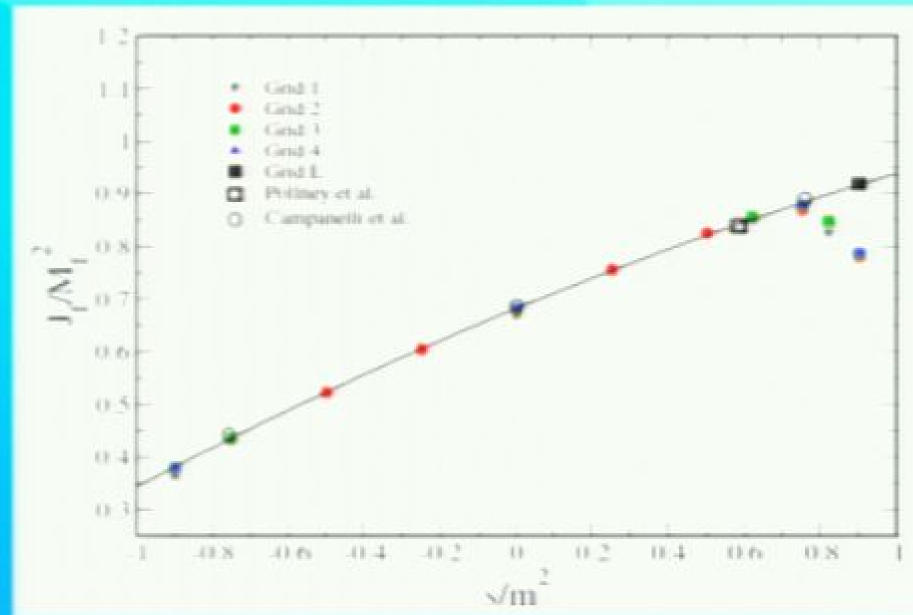
- Can it tell us anything about comparable-mass mergers?

Numerical Relativity



- Pretorius 2005 successfully merged BHs in full numerical relativity for the very first time
- Simulations are limited to
  - Initial data with spins $a_* < 0.9$ (Marronetti *et al.* 2008)
  - Mass ratios $q \equiv m_2/m_1 \geq 0.1$ (Gonzalez *et al.* 2009)
- Cannot simulate the regime of our previous calculations, making direct comparison difficult

Fortune favors the bold ...

- Hughes and Blandford 2003

$$M_{f,\mathrm{HB}} = m_1 + m_2 E_{\mathrm{ISCO}}(\chi_1) ,$$
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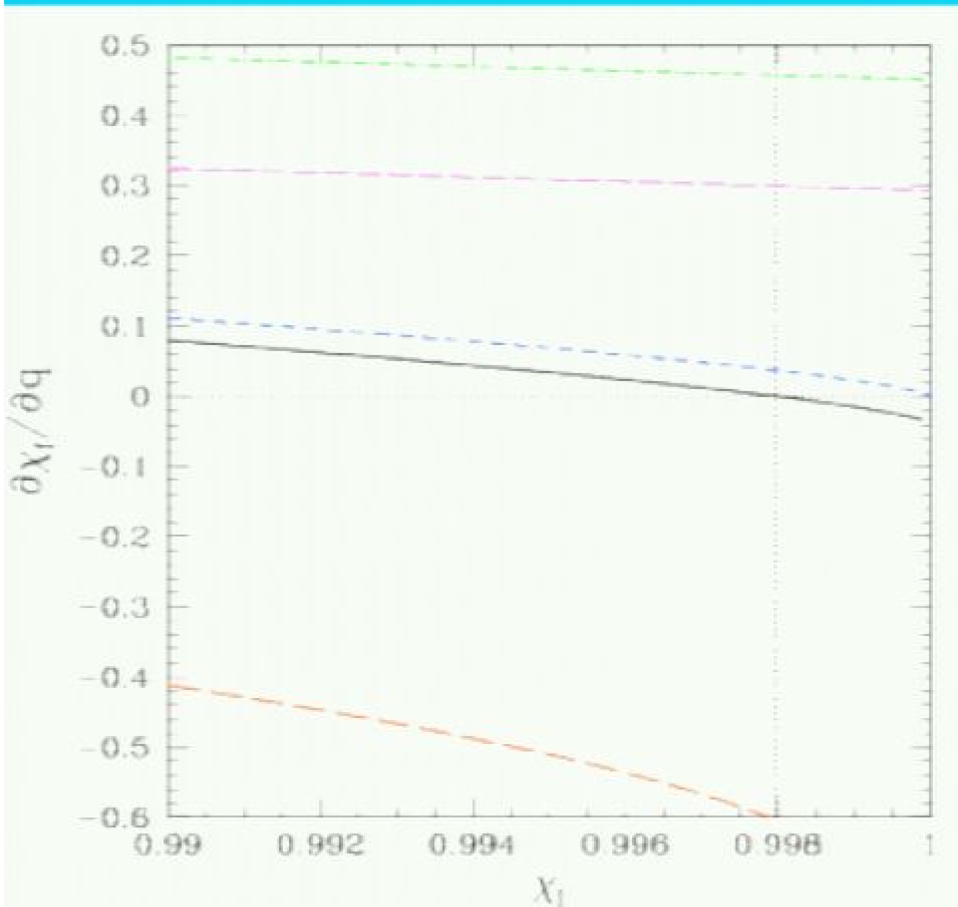
Numerical relativity vs. test-particle extrapolation



χ_i	-0.43757 [13]	0.0 [12]	1.0 [15]
NR	0.547812 ± 0.000009	0.68646 ± 0.00004	0.951 ± 0.004
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Conclusions I

- General relativity allows BHs to have spins up to the Kerr limit $a_* \equiv a/m = 1$
- Astrophysical BHs will have spins that depend sensitively on how they were assembled
- Kip Thorne showed in 1974 that the preferential accretion of photons with negative L limits $a_* < 0.998$
- We showed that the superradiant scattering of GWs sets the *same* limit for BHs formed in binary mergers
- Black hole perturbation theory complements numerical relativity by predicting final spins for small mass ratios and high spins

Spin precession in BH mergers

- Numerical relativity indicates final spin/recoils depend sensitively on BH spin *orientation* at merger.
- Why should astrophysical BH mergers be aligned?
- Tidal torque by a circumbinary disk may align BH spins with disk orbital angular momentum (Bogdanović *et al.* 2007).
- Effectiveness of this alignment depends on the accretion flow, 10°(30°) for cold (hot) disks (Dotti *et al.* 2009).
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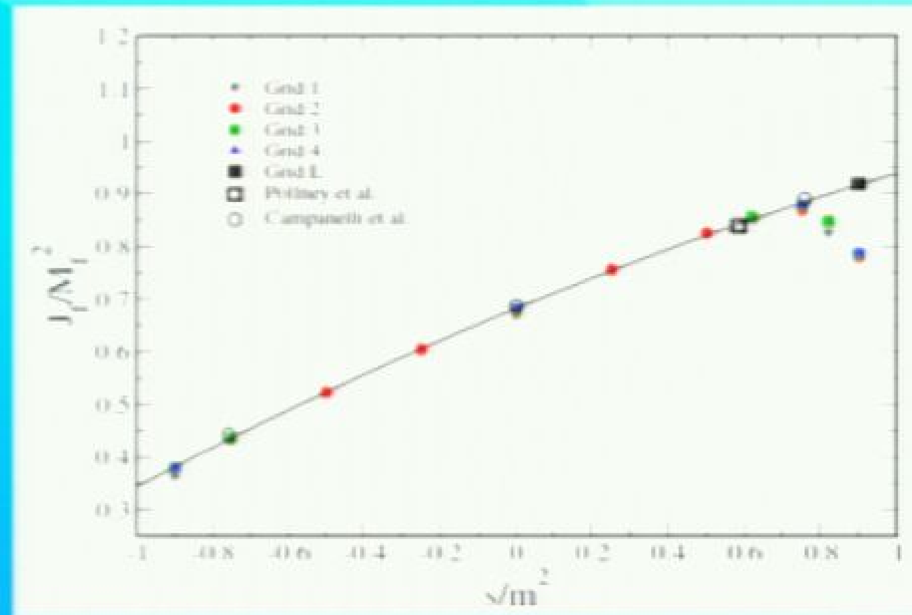

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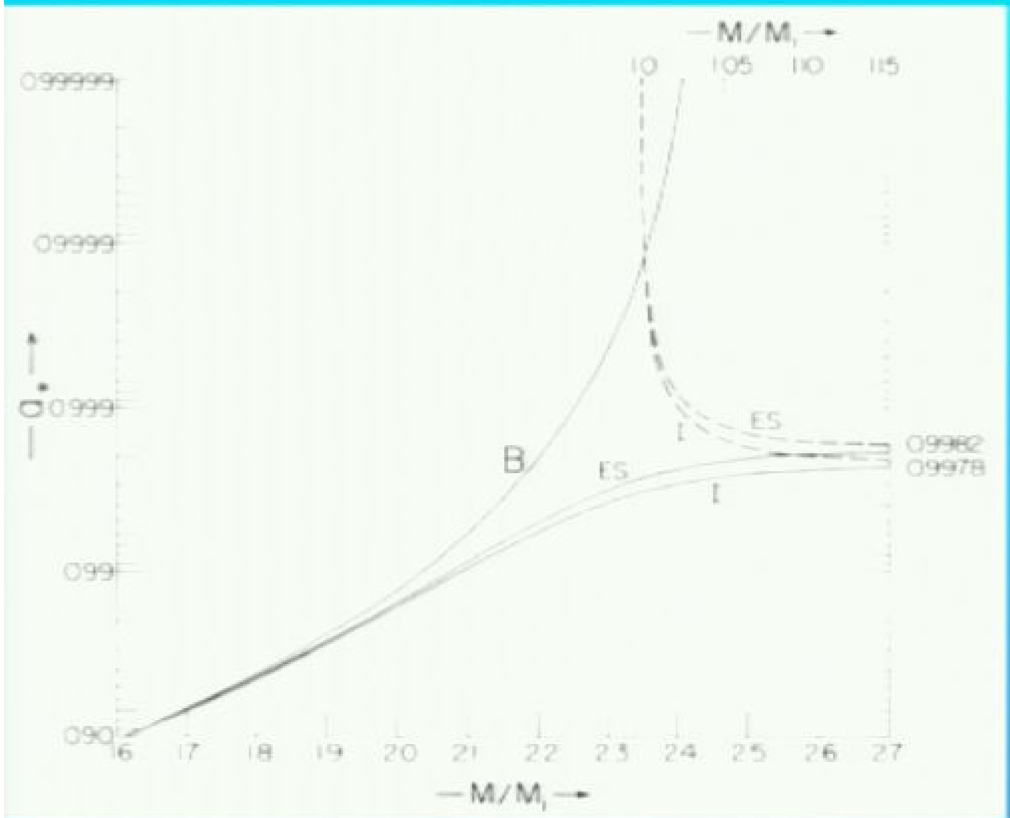

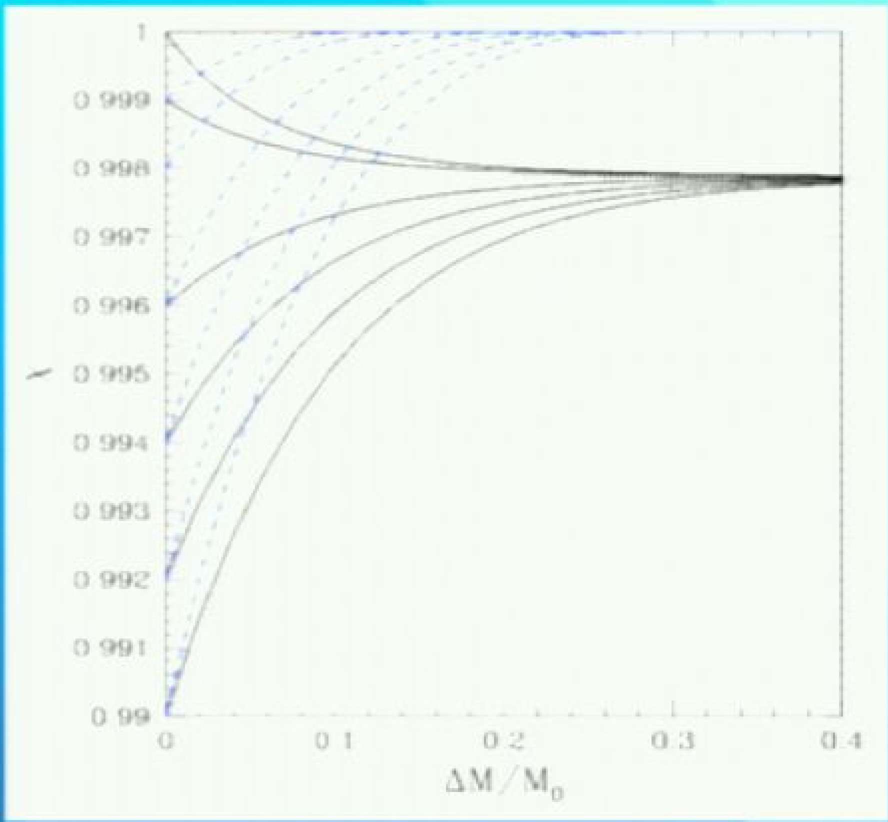

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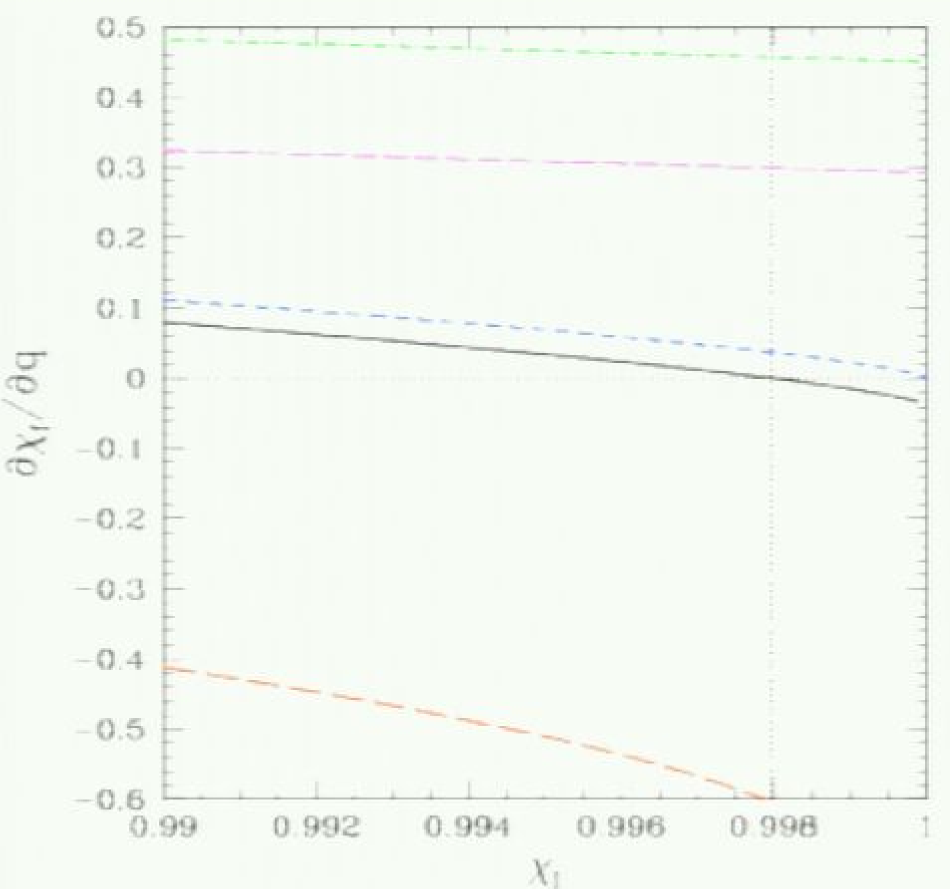

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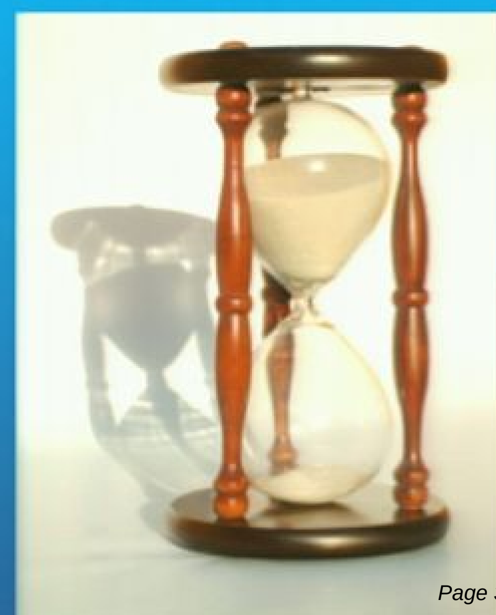
Hierarchy of timescales

- Orbital time: $t_{orb} \sim r^{3/2}$
- Precession time: $t_p \sim r^{5/2}$
- Inspiral time: $t_{GW} = E/(dE/dt)_{GW} \sim r^4$
- $r \gg M \Rightarrow t_{orb} \ll t_p \ll t_{GW}$





Spin precession

- $t_{orb} \ll t_p \Rightarrow$ *orbit-averaged* precession equations:

$$\dot{\mathbf{S}}_1 = \bar{\mathbf{\Omega}}_1 \times \mathbf{S}_1 \,, \tag{2.1a}$$

$$\dot{\mathbf{S}}_2 = \bar{\mathbf{\Omega}}_2 \times \mathbf{S}_2 \,, \tag{2.1b}$$

where

$$\bar{\mathbf{\Omega}}_1 = \frac{1}{2r^3}\left[\left(4 + 3q - \frac{3(\mathbf{S}_2 + q\mathbf{S}_1)\cdot\mathbf{L}_N}{L_N^2}\right)\mathbf{L}_N + \mathbf{S}_2\right] , \tag{2.2a}$$

$$\bar{\mathbf{\Omega}}_2 = \frac{1}{2r^3}\left[\left(4 + \frac{3}{q} - \frac{3(\mathbf{S}_1 + q^{-1}\mathbf{S}_2)\cdot\mathbf{L}_N}{L_N^2}\right)\mathbf{L}_N + \mathbf{S}_1\right] \tag{2.2b}$$

Orbital angular momentum evolution

$$\mathbf{L}_N = \eta M \mathbf{r} \times \mathbf{v} = \frac{\eta M^2}{(M\omega)^{1/3}} \hat{\mathbf{L}}_N$$

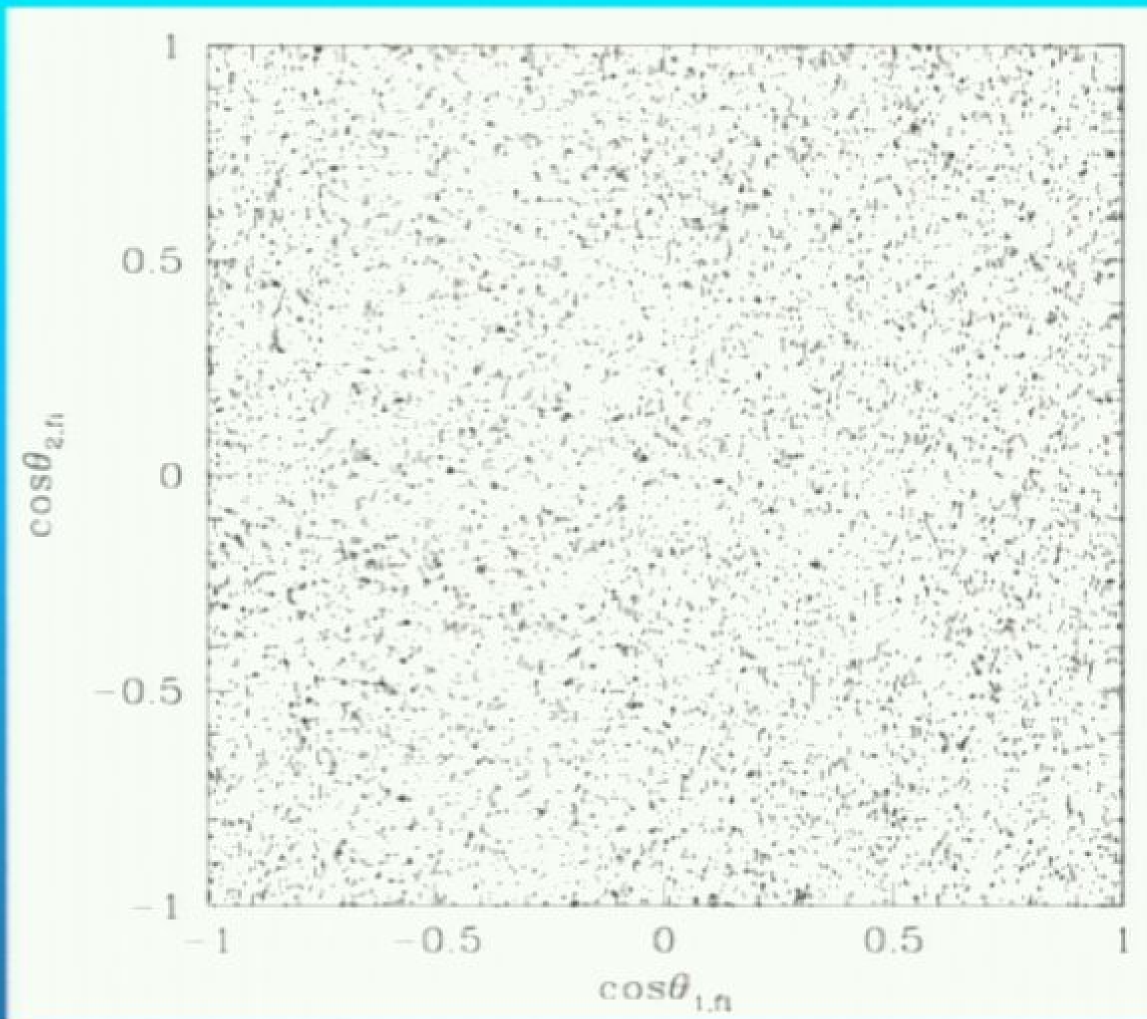
- $t_p \ll t_{GW} \Rightarrow \mathbf{J} \equiv \mathbf{L}_N + \mathbf{S}_1 + \mathbf{S}_2$ and $|\mathbf{L}_N|$ conserved on t_p

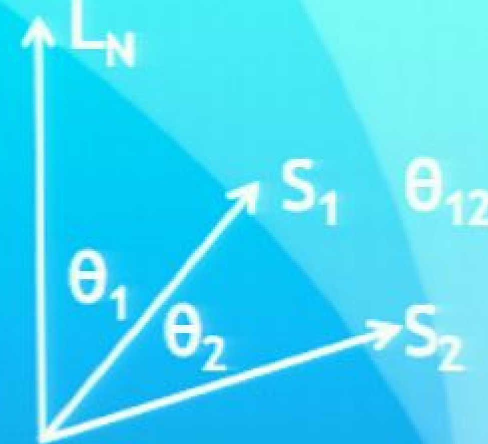
$$\dot{\hat{\mathbf{L}}}_N = -\frac{(M\omega)^{1/3}}{\eta M^2} \frac{d\mathbf{S}}{dt}$$

- Orbital frequency increases on t_{GW}

$$\begin{aligned}
\dot{\omega} = \omega^2 \frac{96}{5} \eta (M\omega)^{5/3} \Big\{ & 1 - \frac{743 + 924\eta}{336} (M\omega)^{2/3} + \left[\left(\frac{19}{3}\eta - \frac{113}{12} \right) \boldsymbol{\chi}_s \cdot \hat{\mathbf{L}}_N - \frac{113\delta}{12} \boldsymbol{\chi}_a \cdot \hat{\mathbf{L}}_N + 4\pi \right] (M\omega) \\
& + \left\{ \left(\frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 \right) - \frac{\eta \chi_1 \chi_2}{48} \left(247 \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 - 721 (\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_1)(\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_2) \right) \right. \\
& + \left. \sum_{i=1}^{2} \frac{(m_i \chi_i)^2}{M^2} \left[\frac{5}{2} \left(3 (\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_i)^2 - 1 \right) + \frac{1}{96} \left(7 - (\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_i)^2 \right) \right] \right\} (M\omega)^{4/3} \\
& - \frac{4159 + 15876\eta}{672} \pi (M\omega)^{5/3} + \left[\left(\frac{16447322263}{139708800} - \frac{1712\gamma_E}{105} + \frac{16\pi^2}{3} \right) + \left(-\frac{273811877}{1088640} + \frac{451\pi^2}{48} - \frac{88}{3}\hat{\theta}\eta \right) \eta \right. \\
& + \left. \frac{541}{896}\eta^2 - \frac{5605}{2592}\eta^3 - \frac{856}{105} \log[16 (M\omega)^{2/3}] \right] (M\omega)^2 + \left(-\frac{4415}{4032} + \frac{358675}{6048}\eta + \frac{91495}{1512}\eta^2 \right) \pi (M\omega)^{7/3} \Big\}
\end{aligned} \qquad (2.6)$$

Isotropic spin distributions





$$\Delta\varphi \equiv \varphi_2 - \varphi_1$$

- Isotropic spins remain isotropic (Bogdanović *et al.* 2007)

Orbital angular momentum evolution

$$\mathbf{L}_N = \eta M \mathbf{r} \times \mathbf{v} = \frac{\eta M^2}{(M\omega)^{1/3}} \hat{\mathbf{L}}_N$$

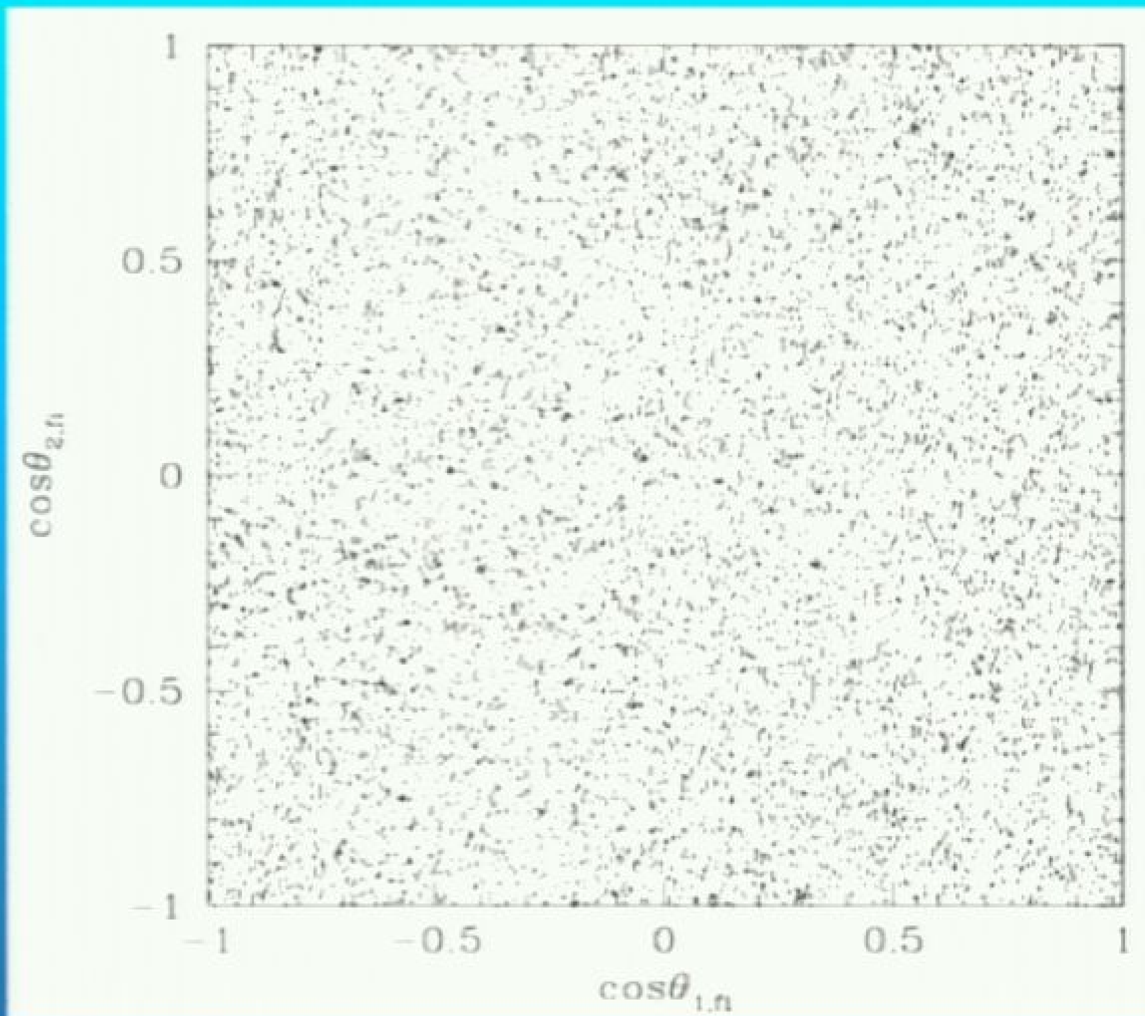
- $t_p \ll t_{GW} \Rightarrow \mathbf{J} \equiv \mathbf{L}_N + \mathbf{S}_1 + \mathbf{S}_2$ and $|\mathbf{L}_N|$ conserved on t_p

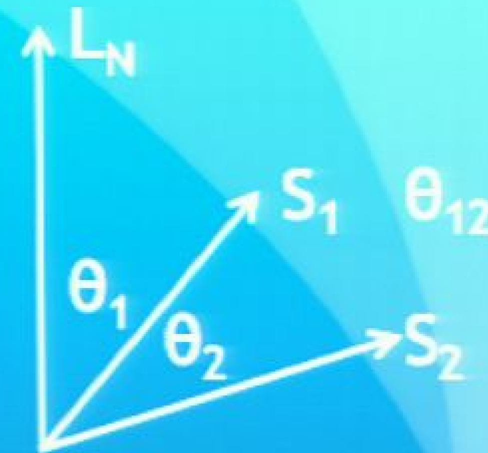
$$\dot{\hat{\mathbf{L}}}_N = -\frac{(M\omega)^{1/3}}{\eta M^2} \frac{d\mathbf{S}}{dt}$$

- Orbital frequency increases on t_{GW}

$$\begin{aligned}
\dot{\omega} = {} & \omega^2 \frac{96}{5} \eta (M\omega)^{5/3} \left\{ 1 - \frac{743 + 924\eta}{336} (M\omega)^{2/3} + \left[\left(\frac{19}{3}\eta - \frac{113}{12} \right) \boldsymbol{\chi}_s \cdot \hat{\mathbf{L}}_N - \frac{113\delta}{12} \boldsymbol{\chi}_a \cdot \hat{\mathbf{L}}_N + 4\pi \right] (M\omega) \right. \qquad (2.6) \\
& + \left\{ \left(\frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 \right) - \frac{\eta \chi_1 \chi_2}{48} \left(247 \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 - 721 (\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_1)(\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_2) \right) \right. \\
& \left. + \sum_{i=1}^{2} \frac{(m_i \chi_i)^2}{M^2} \left[\frac{5}{2} \left(3 (\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_i)^2 - 1 \right) + \frac{1}{96} \left(7 - (\hat{\mathbf{L}}_N \cdot \hat{\mathbf{S}}_i)^2 \right) \right] \right\} (M\omega)^{4/3} \\
& - \frac{4159 + 15876\eta}{672} \pi (M\omega)^{5/3} + \left[\left(\frac{16447322263}{139708800} - \frac{1712\gamma_E}{105} + \frac{16\pi^2}{3} \right) + \left(-\frac{273811877}{1088640} + \frac{451\pi^2}{48} - \frac{88}{3}\hat{\theta}\eta \right) \eta \right. \\
& \left. \left. + \frac{541}{896}\eta^2 - \frac{5605}{2592}\eta^3 - \frac{856}{105} \log\left[16 (M\omega)^{2/3}\right] \right] (M\omega)^2 + \left(-\frac{4415}{4032} + \frac{358675}{6048}\eta + \frac{91495}{1512}\eta^2 \right) \pi (M\omega)^{7/3} \right\}
\end{aligned}$$

Isotropic spin distributions

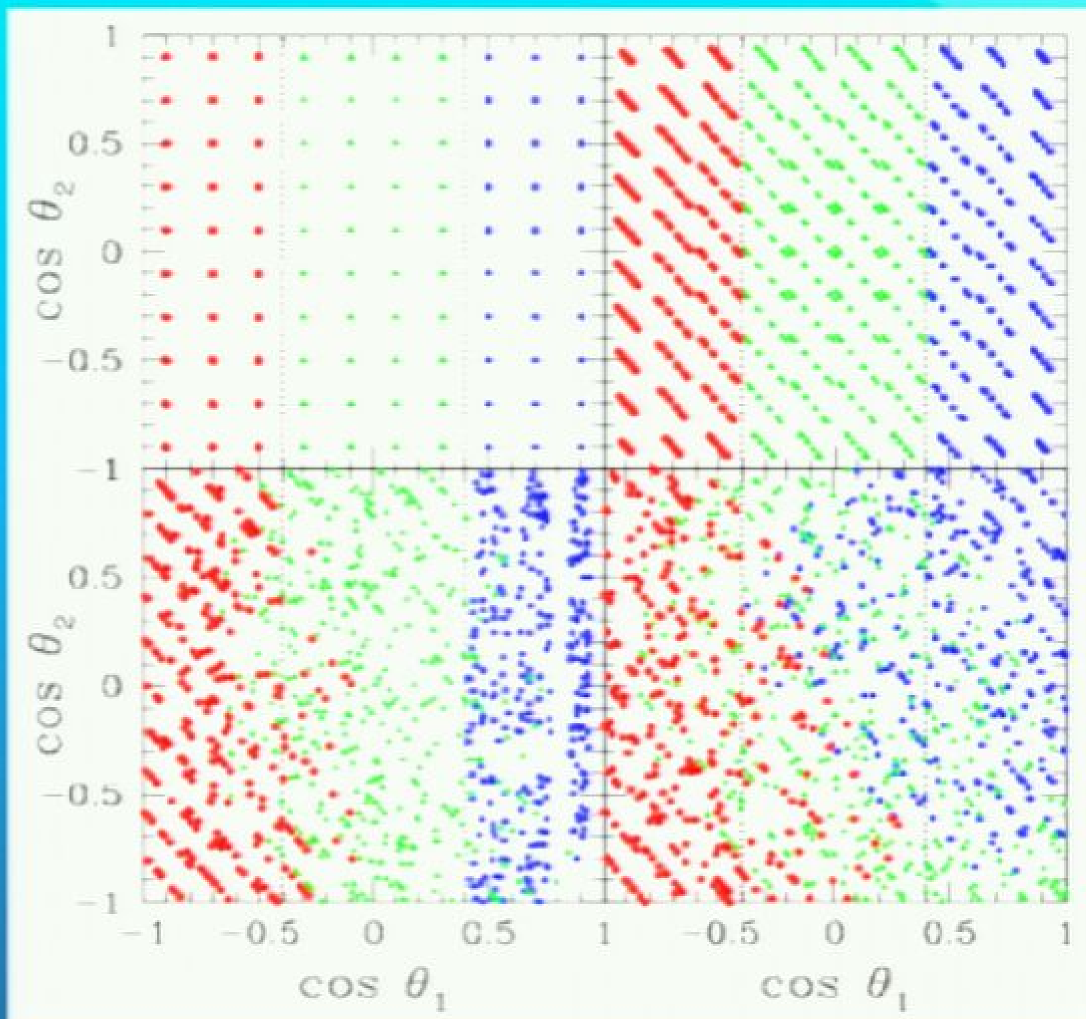




$\Delta\varphi \equiv \varphi_2 - \varphi_1$

- Isotropic spins remain isotropic (Bogdanović *et al*. 2007)

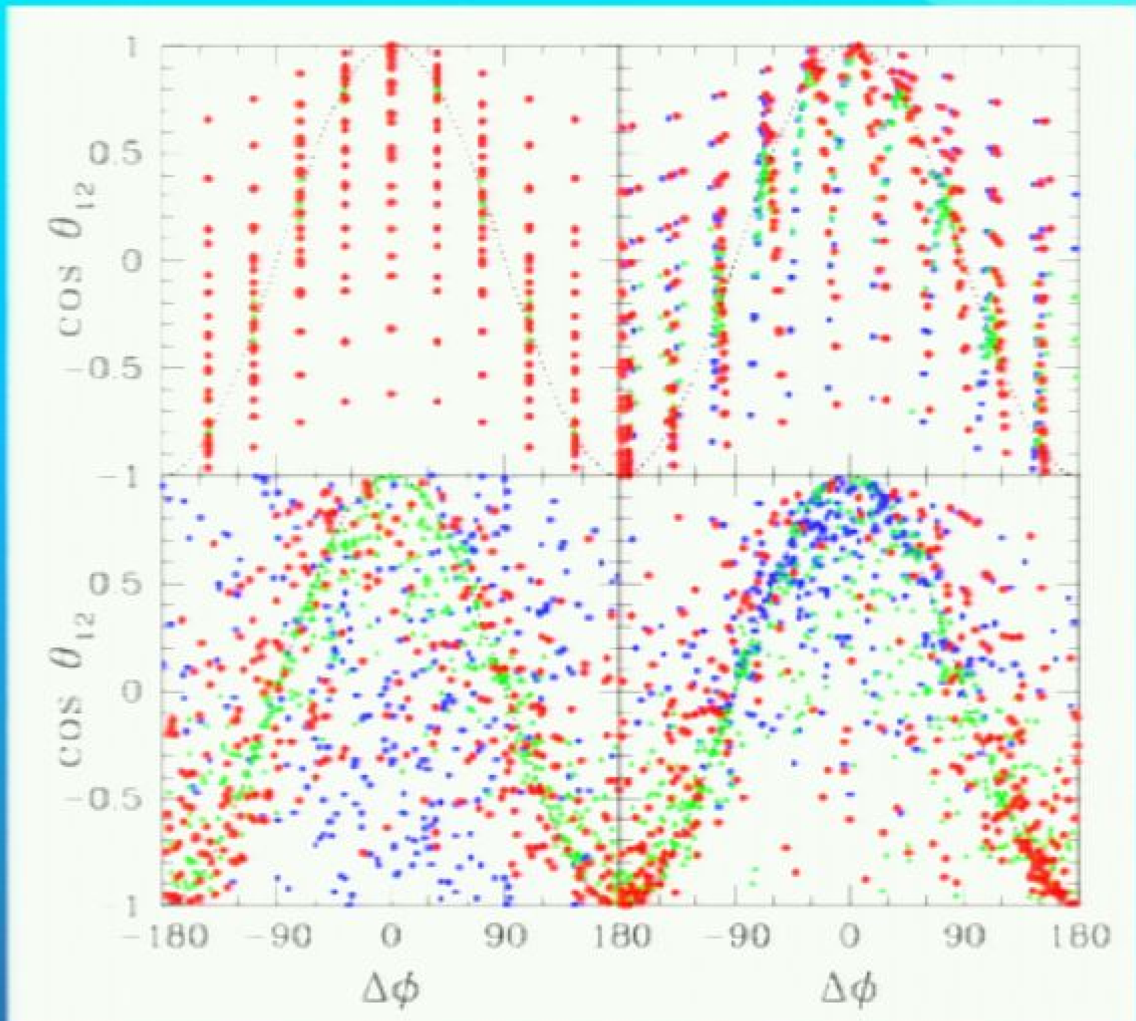
Non-isotropic distributions



- $S_0 \cdot L_N$ conserved (Racine 2008)

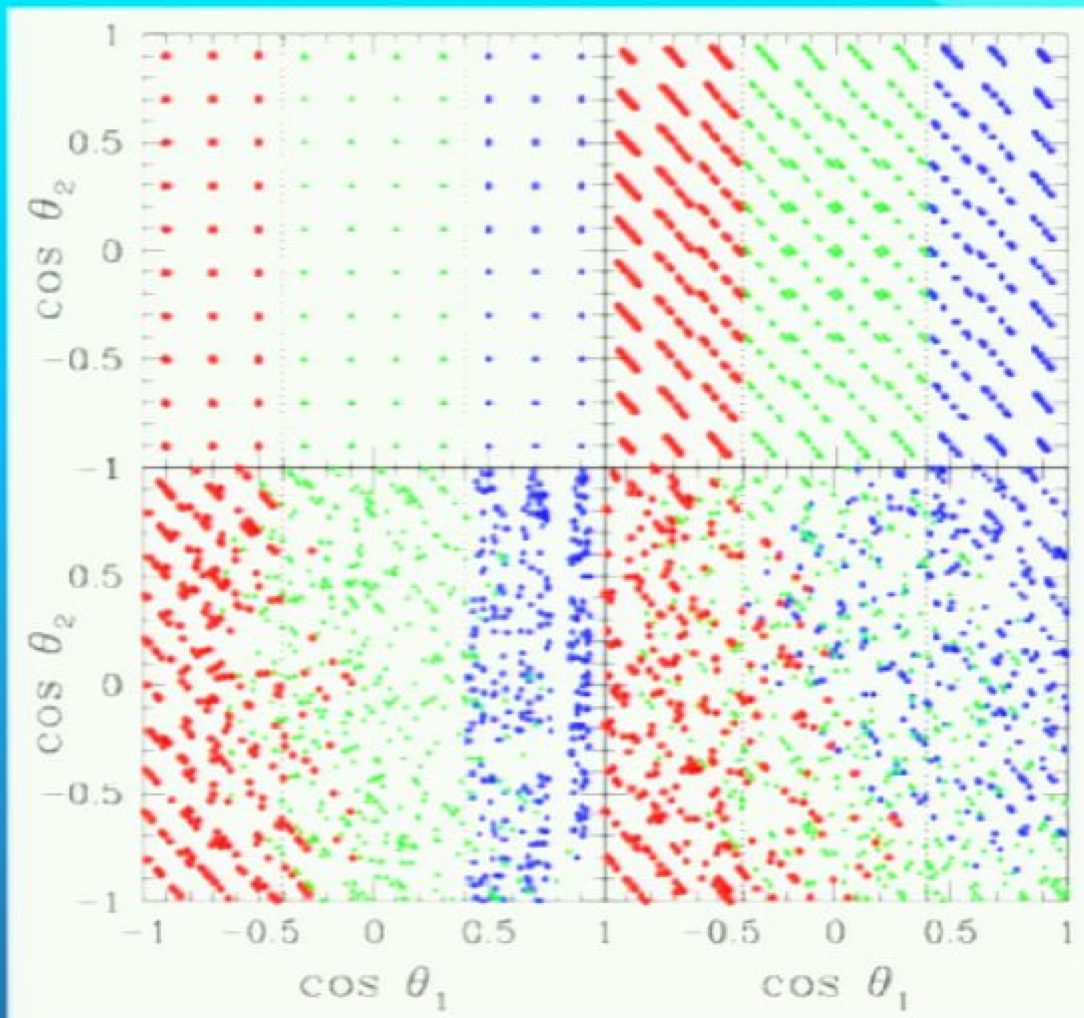
$$\mathbf{S}_0 \equiv (1+q)\mathbf{S}_1 + (1+q^{-1})\mathbf{S}_2$$

Spin alignment I



- Partial alignment of S_1 with L_N at r_i = 1000 M ⇒ alignment of S_1 and S_2 at r_f = 10 M (Schnittman 2004)

Non-isotropic distributions



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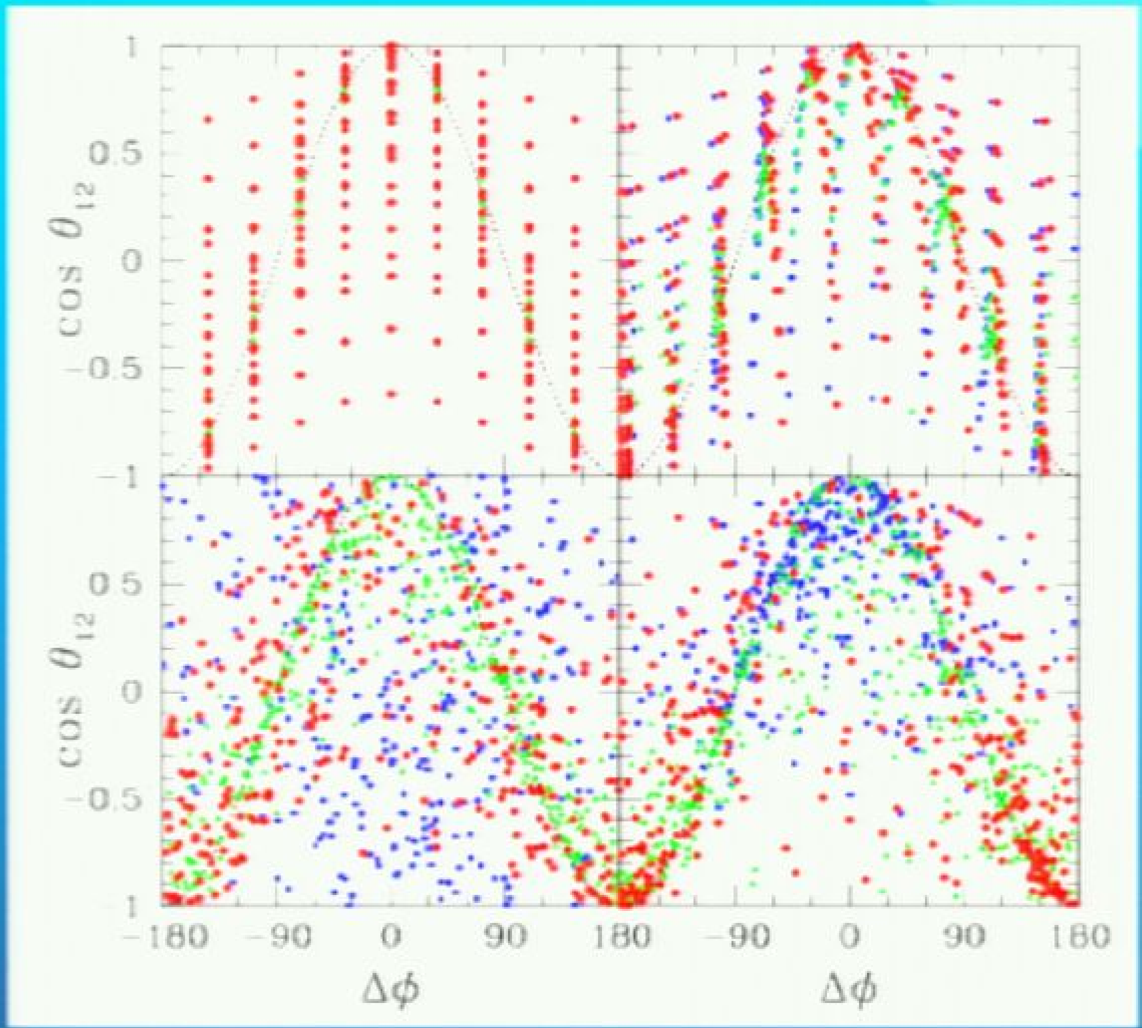
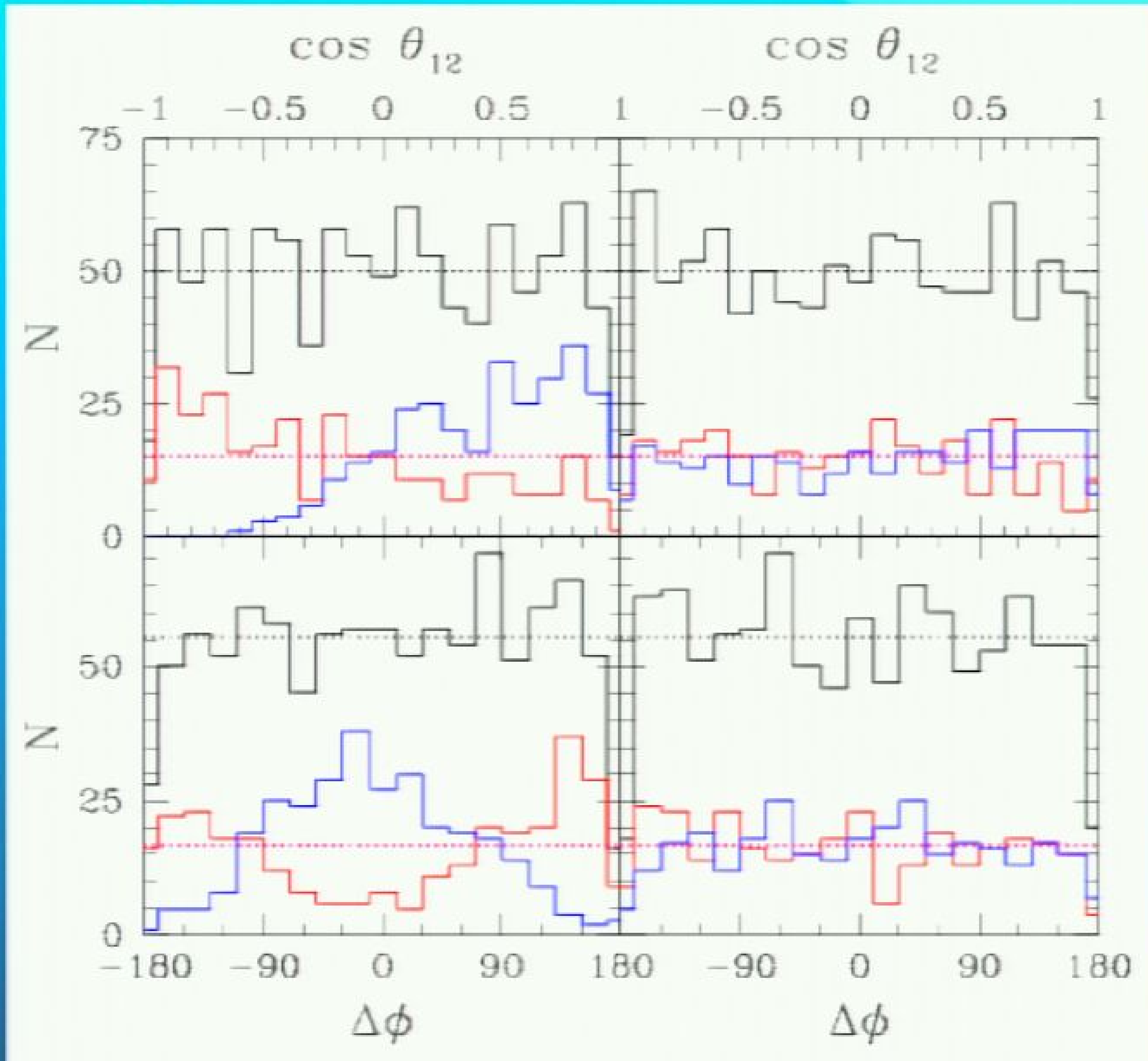
Spin alignment I

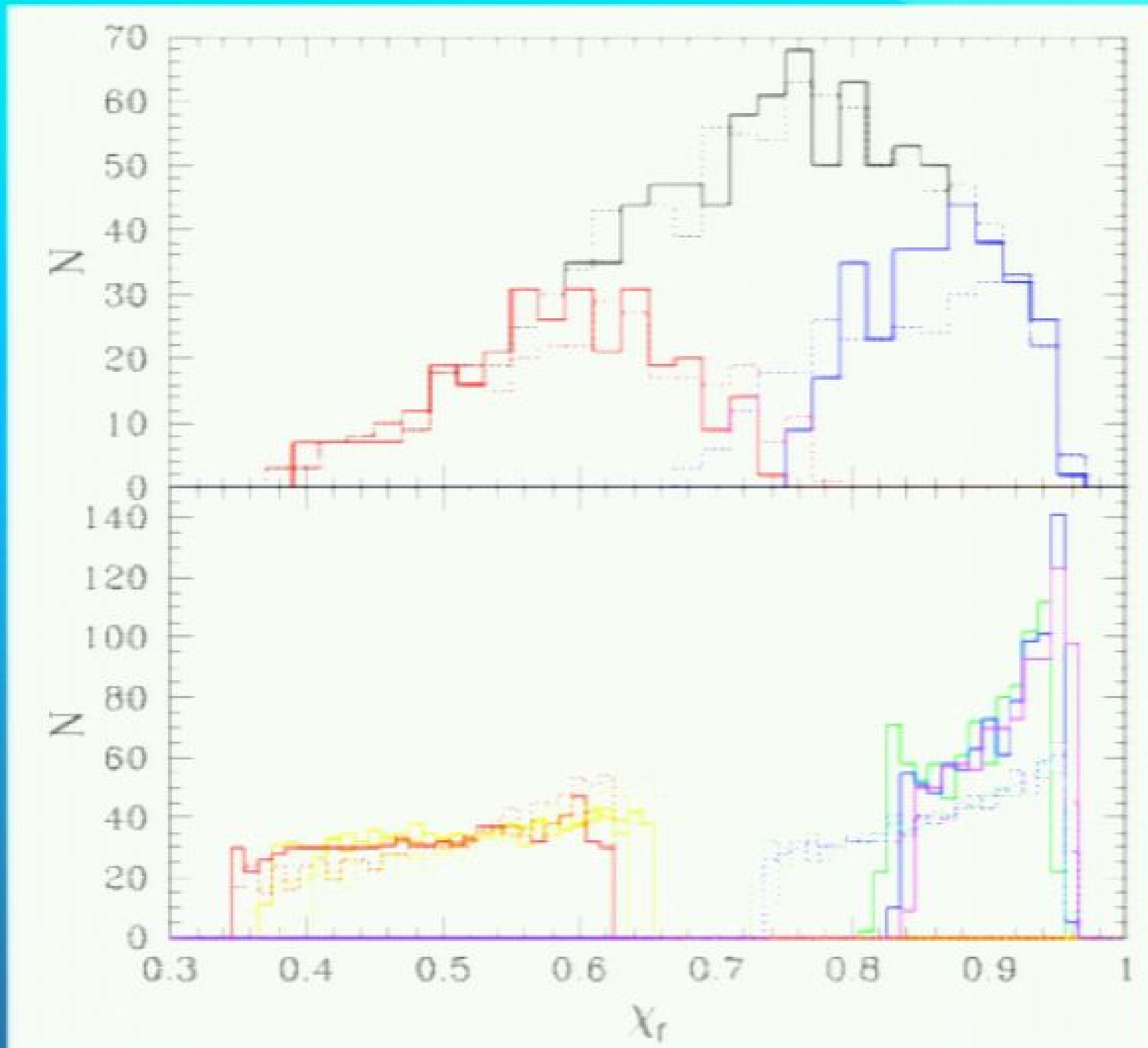


- Partial alignment of S_1 with L_N at r_i = 1000 M ⇒ alignment of S_1 and S_2 at r_f = 10 M (Schnittman 2004)

Spin alignment II

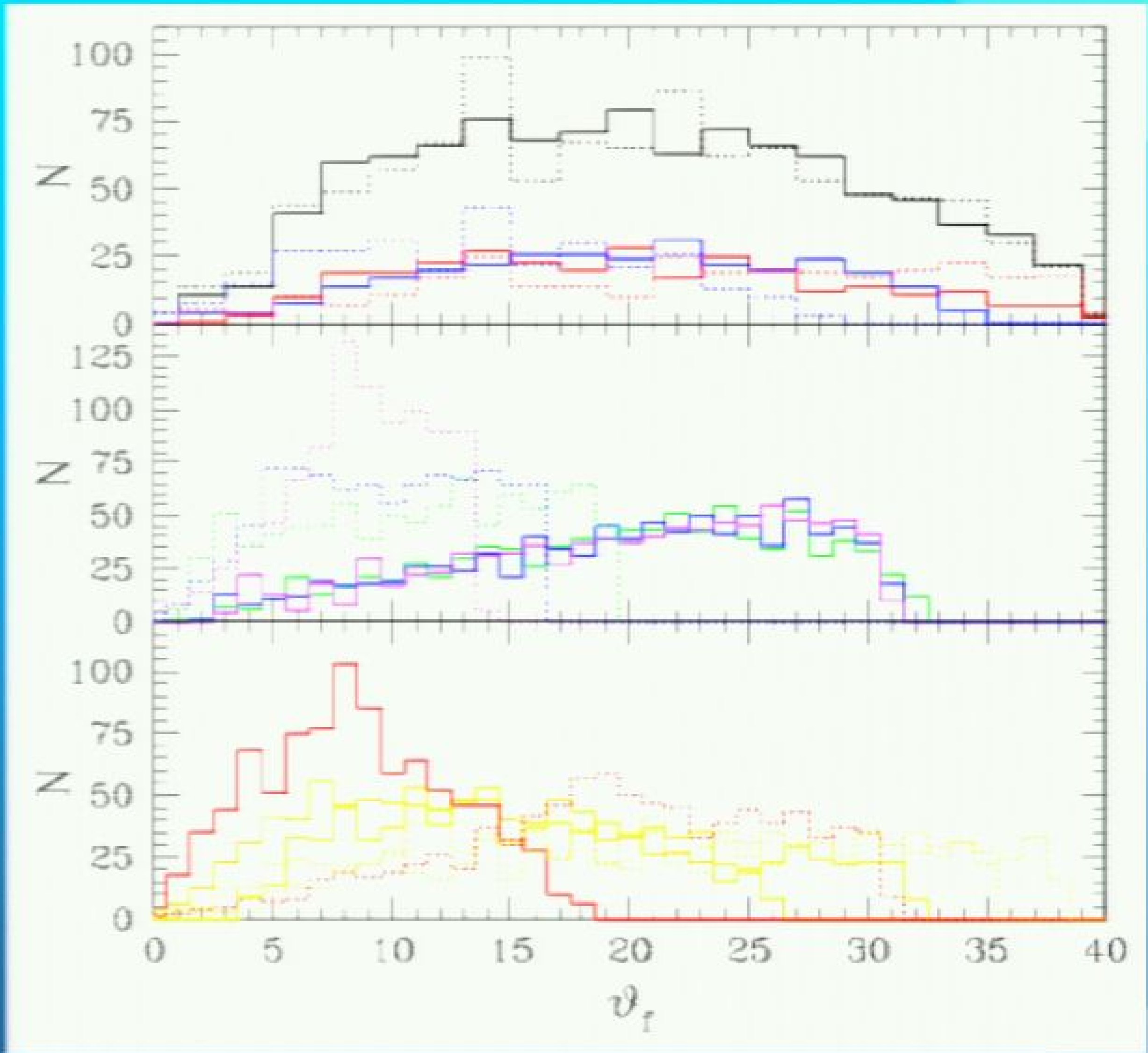


Final spin magnitudes



- Apply Barausse and Rezzolla 2009 spin formula at r_i = 1000 M (dotted) and r_f = 10 M (solid)

Final spin directions



Recoil velocities

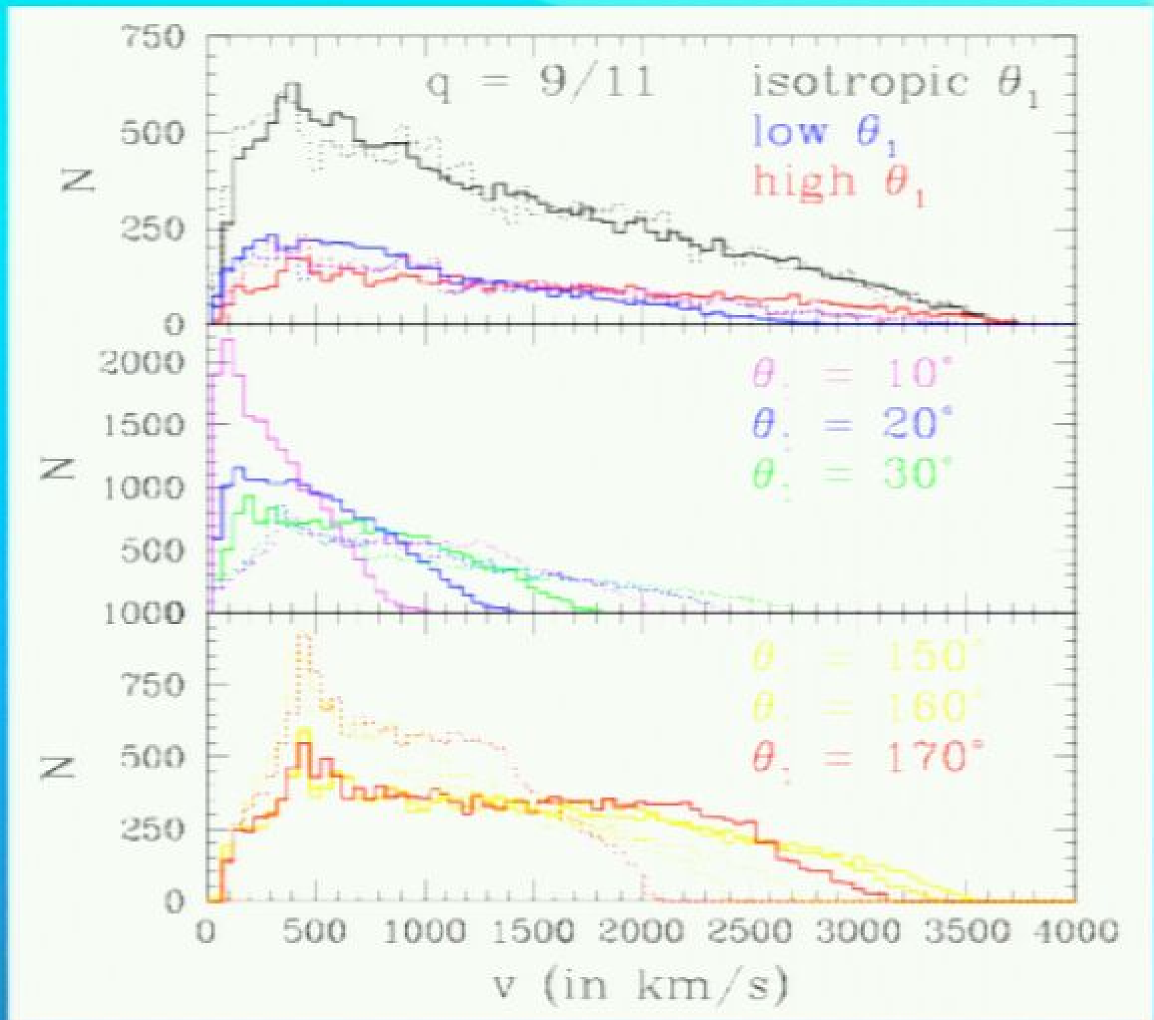
- The final BH can *recoil* with velocities ≤ 4,000 km/s (Campanelli *et al*. 2007)

$$\vec{V}_{\rm recoil}(q,\vec{\alpha}_i) = v_m\,\hat{e}_1 + v_\perp(\cos(\xi)\,\hat{e}_1 + \sin(\xi)\,\hat{e}_2) + v_\parallel\,\hat{e}_z,$$

$$v_m = A\frac{q^2(1-q)}{(1+q)^5}\left(1 + B\,\frac{q}{(1+q)^2}\right),$$

$$v_\perp = H\,\frac{q^2}{(1+q)^5}\left(\alpha_2^\parallel - q\alpha_1^\parallel\right),$$

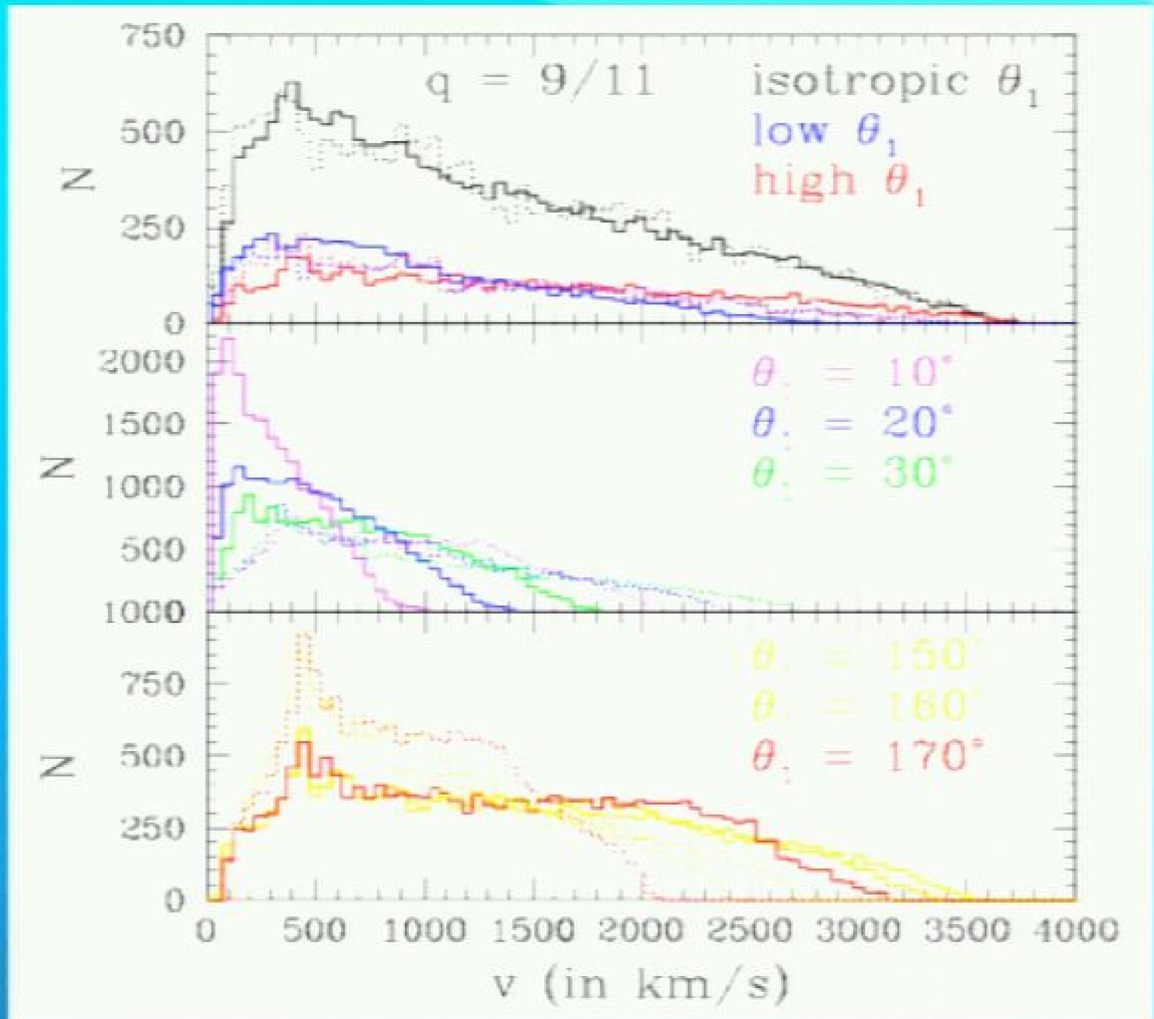
$$v_\parallel = K\cos(\Theta-\Theta_0)\,\frac{q^2}{(1+q)^5}\left(\alpha_2^\perp - q\alpha_1^\perp\right), \quad (1)$$



model	q	v	$\theta_1 = 10°$ 1000M	$\theta_1 = 10°$ 10M	$\theta_1 = 20°$ 1000M	$\theta_1 = 20°$ 10M	$\theta_1 = 30°$ 1000M	$\theta_1 = 30°$ 10M	$\theta_1 = 150°$ 1000M	$\theta_1 = 150°$ 10M	$\theta_1 = 160°$ 1000M	$\theta_1 = 160°$ 10M	$\theta_1 = 170°$ 1000M	$\theta_1 = 170°$ 10M
RIT	9/11	v_{50}	864	273	865	474	889	670	910	1,354	890	1,355	890	1,333
RIT	9/11	v_{90}	1,587	611	1,802	947	2,037	1,302	2,047	2,699	1,813	2,567	1,600	2,120
RIT	2/3	v_{50}	724	283	724	519	801	707	847	1,031	777	1,035	781	1,060
RIT	2/3	v_{90}	1,364	538	1,602	892	1,854	1,252	1,874	2,257	1,627	2,039	1,394	1,930
RIT	1/3	v_{50}	290	206	382	384	520	562	601	619	195	521	135	[illegible]
RIT	1/3	v_{90}	621	364	834	617	1,050	878	1,093	1,183	891	996	699	810

Conclusions II

- Misaligned BH spins will precess many times as they inspiral from $r_i \sim 1000$ M to $r_f = 10$ M
- Isotropic distributions remain isotropic, but partial alignment between the BH spins and orbital angular momentum at r_i is expected in astrophysical mergers
- PN equations of spin precession imply that the BH spins will become substantially aligned by r_f for $q \sim 1$
- This alignment can significantly alter the expected distribution of final spins and recoil velocities



model	q	v	$\theta_1 = 10°$ 1000M	10M	$\theta_1 = 20°$ 1000M	10M	$\theta_1 = 30°$ 1000M	10M	$\theta_1 = 150°$ 1000M	10M	$\theta_1 = 160°$ 1000M	10M	$\theta_1 = 170°$ 1000M	10M
RIT	9/11	v_{50}	864	273	865	171	889	670	910	1,351	890	1,355	890	1,333
RIT	9/11	v_{90}	1,587	611	1,802	947	2,037	1,302	2,047	2,699	1,813	2,567	1,600	2,120
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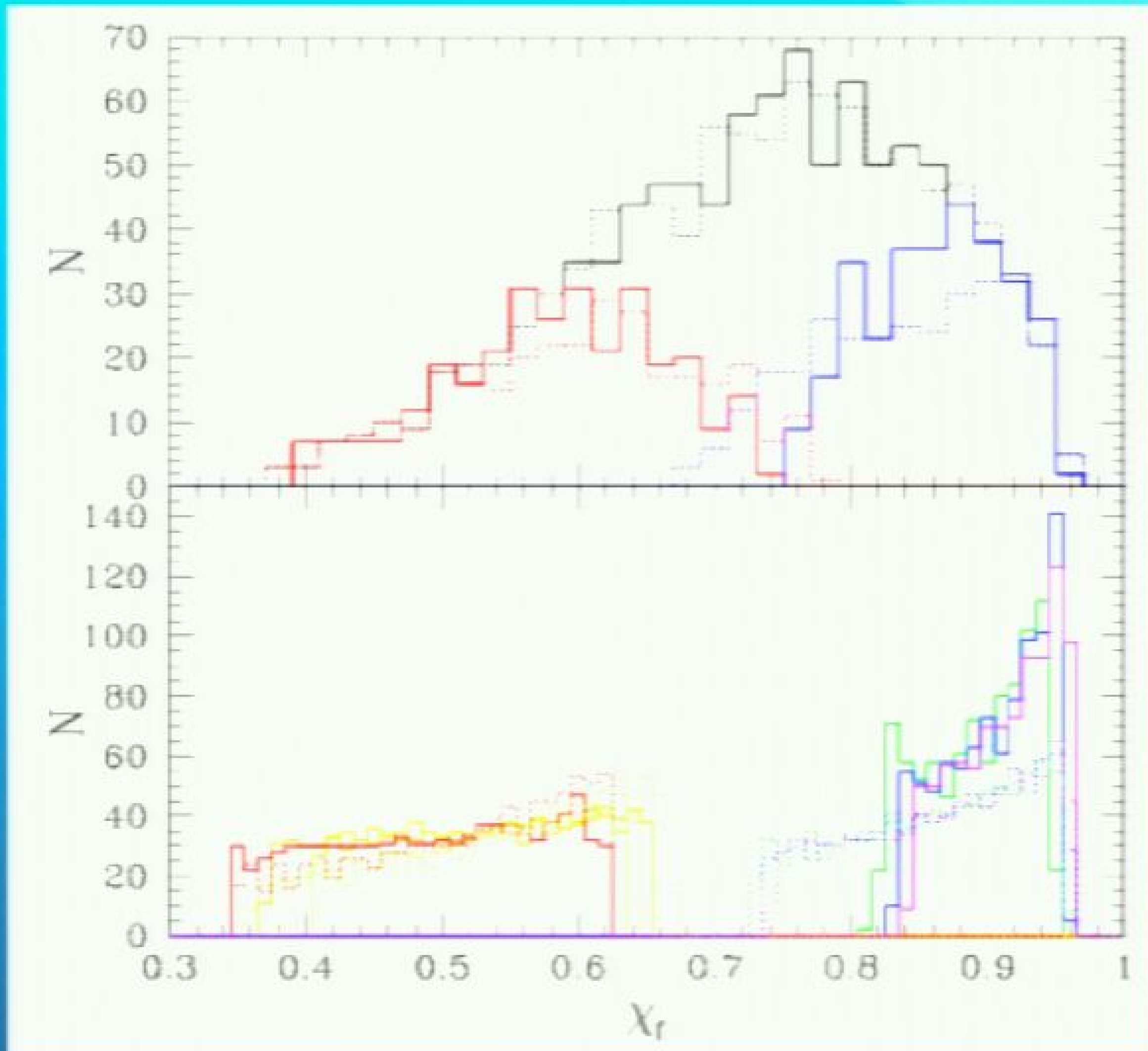
Final spin magnitudes



- Apply Barausse and Rezzolla 2009 spin formula at r_i = 1000 M (dotted) and r_f = 10 M (solid)

Spin alignment II