Title: Instantons and conformal blocks

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Abstract: A systematic method to construct 4d N=2 supersymmetric theories by compactifying M5-branes on a Riemann surface was found by Gaiotto last year.
This suggests that any physical quantity of the 4d theory should be reflected in another physical quantity of the 2d theory living on the Riemann surface.
Indeed, one finds that the instanton partition function of the 4d theories equals the conformal blocks of the 2d theory.
I would like to illustrate this construction through explicit examples.

Instantons and Conformal blocks

Yuji Tachikawa

based on **L. F. Alday, D. Gaiotto, YT** 0906.3219

and on **L. F. Alday, YT** in preparation

April 2010

6d, 4d and 2d

- N M5-branes wrapped on $X_4 \times C_2$
- Consider a physical quantity Z of the 6d theory,
- Assume Z is protected in some sense, so that it can be evaluated by KK reduction.

$$Z[\text{ 6d theory on } X_4 \times C_2\ ]$$

KK-reduce along C_2 ↙ ↘ KK-reduce along X_4

$$Z[\text{ 4d theory on } X_4\ ] = Z[\text{ 2d theory on } C_2]$$

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4d vs 2d

- **Nekrasov's instanton partition function**
  = the **Virasoro/W_N conformal block.**
- **Full partition function**
  = the **Liouville/Toda correlation function.**
- **Superconformal Index** = a **2d TQFT**
  [Gadde-Pomoni-Rastelli-Razamat]

Contents

1. 4d gauge theory

2. 2d CFT

3. Surface operator

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2. 2d CFT

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$\mathcal{N} = 2$ gauge theory

- Vector multiplets:

$$\phi^{i\bar{j}} \qquad \psi_\alpha^{i\bar{j}} \qquad F_{\mu\nu}^{i\bar{j}}$$

- $\phi = \mathbf{diag}(a_1, a_2, \ldots, a_n)$ is a SUSY configuration
- breaks $\mathbf{SU}(N)$ to $\mathbf{U}(1)^{N-1}$:

$$S = \tau_{\mathrm{UV}} \,\mathbf{tr}\, F_{\mu\nu} F_{\mu\nu} + c.c. \longrightarrow S = \tau_{ij}(a) F_{\mu\nu}^i F_{\mu\nu}^j + c.c.$$

- In $\mathcal{N} = 2$ superspace,

$$S = \int d^4\theta \tau_{\mathrm{UV}} \,\mathbf{tr}\, \Phi^2 + c.c. \longrightarrow S = \int d^4\theta \mathcal{F}(a) + c.c.$$

$\mathcal{F}$ is called the **prepotential**.

$$a^i, \qquad \psi_\alpha^i, \qquad F_{\mu\nu}^i$$

Prepotential

- Perturbatively,

$$\mathcal{F}(a) = \tau_{UV} \sum a_i^2 + \sum_{i>j} (a_i - a_j)^2 \log \frac{a_i - a_j}{\Lambda_{UV}}$$

- Instanton corrections

$$\mathcal{F}(a) = \sum_{i>j} (a_i - a_j)^2 \log \frac{a_i - a_j}{\Lambda} + \sum_k \Lambda^{2Nk} f_k(a)$$

where $f_k(a)$: k-instanton correction.

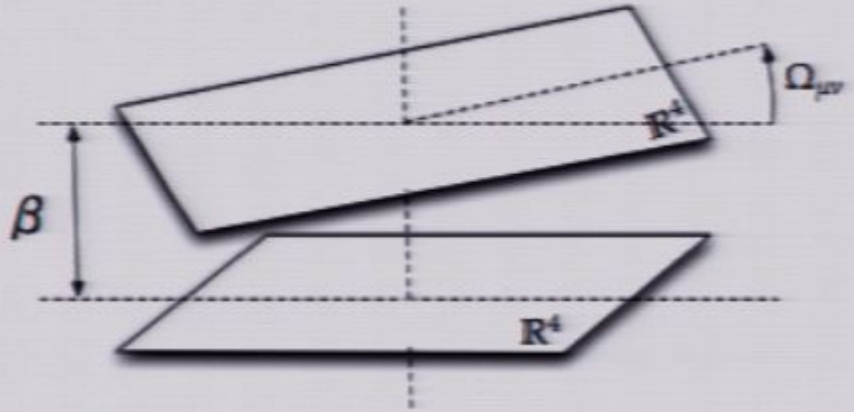
$$f_1(a) = \sum_i \prod_{j \neq i} (a_i - a_j)^{-2}.$$

- Huge literature devoted to calculate f_k.

Nekrasov's instanton partition function

$$Z(a;\epsilon_1,\epsilon_2) = \exp\left(\frac{\mathcal{F}(a)}{\epsilon_1\epsilon_2} + \cdots\right)$$

1. Take 5d version of the theory
2. Put it on a circle of length β.



3. Glue the two ends by

$$\text{4d rotation} \quad \Omega = e^{\beta\epsilon_1 L_{12} + \beta\epsilon_2 L_{34}} \qquad \text{and}$$
$$\text{gauge rotation} \quad g = \mathbf{diag}(e^{\beta a_1}, \ldots, e^{\beta a_N})$$

4. Take $\beta \to 0$.

Localization

$$Z(a) = \sum_k q^k \int_{\mathcal{M}_{N,k}} (\text{something complicated})$$

$$= \sum_k q^k \sum_{\text{fixed point } p \text{ on } \mathcal{M}_{N,k}} (\text{contribution from } p)$$

- $\mathcal{M}_{N,k}$: k-instanton moduli of $\mathbf{U}(N)$ gauge theory.
- $\mathbf{SO}(4)$ and $\mathbf{SU}(N)$ act on $\mathcal{M}_{N,k}$.
- Fixed points on the moduli space =

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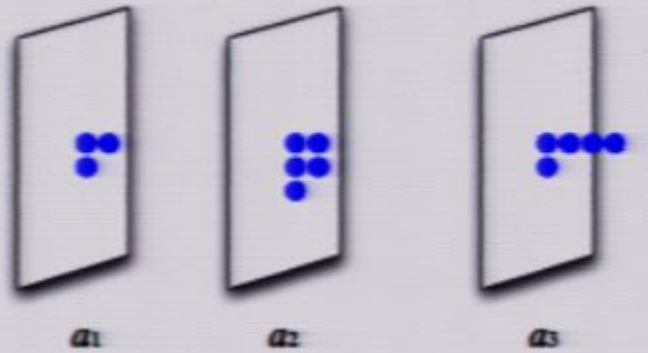

$$Z(a) = \sum_{\vec{Y}} q^{|\vec{Y}|} \cdot (\text{contribution from } \vec{Y})$$

Pure SU(2)

$$Z(a) = \sum_{\vec{Y}} \Lambda^{4|\vec{Y}|} \frac{1}{G_a(\vec{Y})}$$

$G_a(\vec{Y})$: contribution from the **Gauge** multiplet.

Consider the vector space

$$V_a \ni |v\rangle\rangle = (v_{(0,0)};\ v_{(0,\square)},\ v_{(\square,0)};\ v_{(0,\square\square)},\ v_{(0,\square)},\ v_{(\square,\square)},\ \ldots)$$

Introduce the inner product via

$$\langle\langle v|w\rangle\rangle = \sum_{\vec{Y}} \frac{v_{\vec{Y}}\ w_{\vec{Y}}}{G_a(\vec{Y})}$$

Consider this stupid vector

$$|w_a\rangle\rangle = (1;\ \Lambda^2,\ \Lambda^2;\ \Lambda^4,\ \Lambda^4,\ \ldots)$$

then

$$Z(a) = \langle\langle w_a|w_a\rangle\rangle$$

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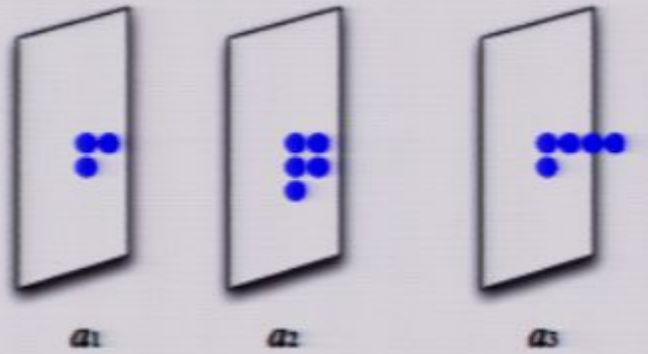

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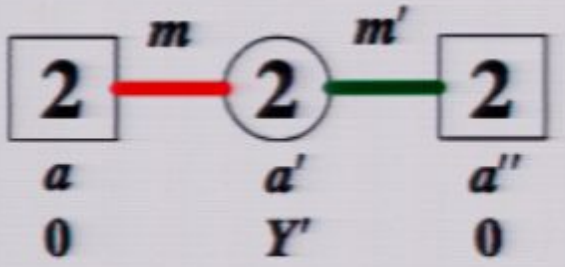
SU(2) with 4 flavors



$$Z = \sum q^{|\vec{Y}'|} \frac{H_a{}^m{}_{a'}(\vec{0},\vec{Y})\, H_{a'}{}^{m'}{}_{a''}(\vec{Y},\vec{0})}{G_a(\vec{0})\, G_{a'}(\vec{Y})\, G_{a''}(\vec{0})}$$

Let $H_m : V_{a'} \to V_a$ be such that for $v \in V_{a'}$

$$(H_m v)_{\vec{Y}} = \sum_{\vec{Y}'} \frac{H_a{}^m{}_{a'}(\vec{Y},\vec{Y}')\, v_{\vec{Y}'}}{G_{a'}(\vec{Y}')}$$

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$$Z = \langle\langle a | H_m\, q^N\, H_{m'} | a'' \rangle\rangle$$

where N counts the number of boxes and

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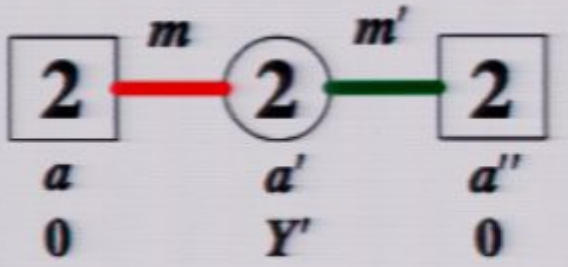

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Recap

- Nekrasov's partition function

$$Z(a;\epsilon_1,\epsilon_2) = \exp\left(\frac{\mathcal{F}(a_1,\ldots,a_N)}{\epsilon_1\epsilon_2} + \cdots\right)$$

has an explicit expression in terms of Young diagrams.

- It can be recast in a nice form:

$$Z_{\text{pure SU(2)}} = \langle\langle w|w\rangle\rangle$$

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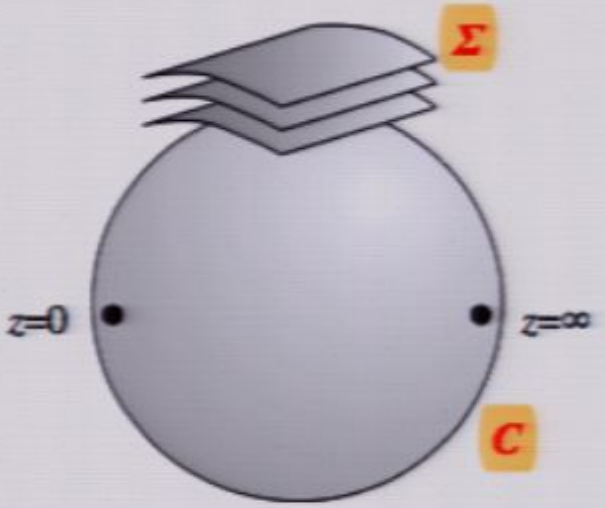
- There's a completely different way to encode $\mathcal{F}(a)$, without any expansion.

Seiberg-Witten curve

$$\Sigma: \quad \Lambda^N(z+\frac{1}{z}) = y^N + u_2 y^{N-2} + \cdots + u_N$$



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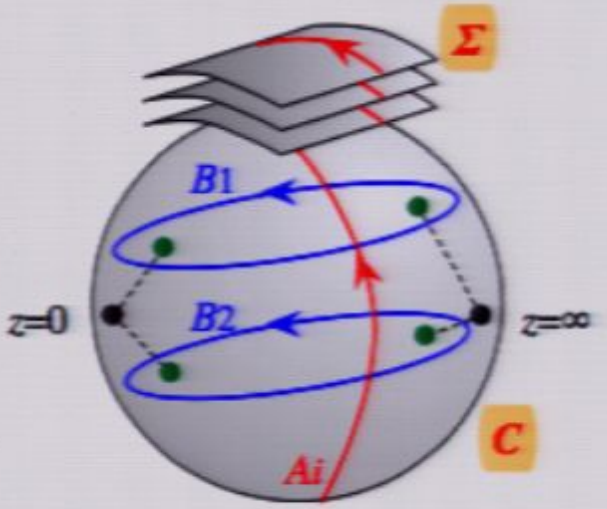

- The sphere parameterized by z is **the base** C, the **Gaiotto curve**.
- Its N-sheeted cover is the **Seiberg-Witten curve** Σ

$$a_i = \int_{A_i} \frac{ydz}{z}, \qquad a_i^D = \int_{B_i} \frac{ydz}{z}, \quad \text{and} \quad a_i^D = \frac{\partial \mathcal{F}(a)}{\partial a_i}.$$

M5-branes

Let $\lambda = ydz/z$. Then

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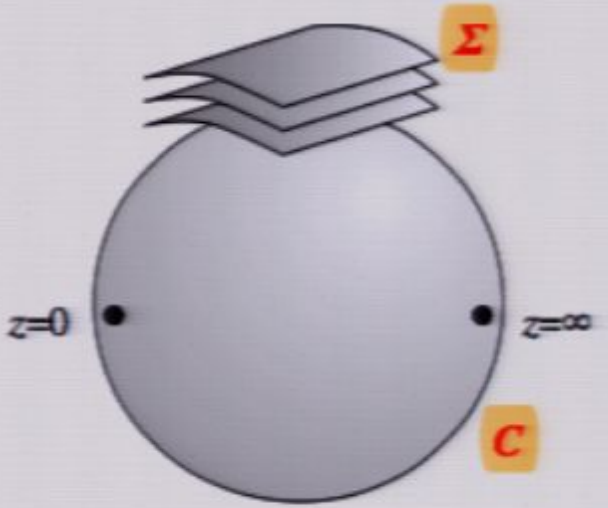

becomes

$$\lambda^N + \phi_2(z)\lambda^{N-2} + \cdots + \phi_N(z) = 0$$

where

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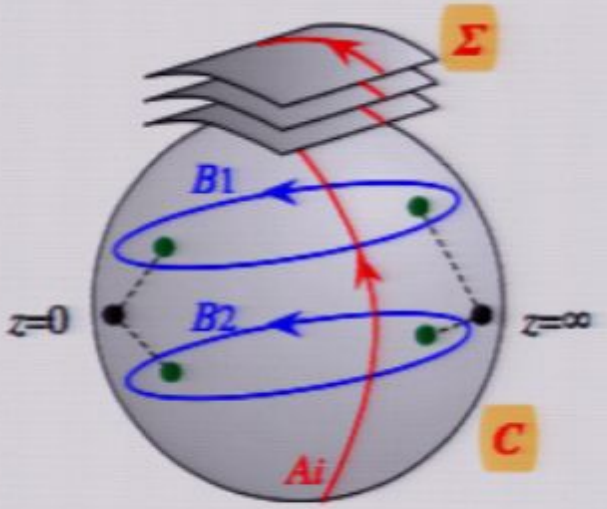

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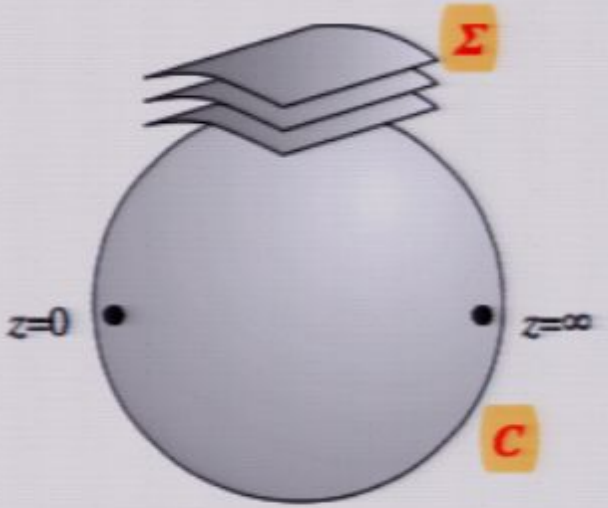

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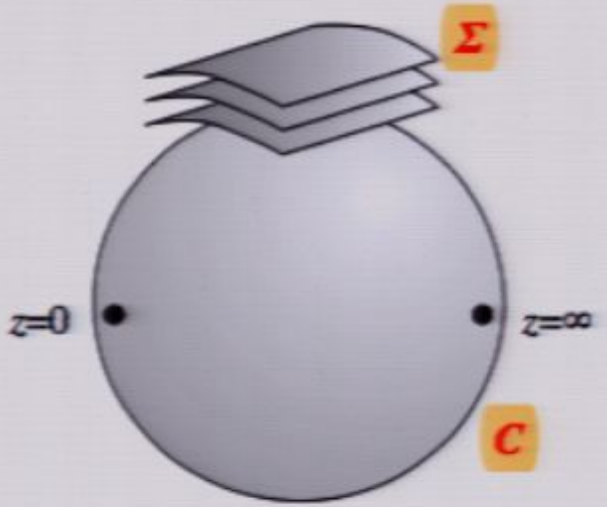
M5-branes

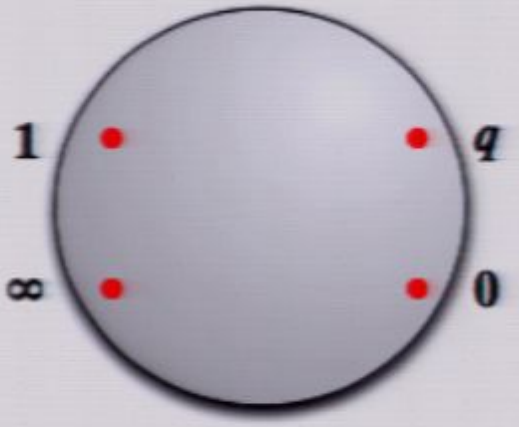
$$\lambda^N + \phi_2(z)\lambda^{N-2} + \cdots + \phi_N(z) = 0$$

- λ: one-form. ϕ_k: degree-k form.
- Determine $\Sigma \subset T^*C$
- Wrapping N M5-branes on Σ inside the hyperkahler T^*C.
- 11d spacetime is $\mathbb{R}^{3,1} \times T^*C \times \mathbb{R}^3$.



Another example

- **SU(2)** with four flavors has the form $\lambda^2 = \phi_2(z)$



- at each of the puncture z_i,

$$\phi_2(z) \sim \frac{m_i^2}{(z - z_i)^2} dz^2$$

- The cross ratio gives the marginal coupling

$$q \sim e^{2\pi i \tau_{\text{UV}}}$$

Summary so far

	Pure **SU(2)**	**SU(2)** with 4 flavors
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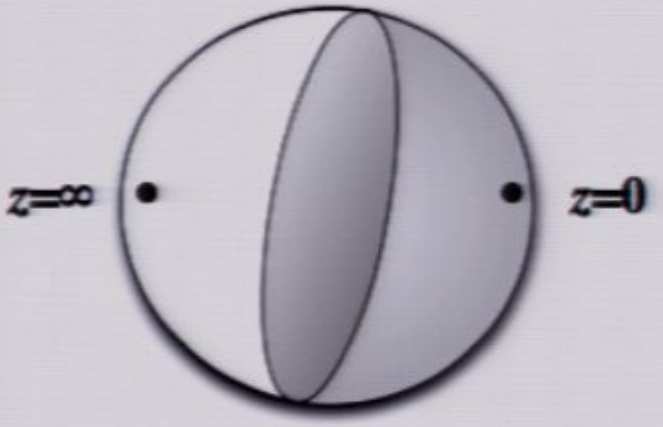
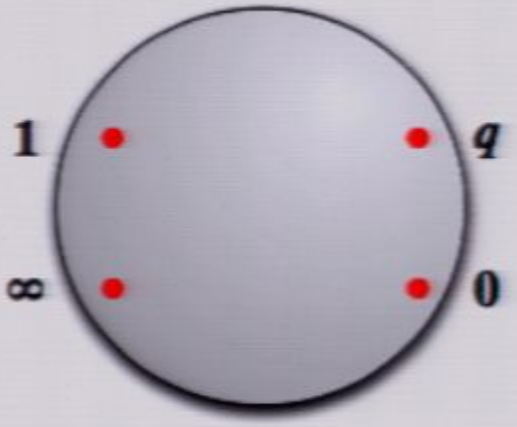
Gaiotto curve		

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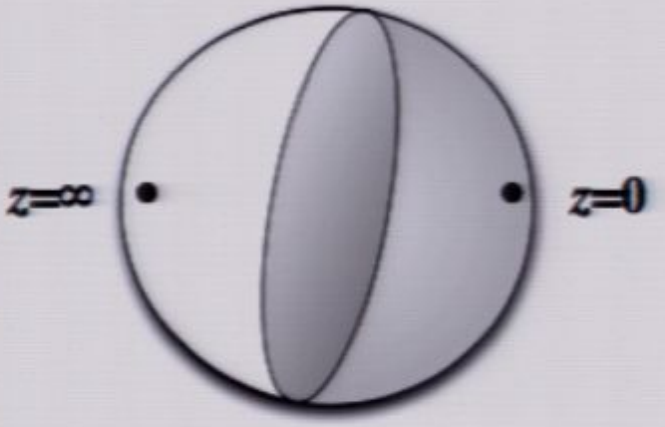
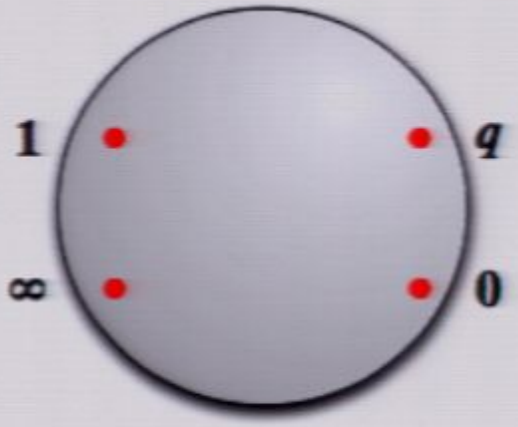
Gaiotto curve		
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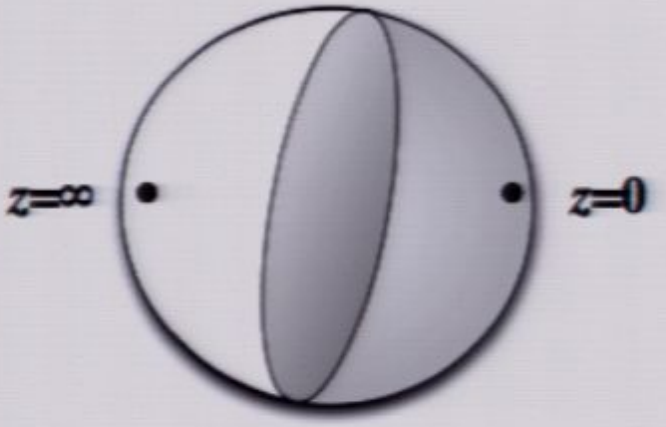
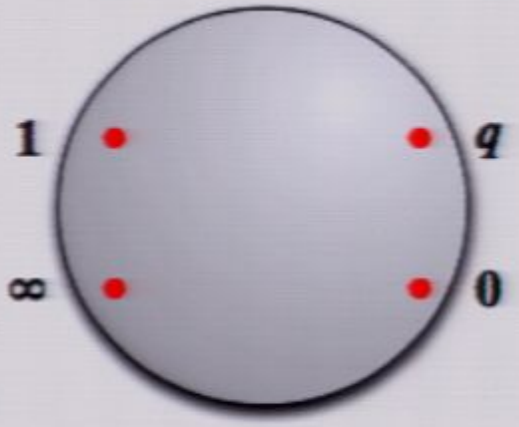
Gaiotto curve		
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Z = conformal block

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4d CFT vs 2d CFT

- We now have a **map**

$$G_N : \text{Riemann surface with punctures} \longrightarrow \text{4d field theory}$$

- G_N **behaves nicely under degenerations** of the Riemann surface Σ

  i.e. any thin, long tube gives a weakly coupled $\mathbf{SU}(N)$ gauge group

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- Take whatever physical quantity Z calculable in 4d:

$$Z : \text{4d field theory} \longrightarrow \text{number}$$

- Then, $Z(G_N(\Sigma))$ **factorizes under degenerations** of Σ,

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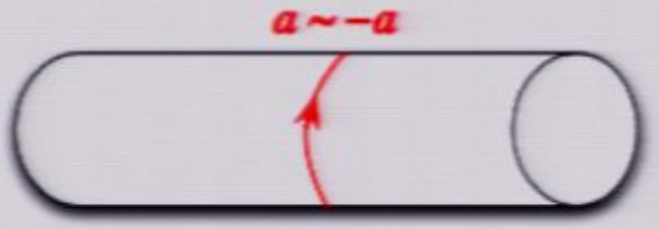
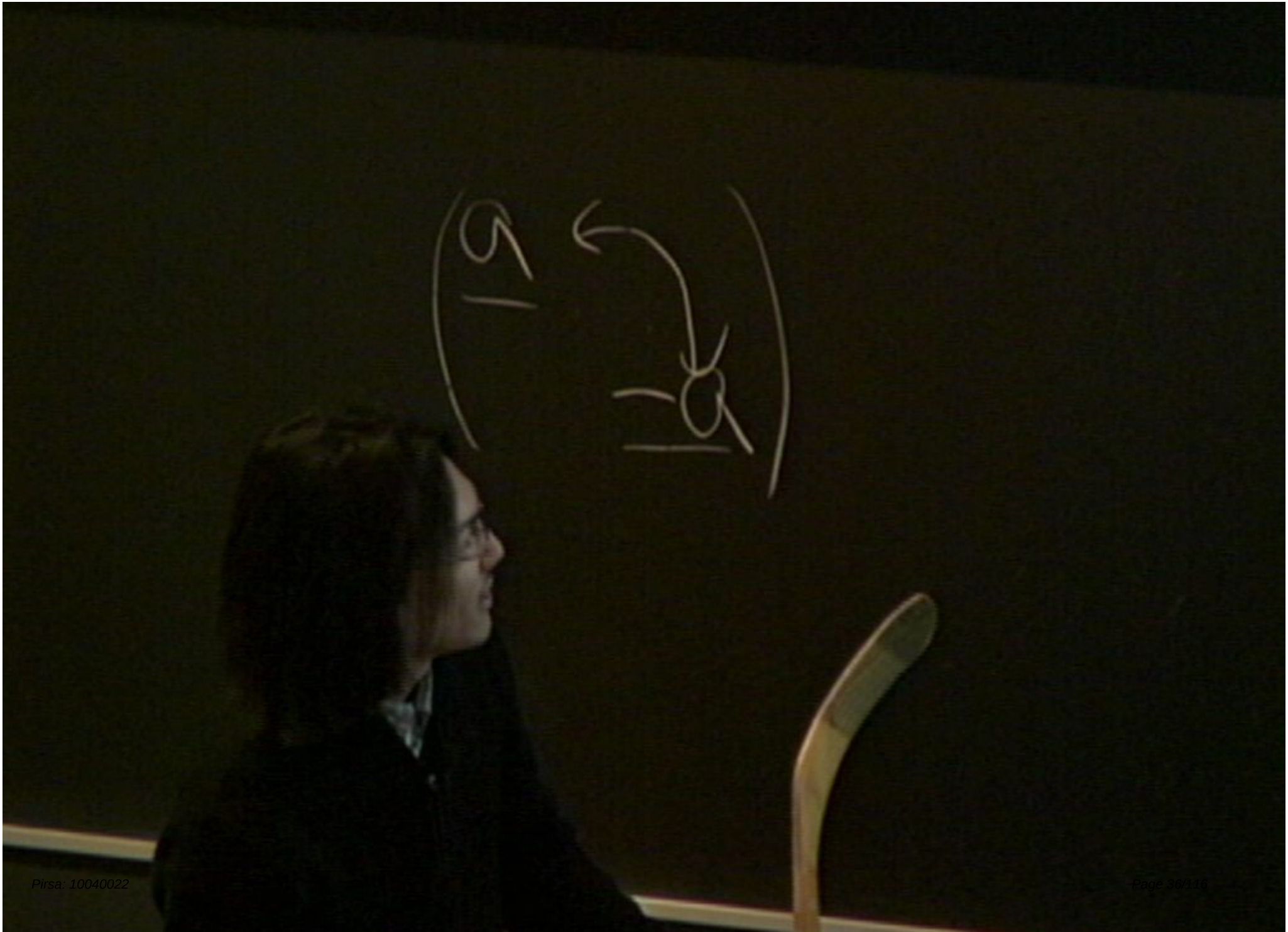
- This morally means that $Z \circ G_N$ gives a **2d CFT**.

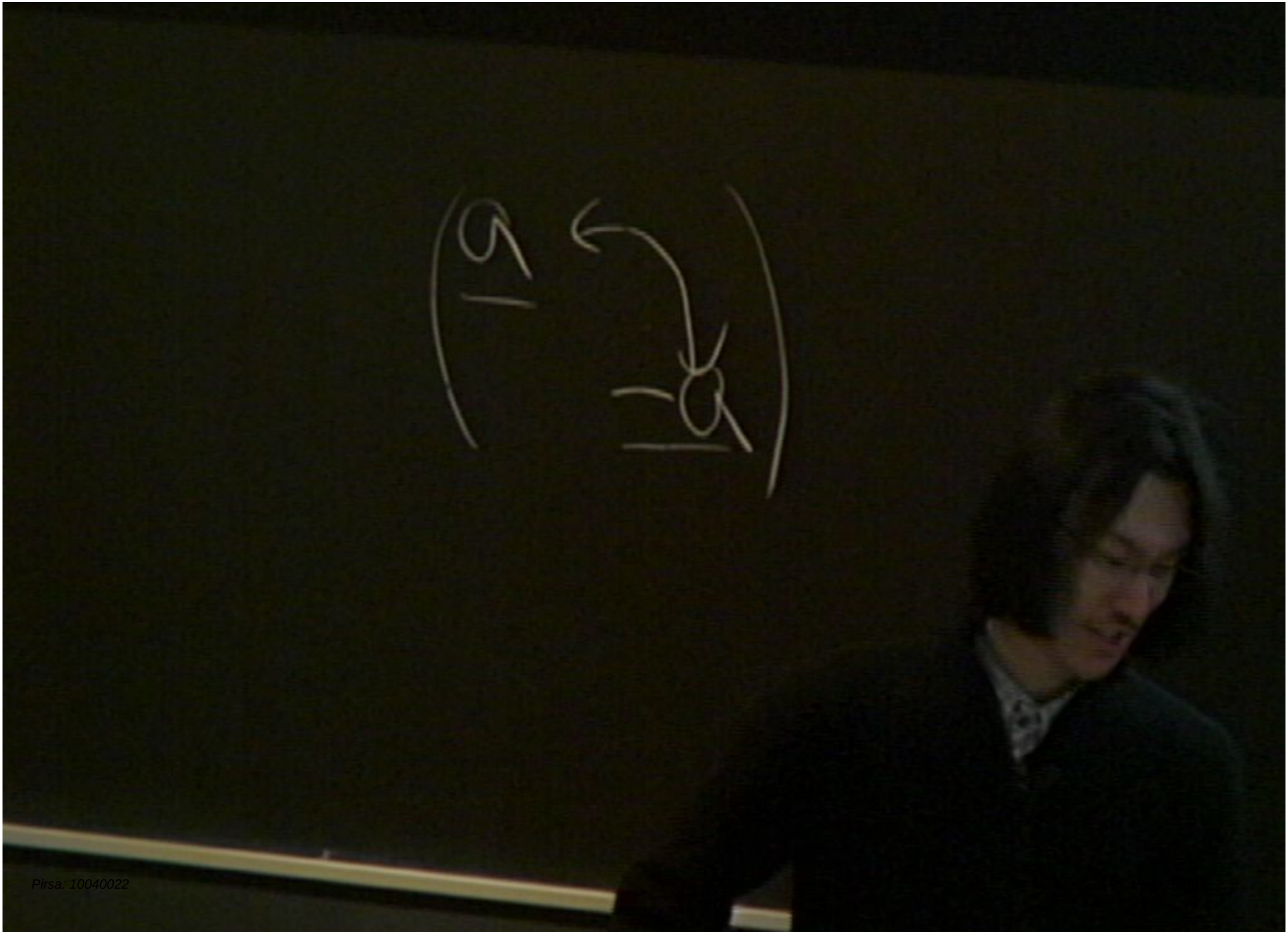
SU(2) vs. Liouville

- What do we get from 2 M5-branes?



- Each channel is labeled by one variable a with the identification $a \sim -a$.





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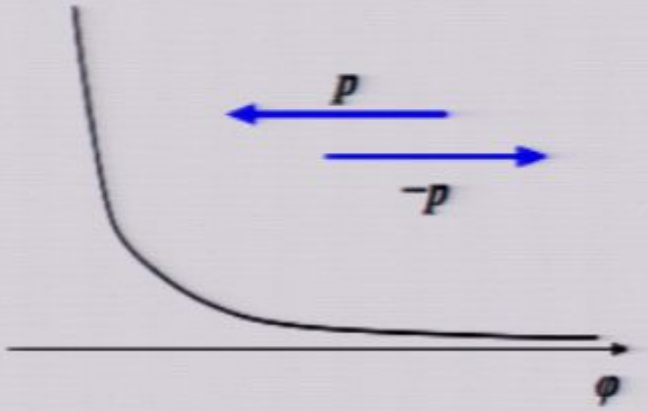
- What do we get from 2 M5-branes?

- Each channel is labeled by one variable a with the identification $a \sim -a$.
- Such 2d CFT is bound to be Liouville [Teschner,Pakman…]

Liouville

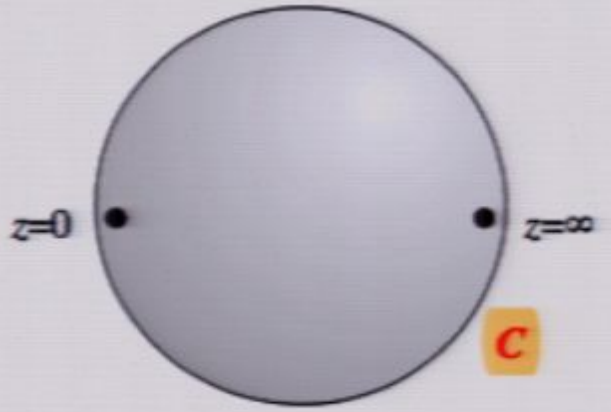


- Reflection off an exponential wall.
- The action is

$$S = \frac{1}{\pi}\int d^2x\sqrt{g}\left(|\partial_\mu\varphi|^2 + \mu e^{2b\varphi} + QR\varphi\right)$$

- $Q = b + 1/b$ and $c = 1 + 6Q^2$.

Proposal: SU(2)



- SW curve was $\lambda^2 = \phi_2(z)$.
- Liouville theory has $T(z)$.
- Both have spin-2.

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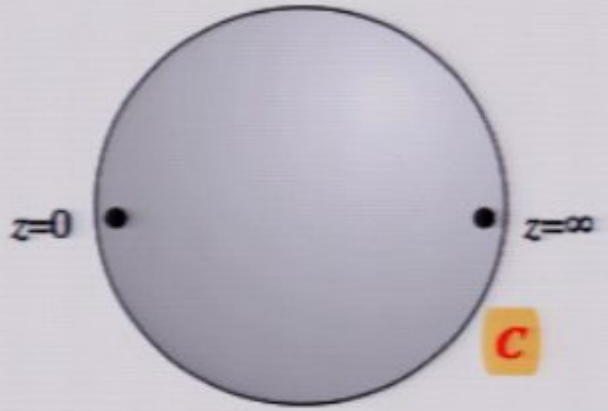

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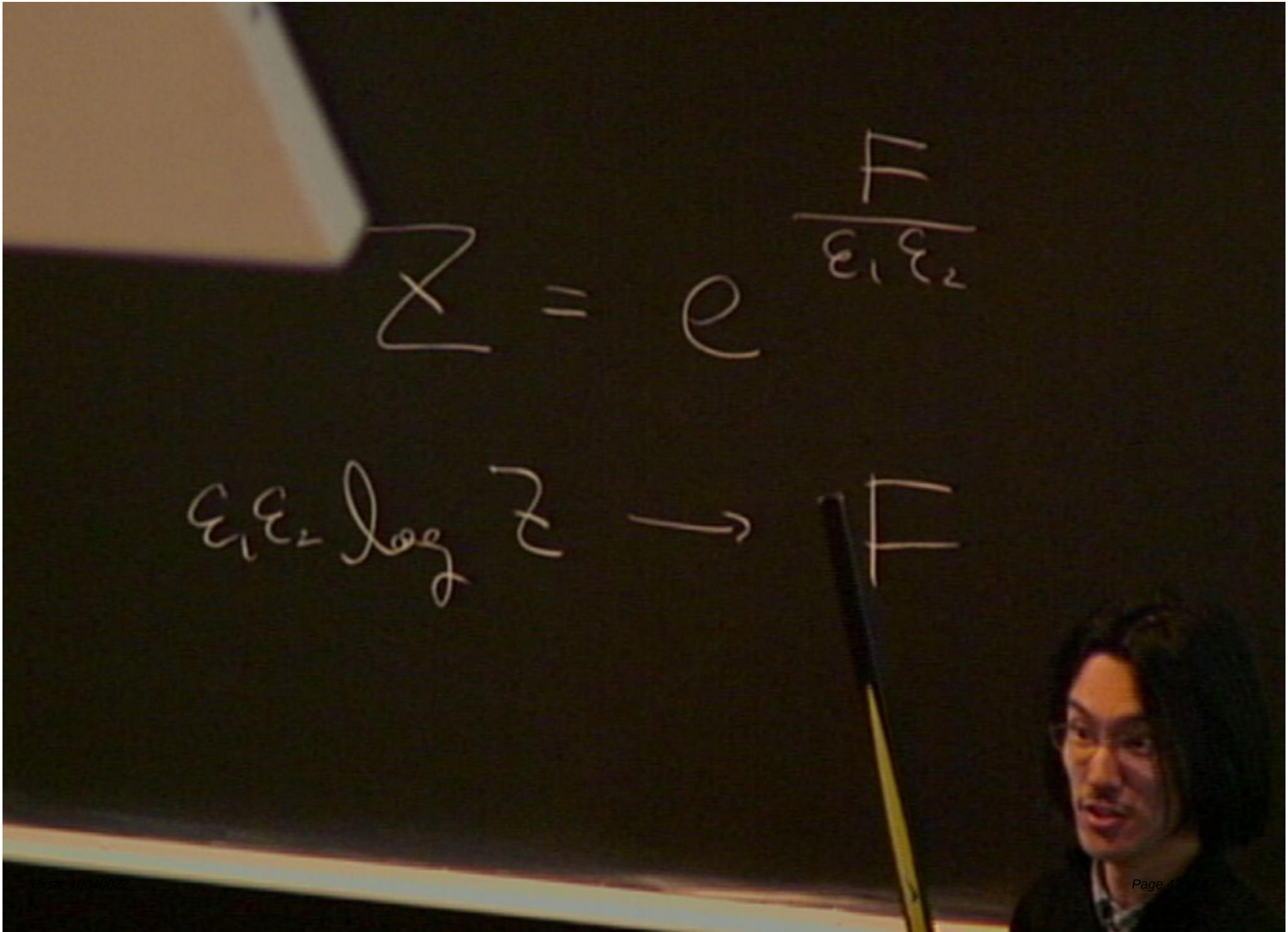
$$\epsilon_1\epsilon_2\langle T(z)\rangle dz^2 \to \phi_2(z) \quad \text{when} \quad \epsilon_{1,2} \to 0$$

under

$$\text{momentum } p^2 \leftrightarrow \text{vev } \frac{a^2}{\epsilon_1\epsilon_2}, \qquad Q^2 \leftrightarrow \frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}$$


$Z = e^{\frac{F}{\epsilon_1 \epsilon_2}}$
$\epsilon_1 \epsilon_2 \log Z \longrightarrow F$

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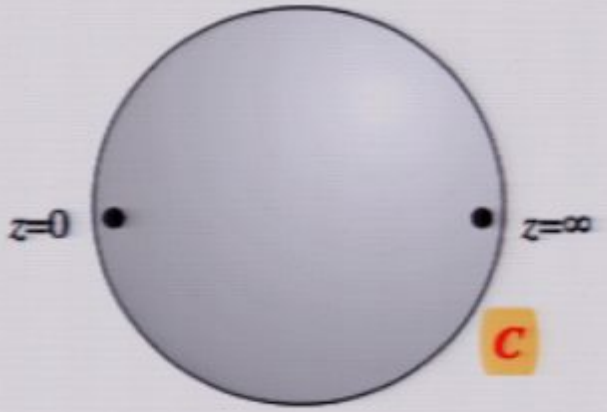

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              If[Delete[L, 1][[i]] + L[[1]] == 0,
               c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
              {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

```
In[6]:= Kac[n_] :=
         Kac[n] = Module[{par = Partitions[n]},
           Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
            {j, 1, Length[par]}]]

        IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
        ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          Λ^(4 n) / (ϵ1 ϵ2)^(2 n)
```

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```
In[3]:= c = 1 + 6 Q2;
```

- B Evaluates <0|LLL ... LLLL|0> by commutation relations. By Davide.

```
In[4]:= B[{}] := 1

In[5]:= B[L_] :=
        B[L] = If[L[[1]] < 0, 0, If[L[[1]] == 0, D0 B[Delete[L, 1]],
            Sum[
             (L[[1]] - Delete[L, 1][[i]])
               B[ReplacePart[Delete[L, 1], i -> Delete[L, 1][[i]] + L[[1]]]] +
              If[Delete[L, 1][[i]] + L[[1]] == 0,
               c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
             {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

```
In[6]:= Kac[n_] :=
         Kac[n] = Module[{par = Partitions[n]},
           Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
            {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
  Λ^(4 n) / (ε1 ε2)^(2 n)
```

Nekrasov parittion functions

```
DualPartition[l_] :=
 Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]
```

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```
In[3]:= c = 1 + 6 Q2;
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In[4]:= B[{}] := 1

In[5]:= B[L_] :=
        B[L] = If[L[[1]] < 0, 0, If[L[[1]] == 0, D0 B[Delete[L, 1]],
            Sum[
             (L[[1]] - Delete[L, 1][[i]])
               B[ReplacePart[Delete[L, 1], i -> Delete[L, 1][[i]] + L[[1]]]] +
              If[Delete[L, 1][[i]] + L[[1]] == 0,
               c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
             {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

```
In[6]:= Kac[n_] :=
        Kac[n] = Module[{par = Partitions[n]},
          Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
           {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
  Λ^ (4 n) / (ε1 ε2) ^ (2 n)
```

Nekrasov parittion functions

```
DualPartition[I_] :=
Module[{i}, Table[Length[Select[I, (# >= i) &]], {i, 1, I[[1]]}]]
```

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```
In[3]:= c = 1 + 6 Q2;
```

- B Evaluates <0|LLL .... LLLL|0> by commutation relations. By Davide.

```
In[4]:= B[{}] := 1

In[5]:= B[L_] :=
        B[L] = If[L[[1]] < 0, 0, If[L[[1]] = 0, D0 B[Delete[L, 1]],
            Sum[
             (L[[1]] - Delete[L, 1][[i]])
               B[ReplacePart[Delete[L, 1], i -> Delete[L, 1][[i]] + L[[1]]]] +
              If[Delete[L, 1][[i]] + L[[1]] = 0,
               c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
             {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

```
In[6]:= Kac[n_] :=
         Kac[n] = Module[{par = Partitions[n]},
           Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
            {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
In[8]:= ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          A^(4 n) / (e1 e2)^(2 n)
```

Nekrasov parittion functions

```
DualPartition[l_] :=
 Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]
```

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```
In[5]:= B[L_] :=
        B[L] = If[L[[1]] < 0, 0, If[L[[1]] == 0, DO B[Delete[L, 1]],
            Sum[
              (L[[1]] - Delete[L, 1][[i]])
                B[ReplacePart[Delete[L, 1], i -> Delete[L, 1][[i]] + L[[1]]]] +
               If[Delete[L, 1][[i]] + L[[1]] == 0,
                c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
              {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

```
In[6]:= Kac[n_] :=
         Kac[n] = Module[{par = Partitions[n]},
           Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
            {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
In[8]:= ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          Λ^ (4 n) / (ε1 ε2) ^ (2 n)
```

Nekrasov parittion functions

```
DualPartition[l_] :=
 Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]

DualPartition[{}] = {};

get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

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```
In[5]:= B[L_] :=
        B[L] = If[L[[1]] < 0, 0, If[L[[1]] == 0, 00 B[Delete[L, 1]],
            Sum[
              (L[[1]] - Delete[L, 1][[i]])
                B[ReplacePart[Delete[L, 1], i -> Delete[L, 1][[i]] + L[[1]]]] +
               If[Delete[L, 1][[i]] + L[[1]] == 0,
                c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
              {i, 1, Length[Delete[L, 1]]}]]]
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- Kac matrix.

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In[6]:= Kac[n_] :=
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           Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
            {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
In[8]:= ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          A^ (4 n) / (e1 e2) ^ (2 n)
```

Nekrasov parittion functions

```
In[9]:= DualPartition[l_] :=
         Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]

        DualPartition[{}] = {};

        get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

```
        arm[Y_, {i_, j_}] := get[Y, i] - j
```

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```
In[5]:= B[L_] :=
        B[L] = If[L[[1]] < 0, 0, If[L[[1]] == 0, 00 B[Delete[L, 1]],
            Sum[
             (L[[1]] - Delete[L, 1][[i]])
               B[ReplacePart[Delete[L, 1], i -> Delete[L, 1][[i]] + L[[1]]]] +
              If[Delete[L, 1][[i]] + L[[1]] == 0,
               c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
             {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

```
In[6]:= Kac[n_] :=
         Kac[n] = Module[{par = Partitions[n]},
           Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
            {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
In[8]:= ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          Λ^ (4 n) / (ε1 ε2) ^ (2 n)
```

Nekrasov parittion functions

```
In[9]:= DualPartition[l_] :=
         Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]

In[10]:= DualPartition[{}] = {};

         get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

```
         arm[Y_, {i_, j_}] := get[Y, i] - j
```

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```
        If[Delete[L, 1][[i]] + L[[1]] == 0,
         c / 12 L[[1]] (L[[1]]^2 - 1) B[Delete[Delete[L, 1], i]], 0],
        {i, 1, Length[Delete[L, 1]]}]]]
```

- Kac matrix.

In[6]:=
```
Kac[n_] :=
 Kac[n] = Module[{par = Partitions[n]},
   Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
    {j, 1, Length[par]}]]
```

In[7]:=
```
IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

In[8]:=
```
ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
   Λ^(4 n) / (ε1 ε2)^(2 n)
```

Nekrasov parittion functions

In[9]:=
```
DualPartition[l_] :=
 Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]
```

In[10]:=
```
DualPartition[{}] = {};
```

In[11]:=
```
get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

In[12]:=
```
arm[Y_, {i_, j_}] := get[Y, i] - j
```

```
leg[Y_, {i_, j_}] := get[DualPartition[Y], j] - i

Boxes[Y_] :=
 Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]
```

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```
Kac[n] = Module[{par = Partitions[n]},
  Table[B[Join[Reverse[ par[[i]]], -par[[j]]]], {i, 1, Length[par]},
   {j, 1, Length[par]}]]

In[7]:= IKac[n_] := Inverse[Kac[n]]
```

- <w|w> as the entry Kac^(-1)({1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
In[8]:= ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          Λ^(4 n) / (ϵ1 ϵ2)^(2 n)
```

Nekrasov parittion functions

```
In[9]:= DualPartition[λ_] :=
         Module[{i}, Table[Length[Select[λ, (# >= i) &]], {i, 1, λ[[1]]}]]

In[10]:= DualPartition[{}] = {};

In[11]:= get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

```
In[12]:= arm[Y_, {i_, j_}] := get[Y, i] - j

In[13]:= leg[Y_, {i_, j_}] := get[DualPartition[Y], j] - i

In[14]:= boxes[Y_] :=
          Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

         e[a_, Y1_, Y2_, s_] := a - ϵ1 leg[Y2, s] + ϵ2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
         fromW[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ϵ1 + ϵ2 - e[a - bb, Y1, Y2, #])) & /@
```

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- <w|w> as the entry Kac^(-1)[{1^n} , {1^n}]. Uses the fact {1^n} appears last in Partitions[n]. ww[i] is the contribution from i-th descendants.

```
In[8]:= ww[n_] := Module[{par = Partitions[n]}, IKac[n][[Length[par], Length[par]]]
          Λ^(4 n) / (ε1 ε2)^(2 n)
```

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Nekrasov parittion functions

```
In[9]:= DualPartition[λ_] :=
         Module[{i}, Table[Length[Select[λ, (# >= i) &]], {i, 1, λ[[1]]}]]

In[10]:= DualPartition[{}] = {};

In[11]:= get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

```
In[12]:= arm[Y_, {i_, j_}] := get[Y, i] - j

In[13]:= leg[Y_, {i_, j_}] := get[DualPartition[Y], j] - i

In[14]:= boxes[Y_] :=
         Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
fromSU2V[a_, Y1_, Y2_] :=
```

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asymptotically-free.nb

```
In[9]:= DualPartition[l_] :=
         Module[{i}, Table[Length[Select[l, (# >= i) &]], {i, 1, l[[1]]}]]

In[10]:= DualPartition[{}] = {};

In[11]:= get[Y_, i_] := If[i > Length[Y], 0, Y[[i]]]
```

- arm and leg lengths

```
In[12]:= arm[Y_, {i_, j_}] := get[Y, i] - j

In[13]:= leg[Y_, {i_, j_}] := get[DualPartition[Y], j] - i

In[14]:= boxes[Y_] :=
         Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
              fromWa[-a, -a, Y2, Y2])

         youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- i-instanton contribution

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- arm and leg lengths

```
In[12]:= arm[Y_, {i_, j_}] := get[Y, i] - j

In[13]:= leg[Y_, {i_, j_}] := get[DualPartition[Y], j] - i

In[14]:= boxes[Y_] :=
          Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
             fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- i-instanton contribution

```
pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^ (4 i)
```

Comparison

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- arm and leg lengths

```
In[12]:= arm[Y_, {i_, j_}] := get[Y, i] - j

In[13]:= leg[Y_, {i_, j_}] := get[DualPartition[Y], j] - i

In[14]:= boxes[Y_] :=
          Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
              fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- i-instanton contribution

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^ (4 i)
```

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Comparison

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```
In[14]:= boxes[Y_] :=
          Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
              fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- i-instanton contribution

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
Δ[a_] := Q2 / 4 - a^2 / (ε1 ε2);

k = 1;

pure[k]
```

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```
In[14]:= boxes[Y_] :=
          Join @@ Module[{i, j}, Table[Table[{i, j}, {j, 1, Y[[i]]}], {i, 1, Length[Y]}]]

In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
              fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- i-instanton contribution

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
Λ[a_] := Q2 / 4 - a^2 / (ε1 ε2);

k = 1;

pure[k]
```

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```
In[15]:= e[a_, Y1_, Y2_, s_] := a - ε1 leg[Y2, s] + ε2 (arm[Y1, s] + 1)
```

- Contribution of a W-boson

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
```

- Contribution of a U(2) vector multiplet

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
             fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- i-instanton contribution

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
In[20]:= Δ[a_] := Q2 / 4 - a^2 / (ε1 ε2);

         k = 1;

         pure[k]

         nw[k] /. Q0 → Δ[a]
```

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- **Contribution of a W-boson**

```
In[16]:= fromWa[a_, bb_, Y1_, Y2_] :=
          (Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
              boxes[Y1]))
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- **Contribution of a U(2) vector multiplet**

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
              fromWa[-a, -a, Y2, Y2])
```

```
In[18]:= youngPairs[k_] :=
          youngPairs[k] =
           Join @@
            Module[{i},
             Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
               {i, 0, k}]]
```

- **i-instanton contribution**

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
In[20]:= Δ[a_] := Q2 / 4 - a^2 / (ε1 ε2);
```

```
In[21]:= k = 1;

         pure[k]

         ww[k] /. D0 → Δ[a]

         % - %% // FullSimplify

         0
```

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```
(Times @@ ( (e[a - bb, Y1, Y2, #] (ε1 + ε2 - e[a - bb, Y1, Y2, #])) & /@
    boxes[Y1]))
```

- **Contribution of a U(2) vector multiplet**

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
  1 / (fromNa[a, a, Y1, Y1] fromNa[a, -a, Y1, Y2] fromNa[-a, a, Y2, Y1]
    fromNa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
  youngPairs[k] =
   Join @@
    Module[{i},
     Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
       {i, 0, k}]]
```

- **i-instanton contribution**

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
In[20]:= Δ[a_] := Q2 / 4 - a^2 / (ε1 ε2);

In[21]:= k = 1;

In[22]:= pure[k]
```

$$\text{Out[22]}= \Lambda^4 \left(\frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(-2\,a+\epsilon_1+\epsilon_2)} - \frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(2\,a+\epsilon_1+\epsilon_2)} \right)$$

```
ww[k] /. D0 → Δ[a]

% - %% // FullSimplify

0
```

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- **Contribution of a U(2) vector multiplet**

```
In[17]:= fromSU2V[a_, Y1_, Y2_] :=
          1 / (fromWa[a, a, Y1, Y1] fromWa[a, -a, Y1, Y2] fromWa[-a, a, Y2, Y1]
              fromWa[-a, -a, Y2, Y2])

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```

- **i-instanton contribution**

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[1], #[2]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
In[20]:= Δ[a_] := Q2 / 4 - a^2 / (ϵ1 ϵ2);

In[21]:= k = 1;

In[22]:= pure[k]
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```
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% - %% // FullSimplify
```

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```
    fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
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In[22]:= pure[k]
```

Out[22]= $\Lambda^4 \left(\frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(-2\,a+\epsilon_1+\epsilon_2)} - \frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(2\,a+\epsilon_1+\epsilon_2)} \right)$

```
In[23]:= ww[k] /. D0 → Δ[a]
```

Out[23]= $\frac{\Lambda^4}{2\,\epsilon_1^2\,\epsilon_2^2 \left(-\frac{a^2}{\epsilon_1\,\epsilon_2} + \frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2} \right)}$

```
% - %% // FullSimplify
```

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```
    fromWa[-a, -a, Y2, Y2])

In[18]:= youngPairs[k_] :=
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Out[22]= $\Lambda^4 \left(\frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(-2\,a+\epsilon_1+\epsilon_2)} - \frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(2\,a+\epsilon_1+\epsilon_2)} \right)$

```
In[23]:= ww[k] /. D0 → Δ[a]
```

Out[23]= $\frac{\Lambda^4}{2\,\epsilon_1^2\,\epsilon_2^2 \left(-\frac{a^2}{\epsilon_1\,\epsilon_2} + \frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2} \right)}$

```
% - %% // FullSimplify
```

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```
        fromNa[-a, -a, Y2, Y2])

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```
In[23]:= ww[k] /. D0 → Δ[a]
```

$$\text{Out[23]}= \frac{\Lambda^4}{2\,\epsilon_1^2\,\epsilon_2^2 \left(-\frac{a^2}{c_1\,c_2} + \frac{(c_1+c_2)^2}{4\,c_1\,c_2} \right)}$$

```
        % - %% // FullSimplify
Out[24]= 0
```

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```
youngPairs[k] =
 Join @@
  Module[{i},
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In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[1], #[2]] & /@ youngPairs[i])) Λ^(4 i)
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$$\text{Out[23]}= \frac{\Lambda^4}{2\,\epsilon_1^2\,\epsilon_2^2 \left(-\frac{a^2}{c_1\,c_2} + \frac{(c_1+c_2)^2}{4\,c_1\,c_2} \right)}$$

```
In[24]:= % - %% // FullSimplify
Out[24]= 0
```

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youngPairs[k] =
 Join @@
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   Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
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```
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```

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$$\text{Out[23]}= \frac{\Lambda^4}{2\,\epsilon_1^2\,\epsilon_2^2 \left(-\frac{a^2}{c_1\,c_2} + \frac{(c_1+c_2)^2}{4\,c_1\,c_2} \right)}$$

```
In[24]:= % - %% // FullSimplify
Out[24]= 0
```

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```
youngPairs[k] =
 Join @@
  Module[{i},
   Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
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```

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```
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In[20]:= Δ[a_] := Q2/4 - a^2/(ϵ1 ϵ2);

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```

$$\text{Out[22]}= \Lambda^4 \left(\frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(-2\,a+\epsilon_1+\epsilon_2)} - \frac{1}{2\,a\,\epsilon_1\,\epsilon_2\,(2\,a+\epsilon_1+\epsilon_2)} \right)$$

```
In[23]:= ww[k] /. D0 → Δ[a]
```

$$\text{Out[23]}= \left| \frac{\Lambda^4}{2\,\epsilon_1^2\,\epsilon_2^2 \left(-\frac{a^2}{c_1\,c_2} + \frac{(c_1+c_2)^2}{4\,c_1\,c_2} \right)} \right|$$

```
In[24]:= % - %% // FullSimplify
Out[24]= 0
```

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```
Module[{i},
 Join @@ Table[Outer[List, Partitions[i], Partitions[k - i], 1],
   {i, 0, k}]]
```

- i-instanton contribution

```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[1], #[2]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
In[20]:= Δ[a_] := Q2/4 - a^2/(ϵ1 ϵ2);

In[25]:= k = 2;

In[26]:= pure[k]
```

$$\text{Out[26]}= \Lambda^8 \left(\frac{1}{\epsilon_1^2 (-2a+\epsilon_1)(2a+\epsilon_1)\epsilon_2^2 (-2a+\epsilon_2)(2a+\epsilon_2)} + \frac{1}{4a(2a-\epsilon_1)\epsilon_1^2\epsilon_2(-\epsilon_1+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_1+\epsilon_2)} - \frac{1}{4a(-2a-\epsilon_1)\epsilon_1^2\epsilon_2(-\epsilon_1+\epsilon_2)(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_1+\epsilon_2)} + \frac{1}{4a\epsilon_1(2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^2(-2a+\epsilon_1+\epsilon_2)(-2a+\epsilon_1+2\epsilon_2)} - \frac{1}{4a\epsilon_1(-2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^2(2a+\epsilon_1+\epsilon_2)(2a+\epsilon_1+2\epsilon_2)} \right)$$

```
In[23]:= ww[k] /. D0 → Δ[a]
```

$$\text{Out[23]}= \frac{\Lambda^4}{\epsilon_1^2\epsilon_2^2\left(\frac{a^2}{\epsilon_1\epsilon_2} - \frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}\right)}$$

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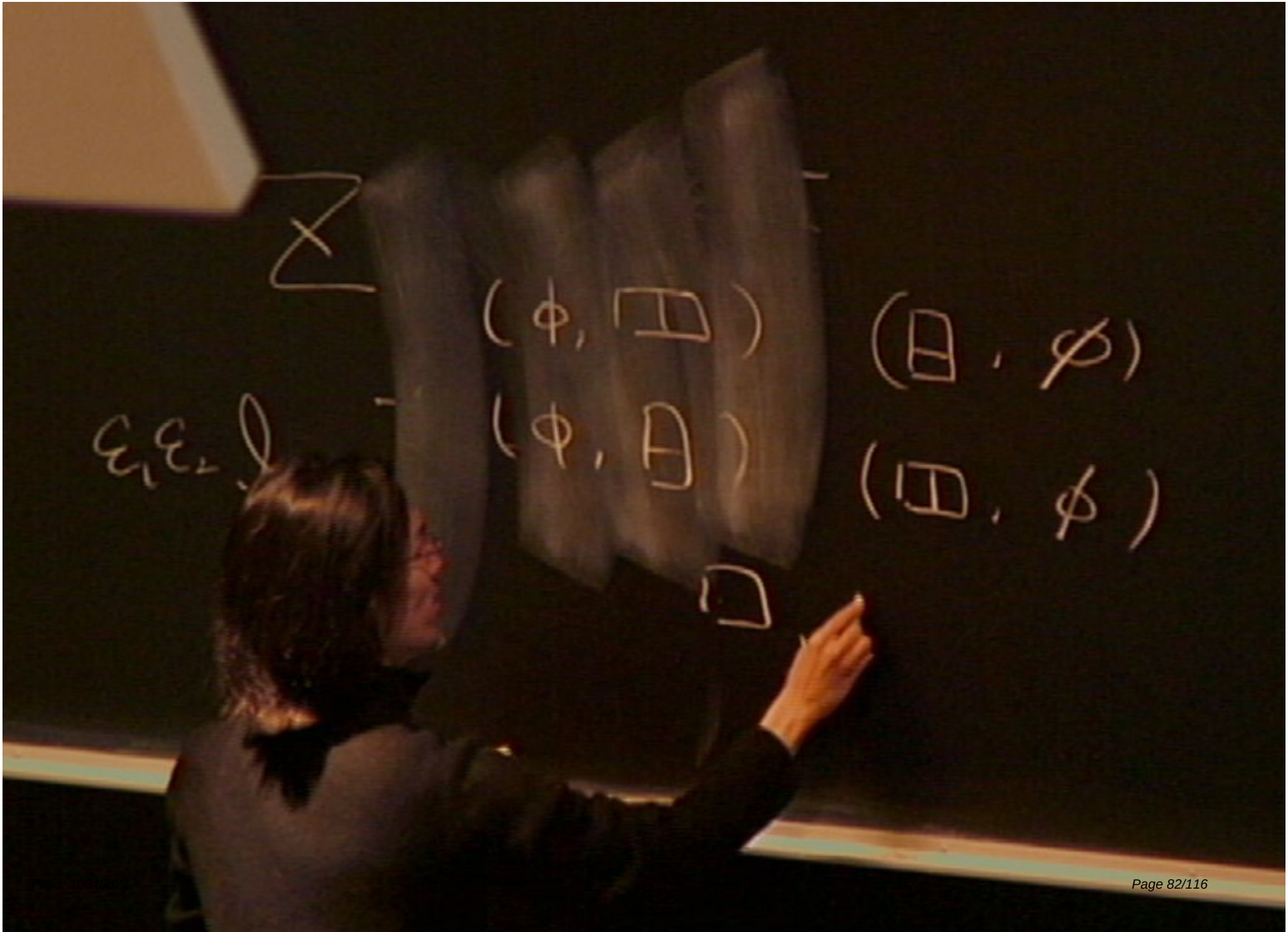
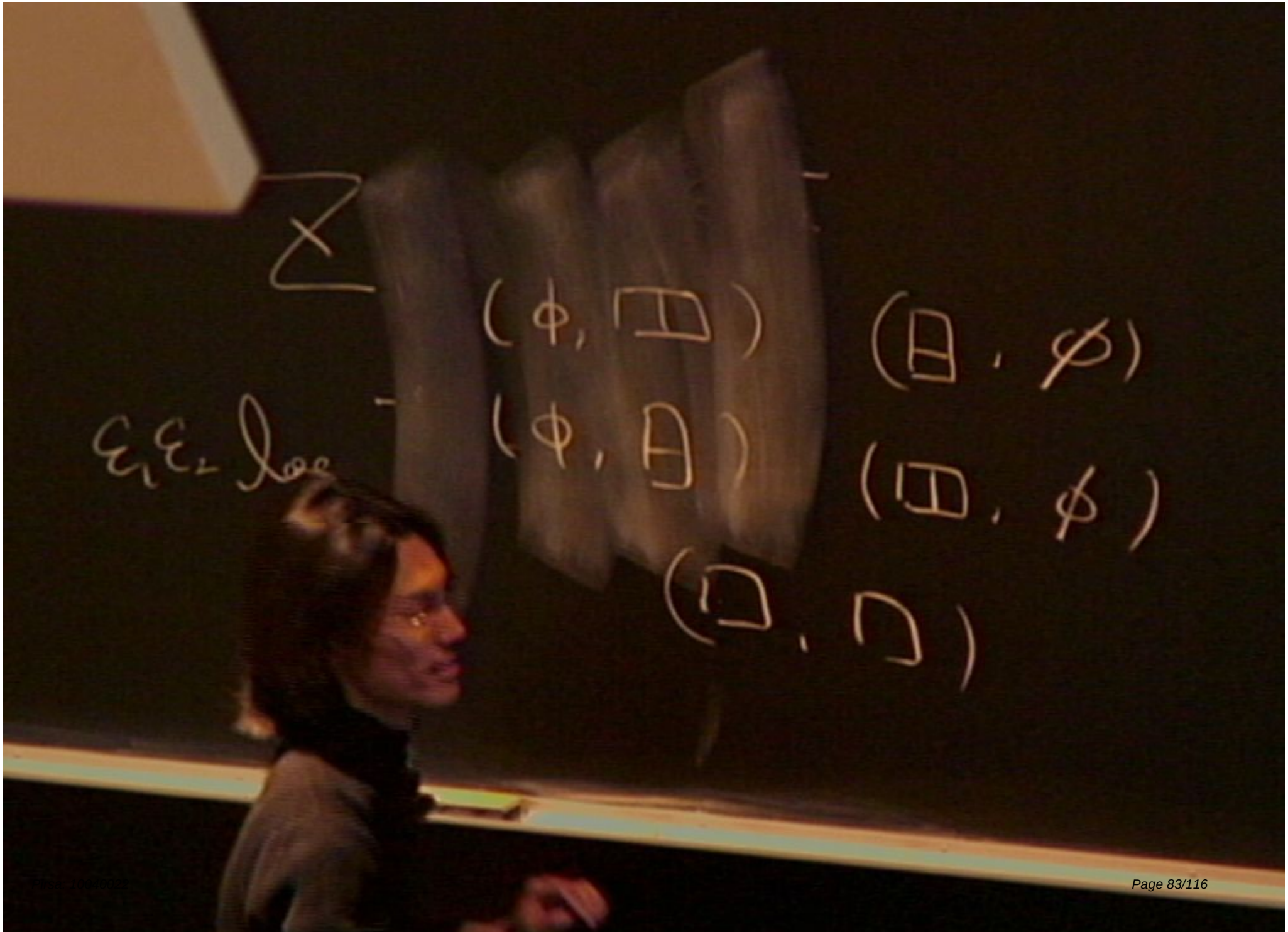

Σ
(φ, 口)
(日, φ)
(φ, 日)
(口, φ)
口


Σ
(φ, 日)
(日, φ)
(φ, 日)
(日, φ)
(口, 口)

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$$\frac{1}{4\,a\,\epsilon_1\,(2\,a-\epsilon_2)\,(\epsilon_1-\epsilon_2)\,\epsilon_2^2\,(-2\,a+\epsilon_1+\epsilon_2)\,(-2\,a+\epsilon_1+2\,\epsilon_2)} -$$

$$\frac{1}{4\,a\,\epsilon_1\,(-2\,a-\epsilon_2)\,(\epsilon_1-\epsilon_2)\,\epsilon_2^2\,(2\,a+\epsilon_1+\epsilon_2)\,(2\,a+\epsilon_1+2\,\epsilon_2)}\Bigg)$$

In[27]:= `ww[k] /. D0 → Δ[a]`

Out[27]=

$$\left(\Lambda^8\left(4\left(-\frac{a^2}{\epsilon_1\,\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2}\right)+\frac{1}{2}\left(1+\frac{6\,(\epsilon_1+\epsilon_2)^2}{\epsilon_1\,\epsilon_2}\right)\right)\right)\Bigg/\left(\epsilon_1^4\,\epsilon_2^4\right.$$

$$\left(26\left(-\frac{a^2}{\epsilon_1\,\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2}\right)+\frac{12\,\epsilon_1\left(-\frac{a^2}{c_1\,c_2}+\frac{(c_1+c_2)^2}{4\,c_1\,c_2}\right)}{\epsilon_2}+\frac{12\,\epsilon_2\left(-\frac{a^2}{c_1\,c_2}+\frac{(c_1+c_2)^2}{4\,c_1\,c_2}\right)}{\epsilon_1}+\right.$$

$$32\left(-\frac{a^2}{\epsilon_1\,\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2}\right)^2+\frac{24\,\epsilon_1\left(-\frac{a^2}{c_1\,c_2}+\frac{(c_1+c_2)^2}{4\,c_1\,c_2}\right)^2}{\epsilon_2}+$$

$$\left.\left.\frac{24\,\epsilon_2\left(-\frac{a^2}{c_1\,c_2}+\frac{(c_1+c_2)^2}{4\,c_1\,c_2}\right)^2}{\epsilon_1}+32\left(-\frac{a^2}{\epsilon_1\,\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2}\right)^3\right)\right)$$

In[28]:= `% - %% // FullSimplify`

Out[28]= 0

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```
In[19]:= pure[i_] := (Plus @@ (fromSU2V[a, #[[1]], #[[2]]] & /@ youngPairs[i])) Λ^(4 i)
```

Comparison

```
In[20]:= Δ[a_] := Q2 / 4 - a^2 / (ϵ1 ϵ2);

In[25]:= k = 2;

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```

Out[26]=

$$\Lambda^8 \left(\frac{1}{\epsilon_1^2 (-2a+\epsilon_1)(2a+\epsilon_1)\epsilon_2^2(-2a+\epsilon_2)(2a+\epsilon_2)} + \frac{1}{4a(2a-\epsilon_1)\epsilon_1^2\epsilon_2(-\epsilon_1+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_1+\epsilon_2)} - \frac{1}{4a(-2a-\epsilon_1)\epsilon_1^2\epsilon_2(-\epsilon_1+\epsilon_2)(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_1+\epsilon_2)} + \frac{1}{4a\epsilon_1(2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^2(-2a+\epsilon_1+\epsilon_2)(-2a+\epsilon_1+2\epsilon_2)} - \frac{1}{4a\epsilon_1(-2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^2(2a+\epsilon_1+\epsilon_2)(2a+\epsilon_1+2\epsilon_2)} \right)$$

```
In[27]:= ww[k] /. D0 → Δ[a]
```

Out[27]=

$$\left(\Lambda^8 \left(4\left(-\frac{a^2}{\epsilon_1\epsilon_2} + \frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right) + \frac{1}{2}\left(1 + \frac{6(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}\right)\right)\right) \Big/ \left(\epsilon_1^4\epsilon_2^4\right.$$

$$\left(\left(\frac{a^2}{\ldots} \quad \frac{(\epsilon_1+\epsilon_2)^2}{\ldots} \right) \quad \frac{12\epsilon_1\left(-\frac{a^2}{\epsilon_1\epsilon_2} + \frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right)}{\ldots} \quad \frac{12\epsilon_2\left(-\frac{a^2}{\epsilon_1\epsilon_2} + \frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right)}{\ldots} \right.$$

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$$\frac{1}{4a\epsilon_1^3(2a+\epsilon_1)(-2a+2\epsilon_1)\epsilon_2^2(-2a+\epsilon_2)(-\epsilon_1+\epsilon_2)(2a-\epsilon_1+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)}$$

$$-\frac{1}{4a\epsilon_1^3(-2a+\epsilon_1)(2a+2\epsilon_1)\epsilon_2^2(2a+\epsilon_2)(-\epsilon_1+\epsilon_2)(-2a-\epsilon_1+\epsilon_2)(2a+\epsilon_1+\epsilon_2)}$$

$$+1/\left(12a(2a-2\epsilon_1)(2a-\epsilon_1)\epsilon_1^3\epsilon_2(-2\epsilon_1+\epsilon_2)(-\epsilon_1+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_1+\epsilon_2)(-2a+3\epsilon_1+\epsilon_2)\right)-$$

$$1/\left(12a(-2a-2\epsilon_1)(-2a-\epsilon_1)\epsilon_1^3\epsilon_2(-2\epsilon_1+\epsilon_2)(-\epsilon_1+\epsilon_2)(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_1+\epsilon_2)(2a+3\epsilon_1+\epsilon_2)\right)+$$

$$\frac{1}{4a\epsilon_1^2(-2a+\epsilon_1)(\epsilon_1-\epsilon_2)(2a+\epsilon_1-\epsilon_2)\epsilon_2^3(2a+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_2)}$$

$$-\frac{1}{4a\epsilon_1^2(2a+\epsilon_1)(\epsilon_1-\epsilon_2)(-2a+\epsilon_1-\epsilon_2)\epsilon_2^3(-2a+\epsilon_2)(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_2)}$$

$$+1/\left(2a(2a-\epsilon_1)\epsilon_1^2(2a-\epsilon_2)(2\epsilon_1-\epsilon_2)\epsilon_2^2(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_1+\epsilon_2)(-\epsilon_1+2\epsilon_2)(-2a+\epsilon_1+2\epsilon_2)\right)-$$

$$1/\left(2a(-2a-\epsilon_1)\epsilon_1^2(-2a-\epsilon_2)(2\epsilon_1-\epsilon_2)\epsilon_2^2(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_1+\epsilon_2)(-\epsilon_1+2\epsilon_2)(2a+\epsilon_1+2\epsilon_2)\right)+$$

$$1/\left(12a\epsilon_1(2a-2\epsilon_2)(\epsilon_1-2\epsilon_2)(2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^3(-2a+\epsilon_1+\epsilon_2)(-2a+\epsilon_1+2\epsilon_2)(-2a+\epsilon_1+3\epsilon_2)\right)-$$

$$1/\left(12a\epsilon_1(-2a-2\epsilon_2)(\epsilon_1-2\epsilon_2)(-2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^3(2a+\epsilon_1+\epsilon_2)(2a+\epsilon_1+2\epsilon_2)(2a+\epsilon_1+3\epsilon_2)\right)$$

In[27]:= `ww[k] /. D0 → Δ[a]`

Out[27]= $\left(\Lambda^8\left(4\left(-\frac{a^2}{\dots}+\frac{(\epsilon_1+\epsilon_2)^2}{\dots}\right)+\frac{1}{\dots}\left(1+\frac{6(\epsilon_1+\epsilon_2)^2}{\dots}\right)\right)\right)/\left(\epsilon_1^4\epsilon_2^4\right.$

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$$-\frac{1}{4a\epsilon_1^3(-2a+\epsilon_1)(2a+2\epsilon_1)\epsilon_2^2(2a+\epsilon_2)(-\epsilon_1+\epsilon_2)(-2a-\epsilon_1+\epsilon_2)(2a+\epsilon_1+\epsilon_2)}$$

$$+1/\left(12a(2a-2\epsilon_1)(2a-\epsilon_1)\epsilon_1^3\epsilon_2(-2\epsilon_1+\epsilon_2)\right.$$
$$\left.(-\epsilon_1+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_1+\epsilon_2)(-2a+3\epsilon_1+\epsilon_2)\right)-$$

$$1/\left(12a(-2a-2\epsilon_1)(-2a-\epsilon_1)\epsilon_1^3\epsilon_2(-2\epsilon_1+\epsilon_2)(-\epsilon_1+\epsilon_2)\right.$$
$$\left.(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_1+\epsilon_2)(2a+3\epsilon_1+\epsilon_2)\right)+$$

$$\frac{1}{4a\epsilon_1^2(-2a+\epsilon_1)(\epsilon_1-\epsilon_2)(2a+\epsilon_1-\epsilon_2)\epsilon_2^3(2a+\epsilon_2)(-2a+\epsilon_1+\epsilon_2)(-2a+2\epsilon_2)}$$

$$-\frac{1}{4a\epsilon_1^2(2a+\epsilon_1)(\epsilon_1-\epsilon_2)(-2a+\epsilon_1-\epsilon_2)\epsilon_2^3(-2a+\epsilon_2)(2a+\epsilon_1+\epsilon_2)(2a+2\epsilon_2)}$$

$$+1/\left(2a(2a-\epsilon_1)\epsilon_1^2(2a-\epsilon_2)(2\epsilon_1-\epsilon_2)\epsilon_2^2(-2a+\epsilon_1+\epsilon_2)\right.$$
$$\left.(-2a+2\epsilon_1+\epsilon_2)(-\epsilon_1+2\epsilon_2)(-2a+\epsilon_1+2\epsilon_2)\right)-$$

$$1/\left(2a(-2a-\epsilon_1)\epsilon_1^2(-2a-\epsilon_2)(2\epsilon_1-\epsilon_2)\epsilon_2^2(2a+\epsilon_1+\epsilon_2)\right.$$
$$\left.(2a+2\epsilon_1+\epsilon_2)(-\epsilon_1+2\epsilon_2)(2a+\epsilon_1+2\epsilon_2)\right)+$$

$$1/\left(12a\epsilon_1(2a-2\epsilon_2)(\epsilon_1-2\epsilon_2)(2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^3\right.$$
$$\left.(-2a+\epsilon_1+\epsilon_2)(-2a+\epsilon_1+2\epsilon_2)(-2a+\epsilon_1+3\epsilon_2)\right)-$$

$$1/\left(12a\epsilon_1(-2a-2\epsilon_2)(\epsilon_1-2\epsilon_2)(-2a-\epsilon_2)(\epsilon_1-\epsilon_2)\epsilon_2^3\right.$$
$$\left.(2a+\epsilon_1+\epsilon_2)(2a+\epsilon_1+2\epsilon_2)(2a+\epsilon_1+3\epsilon_2)\right)$$

In[27]:= ww[k] /. D0 → Δ[a]

$$\left(\Delta^8\left(4\left(-\frac{a^2}{\epsilon_1\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right)+\frac{1}{2}\left(1+\frac{6(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}\right)\right)\right)/\left(\epsilon_1^4\epsilon_2^4\right.$$

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$$\epsilon_1^2 \qquad \left(\epsilon_1 \epsilon_2 \qquad 4\,\epsilon_1 \epsilon_2\right)$$

$$\frac{5184\,\epsilon_1^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_2^2}+\frac{18\,720\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_2}+$$

$$\frac{18\,720\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_1}+\frac{5184\,\epsilon_2^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_1^2}+$$

$$24\,912\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^4+$$

$$\frac{3456\,\epsilon_1^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_2^2}+\frac{13\,536\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_2}+$$

$$\frac{13\,536\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_1}+\frac{3456\,\epsilon_2^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_1^2}+$$

$$6912\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^5+\frac{6336\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^5}{\epsilon_2}+$$

$$\frac{6336\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^5}{\epsilon_1}+2304\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^6\Bigg)\Bigg)$$

In[28]:= % - %% // FullSimplify

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$$\frac{5184\,\epsilon_1^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^3}{\epsilon_2^2}+\frac{18\,720\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^3}{\epsilon_2}+$$

$$\frac{18\,720\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^3}{\epsilon_1}+\frac{5184\,\epsilon_2^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^3}{\epsilon_1^2}+$$

$$24\,912\left(-\frac{a^2}{\epsilon_1\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right)^4+$$

$$\frac{3456\,\epsilon_1^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^4}{\epsilon_2^2}+\frac{13\,536\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^4}{\epsilon_2}+$$

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$$6912\left(-\frac{a^2}{\epsilon_1\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right)^5+\frac{6336\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^5}{\epsilon_2}+$$

$$\frac{6336\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4c_1c_2}\right)^5}{\epsilon_1}+2304\left(-\frac{a^2}{\epsilon_1\epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\epsilon_1\epsilon_2}\right)^6\Bigg)\Bigg)$$

In[28]:= % - %% // FullSimplify

Out[28]= 0

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$$\frac{}{\epsilon_2^2} + \frac{}{\epsilon_2} +$$

$$\frac{18\,720\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_1} + \frac{5184\,\epsilon_2^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_1^2} +$$

$$24\,912\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^4 +$$

$$\frac{3456\,\epsilon_1^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_2^2} + \frac{13\,536\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_2} +$$

$$\frac{13\,536\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_1} + \frac{3456\,\epsilon_2^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^4}{\epsilon_1^2} +$$

$$6912\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^5 + \frac{6336\,\epsilon_1\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^5}{\epsilon_2} +$$

$$\frac{6336\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^5}{\epsilon_1} + 2304\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^6\Bigg)\Bigg)$$

In[32]:= % - %% // FullSimplify

Out[32]= 0

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$$\frac{}{\epsilon_2^2} + \frac{}{\epsilon_2} +$$

$$\frac{18\,720\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_1} + \frac{5184\,\epsilon_2^2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^3}{\epsilon_1^2} +$$

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$$\frac{6336\,\epsilon_2\left(-\frac{a^2}{c_1 c_2}+\frac{(c_1+c_2)^2}{4 c_1 c_2}\right)^5}{\epsilon_1} + 2304\left(-\frac{a^2}{\epsilon_1 \epsilon_2}+\frac{(\epsilon_1+\epsilon_2)^2}{4\,\epsilon_1 \epsilon_2}\right)^6\Bigg)\Bigg)$$

In[32]:= `% - %% // FullSimplify`

Out[32]= 0

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$$\frac{}{\epsilon_2^2} + \frac{}{\epsilon_2} +$$

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In[32]:= % - %% // FullSimplify

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auge theory

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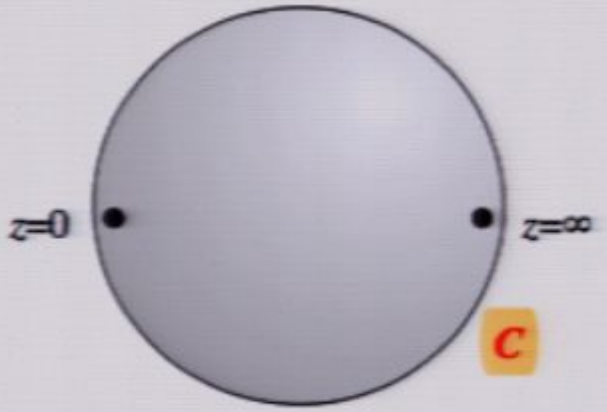
Mathematica

Proposal: SU(N)



- SW curve was $\lambda^N + \phi_2(z)\lambda^{N-2} + \cdots + \phi_N(z) = 0.$
- Toda theory has $T(z), W_3(z), \cdots, W_N(z).$

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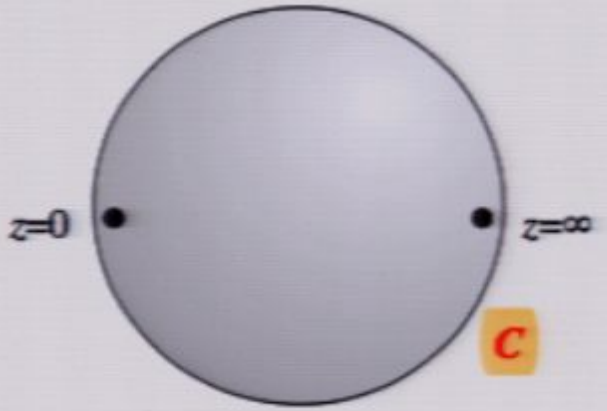

- SW curve was $\lambda^N + \phi_2(z)\lambda^{N-2} + \cdots + \phi_N(z) = 0.$
- Toda theory has $T(z), W_3(z), \cdots, W_N(z).$

$$\epsilon_1\epsilon_2\langle W_k(z)\rangle dz^k \to \phi_k(z) \quad \text{when} \quad \epsilon_{1,2} \to 0$$

$$\text{momentum } \vec{p} \leftrightarrow \text{vev } \frac{\vec{a}}{\epsilon_1\epsilon_2}, \qquad b^2 \leftrightarrow \frac{\epsilon_1}{\epsilon_2}$$

$$c = (N-1) + (N-1)N(N+1)\frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}$$

Status concerning conformal blocks

- Many checks for **SU(2)**. [Marshakov,Mironov^2,Morozov^2], ...
- [Fateev-Litvinov] **proved** the equality

  Z of SU(2) with massive adjoint

  = torus one-point conformal block

  by showing boths sides satisfy the same non-linear relation
  cf. [Poghossian], [Hadasz,Jaskólski,Suchanek]

- **SU(N)** relation is not as much studied.
  [Wyllard], [Mironov,Morozov], [Taki], ...,

How about U(1) ?

- Mathematicians [Carlsson-Okounkov] had done that in **2008**!
- They proved that

Nekrasov function of U(1) quiver

= correlator of a free boson

- We introduced the space

$$V_a \ni |v\rangle\rangle = (v_{(0,0)};\ v_{(0,\square)},\ v_{(\square,0)};\ v_{(0,\square\square)},\ v_{(0,\begin{smallmatrix}\square\\\square\end{smallmatrix})},\ v_{(\square,\square)},\ \ldots).$$

It's called the equivariant cohomology of the instanton moduli.

- Our observation **predicts** that
  1) V_a is naturally the Verma module of Virasoro algebra, and
  2) hypers should give the vertex operator.

Contents

1. 4d gauge theory

2. 2d CFT

3. Surface operator

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Contents

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Defects

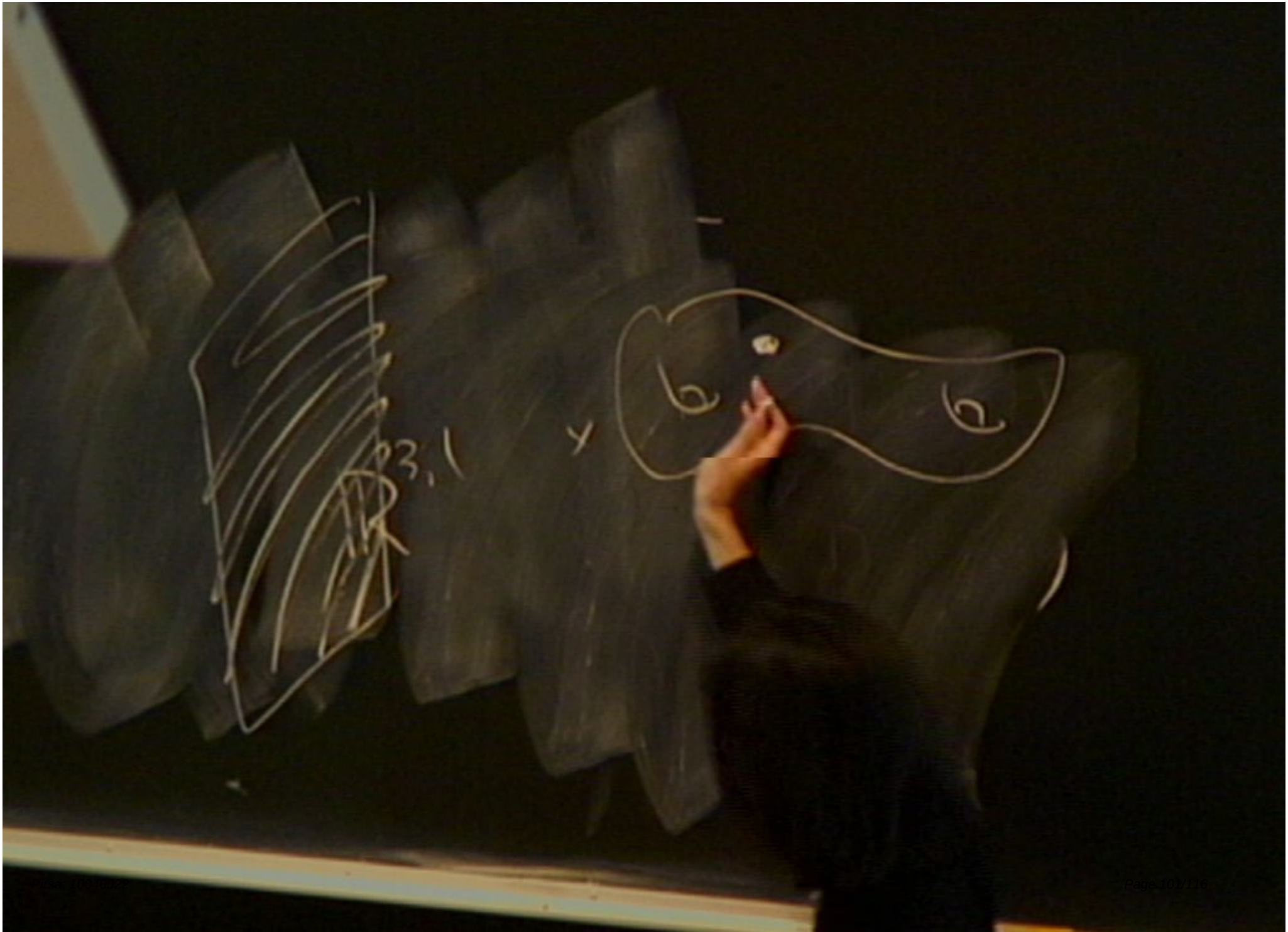
6d theory on M5-branes has two basic defects:

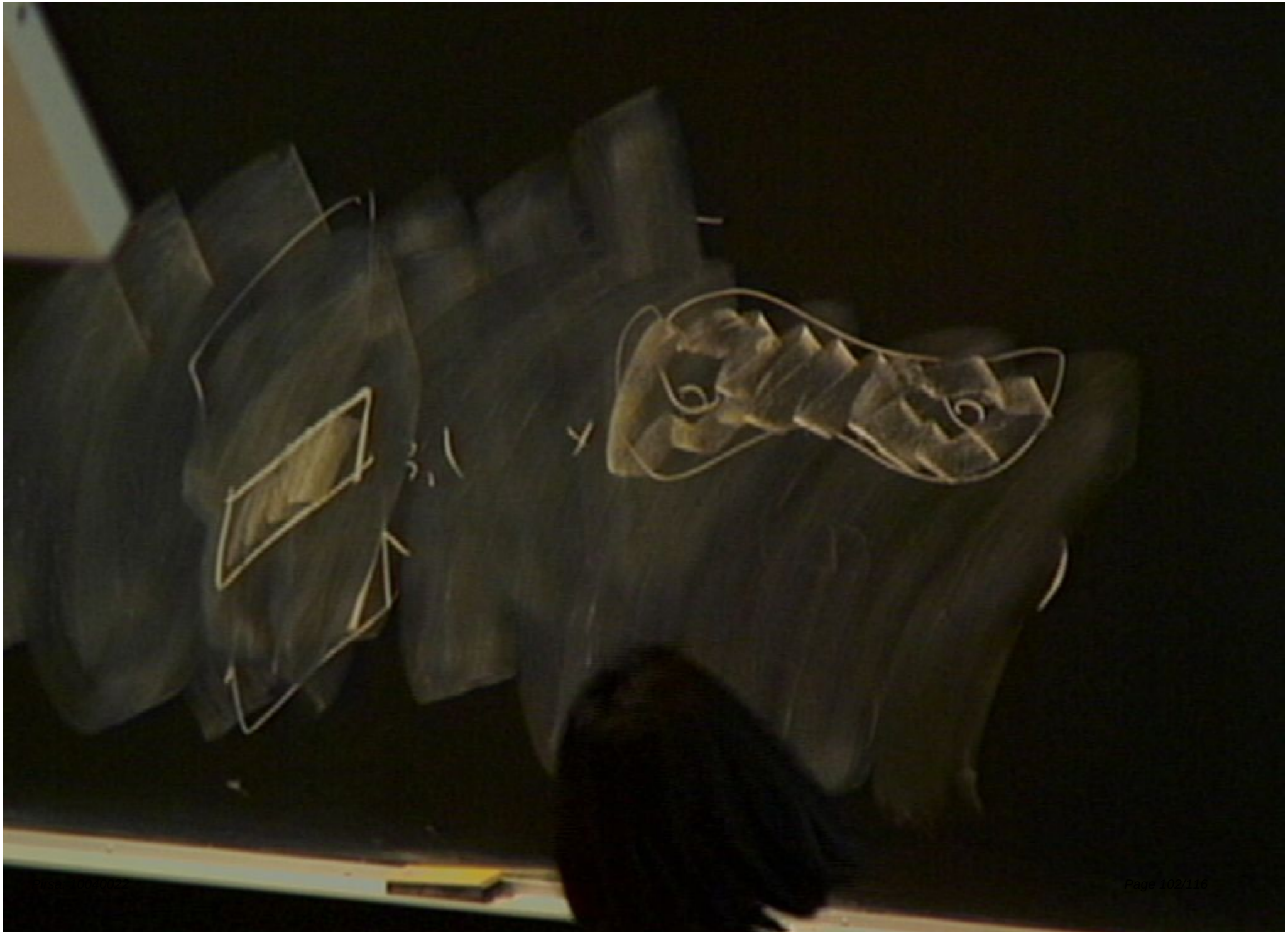
- 4d objects: intersections with other M5-branes
- 2d objects: ends of M2-branes

Various choices in how to wrap these, when 6d = $X_4 \times C_2$.

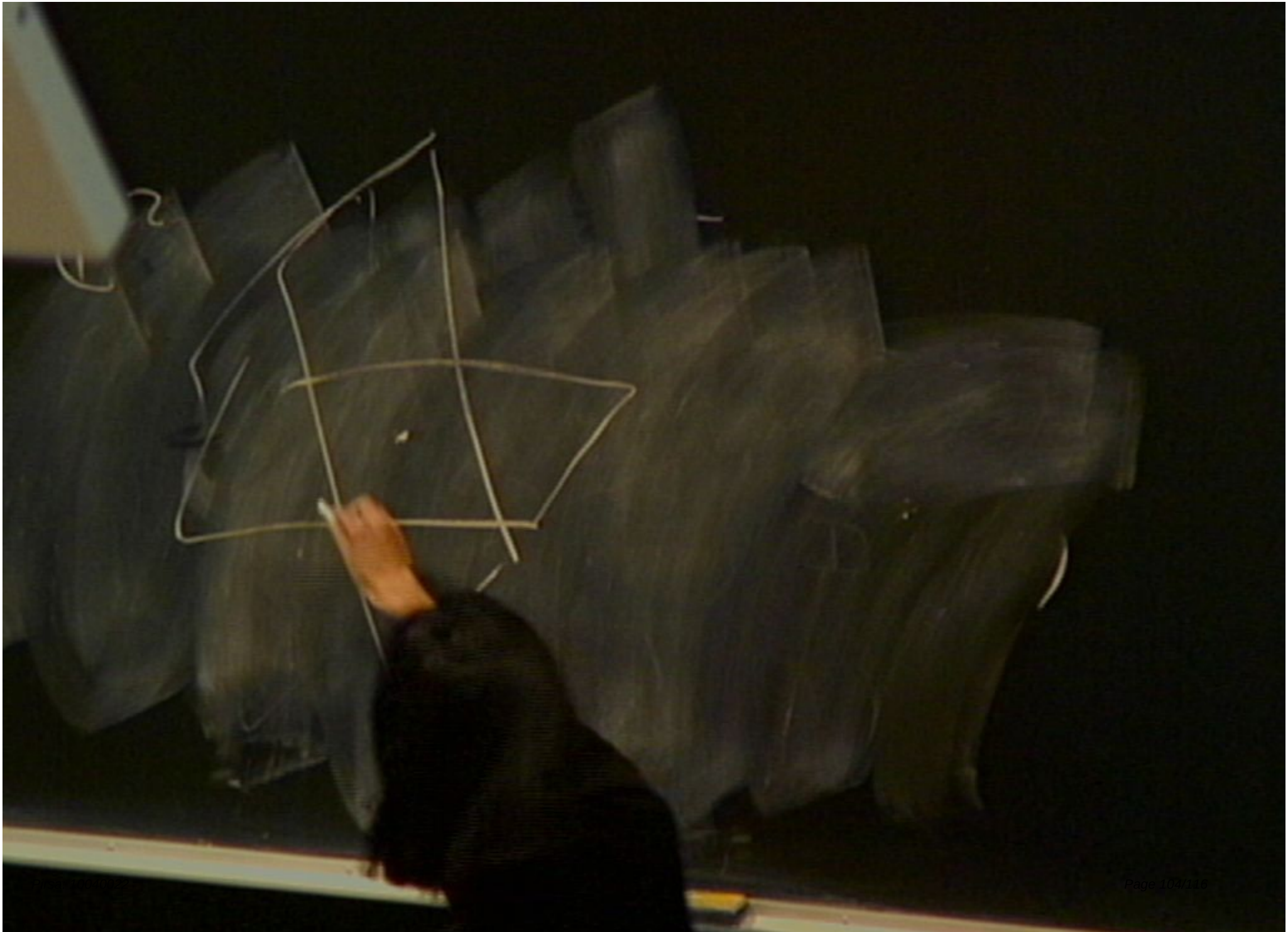
Studied by various people [Drukker-Morrison-Okuda], [Drukker-Gomis-Okuda-Teschner], [Drukker-Gaiotto-Gomis], ...

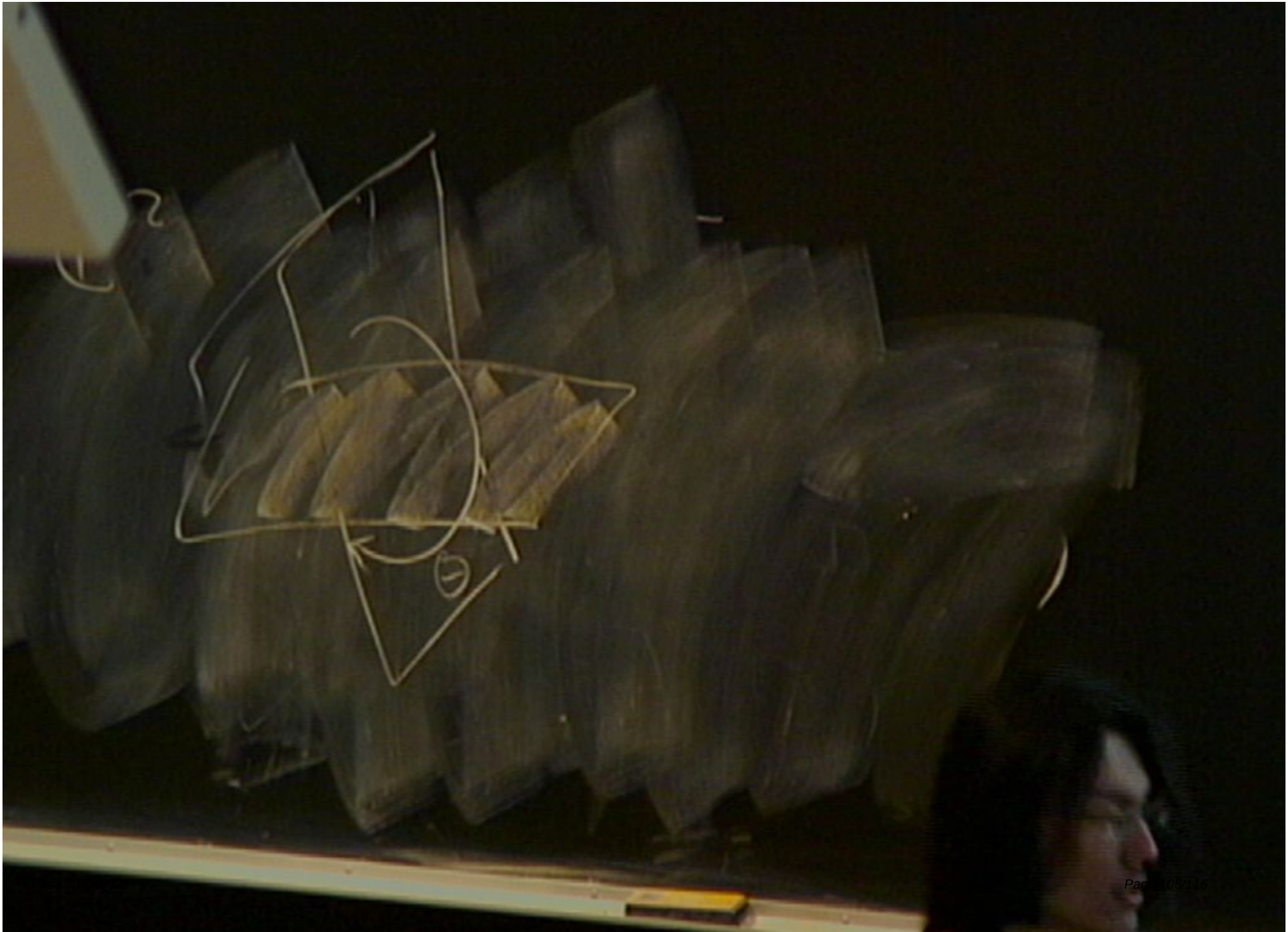
6d	X_4	C_2	
2d	1	1	Loops on X_4
2d	2	0	Surfaces on X_4
4d	4	0	Changes theory on X_4


3,1
y



6d





nstanton partition function with surface perator

Take pure $\mathbf{U}(N)$ theory. Without the surface operator, we considered

$$Z(a) = \sum_k q^k \int_{\mathcal{M}_{N,k}} (\text{something complicated})$$

where $\mathcal{M}_{N,k}$: k-instanton moduli of $\mathbf{U}(N)$.

With the full surface opeartor, we consider

$$Z^S(a) = \sum_{k_1,k_2,\ldots,k_N} q_1^{k_1} \cdots q_N^{k_N} \int_{\mathcal{M}_{N;k_1,\ldots,k_N}} (\text{something complicated})$$

$\mathcal{M}_{N;k_1,\ldots,k_N}$: instanton moduli **with full surface operator**.

k_1: instanton number; $k_i - k_{i+1}$: monopole flux.

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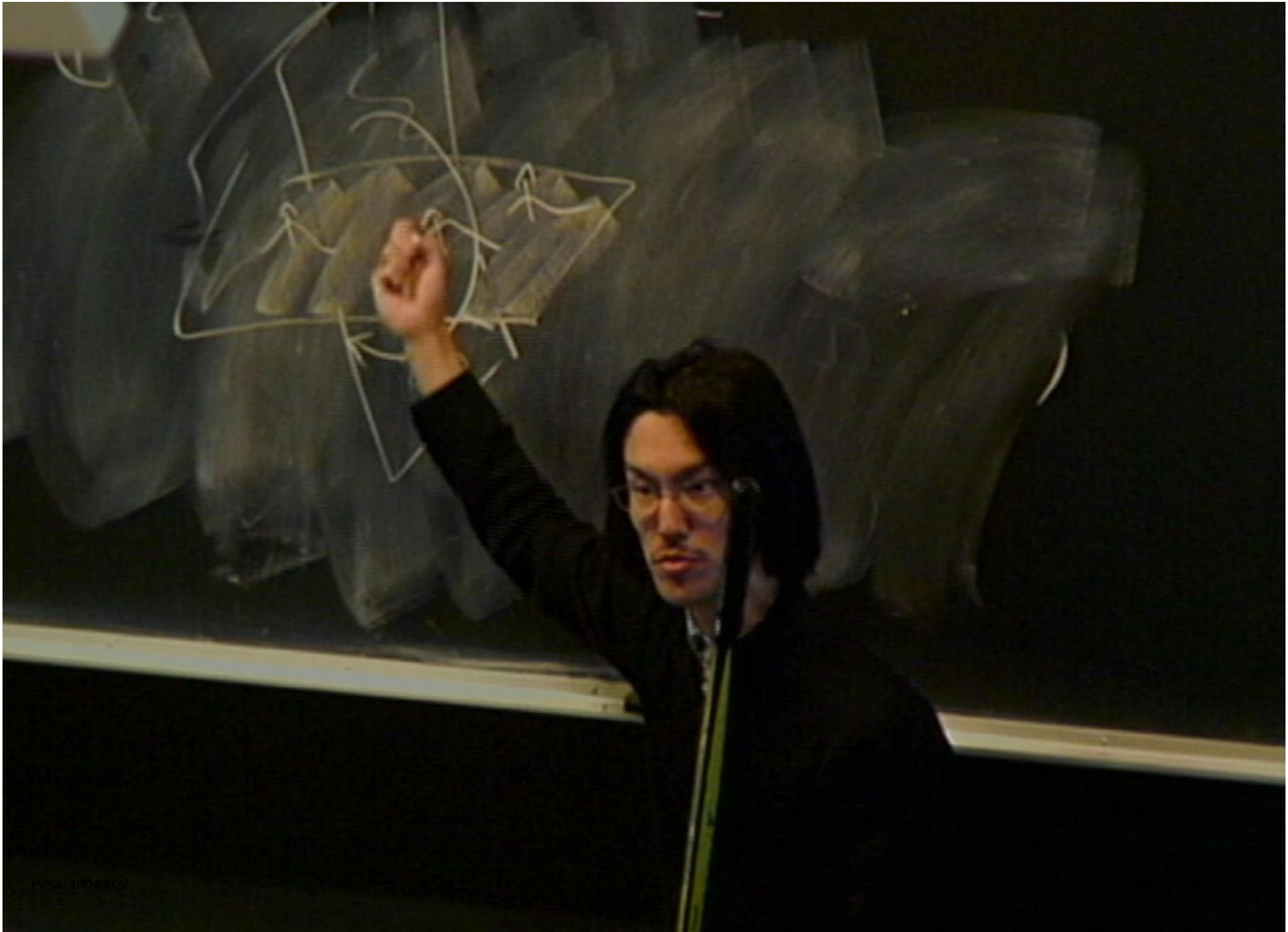

Pirsa: 10040022

$$\langle\langle v|w\rangle\rangle = \sum_{\vec{Y}} \frac{v_{\vec{Y}}\ w_{\vec{Y}}}{G_a(\vec{Y})}$$

Consider the vector

$$|w_a\rangle\rangle = (1;\ \Lambda^2,\ \Lambda^2;\ \Lambda^4,\ \Lambda^4,\ \ldots)$$

then

$$Z(a) = \langle\langle w_a|w_a\rangle\rangle$$

Fixed point formula without surface operator

Fixed points are labeled by a vector $\vec{Y}$ of Young diagrams

$$Z(a) = \sum_{\vec{Y}} \Lambda^{4|\vec{Y}|} \frac{1}{G_a(\vec{Y})}$$

Consider the vector space

$$V_a \ni |v\rangle\rangle = (v_{\vec{Y}})$$

Introduce the inner product via

$$\langle\langle v|w\rangle\rangle = \sum_{\vec{Y}} \frac{v_{\vec{Y}}\, w_{\vec{Y}}}{G_a(\vec{Y})}$$

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then

$$Z(a) = \langle\langle w_a|w_a\rangle\rangle$$

Fixed point formula with surface operator

Fixed points are labeled by $N \times N$ matrix of Young diagrams.

$$Z^S(a) = \sum_{Y_{ij}} \prod_i q_i^{\sum_j |Y_{ij}|} \frac{1}{G_a^S(Y_{ij})}.$$

Consider the vector space

$$V_a^S \ni |v\rangle\rangle = (v_{Y_{ij}}).$$

Introduce the inner product

$$\langle\langle v|w\rangle\rangle = \sum_{Y_{ij}} \frac{v_{Y_{ij}} w_{Y_{ij}}}{G_a^S(Y_{ij})}$$

Then again

$$Z^S(a) = \langle\langle w|w\rangle\rangle$$

where $w \in U_a$ is defined by

$$w_{Y_{ij}} = \prod_i q_i^{\sum_j |Y_{ij}|/2}$$

Fixed point formula without surface operator

Fixed points are labeled by a vector $\vec{Y}$ of Young diagrams

$$Z(a) = \sum_{\vec{Y}} \Lambda^{4|\vec{Y}|} \frac{1}{G_a(\vec{Y})}$$

Consider the vector space

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Consider the vector

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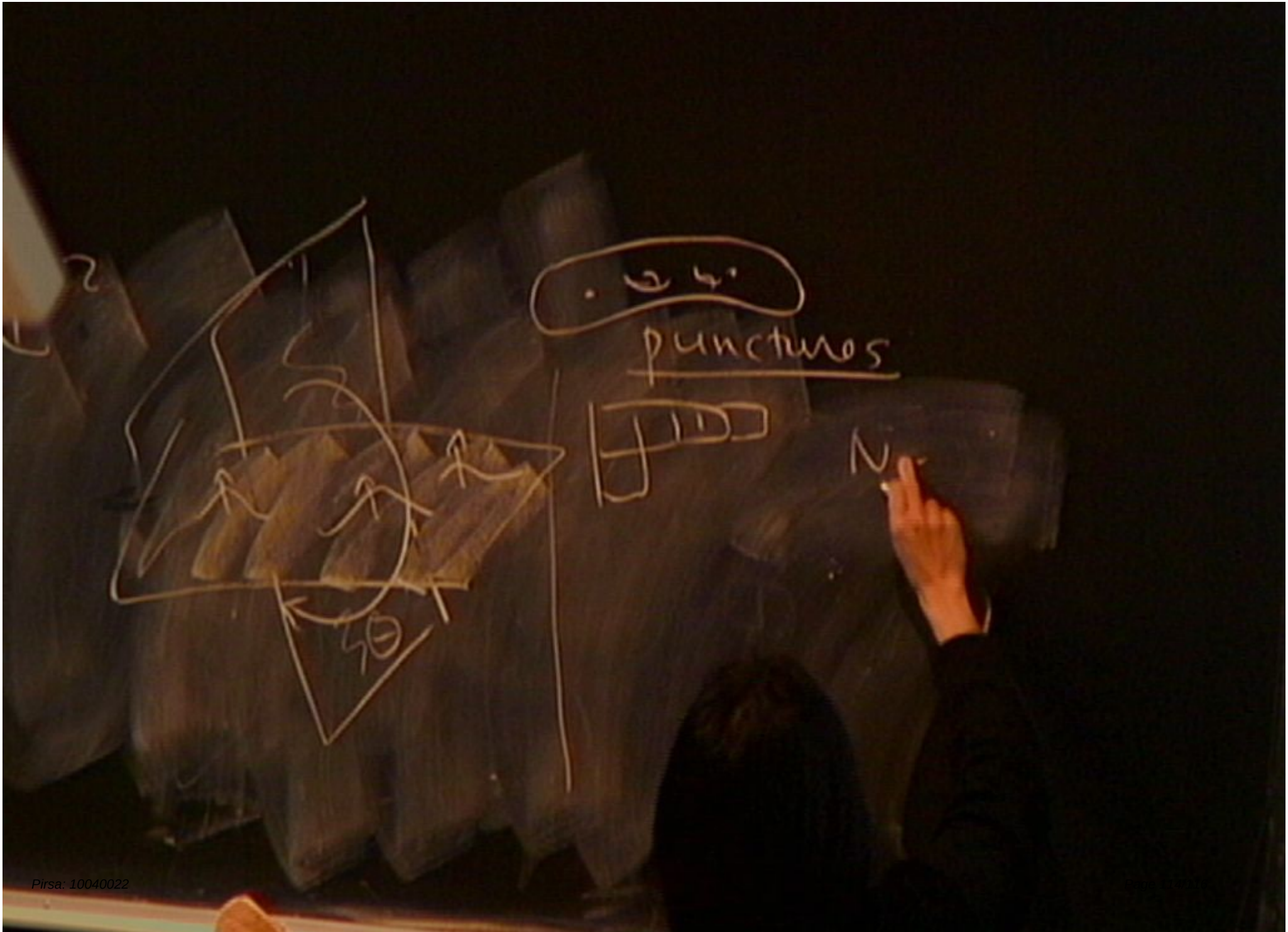
then

$$Z(a) = \langle\langle w_a|w_a\rangle\rangle$$



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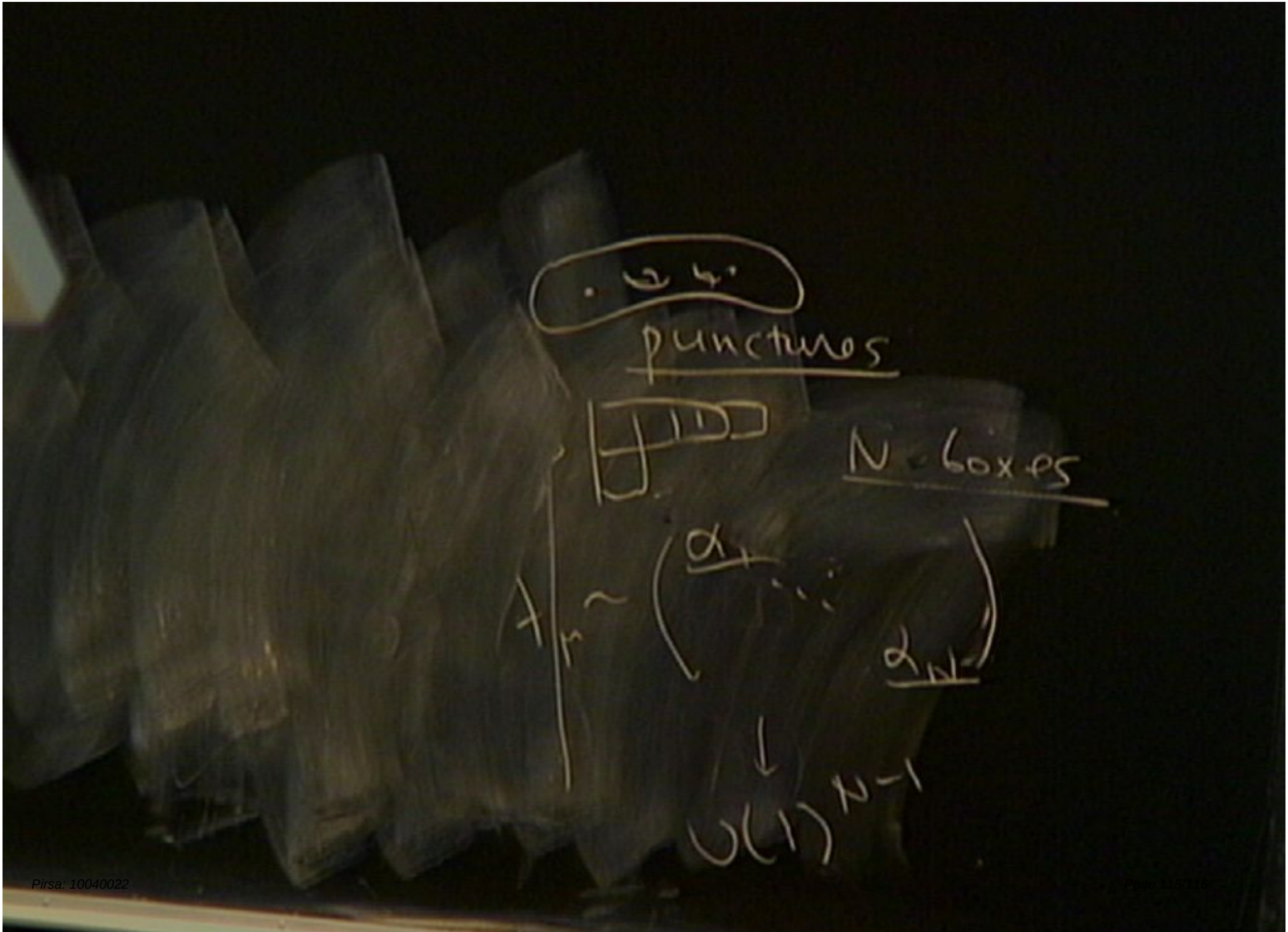

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With / Without surface opeartor

- As explained, V_a is conjectured to be the Verma module of W_N algebra.

$$c = (N-1) + (N-1)N(N+1)\frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}.$$

- What's V_a^S, i.e. with the full surface operator?

- Surprisingly, it had been **proved in 2004** that V_a^S is the Verma module of affine $SL(N)$ algebra, with level

$$k = -N + \frac{\epsilon_2}{\epsilon_1}$$

- This means that there lives $SL(N)$ WZW model on C_2.