Title: The Holographic Universe

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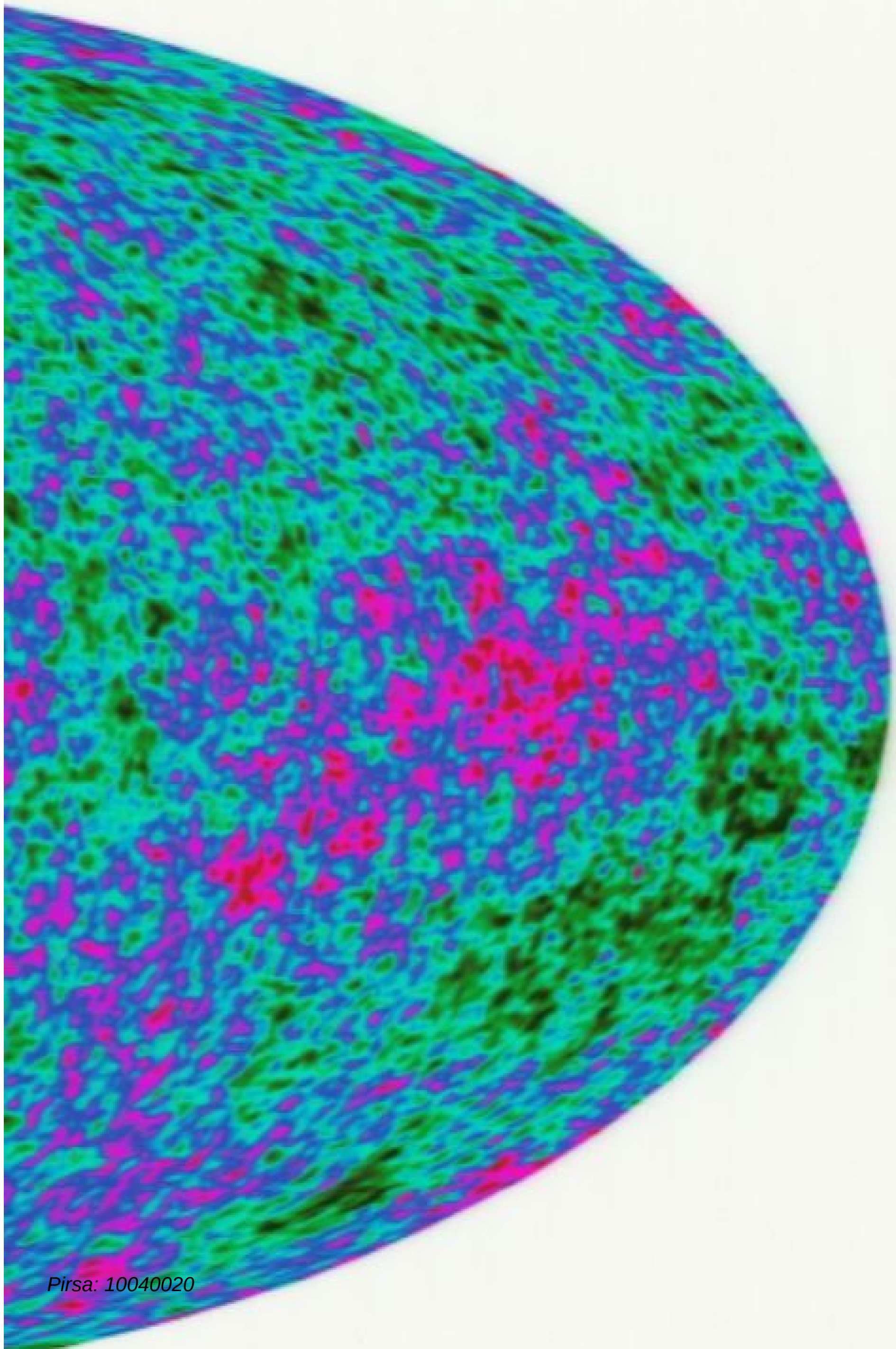
Abstract: We present a holographic description of four-dimensional single-scalar inflationary universes in terms of a three-dimensional quantum field theory. The holographic description correctly reproduces standard inflationary predictions in their regime of applicability. In the opposite case, wherein gravity is strongly coupled at early times, we propose a holographic description in terms of perturbative QFT and present models capable of satisfying the current observational constraints while exhibiting a phenomenology distinct from standard inflation. This provides a qualitatively new method for generating a nearly scale-invariant spectrum of primordial cosmological perturbations.



The Holographic Universe

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work with Kostas Skenderis
arXiv:0907.5542 & 1001.2007

Holography

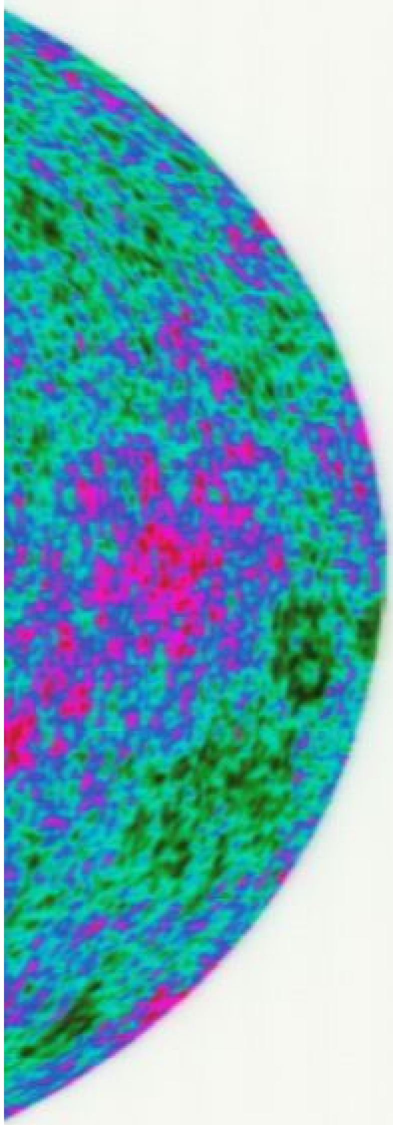
The notion of holography ['t Hooft 1993] emerged from black hole physics as an answer to the question: why is the entropy of a black hole proportional to the area of its horizon rather than its volume?

Holography: Any quantum gravitational system should have a dual description in terms of a quantum field theory (QFT), *without gravity*, in one dimension less.

- Holography became a prominent research direction when precise holographic dualities were found in string theory.

  [Maldacena 1997, Gubser, Klebanov & Polyakov 1998, Witten 1998]

Holography *for Cosmology*



- The holographic dualities found in string theory involve spacetimes with a negative cosmological constant, but the general argument for holography is applicable to any theory of gravity.
- In particular, it should apply to our own universe.
- Here we describe how to set up a holographic framework for inflationary cosmology.

  Specifically, we construct a dual description of four-dimensional inflationary cosmology in terms of a three-dimensional QFT without gravity.

Holography for Cosmology

Any proposed holographic framework for cosmology should specify:

1. The precise nature of the dual QFT.
2. How to compute cosmological observables (e.g. the primordial power spectrum) from the correlation functions of the dual QFT.

Having defined such a duality,

3. Must recover standard inflationary predictions in regime where usual perturbative quantisation of fluctuations is valid (i.e. weakly coupled gravity = strongly coupled QFT).

Holography for Cosmology

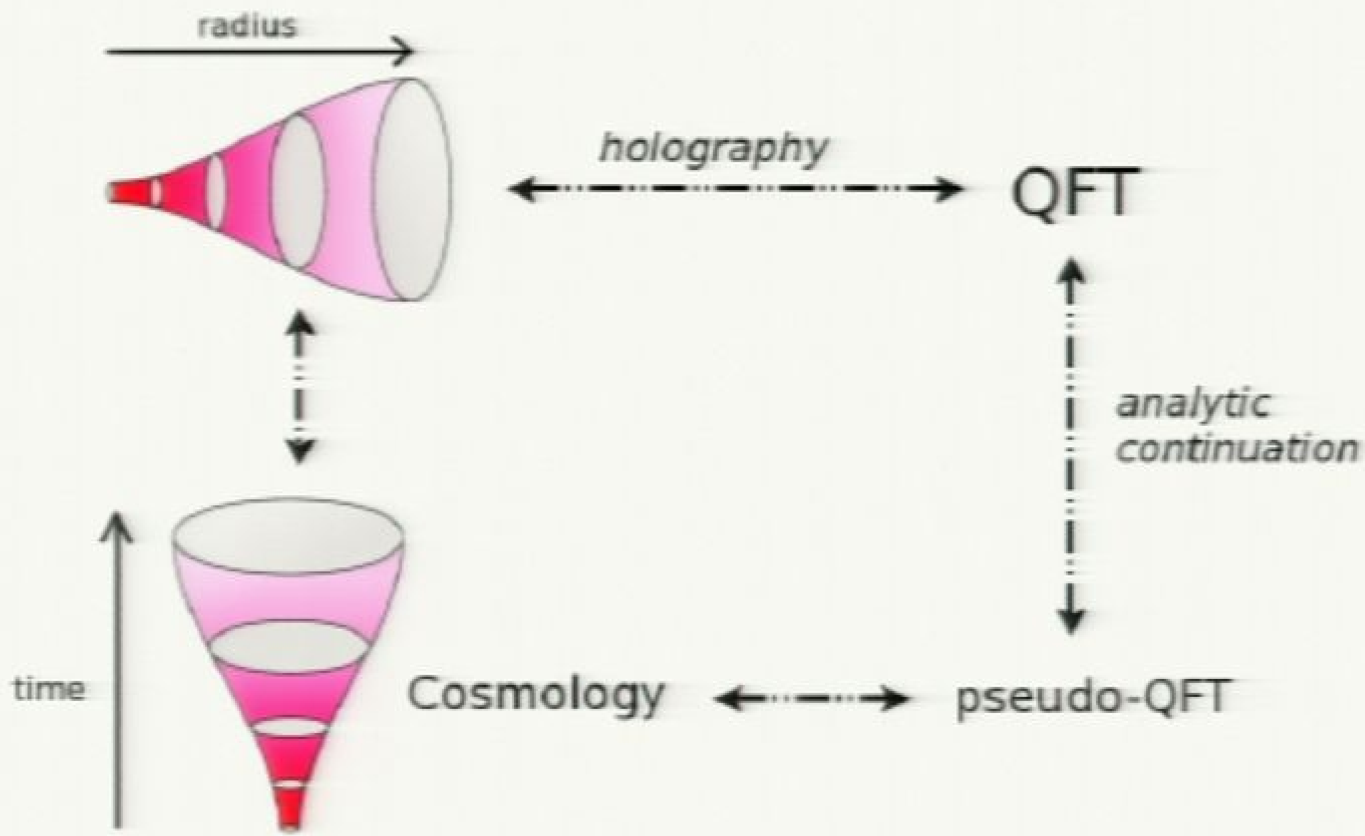
Moreover, since holographic dualities are strong/weak coupling dualities:

4. New results should follow by applying the holographic framework in the opposite regime, where gravity is *strongly coupled* at early times and the usual perturbative quantisation of fluctuations breaks down.

   In this regime the dual QFT is *weakly coupled* and we can use perturbative QFT to make predictions.

Plan of talk

arXiv:1001.2007 & 0907.5542



- Part I: Holography for cosmology.
- Part II: Beyond the weak gravitational description:
  holographic phenomenology, results & predictions.

Part I:
Holography for cosmology

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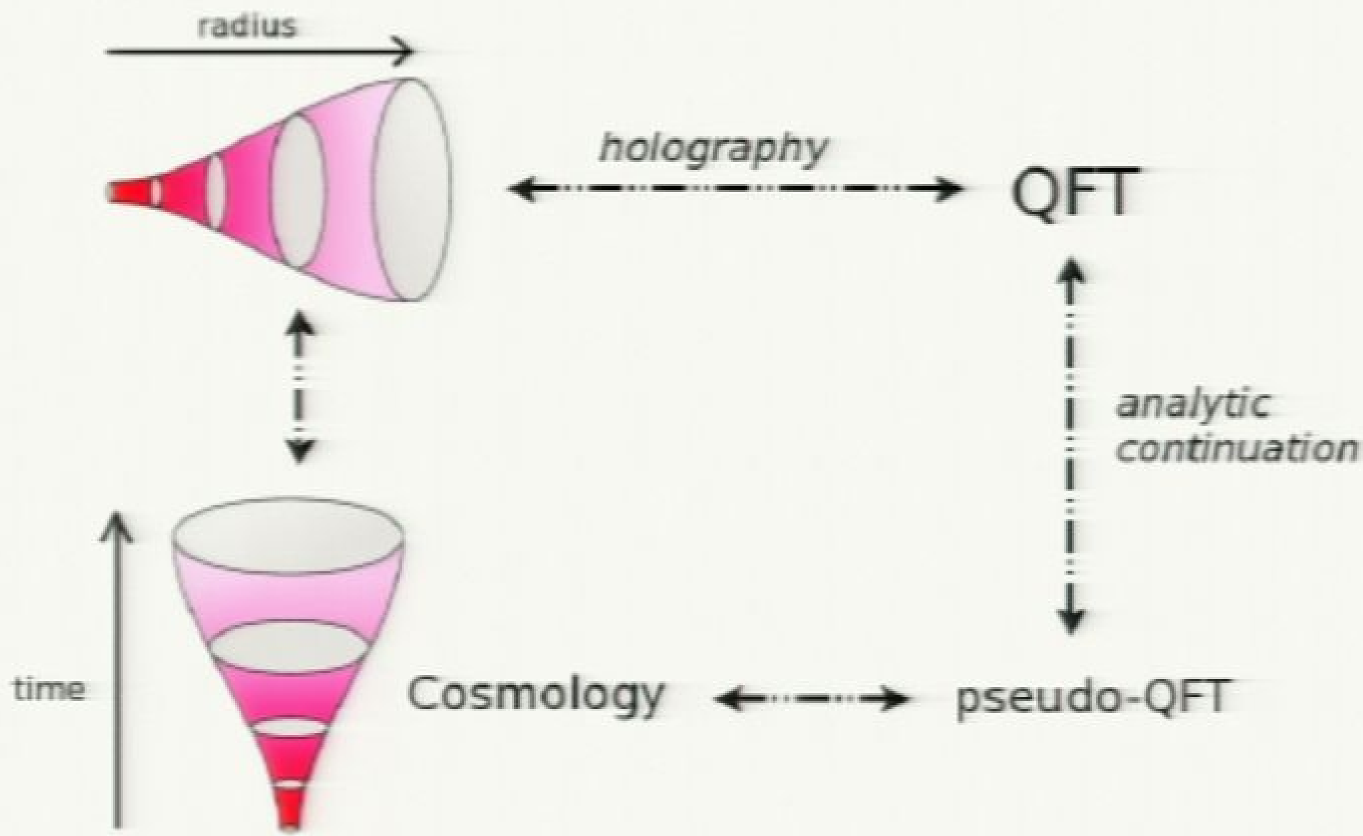

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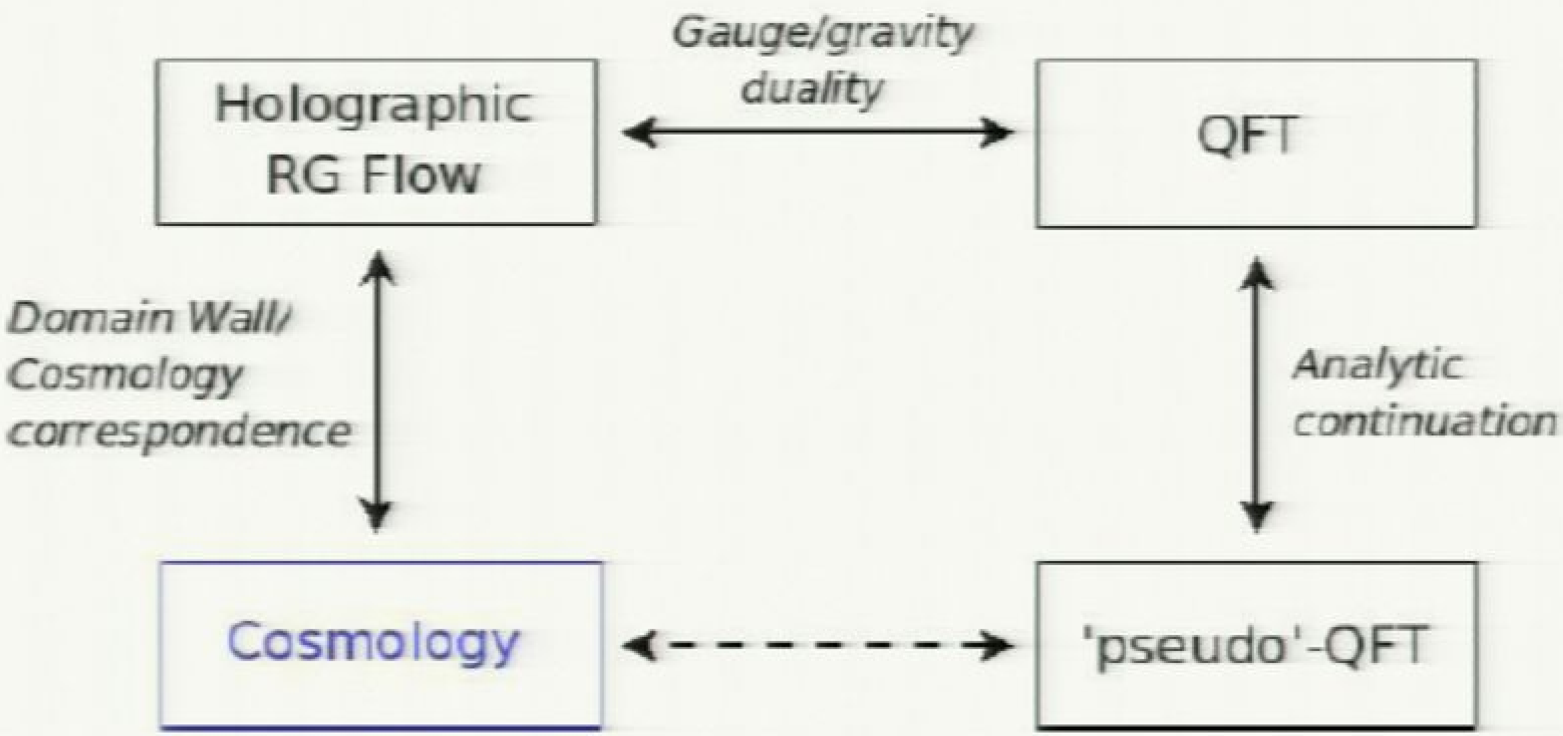
Part I:
Holography for cosmology



Holographic RG Flow
Gauge/gravity duality
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

Cosmological perturbations

We start by reviewing standard inflationary cosmology and the cosmological observables we would like to compute holographically.

- For simplicity, we discuss single-field 4d inflationary models:

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi)].$$

- We assume a spatially flat background and perturb

$$\begin{aligned} \mathrm{d}s^2 &= -\mathrm{d}t^2 + a^2(t)[\delta_{ij} + h_{ij}(t,\vec{x})]\mathrm{d}x^i\mathrm{d}x^j, \\ \Phi &= \varphi(t) + \delta\varphi(t,\vec{x}), \end{aligned}$$

  where $h_{ij} = -2\psi(z,\vec{x})\delta_{ij} + 2\partial_i\partial_j\chi(z,\vec{x}) + \gamma_{ij}(z,\vec{x})$.

- γ_{ij} is transverse traceless and we form the gauge-invariant scalar perturbation $\zeta = \psi + (H/\dot{\varphi})\delta\varphi$.

Power spectra

In the inflationary paradigm, cosmological perturbations are assumed to originate on sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\begin{aligned}\langle \zeta(t,\vec{q})\zeta(t,-\vec{q})\rangle &= |\zeta_q(t)|^2, \\ \langle \gamma_{ij}(t,\vec{q})\gamma_{kl}(t,-\vec{q})\rangle &= 2|\gamma_q(t)|^2\Pi_{ijkl},\end{aligned}$$

  where Π_{ijkl} is the transverse traceless projection operator while $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are then given by

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}|\zeta_{q(0)}|^2, \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}|\gamma_{q(0)}|^2,$$

  where $\gamma_{q(0)}$ and $\zeta_{q(0)}$ are the constant late-time values of the mode functions, with initial conditions set by the Bunch-Davies vacuum.

Power spectra via response functions

In preparation for our holographic discussion, we rewrite the power spectrum as follows.

- We define the *response functions* as

$$\Pi^{(\zeta)} = \Omega\,\zeta, \quad \Pi^{(\gamma)}_{ij} = E\,\gamma_{ij},$$

  where $\Pi^{(\zeta)}$ and $\Pi^{(\gamma)}_{ij}$ are the canonical momenta.

- One can show that

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E(q)],$$

  hence the power spectra may be expressed in terms of the late-time behaviour of the response functions.

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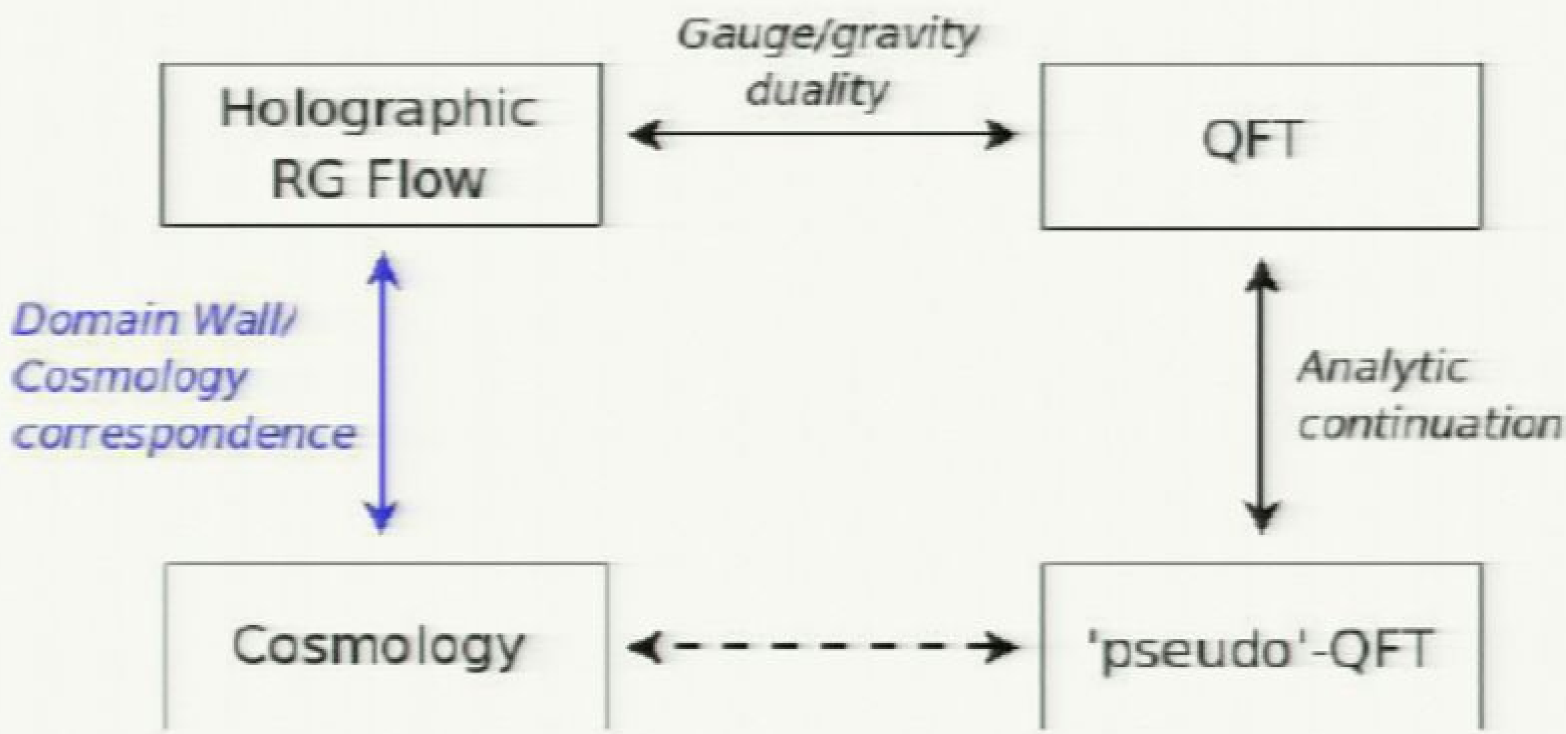
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Domain-wall spacetimes

- 'Domain-wall' spacetimes are closely related to cosmological spacetimes:

$$\mathrm{d}s^2 = \eta\, \mathrm{d}z^2 + a^2(z)\mathrm{d}\vec{x}^2, \qquad \Phi = \varphi(z),$$

where $\eta = +1$ for a (Euclidean) DW and $\eta = -1$ for cosmology.

- They play a prominent role in holography where they describe holographic RG flows (*i.e.* radial evolution of DW $\leftrightarrow$ RG flow of dual QFT).
- The DW action is

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{g}\,[-R + (\partial\Phi)^2 + 2\kappa^2 V(\Phi)].$$

Domain-wall/cosmology correspondence

- Including perturbations, the equations of motion for DW/C read:

$$H = \dot{a}/a = -(1/2)W(\varphi), \quad \dot{\varphi} = W_{,\varphi}, \quad 2\eta\kappa^2 V = (W_{,\varphi})^2 - (3/2)W^2,$$

$$0 = \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \eta a^{-2}q^2\zeta, \quad 0 = \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \eta a^{-2}q^2\gamma_{ij},$$

where $\dot{} = \mathrm{d}/\mathrm{d}z$ and $\epsilon = -\dot{H}/H^2$.

- Defining the analytically continued variables

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq,$$

we see that a cosmological solution written in terms of (κ, q) continues to a DW solution expressed in terms of $(\bar{\kappa}, \bar{q})$.

Domain-wall/cosmology correspondence

- This particular bulk continuation was chosen as it has a clear interpretation in terms of dual QFT variables.
- Our choice of sign in the continuation of q ensures that the Bunch-Davies vacuum on the cosmology side maps to a solution that is *regular* in the interior of the domain-wall:

$$\zeta, \gamma \sim \exp(-iq\tau) \quad \rightarrow \quad \zeta, \gamma \sim \exp(\bar{q}\tau)$$

  where $\tau = \int \mathrm{d}z/a$ and the DW interior is $\tau \to -\infty$.
- One can define response functions $\bar{\Omega}$ and $\bar{E}$ for the DW spacetime. They are related to their cosmological counterparts by the analytic continuations $\bar{\Omega}(-iq) = \Omega(q)$ and $\bar{E}(-iq) = E(q)$.

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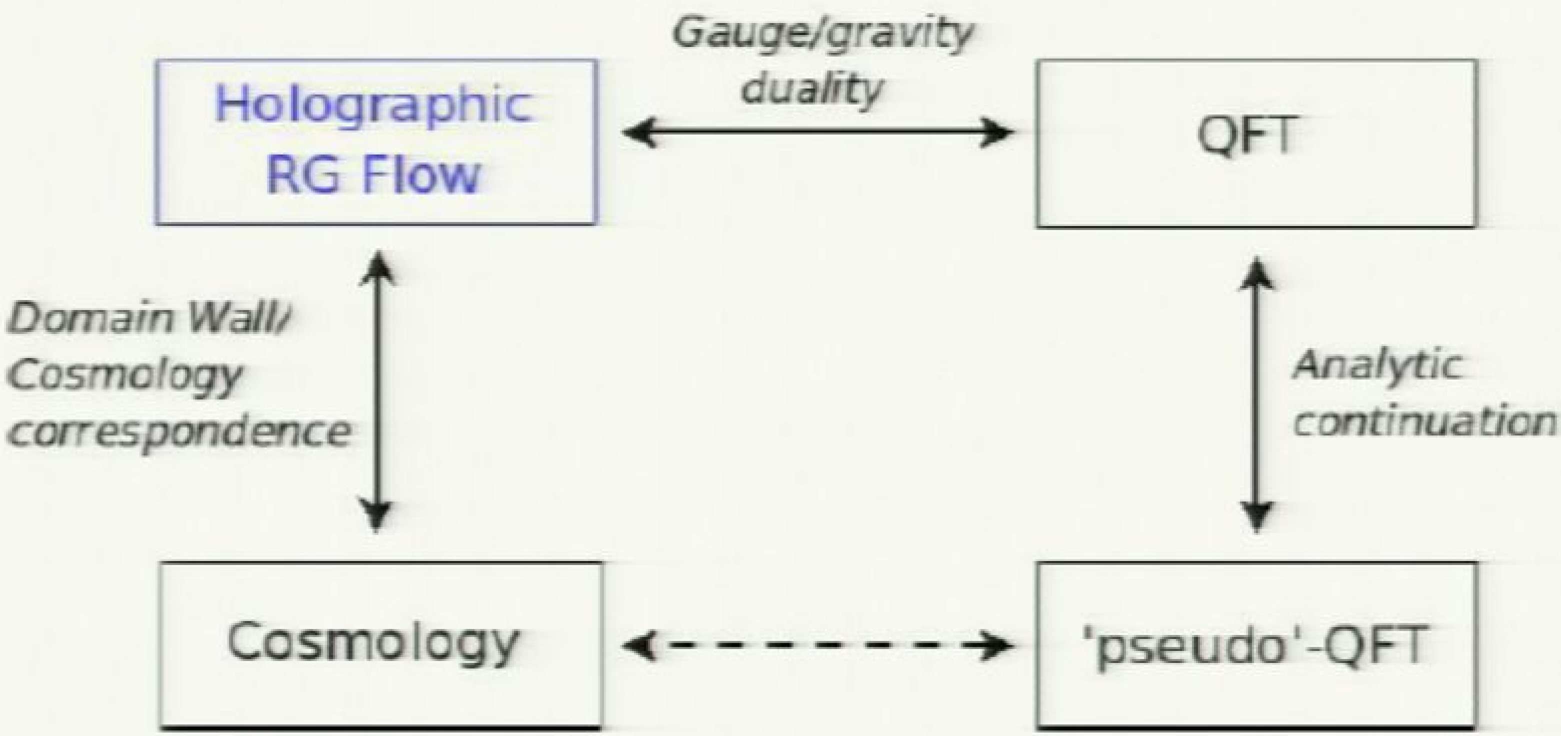
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Holographic RG flows

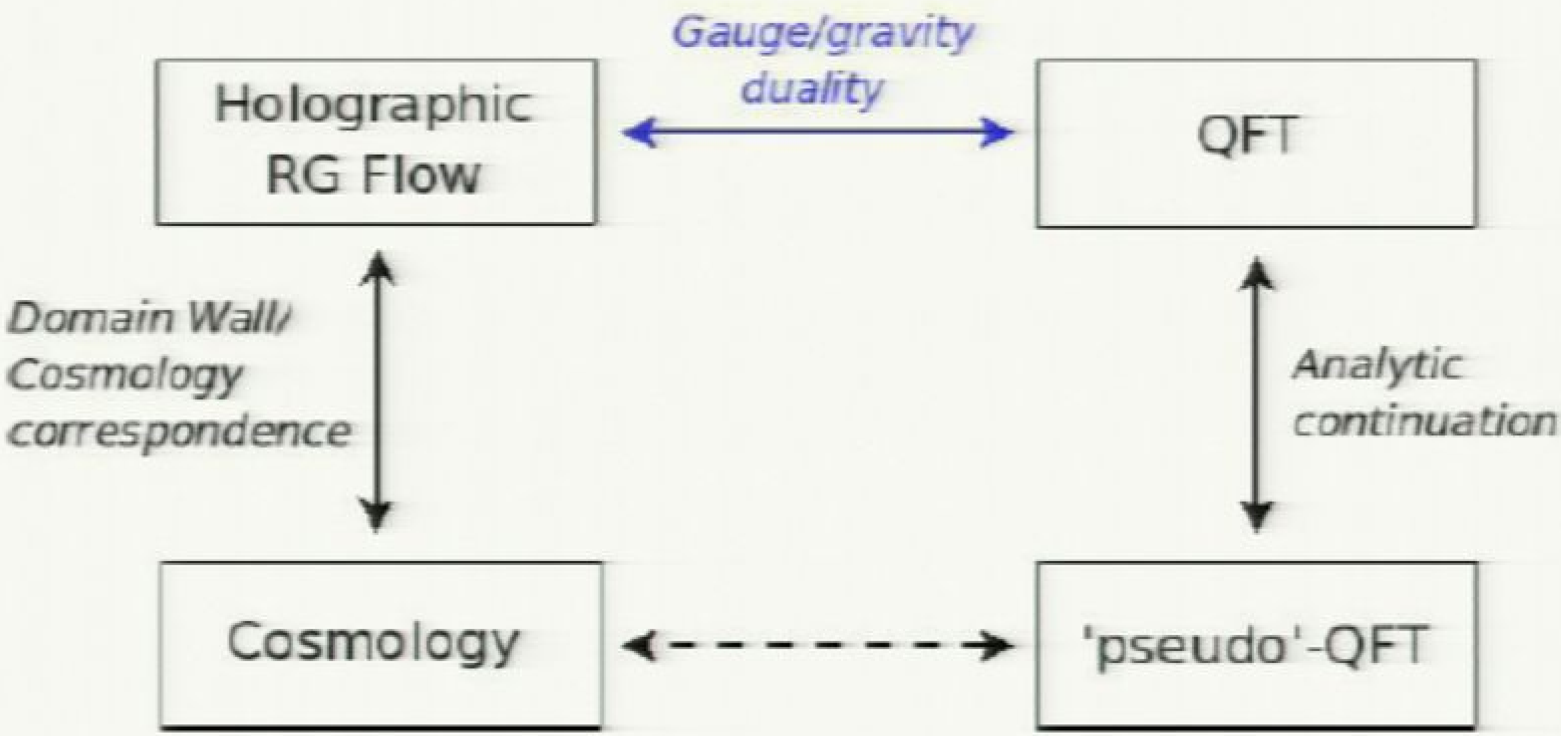
There are two classes of DW spacetimes whose holographic interpretation is well understood:

1. Asymptotically AdS solutions: $a \sim e^z$, $\varphi \sim 0$ as $z \to \infty$.

- These describe QFTs that flow to a CFT in the UV. Under the DW/C correspondence, they are mapped to asymptotically de Sitter cosmologies.

2. Asymptotically power-law solutions: $a \sim (z/z_0)^n$, $\varphi \sim \sqrt{2n}\ln(z/z_0)$ as $z \to \infty$.

- These describe QFTs with a single dimensionful coupling constant, in the regime where the dimensionality of the coupling constant drives the dynamics [arXiv:0807.3324]. Under the DW/C correspondence, they are mapped to asymptotically power-law inflationary cosmologies.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Holography: a primer

Our holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

- There is a 1-to-1 correspondence between local gauge-invariant operators of the boundary QFT and bulk supergravity modes:
  - ⇒ The bulk metric corresponds to the stress-energy tensor T_{ij} of the boundary theory.
  - ⇒ Bulk scalar fields correspond to boundary scalar operators, e.g. $\mathrm{tr}F_{ij}F^{ij}$.
- Correlation functions of the dual QFT may be read off from the asymptotics of the bulk solution. Conversely, given appropriate QFT data, one can reconstruct the bulk asymptotics.

Bulk asymptotics

The general asymptotic solution for the 4d bulk metric reads:

$$ds^2 = dr^2 + e^{2r} g_{ij}(r,x) dx^i dx^j,$$
$$g_{ij}(r,x) = g_{(0)ij}(x) + e^{-2r} g_{(2)ij}(x) + \ldots + e^{-2\sigma r} g_{(2\sigma)ij}(x) + \ldots$$

- $g_{(0)ij}(x)$ is the metric seen by the dual QFT, and hence acts as the *source* for the dual stress tensor T_{ij}.
- The $g_{(2k)ij}(x)$ with $k < \sigma$ are locally determined in terms of $g_{(0)ij}(x)$ via the asymptotic analysis of the field equations.
- $g_{(2\sigma)ij}(x)$ is only partially constrained by the asymptotic analysis of the field equations, and is related to the dual 1-pt function:

$$\langle T_{ij} \rangle = \frac{1}{2\bar{\kappa}^2}(2\sigma g_{(2\sigma)ij}).$$

Bulk asymptotics

- From the bulk asymptotics, we can read off $\langle T_{ij} \rangle$. Equivalently, given $\langle T_{ij} \rangle$, we can reconstruct the bulk asymptotics.
- This remains true even in the regime where gravity is *strongly coupled* and the description in terms of low-energy fields (such as the metric) breaks down deep in the interior.
- The metric description is still valid *asymptotically*, however, and takes the same form as before. Gauge/gravity duality requires the value of $g_{(2\sigma)ij}$ deriving from stringy dynamics to match that derived from the dual weakly coupled QFT.

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- From the bulk asymptotics, we can read off $\langle T_{ij} \rangle$. Equivalently, given $\langle T_{ij} \rangle$, we can reconstruct the bulk asymptotics.
- This remains true even in the regime where gravity is *strongly coupled* and the description in terms of low-energy fields (such as the metric) breaks down deep in the interior.
- The metric description is still valid *asymptotically*, however, and takes the same form as before. Gauge/gravity duality requires the value of $g_{(2\sigma)ij}$ deriving from stringy dynamics to match that derived from the dual weakly coupled QFT.

Two-point functions

- Higher-point functions may be obtained by differentiating the 1-pt function w.r.t. the sources and then setting the sources to their background values,

$$\text{e.g.} \qquad \langle T_{ij}(x) T_{kl}(y)\rangle \sim \frac{\delta g_{(2\sigma)ij}(x)}{\delta g_{(0)kl}(y)}\Big|_{g_{(0)}=\delta}$$

- To compute 2-pt functions one only needs to solve for the bulk fluctuations to *linear* order.
- On general grounds, the 2-pt function for the stress tensor admits the decomposition

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q})\rangle = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl},$$

where the transverse and transverse traceless projection operators are

$$\pi_{ij} = \delta_{ij} - \bar{q}_i\bar{q}_j/\bar{q}^2, \quad \Pi_{ijkl} = \pi_{i(k}\pi_{l)j} - (1/2)\pi_{ij}\pi_{kl}.$$

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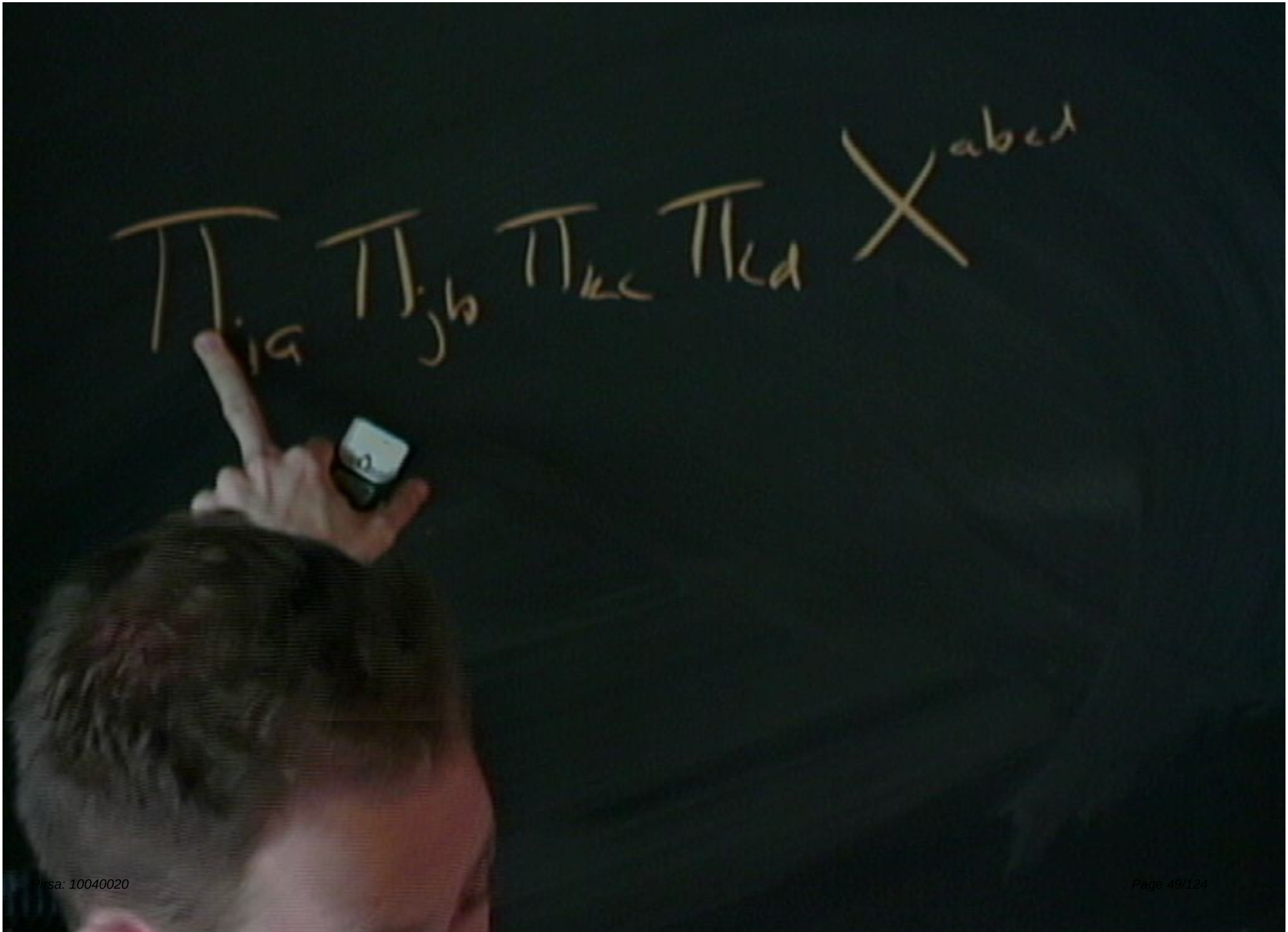
$\Pi_{ia} \Pi_{jb} \Pi_{kc} \Pi_{ld} X^{abcd}$

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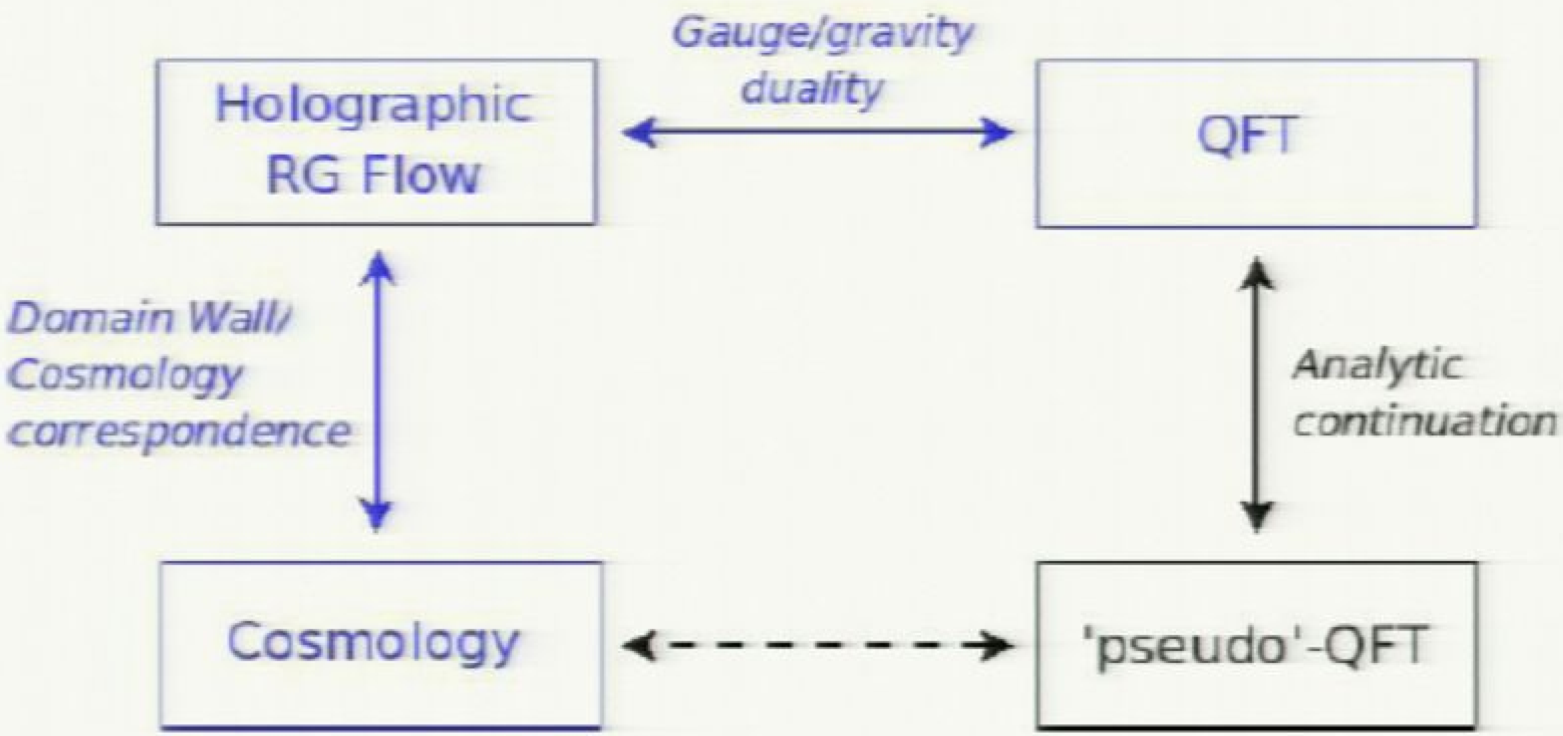
Holographic RG Flow
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Cosmology
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From cosmology to QFT

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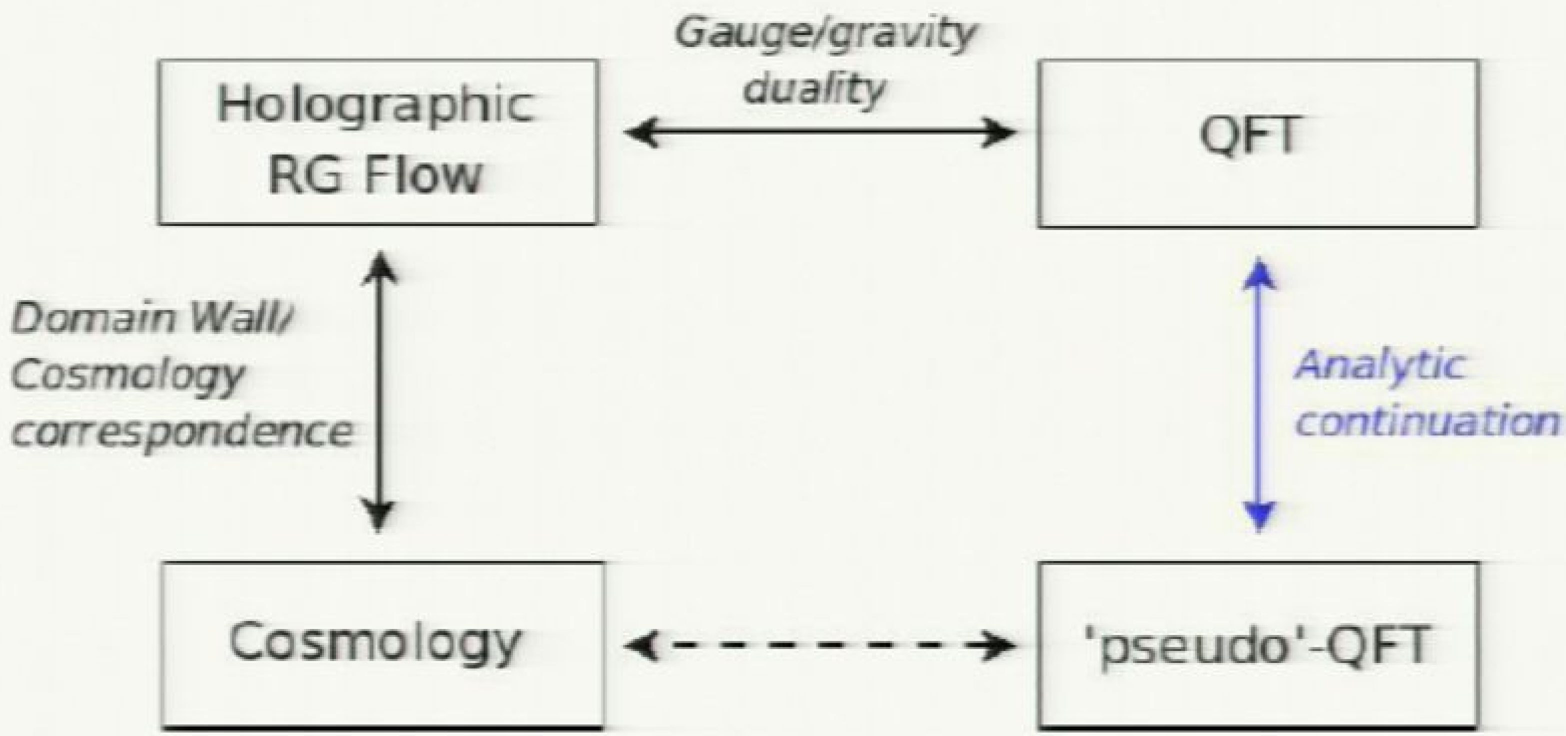
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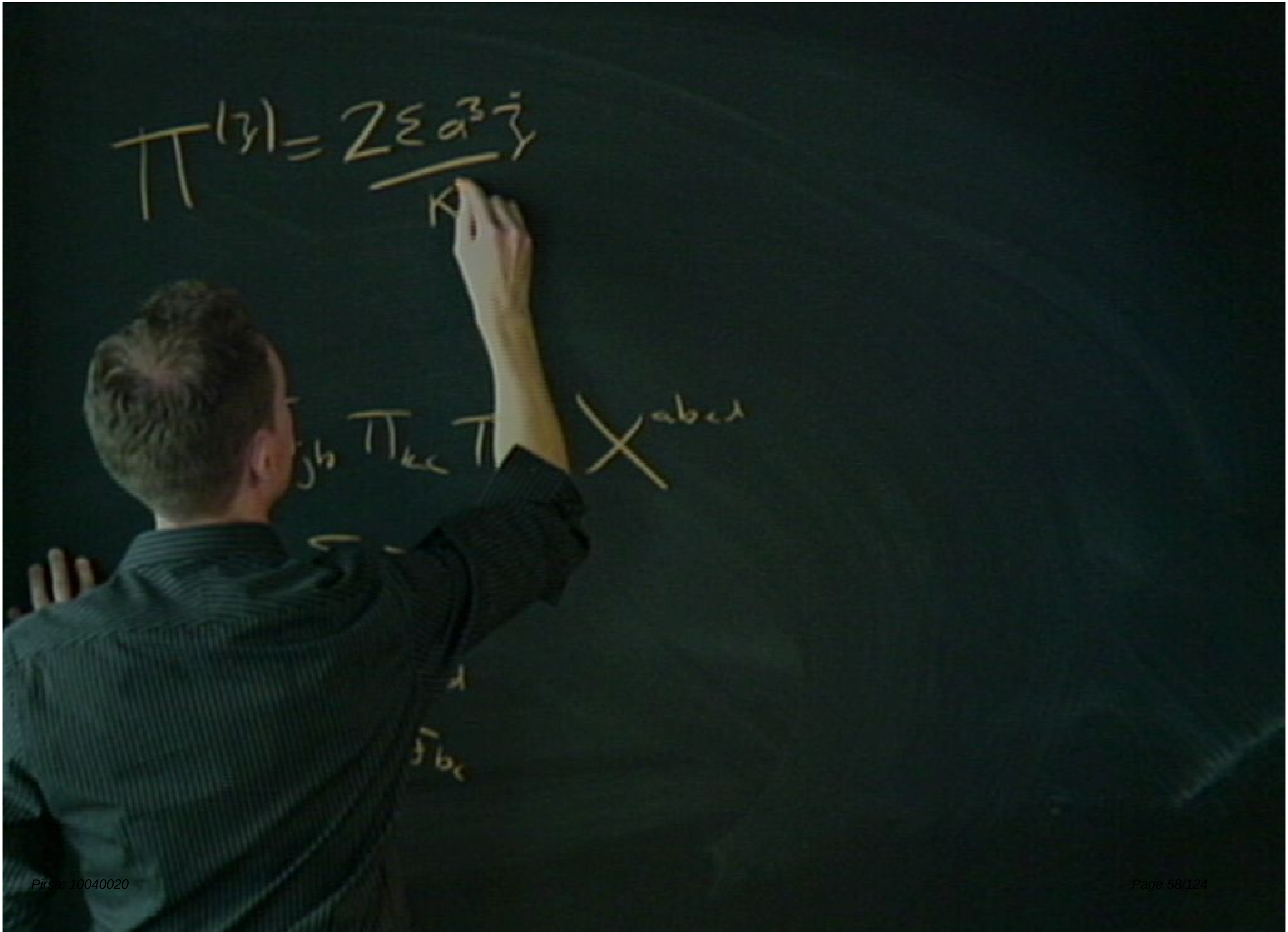

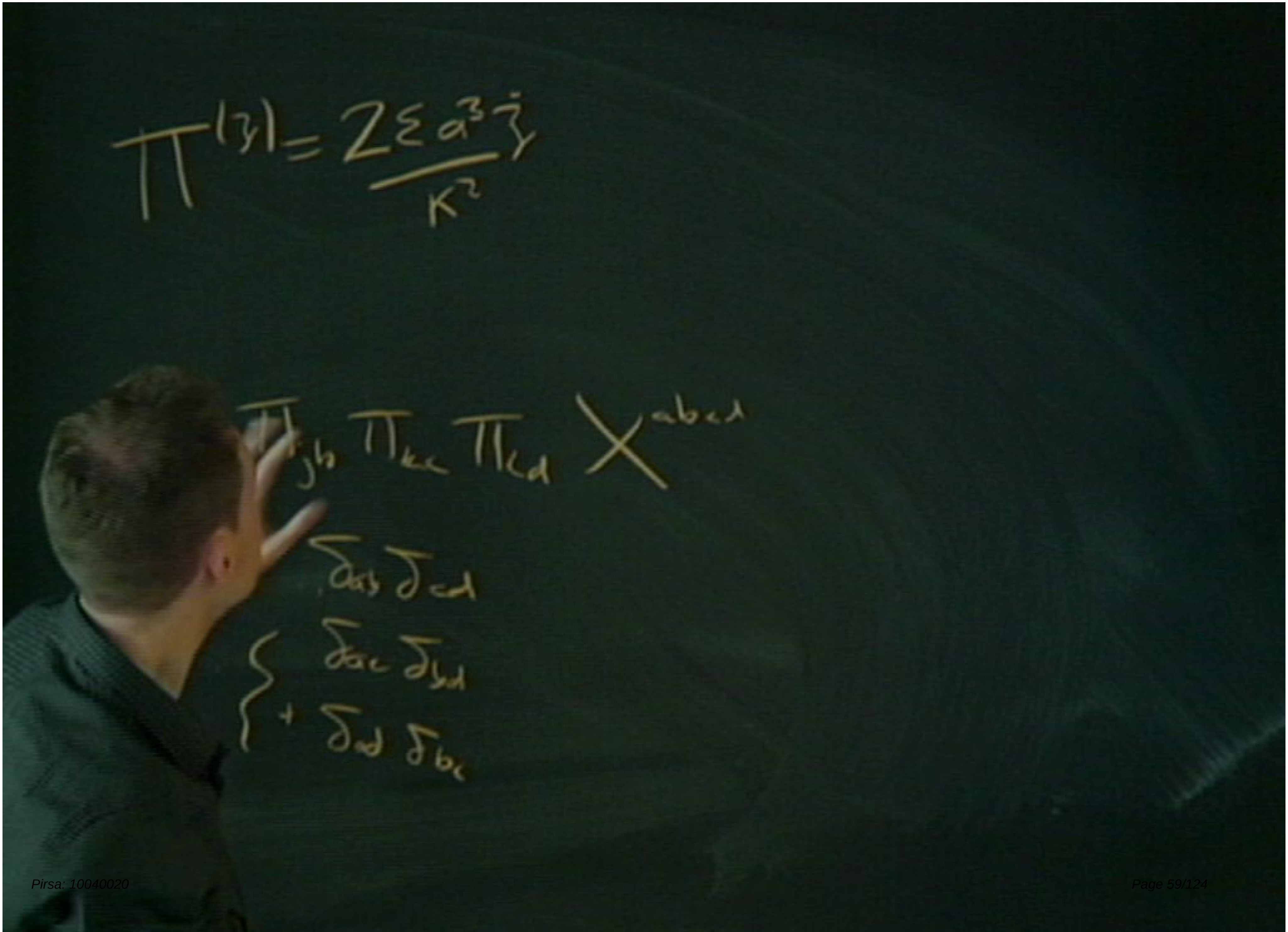
Π(3) = 2εa³ξ̇ / κ²
Π_jb Π_kc Π_ld X^abcd
δab δcd
δac δbd
+ δad δbc

$$\Pi^{(\zeta)} = \frac{2\epsilon a^3 \dot{\zeta}}{k^2}$$

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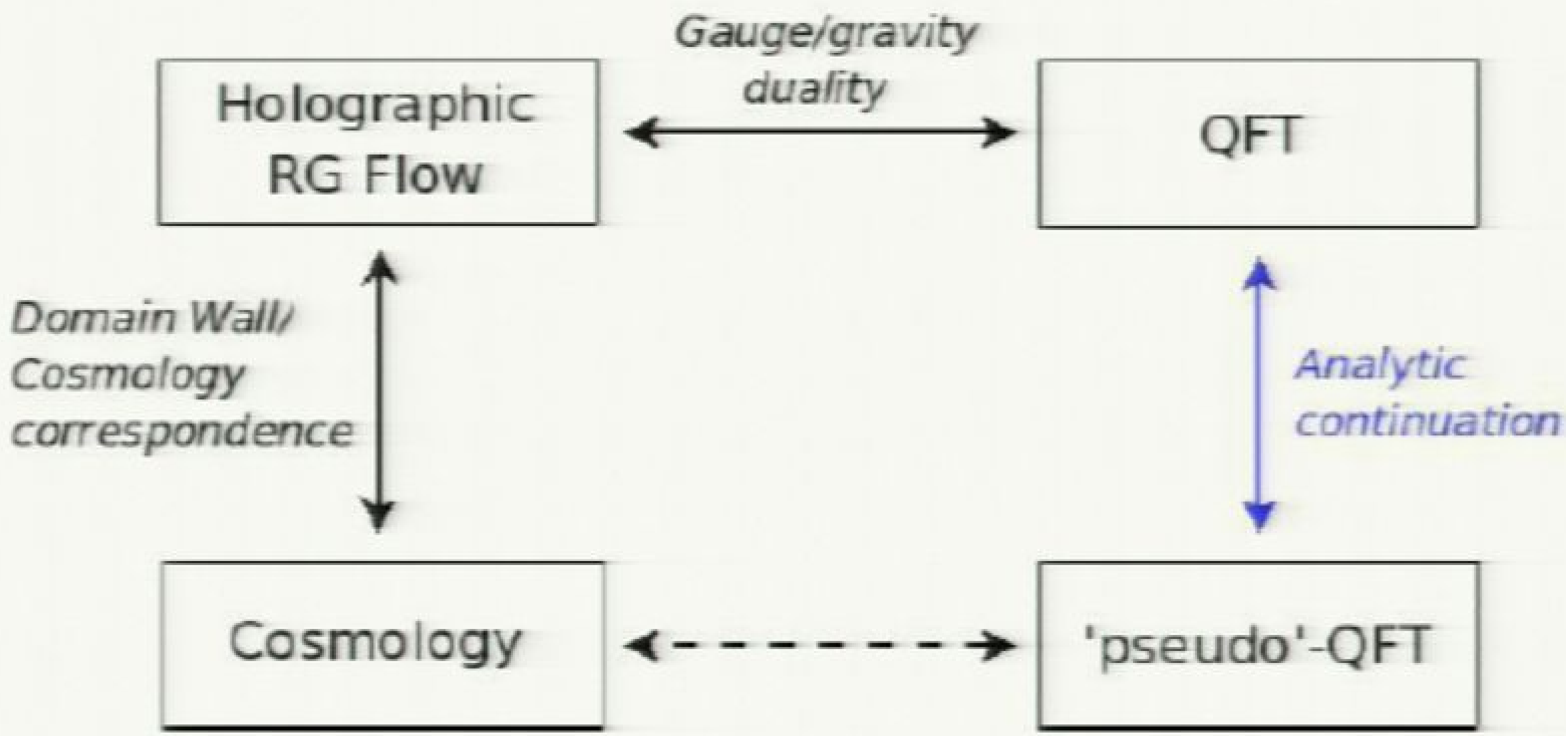
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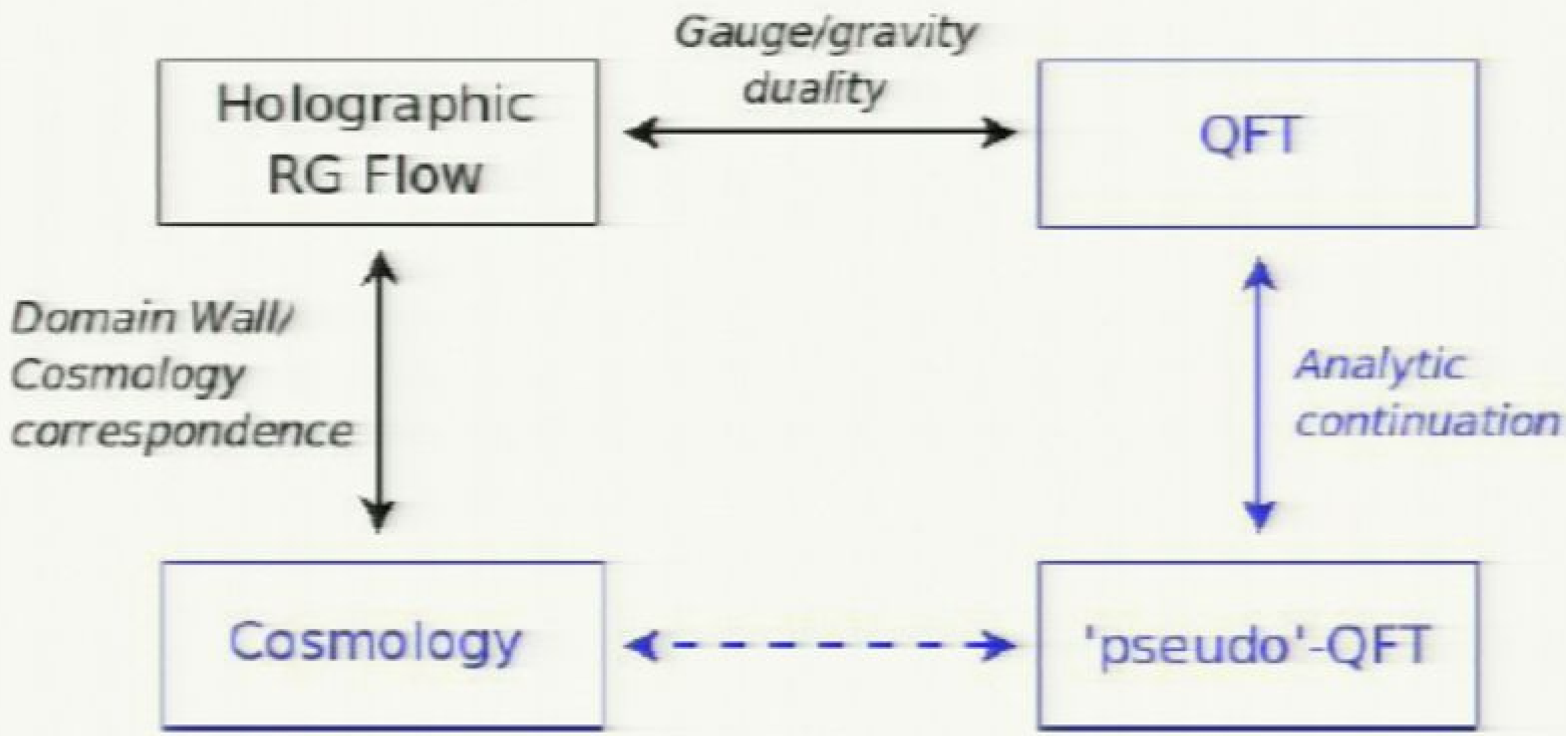
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Beyond the weak gravitational description

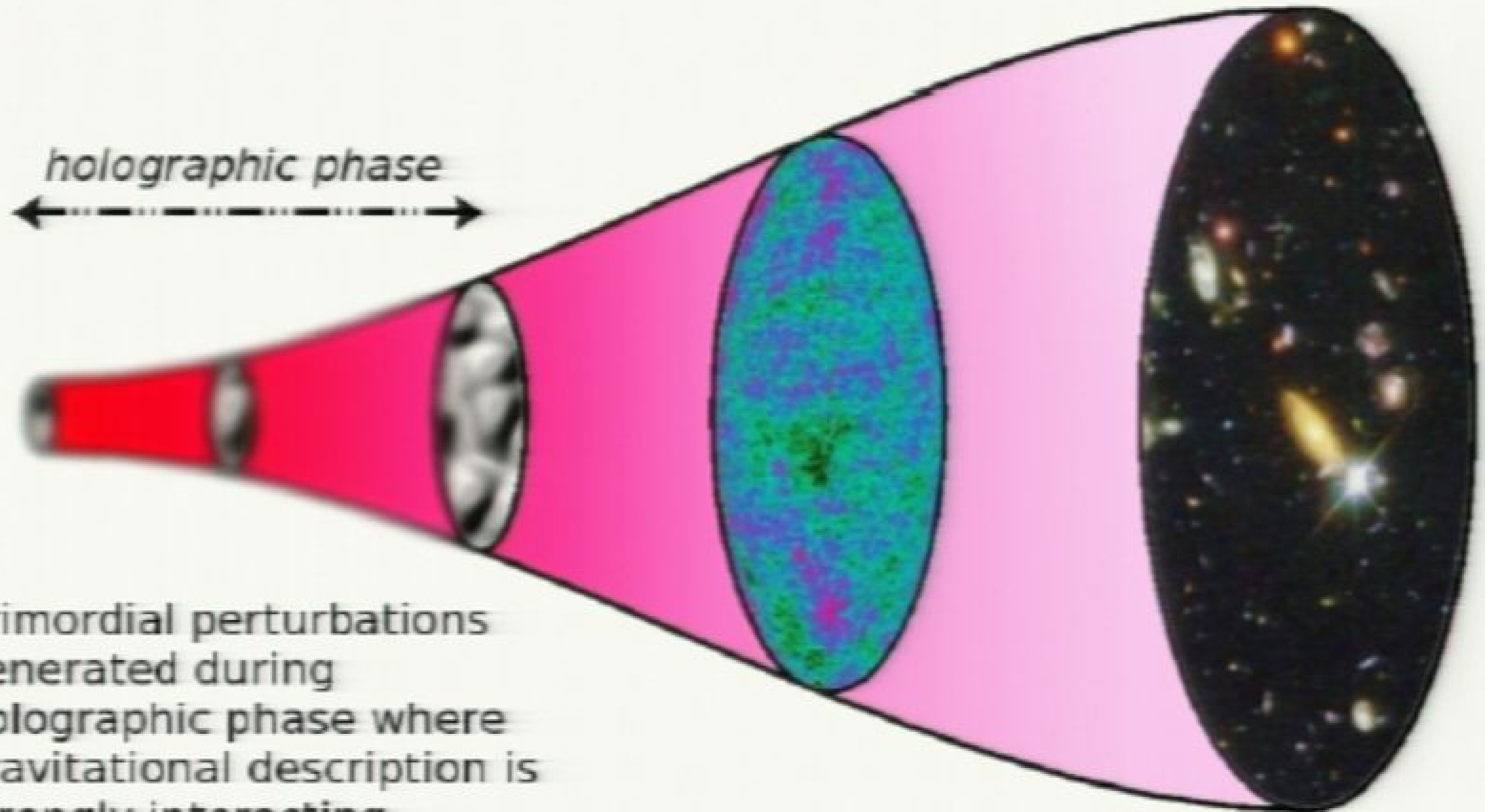
We inferred above a QFT description of inflationary cosmology using gauge/gravity duality and analytic continuation.

- So far, all computations have been performed on the gravity side.

For the very early Universe, however, we should also consider the possibility that the gravitational description might be *strongly coupled*.

- In this regime the conventional inflationary description fails. Nevertheless, we may still reconstruct the late-time asymptotic behaviour of the system *holographically* via the correlators of the dual QFT at weak coupling.

The new scenario



Primordial perturbations generated during holographic phase where gravitational description is strongly interacting.

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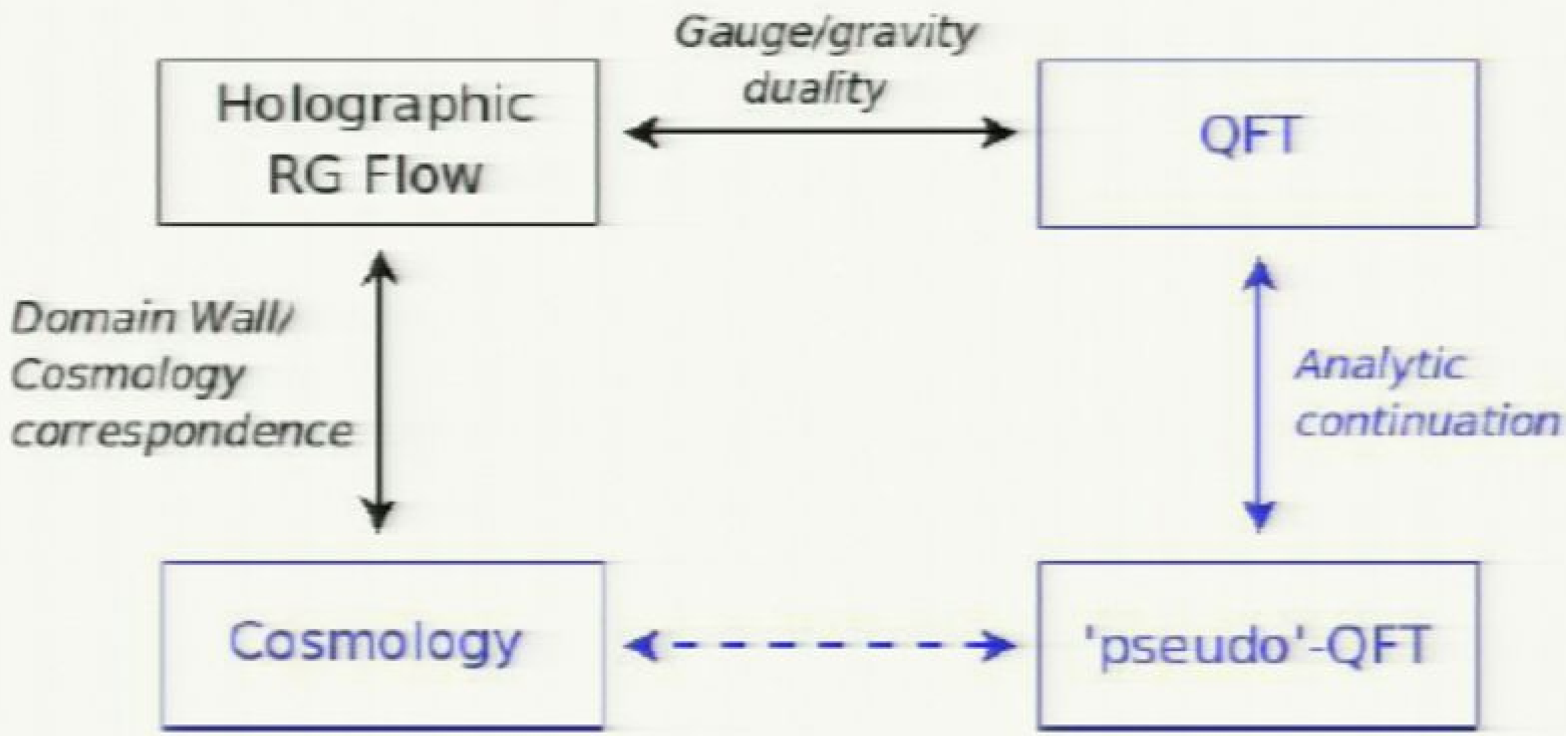
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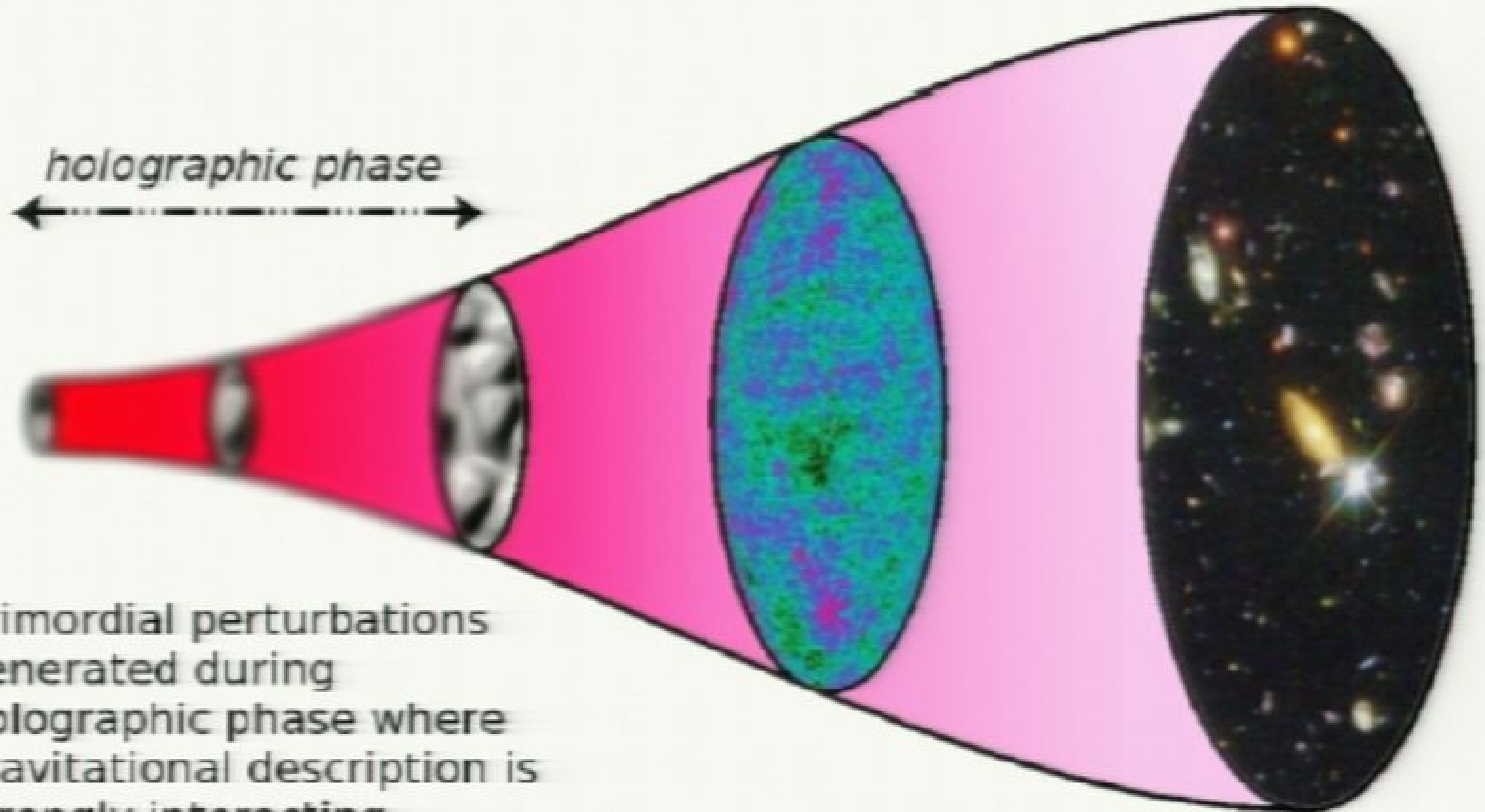

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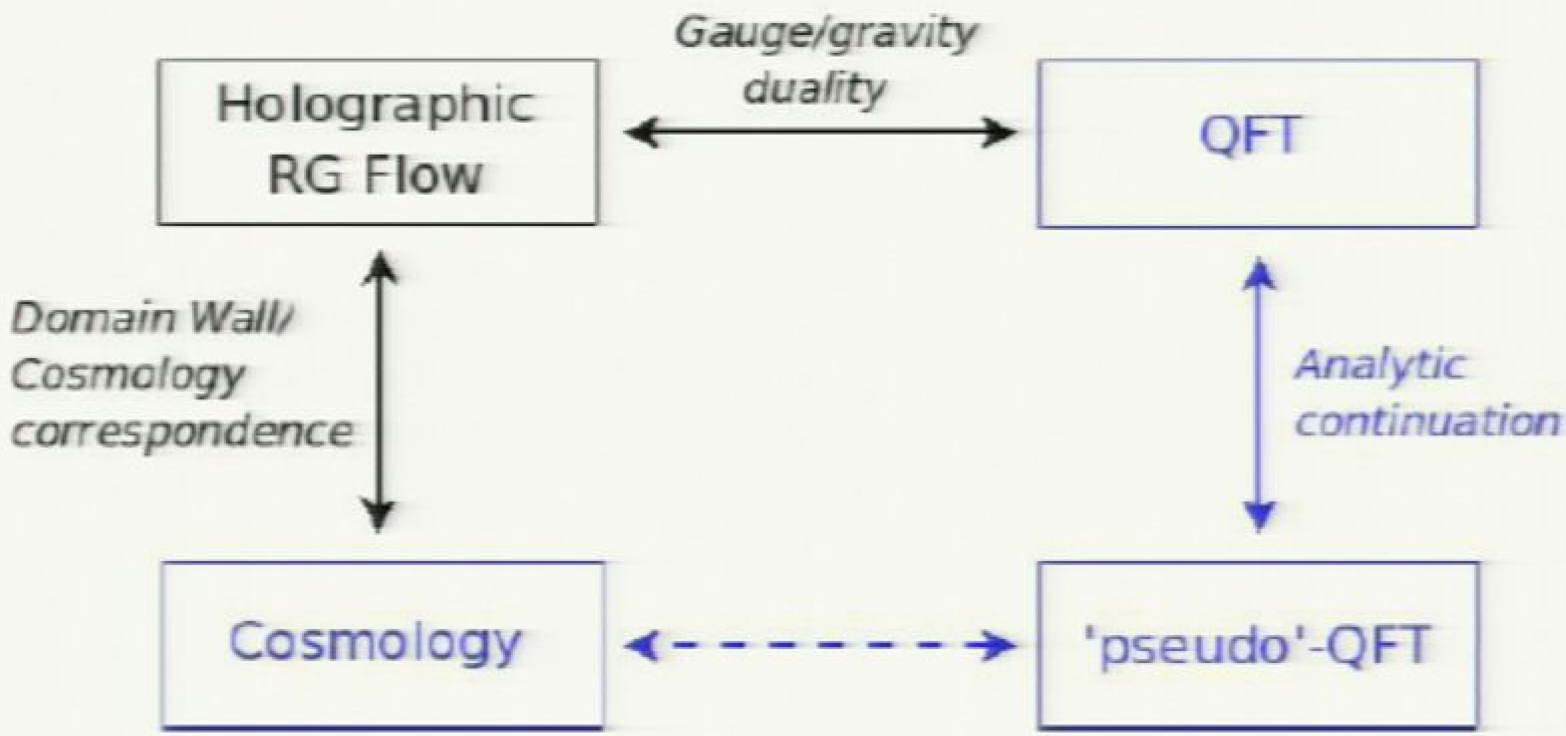
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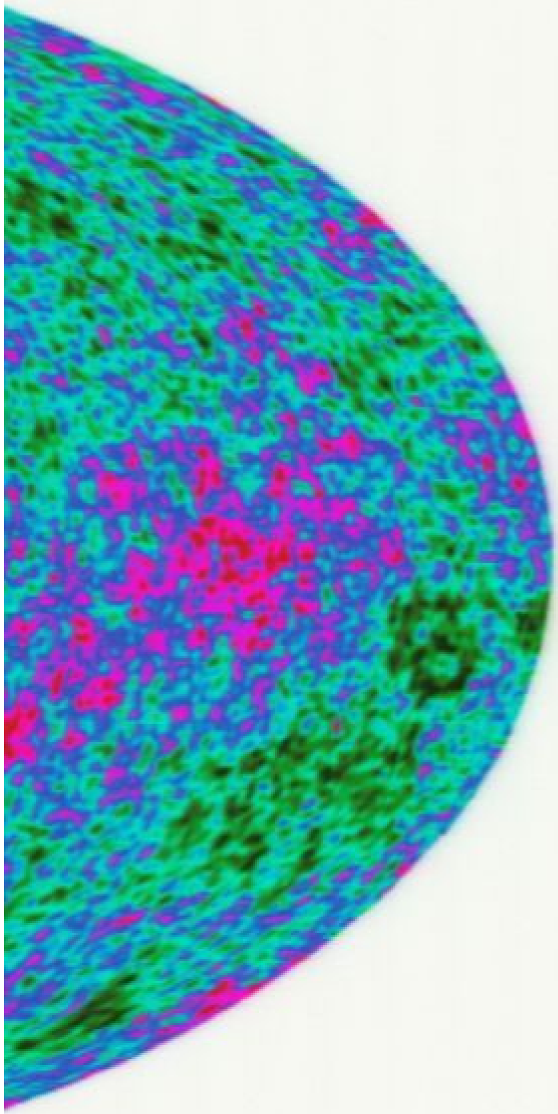
Holographic phenomenology for cosmology



- The boundary theory will be a combination of gauge fields, fermions and scalars, and it should admit a large N expansion.
- To extract predictions, we must compute $\langle T_{ij} T_{kl} \rangle$ and plug the coefficients $A(\bar{q})$ and $B(\bar{q})$ into our holographic formulae for the cosmological power spectra.
- One can then look for holographic theories that fit the observations.

Form of the primordial perturbations

Observationally, the primordial power spectra may be parametrised by an amplitude and tilt according to

$$\Delta_S^2(q) = \Delta_S^2(q_0)\,(q/q_0)^{n_S(q)-1}, \quad \Delta_T^2(q) = \Delta_T^2(q_0)\,(q/q_0)^{n_T(q)}.$$

The WMAP data then yield (for $q_0 = 0.002\mathrm{Mpc}^{-1}$)

$$\Delta_S^2(q_0) = (2.445 \pm 0.096) \times 10^{-9}, \qquad n_S - 1 = -0.040 \pm 0.013,$$

i.e., the scalar perturbations have small amplitude and are nearly scale invariant.

- These two small numbers should appear naturally in any theory that explains the data.

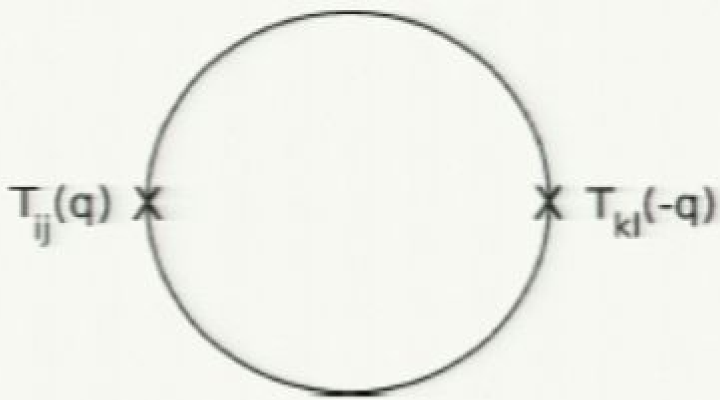
Holographic phenomenology for cosmology

- As a starting point one can consider the strong-gravity version of asymptotically dS and asymptotically power-law cosmologies.
- Here we focus on the latter. These are dual to super-renormalisable QFTs that depend on a single dimensionful coupling, $g^2_{\rm YM}$.
- Prototype dual QFT: 3d $SU(\bar{N})$ Yang-Mills theory coupled to adjoint fermions and scalars (both conformally and minimally coupled).

$$S = \frac{1}{g^2_{\rm YM}} \int d^3x \, {\rm tr} \Big[\frac{1}{2} F^I_{ij} F^{Iij} + \frac{1}{2}(D\phi^J)^2 + \frac{1}{2}(D\chi^K)^2 + \bar{\psi}^L \not{D} \psi^L + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu^{\alpha\beta}_{M L_1 L_2} \Phi^M \psi^{L_1}_\alpha \psi^{L_2}_\beta \Big]$$

1-loop calculation

The leading contribution to $\langle T_{ij} T_{kl} \rangle$ is at 1-loop order. Since T_{ij} has dimension 3, and $g^2_{\rm YM}$ does not appear to this order, it follows that



$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 + O(g^2_{\rm YM}),$$
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Explicit calculation then reveals:

$$C_A = (\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi)/256, \quad C_B = (\mathcal{N}_A + \mathcal{N}_\phi)/256.$$

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- The cosmological power spectra are *scale–invariant* to leading order.
- This is a consequence of simple dimensional analysis and is independent of field content!
- The observed amplitude $\Delta_S^2(q_0) \sim O(10^{-9})$ for the scalar power spectrum implies $N \sim O(10^4)$, justifying our use of the large N limit.

- We can fit the upper bound on the ratio of tensor to scalar power spectra by tuning the field content of the model:

$$r = \Delta_T^2 / \Delta_S^2 = 32 C_B / C_A .$$

A small upper bound on r requires more conformal scalars and massless fermions and/or fewer gauge fields and minimal scalars.

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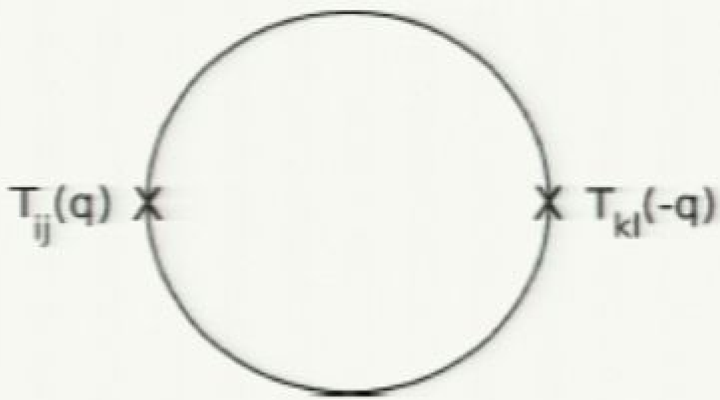

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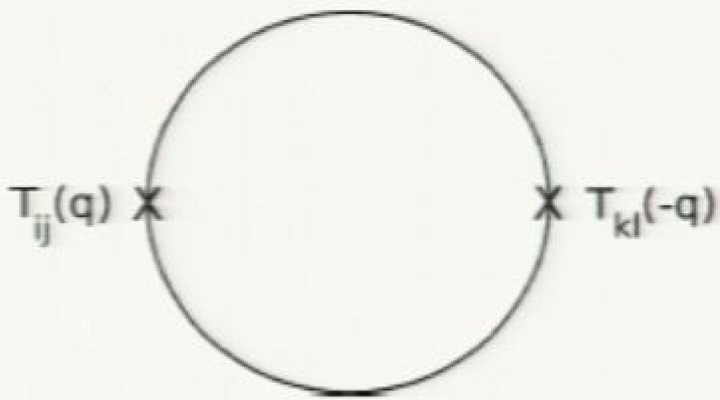

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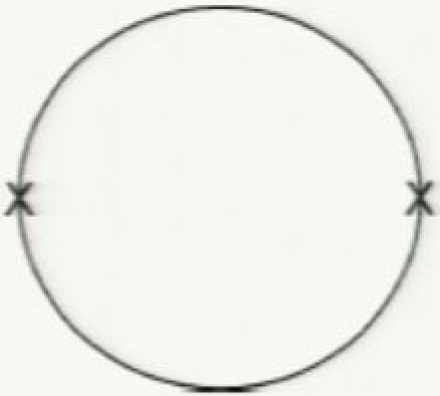

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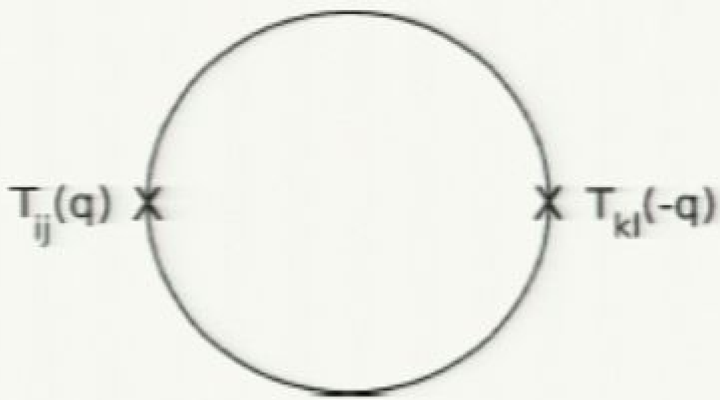

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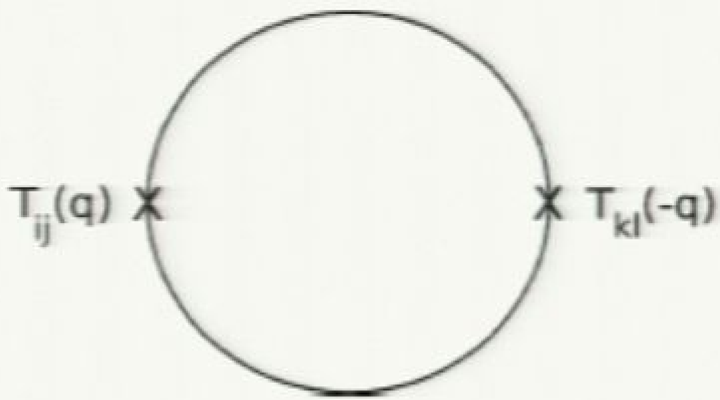

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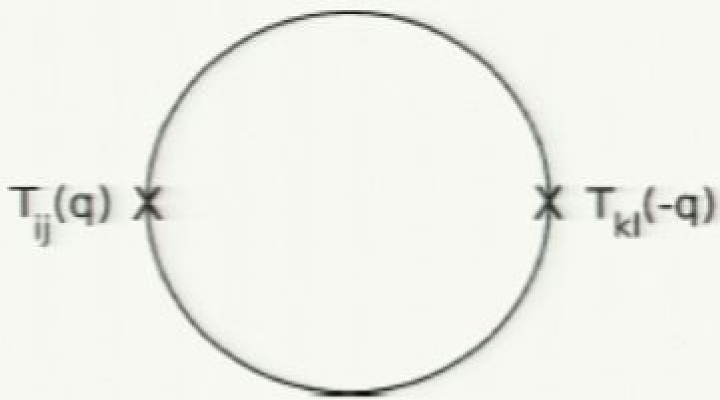

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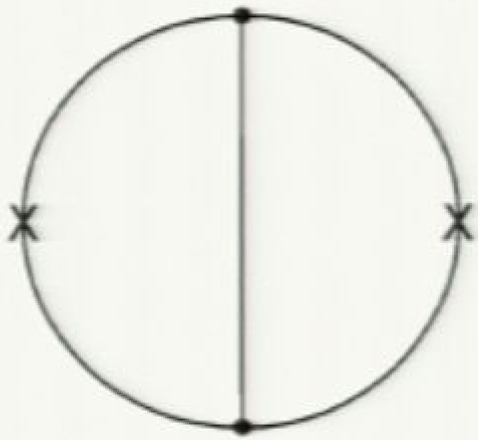

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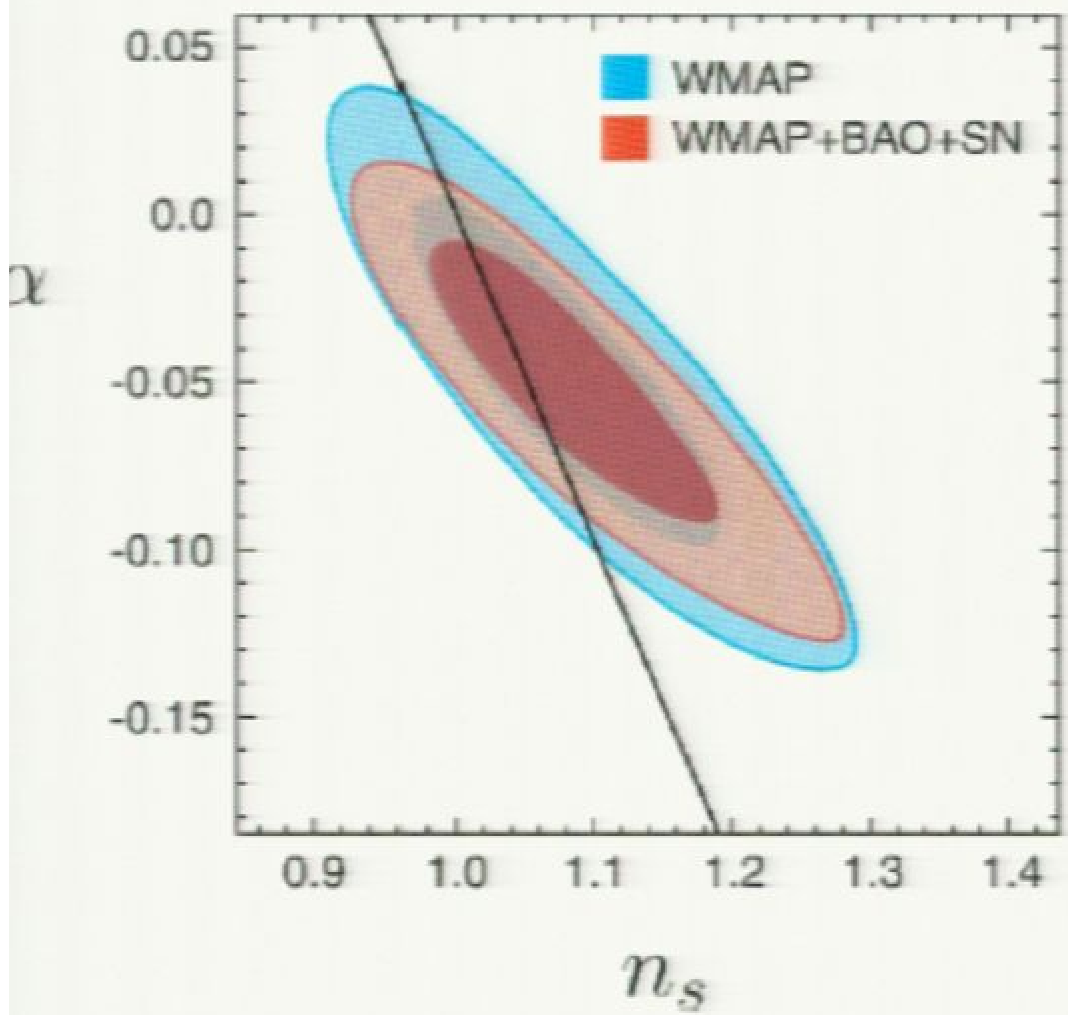

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$$\begin{aligned}\alpha_s &= \mathrm{d}n_s/\mathrm{d}\ln q \\ &= -(n_s - 1) + O(g_{\rm eff}^4).\end{aligned}$$

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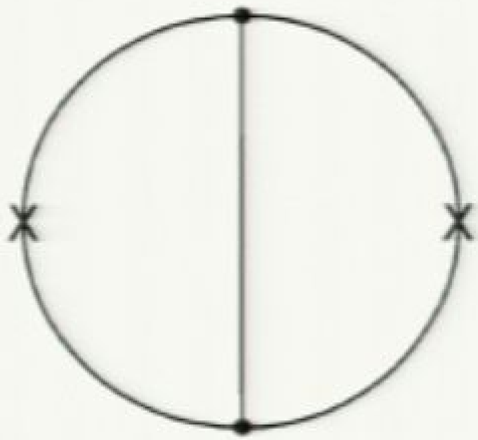

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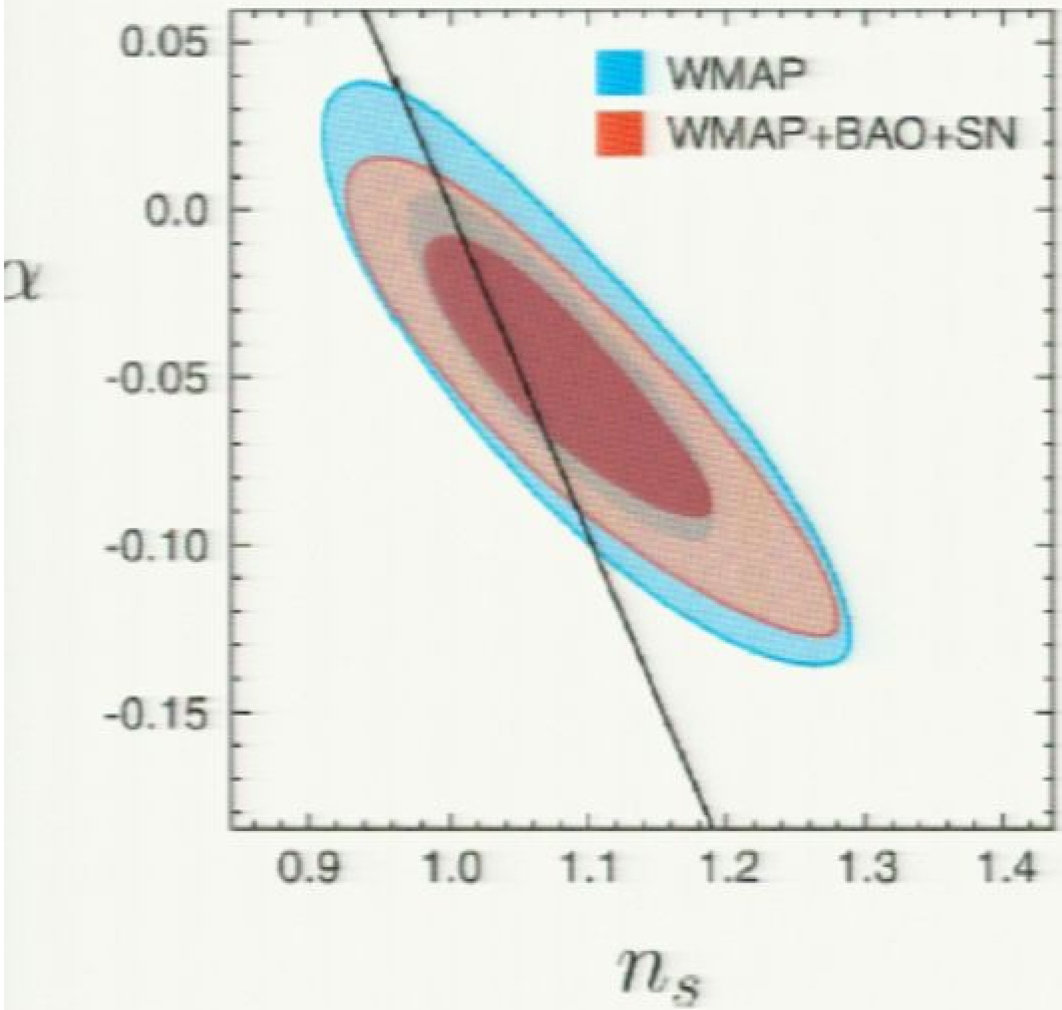

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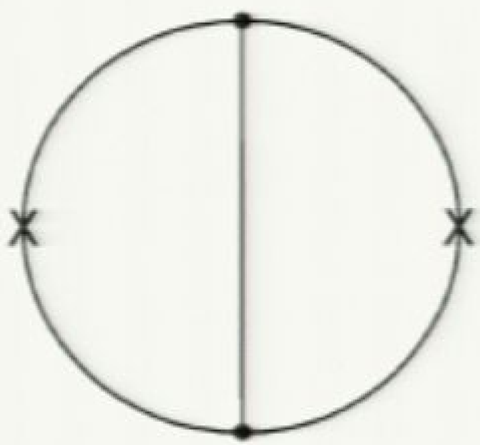

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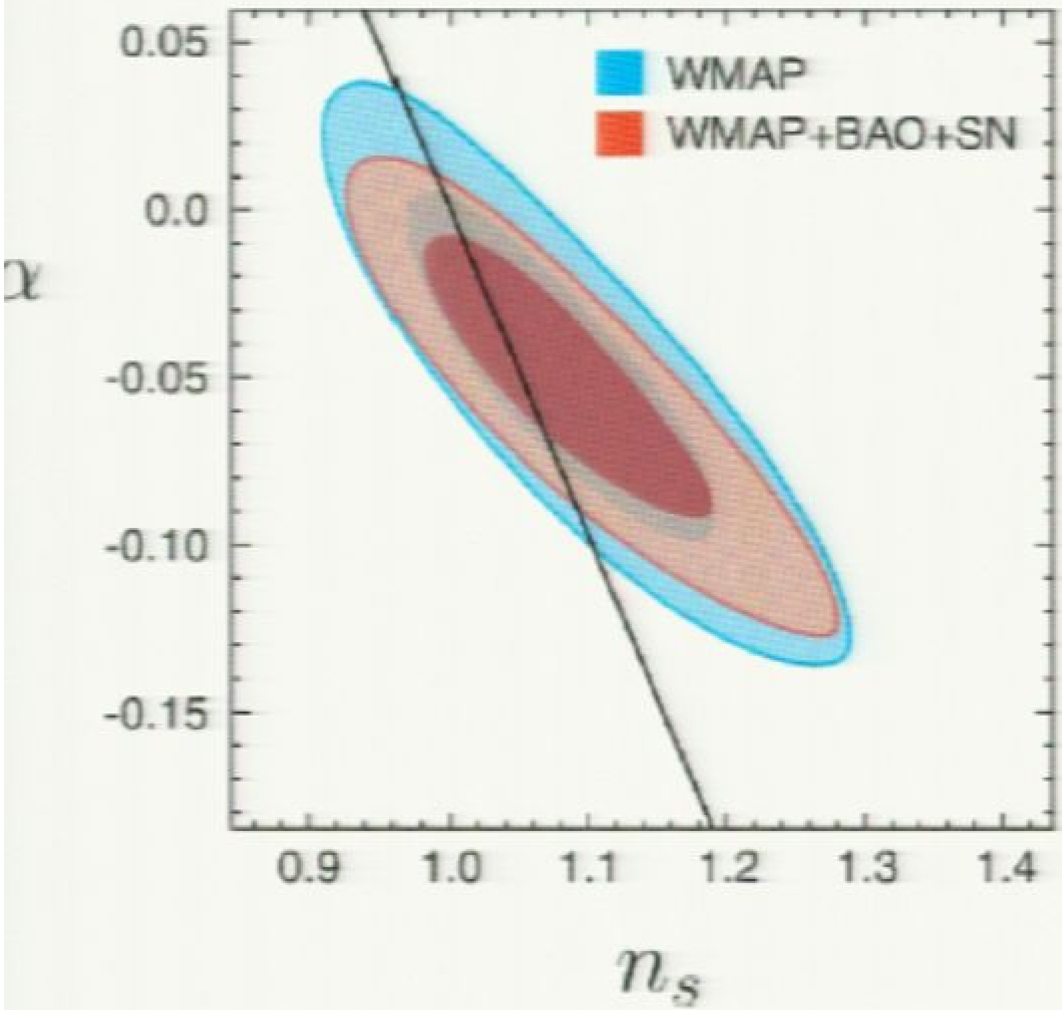

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Non-Gaussianities

Once N, g^2_{YM} and the field content have been fixed, all remaining cosmological observables, including non-Gaussianities, may be directly computed.

- Programme currently underway to extract holographic predictions for non-Gaussianity.
- Should provide stringent test of holographic models.
- An order of magnitude estimate suggests that holographic models predict f_{NL} is independent of N to leading order.

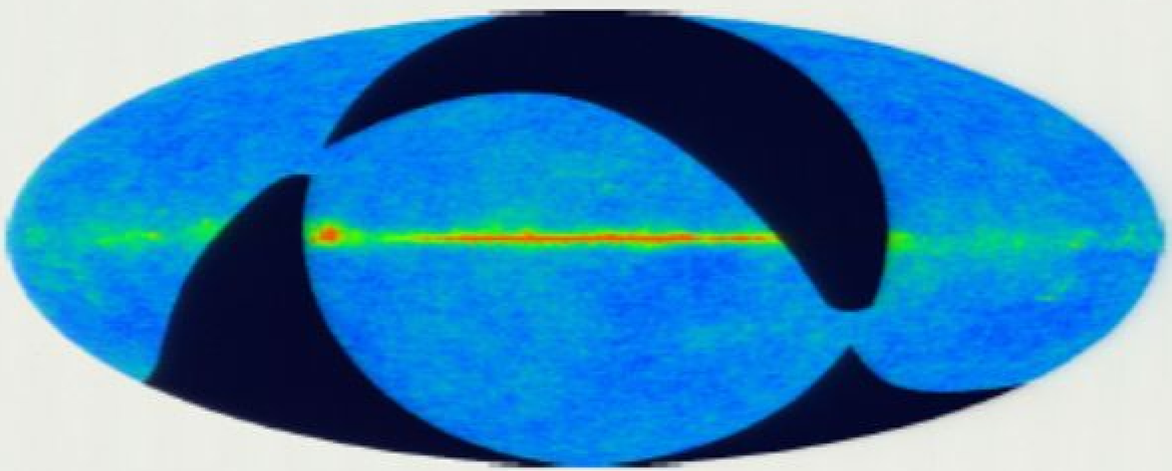
Conclusions

arXiv:0907.5542, 1001.2007

- Presented holographic description of inflationary cosmology in terms of a 3-dimensional dual QFT.
- Reproduces standard inflationary predictions in their regime of applicability.
- In opposite regime, where gravitational description is strongly coupled, we may use weakly coupled dual QFT to make predictions.
- Easy to find models that satisfy current observational constraints, yet make distinct predictions from standard inflation.
- In particular, find new and simple mechanism for obtaining near scale-invariant spectrum.

Outlook

- Forthcoming observations (*e.g.* Planck) should dramatically tighten observational constraints on many key cosmological parameters.



Planck sky coverage as of 15 Dec 2009.

- Might provide the first *observational* evidence for the holographic nature of our universe!

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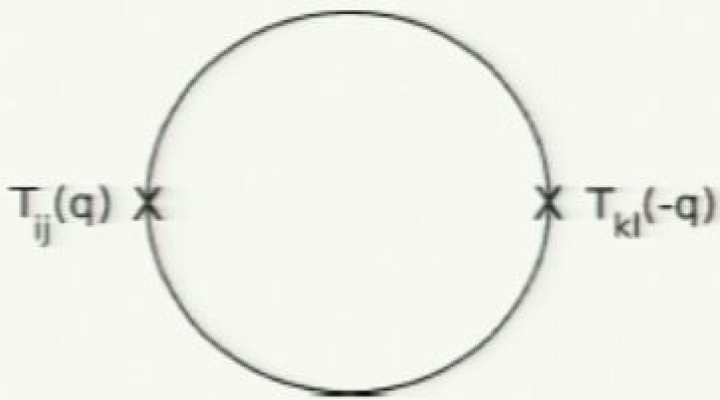

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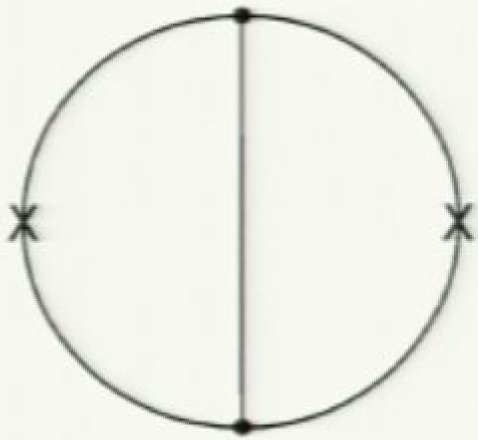

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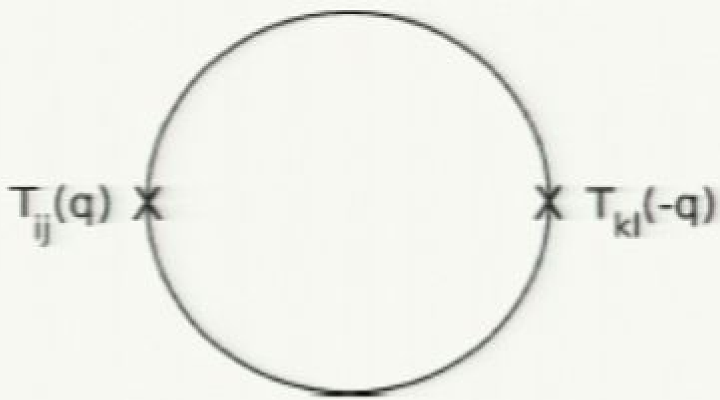

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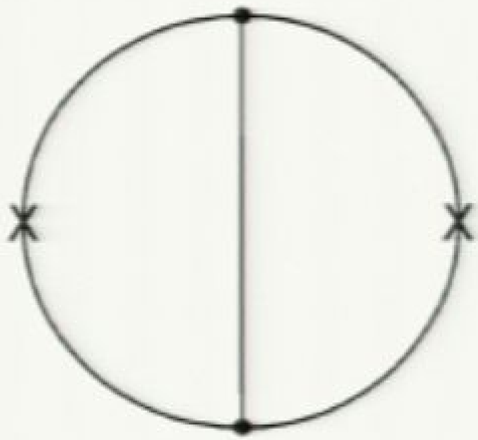

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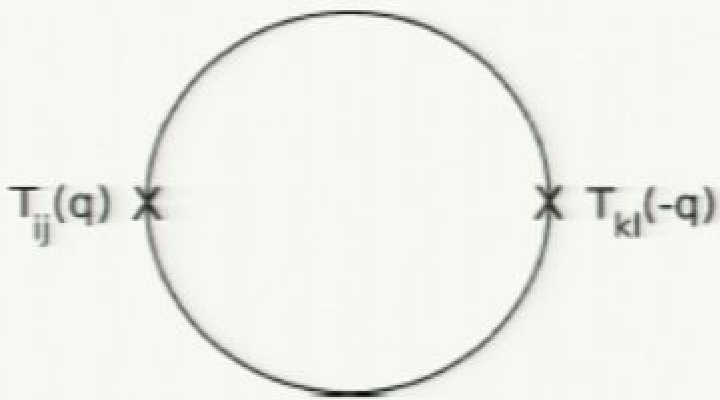

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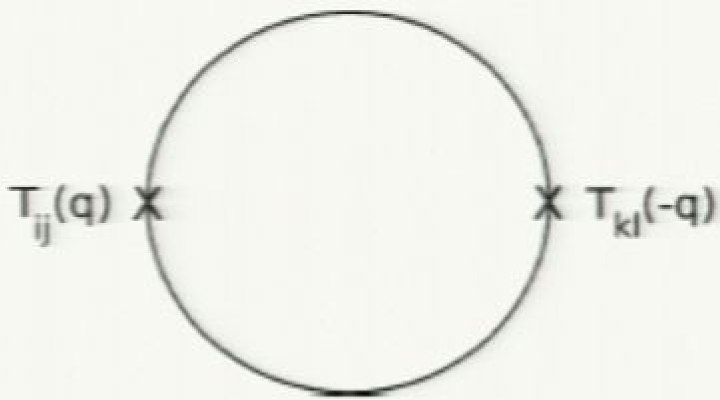

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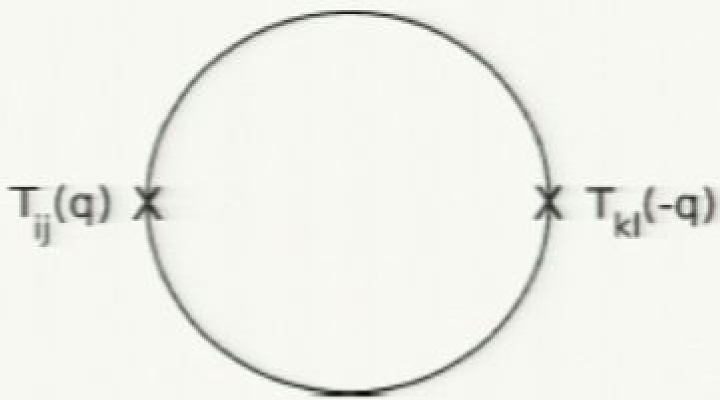

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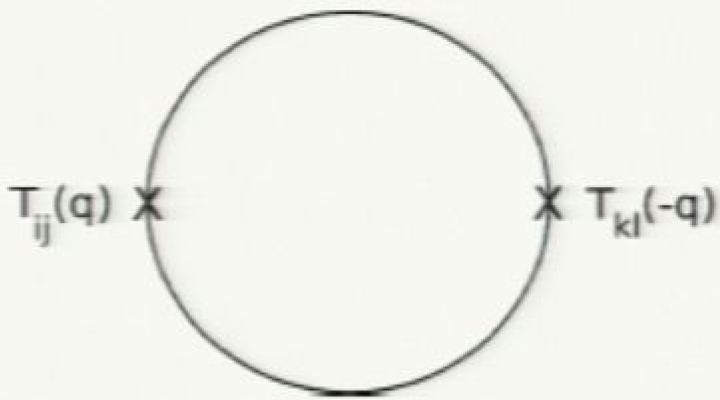

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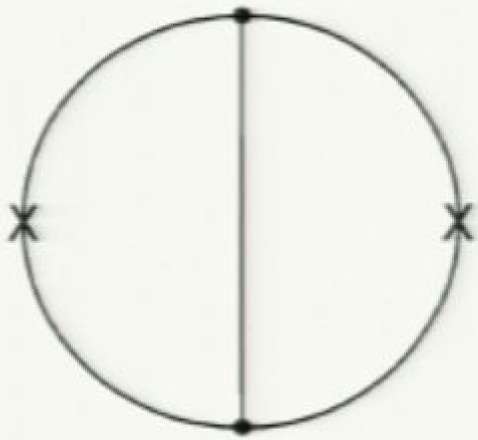

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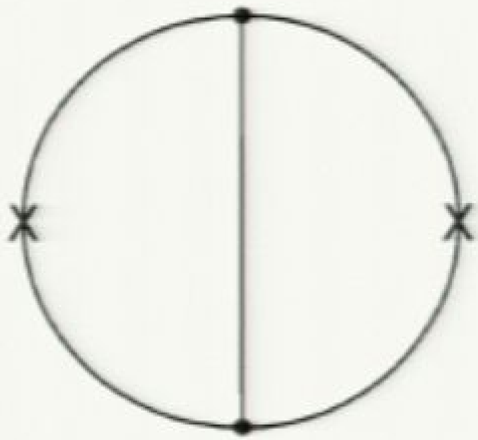

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