Title: Quantum Field Theory for Cosmology - Lecture 22

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Abstract:

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Inflationary generating of scalar and tensor fluctuations

Recall:

- We decompose the inflaton field $\phi(x,\eta)$:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta)$$

where:

* $\phi_0(\eta)$ is assumed large and is treated classically.

* $\varphi(x,\eta) =: \delta\phi(x,\eta)$ describes a field of small inhomogeneities and is to be quantized: $\hat{\varphi}(x,\eta)$

- We decompose the metric $g_{\mu\nu}(x,\eta)$:

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- We decompose the metric $g_{\mu\nu}(x,\eta)$:

$$g_{\mu\nu}(x,\eta) = a_0^2(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$$

treated classically — assumed small, to be quantized

- Here, $\gamma_{\mu\nu}(x,\eta)$ can be decomposed into scalar, vector and tensor-type inhomogeneities, using functions

$$E, B, \Psi, \Phi, V_i, W_i, h_{ij}.$$

- Slicing of spacetime into spacelike hypersurfaces can be done so that all these fctns vanish, except for $\Psi(x,\eta)$, $h_{ij}(x,\eta)$

- We noticed that $\delta\phi(x,\eta) = \varphi(x,t)$ combines with the scalar part of the metric inhomogeneities $\Psi(x,\eta)$, due to the Einstein eqn., to yield one dynamical entity, nam...

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- We noticed that $\delta\phi(x,\eta) = \varphi(x,t)$ combines with the scalar part of the metric inhomogeneities $\Psi(x,\eta)$, due to the Einstein eqn, to yield one dynamical entity, namely:

$$r(x,\eta) = -\Psi(x,\eta) - \frac{a'(\eta)}{a(\eta)} \frac{\varphi(x,\eta)}{\phi_0'(\eta)}$$

Physically, what is $r(x,\eta)$?

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Physically, what is $r(x,\eta)$?

* First term: $\Psi(x,\eta)$ is the (scalar) metric's fluctuation.

* Second term: In $\frac{a'}{a}\frac{1}{\phi_0'}\varphi$, the $\varphi(x,\eta)$ is the scalar field's fluctuatio

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a.) Foliate so that on surfaces of equal time, η, one has: $\varphi \equiv 0$

$\rightsquigarrow$ Equal time hypersurfaces chosen so that all points of equal value of ϕ have equal value of time

Note: Only possible if ϕ decays over time

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$\rightsquigarrow$ Equal time hypersurfaces chosen so that all points of equal value of ϕ have equal value of time

Note: Only possible if ϕ decays over time (e.g. slow roll inflation, but not de Sitter).

$\rightsquigarrow$ We see that $\tau(x,\eta)$ expresses now purely metric fluctuations

$\rightsquigarrow$ Technically, these are fluctuations in the "intrinsic curvature". (Local bloating)

b.) Foliate so that on surfaces of equal time, η, one has: $\Psi \equiv 0$

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In this case, along each equal time surface there is no local bloating - but instead the inflaton field fluctuates

Question:

Why does the contribution of the inflaton in $\Psi(x,\eta)$ take this particular form:

$$\frac{a'(\eta)}{a(\eta)} \frac{\varphi(x,\eta)}{\phi_0'(\eta)} \quad ?$$

Answer:

* The inflaton's inhomogeneities imply locally

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$\Rightarrow$ some regions are ahead, others lag behind in their expansion

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$$\frac{\delta a(x,\eta)}{a(\eta)}$$

into pure inflaton fluctuations $\varphi(x,\eta)$.

* Indeed:

$\delta\eta(x)$ is the time "lag" between slicings a) and b.)

$$\frac{\delta a}{a} = \frac{1}{a}\frac{\delta a}{\delta \phi}\delta\phi = \frac{1}{a}\frac{\delta a}{\delta \eta}\frac{\delta \eta}{\delta \phi}\delta\phi = \frac{a'}{a}\frac{1}{\phi'}\delta\phi$$

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$$= \frac{a'}{a}\,\frac{1}{\phi'}\,\varphi \checkmark$$

(Note: We also have

$$\frac{\delta a}{a} = \frac{a'}{a}\frac{1}{\phi'}\varphi = \frac{a}{z}\varphi$$

where $z = \frac{a^2}{a'}\phi'$ from previous lecture

Ramifications:

□ The intrinsic curvature inhomogeneities

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Ramifications:

- The intrinsic curvature inhomogeneities

$$\mathcal{R} = -\Psi - \frac{a'}{a}\frac{1}{\phi_0'}\varphi$$

can become strongly enhanced, namely, as it happens, for close to de Sitter inflation

i.e., for $a(t) \approx e^{Ht}$

i.e., for $H = \frac{\dot{a}}{a} \approx \text{const}$ $\left(\text{recall: } H \sim \sqrt{V(\phi)}\right)$

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Why? Recall that:

$$\frac{\delta a}{a} = \frac{1}{a}\frac{\delta a}{\delta \eta}\frac{\delta \eta}{\delta \phi}\delta\phi = \frac{a'}{a}\frac{1}{\phi_0'}\varphi$$

Thus: Assume $\phi' = \frac{\delta\phi}{\delta\eta} \ll 1$

$\Rightarrow \delta\eta \gg 1$

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$$\Rightarrow \quad \frac{\delta\eta}{\delta\phi} \gg 1$$

Intuition:

$\frac{\delta\eta}{\delta\phi} \gg 1$ means that the local time-lag $\delta\eta$ between slicings a.) and b.) is large

This could mean large $v(x,\eta)$ against assumpt

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a Could it be a problem?

Observations:

We know the size of $|r|$ from the CMB. The curvature fluctuations r are of order 10^{-5}. Also, there is evidence that the Hubble radius increased during inflation. Namely, the fluctuations of modes that crossed it are smaller. So inflation was significantly different from de Sitter

Is there a preferred slicing of spacetime, say a) or b) in nature?

* Not during inflation, but at its end point!

So it is slicing type a.) * Why? At each point in space, inflation ends the moment the value of ϕ drops to its minimum. Thus, $r(x,y)$ is intrinsic curvature

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Quantization:

- As Hawking, Starobinski and others realized in the early 1980s, the quantization of the inhomogeneity functions $\hat{\tau}(x,y)$ and $\hat{h}_{ij}(x,y)$ yields inhomogeneities as vacuum fluctuations.

- Recall:

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- Recall:

  * Defined auxiliary field for scalar inhomogeneities,

    "Mukhanov variable"

    $$u(x,\eta) := -z(\eta)\,\tau(x,\eta)$$

    Recall: It is related to $\frac{\delta a}{a}$, see comment above.

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* Similarly, we introduced an auxiliary field $p_{ij}(x,\eta)$ for the tensor inhomogeneities:

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$$p_{ij}(x,\eta) := \frac{1}{\sqrt{32\pi G}}\, a(\eta)\, h'_{ij}(x,\eta)$$

* Note: * The components of p_{ij} are not all independent, because h_{ij} obeys:

$h_{ij} = h_{ji}$ and $\sum_{i=1}^{3} h_{ii} = 0$ and in particular:

$$\sum_{i=1}^{3} \frac{\partial}{\partial x^i} h_{ij}(x,\eta) = 0 \quad \text{i.e.} \quad \sum_{i=1}^{3} k_i\, h_{ij}(k,\eta) = 0$$

* But $\vec{k}$ is the vector that points in the direction in which the mode $\vec{k}$ propagates.

$\Rightarrow$ The equation

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⇒ The equation

$$\sum_{i=1}^{3} k_i h_{ij}(k,\eta) = 0$$

(For fixed j, the vectors h_{ij} and k_i are orthogonal) ⟶ means that h_{ij} has no component in the propagation direction:

⇒ h_{ij} describes transversal waves (like e.g. tectonic shear waves), not longitudinal waves (such as e.g. sound waves).

⇝ h_{ij} possesses only 2 degrees of freedom:

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$$p_{ij}(k,\eta) := \sum_{\lambda=1,2} v_{k,\lambda}(\eta)\,\varepsilon_{ij}(k,\lambda)$$

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Here, $\varepsilon_{ij}(k,\lambda)$ are for each k two arbitrary but fixed matrices, obeying $\sum_{i,j=1}^{3}\varepsilon_{ij}(k,1)\,\varepsilon_{ji}(k,2)=0$ and:

$$\varepsilon_{ij}=\varepsilon_{ji}\,,\quad \sum_{i=1}^{3}\varepsilon_{ii}=0\,,\quad \sum_{i=1}^{3}k_i\,\varepsilon_{ij}=0$$

It is convenient to choose

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$$v''_{k,\lambda}(\eta) + \left(k^2 - \frac{a''}{a}\right) v_{k,\lambda}(\eta) = 0$$

(As desired, it has no friction term)

Quantum fluctuations

- We need to solve the quantum wave equations

$$\hat{u}''_k(\eta) + \left(k^2 - \frac{z''(\eta)}{z}\right) \hat{u}_k(\eta) = 0 \qquad \text{scalar}$$

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along with the commutation relations and hermiticity condition

As before, this reduces to solving the eqns of motion for the mode functions, which are complex number-valued, say $\tilde{u}_k(\eta)$, $\tilde{v}_{k,\lambda}(\eta)$:

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along with the Wronskian conditions.

We say we choose the "Bunch Davies vacuum"

Initial conditions?

At early times:

* The k^2 term dominates

⇒ Can choose Minkowski-like init. cond

The mode fctns at late times?

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* The mode k crossed the Hubble horizon:

* The terms $\frac{z''}{z}$ and $\frac{a''}{a}$ dominate.

* The harmonic oscillator is inverted

* Instead of 2 oscillatory basis sol's we now expect one growing and one decaying basis solution.

* Soon after horizon crossing the mode

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The mode fctns at late times?

At late times:

- The mode k crossed the Hubble horizon:
- The terms $\frac{z''}{z}$ and $\frac{a''}{a}$ dominate.
- The harmonic oscillator is inverted
- Instead of 2 oscillatory basis sols we now expect one growing and one decaying basis solution.
- Soon after horizon crossing the mode function consists of essentially only the growing solution.

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Which is the growing solution at late times?

* Eqns of motion:

$$\tilde{u}_k''(\eta) + \left(\cancel{k^2} - \frac{z''(\eta)}{z(\eta)}\right)\tilde{u}_k(\eta) = 0$$

$$\tilde{v}_{k,\lambda}''(\eta) + \left(\cancel{k^2} - \frac{a''}{a}\right)\tilde{v}_{k,\lambda}(\eta) = 0$$

$\Rightarrow$ Growing solution must behave as:

$$\tilde{u}_k(\eta) \sim z(\eta) \quad \text{at late } \eta$$

$$\tilde{v}_{k,\lambda}(\eta) \sim a(\eta) \quad \text{at late } \eta$$

$\Rightarrow$ This means that the mode fctns $\tilde{r}_k(\eta)$ and $\tilde{h}_{ij_k}(\eta)$ become constant at late η, i.e., after the mode k crosses the horizon!

$30\%\nu \longrightarrow 30$

$$u'' - \frac{z''}{z}u = 0$$

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Why? $\tilde{v}_k(\eta) = \frac{1}{z(\eta)}\tilde{u}(\eta) \sim \frac{z(\eta)}{z(\eta)}$ for late η

$$\tilde{h}_{ij_k}(\eta) = \frac{1}{a(\eta)}\tilde{p}_{ij_k}(\eta) \sim \frac{1}{a(\eta)}\tilde{v}_{k,\lambda}(\eta) \sim \frac{a(\eta)}{a(\eta)} \text{ for late } \eta$$

$\Rightarrow$ As expected, the magnitude of the mode k's quantum fluctuations

$$\delta r_k = \underbrace{k^{3/2}|\tilde{r}_k|^2}_{z^{-1}k^{3/2}|\tilde{u}_k|} \quad \text{and} \quad \delta h_{ij_k} = \underbrace{k^{3/2}|\tilde{h}_{ij_k}|}_{=a^{-1}k^{3/2}|\tilde{v}|}$$

stay constant at the value that they possess when the mode crosses the horizon, even as the mode's proper wavelength then continues to increase rapidly.

* **Goal now:** Calculate the magnitude of the fluctuations at horizon crossing

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Realistic example: "Power law inflation"

□ We need an explicit potential $V(\phi)$ in order to

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* $V(\phi)$, and therefore the temporary "cosmological constant" $H \sim \sqrt{V(\phi)}$ must slowly decrease (slow roll).

* In any case, our perturbation assumptions don't allow exact de Sitter, as δv_k would diverge, invalidating the assumption that it is small.

The "slow roll parameters"

Idea:

* We do not know the exact slow roll potentia

E.g.

18 / 2

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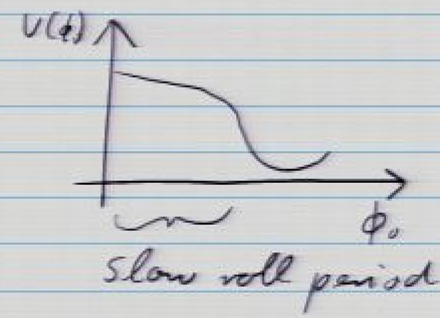

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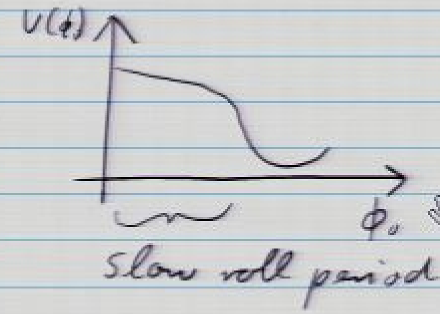

* For all values of ϕ_0 during the inflationary period we can parametrize the slope of the potential by its derivatives.

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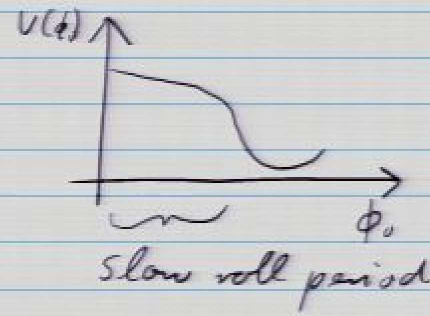

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$$\varepsilon(\phi) := \overbrace{\frac{1}{4\pi G}}^{\text{convenience factor}} \left(\frac{H'(\phi)}{H(\phi)}\right)^2 \qquad \left(= \frac{\frac{3}{2}\dot{\phi}^2}{V + \frac{1}{2}\dot{\phi}^2} \right)$$

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The simplest solvable case:

* The simplest case is that of

$\varepsilon(\phi) = c$ where c is a constant.

* In this case:

$$c = \varepsilon(\phi) := \frac{1}{4\pi G}\left(\frac{H'(\phi)}{H(\phi)}\right)^2$$

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$$H(\phi) \sim e^{\sqrt{4\pi G}\,\phi}$$

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$$V(\phi) = e^{s\phi}$$

* Exercise: What is the value of s ?

* Then, one also finds:

$c = \varepsilon = \eta = \rho = \ldots$

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* The expansion rate is polynomial:

Exercise:

Show that we now have

$$a(t) = a_0\, t^{1/\varepsilon} \qquad (t \text{ is proper time})$$

Exercise:

Show that, in terms of the conformal time η, we now have:

$$a(\eta) = \frac{-1}{\eta H}\,\frac{1}{1-\varepsilon}$$

Note: Still η is always negative and $t \to \infty$ means $\eta \to 0$.

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The mode equations:

Scalar: We can now calculate $z(\eta) = \frac{a^2(\eta)}{a'(\eta)}\phi_0'(\eta)$ and therefore also the mode equation's term z''/z explicitly to obtain

A Bessel differential equation

$$\tilde{u}_k''(\eta) + \left(k^2 - \frac{(\nu^2 - 1/4)}{\eta^2}\right)\tilde{u}_k(\eta) = 0$$

where: $\nu := \frac{3}{2} + \frac{c}{1-c}$

* Solution for Bunch Davies initial conditions:

$$\tilde{u}_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu + 1/2)\frac{\pi}{2}} (-\eta)^{1/2} H_\nu^{(1)}(-k\eta)$$

Hankel fctn of [illegible] kind of order ν.

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* Behavior after horizon crossing:

$$\tilde{u}_k(\eta) \to e^{i(\nu-\frac{1}{2})\frac{\pi}{2}} 2^{\nu-3/2} \frac{\Gamma(\nu)}{\Gamma(3/2)} \frac{1}{\sqrt{2k}} (-k\eta)^{-\nu+1/2}$$

* Thus, the magnitude of intrinsic curvature fluctuations after horizon crossing becomes:

$$\delta r_k(\eta > \eta_{hor}(k)) = G\, 2^{\nu-\frac{1}{2}} \frac{\Gamma(\nu)}{\Gamma(3/2)} (\nu - \tfrac{1}{2})^{1/2-\nu} \left.\frac{H^2}{|H'|}\right|_{k=aH}$$

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* Exercise: Verify.

* **Remark:** Notice that since $|H'|$ must be small, also H must be small. This is important because the tensor fluctuations will come out to be proportional to H, and will therefore be small

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we obtain for the term a''/a

$$\frac{a''}{a} = 2a^2H^2(1-\epsilon/2)$$

which comes out to be (verify):

$$\frac{a''}{a} = \frac{1}{\eta^2}\left(\nu^2 - \frac{1}{4}\right)$$ recall: $\nu = \frac{3}{2} - \frac{c}{1-c}$

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$\Rightarrow$ The tensor fluctuation spectrum:

The tensor fluctuation spectrum:

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proportional to the value of H at horizon crossing.

Observations:

- δ_{T_k} is predicted to have seeded oscillations in the hot plasma after re-heating, leading to a calculable fluctuation spectrum in the CMB.
- The match is very good. The WMAP results show indications that $\varepsilon \neq 0$, namely that $\delta_{T_k} \neq$ const.
- $\delta h_{ij\,k}$ is predicted to have led to polarization fluctuations in the light of the CMB - a pure curl polarization field.

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* Exercise: Verify.

* Remark: Notice that since $|H'|$ must be small, also H must be small. This is important because the tensor fluctuations will come out to be proportional to H, and will therefore be small

Tensor modes:

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⇒ The mode eqn is again solved by the Hankel functio

⇒ The tensor fluctuation spectrum:

$$\delta h_{ij\,k} = \frac{2}{\sqrt{\pi}}\, 2^{\nu-\frac{1}{2}}\, \frac{\Gamma(\nu)}{\Gamma(3/2)}\, (\nu - 1/2)^{1/2-\nu} \sqrt{G}\, H \Big|_{k=aH}$$

proportional to the value of H at horizon crossing.

Observations:

- δT_k is predicted to have seeded oscillations in the hot plasma after re-heating, leading to a calculable fluctuation spectrum in the CMB.
- The match is very good. The WMAP results

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Observations:

- $\delta\tau_k$ is predicted to have seeded oscillations in the hot plasma after re-heating, leading to a calculable fluctuation spectrum in the CMB.
- The match is very good. The WMAP results show indications that $\varepsilon \neq 0$, namely that $\delta\tau_k \neq$ const.
- δh_{ij_k} is predicted to have led to polarization fluctuations in the light of the CMB – a pure curl polarization field.

- Since H' is small, H is small, so δh_{ij} is small. Indeed still undetected

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* Thus, the magnitude of intrinsic curvature fluctuations after horizon crossing becomes:

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Tensor modes:

* In the power law case, in the mode equation

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