Title: Algebraic Bose Liquid and Emergent Lifshitz gravity

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Abstract: We study a novel state of matter: algebraic Bose liquid (ABL). An ABL is a quantum bosonic system on a 2d or 3d lattice that does not break any symmetry in its ground state, but still able to stabilize a gapless spectrum. At high energy these boson systems only have the simplest U(1) global symmetry associated with the conservation of boson number, but at low energy the system is described by self-dual gauge fields. In this talk we will present two new ABL phases emerged from a quantum Boson model on the cubic lattice. At low energy these two ABL phases are described by the linearized z=2 and z=3 Lifshitz gravity respectively. We will show that the self-duality of the field theory is crucial to guarantee the stability of the ABL.

References:

Cenke Xu, arXiv:cond-mat/0602443, Phys. Rev. B. 74, 224433

Cenke Xu and Petr Horava, arXiv:1003.0009

Algebraic Bose Liquid and Emergent Lifshitz Gravity

Cenke Xu

Harvard University & UCSB

Algebraic Bose Liquid and Emergent Lifshitz Gravity

Outline:

1, Introduction,

Define emergent excitation, define ABL phase

2, Known example of ABL

Boson model with photons, Importance of self-duality

3, New example of ABL

emergent z=2, and z=3 Lifshitz gravity, and its self-duality

****4, liquid state in real system**

kappa-(ET) compound, strongly coupled liquid state?

QCD in cold atom system, Z2 gauge theory in Josephson array

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Question: how to guarantee a gapless/massless spectrum in a local quantum system on lattice?

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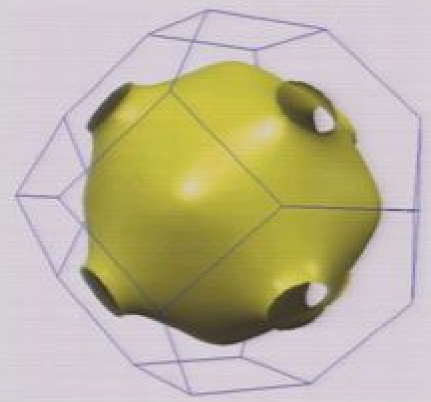

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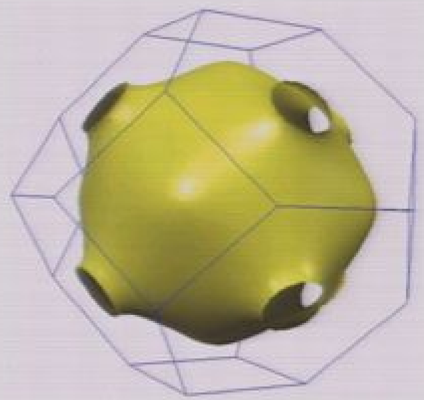

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2, free, or fine-tuned bosons (quantum critical point)

$$L = (\partial_\mu \phi)^2 + r\phi^2 + g\phi^4$$

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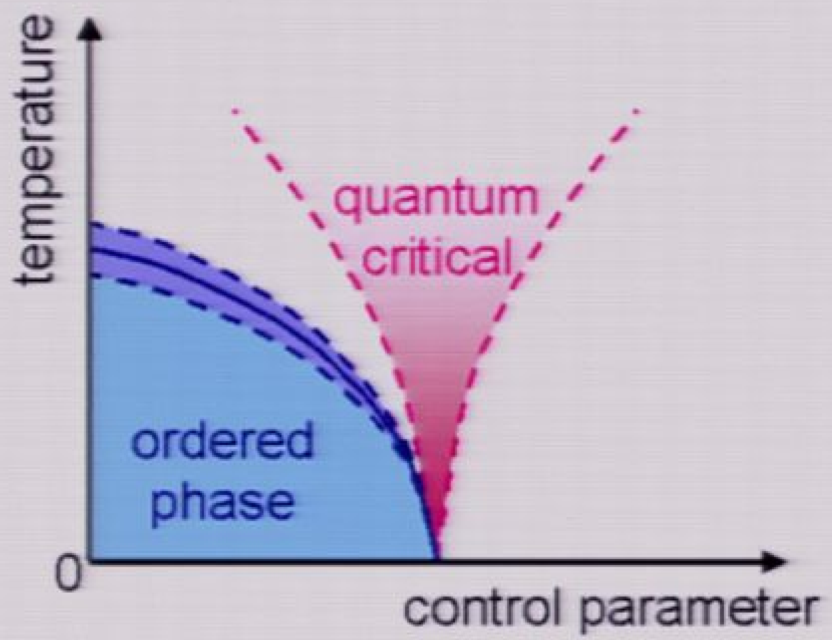
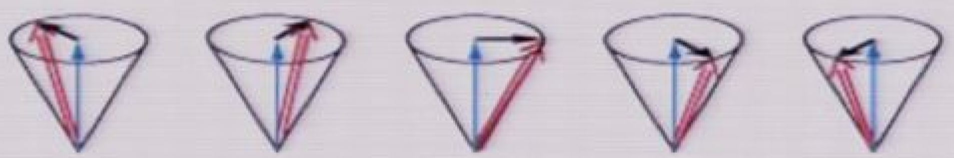


how to guarantee a gapless excitation? ----Goldstone theorem

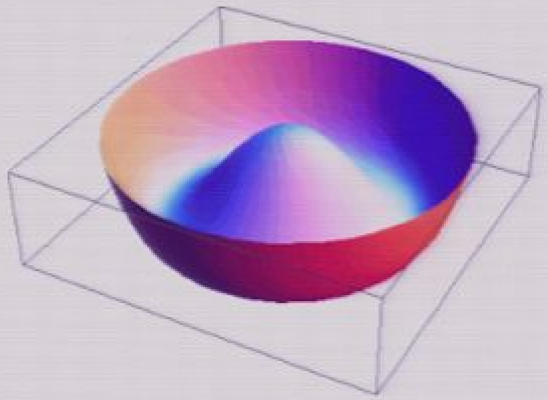
3, **Goldstone theorem:**

Example: magnon, or spin-wave quantum.

$$H = \sum_{i,\mu} -J\vec{S}_i \cdot \vec{S}_{i+\mu}$$



Magnons are Goldstone modes, which always happen with continuous global symmetry breaking.



Define emergent excitation and ABL phase

Refined Question:
how to guarantee a gapless/massless spectrum in an **unfine-tuned**, **interacting**, **"simple"**, **Bosonic** system on lattice with **no symmetry breaking**?

These gapless excitations are called **emergent** excitations, the phase is called **Algebraic Bose Liquid (ABL)**

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Emergence means, at high energy only the simplest symmetry is assumed. For instance the **U(1) global symmetry** of the boson system, corresponding to the particle number conservation.

Define ABL phase on lattice

What is liquid?

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Algebraic Bose liquid on lattice:
does **not** break the global U(1) symmetry, or the lattice symmetry, however, has algebraic density-density correlation, and **gapless** excitations.

Example of ABL, 1d and quasi 1d system

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Example of ABL: 1d interacting boson system, Luttinger liquid, $c=N$ CFT, with no symmetry breaking (Mermin-Wagner theorem).

Enlarge the symmetry/gauge symmetry, all kinds of CFT in 1d, like $SU(N)_k$ WZW, can be realized as k-orbital SU(N) spin chain.

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Refined Question (final):
how to guarantee a gapless/massless spectrum in an **unfine-tuned**, **interacting**, **"simple"**, **Bosonic** system on **2d** or **3d** lattice with **no symmetry breaking**? *i.e.* can we have **ABL**?

Difference between High energy and condensed matter physics

High energy physics: assume large symmetry/gauge symmetry at high energy.

SUSY, SU(5) GUT.... —symmetry breaking→ SU(3)×SU(2) ×U(1) Standard model

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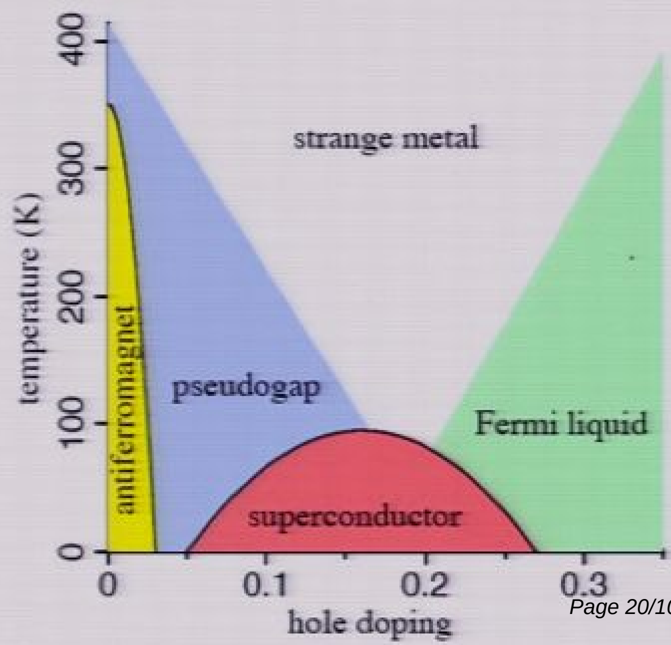
For example, in the mystery High Tc superconductor, we know the high energy model, but we do not know the low energy model.



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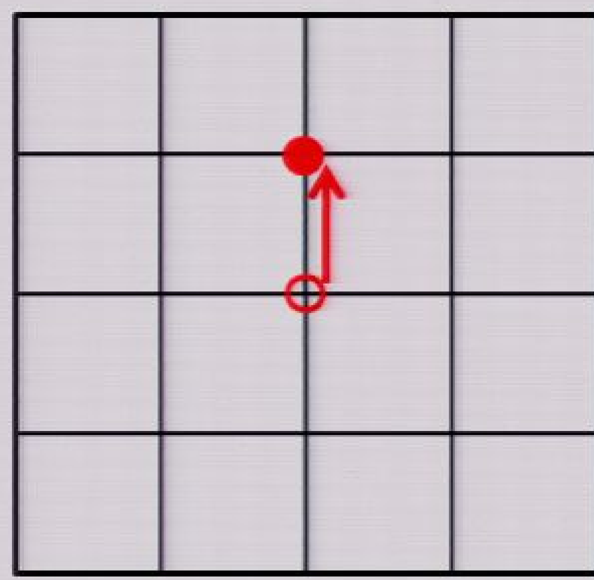
Introducing Bose Hubbard model

A typical quantum boson model: Bose Hubbard model

$$H = -t \sum_{\langle i,j \rangle} \left(b_i^\dagger b_j + b_j^\dagger b_i \right) + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - \mu \sum_i \hat{n}_i$$

The superfluid phase (SF) is gapless, and excitations are Goldstone modes with U(1) global symmetry breaking.

The Mott insulator phase (MI) is gapped, without any symmetry breaking.



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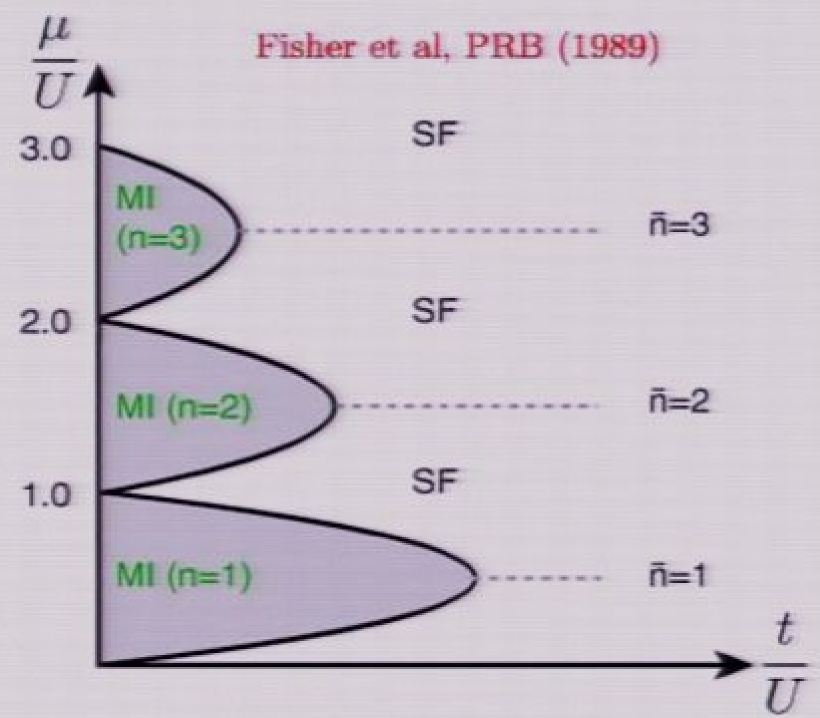

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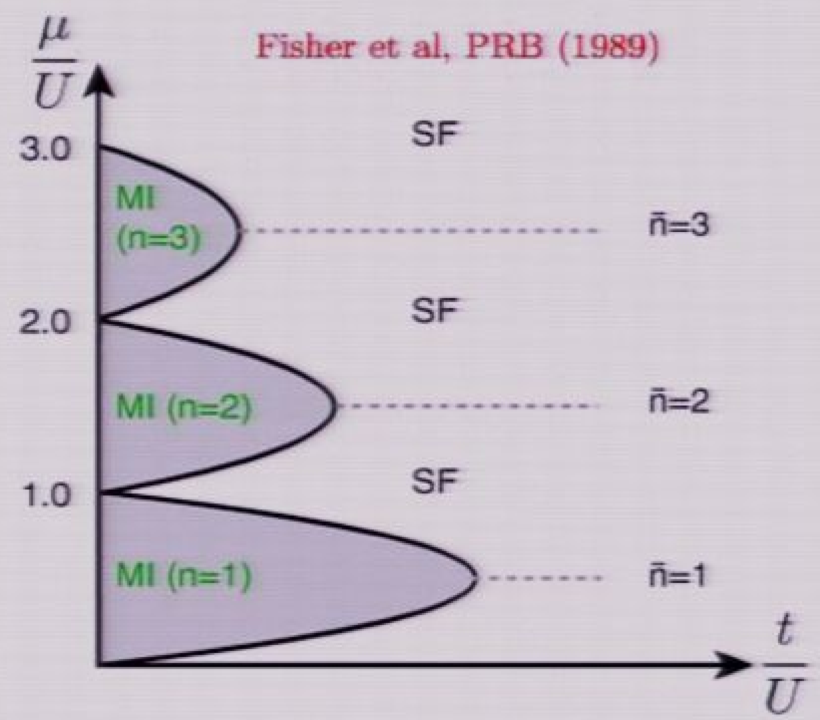

Goal: to get ABL from Bose Hubbard type of model in 2d and 3d

Example of ABL phase, emergent photon model

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An example to warm up:

Define bosons on the links of the square lattice, and Hamiltonian:

$$H = H_u + H_v + H_t,$$

$$H_u = \sum_{i,\mu} U(n_{i,\mu} - \bar{n})^2$$

$$H_v = \sum_i V(\sum_{\mu=\pm x,\pm y} n_{i,\mu} - 4\bar{n})^2,$$

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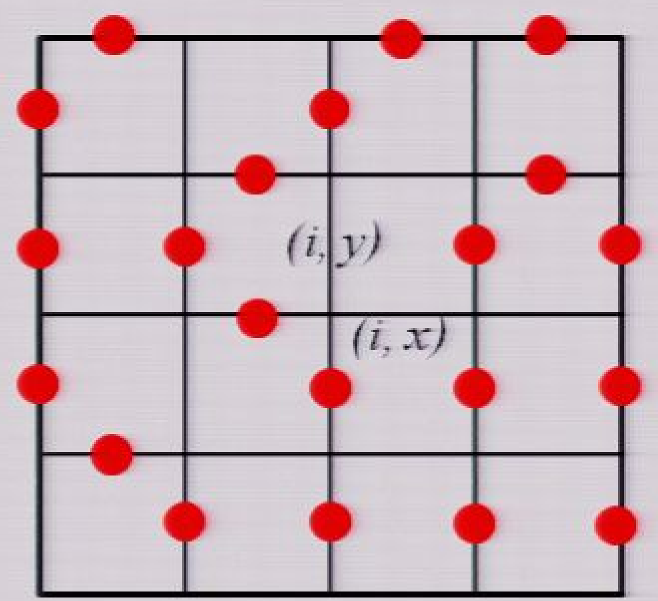

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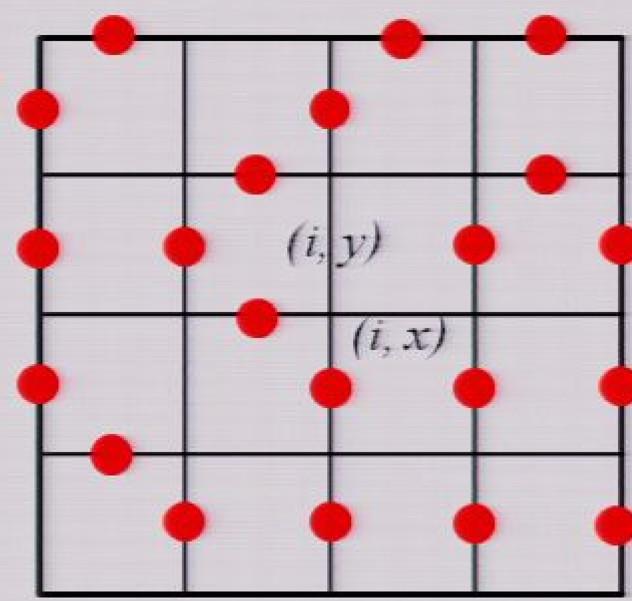

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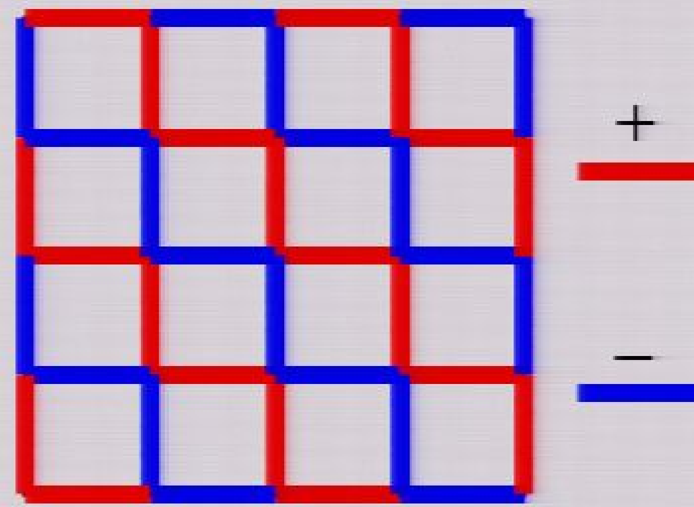

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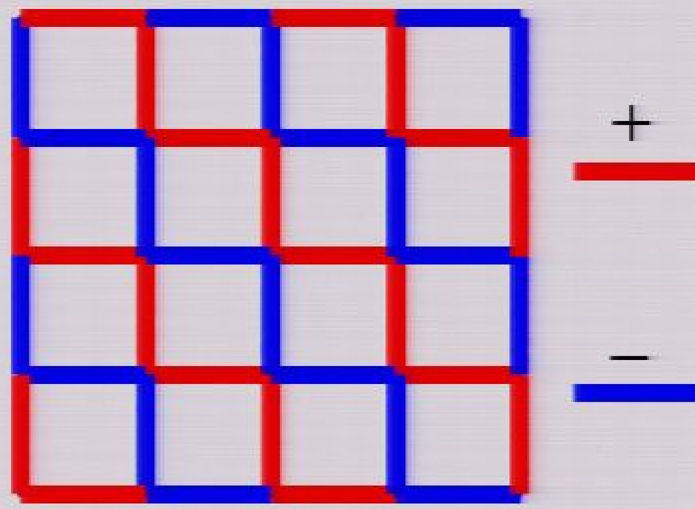

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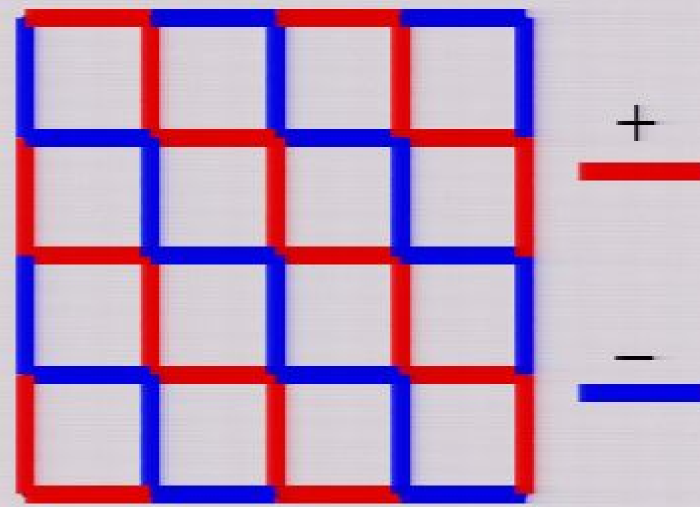

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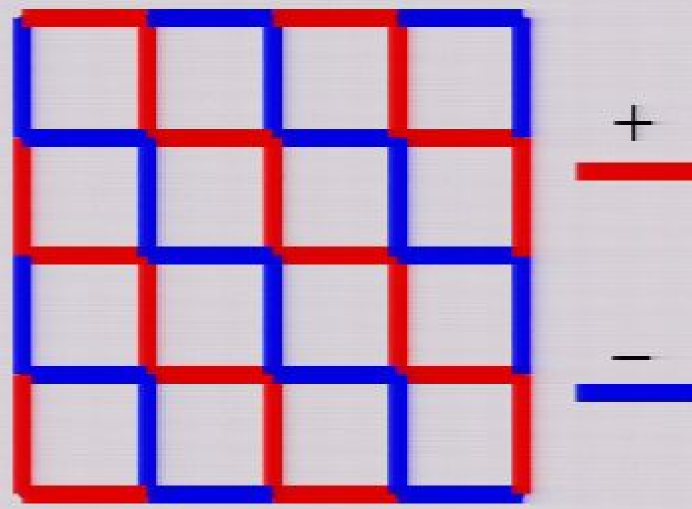

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How do we generate the QED like Hamiltonian?

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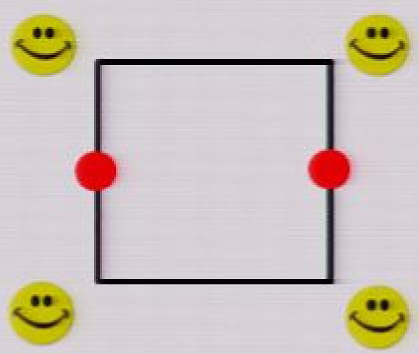

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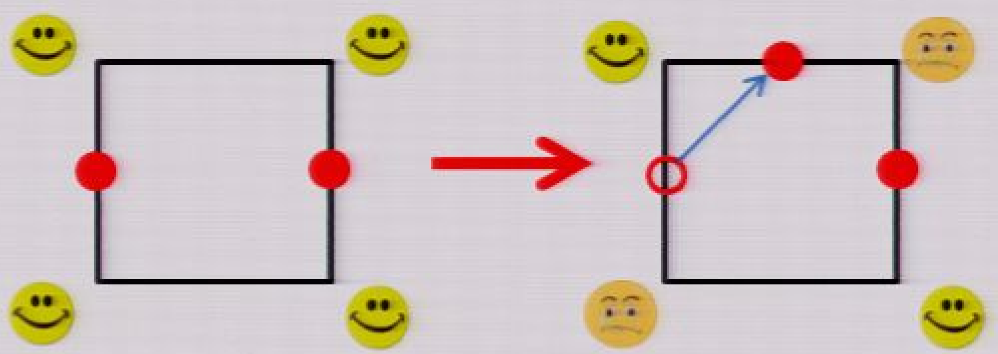

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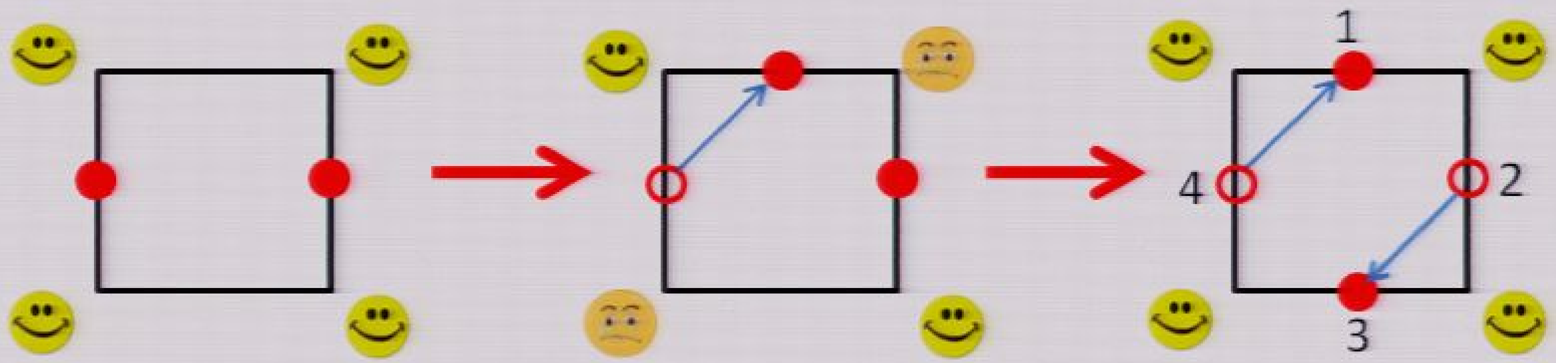

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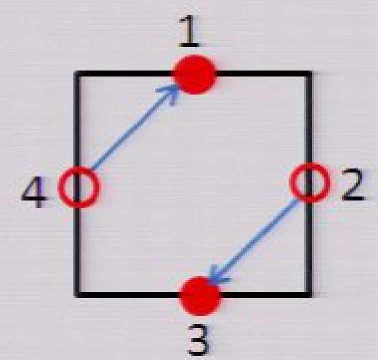
How do we generate the QED like Hamiltonian?

$$H_t = \sum_{i,j,\mu,\nu} -\tilde{t} b^\dagger_{i,\mu} b_{j,\nu} + H.c.$$

First order boson hopping always violates the constraint, the second order boson hopping (ring exchange) is OK.



$$H_{\text{eff}} = -t b_1^\dagger b_3^\dagger b_2 b_4 + H.c. \sim -t\cos(\theta_1 + \theta_3 - \theta_2 - \theta_2) \qquad b \sim \exp(i\theta)$$

$$t \sim \frac{\tilde{t}^2}{V} \qquad A_\mu(i) = \eta_i \theta_{i,\mu} \qquad \boxed{H = U\vec{E}^2 - t\cos(\vec{\nabla}\times\vec{A}),}$$

Photon model and Duality

$$H = U\vec{E}^2 - t\cos(\vec{\nabla}\times\vec{A}), \qquad \vec{A} \to \vec{A} + \vec{\nabla} f$$

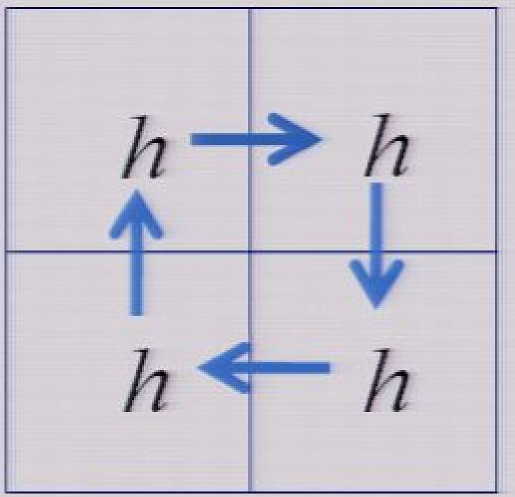
The gauge invariance is emergent, which only exists at low energy.

Does gauge invariance imply that the spectrum is gapless? To answer this question we need to go to the dual picture.

Dual formalism 2+1d:

$$\vec{\nabla}\cdot\vec{E} = 0 \longrightarrow \vec{E} = \hat{z}\times\vec{\nabla}h$$

$$\vec{\nabla}\times\vec{A} = \Pi \qquad H = -t\cos(\Pi) + U(\vec{\nabla}h)^2$$



Ordinary gauge field --- superfluid duality

Photon model and Duality

Dual lagrangian in 2+1d:

$$L = (\partial_\tau h)^2 + (\vec{\nabla} h)^2$$

h seems to give us gapless excitation, but, let us not forget that ***h*** only takes discrete values: $\vec{E} = \hat{z} \times \vec{\nabla} h$

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The vertex operator is a relevant perturbation, which will gap out the ***h*** field excitations.

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Violate gauge invariance, hence **irrelevant** !!

Duality and stability of ABL

$$L = (\partial_\tau \vec{h})^2 + (\vec{\nabla} \times \vec{h})^2 - \boxed{\alpha \cos(2\pi \vec{h})}$$

Vertex operator is only relevant when the dual gauge invariance is spontaneously broken (Higgsed) by condensing topological defects. **Self-duality guarantees a gapless phase!**

The duality is simply the **quantum** and **compact** version of the basic self-duality of the Maxwell equation, without matter fields!

$$H \sim E^2 + B^2$$

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}.$$

Summary of Photon phase

In the gapless phase, the boson density-density correlation is algebraic:

$$\langle n(0)\ n(\vec{r})\rangle \sim (-1)^{\vec{r}} \langle \vec{\nabla} \times \vec{h}(0)\ \vec{\nabla} \times \vec{h}(\vec{r})\rangle \sim \frac{1}{r^3}$$

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Many condensed matter version of it, like
3+1d Quantum dimer model (Moessner, Sondhi)
XXZ model on Pyrochlore lattice (Hermele, Balents, Fisher)
Loop model, or string net model (Wen, Levin)

Summary of Photon phase

Let us summarize what we just did:

Summary of Photon phase

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2, a dominant energy scale separates high energy Hilbert space, and low energy Hilbert space.

3, within the low energy Hilbert space, the only dynamics are ring exchanges that can be calculated through high order perturbation of the original model.

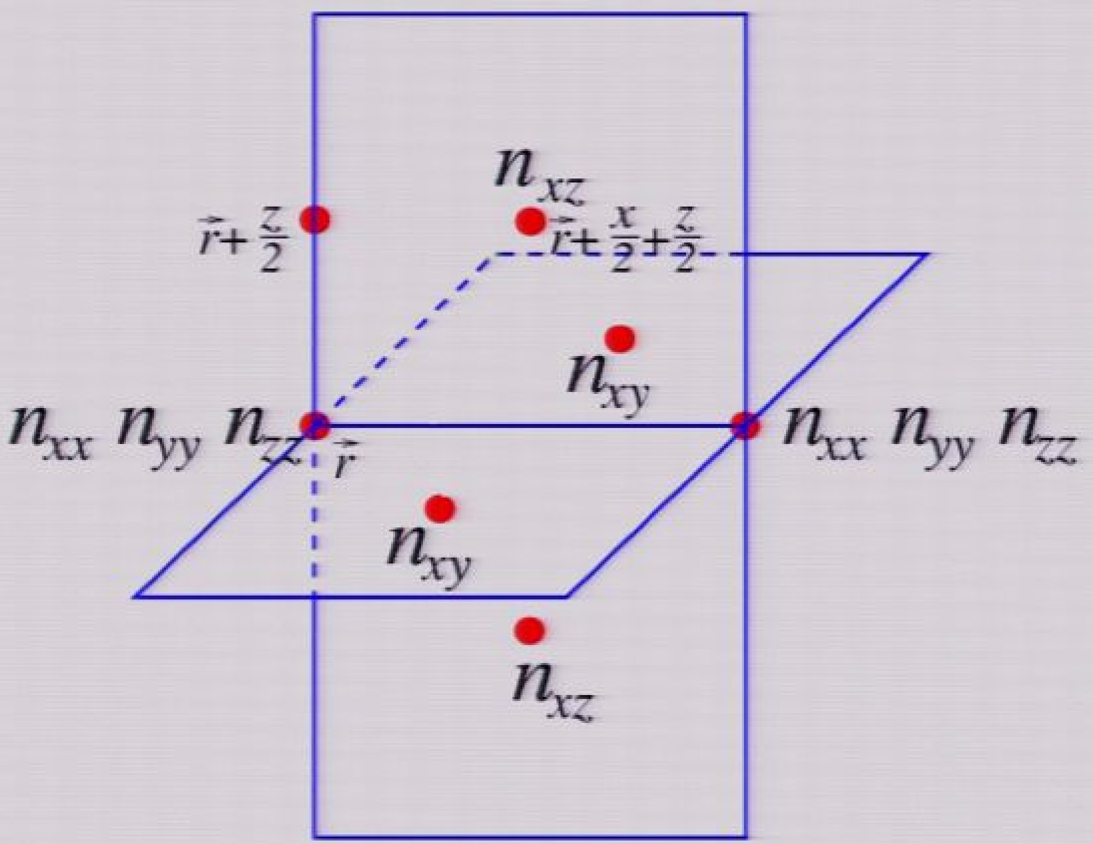
4, the effective low energy model has **emergent** gauge invariance.

5, the self-duality of the 3d model guarantees the gaplessness of the spectrum, *i.e.* we obtain an **ABL** phase

To look for new ABL phase, we want to look for self-dual gauge theory, that can emerge through perturbation of local hopping on lattice.

New ABL phase, Lifshitz gravity model

New example: Lifshitz gravity model



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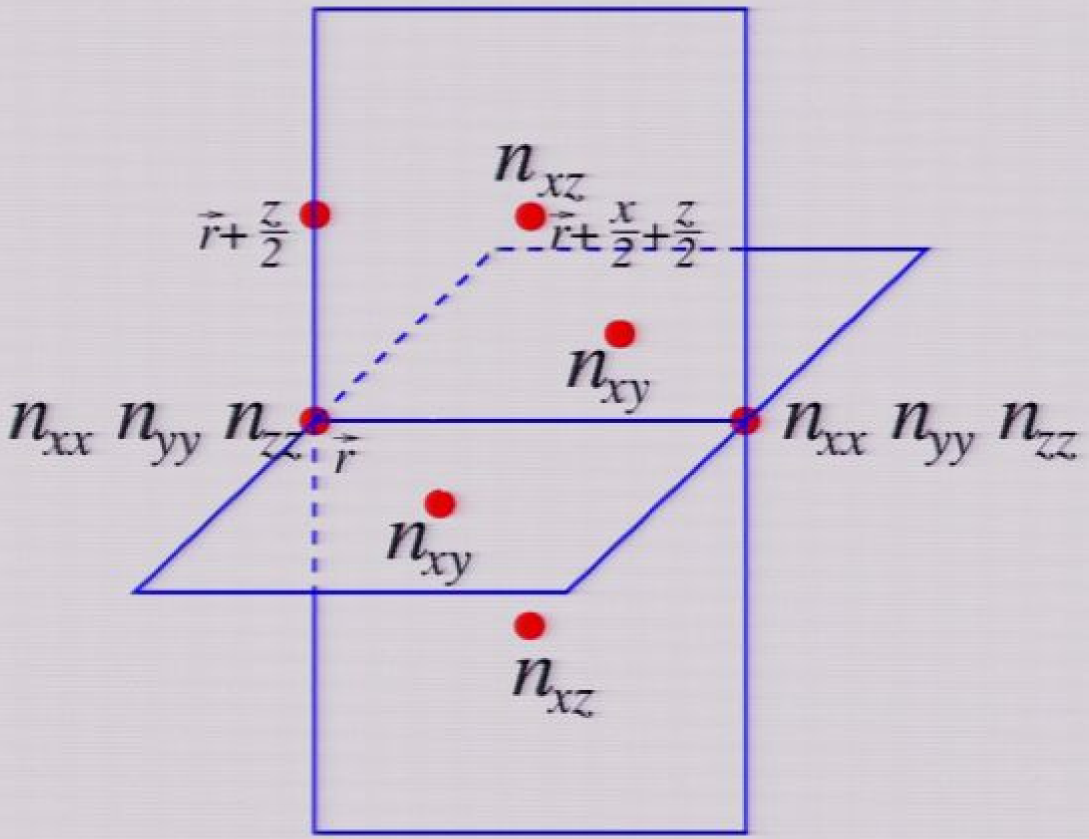

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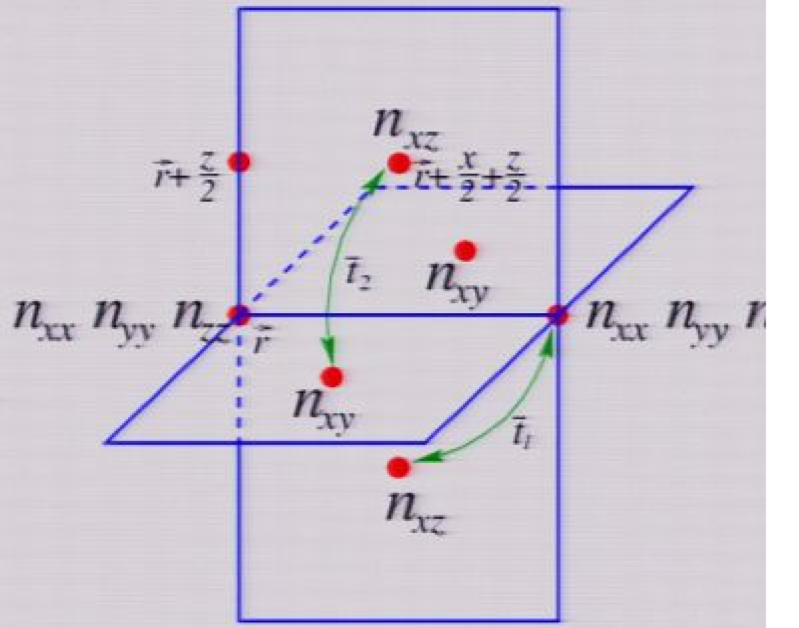
$$H = H_t + H_v + H_u + H_{v'}$$

$$H_t = -\bar{t}_1 H_{sp} - \bar{t}_2 H_{pp}$$

$$H_{v,\ \hat{x}\ \text{link}} = V(2n_{xx,\vec{r}} + 2n_{xx,\vec{r}+\hat{x}} + n_{xy,\vec{r}+\frac{\hat{x}}{2}+\frac{\hat{y}}{2}} + n_{xy,\vec{r}+\frac{\hat{x}}{2}-\frac{\hat{y}}{2}} + n_{xz,\vec{r}+\frac{\hat{x}}{2}+\frac{\hat{z}}{2}} + n_{xz,\vec{r}+\frac{\hat{x}}{2}-\frac{\hat{z}}{2}} - 8\bar{n})^2$$



$$H_u = \sum_{\vec{r}} \sum_{ii} \frac{u_1}{2}(n_{ii,\vec{r}} - \bar{n})^2 + \sum_{i<j} \frac{u_2}{2}(n_{ij,\vec{r}+\frac{i}{2}+\frac{j}{2}} - \bar{n})^2$$

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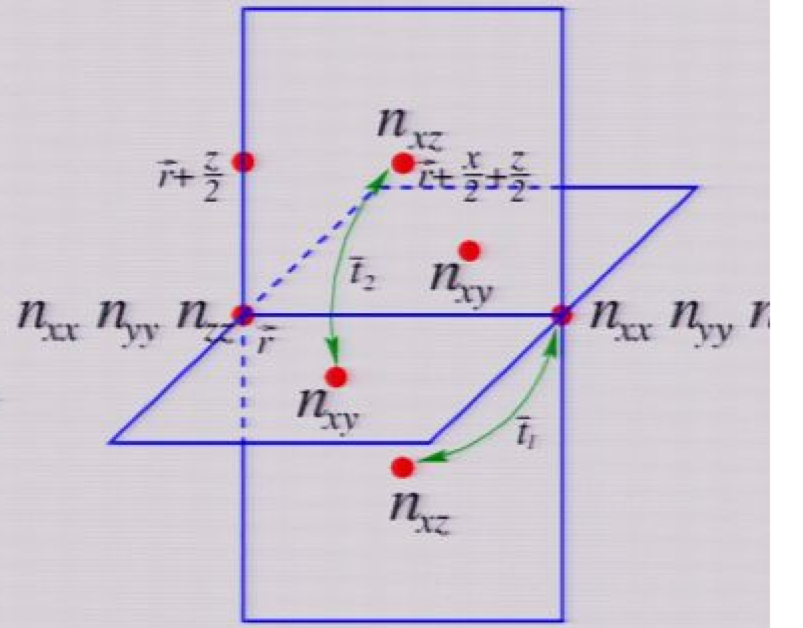

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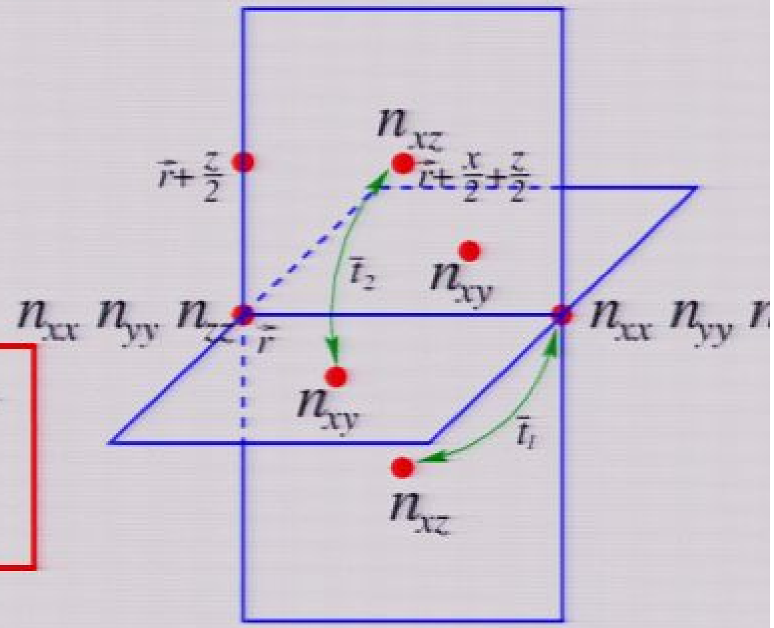

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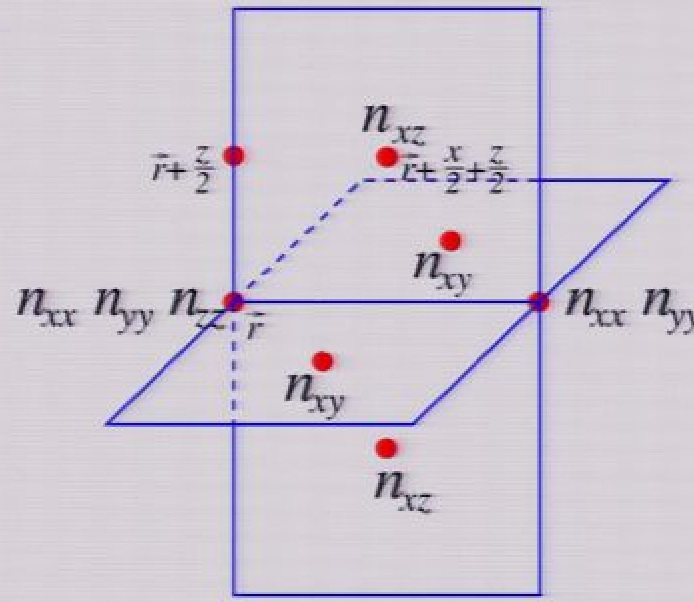

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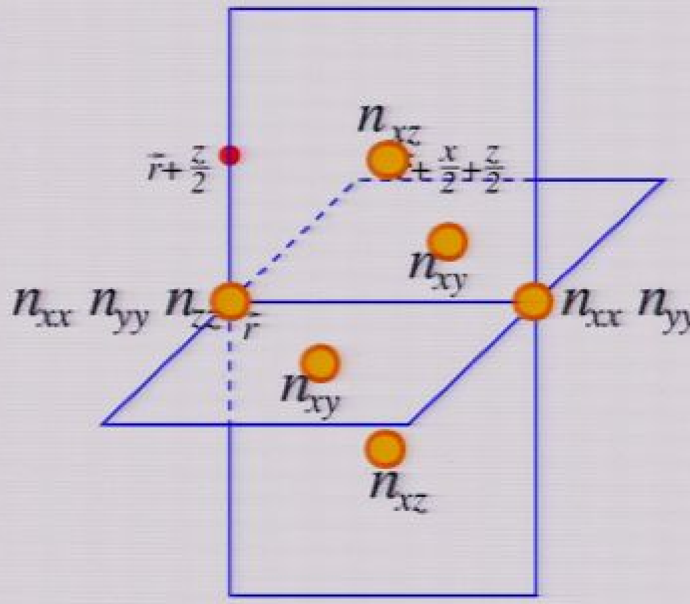

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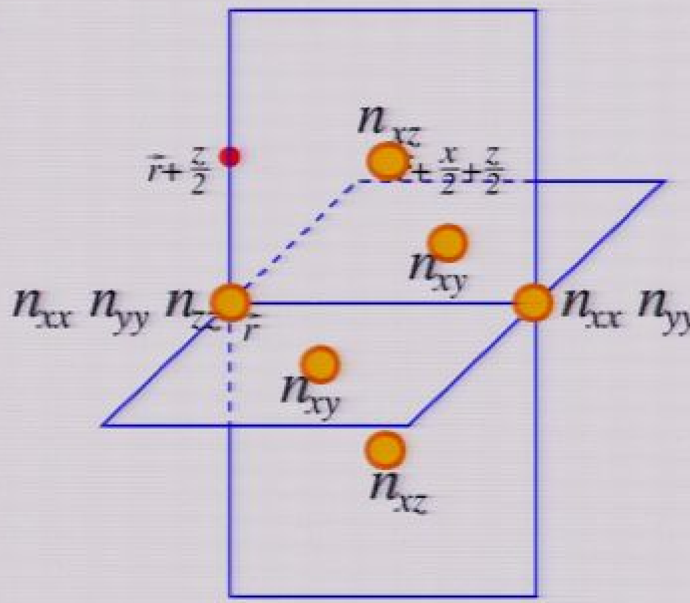

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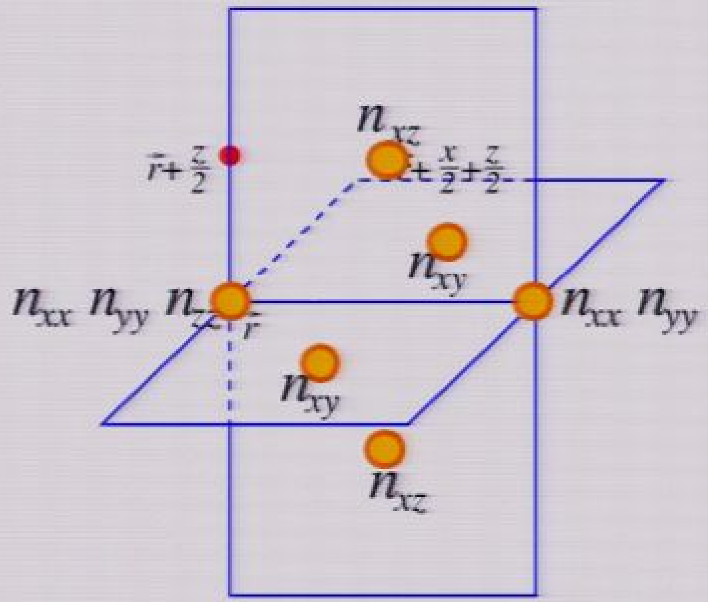

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New ABL phase, Lifshitz gravity model

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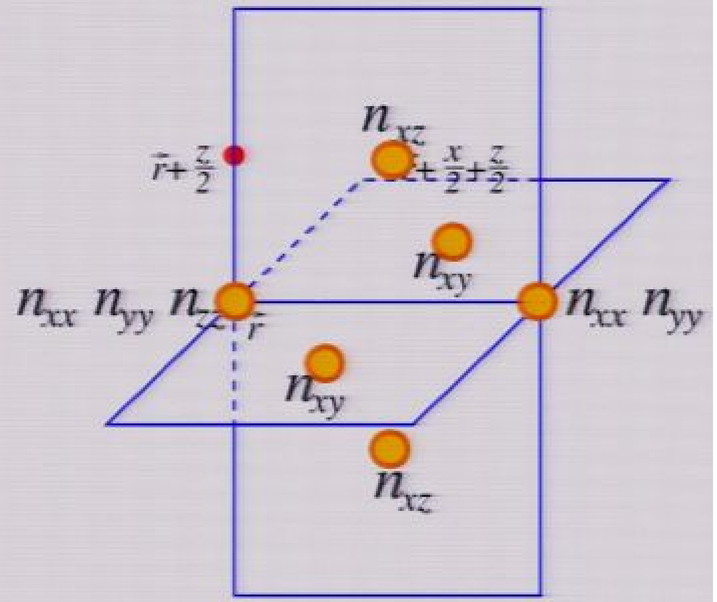

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Emergent linearized gravity gauge symmetry!

New ABL phase, Lifshitz gravity model

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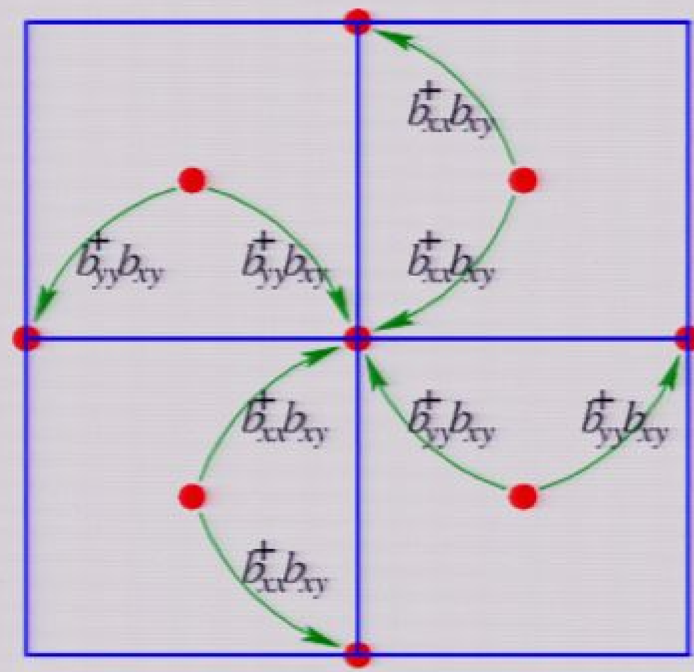
Lowest order ring exchange term that is consistent with the constraint, at the eighth order perturbation:

$$t_1 \sim \frac{\tilde{t}_1^8}{V^7} \qquad t_2 \sim \frac{\tilde{t}_1^4 \tilde{t}_2^4}{V^7}$$



$$H_{\text{eff}} = \sum_{\vec{r}} - \sum_{i \neq j} t_1 \cos(R_{ijij})_{\vec{r}} - \sum_{i \neq j, j \neq k, k \neq i} t_2 \cos(R_{ijik})_{\vec{r}}. \qquad A = (-1)^{\vec{r}} \theta$$

$$b \sim \exp(i\theta)$$

$$R_{\alpha\mu\beta\nu} = 1/2(A_{\alpha\nu,\mu\beta} + A_{\mu\beta,\alpha\nu} - A_{\mu\nu,\alpha\beta} - A_{\alpha\beta,\mu\nu})$$

[redacted]

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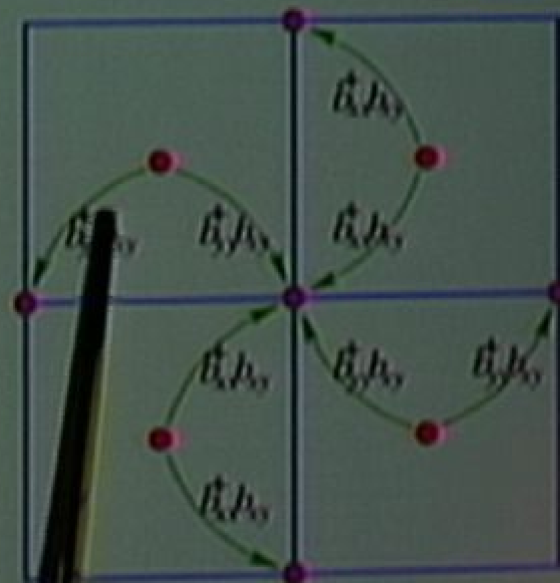

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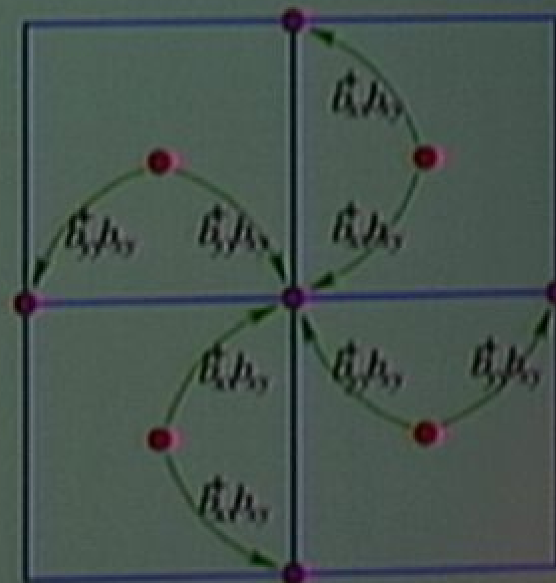

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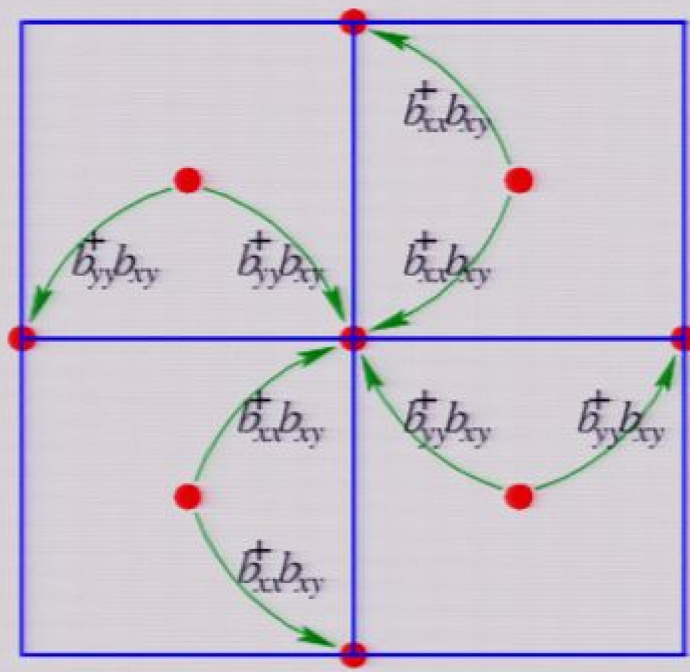

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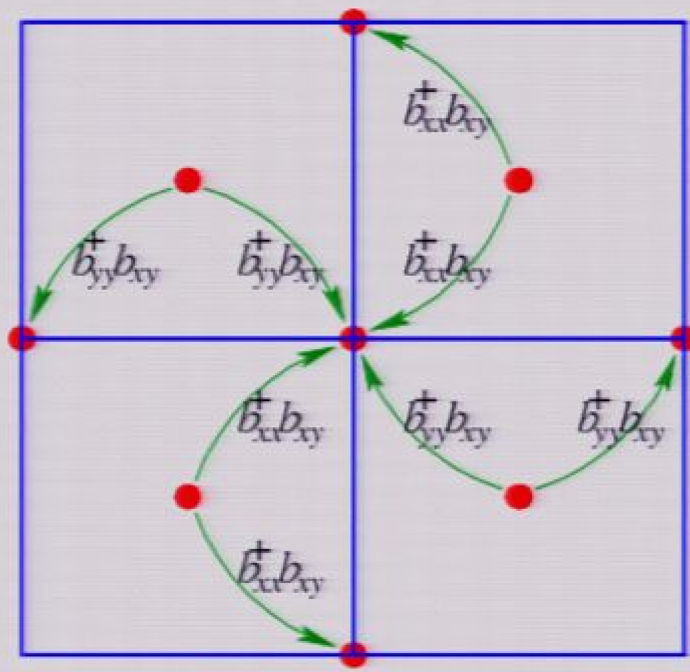

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Expanding cosines, quadratic dispersion, instead of linear.

$$L = \int d^3r \sum_{ij} (\partial_t A_{ij})^2 - \sum_{i\neq j} \frac{t_1}{2u} R^2_{ijij} - \sum_{i\neq j, j\neq k, k\neq i} \frac{t_2}{2u} R^2_{ijik}$$

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To guarantee the stability of the gapless excitation against vertex operators, we now show this model has a self-dual field theory.

Cenke Xu,
cond-mat/0602443, Phys. Rev. B, 74, 224433

Lifshitz gravity model and its self-duality

$$\mathcal{B}_{ii} = \frac{1}{2}\epsilon_{ijk} R_{jkjk} \qquad \mathcal{B}_{ij} = -R_{ikjk}$$

$$H \sim \mathcal{E}^2 + \mathcal{B}^2$$

$$\mathcal{B}_{ij} = \epsilon_{iab}\epsilon_{jcd}\nabla_a\nabla_c A_{bd}$$

$\mathcal{E}$ and $\mathcal{B}$ are both symmetric, covariant tensors.

$$\nabla_i \mathcal{E}_{ij} = \nabla_i \mathcal{B}_{ij} = 0$$

Self-duality again!

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Equation of motion:
New Maxwell equation:

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The self-duality at the quantum level is proved in

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We've got an ABL phase!

More about Lifshitz gravity model

In the gapless phase, the boson density-density correlation is again algebraic:

$$\langle \delta n(0)\ \delta n(\vec{r}) \rangle \sim \frac{1}{r^5}$$

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Z. C. Gu and X. G. Wen
gr-qc/0606100

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$$H = \mathcal{E}_{ij}^2 + A_{ij}\mathcal{B}_{ij}$$

Z. C. Gu and X. G. Wen
gr-qc/0606100

Not completely gauge invariant, there is a boundary term after gauge transformation, therefore can NOT emerge with perturbation theory of local boson hoppings.

More about Lifshitz gravity model

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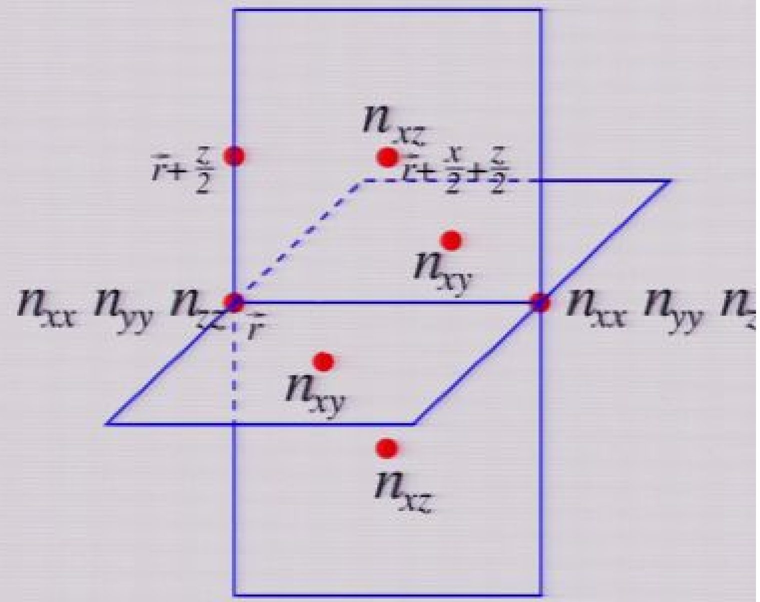
Cenke Xu,
cond-mat/0602443, Phys. Rev. B, 74, 224433

z=3 Lifshitz gravity model

Modification, turn on another large term:

$$H_{v'} = \sum_{\vec{r}} V'(n_{xx,\vec{r}} + n_{yy,\vec{r}} + n_{zz,\vec{r}} - 6\bar{n})^2$$



Constraint: $\nabla_i \mathcal{E}_{ij} = 0, \ \sum_i \mathcal{E}_{ii} = 0,$

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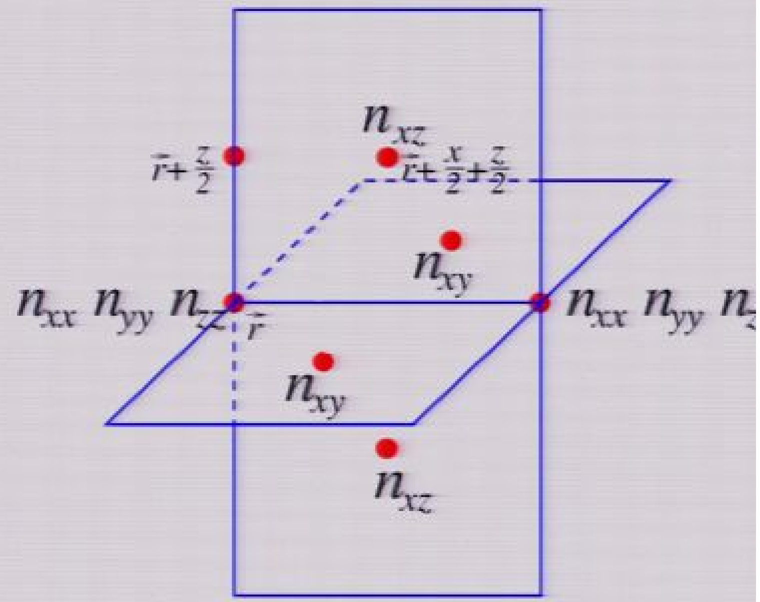

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$$H = \sum_{\vec{r}} \sum_{i,j} u\mathcal{E}^2_{ij,\vec{r}} - \sum_i t_3 \cos(\mathcal{C}_{ii})_{\vec{r}} - \sum_{i \neq j} t_4 \cos(\mathcal{C}_{ij})_{\vec{r}},$$

$$\mathcal{C}_{ij} = \epsilon_{ijk} \nabla_k (R_{jl} - \frac{1}{4} R\delta_{jl})$$

z=3 Lifshitz gravity model and its self-duality

$$H \sim \mathcal{E}^2 + \mathcal{C}^2 \qquad\qquad L \sim (\dot{A})^2 - \mathcal{C}^2$$

$\mathcal{C}$ has three spatial derivatives, the gapless modes have z=3 dispersion.

$\mathcal{E}$ and $\mathcal{C}$ are both symmetric, covariant, and **traceless** tensors

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Cenke Xu, Petr Horava, arXiv: 1003.0009

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The low energy field theory is identical to the Lifshitz gravity theory, **after linearization**.

Petr Horava, Phys. Rev. D, 79, 084008

Phase transitions between Lifshitz gravity phases

Recall, to obtain the z=3 phase, we turned on

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The transition is a "Higgs transition", breaks part of the gauge symmetry, and make the dispersion more "stiff".

Cenke Xu, Petr Horava, arXiv:1003.0009

More about Lifshitz gravity models

Comment 1: nonrelativistic dispersion

In condensed matter system, although Lorentz invariance is absent, most of gapless excitations (Goldstone modes) are linear, as long as time reversal is unbroken.

$$L = (\partial_\tau h)^2 + r(\nabla h)^2 + (\nabla^2 h)^2 + \cdots$$

The critical point with r=0 is called the Lifshitz transition, transition between uniform and modulated phases.

In our case, the emergent local gauge invariance guarantees the nonlinear dispersion (without long range interaction):

$$A_{ij} \to A_{ij} + \nabla_i f_j + \nabla_j f_i \qquad A_{ij} \to A_{ij} + \delta_{ij}\varphi,$$

More about Lifshitz gravity models

Comment 2: can we get interacting theory?

Key: We need emergent real diffeomorphism invariance instead of linearized graviton gauge invariance on the lattice.

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Comment 3: how to gap the photon/graviton?

Confine-deconfine transition:

$$L = |(\partial_\mu - ih_\mu)\psi^{(m)}|^2 + r|\psi^{(m)}|^2 + \cdots$$

Condense monopole, gap out the spectrum, charged matter are confined.

z=3 Lifshitz gravity and RG flow

Petr Horava, Phys. Rev. D, 79, 084008

$$L \sim \dot{A}^2 + \mathcal{C}^2 + R$$

$\mathcal{C}$ has higher derivatives than R, so at low energy the theory flows back to the Hilbert-Einstein action, while at high energy it becomes the Lifshitz gravity.

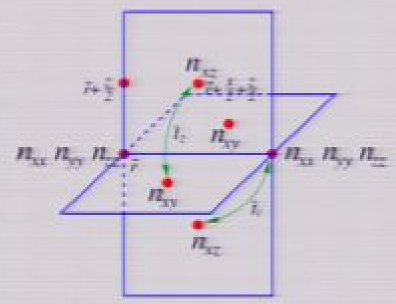
Einstein gravity

$L = R$

Lifshitz gravity

$H = \mathcal{E}^2 + \mathcal{C}^2$

Lattice model With U(1) symmetry



Increasing energy scale

Summary of Lifshitz Gravity ABL

Summary and outlook:

What we did:

Construct lattice model that leads to gapless excitations without breaking any symmetry, at low energy the theory is described by self-dual gauge theory.

What we will try:

1, to obtain emergent diffeomorphism invariance.
2, to construct ABL phase with strongly coupled stable fixed point.
3, liquid phase in real condensed matter system?

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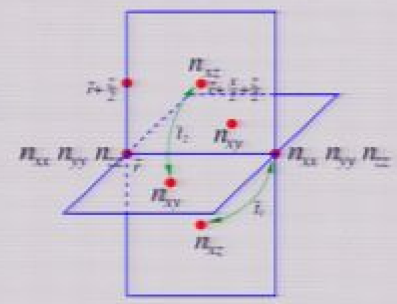

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