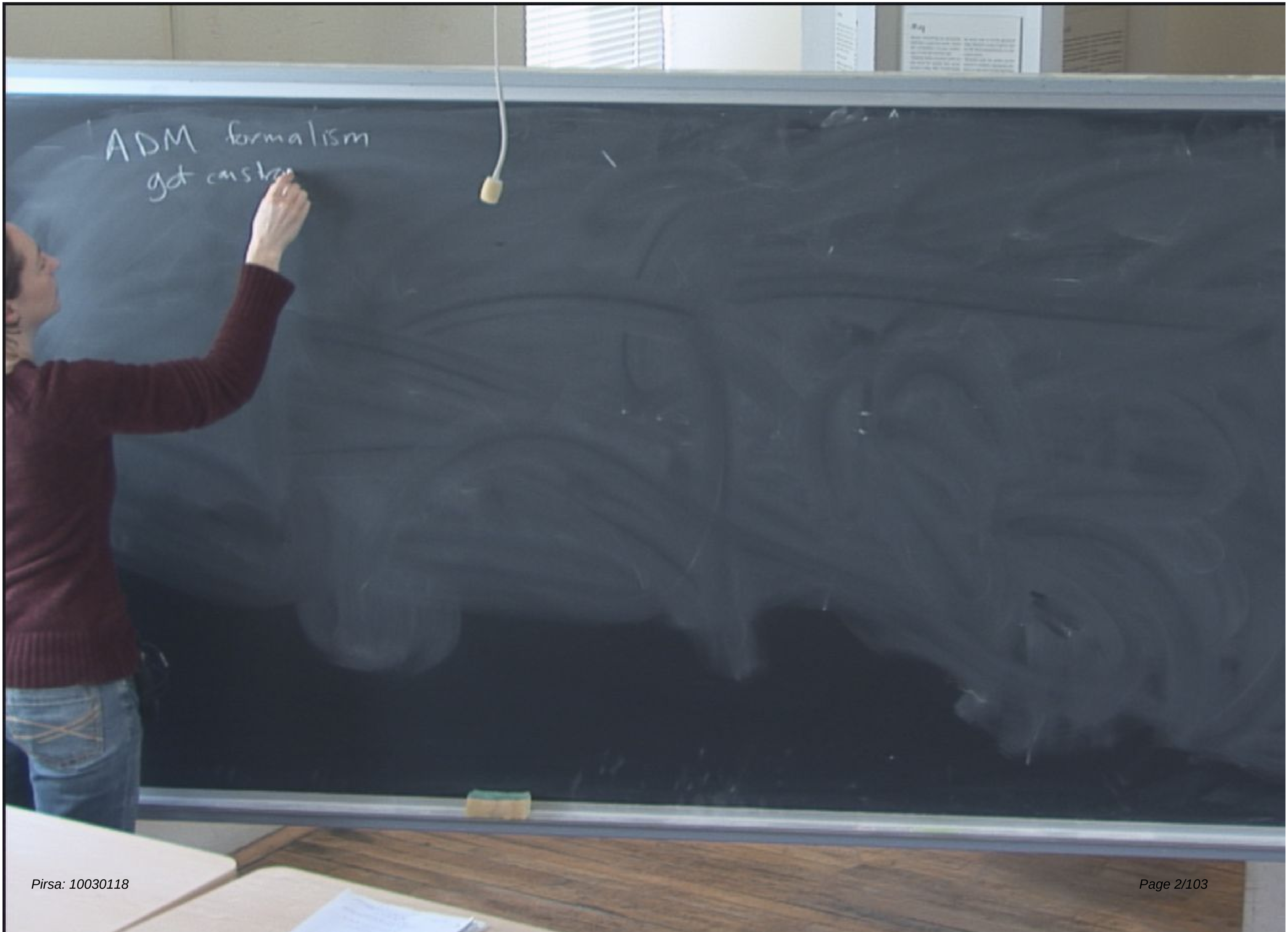
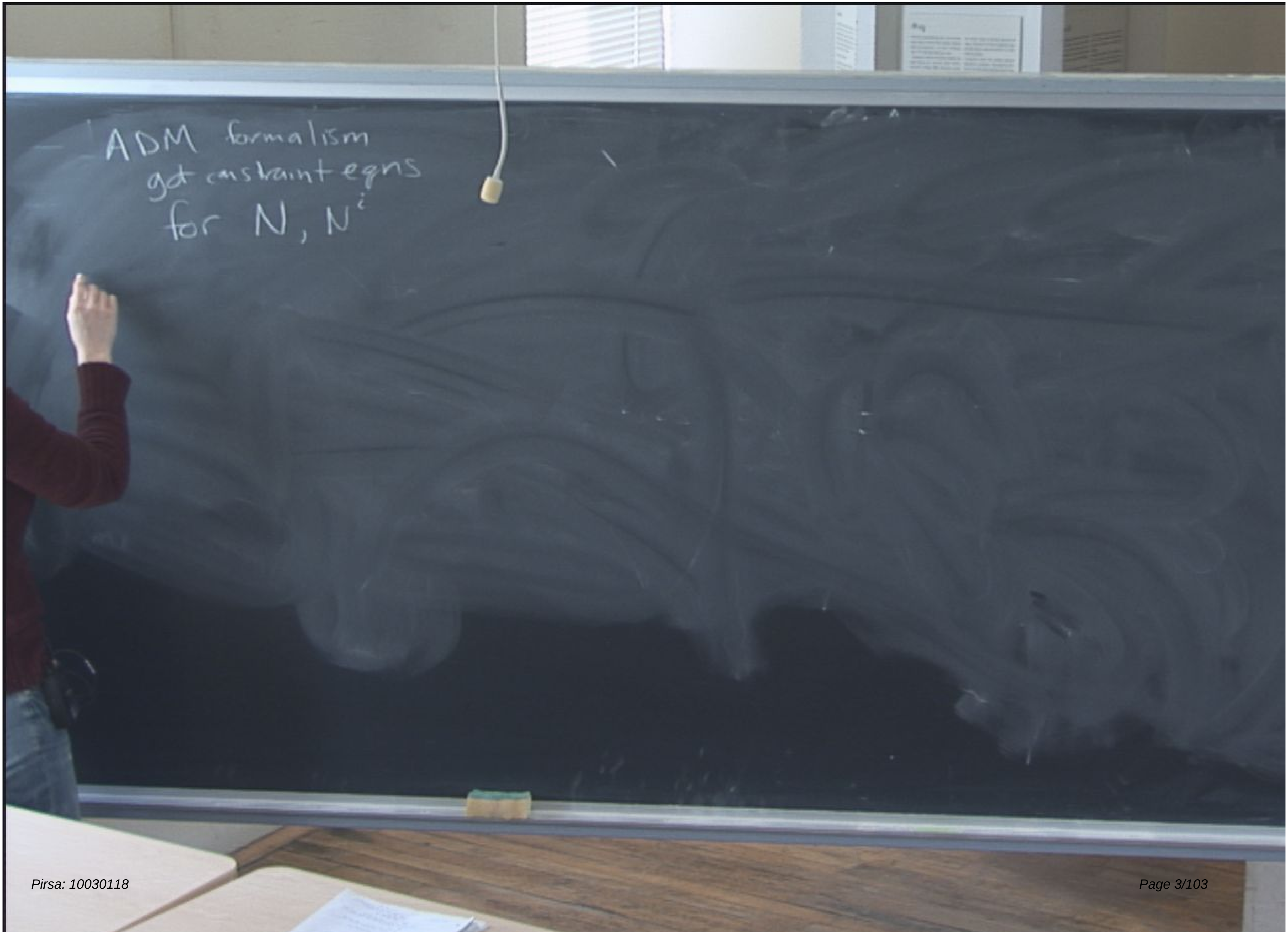
Title: Explorations in Cosmology (PHYS 649) - Lecture 14

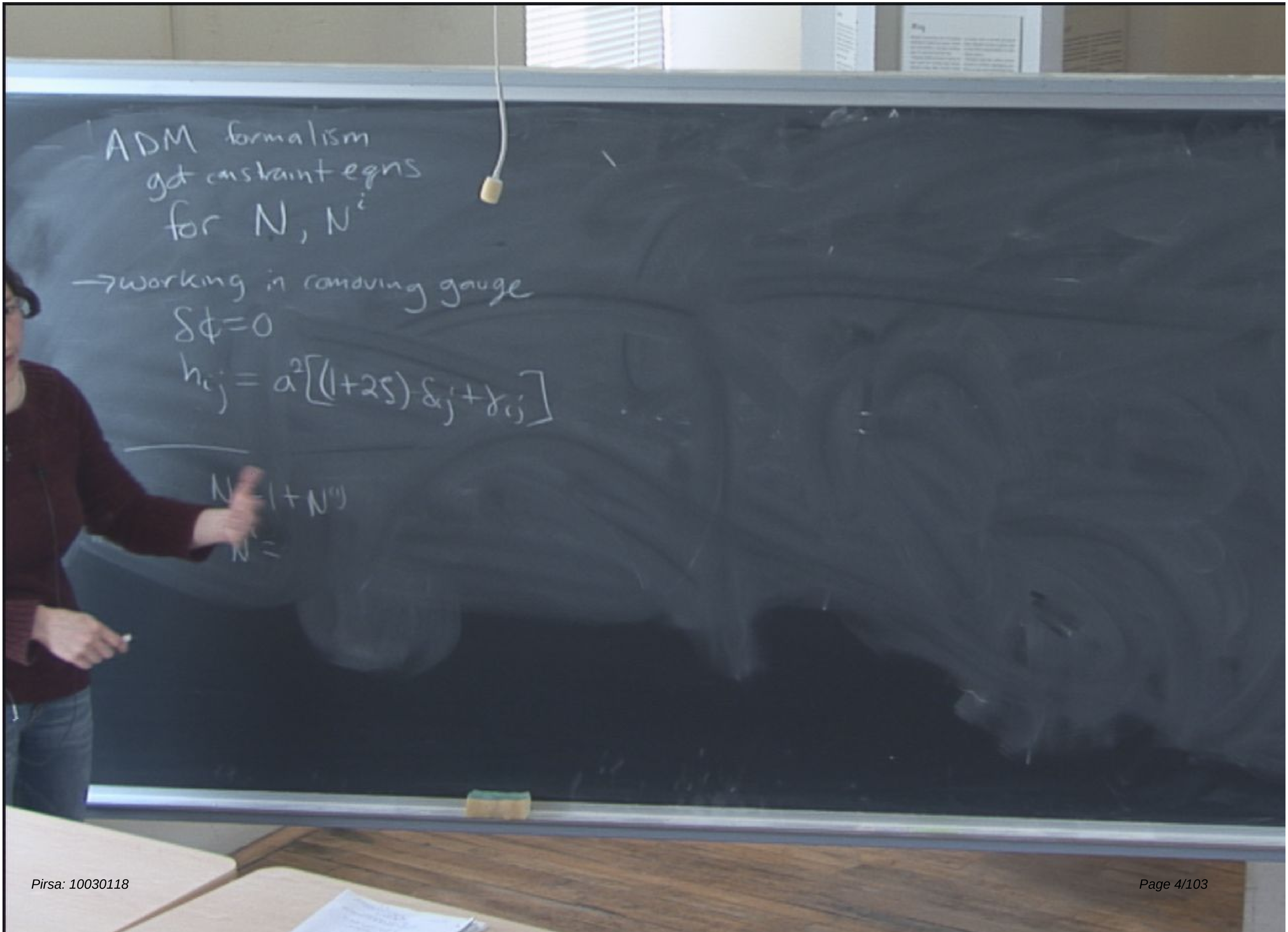
Date: Mar 31, 2010 03:30 PM

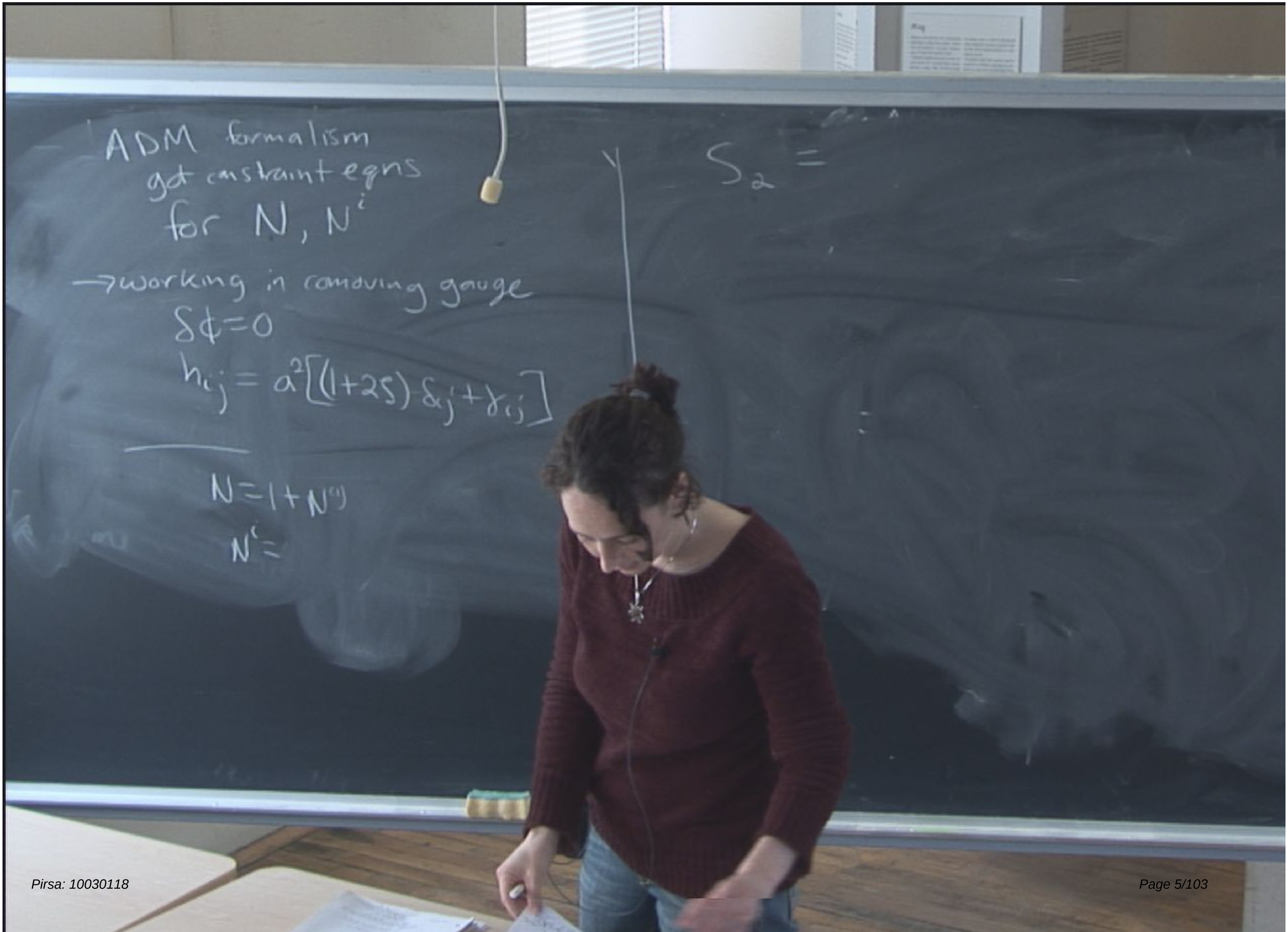
URL: http://pirsa.org/10030118

Abstract:


ADM formalism
get constr


ADM formalism
get constraint eqns
for N, N^i


ADM formalism
get constraint eqns
for N, N^i
→ working in comoving gauge
δφ=0
h_ij = a^2[(1+2ζ)δ_ij + γ_ij]
N = 1+N^(1)
N^i =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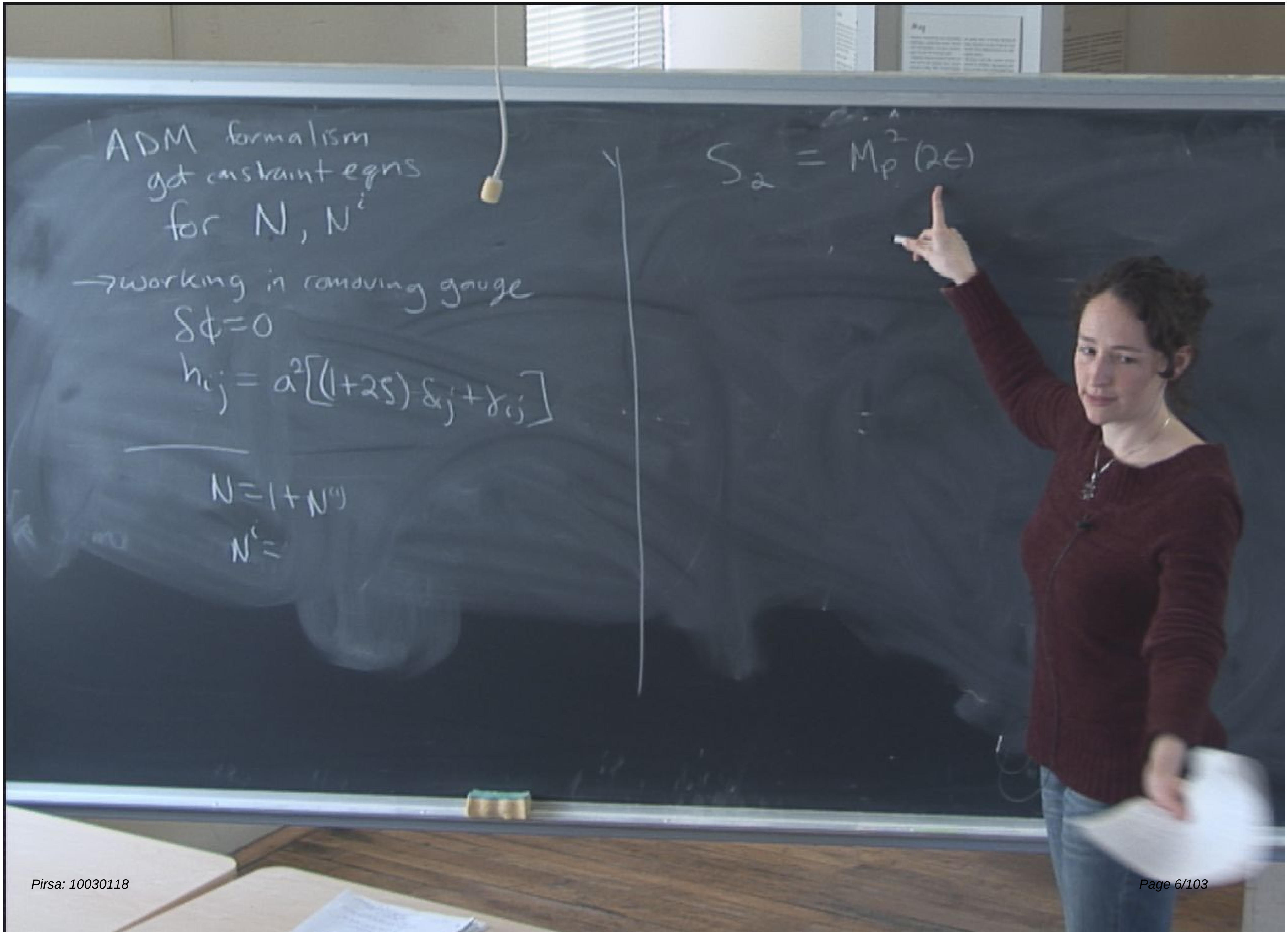
S_2 =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get constraint eqns
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$\delta\phi = 0$
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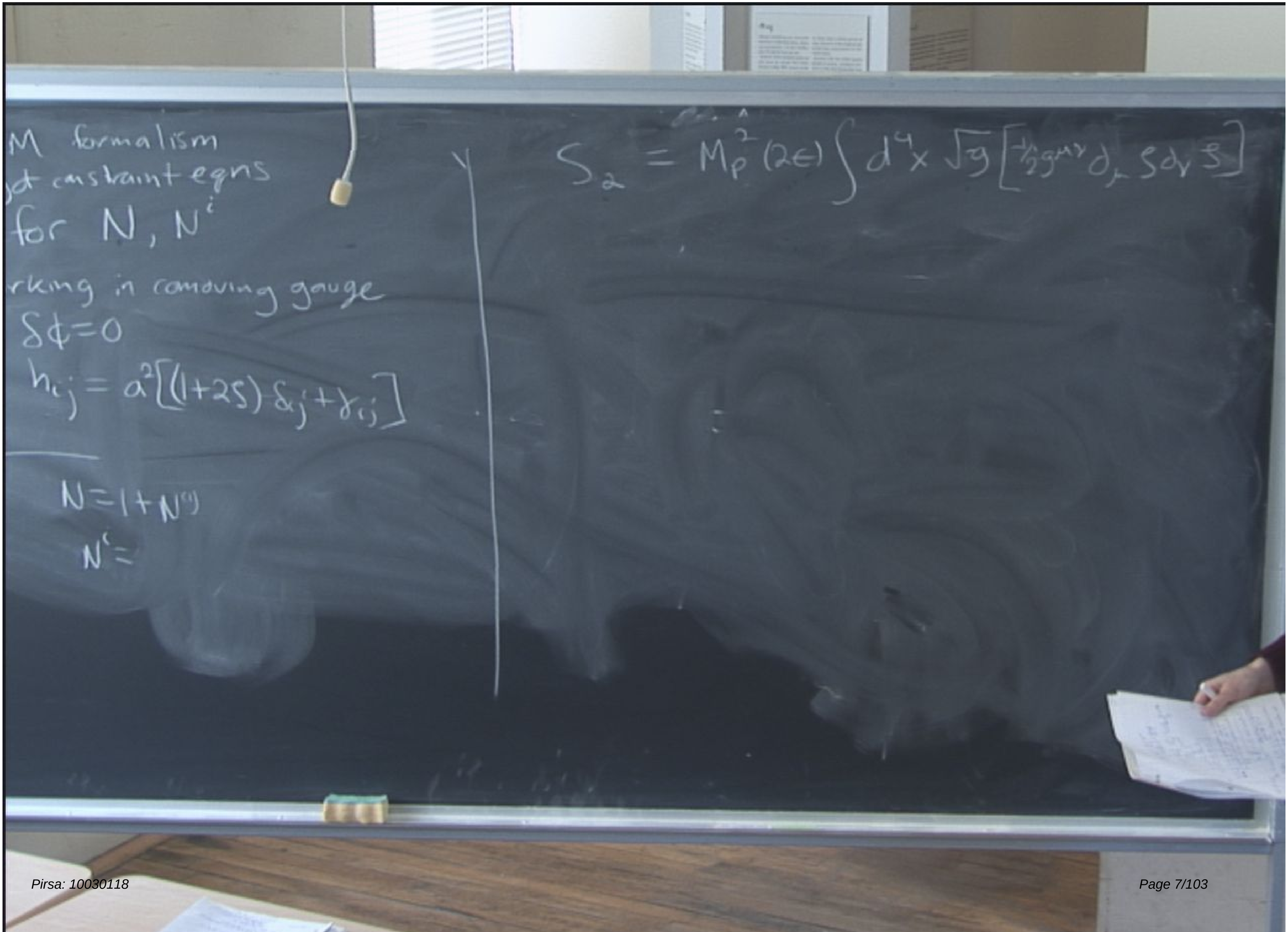
$S_2 = M_P^2 (2\epsilon)$



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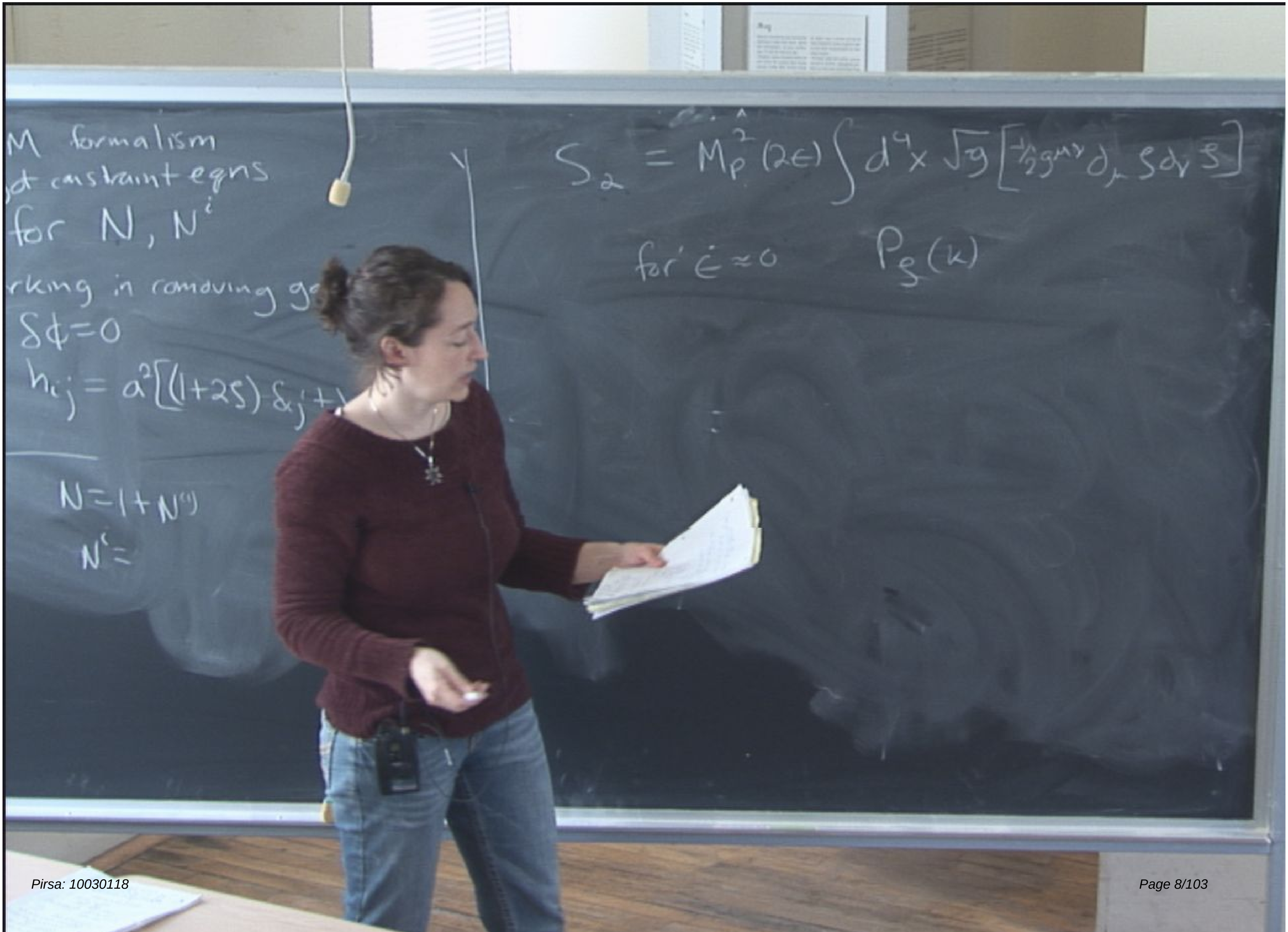
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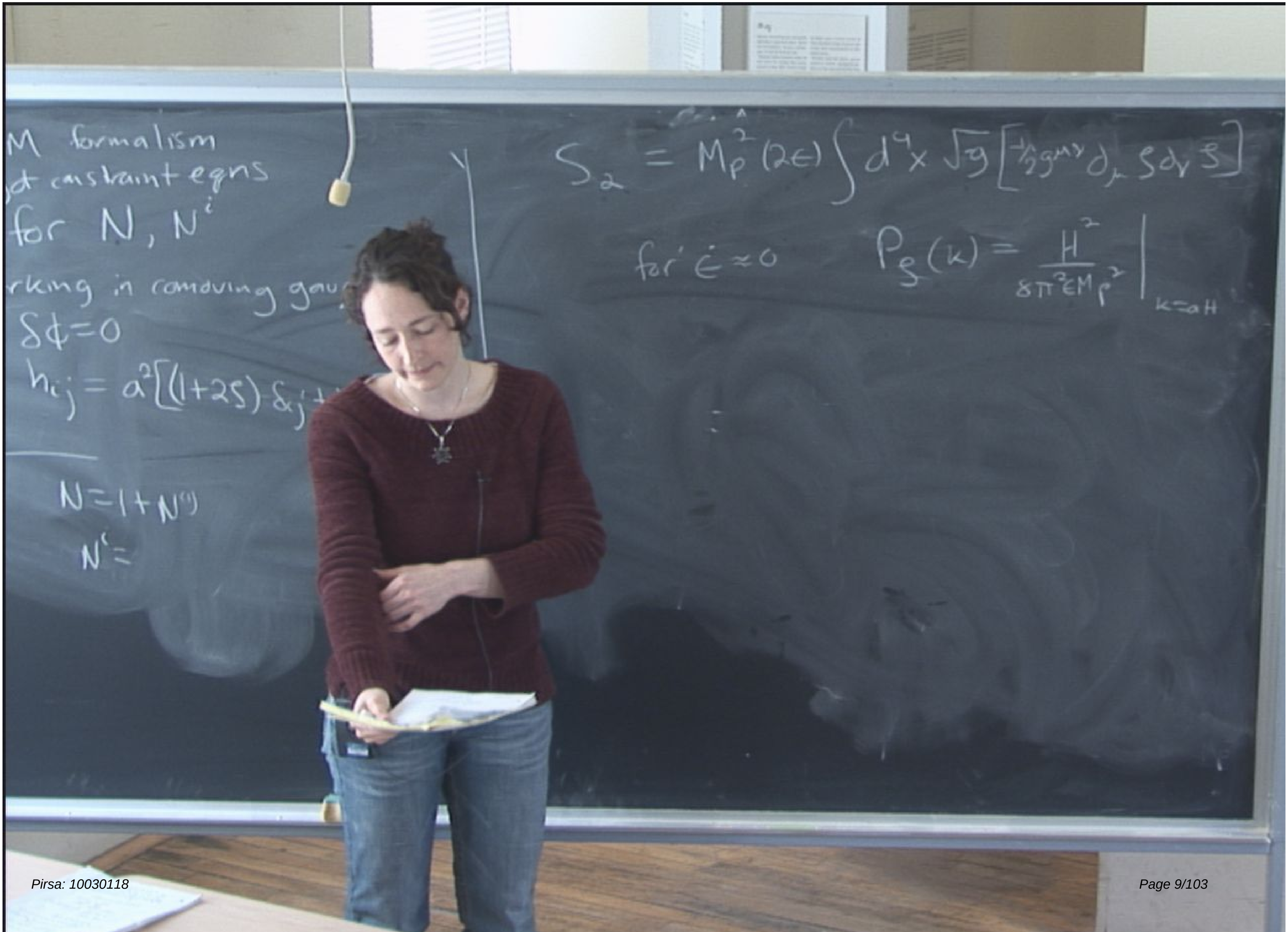
for ε ≈ 0
P_ζ(k)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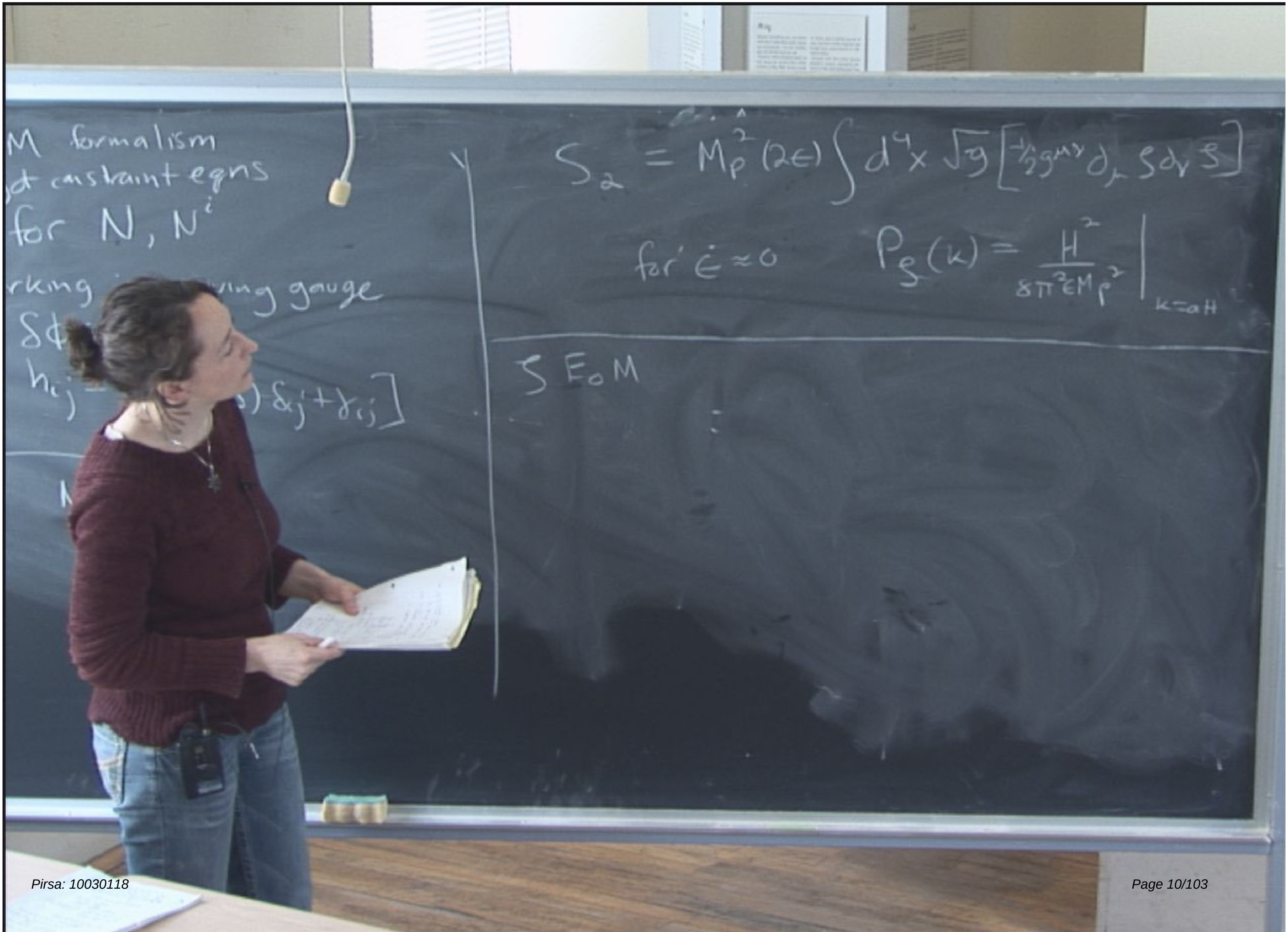
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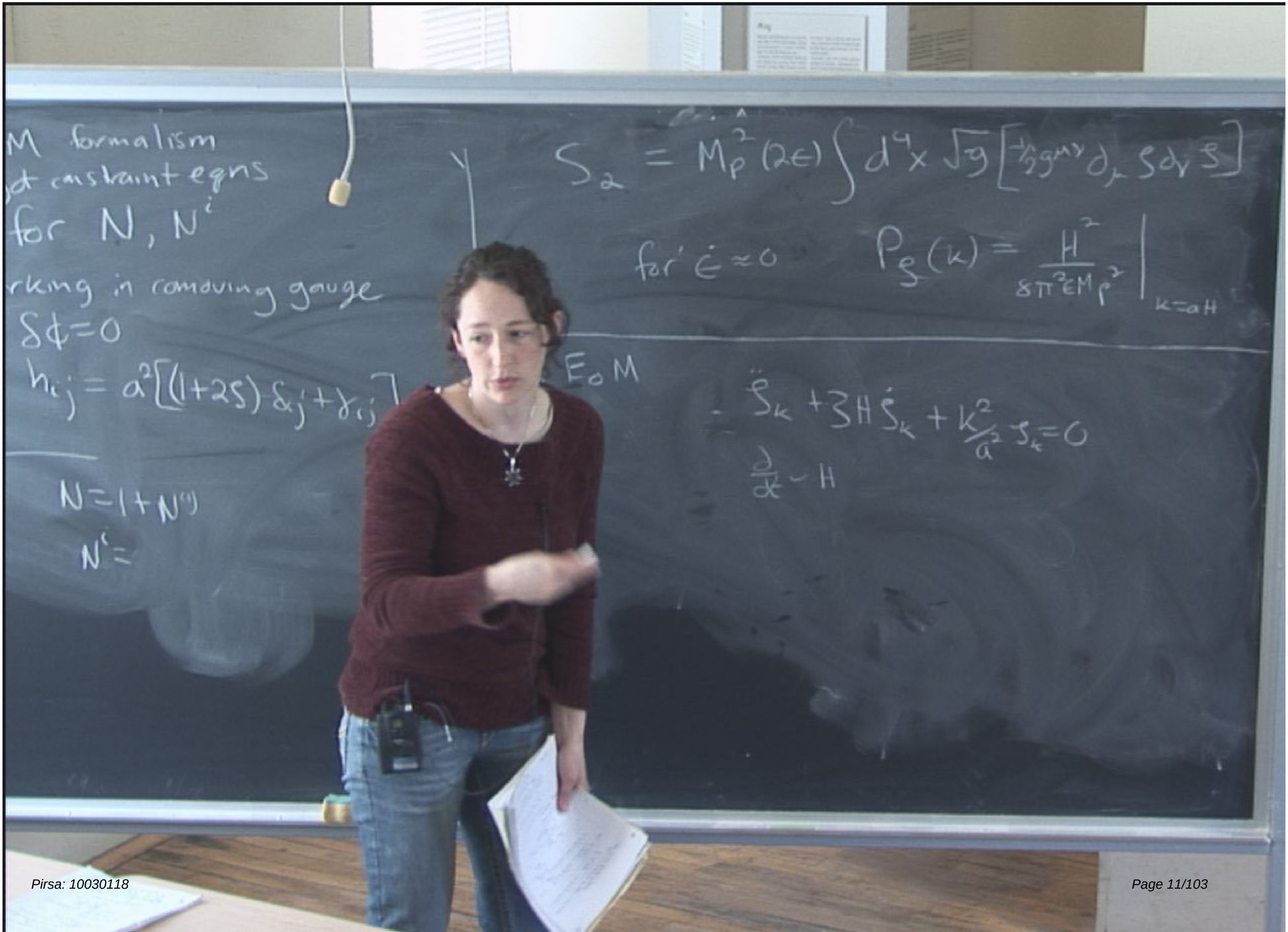
ζ EoM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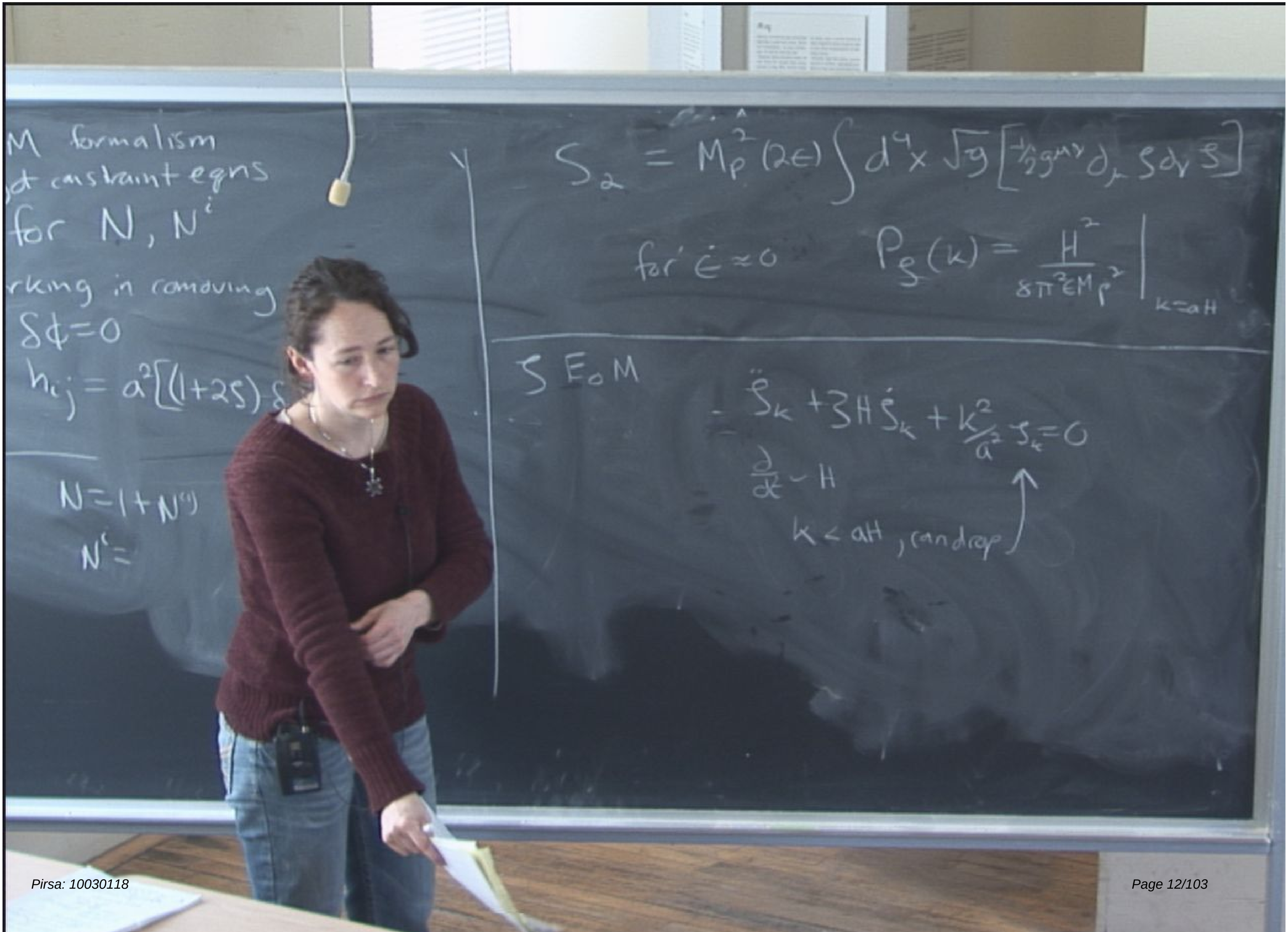
ζ̈_k + 3H ζ̇_k + k²/a² ζ_k = 0
d/dt ~ H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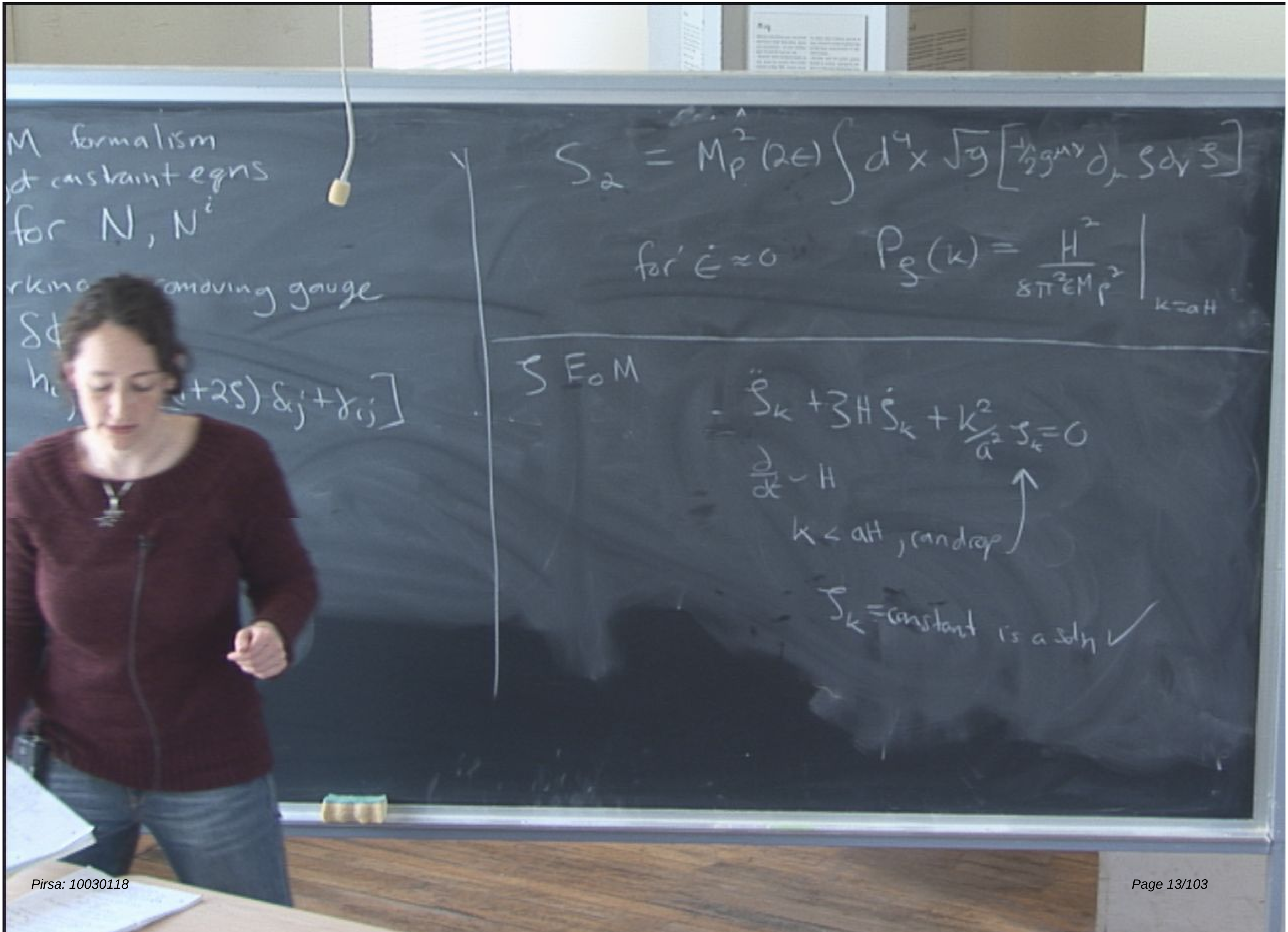
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$S_2 = M_p^2 (2\epsilon) \int d^4x \sqrt{g} [-\frac{1}{2} g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta]$
for $\dot{\epsilon} \approx 0$ $P_\zeta(k) = \frac{H^2}{8\pi^2 \epsilon M_p^2} \Big|_{k=aH}$
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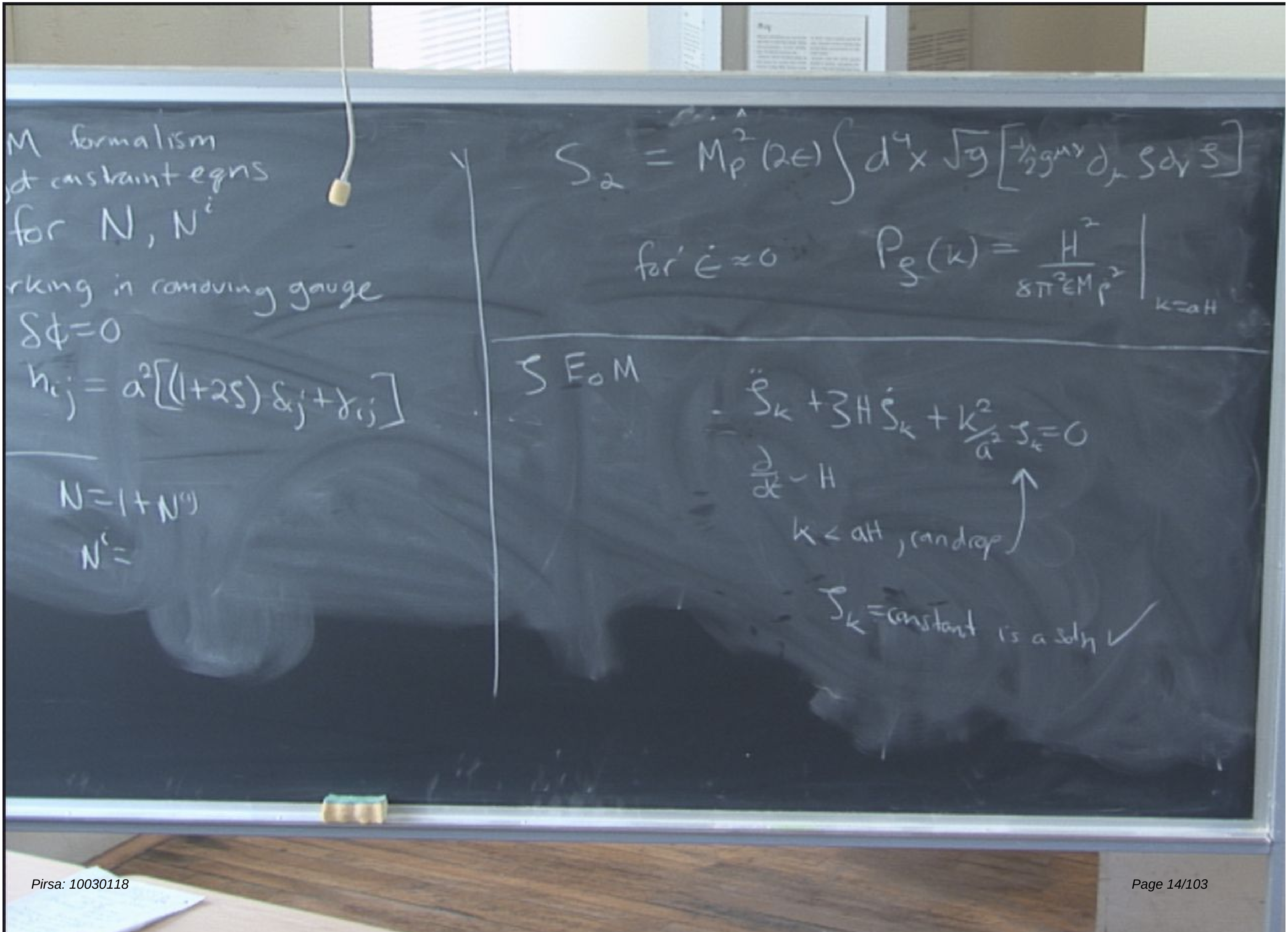
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$
$\frac{d}{dt} \sim H$
k < aH, can drop
ζ_k = constant is a soln ✓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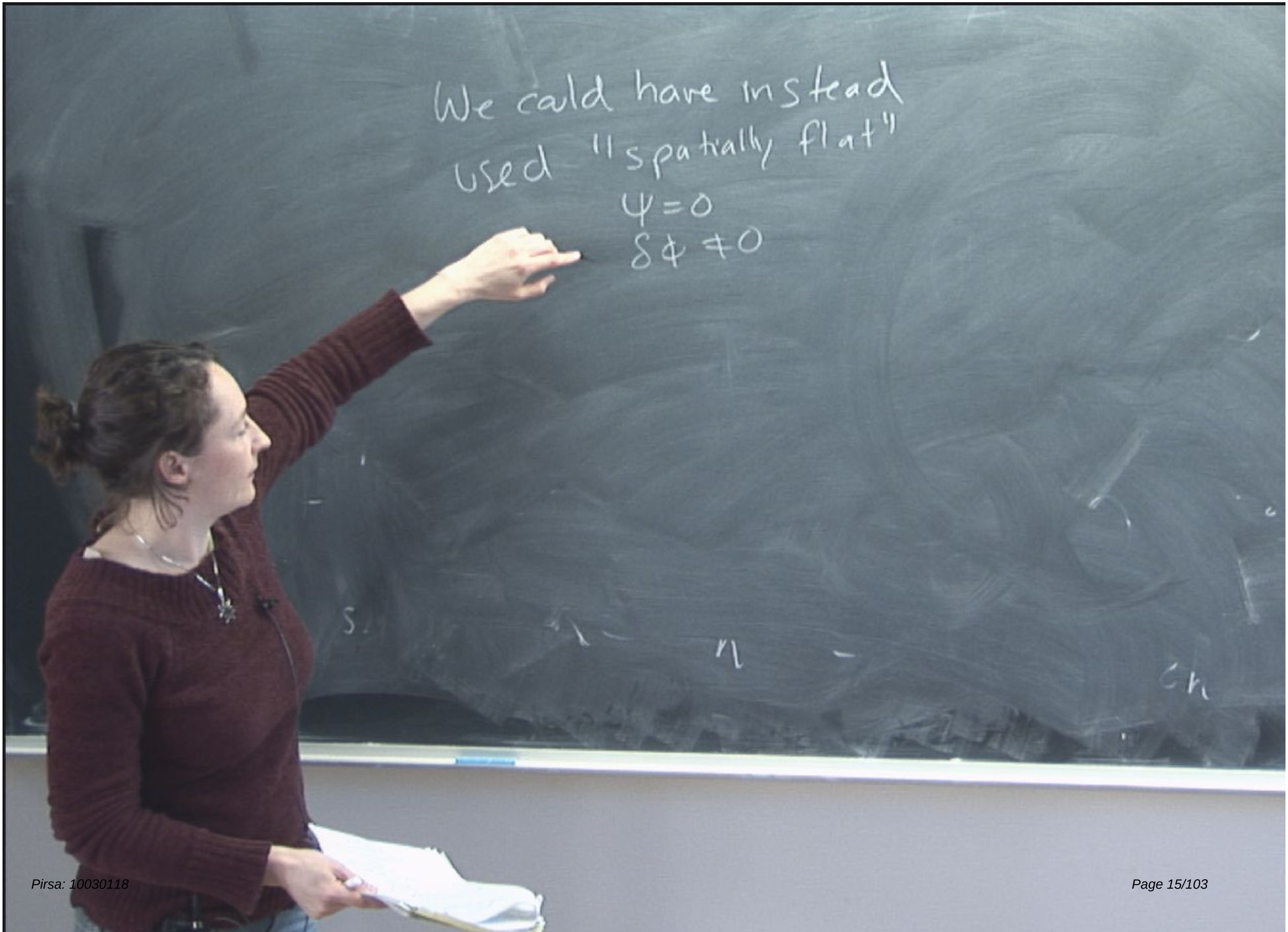
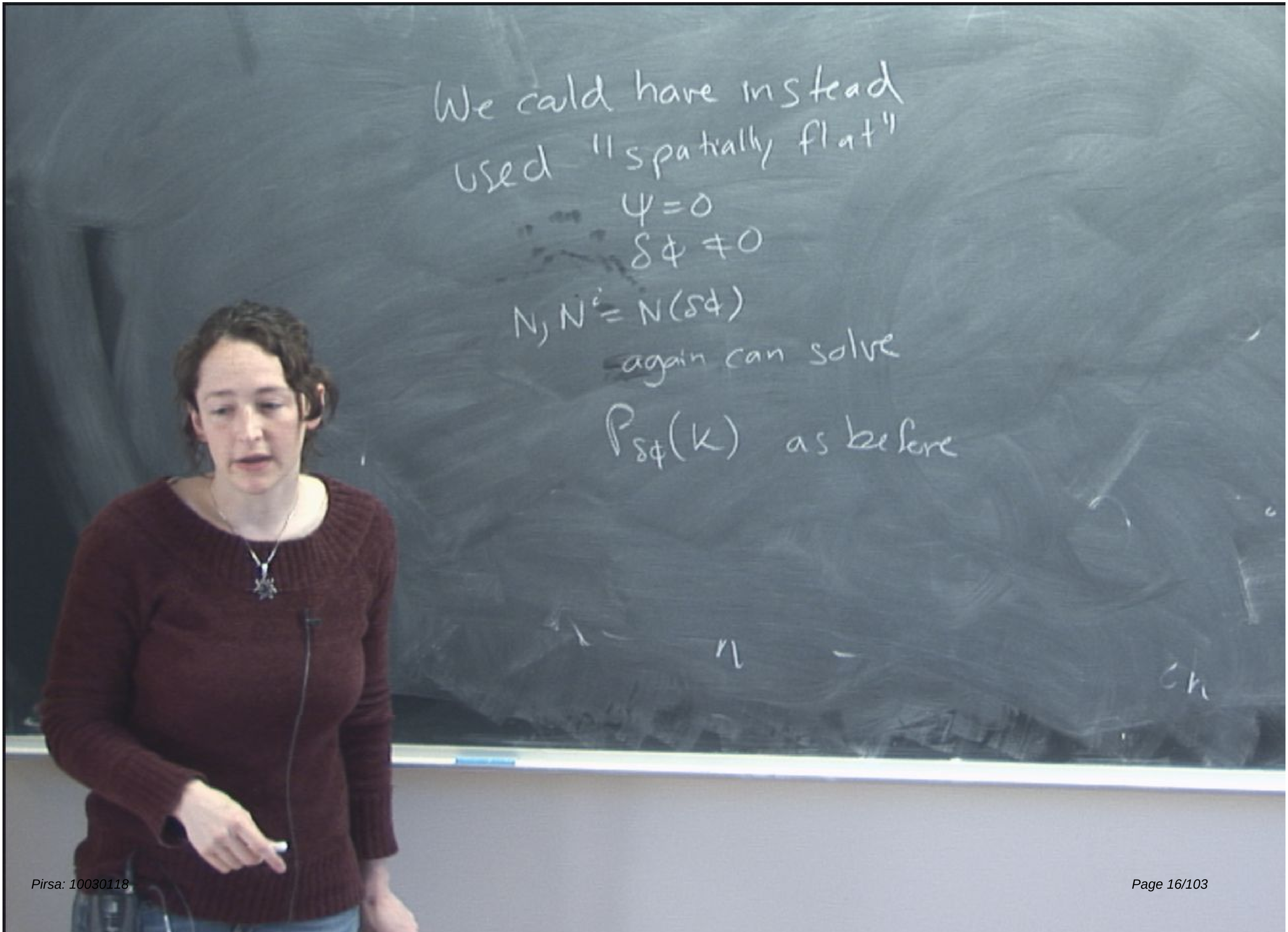

We could have instead
used "spatially flat"
ψ = 0
δφ ≠ 0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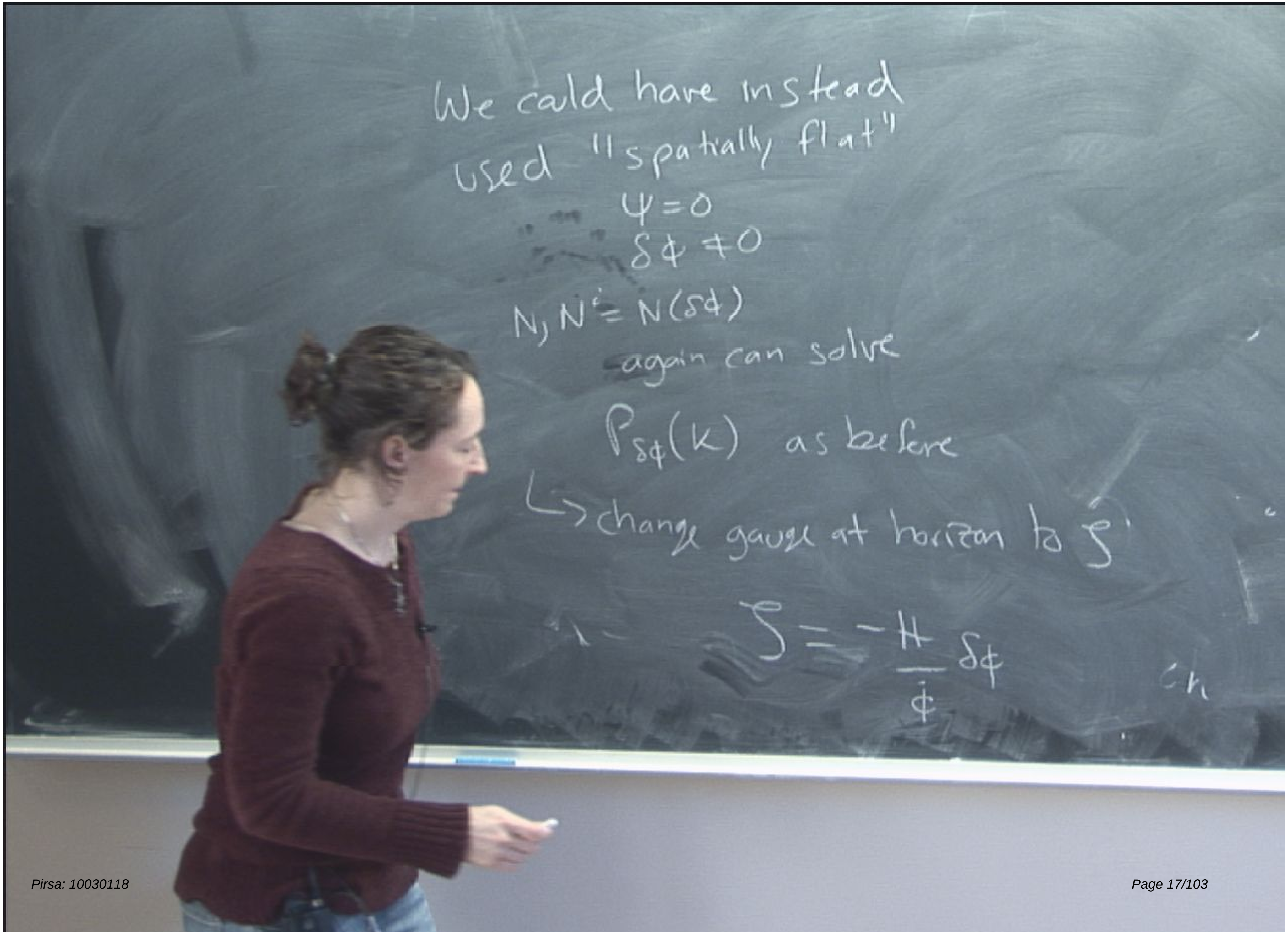
N, N^i = N(δφ)
again can solve
P_δφ(k) as before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$\psi = 0$
$\delta\phi \neq 0$
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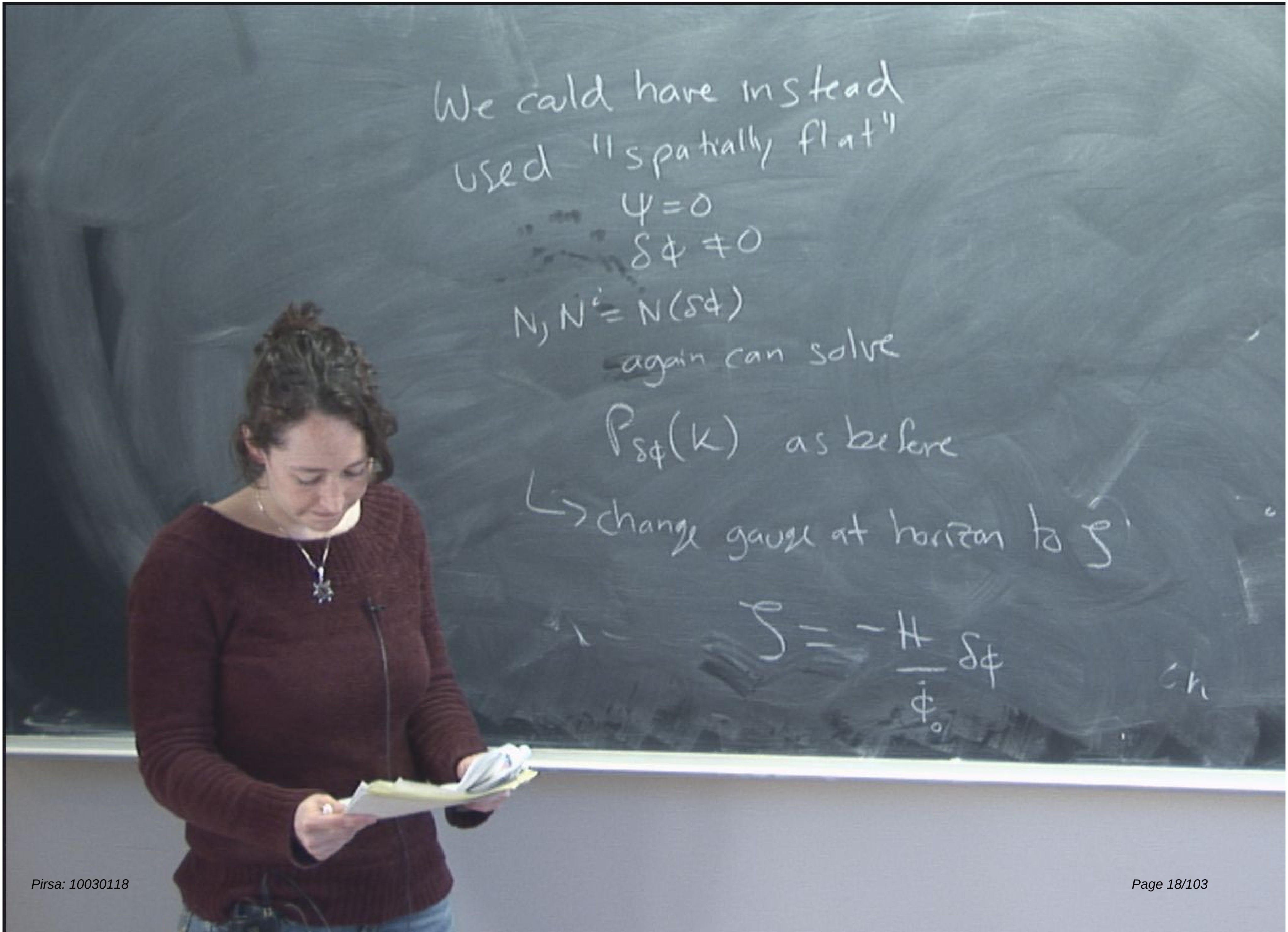
$P_{\delta\phi}(k)$ as before
↳ change gauge at horizon to ζ
$\zeta = -\frac{H}{\dot\phi}\delta\phi$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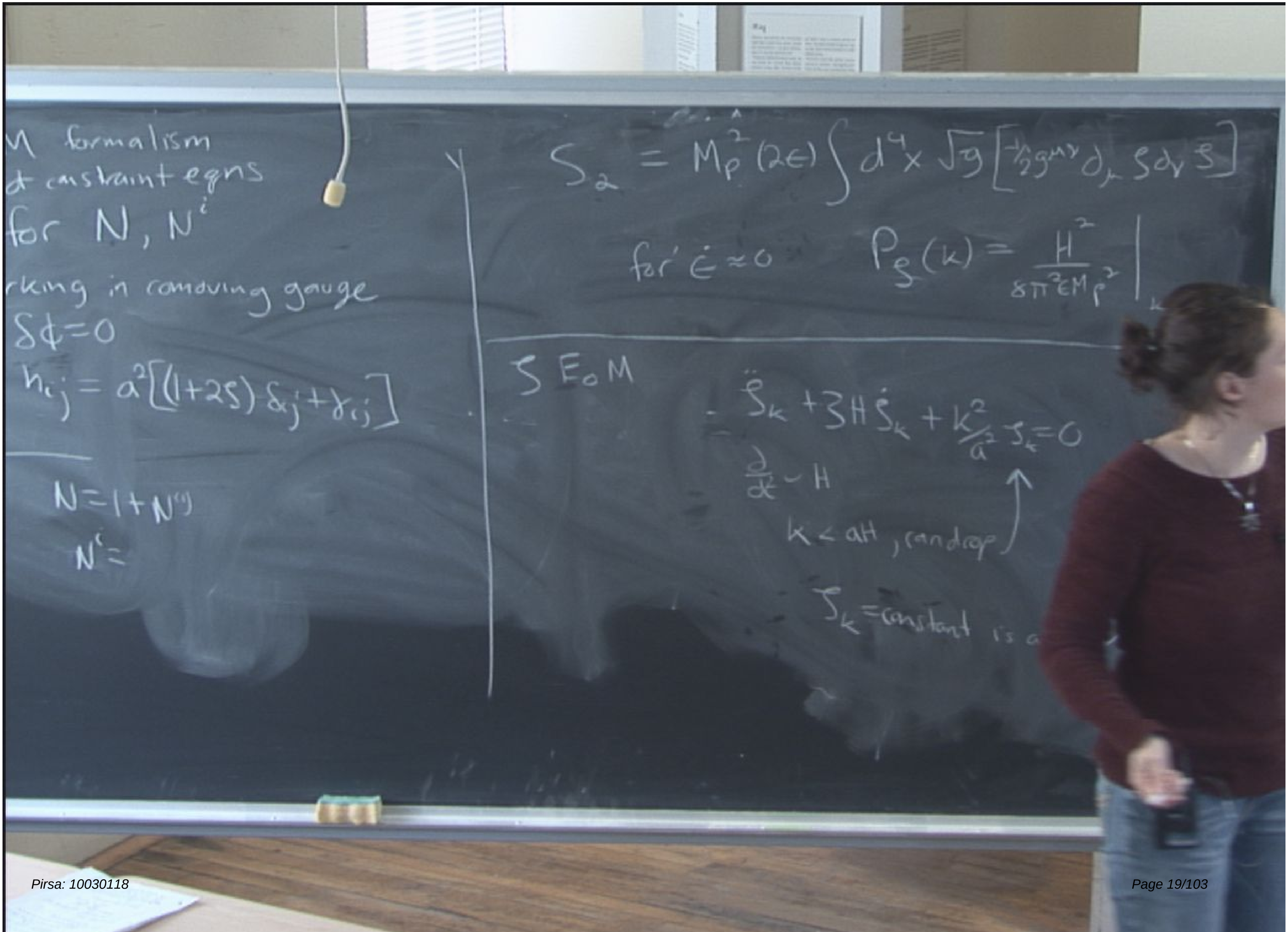
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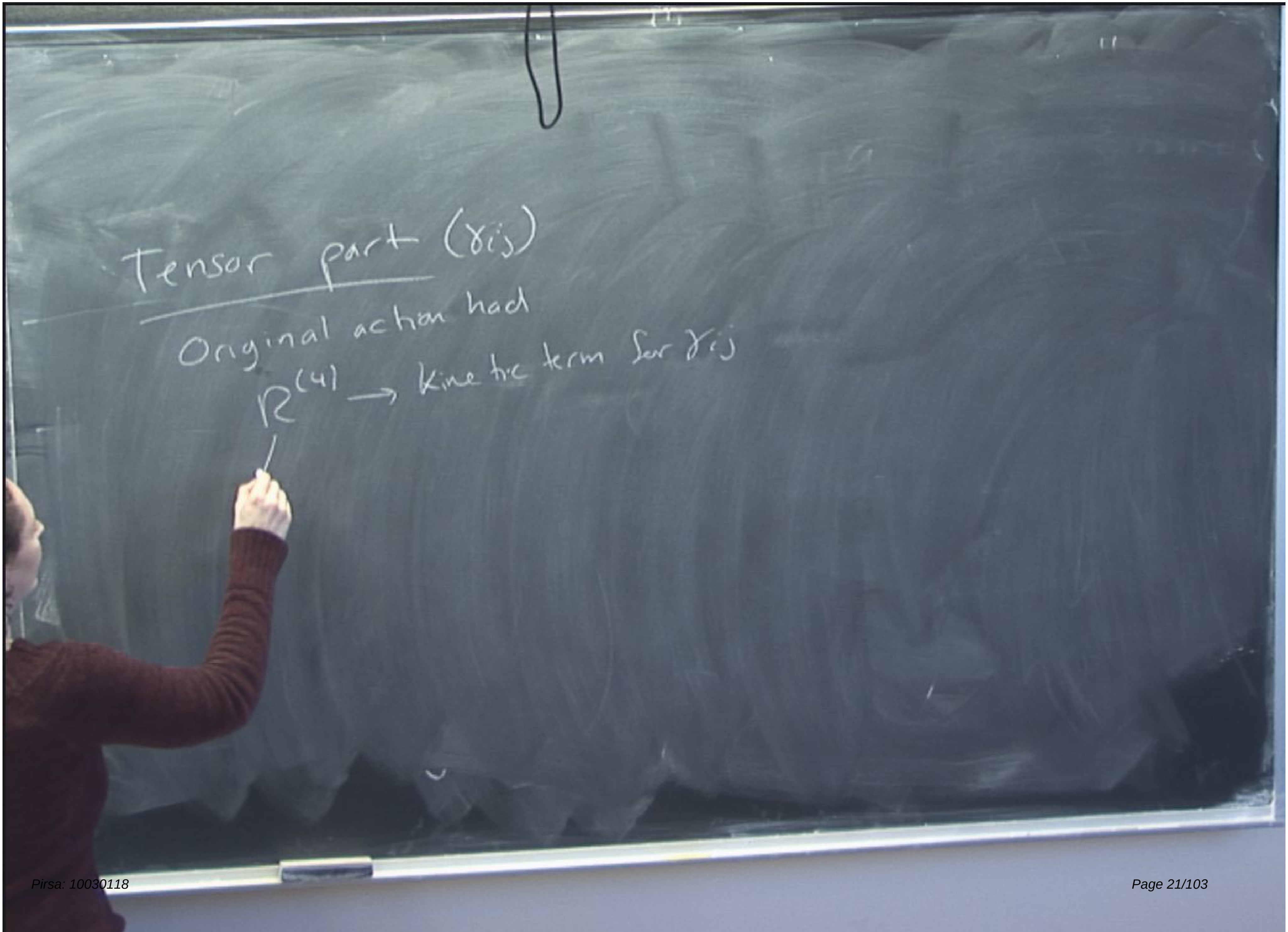
Tensor part (γ_{ij})
Original action had
$R^{(4)}$ → kinetic term for γ_{ij}

Tensor part (γ_{ij})

Original action had

$R^{(4)} \longrightarrow$ kinetic term for γ_{ij}

$\downarrow$

$R^{(3)} + \frac{1}{N}\left(E_{ij}E^{ij} - E^2\right)$

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$R^{(3)} + \frac{1}{N}\left(E_{ij}E^{ij} - E^2\right)$

$E_{ij} = \dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i$

$\downarrow$

$\dot{\gamma}_{ij}$

Collect terms

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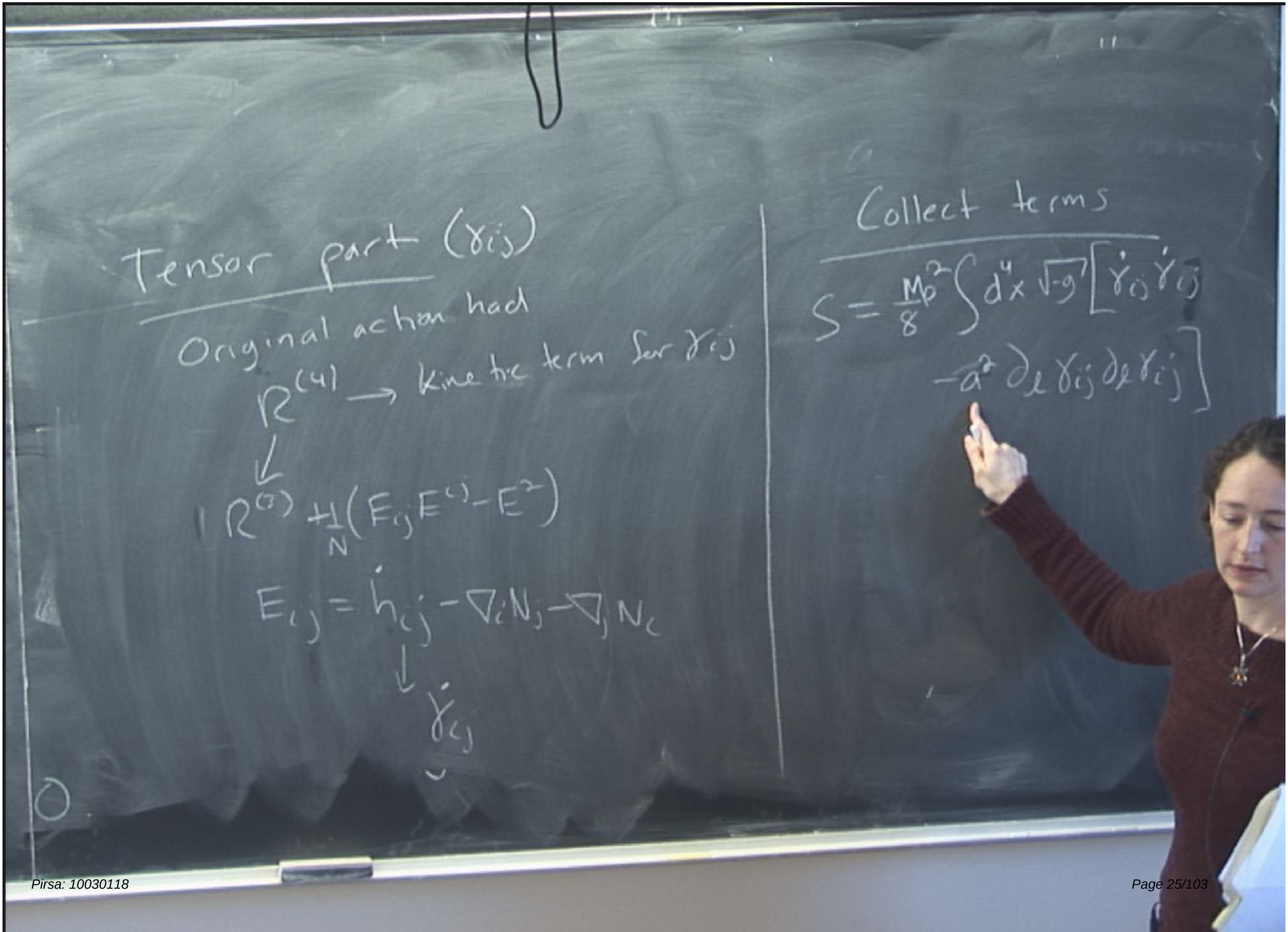
$$S = \frac{M_p^2}{8}\int d^4x \sqrt{-g}\left[\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \partial_l \gamma_{ij}\partial_l \gamma_{ij}\right]$$


Tensor part (γij)
Original action had
R(4) → kinetic term for γij
R(3) + 1/N (Eij E^ij − E²)
Eij = ḣij − ∇i Nj − ∇j Ni
γ̇ij
Collect terms
S = Mp²/8 ∫ d⁴x √−g [γ̇ij γ̇ij − a⁻² ∂l γij ∂l γij]

Tensor part (γ_{ij})

Original action had

$R^{(4)} \longrightarrow$ kinetic term for γ_{ij}

$\downarrow$

$R^{(3)} + \frac{1}{N}\left(E_{ij}E^{ij} - E^2\right)$

$E_{ij} = \dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i$

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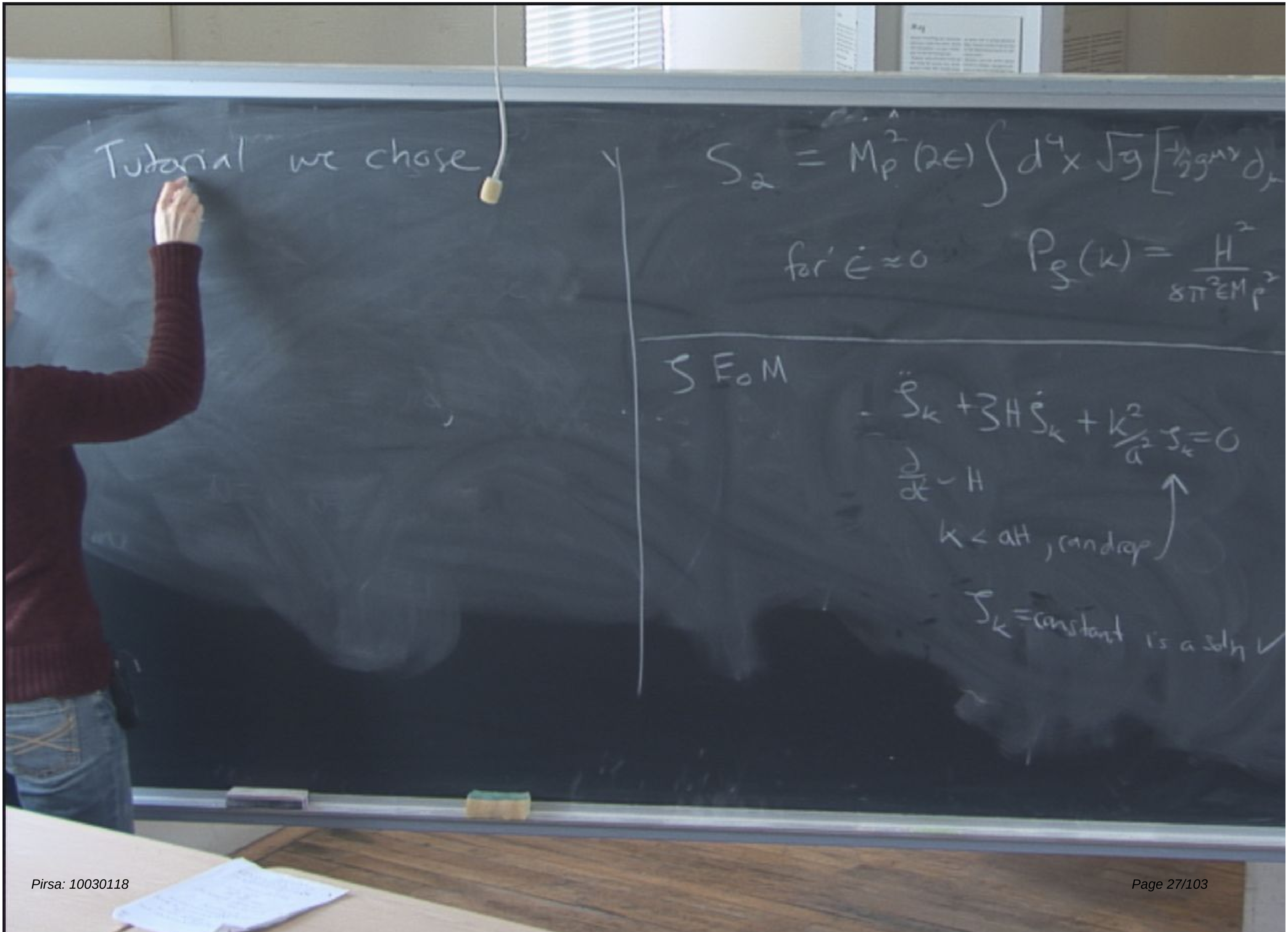

Tutorial we chose
$S_2 = M_p^2 (2\epsilon) \int d^4x \sqrt{g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \right.$
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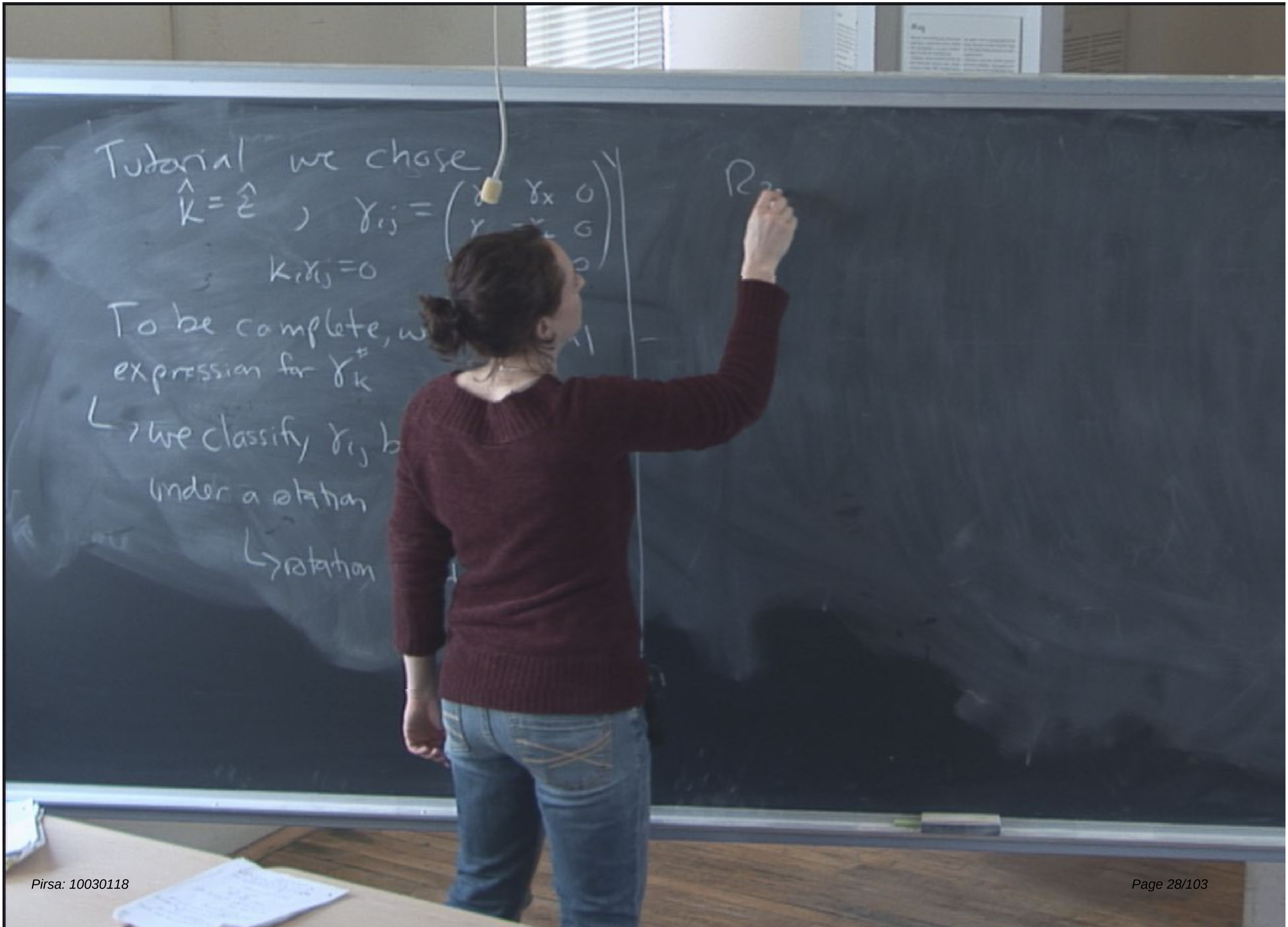
Tutorial we chose
To be complete, w
expression for
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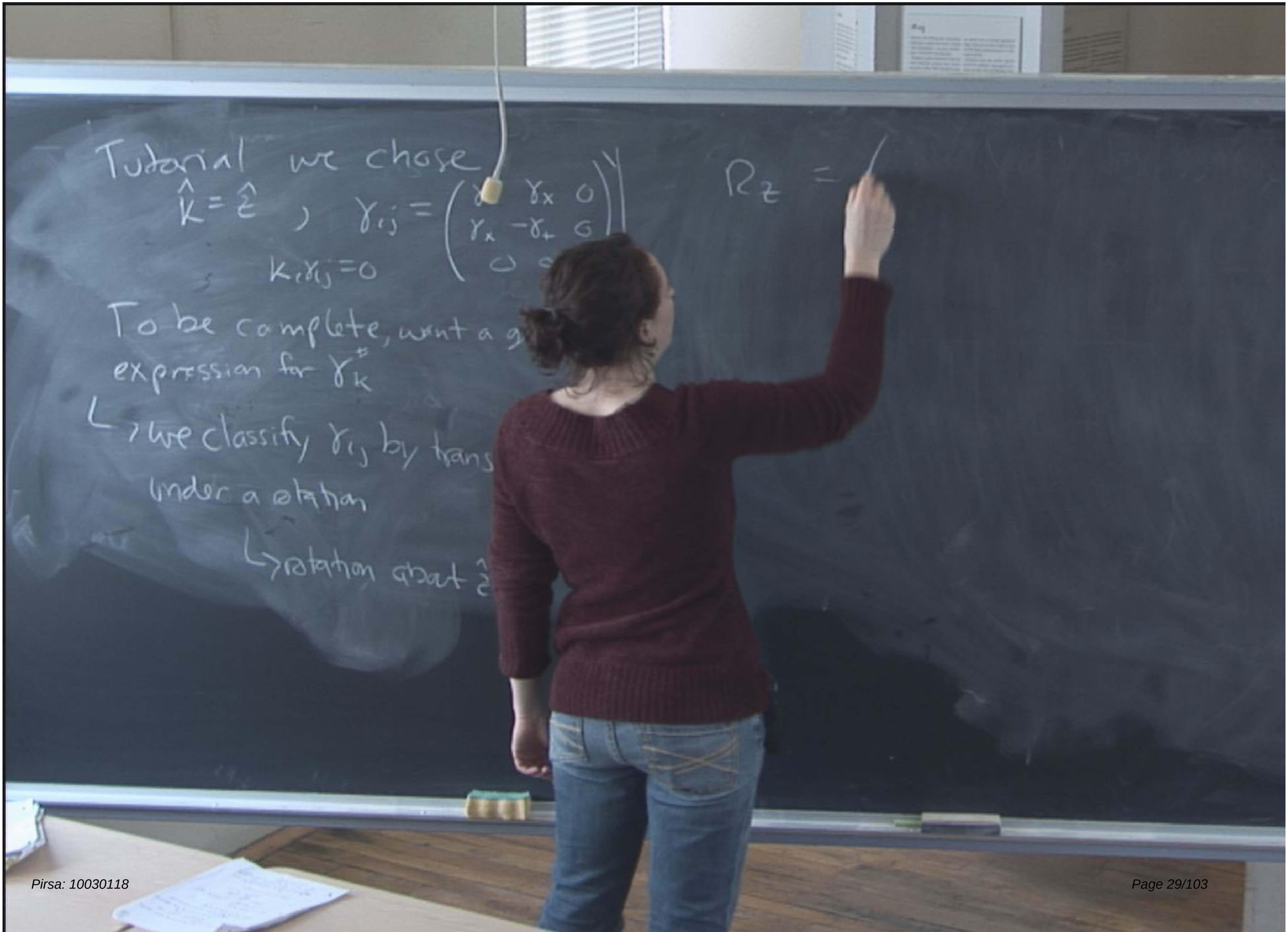
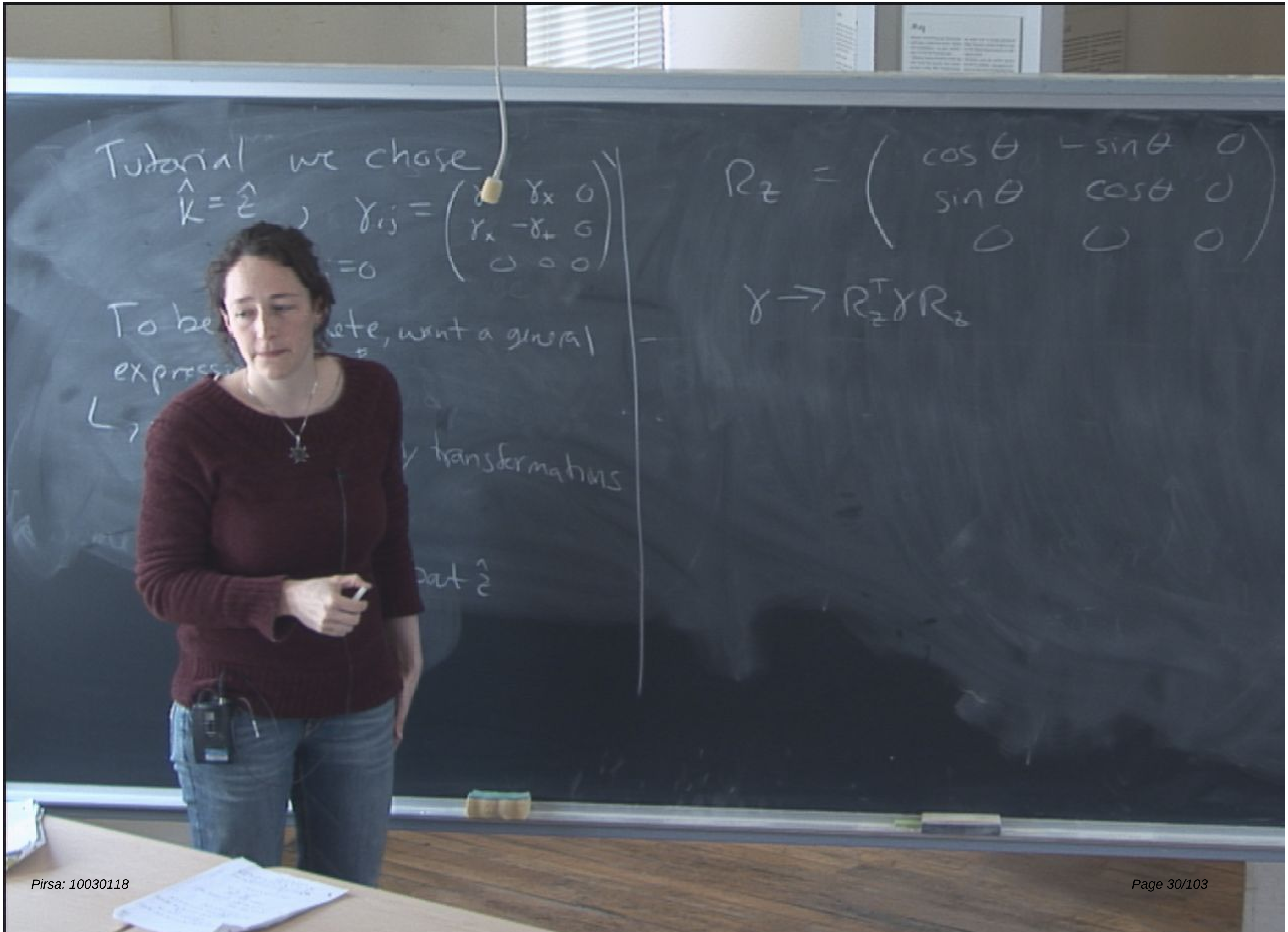
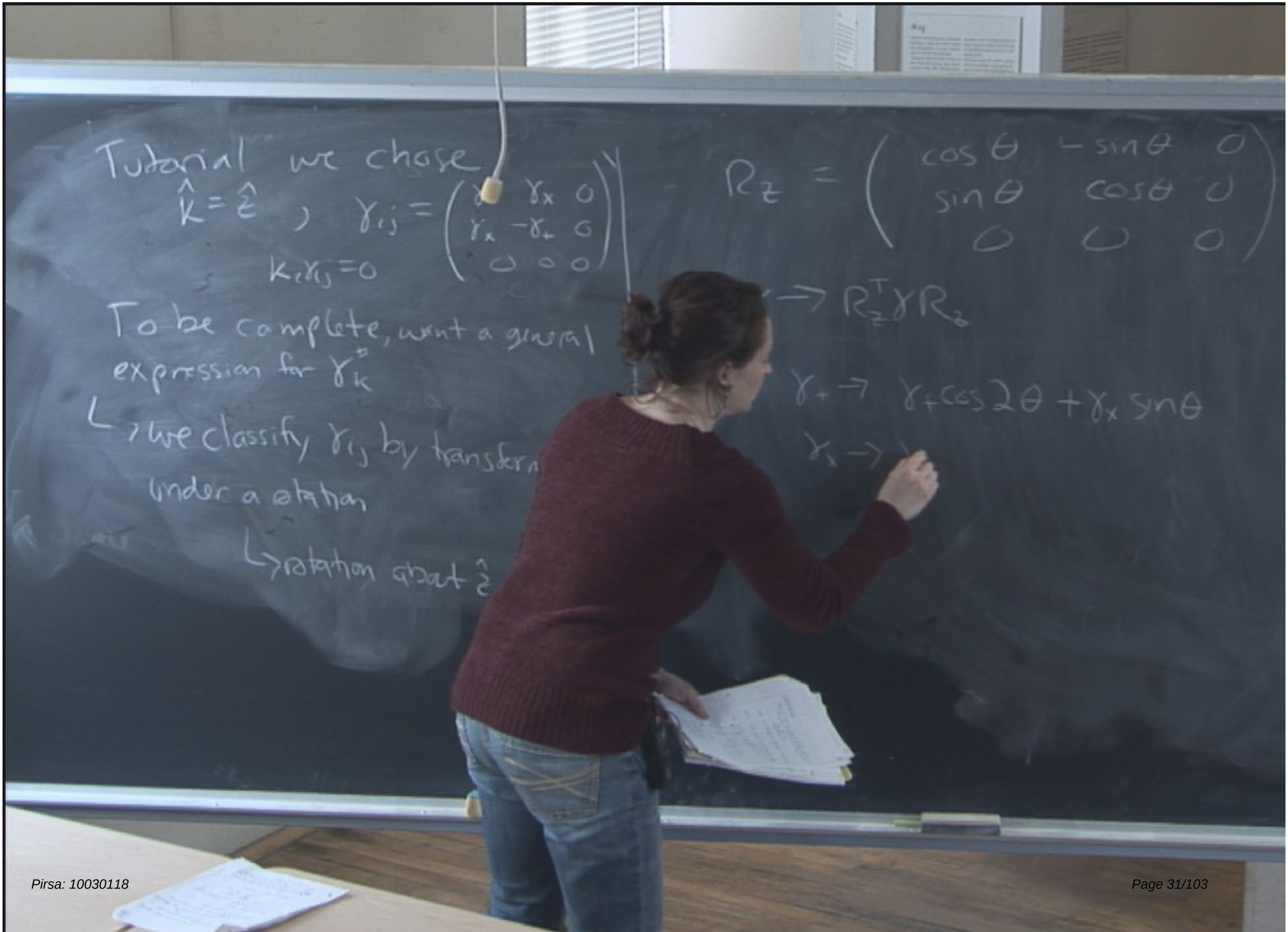

Tutorial we chose
k̂ = ẑ , γij =
kiγij = 0
Rz =
To be complete, want a g
expression for γk
we classify γij by trans
under a otation
rotation about ẑ


Tutorial we chose
k̂ = ẑ , γ_ij = (γ+ γx 0 / γx −γ+ 0 / 0 0 0)
To be ...ete, want a general
expres...
transformations
...out ẑ
R_z = (cos θ −sin θ 0 / sin θ cos θ 0 / 0 0 0)
γ → R_z^T γ R_z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↳ rotation about ẑ
Rz =
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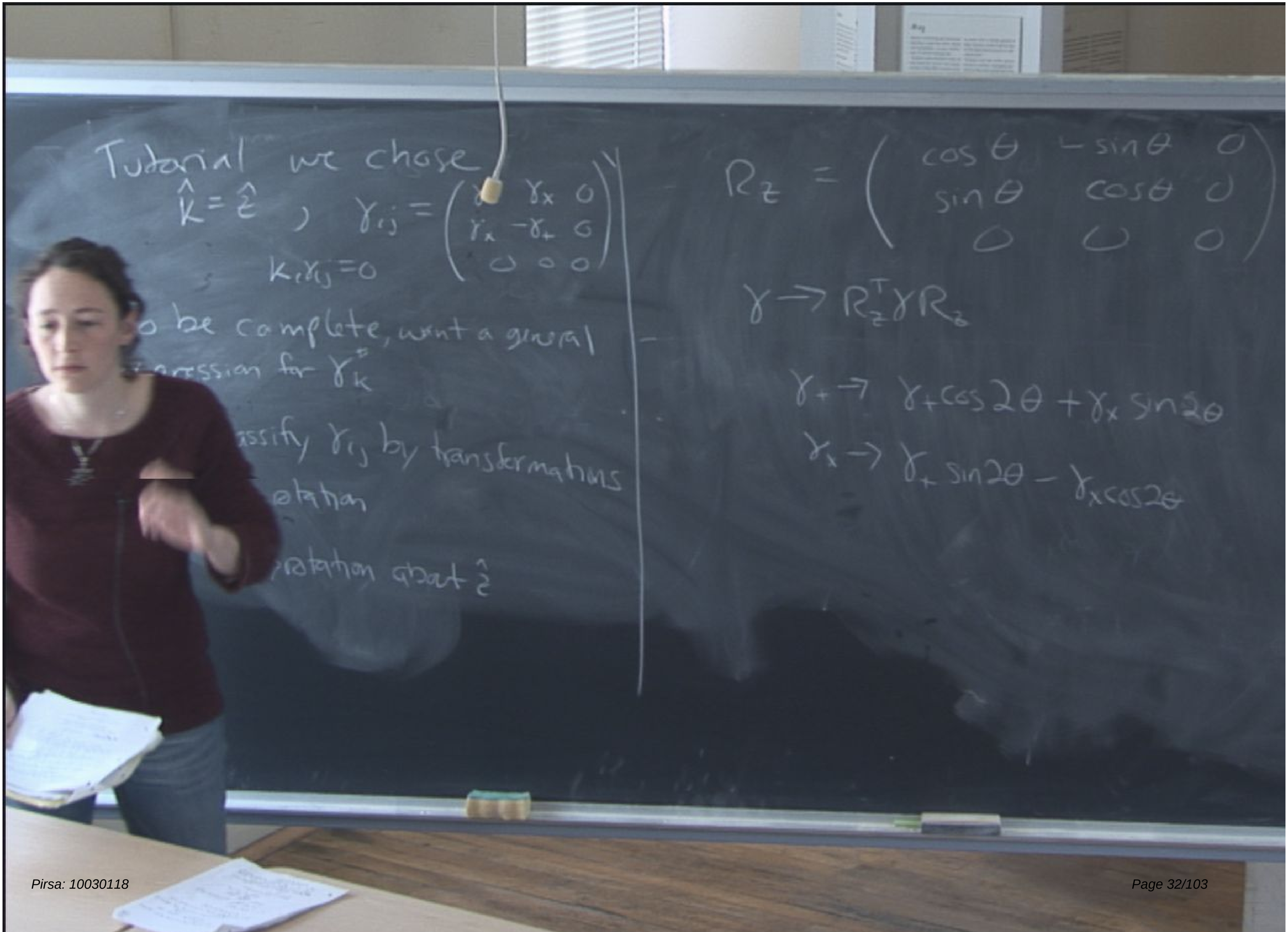
sin θ cos θ 0
0 0 0
γ → RzT γ Rz
γ+ → γ+ cos 2θ + γx sin θ
γx →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$\hat{k} = \hat{z}$, $\gamma_{ij} = \begin{pmatrix} \gamma_+ & \gamma_x & 0 \\ \gamma_x & -\gamma_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
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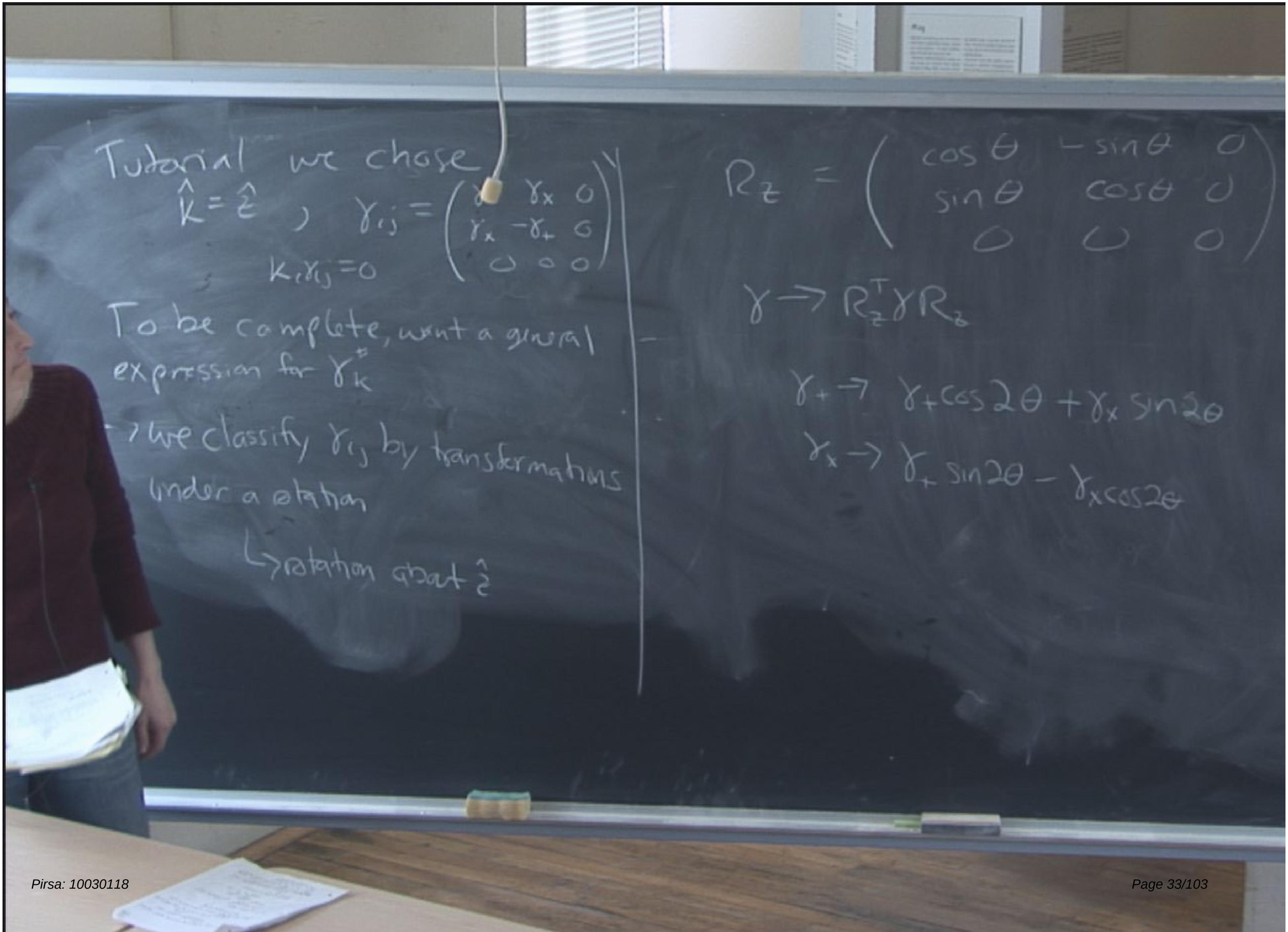
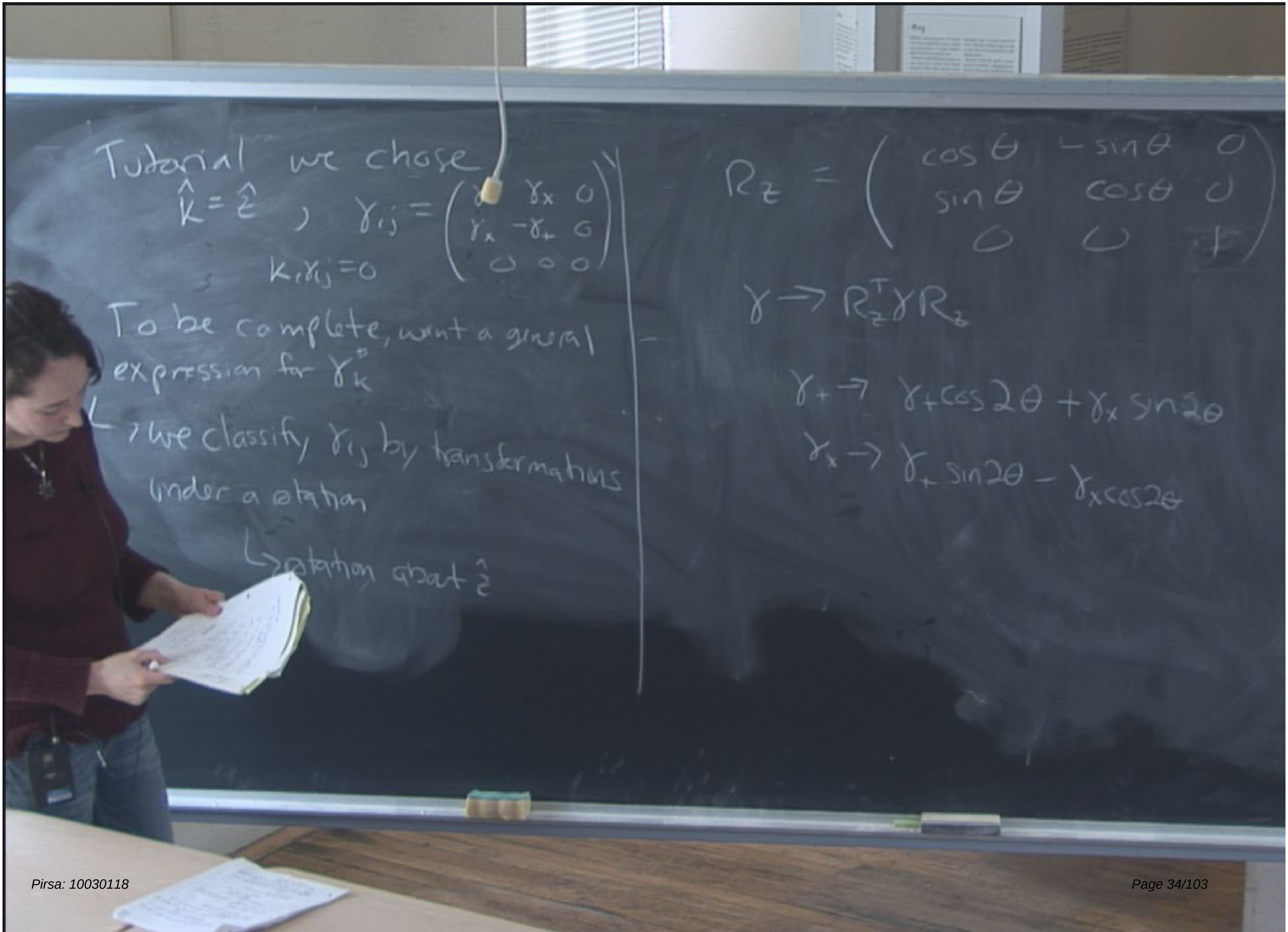
$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\gamma \to R_z^T \gamma R_z$
$\gamma_+ \to \gamma_+ \cos 2\theta + \gamma_x \sin 2\theta$
$\gamma_x \to \gamma_+ \sin 2\theta - \gamma_x \cos 2\theta$



Tutorial we chose
k̂ = ẑ , γ_ij = (γ+ γx 0 ; γx −γ+ 0 ; 0 0 0)
k_i γ_ij = 0
To be complete, want a general expression for γ_k*
→ we classify γ_ij by transformations under a etation
↳ rotation about ẑ
R_z = (cos θ −sin θ 0 ; sin θ cos θ 0 ; 0 0 0)
γ → R_z^T γ R_z
γ+ → γ+ cos 2θ + γx sin 2θ
γx → γ+ sin 2θ − γx cos 2θ


Tutorial we chose
$\hat{k} = \hat{z}$, $\gamma_{ij} = \begin{pmatrix} \gamma_+ & \gamma_x & 0 \\ \gamma_x & -\gamma_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$k_i \gamma_{ij} = 0$
To be complete, want a general expression for $\gamma^{\pm}_k$
↳ we classify γ_{ij} by transformations under a rotation
↳ rotation about $\hat{z}$
$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$\gamma \to R_z^T \gamma R_z$
$\gamma_+ \to \gamma_+ \cos 2\theta + \gamma_x \sin 2\theta$
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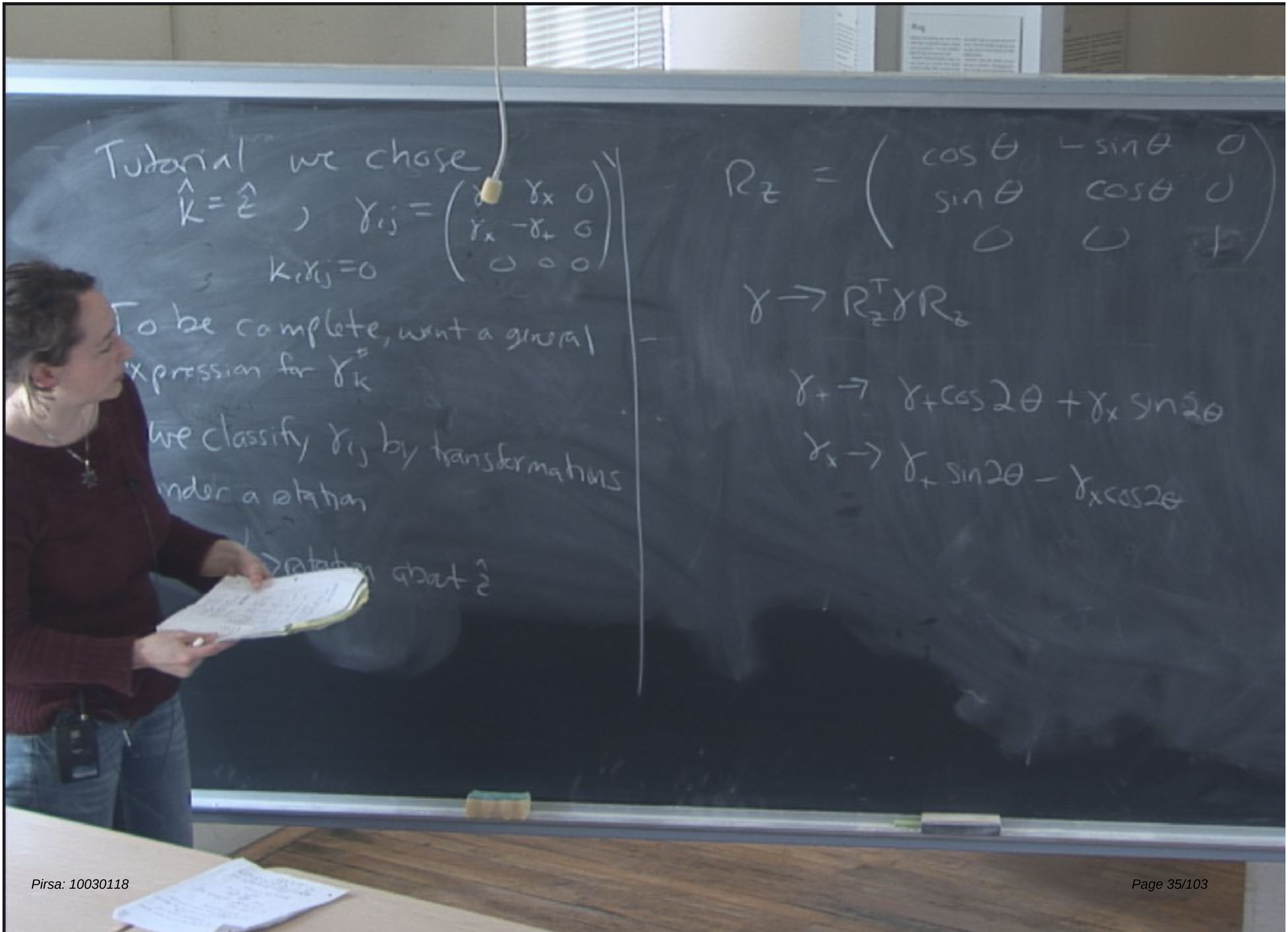
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$\gamma \to R_z^T \gamma R_z$
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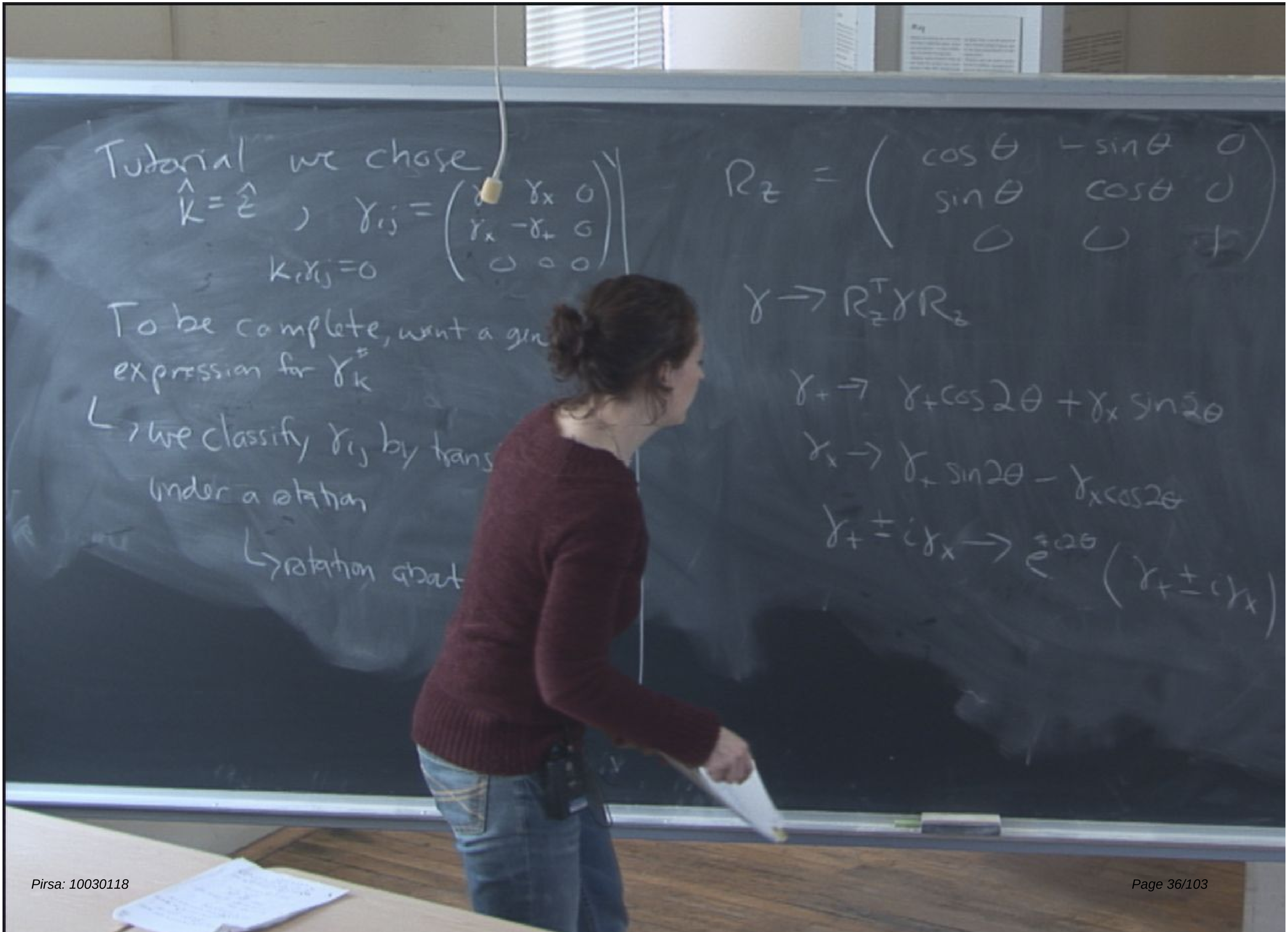
$\gamma_x \to \gamma_+ \sin 2\theta - \gamma_x \cos 2\theta$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↳ we classify γij by trans
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↳ rotation about
Rz = (cos θ −sin θ 0 ; sin θ cos θ 0 ; 0 0 1)
γ → RzT γ Rz
γ+ → γ+ cos 2θ + γx sin 2θ
γx → γ+ sin 2θ − γx cos 2θ
γ+ ± iγx → e±i2θ (γ+ ± iγx)

Tutorial we chose

$\hat{k} = \hat{z}$, $\gamma_{ij} = \begin{pmatrix} \gamma_+ & \gamma_\times & 0 \\ \gamma_\times & -\gamma_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$k_i \gamma_{ij} = 0$

To be complete, want a general expression for $\gamma_k^{\pm}$

↳ we classify γ_{ij} by transformations under a rotation

↳ rotation about $\hat{z}$

$$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

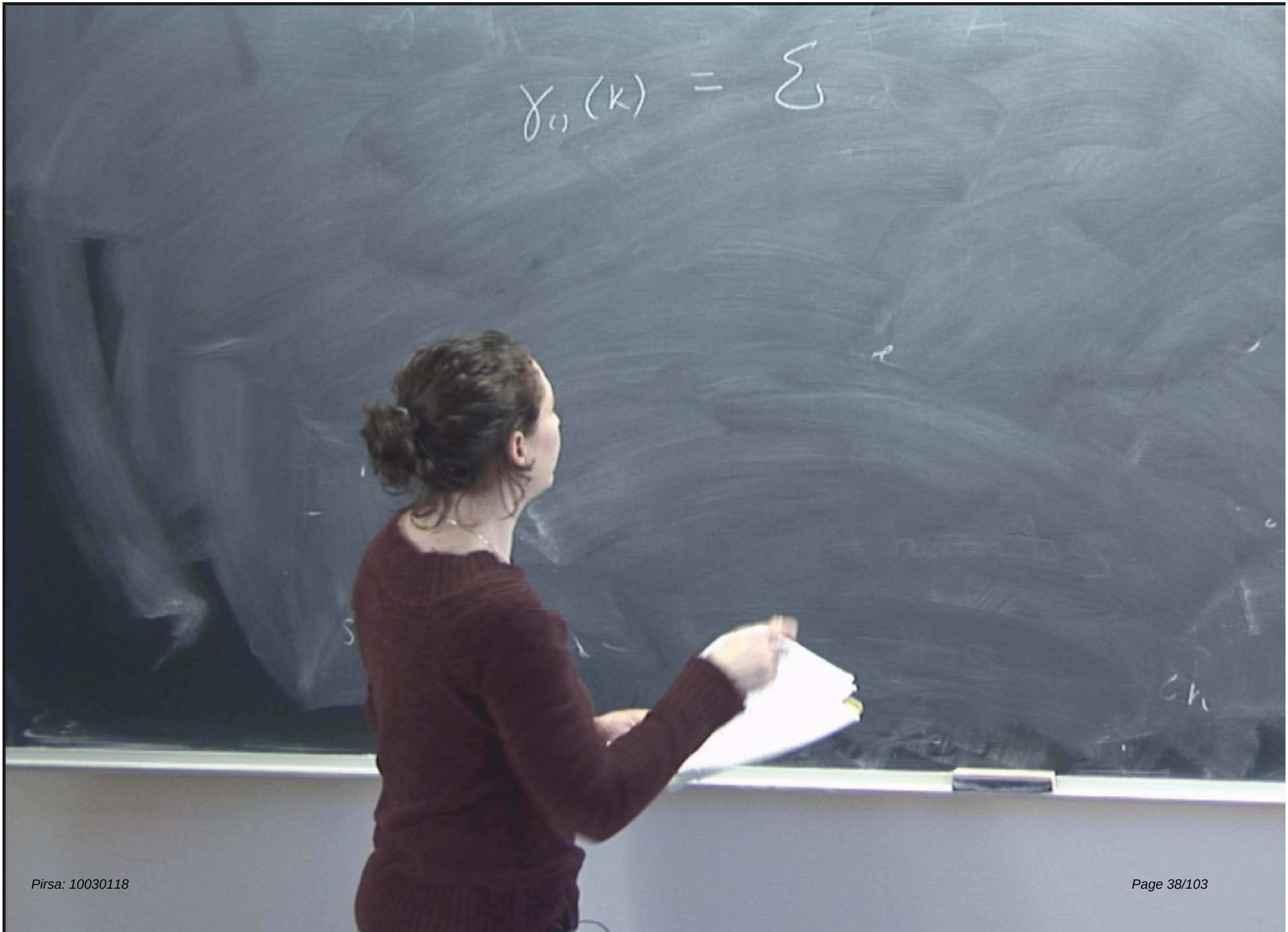
$$\gamma \to R_z^T \gamma R_z$$

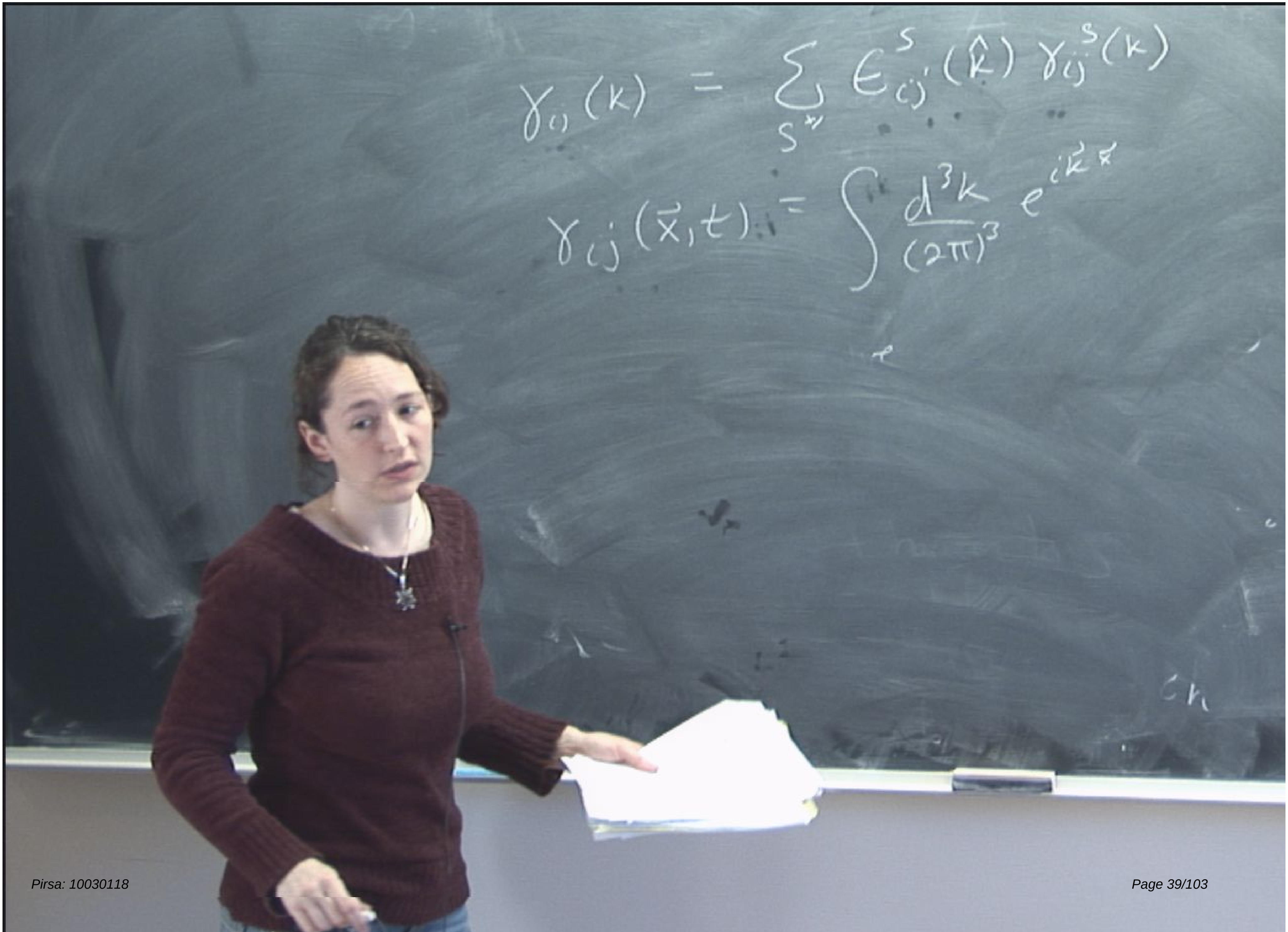
$$\gamma_+ \to \gamma_+ \cos 2\theta + \gamma_\times \sin 2\theta$$

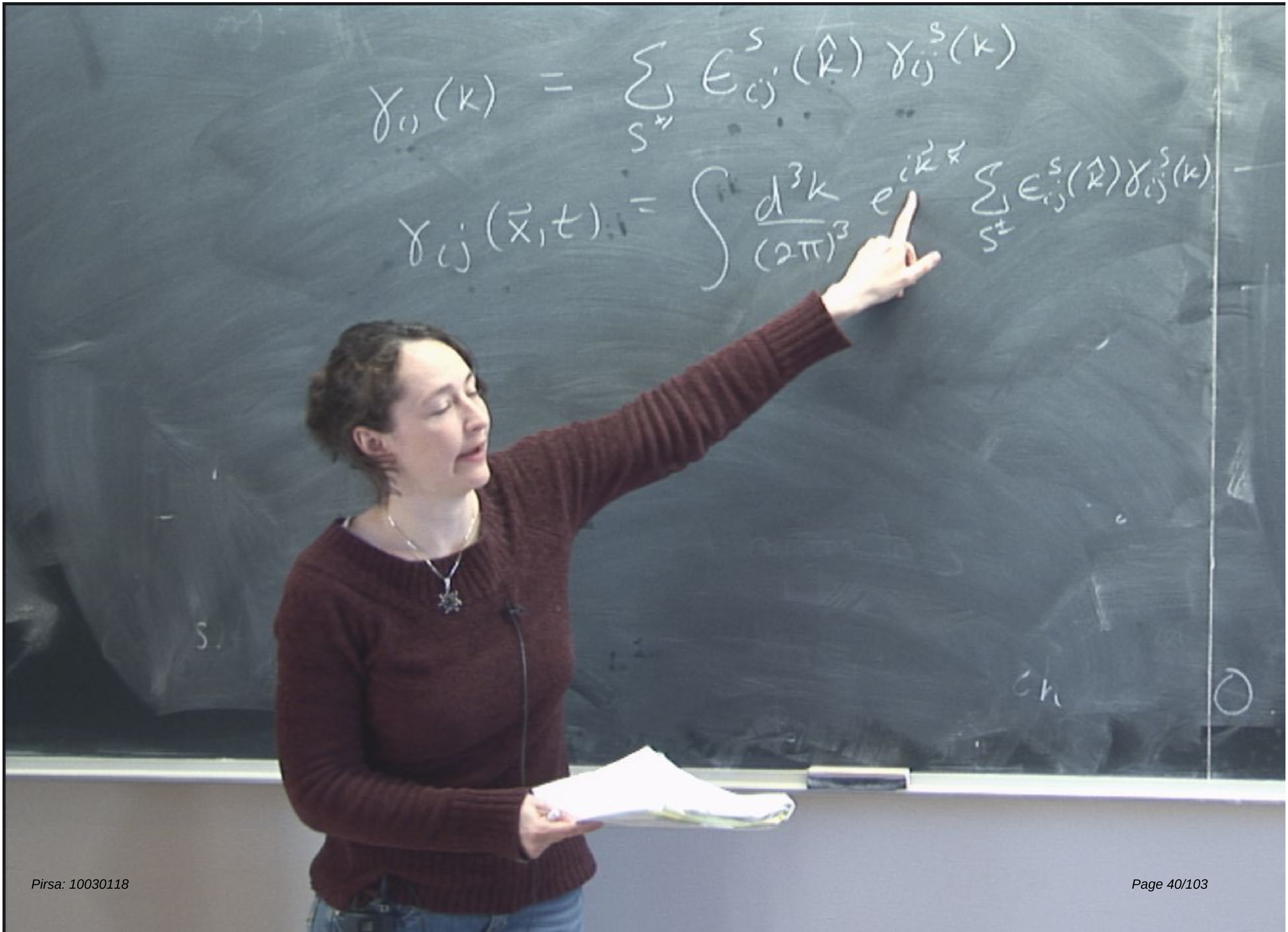
$$\gamma_\times \to \gamma_+ \sin 2\theta - \gamma_\times \cos 2\theta$$

$$\gamma_+ \pm i\gamma_\times \to e^{\pm i 2\theta} (\gamma_+ \pm i \gamma_\times)$$

"helicity two"


$\gamma_{(1)}(k) = \Sigma$

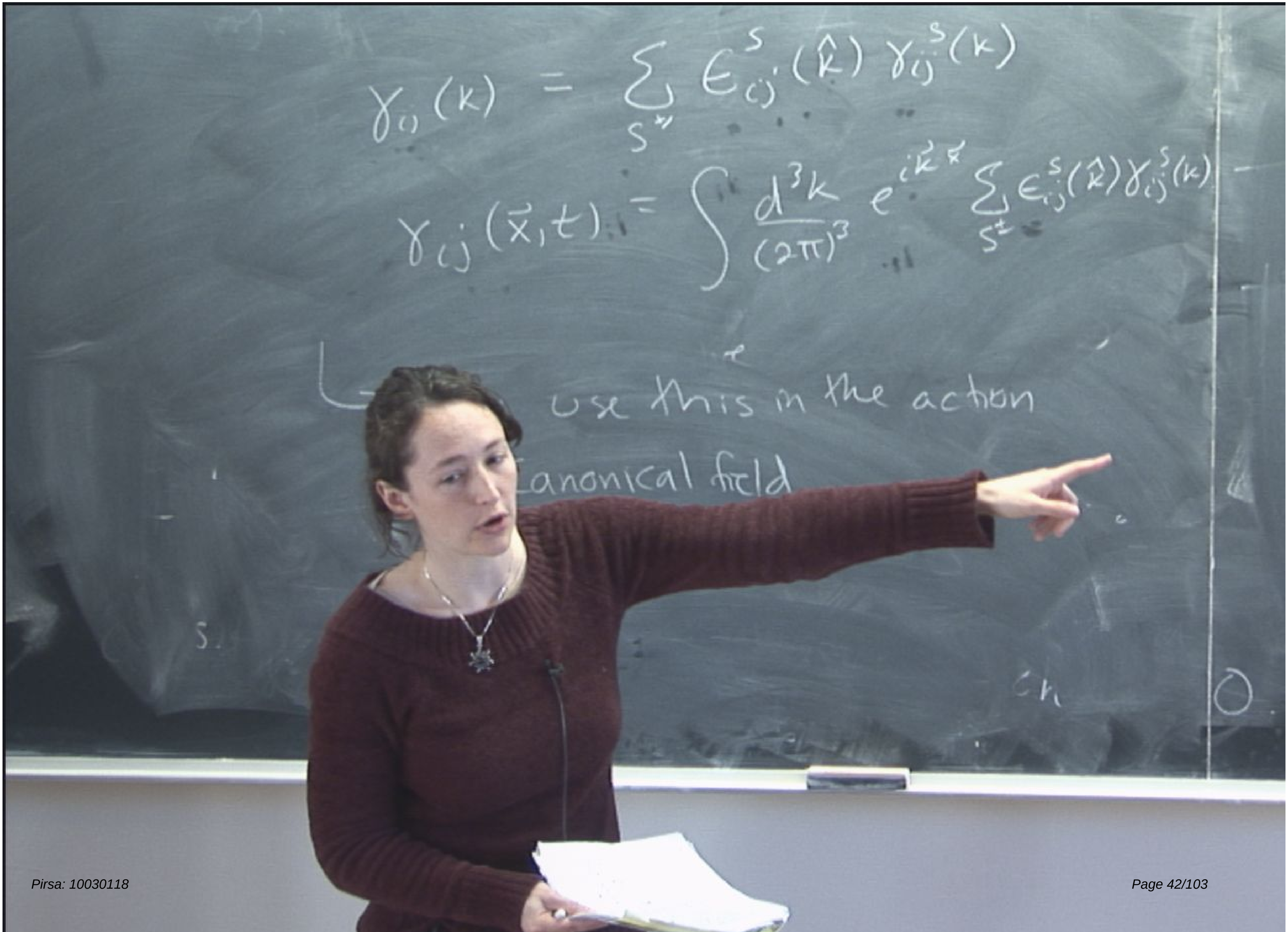

$\gamma_{ij}(k) = \sum_{s} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$
$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}}$


$\gamma_{ij}(k) = \sum_{s=+,\times} \epsilon^s_{ij}(\hat{k})\,\gamma^s_{ij}(k)$
$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \sum_{s=\pm} \epsilon^s_{ij}(\hat{k})\,\gamma^s_{ij}(k)$

$$\gamma_{ij}(k) = \sum_{s=\pm} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$$

$$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \sum_{s=\pm} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$$

↳ Now use this in the action

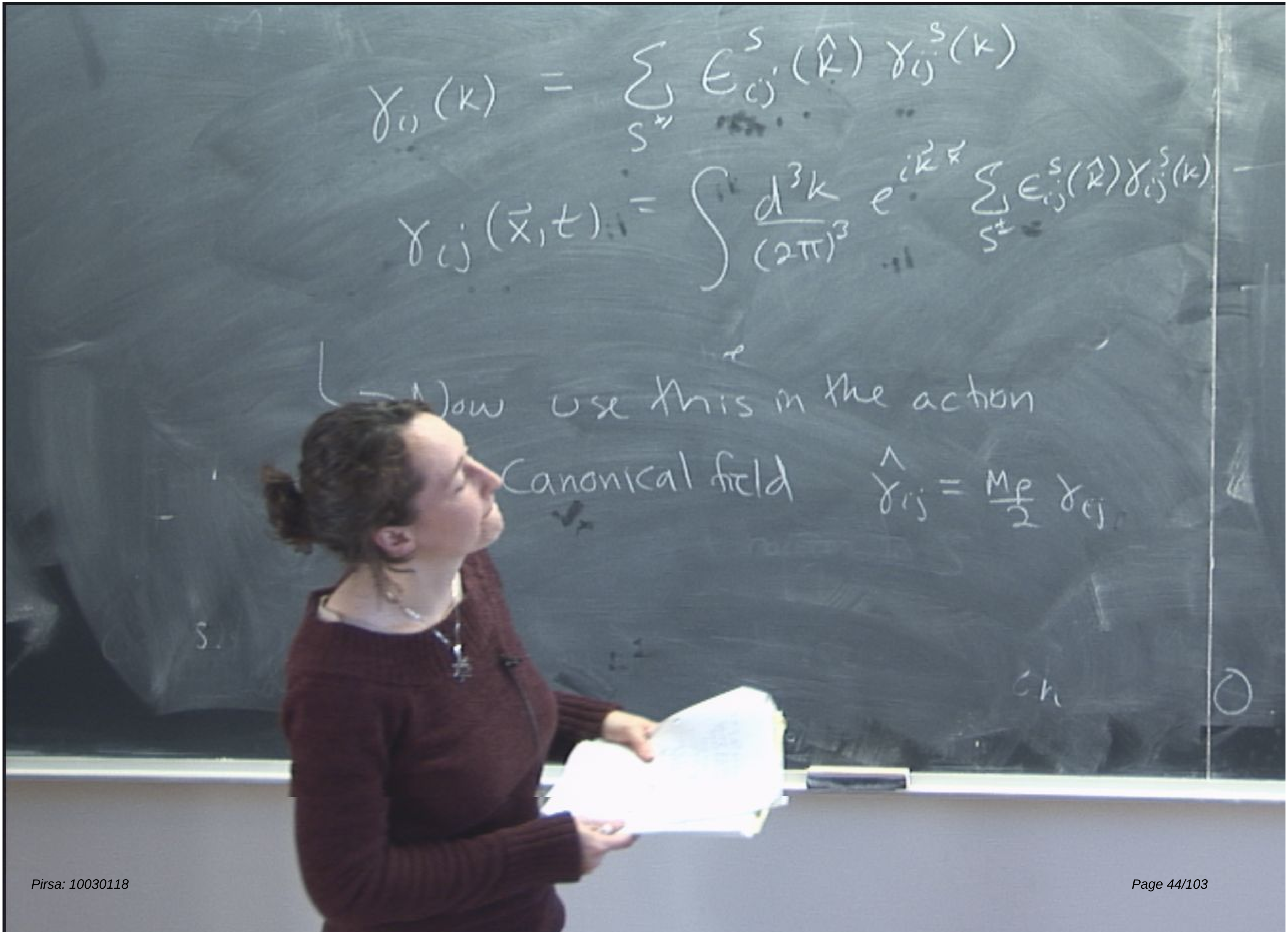

$\gamma_{ij}(k) = \sum_{s=\pm} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$
$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \sum_{s=\pm} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$
use this in the action
anonical field

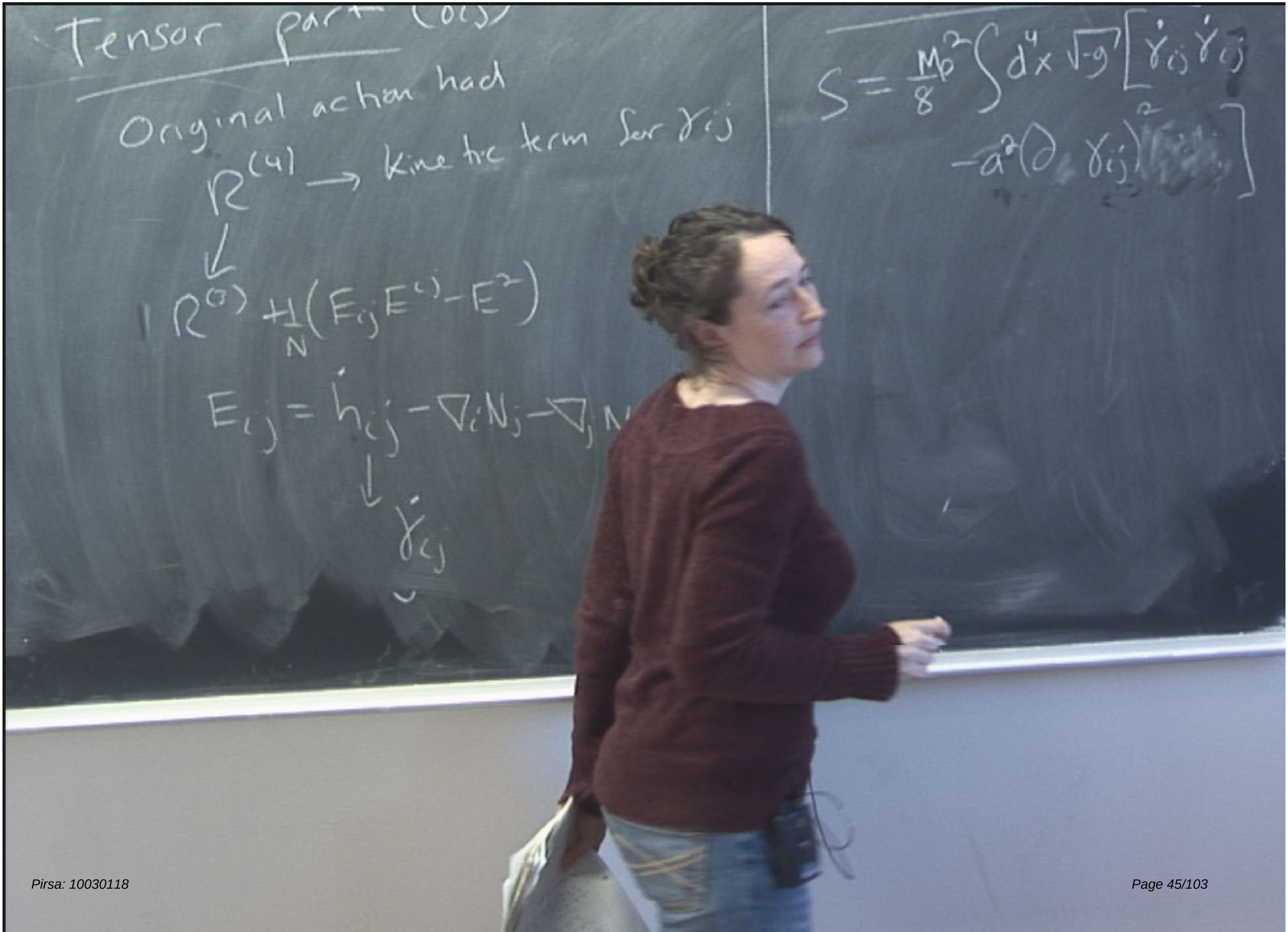
$$\gamma_{ij}(k) = \sum_{s^{+,\times}} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$$

$$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \sum_{s^{\pm}} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$$

↳ Now use this in the action

Canonical field $\hat{\gamma}_{ij} = \frac{M_p}{2}\gamma_{ij}$


$\gamma_{ij}(k) = \sum_{s=+,\times} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$
$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \sum_{s^{\pm}} \epsilon^{s}_{ij}(\hat{k})\gamma^{s}_{ij}(k)$
→ Now use this in the action
Canonical field $\hat{\gamma}_{ij} = \frac{M_P}{2}\gamma_{ij}$


Tensor part
Original action had
Kinetic term for γij
S = Mp²/8 ∫ d⁴x √-g [γ̇ij γ̇ij
−a²(∂ γij)²]
Eij = ḣij − ∇iNj − ∇jN

$$\gamma_{ij}(k) = \sum_{s=\pm} \epsilon_{ij}^{s}(\hat{k})\, \gamma_{ij}^{s}(k)$$

$$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \sum_{s=\pm} \epsilon_{ij}^{s}(\hat{k})\, \gamma_{ij}^{s}(k)$$

↳ Now use this in the action

Canonical field $\hat{\gamma}_{ij} = \frac{M_P}{2}\gamma_{ij}$

$$\langle \hat{\gamma}_+ \hat{\gamma}_+ \rangle = \langle \hat{\gamma}_- \hat{\gamma}_- \rangle = \frac{H^2}{2k^3}$$

$$\langle \gamma_+ \gamma_+ \rangle = \frac{H^2}{M_P^2}\frac{2}{k^3}$$

$(\hat{k})\gamma^{s}_{ij}(k)$

→ total power in tensor modes

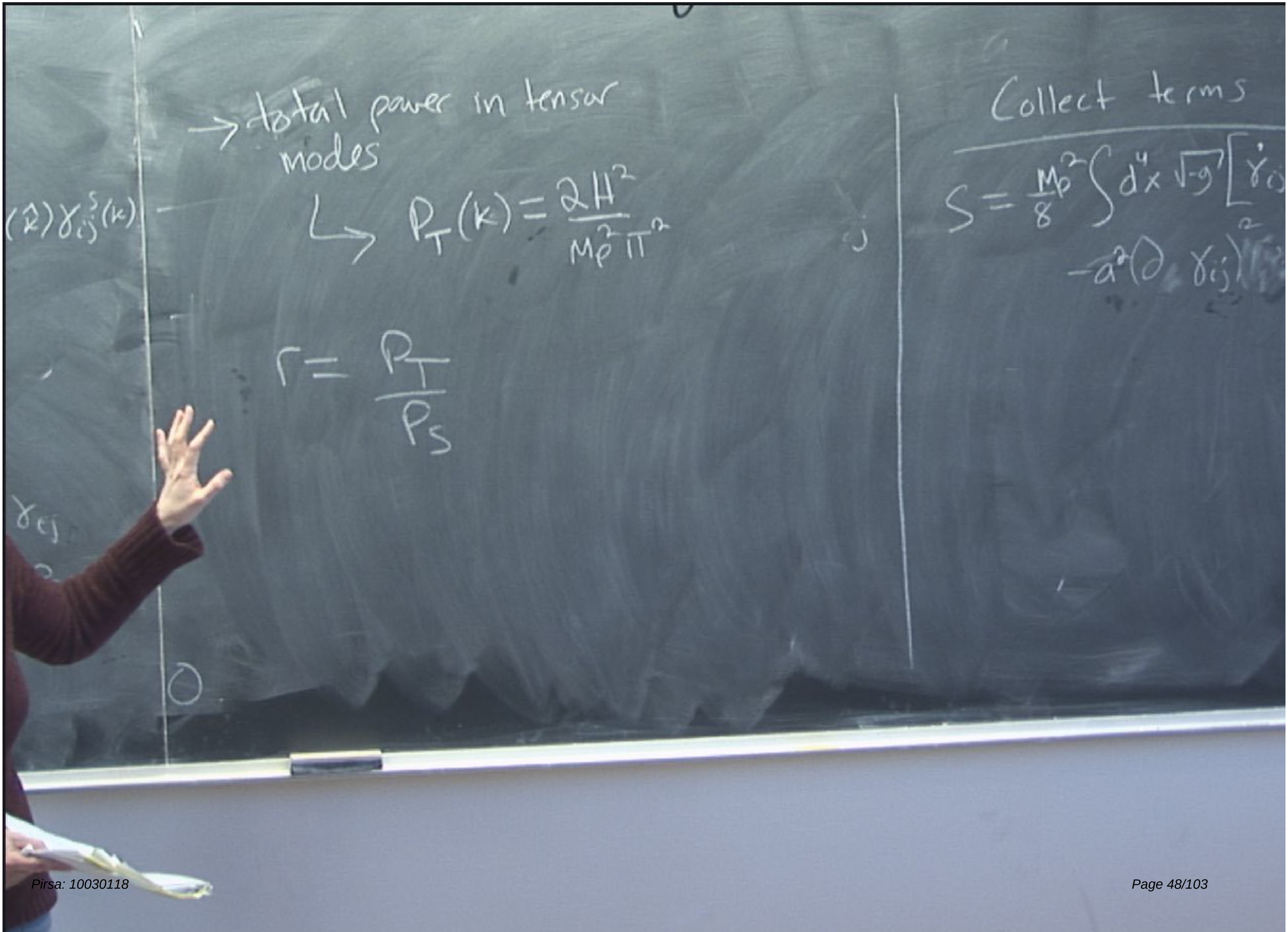
$$\hookrightarrow P_T(k) = \frac{2H^2}{M_p^2 \pi^2}$$

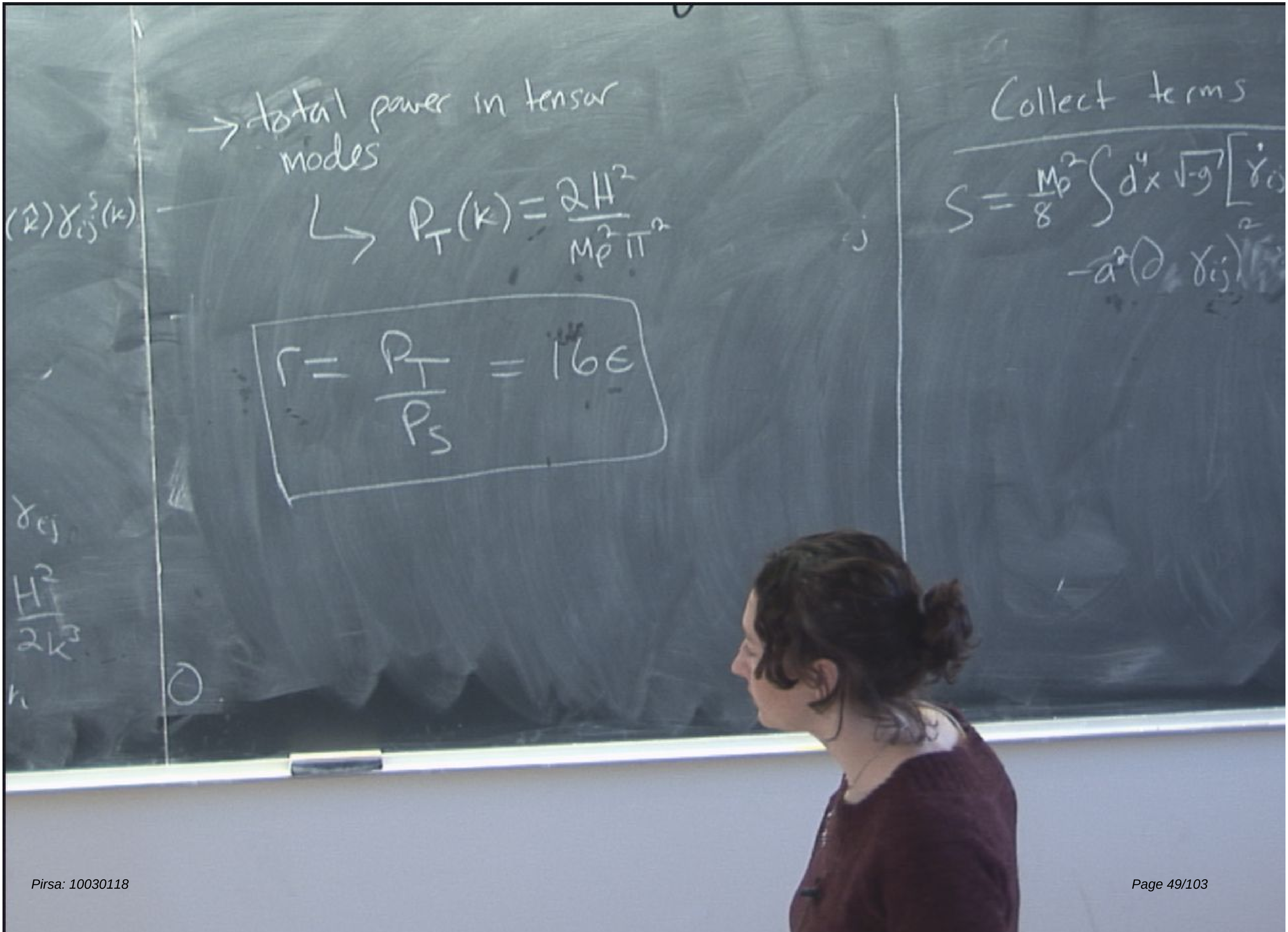
$$= \frac{P_T}{P_S}$$

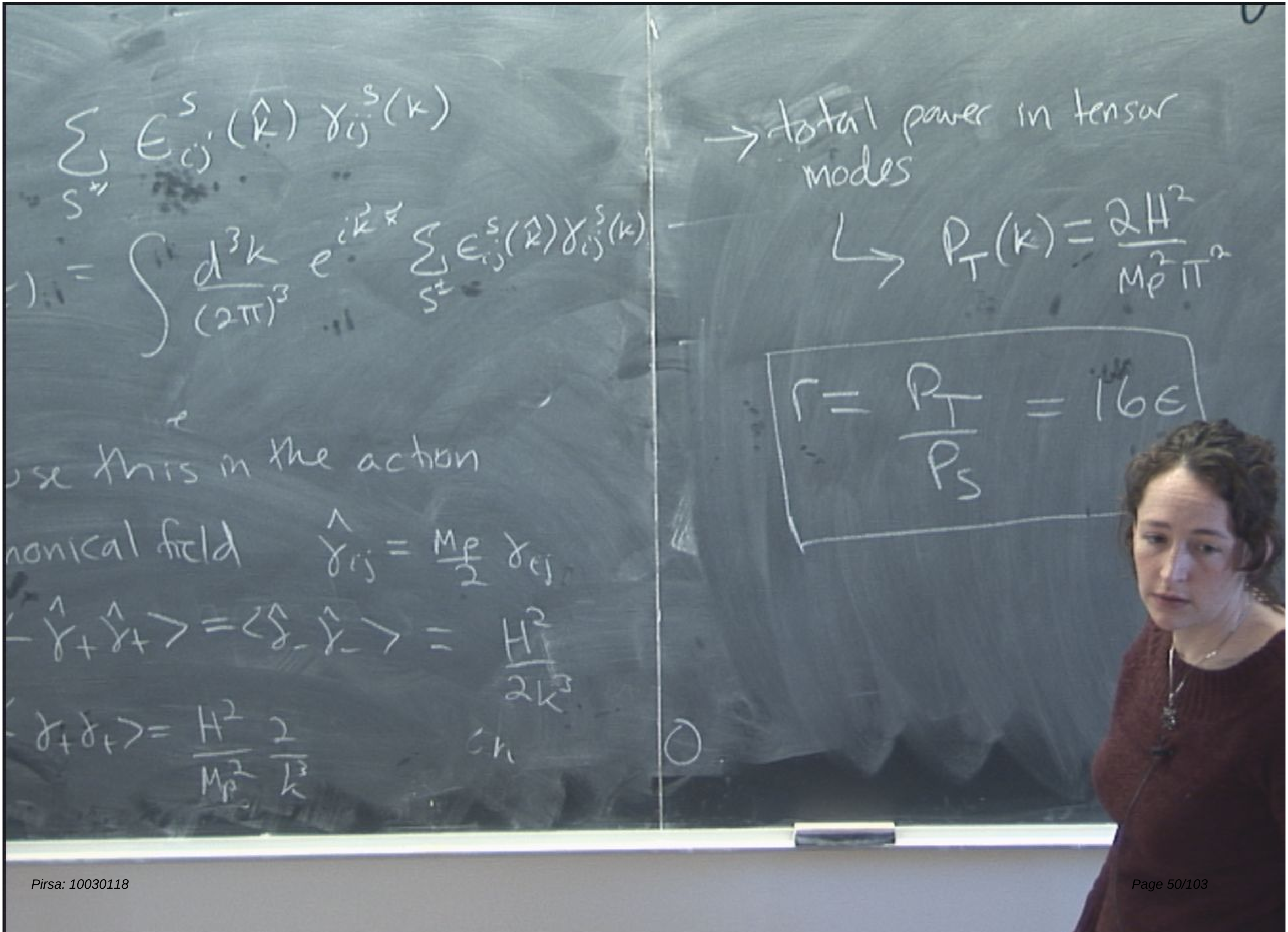
$\frac{H^2}{2k^3}$

Collect terms

$$S = \frac{M_p^2}{8}\int d^4x \sqrt{-g}\,\Big[\dot{\gamma}_{ij}^2 \ldots$$
$$-a^2(\partial\,\gamma_{ij})^2 \ldots$$


→ total power in tensor modes
↳ $P_T(k) = \frac{2H^2}{M_p^2 \pi^2}$
$r = \frac{P_T}{P_S}$
Collect terms
$S = \frac{M_p^2}{8} \int d^4x \sqrt{-g} [\dot{\gamma}_{ij}$
$-a^2(\partial \gamma_{ij})^2$


→ total power in tensor modes
↳ $P_T(k) = \frac{2H^2}{M_p^2 \pi^2}$
$r = \frac{P_T}{P_s} = 16\epsilon$
Collect terms
$S = \frac{M_p^2}{8}\int d^4x \sqrt{-g}\,[$


$\sum_{s} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$
$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \sum_{s} \epsilon^{s}_{ij}(\hat{k})\, \gamma^{s}_{ij}(k)$
use this in the action
nonical field $\hat{\gamma}_{ij} = \frac{M_p}{2} \gamma_{ij}$
$\langle \hat{\gamma}_+ \hat{\gamma}_+ \rangle = \langle \hat{\gamma}_- \hat{\gamma}_- \rangle = \frac{H^2}{2k^3}$
$\langle \gamma_+ \gamma_+ \rangle = \frac{H^2}{M_p^2} \frac{2}{k^3}$
→ total power in tensor modes
$P_T(k) = \frac{2H^2}{M_p^2 \pi^2}$
$r = \frac{P_T}{P_S} = 16\epsilon$

→ total power in tensor modes

↳ $$P_T(k) = \frac{2H^2}{M_p^2 \pi^2}$$

$$\boxed{r = \frac{P_T}{P_S} = 16\epsilon} \leq 0.2$$

Collec

$$S = \frac{M_p^2}{8}$$

$$\gamma_{ij}(k) = \sum_{s=+,\times} \epsilon_{ij}^{s}(\hat{k})\, \gamma_{ij}^{s}(k)$$

$$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \sum_{s=\pm} \epsilon_{ij}^{s}(\hat{k})\, \gamma_{ij}^{s}(k)$$

↳ Now use this in the action

Canonical field $\hat{\gamma}_{ij} = \frac{M_p}{2}\gamma_{ij}$

$$\langle \hat{\gamma}_+ \hat{\gamma}_+ \rangle = \langle \hat{\gamma}_- \hat{\gamma}_- \rangle = \frac{H^2}{2k^3}$$

$$\langle \gamma_+ \gamma_+ \rangle = \frac{H^2}{M_p^2} \frac{2}{k^3}$$

$$\gamma_{ij}(k) = \sum_{s=+,\times} \epsilon^s_{ij}(\hat{k})\, \gamma^s_{ij}(k)$$

$$\epsilon_{ij} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \sum_{s=\pm} \epsilon^s_{ij}(\hat{k})\, \gamma^s_{ij}(k)$$

→ Now use this in the action

Canonical field $\hat{\gamma}_{ij} = \frac{M_p}{2}\gamma_{ij}$

$$\langle \hat{\gamma}_+ \hat{\gamma}_+ \rangle = \langle \hat{\gamma}_- \hat{\gamma}_- \rangle = \frac{H^2}{2k^3}$$

$$\langle \gamma_+ \gamma_+ \rangle = \frac{H^2}{M_p^2}\frac{2}{k^3}$$

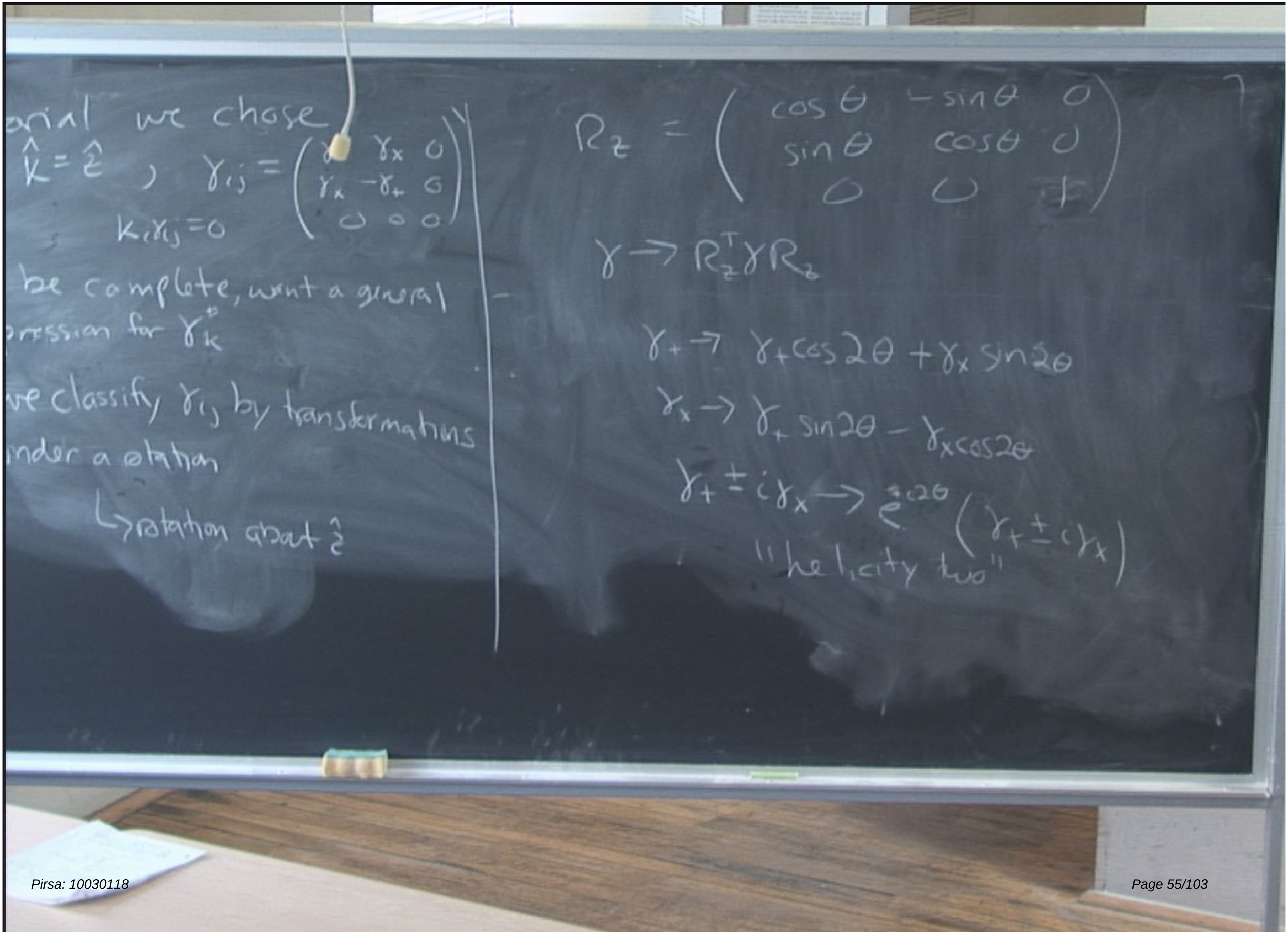
→ total power in tensor modes

$\hookrightarrow P_T(k) = \frac{2H^2}{M_p^2 \pi^2}$

$$r = \frac{P_T}{P_S} = 16\epsilon \leq 0.2$$

Collect terms

$$S = \frac{M_p^2}{8}\int d^4x \sqrt{-g}\,\left[\dot{\gamma}_{ij}\dot{\gamma}_{ij} - a^2(\partial\, \gamma_{ij})^2\right]$$


orial we chose
$\hat{k} = \hat{z}$, $\gamma_{ij} = \begin{pmatrix} \gamma_+ & \gamma_\times & 0 \\ \gamma_\times & -\gamma_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$k_i \gamma_{ij} = 0$
be complete, want a general
pression for γ^*_k
we classify γ_{ij} by transformations
nder a rotation
↳ rotation about $\hat{z}$
$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$\gamma \to R_z^T \gamma R_z$
$\gamma_+ \to \gamma_+ \cos 2\theta + \gamma_\times \sin 2\theta$
$\gamma_\times \to \gamma_+ \sin 2\theta - \gamma_\times \cos 2\theta$
$\gamma_+ \pm i\gamma_\times \to e^{\pm i 2\theta} (\gamma_+ \pm i\gamma_\times)$
"helicity two"

...orial we chose

$\hat{k} = \hat{z}$, $\gamma_{ij} = \begin{pmatrix} \gamma_+ & \gamma_x & 0 \\ \gamma_x & -\gamma_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$k_i \gamma_{ij} = 0$

...be complete, want a general ...pression for $\gamma_k^{\pm}$

...we classify γ_{ij} by transformations ...nder a rotation

↳ rotation about $\hat{z}$

$$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

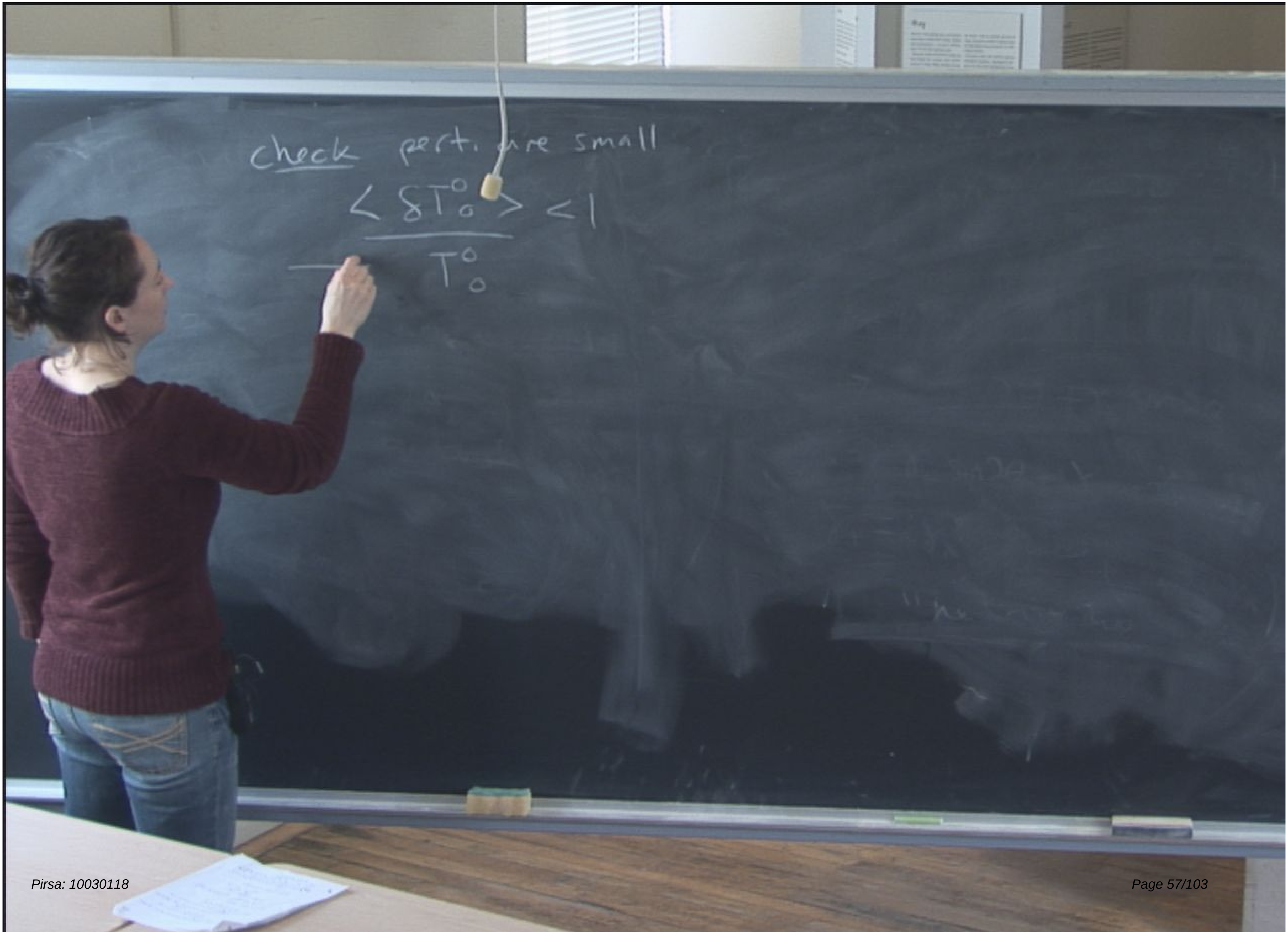
$$\gamma \to R_z^T \gamma R_z$$

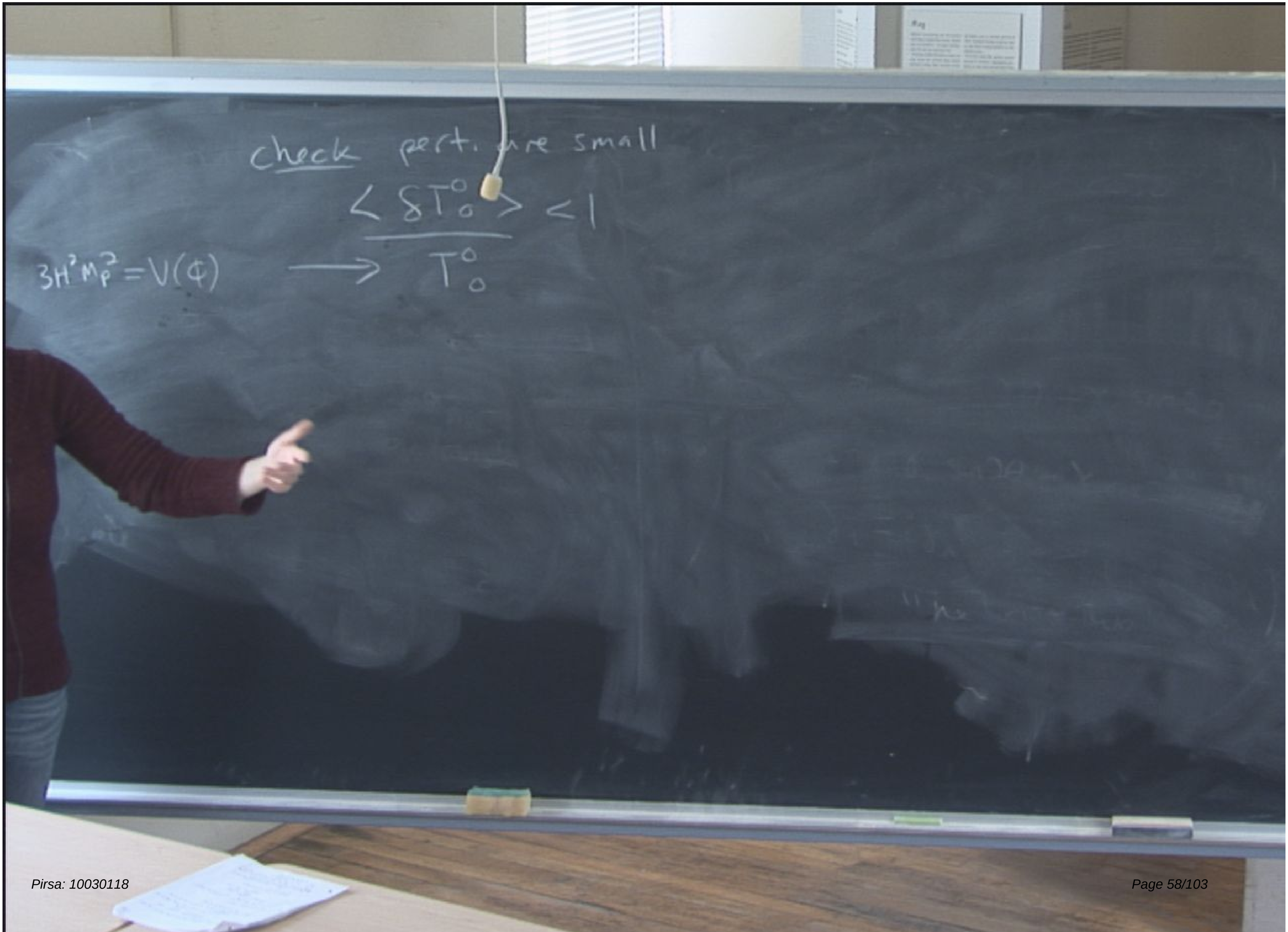
$$\gamma_+ \to \gamma_+ \cos 2\theta + \gamma_x \sin 2\theta$$

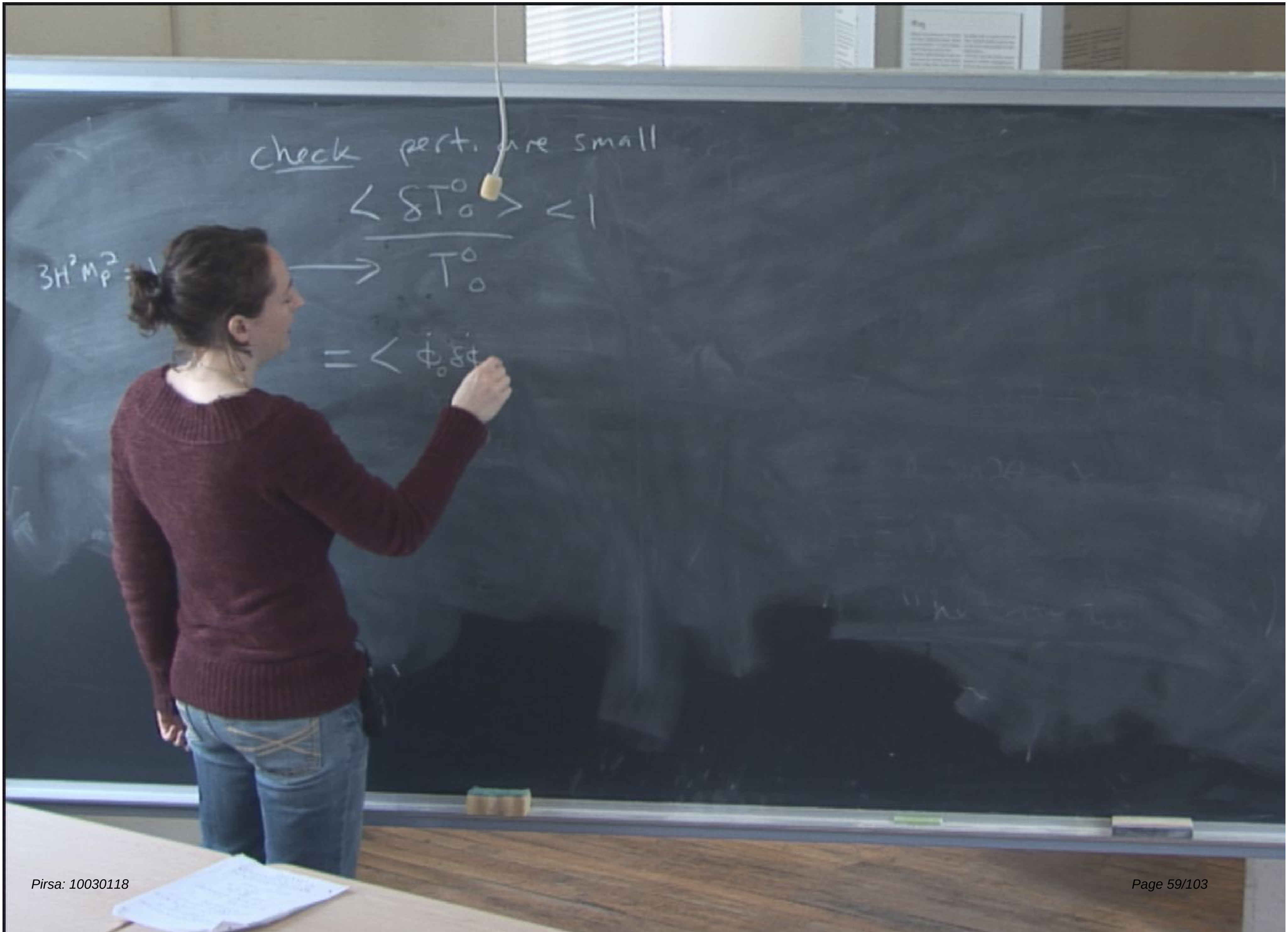
$$\gamma_x \to \gamma_+ \sin 2\theta - \gamma_x \cos 2\theta$$

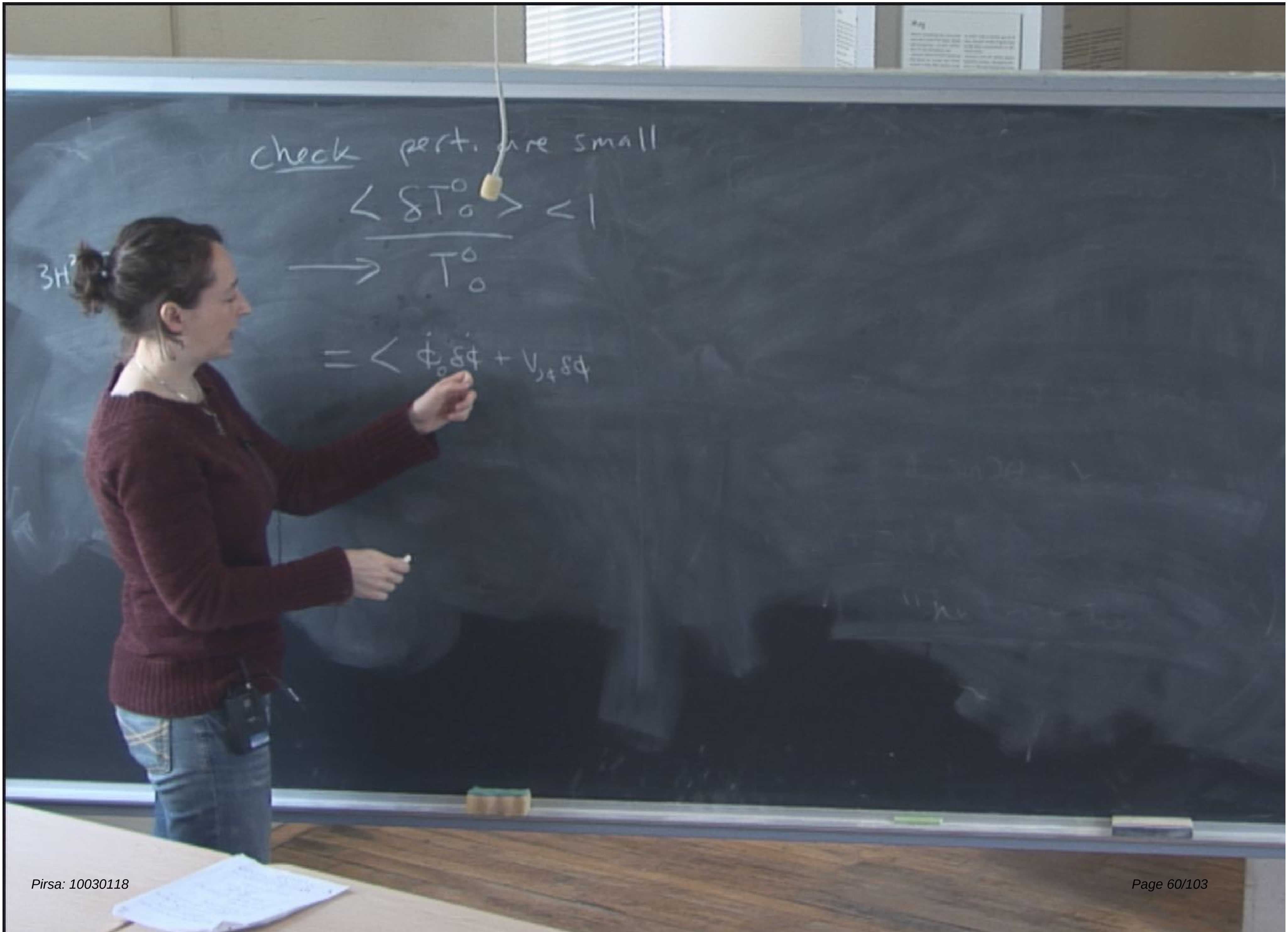
$$\gamma_+ \pm i\gamma_x \to e^{\pm i 2\theta} (\gamma_+ \pm i \gamma_x)$$

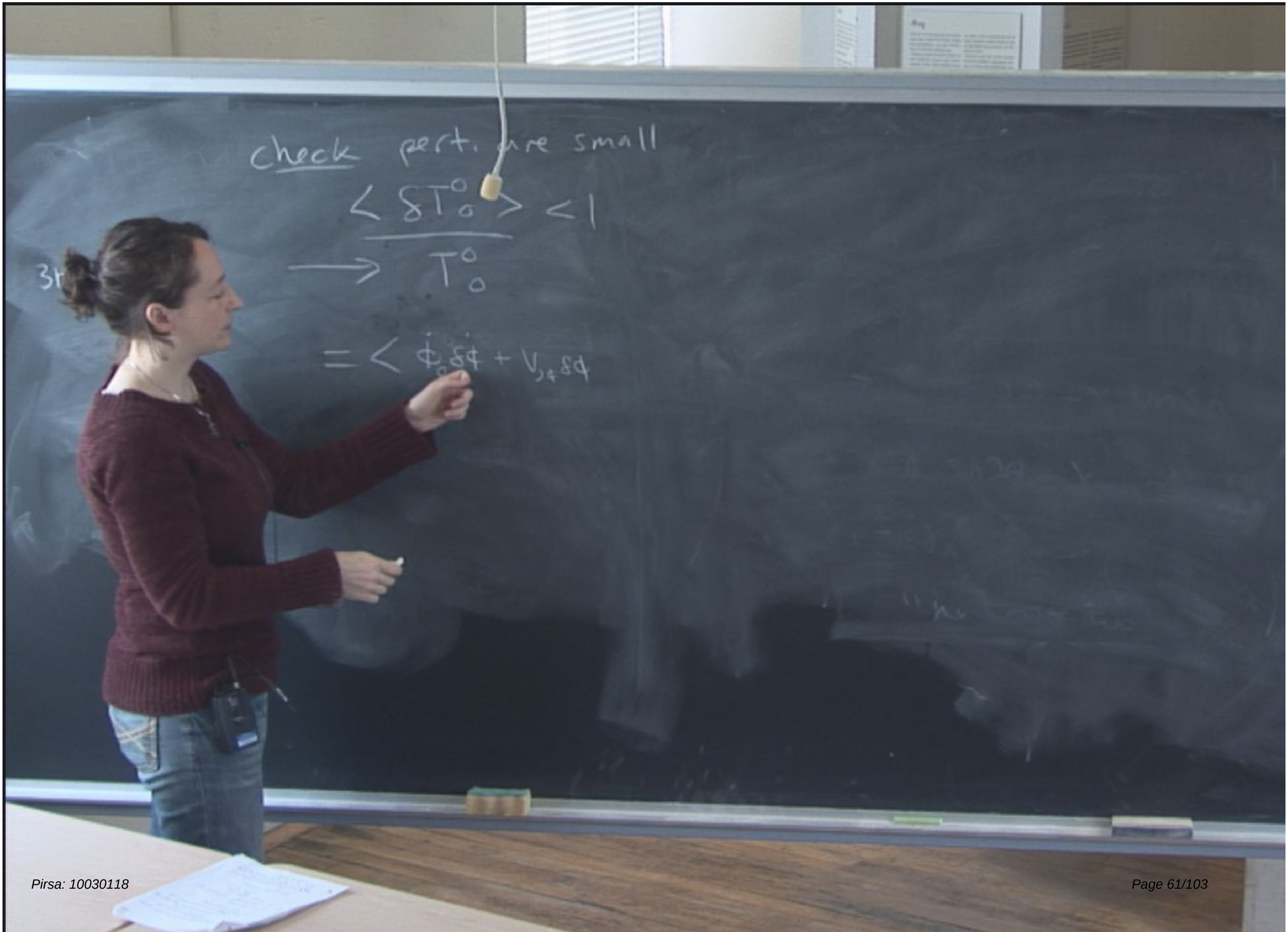
"helicity two"

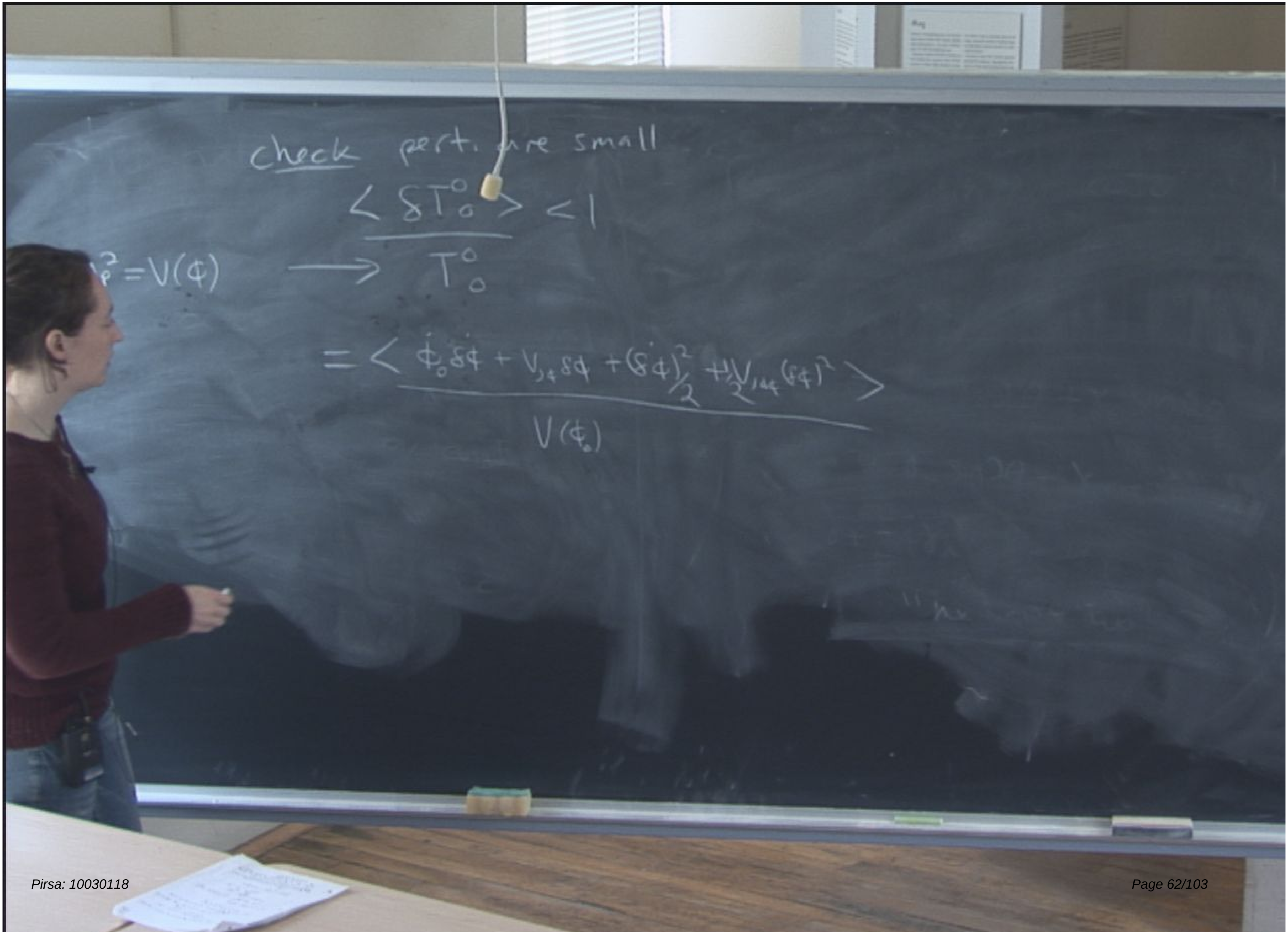

check pert. are small
$\frac{\langle \delta T^0_0 \rangle}{T^0_0} < 1$

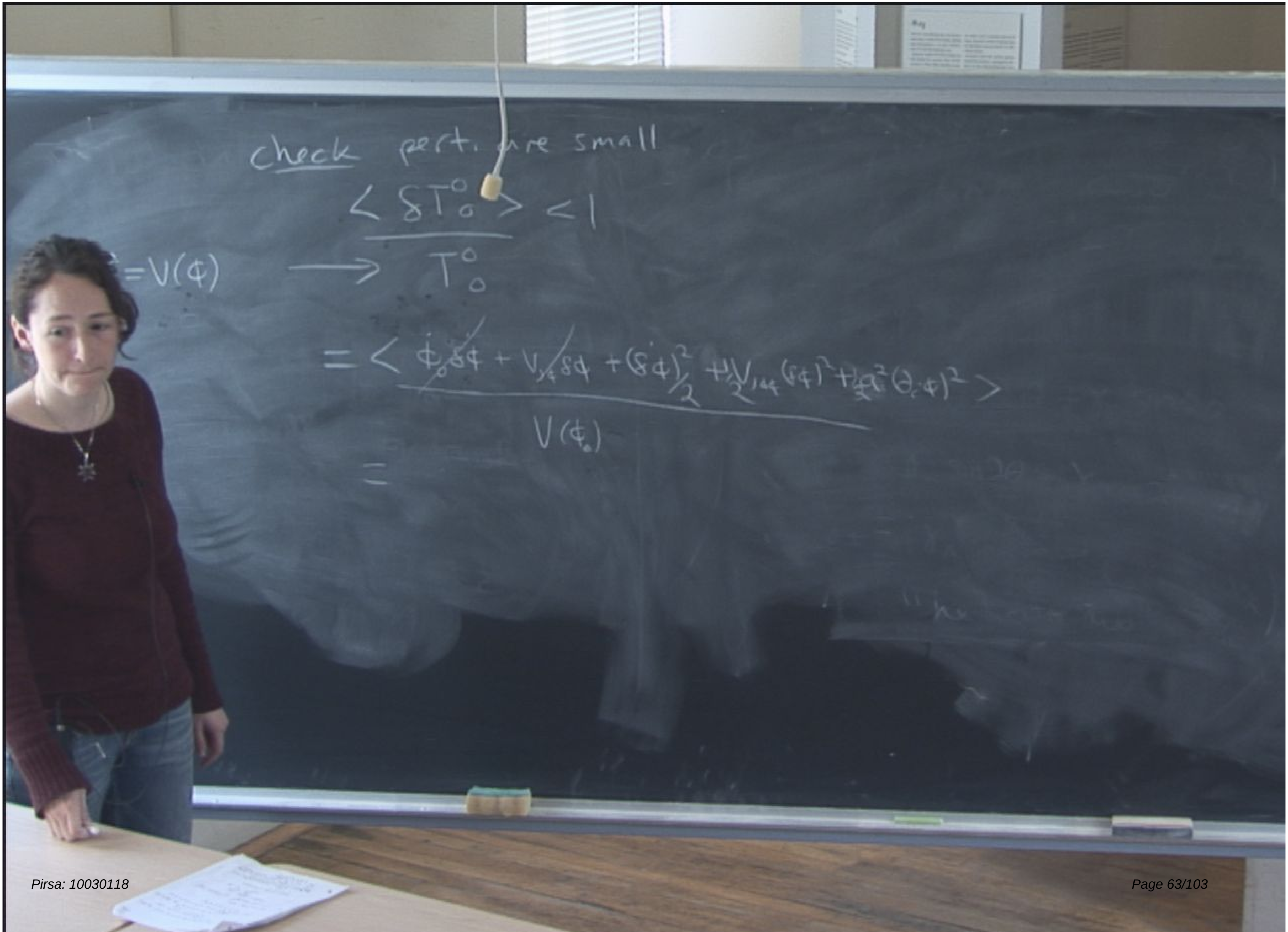

check pert. are small
$\frac{\langle \delta T^0_0 \rangle}{T^0_0} < 1$
$3H^2 M_p^2 = V(\phi) \longrightarrow$

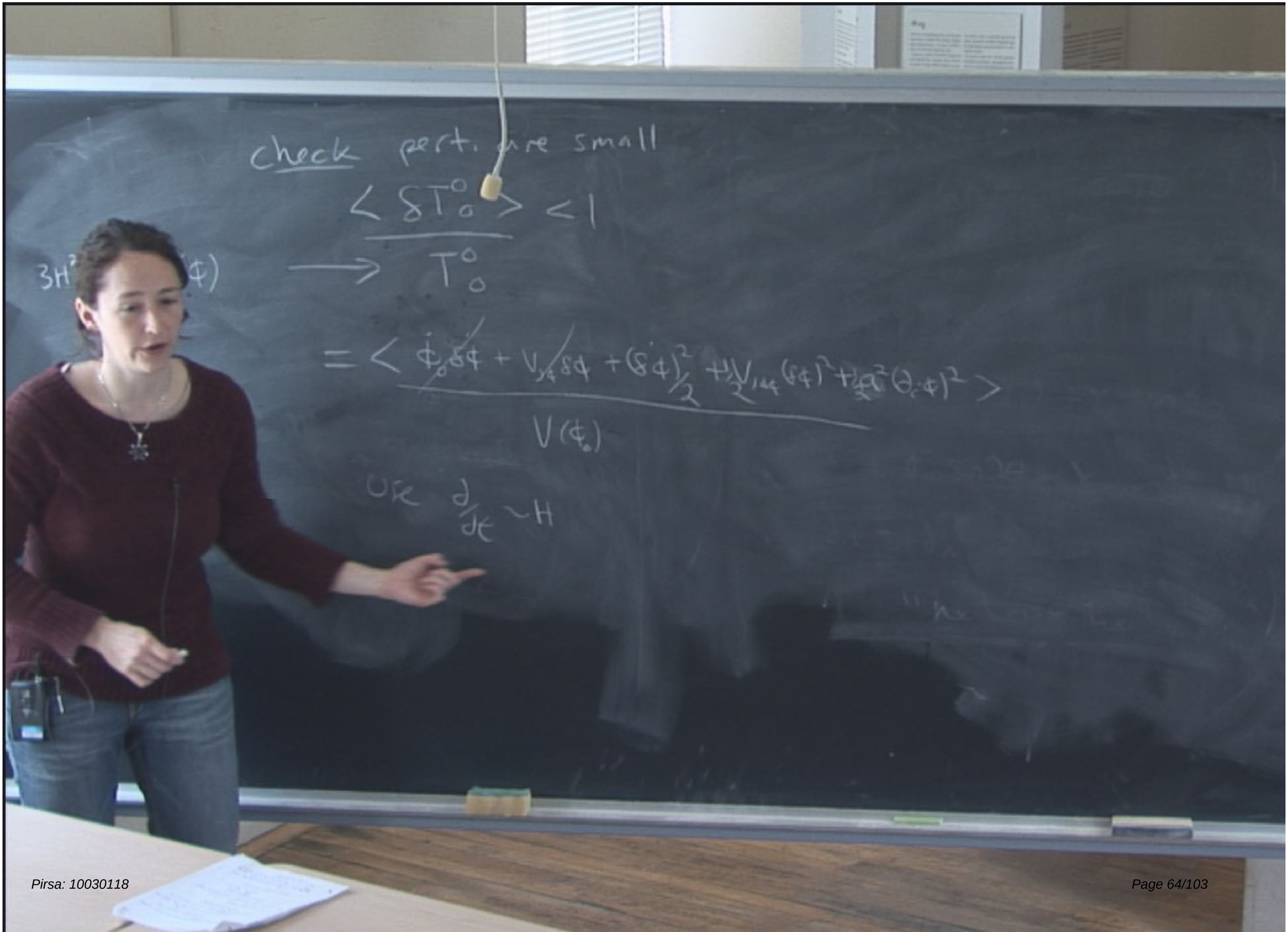

check pert. are small
< δT⁰₀ > < 1
T⁰₀
3H²Mp²
= < φ̇₀ δφ̇

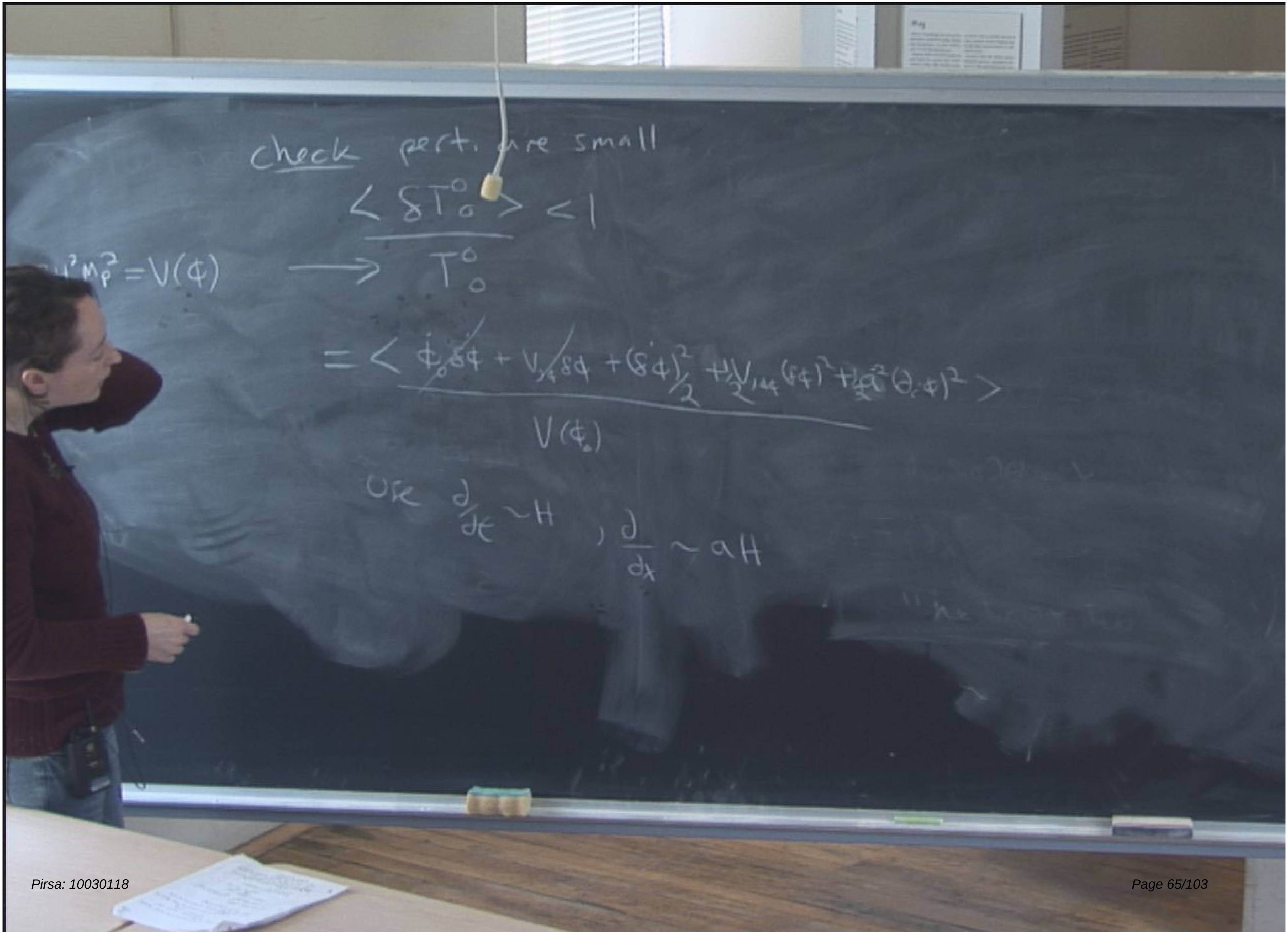

check pert. are small
< δT^0_0 > / T^0_0 < 1
= < φ̇_0 δφ̇ + V_{,φ} δφ
3H

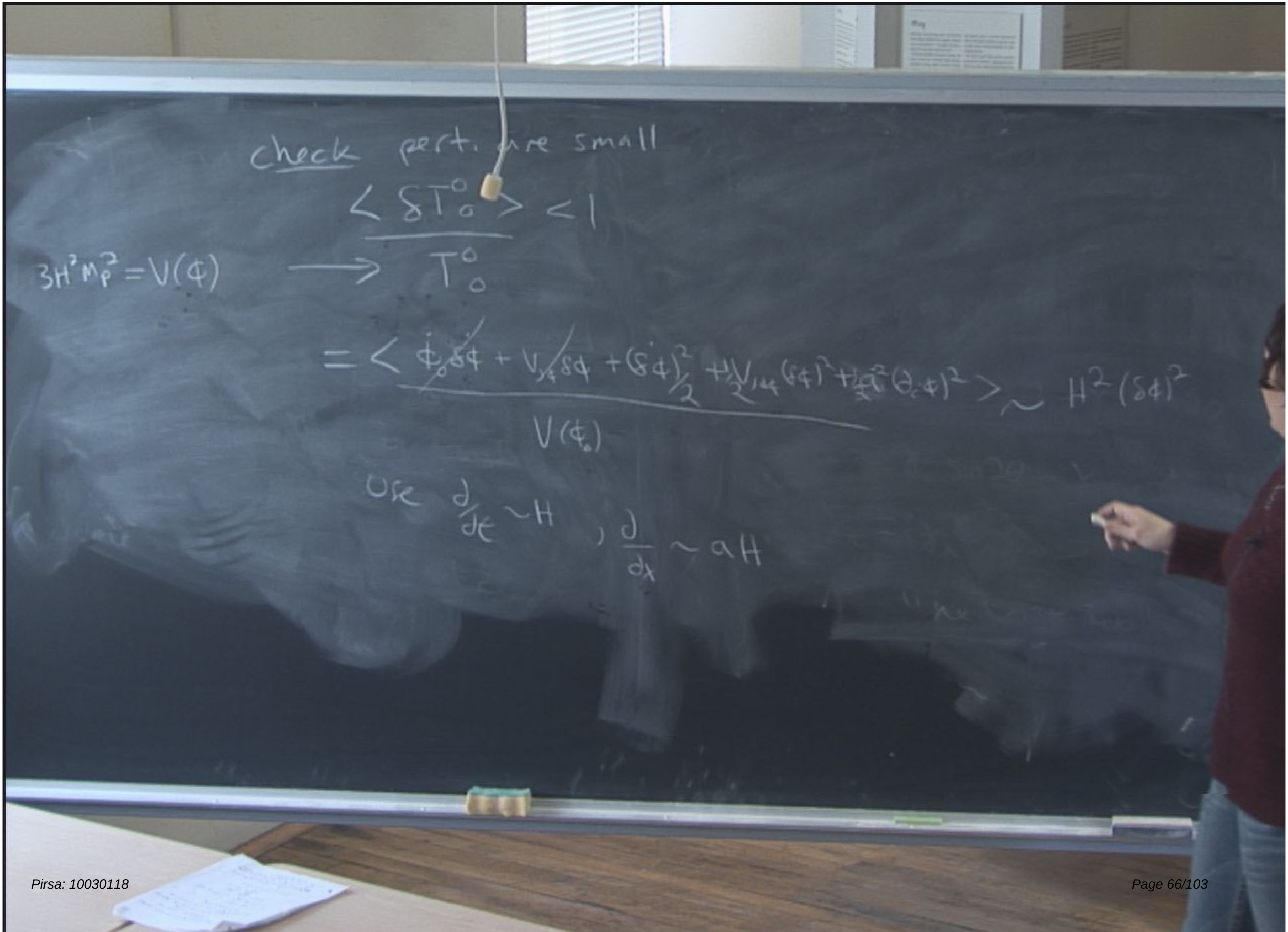

check pert. are small
< δT°o > < 1
T°o
= < φ̇o δφ̇ + V,φ δφ

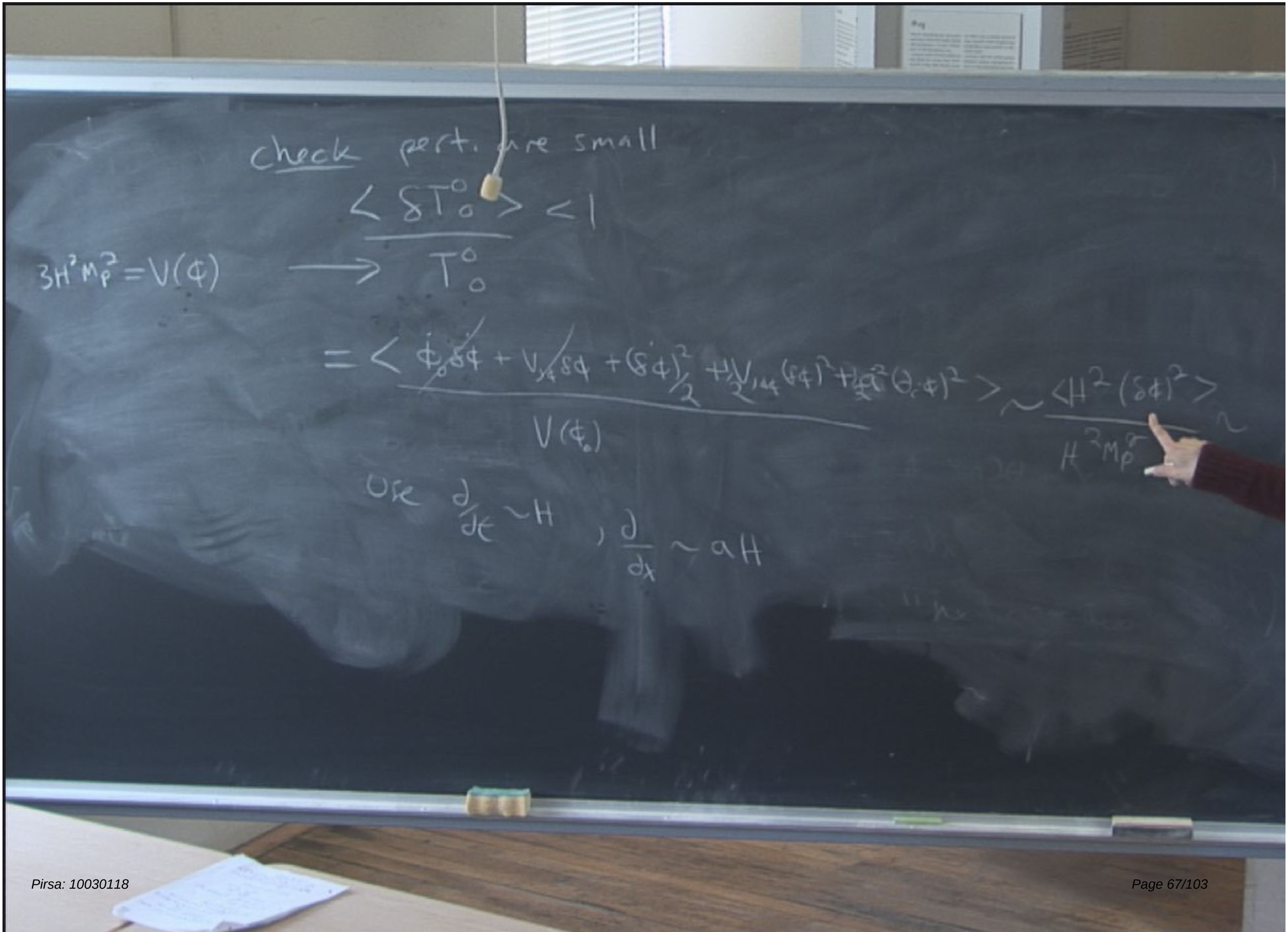

check pert. are small
$\frac{\langle \delta T^0_0 \rangle}{T^0_0} < 1$
$= V(\phi)$
$= \frac{\langle \dot{\phi}_0 \delta\dot{\phi} + V_{,\phi}\delta\phi + (\delta\dot{\phi})^2/2 + \frac{1}{2}V_{,\phi\phi}(\delta\phi)^2 \rangle}{V(\phi_0)}$

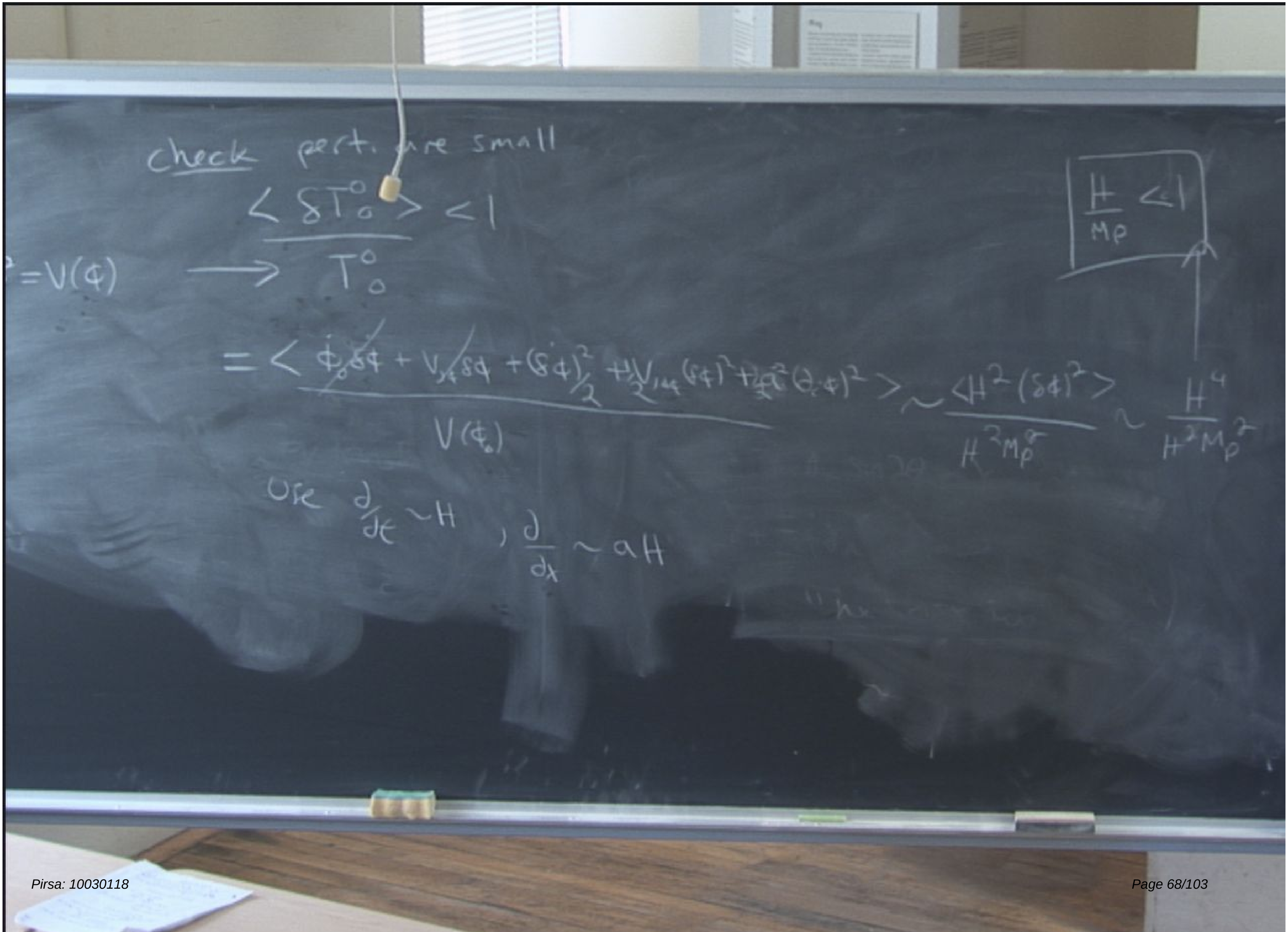

check pert. are small
< δT⁰₀ > < 1
T⁰₀
V(ϕ)


check pert. are small
< δT^0_0 > < 1
T^0_0
V(ϕ_0)
use ∂/∂t ~ H


check pert. are small
< δT^0_0 > < 1
T^0_0
H^2 M_p^2 = V(φ)
= < φ̇_0 δφ̇ + V_φ δφ + (δφ̇)^2/2 + 1/2 V_φφ (δφ)^2 + 1/2 a^-2 (∂_i φ)^2 >
V(φ_0)
Use ∂/∂t ~ H , ∂/∂x ~ aH


check pert. are small
$\frac{\langle \delta T^0_0 \rangle}{T^0_0} < 1$
$3H^2 M_p^2 = V(\phi)$
$= \frac{\langle \dot\phi_0 \dot{\delta\phi} + V_{,\phi}\delta\phi + (\dot{\delta\phi})^2/2 + \frac{1}{2}V_{,\phi\phi}(\delta\phi)^2 + \frac{1}{2}a^{-2}(\partial_i\phi)^2 \rangle}{V(\phi_0)} \sim H^2(\delta\phi)^2$
use $\partial/\partial t \sim H$, $\partial/\partial x \sim aH$


check pert. are small
$\frac{\langle \delta T^0_0 \rangle}{T^0_0} < 1$
$3H^2 M_P^2 = V(\phi)$
$= \frac{\langle \dot\phi_0 \dot{\delta\phi} + V_{,\phi}\delta\phi + (\dot{\delta\phi})^2/2 + \frac{1}{2}V_{,\phi\phi}(\delta\phi)^2 + \frac{1}{2}a^{-2}(\partial_i\phi)^2 \rangle}{V(\phi_0)} \sim \frac{\langle H^2 (\delta\phi)^2 \rangle}{H^2 M_P^2}$
Use $\partial/\partial t \sim H$, $\partial/\partial x \sim aH$


check pert. are small
< δT00 > < 1
T00
= V(φ) →
= < φ̇0 δφ̇ + Vφ δφ + (δφ̇)²/2 + ½ Vφφ (δφ)² + … >
V(φ0)
~ <H² (δφ)²> / H² Mp² ~ H⁴ / H² Mp²
use ∂/∂t ~ H , ∂/∂x ~ aH
H/Mp << 1

$$\gamma_{ij}(k) = \sum_{s=+} \epsilon^s_{ij}(\hat{k})\,\gamma^s_{ij}(k)$$

$$\epsilon_{ij} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

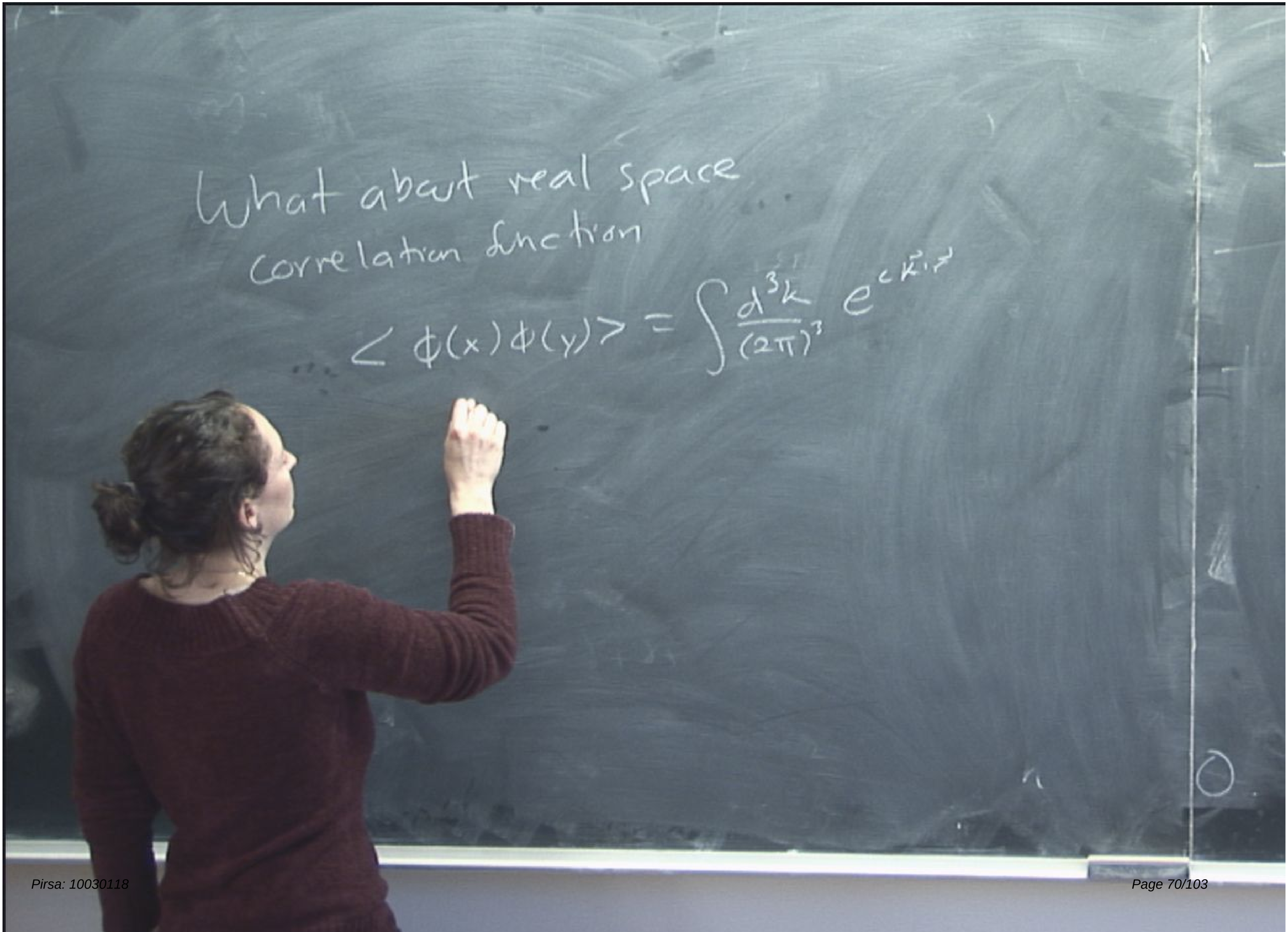
$$\gamma_{ij}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\vec{x}} \sum_{s\pm} \epsilon^s_{ij}(\hat{k})\,\gamma^s_{ij}(k)$$

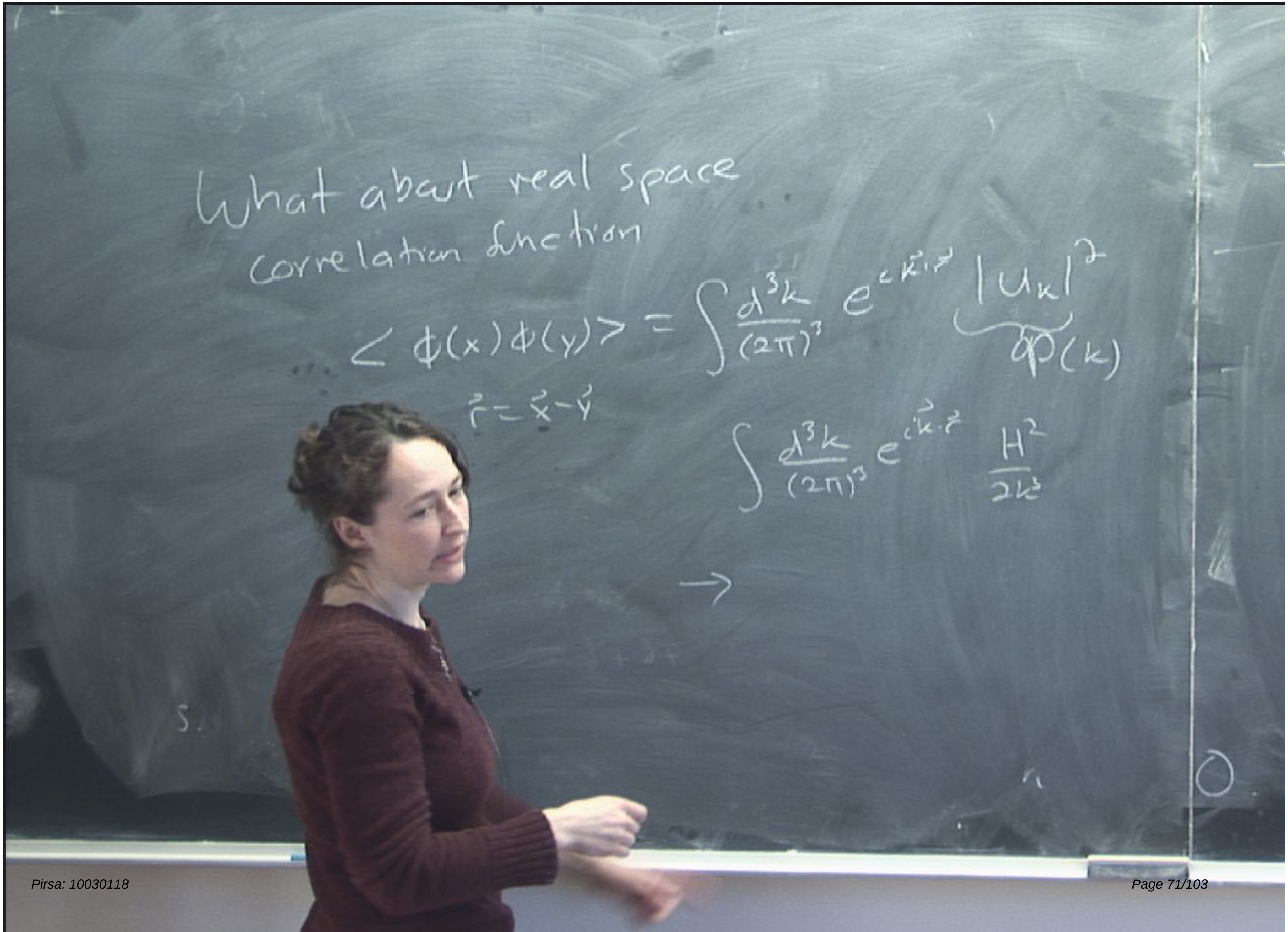
↳ Now use this in the action

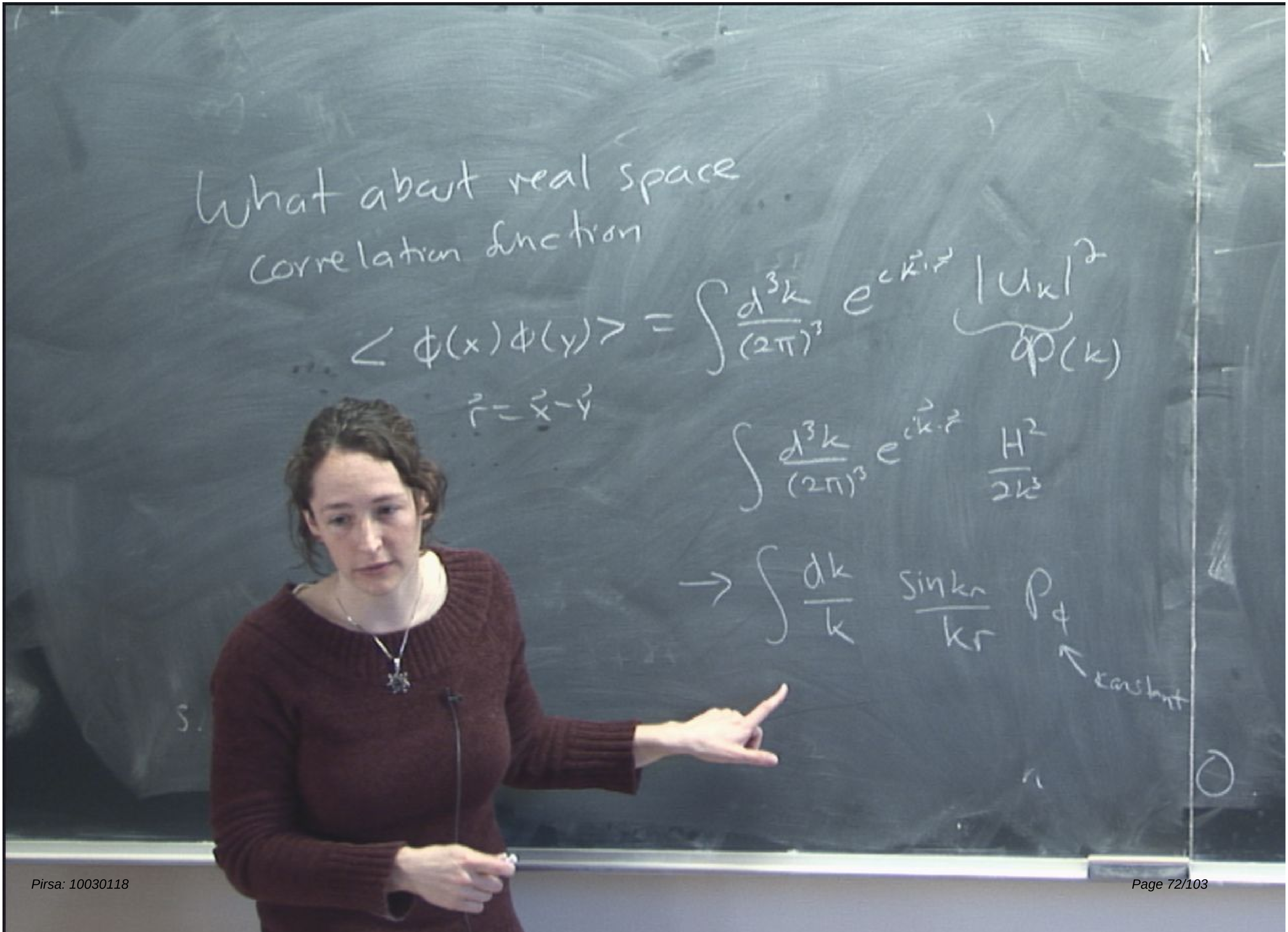
Canonical field $\hat{\gamma}_{ij} = \frac{M_p}{2}\gamma_{ij}$

$$\langle \hat{\gamma}_+^{(k_1)} \hat{\gamma}_+^{(k_2)} \rangle = \langle \hat{\gamma}_- \hat{\gamma}_- \rangle = \frac{H^2}{2k^3}$$

$$\langle \gamma_+^1 \gamma_+^2 \rangle = \frac{H^2}{M_p^2}\frac{2}{k^3}$$


What about real space
correlation function
$\langle \phi(x)\phi(y) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}}$


What about real space
correlation function
$\langle \phi(x)\phi(y) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} |u_k|^2$
$P(k)$
$\vec{r} = \vec{x} - \vec{y}$
$\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{H^2}{2k^3}$


What about real space
correlation function
$\langle \phi(x)\phi(y)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} |u_k|^2$
$\mathcal{P}(k)$
$\vec{r} = \vec{x} - \vec{y}$
$\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{H^2}{2k^3}$
$\rightarrow \int \frac{dk}{k} \frac{\sin kr}{kr} \mathcal{P}_\phi$
constant

What about real space correlation function

$$\langle \phi(x)\phi(y)\rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{r}}\, \underbrace{|u_k|^2}_{\wp(k)}$$

$\vec{r} = \vec{x} - \vec{y}$

$$\int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{r}}\, \frac{H^2}{2k^3}$$

$$\rightarrow \int \frac{dk}{k}\, \frac{\sin kr}{kr}\, \rho_\phi$$

$\uparrow$ constant

? $k \to \infty$?

$k \to 0$

What about real space correlation function

$$\langle \phi(x)\phi(y)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \underbrace{|u_k|^2}_{\mathcal{P}(k)}$$

$\vec{r} = \vec{x} - \vec{y}$

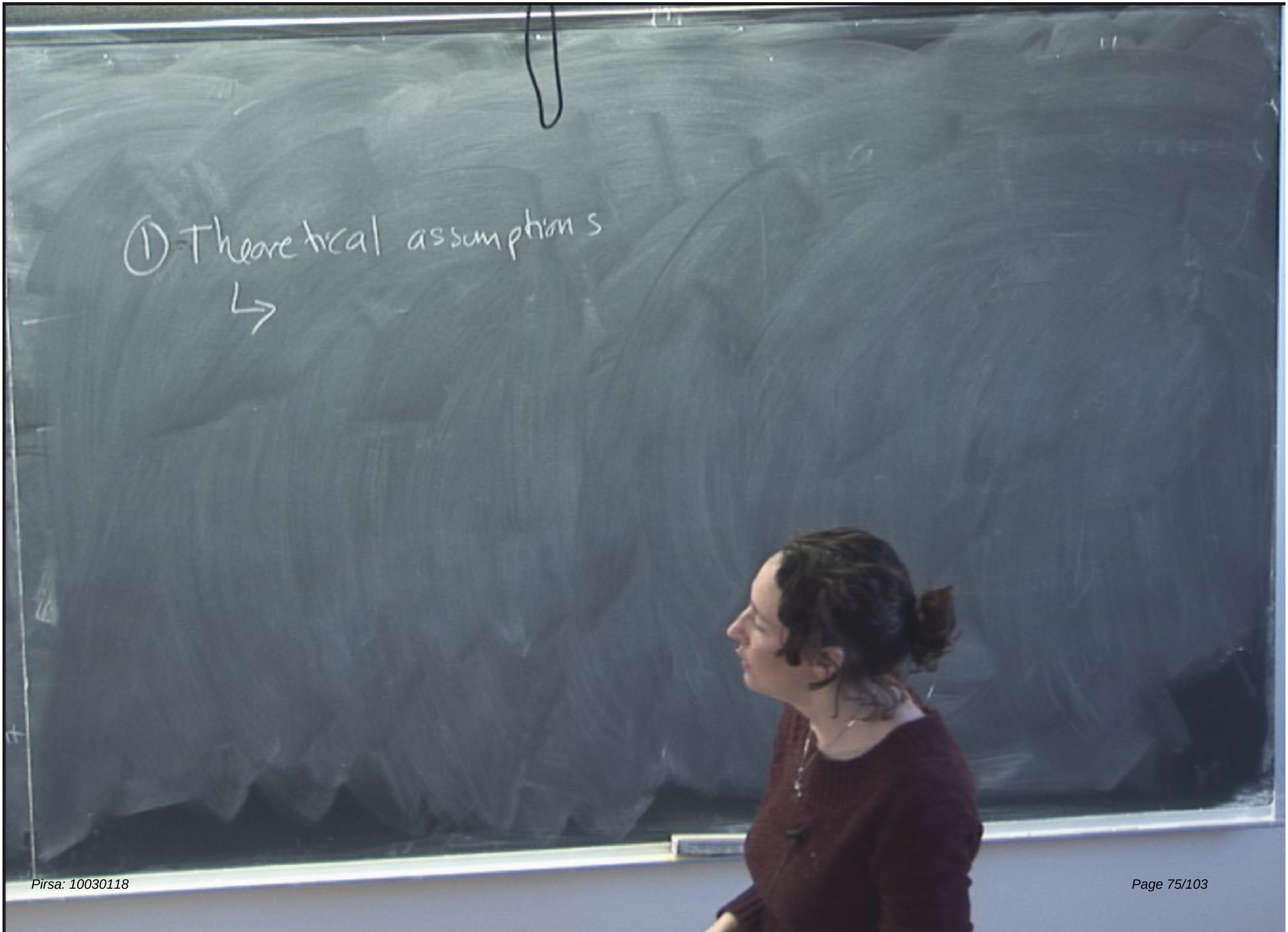
$$\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{H^2}{2k^3}$$

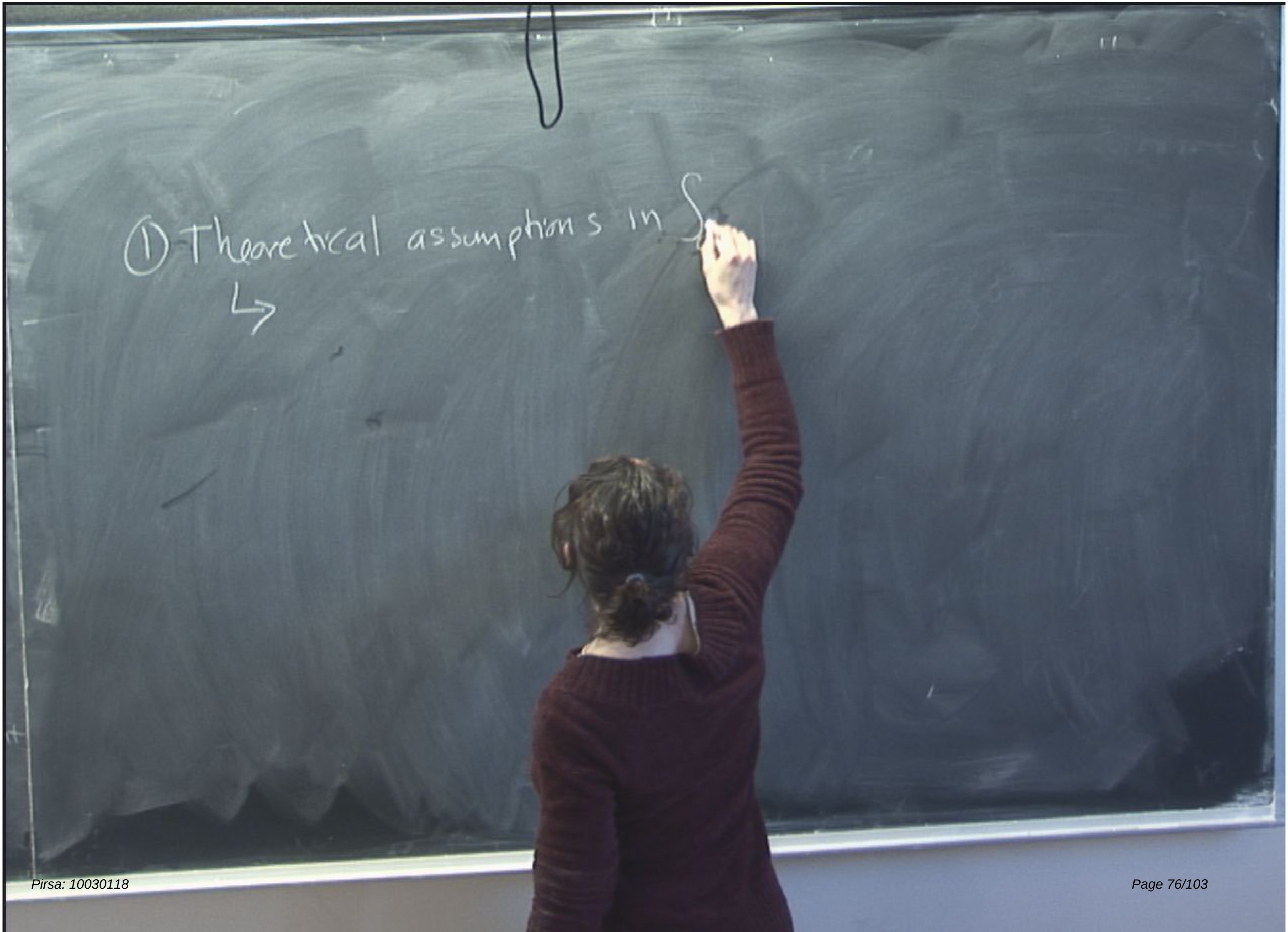
$$\rightarrow \int \frac{dk}{k} \frac{\sin kr}{kr} P_\phi$$

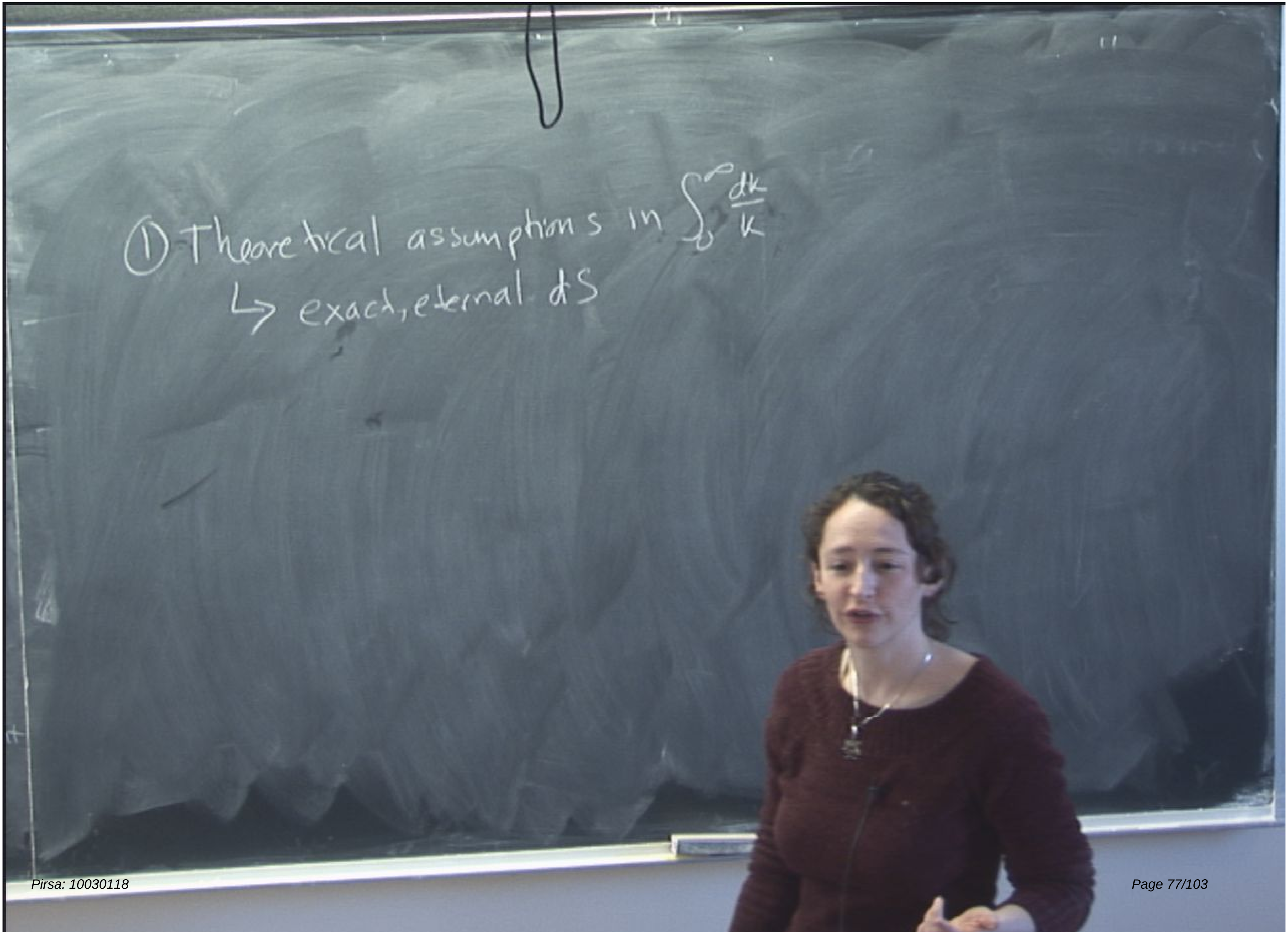
← constant

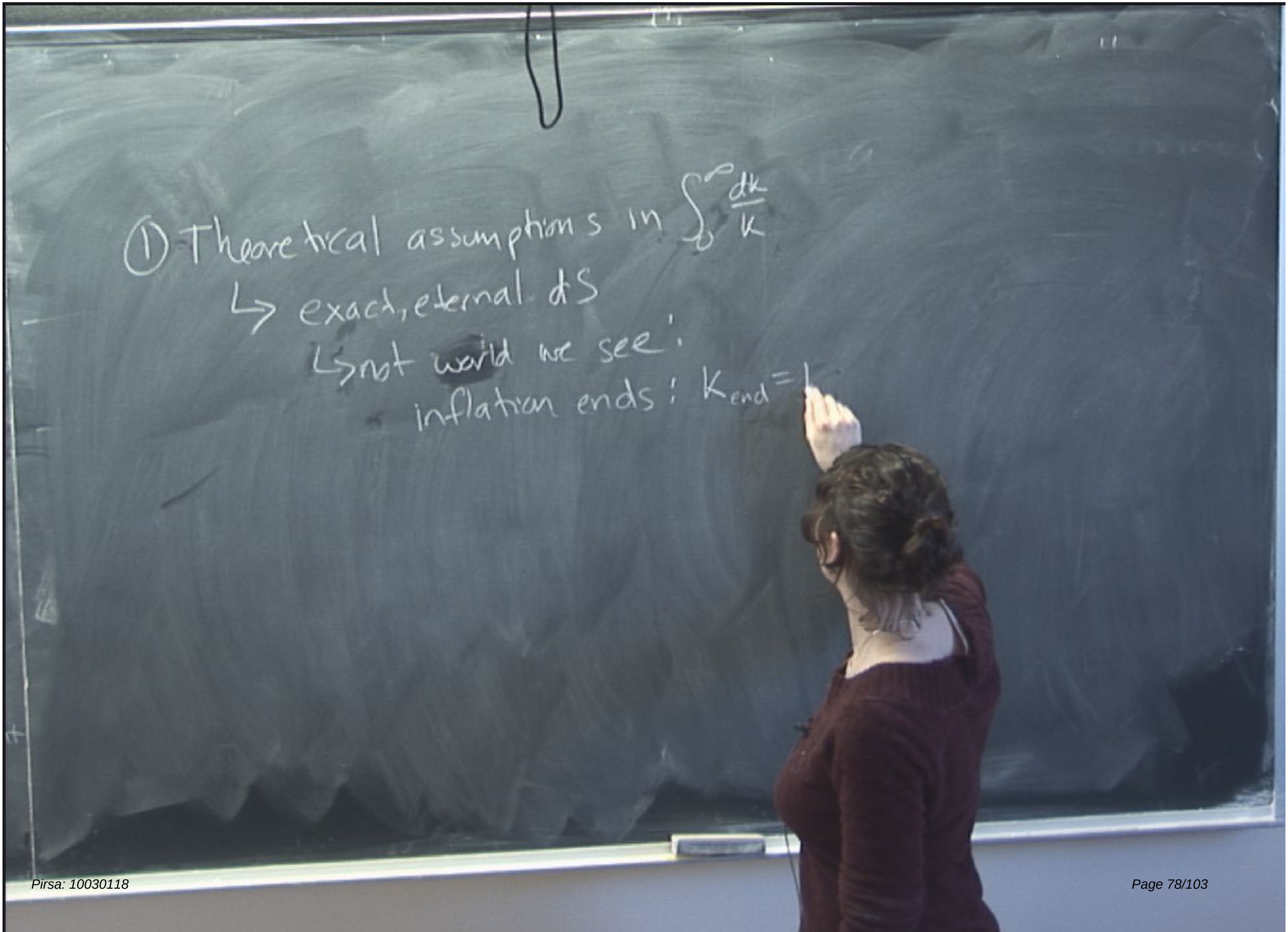
? $k \to \infty$?

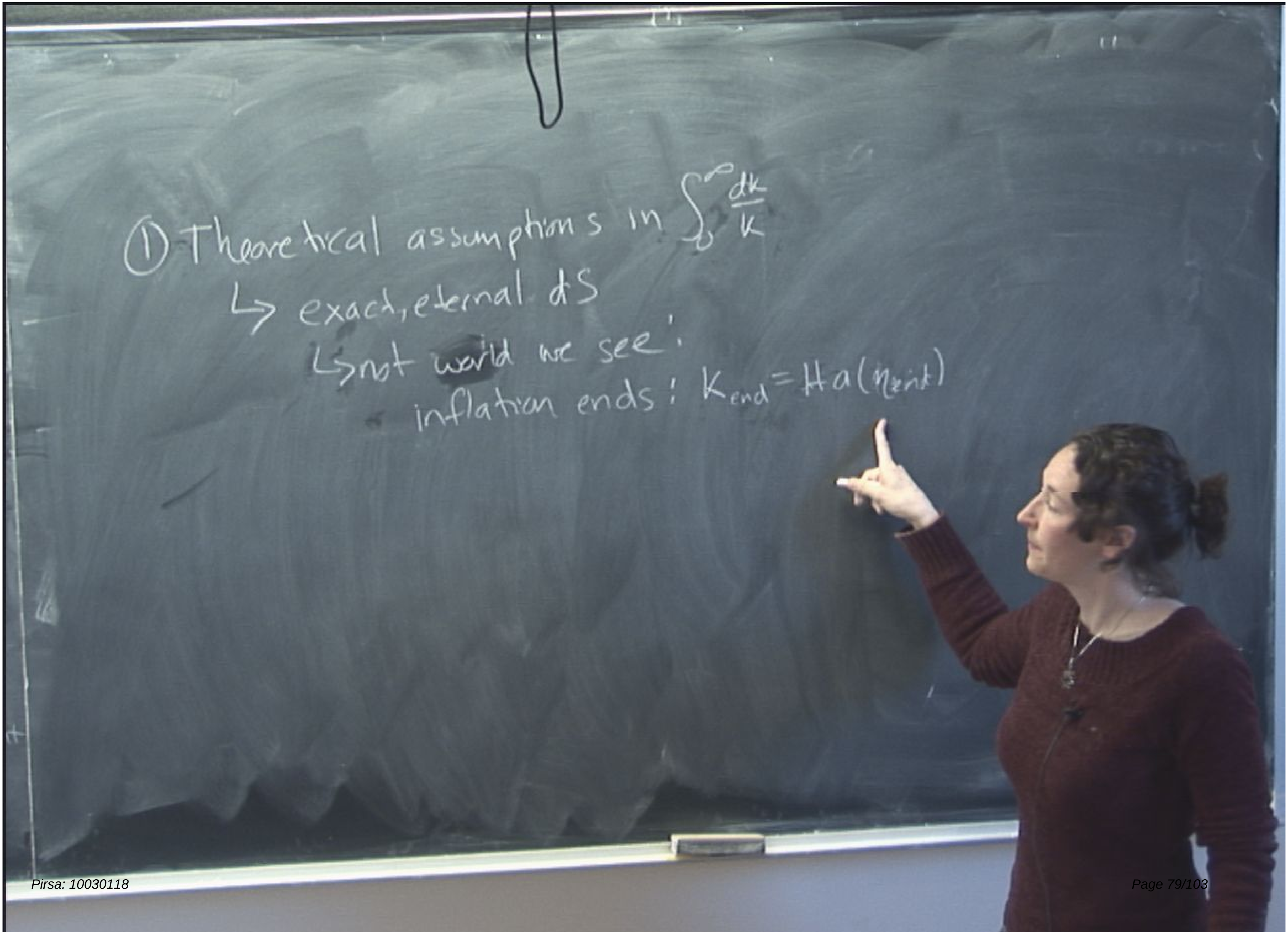
$k \to 0$?

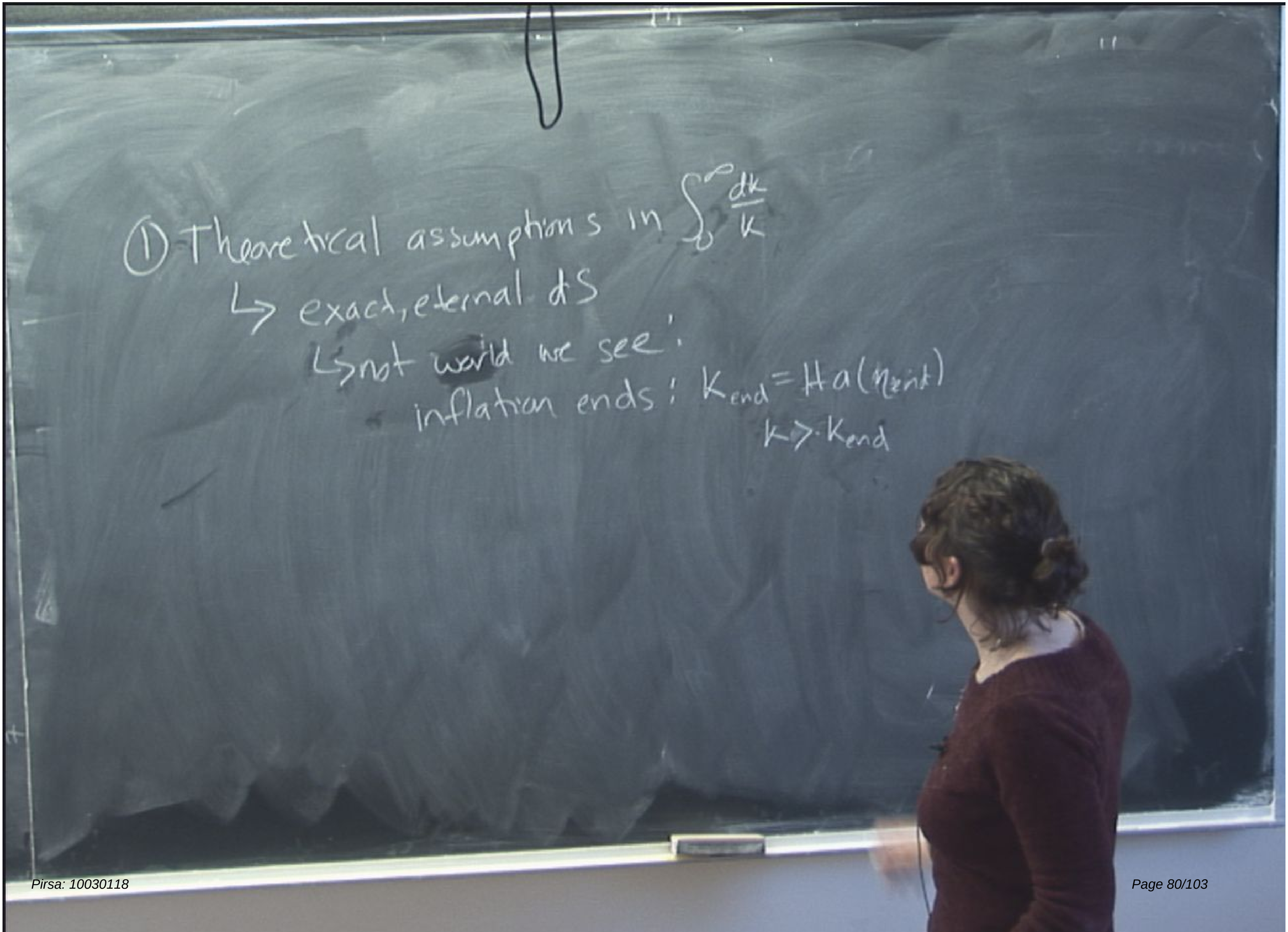

① Theoretical assumptions
↳

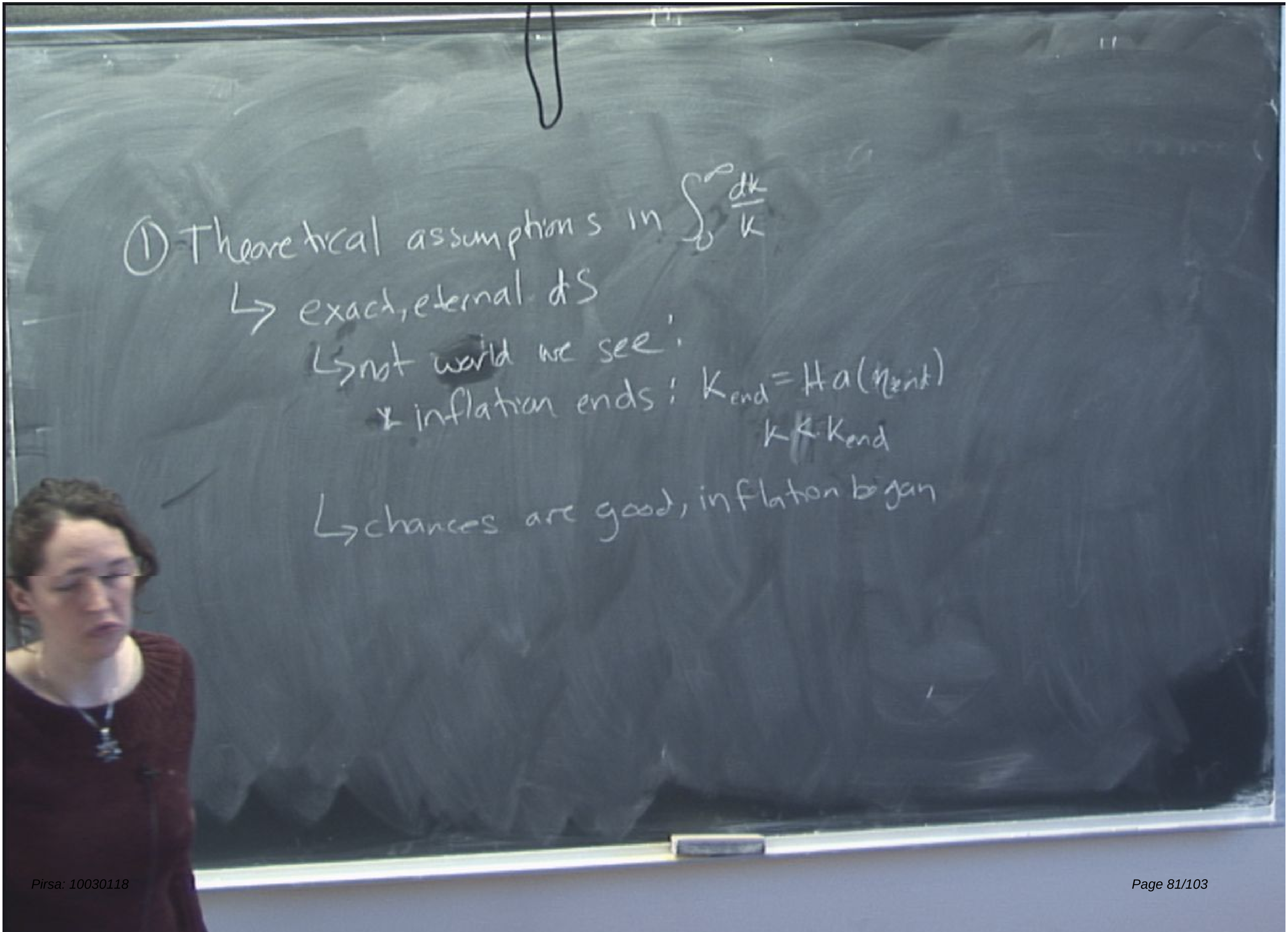

① Theoretical assumptions in ∫
↳

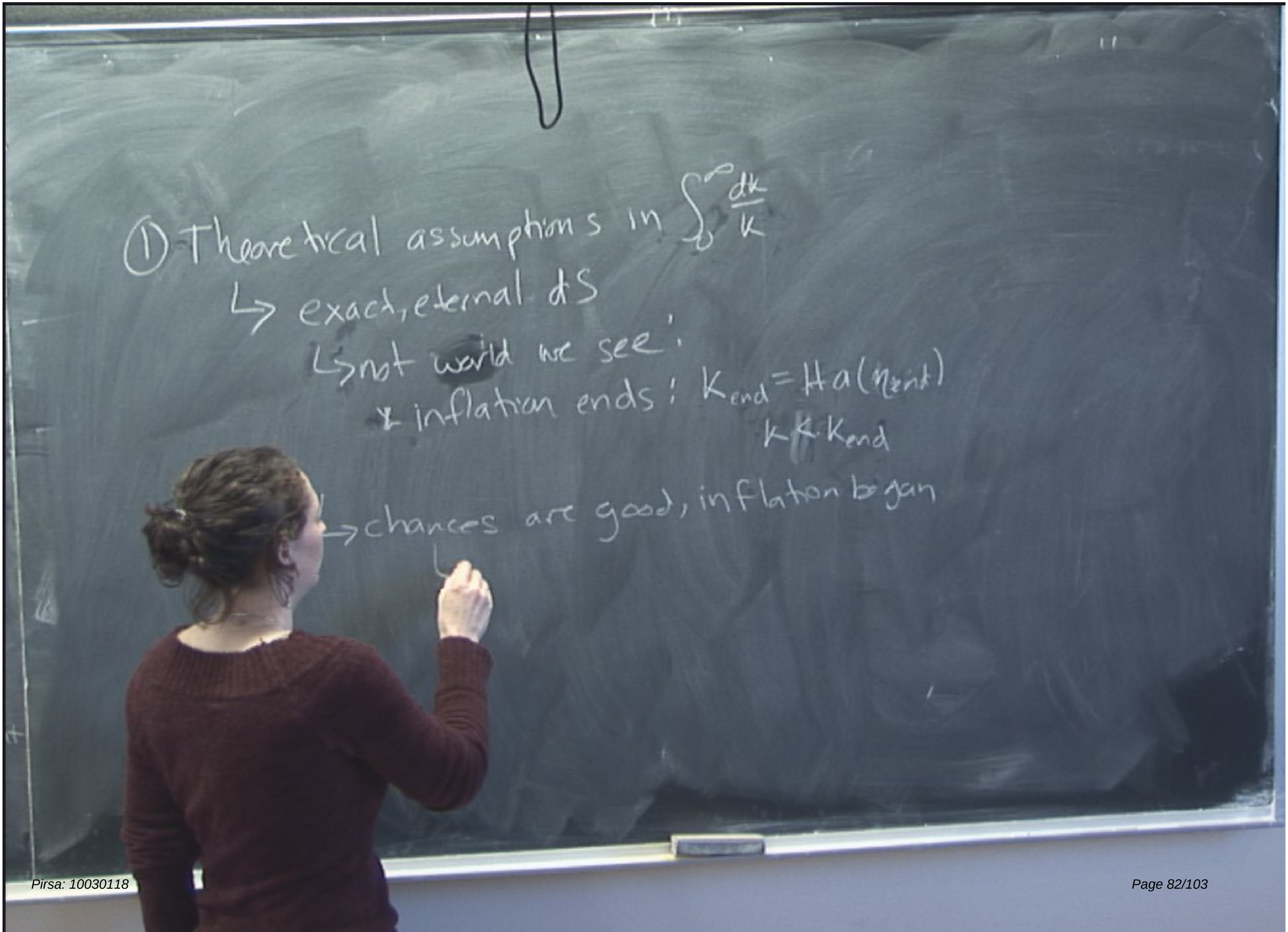

① Theoretical assumptions in $\int_0^\infty \frac{dk}{k}$
↳ exact, eternal dS


① Theoretical assumptions in $\int_0^\infty \frac{dk}{k}$
↳ exact, eternal dS
↳ not world we see:
inflation ends: k_{end} =


① Theoretical assumptions in $\int_0^\infty \frac{dk}{k}$
↳ exact, eternal dS
↳ not world we see:
inflation ends: $K_{end} = Ha(\eta_{end})$


① Theoretical assumptions in $\int_0^\infty \frac{dk}{k}$
↳ exact, eternal dS
↳ not world we see:
inflation ends: $k_{end} = Ha(\eta_{end})$
$k > k_{end}$


① Theoretical assumptions in ∫₀^∞ dk/k
↳ exact, eternal dS
↳ not world we see:
* inflation ends: K_end = H a(η_end)
k ≪ K_end
↳ chances are good, inflation began


① Theoretical assumptions in $\int_0^\infty \frac{dk}{k}$
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↳ not world we see!
* inflation ends: $k_{end} = Ha(\eta_{end})$
$k \ll k_{end}$
↳ chances are good, inflation began

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↳ not world we see:

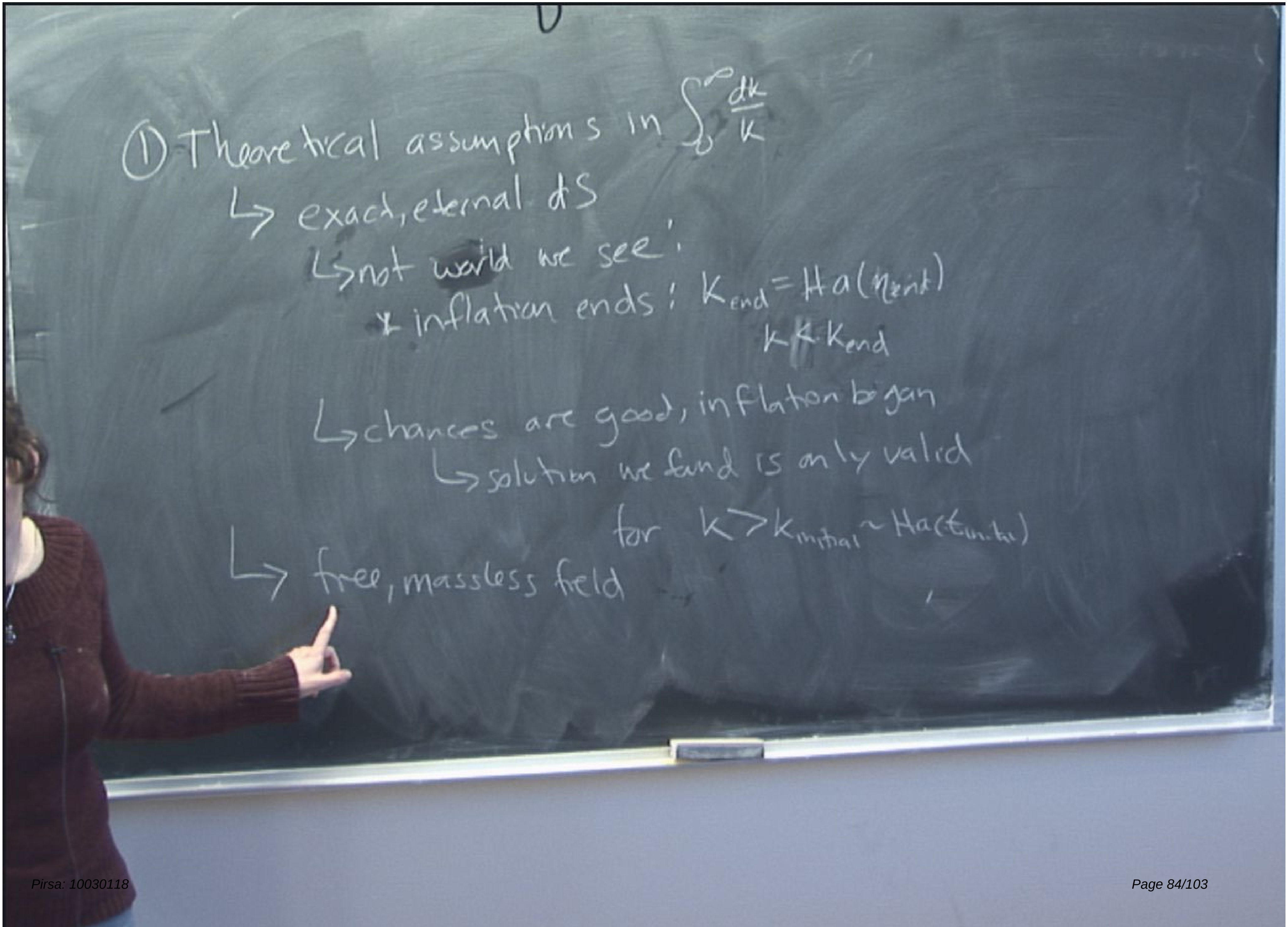
& inflation ends: $k_{end} = Ha(\eta_{end})$

$k \ll k_{end}$

↳ chances are good, inflation began

↳ solution we found is only valid

for $k > k_{initial} \sim Ha(t_{initial})$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$k \ll K_{end}$

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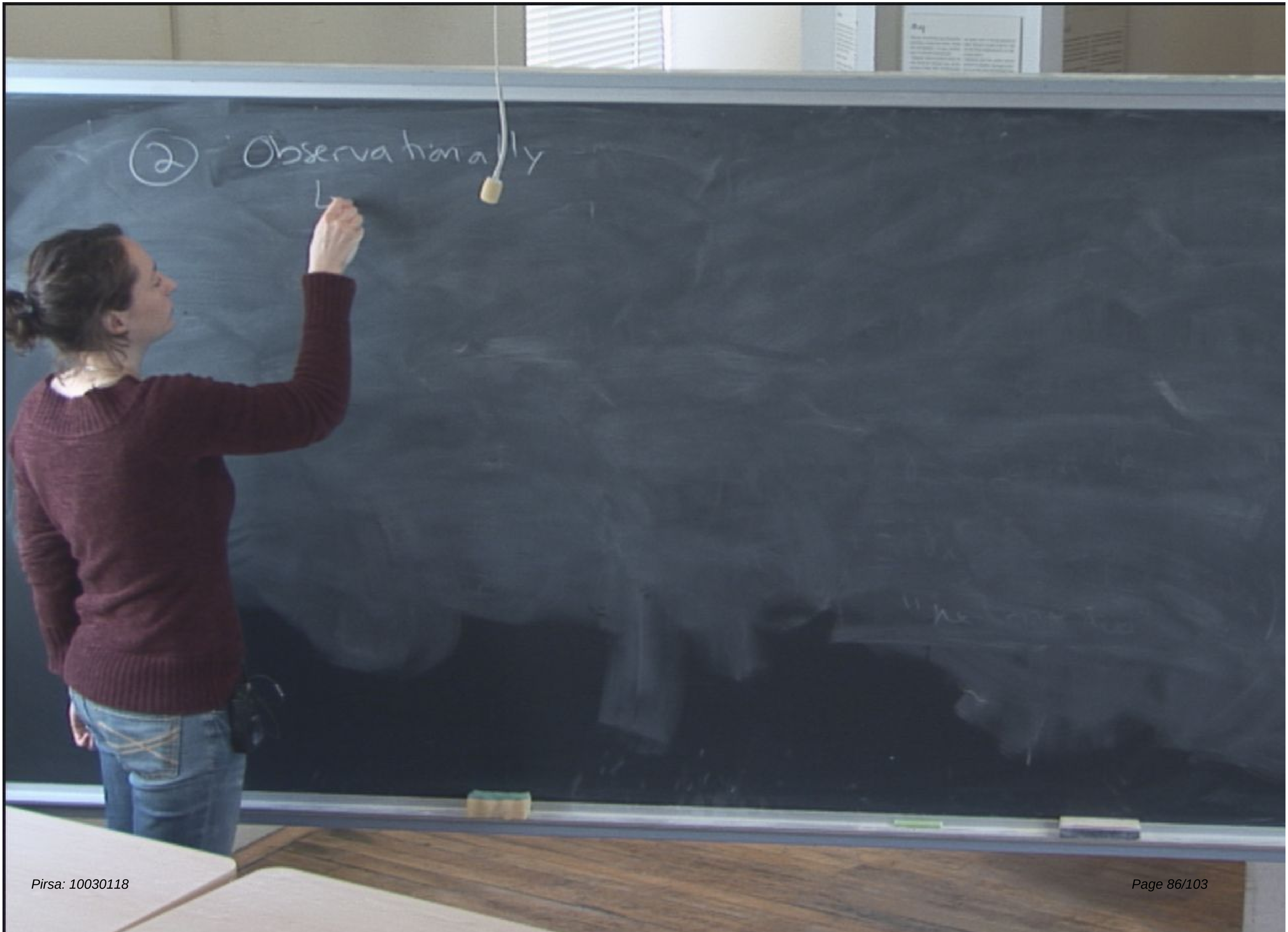
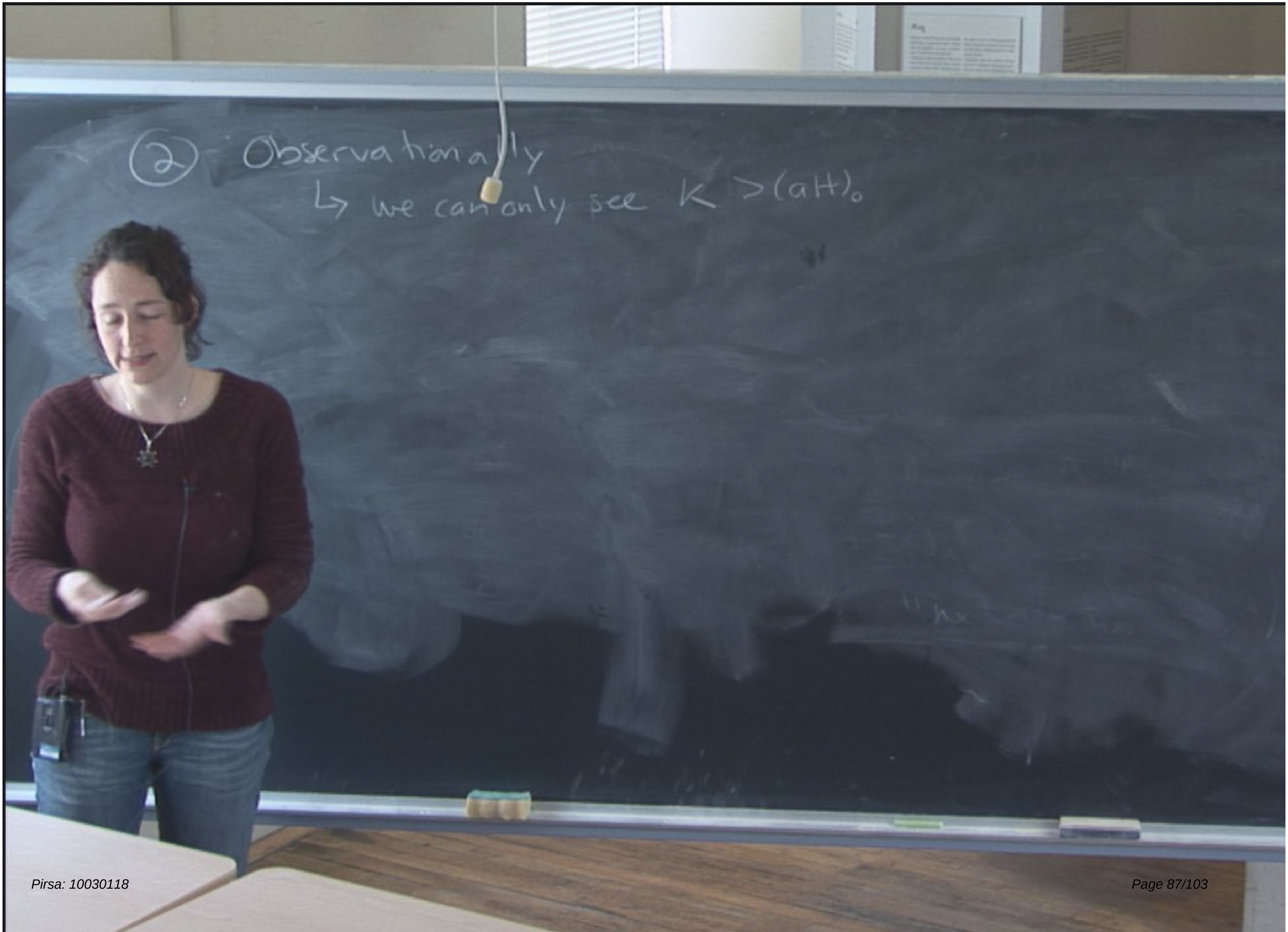
↳ solution we found is only valid

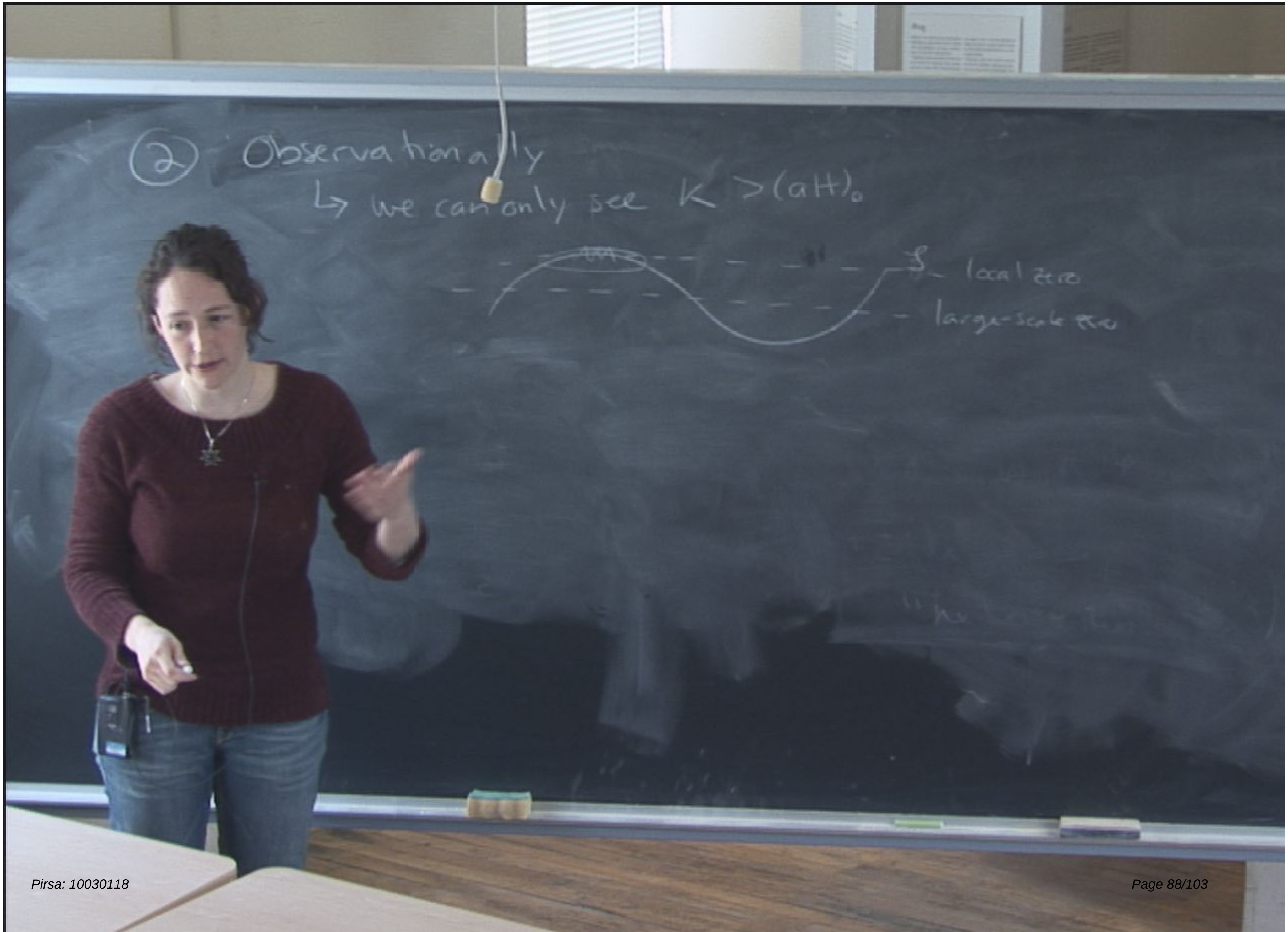
for $k > k_{initial} \sim Ha(t_{initial})$

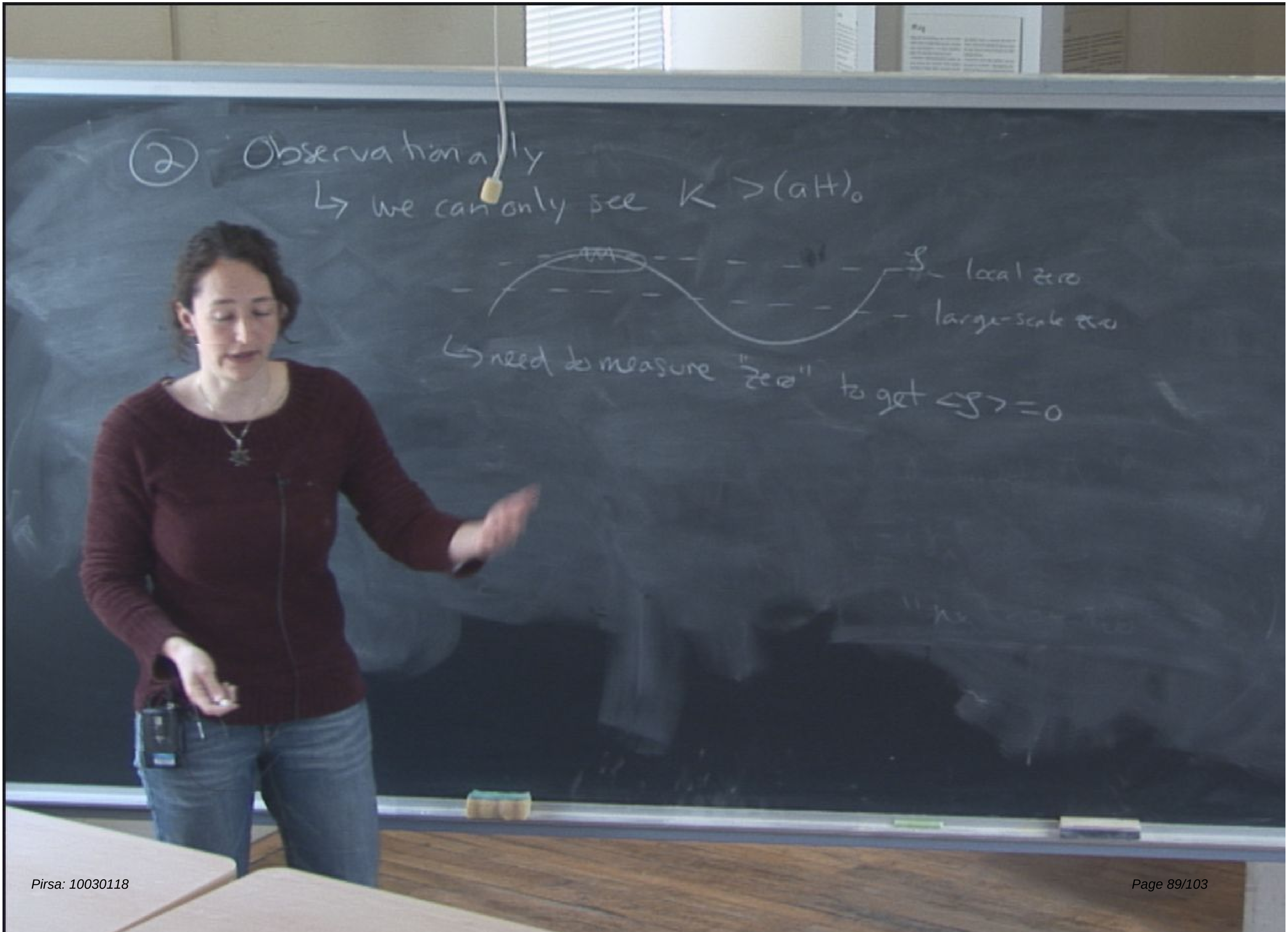
↳ free, massless field

saw, massive field $\int \frac{dk}{k} P_\phi'(-k\eta)^\delta$, $\delta = \frac{m^2}{3H^2} > 0$

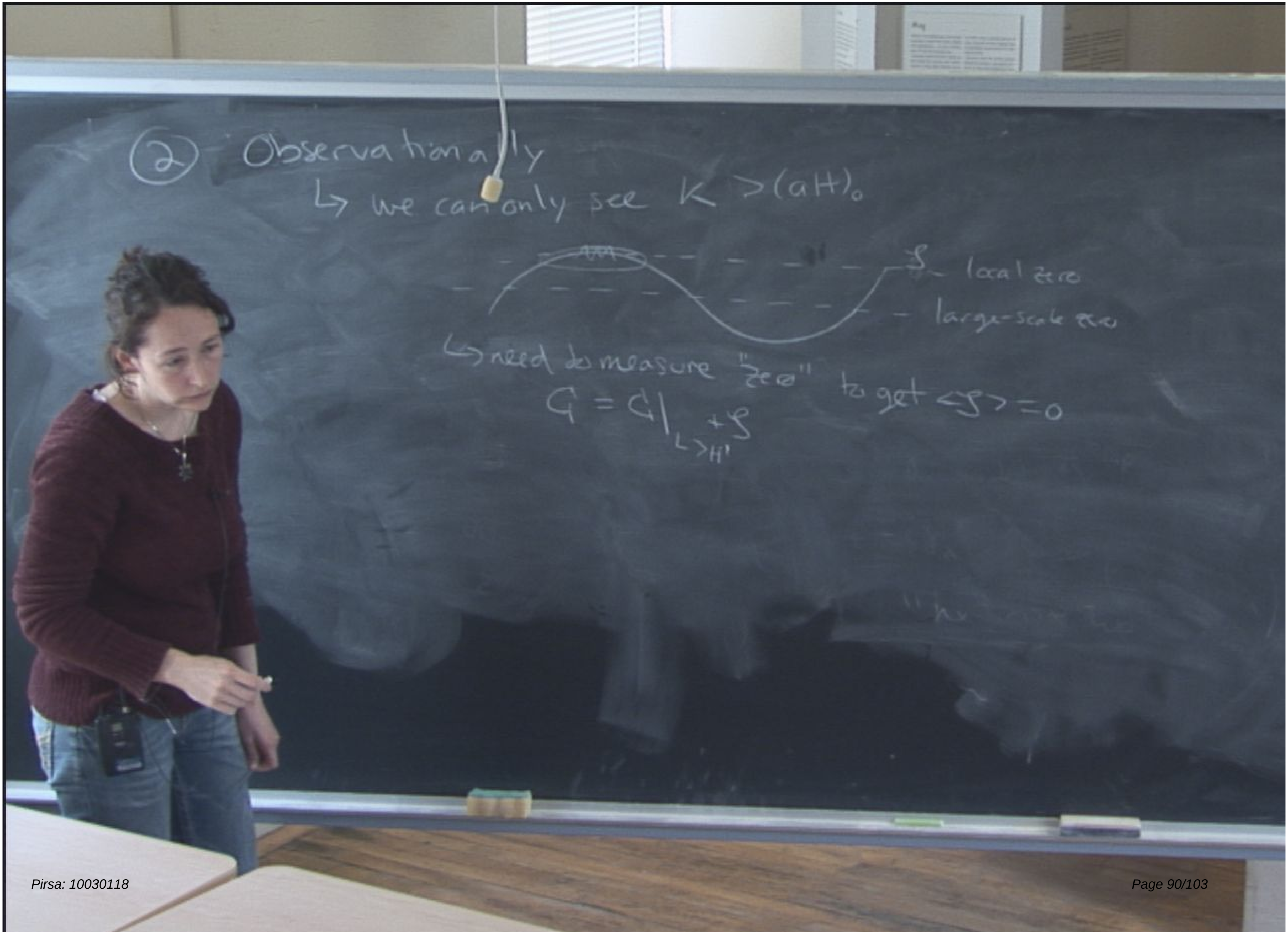

② Observationally


② Observationally
↳ we can only see K > (aH)₀


② Observationally
↳ we can only see K >(aH)₀
local zero
large-scale zero



② Observationally
↳ we can only see K > (aH)₀
local zero
large-scale zero
↳ need to measure "zero" to get <ζ> = 0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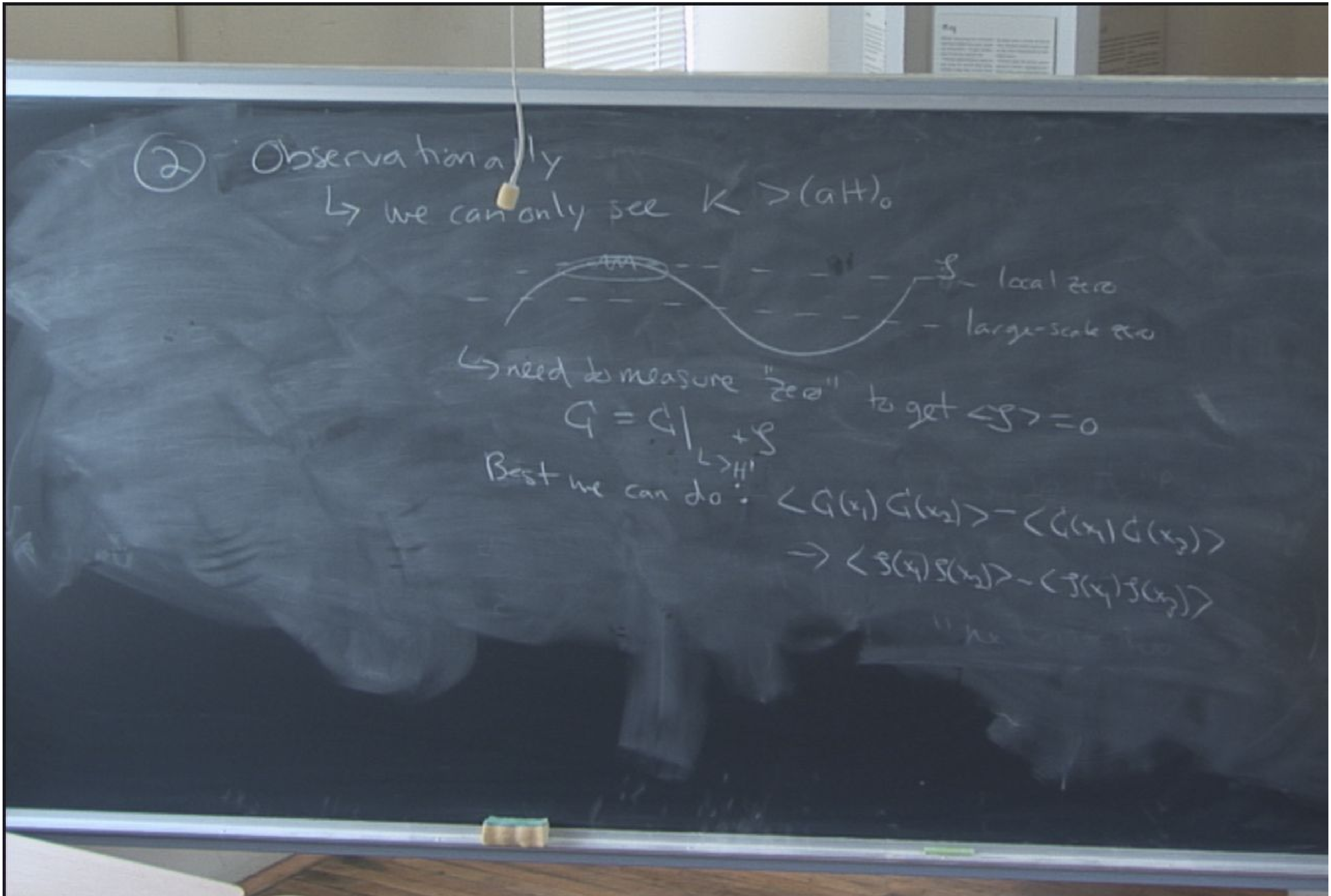
② Observationally
↳ we can only see K > (aH)₀
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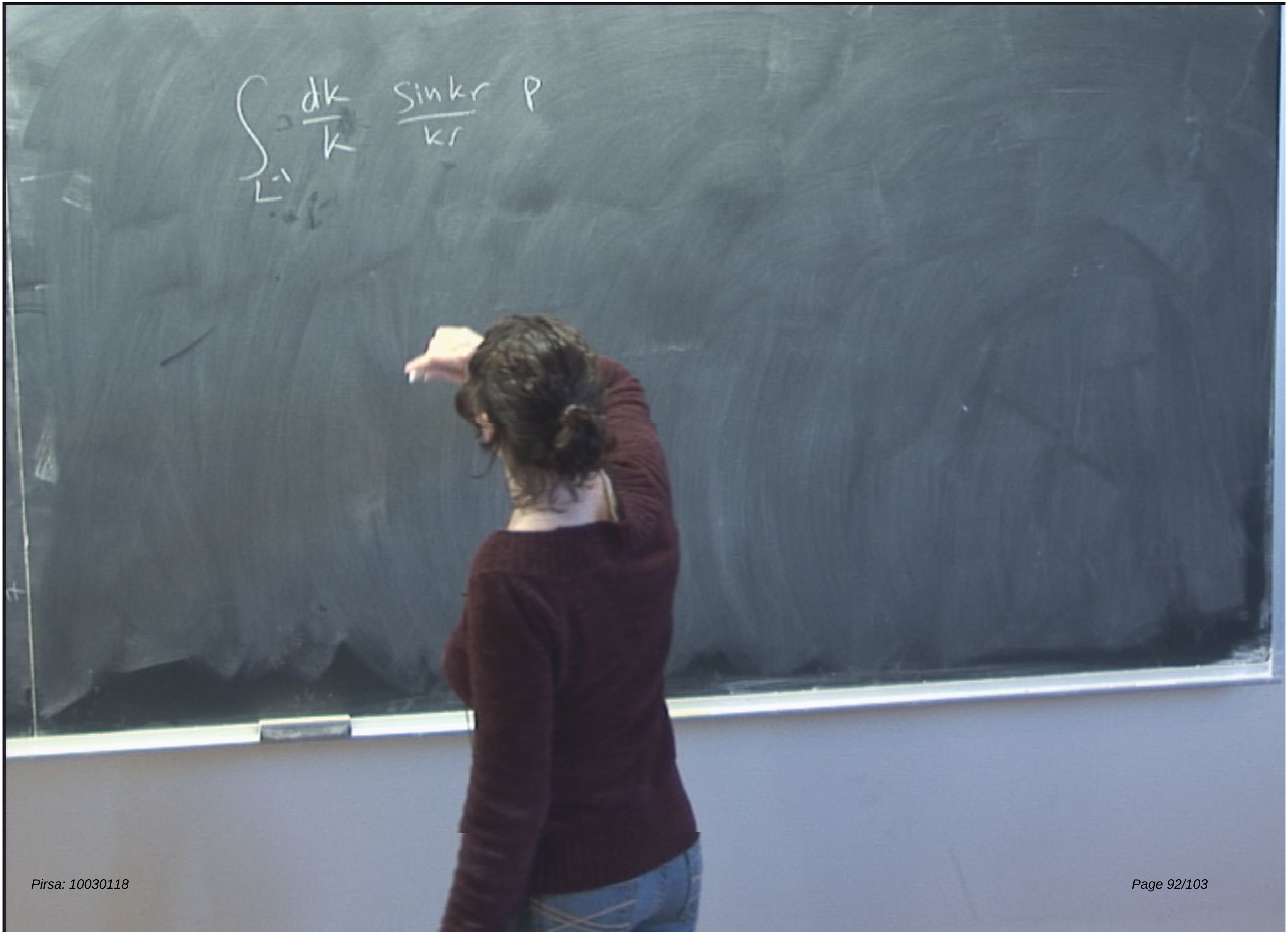
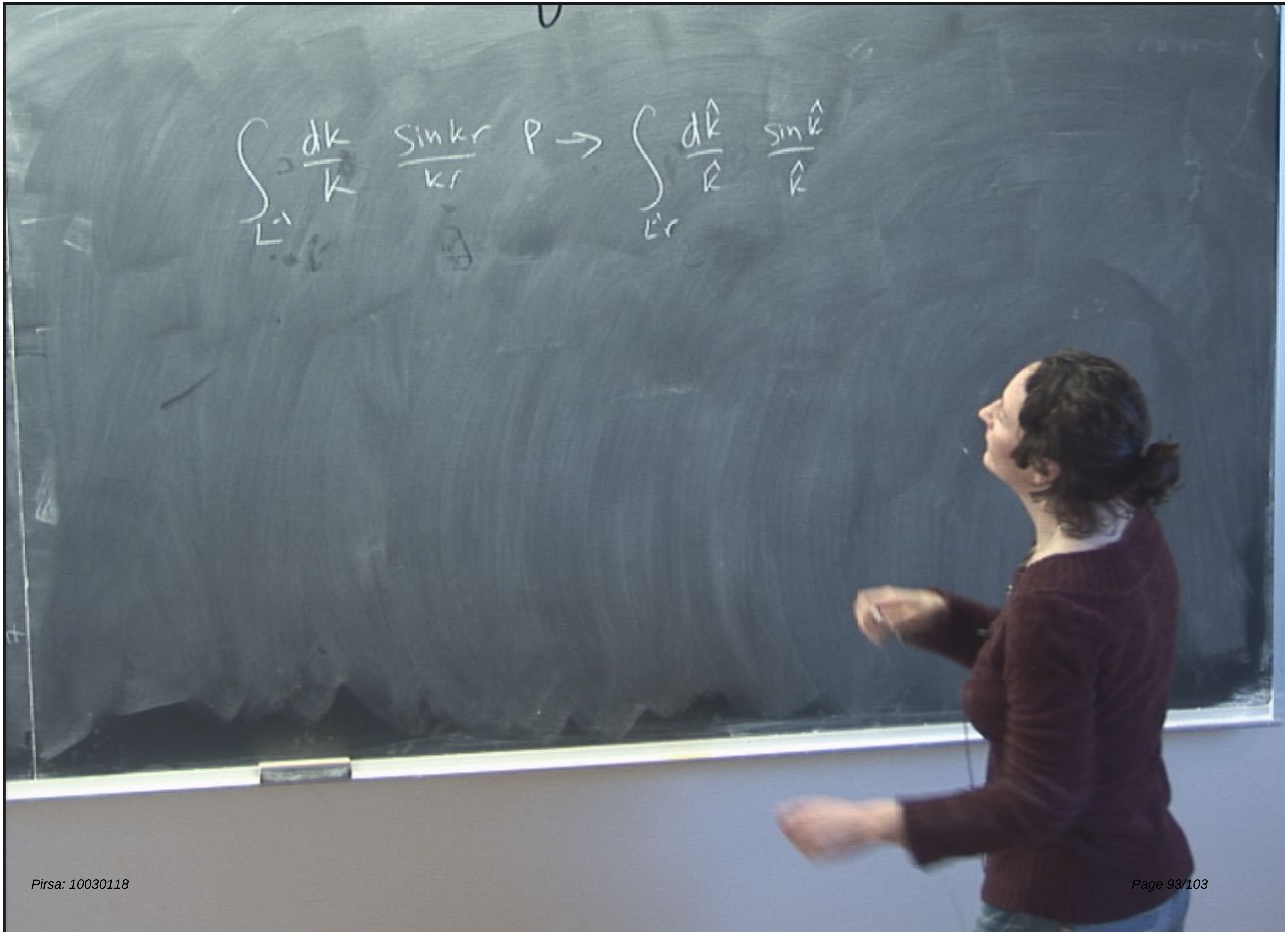
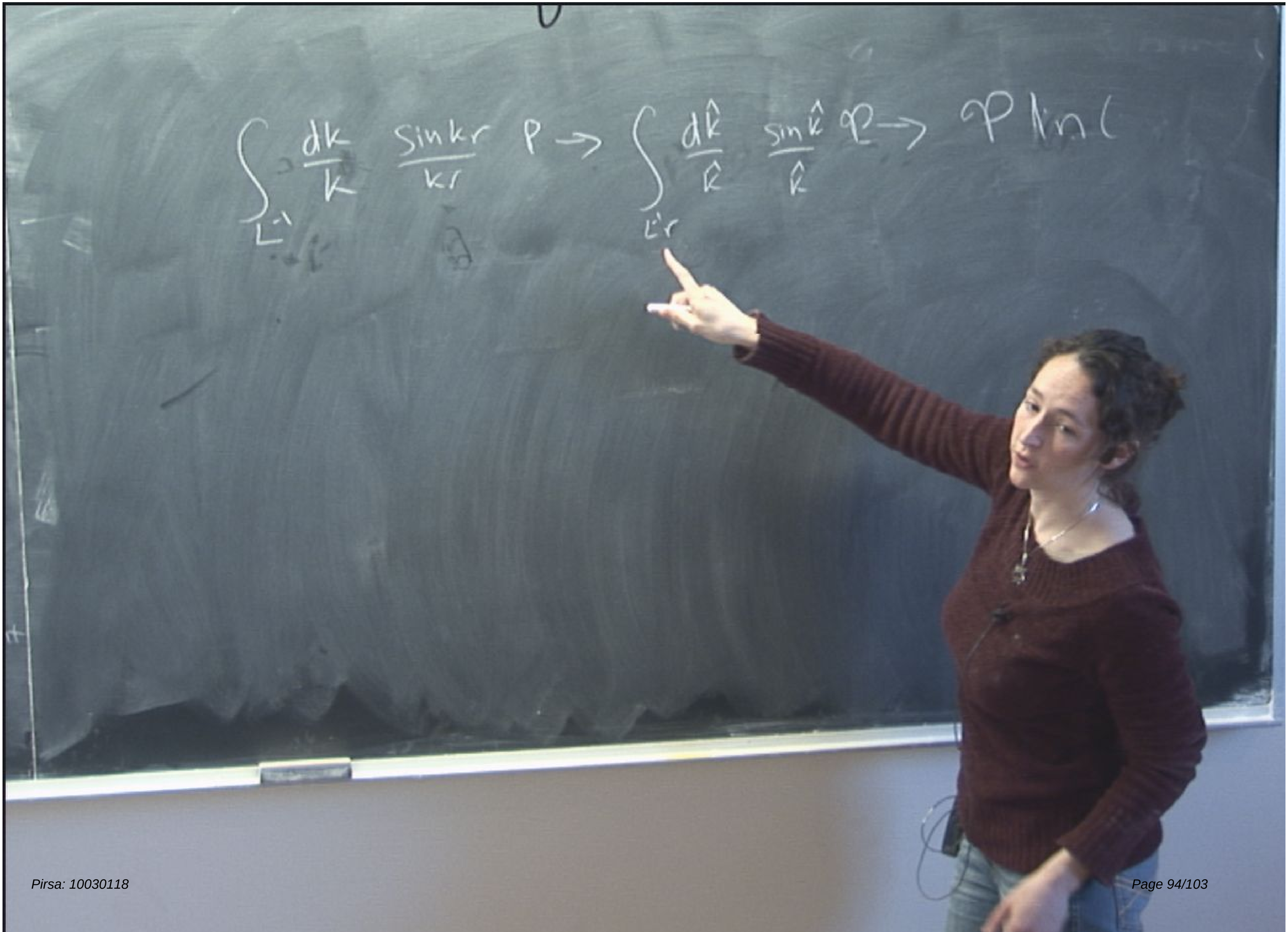
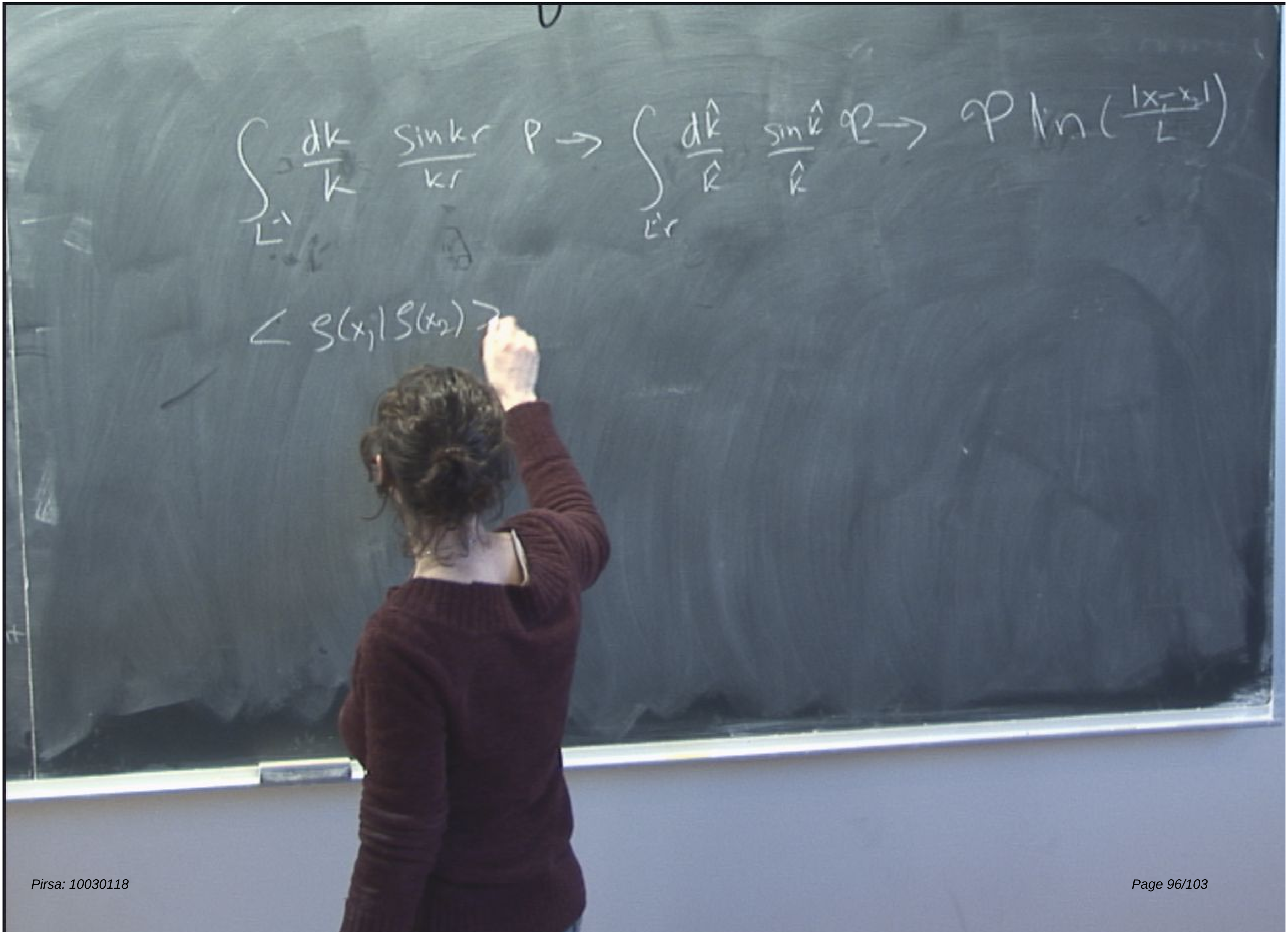
— large-scale zero
↳ need to measure "zero" to get <ζ> = 0
G = G| + ζ
L>H'
Best we can do: <G(x1) G(x2)> − <G(x1) G(x2)>
→ <ζ(x1) ζ(x2)> − <ζ(x1) ζ(x2)>

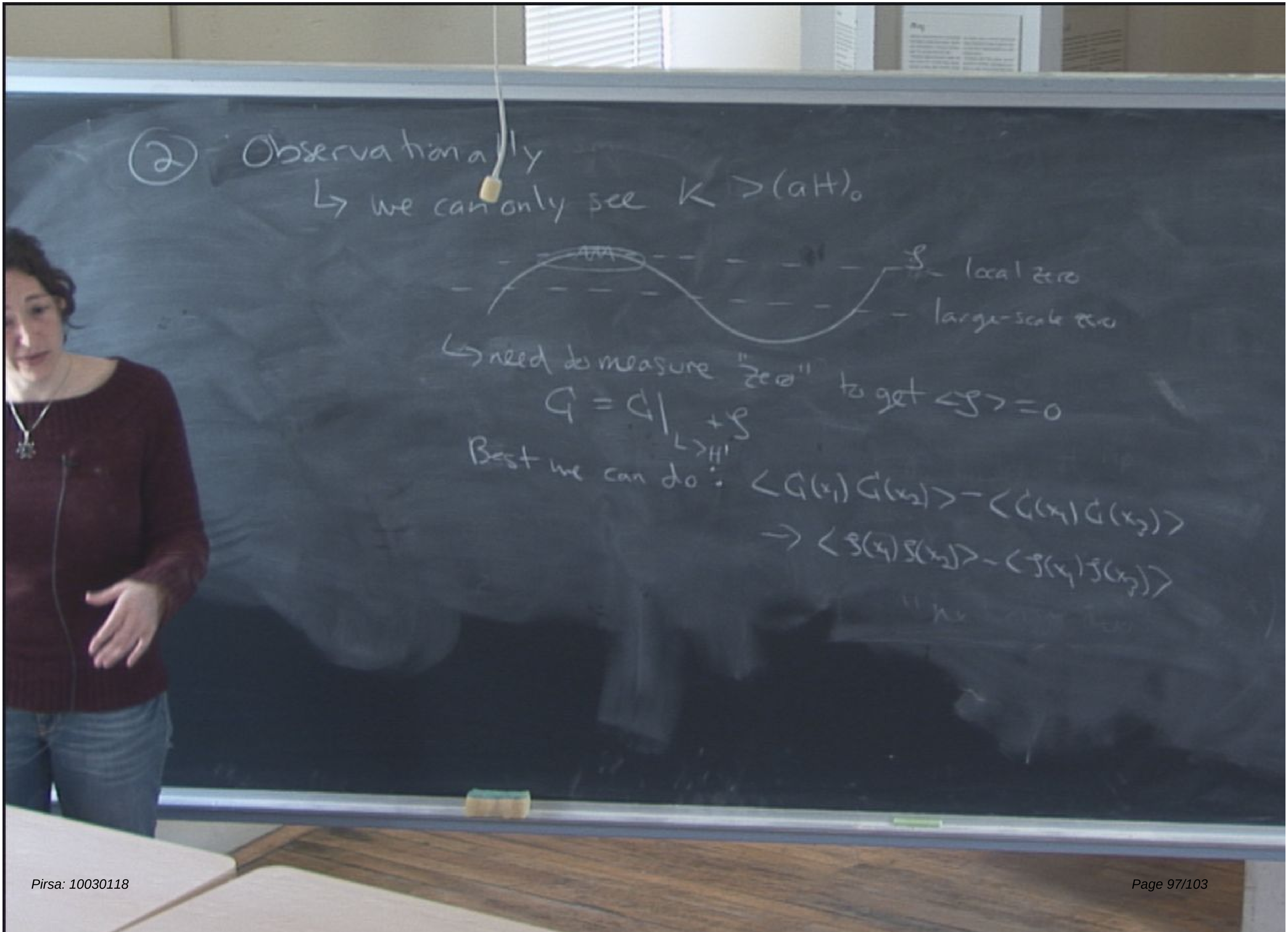

$\int_{L^{-1}} \frac{dk}{k} \frac{\sin kr}{kr} P$


$\int_{L^{-1}} \frac{dk}{k} \frac{\sin kr}{kr} P \to \int_{L^{-1}r} \frac{d\hat{k}}{\hat{k}} \frac{\sin \hat{k}}{\hat{k}}$


$\int_{L^{-1}} \frac{dk}{k} \frac{\sin kr}{kr} P \rightarrow \int_{L^{-1}r} \frac{d\hat{k}}{\hat{k}} \frac{\sin \hat{k}}{\hat{k}} P \rightarrow P \ln($

$$\int_{L^{-1}} \frac{dk}{k}\,\frac{\sin kr}{kr}\,P \to \int_{L^{-1}r} \frac{d\hat{k}}{\hat{k}}\,\frac{\sin \hat{k}}{\hat{k}}\,P \to P \ln\left(\frac{|x_1 - x_2|}{L}\right)$$


$\int_{L^{-1}} \frac{dk}{k} \frac{\sin kr}{kr} P \to \int_{L^{-1}r} \frac{d\hat{k}}{\hat{k}} \frac{\sin \hat{k}}{\hat{k}} P \to P \ln\left(\frac{|x_1 - x_2|}{L}\right)$
$\langle \rho(x_1) \rho(x_2) \rangle$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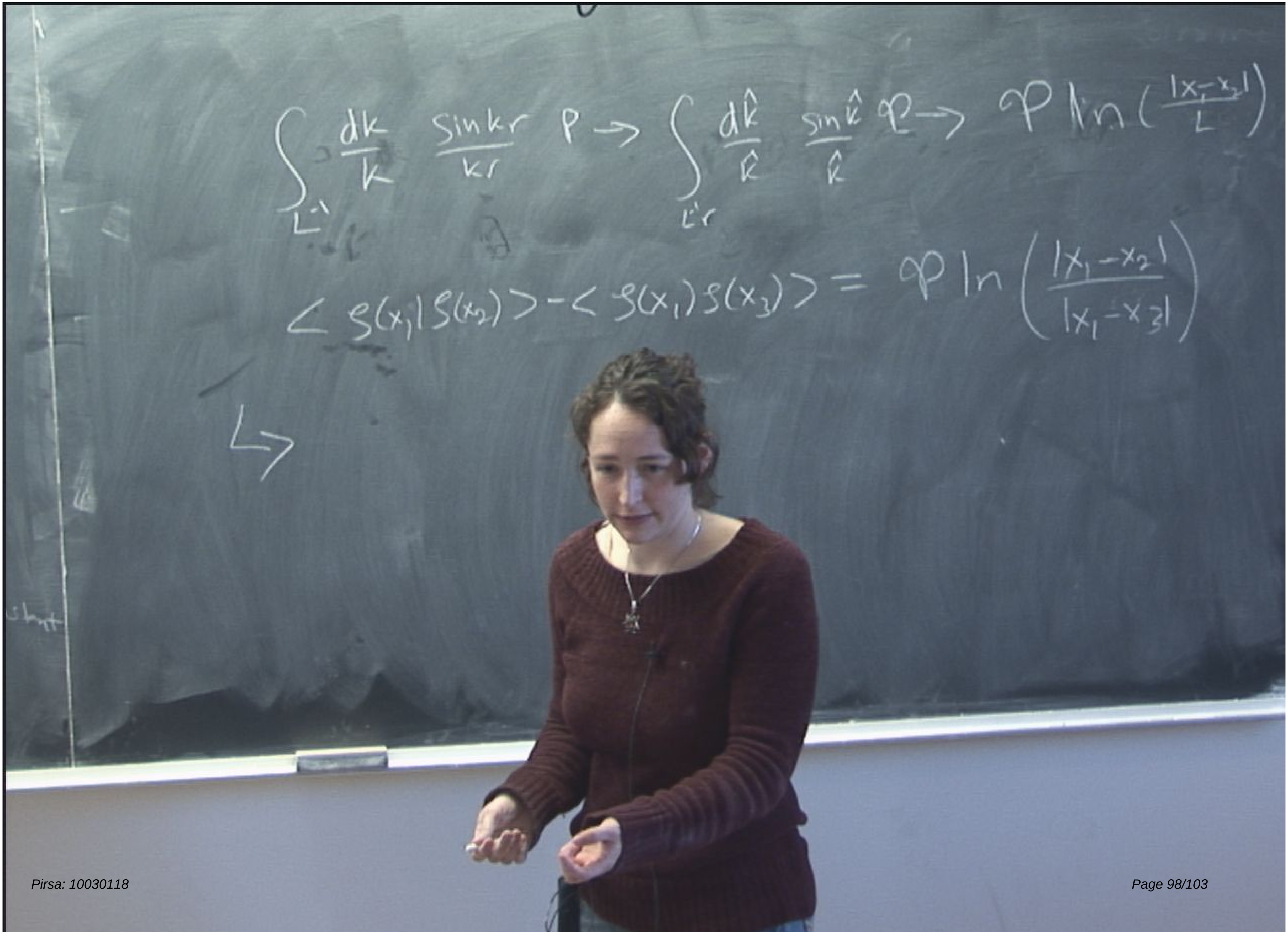
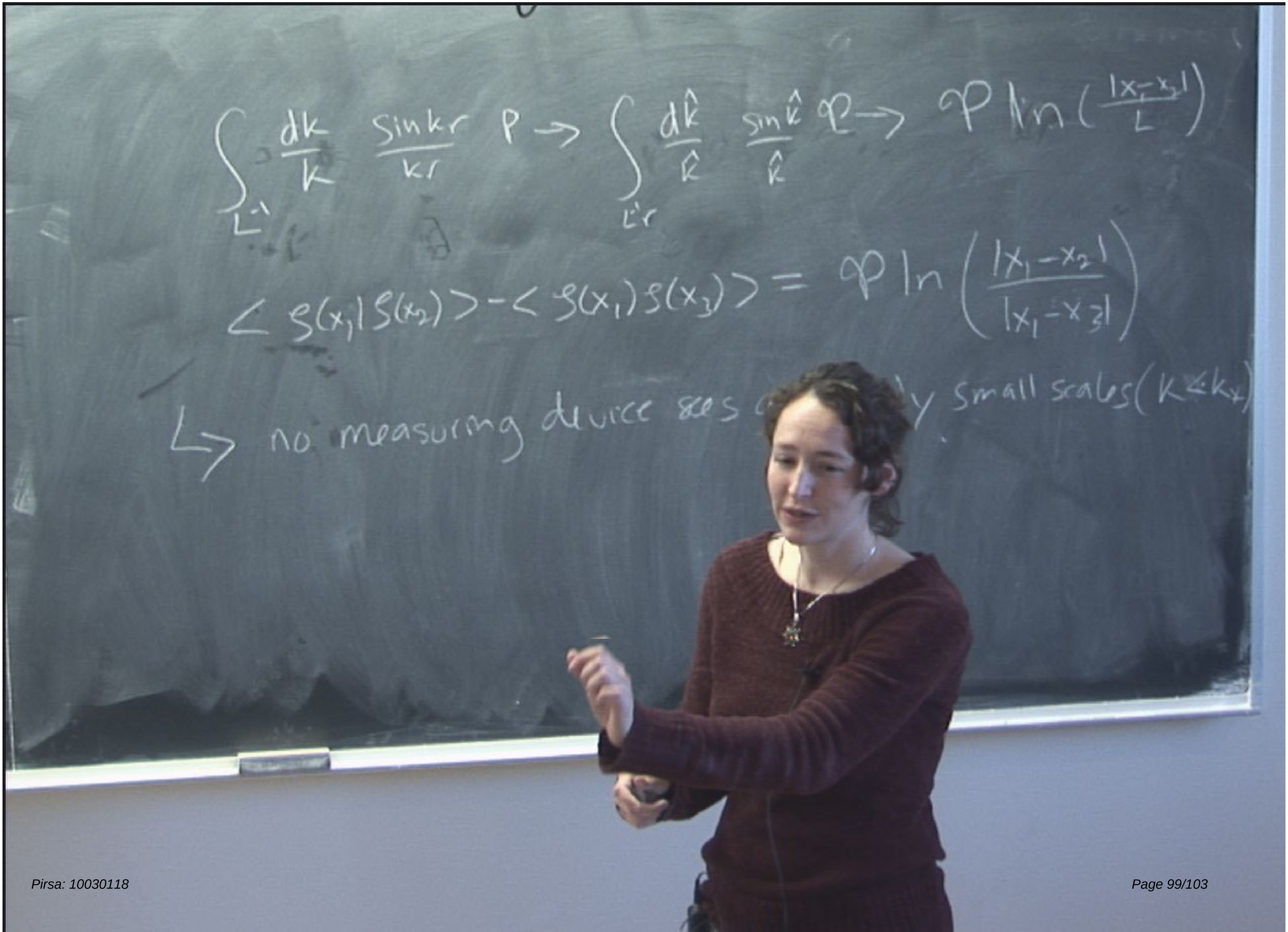

$\int_{L^{-1}} \frac{dk}{k} \frac{\sin kr}{kr} P \to \int_{L^{-1}r} \frac{d\hat{k}}{\hat{k}} \frac{\sin \hat{k}}{\hat{k}} P \to P \ln\left(\frac{|x_1 - x_2|}{L}\right)$
$\langle \rho(x_1)\rho(x_2)\rangle - \langle \rho(x_1)\rho(x_3)\rangle = P \ln\left(\frac{|x_1 - x_2|}{|x_1 - x_3|}\right)$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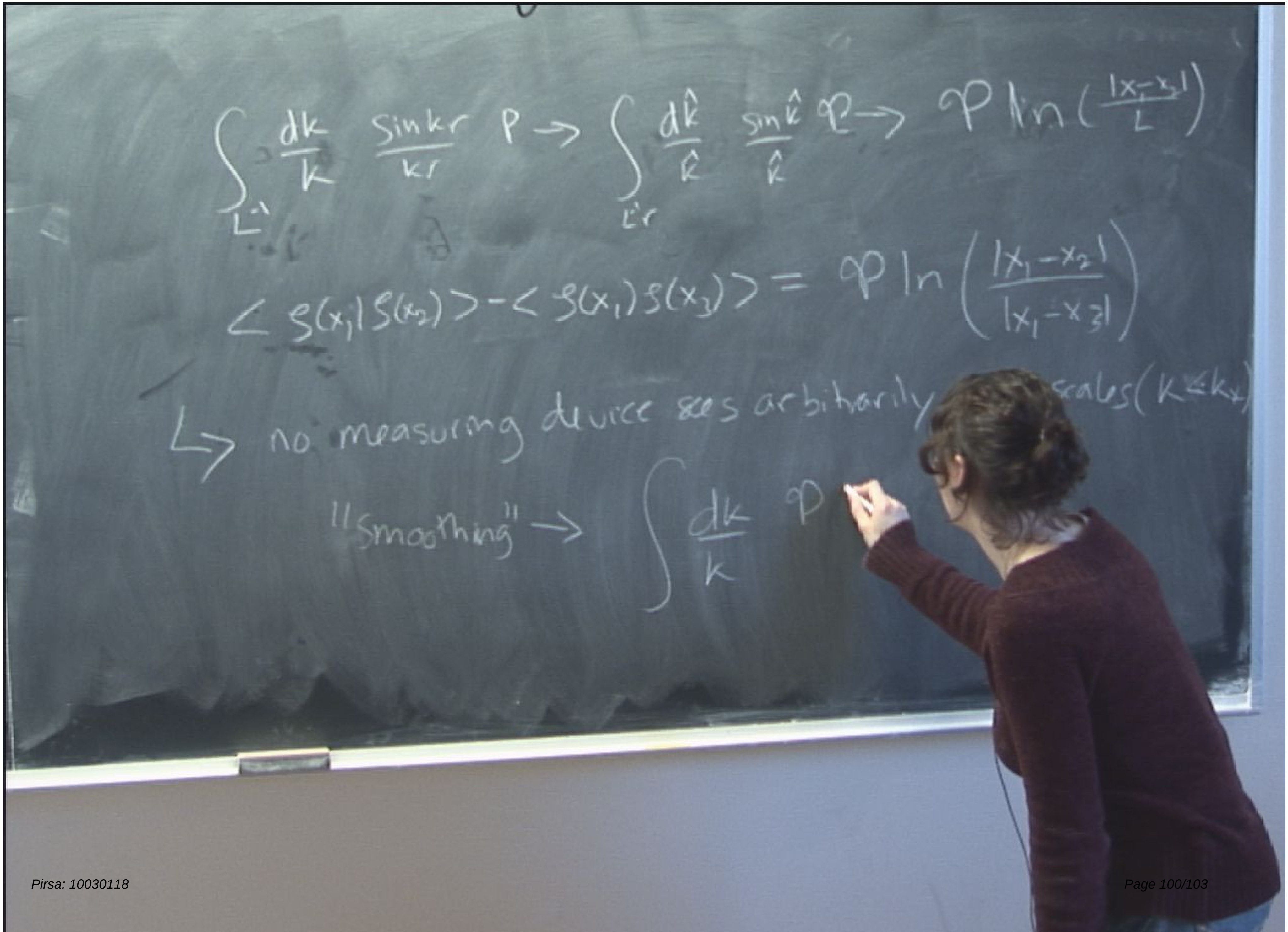
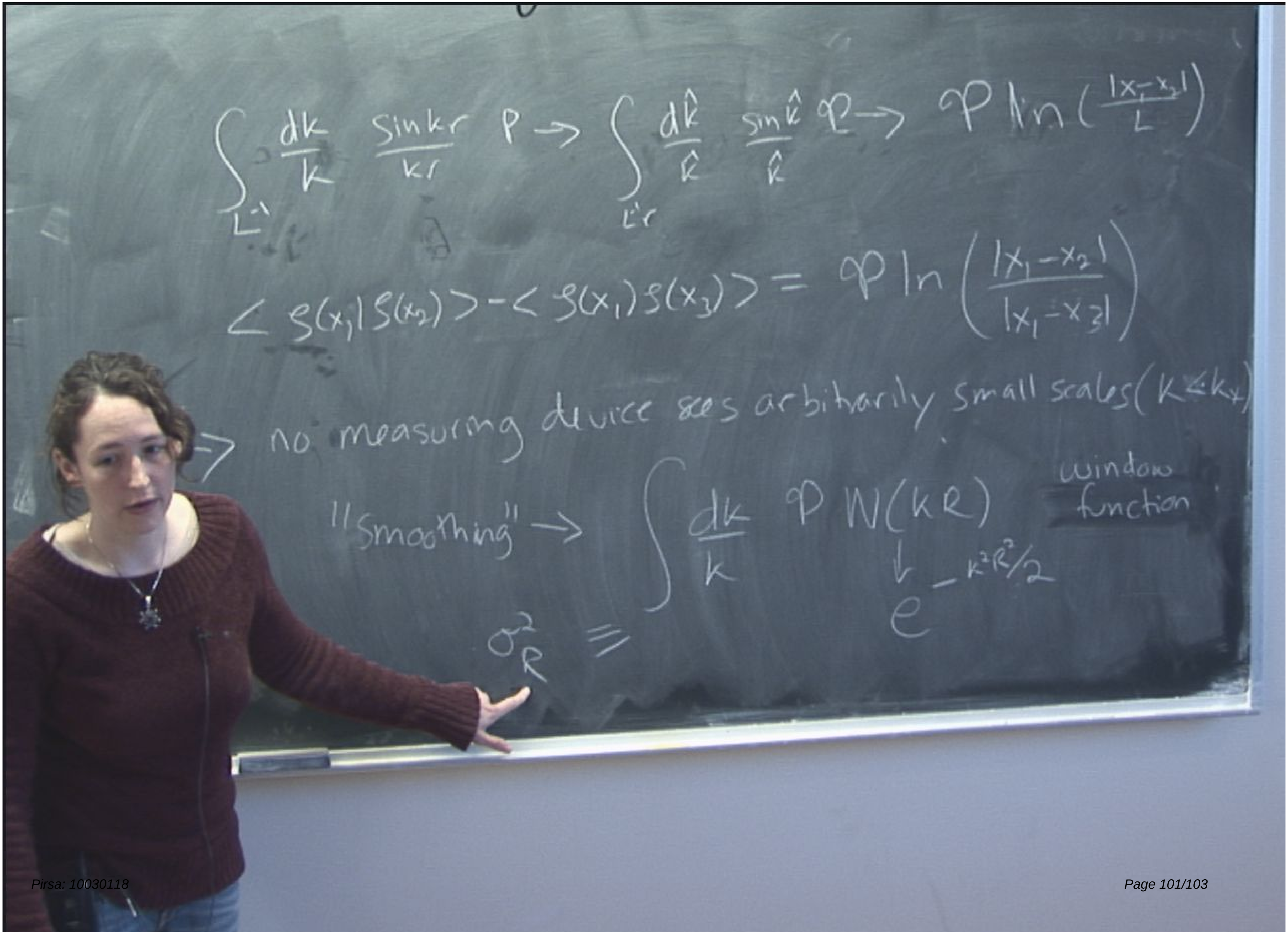
↳ no measuring device sees
small scales $(k < k_*)$


no measuring device sees arbitrarily
scales
"smoothing" →


∫ dk/k sin kr/kr P → ∫ dk̂/k̂ sin k̂/k̂ P → P ln(|x1−x2|/L)
< ρ(x1) ρ(x2) > − < ρ(x1) ρ(x3) > = P ln(|x1−x2| / |x1−x3|)
→ no measuring device sees arbitrarily small scales (k ≤ k*)
"smoothing" → ∫ dk/k P W(kR)
window function
e^(−k²R²/2)
σ_R² ≡

$$\int_{L^{-1}} \frac{dk}{k} \frac{\sin kr}{kr} P \to \int_{L^{-1}r} \frac{d\hat{k}}{\hat{k}} \frac{\sin \hat{k}}{\hat{k}} \mathcal{P} \to \mathcal{P} \ln\left(\frac{|x_1 - x_2|}{L}\right)$$

$$\langle \rho(x_1)\rho(x_2)\rangle - \langle \rho(x_1)\rho(x_3)\rangle = \mathcal{P} \ln\left(\frac{|x_1 - x_2|}{|x_1 - x_3|}\right)$$

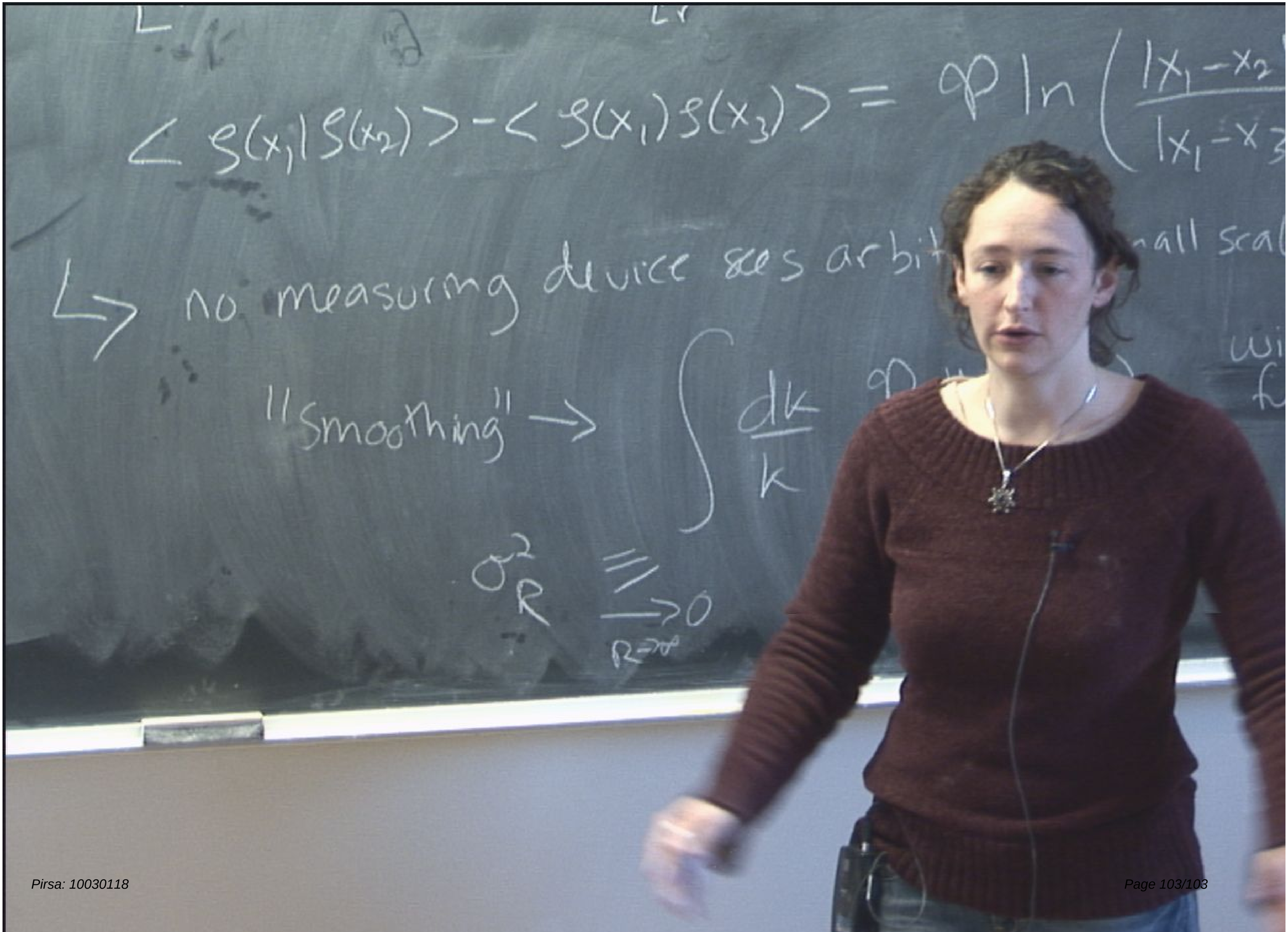
↳ no measuring device sees arbitrarily small scales ($k \le k_*$)

"smoothing" →

$$\sigma_R^2 \equiv \int \frac{dk}{k} \mathcal{P} \, W(kR)$$

window function

$$W(kR) \to e^{-k^2R^2/2}$$


no measuring device sees
"smoothing" →