Title: Explorations in Particle Theory (PHYS 646) - Lecture 9

Date: Mar 24, 2010 03:30 PM

URL: http://pirsa.org/10030117

Abstract:

Explicit representation of operators $Q, \bar{Q}$ on superfields: ① Q, Explicit representation of operators $Q, \bar{Q}$ on superfields: (1) Q,

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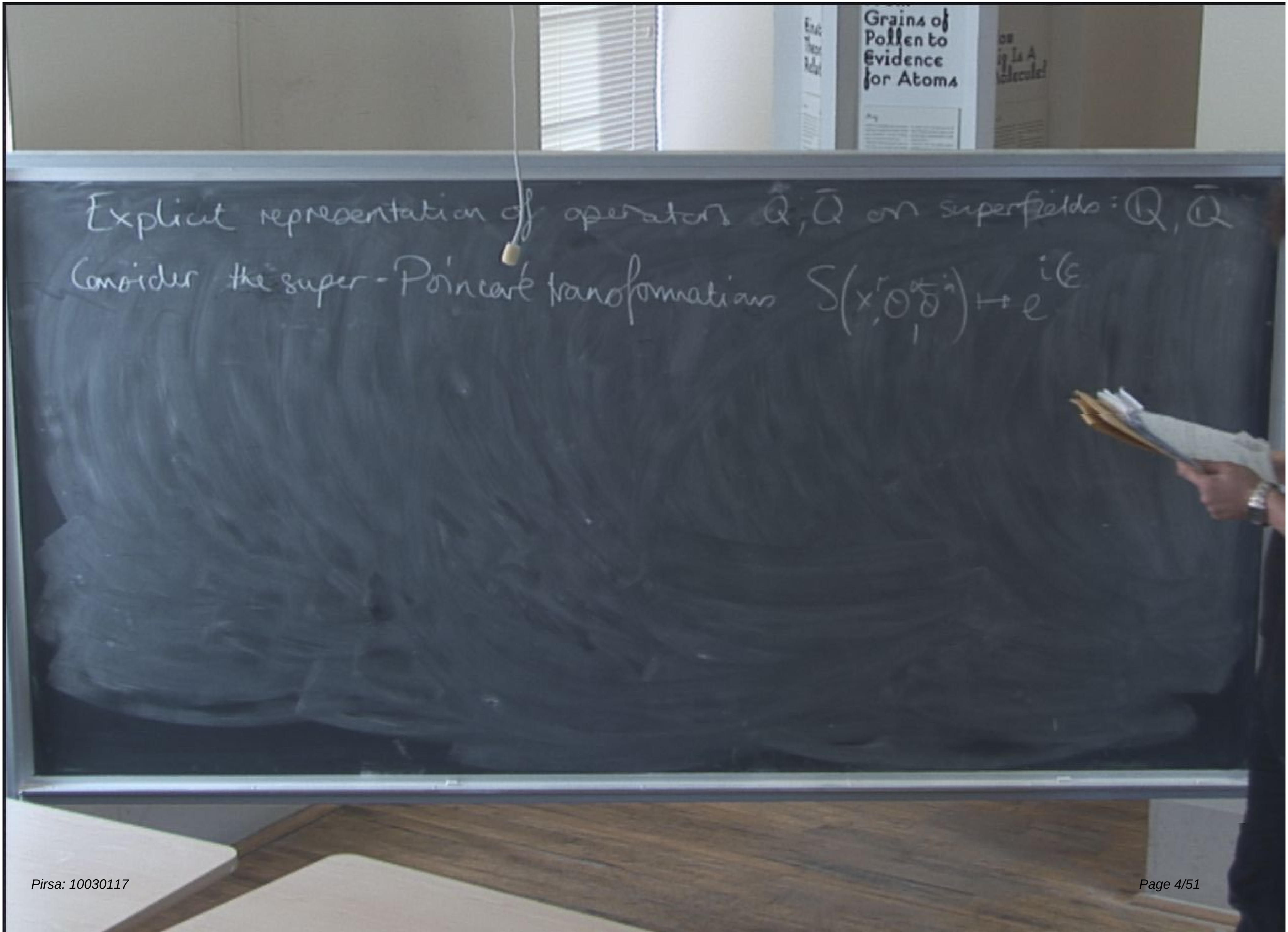
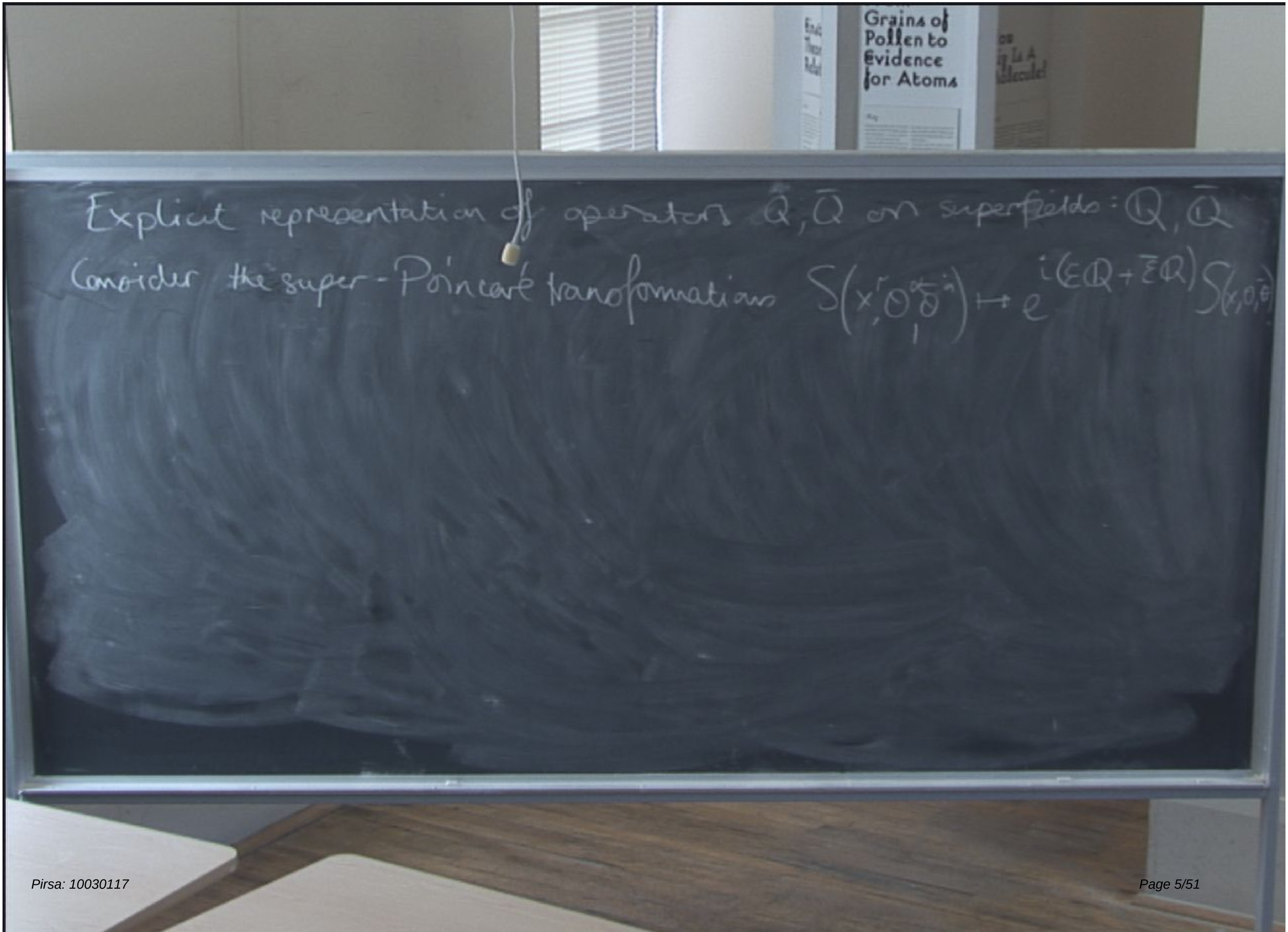

Grains of Pollen to Evidence for Atoms
Explicit representation of operators $Q, \bar{Q}$ on superfields: $\mathbb{Q}, \bar{\mathbb{Q}}$
Consider the super-Poincaré transformations $S(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) \mapsto e^{i(\varepsilon}$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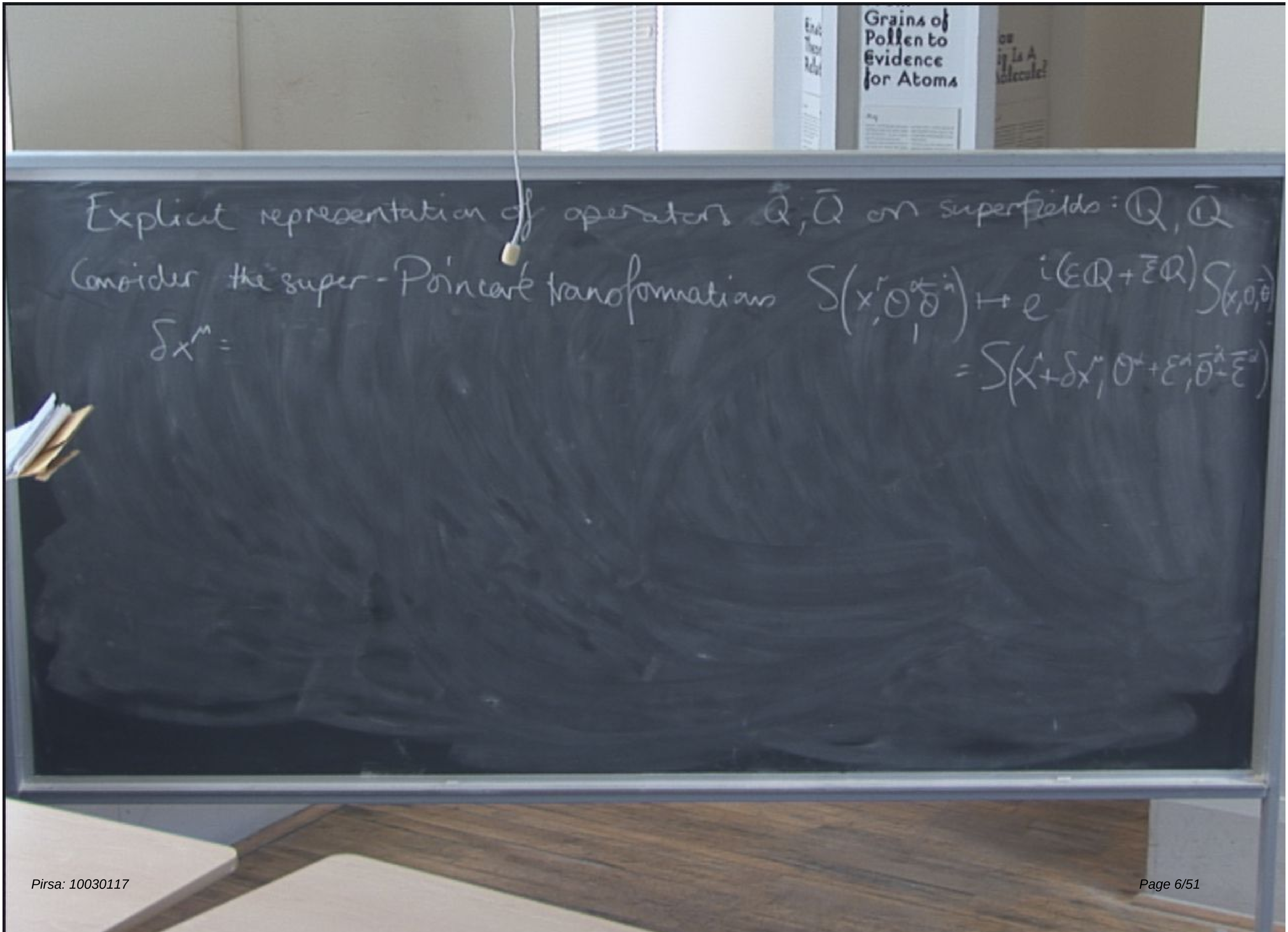
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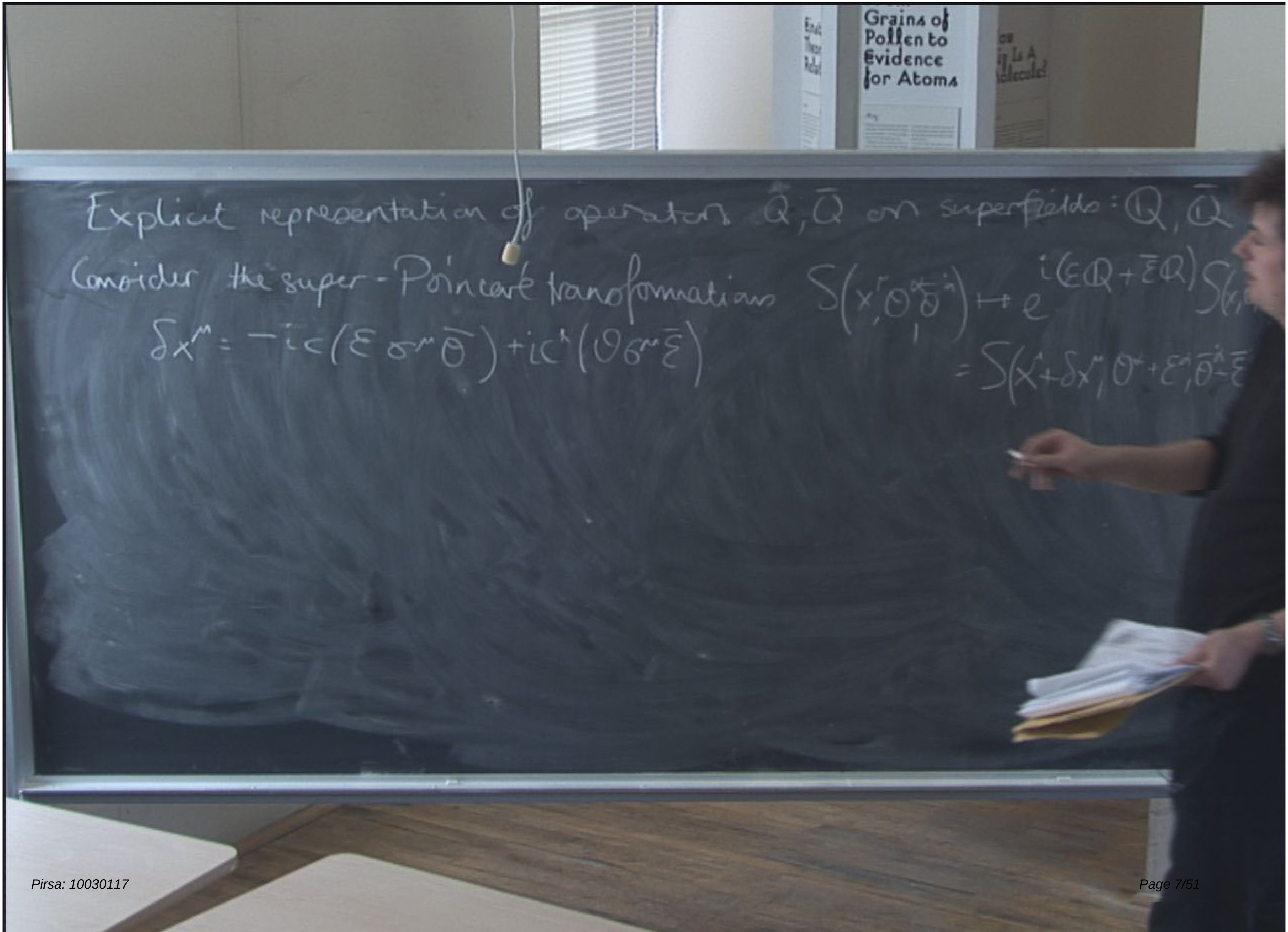
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Explicit representation of operators Q, Q̄ on superfields: Q, Q̄
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$\delta x^\mu = -ic(\varepsilon \sigma^\mu \bar\theta) + ic^*(\theta \sigma^\mu \bar\varepsilon)$
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$$\Rightarrow \quad \mathbb{Q}_\alpha = -i \frac{\partial}{\partial \theta^\alpha} - c(\sigma^\mu)_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}} \partial_\mu$$

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(*) in terms of $Q^{\alpha}, \bar{Q}^{\dot\alpha}, P_\mu$:

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \, \mathrm{Re}(c).$$

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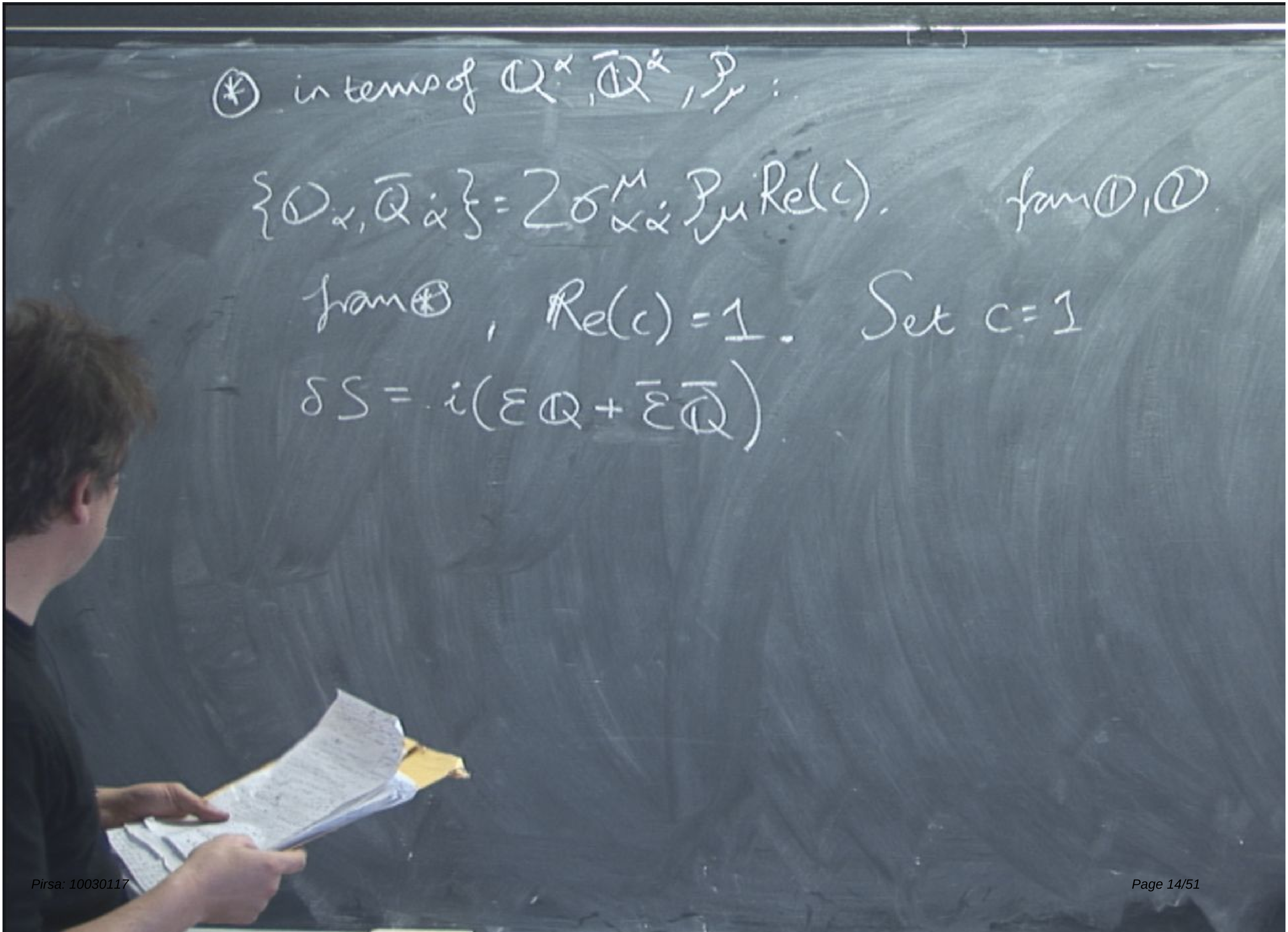
from (1), (2).



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$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \mathrm{Re}(c)$. from (1), (2)
from (*), $\mathrm{Re}(c) = 1$. Set $c = 1$
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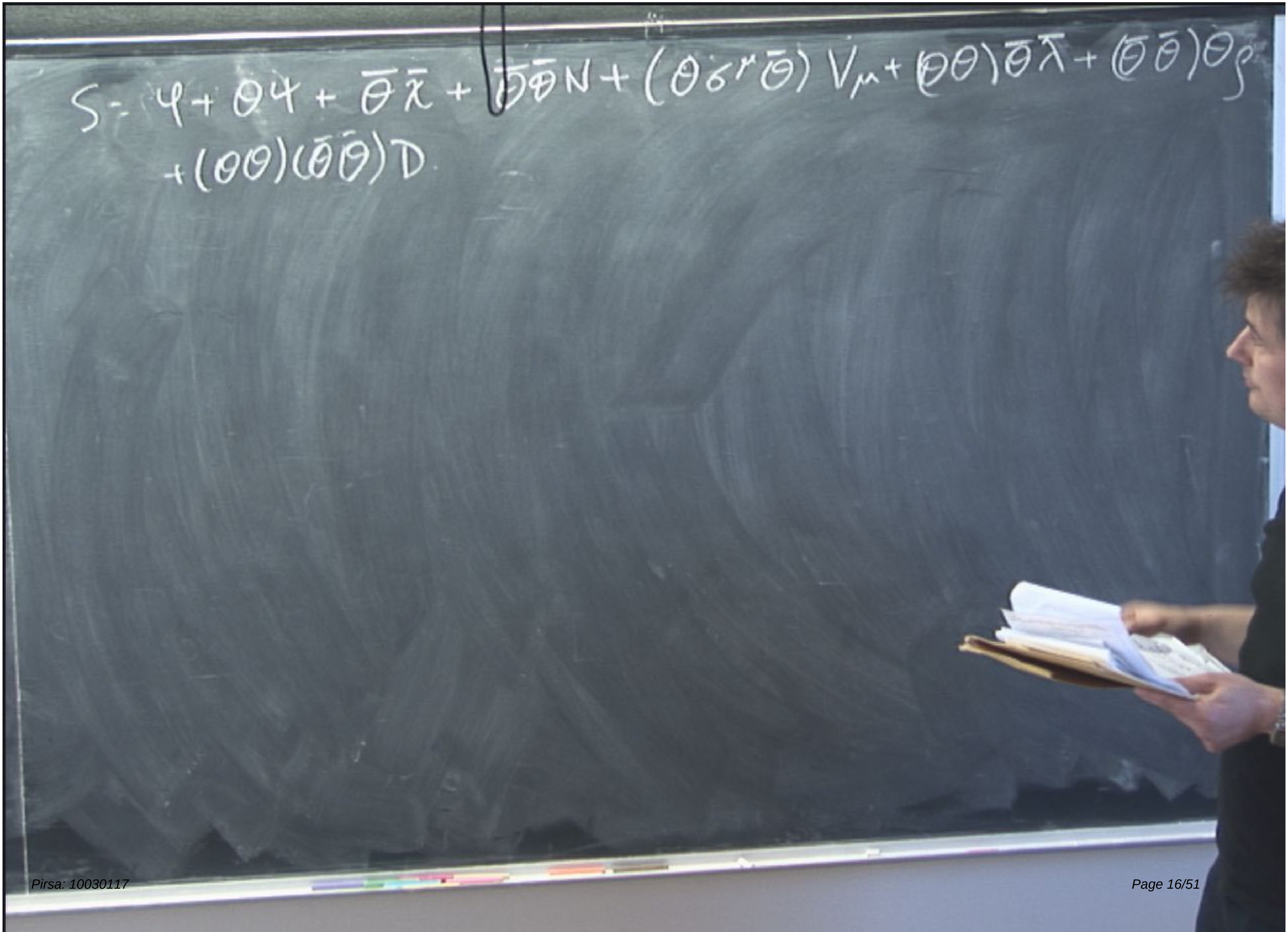
$S = \varphi + \theta\psi + \bar\theta\bar\chi + \bar\theta\bar\theta N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\rho$
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$$\Rightarrow \delta\varphi = \varepsilon\psi + \bar\varepsilon\bar\psi$$

$$\delta\psi = 2\varepsilon M + \sigma^\mu \bar\varepsilon V_\mu + i\sigma^\mu\bar\varepsilon\,\partial_\mu\varphi$$

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$\delta M = \bar\varepsilon \bar\lambda - \frac{i}{2}(\partial_\mu \psi)\sigma^\mu \bar\varepsilon$

$\delta N = \varepsilon\rho + \frac{i}{2}\varepsilon\sigma^\mu \partial_\mu \bar\chi$

$\delta V_\mu = \varepsilon\sigma_\mu \bar\lambda + \rho\sigma_\mu \bar\varepsilon - \frac{i}{2}(\bar\varepsilon\bar\sigma_\nu \sigma_\mu \partial^\nu \bar\chi) + \frac{i}{2}(\partial^\nu \psi\, \sigma_\mu \bar\sigma_\nu\, \varepsilon)$

$S = \varphi + \theta\psi + \ldots$

$+ (\theta\theta)(\bar\theta\bar\theta)\ldots$

$\delta\bar\lambda = 2\ldots$

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bosons $\leftrightarrow$ fermions

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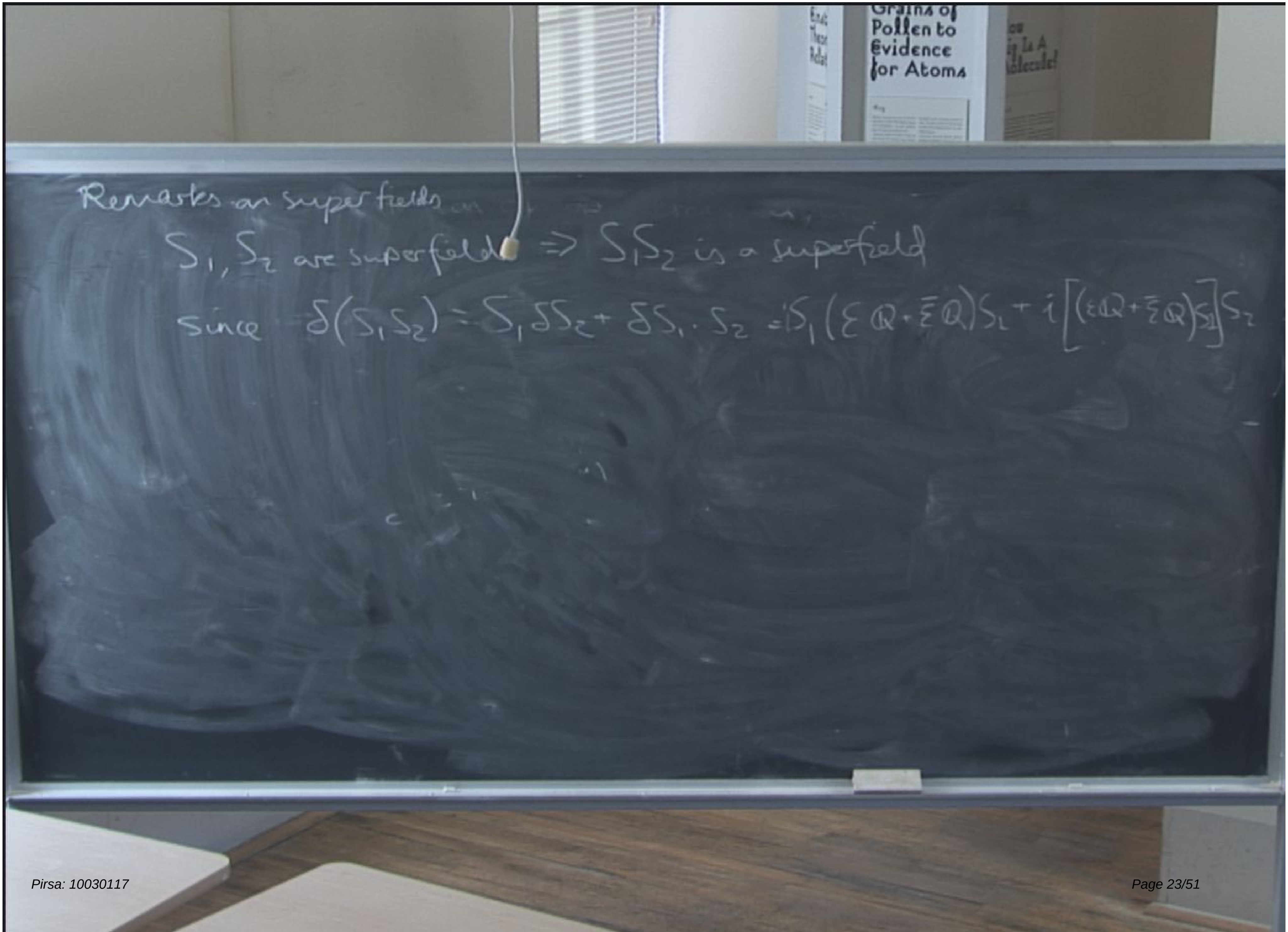
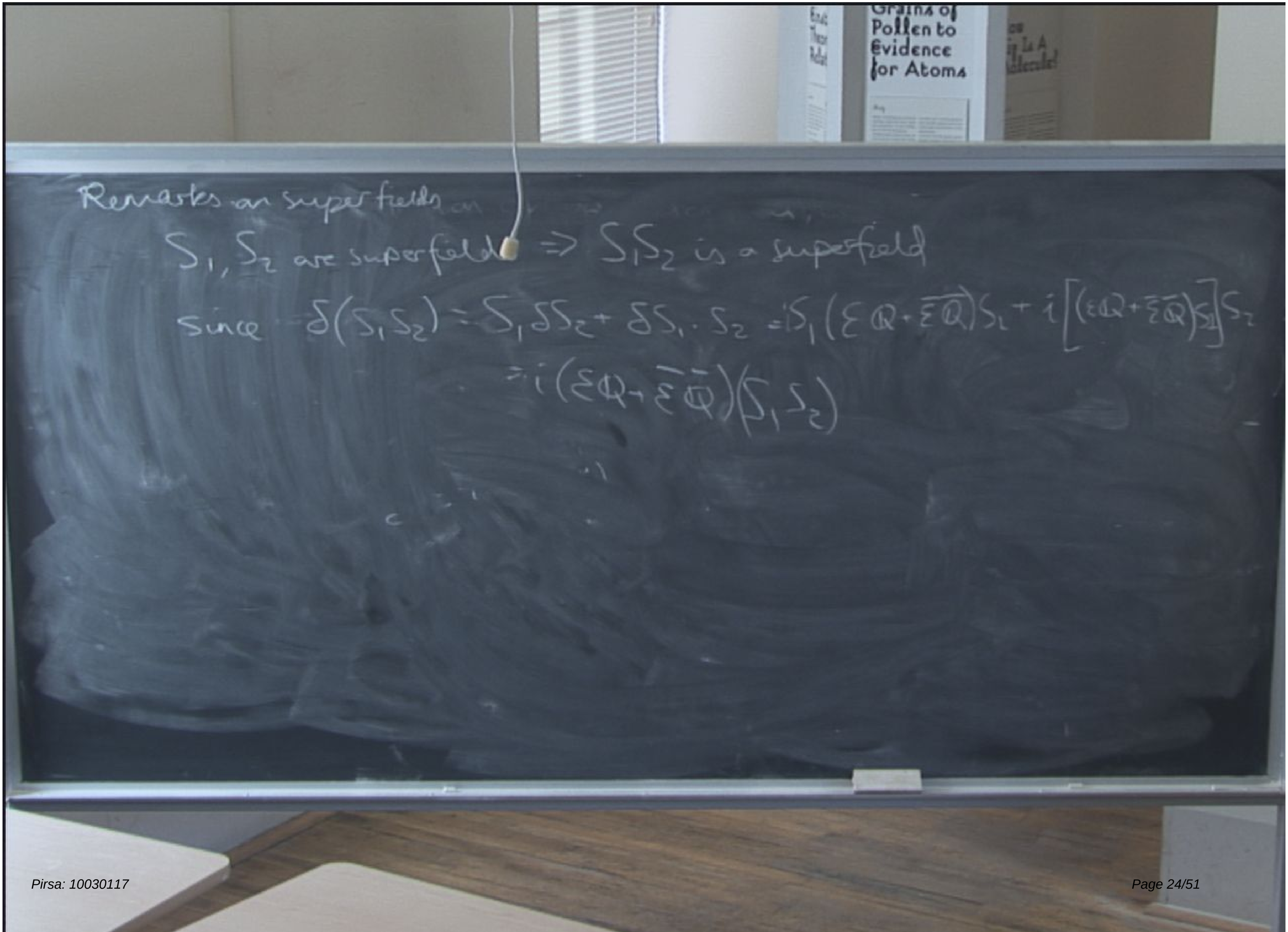


Grains of Pollen to Evidence for Atoms
Remarks on superfields
S_1, S_2 are superfields $\Rightarrow S_1 S_2$ is a superfield
since $\delta(S_1 S_2) = S_1 \delta S_2 + \delta S_1 \cdot S_2 = i S_1 (\varepsilon Q + \bar{\varepsilon} \bar{Q}) S_2 + i \left[(\varepsilon Q + \bar{\varepsilon} \bar{Q}) S_1 \right] S_2$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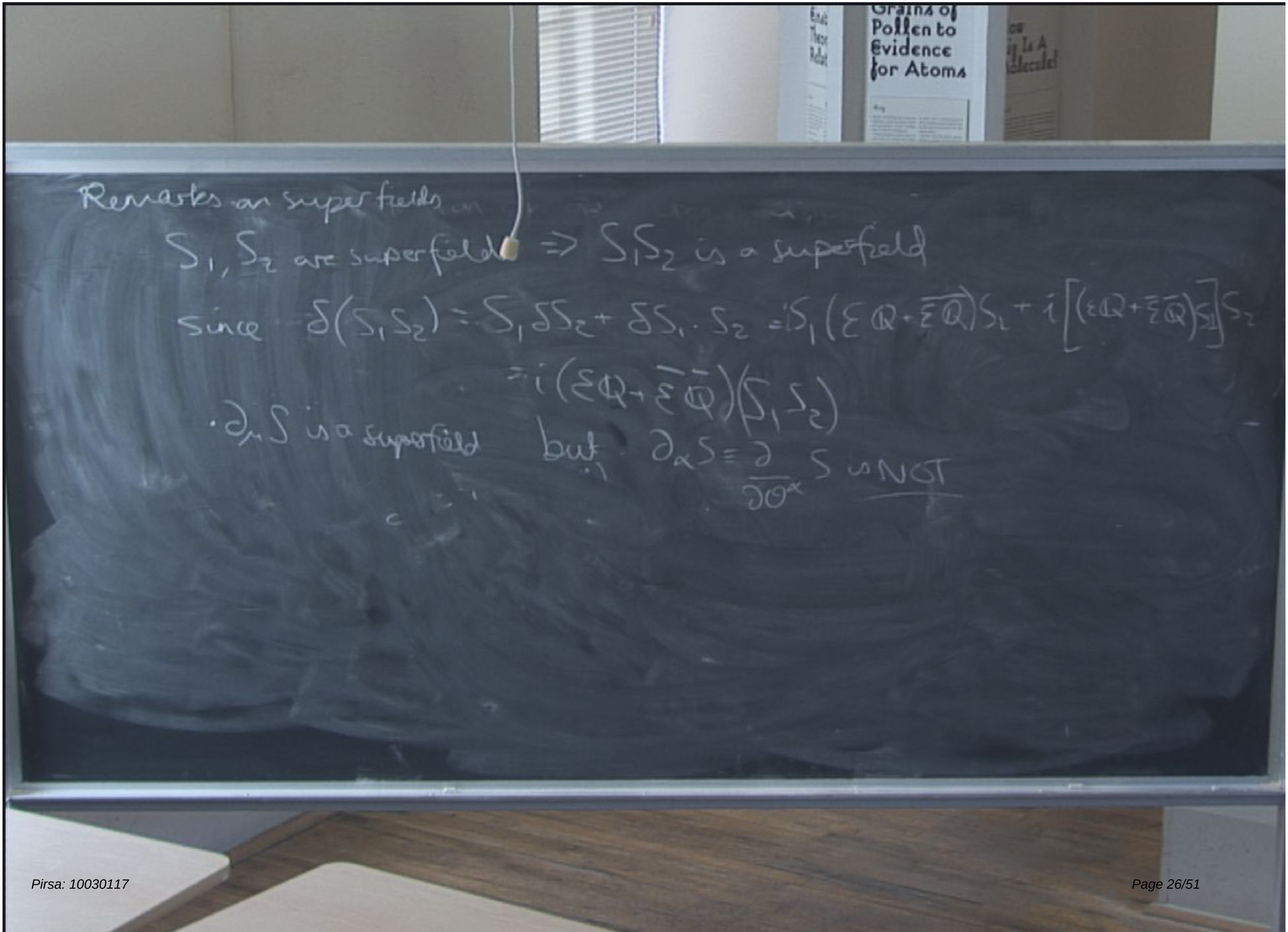
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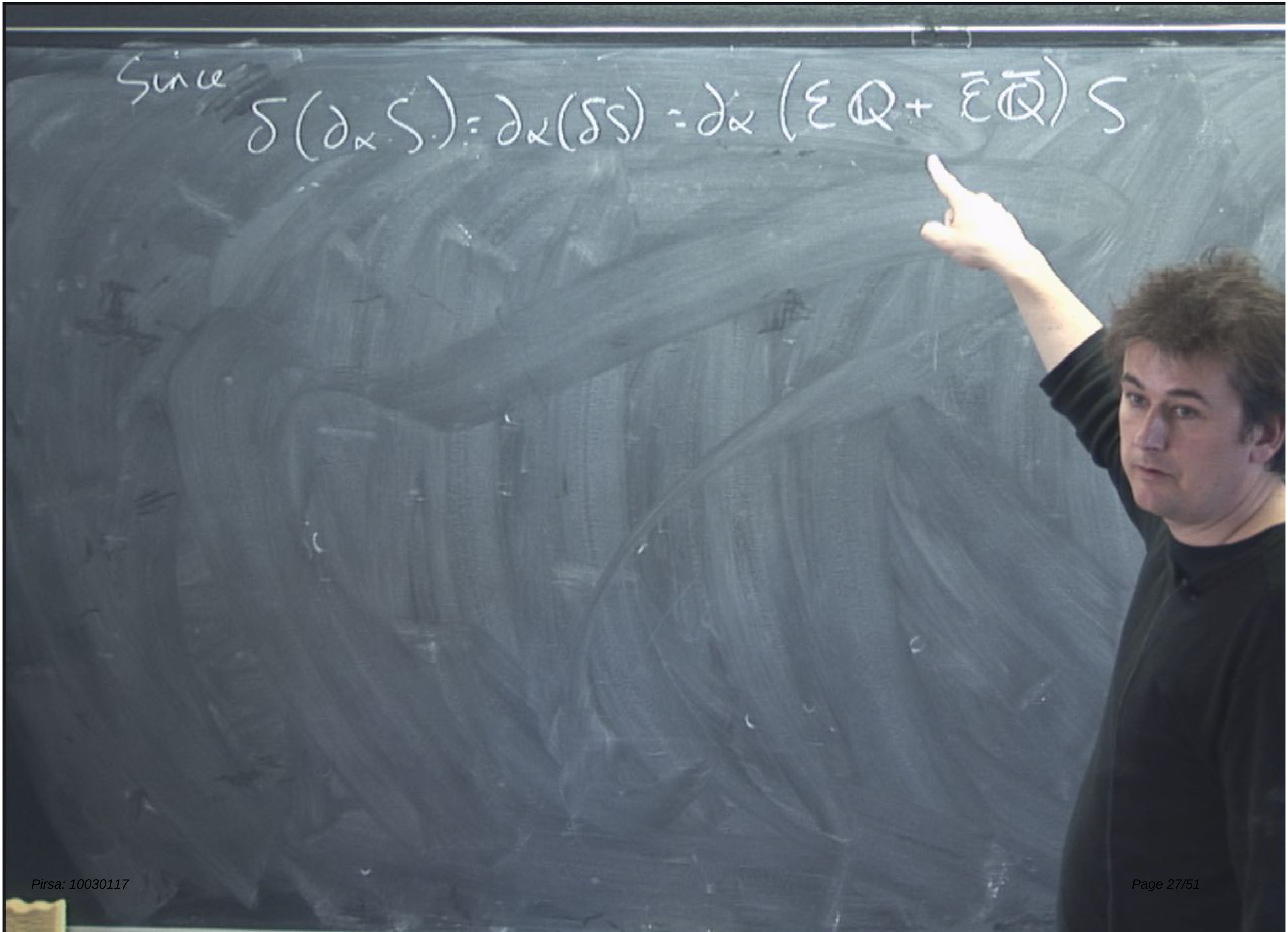
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– obviously, we must define a covariant derivative

Hence, define $D_\alpha \equiv \dfrac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$

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$$\{D_\alpha, \bar{Q}_{\dot{\beta}}\} = \{D_\alpha, Q_\beta\} = 0.$$

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Since $\delta(\partial_\alpha S) = \partial_\alpha(\delta S) = \partial_\alpha\left(\varepsilon Q + \bar\varepsilon \bar Q\right) S$

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S

Can't have $S = c\psi(x)$, since then $\delta\varphi = \varepsilon\psi$

S is a reducible rep. – can eliminate some components + keep it a superfield.

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Can't have $S = c\varphi(x)$, since then $\delta\varphi = \varepsilon\varphi$

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- chiral superfield $\bar{D}_{\dot\alpha} S = 0$
- anti " " $D_\alpha S = 0$
- vector $S = S^\dagger$
- linear $S = S^\dagger$ and $DDS = 0$

χral superfields

Define $y^\mu \equiv x^\mu + i\theta\sigma^\mu\bar{\theta}$

If $\Phi(y,\theta,\bar{\theta})$ since $\bar{D}_{\dot{\alpha}}$ is a differential op.

$$\bar{D}_{\dot{\alpha}}\Phi(y,\theta,\bar{\theta}) = (\bar{D}_{\dot{\alpha}}\theta^\beta)\frac{\partial\Phi}{\partial\theta^\beta}\Big|_{y,\bar{\theta}} + (\bar{D}_{\dot{\alpha}}y^\mu)\frac{\partial\Phi}{\partial y^\mu}$$

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$$+ (\bar D_{\dot\alpha}\bar\theta^{\dot\beta})\frac{\partial\Phi}{\partial\bar\theta^{\dot\beta}}\Big|_{y}$$

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Hence

$$\underline{\Phi}(y^\mu, \theta^\alpha) = \varphi(y^\mu) + \theta\,\psi(y^\mu)$$

$S = \varphi + \theta\psi + \bar\theta\bar\chi + \theta\bar\theta N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\rho$

$+ (\theta\theta)(\bar\theta\bar\theta)D + \theta\theta M$

Hence

$$\Phi(y^\mu, \theta^\alpha) = \varphi(y^\mu) + \sqrt{2}\,\theta\,\psi(y^\mu) + \theta^\alpha\theta_\alpha F(y^\mu)$$

expand RHS into x^μ

$$\Phi(y, \theta^\alpha) = \varphi(x) + i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x) - \frac{1}{2}(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\partial_\mu\partial_\nu\varphi(x)$$

$$S = \varphi + \theta\psi + \bar\theta\bar\chi + \bar\theta\bar\theta N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\rho$$
$$+ (\theta\theta)(\bar\theta\bar\theta)D + \theta\theta M$$

Hence
$$\Phi(y^\mu,\theta^\alpha) = \varphi(y^\mu) + \sqrt{2}\theta\,\psi(y^\mu) + \theta^\alpha\theta_\alpha F(y^\mu)$$

expand RHS into x^μ

$$\Phi(y,\theta) = \varphi(x) + i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x) - \frac{1}{2}(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\partial_\mu\partial_\nu\varphi(x)$$

$$S = \varphi + \theta\psi + \bar\theta\bar\chi + \bar\theta\bar\theta N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\rho$$
$$+ (\theta\theta)(\bar\theta\bar\theta)D + \theta\theta M$$

Hence

$$\Phi(y^\mu, \theta^\alpha) = \varphi(y^\mu) + \sqrt{2}\,\theta\,\psi(y^\mu) + \theta^\alpha\theta_\alpha F(y^\mu)$$

expand RHS into x^μ

$$\Phi(y^\mu, \theta^\alpha) = \varphi(x^\mu) + i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x) - \frac{1}{2}(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\partial_\mu\partial_\nu\varphi(x)$$
$$+ \sqrt{2}\,\theta^\alpha\left(\psi_\alpha(x^\mu) + i\,\theta^\beta\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}\partial_\mu\psi_\alpha(x^\mu)\right)$$
$$+ \theta\theta F(x^\mu)$$

$$S = \varphi + \theta\psi + \bar\theta\bar\chi + \bar\theta\bar\theta N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\rho$$
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expand RHS into x^μ

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$$+ \theta\theta F(x^\mu)$$
$$= \varphi(x) + i(\theta\sigma^\mu\bar\theta)\partial_\mu\varphi(x) - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box\varphi(x) + \sqrt{2}\theta\psi(x)$$
$$- \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi^\beta\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}$$
$$+ \theta\theta F(x)$$

$$S = \varphi + \theta\psi + \bar\theta\bar\chi + \bar\theta\bar\theta N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\rho$$
$$+ (\theta\theta)(\bar\theta\bar\theta)D + \theta\theta M$$

Hence

$$\Phi(y^\mu, \theta^\alpha) = \varphi(y^\mu) + \sqrt{2}\,\theta\,\psi(y^\mu) + \theta^\alpha\theta_\alpha F(y^\mu)$$

expand RHS into x^μ

$$\Phi(y^\mu,\theta^\alpha) = \varphi(x^\mu) + i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x) - \frac{1}{2}(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\partial_\mu\partial_\nu\varphi(x)$$
$$+ \sqrt{2}\,\theta^\alpha\left(\psi_\alpha(x^\mu) + i\,\theta^\beta\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}\partial_\mu\psi_\alpha(x^\mu)\right)$$
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$$= \varphi(x) + i(\theta\sigma^\mu\bar\theta)\partial_\mu\varphi(x) - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box\varphi(x) + \sqrt{2}\,\theta\psi(x)$$
$$- \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi^\beta\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}$$
$$+ \theta\theta F(x)$$

$$\Rightarrow \quad \delta\varphi = \sqrt{2}\,\varepsilon\psi$$

$$\delta\psi = i\sqrt{2}\,\sigma^\mu\bar\varepsilon\,\partial_\mu\varphi + \sqrt{2}\,\varepsilon F$$

$$\boxed{\delta F = i\sqrt{2}\,\bar\varepsilon\,\bar\sigma^\mu\,\partial_\mu\psi} \longrightarrow \text{a total deriv}$$

$$S = \varphi + \theta\psi + \bar\theta\bar\chi + (\bar\theta\bar\theta)N + (\theta\sigma^\mu\bar\theta)V_\mu + (\theta\theta)(\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}) + (\bar\theta\bar\theta)\theta\rho$$

$$+(\theta\theta)(\bar\theta\bar\theta)D + \theta\theta M$$

scalar   scalar.

Hence

$$\Phi(y^\mu, \theta^\alpha) = \varphi(y^\mu) + \sqrt{2}\,\theta\,\psi(y^\mu) + \theta^\alpha\theta_\alpha F(y^\mu)$$

expand RHS into x^μ

$$\Phi(y^\mu,\theta^\alpha) = \varphi(x^\mu) + i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x) - \frac{1}{2}(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)\partial_\mu\partial_\nu\varphi(x)$$

$$+\sqrt{2}\,\theta^\alpha\left(\psi_\alpha(x^\mu) + i\,\theta^\beta\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}\partial_\mu\psi_\alpha(x^\mu)\right)$$

$$+\theta\theta F(x^\mu).$$

$$= \varphi(x) + i(\theta\sigma^\mu\bar\theta)\partial_\mu\varphi(x) - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box\varphi(x) + \sqrt{2}\,\theta\psi(x)$$

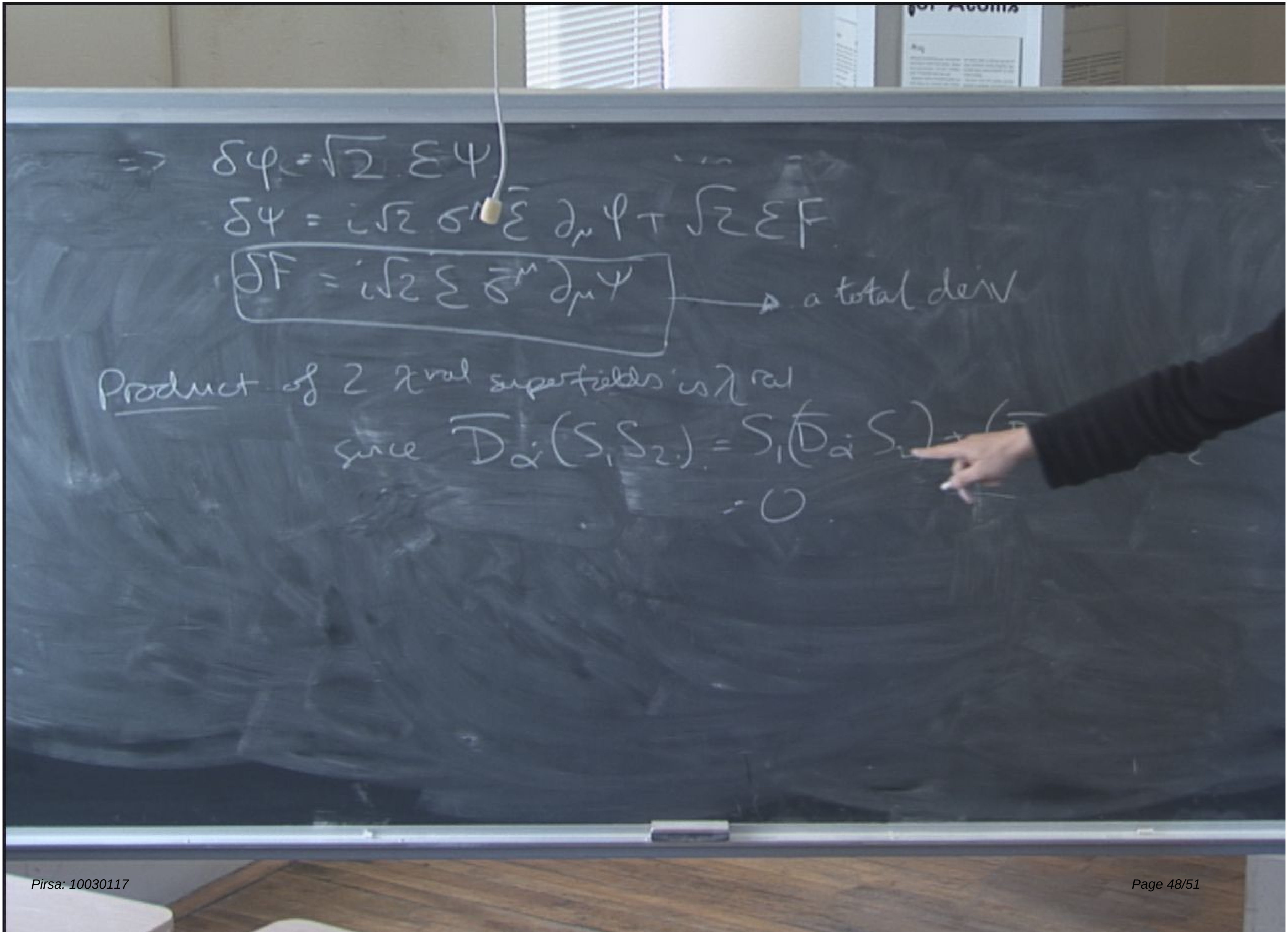
$$-\frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi^\beta\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}$$

$$+\theta\theta F(x).$$

$$\Rightarrow \quad \delta\varphi = \sqrt{2}\,\varepsilon\psi$$

$$\delta\psi = i\sqrt{2}\,\sigma^\mu\bar{\varepsilon}\,\partial_\mu\varphi + \sqrt{2}\,\varepsilon F$$

$$\boxed{\delta F = i\sqrt{2}\,\varepsilon\,\bar{\sigma}^\mu\partial_\mu\psi} \longrightarrow \text{a total deriv}$$


⇒ δφ = √2 εψ
δψ = i√2 σ^μ ε̄ ∂_μ φ + √2 ε F
δF = i√2 ε̄ σ̄^μ ∂_μ ψ → a total deriv
Product of 2 χral superfields is χral
since D̄_α̇ (S_1 S_2) = S_1 (D̄_α̇ S_2) + (D̄
= 0

$\Rightarrow \quad \delta\varphi = \sqrt{2}\,\varepsilon\psi$

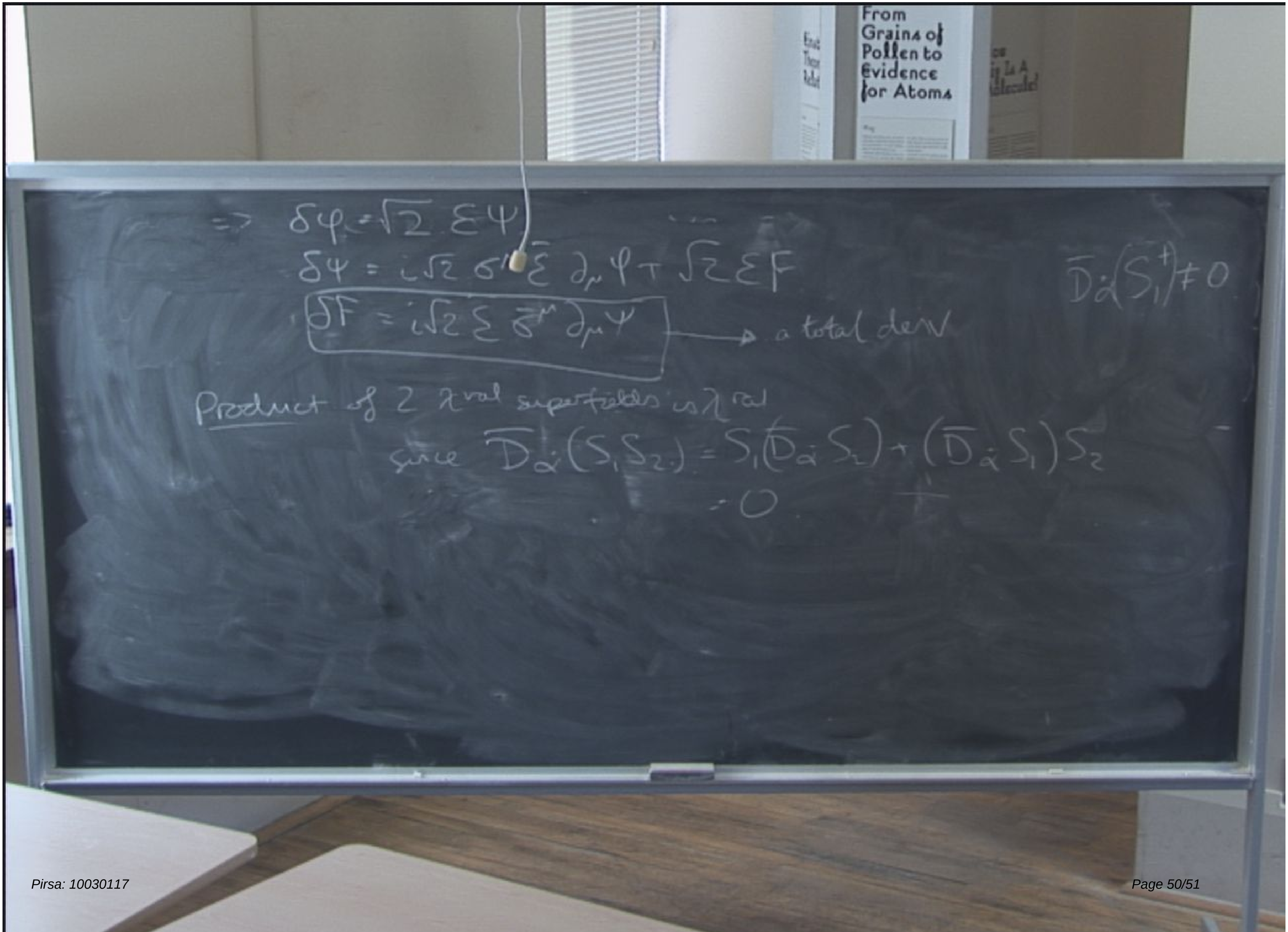
$\delta\psi = i\sqrt{2}\,\sigma^\mu\bar{\varepsilon}\,\partial_\mu\varphi + \sqrt{2}\,\varepsilon F$

$\boxed{\delta F = i\sqrt{2}\,\bar{\varepsilon}\,\bar{\sigma}^\mu\partial_\mu\psi} \longrightarrow$ a total deriv

<u>Product</u> of 2 χiral superfields is χiral

since $\bar{D}_{\dot\alpha}(S_1S_2) = S_1(\bar{D}_{\dot\alpha}S_2) + (\bar{D}_{\dot\alpha}S_1)S_2$

$= 0$


From Grains of Pollen to Evidence for Atoms
$\Rightarrow \delta\varphi = \sqrt{2}\,\varepsilon\psi$
$\delta\psi = i\sqrt{2}\,\sigma^\mu\bar{\varepsilon}\,\partial_\mu\varphi + \sqrt{2}\,\varepsilon F$
$\boxed{\delta F = i\sqrt{2}\,\bar{\varepsilon}\,\bar{\sigma}^\mu\,\partial_\mu\psi} \longrightarrow$ a total deriv
$\bar{D}_{\dot\alpha}(S_1^\dagger) \neq 0$
Product of 2 χral superfields is χral
since $\bar{D}_{\dot\alpha}(S_1 S_2) = S_1(\bar{D}_{\dot\alpha} S_2) + (\bar{D}_{\dot\alpha} S_1) S_2$
$= 0$

If Φ is χral s-field,

$\Phi^+ \Phi$ and $\Phi^+ + \Phi$ are real superfield

<u>not</u> χral or anti-chiral