Title: Geometro-kinematics and dynamics in a discrete setting

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Abstract: A quantum theory of gravity implies a quantum theory of geometries. To this end we will introduce different phases spaces and choices for the space of discretized geometries. These are derived through a canonical analysis of simplicity constraints - which are central for spin foam models - and gluing constraints. We will discuss implications for spin foam models and map out how to obtain a path integral quantization starting from a canonical quantization.

Spin Foams and Plebanski action

$$S_{Pleb}[E,A] \;=\; \int_{\mathcal{M}} E^A \wedge F^A + \frac{1}{\gamma} \star E^A \wedge F^A + \phi^{AB} E^A \wedge E^B$$

SO(4) 'BF' term

Holst term, Barbero-Immirzi parameter

simplicity constraints ensure

$$E^A = \begin{cases} \pm {}^{\star} e \wedge e^A & \text{gravitational sector} \\ \pm e \wedge e^A & \text{topological sector} \end{cases}$$

Spinfoams:

- consider partition function based on BF (and Holst)
- central problem: impose simplicity constraints

- Barrett-Crane model ['98] : without Holst term (will be important later)
- 'master constraint method' [Engle, Pereira, Rovelli + Livine '07]
- coherent state method [Freidel, Krasnov '07, Livine, Speziale '07, Conrady, Hnybida '10]

Spin Foams and Plebanski action

Questions:

- relation to Dirac quantization?
- secondary simplicity constraints (Faddeev Popov determinant)? [Alexandrov '08]
- relation to LQG phase space?
- relation to Regge phase space [length Regge calculus Regge '61, area-angle Regge calculus BD, Speziale '08, phase space Bahr, BD 09, BD, Hoehn 09]

Aim:

- analysis of discrete primary and secondary simplicity constraints: Dirac brackets
- continuum [Buffenoir, Henneaux, Noui, Roche '04, Krasnov, Alexandrov '08]

- spin foam amplitudes from canonical quantization [Alexandrov '07, to do]
- will find a few surprises

Discrete phase space

•discretize 3d spatial hypersurface via a triangulation: [Waelbroeck, Zapata '93]

$\{i\}$ labels tetrahedra

$\{ij\}$ labels triangles

$\{ijk\}$ labels edges

•associate to triangles (dual edges) $\{ij\}$:

M_{ij} SO(4) holonomies

E_{ij} bivectors (SO(4) algebra)

•split into dual and anti-selfdual parts:

$$E^A_{ij\pm} = P^{AB}_\pm E^B_{ij} \qquad P^{AB}_\pm = \frac{1}{2}(\delta^{AB} \pm \epsilon^{AB})$$

Symplectic Structure

$$\{E^A_{ij\pm}, M^{BC}_{ij\pm}\} = \frac{\gamma}{\gamma \pm 1} C^{ABD}_{\pm} M^{DC}_{ij\pm}$$

$$\{M^{AB}_{ij\pm}, M^{CD}_{kl\pm}\} = 0$$

$$\{E^A_{ij\pm}, E^B_{ij\pm}\} = \frac{\gamma}{\gamma \pm 1} C^{ABC}_{\pm} E^C_{ij\pm}$$

$$E^A_{ji} = -M^{AB}_{ji} E^B_{ij} \qquad M^{AB}_{ij} = m^{BA}_{ji}$$

- Gauss constraints: closure for tetrahedra

$$g_i = \sum_j E_{ij}$$

Gauge invariant phase space

[Dittrich, Ryan '08]

- reduce first by Gauss constraints - prevents any (time) gauge fixing

$$A_{ij\pm} := E_{ij\pm} \cdot E_{ij\pm}$$ squared areas

$$\cos\phi_{ijk\pm} := \frac{E_{ij\pm} \cdot E_{ik\pm}}{\sqrt{E_{ij\pm} \cdot E_{ij\pm}\; E_{ik\pm} \cdot E_{ik\pm}}}$$ 3d dihedral angles

$$\cos\theta_{ik,jl\pm} := \frac{N_{ijk\pm} \cdot \left(M_{ij\pm} N_{jil\pm}\right)}{\sqrt{N_{ijk\pm} \cdot N_{ijk\pm}\; N_{jil\pm} \cdot N_{jil\pm}}}$$ 4d dihedral angles

$$N^A_{ijk\pm} = C^{ABC}_{\pm} E^B_{ij\pm} E^C_{ik\pm}$$

- for 4-simplex 60 variables

Simplicity constraints

diagonal simplicity	$d_{ij} = \epsilon E_{ij} \cdot E_{ij}$	$A_{ij+} \;=\; A_{ij-}$
cross simplicity	$c_{ij} = \epsilon E_{ij} \cdot E_{ik}$	$\cos\phi_{ijk+} \;=\; \cos\phi_{ijk-}$
edge simplicity	$e_{ij} = \epsilon E_{ij} \cdot M_{ij} E_{jl}$	$\cos\theta_{ikjl+} \;=\; \cos\theta_{ikjl-}$

self-dual geometry = antiself-dual geometry

Theorem: [BD, Ryan '08]

For non-vanishing 3d-volume and non-parallel 4d normals for neighboring tetrahedra one can reconstruct consistent tetrad assignments to edges on constraint hypersurface.

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Reduction I

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vanishes for degenerate 3-vol

vanishes on gravitational sector

Remark

$$b_i = \frac{\gamma^2}{\gamma^2 - 1}\left[(V_{ijkl+} - V_{ijkl-}) - \frac{1}{\gamma}(V_{ijkl-} + V_{ijkl-})\right]$$

vanishes on gravitational sector

- leads to first class subalgebra of cross simplicity for infinite Immirzi Parameter (without Holst term)
  [Engle, Pereira '08]

- also Dirac matrix not invertible in this case

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in full beauty ...

Dirac brackets I

$$\{f,g\}_1 = \{f,g\} - \{f,\Phi_\alpha\}\, X^{-1}_{\alpha\beta}\, \{\Phi_\beta, g\}$$

$$\{\sqrt{A_{ij}}, \theta_{ijkl}\}$$

areas conjugated to extrinsic curvature

$$\{\sqrt{A_{ij}A_{ik}}\cos\phi_{ijk}, \sqrt{A_{ij}A_{il}}\cos\phi_{ijl}\} = \frac{\gamma}{2}V_{ijkl}$$

3d angles do not commute, singular without Holst term

Also the other brackets can be computed.

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Match to LQG phase space

Reduction from Plebanski.

LQG Phase space.

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$$M \sim \exp A \qquad A^a_i = \Gamma^a_i + \gamma K^a_i$$

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Complete agreement with (discrete) LQG phase space.

But ...

Both phase spaces larger than Regge phase space!

reduced (I) Plebanski, LQG

30 variables= 10 areas, 10 3d angles, 10 4d angles

Regge on a simplex [BD, Ryan 08]

20 variables=10 length, 10 conjugated momenta

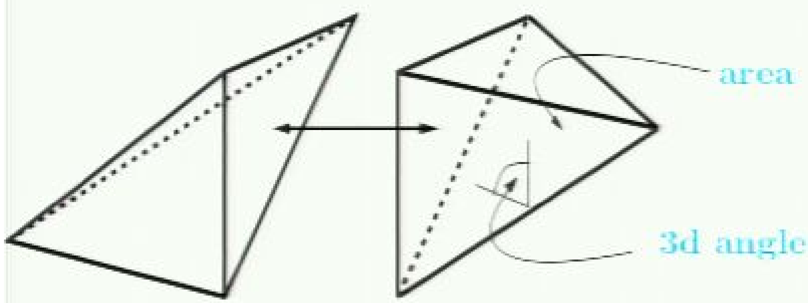
But we should have had obtained consistent tetrad assignment to the edges.
At least for 'generic configurations'.

What is missing?

Situation resembles Area-angle Regge calculus [BD, Speziale '08]:
areas and 3d angles angles are free variables.

There we needed to add gluing constraints. These ensure that two triangles shared by two tetrahedra, have not only the same area but also the same shape!

$$\cos\alpha_{ijkl} = \cos\alpha_{jikl}$$



where $\cos\alpha_{ijkl} = \dfrac{N_{ijk}\cdot N_{ijl}}{\sqrt{N_{ijk}\cdot N_{ijk}\; N_{ijl}\cdot N_{ijl}}} = \dfrac{\cos\theta_{ikl} - \cos\theta_{ijk}\cos\theta_{ijl}}{\sin\theta_{ijk}\sin\theta_{ijl}}$

'Twisted geometries' [Freidel, Speziale '10] from the full LQG phase space, do not satisfy gluing.

Do we have to worry?

- seem to come from degenerate configurations, general problem of first order formal.?

- 'configuration space analysis': obtain exactly 5 additional degrees of freedom if 4d volume is zero [Conrady, Freidel '08]
- turn up in asymptotic analysis [Barrett, Fairbairn Hellmann et al '08]
  vector geometries or BF contribution
- extrinsic curvature (phase for coherent state) cannot be determined
- difficult to impose: second class (plus first class for bigger triangulations)

Area-angle Regge calculus: need the gluing constraints to obtain correct dynamics of GR.
Otherwise all the areas are free variables: leads to flat geometries.
Gluing constraints impose (non-local) restrictions on spins.

Are the gluing constraints preserved by the dynamics?

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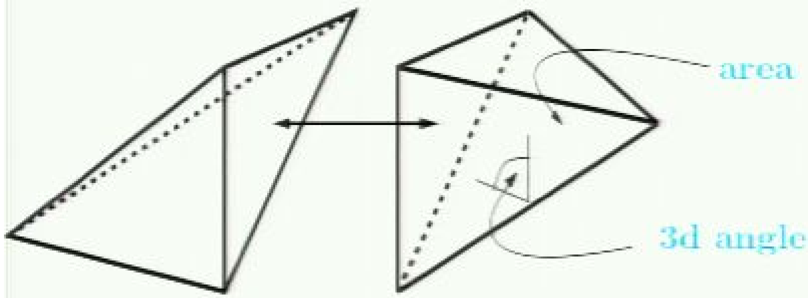

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Different formulations for gluing constraints

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2d angles computed for the same triangle in the two different tetrahedra agree.

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3d angles and areas

$$\phi_{ijk} = \phi_{ijk}\,A$$
$$C\;A \;=$$

3d angles completely fixed as functions of areas. Further constraints between areas for bigger triangulations.

Easiest to compute Dirac matrix.

Area constraints first class. Need further constraints!

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4d angles

$$\cos \theta_{ijkl} = \cos \theta_{ijmn}$$

4d angles computed for the same triangle in different ways agree.

Reduction II (for simplex)

$$g_{ijk} = \phi_{ijk} - \phi_{ijk}(A)$$

- two constraints per tetrahedron
- Dirac matrix block diagonal with 2x2 blocks
- area commutes with gluing constraints

$$\{\sqrt{A_{ij}}, \theta_{ijkl}\}_2 = 1$$

- Barbero-Immirzi parameter disappears
- 'non-commutativity of spatial geometry' disappears (parametrized by Barbero-Immirzi parameter)
- match to Regge phase space

Reduction II (bigger)

- additional nonlocal constraints between areas
- are first class, therefore further constraints required (eample with two simplices [BD,Ryan'08])
- than complete match to Regge phase space: length and conjugated variables

- on this phase space one can impose Hamiltonian and Diffeomorphism (pseudo) constraints or Pachner moves

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Summary

- first analysis of simplicity constraints in discrete setting
- surprises: degenerate sector seems to play big role
- Holst term seems to be essential
- LQG phase space is not the space of simplicial geometries, but much bigger

Outlook

- reduction with gauge variables
- understand embedding of SU(2) variables into SO(4)
- reconsider quantization of a simplex (dynamics known) and derive spin foam amplitude
- go to bigger triangulation ...

- do gluing conditions propagate? [consider three simplices]

- more careful analysis [as Sergej complains] : irregular constraint system requires to consider degenerate points separately.
- understand additional constraints for bigger triangulations on dihedral angles

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Non-commutative flux representation for LQG

[Baratin, BD, Oriti, Tambornino: to appear really really soon]

Idea:

- take 'non-commutative Fourier transform' on SU(2) [Freidel, Livine '05, Freidel, Majid '06, Joung, Mourad, Noui '08]
- and apply to every edge in graph: triads act by non-commutative star multiplicatino
- make this transformation cylindrically consistent
- so that one can take the continuum limit

- Geometrical interpretation of resulting space? Spectral triple construction?
- Non-geometric part?

A sketch

$$\hat{f}(x) := \int d\mu(g) f(g) \exp(\mathrm{tr} x|g|)$$

Maps (even) function on the group SU(2) to a certain class of functions on R^3.

Plane waves $e_g(x)$

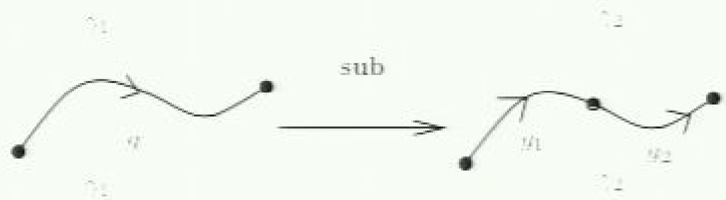
$$(e_g(x) \star e_{g'})(x) = e_{gg'}(x)$$

define non-commutative multiplication for x variables.

$$(E^i \ \hat{f})(x) = (x^i \star \hat{f})(x)$$

Flux operators act as non-commutative multiplication.

A sketch

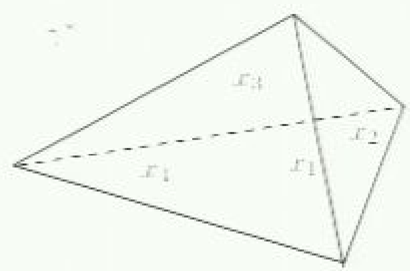


Map extends as unitary map to full (continuum) Hilbert space of LQG.

$$f_{\gamma_2}(g_1, g_2) = f_{\gamma_1}(g_1 g_2)$$

$$\hat{f}_{\gamma_2}(x\ .x\ ) = \delta_{x_1} \star \hat{f}_{\gamma_1}(x\ )$$

$$\delta_x(y) := \frac{1}{8\pi} \int \mathrm{d}g\, \mathrm{e}_{g^{-1}}(x) \mathrm{e}_g(y)$$



$$\hat{f}(x\ .x\ .x\ .x\ ) = \delta(x\ .x\ .x\ .x\ ) \star_{x_i} f(x\ .x\ .x\ .x\ )$$

Immediate geometrical interpretation of Gauss law.

Outlook

- Does this provide better understanding of quantum geometries?
- relation to non-commutative geometry?
- semiclassical states in x-space
- expansion of non-commutative product: large j expansion?
- implementation of simplicity constraints [Baratin, Oriti]

Do we have to worry?

- seem to come from degenerate configurations, general problem of first order formal.?

- 'configuration space analysis': obtain exactly 5 additional degrees of freedom if 4d volume is zero [Conrady, Freidel '08]
- turn up in asymptotic analysis [Barrett, Fairbairn Hellmann et al '08]
  vector geometries or BF contribution
- extrinsic curvature (phase for coherent state) cannot be determined
- difficult to impose: second class (plus first class for bigger triangulations)

Area-angle Regge calculus: need the gluing constraints to obtain correct dynamics of GR.
Otherwise all the areas are free variables: leads to flat geometries.
Gluing constraints impose (non-local) restrictions on spins.

Are the gluing constraints preserved by the dynamics?

Dirac brackets I

$$\{f,g\}_1 = \{f,g\} - \{ f, \Phi_\alpha\} \, X^{-1}_{\alpha\beta} \, \{\Phi_\beta, g\}$$

$$\{ \sqrt{A_{ij}}, \theta_{ijkl}\}$$

areas conjugated to extrinsic curvature

$$\{ \sqrt{A_{ij} A_{ik}} \cos \phi_{ijk}, \sqrt{A_{ij} A_{il}} \cos \phi_{ijl}\} = \frac{\gamma}{2} V_{ijkl}$$

3d angles do not commute, singular without Holst term

Also the other brackets can be computed.

Match to LQG phase space

Reduction from Plebanski.		LQG Phase space.
$\{\sqrt{A_{ij}A_{ik}}\cos\phi_{ijk}, \sqrt{A_{ij}A_{il}}\cos\phi_{ijl}\} = \frac{\gamma}{2}V_{ijkl}$	$\longrightarrow$	$\{E^a_{ij}, E^b_{ij}\} = \gamma\epsilon^{abc}E^c_{ij}$
$\{\sqrt{A_{ij}}, \theta_{ijkl}\}$	$\xrightarrow{?}$	$\{E^a_{ij}, M^{bd}_{ij}\} = \gamma\epsilon^{abc}M^{cd}_{ij}$

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But (because reduced conjugated variable is not a connection anymore) one introduces the Ashtekar (Barbero-Immirzi) connection

$$M \sim \exp A \qquad A^a_i = \Gamma^a_i + \gamma K^a_i$$

$$M \sim \exp \gamma\theta$$

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Reduction from Plebanski: ... e space.

You are now running on reserve battery power. Your slideshow has been suspended so you can address the

$\{\sqrt{A_{ij}A_{ik}}\cos\theta_{ijk}, \sqrt{A_{ij}A_{il}}$ ... $\epsilon^{abc}E^c_{ij}$

You are now running on reserve battery power.

Please connect your computer to AC power. If you do not, your computer will go to sleep in a few minutes to preserve the contents of memory.

OK

$\{\sqrt{A_{ij}}, \theta_{ijkl}\}$ ... $\gamma\epsilon^{abc}M^{cd}_{ij}$

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Complete agreement with (discrete) LQG phase space.

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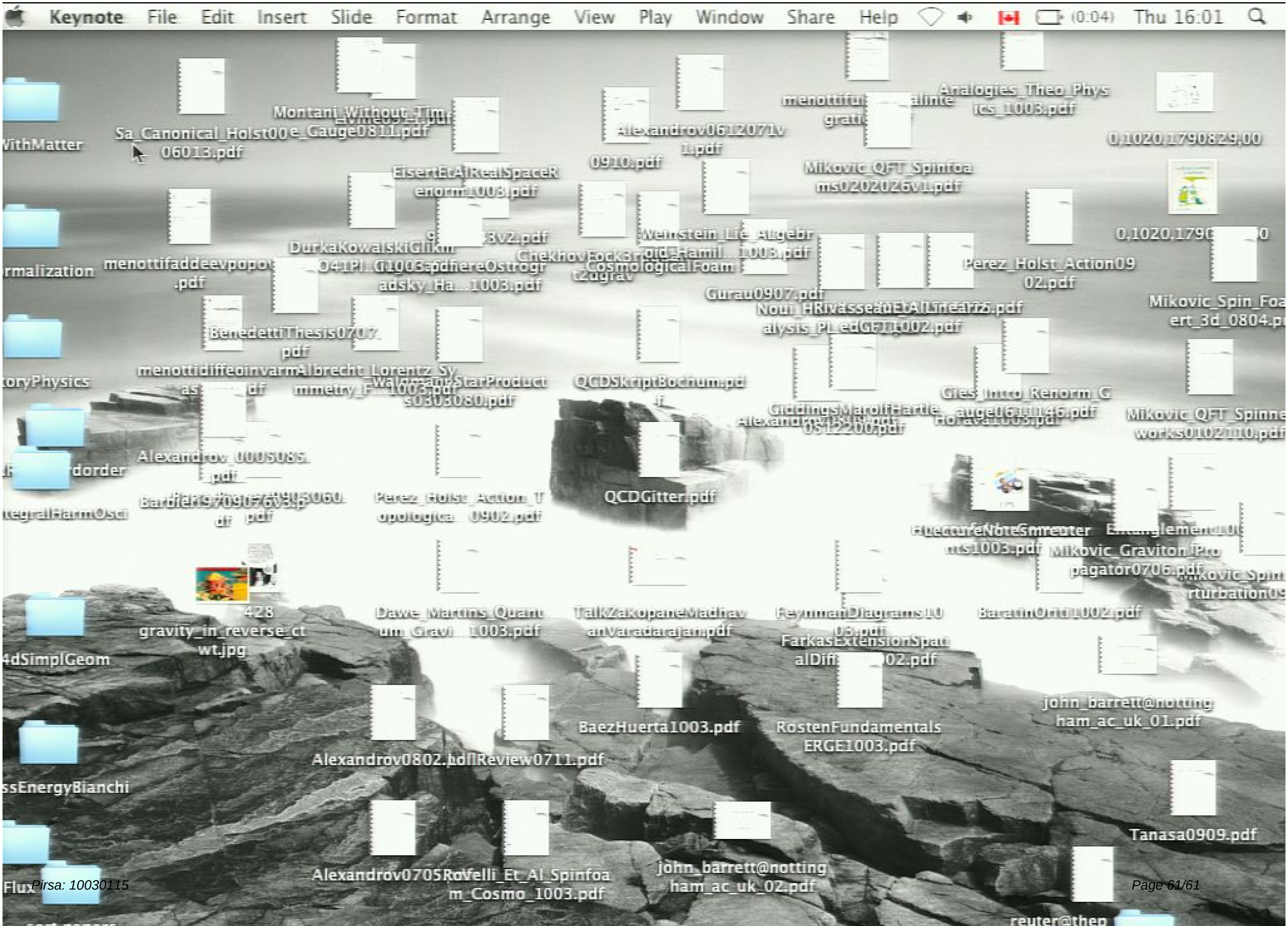
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