Title: The compositional structure of multipartite quantum entanglement

Date: Mar 29, 2010 04:00 PM

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Abstract: Multipartite quantum states constitute a (if not the) key resource for quantum computations and protocols. However obtaining a generic, structural understanding of entanglement in N-qubit systems is still largely an open problem. Here we show that multipartite quantum entanglement admits a compositional structure. The two SLOCC-classes of genuinely entangled 3-qubit states, the GHZ-class and the W-class, exactly correspond with the two kinds of commutative Frobenius algebras on C^2, namely `special' ones and `anti-special' ones. Within the graphical language of symmetric monoidal categories, the distinction between `special' and `anti-special' is purely topological, in terms of `connected' vs.~`disconnected'. These GHZ and W Frobenius algebras form the primitives of a graphical calculus which is expressive enough to generate and reason about representatives of arbitrary N-qubit states.

This calculus induces a generalised graph state paradigm for measurement-based quantum computing, and refines the graphical calculus of complementary observables due to Duncan and one of the authors [ICALP'08], which has already shown itself to have many applications and admit automation.

References: Bob Coecke and Aleks Kissinger, http://arxiv.org/abs/1002.2540

The Compositional Structure of Multipartite Quantum Entanglement

Bob Coecke and Aleks Kissinger
Oxford University Computing Laboratory

EPSRC

Engineering and Physical Sciences Research Council

ONR

QICS

fq(x)

SLOCC

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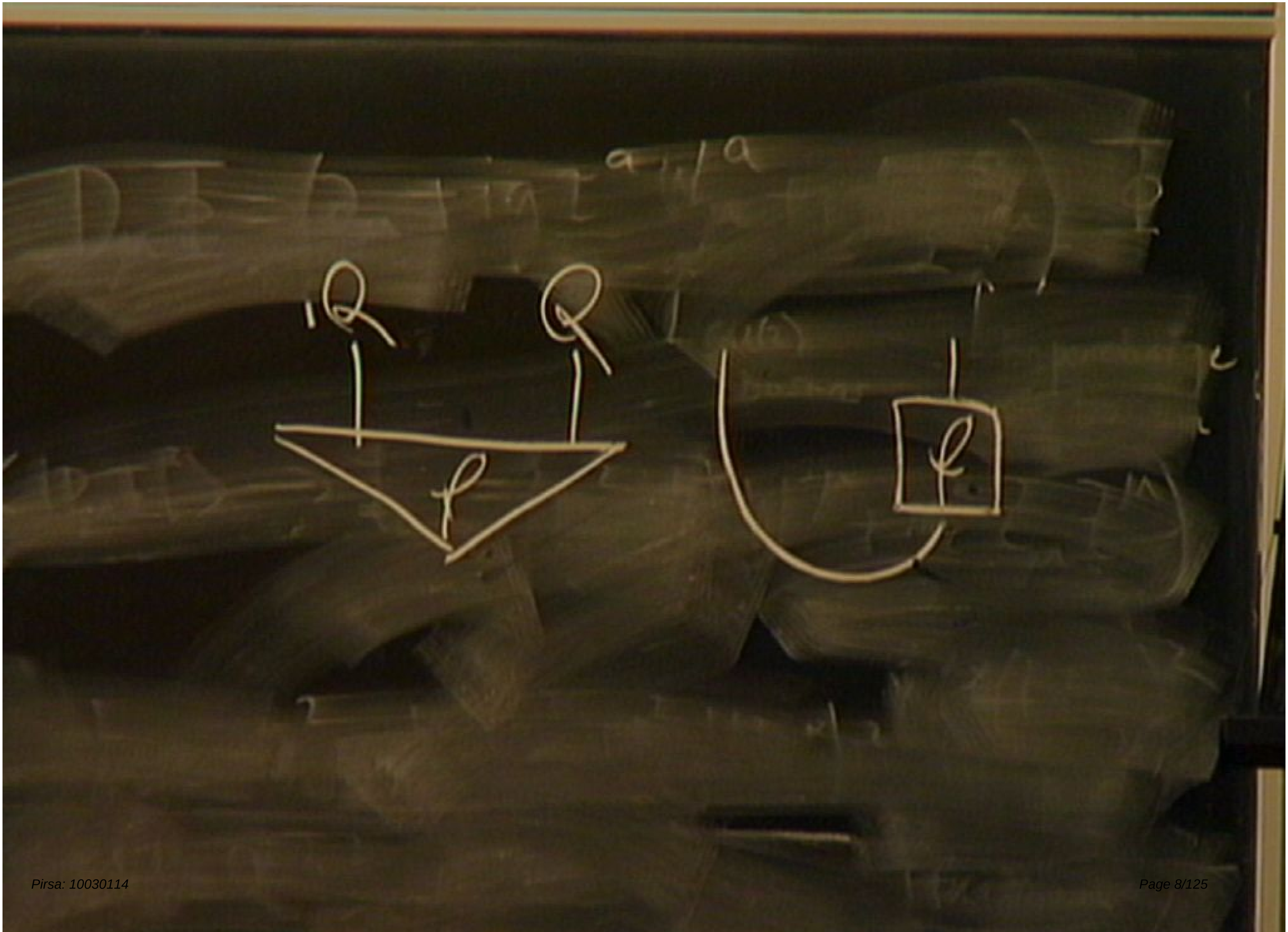
Proof: A linear map either has an inverse or not.



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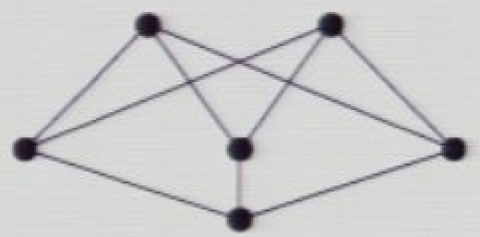

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$$GHZ = |000\rangle + |111\rangle$$

Many applications in quantum computing e.g. fault-tolerance; canonical witness of quantum non-locality.

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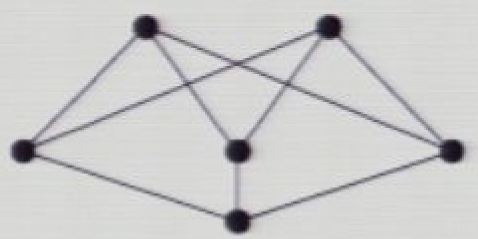

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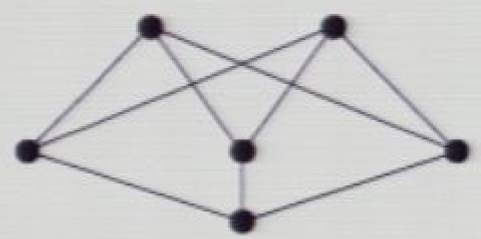

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  - GHZ/W-duality more fundamental than complementarity?

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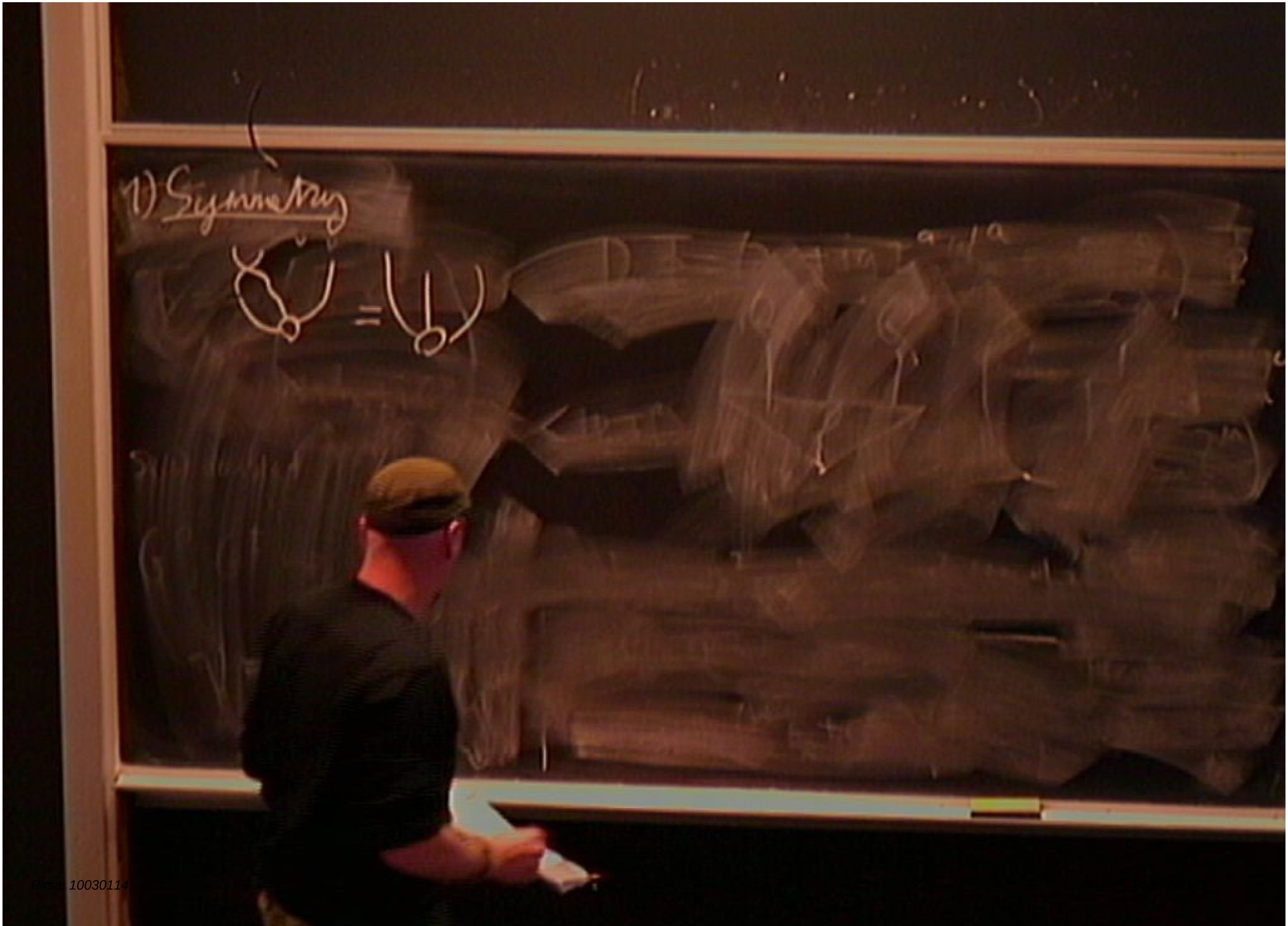
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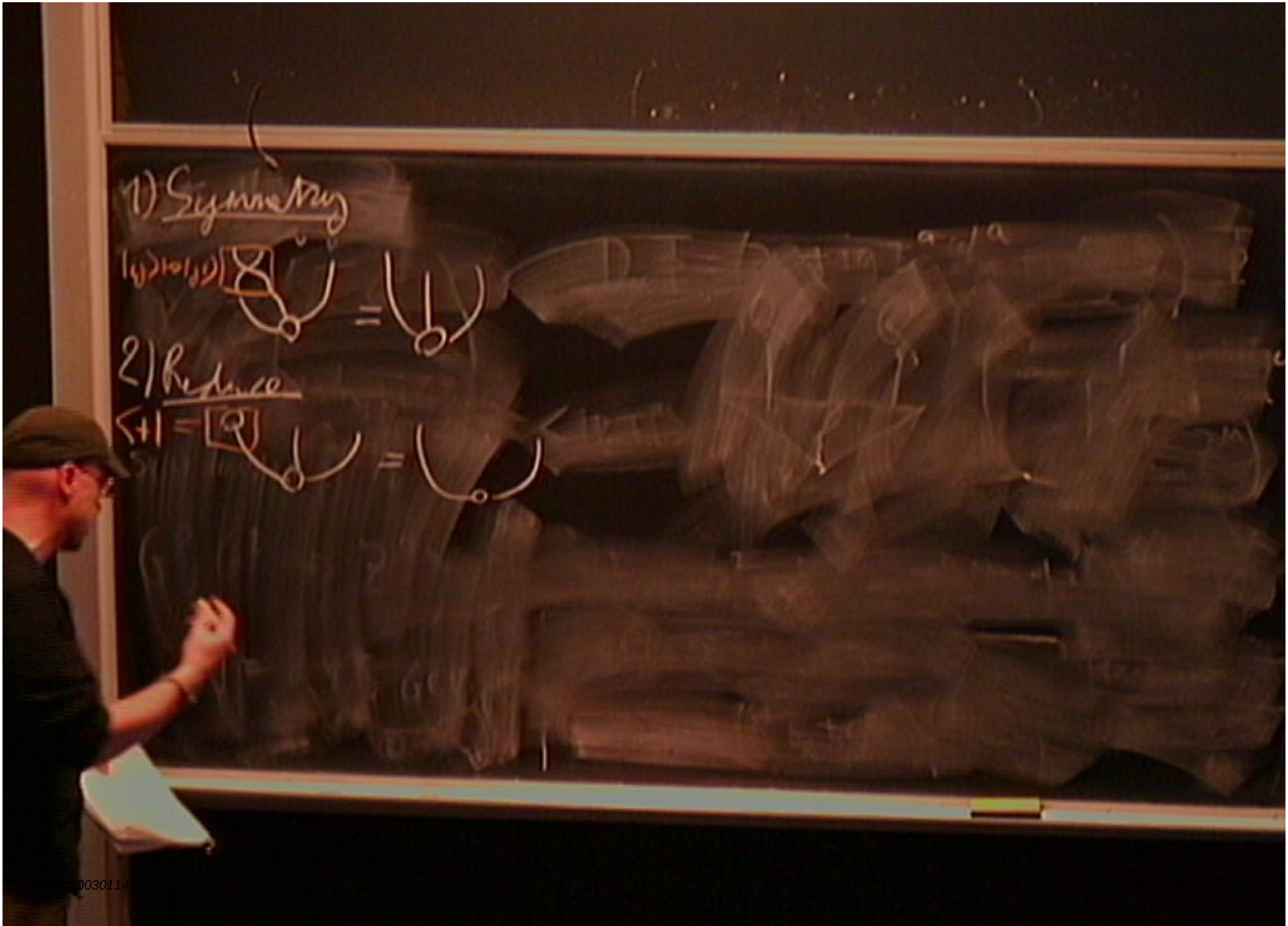
1) Symmetry
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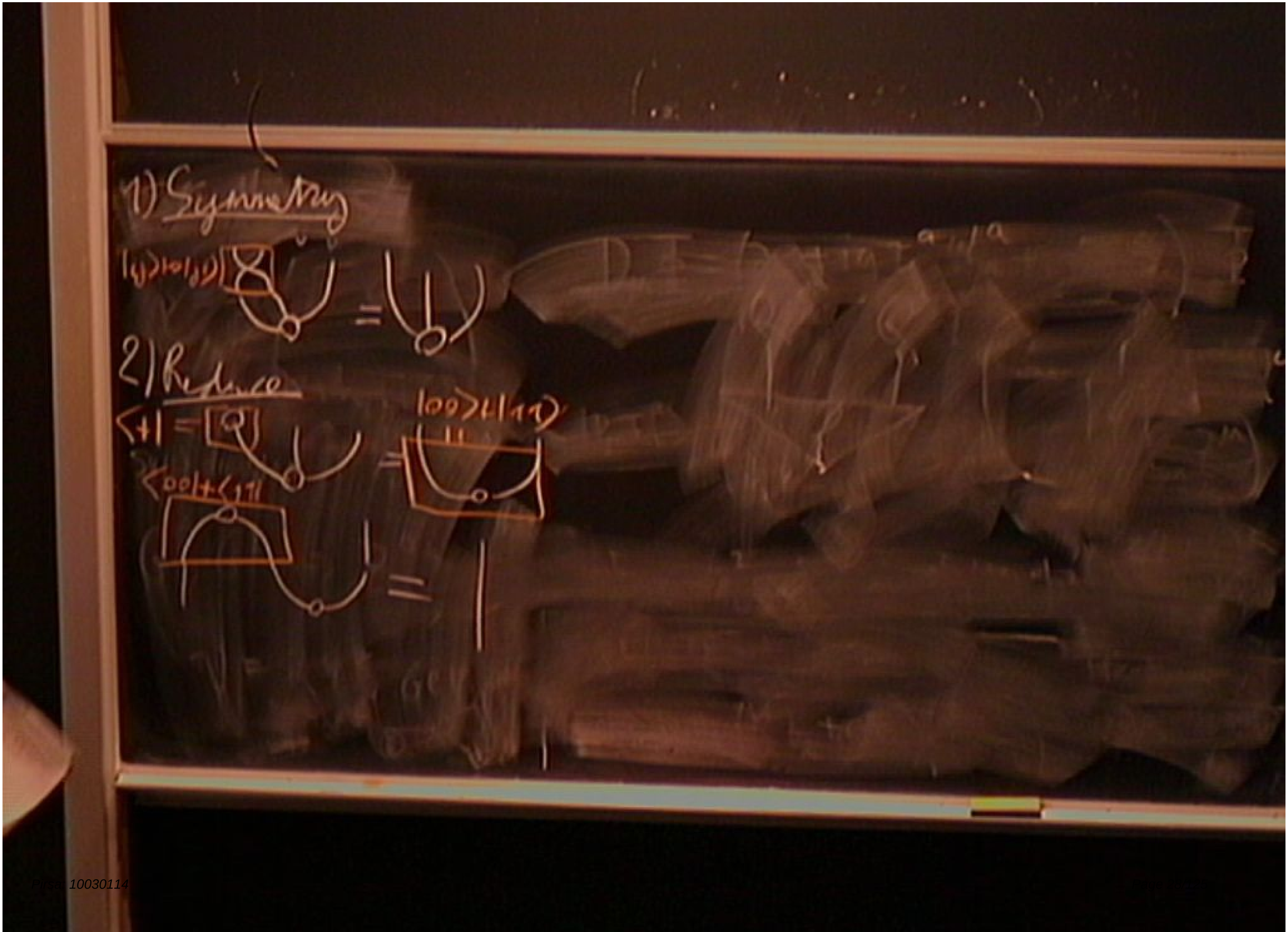
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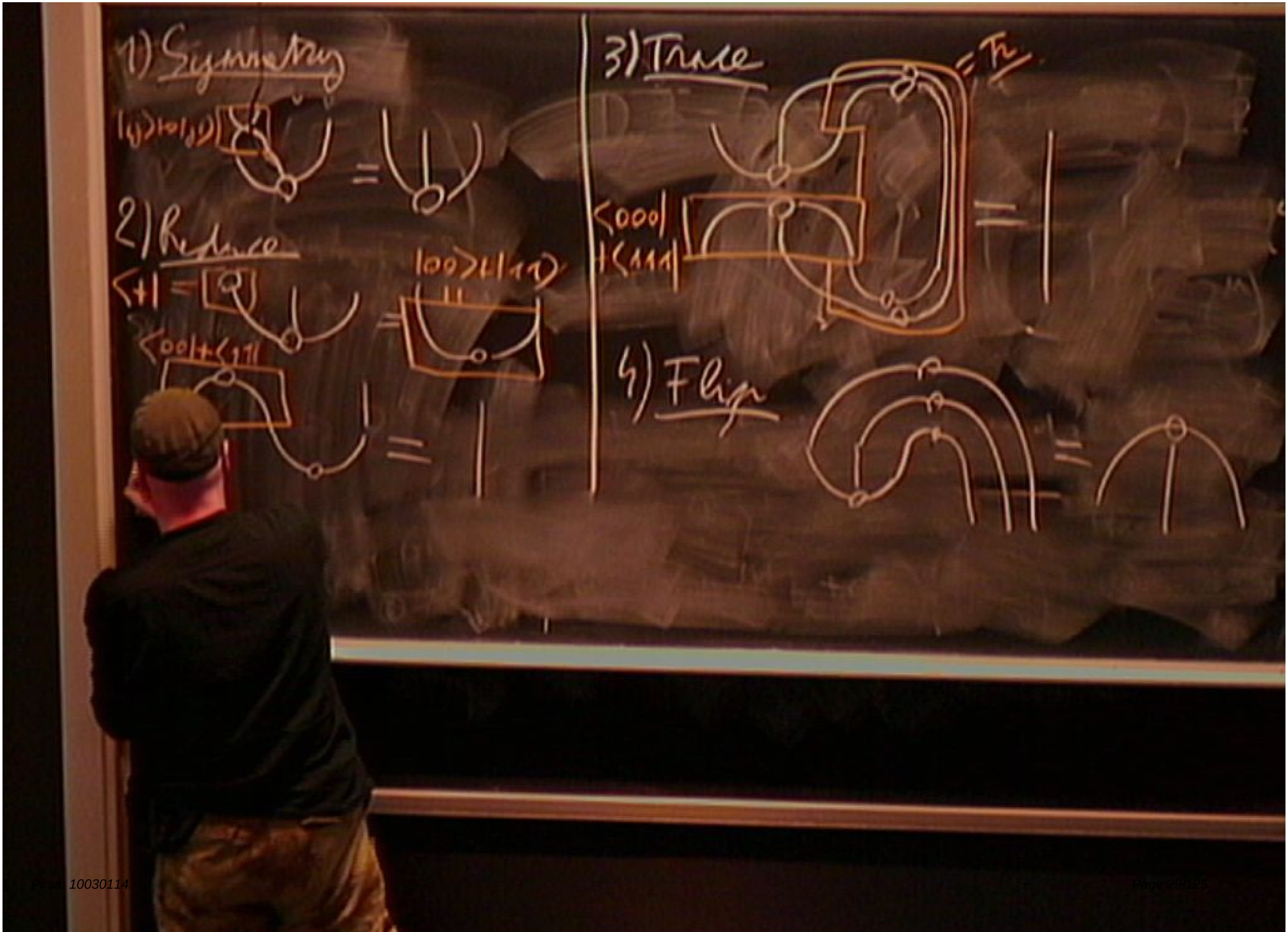
1) Symmetry
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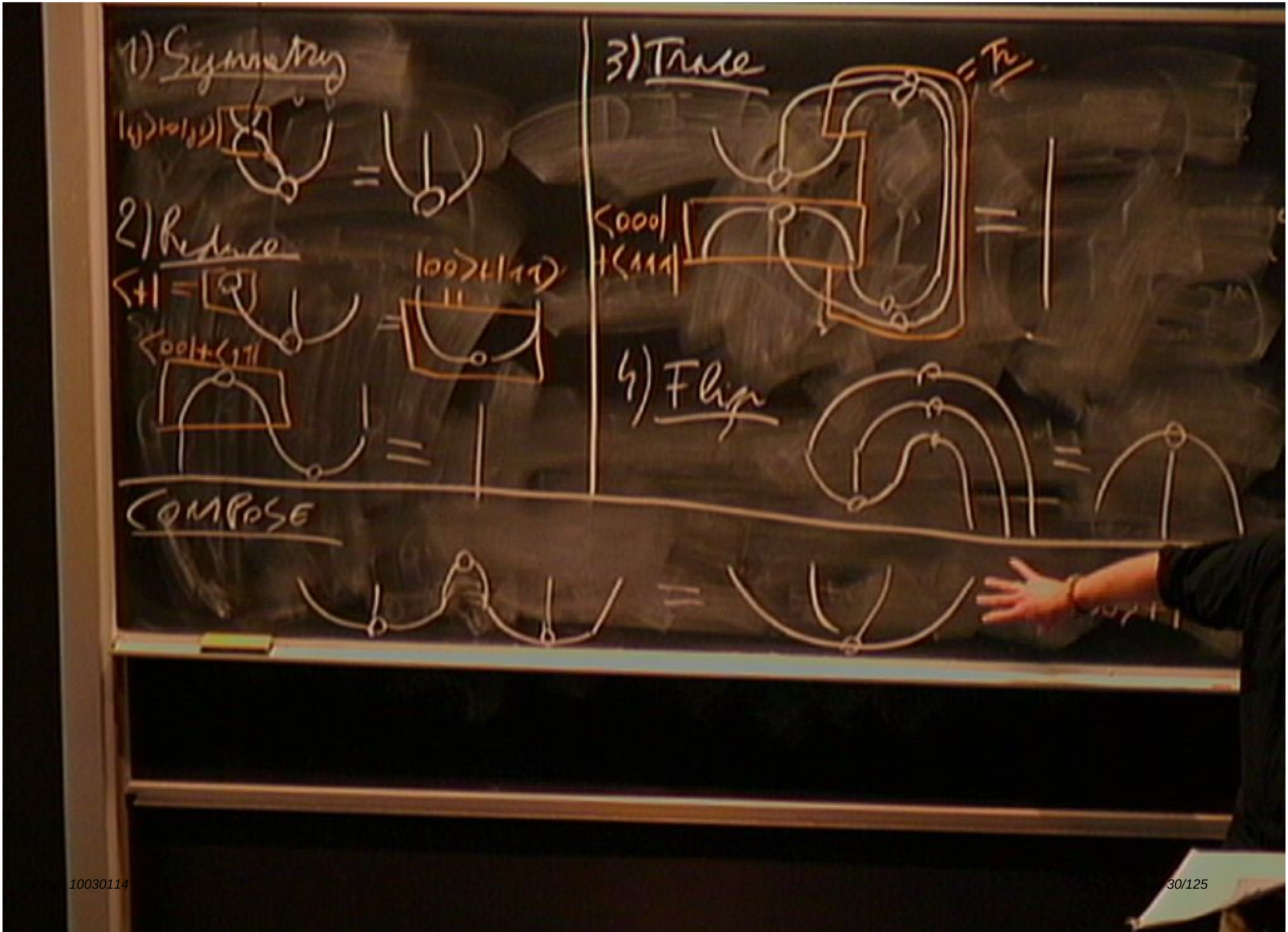
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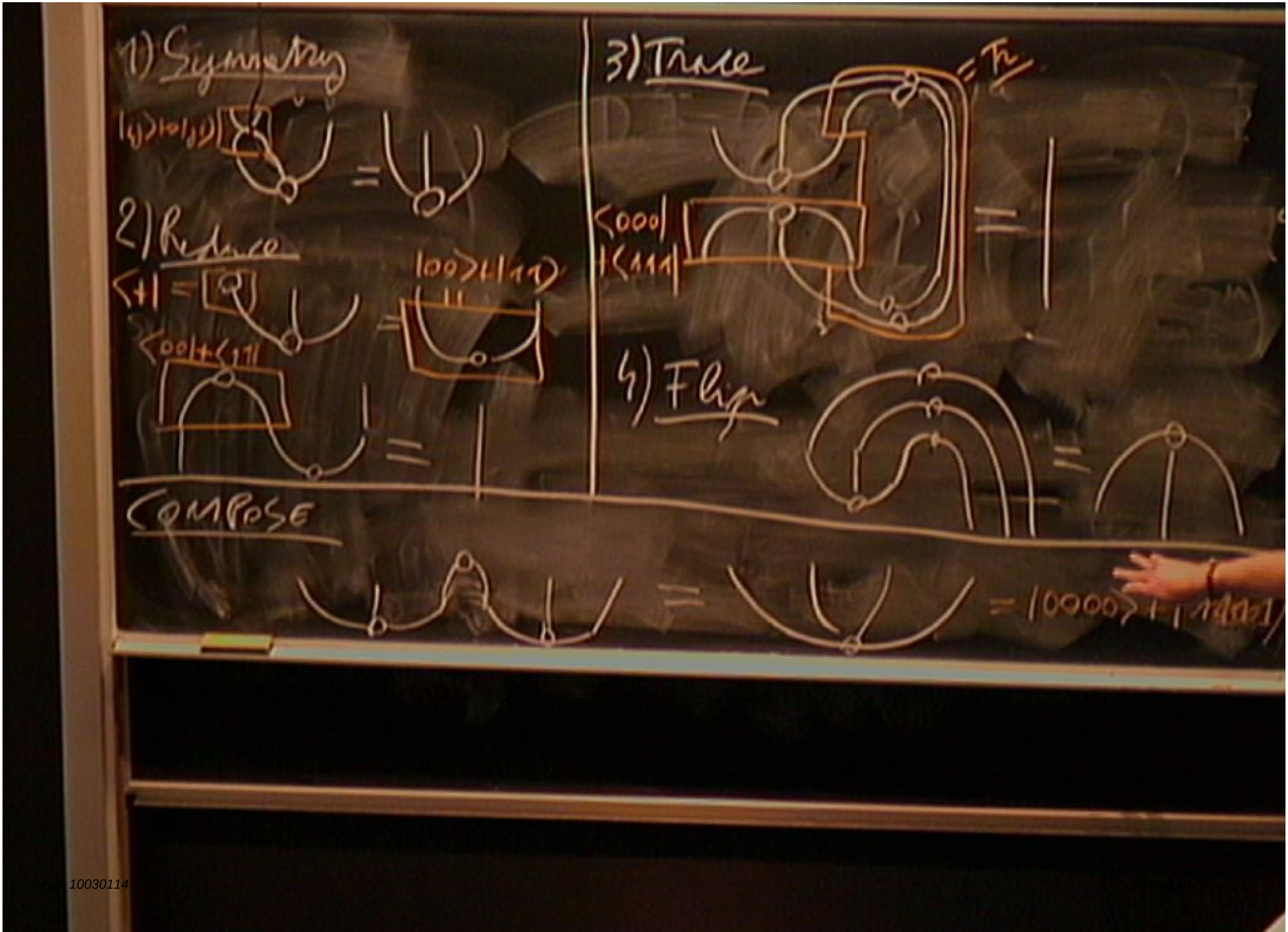
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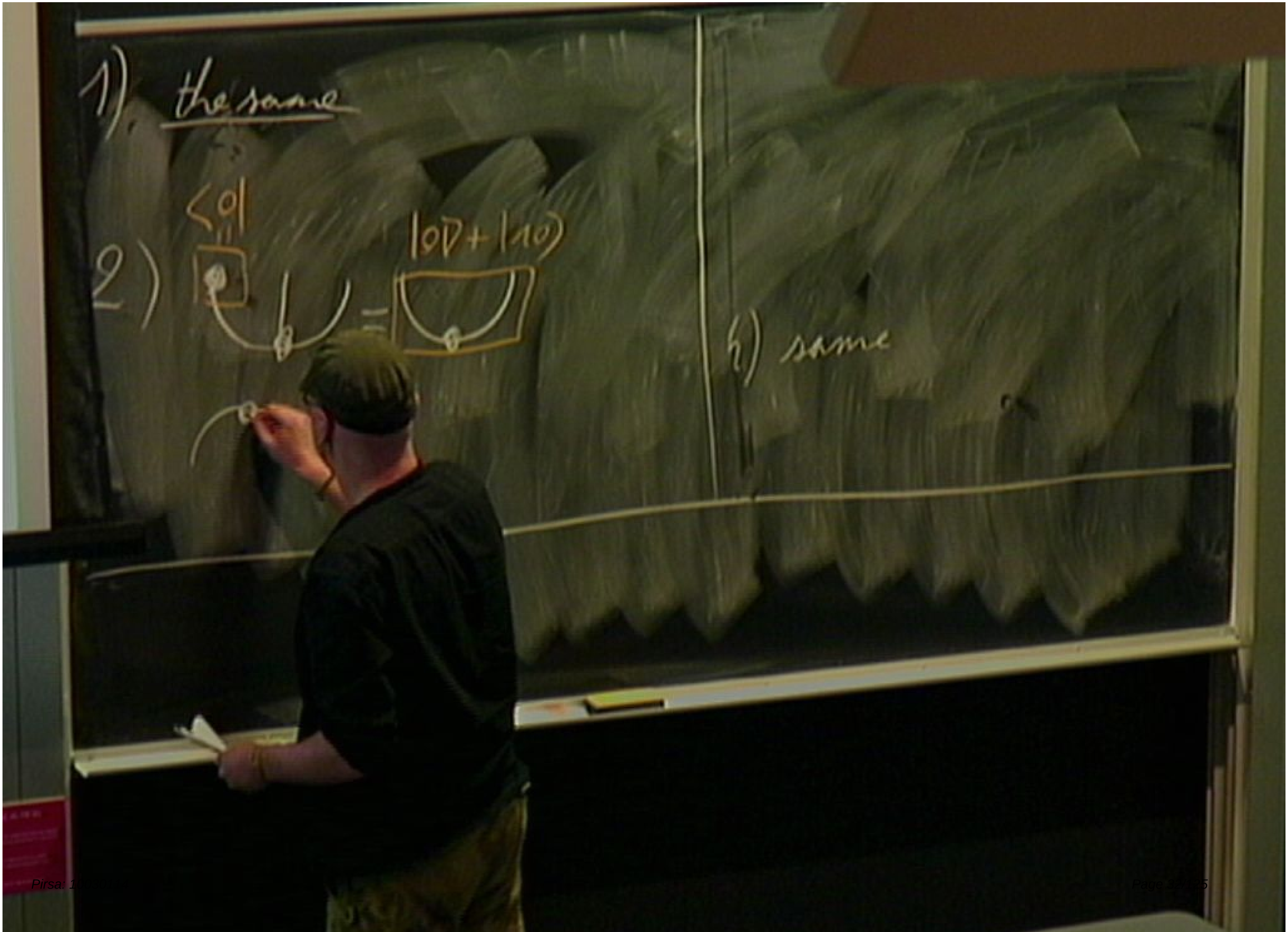

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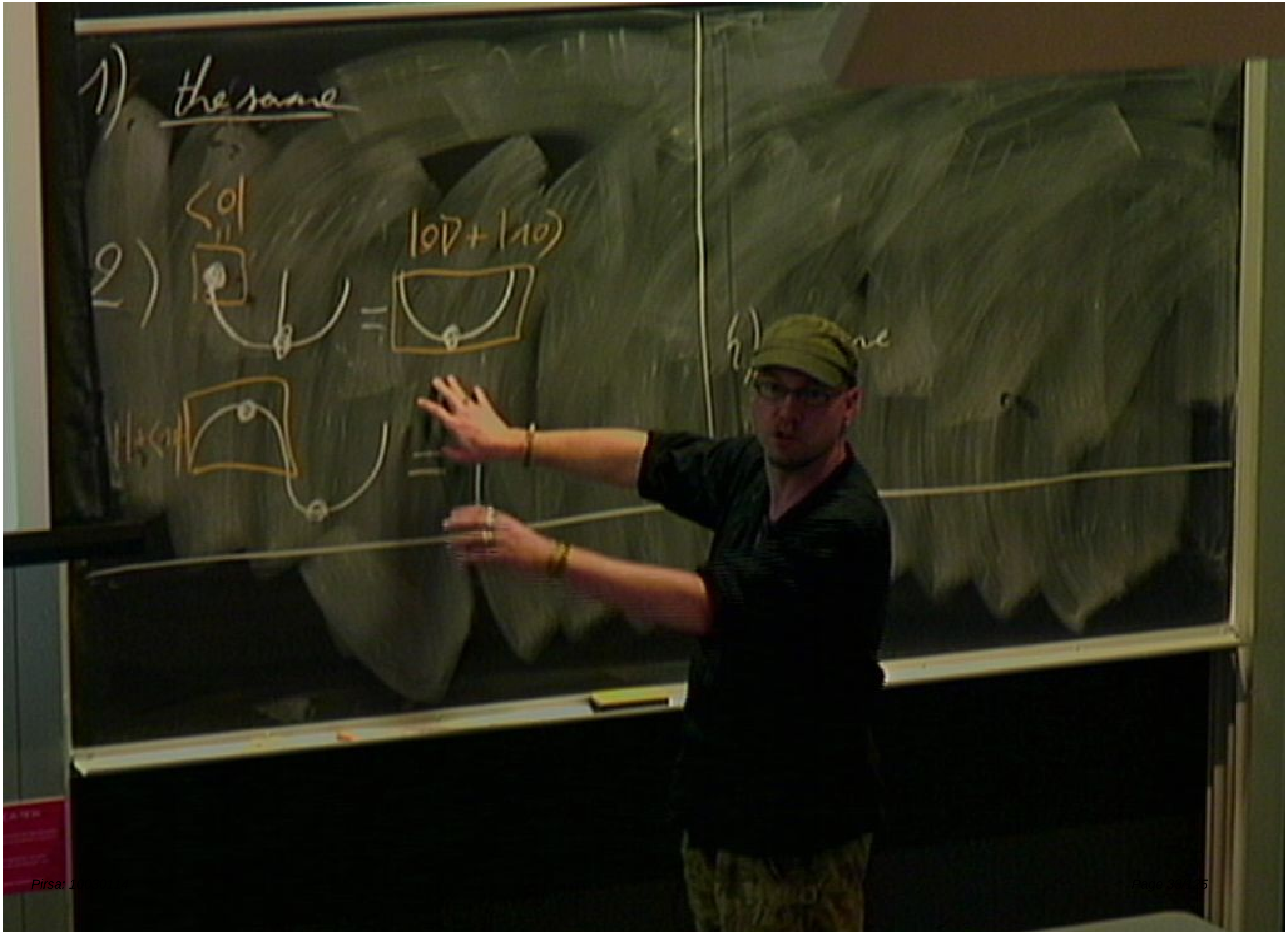
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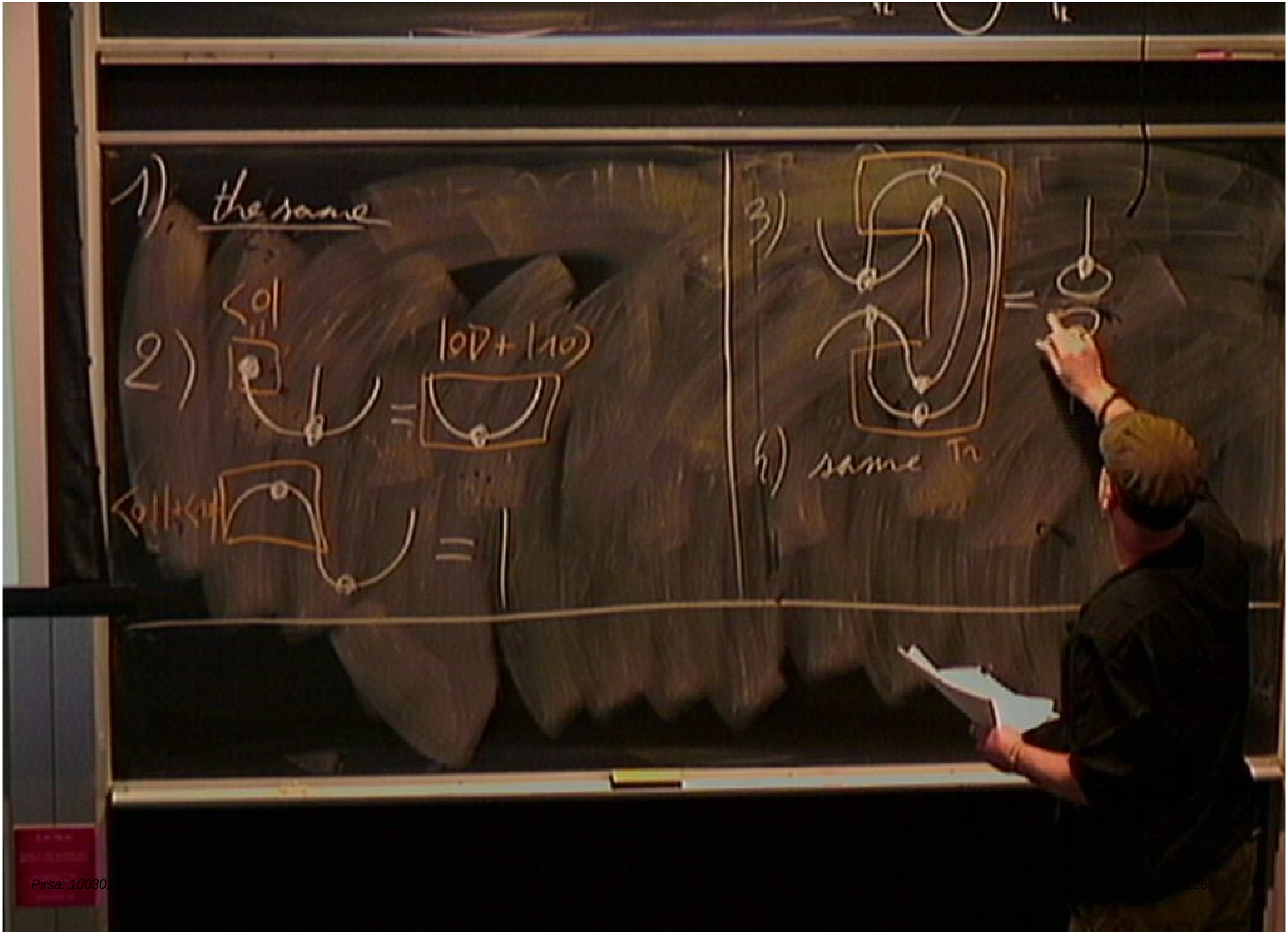
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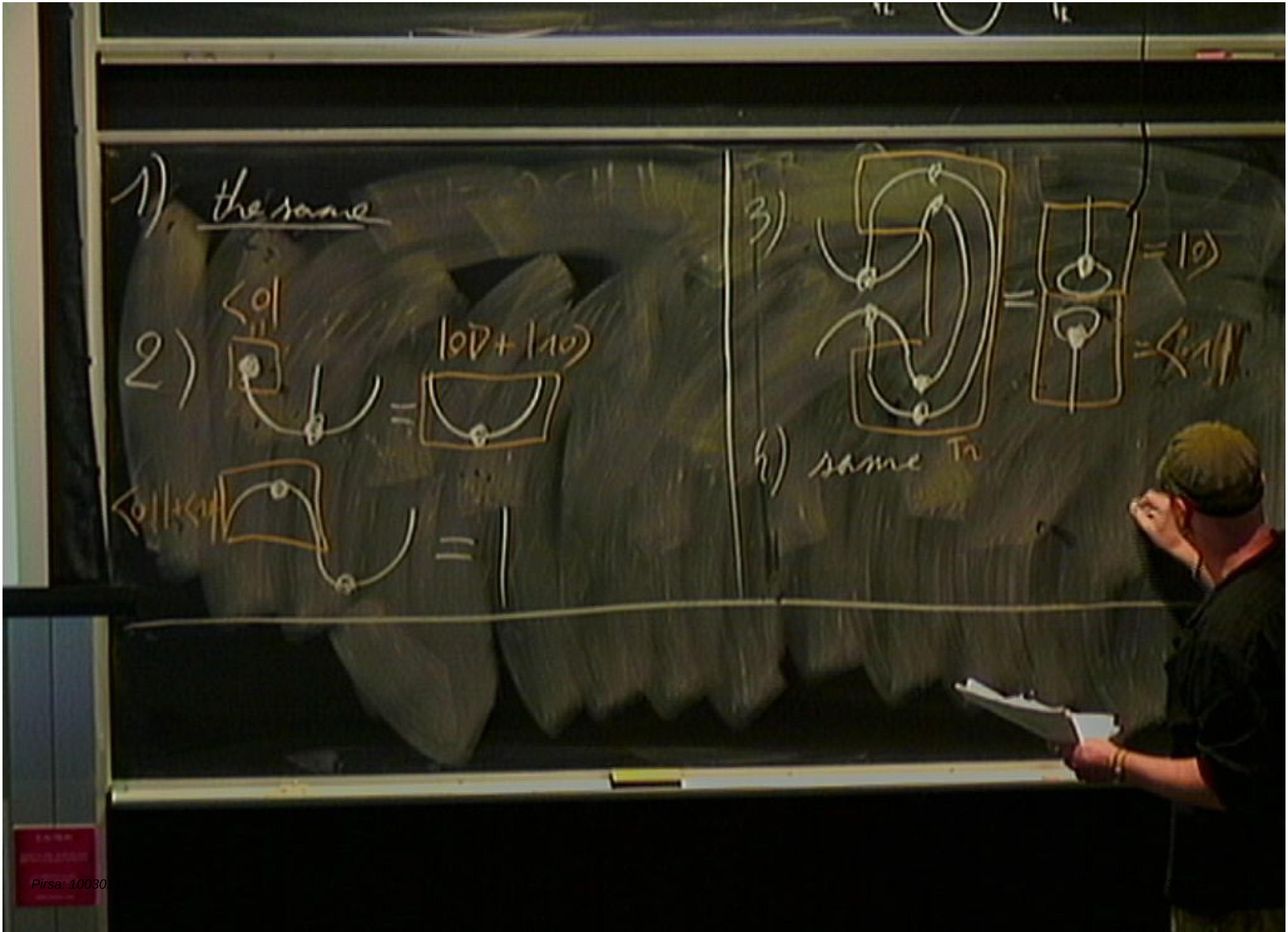
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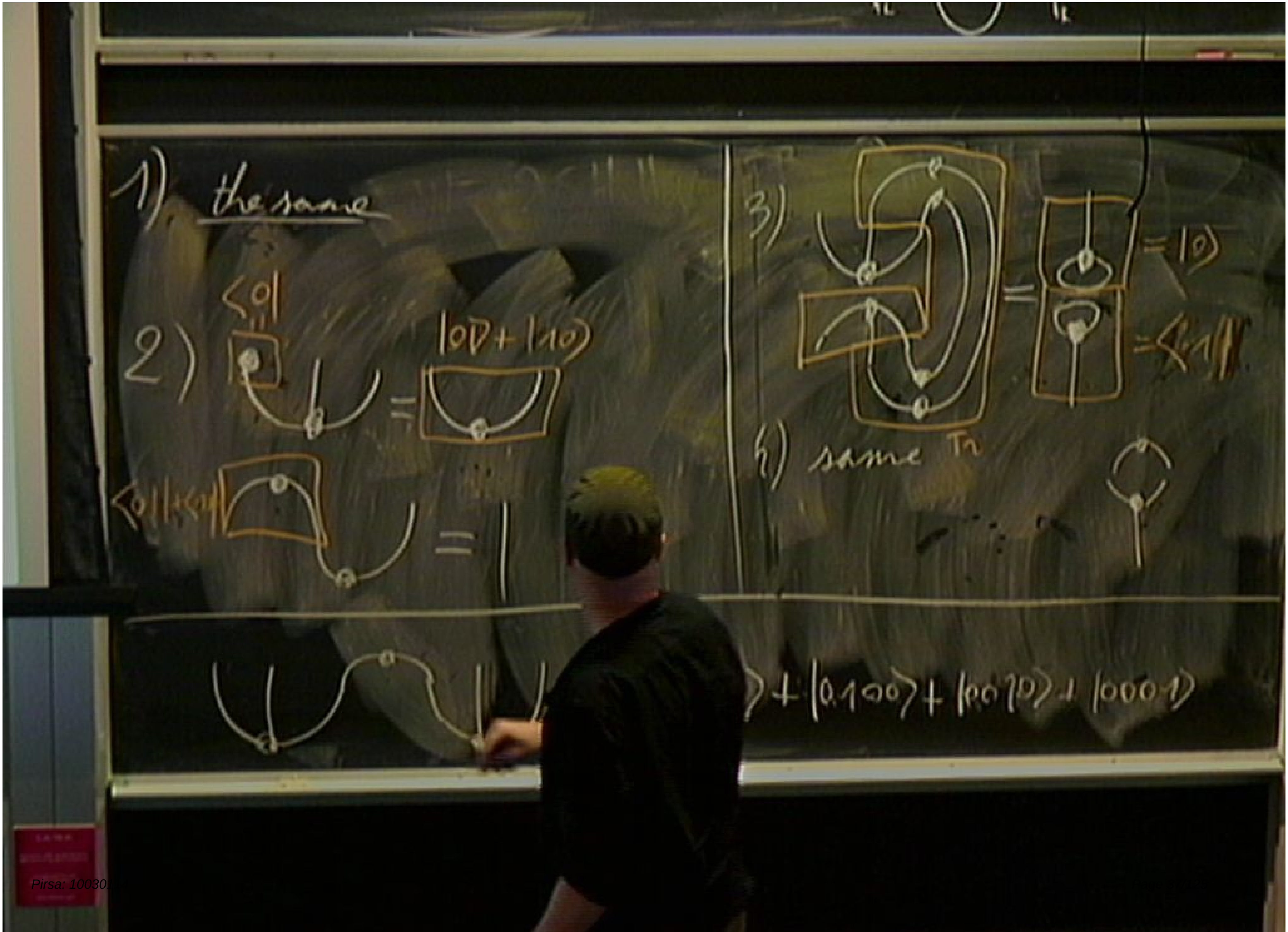
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A commutative monoid is a set A with a binary map

$$\mu(-,-) : A \times A \to A$$

which is commutative, associative and unital i.e

$$\mu(\mu(a,b),c) = \mu(a,\mu(b,c)) \quad \mu(a,b) = \mu(b,a) \quad \mu(a,1) = a$$

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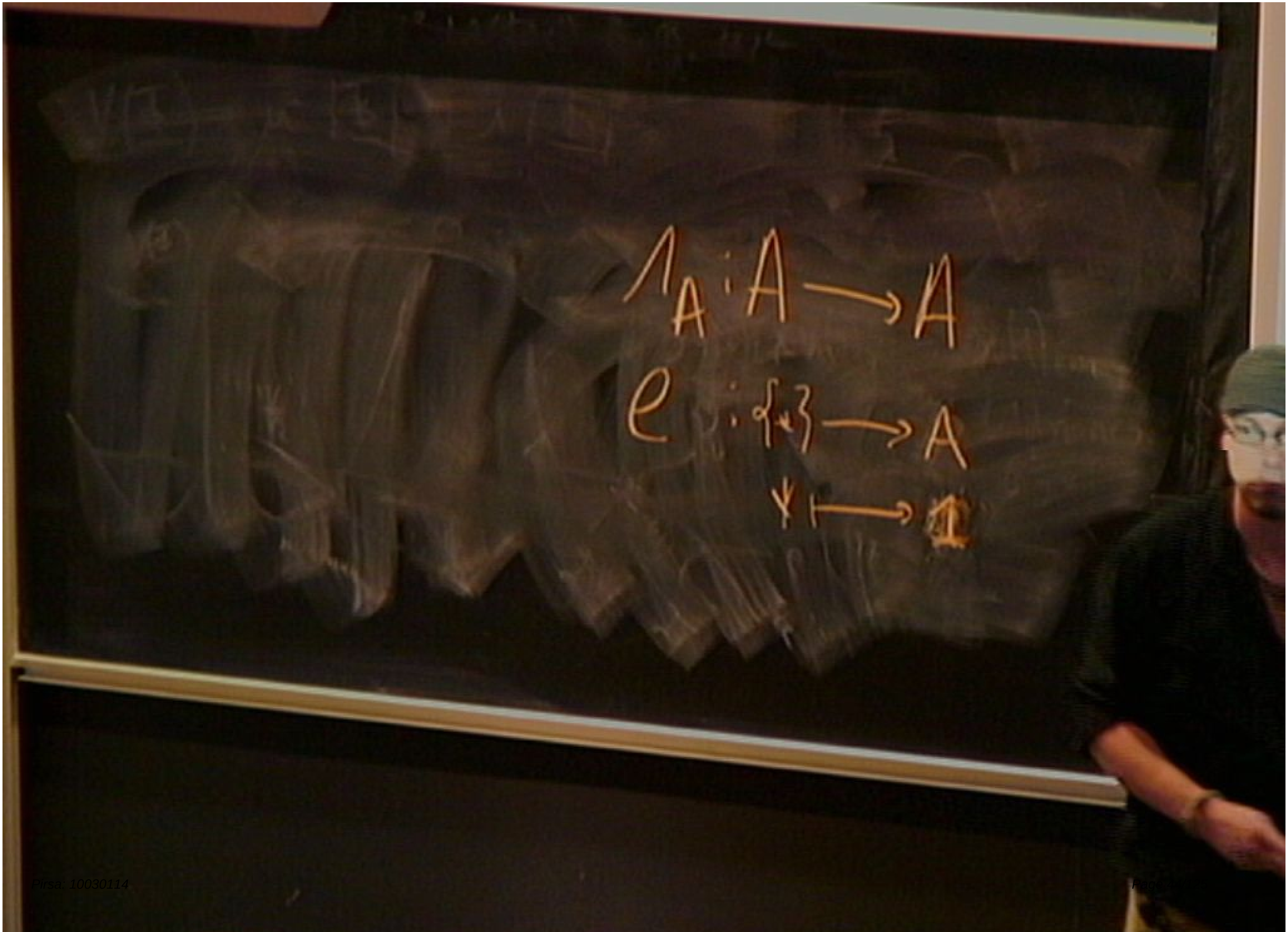
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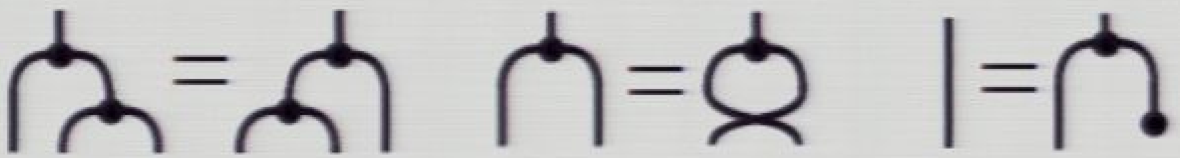

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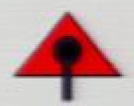
 $: A \to A \otimes A$  $: A \to \mathrm{I}$

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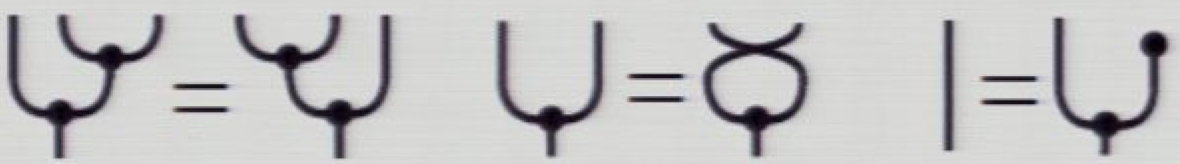

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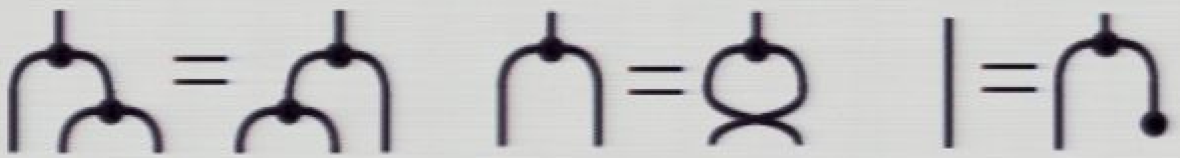

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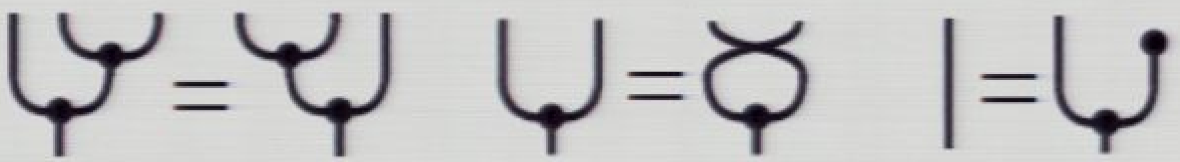

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$$:: \begin{cases} |++\rangle \mapsto |+\rangle \\ |--\rangle \mapsto |-\rangle \end{cases}$$

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$$:: \begin{cases} |\sharp\,\sharp\rangle \mapsto |\sharp\rangle \\ |==\rangle \mapsto |=\rangle \end{cases}$$

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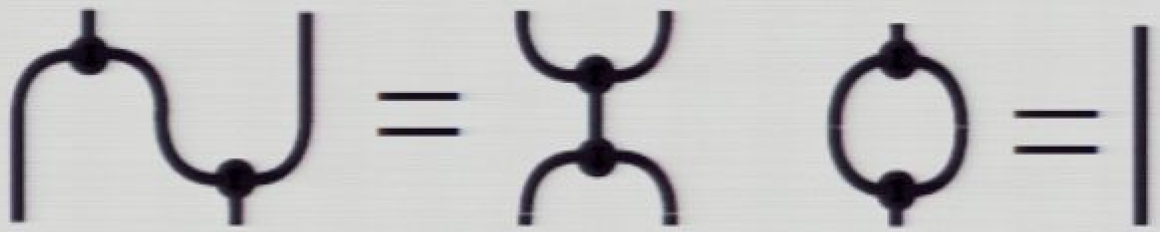
Thm. In FHilb special CFAs, i.e



Frobenius special

exactly correspond with (arbitrary i.e. non-ONB) bases on the underlying Hilbert space via the correspondence:

$$\{|i\rangle\}_i \quad \longleftrightarrow \quad |i\rangle \mapsto |ii\rangle$$

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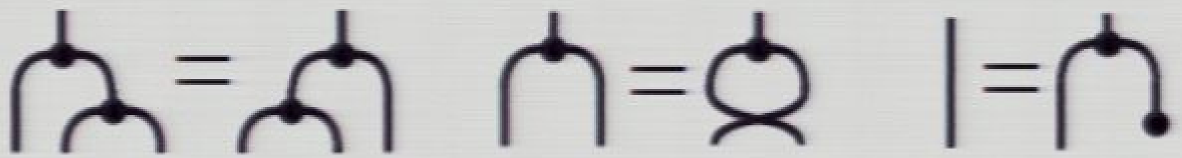

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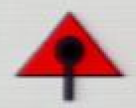
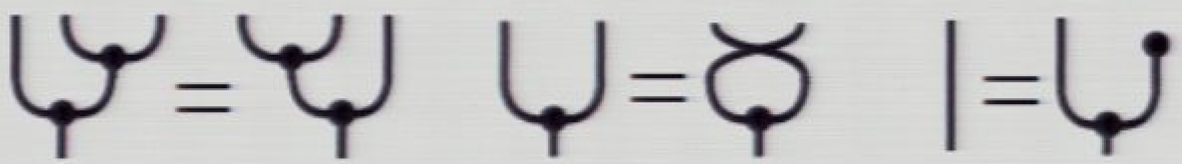
 $: A \rightarrow A \otimes A$  $: A \rightarrow \mathrm{I}$

s.t.



— *commutative Frobenius algebras* —

FSet:

$$:: \begin{cases} |00\rangle, |01\rangle, |10\rangle \mapsto |0\rangle \\ |11\rangle \mapsto |1\rangle \end{cases}$$

$$:: \begin{cases} |0\rangle \mapsto |00\rangle \\ |1\rangle \mapsto |11\rangle \end{cases}$$

— *commutative Frobenius algebras* —

FdHilb:

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$$:: \begin{cases} |0\rangle \mapsto |00\rangle \\ |1\rangle \mapsto |11\rangle \end{cases}$$

$$:: \begin{cases} |++\rangle \mapsto |+\rangle \\ |--\rangle \mapsto |-\rangle \end{cases}$$

$$:: \begin{cases} |+\rangle \mapsto |++\rangle \\ |-\rangle \mapsto |--\rangle \end{cases}$$

$$:: \begin{cases} |\sharp\sharp\rangle \mapsto |\sharp\rangle \\ |==\rangle \mapsto |=\rangle \end{cases}$$

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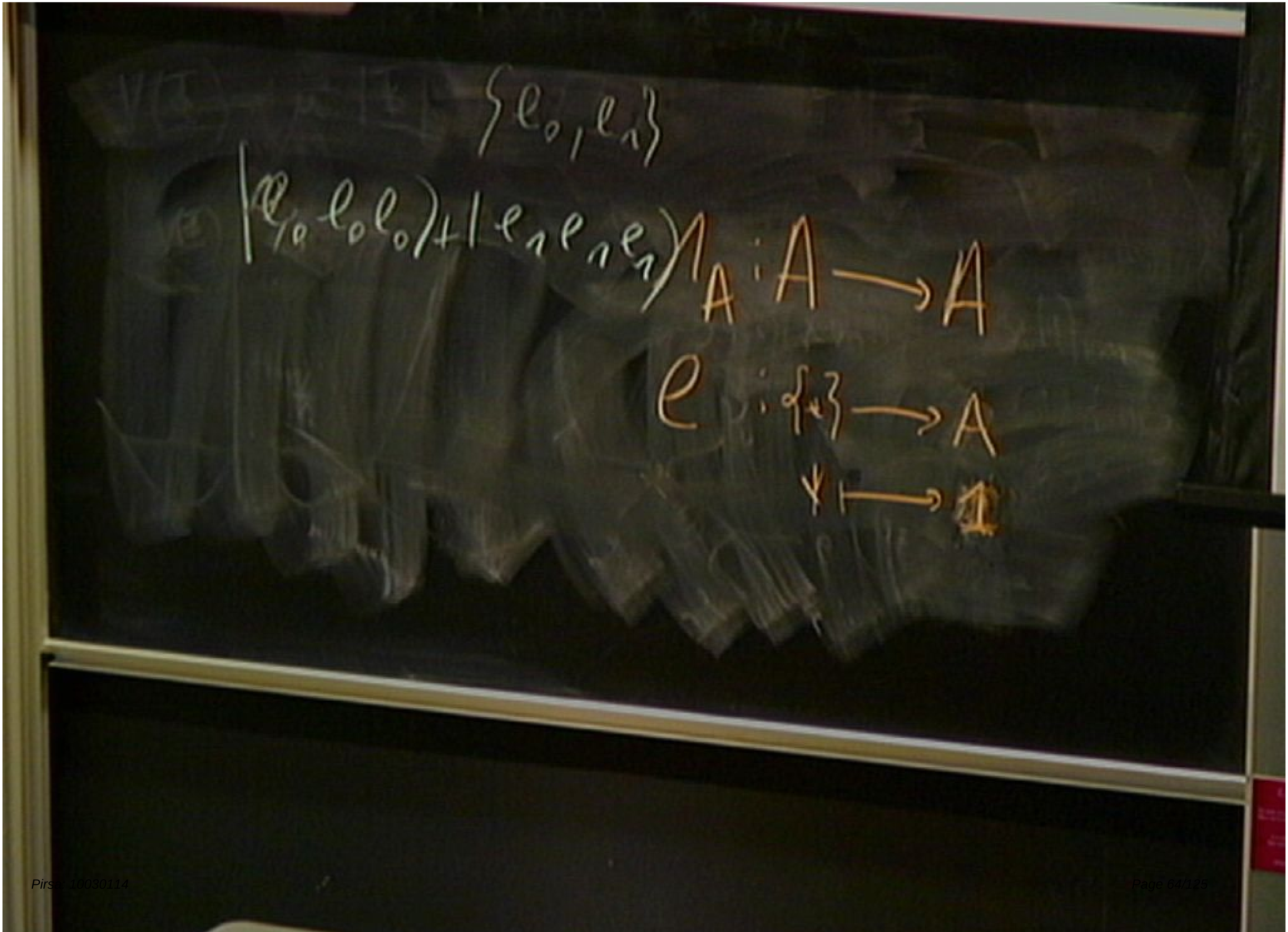
— *commutative Frobenius algebras* —

Thm. In FHilb special CFAs, i.e

Frobenius special

exactly correspond with (arbitrary i.e. non-ONB) bases on the underlying Hilbert space via the correspondence:

$$\{|i\rangle\}_i \quad \longleftrightarrow \quad |i\rangle \mapsto |ii\rangle$$


{e0, e1}
|e0 e0 e0⟩ + |e1 e1 e1⟩
ΔA : A → A
e : {*} → A
* ↦ 1

GHZ AND W:

ALGEBRAIC SIMILARITY

and

TOPOLOGICAL DIFFERENCE

— *commutative Frobenius algebras* —

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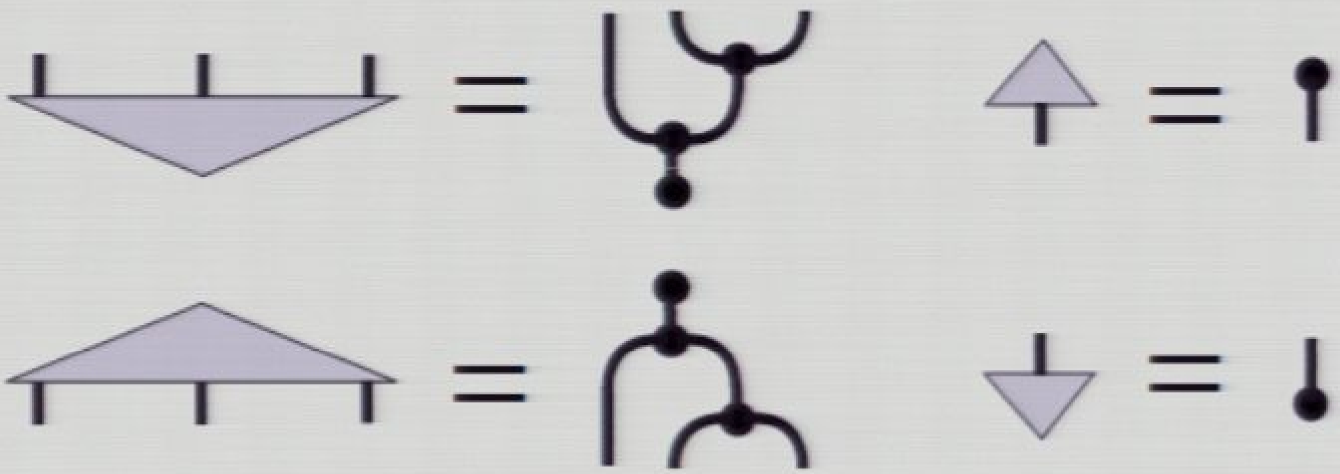
GHZ AND W:
ALGEBRAIC SIMILARITY
and
TOPOLOGICAL DIFFERENCE

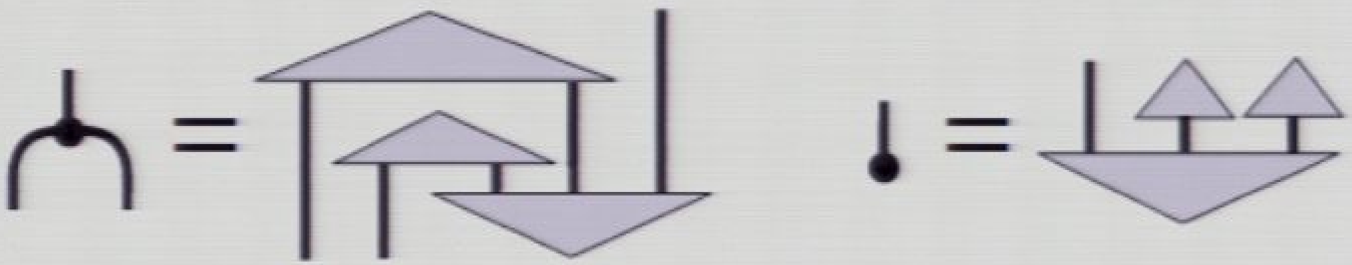
— C(F)As ⟷ tripartite states —

From (co)monoid and (co)unit we build tripartite (co)state:



— *C(F)As ⟷ tripartite states* —

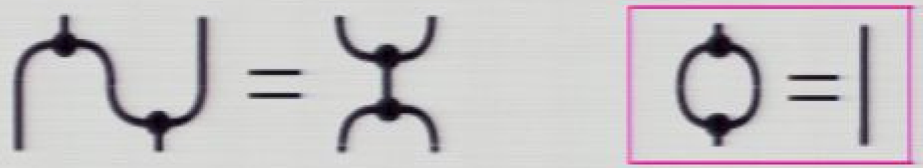
From tripartite state and unit we build:



with  *s.t.* 

and via transposition we obtain comonoid.

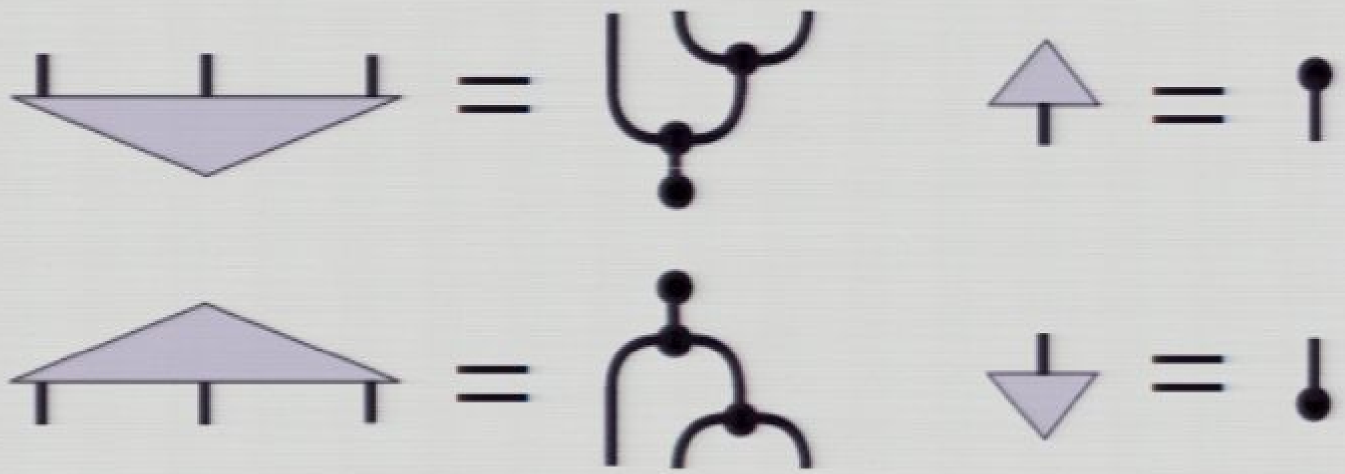
Proposition. A special CFA on $\mathbb{C}^2$, i.e.



induces a symmetric GHZ-class state, **and vice versa.**

— *C(F)As* $\longleftrightarrow$ *tripartite states* —

From (co)monoid and (co)unit we build tripartite (co)state:



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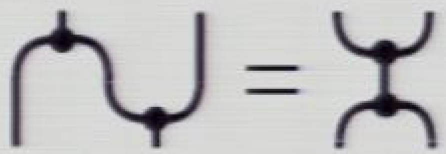

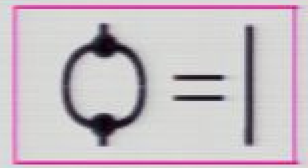

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From tripartite state and unit we build:

with

s.t.

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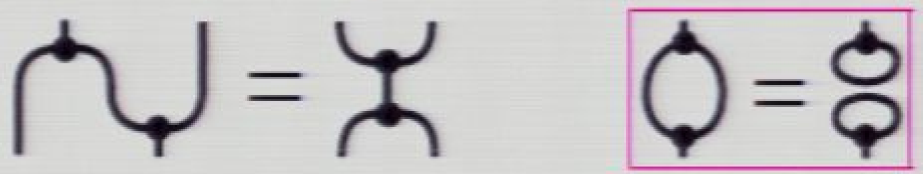
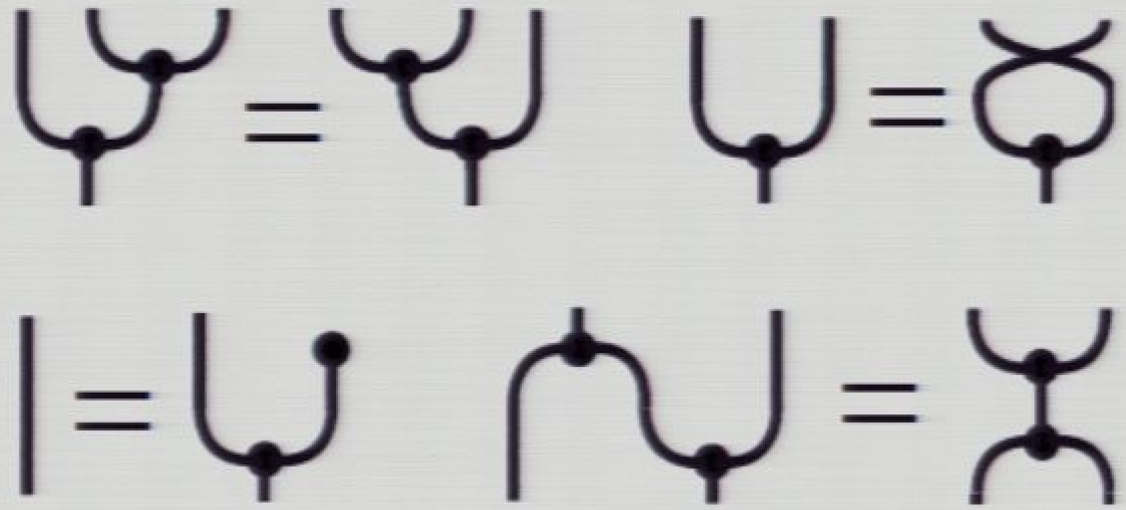
Proposition. An anti-special CFA on $\mathbb{C}^2$, i.e.

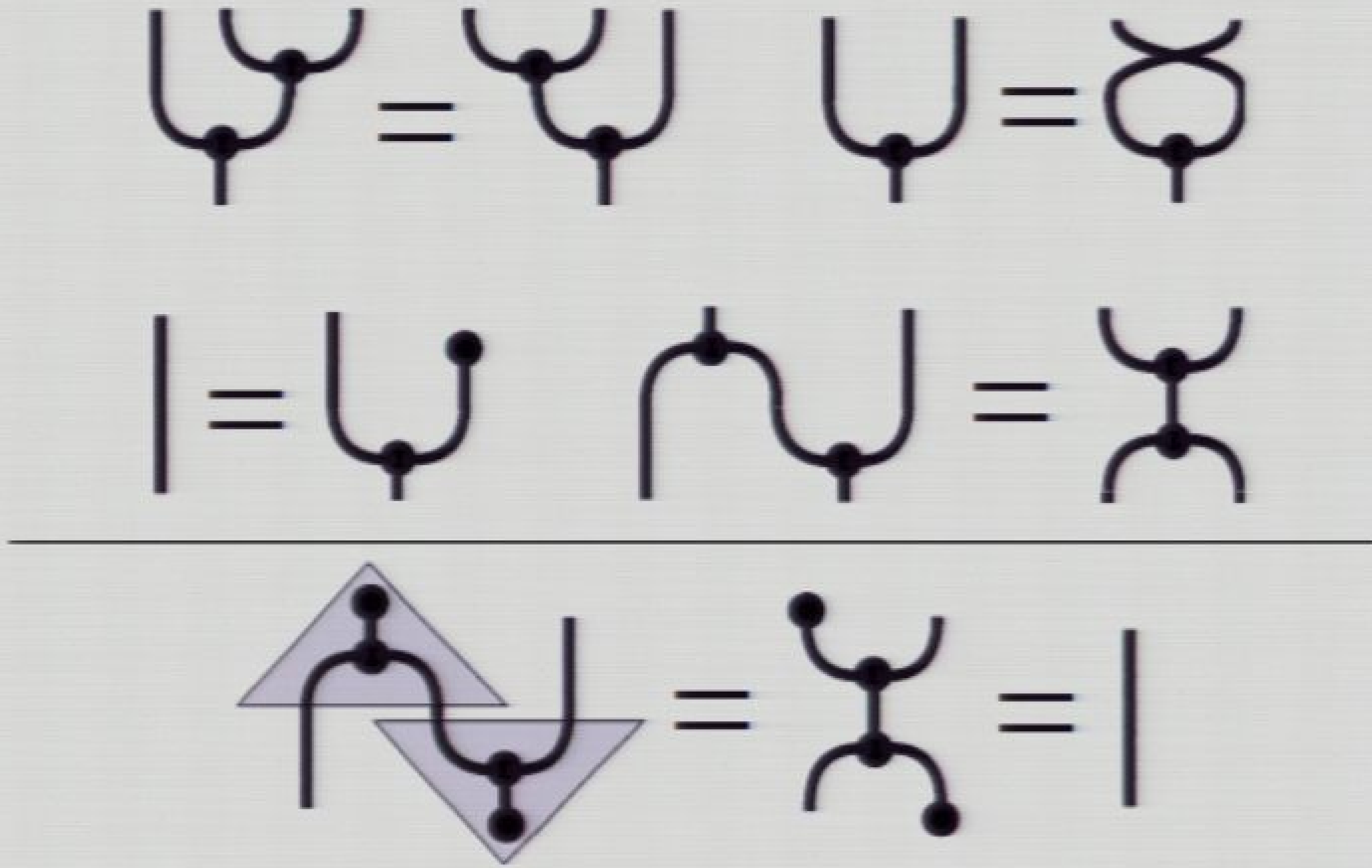


induces a symmetric W-class state, **and vice versa**.

— CFA axioms —

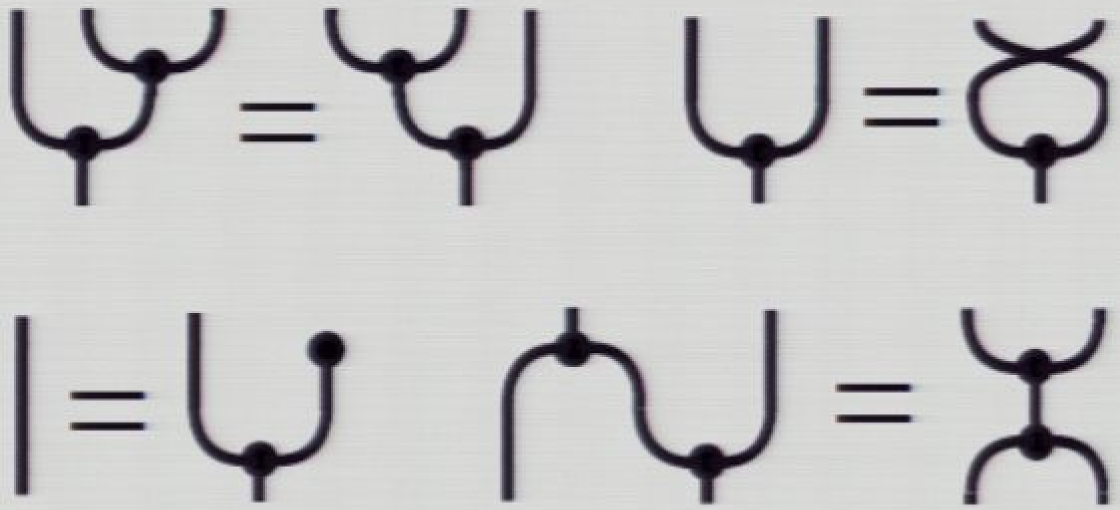


— CFA axioms —



GRAPHICAL REASONING
WITH SCFAs and ACFAs

— CFA axioms —



Proposition. A special CFA on $\mathbb{C}^2$, i.e.

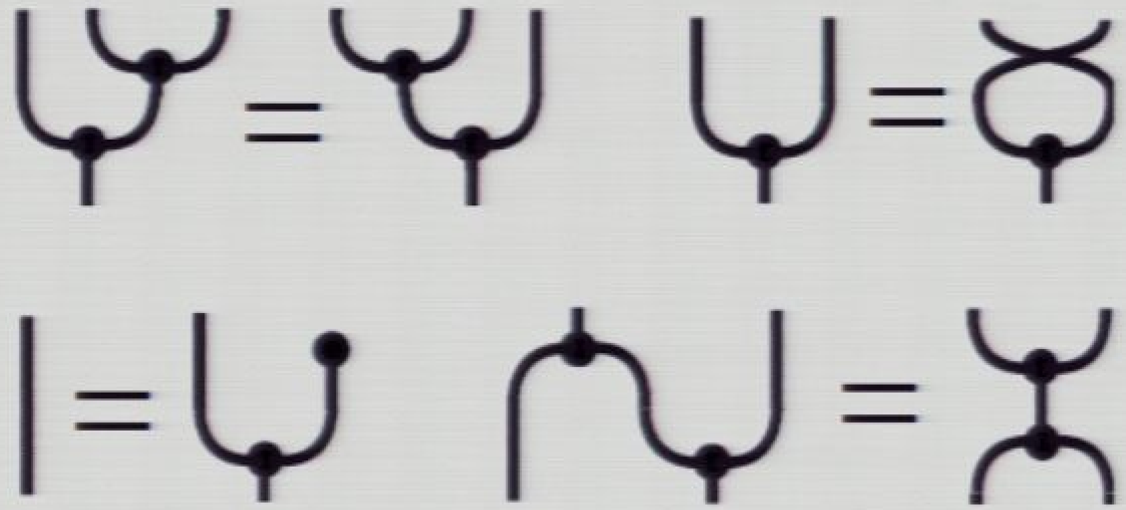
induces a symmetric GHZ-class state, **and vice versa**.

Proposition. An anti-special CFA on $\mathbb{C}^2$, i.e.

induces a symmetric W-class state, **and vice versa**.

Proposition. Every CFA on $\mathbb{C}^2$ is either special or anti-special; every monoid on $\mathbb{C}^2$ extends to an CFA.

— CFA axioms —



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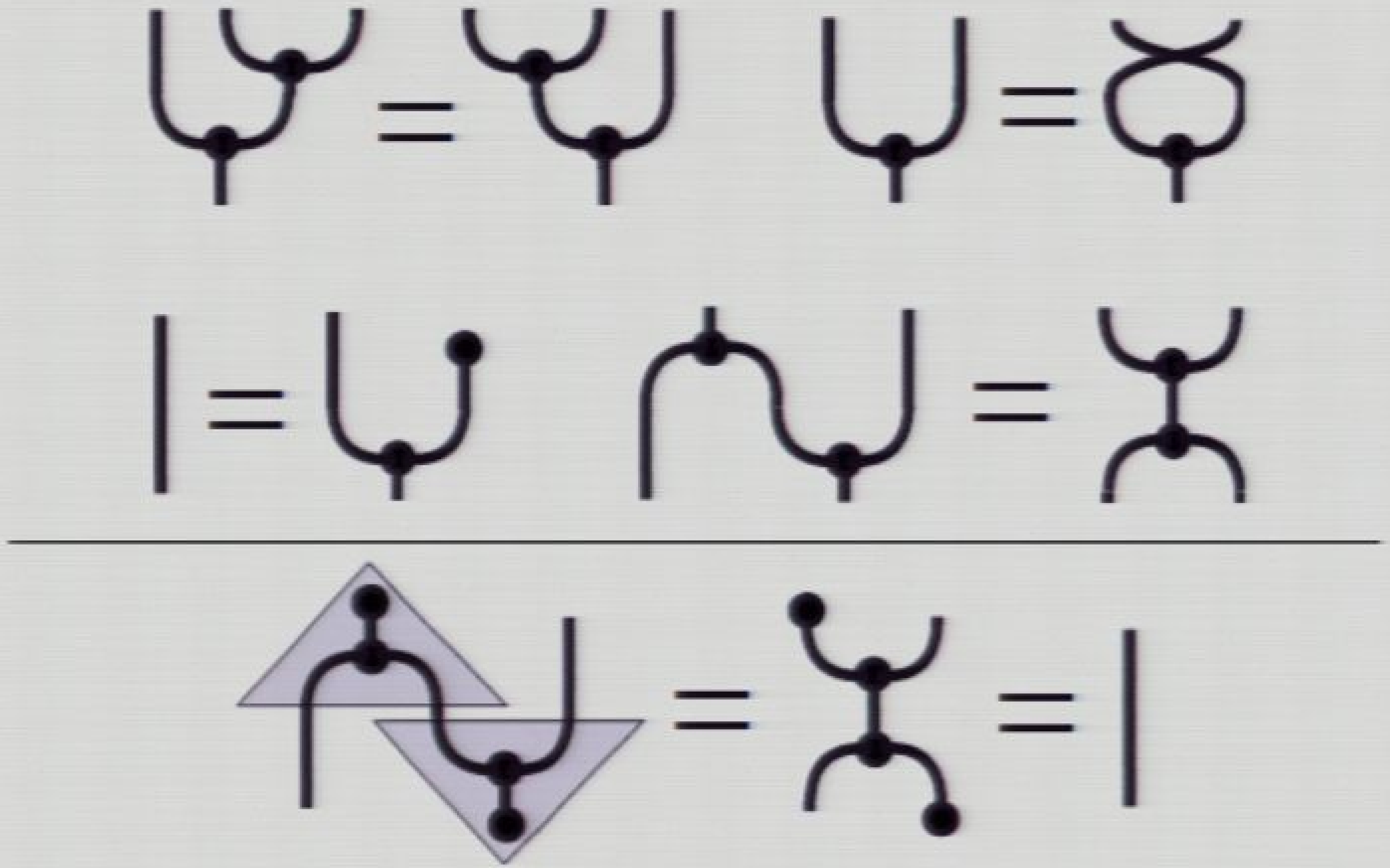

GRAPHICAL REASONING WITH SCFAs and ACFAs

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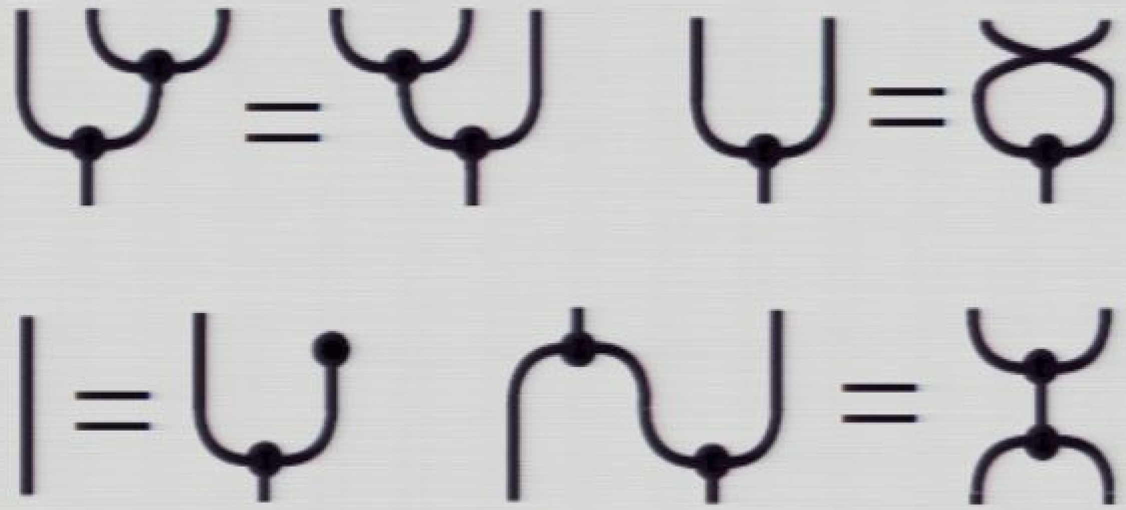
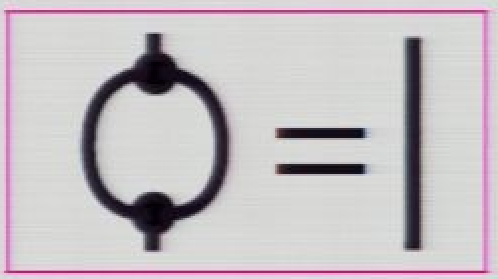
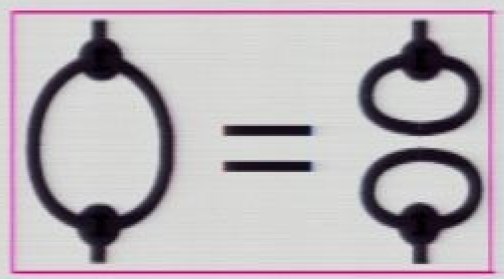
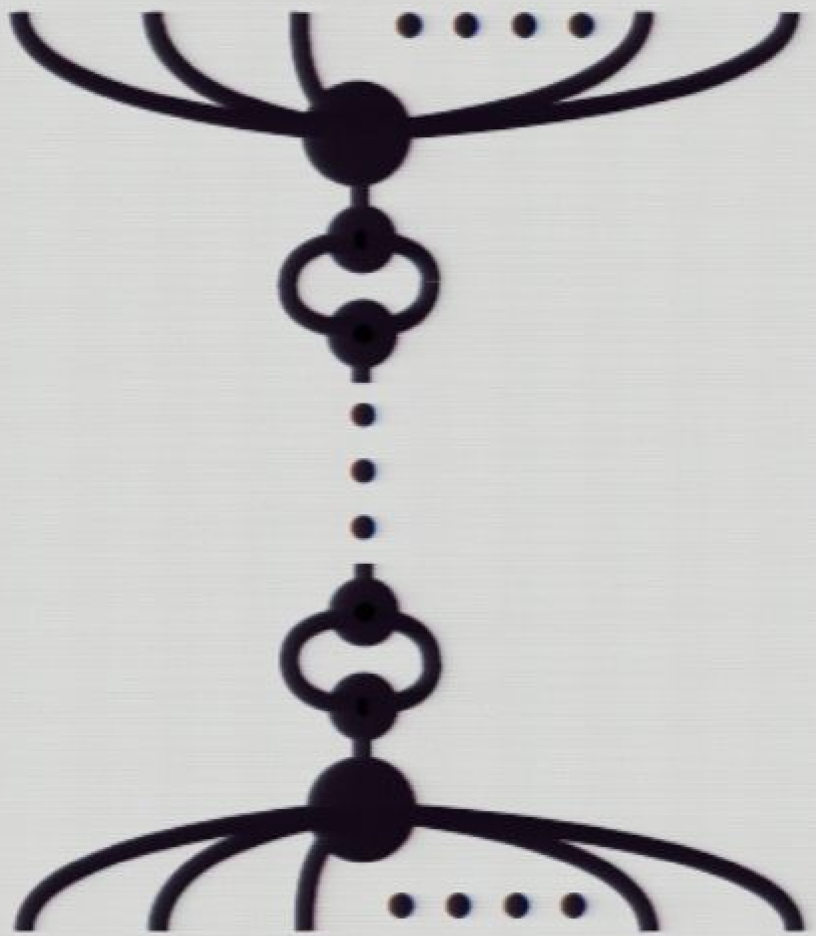




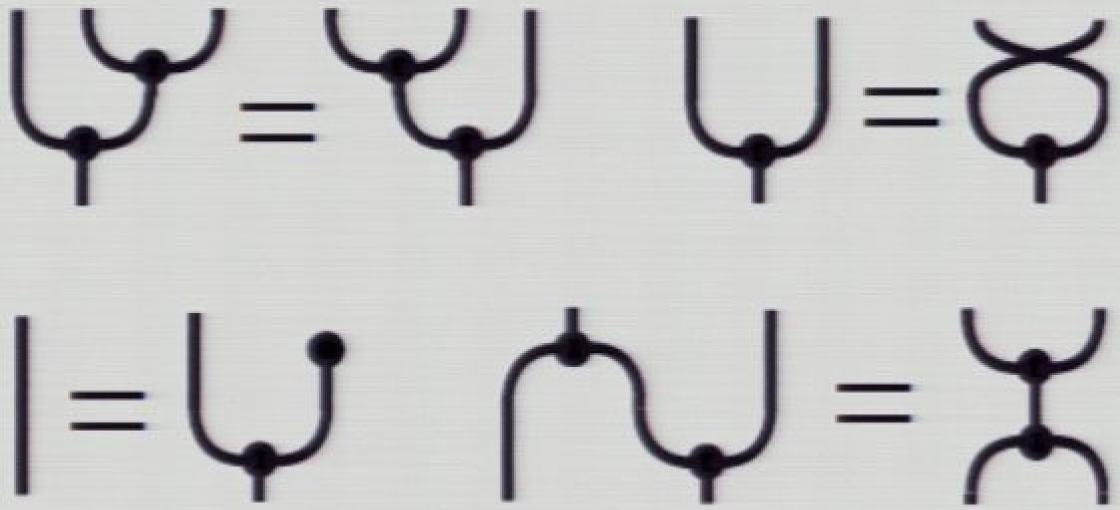
or

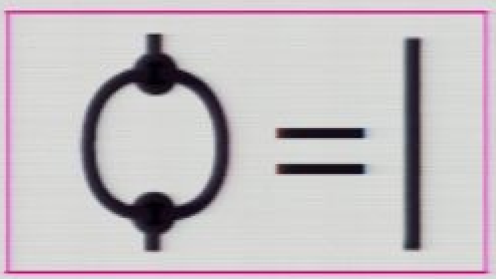


— CFA normal form —

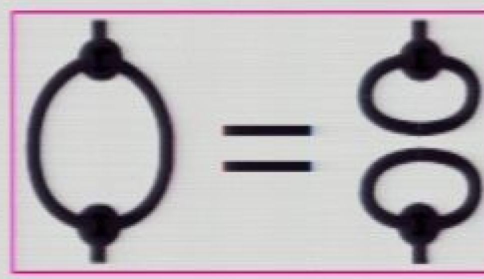


— CFA axioms —

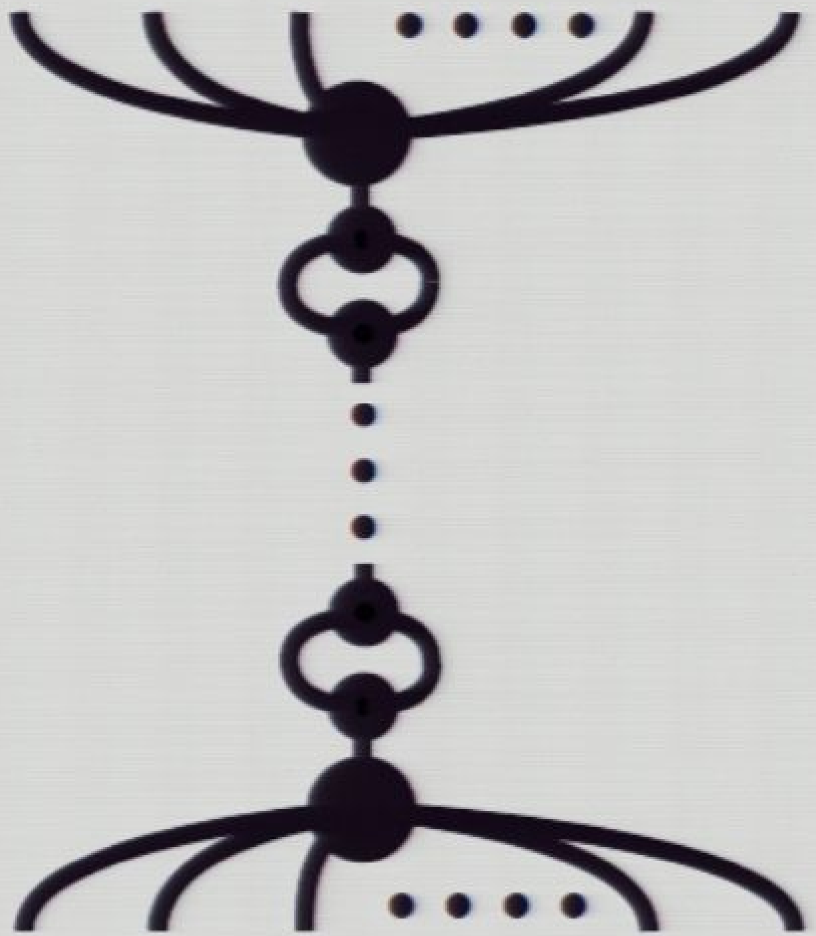




or



— CFA normal form —



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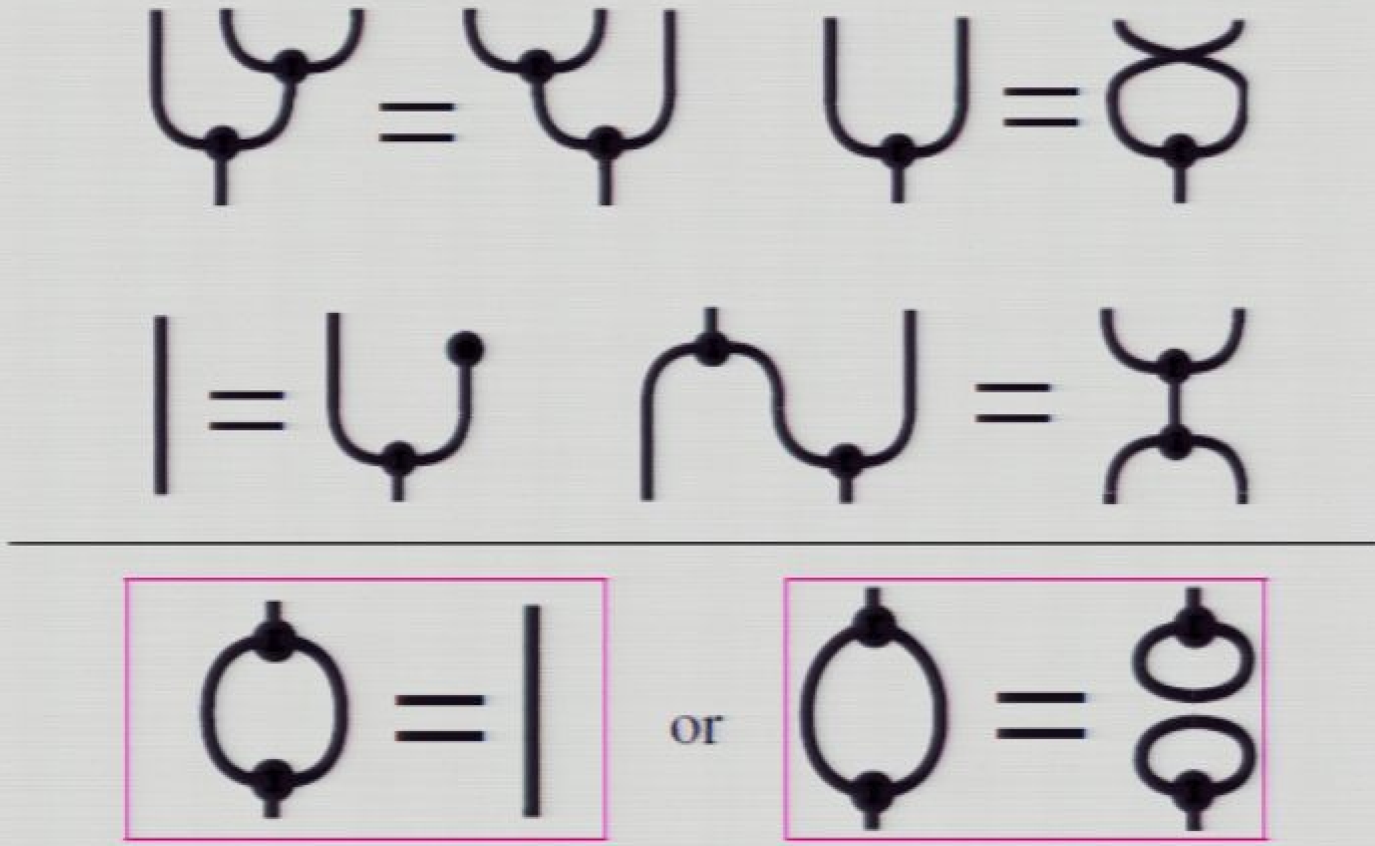

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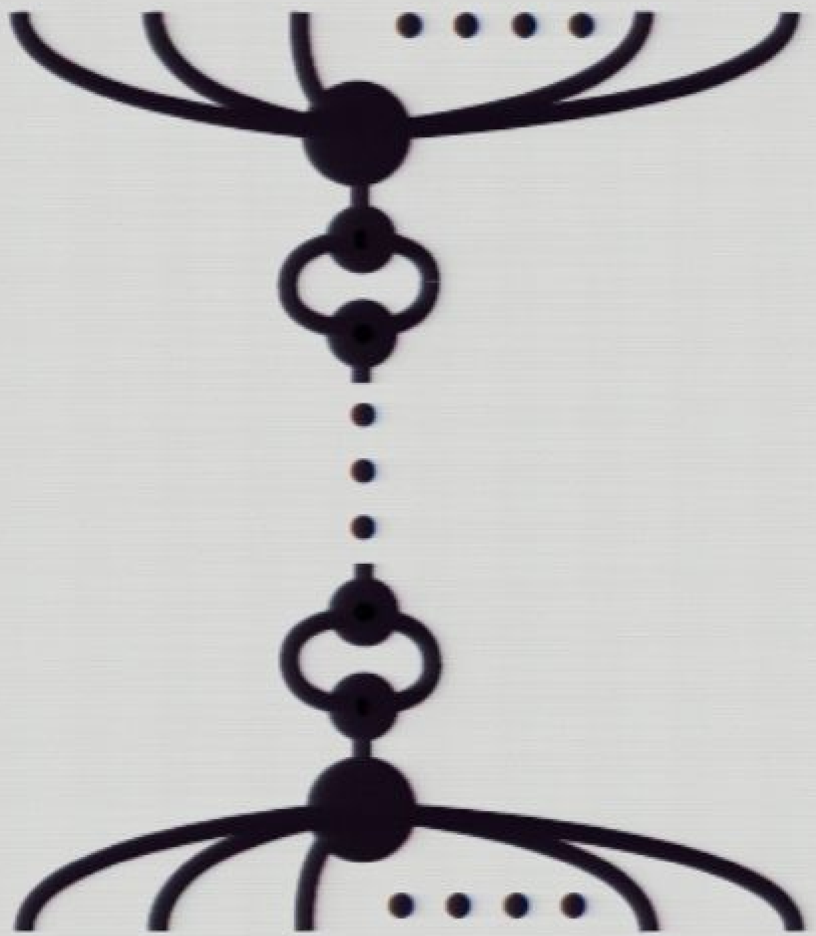
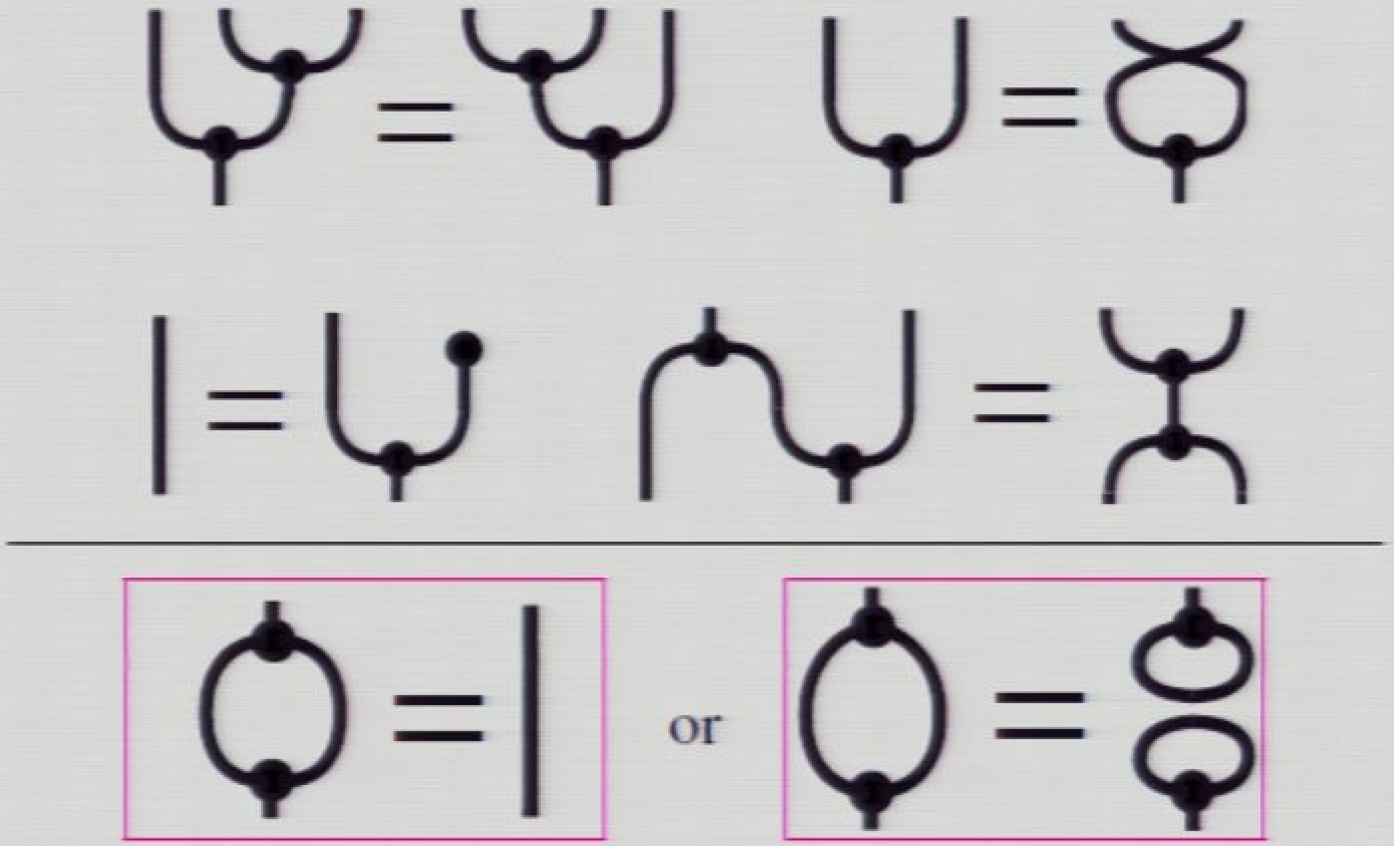
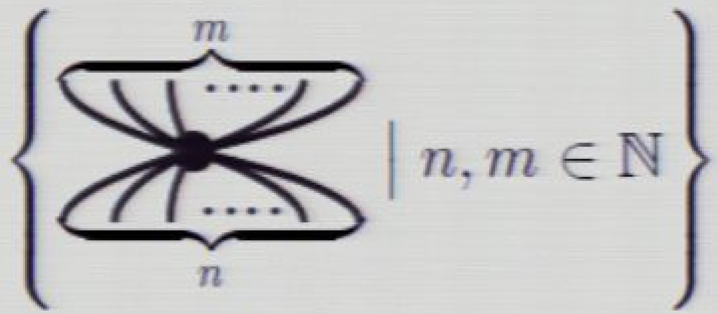


— *CFA axioms* —

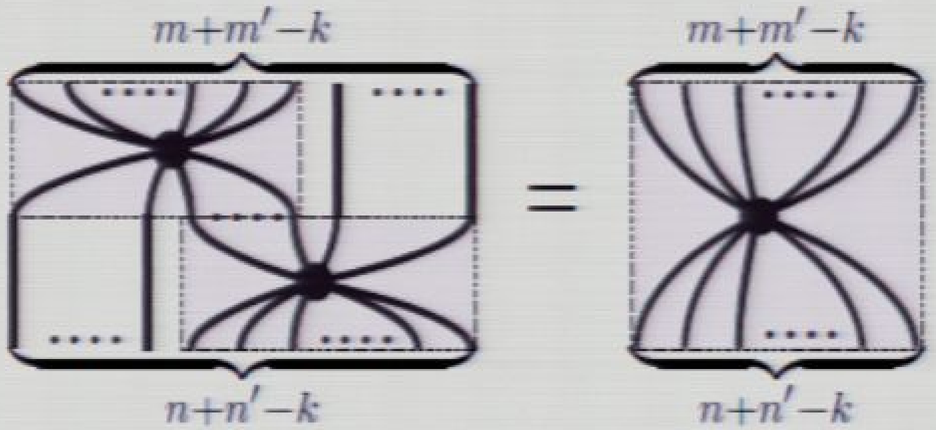


— *GHZ-spiders* —

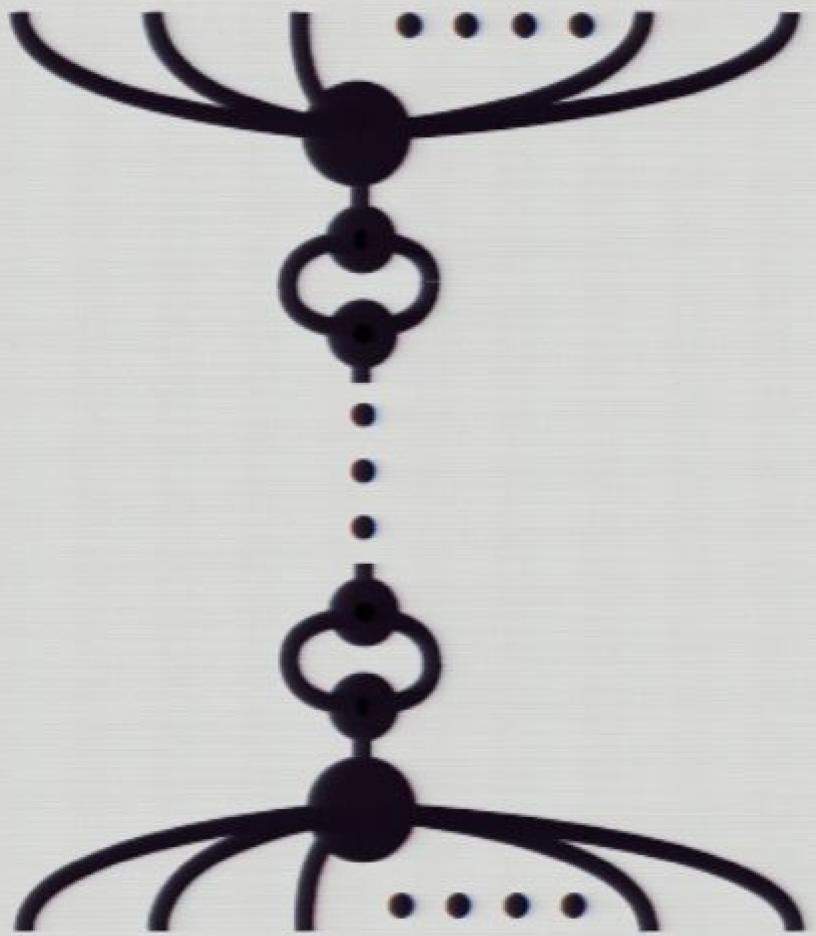
Data:



Rules:



— CFA normal form —



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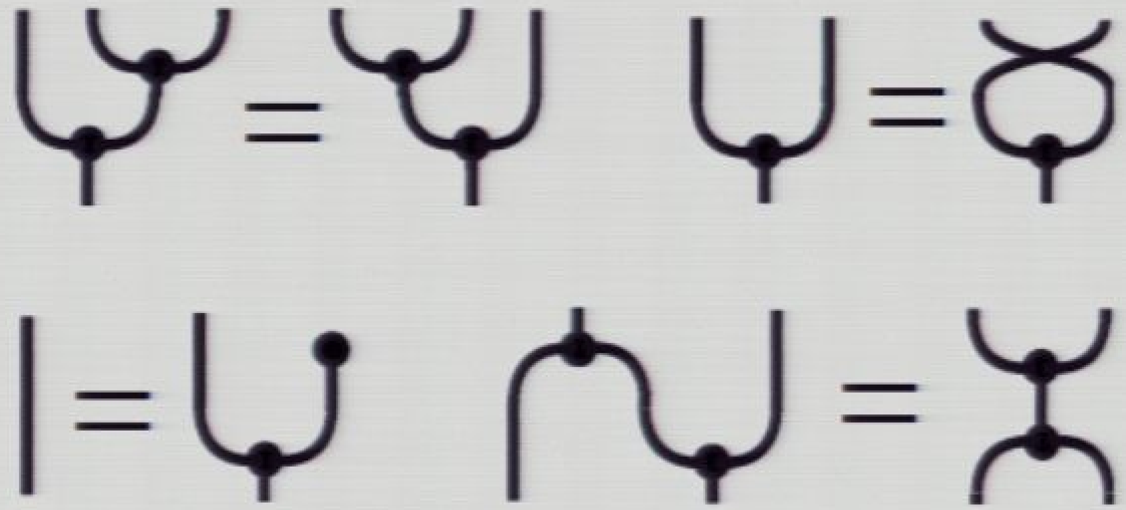
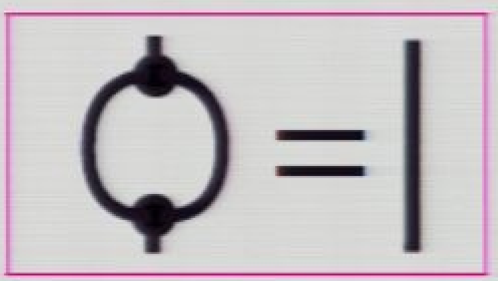
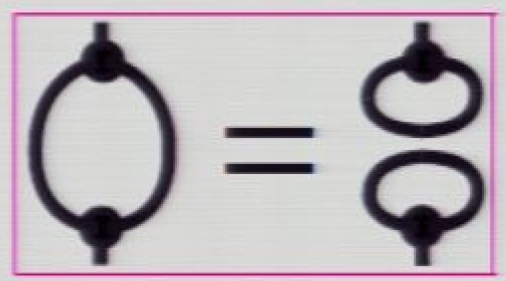


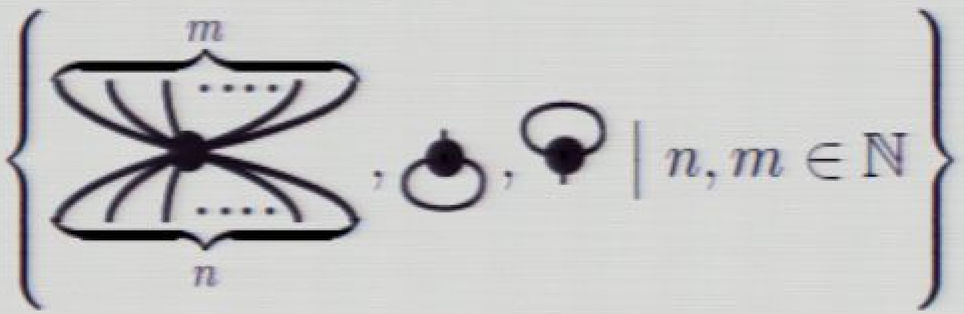


or

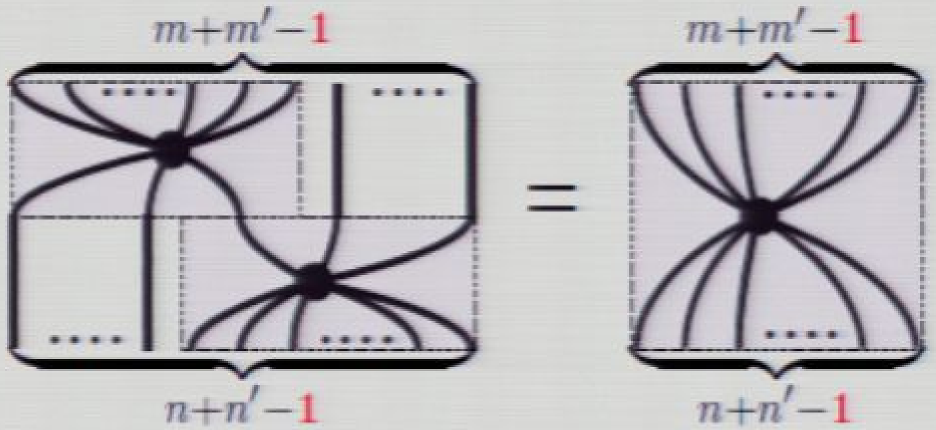


— *W-spiders* —

Data:



Rules:



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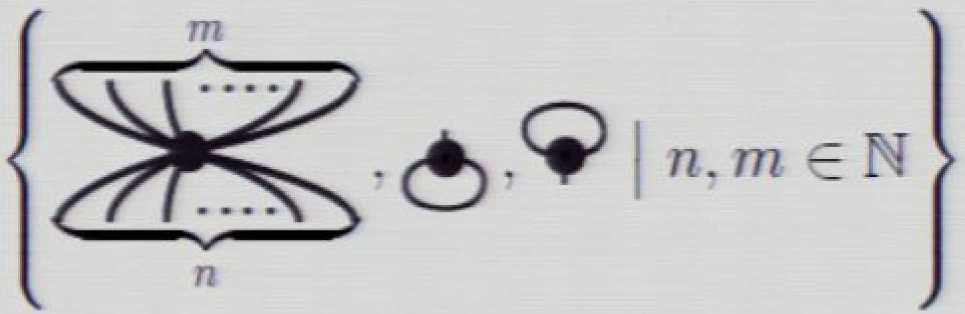
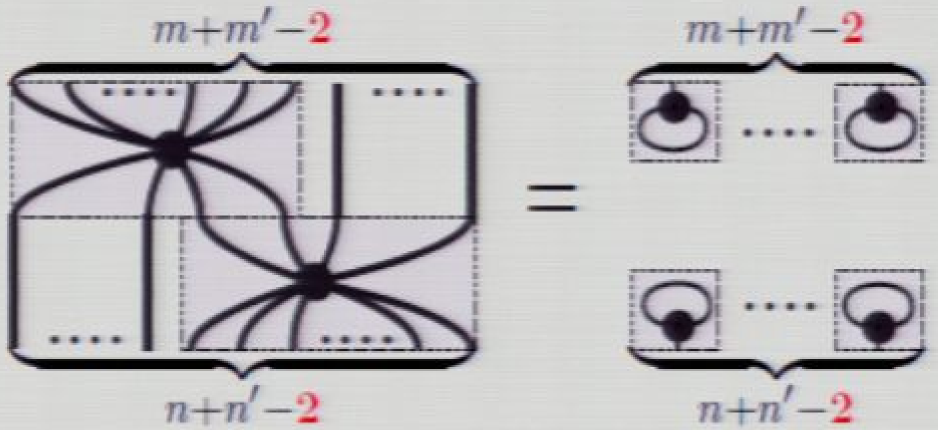
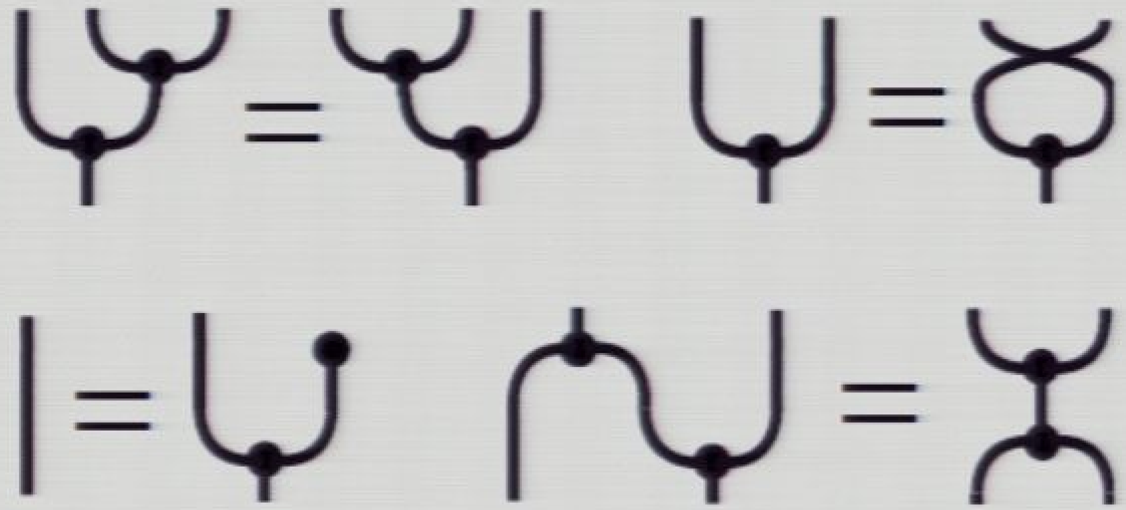
Data:

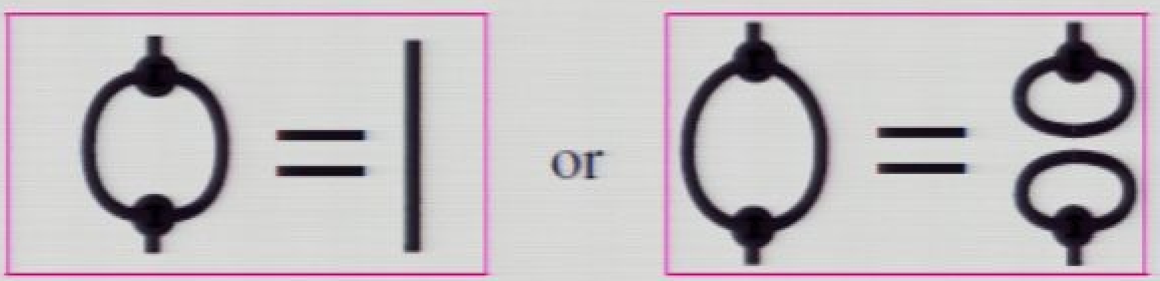


Rules:



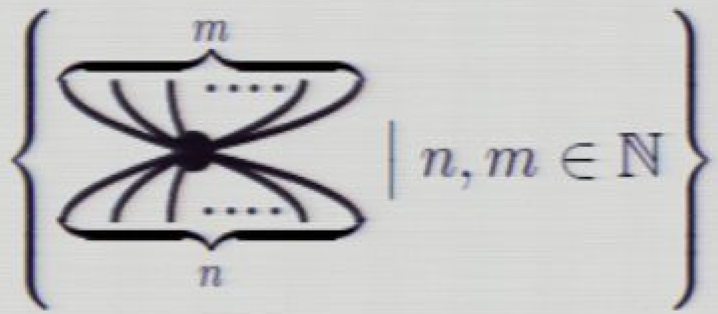
— CFA axioms —



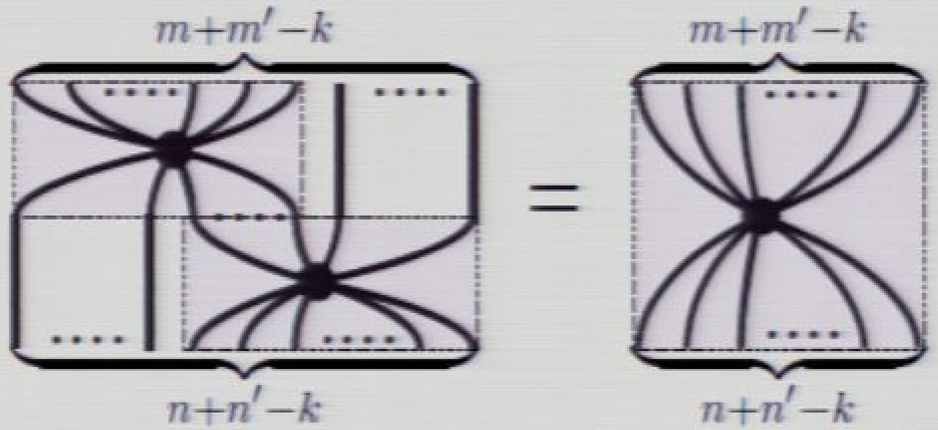


— *GHZ-spiders* —

Data:

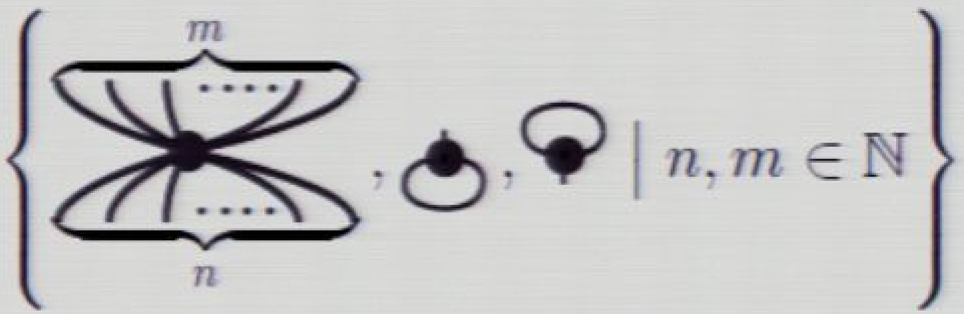


Rules:

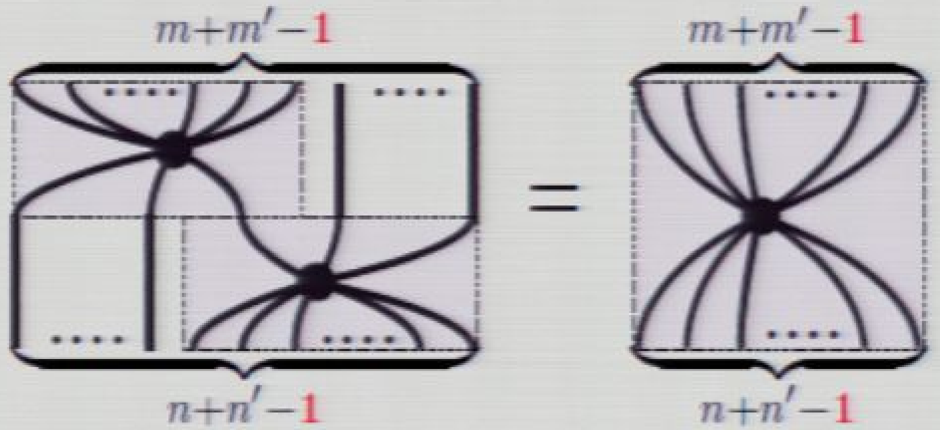


— *W-spiders* —

Data:

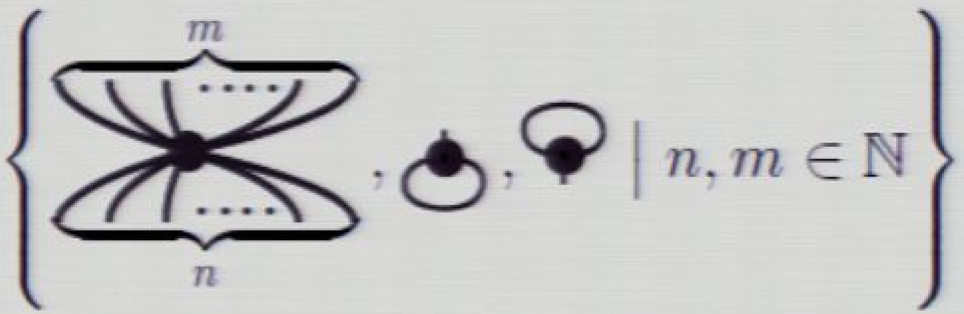


Rules:



— *W-spiders* —

Data:



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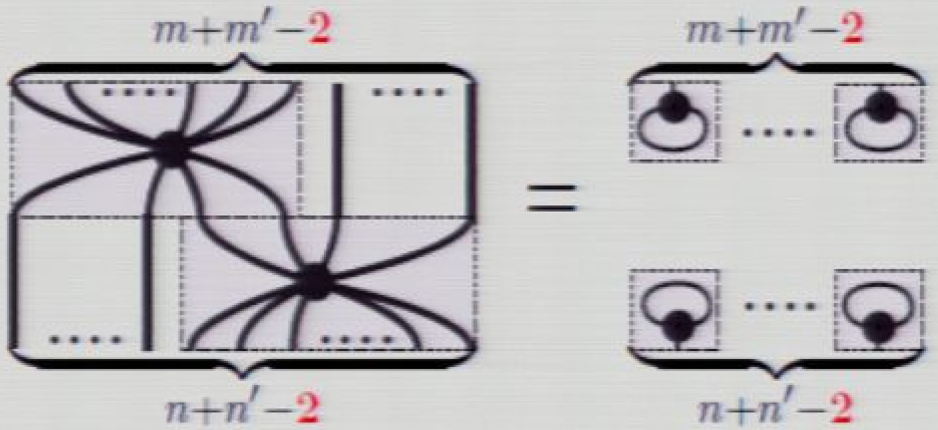
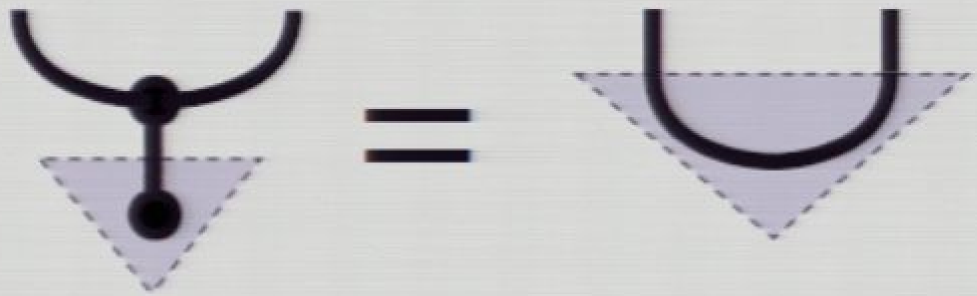
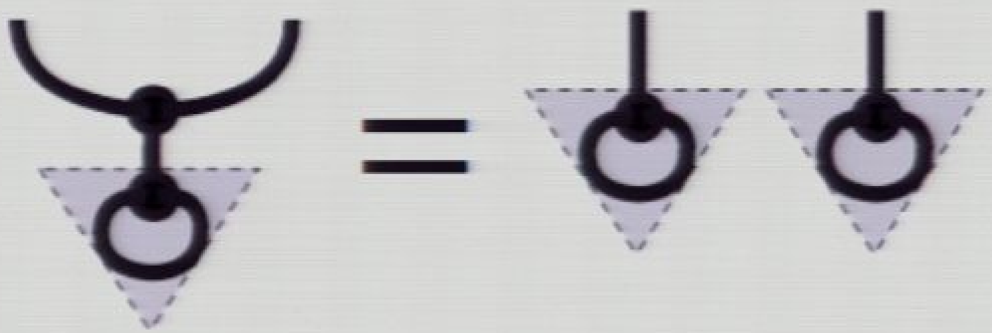


— *W-spider example* —

Examples:





COMPOSING W AND GHZ

— *composition of structures* —

Examples:

- GHZ-states $\Rightarrow$ multi-qubit GHZ

- W-states $\Rightarrow$ multi-qubit W

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— *composition of different structures* —

Examples:

- $+$ & $\times \Rightarrow$ general polynomials.
  - Interact via distributive law
  - Interconvert via exponential

— *composition of different structures* —

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- $\{|0\rangle, |1\rangle\}$- & $\{|+\rangle, |-\rangle\}$-bases $\Rightarrow$ graph states.
  - Interact via bialgebra-like laws
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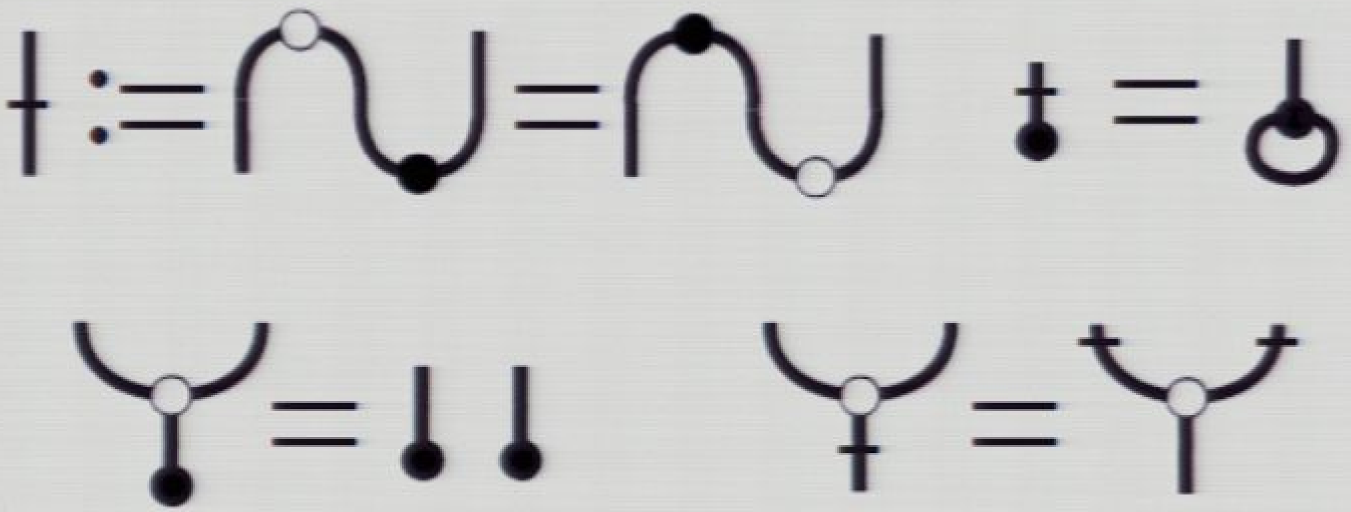
  - Interact via Hopf-like law
  - Interconvert via Hadamard gate
- GHZ- & W-states $\Rightarrow$ **???**
  - Interact via **???**
  - Interconvert via **???**

— *composition of W- and GHZ-CFAs* —

Interaction rules:



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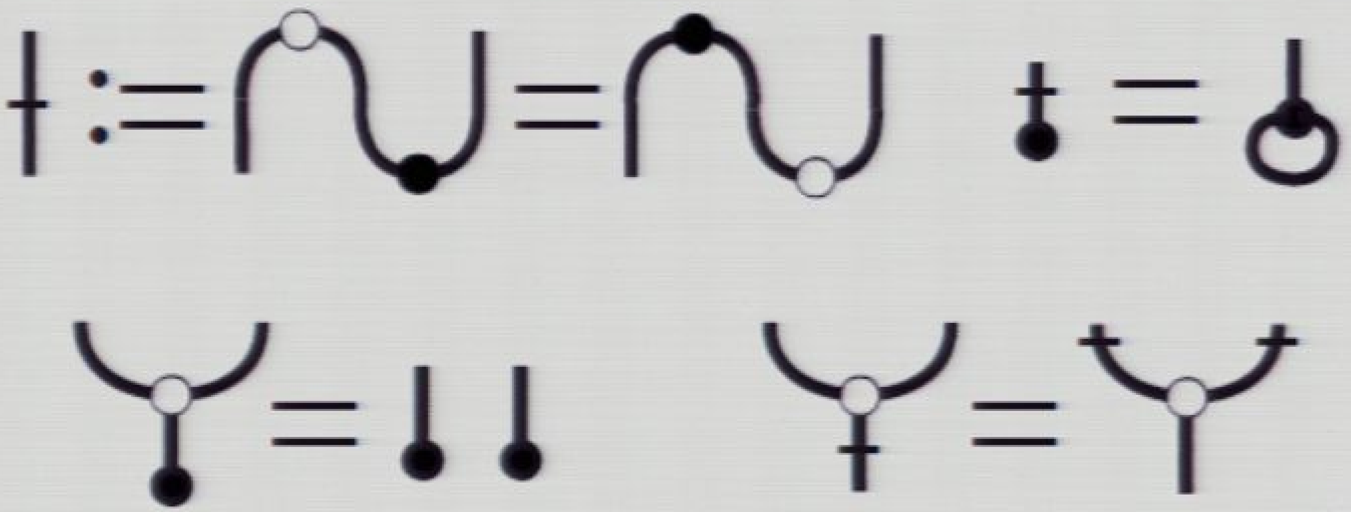
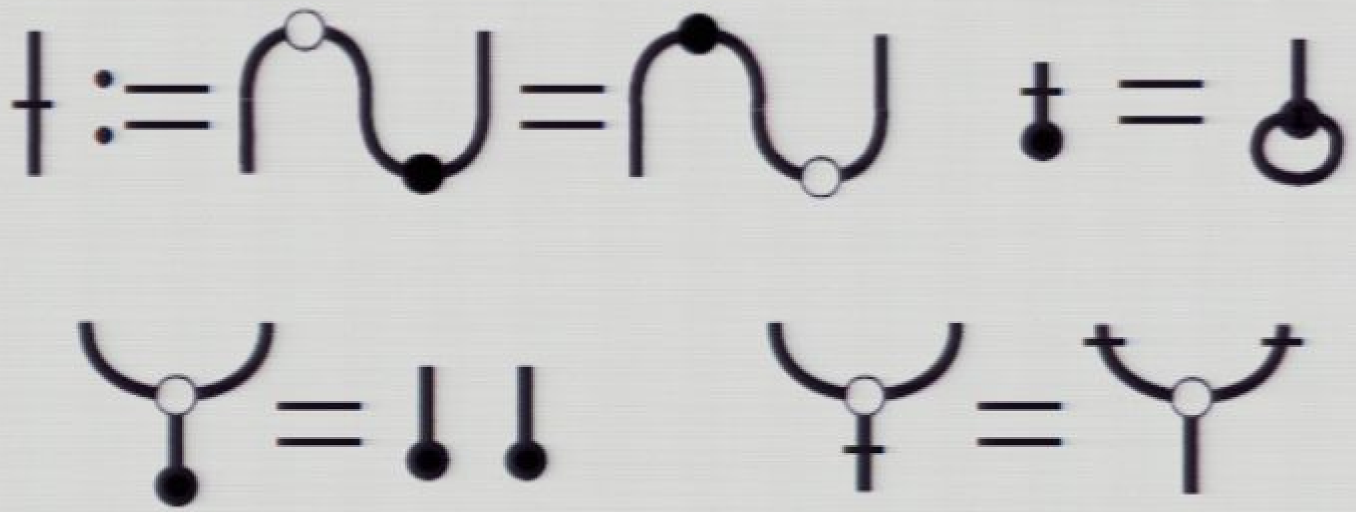


— *composition of W- and GHZ-CFAs* —

Interaction rules:



Correspondence:

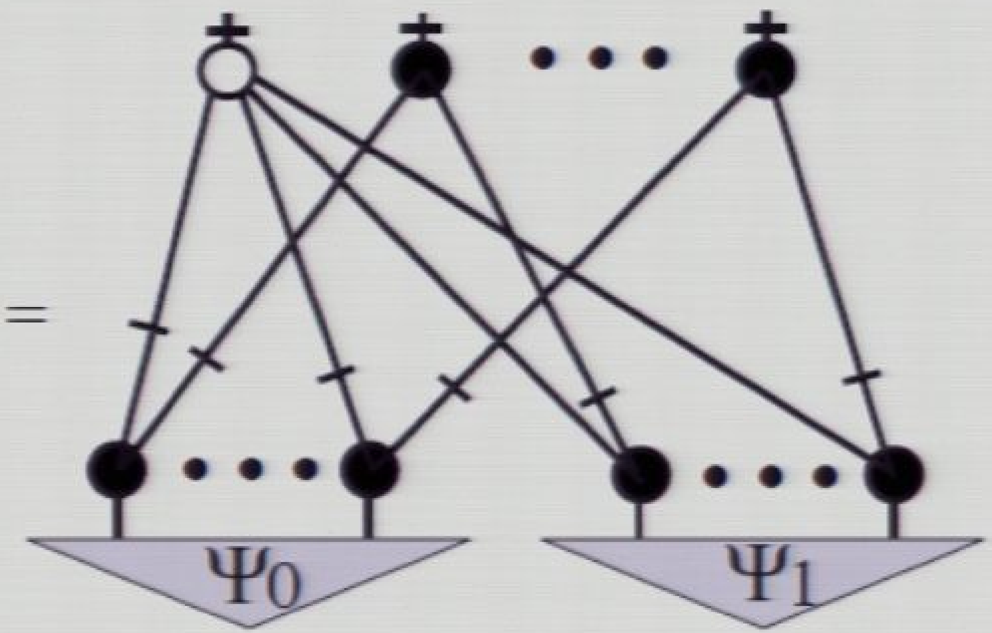
- Structural points of W are copyable points of GHZ

— *composition of W- and GHZ-CFAs* —

Generating power:

Emulating SLOCC-superclass generation:

$$|0\Psi_0\rangle + |1\Psi_1\rangle =$$



— *composition of W- and GHZ-CFAs* —

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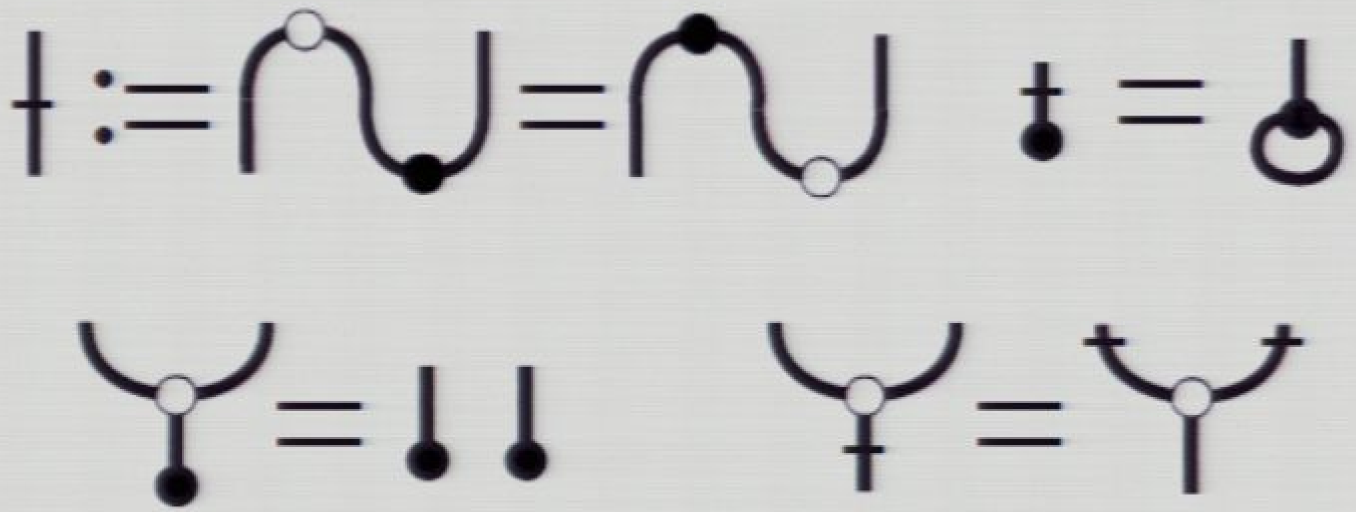

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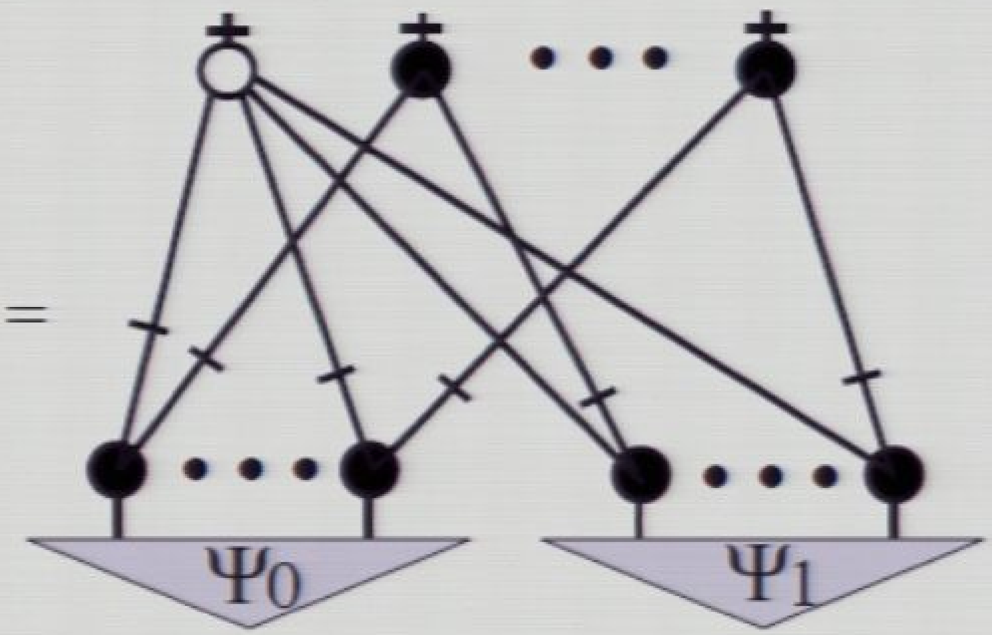
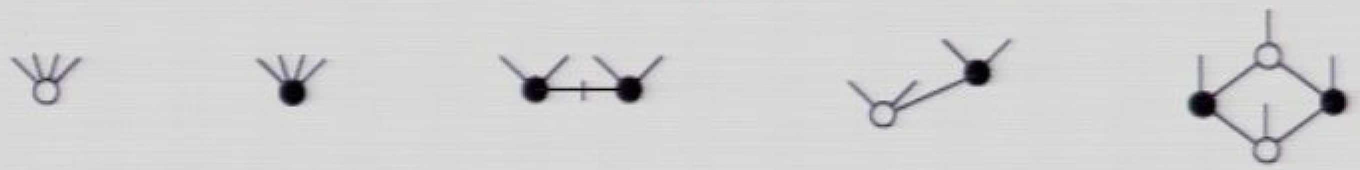


— *composition of W- and GHZ-CFAs* —

Generating power:

Some four qubit SLOCC-superclass representatives:



— *composition of W- and GHZ-CFAs* —

Generating power:

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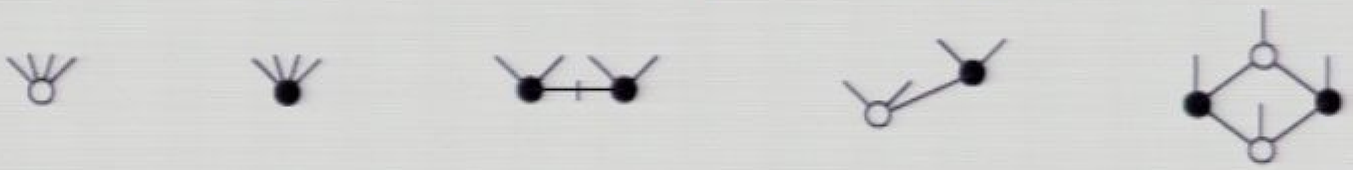
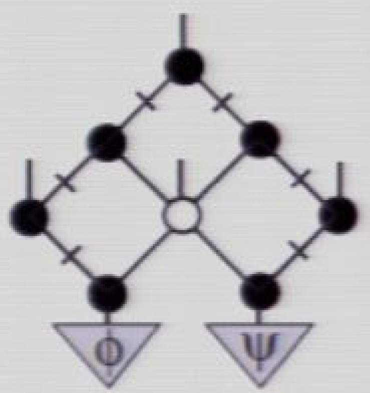
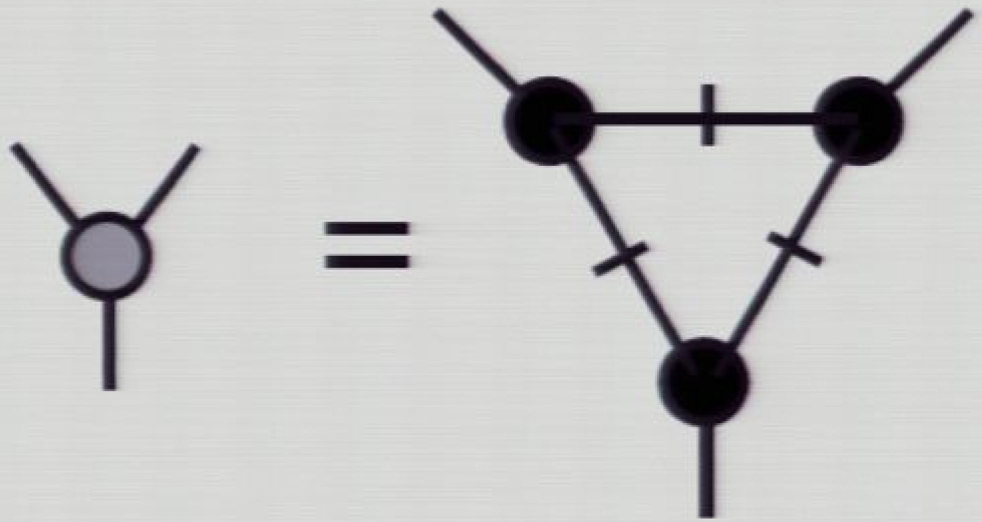


A four qubit continuous SLOCC-superclass:



COMPLEMENTARY IS SUBSUMED

Thm. Under the assumption that one-input operations are determined by their action of W structural points,



together with the GHZ CFA define a pair of complementary observables.

— *this talk* —

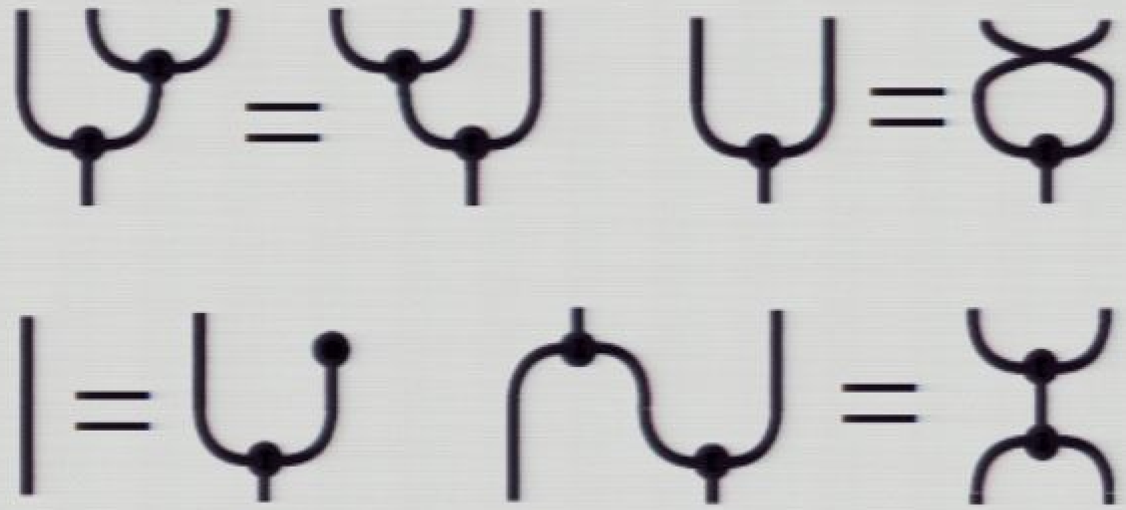
- The **algebraic similarity** and **purely topological** difference for GHZ and W SLOCC-class states.

- A **compositional** paradigm for multipartite quantum entanglement, with GHZ and W as generators.
  - Discreteness supports automation e.g. protocol design

- **Quantum structural paradigm** and **graphical calculus** which subsumes complementary observables:
  - GHZ/W-duality more fundamental than complementarity?

Ref: B. Coecke and A. Kissinger (2010) *The compositional structure of multipartite quantum entanglement*. `arXiv:1002.2540`

— CFA axioms —



Classifying entanglement: Two multipartite quantum states **compare** if by (possibly probabilistic) either local or classical means one can be turned into the other.

The Compositional Structure of Multipartite Quantum Entanglement

Bob Coecke and Aleks Kissinger
Oxford University Computing Laboratory

EPSRC

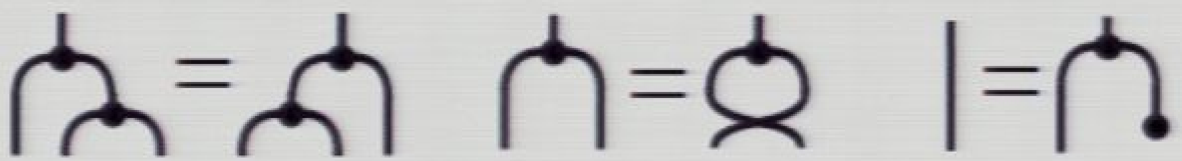
SLOCC

— *commutative Frobenius algebras* —

A commutative monoid is object A with morphisms

 $: A \otimes A \to A$  $: \mathrm{I} \to A$

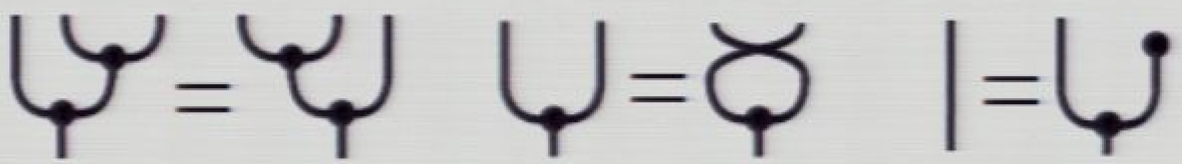
s.t.



A cocommutative comonoid is object A with morphisms

 $: A \to A \otimes A$  $: A \to \mathrm{I}$

s.t.



— *composition of structures* —

Examples:

- GHZ-states $\Rightarrow$ multi-qubit GHZ

- W-states $\Rightarrow$ multi-qubit W

— *W-spider example* —

Examples: