Title: Explorations in Numerical Relativity (PHYS 642) - Lecture 13

Date: Mar 31, 2010 11:20 AM

URL: http://pirsa.org/10030112

Abstract:

$$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$$

$$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_{\delta} \Gamma^{\delta}_{\alpha\beta} - \Gamma^{\delta}_{\beta\gamma} \Gamma^{\gamma}_{\alpha\delta}$$

$$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right)$$


$32^r \times 16^\infty$

$3\cancel{4}^{r} \times 1\overset{8}{6}$

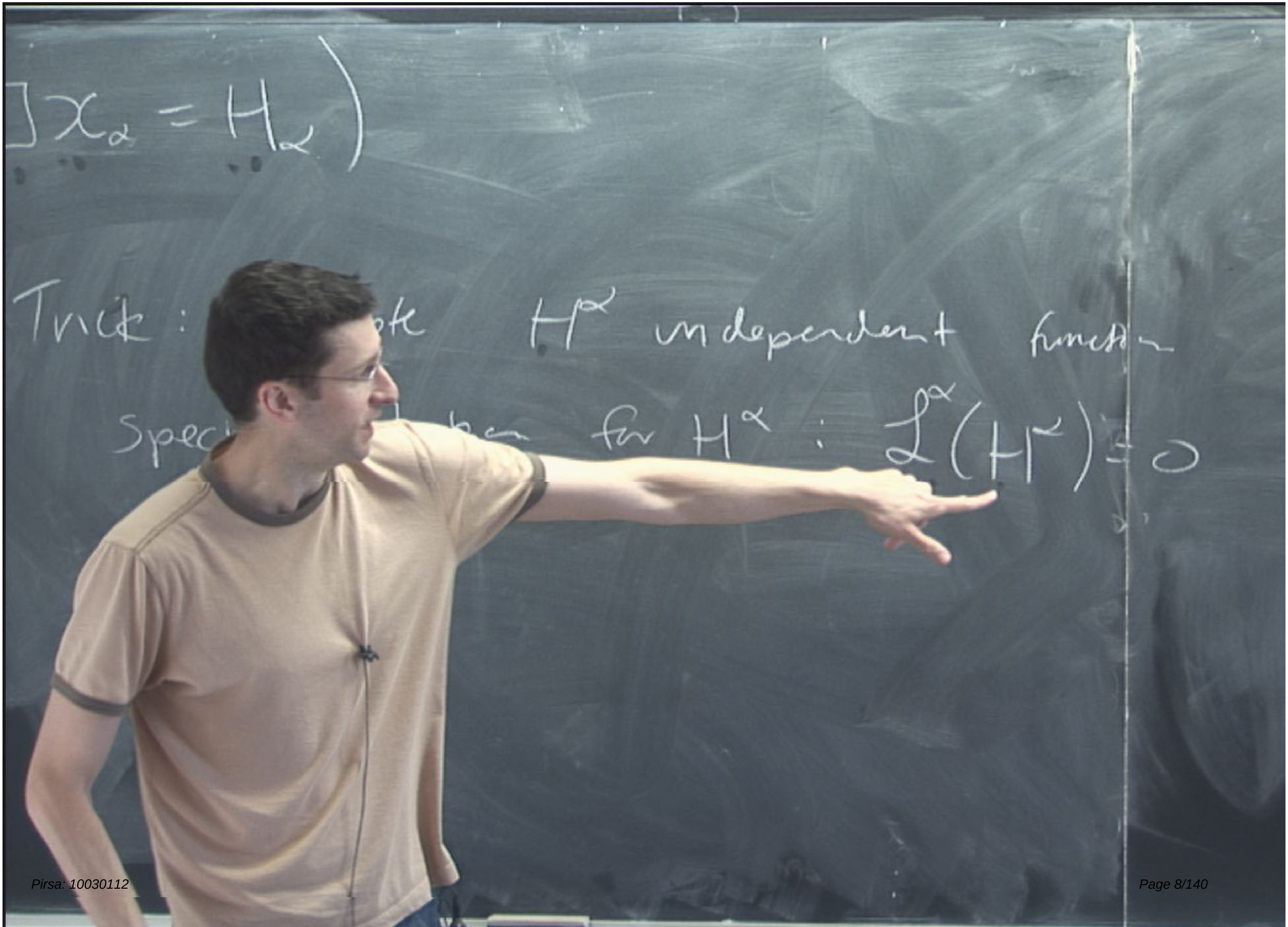
6

64

$$\Box x_\alpha = H_\alpha$$

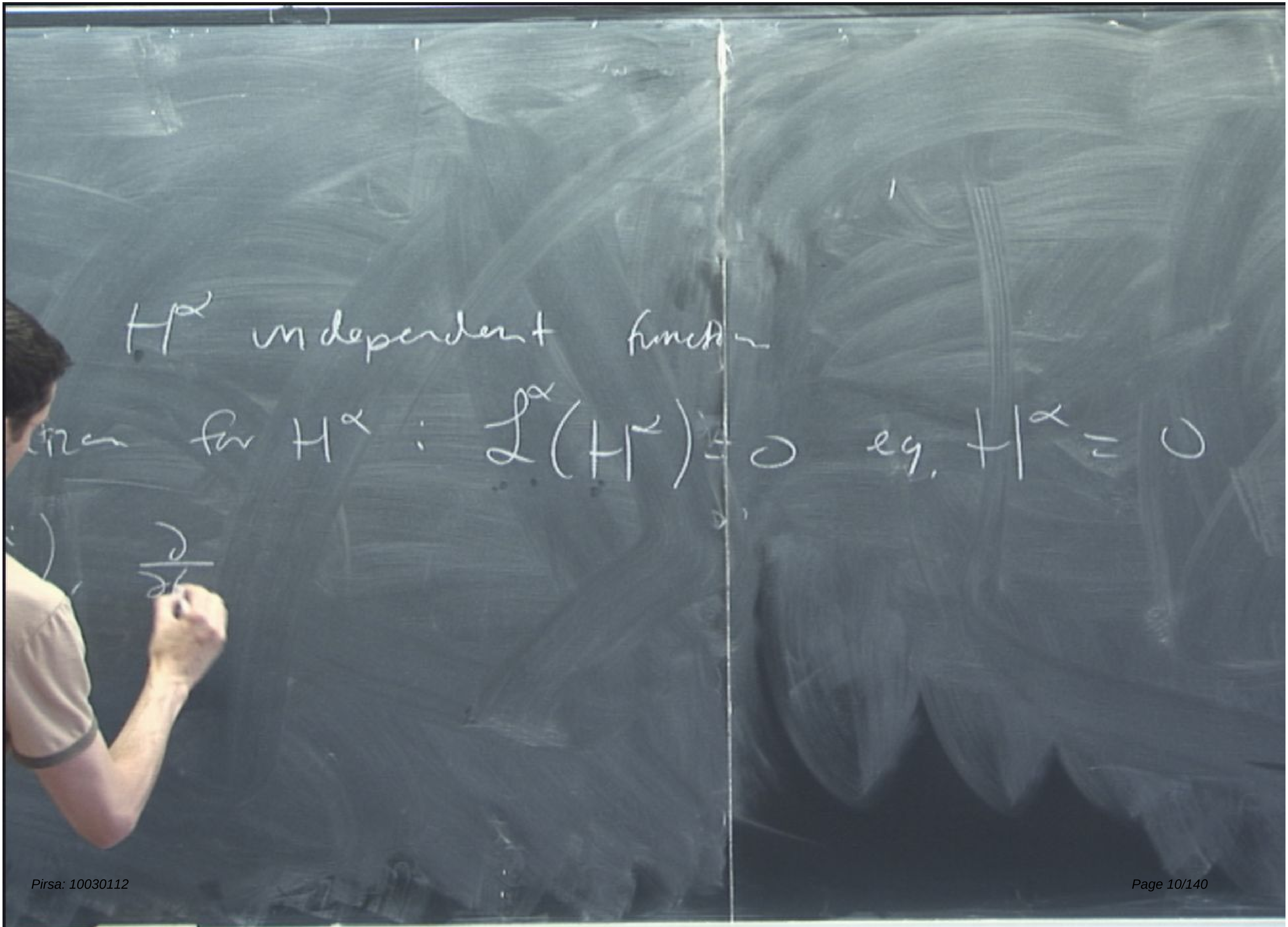
$$\left(\Box X_\alpha = H_\alpha \right)$$

Trick: promote H^α independent function


$]x_\alpha = H_\alpha)$
Trick:
H^α independent function
for H^α : $\mathcal{L}^\alpha(H^\alpha) = 0$

H^α independent function

-ntion for H^α : $\mathcal{L}^\alpha(H^\alpha) = 0$ e.g. $H^\alpha = 0$


H^α independent function
for H^α : $\mathcal{L}^\alpha(H^\alpha) = 0$ eg. $H^\alpha = 0$
$\frac{\partial}{\partial t}$

H^α independent function

for H^α : $\mathcal{L}^\alpha(H^\alpha) = 0$ eg. $H^\alpha = 0$

, $\frac{\partial}{\partial t} g_{\alpha\beta}(t=0, x^i)$, $H^\alpha(t=0, x^i)$, $\frac{\partial}{\partial t} H$

H^α independent function

...ntion for H^α : $\mathcal{L}^\alpha(H^\alpha) = 0$ eg. $H^\alpha = 0$

$), \frac{\partial}{\partial t} g_{\alpha\beta}(t=0, x^i)$, $H^\alpha(t=0, x^i)$, $\frac{\partial}{\partial t} H^\alpha($

...dent function

$$\mathcal{L}^\alpha(H^\alpha) = 0 \quad \text{eg. } H^\alpha = 0$$

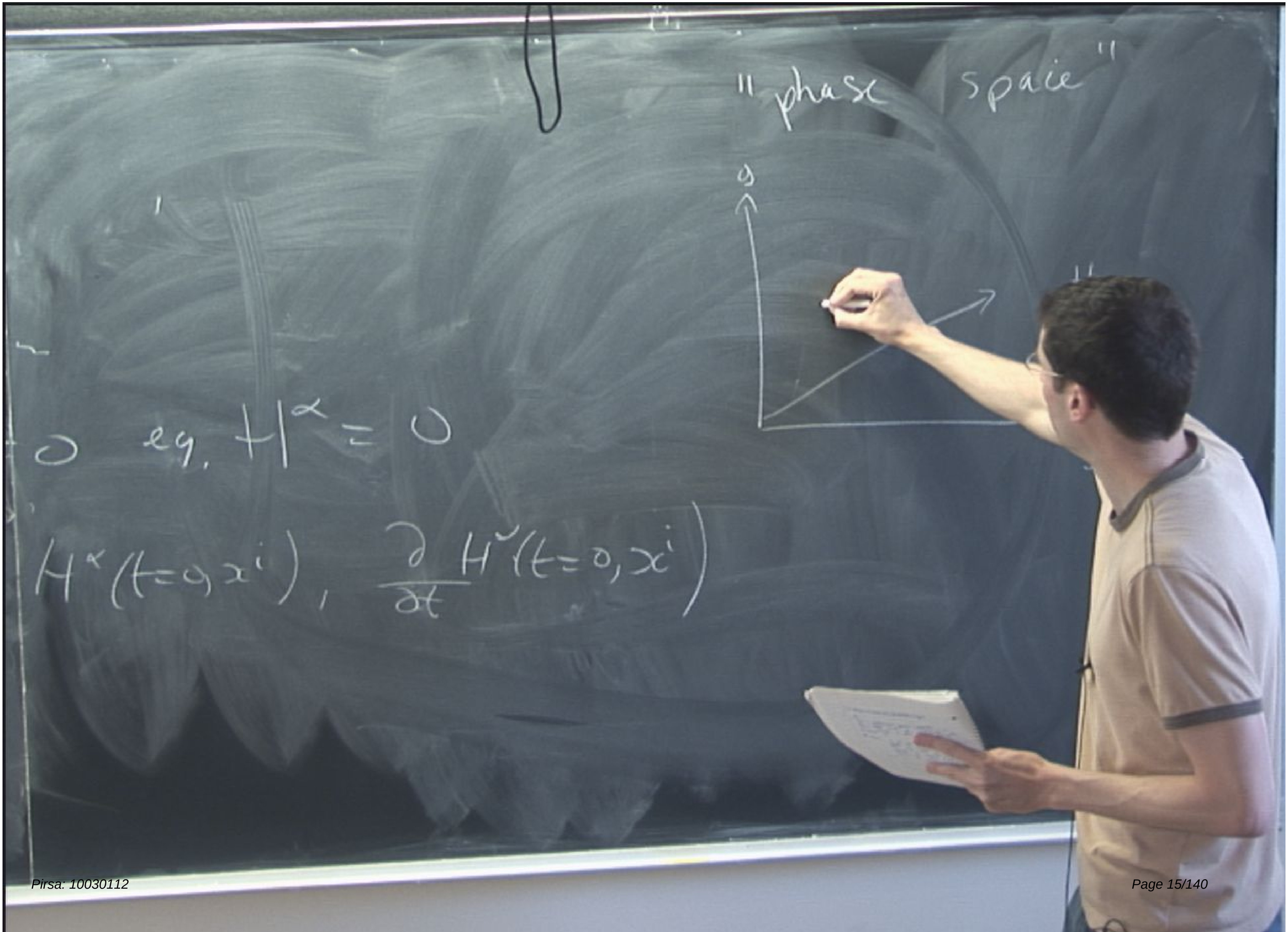
$$x^i), \quad H^\alpha(t=0, x^i), \quad \frac{\partial}{\partial t} H^\alpha(t=0, x^i)$$

$$\left(\Box x_\alpha = H_\alpha \right)$$

Trick: promote H^α independent f

Specify evolution for H^α : $\mathcal{L}^\alpha(H$

I.D. $g_{\alpha\beta}(t=0, x^i)$, $\frac{\partial}{\partial t} g_{\alpha\beta}(t=0, x^i)$,


"phase space"
eq. H^α = 0
H^α(t=0,x^i), ∂/∂t H^α(t=0,x^i)

"phase space"

$\Box$ eg. $H^\alpha = 0$

$H^\alpha(t=0, x^i), \ \frac{\partial}{\partial t} H^\alpha(t=0, x^i))$

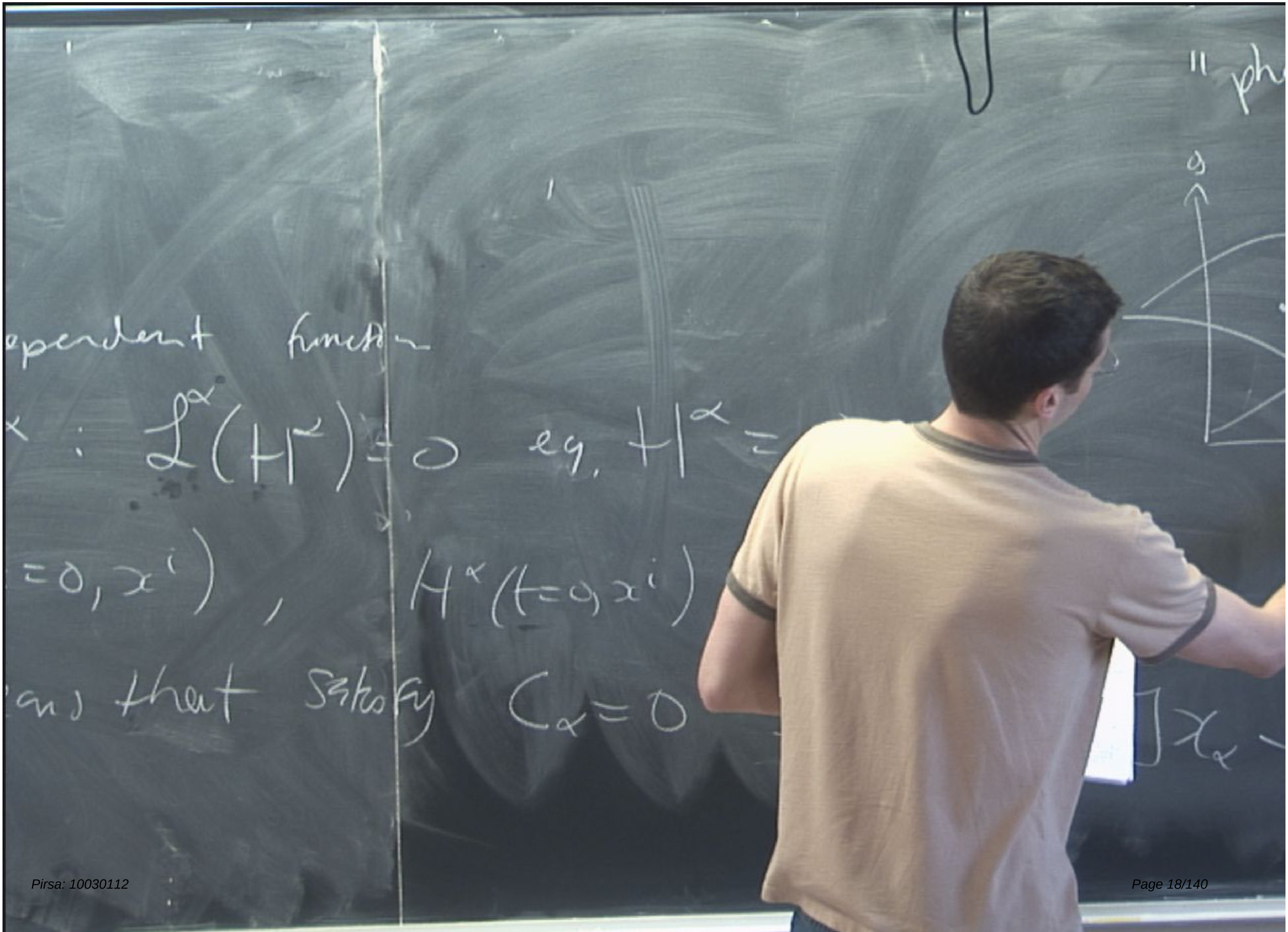
$_\alpha = H_\alpha$)

...t: promote H^α independent function

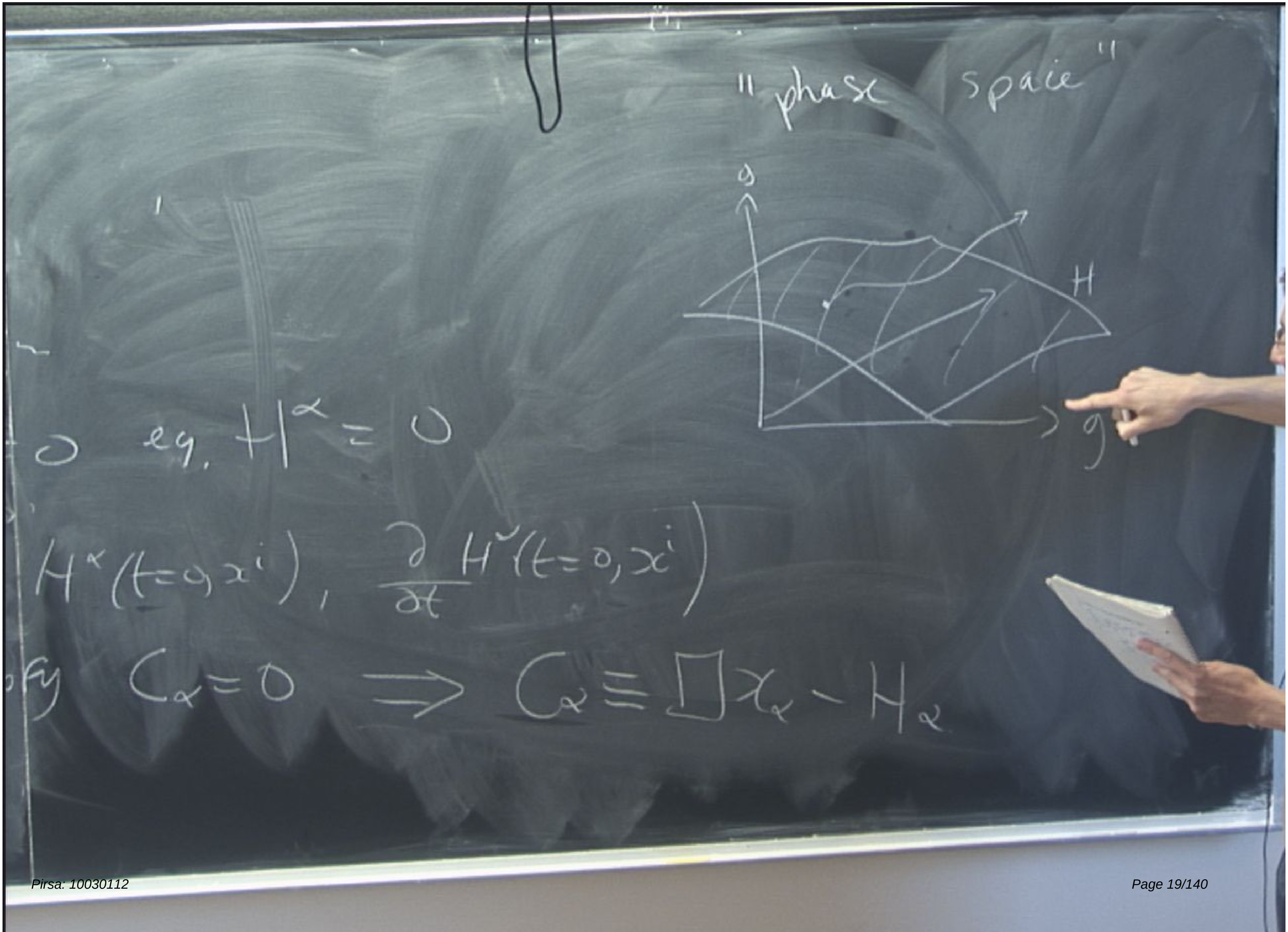
Specify evolution for H^α: $\mathcal{L}^\alpha(H^\alpha) = 0$ eg. ...

$g_{\alpha\beta}(t=0, x^i)$, $\frac{\partial}{\partial t} g_{\alpha\beta}(t=0, x^i)$, $H^\alpha(t=0, x^i$ ...

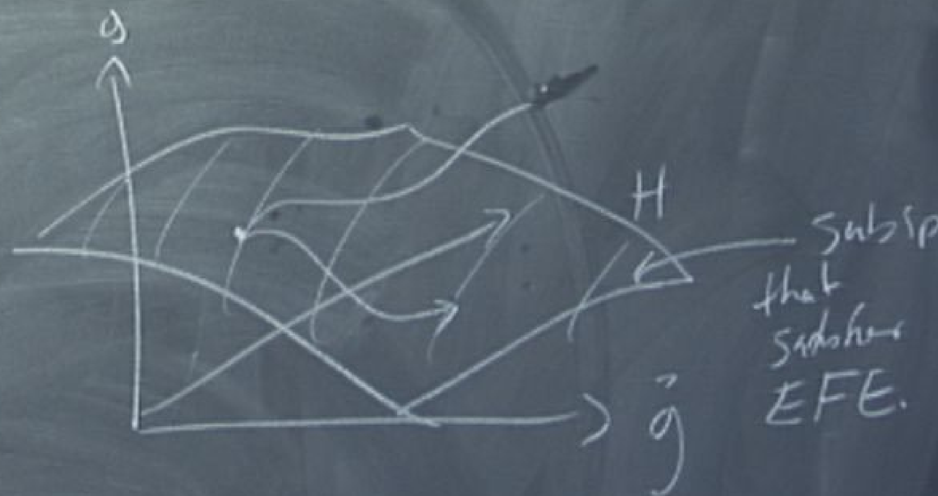
...nterested subset of solutions that satisfy $C_\alpha = 0$ ...



ependent function
L^α(H^α) = 0 eg. H^α =
=0, x^i) , H^α(t=0, x^i)
ans that satisfy C_α = 0


"phase space"
g
H
g
eq. $H^\alpha = 0$
$H^\alpha(t=0,x^i)$, $\frac{\partial}{\partial t} H^\alpha(t=0,x^i)$
$C_\alpha = 0 \Longrightarrow C_\alpha \equiv \Box x_\alpha - H_\alpha$

"phase space"

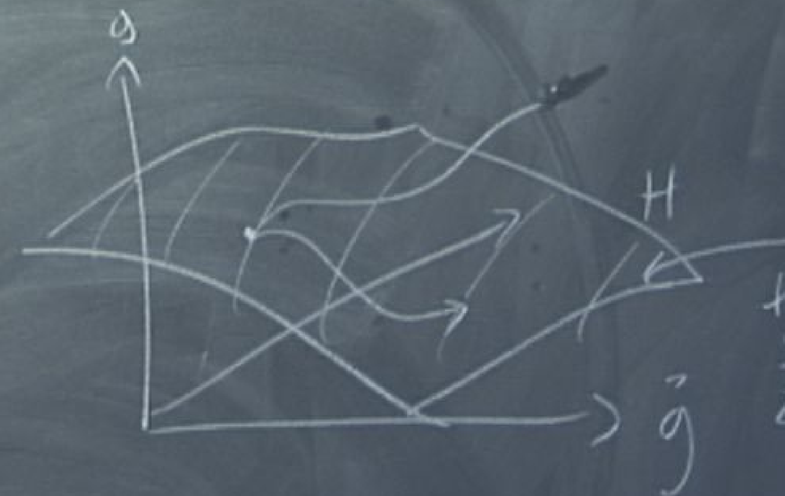


$\circ$ e.g. $H^\alpha = 0$

$\left(H^\alpha(t=0, x^i), \frac{\partial}{\partial t} H^\alpha(t=0, x^i) \right)$

$\underline{C_\alpha = 0} \Longrightarrow C_\alpha \equiv \Box x_\alpha - H_\alpha$

"phase space"

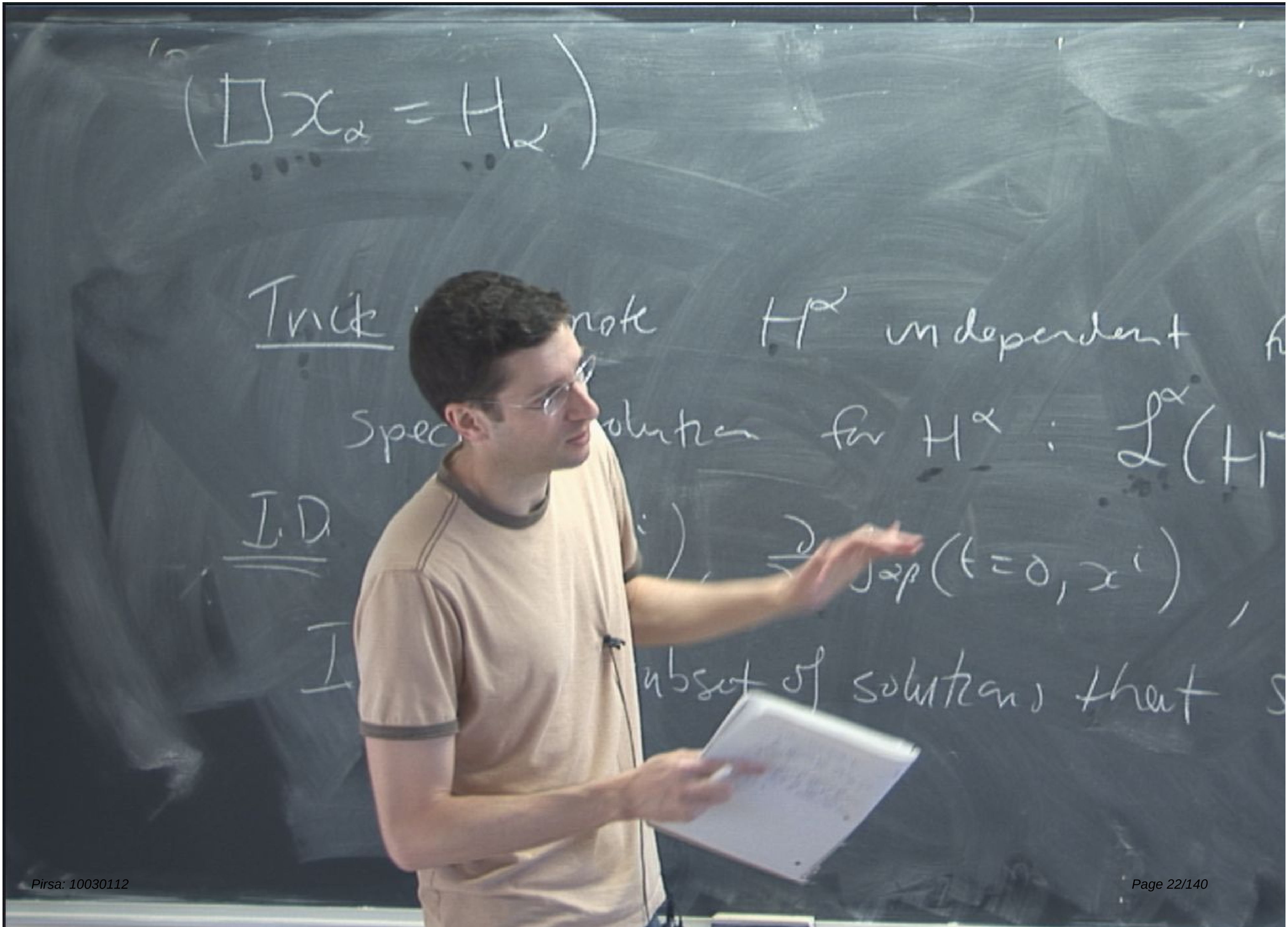


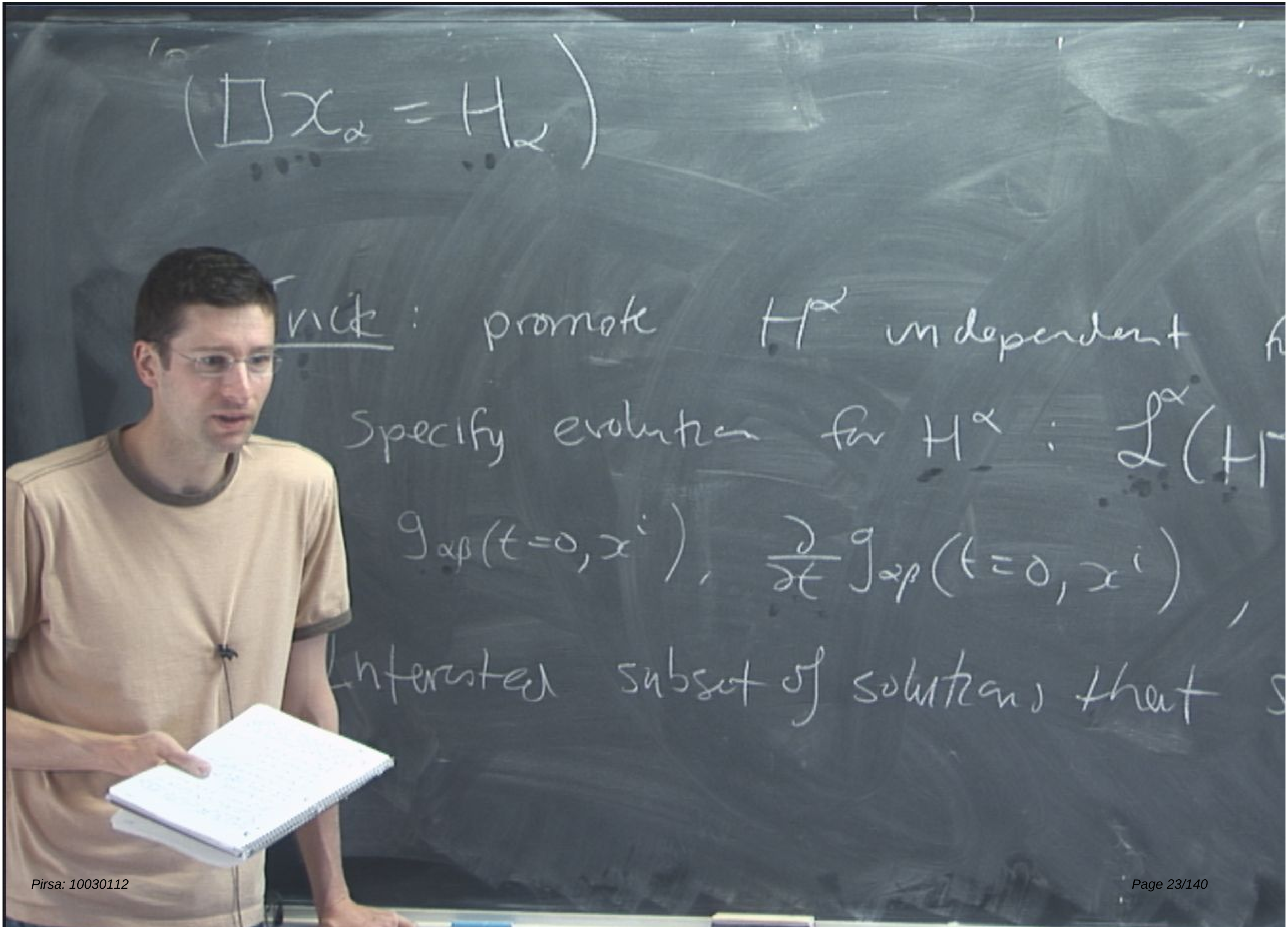
subspace that satisfies EFE.

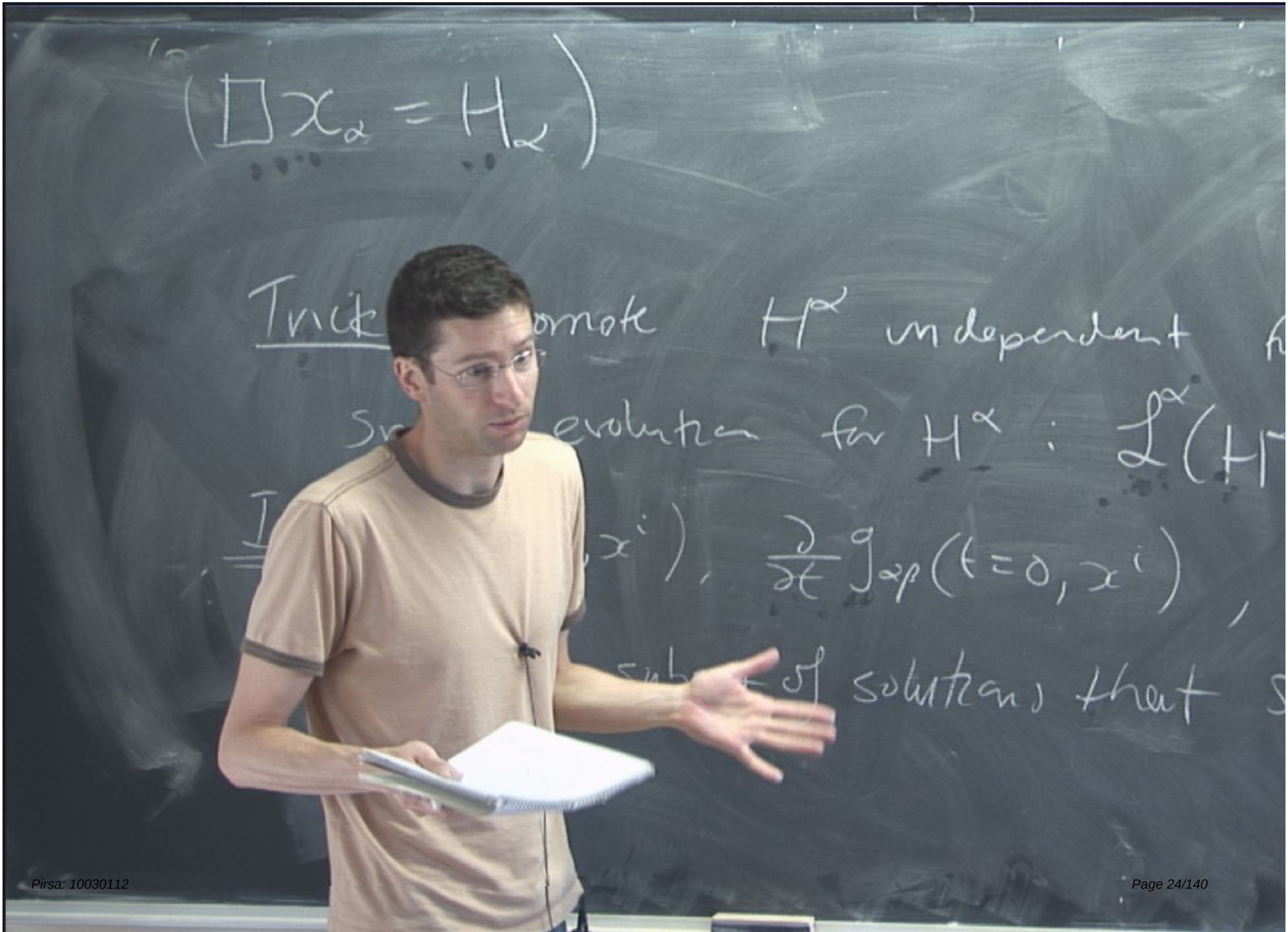
eq. $H^\alpha = 0$

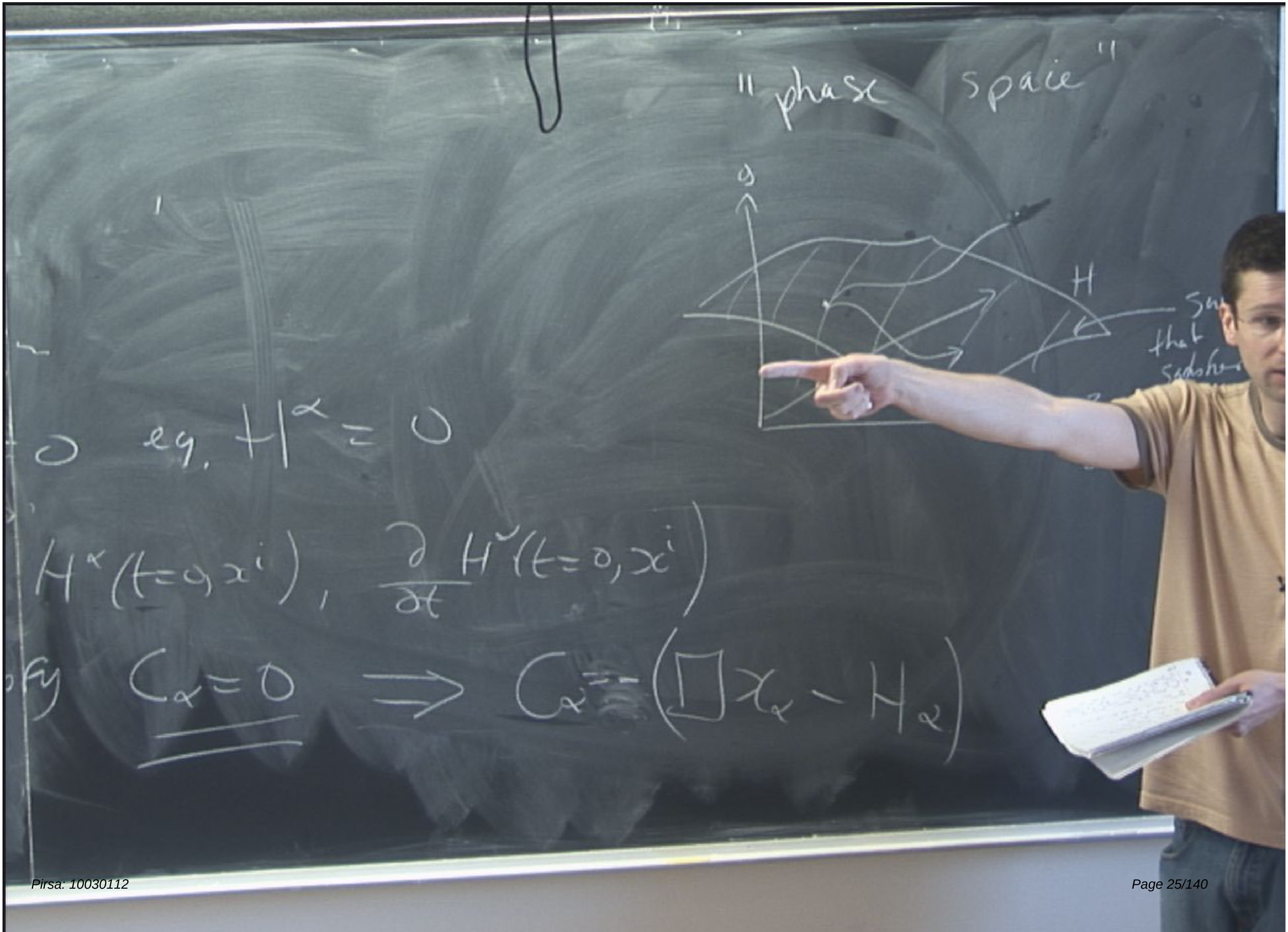
$H^\alpha(t=0,x^i)$, $\frac{\partial}{\partial t}H^\alpha(t=0,x^i)$

$\underline{C_\alpha = 0} \Longrightarrow C_\alpha = -(\Box x_\alpha - H_\alpha)$

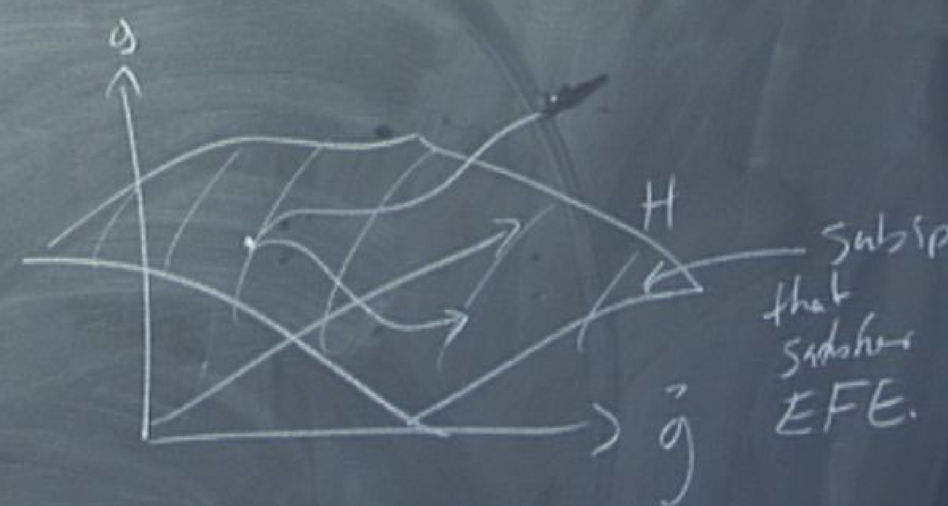

(□x_α = H_α)
Trick
note
H^α independent
Spec
olution for H^α :
I.D.
subset of solutions that


$(\Box x_\alpha = H_\alpha)$
Trick: promote H^α independent
Specify evolution for H^α :
$g_{\alpha\beta}(t=0,x^i)$, $\frac{\partial}{\partial t} g_{\alpha\beta}(t=0,x^i)$,
Interested subset of solutions that


(□x_α = H_α)
Trick
H^α independent
evolution for H^α :
∂/∂t g_αβ(t=0, x^i),
solutions that


"phase space"
g
H
eg. $H^\alpha = 0$
$H^\alpha(t=0,x^i)$, $\frac{\partial}{\partial t} H^\alpha(t=0,x^i)$
$C_\alpha = 0 \Longrightarrow C_\alpha = -(\Box x_\alpha - H_\alpha)$

"phase space"



eq. $H^\alpha = 0$

$H^\alpha(t=0,x^i)$, $\frac{\partial}{\partial t} H^\alpha(t=0,x^i)$

fy $\underline{\underline{C_\alpha = 0}} \Longrightarrow C_\alpha = -(\Box x_\alpha - H_\alpha)$

$$\left(\Box x_\alpha = H_\alpha \right)$$

Trick: promote H^α independent f...

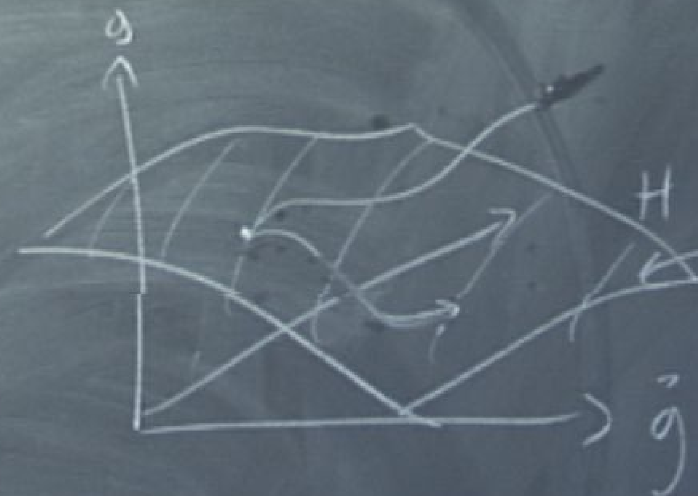
Specify evolution for H^α : $\mathcal{L}^\alpha(H$...

I.D. $g_{\alpha\beta}(t=0, x^i)$, $\frac{\partial}{\partial t} g_{\alpha\beta}(t=0, x^i)$,

Interested subset of solutions that s...

consistent initial condition : $C^\alpha = 0$ $\frac{\partial}{\partial}$...

"phase space"



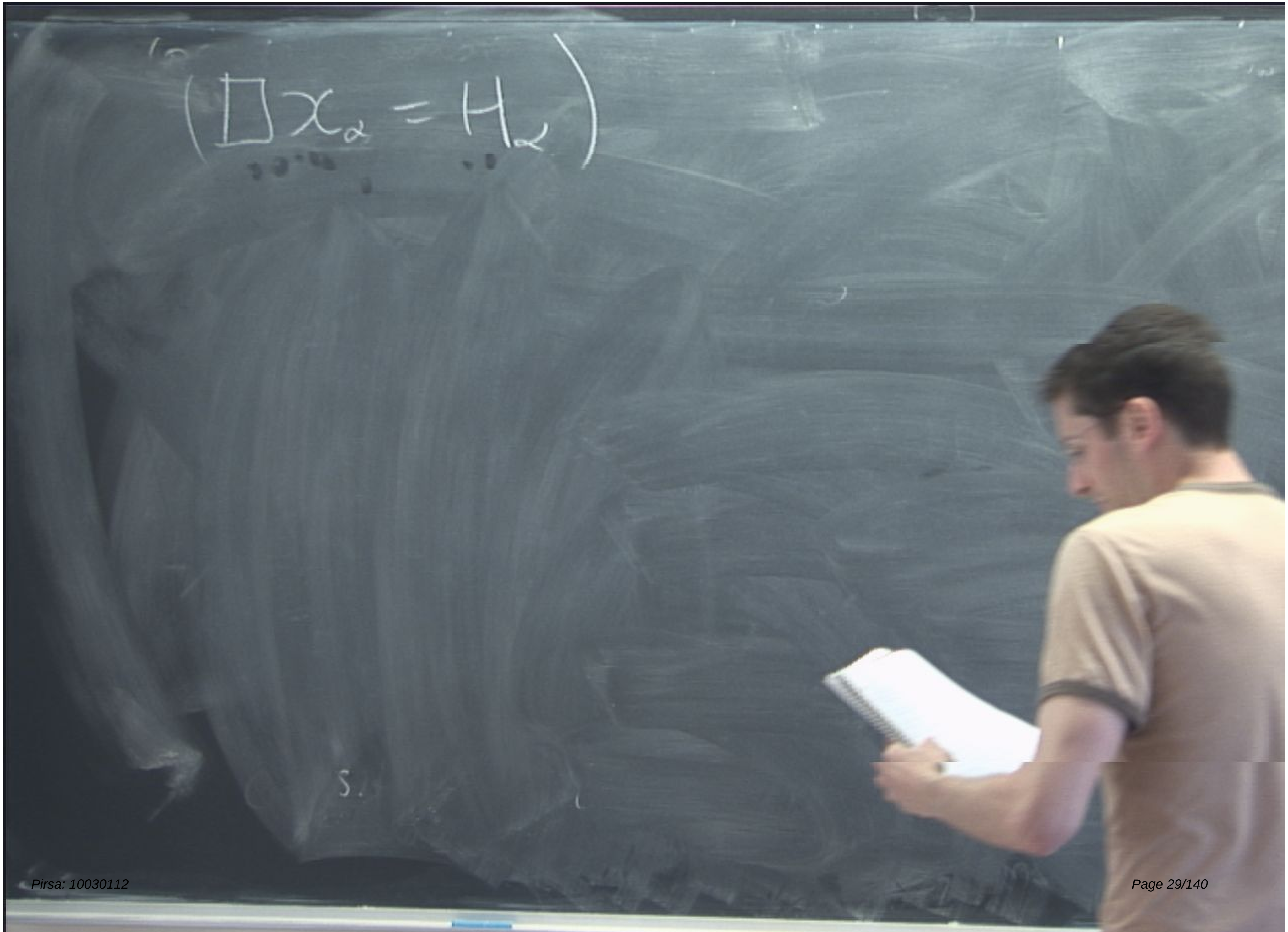
subspace that satisfies EFE.

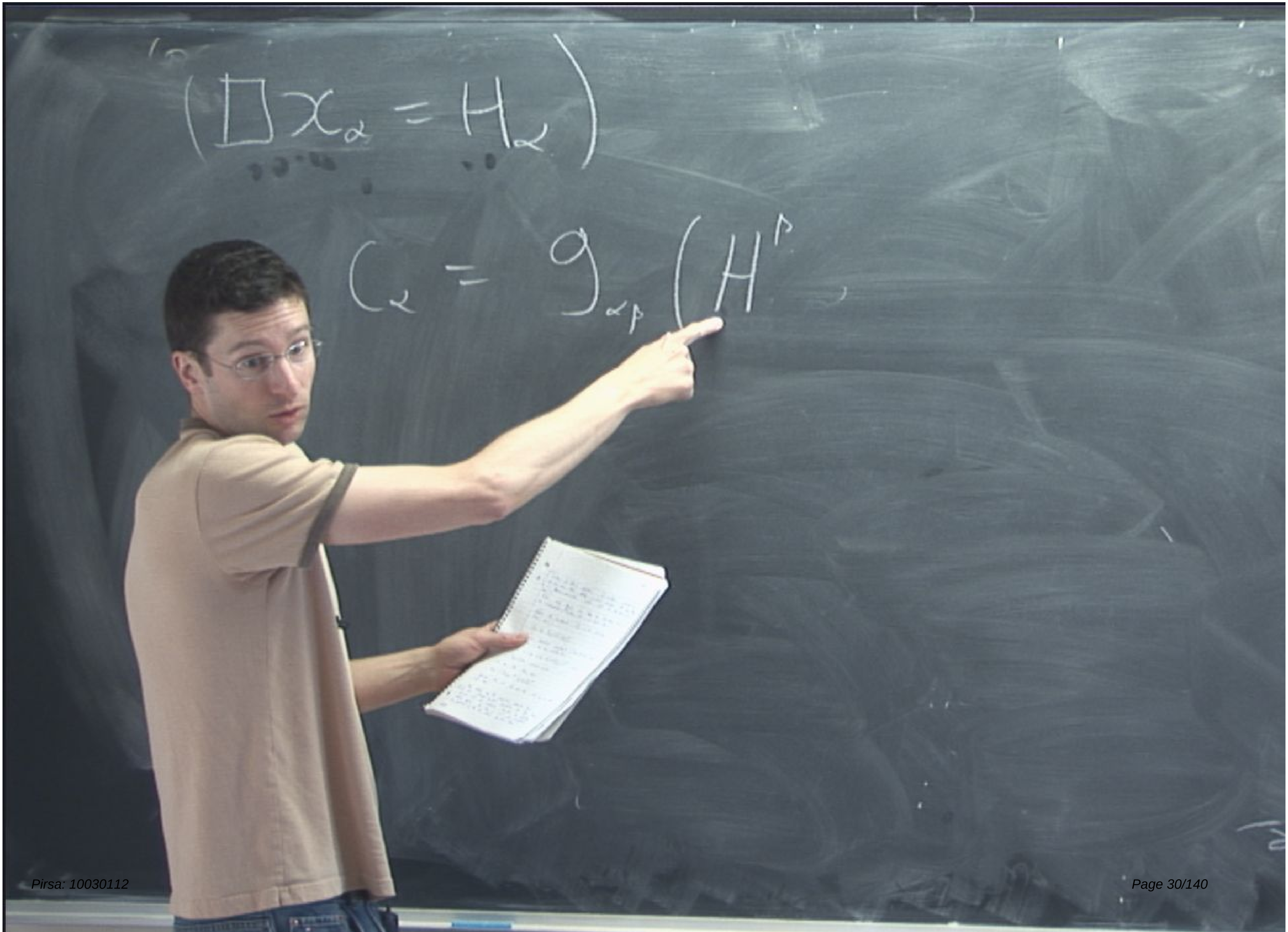
$\mathcal{O}$ eg. $H^\alpha = 0$

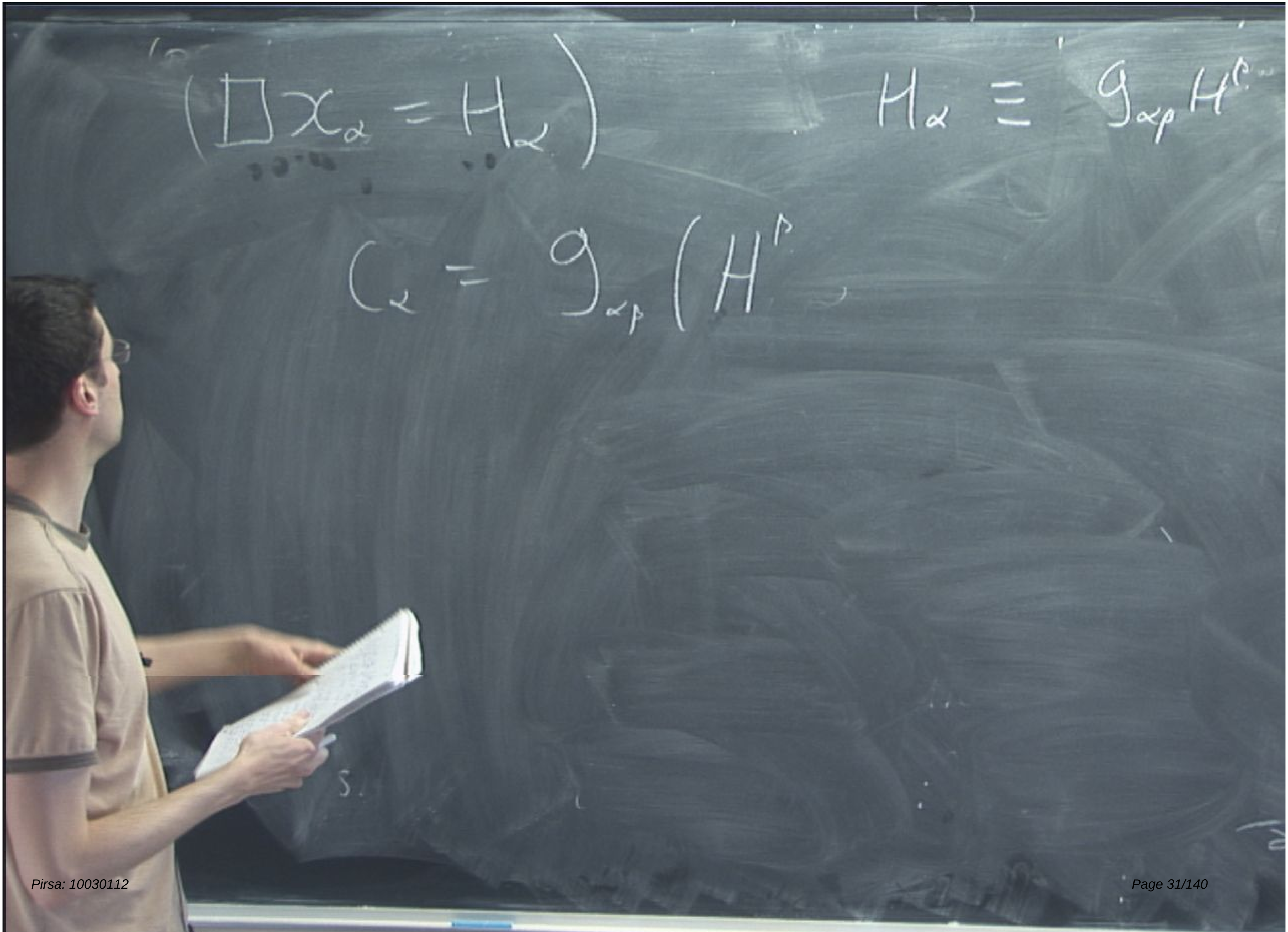
$H^\alpha(t=0,x^i)$, $\frac{\partial}{\partial t} H^\alpha(t=0,x^i)$

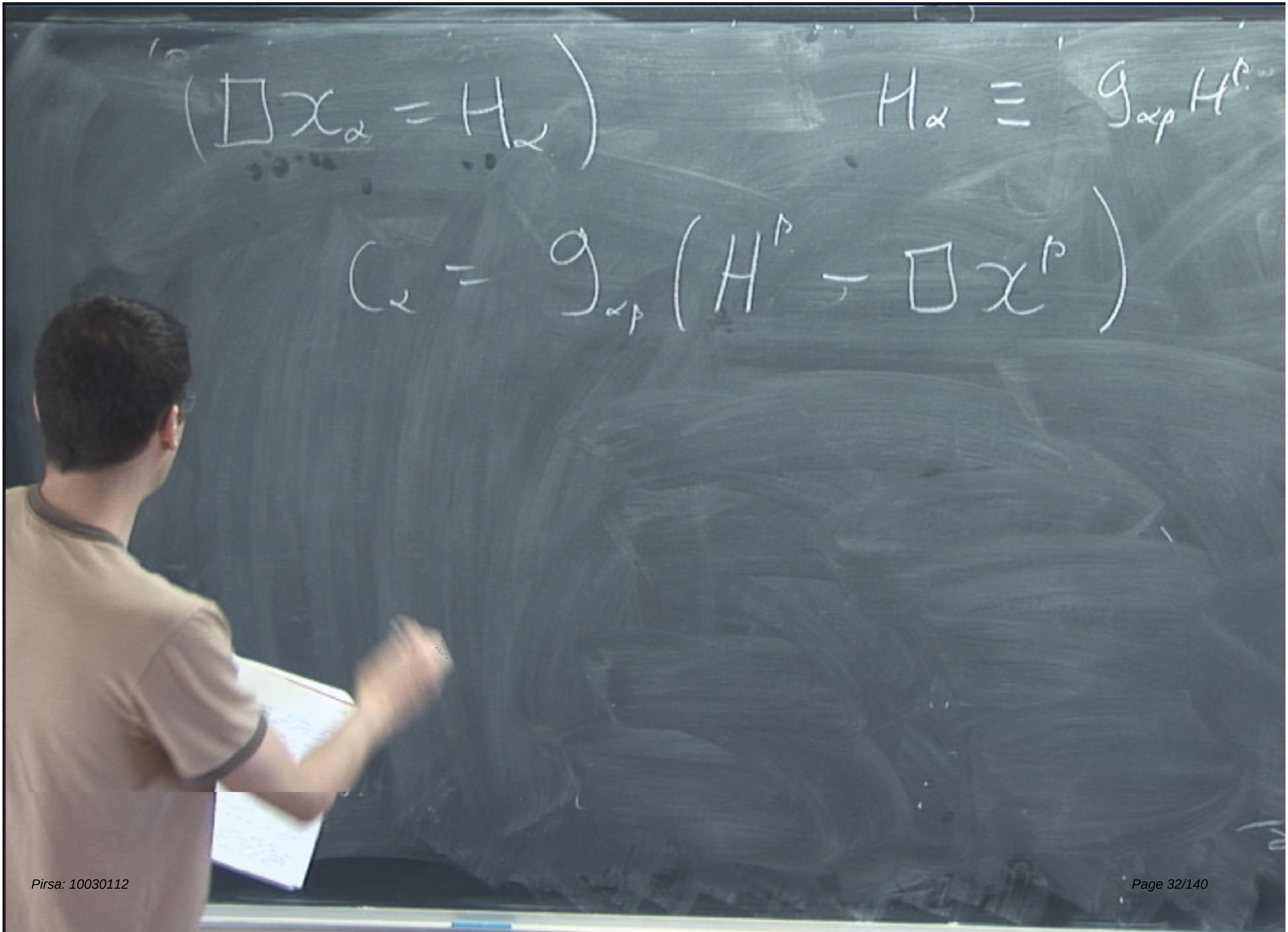
fy $\underline{\underline{C_\alpha = 0}} \Longrightarrow C_\alpha = -(\Box x_\alpha - H_\alpha)$

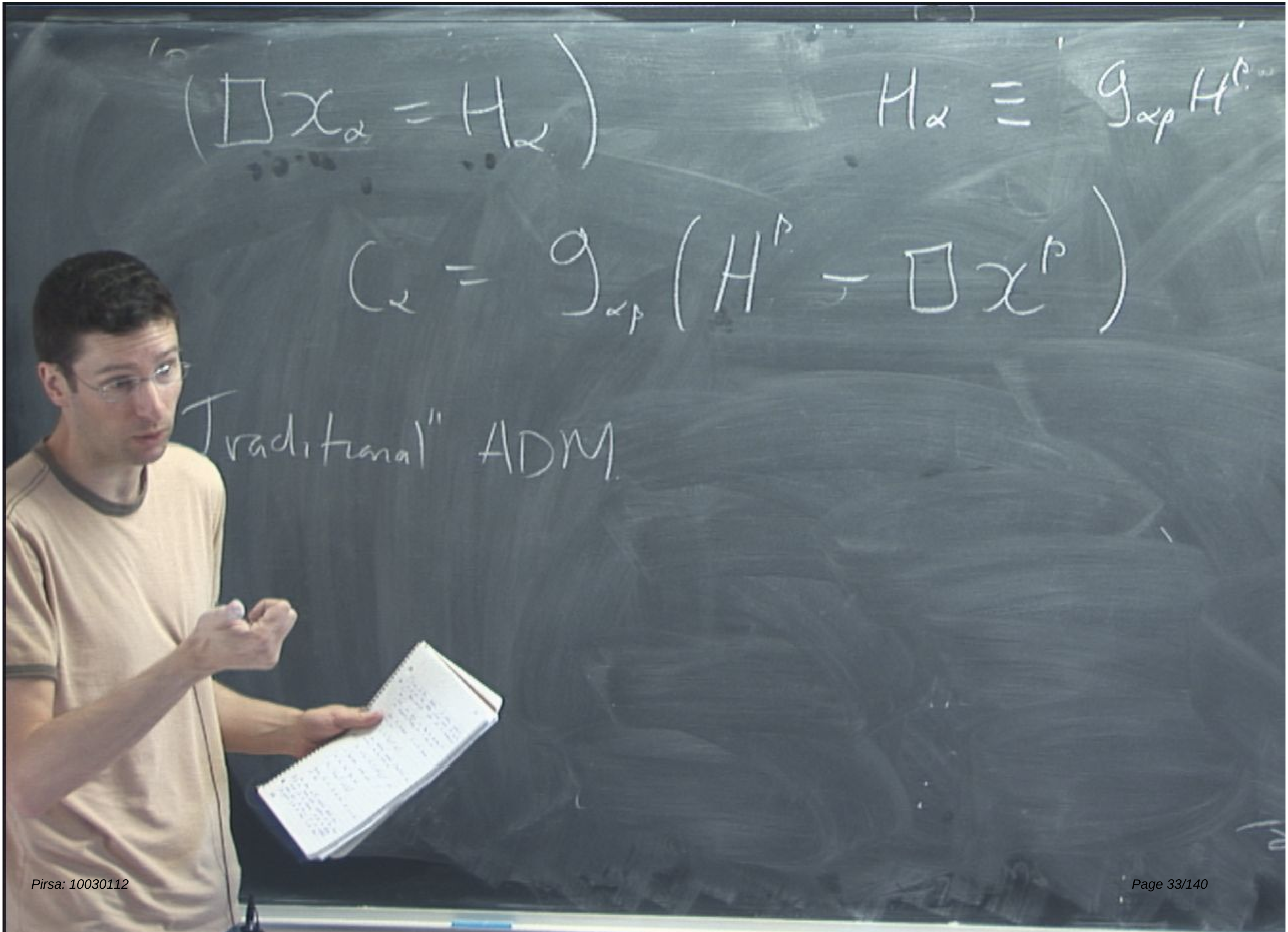
$C^\alpha = 0$


$$\left(\Box x_\alpha = H_\alpha \right)$$


($\Box x_\alpha = H_\alpha$)
$C_\alpha = g_{\alpha\rho}(H^\rho$


$(\Box x_\alpha = H_\alpha)$
$H_\alpha \equiv g_{\alpha\rho} H^\rho$
$C_\alpha = g_{\alpha\rho} (H^\rho$


$(\Box x_\alpha = H_\alpha)$
$H_\alpha \equiv g_{\alpha\rho} H^\rho$
$C_\alpha = g_{\alpha\rho}(H^\rho - \Box x^\rho)$


$(\Box x_\alpha = H_\alpha)$
$H_\alpha \equiv g_{\alpha\rho} H^\rho$
$C_\alpha = g_{\alpha\rho}(H^\rho - \Box x^\rho)$
"Traditional" ADM

$$\left(\Box x_\alpha = H_\alpha\right) \qquad H_\alpha \equiv g_{\alpha\rho} H^\rho$$

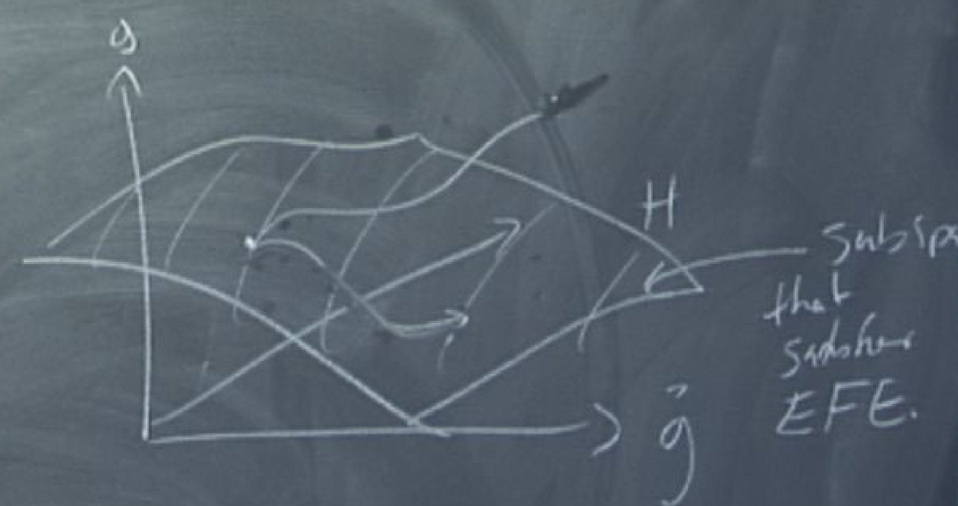
$$C_\alpha = g_{\alpha\rho}\left(H^\rho - \Box x^\rho\right)$$

"Traditional" ADM

$t = \text{const.}$

n^α unit, timelike normal vector to $t = \text{const.}$

"phase space"

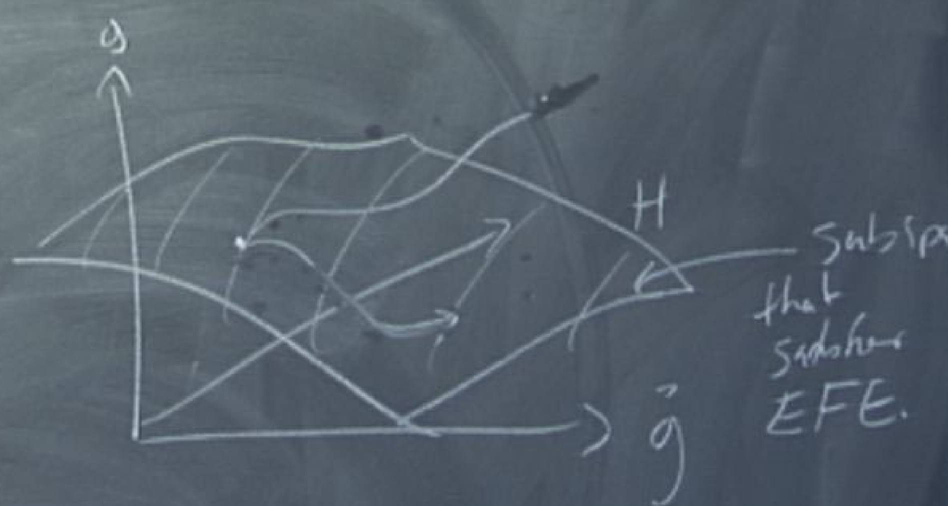


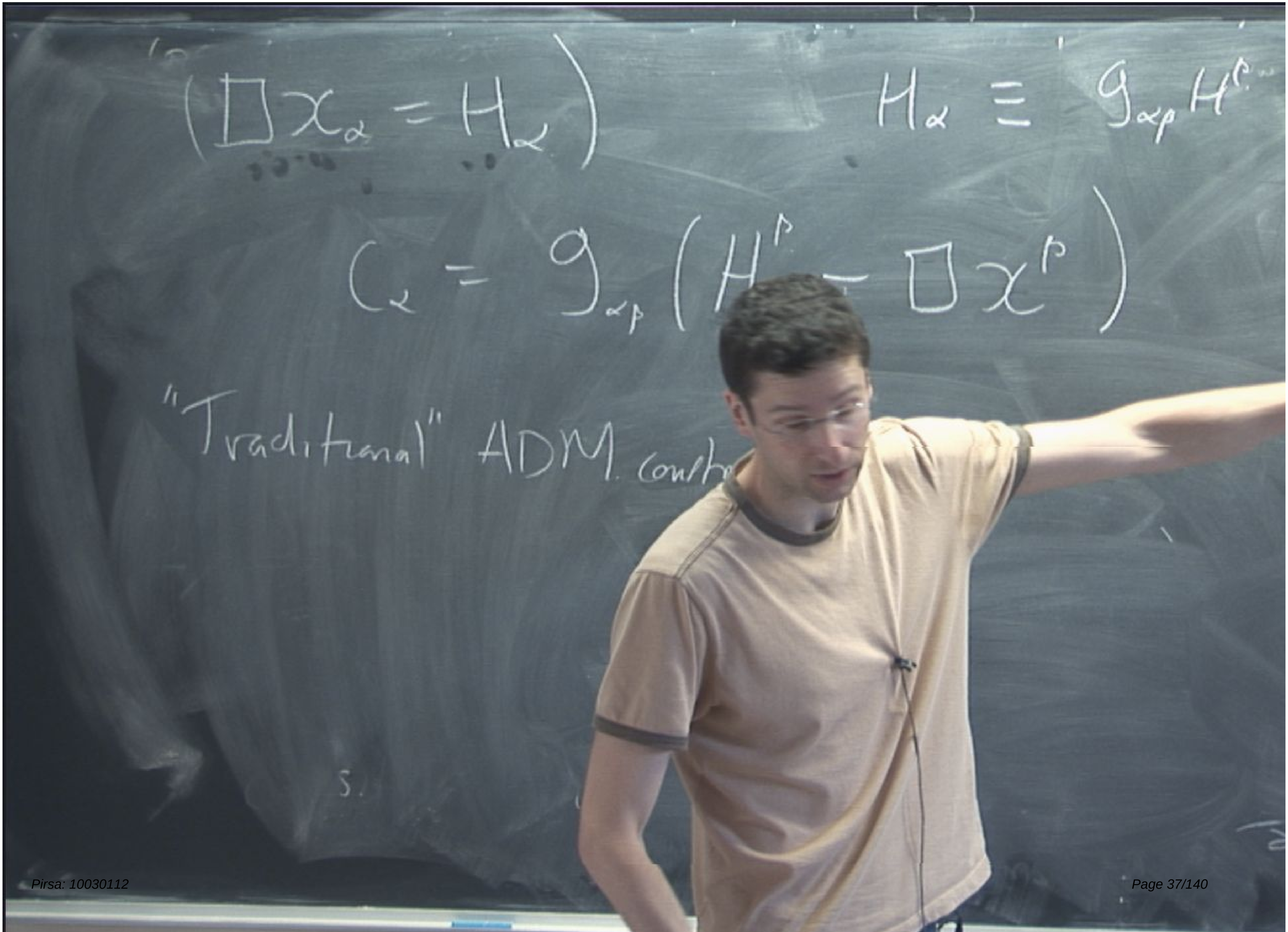
$t = \text{const.}$

n^α unit, timelike normal vector to $t = \text{const}$

$\perp^\alpha{}_\beta$

"phase space"

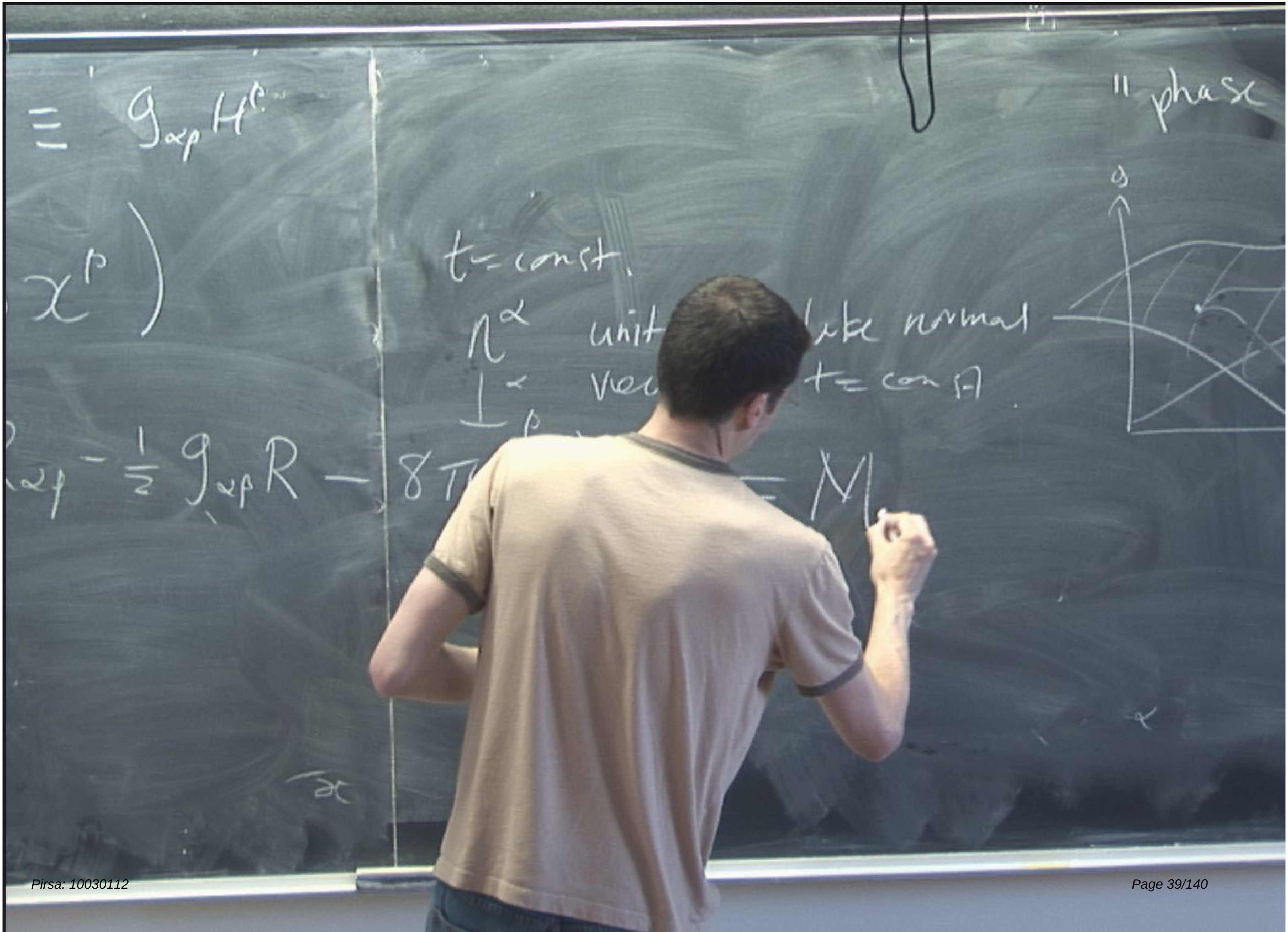



$(\Box x_\alpha = H_\alpha)$
$H_\alpha \equiv g_{\alpha\rho} H^\rho$
$C_\alpha = g_{\alpha\rho}(H^\rho - \Box x^\rho)$
"Traditional" ADM.

$$\left(\Box x_\alpha = H_\alpha\right) \qquad H_\alpha \equiv g_{\alpha\rho} H^\rho$$

$$C_\alpha = g_{\alpha\rho}\left(H^\rho - \Box x^\rho\right)$$

"Traditional" ADM constraints: $\left(R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R\right.$


≡ g_αβ H^β
x^β)
t = const
n^α unit
like normal
vec
t = const
- 1/2 g_αβ R - 8π
M
"phase

"phase

$\equiv g_{\alpha\beta} H^{\beta \cdots}$

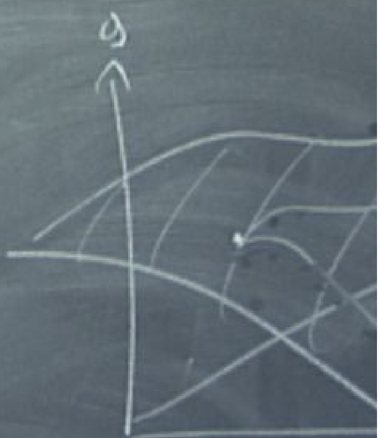
$x^{\beta}\,)$

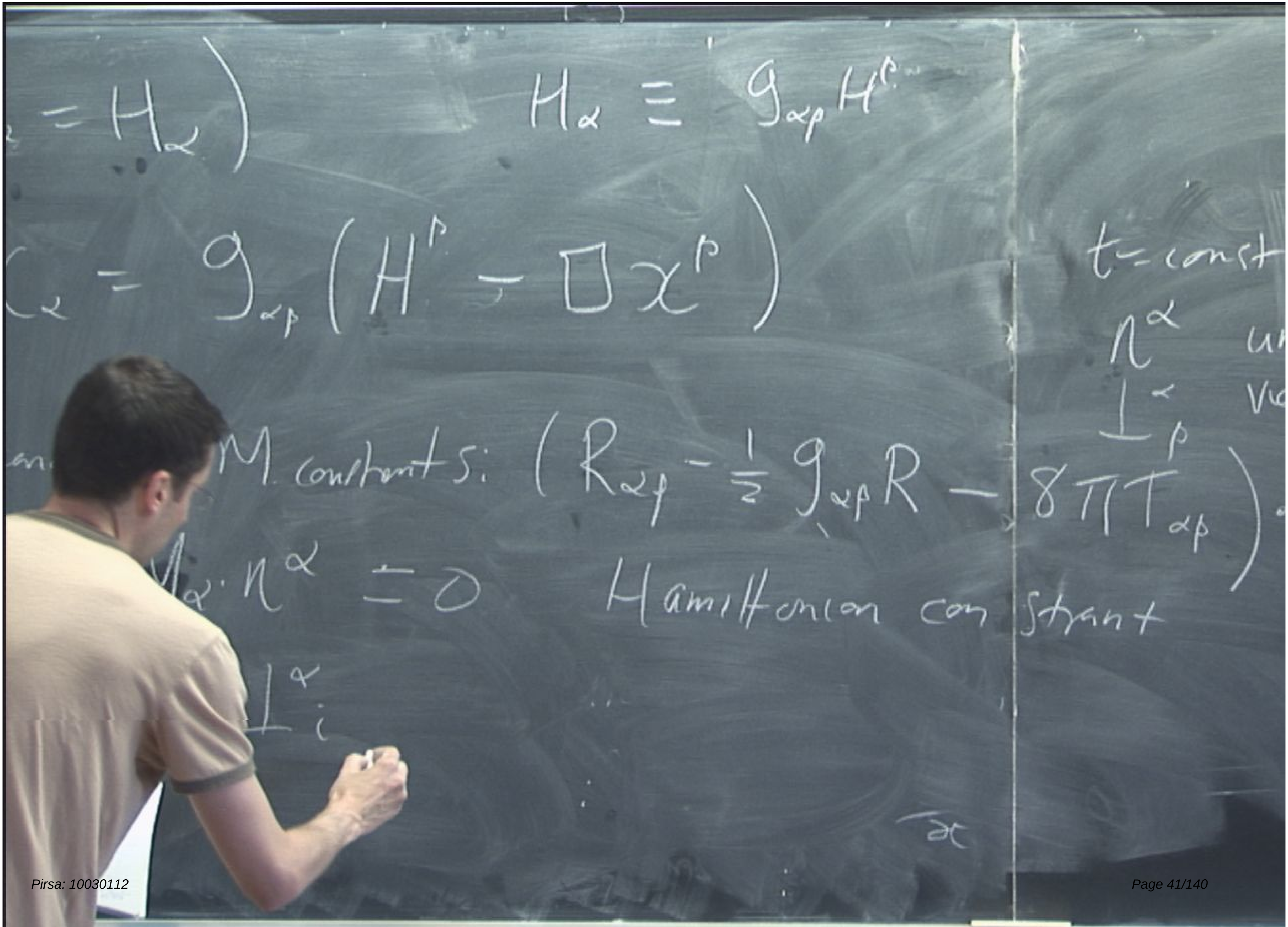
$t = \text{const.}$

n^{α} unit, timelike normal vector to $t = \text{const}$

$\perp^{\alpha}{}_{\beta}$

$$\left(R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R - 8\pi T_{\alpha\beta}\right) \cdot n^{\beta} = M_{\alpha}$$




$= H_\alpha)$
$H_\alpha \equiv g_{\alpha\rho} H^\rho$
$C_\alpha = g_{\alpha\rho}(H^\rho - \Box x^\rho)$
$t = \text{const}$
n^α
$\perp^\alpha_\rho$
M. constraints: $(R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R - 8\pi T_{\alpha\beta})$
$M_\alpha \cdot n^\alpha = 0$ Hamiltonian constraint
$\perp^\alpha_i$

$$= H_\alpha)$$

$$H_\alpha \equiv g_{\alpha\rho} H^\rho$$

$$C_\alpha = g_{\alpha\rho}\left(H^\rho - \Box x^\rho\right)$$

$t = \text{const}$

n^α un

$\perp^\alpha_\rho$ ve

"...nal" ADM constraints: $\left(R_{\alpha\rho} - \frac{1}{2} g_{\alpha\rho} R - 8\pi T_{\alpha\rho}\right)$

$M_\alpha \cdot n^\alpha = 0$ Hamiltonian constraint

$M_\alpha \cdot \perp^\alpha_i = 0$

x

$= H_\alpha)$ $\quad H_\alpha \equiv g_{\alpha\rho} H^\rho$

$C_\alpha = g_{\alpha\rho}\left(H^\rho - \Box x^\rho\right)$

t = const

n^α un

$\perp^\alpha_\rho$ ve

"...nal" ADM constraints: $\left(R_{\alpha\rho} - \frac{1}{2} g_{\alpha\rho} R - 8\pi T_{\alpha\rho}\right)$

$M_\alpha \cdot n^\alpha = 0$ Hamiltonian constraint

$M_\alpha \cdot \perp^\alpha_i = 0$ Momentum constraints.

∂x

show : $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$

$t = \text{const}$

n^α u

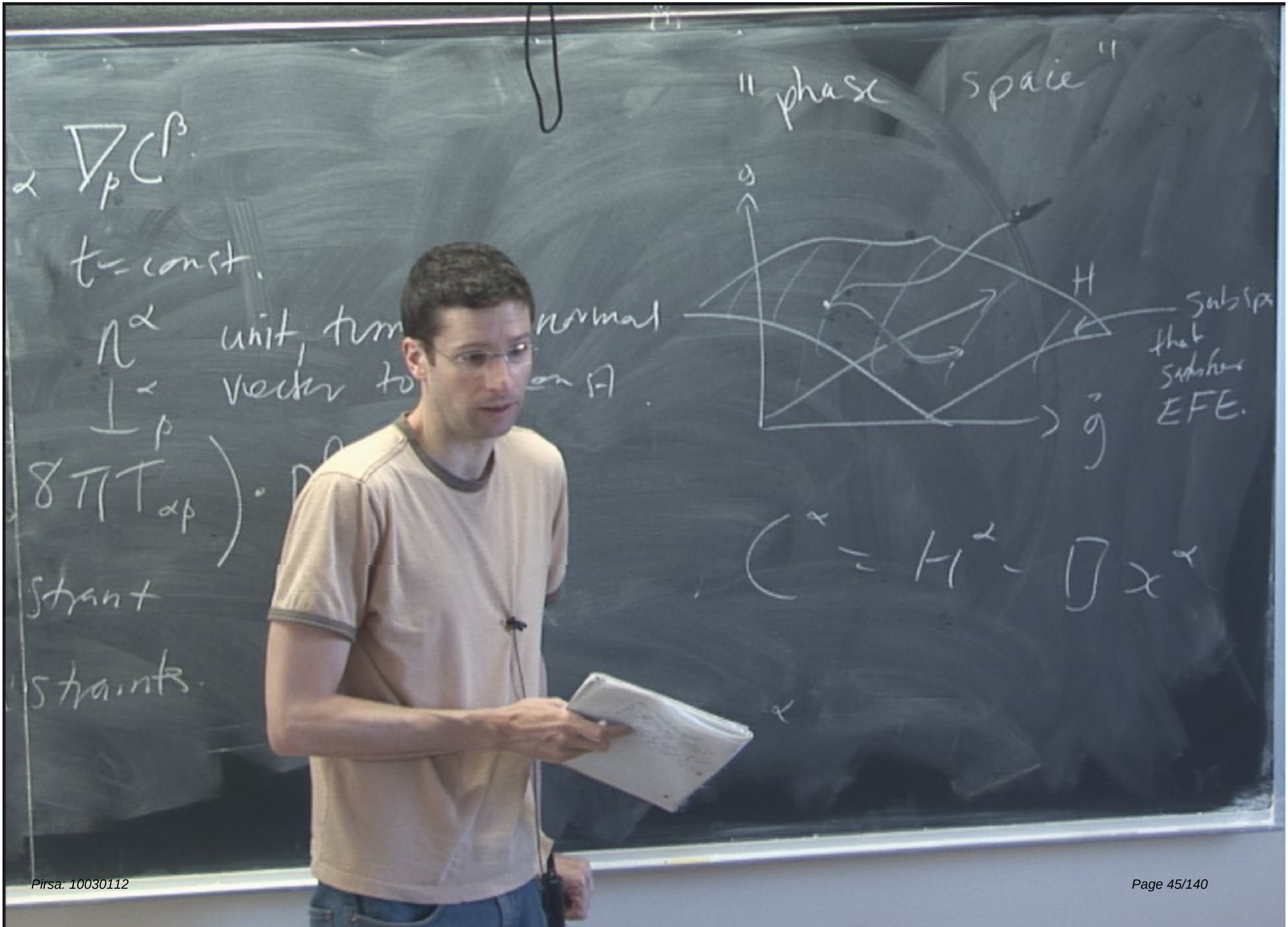
$\perp^\alpha_\beta$ v

ADM constraints: $\left(R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R - 8\pi T_{\alpha\beta} \right)$

$M_\alpha \cdot n^\alpha = 0$ Hamiltonian constraint

$M_\alpha \cdot \perp^\alpha_i = 0$ Momentum constraints.

∂t


"phase space"
$\nabla_\rho C^\beta$
t=const.
n^α unit, tim... normal
vector to
$8\pi T_{\alpha\rho}$
H
that
EFE.
$C^\alpha = H^\alpha - \Box x^\alpha$

$\alpha \nabla_\beta C^\beta$.

$t = \text{const.}$

n^α unit, timelike normal vector to $t = \text{const}$.

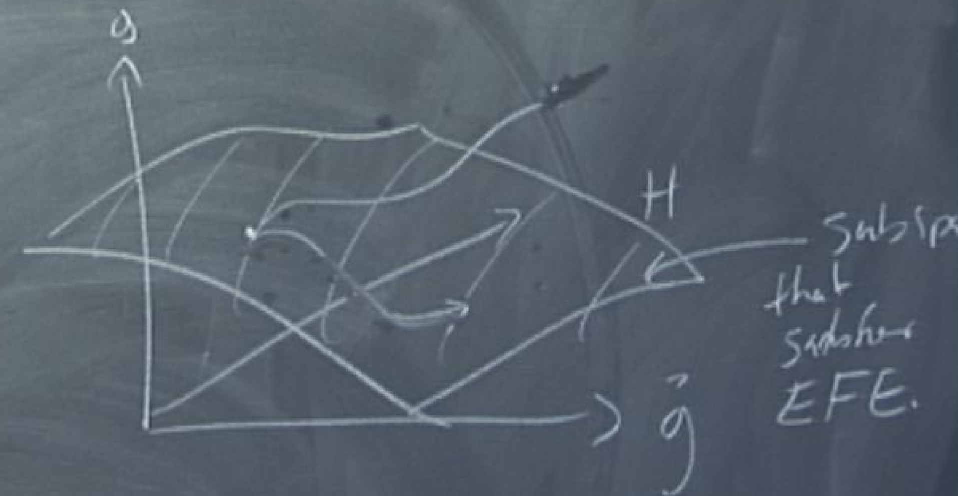
$\perp^\alpha{}_\beta$

$8\pi T_{\alpha\beta}) \cdot n^\beta = M_\alpha$

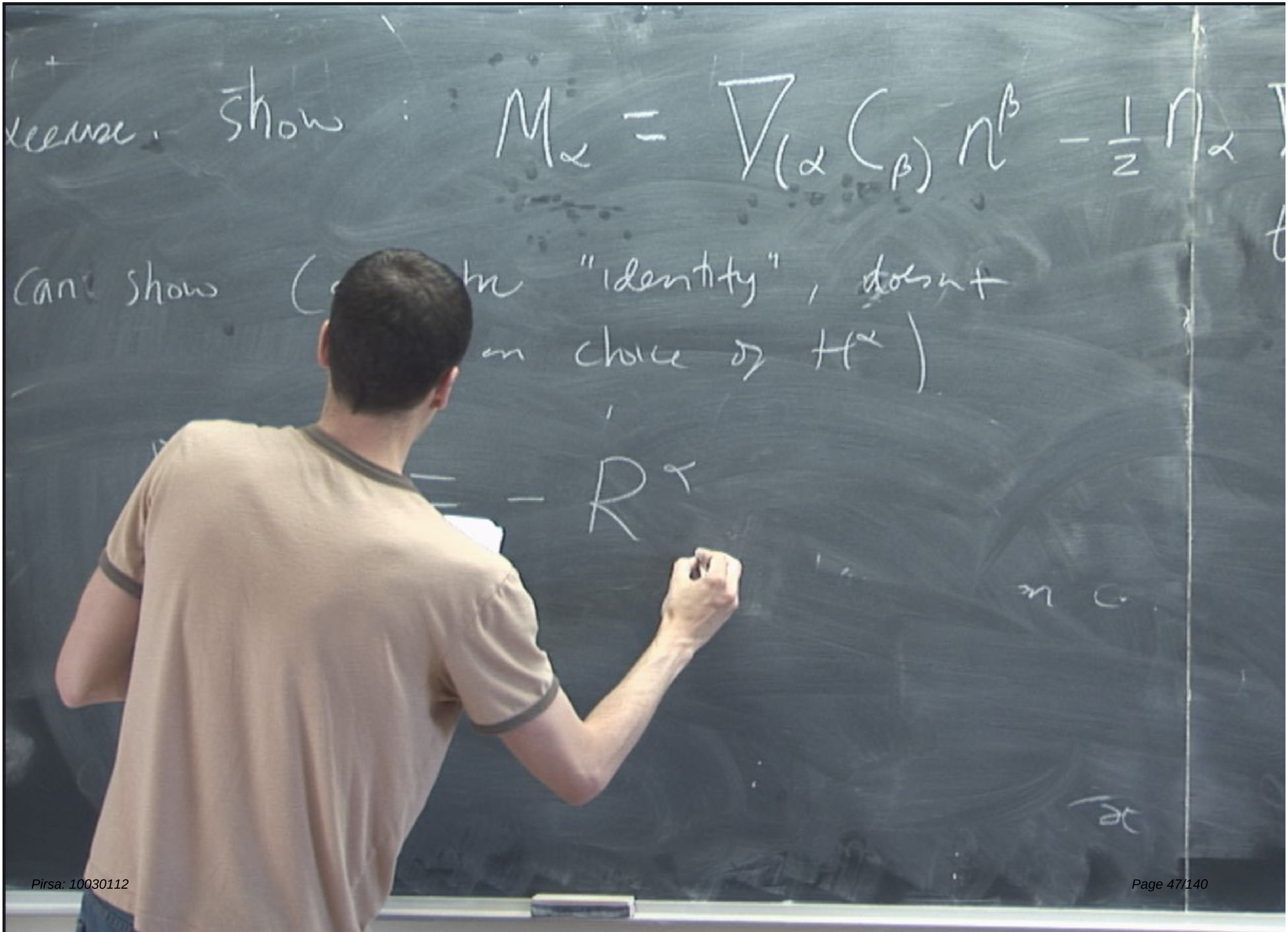
straint

straints.

"phase space"



$C^\alpha = H^\alpha - \Box x^\alpha$


show : $M_\alpha = \nabla_{(\alpha}C_{\beta)}n^\beta - \frac{1}{2}n_\alpha$
can show
"identity", doesn't
choice of H^α)
$- R^\alpha$

xercise. show: $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha$

can't show (geometric "identity", doesn't depend on choice of H^α)

$$\Box C^\alpha = - R^\alpha{}_\beta C^\beta$$

show : $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$

$t = \text{con}$

n^α

n_α

how (geometric "identity", doesn't depend on choice of H^α)

$\Box C^\alpha = - R^\alpha{}_\beta C^\beta$,

If we choose $C^\alpha = 0$, $\partial_t C^\alpha = 0$, and

∂t

$n_\alpha \nabla_\beta C^\beta$

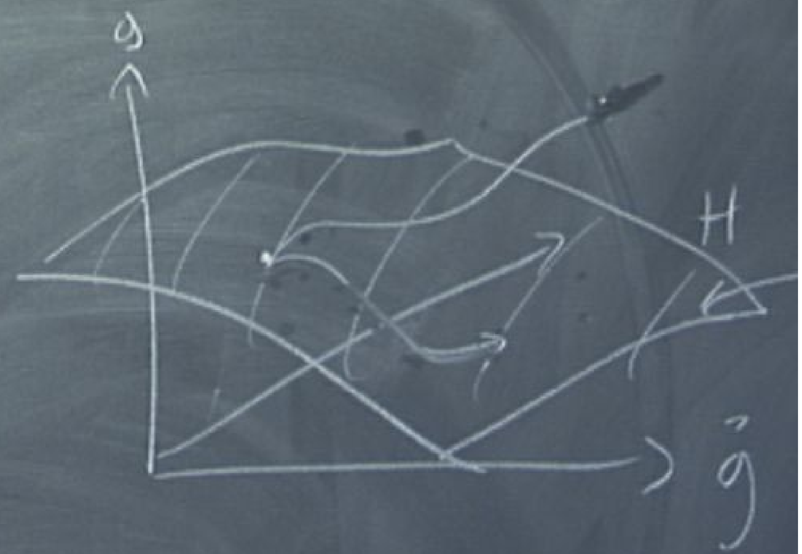
$t = \text{const.}$

n^α unit, timelike normal vector to $t = \text{const}$

n_α

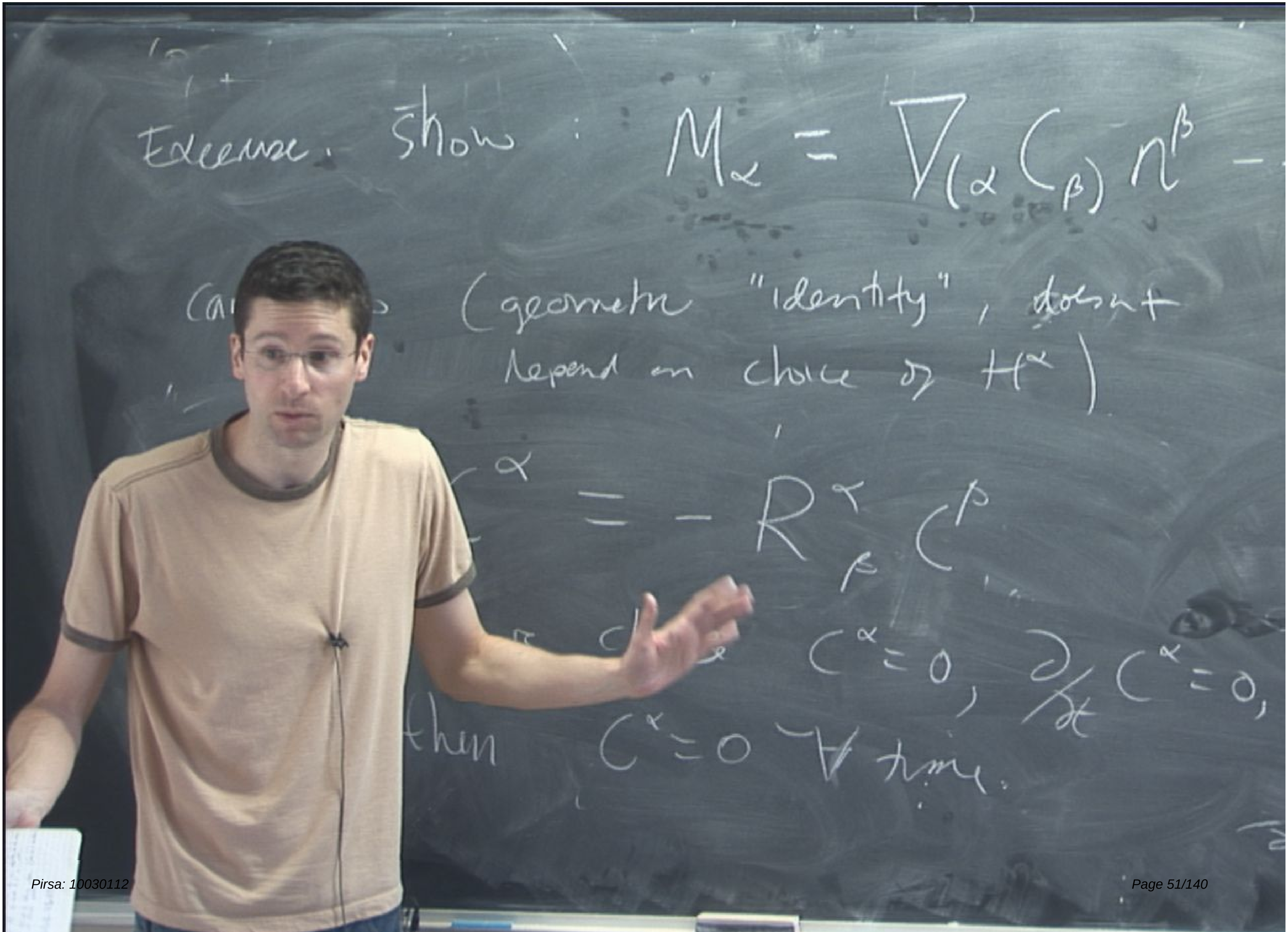
$n^\beta = M_\alpha$

" phase space



$C^\alpha = H^\alpha - [\,]$

and impose b.c. such that $C^\alpha = 0$ on boundary


Exercise. Show: $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta$ -
(geometric "identity", doesn't depend on choice of N^α)
$= -R^\alpha{}_\beta C^\beta$
$C^\alpha = 0$, $\partial_t C^\alpha = 0$,
then $C^\alpha = 0$ $\forall$ time.

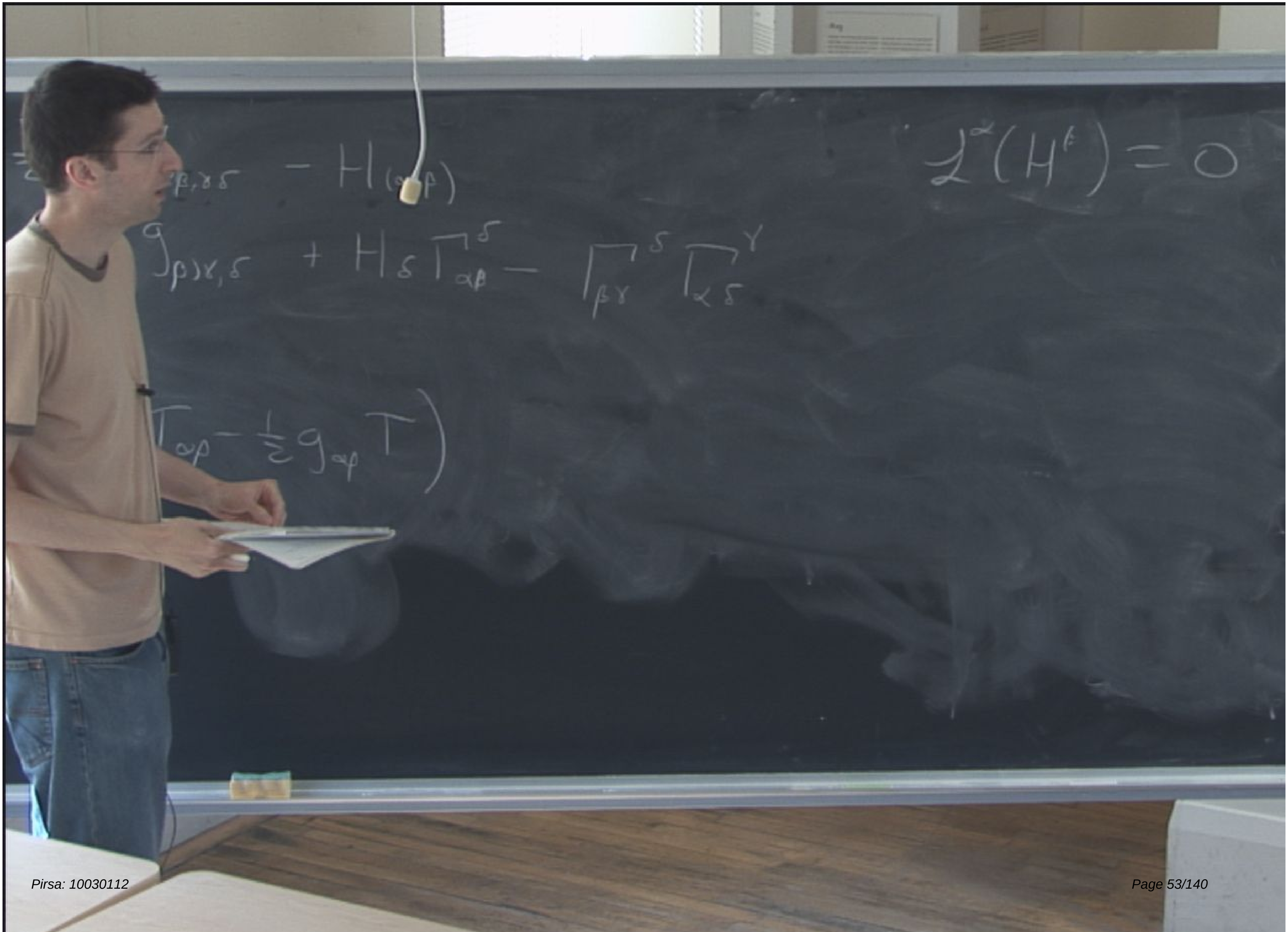
Exercise: show: $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta -$

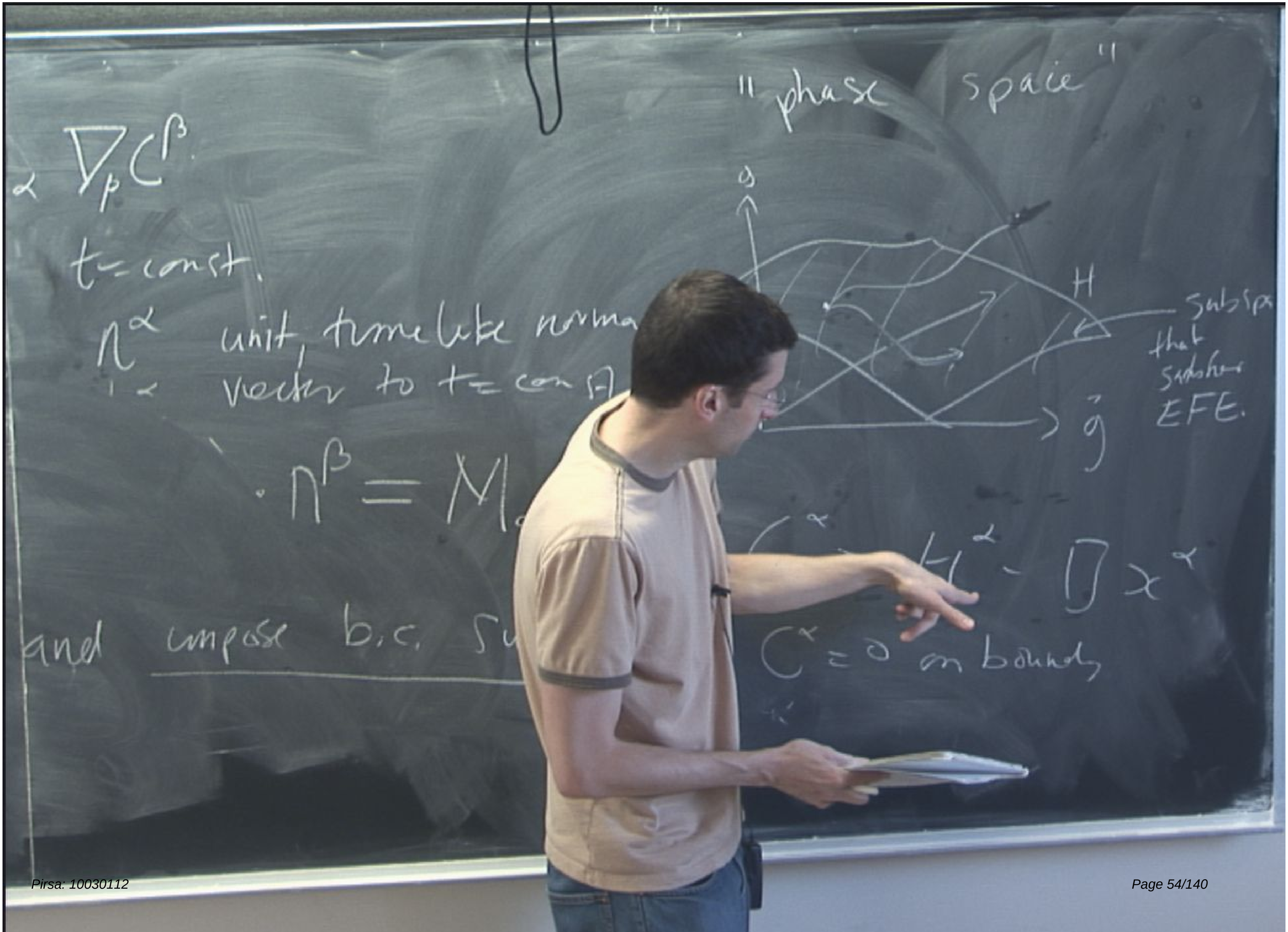
Can show (geometric "identity", doesn't depend on choice of H^α)

$$\Box C^\alpha = - R^\alpha{}_\beta C^\beta$$

If we choose $C^\alpha = 0$, $\partial_t C^\alpha = 0$,

then $C^\alpha = 0 \;\forall$ time.


$-H_{(\alpha\beta)}$
$g_{\beta)\gamma,\delta} + H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma}\Gamma^\gamma_{\alpha\delta}$
$T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T)$
$\mathcal{L}^\alpha(H^\beta) = 0$


"phase space"
∇ρ C^β
t = const.
n^α unit, timelike norma
vector to t = const
n^β = M
H
subspa
that
satisfies
EFE.
C^α = H^α − □x^α
and impose b.c. su
C^α = 0 on boundary

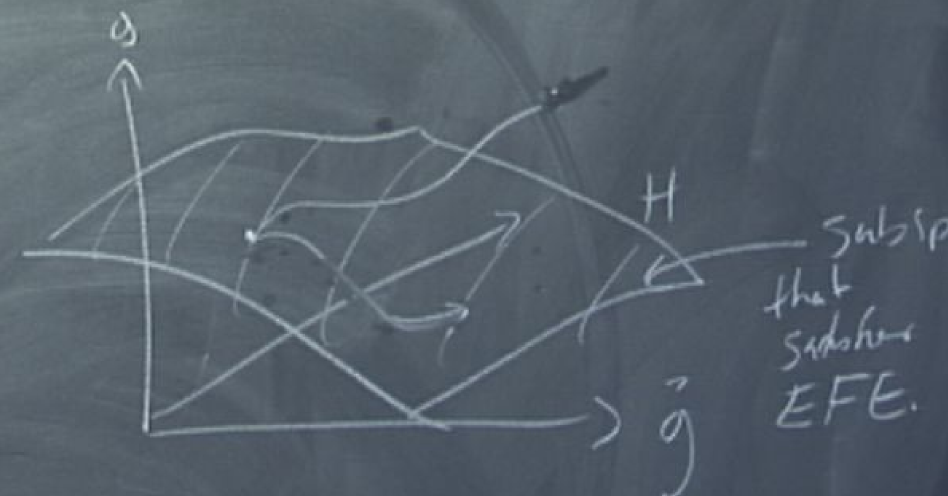
$\alpha \quad \nabla_\beta C^\beta$.

$t = \text{const}$.

n^α unit, timelike normal vector to $t = \text{const}$

$\cdot\, n^\beta = M_\alpha$.

"phase space"



$$C^\alpha = H^\alpha - \Box x^\alpha$$

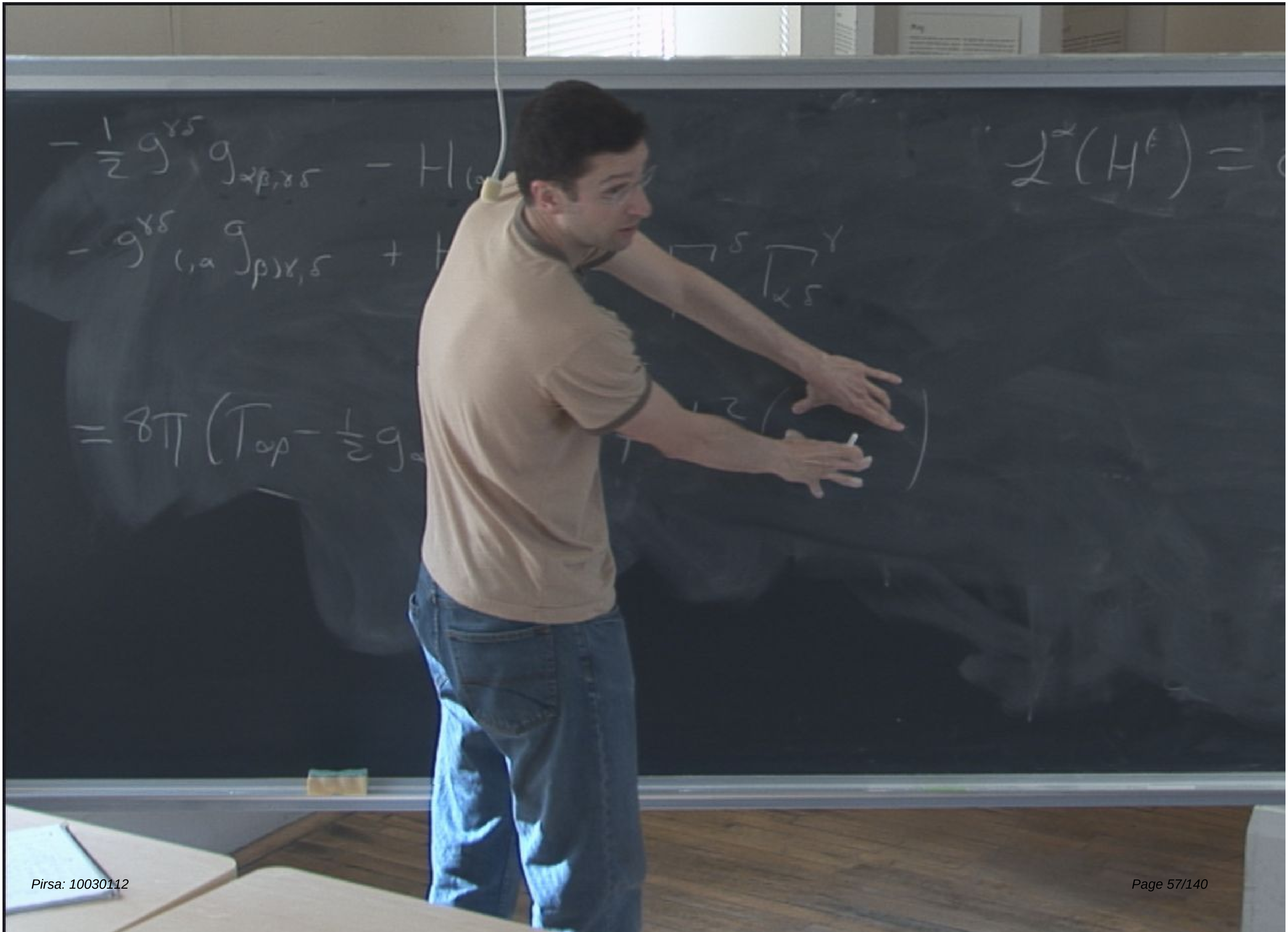
and impose b.c. such that $C^\alpha = 0$ on boundary

Exercise. Show: $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta -$

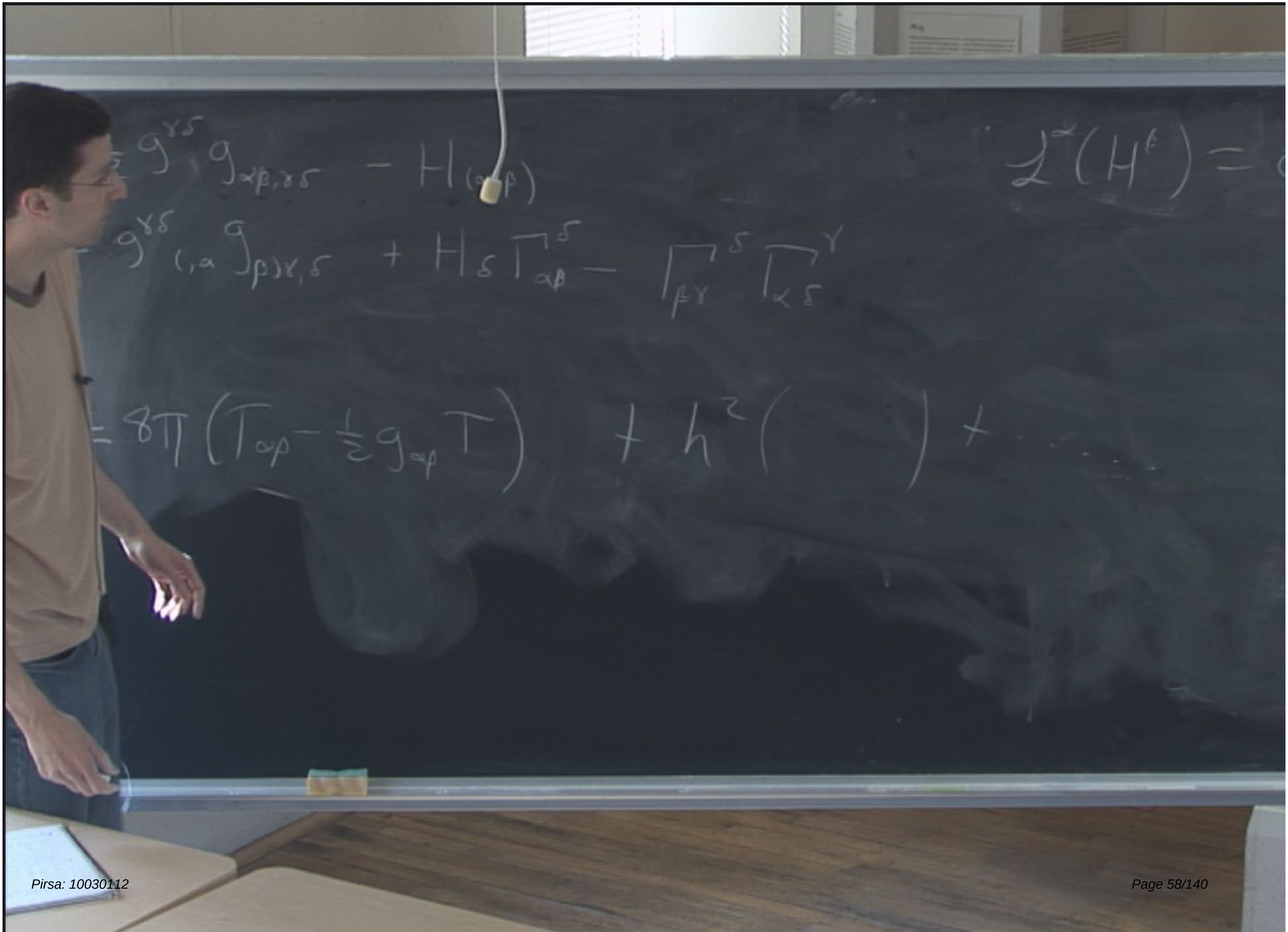
Can show (geometric "identity", doesn't depend on choice of H^α)

$$\Box C^\alpha = - R^\alpha{}_\beta C^\beta$$

If we choose $C^\alpha = 0$, $\partial_t C^\alpha = 0$,
then $C^\alpha = 0$ $\forall$ time.



$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha}$
$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H$
$\Gamma^{\delta} \Gamma^{\gamma}_{\alpha\delta}$
$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha}\right.$
$\mathcal{L}^{\alpha}(H^{\epsilon}) =$

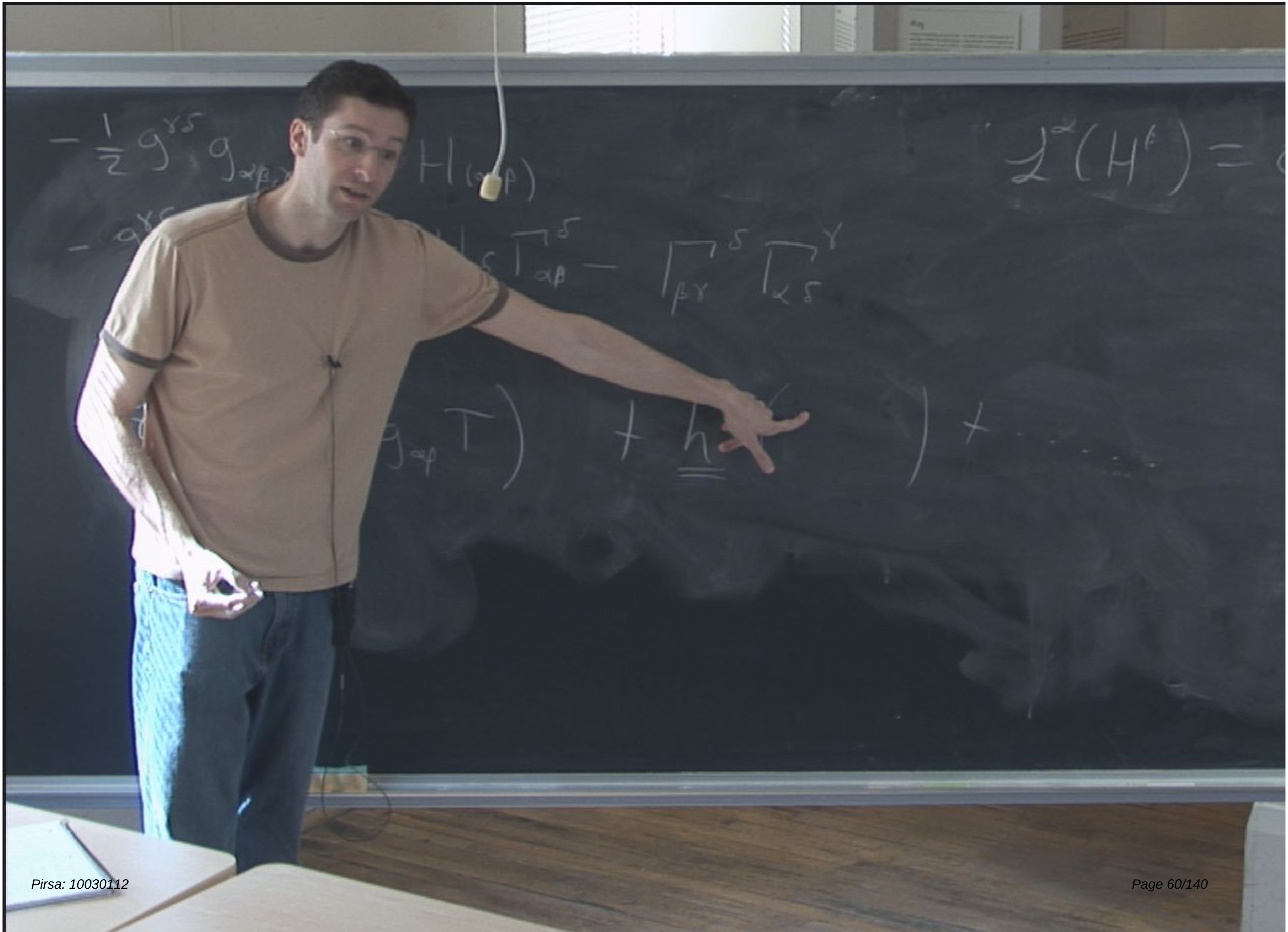

$= g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$
$g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$
$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right) + h^2 (\quad) + \ldots$
$\mathcal{L}^\alpha (H^\beta) =$

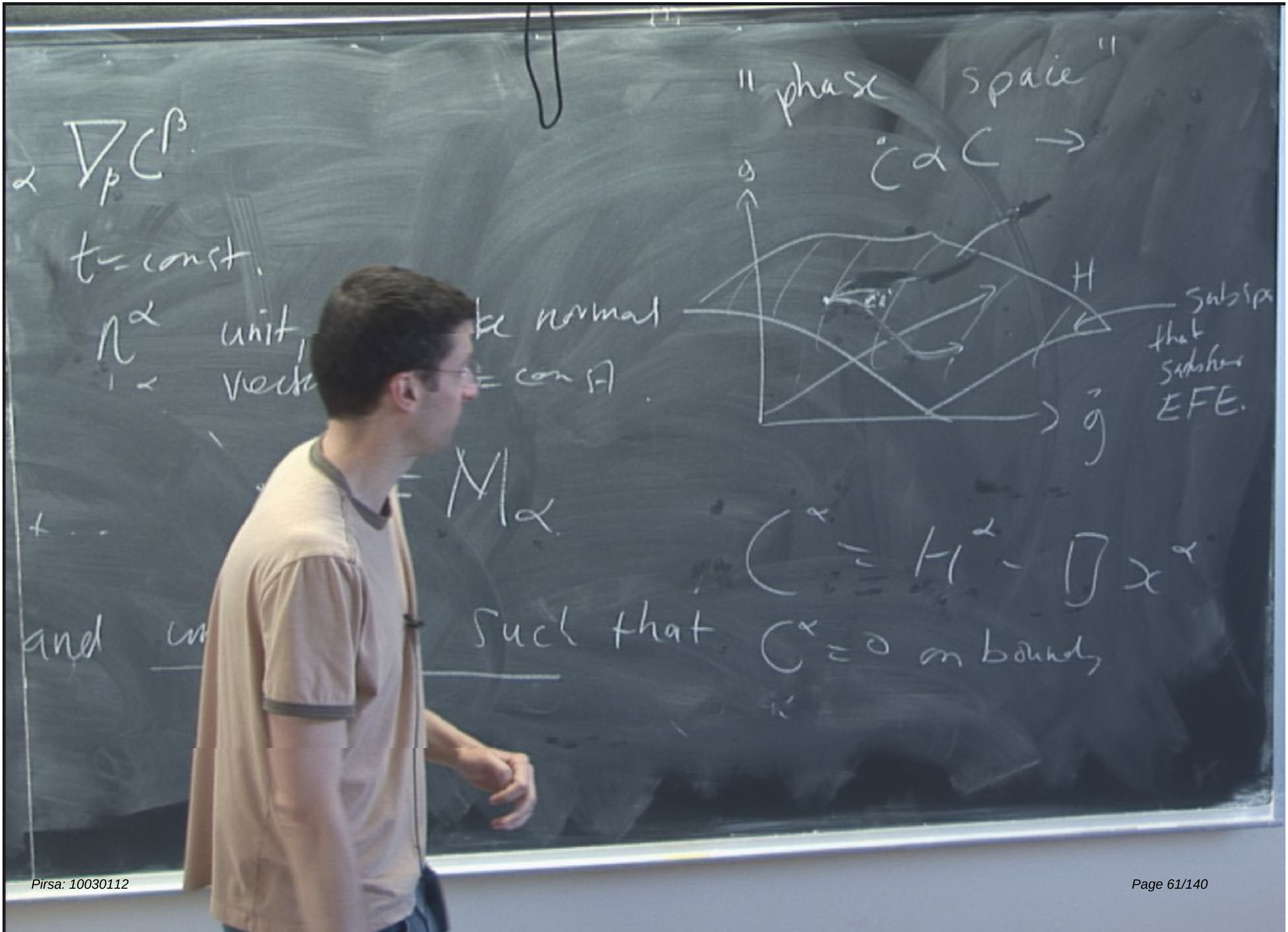
$$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha;\beta)}$$

$$\mathcal{L}^{\alpha}(H^{\epsilon}) =$$

$$- g^{\gamma\delta}{}_{,(\alpha}\, g_{\beta)\gamma,\delta} + H_{\delta} \Gamma^{\delta}_{\alpha\beta} - \Gamma^{\delta}_{\beta\gamma} \Gamma^{\gamma}_{\alpha\delta}$$

$$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right) + \underline{\underline{h}}^{2} \left(\qquad \right) + \dots$$




"phase space"
t = const.
unit
normal
such that
subspace
that
satisfies
EFE.
$C^\alpha = H^\alpha - \Box x^\alpha$
$C^\alpha = 0$ on boundary

$\alpha \quad \nabla_\rho C^\beta$

$t = \text{const.}$

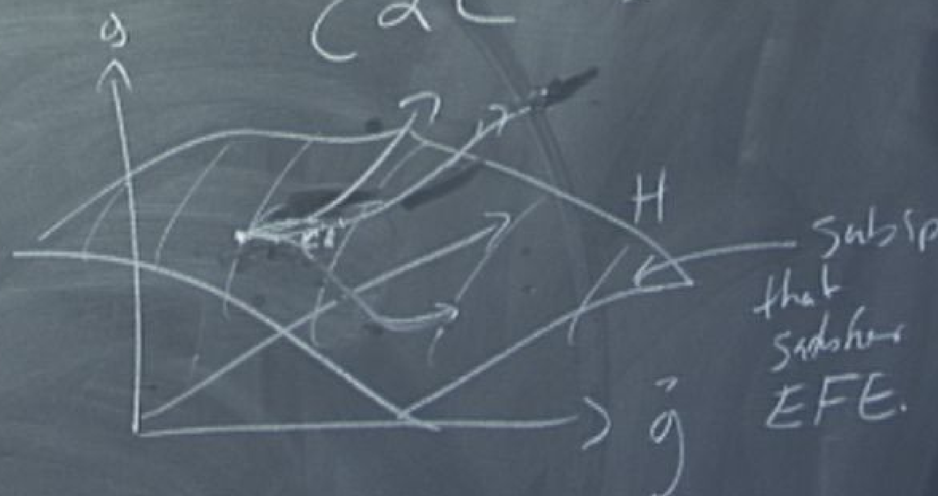
n^α unit, timelike normal vector to $t = \text{const}$

$n^\beta = M_\alpha$

$+ \ldots$

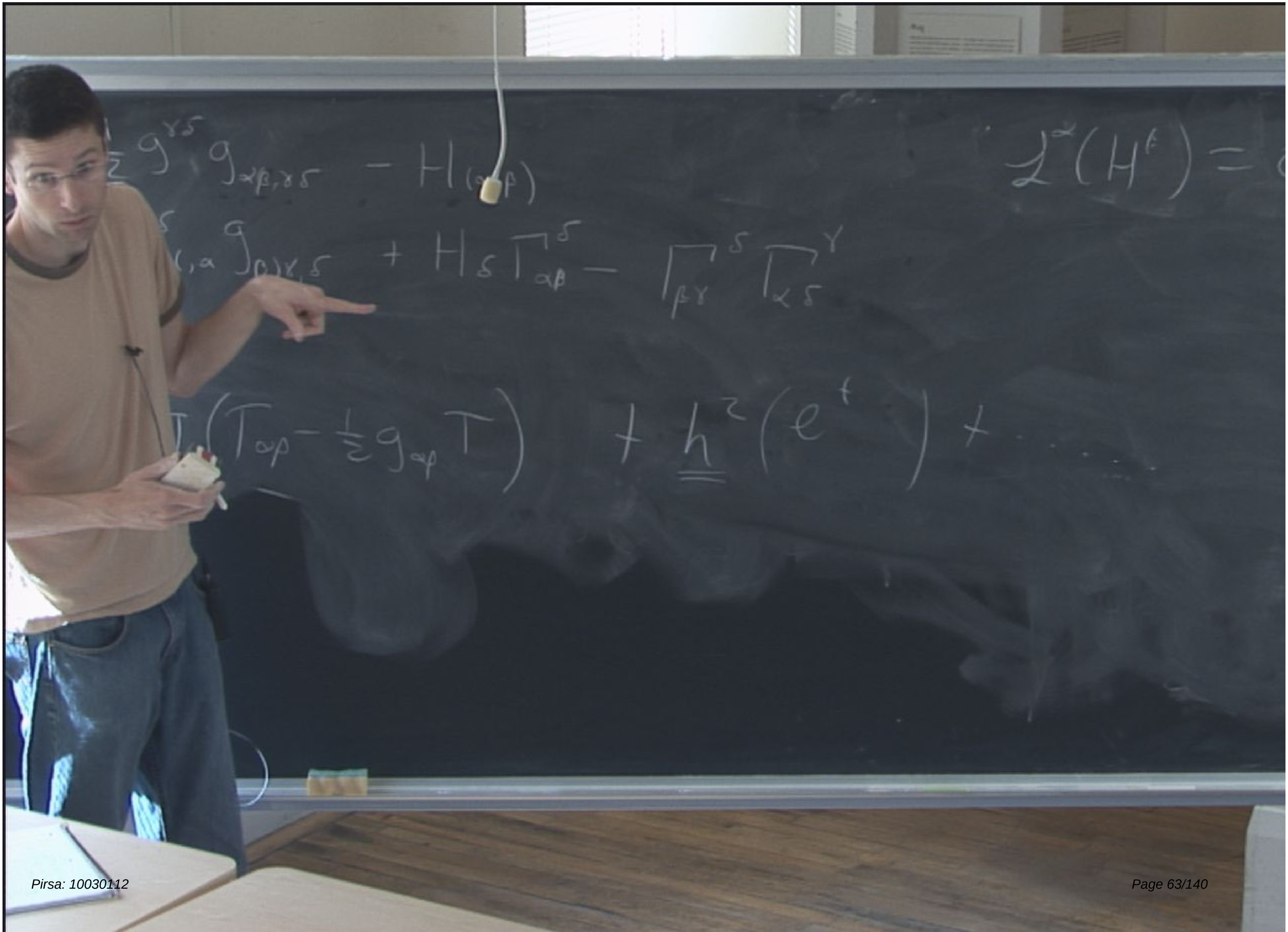
"phase space"

$\dot{C} \propto C \rightarrow$



$C^\alpha = H^\alpha - \Box x^\alpha$

and impose b.c. such that $C^\alpha = 0$ on boundary



$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$
$+ H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$
$\mathcal{L}^2(H^\epsilon) =$
$\left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T\right) + \underline{\underline{h}}^2(e^t) + \ldots$

$\alpha \quad \nabla_\beta C^\beta.$

$t = \text{const.}$

n^α unit, timelike normal

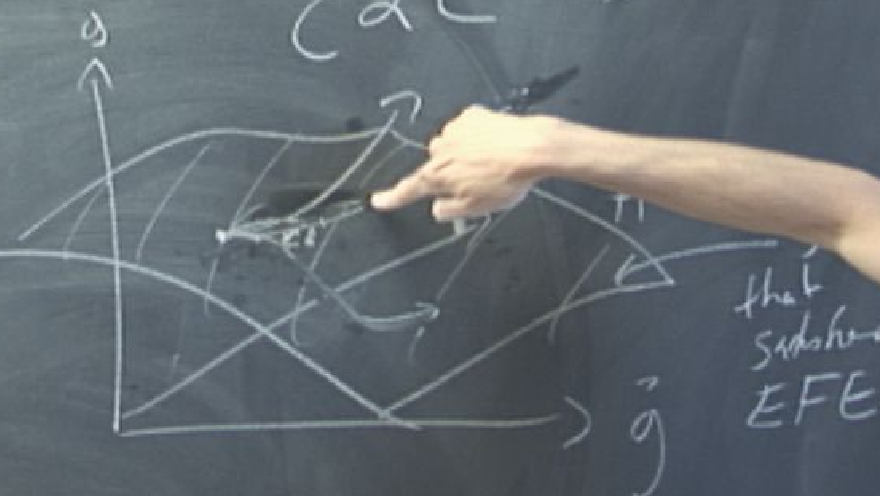
n_α vector to $t = \text{const.}$

$$\cdot\, n^\beta = M_\alpha$$

$+ \ldots$

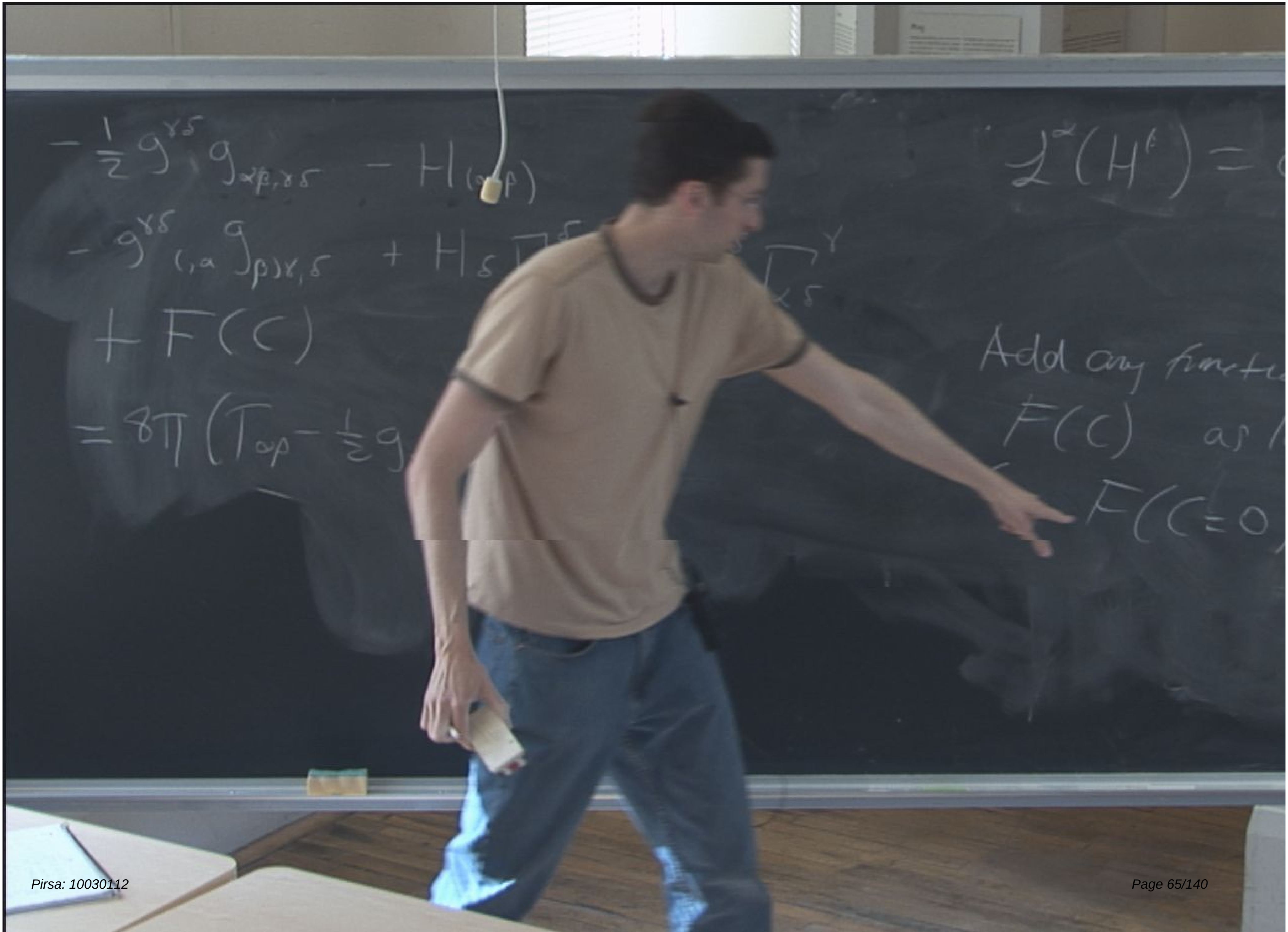
"phase space"

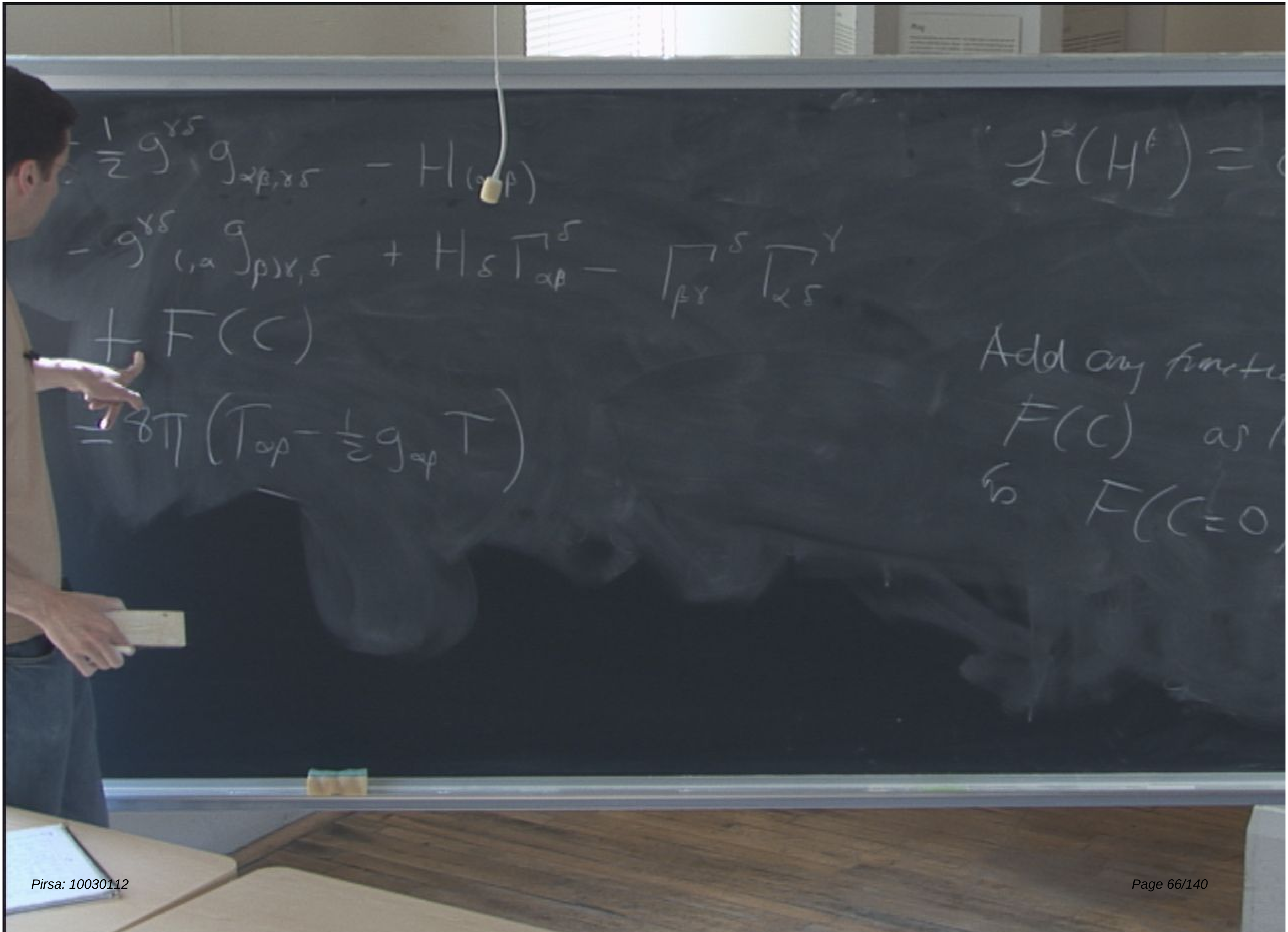
$\dot{c} \propto C \rightarrow$



$$C^\alpha = H^\alpha - \Box x^\alpha$$

and impose b.c. such that $C^\alpha = 0$ on boundary


+ F(C)
Add any
F(C)
as


$\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha\beta)}$
$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$
$+ F(C)$
$= 8\pi (T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T)$
$\mathcal{L}^\alpha (H^\epsilon) =$
Add any functi
F(C) as l
So F(C=0

$$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$$

$$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_{\delta} \Gamma^{\delta}_{\alpha\beta} - \Gamma^{\delta}_{\beta\gamma} \Gamma^{\gamma}_{\alpha\delta}$$

$$+ F(C)$$

$$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right)$$

$\mathcal{L}^{\alpha}(H^{\beta}) =$

Add any functi

$F(C)$ as l

so $F(C=0$

"phase space"

$\mathring{C} \propto C \to$

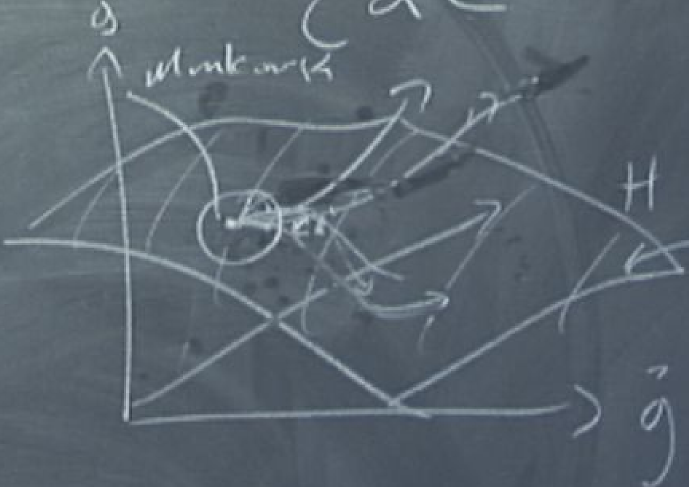
$\alpha \quad \nabla_\rho C^\beta .$

$t = \text{const.}$

n^α unit, timelike normal vector to $t = \text{const.}$

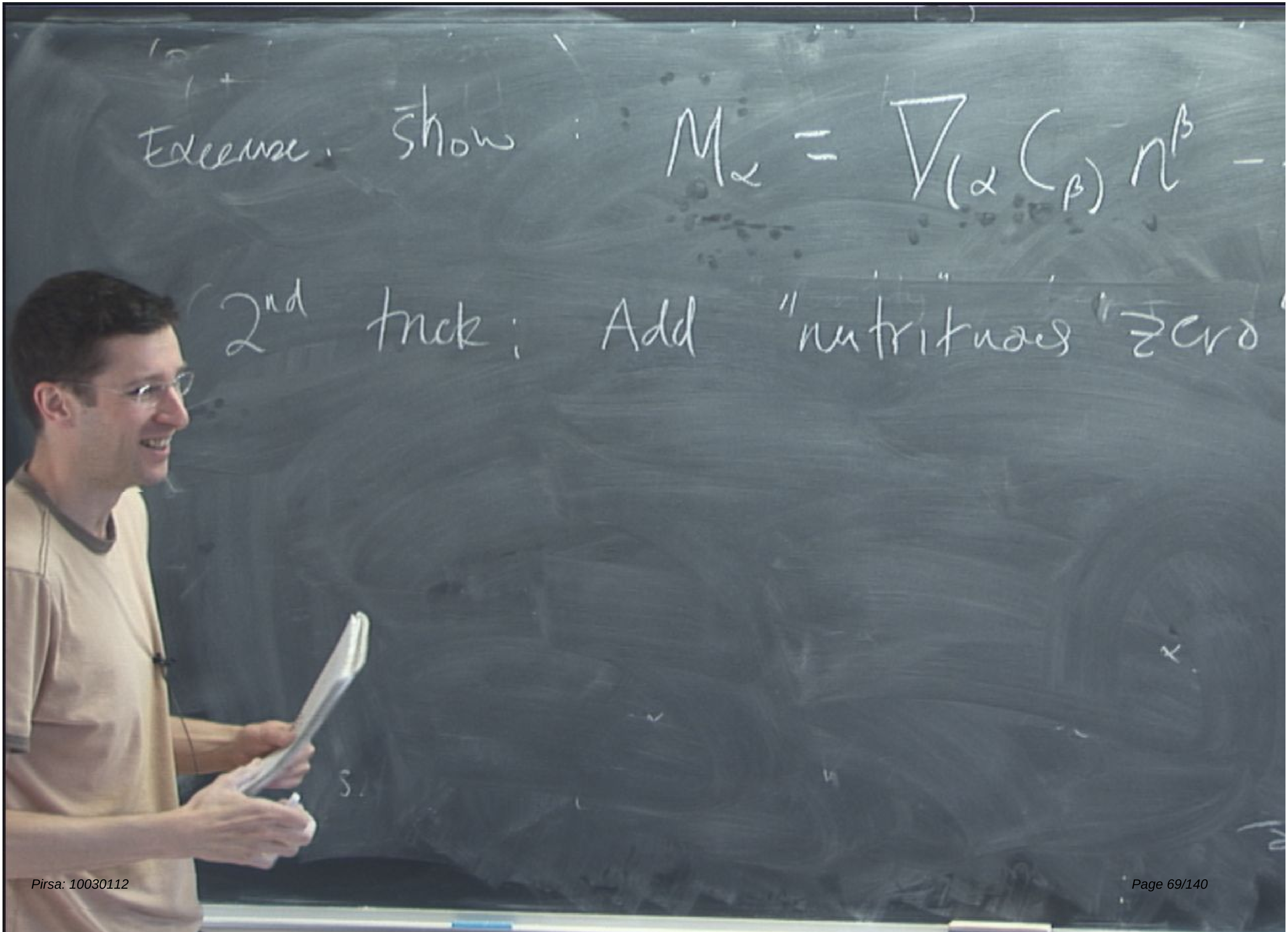
$\cdot\, n^\beta = M_\alpha$

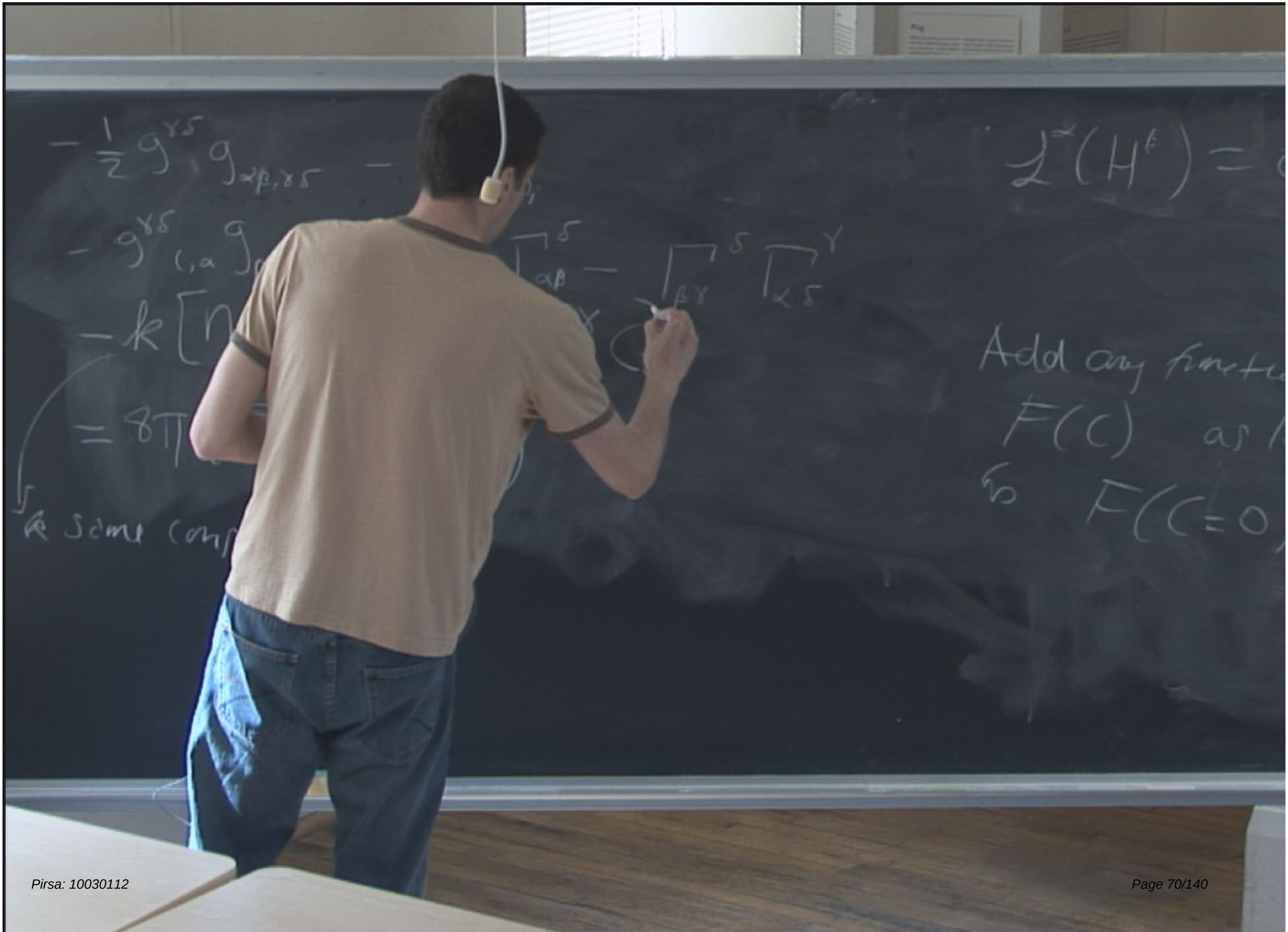
$+ \ldots$



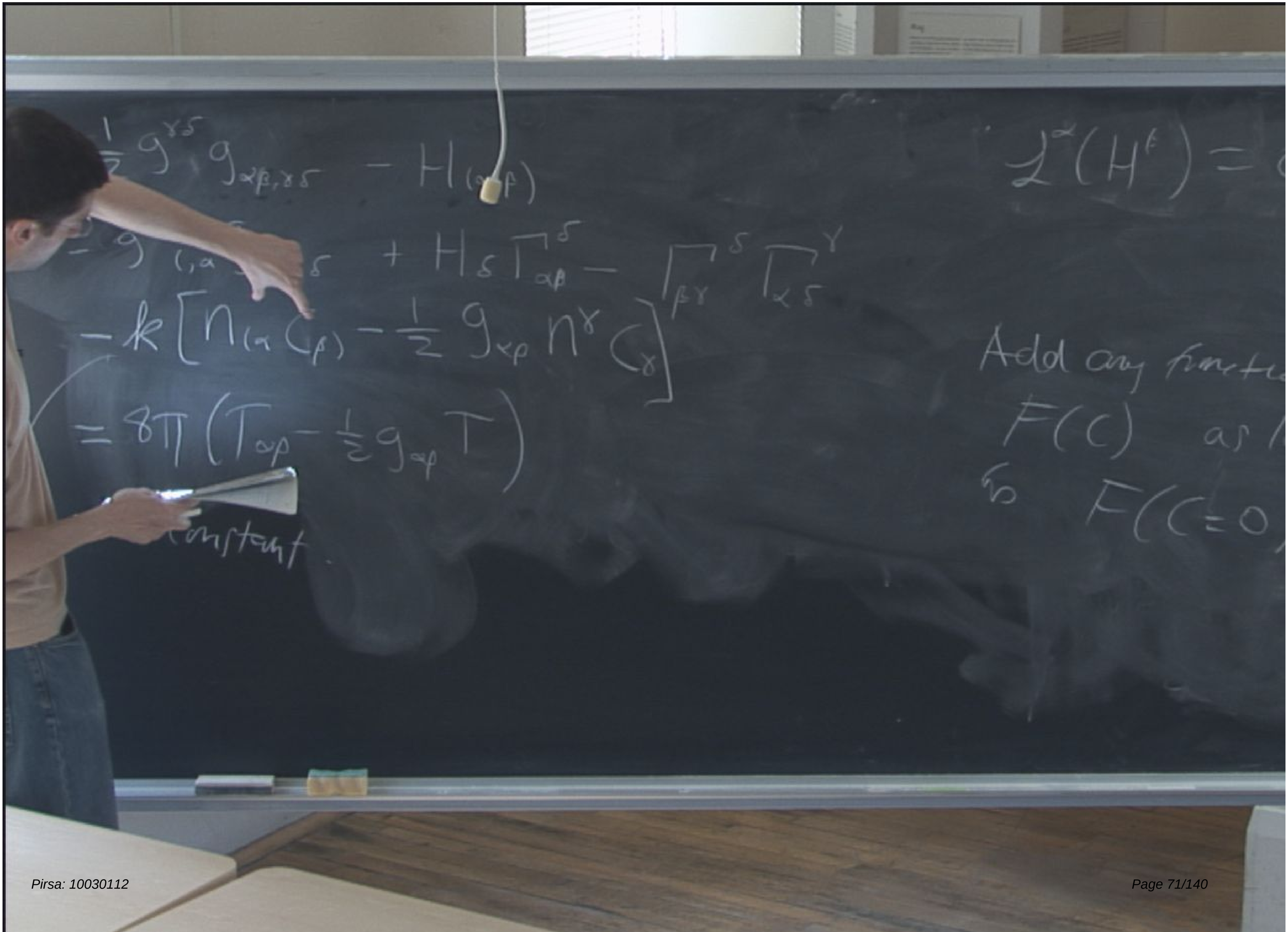
$C^\alpha = H^\alpha - \Box x^\alpha$

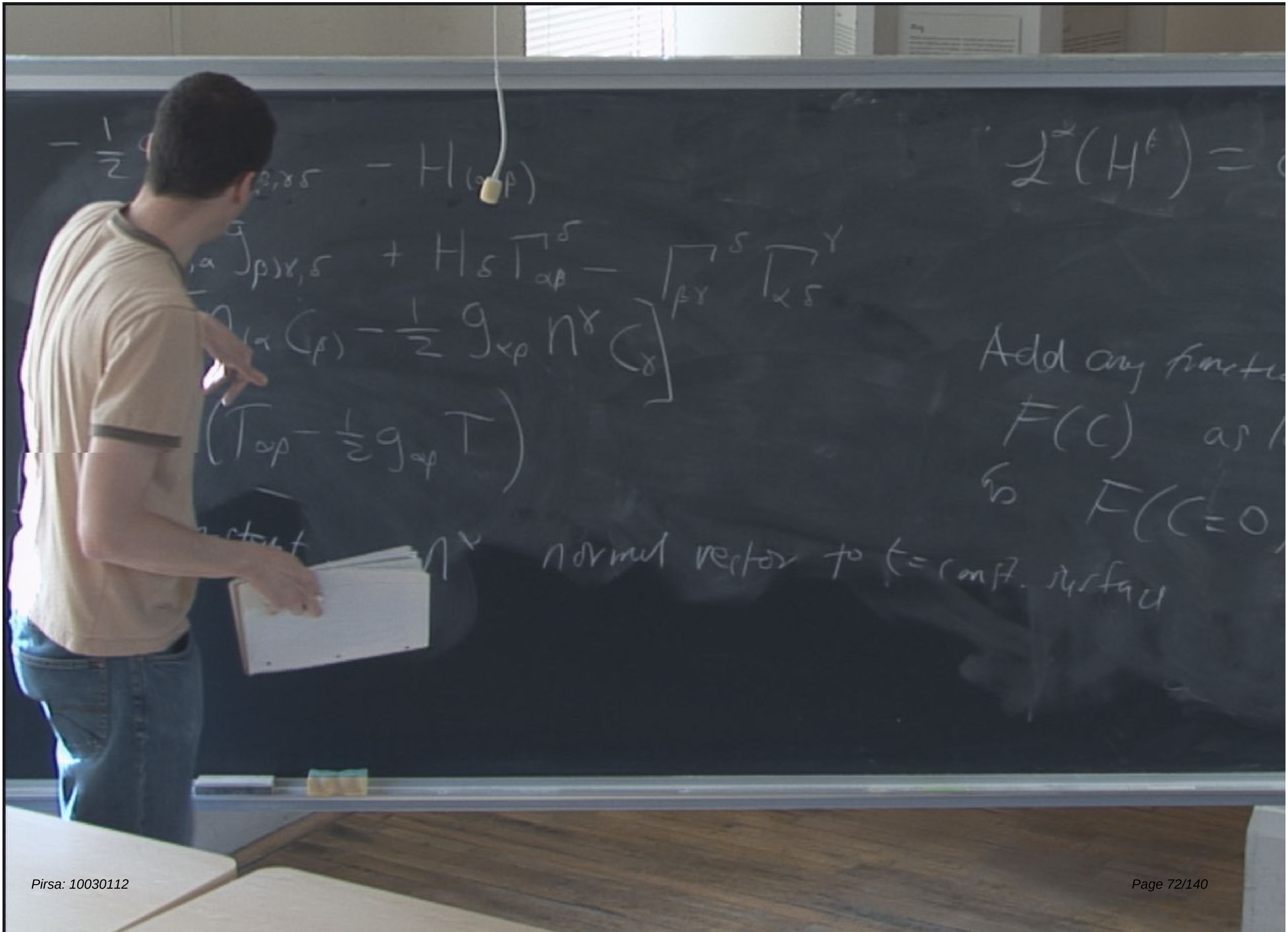
and impose b.c. such that $C^\alpha = 0$ on boundary

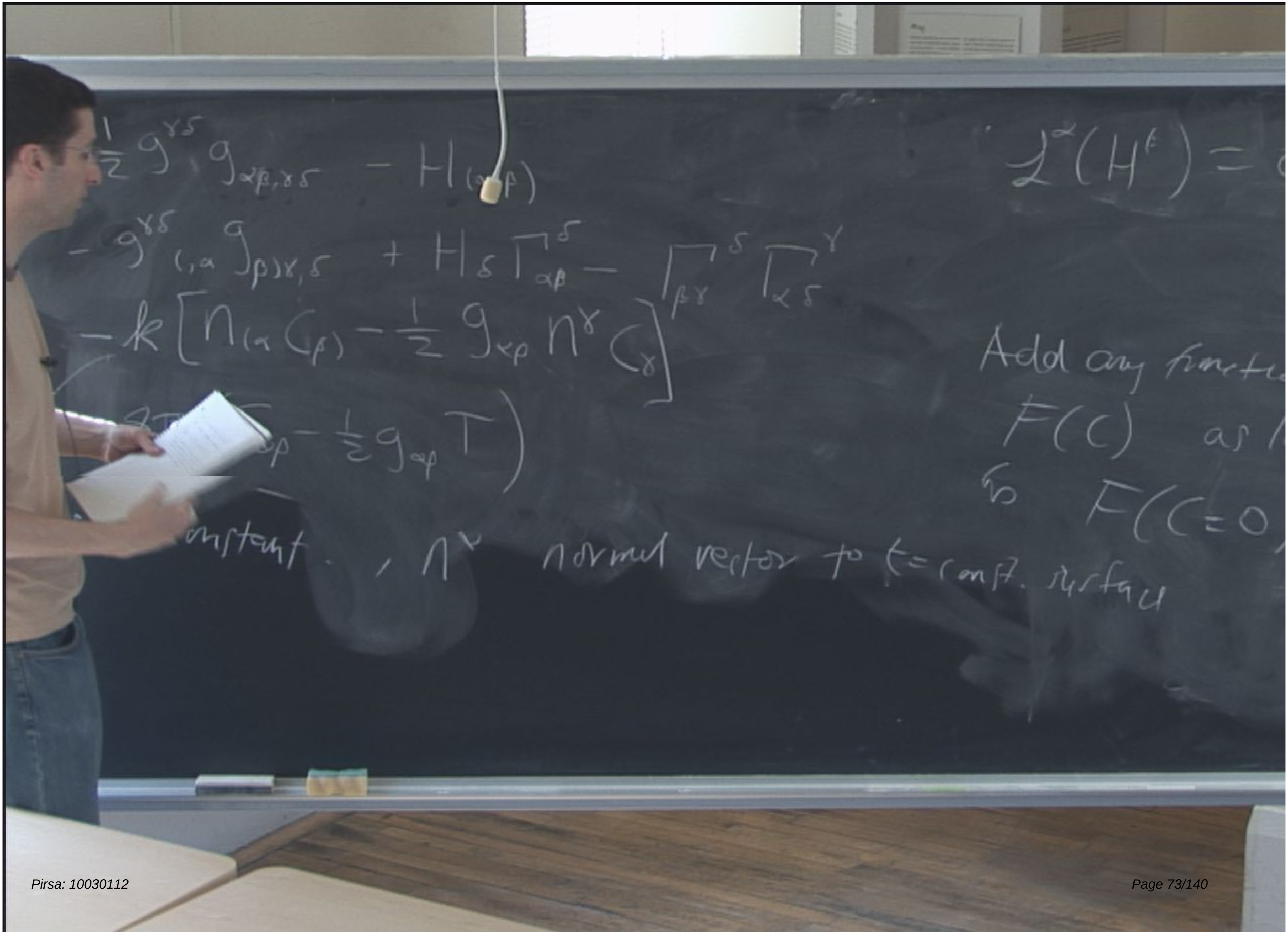

Exercise: show: $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta -$
2^{nd} trick: Add "nutritious" zero



$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta}$
Add any
F(C) as
F(C=0


$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$
$+ H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$
$-k\left[n_{(\alpha} C_{\beta)} - \frac{1}{2} g_{\alpha\beta} n^\gamma C_\gamma\right]$
$= 8\pi\left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T\right)$
constant
$\mathcal{L}^\alpha(H^\beta) =$
Add any
$F(C)$ as


Add any
normal vector to t=const. surface


$\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha\beta)}$
$- g^{\gamma\delta}{}_{(,\alpha} g_{\beta)\gamma,\delta} + H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$
$- k \left[n_{(\alpha} C_{\beta)} - \frac{1}{2} g_{\alpha\beta} n^\gamma C_\gamma \right]$
$- \frac{1}{2} g_{\alpha\beta} T \Big)$
onstant , n^γ normal vector to t = const. surface
$\mathcal{L}^\alpha (H^\epsilon) =$
Add any functi
$F(C)$ as
to $F(C=0$

$$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$$

$$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_{\delta} \Gamma^{\delta}_{\alpha\beta} - \Gamma^{\delta}_{\beta\gamma} \Gamma^{\gamma}_{\alpha\delta}$$

$$- k \left[n_{(\alpha} C_{\beta)} - \frac{1}{2} g_{\alpha\beta} n^{\gamma} C_{\gamma} \right]$$

$$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right)$$

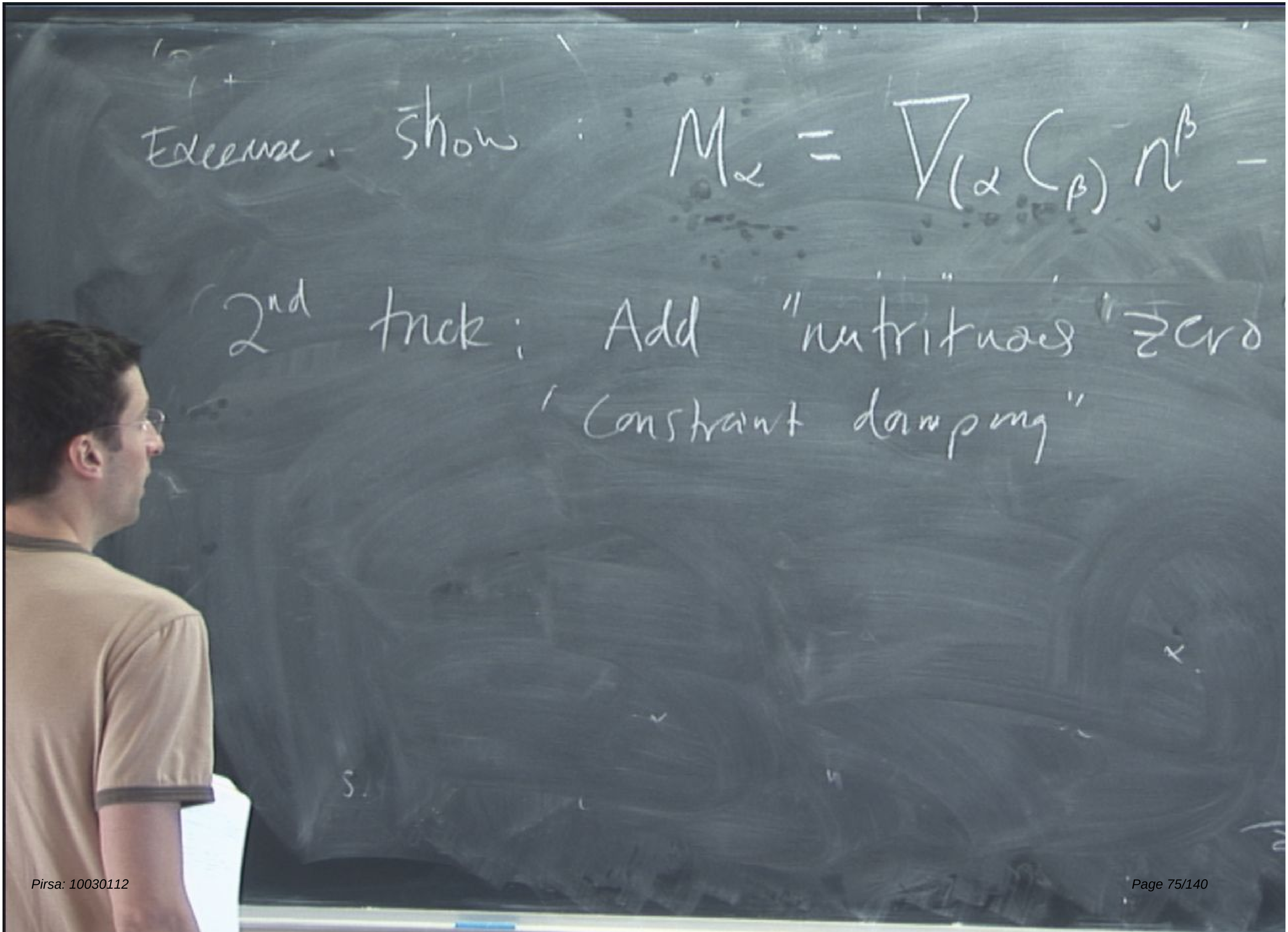
k some constant, n^{γ} normal vector to $t = \text{const.}$ surface

$\mathcal{L}^{\alpha}(H^{\beta}) =$

Add any functi

$F(C)$ as

to $F(C=0$


Exercise: show : $M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta -$
2nd trick: Add "nutritious" zero
"Constraint damping"

$$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$$

$t = \text{const.}$

Add "nutritious" "zero" $\quad n^\alpha$ unit, timelike

$\quad \perp$ vector to $t=$

onstraint damping"

$$\Box C^\alpha = - R^\alpha{}_\beta C^\beta + 2k \nabla_\beta \left[n^{(\beta} C^{\alpha)} \right]$$

$$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$$

$t = $ const.

Add "nutritious "zero"" ... "constraint damping"

n^α unit, timelike vector to $t =$

$$\Box C^\alpha = - R^\alpha{}_\beta C^\beta + 2k \nabla_\beta \left[n^{(\beta} C^{\alpha)} \right]$$

$$\Box f =$$

$$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$$

$t = \text{const}$.

Add "nutritious "zero" n^α unit, timelike

constraint damping" vector to $t =$

$$\Box C^\alpha = -R^\alpha{}_\beta C^\beta + 2k \nabla_\beta \left[n^{(\beta} C^{\alpha)} \right]$$

$$\Box f = k \nabla_\beta f \cdot n^\beta \ldots$$

∂c

$$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$$

$t = \text{const}$

Add "nutritious "zero" n^α unit, timelike

"constraint damping" n_α vector to $t =$

$$\Box C^\alpha = -R^\alpha{}_\beta C^\beta + 2k \nabla_\beta \left[n^{(\beta} C^{\alpha)} \right]$$

$\Box f = k \nabla_\beta \ldots^\beta$ damped wave eqn

$$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$$

$t = \text{const.}$

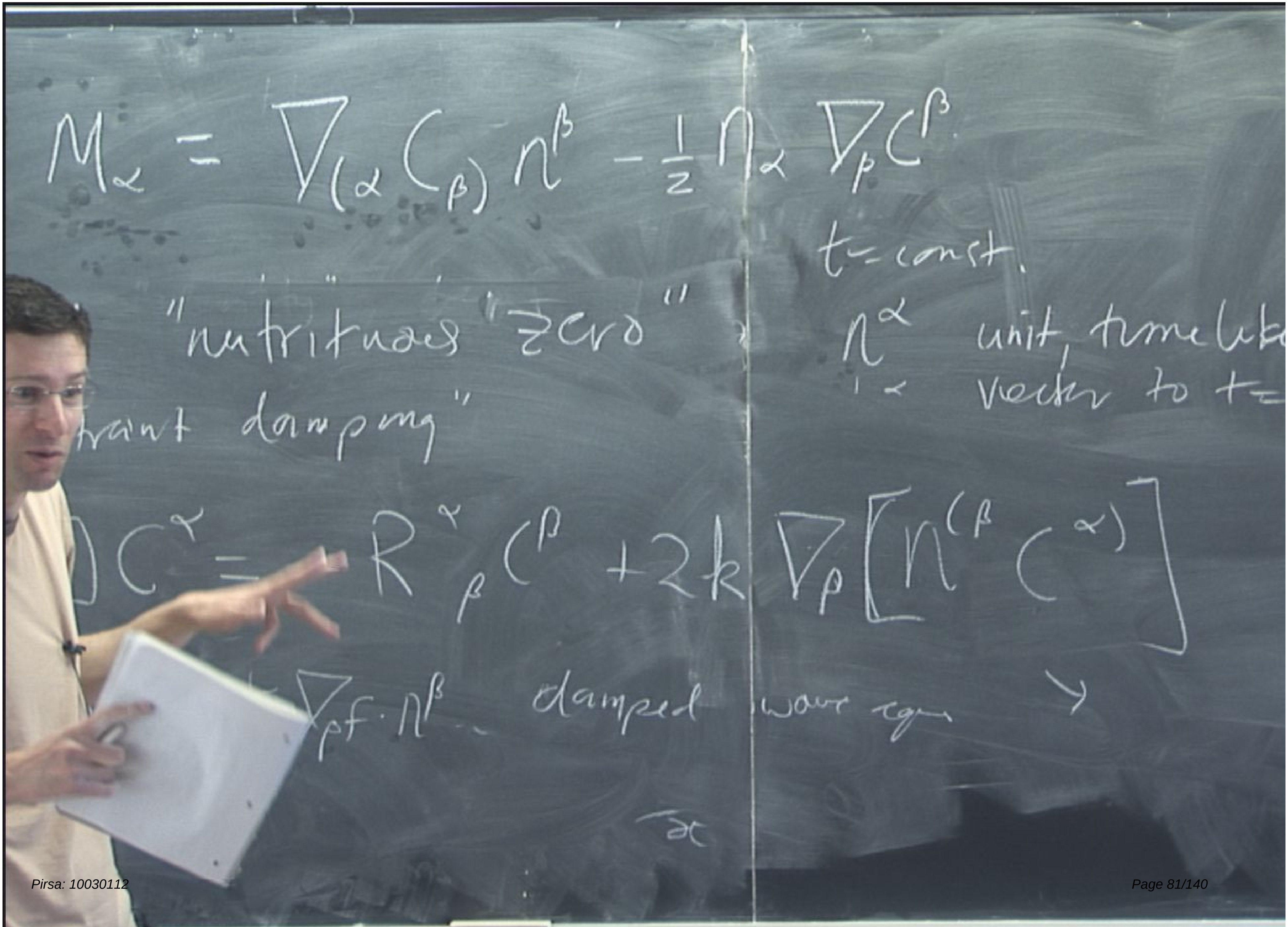
Add "nutritiuous "zero" n^α unit, timelike

onstraint damping" n_α vector to $t =$

$$\Box C^\alpha = -R^\alpha{}_\beta C^\beta + 2k \nabla_\beta \left[n^{(\beta} C^{\alpha)} \right]$$

$\Box f = k \nabla_\beta f \cdot n^\beta$ damped wave eqn

∂c


$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$
t = const.
"nutritious" "zero"
n^α unit, timelik
vector to t=
traint damping"
$\Box C^\alpha = R^\alpha{}_\beta C^\beta + 2k \nabla_\beta [n^{(\beta} C^{\alpha)}]$
$\nabla_\beta f \cdot n^\beta$
damped wave eqn

$$M_\alpha = \nabla_{(\alpha} C_{\beta)} n^\beta - \frac{1}{2} n_\alpha \nabla_\beta C^\beta$$

$t = \text{const.}$

Add "nutritious "zero""

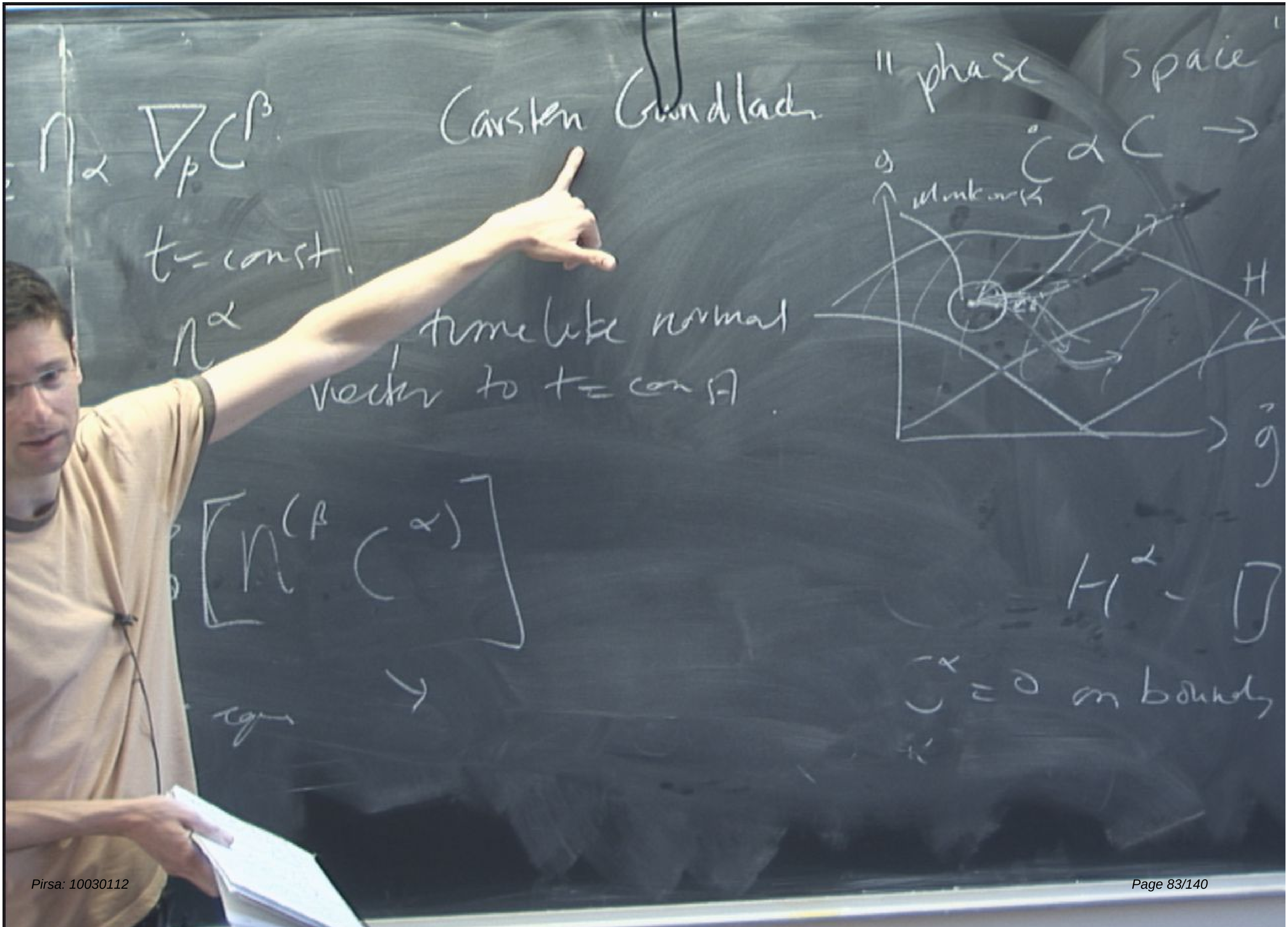
n^α unit, timelike vector to $t =$

onstraint damping"

$$\Box C^\alpha = - R^\alpha{}_\beta C^\beta + 2k \nabla_\beta \left[n^{(\beta} C^{\alpha)} \right]$$

$$\Box f = k \nabla_\beta f \cdot n^\beta$$

damped wave eqn


Carsten Gundlach
"phase space"
t = const.
timelike normal
vector to t = const.
Minkowski
H
= 0 on boundary

$n_\alpha \nabla_p C^\beta$

$t = \text{const}$.

n^α unit, timelike normal vector to $t = \text{const}$

n_α

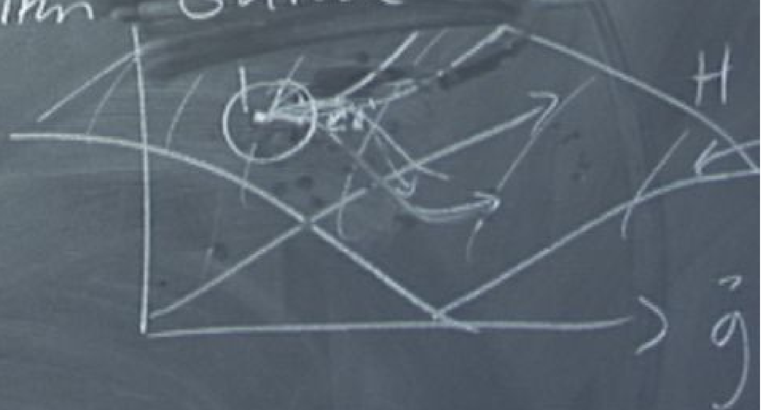
Carsten Gundlach

Ian Hinder

Jose Maria Martin Garcia

"phase space"

$C \rightarrow$



$\nabla_\rho \left[n^{(\beta} C^{\alpha)} \right]$

$\rightarrow$

wave eqn

$H^\alpha \sim \square$

$C^\alpha = 0$ on boundary

$$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha\beta)}$$

$$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$$

$$- k \left[n_{(\alpha} C_{\beta)} - \frac{1}{2} g_{\alpha\beta} n^\gamma C_\gamma \right]$$

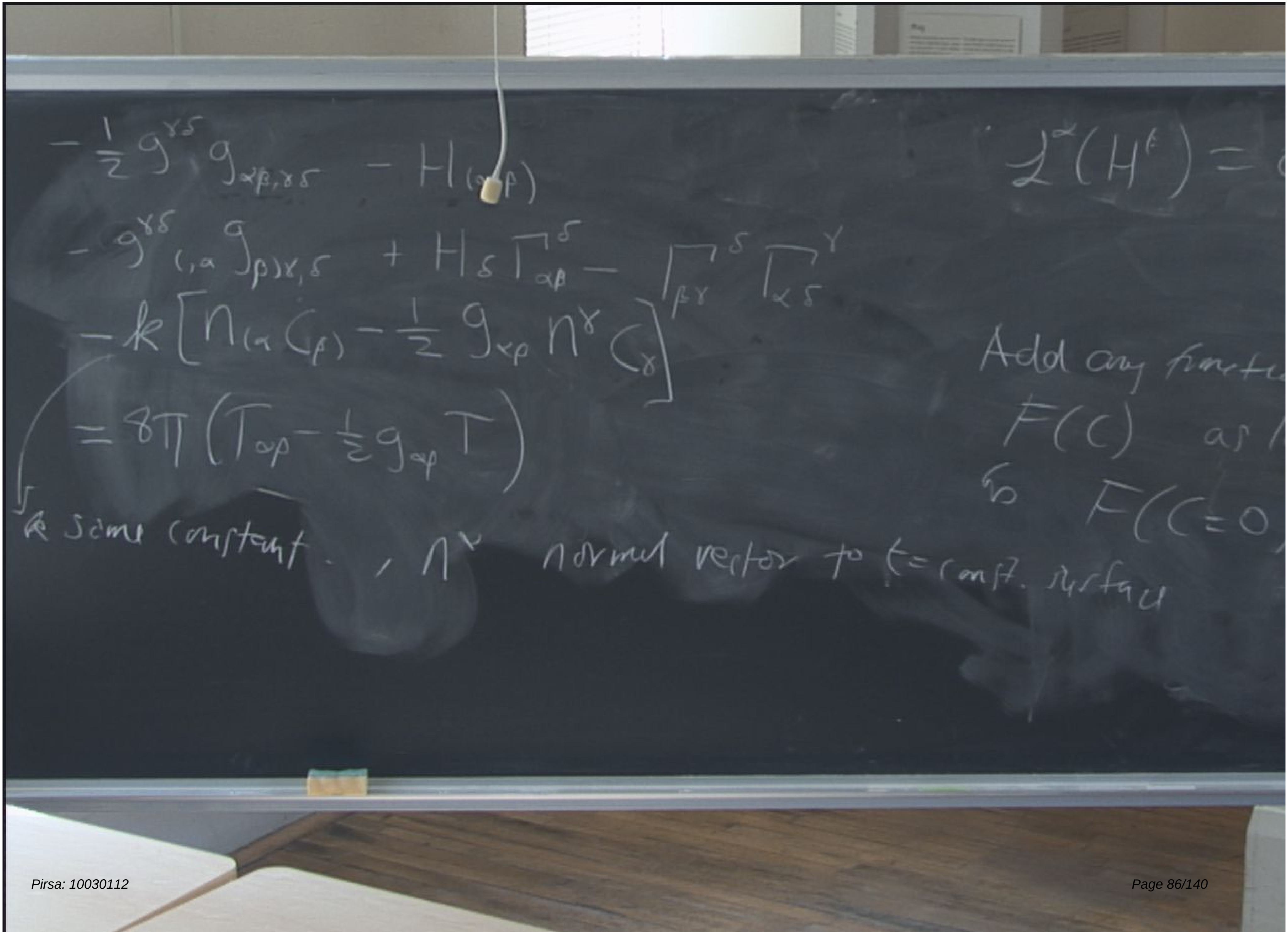
$$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right)$$

k some constant, n^γ normal vector to $t=$ const. surface

$$\mathcal{L}^\alpha (H^\epsilon) = \ldots$$

Add any function $F(C)$ as ...

so $F(C=0)$ ...


$-\frac{1}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} - H_{(\alpha,\beta)}$
$- g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\gamma,\delta} + H_\delta \Gamma^\delta_{\alpha\beta} - \Gamma^\delta_{\beta\gamma} \Gamma^\gamma_{\alpha\delta}$
$- k \left[n_{(\alpha} C_{\beta)} - \frac{1}{2} g_{\alpha\beta} n^\gamma C_\gamma \right]$
$= 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right)$
k some constant, n^γ normal vector to t = const. surface
$\mathcal{L}^\alpha(H^\epsilon) =$
Add any
$F(C)$ as
so $F(C=0$

linearised Einstein equations.   GW in TT

W $_{\mu\nu}$ TT (Transverse, traceless).

Linearised Einstein equations. GW

Assume: $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$

Einstein equations. GW in TT (Transvers

$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$

$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$

$h_{\alpha\beta}$

l Einstein equations. GW in TT (Trans

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & \\ & 1 & \\ 0 & & 1 \end{pmatrix}$$

$$|h_{\alpha\beta}| \ll 1$$

l Einstein equations. GW in TT (Trans

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & \\ & 1 & \\ 0 & & 1 \end{pmatrix}$$

$$|h_{\alpha\beta}| \ll 1$$

vacuum $T_{\alpha\beta} = 0$

Harmonic gauge $H_\alpha = 0$

$R = 0$

...l Einstein equations. GW in TT (Trans...

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & \\ & 1 & \\ 0 & & 1 \end{pmatrix}$$

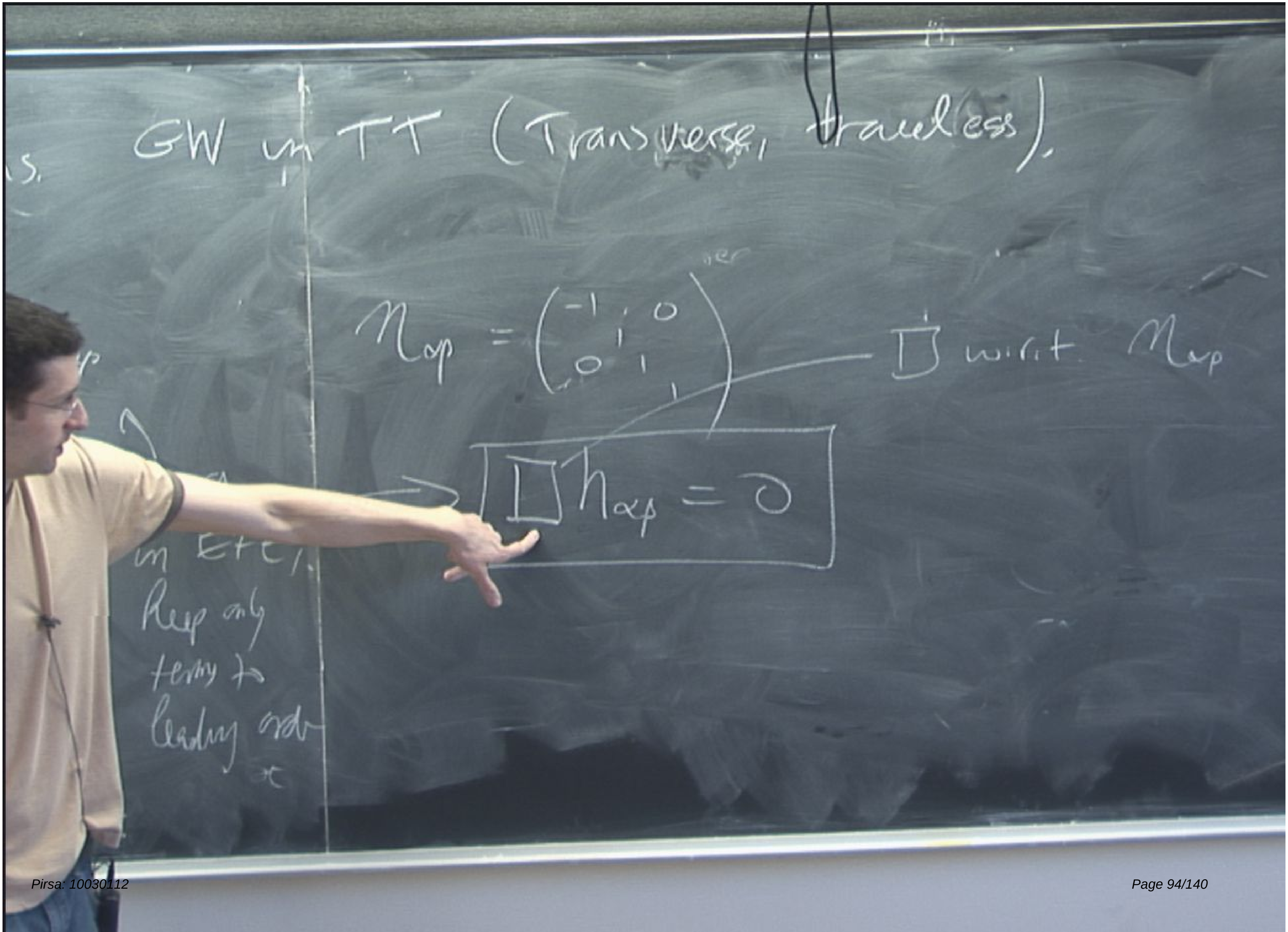
$$|h_{\alpha\beta}| \ll 1$$

Sub $g_{\alpha\beta}$ in EFE, keep only terms to leading order

vacuum $T_{\alpha\beta} = 0$

Harmonic gauge $H_\alpha = 0$

$R = 0$


GW in TT (Transverse, traceless).
η_αβ =
□ w.r.t. η_αβ
□ h_αβ = 0
in EFE.
keep only
terms to
leading order

s. GW in TT (Transverse, traceless).

$h_{\alpha\beta}$

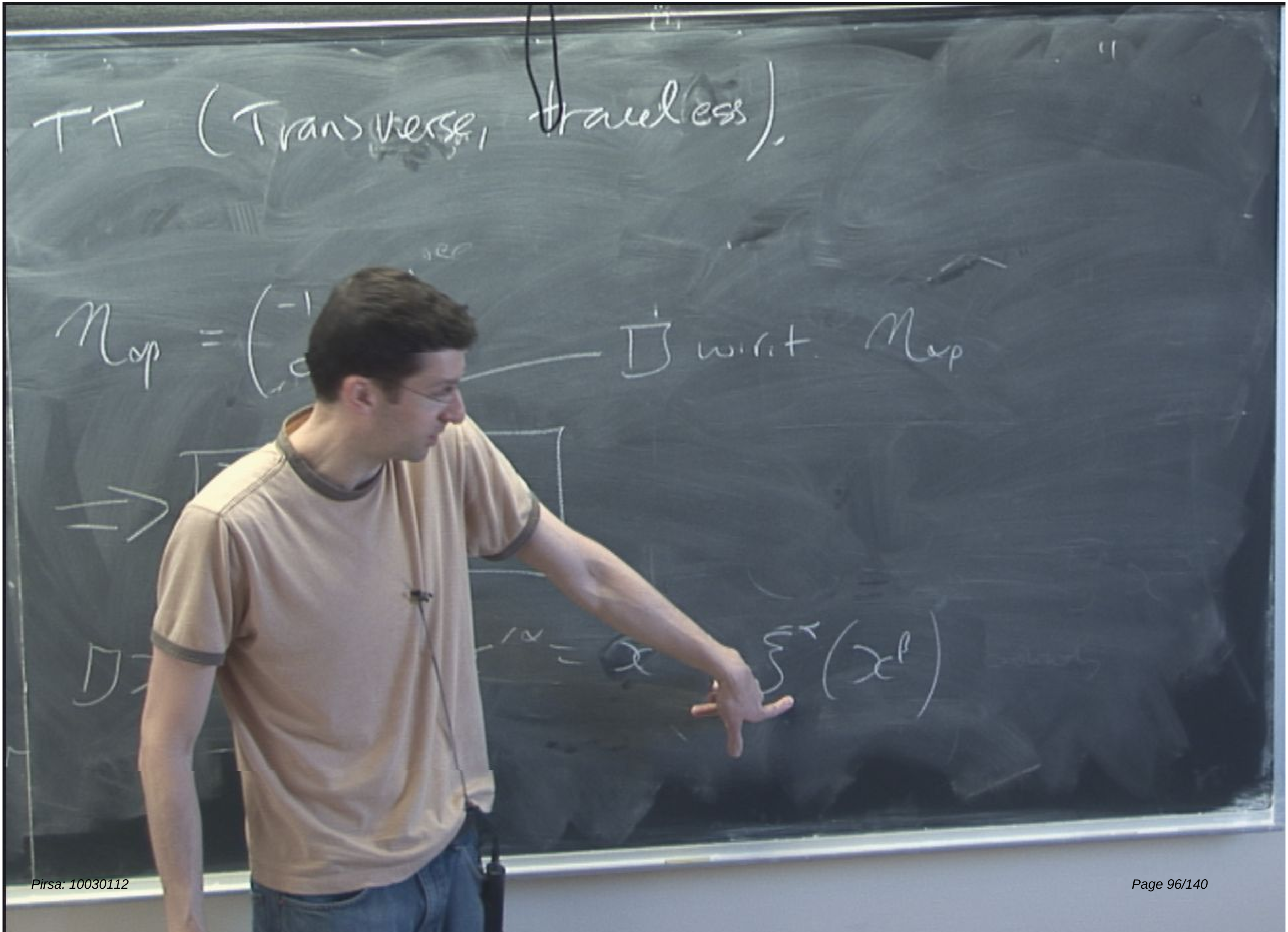
Sub $g_{\alpha\beta}$ in EFE, keep only terms to leading order

$= 0$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

$\square$ w.r.t. $\eta_{\alpha\beta}$

$$\Rightarrow \boxed{\square h_{\alpha\beta} = 0}$$


TT (Transverse, traceless).
η_{op} =
□ w.r.t. η_{op}
$\xi^\alpha(x^\beta)$

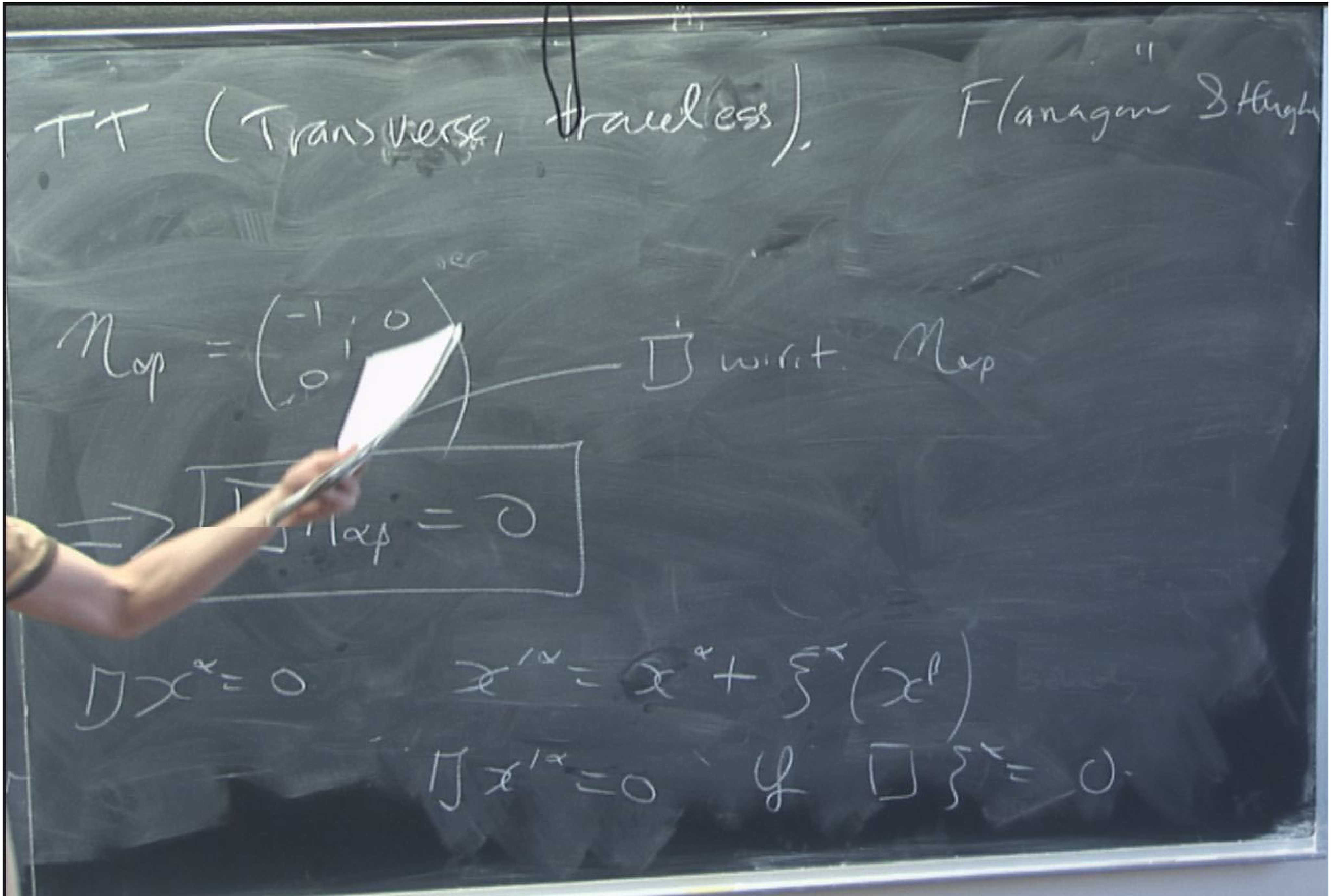
TT (Transverse, traceless).

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} \qquad \Box \text{ w.r.t. } \eta_{\alpha\beta}$$

$$\Rightarrow \boxed{\Box h_{\alpha\beta} = 0}$$

$$\Box x^\alpha = 0 \qquad x'^\alpha = x^\alpha + \xi^\alpha(x^\beta)$$

$$\Box x'^\alpha = 0 \quad \text{if} \quad \Box \xi^\alpha = 0.$$


TT (Transverse, traceless).
Flanagan & Hughes
$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
$\Box$ w.r.t. $\eta_{\alpha\beta}$
$\Rightarrow \boxed{\Box h_{\alpha\beta} = 0}$
$\Box x^\alpha = 0$
$x'^\alpha = x^\alpha + \xi^\alpha(x^\beta)$
$\Box x'^\alpha = 0$ if $\Box \xi^\alpha = 0$.

TT (Transverse, traceless). Flanagan & Hughes

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} \quad \square \text{ w.r.t. } \eta_{\alpha\beta}$$

$$\Rightarrow \boxed{\square h_{\alpha\beta} = 0} \rightarrow \text{subject to } \square x^\alpha = 0$$

$$\square x^\alpha = 0 \qquad x'^\alpha = x^\alpha + \xi^\alpha(x^\beta)$$

$$\square x'^\alpha = 0 \quad \text{if} \quad \square \xi^\alpha = 0.$$

Linearised Einstein equations. GW in TT

$\frac{1}{2}$ more "gauge choices" let:

Choose $h_{\mu\nu}$ is traceless $h^\alpha{}_\alpha = 0$

$\eta_{\alpha\beta}$

$\Rightarrow$

$\Box$

Linearised Einstein equations. GW in TT

4 more "gauge choices" left:

1) Choose $h_{\mu\nu}$ is traceless $h^{\alpha}{}_{\alpha} = 0$

2) Cho[illegible] $h^{i0} = 0$ "t" $= 0$

x^i, $i \in 1,2,3$.

$\eta_{\alpha\beta}$

$\Rightarrow$

$\square x$

Linearised Einstein equations.

GW in TT

4 more "gauge choices", let:

1) Choose $h_{\mu\nu}$ is traceless $h^{\alpha}{}_{\alpha} = 0$

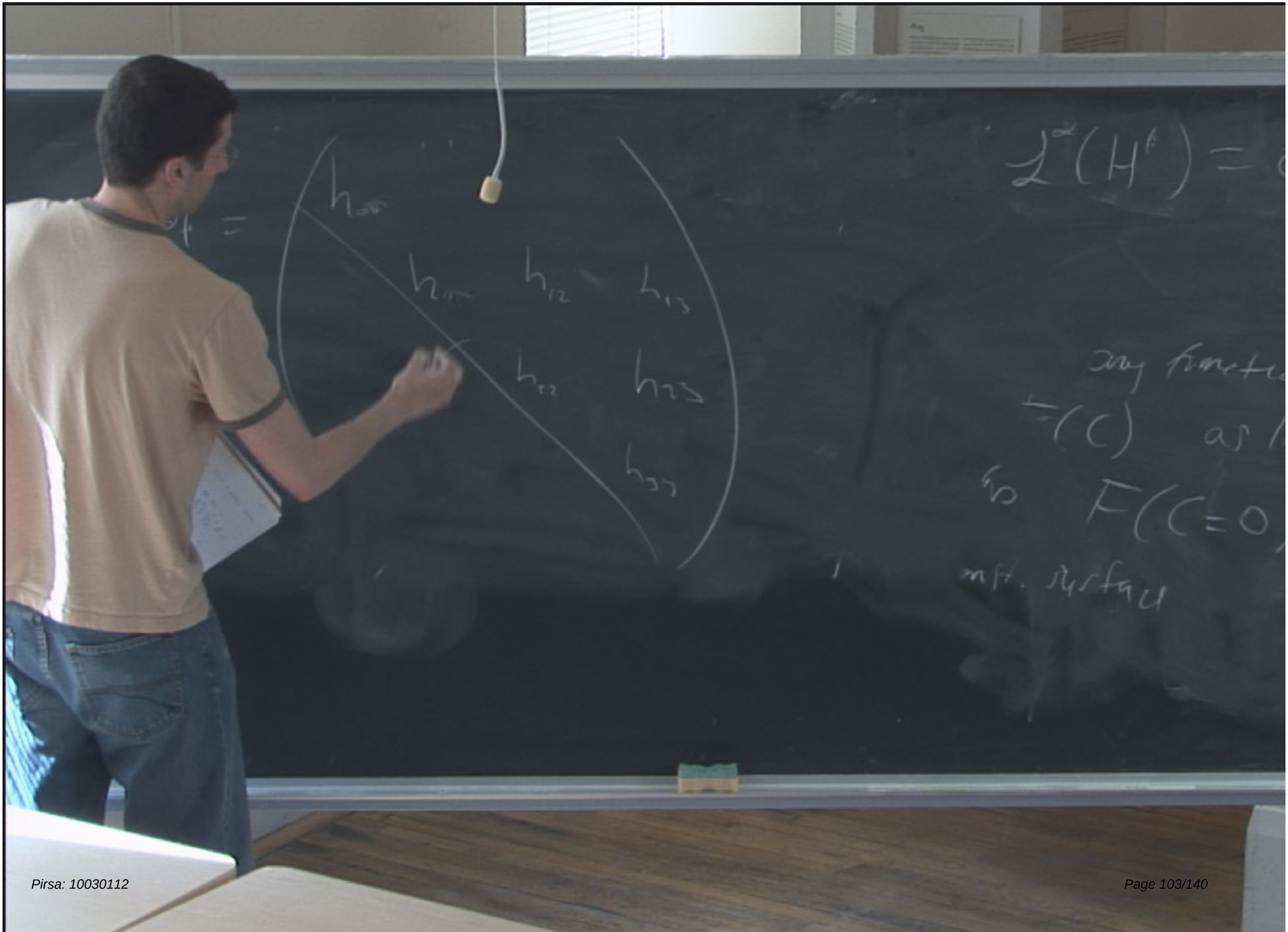
2) Choose $h^{i0} = 0$ "t" $= 0$
x^i, $i \in 1,2,3$.

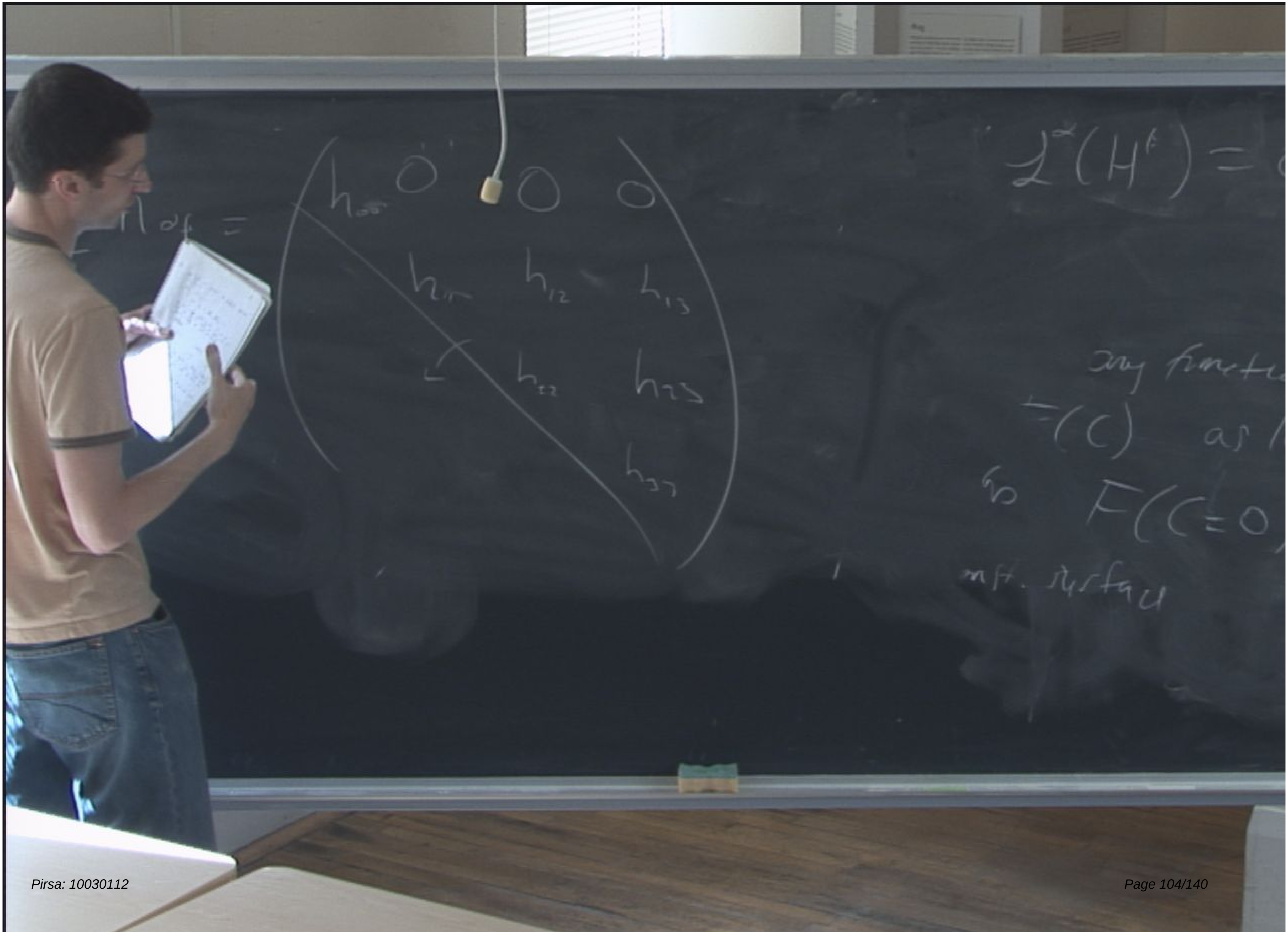
$\eta_{\alpha\beta}$

$\Rightarrow$

$\square x$

s.


h₀₀
h₁₁ h₁₂ h₁₃
h₂₂ h₂₃
h₃₃
any functi
(C) as
F(C=0
inf. surface


h00 0 0 0
h11 h12 h13
h22 h23
h33
F(C=0

Linearised Einstein equations. GW

4 more "gauge choices", let:

(1) Choose $h_{\alpha\beta}$ is traceless $h^{\alpha}{}_{\alpha} = 0$

(2) choose $h^{i0} = 0$ "t" $\Leftrightarrow$ 0

x^i, $i \in 1, 2,$

Linearised Einstein equations. GW

4 more "gauge choices" let:

① Choose $h_{\mu\nu}$ is traceless $h^{\alpha}{}_{\alpha} = 0$

② choose $h^{i0} = 0$ "t" $= 0$

x^i, $i \in 1$

$\downarrow$ + harmonic $\Rightarrow$

Linearised Einstein equations. GW

4 more "gauge choices", let:

① Choose $h_{\alpha\beta}$ is traceless $h^{\alpha}{}_{\alpha} = 0$

② Choose $h^{i0} = 0$ "t" $= 0$

$x^i,\ i \in 1, 2,$

$\downarrow$ + harmonic $\Rightarrow -\partial_t h^{00} + \partial_{x^i} \cancel{h}^{i0} =$

$h^{00} = \text{const} \Rightarrow$ choose

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ & h_{11} & h_{12} & h_{13} \\ & & h_{22} & h_{23} \\ & & & h_{33} \end{pmatrix} + \Box x = 0$$

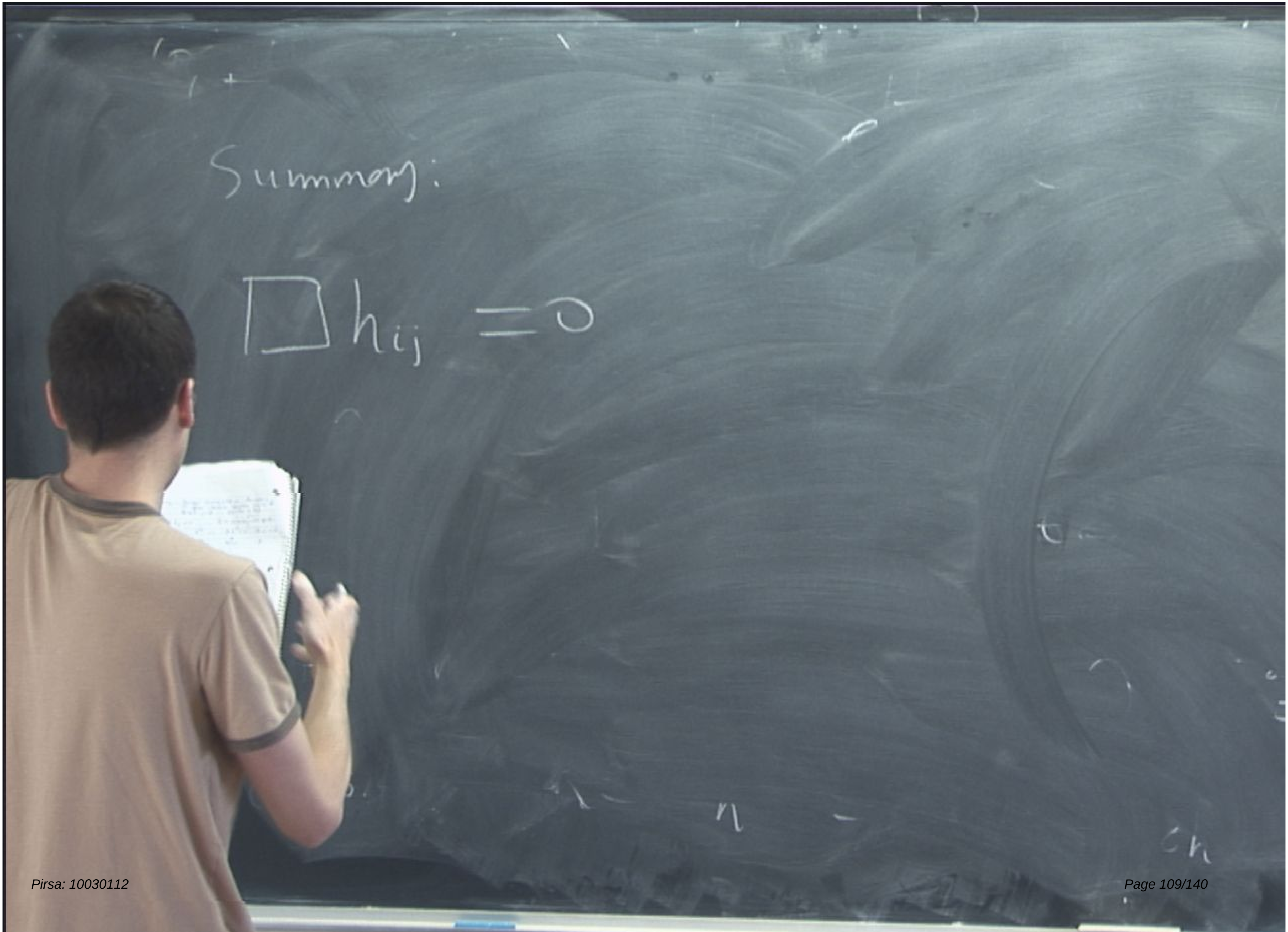
$$\mathcal{L}^2(H^t) =$$

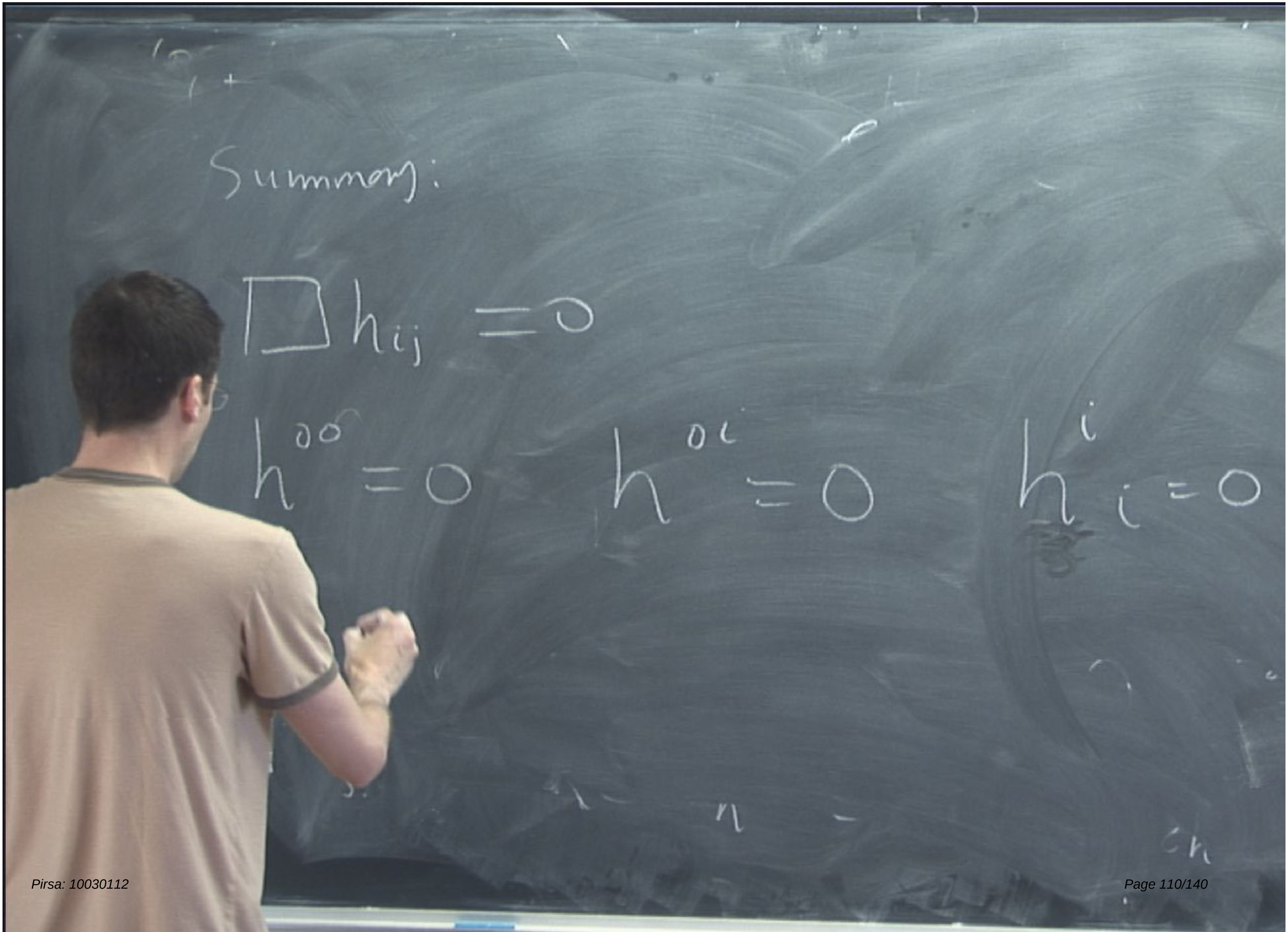
any functi

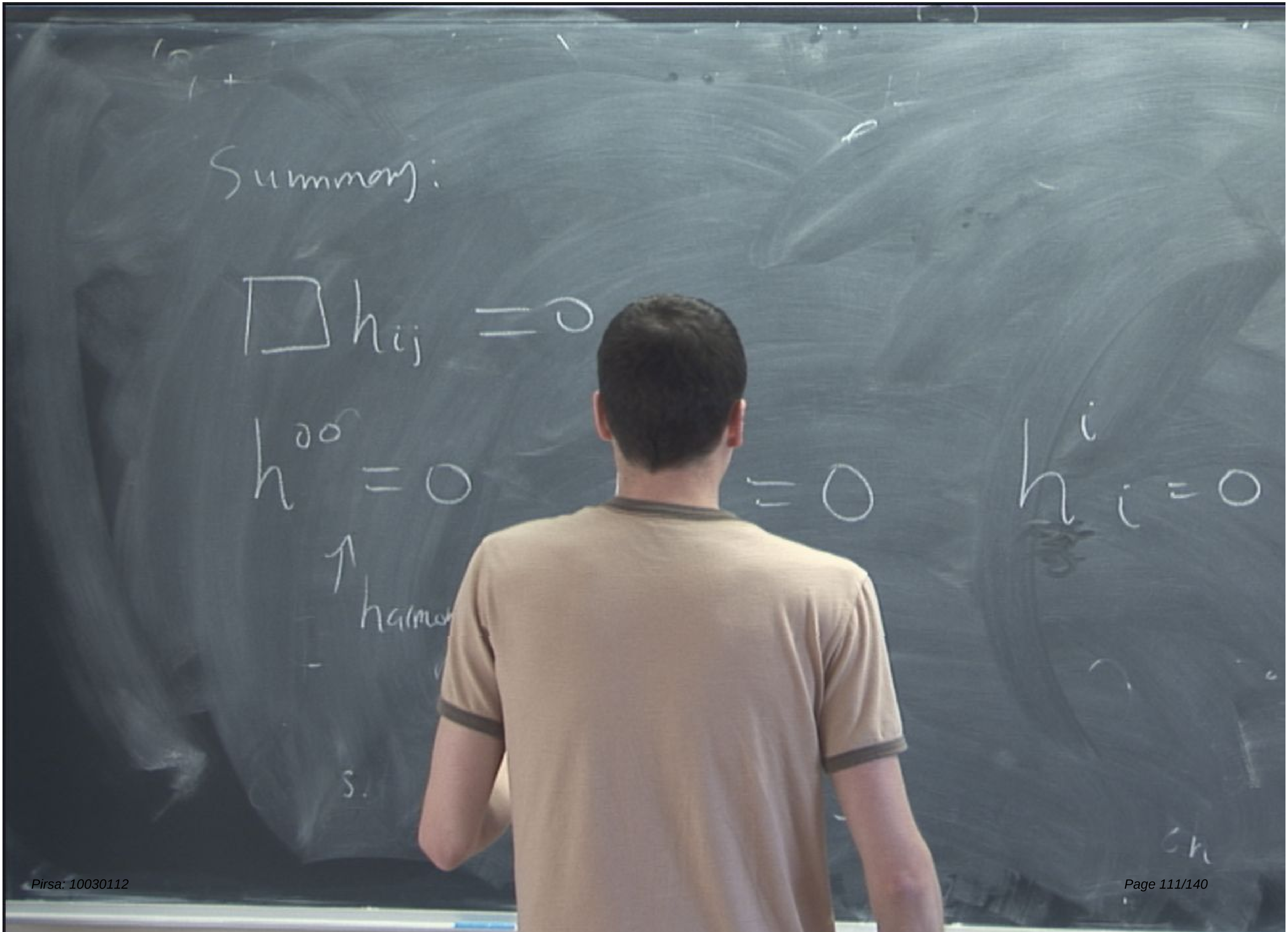
$= (C)$ as

so $F(C = 0$

mft. surface


Summary:
$\Box h_{ij} = 0$


Summary:
$\Box h_{ij} = 0$
$h^{00} = 0$
$h^{0i} = 0$
$h^{i}_{i} = 0$


Summary:
$\Box h_{ij} = 0$
$h^{00} = 0$
$= 0$
$h^{i}{}_{i} = 0$
harmon

Summary:

$$\Box h_{ij} = 0$$

$$h^{00} = 0 \qquad h^{0i} = 0 \qquad h^i{}_i = 0$$

harmonic + TT

TT

Summary:

EFE: $\Box h_{ij} = 0$

$h^{00} = 0$ $\quad h^{0i} = 0$ $\quad h^{i}{}_{i} = 0$

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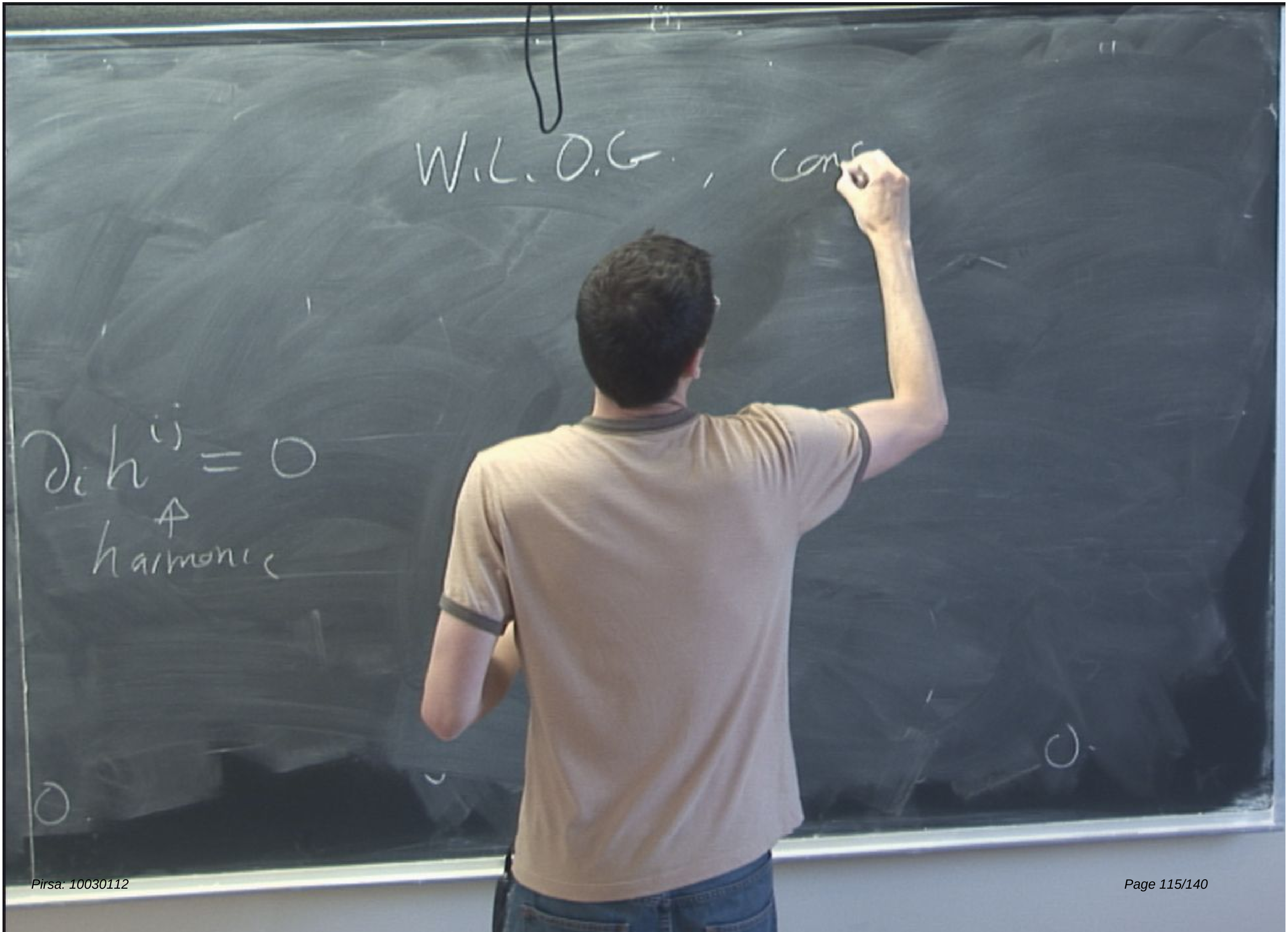
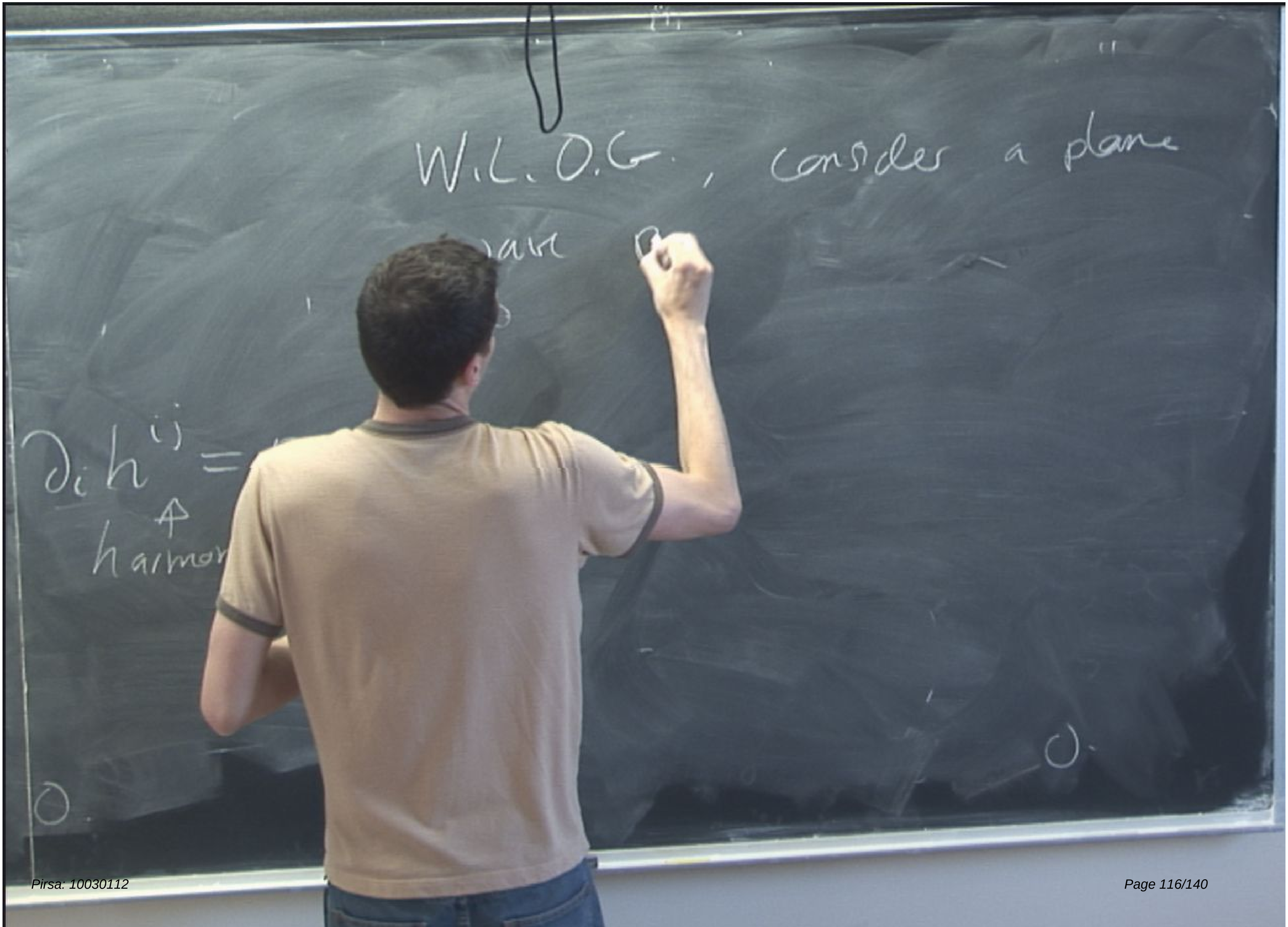
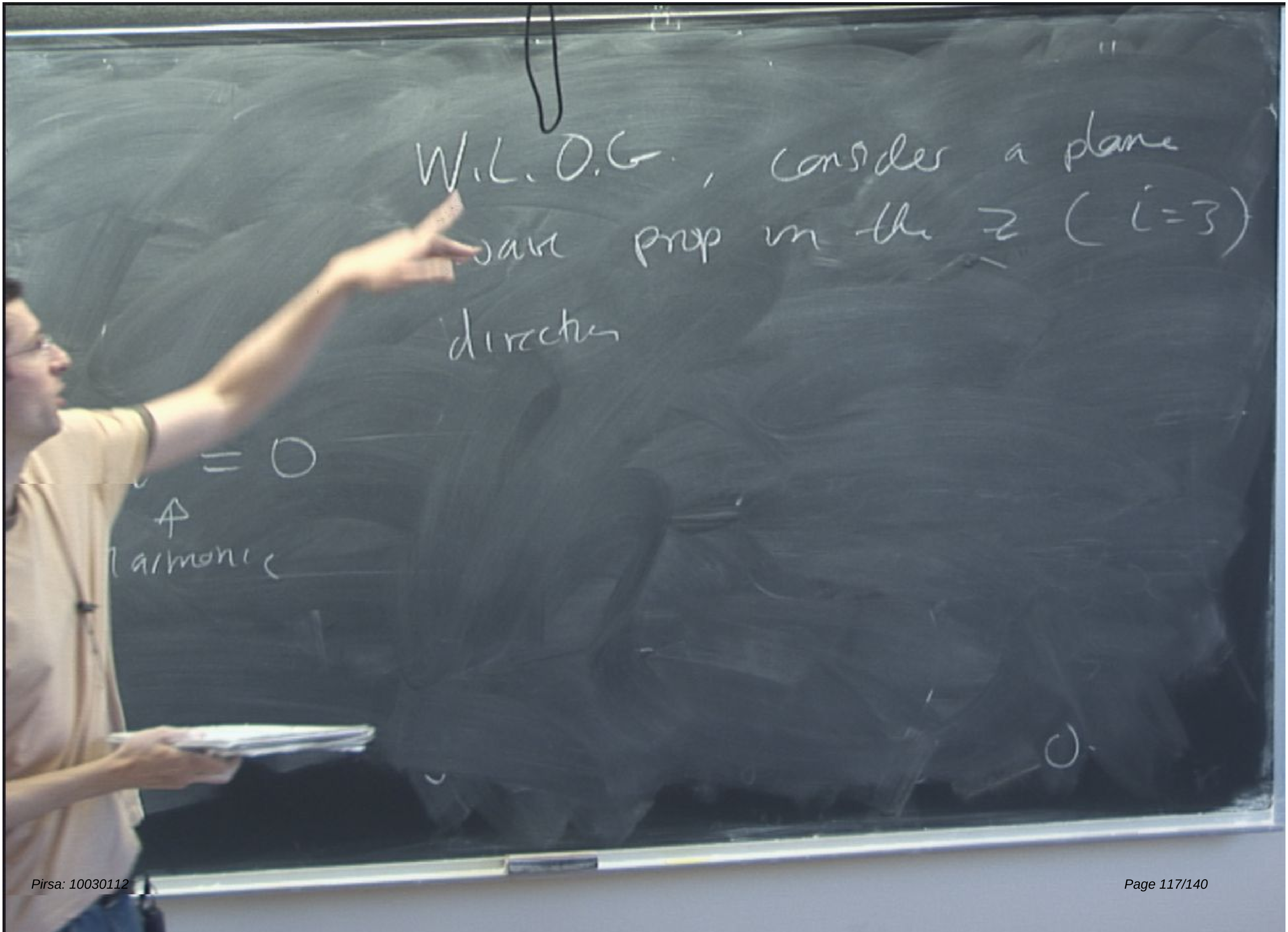
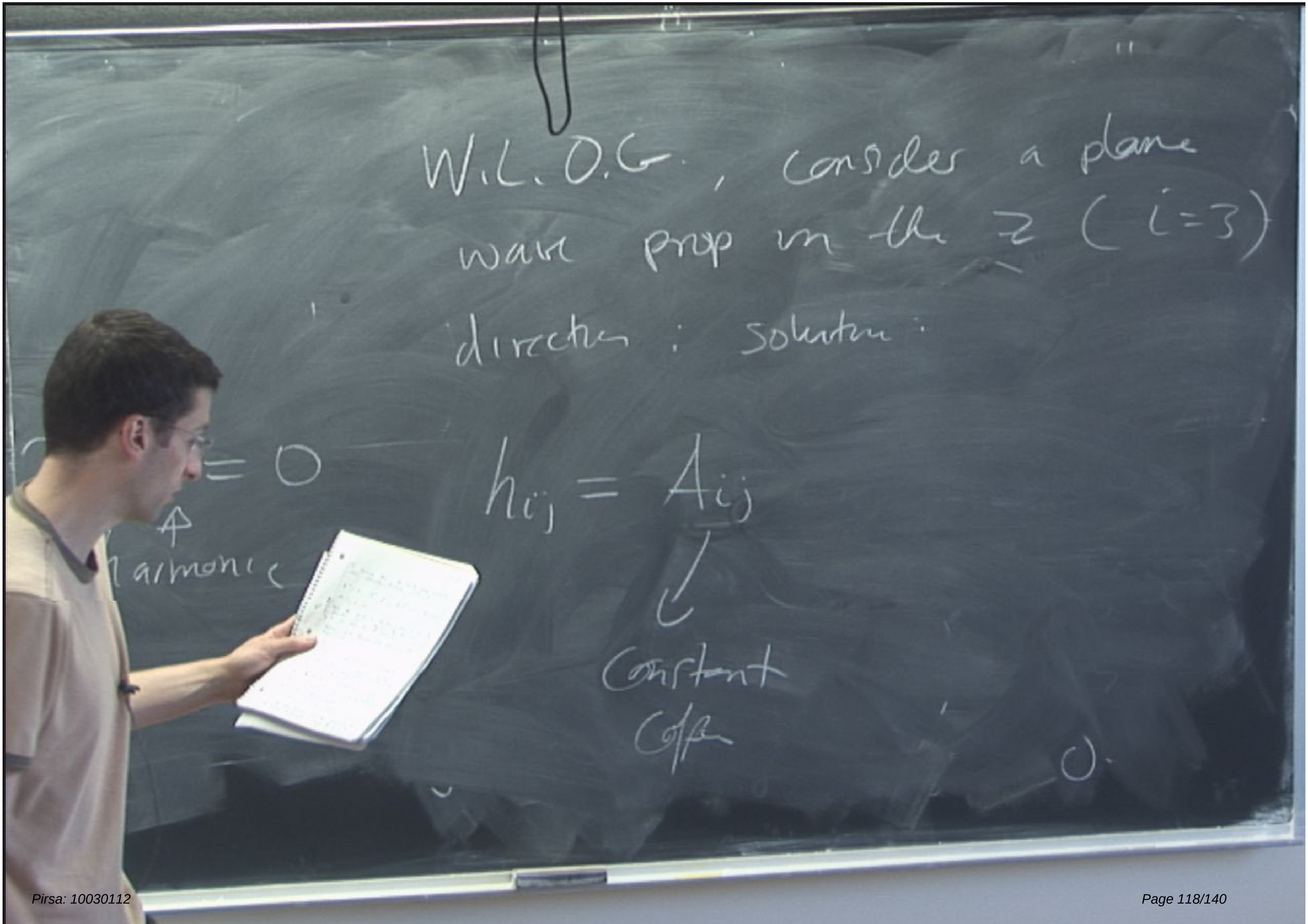

W.L.O.G., cons
$\partial_i h^{ij} = 0$
harmonic


W.L.O.G. , consider a plane
$\partial_i h^{ij} =$


W.L.O.G., consider a plane
ave prop in the z (i=3)
direction
= 0
armonic


W.L.O.G., consider a plane
wave prop in the z (i=3)
direction: solution:
= 0
harmonic
$h_{ij} = A_{ij}$
constant
coeff

W.L.O.G., consider a plane wave prop in the z ($i=3$) direction: solution:

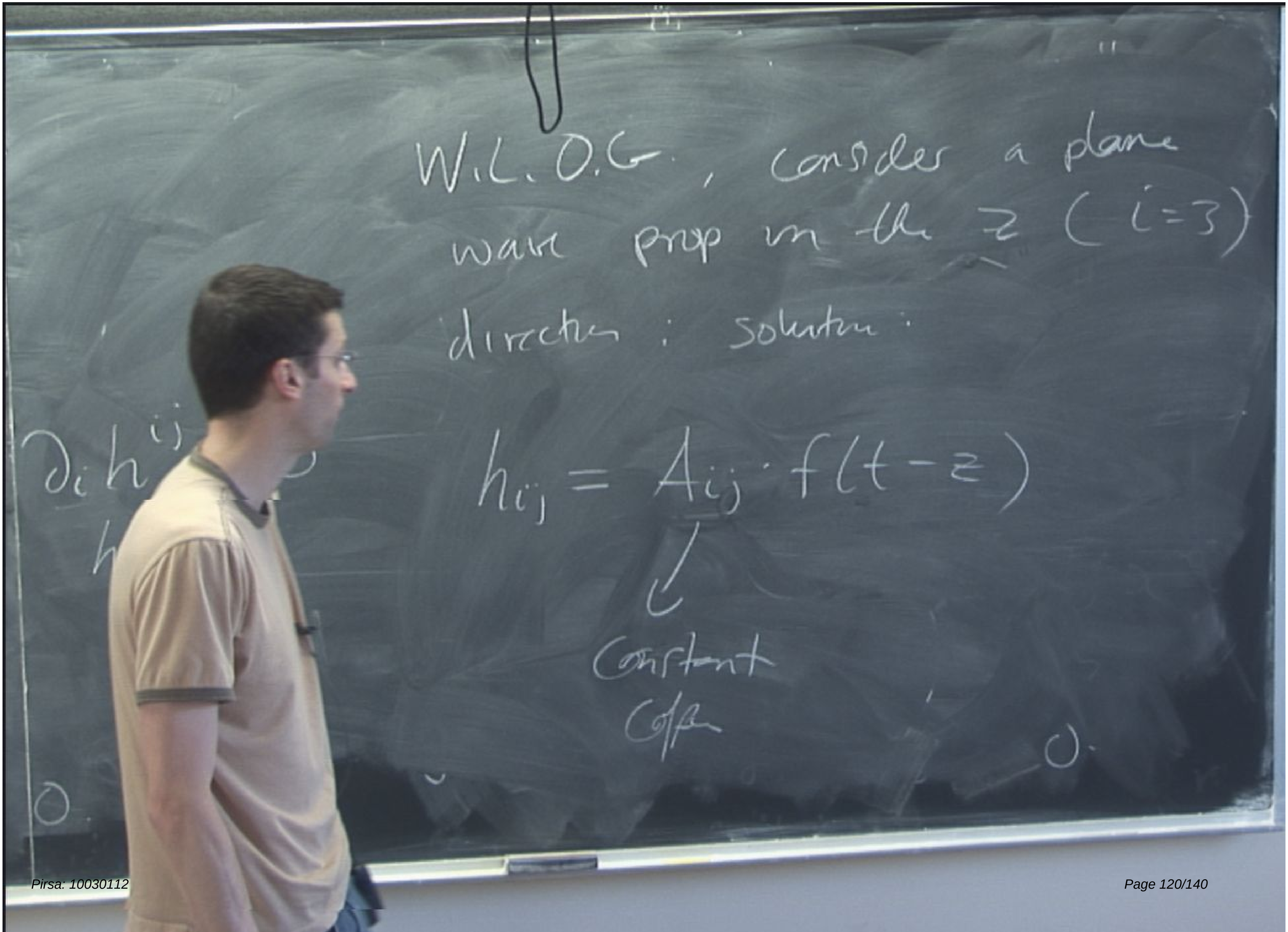
$\partial_i h^{ij} = 0$

harmonic

$$h_{ij} = A_{ij} \cdot f(t-z)$$

constant

coeffs


W.L.O.G., consider a plane
wave prop in the z (i=3)
direction: solution:
$h_{ij} = A_{ij} \cdot f(t-z)$
Constant

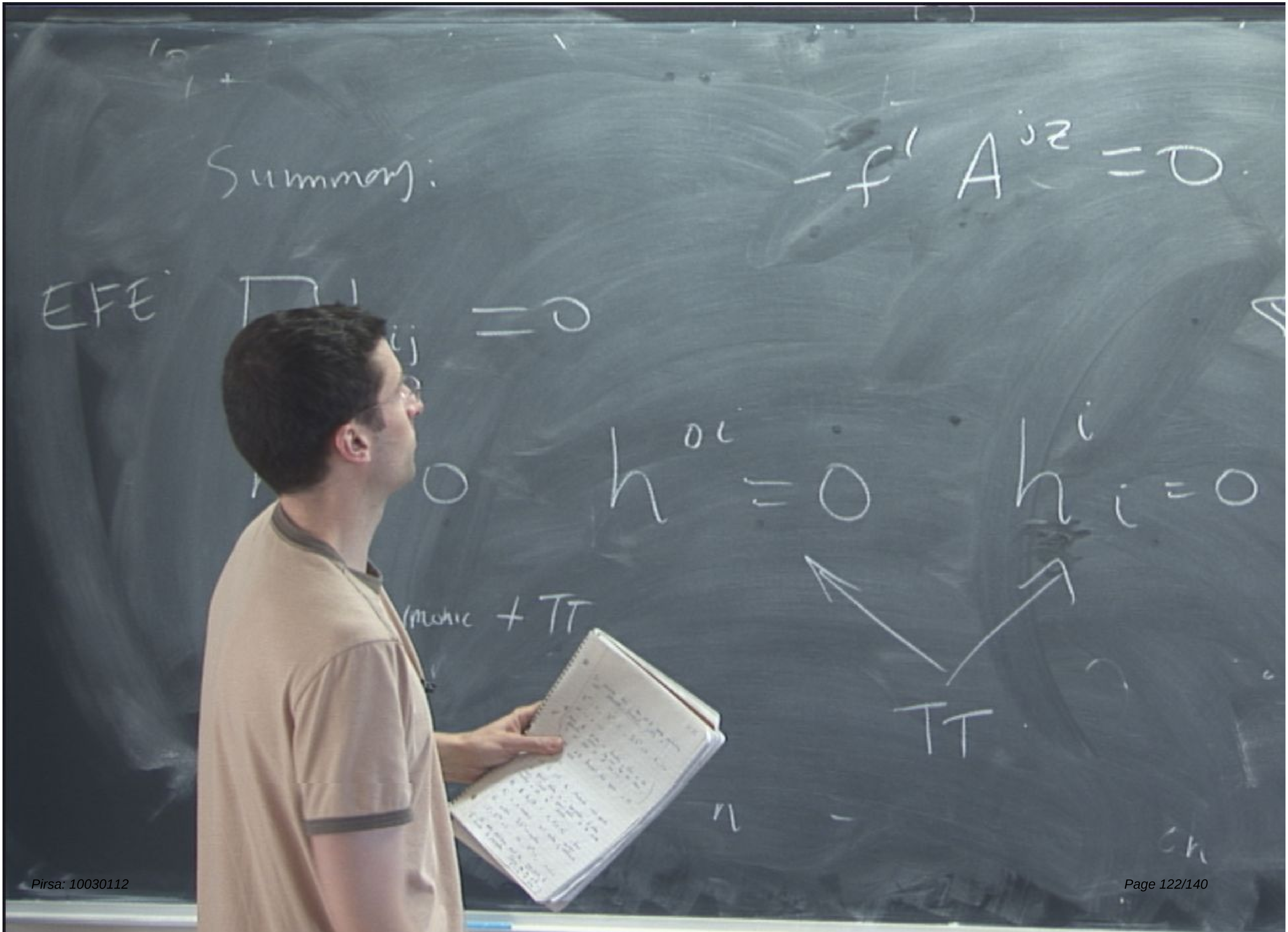
W.L.O.G., consider a plane wave prop in the z ($i=3$) direction : <u>solution</u>:

$$\partial_i h^{ij} = 0$$

harmonic

$$h_{ij} = A_{ij} \cdot f(t-z)$$

constant coefs


Summary:
$-f' A^{jz} = 0.$
EFE
$h^{0i} = 0$
$h^{i}_{i} = 0$
TT

Summary:

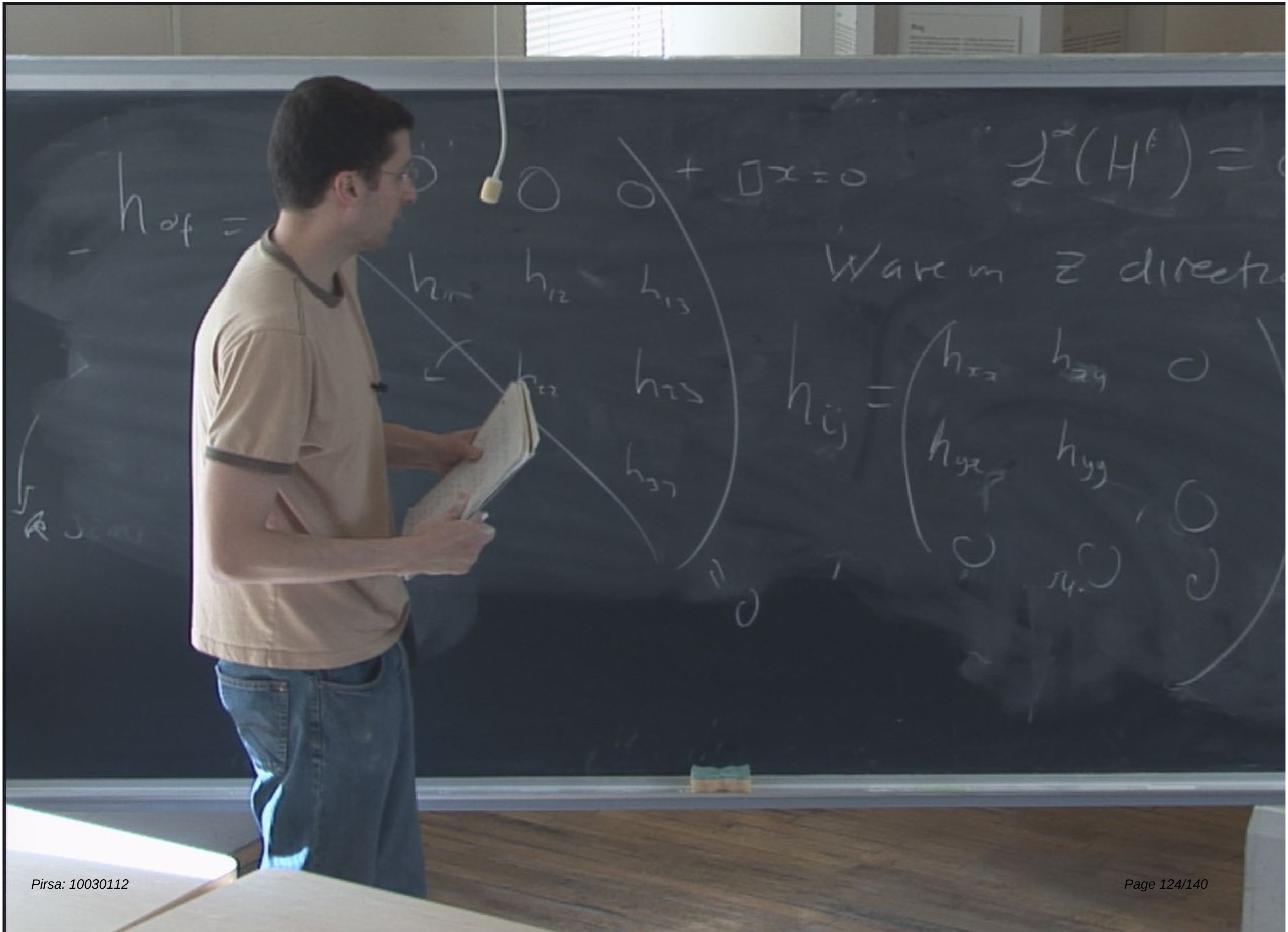
$$-f' A^{iz} = 0.$$

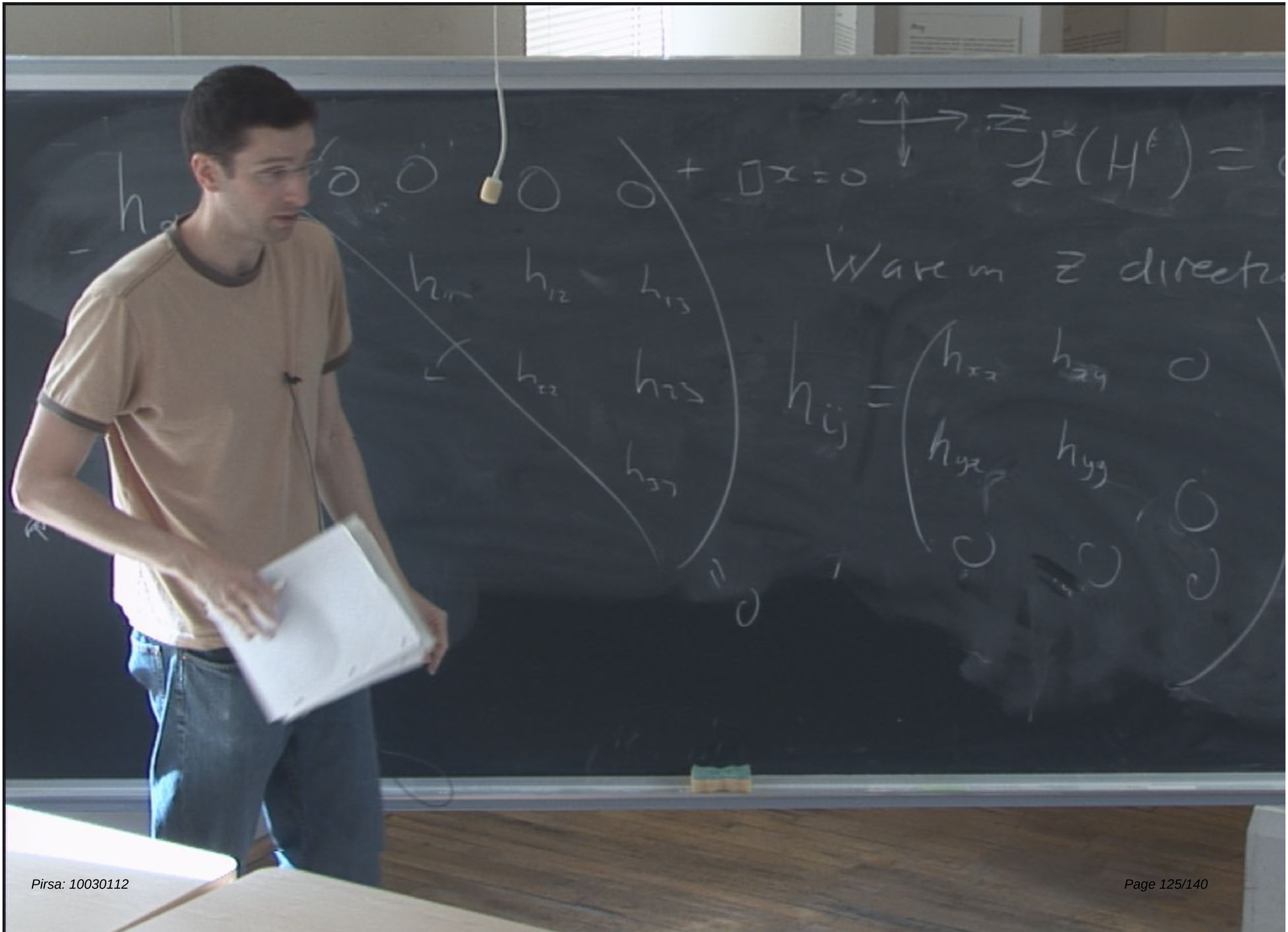
EFE: $\Box h_{ij} = 0$

$h^{00} = 0$ $\quad h^{0i} = 0$ $\quad h^{i}{}_{i} = 0$

harmonic + TT

TT


□x = 0
Wave in z


Wave in z direct

left with a 2×2, symmetric traceless matrix A_{IJ}

$\partial_i h^{ij} = 0$

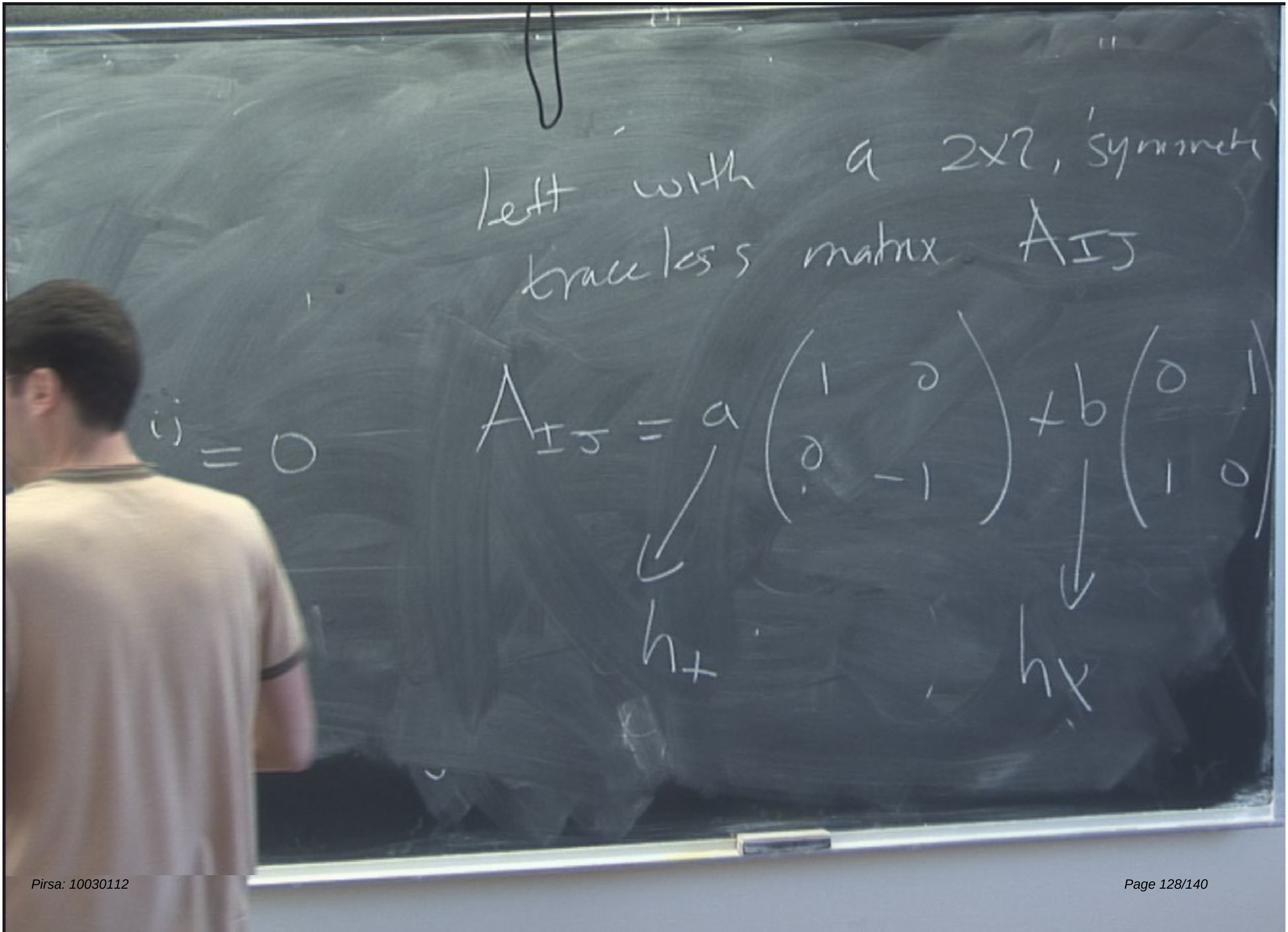
harmonic

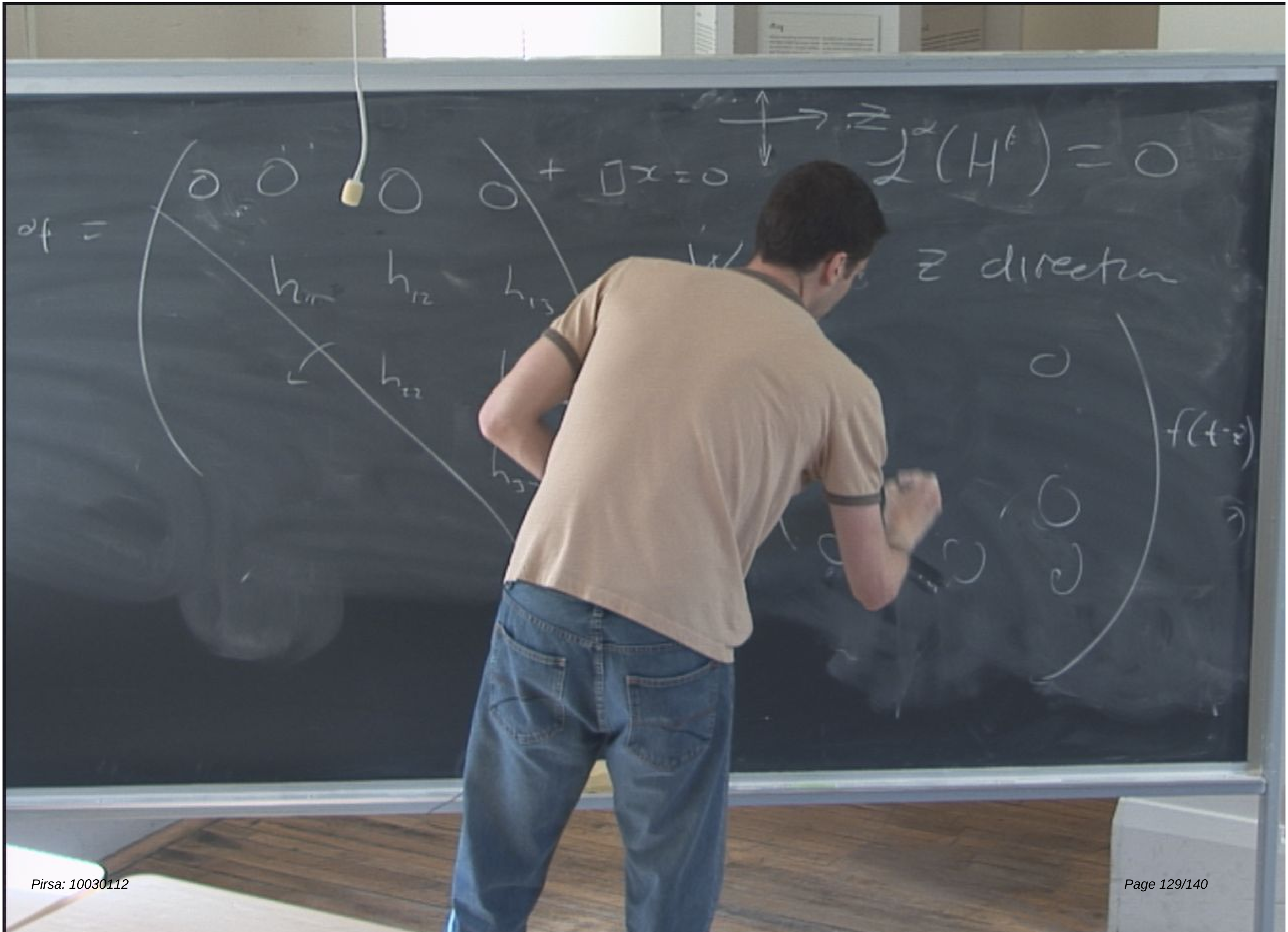
left with a 2x2, symmetric traceless matrix A_{IJ}

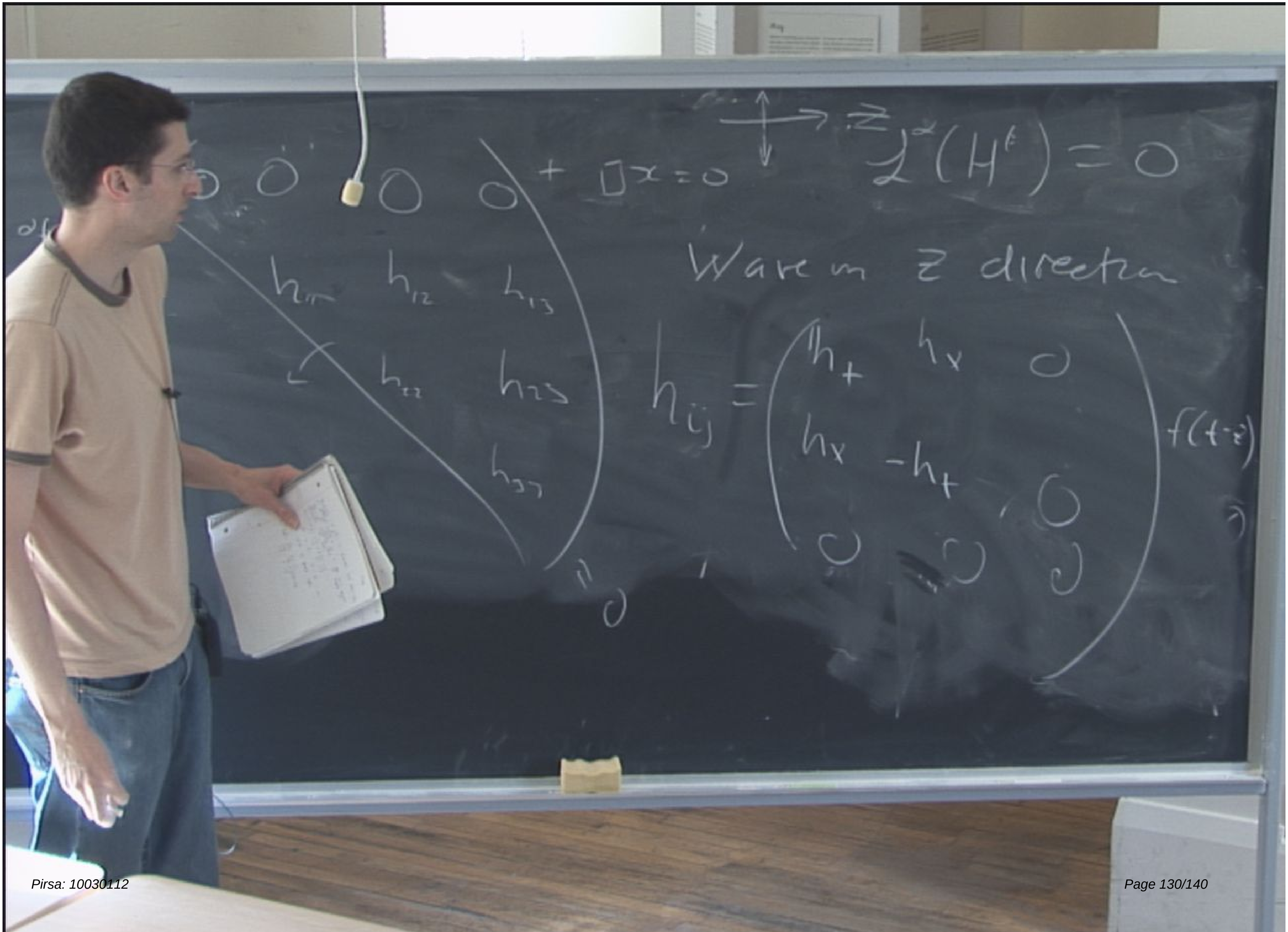
$$\partial_i h^{ij} = 0$$

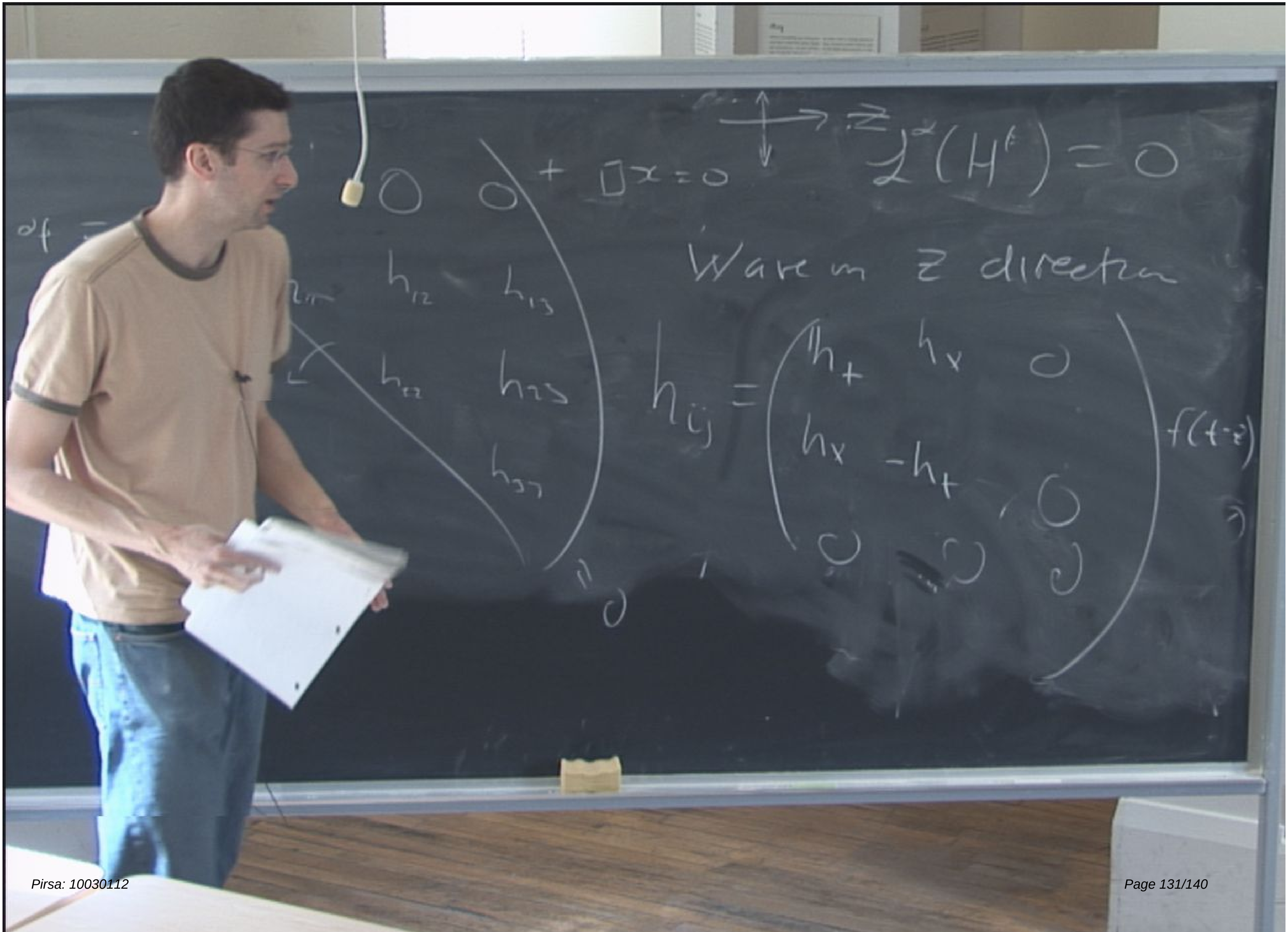
harmonic

$$A_{IJ} = a\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$


left with a 2x2, symmetric
traceless matrix A_{IJ}
$ij = 0$
$A_{IJ} = a\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + b\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
h_+
$h_\times$


z direction
f(t-z)


Wave in z direction
h_ij = h_+ h_x 0 h_x -h_+ 0 0 0 0 f(t-z)
□x = 0
h_11 h_12 h_13 h_22 h_23 h_33

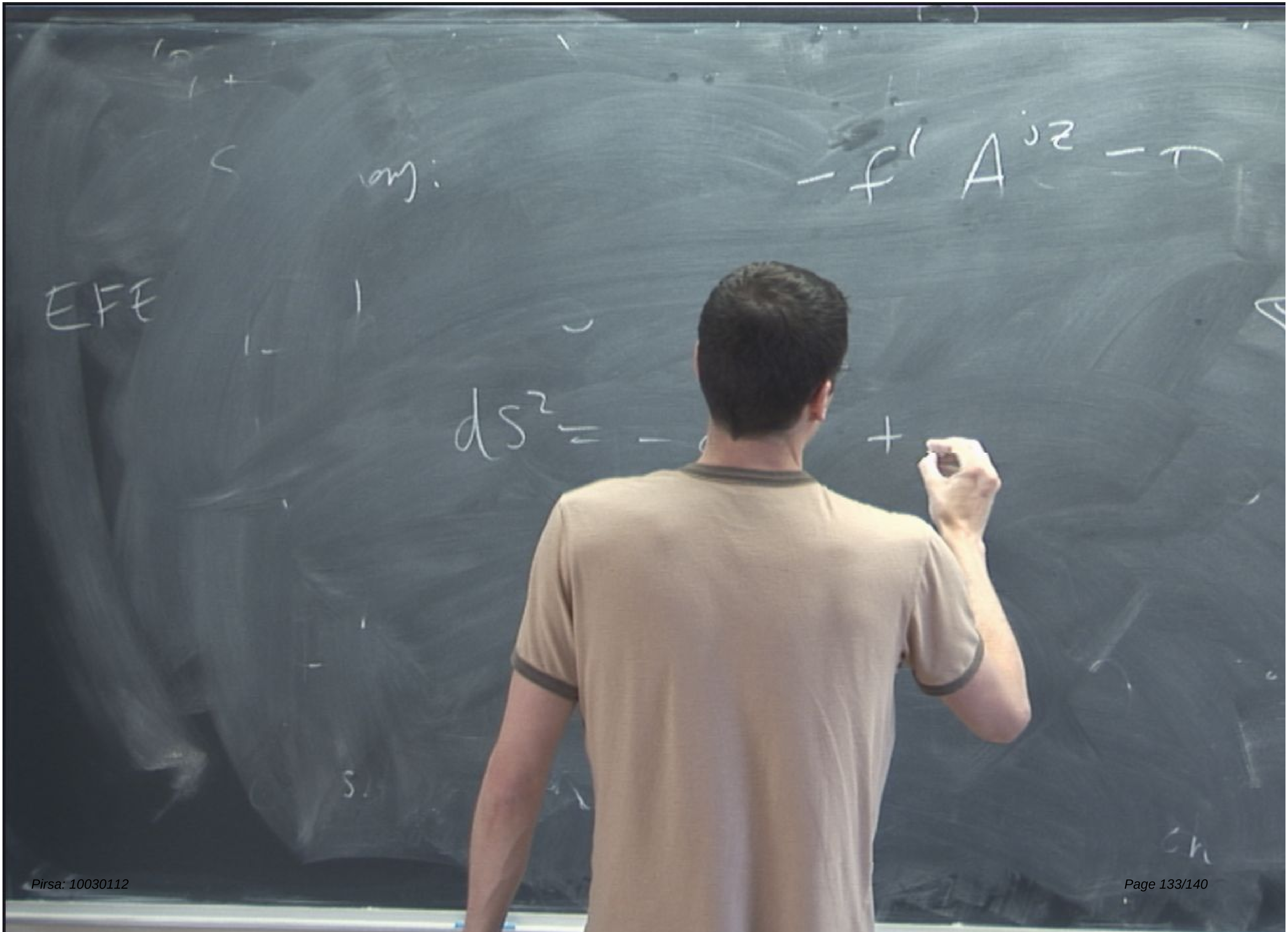

+ □x = 0
$\mathcal{L}^\alpha(H^\epsilon) = 0$
Wave in z direction
$h_{ij} = \begin{pmatrix} h_+ & h_x & 0 \\ h_x & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} f(t-z)$

$$\partial f = \begin{pmatrix} 0 & 0 & 0 & 0 \\ & h_{11} & h_{12} & h_{13} \\ & & h_{22} & h_{23} \\ & & & h_{33} \end{pmatrix} + \Box x = 0$$

$$\mathcal{L}^{\alpha}(H^{t}) = 0$$

Wave in z direction

$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} f(t-z)$$


EFE
ds² = −
− f′ A

$-f' A^{jz} - \ldots$

EFE

$$ds^2 = -dt^2 + (1+h_+)dx^2$$

$z - T$

left with

traceless

$$+ h_+) dx^2 + (1 - h_+) dy^2 + dz^2$$

h_+

0

$\partial z - T$

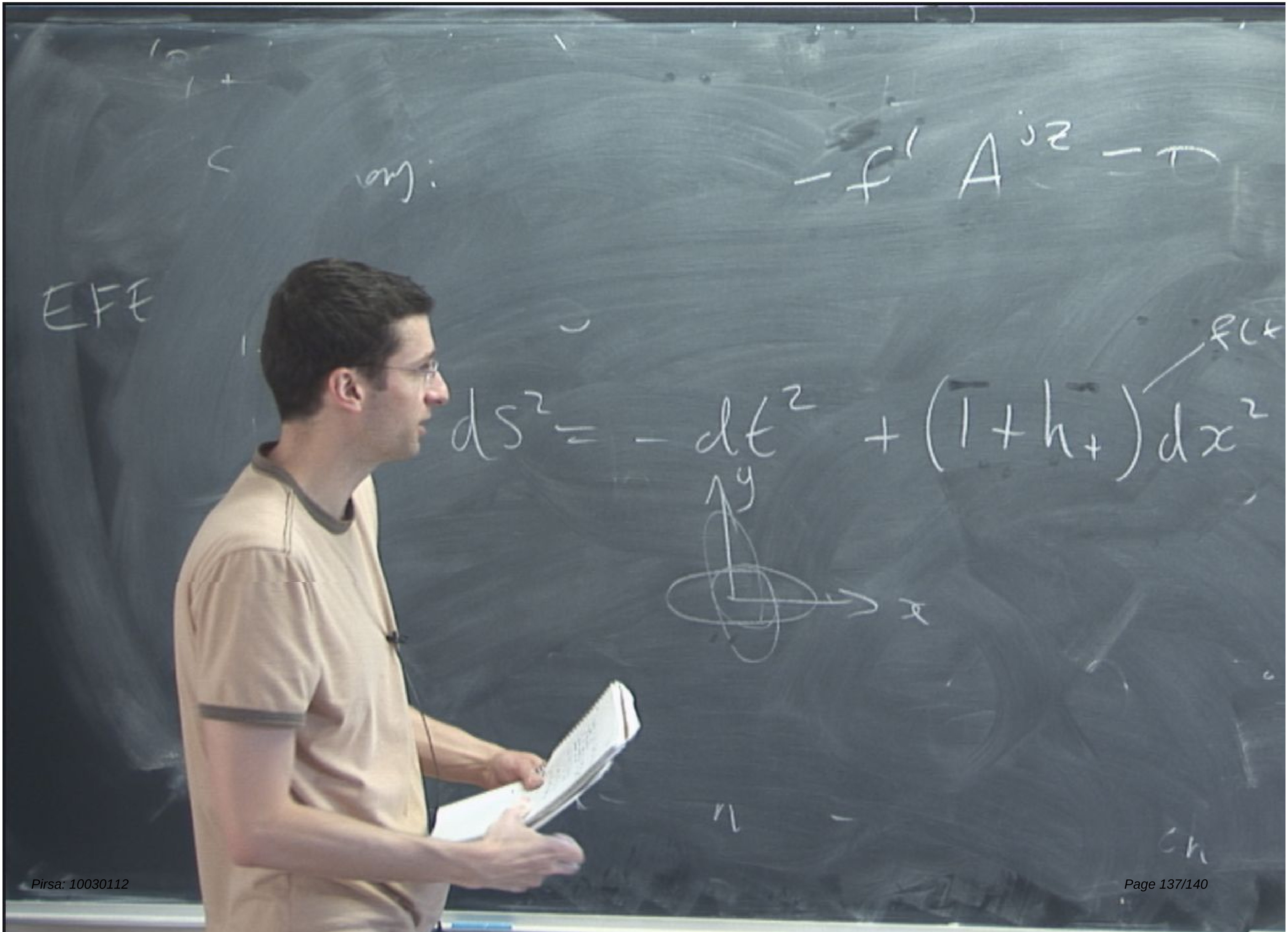
left with

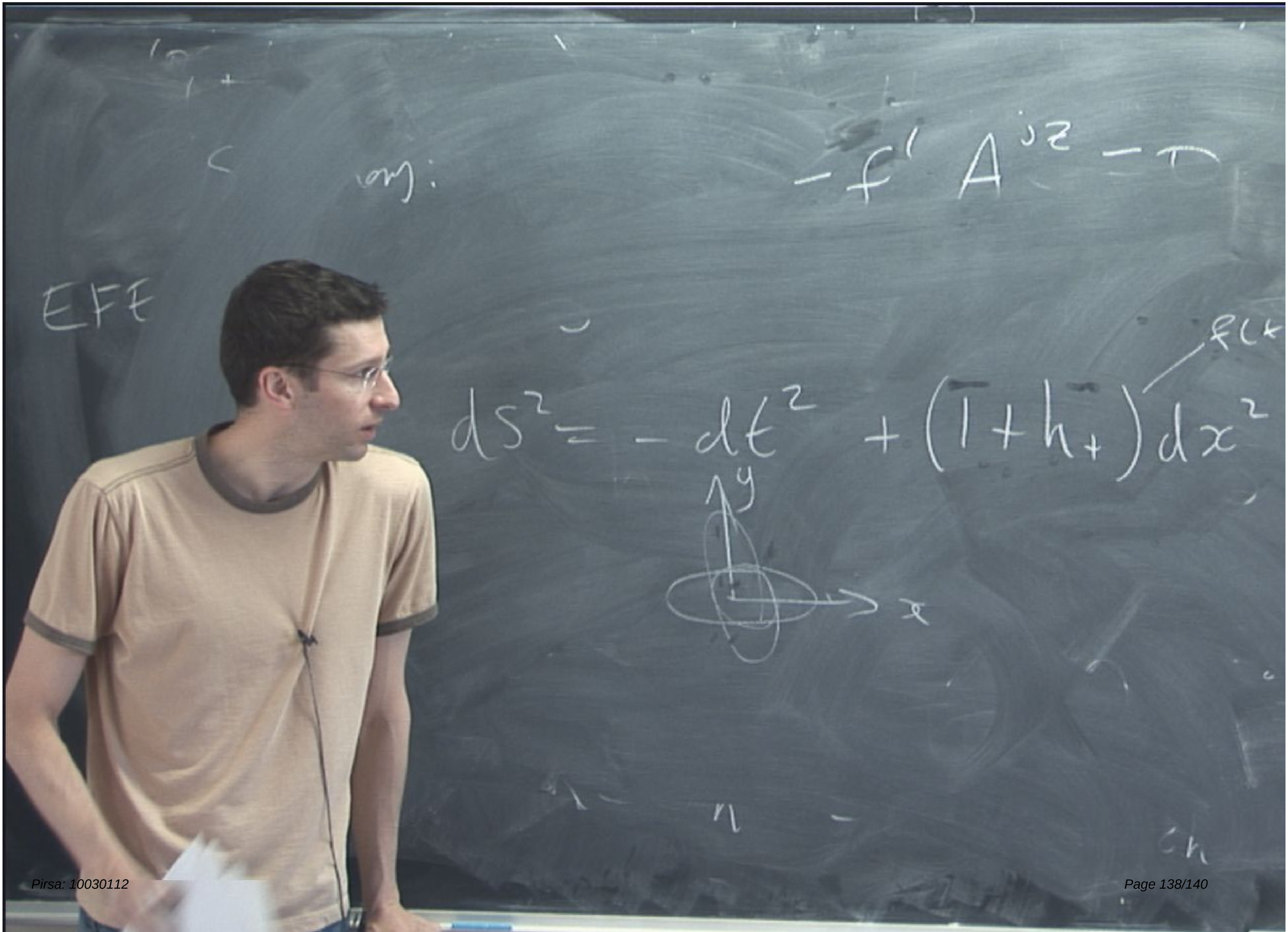
traceless

$f(t-z)$

$+h_+)dx^2 \quad + (1-h_+)dy^2 + dz^2$

h_+


EFE
$ds^2 = -dt^2 + (1+h_+)dx^2$
y
x


EFE
$-f' A^{iz}$
$ds^2 = -dt^2 + (1+h_+)dx^2$
y
x

left with a
traceless matrix

$f(t-z)$

$) dx^2 + (1-h_+) dy^2 + dz^2$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\Psi_4 \propto (\ddot{h}_+ - i\ddot{h}_x) h_+$

left with a
traceless matrix

$f(x-z)$

$) dx^2 + (1-h_+) dy^2 + dz^2 \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\Psi_4 \propto (\ddot{h}_+ - i \ddot{h}_\times) \quad h_+$