Title: Explorations in Numerical Relativity - Lecture 11

Date: Mar 29, 2010 11:20 AM

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Abstract:

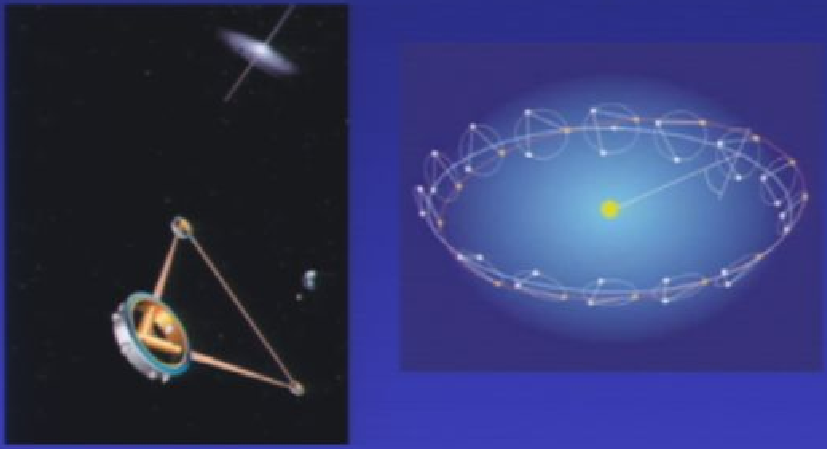
Gravitational Wave Astrophysics

LIGO/VIRGO/GEO/TAMA
ground based laser interferometers

LIGO Hanford

LIGO Livingston

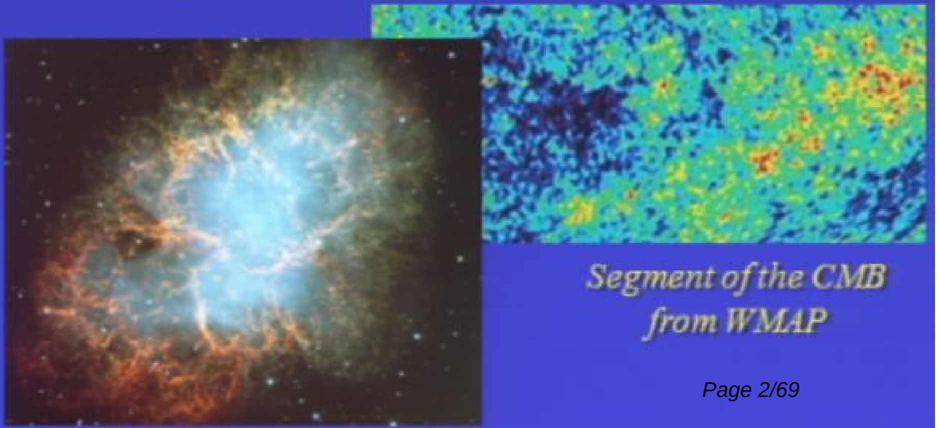
LISA
space-based laser interferometer (hopefully with get funded for a 20?? Launch)



ALLEGRO/NAUTILUS/AURIGA/…
resonant bar detectors

AURIGA

Pulsar timing network, CMB anisotropy



Segment of the CMB from WMAP

The Crab nebula … a supernovae

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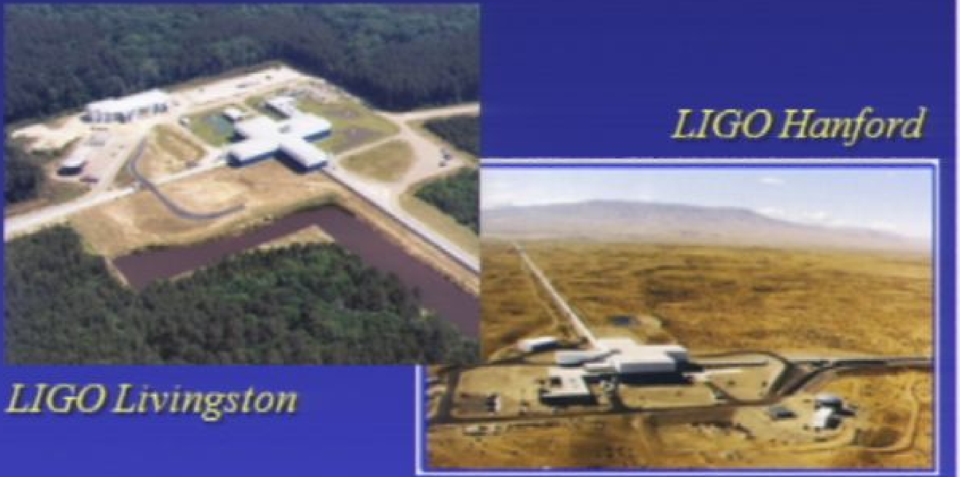

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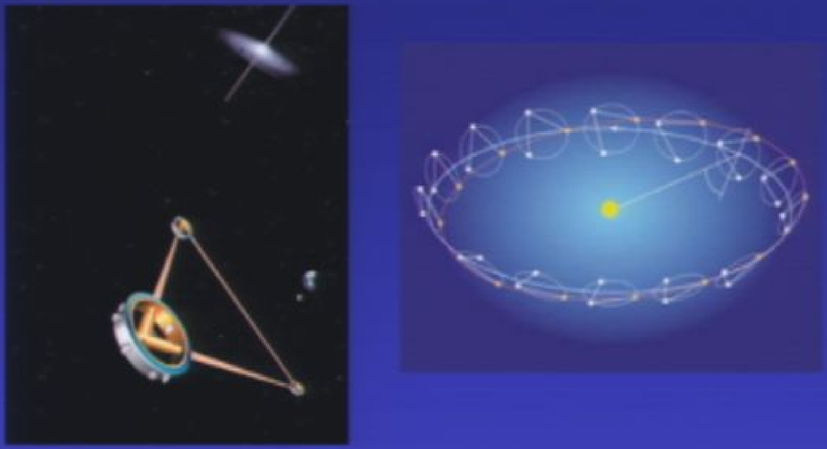

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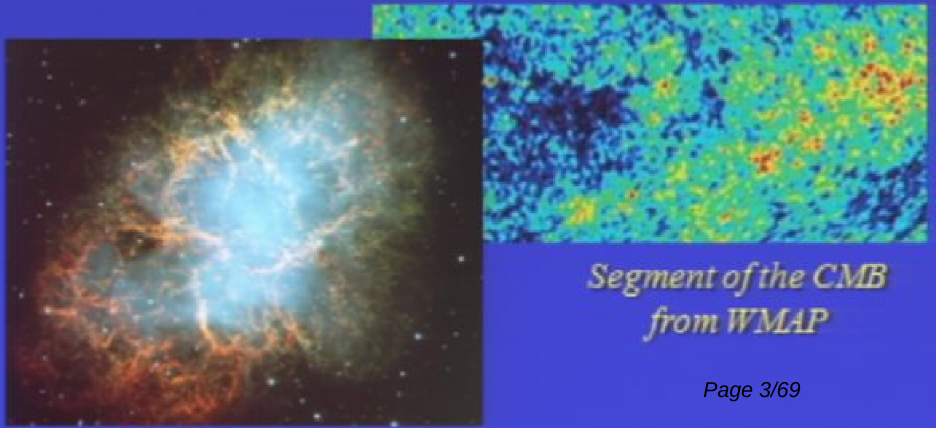

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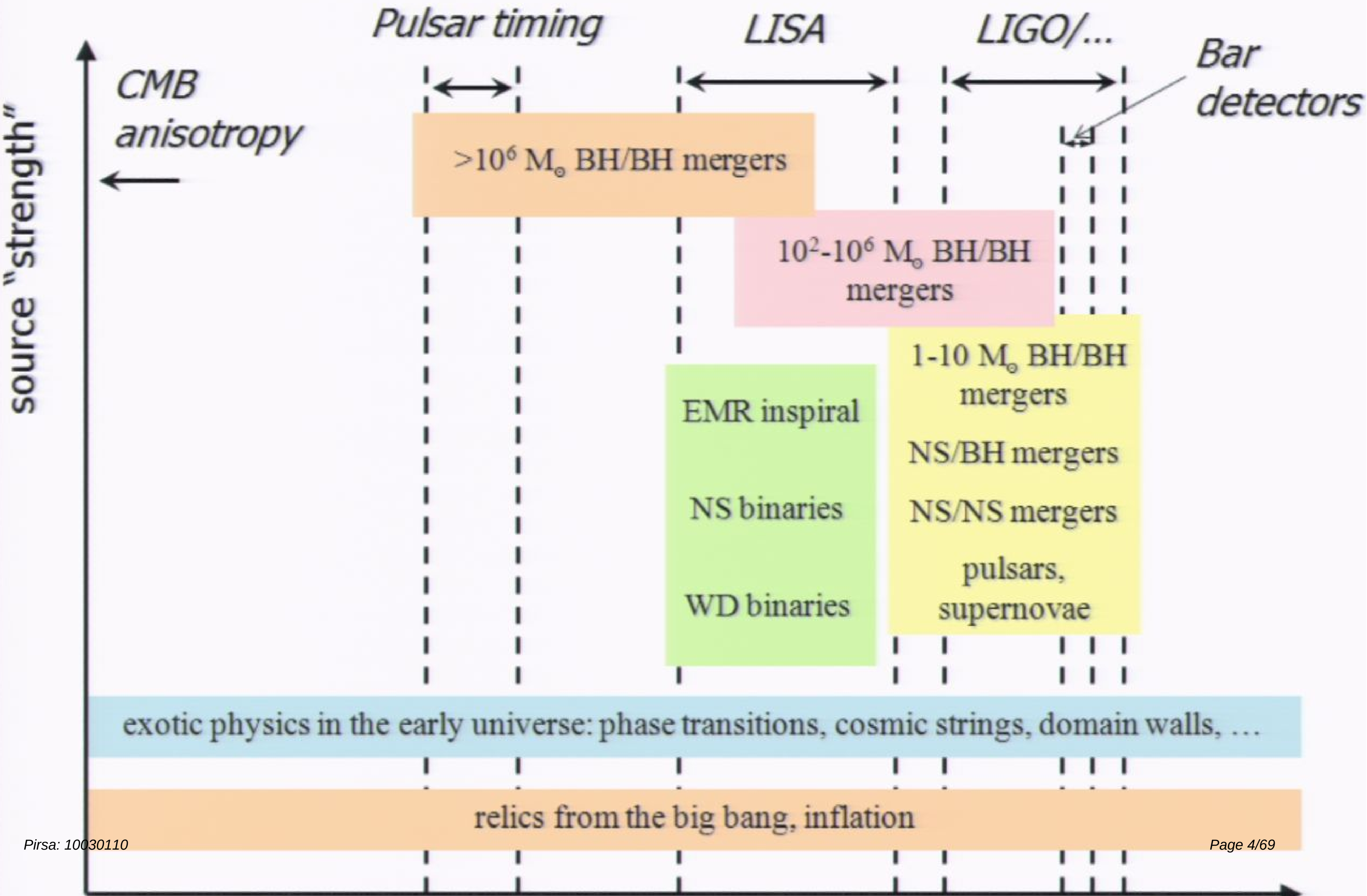
Overview of expected gravitational wave sources



Pulsar timing

LISA

LIGO/...

Bar detectors

CMB anisotropy

source "strength"

$>10^6$ $M_\odot$ BH/BH mergers

10^2-10^6 $M_\odot$ BH/BH mergers

EMR inspiral

NS binaries

WD binaries

1-10 $M_\odot$ BH/BH mergers

NS/BH mergers

NS/NS mergers

pulsars, supernovae

exotic physics in the early universe: phase transitions, cosmic strings, domain walls, …

relics from the big bang, inflation

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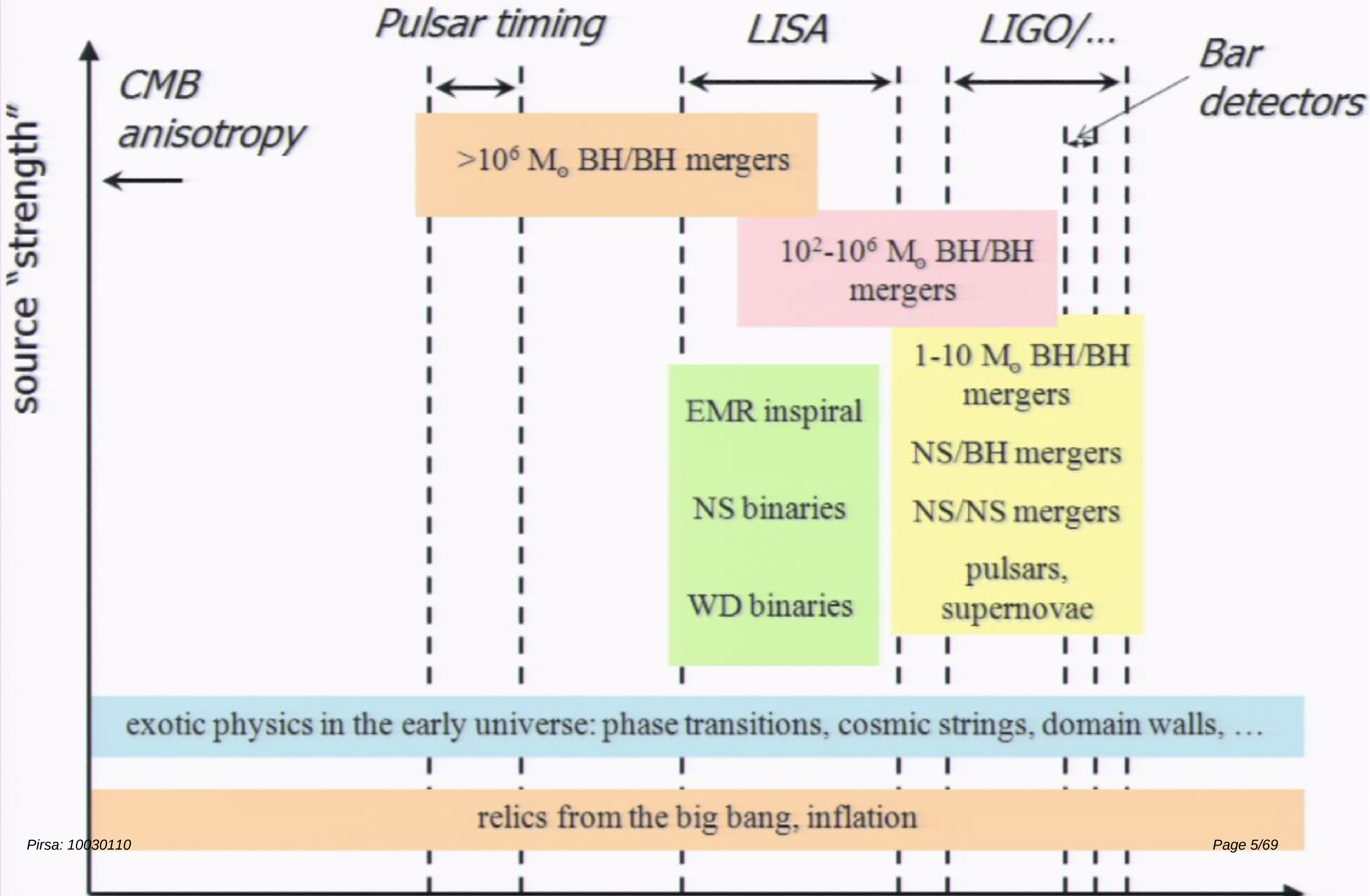

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Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - expected be the strongest source of gravitational waves in the universe
  - The binary BH is the "cleanest" simplest system, described exactly by vacuum GR
    - In the astrophysical system, will be a certain amount of matter (possibly circumbinary disk, ambient gas, CMB photons, …), though for the most part is expected very low mass relative to the black holes and hence dynamically insignificant
  - With NS's, microphysics must be approximated, and the sky's the limit as to how complicated this can be
    - neutron star structure (crust, multi-component superfluid core, at nuclear densities with uncertain equations of state, magnetic fields, etc.)
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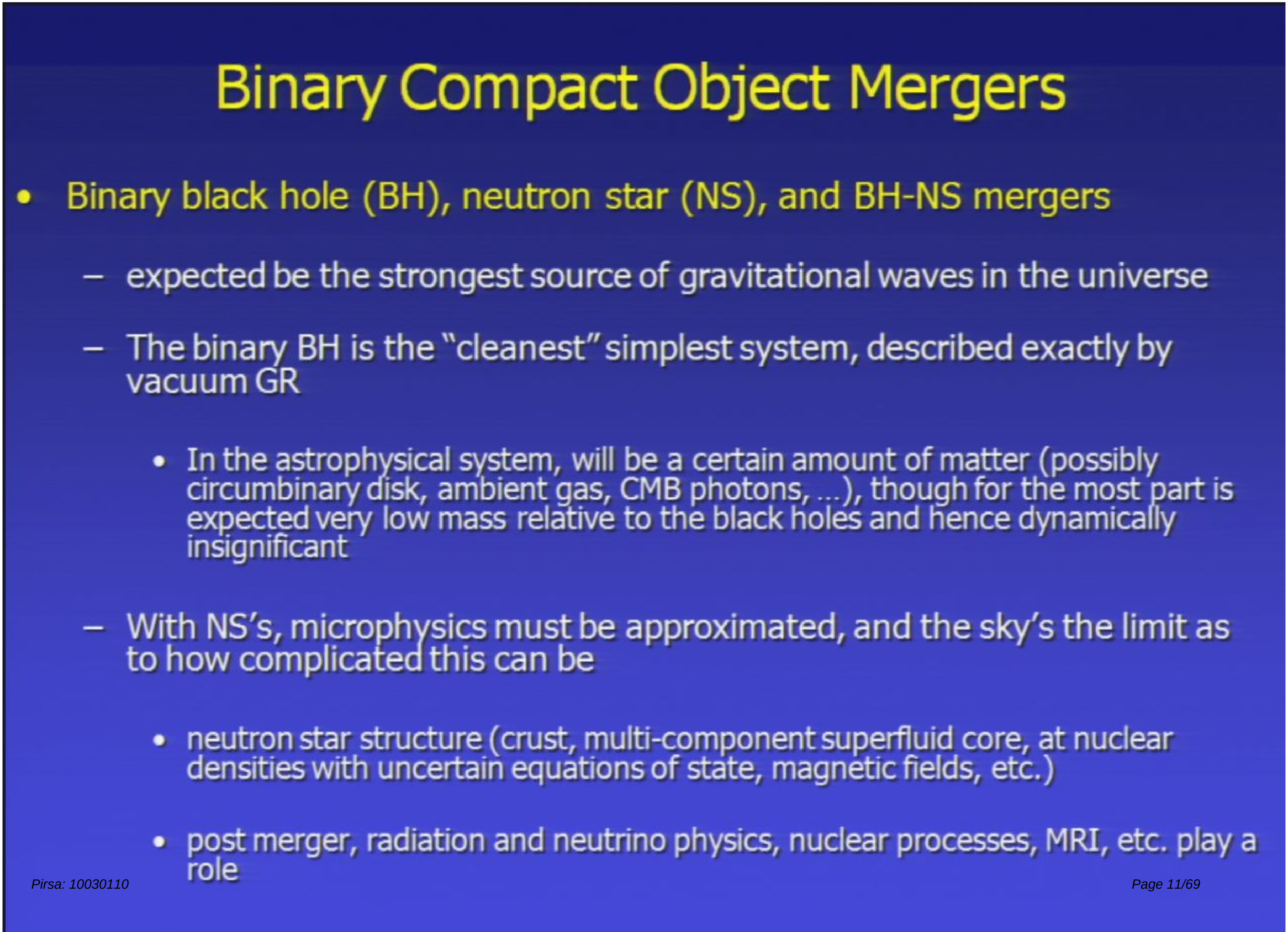

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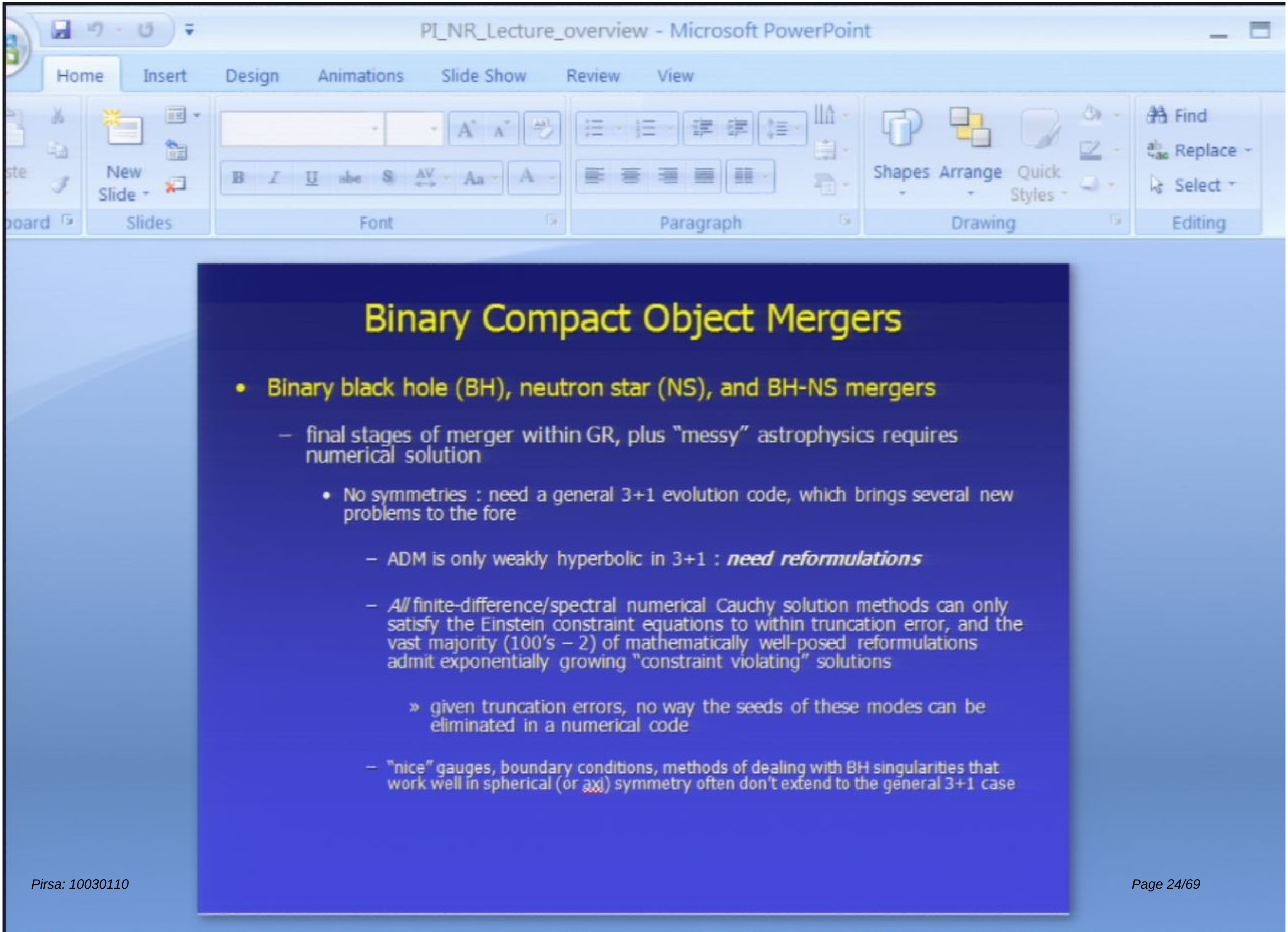

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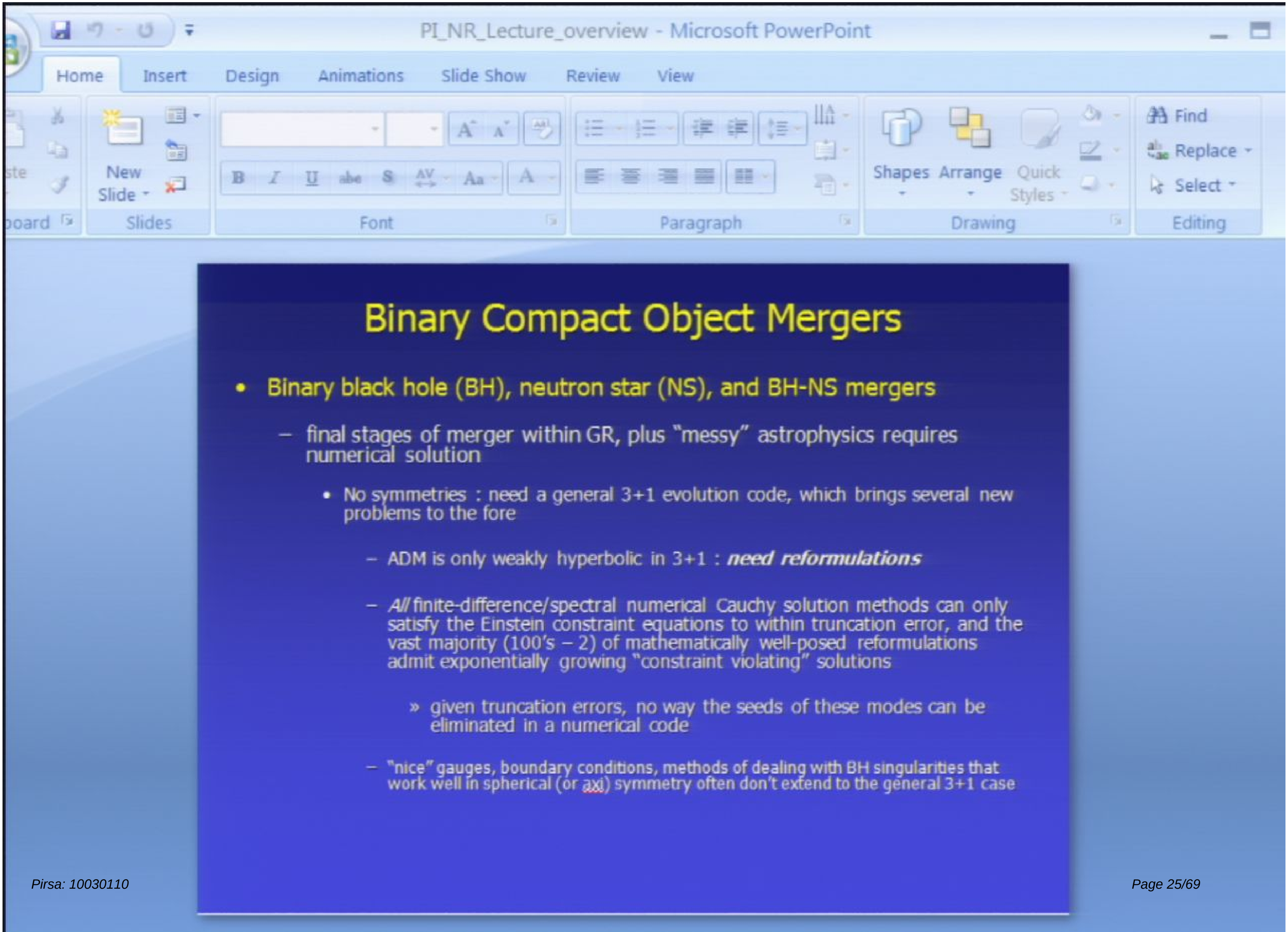

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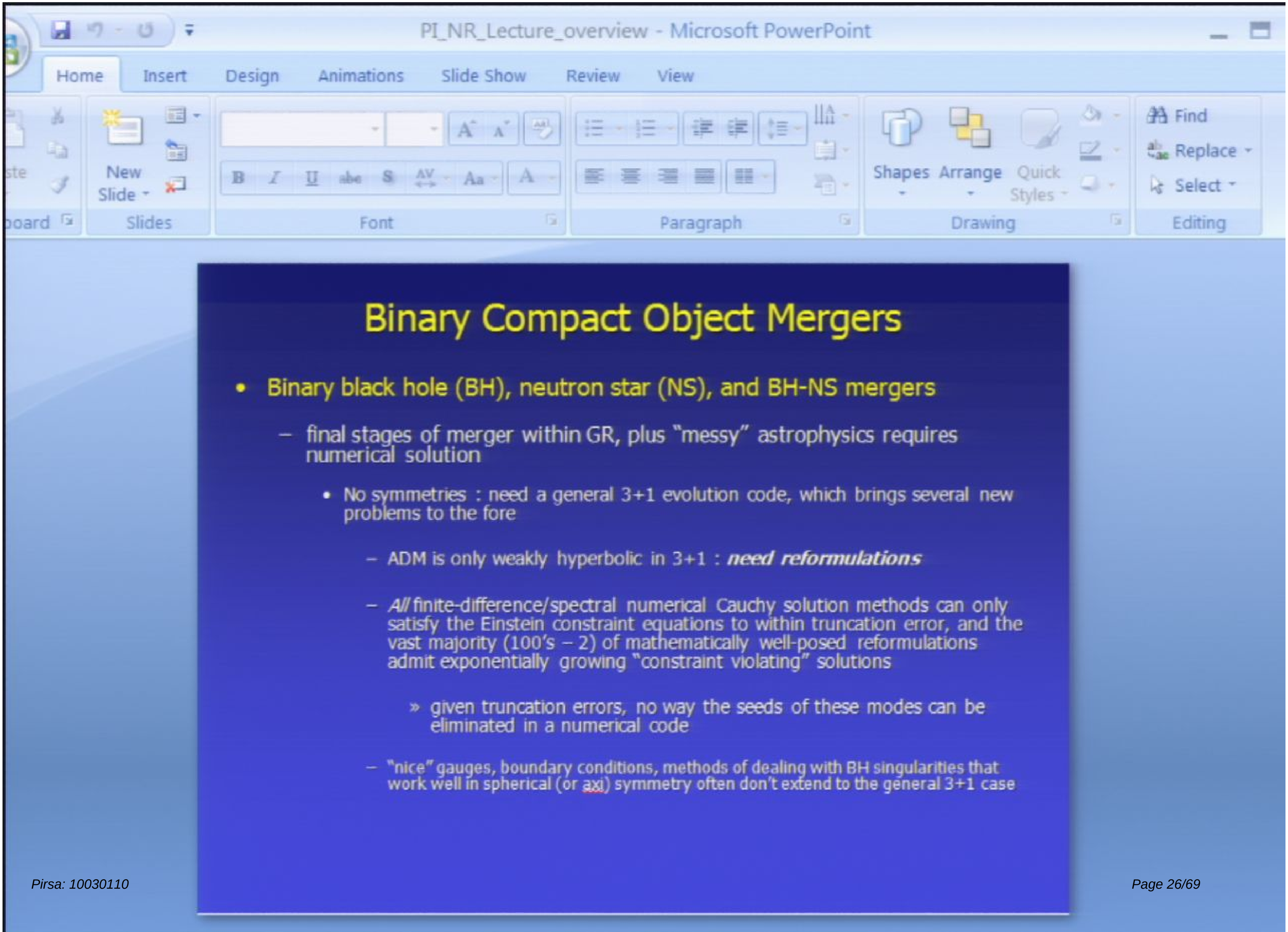

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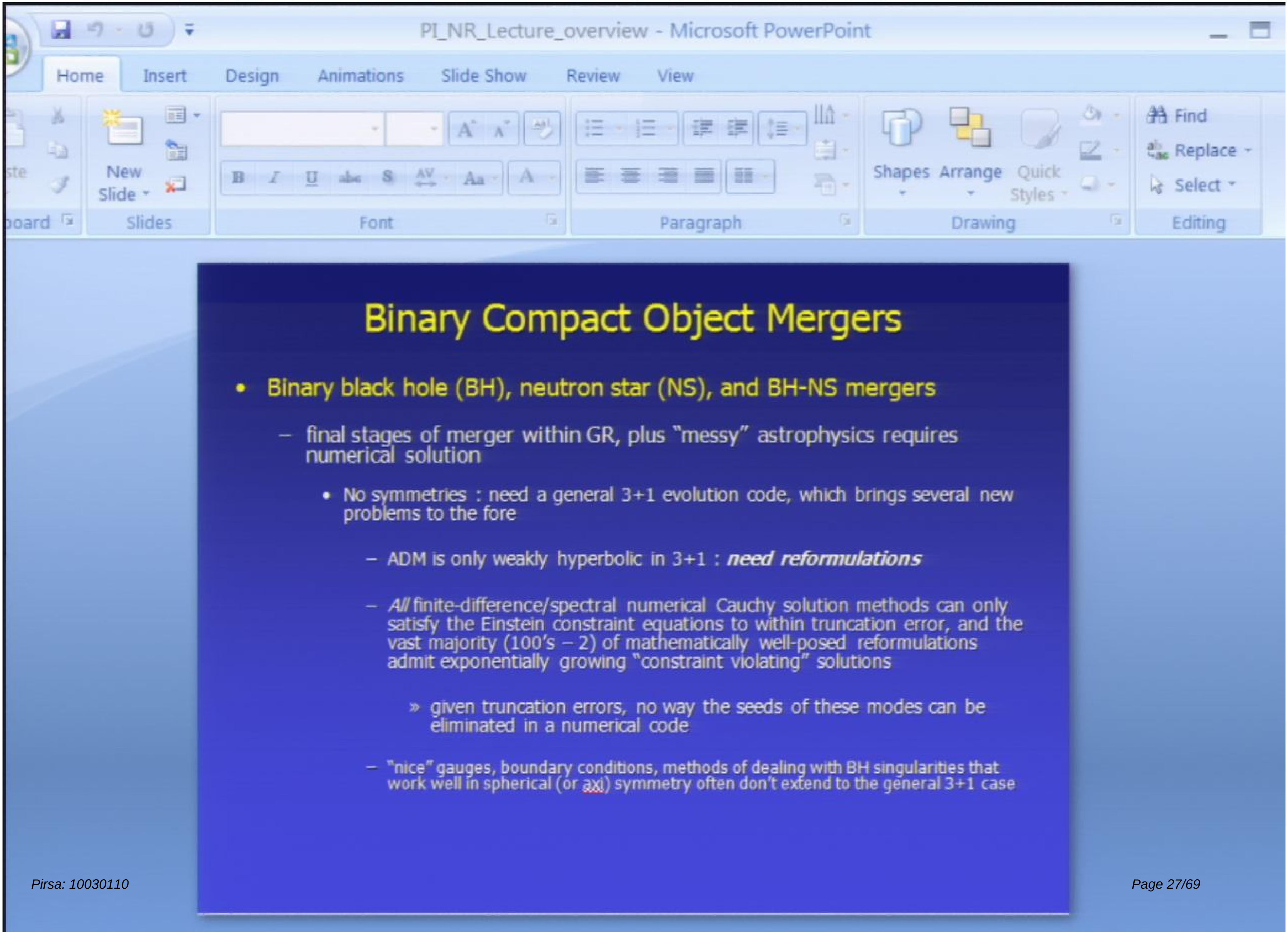

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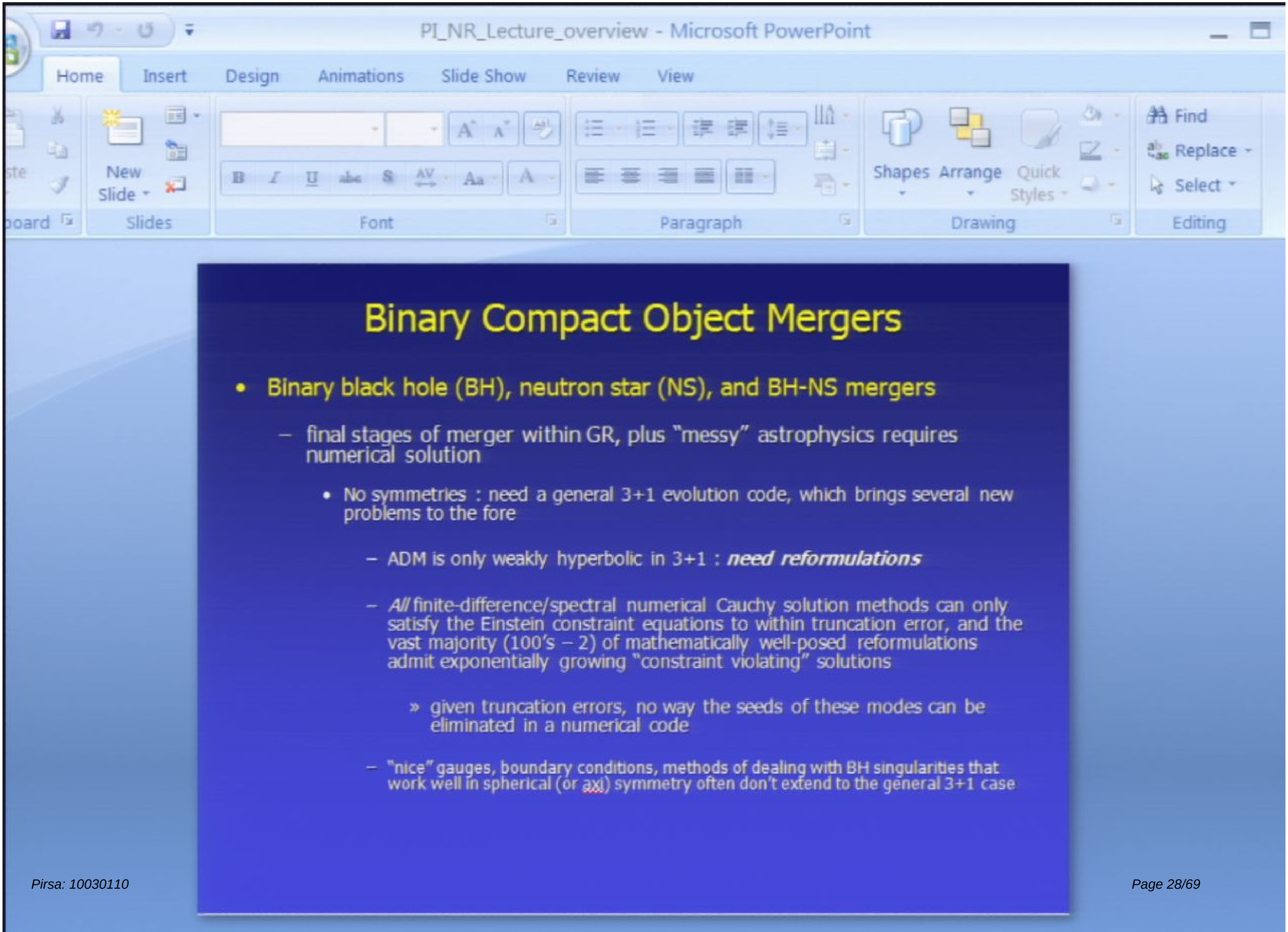

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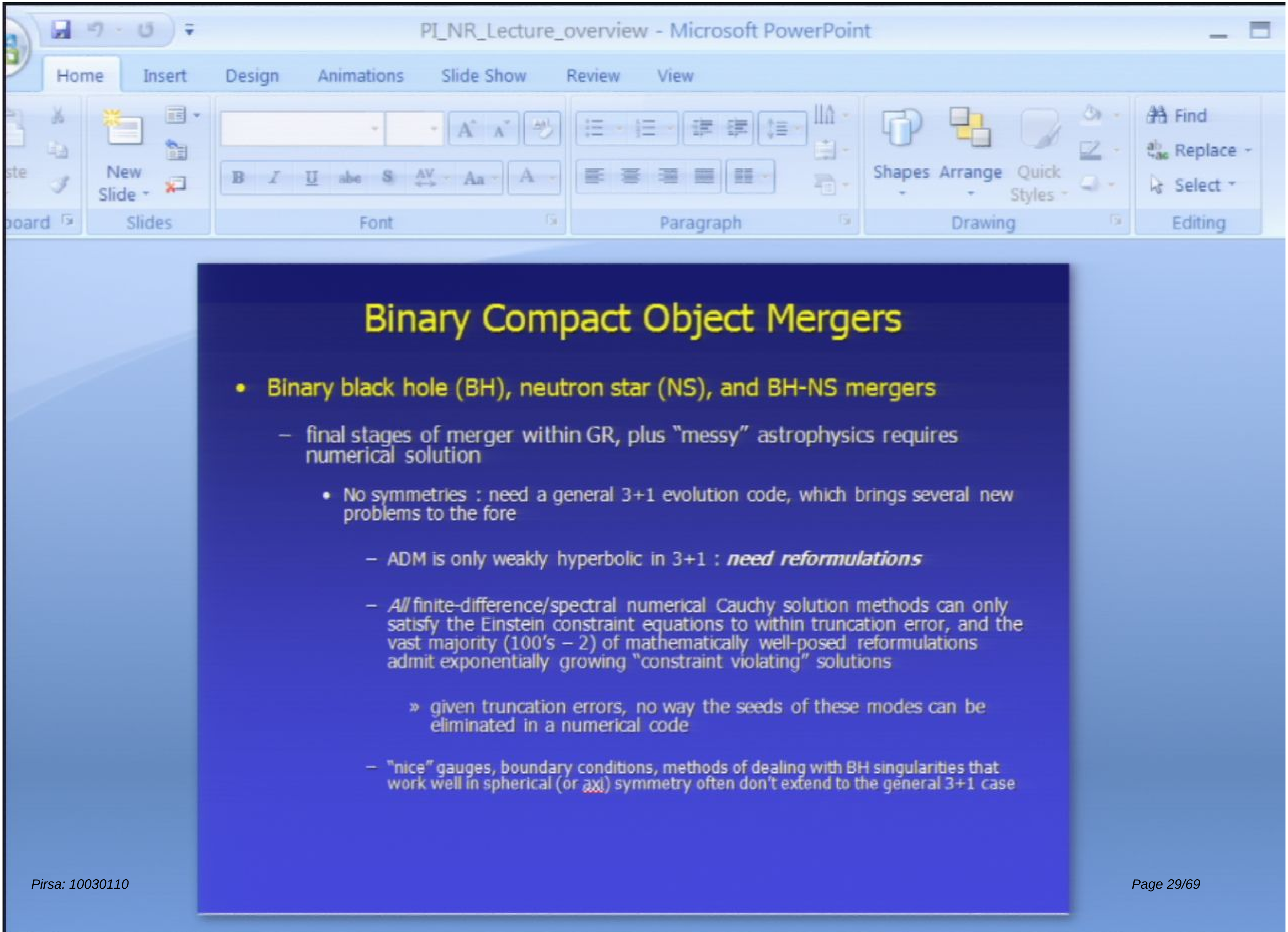

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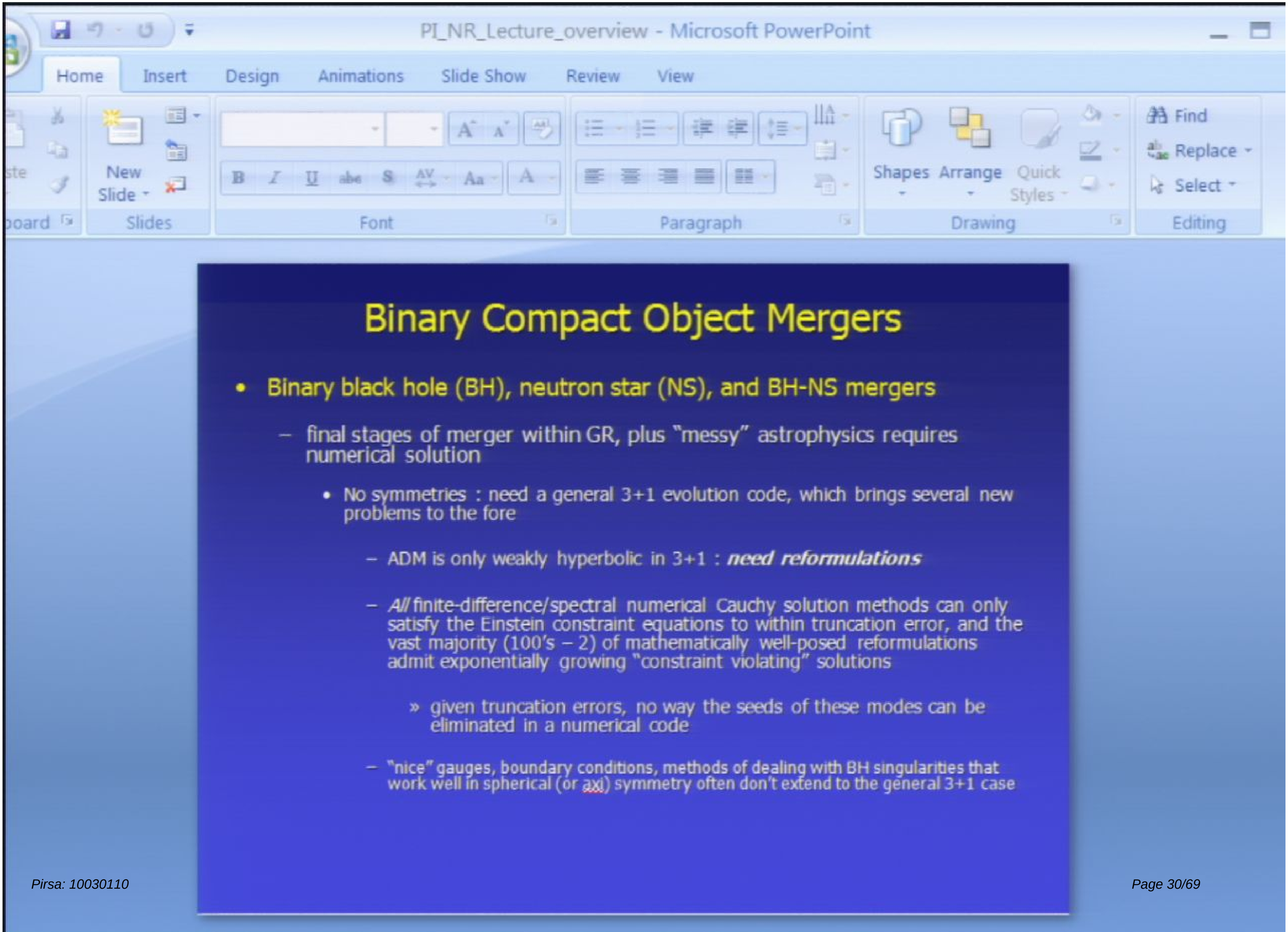

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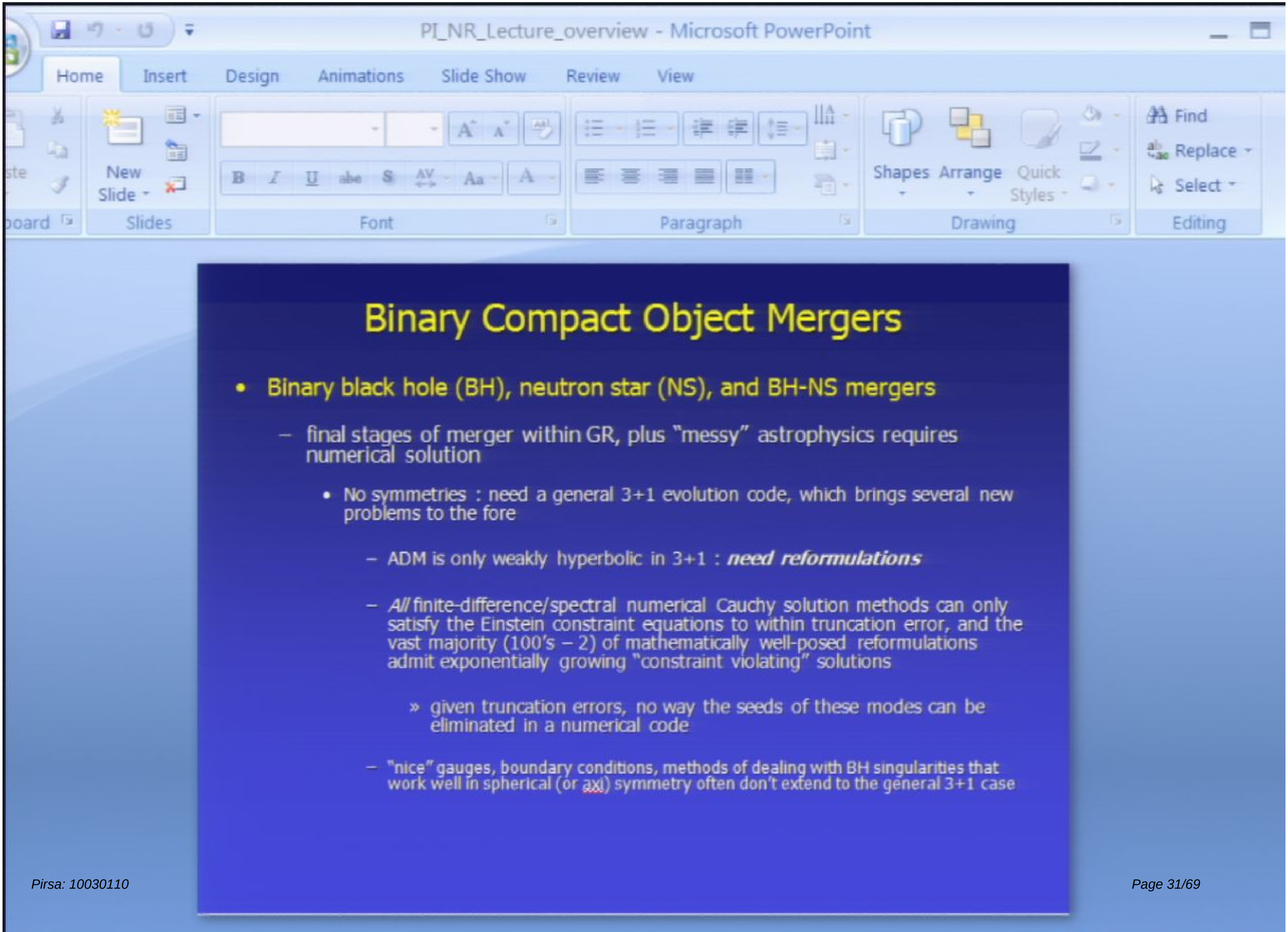

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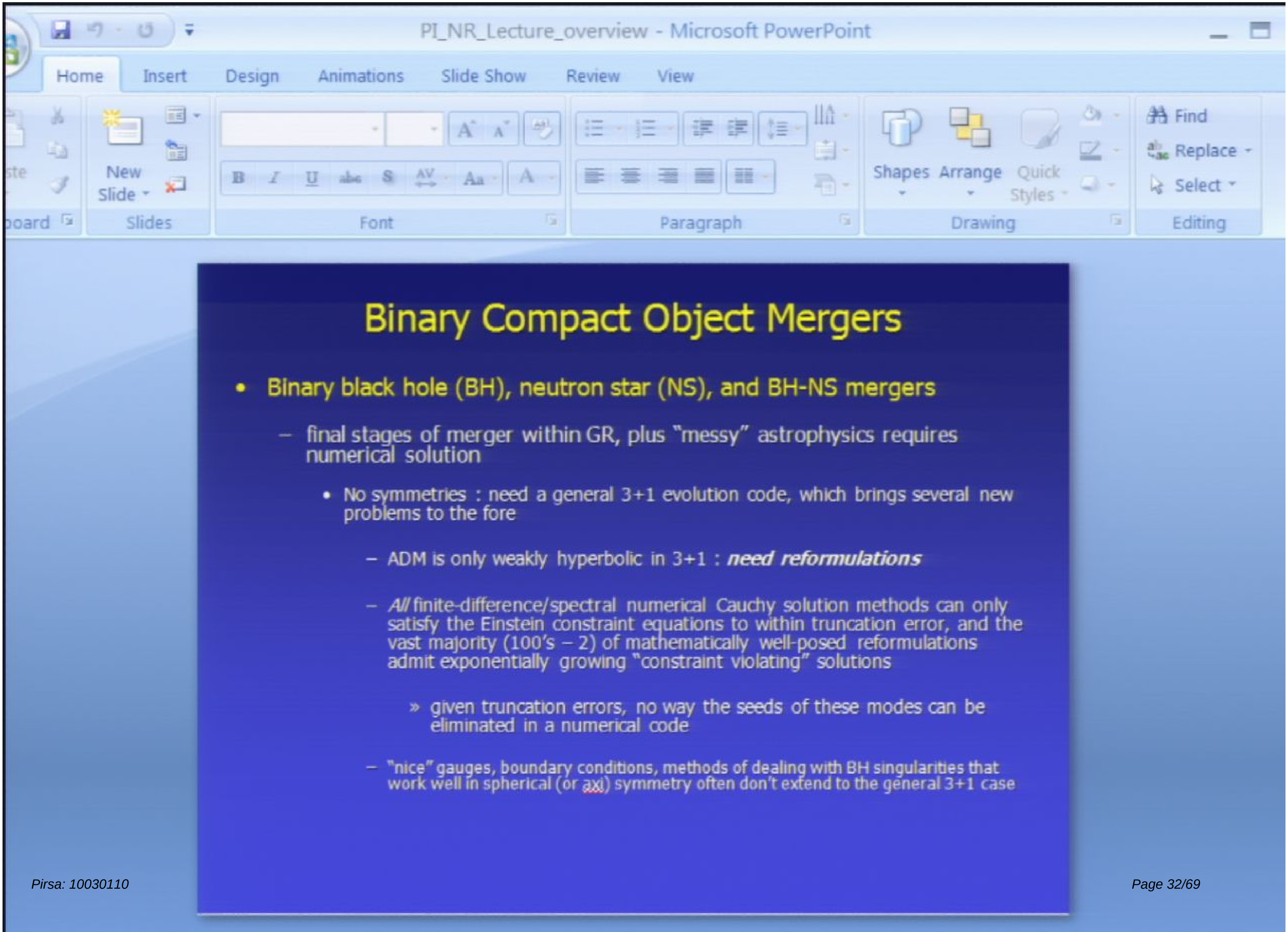

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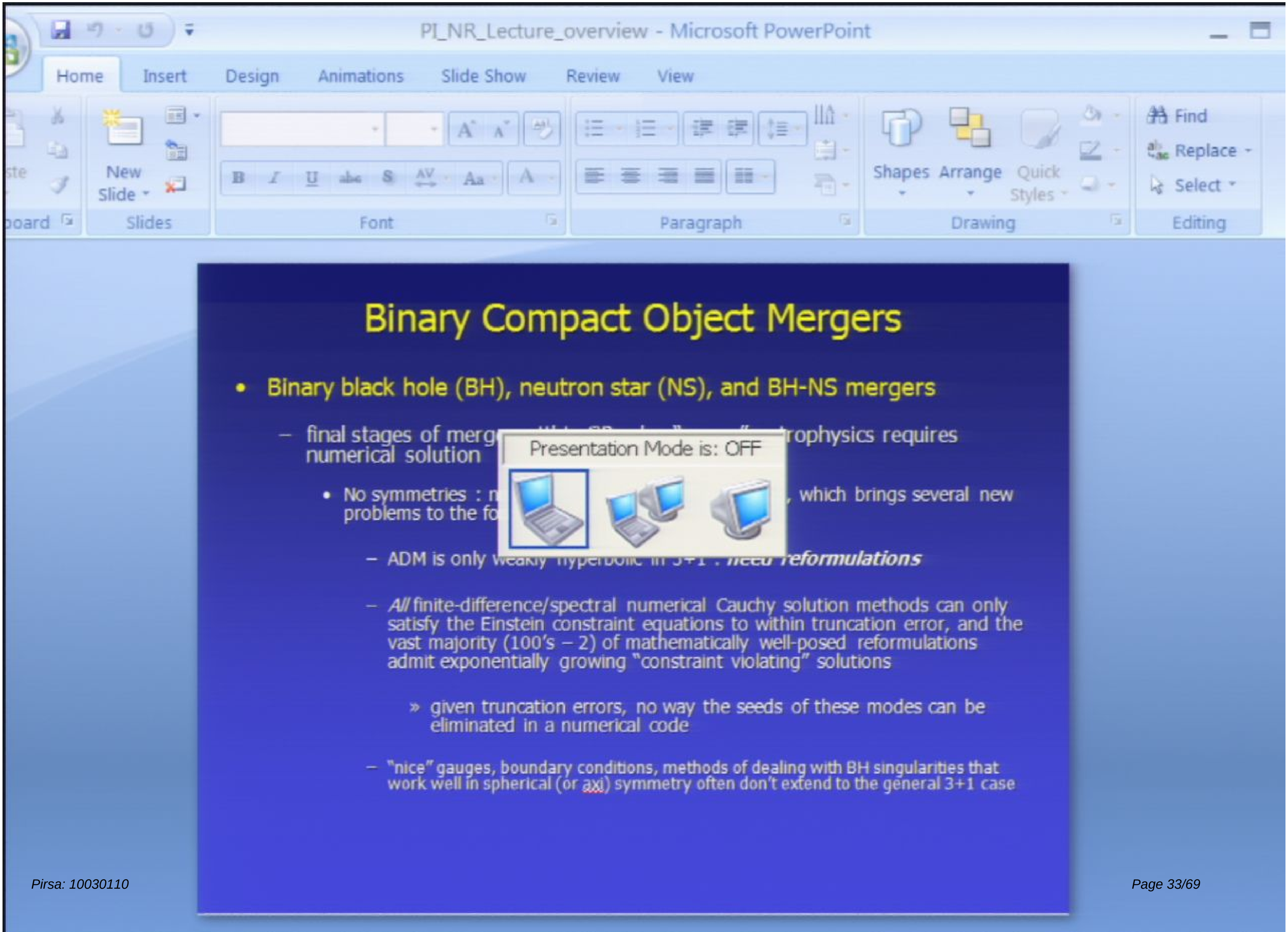

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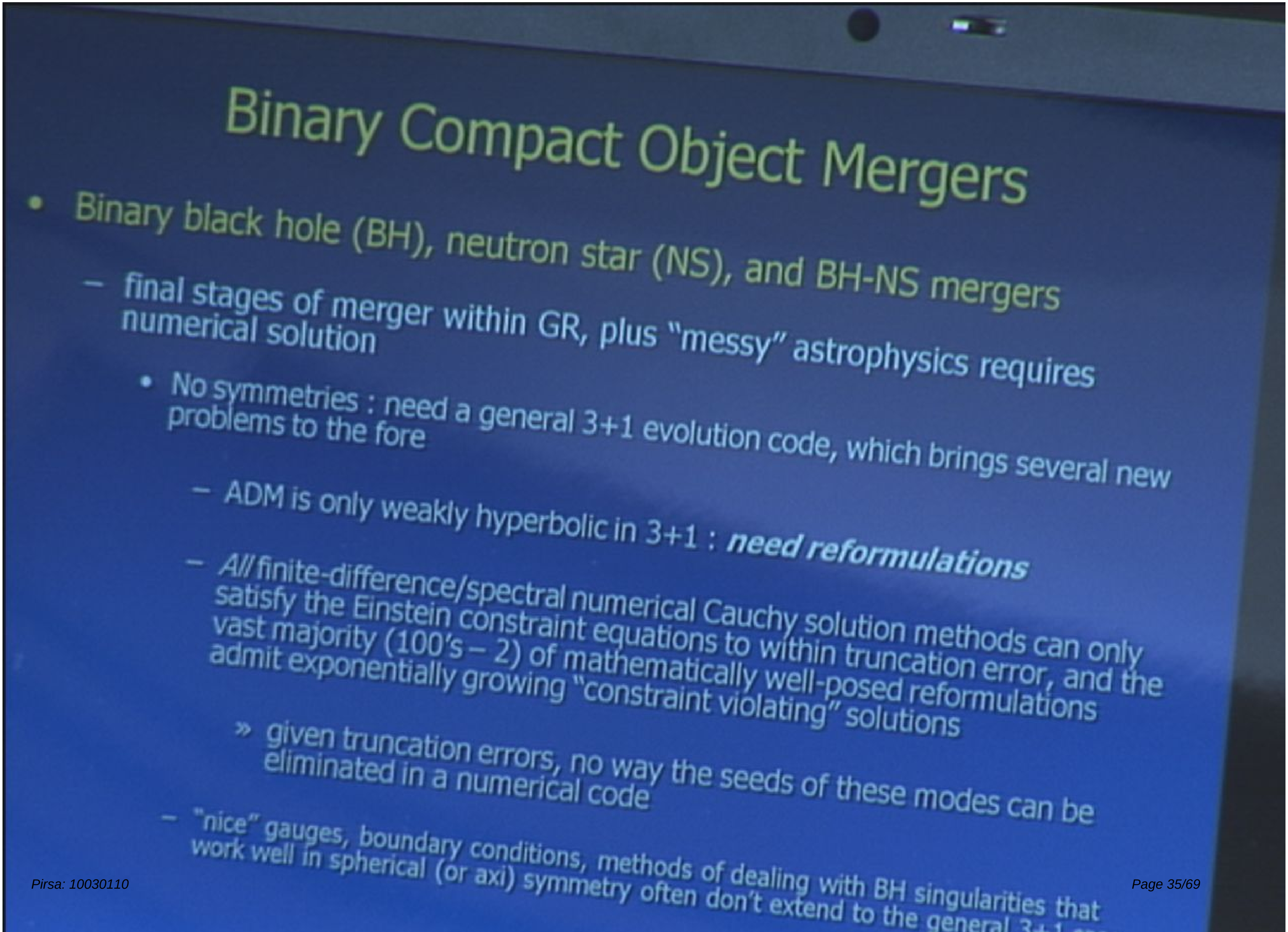
      - "nice" gauges, boundary conditions, methods of dealing with BH singularities that work well in spherical (or axi) symmetry often don't extend to the general 3+1 case



Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - final stages of merger within GR, plus "messy" astrophysics requires numerical solution
    - No symmetries : need a general 3+1 evolution code, which brings several new problems to the fore
      - ADM is only weakly hyperbolic in 3+1 : ***need reformulations***
      - *All* finite-difference/spectral numerical Cauchy solution methods can only satisfy the Einstein constraint equations to within truncation error, and the vast majority (100's – 2) of mathematically well-posed reformulations admit exponentially growing "constraint violating" solutions
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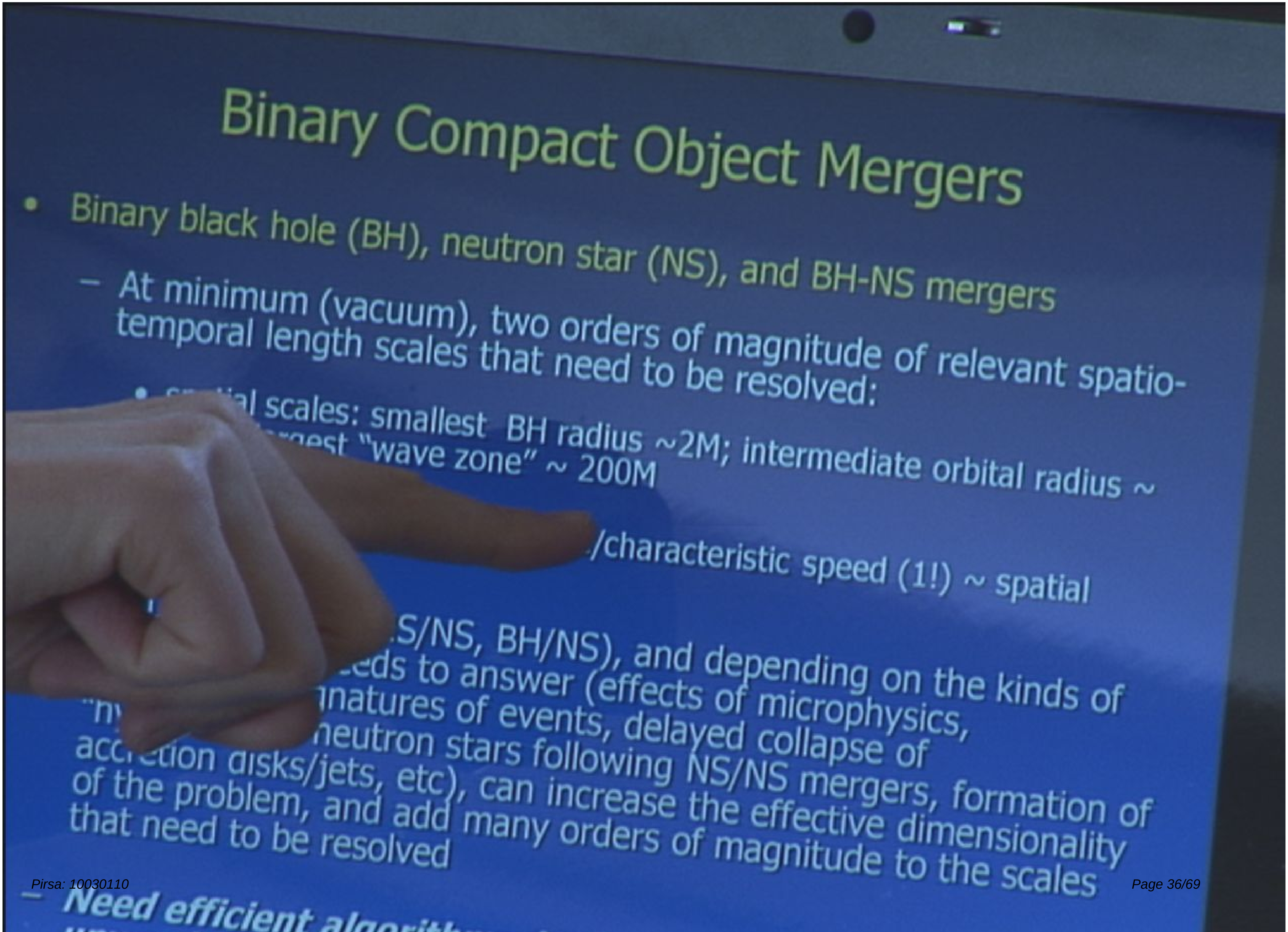

Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - At minimum (vacuum), two orders of magnitude of relevant spatio-temporal length scales that need to be resolved:
    - al scales: smallest BH radius ~2M; intermediate orbital radius ~ gest "wave zone" ~ 200M
    - /characteristic speed (1!) ~ spatial

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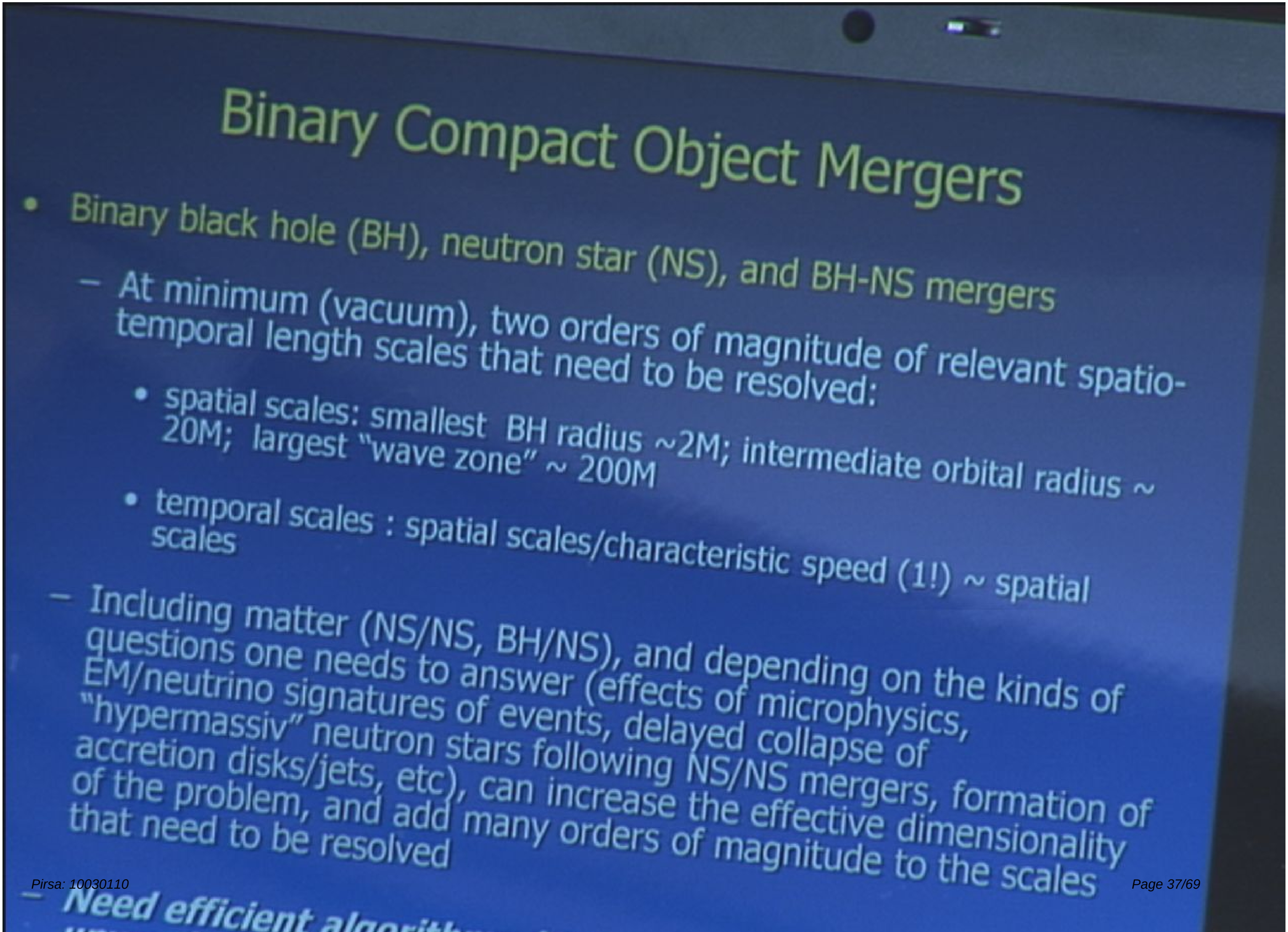
  - ***Need efficient algorit***



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  - Including matter (NS/NS, BH/NS), and depending on the kinds of questions one needs to answer (effects of microphysics, EM/neutrino signatures of events, delayed collapse of "hypermassiv" neutron stars following NS/NS mergers, formation of accretion disks/jets, etc), can increase the effective dimensionality of the problem, and add many orders of magnitude to the scales that need to be resolved
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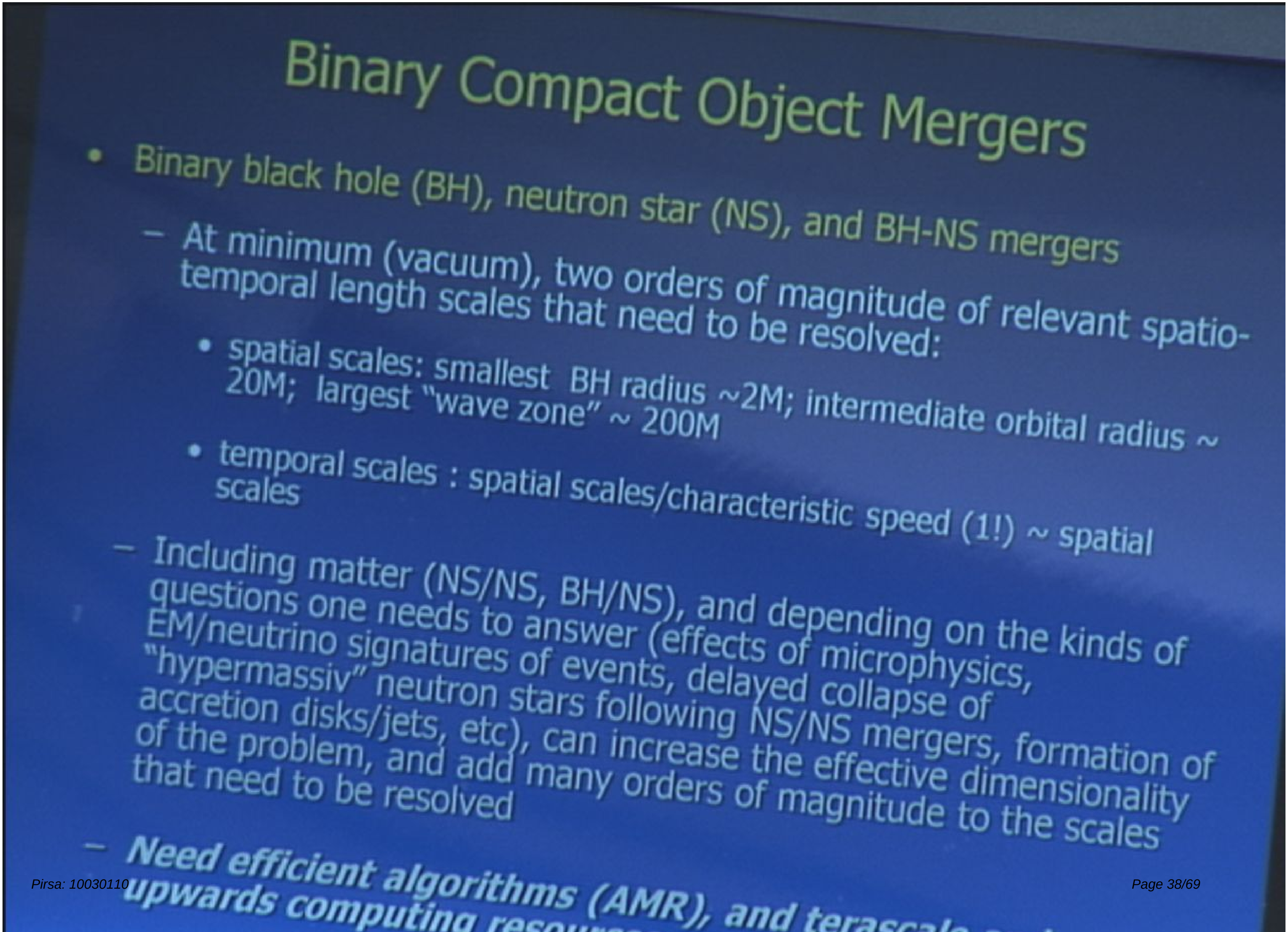

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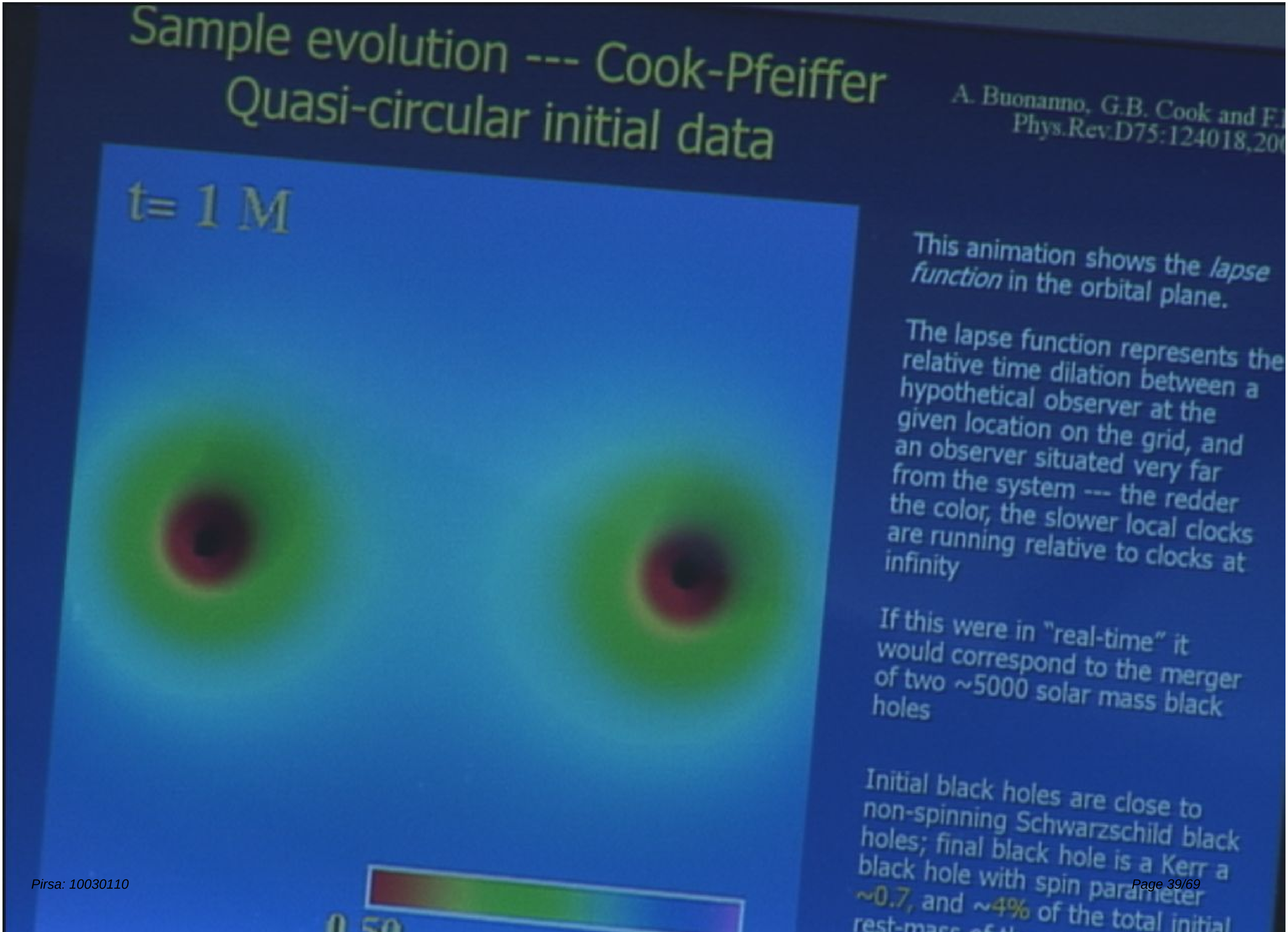
  - ***Need efficient algorithms (AMR), and terasca***
    upwards computing reso



Sample evolution --- Cook-Pfeiffer Quasi-circular initial data

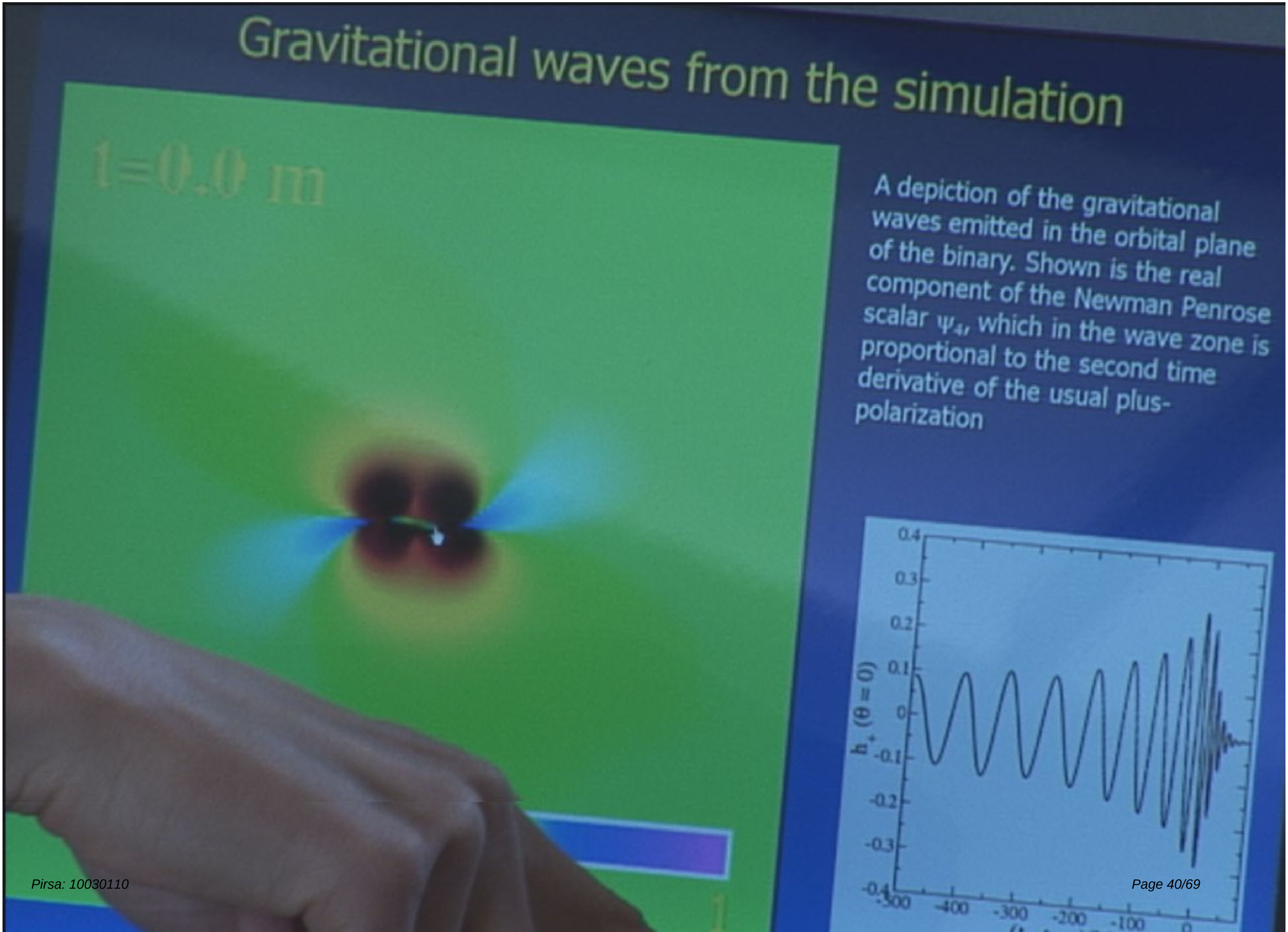
A. Buonanno, G.B. Cook and F.
Phys.Rev.D75:124018,20

This animation shows the *lapse function* in the orbital plane.

The lapse function represents the relative time dilation between a hypothetical observer at the given location on the grid, and an observer situated very far from the system --- the redder the color, the slower local clocks are running relative to clocks at infinity

If this were in "real-time" it would correspond to the merger of two ~5000 solar mass black holes

Initial black holes are close to non-spinning Schwarzschild black holes; final black hole is a Kerr a black hole with spin parameter ~0.7, and ~4% of the total initial rest-mass of th



Gravitational waves from the simulation

A depiction of the gravitational waves emitted in the orbital plane of the binary. Shown is the real component of the Newman Penrose scalar ψ_4, which in the wave zone is proportional to the second time derivative of the usual plus-polarization

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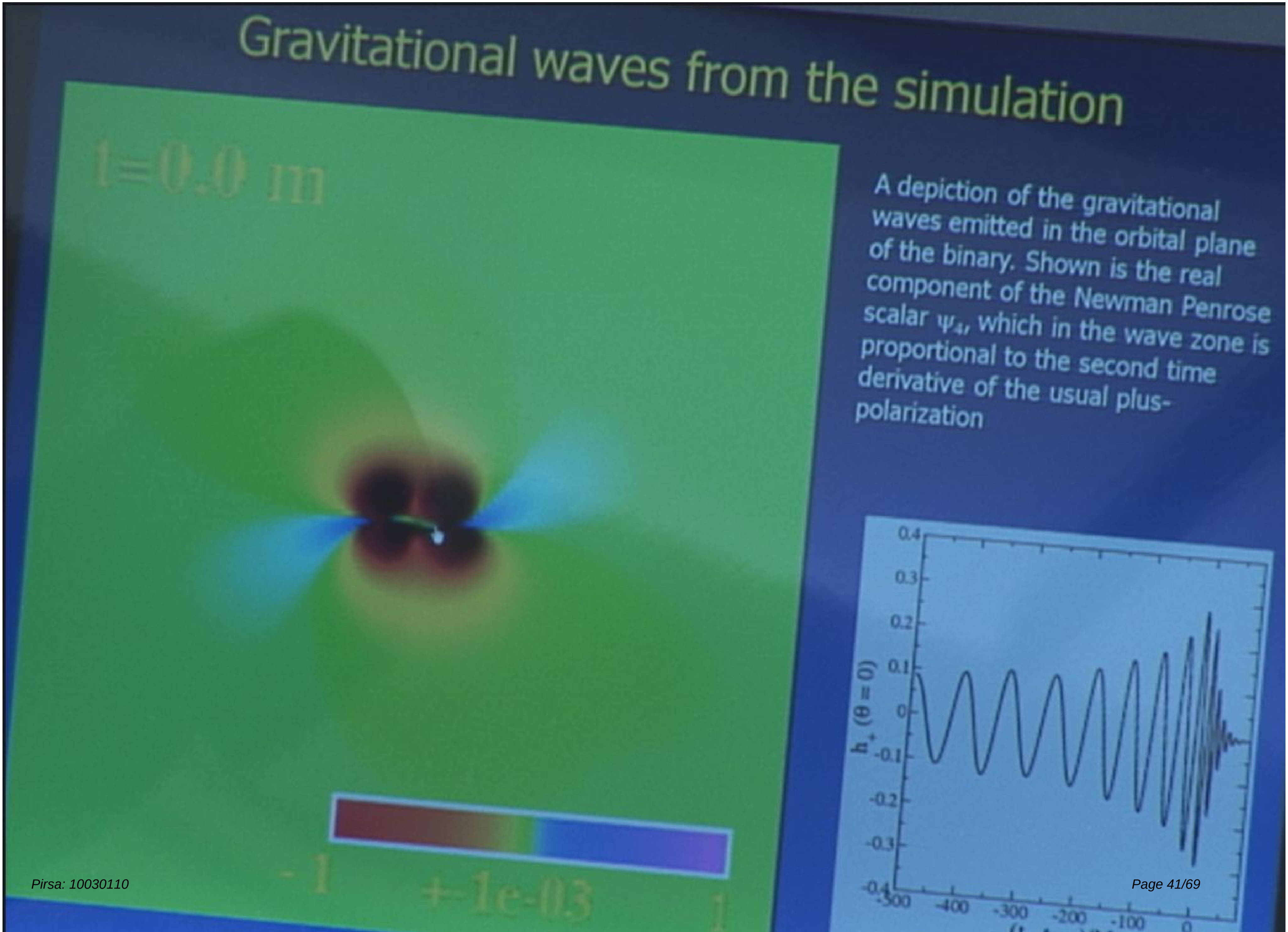

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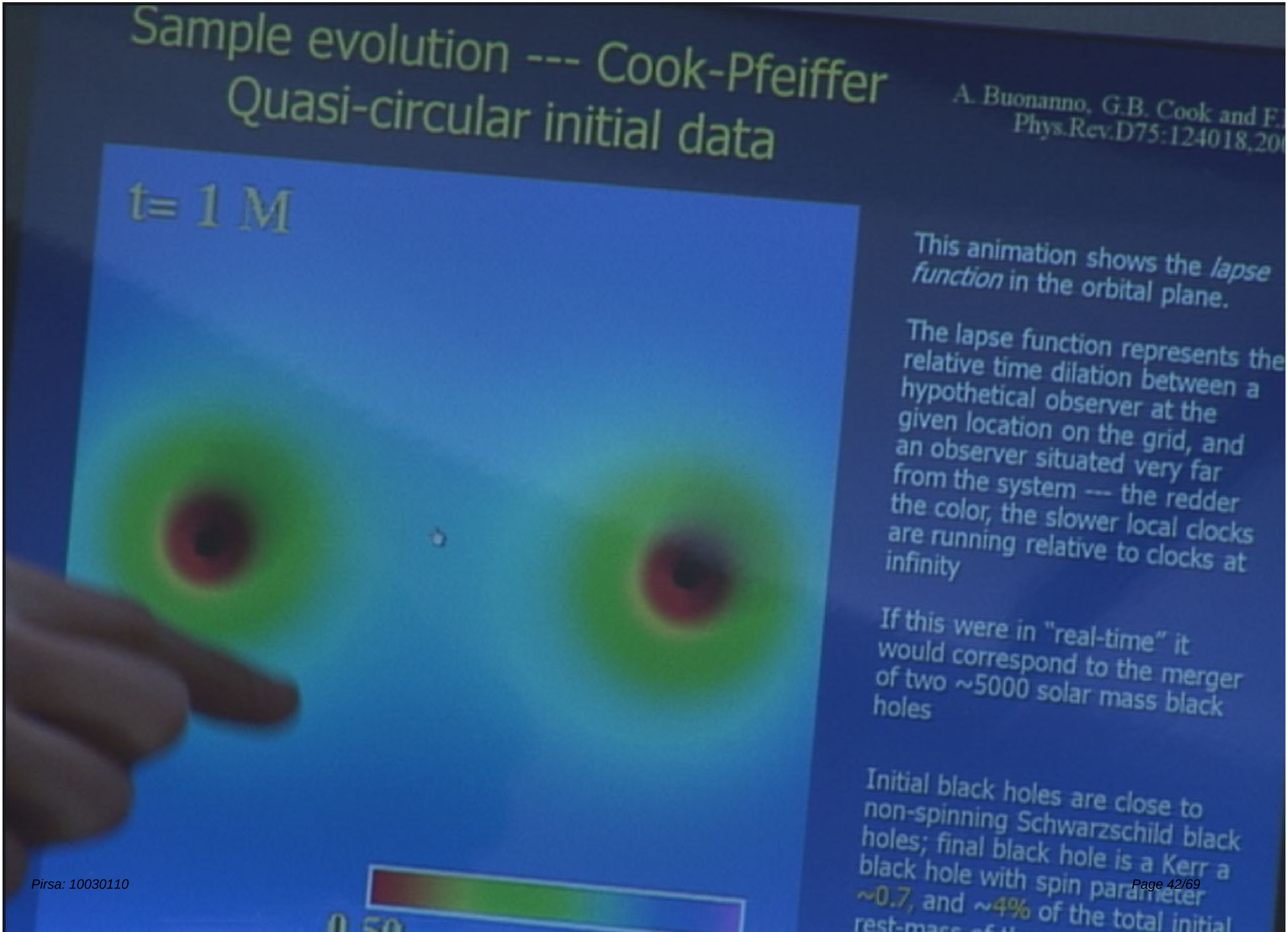

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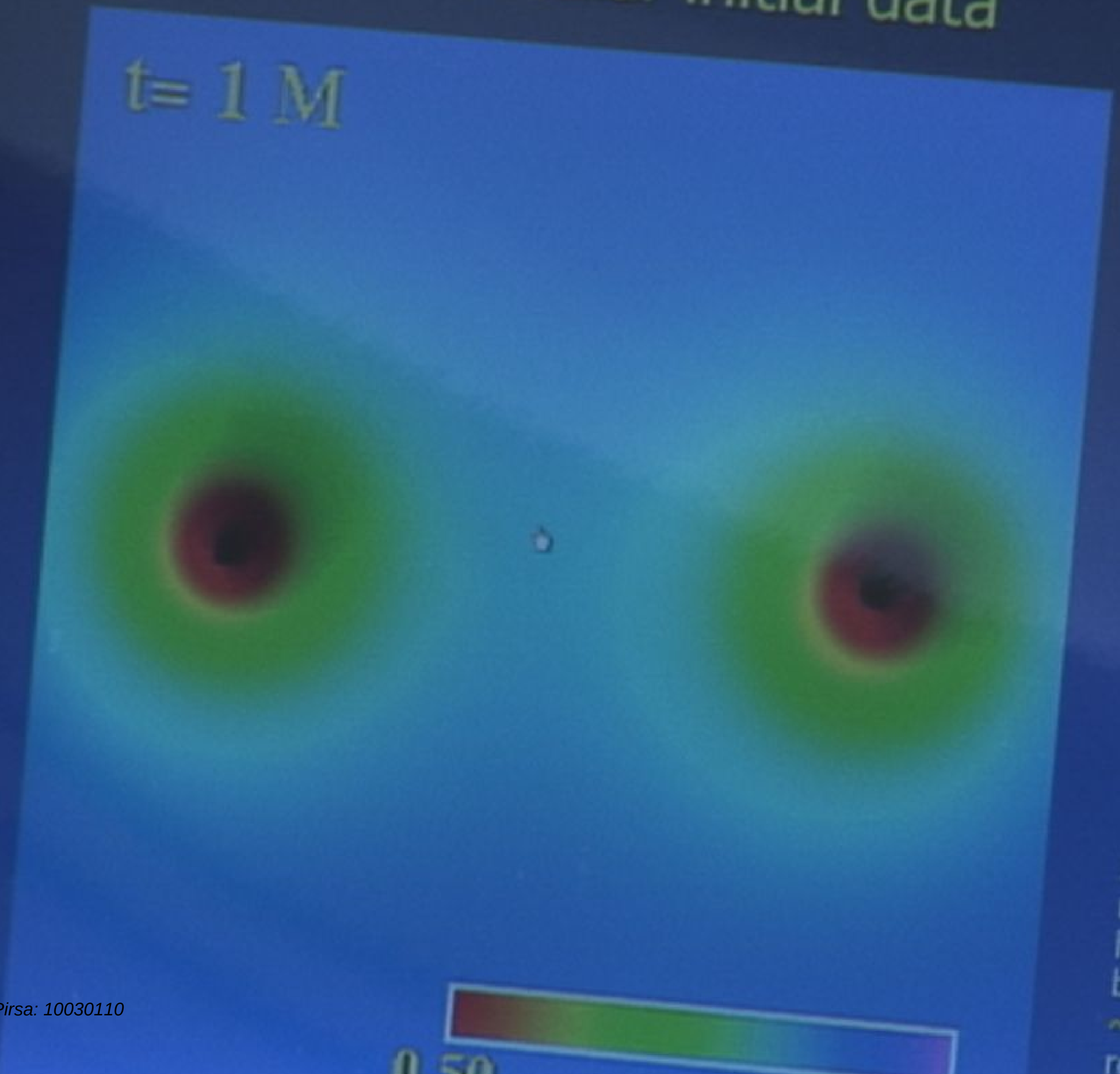

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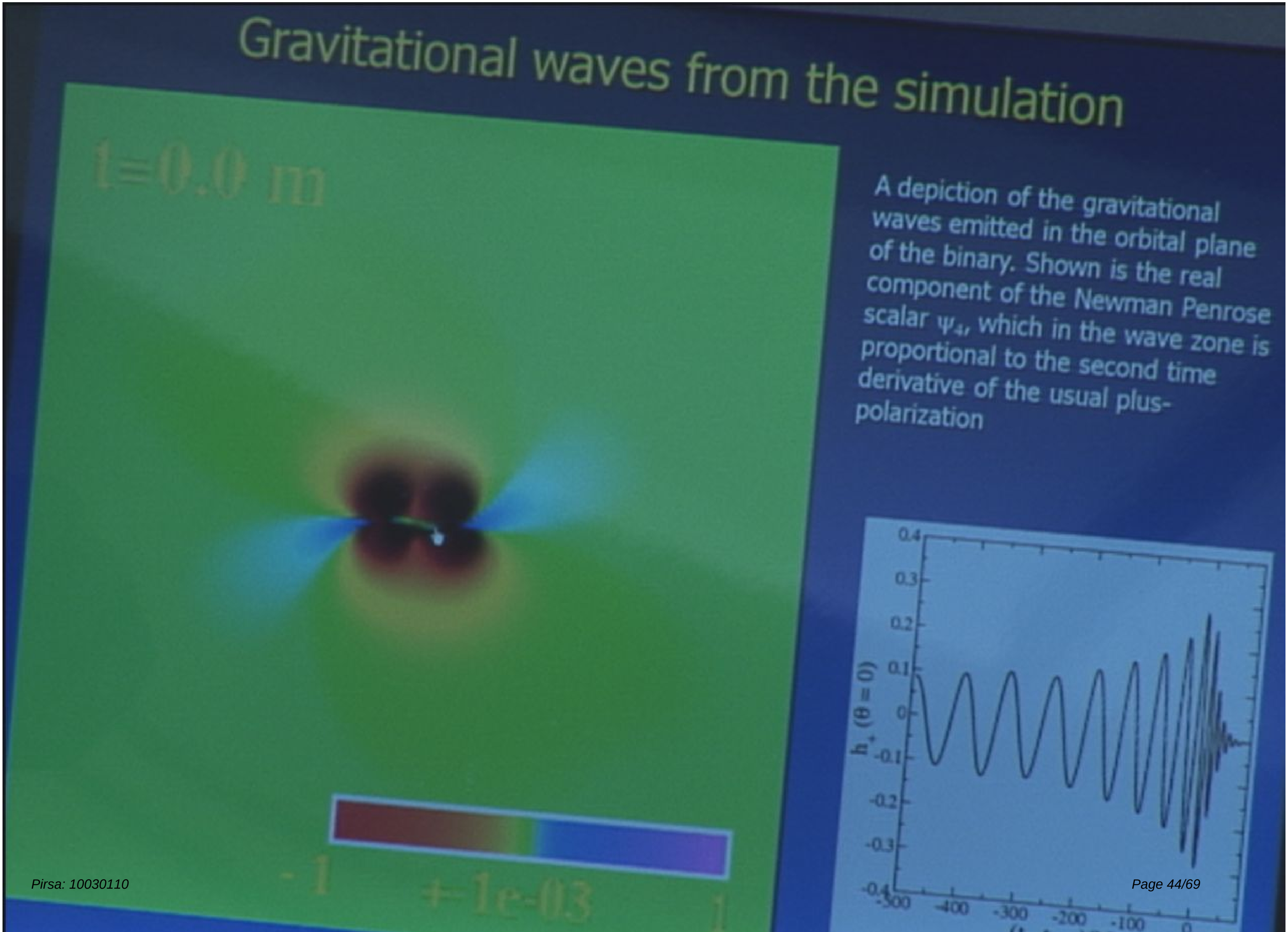

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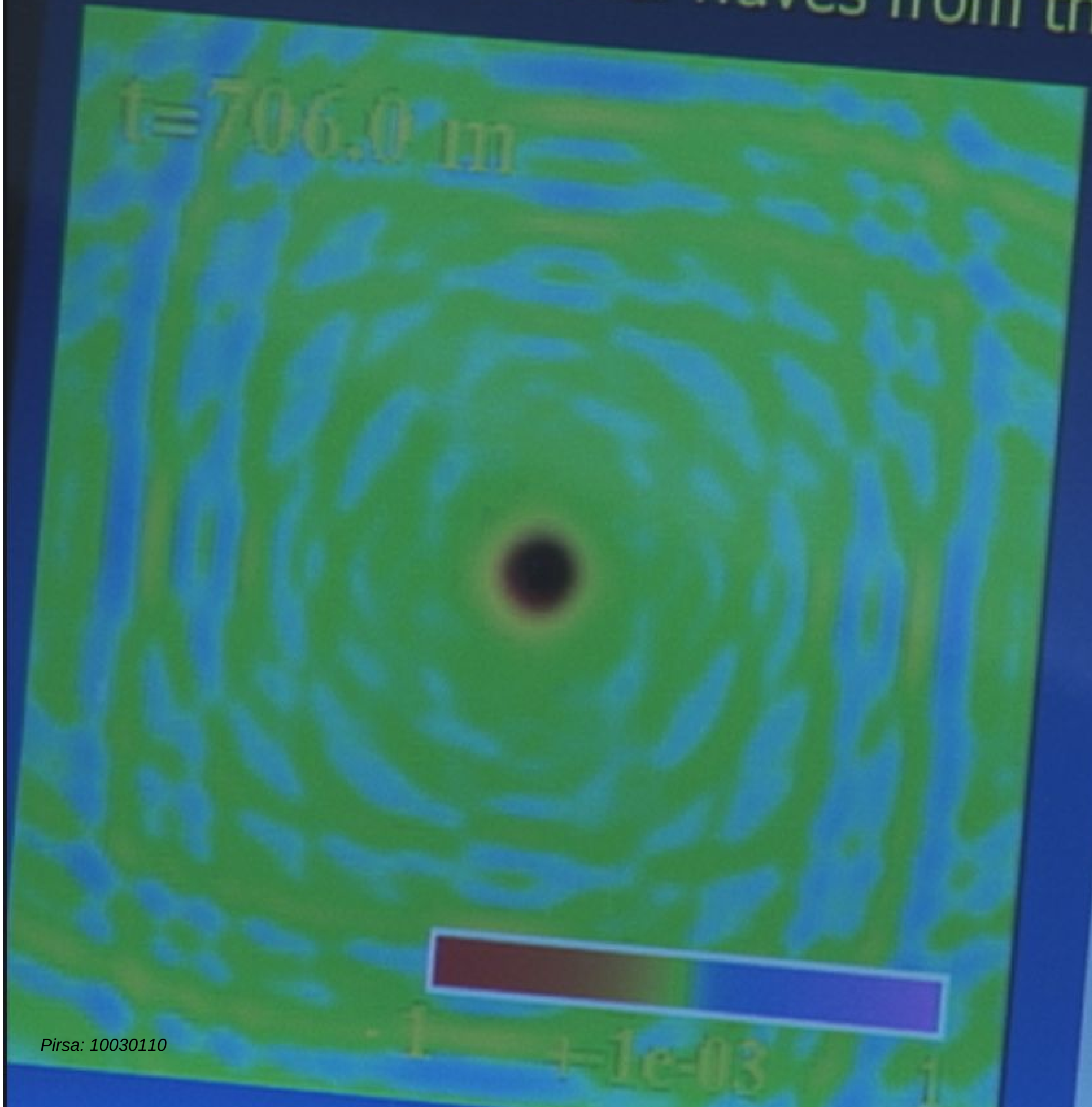

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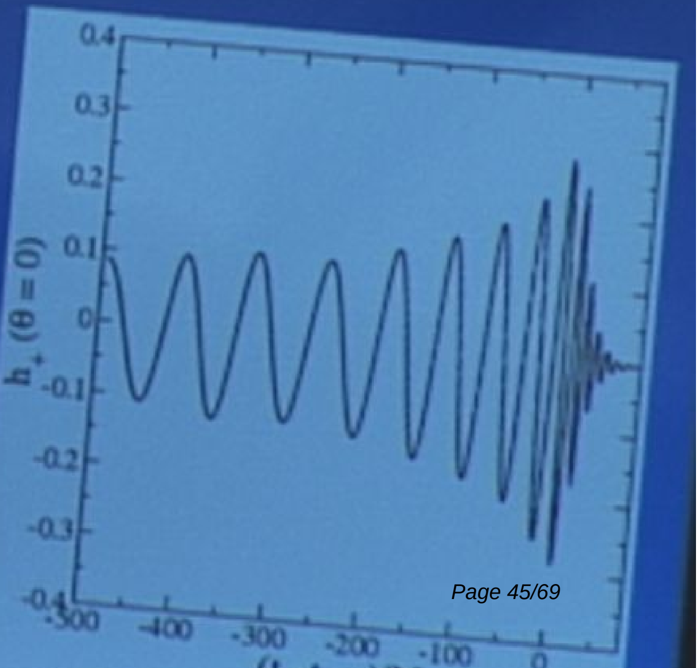

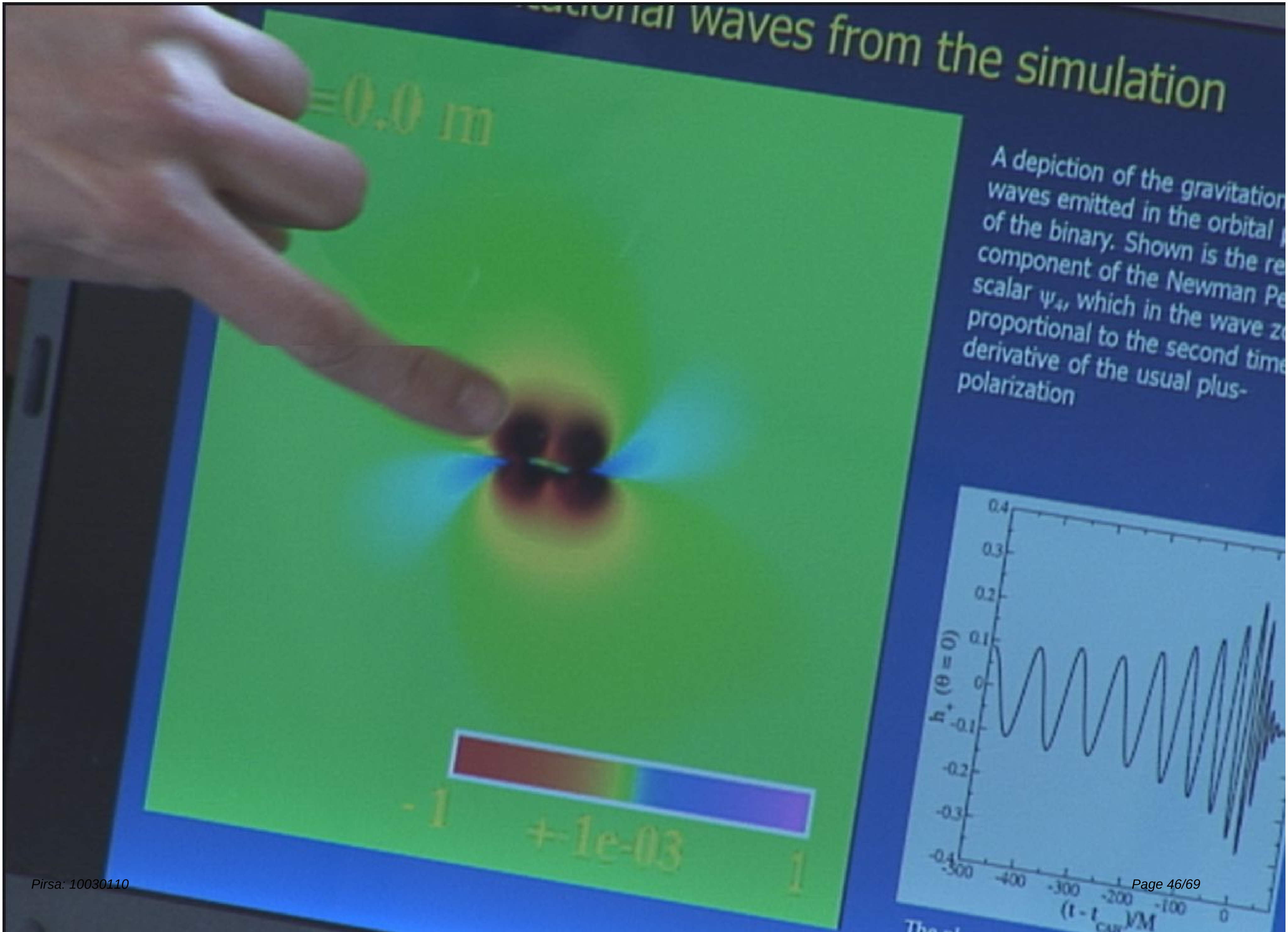

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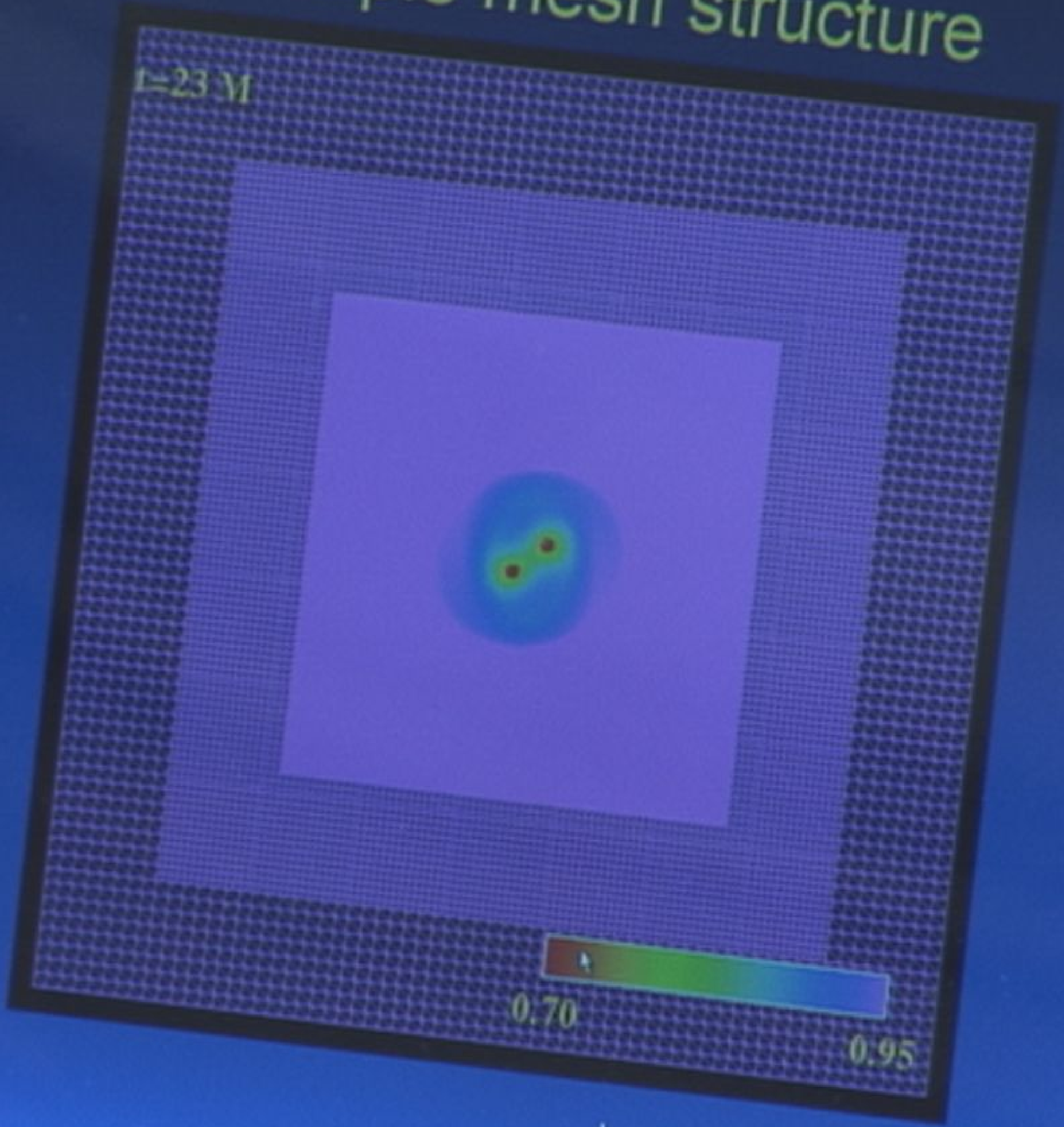
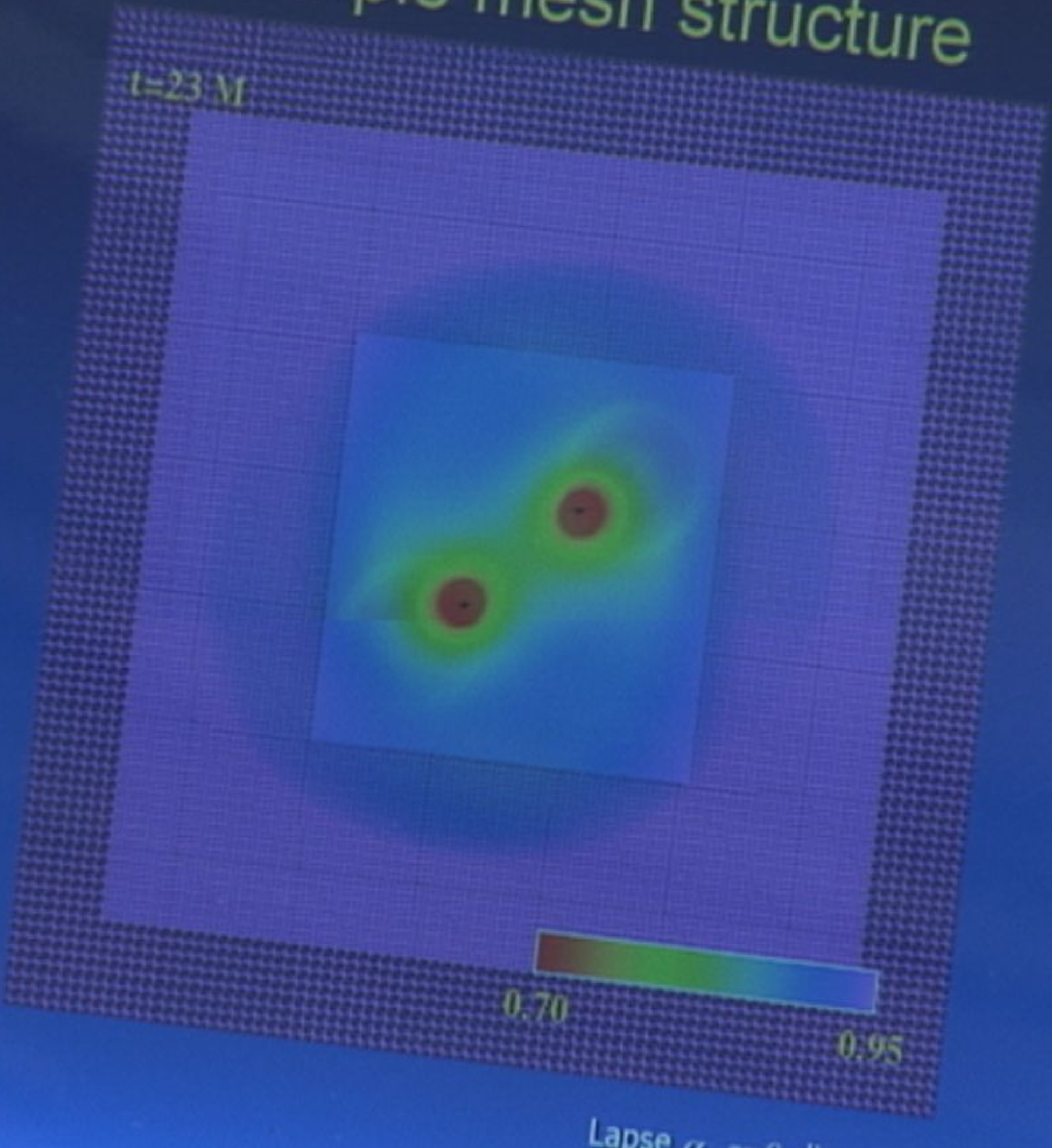
Sample mesh structure



Lapse

Sample mesh structure



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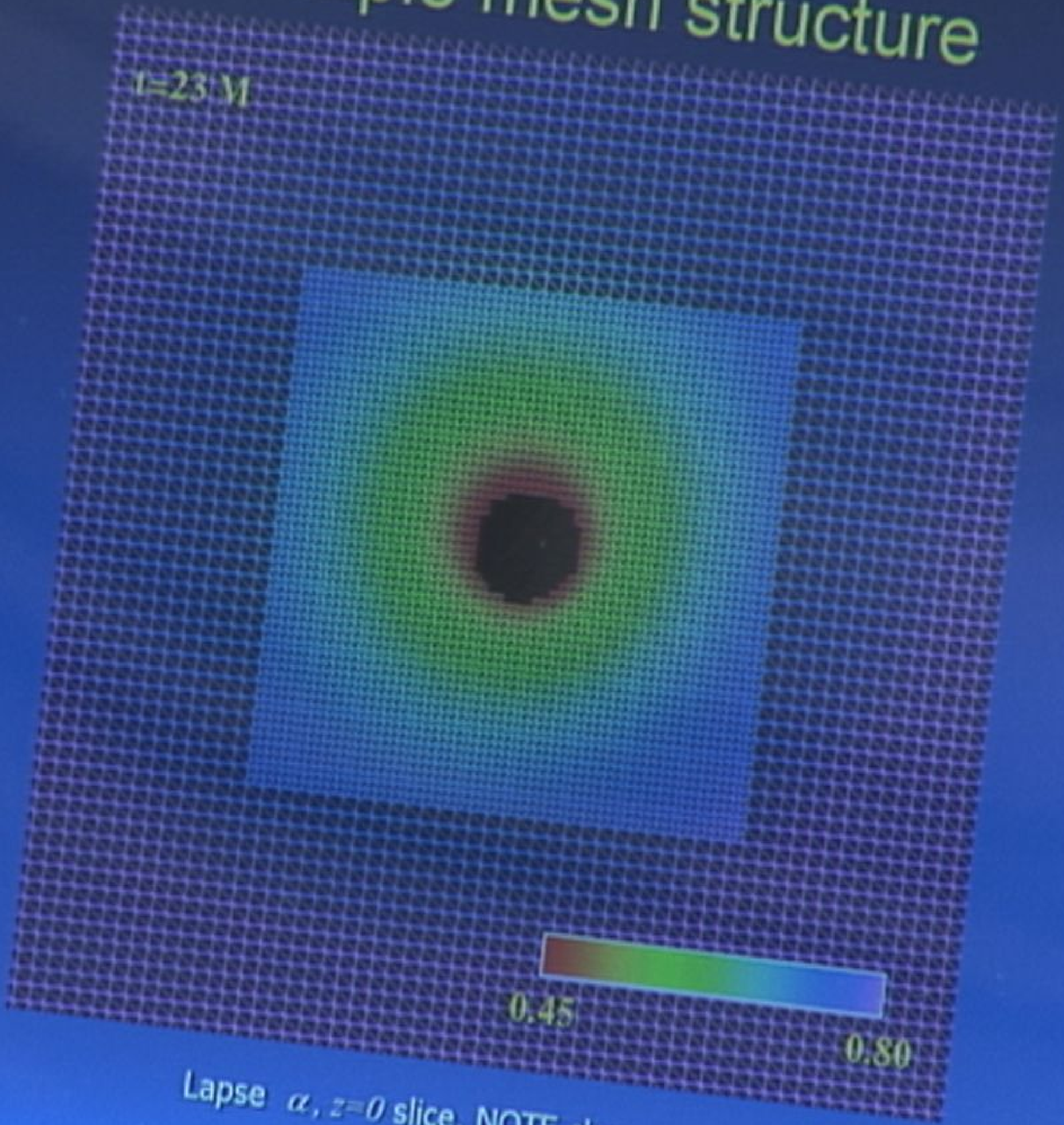
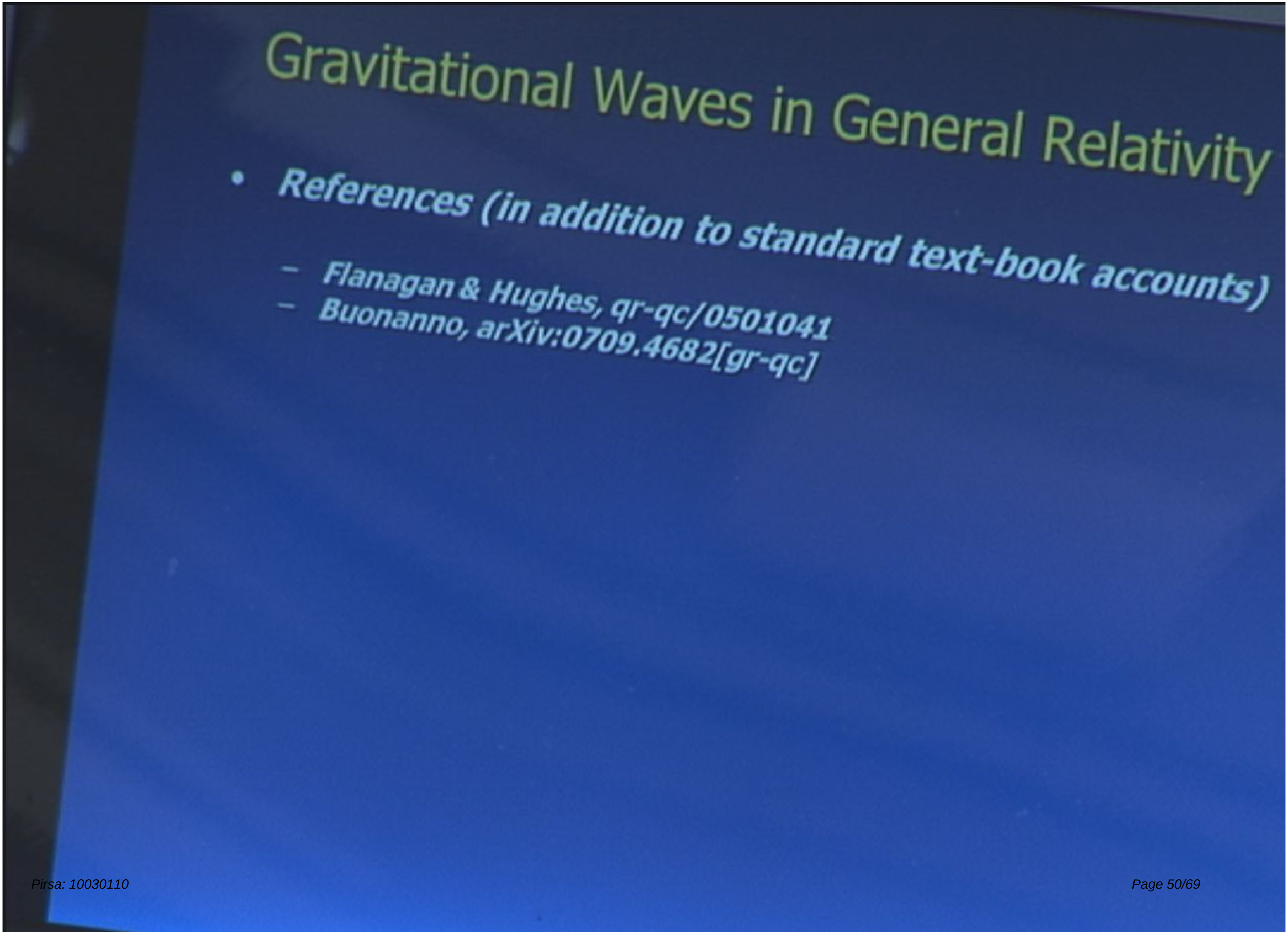
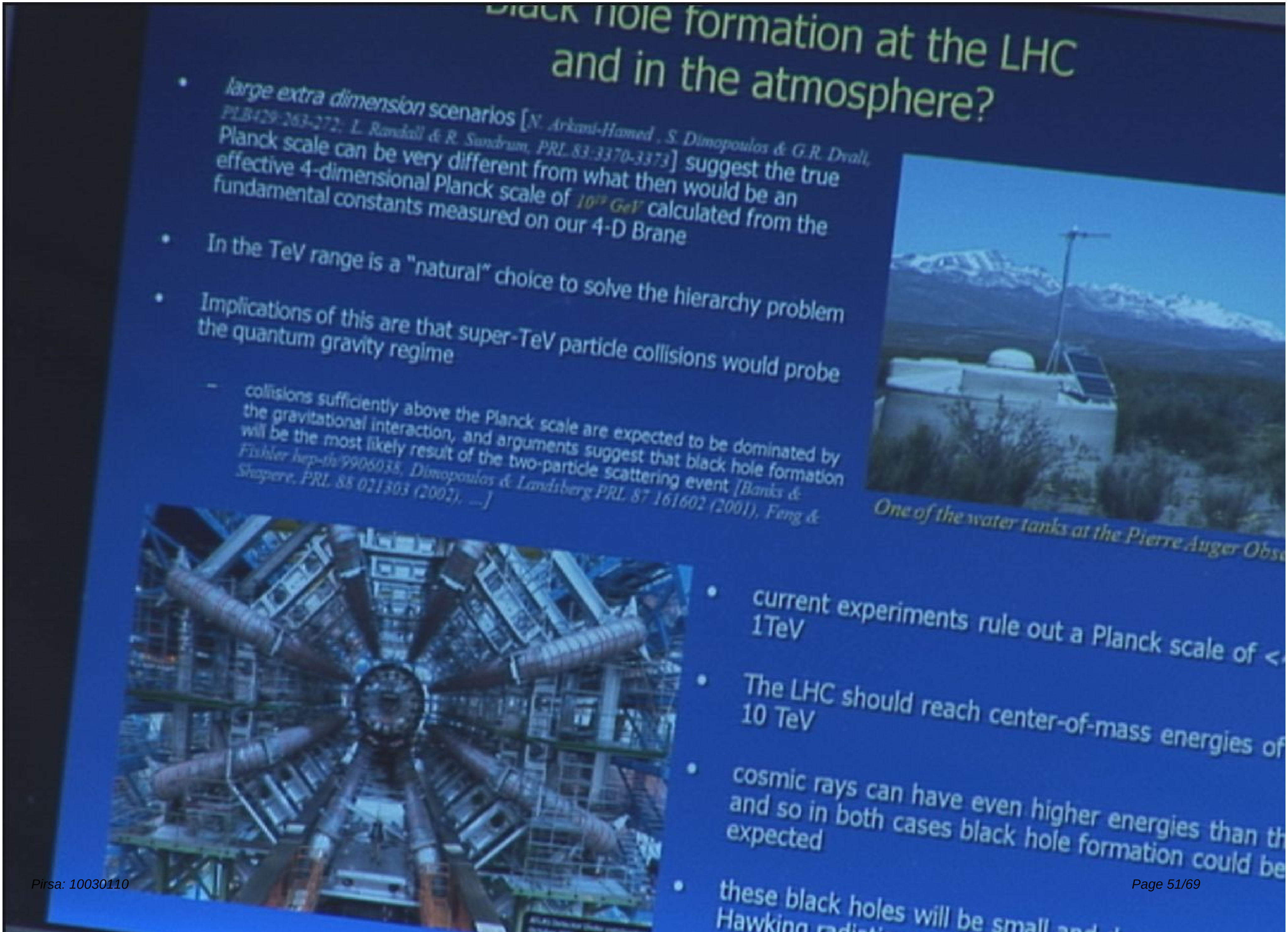


Lapse α, $z=0$ slice, NOTE change of



Gravitational Waves in General Relativity

- ***References (in addition to standard text-book accounts)***
  - *Flanagan & Hughes, qr-qc/0501041*
  - *Buonanno, arXiv:0709.4682[gr-qc]*

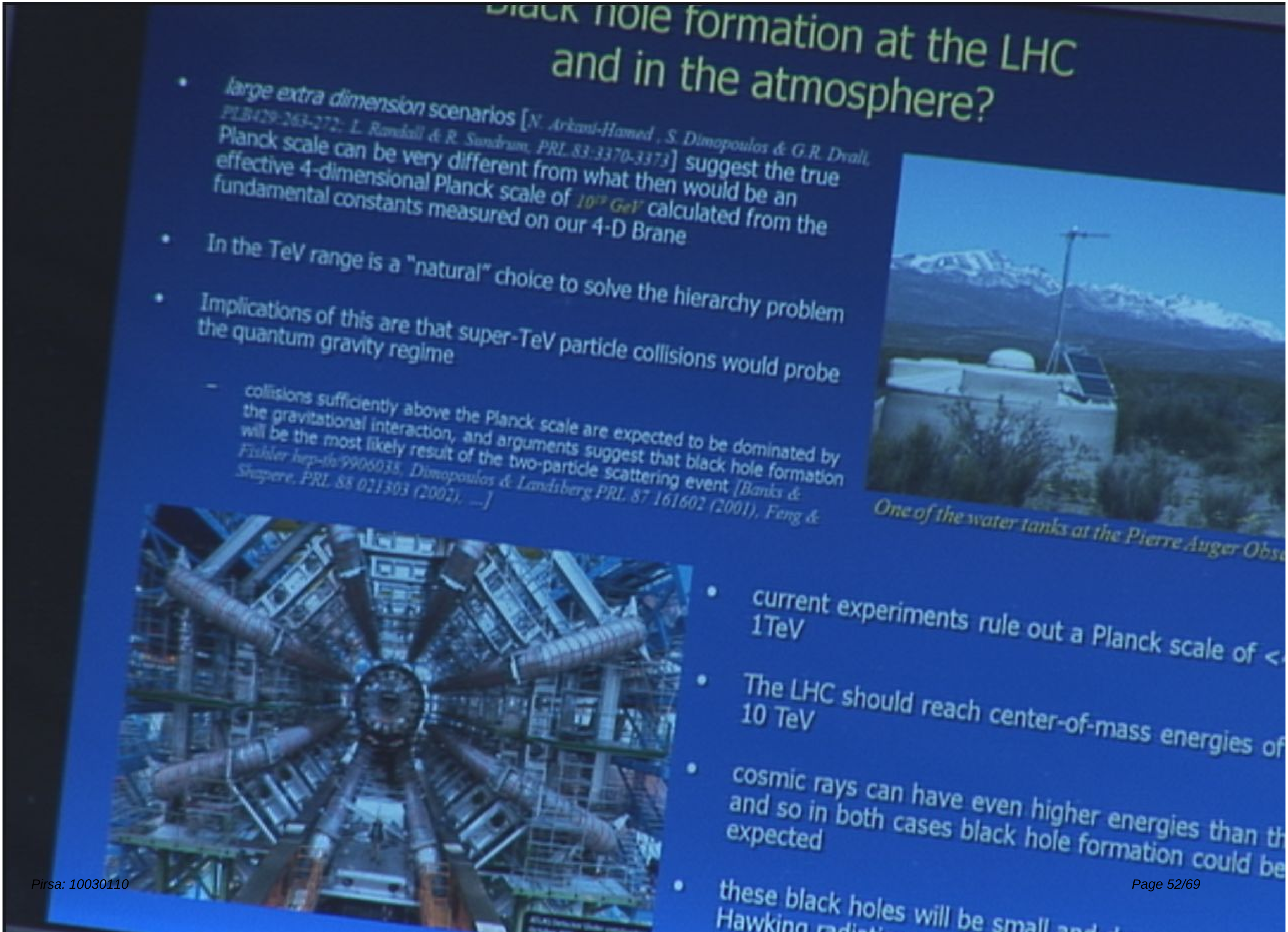


lack hole formation at the LHC and in the atmosphere?

- *large extra dimension* scenarios [*N. Arkani-Hamed , S. Dimopoulos & G.R. Dvali, PLB429:263-272; L. Randall & R. Sundrum, PRL.83:3370-3373*] suggest the true Planck scale can be very different from what then would be an effective 4-dimensional Planck scale of 10^{19} *GeV* calculated from the fundamental constants measured on our 4-D Brane
- In the TeV range is a "natural" choice to solve the hierarchy problem
- Implications of this are that super-TeV particle collisions would probe the quantum gravity regime
  - collisions sufficiently above the Planck scale are expected to be dominated by the gravitational interaction, and arguments suggest that black hole formation will be the most likely result of the two-particle scattering event [*Banks & Fishler hep-th/9906038, Dimopoulos & Landsberg PRL 87 161602 (2001), Feng & Shapere, PRL 88 021303 (2002), ...*]

One of the water tanks at the Pierre Auger Obs

- current experiments rule out a Planck scale of <
  1TeV
- The LHC should reach center-of-mass energies of
  10 TeV
- cosmic rays can have even higher energies than th
  and so in both cases black hole formation could be
  expected
- these black holes will be small an
  Hawking radi



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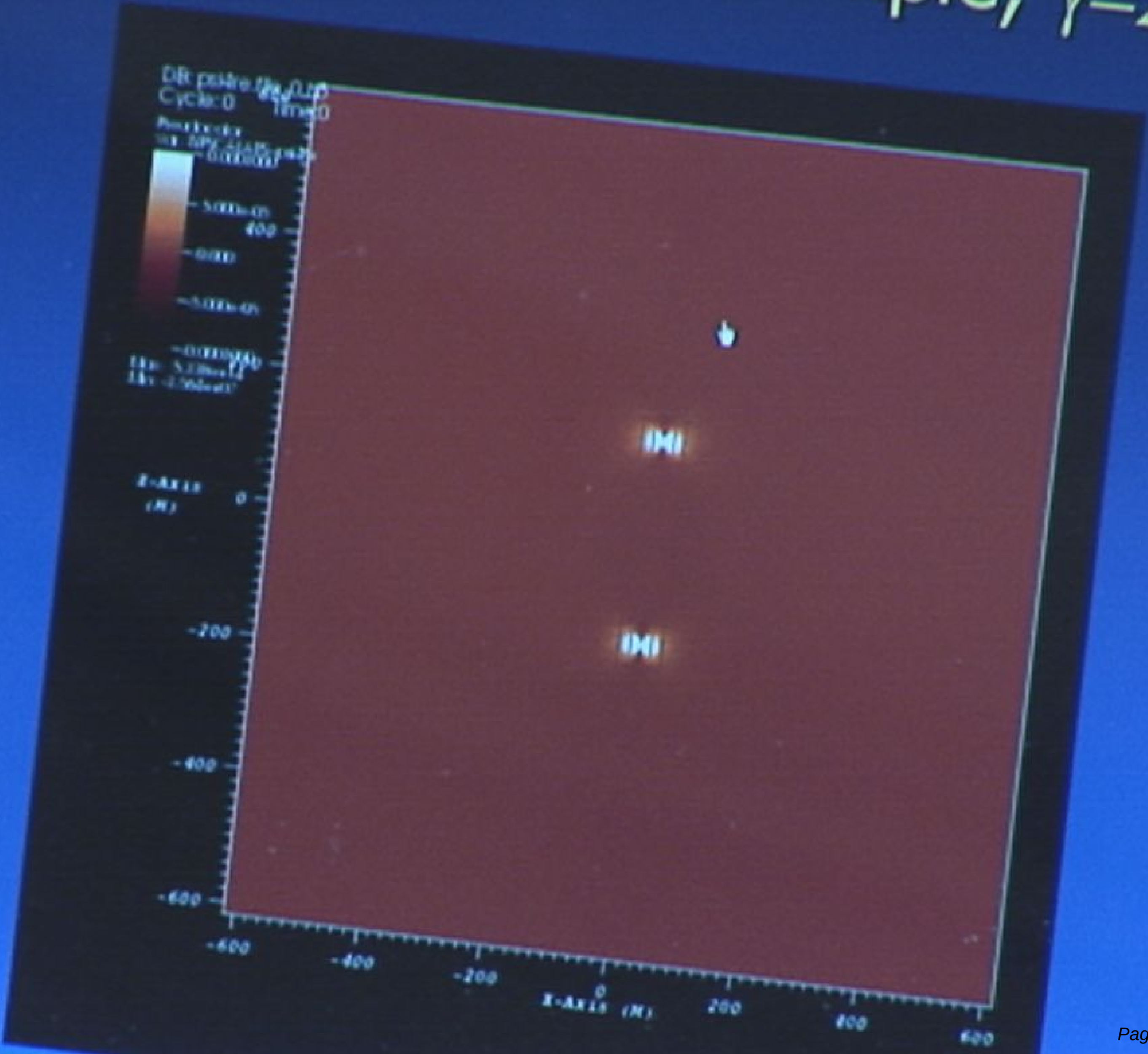
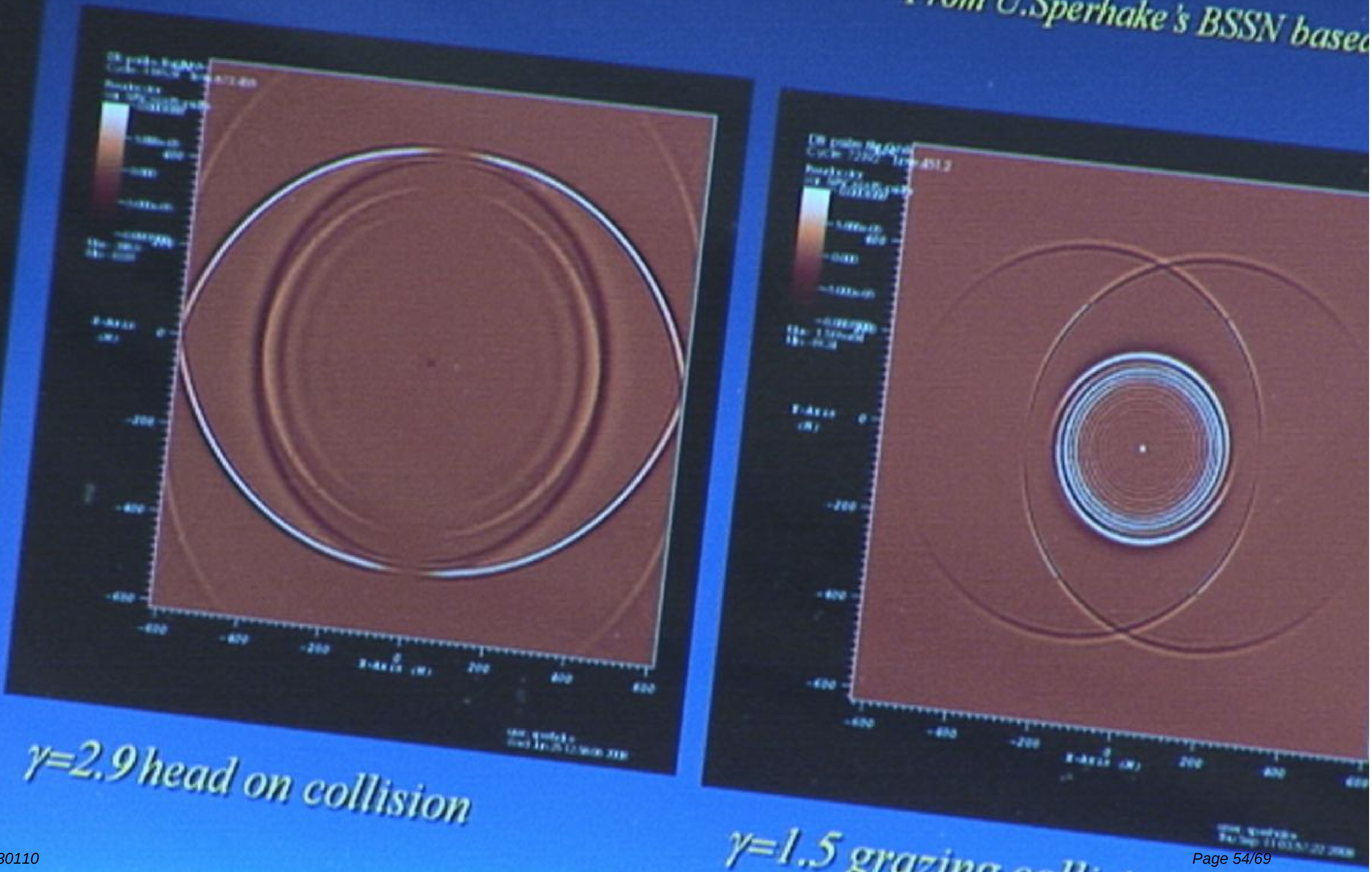
head-on collision example, $\gamma=2.9$



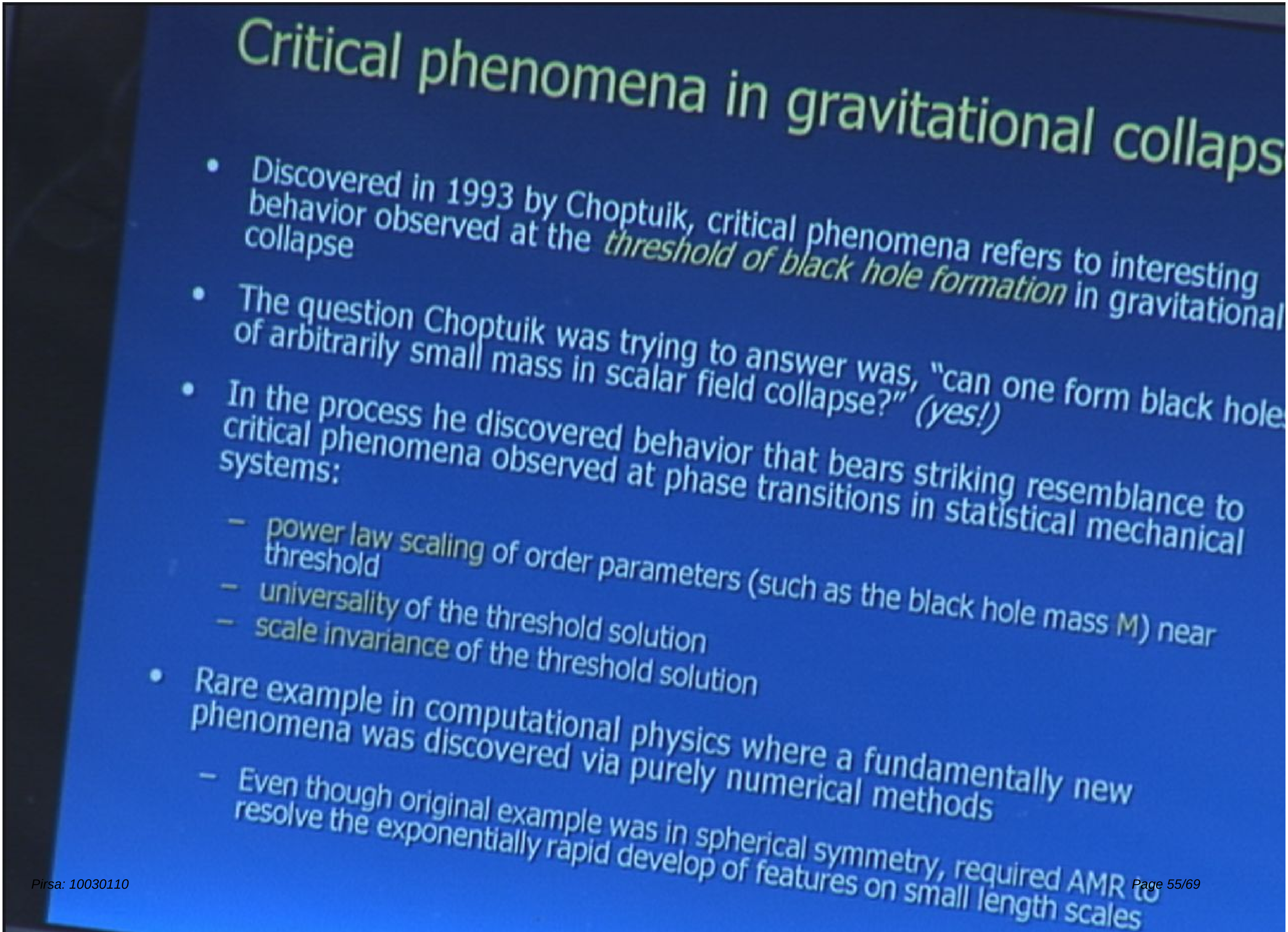
Sample BH collisions

From U.Sperhake's BSSN based



γ=2.9 head on collision

γ=1.5 grazing collision



Critical phenomena in gravitational collaps

- Discovered in 1993 by Choptuik, critical phenomena refers to interesting behavior observed at the *threshold of black hole formation* in gravitational collapse
- The question Choptuik was trying to answer was, "can one form black hole of arbitrarily small mass in scalar field collapse?" *(yes!)*
- In the process he discovered behavior that bears striking resemblance to critical phenomena observed at phase transitions in statistical mechanical systems:
  - power law scaling of order parameters (such as the black hole mass M) near threshold
  - universality of the threshold solution
  - scale invariance of the threshold solution
- Rare example in computational physics where a fundamentally new phenomena was discovered via purely numerical methods
  - Even though original example was in spherical symmetry, required AMR to resolve the exponentially rapid develop of features on small length scales

Finding the threshold of black hole formation

- Consider a smooth, one parameter (p) family of initial data, where p is, in some sense, related to the energy density of the initial configuration

- Then for $p<p^*$, evolution will lead to dispersal, while for $p>p^*$ a black hole will form — p^* labels the critical solution for this family
- In a numerical "experiment", p^* can be found via a bisection search

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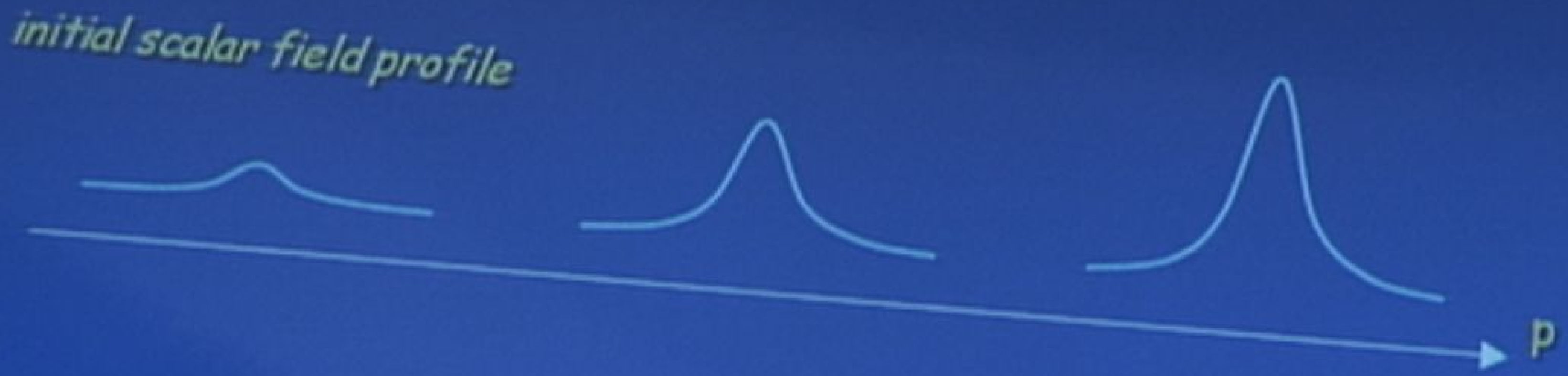

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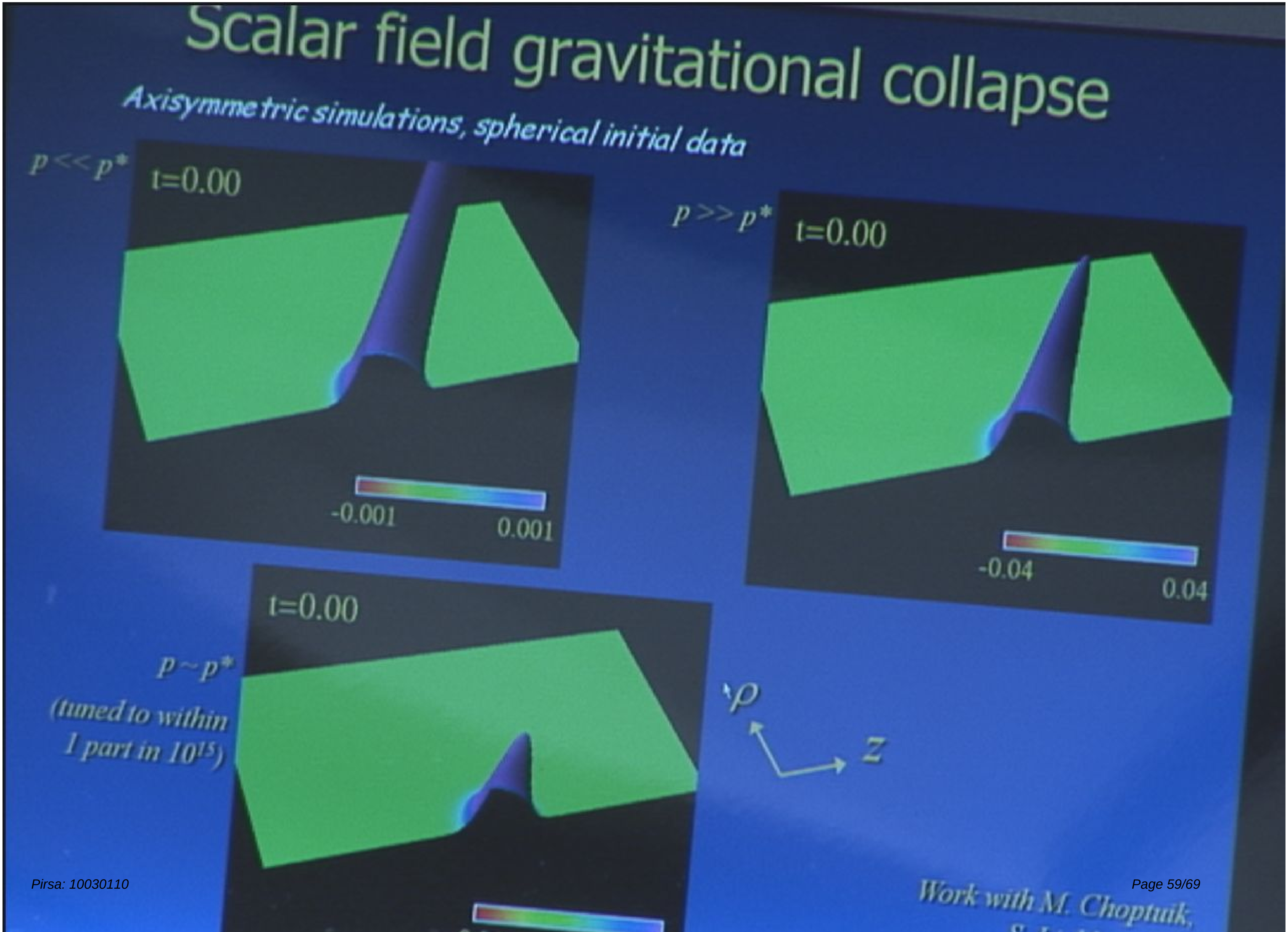


Scalar field gravitational collapse

Axisymmetric simulations, spherical initial data

$p << p^*$

$p >> p^*$

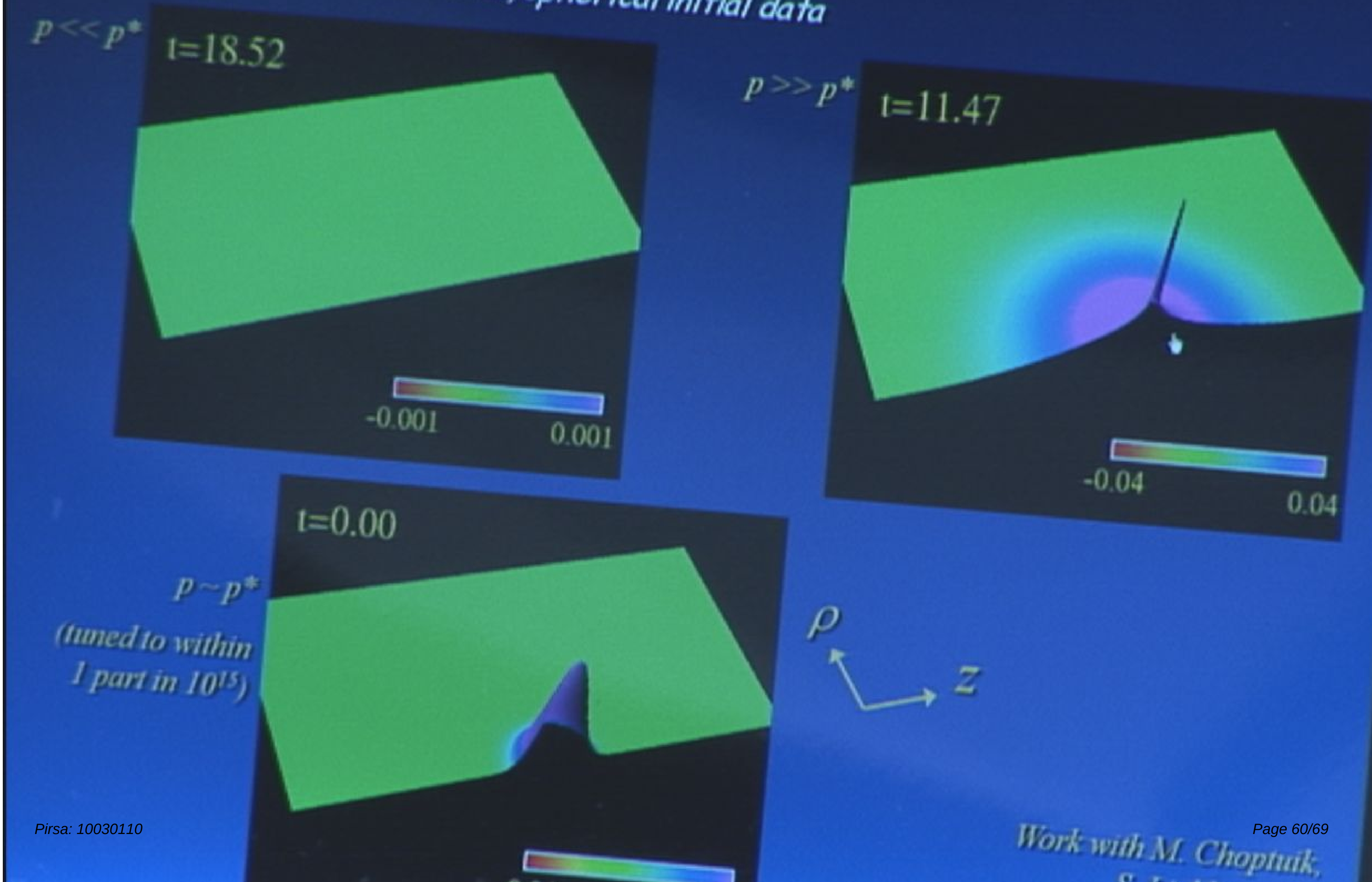
$p \sim p^*$

(tuned to within 1 part in 10^{15})

Work with M. Choptuik,

Scalar field gravitational collapse

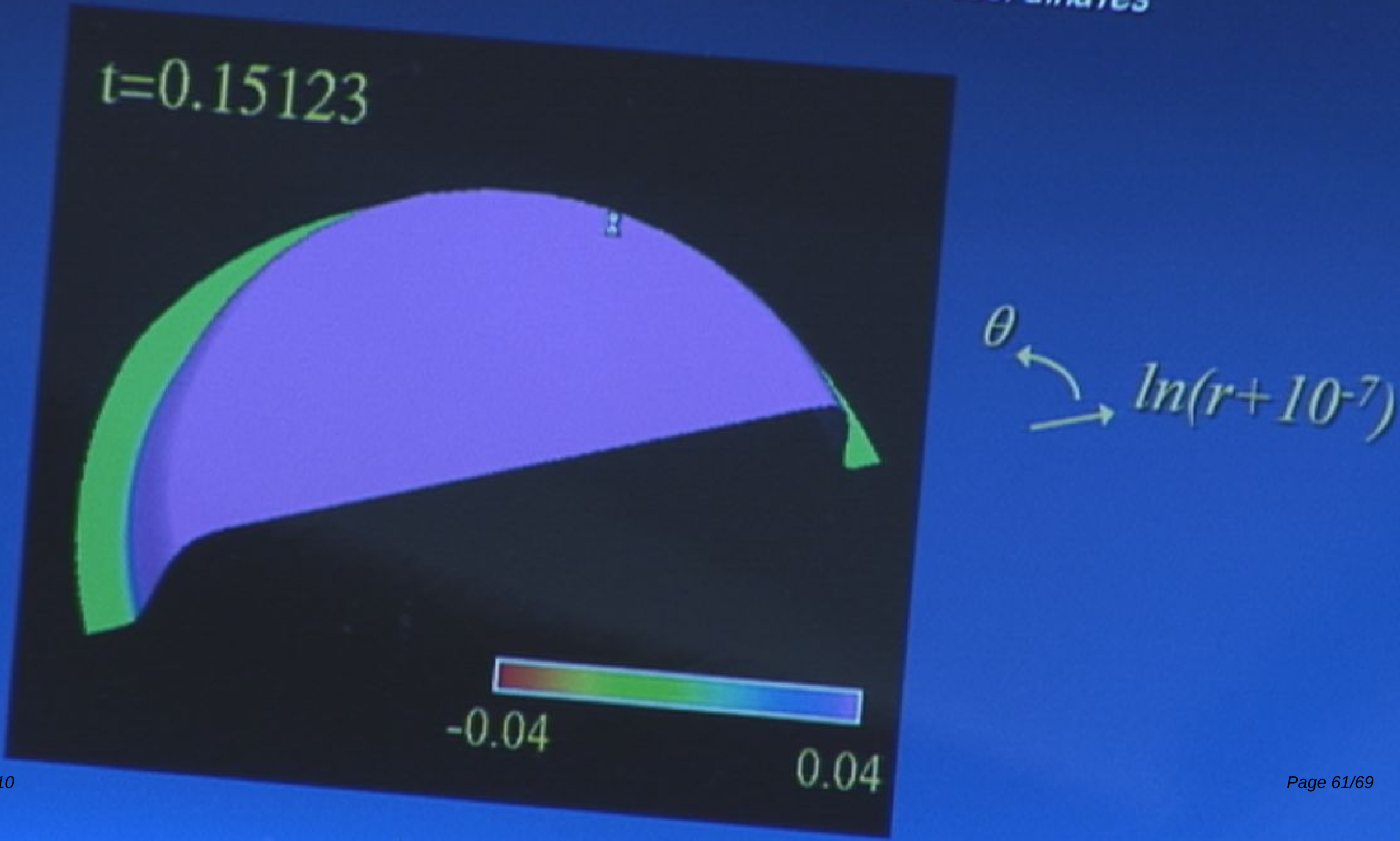
Axisymmetric simulations, spherical initial data



Work with M. Choptuik,

The scalar field threshold solution

Same near critical solution, transformed to spherical polar coordinates, and using logarithmic radial and time coordinates

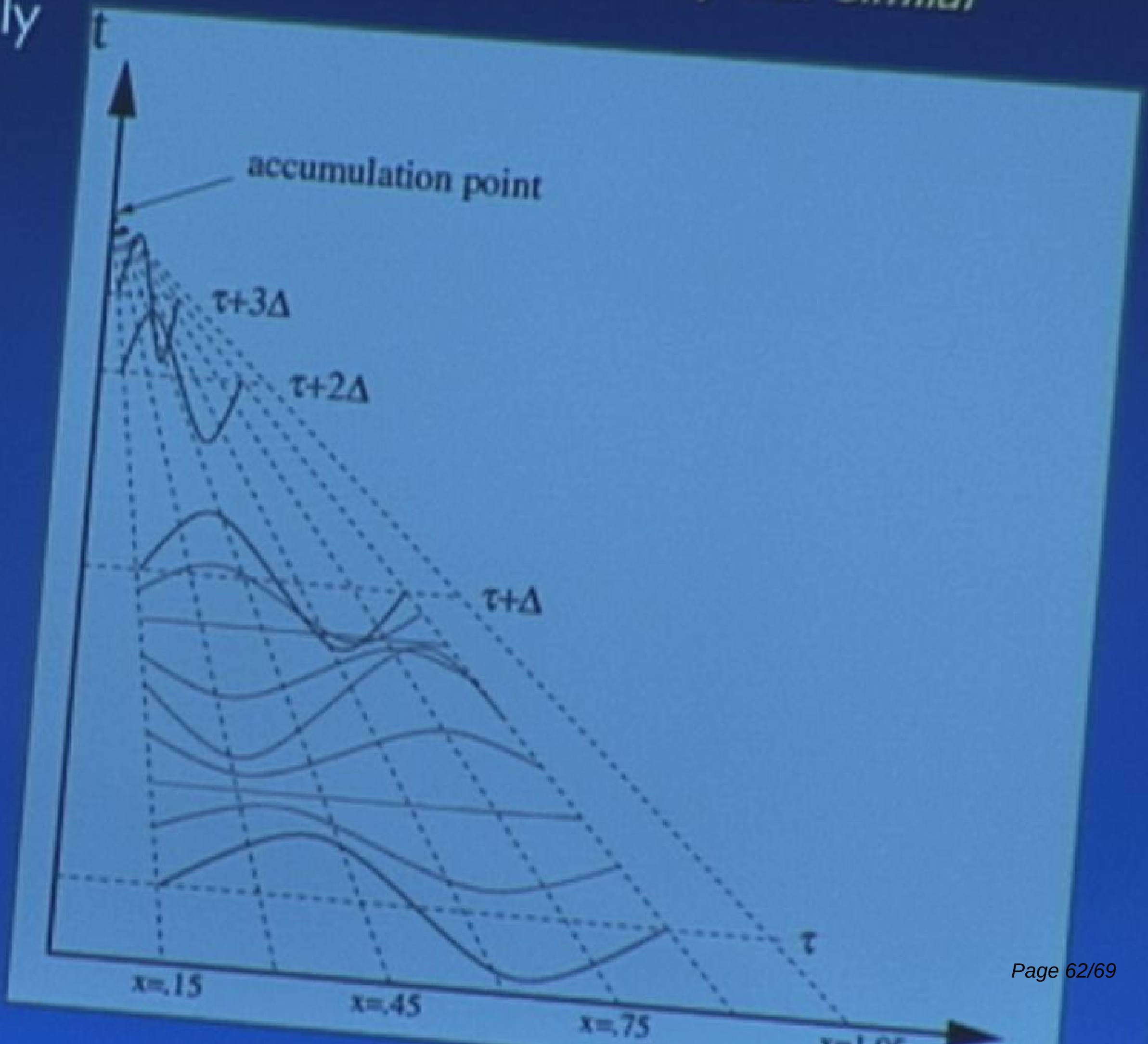


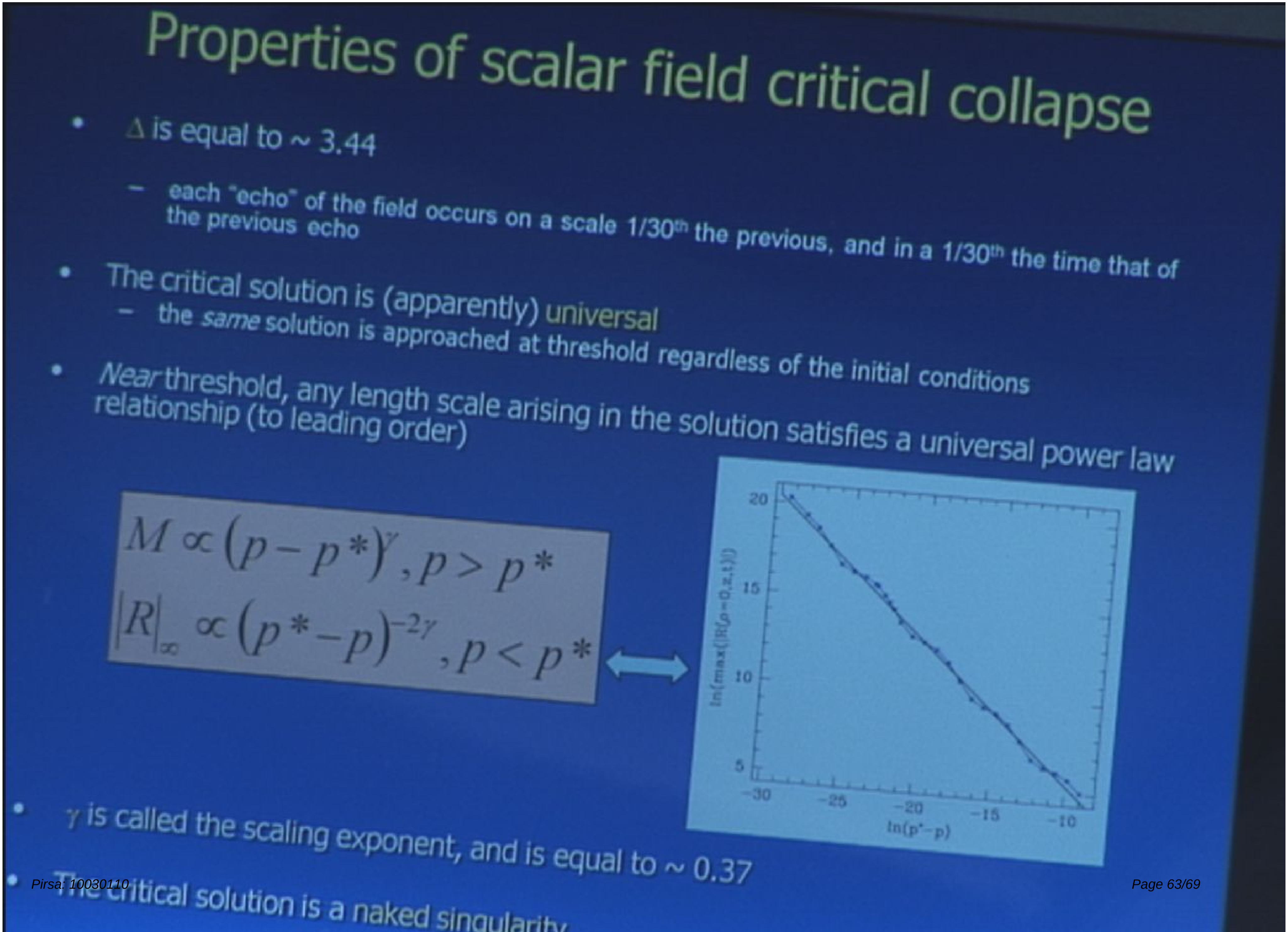
Properties of scalar field critical collapse

- the critical solution (scalar field and spacetime geometry) is spherically symmetric and scale invariant — specifically it is *discretely self-similar*
- example of a discretely self similar function $f(x,\tau)$
  - $f(x,t)$ is periodic in time τ with *echoing period* Δ
  - τ is related to the proper time t measured by a central observer (at radius $r=0$) via

$$\tau=-\ln(-t)$$

  - x is a *dimensionless* variable, related to r and t via





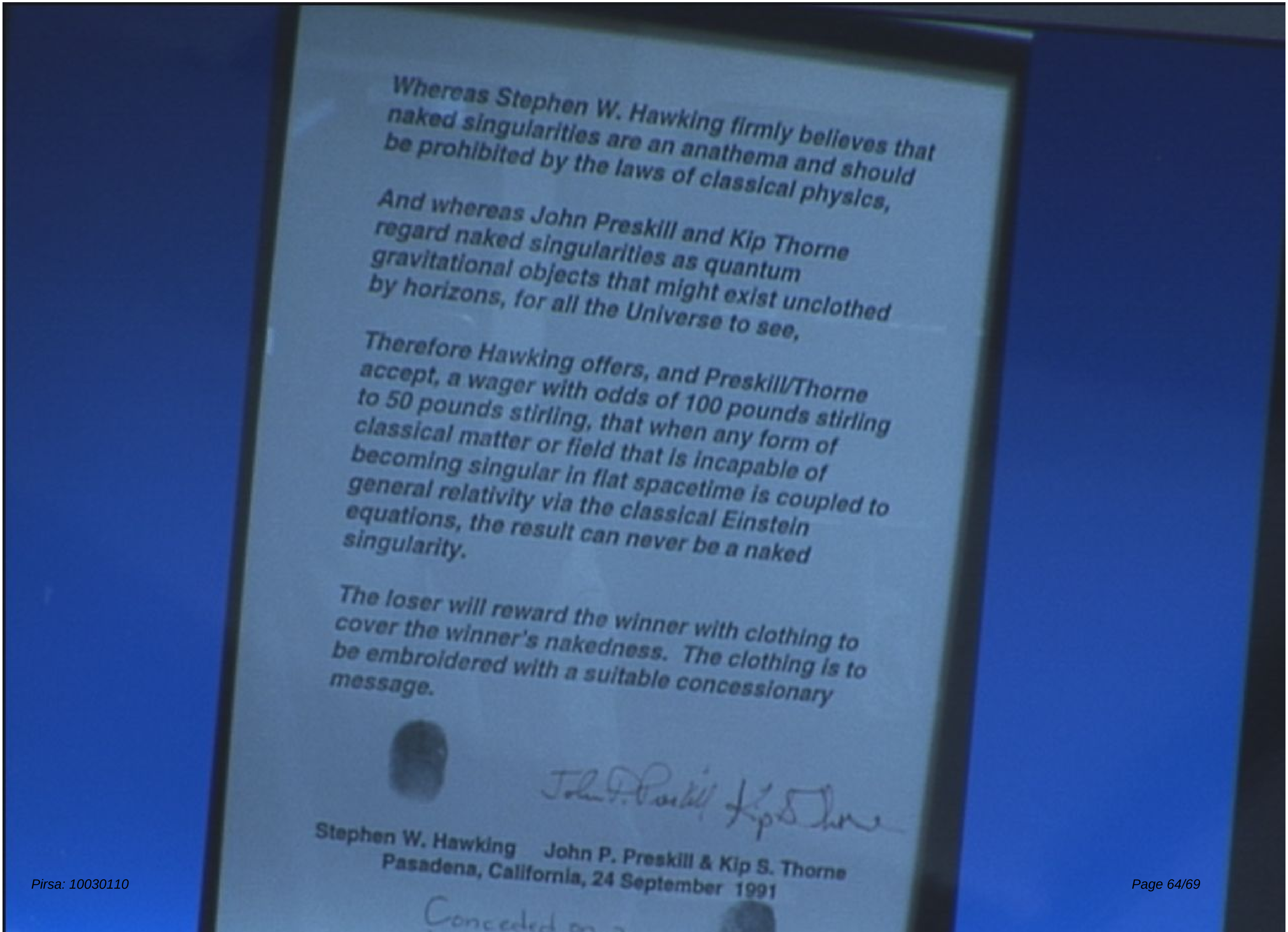
Properties of scalar field critical collapse

- Δ is equal to ~ 3.44
  - each "echo" of the field occurs on a scale 1/30th the previous, and in a 1/30th the time that of the previous echo
- The critical solution is (apparently) universal
  - the *same* solution is approached at threshold regardless of the initial conditions
- *Near* threshold, any length scale arising in the solution satisfies a universal power law relationship (to leading order)

$$M \propto (p-p^*)^{\gamma}, p>p^*$$

$$|R|_{\infty} \propto (p^*-p)^{-2\gamma}, p<p^*$$

- γ is called the scaling exponent, and is equal to ~ 0.37
- The critical solution is a naked singularity



Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all the Universe to see,

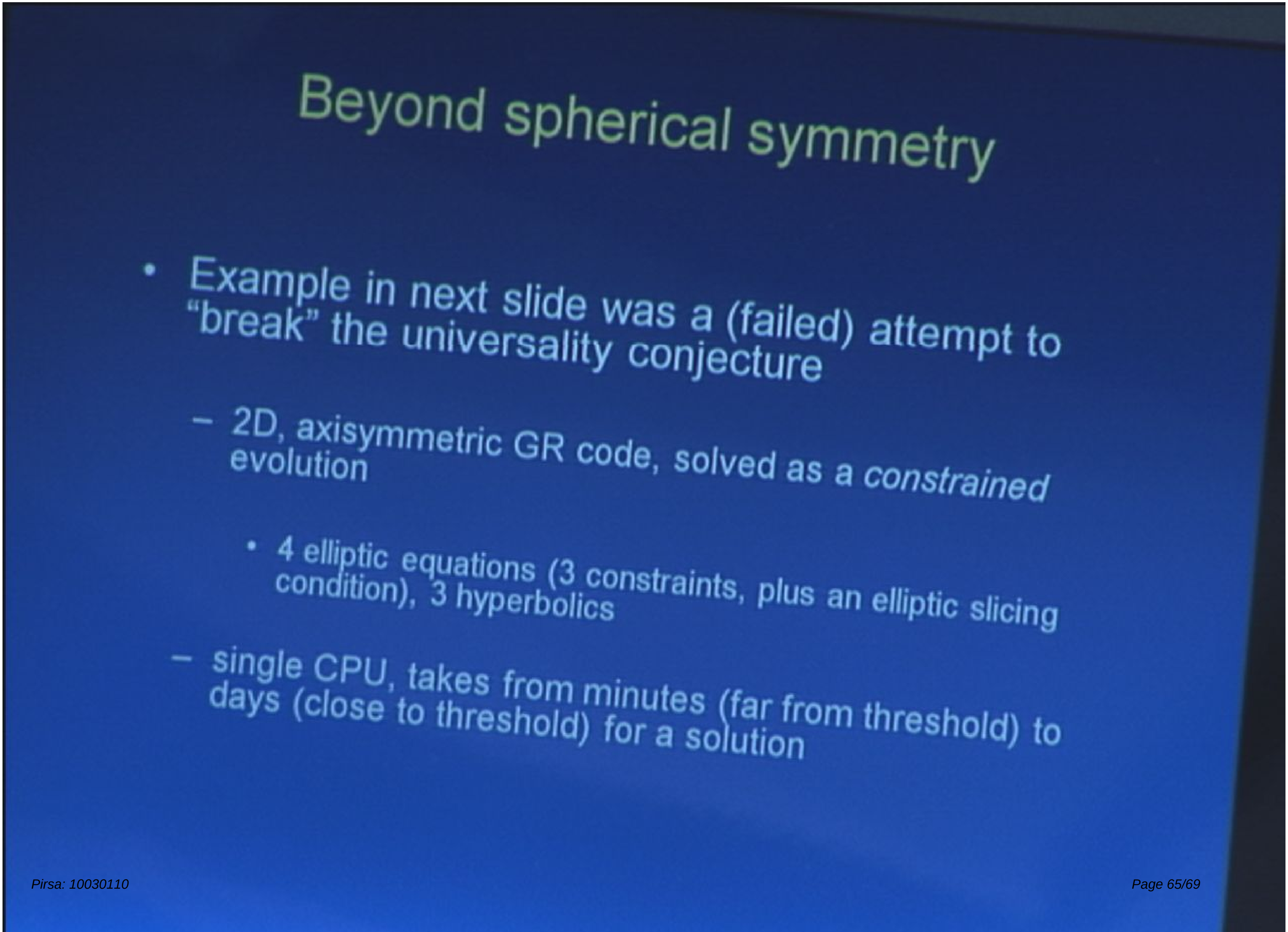
Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds stirling to 50 pounds stirling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.

John P. Preskill Kip S Thorne

Stephen W. Hawking **John P. Preskill & Kip S. Thorne**
Pasadena, California, 24 September 1991

Conceded on



Beyond spherical symmetry

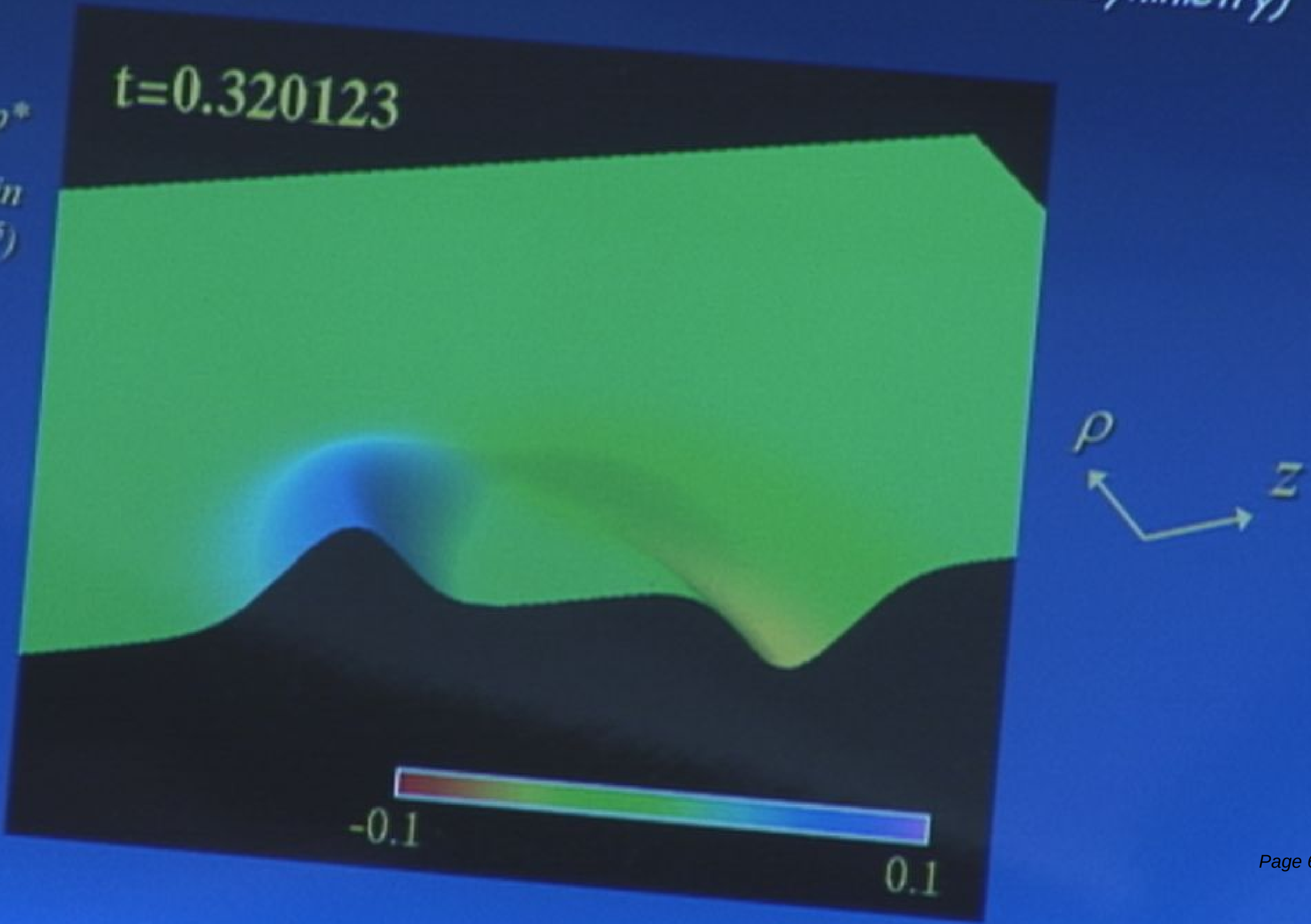
- Example in next slide was a (failed) attempt to “break” the universality conjecture
  - 2D, axisymmetric GR code, solved as a *constrained* evolution
    - 4 elliptic equations (3 constraints, plus an elliptic slicing condition), 3 hyperbolics
  - single CPU, takes from minutes (far from threshold) to days (close to threshold) for a solution

Evolution of plane anti-symmetric initial data

Initial data that is reflection anti-symmetric about z=0 (a conserved symmetry)

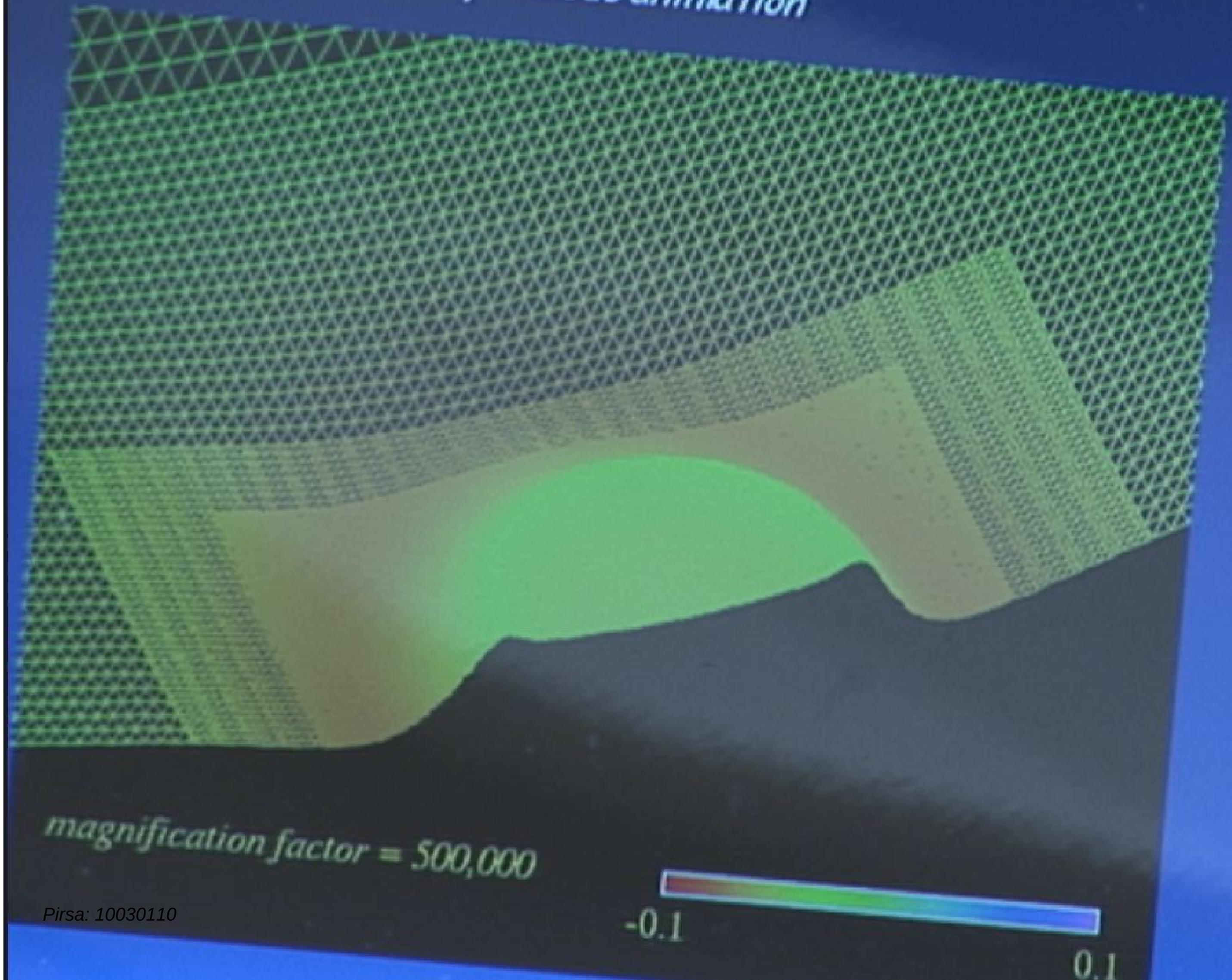
$p \sim p^*$

(tuned to within 1 part in 10^{15})



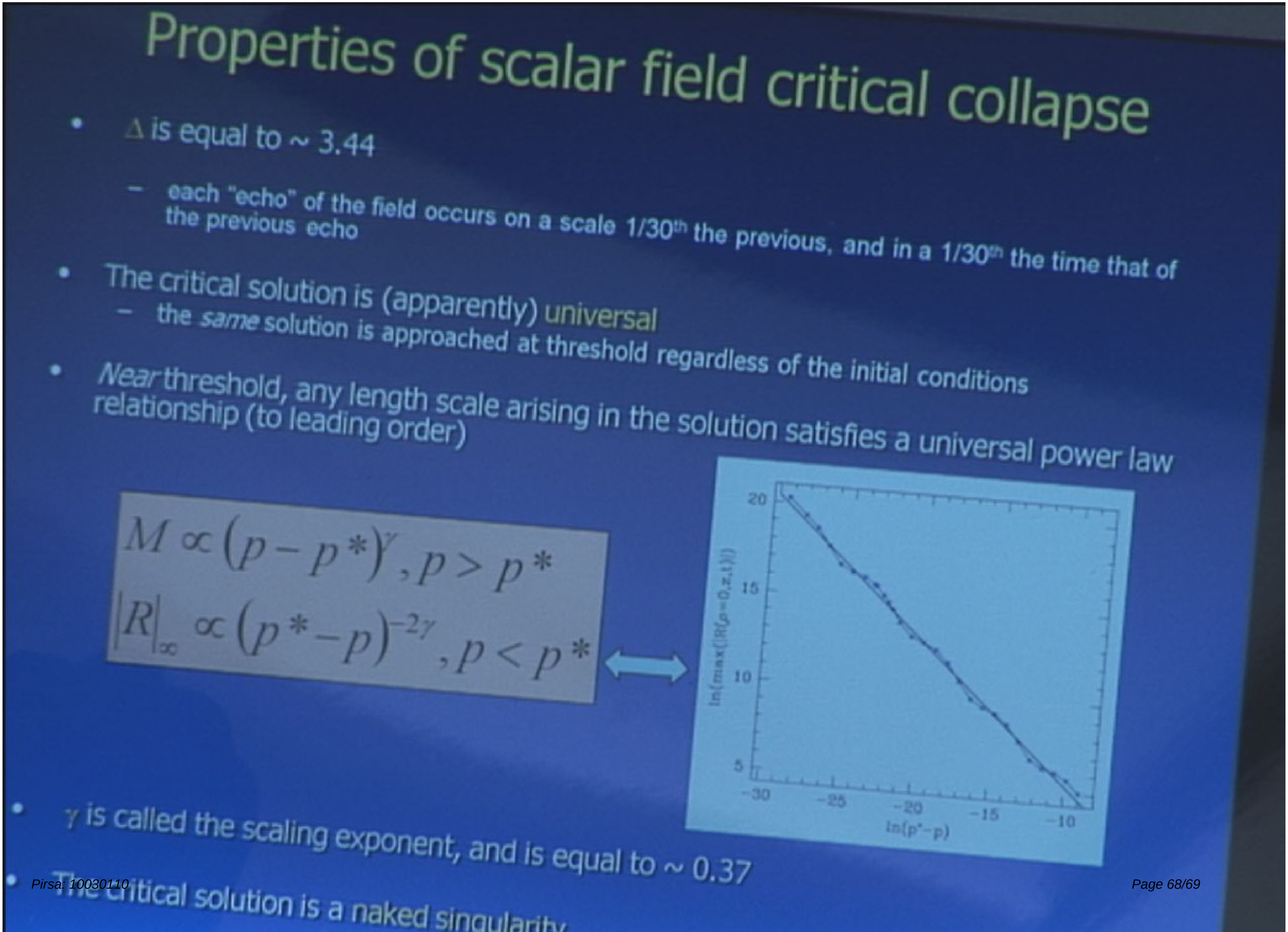
Aside: AMR grid hierarchy sample

Last frame from the previous animation



25, 2:1 refined levels

(2:1 coarsened in figure)



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