Title: Explorations in Numerical Relativity (PHYS 642) - Lecture 9

Date: Mar 25, 2010 11:20 AM

URL: http://pirsa.org/10030108

Abstract:

Burger's eqn, solns and *solns*...

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} q_i^n (q_i^{n+1/2} - q_{i-1}^{n+1/2})$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x 2}((q_i^{n+1/2})^2 - (q_{i-1}^{n+1/2})^2) = \frac{\Delta t}{\Delta x}(q_i^{n+1/2} + q_{i-1}^{n+1/2})(q_i^{n+1/2} - q_{i-1}^{n+1/2})$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x 4}(3(q_i^{n+1/2})^2 - 4(q_{i-1}^{n+1/2})^2 + (q_{i-1}^{n+1/2})^2)$$

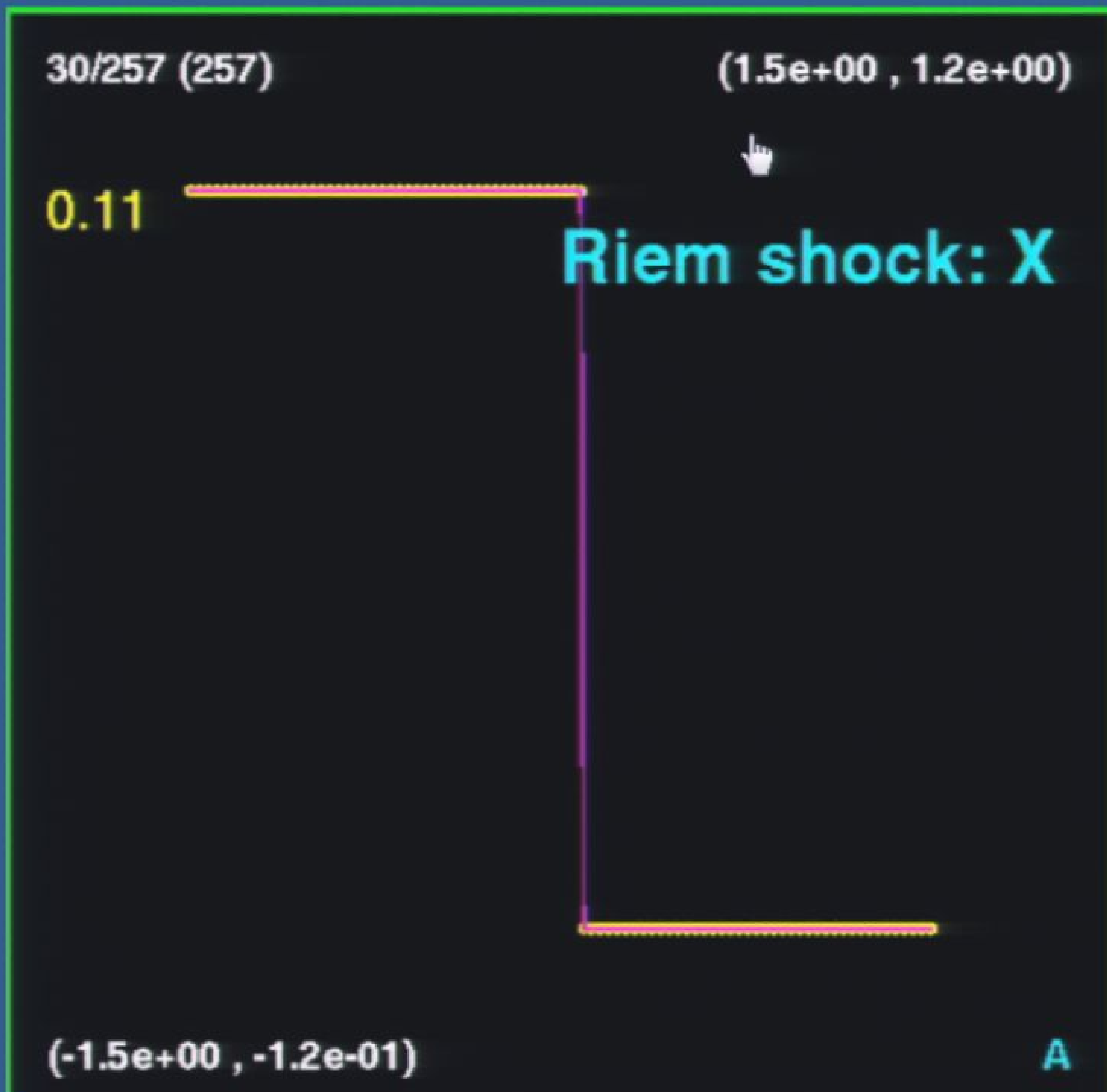
$$+ Godunov$$

Boundary conditions: 'outflow' $q_1=q_2=q_3$; $q_{Nx} = q_{Nx-1}= q_{Nx-2}$ (domain 3—Nx-2)

Initial data: "Shock tube problem" $q = q_L$ $(x<0)$; $q = q_R$ $(x>=0)$
"Gaussian profile"

→ Time of shock: -1/(min(q(t=0,x)')


1/257 (257)
(1.5e+00 , 1.2e+00)
0.00
Riem shock: X
(-1.5e+00 , -1.2e-01)
A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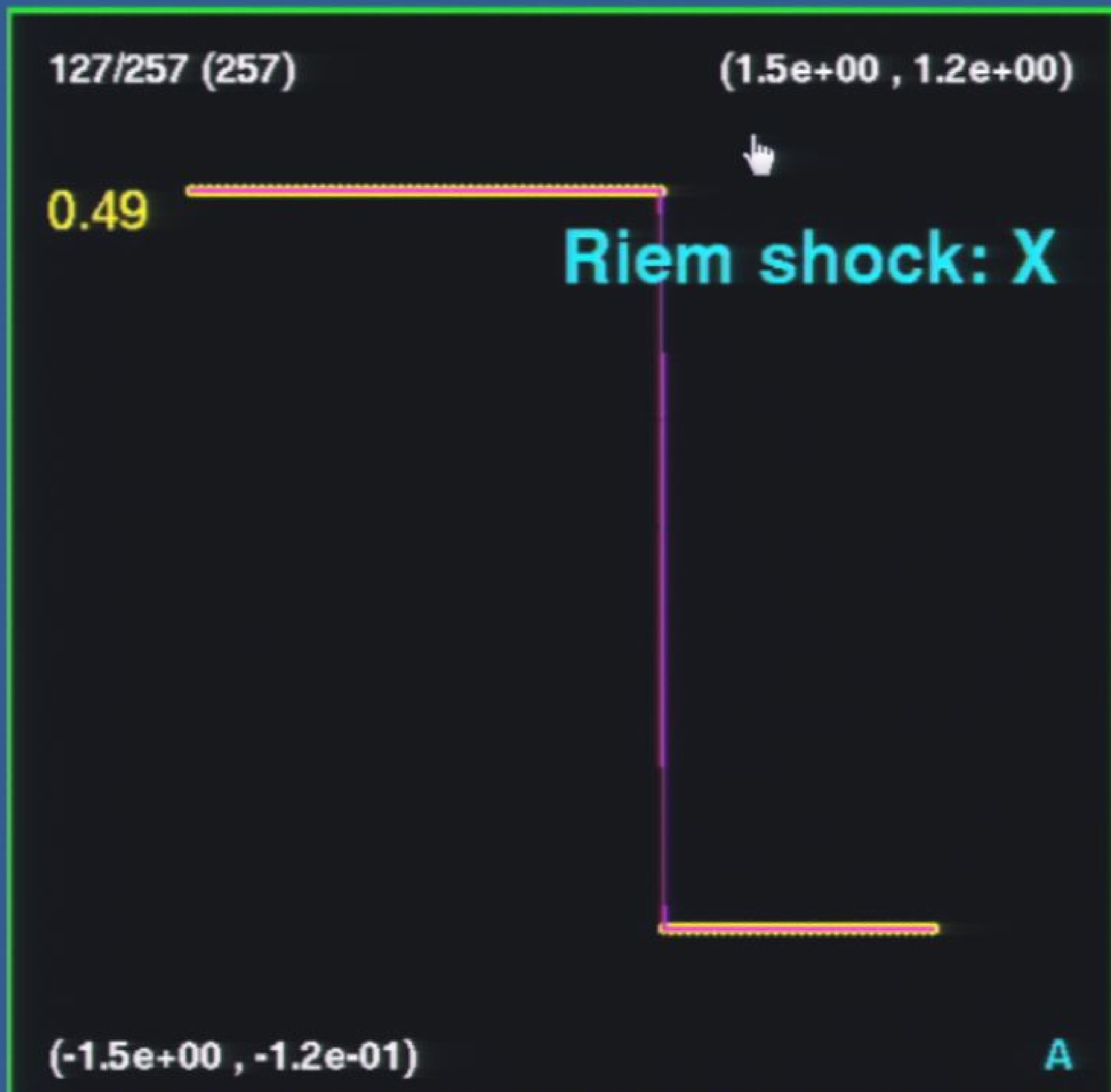
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203/257 (257)
(1.5e+00 , 1.2e+00)
0.79
Riem shock: X
(-1.5e+00 , -1.2e-01)
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237/257 (257)
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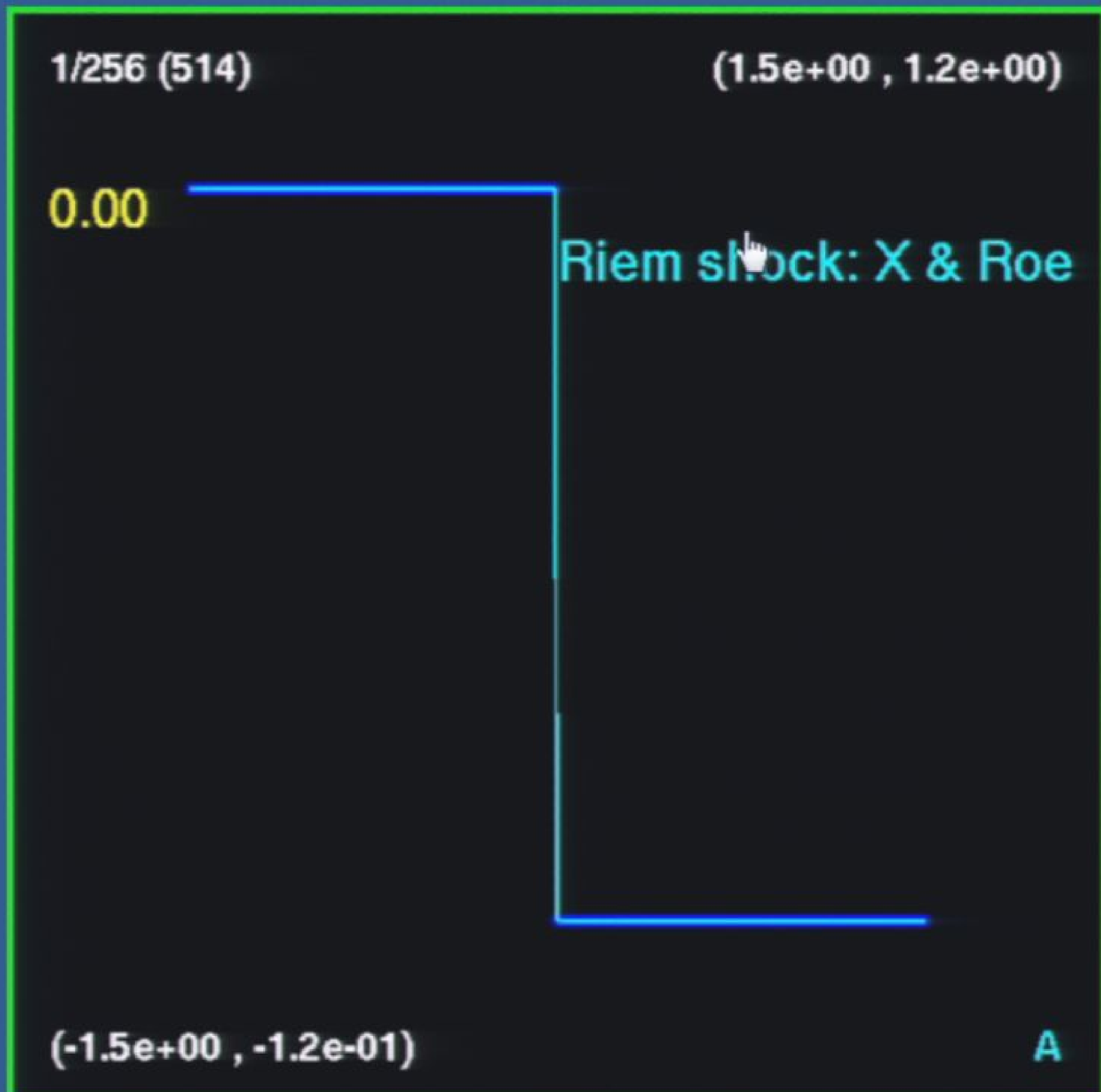
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Riem shock: X & Roe
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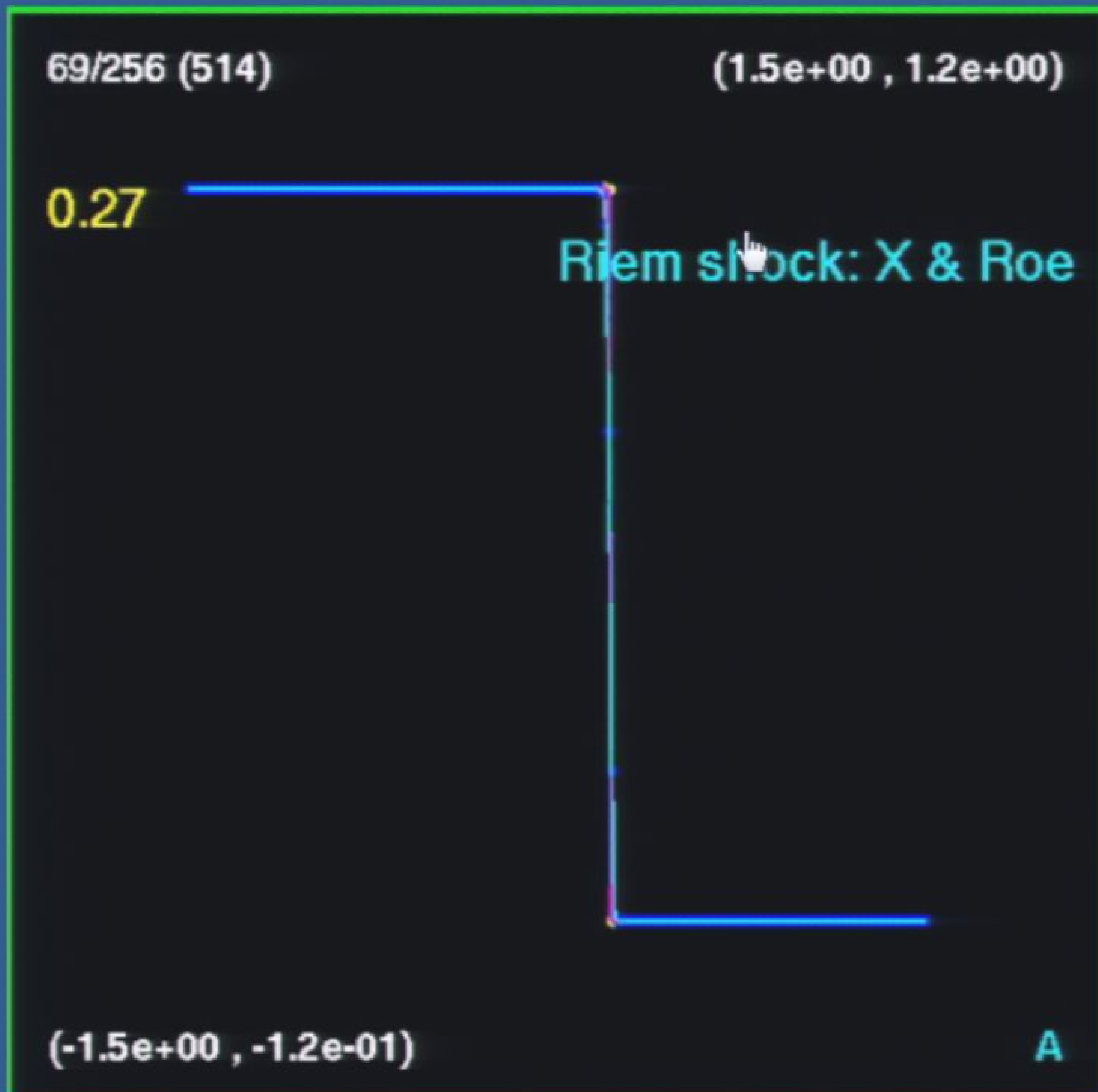
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117/256 (514)
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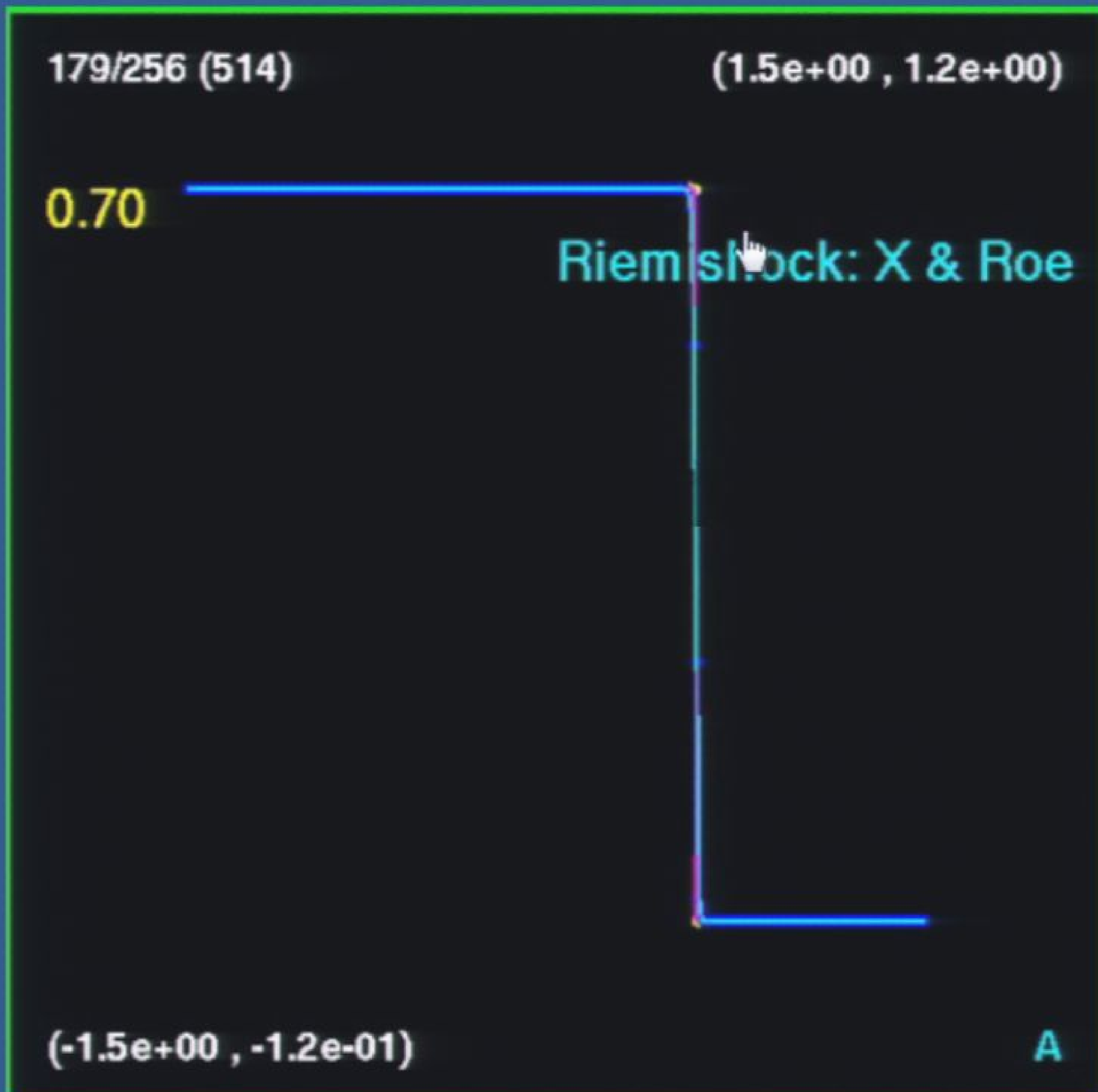
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254/256 (514)
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0.99
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(-1.5e+00 , -1.2e-01)
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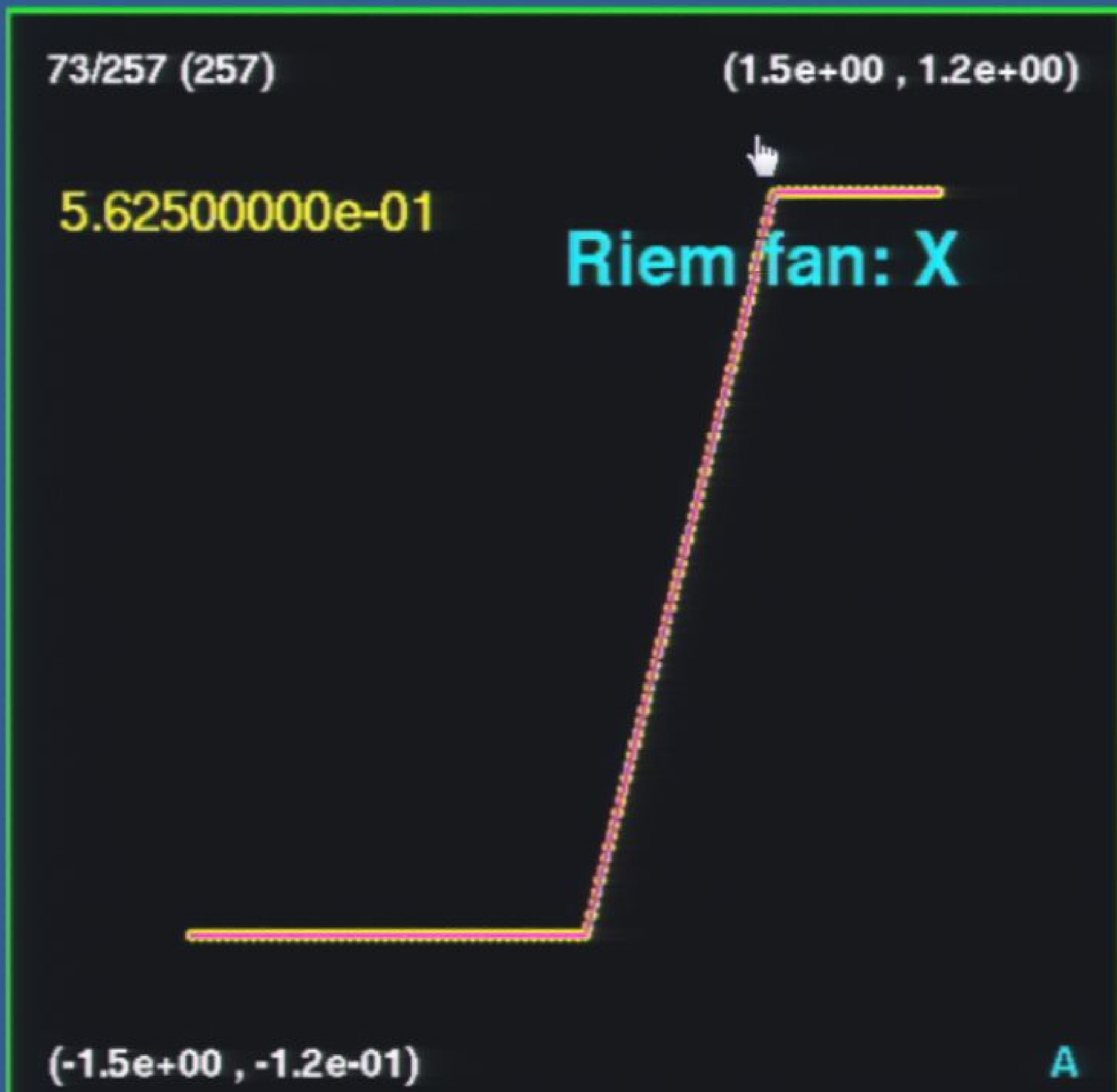
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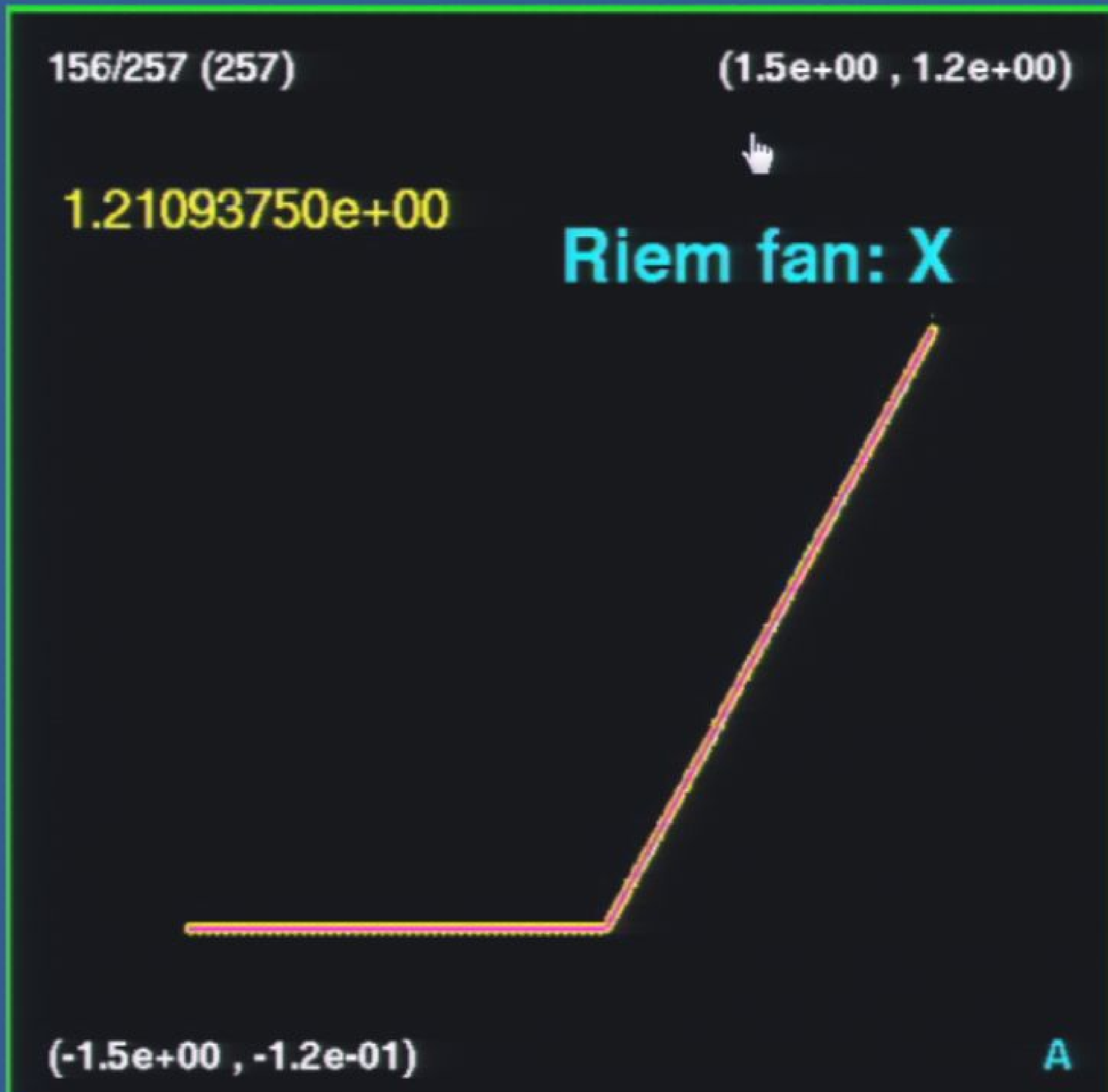
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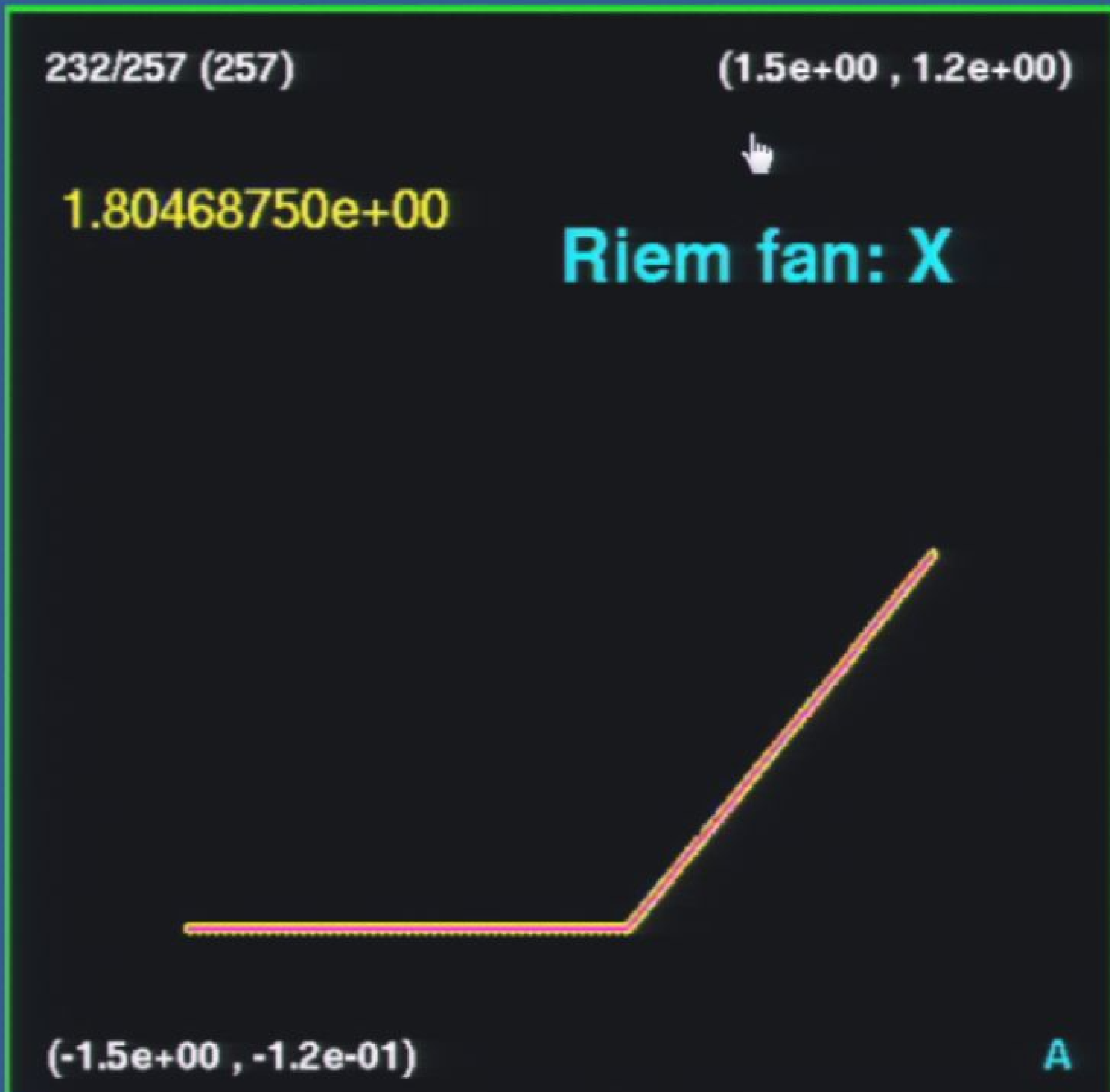
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255/257 (257)
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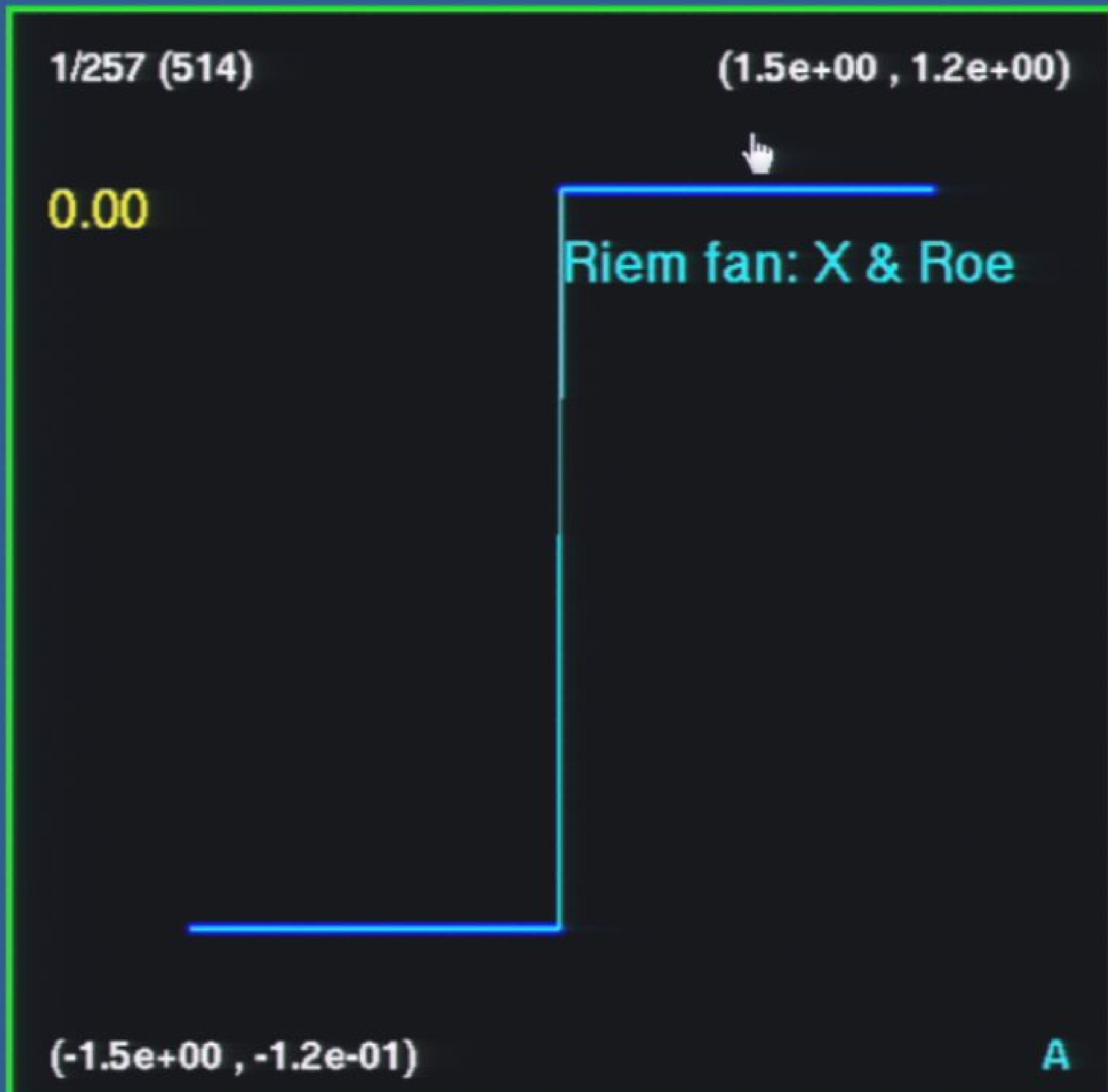
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Riem fan: X & Roe
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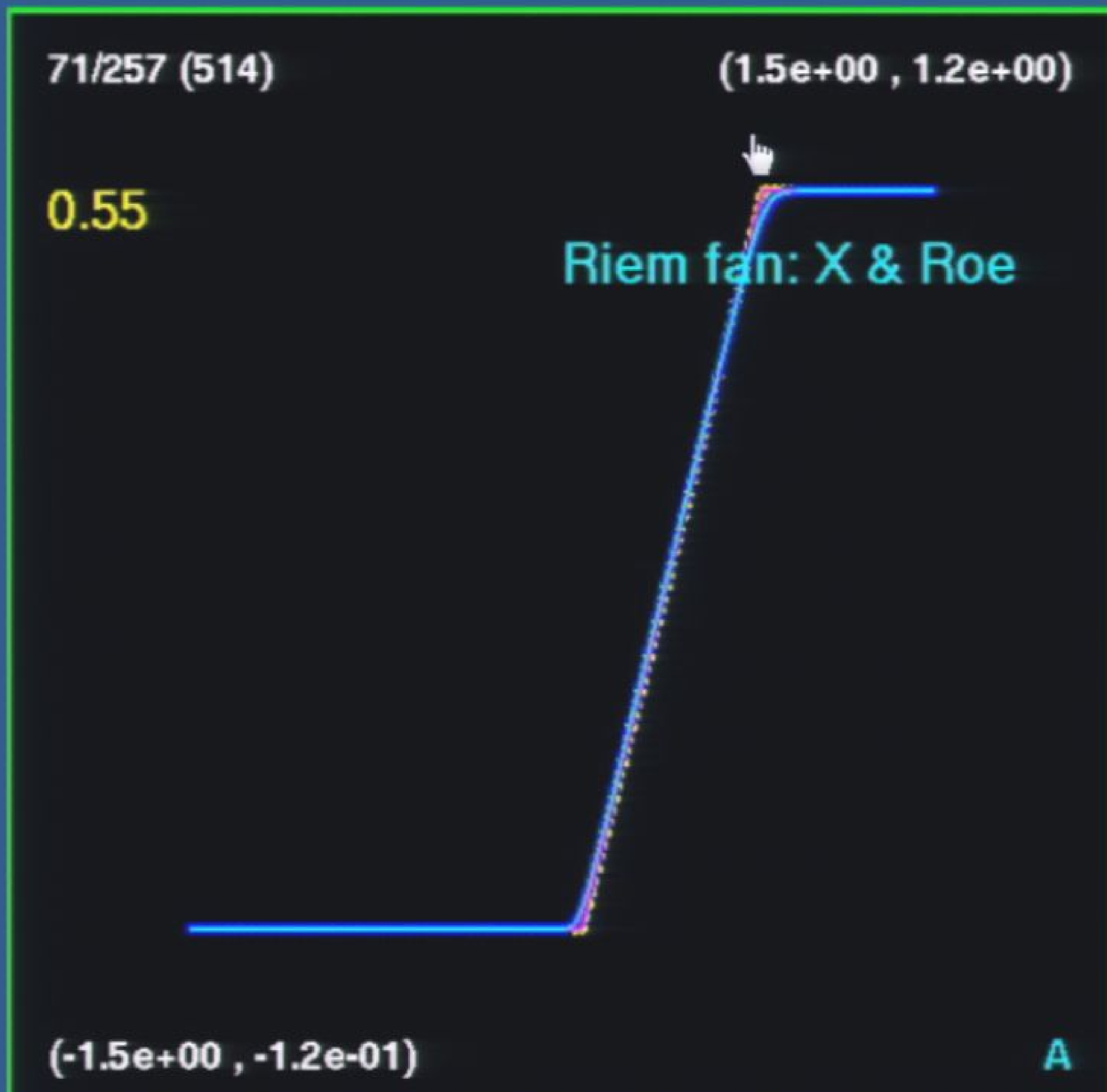
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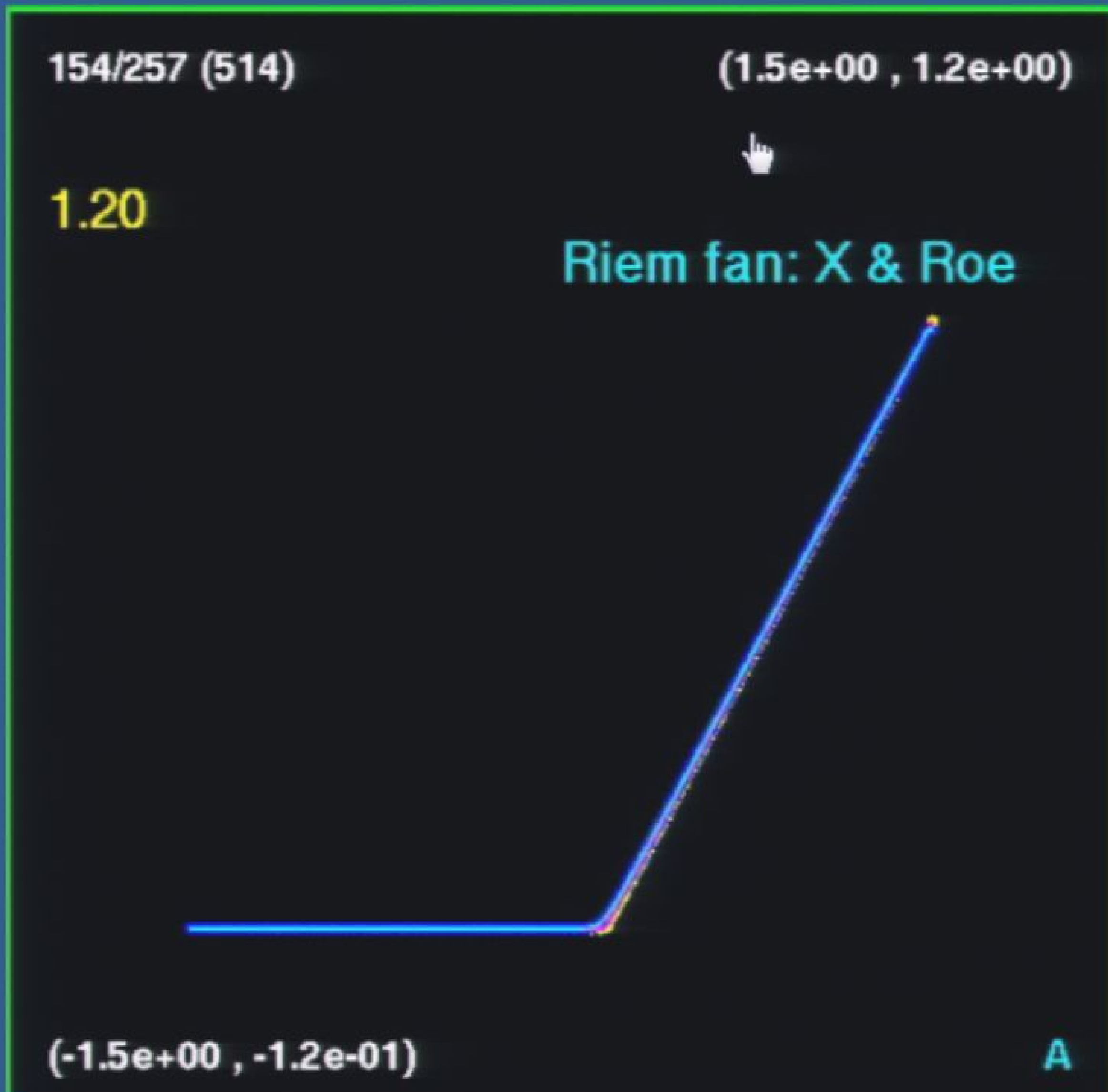
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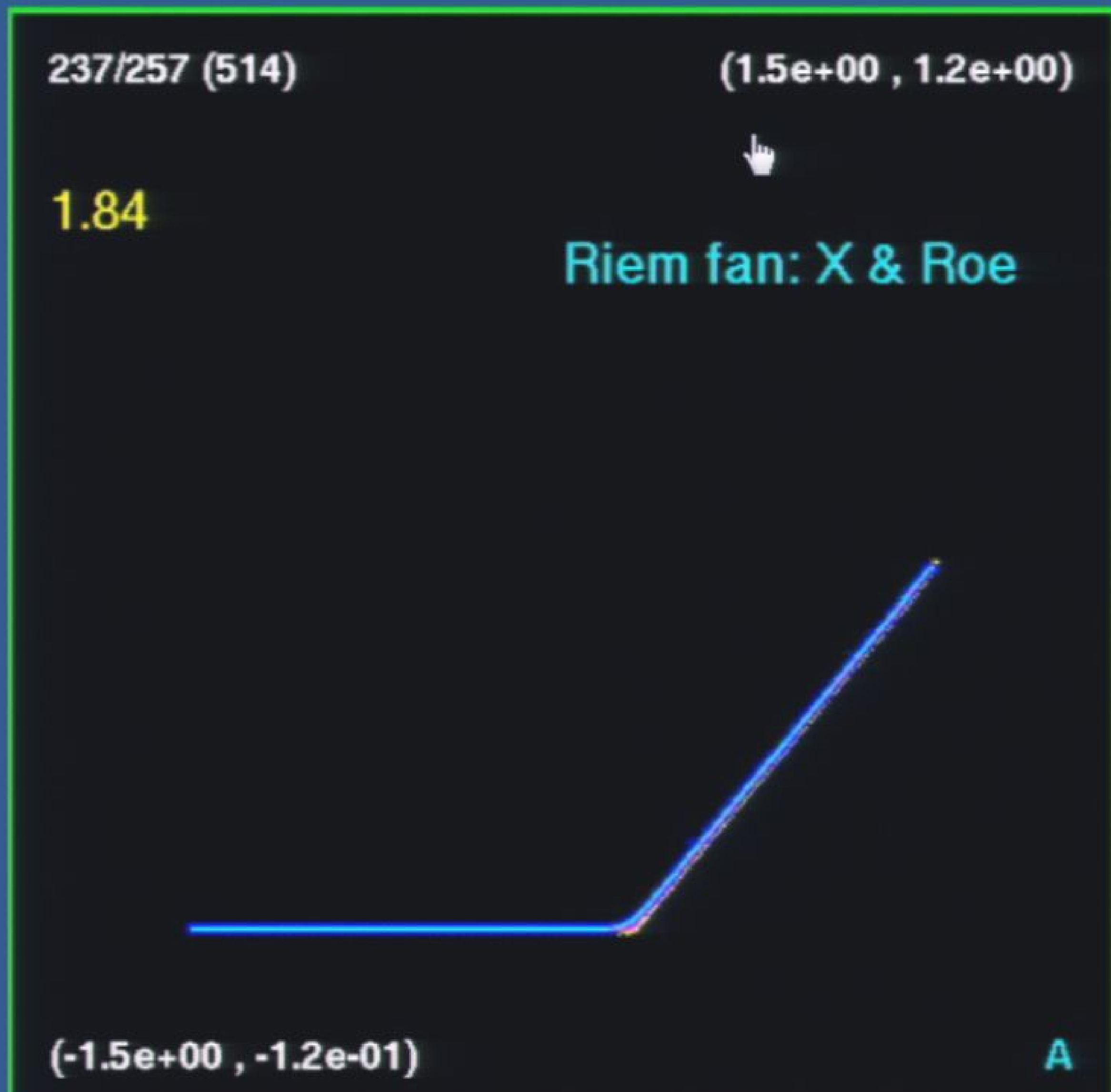
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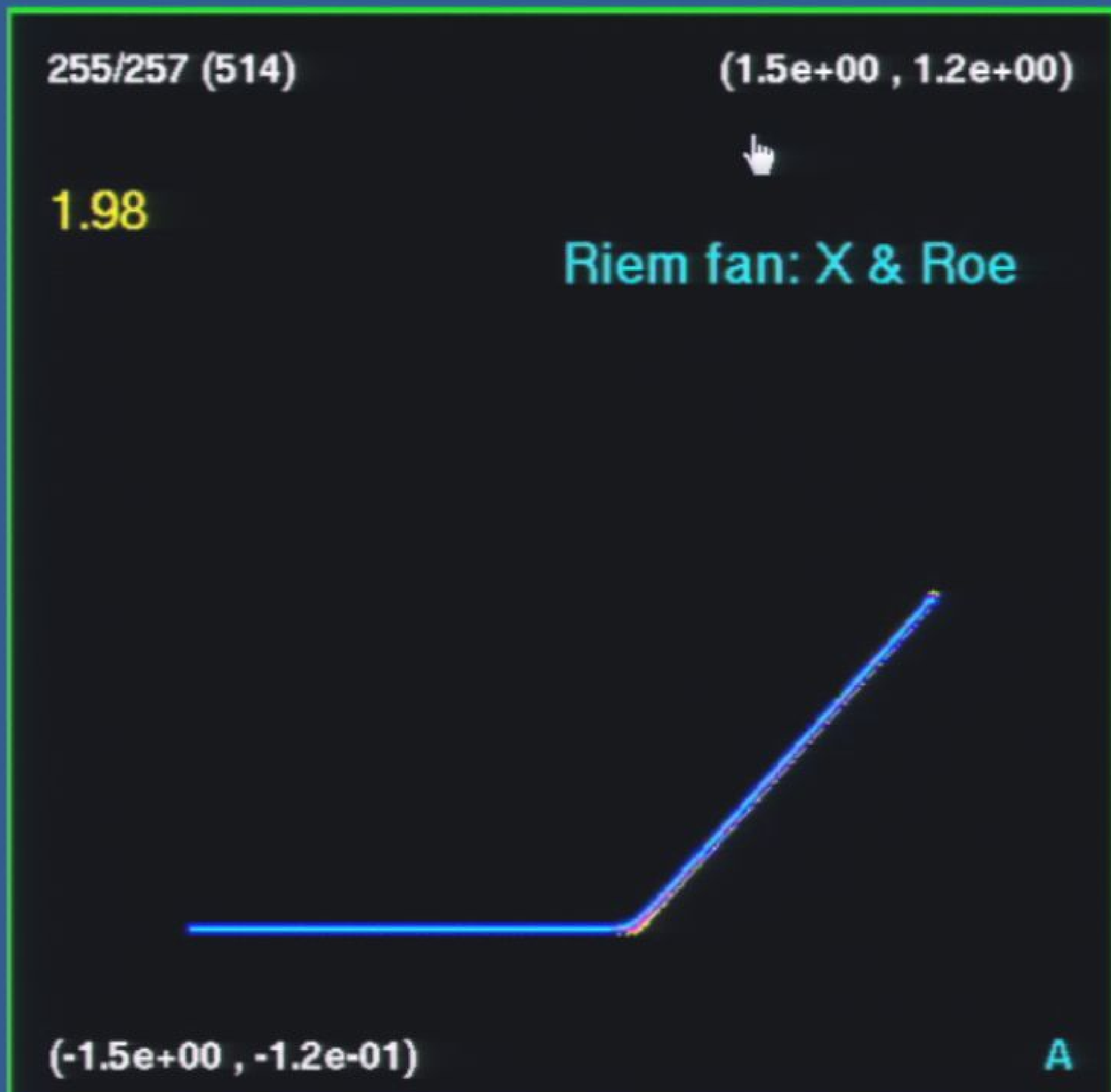

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Riem fan: X & Roe
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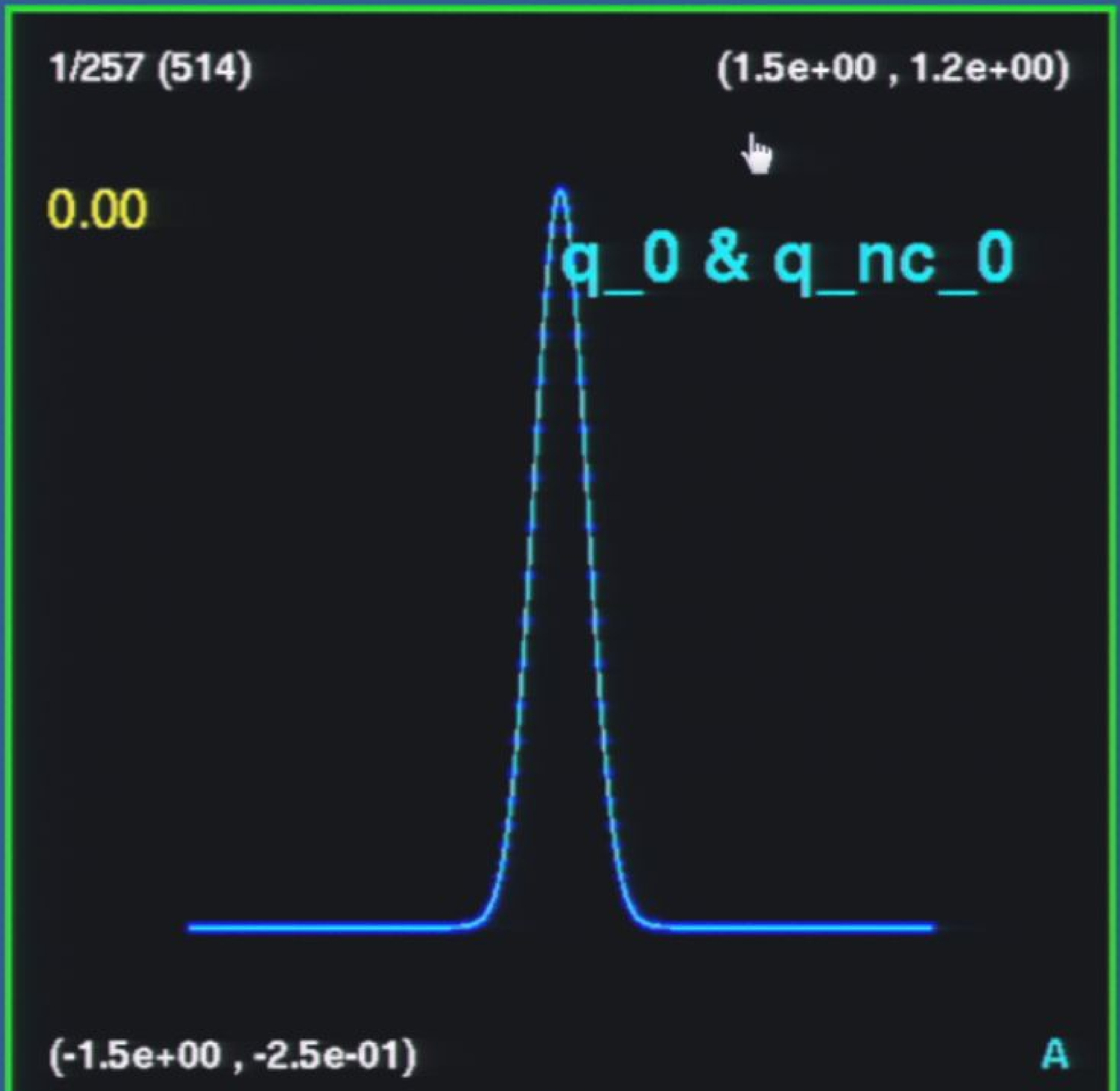

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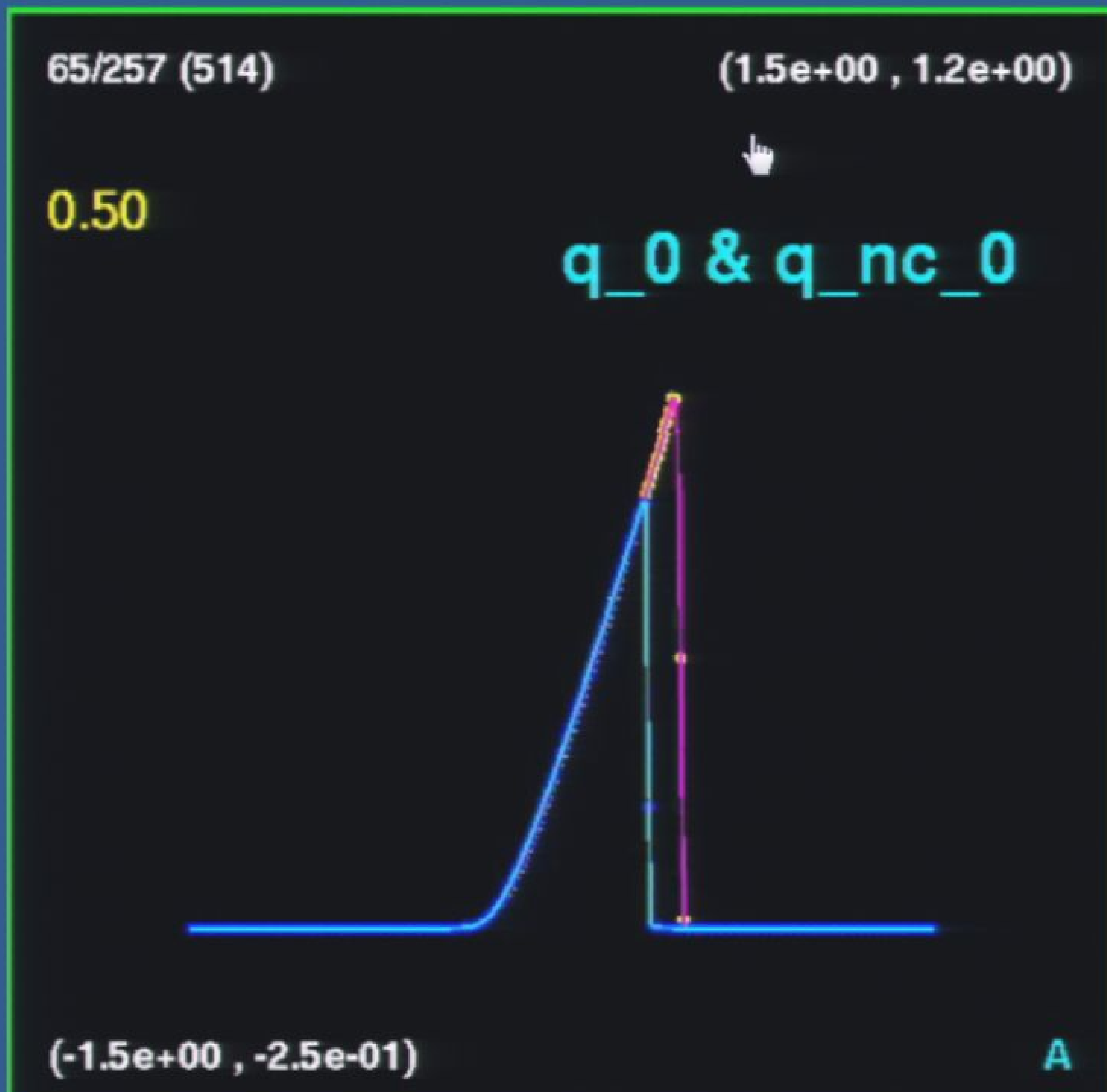
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A


148/257 (514)
(1.5e+00 , 1.2e+00)
1.16
q_0 & q_nc_0
(-1.5e+00 , -2.5e-01)
A


175/257 (514)
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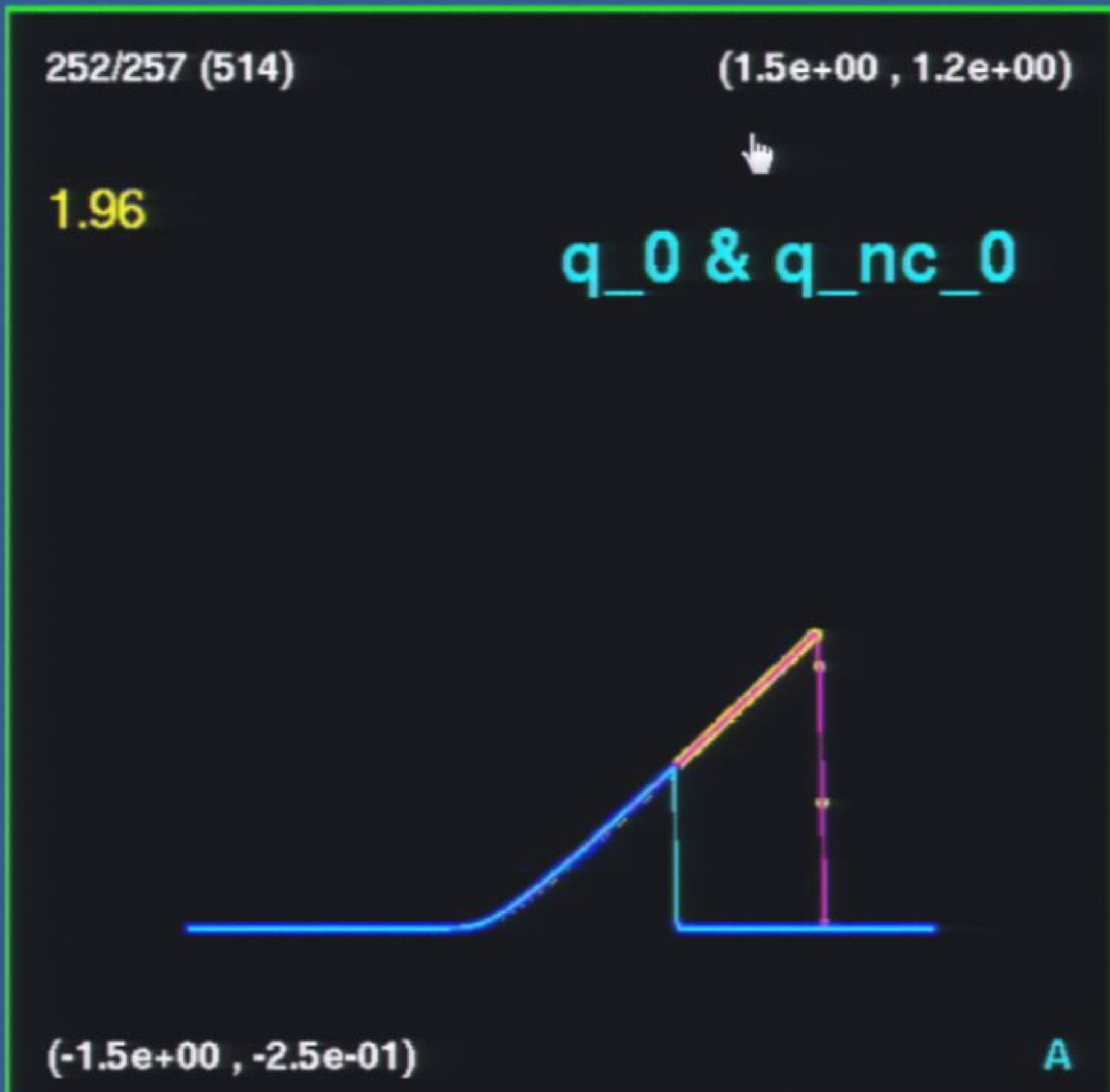
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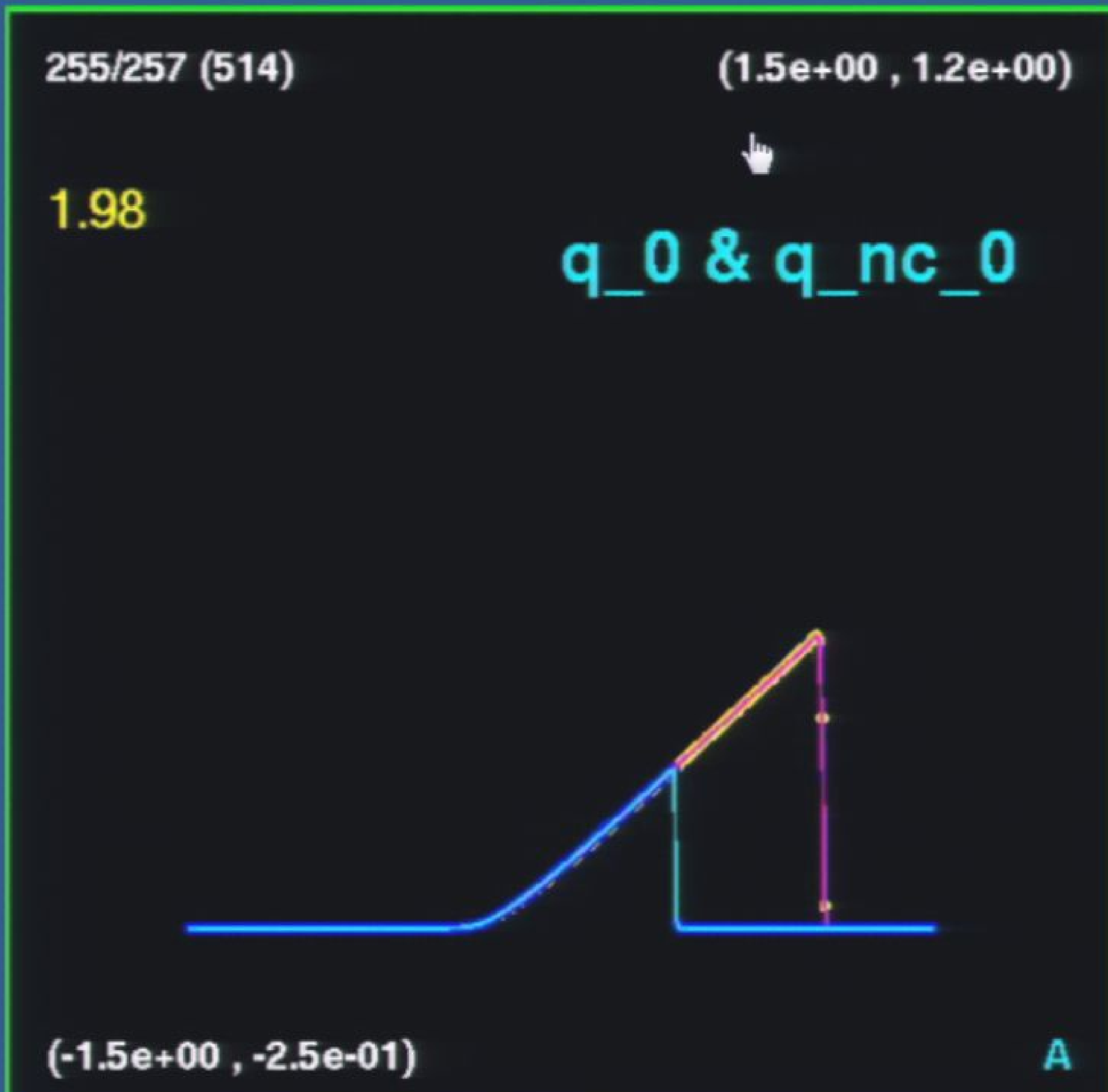
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q_0 & q_nc_0
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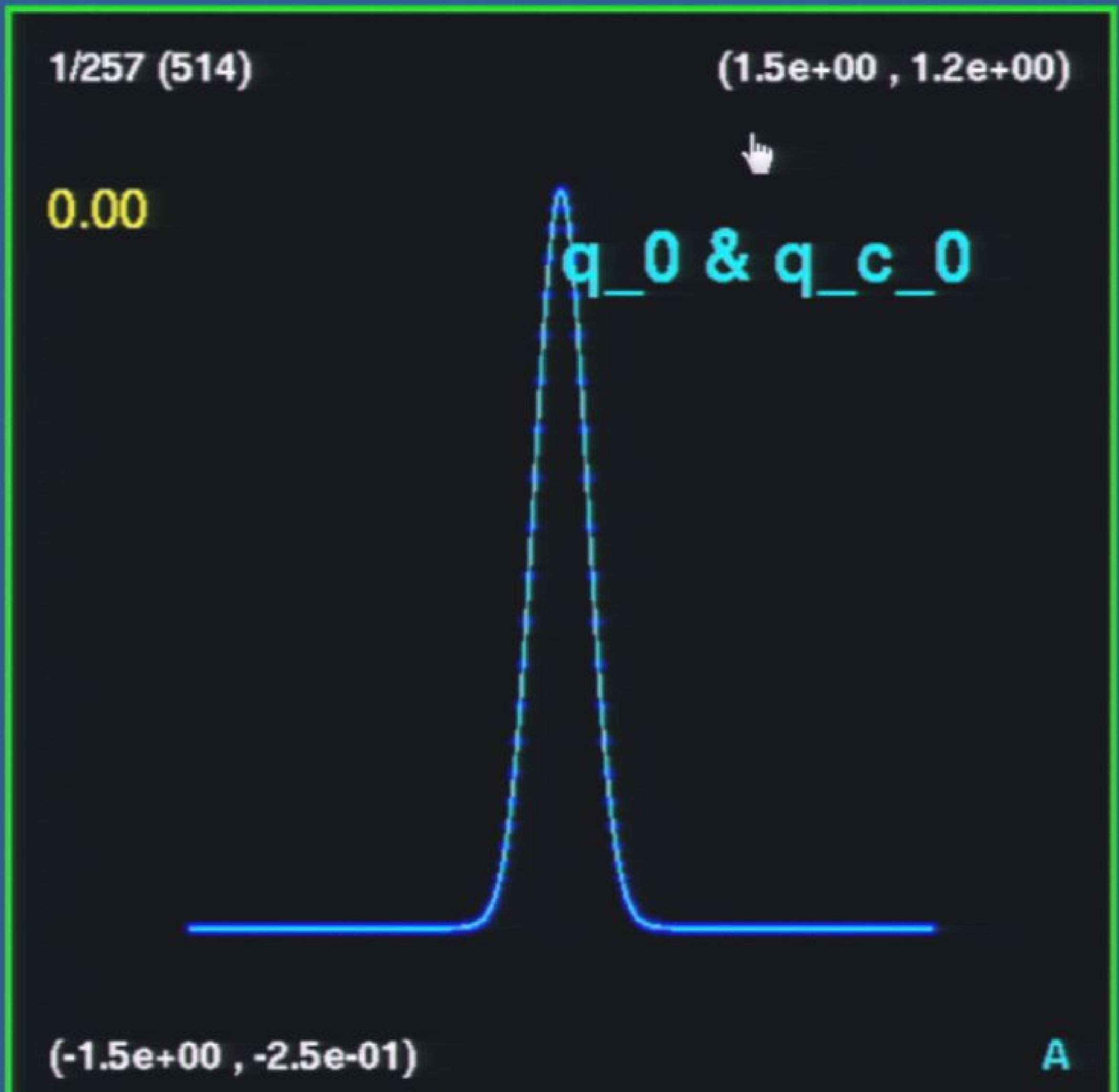
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q_0 & q_c_0
(-1.5e+00 , -2.5e-01)
A


71/257 (514)
(1.5e+00 , 1.2e+00)
0.55
q_0 & q_c_0
(-1.5e+00 , -2.5e-01)
A


153/257 (514)
(1.5e+00 , 1.2e+00)
1.20
q_0 & q_c_0
(-1.5e+00 , -2.5e-01)
A

229/257 (514)
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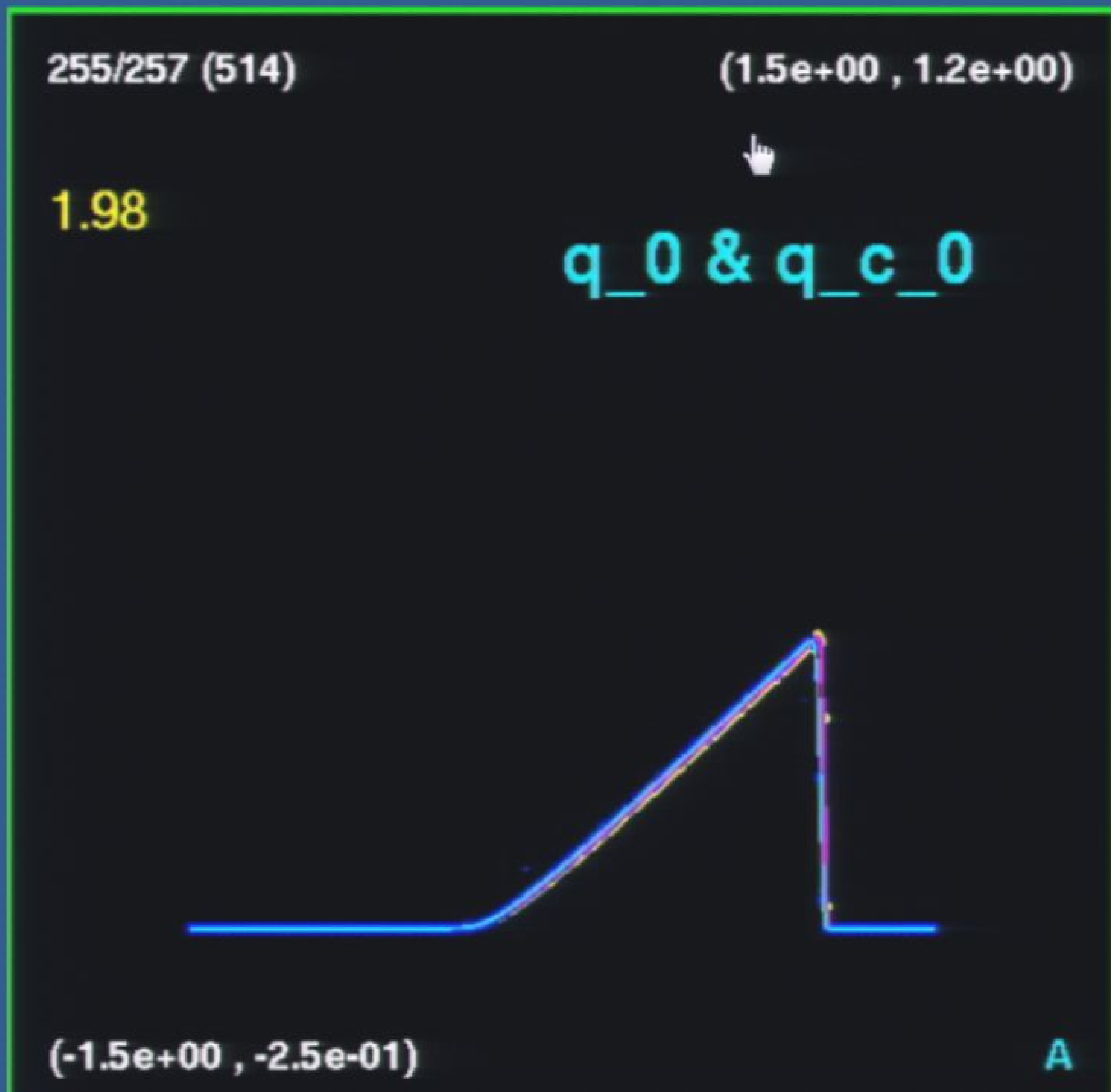
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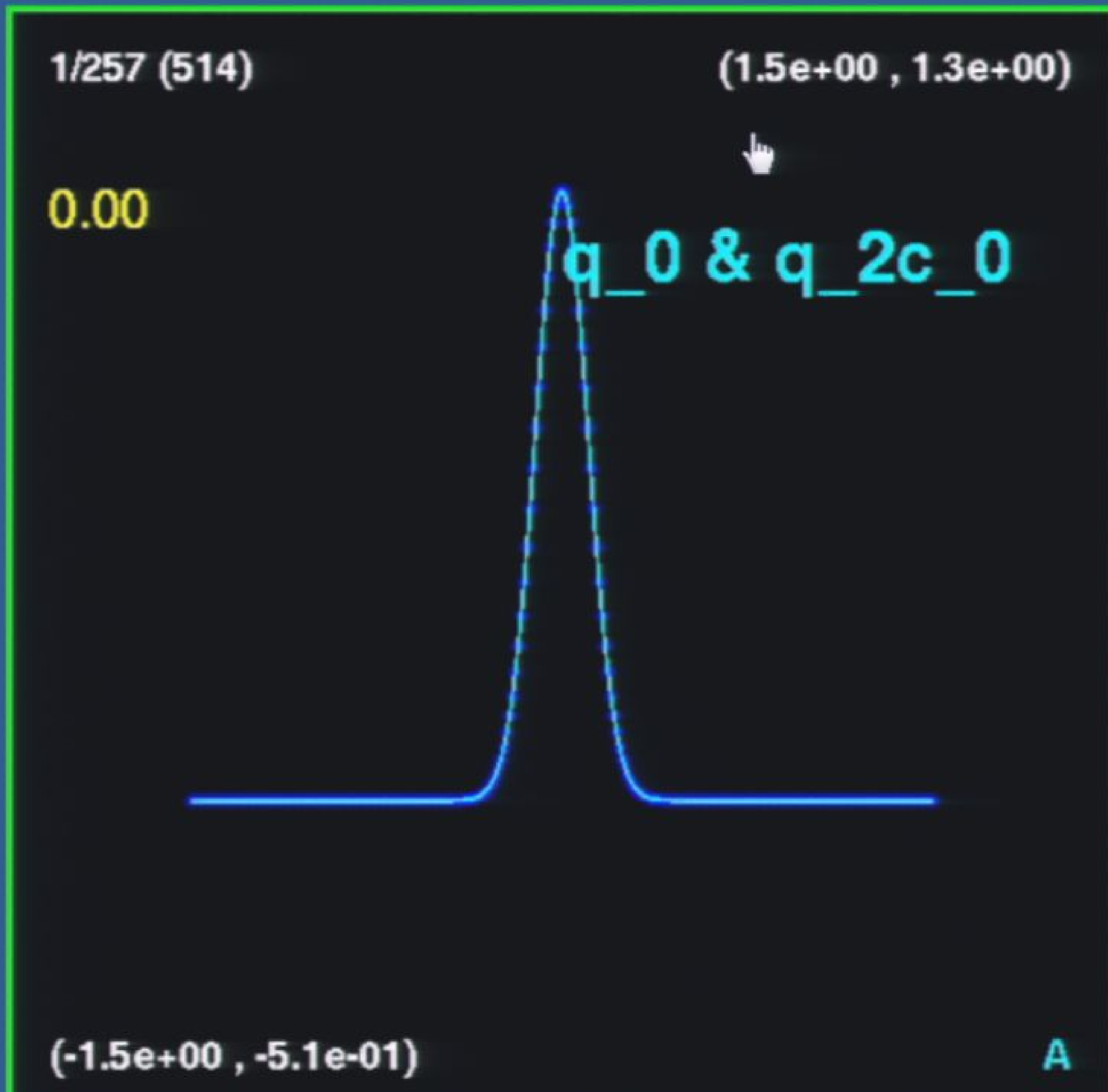

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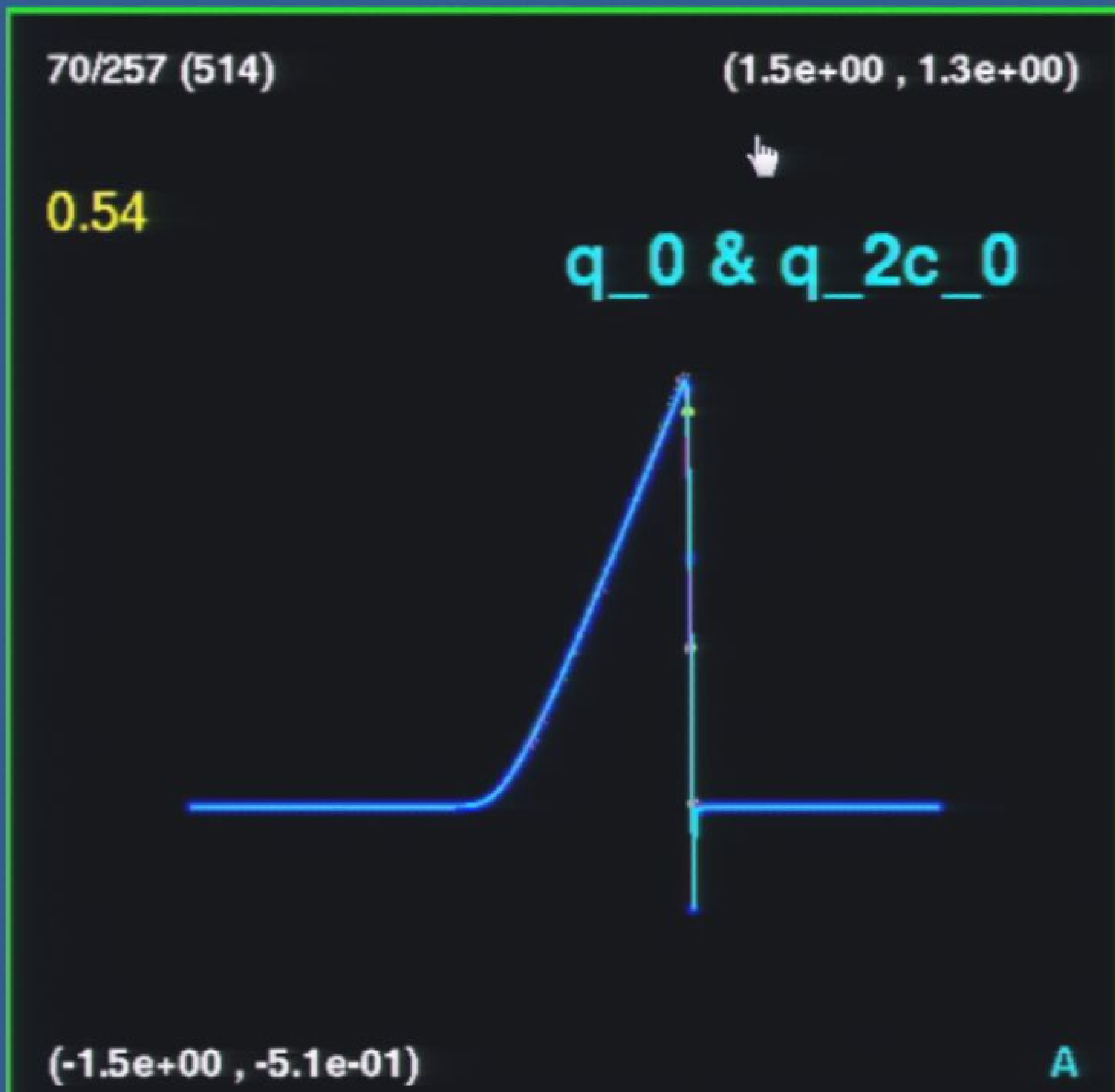
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A

146/257 (514)
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q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
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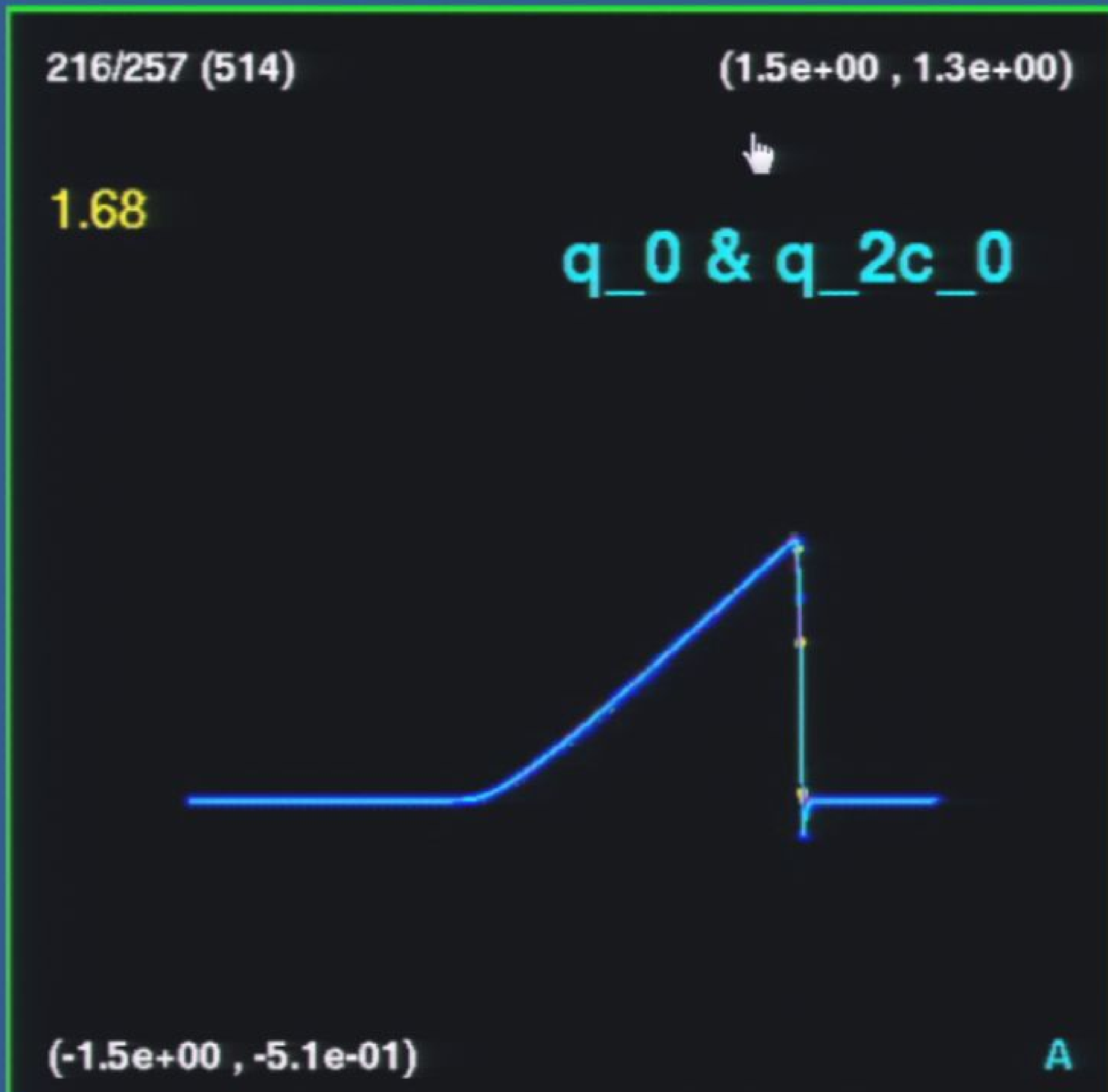
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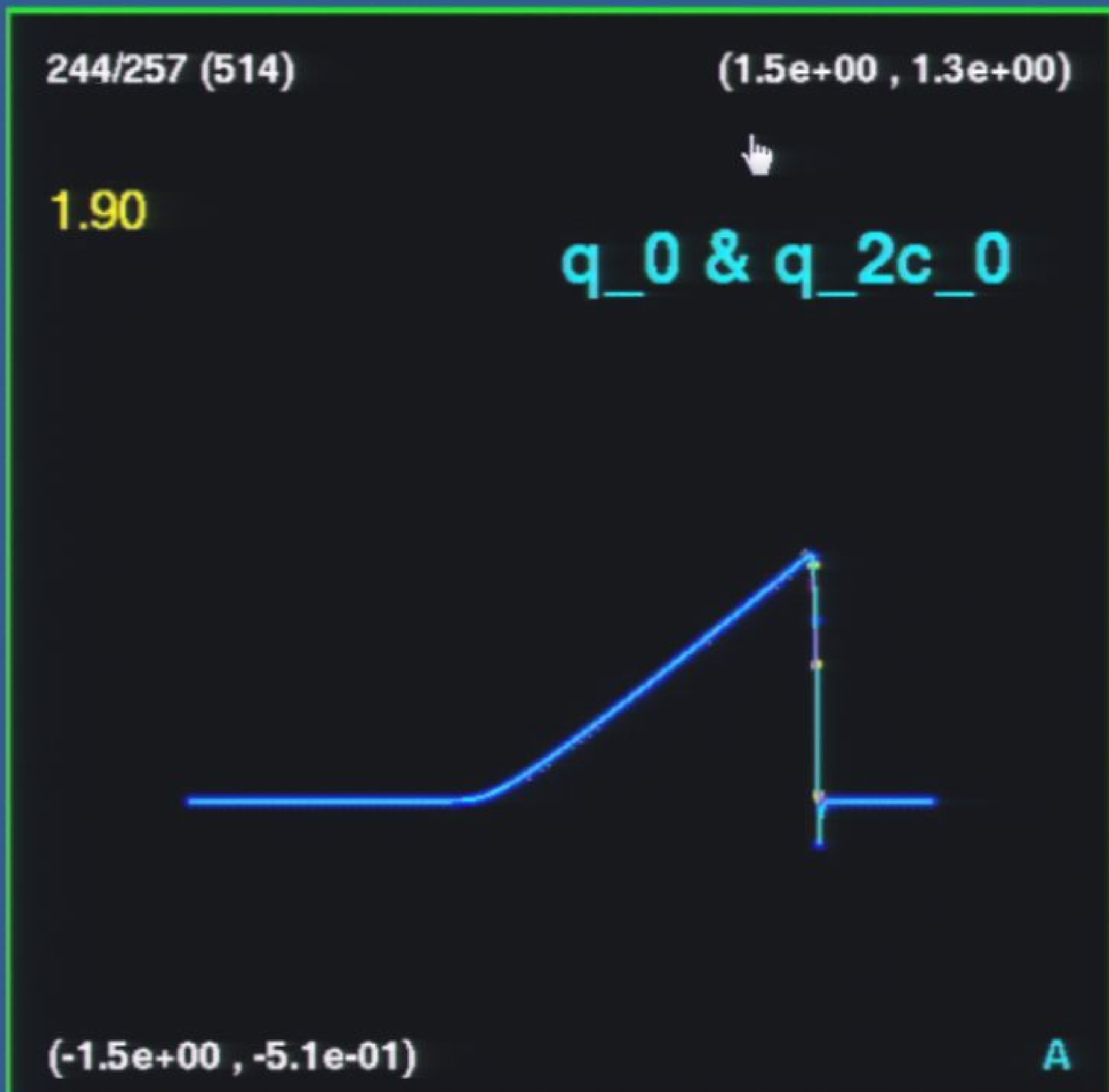
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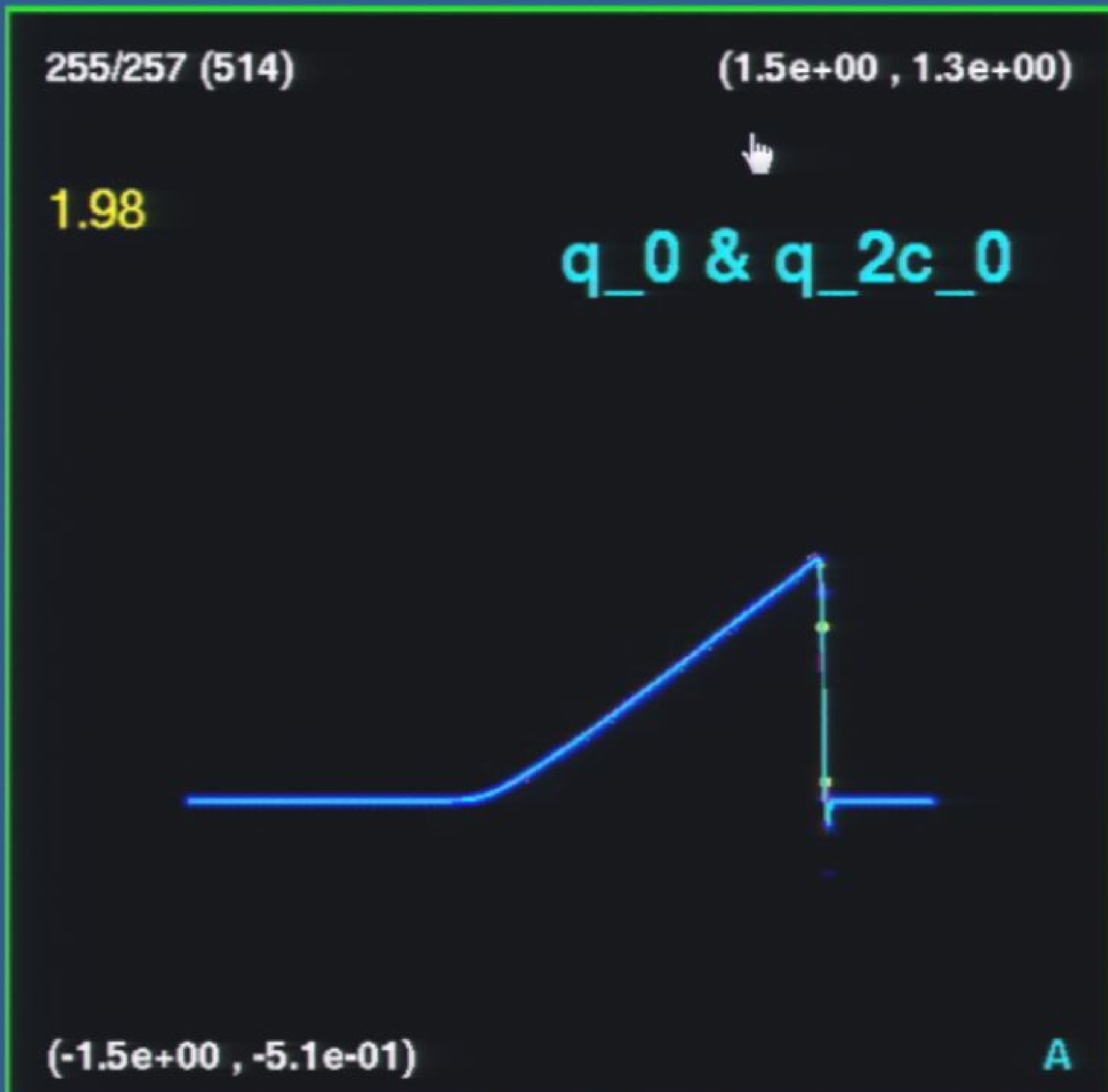
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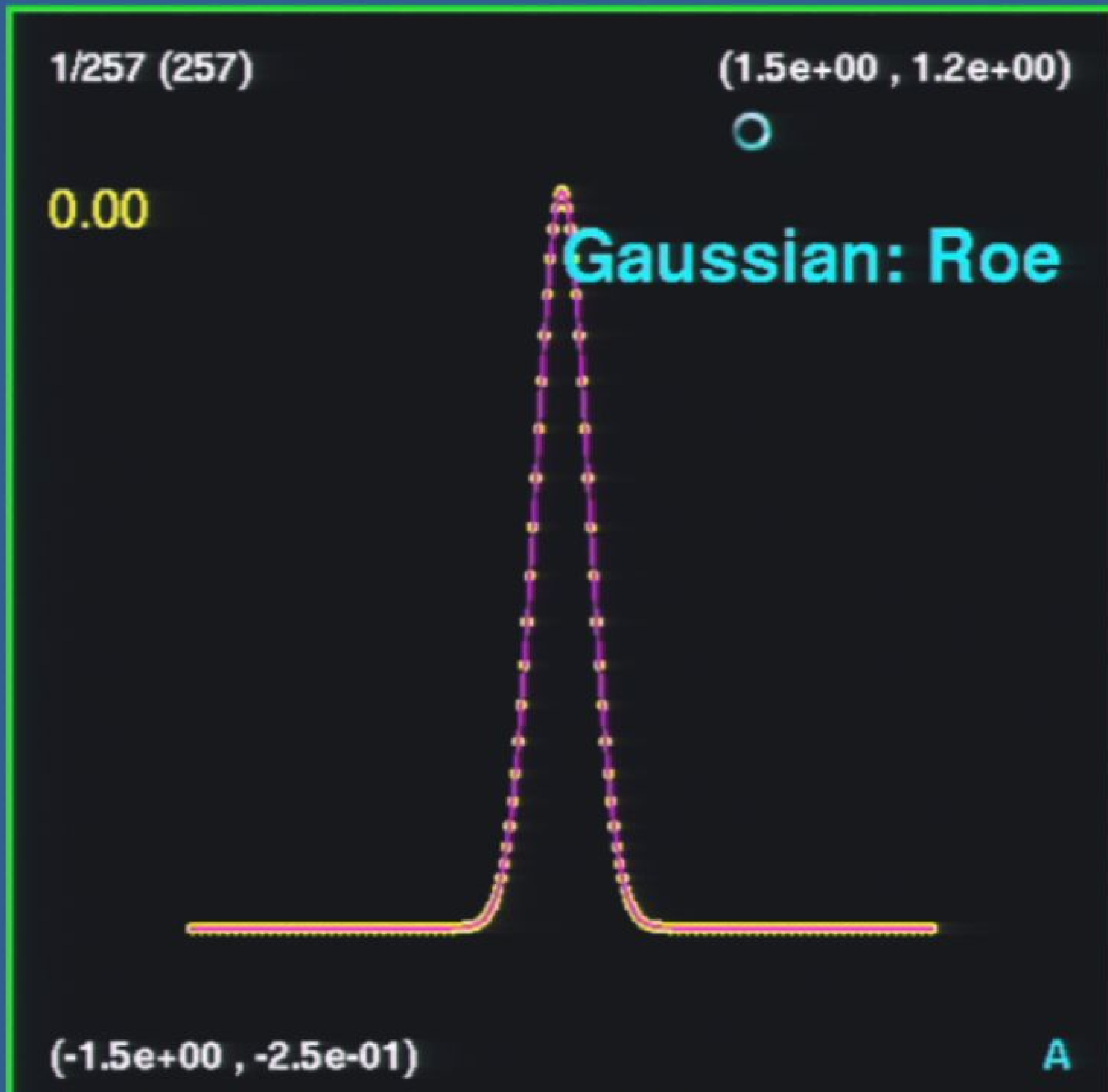
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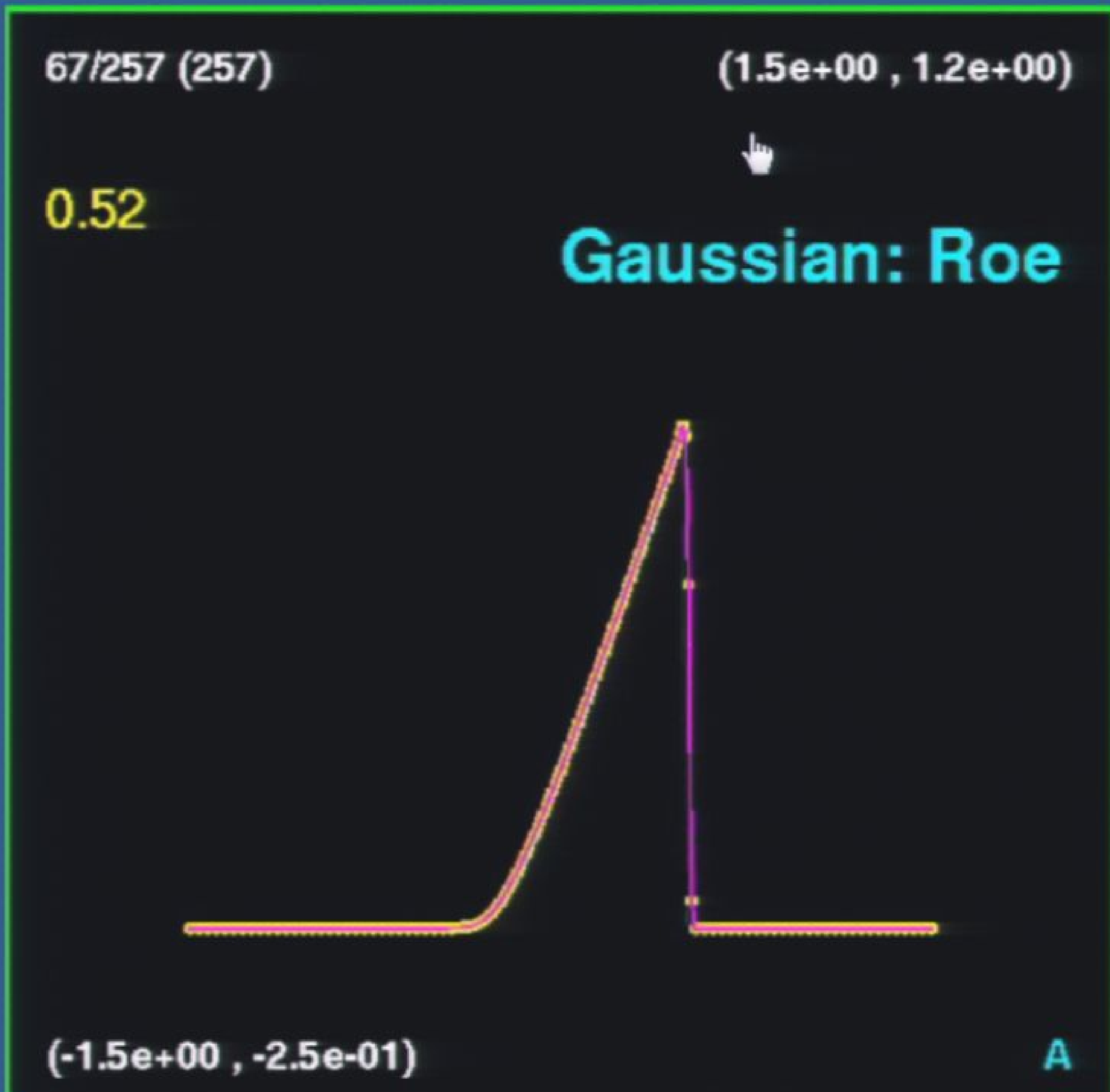
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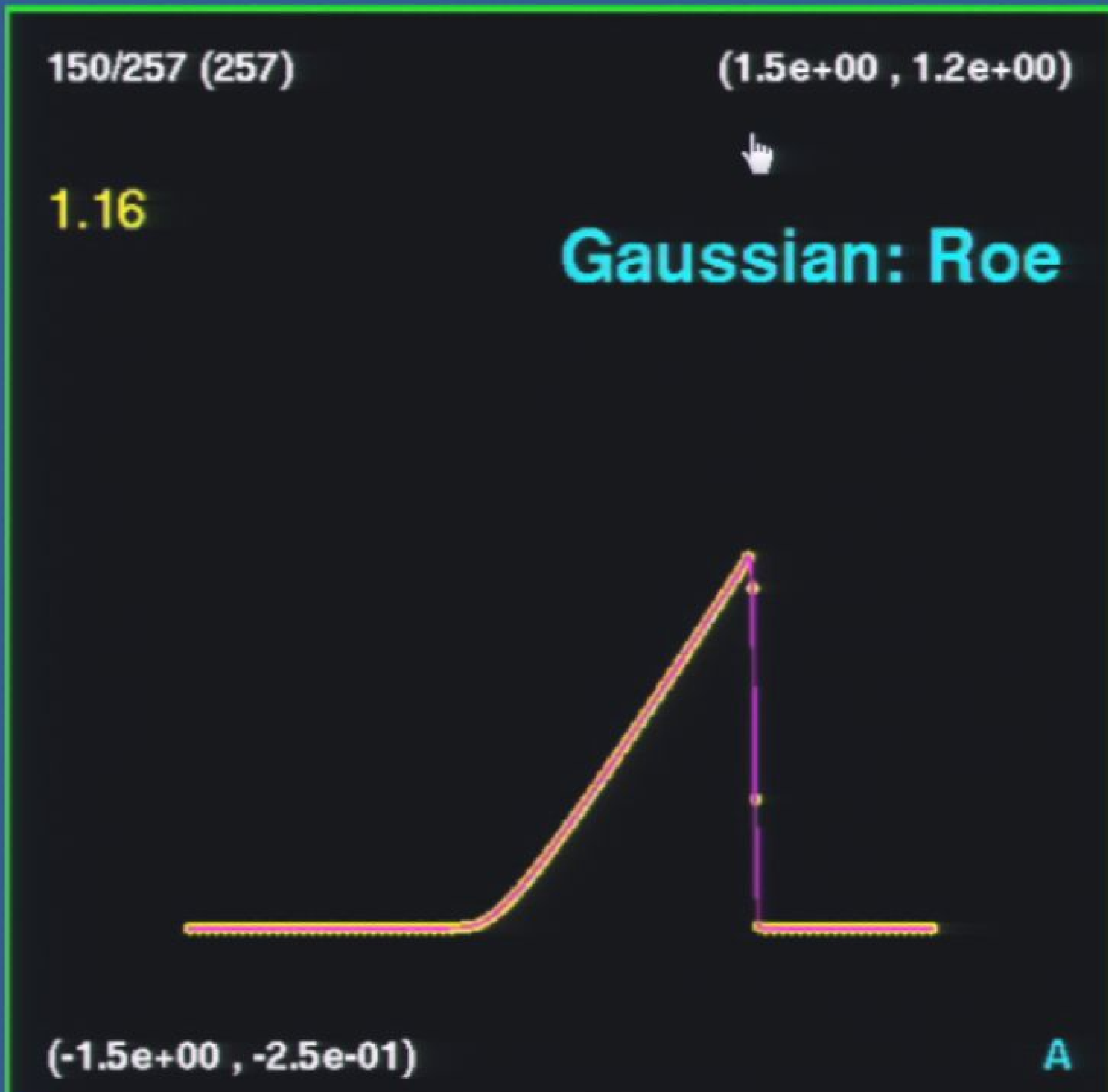
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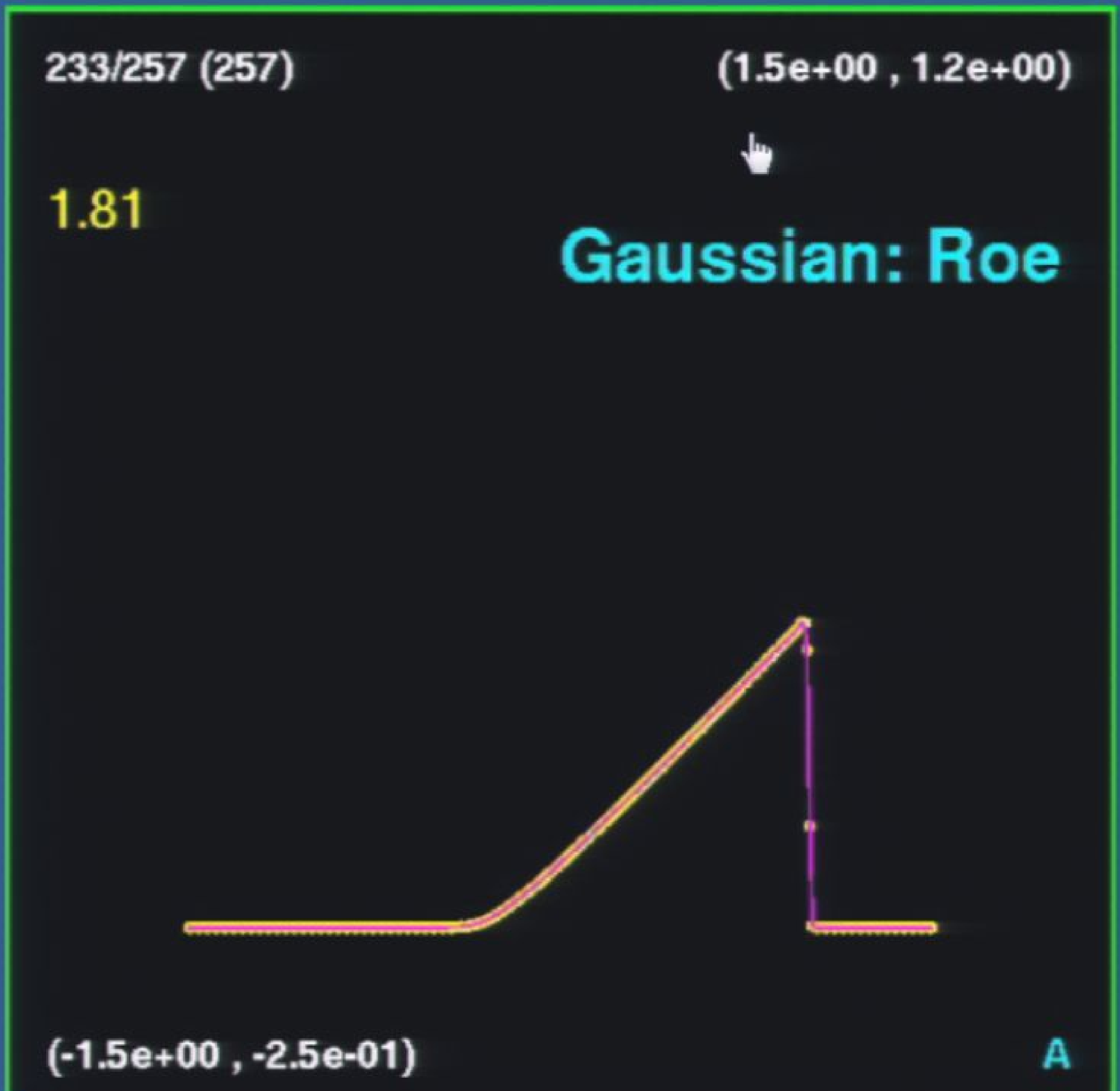

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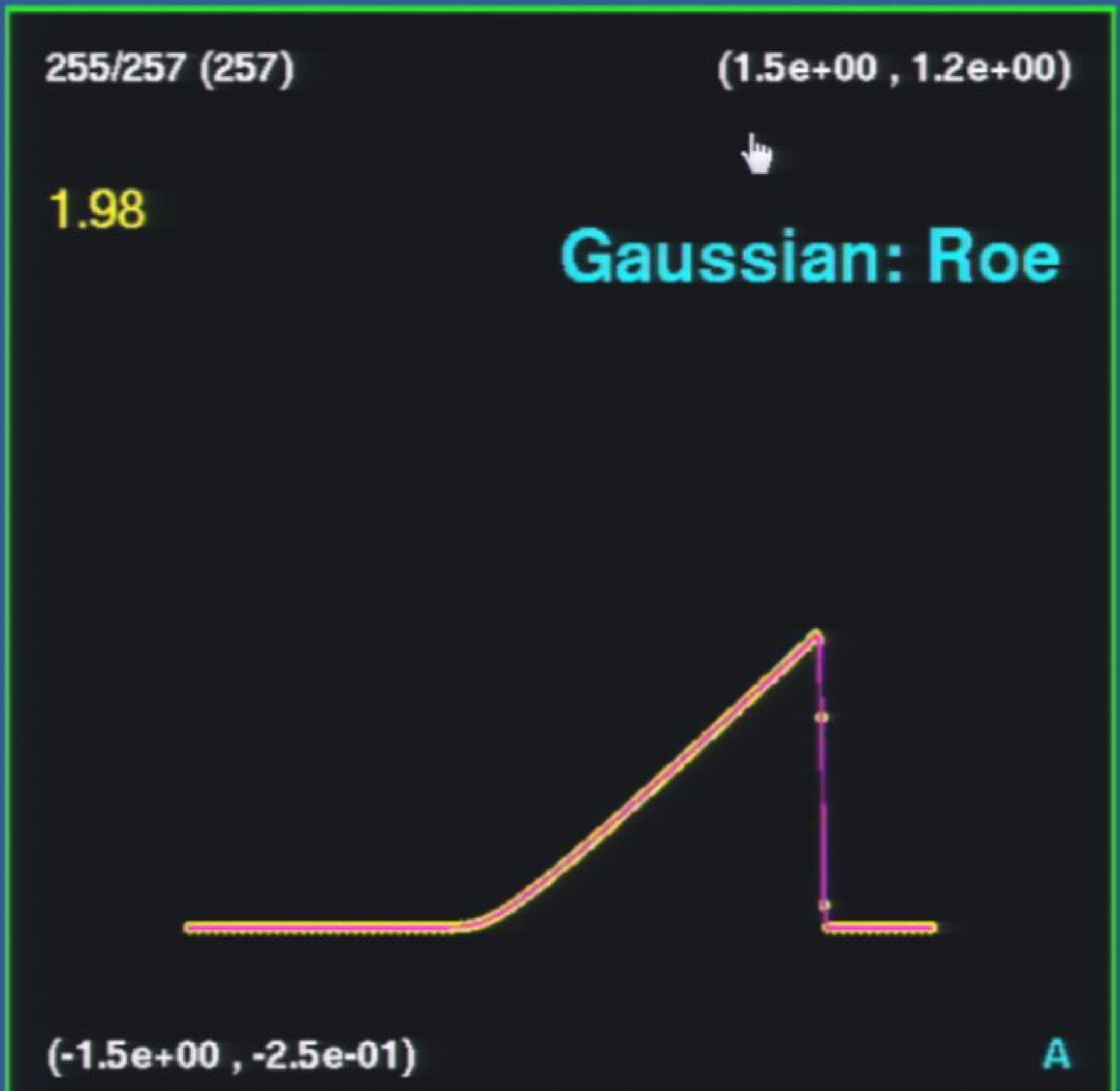
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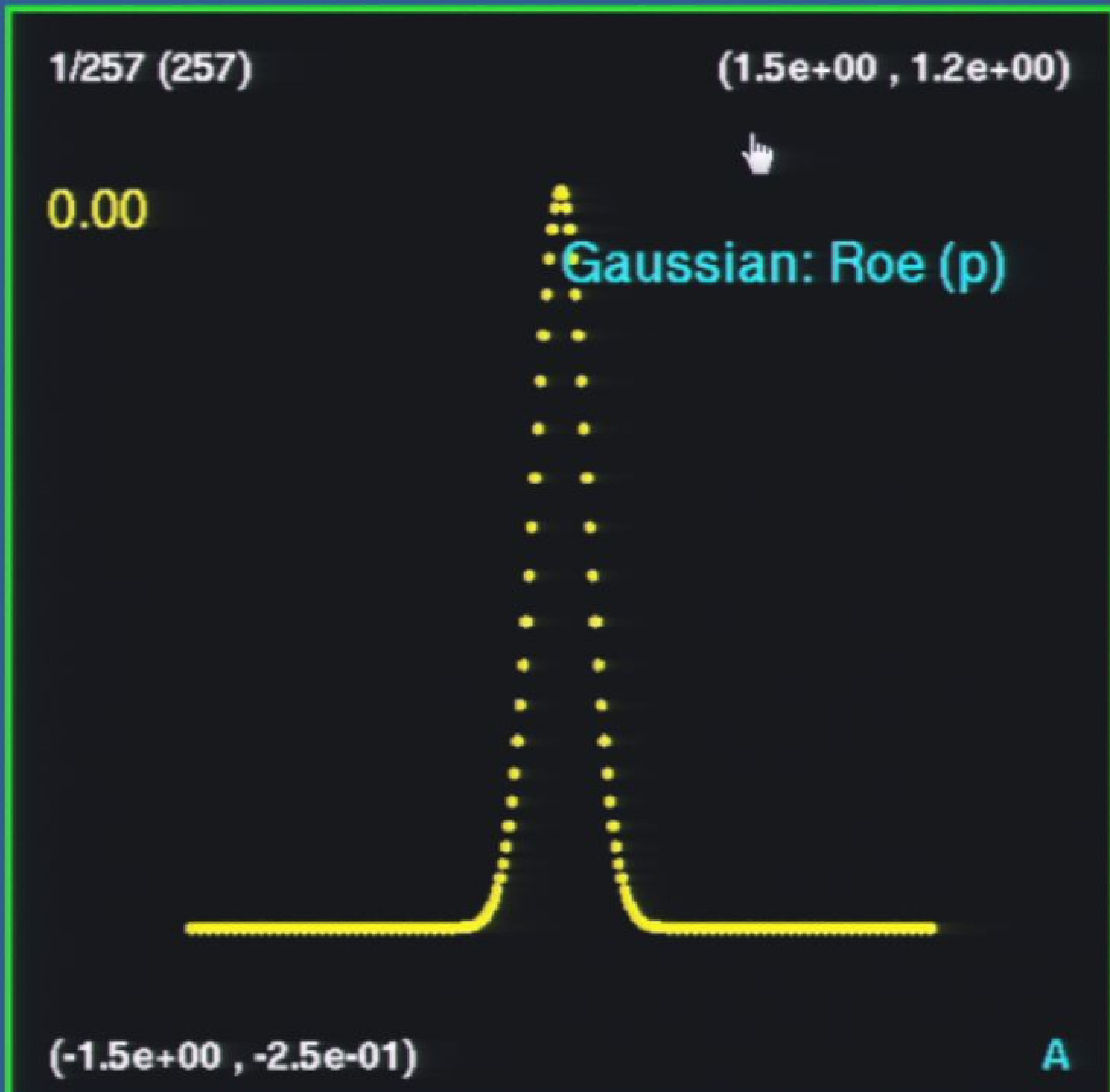
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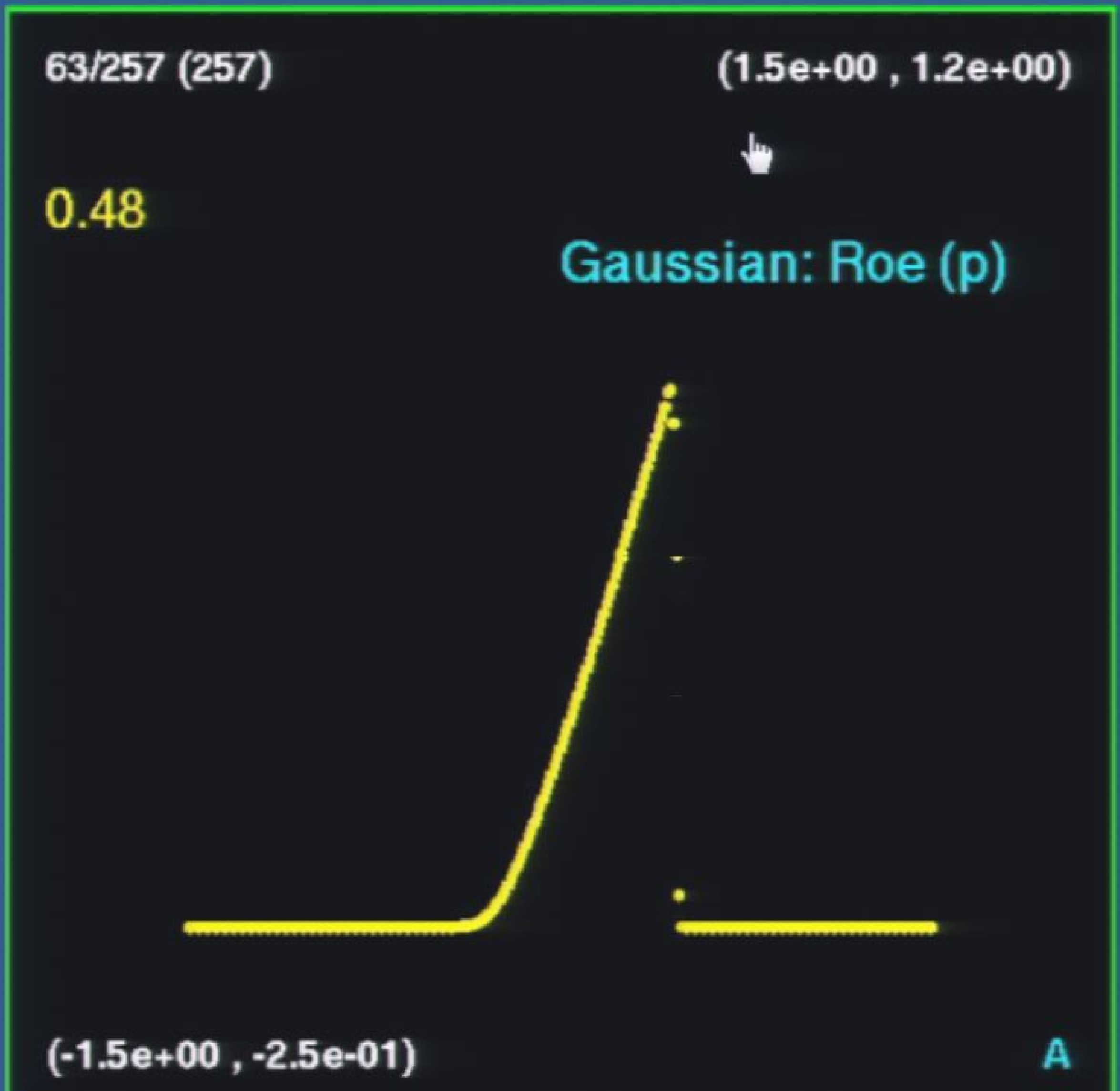
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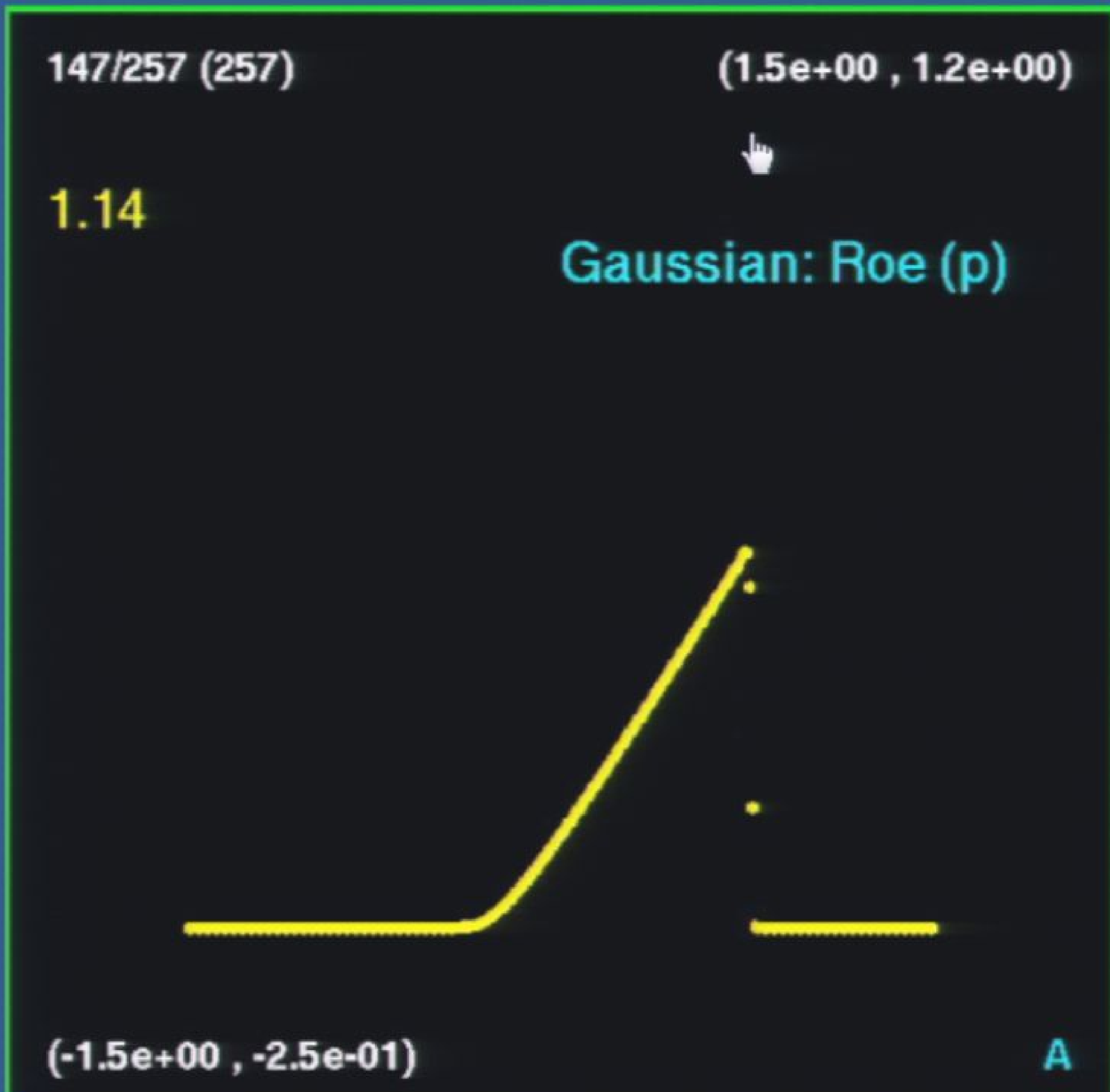
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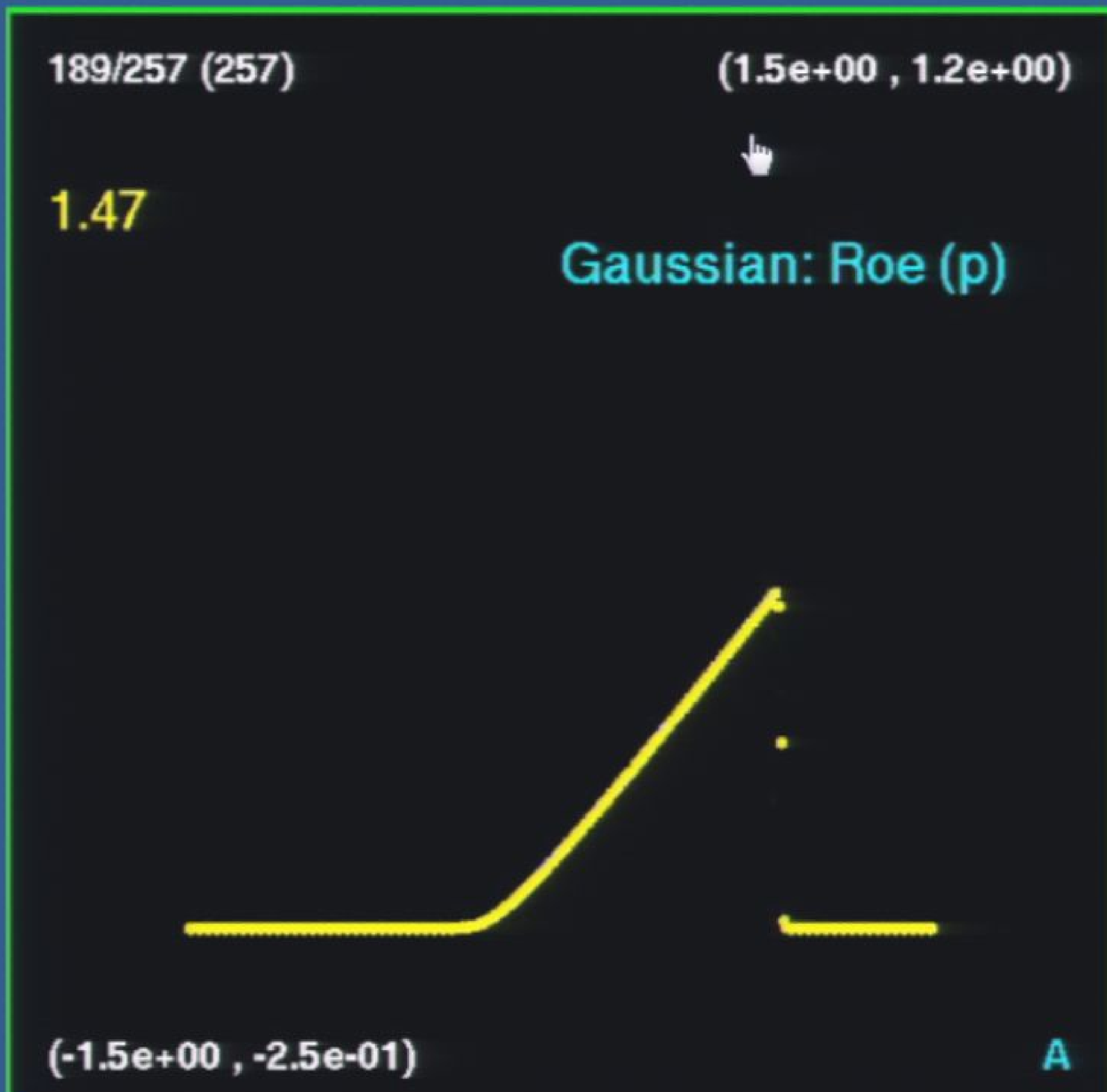
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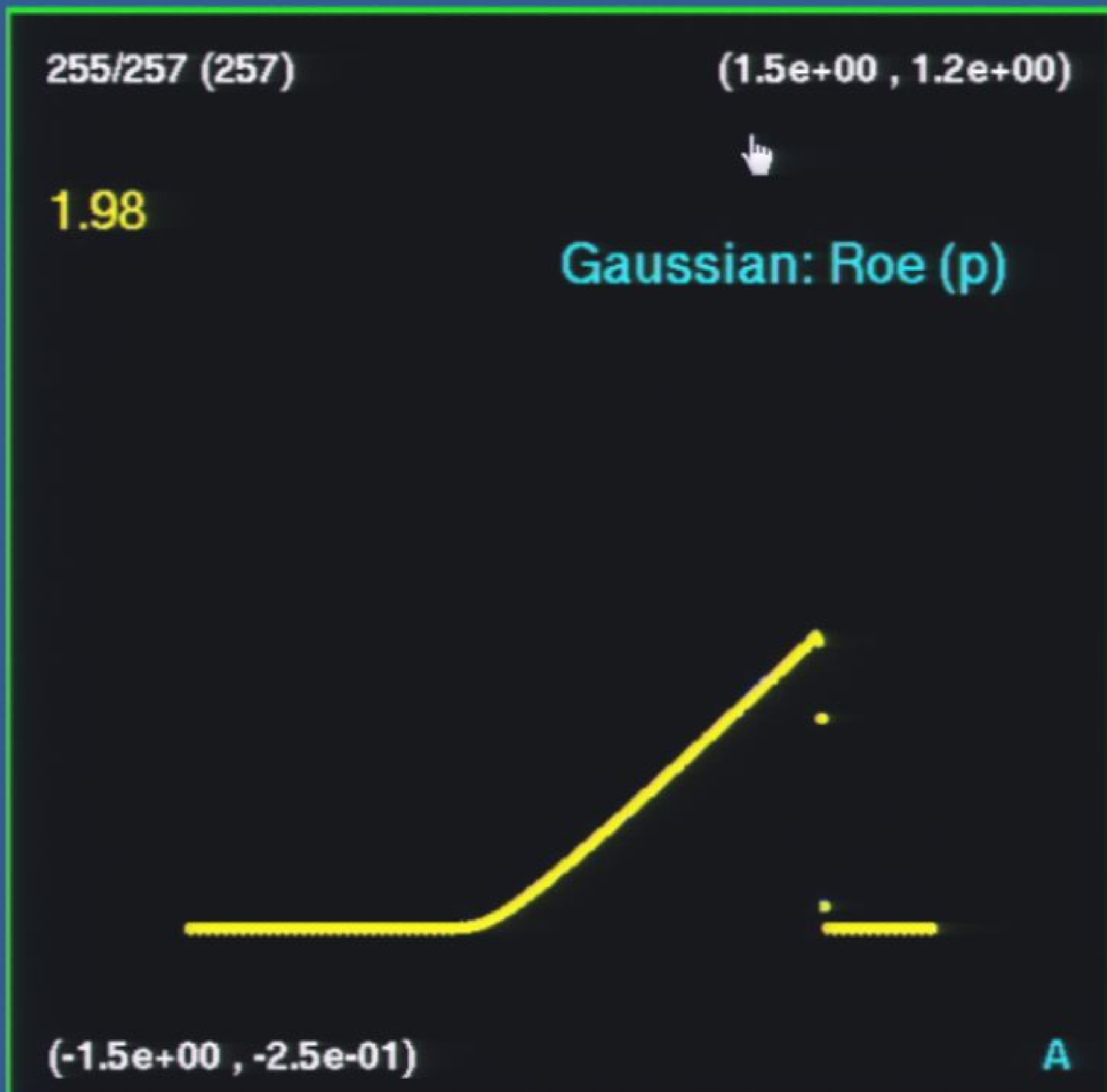
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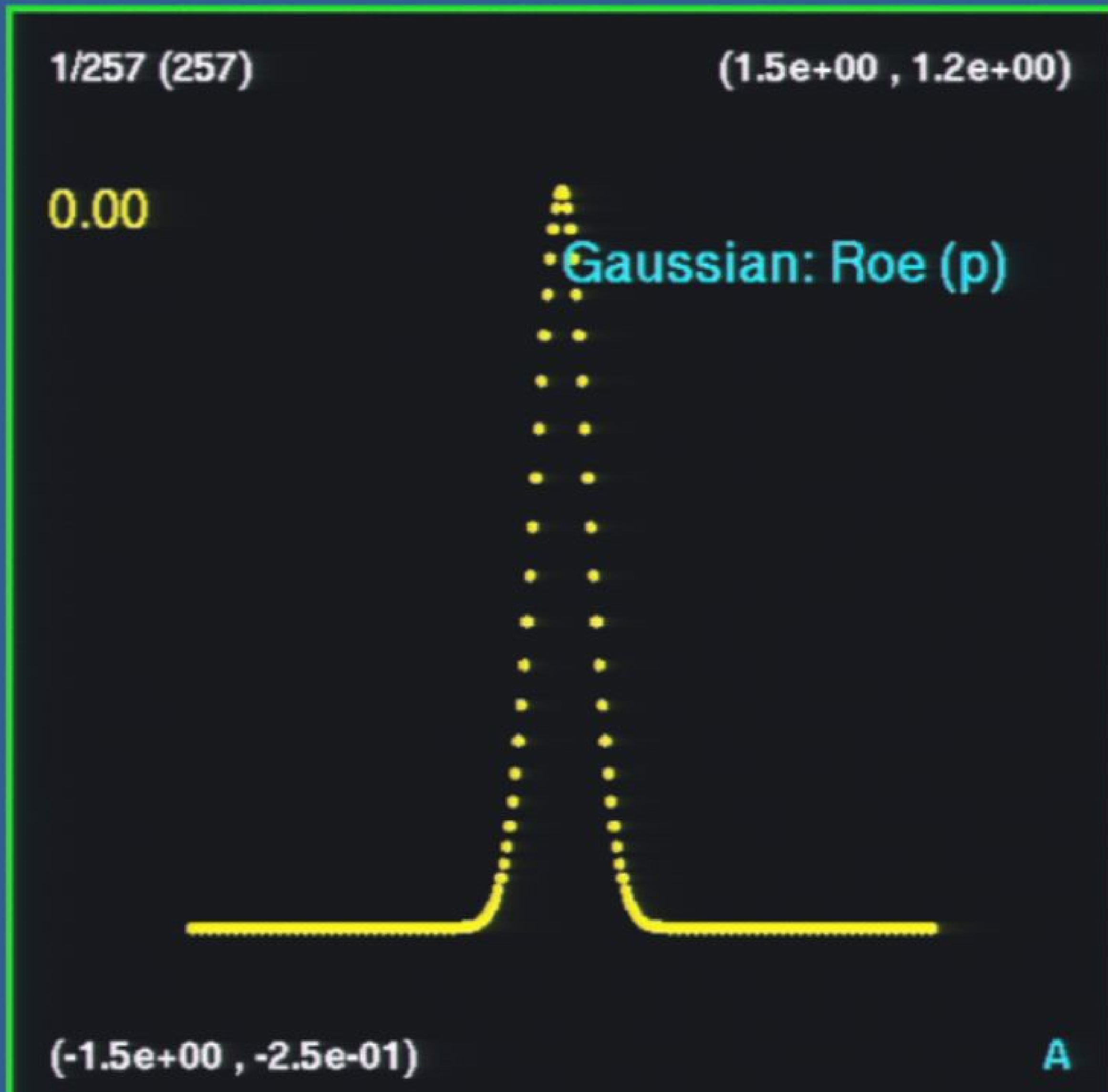
Gaussian: Roe (p)
(-1.5e+00 , -2.5e-01)
A

- Futurology: graves waves to be 'heard' this decade
- Detection/physics extraction: require at least some theoretical understanding of possible phenomenology
  - For binary black holes: status is pretty good [Pretorius next week]
  - For bh-ns / ns-ns binaries: far from being so
    - Parameter space (physics ingredients) is quite messy/more difficult to explore. Still, rich pay off GW, NS eqn of state, probe of mag fields
  - *Exciting possibility: catch systems in both GW and EM bands*
    - Must understand different systems deep enough (to the extent possible) → Cosmology; Astrophysics; etc..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Gaussian: Roe (p)
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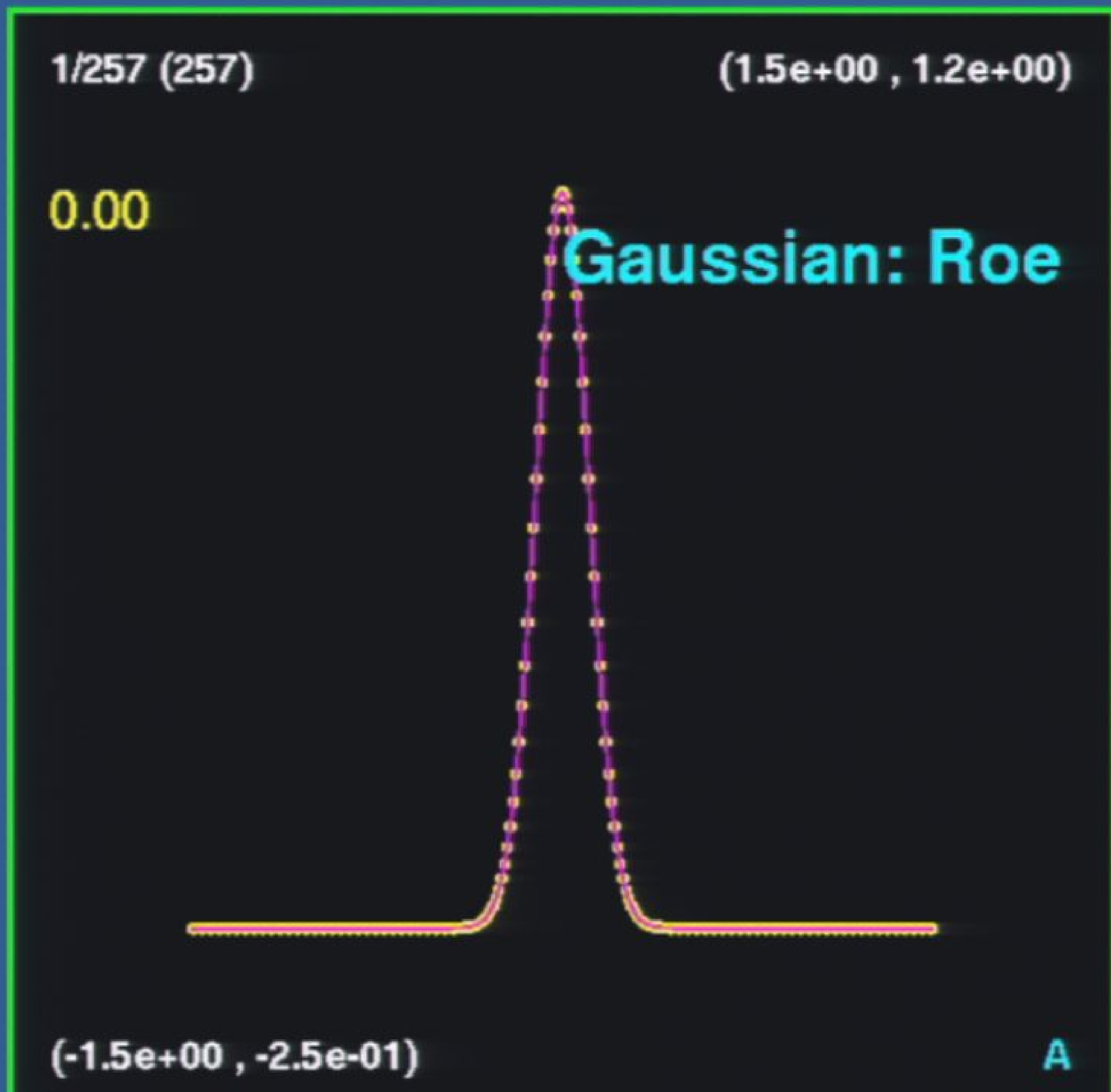
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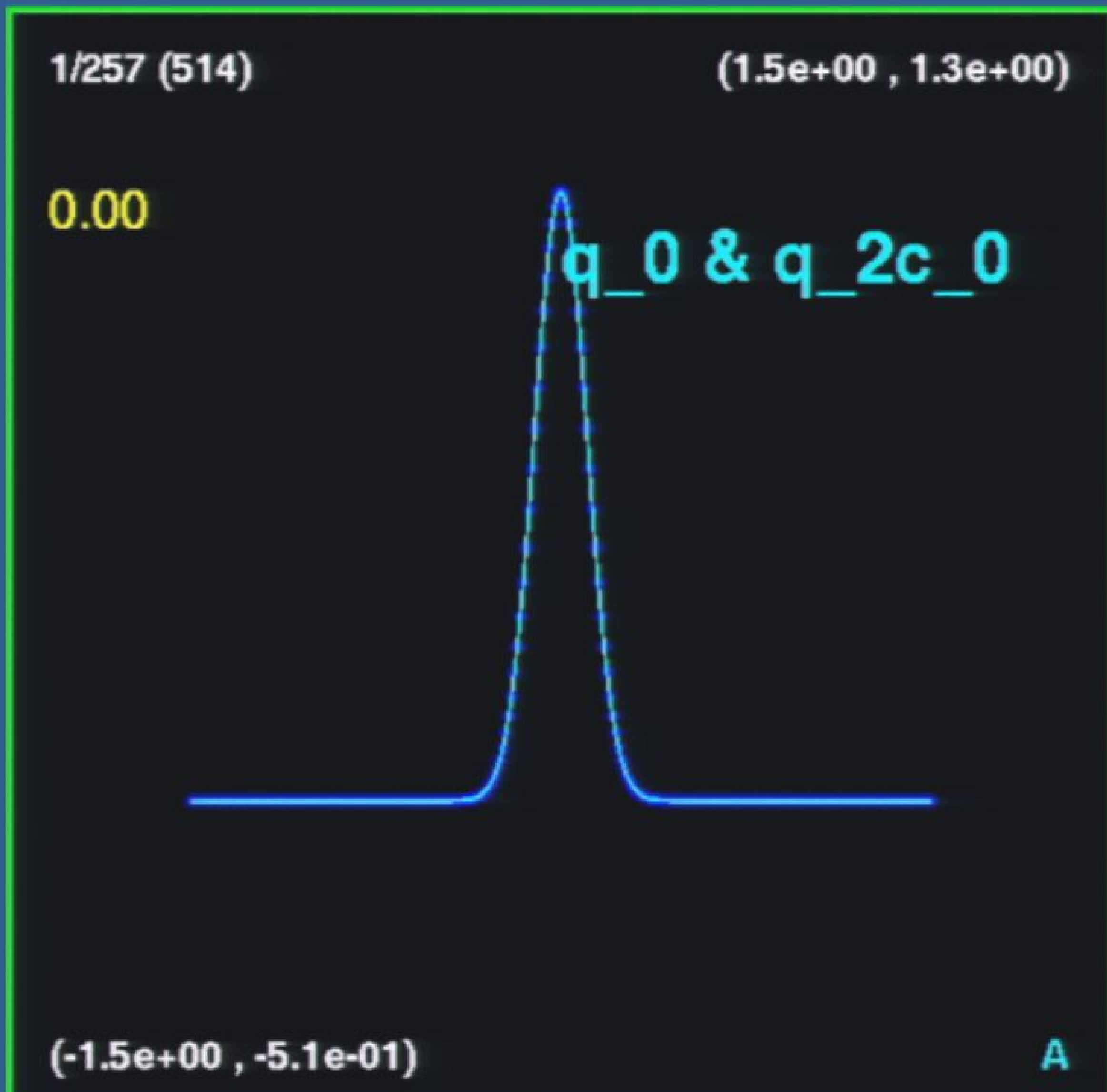
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q_0 & q_2c_0
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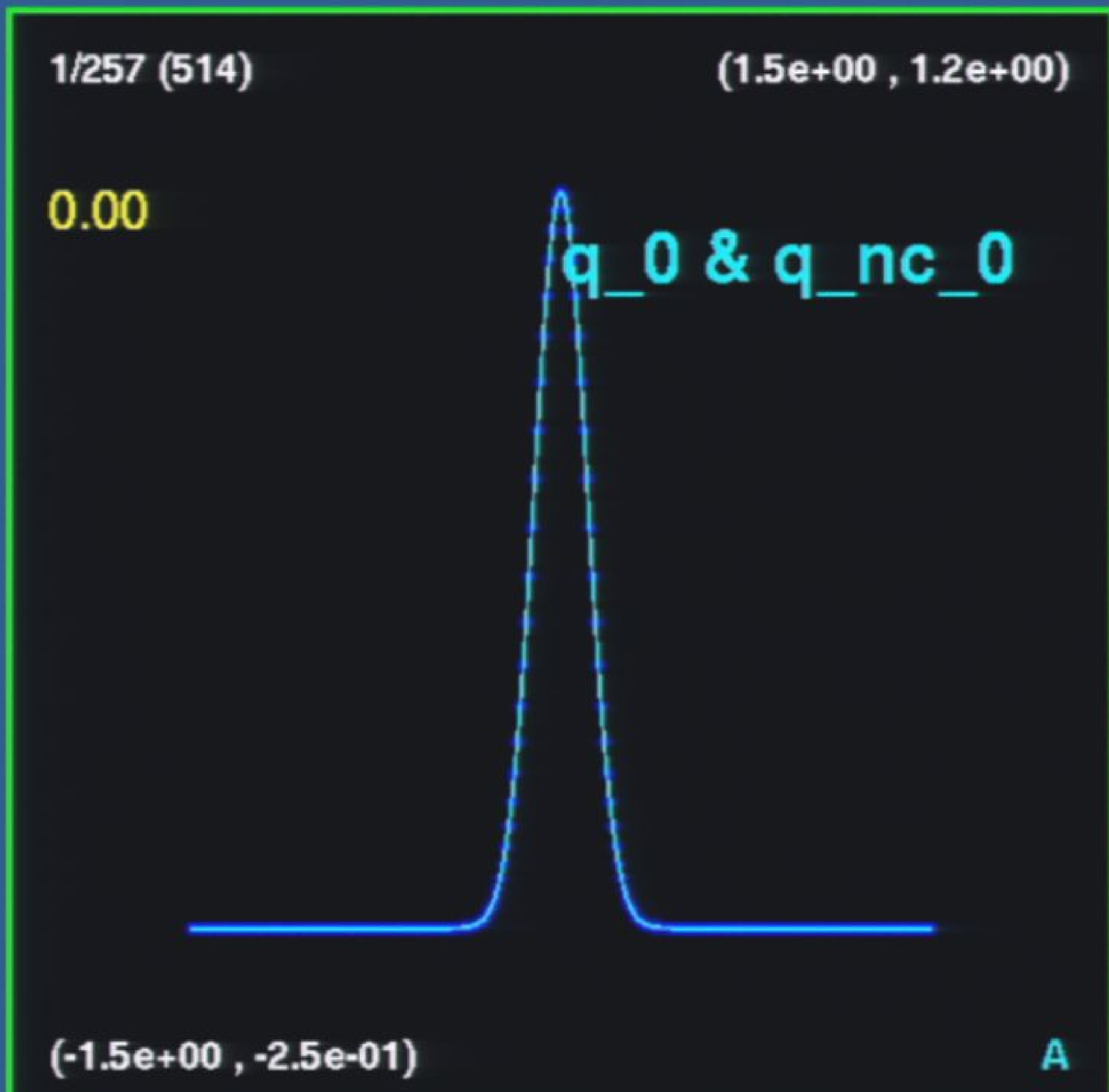
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q_0 & q_nc_0
(-1.5e+00 , -2.5e-01)
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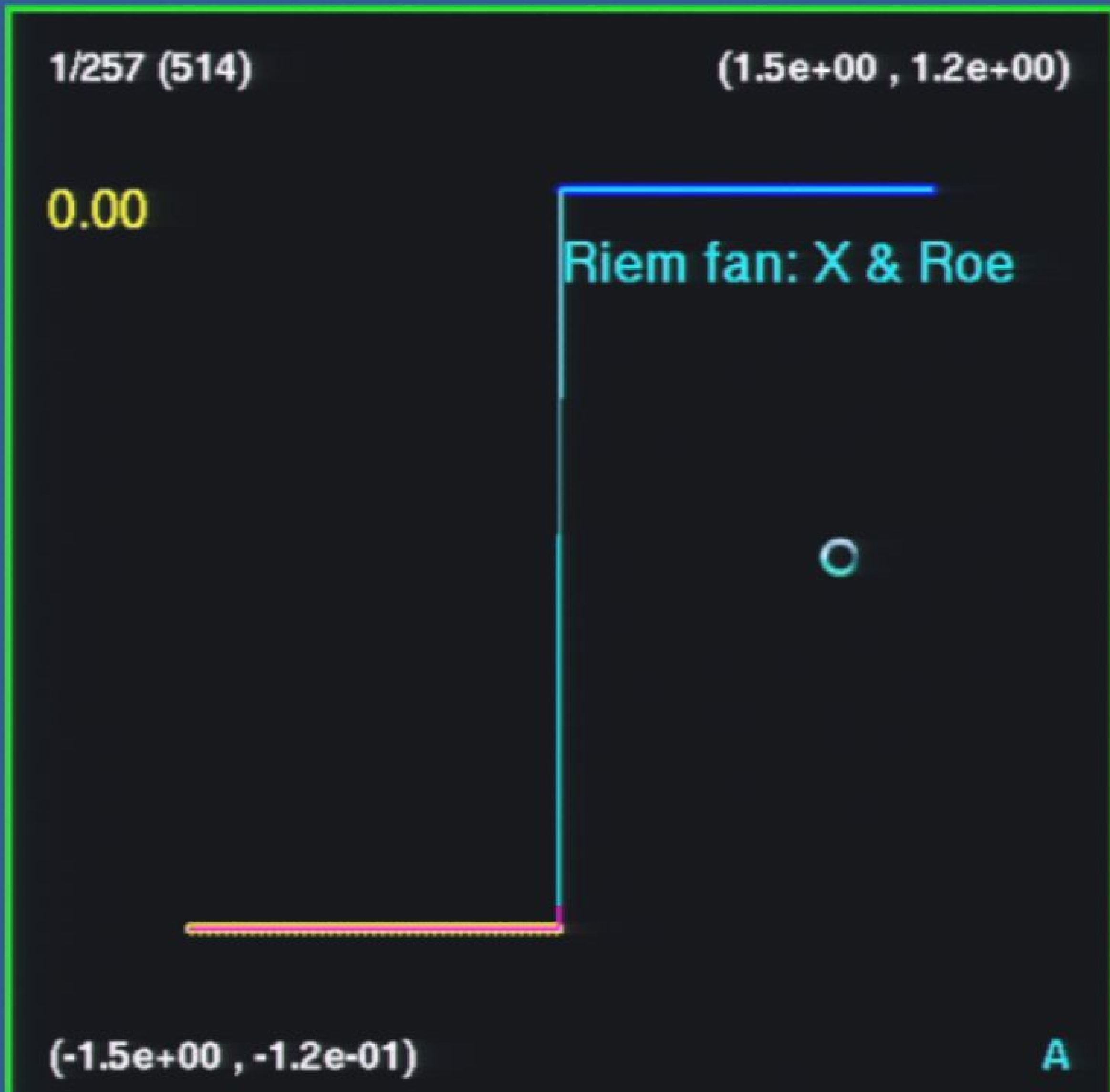
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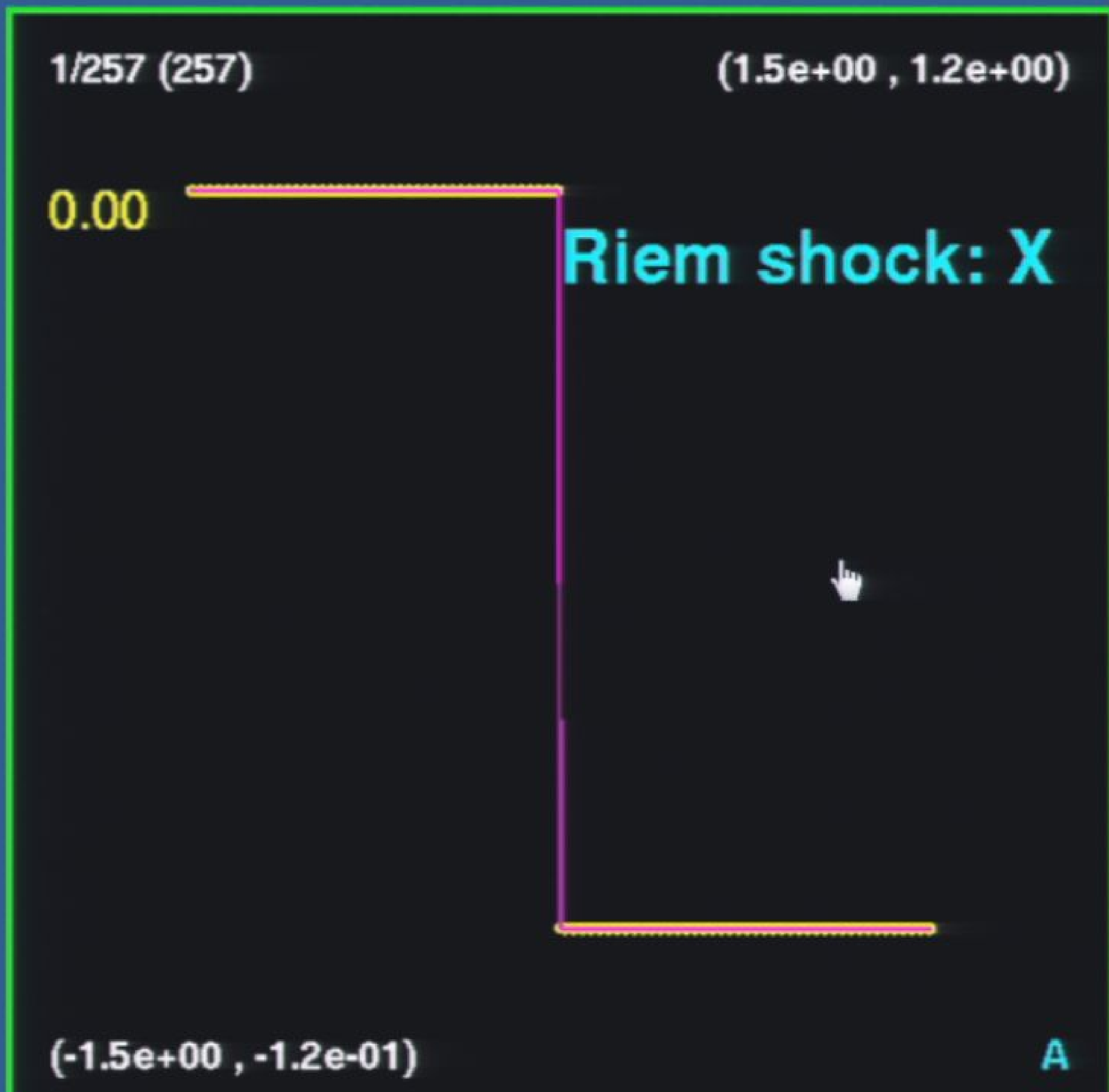
0.00
Riem fan: X & Roe
(-1.5e+00 , -1.2e-01)
A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0.00
Riem shock: X
(-1.5e+00 , -1.2e-01)
A


1/257 (257)
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0.00000000e+00
Riem fan: X
(-1.5e+00 , -1.2e-01)
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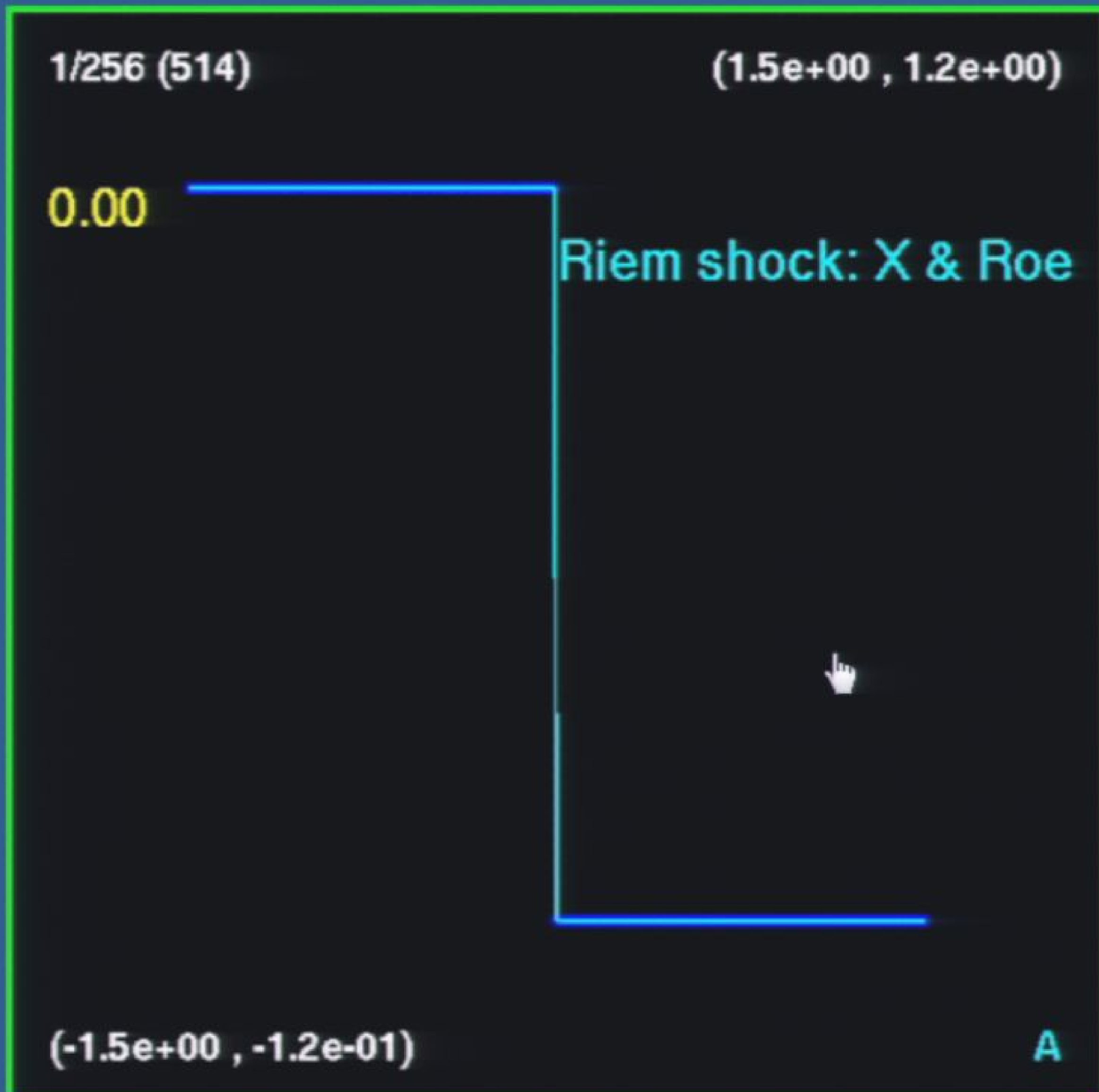
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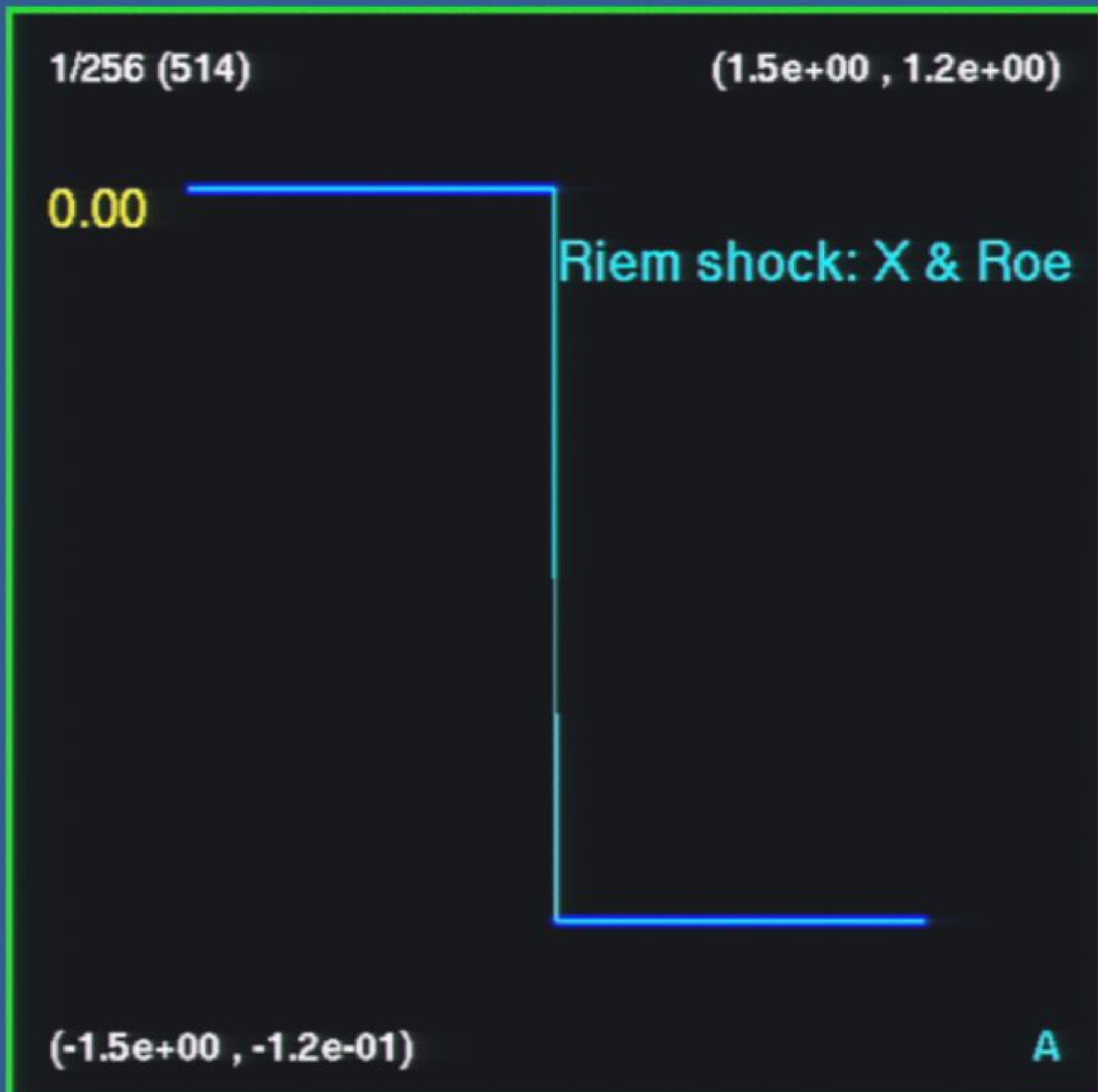
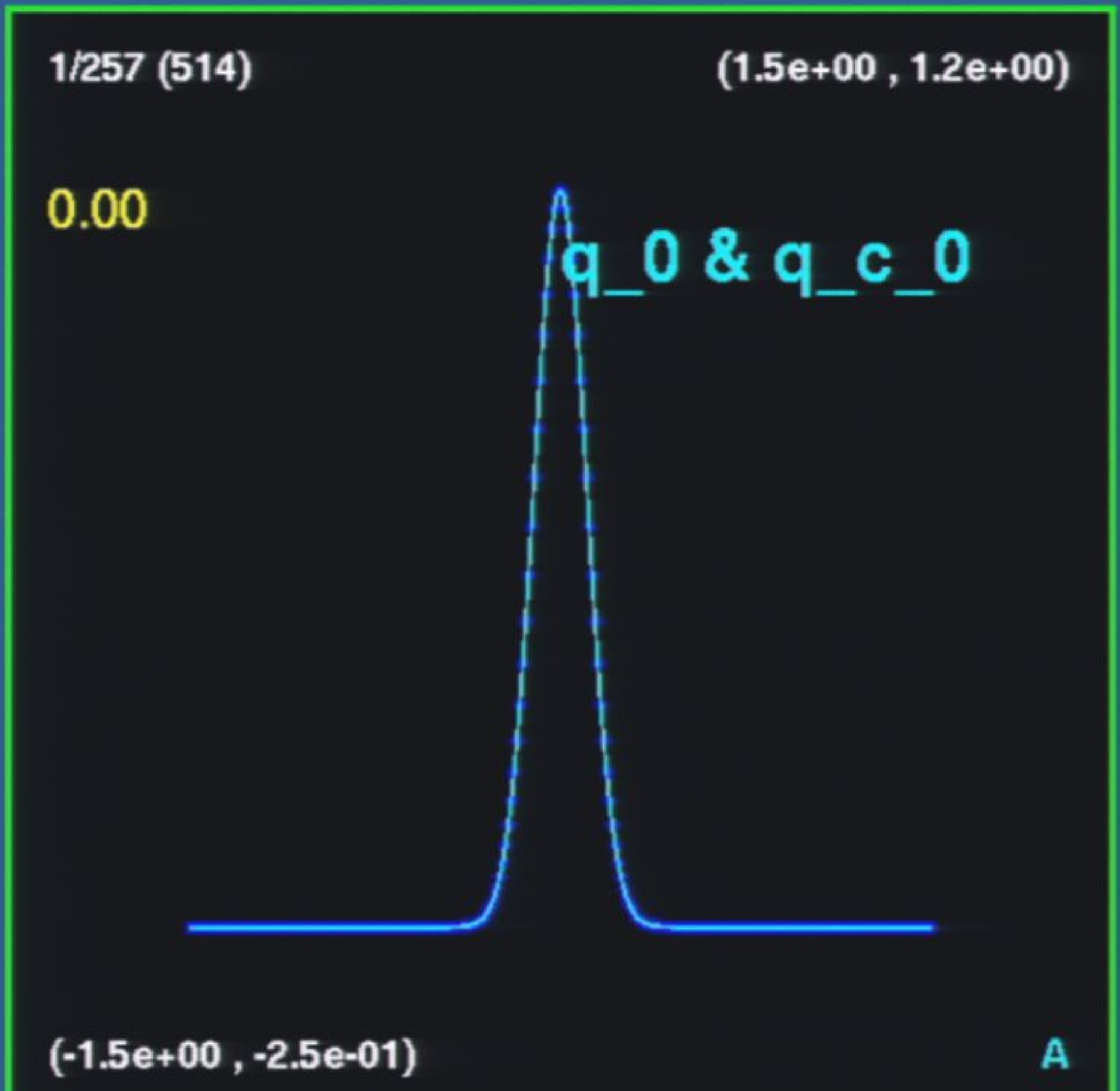
(1.5e+00 , 1.2e+00)
0.00
Riem shock: X & Roe
(-1.5e+00 , -1.2e-01)
A

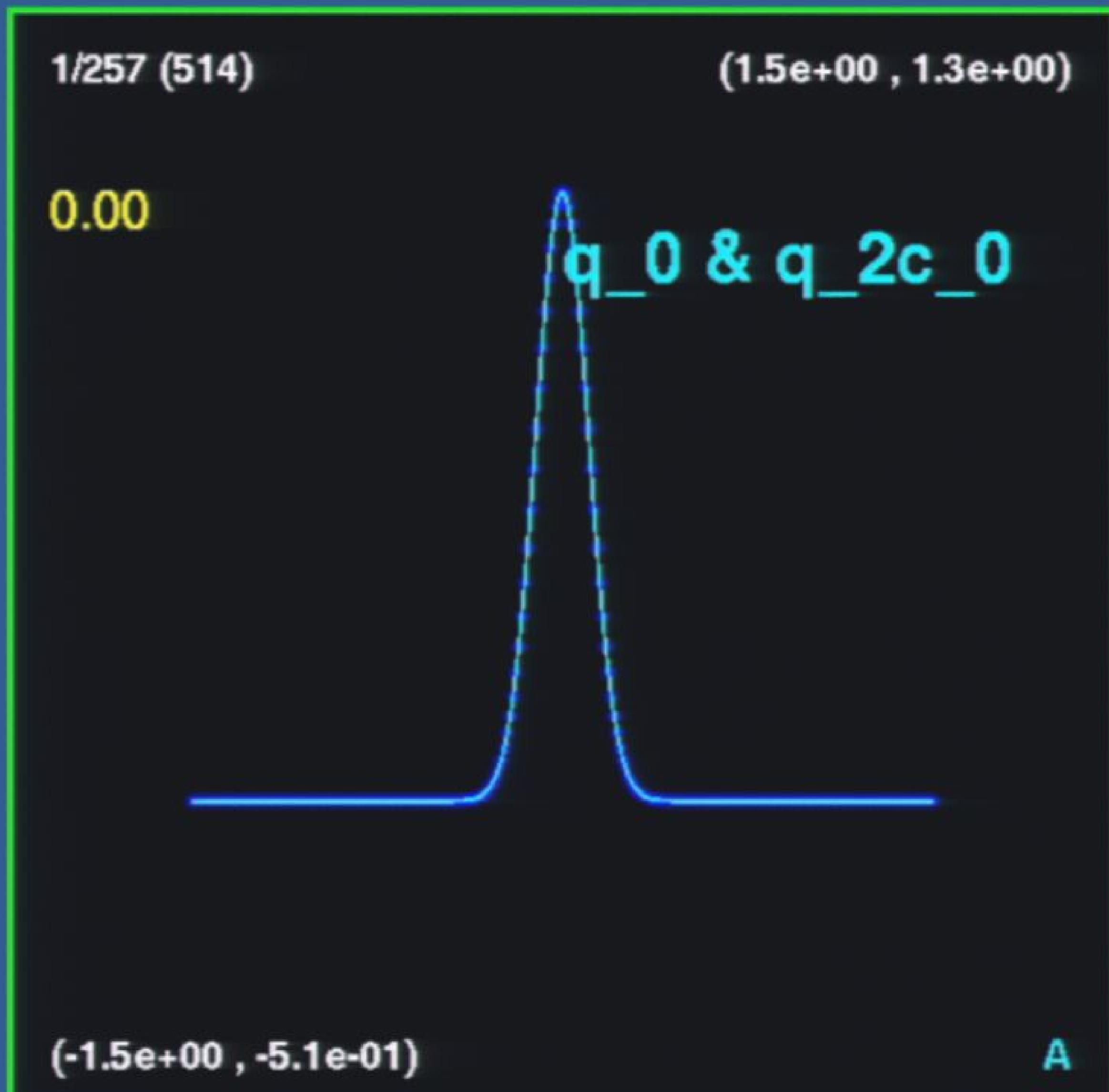

1/257 (257)
(1.5e+00 , 1.2e+00)
0.00000000e+00
Riem fan: X
(-1.5e+00 , -1.2e-01)
A

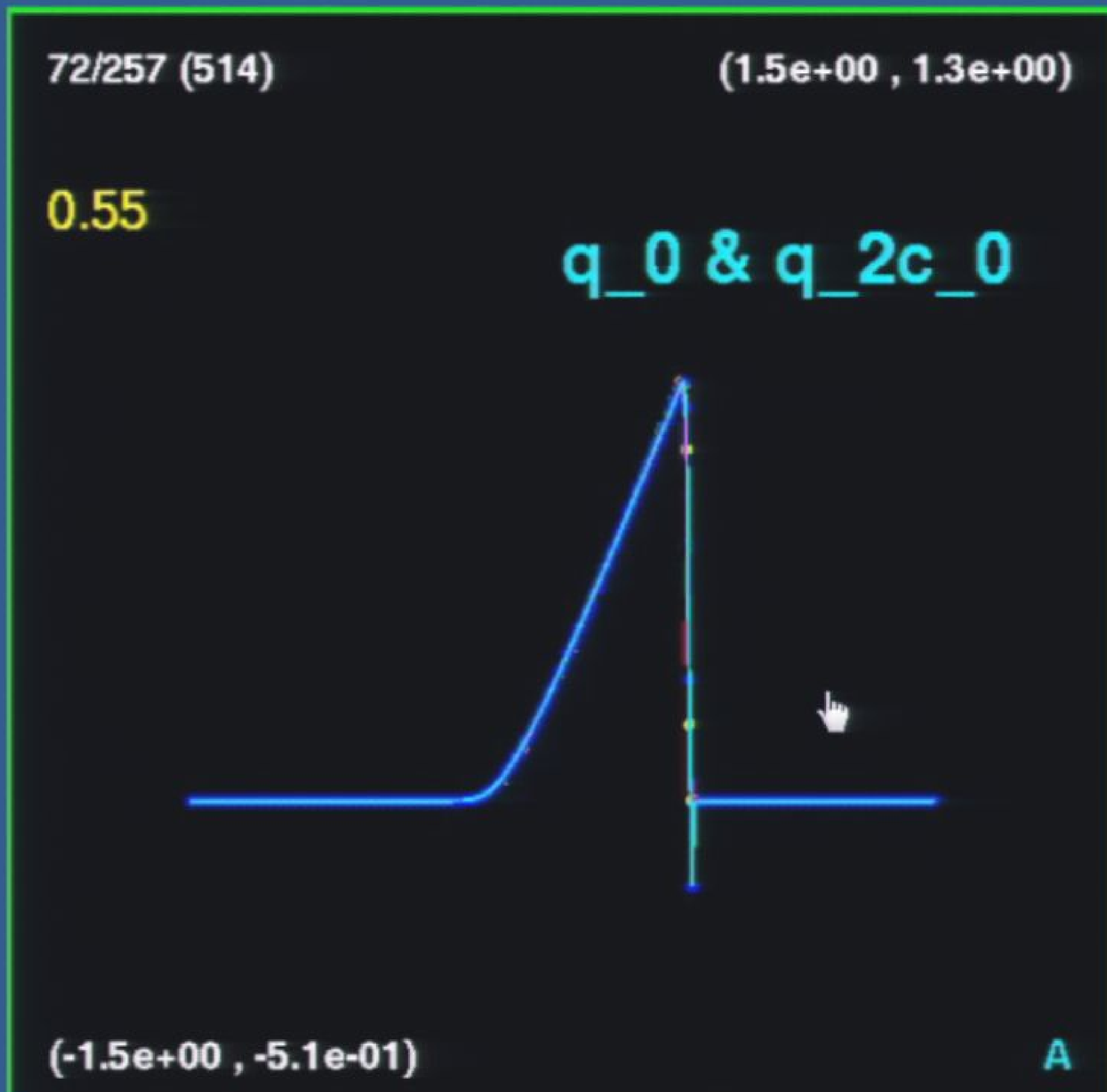

1/257 (257)
(1.5e+00 , 1.2e+00)
0.00
Riem shock: X
(-1.5e+00 , -1.2e-01)
A


1/256 (514)
(1.5e+00 , 1.2e+00)
0.00
Riem shock: X & Roe
(-1.5e+00 , -1.2e-01)
A

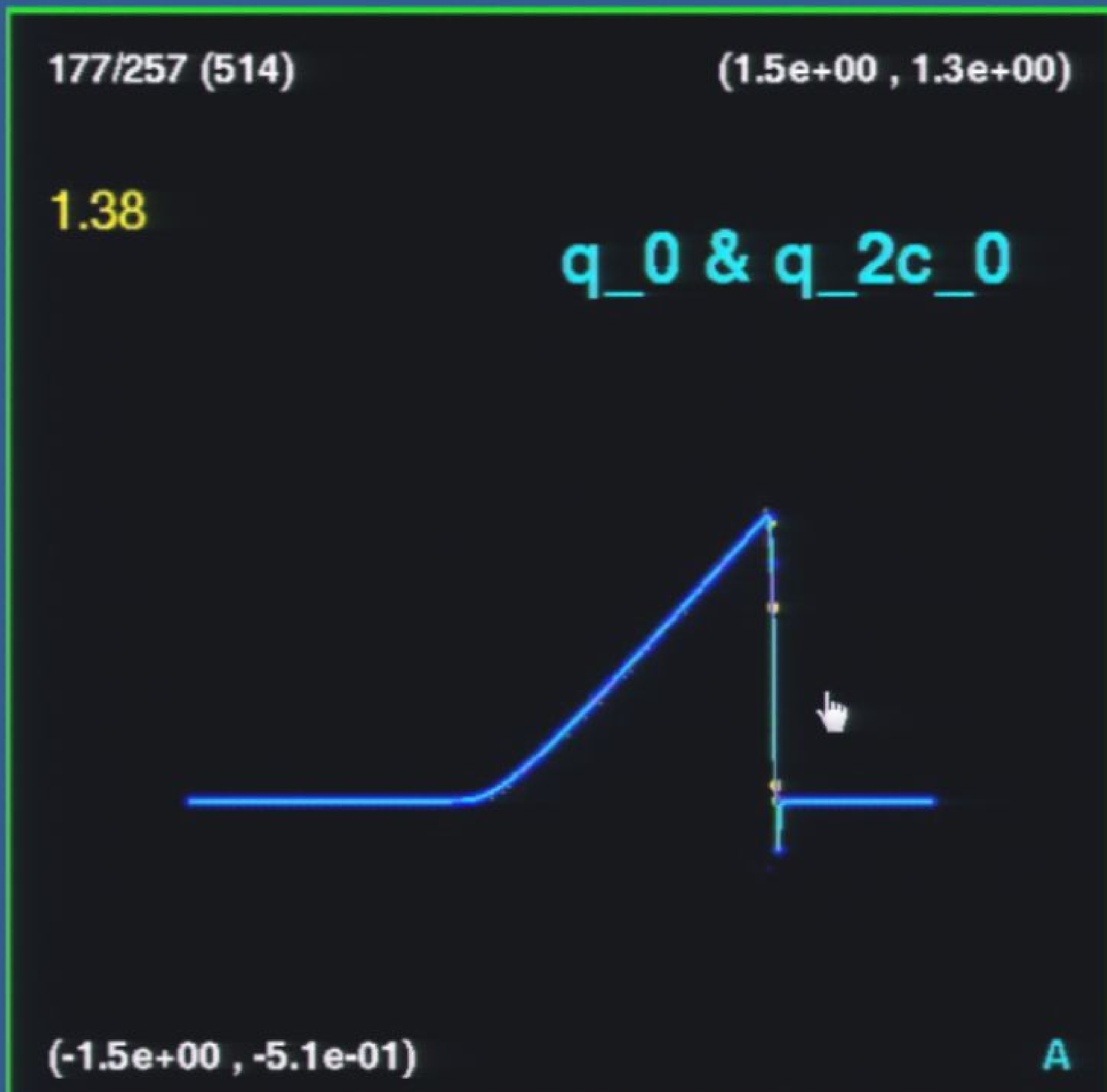

1/257 (257)
(1.5e+00 , 1.2e+00)
0.00000000e+00
Riem fan: X
(-1.5e+00 , -1.2e-01)
A


1/257 (514)
(1.5e+00 , 1.2e+00)
0.00
q_0 & q_c_0
(-1.5e+00 , -2.5e-01)
A

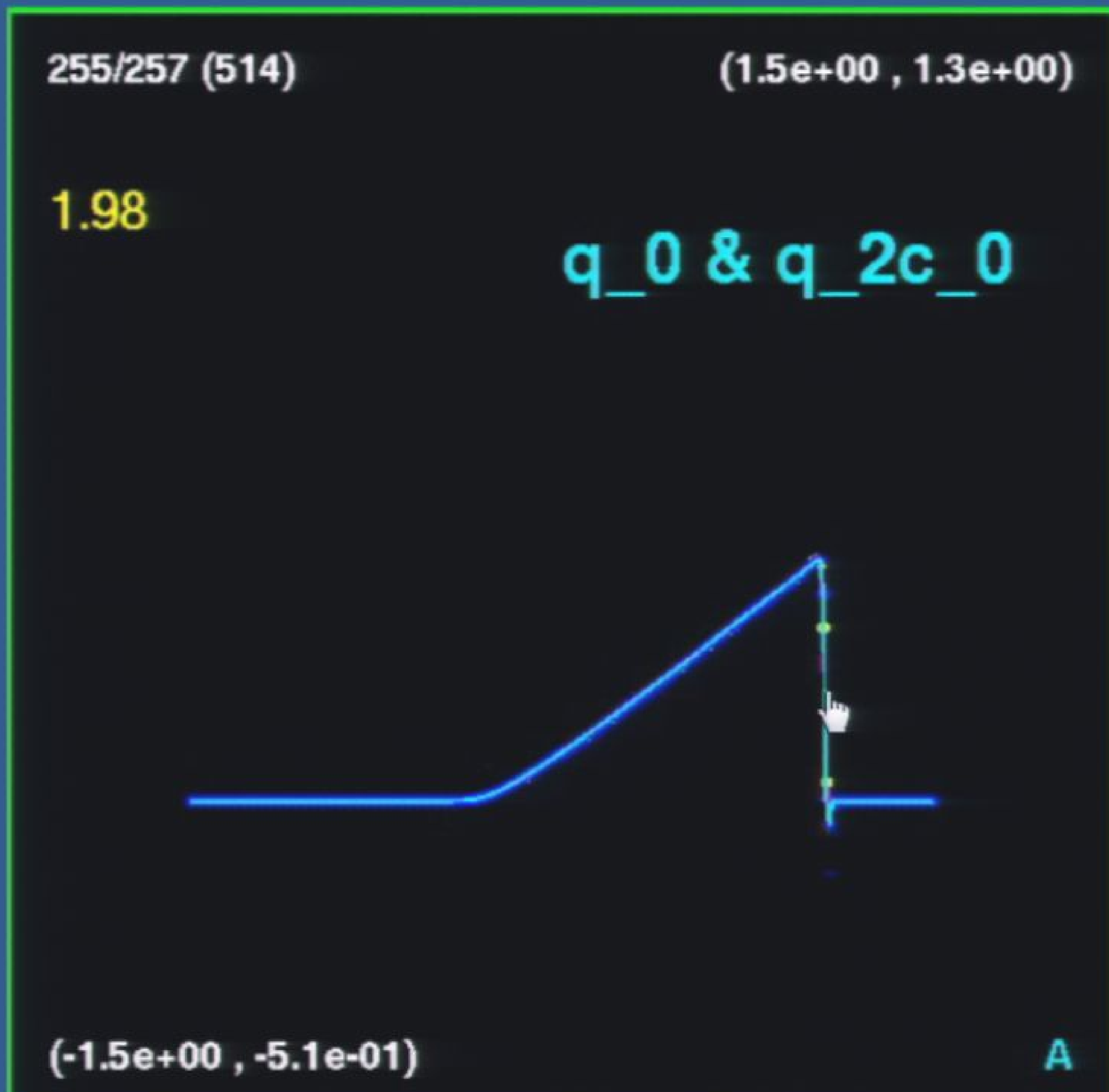

1/257 (514)
(1.5e+00 , 1.3e+00)
0.00
q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
A


72/257 (514)
(1.5e+00 , 1.3e+00)
0.55
q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
A

114/257 (514)
(1.5e+00 , 1.3e+00)
0.88
q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
A

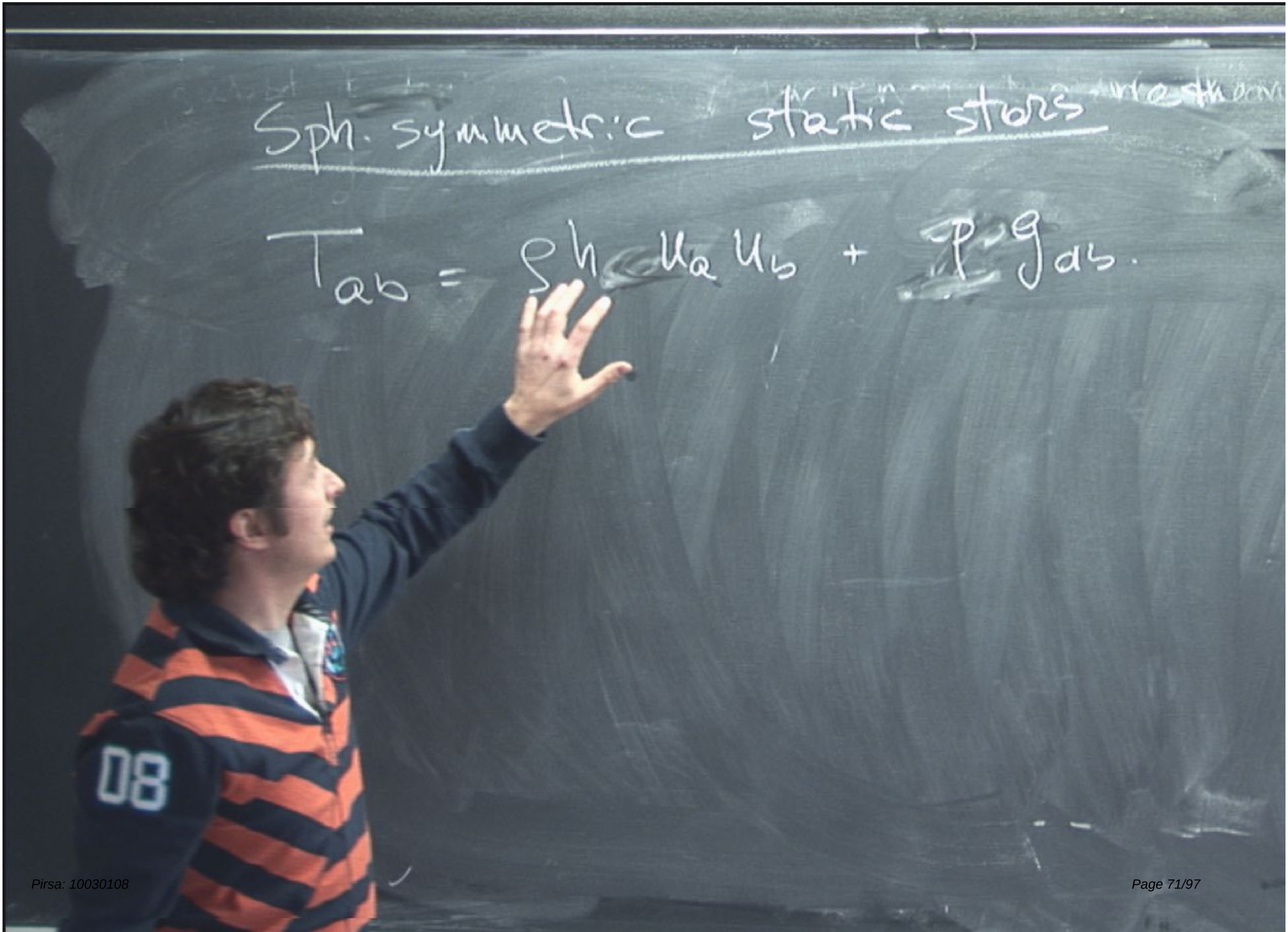

177/257 (514)
(1.5e+00 , 1.3e+00)
1.38
q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
A


232/257 (514)
(1.5e+00 , 1.3e+00)
1.80
q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
A


255/257 (514)
(1.5e+00 , 1.3e+00)
1.98
q_0 & q_2c_0
(-1.5e+00 , -5.1e-01)
A

Sph. symmetric static stars

$T_{ab} =$


Sph. symmetric static stars
$T_{ab} = \rho h u_a u_b + P g_{ab}$.

Sph. symmetric static stars

$$T_{ab} = \rho h\, u_a u_b + p\, g_{ab}.$$

$h = (1 + \varepsilon + P/\rho)$

$\nabla_a (\rho u^a) = 0$

$\nabla_a (T^{ab}) = 0$

$$h = (1 + \varepsilon + P/\rho)$$

$$\nabla_a(\rho u^a) = 0$$

$$\nabla_a(T^{ab}) = 0$$

× Sph. symmetric static stars

$$T_{ab} = \rho h \, u_a u_b + P g_{ab}.$$

$$ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2 d\Omega$$

$$h = \left(1 + \varepsilon + P/\rho\right)$$

$$\hat{\rho} = (\rho + \rho\varepsilon)$$

$$+ \nabla_a(\rho u^a) = 0$$

$$\times \nabla_a(T^{ab}) = 0$$

× Sph. symmetric static stars

$$T_{ab} = \rho h \, u_a u_b + P g_{ab}.$$

$$ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2 d\Omega$$

$$T_{ab} = (\hat{\rho} + P)\, u_a u_b + P g_{ab}.$$

$$h = (1 + \varepsilon + P/\rho)$$

$$\hat{\rho} = (\rho + \rho\varepsilon)$$

$$\nabla_a(\rho u^a) = 0$$

$$\nabla_a(T^{ab}) = 0$$

$$u^a = \alpha(1, \vec{0}) \qquad u^a u_a = -1 \Rightarrow u_a = (-\sqrt{B}, \vec{0})$$

$$h = \left(1 + \varepsilon + P/\rho\right)$$

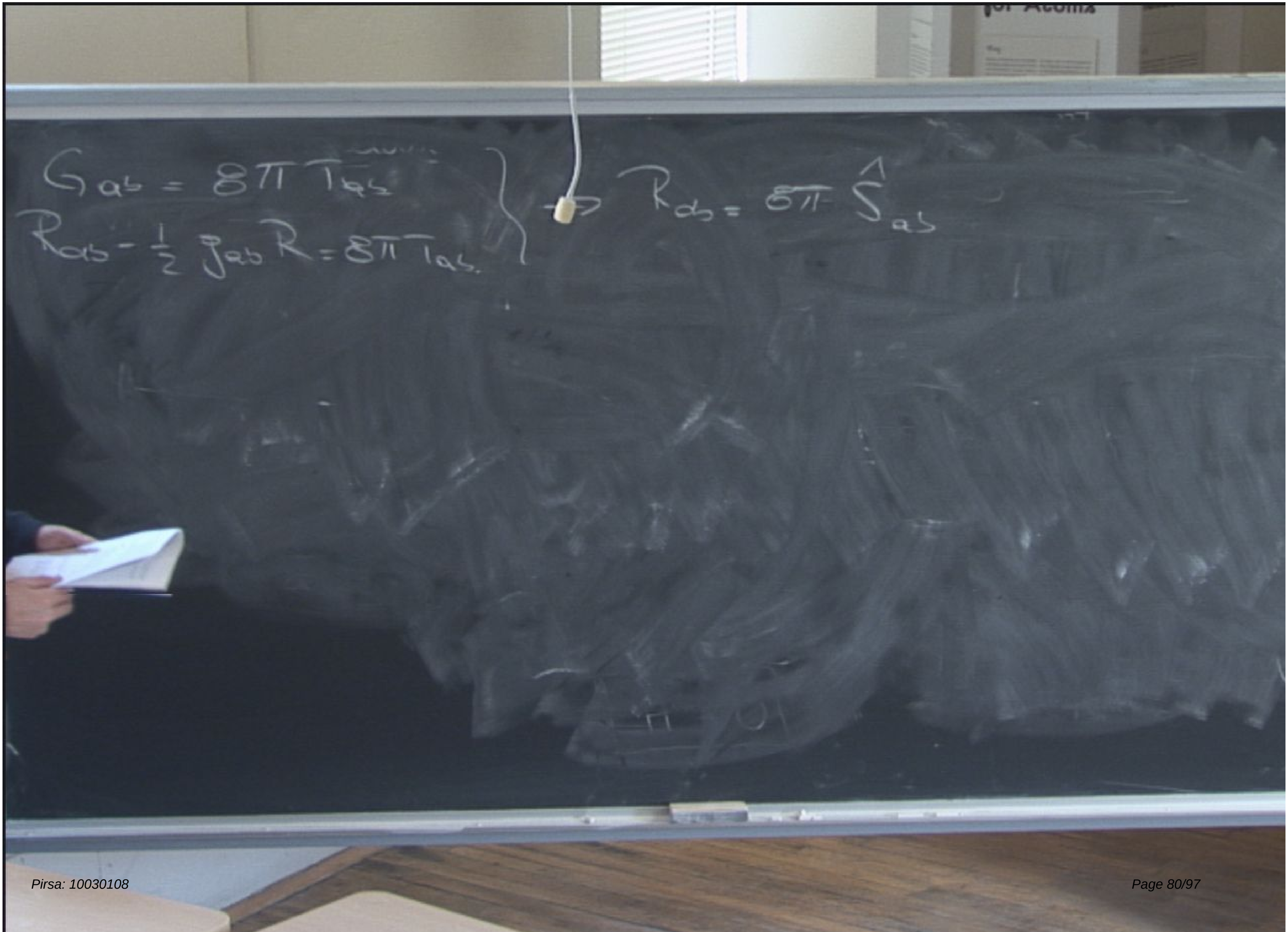
$$\hat{\rho} = (\rho + \rho\varepsilon)$$

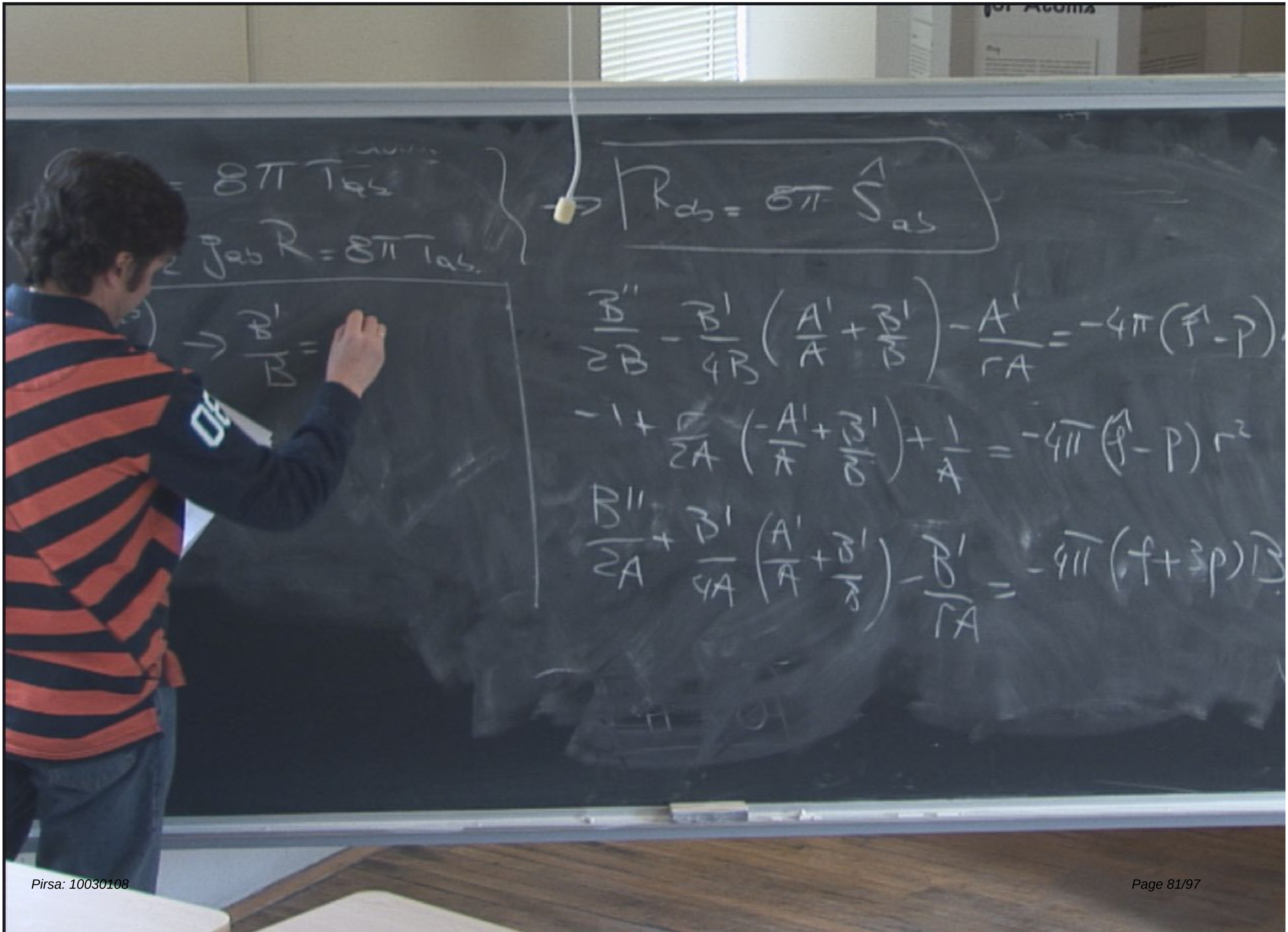
$$\nabla_a (\rho u^a) = 0$$

$$\nabla_a (T^{ab}) = 0$$

$$u^a \propto (1, \vec{0})$$

$$u^a u_a = -1 \Rightarrow u_a = (-\sqrt{B}, \vec{0})$$


$G_{ab} = 8\pi T_{ab}$
$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi T_{ab}.$
$\Rightarrow R_{ab} = 8\pi \hat{S}_{ab}$


$= 8\pi T_{ab}$
$g_{ab} R = 8\pi T_{ab}.$
$\boxed{R_{ab} = 8\pi \hat{S}_{ab}}$
$\frac{B'}{B} =$
$\frac{B''}{2B} - \frac{B'}{4B}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{A'}{rA} = -4\pi(\rho - p)$
$-1 + \frac{r}{2A}\left(-\frac{A'}{A} + \frac{B'}{B}\right) + \frac{1}{A} = -4\pi(\rho - p) r^2$
$\frac{B''}{2A} + \frac{B'}{4A}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{B'}{rA} = -4\pi(\rho + 3p) B$

$$G_{ab} = 8\pi T_{ab}$$

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi T_{ab}.$$

$$\Rightarrow \boxed{R_{ab} = 8\pi \hat{S}_{ab}}$$

$$\nabla_a (T^{ab}) \rightarrow \frac{B'}{B} = \frac{-2p'}{p + \hat{\rho}}$$

$$\frac{B''}{2B} - \frac{B'}{4B}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{A'}{rA} = -4\pi(\hat{\rho} - p)$$

$$-1 + \frac{r}{2A}\left(-\frac{A'}{A} + \frac{B'}{B}\right) + \frac{1}{A} = -4\pi(\hat{\rho} - p) r^2$$

$$\frac{B''}{2A} + \frac{B'}{4A}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{B'}{rA} = -4\pi(\rho + 3p) B$$

× Sph. symmetric static stars

$$\left(\frac{r}{A}\right)' = 1 - 8\pi \hat{\rho}\, r^2$$

$$\hookrightarrow A(r) = \left(1 - 2\,\frac{M(r)}{r}\right)^{-1}$$

$$M(r) \equiv \int_0^r 4\pi \tilde{r}^2\, \hat{\rho}(\tilde{r})\, d\tilde{r}$$

$$-r\,p' = M\,\rho(r)\left(1+\frac{p}{\rho}\right)\left(1+\frac{4\pi p}{M(r)}\right)\left(1-\frac{2M}{r}\right)^{-1}$$

$$-r\,p' = M(r)\,\rho(r)\left(1+\frac{p}{\rho}\right)\left(1+\frac{4\pi p}{M(r)}\right)\left(1-\frac{2M}{r}\right)^{-1}$$

ρ

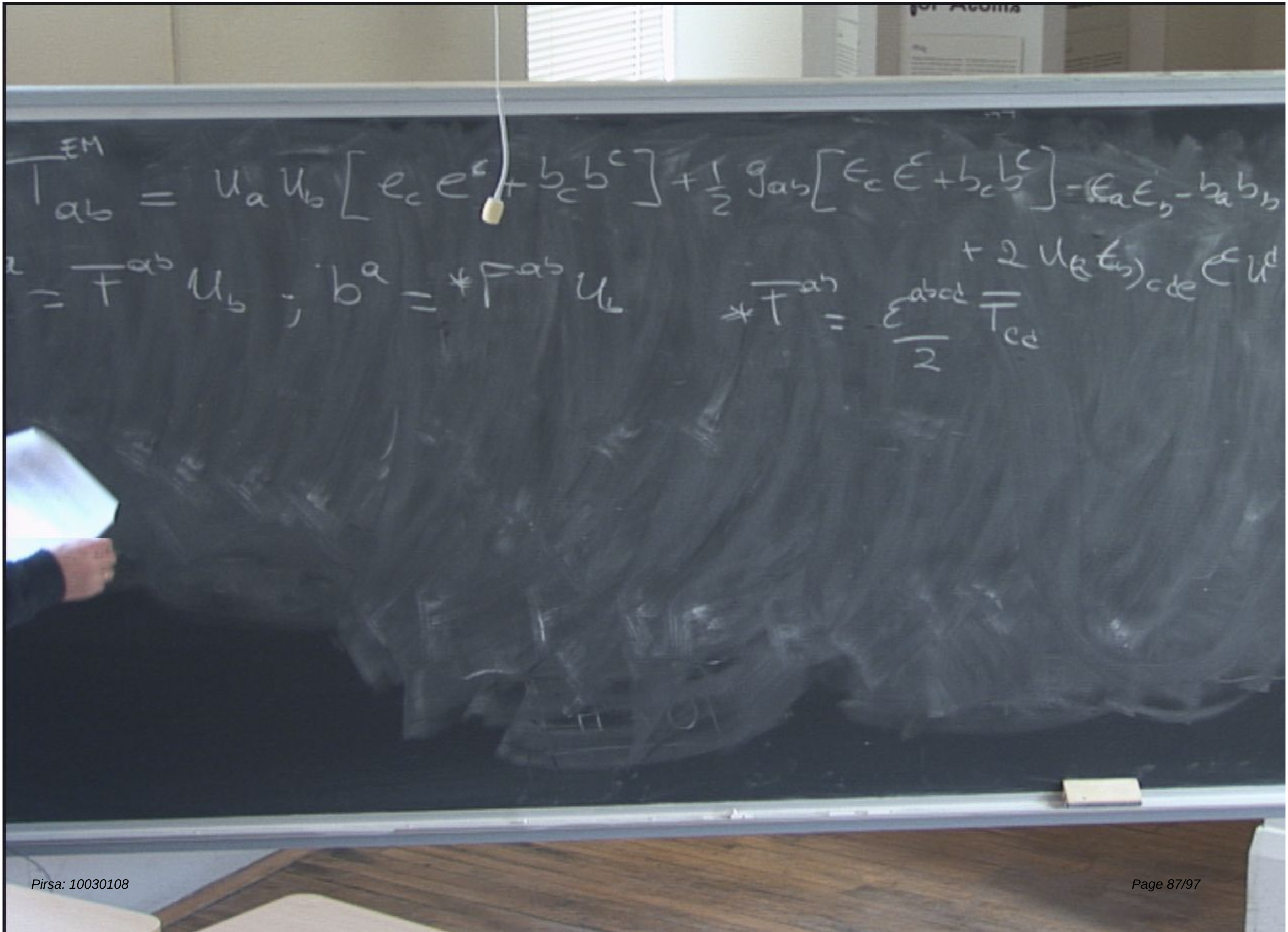
$$-r\,p' = M(r)\,\rho(r)\left(1+\frac{p}{\rho}\right)\left(1+\frac{4\pi p}{M(r)}\right)\left(1-\frac{2M}{r}\right)^{-1}$$

ρ

$$\nabla_a(\rho u^a) = 0$$

$$\nabla_a(T^{ab}) = 0$$

T^F_{ab} ; T^{EM}_{ab}



$T^{EM}_{ab} = u_a u_b [e_c e^c + b_c b^c] + \frac{1}{2} g_{ab} [e_c e^c + b_c b^c] - e_a e_b - b_a b_b$
$+ 2 u_{(a} \epsilon_{b)cde} e^c u^d$
$e^a = F^{ab} u_b \; ; \; b^a = {*F}^{ab} u_b$
${*F}^{ab} = \frac{\epsilon^{abcd}}{2} F_{cd}$

$$T^{EM}_{ab} = u_a u_b \left[e_c e^c + b_c b^c \right] + \frac{1}{2} g_{ab} \left[e_c e^c + b_c b^c \right] - e_a e_b - b_a b_b + 2 u_{(a} e_{b)} \epsilon_{cde} e^c u^d$$

$$e^a = F^{ab} u_b \; ; \; b^a = {}^*F^{ab} u_b \qquad {}^*F^{ab} = \frac{\epsilon^{abcd}}{2} F_{cd}$$

$$T^F_{ab} = \left[\rho (1 + \varepsilon + P/\rho) u_a u_b + P g_{ab} \right.$$

$$T^{EM}_{ab} = u_a u_b \left[e_c e^c + b_c b^c \right] + \frac{1}{2} g_{ab} \left[e_c e^c + b_c b^c \right] - e_a e_b - b_a b_b + 2 u_{(a} e_{b)cde}\, e\, u^d$$

$$e^a = F^{ab} u_b \;;\; b^a = {}^*F^{ab} u_b \qquad {}^*F^{ab} = \frac{\epsilon^{abcd}}{2} F_{cd}$$

$$T^{F}_{ab} = \left[\rho\left(1 + \varepsilon + P/\rho\right) \right] u_a u_b + P g_{ab}$$

$$T_{ab} = \left[\rho(1+\varepsilon) + P + b_c b^c \right] u_a u_b + \left[P + \frac{1}{2} b_c b^c \right] g_{ab} - b_a b_b$$

$\nabla_a(\rho u^a)$; $\nabla_a(T^{ab}) = 0$; $\nabla_a(F^{ab})$

$$\nabla_a(\rho u^a)\,;\quad \nabla_a(T^{ab}) = 0 \,;\quad \nabla_a(F^{ab})$$

$$\hookrightarrow \frac{1}{\sqrt{-g}}\,\partial_a\left(\sqrt{-g}\,\rho u^a\right) = 0 \qquad \sqrt{-g}\,\rho u^t$$

$$\hookrightarrow \frac{1}{\sqrt{-g}}\left[\partial_t\left(\sqrt{-g}\,\rho u^t\right) + \partial_i\left(\sqrt{-g}\,\rho u^i\right)\right.$$

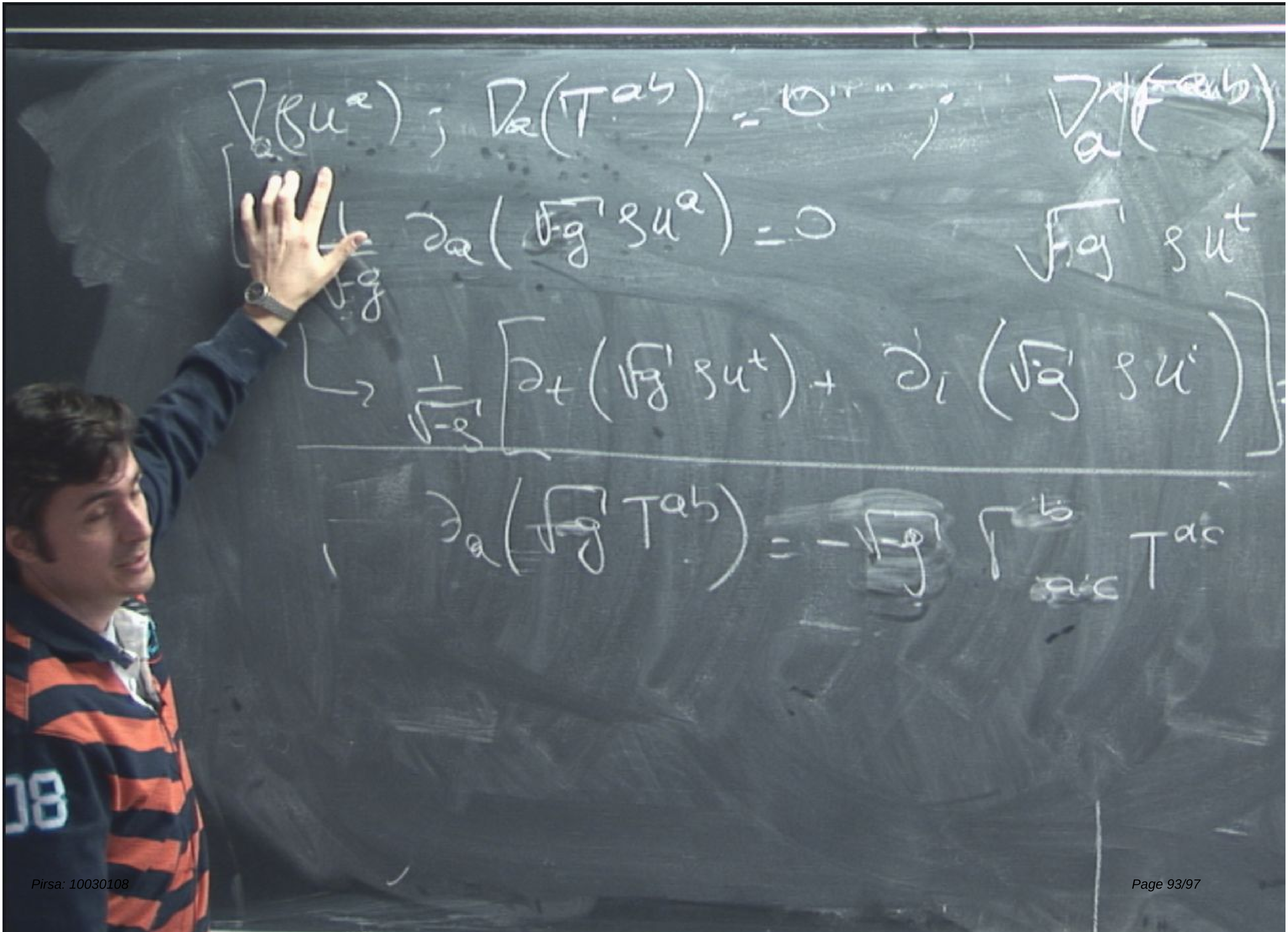
$$D \equiv \sqrt{g}\, \rho\, u^t$$

$$= \alpha \sqrt{\gamma}\, \rho \frac{1}{\sqrt{1-v^2}}$$

$$u^a u_a = -1$$

$$\underline{v^i}$$

$$W = \frac{1}{\sqrt{1-v^2}}$$


$\nabla_a(\rho u^a)$; $\nabla_a(T^{ab}) = 0$; $\nabla_a(F^{ab})$
$\frac{1}{\sqrt{-g}} \partial_a(\sqrt{-g}\,\rho u^a) = 0$
$\sqrt{-g}\,\rho u^t$
$\frac{1}{\sqrt{-g}}\left[\partial_t(\sqrt{-g}\,\rho u^t) + \partial_i(\sqrt{-g}\,\rho u^i)\right] =$
$\partial_a(\sqrt{-g}\,T^{ab}) = -\sqrt{-g}\,\Gamma^b_{ac} T^{ac}$

$$\nabla_a(\rho u^a) \; ; \; \nabla_a(T^{ab}) = 0 \; ; \; \nabla_a(F^{ab})$$

$$\hookrightarrow \frac{1}{\sqrt{-g}}\,\partial_a\left(\sqrt{-g}\,\rho u^a\right) = 0 \qquad \sqrt{-g}\,\rho u^t$$

$$\hookrightarrow \frac{1}{\sqrt{-g}}\left[\partial_t\left(\sqrt{-g}\,\rho u^t\right) + \partial_i\left(\sqrt{-g}\,\rho u^i\right)\right] =$$

$$\partial_a\left(\sqrt{-g}\,T^{ab}\right) = -\sqrt{-g}\,\Gamma^b_{ac}\,T^{ac}$$

$$\partial_t\left(\sqrt{-g}\,T^{tb}\right) = \dots$$

$$D \equiv \sqrt{-g}\,\rho\, u^t$$

$$D = \alpha \sqrt{\gamma}\,\rho \frac{1}{\sqrt{1-v^2}}$$

$$\nabla_a S^a = \frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}\, S^a\right)$$

$$P = (\Gamma - 1)\rho\,\varepsilon$$

$$S_i = \rho\left(1+\varepsilon+P/\rho\right) W^2 v^i$$

$$E = \rho\left(1+\varepsilon+P/\rho\right) W^2 - P$$

$$D \equiv \sqrt{-g}\, \rho\, u^t$$

$$D = \alpha \sqrt{\gamma}\, \rho \frac{1}{\sqrt{1-v^2}}$$

$$\nabla_a S^a = \frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g}\, S^a \right)$$

$$P = (\Gamma - 1) \rho \epsilon$$

$$S_i = \rho \left(1 + \varepsilon + P/\rho \right) W^2 v^i$$

$$E = \rho \left(1 + \varepsilon + P/\rho \right) W^2 - P$$

$$\dot{C} = \left(D, B^i \right)$$

$$\dot{\vec{E}} = \vec{\nabla} \wedge \vec{B}$$

$$\dot{\vec{B}} = - \vec{\nabla} \wedge \vec{E}$$

$$C = \left(\partial_i B^i \right) = \partial_i B^i$$

$$\lambda_0 = \alpha v^n - \beta^n$$

$$\lambda_\pm = \frac{\alpha}{(1-v^2 c_s^2)} \Big\{ v^n (1 - c_s^2) \pm c_s \sqrt{1 - v^2 + (\gamma^{nn} - v^{n2}) \times (1 + c_s^2)}$$

$$S_i = \rho \left(1 + \varepsilon + \tfrac{P}{\rho}\right) W^2 v^i$$

$$E = \rho \left(1 + \varepsilon + \tfrac{P}{\rho}\right) W^2 - P$$

$$\dot{C} = (D_i B^i)$$

$$\dot{\vec{E}} = \vec{\nabla} \wedge \vec{B} + \vec{E}(\vec{\nabla} \cdot \vec{E})$$

$$\dot{\vec{B}} = -\vec{\nabla} \wedge \vec{E} + \vec{B}(\vec{\nabla} \cdot \vec{B})$$

$$v^n = v^i \cdot \hat{n}_i$$

$$\lambda_0 = v^n \; ; \; \lambda_\pm = v^n \pm c_s$$

$$\dot{C} = (\partial_i B^i) = \partial_i \dot{B}^i$$