Title: Explorations in Numerical Relativity (PHYS 642) - Lecture 7

Date: Mar 23, 2010 11:20 AM

URL: http://pirsa.org/10030106

Abstract:

Linearly deg. $q_{,t} + u\, q_{,x} = 0$

$u \neq u(q)$

truly nonlinear $q_{,t} + v(q)\, q_{,x} = 0$

Burger's eqn $q_{,t} + q\, q_{,x} = 0$

$q_0(x,t)$

$q_0(x,t) + \delta$

$\delta_{,t} + q_0 \delta_{,x} = 0$

$$q_0(x,t) + \underline{\delta} \longrightarrow \text{sdu} \left(q_{0,t} + q_0 \, q_{0,x} = 0 \right)$$

$$\delta_{,t} + q_0 \, \delta_{,x} = 0$$

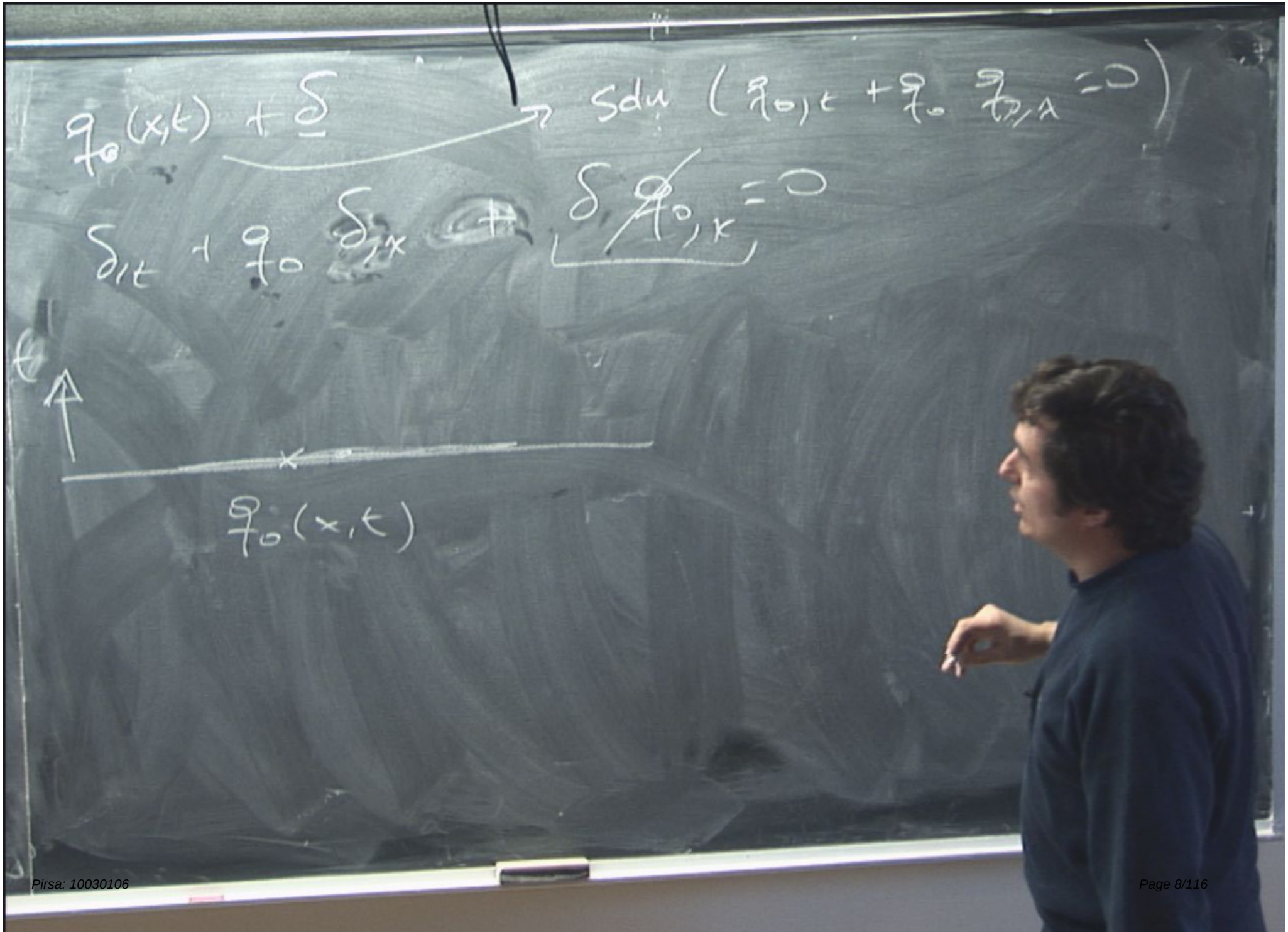
$$q_0(x,t) + \underline{\delta} \longrightarrow \text{sdu} \left(q_{0,t} + q_0 \, q_{0,x} = 0 \right)$$

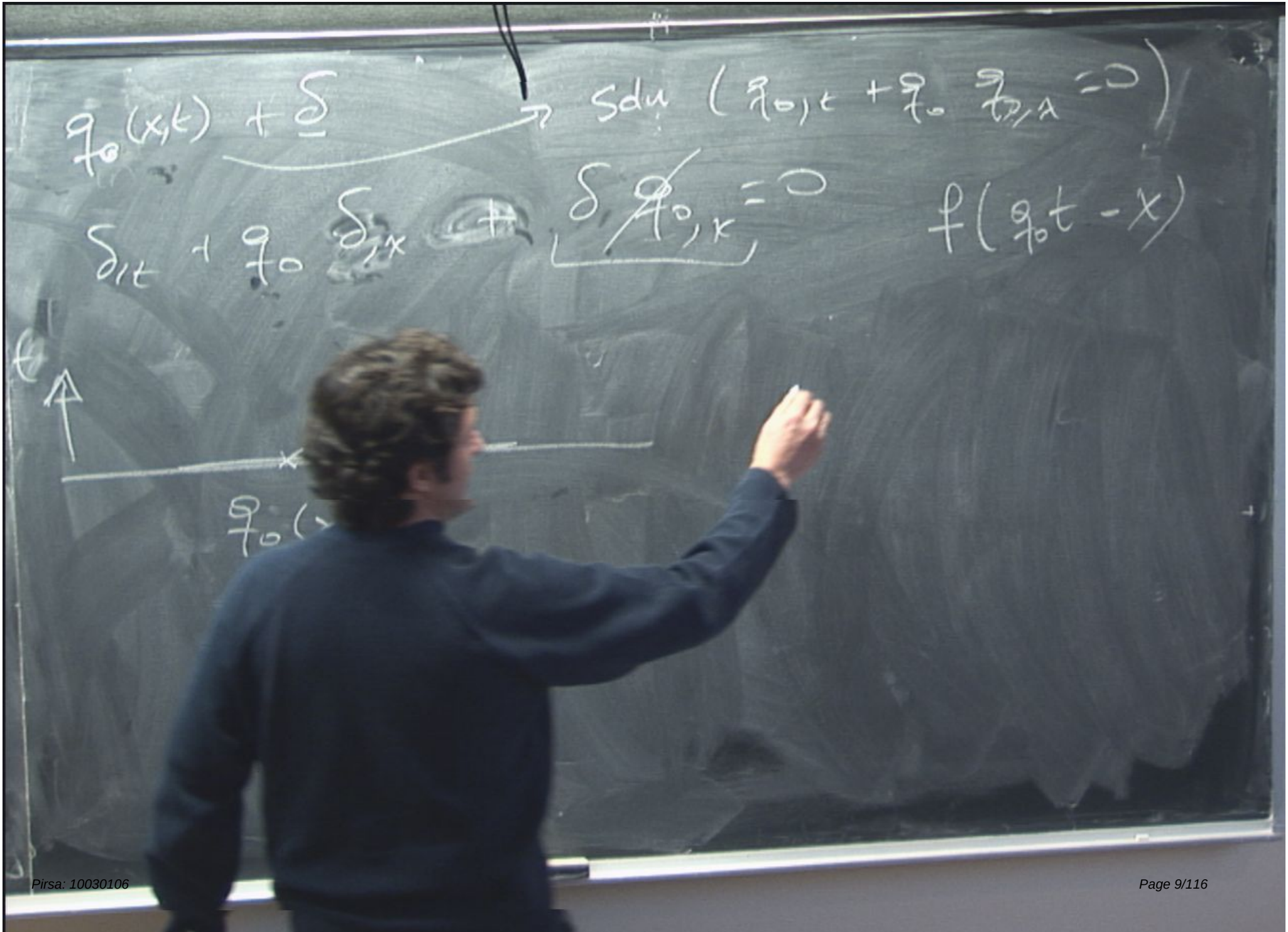
$$\delta_{,t} + q_0 \, \delta_{,x} + \underbrace{\delta \, q_{0,x}} = 0$$

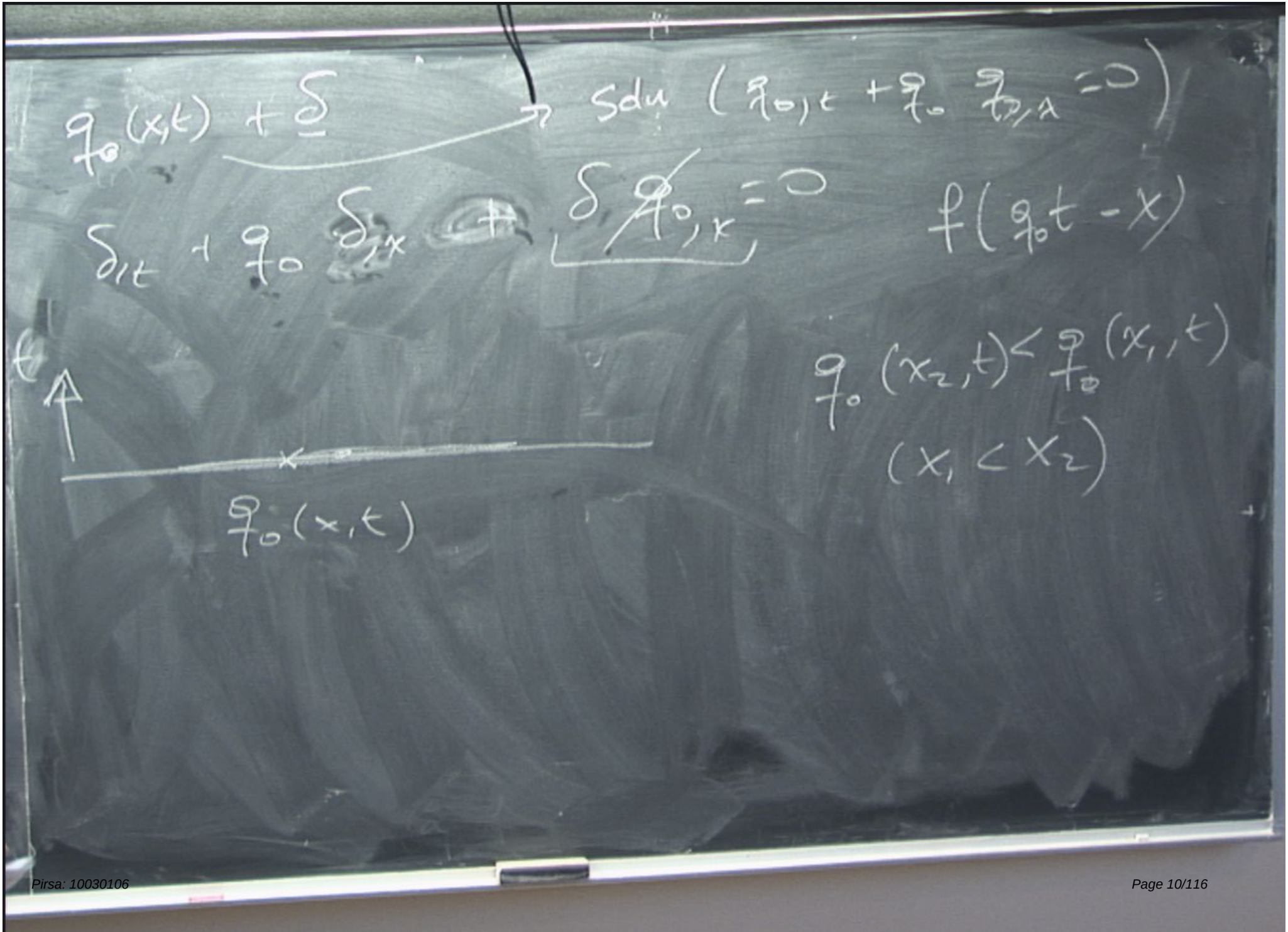
$$q_0(x,t) + \underline{\delta} \longrightarrow \text{sdu} \quad (q_{0,t} + q_0\, q_{0,x} = 0)$$

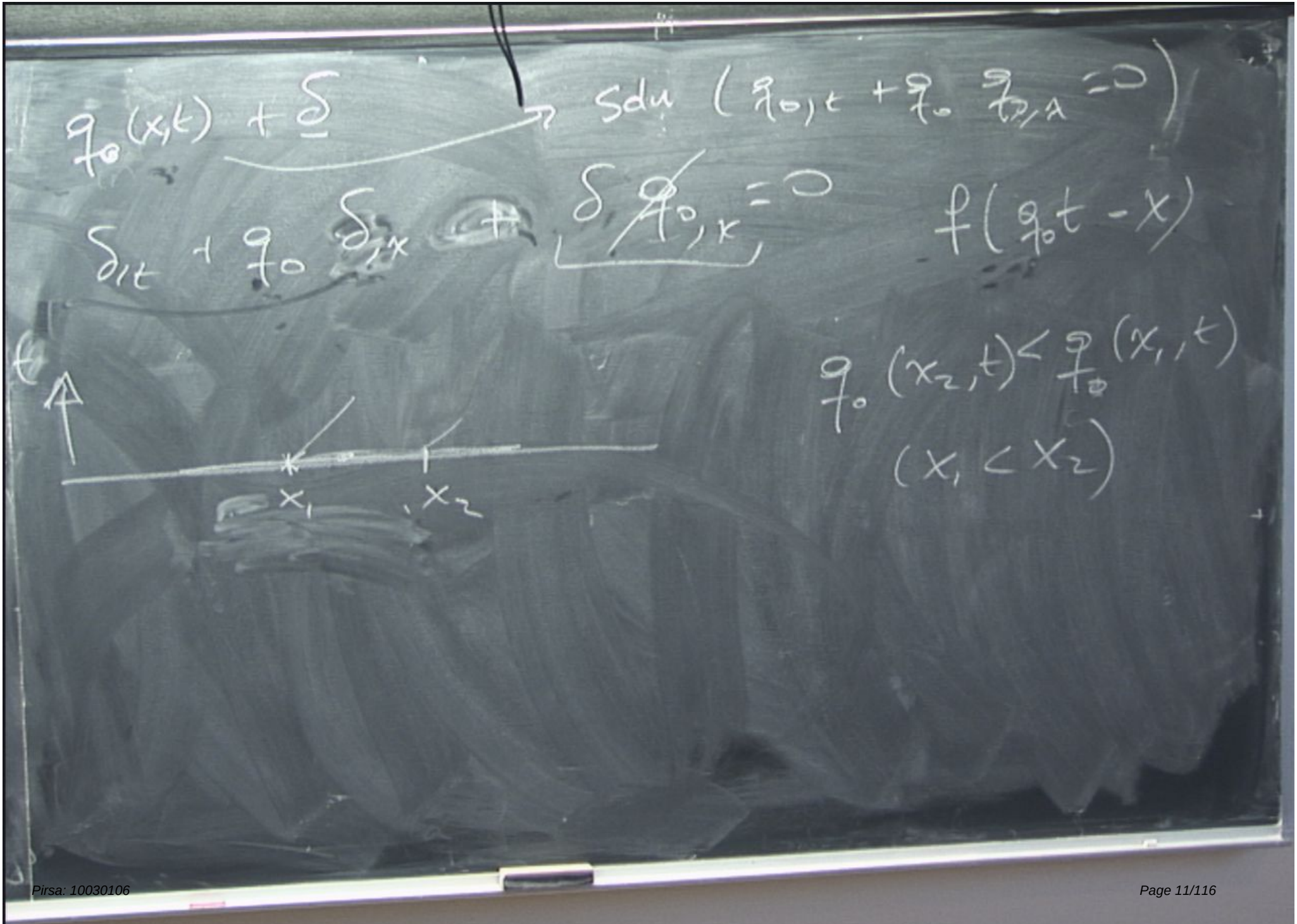
$$\delta_{,t} + q_0\, \delta_{,x} + \underbrace{\delta\, \cancel{q_{0,x}}} = 0$$

$$q_0(x,t)$$


$q_0(x,t) + \delta$
sdu $(q_{0,t} + q_0 q_{0,x} = 0)$
$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$
t
$q_0(x,t)$


$q_0(x,t) + \delta$
$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$
$f(q_0 t - x)$


$q_0(x,t) + \delta$
$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$
$f(q_0 t - x)$
$q_0(x_2,t) < q_0(x_1,t)$
$(x_1 < x_2)$
$q_0(x,t)$


$q_0(x,t) + \delta$
$\delta u\,(q_{0,t} + q_0 q_{0,x} = 0)$
$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$
$f(q_0 t - x)$
t
x_1
x_2
$q_0(x_2,t) < q_0(x_1,t)$
$(x_1 < x_2)$

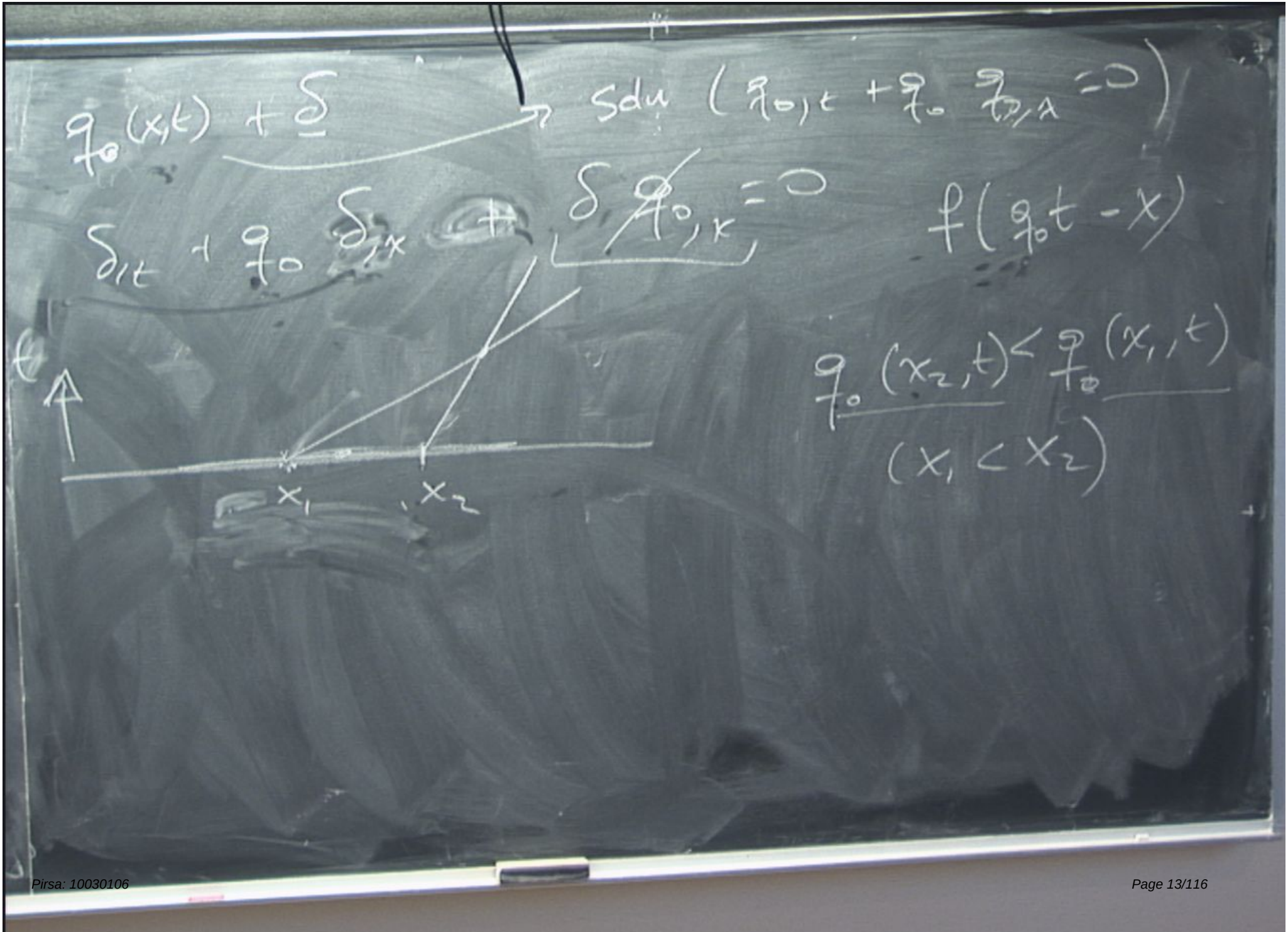
$q_0(x,t) + \underline{\delta}$ $\rightarrow$ sdu $(q_{0,t} + q_0\, q_{0,x} = 0)$

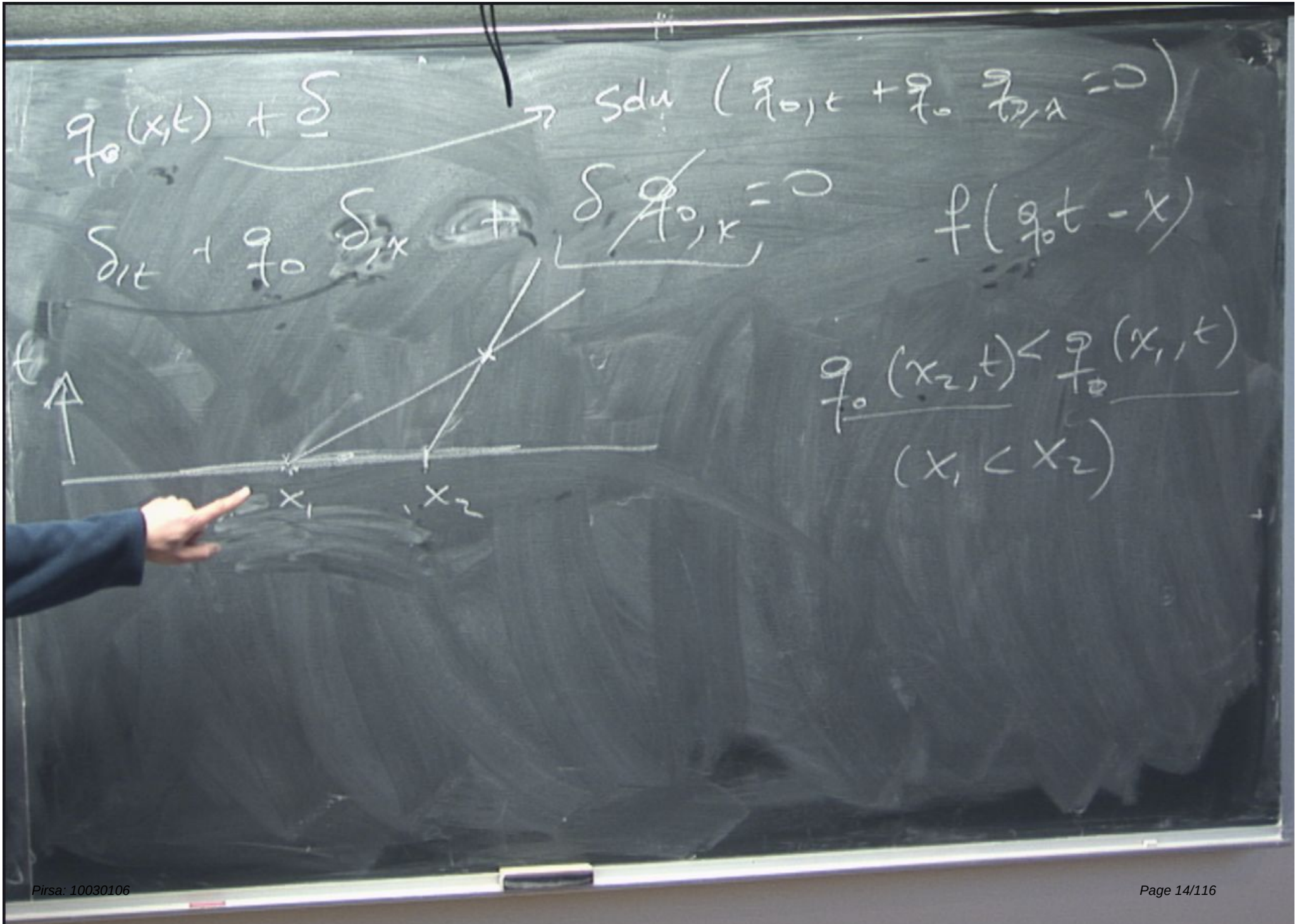
$\delta_{,t} + q_0\, \delta_{,x} + \underbrace{\delta\, \cancel{q_{0,x}}}_{} = 0$

$f(q_0 t - x)$

$\underline{q_0(x_2,t)} < \underline{q_0(x_1,t)}$

$(x_1 < x_2)$


$q_0(x,t) + \delta$
$\delta q_{0,t} + q_0 \delta q_{0,x} = 0$
$\delta q_{0,t} + q_0 \delta q_{0,x} = 0$
$f(q_0 t - x)$
$q_0(x_2,t) < q_0(x_1,t)$
$(x_1 < x_2)$
t
x_1
x_2


$q_0(x,t) + \delta$
$\delta q_{0,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$
$f(q_0 t - x)$
$q_0(x_2,t) < q_0(x_1,t)$
$(x_1 < x_2)$
t
x_1
x_2

$q_0(x,t) + \delta \rightarrow \int du \left(q_{0,t} + q_0 q_{0,x} = 0 \right)$

$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$

$f(q_0 t - x)$

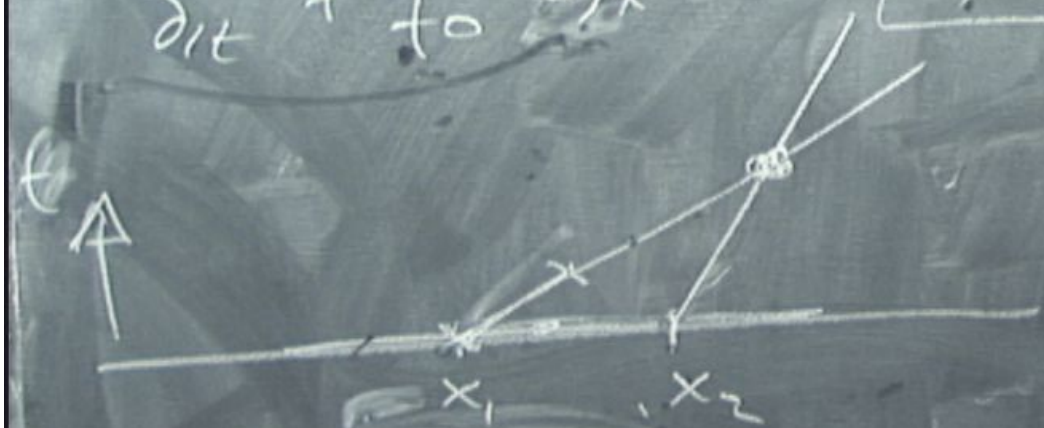
$q_0(x_2,t) < q_0(x_1,t)$

$(x_1 < x_2)$

$q_0(x,t) + \underline{\delta} \longrightarrow$ sdu $\left(q_{0,t} + q_0\, q_{0,x} = 0 \right)$

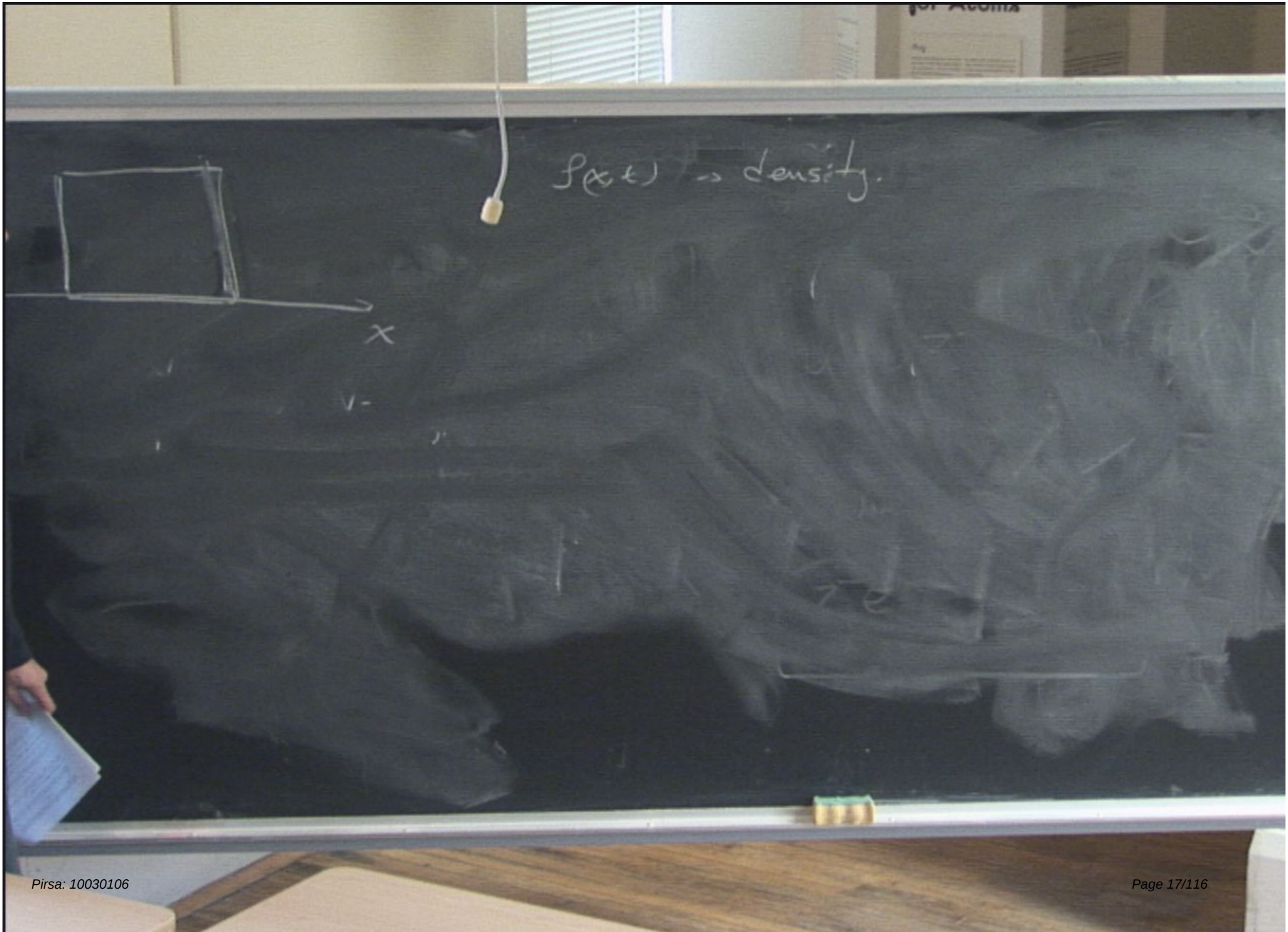
$\delta_{,t} + q_0\, \delta_{,x} + \underbrace{\delta\, \cancel{q_{0,x}}} = 0$

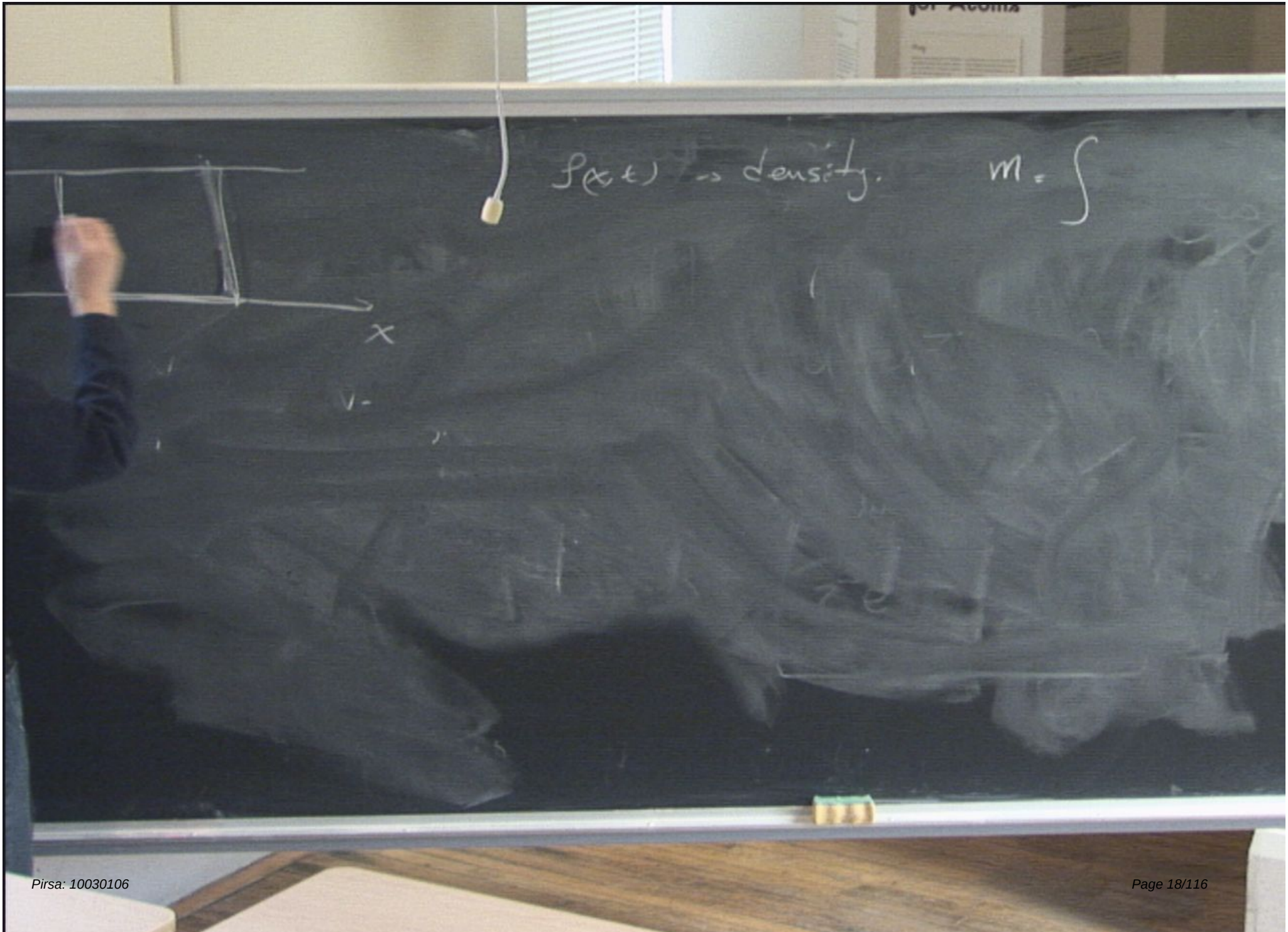
$f(q_0 t - x)$

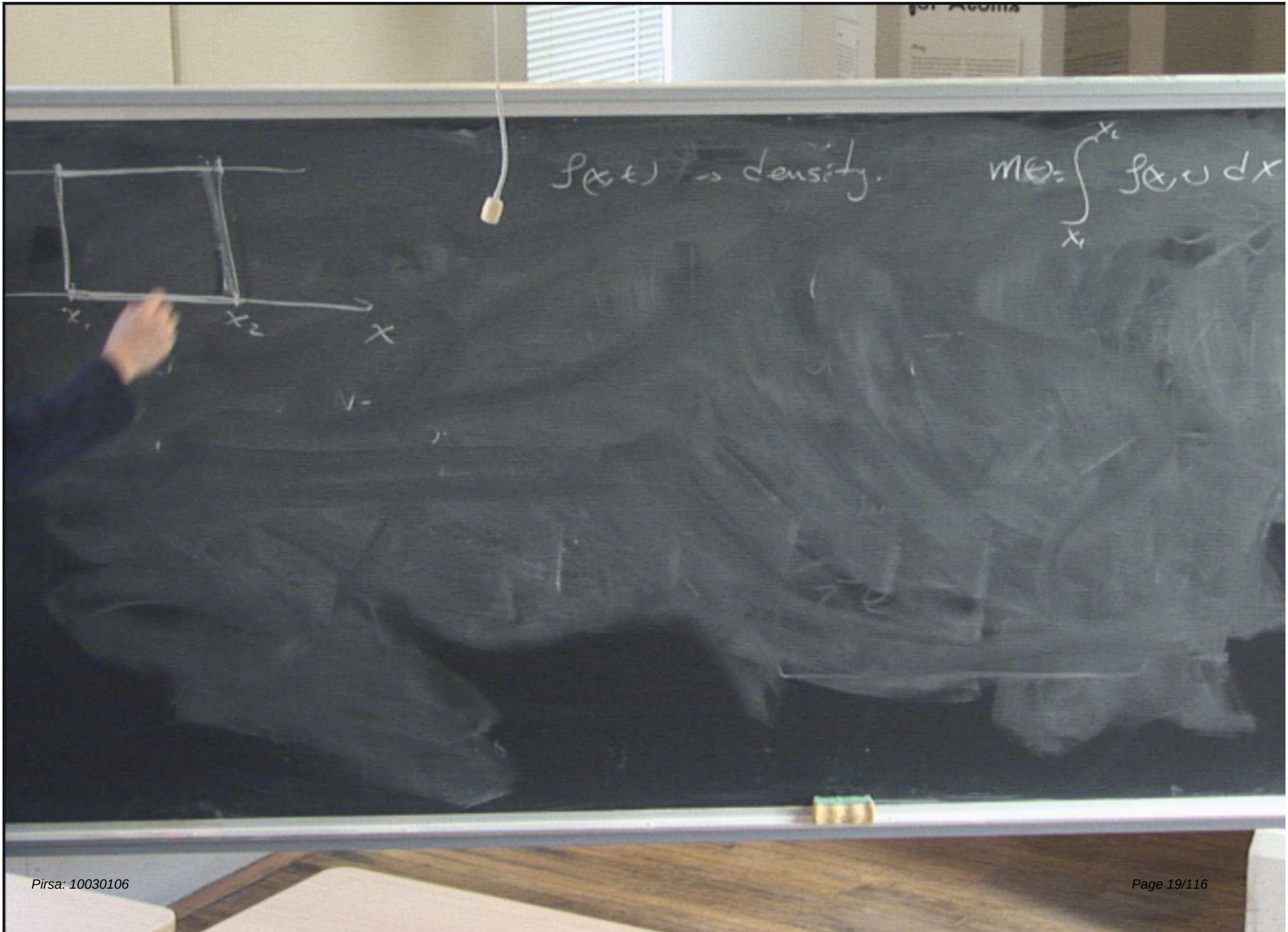


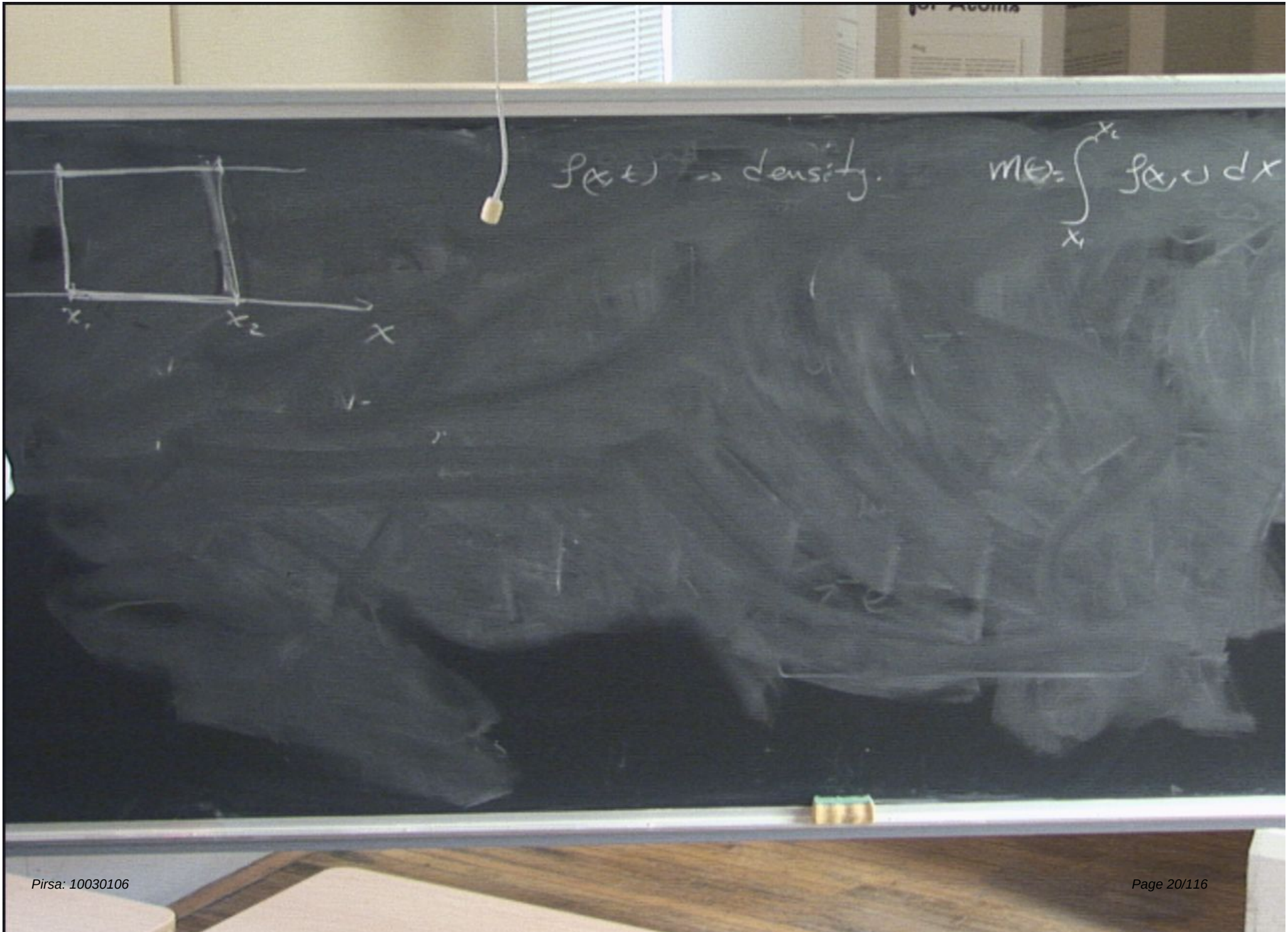
$\underline{q_0(x_2,t)} < \underline{q_0(x_1,t)}$

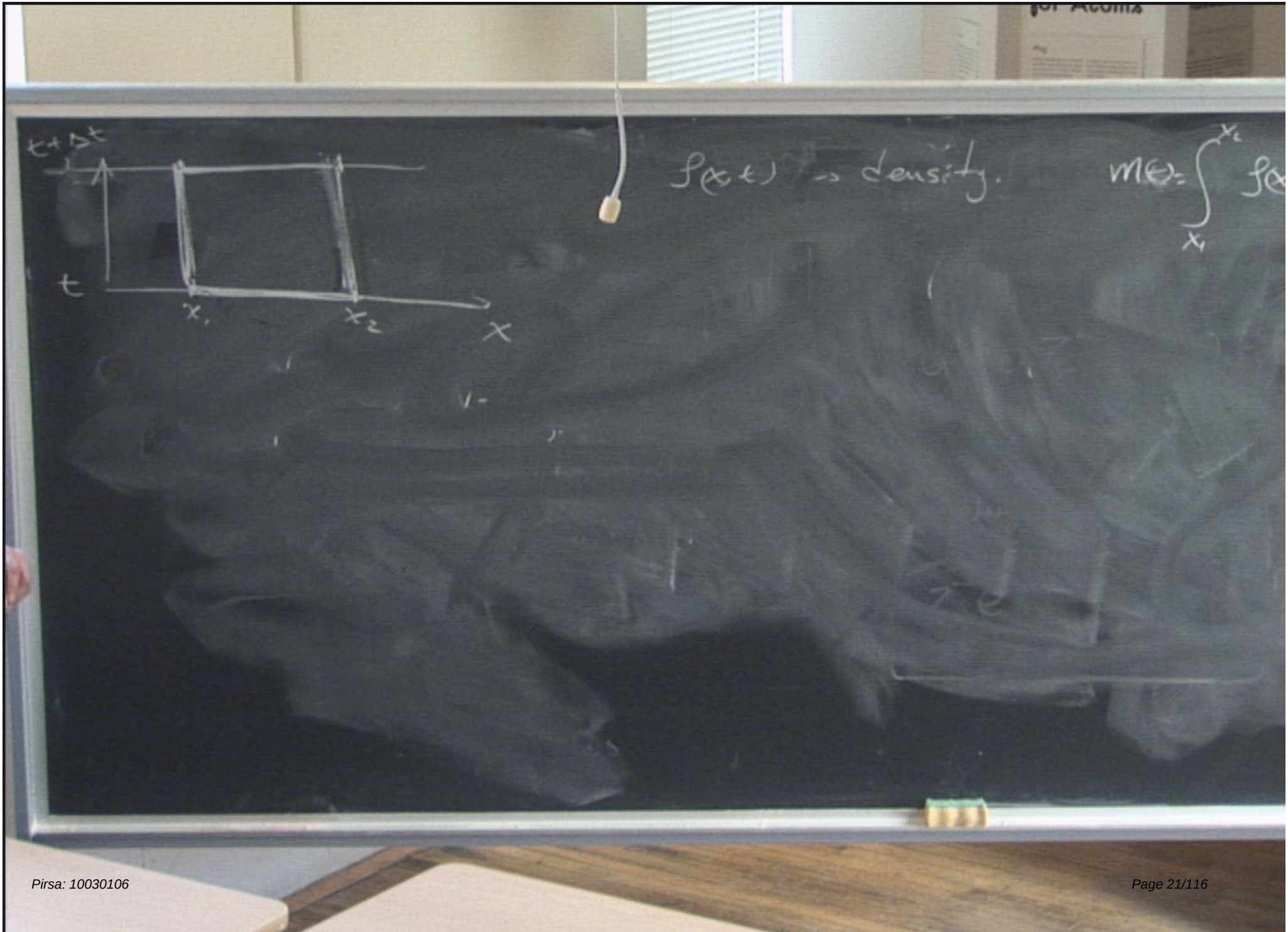
$(x_1 < x_2)$

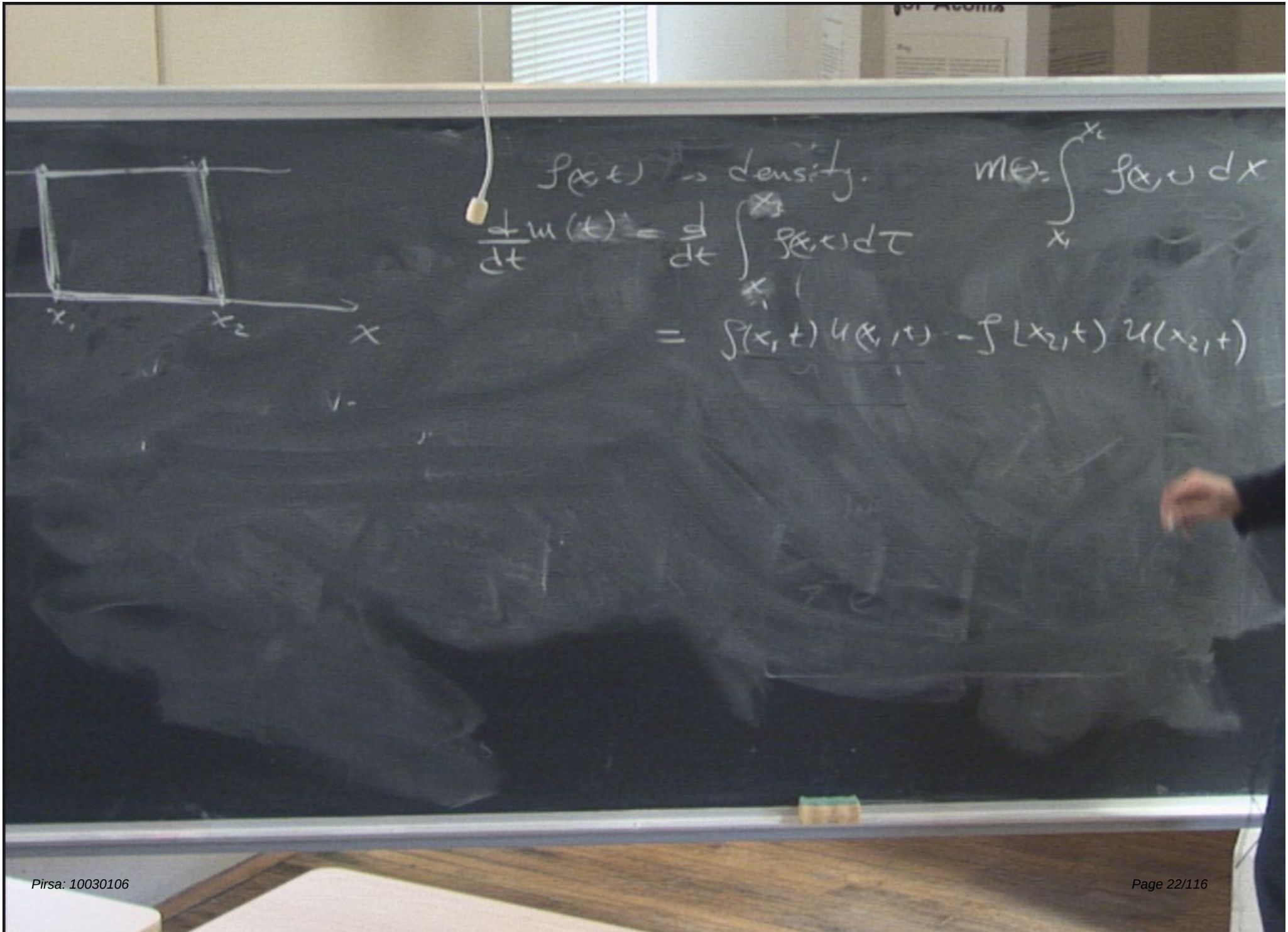

ρ(x,t) → density.
x

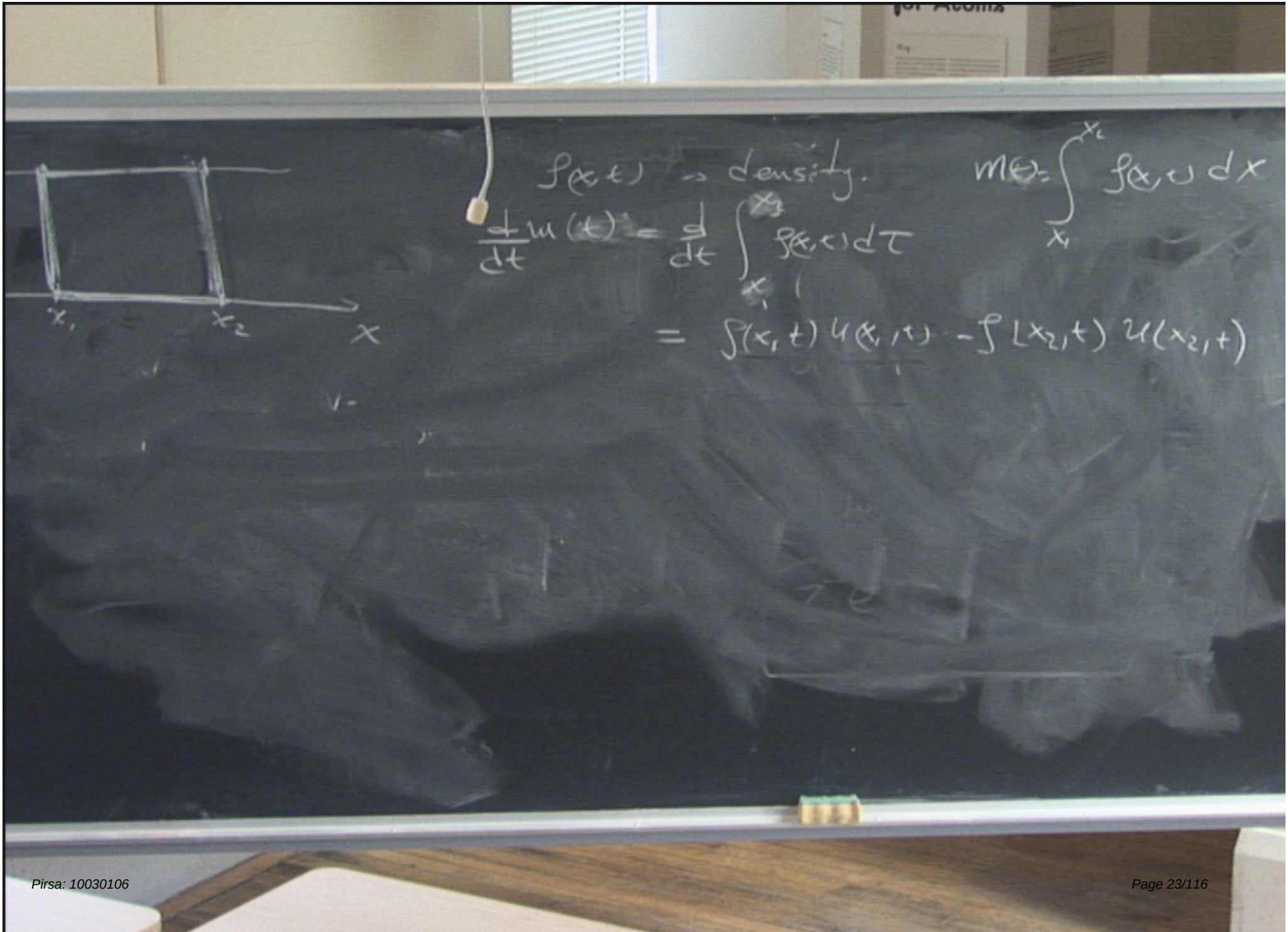

ρ(x,t) -> density.
m = ∫
x

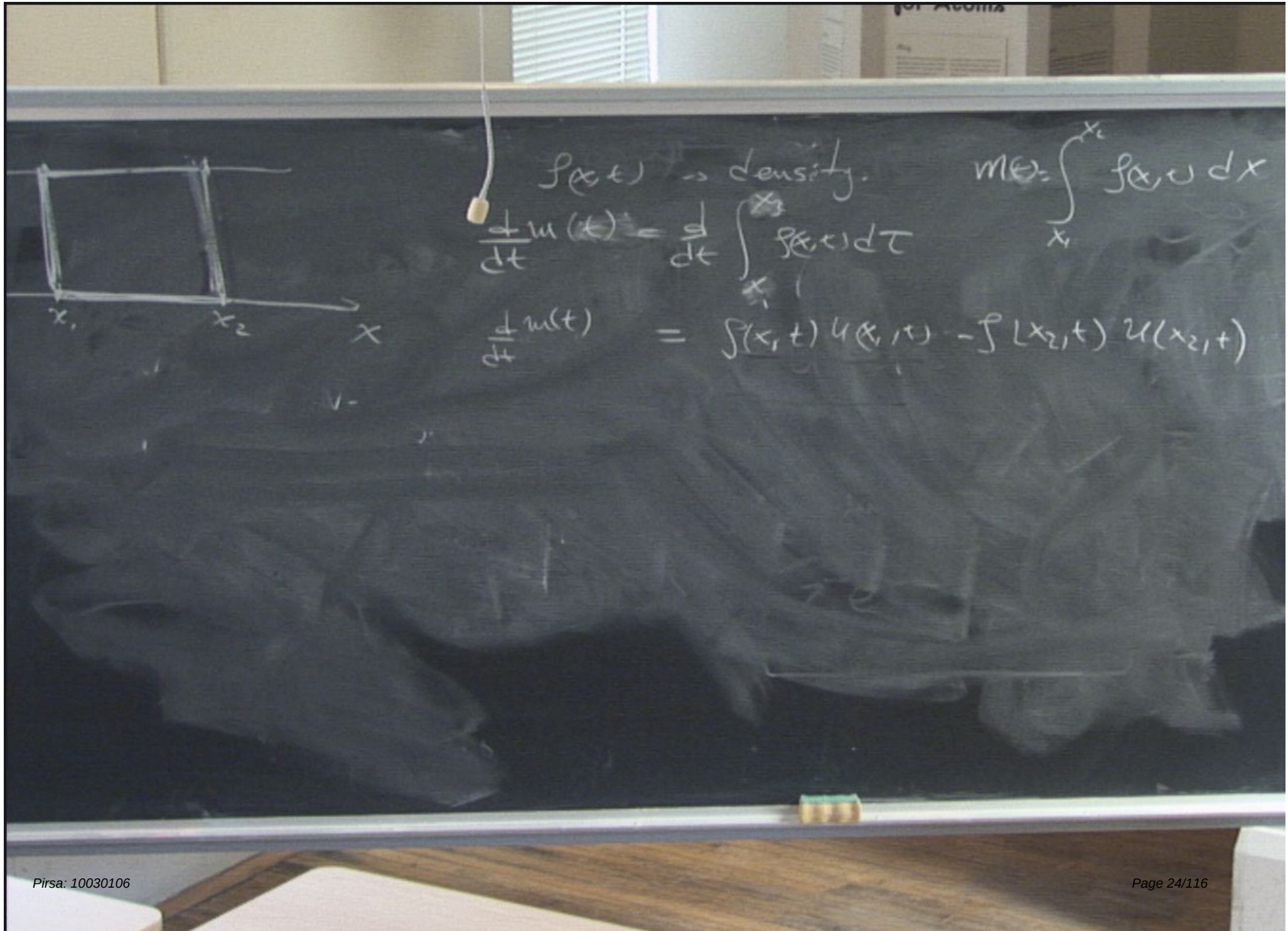

$\rho(x,t)$ -> density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$
x_1
x_2
x

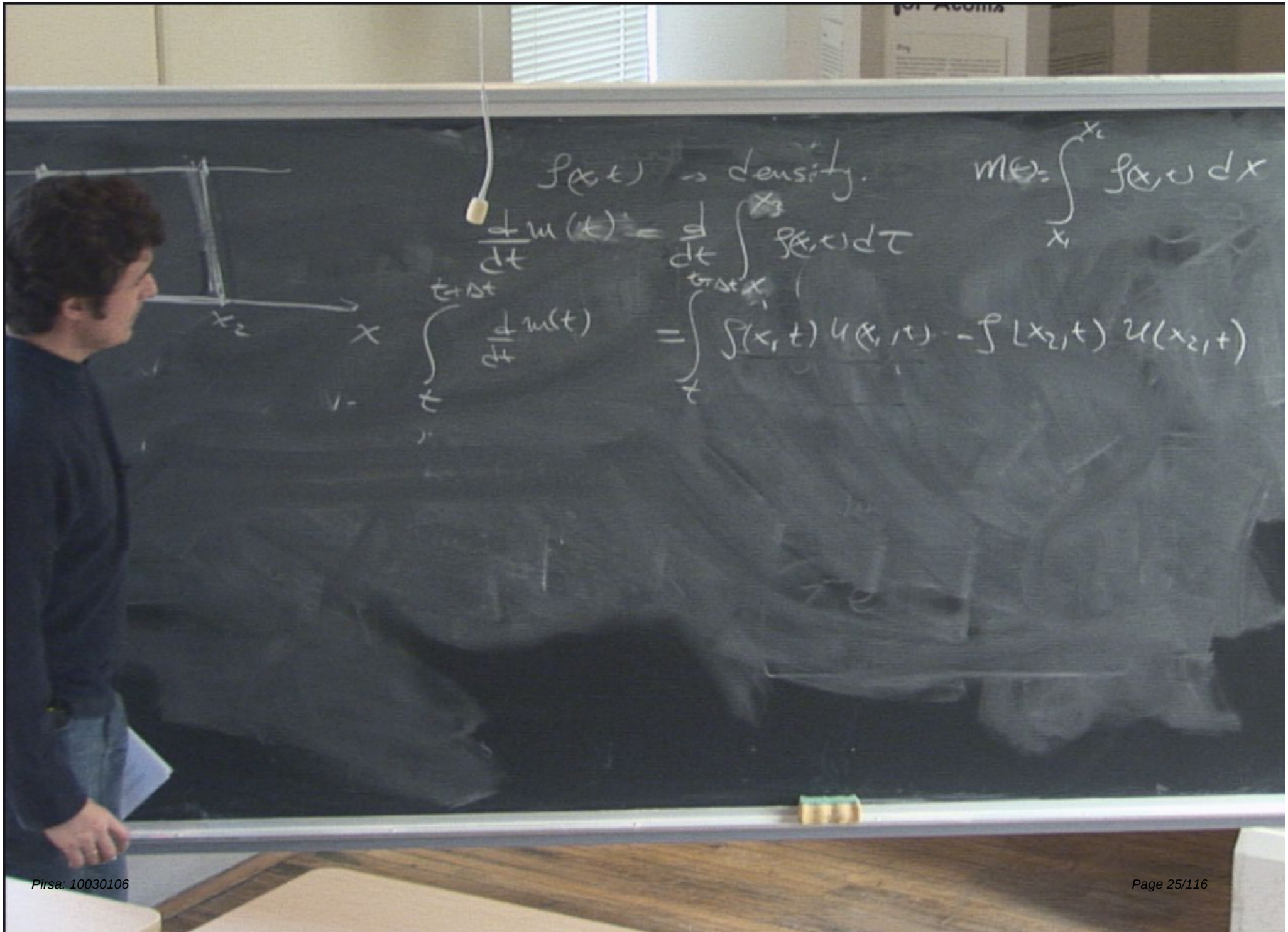

$\rho(x,t)$ -> density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$
x_1
x_2
x


t+Δt
t
x1
x2
x
ρ(x,t) → density.
m(t) = ∫ ρ
x1
x2

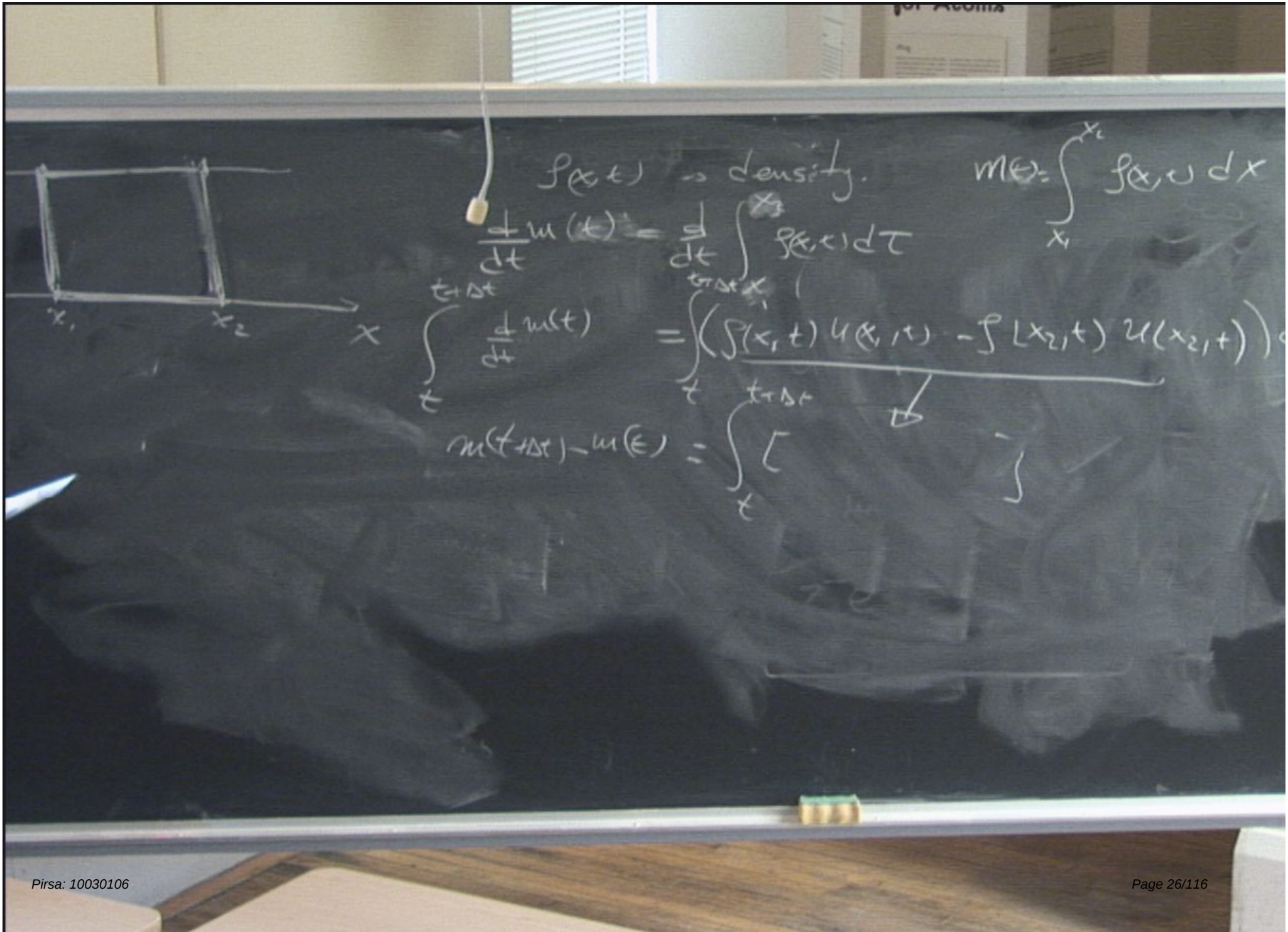

$\rho(x,t)$ → density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$
$\frac{d}{dt} m(t) = \frac{d}{dt} \int_{x_1}^{x_2} \rho(x,\tau)\,d\tau$
$= \rho(x_1,t)\,u(x_1,t) - \rho(x_2,t)\,u(x_2,t)$
x_1
x_2
x

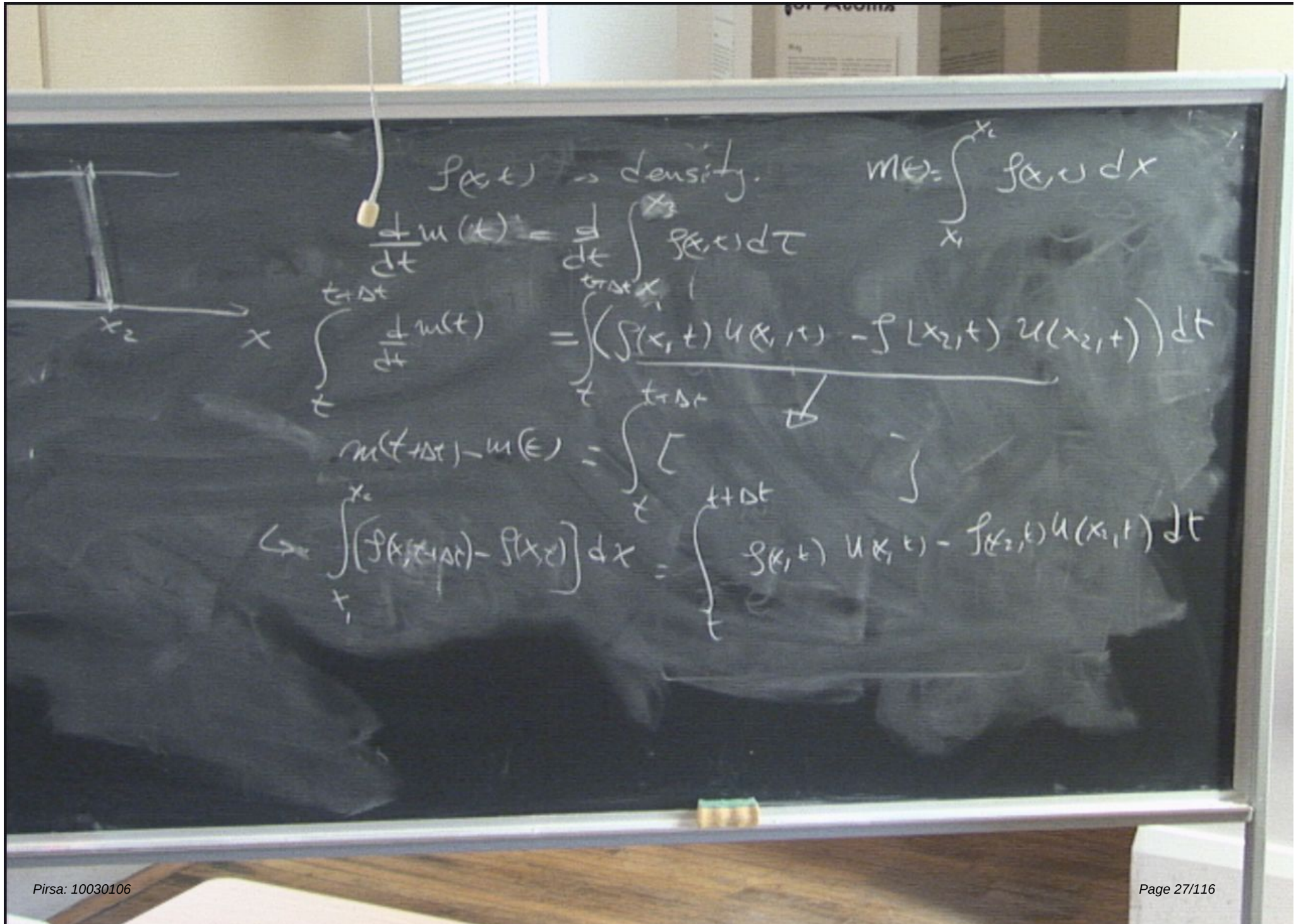

$\rho(x,t)$ → density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$
$\frac{d}{dt} m(t) = \frac{d}{dt} \int_{x_1}^{x_2} \rho(x,\tau)\,d\tau$
$= \rho(x_1,t)\,u(x_1,t) - \rho(x_2,t)\,u(x_2,t)$
x_1
x_2
x

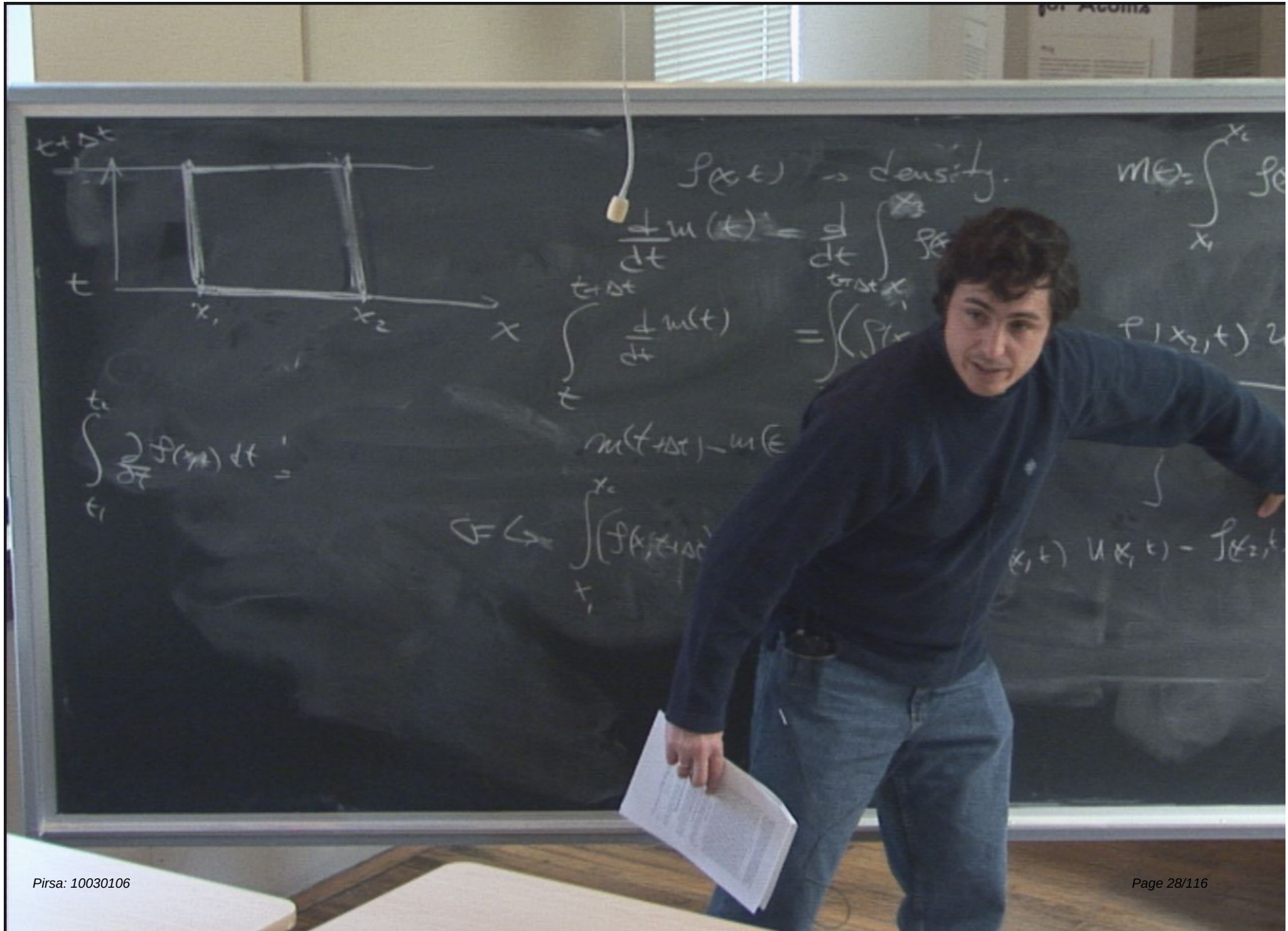

$\rho(x,t)$ → density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$
$\frac{d}{dt}m(t) = \frac{d}{dt}\int_{x_1}^{x_2} \rho(x,t)\,dx$
$\frac{d}{dt}m(t) = \rho(x_1,t)\,u(x_1,t) - \rho(x_2,t)\,u(x_2,t)$
x_1
x_2
x

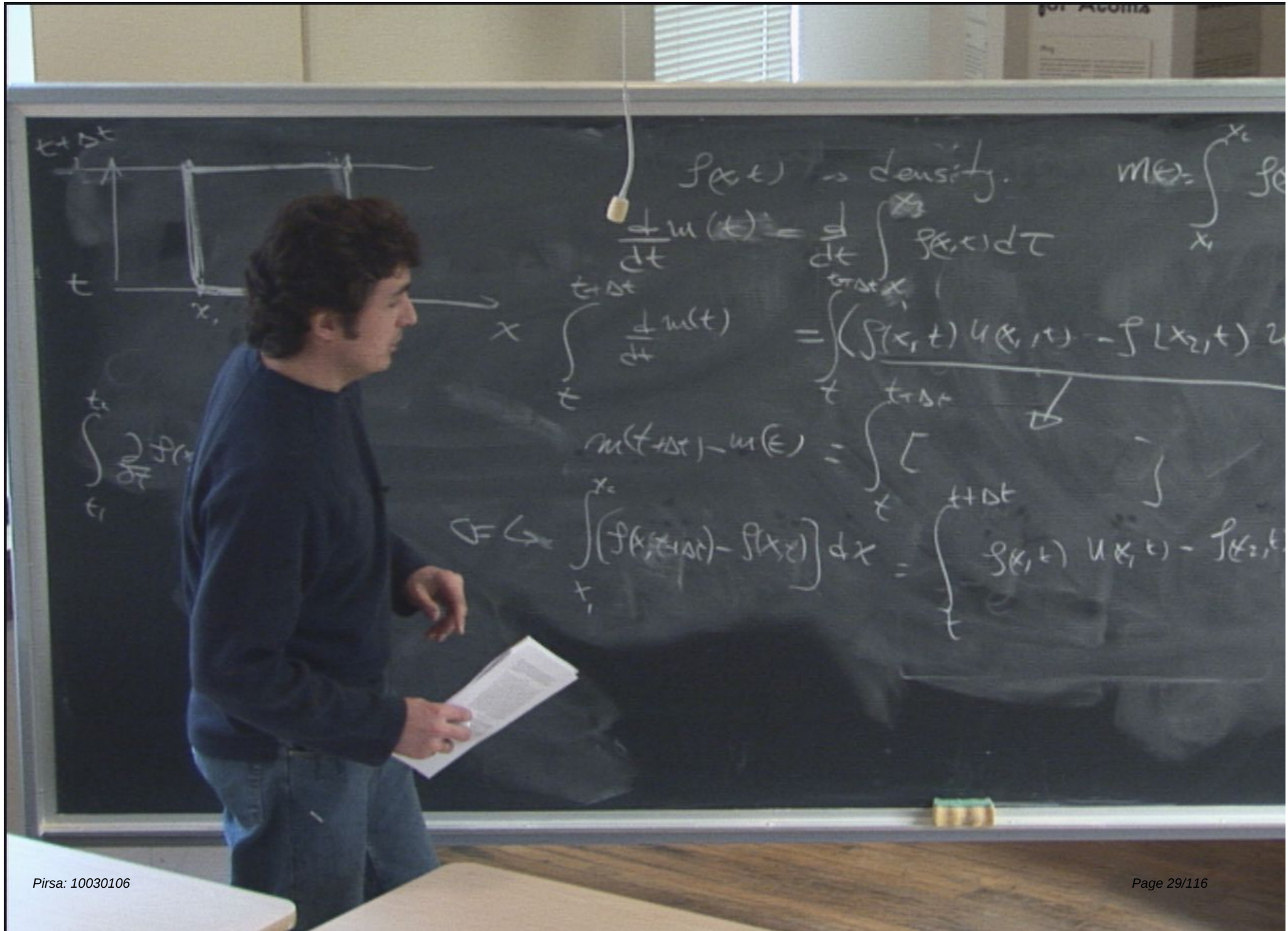


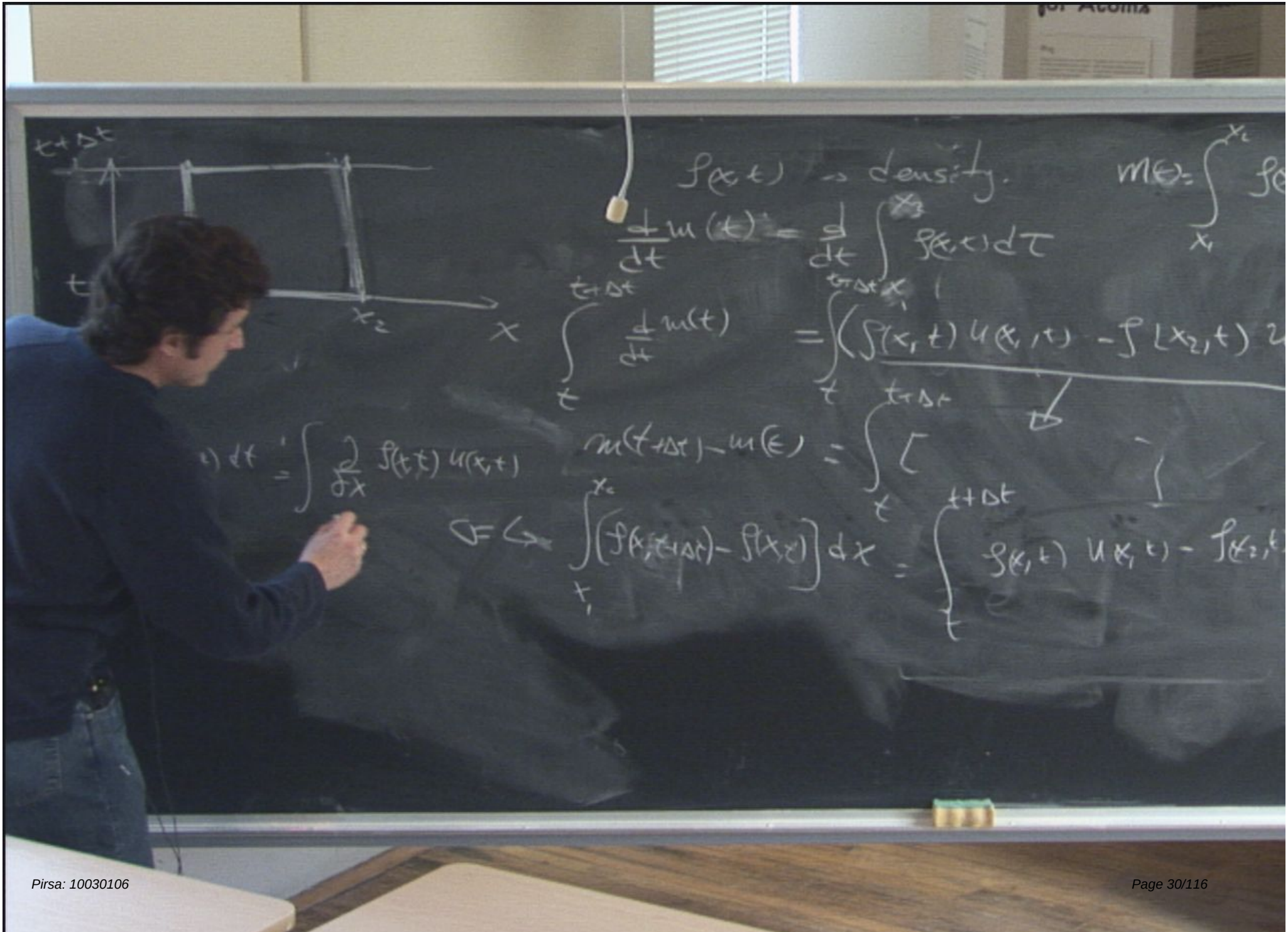
$\rho(x,t) \to$ density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\, dx$
$\frac{d}{dt} m(t) = \frac{d}{dt} \int_{x_1}^{x_3} \rho(x,t)\, d\tau$
$\int_t^{t+\Delta t} \frac{d}{dt} m(t) = \int_t^{t+\Delta t} \rho(x_1,t)\, u(x_1,t) - \rho(x_2,t)\, u(x_2,t)$
x_2
x

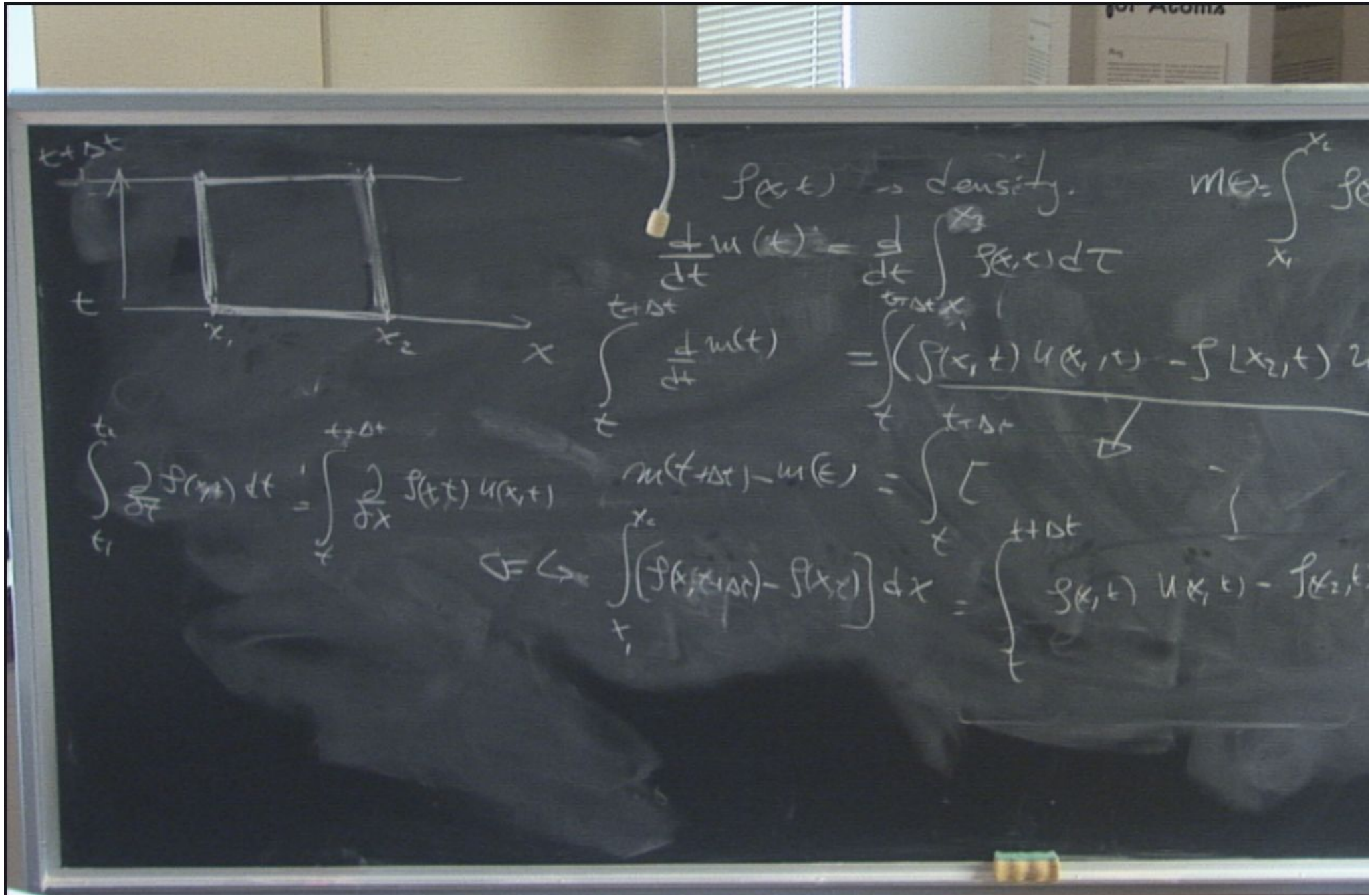

ρ(x,t) → density.
m(t) = ∫ ρ(x,t) dx
d/dt m(t) = d/dt ∫ ρ(x,t) dτ
x₁
x₂
x
∫ d/dt m(t) = ∫ (ρ(x₁,t) u(x₁,t) − ρ(x₂,t) u(x₂,t))
m(t+Δt) − m(t) = ∫ [ ]

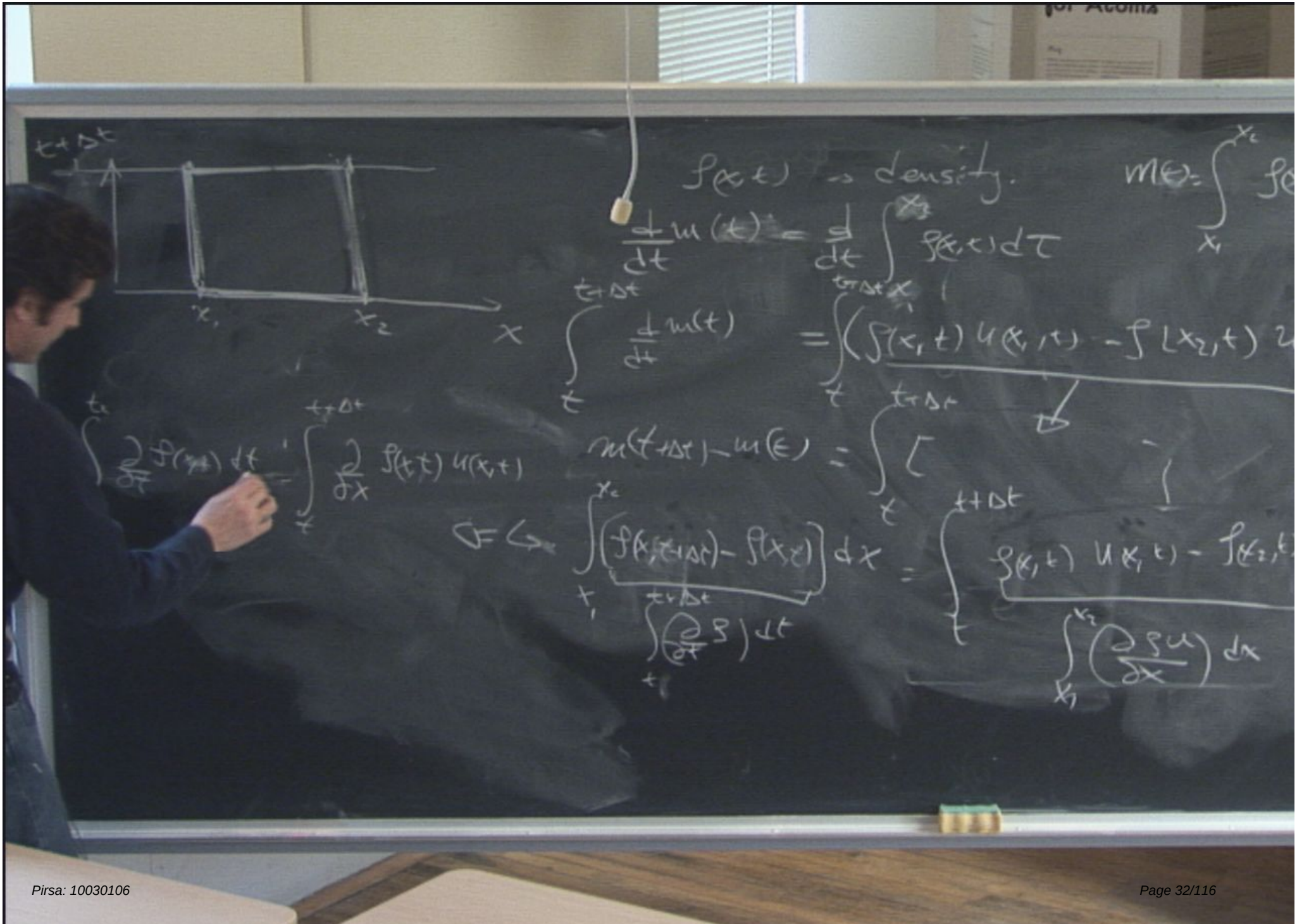

$\rho(x,t)$ → density.
$m(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$
$\frac{d}{dt} m(t) = \frac{d}{dt} \int_{x_1}^{x_2} \rho(x,t)\,dx$
x_1
x_2
x
$\int_t^{t+\Delta t} \frac{d}{dt} m(t) = \int_t^{t+\Delta t} \left(\rho(x_1,t)\,u(x_1,t) - \rho(x_2,t)\,u(x_2,t) \right) dt$
$m(t+\Delta t) - m(t) = \int_t^{t+\Delta t} [\quad]$
$\int_{x_1}^{x_2} \left[\rho(x,t+\Delta t) - \rho(x,t) \right] dx = \int_t^{t+\Delta t} \rho(x_1,t)\,u(x_1,t) - \rho(x_2,t)\,u(x_2,t)\,dt$

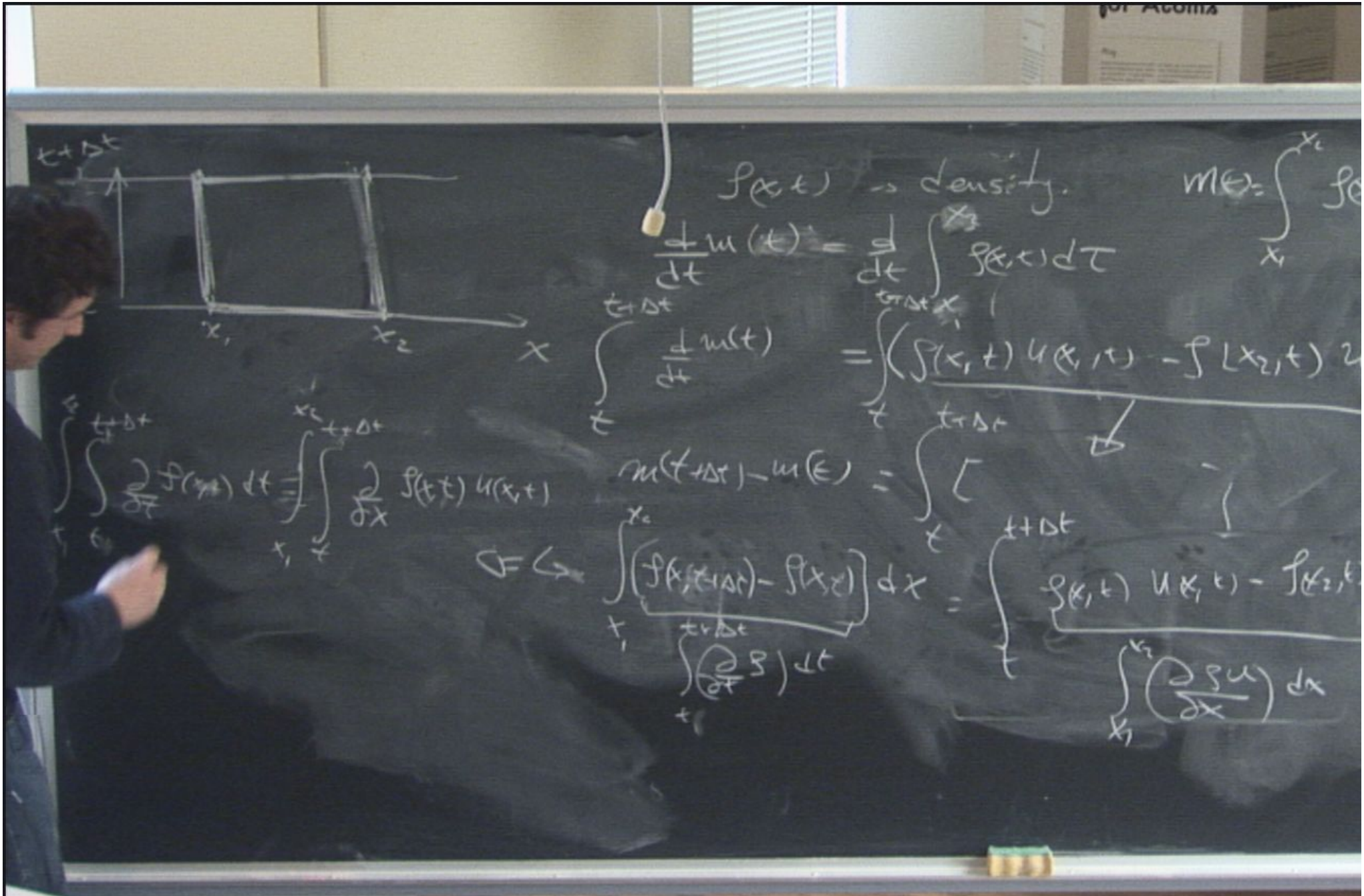

density.

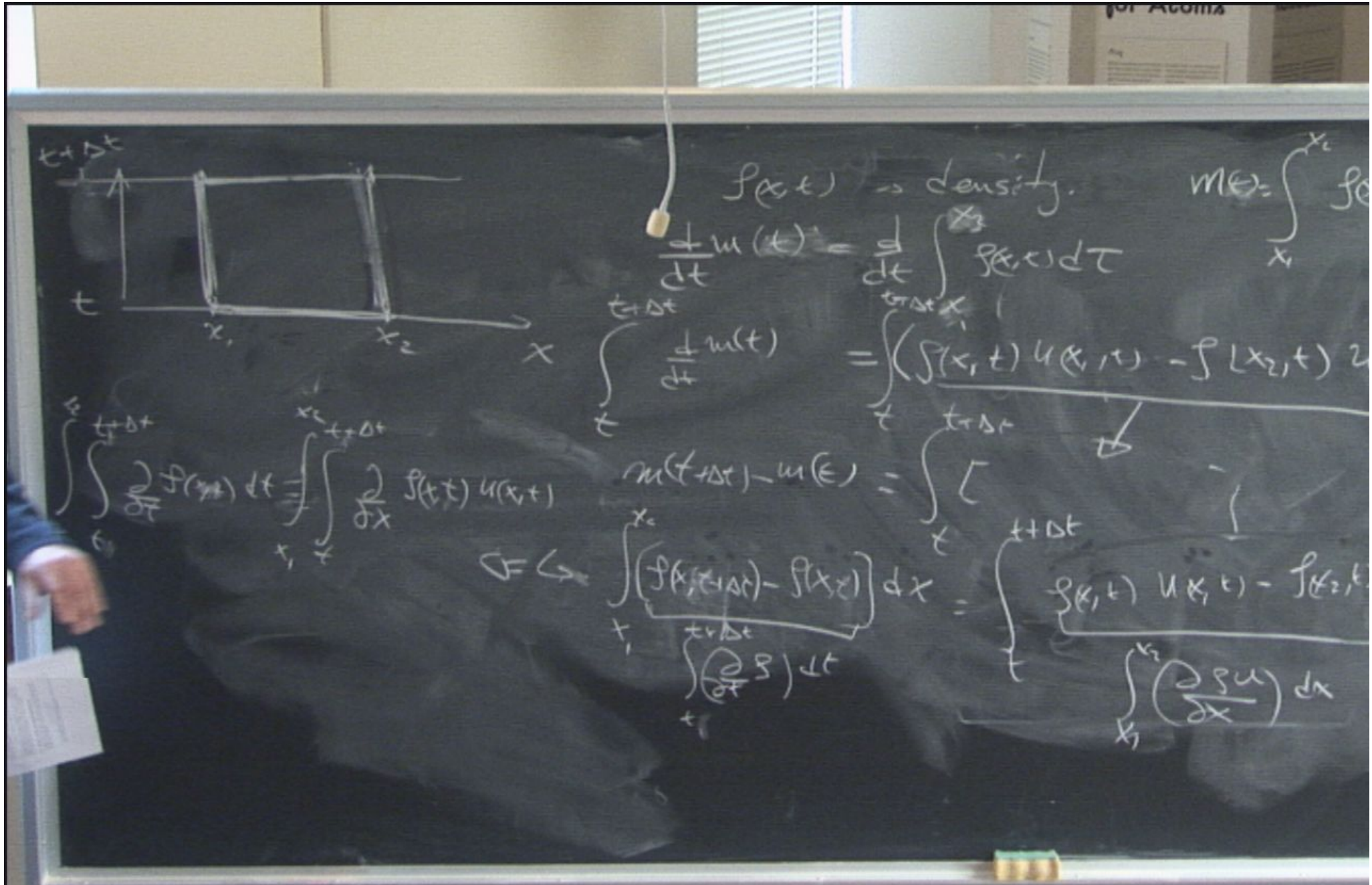

ρ(x,t) → density.
m(t) = ∫ ρ(
d/dt m(t) = d/dt ∫ ρ(x,τ) dτ
∫ d/dt m(t) = ∫ (ρ(x1,t) u(x1,t) − ρ(x2,t) u
m(t+Δt) − m(t) = ∫ [ ]
∫ [ρ(x,t+Δt) − ρ(x,t)] dx = ∫ ρ(x1,t) u(x1,t) − ρ(x2,t
t+Δt
t
x1
x
t1
t2

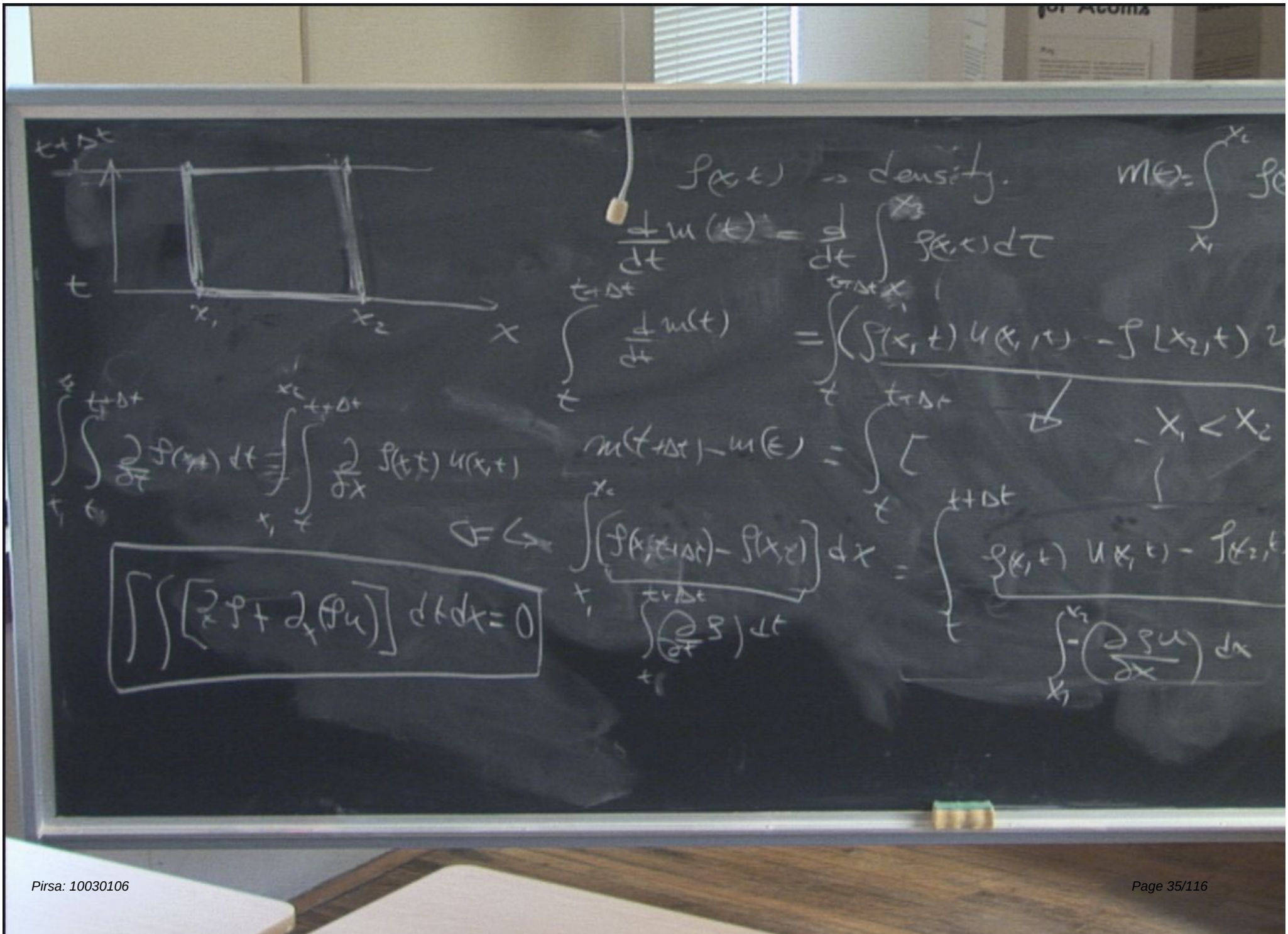

density.


t+Δt
t
x₁
x₂
x
ρ(x,t) → density.
m(t) = ∫ ρ
d/dt m(t) = d/dt ∫ ρ(x,t) dτ
d/dt m(t)
= ∫ (ρ(x₁,t) u(x₁,t) - ρ(x₂,t)
m(t+Δt) - m(t) = ∫
∫ ∂/∂t ρ(x,t) dt = ∫ ∂/∂x ρ(x,t) u(x,t)
∫ [ρ(x,t+Δt) - ρ(x,t)] dx
ρ(x,t) u(x,t) - ρ(x₂,t

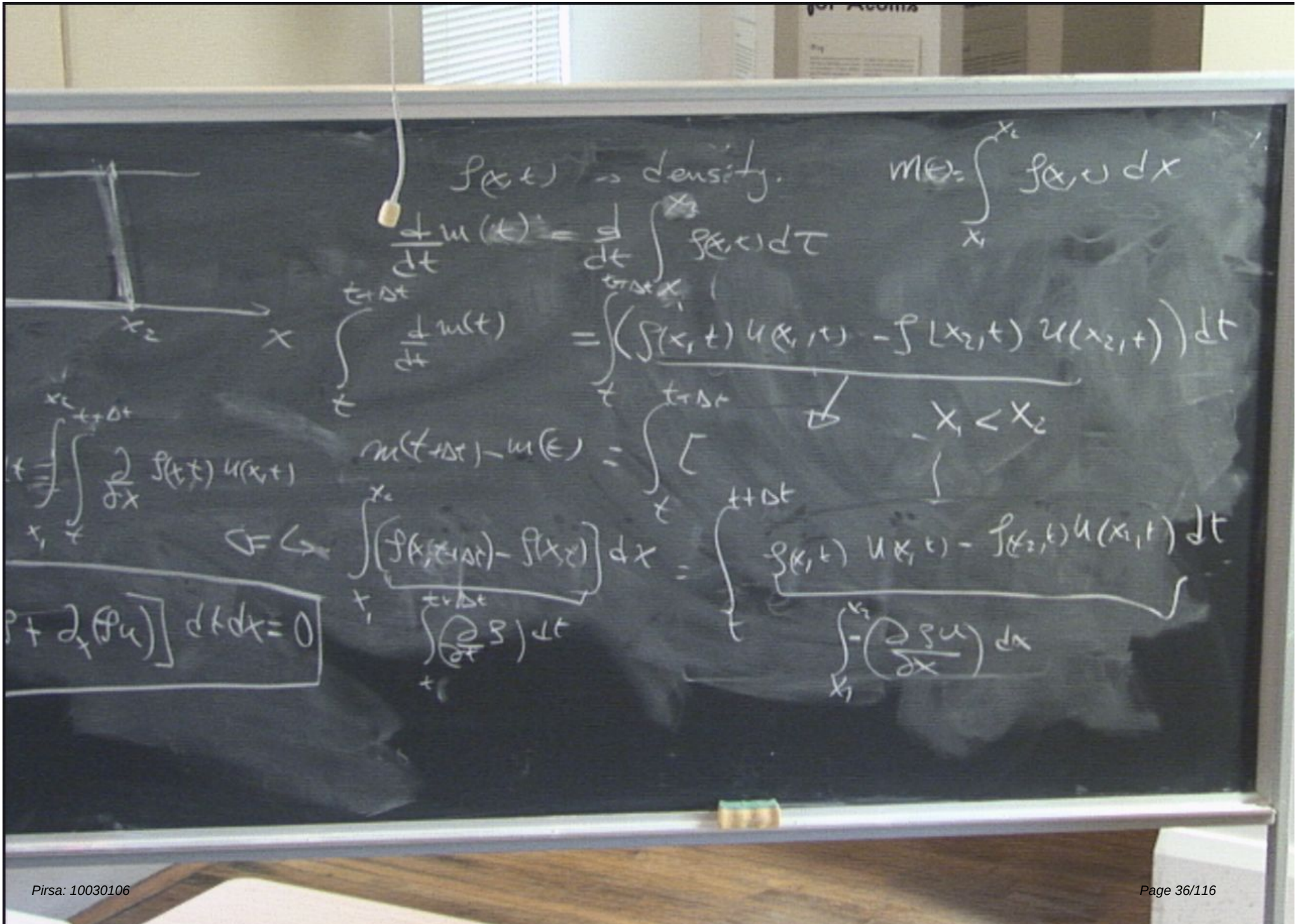

density.

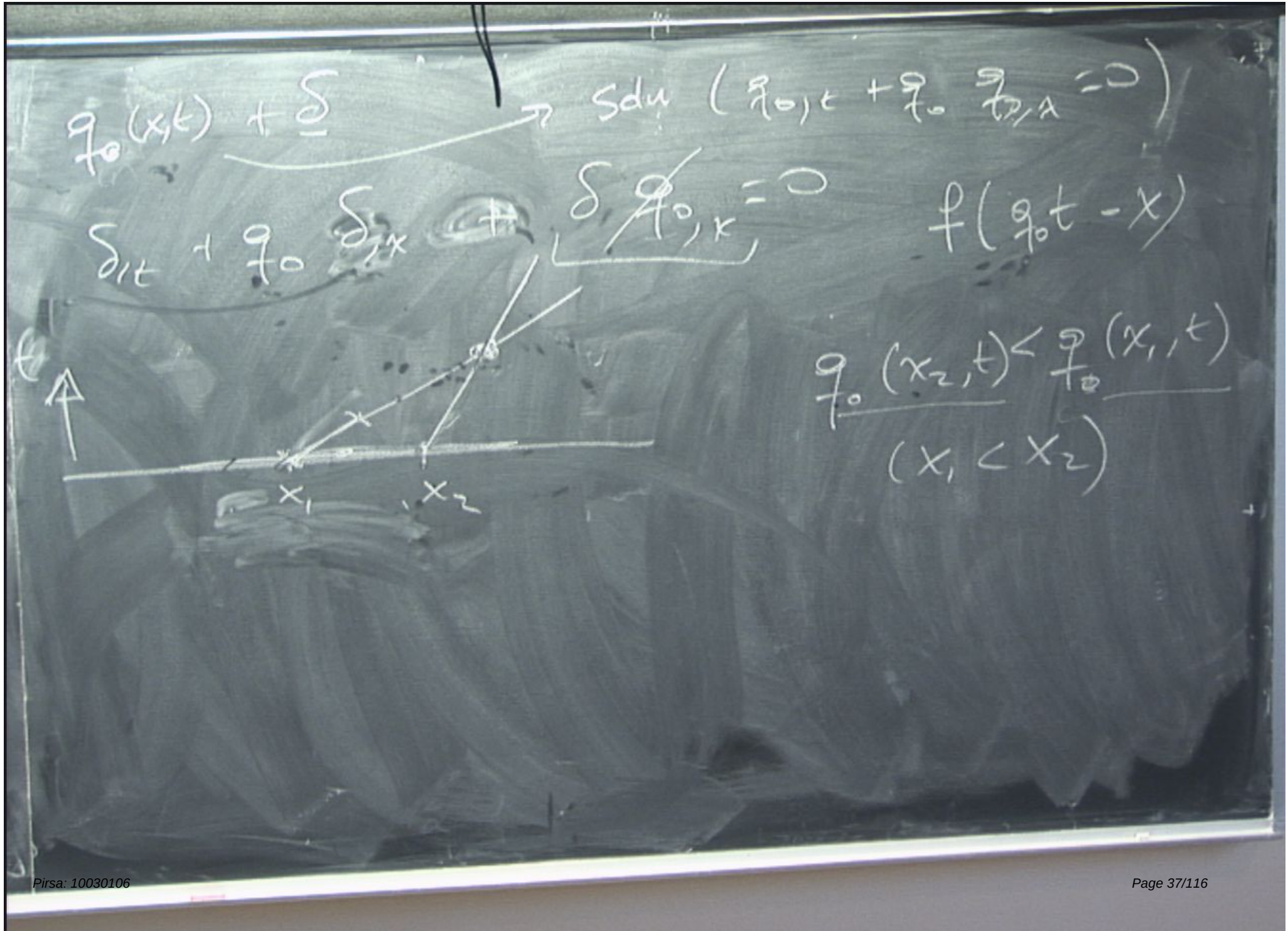

$\rho(x,t)$ -> density.
$\frac{d}{dt} m(t) = \frac{d}{dt} \int \rho(x,\tau)\, d\tau$
$m(t) = \int$
$\int_t^{t+\Delta t} \frac{d}{dt} m(t) = \int_t^{t+\Delta t} (\rho(x_1,t)\, u(x_1,t) - \rho(x_2,t)$
$m(t+\Delta t) - m(t) = \int_t^{t+\Delta t}$
$\int_{x_1}^{x_2} \left(\rho(x,t+\Delta t) - \rho(x,t)\right) dx$
$\int_t^{t+\Delta t} \left(\frac{\partial}{\partial t}\rho\right) dt$
$\rho(x,t)\, u(x,t) - \rho(x_2,t)$
$\int_{x_1}^{x_2} \left(\frac{\partial \rho u}{\partial x}\right) dx$


density.



t+Δt
t
x1
x2
x
ρ(x,t) → density.
m(t) = ∫ ρ(
d/dt m(t) = d/dt ∫ ρ(x,t) dτ
∫ d/dt m(t) = ∫ (ρ(x1,t) u(x1,t) − ρ(x2,t) u
x1 < x2
m(t+Δt) − m(t) = ∫ [
∫∫ [∂t ρ + ∂x (ρu)] dt dx = 0
∫ (∂/∂t ρ) dt
∫ −(∂ρu/∂x) dx

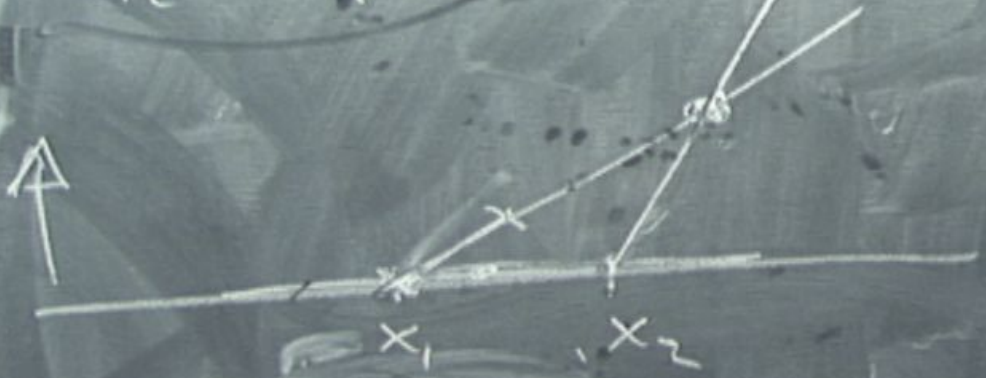

ρ(x,t) → density.
m(t) = ∫_{x_1}^{x_2} ρ(x,t) dx
d/dt m(t) = d/dt ∫_{x_1}^{x_2} ρ(x,τ) dτ
∫_t^{t+Δt} d/dt m(t) = ∫_t^{t+Δt} (ρ(x_1,t) u(x_1,t) − ρ(x_2,t) u(x_2,t)) dt
x_1 < x_2
m(t+Δt) − m(t) = ∫_t^{t+Δt} [
∫_{x_1}^{x_2} [ρ(x,t+Δt) − ρ(x,t)] dx = ∫_t^{t+Δt} ρ(x_1,t) u(x_1,t) − ρ(x_2,t) u(x_2,t) dt
∫_t^{t+Δt} (∂/∂t ρ) dt
∫_{x_1}^{x_2} −(∂ρu/∂x) dx
∫_{x_1}^{x_2} ∫_t^{t+Δt} ∂/∂x ρ(x,t) u(x,t)
ρ + ∂_x(ρu)] dt dx = 0
x_2
x


$q_0(x,t) + \delta$
$\int du\ (q_{0,t} + q_0 q_{0,x} = 0)$
$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$
$f(q_0 t - x)$
t
x
x_1
x_2
$q_0(x_2,t) < q_0(x_1,t)$
$(x_1 < x_2)$

$q_0(x,t) + \delta$ → $\delta du\ (q_{0,t} + q_0 q_{0,x} = 0)$

$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$

$f(q_0 t - x)$



$q_0(x_2,t) < q_0(x_1,t)$

$(x_1 < x_2)$

1) $q_{,t} + q\, q_{,x} = 0$

$q_0(x,t) + \delta \rightarrow \int du\,(q_{0,t} + q_0 q_{0,x} = 0)$

$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$

$f(q_0 t - x)$

$q_0(x_2,t) < q_0(x_1,t)$

$(x_1 < x_2)$

$\int \int [q_{,t} + q\, q_{,x}]\, \phi\, dt\, dx = 0$

$\phi(x,t) =$ known function or arbitrary

$q_0(x,t) + \delta \rightarrow \int du\,(q_{0,t} + q_0 q_{0,x} = 0)$

$\delta_{,t} + q_0 \delta_{,x} + \delta q_{0,x} = 0$

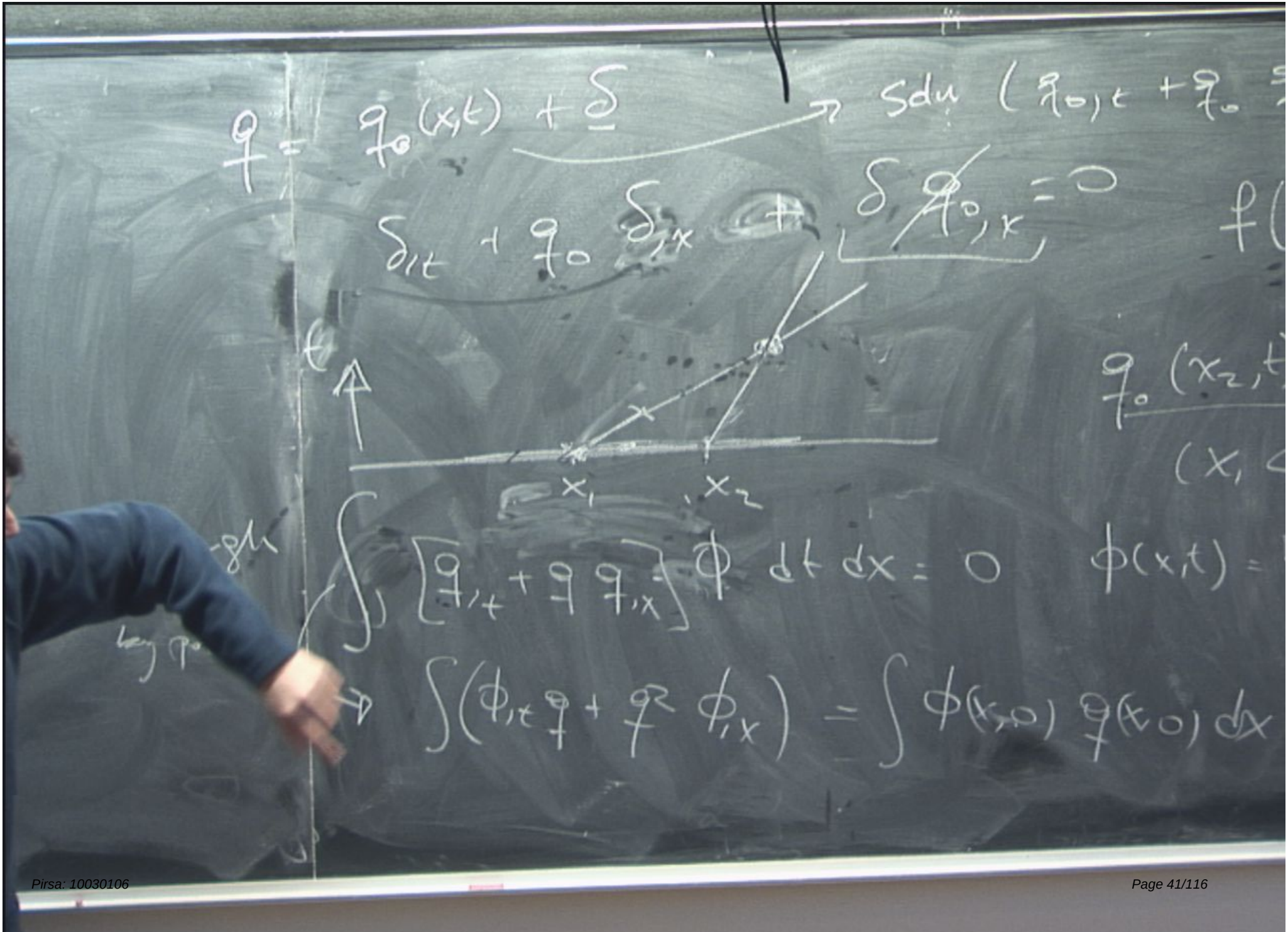
$f(q_0 t - x)$

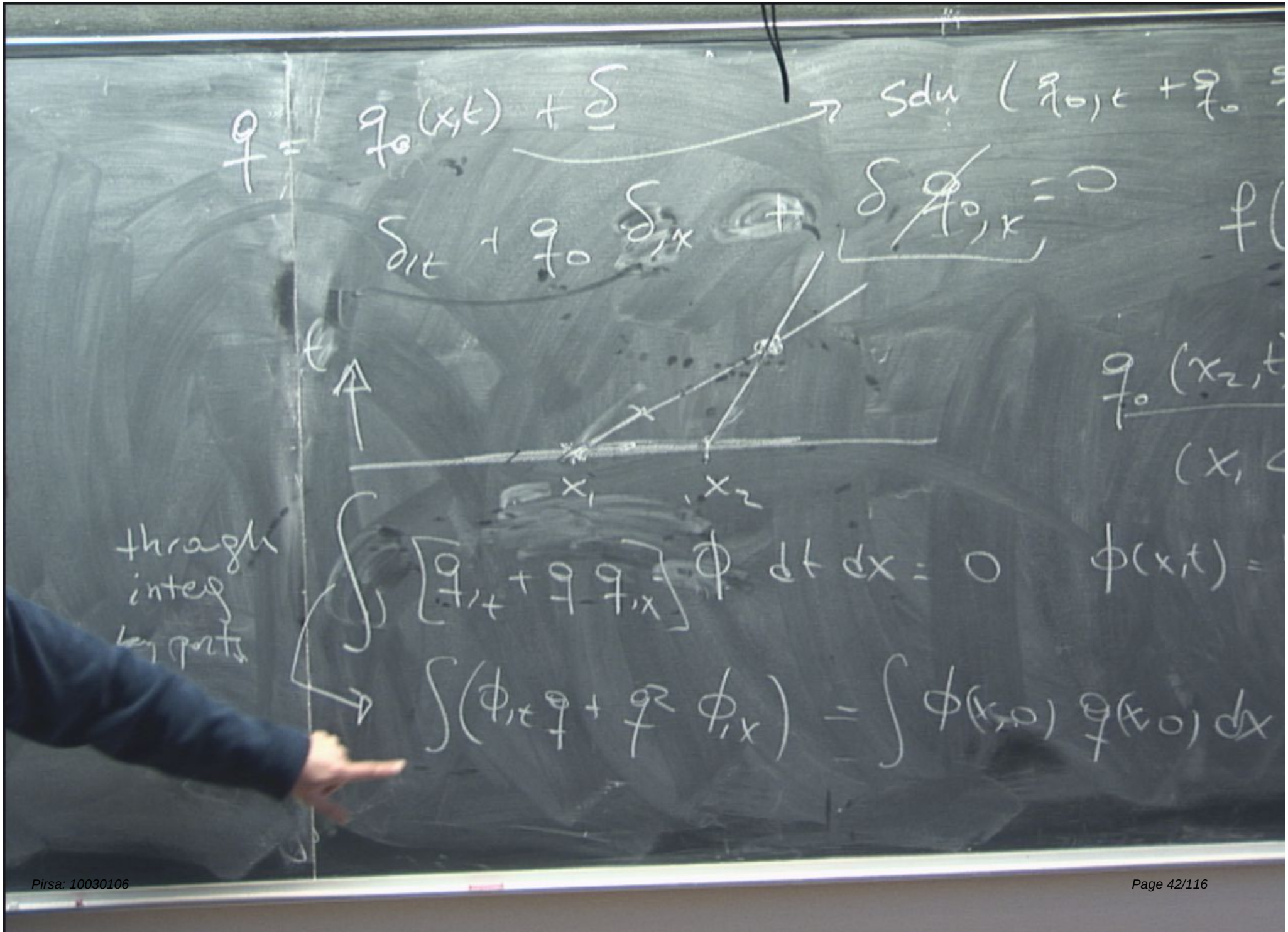
$q_0(x_2,t) < q_0(x_1,t)$

$(x_1 < x_2)$

$\int\int [q_{,t} + q\,q_{,x}]\,\phi\,dt\,dx = 0$

$\phi(x,t)$ = known function or arbitrary

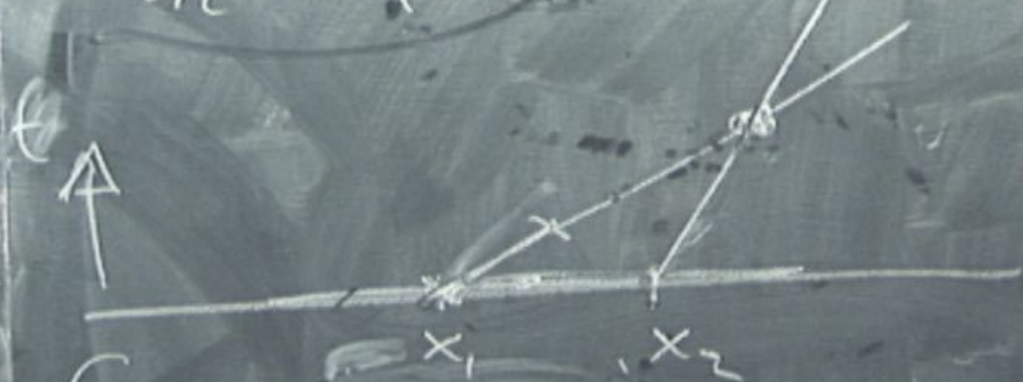

q = q0(x,t) + δ
δ,t + q0 δ,x + δ q0,x = 0
x1
x2
∫ [q,t + q q,x] φ dt dx = 0
φ(x,t) =
∫(φ,t q + q² φ,x) = ∫ φ(x,0) q(x,0) dx


q = q0(x,t) + δ
δ,t + q0 δ,x + δ q0,x = 0
x1
x2
through
integ
by parts
∫ [q,t + q q,x] φ dt dx = 0
φ(x,t) =
∫(φ,t q + q² φ,x) = ∫ φ(x,0) q(x,0) dx

$$q = q_0(x,t) + \underline{\delta} \longrightarrow \text{sdu} \; (q_{0,t} + q_0 \; \ldots$$

$$\delta_{,t} + q_0 \, \delta_{,x} + \underbrace{\delta \, \cancel{q_{0,x}}} = 0$$

$f($



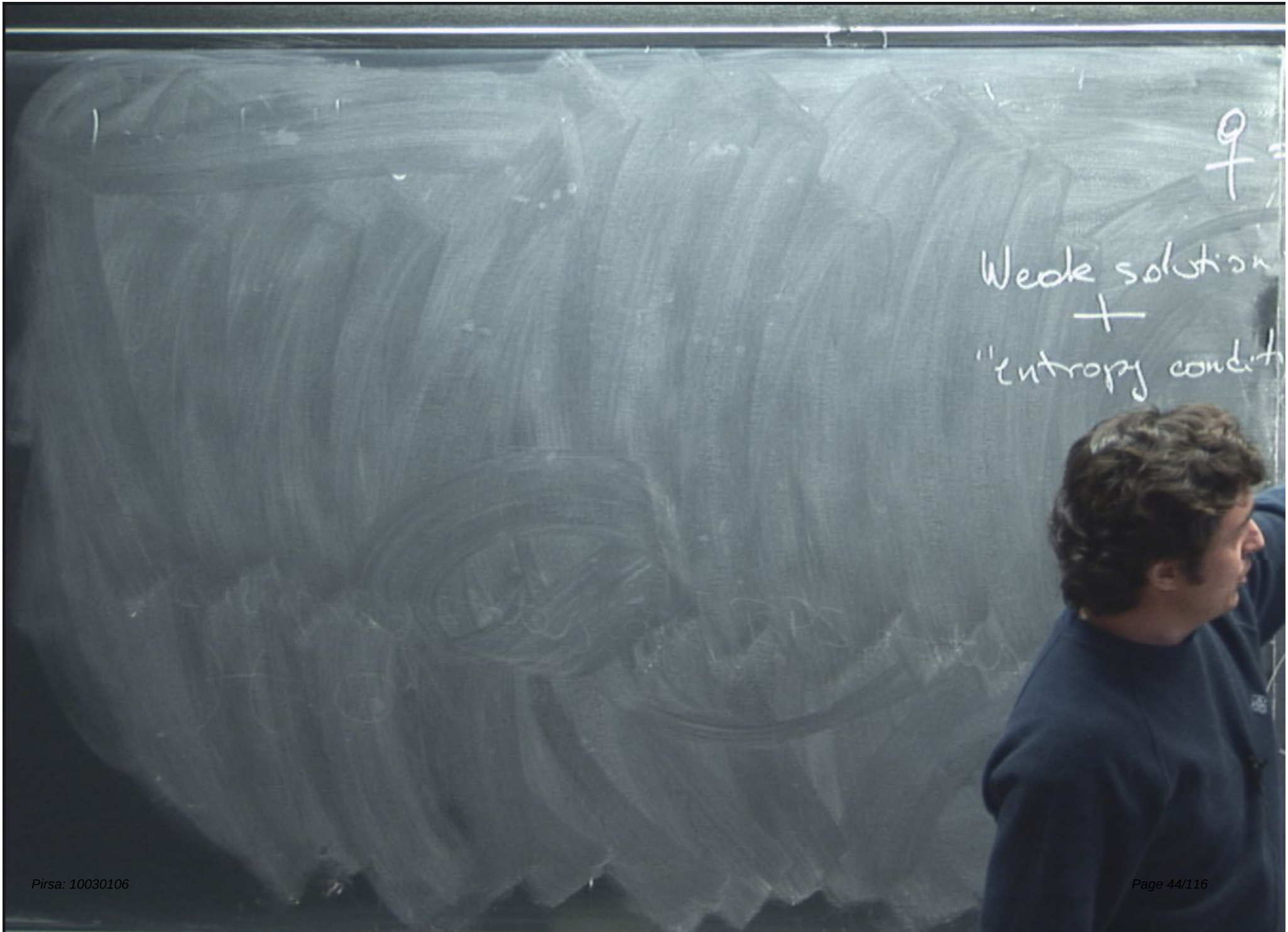
$q_0 \, (x_2, t \ldots$

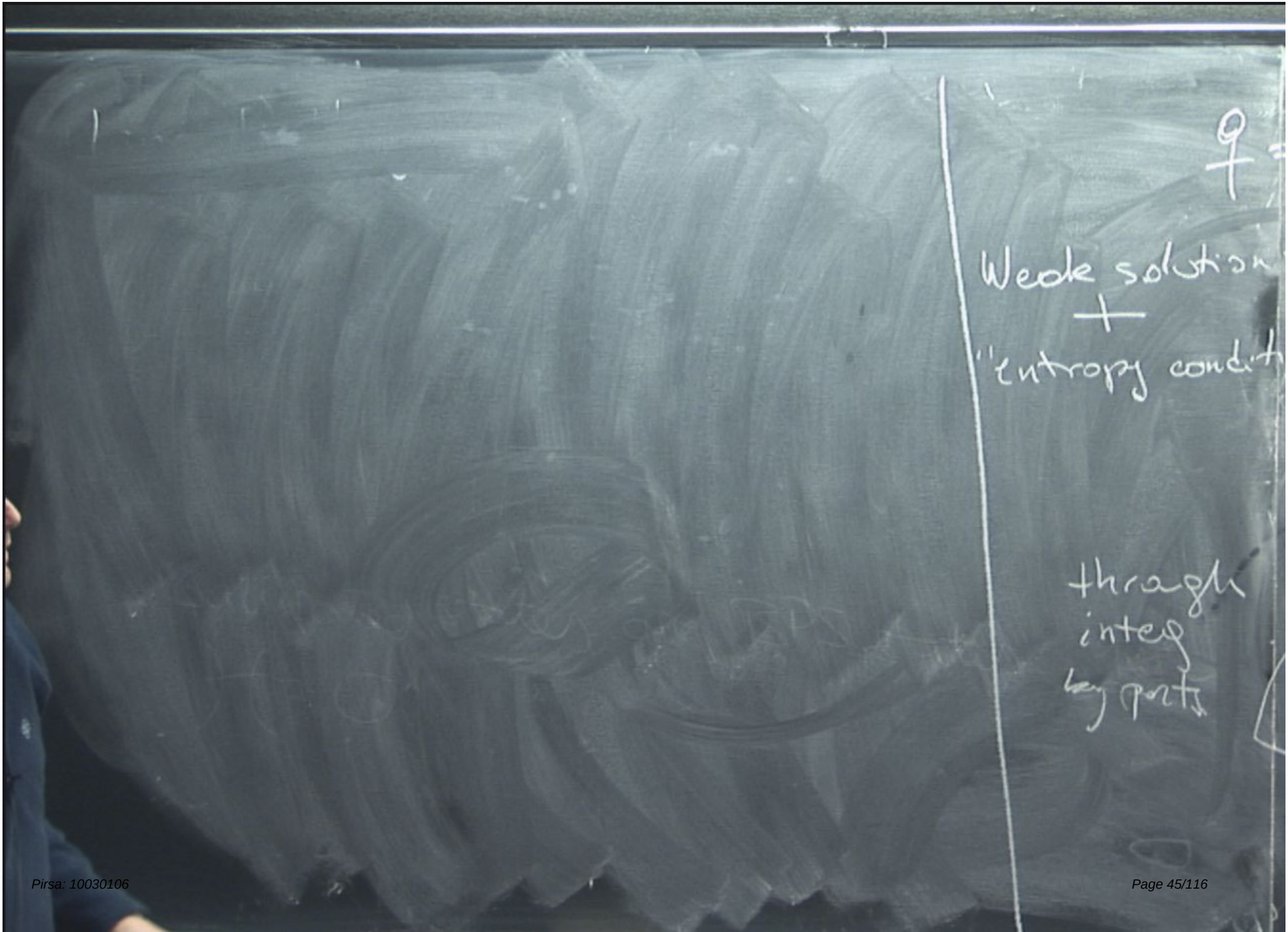
$(x, \ldots$

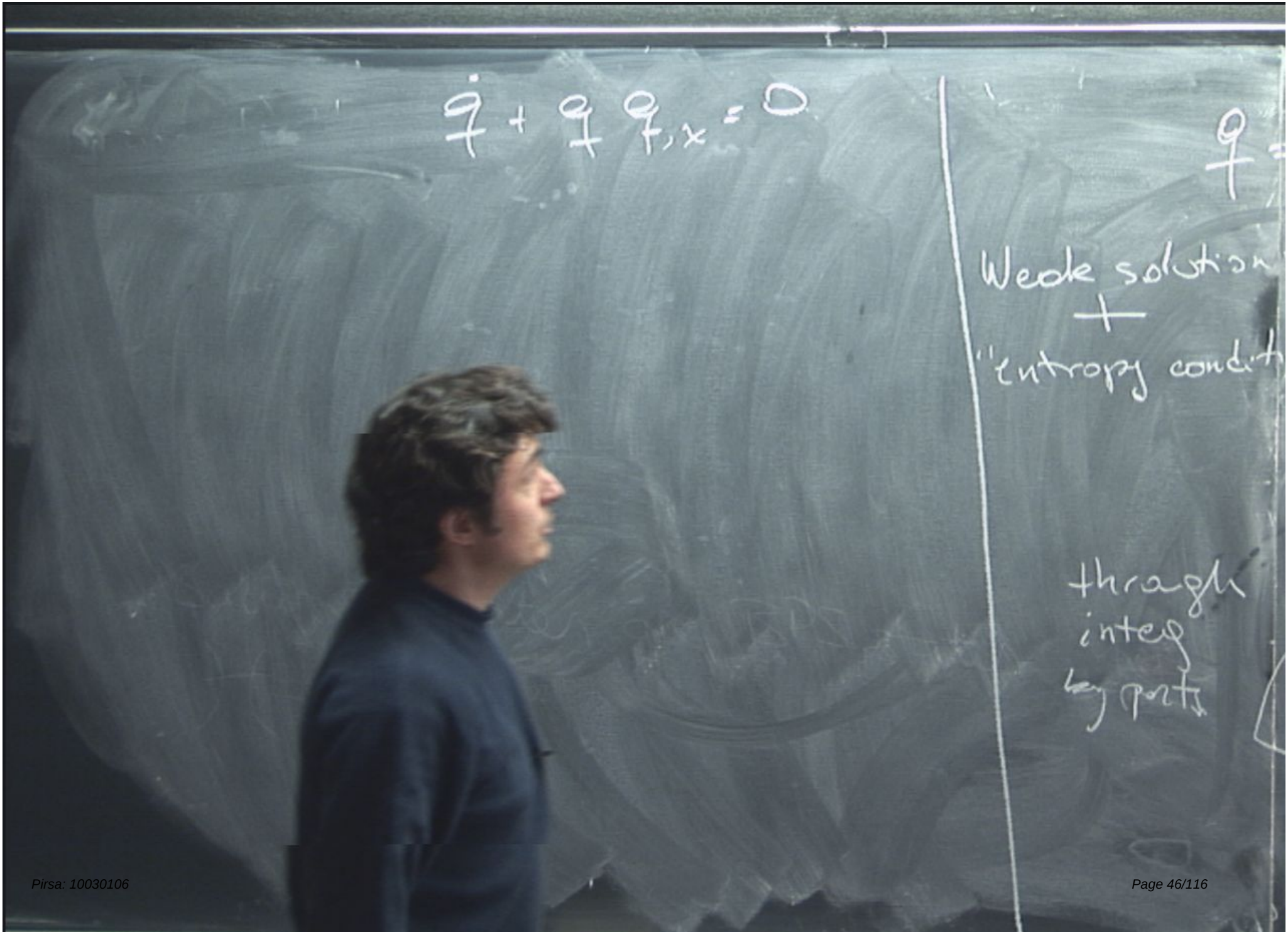
through integ by parts

$$\int \left[q_{,t} + q \, q_{,x} \right] \phi \, dt \, dx = 0 \qquad \phi(x,t) = \ldots$$

$$\int \left(\phi_{,t} \, q + q^2 \, \phi_{,x} \right) = \int \phi(x,0) \, q(x,0) \, dx$$


Weak solution
+
"entropy condit


Weak solution
+
"entropy condit
through
integ
by parts


$\dot{q} + q q_{,x} = 0$
$q =$
Weak solution
+
"entropy condit
through
integ
by parts

$\dot{q} + q\, q_{,x} = 0$

Riemann problem

$q(x, t=0) = \begin{cases} q_L \\ \end{cases}$

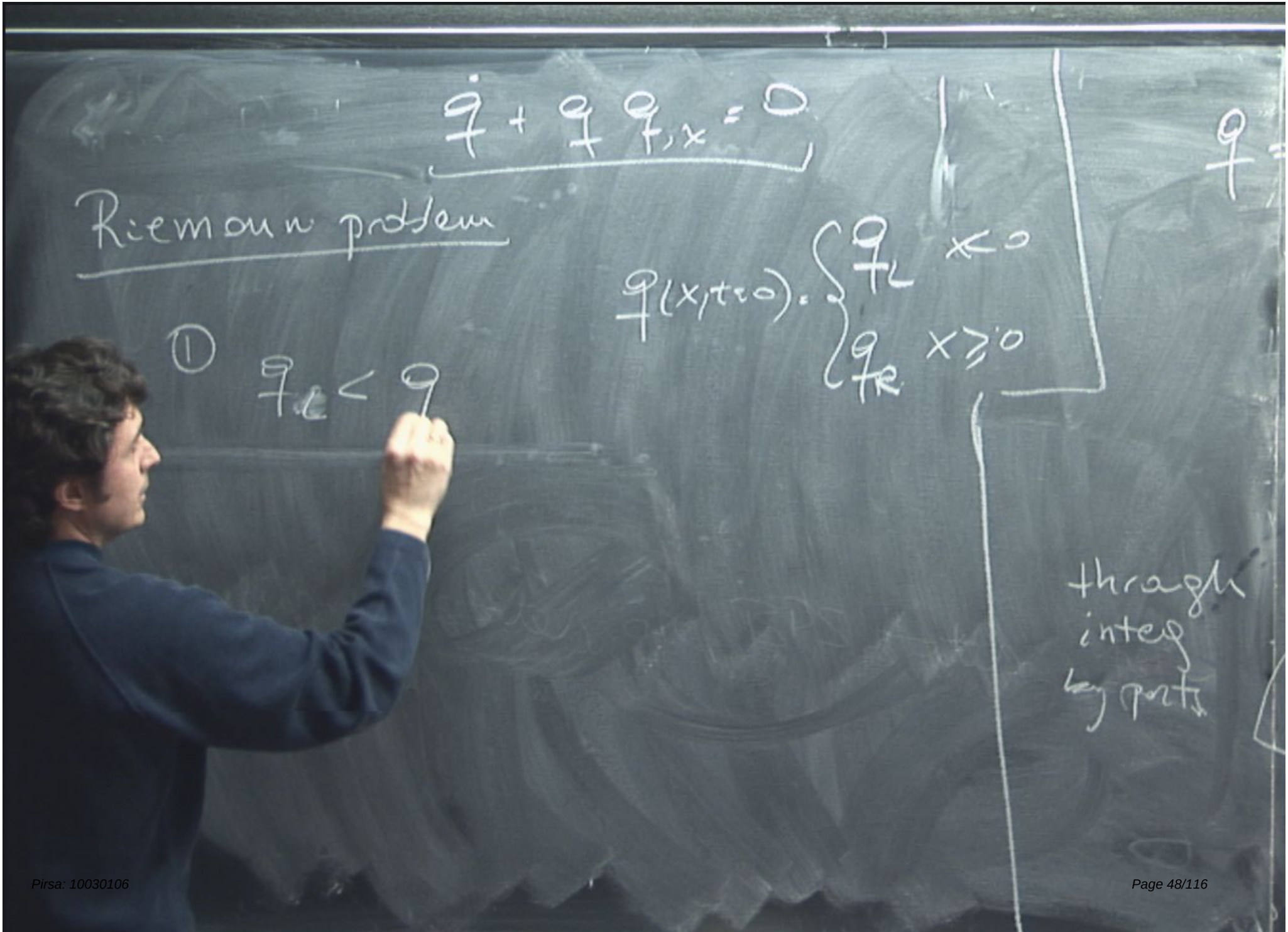
$q =$

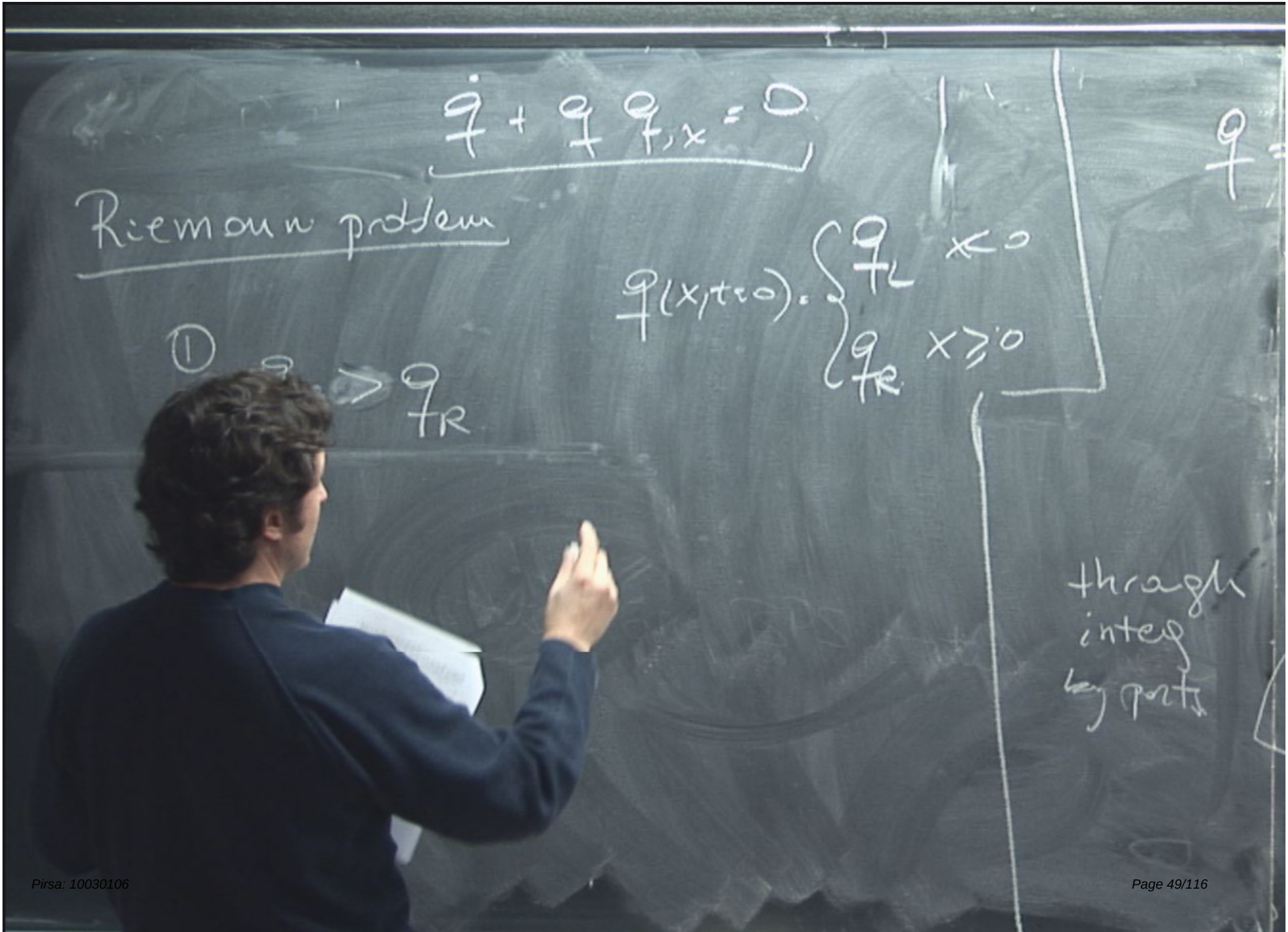
Weak solution

+

"entropy condit

through integ by parts


$\dot{q} + q q_{,x} = 0$
Riemann problem
$q(x,t=0) = \begin{cases} q_L & x<0 \\ q_R & x \geq 0 \end{cases}$
① $q_L < q$
through
integ
by parts


$\dot{q} + q\,q_{,x} = 0$
Riemann problem
$q(x,t=0) = \begin{cases} q_L & x<0 \\ q_R & x \geq 0 \end{cases}$
① $q > q_R$
through integ by parts

$\dot{q} + q\, q_{,x} = 0$

Riemann problem

$q(x,t=0) = \begin{cases} q_L & x < 0 \\ q_R & x \geq 0 \end{cases}$

① $q_L > q_R$

Soln q_L if $x < St$

q_R if $x > St$

through integ by parts

$\dot{q} + q\, q_{,x} = 0$

Riemann problem

$q(x, t=0) = \begin{cases} q_L & x < 0 \\ q_R & x \geq 0 \end{cases}$

① $q_L > q_R$

q_L if $x < St$

q_R if $x > St$

through integ by parts

$$\dot{q} + q\, q_{,x} = 0$$

Riemann problem

$$q(x,t=0) = \begin{cases} q_L & x<0 \\ q_R & x \geq 0 \end{cases}$$

① $q_L > q_R$

q_L if $x < St$

q_R if $x > St$

$$S = \frac{q_L + q_R}{2}$$

through integ by parts

$q_t + q\,q_x = 0$

Riemann problem

$q(x,t=0) = \begin{cases} q_L & x<0 \\ q_R & x \geq 0 \end{cases}$

① $q_L > q_R$

Soln q_L if $x < St$

q_R if $x > St$

$S = \dfrac{q_L + q_R}{2}$

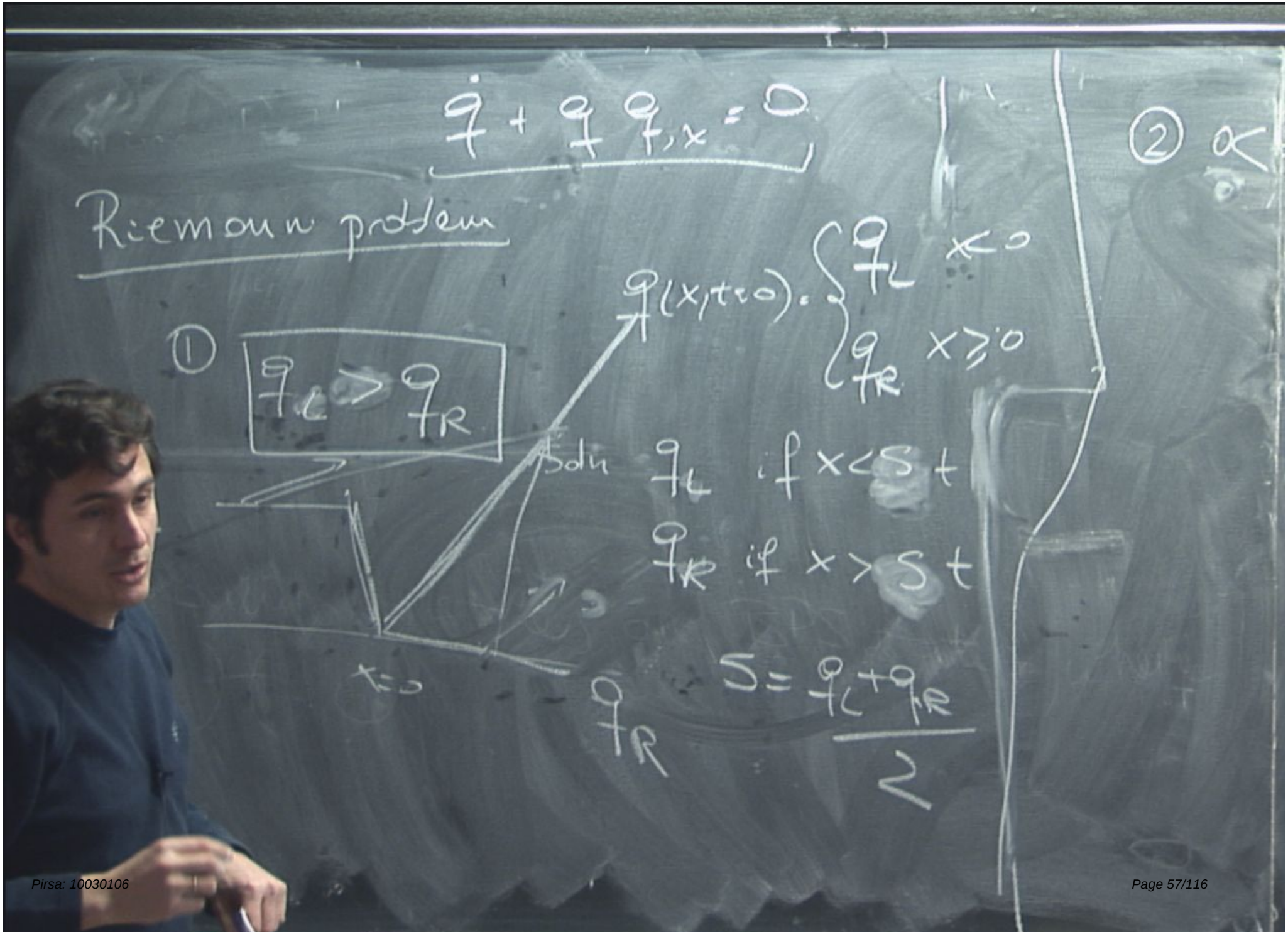
through integ by parts

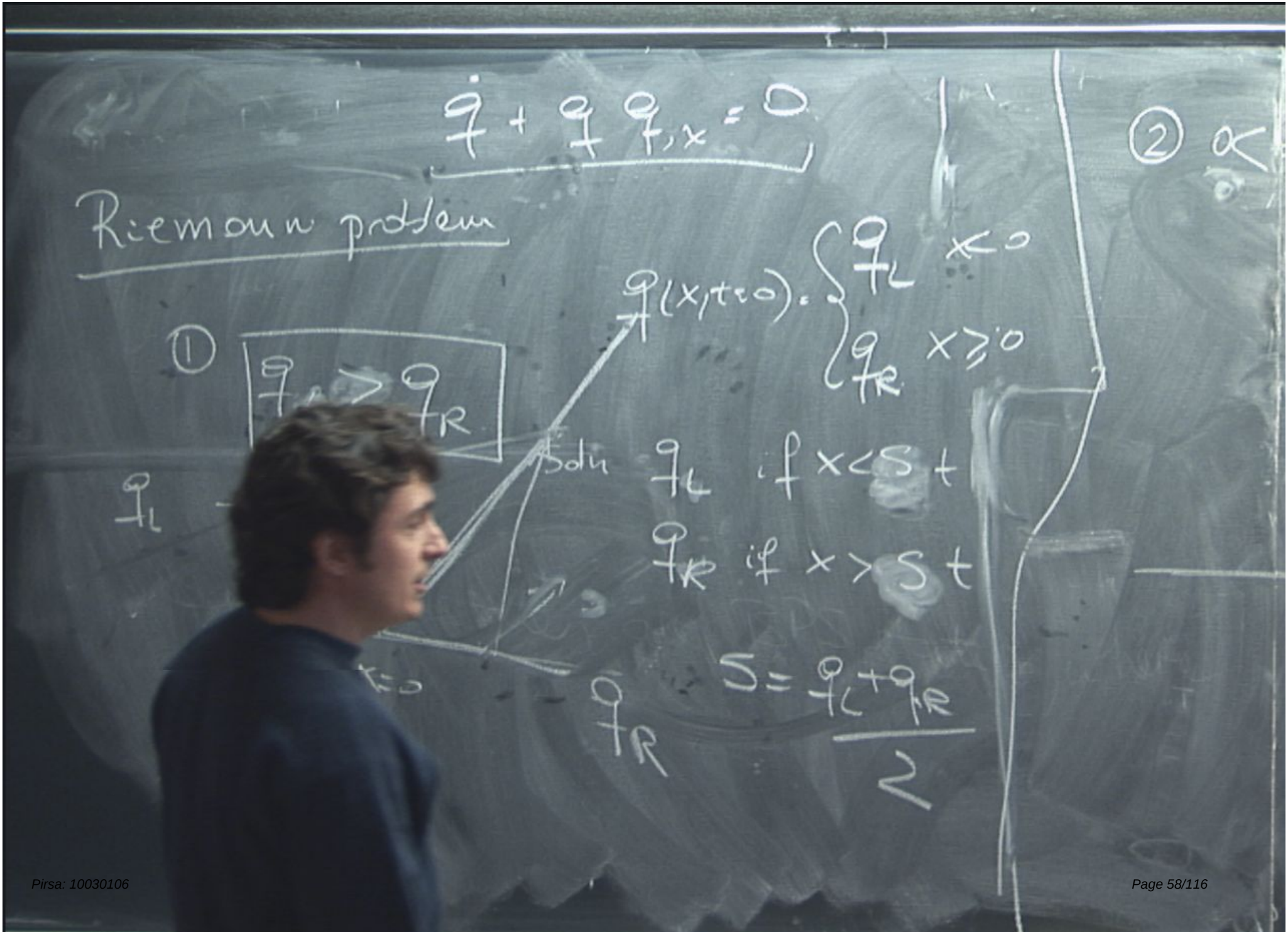
② $q_L < q_R$

② $q_L < q_R$

$q_L < q_R$

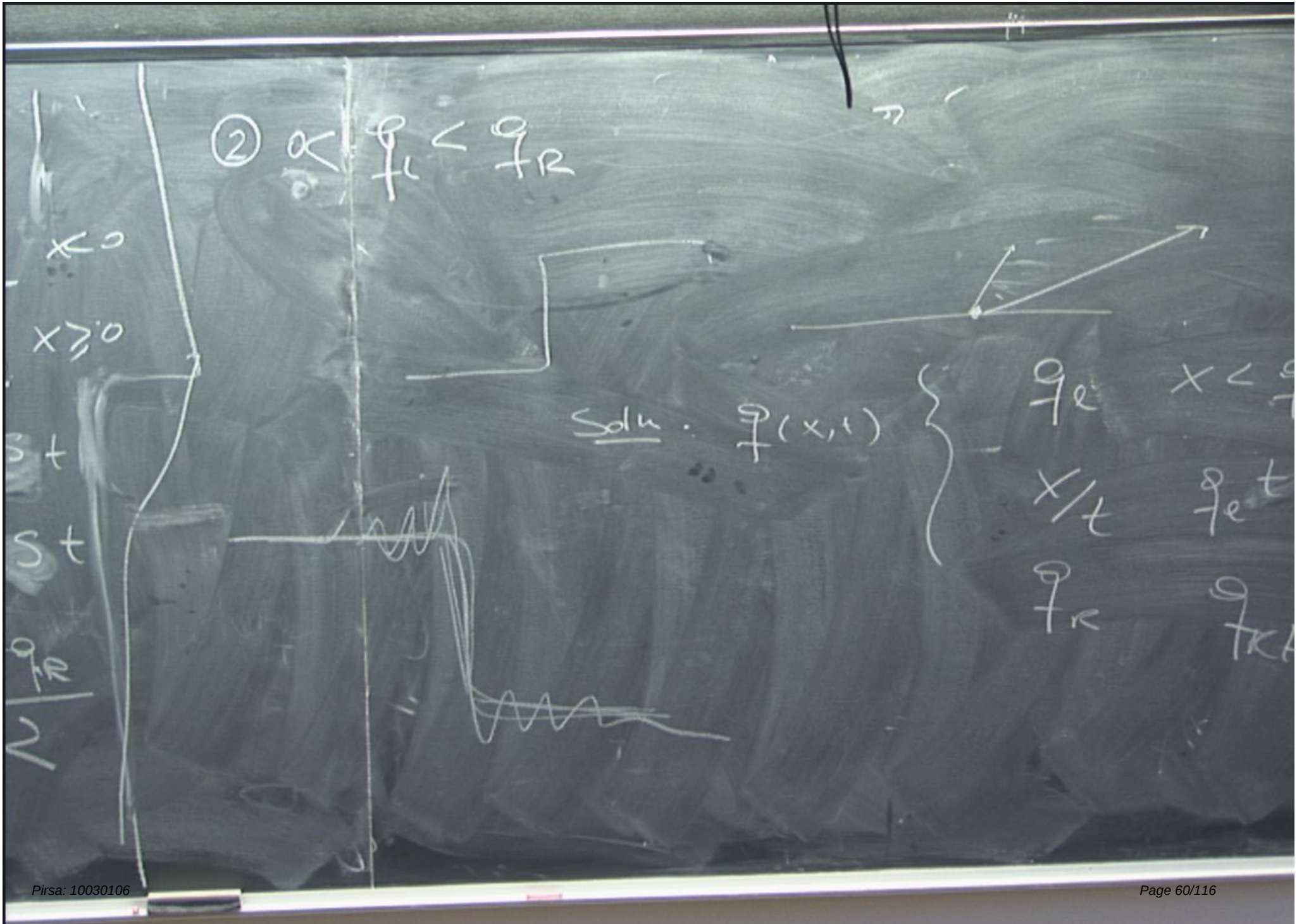
Soln. $q(x,t)$ $\begin{cases} q_L & x < q_L t \\ x/t & q_L t < x < q_R t \\ q_R & q_R t < x \end{cases}$

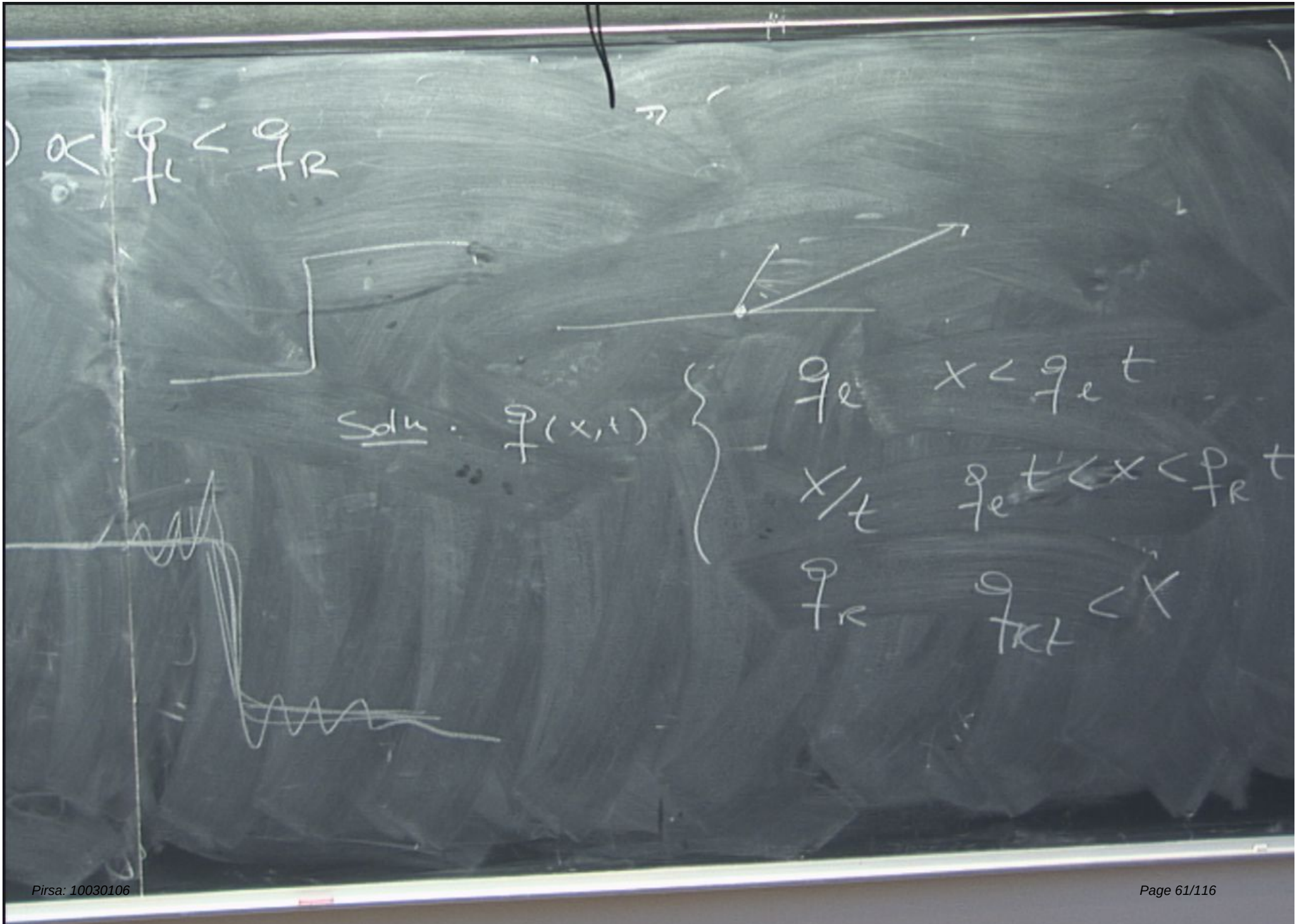

$\dot{q} + q q_x = 0$
Riemann problem
$q(x,t=0) = \begin{cases} q_L & x<0 \\ q_R & x>0 \end{cases}$
① $q_L > q_R$
Soln q_L if $x < St$
q_R if $x > St$
$S = \frac{q_L + q_R}{2}$
$t=0$
q_R
② α

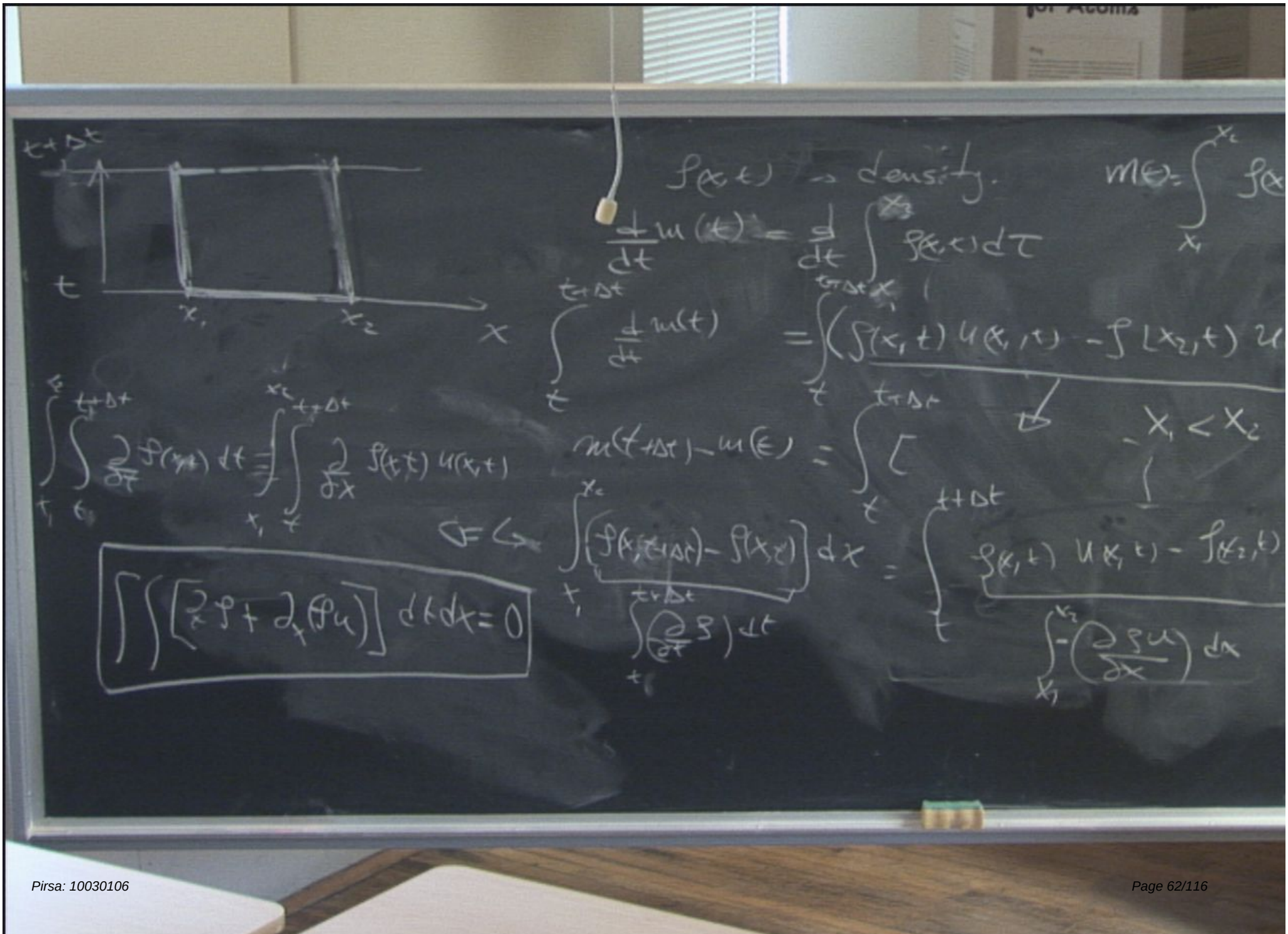

$\dot{q} + q\,q_{,x} = 0$
Riemann problem
$q(x,t=0) = \begin{cases} q_L & x<0 \\ q_R & x \geq 0 \end{cases}$
①
$q_L > q_R$
Soln q_L if $x < St$
q_R if $x > St$
$S = \frac{q_L + q_R}{2}$
q_L
q_R
$x=0$
②

$q_L < q_R$

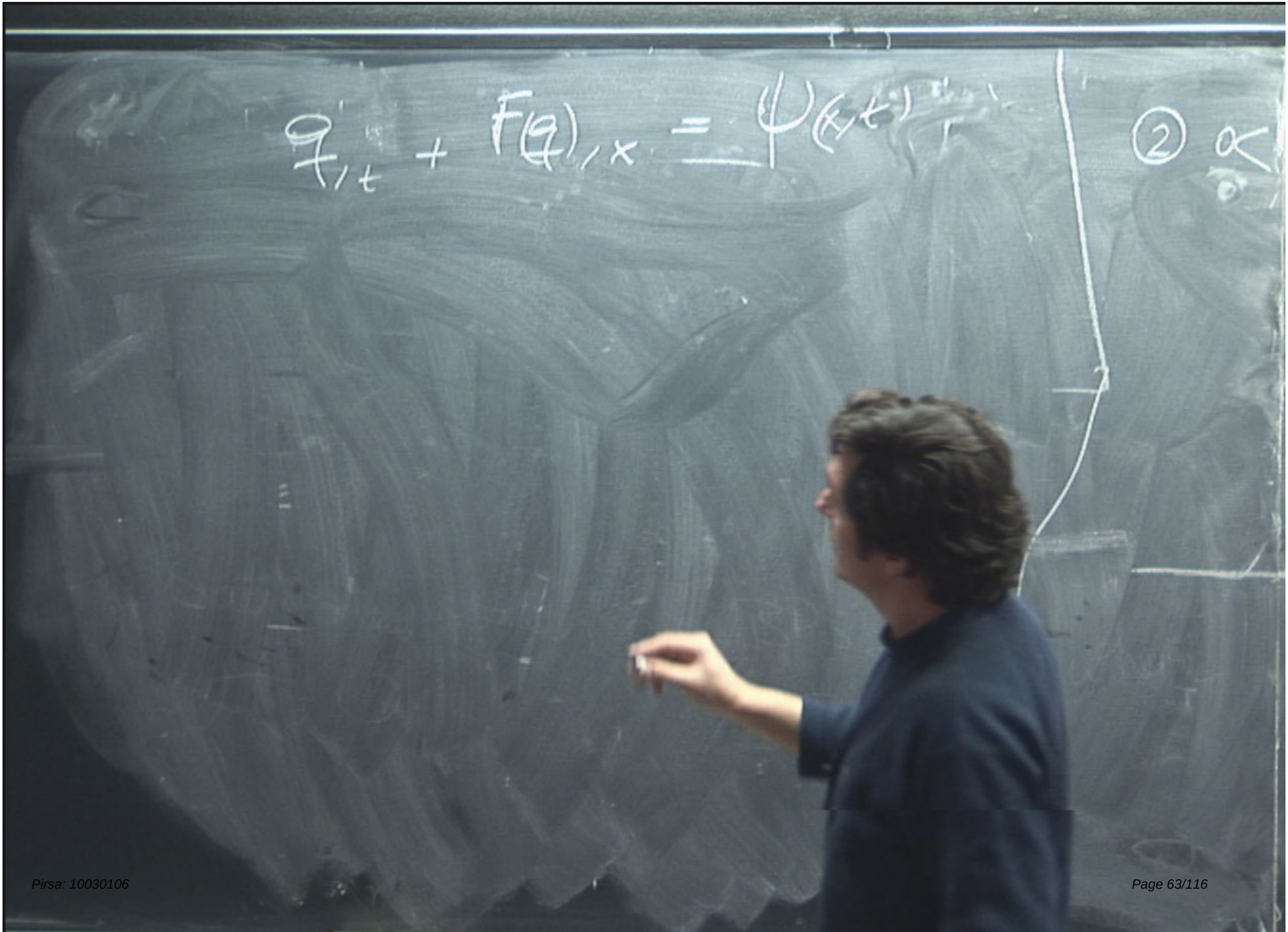
Soln. $q(x,t)$ $\begin{cases} q_L & x < q_L t \\ x/t & q_L t < x < q_R t \\ q_R & q_R t < x \end{cases}$


② 0 < q_L < q_R
x < 0
x ≥ 0
Soln. q(x,t)
q_l x <
x/t q_l t
q_R q_R


$0 < q_L < q_R$
Soln. $q(x,t)$
q_L $x < q_L t$
x/t $q_L t < x < q_R t$
q_R $q_{RL} < x$



t+Δt
t
x₁
x₂
x
ρ(x,t) → density.
m(t) = ∫ ρ(x
d/dt m(t) = d/dt ∫ ρ(x,τ) dτ
∫ d/dt m(t) = ∫ (ρ(x₁,t) u(x₁,t) − ρ(x₂,t) u
m(t+Δt) − m(t) = ∫ [
x₁ < x₂
∫∫ [∂ρ + ∂(ρu)] dt dx = 0
∫ (∂ρ/∂t) dt
∫ −(∂ρu/∂x) dx


$q_{,t} + F(q)_{,x} = \psi(x,t)$
②

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

②

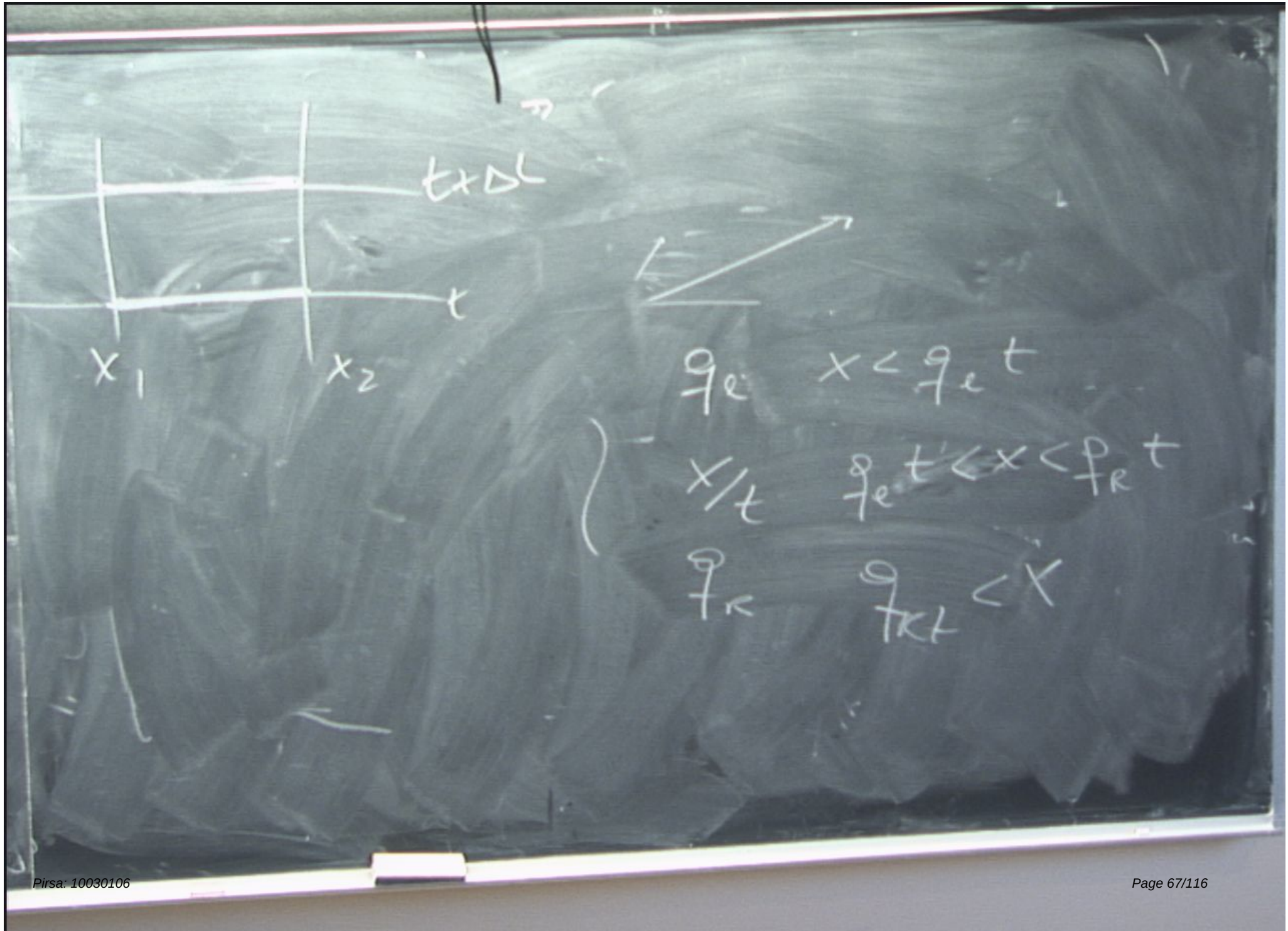
$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

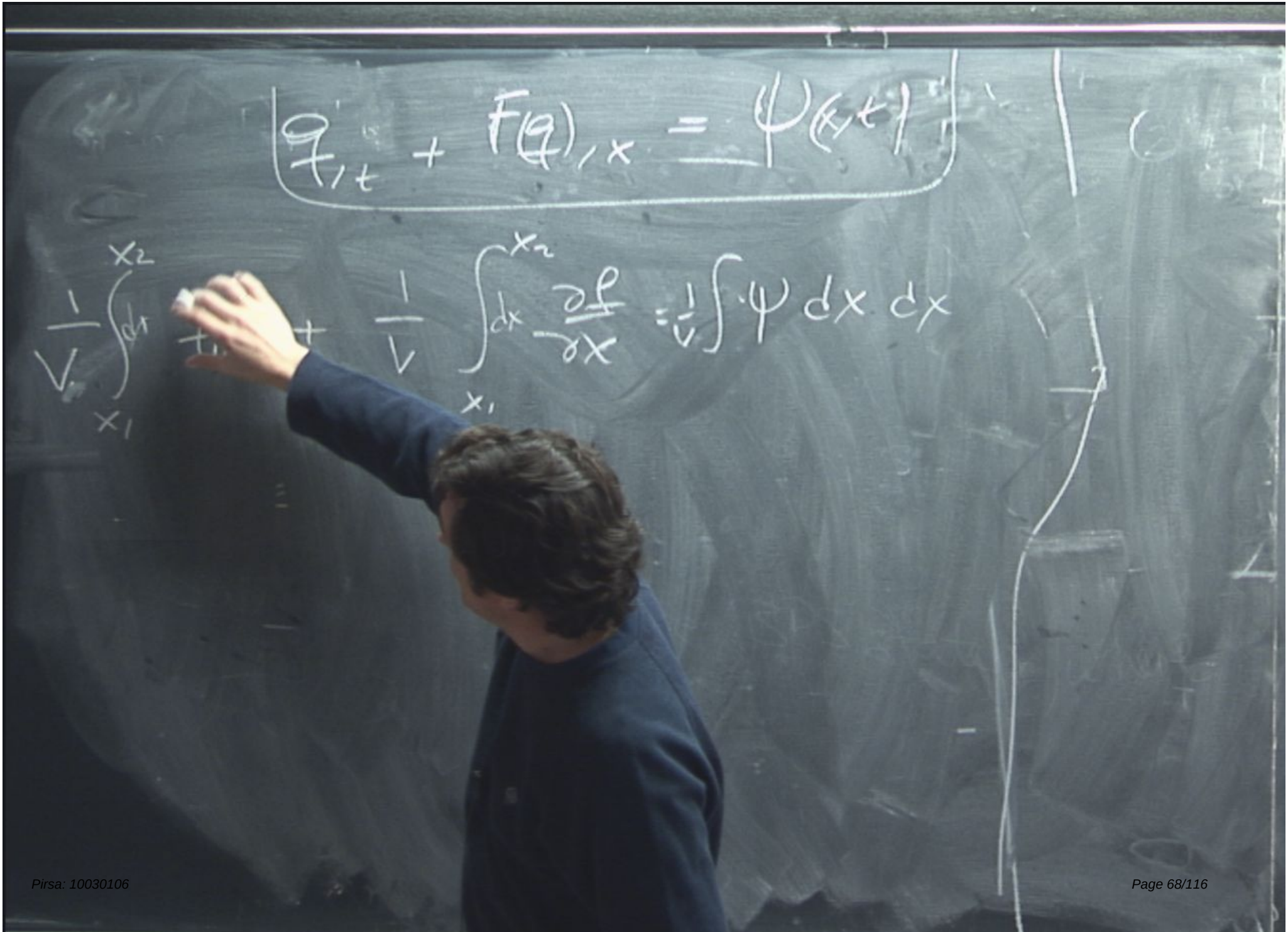
② α

$$\frac{1}{V}\int_{x_1}^{x_2} q_{,t} + \frac{1}{V}\int_{x_1}^{x_2} \frac{\partial f}{\partial x} = \frac{1}{V}\int \psi \, dx$$

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$\frac{1}{V}\int_{x_1}^{x_2} q_{,t}\,dx + \frac{1}{V}\int_{x_1}^{x_2} \frac{\partial f}{\partial x}\,dx = \frac{1}{V}\int \psi\,dx\,dx$$

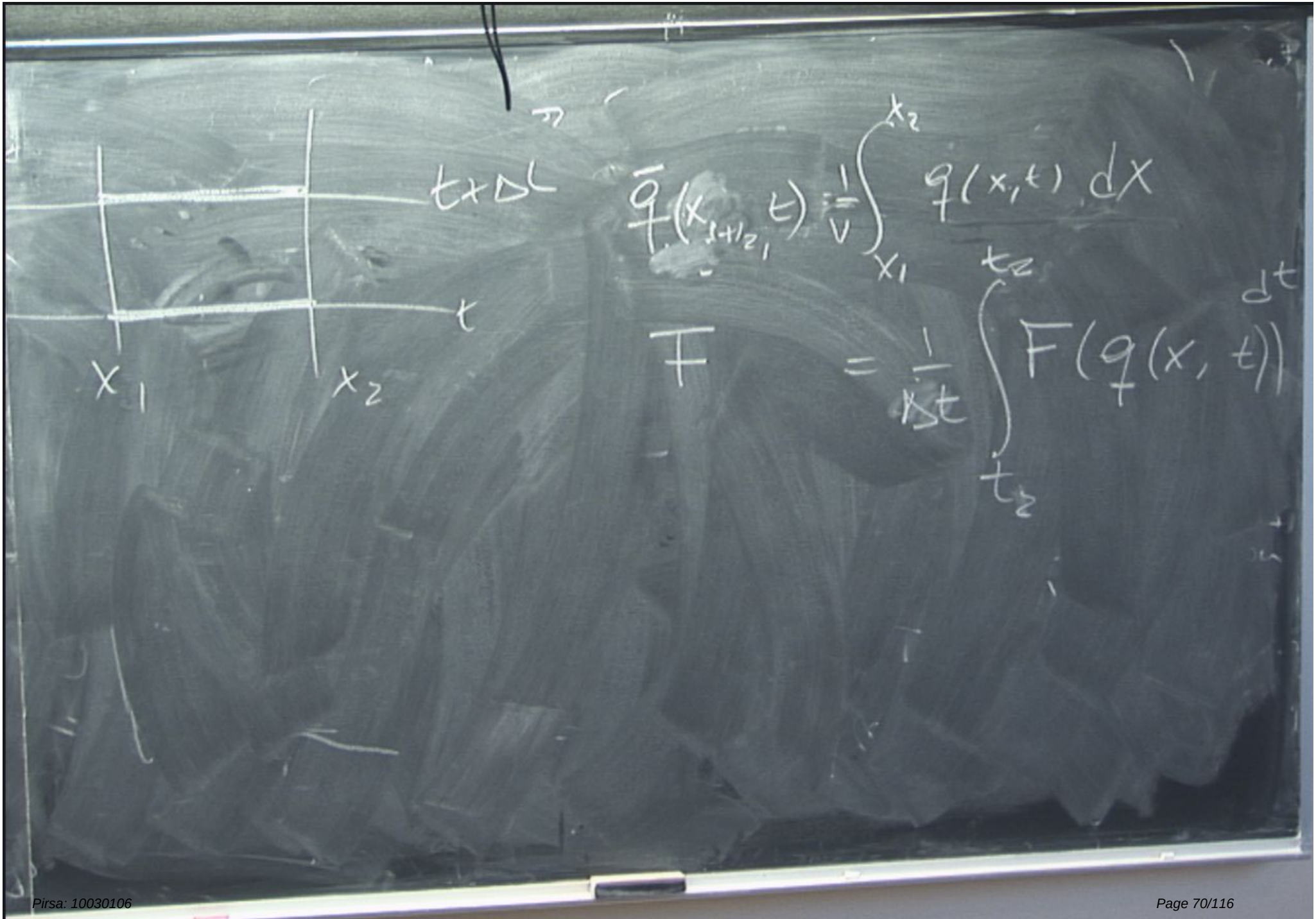

t+Δt
t
x1
x2
qe x < qe t
x/t qe t' < x < qR t
qR qRt < x

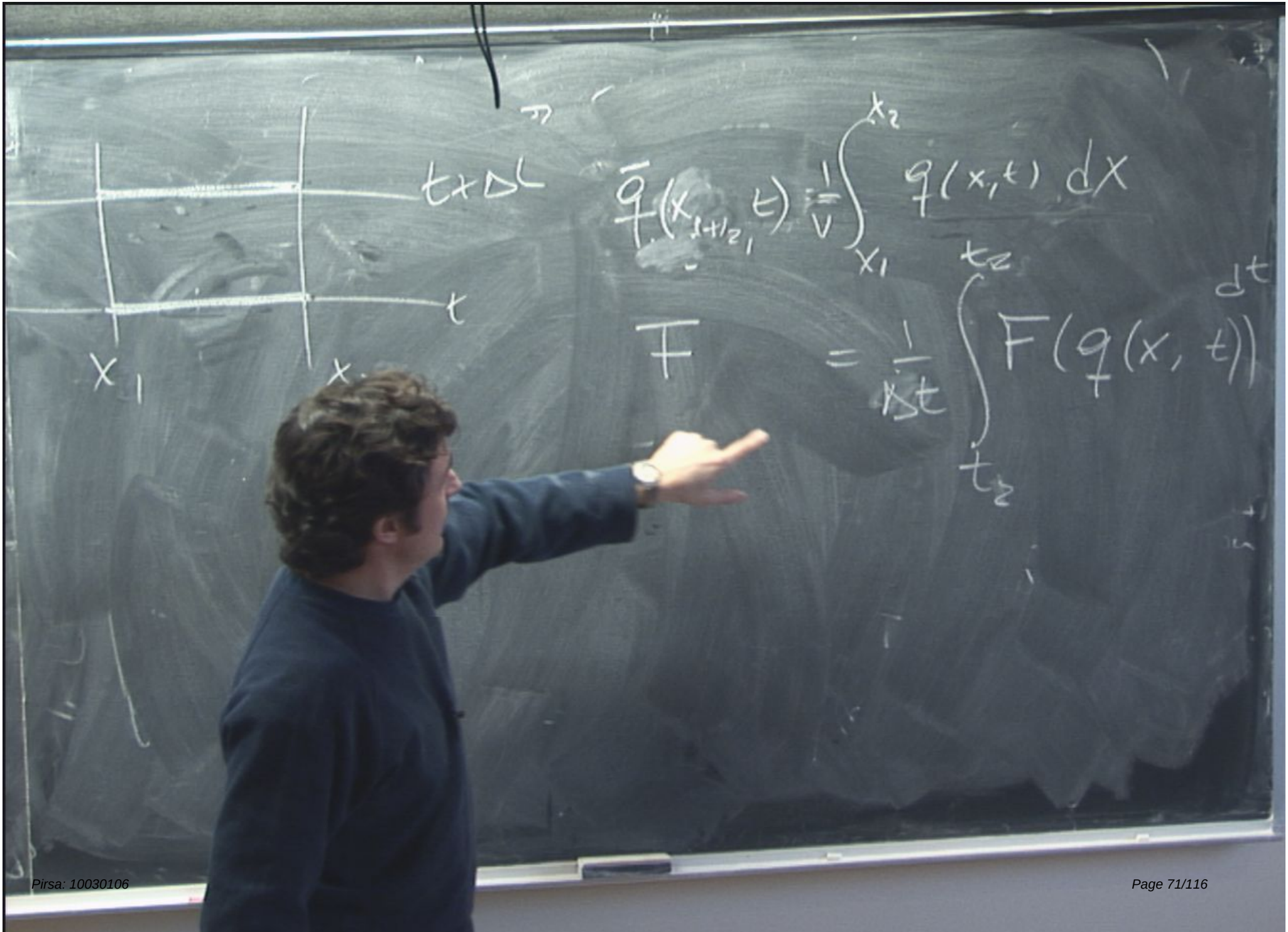

$q_{,t} + F(q)_{,x} = \psi(x,t)$
$\frac{1}{V}\int_{x_1}^{x_2} dx \; q_{,t} + \frac{1}{V}\int_{x_1}^{x_2} dx \frac{\partial F}{\partial x} = \frac{1}{V}\int \psi \, dx \, dx$

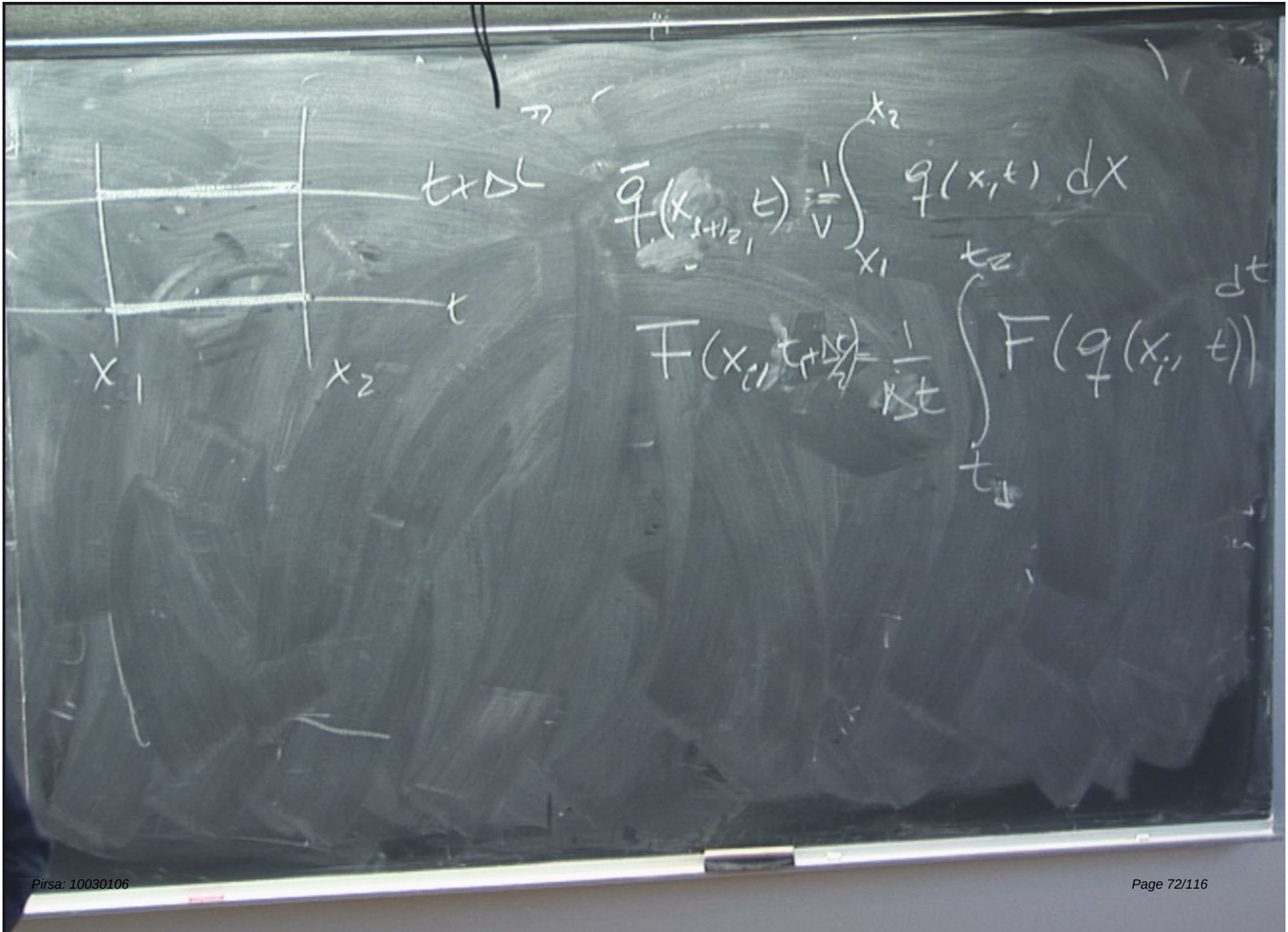
$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

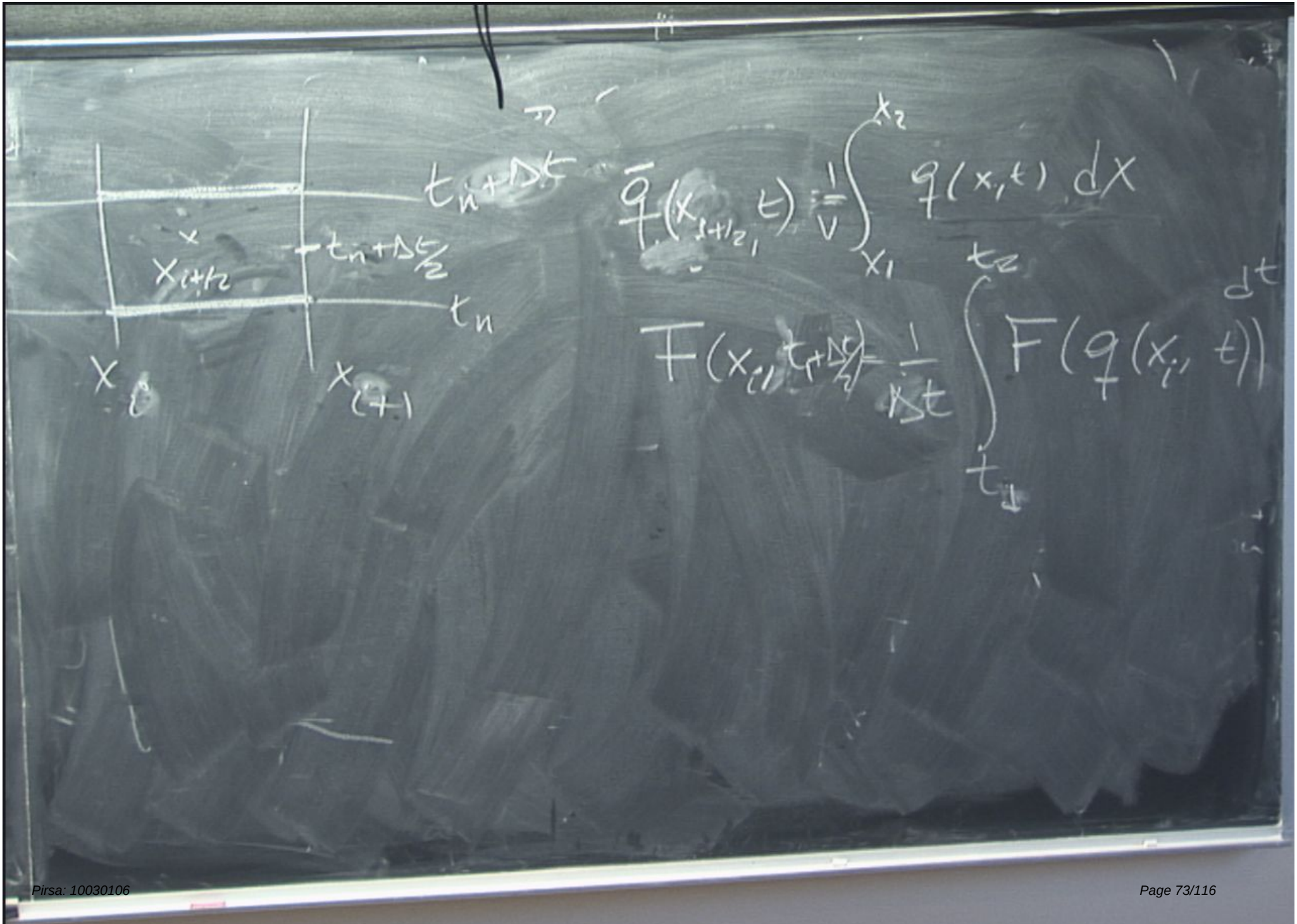
$$t_2 = t + \Delta$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$


t+Δt
t
x1
x2
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V}\int_{x_1}^{x_2} q(x,t)\,dx$
$\bar{F} = \frac{1}{\Delta t}\int_{t_2}^{t_2} F(q(x,t))\,dt$


t+Δt
t
x_1
x_2
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V}\int_{x_1}^{x_2} q(x,t)\,dx$
$F = \frac{1}{\Delta t}\int_{t_2}^{t_2} F(q(x,t))\,dt$

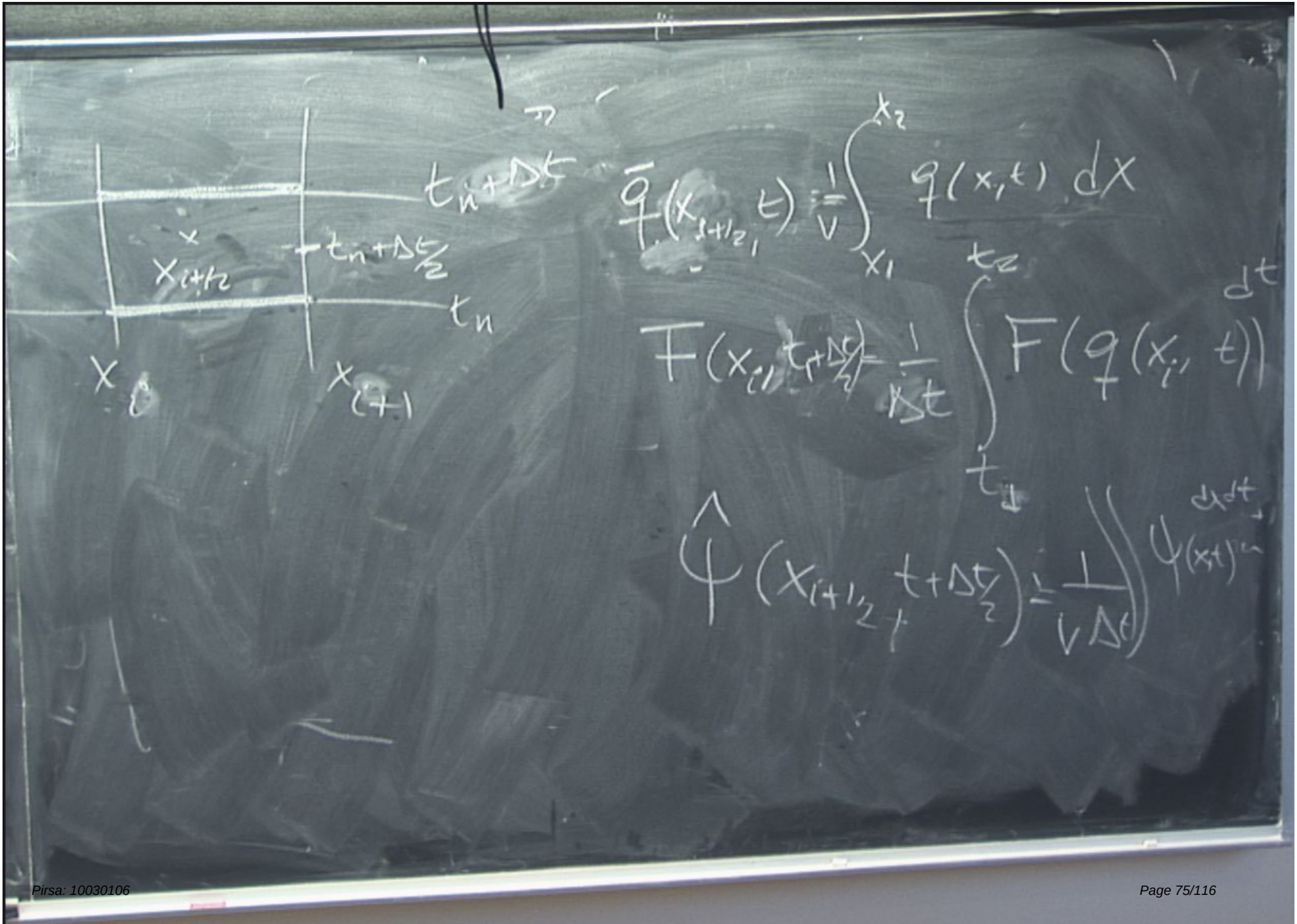

t+Δt
t
x1
x2
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V}\int_{x_1}^{x_2} q(x,t)\,dx$
$F(x_i, t+\frac{\Delta t}{2}) = \frac{1}{\Delta t}\int_{t_1}^{t_2} F(q(x_i, t))\,dt$


$t_n + \Delta t$
$t_n + \Delta t/2$
t_n
$x_{i+1/2}$
x_i
$x_{(i+1)}$
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V} \int_{x_1}^{x_2} q(x,t)\, dx$
$F(x_i, t_n + \frac{\Delta t}{2}) = \frac{1}{\Delta t} \int_{t_1}^{t_2} F(q(x_i, t))\, dt$

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$

$$\frac{\bar{q}^{n+1} - \bar{q}^{n}}{\Delta t} + \frac{F_i^{n+1/2} - F_{i+1}^{n}}{\Delta x} =$$


$t_n + \Delta t$
$t_n + \Delta t/2$
t_n
$x_{i+1/2}$
x_i
x_{i+1}
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V} \int_{x_1}^{x_2} q(x,t)\, dx$
$F(x_i, t + \Delta t/2) = \frac{1}{\Delta t} \int_{t_1}^{t_2} F(q(x_i, t))\, dt$
$\hat{\psi}(x_{i+1/2}, t + \Delta t/2) = \frac{1}{V \Delta t} \iint \psi(x,t)\, dx\, dt$

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$t_2 = t + \Delta$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$

$$\frac{\bar{q}^{n+1} - \bar{q}^{n}}{\Delta t} + \frac{F_i^{n+1/2} - F_{i+1}^{n+1/2}}{\Delta x} = \bar{\psi}$$

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$

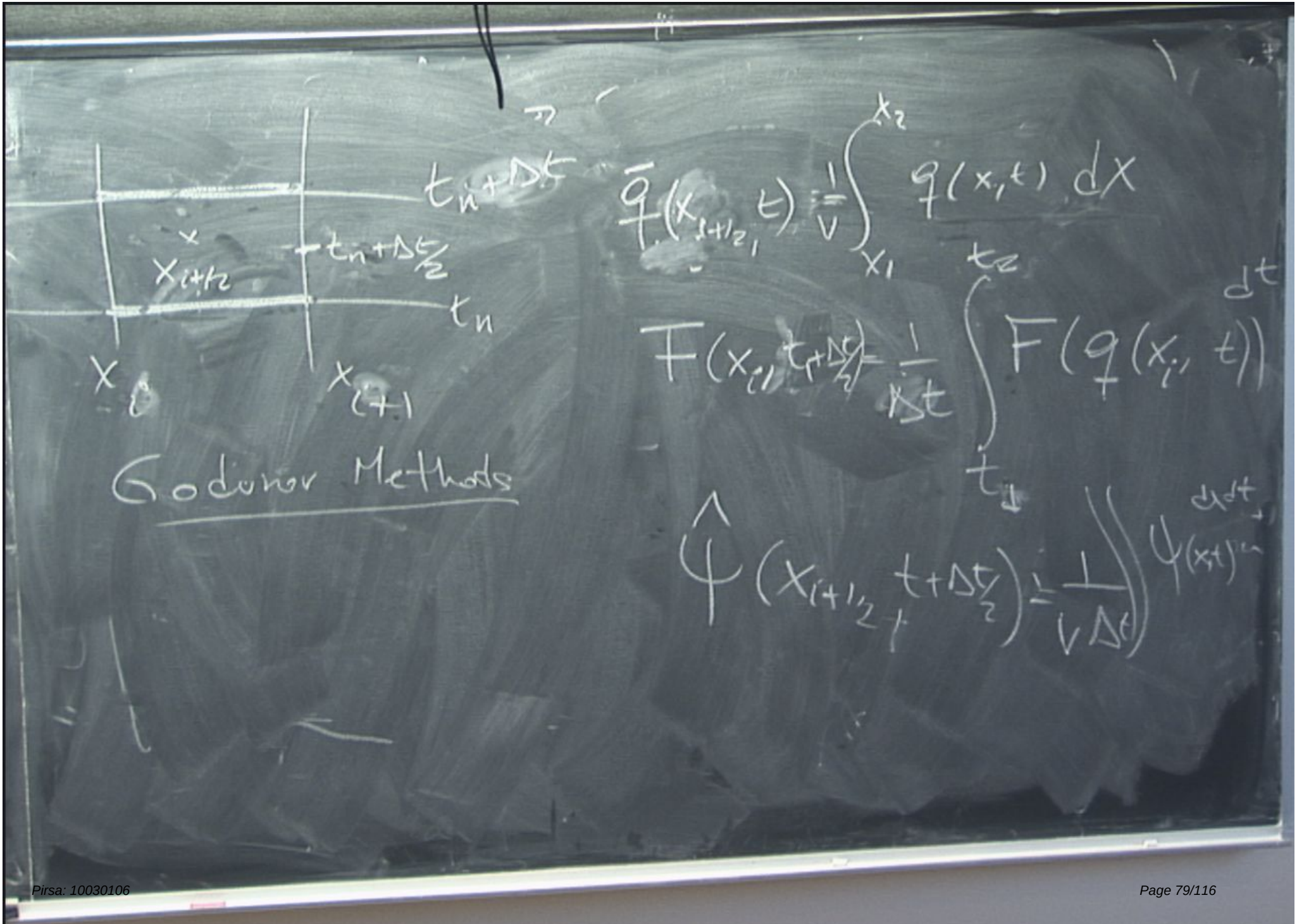
$$\frac{\bar{q}^{n+1} - \bar{q}^{n}}{\Delta t} + \frac{F_i^{n+1/2} - F_{i+1}^{n+1/2}}{\Delta x} = \bar{\psi}$$

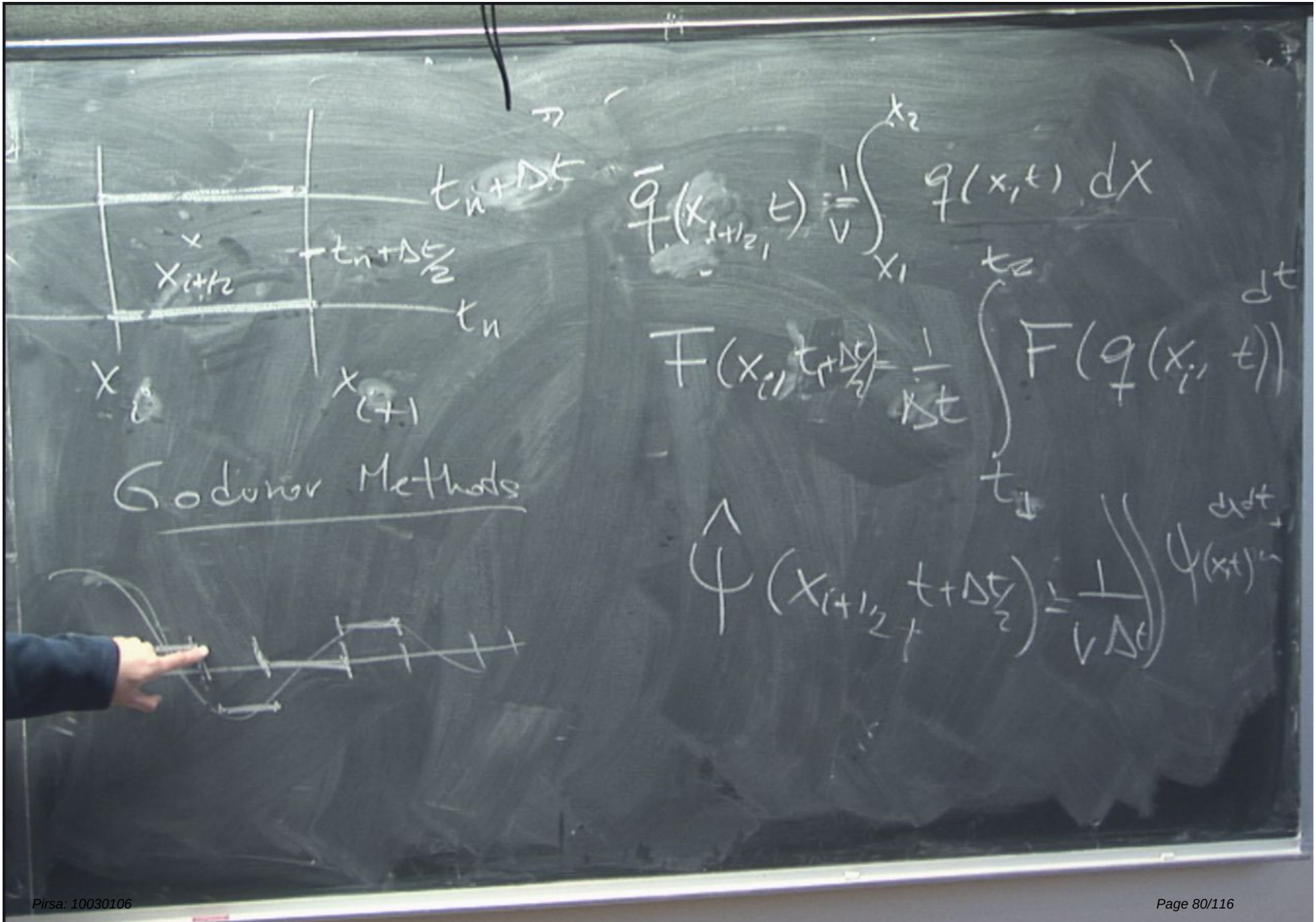
$t_2 = t + \Delta$

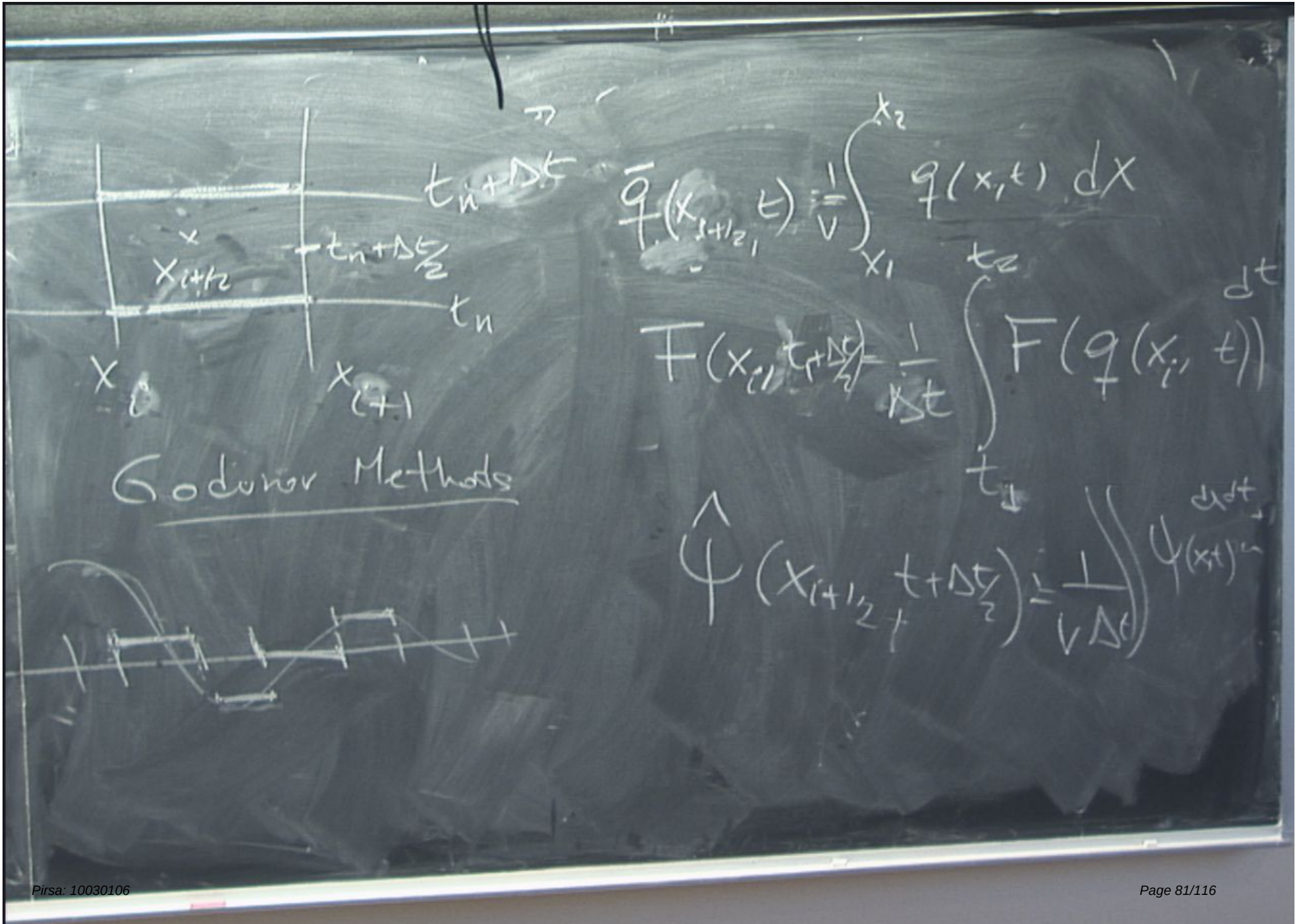
$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$

$$\frac{\bar{q}^{n+1} - \bar{q}^{n}}{\Delta t} + \frac{F_i^{n+1/2} - F_{i+1}^{n+1/2}}{\Delta x} = \bar{\psi}$$


$t_n + \Delta t$
$t_n + \Delta t/2$
t_n
$x_{i+1/2}$
x_i
x_{i+1}
Godunov Methods
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V} \int_{x_1}^{x_2} q(x,t)\, dx$
$F(x_i, t+\Delta t/2) = \frac{1}{\Delta t} \int_{t_1}^{t_2} F(q(x_i, t))\, dt$
$\hat{\psi}(x_{i+1/2}, t+\Delta t/2) = \frac{1}{V \Delta t} \iint \psi(x,t)\, dx\, dt$


$t_n + \Delta t$
$t_n + \Delta t/2$
t_n
$x_{i+1/2}$
x_i
x_{i+1}
Godunov Methods
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V} \int_{x_1}^{x_2} q(x,t)\, dx$
$F(x_i, t+\frac{\Delta t}{2}) = \frac{1}{\Delta t} \int_{t_1}^{t_2} F(q(x_i, t))\, dt$
$\hat{\psi}(x_{i+1/2}, t+\frac{\Delta t}{2}) = \frac{1}{V \Delta t} \iint \psi(x,t)\, dx\, dt$


$t_n + \Delta t$
$t_n + \Delta t/2$
t_n
$x_{i+1/2}$
x_i
x_{i+1}
Godunov Methods
$\bar{q}(x_{i+1/2}, t) = \frac{1}{V}\int_{x_1}^{x_2} q(x,t)\,dx$
$F(x_i, t+\frac{\Delta t}{2}) = \frac{1}{\Delta t}\int_{t_1}^{t_2} F(q(x_i,t))\,dt$
$\hat{\psi}(x_{i+1/2}, t+\frac{\Delta t}{2}) = \frac{1}{V\Delta t}\iint \psi(x,t)\,dx\,dt$

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$

$$\frac{\bar{q}^{n+1} - \bar{q}^{n}}{\Delta t} + \frac{F_{i+1}^{n+1/2} - F_{i}^{n+1/2}}{\Delta x} =$$

$$q_{,t} + F(q)_{,x} = \psi(x,t)$$

$$\frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} q_{,t}\,dx\,dt + \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \frac{\partial F}{\partial x}\,dx\,dt = \frac{1}{V}\int_{t_1}^{t_2}\int_{x_1}^{x_2} \psi(x,t)\,dx\,dt$$

$$\frac{\bar{q}^{n+1} - \bar{q}^{n}}{\Delta t} + \frac{F_{i+1}^{n+1/2} - F_{i}^{n+1/2}}{\Delta x} = \cdots$$

$\psi(x,t)$

$\psi(x')dx'$

$F^{n+1/2}$

Δx

Roe Solver

$$\frac{\partial}{\partial t} q + f(q) = 0$$

$\psi(x,t)$

$\psi(x')dx'$

$F^{n+1/2}$

Δx

Roe Solver

$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = 0$$

ver

$\frac{\partial}{\partial x} f(q) = 0$

→ Burgers eqn $f(q) = \frac{1}{2} q^2$

q: can be a vector

Roe Solver

$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = 0$$

→ Burgers eqn

q: can be a

→ Linearize the problem

Roe Solver

$$\frac{\partial}{\partial t} q + \frac{\partial}{\partial x} f(q) = 0$$

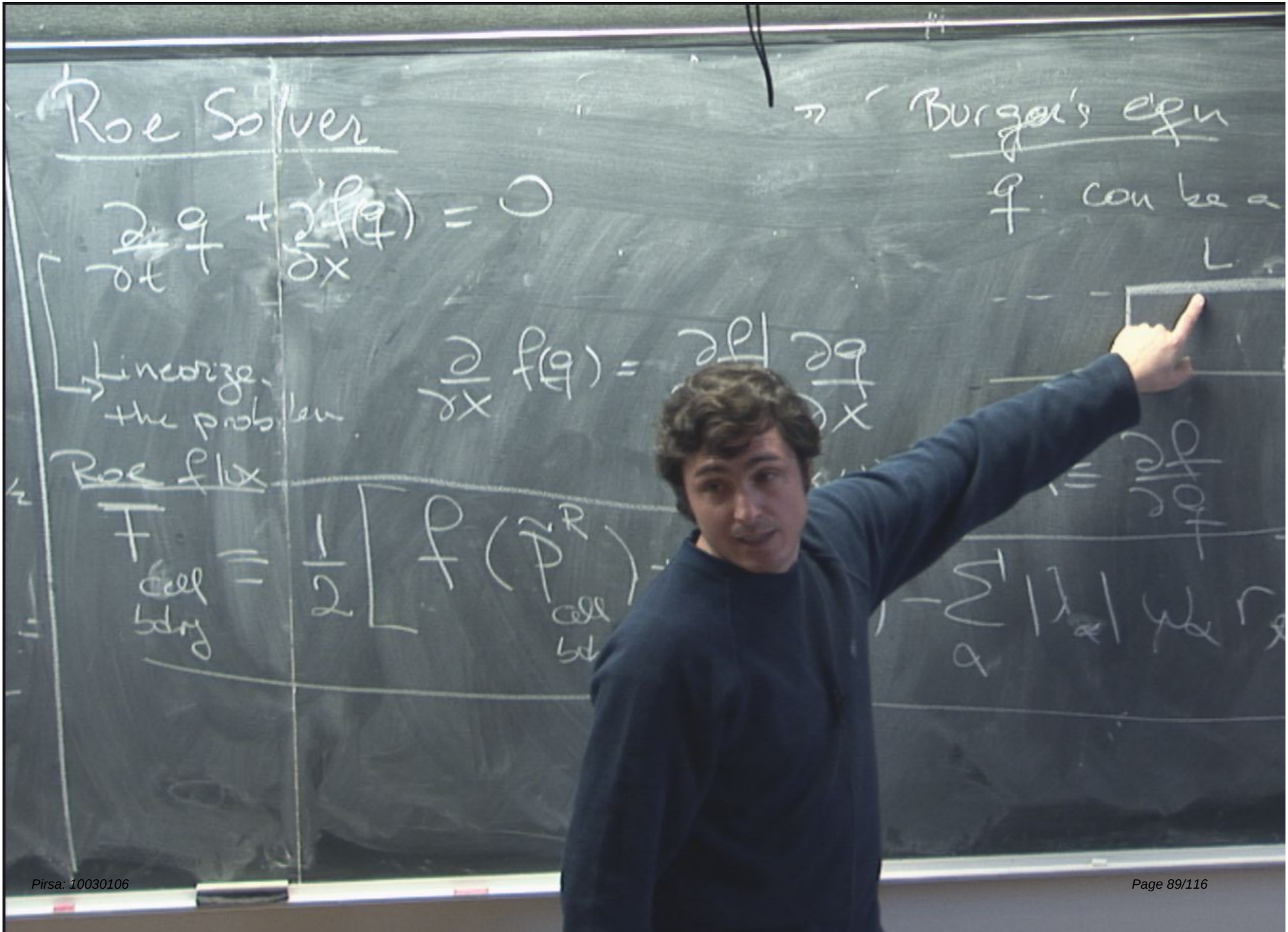
Linearize the problem

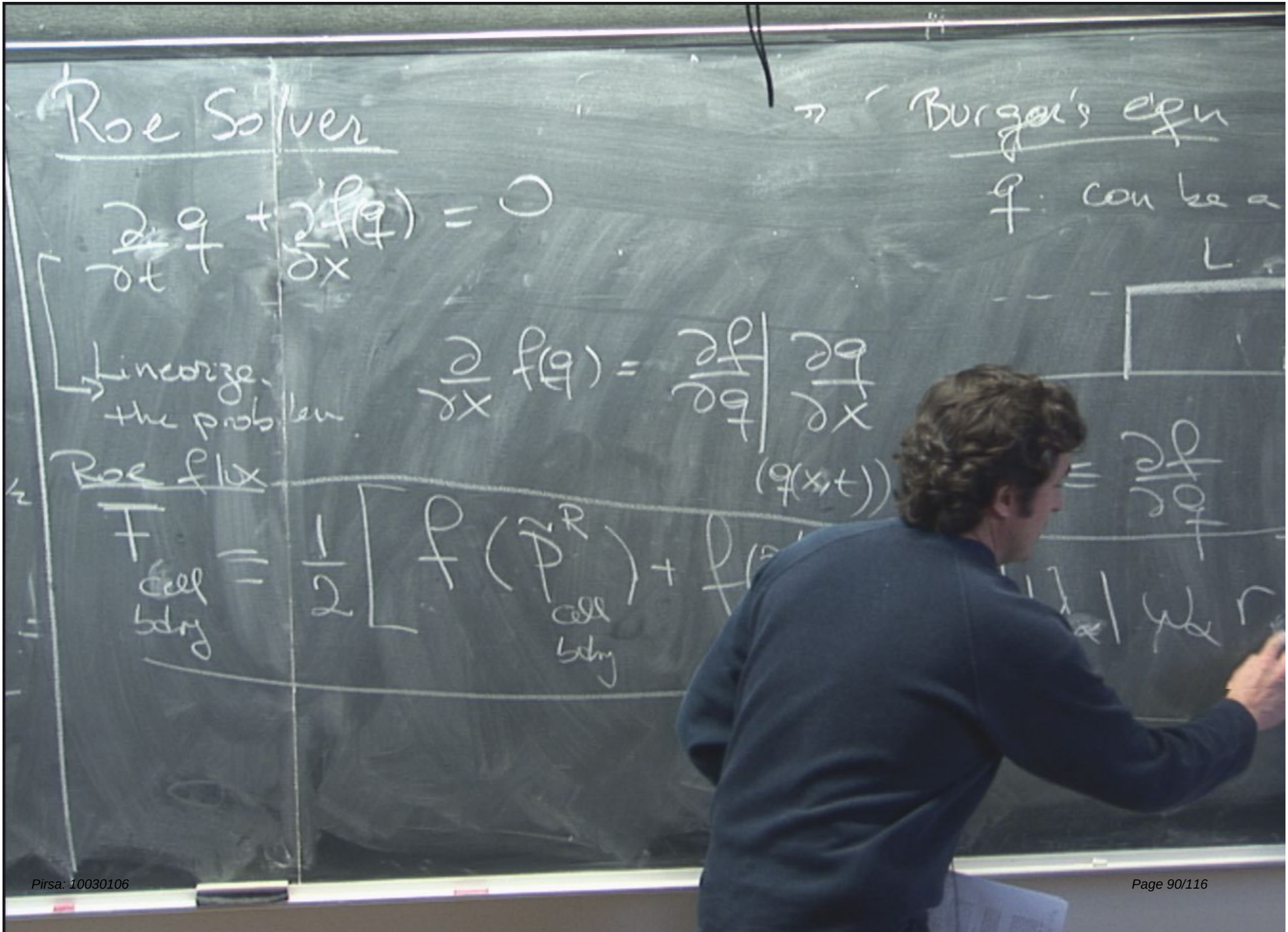
$$\frac{\partial}{\partial x} f(q) = \left.\frac{\partial f}{\partial q}\right|_{(q(x,t))} \frac{\partial q}{\partial x}$$

$\Rightarrow$ Burger's eqn

q: can be a

$$A \equiv \frac{\partial f}{\partial q}$$


Roe Solver
Burger's eqn
q: can be a
Linearize the problem
Roe flux
cell bdry

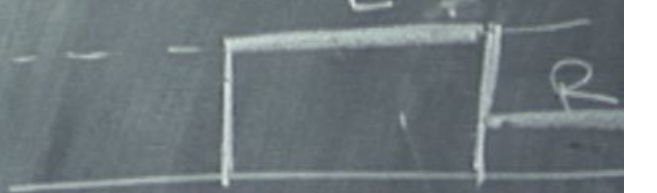

Roe Solver
Burger's eqn
q: can be a
Linearize the problem
Roe flux
cell bdry
cell bdry

oe Solver

$$\frac{\partial}{\partial t} q + \frac{\partial}{\partial x} f(q) = 0$$

→ Burgers eqn $f(q)$

q: can be a vec



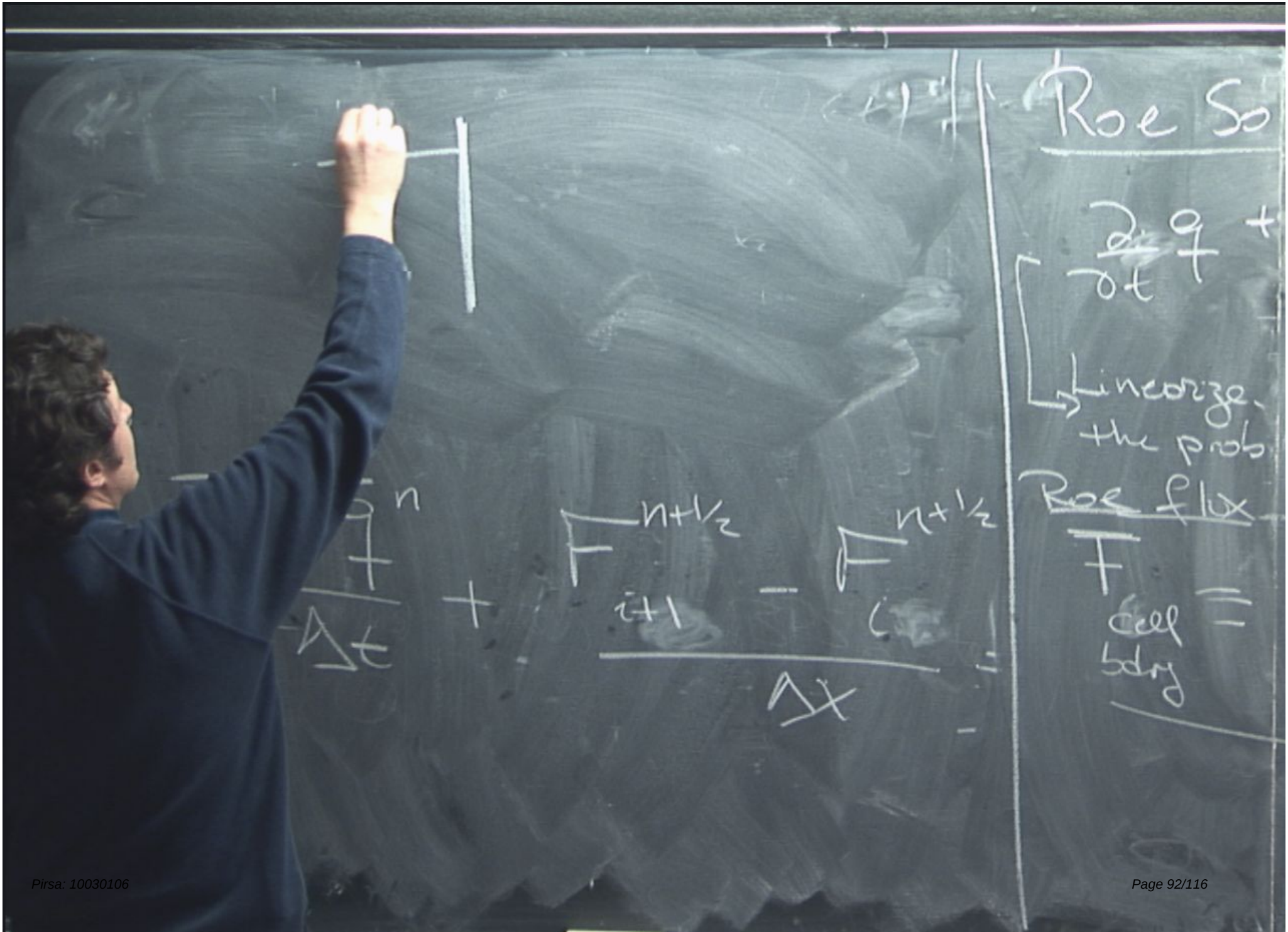
inearize.
the prob.lem

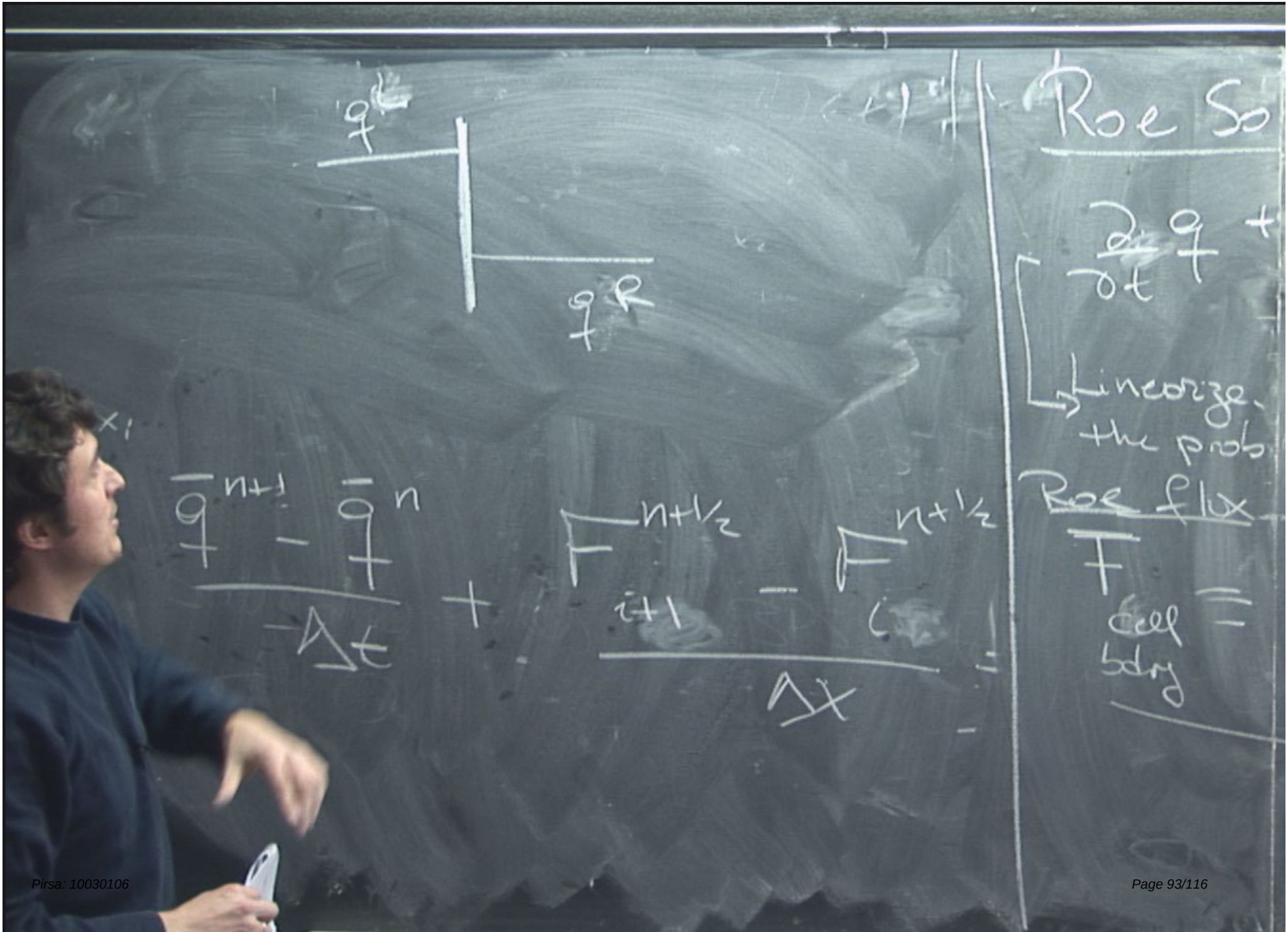
$$\frac{\partial}{\partial x} f(q) = \frac{\partial f}{\partial q}\bigg|_{(q(x,t))} \frac{\partial q}{\partial x}$$

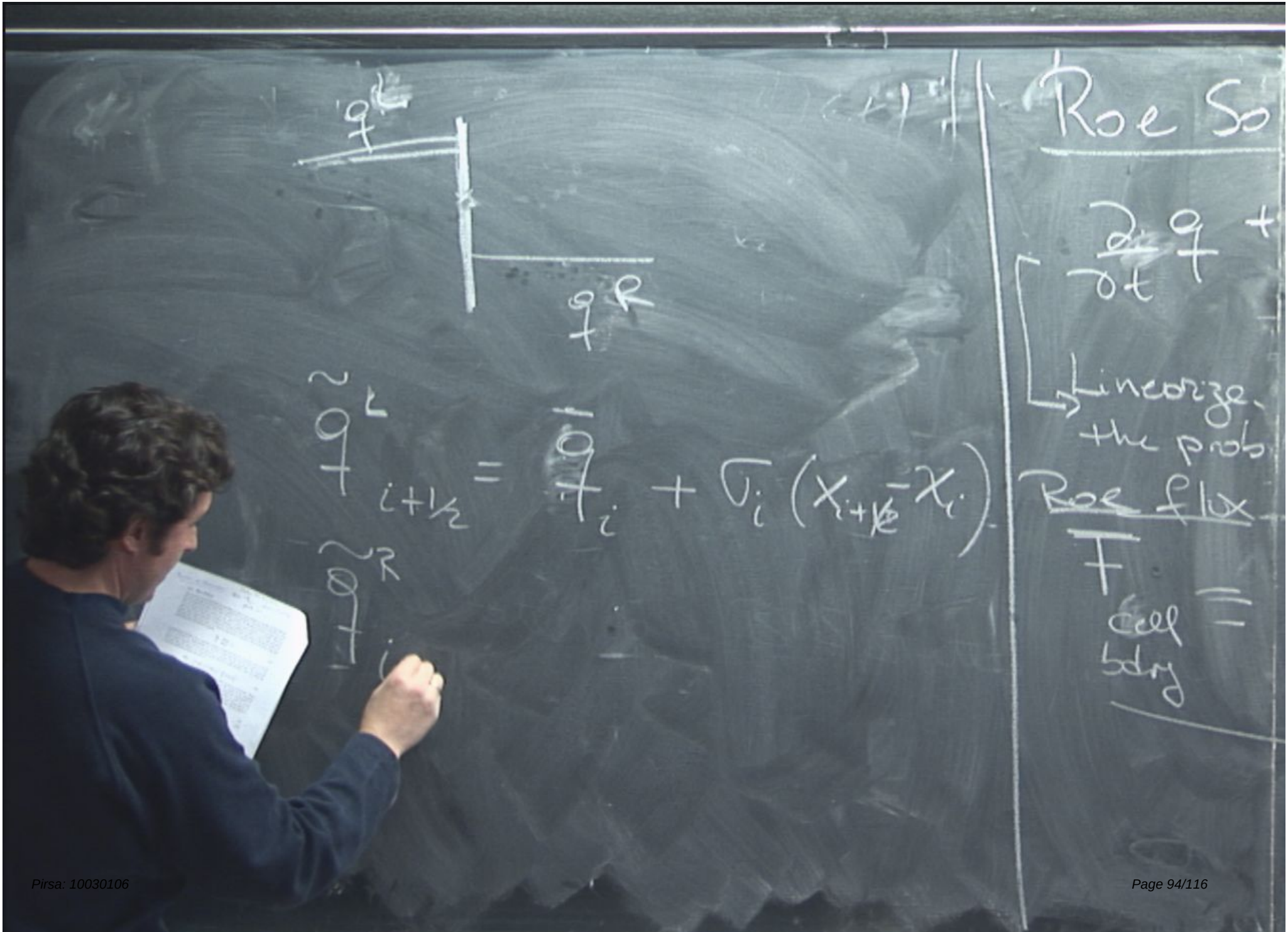
$$A \equiv \frac{\partial f}{\partial q}$$

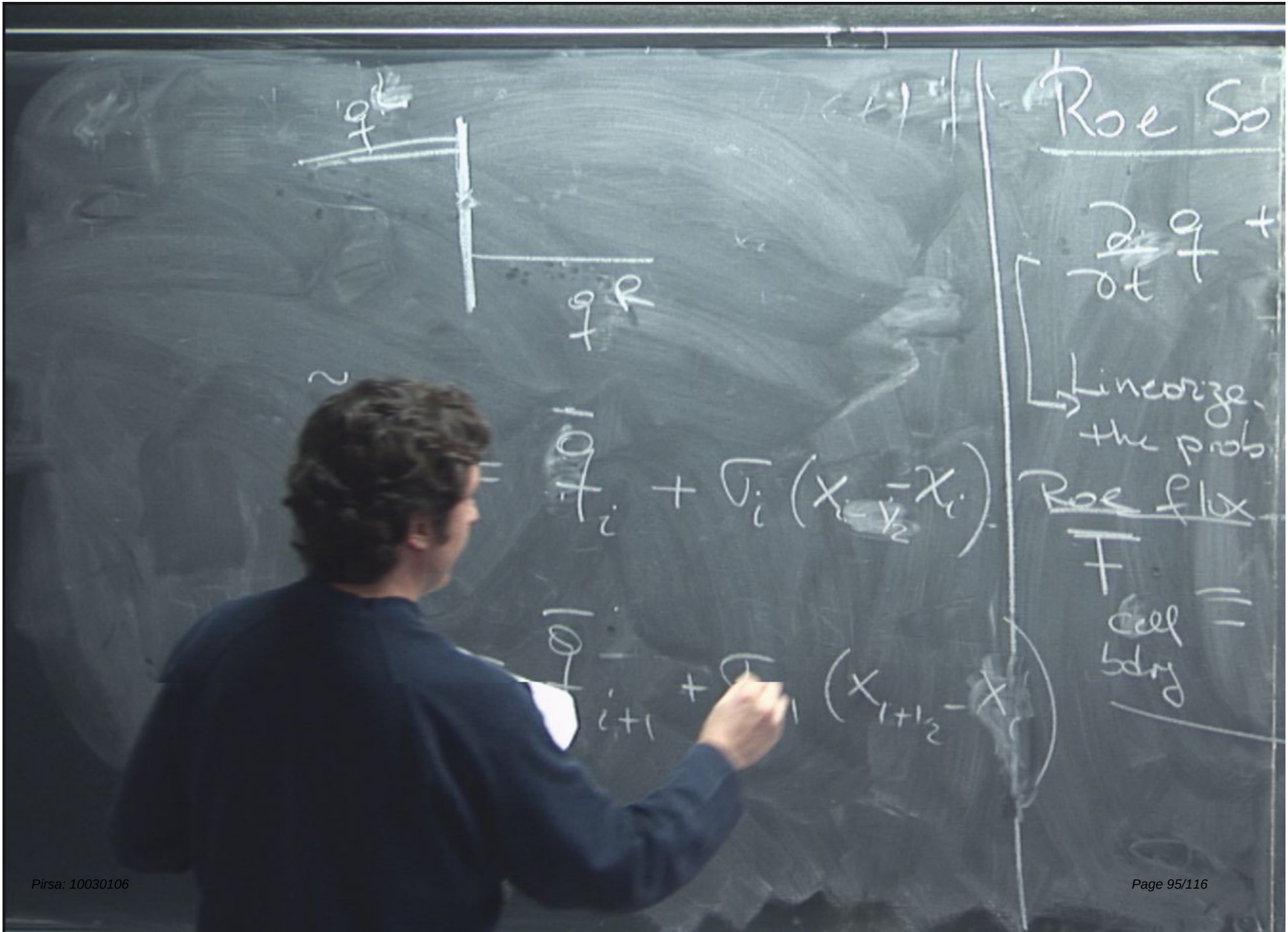
oe flux

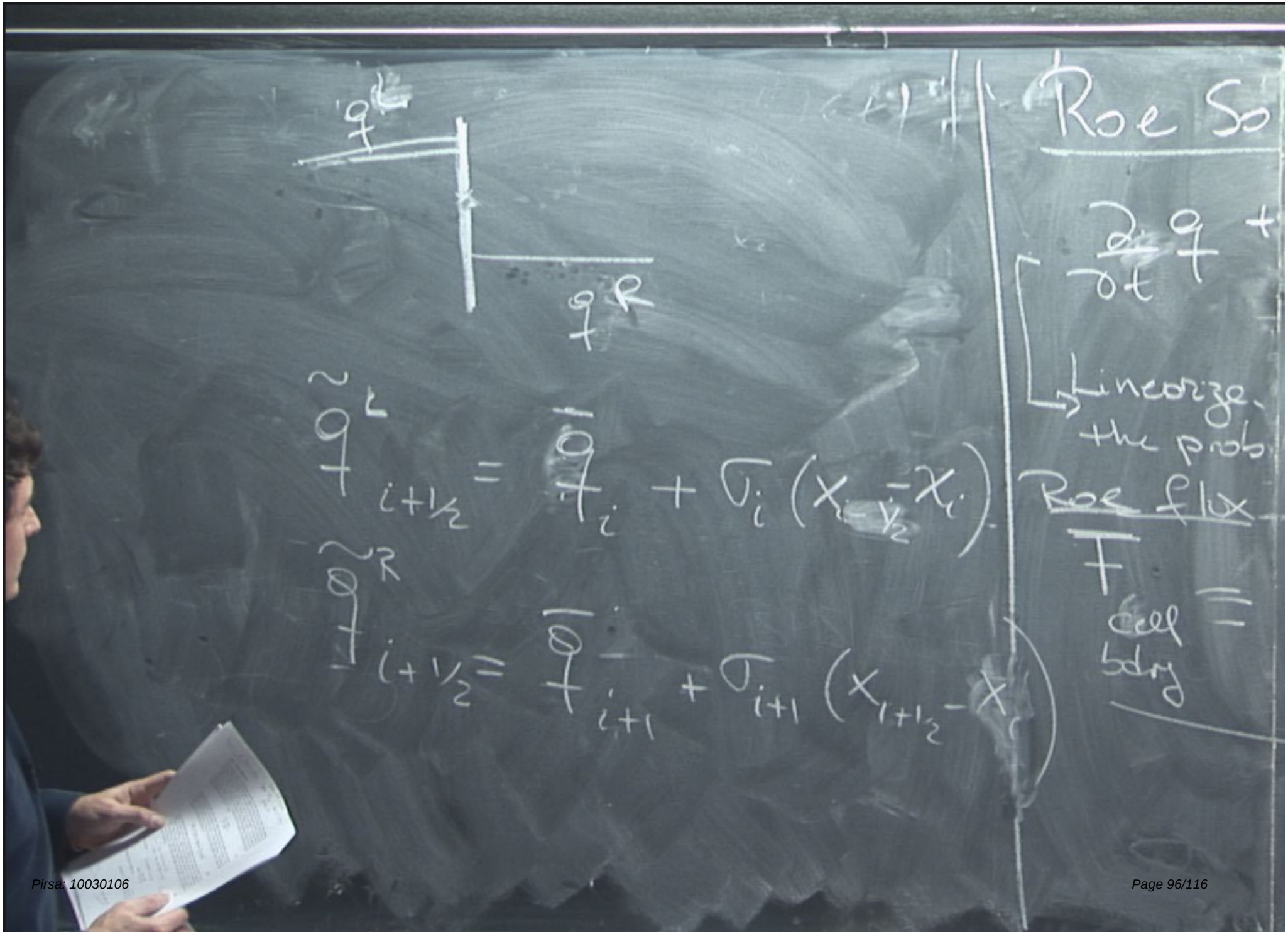
$$F_{\substack{\text{cell} \\ \text{bdry}}} = \frac{1}{2}\left[f\left(\tilde{P}^{R}_{\substack{\text{cell} \\ \text{bdry}}}\right) + f\left(\tilde{P}^{L}_{\text{cell}}\right) - \sum_{\alpha} |\lambda_\alpha| \, w_\alpha r_\alpha \right]$$

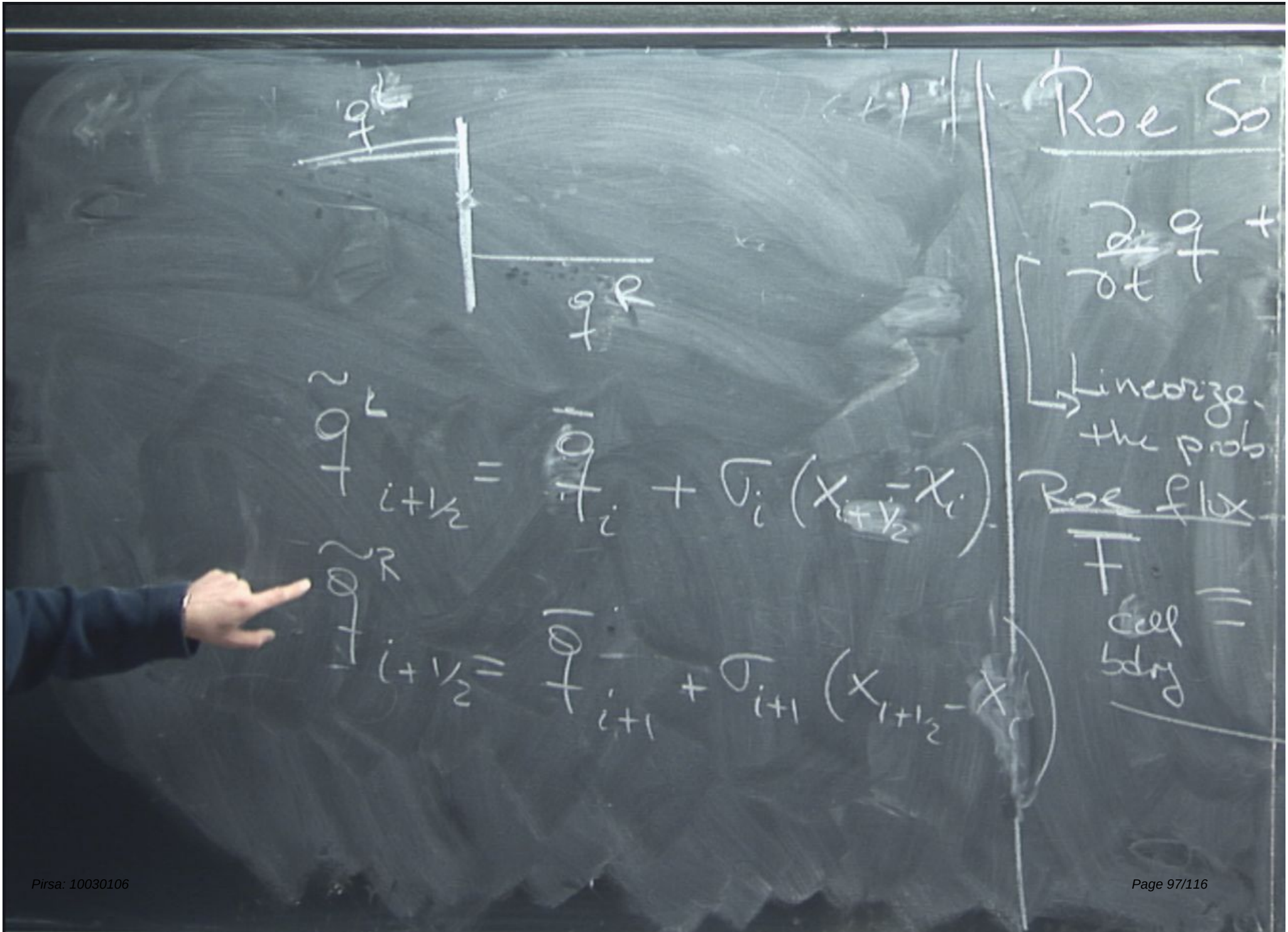

Roe So
Linearize the prob
Roe flux
cell
bdry


Roe So
Linearize the prob
Roe flux
cell
bdry


Roe So
Linearize
the prob
Roe flux
cell
bdry

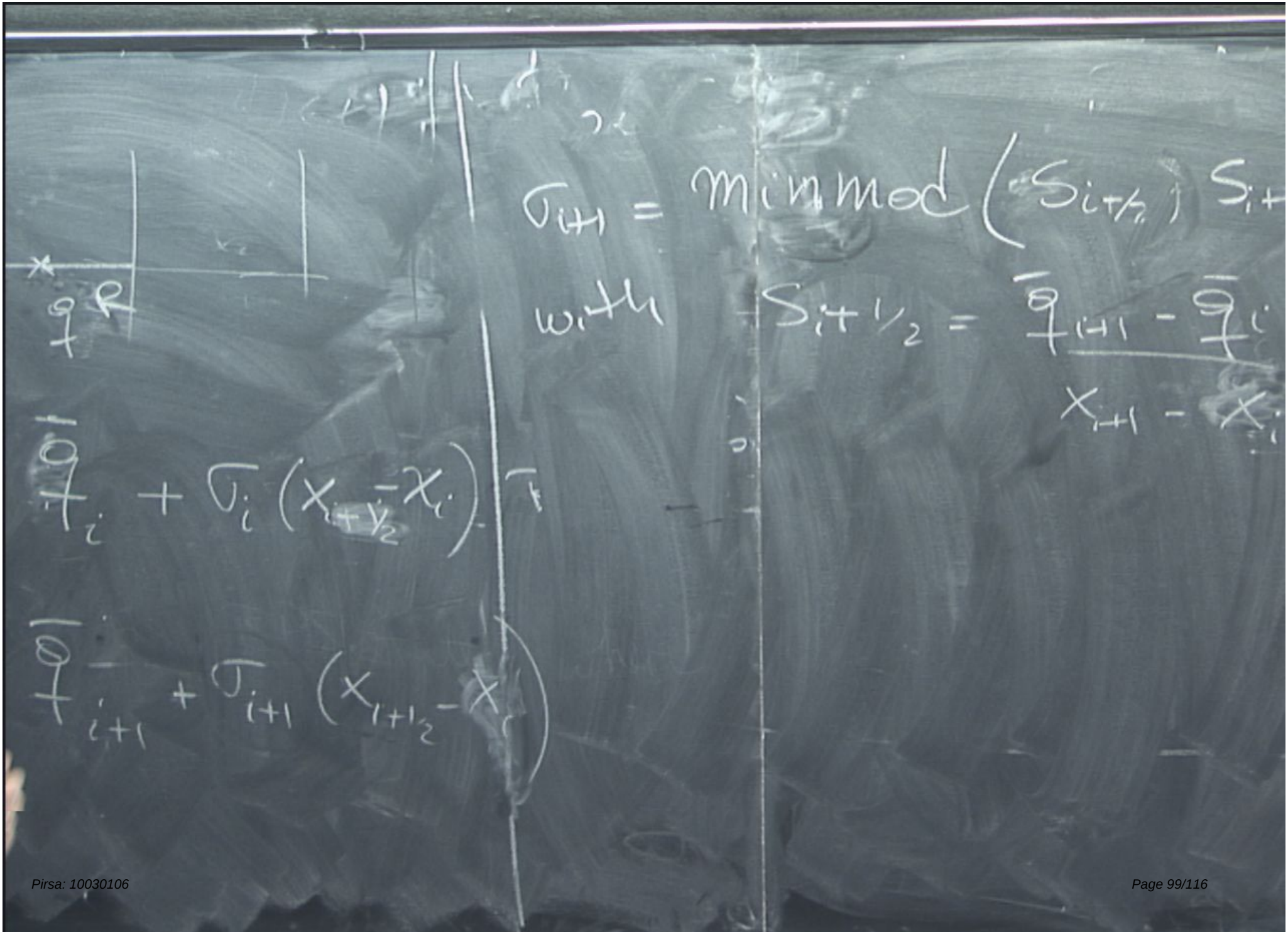

Roe So
Linearize the prob
Roe flux
cell bdry

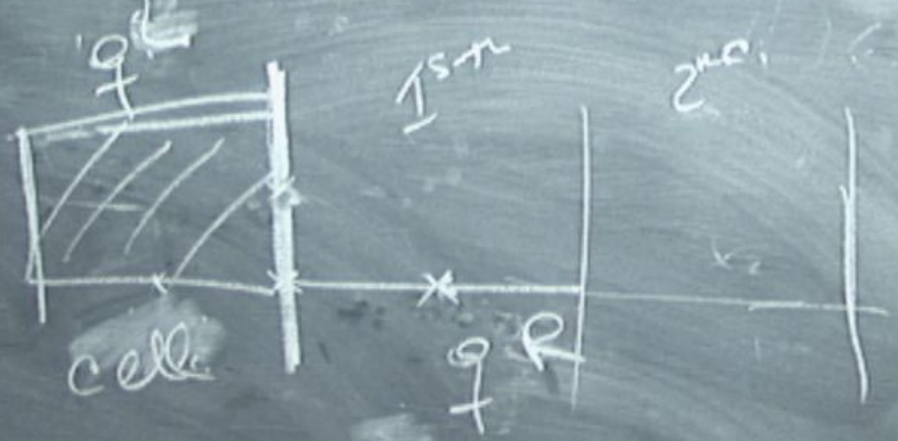

Roe So
Linearize the prob
Roe flux
cell
bdry


Roe So
Linearize the prob
Roe flux
cell bdry

$$q^L_{i+1/2} = q_i + \sigma_i \left(x_{i+1/2} - x_i \right)$$

$$q^R_{i+1/2} = q_{i+1} + \sigma_{i+1} \left(x_{i+1/2} - x_{i+1} \right)$$


$\sigma_{i+1} = \text{minmod}(s_{i+1/2}, s_{i+$
with $s_{i+1/2} = \frac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$
$\bar{q}_i + \sigma_i (x_{i+1/2} - x_i)$
$\bar{q}_{i+1} + \sigma_{i+1} (x_{i+1/2} - x_i)$

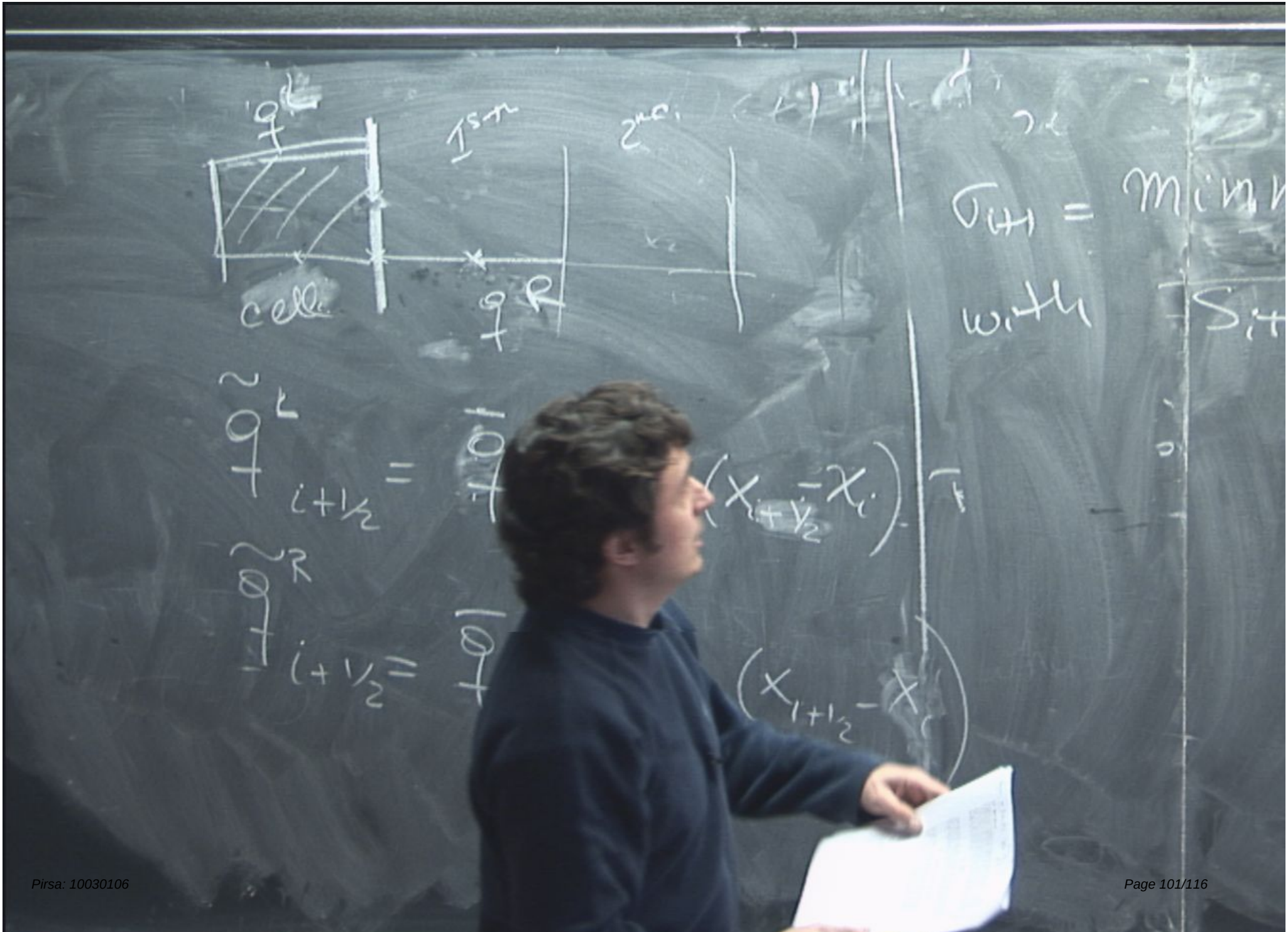


$\sigma_{i+1} = \min$

with S_{i+

$$\tilde{q}^{L}_{i+1/2} = \bar{q}_i + \sigma_i \left(x_{i+1/2} - x_i \right)$$

$$\tilde{q}^{R}_{i+1/2} = \bar{q}_{i+1} + \sigma_{i+1} \left(x_{i+1/2} - x_{i+1} \right)$$


1st
2nd
cell
with

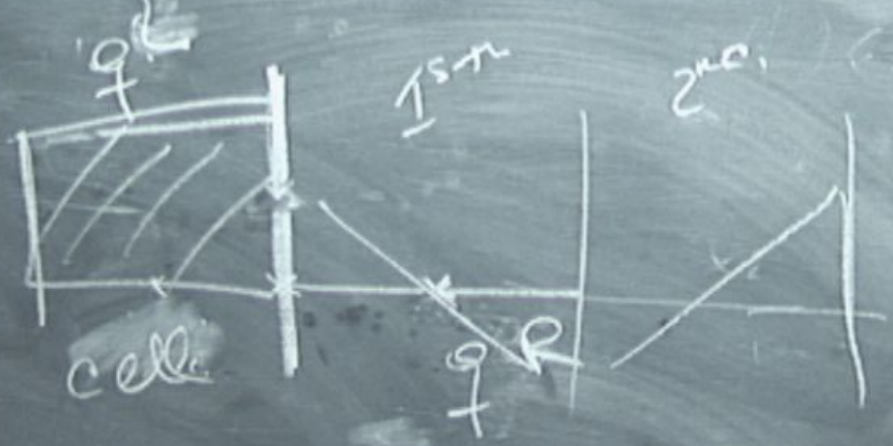
$$\sigma_{i+1} = \text{minmod}\left(s_{i+1/2}, s_{i+3/2}\right)$$

with

$$s_{i+1/2} = \frac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$$

$$\sigma_i \left(x_{i+1/2} - x_i\right)$$

$$\sigma_{i+1} \left(x_{i+1/2} - x_i\right)$$



$$\sigma_{i+1} = m$$

with

$$\tilde{q}^L_{i+1/2} = \bar{q}_i + \sigma_i \left(x_{i+1/2} - x_i \right)$$

$$\tilde{q}^R_{i+1/2} = \bar{q}_{i+1} + \sigma_{i+1} \left(x_{i+1/2} - x_{i+1} \right)$$

Burger's eqn

$$\sigma_{i+1} = \text{minmod}\left(s_{i+1/2}, s_{i+3/2}\right)$$

f: con

L

with $s_{i+1/2} = \dfrac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$

$$\text{minmod}(a,b) = \begin{cases} 0 : f \end{cases}$$

Burger's eqn

$\sigma_{i+1} = \text{minmod}\left(s_{i+1/2}, s_{i+3/2}\right)$

f: con

L

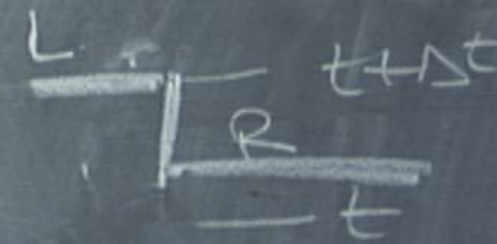
with $s_{i+1/2} = \dfrac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$

$\text{minmod}(a,b) = \begin{cases} 0 : \text{if} \end{cases}$

$\text{minmod}(S_{i+1/2}, S_{i+3/2})$

Burger's eqn $f(q) = q^2$

q: can be a vector



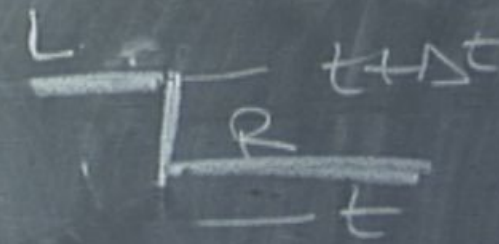
$$S_{i+1/2} = \frac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$$

$$\text{minmod}(a, b) = \begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \text{ if } ab > 0 \\ b & \text{if } |a| > |b| \text{ if } ab > 0 \end{cases}$$

$\text{minmod}(S_{i+1/2}, S_{i+3/2})$

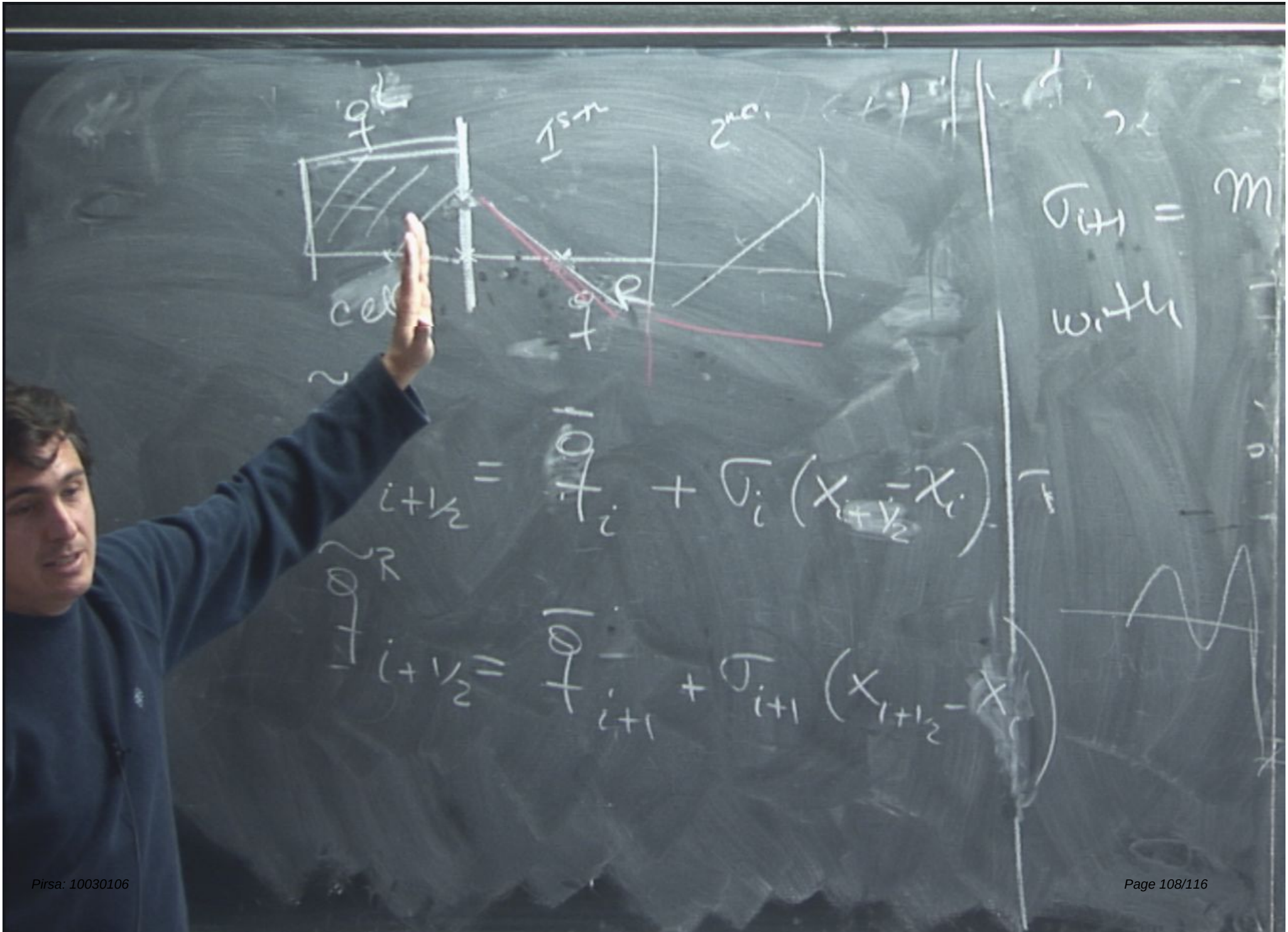
Burger's eqn $f(q) = q^2$

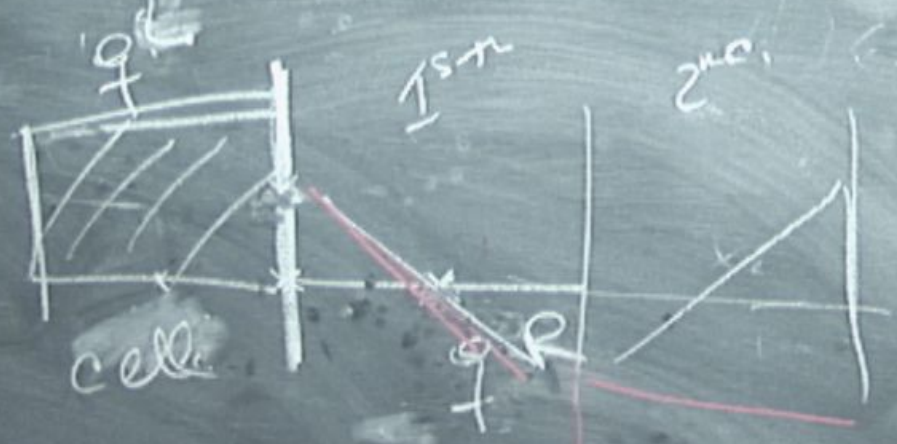
q: can be a vector



$$S_{i+1/2} = \frac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$$

$$\text{minmod}(a, b) = \begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \text{ if } ab > 0 \\ b & \text{if } |a| > |b| \text{ if } ab > 0 \end{cases}$$


1st
2nd
cell
$\sigma_{i+1} = m$
with
$\tilde{q}_{i+1/2} = \bar{q}_i + \sigma_i (x_{i+1/2} - x_i)$
$\tilde{q}_{i+1/2} = \bar{q}_{i+1} + \sigma_{i+1} (x_{i+1/2} - x_{i+1})$



$\sigma_{i+1} = m$

with

$$\tilde{q}^L_{i+1/2} = \bar{q}_i + \sigma_i \left(x_{i+1/2} - x_i \right)$$

$$\tilde{q}^R_{i+1/2} = \bar{q}_{i+1} + \sigma_{i+1} \left(x_{i+1/2} - x_i \right)$$

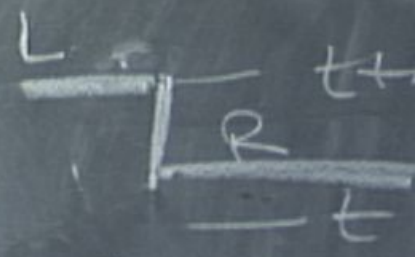
$$= \text{minmod}\left(S_{i+1/2}, S_{i+3/2}\right)$$

$$S_{i+1/2} = \frac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$$

$$\partial_t (\quad = \begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \\ b & \text{if } |a| > |b| \end{cases}$$

Burger's eqn $f(q) = q$

q: can be a vector



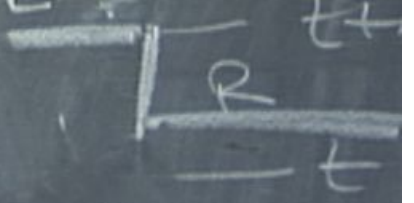
minmod $\left(s_{i+1/2}, s_{i+3/2}\right)$

$s_{i+1/2} = \dfrac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$

$\partial_t(q) + \partial_x(fu) = 0$

Burgers eqn $f(q) = q$

q: can be a vector



0 : if $ab < 0$

a : if $|a| < |b|$

b : if $|a| > |b|$

$= \text{minmod}\left(s_{i+1/2}, s_{i+3/2}\right)$

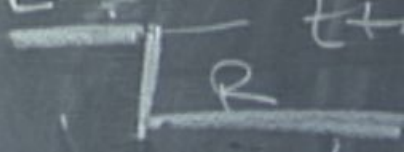
$s_{i+1/2} = \dfrac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$

$\partial_t(\rho) + \partial_x(\rho u) = 0$

$\partial_t(\rho u) + \partial_x(\quad) = 0$

Burgers' eqn $f(q) =$

q: can be a vector



$$\begin{cases} 0 & \text{if } ab<0 \\ a & \text{if } |a|<|b| \\ b & \text{if } |a|>|b| \end{cases}$$

$= \text{minmod}\left(s_{i+1/2}, s_{i+3/2}\right)$

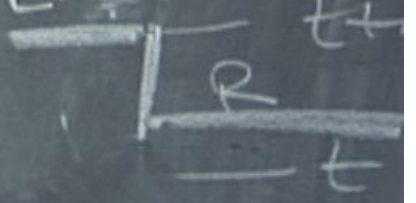
$s_{i+1/2} = \dfrac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$

$\partial_t(q) + \partial_x(fu) = 0$

$\partial_t(qu) + \partial_x(\quad) = 0$

Burgers' eqn $f(q) =$

q: can be a vector



$\begin{cases} 0 & \text{if } ab<0 \\ a & \text{if } |a|<|b| \\ b & \text{if } |a|>|b| \end{cases}$

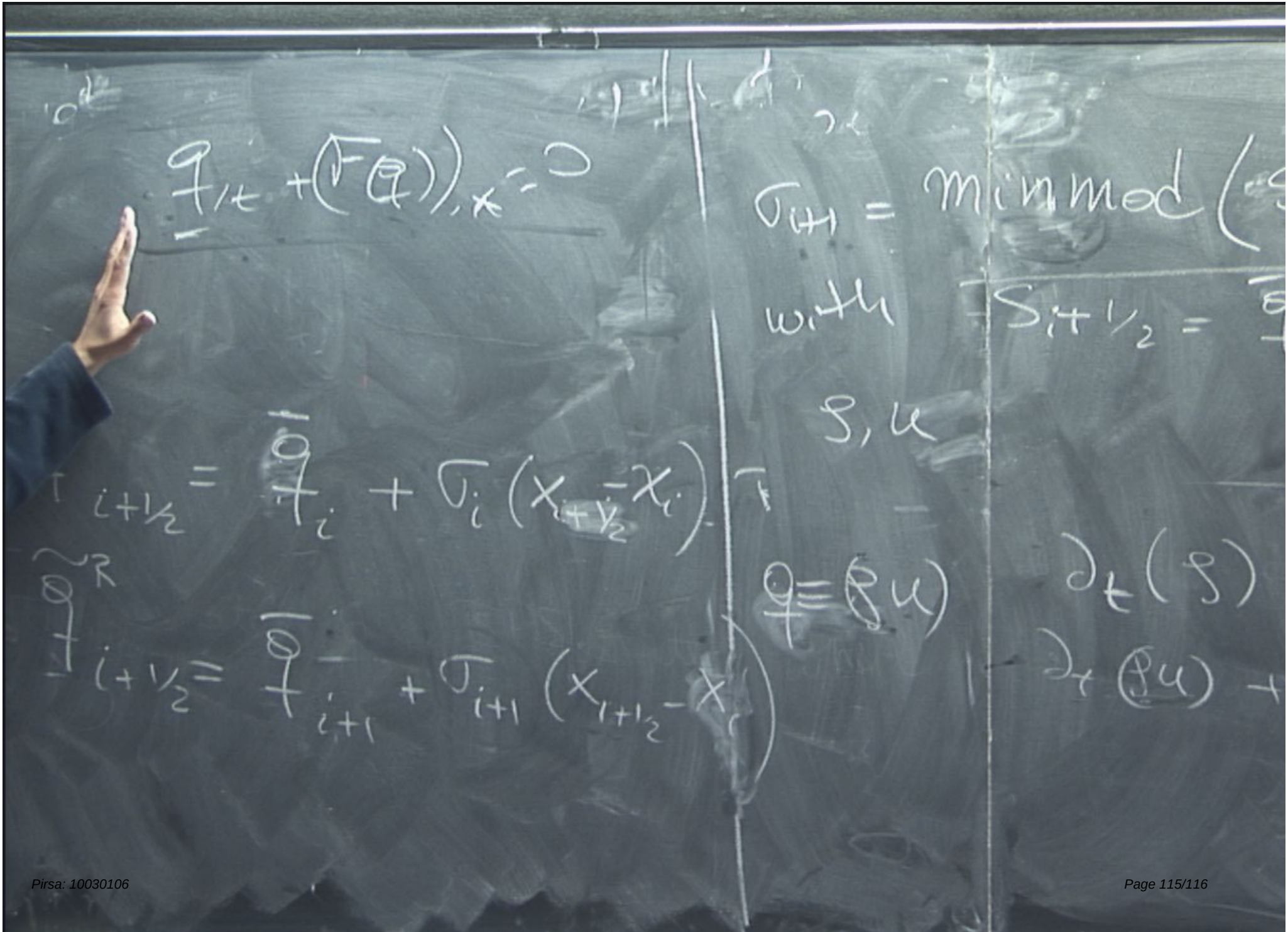
$$\underline{q}_t + (F(q))_x = 0$$

$$\sigma_{i+1} = m$$

with

$$\tilde{q}^L_{i+1/2} = \bar{q}_i + \sigma_i (x_{i+1/2} - x_i)$$

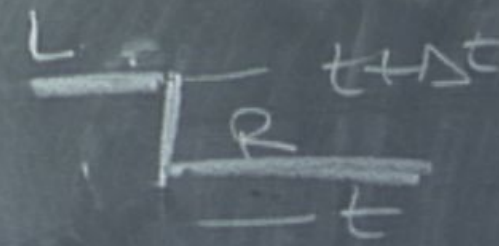
$$\tilde{q}^R_{i+1/2} = \bar{q}_{i+1} + \sigma_{i+1} (x_{i+1/2} - x_i)$$


$\bar{q}_t + (\bar{F}(q))_x = 0$
$\sigma_{i+1} = \text{minmod}($
with
$s_{i+1/2} =$
q, u
$\bar{q}_{i+1/2} = \bar{q}_i + \sigma_i (x_{i+1/2} - x_i)$
$q \equiv (\rho u)$
$\partial_t(\rho)$
$\bar{q}_{i+1/2} = \bar{q}_{i+1} + \sigma_{i+1} (x_{i+1/2} - x_i)$
$\partial_t(\rho u) +$

$\Rightarrow$ Burger's eqn $f(q) = q^2$

$\text{inmod}\left(S_{i+1/2}, S_{i+3/2}\right)$

q: can be a vector



$$S_{i+1/2} = \frac{\bar{q}_{i+1} - \bar{q}_i}{x_{i+1} - x_i}$$

$$\partial_t(\rho) + \partial_x(\rho u) = 0$$

$$\partial_t(\rho u) + \partial_x(\quad) = 0$$

$$\begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \text{ if } ab > 0 \\ b & \text{if } |a| > |b| \text{ if } ab > 0 \end{cases}$$