

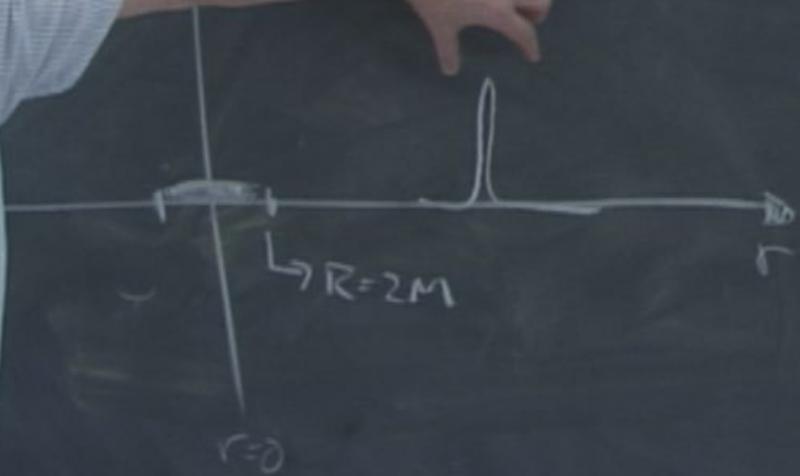
Title: Explorations in Numerical Relativity (PHYS 642) - Lecture 4

Date: Mar 18, 2010 11:20 AM

URL: <http://pirsa.org/10030103>

Abstract:

$$\phi(\theta, r)$$



$$\phi(0, r)$$

$$R=2M$$

$$r=0$$

$$r$$

$$\phi(\theta, r)$$

$$R=2M$$

$$r=0$$

□

$$r$$



$$\phi(\theta, r)$$

$$R=2M$$

$$r=0$$

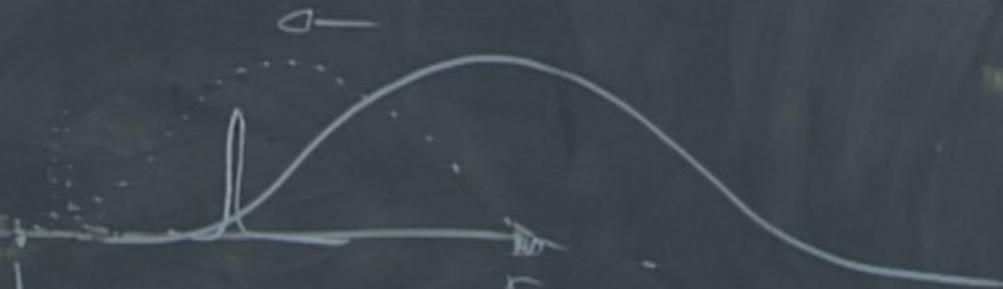


$$\phi(\theta, r)$$

$$R=2M$$

$$r=0$$

$$\phi(u, r)$$



$$\phi(0, r)$$

$$\phi(+, r)$$

$$R=2M$$

$$r=0$$

$$\phi(\theta, r)$$

$$\phi(t, r) f_A$$

$$R=2M$$

δ

$$\phi(u, r)$$

$$\phi(t, r) f_A$$

$$R = 2M$$

$$r=0$$

$$\phi(0, r)$$

$$R = 2M$$

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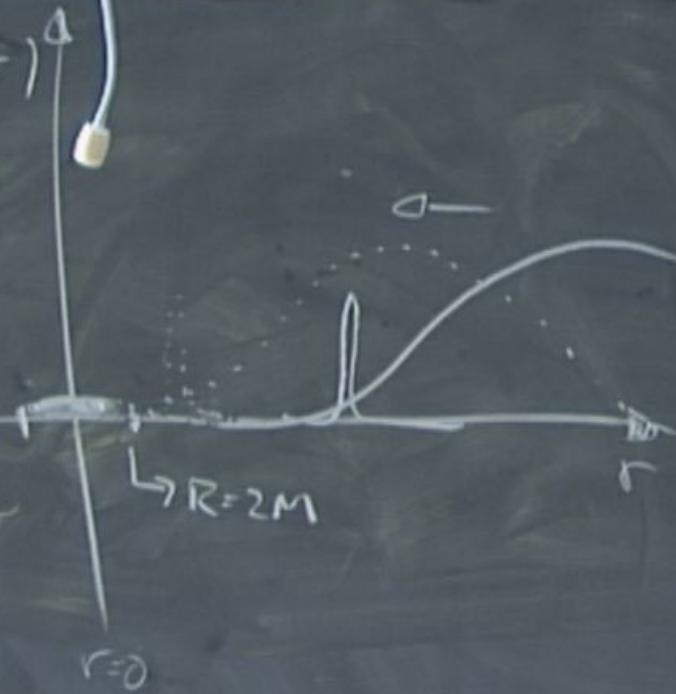
$$\phi(+, r) f_A$$

$$\delta \sim R^0$$

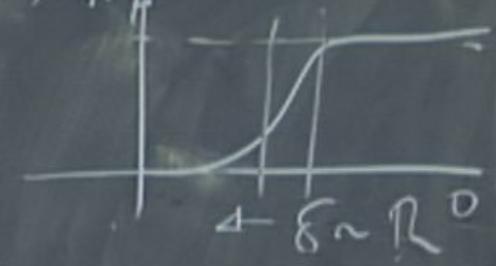
$$(+, r) = 0$$

MASSLESS
K.G.

$$\phi(+, r)$$



$$\phi(+, r) f_A$$



4D

ω_{MP}

$(\beta+1)D$

$${}^{(z)}g_{ij} = \gamma_{ij}$$

4D

$g_{\mu\nu} \Rightarrow 10 \text{ IND } \omega \text{ MP}$

(3+1)D

(3) $g_{ij} = \gamma_{ij} \Rightarrow 6$

2 (LAPSE) 1

3 (SHIFT VEC) 3

10

4D

(3+1)D

\Rightarrow 10 IND ω MP

$\Pi \bar{T}_{\mu\nu} =$

(3)
 $g_{ij} = \gamma_{ij} \Rightarrow 6$

α (LAPSE) 1

β (SHIFT VEC) 3

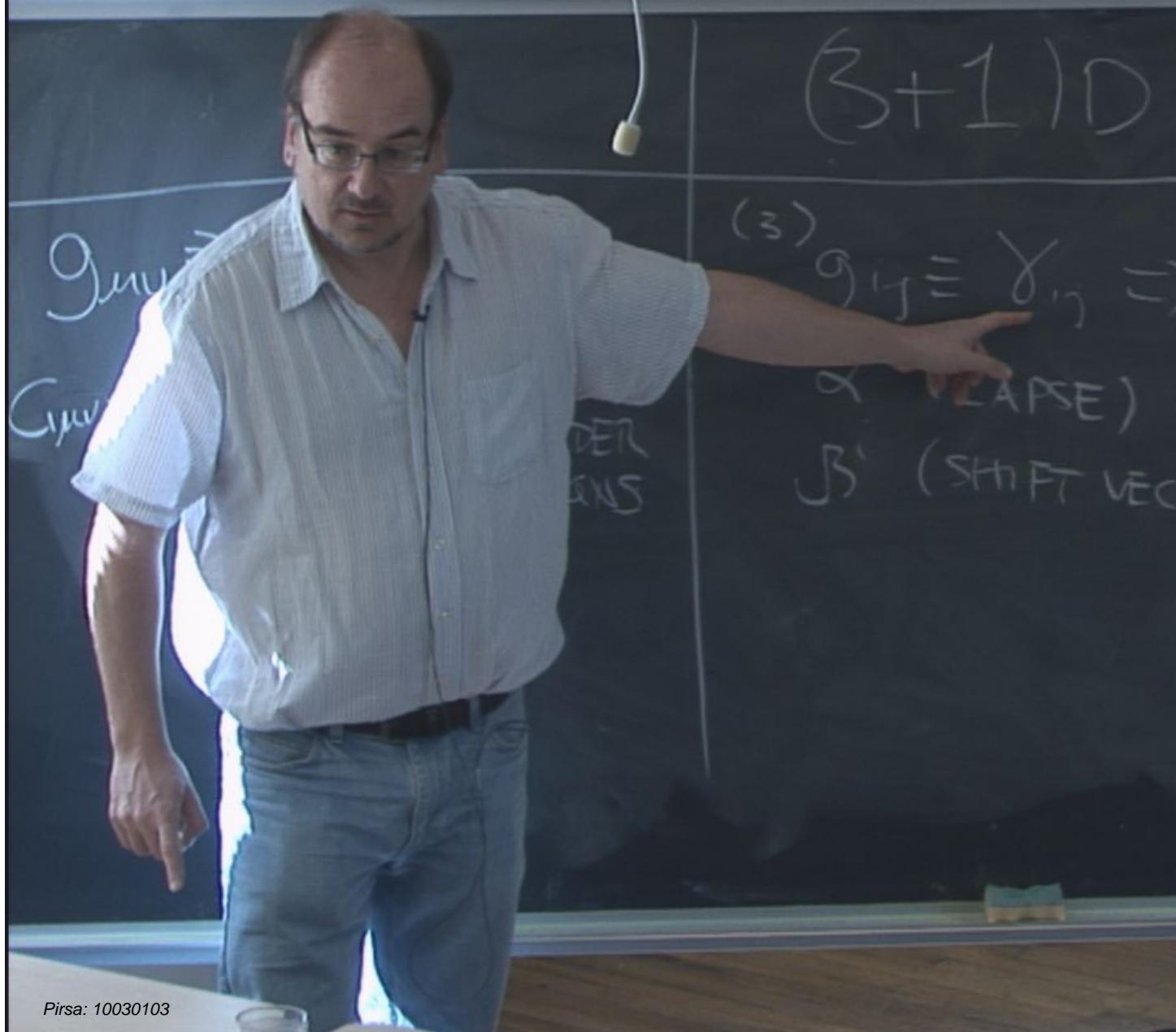
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$\beta + 1)D$

$$(z) \gamma_{ij} = \gamma_{ij} \Rightarrow 6$$

α (CAUSE) 1

β (SHIFT VEC) 3
—
10



4D

$uv \Rightarrow 10 \text{ IND } \omega MP$

$\beta \Pi \bar{t}_{\mu\nu} \Rightarrow 10 \text{ 2ND ORDER EQUATIONS}$
- 4 WNS

(3+1)D

$\mathcal{G}_{ij} = \gamma_{ij} \Rightarrow 6$
 α (LAPSE) 1
 β (SHIFT VEC) 3
10

4D

0 IND COMP

\Rightarrow 10 2ND ORDER EQUATIONS

- 4 CONS

6 IND ORDER EQUATIONS

(3+1)D

(3)
 $g_{ij} = \gamma_{ij} \Rightarrow 6$

α (LAPSE) 1

β (SHIFT VEC) 3

10

4D

\Rightarrow 10 IND. EQUATIONS

\Rightarrow 10 2ND ORDER EQUATIONS

- 4 WNS

6 1ST ORDER EQUATIONS

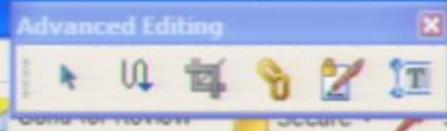
(3+1)D

(3) $g_{ij} = \gamma_{ij} \Rightarrow 6$

α (LAPSE) 1

β (SHIFT VEC) 3

10



THE 3+1 EINSTEIN EQUATIONS

These notes rework the calculation of the 3+1 equations as presented in *Kinematics and Dynamics of General Relativity*, by J. W. York, Jr., which itself is contained in the volume *Sources of Gravitational Radiation*, edited by L. Smarr. Many calculational details omitted from that source are included here.

(Note: A colleague of mine, who will remain anonymous this week, has used these notes to demonstrate the power of his symbolic manipulation software and has found at least one error amongst the many “intermediate” results that are derived below. There is a \$234 Argentinian peso reward for the PSI student who first identifies such an error and who submits a hand-written explanation of the gaffe, along with the corrected expression, to the tutor.)

1) Foliations and Normals

As before, we consider a spacetime M with metric g_{ab} which is sliced into a foliation $\{\Sigma\}$ defined by the isosurfaces of a scalar field τ (the time parameter). Then the spacelike hypersurfaces are, at least locally, described by a *closed* one-form (dual vector field), Ω_a :

$$\Omega_a = \nabla_a \tau. \tag{1}$$

Note that since Ω_a is the gradient of a scalar function, and ∇_a is torsion-free, we have



THE 3+1 EINSTEIN EQUATIONS

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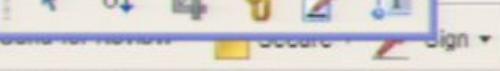


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PHYS 357N

SPHERICAL SYMMETRY (SR)

- IN 3D SPACETIME NEED MATTER FOR DYNAMICS
(BIRKHOFF'S THM, UNIQUENESS & SCHWARZSCHILD
SOL'N AS SOL'N of Einstein = 0)
- WILL RESTRICT MATTER CONTENT TO SIMPLE, MASSLESS,
MINIMALLY-COUPLED SCALAR FIELD, ϕ
 - GOOD MODEL PROBLEM FOR STUDYING STRONG-
FIELD, RADIATIVE BT'S - INCLUDING BLACK
HOLE FORMATION
 - EXHIBITS INTERESTING PHYSICAL BEHAVIOUR →
CRITICAL PHENOMENA aka BLACK HOLE PIPE-SHEET
PHENOMENA



LAGRANGIAN DENSITY FOR ENTIRE SYSTEM

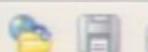
$$L = L_{\text{mat}} + L_{\phi}$$

$$= \sqrt{-g} \left(R - \frac{1}{2} \nabla_a \phi \nabla^a \phi \right)$$

"Consistent" E.O.M

$$G_{ab} = g^{cd} T_{ab} = g^{cd} (\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi)$$

$$\square \phi \equiv \nabla^a \nabla_a \phi = 0$$



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"Consistent" E.O.M

$$G_{ab} = 8\pi T_{ab} = 8\pi (\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi)$$

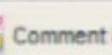
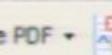
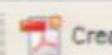
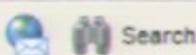
$$\Box \phi = \nabla^a \nabla_a \phi = 0$$

?

3.1 FORM OF SPACETIME METRIC IN S.S.

COORDINATES (t, r, θ, ϕ) ADAPTED TO S.S.

METRIC ON UNIT 2-SPHERE $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$



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3.1 FORM OF SPACETIME METRIC IN SS

- COORDINATES (t, r, θ, ϕ) ADAPTED TO S.S.

METRIC ON UNIT 2-SPHERE $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$

- THEM MOST GENERAL 3-METRIC IS

$$\gamma_{ij} = \text{diag}(a^2(r,t), r^2 b^2(r,t), r^2 b^2 \sin^2\theta) \quad (1)$$

THE LAPSE FUNCTION IS $\alpha(r,t)$, AND THE SHIFT VECTOR $\beta^i(r,t)$ HAS ONLY A RADIAL COMPONENT, $\beta^r(r,t)$



THE MOST GENERAL 3-METRIC IS

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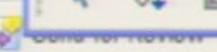
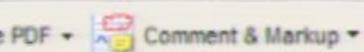
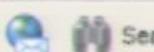
THE LAPSE FUNCTION IS $\alpha(r,t)$, AND THE SHIFT VECTOR $\beta^i(r,t)$ HAS ONLY A RADIAL COMPONENT, $\beta^r(r,t)$

$$\beta^i = (\beta^r, 0, 0) \quad (2)$$

$$\beta_i = \gamma_{ij} \beta^j = (a^2 \beta^r, 0, 0) \quad (3)$$

THE MOST GENERAL 4-METRIC IS THEN

$$ds^2 = (-\alpha^2 + \beta^i \beta_i) dt^2 + 2\beta^i dt dx^i + \gamma_{ij} dx^i dx^j$$



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X!

Help

THE MOST GENERAL 3-METRIC IS

$$\gamma_{ij} = \text{diag}(a^2(r,t), r^2 b^2(r,t), r^2 b^2 \sin^2\theta) \quad (1)$$

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$$ds^2 = (-\alpha^2 + \beta^i \beta_i) dt^2 + 2 \beta^i dt dx^i + \gamma_{ij} dx^i dx^j$$

4D

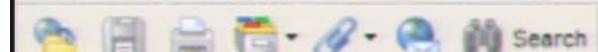
→ 10 IND OMP

$T_{\text{tot}} \Rightarrow 10 \text{ 2ND ORDER EQUATIONS}$

- 4 QNS

6 2ND ORDER EQUATIONS

$$P: x^u \in (\cdot, x^i)$$
$$Q: x^n + dx^n = (\cdot + dt, x^i + dx^i)$$
$$\sum(\cdot + dt)$$
$$\alpha dt$$
$$\beta' dt$$
$$dx^i$$
$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta' dt)^2$$



THE MOST GENERAL A-METRIC IS THEN

$$ds^2 = (-\alpha^2 + \beta^i \beta_i) dt^2 + 2\beta^i dt dx^i + \gamma_{ij} dx^i dx^j$$

$$= (-\alpha^2 + a^2 r^2) dt^2 + 2a^2 \beta^r dt dr + a^2 dr^2 + r^2 b^2 d\Omega^2 \quad (a)$$

?

CORRESPONDING EXTRINSIC CURVATURE TENSOR, K^i_j ,
LIKE γ_{ij} , HAS ONLY TWO INDEPENDENT COMPONENTS

$$K^i_j = \text{diag}(K^r_r(r,t), K^{\theta}_{\theta}(r,t), K^{\phi}_{\phi}(r,t)) \quad (5)$$

EASIER TO SHOW EQUALITY
FROM

$$K_{ij} = (2\alpha)^{-1} (-\alpha + \gamma_{ij} + \partial_i \gamma_j + \partial_j \gamma_i)$$



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 $R_{ij} = \dots$ PLANE SEPARATIONSPHERICAL SYMMETRY

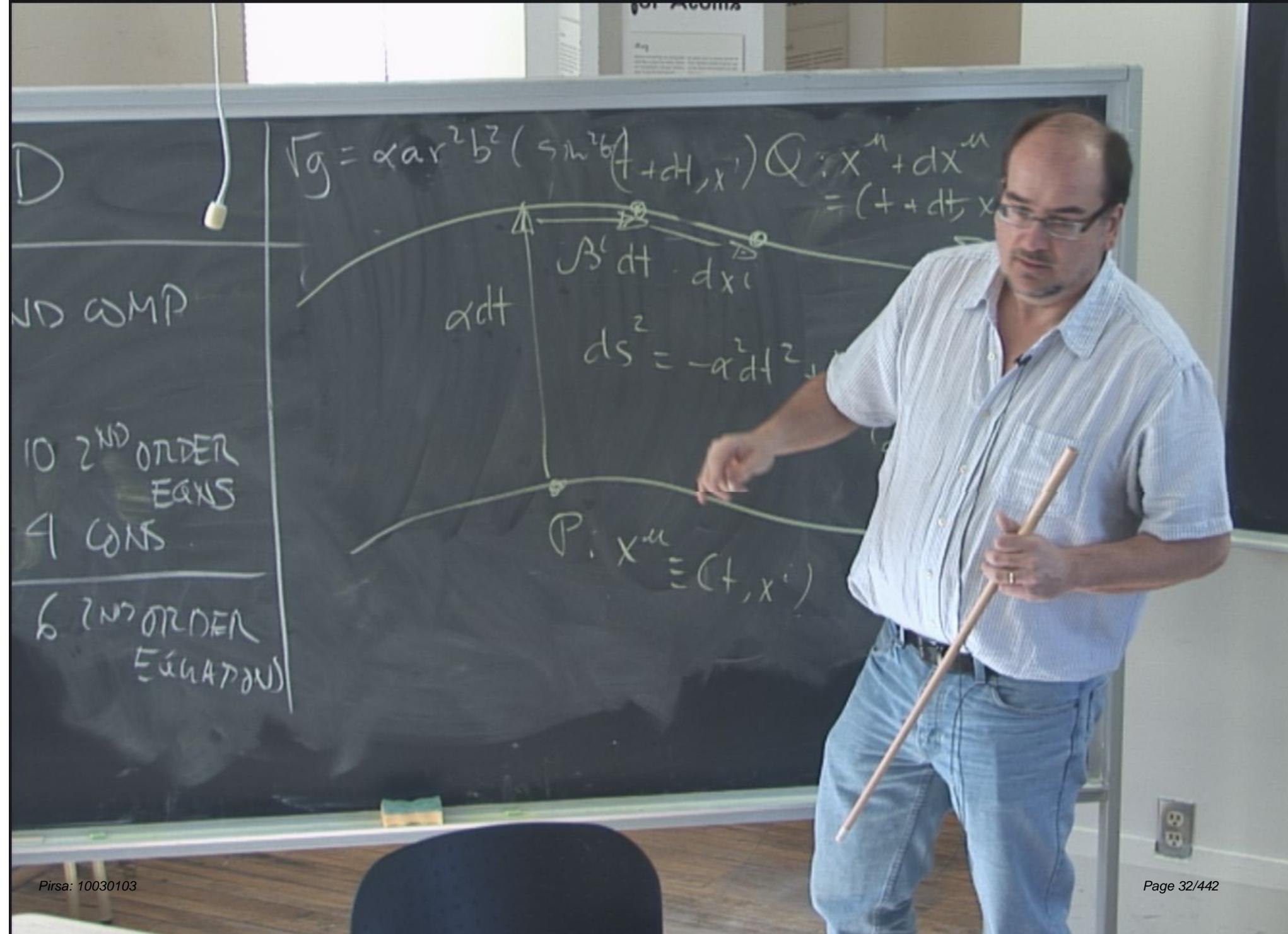
(2)

so, have reduced total # of gravitational kin.

DIM VBLS from 16 to 6, AND, OF COURSE, THESE VBLS ARE FUNCTIONS ONLY of (r, t) RATHER THAN (x, y, z, t)

EINSTEIN EQUATIONS(1) CONSTRAINTS

$$R - K^i_{;j} K^j_{;i} - K^2 = 16\pi G \quad (c)$$



D

ND WMP

10 2ND ORDER
EQNS

4 WNS

6 1ST ORDER
EQUATIONS

$$\sqrt{g} = \alpha a r^2 b^2 (\sin^2 b(t+dt, x^i)) Q : x^n + dx^n = (t + dt, x^i + dx^i) \sum (t + dt)$$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)^2$$



EINSTEIN EQUATIONS

(1) CONSTRAINTS

$$R - K^i_{;i} K^i_{;i} - K^2 = 16\pi J \quad (1)$$

$$D_j K^i_{;i} - D_i K = 8\pi j_i \quad (2)$$

(NOTE INDEX SHIFT RELATIVE TO PREVIOUS FORM)

WHERE

$$J = n_m n^u T^{uv} \quad (3)$$

$$j_i = \gamma_{ik} j^k = -n_u T^u_{;i} \quad (4)$$

$$\text{RECALL: } n_u = (-\alpha, 0, 0, 0)$$



$$D_j K_i^j - D_i K = \partial \pi_j^i \quad (7)$$

(NOTE INDEX SHIFT RELATIVE TO PREVIOUS FORM)

WHERE

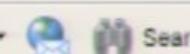
$$\mathcal{J} = n_m n^m T^{uv} \quad (8)$$

$$j_i^i = Y_{ik} j^k = -n_m T^m{}_i \quad (9)$$

$$\text{RECALL: } n_m = (-\alpha, 0, 0, 0) \quad (10)$$

EQUATION EQUATIONS ($\cdot \equiv \frac{\partial}{\partial t} \equiv \partial_t$)

$$Y_{ij} = -2\alpha Y_{ik} K_j^k + \beta^k \partial_k Y_{ij} + Y_{ik} \partial_j \beta^k + Y_{kj} \partial_i \beta^k \quad (11)$$



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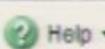
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EQUATION EQUATIONS

$$(\cdot = \frac{\partial}{\partial t} = \partial_t)$$

$$\dot{\gamma}_{ij} = -2\alpha \gamma_{ik} K^k{}_{;j} + \beta^k \partial_k \gamma_{ij} + \gamma_{ik} \partial_j \beta^k + \gamma_{kj} \partial_i \beta^k \quad (1)$$

$$\dot{K}^i{}_{;j} = \beta^k \partial_k K^i{}_{;j} - 2\alpha \beta^i K^k{}_{;k} + \partial_j \beta^k K^i{}_{;k} - D^i D_j \alpha$$

$$+ \alpha (R^i{}_{;j} + K K^i{}_{;j} + 4\pi (S_{-}) \delta^i{}_{;j} - 8\pi S^i{}_{;j}) \quad (2)$$

WHERE $S_{ij} = \gamma_{ij}$, $S^i{}_{;j} = \gamma^{ik} S_{kj}$, $S = S^i{}_{;i}$

NEED CHRISTOFFEL SYMBOLS $\Gamma^i{}_{jk}$, RICCI COMPONENTS $R^i{}_{;j}$ AND RICCI SCALAR R ASSOCIATED WITH γ_{ij} (1). USE

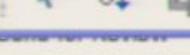
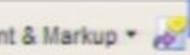
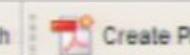
TO FORMULATE EMO FOLLOWING NOETHERIANITY $\Gamma^i{}_{jk}$
 $(\cdot = \frac{\partial}{\partial t} = \partial_t)$

NEED CHRISTOFFEL SYMBOLS Γ^i_{jk} , RICCI COMPONENTS R^i_{ij}
 AND RICCI SCALAR R ASSOCIATED WITH Γ^i_{jk} (1). USE
 STANDARD FORMULAE FROM FOLLOWING NON-VANISHING Γ^i_{jk}
 $(' = \frac{\partial}{\partial r} \equiv \partial_r)$

$$\Gamma^r_{rr} = \frac{a'}{a} \quad \Gamma^r_{\theta\theta} = -\frac{(r^2 b^2)'}{2a^2} \quad \Gamma^r_{\phi\phi} = \sin^2 \theta \Gamma^r_{\theta\theta}$$

$$\Gamma^{\theta}_{r\theta} = \Gamma^{\theta}_{\theta r} = \frac{(r^2 b^2)'}{2(r^2 b^2)} \quad \Gamma^{\theta}_{\phi\phi} = -\sin \theta \cos \theta \quad (16a-g)$$

$$\Gamma^{\phi}_{r\phi} = \Gamma^{\phi}_{\phi r} = \Gamma^{\phi}_{\theta\phi} \quad \Gamma^{\phi}_{\phi\phi} = \Gamma^{\phi}_{\theta\theta} = \cot \theta$$



$$\Gamma_{rr}^r = \frac{a'}{a}$$

$$\Gamma_{rr}^r = -\frac{(r^2 b^2)'}{2a^2}$$

$$\Gamma_{\theta\theta}^r = \sin^2 \theta \Gamma_{rr}^r$$

$$\Gamma_{r\theta}^r = \Gamma_{\theta r}^r = \frac{(r^2 b^2)'}{2(r^2 b^2)} \quad \Gamma_{\theta\phi}^r = -\sin \theta \cos \theta \quad (16a-g)$$

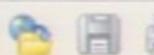
$$\Gamma_{rd}^d = \Gamma_{dr}^d = \Gamma_{\theta r}^r$$

$$\Gamma_{ed}^d = \Gamma_{de}^d = \cot \theta$$

From THESE WE COMPUTE ~~the~~ QUASI-INVARIANTA R^i_j

$$R_{rr}^r = -\frac{2}{arb} \left(\frac{(rb)'}{a} \right)' \quad (17a)$$

$$R_{\theta\theta}^r = R_{\phi\phi}^r = \frac{1}{a(rb)^2} \left(a - \left(\frac{rb}{a} \frac{(rb)'}{a} \right)' \right) \quad (17b)$$



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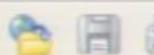
$$\overline{a(r_b)^2} \left(\frac{a}{r_b} - \left(\frac{a}{r_b} \right)' \right)$$

AND FINALLY, THE SCALAR CURVATURE, R , IS

$$R = R^r_r + R^e_e + R^\phi_\phi = R^r_r + 2R^e_e$$

$$\boxed{\boxed{-\frac{2}{ar_b} \left(\left(\frac{(rb)'}{a} \right)' + \frac{1}{rb} \left(\left(\frac{rb}{a} (rb)' \right)' - a \right) \right)}} \quad (18)$$

IN THE EVOLUTION EQUATION FOR K_{ij} WE NEED TO
EVALUATE $D^i D_j \alpha$:



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EVALUATE $D^i D_j \alpha$:DEFINITIONSUPERICAL SYMMETRY

(5)

$$\begin{aligned}
 D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\
 &= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha)
 \end{aligned}$$

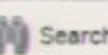
USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)'$$

(13a)

$$D^r D_6 \alpha = D^r D_4 \alpha = \underline{\alpha' (rb)}'$$

(13b)



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EVALUATE $D^i D_j \alpha$:

DEM DEM

SUPERICAL SYMMETRY

(5)

$$\begin{aligned} D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\ &= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha) \end{aligned}$$

USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)' \quad (19a)$$

$$D^r D_6 \alpha = D^r D_4 \alpha = \underline{\alpha' (rb)}' \quad (19b)$$



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EVALUATE $D^i D_j \alpha$:

DEFINITIONSUPERICAL SYMMETRY

(5)

$$\begin{aligned}
 D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\
 &= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha)
 \end{aligned}$$

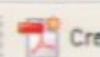
USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)'$$

(13a)

$$D^r D_\theta \alpha = D^r D_\phi \alpha = \underline{\alpha' (rb)}'$$

(13b)



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EVALUATE $D^i D_j \alpha$:

DEFINITION

SUPERICAL SYMMETRY

(5)

$$\begin{aligned} D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\ &= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha) \end{aligned}$$

USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)'$$

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(19b)



EVALUATE $D^i D_j \alpha$:

DIM 3 ELEM

SUPERICAL SYMMETRY

(1)

$$\begin{aligned}
 D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\
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 \end{aligned}$$

USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)' \quad (13a)$$

$$D^r D_\theta \alpha = D^r D_\theta \alpha = \underline{\alpha' (rb)}' \quad (13b)$$



EVALUATE $D^i D_j \alpha$:

DEFINITION

SUPERICAL SYMMETRY

(5)

$$\begin{aligned}
 D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\
 &= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha)
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USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)' \quad (13a)$$

$$D^r D_\theta \alpha = D^r D_\phi \alpha = \underline{\alpha' (rb)}' \quad (13b)$$



Object Data Tool

EVALUATE $D^i D_j \alpha$:

DIM 3 FM

SPHERICAL SYMMETRY

(1)

$$D^i D_j \alpha = \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha$$

$$= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha)$$

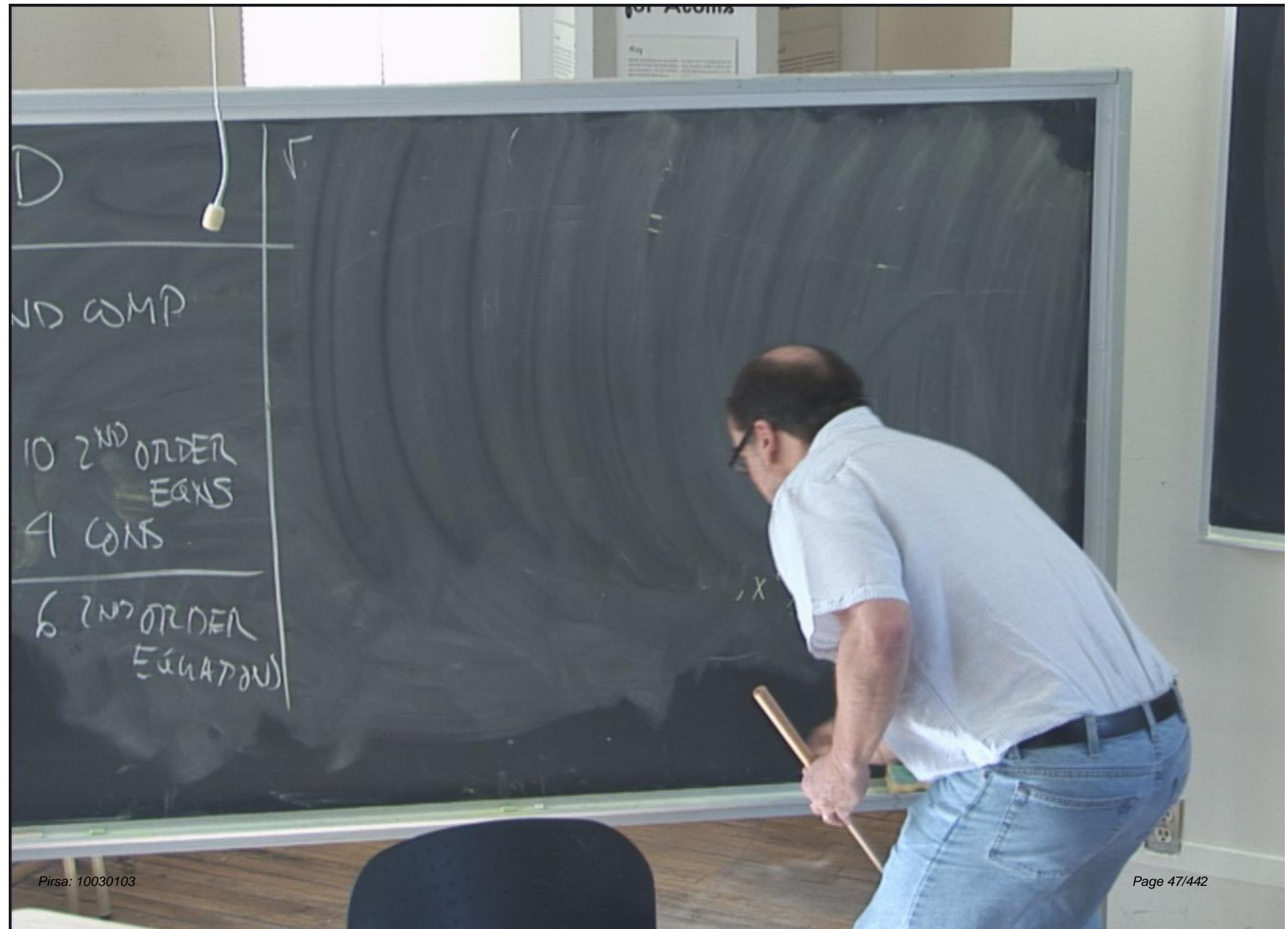
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$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)'$$

(13a)

$$D^r D_\theta \alpha = D^r D_\phi \alpha = \underline{\alpha' (rb)}$$

(13b)



D

ND ω_{MP}

10 2ND ORDER
EQUATIONS

4 WNS

6 1ST ORDER
EQUATIONS

R

$$Y_{ij} = \text{diag}(\bar{a}^2, r^2 b^2, r^2 b^2 \sin^2 \theta)$$

$$Y^{(1)} = \text{diag} \left(\quad \right)$$

, X⁽¹⁾

D

ND WMP

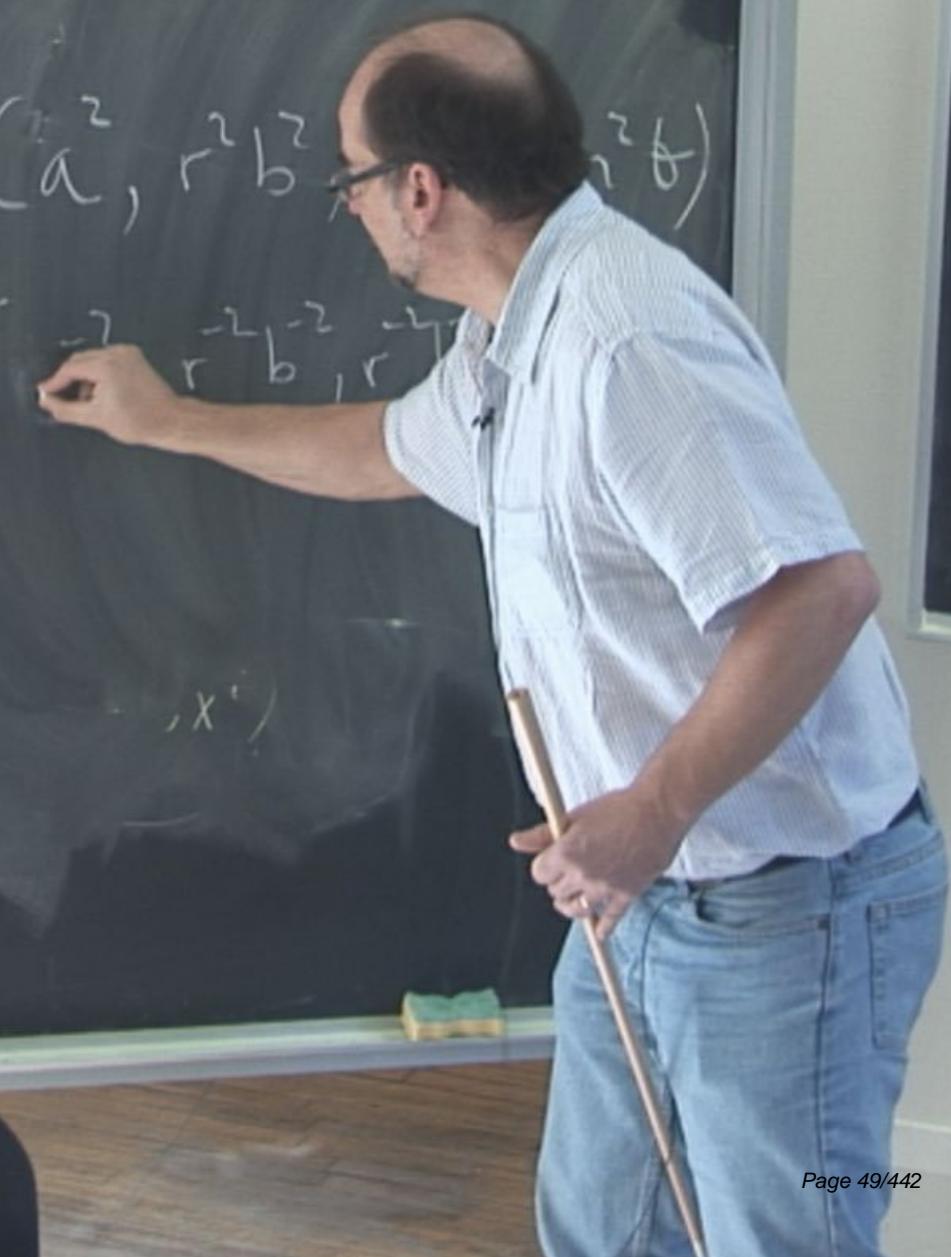
10 2ND ORDER
EQUATIONS

4 WNS

6 1ST ORDER
EQUATIONS

$$Y_{ij} = \text{diag}(r^2, r^2 b^2, r^2)$$

$$Y^{ij} = \text{diag}(r^{-2}, r^{-2} b^{-2}, r^{-2})$$



D

ND OMP

10 2ND ORDER
EQNS

4 WNS

6 1ST ORDER
EQUATIONS

R

$$Y_{12} = \text{diag}(a^2, r^2 b^2, r^2 b^2 \sin^2 \theta)$$

$$Y^{(1)} = \text{diag}(a^{-2}, r^{-2} b^{-2}, r^{-2} b^{-2} \sin^2 \theta)$$



D

ND OMP

10 2ND ORDER
EQUATIONS

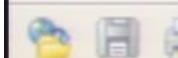
4 WNS

6 1ST ORDER
EQUATIONS

$$Y_{ij} = \text{diag}(a^2, r^2 b^2, r^2 b^2 \sin^2 \theta)$$

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$$\begin{aligned} D^i D_j \alpha &= \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha \\ &= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m{}_{jk} \partial_m \alpha) \end{aligned}$$

USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)' \quad (13a)$$

$$D^a D_a \alpha = D^a D_a \alpha = \frac{\alpha' (rb)}{a^2 rb} \quad (13b)$$

ALSO NEED STRESS-TENSOR "COMPONENTS" σ_{ij}, S_{ij}

- ALSO NEED STRESS-TENSOR "COMPONENTS" σ_{ij}, S^i_j
- IN SPIRIT OF HAMILTONIAN APPROACH, IT IS CONVENIENT TO INTRODUCE AUXILIARY FUNCTIONS

$$\bar{\Phi}(r,t) \equiv \phi'(r,t) = \partial_r \phi(r,t) \quad (20)$$

$$\bar{\pi}(r,t) \equiv \frac{\alpha}{\omega} (\dot{\phi} - \beta \phi') \quad (21)$$

VIEW $\bar{\Phi}, \bar{\pi}$ AS "REAL" DYNAMICAL VOLS FOR SCALAR FIELDS; NOTE: FOR MASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \sim \phi + const$) BUT

SATISFIES $\bar{\Pi}\phi = 0$) ALL "ACTION" IS IN GRADIENT

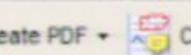


TO INTRODUCE AUXILIARY FUNCTIONS

$$\Phi(r,t) \equiv \dot{\phi}(r,t) = \partial_r \phi(r,t) \quad (20)$$

$$\Pi(r,t) \equiv \frac{\alpha}{\omega} (\dot{\phi} - \omega \phi') \quad (21)$$

VIEW Φ, Π AS "REAL" DYNAMICAL VOLS FOR SCALAR FIELDS; NOTE: FOR MASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \rightarrow \phi + \text{const}$ STILL SATISFIES $\Box \phi = 0$) ALL "ACTION" IS IN GRADIENTS OF ϕ (I.E. IN Φ AND Π)



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SATISFIES $\nabla d = 0$) ALL "action" IS IN GRADIENTS
 αd (i.e. in E AND Π)

ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{\perp\perp} = -\alpha^2 + a^2 \beta^2$$

$$g_{\perp r} = g_{r\perp} = a^2 \beta$$

(22a-e)

$$g_{rr} = a^2$$

$$g_{\theta\theta} = r^2 b^2$$

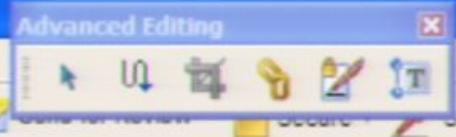
$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

AND

$$g^{tt} = -\alpha^{-2}$$

$$g^{tr} = g^{rt} = \beta \alpha^{-2}$$

(22a-e) Page 55/442



$$\dot{\Phi}(r,t) = \dot{\phi}'(r,t) = \partial_r \phi(r,t) \quad (20)$$

$$\Pi(r,t) = \frac{a}{\alpha} (\dot{\phi} - \beta \dot{\phi}') \quad (21)$$

VIEW $\dot{\Phi}, \Pi$ AS "REAL" DYNAMICAL VOLS FOR SCALAR FIELDS; NB: FOR MASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \sim \phi + \text{const}$ still satisfies $\Box \phi = 0$) ALL "ACTION" IS IN GRADIENTS OF ϕ (I.E. IN $\dot{\Phi}$ AND Π)

ALSO NOTE THAT WE HAVE $(c/f(a))$



TO INTRODUCE AUXILIARY FUNCTIONS

$$\dot{\Phi}(r,t) \equiv \dot{\phi}'(r,t) = \partial_r \phi(r,t) \quad (20)$$

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BY INTRODUCING THESE, IT IS CONVENIENT

TO INTRODUCE AUXILIARY FUNCTIONS

$$\dot{\Phi}(r,t) \equiv \dot{\phi}'(r,t) = \partial_r \phi(r,t) \quad (20)$$

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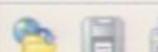
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TO INTRODUCE AUXILIARY FUNCTIONS

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VIEW $\dot{\Phi}, \Pi$ AS "REAL" DYNAMICAL VOLS FOR SCALAR FIELDS; NOTE: FOR MASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \rightarrow \phi + \text{const}$ still satisfies $\Box \phi = 0$) ALL "ACTION" IS IN GRADIENTS $\dot{\phi}$ & $\dot{\phi}'$ (i.e. IN $\dot{\Phi}$ AND Π)



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TO INTRODUCE AUXILIARY FUNCTIONS

$$\Phi(r,t) \equiv \dot{\phi}(r,t) = \partial_r \phi(r,t) \quad (20)$$

$$\Pi(r,t) \equiv \frac{\alpha}{\omega} (\dot{\phi} - \omega \phi') \quad (21)$$

VIEW Φ, Π AS "REAL" DYNAMICAL VELS FOR SCALAR FIELDS; NOTE: FOR MASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \rightarrow \phi + \text{const}$ STILL SATISFIES $\Box \phi = 0$) ALL "ACTION" IS IN GRADIENTS OF ϕ (I.E. IN Φ AND Π)

VIEW ϕ, π AS "DEAL" DYNAMICAL VOLS FOR SCALAR FIELDS; NOTE: FOR CLASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \rightarrow \phi + \text{const}$ still satisfies $\nabla \phi = 0$) ALL "ACTION" IS IN GRADIENTS OF ϕ (I.E. IN \mathcal{S} AND Π)

ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{\perp\perp} = -\alpha^2 + a^2 \beta^2$$

$$g_{\perp r} = g_{r\perp} = a^2 \beta \dot{\beta}$$

$(22a-e)$

$$g_{rr} = a^2$$

$$g_{\theta\theta} = r^2 b^2$$

$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$



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$\alpha \neq 0$ (i.e. in \mathbb{R} and \mathbb{H})

ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{tt} = -\alpha^2 + \alpha^2 \beta^2 \quad g^{tr} = g_{rt} = \alpha^2 \beta$$

(22a-e)

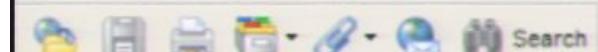
$$g_{rr} = \alpha^2 \quad g_{\theta\theta} = r^2 b^2 \quad g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

AND

$$g^{tt} = -\alpha^{-2}$$

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(23a-e)



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(22a-e)

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(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

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ALSO NOTE THAT WE HAVE $(c/f(a))$

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(22a-e)

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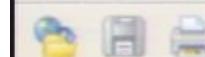
$$g^{tr} = g^{rt} = \beta \alpha^{-2}$$

(23a-e)

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X

ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{zz} = -\alpha^2 + \alpha^2 \beta^2$$

$$g_{rr} = g_{tt} = \alpha^2$$

(22a-e)

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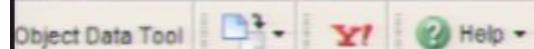
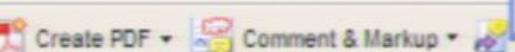
(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

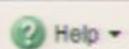
$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEIR WE FIND



Y!



Help

ALSO NOTE THAT WE HAVE $(c/f(a))$

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(22a-e)

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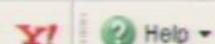
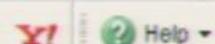
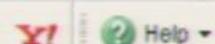
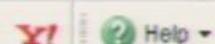
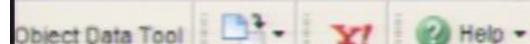
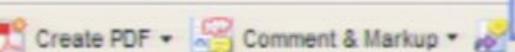
(23a-e)

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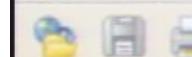
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THEN WE FIND



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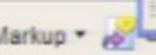
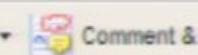
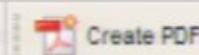
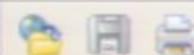
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THEN WE FIND



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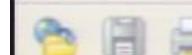
(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEN WE FIND



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X

ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{tt} = -\alpha^2 + \alpha^2 \beta^2$$

$$g_{tr} = g_{rt} = \alpha^2 \beta$$

(22a-e)

$$g_{rr} = \alpha^2$$

$$g_{\theta\theta} = r^2 b^2$$

$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

AND

$$g^{tt} = -\alpha^{-2}$$

$$g^{tr} = g^{rt} = \beta \alpha^{-2}$$

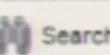
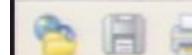
(23a-e)

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THEIR WE FIND



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(22a-e)

$$g_{rr} = \alpha^2$$

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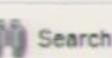
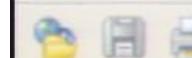
(23a-e)

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$$g^{\theta\theta} = (rb)^{-2}$$

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(22a-e)

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(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

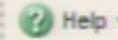
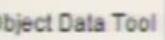
$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEN WE FIND



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ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{tt} = -\alpha^2 + \alpha^2 \beta^2$$

$$g_{tr} = g_{rt} = \alpha^2 \beta$$

(22a-e)

$$g_{rr} = \alpha^2$$

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$$g^{tt} = -\alpha^{-2}$$

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(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEN WE FIND



ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{zz} = -\alpha^2 + \alpha^2 \beta^2$$

$$g_{rr} = g_{tt} = \alpha^2$$

(22a-e)

$$g_{rr} = \alpha^2$$

$$g_{\theta\theta} = r^2 b^2$$

$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

AND

$$g^{tt} = -\alpha^{-2}$$

$$g^{rr} = g^{tt} = \beta \alpha^{-2}$$

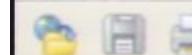
(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEN WE FIND



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ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{tt} = -\alpha^2 + \alpha^2 \beta^2$$

$$g_{tr} = g_{rt} = \alpha^2 \beta$$

(22a-e)

$$g_{rr} = \alpha^2$$

$$g_{\theta\theta} = r^2 b^2$$

$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

AND

$$g^{tt} = -\alpha^{-2}$$

$$g^{tr} = g^{rt} = \beta \alpha^{-2}$$

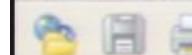
(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEN WE FIND



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ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{zz} = -\alpha^2 + \alpha^2 \beta^2$$

$$g_{rr} = g_{tt} = \alpha^2$$

(22a-e)

$$g_{rr} = \alpha^2$$

$$g_{\theta\theta} = r^2 b^2$$

$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

AND

$$g^{tt} = -\alpha^{-2}$$

$$g^{rr} = g^{tt} = \beta \alpha^{-2}$$

(23a-e)

$$g^{rr} = \alpha^{-2} - \beta^2 \alpha^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

THEIR WE FIND



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$$g^{tt} = -a^2 - c^2 r^2$$

$$g^{tr} = g^{rt} = a \cos \theta$$

(22a-e)

$$g_{rr} = a^2$$

$$g_{\theta\theta} = r^2 b^2$$

$$g_{\phi\phi} = r^2 b^2 \sin^2 \theta$$

And

$$g^{tt} = -a^{-2}$$

$$g^{tr} = g^{rt} = b a^{-2}$$

(23a-e)

$$g^{rr} = a^{-2} - b^2 a^{-2}$$

$$g^{\theta\theta} = (rb)^{-2}$$

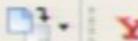
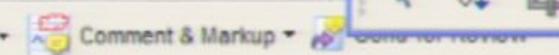
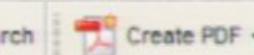
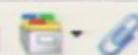
$$g^{\phi\phi} = (rb \sin \theta)^{-2}$$

Then we find

$$\nabla^t \phi = \partial^t \phi = g^{tt} \partial_r \phi + g^{tr} \partial_\theta \phi = -\frac{\pi}{a} \quad (2a)$$

$$(\nabla^t \phi)(\nabla_r \phi) = \partial^t \partial_r \phi = \frac{\pi^2 - \pi^2}{a^2}$$

(25)



$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb \sin \theta)^{-2}$$

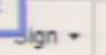
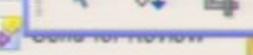
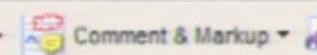
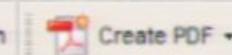
THEN WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ t + g^{rr} \partial_r t = -\frac{\pi}{x a} \quad (24)$$

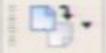
$$(\nabla^+ \phi)(\nabla_- \phi) = \partial^+ \phi_{,r} = \frac{\pi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND

$$\nabla^\mu n_\mu n_\nu T^{\mu\nu} = x^2 T^{tt} = x^2 (\partial^+ \phi^{tt} - \frac{1}{2} g^{++} \partial^+ \phi_{,r})$$



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THEIR WE FIND

$$\nabla^+ \phi = \partial^+ \phi = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{x a} \quad (24)$$

$$(\nabla^+ \phi)(\nabla_- \phi) = \partial^+ \partial_- \phi = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, 0)$, WE FIND

$$\partial_\mu n_\mu n_\nu T^{\mu\nu} = x^2 T^{tt} = x^2 (\partial^+ \partial^- - \frac{1}{2} g^{++} \partial^+ \partial_{,r})$$



$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THEN WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{xa} \quad (24)$$

$$(\nabla^+ \phi)(\nabla_+ \phi) = \partial^+ \phi_{,++} = \frac{\pi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND

$$\Delta n_\mu n_\nu T^{\mu\nu} = x^2 T^{tt} = x^2 (\partial^+ \phi \partial^{+t} \phi - \frac{1}{2} g^{++} \partial^+ \phi_{,++})$$



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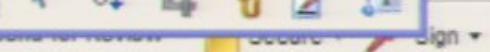
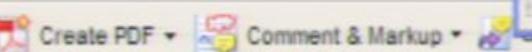
$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THEL WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{x a} \quad (24)$$

$$(\nabla^+ \phi)(\nabla_+ \phi) = \partial^+ \partial_+ \phi = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



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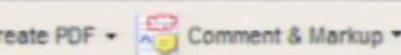
$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THELL WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = - \frac{\pi}{x a} \quad (24)$$

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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



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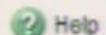


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$$g^{rr} = a^{-2} - b^2 \alpha^2 \quad g^{tt} = (rb)^{-2} \quad g^{\theta\theta} = (rb\sin\theta)^{-2}$$

THEL WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = - \frac{\pi}{\alpha} \quad (24)$$

$$(\nabla^+ \phi)(\nabla_- \phi) = \partial^+ \partial_- \phi = \frac{\Phi^2 - \pi^2}{\alpha^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-\alpha, \omega, 0, 0)$, WE FIND



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THEL WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{x a} \quad (24)$$

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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



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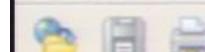
$$g^{rr} = a^{-2} - b^2 x^{-2} \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THESE WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{xa} \quad (24)$$

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$$\Delta = n_\mu n^\nu T^{\mu\nu} = x^2 T^{++} = x^2 (\partial^+ \partial^{+t} - \frac{1}{2} g^{++} \partial^+ \partial_{+t})$$



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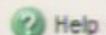
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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THEL WE FIND

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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



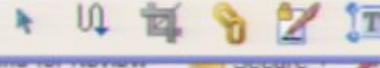
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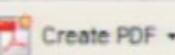
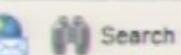
$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{\theta\theta} = (rb \sin \theta)^{-2}$$

THELL WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = - \frac{\pi}{x a} \quad (24)$$

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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THEIR WE FIND

$$\nabla^+ \phi = \partial^+ \phi = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{xa} \quad (24)$$

$$(\nabla^+ \phi)(\nabla_- \phi) = \partial^+ \partial_- \phi = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

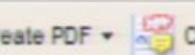
THEN WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{x a} \quad (24)$$

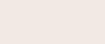
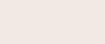
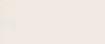
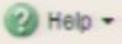
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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND

$$\nabla^\mu n_\mu n_\nu T^{\mu\nu} = x^2 T^{tt} = x^2 (\partial^+ \partial^+ - \frac{1}{2} g^{++} \partial^+ \partial_{,r})$$



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

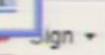
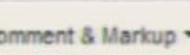
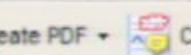
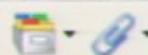
THEL WE FIND

$$\nabla^+ \phi = \partial^+ t = g^{++} \partial_+ \phi + g^{rr} \partial_r \phi = -\frac{\pi}{x a} \quad (24)$$

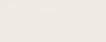
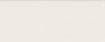
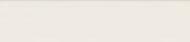
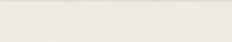
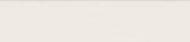
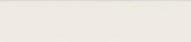
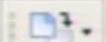
$$(\nabla^+ \phi)(\nabla_+ \phi) = \partial^+ \partial_+ \phi = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND

$$\nabla_\mu n_\nu T^{\mu\nu} = x^2 T^{tt} = x^2 (\partial^+ \partial^{+t} - \frac{1}{2} g^{++} \partial^+ \partial_{+t})$$



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

THELL WE FIND

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$$(\nabla^+ \phi)(\nabla_- \phi) = \partial^+ \partial_- t_{,rr} = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND

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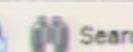
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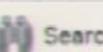
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AND, AGAIN RECALLING THAT $n_\mu = (-x, u, 0, v)$, WE FIND

$$\Delta n_\mu n_\nu T^{\mu\nu} = x^2 T^{tt} = x^2 (\partial^+ \partial^- t - \frac{1}{2} g^{++} \partial^+ \partial_- t) \quad (26)$$



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$$g^{rr} = a^{-2} - b^2 x^2 \quad g^{tt} = (rb)^{-2} \quad g^{xx} = (rb\sin\theta)^{-2}$$

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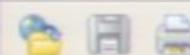
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$$(\nabla^2 \phi)(\nabla_\mu \phi) = d^{1+} \phi_{,1\mu} = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-\alpha, \omega_0, 0, 0)$, WE FIND

$$\begin{aligned} J = n_\mu n_\nu T^{\mu\nu} &= \alpha^2 T^{tt} = \alpha^2 (\phi^{1+} \phi^{1+} - \frac{1}{2} g^{++} \phi^{1+} \phi_{,1\mu}) \\ &= \frac{\Phi^2 + \pi^2}{2a^2} \end{aligned} \quad (26)$$

$$j_i = (j_r, 0, 0)$$

$$j_r = -n_\mu T^{\mu r} = \alpha T^r; = \alpha \phi^{1+} \phi_{,1r} = -\frac{\Phi \pi}{a} \quad (27)$$



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$$\begin{aligned} j^\mu &= n_\mu n_\nu T^{\mu\nu} = \alpha^2 T^{tt} = \alpha^2 (\partial^+ \partial^- - \frac{1}{2} g^{++} \partial^\mu \partial_\mu) \\ &= \frac{\Phi^2 + \pi^2}{2a^2} \end{aligned} \quad (26)$$

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$$j_i = (j_r, 0, 0)$$

$$j_r = -n_\mu T^r_i = x T^r_i = x \phi^{rr} \phi_{,rr} = -\frac{\Phi \pi}{a} \quad (27)$$



$$(\nabla^2 \phi)(\nabla_\mu \phi) = \phi^{''\mu} \phi_{,\mu} = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_\mu = (-\alpha, \omega_0, 0)$, WE FIND

$$\begin{aligned} J^\mu n_\mu n_\nu T^{\nu\nu} &= \alpha^2 T^{tt} = \alpha^2 (\phi^{'+} \phi^{'+} - \frac{1}{2} g^{++} \phi^{''\mu} \phi_{,\mu}) \\ &= \frac{\Phi^2 + \pi^2}{2a^2} \end{aligned} \quad (26)$$

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$$j^t = (j^t, 0, 0, 0)$$

$$j^t = -n_\mu T^{\mu t} = \alpha T^0; = \alpha \phi^{1+} \phi_{,1r} = -\frac{\Phi \pi}{a} \quad (27)$$



$$(\nabla^2 \phi)(\nabla_\mu \phi) = d^{1+} \phi_{,1\mu} = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

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$$(\nabla^{\perp} \phi)(\nabla_{\perp} \phi) = d^{\perp \perp} \phi_{,\perp} = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

AND, AGAIN RECALLING THAT $n_{\mu} = (-x, u, 0, v)$, WE FIND

$$\begin{aligned} J &= n_{\mu} n_{\nu} T^{\mu\nu} = x^2 T^{tt} = x^2 (\phi^{tt} \phi^{tt} - \frac{1}{2} g^{tt} \phi^{rr} \phi_{,rr}) \\ &= \frac{\Phi^2 + \pi^2}{2a^2} \end{aligned} \quad (26)$$

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$$(\nabla^{\mu} \phi)(\nabla_{\mu} \phi) = d^{\mu} \phi_{,\mu} = \frac{\Phi^2 - \pi^2}{a^2} \quad (25)$$

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$$j^i = (j^r, 0, 0)$$

$$j^r = -n_{\mu} T^{\mu r} = \alpha T^r t = \alpha \phi^{tt} \phi_{,r} = -\frac{\Phi \pi}{a} \quad (27)$$



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$$= \frac{\phi^2 + \tau^2}{2a^2} \quad (26)$$

 $\sigma_{ij} = \epsilon_{ijk} T_{kj}$

$$\sigma_{ij} = -\epsilon_{ijk} T_{ki} = \alpha T^0_i = \alpha \phi^+ d_{ir} = -\frac{e^2 \pi}{a^2} \quad (27)$$

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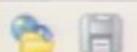
SPHERICAL SYMMETRY

④

FINALLY, WE HAVE THE RADIAL STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} S_{kj} = \gamma^{ik} T_{ki}$$

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$$\frac{1}{2a^2}$$

(26)

$$j_i = (j_r, 0, 0)$$

$$j_r = -n_r T^r_i = \alpha T^r_i = \alpha \phi^+ d_r r = -\frac{\alpha r}{2} \pi \quad (27)$$

PBM BEAM

SPHERICAL SYMMETRY

④

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$$\frac{1}{2a^2}$$

(26)

$$j_i = (j_r, 0, 0)$$

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PHYSICALSPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$S_{ij}^r = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$\frac{1}{2a^2}$$

(26)

$$\mathbf{j}_i = (j_r, 0, 0)$$

$$j_r = -n_r T^r_i = \alpha T^r_i = \alpha \phi^+ d_r r = -\frac{\alpha r}{a} \pi \quad (27)$$

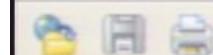
PHYSICALSPIRICAL SYMMETRY

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$$\frac{1}{2a^2}$$

(26)

$$\mathbf{j}_i = (j_r, 0, 0)$$

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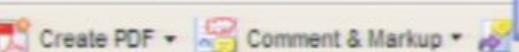
PHYSICALSPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$S_{ij}^i = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND



$$\dot{r} = -n_m T^m_i = \alpha T^0_i = \alpha \phi' dr = -\frac{\alpha}{r} \ddot{\phi} \pi \quad (27)$$

PHY 3E7H

SPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$S_{ij}^i = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

From which we find

$$S_r^r = \rho = \frac{\dot{\phi}^2 + T^1_1}{2a^2} \quad (28)$$



PHM 3E7H SPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} \sigma_{kj} = \gamma^{ik} T_{kj}$$

From which we find

$$\sigma_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (2e)$$

$$\sigma_\theta = \sigma_\phi = \frac{\pi^2 - \Phi^2}{2a^2} \quad (2g)$$



PHYSICAL SUPERICAL SYMMETRY

(4)

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} \sigma_{kj} = \gamma^{ik} T_{kj}$$

From which we find

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$$\sigma_\theta = \sigma_\phi = \frac{\pi^2 - \Phi^2}{2a^2} \quad (2g)$$

PHYSICAL SUPERICAL SYMMETRY

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PIN BEAM SPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

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FROM WHICH WE FIND

$$\sigma_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (2e)$$

$$\sigma_\theta = \sigma_\phi = \frac{\pi^2 - \Phi^2}{2a^2} \quad (2g)$$

PHM BEAM SPIRICAL SYMMETRY

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} \sigma_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$\sigma_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (2e)$$

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PHY 3E7M SPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} \sigma_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$\sigma_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (28)$$

$$\sigma_\theta = \sigma_\phi = \frac{\pi^2 - \Phi^2}{2a^2} \quad (29)$$



PHM 3E7H SPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL PRESSURE COMPONENTS

$$S_{ij}^i = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$S_r^r = \rho = \frac{\phi^2 + \tau^2}{2a^2} \quad (2e)$$

$$S_\theta^\theta = S_\phi^\phi = \frac{\pi^2 - \phi^2}{2a^2} \quad (2g)$$

POLY BEAMSPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} \sigma_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$\sigma_r = \rho = \frac{\pi^2 + \bar{\pi}^2}{2a^2} \quad (2e)$$

$$\sigma_\theta = \sigma_\phi = \frac{\pi^2 - \bar{\pi}^2}{2a^2} \quad (2g)$$

POLY 3E7MSPIRICAL SYMMETRY

④

FINALLY, WE HAVE THE STRESS COMPONENTS

$$\sigma_{ij} = \gamma^{ik} \sigma_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$\sigma_r = \rho = \frac{\pi^2 + \tau^2}{2a^2} \quad (2e)$$

$$\sigma_\theta = \sigma_\phi = \frac{\pi^2 - \tau^2}{2a^2} \quad (2g)$$



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$$S^i_j = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

From which we find

$$S_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (2e)$$

$$S_e = S_d = \frac{\pi^2 - \Phi^2}{2a^2} \quad (2g)$$

$$S_\rho = 2S_e = \frac{\pi^2 - \Phi^2}{a^2} \quad (3h)$$



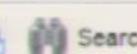
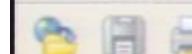
$$S^i_j = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

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$$S_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (2e)$$

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$$S^i_j = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

From which we find

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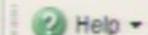


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$$S^i_j = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

From which we find

$$S^r_r = \rho = \frac{\Phi^2 + \tau^2}{2a^2} \quad (28)$$

$$S^e_e = S^d_d = \frac{\pi^2 - \Phi^2}{2a^2} \quad (29)$$

$$S_{\rho\rho} = 2S^e_e = \frac{\pi^2 - \Phi^2}{a^2} \quad (30)$$



$$S^i_j = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

From which we find

$$S_r = \rho = \frac{\Phi^2 + \pi^2}{2a^2} \quad (28)$$

$$S_e = S_d = \frac{\pi^2 - \Phi^2}{2a^2} \quad (29)$$

$$S_\rho = 2S_e = \frac{\pi^2 - \Phi^2}{a^2} \quad (30)$$



$$\frac{1}{2a^2}$$

$$S^e = S^d = \frac{\pi^2 - \Phi^2}{2a^2} \quad (29)$$

$$S_{-p} = 2S^e = \frac{\pi^2 - \Phi^2}{a^2} \quad (30)$$

* WE CAN NOW ASSEMBLE THE ABOVE RESULTS TO REACH THE SPHERICALLY-SYMMETRIC SPECIALIZATION OF THE GENERAL 3+1 EQUATIONS (6), (7), (11) ; (12)

A) HAMILTONIAN CONSTRAINT

$$R - K^i_K^j + K^2 = 16\pi G$$



THE SPHERICALLY-SYMMETRIC SPECIALIZATION of THE
GENERAL 3+1 EQUATIONS (6), (7), (11) & (12)

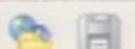
A) HAMILTONIAN CONSTRAINT

$$R - K^i_{;j} K^{j;i} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K^i_{;j} K^{j;i} + K^2 &= -(K_r^r{}^2 + 2K_e^e{}^2) + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)



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A) HAMILTONIAN CONSTRAINT

$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

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$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

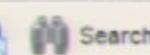
$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K^i_{;j} K^j_{;i} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



A) HAMILTONIAN CONSTRAINT

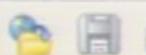
$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K^i_{;j} K^j_{;i} + K^2 &= -(K_r^r{}^2 + 2K_\theta^\phi{}^2) + (K_r^r + 2K_\theta^\phi)^2 \\ &= 4K_r^r K_\theta^\phi + 2K_\theta^\phi{}^2 \end{aligned}$$

$$R + 4K_r^r K_\theta^\phi + 2K_\theta^\phi{}^2 = 8\pi \frac{\dot{\phi}^2 + \dot{r}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi \rho$$

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$$R + 4K_r^r K_\theta^\phi + 2K_\theta^\phi{}^2 = 8\pi \frac{\dot{\ell}^2 + \ell l^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

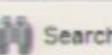
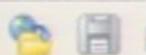
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SEE (18)

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\ell}^2 + \tau l^2}{a^2} \quad (31)$$

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



A) HAMILTONIAN CONSTRAINT

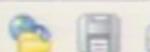
$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

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$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

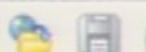
$$R - K^i_{;j} K^{j;i} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K^i_{;j} K^{j;i} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\phi}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

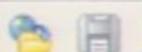
$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi \rho$$

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$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

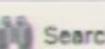
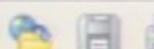
$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

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B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$

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B) MOMENTUM CONSTRAINT (only r -component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi G$$

$$\begin{aligned} -K^i_{;j} K^j_{;i} + K^2 &= -(K_r^r{}^2 + 2K_e^e{}^2) + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\phi}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

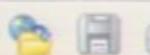
$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\phi}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

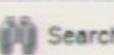
$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_\theta^\theta)^2 + (K_r^r + 2K_\phi^\phi)^2 \\ &= 4K_r^r K_\theta^\theta + 2K_\phi^\phi \end{aligned}$$

$$R + 4K_r^r K_\theta^\theta + 2K_\phi^\phi = 8\pi \frac{\dot{\theta}^2 + \dot{\phi}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



A) HAMILTONIAN CONSTRAINT

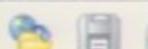
$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K^i_{;j} K^j_{;i} + K^2 &= -(K_r^r{}^2 + 2K_e^e{}^2) + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

SEE (18)

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

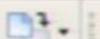
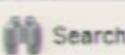
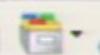
$$R - K^i_{;j} K^{j;i} + K^2 = 16\pi \mathcal{J}$$

$$\begin{aligned} -K^i_{;j} K^{j;i} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\mathcal{J}}^2 + \mathcal{T}^2}{a^2} \quad (31)$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



A) HAMILTONIAN CONSTRAINT

$$R - K^i_{;j} K^{j;} + K^2 = 16\pi \rho$$

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SEE (18)

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

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SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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A) HAMILTONIAN CONSTRAINT

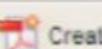
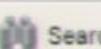
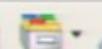
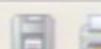
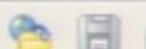
$$R - K^i_{;j} K^j_{;i} + K^2 = 16\pi G$$

$$\begin{aligned} -K^i_{;j} K^j_{;i} + K^2 &= -(K_r^r{}^2 + 2K_e^e{}^2) + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

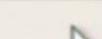
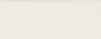
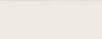
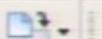
SEE (18)

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



Object Data Tool



A) HAMILTONIAN CONSTRAINT

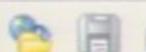
$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r -component is non-trivial)



Object Data Tool



A) HAMILTONIAN CONSTRAINT

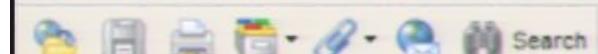
$$R - K_{,j}^i K^{ji} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K_{,j}^i K^{ji} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

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SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



A) HAMILTONIAN CONSTRAINT

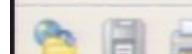
$$R - K^i_{;j} K^{j;} + K^2 = 16\pi \rho$$

$$\begin{aligned} -K^i_{;j} K^{j;} + K^2 &= -(K_r^r + 2K_e^e)^2 + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

SEE (18)

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



A) HAMILTONIAN CONSTRAINT

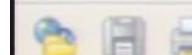
$$R - K^i_{;j} K^{j;i} + K^2 = 16\pi \rightarrow$$

$$\begin{aligned} -K^i_{;j} K^{j;i} + K^2 &= -(K_r^r{}^2 + 2K_e^e{}^2) + (K_r^r + 2K_e^e)^2 \\ &= 4K_r^r K_e^e + 2K_e^e{}^2 \end{aligned}$$

SEE (18)

$$R + 4K_r^r K_e^e + 2K_e^e{}^2 = 8\pi \frac{\dot{\theta}^2 + \dot{\tau}^2}{a^2} \quad (31)$$

B) MOMENTUM CONSTRAINT (only r-component is non-trivial)



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B) Momentum constraint (only r-component is non-trivial)

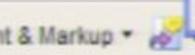
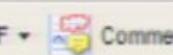
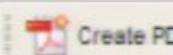
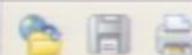
• FIRST NOTE THAT

$$\begin{aligned} D_r K^r_{\perp} &= \partial_r K^r_{\perp} + \nabla^r_{\perp} K^{\perp}_{\perp} - \nabla^{\perp}_{\perp} K^r_{\perp} \\ &= K^r_{rr} + 2 \nabla^{\perp}_{\perp} (K^r_{\perp} - K^{\perp\perp}) \end{aligned}$$

$$D_r K = (K^r_{rr} + 2 K^{\perp\perp})'$$

THEN WE HAVE FROM (1) AND (2)

$$K^{\perp\perp}' + (r b)' (K^{\perp\perp} - K^r_{\perp}) = 4 \pi \rho \bar{\rho} \pi$$



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$$D_r K = (K^r_r + 2K^e_e)'$$

THESE WE HAVE FROM (1) AND (2a)

$$K^e_e' + \frac{(rb)}{rb} (K^e_e - K^r_r) = 4\pi \frac{\rho \pi}{a} \quad (2)$$

c) Evolution Equations for \dot{x}_{ij} (a, b)

- FOLLOW DIRECTLY FROM (1), RECALL, CAN BE VIEWED AS DEF'N OF K^i_j :

$$\dot{a} = -daK^r_r + (a\beta)^i_i \quad (3)$$



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THESE WE HAVE FROM (1) AND (2)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = \alpha \frac{\partial \Pi}{\partial a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR \dot{x}_{ij} (a, b)

- FOLLOW DIRECTLY FROM (1), RECALL, CAN BE VIEWED AS DEFN OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)' \quad (34)$$



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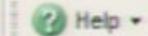


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THESE WE HAVE FROM (1) AND (2)

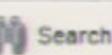
$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = m \frac{\partial \pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR \dot{x}_{ij} (a, b)

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$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$



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THESE WE HAVE FROM (1) AND (2)

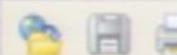
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THEN WE HAVE FROM (1) AND (2)

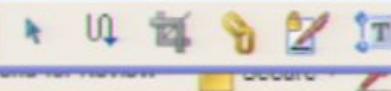
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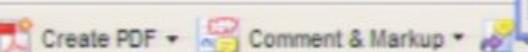
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THEN WE HAVE FROM (1) AND (2)

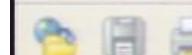
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THESE WE HAVE FROM (1) AND (2)

$$K^e_e' + \frac{(rb)}{rb} (K^e_e - K^r_r) = m \frac{\partial \Pi}{a} \quad (32)$$

c) EQUATION EQUATIONS FOR \dot{x}_{ij} (a, b)

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THESE WE HAVE FROM (1) AND (2)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = m \frac{\partial \Pi}{a} \quad (32)$$

c) Evolution Equations for \dot{x}_{ij} (a, b)

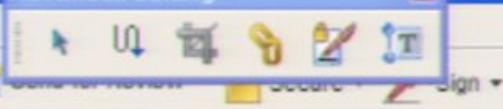
- Follow directly from (1), recall, can be viewed as DEFⁿ of K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + (rb)' \quad (34)$$



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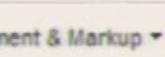
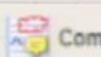
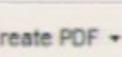
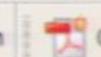
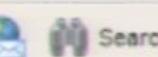
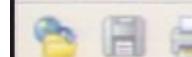
$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = m \frac{\partial \Pi}{a} \quad (32)$$

c) Evolution Equations for γ_{ij} (a, b)

- Follow DIRECTLY from (1), RECALL, CAN BE VIEWED AS DEF'N OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)' \quad (34)$$



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THESE WE HAVE FROM (1) AND (2)

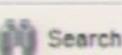
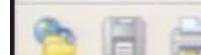
$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = m \frac{\Phi \pi}{a} \quad (32)$$

c) EQUATION EQUATIONS FOR \dot{x}_{ij} (a, b)

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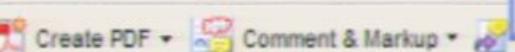
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$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)' \quad (34)$$



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$$\dot{r}^e + \frac{(rb)}{r^2} (K^e_r - K^e_r) = -\frac{a}{r}$$

c) EVOLUTION EQUATIONS FOR \dot{x}_{ij} (a, b)

- FOLLOW DIRECTLY FROM (1), RECALL, CAN BE VIEWED AS DEF'N OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_j (K^r_r, K^e_e)

$$\dot{r}^e + \frac{(rb)'}{rb} (K^e_r - K^e_r) = -\frac{\alpha}{a} \dot{a}$$

c) EQUATION EQUATIONS FOR \dot{x}_{ij} (a, b)

• FOLLOW DIRECTLY FROM (1), RECALL, CAN BE VIEWED AS "DEF" OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

d) EQUATION EQUATIONS FOR K^i_j (K^r_r, K^e_e)

c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

• FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS
DEF["] OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEF" OF K^i_j :

$$\dot{a} = -\alpha a K^r_{\text{rr}} + (a \beta)^r \quad (33)$$

$$\dot{b} = -\alpha b K^e_{\text{ee}} + \sum_r (rb)^r \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} ($K^r_{\text{rr}}, K^e_{\text{ee}}$)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEF" OF K^i :

$$\dot{a} = -\alpha a K^r_r + (a \beta)^' \quad (33)$$

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d) EVOLUTION EQUATIONS FOR K^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS DEF["] OF K^i :

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$$\dot{b} = -\alpha b K^e_e + \sum_r (r b)^' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}_{ij}^i (K_r^r, K_e^e)

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEFⁿ" OF K^i_j :

$$\dot{a} = -\alpha a K^r_{\text{rr}} + (a \beta)^{\prime} \quad (33)$$

$$\dot{b} = -\alpha b K^e_{\text{ee}} + \sum_r (rb)^{\prime} \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

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$$\dot{a} = -\alpha a K^r_{\text{rr}} + (a \beta)^{\prime} \quad (33)$$

$$\dot{b} = -\alpha b K^e_{\text{ee}} + \sum_r (rb)^{\prime} \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

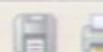
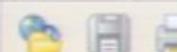
- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS DEF["] OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEFⁿ" OF K^i_j :

$$\dot{a} = -\alpha a K^r_n + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_n + \sum_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR K^i_j (K^r_n, K^e_n)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR \dot{r}_{ij} (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS DEF["] OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)^' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)^' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

- FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEF["] OF K^i :

$$\dot{a} = -\alpha a K^r_r + (a \beta)^' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)^' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}_{ij}^i (K_r^r, K_e^e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

c) EVOLUTION EQUATIONS FOR \dot{r}_{ij} (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEFⁿ" OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

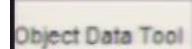
$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS DEF["] OF K^i_j :

$$\dot{a} = -\alpha a K^r_r + (a \beta)^r \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)^r \quad (34)$$

D) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

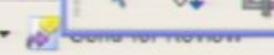
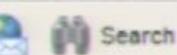
- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEF" OF K^i :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

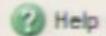
$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR K^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

- FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS "DEFⁿ" OF K^i :

$$\dot{a} = -\alpha a K^r_r + (a \beta)^r \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (r b)^r \quad (34)$$

d) EVOLUTION EQUATIONS FOR \dot{K}_{ij}^i (K_r^r, K_e^e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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c) EVOLUTION EQUATIONS FOR $\dot{\gamma}_{ij}$ (a, b)

- FOLLOW DIRECTLY FROM (11). RECALL, CAN BE VIEWED AS "DEFⁿ" OF K^i :

$$\dot{a} = -\alpha a K^r_r + (a \beta)' \quad (33)$$

$$\dot{b} = -\alpha b K^e_e + \sum_r (rb)' \quad (34)$$

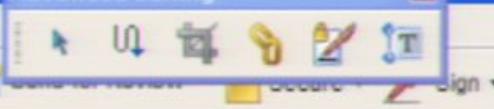
d) EVOLUTION EQUATIONS FOR \dot{K}^i_{ij} (K^r_r, K^e_e)

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE



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$$a = -\alpha a K_r + (a)^3 \quad (33)$$

$$b = -\alpha b K_e^e + \frac{1}{r} (rb)' \quad (34)$$

D) EVOLUTION EQUATIONS FOR K_{ij} (K_r, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\ddot{K}_r = \beta K_r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + KK_r - \frac{\dot{\alpha}^2}{a^2} \right) \quad (35)$$

PHY 3E7NSpherical Symmetry

⑤

$$\ddot{K}_e^e = \beta K_e^e' + K_r - \frac{1}{r} (\alpha rb (rb)')' + \alpha K K_e^e$$



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \frac{\dot{\alpha}^2}{a^2} \quad (35)$$

PHYSICSSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



d) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

e) FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

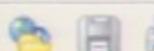
PHYSICS

Spherical Symmetry

5

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{1}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)'}{a} \right)' + K K_r^r - \frac{\dot{\alpha}^2}{a^2} \right) \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



b) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

c) FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

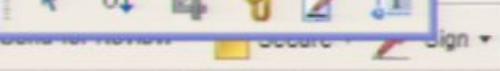
$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICAL

SYMMETRY

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \sin \left(\frac{\alpha}{a^2} \right) \quad (35)$$

PHYSICAL

SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \xi \pi \frac{\dot{a}^2}{a^2} \quad (35)$$

PHYSICAL

SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{\kappa}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (12) and other results above

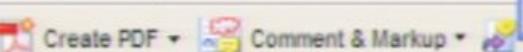
$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \frac{\dot{\alpha}^2}{a^2} \quad (35)$$

PHYSICS

Spherical Symmetry

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

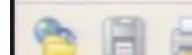
FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \sin \frac{\alpha^2}{a^2} \quad (35)$$

PHYSICAL
Spherical Symmetry

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{ir}^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi}{a^2} \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{1}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \text{err} \frac{\dot{\alpha}^2}{a^2} \quad (35)$$

PHYSICAL

Spherical Symmetry

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{r_{ab} (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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b) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

c) Follow directly from (32) and other results above

$$K_{ir}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \frac{\alpha^2}{a^2} \quad (35)$$

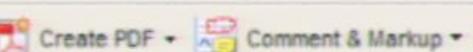
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BETWEEN

Spherical Symmetry

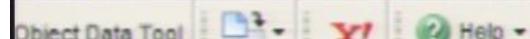
⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{\alpha}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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d) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

e) FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r - \frac{2\pi}{a^2} \left(\frac{\dot{\phi}}{a^2} \right)^2 \quad (35)$$

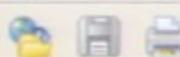
PHY 3E7N

SPHERICAL SYMMETRY

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

- FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha}^i)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r - \frac{\dot{\alpha}^i}{a^2} \quad (35)$$

PHYSICSSpherical Symmetry

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_{rr}' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_{rr} - \frac{2\pi}{a^2} \left(\frac{\dot{\alpha}}{a^2} \right)' \quad (35)$$

PHYSICAL

SYMMETRY

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



d) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

e) Follow directly from (12) and other results above

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYS 367N

Spherical Symmetry

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{1}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



b) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

c) FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICAL

SYMMETRY

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{1}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{r_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



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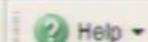


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D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (12) and other results above

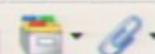
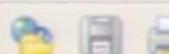
$$K_r^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' + K K_r - \frac{\dot{\alpha}^2}{a^2} \right) \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{\kappa}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (32) and other results above

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICAL
BETWEEN

Spherical Symmetry

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \frac{2\pi}{a^2} \left(\frac{\dot{\alpha}}{a^2} \right)' \quad (35)$$

PHYSICAL

SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (31) and other results above

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)'}{a} \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \right) \quad (35)$$

PHYSICAL
BETWEEN

Spherical Symmetry

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (32) and other results above

$$K_{ir}^i = \beta K_r^r' - \frac{1}{a} (\bar{a})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' + K K_r - \frac{\dot{\alpha}^2}{a^2} \right) \quad (35)$$

PHYSICAL

SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (12) and other results above

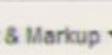
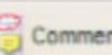
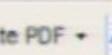
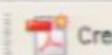
$$K_r^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r - \frac{2}{a^2} \quad (35)$$

PHYSICAL

SYMMETRY

(5)

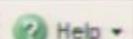
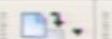
$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{r_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

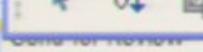
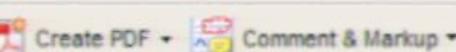
$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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Help

D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{ir}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \text{STI} \frac{\dot{\alpha}^2}{a^2} \quad (35)$$

PHYSICAL

SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



b) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICAL

SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r - \xi \pi \frac{\dot{a}^2}{a^2} \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{\kappa}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

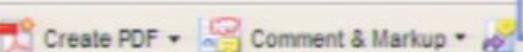
Follow directly from (12) and other results above

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICAL SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



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Help

D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \text{STI} \frac{\dot{\alpha}^2}{a^2} \quad (35)$$

PHYSICALSYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



d) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

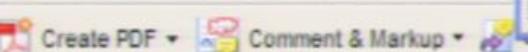
e) FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \frac{\alpha^2}{a^2} \quad (35)$$

PHYSICAL SYMMETRY

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{\kappa}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



b) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

c) FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \bar{\epsilon}_T \frac{\dot{a}^2}{a^2} \quad (35)$$

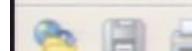
PHYSICAL

SYMMETRY

5

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{1}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

5



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D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

FOLLOW DIRECTLY FROM (32) AND OTHER RESULTS ABOVE

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r - \sin \frac{\phi^2}{a^2} \quad (35)$$

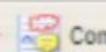
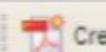
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BETWEEN

Spherical Symmetry

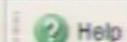
(35)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$

(36)



Object Data Tool



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_a^a)$

FOLLOW DIRECTLY FROM (31) AND OTHER RESULTS ABOVE

$$K_{ir}^i = \beta K_r^r - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)'}{a} \right)' + K K_r - \frac{\dot{\alpha}^2}{a^2} \right) \quad (35)$$

PHYSICAL Spherical Symmetry

(5)

$$K_a^a = \beta K_a^a + \frac{\kappa}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_a^a \quad (36)$$

(36)



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D) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

Follow directly from (32) and other results above

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K_r^r - \frac{\alpha^2}{a^2} \quad (35)$$

PHYSICAL
BETWEEN

Spherical Symmetry

(35)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



d) Evolution Equations for $K_{ij}^i (K_r^r, K_\theta^\theta)$

e) Follow directly from (32) and other results above

$$K_r^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYS 367N

Spherical Symmetry

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{1}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{r_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{r_b}{a} \right)' \right)' + K K_r^r - \sin \frac{\phi^2}{a^2} \quad (35)$$

PHYSICAL Spherical Symmetry

(5)

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{r_b}{a} (r_b)' \right)' + \alpha K K_\theta^\theta \quad (36)$$



⑦) EVOLUTION EQUATIONS FOR $K_{ij}^i (K_r^r, K_\theta^\theta)$

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$K_{rr}^i = \beta K_r^r' - \frac{1}{a} (\bar{a}')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' + K K_r^r - \frac{\dot{\alpha}^2}{a^2} \right) \quad (35)$$

PHYSICAL

SYMMETRY

⑤

$$K_\theta^\theta = \beta K_\theta^\theta' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{a r_b (r_b)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$



D) Evolution Equations for K^i_r (K^r_r, K^e_e)

Follow directly from (12) and other results above

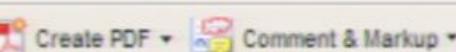
$$K^e_r = \beta K^r_r' - \frac{1}{a} (\dot{\alpha})' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' \right)' + K K^r_r - \sin \frac{\dot{\phi}}{a^2} \quad (35)$$

PHYSICAL

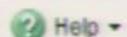
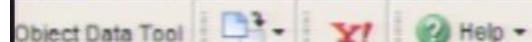
SYMMETRY

(5)

$$K^e_e = \beta K^e_e' + \frac{K}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K^e_e \quad (36)$$



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$$\dot{K}_r = \beta K_r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(r_b)}{a} \right)' + k_0 K_r - \sin \frac{\phi}{a^2} \right) \quad (35)$$

PHY 3E7N

Spherical Symmetry

⑤

$$\dot{K}_\theta = \beta K_\theta' + \frac{k}{(r_b)^2} - \frac{1}{a(r_b)^2} \left(\frac{dr_b}{a} (r_b)' \right)' + \alpha K K_\theta \quad (36)$$

• (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE GEOMETRIC VARIABLES (NOTE: WE HAVE MADE NOTHING YET DE COORDINATE CHOICES, I.E. DE SPECIFICATIONS OF α AND β)



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$$\ddot{K}_r = \beta K_r' - \frac{1}{a} (\alpha')' + \alpha \left(-\frac{2}{r_{ab}} \left(\frac{(rb)}{a} \right)' + k_0 K_r - \sin \frac{\alpha'}{a^2} \right) \quad (35)$$

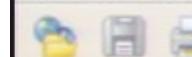
PHY 3E7N

Spherical Symmetry

⑤

$$\ddot{K}_0 = \beta K_0' + \frac{k}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{arb}{a} (rb)' \right)' + \alpha K K_0 \quad (36)$$

• (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE GEOMETRIC VARIABLES (NOTE: WE HAVE MADE NOTHING YET DE COORDINATE CHOICES, I.E. NO SPECIFICATIONS OF α AND β)



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(31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE GEOMETRIC VARIABLES (NOTE: WE HAVE MADE NOTHING YET RE COORDINATE CHOICES, I.E. RE SPECIFICATIONS OF α AND β)

MASSLESS KLEIN-GORDON EQUATION

NEED E.O.R. FOR $\bar{\phi}$ AND Π

* RECALL DEF'N OF Π , (23)

$$\Pi = \frac{\partial}{\partial t} (\dot{\phi} - j\phi')$$

$$\rightarrow \dot{\phi} = \frac{\partial}{\partial t} \Pi + j\phi' = \frac{\partial}{\partial t} \Pi + j\beta \bar{\phi}$$

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MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

* RECALL DEFⁿ OF π , (22)

$$\pi = \frac{\alpha}{\dot{x}} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{\alpha}{\dot{x}} \pi + \beta \phi' = \frac{\alpha}{\dot{x}} \pi + \beta \bar{\pi}$$

* BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\pi} + \frac{\alpha}{\dot{x}} \pi)' \quad (22)$$



MASSLESS KLEIN-GORDON EQUATION

NEED E.O.N. FOR $\bar{\psi}$ AND π

* RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{i} (\dot{\phi} - i\phi')$$

$$\rightarrow \dot{\phi} = \frac{i}{\alpha} \pi + \beta \phi' = \frac{i}{\alpha} \pi + \beta \bar{\psi}$$

* BUT $\dot{\phi}' = \bar{\psi}$, so

$$\dot{\bar{\psi}} = (\beta \bar{\psi} + \frac{\alpha}{i} \pi)'$$
(22)



MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

* RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{x} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{x}{\alpha} \pi + \beta \phi' = \frac{x}{\alpha} \pi + \beta \bar{\phi}$$

* BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\phi} + \frac{\alpha}{x} \pi)' \quad (22)$$



MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

* RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{x} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{x}{\alpha} \pi + \beta \phi' = \frac{x}{\alpha} \pi + \beta \bar{\pi}$$

* BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\pi} + \frac{\alpha}{x} \pi)'$$
(22)



MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

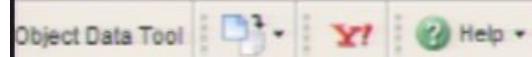
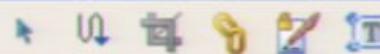
* RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{x} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{x}{\alpha} \pi + \beta \phi' = \frac{x}{\alpha} \pi + \beta \bar{\pi}$$

* BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\pi} + \frac{\alpha}{x} \pi)'$$
(22)



MASSLESS KLEIN-GORDON EQUATION

SEARCH E.Q.N. FOR $\bar{\phi}$ AND π

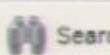
* RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{x} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{x}{\alpha} \pi + \beta \phi' = \frac{x}{\alpha} \pi + \beta \bar{\phi}$$

* BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\phi} + \frac{\alpha}{x} \pi)' \quad (22)$$



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MASSLESS KLEIN-GORDON EQUATIONSHEED E.QN. FOR $\bar{\phi}$ AND π • RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{\dot{x}} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{\alpha}{\dot{x}} \pi + \beta \phi' = \frac{\alpha}{\dot{x}} \pi + \beta \bar{\pi}$$

• BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\pi} + \frac{\alpha}{\dot{x}} \pi)' \quad (22)$$



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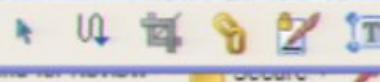


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MASSLESS KLEIN-GORDON EQUATION

SEARCH E.Q.N. FOR $\bar{\phi}$ AND π

* RECALL DEF'N OF π , (22)

$$\pi = \frac{\alpha}{x} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{x}{\alpha} \pi + \beta \phi' = \frac{x}{\alpha} \pi + \beta \bar{\pi}$$

* BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\bar{\phi}} = (\beta \bar{\pi} + \frac{\alpha}{x} \pi)' \quad (22)$$



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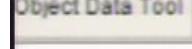
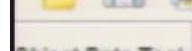
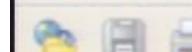
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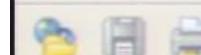
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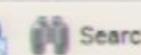
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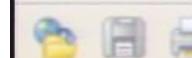
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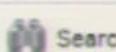
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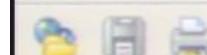
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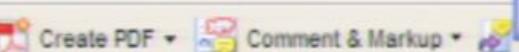
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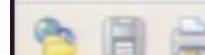
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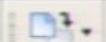


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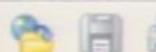
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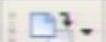
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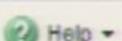
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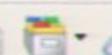
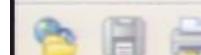
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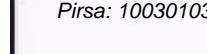
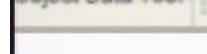
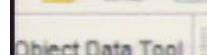
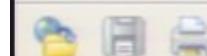
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MASSLESS KLEIN-GORDON EQUATIONSHEED E.O.N. FOR $\bar{\phi}$ AND π • RECALL DEFⁿ OF π , (22)

$$\pi = \frac{\alpha}{x} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{x}{\alpha} \pi + \beta \phi' = \frac{x}{\alpha} \pi + \beta \bar{\pi}$$

• BUT $\dot{\phi}' = \bar{\phi}$, so

$$\dot{\phi} = (\beta \bar{\pi} + \frac{\alpha}{x} \pi)'$$
 (32)



MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

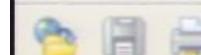
* RECALL DEFⁿ OF π , (22)

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(32)



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MASSLESS KLEIN-GORDON EQUATION

SEARCH E.Q.N. FOR $\bar{\phi}$ AND π

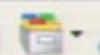
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(22)



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MASSLESS KLEIN-GORDON EQUATIONNEED E.O.R. FOR $\bar{\psi}$ AND π • RECALL DEF'N OF π , (23)

$$\pi = \frac{\alpha}{i} (\dot{\phi} - i\phi')$$

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• BUT $\dot{\phi}' = \bar{\psi}$, so

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MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

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MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

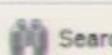
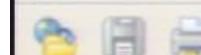
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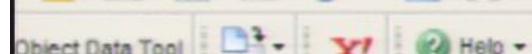
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MASSLESS KLEIN-GORDON EQUATION

SEARCH E.QN. FOR $\bar{\phi}$ AND π

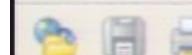
* RECALL DEF'N OF π , (23)

$$\pi = \frac{\partial}{\partial t} (\dot{\phi} - \beta \dot{\phi}')$$

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MASSLESS KLEIN-GORDON EQUATION

NEED E.O.M. FOR $\bar{\psi}$ AND π

* RECALL DEFⁿ OF π , (23)

$$\pi = \frac{\partial}{\partial t} (\dot{\phi} - J\phi')$$

$$\rightarrow \dot{\phi} = \frac{\partial}{\partial t} \pi + J\phi' = \frac{\partial}{\partial t} \pi + J\bar{\psi}$$

* BUT $\dot{\phi}' = \bar{\psi}$, so

$$\dot{\bar{\psi}} = (J\bar{\psi} + \frac{\partial}{\partial t} \pi)' \quad (32)$$



MASSLESS KLEIN-GORDON EQUATION

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MASSLESS KLEIN-GORDON EQUATION

NEED E.O.R. FOR $\bar{\phi}$ AND π

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MASSLESS KLEIN-GORDON EQUATION

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* BUT $\dot{\phi}' = \bar{\phi}$, so

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(22)



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* RECALL DEF' OF Π , (21)

$$\Pi = \frac{\alpha}{2} (\dot{\phi}^2 - \omega^2 \phi^2)$$

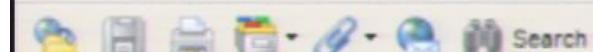
$$\rightarrow \dot{\phi} = \frac{\partial}{\partial t} \Pi + \omega \phi = \frac{\partial}{\partial t} \Pi + \omega \bar{\phi}$$

* BUT $\dot{\phi}' = \dot{\bar{\phi}}$, so

$$\dot{\bar{\phi}} = (\omega \bar{\phi} + \frac{\partial}{\partial t} \Pi)' \quad (32)$$

* TO FIND $\bar{\Pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{c^2} \nabla^2 (\text{Lagrangian})$$



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$$\rightarrow \dot{\phi} = \frac{\alpha}{a}\pi + \beta\dot{e}^t = \frac{\alpha}{a}\pi + \beta\dot{E}$$

* BUT $\dot{\phi}' = \dot{\bar{E}}$, so

$$\dot{\bar{E}} = (\beta\dot{E} + \frac{\alpha}{a}\pi)' \quad (32)$$

* TO FIND $\dot{\pi}$ EQU, RECALL THAT

$$\square\phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$\square\phi = 0 \rightarrow \partial_i (\Gamma_{ij} g^{uv} \partial_u \phi) = 0$$



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$$\dot{\phi} = (\omega \bar{e} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau) \quad (32)$$

TO FIND $\dot{\tau}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_g} \tau_u (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \tau_u (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \tau_u (F_g g^{\mu\nu} \partial_\nu \phi) + \tau_v (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$= \tau_u (\alpha a r^2 b^2 (-\omega^2 \phi + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau) \quad (32)$$

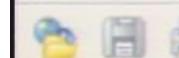
→ TO FIND $\dot{\tau}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

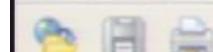
→ TO FIND $\dot{\pi}$ EQU, RECALL THAT

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$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square d = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v d)$$

$$\square d = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v d) + \partial_v (\Gamma_{ij} g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 d'))$$



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$$\dot{\phi} = (\omega \bar{z} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\bar{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_g} \bar{\pi} (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \bar{\pi} (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \partial_\mu (\bar{\pi} F_g g^{\mu\nu} \partial_\nu \phi) + \partial_\nu (\bar{\pi} F_g g^{\mu\nu} \partial_\mu \phi)$$

$$= \bar{\pi} (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
(32)

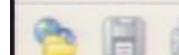
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square d = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v d)$$

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$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v d) + \partial_v (\Gamma_{ij} g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 d'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau) \quad (32)$$

→ TO FIND τ EQU, RECALL THAT

$$\nabla d = \frac{1}{F_g} \times (F_g g^{\mu\nu} \partial_\nu d)$$

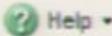
$$\nabla d = 0 \rightarrow \partial_\mu (F_g g^{\mu\nu} \partial_\nu d) = 0$$

$$\rightarrow \partial_\mu (F_g g^{\mu\nu} \partial_\nu d) + \partial_\nu (F_g g^{\mu\nu} \partial_\mu d) = 0$$

$$= \partial_\mu (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \dot{d}))$$



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$$\dot{\phi} = (\lambda \bar{e} + \frac{\alpha}{a} \pi)'$$

(32)

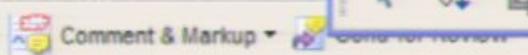
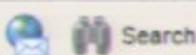
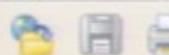
→ TO FIND $\bar{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\bar{F}_{jk} g^{jk} \partial_k \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\bar{F}_{jk} g^{jk} \partial_k \phi) = 0$$

$$\rightarrow \partial_i (\bar{F}_{jk} g^{jk} \partial_k \phi) + \partial_r (\bar{F}_{jk} g^{jk} \partial_r \phi) = 0$$

$$= \partial_1 (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\lambda \dot{x} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

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$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \bar{z} + \frac{\alpha}{a} \pi)'$$
(32)

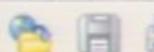
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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\ddot{\pi}$ EQU, RECALL THAT

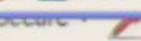
$$\square \phi = \frac{1}{F_g} \nabla \cdot (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \nabla \cdot (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \partial_\mu (F_g g^{\mu\nu} \partial_\nu \phi) + \partial_\nu (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$

(32)

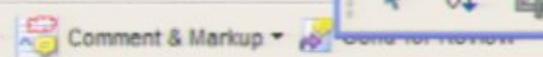
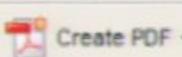
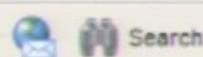
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

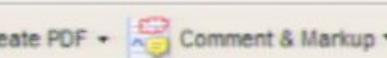
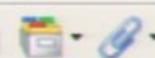
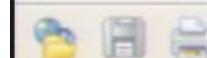
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

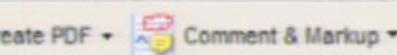
$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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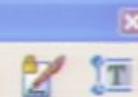


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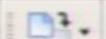
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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square d = \frac{1}{F_g} \rightarrow (F_g g^{\mu\nu} \partial_\nu d)$$

$$\square d = 0 \rightarrow \partial_\mu (F_g g^{\mu\nu} \partial_\nu d) = 0$$

$$\rightarrow \partial_\mu (F_g g^{\mu\nu} \partial_\nu d) + \partial_\nu (F_g g^{\mu\nu} \partial_\mu d) = 0$$

$$= \partial_\mu (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 d'))$$



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$$\dot{\phi} = (\lambda \bar{e} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\bar{\pi}$ EASILY, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \lambda \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
(32)

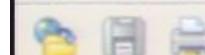
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 l^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND π EQU, RECALL THAT

$$\square \phi = \frac{1}{F_g g} \partial_\mu (\bar{F}_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \partial_\mu (\bar{F}_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \partial_\mu (\bar{F}_g g^{\mu\nu} \partial_\nu \phi) + \partial_\nu (\bar{F}_g g^{\mu\nu} \partial_\mu \phi) = 0$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\dot{\tau}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{gg}} \Im (F_{gg}^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \Im (F_{gg}^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \Im (F_{gg}^{\mu\nu} \partial_\nu \phi) + \Re (F_{gg}^{\mu\nu} \partial_\nu \phi)$$

$$= \Im (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

• TO FIND $\dot{\tau}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



$$\dot{\phi} = (\omega \bar{z} + \frac{\alpha}{a} \pi)'$$
(32)

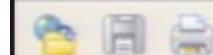
→ TO FIND $\bar{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

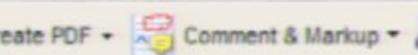
$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\dot{\tau}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\lambda \dot{z} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \lambda \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$

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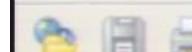
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\dot{\alpha}^2 \dot{\phi} + \lambda \dot{\alpha}^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\dot{\tau}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$

(32)

→ TO FIND π EQU, RECALL THAT

$$\square d = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v d)$$

$$\square d = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v d) + \partial_v (\Gamma_{ij} g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 d'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



$$\dot{\phi} = (\omega \pm i\frac{\alpha}{a}\pi)'$$
(32)

→ TO FIND $\vec{\pi}$ EOM, RECALL THAT

$$\square d = \frac{1}{F_g} \vec{\pi} \cdot (\vec{F}_g g^{\mu\nu} \partial_\nu d)$$

$$\square d = 0 \rightarrow \vec{\pi} \cdot (\vec{F}_g g^{\mu\nu} \partial_\nu d) = 0$$

$$\rightarrow \vec{\pi} \cdot (\vec{F}_g g^{\mu\nu} \partial_\nu d) + \vec{\pi} \cdot (\vec{F}_g g^{\nu\mu} \partial_\nu d)$$

$$= \vec{\pi} \cdot (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 d'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi) t \quad (32)$$

→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \dot{\phi}'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
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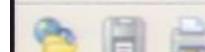
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\lambda \dot{x} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \lambda \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_r (\Gamma_{ij} g^{rv} \partial_v \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$

$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

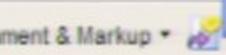
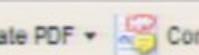
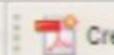
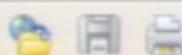
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \bar{e} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU, RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \bar{e} + \frac{\alpha}{a} \pi)'$$
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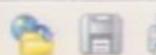
→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
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→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

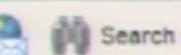
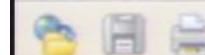


$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
(32)

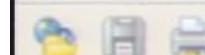
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau) \quad (32)$$

• TO FIND $\ddot{\phi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_g} \nabla \cdot (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \nabla \cdot (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \partial_\mu (F_g g^{\mu\nu} \partial_\nu \phi) + \partial_\nu (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$= \partial_\mu (\partial_\nu \phi \partial^\nu \phi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi)$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm + \frac{\alpha}{a} \pi)'$$
(32)

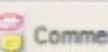
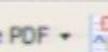
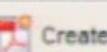
→ TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \lambda \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau) \quad (32)$$

→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_r (\Gamma_{ij} g^{rv} \partial_v \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi) = 0$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)^\dagger \quad (32)$$

→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_g} \Im \left(F_g g^{\mu\nu} \partial_\nu \phi \right)$$

$$\square \phi = 0 \rightarrow \Im \left(F_g g^{\mu\nu} \partial_\nu \phi \right) = 0$$

$$\rightarrow \Im \left(F_g g^{\mu\nu} \partial_\nu \phi \right) + \Re \left(F_g g^{\mu\nu} \partial_\nu \phi \right)$$

$$= \Im \left(\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi') \right)$$



$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \pi)'$$
(32)

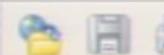
→ TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi)$$

$$\square \phi = 0 \rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = 0$$

$$\rightarrow \partial_i (\sqrt{g} g^{ij} \partial_j \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi)$$

$$= \partial_i (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$

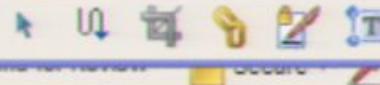


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$$\dot{\phi} = (\omega \bar{z} + \frac{\alpha}{a} \pi)'$$
(32)

→ TO FIND $\bar{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_g} \bar{\pi} (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \bar{\pi} (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \bar{\pi} (F_g g^{\mu\nu} \partial_\nu \phi) + \bar{\pi}_r (F_g g^{rr} \partial_r \phi)$$

$$= \bar{\pi} (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau) \quad (32)$$

• TO FIND $\ddot{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\Gamma_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\Gamma_{ij} g^{uv} \partial_v \phi) + \partial_v (\Gamma_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



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$$\dot{\phi} = (\omega \pm \frac{\alpha}{a} \tau)'$$
(32)

→ TO FIND $\dot{\tau}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_g} \Im (F_g g^{\mu\nu} \partial_\nu \phi)$$

$$\square \phi = 0 \rightarrow \Im (F_g g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\rightarrow \partial_t (\Im g^{\mu\nu} \partial_\nu \phi) + \partial_r (\Im g^{\mu\nu} \partial_\nu \phi)$$

$$= \partial_t (\alpha r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \hat{e} + \frac{\alpha}{a} \pi)'$$
(32)

• TO FIND $\dot{\pi}$ EQU. RECALL THAT

$$\square d = \frac{1}{F_g} \Im (F_g g^{\mu\nu} \partial_\nu d)$$

$$\square d = 0 \rightarrow \Im (F_g g^{\mu\nu} \partial_\nu d) = 0$$

$$\rightarrow \partial_\mu (\Im F_g g^{\mu\nu} \partial_\nu d) + \partial_\nu (\Re F_g g^{\mu\nu} \partial_\nu d)$$

$$= \partial_\mu (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 d'))$$



$$\dot{\phi} = (\omega \pm i\frac{\alpha}{a}\tau)^\frac{1}{2} \quad (32)$$

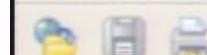
→ TO FIND $\dot{\pi}$ EQU, RECALL THAT

$$\square\phi = \frac{1}{F_g} \nabla^2 (\nabla_g g^{uv} \partial_v \phi)$$

$$\square\phi = 0 \rightarrow \nabla^2 (\nabla_g g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\nabla_g g^{v\perp} \partial_v \phi) + \partial_v (\nabla_g g^{u\perp} \partial_v \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\omega^2 \dot{\phi} + J \omega^2 \phi'))$$



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$$\dot{\phi} = (\omega \bar{z} + \frac{\alpha}{a} \pi)'$$

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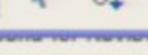
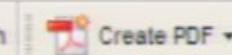
→ TO FIND $\bar{\pi}$ EQU. RECALL THAT

$$\square \phi = \frac{1}{F_{ij}} \partial_i (\bar{F}_{ij} g^{uv} \partial_v \phi)$$

$$\square \phi = 0 \rightarrow \partial_u (\bar{F}_{ij} g^{uv} \partial_v \phi) = 0$$

$$\rightarrow \partial_u (\bar{F}_{ij} g^{uv} \partial_v \phi) + \partial_v (\bar{F}_{ij} g^{uv} \partial_u \phi)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + J \alpha^2 \phi'))$$



→ TO FIND Π EQU, RECALL THAT

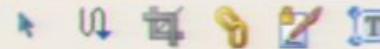
$$\square \phi = \frac{1}{F_g} \sum (\bar{F}_g g^{\mu\nu} \partial_\mu \phi)$$

$$\square \phi = 0 \rightarrow \sum (\bar{F}_g g^{\mu\nu} \partial_\mu \phi) = 0$$

$$\rightarrow \partial_t (\bar{F}_g g^{tt} \partial_t \phi) + \partial_r (\bar{F}_g g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\dot{x}^2 \dot{\phi} + \lambda \dot{x}^2 \dot{\phi}'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\dot{a}^2 - \lambda^2 \dot{x}^2) \dot{\phi}' + \lambda \dot{x}^2 \dot{\phi}'))$$



$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

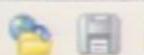
$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \beta \alpha^2 d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^2 - \beta^2 \alpha^2) d' + \beta \alpha^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{c} \dot{\pi}))'$$



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$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

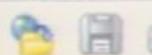
$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{d}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{2} \dot{\pi}))'$$



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$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{d}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{c} \dot{\pi}))'$$



$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{d}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$



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$$\bar{F}_g = \left(\cdot \cdot \cdot \right)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{d}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$



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$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\phi}))'$$



$$\bar{F}_g = (\dots)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

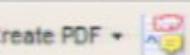
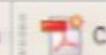
$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

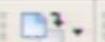
$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{r} \dot{\pi}))'$$



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$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \beta \alpha^2 d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^2 - \beta^2 \alpha^2) d' + \beta \alpha^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$



$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^2 \dot{\phi} + \beta \alpha^2 d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^2 - \beta^2 \alpha^2) d' + \beta \alpha^2 \dot{d}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{r} \dot{\pi}))'$$



$$\bar{F}_g = (\cdot \cdot \cdot)$$

$$\square d = 0 \rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) = 0$$

$$\rightarrow \partial_u (\bar{F}_g g^{uv} \partial_v d) + \partial_v (\bar{F}_g g^{uv} \partial_u d)$$

$$= \partial_u (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} d'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) d' + \beta \alpha^{-2} \dot{d}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{r} \dot{\pi}))'$$



$$\rightarrow \partial_t (\sqrt{g} g^{vt} \partial_v \phi) + \partial_v (\sqrt{g} g^{vt} \partial_v \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\dot{\alpha} r^2 \dot{\phi} + \lambda r^2 \dot{\phi}'))$$

$$+ \partial_v (\alpha a r^2 b^2 ((\dot{\alpha} - \lambda^2 \dot{\alpha}^2) \dot{\phi}' + \lambda \dot{\alpha}^2 \dot{\phi}''))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

↪ WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Evaluation equation (39) FOR b



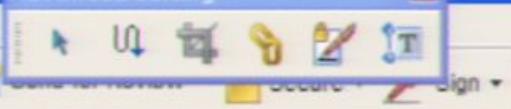
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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

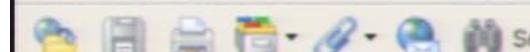
$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

↪ WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EVALUATION EQUATION (39) FOR b

$$\ddot{\pi} = - \frac{1}{r^2} (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

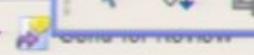
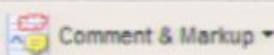
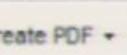
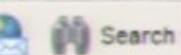
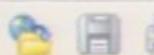
$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$



$$\begin{aligned}
 & \rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\
 & = \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\
 & + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\
 & = 0 \\
 & \rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))' \\
 & \hookrightarrow \text{WRITE AS } (r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi} \text{ AND USE} \\
 & \text{Euler-Lagrange eqn (39) FOR } b
 \end{aligned}$$



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm \left(r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}) \right)'$$



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$$\begin{aligned}
 & \rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\
 & = \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\
 & + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\
 & = 0 \\
 & \rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))' \\
 & \hookrightarrow \text{WRITE AS } (r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi} \text{ AND USE} \\
 & \text{Euler-Lagrange eqn (39) for } b
 \end{aligned}$$



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
Euler-Lagrange eqn (39) for b



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

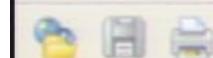
$$= \partial_t (x a r^2 b^2 (-\alpha^{rr} \dot{\phi} + \beta \alpha^{rr} \dot{\phi}'))$$

$$+ \partial_r (x a r^2 b^2 ((\alpha^{rr} - \beta^2 \alpha^{rr}) \dot{\phi}' + \beta \alpha^{rrr} \dot{\phi})) = 0$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

↪ write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for $\dot{\pi}$



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

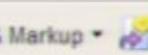
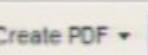
$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

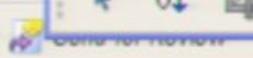
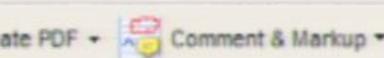
$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

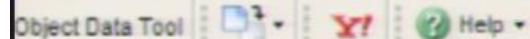
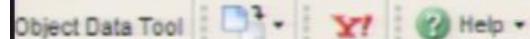
$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

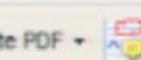
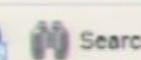
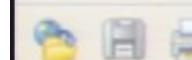
$$+ \partial_r (x a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi'$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$



$$\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha^2 \dot{\phi} + \beta \alpha^2 \dot{\phi}'))$$

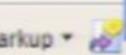
$$+ \partial_r (x a r^2 b^2 ((\alpha^2 - \beta^2 \alpha^2) \dot{\phi}' + \beta \alpha^2 \dot{\phi}))$$

$$= 0$$

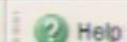
$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm \left(r^2 b^2 \left(\beta \pi + \frac{x}{a} \dot{\pi} \right) \right)'$$



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

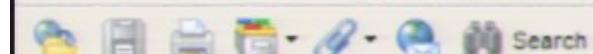
$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

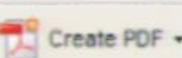
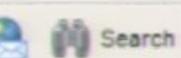
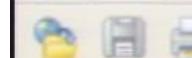
$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

↪ write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
Euler-Lagrange eqn (39) for b



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$$\begin{aligned}
 & \rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\
 & = \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\
 & + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\
 & = 0
 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

↶ write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
 Evolution equation (39) for \dot{b}



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$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

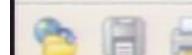
$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (x a r^2 b^2 (-\alpha'^2 \dot{\phi} + \beta \alpha'^2 \dot{\phi}'))$$

$$+ \partial_r (x a r^2 b^2 ((\alpha'^2 - \beta^2 \alpha'^2) \dot{\phi}' + \beta \alpha'^2 \dot{\phi})) = 0$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

↪ write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for $\dot{\pi}$

$$\ddot{\pi} = \pm \left(r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}) \right)'$$



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b

$$\ddot{\pi} = \pm (r^2 b^2 (\beta \pi + \frac{x}{a} \dot{\pi}))'$$



$$\rightarrow \partial_t (\sqrt{g} g^{rr} \partial_r \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi)$$

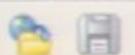
$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

\hookrightarrow write as $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Euler-Lagrange eqn (39) for b



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$$= 0$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \bar{z}))'$$

↪ write as $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
Evaluation equation (39) FOR b

$$\dot{\pi} = \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \bar{z} \right) \right)'$$

(39)

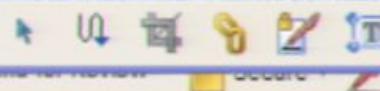
$$+ 2 \left(\alpha K^e - \beta \frac{(r h)'}{r b} \right) \pi$$



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$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \bar{E}))'$$

↪ WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
Evaluation equation (39) FOR \dot{b}

$$\dot{\pi} = \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \bar{E} \right) \right)'$$

(39)

$$+ 2 \left(\alpha K^* e - \beta \frac{(rb)'}{rb} \right) \pi$$



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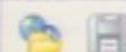
$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \bar{E}))'$$

↪ WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
Evaluation equation (34) FOR \dot{b}

$$\dot{\pi} = \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \bar{E} \right) \right)'$$

(35)

$$+ 2 \left(\alpha K^* e - \beta \frac{(r \dot{b})'}{r b} \right) \pi$$



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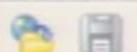
$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{\pi}))'$$

↪ WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
Evaluation equation (34) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \dot{\pi} \right) \right)'$$

(35)

$$+ 2 \left(\alpha K^* e - \beta \left(\frac{rh}{rb} \right)' \right) \pi$$



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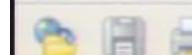
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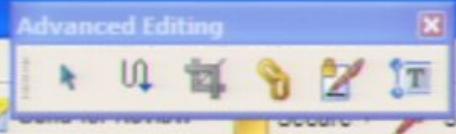
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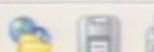
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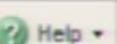
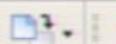
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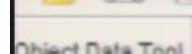
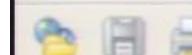
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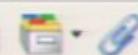
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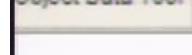
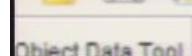
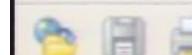
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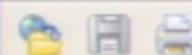


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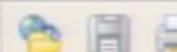
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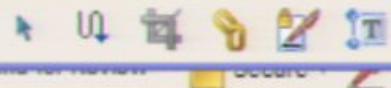
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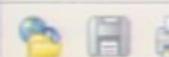
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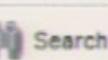
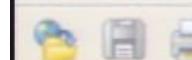
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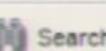
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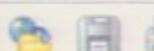
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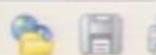
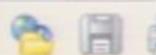
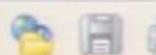
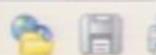
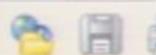
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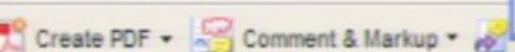
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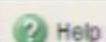
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$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \bar{z}))'$$

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(39)

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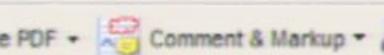
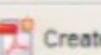
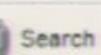
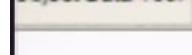
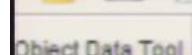
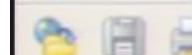
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↪ WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
Evaluation equation (34) FOR \dot{b}

$$\dot{\pi} = \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \bar{E} \right) \right)'$$

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$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \dot{b}))'$$

↪ WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
Evaluation equation (39) FOR \dot{b}

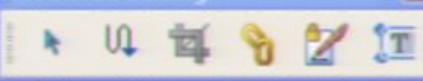
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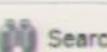
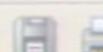
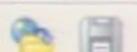
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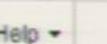
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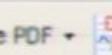
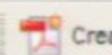
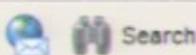
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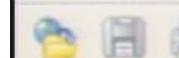
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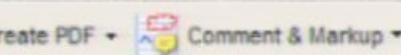
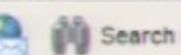
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