

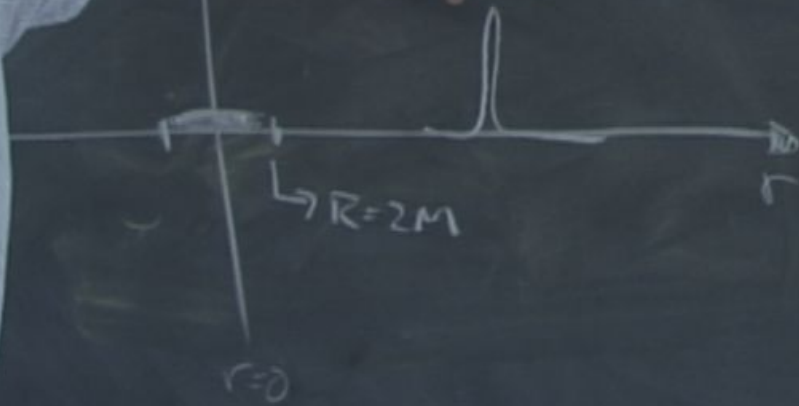
Title: Explorations in Numerical Relativity (PHYS 642) - Lecture 4

Date: Mar 18, 2010 11:20 AM

URL: <http://pirsa.org/10030103>

Abstract:

$$\phi(0, r)$$



$$\phi(t, r)$$

$$R=2M$$

$$r=0$$

t

$$\phi(t, r)$$

$$\rightarrow R=2M$$

$$r=0$$

ϕ

r

$$\phi(a, r)$$

Δ

$$\rightarrow R=2M$$

$$r=0$$

Δ

$$\phi(a, r)$$

 Δ Δ

$$\rightarrow R=2M$$

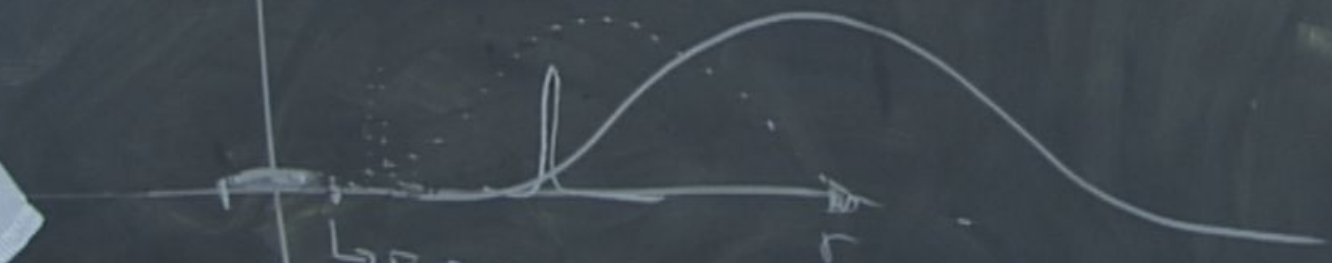
$$r=0$$

 Δ

$$\phi(\omega, r)$$



$$\rightarrow R=2M$$



$$\phi(t, r)$$

$$\phi(t, r)$$

$$\rightarrow R=2M$$

$$r=0$$

$$\phi(0, r)$$

$$\phi(1, r) \neq A$$

$$R = 2M$$

$$r = 0$$

 δ

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$$\phi(1,r) \neq A$$

$$R=2M$$

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$$\phi(0, r)$$

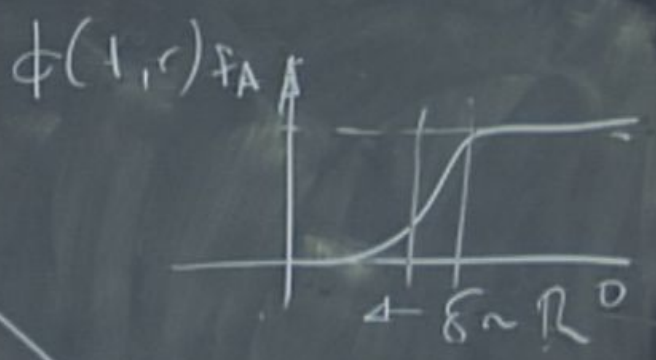
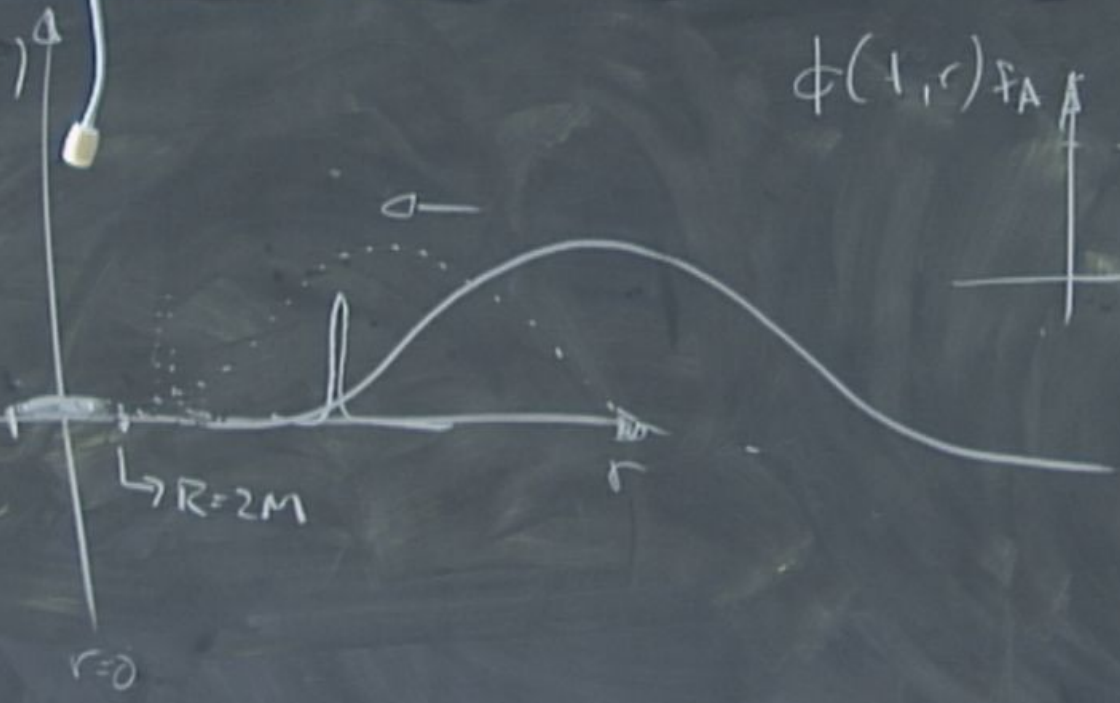
$$\phi(1, r) \neq A$$

$$\rightarrow R=2M$$

$$r=0$$

$$\delta \sim R^0$$

$(t, r) = 0$ $\phi(t, r)$
 MASSLESS
 K.G.



4D

$(3+1)D$

ω_{MP}

$${}^{(3)}g_{ij} \equiv \gamma_{ij}$$

4D

$g_{\mu\nu} \Rightarrow 10$ IND COMP

$(3+1)D$

$(3) \quad g_{ij} \equiv \gamma_{ij} \Rightarrow 6$

α (LAPSE) 1

β^i (SHIFT VEC) 3

10

4D

(3+1)D

$\Rightarrow 10$ IND COMP

\Rightarrow

$(3) \quad g_{ij} \equiv \gamma_{ij} \Rightarrow 6$

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$$(3+1)D$$

$$(3) \quad g_{ij} \equiv \gamma_{ij} \Rightarrow 6$$

$$\alpha \text{ (LAPSE)} \quad 1$$

$$\beta^i \text{ (SHIFT VEC)} \quad 3$$

$$\underline{10}$$

4D

(3+1)D

$\mu \Rightarrow 10$ IND COMP

$\beta \Pi T_{\mu\nu} \Rightarrow 10$ 2ND ORDER
EQNS
- 4 CONS

(3) $g_{ij} \equiv \gamma_{ij} \Rightarrow 6$

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β^i (SHIFT VEC) 3

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4D

10 IND COMP

\Rightarrow 10 2ND ORDER
EQNS
- 4 CONS

6 (2ND ORDER
EQUATIONS)

(3+1)D

(3) $g_{ij} \equiv \gamma_{ij} \Rightarrow 6$

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4D

\Rightarrow 10 IND COMP

\Rightarrow 10 2ND ORDER
EQNS
- 4 CONS

6 2ND ORDER
EQUATIONS

(3+1)D

(3) $g_{ij} \equiv \gamma_{ij} \Rightarrow 6$

α (LAPSE) 1

β^i (SHIFT VEC) 3

10

Advanced Editing

THE 3+1 EINSTEIN EQUATIONS

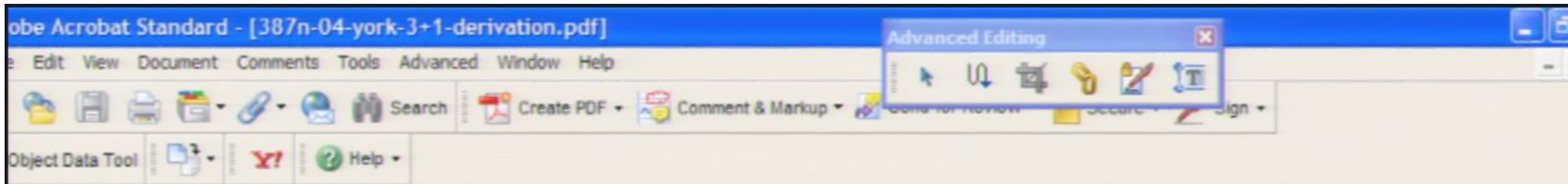
These notes rework the calculation of the 3+1 equations as presented in *Kinematics and Dynamics of General Relativity*, by J. W. York, Jr., which itself is contained in the volume *Sources of Gravitational Radiation*, edited by L. Smarr. Many calculational details omitted from that source are included here.

(**Note:** A colleague of mine, who will remain anonymous this week, has used these notes to demonstrate the power of his symbolic manipulation software and has found at least one error amongst the many “intermediate” results that are derived below. There is a \$234 Argentinian peso reward for the PSI student who first identifies such an error and who submits a hand-written explanation of the gaffe, along with the corrected expression, to the tutor.)

1) Foliations and Normals

As before, we consider a spacetime M with metric g_{ab} which is sliced into a foliation $\{\Sigma\}$ defined by the isosurfaces of a scalar field τ (the time parameter). Then the spacelike hypersurfaces are, at least locally, described by a *closed* one-form (dual vector field), Ω_a :

$$\Omega_a = \nabla_a \tau . \tag{1}$$



THE 3+1 EINSTEIN EQUATIONS

These notes rework the calculation of the 3+1 equations as presented in *Kinematics and Dynamics of General Relativity*, by J. W. York, Jr., which itself is contained in the volume *Sources of Gravitational Radiation*, edited by L. Smarr. Many calculational details omitted from that source are included here.

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$$\Omega_a = \nabla_a \tau. \quad (1)$$



PH 387N SPHERICAL SYMMETRY (SS) (9)

- IN SS SPACETIME NEED MATTER FOR DYNAMICS
(BIRKHOFF'S THM, UNIQUENESS OF SCHWARZSCHILD
SOLⁿ AS SOLⁿ of $G_{ab} = 0$)
- WILL RESTRICT MATTER CONTENT TO SINGLE, MASSLESS,
MINIMALLY-COUPLED SCALAR FIELD, ϕ
 - GOOD MODEL PROBLEM FOR STUDYING STAB-
FIELD, RADIATIVE S.T.'S - INCLUDING BLACK
HOLE FORMATION
 - EXHIBITS INTERESTING PHYSICAL BEHAVIOUR -
CRITICAL PHENOMENA - aka BLACK HOLE THRESHOLD
PHENOMENA

LAGRANGIAN DENSITY FOR EMKG SYSTEM

$$L = L_{\text{ADM}} + L_{\phi}$$

$$= \sqrt{-g} \left(R - \frac{1}{2} \nabla_a \phi \nabla^a \phi \right)$$

"CONSTRAINT" E.O.M 

$$G_{ab} = 8\pi T_{ab} = 8\pi \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi \right)$$

$$\square \phi = \nabla^a \nabla_a \phi = 0$$

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3+1 FORM of SPACETIME METRIC IN SS

o COORDINATES (t, r, θ, ϕ) ADAPTED TO S.S.

METRIC ON UNIT 2-SPHERE $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$

3.1 FORM of SPACETIME METRIC IN SS

o COORDINATES (t, r, θ, ϕ) ADAPTED TO S.S.

METRIC ON UNIT 2-SPHERE $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$

o THEN MOST GENERAL 3-METRIC IS

$$h_{ij} = \text{diag}(a^2(r,t), r^2 b^2(r,t), r^2 b^2 \sin^2\theta) \quad (1)$$

THE LAPSE FUNCTION IS $\alpha(r,t)$, AND THE SHIFT VECTOR $\beta^i(r,t)$ HAS ONLY A RADIAL COMPONENT, $\beta(r,t)$

• THEN MOST GENERAL 3-METRIC IS

$$\gamma_{ij} = \text{diag} (a^2(r,t), r^2 b^2(r,t), r^2 b^2 \sin^2 \epsilon) \quad (1)$$

THE LAPSE FUNCTION IS $\alpha(r,t)$, AND THE SHIFT VECTOR $\beta^i(r,t)$ HAS ONLY A RADIAL COMPONENT, $\beta(r,t)$

$$\beta^i = (\beta, 0, 0) \quad (2)$$

$$\beta_i = \gamma_{ij} \beta^j = (a^2 \beta, 0, 0) \quad (3)$$

• THE MOST GENERAL 4-METRIC IS THEN

$$ds^2 = (-\alpha^2 + \beta^i \beta_i) dt^2 + 2 \beta_i dt dx^i + \gamma_{ij} dx^i dx^j$$

THE MOST GENERAL 3-METRIC IS

$$\gamma_{ij} = \text{diag} (a^2(r,t), r^2 b^2(r,t), r^2 b^2 \sin^2 \epsilon) \quad (1)$$

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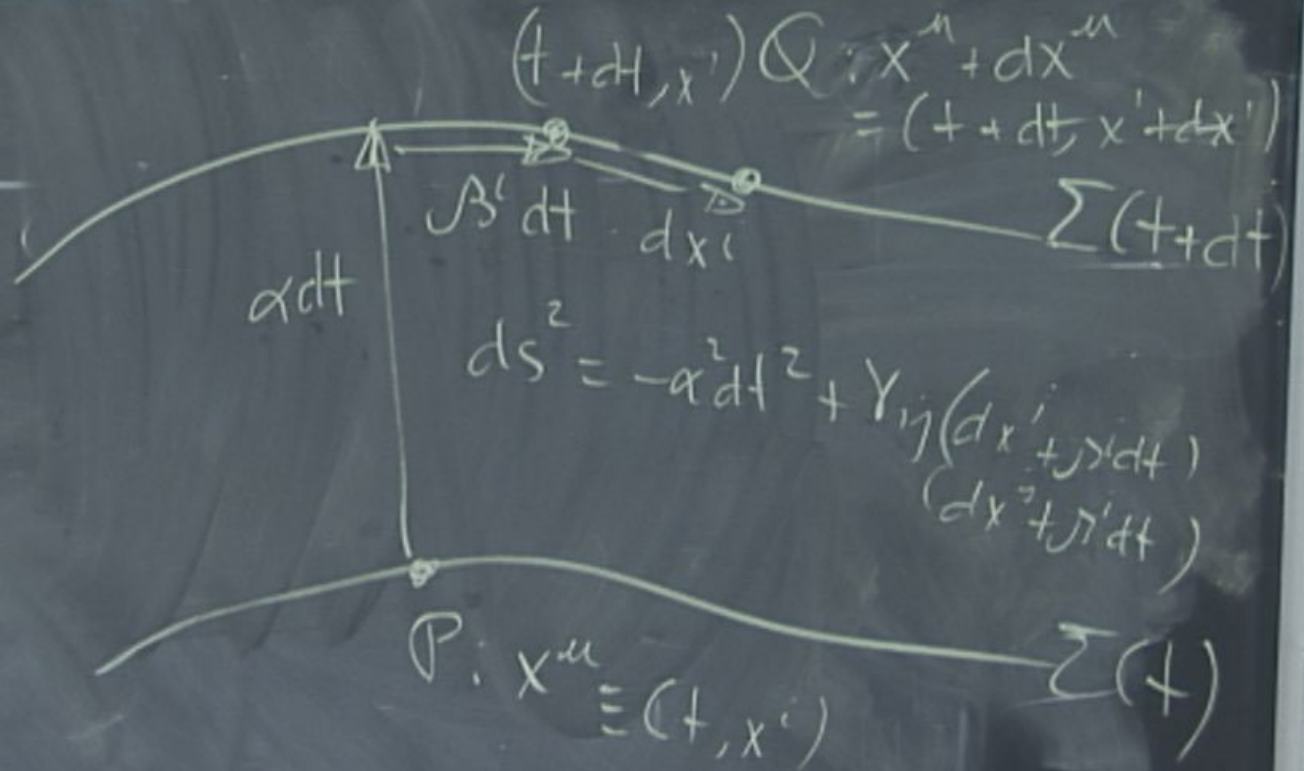
4D

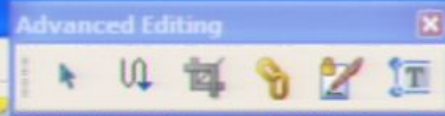
10 IND WMP

$T_{\mu\nu} \Rightarrow$ 10 2ND ORDER
EQNS

- 4 CONS

6 2ND ORDER
EQUATIONS





THE MOST GENERAL 4-METRIC IS THEN

$$ds^2 = (-\alpha^2 + \beta^i \beta_i) dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j$$

$$= (-\alpha^2 + a^2 \beta^2) dt^2 + 2a^2 \beta dr dt + a^2 dr^2 + r^2 b^2 d\theta^2 \quad (a)$$



CORRESPONDING EXTRINSIC CURVATURE TENSOR, K^i_j , LIKE γ_{ij} , HAS ONLY TWO INDEPENDENT COMPONENTS

$$K^i_j = \text{diag}(K^r_r(r,t), K^e_e(r,t), K^e_e) \quad (5)$$

EASY TO SHOW EQUALITY FROM

$$K_{ij} = (2\alpha)^{-1} (-\partial_t \gamma_{ij} + D_i \beta_j + D_j \beta_i)$$

$K_{ij} = (2\pi)^{-1} (\dots)$

PHYS 321

SPHERICAL SYMMETRY

(2)

SO, HAVE REDUCED TOTAL # OF GRAVITATIONAL KIN. ...
 DIM VOLS FROM 16 TO 6, AND, OF COURSE, THESE
 VOLS ARE FUNCTIONS ONLY OF (r, t) RATHER THAN
 (x, y, z, t)

EINSTEIN EQUATIONS

(1) CONSTRAINTS

$$R - K^i_i; K^i_i; -K^2 = 16\pi J \quad (c)$$

D

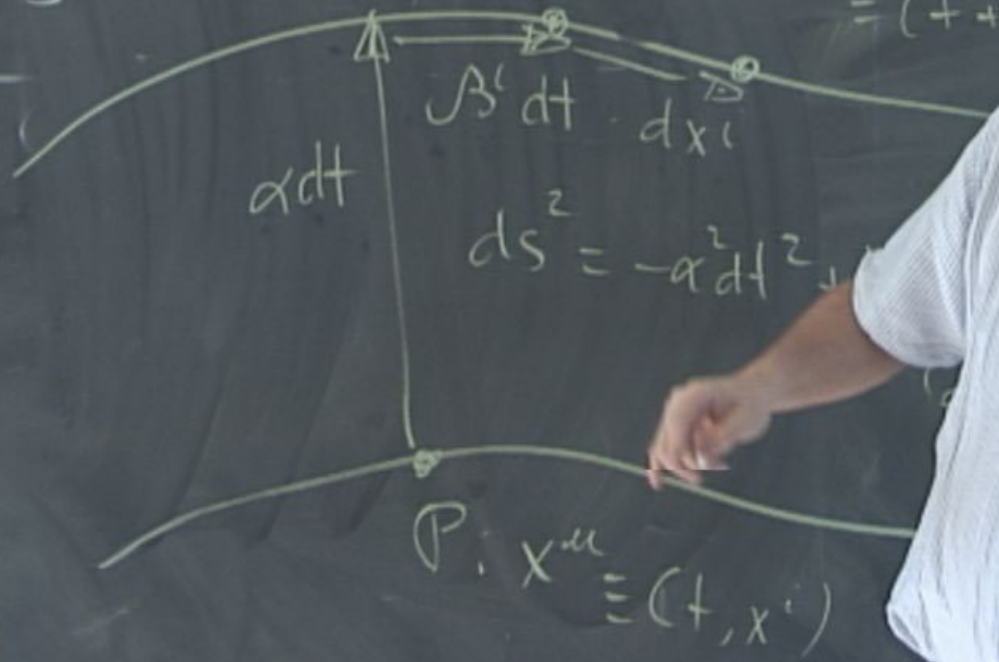
$$\sqrt{g} = \alpha a r^2 b^2 (\sin^2 \theta) Q: x^\mu + dx^\mu = (t + dt, x^i)$$

ND WMP

10 2ND ORDER EQNS

4 CONS

6 2ND ORDER EQUATIONS



D

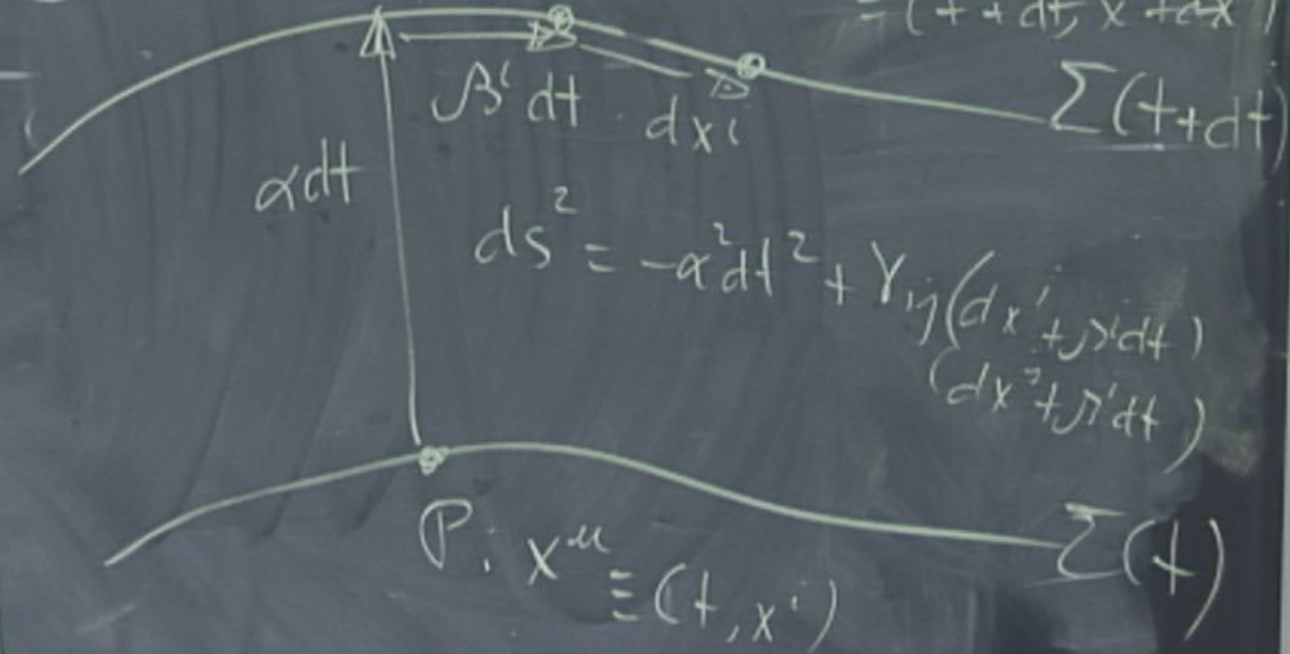
ND WMP

10 2ND ORDER EQNS

4 CONS

6 2ND ORDER EQUATIONS

$$\sqrt{g} = \alpha a r^2 b^2 (\sin^2 \theta) \left(\frac{dt}{\alpha} + dx^i \right) \quad Q: x^{\mu} + dx^{\mu} = (t + dt, x^i + dx^i)$$



$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt)$$

EINSTEIN EQUATIONS

(1) CONSTRAINTS

$$R - K^i{}_i - K^2 = 16\pi J \quad (6)$$

$$D_j K^i{}_j - D_i K = 8\pi j_i \quad (7)$$

(NOTE INDEX SHIFT RELATIVE TO PREVIOUS FORM)

WHERE $J = n_\mu n_\nu T^{\mu\nu} \quad (8)$

$$j_i = \gamma_{ik} j^k = -n_\mu T^{\mu}{}_i \quad (9)$$

RECALL: $n_\mu = (-\alpha, 0, 0, 0)$

$$D_j K_i^j - D_i K = \beta \pi_j^i \quad (7)$$

(NOTE INDEX SHIFT RELATIVE TO PREVIOUS PART)

WHERE $\beta = n_\mu n_\nu T^{\mu\nu} \quad (8)$

$$j_i = \gamma_{ik} j^k = -n_\mu T^{\mu i} \quad (9)$$

RECALL: $n_\mu = (-\alpha, 0, 0, 0)$ (10)

EVOLUTION EQUATIONS $(\cdot \equiv \frac{\partial}{\partial t} \equiv \partial_t)$

$$\dot{\gamma}_{ij} = -2\alpha \gamma_{ik} K^k_j + \beta^k \partial_k \gamma_{ij} + \gamma_{ik} \partial_j \beta^k + \gamma_{kj} \partial_i \beta^k \quad (11)$$

$$\dot{K}^i_j = \beta^k \partial_k K^i_j - \partial_k \beta^i K^k_j + \partial_j \beta^k K^i_k - D^i D_j \alpha$$

EVOLUTIONAL EQUATIONS ($\cdot \equiv \frac{\partial}{\partial t} \equiv \partial_t$)

$$\dot{\gamma}_{ij} = -2\alpha \gamma_{ik} K^k_j + \beta^k \partial_k \gamma_{ij} + \gamma_{ik} \partial_j \beta^k + \gamma_{ki} \partial_i \beta^k \quad (11)$$

$$\dot{K}^i_j = \beta^k \partial_k K^i_j - \partial_k \beta^i K^k_j + \partial_j \beta^k K^i_k - D^i D_j \alpha$$

$$(12) \quad + \alpha (R^i_j + K K^i_j + 4\pi(S-\rho)\delta^i_j - 8\pi S^i_j)$$

WHERE $S_{ij} = T_{ij}$ (12), $S^i_j = \gamma^{ik} S_{kj}$ (14), $S = S^i_i$ (15)

NEED CHRISTOFFEL SYMBOLS Γ^i_{jk} , RICCI COMPONENTS R^i_j AND RICCI SCALAR R ASSOCIATED WITH γ_{ij} (1). USING

FORMULAE FIND FOLLOWING NON-VANISHING Γ^i_{jk}

($\cdot \equiv \frac{\partial}{\partial t} \equiv \partial_t$)

NEED CHRISTOFFEL SYMBOLS Γ^i_{jk} , RICCI COMPONENTS R^i_j AND RICCI SCALAR R ASSOCIATED WITH g_{ij} (1). USING STANDARD FORMULAE FIND FOLLOWING NON-VANISHING Γ^i_{jk} ($' \equiv \frac{\partial}{\partial r} \equiv \partial_r$)

$$\Gamma^r_{rr} = \frac{a'}{a} \quad \Gamma^r_{\theta\theta} = -\frac{(r^2 b^2)'}{2a^2} \quad \Gamma^r_{\phi\phi} = \sin^2\theta \Gamma^r_{\theta\theta}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{(r^2 b^2)'}{2(r^2 b^2)} \quad \Gamma^\theta_{\phi\phi} = -\sin\theta \cos\theta \quad (16a-g)$$

$$\Gamma^\phi_{r\phi} = \Gamma^\phi_{\phi r} = \Gamma^\theta_{\theta r} \quad \Gamma^\phi_{\theta\phi} = \Gamma^\phi_{\phi\theta} = \cot\theta$$

FROM THESE WE COMPUTE NON-VANISHING R^i_j

$$\Gamma_{rr}^r = \frac{a'}{a} \quad \Gamma_{\theta\theta}^r = -\frac{(r^2 b^2)'}{2a^2} \quad \Gamma_{\phi\phi}^r = \sin^2 \theta \Gamma_{\theta\theta}^r$$

$$\Gamma_{r\theta}^e = \Gamma_{\theta r}^e = \frac{(r^2 b^2)'}{2(r^2 b^2)} \quad \Gamma_{\phi\phi}^e = -\sin \theta \cos \theta \quad (16a-g)$$

$$\Gamma_{rd}^d = \Gamma_{dr}^d = \Gamma_{\theta r}^e \quad \Gamma_{\phi d}^d = \Gamma_{d\phi}^d = \cot \theta$$

FROM THESE WE COMPUTE Ricci-tensor R^i_j

$$R^r_r = -\frac{2}{arb} \left(\frac{(rb)'}{a} \right)' \quad (17a)$$

$$R^e_e = R^d_d = \frac{1}{a(rb)^2} \left(a - \left(\frac{rb}{a} (rb)' \right)' \right) \quad (17b)$$

$$a(rb)^2 \left(a - \left(\frac{rb}{a} \right) \right)$$

AND FINALLY, THE SCALAR CURVATURE, R , IS

$$R = R^r_r + R^e_e + R^t_t = -R^r_r + 2R^e_e$$

$$= -\frac{2}{arb} \left(\left(\frac{(rb)'}{a} \right)' + \frac{1}{rb} \left(\left(\frac{rb}{a} (rb)' \right)' - a \right) \right) \quad (1E)$$

IN THE EVALUATION EQUATION FOR K^i_j , WE NEED TO EVALUATE $D^i D_j \alpha$:

EVALUATE $D^i D_j \alpha$:

DIH 3E7K SPHERICAL SYMMETRY

①

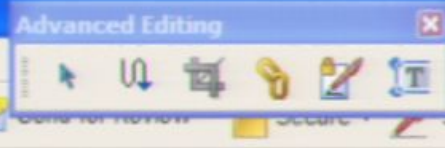
$$D^i D_j \alpha = \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha$$

$$= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha)$$

USING RESULTS FROM ABOVE, WE FIND

$$D^r D_r \alpha = \frac{1}{a} \left(\frac{\alpha'}{a} \right)' \tag{19a}$$

$$D^\theta D_\theta \alpha = D^\phi D_\phi \alpha = \alpha' (r\theta)' \tag{19b}$$



EVALUATE $D^i D_j \alpha$:

ДІЯ ЗЕРКА SPHERICAL SYMMETRY ①

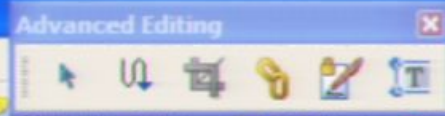
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ДАН ЗЕРКА SPHERICAL SYMMETRY

①

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ДІЯ ЗЕРКА SPHERICAL SYMMETRY ①

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EVALUATE $D^i D_j \alpha$:

DIAMETER SPHERICAL SYMMETRY ①

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USING RESULTS FROM ABOVE, WE FIND

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EVALUATE $D^i D_j \alpha$:

ДІЯ ЗЕРКА SPHERICAL SYMMETRY

①

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ДІЯ ЗЕРКА SPHERICAL SYMMETRY

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D

ND OMP

10 2ND ORDER
EQNS

4 ONS

6 2ND ORDER
EQUATIONS

D

ND WMP

10 2ND ORDER
EQNS

4 CONS

6 2ND ORDER
EQUATIONS

$$\gamma_{ij} = \text{diag}(a^2, r^2 b^2, r^2 b^2 \sin^2 \theta)$$

$$\gamma'^{ij} = \text{diag}(\dots)$$

x'

D

ND OMP

10 2ND ORDER
EQNS

4 ONS

6 2ND ORDER
EQUATIONS

$$\gamma_{ij} = \text{diag}(a^2, r^2 b^2, r^2 b^2, r^2 b^2)$$

$$\gamma'_{ij} = \text{diag}(r^2, r^2 b^2, r^2 b^2, r^2 b^2)$$

x'

D

ND WMP

10 2ND ORDER
EQNS

4 CONS

6 2ND ORDER
EQUATIONS

$$\gamma_{ij} = \text{diag}(a^2, r^2 b^2, r^2 b^2 \sin^2 \theta)$$

$$\gamma^{ij} = \text{diag}(a^{-2}, r^{-2} b^{-2}, r^{-2} b^{-2} \sin^{-2} \theta)$$

x^i

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x^i

$$D^i D_i \alpha = \gamma^{ik} D_k D_j \alpha = \gamma^{ik} D_k \partial_j \alpha$$

$$= \gamma^{ik} (\partial_k \partial_j \alpha - \Gamma^m_{jk} \partial_m \alpha)$$

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$$D^b D_b \alpha = D^d D_d \alpha = \frac{\alpha'(rb)'}{a^2 rb} \tag{19b}$$

ALSO NEED STRESS-TENSOR "COMPONENTS" μ_i
 j_i, S^i_j

Advanced Editing

• ALSO NEED STRESS-TENSOR "COMPONENTS" j_i, S^i_j

• IN SPIRIT OF HAMILTONIAN APPROACH, IT IS CONVENIENT TO INTRODUCE AUXILIARY FUNCTIONS

$$\Phi(r,t) \equiv \phi'(r,t) = \partial_r \phi(r,t) \quad (20)$$

$$\pi(r,t) \equiv \frac{a}{\alpha} (\dot{\phi} - \beta \phi') \quad (21)$$

VIEW Φ, π AS "REAL" DYNAMICAL VOLS FOR SCALAR FIELD; NOTE: FOR MASSLESS FIELD, VALUE OF ϕ IS MEANINGLESS ($\phi \rightarrow \phi + \text{const}$ STILL SATISFIES $\square \phi = 0$) ALL "ACTION" IS IN GRADIENTS

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ALSO NOTE THAT WE HAVE $(c/f(a))$

$$g_{\theta\theta} = -a^2 + a^2 \beta^2 \quad g_{\theta r} = g_{r\theta} = a^2 \beta \quad (22a-c)$$

$$g_{rr} = a^2 \quad g_{\phi\phi} = r^2 b^2 \quad g_{\phi\theta} = r^2 b^2 \sin^2 \theta$$

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$$\nabla^t \phi \equiv \partial^t \phi = g^{tt} \partial_t \phi + g^{tr} \partial_r \phi = -\frac{\pi}{\alpha a} \quad (24)$$

$$(\nabla^t \phi)(\nabla_t \phi) = \partial^{i\mu} \phi \partial_{i\mu} \phi = \Phi^2 - \pi^2 \quad (25)$$

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$$\nabla^t \phi \equiv \partial^{it} = g^{tt} \partial_t \phi + g^{tr} \partial_r \phi = -\frac{\pi}{2a} \quad (24)$$

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$\sigma = (\dots)$

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PHY 382M SPHERICAL SYMMETRY

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

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FROM WHICH WE FIND

$$= \frac{1}{2a^2}$$

(25)

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PHY 387M SPHERICAL SYMMETRY

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$$j_r = -n_\mu T^\mu{}_i = \alpha T^0{}_i = \alpha \phi'^+ d_{,r} = -\frac{\phi'}{\rho} \quad (27)$$

PHY 387M SPHERICAL SYMMETRY

④

FINALLY, WE HAVE THE SPATIAL STRESS COMPONENTS

$$S^i{}_j = \gamma^{ik} S_{kj} = \gamma^{ik} T_{kj}$$

FROM WHICH WE FIND

$$= \frac{2a^2}{2a^2} \quad (26)$$

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Advanced Editing

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PHY 3E2W SPHERICAL SYMMETRY

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PHY 3E7M SPHERICAL SYMMETRY

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PHYSICS SPHERICAL SYMMETRY

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$$2a^2$$

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WE CAN NOW ASSEMBLE THE ABOVE RESULTS TO PRODUCE THE SPHERICALLY-SYMMETRIC SPECIALIZATION OF THE GENERAL 3+1 EQUATIONS (6), (7), (11) ; (12)

A) HAMILTONIAN CONSTRAINT

$$R - K^i_j K^j_i + K^2 = 16\pi \rho$$

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B) MOMENTUM CONSTRAINT (ONLY r-COMPONENT IS NON-TRIVIAL)

FIRST NOTE THAT

$$D_i K^i_r = \partial_i K^i_r + \Gamma^i_{mi} K^m_r - \Gamma^m_{ri} K^i_m$$

$$= K^r_r' + 2\Gamma^e_{re} (K^r_r - K^e_e)$$

$$D_r K = (K^r_r + 2K^e_e)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi}{\pi} \quad (32)$$

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$$K_e^e' + \frac{(rb)'}{rb} (K_e^e - K_r^r) = 4\pi \frac{\Phi \Pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

° FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^r + (a\beta)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$
(33)

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$
(34)

THEN WE HAVE FROM (2) AND (29)

$$K^e e' + \frac{(rb)'}{rb} (K^e e - K^r r) = 4\pi \frac{\Phi \Pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi \Pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i_i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$
(33)

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$
(34)

THEN WE HAVE FROM (2) AND (29)

$$K^e e' + \frac{(rb)'}{rb} (K^e e - K^r r) = 4\pi \frac{\Phi \Pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i_j ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

Advanced Editing

THEN WE HAVE FROM (2) AND (29)

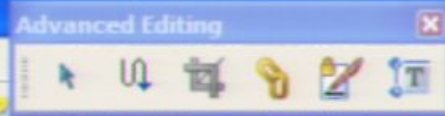
$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$



THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i_i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

(34)

THEN WE HAVE FROM (2) AND (29)

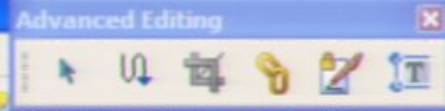
$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$



THEN WE HAVE FROM (2) AND (29)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i_j ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (27)

$$K^e_e' + \frac{(rb)'}{rb} (K^e_e - K^r_r) = 4\pi \frac{\Phi\pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

THEN WE HAVE FROM (2) AND (29)

$$K^e e' + \frac{(rb)'}{rb} (K^e e - K^r r) = 4\pi \frac{\Phi \Pi}{a} \quad (32)$$

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K^r_r + (a\beta)'$$

$$\dot{b} = -\alpha b K^e_e + \frac{1}{r} (rb)'$$

Advanced Editing

$$\dot{a} = -\alpha a K_r^r + (a\beta)' \quad (32)$$

c) EVOLUTION EQUATIONS FOR $\gamma_{ij}(a, b)$

• FOLLOW DIRECTLY FROM (ii), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^r + (a\beta)' \quad (33)$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR K^i ; (K_r^r, K_e^e)

Advanced Editing

$$\dot{a} = -\alpha a K_r^r + (\alpha\beta)' \quad (32)$$

c) EVOLUTION EQUATIONS FOR $\gamma_{ij}(a, b)$

• FOLLOW DIRECTLY FROM (ii), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^r + (\alpha\beta)' \quad (33)$$

$$\dot{b} = -\alpha b K_e^e + \frac{1}{r} (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR K^i ; (K_r^r, K_e^e)

Advanced Editing

c) EVOLUTIONAL EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTIONAL EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 / (rb)')'$... Φ^2

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

° FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

° FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 / (rb)')'$... ϕ^2

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \quad (33)$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \quad (34)$$

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha^i)'$... $(-2 / (rb)')'$... ϕ^2

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \sum_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$\dot{K}_r^n = \dots$ $\dot{K}_e^e = \dots$

Advanced Editing

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \sum_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 \sum_r (rb)')'$... ϕ^2

Advanced Editing

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$\dot{K}_r^n = -2 \int_r (rb)'$... $\dot{K}_e^e = \dots$

Advanced Editing

c) EVOLUTIONAL EQUATIONS FOR δ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)'$$

(34)

d) EVOLUTIONAL EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha^i)'$... $(-2 \int (rb)')'$... ϕ^2

c) EVOLUTION EQUATIONS FOR δ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 \int (rb)')$... Φ^2

c) EVOLUTION EQUATIONS FOR δ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha^i)'$... $(-2 \int (rb)')'$... Φ^2

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 \int (rb)')'$... Φ^2

c) EVOLUTION EQUATIONS FOR γ_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)'$$

(33)

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)'$$

(34)

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

c) EVOLUTION EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTION EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 \int (rb)')'$... Φ^2

c) EVOLUTIONAL EQUATIONS FOR x_{ij} (a, b)

• FOLLOW DIRECTLY FROM (11), RECALL, CAN BE VIEWED AS DEFⁿ OF K^i ;

$$\dot{a} = -\alpha a K_r^n + (a\beta)' \tag{33}$$

$$\dot{b} = -\alpha b K_e^e + \int_r (rb)' \tag{34}$$

d) EVOLUTIONAL EQUATIONS FOR K^i (K_r^n, K_e^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

... $(\alpha)'$... $(-2 \int (rb)')'$... Φ^2

$$\dot{a} = -\alpha a K_r + (\alpha)^3 \quad (33)$$

$$\dot{b} = -\alpha b K_e + \frac{2}{r} (rb)' \quad (34)$$

D) EVOLUTIONAL EQUATIONS FOR K_i ; (K_r, K_e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\ddot{K}_r = \beta K_r' - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K_r - 5\pi \frac{\phi^2}{a^2} \right) \quad (35)$$

PHYSICAL SPHERICAL SYMMETRY

$$\ddot{K}_e = \beta K_e' + \frac{1}{r} \left(\alpha rb (rb)' \right)' + \alpha K K_e$$

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

→ FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - \frac{1}{a^2} \right) \quad (35)$$

PHYSICAL SPHERICAL SYMMETRY

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

→ (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

→ FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - \frac{1}{a^2} \right) \quad (35)$$

PHYSICAL SPHERICAL SYMMETRY

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

→ (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^{θ})

← FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - 8\pi \frac{\rho}{a^2} \right) \quad (35)$$

PHY 3E7N

SPHERICAL SYMMETRY

⑤

$$\dot{K}^{\theta} = \beta K^{\theta} + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^{\theta} \quad (36)$$

← (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

← FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - 3\pi \frac{\dot{\phi}^2}{a^2} \right) \quad (35)$$

PHY 3E7N SPHERICAL SYMMETRY ⑤

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

← (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

← FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - 3\pi \frac{\dot{\phi}^2}{a^2} \right) \quad (35)$$

PHY 3E7N SPHERICAL SYMMETRY ⑤

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

← (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - \frac{1}{a^2} \right) \quad (35)$$

PHYSICAL SPHERICAL SYMMETRY

⑤

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

• (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - \frac{3}{2} \frac{\dot{\phi}^2}{a^2} \right) \quad (35)$$

PHYSICAL SPHERICAL SYMMETRY

$$\dot{K}^e = \beta K^e + \frac{\dot{\phi}}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

• (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - \frac{5}{2} \pi \frac{\dot{\phi}^2}{a^2} \right) \quad (35)$$

PHY 3E7N SPHERICAL SYMMETRY

⑤

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

(31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

• FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} (\frac{\alpha'}{a})' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - \frac{1}{a^2} \right) \quad (35)$$

PHYSICAL SPHERICAL SYMMETRY

⑤

$$\dot{K}^e = \beta K^e + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^e \quad (36)$$

• (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K_r^r, K_θ^θ)

→ FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}_r^r = \beta K_r^r - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K_r^r - 8\pi \frac{\rho}{a^2} \right) \quad (35)$$

PHY 3E7N SPHERICAL SYMMETRY ⑤

$$\dot{K}_\theta^\theta = \beta K_\theta^\theta + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K_\theta^\theta \quad (36)$$

→ (31)-(36) ARE THE COMPLETE 3+1 EQUATIONS FOR THE

D) EVOLUTIONAL EQUATIONS FOR K^i ; (K^r, K^e)

← FOLLOW DIRECTLY FROM (12) AND OTHER RESULTS ABOVE

$$\dot{K}^r = \beta K^r - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K K^r - 3\pi \frac{\dot{\phi}^2}{a^2} \right) \quad (35)$$

PHY 3E7N SPHERICAL SYMMETRY ⑤

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PHY 3E7N SPHERICAL SYMMETRY ⑤

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PHYSICAL SPHERICAL SYMMETRY

$$\dot{K}^{\theta} = \beta K^{\theta} + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^{\theta} \quad (36)$$

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PHYSICAL SPHERICAL SYMMETRY

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PHY 3E7N SPHERICAL SYMMETRY ⑤

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PHYSICAL SPHERICAL SYMMETRY

⑤

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PHYSICAL SPHERICAL SYMMETRY

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PHYSICAL SPHERICAL SYMMETRY

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PHY 3E7N SPHERICAL SYMMETRY

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PHYSICS SPHERICAL SYMMETRY

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PHYSICAL SPHERICAL SYMMETRY

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PHYSICAL SPHERICAL SYMMETRY

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PHYSICAL SPHERICAL SYMMETRY

⑤

$$\dot{K}^{\theta} = \beta K^{\theta} + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K^{\theta} \quad (36)$$

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PHYSICAL SPHERICAL SYMMETRY

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PHY 3E7N SPHERICAL SYMMETRY

⑤

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$$\dot{K}_r^0 = \beta \dot{K}_r^0 - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(-\frac{2}{rab} \left(\frac{(rb)'}{a} \right)' + K_r^0 - \sigma \pi \frac{H^2}{a^2} \right) \quad (35)$$

PHY 387N SPHERICAL SYMMETRY

⑤

$$\dot{K}_0^0 = \beta \dot{K}_0^0 + \frac{\kappa}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha K K_0^0 \quad (36)$$

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PHY 387N SPHERICAL SYMMETRY

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$$\dot{K}_0^0 = \beta \dot{K}_0^0 + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha rb (rb)'}{a} \right)' + \alpha \dot{K}_0^0 \quad (36)$$

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MASSLESS KLEIN GORDON EQUATION

NEED E.O.M. FOR Φ AND Π

RECALL DEFⁿ OF Π , (29)

$$\Pi \equiv \frac{\alpha}{2} (\dot{\Phi} - \beta \Phi')$$

$$\rightarrow \dot{\Phi} = \frac{\alpha}{2} \Pi + \beta \Phi' = \frac{\alpha}{2} \Pi + \beta \underline{\Phi}$$

MASSLESS KLEIN GORDON EQUATION

WANTED E.O.M. FOR $\bar{\phi}$ AND π

RECALL DEFⁿ OF π , (28)

$$\pi = \frac{\alpha}{2} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{\alpha}{2} \pi + \beta \phi' = \frac{\alpha}{2} \pi + \beta \bar{\phi}$$

BUT $\dot{\phi}' = \dot{\bar{\phi}}$, so

$$\dot{\bar{\phi}} = \left(\beta \bar{\phi} + \frac{\alpha}{2} \pi \right)' \quad (32)$$

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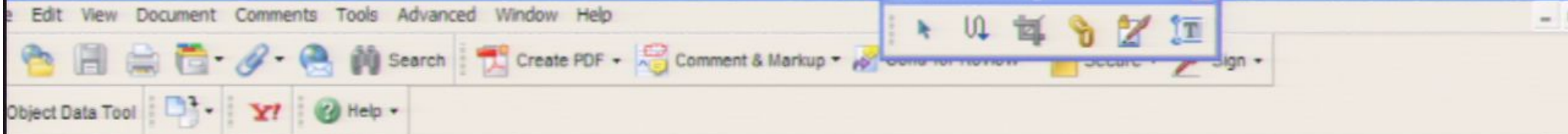
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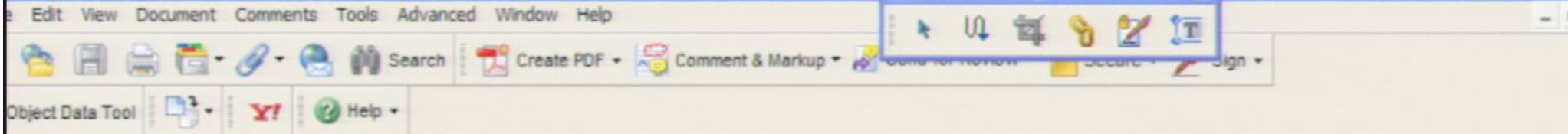
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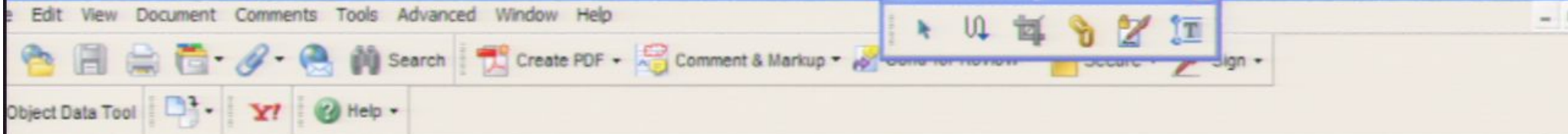
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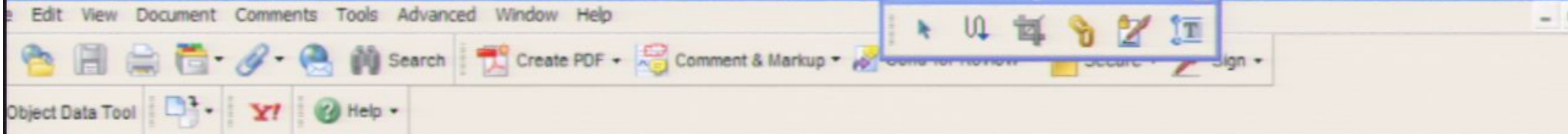
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$$\pi = \frac{\alpha}{2} (\dot{\phi} - \beta \phi')$$

$$\rightarrow \dot{\phi} = \frac{\alpha}{2} \pi + \beta \phi' = \frac{\alpha}{2} \pi + \beta \bar{\phi}$$

BUT $\dot{\phi}' = \dot{\bar{\phi}}$, SO

$$\dot{\bar{\phi}} = \left(\beta \bar{\phi} + \frac{\alpha}{2} \pi \right)' \quad (32)$$

MASSLESS KLEIN GORDON EQUATION

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$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \mathcal{L} \alpha^{-2} \phi'))$$

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TO FIND THE EQU, RECALL THAT

$$\square \phi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi)$$

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$$= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi'))$$

$$+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi}))$$

$$= 0$$

$$\sqrt{-g} \rightarrow (\dots)$$

$$\square d = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

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$\sqrt{-g} \rightarrow (r^2 \sin^2 \theta)$

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$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}))'$$

$\sqrt{-g} \rightarrow (\dots)$

$$\square d = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}))'$$

$\sqrt{-g} \rightarrow (\dots)$

$$\square d = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}))'$$

$\sqrt{-g} \rightarrow (\dots)$

$$\square \phi = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}))'$$

$$\sqrt{-g} \rightarrow (r^2 \sin^2 \theta)$$

$$\square d = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}'))'$$

$\sqrt{-g} \rightarrow (\dots)$

$$\square d = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}))'$$

$\sqrt{-g} \rightarrow (\dots)$

$$\square d = 0 \rightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu d) = 0$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{-g} g^{tt} \partial_t d) + \partial_r (\sqrt{-g} g^{rr} \partial_r d) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\phi}'))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \bar{\phi}))'$$

↳ WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACTLY EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \dots (r^2 b^2 (\beta \pi + \alpha \bar{\phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &+ \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACT SAME EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACT SAME EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EULER-LAGRANGE EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
EINSTEIN EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE EXACTLY EQUATION (39) FOR \dot{b}

$$\pi = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EVACTION EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACT SAME EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \dot{\pi} + \frac{\alpha}{a} \ddot{\phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE
EINSTEIN EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \dot{\pi} + \frac{\alpha}{a} \ddot{\phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACT SAME EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EINSTEIN EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE EXACTLY EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EVACTION EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EINSTEIN EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EVACTION EQUATION (39) FOR \dot{b}

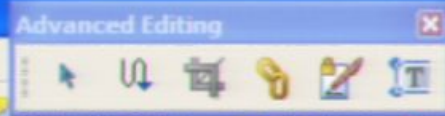
$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACTLY EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$



$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE
EINSTEIN EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \dot{\pi} + \frac{\alpha}{a} \ddot{\phi}))'$$

WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \dot{\pi}$ AND USE EXACTLY EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \dot{\pi} + \frac{\alpha}{a} \ddot{\phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EULER-LAGRANGE EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{tt} \partial_t \phi) + \partial_r (\sqrt{g} g^{rr} \partial_r \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \ddot{\pi} + \frac{\alpha}{a} \ddot{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \ddot{\pi}$ AND USE
EVACTION EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \ddot{\pi} + \frac{\alpha}{a} \ddot{\Phi}))'$$

$$\begin{aligned} &\rightarrow \partial_t (\sqrt{g} g^{vr} \partial_v \phi) + \partial_r (\sqrt{g} g^{vr} \partial_v \phi) \\ &= \partial_t (\alpha a r^2 b^2 (-\alpha^{-2} \dot{\phi} + \beta \alpha^{-2} \phi')) \\ &\quad + \partial_r (\alpha a r^2 b^2 ((\alpha^{-2} - \beta^2 \alpha^{-2}) \phi' + \beta \alpha^{-2} \dot{\phi})) \\ &= 0 \end{aligned}$$

$$\rightarrow (r^2 b^2 \ddot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

(WRITE AS $(r^2 b^2) \ddot{\pi} + (r^2 b^2) \pi$ AND USE EXACT SAME EQUATION (39) FOR \dot{b}

$$\ddot{\pi} = \frac{1}{r^2 b^2} (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \underline{\Phi}))'$$

= 0

$$\rightarrow (r^2 b^2 \dot{\pi}) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \Phi))'$$

WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE EXACTLY EQUATION (39) FOR b

$$\dot{\pi} = \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \Phi \right) \right)' + 2 \left(\alpha K e - \beta \frac{(rb)'}{rb} \right) \pi \tag{32}$$

$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \Phi))'$
 (WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
 EXACTLY EQUATION (39) FOR \dot{b}

$$\begin{aligned}
 \dot{\pi} &= \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \Phi \right) \right)' \\
 &+ 2 \left(\alpha K^0 e - \beta \frac{(rb)'}{rb} \right) \pi
 \end{aligned}
 \tag{32}$$

$$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \Phi))'$$

(WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
 EXACTLY EQUATION (39) FOR \dot{b})

$$\begin{aligned} \dot{\pi} &= \frac{1}{r^2 b^2} \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \Phi \right) \right)' \\ &+ 2 \left(\alpha K^0 e - \beta \frac{(rb)'}{rb} \right) \pi \end{aligned} \tag{39}$$

$\rightarrow (r^2 b^2 \pi) = (r^2 b^2 (\beta \pi + \frac{\alpha}{a} \Phi))'$
 (WRITE AS $(r^2 b^2) \dot{\pi} + (r^2 b^2) \pi$ AND USE
 EXACTLY EQUATION (39) FOR \dot{b}

$$\begin{aligned}
 \dot{\pi} = \frac{1}{r^2 b^2} & \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \Phi \right) \right)' \\
 & + 2 \left(\alpha K^0 e - \beta \frac{(rb)'}{rb} \right) \pi
 \end{aligned}
 \tag{39}$$

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(38)

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$$\begin{aligned} \dot{\pi} = \frac{1}{r^2 b^2} & \left(r^2 b^2 \left(\beta \pi + \frac{\alpha}{a} \Phi \right) \right)' \\ & + 2 \left(\alpha K^0 e - \beta \frac{(rb)'}{rb} \right) \pi \end{aligned} \tag{38}$$

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Advanced Editing

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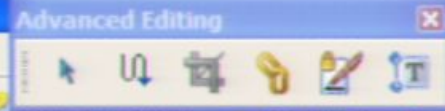
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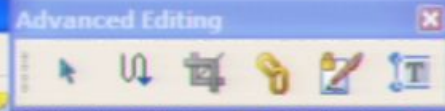
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(32)

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