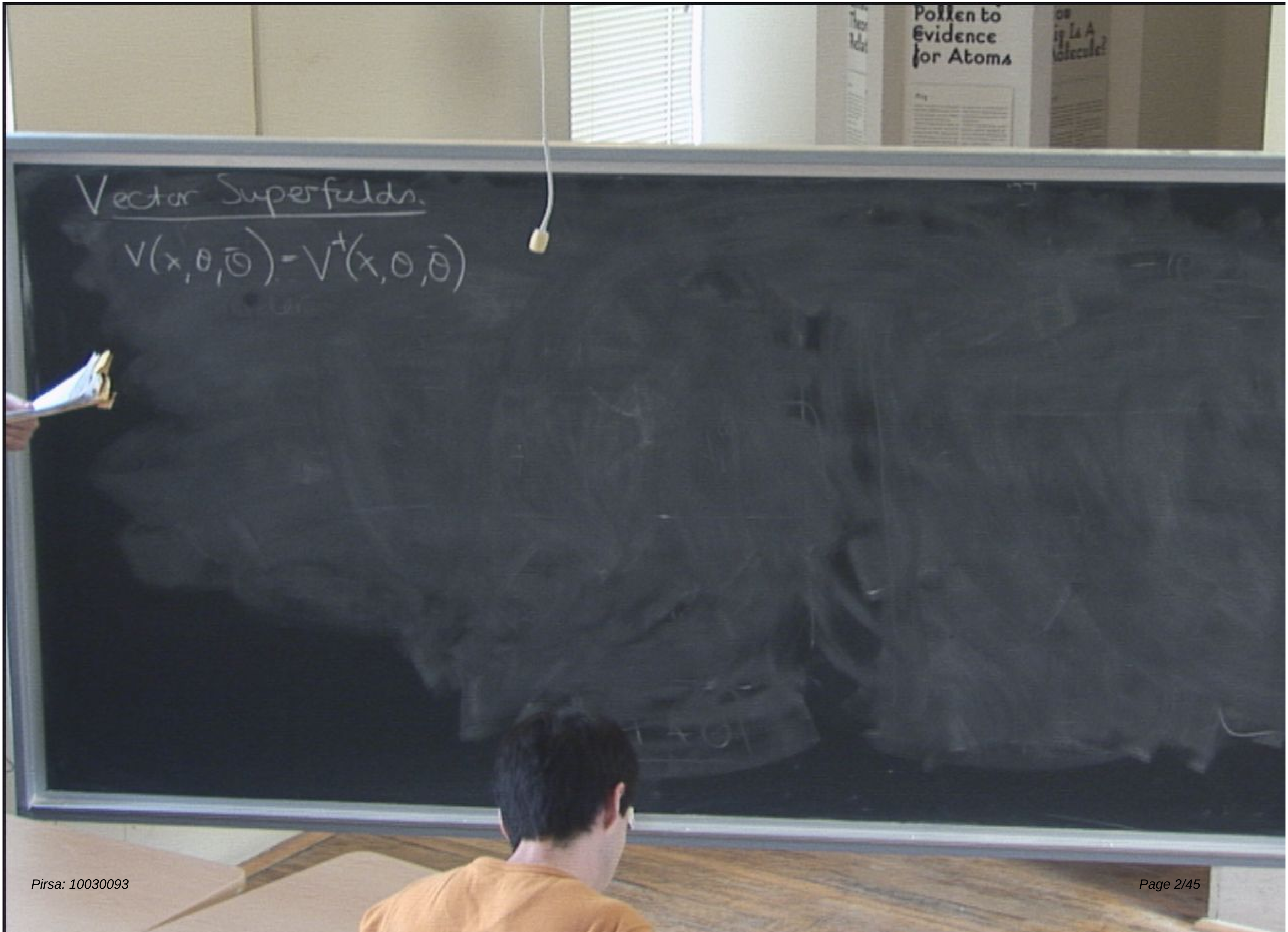
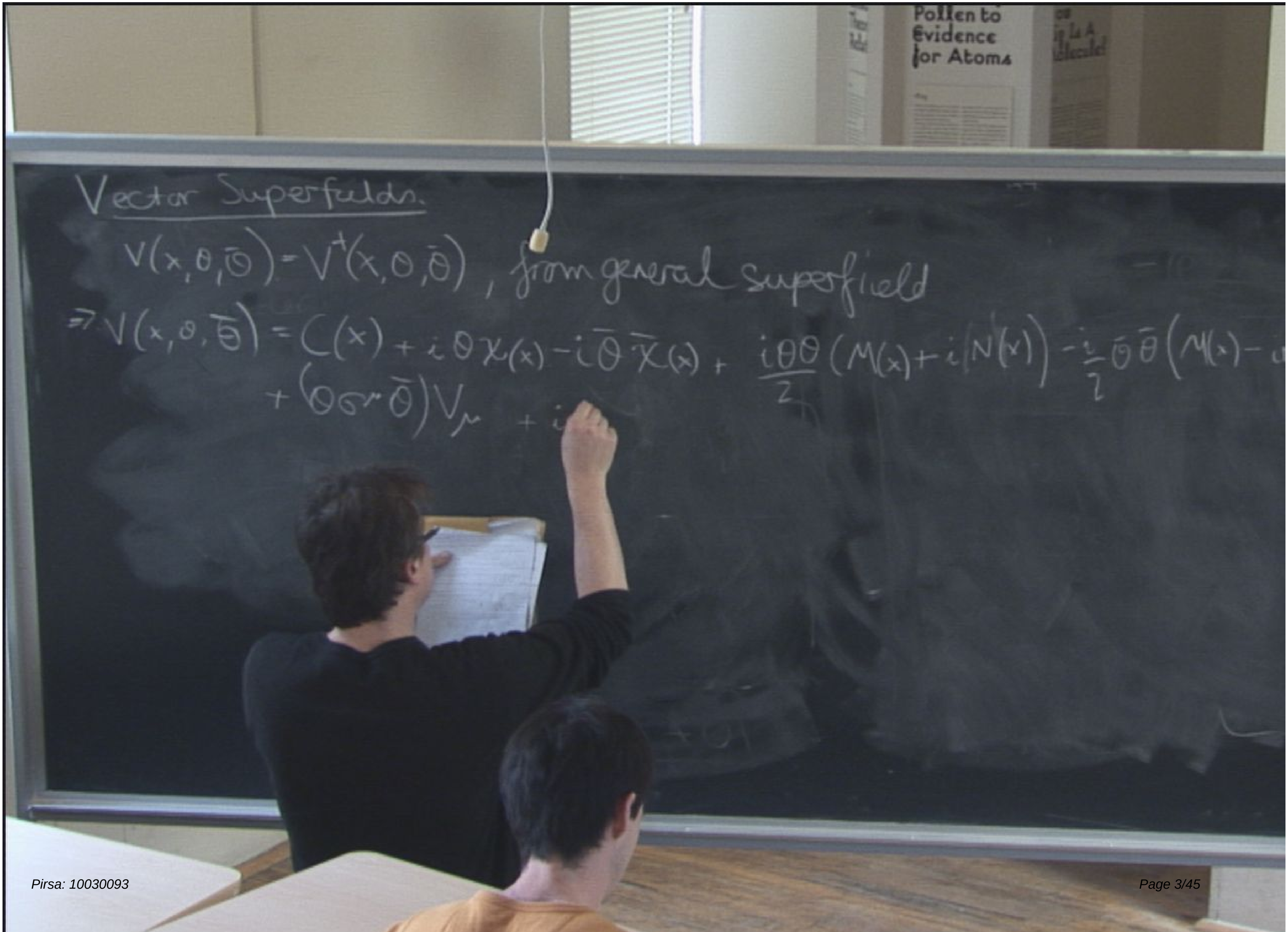
Title: Explorations in Particle Theory (PHYS 646) - Lecture 10

Date: Mar 25, 2010 10:10 AM

URL: http://pirsa.org/10030093

Abstract:


Vector Superfields.
$V(x,\theta,\bar{\theta}) = V^{\dagger}(x,\theta,\bar{\theta})$
Pollen to
Evidence
for Atoms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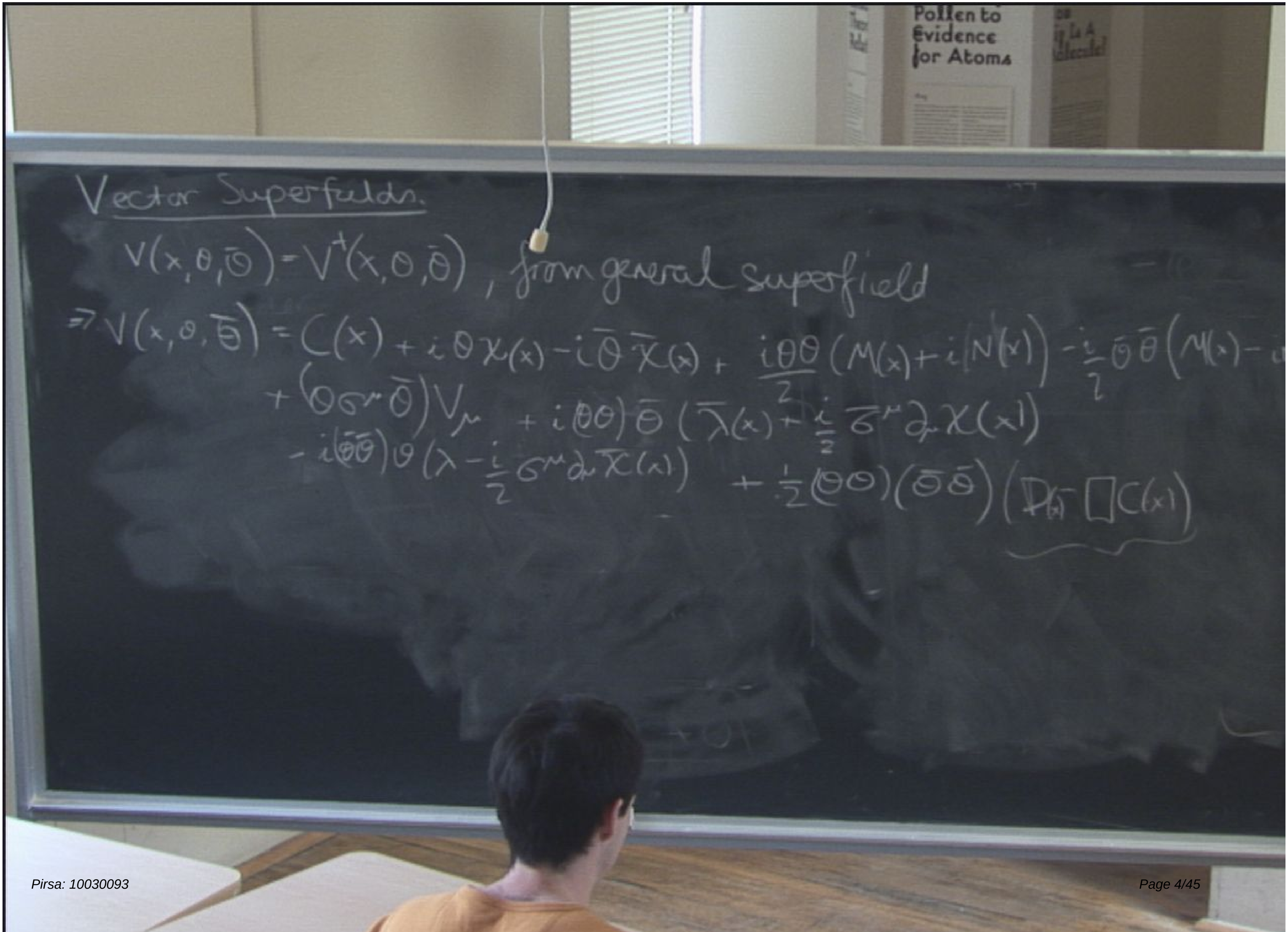
$V(x,\theta,\bar\theta) = V^{\dagger}(x,\theta,\bar\theta)$, from general superfield
$\Rightarrow V(x,\theta,\bar\theta) = C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \frac{i\theta\theta}{2}(M(x)+iN(x)) - \frac{i}{2}\bar\theta\bar\theta(M(x) -$
$+ (\theta\sigma^{\mu}\bar\theta)V_{\mu} \quad + i$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$+ (\theta\sigma^\mu\bar\theta)V_\mu + i(\theta\theta)\bar\theta(\bar\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x))$
$- i(\bar\theta\bar\theta)\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)(D(x) - \Box C(x))$

IR d.o.f: 8 bosonic cpts C, M, D, N, V_μ

4+4 fermionic ones $\chi_\alpha, \lambda_\alpha$

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Introduce a χral s. field Λ

$i(\Lambda - \Lambda^+)$ is $\mathbb{R}$ $\therefore$ it's an example of a vector superfield.

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$C = i(\varphi - \varphi^\dagger)$

$\chi = \sqrt{2}\,\psi$

$M + iN = F$

$V_\mu = -\partial_\mu(\varphi + \varphi^\dagger)$

$\lambda = D = 0$

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Define a generalised gauge transformation

$$V \longmapsto V + i(\Lambda - \Lambda^\dagger)$$

$V_\mu \mapsto V_\mu - \partial_\mu \alpha(x)$ in QED.

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We may choose φ, ψ, F in Λ to "gauge away" some cpts of V.

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Wess-Zumino Gauge $\quad C = 0 = \chi = M = N \ldots$

$V_{WZ}(x,\theta,\bar\theta) = (\theta\sigma^\mu\bar\theta)V_\mu \pm i(\theta\theta)(\bar\theta\bar\lambda) - i(\bar\theta\bar\theta)\theta\lambda$

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gauge boson

gaugino

auxiliary

N.B. $V_{WZ}^2 = \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)V_\mu V^\mu$

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Z gauge isn't SUSY

under SUSY $V_{WZ} \not\to V'_{\cancel{WZ}}$

Under a combination of a SUSY + gauge transf.,

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Non-SUSY

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Field Strength Superfield

Non-SUSY: charged scalar $\varphi(x) \xmapsto{U(1)} e^{i\alpha(x)}\varphi$

$V_\mu(x) \mapsto V_\mu(x) - \partial_\mu\alpha$

Under SUSY,

$$V \mapsto V + i(\Lambda - \Lambda^{+}).$$

$$\Phi :$$

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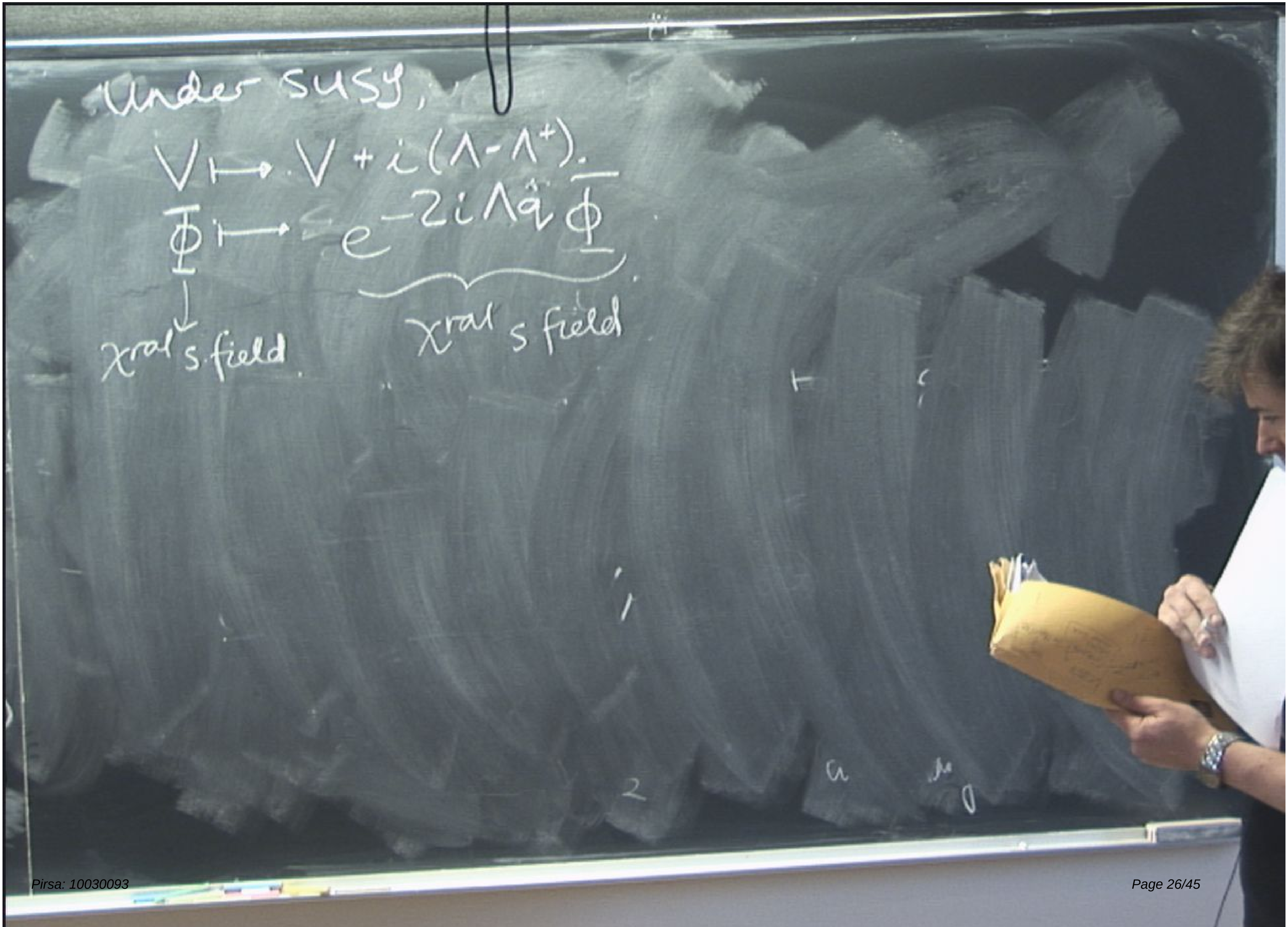
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Φ $\downarrow$ χral s. field

Field strength $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$.

SUSY analogy $W_\alpha = -\frac{1}{4}(\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha})\,D_\alpha V.$

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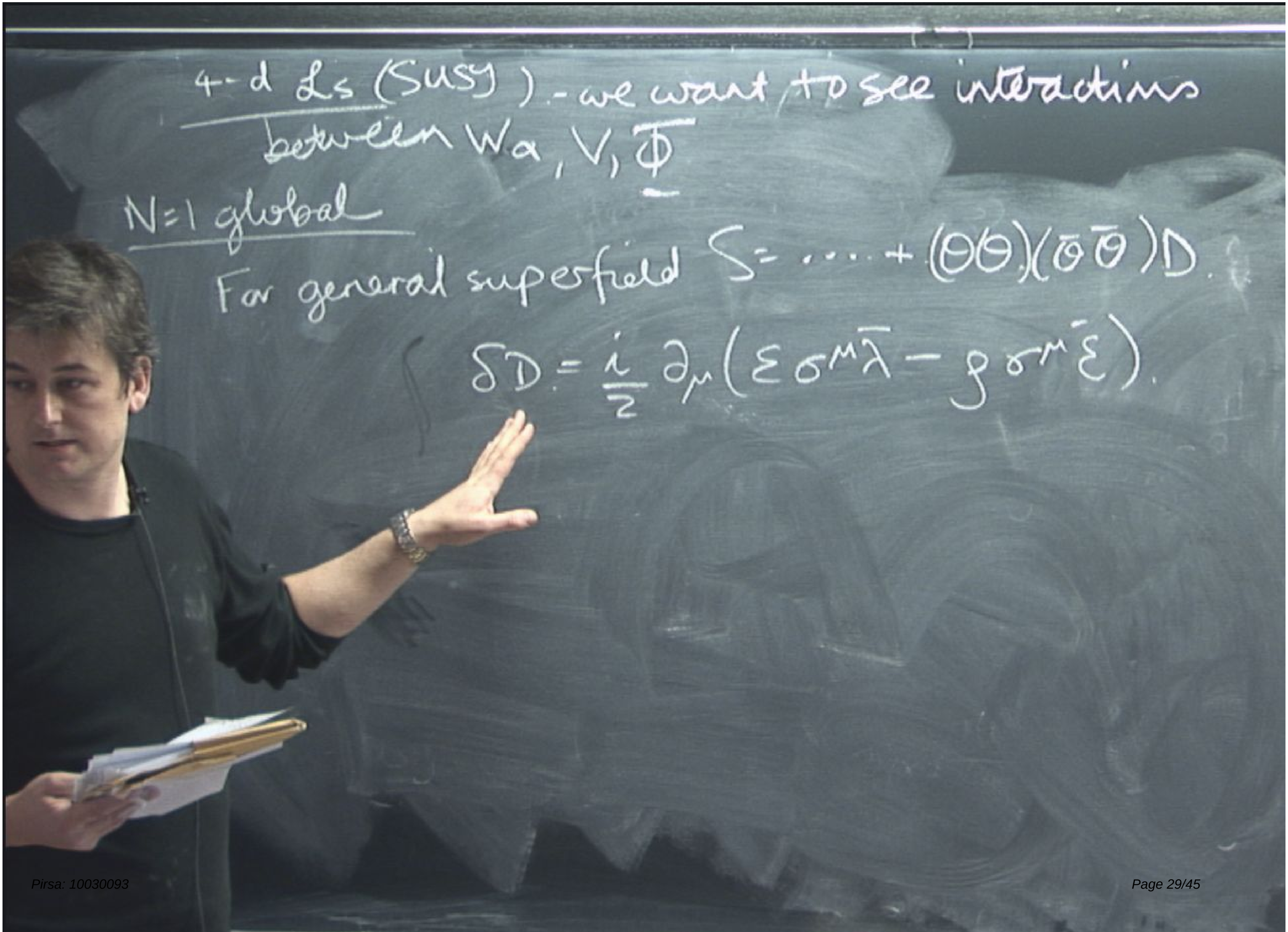
is χ^{ral} and inv. under a generalised gauge transformation.

$$W_\alpha(y^\mu) = -i\lambda_\alpha(y^\mu) + \theta_\alpha D(y^\mu) - \frac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu}(y^\mu)$$

$$+ (\theta\theta)\sigma^\mu_{\alpha\dot\beta}\partial_\mu\bar\lambda^{\dot\beta}(y^\mu)$$


4-d $\mathcal{L}$s (SUSY) - we want to see interactions
between W_α, V, Φ
N=1 global
For general superfield $S = \dots + (\theta\theta)(\bar{\theta}\bar{\theta})D$.
$\delta D = \frac{i}{2}\partial_\mu(\varepsilon\sigma^\mu\bar{\lambda} - \rho\sigma^\mu\bar{\varepsilon})$.

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$$\therefore \text{Susy } \mathcal{L} = K(\Phi_i, \Phi_j^+)\Big|_D + [W(\Phi_i)|_F + \ldots$$

$|_D \rightarrow$ coeff of $(\theta\theta)(\bar\theta\bar\theta)$

$|_F \rightarrow$ coef $(\theta\theta)$

K is the "Kähler potential", $\mathbb{R}$ f^n:

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Taking renormalisable QFT, all products of fields should dim ≤ 4 in $\mathcal{L}$

$$\Phi = \varphi + \sqrt{2}\,\theta\psi + \theta\theta F$$

$$[\mathcal{L}] = 4 \Rightarrow [\varphi] = 1$$

$$\therefore [\Phi] = 1, \ [\psi] = 3/2$$

$$\Rightarrow [\theta] = -1/2, \ [F] = 2, \ [\bar{\theta}] = -1/2$$

$$[K|_D] \leq 4 \Rightarrow [K] \leq 2$$

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$$[K|_D] \leq 4 \Rightarrow [K] \leq 2 \quad \text{i.e. } K = \Phi^\dagger \Phi$$

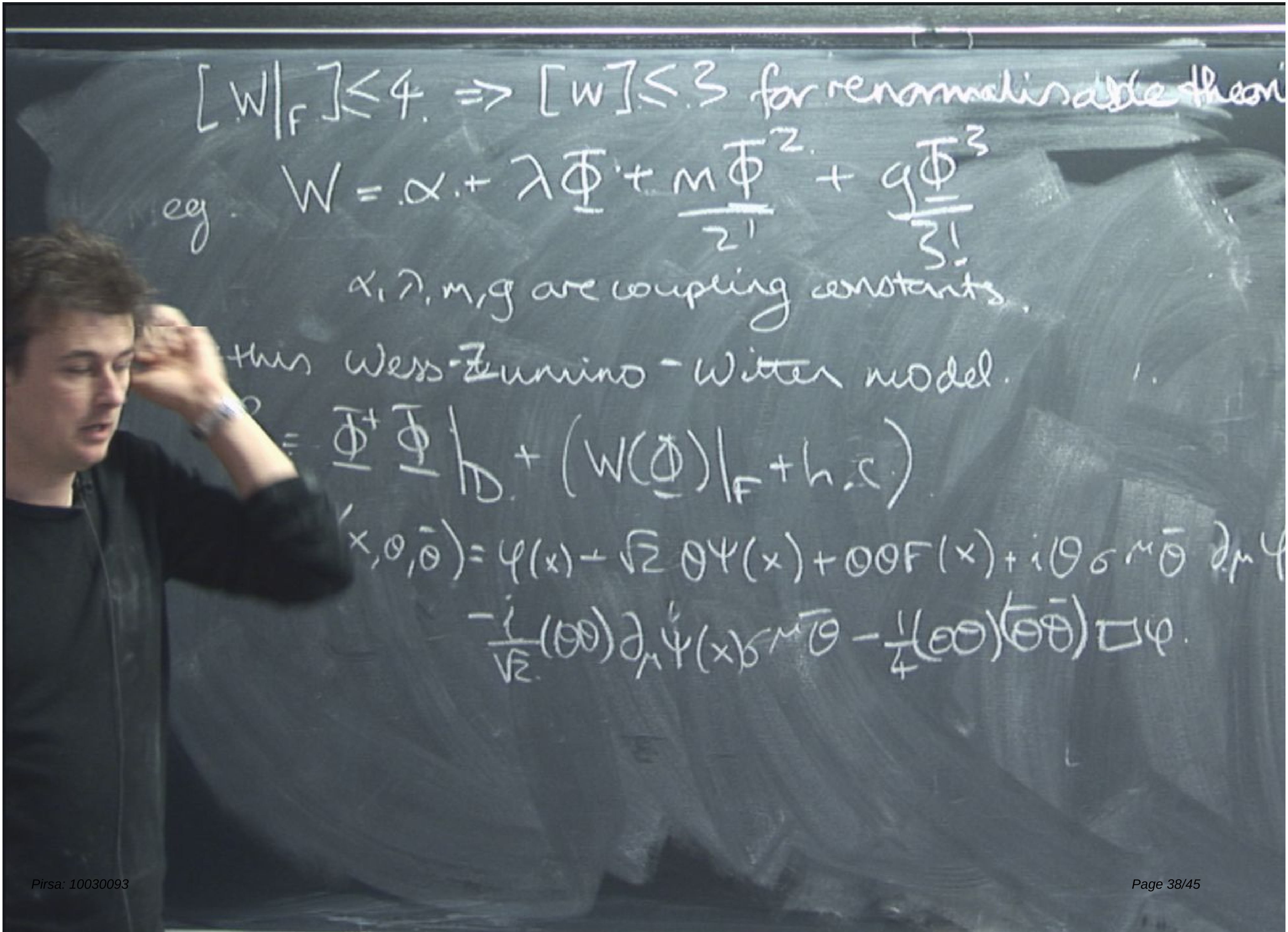
($\Phi + \Phi^\dagger$, $\Phi\Phi + \Phi^\dagger\Phi^\dagger$ have no D-term)

$[W|_F] \leq 4 \Rightarrow [W] \leq 3$ for renormalisable theori

eg. $W = \alpha + \lambda \Phi + \frac{m \Phi^2}{2!} + \frac{g \Phi^3}{3!}$

α, λ, m, g are coupling constants.

– this Wess-Zumino-Witten model.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this Wess-Zumino-Witten model.
= Φ⁺Φ|_D + (W(Φ)|_F + h.c)
(x, θ, θ̄) = φ(x) − √2 θψ(x) + θθF(x) + iθσ^μθ̄ ∂_μφ
− i/√2 (θθ) ∂_μψ(x)σ^μθ̄ − 1/4 (θθ)(θ̄θ̄) □φ.

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$\mathcal{L} = \Phi^\dagger \Phi |_D + (W(\Phi)|_F + h.c.)$

Remember $\Phi(x,\theta,\bar\theta) = \varphi(x) - \sqrt{2}\,\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi$

$- \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi(x)\sigma^\mu\bar\theta - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box\varphi.$

$\partial_\mu\varphi^*\partial^\mu\varphi - i\bar\psi\bar\sigma^\mu\partial_\mu\psi + FF^*.$

Taylor expand around $\Phi = \varphi$. $\left(\frac{\partial W}{\partial \varphi} \equiv \frac{\partial W(\Phi)}{\partial \Phi}\Big|_{\Phi=\varphi} \right)$

$$W(\Phi) = W(\varphi) + (\Phi - \varphi)\frac{\partial W}{\partial \varphi} + \frac{1}{2}(\Phi - \varphi)^2 \frac{\partial^2 W}{\partial \varphi^2}.$$

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$$\to \left(\frac{\partial W}{\partial \varphi} F + h.c. \right) - \left[\frac{1}{2} \frac{\partial^2 W}{\partial \varphi^2} (\psi\psi) + h.c. \right]$$

$$\mathcal{L}_F = FF^* + \frac{\partial W}{\partial \varphi} F + \left(\frac{\partial W}{\partial \varphi} \right)^* F^*$$

is a quadratic in F. $\exists$ NO Kinetic terms for F $\Rightarrow$ no propagator!

"auxiliary fields": can be eliminated by using eq^{n}s of motion

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$$\Rightarrow \left(\frac{\partial W}{\partial \varphi} F + h.c.\right) - \left[\frac{1}{2}\frac{\partial^2 W}{\partial \varphi^2}(\psi\psi) + h.c.\right]$$

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"auxiliary fields": can be eliminated by using eq^{n}s of motion

$$\frac{\delta \mathcal{L}}{\delta F} = 0 \Rightarrow \frac{\partial W}{\partial \varphi} = -F^* \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial F^*} = 0 \Rightarrow F + \left(\frac{\partial W}{\partial \varphi}\right)^* = 0.$$

subst. F back into $\mathcal{L}_F$

$$\Rightarrow \mathcal{L}_F = -\left|\frac{\partial W}{\partial \varphi}\right|^2 = -V_F(\varphi), \text{ the scalar potential.}$$

Comments

- $N=1$ SUSY is a special case of $N=0$ SUSY

$\mathcal{L}$ couples φ, ψ with $V_F(\varphi) \geq 0$

- $m_\varphi = m_\psi$ for WZW model.
- quartic scalar coupling for WZW model is related to Yukawa coupling $\rightarrow \mathcal{L} \supset -\frac{g^2|\varphi|^4}{4} - \frac{g}{2}\varphi\psi\psi$