Title: Explorations in Particle Theory (PHYS 646) - Lecture 8

Date: Mar 24, 2010 10:10 AM

URL: http://pirsa.org/10030092

Abstract:

Superspace and Superfields

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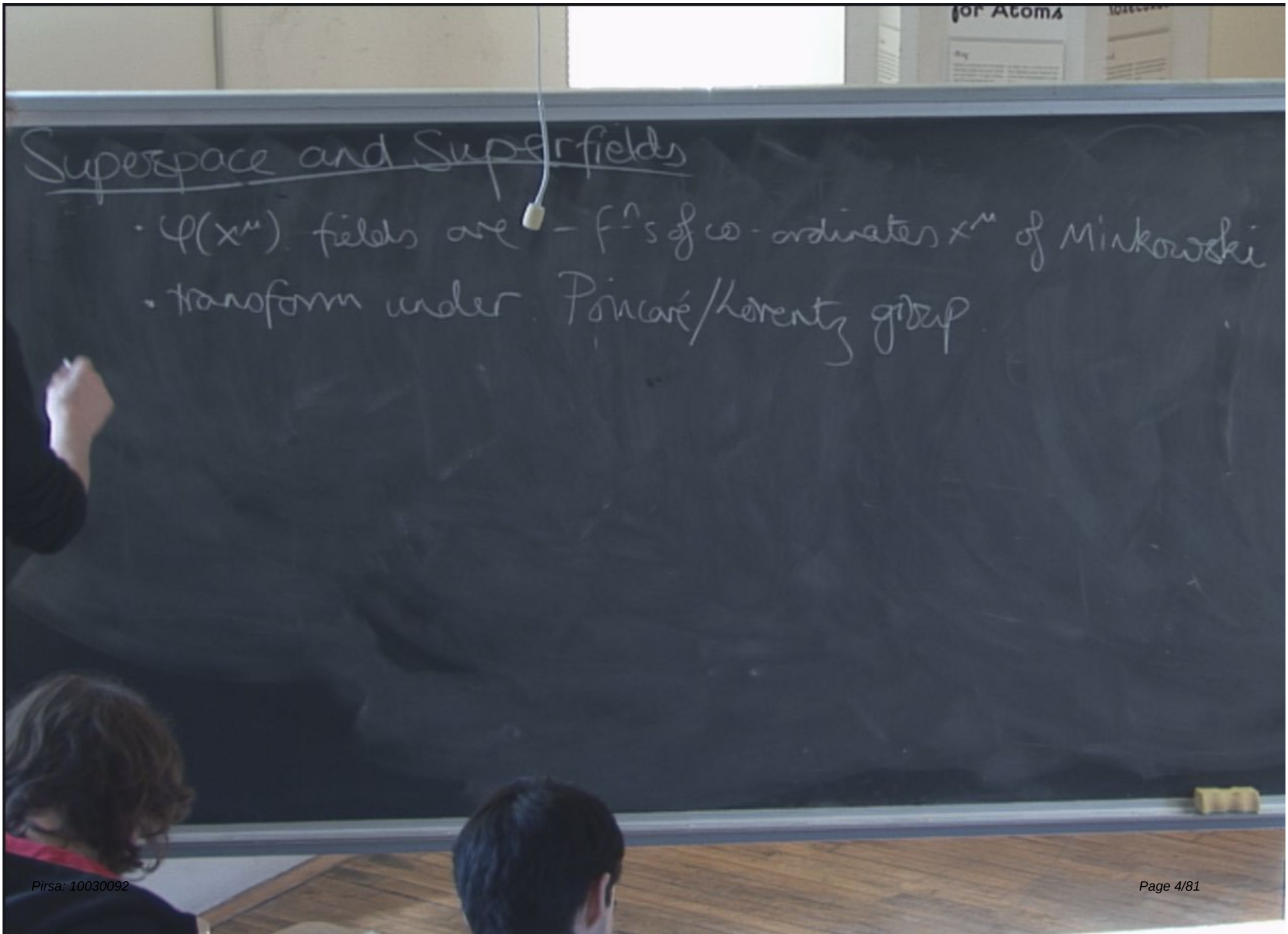
- $\varphi(x^\mu)$ fields are – f^{n}s of co-ordinates x^μ of Minkowski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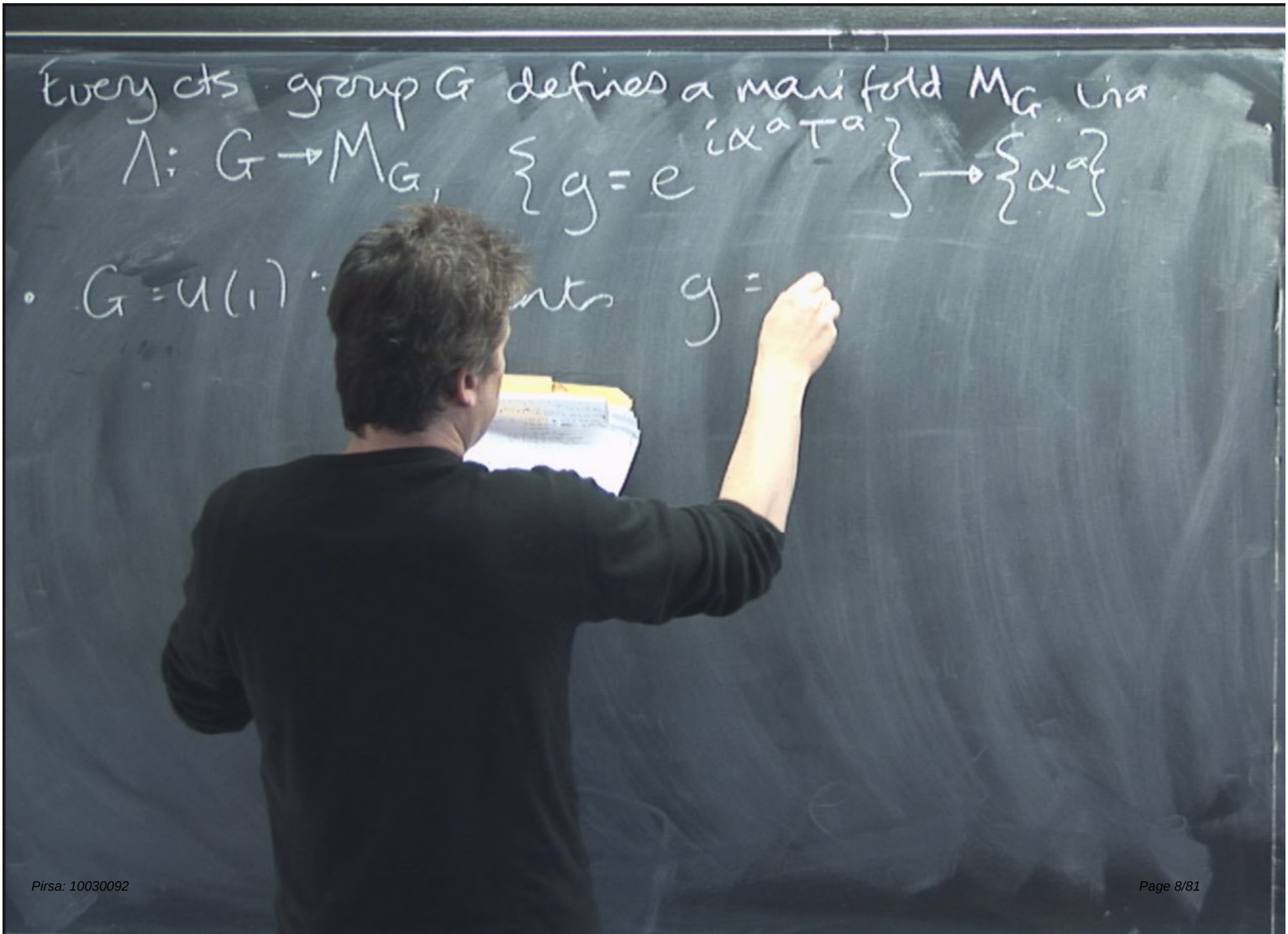
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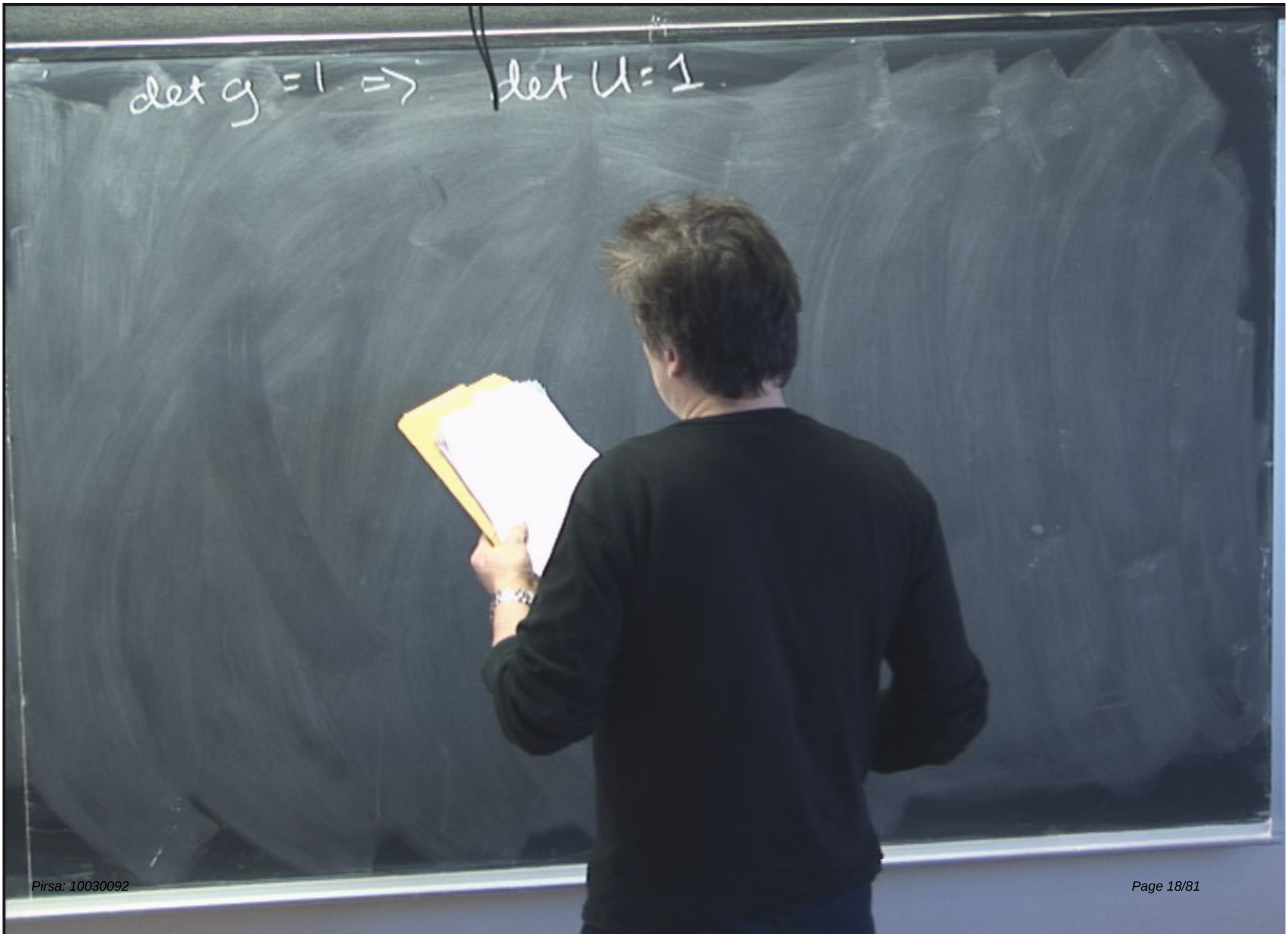
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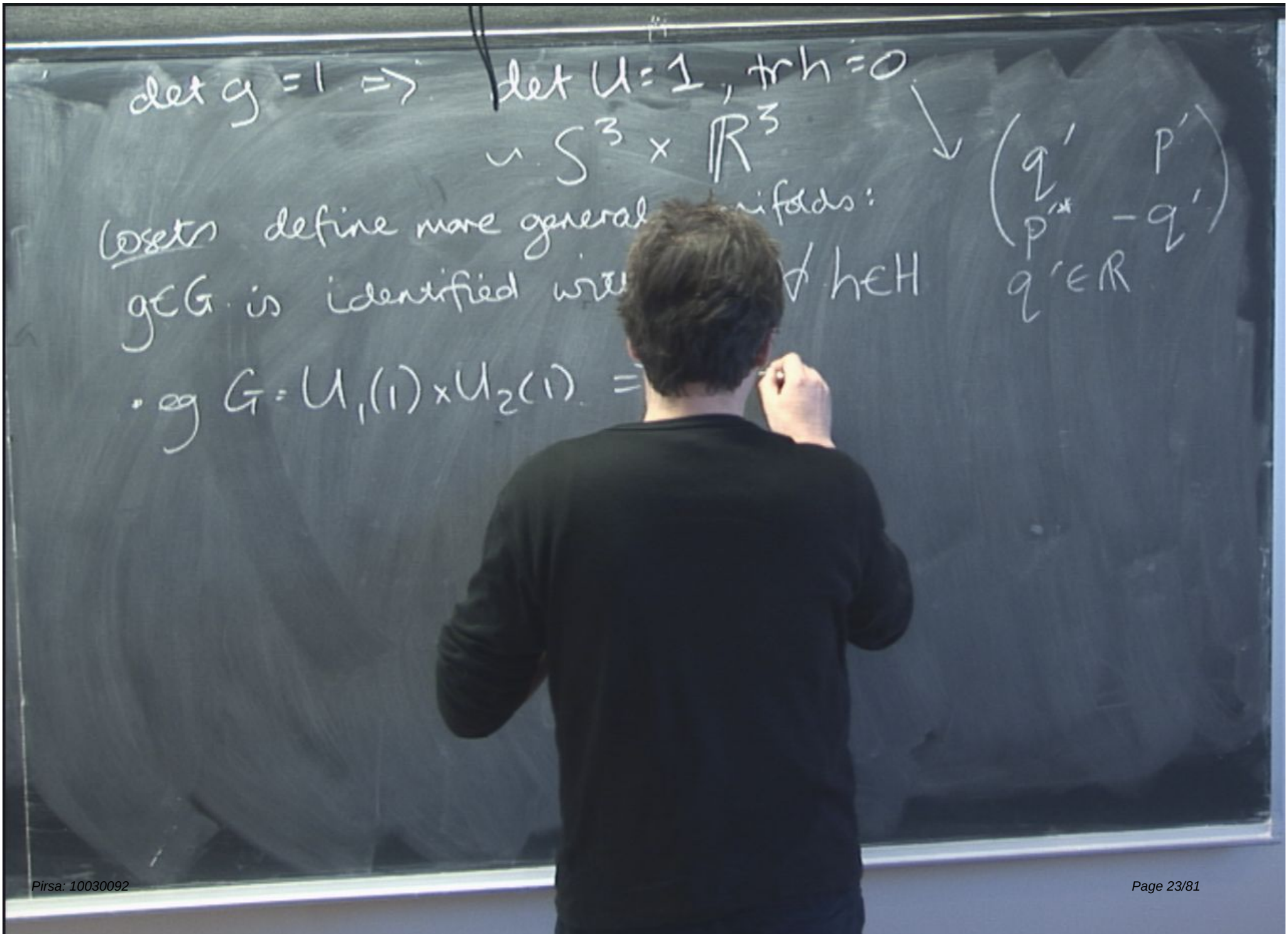
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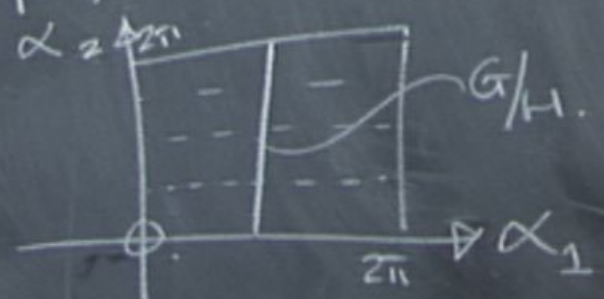

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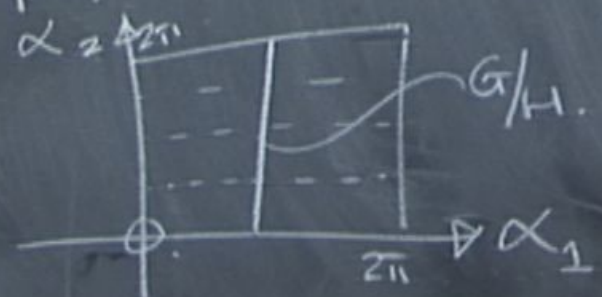

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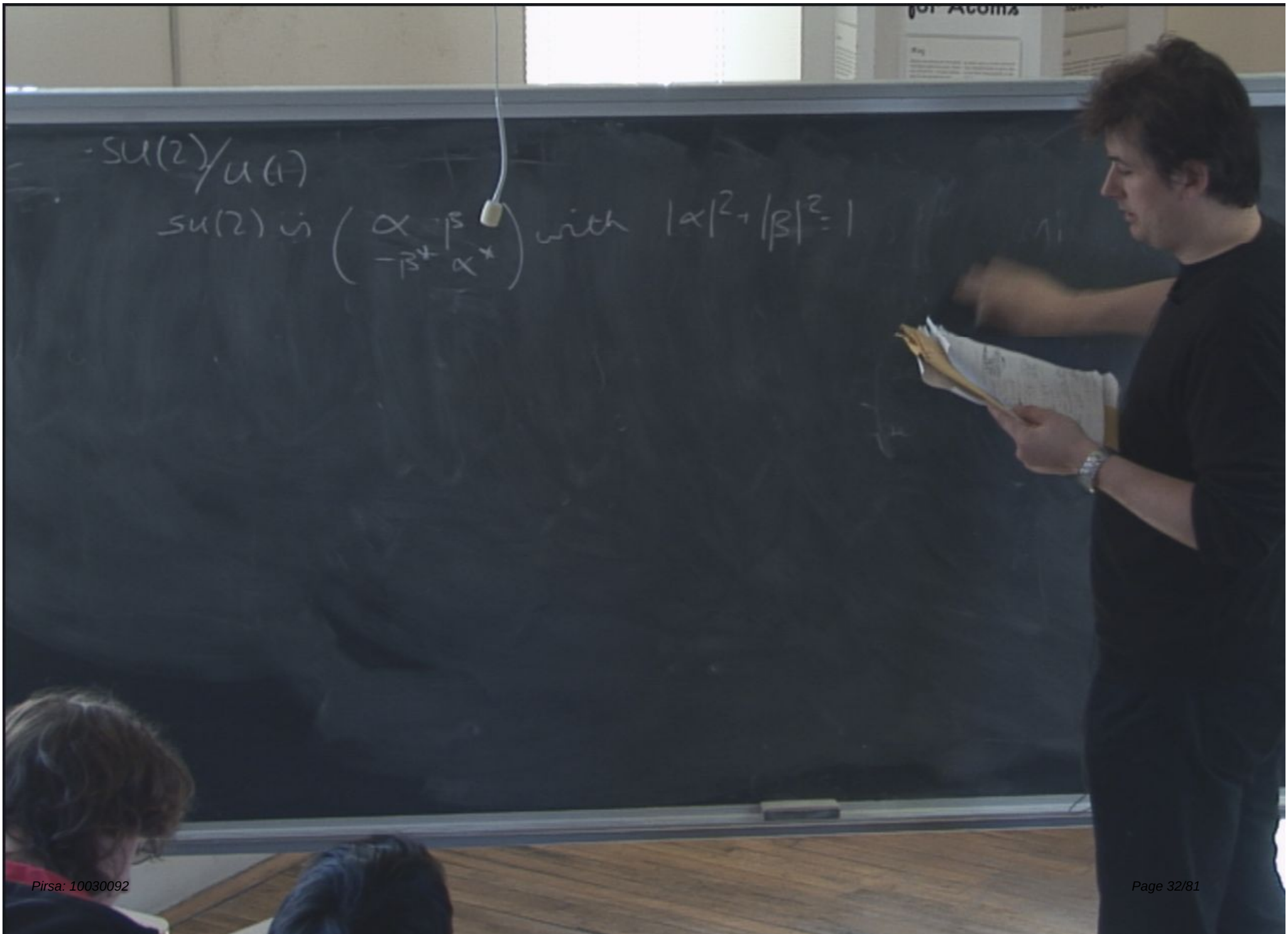
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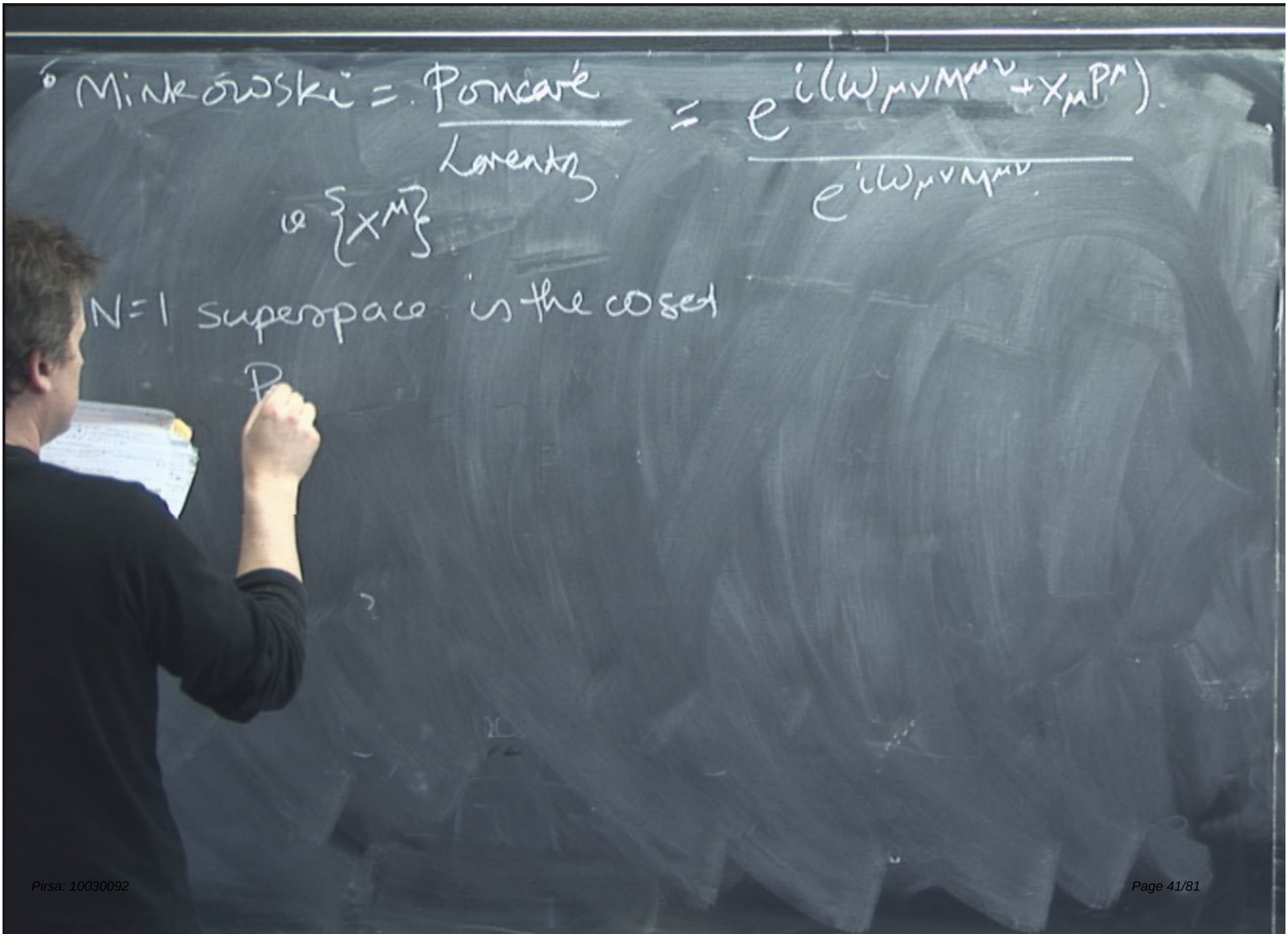
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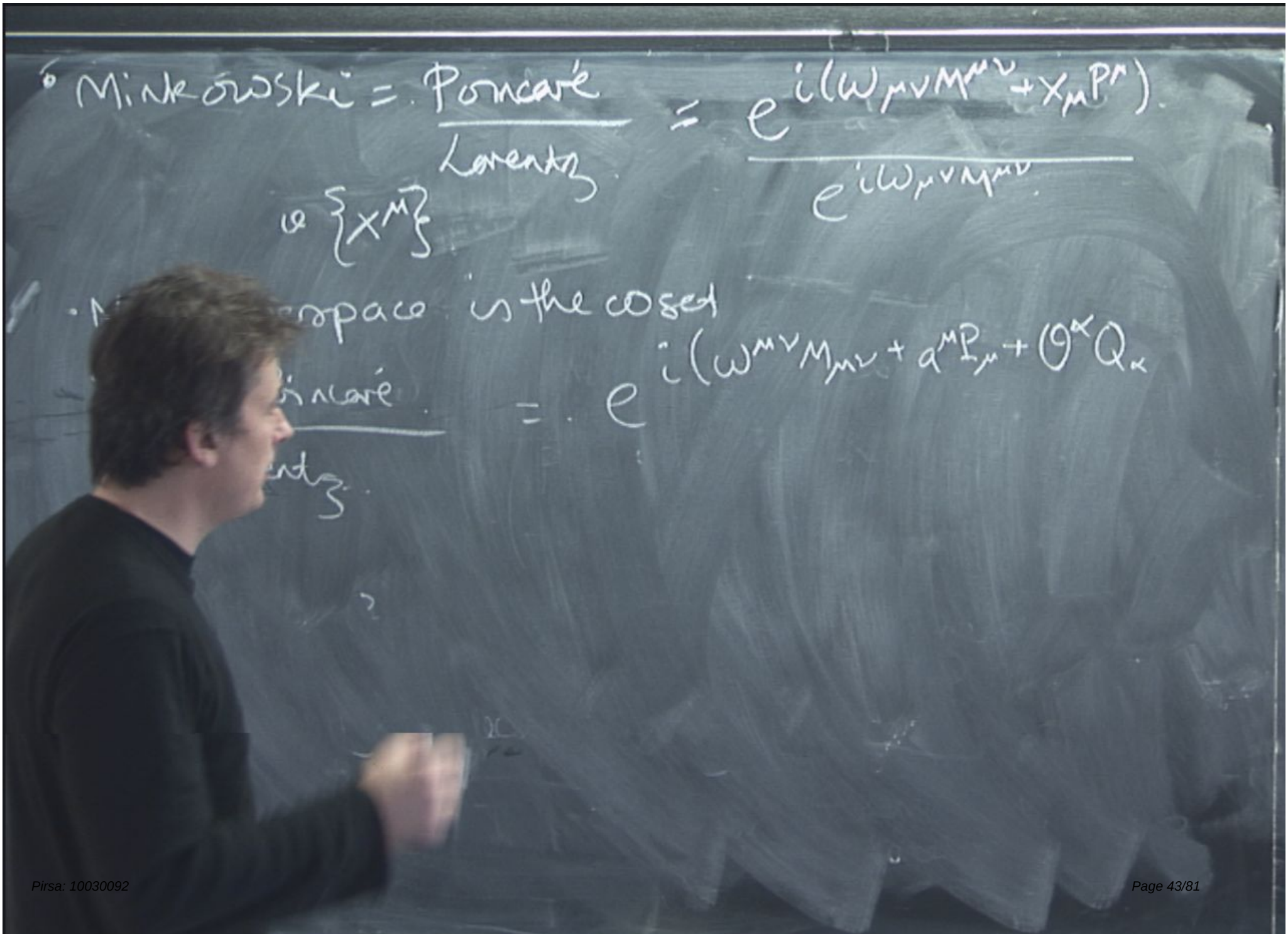
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Grassmann-valued

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$\theta^\alpha, \bar\theta_{\dot\alpha}$ are spinors of Grassmann valued quantities

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$$\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = +\frac{1}{2}\varepsilon^{\dot\alpha\dot\beta}\bar\theta\bar\theta$$

$$\theta^\alpha\theta^\beta = -\frac{1}{2}\varepsilon^{\alpha\beta}\theta\theta$$

$$\frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha \Rightarrow \frac{\partial \theta_\beta}{\partial \theta_\alpha} = -\delta^\alpha_\beta \qquad \frac{\partial \bar\theta^{\dot\beta}}{\partial \bar\theta^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \quad \frac{\partial \bar\theta_{\dot\beta}}{\partial \bar\theta_{\dot\alpha}} = -\delta^{\dot\alpha}_{\dot\beta}$$

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$$\int d\theta^1 \left(\int d\theta^2 \theta^2 \right) \theta^1 = 1 = \frac{1}{2} \int d\theta^1 \int d\theta^2 \, \theta\theta$$

$$\frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha \Rightarrow \frac{\partial \theta_\beta}{\partial \theta_\alpha} = -\delta^\alpha_\beta \qquad \frac{\partial \bar\theta^{\dot\beta}}{\partial \bar\theta^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \quad \frac{\partial \bar\theta_{\dot\beta}}{\partial \bar\theta_{\dot\alpha}} = -\delta^{\dot\alpha}_{\dot\beta}$$

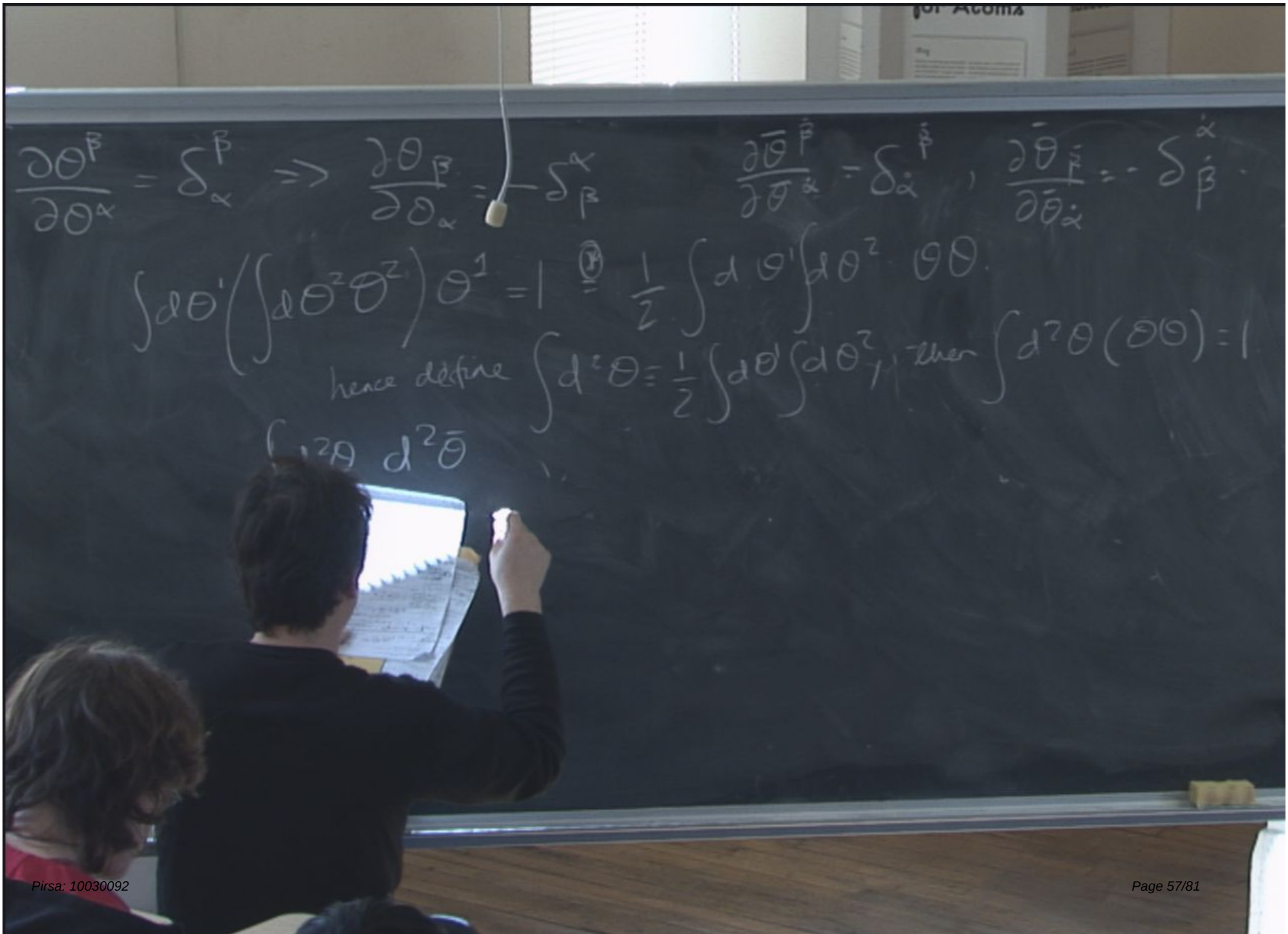
$$\int d\theta^1 \left(\int d\theta^2 \theta^2 \right) \theta^1 = 1 \overset{?}{=} \frac{1}{2} \int d\theta^1 \int d\theta^2 \, \theta\theta$$

hence define $\int d^2\theta = \frac{1}{2}$

$$\frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha \Rightarrow \frac{\partial \theta_\beta}{\partial \theta_\alpha} = -\delta^\alpha_\beta \qquad \frac{\partial \bar\theta^{\dot\beta}}{\partial \bar\theta^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \quad \frac{\partial \bar\theta_{\dot\beta}}{\partial \bar\theta_{\dot\alpha}} = -\delta^{\dot\alpha}_{\dot\beta}$$

$$\int d\theta^1 \left(\int d\theta^2 \theta^2 \right) \theta^1 = 1 \overset{?}{=} \frac{1}{2} \int d\theta^1 \int d\theta^2 \, \theta\theta$$

hence define $\int d^2\theta \equiv \frac{1}{2} \int d\theta^1 \int d\theta^2$, then $\int d^2\theta \, (\theta\theta) = 1$


$\frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha \Rightarrow \frac{\partial \theta_\beta}{\partial \theta_\alpha} = -\delta^\alpha_\beta$
$\frac{\partial \bar\theta^{\dot\beta}}{\partial \bar\theta^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \quad \frac{\partial \bar\theta_{\dot\beta}}{\partial \bar\theta_{\dot\alpha}} = -\delta^{\dot\alpha}_{\dot\beta}$
$\int d\theta^1 \left(\int d\theta^2 \theta^2 \right) \theta^1 = 1 \stackrel{?}{=} \frac{1}{2} \int d\theta^1 \int d\theta^2 \, \theta\theta$
hence define $\int d^2\theta = \frac{1}{2} \int d\theta^1 \int d\theta^2$, then $\int d^2\theta \, (\theta\theta) = 1$
$\int d^2\theta \, d^2\bar\theta$

$$\frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha \Rightarrow \frac{\partial \theta_\beta}{\partial \theta_\alpha} = -\delta^\alpha_\beta \qquad \frac{\partial \bar\theta^{\dot\beta}}{\partial \bar\theta^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \quad \frac{\partial \bar\theta_{\dot\beta}}{\partial \bar\theta_{\dot\alpha}} = -\delta^{\dot\alpha}_{\dot\beta}$$

$$\int d\theta^1 \left(\int d\theta^2 \theta^2 \right) \theta^1 = 1 \quad \frac{1}{2} \int d\theta^1 \int d\theta^2 \, \theta\theta$$

hence define $\int d^2\theta \equiv \frac{1}{2} \int d\theta^1 \int d\theta^2$, then $\int d^2\theta \, (\theta\theta) = 1$

$$\int d^2\theta \, d^2\bar\theta \, (\theta\theta)(\bar\theta\bar\theta) = 1$$

so $d^2\theta$

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$$\int d^2\theta \, d^2\bar\theta \, (\theta\theta)(\bar\theta\bar\theta) = 1$$

so $d^2\theta = +\frac{1}{4} d\theta^\alpha d\theta^\beta \varepsilon_{\beta\alpha}$ an

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$$\frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha \Rightarrow \frac{\partial \theta_\beta}{\partial \theta_\alpha} = -\delta^\alpha_\beta \qquad \frac{\partial \bar\theta^{\dot\beta}}{\partial \bar\theta^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \quad \frac{\partial \bar\theta_{\dot\beta}}{\partial \bar\theta_{\dot\alpha}} = -\delta^{\dot\alpha}_{\dot\beta}$$

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$$\int d^2\theta = \frac{\varepsilon^{\alpha\beta}}{4} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

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Definition and transformations of a general superfield

$$\int d^2\theta = \frac{\varepsilon^{\alpha\beta}}{4} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

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Definition and transformations of a general superfield

Recall properties of $\varphi(x^\mu)$:

- treat translation by a_μ as an operator, $\varphi \to e^{ia_\mu P^\mu}\varphi$

$$\int d^2\theta = \frac{\varepsilon^{\alpha\beta}}{4} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

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Definition and transformations of a general superfield

Recall properties of $\varphi(x^\mu)$:

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$$\int d^2\theta = \frac{\varepsilon^{\alpha\beta}}{4} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

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Definition and transformations of a general superfield

Recall properties of $\varphi(x^\mu)$:

- treat translation by a_μ as an operator, $\varphi \to e^{ia_\mu P^\mu} \varphi e^{-ia_\nu P^\nu}$

but $\varphi(x^\mu)$ is also a Hilbert vector in f^n space $\mathcal{F}$, so

$$\varphi(x^\mu) \mapsto e^{-ia_\mu P^\mu} \varphi(x^\mu) \equiv \varphi(x^\mu - a^\mu).$$

$$\int d^2\theta = \frac{\varepsilon^{\alpha\beta}}{4} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

$$\int d^2\bar\theta = -\frac{1}{4} \varepsilon^{\dot\alpha\dot\beta} \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \frac{\partial}{\partial\bar\theta^{\dot\beta}}$$

Definition and transformations of a general superfield

Recall properties of $\varphi(x^\mu)$:

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$$\varphi(x^\mu) \mapsto e^{-ia_\mu P^\mu} \varphi(x^\mu) = \varphi(x^\mu - a^\mu).$$

Taylor's,

$$P_\mu = -i\partial$$

$$\int d^2\theta = \frac{\varepsilon^{\alpha\beta}}{4} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

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Definition and transformations of a general superfield

Recall properties of $\varphi(x^\mu)$:

- treat translation by a_μ as an operator, $\varphi \to e^{ia_\mu \underline{P}^\mu} \varphi e^{-ia_\nu P^\nu}$ — ①

but $\varphi(x^\mu)$ is also a Hilbert vector in f^n space $\mathcal{F}$, so

$$\varphi(x^\mu) \mapsto e^{-ia_\mu P^\mu} \varphi(x^\mu) = \varphi(x^\mu - a^\mu).$$

by Taylor's,

$$P_\mu = -i\partial_\mu.$$

$$① : \left(1 - ia_\mu \underline{P}^\mu\right) \varphi \left(1 + ia_\nu \underline{P}^\nu\right) = \left(1 - a^\mu \partial_\mu\right) \varphi$$

$$\Longleftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi.$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar{\theta}^{\dot\alpha})$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$$ - expand in $\theta^\alpha, \bar\theta^{\dot\alpha}$.

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$$ - expand in $\theta^\alpha, \bar\theta^{\dot\alpha}$

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta \cdot \psi(x) + \bar\theta \bar\chi$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$$ - expand in $\theta^\alpha, \bar\theta^{\dot\alpha}$

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$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$$ - expand in $\theta^\alpha, \bar\theta^{\dot\alpha}$

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta\psi(x) + \bar\theta\bar\chi(x^\mu) + \theta\theta M(x)$$

$$+ \bar\theta\bar\theta N(x) + (\theta\sigma^\mu\bar\theta) V_\mu$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$$ - expand in $\theta^\alpha, \bar\theta^{\dot\alpha}$

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta\psi(x) + \bar\theta\bar\chi(x^\mu) + \theta\theta M(x)$$

$$+ \bar\theta\bar\theta N(x) + (\theta\sigma^\mu\bar\theta) V_\mu + \theta\theta\,\bar\theta\bar\lambda(x)$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) \text{ - expand in } \theta^\alpha, \bar\theta^{\dot\alpha}$$

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta\psi(x) + \bar\theta\bar\chi(x^\mu) + \theta\theta M(x)$$

$$+ \bar\theta\bar\theta N(x) + (\theta\sigma^\mu\bar\theta) V_\mu + (\theta\theta)\bar\theta\bar\lambda(x^\mu) + (\bar\theta\bar\theta)\theta\rho(x^\mu)$$

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$$+ \bar\theta\bar\theta N(x) + (\theta\sigma^\mu\bar\theta) V_\mu + (\theta\theta)\bar\theta\bar\lambda(x^\mu) + (\bar\theta\bar\theta)\theta\rho(x^\mu)$$

$$+ (\theta\theta)(\bar\theta\bar\theta) D(x).$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) \text{ - expand in } \theta^\alpha, \bar\theta^{\dot\alpha}$$

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta\psi(x) + \bar\theta\bar\chi(x^\mu) + \theta\theta M(x)$$

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$$+ (\theta\theta)(\bar\theta\bar\theta) D(x)$$

$$\theta\theta\theta = 0$$

$$\Leftrightarrow \; i a_\mu [\varphi, P^\mu] = -a^\mu \partial_\mu \varphi = \delta\varphi$$

well do the same thing on a general superfield

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) \text{ - expand in } \theta^\alpha, \bar\theta^{\dot\alpha}$$

$$S(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) = \varphi(x^\mu) + \theta\psi(x) + \bar\theta\bar\chi(x^\mu) + \theta\theta M(x)$$

$$+ \bar\theta\bar\theta N(x) + (\theta\sigma^\mu\bar\theta) V_\mu + (\theta\theta)\bar\theta\bar\lambda(x^\mu) + (\bar\theta\bar\theta)\theta\rho(x^\mu)$$

$$+ (\theta\theta)(\bar\theta\bar\theta) D(x)$$

$$\theta^\alpha\theta^\beta\theta^\gamma = 0$$