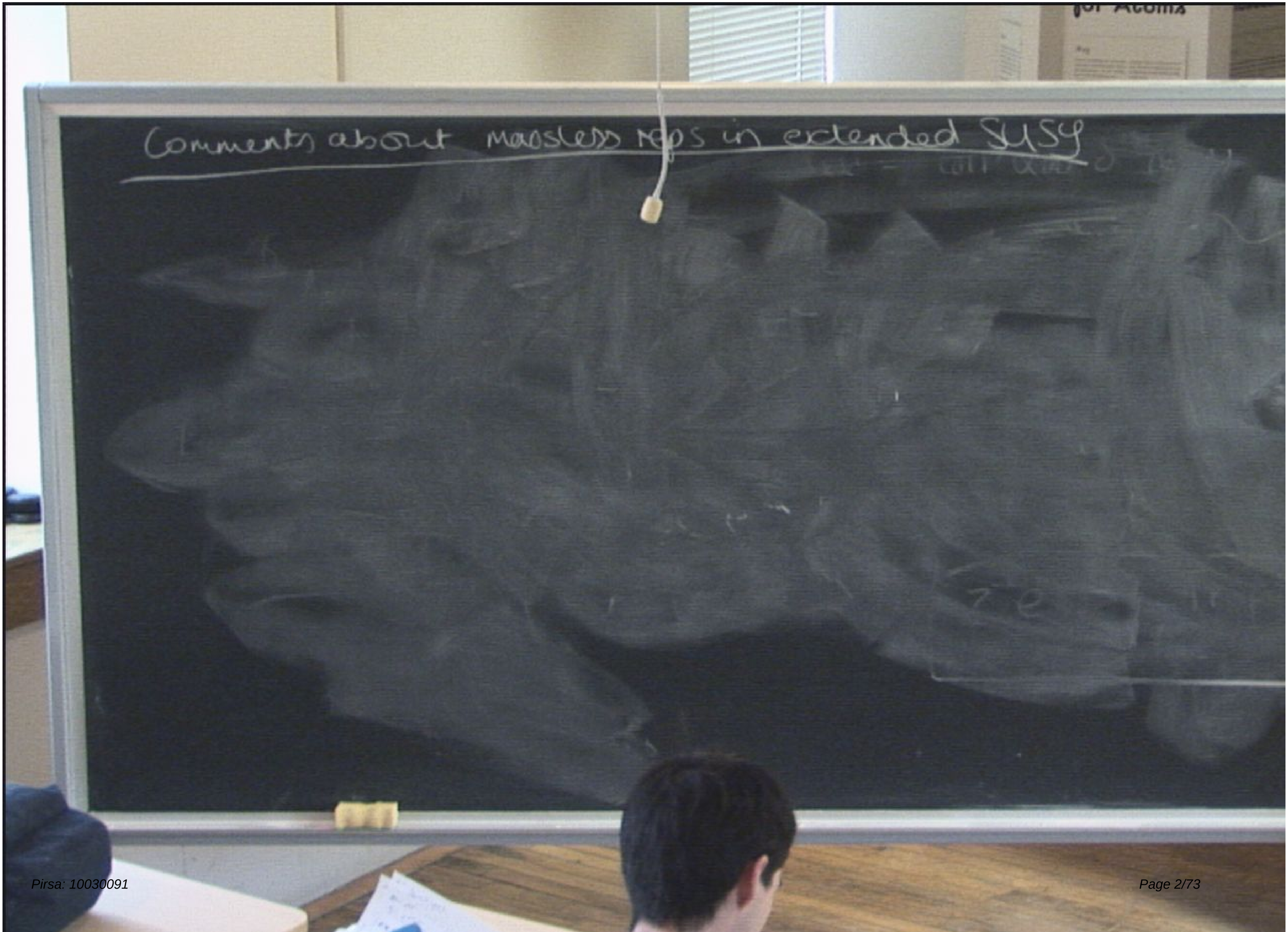
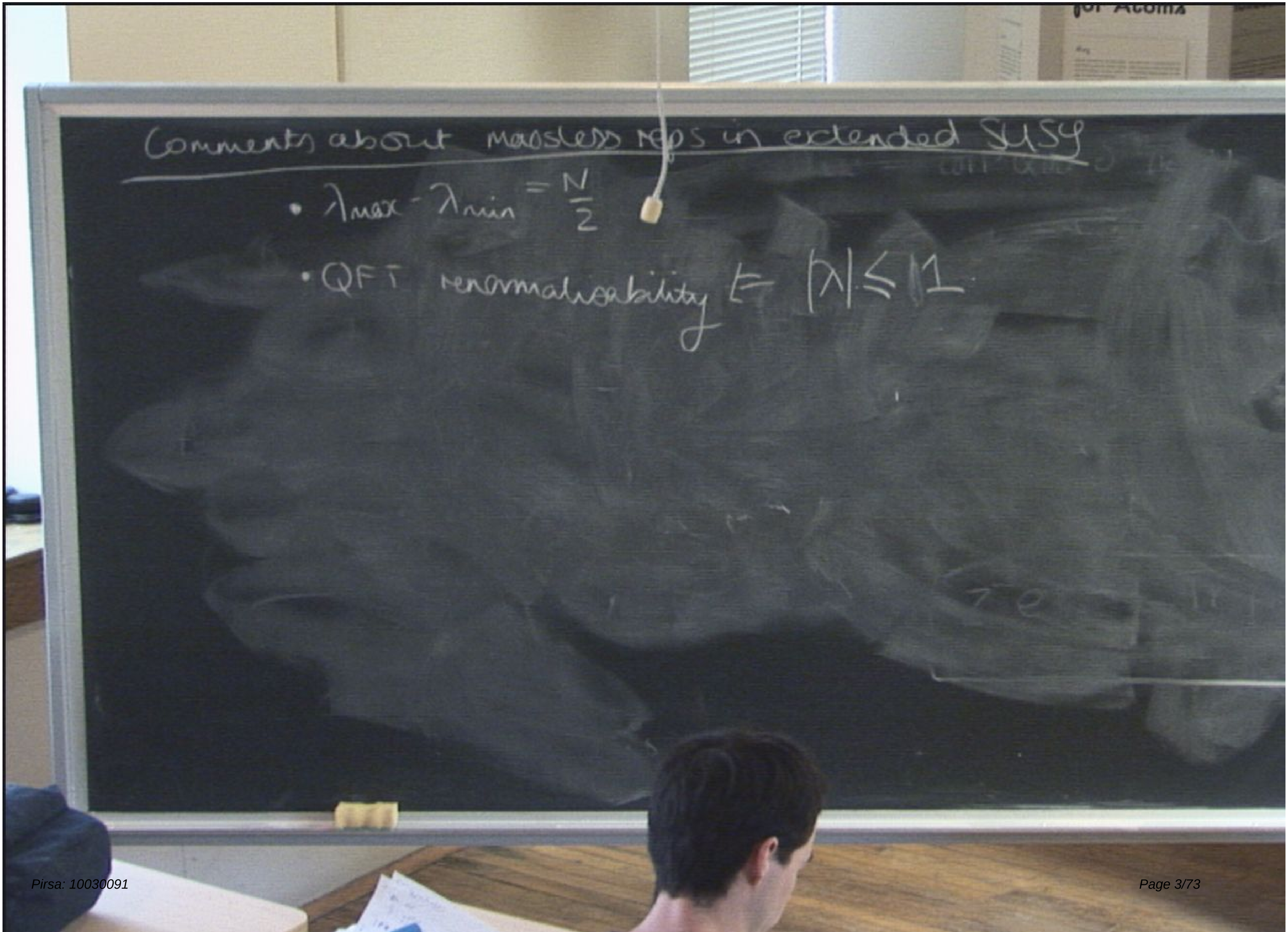
Title: Explorations in Particle Theory (PHYS 646) - Lecture 7

Date: Mar 23, 2010 10:10 AM

URL: http://pirsa.org/10030091

Abstract:


Comments about massless reps in extended SUSY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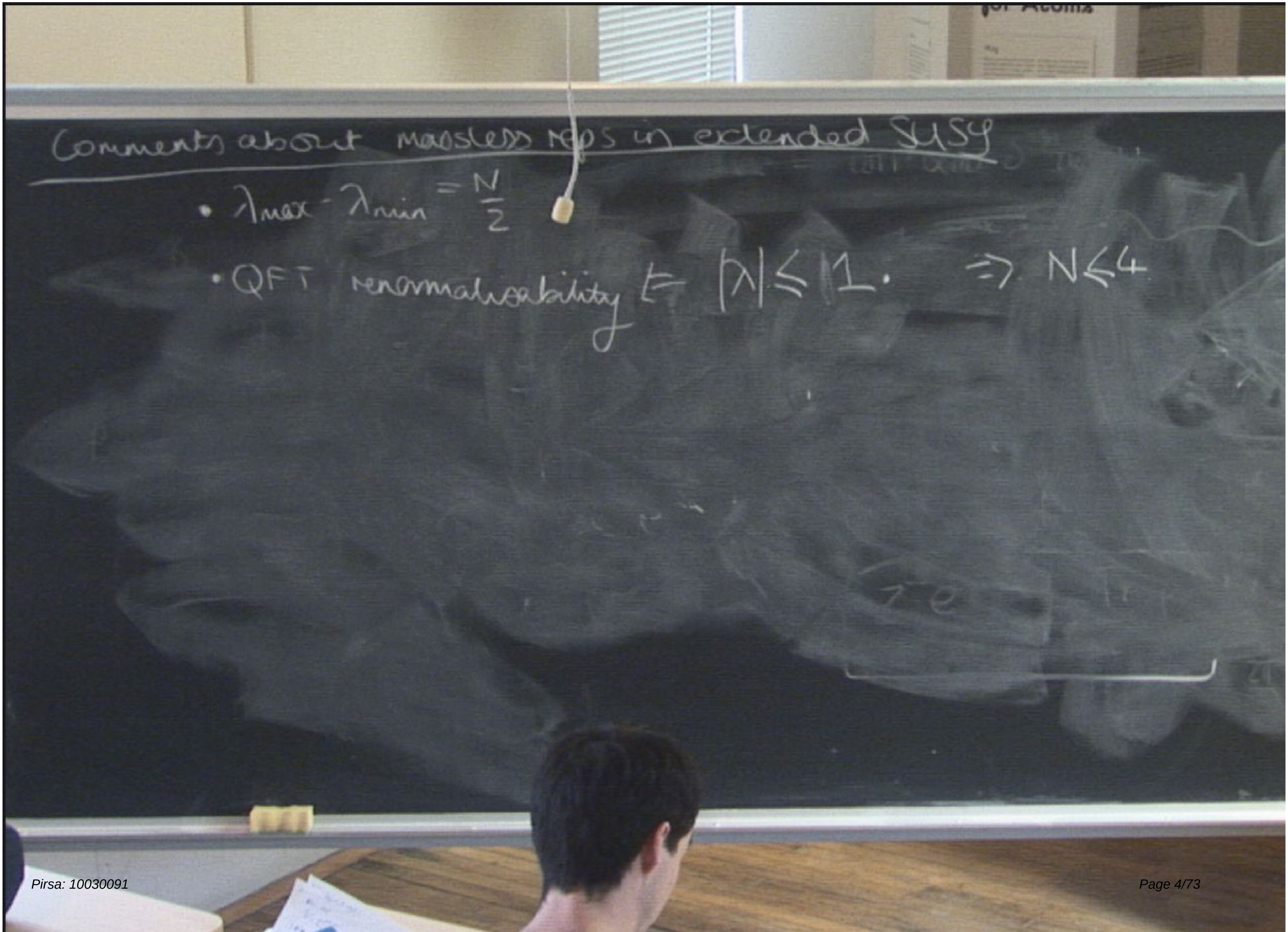
• $\lambda_{max} - \lambda_{min} = \frac{N}{2}$
• QFT renormalizability $\Rightarrow$ $|\lambda| \leq 1$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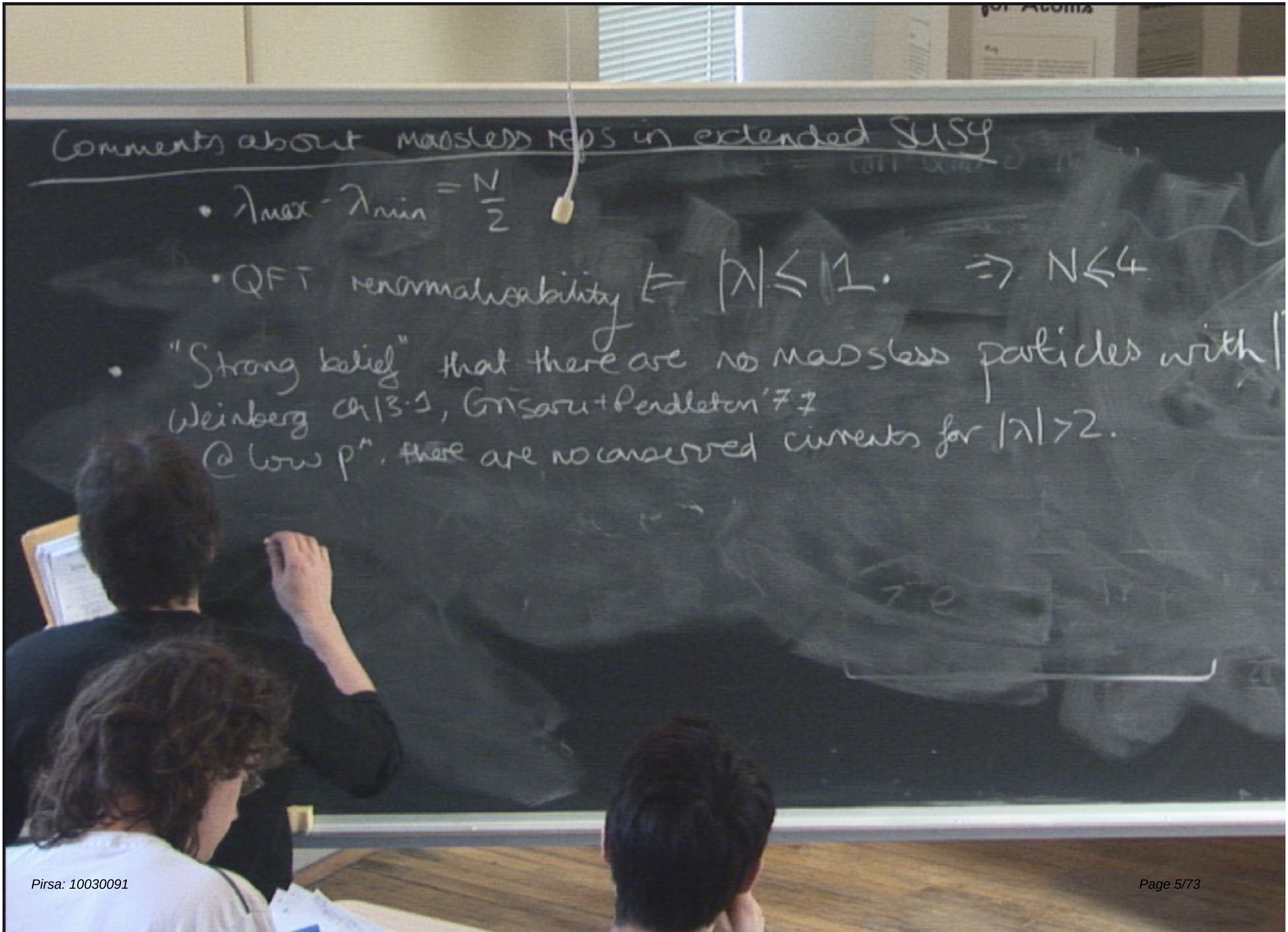
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Weinberg Ch 13.1, Grisaru+Pendleton '77
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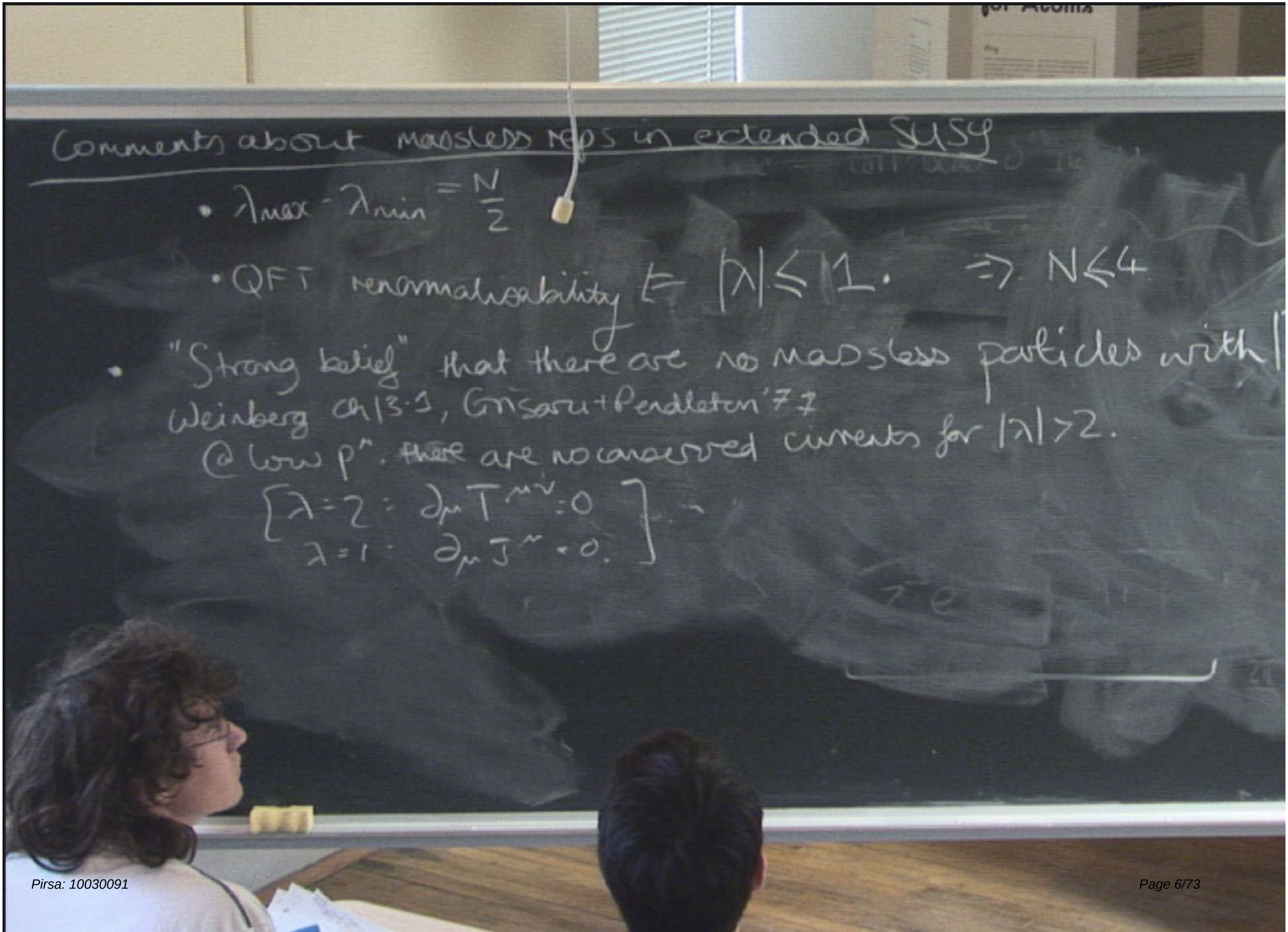
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$\mathcal{L}$? ..

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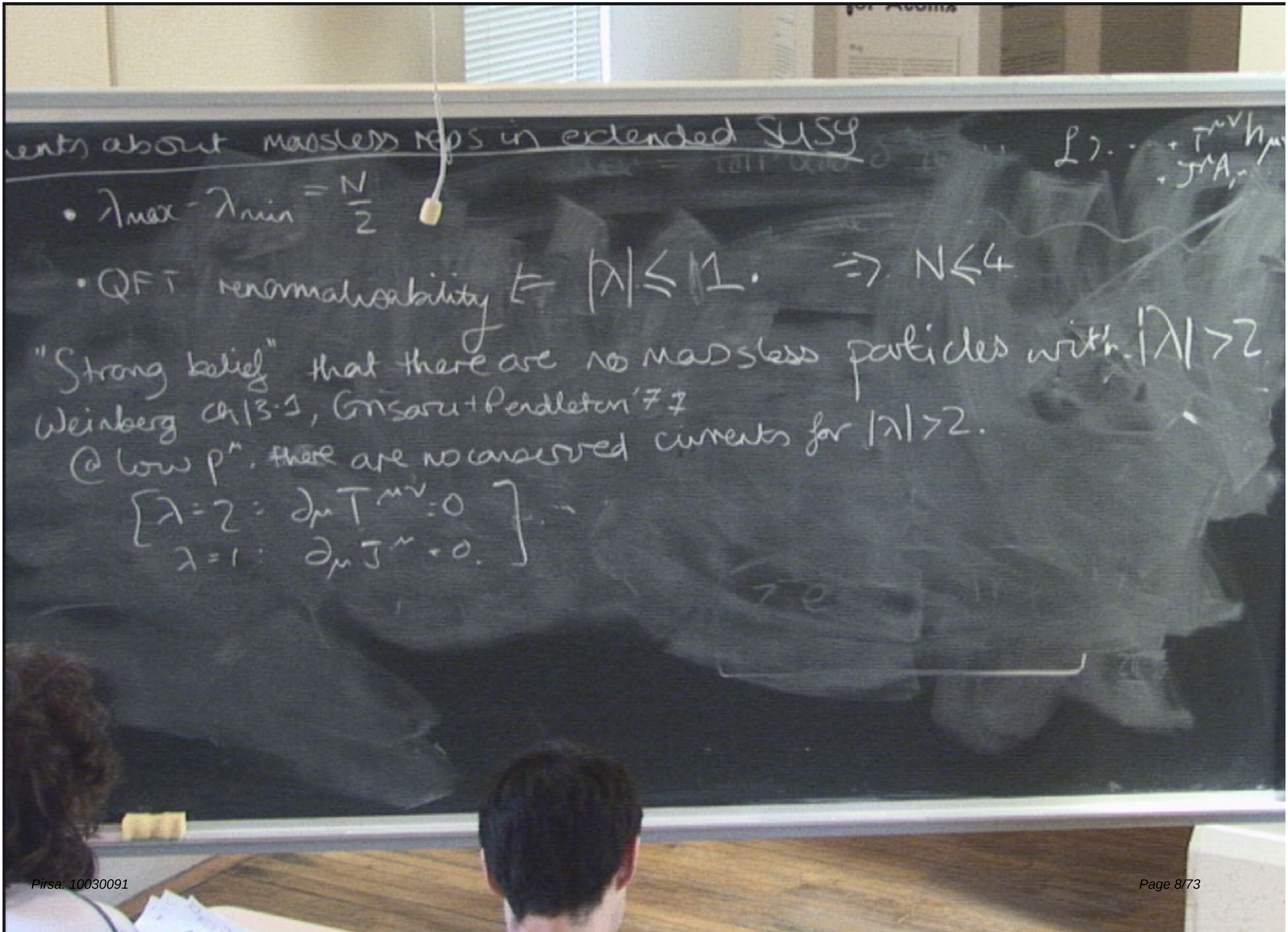
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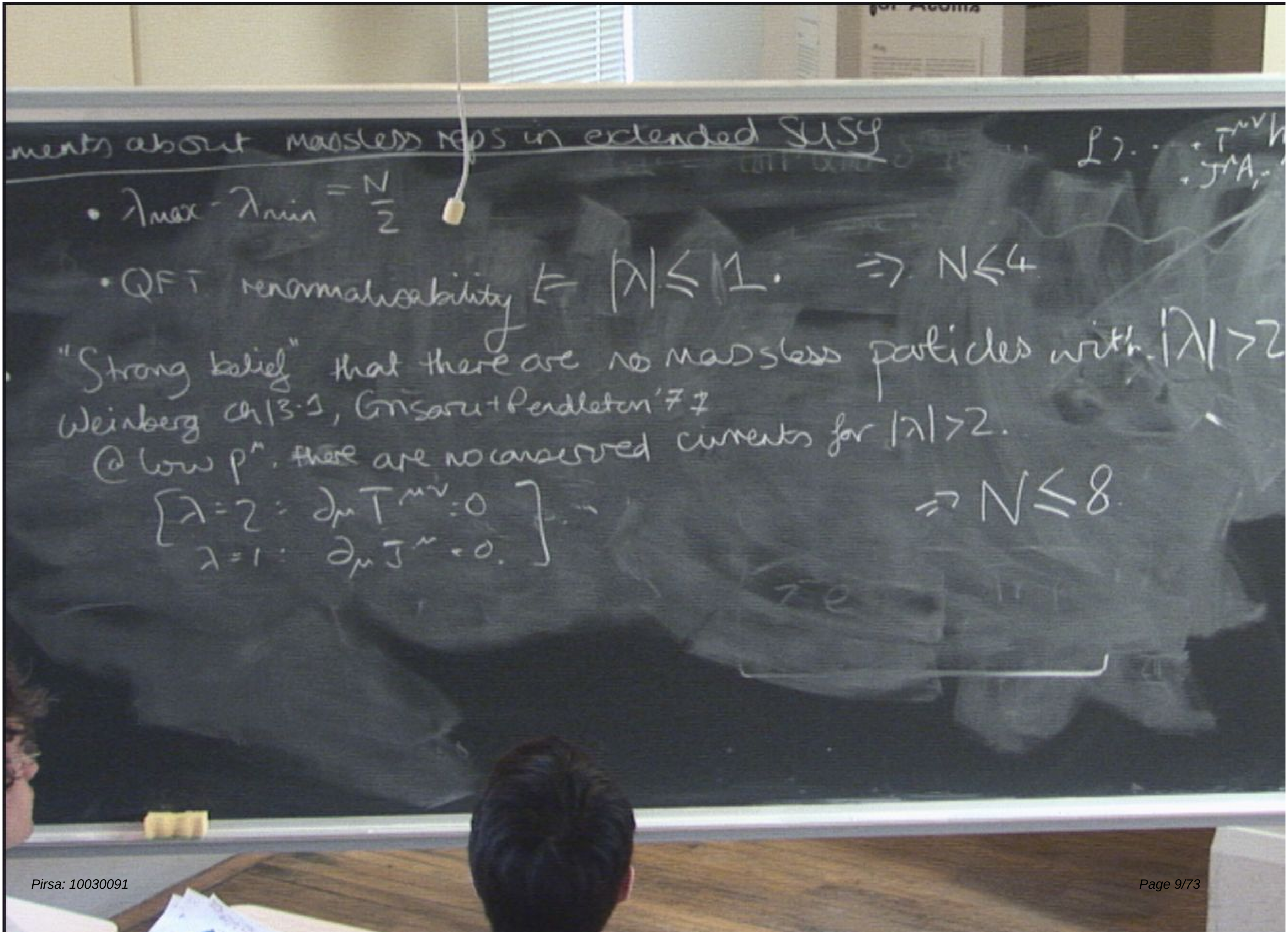
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• $N > 1$ SUSY : supermultiplets are non-χral.

(SM needs χral reps).

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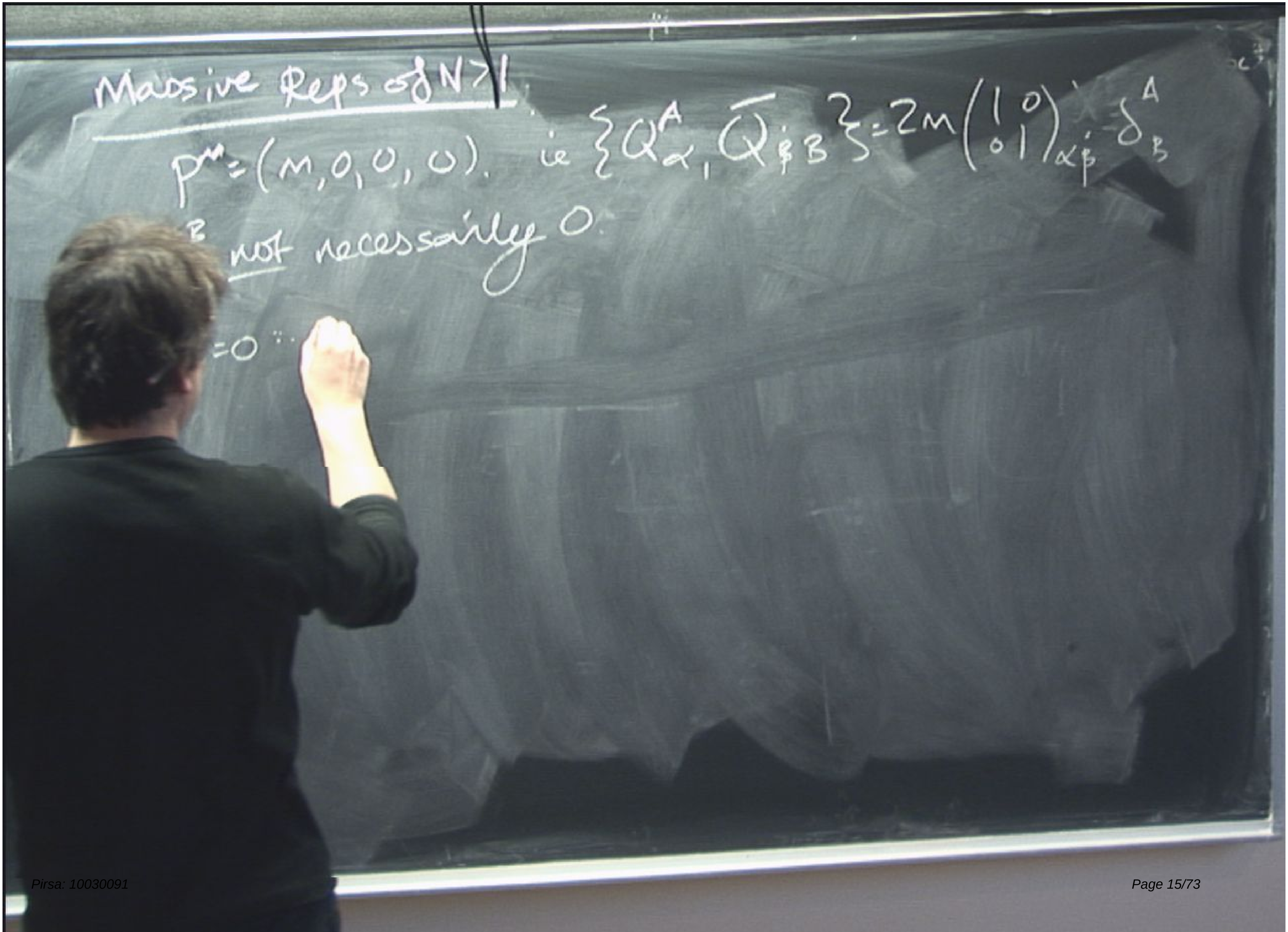
- Only $N = 1$, $N = 0$ has chiral supermultiplets.

Massive Reps of $N>1$


Massive Reps of N>1
$p^\mu = (M,0,0,0)$. ie $\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2M \begin{pmatrix}1&0\\0&1\end{pmatrix}_{\alpha\dot\beta} \delta^A_B$
B not necessarily 0.
=0 ∴

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Z^{AB} <u>not</u> necessarily 0.

$Z^{AB}=0$: 2N raising/lowering op^s

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eg $N=2, y=0$

$|\Omega\rangle$ $1\times$ spin 0

$a^{\dagger A}_{\dot\alpha}|\Omega\rangle$ $4\times$ spin $1/2$

$a^{\dagger A}_{\dot\alpha} a^{\dagger B}_{\dot\beta}|\Omega\rangle$ 3 spin 0, 3 spin 1

$a^{A\dagger}_{\alpha} a^{B\dagger}_{\dot\beta} a^{C\dagger}_{\dot\gamma}|\Omega\rangle$ $4\times$ spin $1/2$

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16 states.

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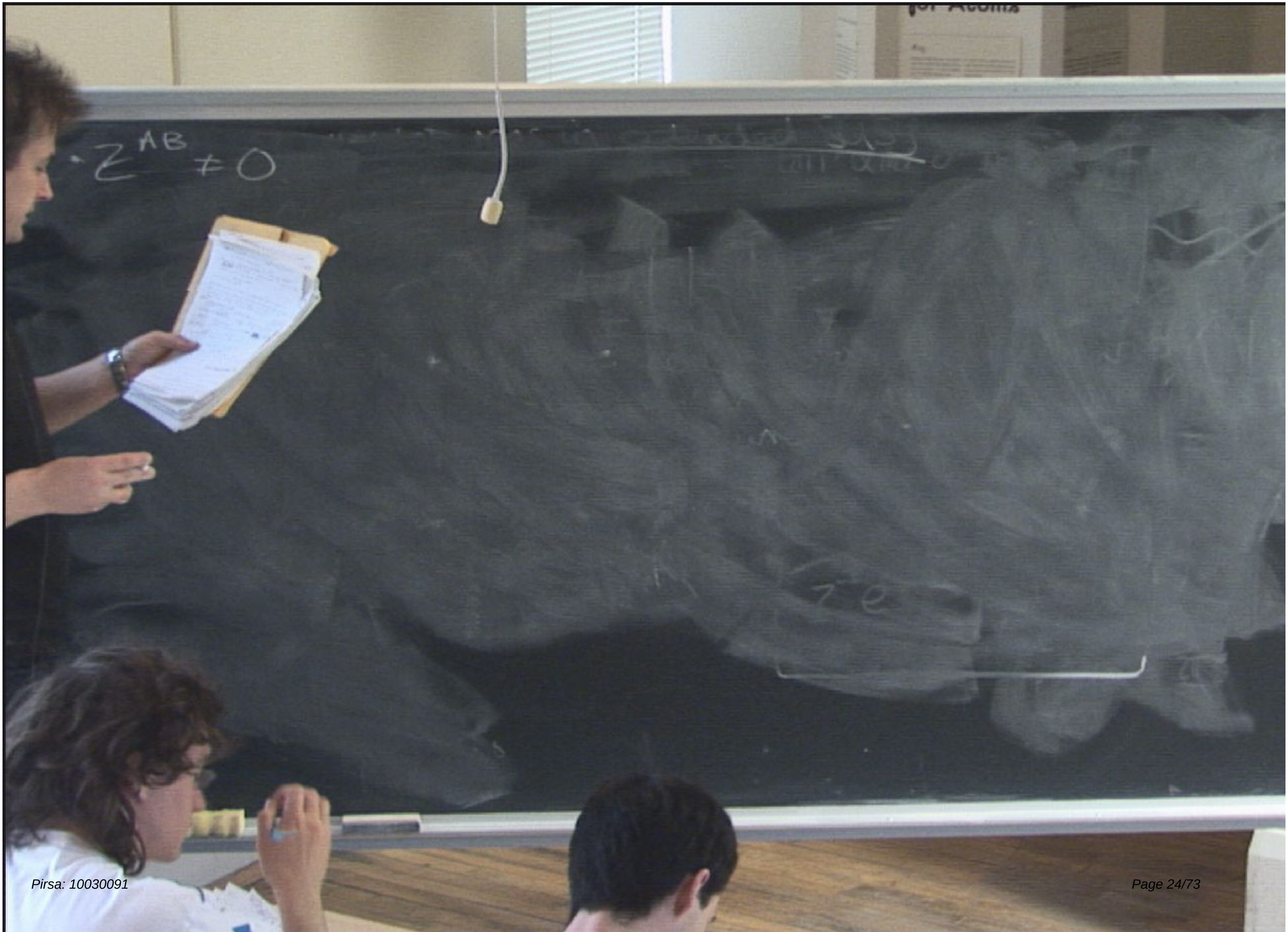
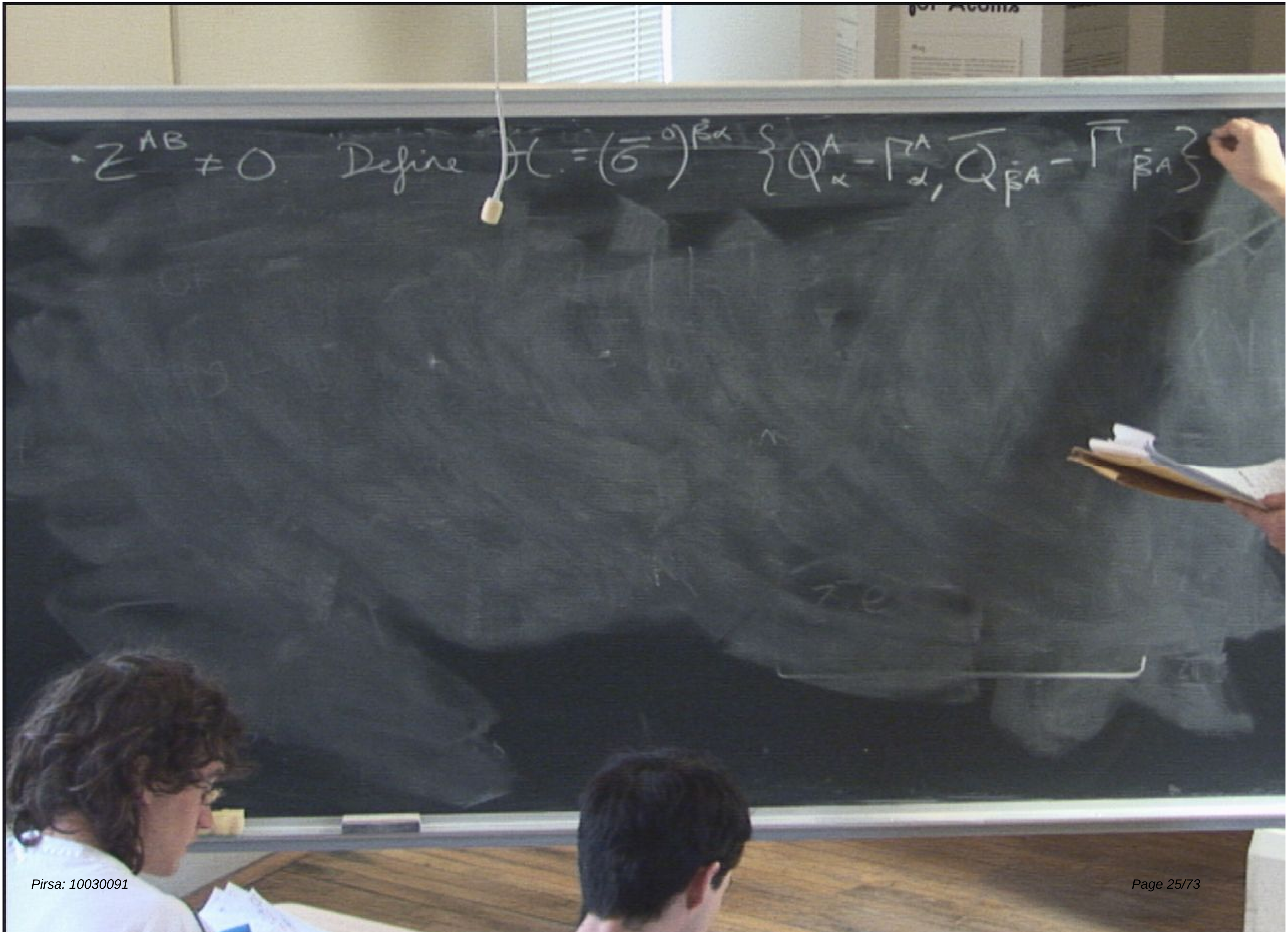
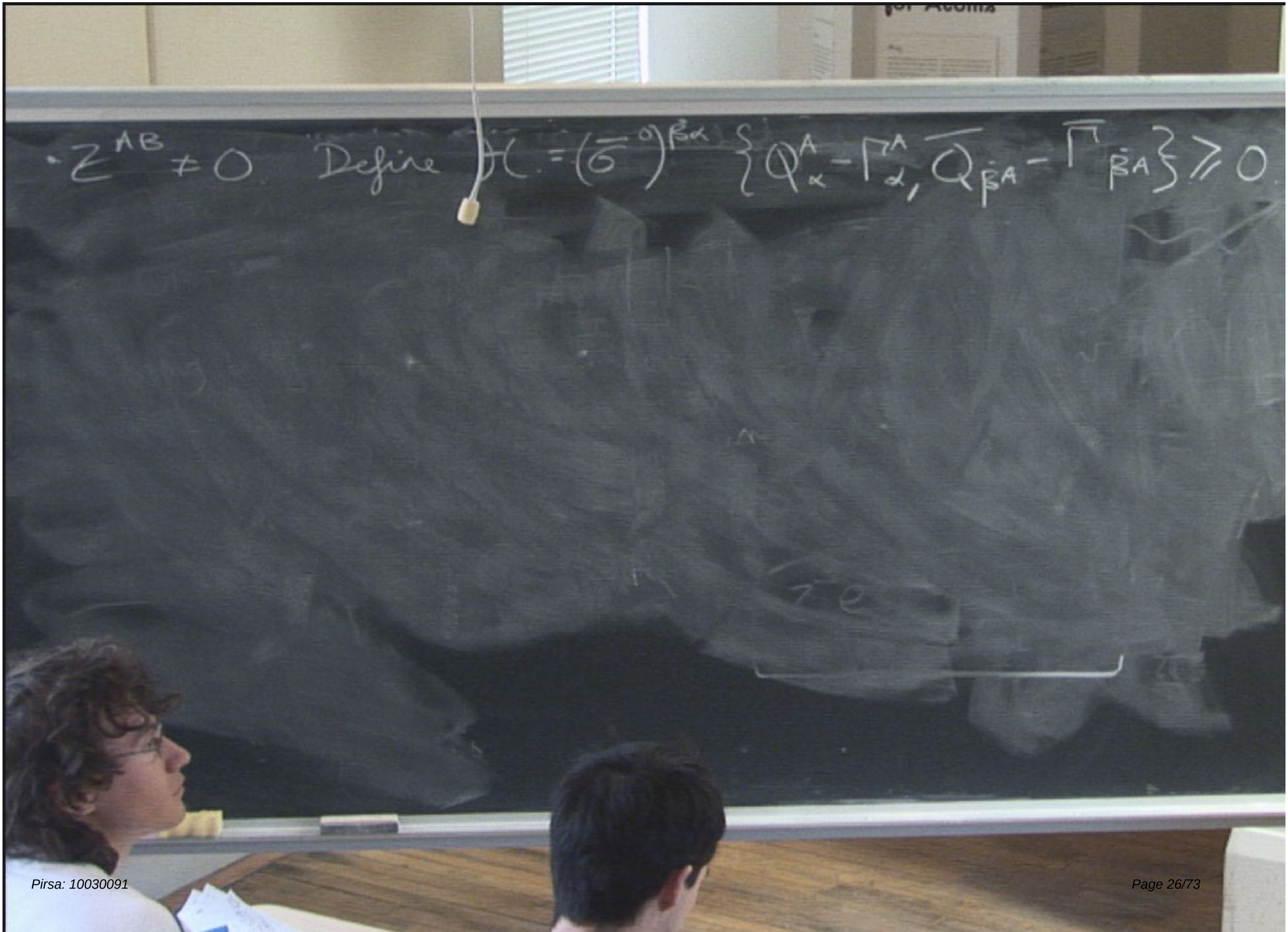

Z^AB ≠ 0


$\cdot Z^{AB} \neq 0$ Define $\mathcal{H} = (\bar{\sigma}^{0})^{\dot{\beta}\alpha} \{ Q^{A}_{\alpha} - \Gamma^{A}_{\alpha}, \bar{Q}_{\dot{\beta}A} - \bar{\Gamma}_{\dot{\beta}A} \}$



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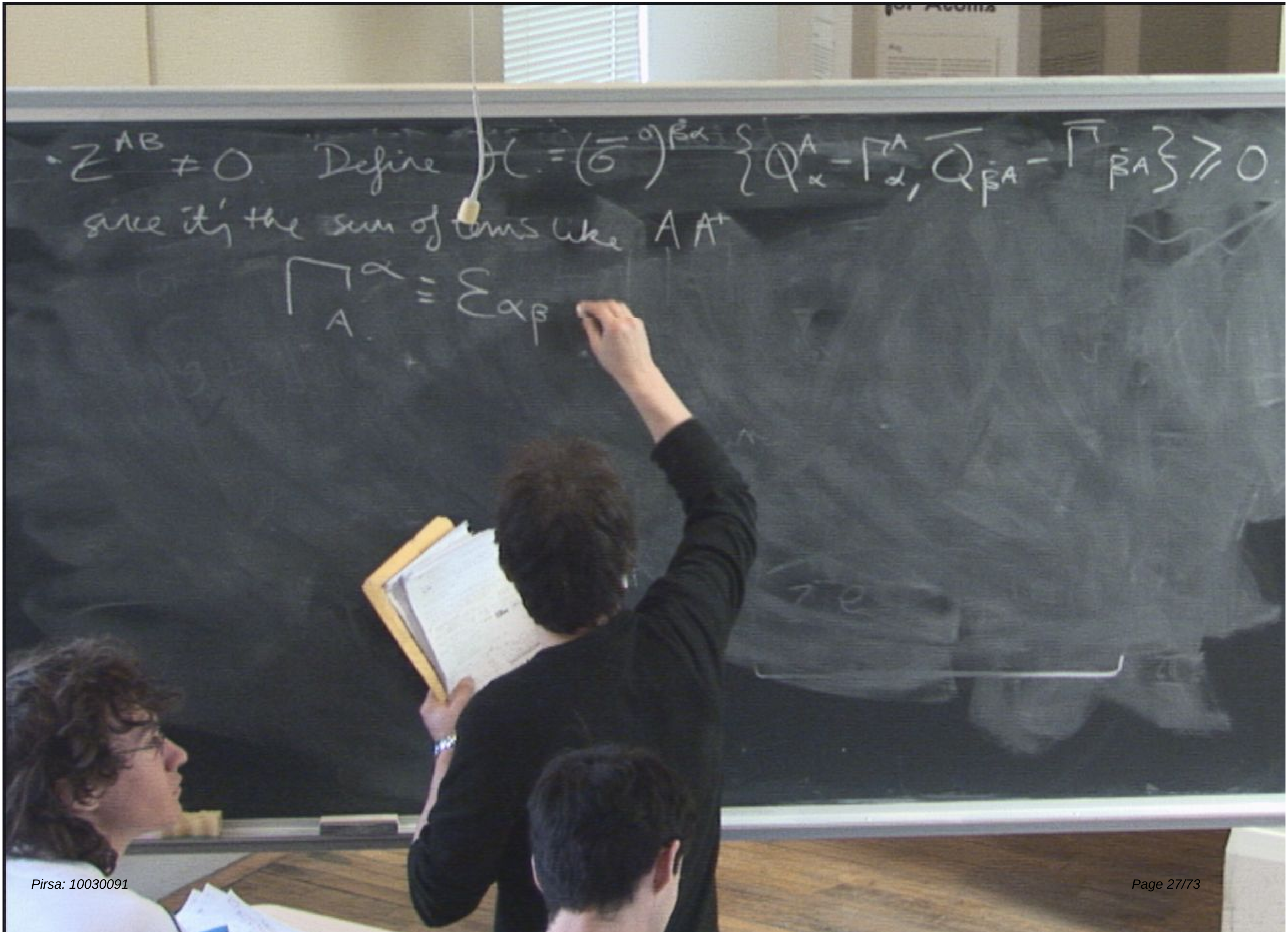

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since it's the sum of terms like $A A^\dagger$
$\Gamma^\alpha_A = \varepsilon_{\alpha\beta}$

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$$\Gamma^\alpha_A \equiv \epsilon_{\alpha\beta} U^{AB} \bar{Q}^\beta_{\dot{\gamma}} (\bar{\sigma}^0)^{\dot{\gamma}\beta}$$

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16 states
$n_B = n_F$ ✓.

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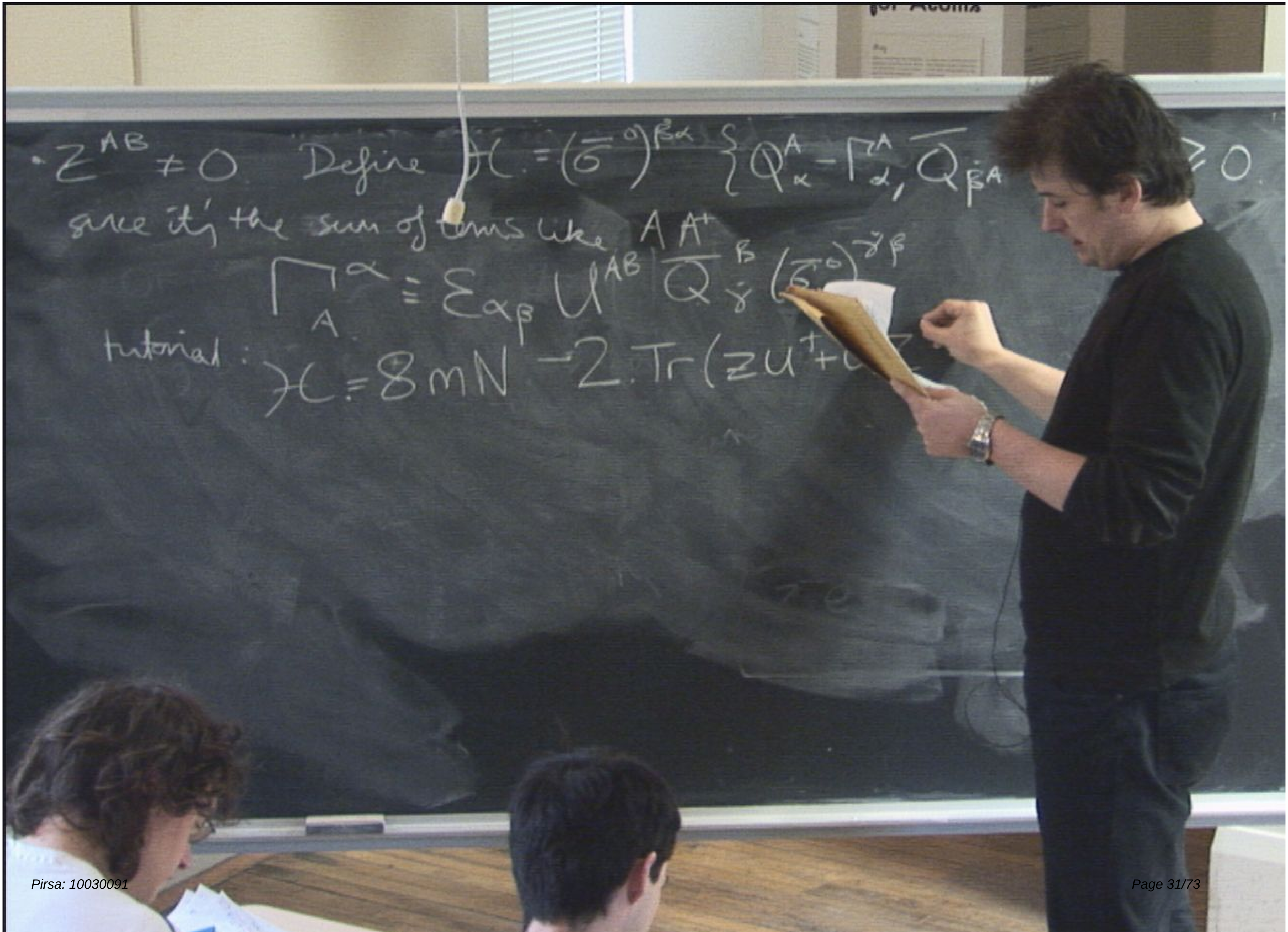
Z^AB ≠ 0 Define H = (σ̄^0)^βα {Q^A_α − Γ^A_α, Q̄_β̇A} ≥ 0
since it's the sum of terms like AA†
Γ^α_A = ε_αβ U^AB Q̄^β_γ̇ (σ̄^0)^γ̇β
tutorial : H = 8mN − 2 Tr(ZU† +

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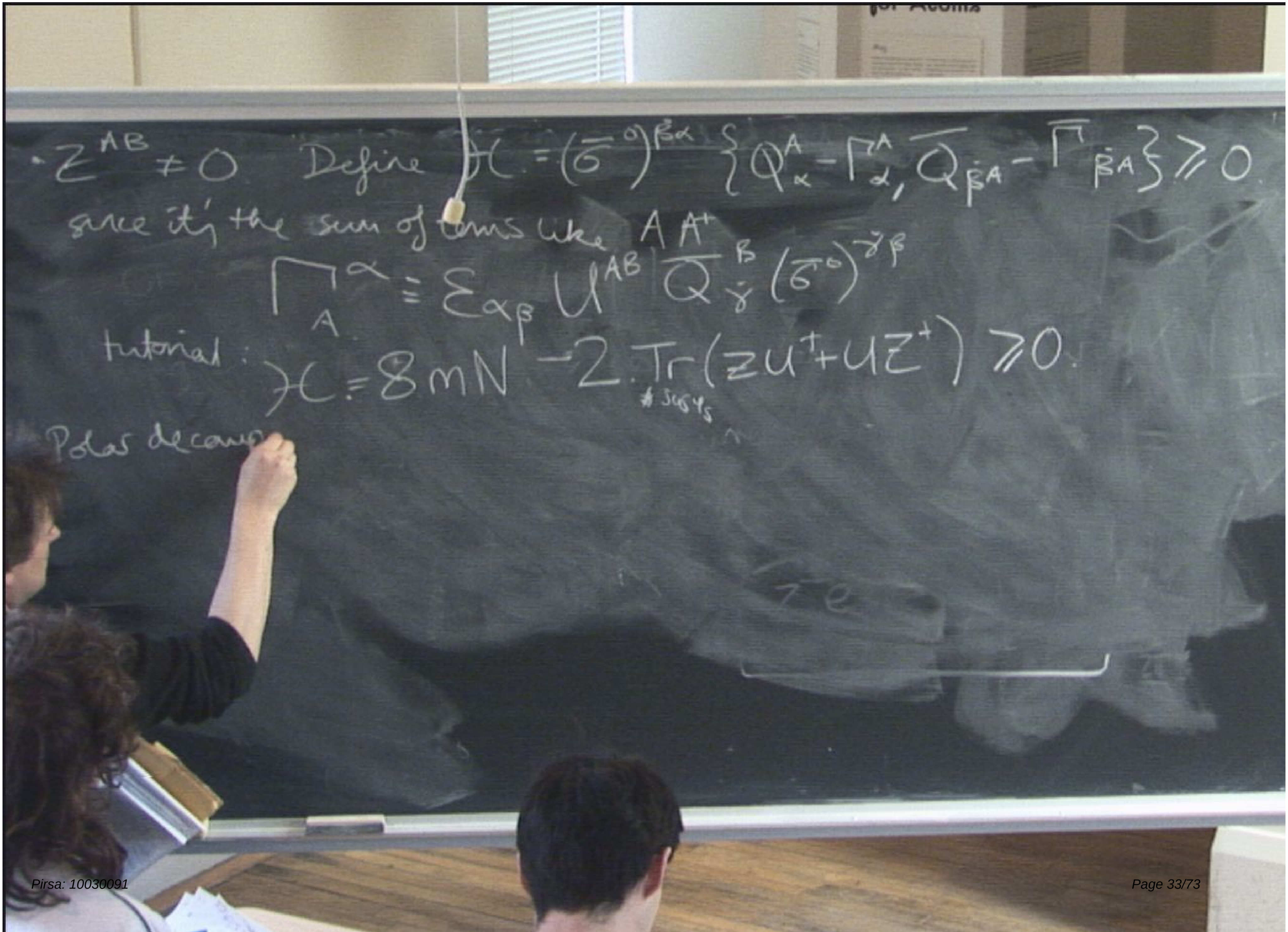
since it's the sum of terms like $A A^{\dagger}$

$$\Gamma_{A}^{\alpha} \equiv \varepsilon_{\alpha\beta} U^{AB} \bar{Q}^{\beta}_{\dot{\gamma}} (\bar{\sigma}^{0})^{\dot{\gamma}\beta}$$

tutorial: $\mathcal{H} = 8mN - 2\,\mathrm{Tr}(ZU^{\dagger} + UZ^{\dagger}) \geq 0$.


• Z^{AB} ≠ 0 Define H = (σ̄^0)^{β̇α} {Q^A_α − Γ^A_α, Q̄_{β̇A} − Γ̄_{β̇A}} ≥ 0
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tutorial: H = 8mN − 2 Tr(ZU^+ + UZ^+) ≥ 0
SUSYs
Polar decomp

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$$\Gamma^\alpha_A \equiv \varepsilon_{\alpha\beta} U^{AB} \bar{Q}^\beta_{\dot\gamma} (\bar{\sigma}^0)^{\dot\gamma\beta}$$

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Polar decomposition theorem:

$Z = HV$ — V → unitary matrix.

H → Hermitian matrix, +ve e'values

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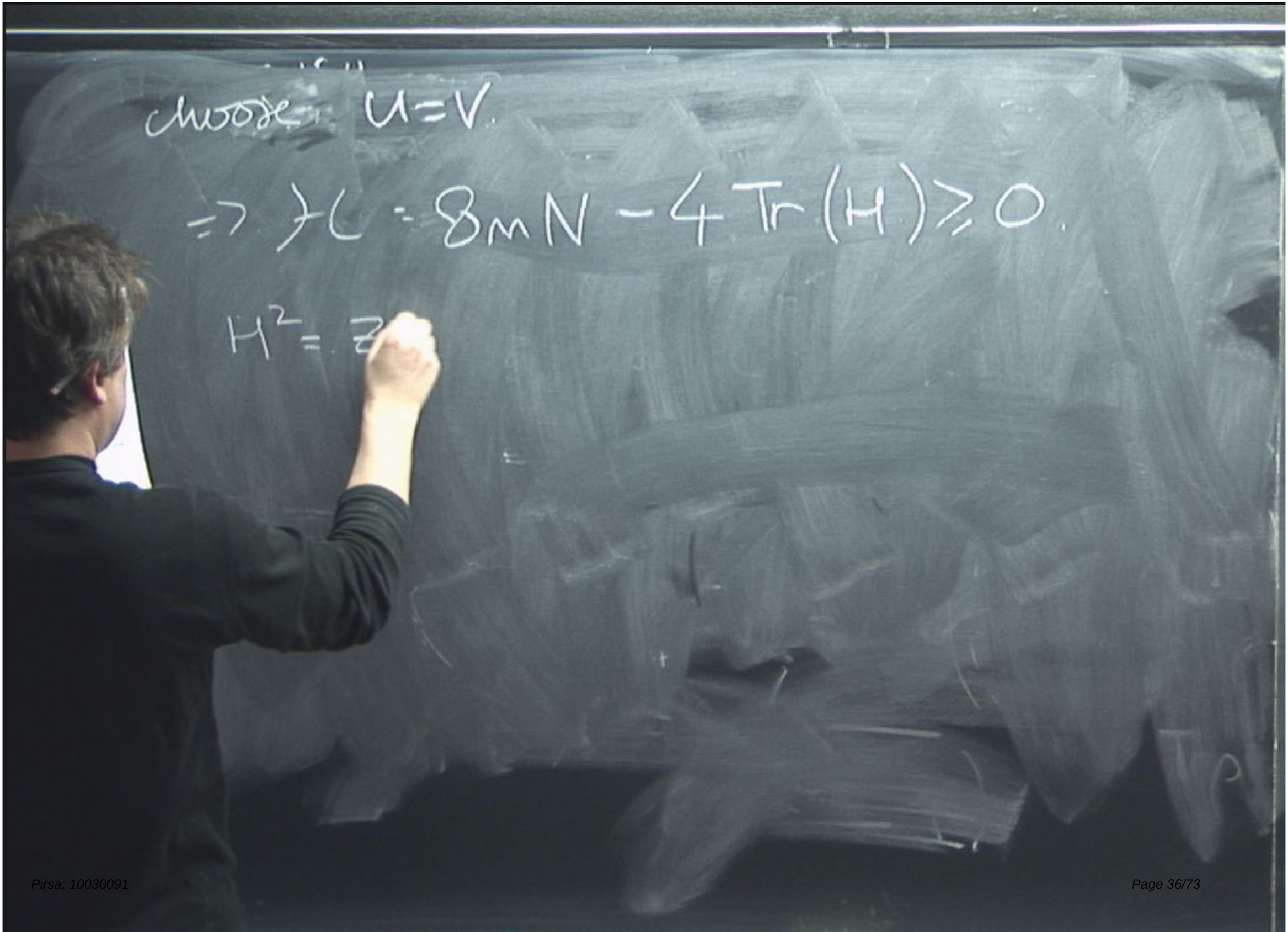
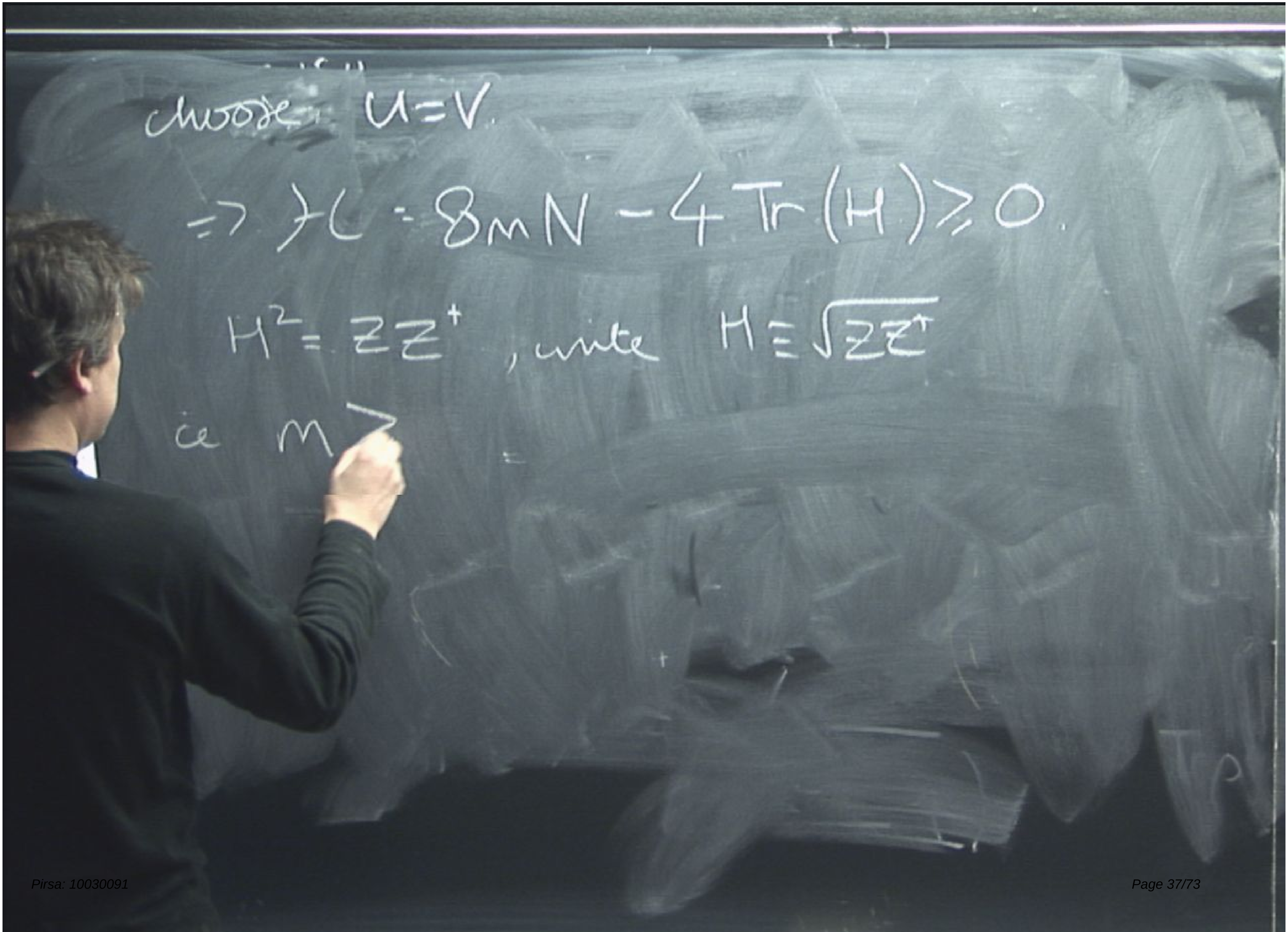
Polar decomposition theorem:

$Z = H V$ → unitary matrix.

↳ Hermitian matrix, i.e. real e'values


choose: U=V.
⇒ ℋ = 8mN − 4 Tr(H) ≥ 0.
H² = Z


choose: $U=V$
$\Rightarrow \mathcal{H} = 8mN - 4\,\mathrm{Tr}(H) \geq 0$
$H^2 = ZZ^\dagger$, write $H \equiv \sqrt{ZZ^\dagger}$
ie $m \geq$

choose: $U = V$.

$$\Rightarrow \mathcal{H} = 8mN - 4\,\mathrm{Tr}(H) \geq 0.$$

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ie $$\boxed{m \geq \frac{1}{2N}\,\mathrm{Tr}\sqrt{ZZ^\dagger}}$$

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Bogomolnyi, Prasad, Sommerfeld.

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Saturated states are BPS states

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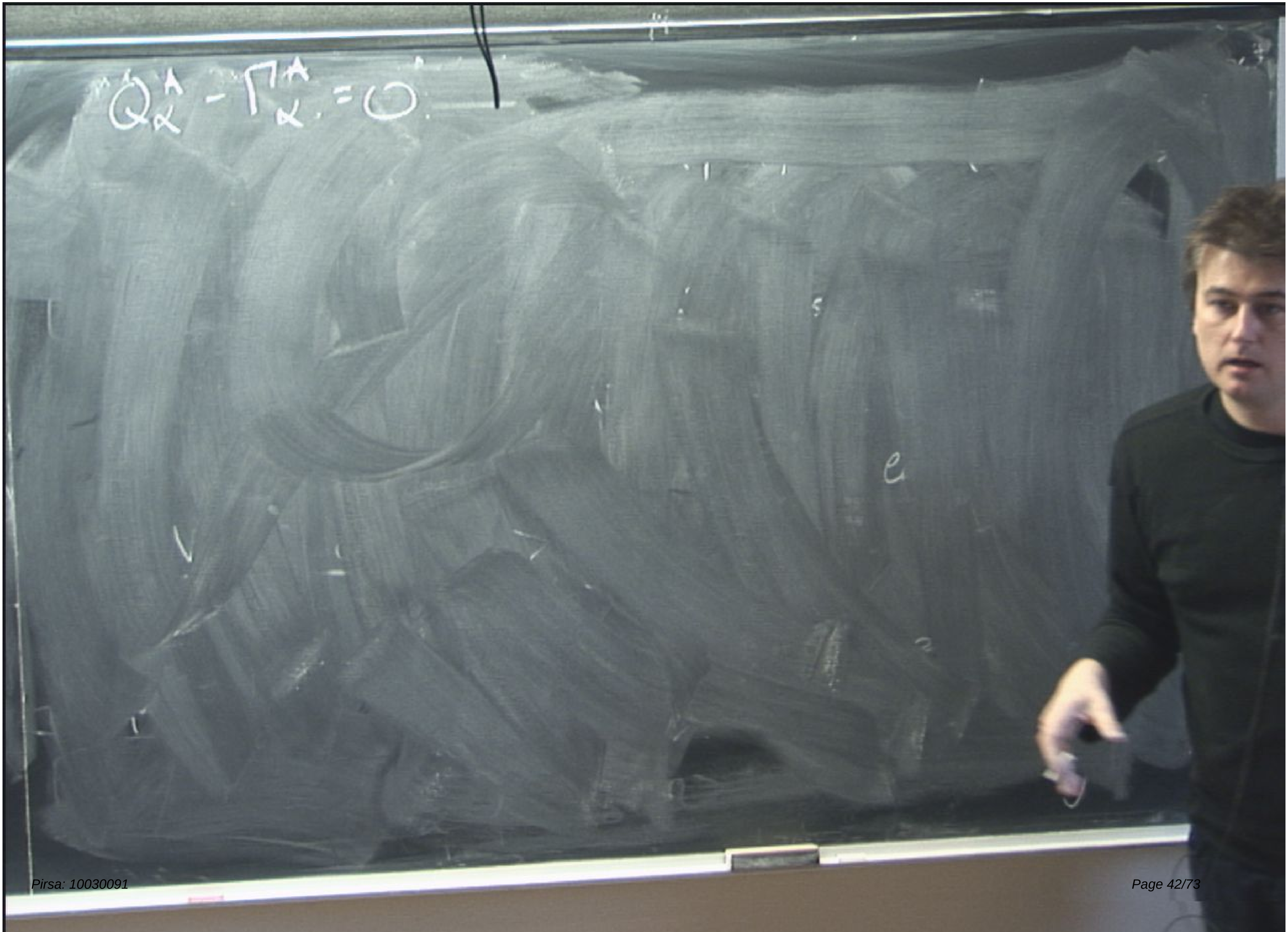
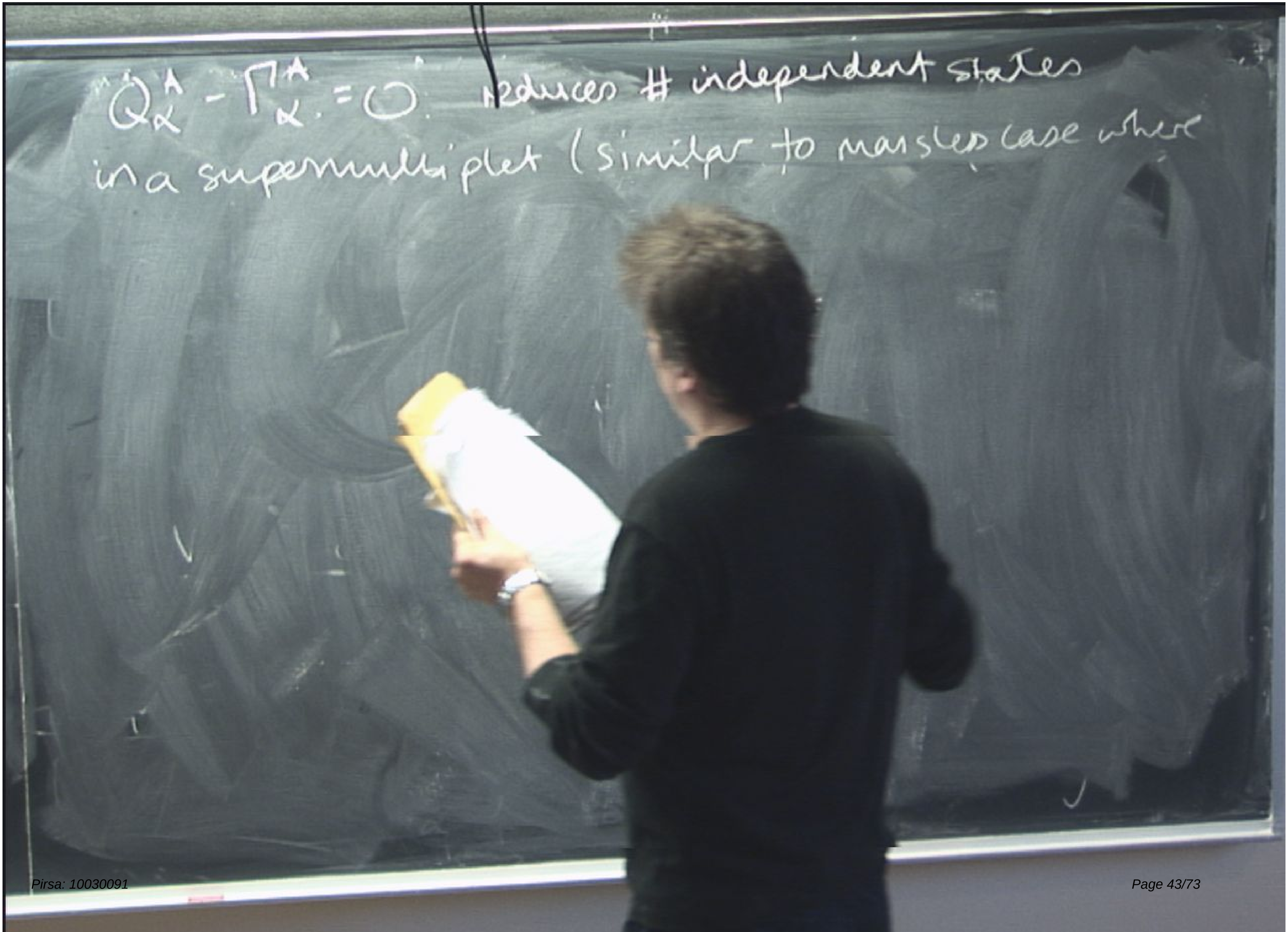
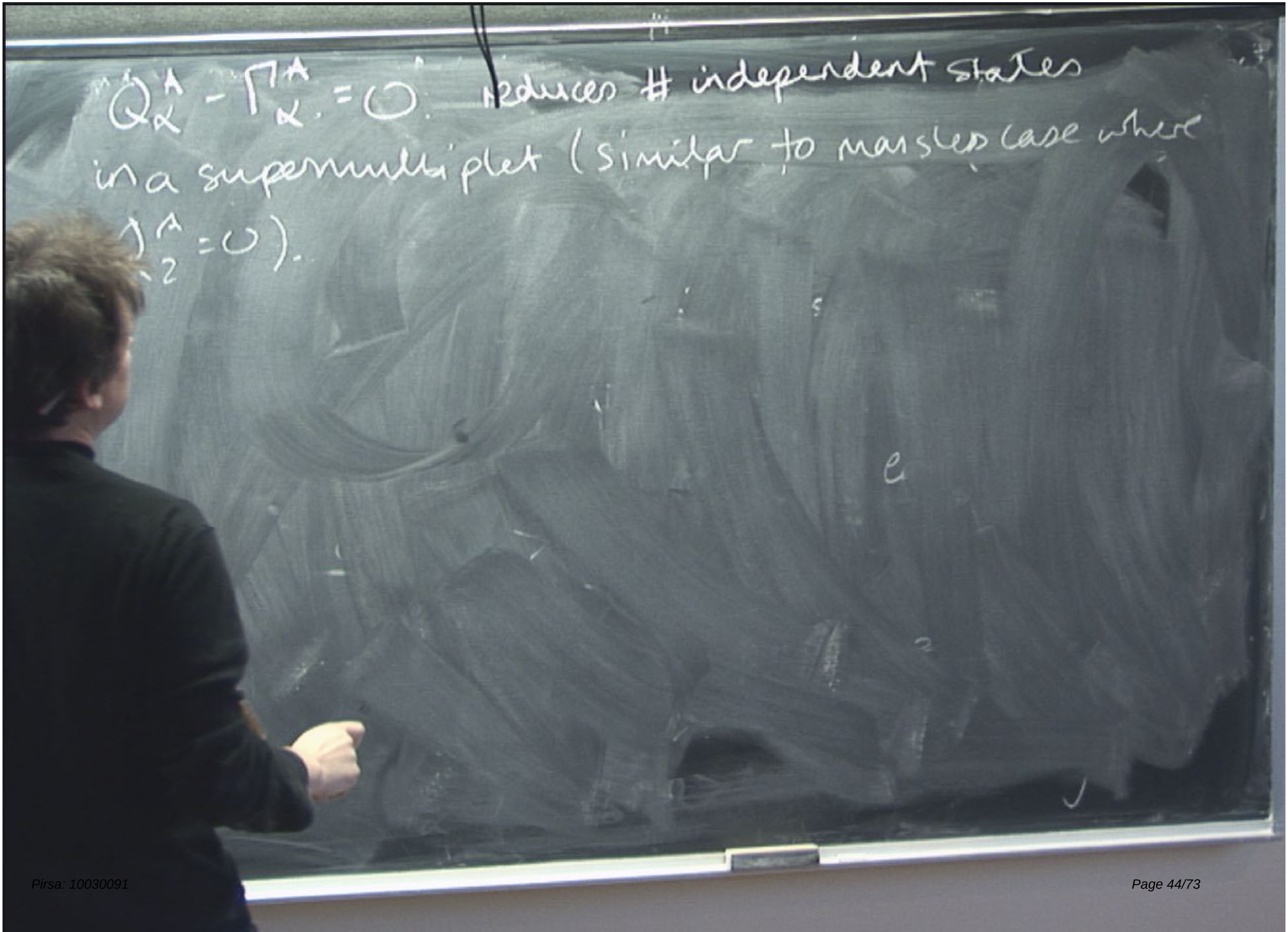
ie $\boxed{m \geq \frac{1}{2N} \mathrm{Tr}\sqrt{ZZ^\dagger}}$ Bogomolnyi, Prasad, Sommerfeld.

Saturated states are BPS states, where $\mathcal{H} = 0$

Tr


$Q^A_\alpha - \Gamma^A_\alpha = 0$


$Q^A_\alpha - \Gamma^A_\alpha = 0$ reduces # independent states
in a supermultiplet (similar to massless case where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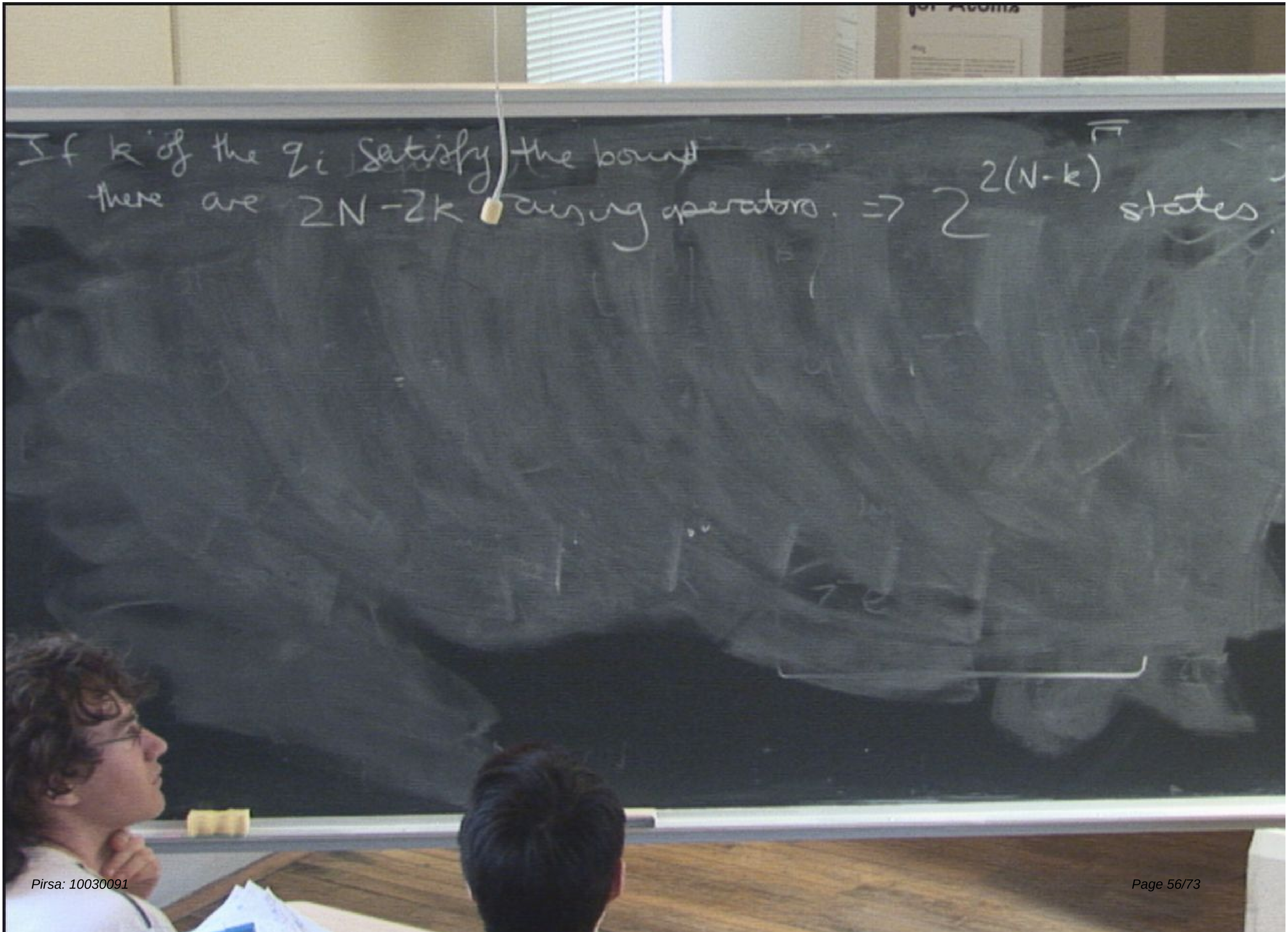
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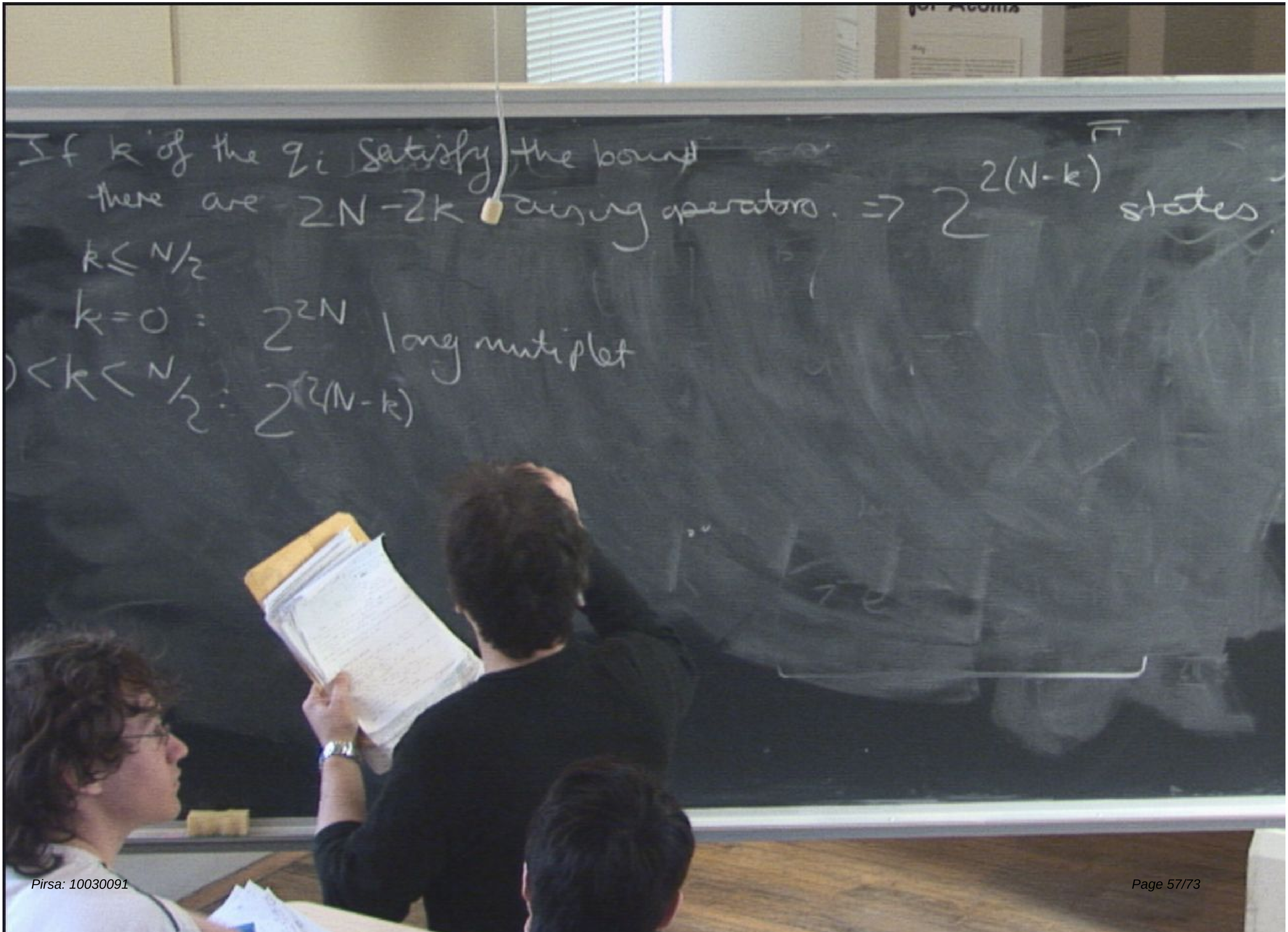
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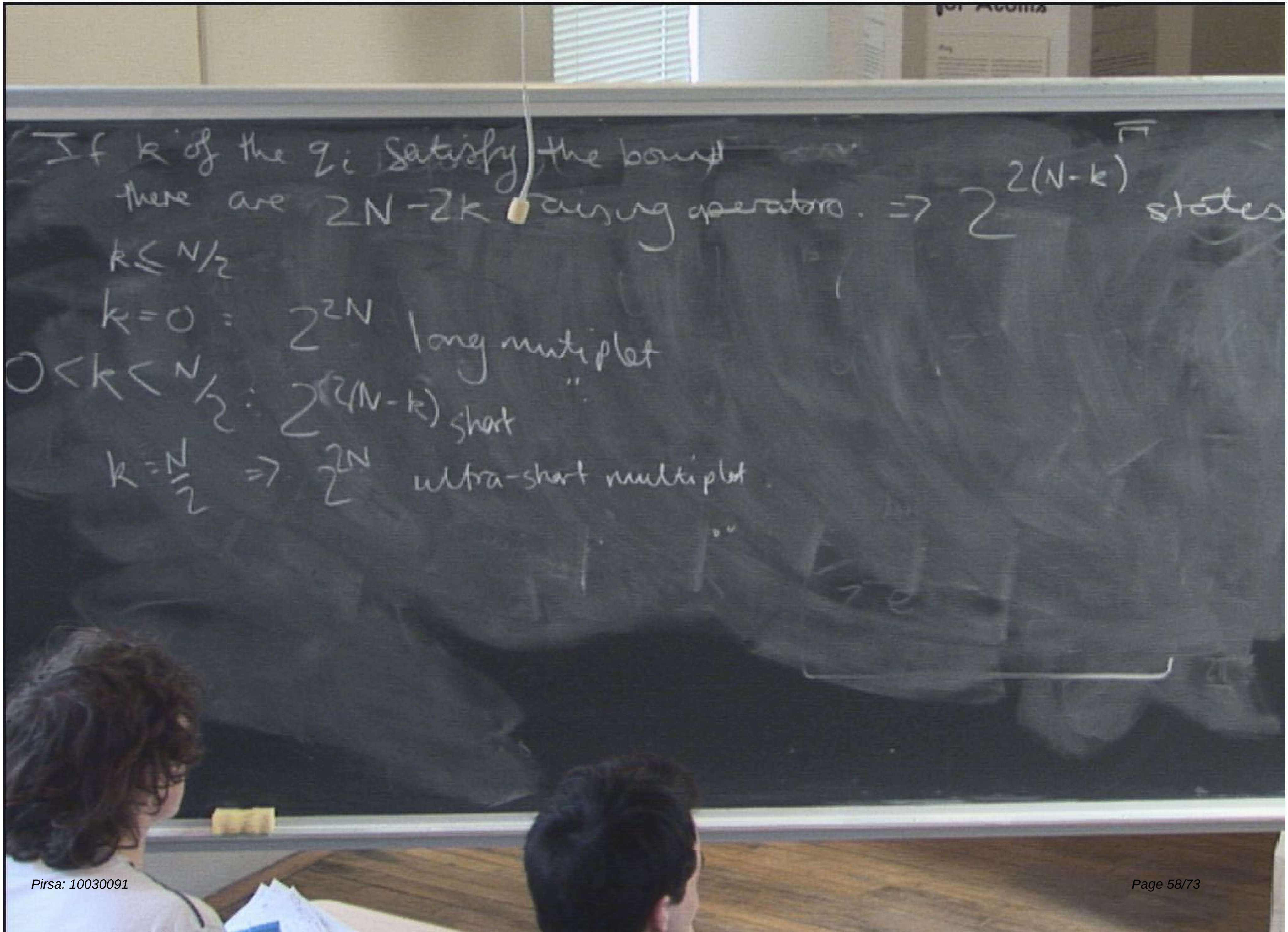
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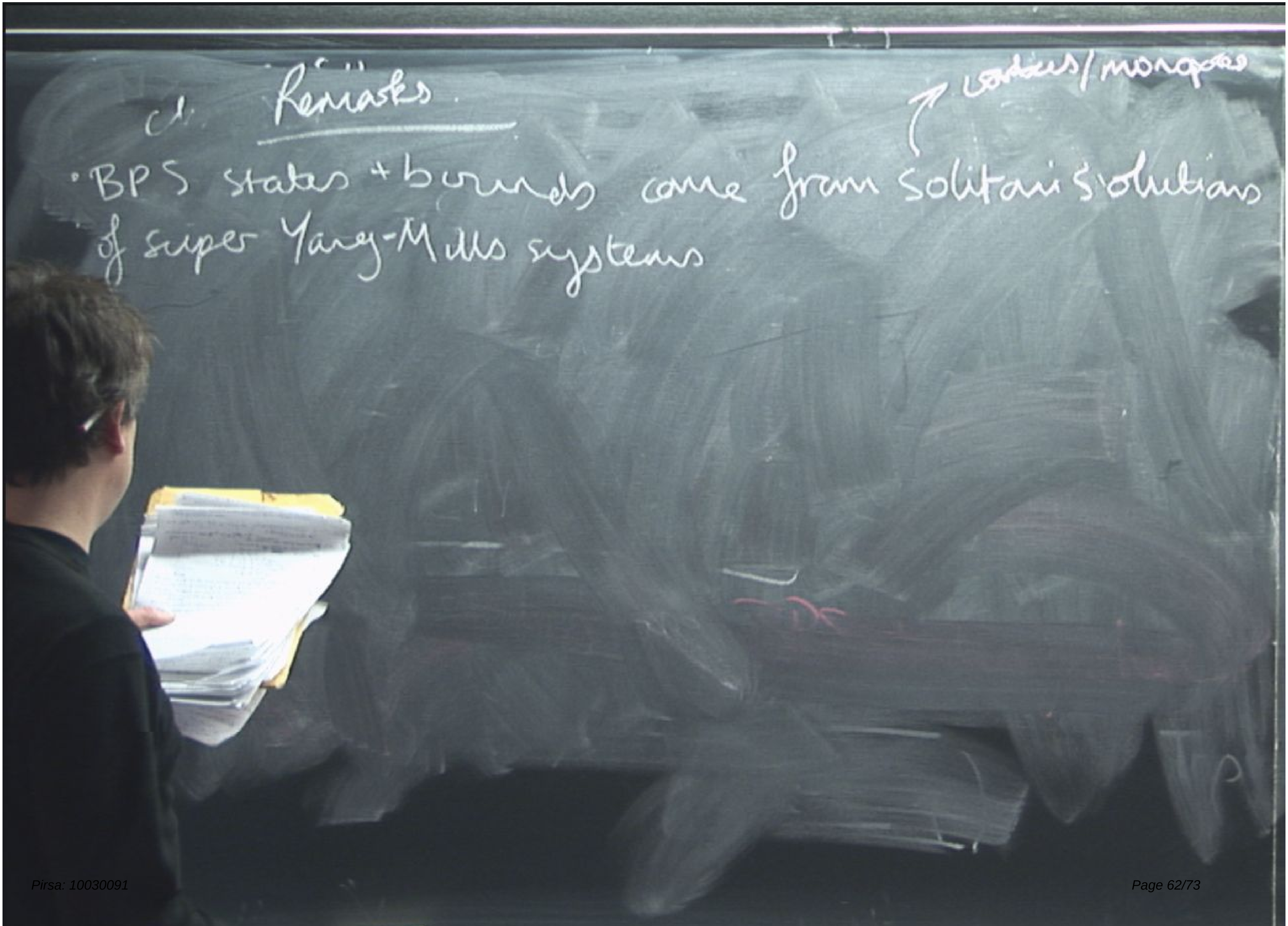
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vortices/monopoles
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↗ vortices / monopoles

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cf. Remarks

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- BPS states are important for understanding weak/strong coupling dualities in QFT/strings

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• In super strings, extended SUSY objects "D-branes"
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