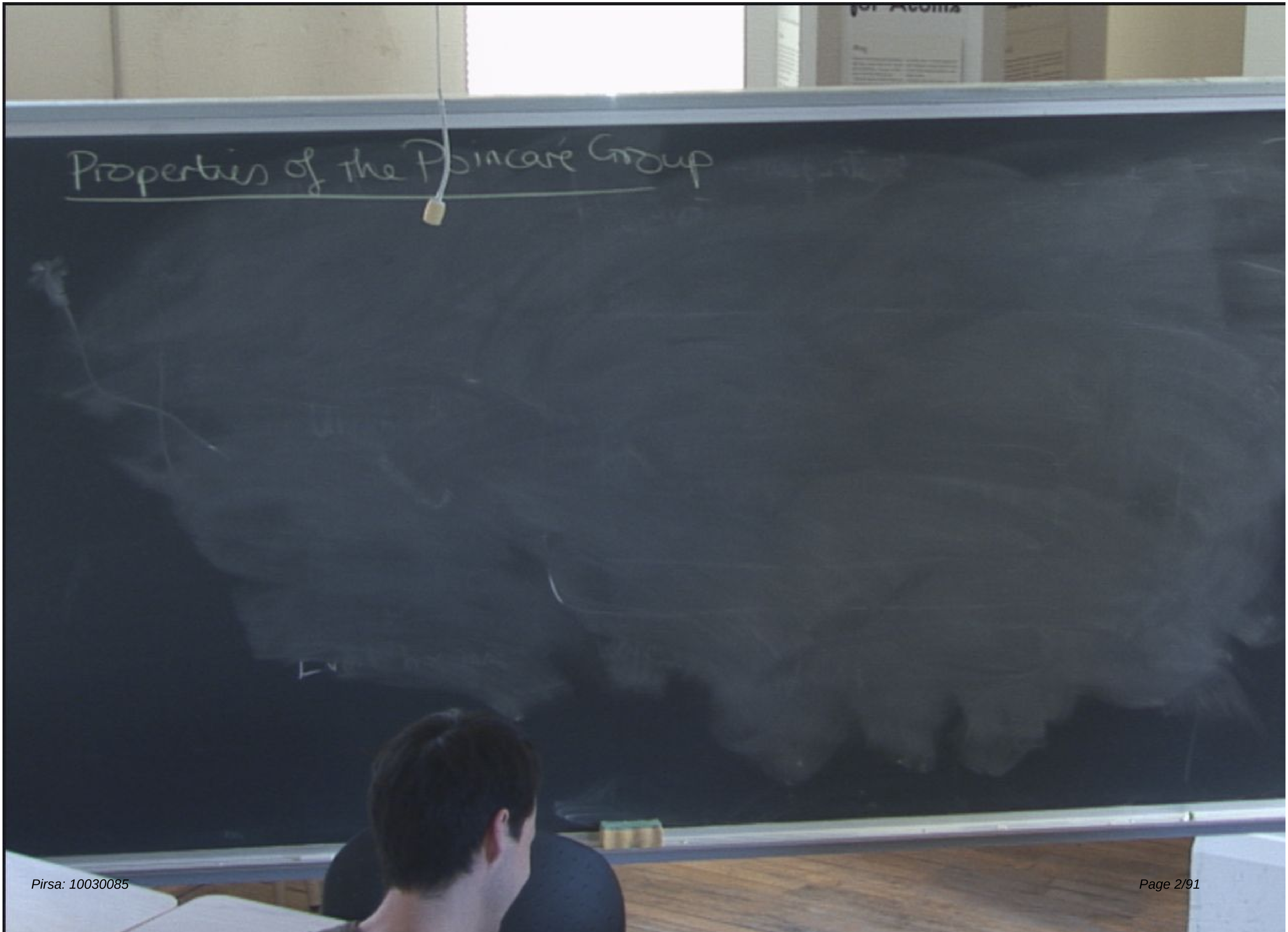
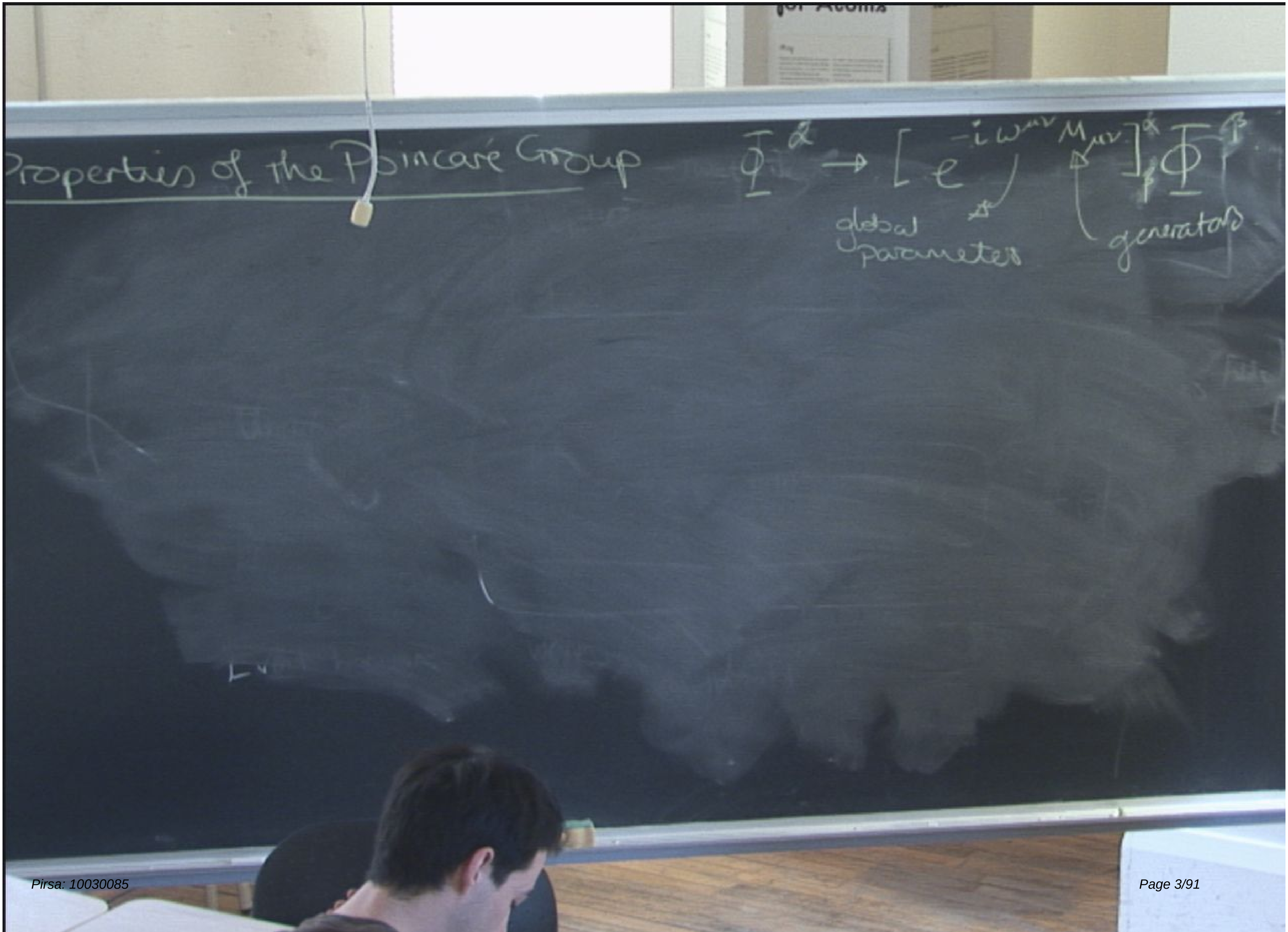
Title: Explorations In Particle Theory (PHYS 646) - Lecture 3

Date: Mar 17, 2010 10:10 AM

URL: http://pirsa.org/10030085

Abstract:


Properties of the Poincaré Group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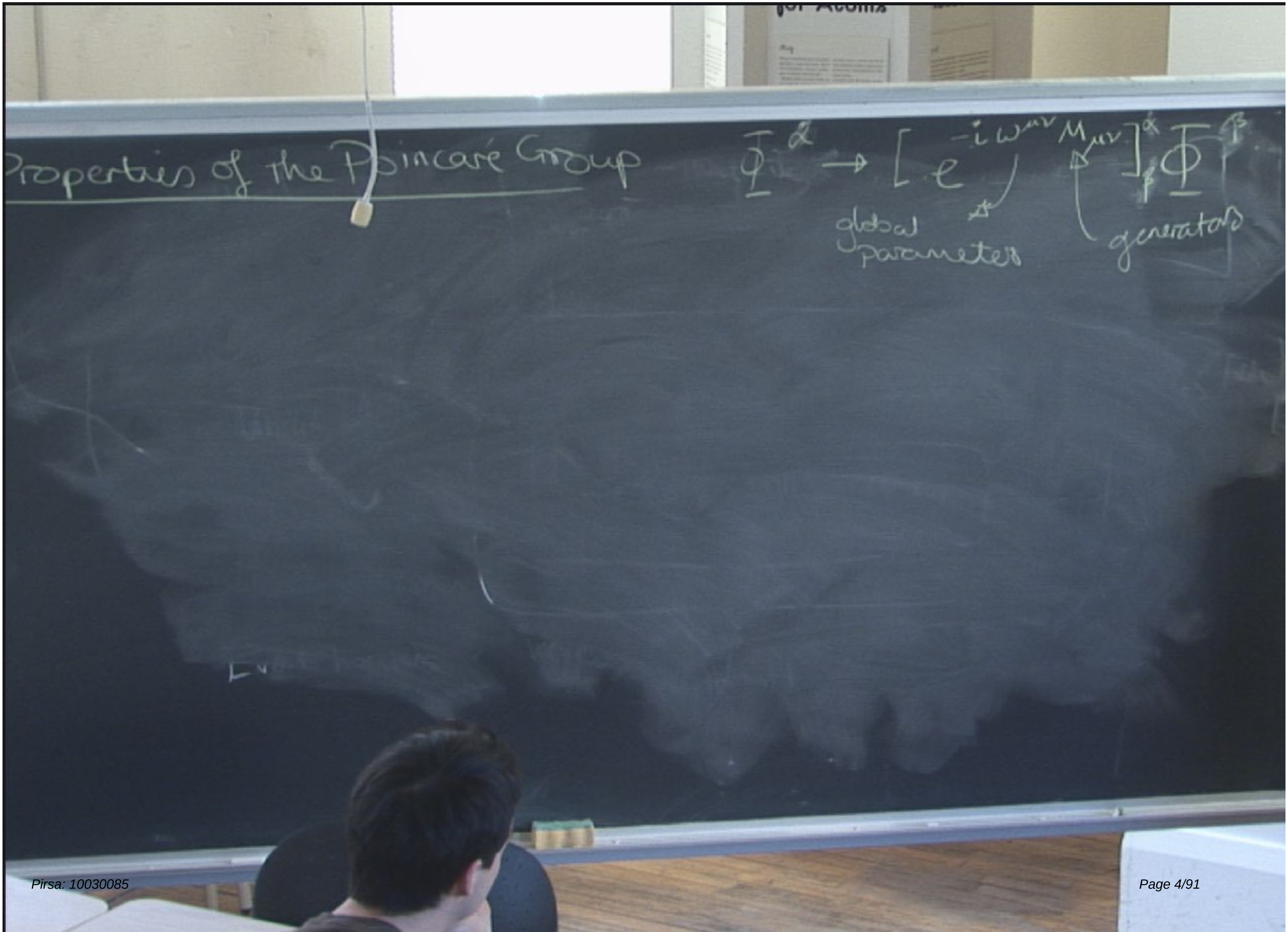
$\Phi^\alpha \to [e^{-i\omega^{\mu\nu} M_{\mu\nu}}]^\alpha{}_\beta \Phi^\beta$
global parameters
generators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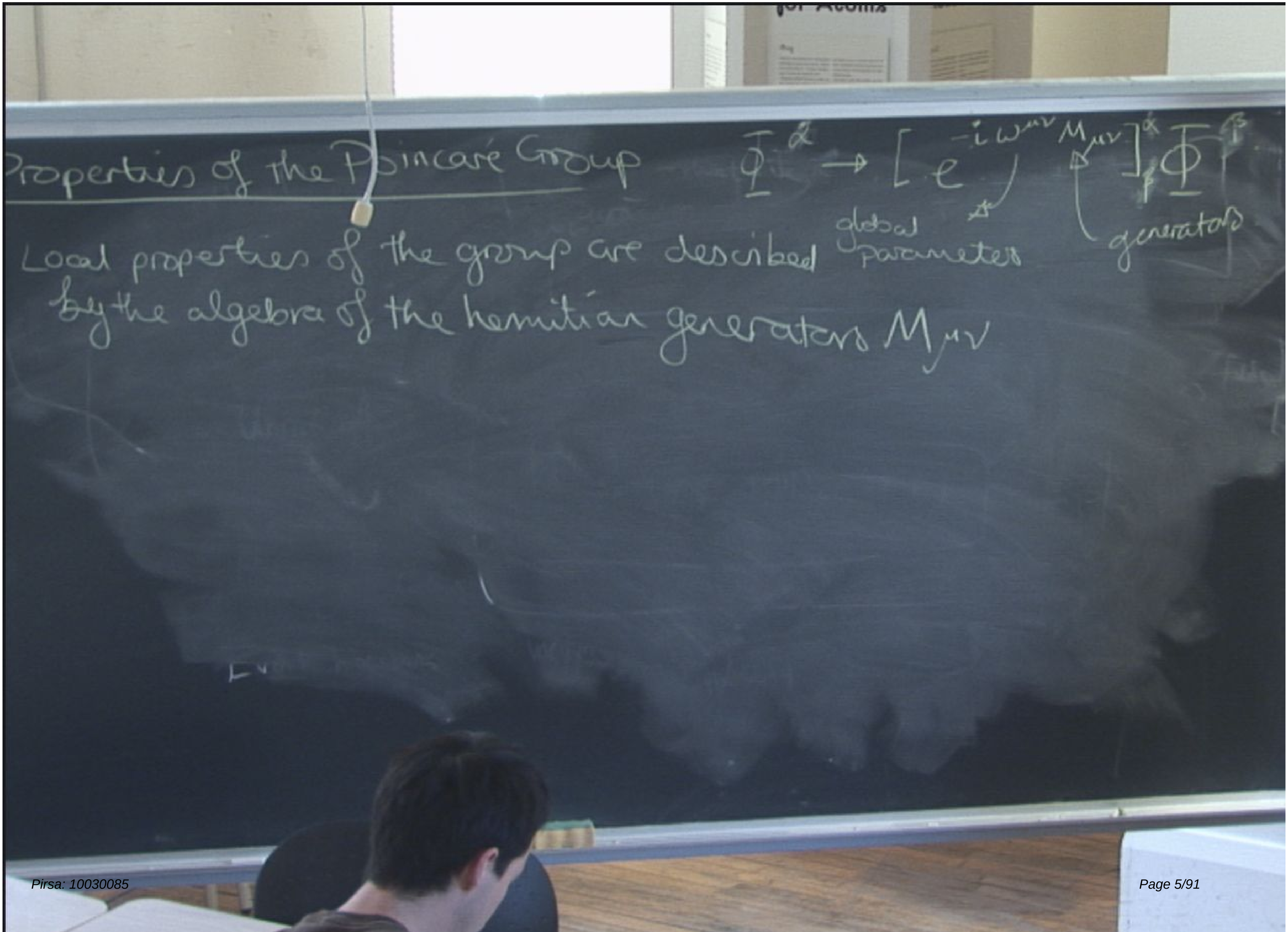
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Local properties of the group are described by the algebra of the hemitian generators $M_{\mu\nu}$

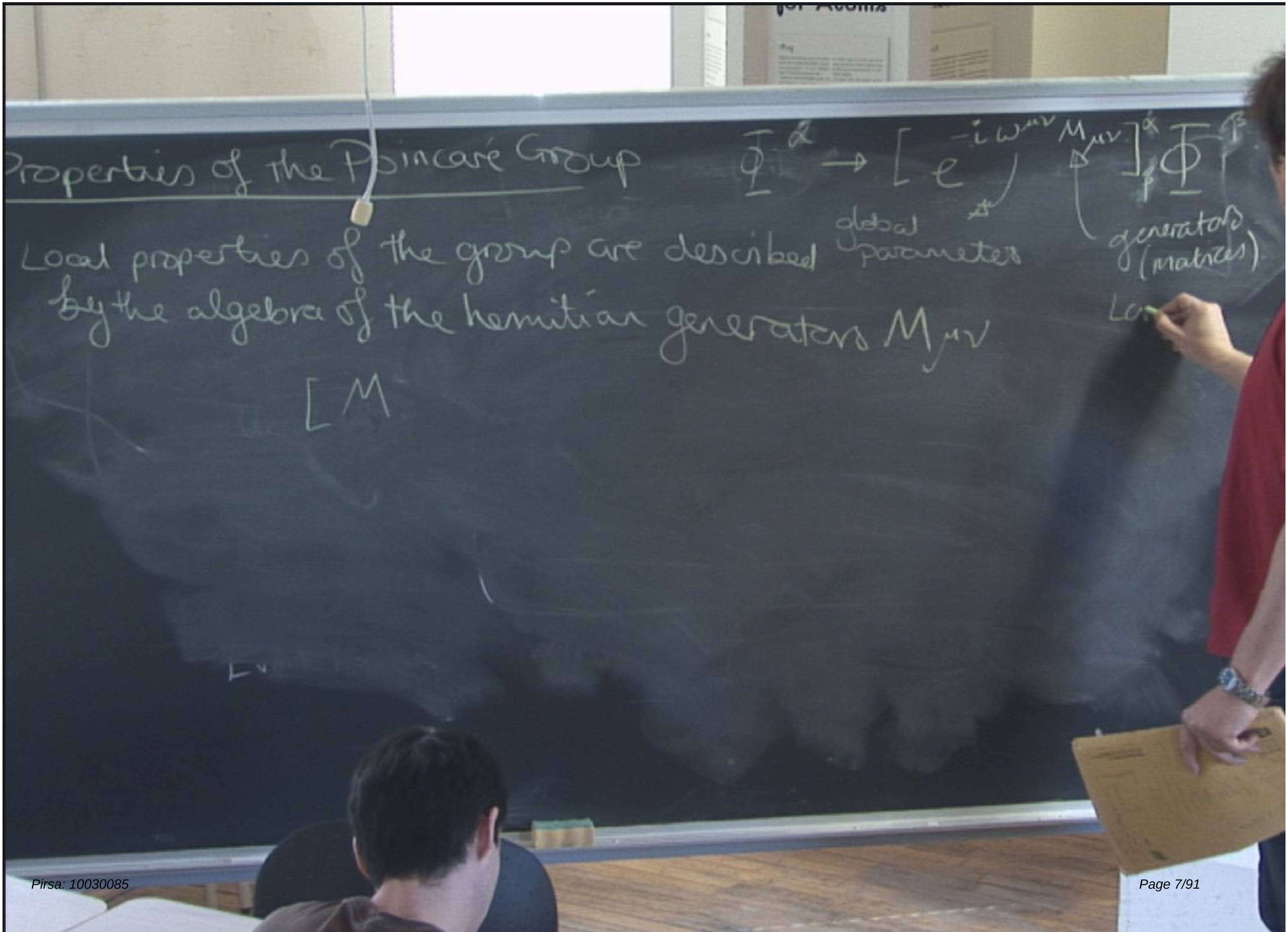
Properties of the Poincaré Group

$$\Phi^{\alpha} \to \left[e^{-i\omega^{\mu\nu}M_{\mu\nu}}\right]^{\alpha}{}_{\beta}\,\Phi^{\beta}$$

global parameters

generators (matrices)

Local properties of the group are described by the algebra of the hermitian generators $M_{\mu\nu}$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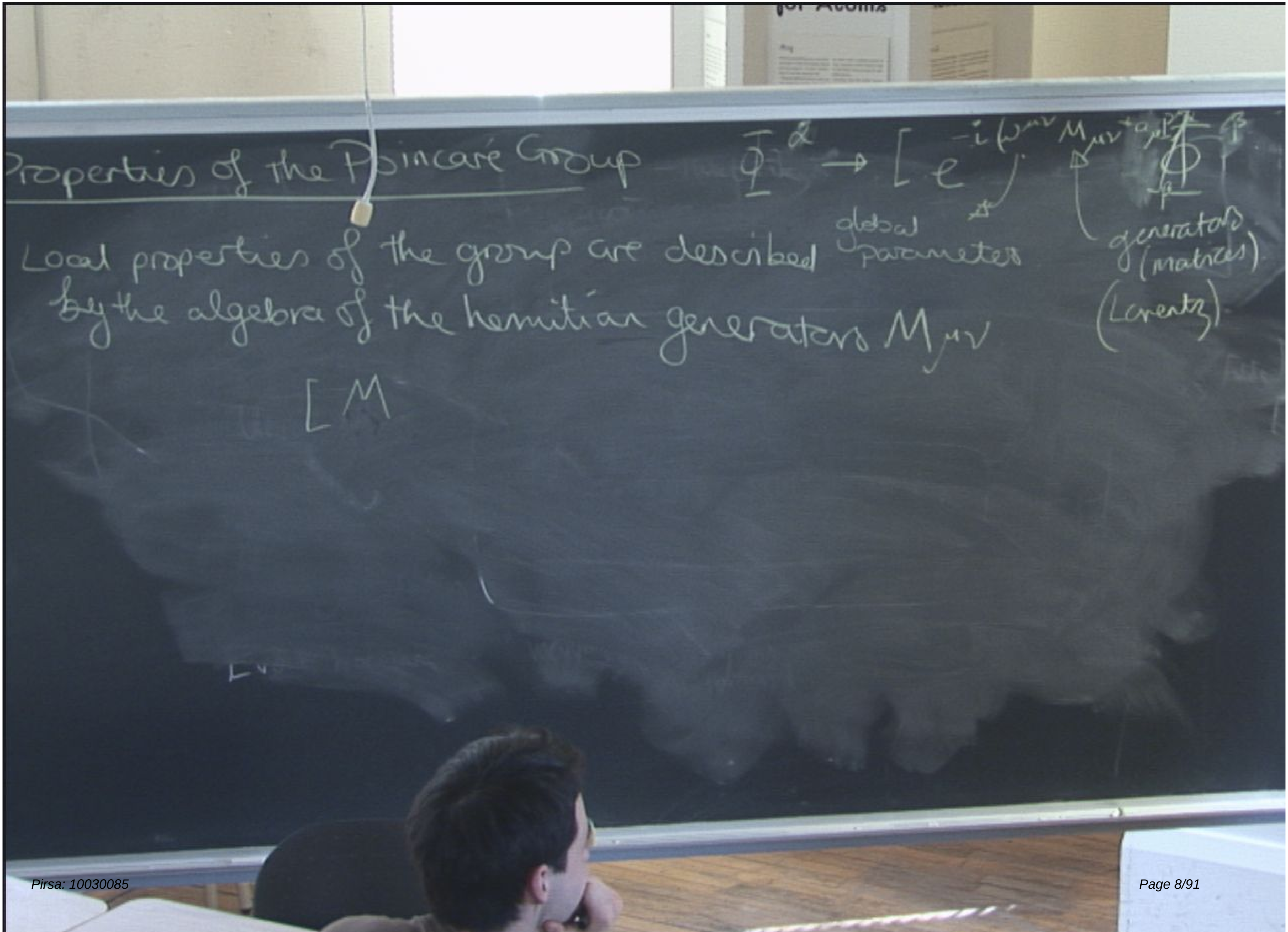
Properties of the Poincaré Group
$\Phi^\alpha \to [e^{-i(\omega^{\mu\nu} M_{\mu\nu} + a_\mu P^\mu)}]^\alpha{}_\beta \Phi^\beta$
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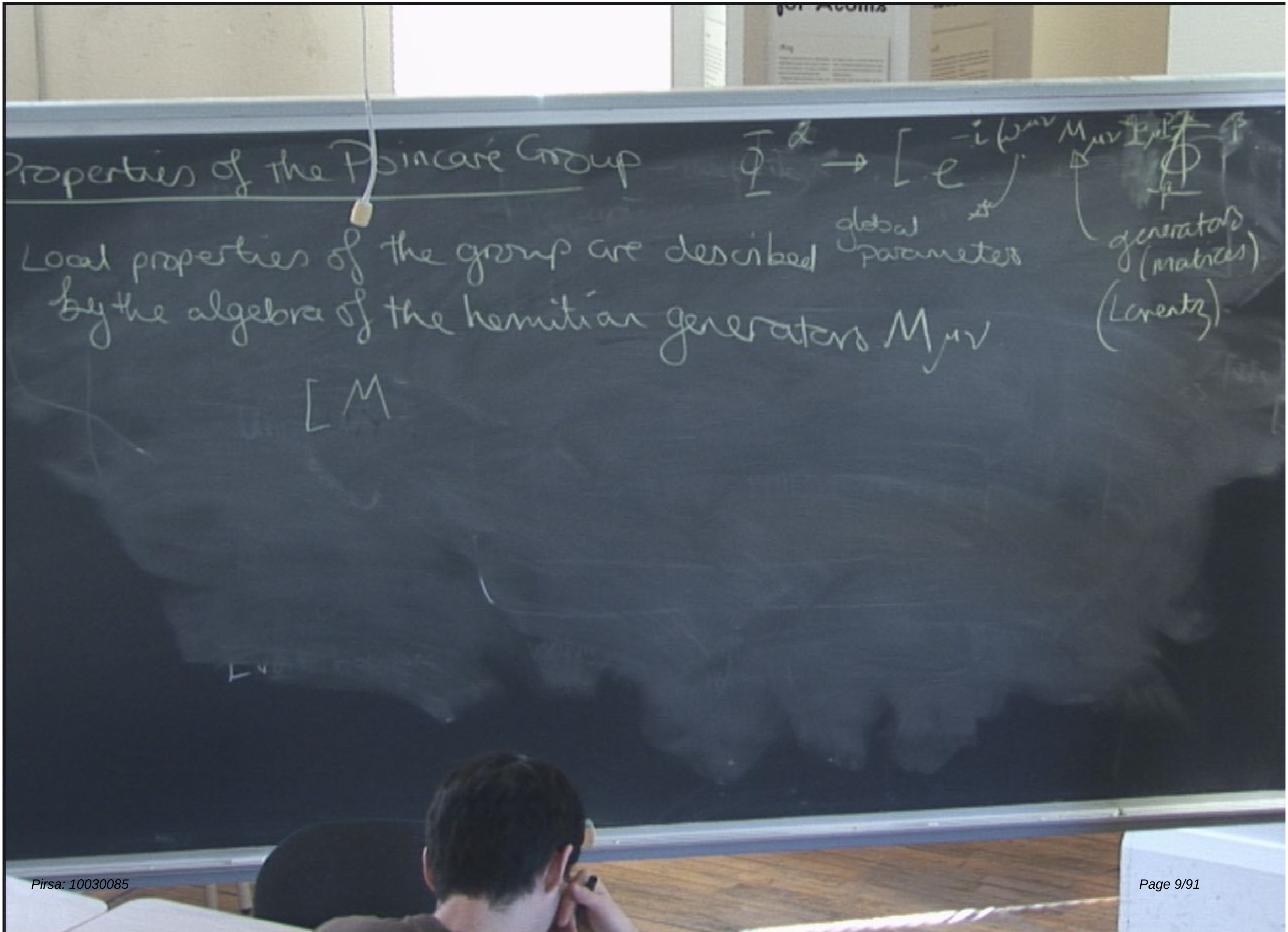
[M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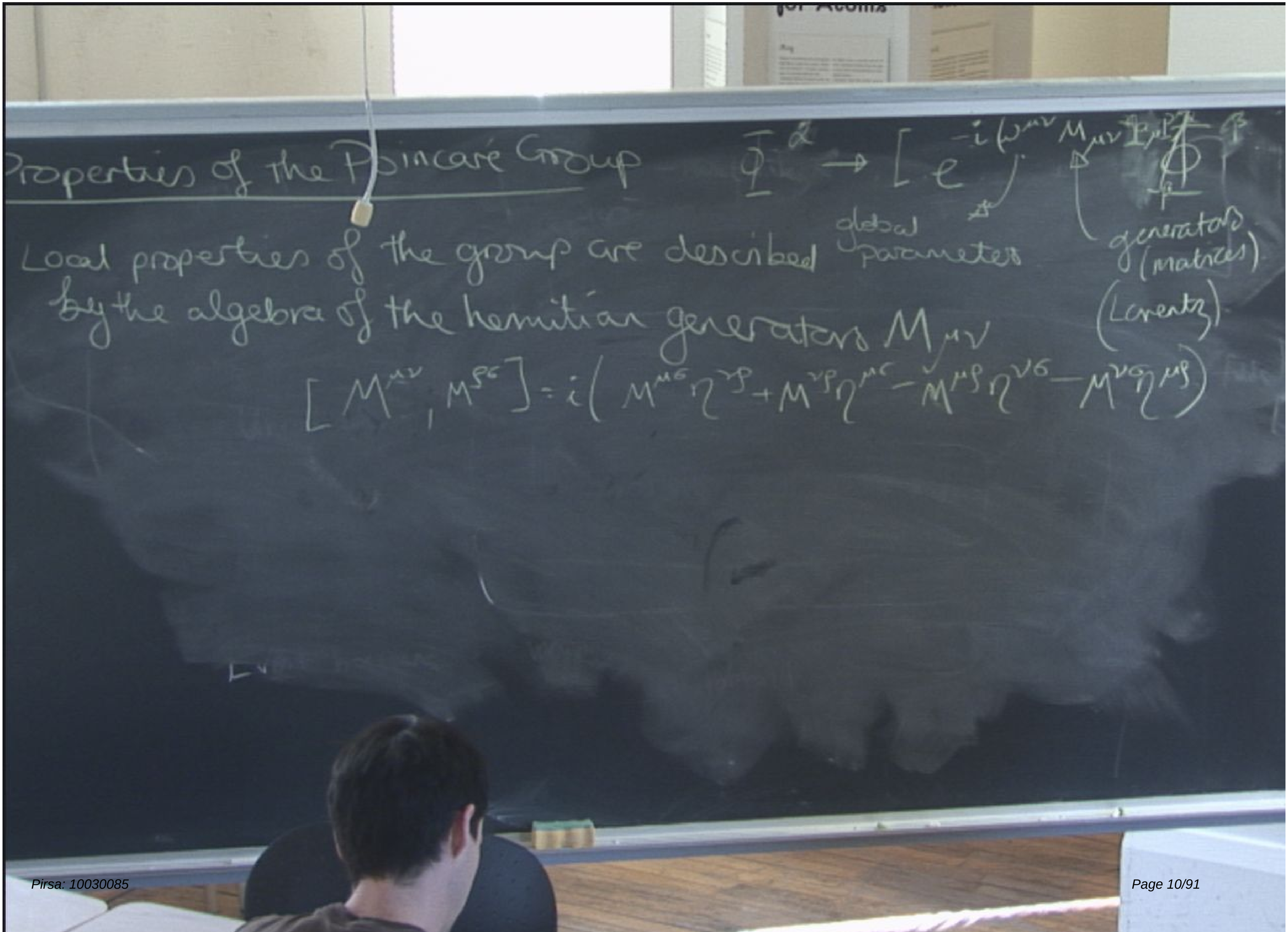
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$[M^{\mu\nu}, M^{\rho\sigma}] = i(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho})$

...operties of the Poincaré Group

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$$[P^{\mu}, P^{\nu}] = 0$$

$$[M^{\mu\nu}, P^{\sigma}] = i[P^{\mu}\eta^{\nu\sigma} - P^{\nu}\eta^{\mu\sigma}].$$

Properties of the Poincaré Group

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global parameters

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Local properties of the group are described by the algebra of the hermitian generators $M_{\mu\nu}$

generator of translations

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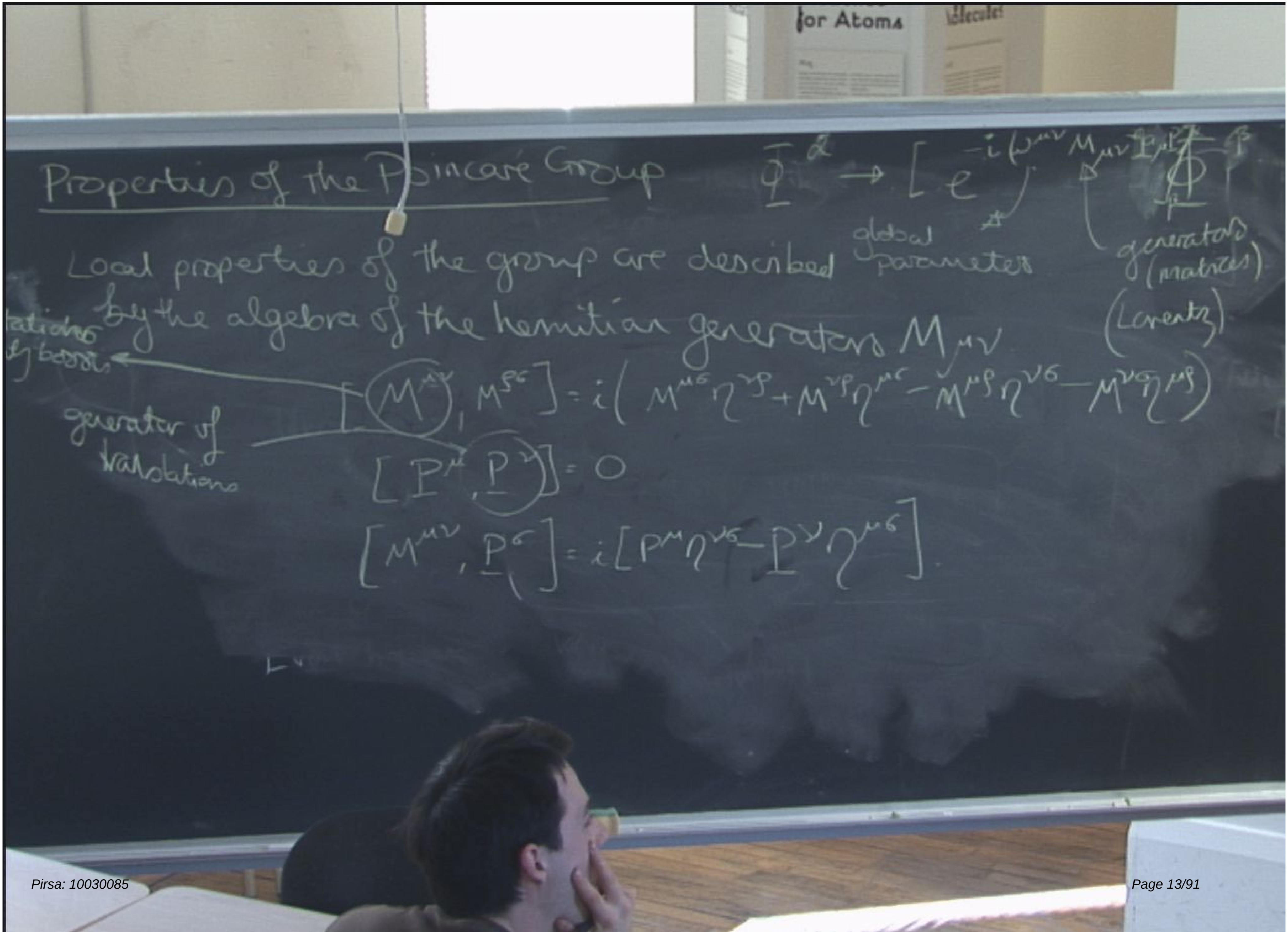
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rotations & boosts
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$SO(1,3) \simeq SU(2)\times SU(2)$

$i=1,2,3$ $\quad J_i \equiv \frac{1}{2}\varepsilon_{ijk}M^{jk}$ are the generators of rotations.

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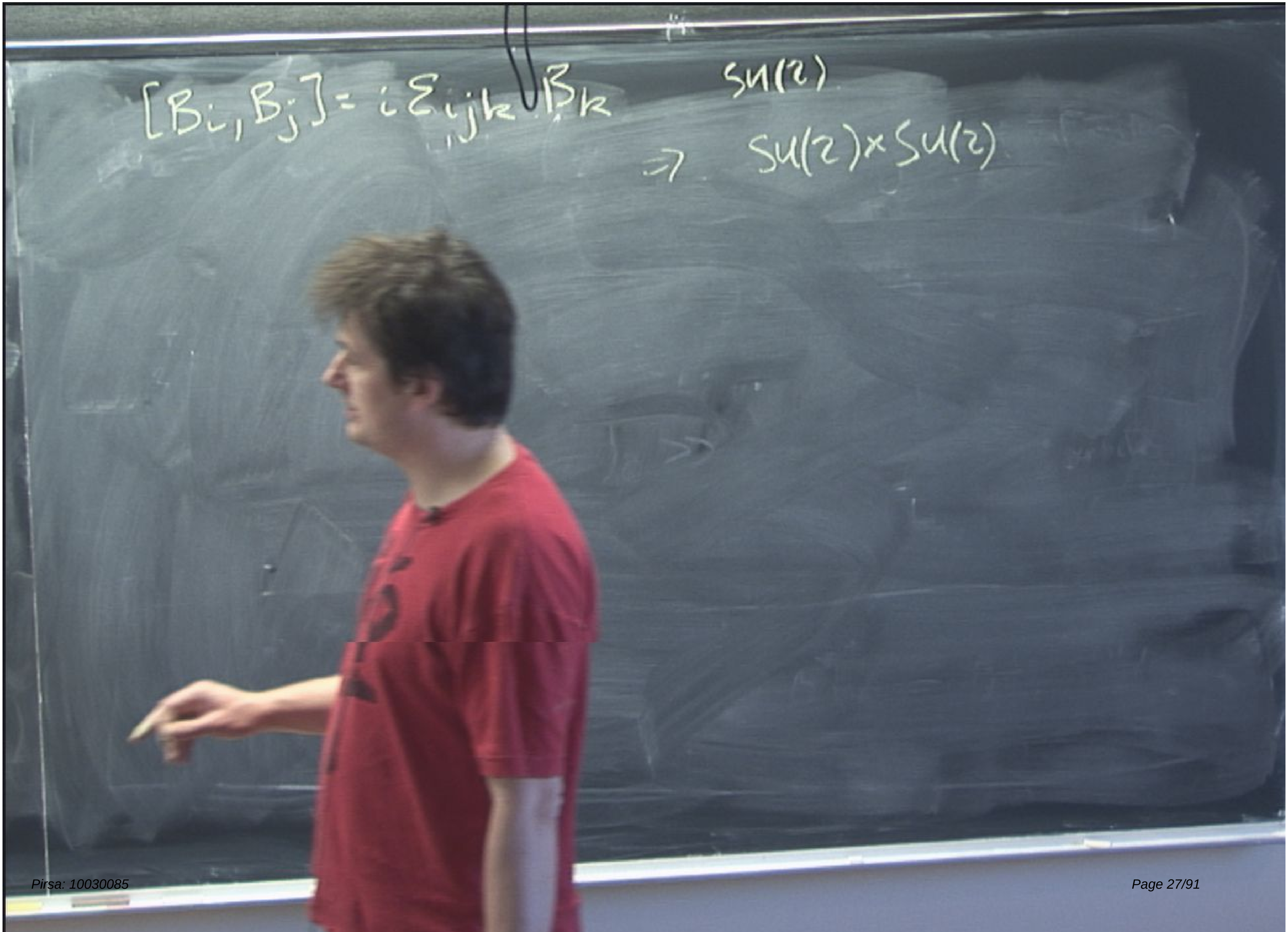

$[B_i, B_j] = i\varepsilon_{ijk} B_k$
SU(2)
$\Rightarrow$ SU(2)×SU(2)

$[B_i, B_j] = i\varepsilon_{ijk} B_k$ SU(2)

Under parity reversal, $\Rightarrow$ SU(2)×SU(2)

$J_i \to \bar{J}_i$ $(\underline{J} = \underline{x} \wedge \underline{p})$

$$[B_i, B_j] = i\varepsilon_{ijk} B_k \qquad SU(2)$$

$$\Rightarrow \quad SU(2) \times SU(2)$$

Under parity reversal,

$$(\underline{J} = \underline{x} \wedge \underline{p})$$

$$J_i \to J_i$$

$$K_i \to -K_i \quad \text{so} \quad A_i \leftrightarrow B_i.$$

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$\underline{J} = \underline{A} + \underline{B}$ is the physical spin of a state.

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Label reps by (A, B) then Spin $= A + B$.

$[B_i, B_j] = i\epsilon_{ijk} B_k$ SU(2)

$\Rightarrow$ SU(2) × SU(2)

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$SO(3,1) \cong SL(2, \mathbb{C})$ $\to$ handy for massless fermions

det $= 1$ matrix mult + inversion.

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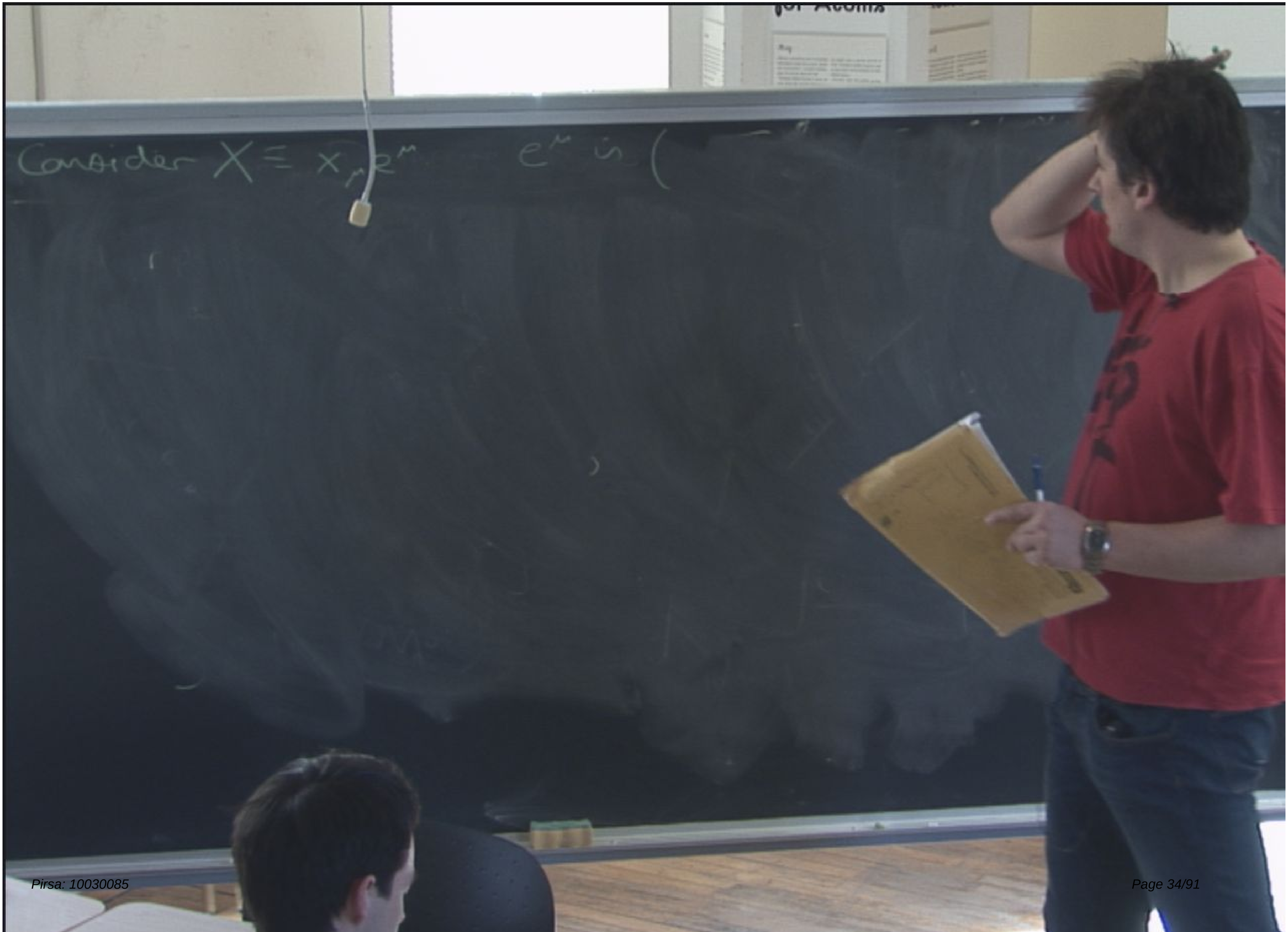
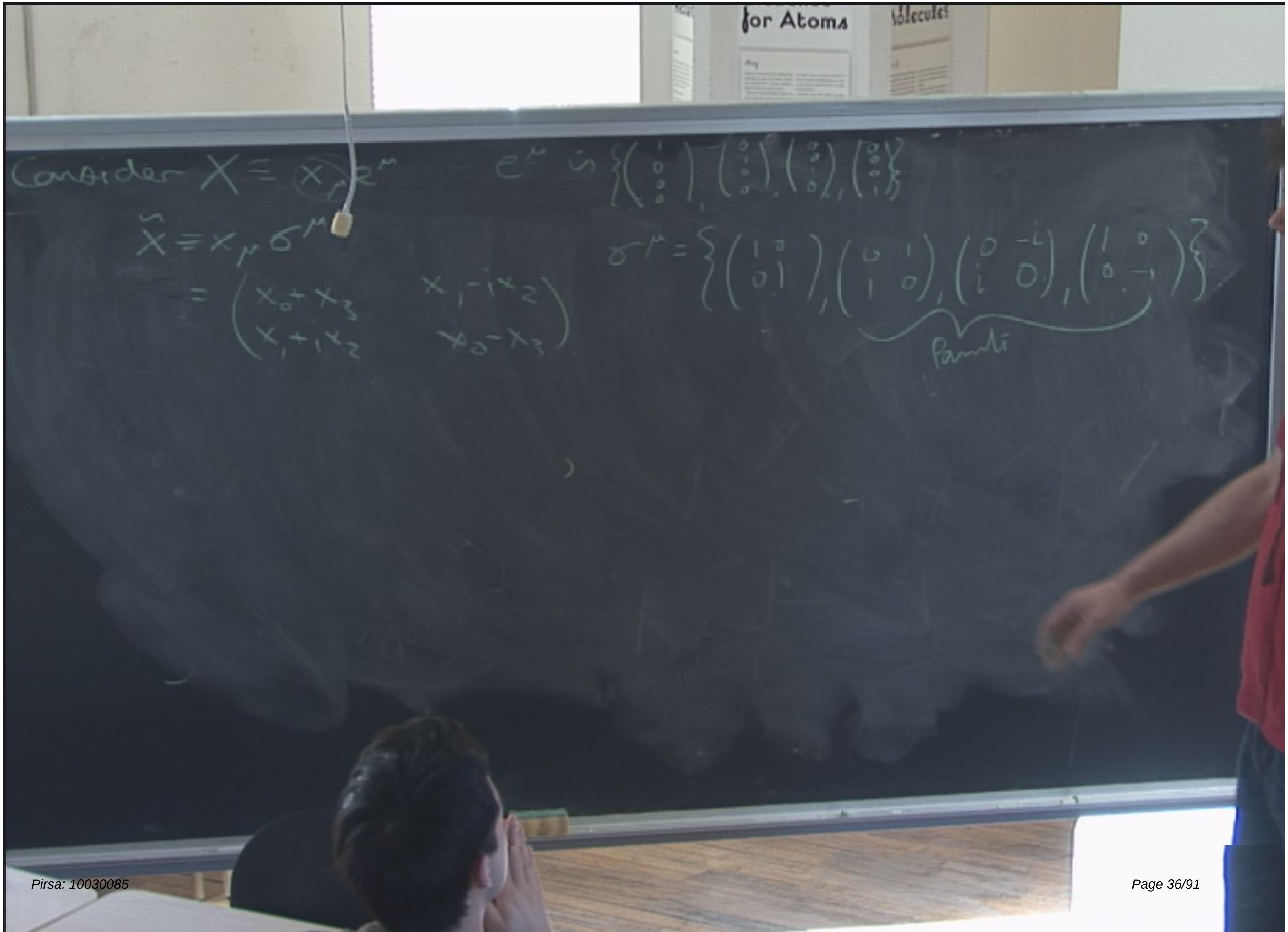

Consider X ≡ x_μ e^μ
e^μ is (

Consider $X \equiv x_\mu e^\mu$ $\quad$ e^μ is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\tilde{X} \equiv x_\mu \sigma^\mu$$

$$= \begin{pmatrix} x_0 + x_3 & x_1 - i x_2 \\ x_1 + i x_2 & x_0 - x_3 \end{pmatrix}$$

$$\sigma^\mu = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\text{Pauli}} \right\}$$


for Atoms
Molecules
Consider $X \equiv x_\mu e^\mu$
e^μ is $\left\{ \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\}$
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$$= \begin{pmatrix} x_0+x_3 & x_1 - i x_2 \\ x_1 + i x_2 & x_0 - x_3 \end{pmatrix}$$

Lorentz transf preserves the "size" of X

$$|X|^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2$$

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$$\tilde{X} \equiv x_\mu \sigma^\mu \; (SL(2,\mathbb{C})) \qquad \sigma^\mu = \left\{ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \underbrace{\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}}_{\text{Pauli}} \right\}$$

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and <u>determinant</u> of $\tilde{X}$.

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Lorentz transf preserves the "size" of X

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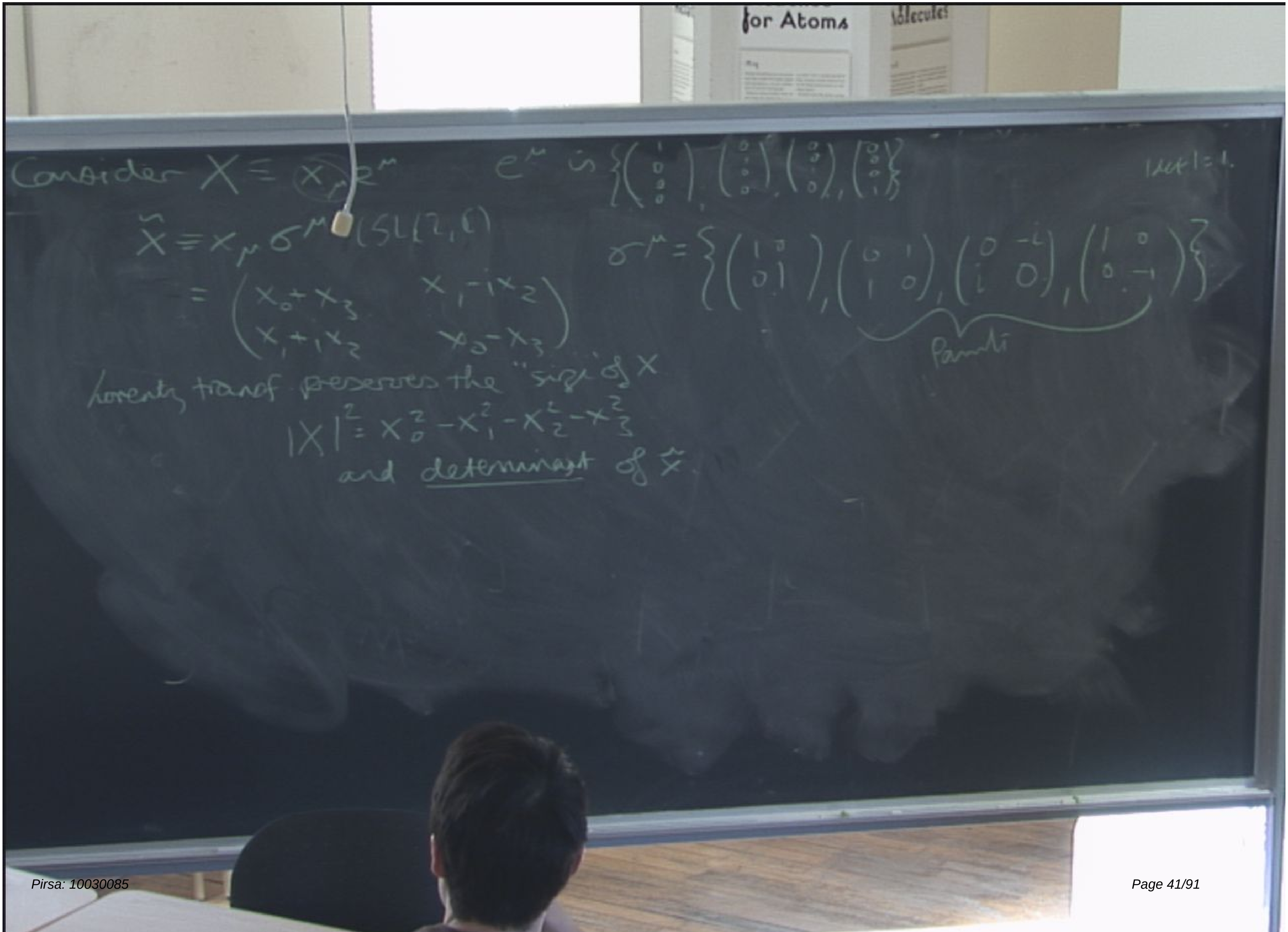
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$|\det| = 1.$
$\tilde{X} \equiv x_\mu \sigma^\mu \; (SL(2,C))$
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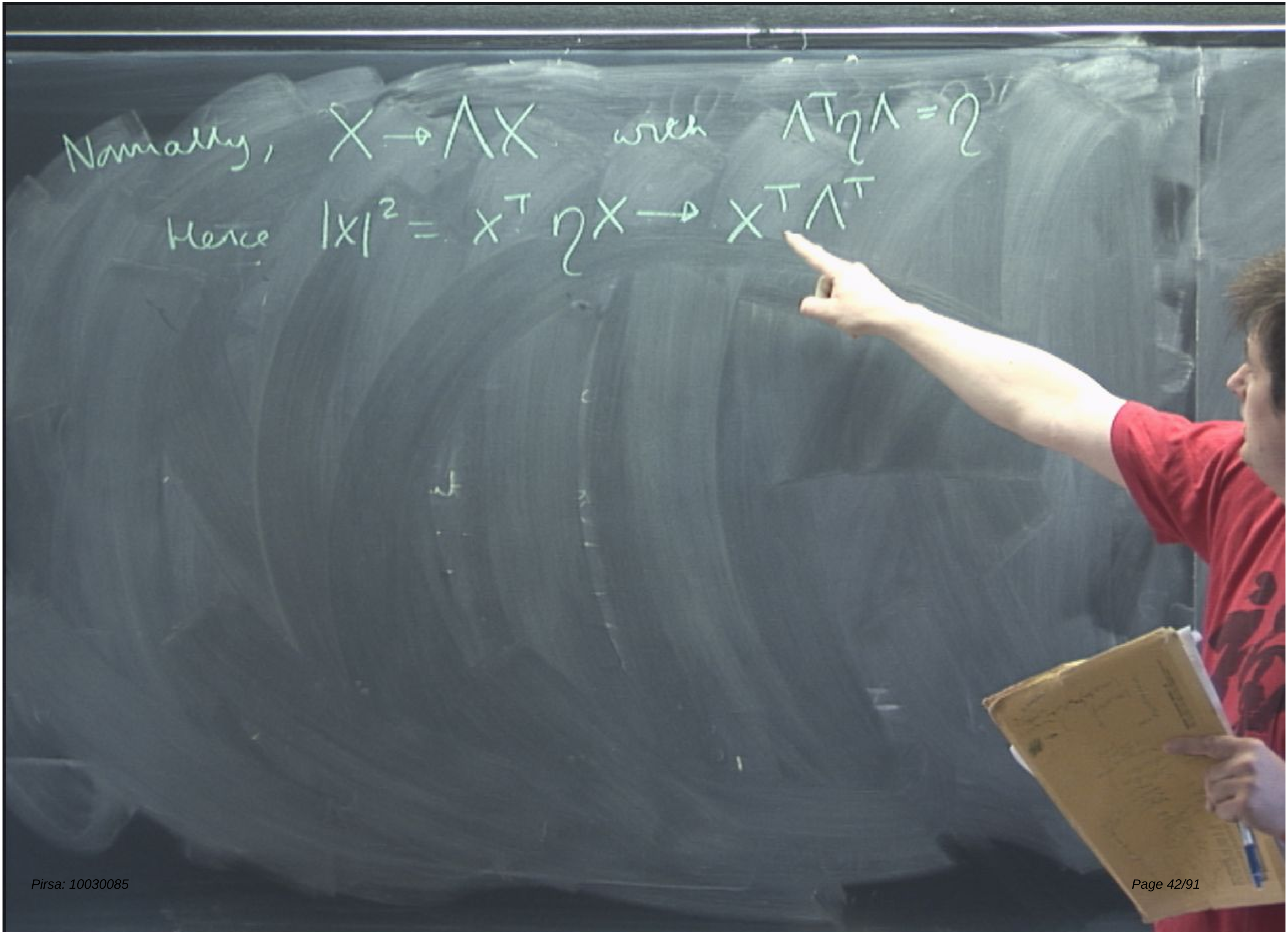
$= \begin{pmatrix}x_0+x_3 & x_1-ix_2\\ x_1+ix_2 & x_0-x_3\end{pmatrix}$
Lorentz transf preserves the "size" of X
$|X|^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2$
and determinant of $\tilde{X}$.


Normally, $X \to \Lambda X$ with $\Lambda^T \eta \Lambda = \eta$
Hence $|X|^2 = X^T \eta X \to X^T \Lambda^T$

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Hence $|X|^2 = X^T \eta X \to X^T \underbrace{\Lambda^T \eta \Lambda}_{\eta} X$.

hence $|X|^2$ preserved under Lorentz transformation.

For $N \in SL(2, \mathbb{C})$

$\tilde{X} \mapsto N$

Normally, $X \to \Lambda X$ with $\Lambda^T \eta \Lambda = \eta$

Hence $|X|^2 = X^T \eta X \to X^T \Lambda^T \eta \Lambda X$

hence $|X|^2$ preserved under Lorentz transformation.

For $N \in SL(2,\mathbb{C})$

$$\tilde{X} \mapsto N \tilde{X} N^{+}$$

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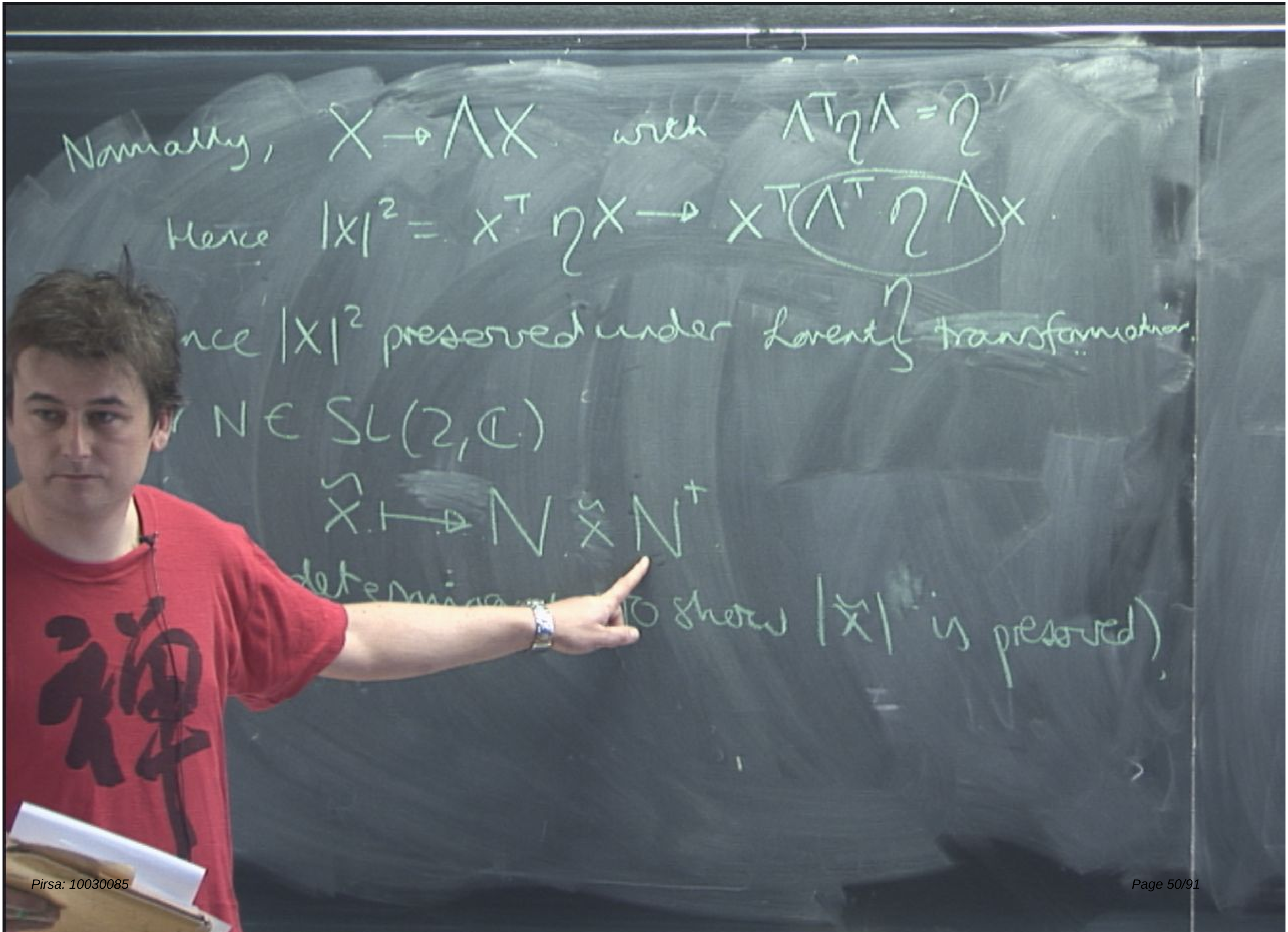
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Reps of $SL(2, \mathbb{C})$:

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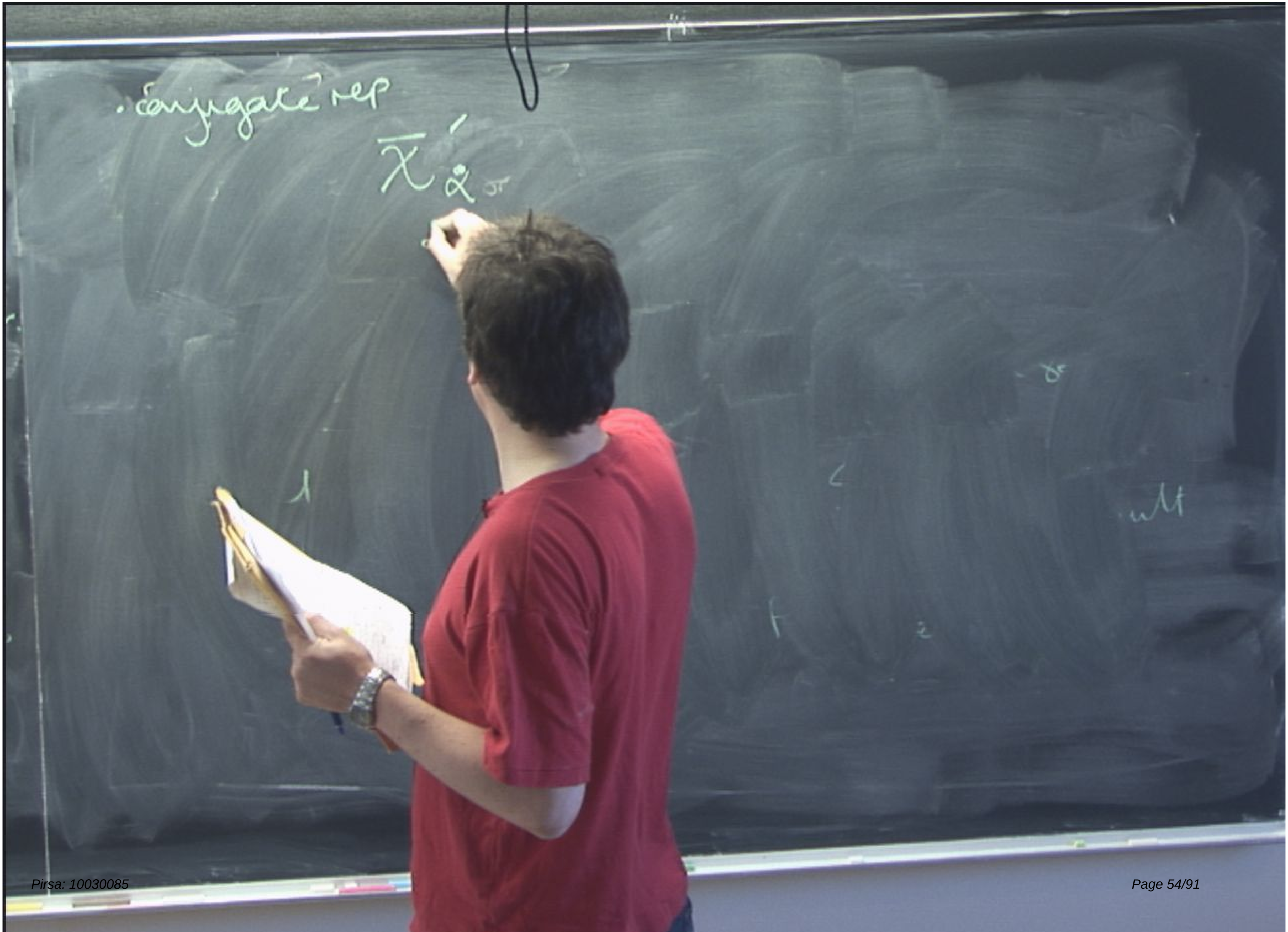
· conjugate rep
$\bar{\chi}'_{\dot\alpha}$

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Contra variant reps

$$\psi'^{\alpha} = \psi^{\beta} (N^{-1})_{\beta}{}^{\alpha}, \quad \bar{\chi}'^{\dot{\alpha}} = \bar{\chi}^{\dot{\beta}} (N^{*-1})$$

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(dotted + undotted notation)

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Invariant Tensors · $\eta_{\mu\nu}$ is inv under

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Invariant Tensors

· $\eta_{\mu\nu}$ is inv under $SO(1,3)$

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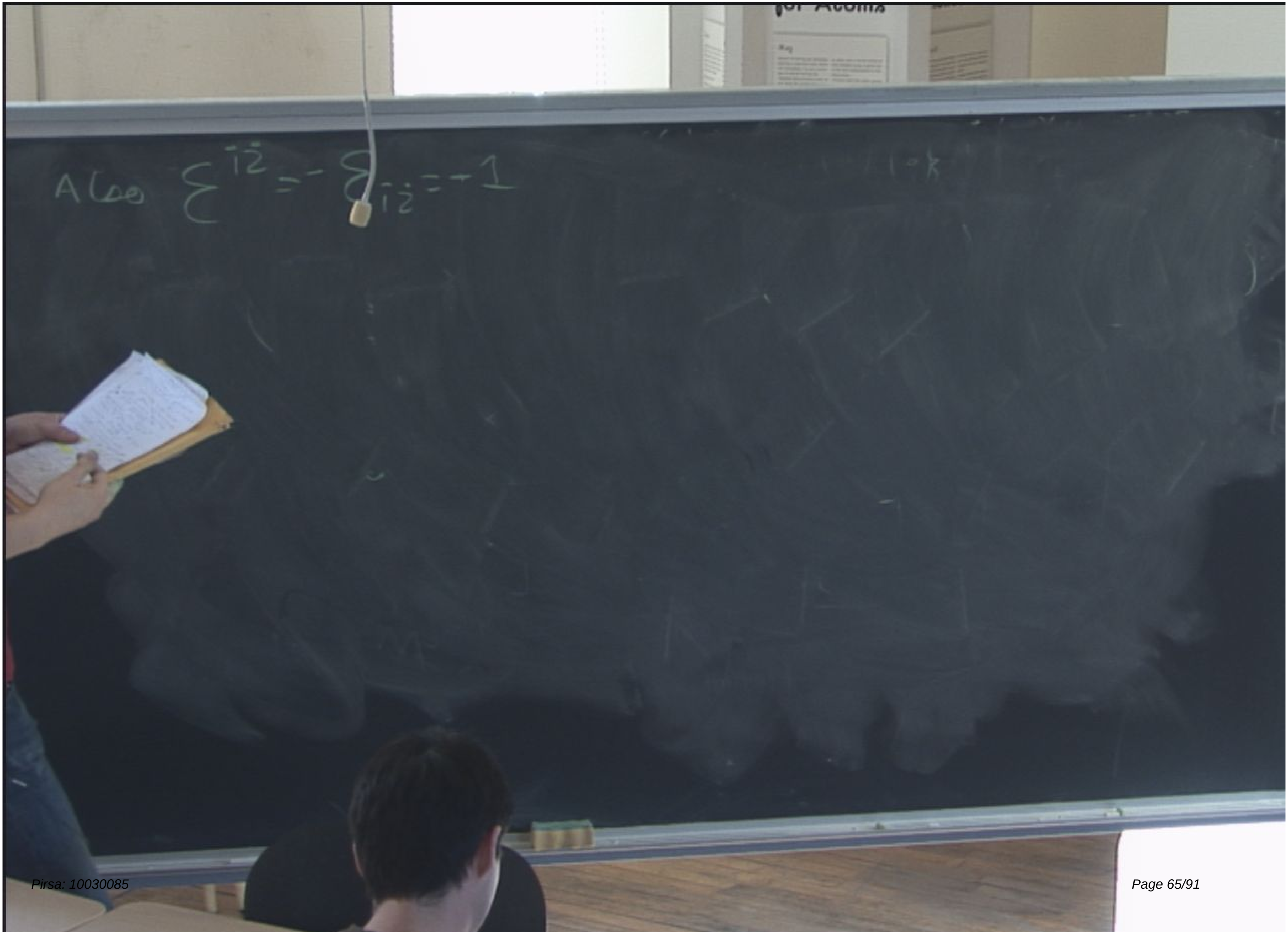
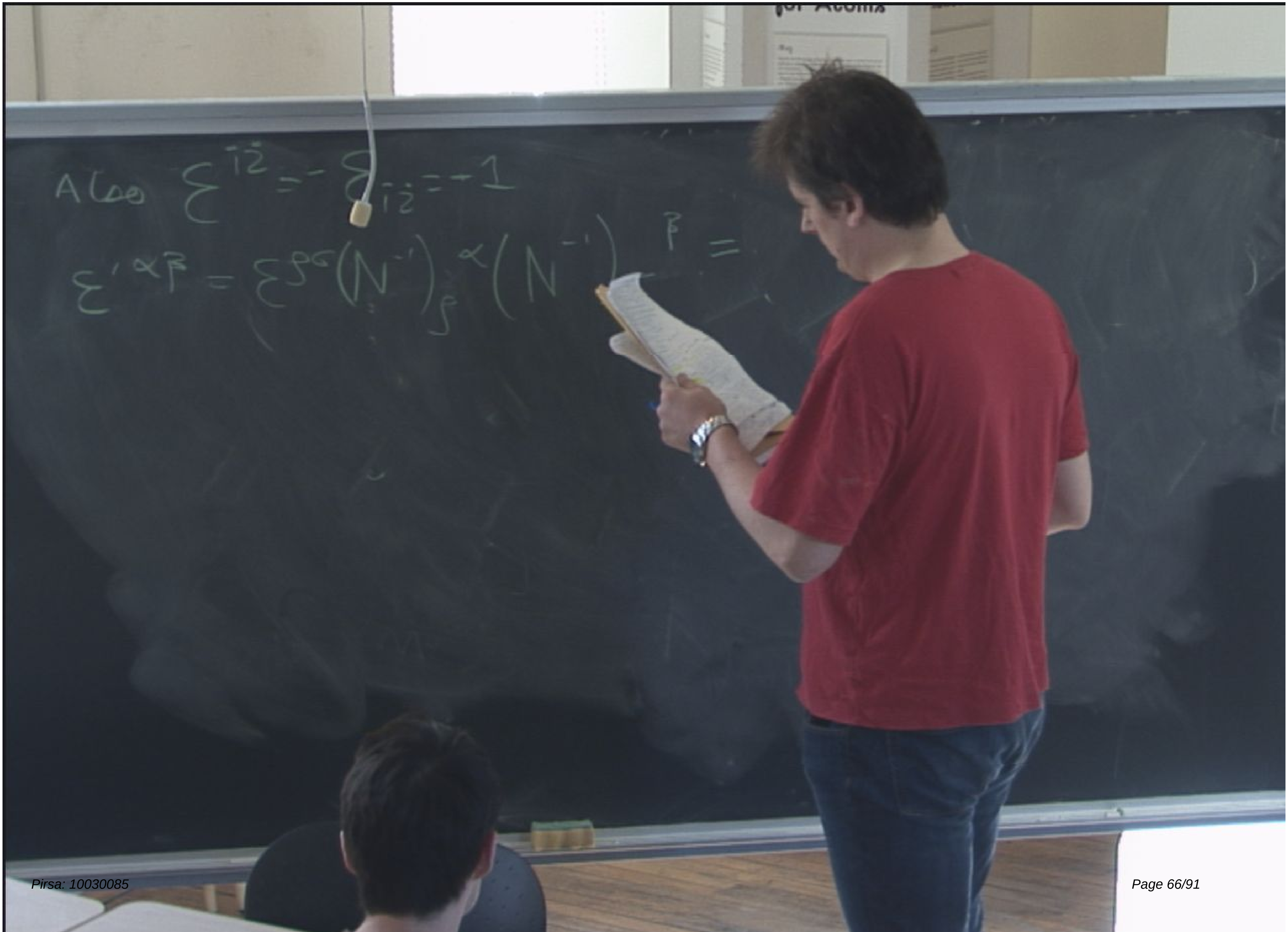

Also $\epsilon^{12} = -\epsilon_{12} = -1$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$$\varepsilon'^{\dot{\alpha}\dot{\beta}} = \varepsilon^{\dot{\rho}\dot{\sigma}} (N^{-1})_{\dot{\rho}}{}^{\dot{\alpha}} (N^{-1})_{\dot{\sigma}}{}^{\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}} (\det N^{-1})$$

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For handling mixed indices use

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hence $\sigma^\mu_{\alpha\dot{\alpha}} = N_\alpha{}^\beta(\sigma^\nu)_{\beta\dot{\gamma}}(\Lambda^{-1})_\nu{}^\mu N^*{}^{\dot{\gamma}}{}_{\dot{\alpha}}$

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Generators of $SL(2, \mathbb{C})$ in spinor rep

$$(\sigma^{\mu\nu})_{\alpha}{}^{\beta} \equiv \frac{i}{4}\left(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right)_{\alpha}{}^{\beta}$$

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These satisfy the Lorentz algebra (*).

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These satisfy the Lorentz algebra (*)

and $\psi_{\alpha} \rightarrow \left[e^{-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}}\right]_{\alpha}{}^{\beta}\psi_{\beta}$

$$\bar{\chi}^{\dot{\alpha}} \rightarrow \left[e^{-\frac{i}{2}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}}\right]^{\dot{\alpha}}{}_{\dot{\beta}}\bar{\chi}^{\dot{\beta}}$$

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$$(\bar\sigma^\mu)^{\dot\alpha\alpha} \equiv \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}(\sigma^\mu)_{\beta\dot\beta} = (\mathbb{1}, -\underline{\sigma}) \quad \mu = 0, 1, 2, 3$$

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• Spins w.r.t. A, B:

ψ_α : $(A,B) = (\frac{1}{2}, 0) \Rightarrow J_i = \frac{1}{2}\sigma_i$ and $K_i = -\frac{i}{2}\sigma_i$

ε

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$\bar{\chi}^{\dot{\alpha}}$: $(A,B)=(0,\frac{1}{2})$ $\Rightarrow$ $J_i=\frac{1}{2}\sigma_i$ and $K_i=+\frac{i}{2}\sigma_i$.

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Useful identities

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u[illegible]ntities

$$\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu = 2\eta^{\mu\nu}\mathbb{1}.$$

$$\mathrm{Tr}(\sigma^\mu \bar{\sigma}^\nu) = 2\eta^{\mu\nu}$$

$$(\sigma^\mu)_{\alpha\dot\alpha}(\bar{\sigma}_\mu)^{\dot\beta\beta} = 2\delta_\alpha{}^\beta\,\delta_{\dot\alpha}{}^{\dot\beta}$$

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$$(\sigma^\mu)_{\alpha\dot{\alpha}} (\bar{\sigma}_\mu)^{\dot{\beta}\beta} = 2\delta_\alpha{}^\beta \delta_{\dot{\alpha}}{}^{\dot{\beta}}$$

$$\sigma^{\mu\nu} = \frac{1}{2i} \varepsilon^{\mu\nu\rho\kappa} \sigma_{\rho\kappa}$$

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anti-sym

normally, you'd think there are

$$\sigma^{\mu\nu} = \frac{1}{2i}\varepsilon^{\mu\nu\rho\chi}\sigma_{\rho\chi}$$

$$\bar\sigma^{\mu\nu} = -\frac{1}{2i}\varepsilon^{\mu\nu\rho\chi}\bar\sigma_{\rho\chi}.$$

ε

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$\bar{\chi}^{\dot{\alpha}}$: $(A,B) = (0, \frac{1}{2}) \Rightarrow J_i = \frac{1}{2}\sigma_i$ and $K_i = +\frac{i}{2}\sigma_i$.

Useful identities

$$\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu = 2\eta^{\mu\nu}\mathbb{1}.$$

$$\mathrm{Tr}(\sigma^\mu \bar{\sigma}^\nu) = 2\eta^{\mu\nu}$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma}_\mu)^{\dot{\beta}\beta} = 2\delta_\alpha{}^\beta\, \delta_{\dot{\alpha}}{}^{\dot{\beta}}$$

anti-sym

normally, you'd think there are $\frac{4\times 3}{2}$ components, but self-duality reduces them by a further factor of 2.

$$\left.\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{2i}\varepsilon^{\mu\nu\rho\lambda}\sigma_{\rho\lambda} \\ \bar{\sigma}^{\mu\nu} &= -\frac{1}{2i}\varepsilon^{\mu\nu\rho\lambda}\bar{\sigma}_{\rho\lambda} \end{aligned}\right\} \text{self-duality}$$