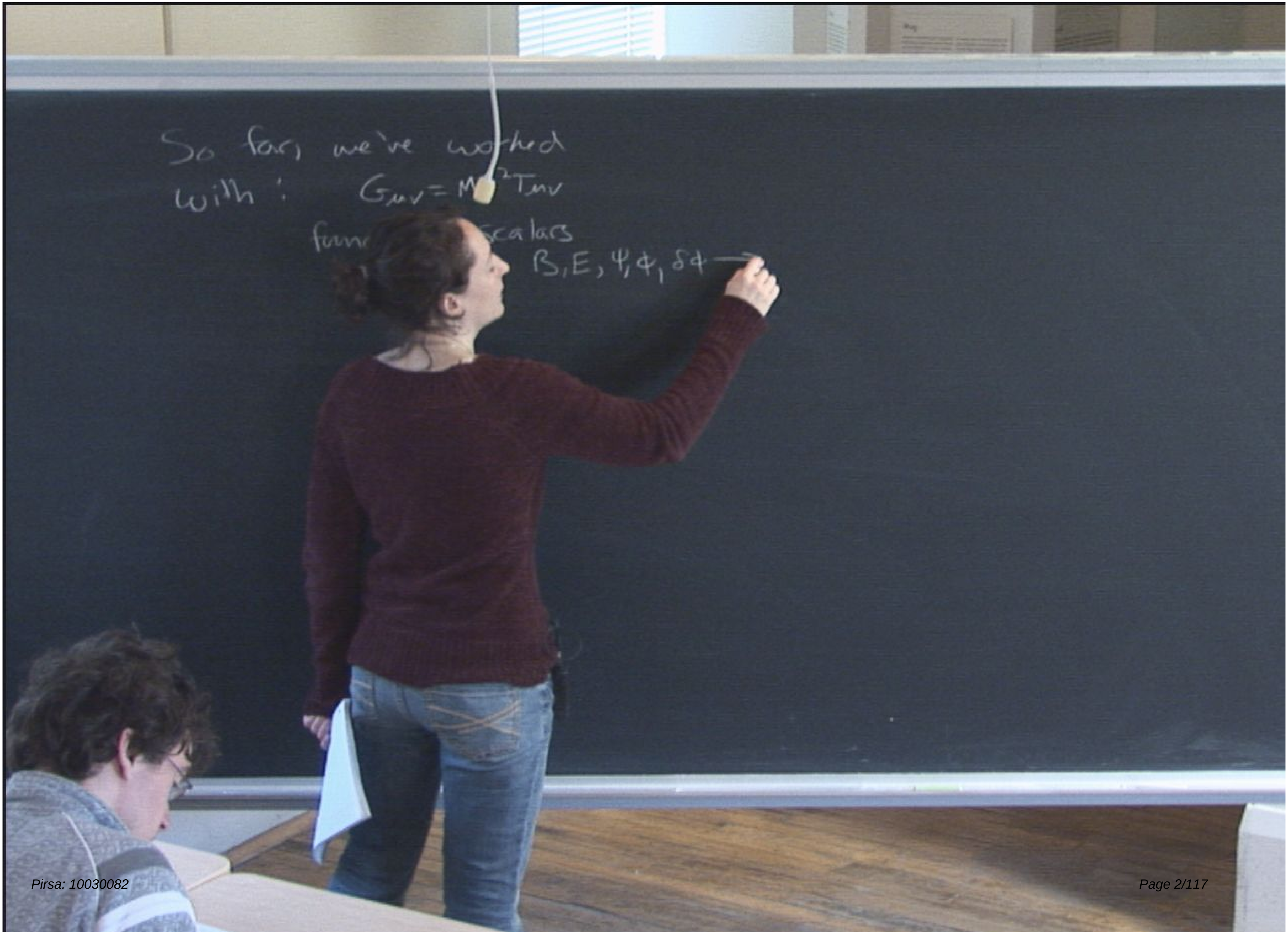
Title: Explorations in Cosmology - Lecture 13

Date: Mar 31, 2010 09:00 AM

URL: http://pirsa.org/10030082

Abstract:

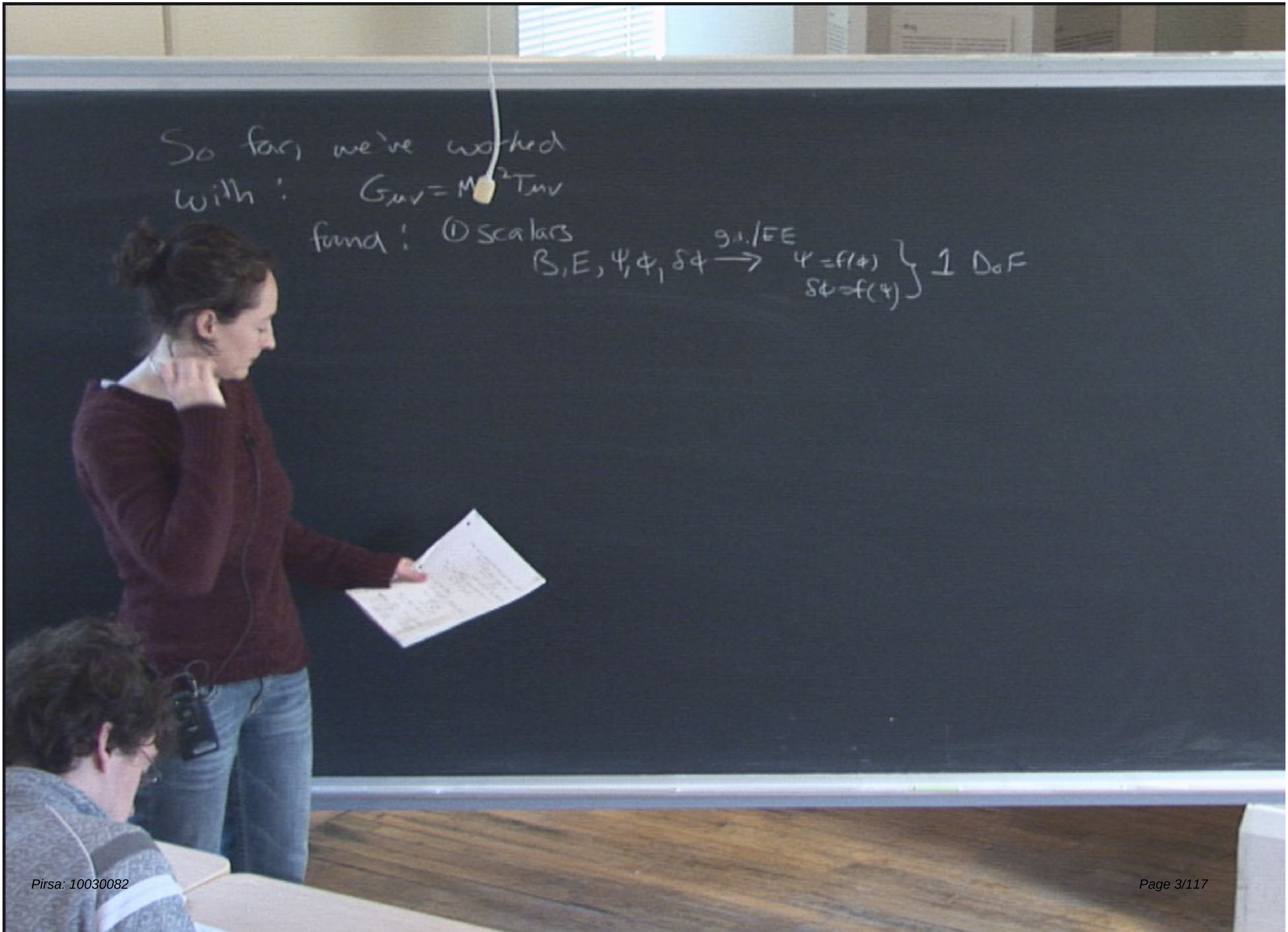


So far, we've worked

with: $G_{\mu\nu} = M^{2} T_{\mu\nu}$

fun scalars

$B, E, \psi, \phi, \delta\phi \rightarrow$



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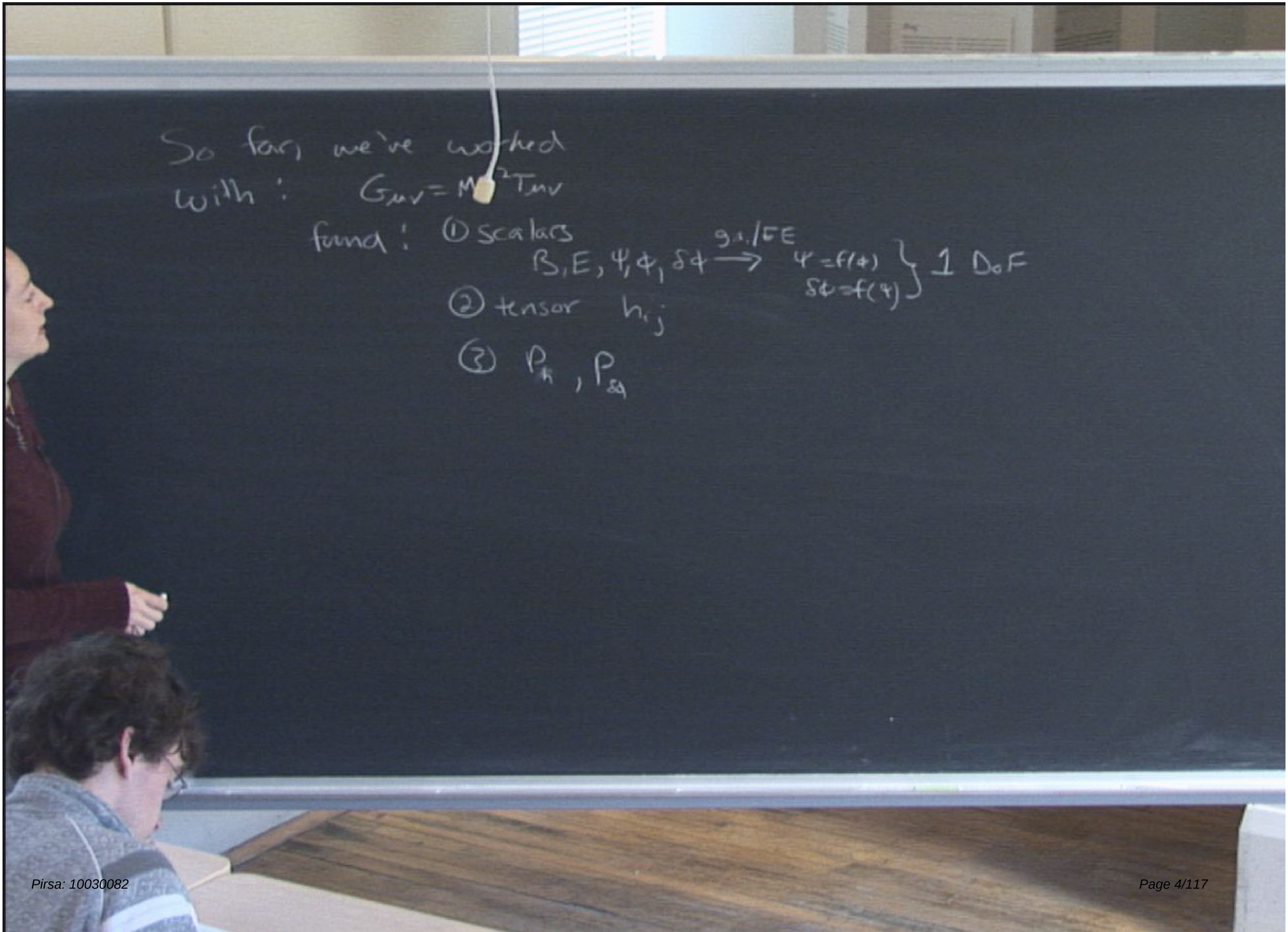
$$B, E, \Psi, \Phi, \delta\phi \xrightarrow{g.t./EE} \left.\begin{aligned} \Psi &= f(\Phi) \\ \delta\phi &= f(\Psi) \end{aligned}\right\} 1 \text{ DoF}$$



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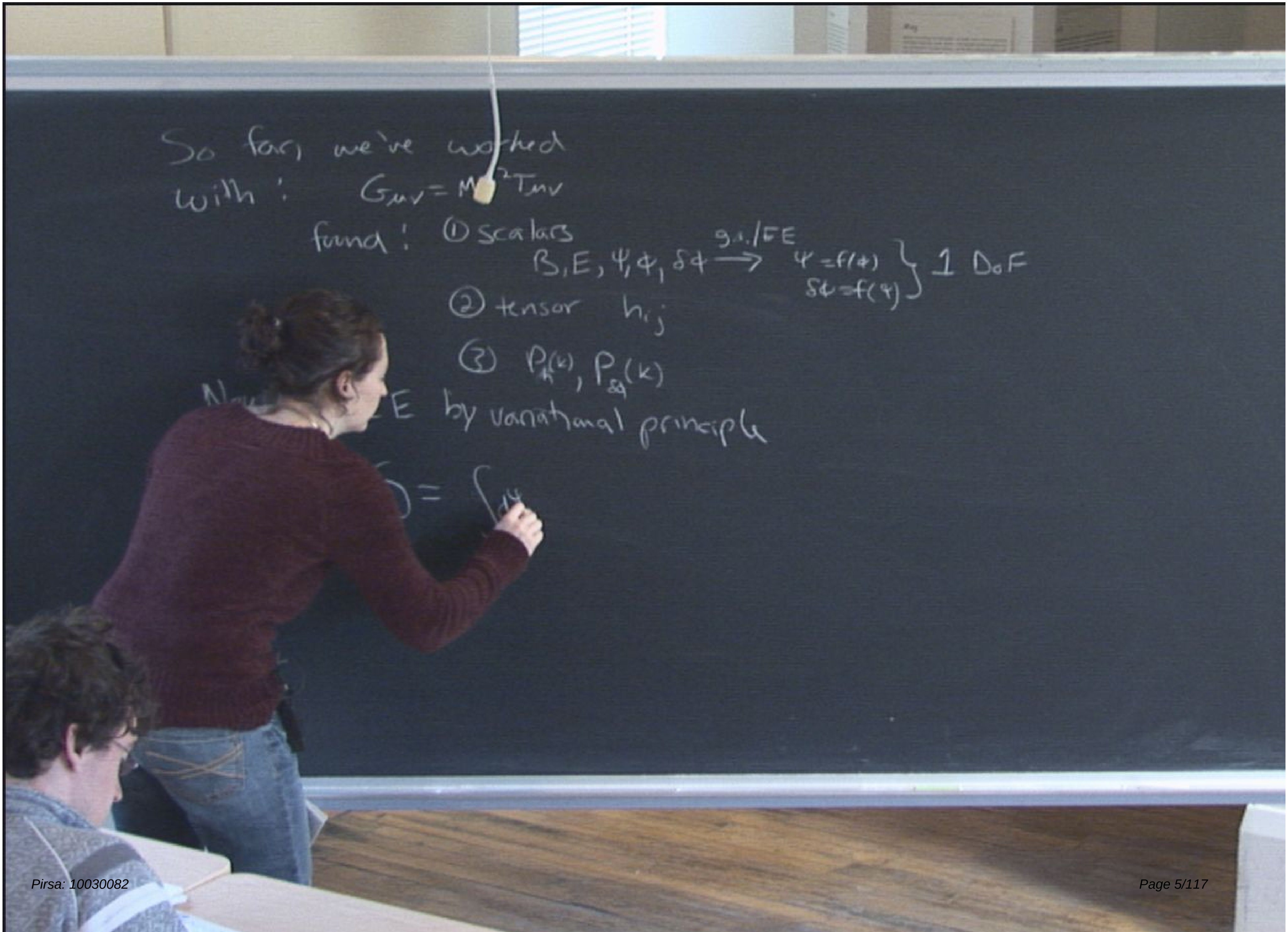
② tensor h_{ij}

③ P_h , $P_{\delta\phi}$



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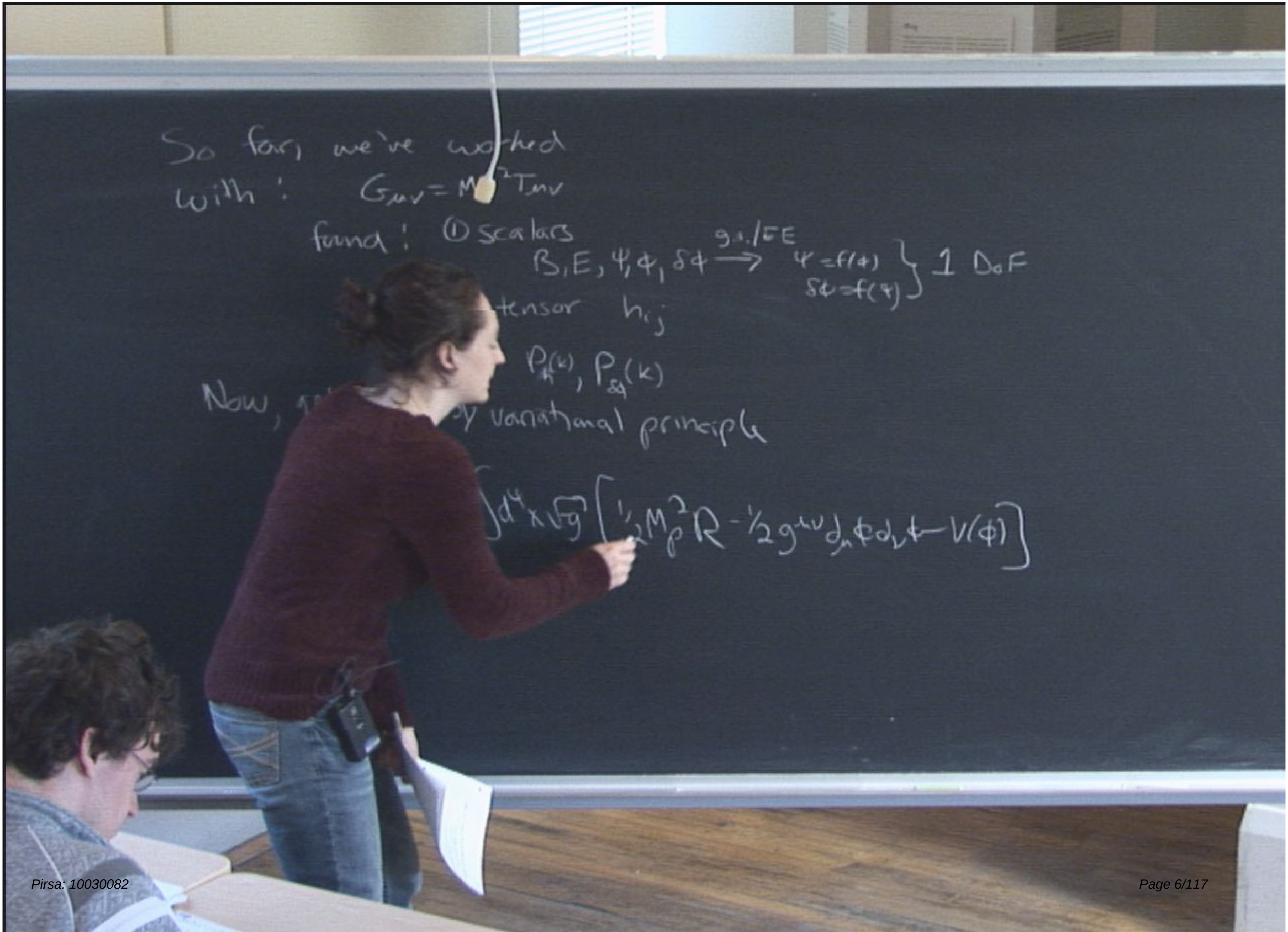
② tensor h_{ij}

③ $P_{\phi}(k), P_{\delta\phi}(k)$

N[illegible] E by variational principle

[illegible]$) = \int d^4$



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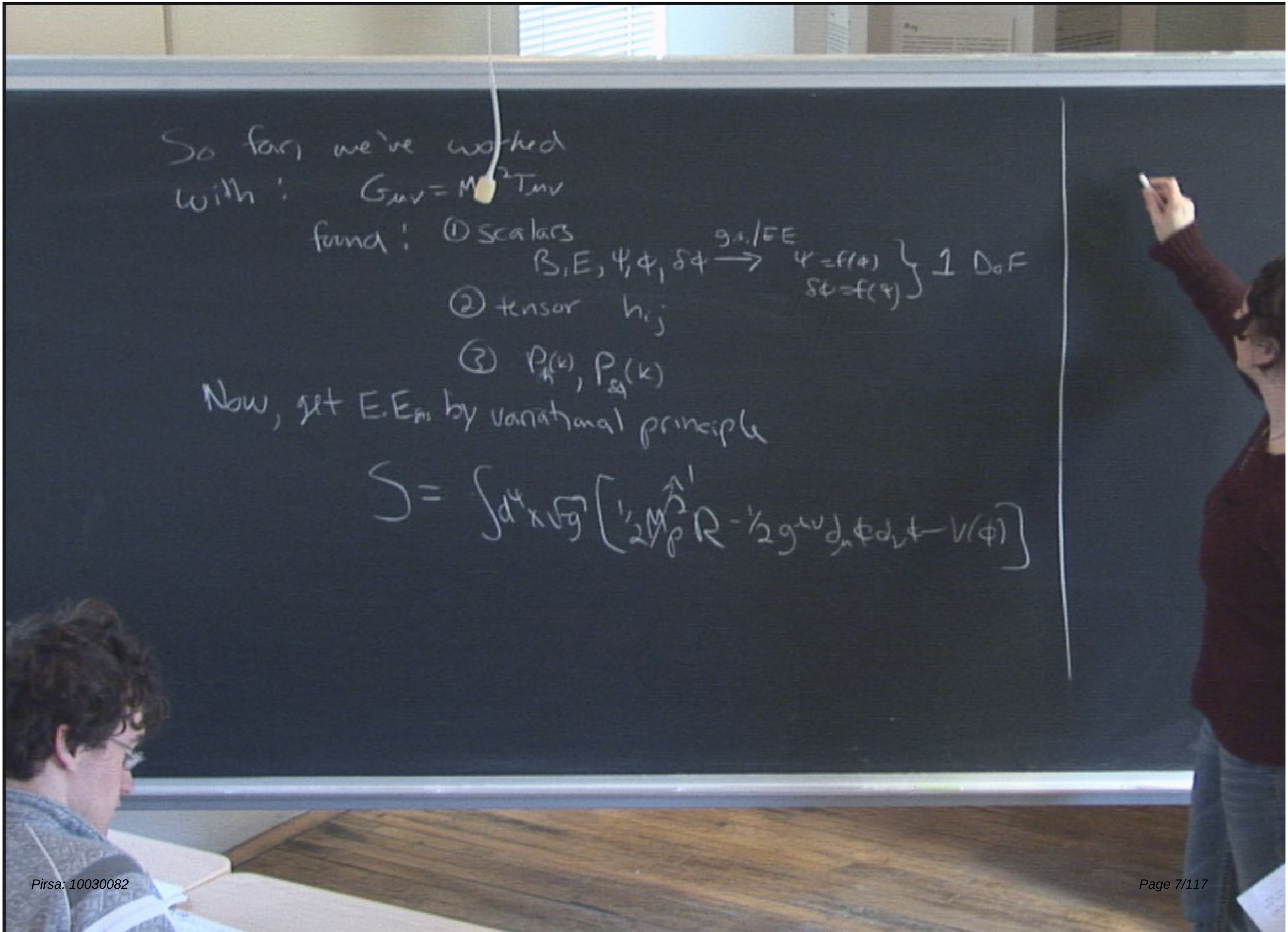
tensor h_{ij}

$P_h(k)$, $P_{\delta\phi}(k)$

Now, [illegible] by variational principle

$$\int d^4x \sqrt{g} \left[\frac{1}{2} M_p^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$



So far, we've worked with: $G_{\mu\nu} = M^{\ \ 2} T_{\mu\nu}$

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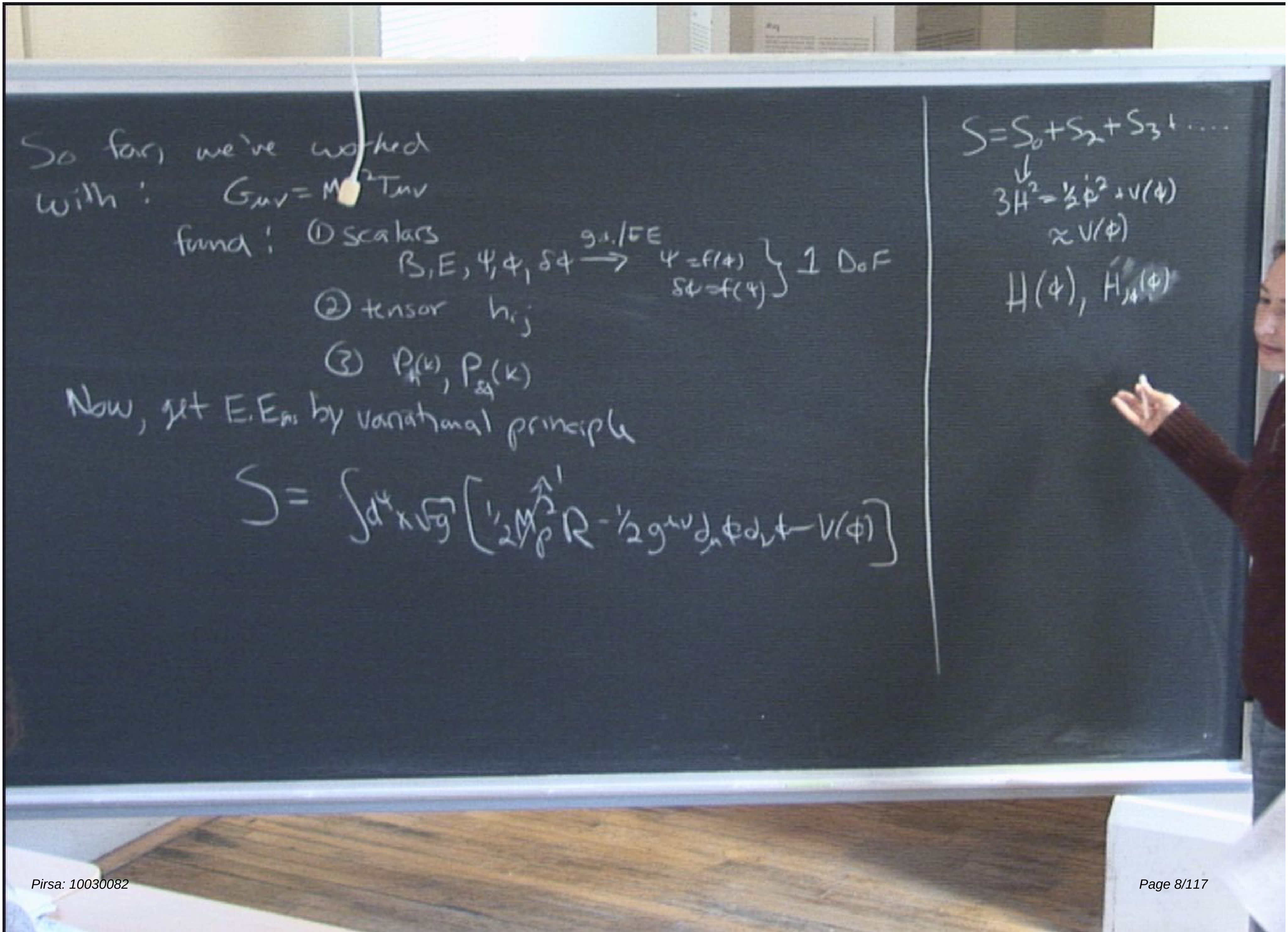
Now, get E.E.ms by variational principle

$$S = \int d^4x \sqrt{g} \left[\frac{1}{2} \overset{\wedge 1}{M_p^{2}} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$



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$$S = S_0 + S_2 + S_3 + \ldots$$

$\downarrow$

$$3H^2 = \tfrac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\approx V(\phi)$$

$$H(\phi),\ H_{,\phi}(\phi)$$

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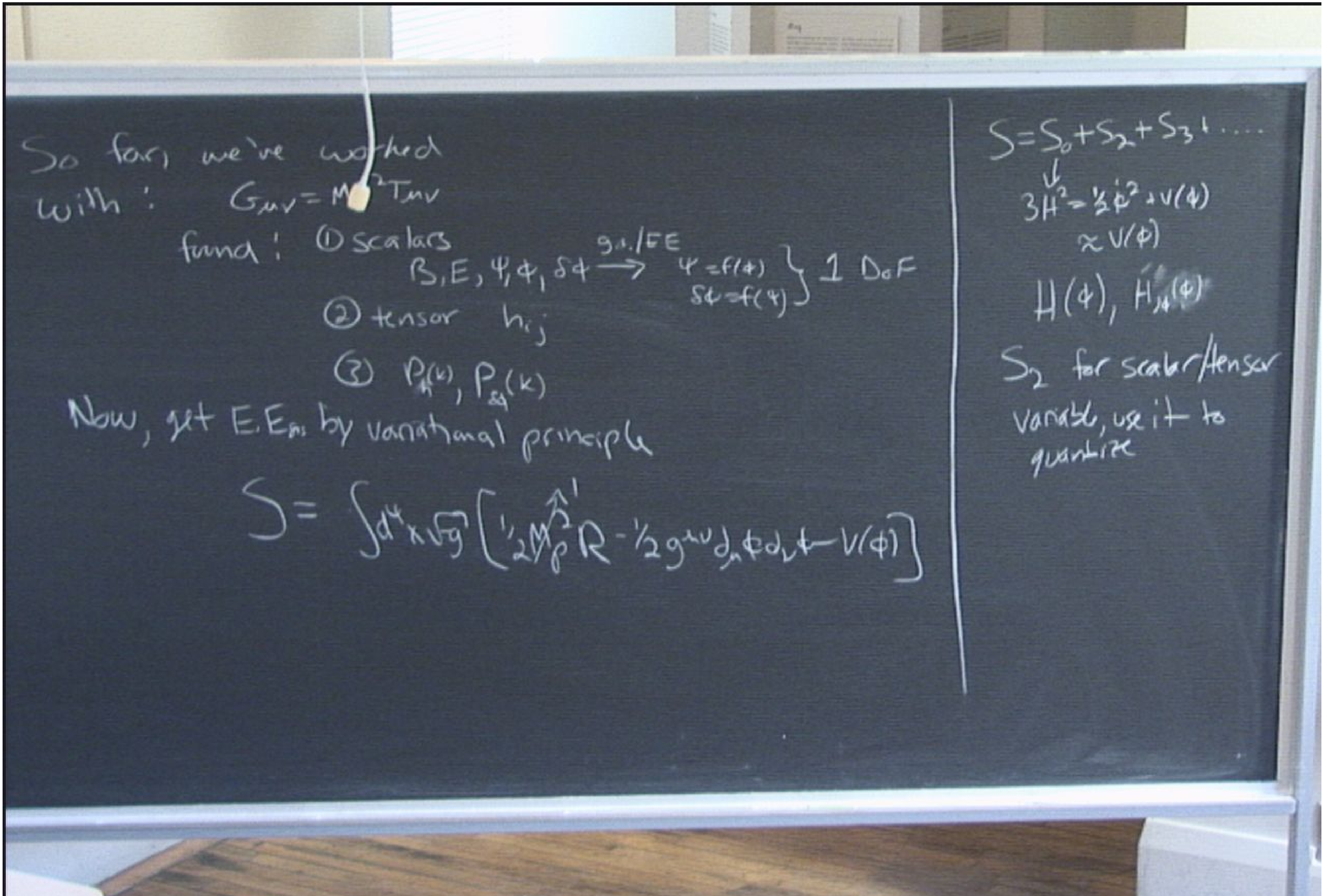
S_2 for scalar/tensor variable, use it to quantize



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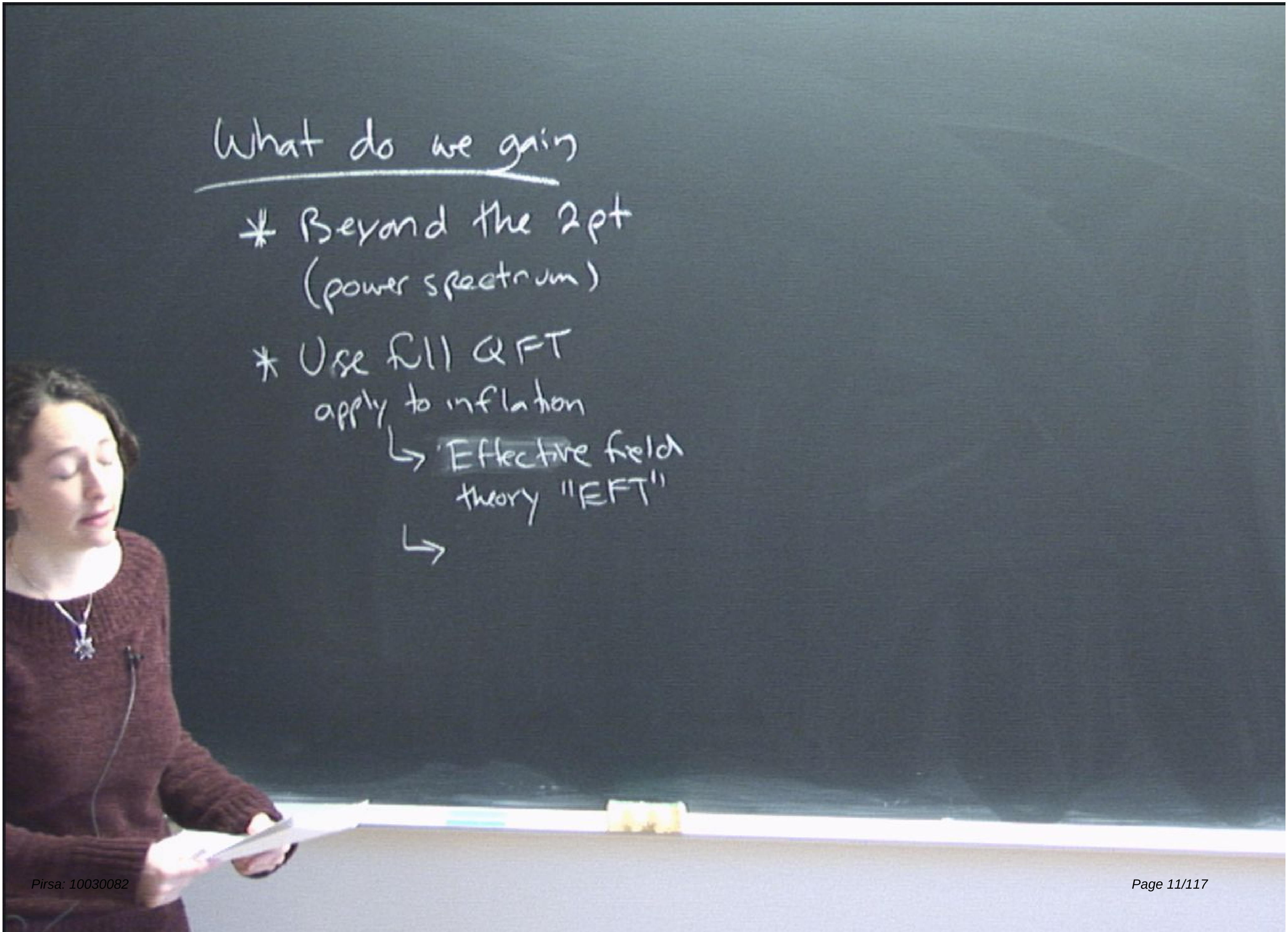
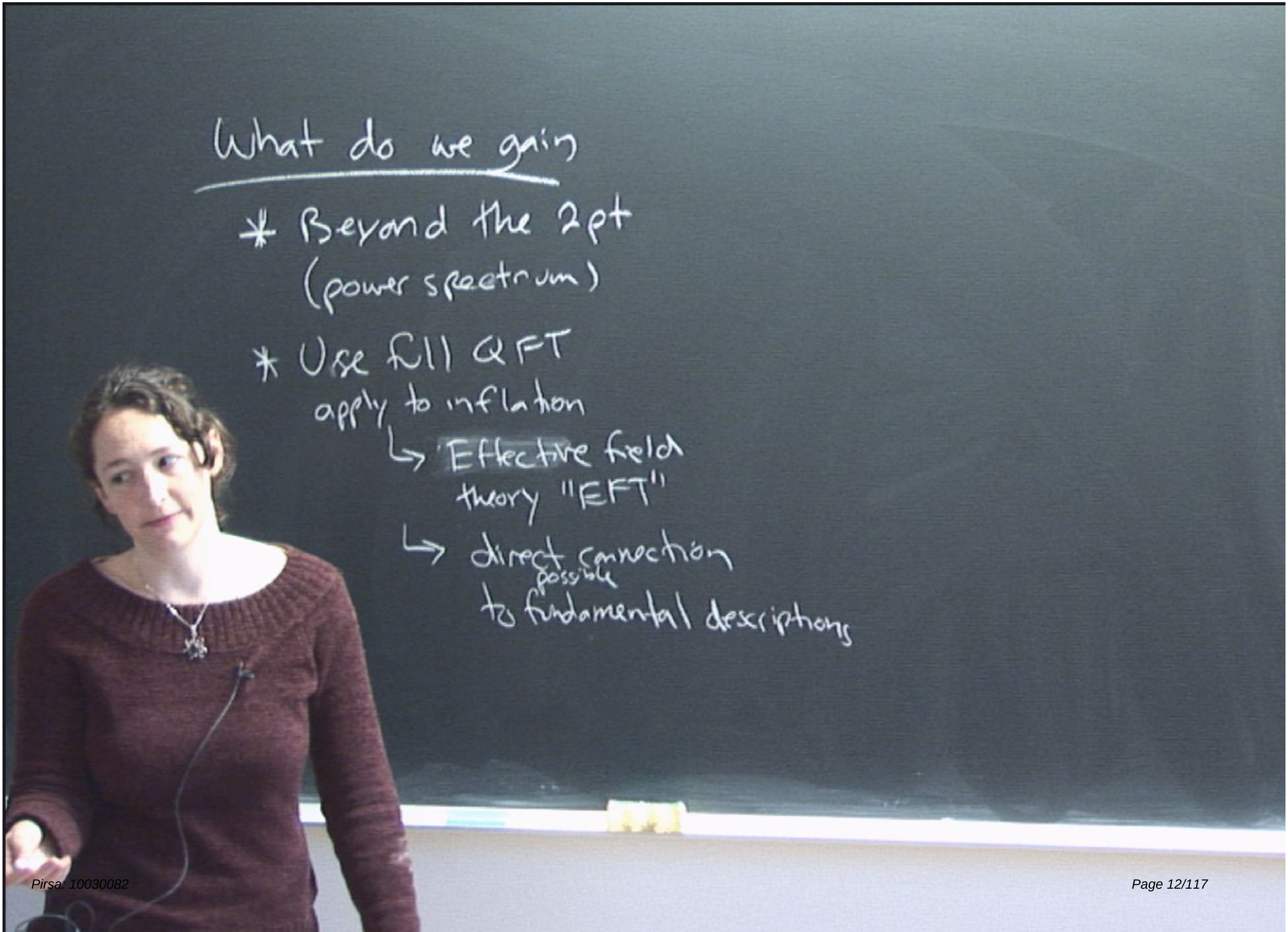


What do we gain

* Beyond the 2pt
  (power spectrum)

* Use full QFT
  apply to inflation
  ↳ 'Effective field
  theory "EFT"
  ↳



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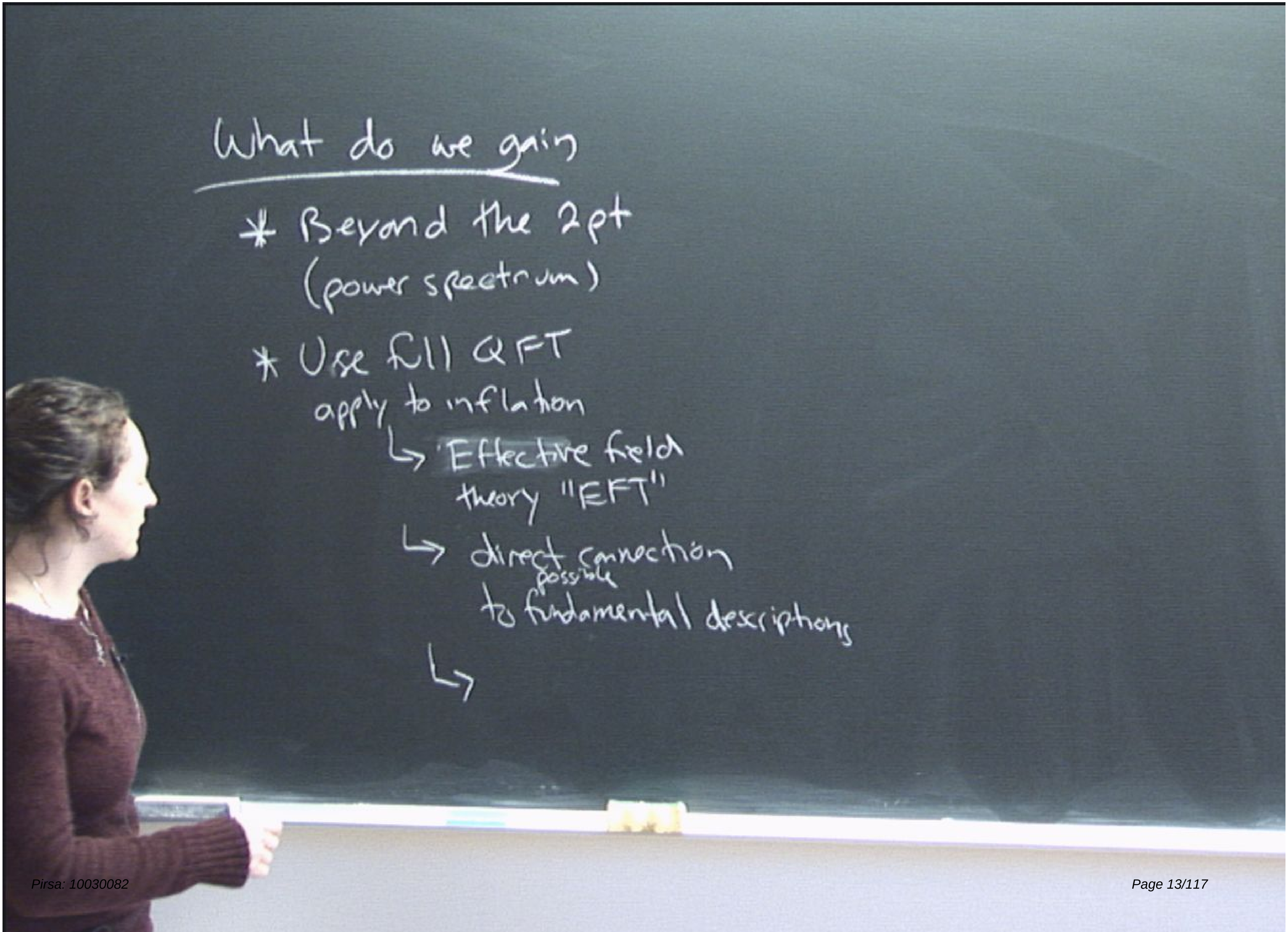

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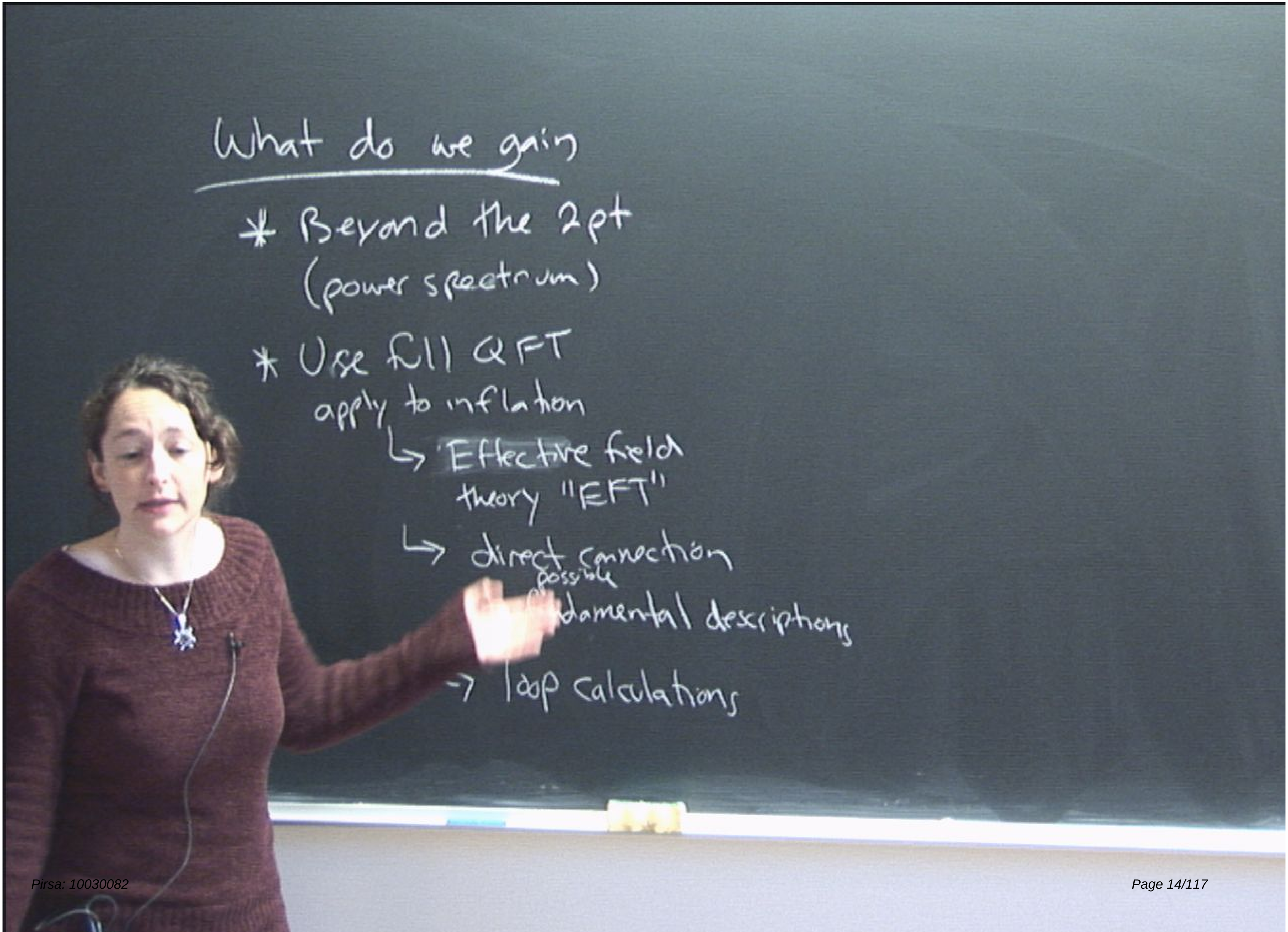

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- * Beyond the 2pt
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- * Use full QFT
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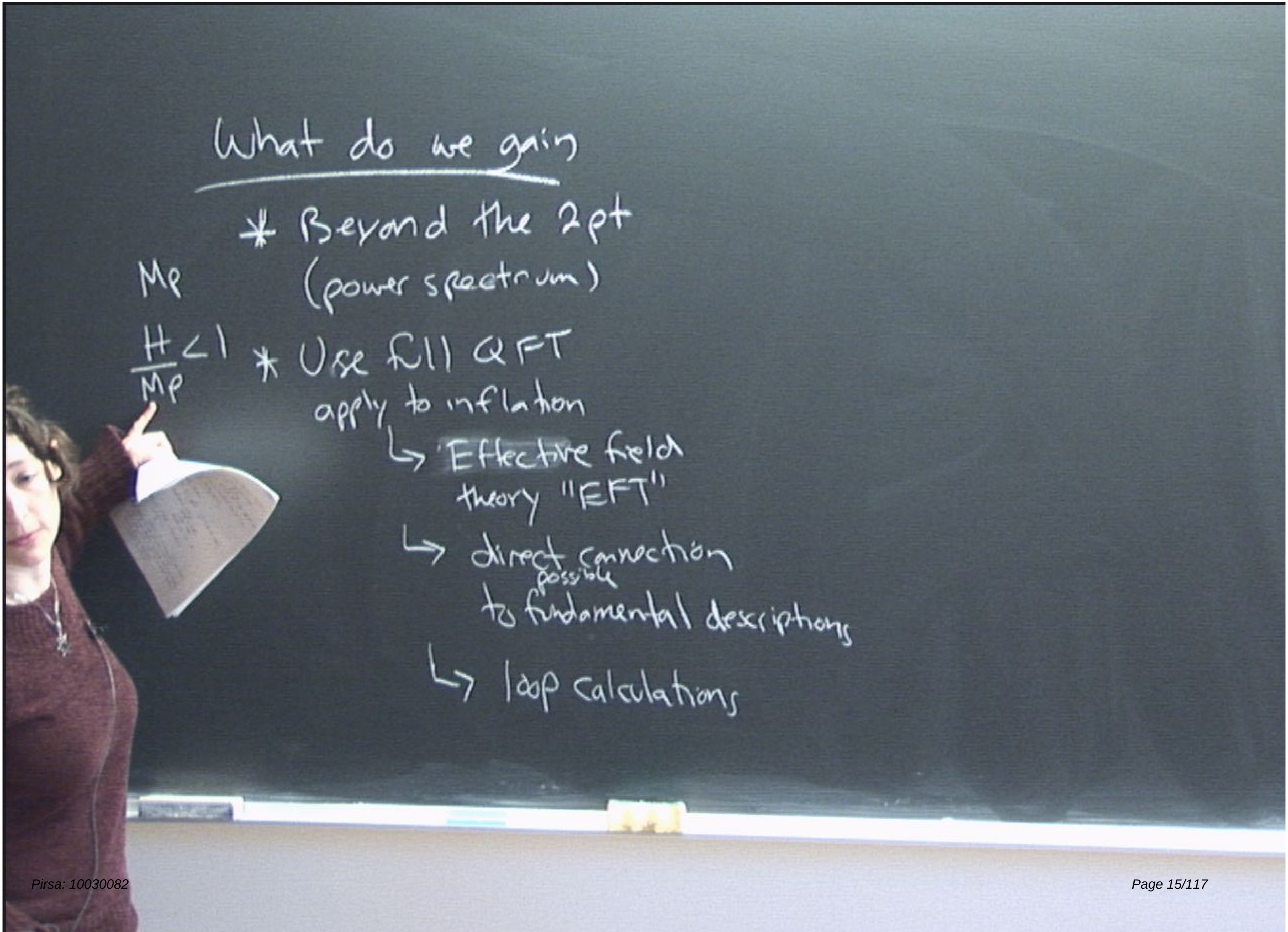
  - ↳ "Effective field theory "EFT"
  - ↳ direct connection possible
    damental descriptions
  - → loop calculations



What do we gain

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M_p

$\frac{H}{M_p} < 1$

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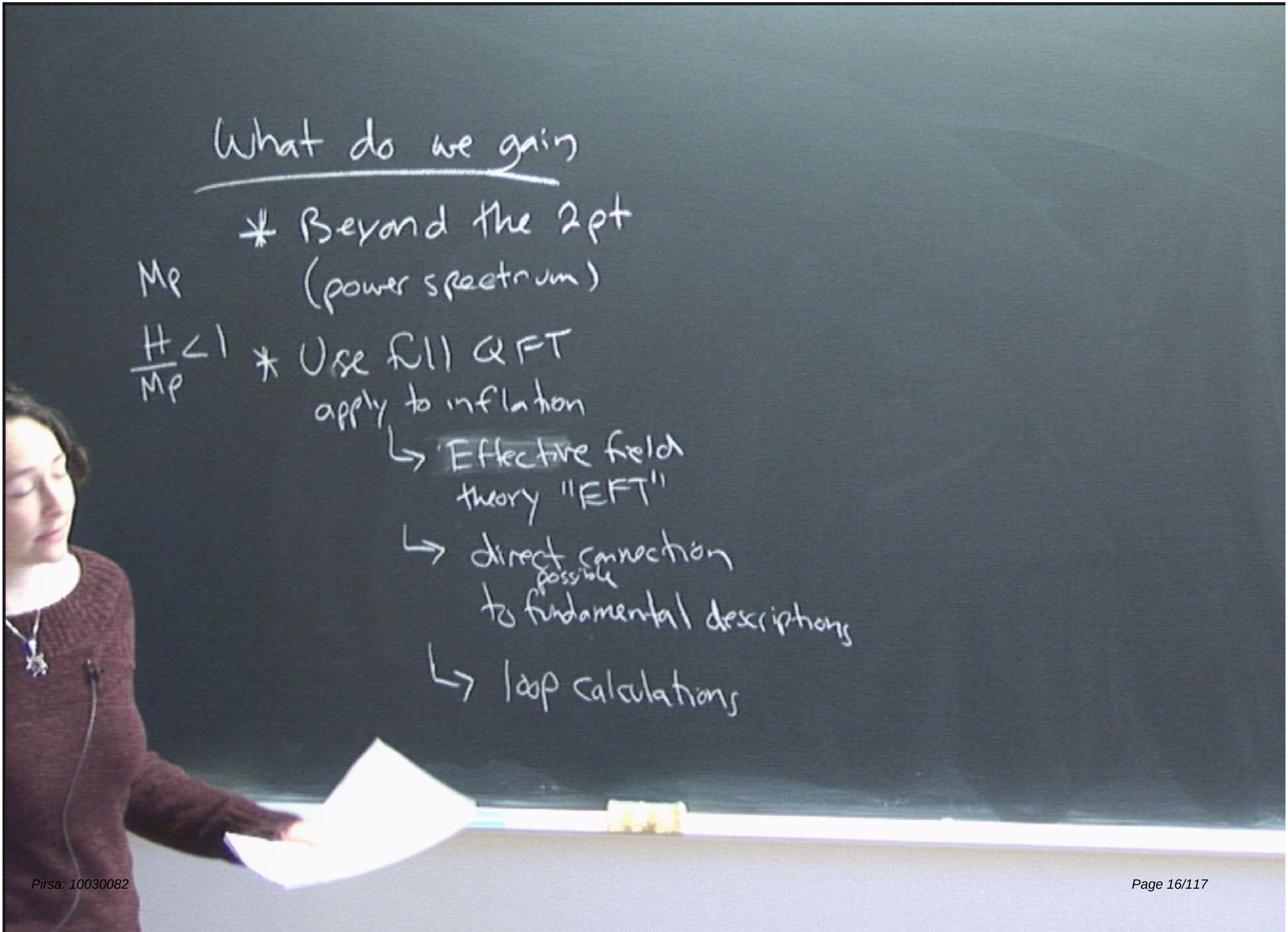
  - ↳ loop calculations



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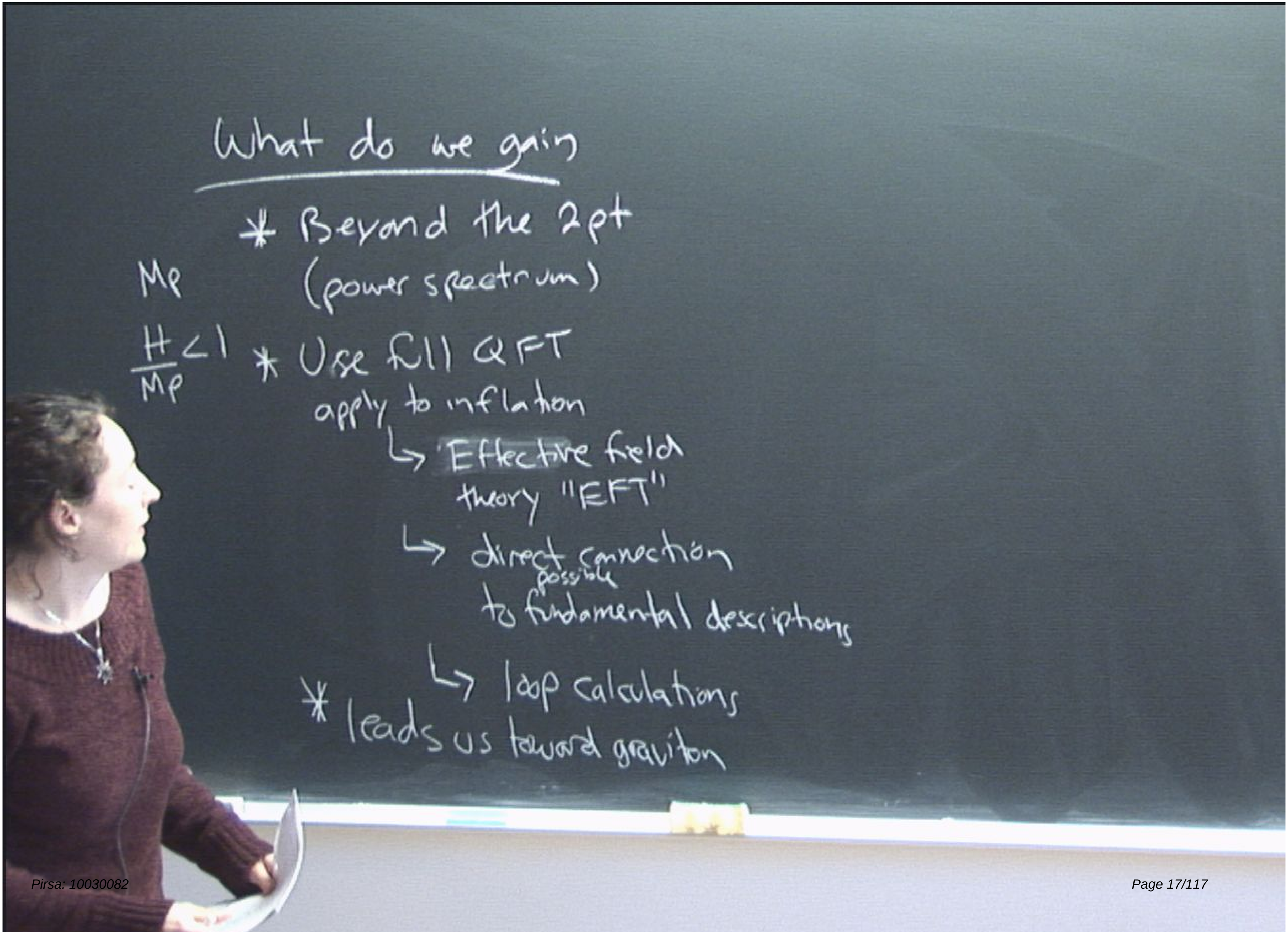

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- Beyond the 2pt (power spectrum)
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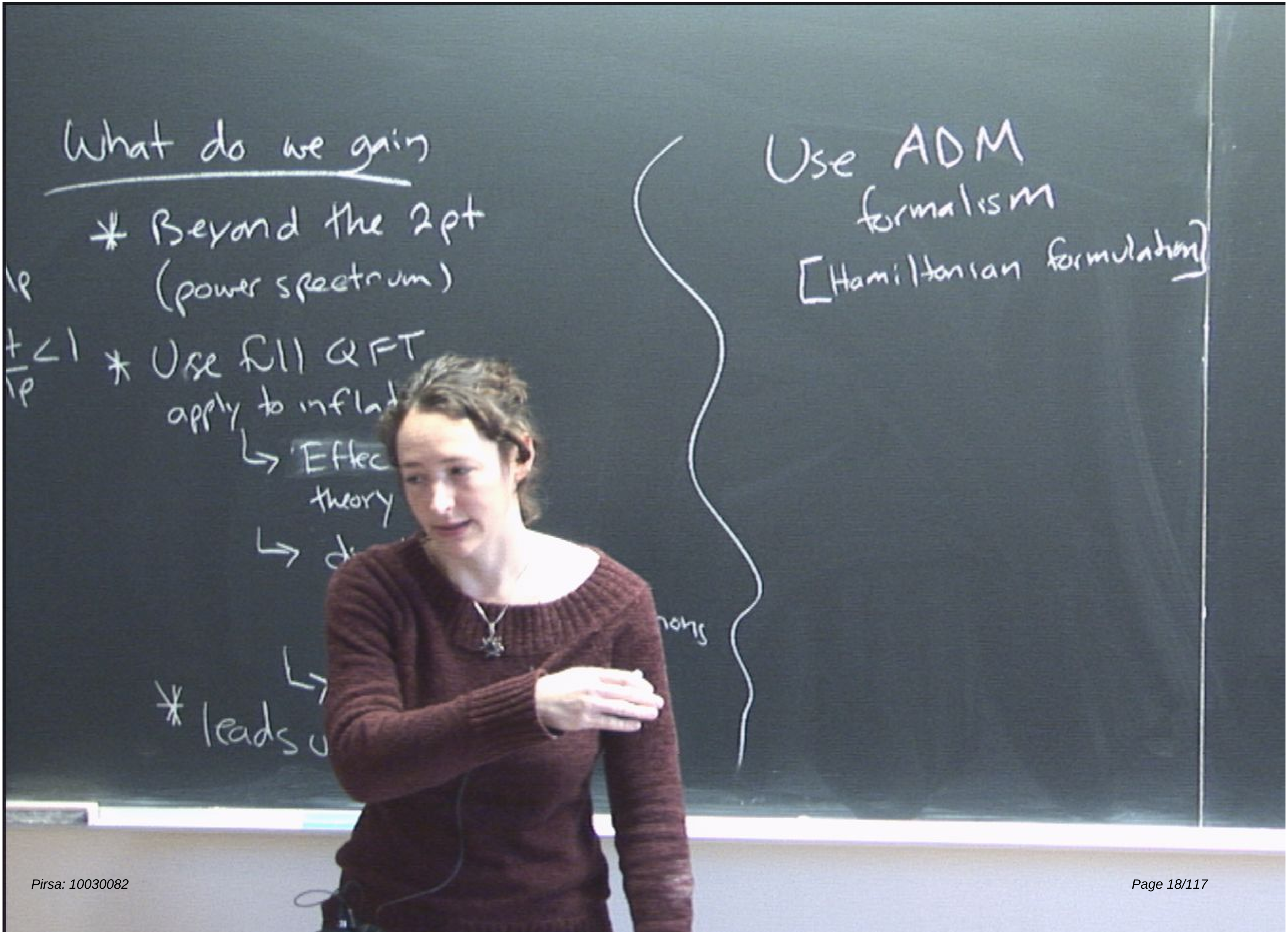
  - ↳ Effective field theory "EFT"
  - ↳ direct possible connection to fundamental descriptions
  - ↳ loop calculations
- leads us toward graviton



What do we gain

* Beyond the 2pt
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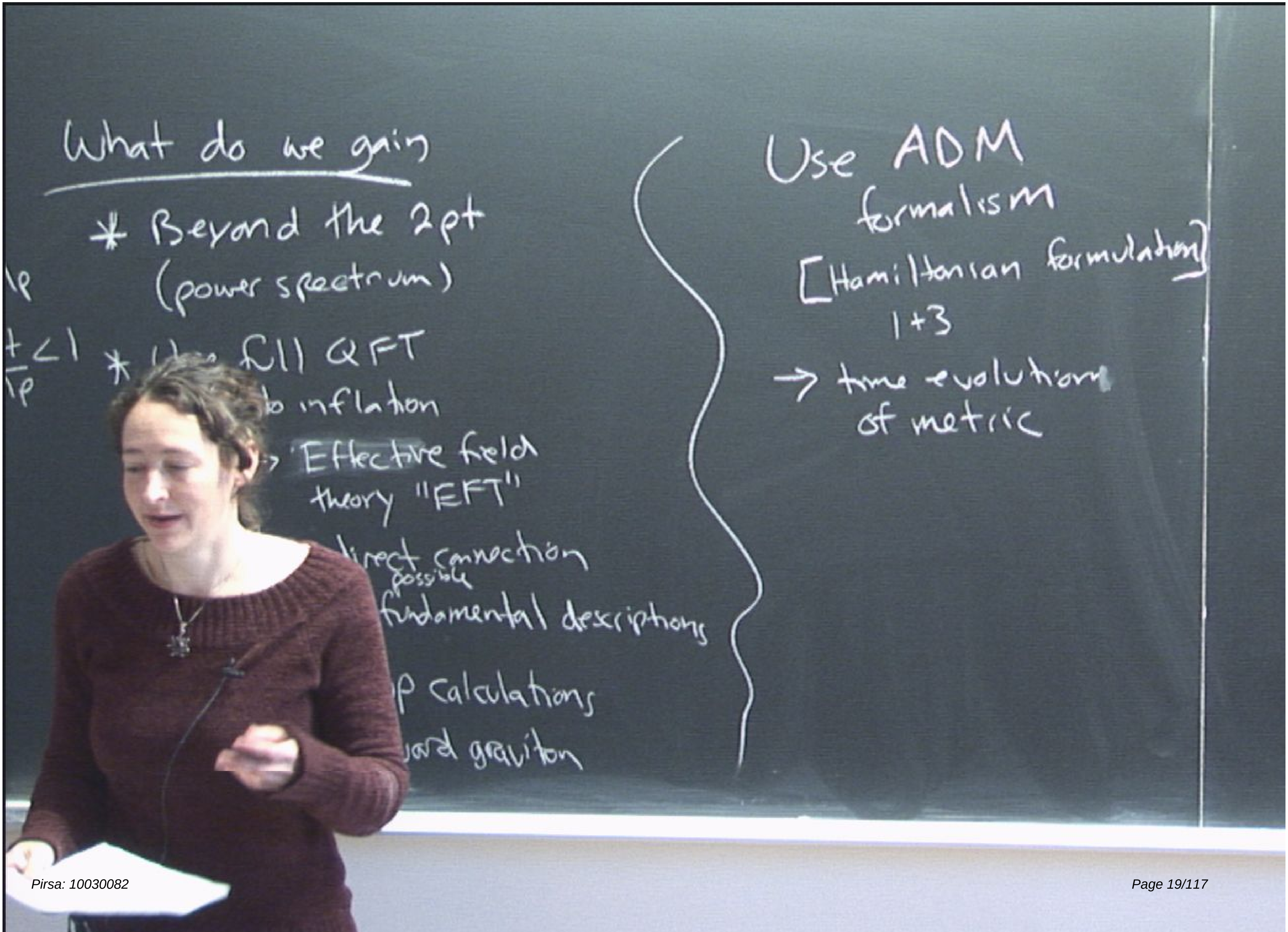
* Use full QFT
apply to infla
↳ "Effec
theory
↳ d
nons
↳

* leads u

Use ADM
formalism
[Hamiltonian formulation]



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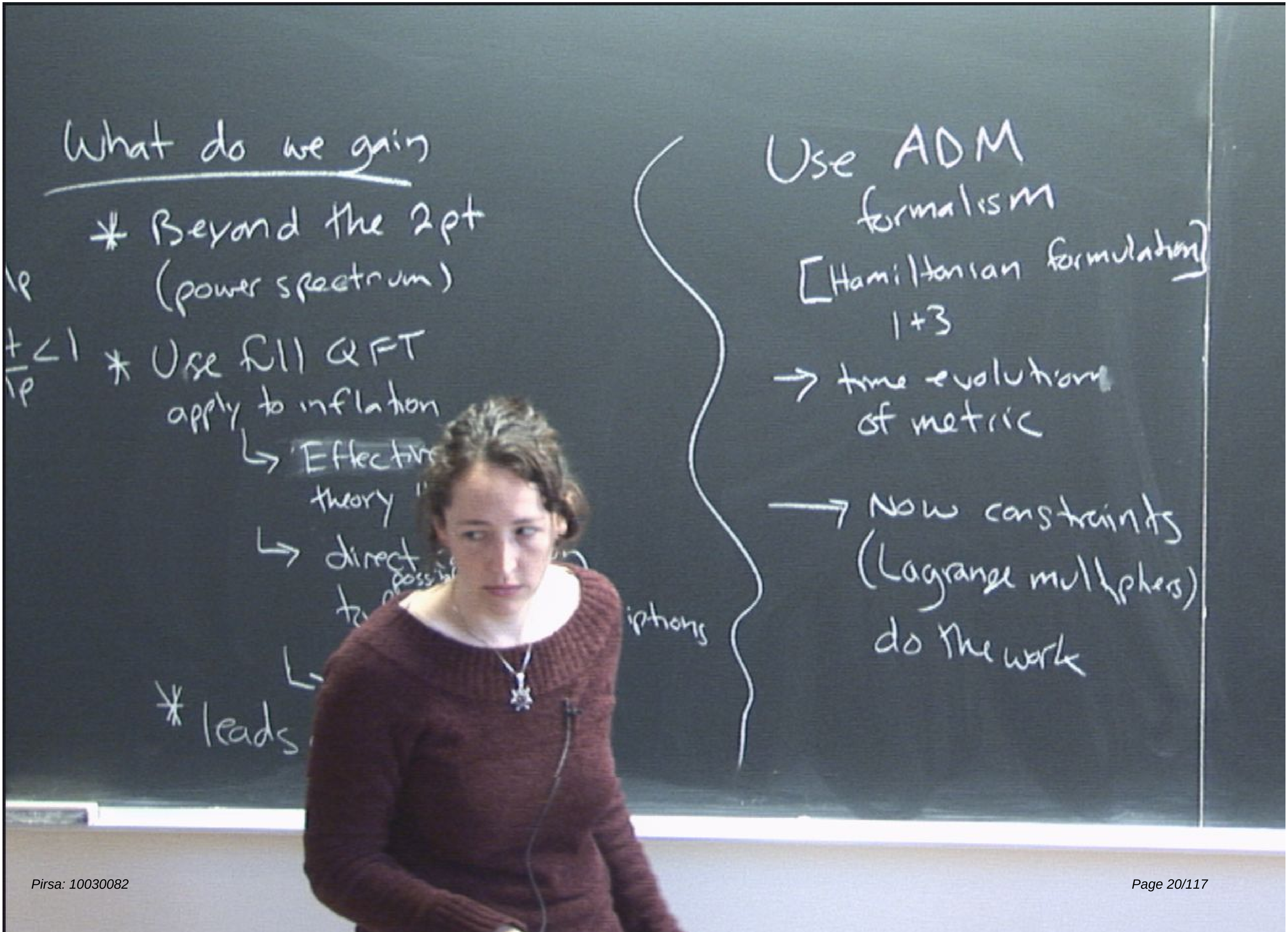
[Hamiltonian formulation]
1+3

→ time evolution
of metric



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↳

* leads

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1+3
→ time evolution
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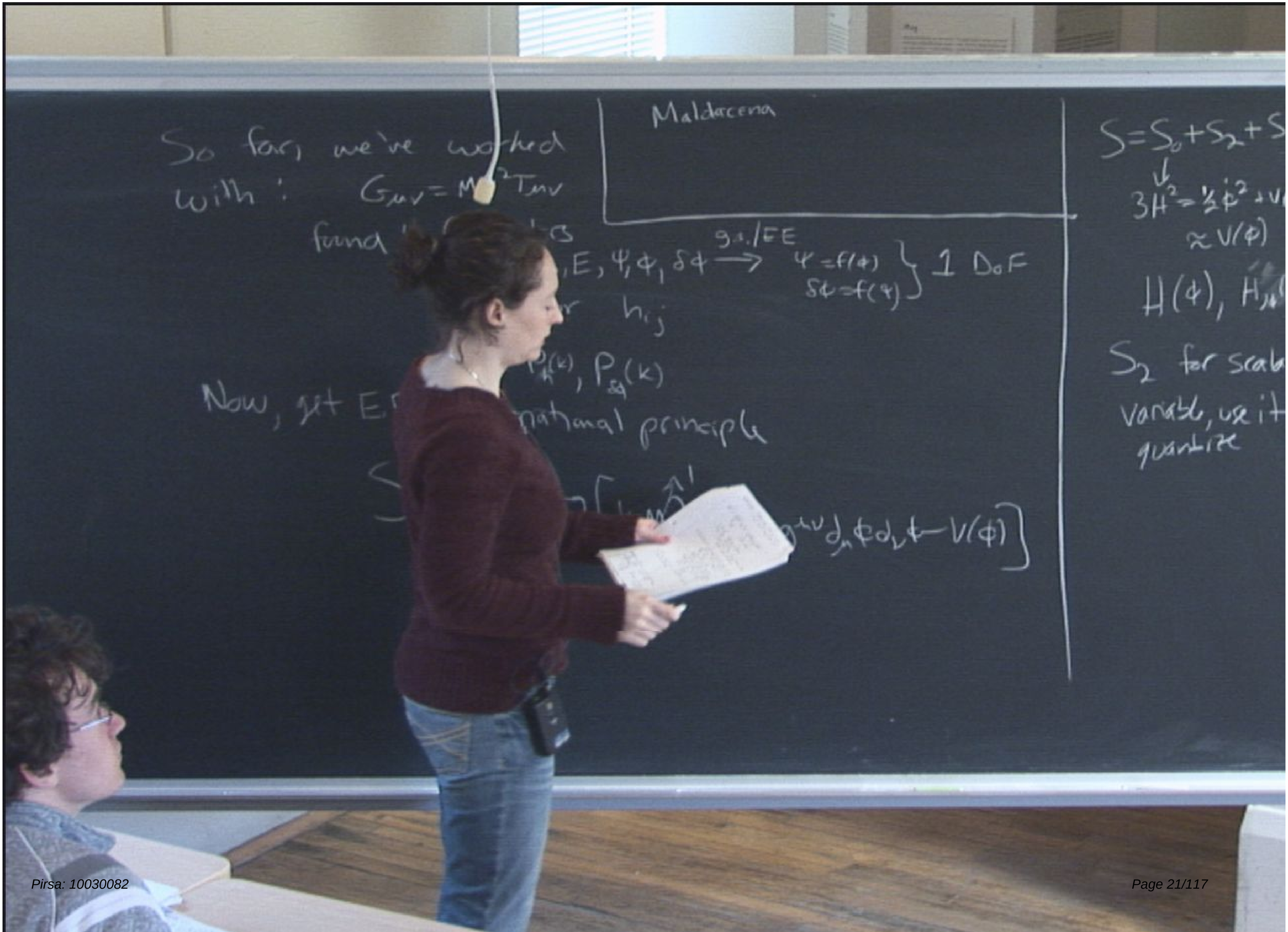
→ Now constraints
(Lagrange multiplers)
do the work



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Maldacena

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S_2 for scala

variable, use it

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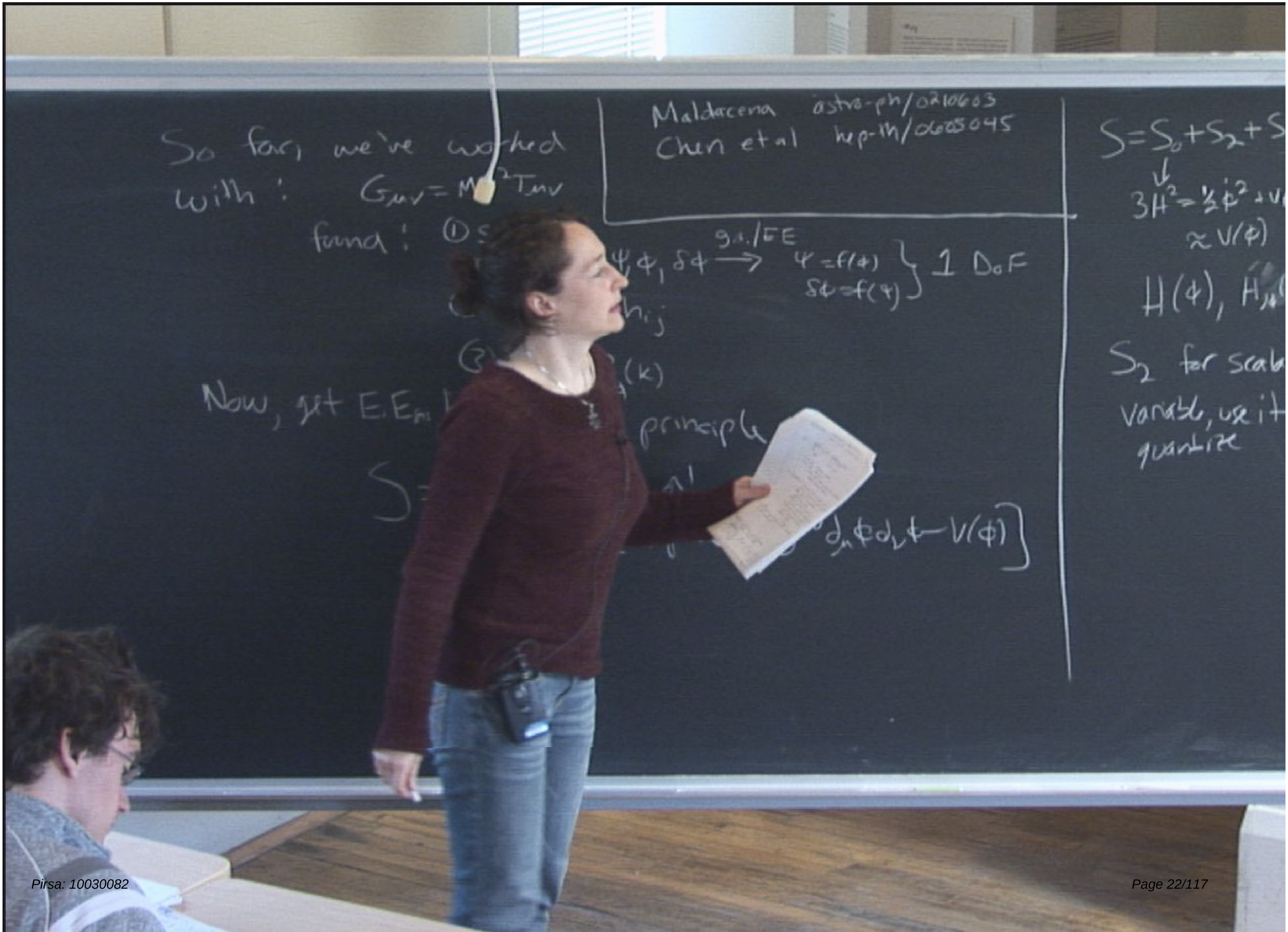
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h_{ij}
③ (k)
Now, get E.E.
principle
S =
$\partial_\mu \phi \partial_\nu \phi - V(\phi)$]
Maldacena astro-ph/0210603
Chen et al hep-th/0605045
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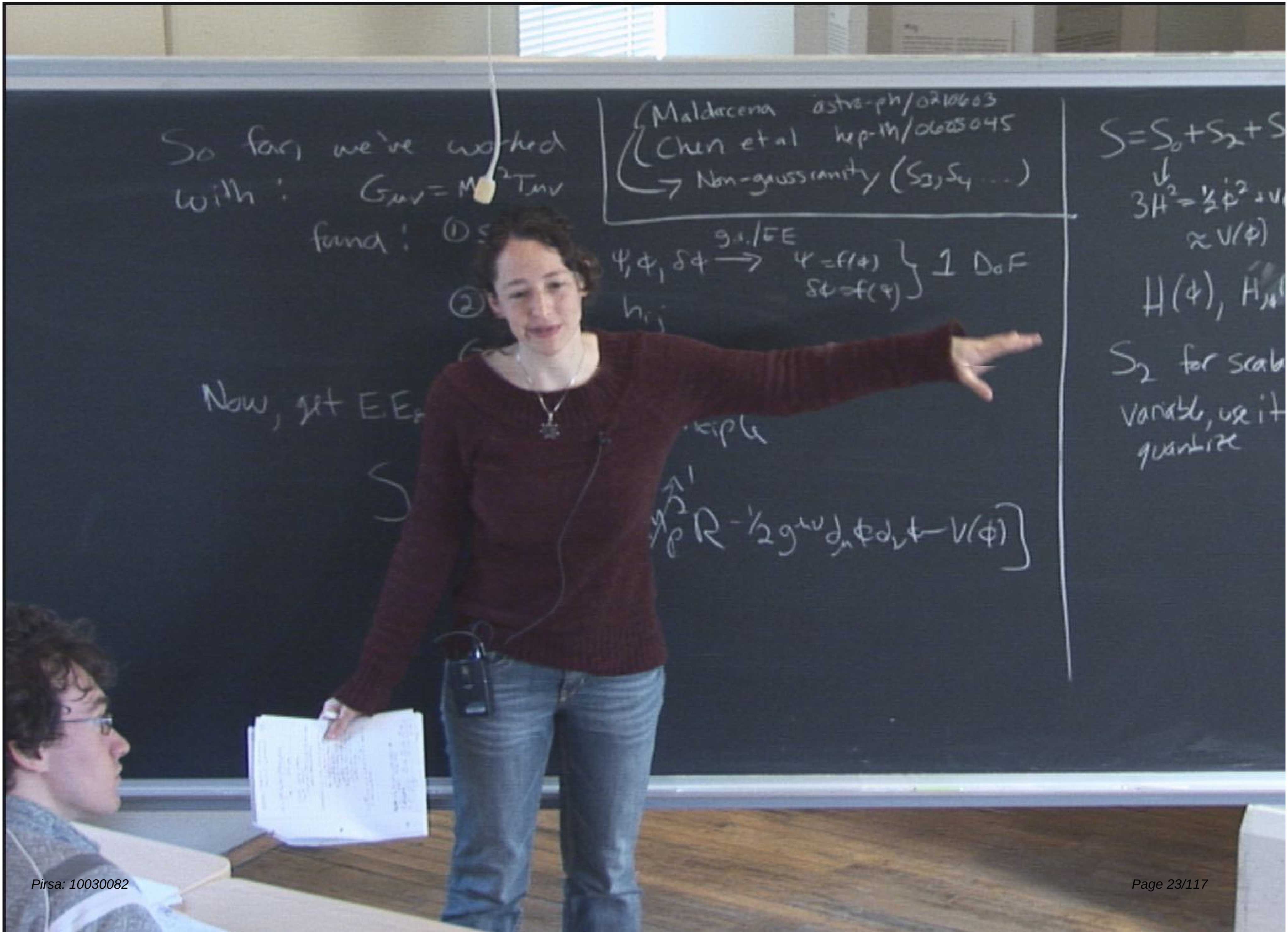
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②
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Maldacena astro-ph/0210603
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Non-gaussianity (S3, S4 ...)
g.i./EE
ψ, φ, δφ → ψ = f(φ)
δφ = f(φ) } 1 DoF
hij
ciple
S
½ Mp² R − ½ gμν ∂μφ ∂νφ − V(φ)]
S = S0 + S2 + S
3H² = ½φ̇² + V
≈ V(φ)
H(φ), H,φ
S2 for scala
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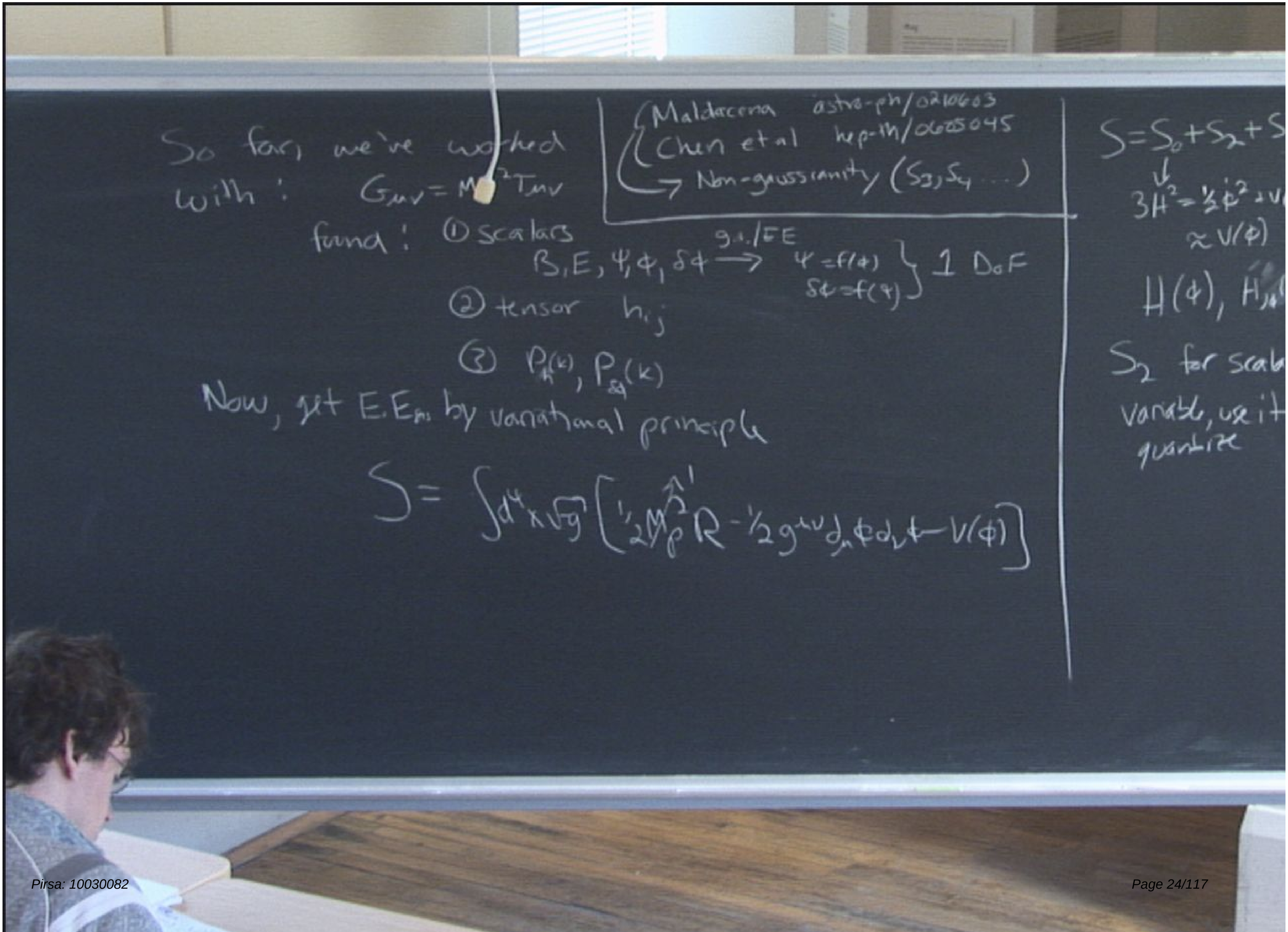

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→ Non-gaussianity ($S_3, S_4 \ldots$)

Now, get E.Eqs by variational principle

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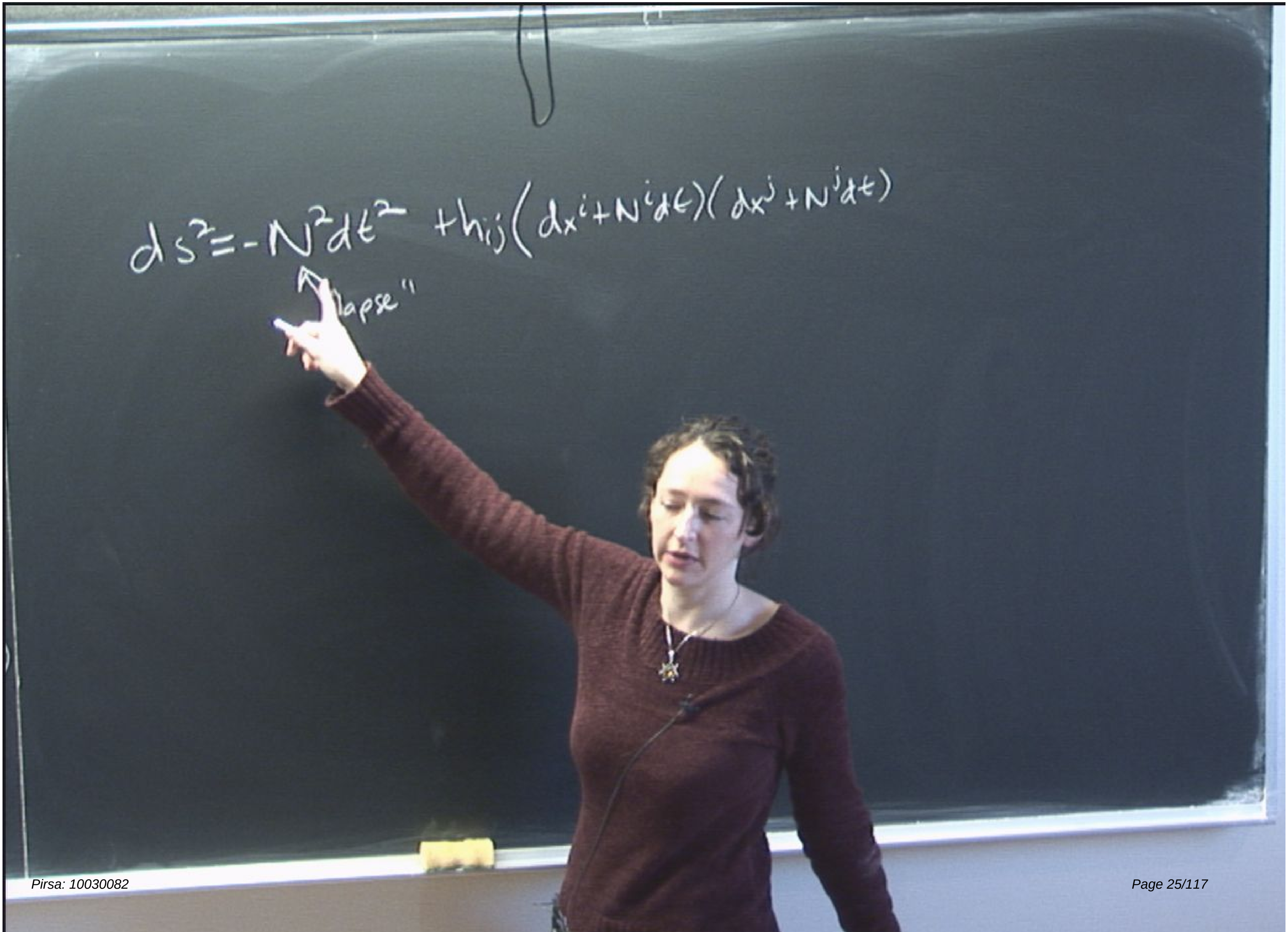
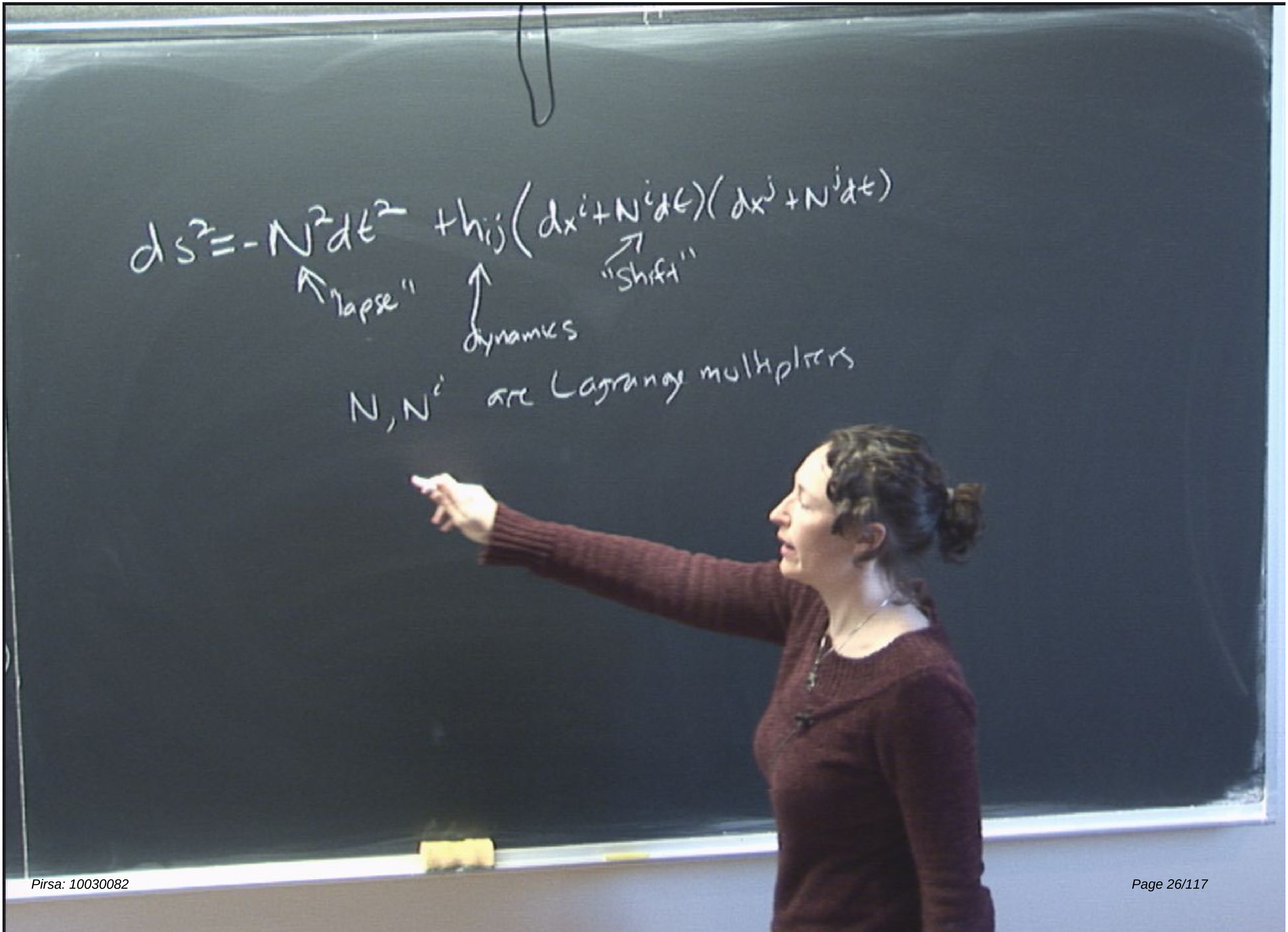


$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

"lapse"



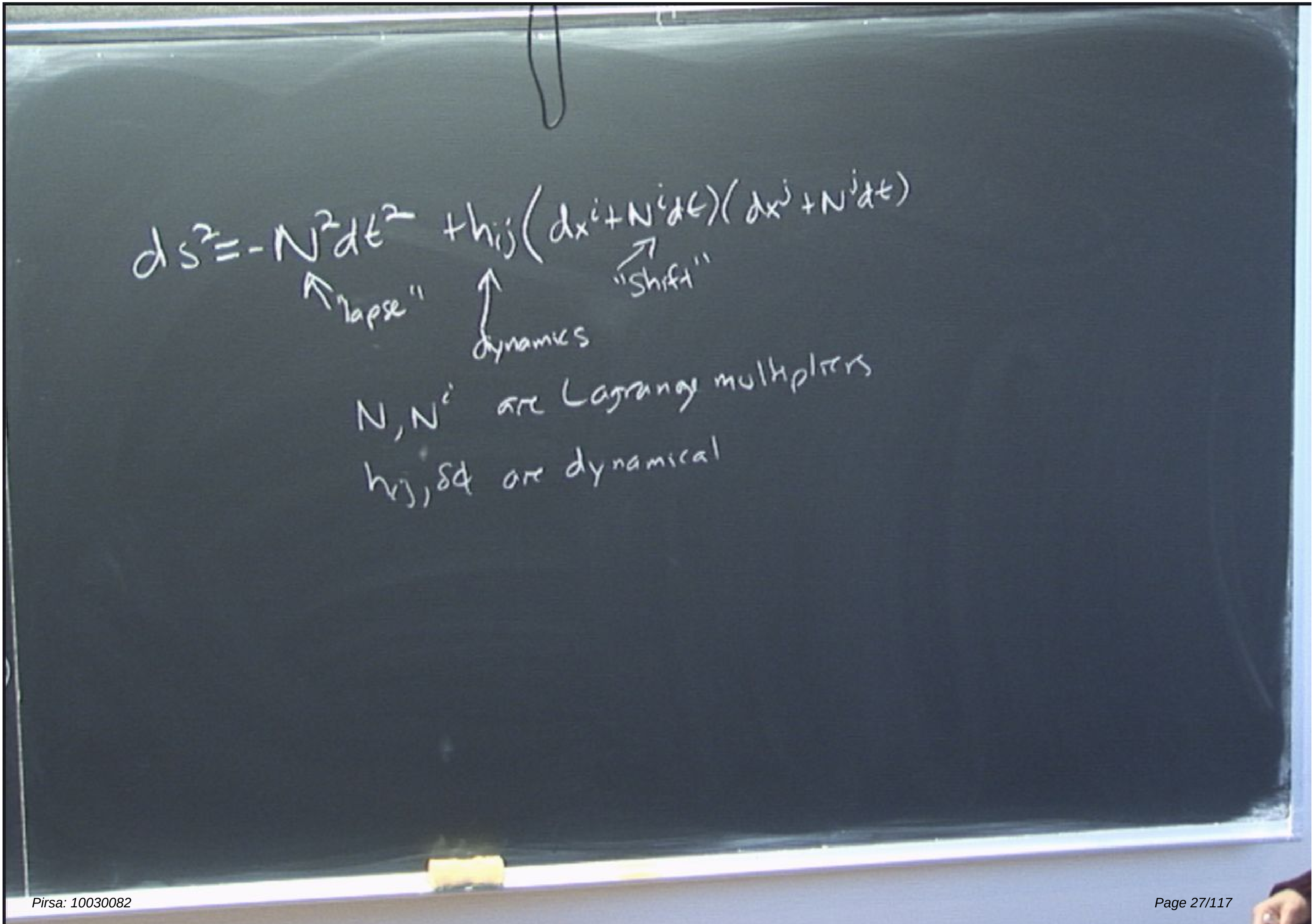
$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

"lapse"

dynamics

"shift"

N, N^i are Lagrange multipliers



$$ds^2 = -\underset{\uparrow \text{"lapse"}}{N^2} dt^2 + \underset{\underset{\text{dynamics}}{\uparrow}}{h_{ij}}(dx^i + \underset{\nearrow \text{"shift"}}{N^i} dt)(dx^j + N^j dt)$$

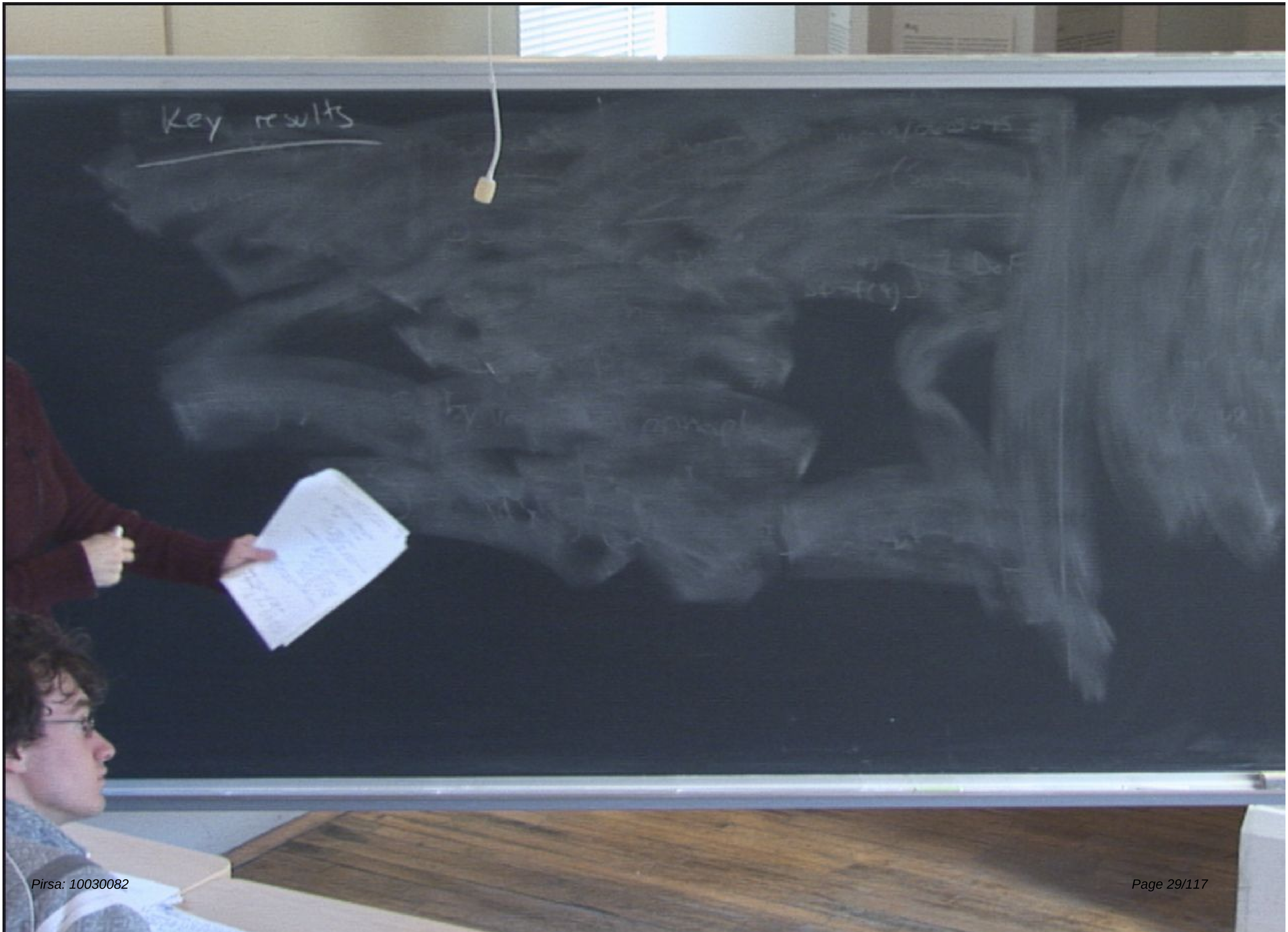
N, N^i are Lagrange multipliers

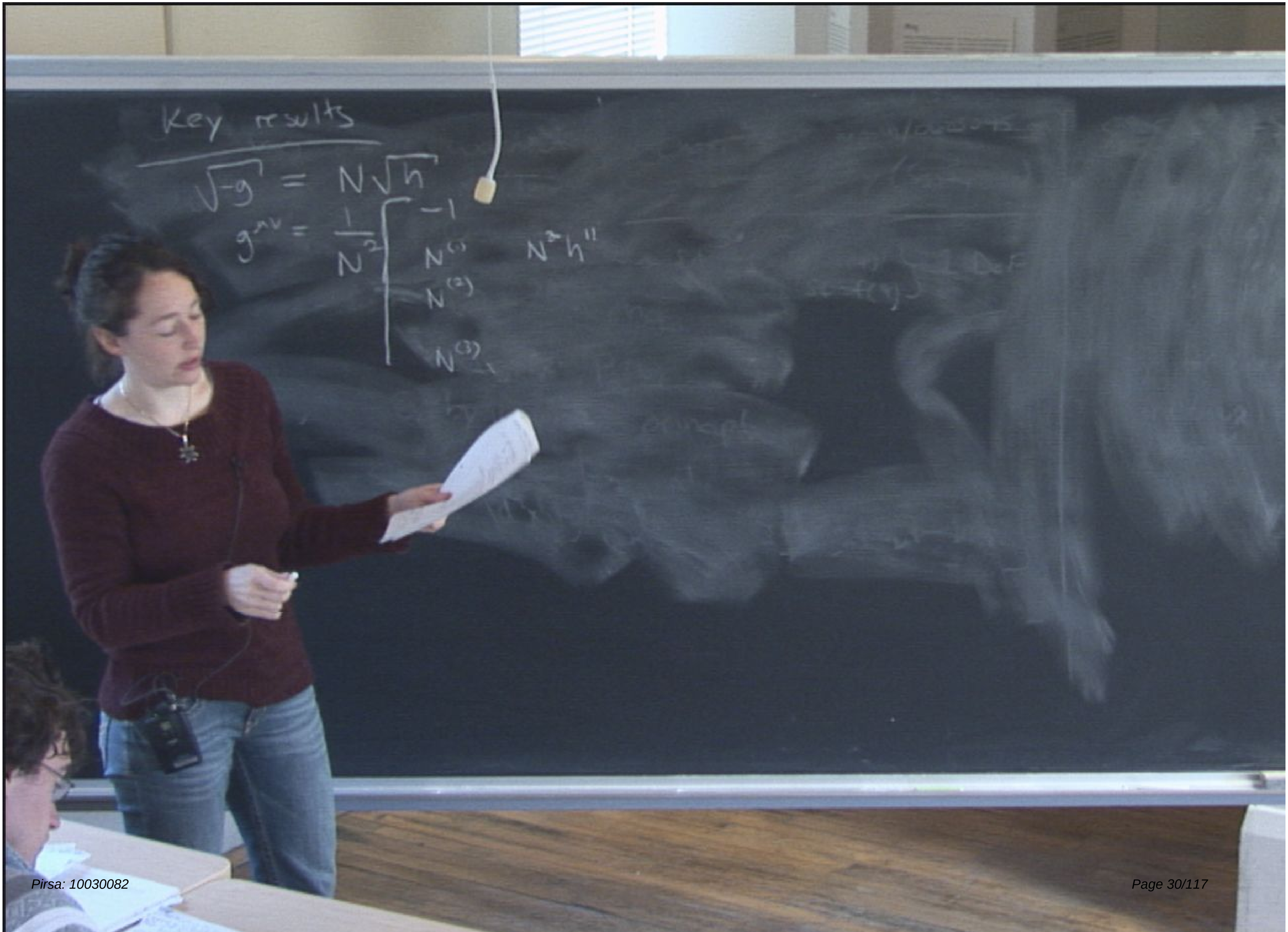
$h_{ij}, \delta\phi$ are dynamical

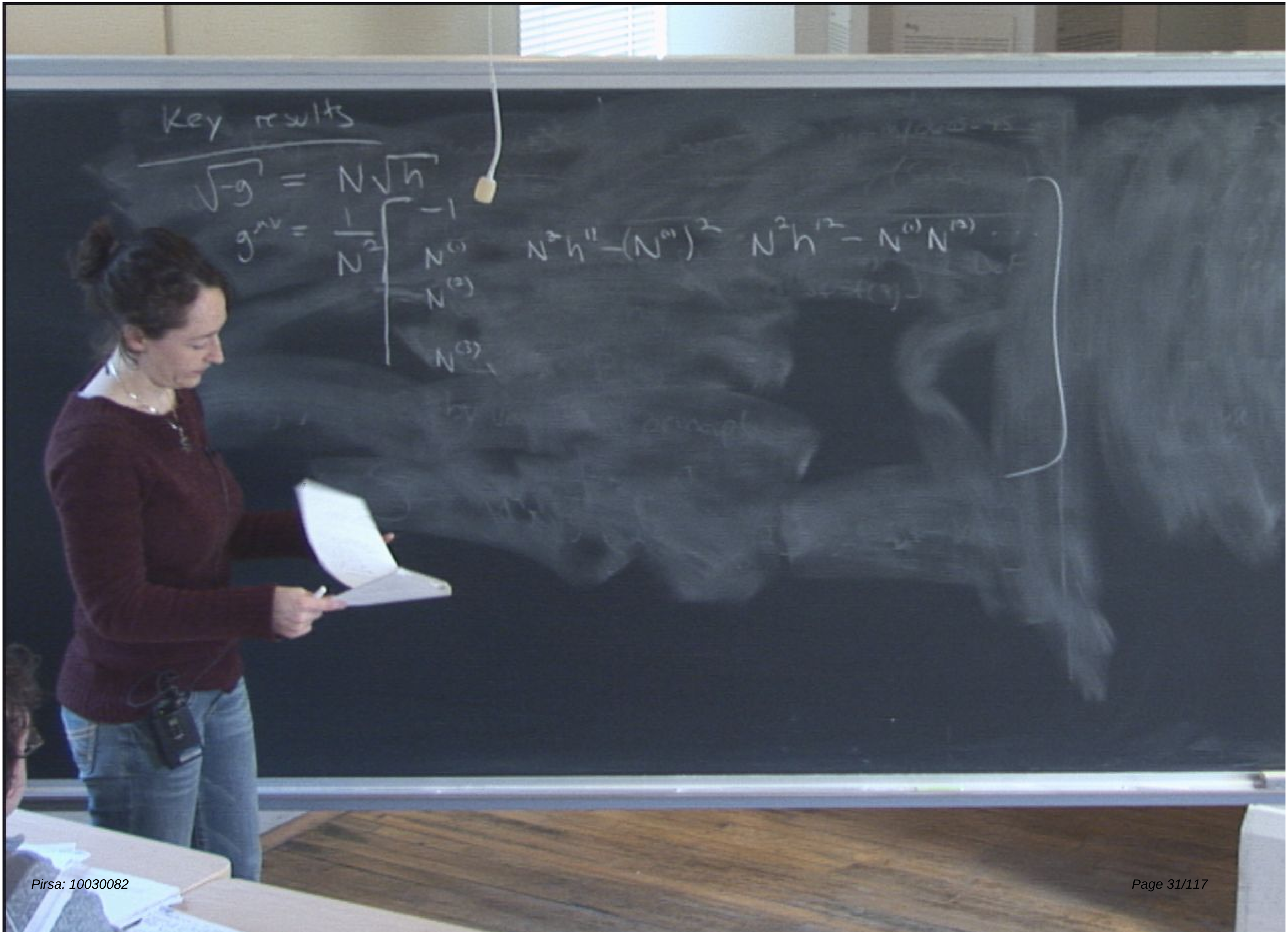
$$ds^2 = -\underset{\uparrow \text{"lapse"}}{N^2} dt^2 + \underset{\underset{\text{dynamics}}{\uparrow}}{h_{ij}}\left(dx^i + \underset{\nearrow\text{"shift"}}{N^i} dt\right)\left(dx^j + N^j dt\right)$$

N, N^i are Lagrange multipliers $\Big\}$ still have gauge issues

$h_{ij}, \delta\phi$ are dynamical


Key results

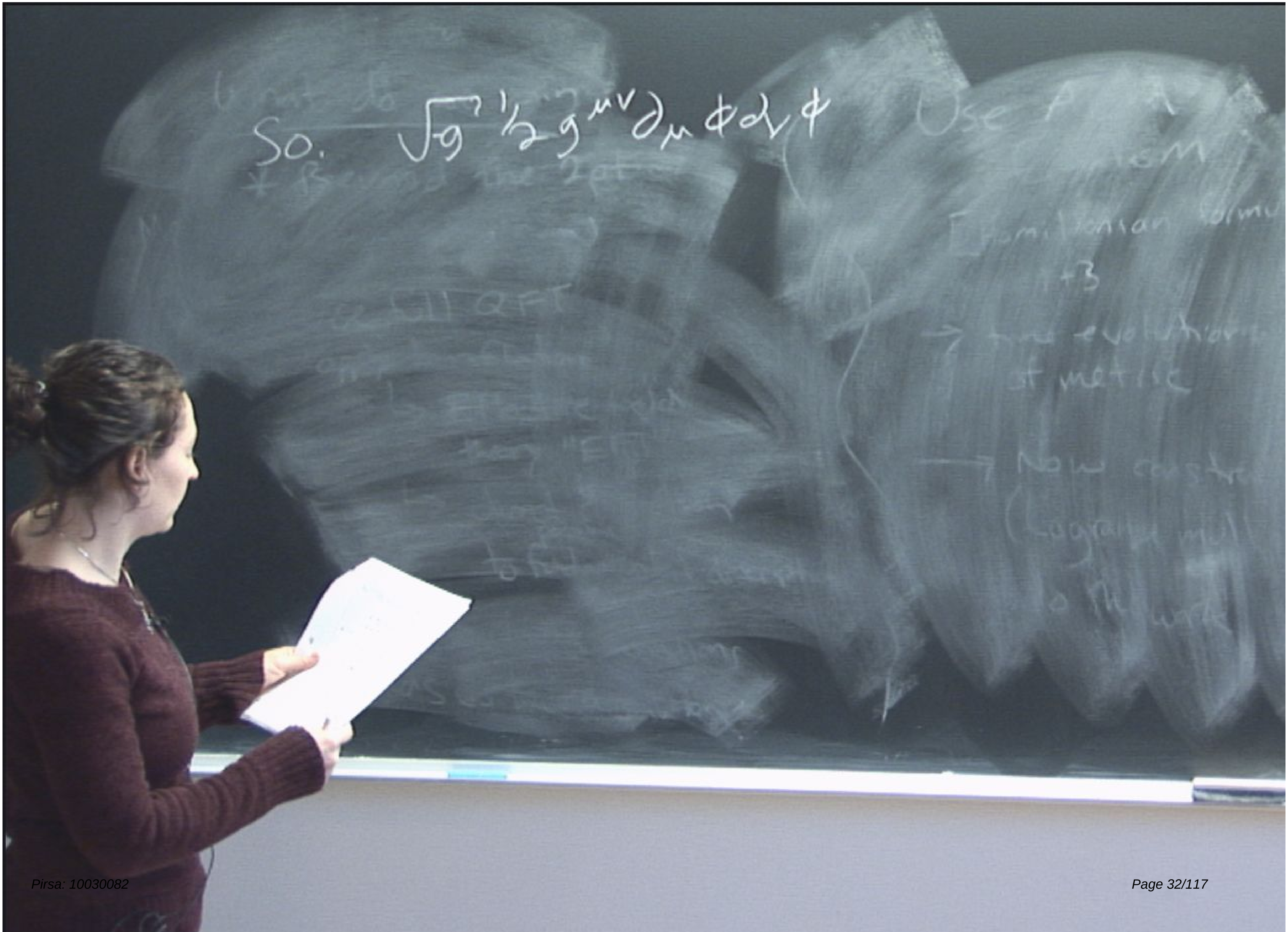

Key results
$\sqrt{-g} = N\sqrt{h}$
$g^{\mu\nu} = \frac{1}{N^2}$
-1
$N^{(1)}$
$N^2 h^{11}$
$N^{(2)}$
$N^{(3)}$



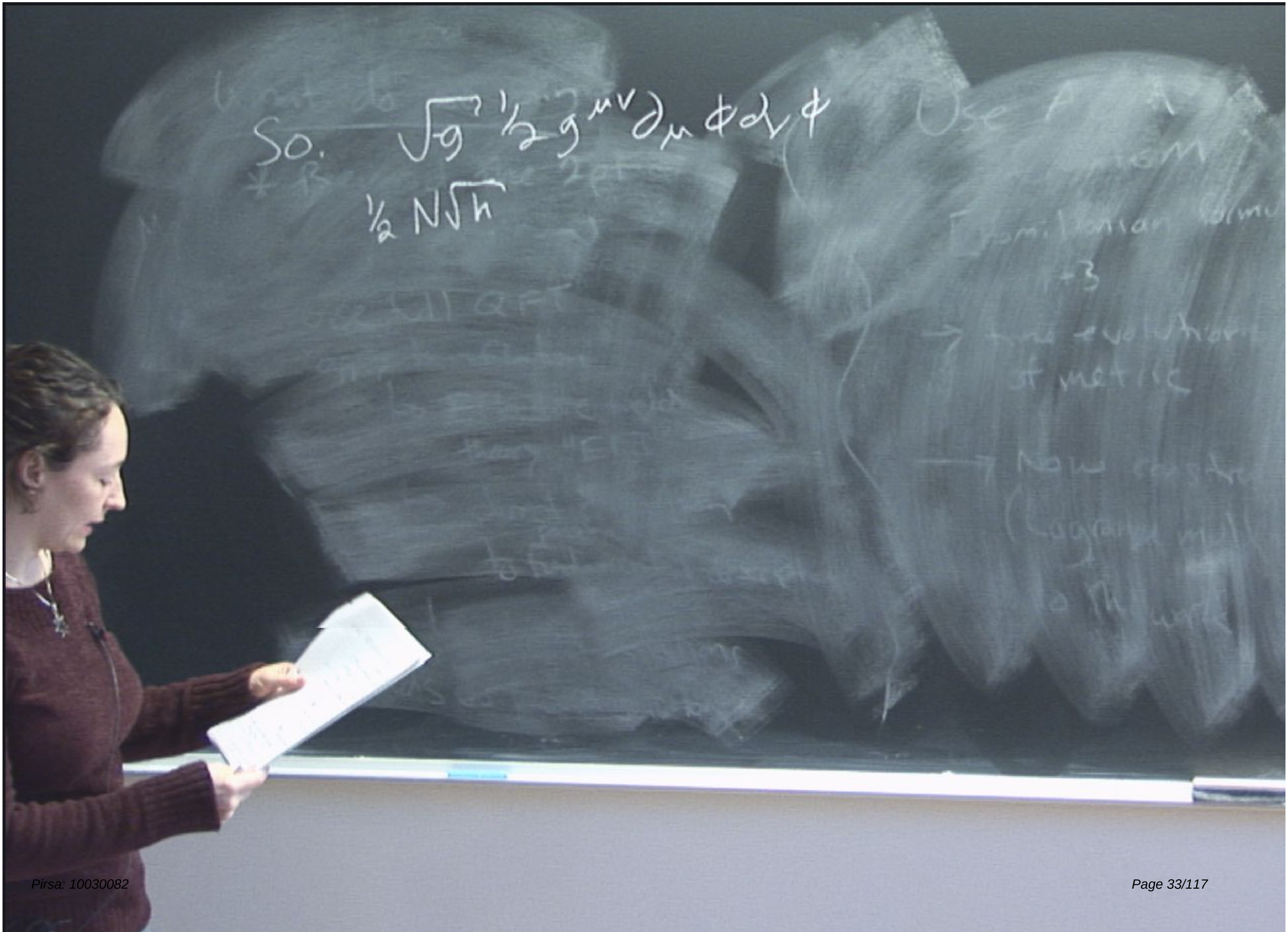
Key results

$$\sqrt{-g} = N\sqrt{h}$$

$$g^{\mu\nu} = \frac{1}{N^2}\begin{bmatrix} -1 & & & \\ N^{(1)} & N^2h^{11}-(N^{(1)})^2 & N^2h^{12}-N^{(1)}N^{(2)} & \cdots \\ N^{(2)} & & & \\ N^{(3)} & & & \end{bmatrix}$$



So. $\sqrt{g}\,\frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$



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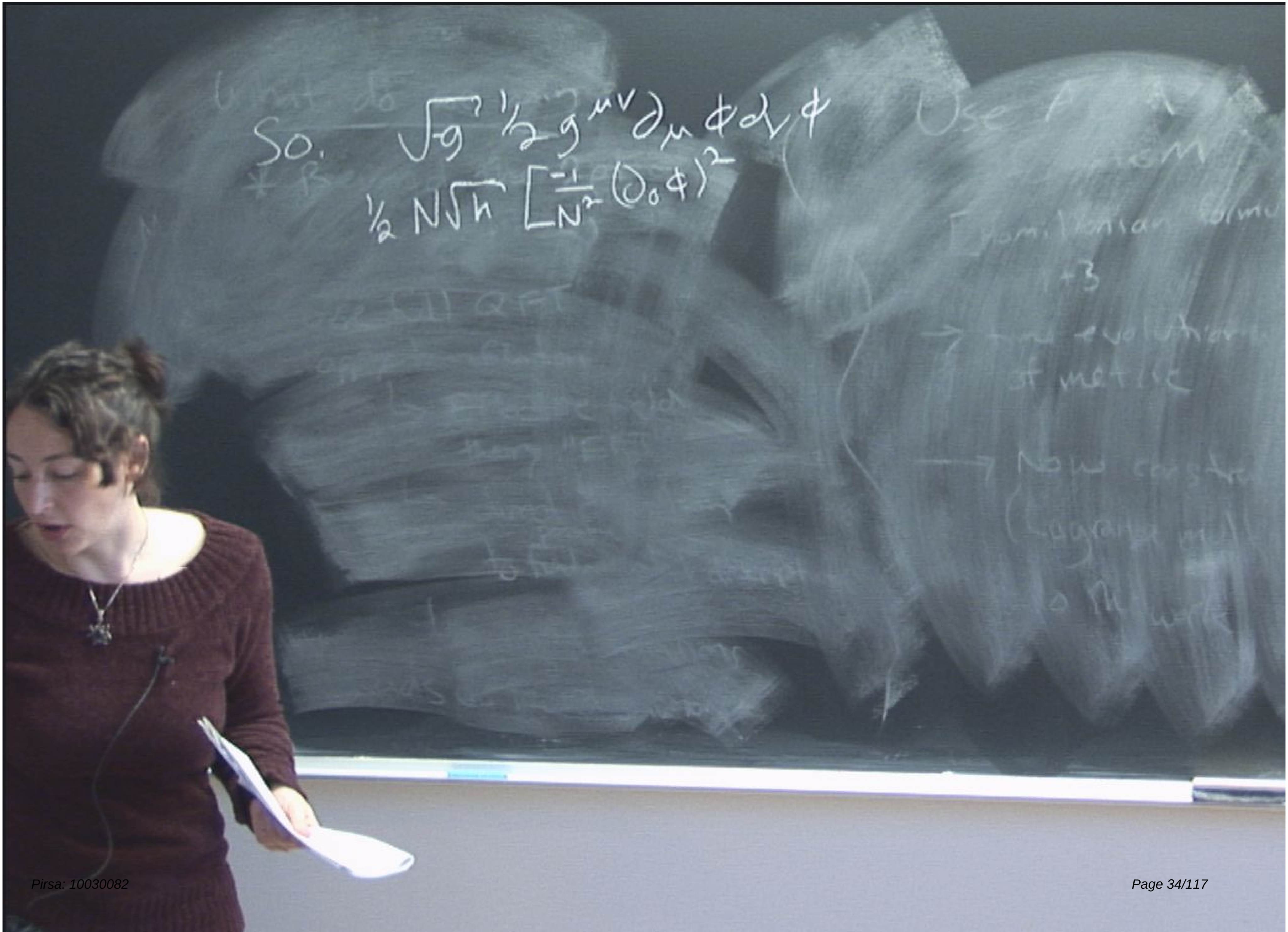
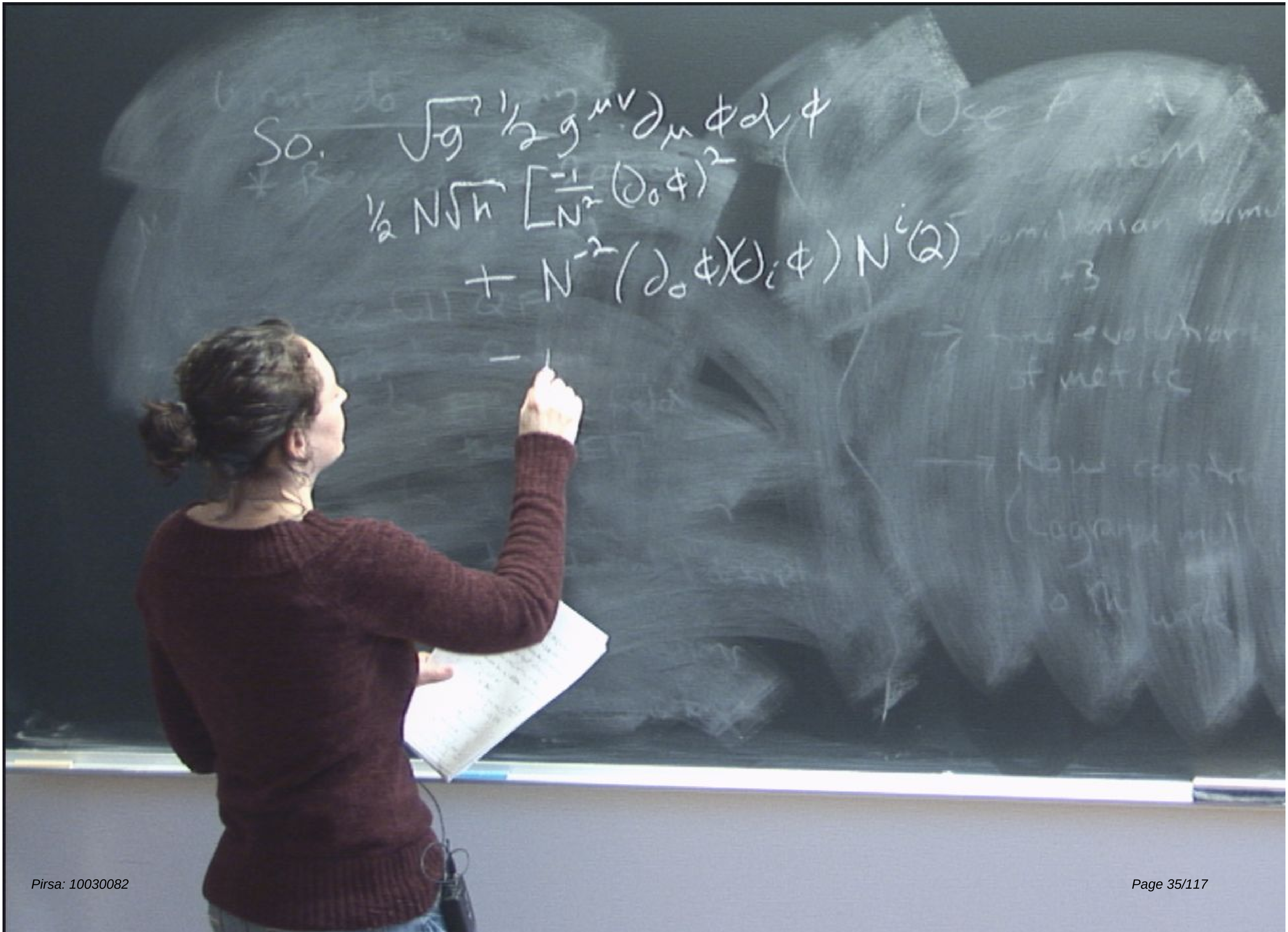
$\frac{1}{2} N\sqrt{h}$



So. $\sqrt{g}\,\frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$

$\frac{1}{2}\,N\sqrt{h}\,\Big[\frac{-1}{N^2}(\partial_0\phi)^2$



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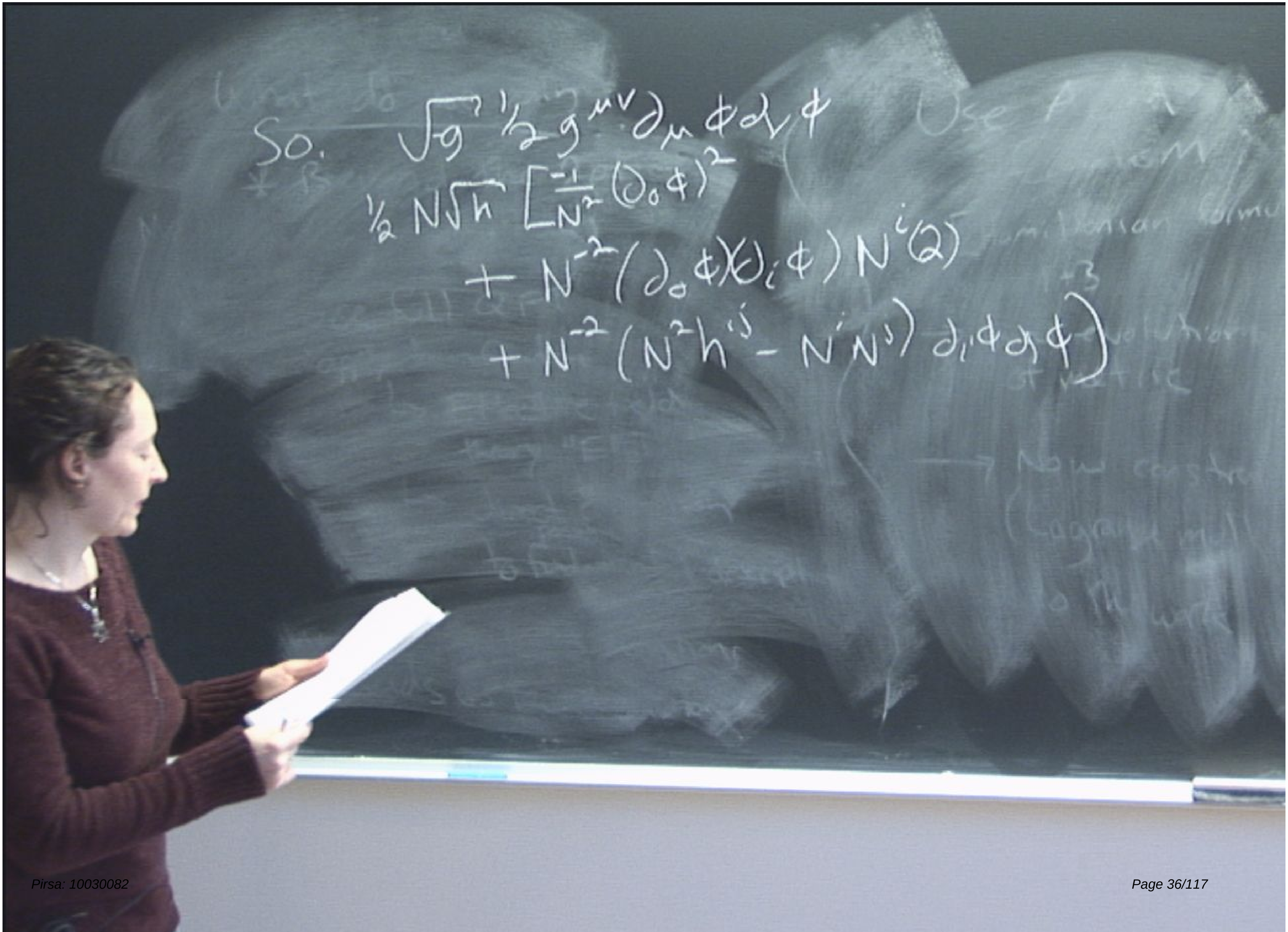
$\frac{1}{2} N\sqrt{h}\,\Big[\frac{-1}{N^2}(\partial_0\phi)^2$

$+ N^{-2}(\partial_0\phi)(\partial_i\phi)N^i(2)$

$-$

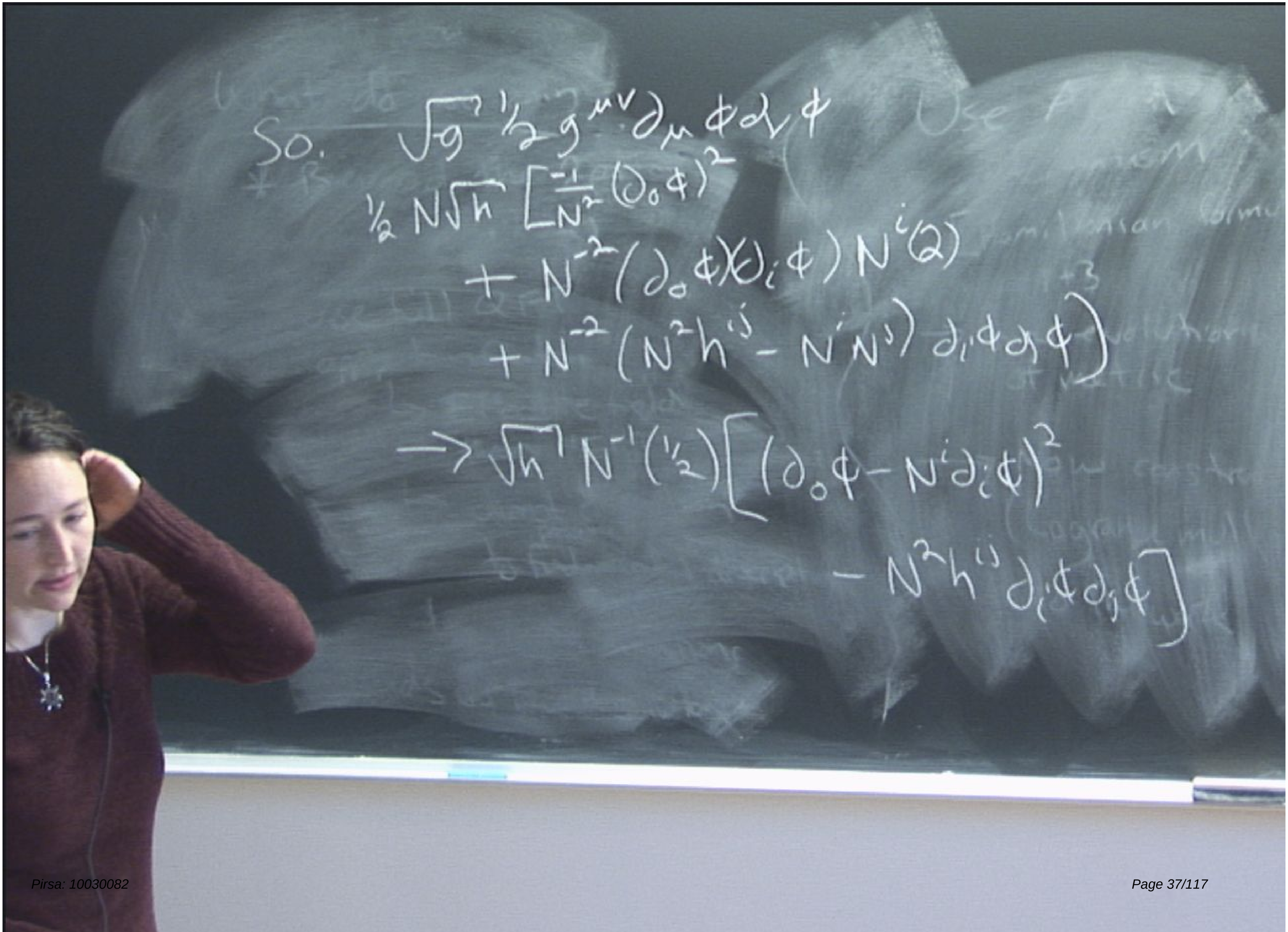


So. $\sqrt{g}\,\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

$$\frac{1}{2}N\sqrt{h}\Big[\frac{-1}{N^2}(\partial_0\phi)^2$$

$$+N^{-2}(\partial_0\phi)(\partial_i\phi)N^i(2)$$

$$+N^{-2}(N^2h^{ij}-N^iN^j)\,\partial_i\phi\,\partial_j\phi\Big]$$



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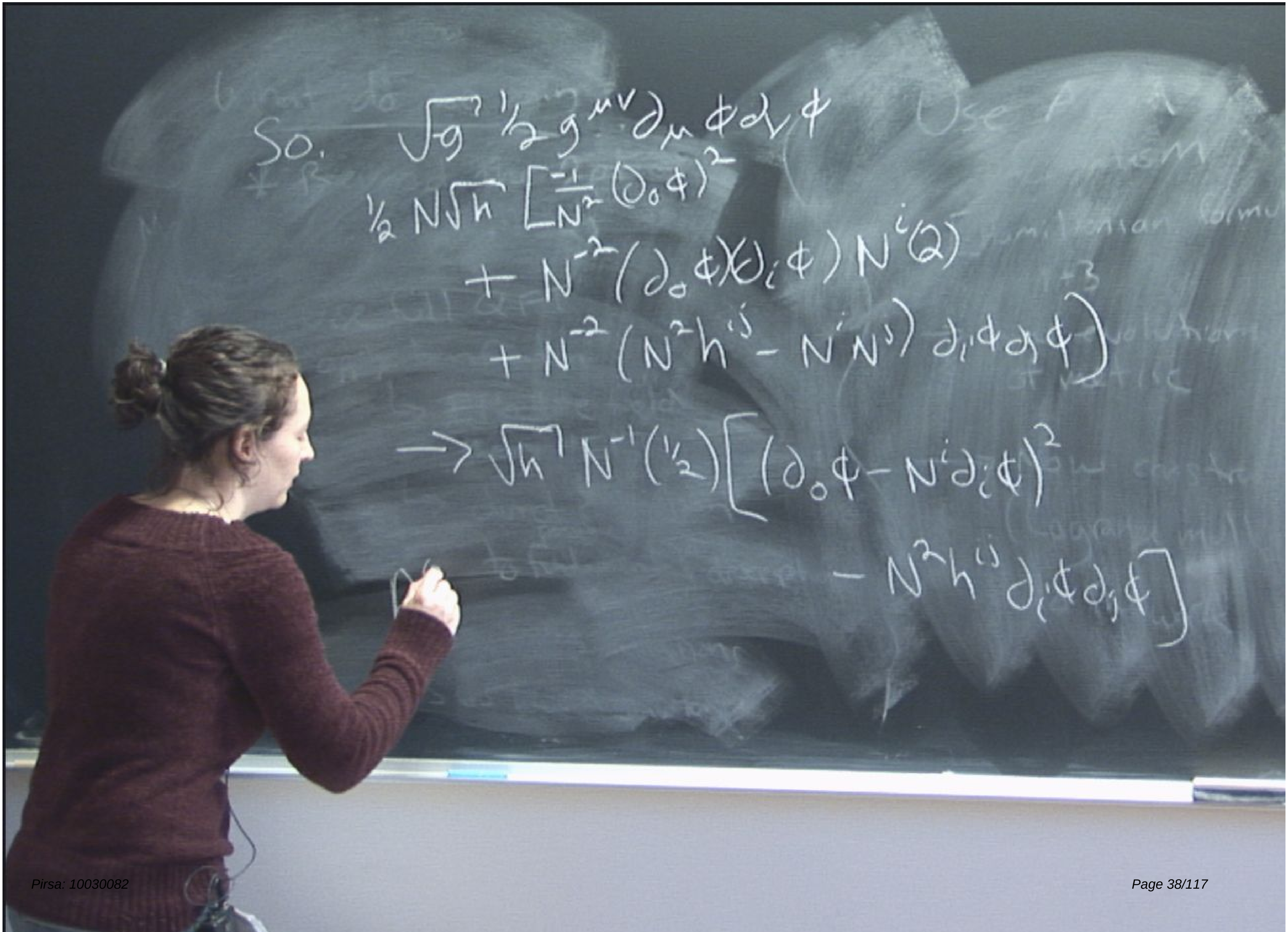
$\frac{1}{2}N\sqrt{h}\Big[\frac{-1}{N^2}(\partial_0\phi)^2$

$+N^{-2}(\partial_0\phi)(\partial_i\phi)N^i(2)$

$+N^{-2}(N^2h^{ij}-N^iN^j)\partial_i\phi\,\partial_j\phi\Big)$

$\rightarrow \sqrt{h}\,N^{-1}(\tfrac{1}{2})\Big[(\partial_0\phi-N^i\partial_i\phi)^2$

$-N^2h^{ij}\partial_i\phi\,\partial_j\phi\Big]$



So. $\sqrt{g}\,\frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$

$$\frac{1}{2}N\sqrt{h}\Big[\frac{-1}{N^2}(\partial_0\phi)^2$$

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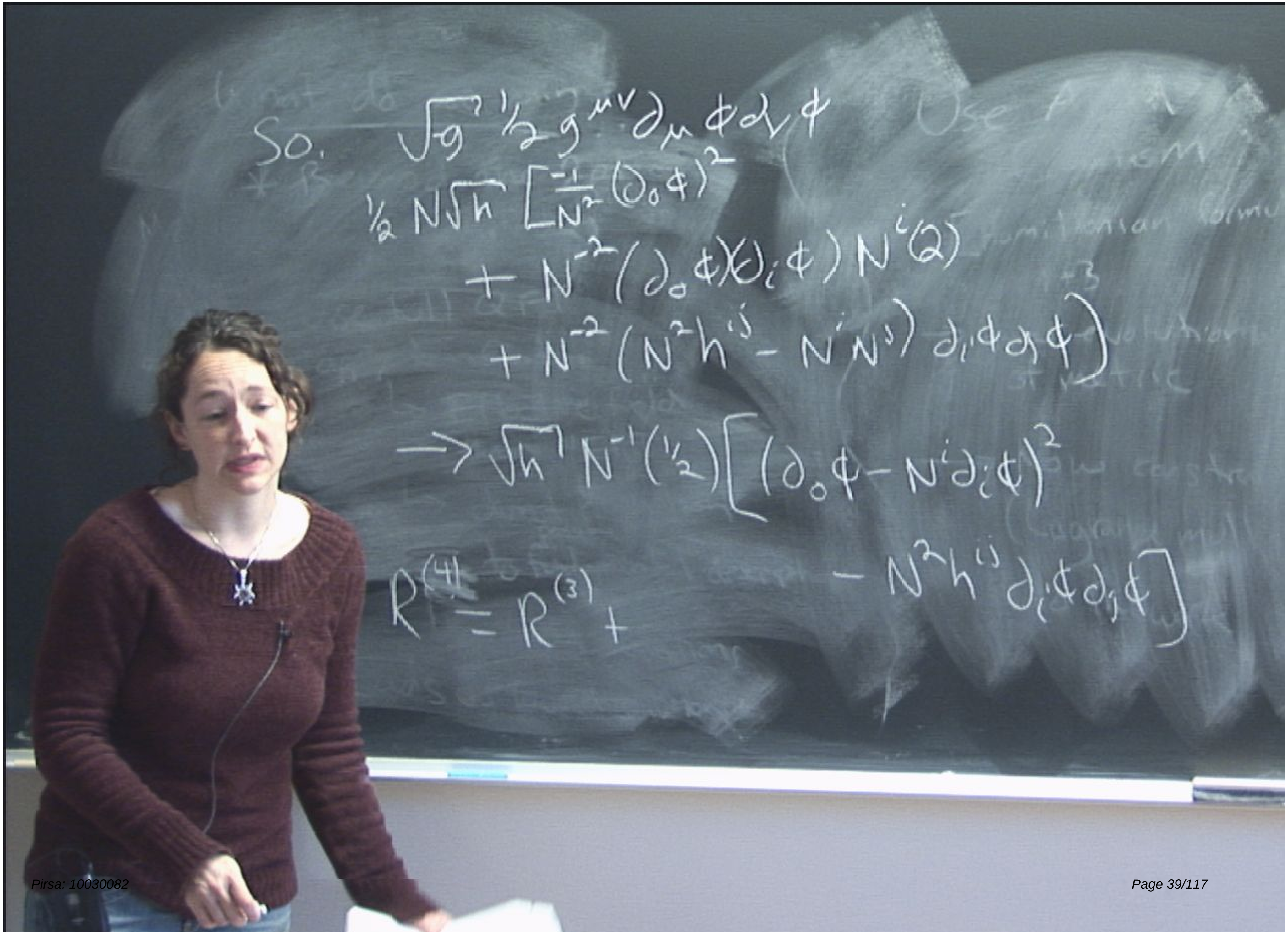

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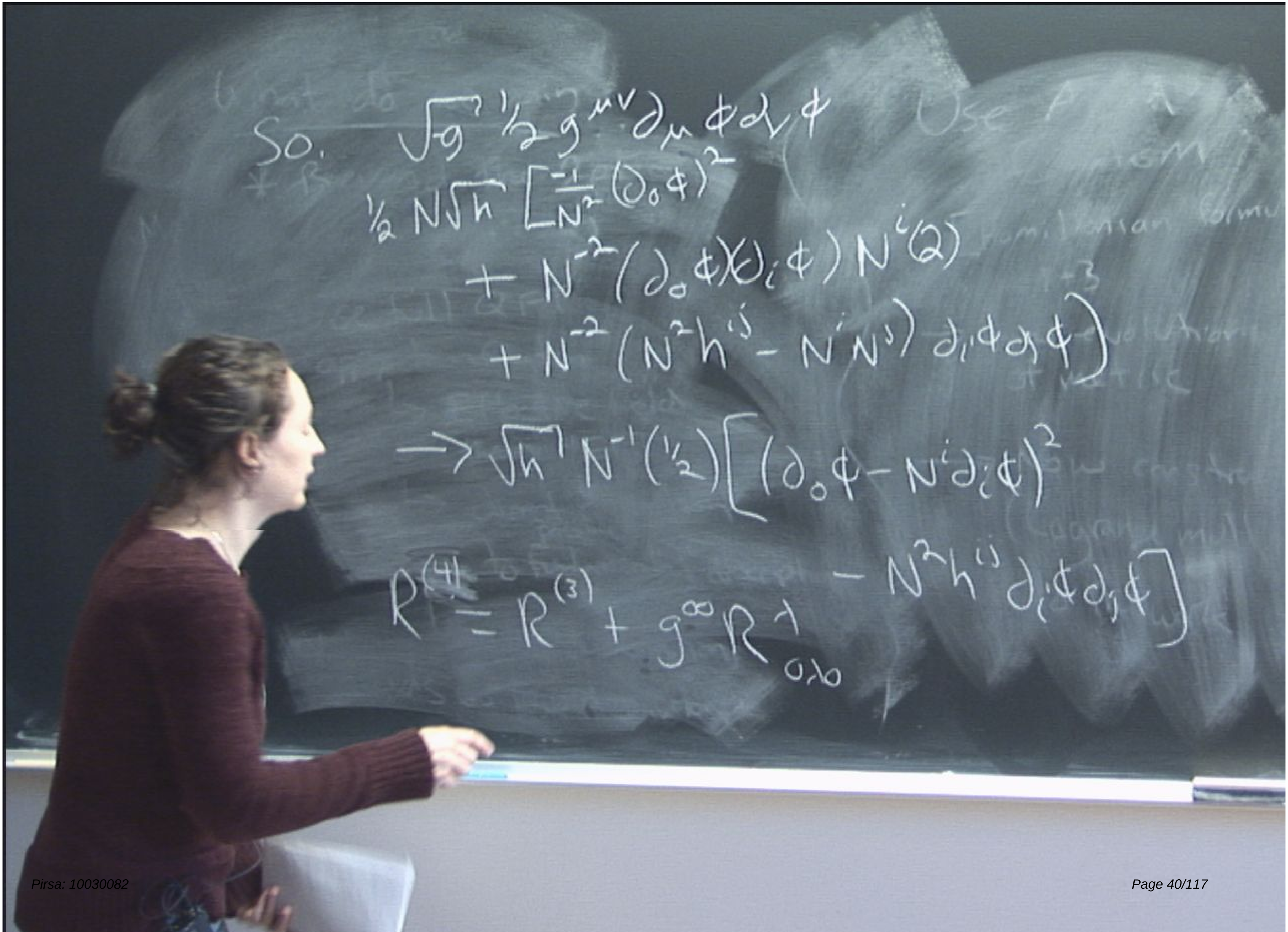
$$R^{(4)} = R^{(3)} +$$



So. $\sqrt{g}\,\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$

$$\frac{1}{2} N\sqrt{h}\Big[\frac{-1}{N^2}(\partial_0\phi)^2$$

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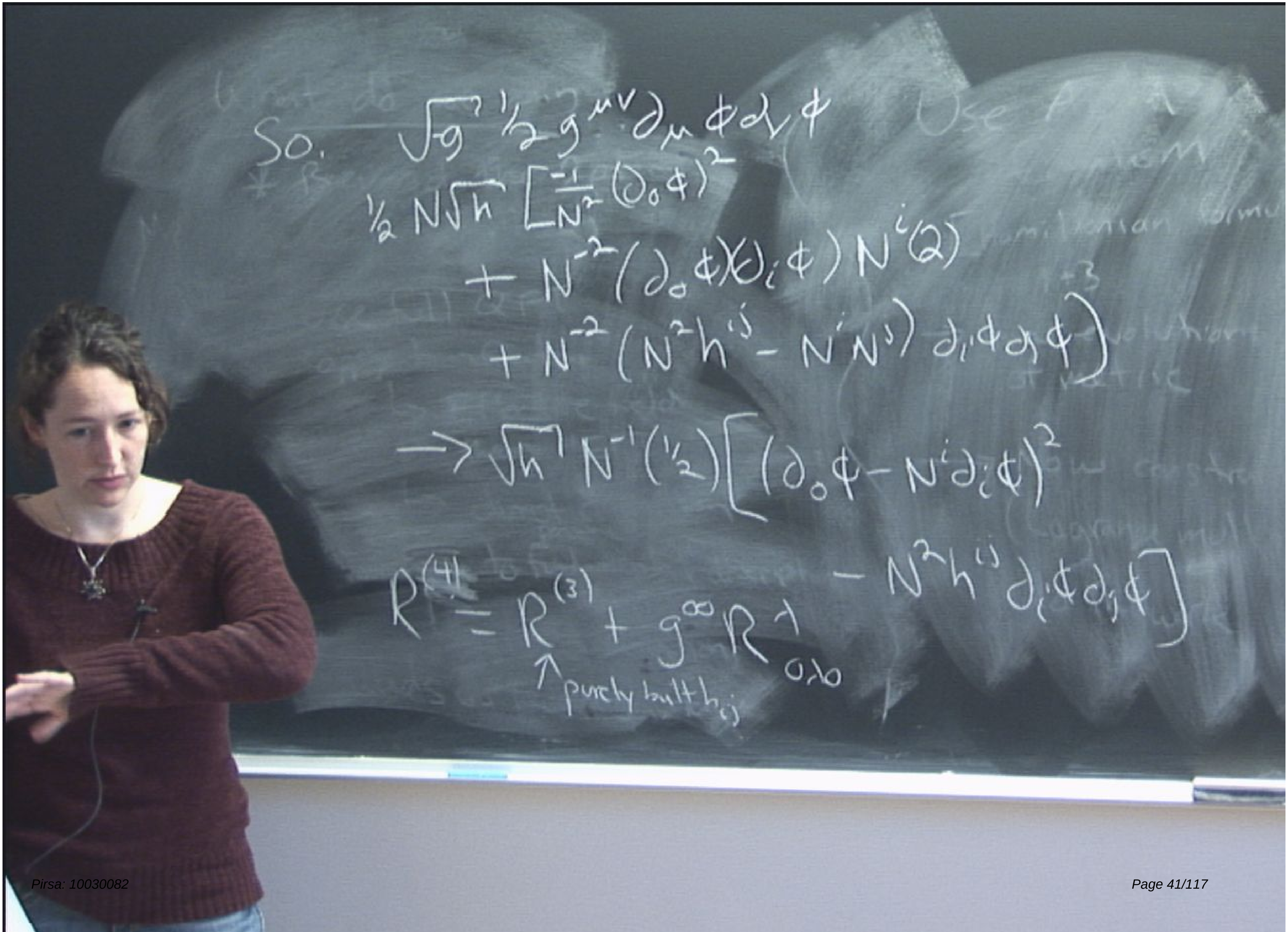
$$+ N^{-2}(N^2h^{ij} - N^iN^j)\,\partial_i\phi\,\partial_j\phi\Big]$$

$$\rightarrow \sqrt{h}\,N^{-1}\left(\tfrac{1}{2}\right)\Big[(\partial_0\phi - N^i\partial_i\phi)^2 - N^2h^{ij}\,\partial_i\phi\,\partial_j\phi\Big]$$

$$R^{(4)} = R^{(3)} + g^{00}R^{\lambda}{}_{0\lambda0}$$



So. $\sqrt{g}\,\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$

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$$\rightarrow\sqrt{h}\,N^{-1}\left(\tfrac{1}{2}\right)\Big[(\partial_0\phi-N^i\partial_i\phi)^2-N^2h^{ij}\partial_i\phi\,\partial_j\phi\Big]$$

$$R^{(4)}=\underset{\uparrow\text{ purely built } h_{ij}}{R^{(3)}}+g^{00}R^{\lambda}{}_{0\lambda 0}$$

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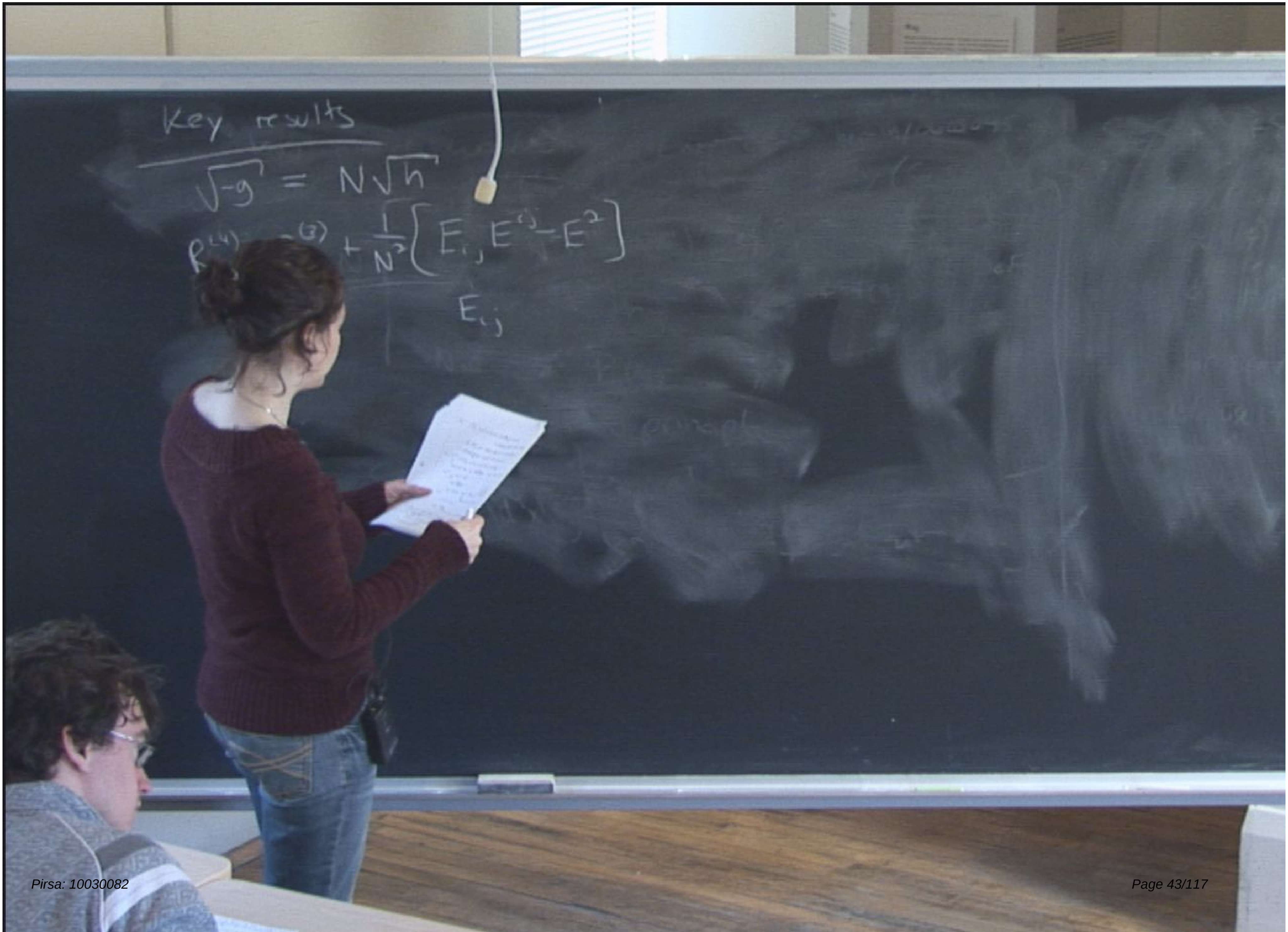
$$R^{(4)} = \underset{\uparrow\ \text{purely built } h_{ij}}{R^{(3)}} + \underbrace{g^{00}R^{\lambda}{}_{0\lambda0} + \dots}_{\rightarrow\ \text{extrinsic curvature}}$$

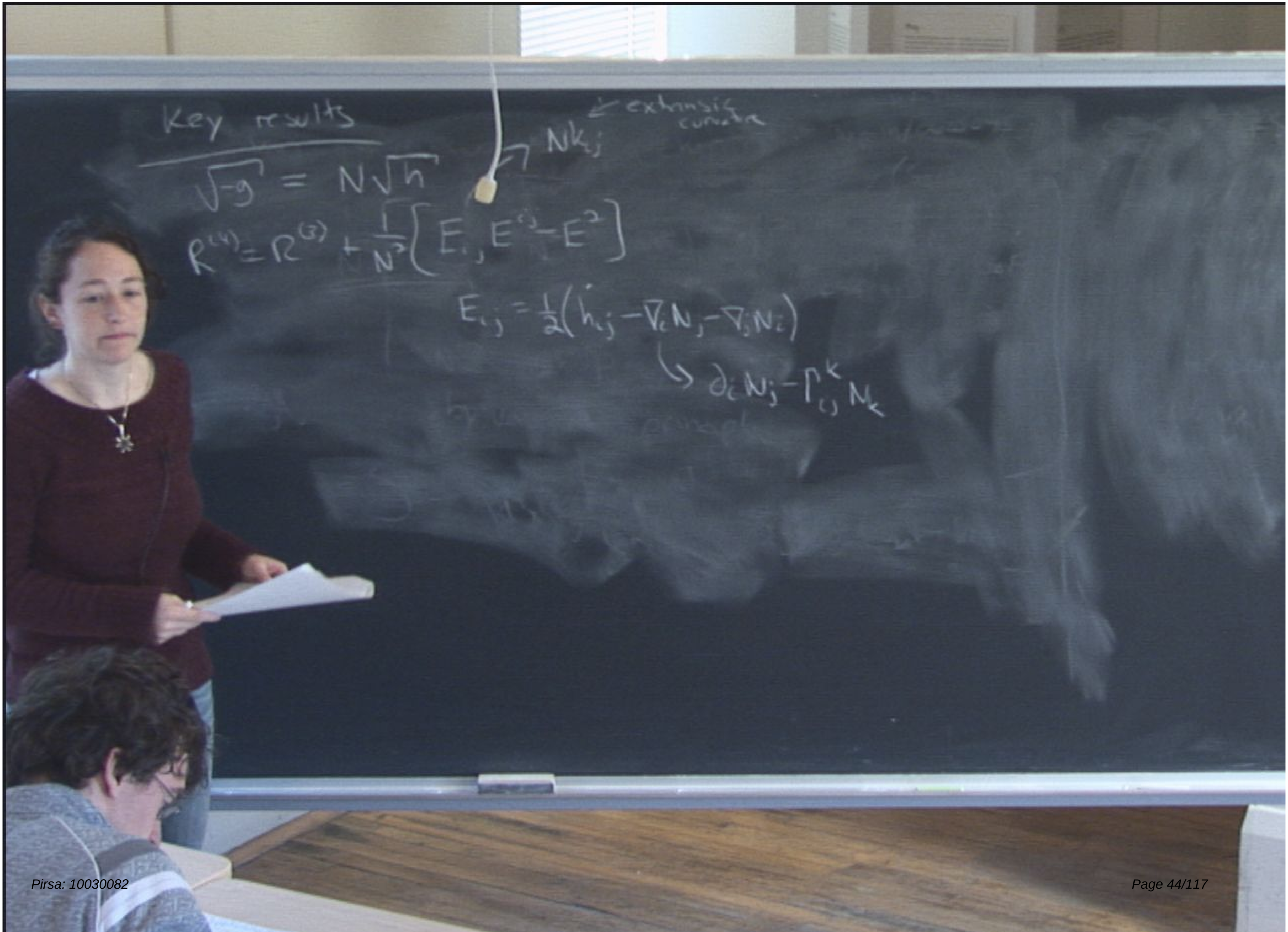


Key results

$$\sqrt{-g} = N\sqrt{h}$$

$$R^{(4)} \quad {}^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$$

$$E_{ij}$$



Key results

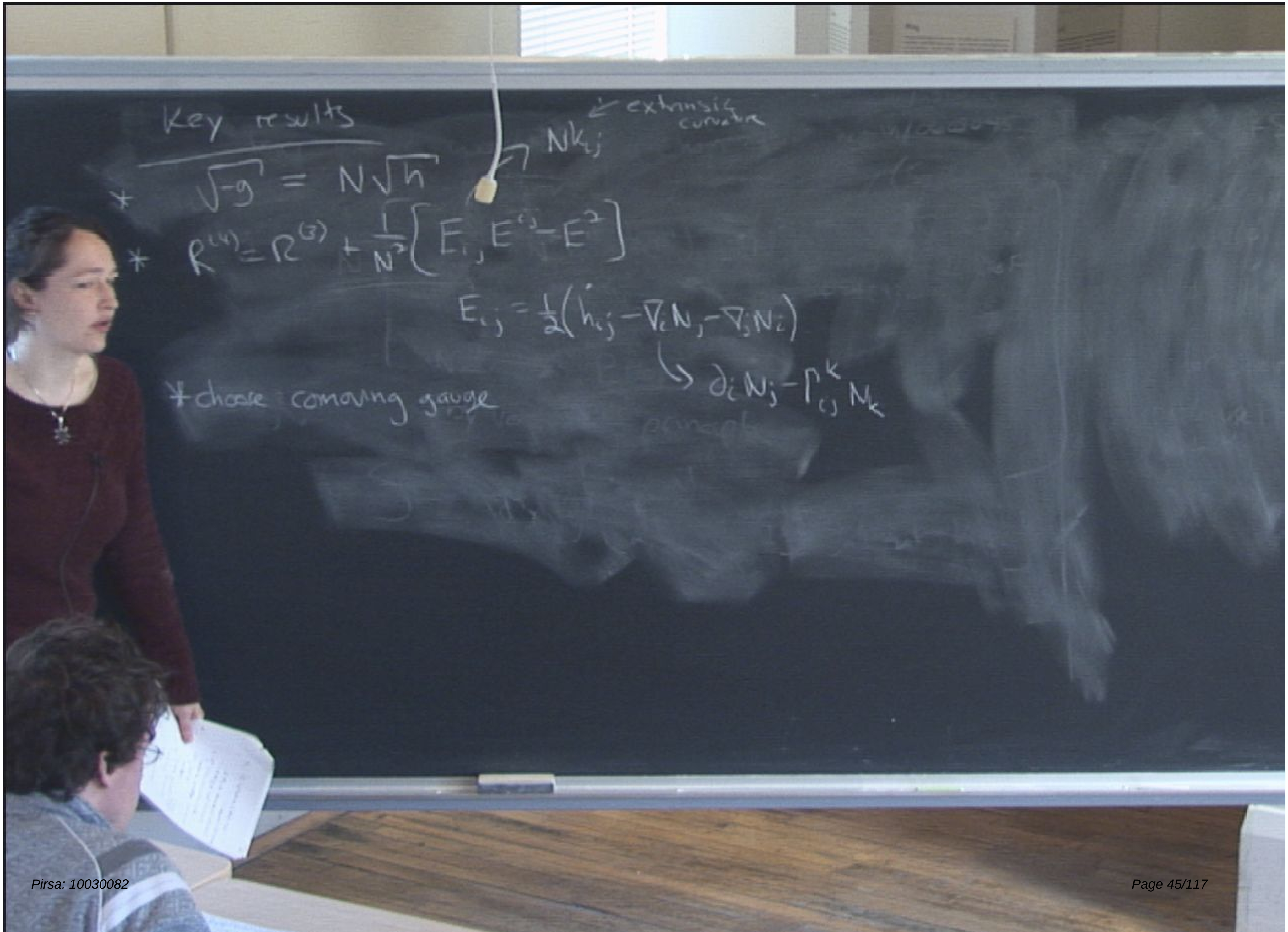
$\sqrt{-g} = N\sqrt{h}$

$R^{(4)} = R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$\nearrow Nk_{ij}$ ← extrinsic curvature

$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

$\hookrightarrow \partial_i N_j - \Gamma^k_{ij} N_k$



Key results

Nk_{ij} ← extrinsic curvature

* $\sqrt{-g} = N\sqrt{h}$
* $R^{(4)} = R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

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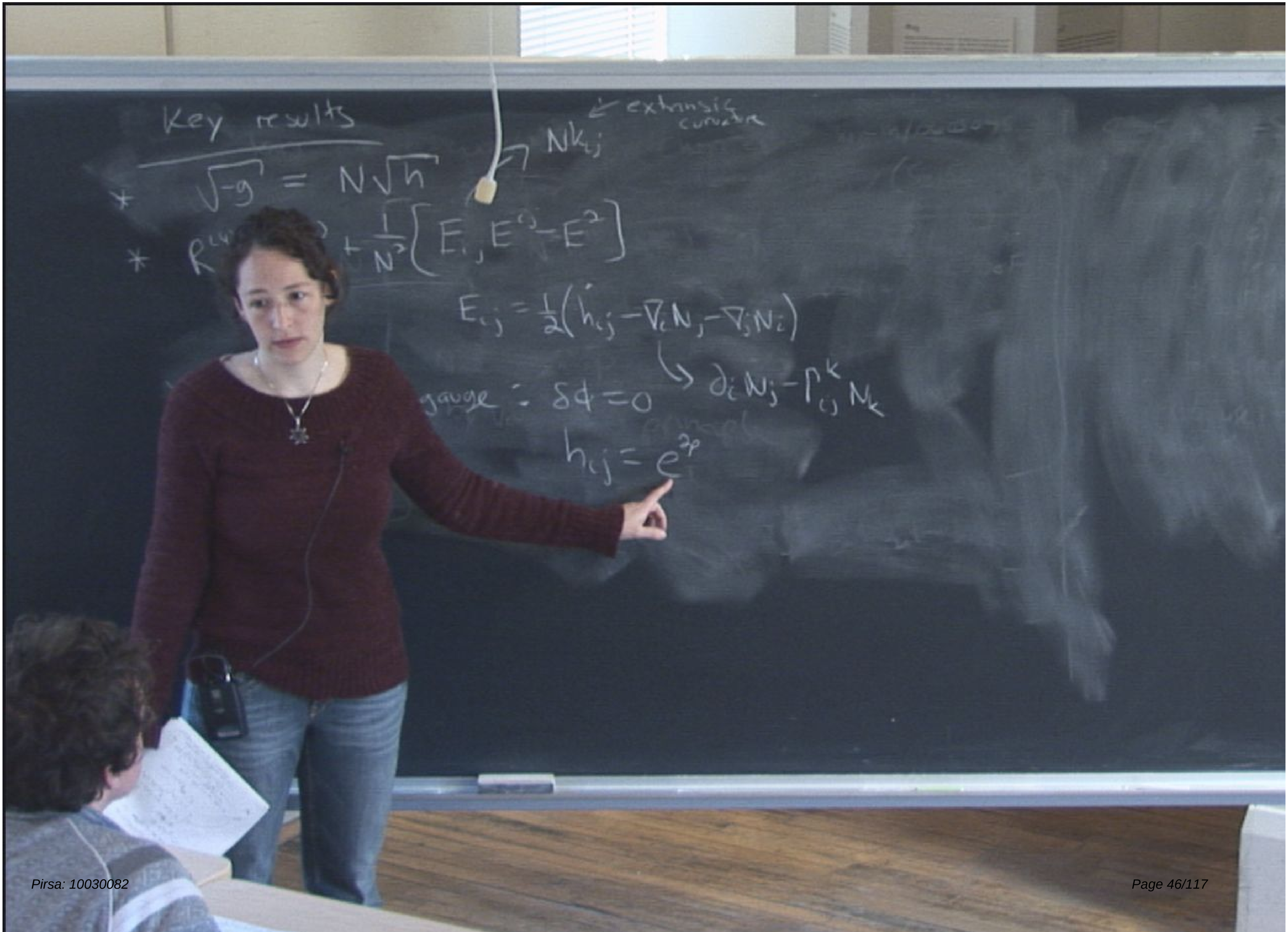
* choose comoving gauge



Key results

* $\sqrt{-g} = N\sqrt{h}$

* $R^{(4)}$ [illegible] $+\frac{1}{N^2}\left[E_{ij}E^{ij}-E^2\right]$

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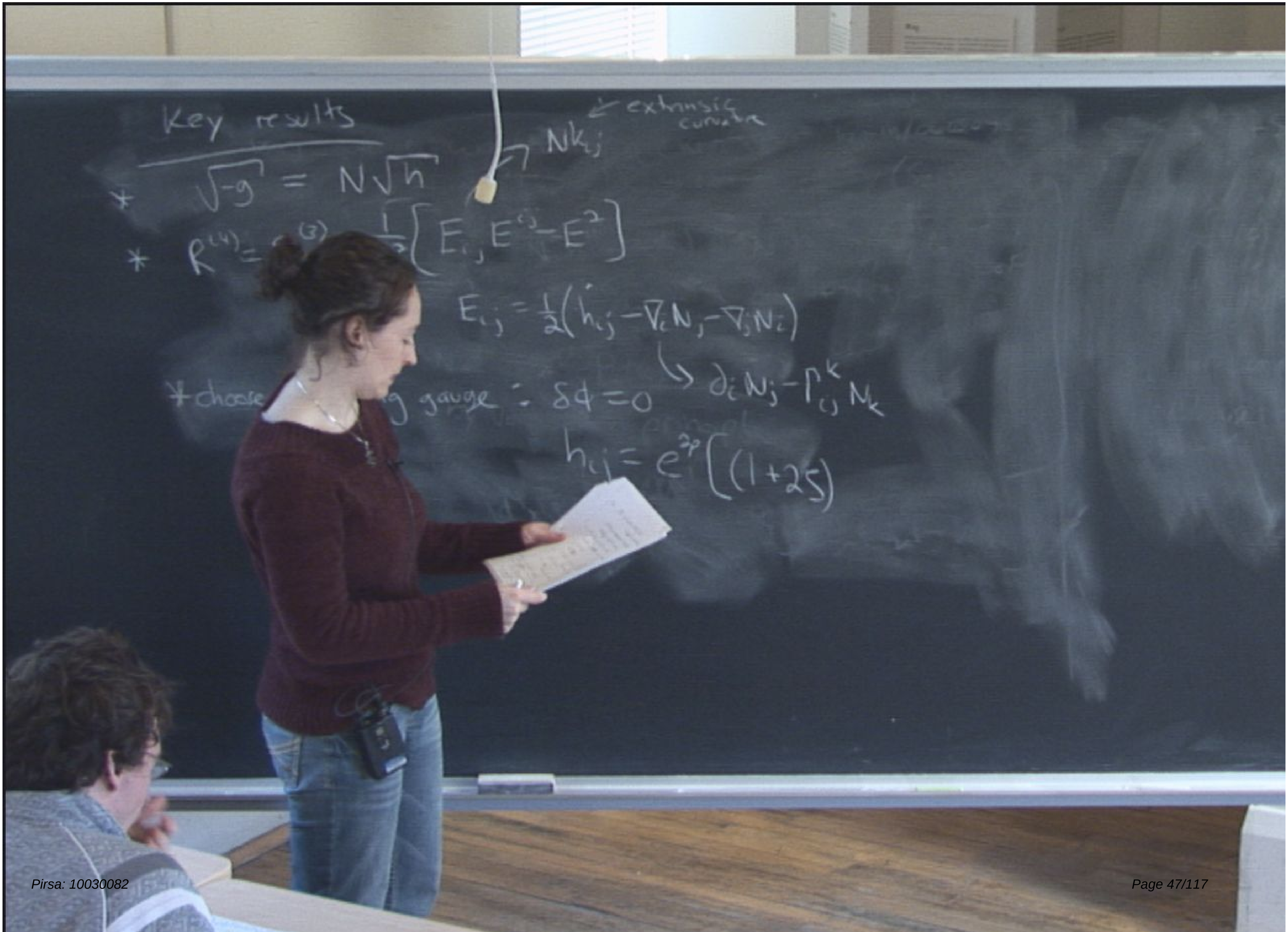
$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

↳ $\partial_i N_j - \Gamma^k_{ij} N_k$

gauge: $\delta\phi = 0$

$h_{ij} = e^{2\rho}$



Key results

* $\sqrt{-g} = N\sqrt{h}$

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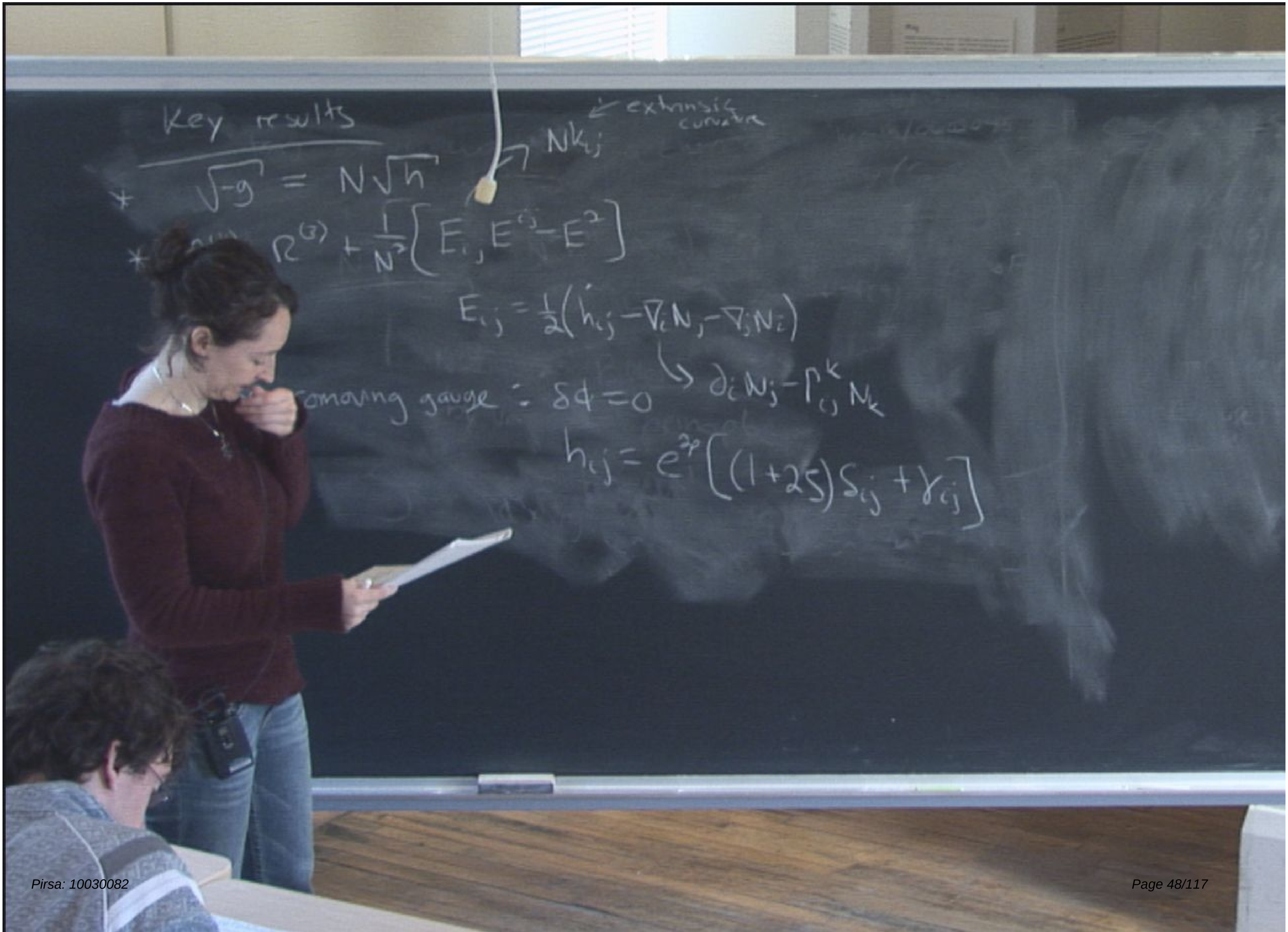
$\to N k_{ij}$ ← extrinsic curvature

*choose g gauge : $\delta\phi = 0$

$h_{ij} = e^{2\rho}\left[(1+25) \right.$



Key results

* $\sqrt{-g} = N\sqrt{h}$ $\rightarrow Nk_{ij}$ ← extrinsic curvature

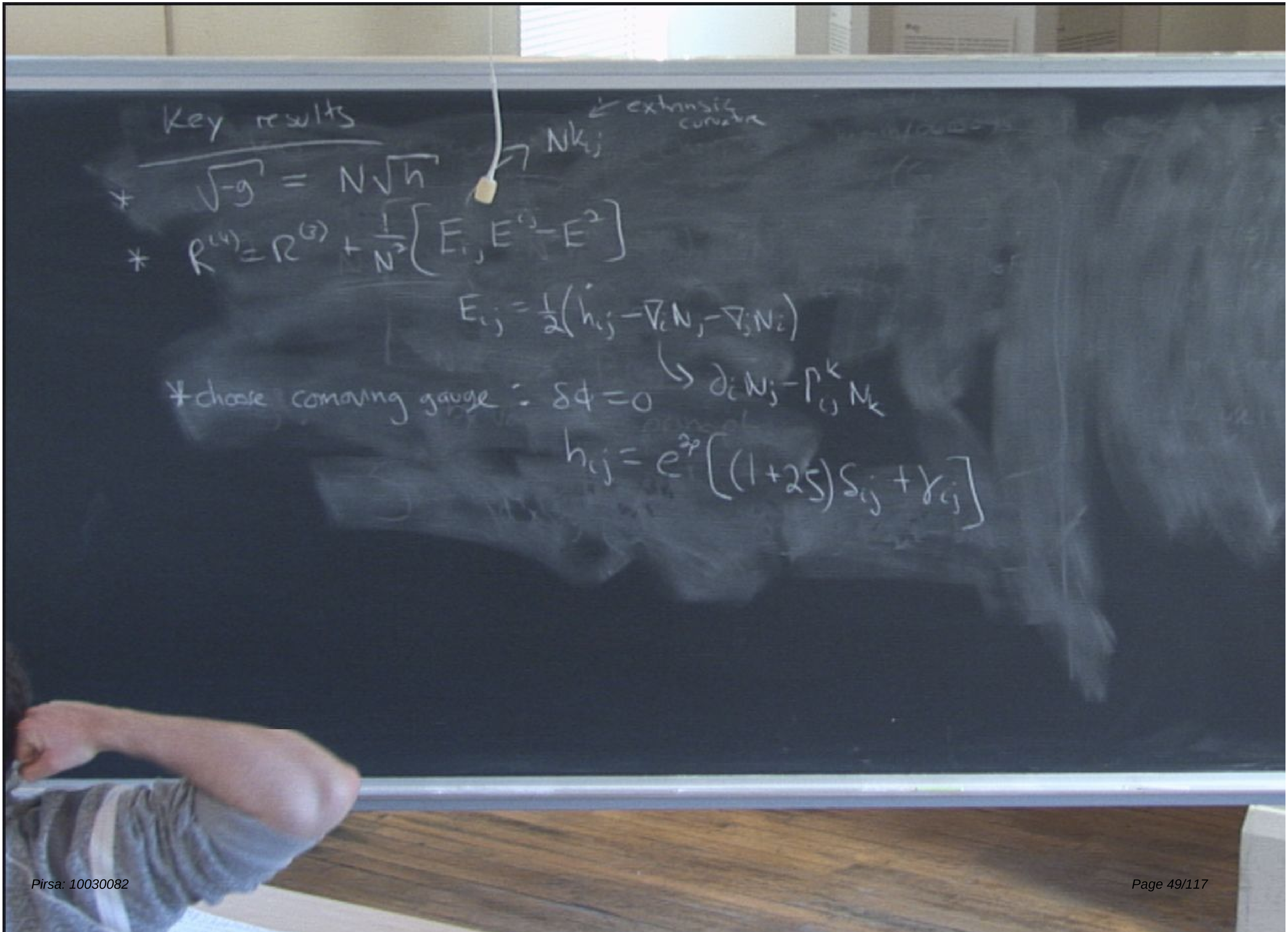
* $R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

↳ $\partial_i N_j - \Gamma^k_{ij} N_k$

comoving gauge: $\delta\phi = 0$

$h_{ij} = e^{2\rho}\left[(1+2\zeta)\delta_{ij} + \gamma_{ij}\right]$



Key results

$\leftarrow$ extrinsic curvature

$\nearrow N k_{ij}$

- $\sqrt{-g} = N\sqrt{h}$
- $R^{(4)} = R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

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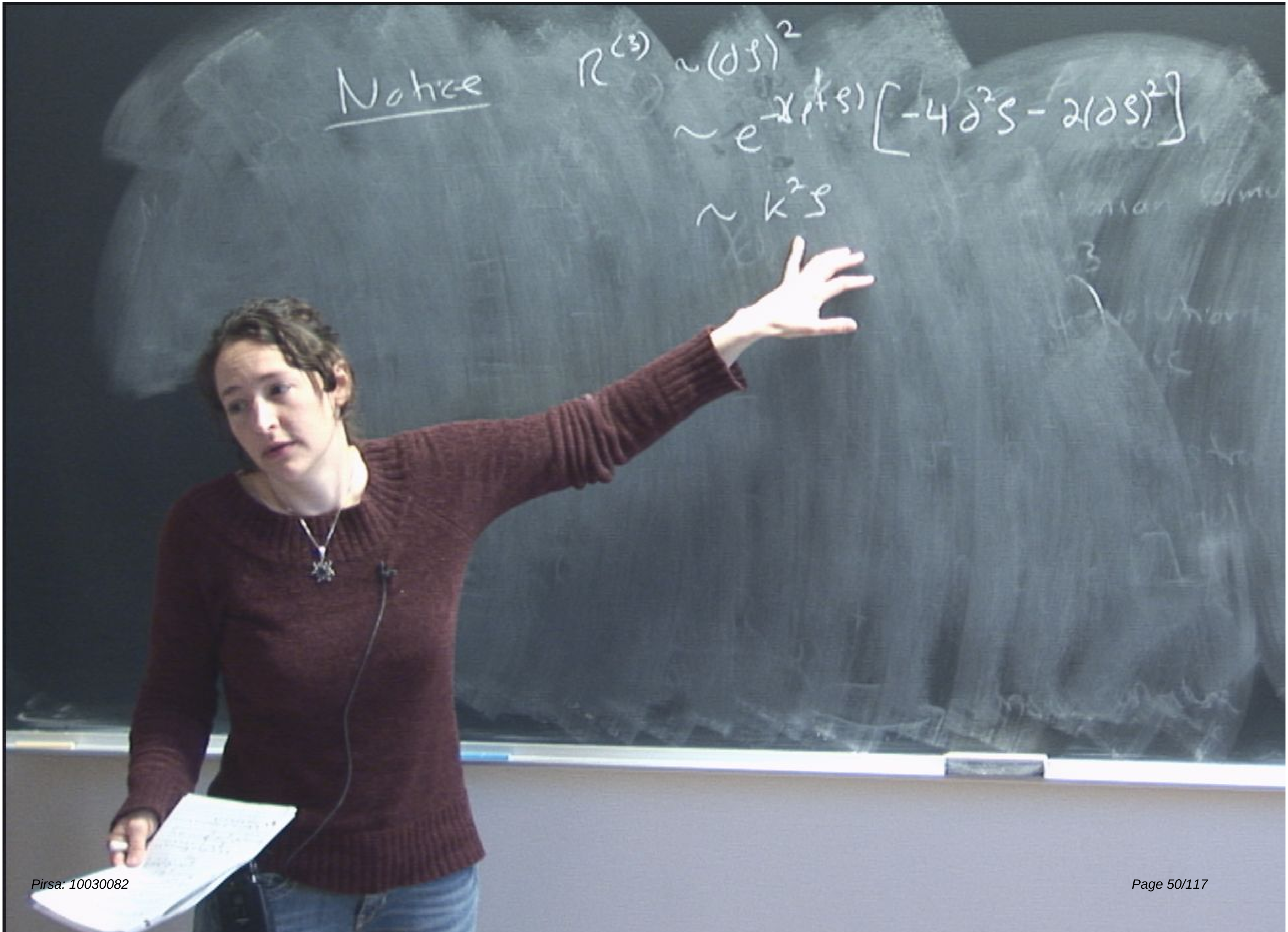
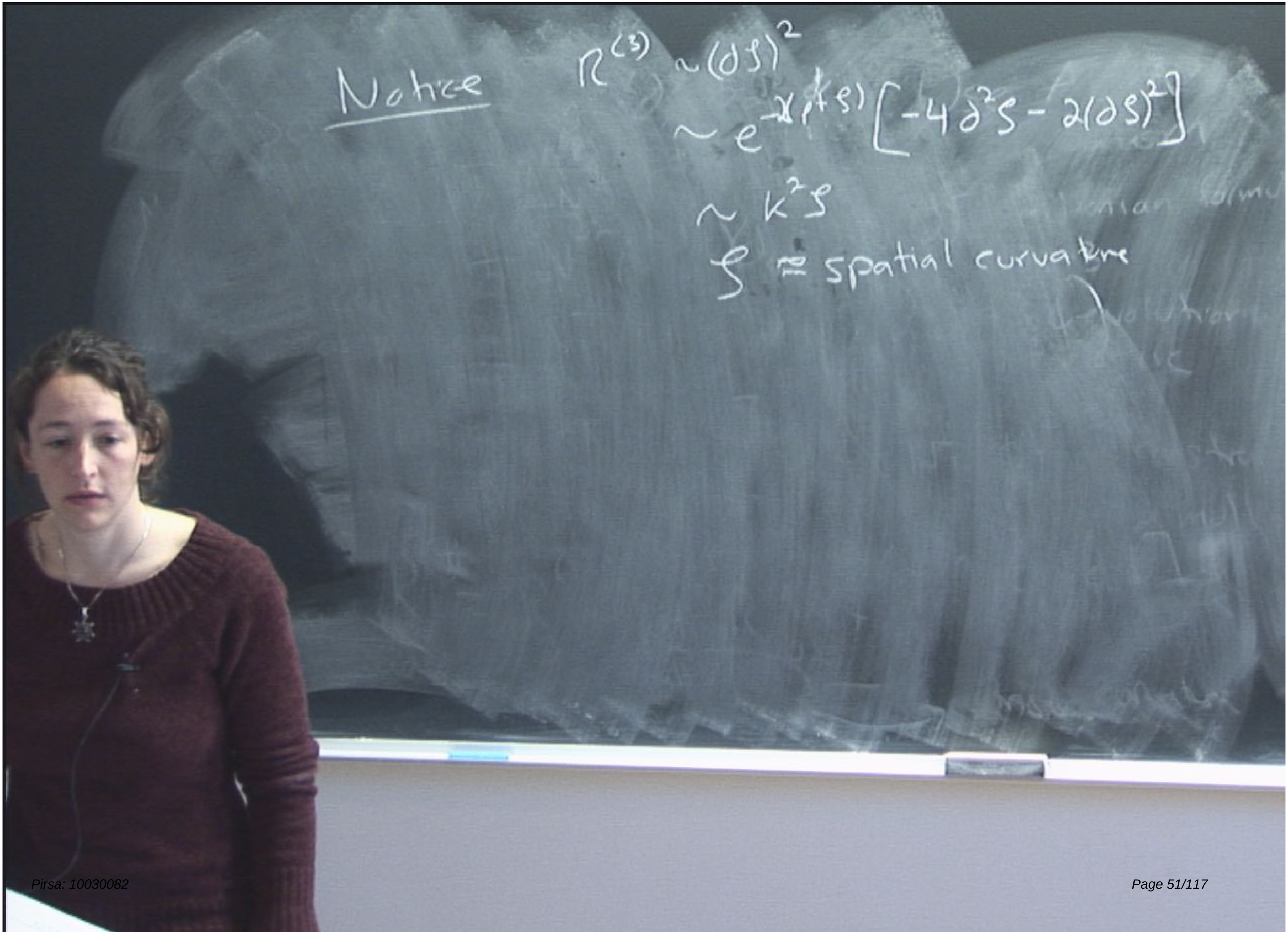


Notice $R^{(3)} \sim (\partial \zeta)^2$

$$\sim e^{-2(\rho+\zeta)}\left[-4\partial^2\zeta - 2(\partial\zeta)^2\right]$$

$$\sim k^2 \zeta$$



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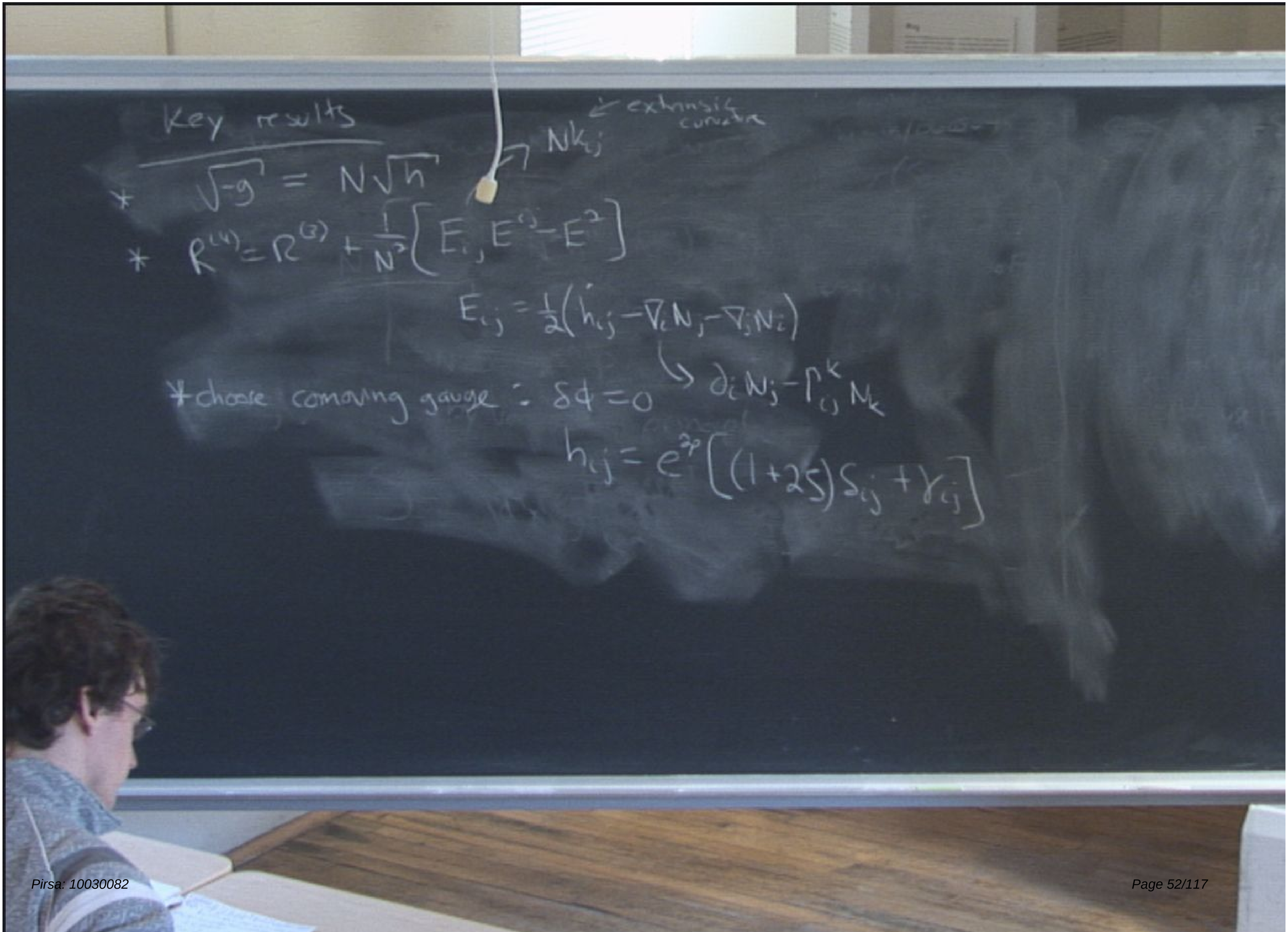
$\zeta \approx$ spatial curvature



Key results

* $\sqrt{-g} = N\sqrt{h}$

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$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

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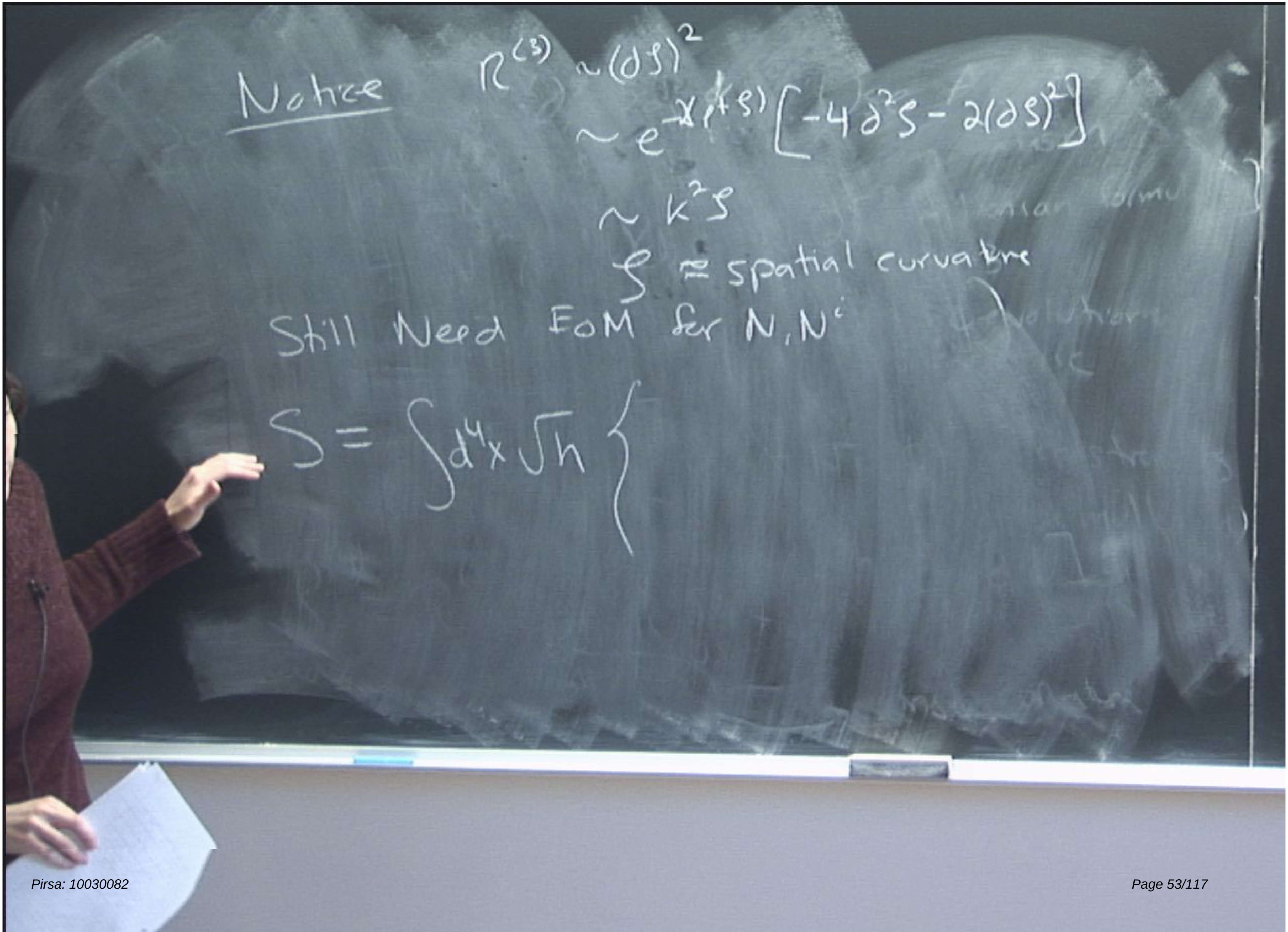

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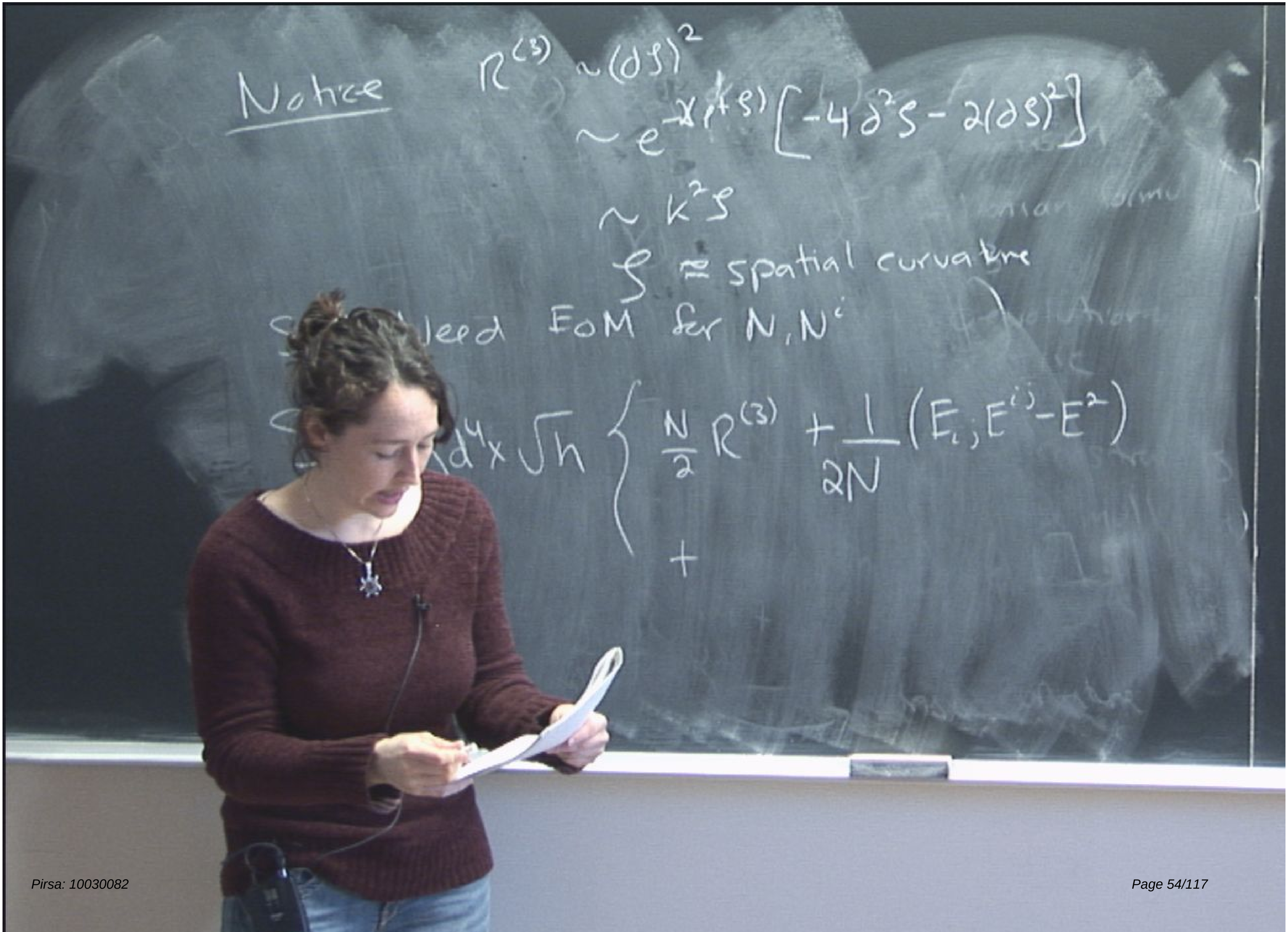
$\zeta \approx$ spatial curvature

Still Need EoM for N, N^i

$$S = \int d^4x \sqrt{h}\,\Big\{$$



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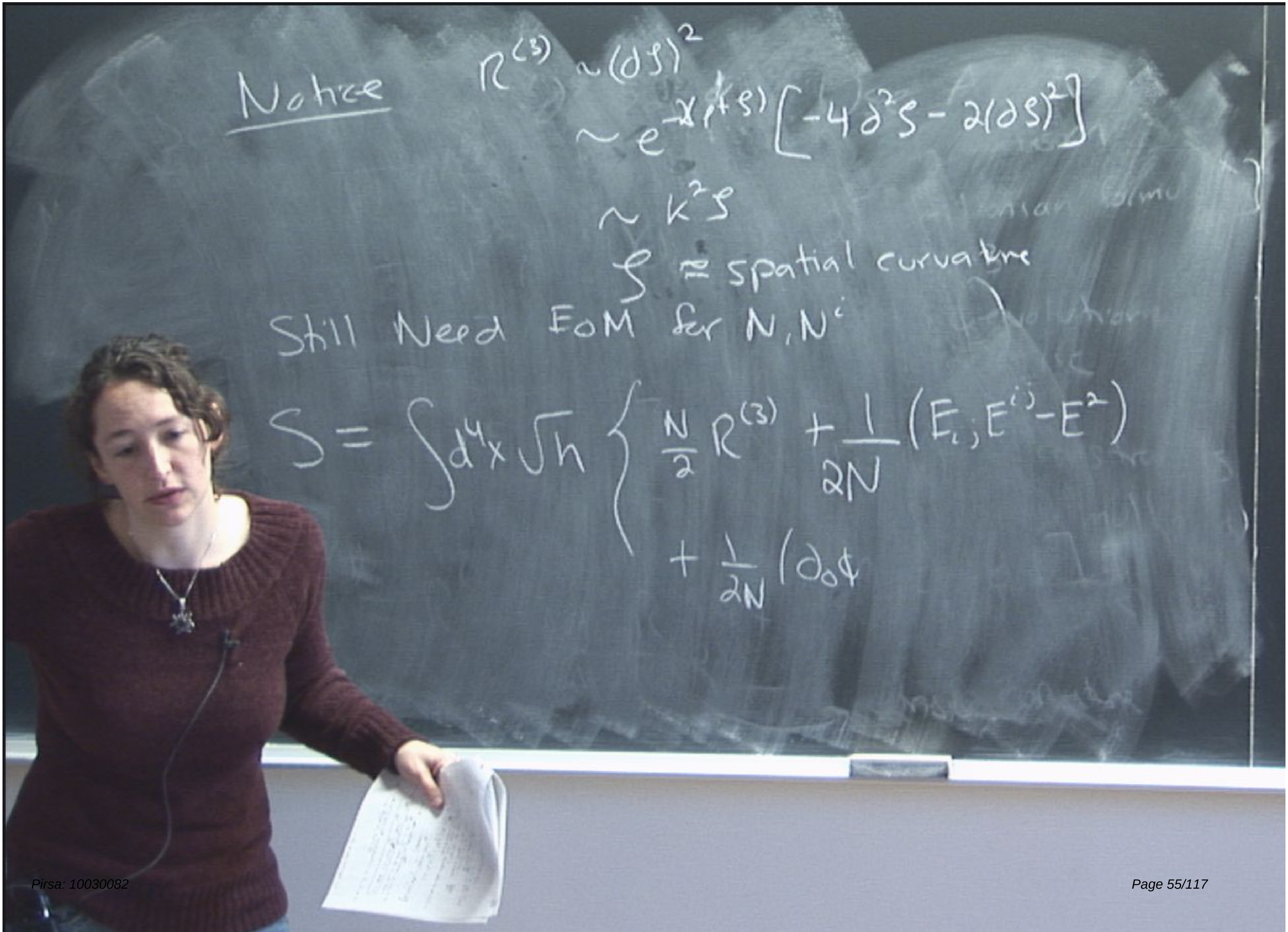
$\zeta \approx$ spatial curvature

S leed EoM for N, N^i

$$S \ldots d^4x\sqrt{h}\left\{\frac{N}{2}R^{(3)} + \frac{1}{2N}\left(E_{ij}E^{ij} - E^2\right) + \right.$$



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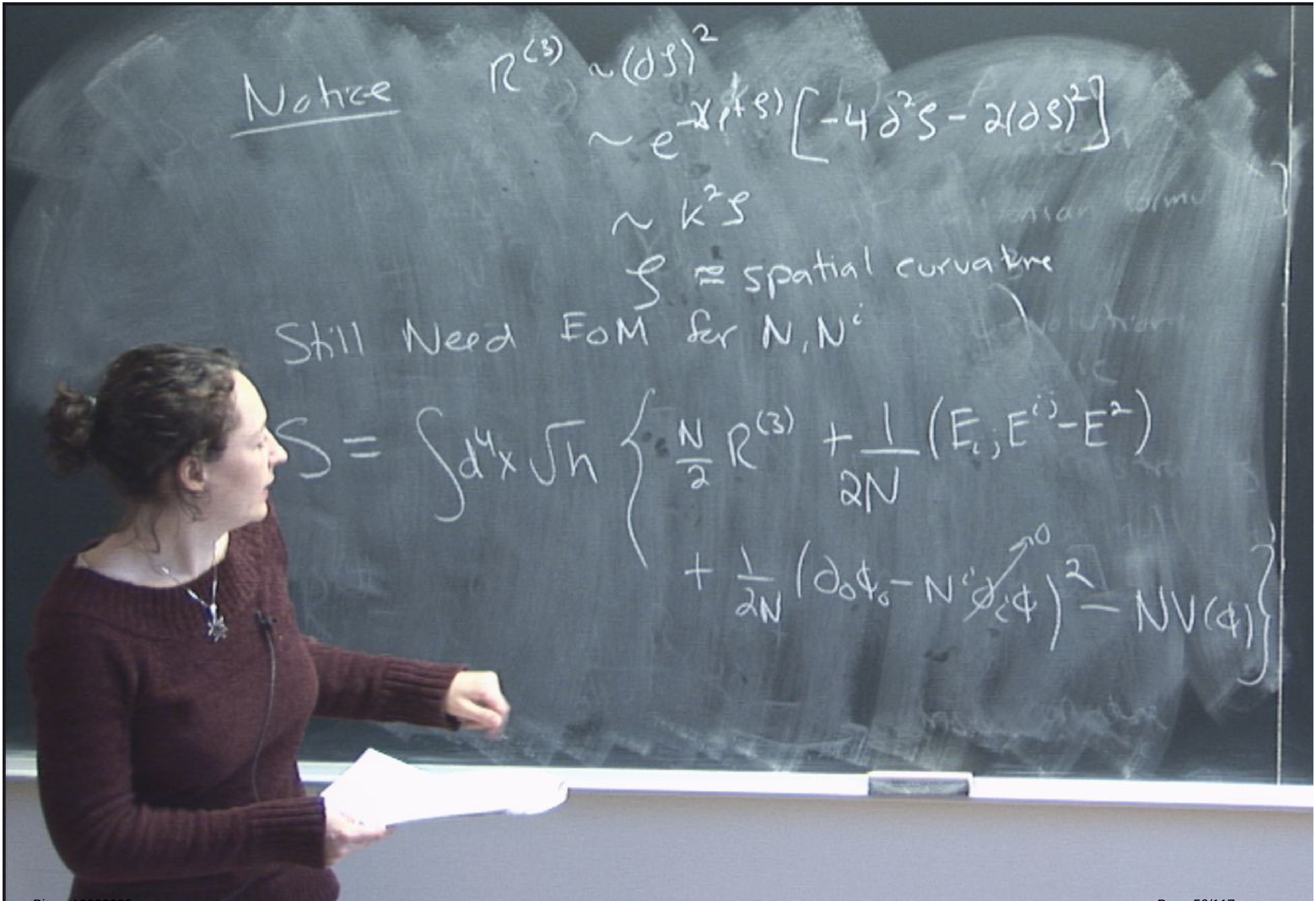
Still Need EoM for N, N^i

$$S = \int d^4x \sqrt{h} \left\{ \frac{N}{2} R^{(3)} + \frac{1}{2N}\left(E_{ij}E^{ij} - E^2\right) \right.$$

$$+ \frac{1}{2N}\left(\partial_0\phi\right.$$



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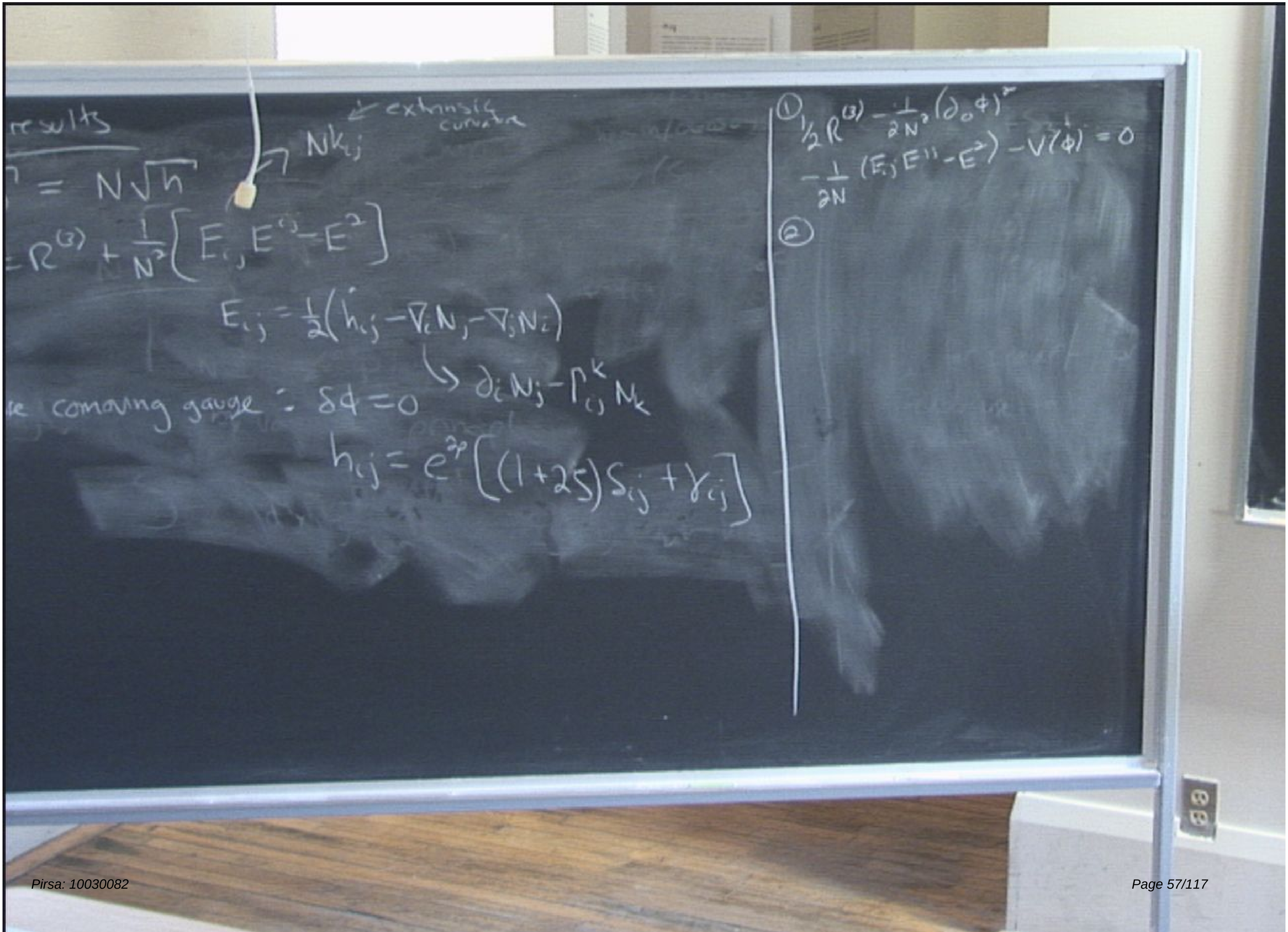
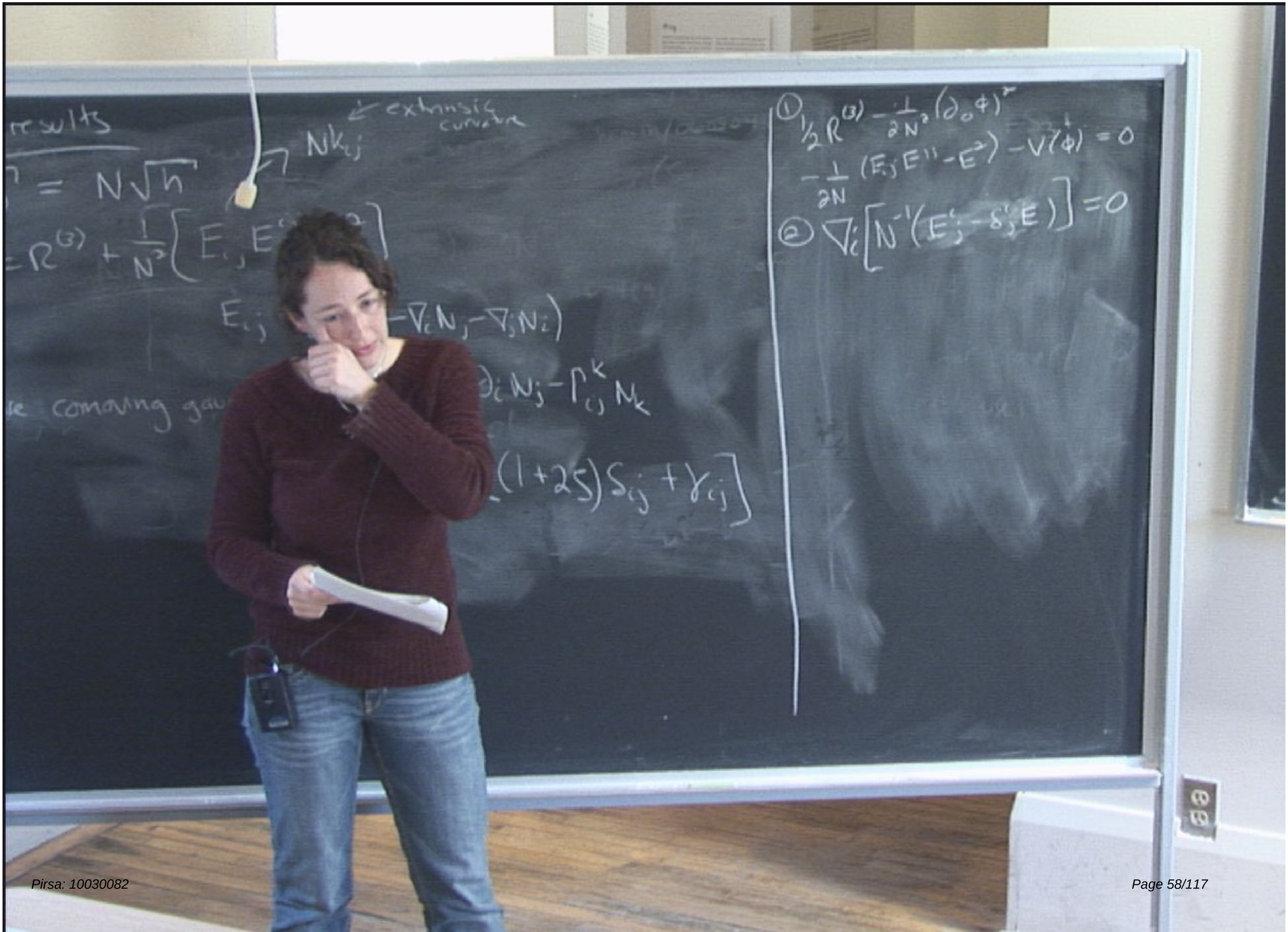
$\sim k^2\zeta$

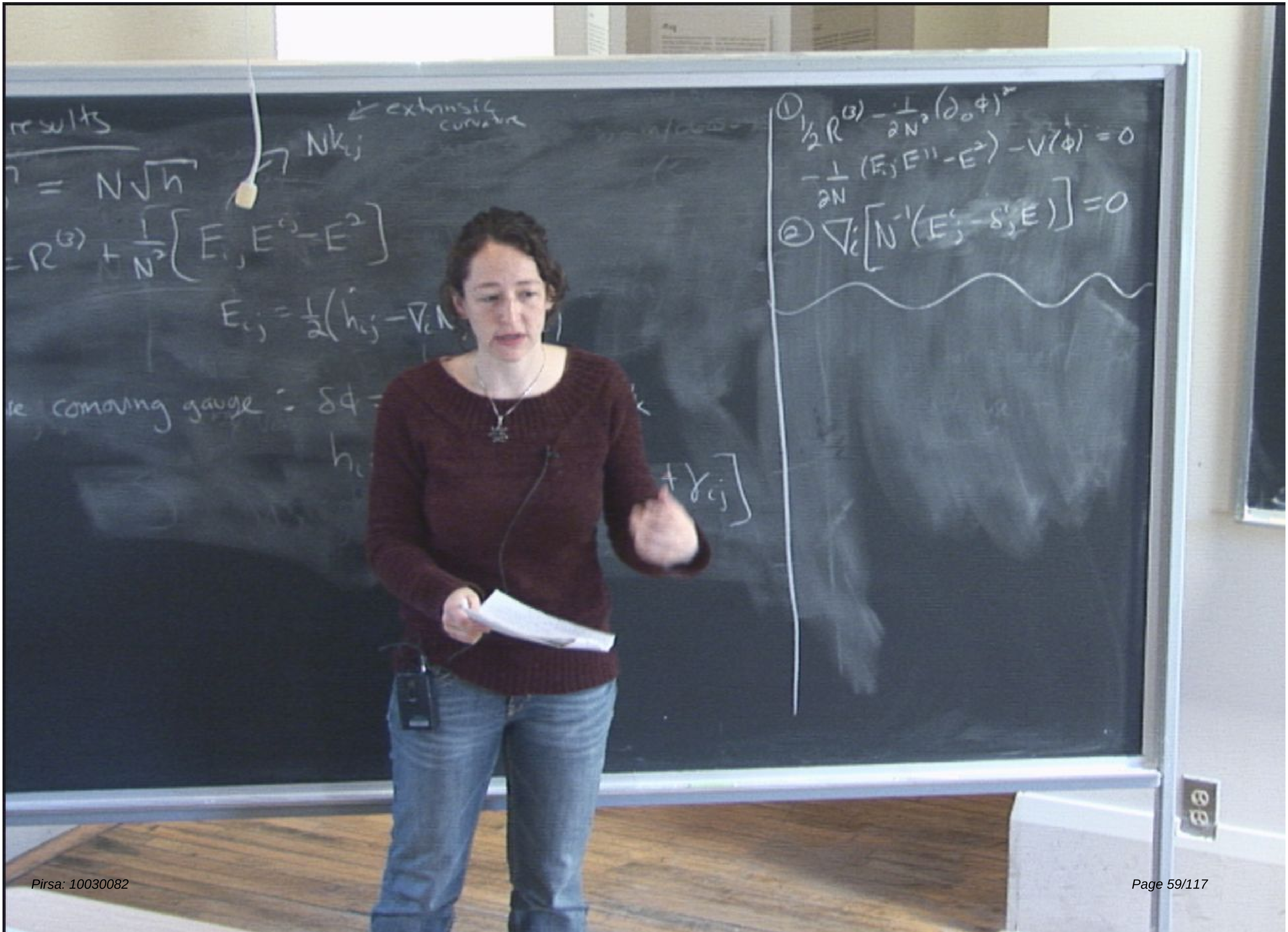
$\zeta \simeq$ spatial curvature

Still Need EoM for N, N^i

$$S = \int d^4x \sqrt{h}\left\{\frac{N}{2}R^{(3)} + \frac{1}{2N}\left(E_{ij}E^{ij} - E^2\right) + \frac{1}{2N}\left(\partial_0\phi_0 - N^i \overset{\nearrow 0}{\cancel{\partial_i}\phi}\right)^2 - NV(\phi)\right\}$$




results
extrinsic curvature
Nk_{ij}
$= N\sqrt{h}$
$R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij}\right.$
E_{ij} $-\nabla_i N_j - \nabla_j N_i)$
$\partial_i N_j - \Gamma^k_{ij} N_k$
comoving gau
$(1+2\zeta)S_{ij} + \gamma_{ij}]$
① $\frac{1}{2}R^{(3)} - \frac{1}{2N^2}(\partial_0\phi)^2 - V(\phi) = 0$
$-\frac{1}{2N}(E_{ij}E^{ij} - E^2)$
② $\nabla_i\left[N^{-1}(E^i_j - \delta^i_j E)\right] = 0$



results
extrinsic curvature
Nk_{ij}
$= N\sqrt{h}$
$= R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$
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comoving gauge : $\delta\phi =$
h_{i}
$+\gamma_{ij}]$
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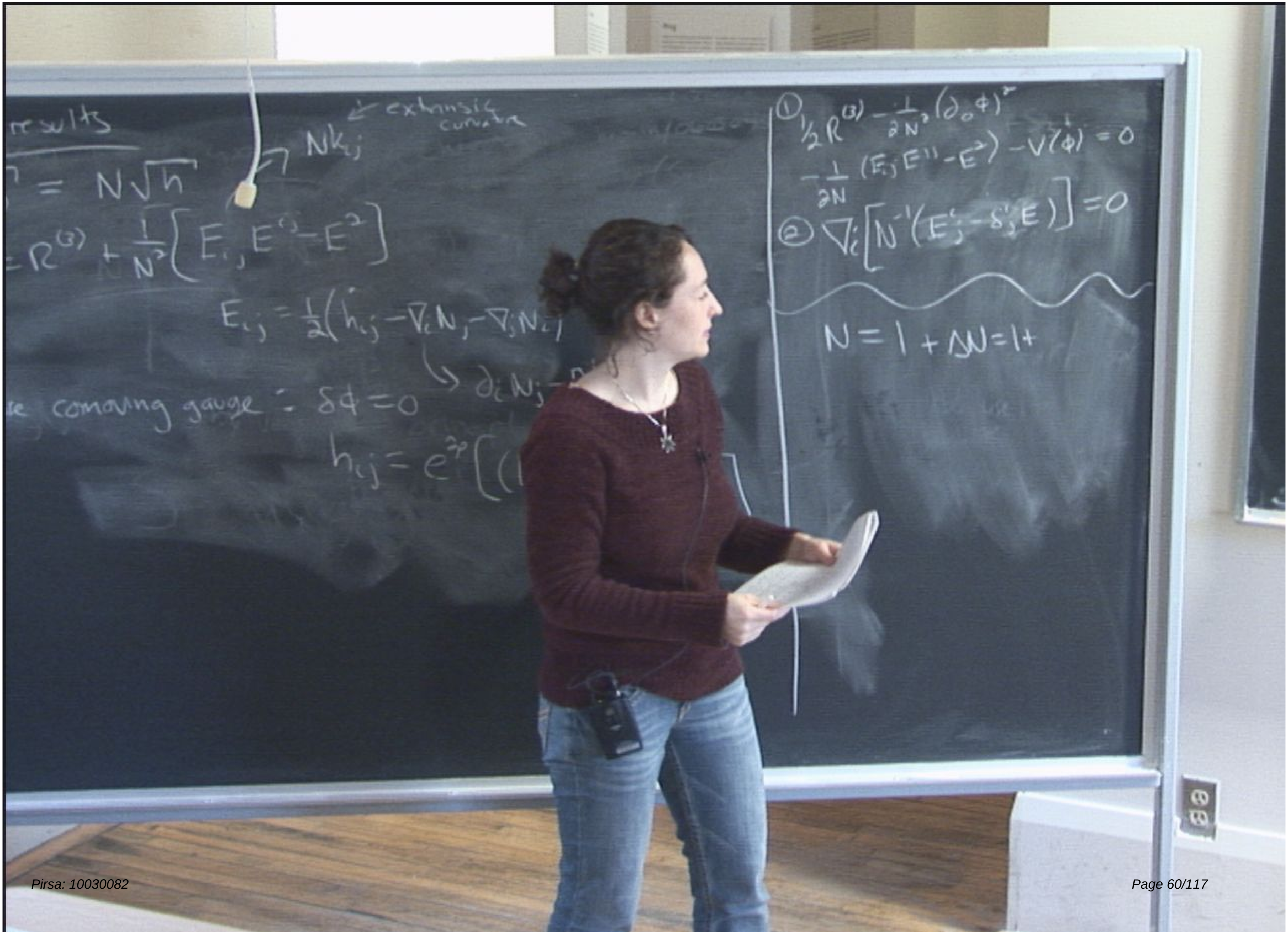
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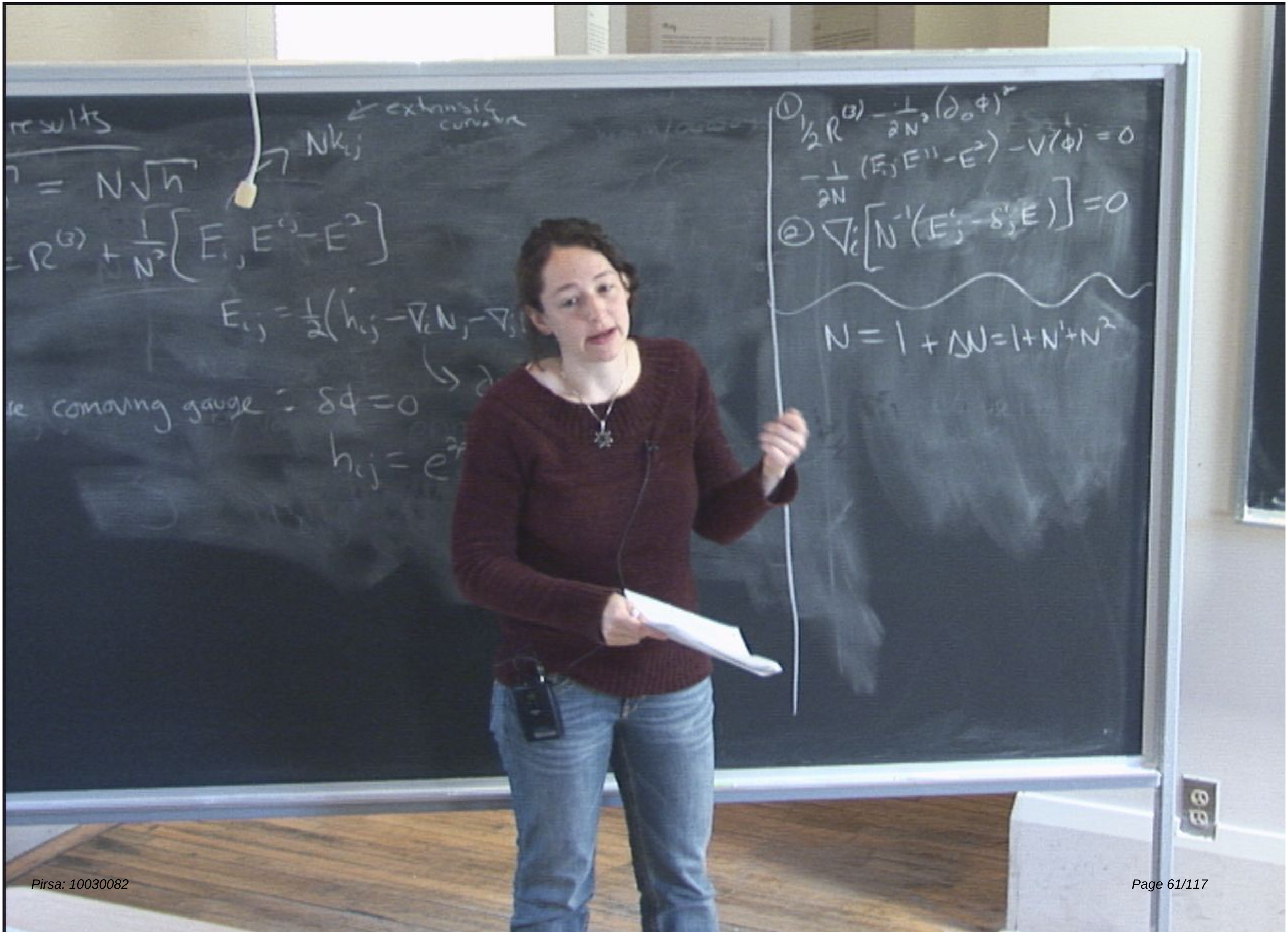
$N = 1 + \Delta N = 1 +$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$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j$
comoving gauge : $\delta\phi = 0$
$h_{ij} = e^{2}$
① $\frac{1}{2}R^{(3)} - \frac{1}{2N^2}(\partial_0\phi)^2 - V(\phi) = 0$
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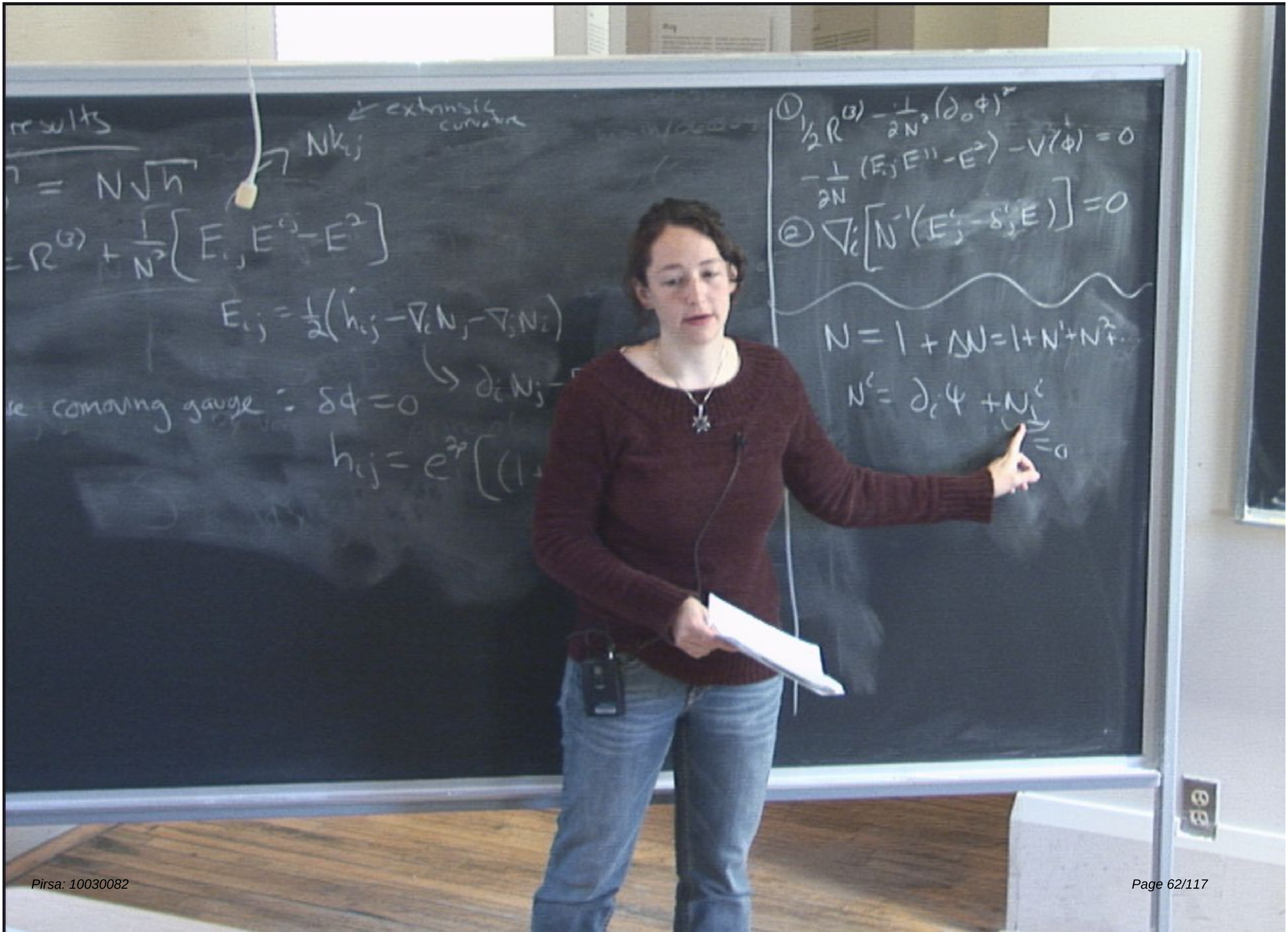
② $\nabla_i\left[N^{-1}(E^i_j - \delta^i_j E)\right] = 0$
$N = 1 + \Delta N = 1 + N^1 + N^2$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Nk_{ij}
extrinsic curvature
$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
$\partial_i N_j -$
comoving gauge : $\delta\phi = 0$
$h_{ij} = e^{2\rho}[(1+$
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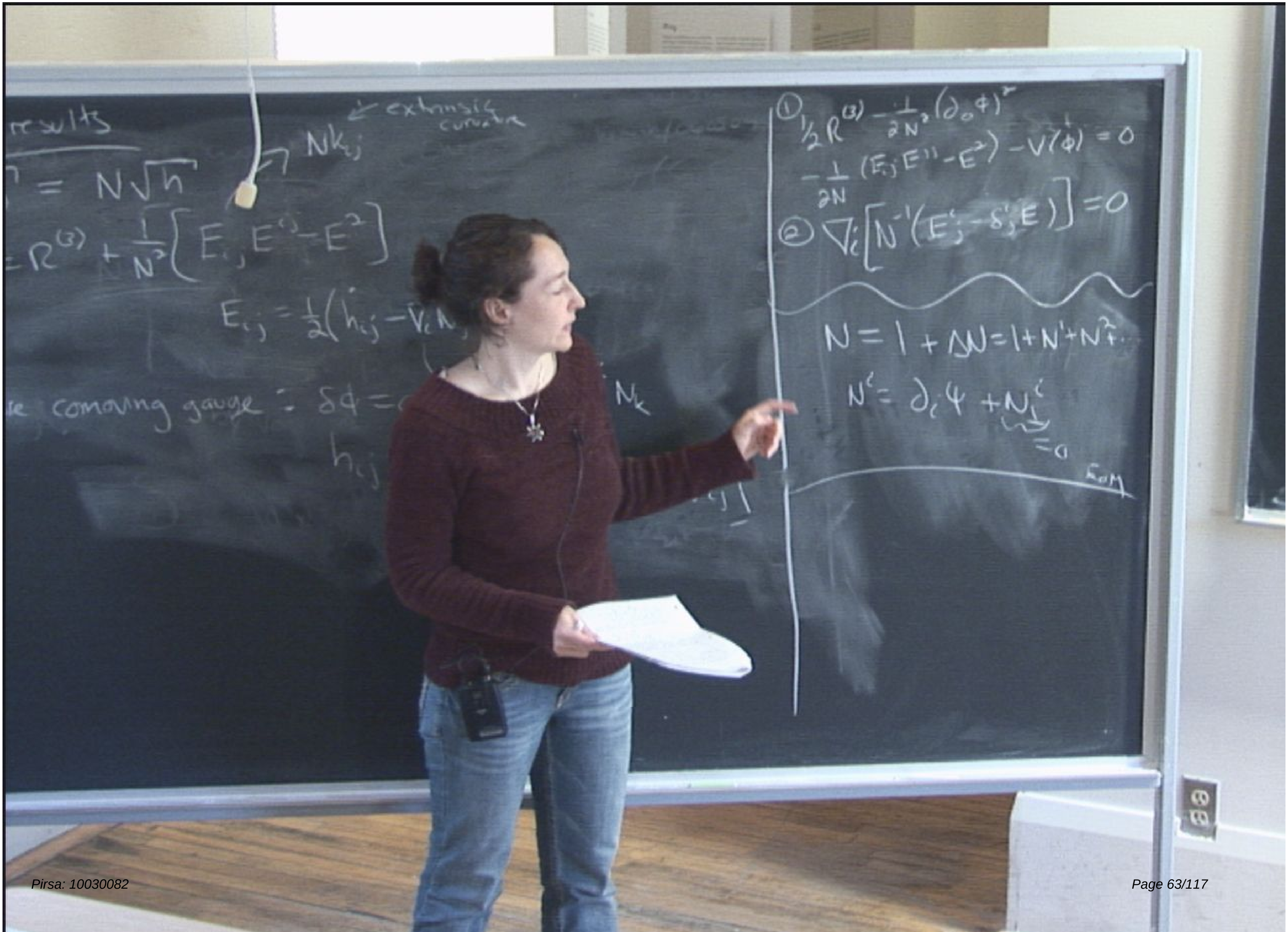
② $\nabla_i\left[N^{-1}(E^i_j - \delta^i_j E)\right] = 0$
$N = 1 + \Delta N = 1 + N^1 + N^2 + \dots$
$N^i = \partial_i\psi + N_1^i$
$= 0$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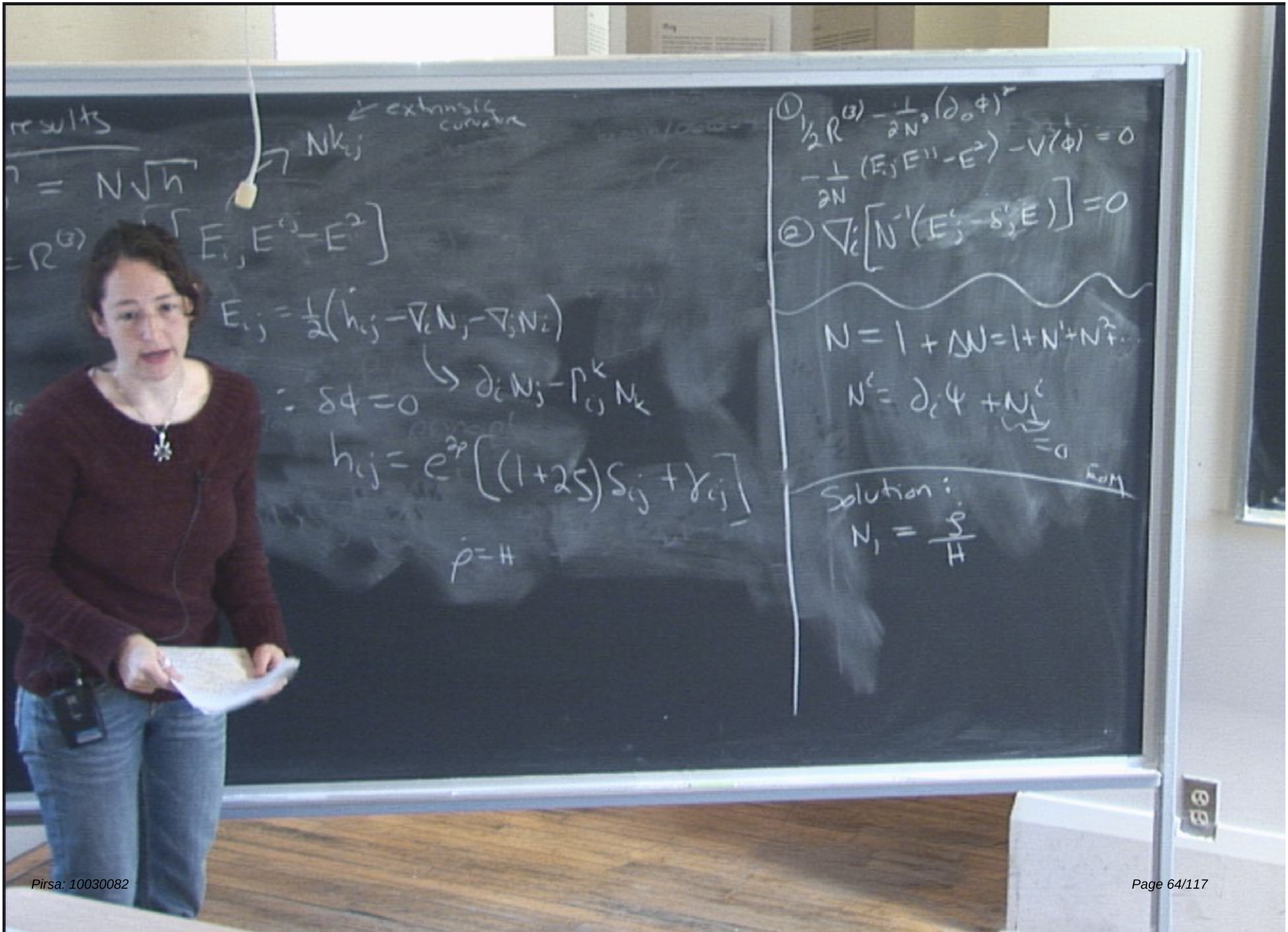
EoM



results

extrinsic curvature

Nk_{ij}

$= N\sqrt{h}$

$R^{(3)}$ $[E_{ij}E^{ij} - E^2]$

$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$

$\hookrightarrow \partial_i N_j - \Gamma^k_{ij} N_k$

$\delta\phi = 0$

$h_{ij} = e^{2\rho}[(1+2\zeta)\delta_{ij} + \gamma_{ij}]$

$\dot{\rho} = H$

① $\frac{1}{2}R^{(3)} - \frac{1}{2N^2}(\partial_0\phi)^2 - \frac{1}{2N}(E_{ij}E^{ij} - E^2) - V(\phi) = 0$

② $\nabla_i[N^{-1}(E^i_j - \delta^i_j E)] = 0$

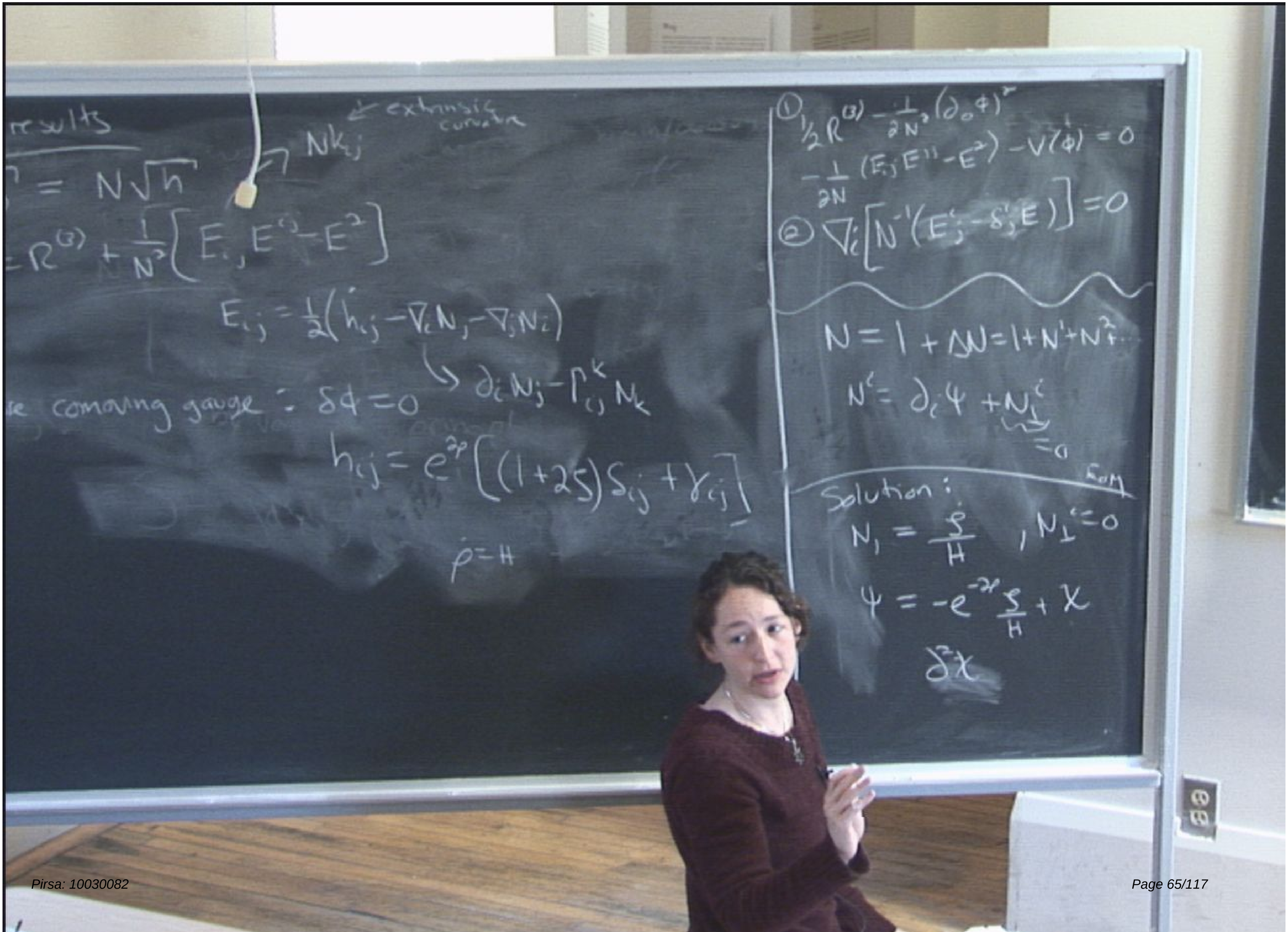
$N = 1 + \Delta N = 1 + N^1 + N^2 + \dots$

$N^i = \partial_i \psi + \underbrace{N^i_\perp}_{=0}$

EoM

Solution:

$N_1 = \frac{\dot{\zeta}}{H}$



results

$= N\sqrt{h}$

Nk_{ij} ← extrinsic curvature

$= R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

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① $\frac{1}{2}R^{(3)} - \frac{1}{2N^2}(\partial_0\phi)^2 - \frac{1}{2N}\left(E_{ij}E^{ij} - E^2\right) - V(\phi) = 0$

② $\nabla_i\left[N^{-1}\left(E^i_j - \delta^i_j E\right)\right] = 0$

$N = 1 + \Delta N = 1 + N^1 + N^2 + \dots$

$N^i = \partial_i \psi + \underbrace{N^i_\perp}_{=0}$

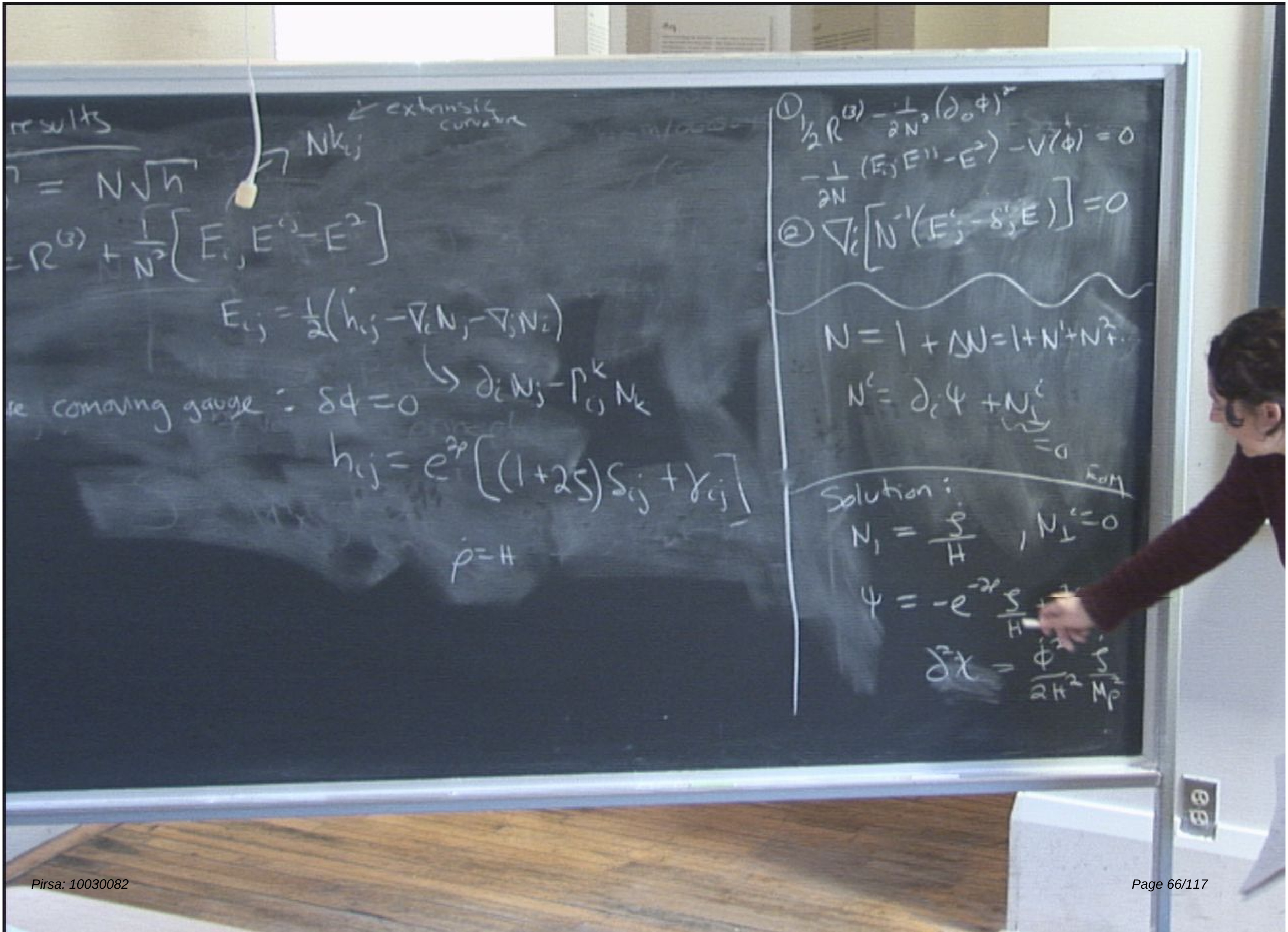
EoM

Solution:

$N_1 = \frac{\dot{\zeta}}{H}, \quad N^i_\perp = 0$

$\psi = -e^{-2\rho}\frac{\zeta}{H} + \chi$

$\partial^2\chi$



results

$N k_{ij}$ ← extrinsic curvature

$= N\sqrt{h}$

$= R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

$\hookrightarrow \partial_i N_j - \Gamma^k_{ij} N_k$

comoving gauge : $\delta\phi = 0$

$h_{ij} = e^{2\rho}\left[(1+2\zeta)\delta_{ij} + \gamma_{ij}\right]$

$\dot{\rho} = H$

① $\frac{1}{2}R^{(3)} - \frac{1}{2N^2}(\partial_0\phi)^2 - V(\phi) = 0$

$- \frac{1}{2N}(E_{ij}E^{ij} - E^2)$

② $\nabla_i\left[N^{-1}(E^i_j - \delta^i_j E)\right] = 0$

$N = 1 + \Delta N = 1 + N^1 + N^2 + \dots$

$N^i = \partial_i \psi + \underbrace{N^i_\perp}_{=0}$

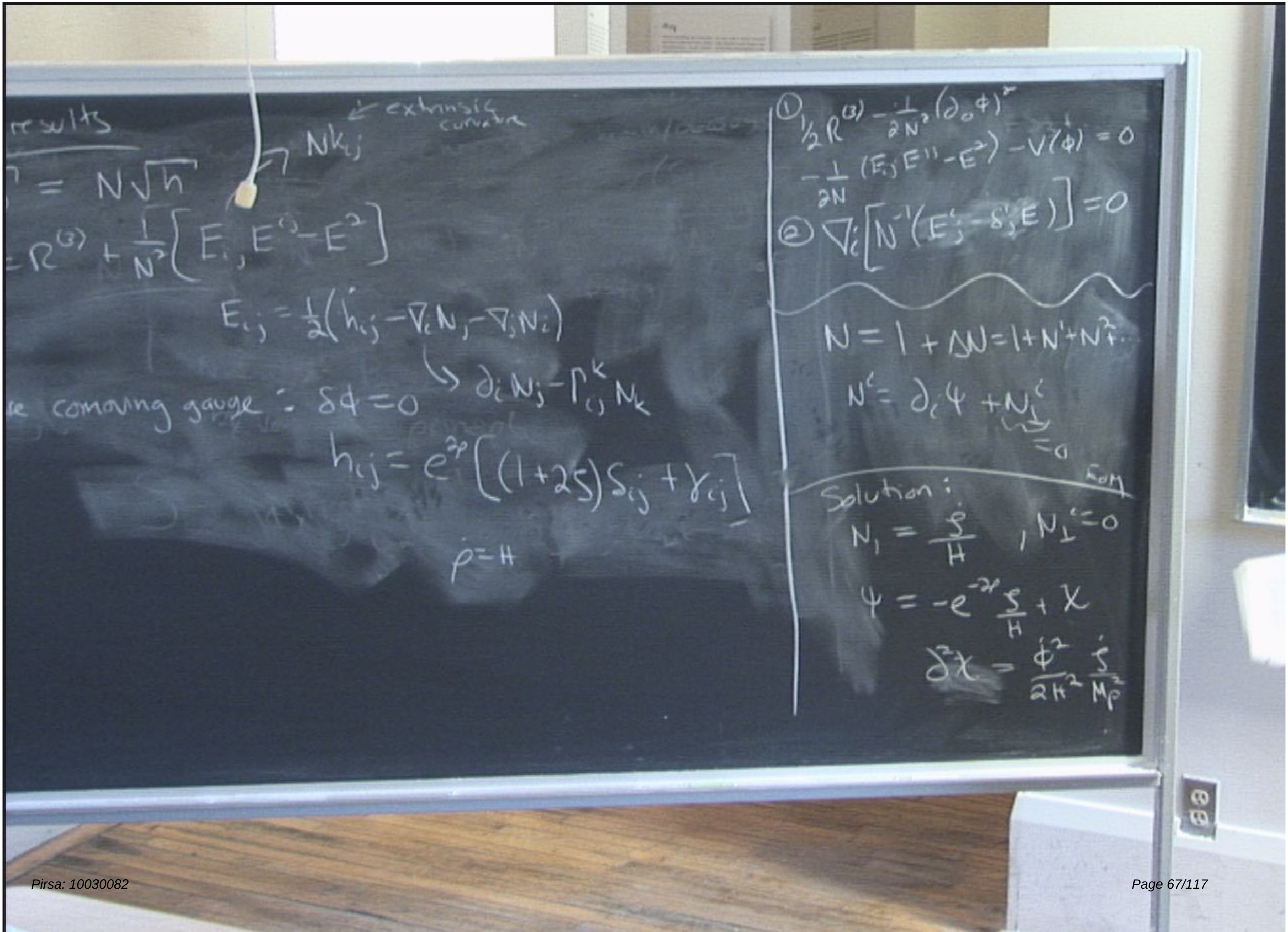
EoM

Solution :

$N_1 = \frac{\dot{\zeta}}{H} \quad , N^i_\perp = 0$

$\psi = -e^{-2\rho}\frac{\zeta}{H}$

$\partial^2\chi = \frac{\dot{\phi}^2}{2H^2}\frac{\dot{\zeta}}{M_p^2}$



results

$$= N\sqrt{h}$$

$\to Nk_{ij}$ ← extrinsic curvature

$$= R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$$

$$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

$$\hookrightarrow \partial_i N_j - \Gamma^k_{ij} N_k$$

comoving gauge : $\delta\phi = 0$

$$h_{ij} = e^{2\rho}\left[(1+2\zeta)\delta_{ij} + \gamma_{ij}\right]$$

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② $\nabla_i\left[N^{-1}(E^i_j - \delta^i_j E)\right] = 0$

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$$N^i = \partial_i \psi + \underbrace{N^i_\perp}_{=0}$$

EoM

Solution:

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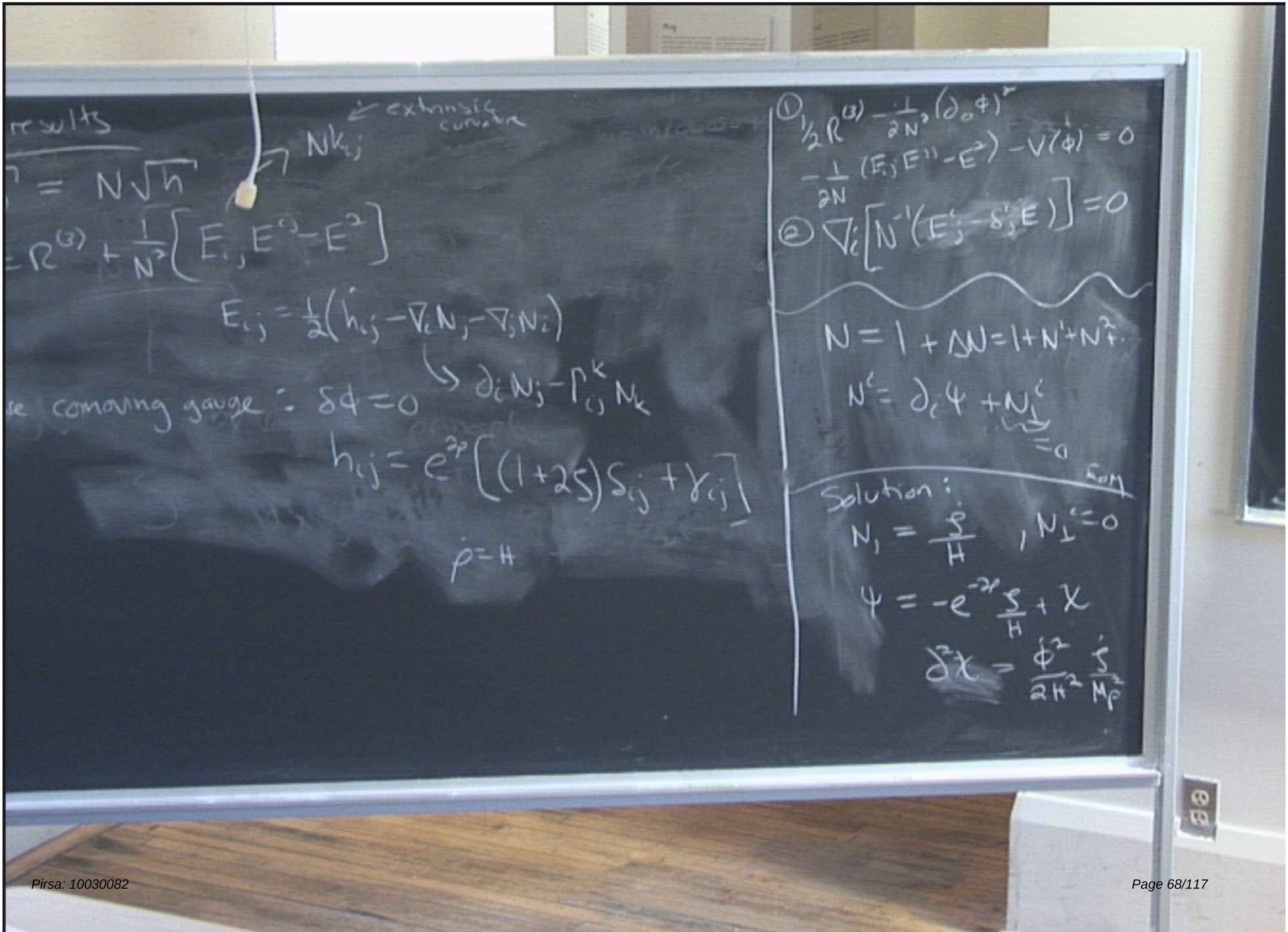
$$\psi = -e^{-2\rho}\frac{\zeta}{H} + \chi$$

$$\partial^2\chi = \frac{\dot{\phi}^2}{2H^2}\frac{\dot{\zeta}}{M_p^2}$$



results

extrinsic curvature

Nk_{ij}

$= N\sqrt{h}$

$R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$

$\hookrightarrow \partial_i N_j - \Gamma^k_{ij} N_k$

comoving gauge : $\delta\phi = 0$

$h_{ij} = e^{2\rho}\left[(1+2\zeta)\delta_{ij} + \gamma_{ij}\right]$

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Solution:

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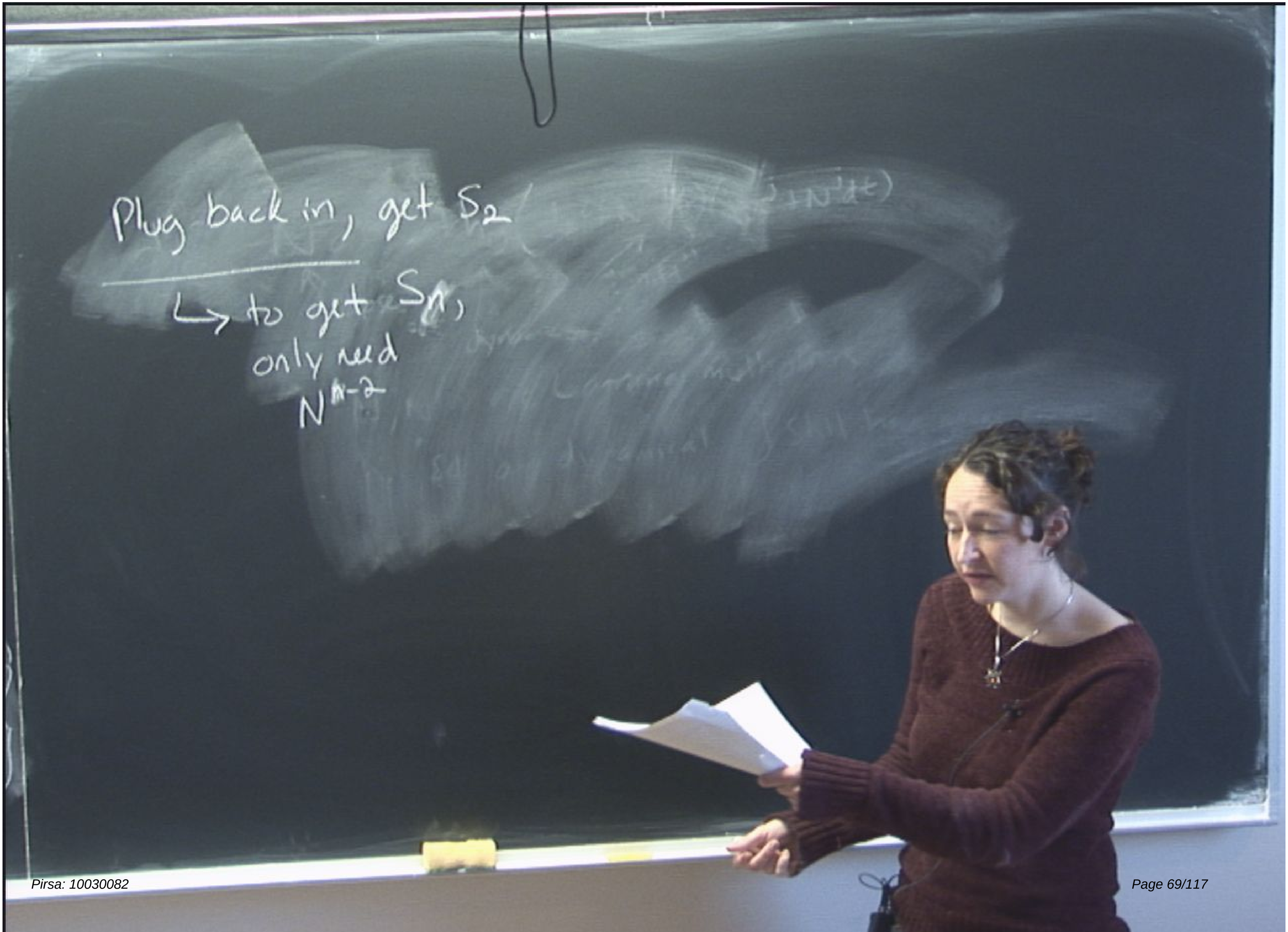
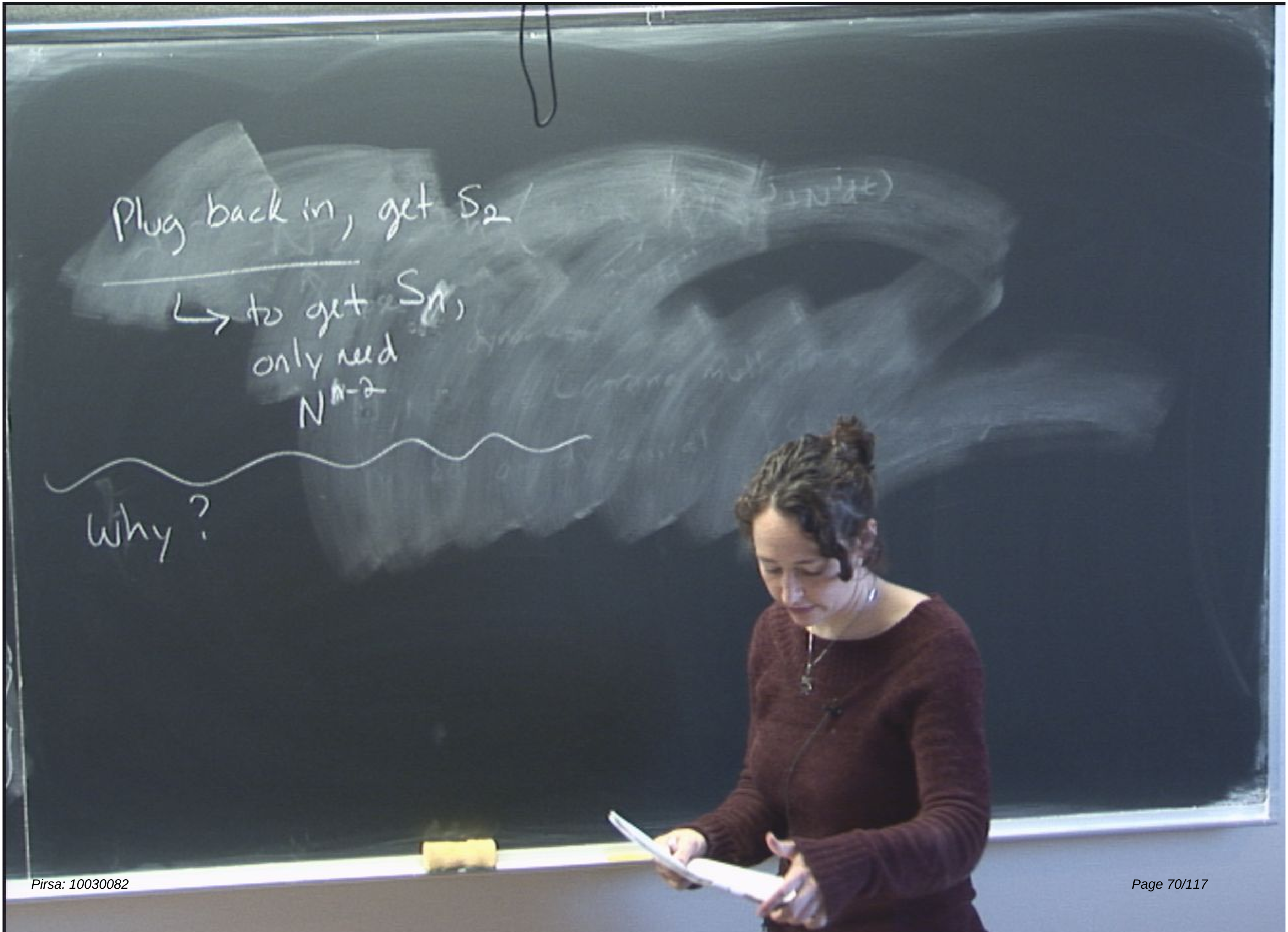
$\partial^2\chi = \frac{\dot{\phi}^2}{2H^2}\frac{\dot{\zeta}}{M_p^2}$



Plug back in, get S_2

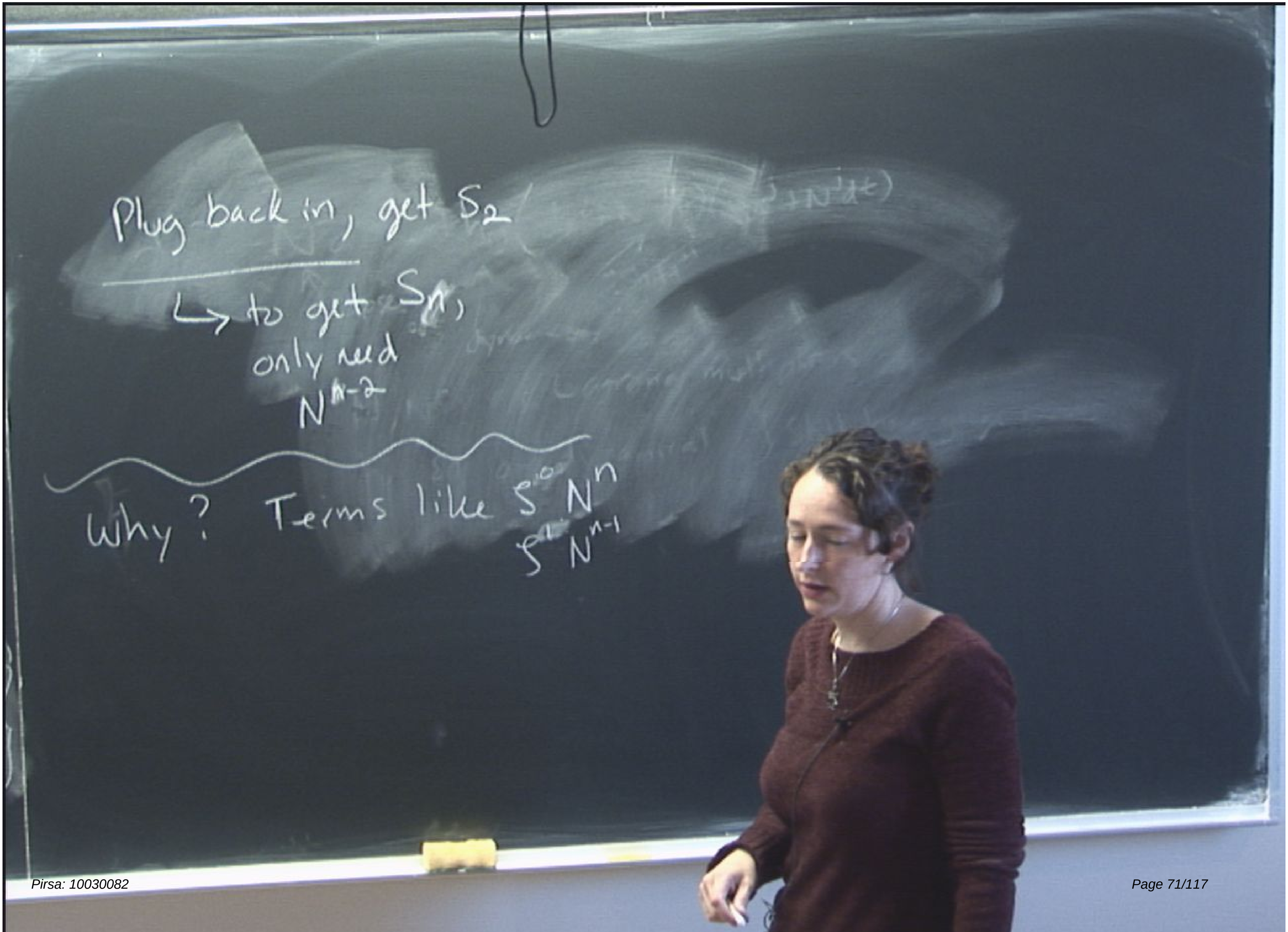
↳ to get S_n,
only need
N^{n-2}



Plug back in, get S_2

↳ to get S_n, only need N^{n-2}

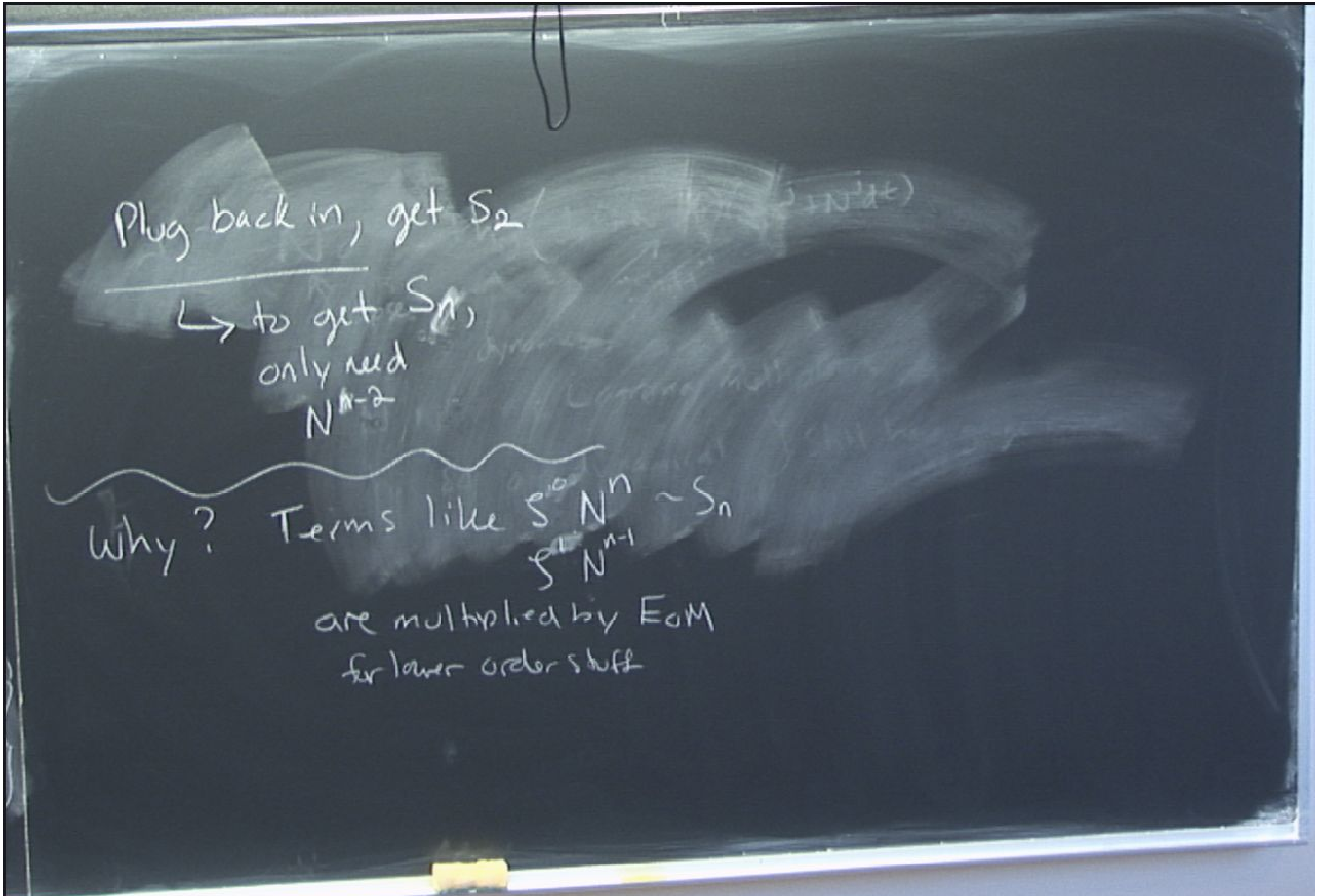
Why?



Plug back in, get S_2

↳ to get S_n,
only need
N^{n-2}

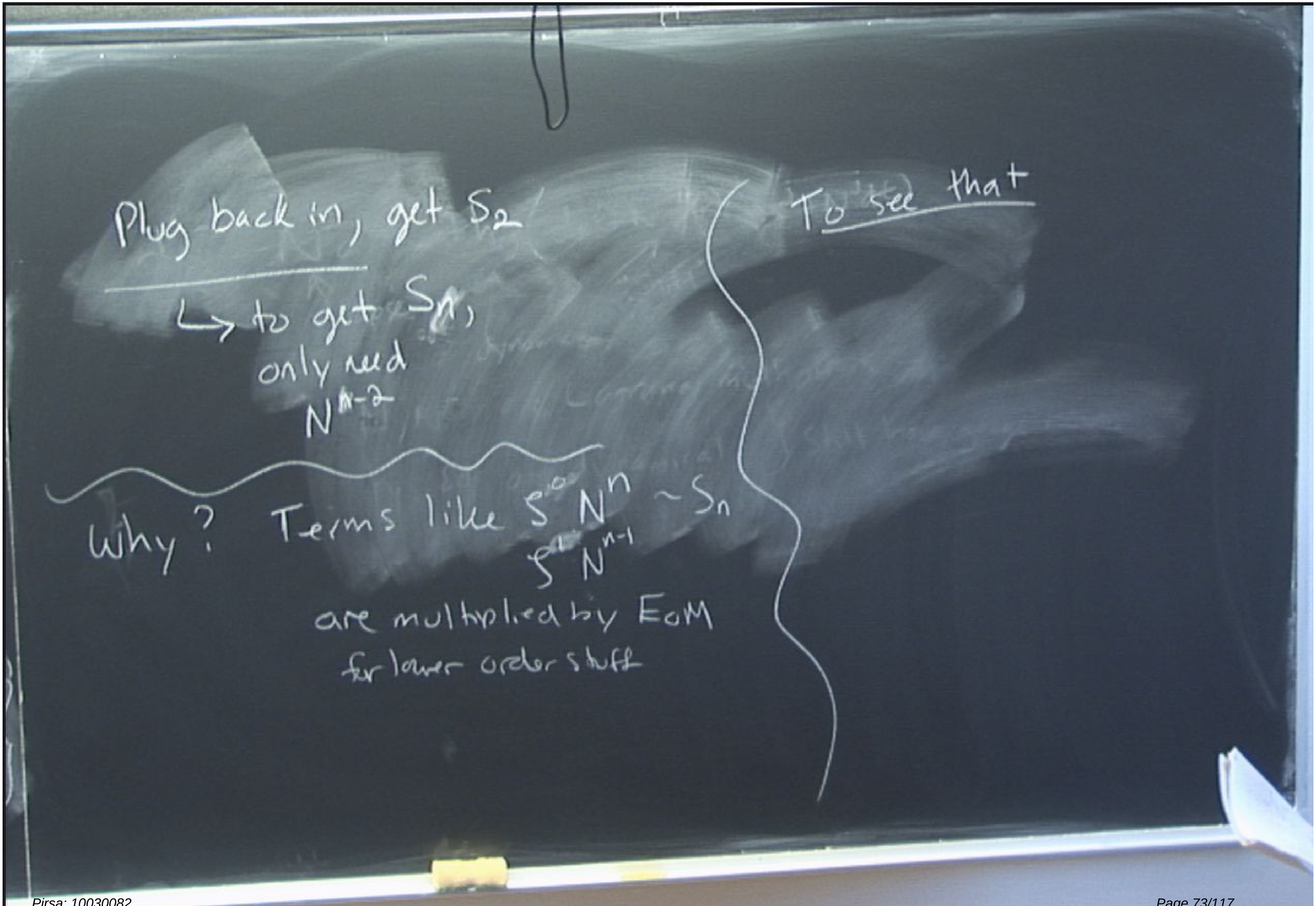
Why? Terms like $S^{(0)} N^n$
$S^{(1)} N^{n-1}$



Plug back in, get S_2

$\hookrightarrow$ to get S_n,
only need
N^{n-2}

Why? Terms like $\zeta^0 N^n \sim S_n$
$\zeta^1 N^{n-1}$
are multiplied by EoM
for lower order stuff



Plug back in, get S_2

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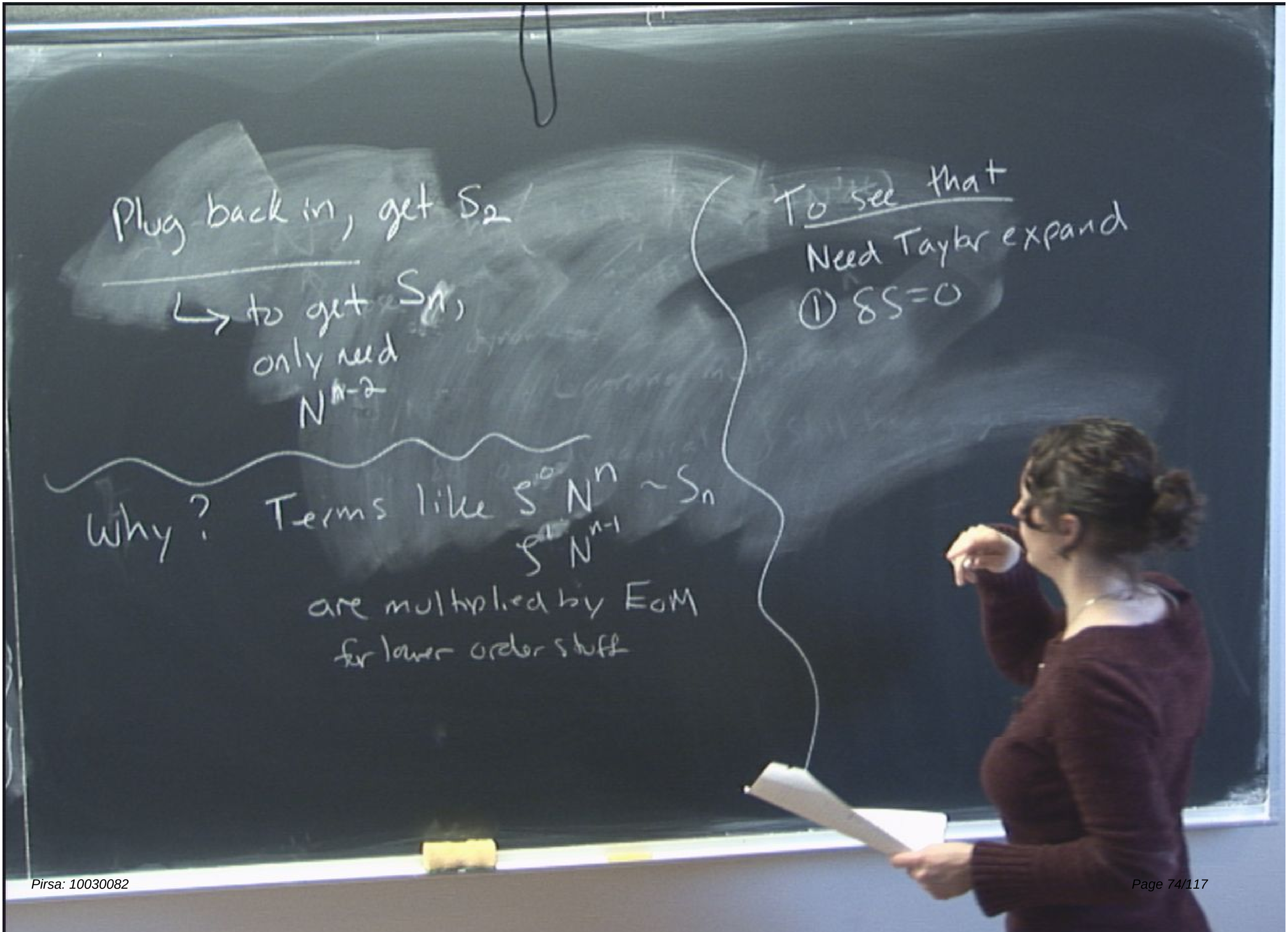
Why? Terms like $\xi^{0} N^{n} \sim S_n$

$\xi^{1} N^{n-1}$

are multiplied by EoM for lower order stuff

To see that



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Why? Terms like $\xi^0 N^n \sim S_n$
$\xi^1 N^{n-1}$
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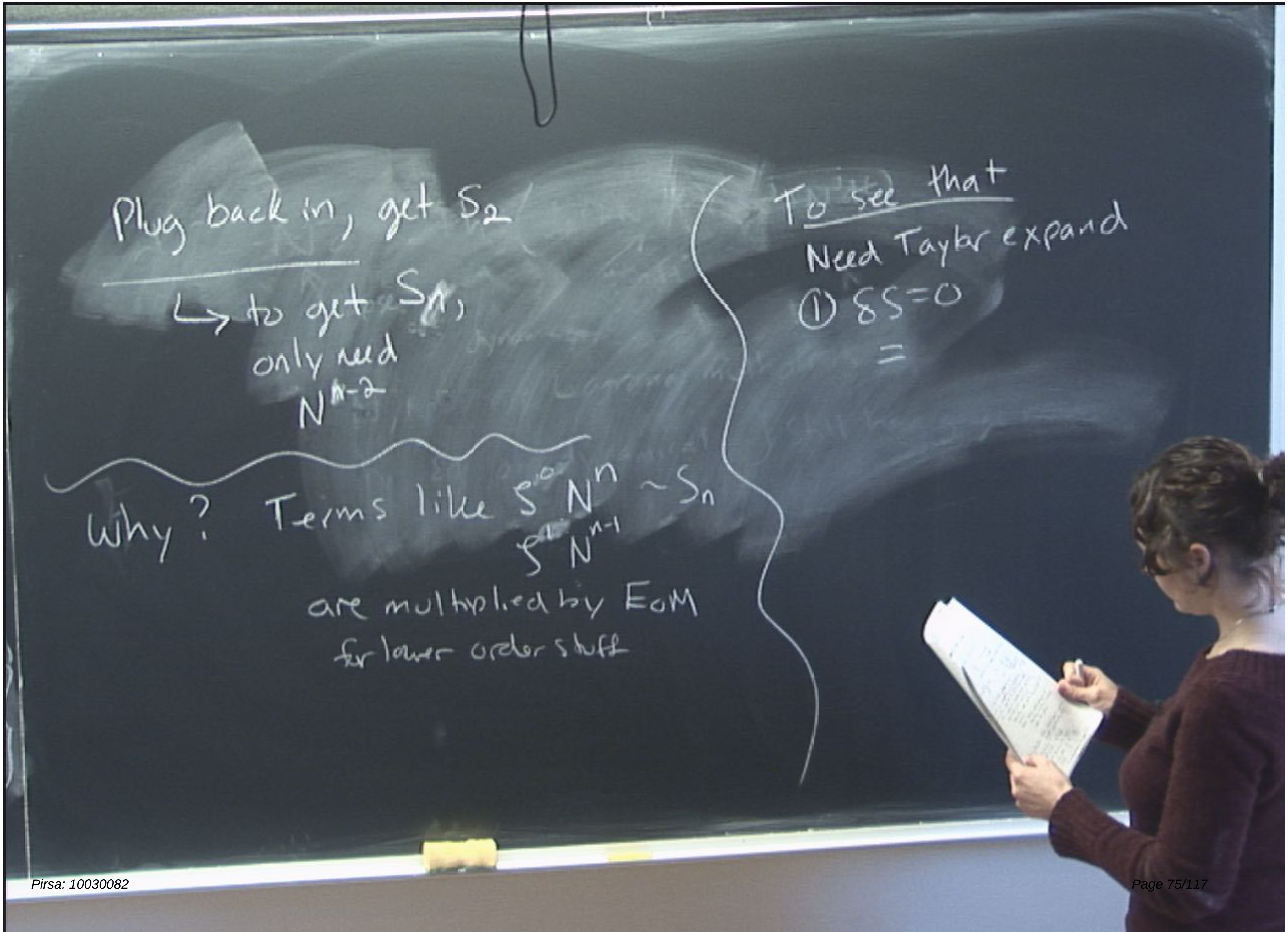
for lower order stuff

To see that

Need Taylor expand

① $\delta S = 0$



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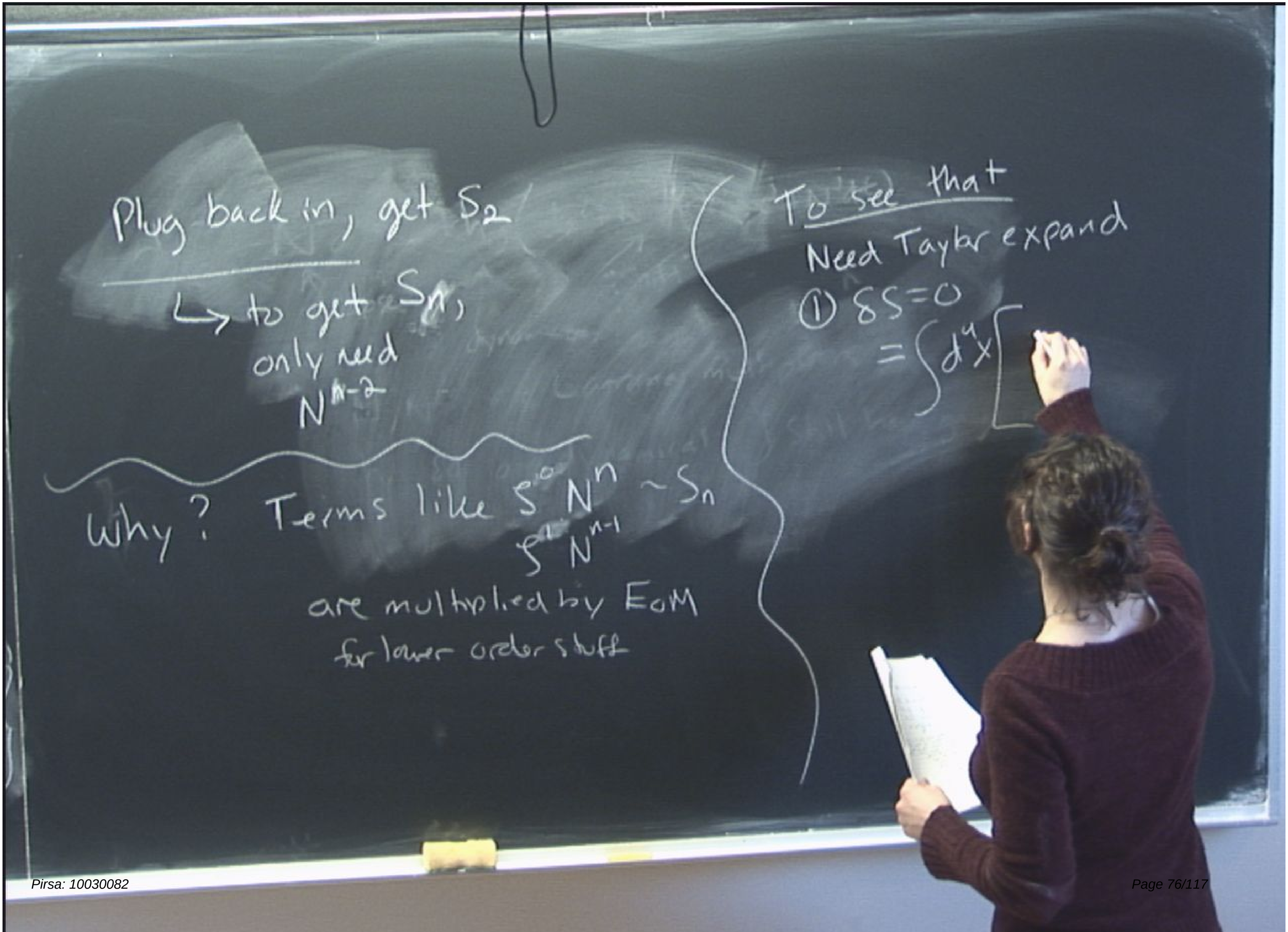
$=$



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Why? Terms like $\xi^0 N^n \sim S_n$

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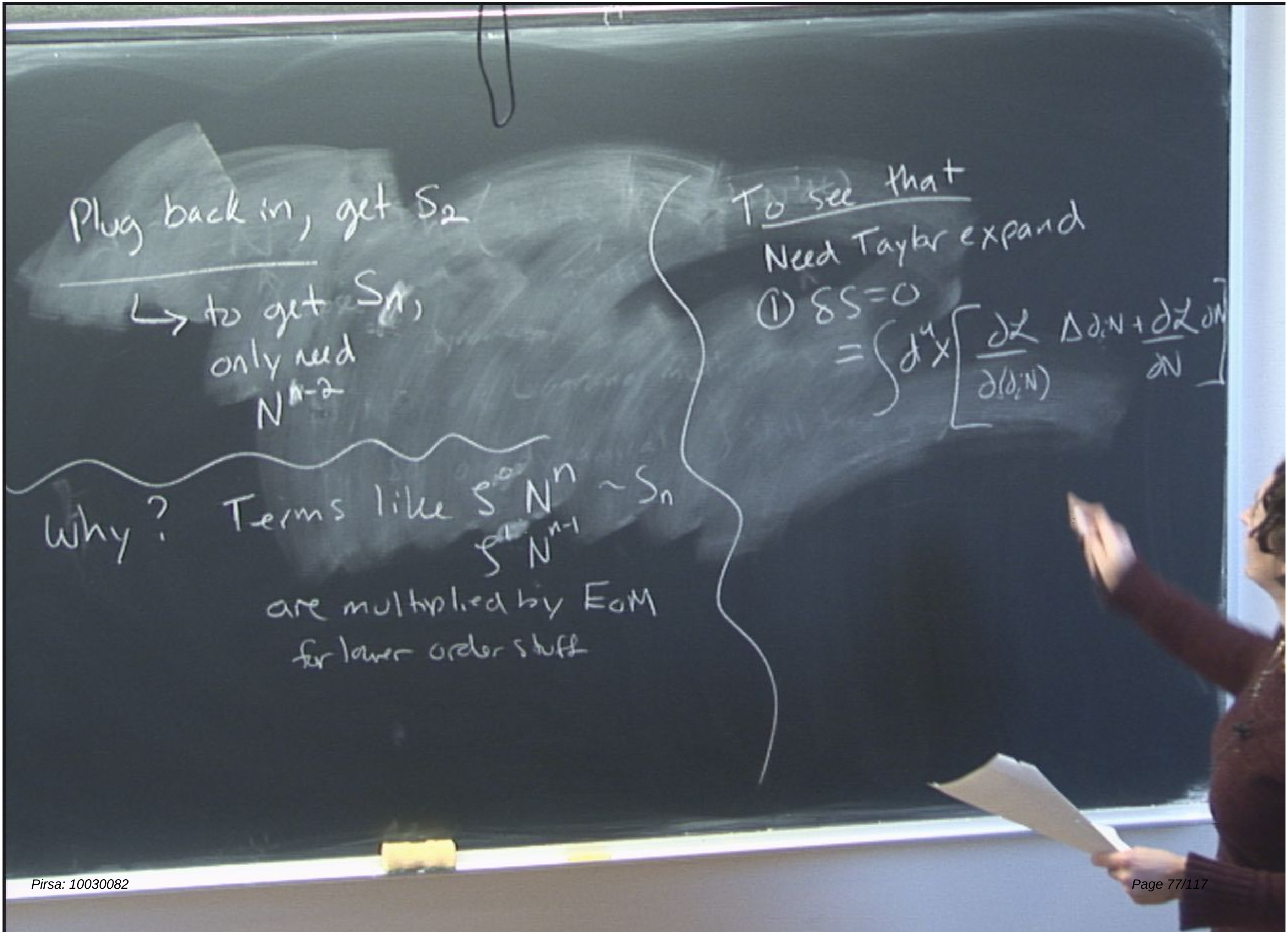
To see that

Need Taylor expand

① $\delta S = 0$

$= \int d^4x \Big[$



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Why? Terms like $\xi^{0} N^{n} \sim S_n$

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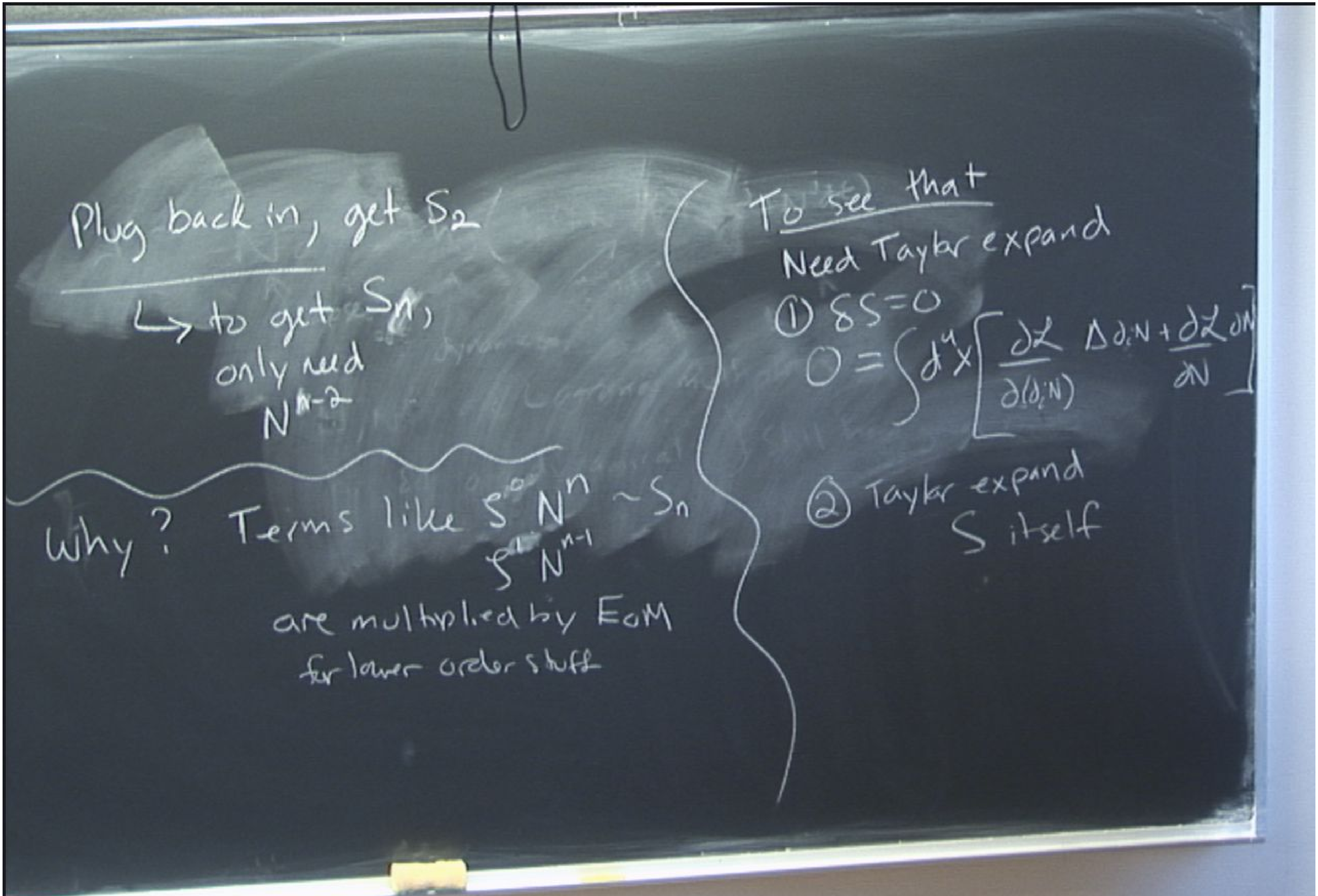
To see that

Need Taylor expand

① $\delta S = 0$

$$= \int d^4x \left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)} \Delta \partial_i N + \frac{\partial \mathcal{L}}{\partial N} \delta N \right]$$



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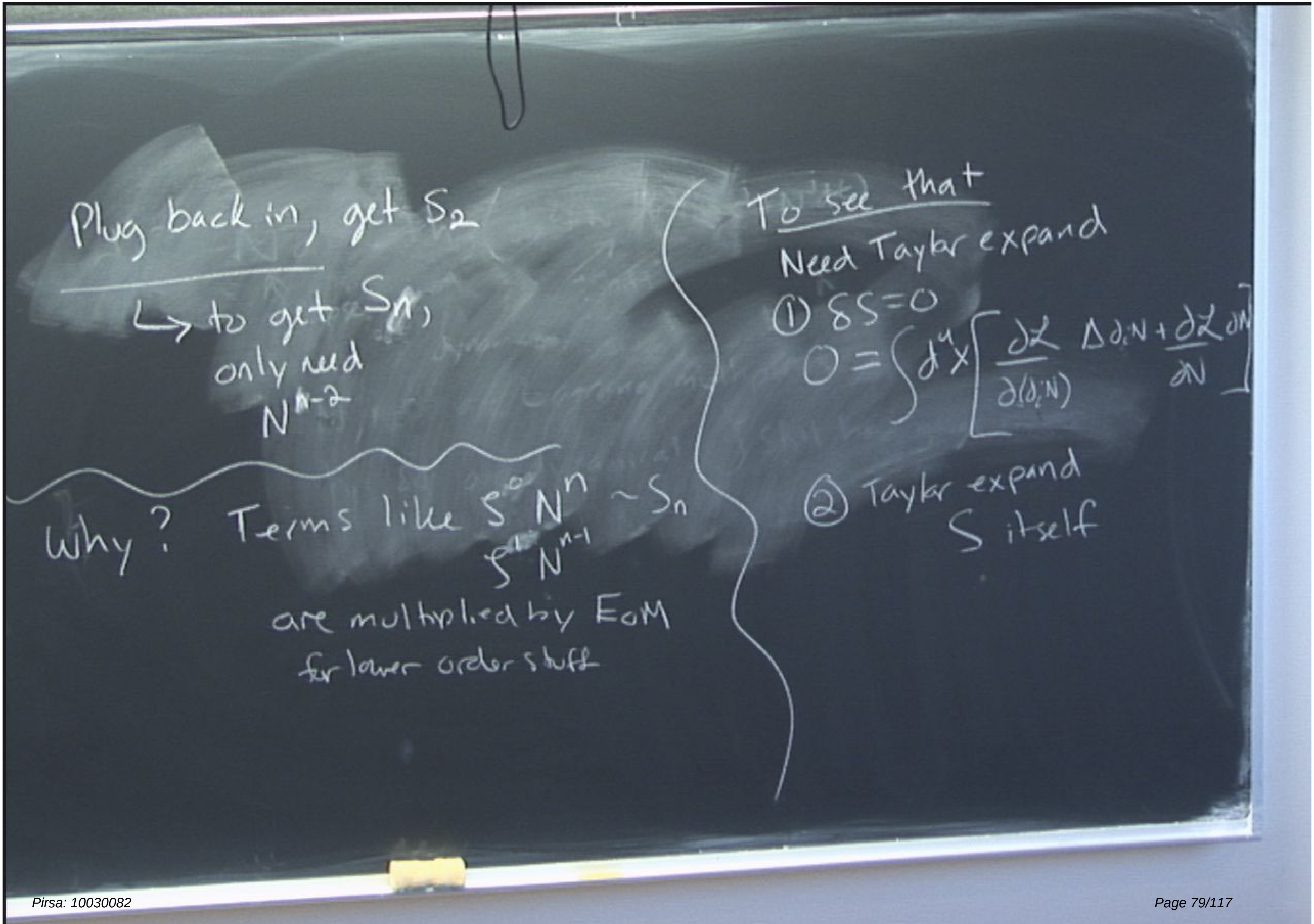
① $\delta S = 0$

$$0 = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)} \Delta \partial_i N + \frac{\partial \mathcal{L}}{\partial N} \partial N \right]$$

② Taylor expand S itself



Plug back in, get S_2

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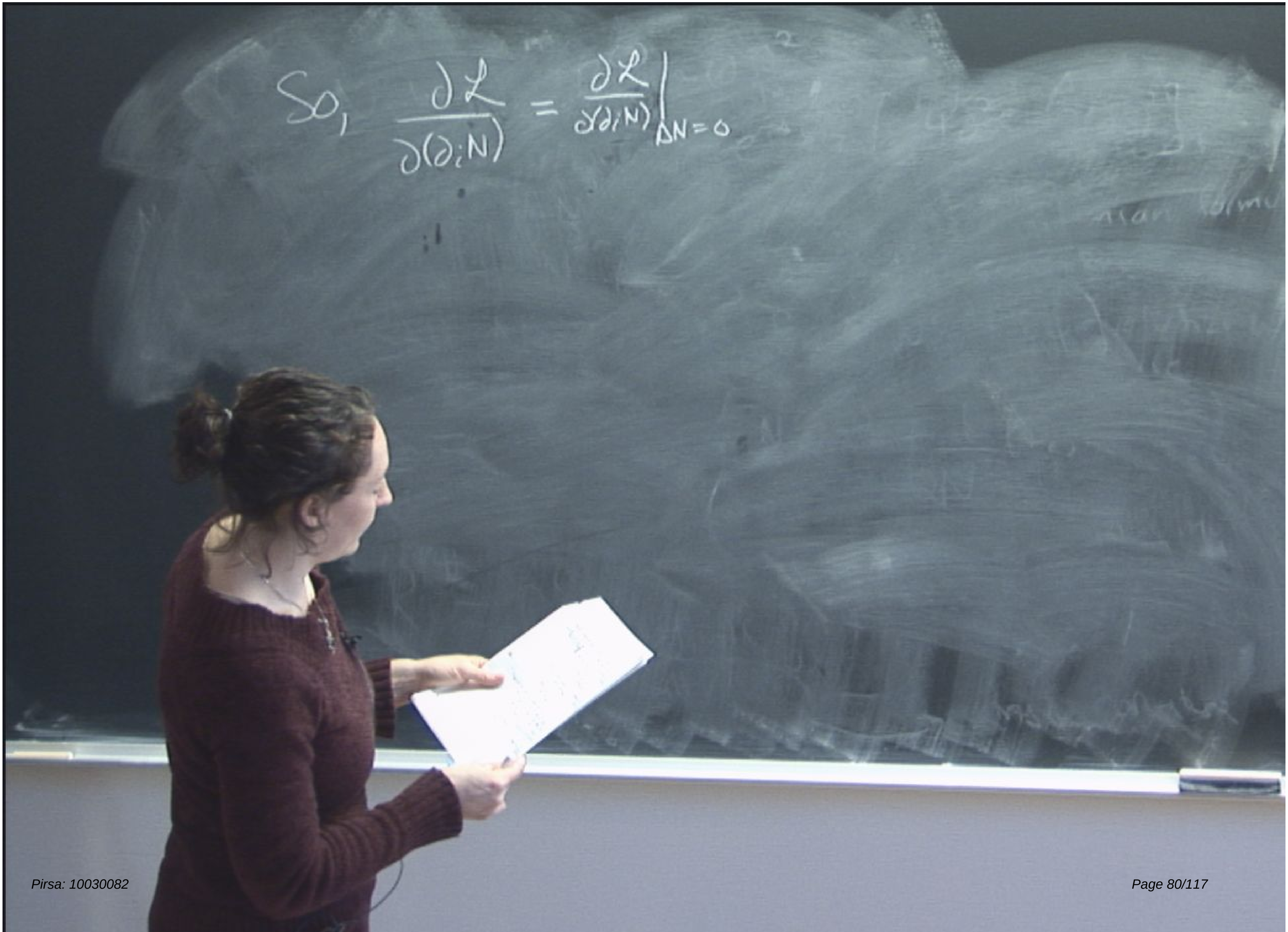
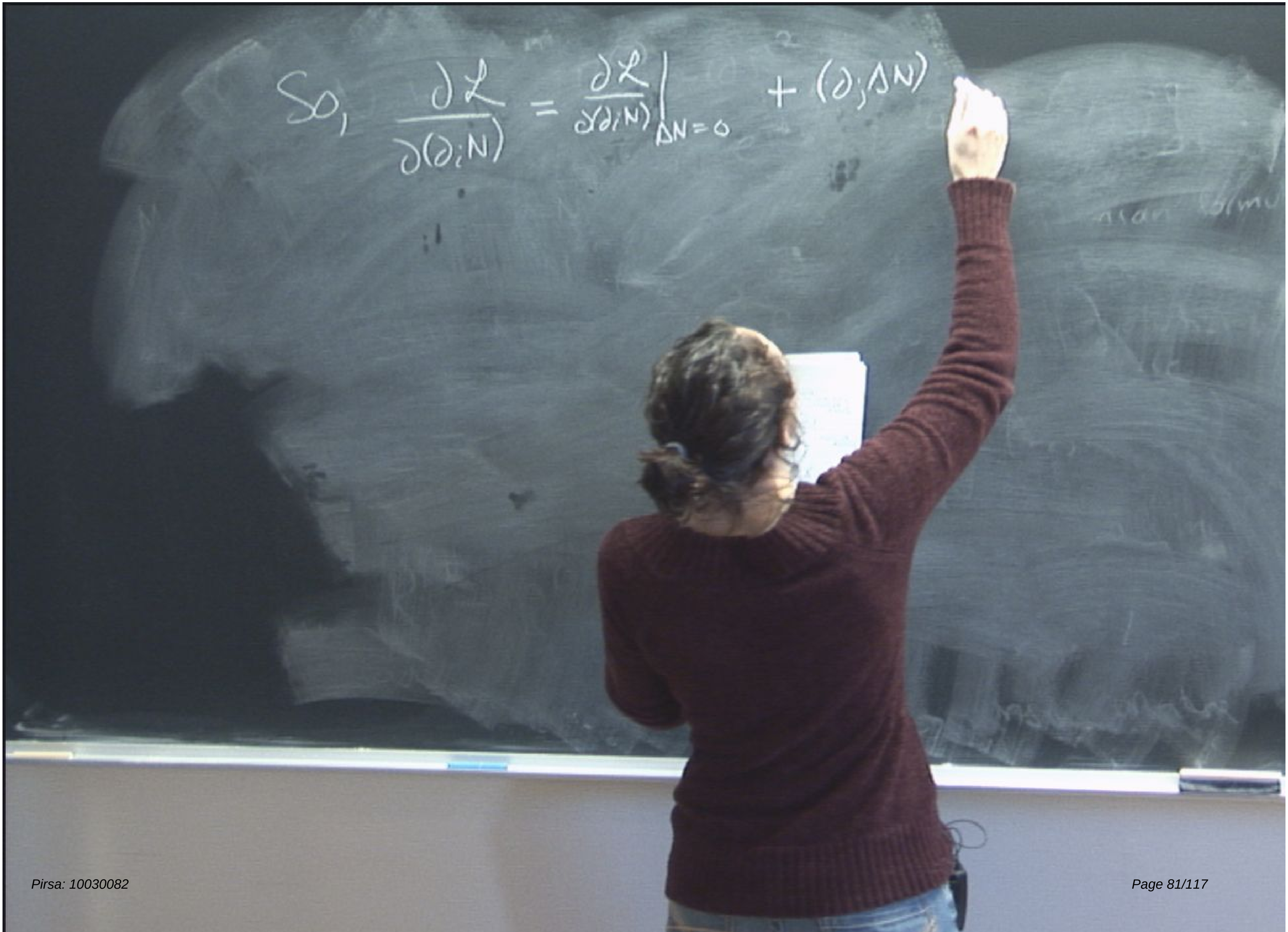
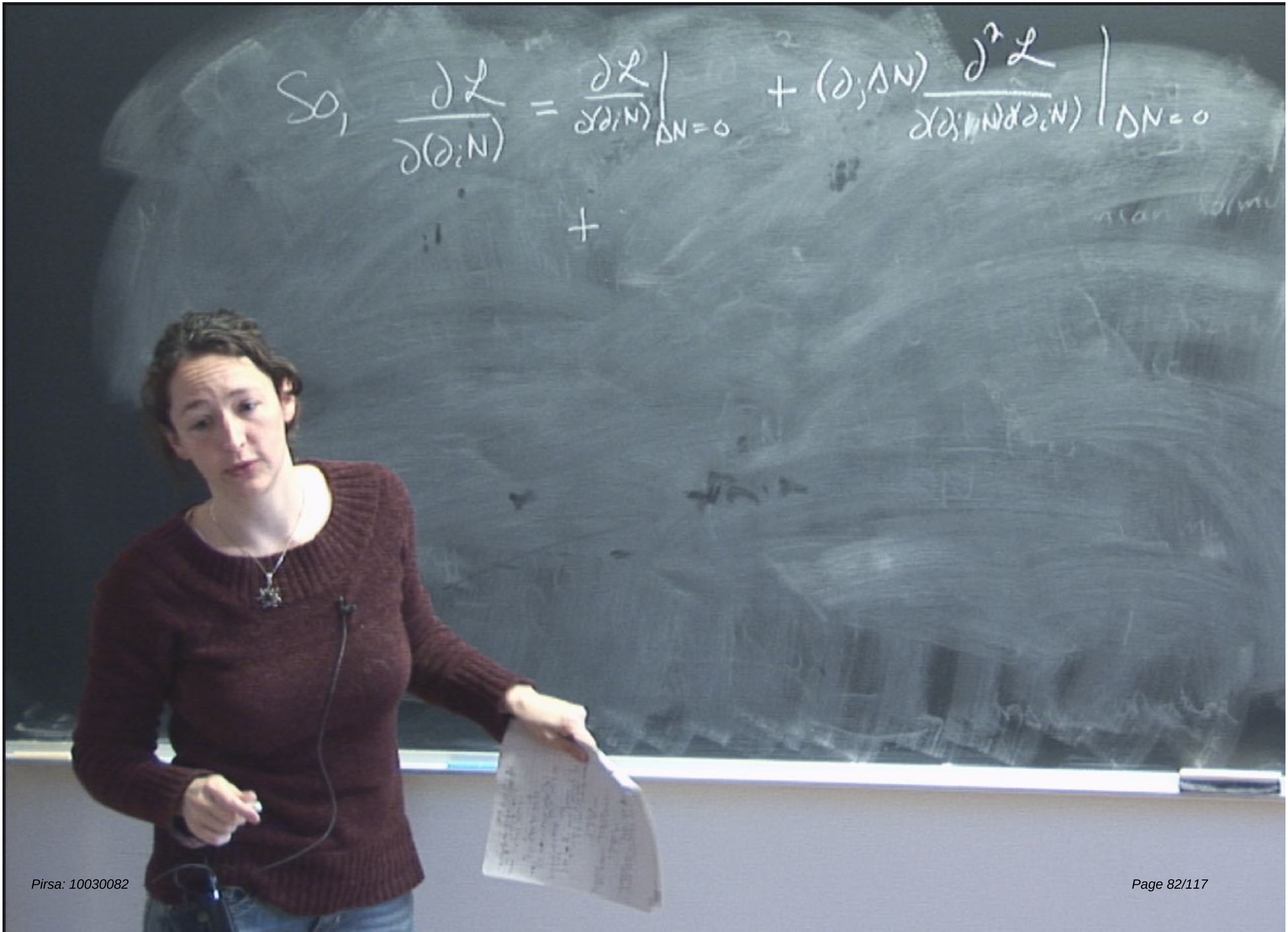
② Taylor expand
S itself



So, $\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0}$

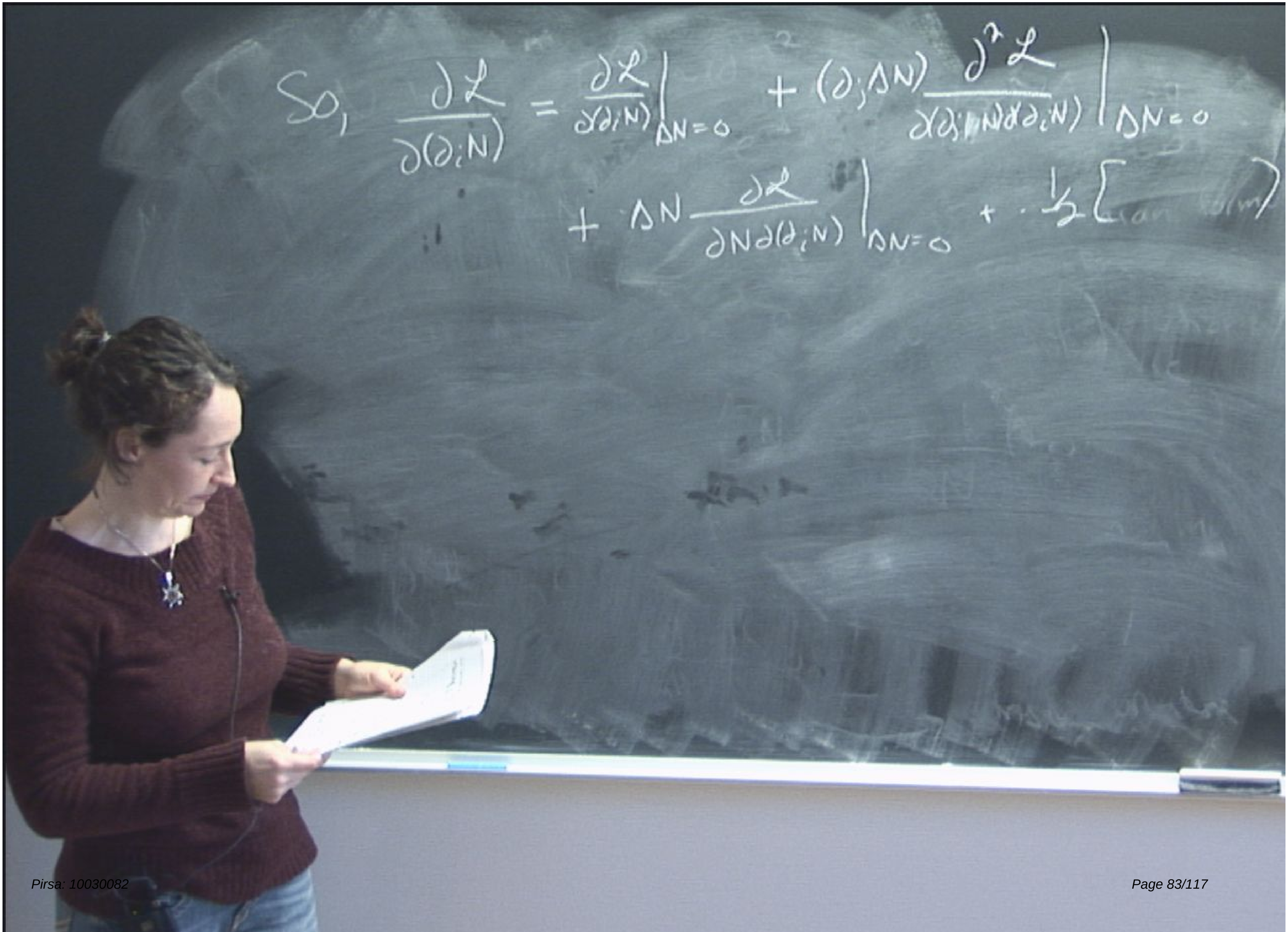


So, $\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)$



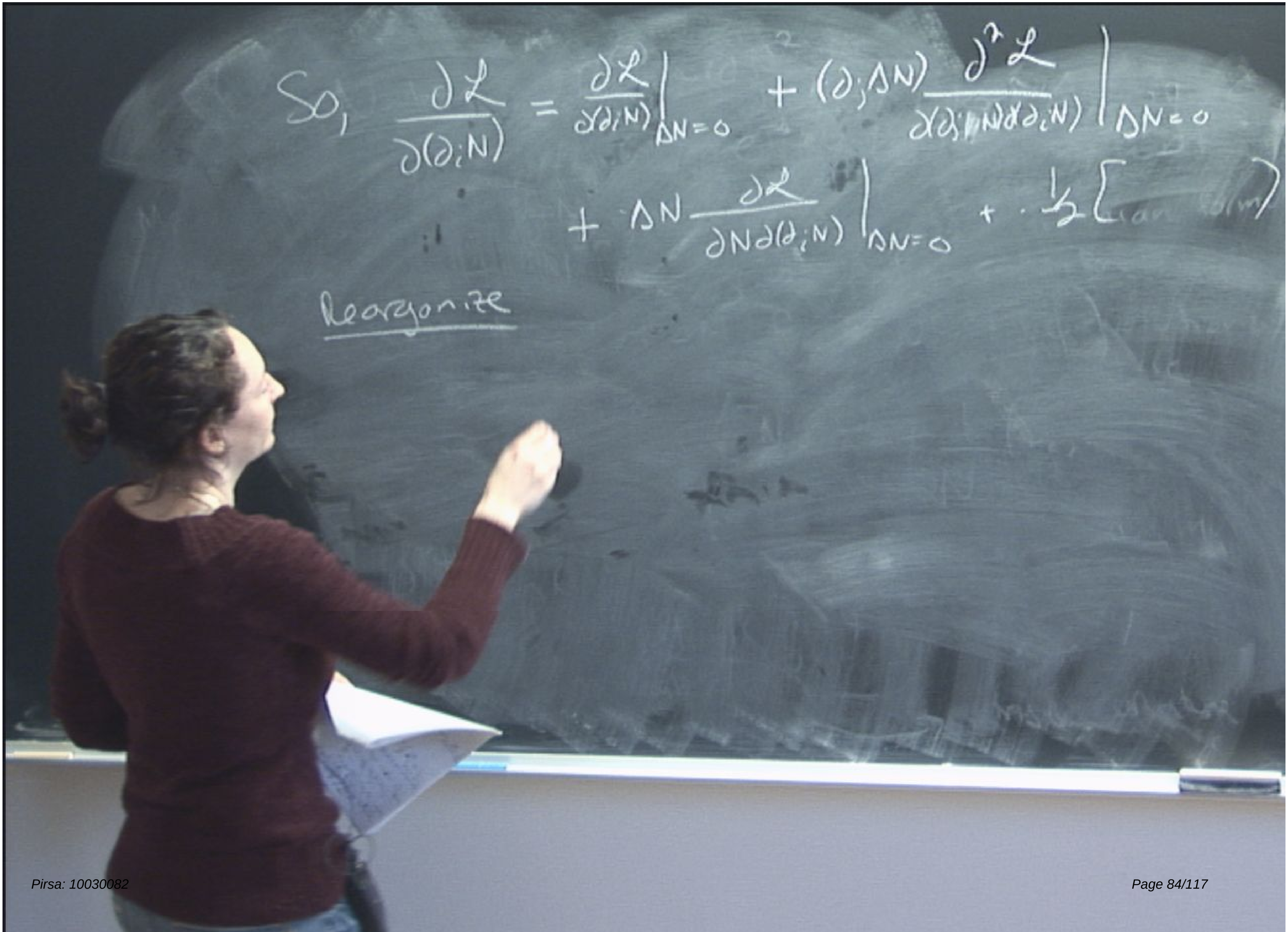
So, $\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\Big|_{\Delta N=0}$

$+$



So, $$\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\Big|_{\Delta N=0}$$

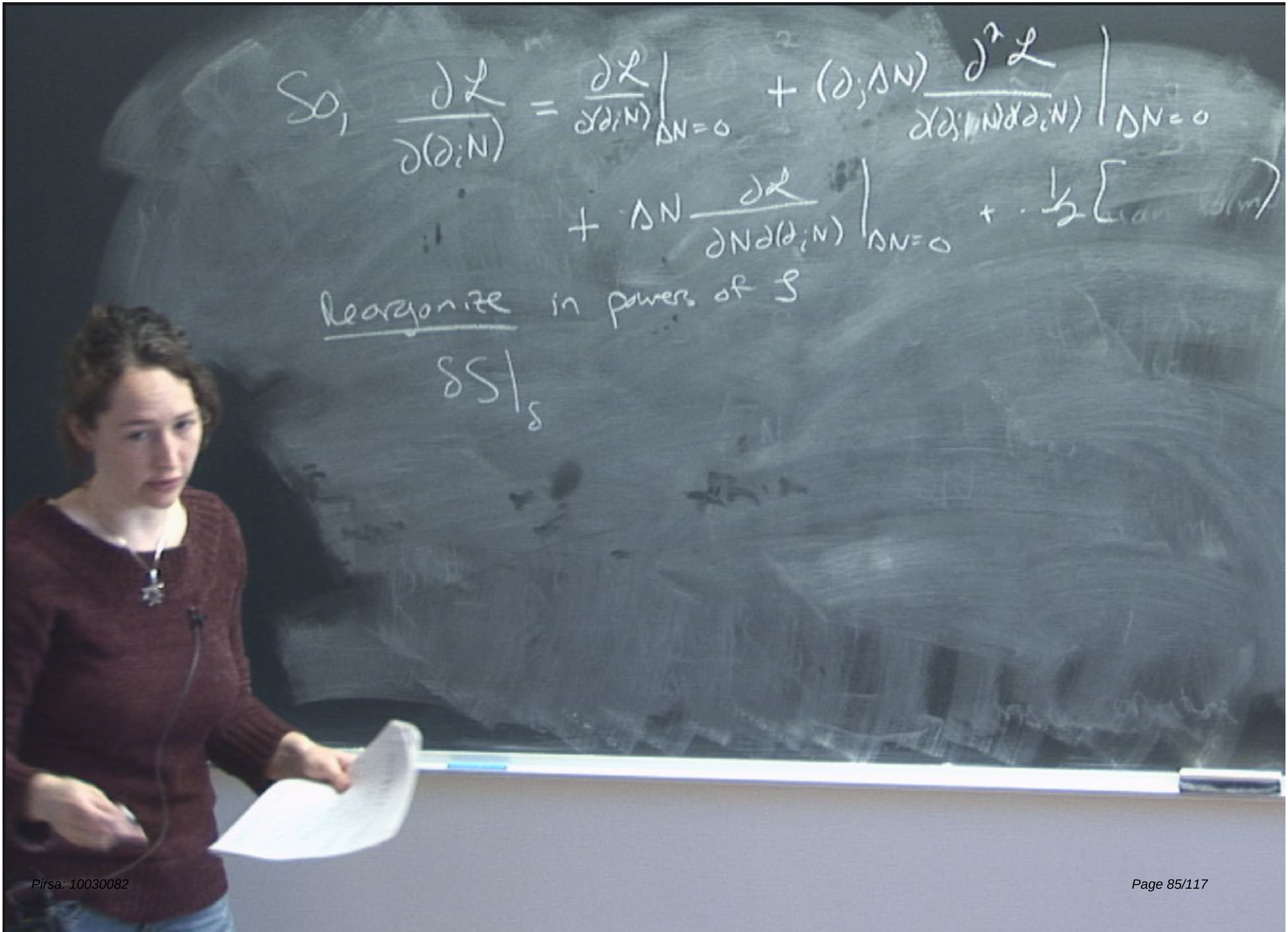
$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\Big|_{\Delta N=0} + \frac{1}{2}\Big[$$ [illegible] $$\Big)$$



$$\text{So, } \frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\bigg|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\bigg|_{\Delta N=0}$$

$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\bigg|_{\Delta N=0} + \frac{1}{2}\Big[$$ [illegible]

Reorganize

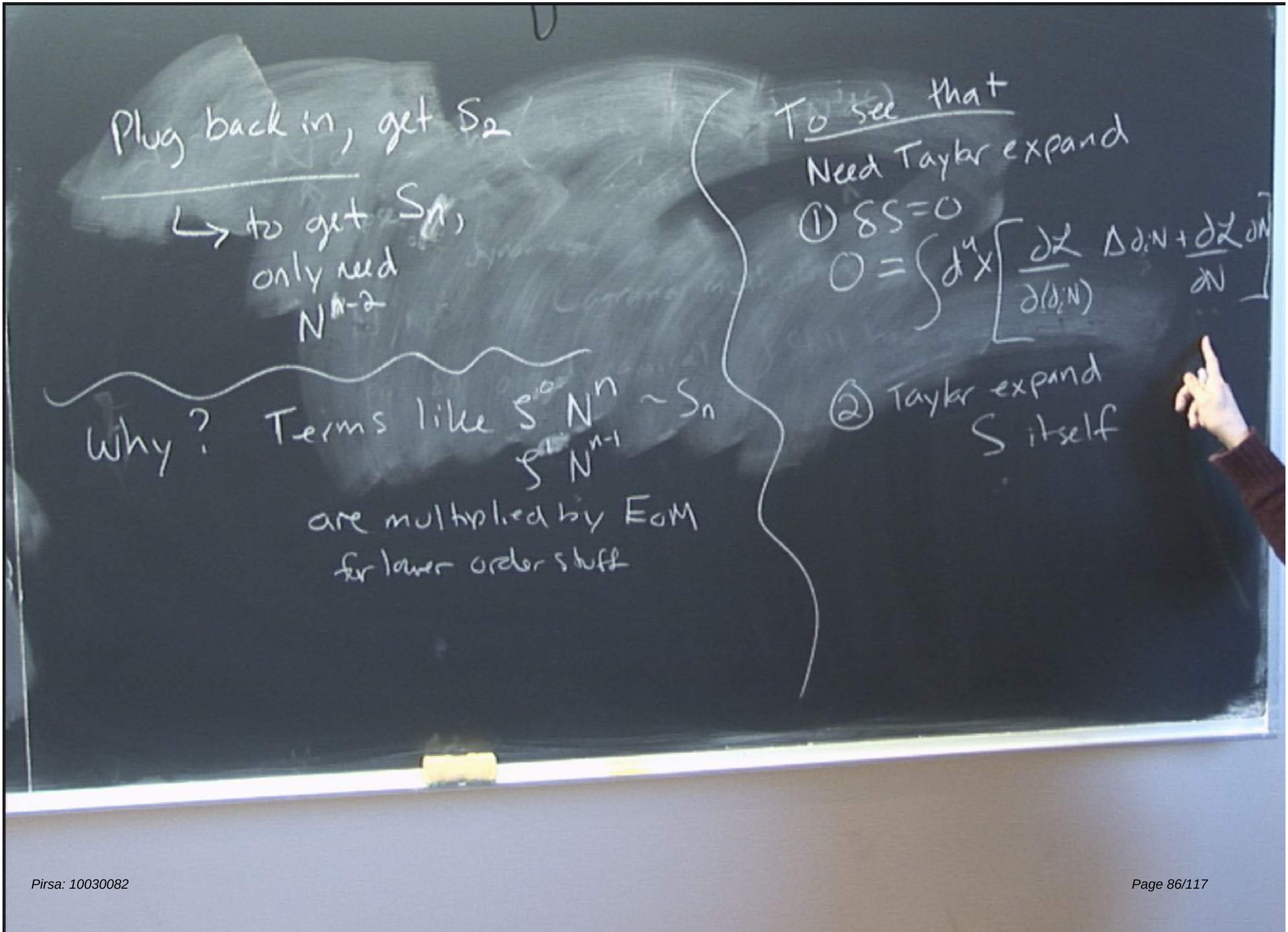


So, $$\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\bigg|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\bigg|_{\Delta N=0}$$

$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\bigg|_{\Delta N=0} + \tfrac{1}{2}\big[\quad \big)$$

<u>Reorganize</u> in powers of δ

$$\delta S\big|_{\delta}$$



Plug back in, get S_2

↳ to get S_n, only need N^{n-2}

Why? Terms like $\xi^0 N^n \sim S_n$

$\xi^1 N^{n-1}$

are multiplied by EoM for lower order stuff

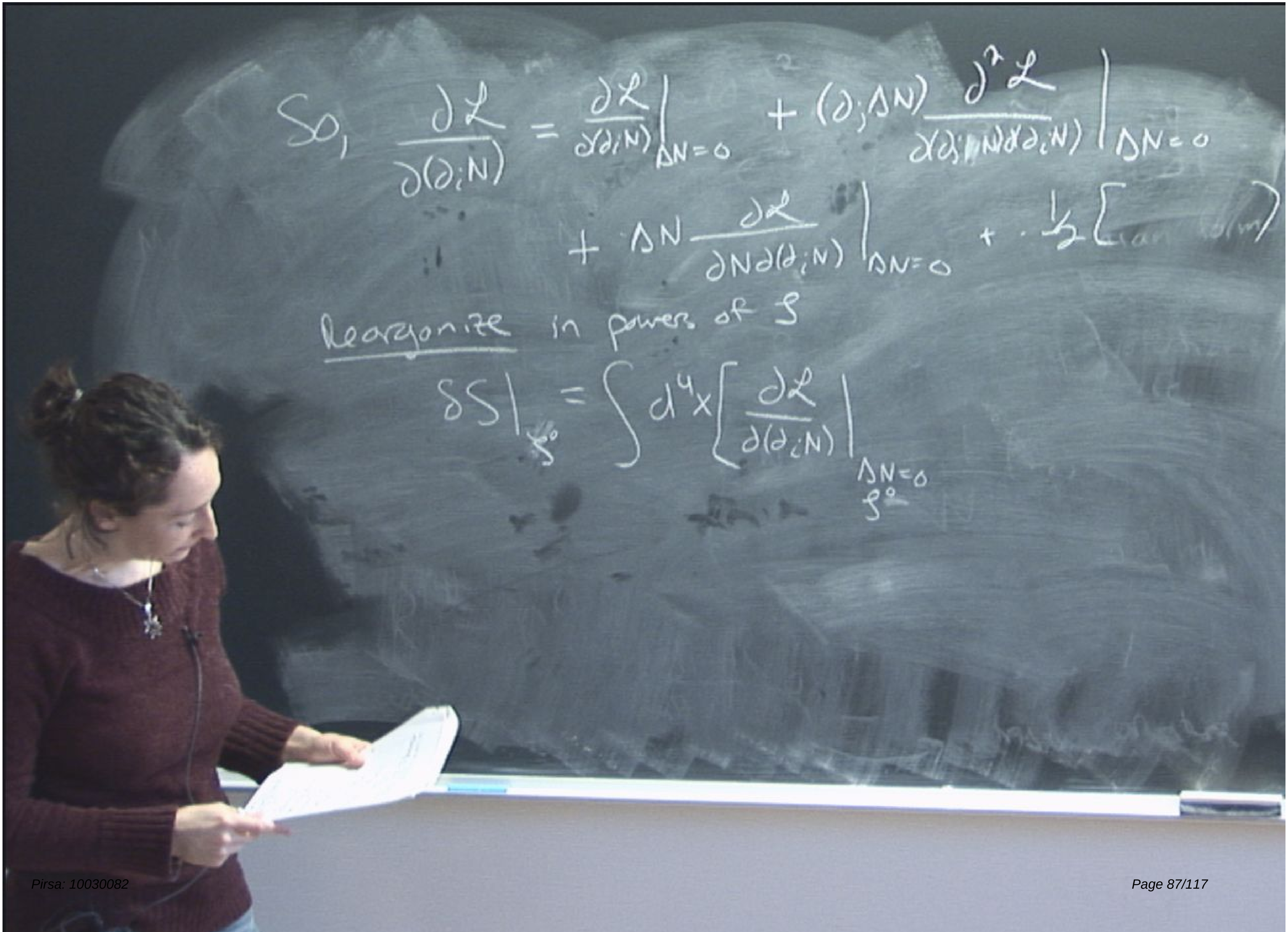
To see that

Need Taylor expand

① $\delta S = 0$

$$0 = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)} \Delta \partial_i N + \frac{\partial \mathcal{L}}{\partial N} \Delta N \right]$$

② Taylor expand S itself

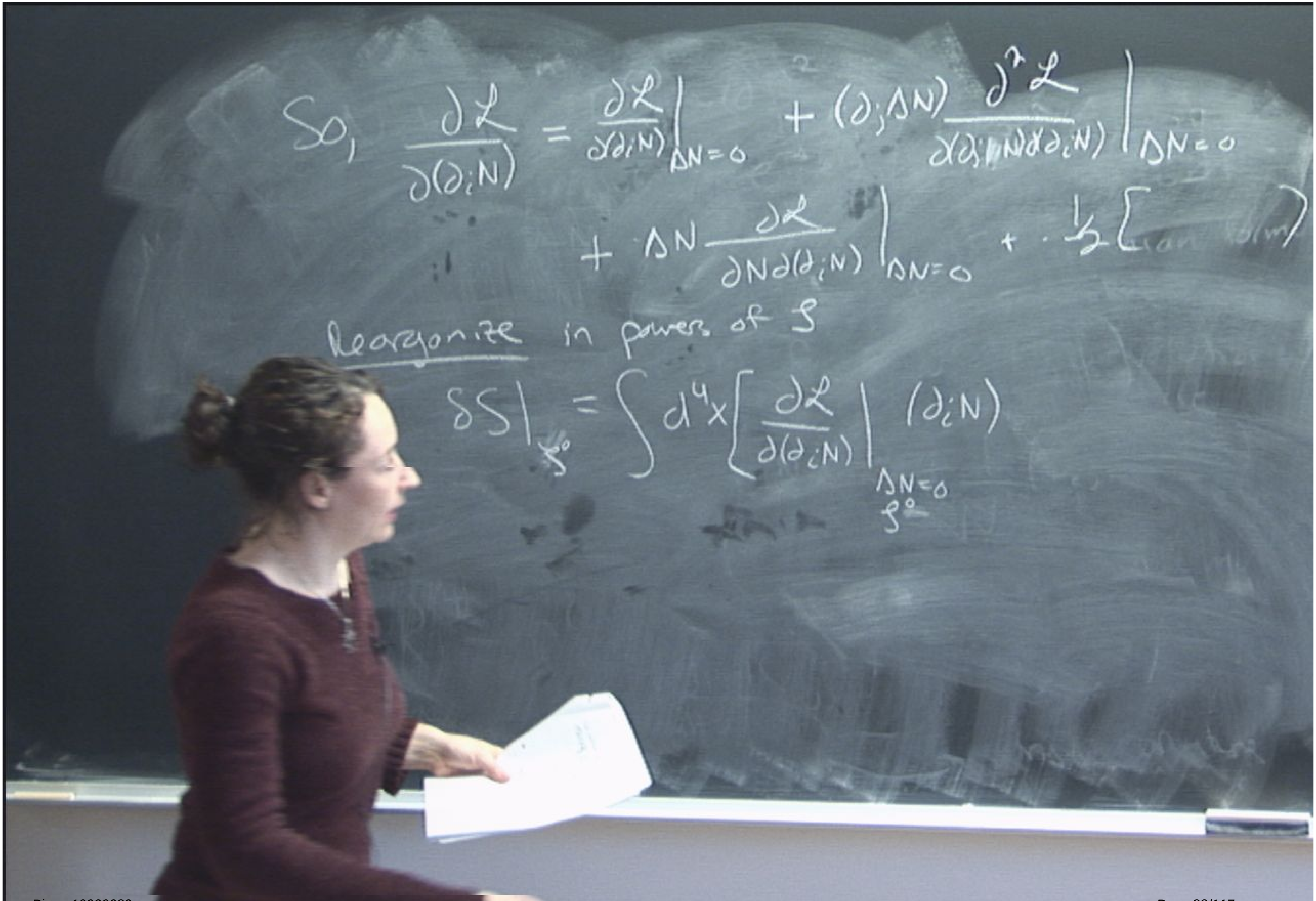


$$\text{So, } \frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\Big|_{\Delta N=0}$$

$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\Big|_{\Delta N=0} + \frac{1}{2}\Big[$$ [illegible] $$\Big)$$

<u>Reorganize</u> in powers of ξ

$$\delta S\Big|_{\xi^0} = \int d^4x \Big[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\substack{\Delta N=0 \\ \xi^0}}$$

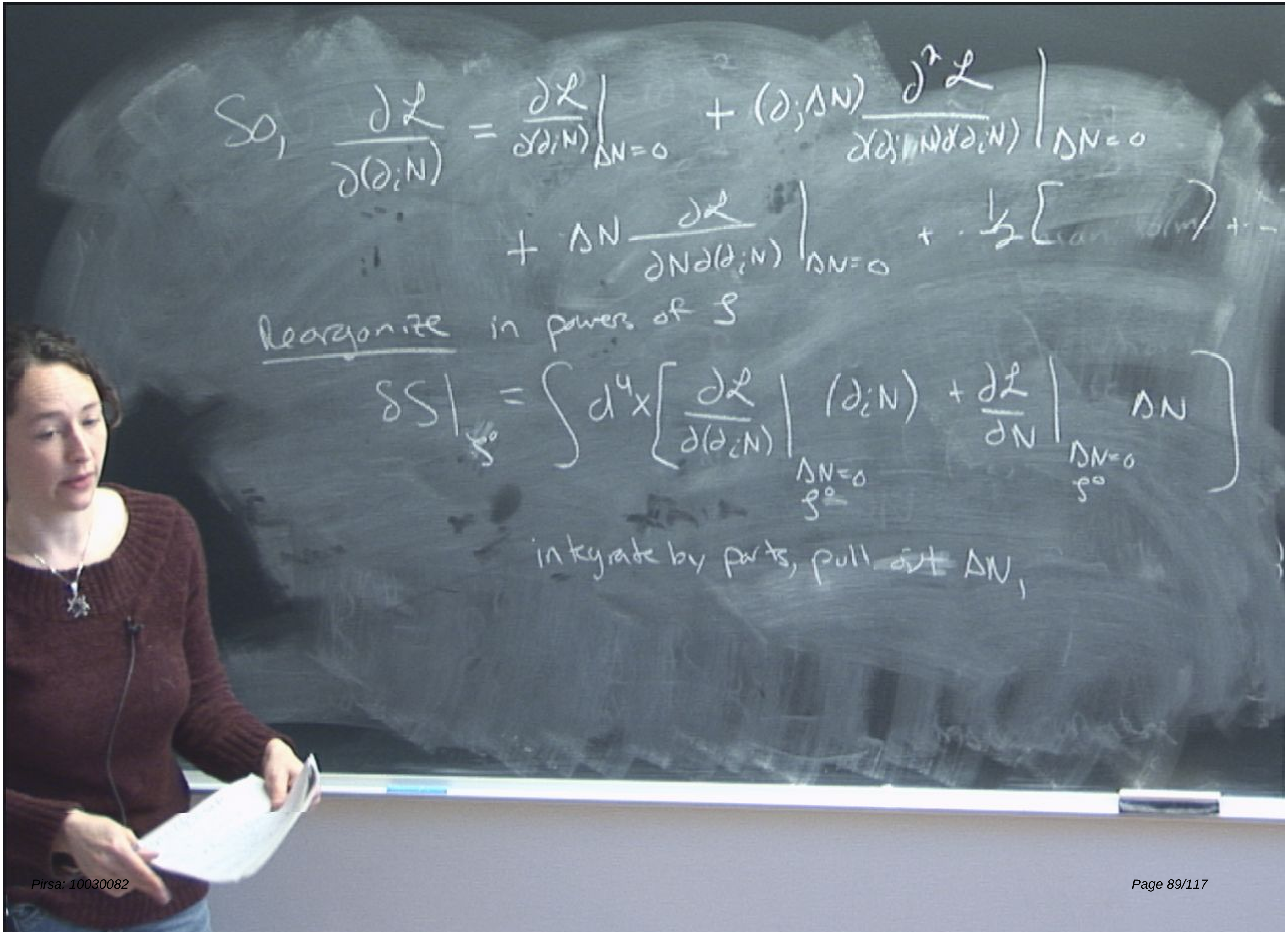


So, $$\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\Big|_{\Delta N=0}$$

$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\Big|_{\Delta N=0} + \frac{1}{2}\Big[\text{[illegible]} \Big)$$

Reargonize in powers of ξ

$$\delta S\Big|_{\xi^0} = \int d^4x \Big[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\substack{\Delta N=0 \\ \xi^0}} (\partial_i N)$$

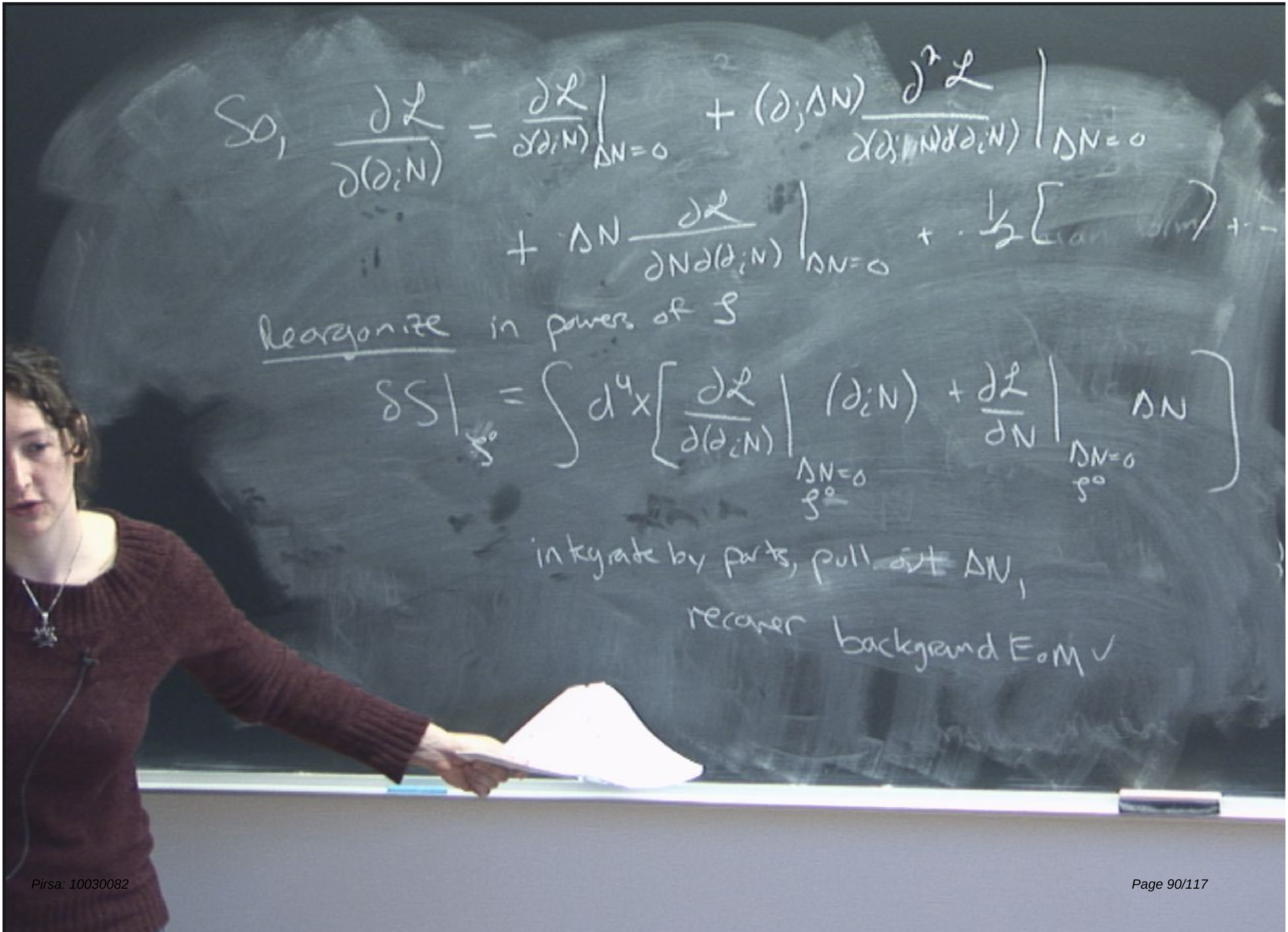


So,
$$\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\bigg|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\bigg|_{\Delta N=0}$$
$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\bigg|_{\Delta N=0} + \frac{1}{2}\big[\quad\big) + \cdots$$

<u>Reorganize</u> in powers of ξ

$$\delta S\big|_{\xi^0} = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\bigg|_{\substack{\Delta N=0 \\ \xi^0}} (\partial_i N) + \frac{\partial \mathcal{L}}{\partial N}\bigg|_{\substack{\Delta N=0 \\ \xi^0}} \Delta N \right]$$

integrate by parts, pull out ΔN,



So, $$\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\Big|_{\Delta N=0}$$

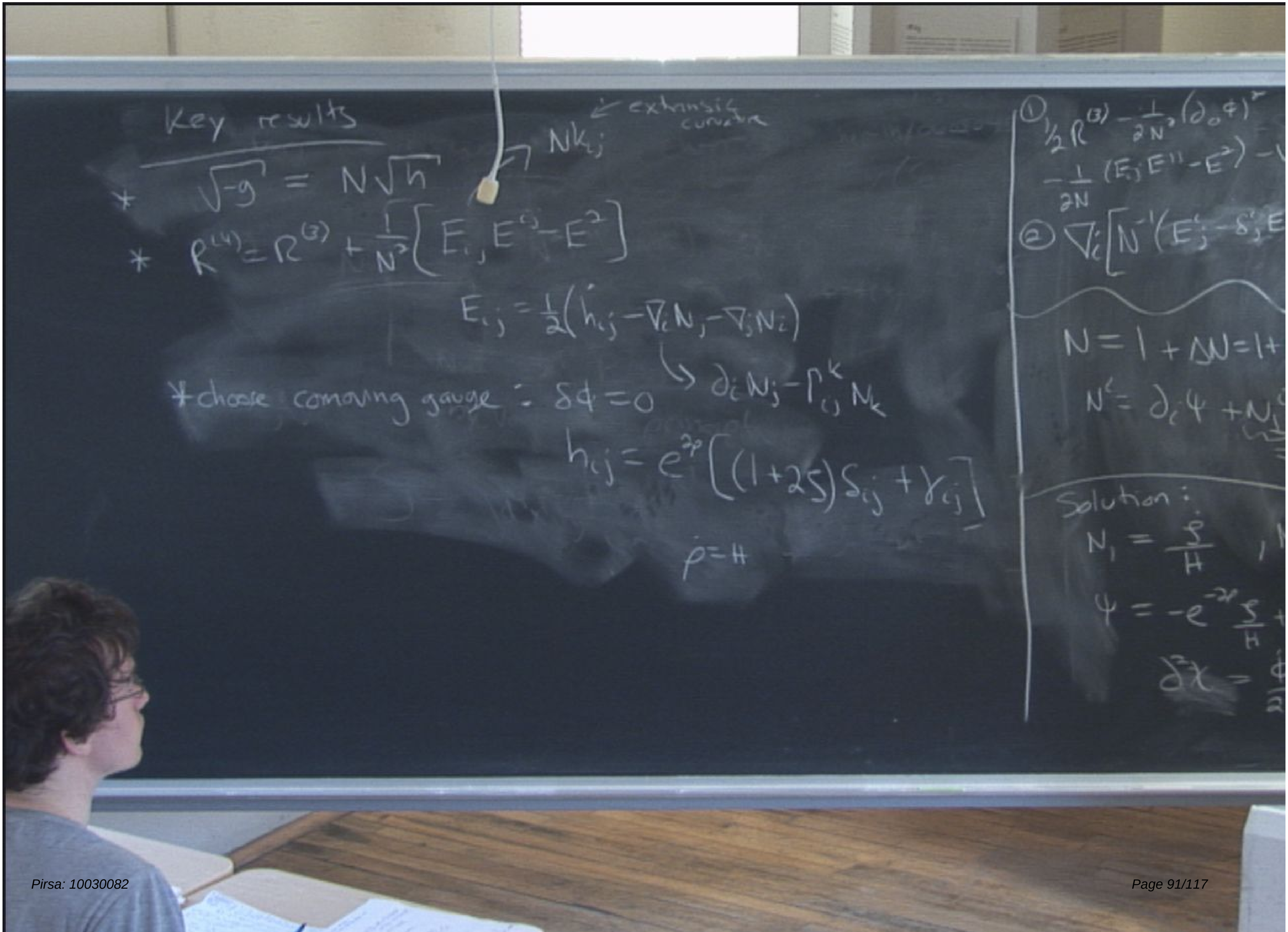
$$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\Big|_{\Delta N=0} + \cdot \tfrac{1}{2}\Big[\quad\Big) + \cdots$$

Reorganize in powers of ρ

$$\delta S\Big|_{\rho^0} = \int d^4x\left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\substack{\Delta N=0\\ \rho^0}} (\partial_i N) + \frac{\partial \mathcal{L}}{\partial N}\Big|_{\substack{\Delta N=0\\ \rho^0}} \Delta N\right]$$

integrate by parts, pull ~~out~~ ΔN,

recover background EoM ✓



Key results

$* \quad \sqrt{-g} = N\sqrt{h}$

$* \quad R^{(4)} = R^{(3)} + \frac{1}{N^2}\left[E_{ij}E^{ij} - E^2\right]$

$\nearrow Nk_{ij} \swarrow$ extrinsic curvature

$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

$\hookrightarrow \partial_i N_j - \Gamma^k_{ij} N_k$

$*$ choose comoving gauge : $\delta\phi = 0$

$h_{ij} = e^{2\rho}\left[(1+2\zeta)\delta_{ij} + \gamma_{ij}\right]$

$\dot{\rho} = H$

① $\frac{1}{2}R^{(3)} - \frac{1}{2N^2}(\partial_0\phi)^2$

$-\frac{1}{2N}(E_{ij}E^{ij} - E^2) -$

② $\nabla_i\left[N^{-1}(E^i_j - \delta^i_j E\right.$

$N = 1 + \Delta N = 1 +$

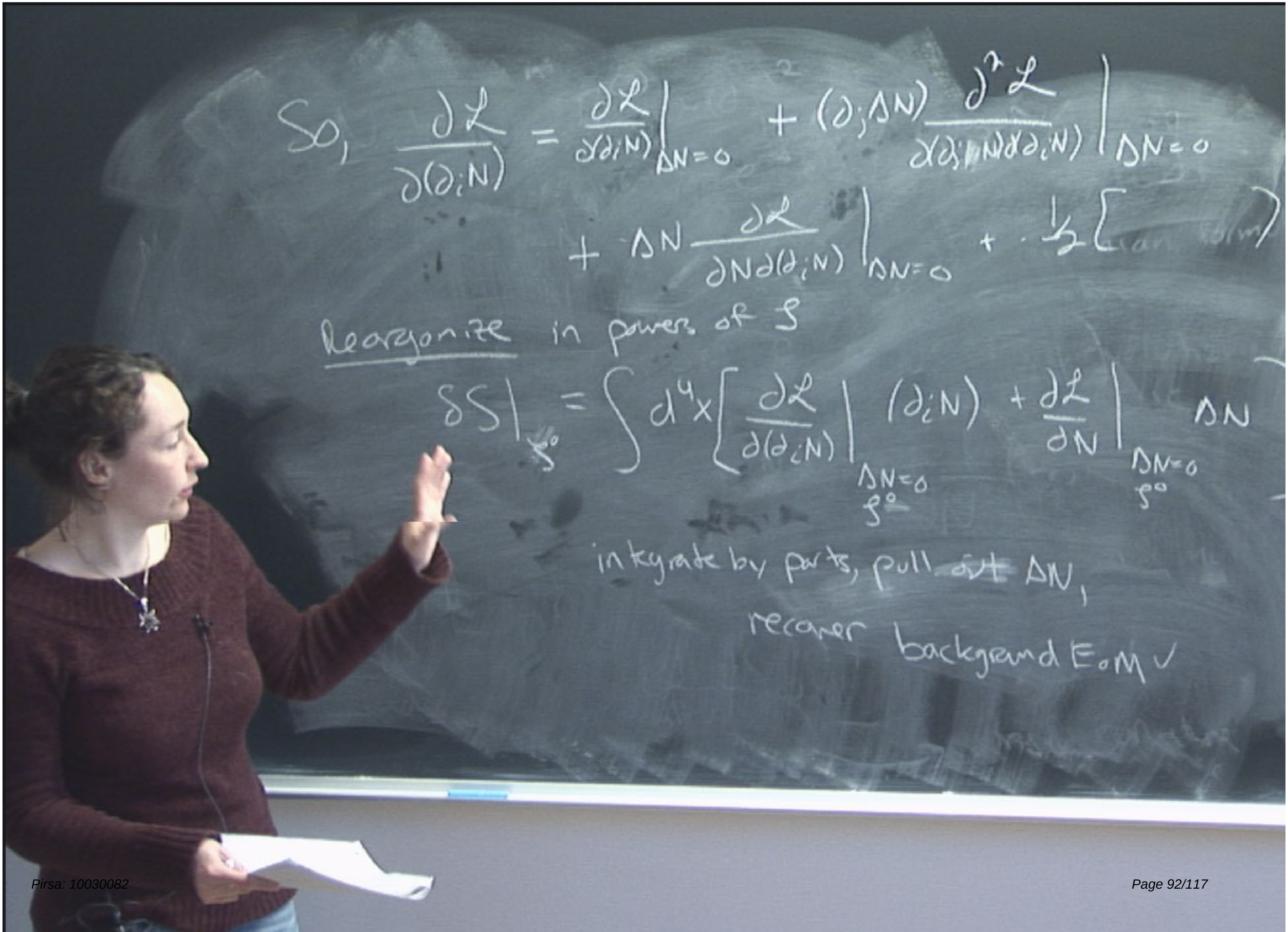
$N^i = \partial_i \psi + N_1$

Solution :

$N_1 = \frac{\dot{\zeta}}{H}$,

$\psi = -e^{-2\rho}\frac{\zeta}{H} +$

$\partial^2\chi = \frac{\dot{\phi}}{2}$



So, $\frac{\partial \mathcal{L}}{\partial(\partial_i N)} = \frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\Delta N=0} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\partial(\partial_i N)}\Big|_{\Delta N=0}$

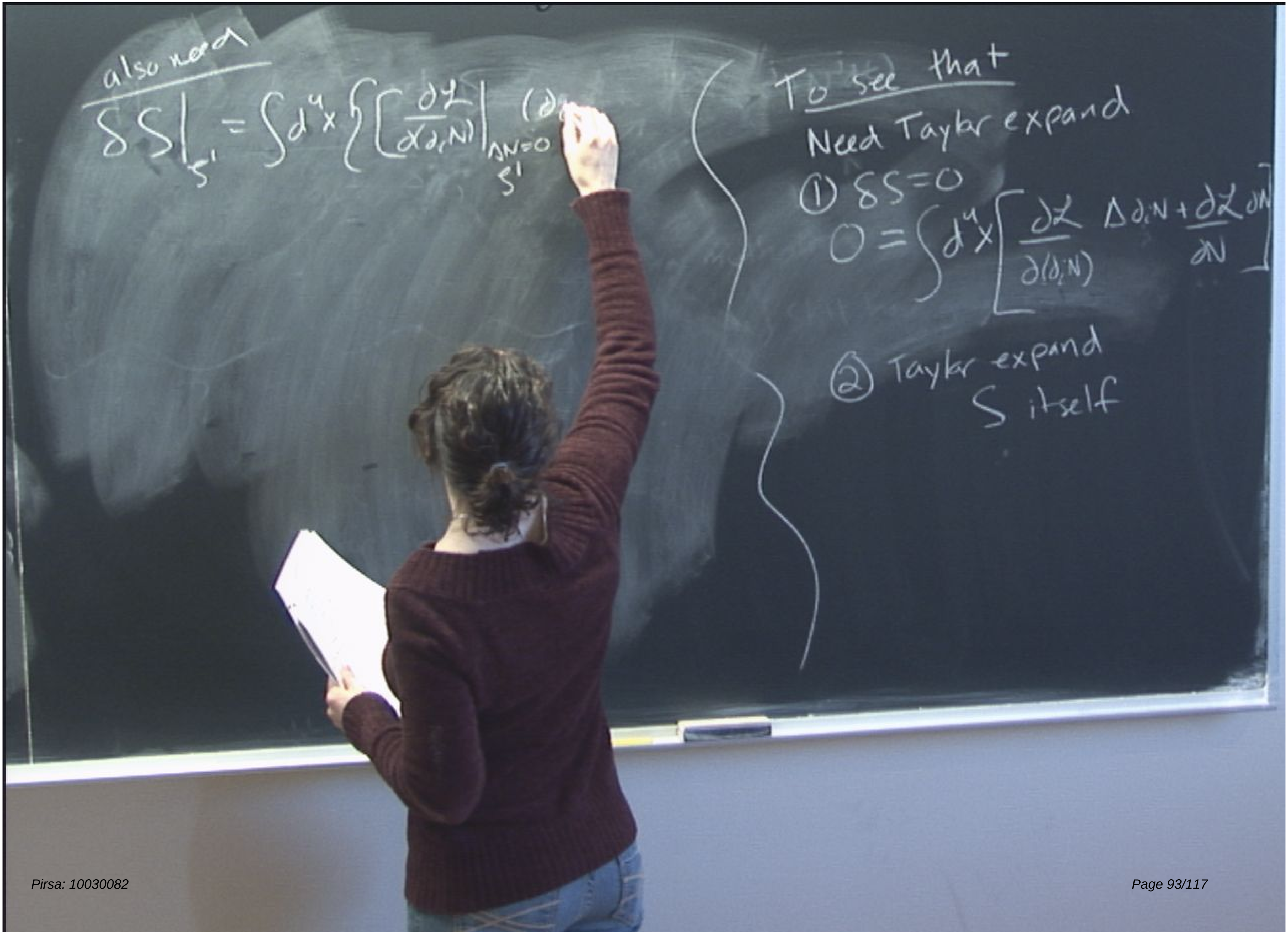
$+ \Delta N \frac{\partial \mathcal{L}}{\partial N \partial(\partial_i N)}\Big|_{\Delta N=0} + \frac{1}{2}\Big[$ [illegible] $\Big)$

<u>Reorganize</u> in powers of ξ

$$\delta S\Big|_{\xi^0} = \int d^4x \Big[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\substack{\Delta N=0 \\ \xi^0}} (\partial_i N) + \frac{\partial \mathcal{L}}{\partial N}\Big|_{\substack{\Delta N=0 \\ \xi^0}} \Delta N$$

integrate by parts, pull ~~out~~ ΔN,

recover background EoM ✓



also need

$$\delta S\Big|_{S'} = \int d^4x \Big\{ \Big[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\substack{\Delta N=0\\ S'}} (\partial_i$$

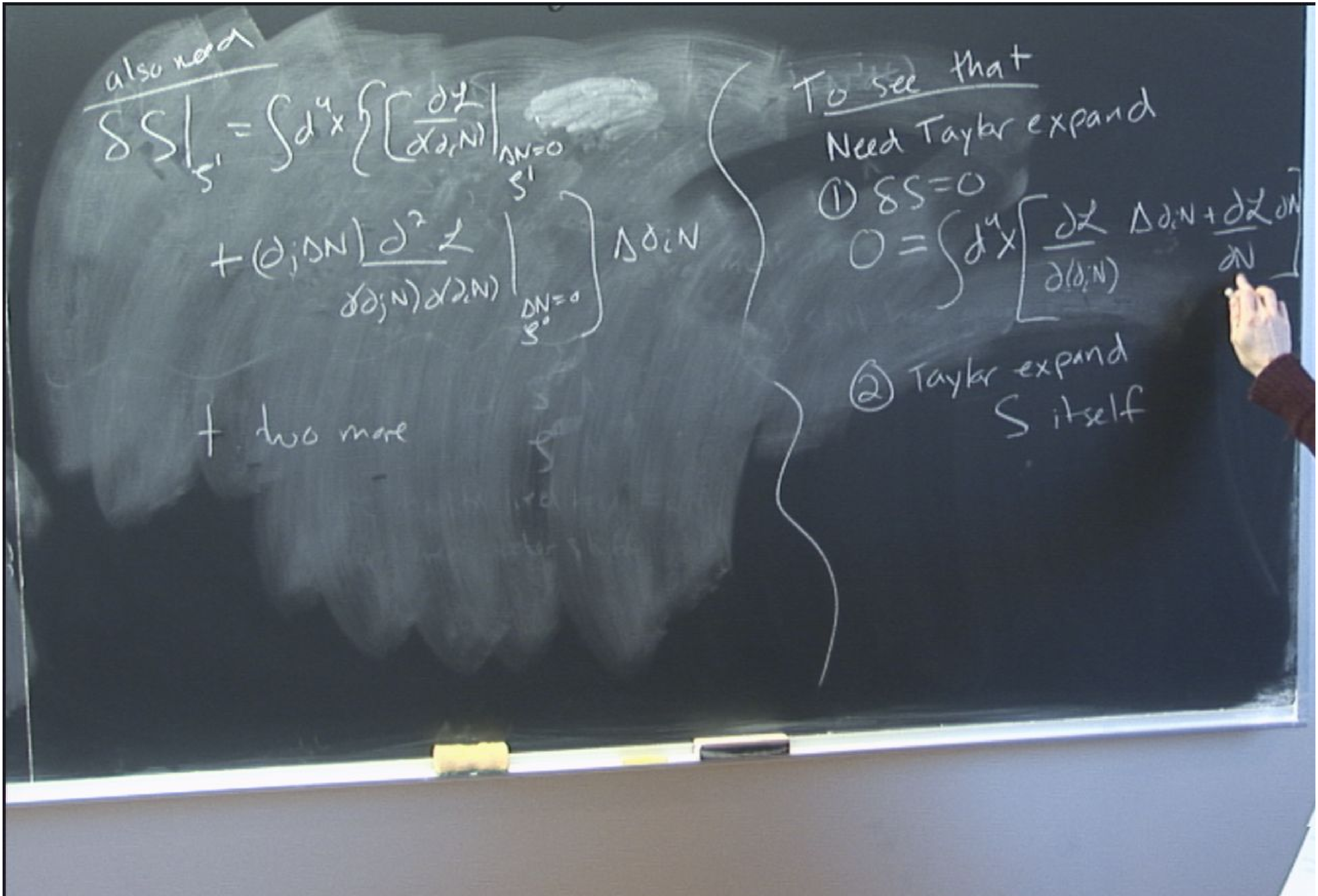
To see that

Need Taylor expand

① $\delta S = 0$

$$0 = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)} \Delta \partial_i N + \frac{\partial \mathcal{L}}{\partial N} \partial N \right]$$

② Taylor expand
S itself



also need

$$\delta S\Big|_{S'} = \int d^4x \left\{ \left[\frac{\partial \mathcal{L}}{\partial(\partial_i N)}\Big|_{\substack{\Delta N=0\\ S'}} + (\partial_j \Delta N)\frac{\partial^2 \mathcal{L}}{\partial(\partial_j N)\,\partial(\partial_i N)}\Big|_{\substack{\Delta N=0\\ S'}} \right] \Delta\partial_i N \right.$$

+ two more

To see that

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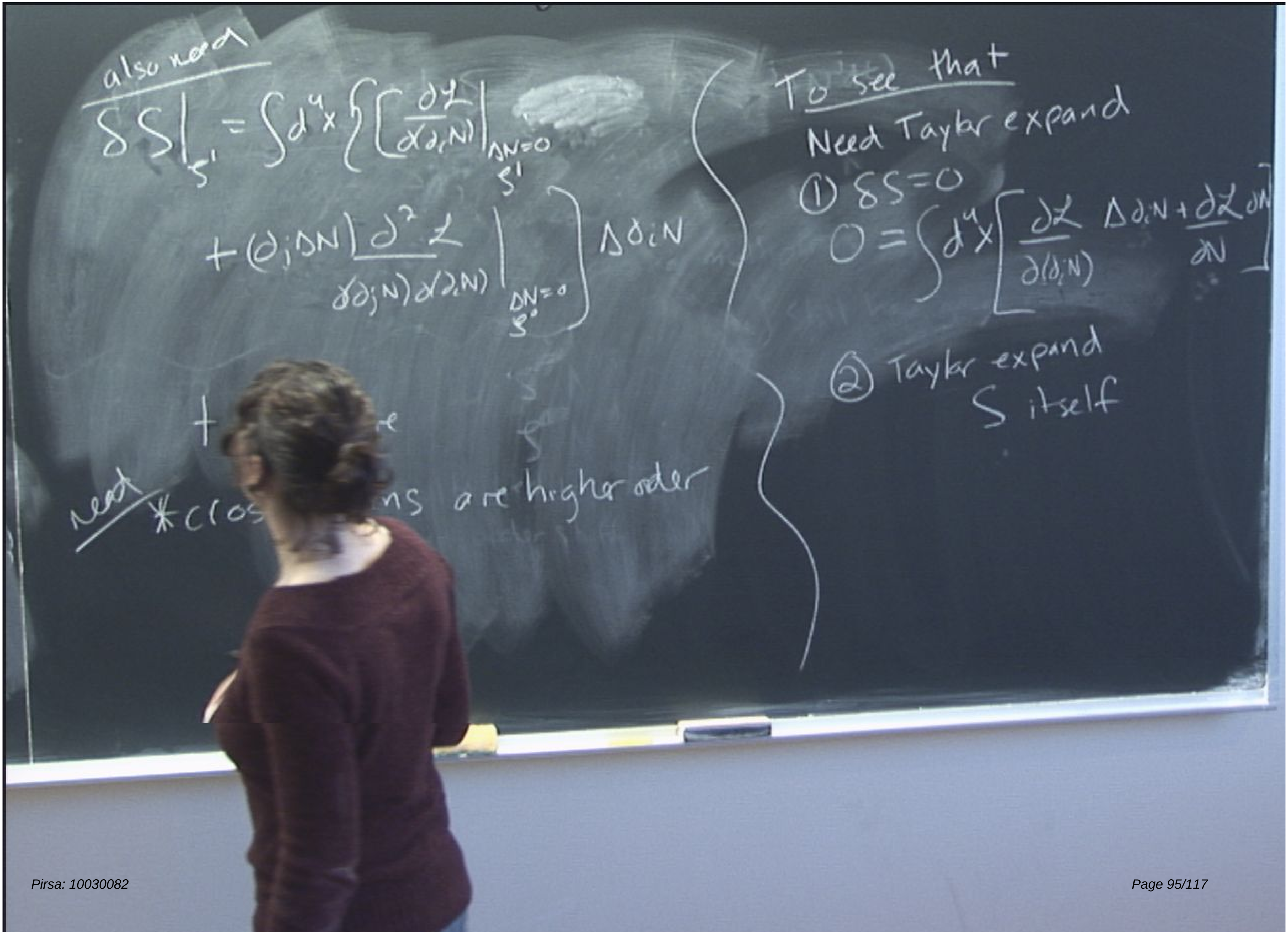
② Taylor expand S itself



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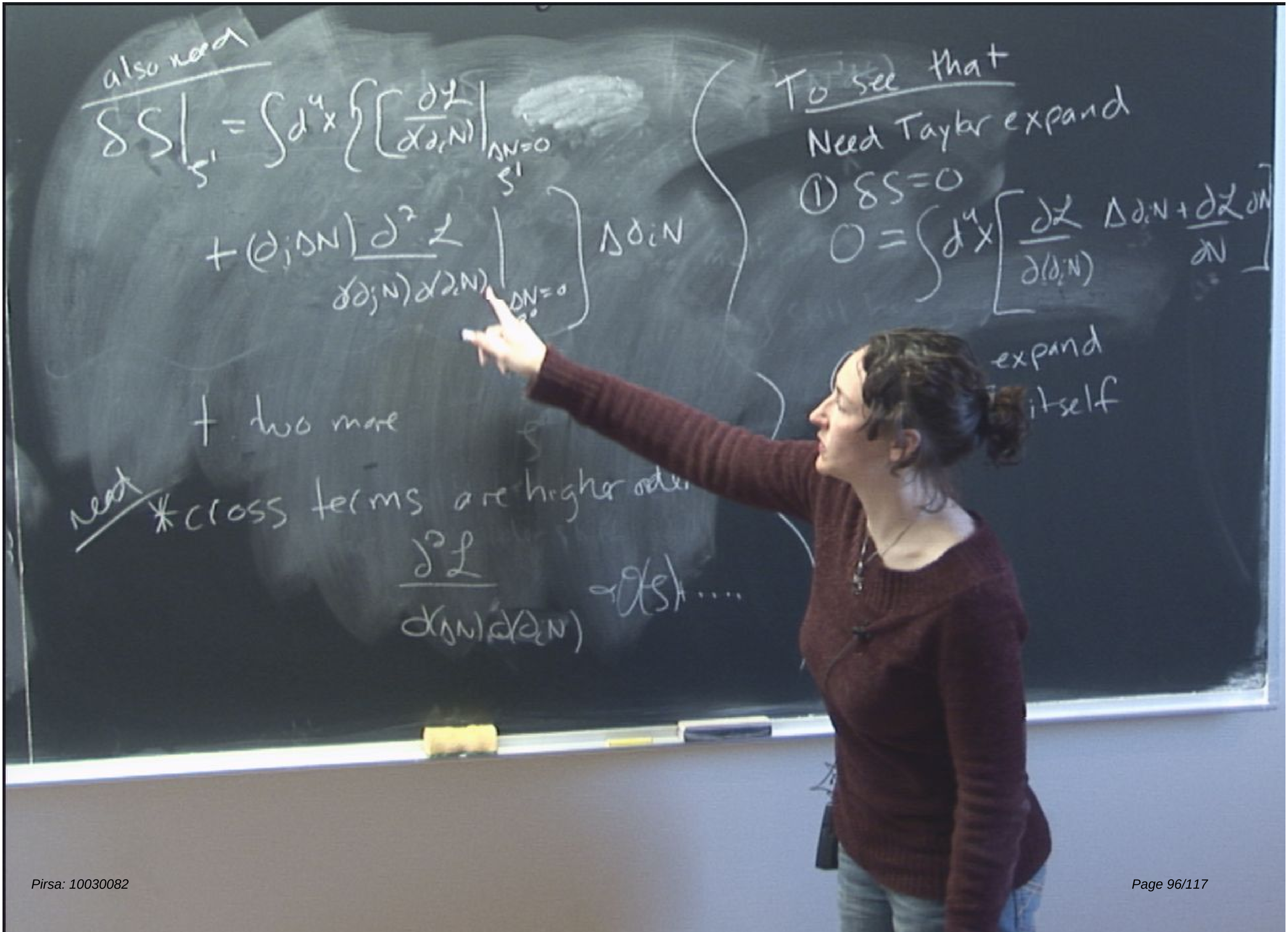

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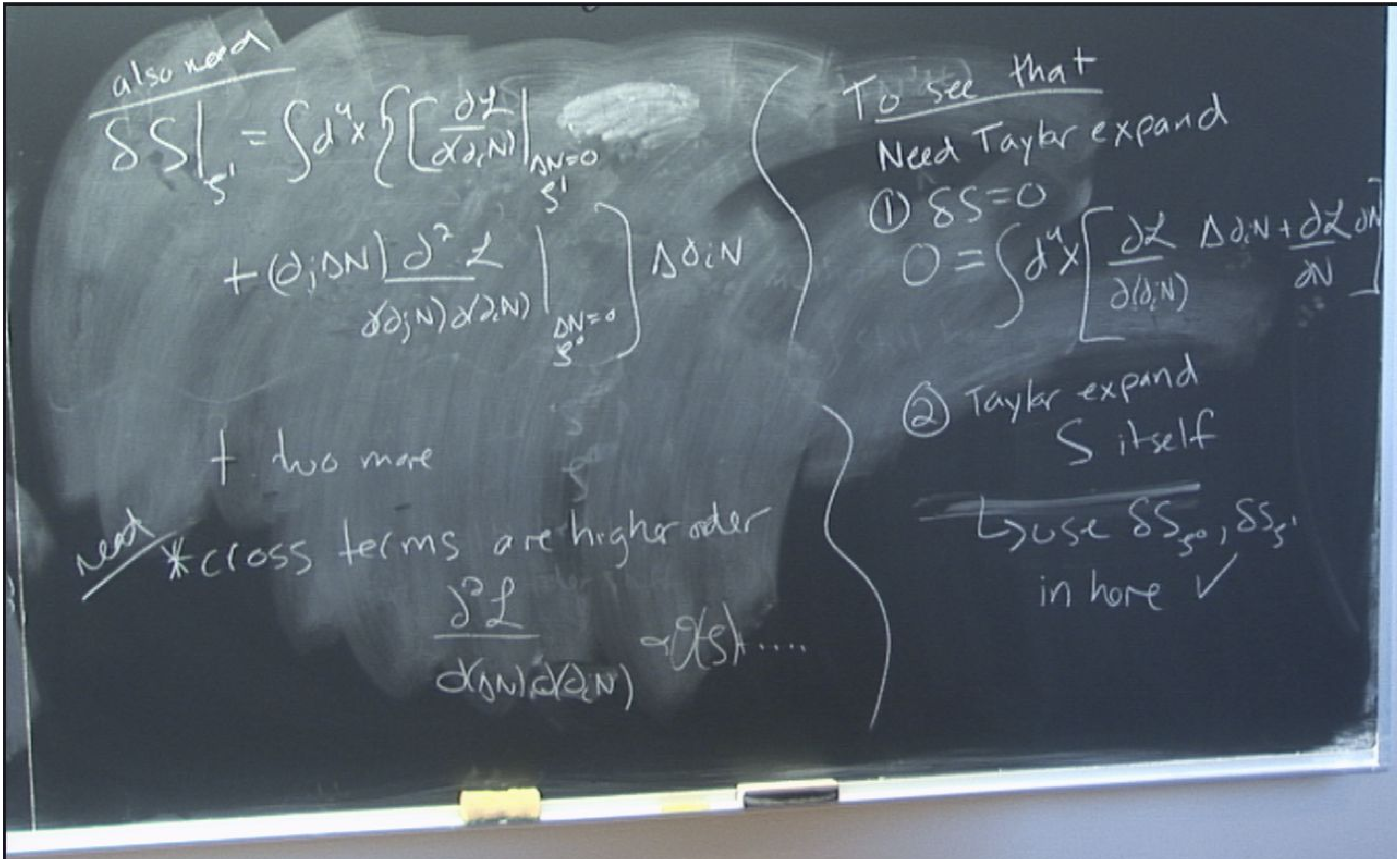

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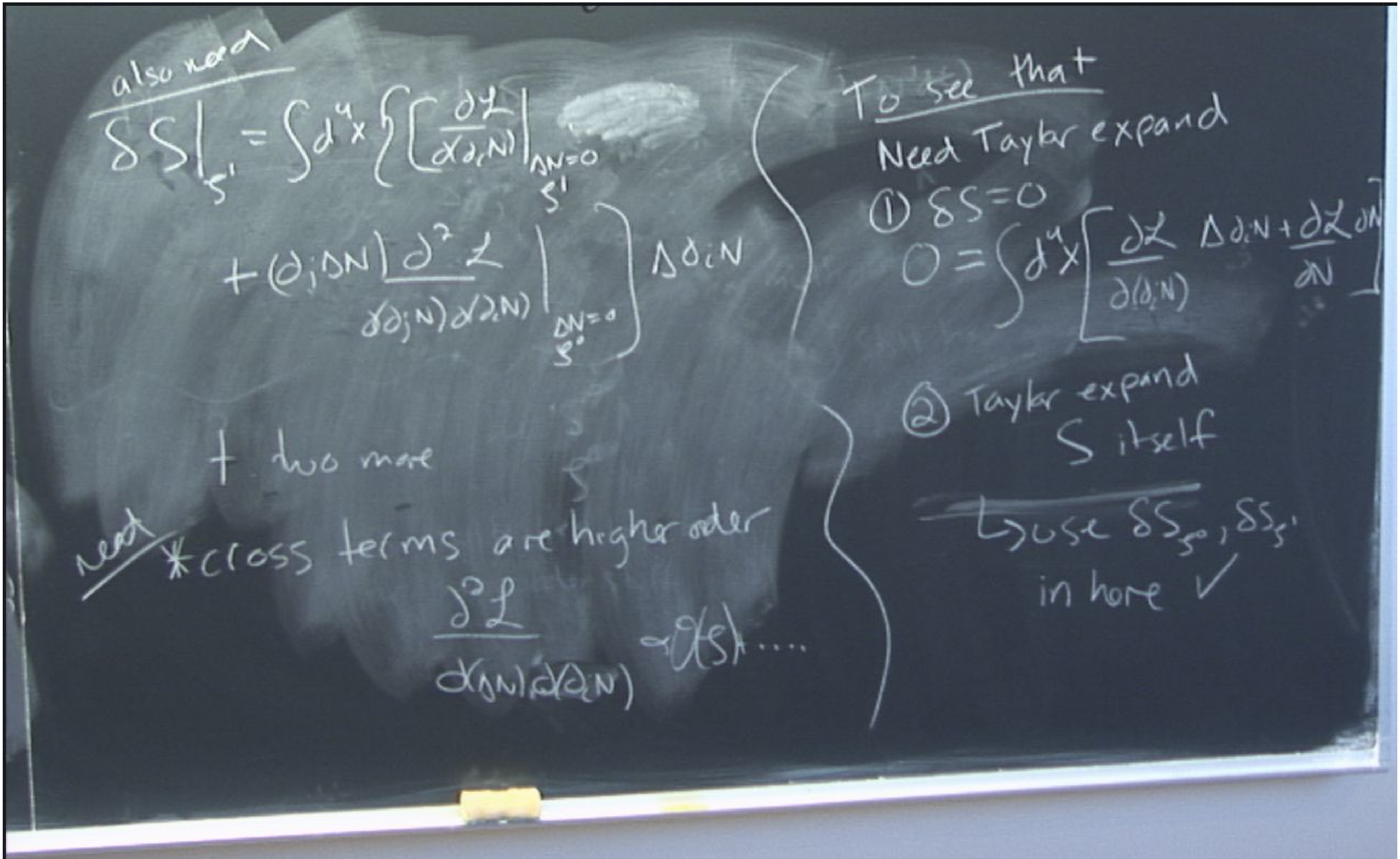

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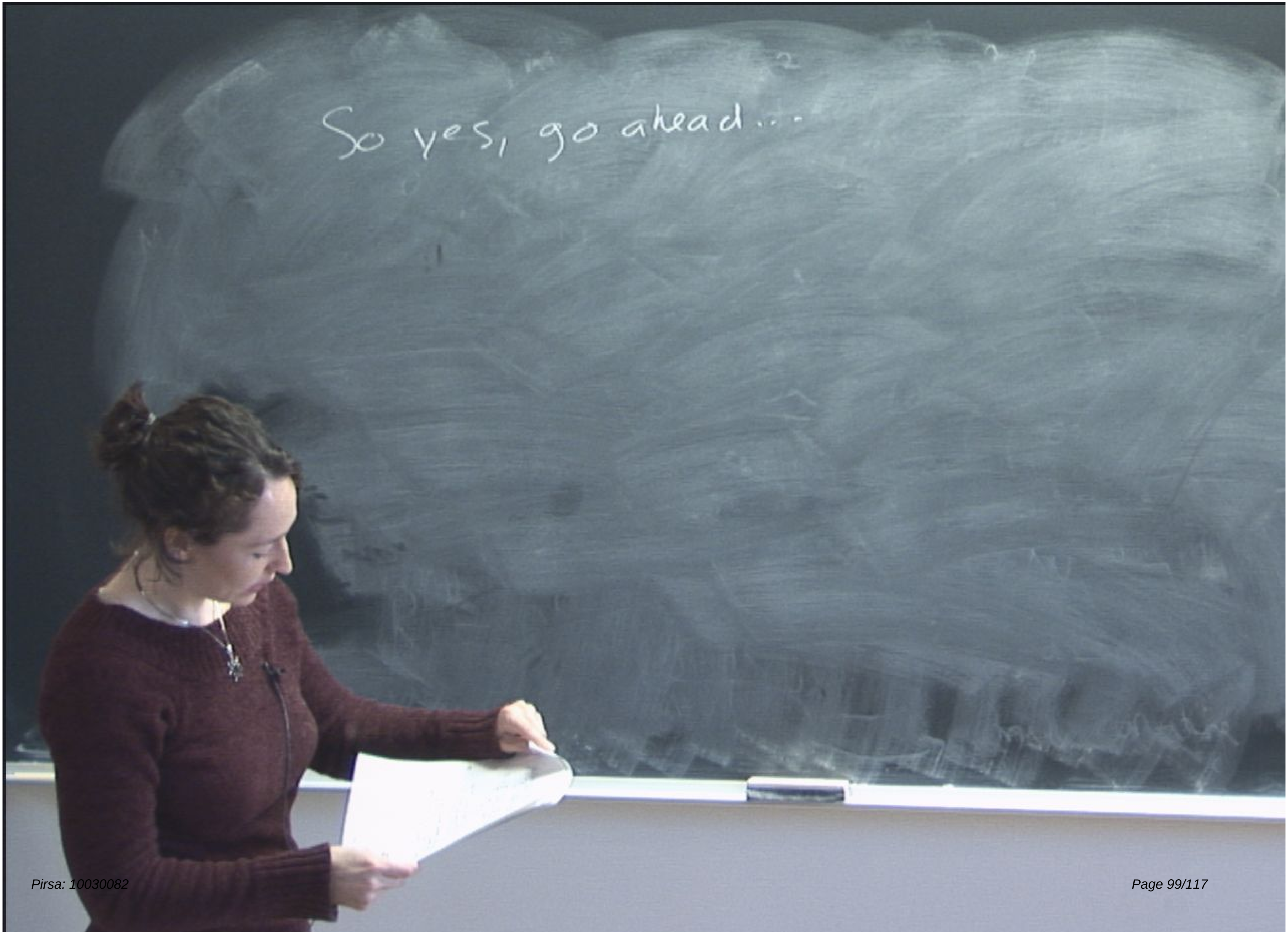

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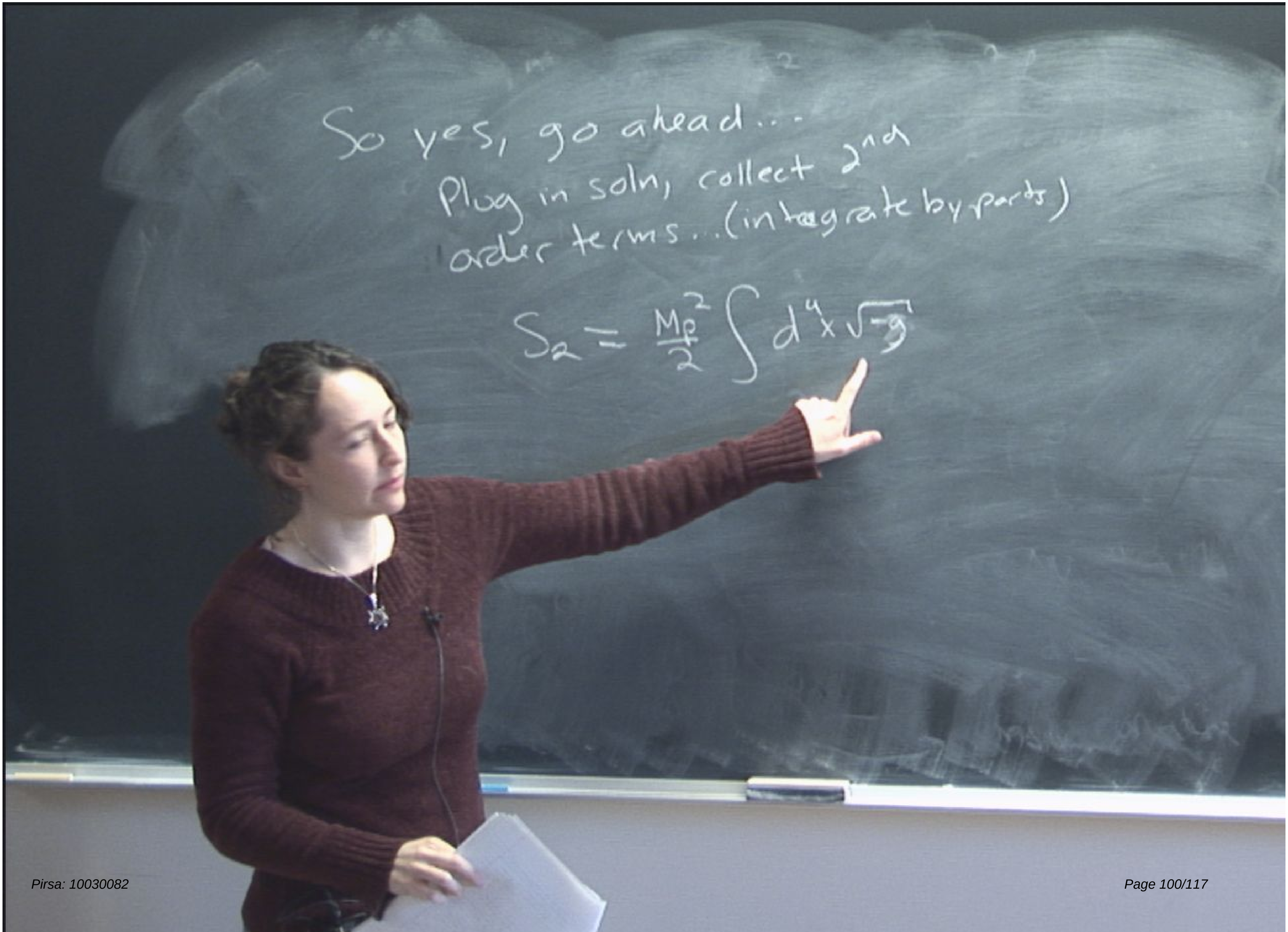

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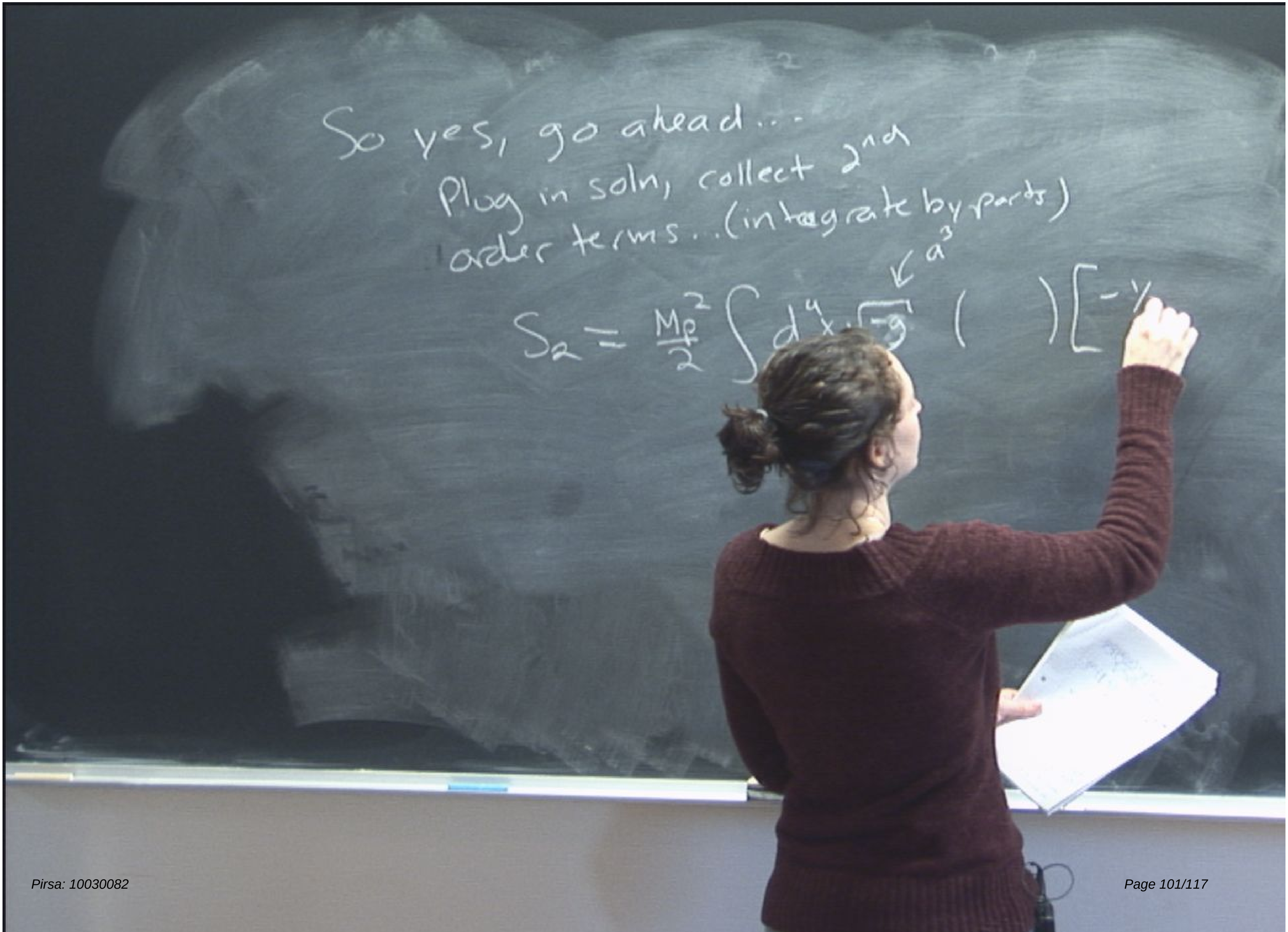
Plug in soln, collect 2nd order terms... (integrate by parts)

$$S_2 = \frac{M_p^2}{2}\int d^4x\sqrt{-g}$$



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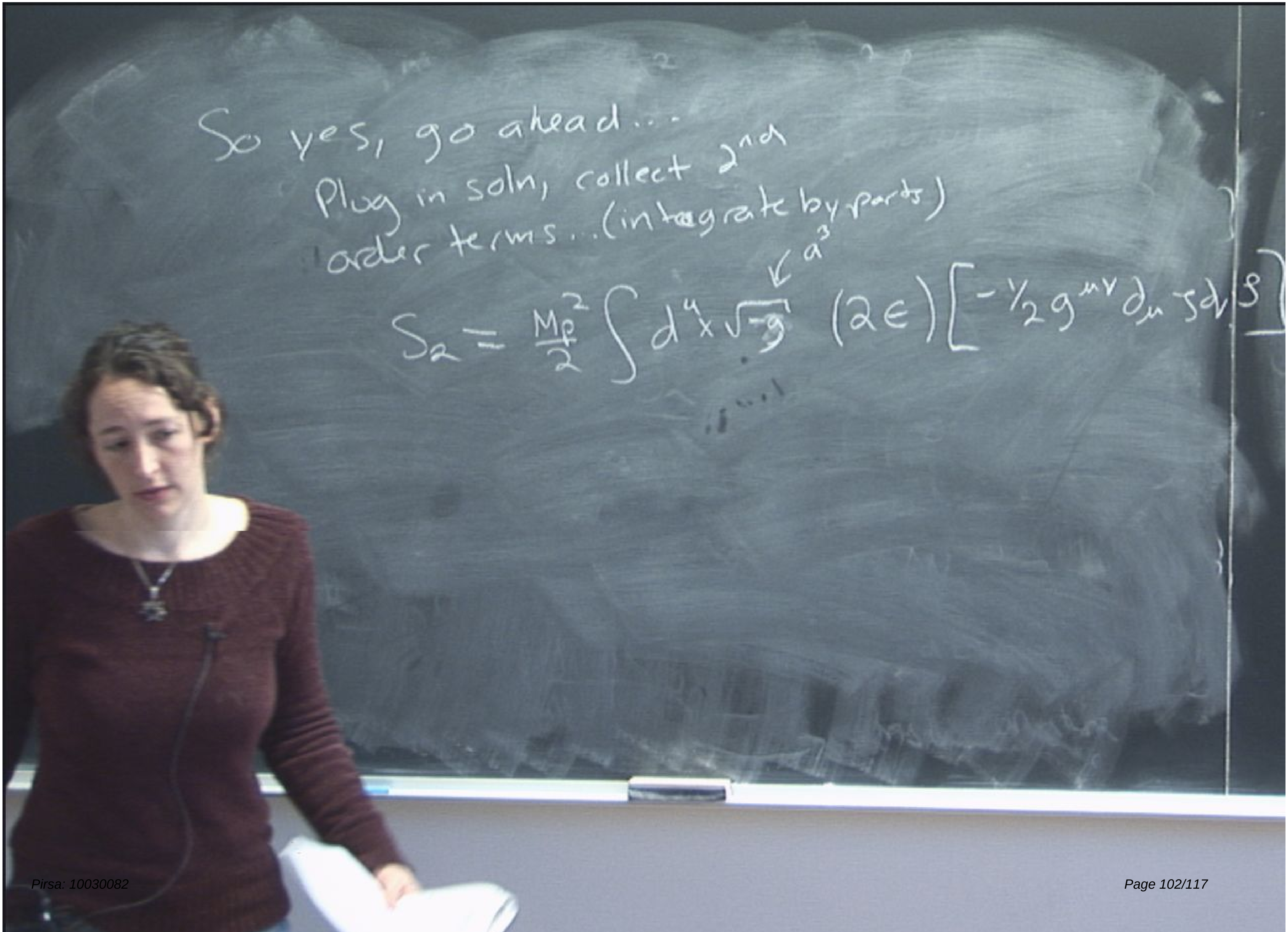

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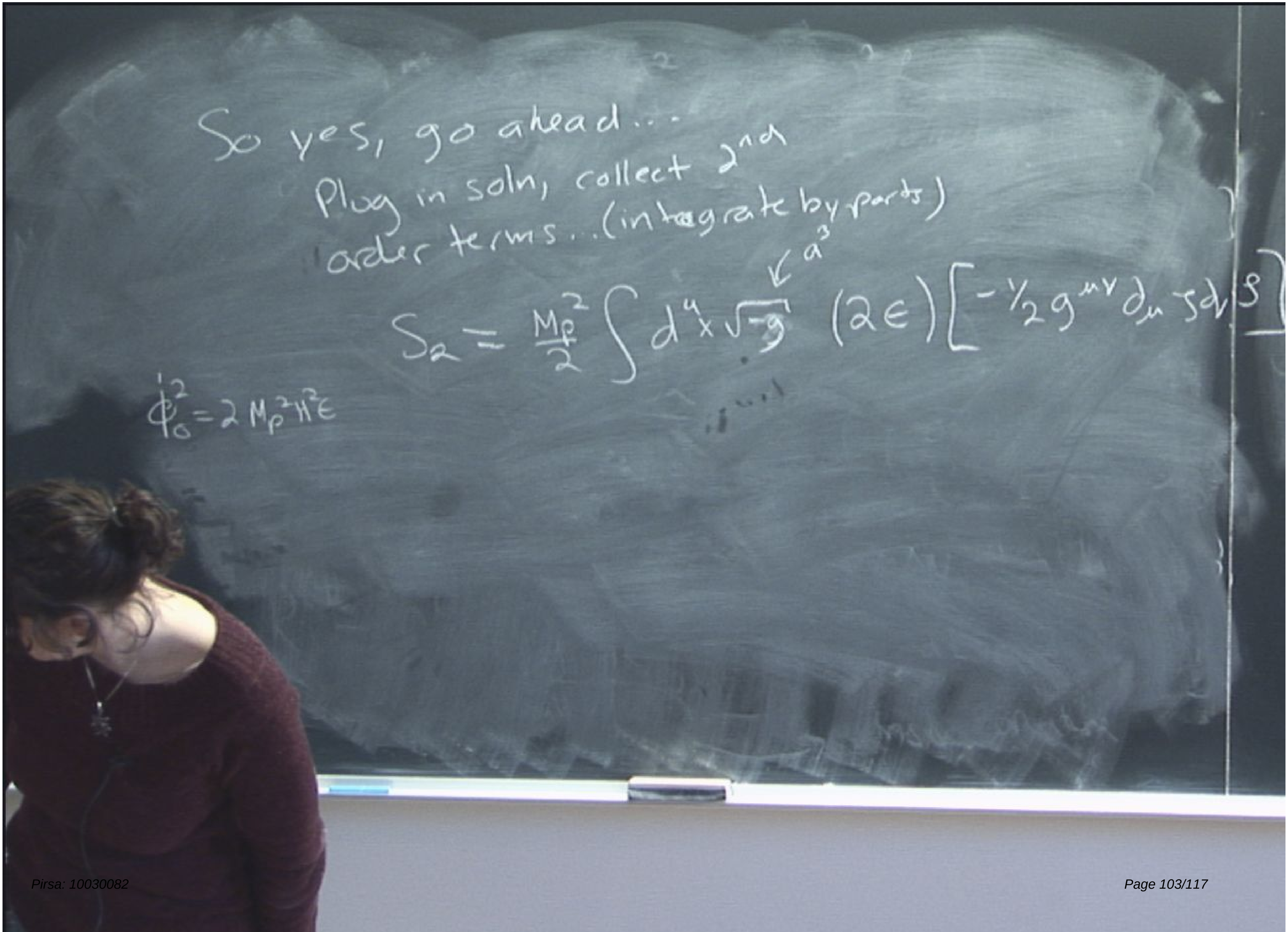
$$S_2 = \frac{M_p^2}{2}\int d^4x \overset{\swarrow a^3}{\sqrt{-g}}\,(2\epsilon)\left[-\tfrac{1}{2}g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta\right.$$



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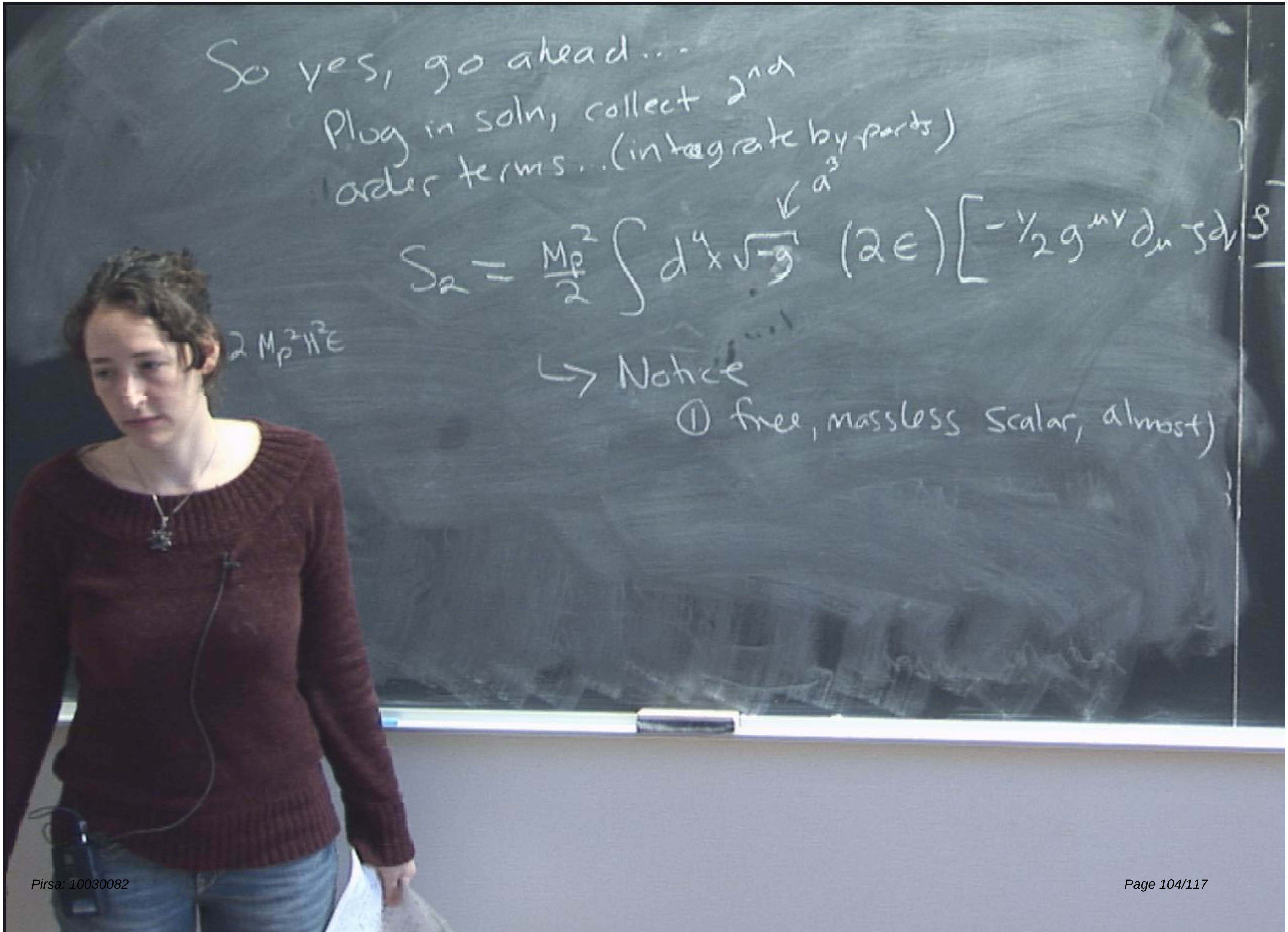

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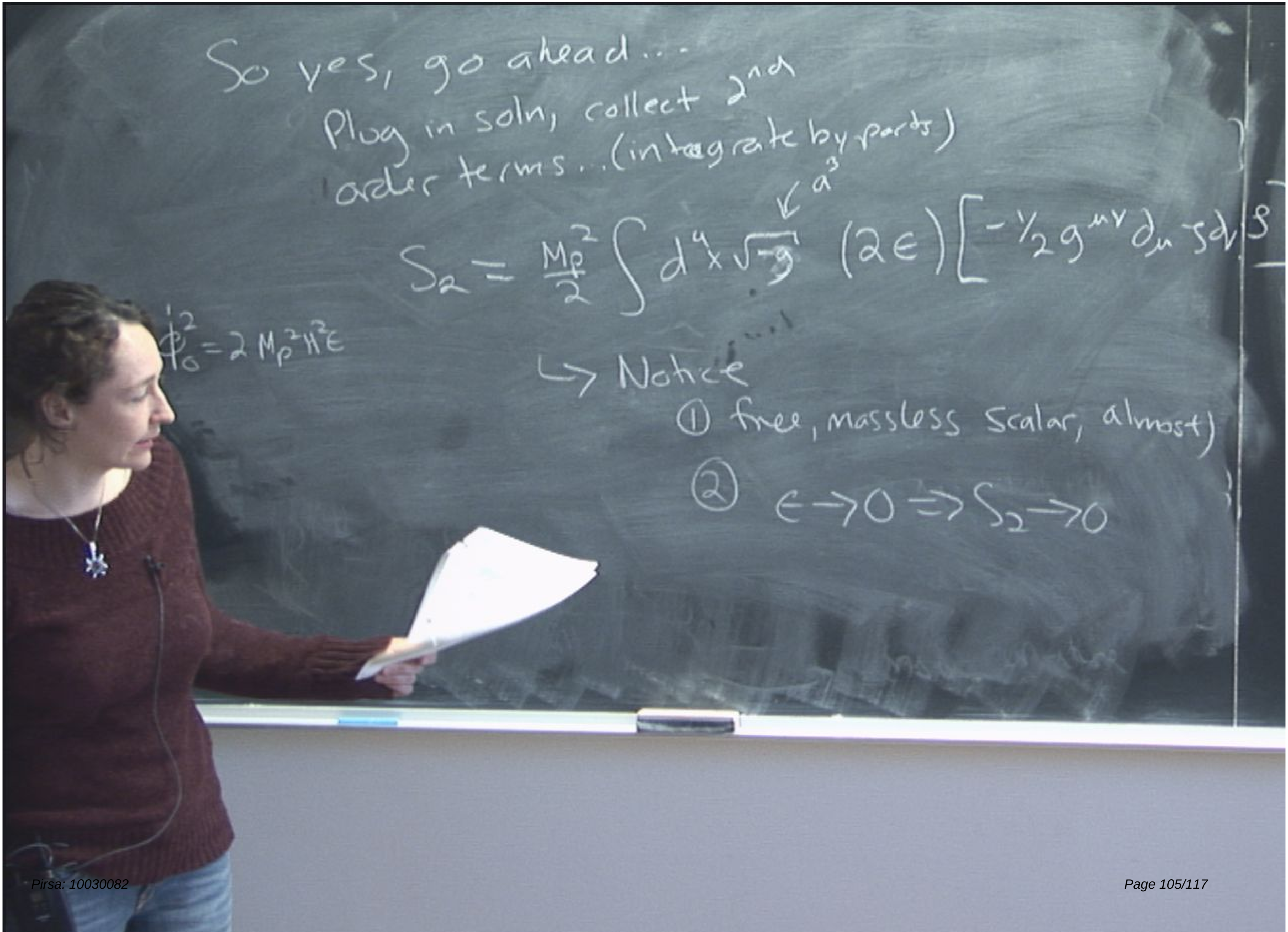
↳ Notice

① free, massless scalar, almost)



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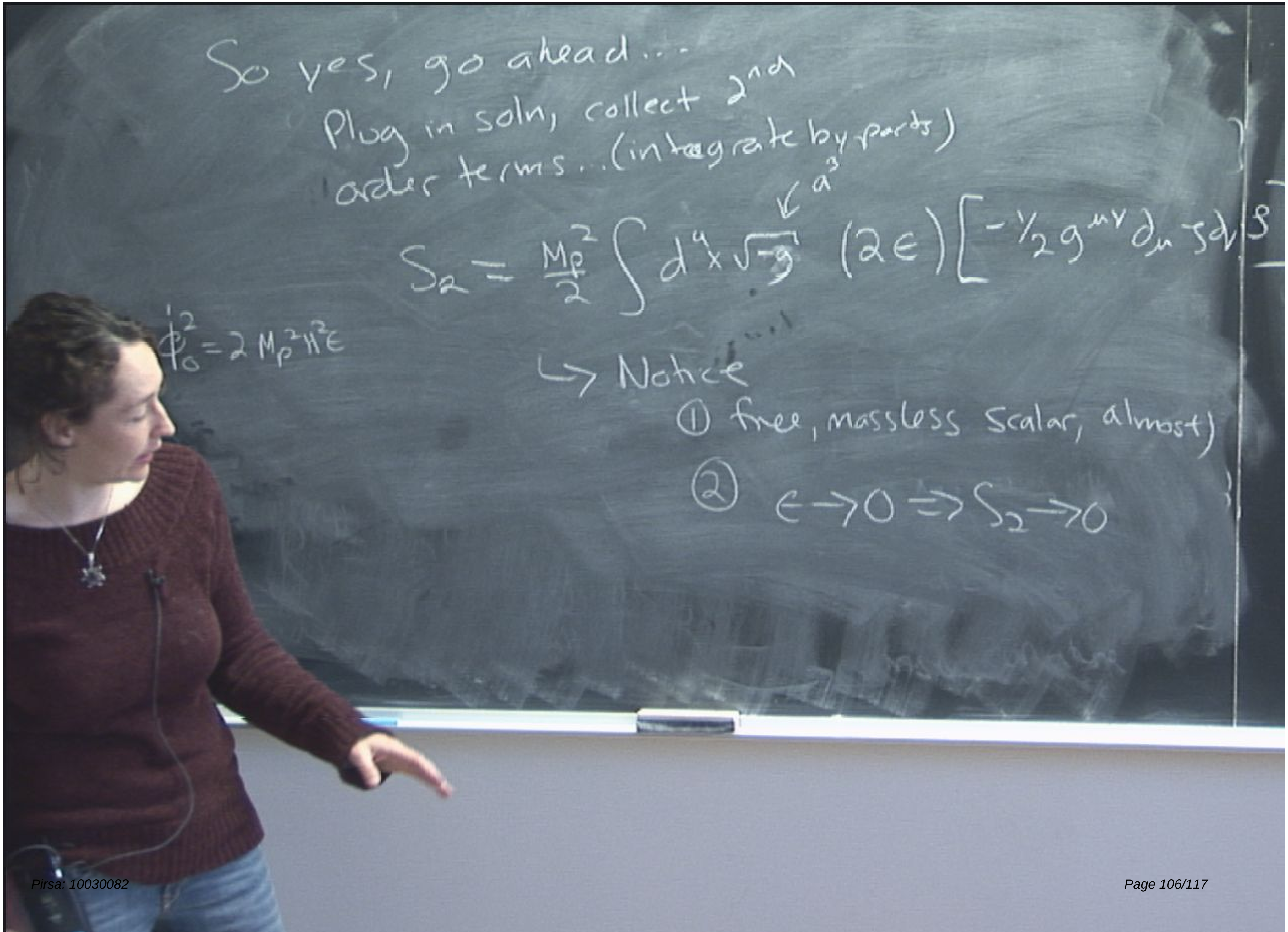

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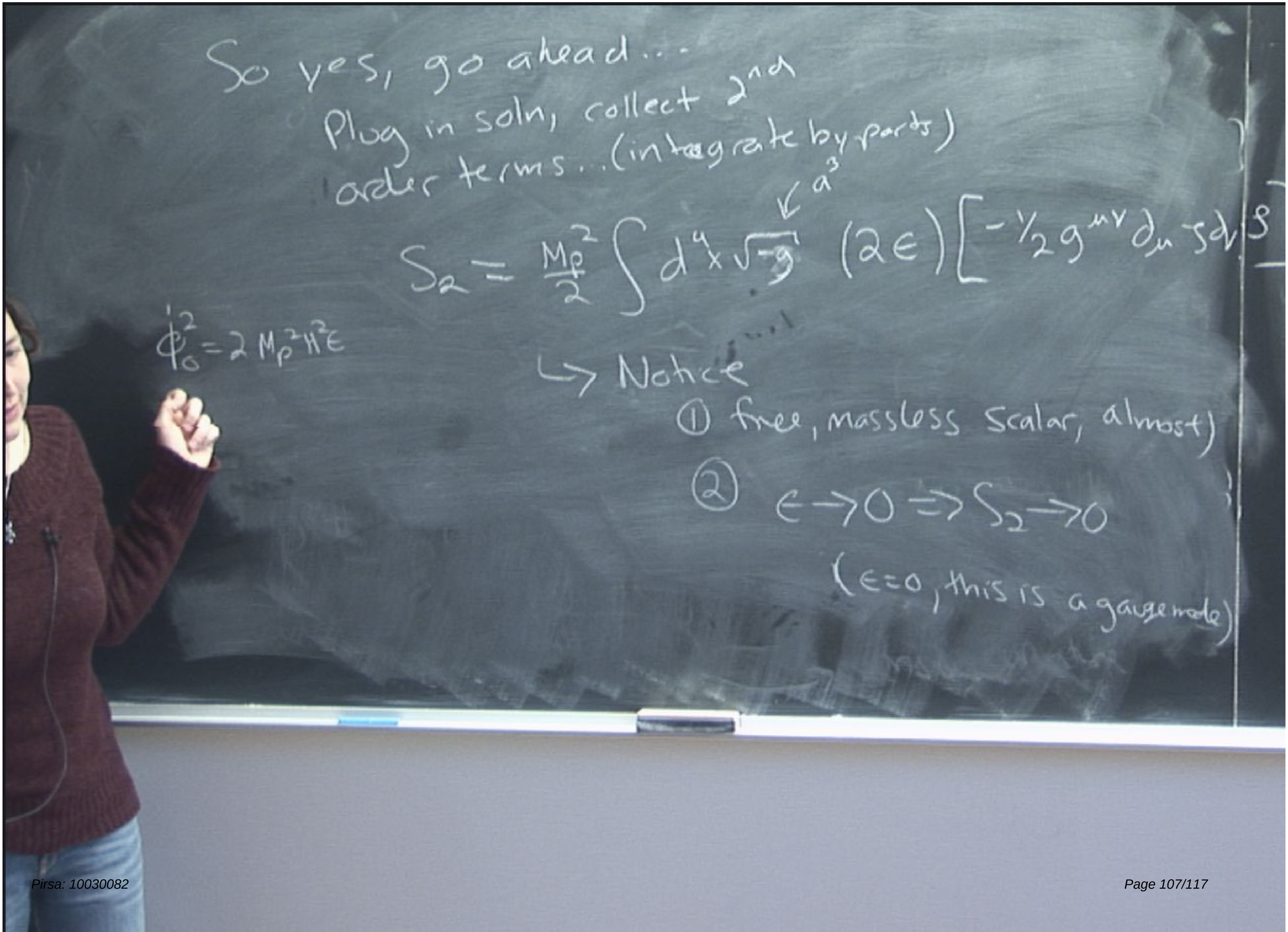

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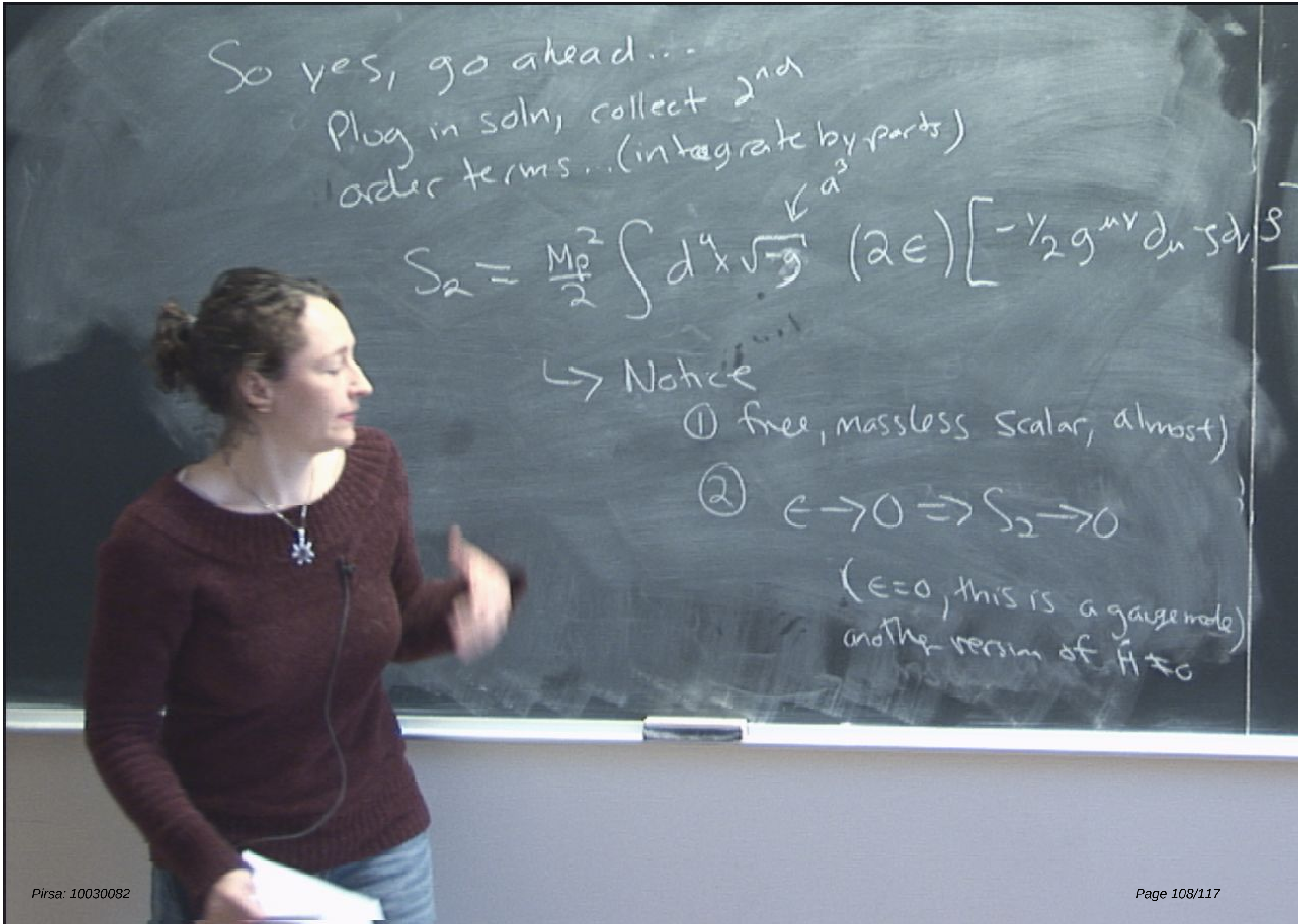

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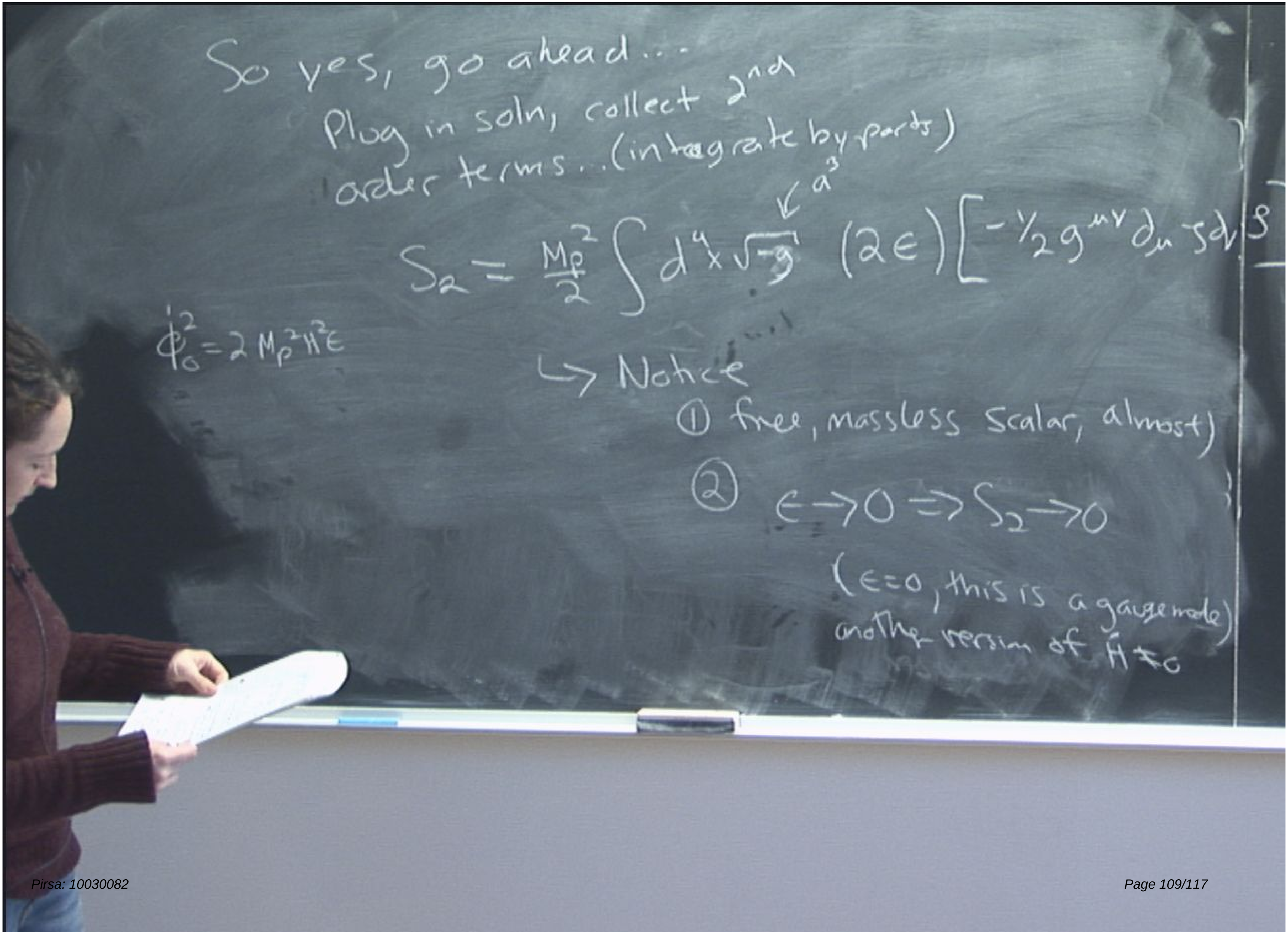

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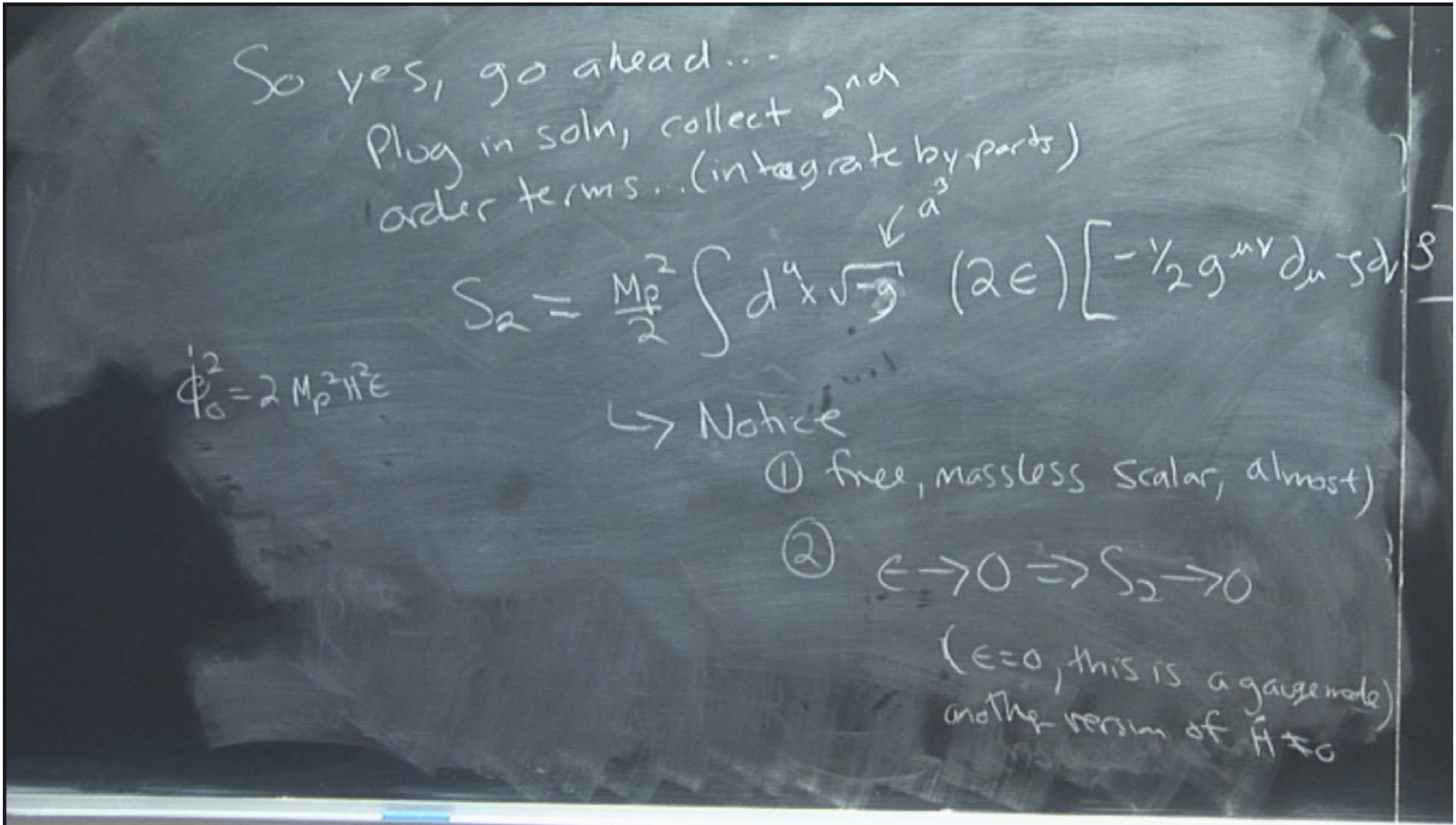

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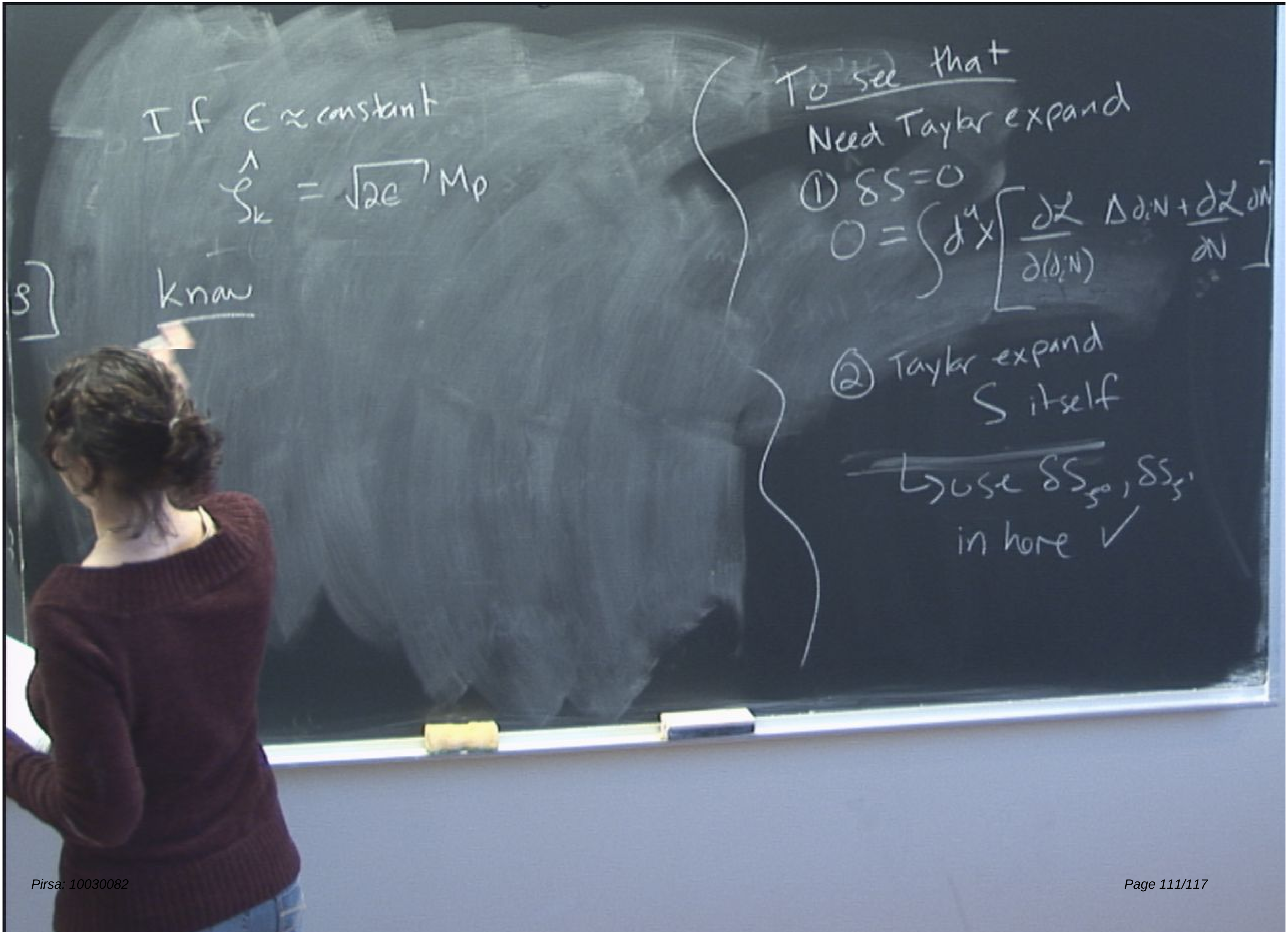

If $\epsilon \simeq$ constant

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② Taylor expand
S itself

↳ use $\delta S_{\zeta^0}, \delta S_{\zeta^1}$
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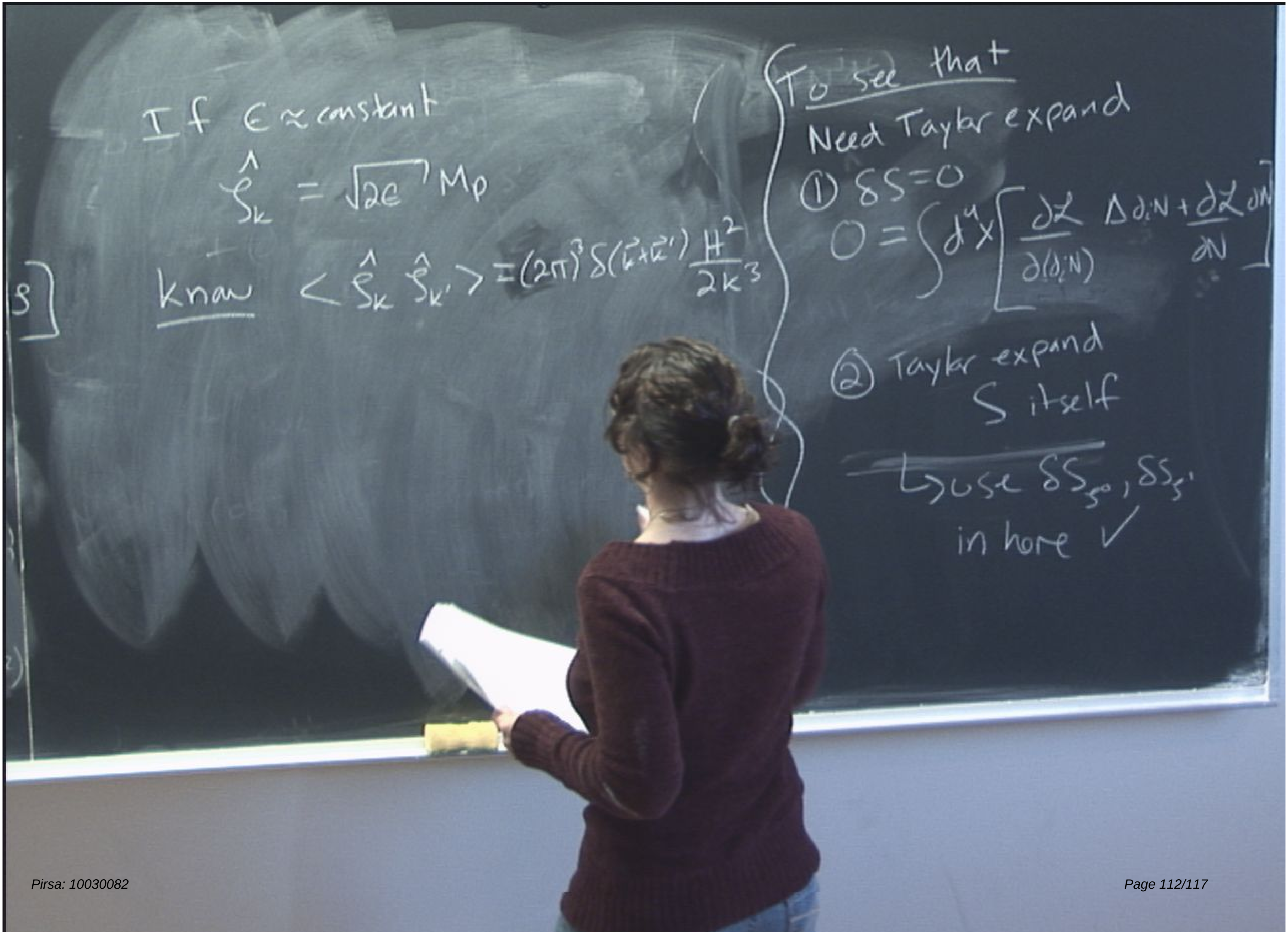

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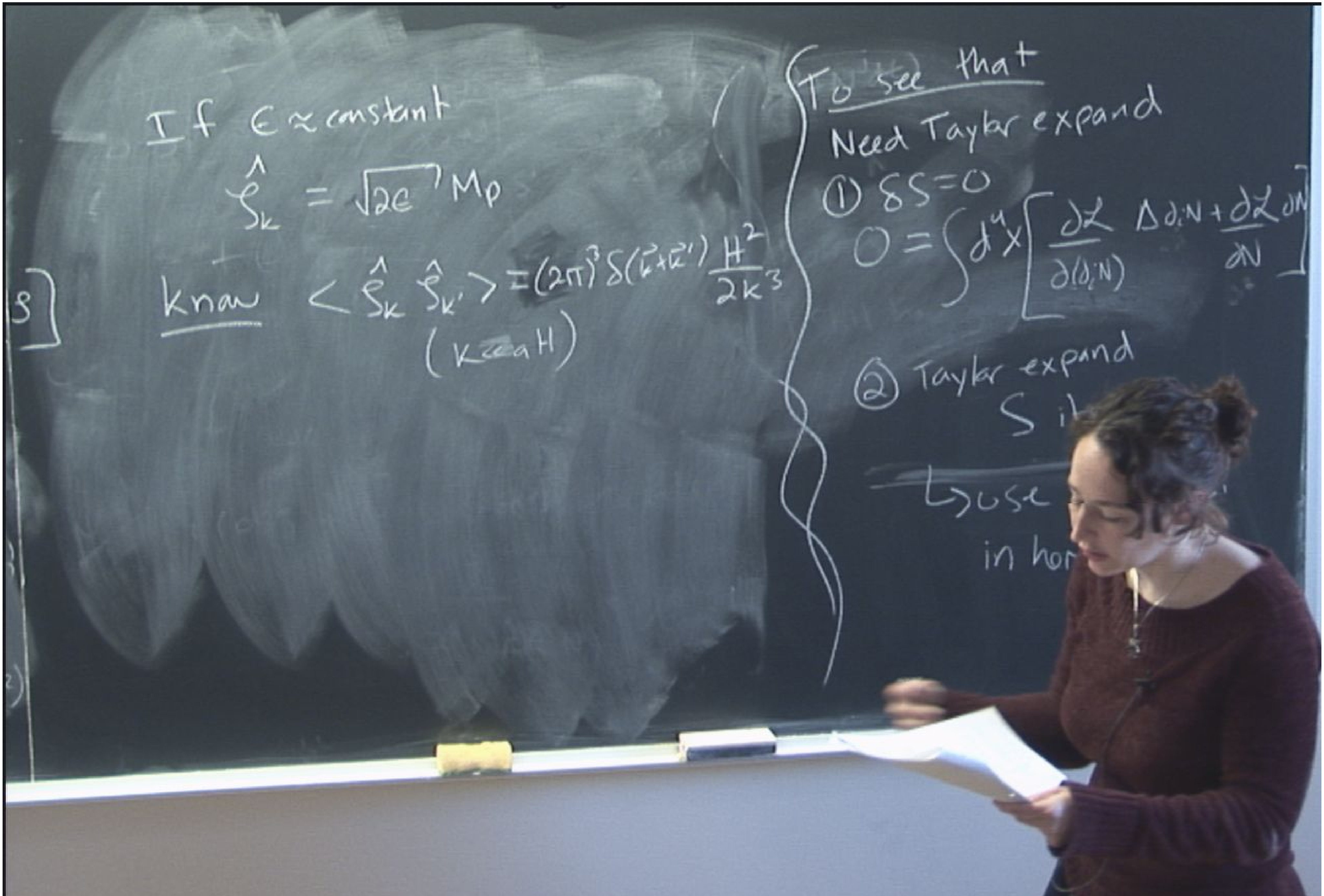

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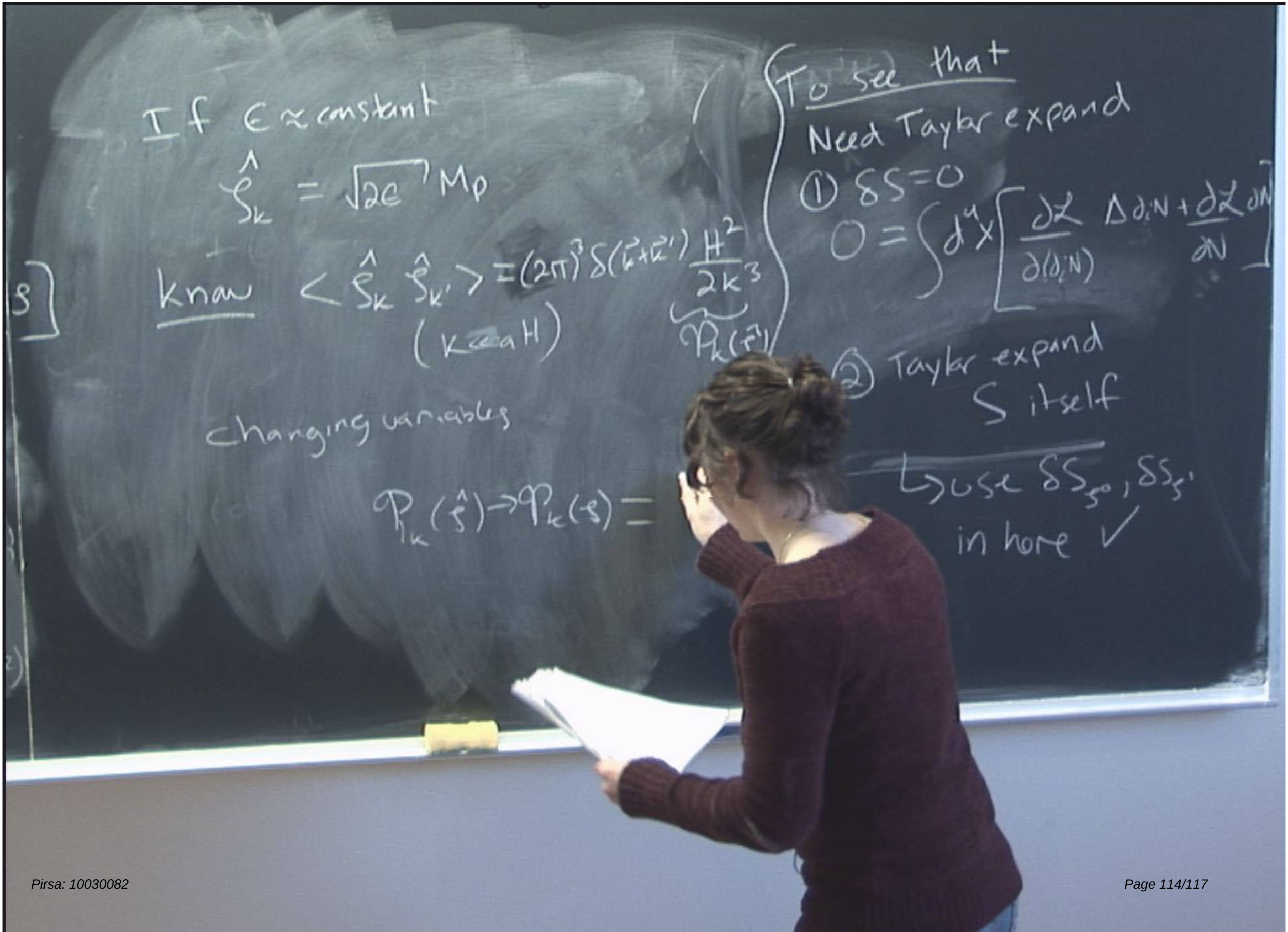

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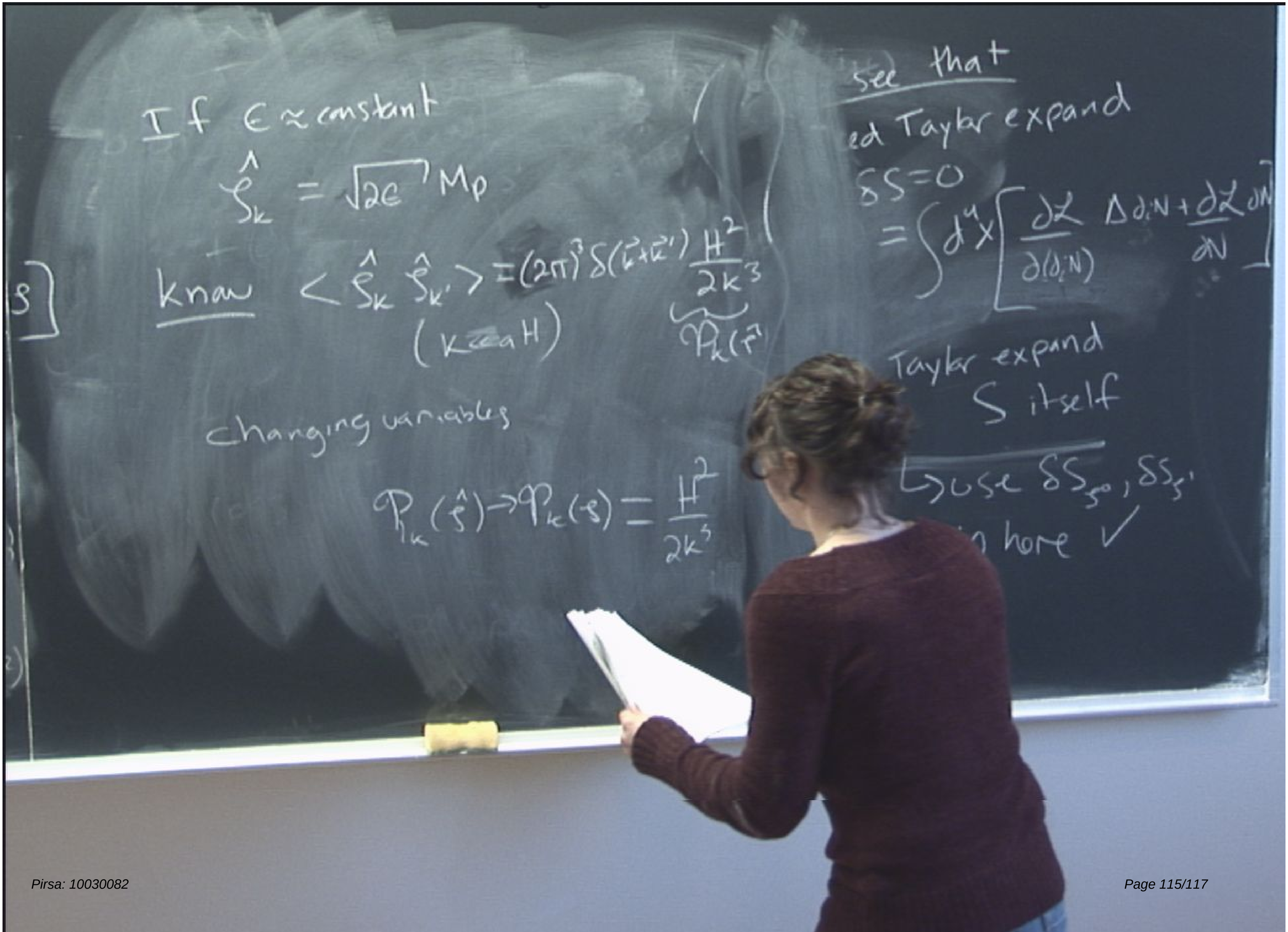

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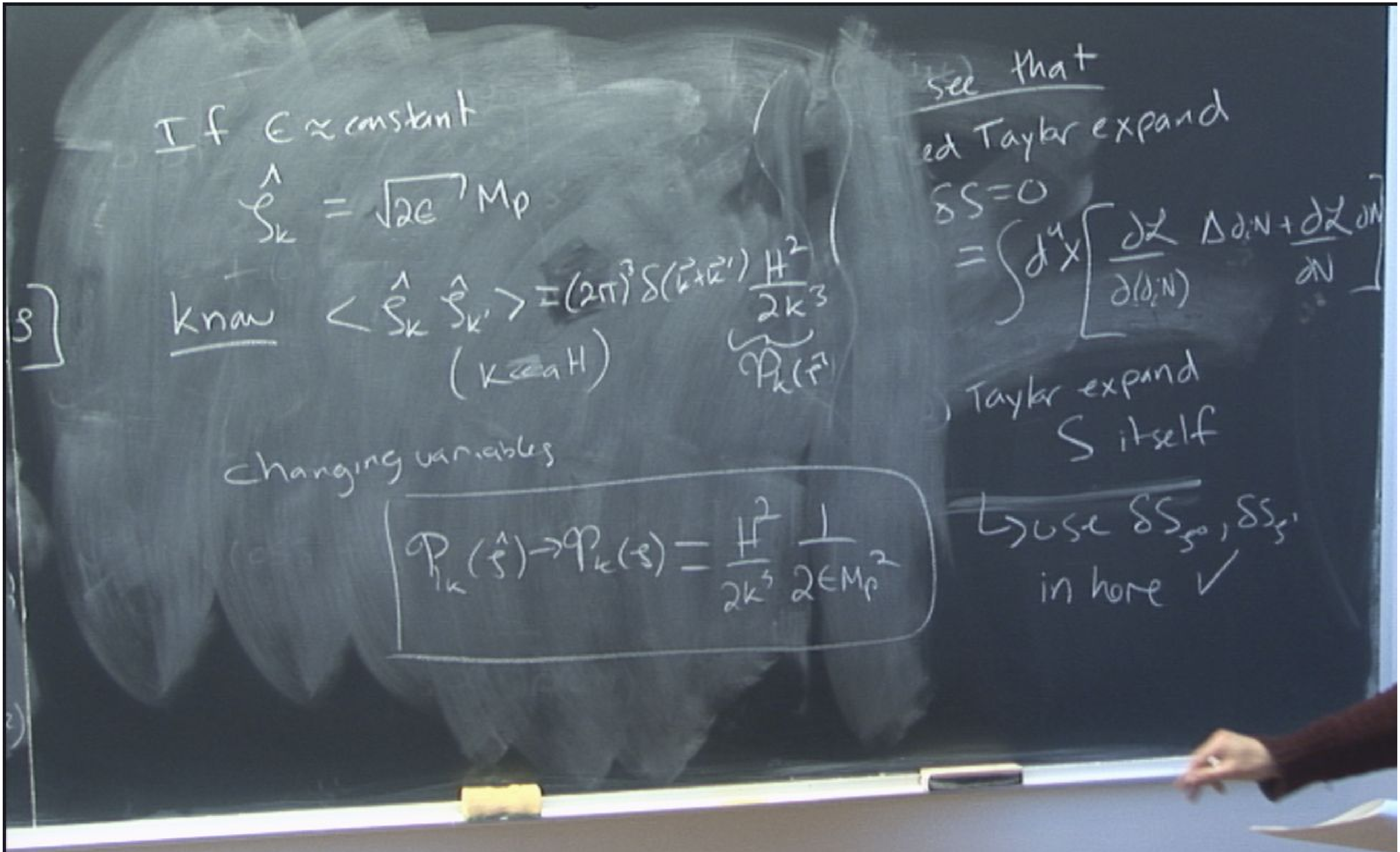

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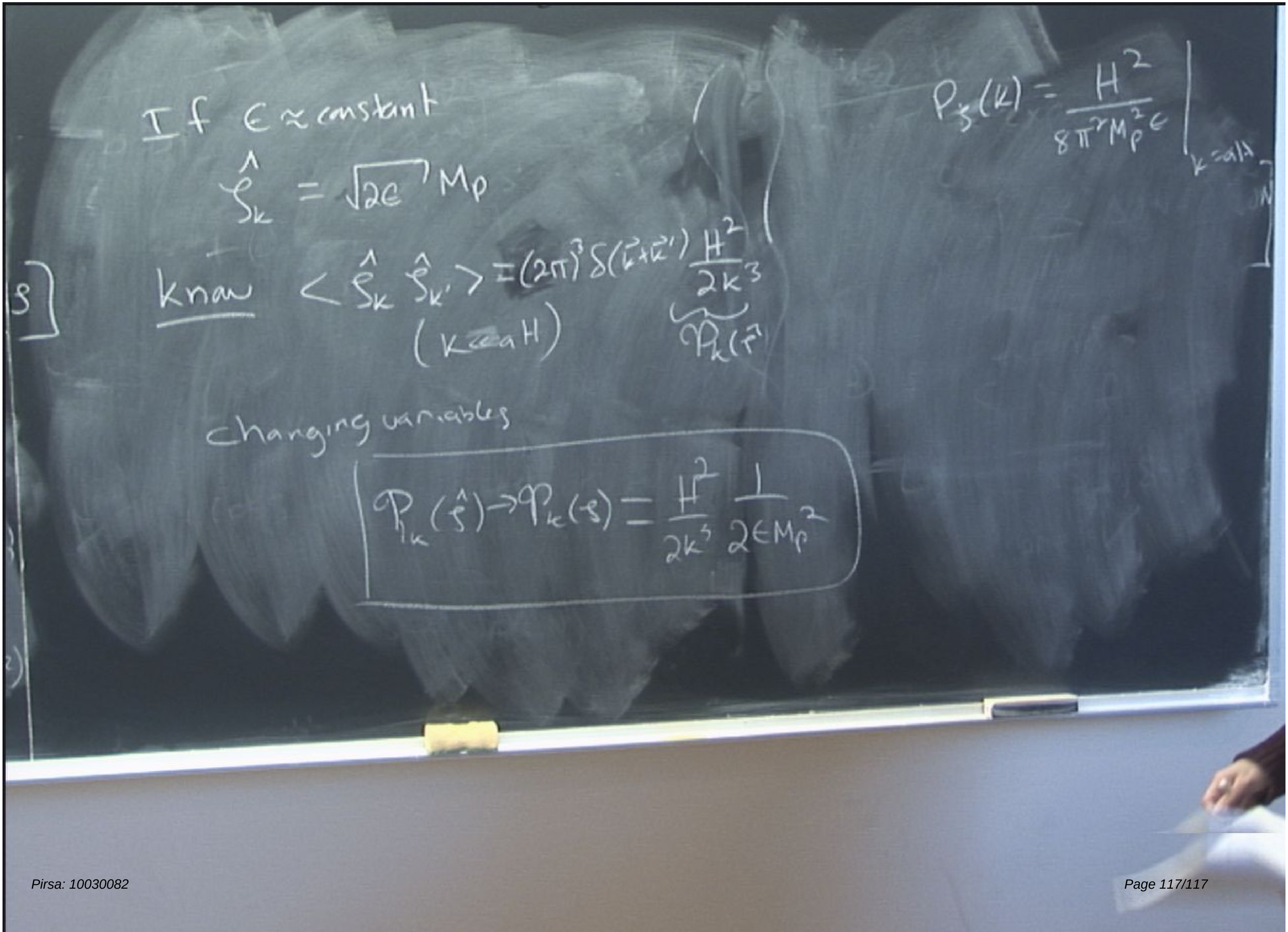

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$$P_\zeta(k) = \frac{H^2}{8\pi^2 M_p^2 \epsilon}\Big|_{k=aH}$$

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