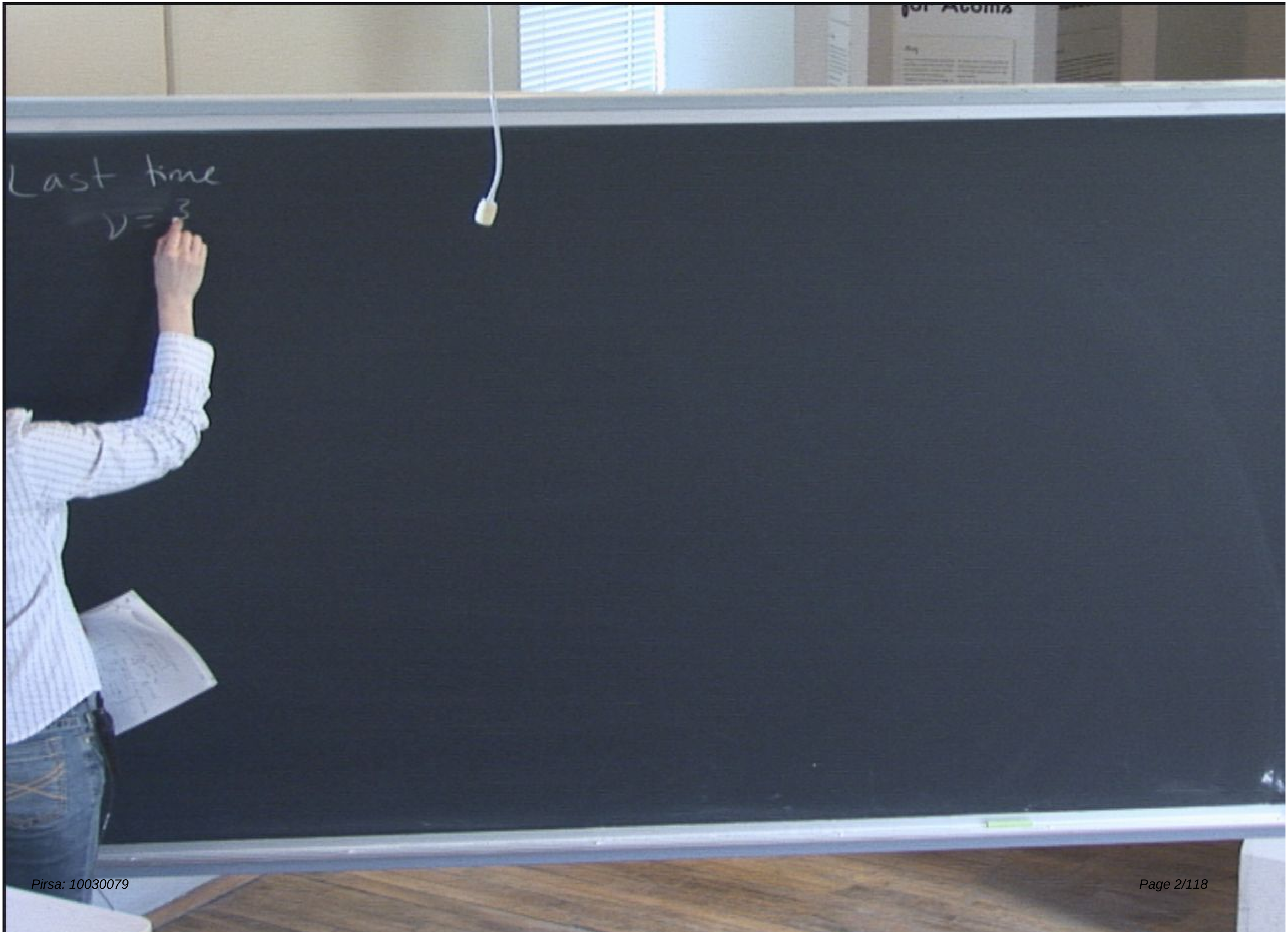
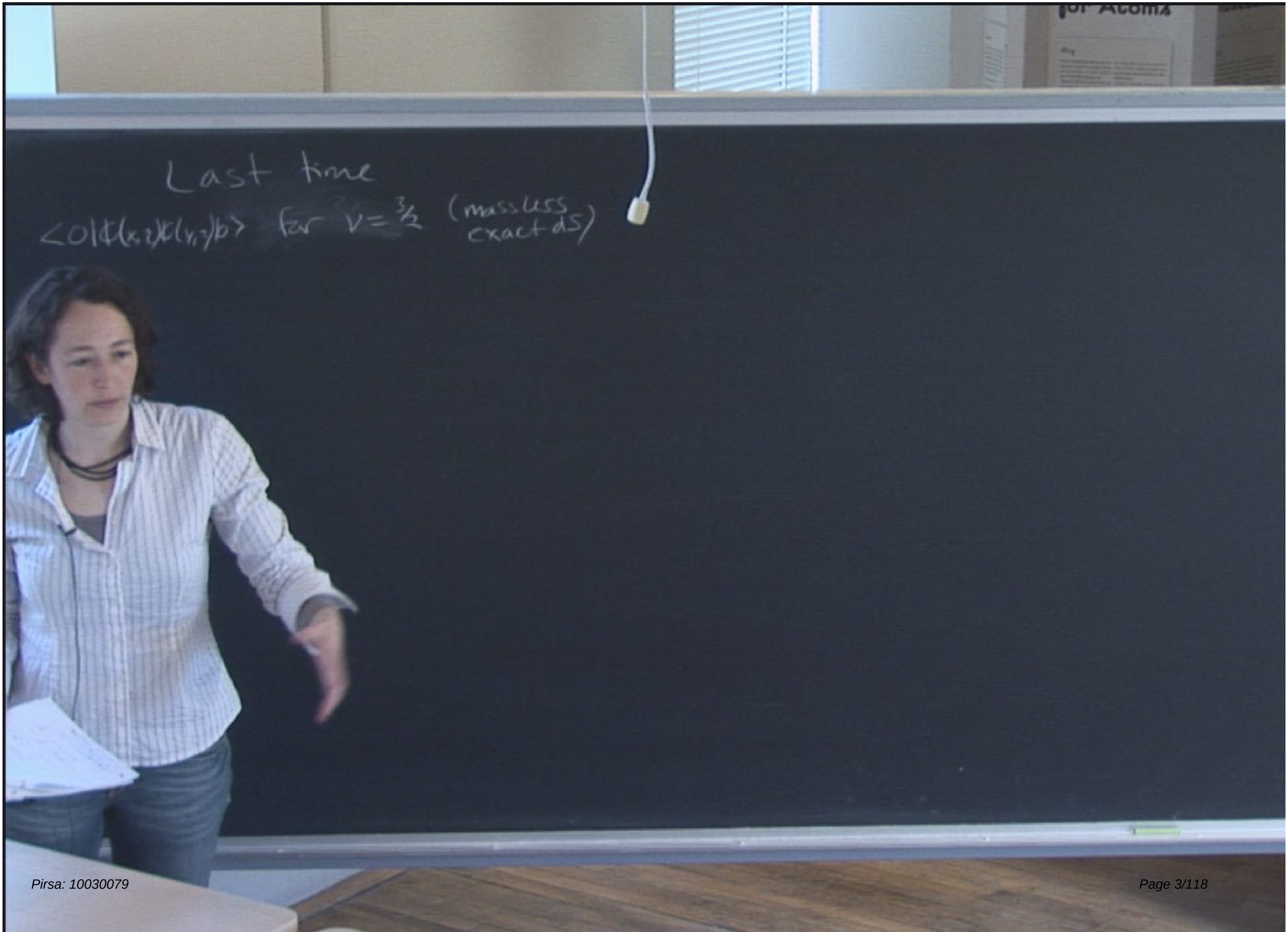
Title: Explorations in Cosmology - Lecture 10

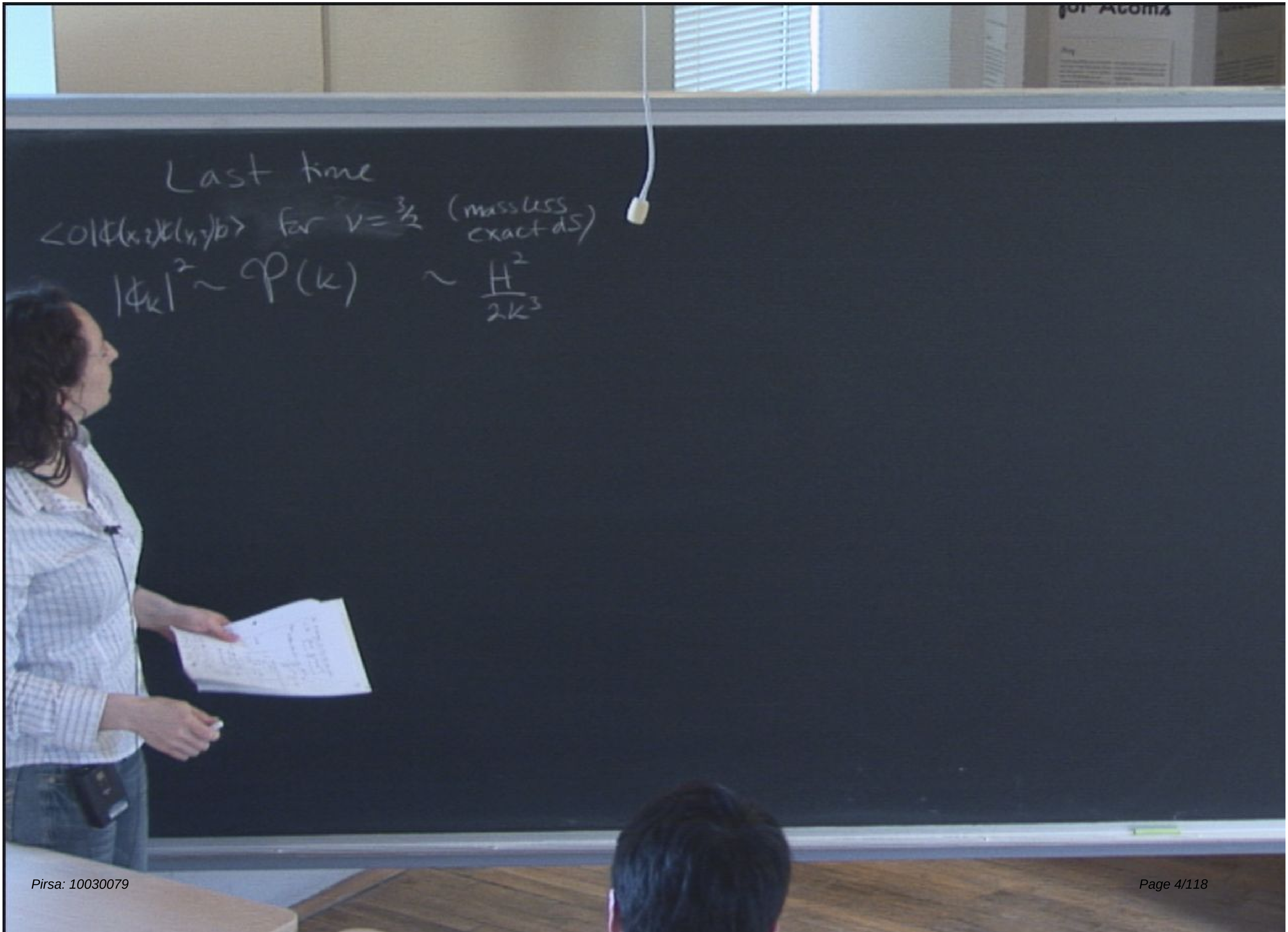
Date: Mar 26, 2010 09:00 AM

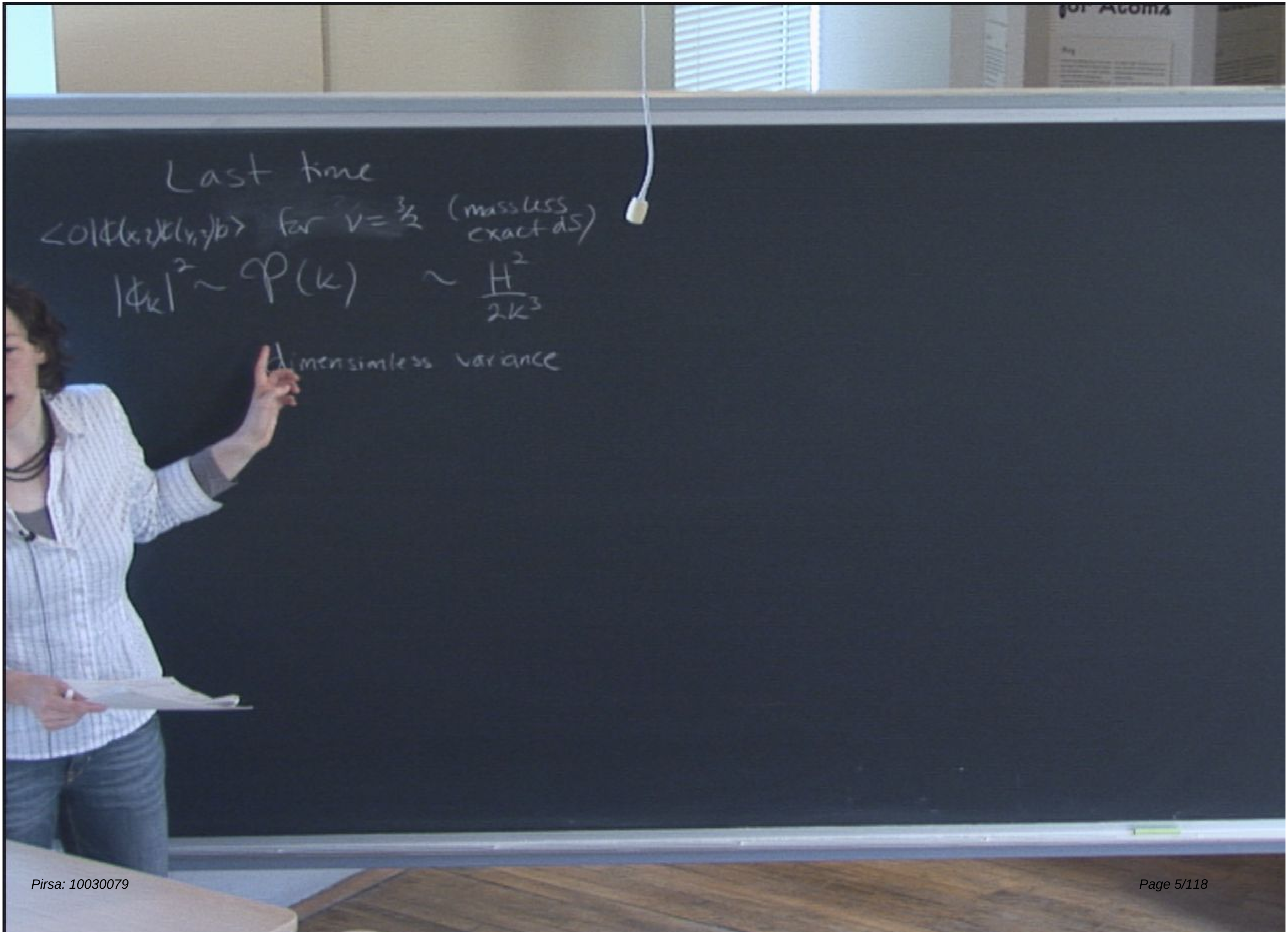
URL: http://pirsa.org/10030079

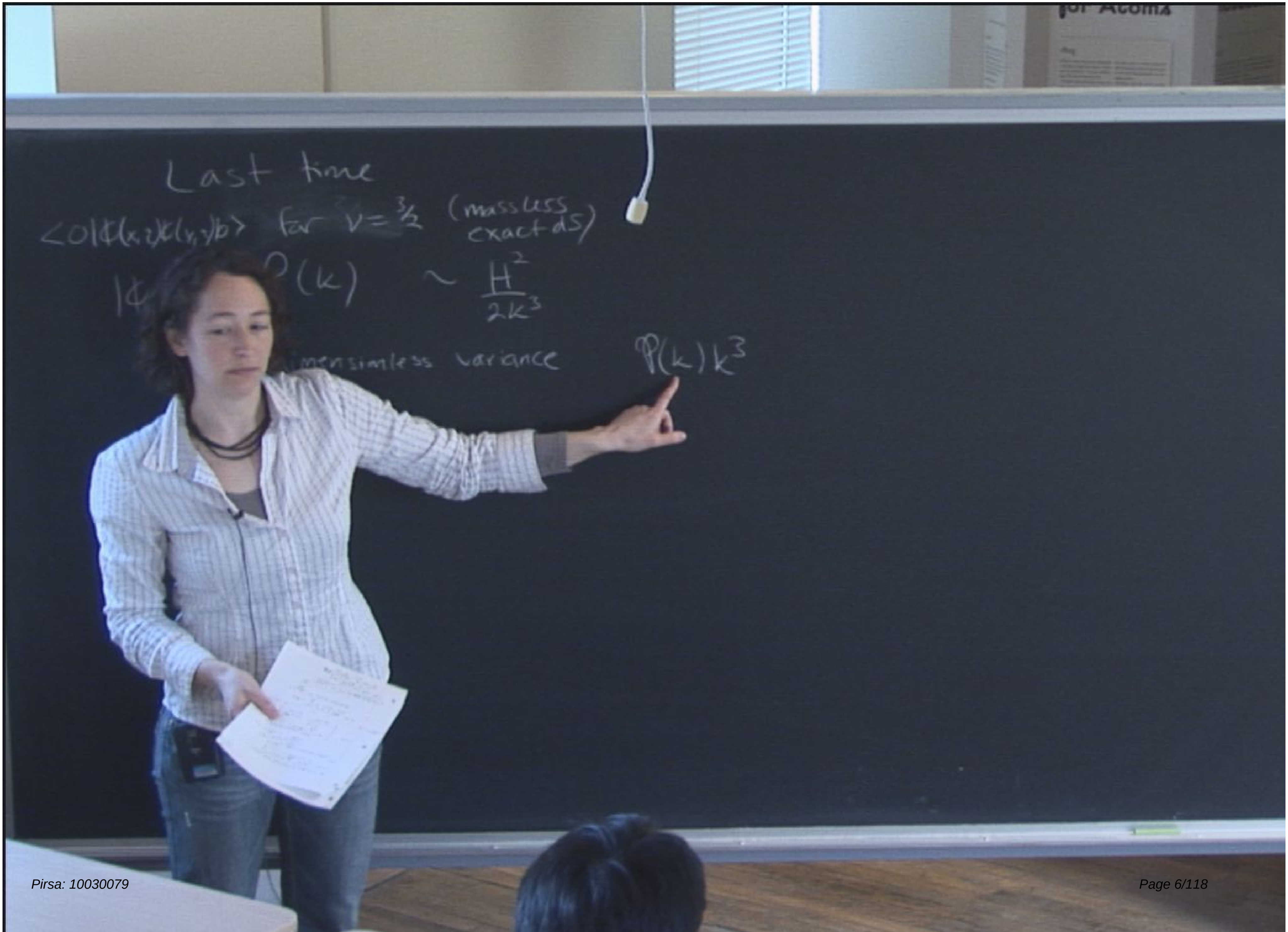
Abstract:

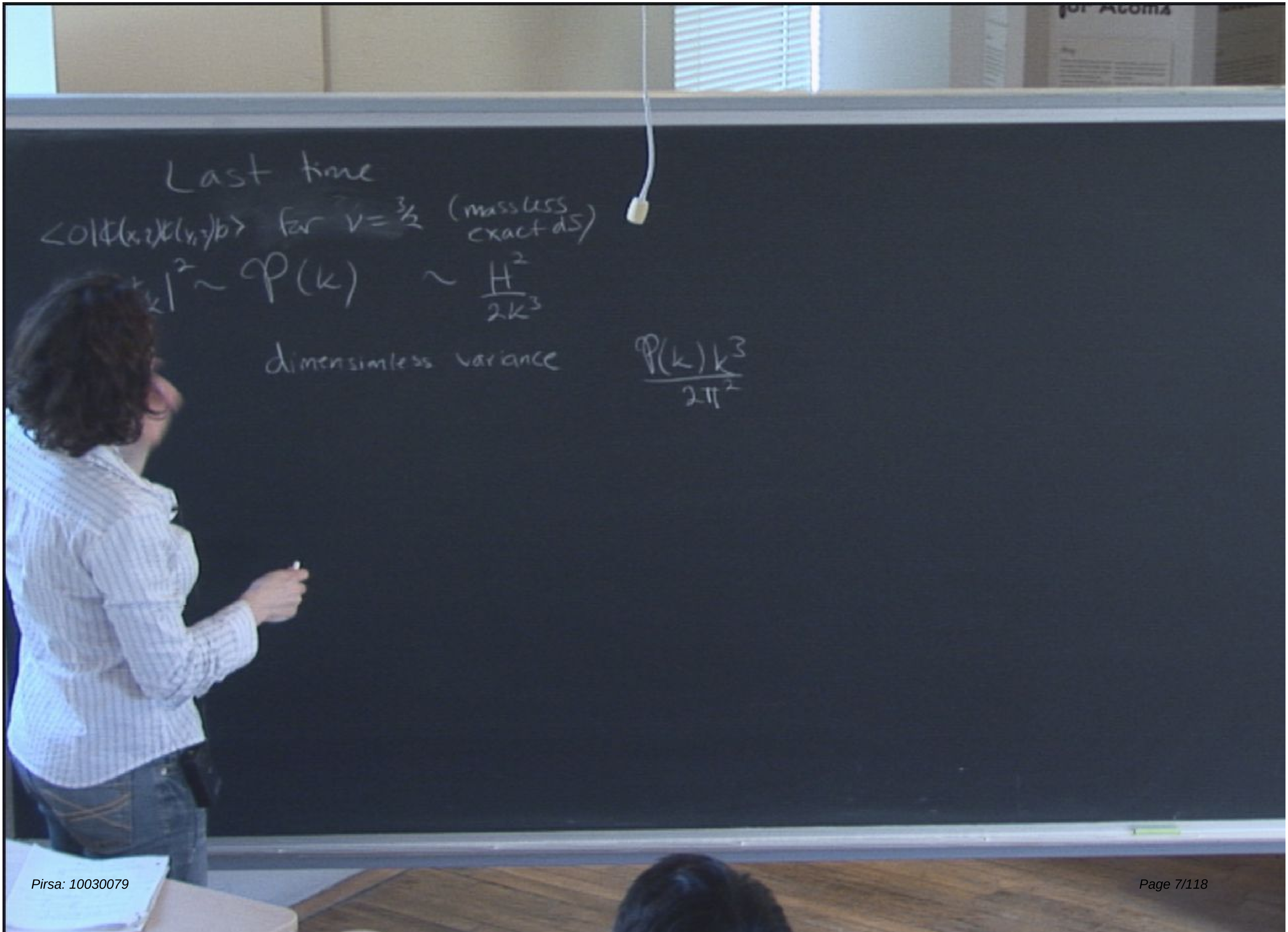

Last time
ν = 3

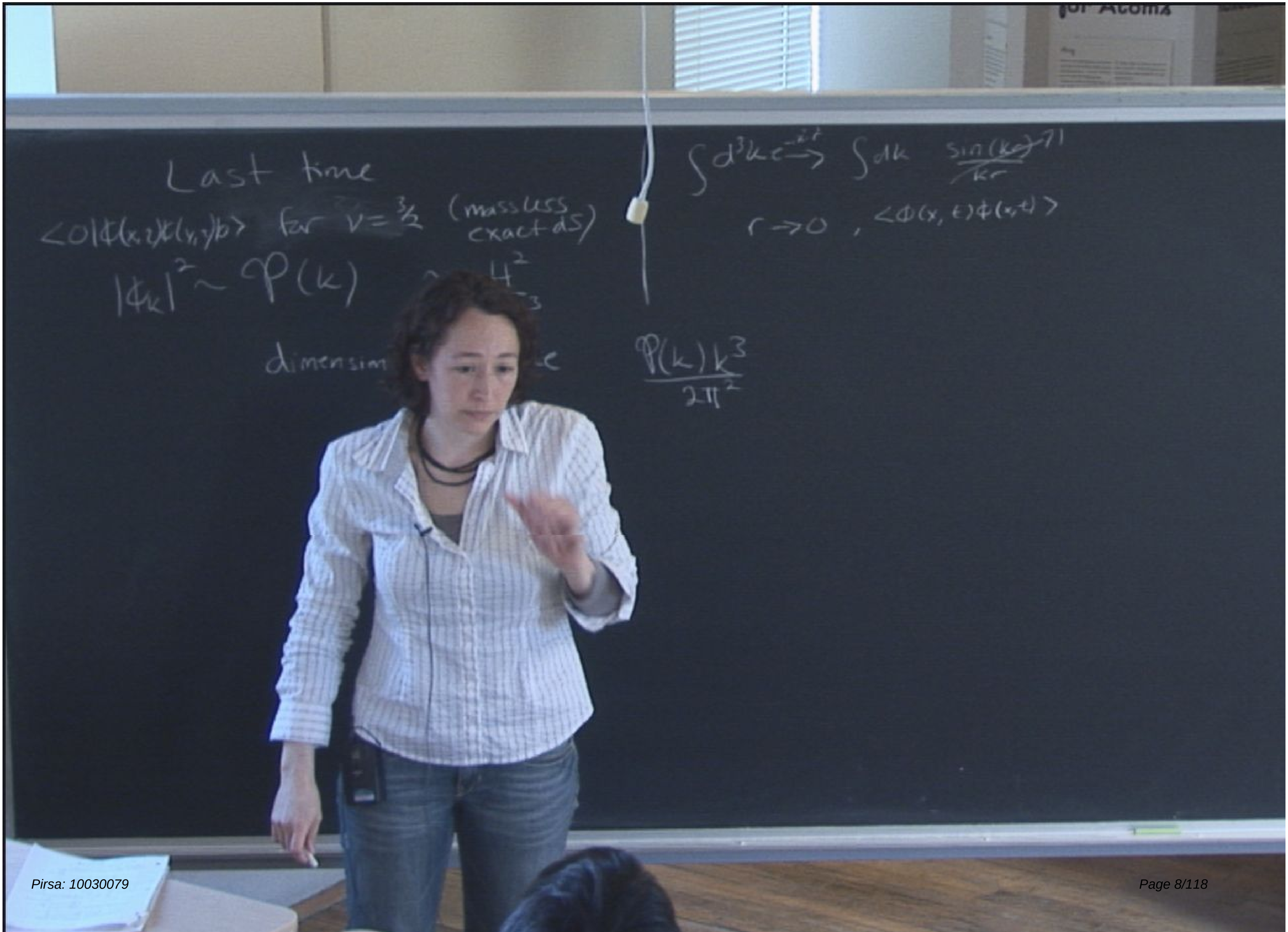

Last time
<0|Φ(x,η)Φ(y,η)|0> for ν = 3/2 (massless exact dS)

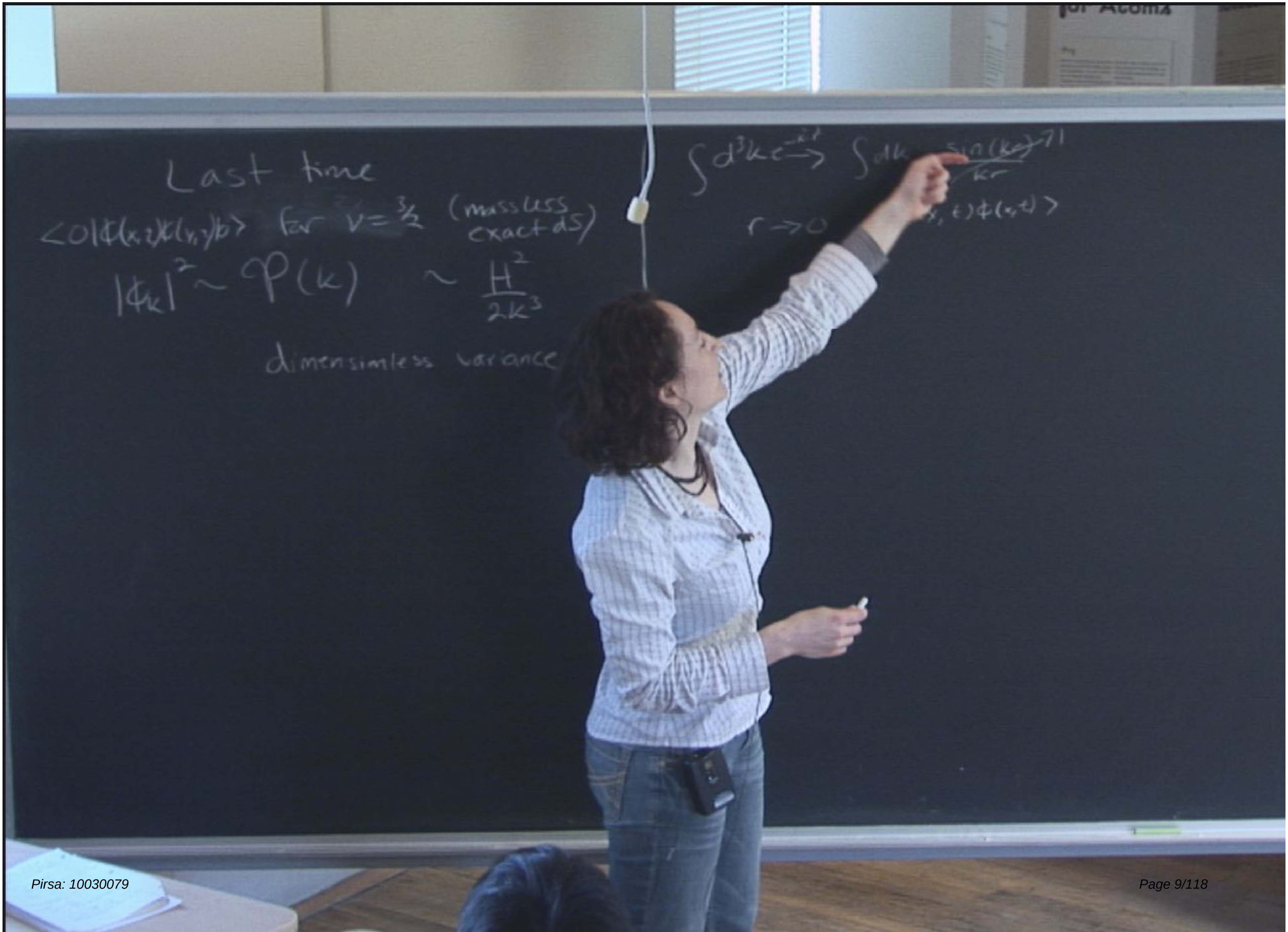

Last time
<0|φ(x,τ)φ(y,τ)|0> for ν = 3/2 (massless exact dS)
|φ_k|^2 ~ P(k) ~ H^2/2k^3

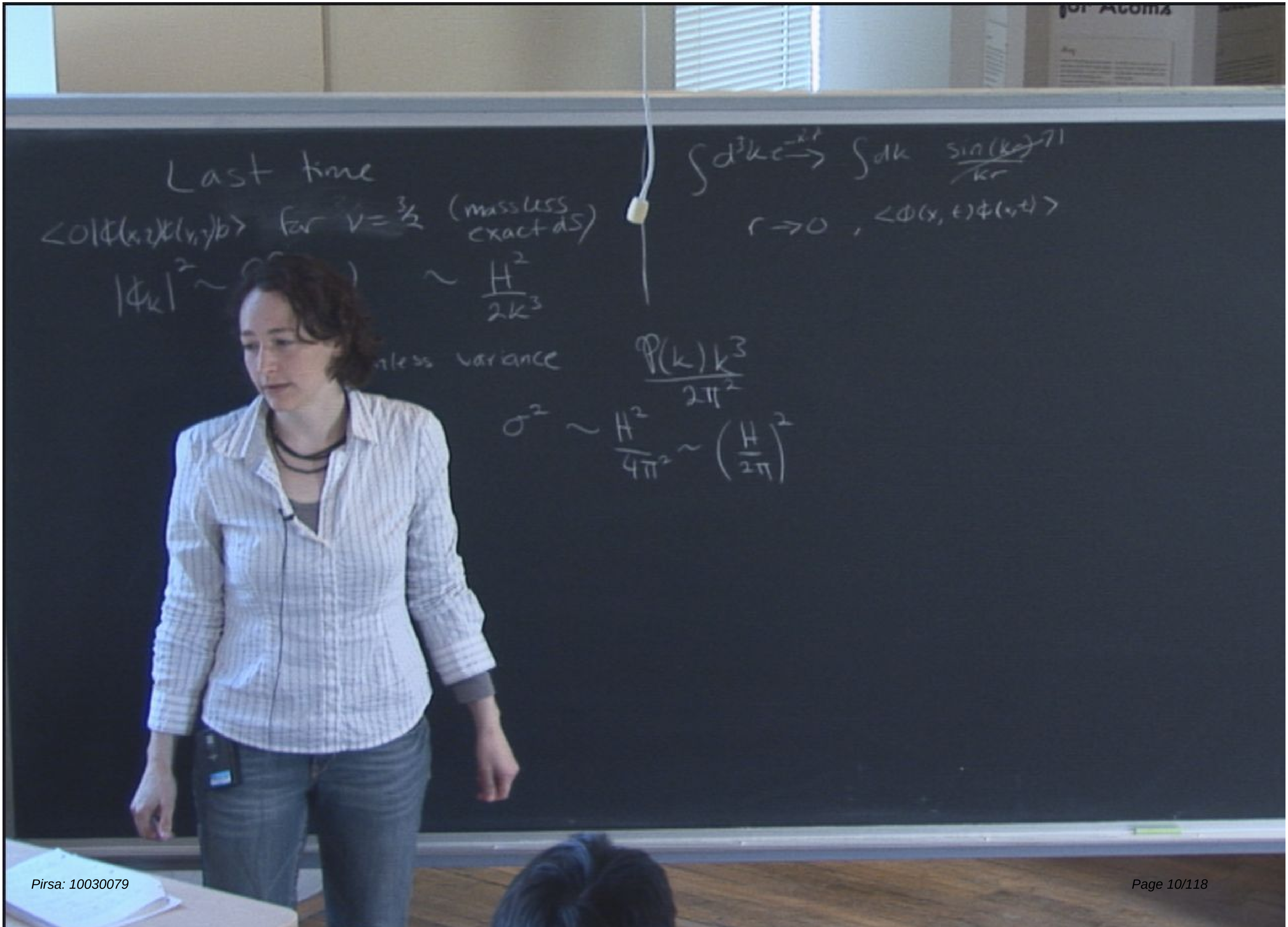

Last time
<0|Φ(x,η)Φ(y,η)|0> for ν = 3/2 (massless exact dS)
|Φk|² ~ P(k) ~ H²/2k³
dimensionless variance

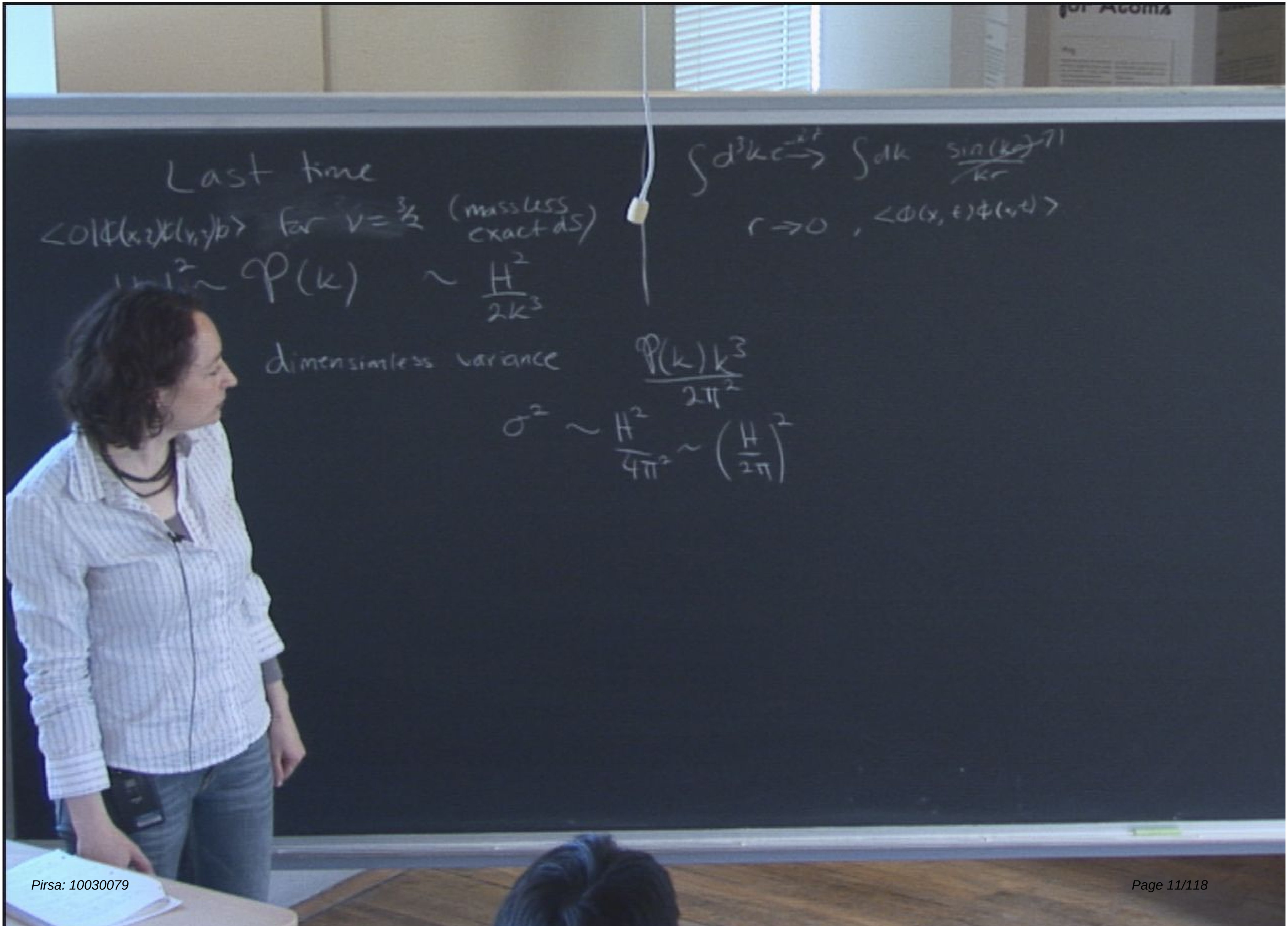

Last time
<0|φ(x,η)φ(y,η)|0> for ν = 3/2 (massless exact dS)
P(k) ~ H^2 / 2k^3
variance P(k)k^3

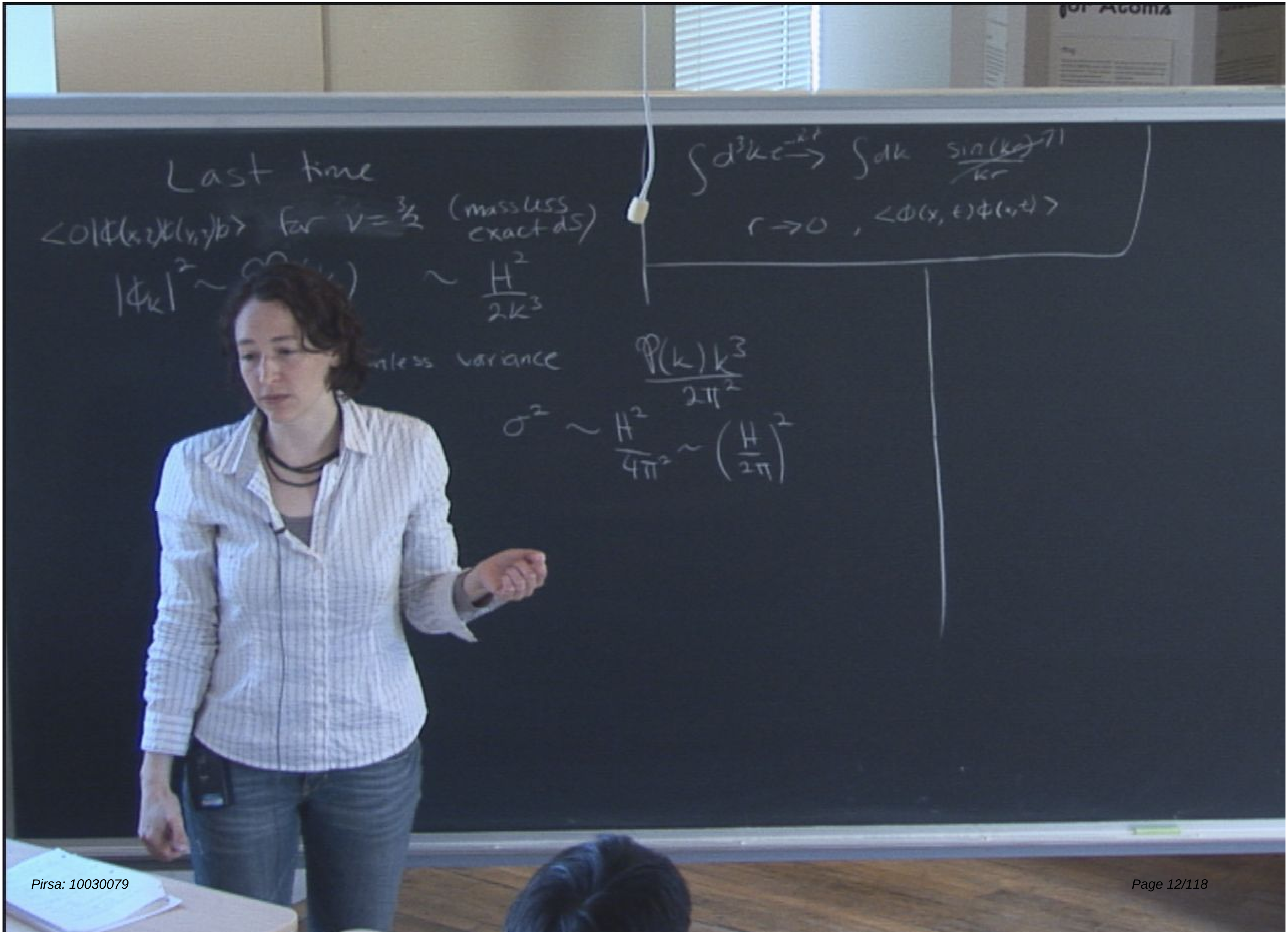

Last time
<0|Φ(x,t)Φ(y,t)|0> for ν = 3/2 (massless exact dS)
k|^2 ~ P(k) ~ H^2 / 2k^3
dimensionless variance P(k)k^3 / 2π^2

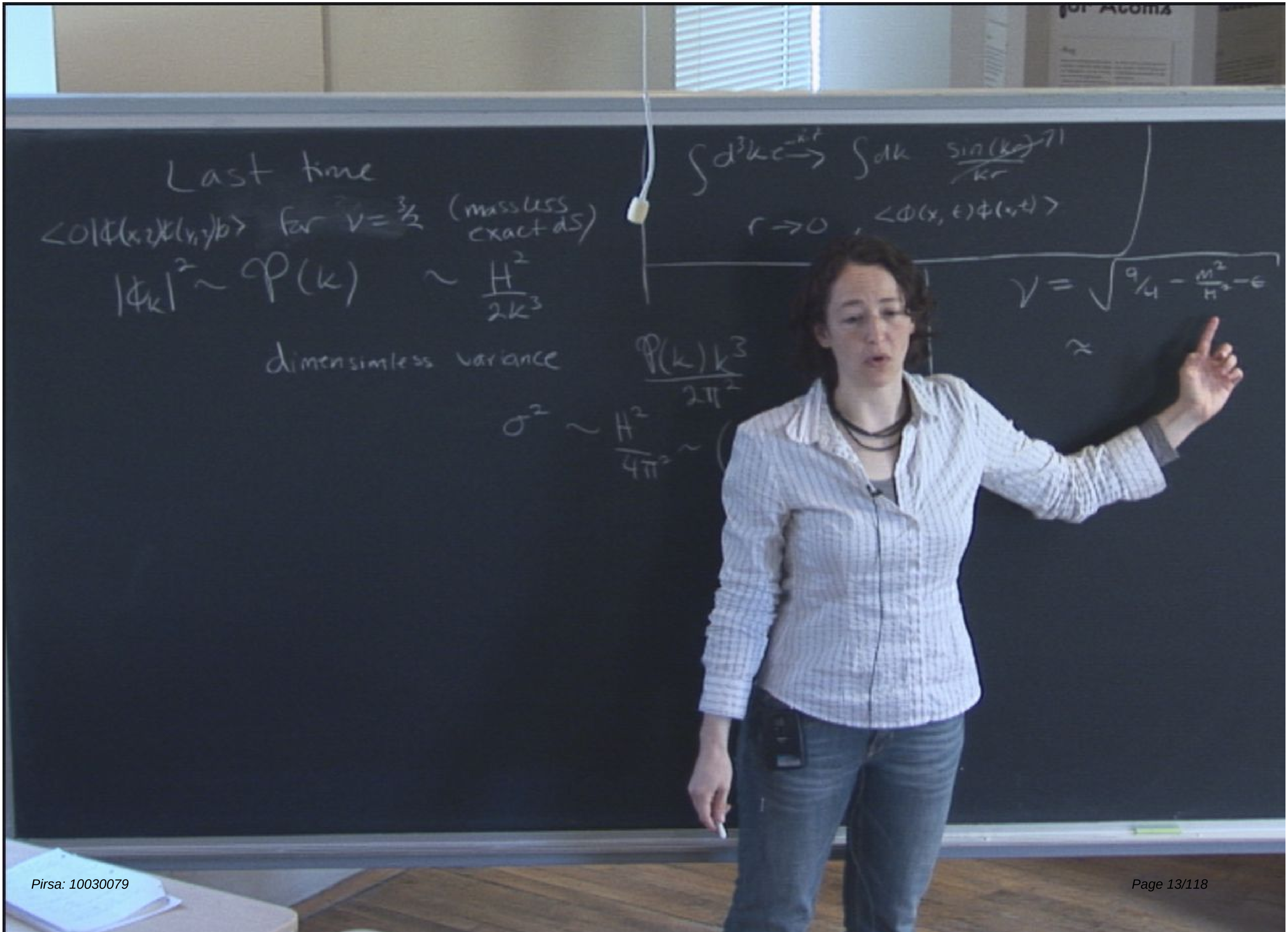

Last time
for ν = 3/2 (massless exact dS)
dimension

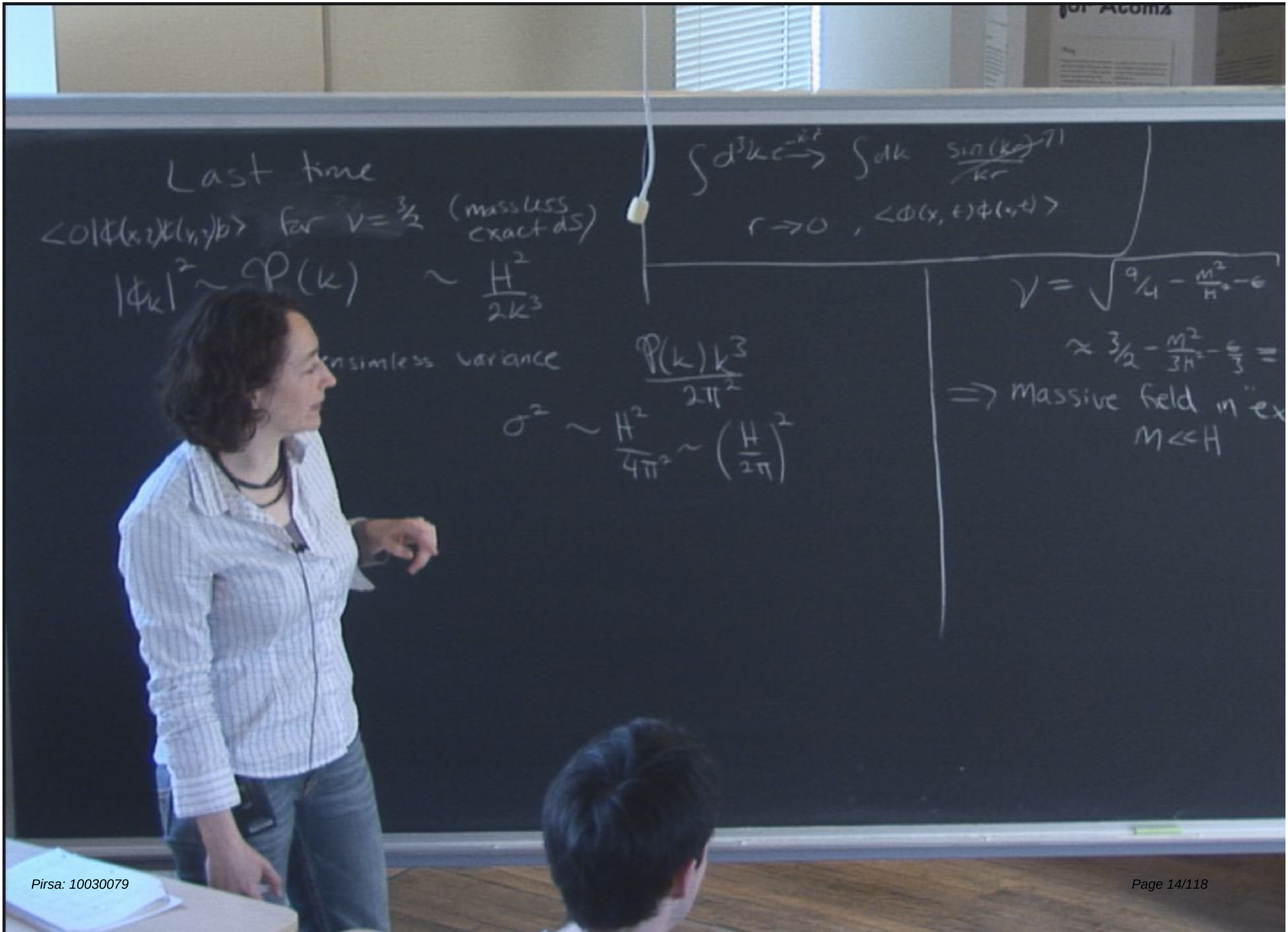

Last time
(massless exact dS)
variance

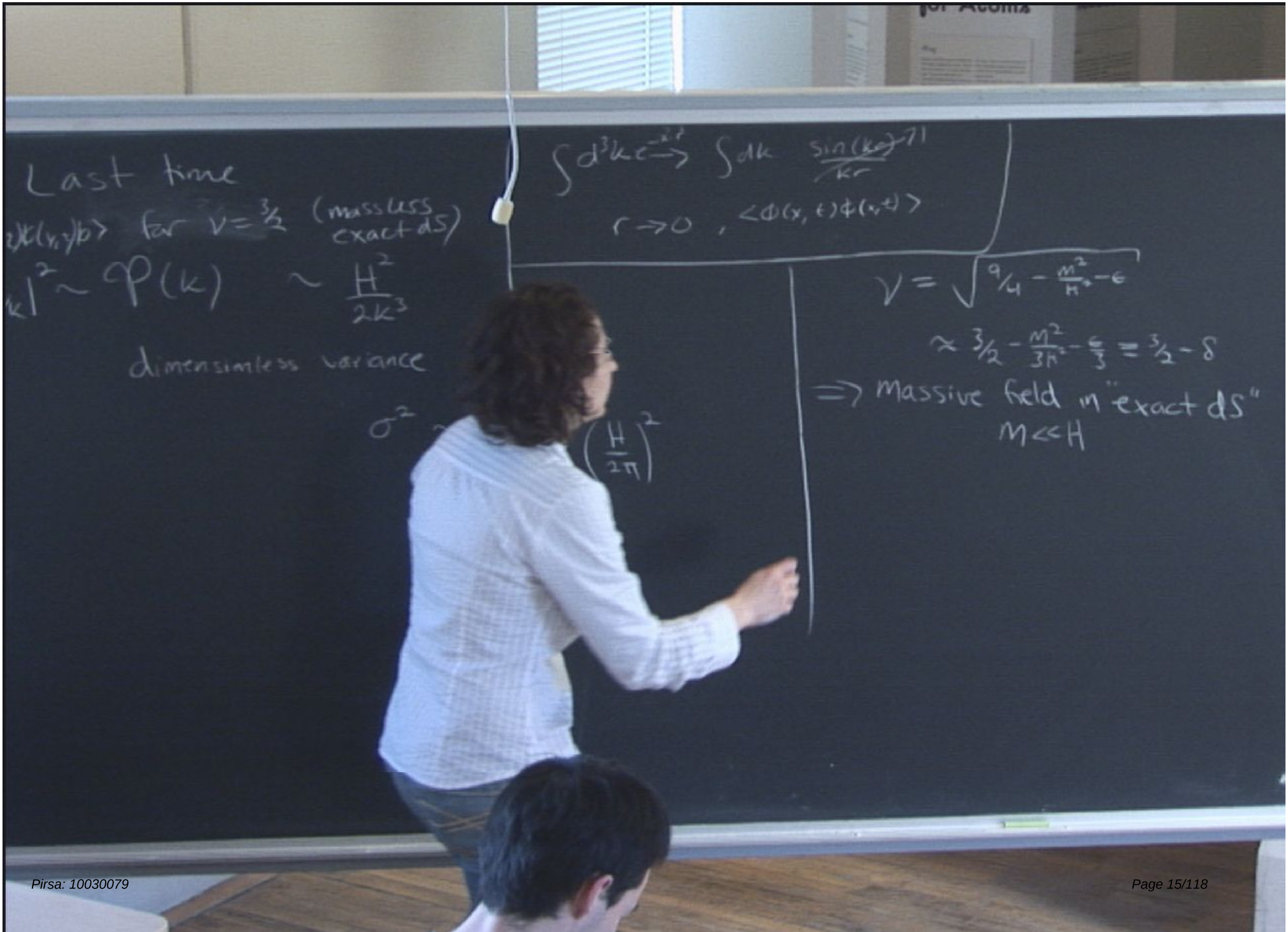

Last time
for ν = 3/2 (massless exact dS)
variance
r → 0

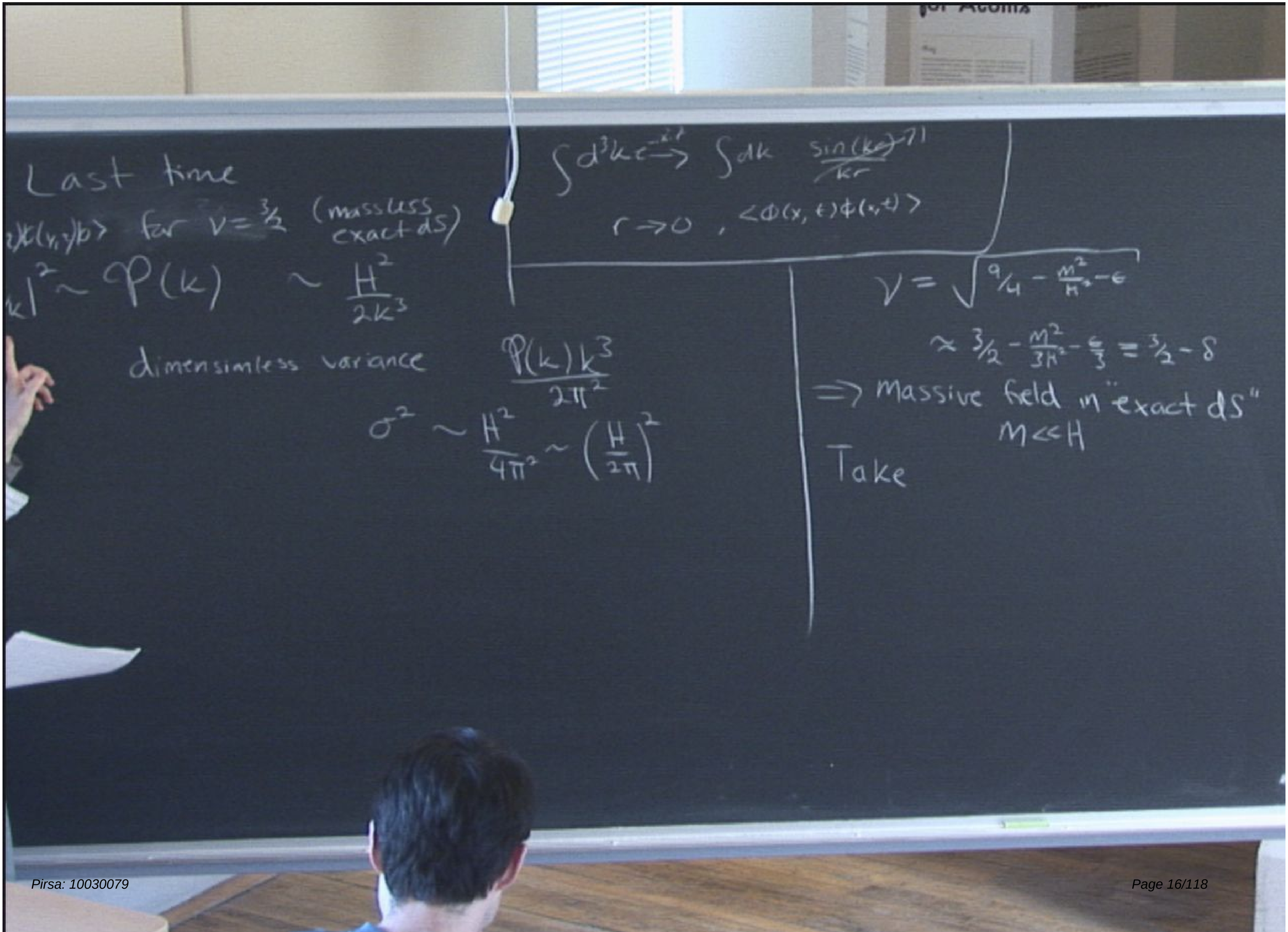

Last time
<0|Φ(x,t)Φ(y,t)|0> for ν = 3/2 (massless exact dS)
~ P(k) ~ H²/2k³
dimensionless variance P(k)k³/2π²
σ² ~ H²/4π² ~ (H/2π)²
∫d³k → ∫dk sin(kr)/kr → 1
r → 0 , <Φ(x,t)Φ(x,t)>

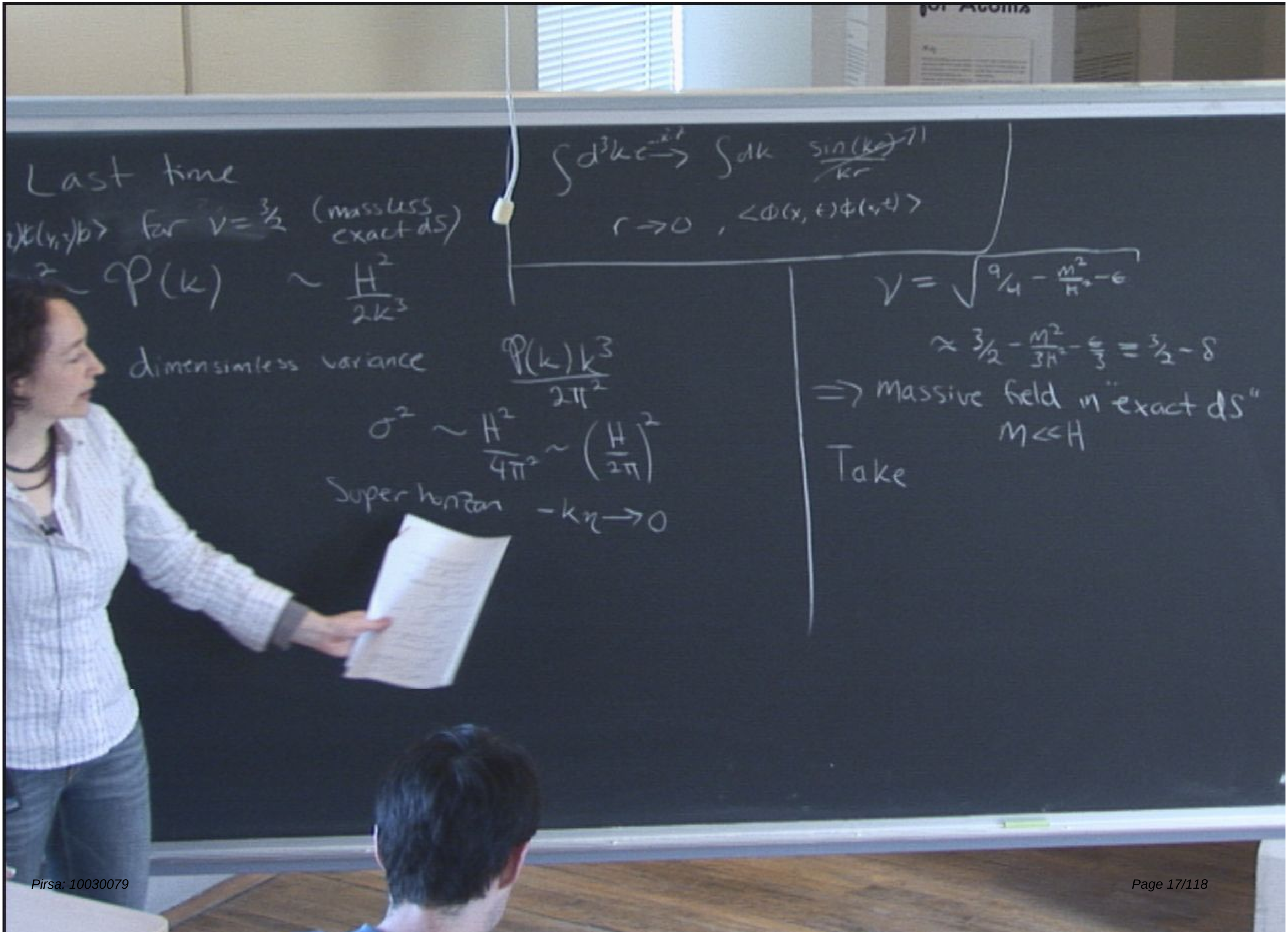

Last time
(massless
exact dS)
variance

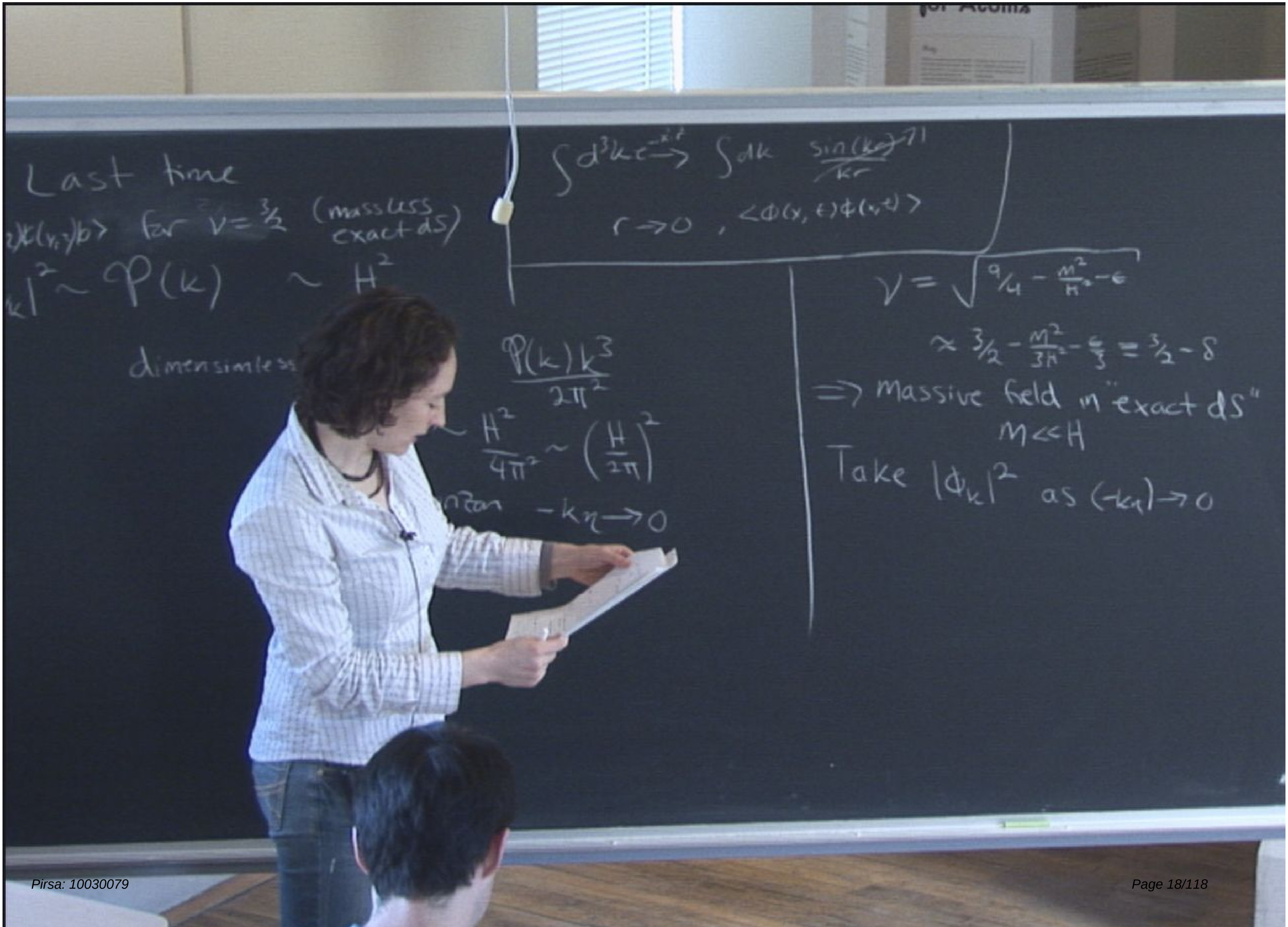

Last time
for ν = 3/2 (massless exact dS)
|φ_k|² ~ P(k) ~ H²/2k³
dimensionless variance P(k)k³/2π²
σ² ~ H²/4π²
r → 0
ν = √(9/4 − m²/H² − ε)

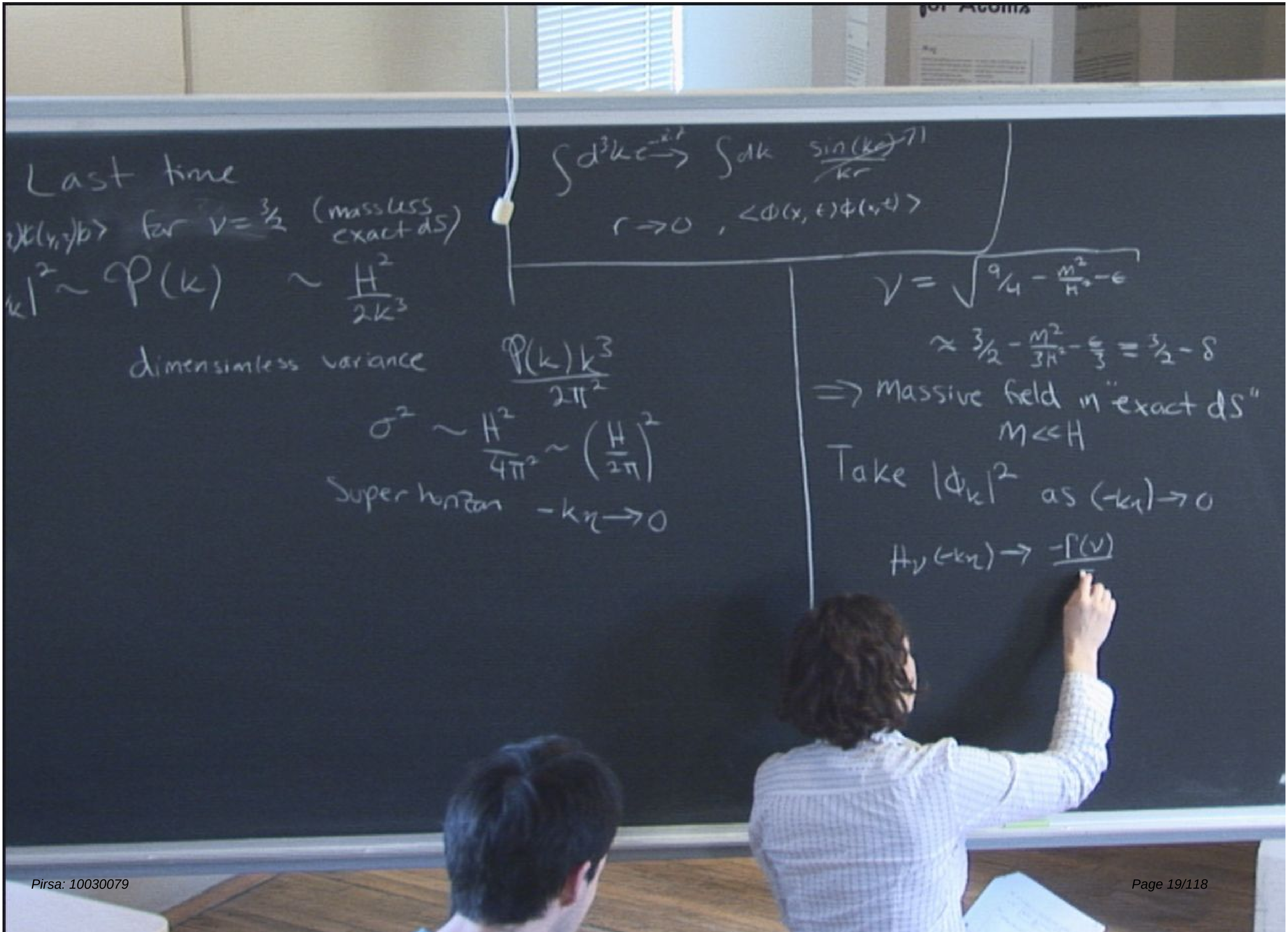

Last time
(massless exact dS)
variance
massive field in
M<<H

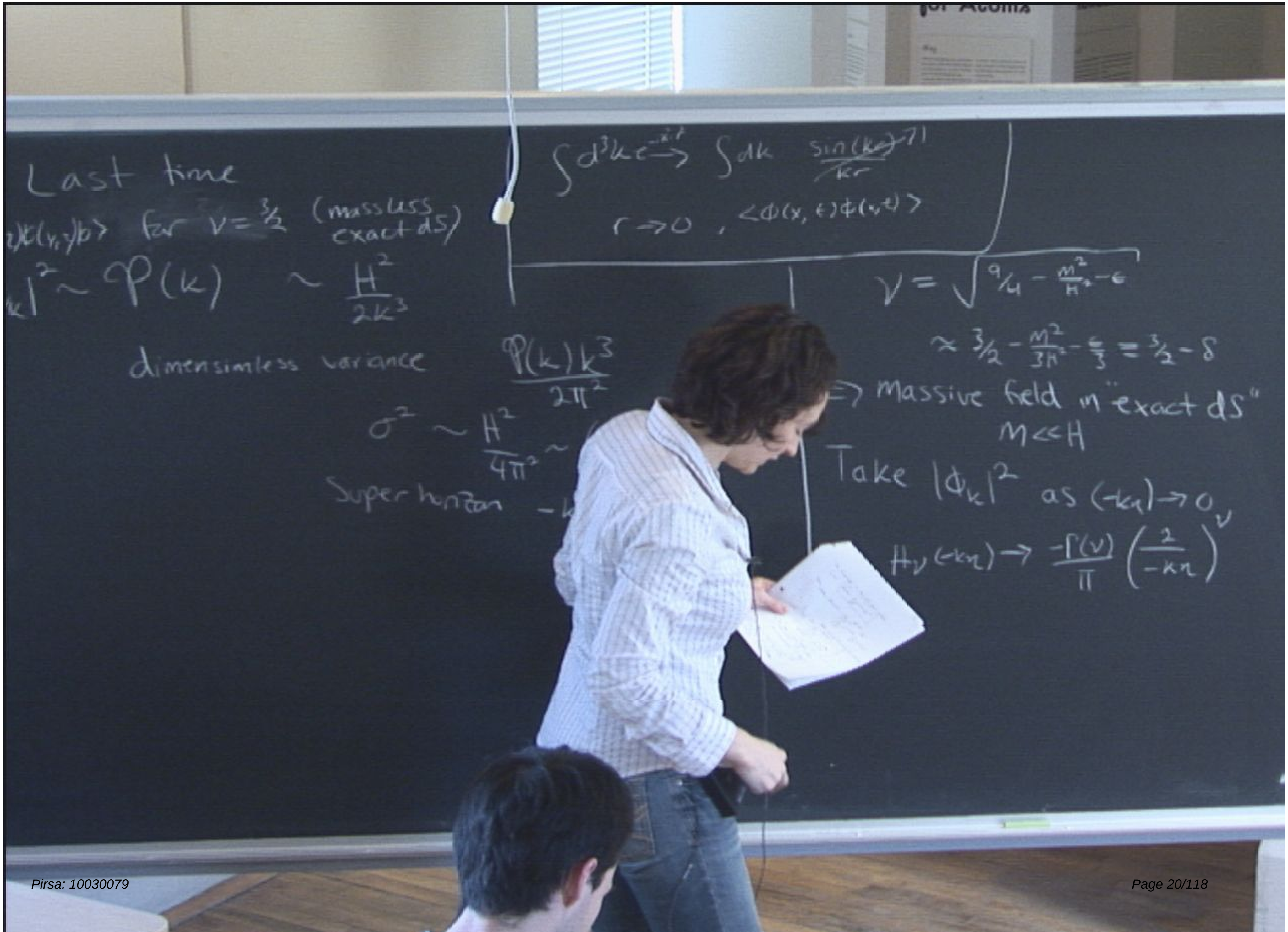

Last time
(massless exact dS)
dimensionless variance
r → 0
⇒ massive field in "exact dS"
m << H

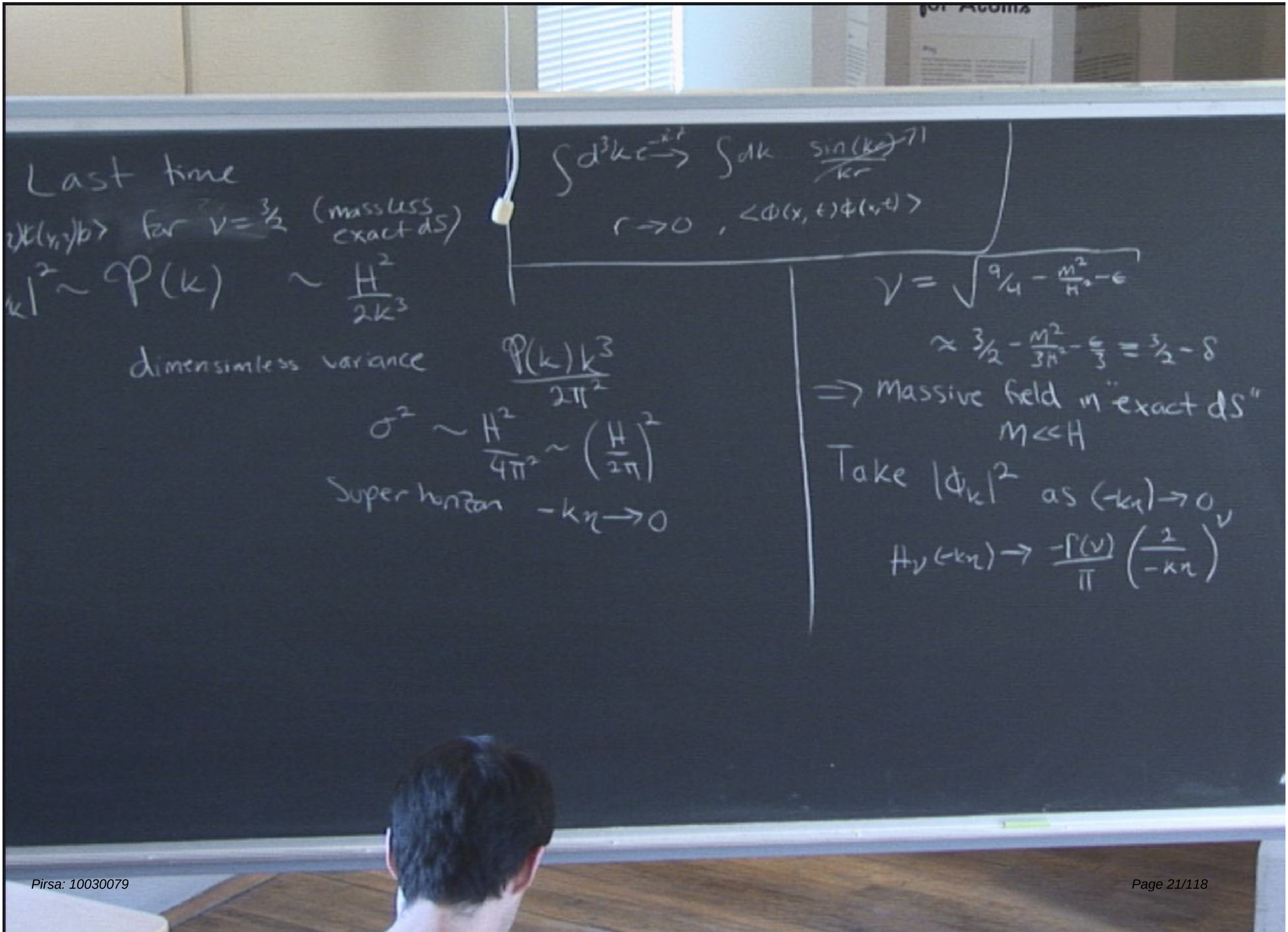

Last time
for ν = 3/2 (massless exact dS)
~ P(k) ~ H²/2k³
dimensionless variance P(k)k³/2π²
σ² ~ H²/4π² ~ (H/2π)²
∫d³k → ∫dk sin(kr)/kr
r → 0 , <Φ(x,t)Φ(x,t)>
ν = √(9/4 − m²/H² − ε)
≈ 3/2 − m²/3H² − ε/3 ≡ 3/2 − δ
⇒ massive field in "exact dS"
m << H
Take

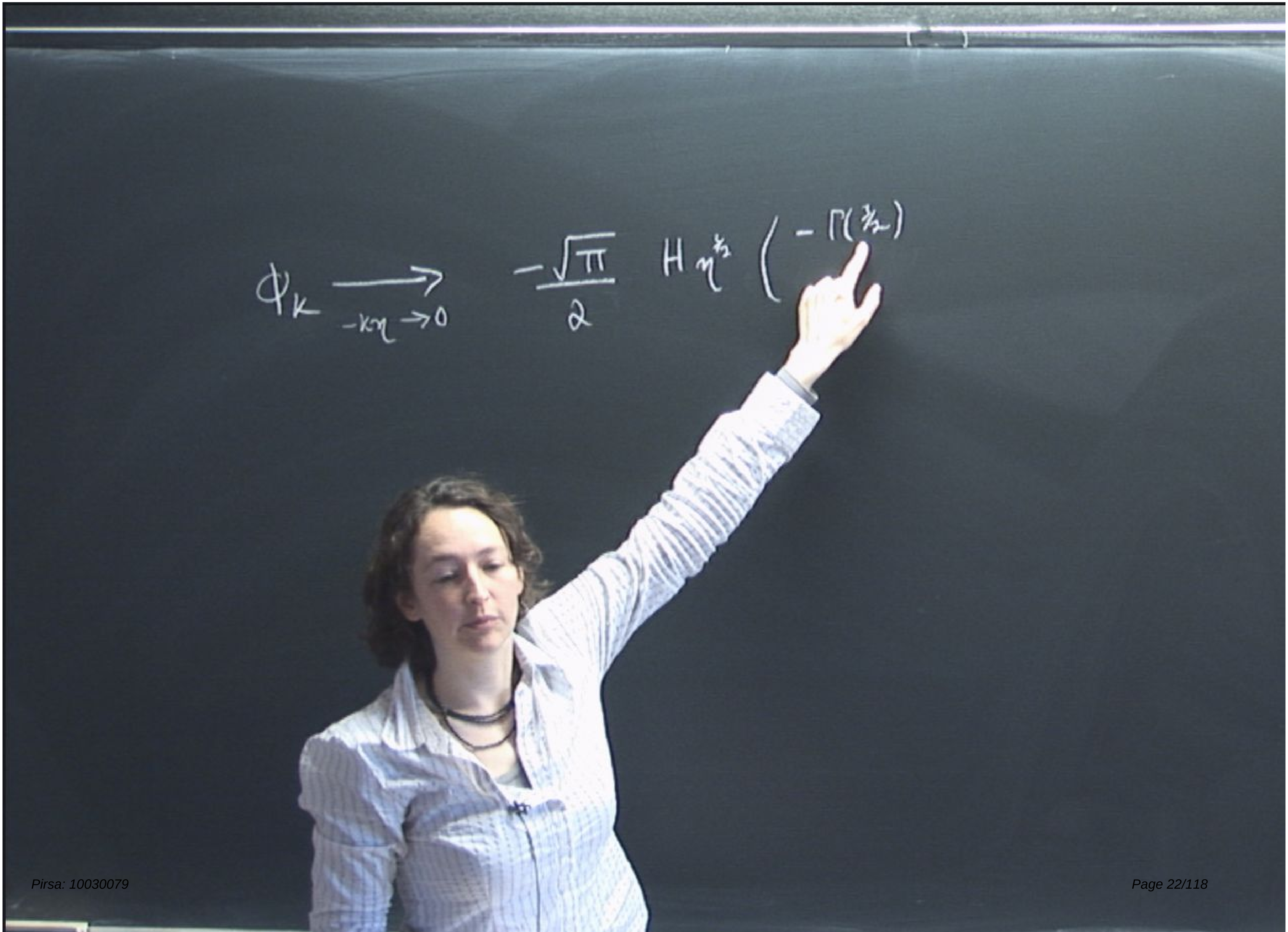

Last time
for ν = 3/2 (massless exact dS)
P(k) ~ H²/2k³
dimensionless variance P(k)k³/2π²
σ² ~ H²/4π² ~ (H/2π)²
Superhorizon −kη → 0
∫d³k e^{ik·x} → ∫dk sin(kr)/kr
r → 0 , ⟨Φ(x, t)Φ(x, t)⟩
ν = √(9/4 − m²/H² − ε)
≈ 3/2 − m²/3H² − ε/3 = 3/2 − δ
⟹ massive field in "exact dS"
m ≪ H
Take

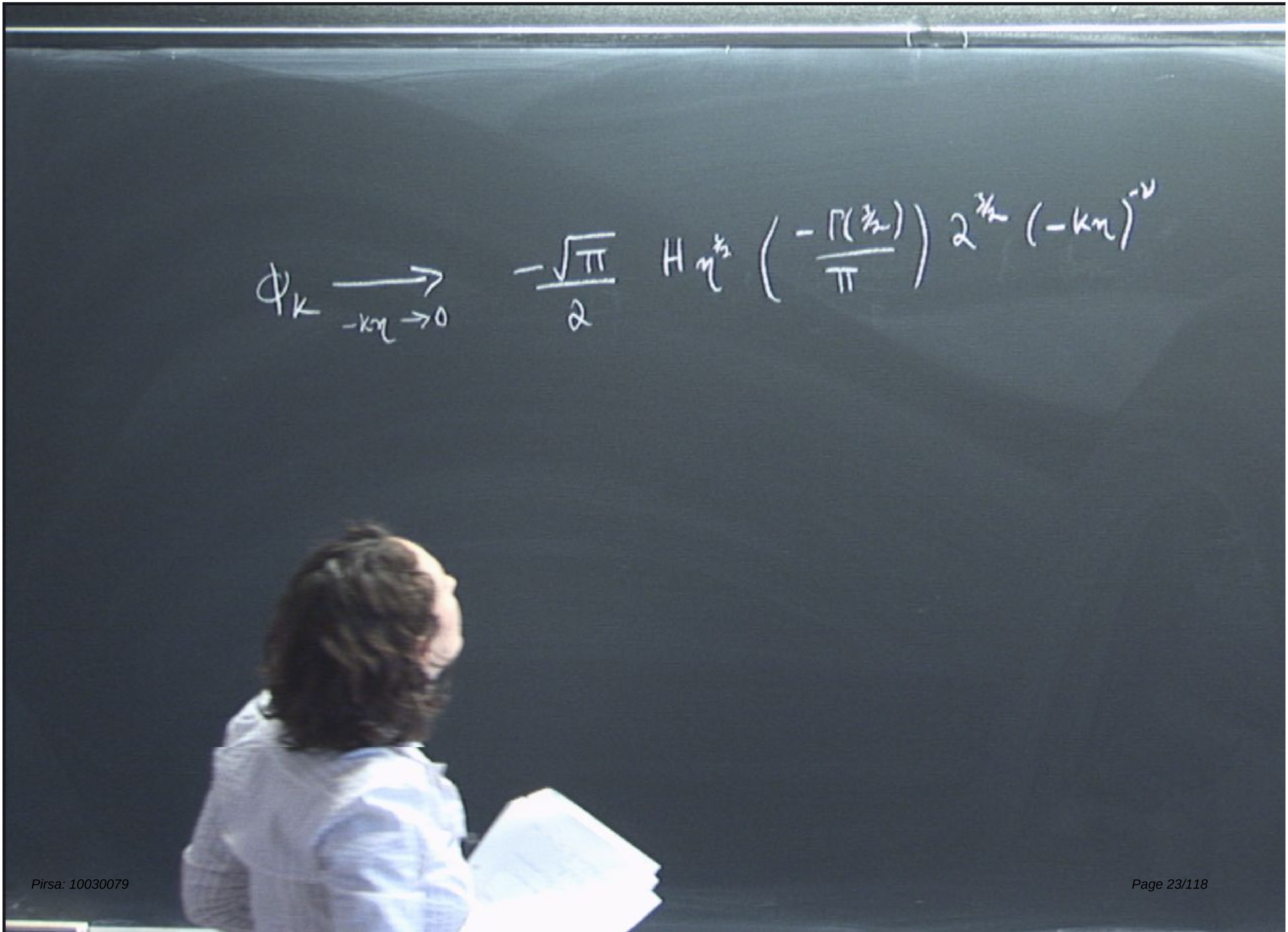

Last time
for ν = 3/2 (massless exact dS)
~ P(k) ~ H²
dimensionless
P(k) k³ / 2π²
~ H² / 4π² ~ (H / 2π)²
−kη → 0
∫d³k e^{-ik·r} → ∫dk sin(kr)/kr
r → 0 , ⟨Φ(x, t) Φ(x, t)⟩
ν = √(9/4 − m²/H² − ε)
≈ 3/2 − m²/3H² − ε/3 = 3/2 − δ
⇒ massive field in "exact dS"
m ≪ H
Take |Φ_k|² as (−kη) → 0

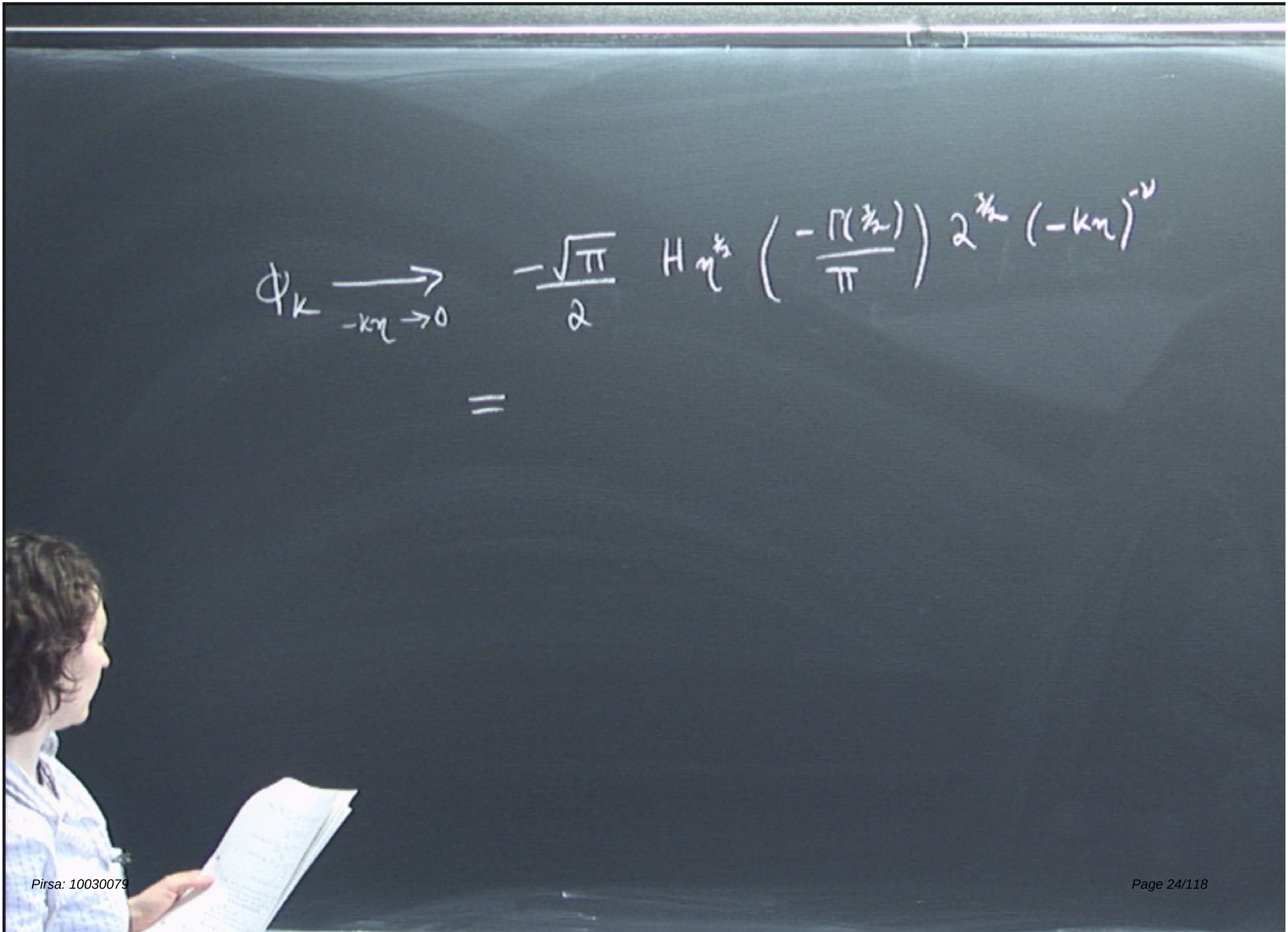

Last time
for ν = 3/2 (massless exact dS)
dimensionless variance
Super horizon −kη → 0
r → 0
⇒ massive field in "exact dS"
m << H
Take |Φk|2 as (−kη) → 0

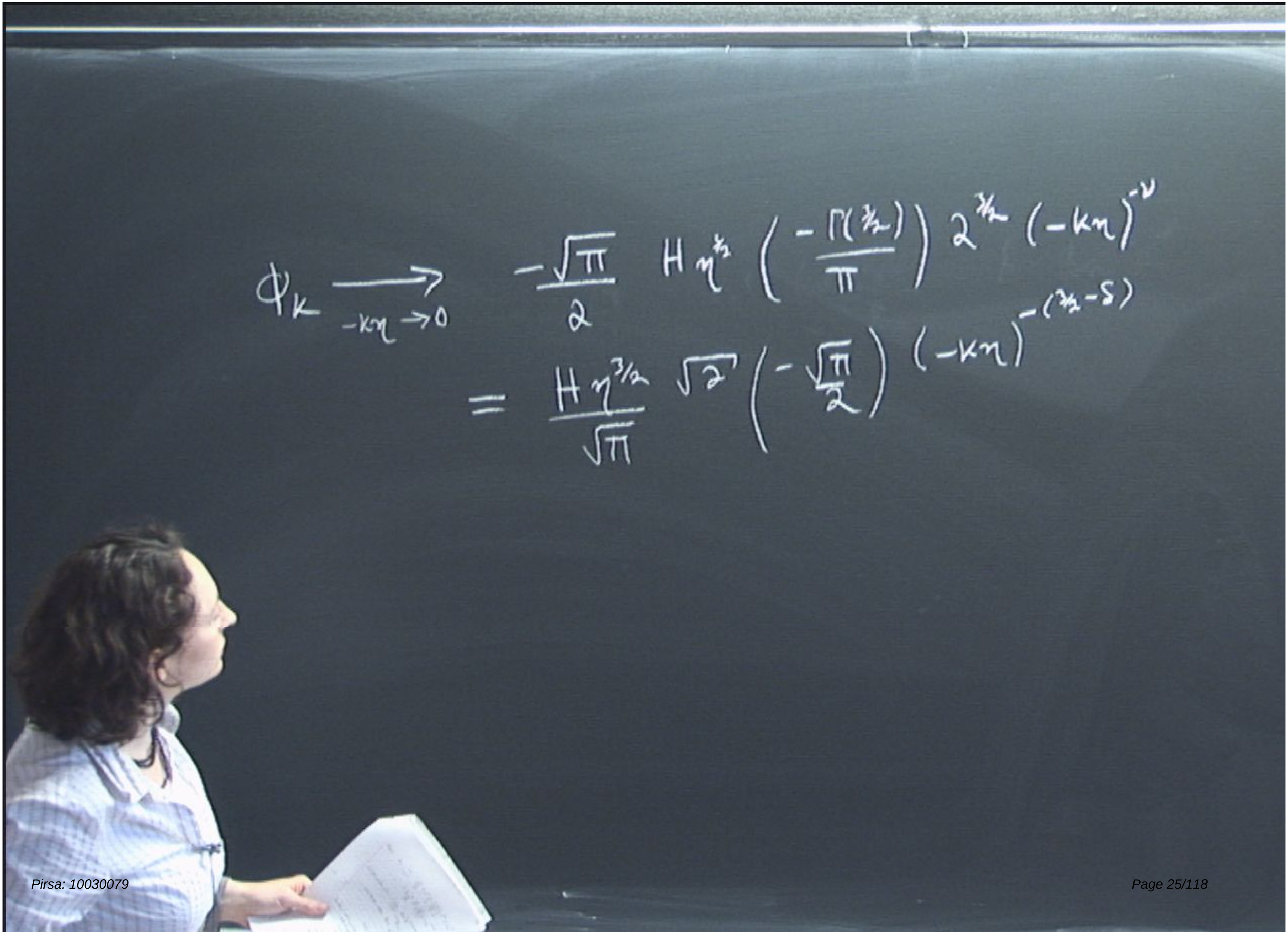

Last time
(massless exact dS)
dimensionless variance
Super horizon
massive field in "exact dS"
M<<H
Take |Φk|² as (-kη)→0

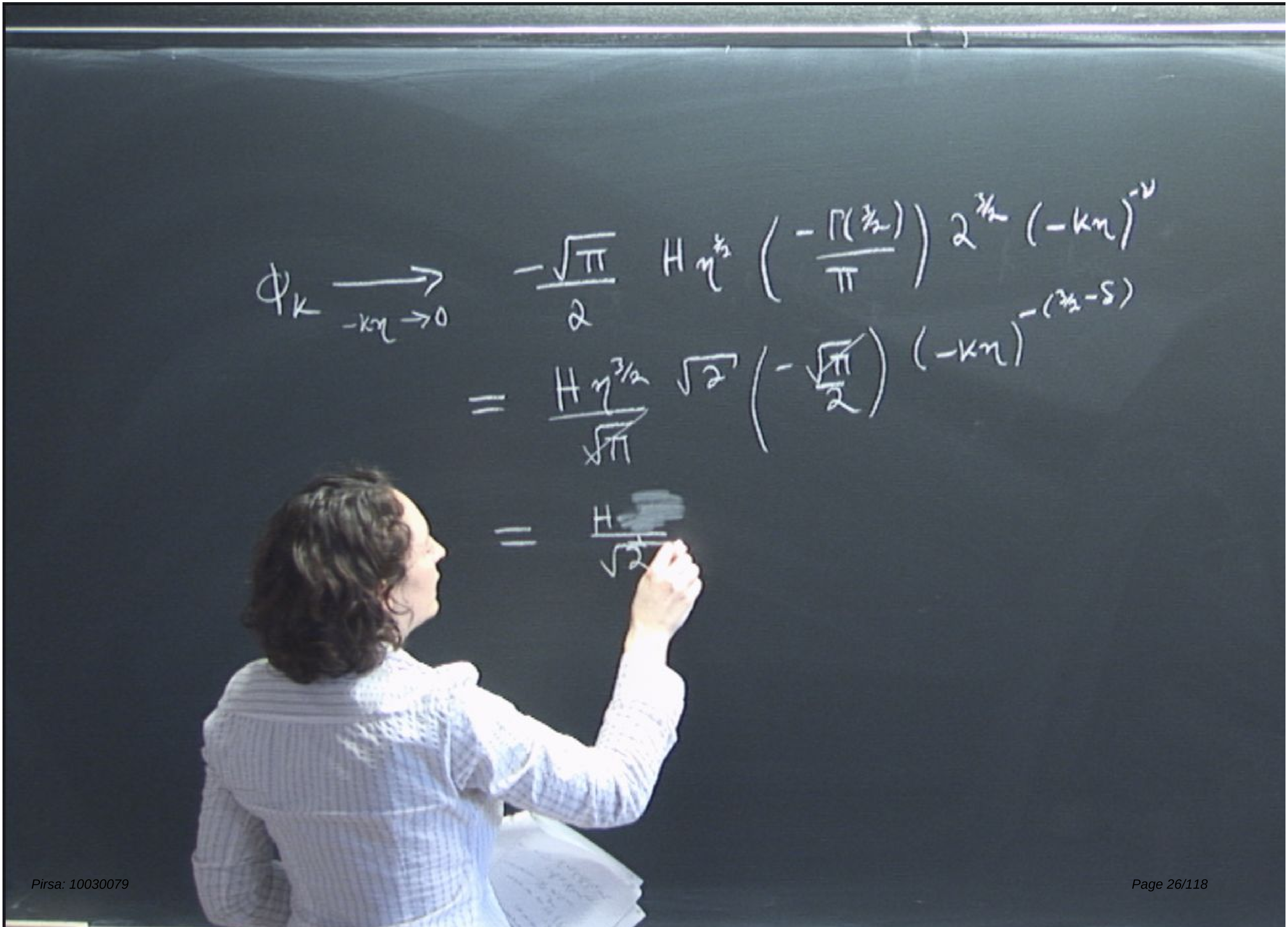

Last time
for ν = 3/2 (massless exact dS)
~ P(k) ~ H^2 / 2k^3
dimensionless variance P(k) k^3 / 2π^2
σ^2 ~ H^2 / 4π^2 ~ (H / 2π)^2
Super horizon −kη → 0
∫d^3k e^{-ik·r} → ∫dk sin(kr)/kr → 1
r → 0 , <Φ(x, t) Φ(x, t)>
ν = √(9/4 − m^2/H^2 − ε)
≈ 3/2 − m^2/3H^2 − ε/3 ≡ 3/2 − δ
⇒ massive field in "exact dS"
m << H
Take |Φ_k|^2 as (−kη) → 0
H_ν(−kη) → −Γ(ν)/π (2/−kη)^ν


$\Phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\Gamma(3/2) \right.$


$\phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$


$\phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$
$=$


$\Phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$
$= \frac{H \eta^{3/2}}{\sqrt{\pi}} \sqrt{2} \left(-\frac{\sqrt{\pi}}{2} \right) (-k\eta)^{-(3/2-\delta)}$

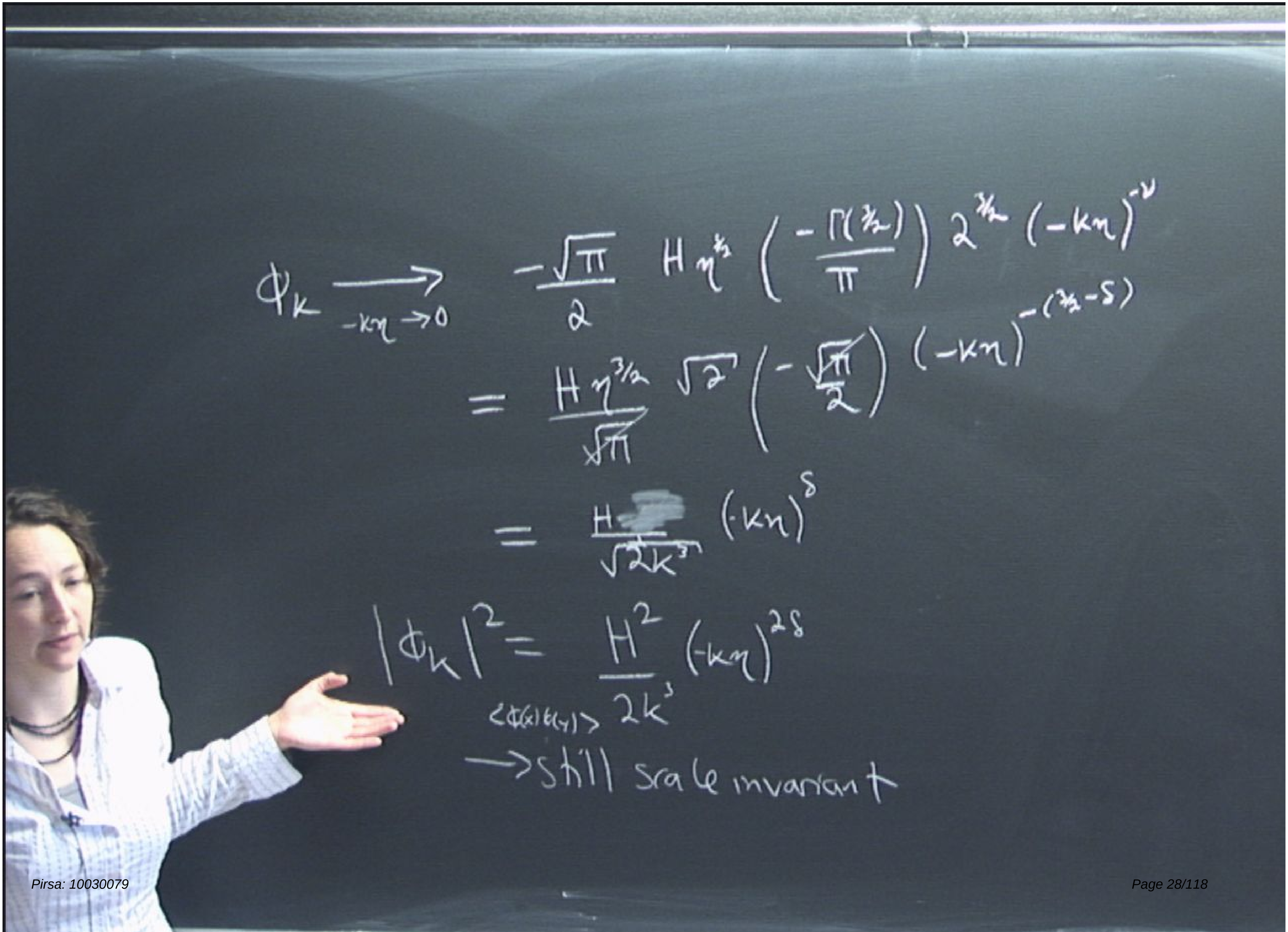

$\Phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$
$= \frac{H \eta^{3/2}}{\sqrt{\pi}} \sqrt{2} \left(-\frac{\sqrt{\pi}}{2} \right) (-k\eta)^{-(3/2-\delta)}$
$= \frac{H}{\sqrt{2}}$

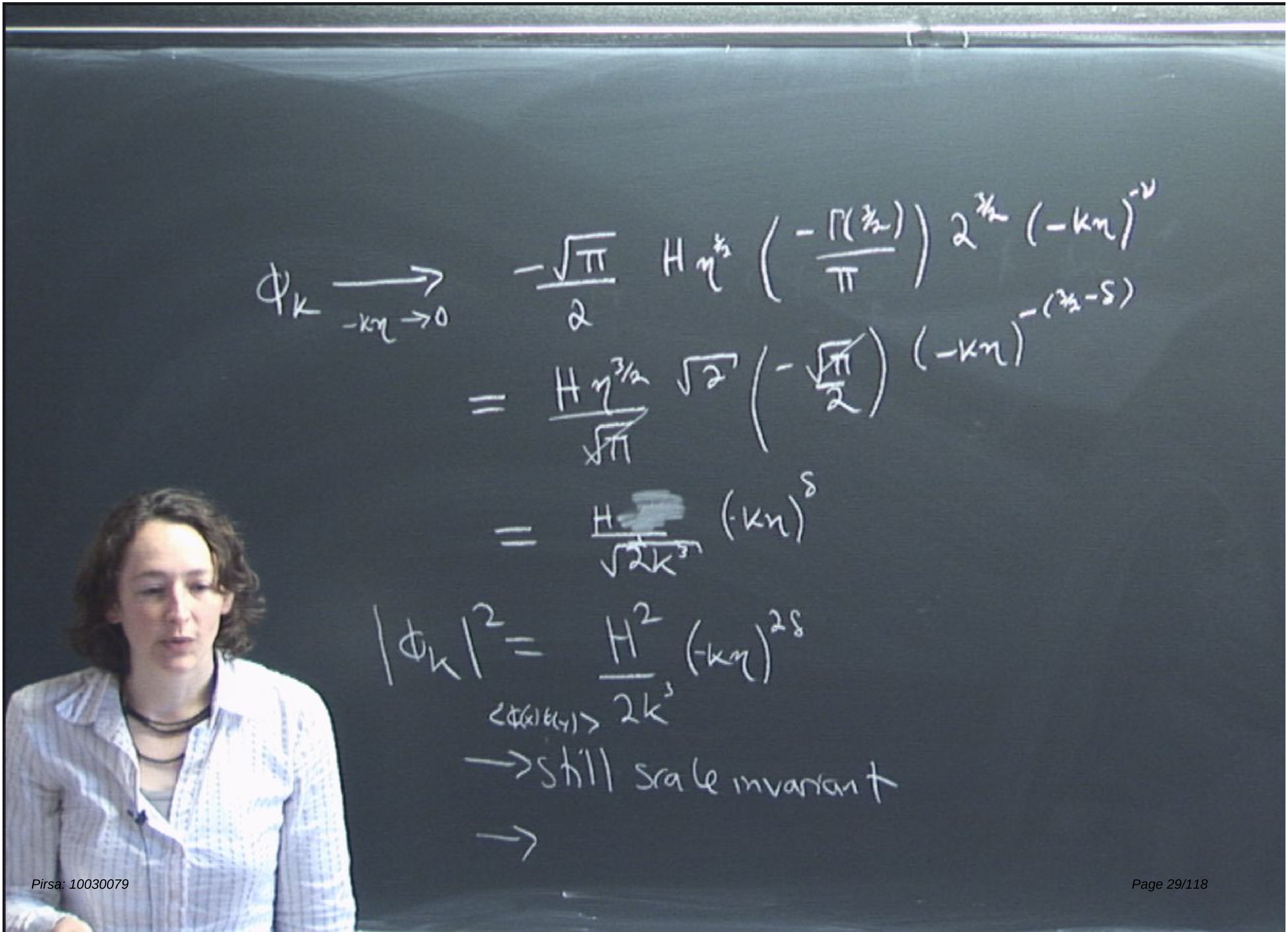
$$\Phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$$

$$= \frac{H \eta^{3/2}}{\sqrt{\pi}} \sqrt{2} \left(-\frac{\sqrt{\pi}}{2} \right) (-k\eta)^{-(3/2-\delta)}$$

$$= \frac{H}{\sqrt{2k^3}} (-k\eta)^{\delta}$$

$$|\Phi_k|^2 = \frac{H^2}{2k^3} (-k\eta)^{2\delta}$$


$\phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$
$= \frac{H \eta^{3/2}}{\sqrt{\pi}} \sqrt{2} \left(-\frac{\sqrt{\pi}}{2} \right) (-k\eta)^{-(3/2-\delta)}$
$= \frac{H}{\sqrt{2k^3}} (-k\eta)^{\delta}$
$|\phi_k|^2 = \frac{H^2}{2k^3} (-k\eta)^{2\delta}$
$\langle \phi(x) \phi(y) \rangle$
$\rightarrow$ still scale invariant


$\Phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi}\right) 2^{3/2} (-k\eta)^{-\nu}$
$= \frac{H\eta^{3/2}}{\sqrt{\pi}} \sqrt{2} \left(-\frac{\sqrt{\pi}}{2}\right) (-k\eta)^{-(3/2-\delta)}$
$= \frac{H}{\sqrt{2k^3}} (-k\eta)^{\delta}$
$|\Phi_k|^2 = \frac{H^2}{2k^3} (-k\eta)^{2\delta}$
$\langle \Phi(x) \Phi(y) \rangle$
→ still scale invariant
→

$$\phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2}\, H\eta^{3/2}\left(-\frac{\Gamma(3/2)}{\pi}\right) 2^{3/2}(-k\eta)^{-\nu}$$

$$= \frac{H\eta^{3/2}}{\sqrt{\pi}}\sqrt{2}\left(-\frac{\sqrt{\pi}}{2}\right)(-k\eta)^{-(3/2-\delta)}$$

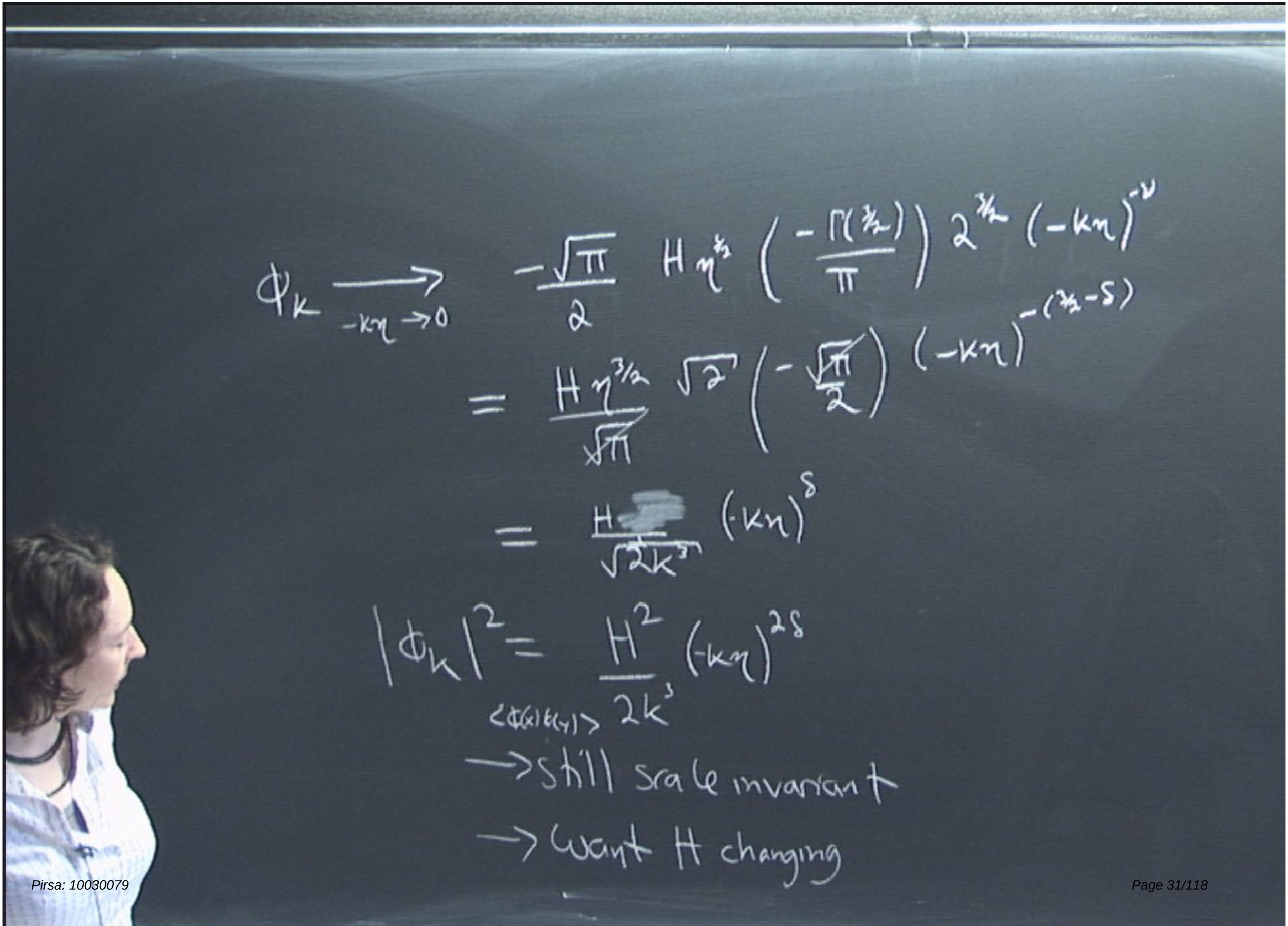
$$= \frac{H}{\sqrt{2k^3}}(-k\eta)^{\delta}$$

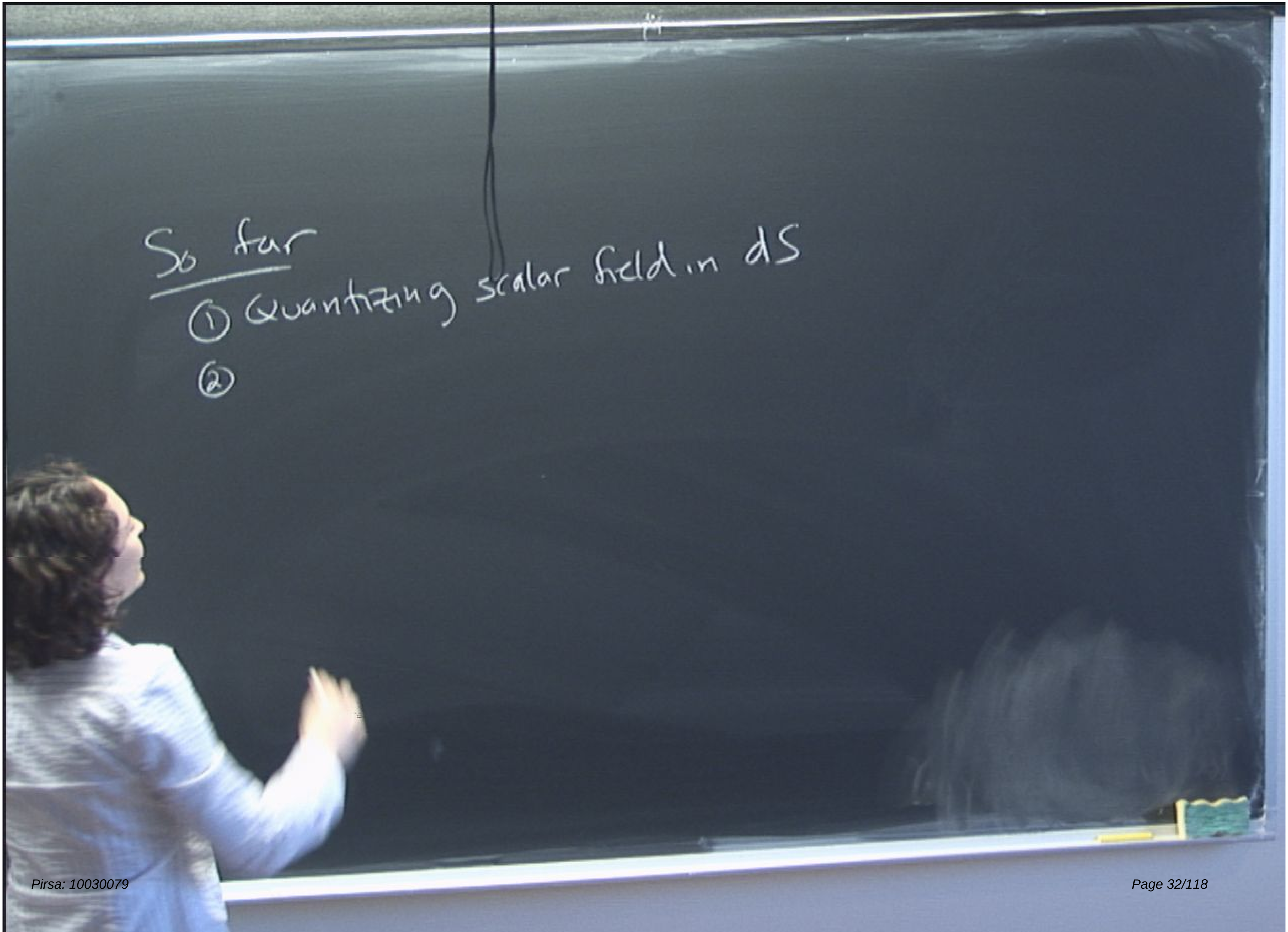
$$|\phi_k|^2 = \frac{H^2}{2k^3}(-k\eta)^{2\delta}$$

$\langle\phi(x)\phi(y)\rangle$

→ still scale invariant

→ want H changing


$\Phi_k \xrightarrow[-k\eta \to 0]{} -\frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(-\frac{\Gamma(3/2)}{\pi} \right) 2^{3/2} (-k\eta)^{-\nu}$
$= \frac{H \eta^{3/2}}{\sqrt{\pi}} \sqrt{2} \left(-\frac{\sqrt{\pi}}{2} \right) (-k\eta)^{-(3/2-\delta)}$
$= \frac{H}{\sqrt{2k^3}} (-k\eta)^{\delta}$
$|\Phi_k|^2 = \frac{H^2}{2k^3} (-k\eta)^{2\delta}$
$\langle \Phi(x) \Phi(y) \rangle$
→ still scale invariant
→ want H changing


So far
① Quantizing scalar field in dS
②

So far

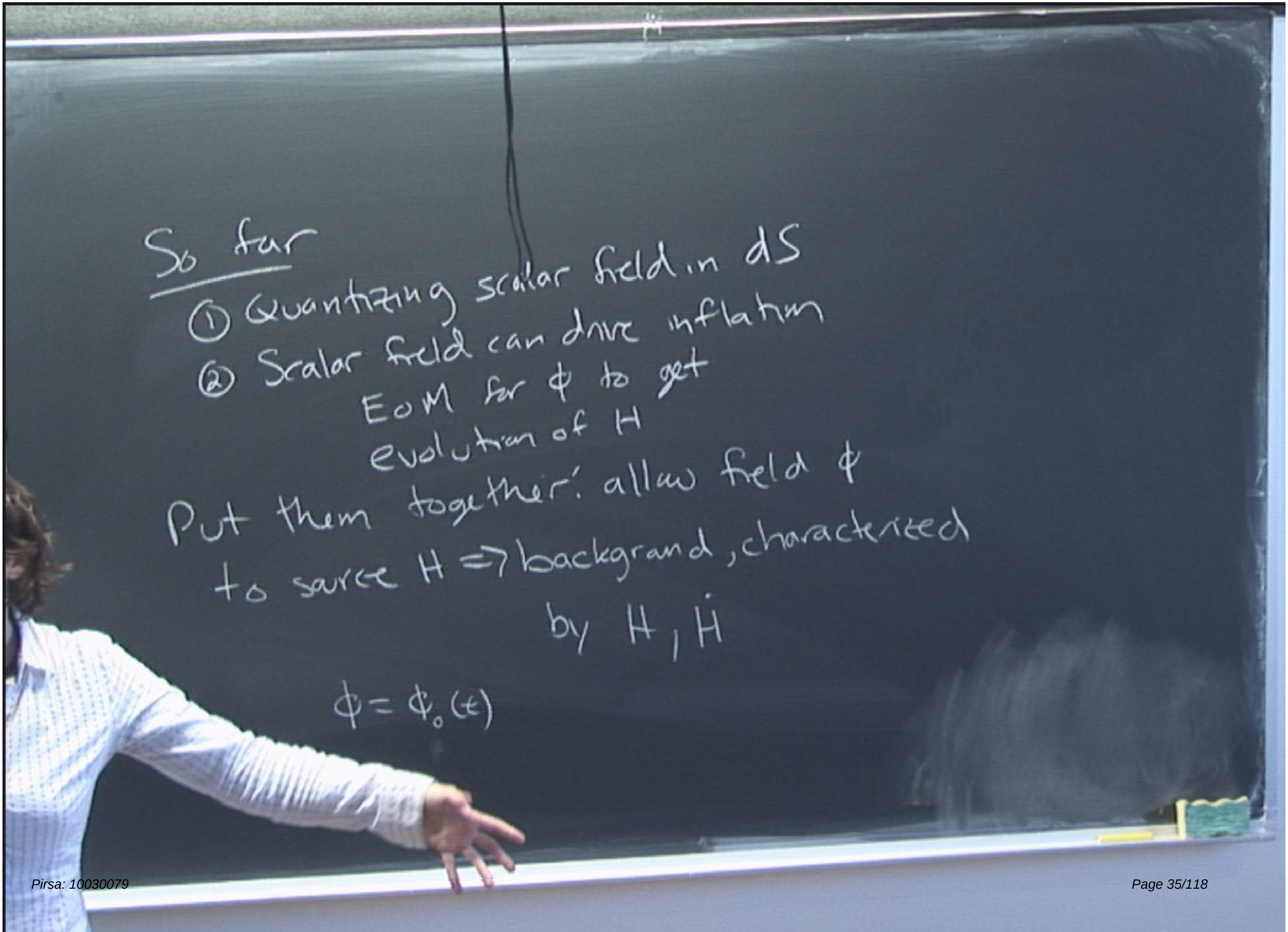
1. Quantizing scalar field in dS
2. Scalar field can drive inflation

   EoM for ϕ to get evolution of H

So far

1. Quantizing scalar field in dS
2. Scalar field can drive inflation
   EoM for ϕ to get
   evolution of H

Put them together: allow field ϕ
to source $H \Rightarrow$ background, characterized
by $H, \dot{H}$

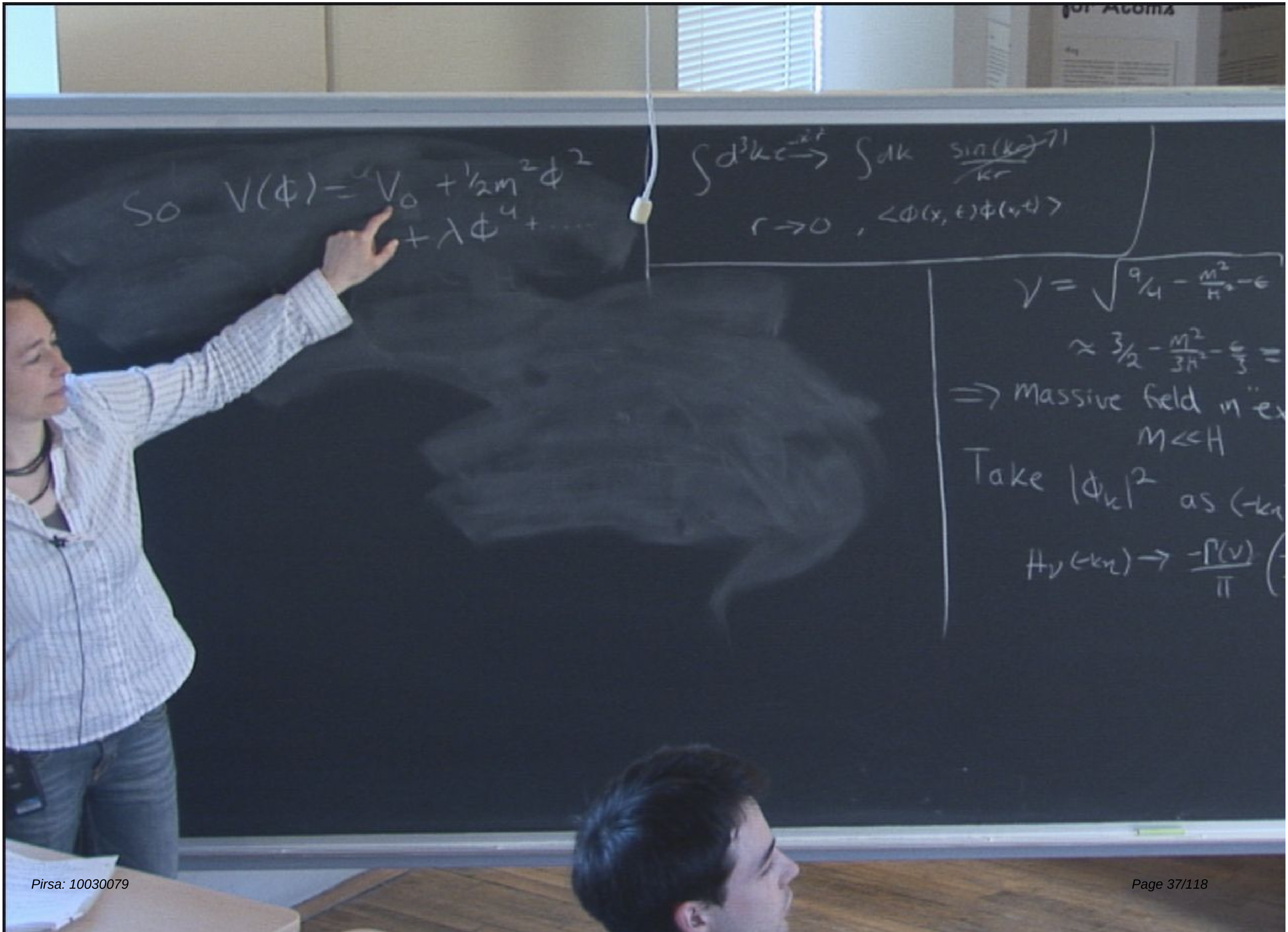

So far
① Quantizing scalar field in dS
② Scalar field can drive inflation
EoM for φ to get
evolution of H
Put them together: allow field φ
to source H ⇒ background, characterized
by H, Ḣ
φ = φ₀(t)

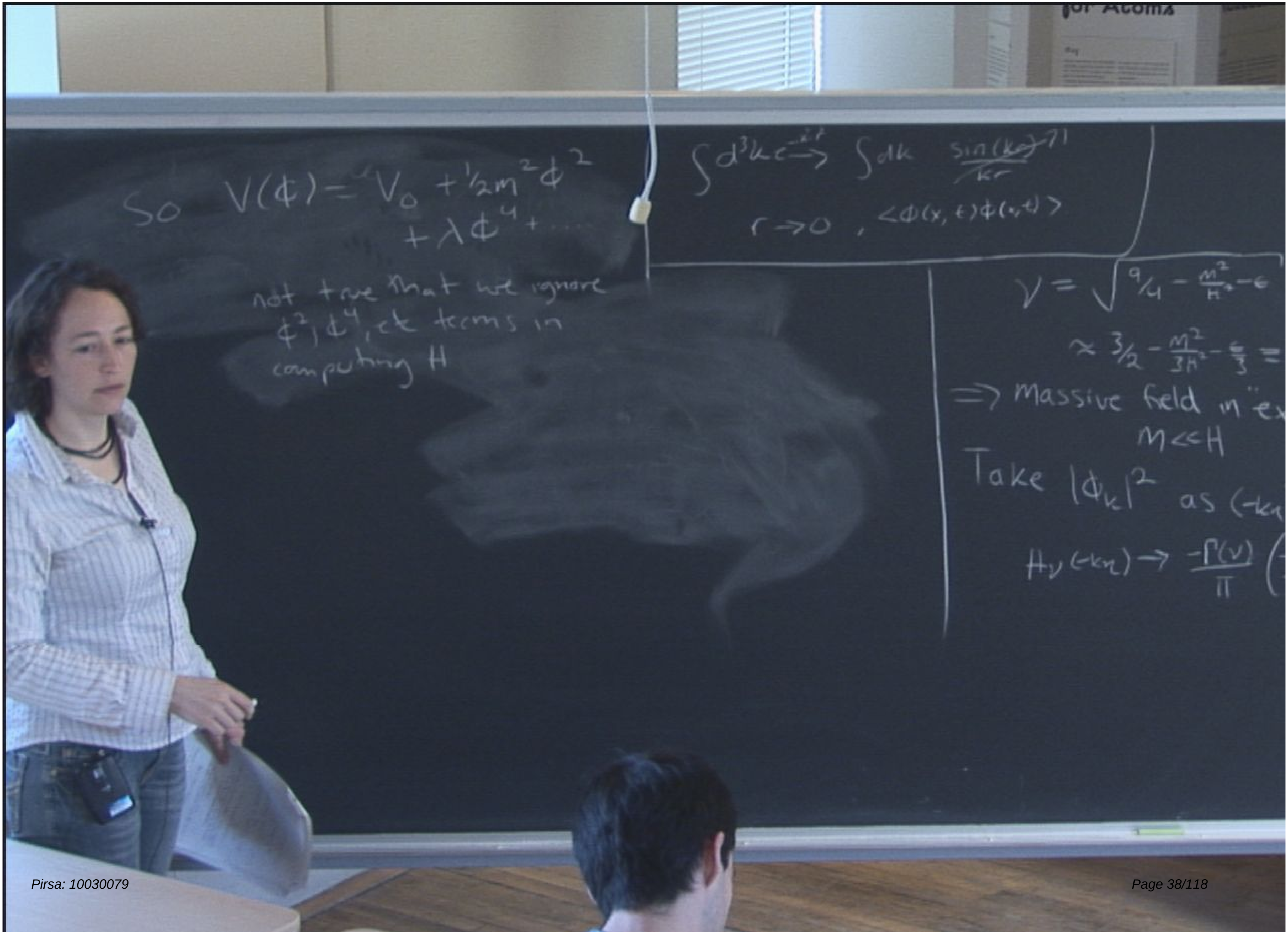
So far

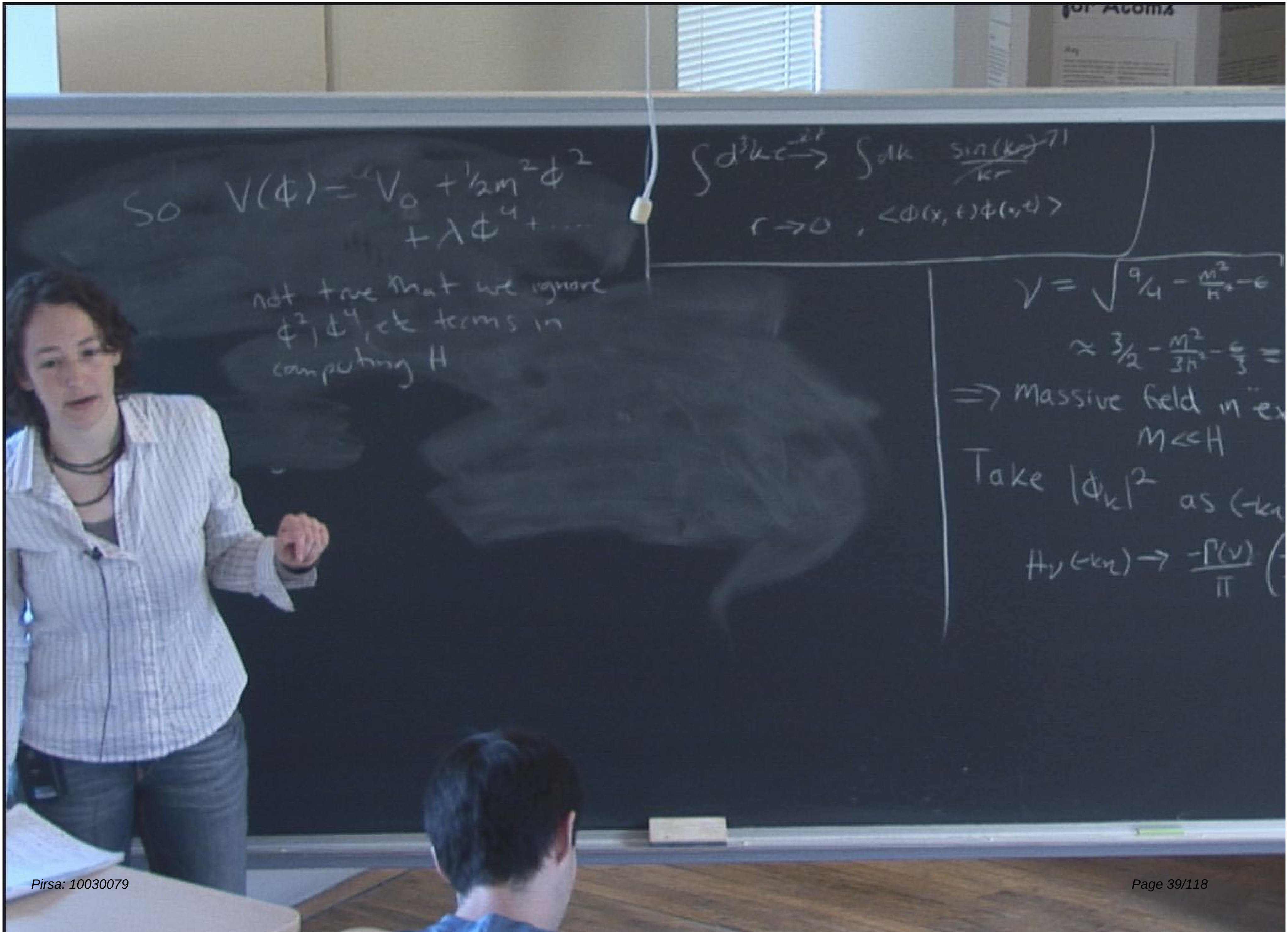
1. Quantizing scalar field in dS
2. Scalar field can drive inflation
   EoM for ϕ to get
   evolution of H

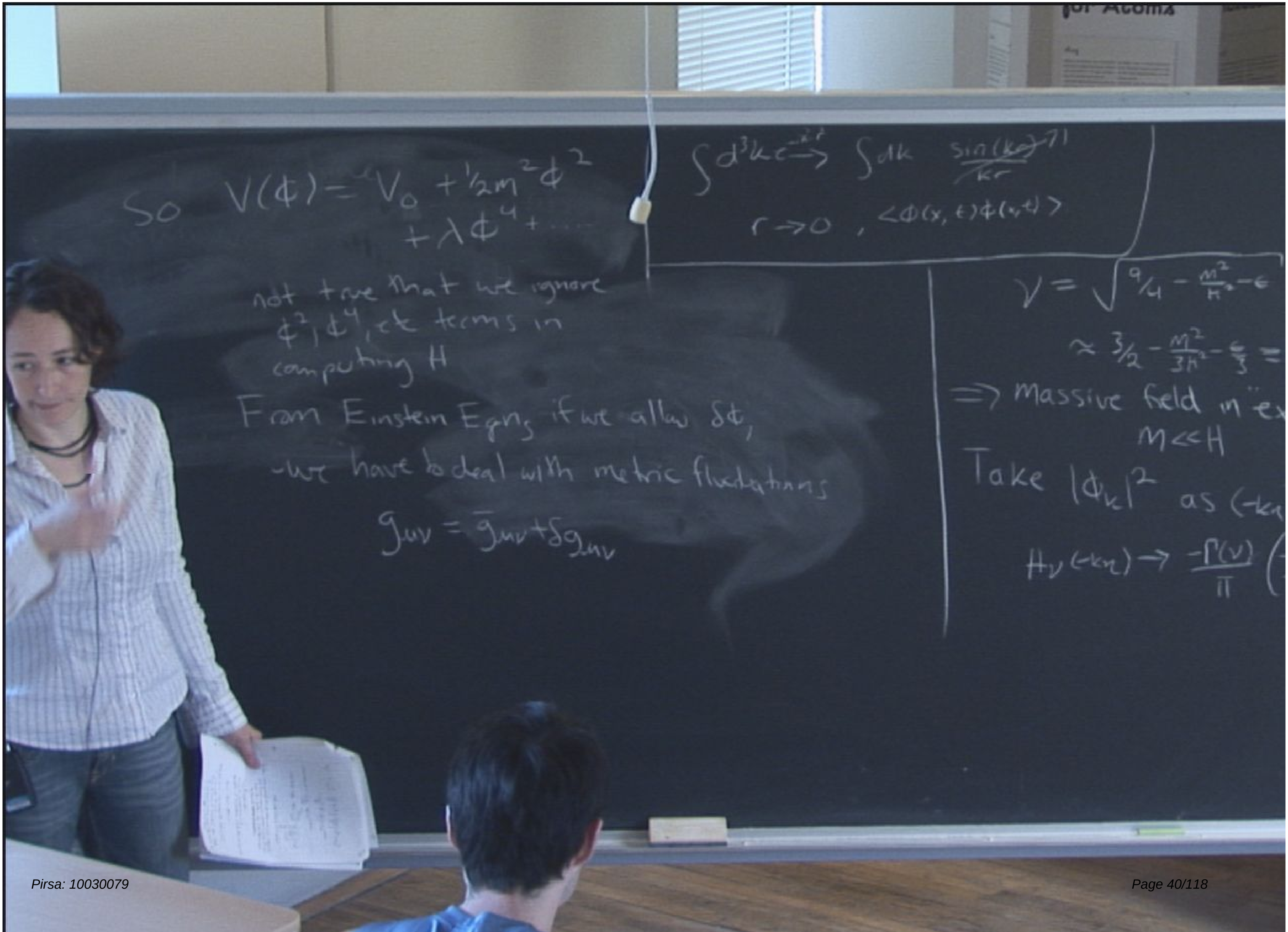
Put them together: allow field ϕ to source $H \Rightarrow$ background, characterized by $H, \dot{H}$

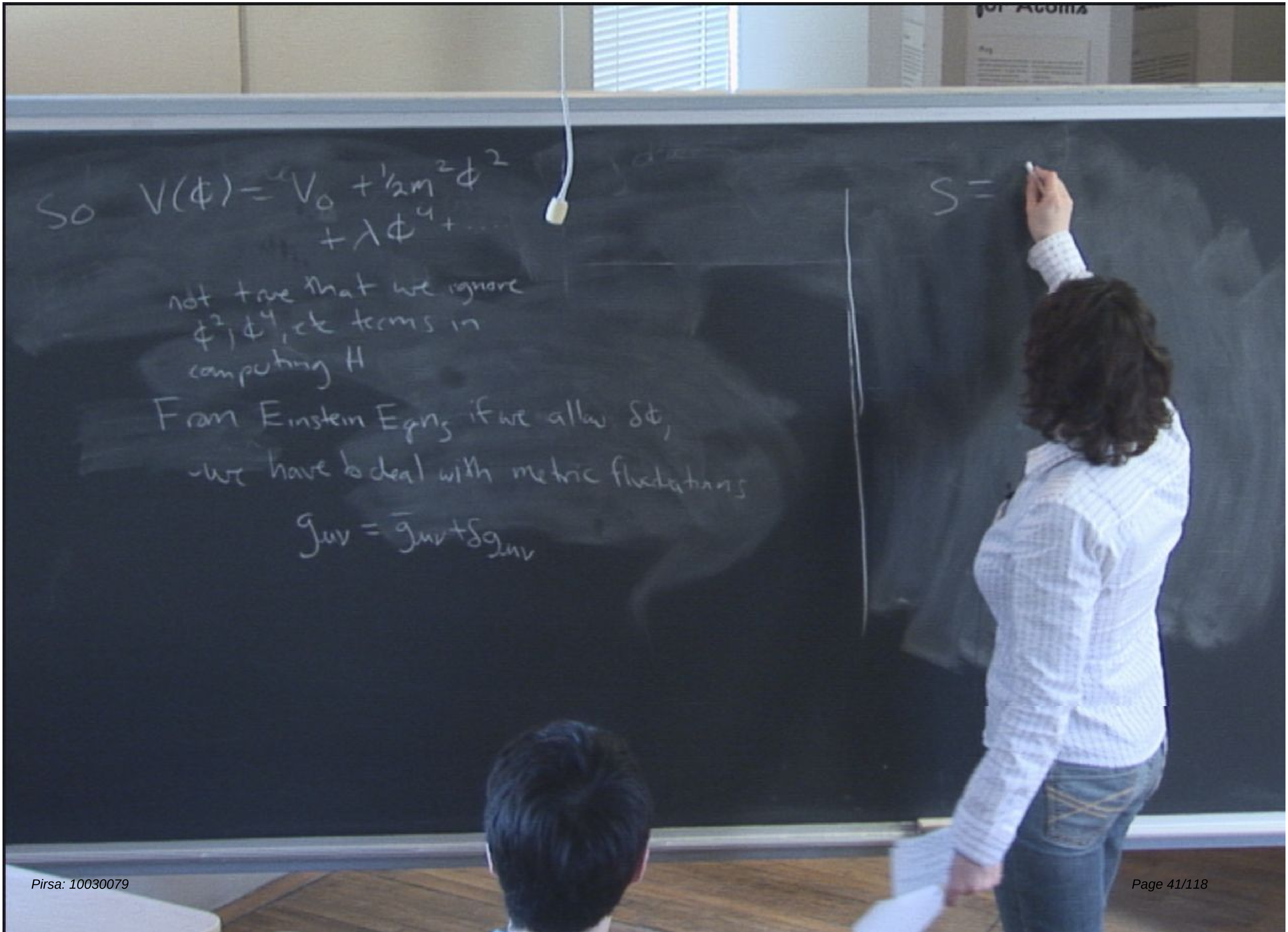
$$\phi = \phi_0(t) + \delta\phi(x,t)$$

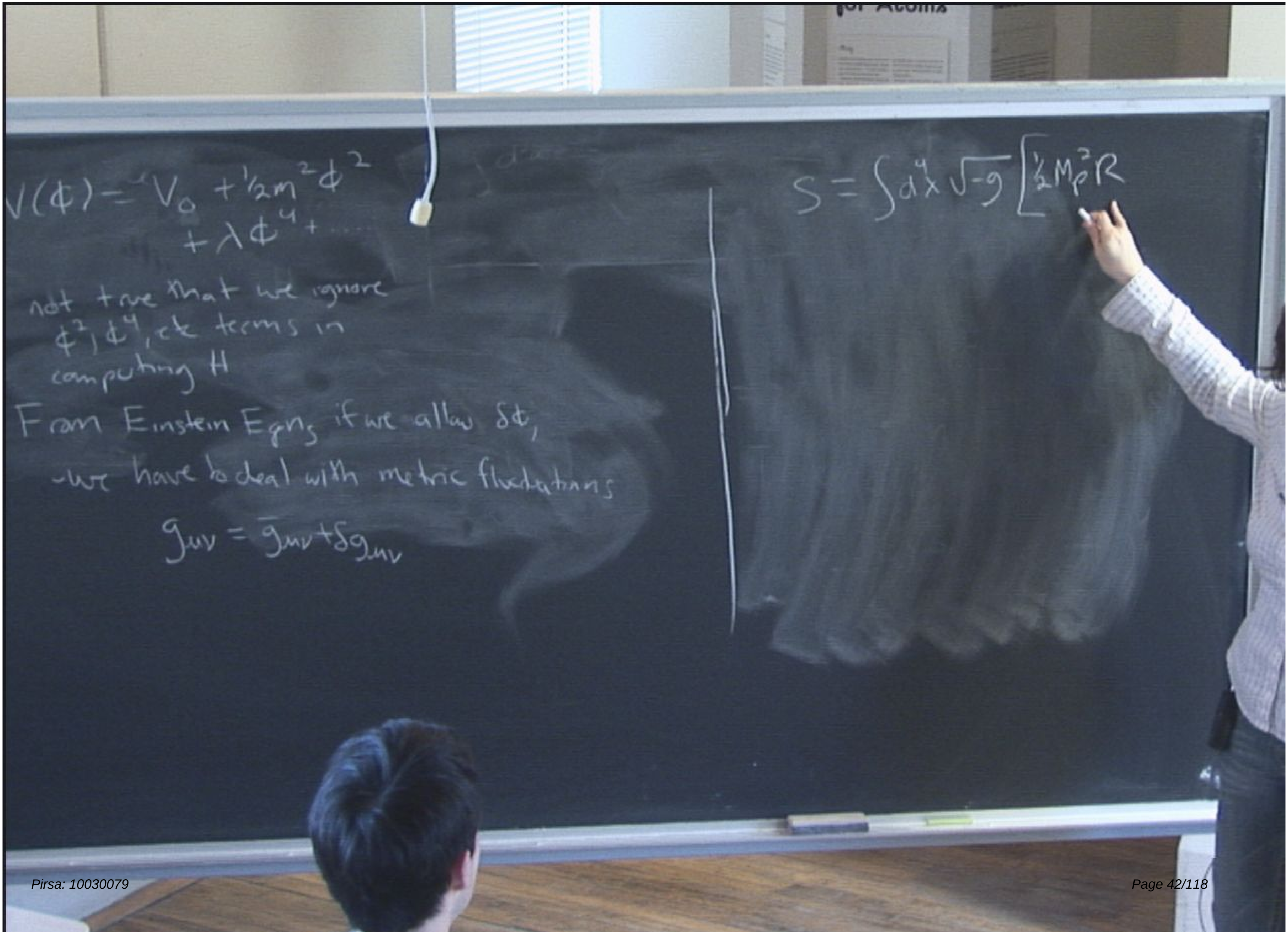

So V(ϕ) = V0 + ½m²ϕ² + λϕ⁴ + ...
∫d³k e^{-ik·r} → ∫dk sin(kr)/kr
r → 0 , <ϕ(x,t)ϕ(x,t)>
ν = √(9/4 − m²/H² − ϵ)
≈ 3/2 − m²/3H² − ϵ/3 =
⇒ massive field in "ex
m << H
Take |ϕk|² as (−kη
Hν(−kη) → −Γ(ν)/π (

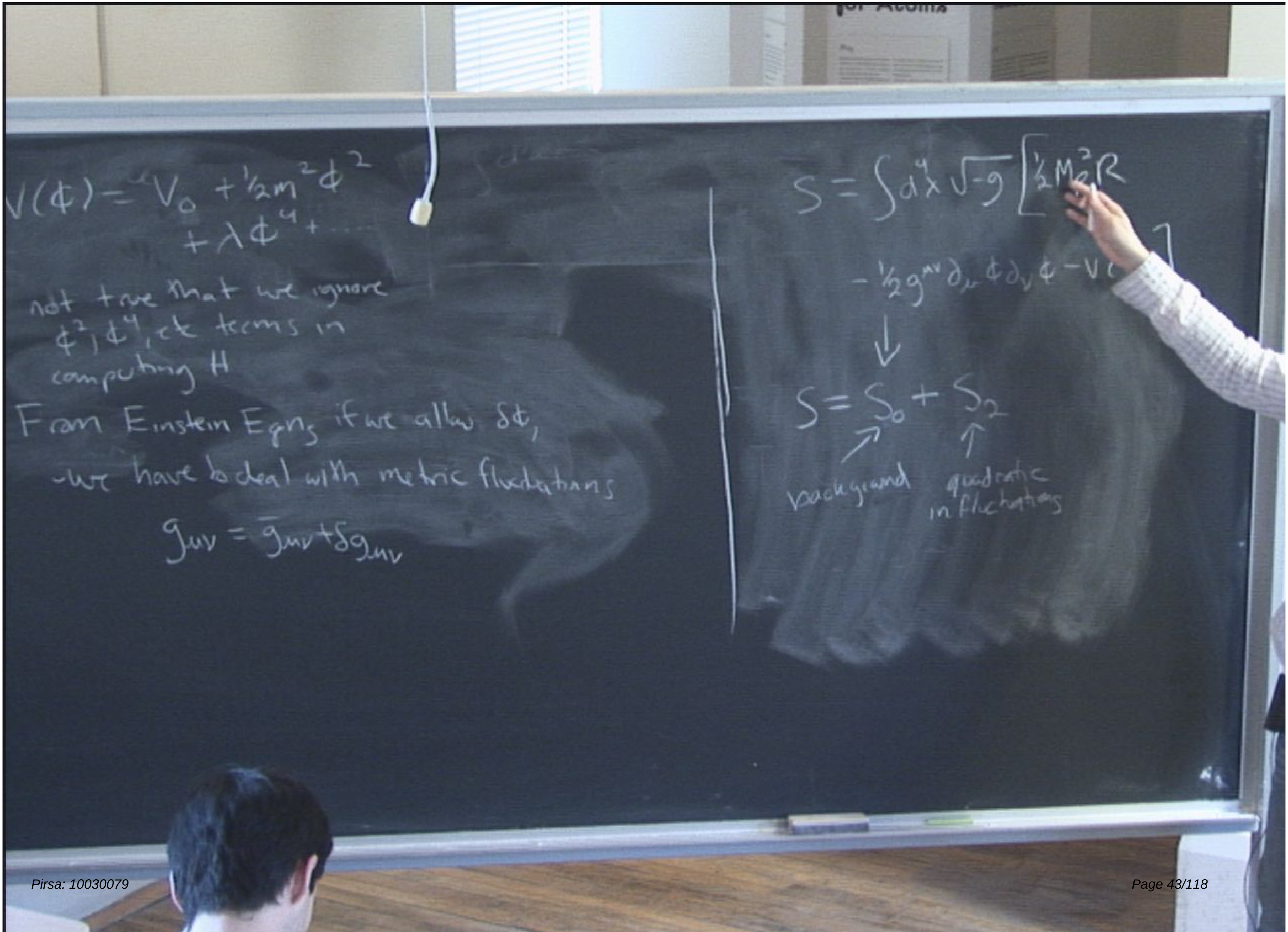

So V(ϕ) = V₀ + ½m²ϕ² + λϕ⁴ + ...
not true that we ignore ϕ², ϕ⁴, etc terms in computing H
r → 0 , <ϕ(x,t)ϕ(x,t)>
⇒ massive field in "ex
m << H
Take |ϕk|² as (-kη

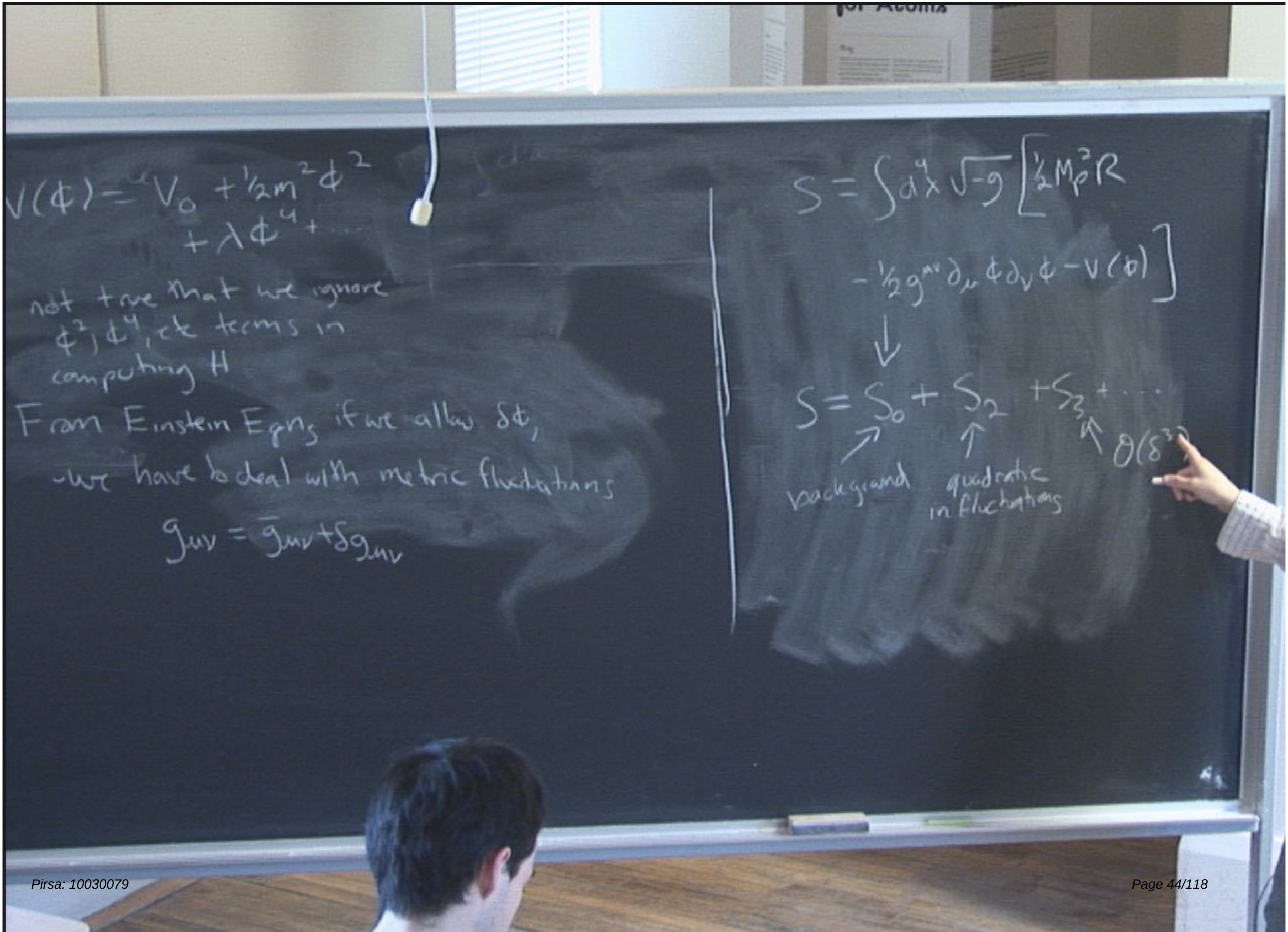

So V(ϕ) = V0 + ½m²ϕ²
+ λϕ⁴ + ...
not true that we ignore
ϕ², ϕ⁴, etc terms in
computing H
∫d³k ⟶ ∫dk sin(kr)/kr
r → 0 , ⟨ϕ(x,t)ϕ(x,t)⟩
ν = √(9/4 − m²/H² − ε
≈ 3/2 − m²/3H² − ε/3
⟹ massive field in "ex
m ≪ H
Take |ϕk|² as (−kη
Hν(−kη) → −Γ(ν)/π

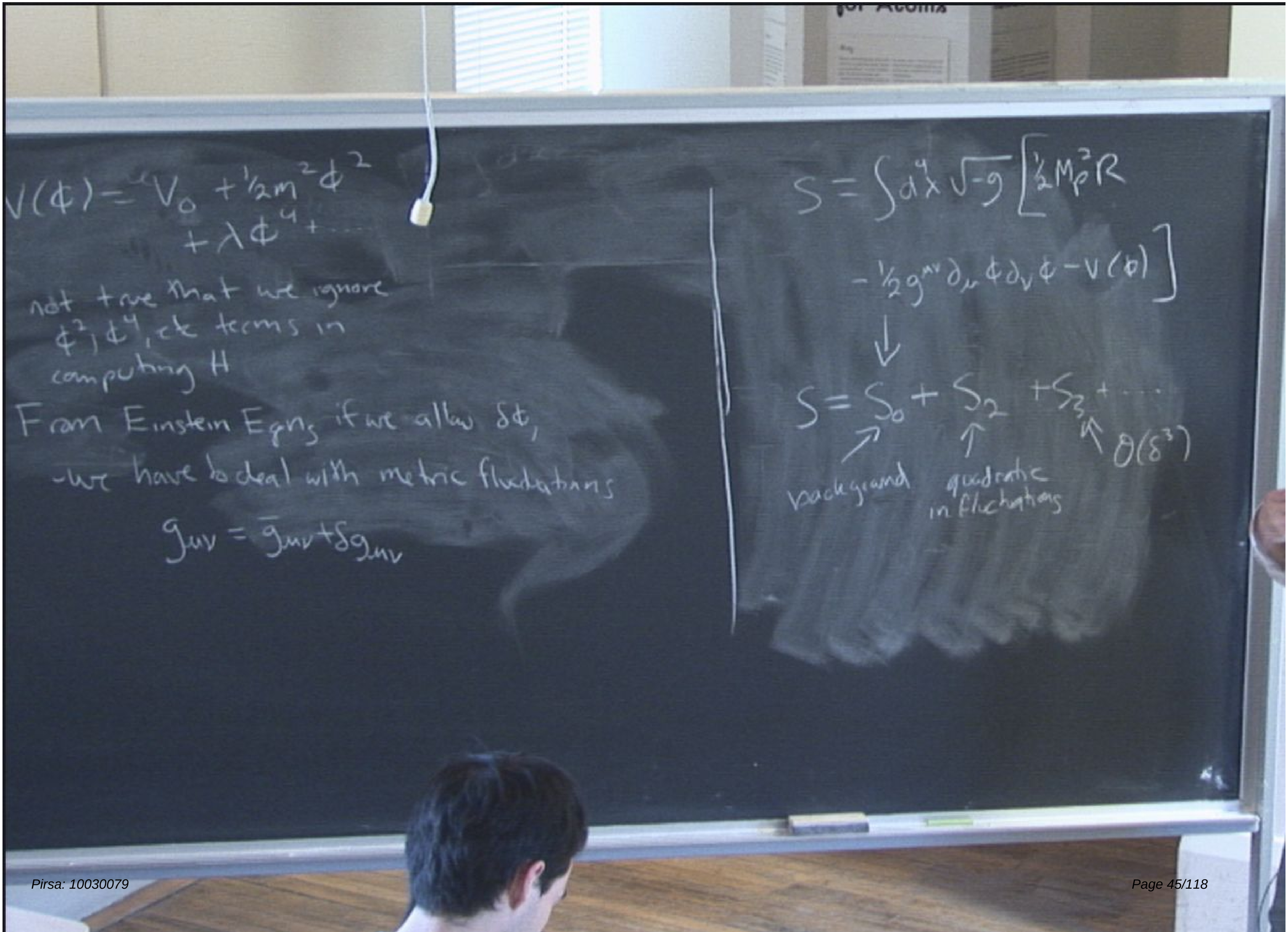

So V(ϕ) = V0 + ½m²ϕ² + λϕ⁴ + ...
not true that we ignore ϕ², ϕ⁴, etc terms in computing H
From Einstein Eqns, if we allow δϕ, we have to deal with metric fluctuations
gμν = ḡμν + δgμν
∫d³k e^{ikx} → ∫dk sin(kr)/kr
r → 0 , <ϕ(x,t)ϕ(x,t)>
ν = √(9/4 − m²/H² − ε)
≈ 3/2 − m²/3H² − ε/3
⟹ massive field in "ex
M << H
Take |ϕk|² as (−kη
Hν(−kη) → −Γ(ν)/π


So V(ϕ) = V_0 + ½m²ϕ² + λϕ⁴ + ...
not true that we ignore
ϕ², ϕ⁴, etc terms in
computing H
From Einstein Eqns if we allow δϕ,
we have to deal with metric fluctuations
g_μν = ḡ_μν + δg_μν
S =

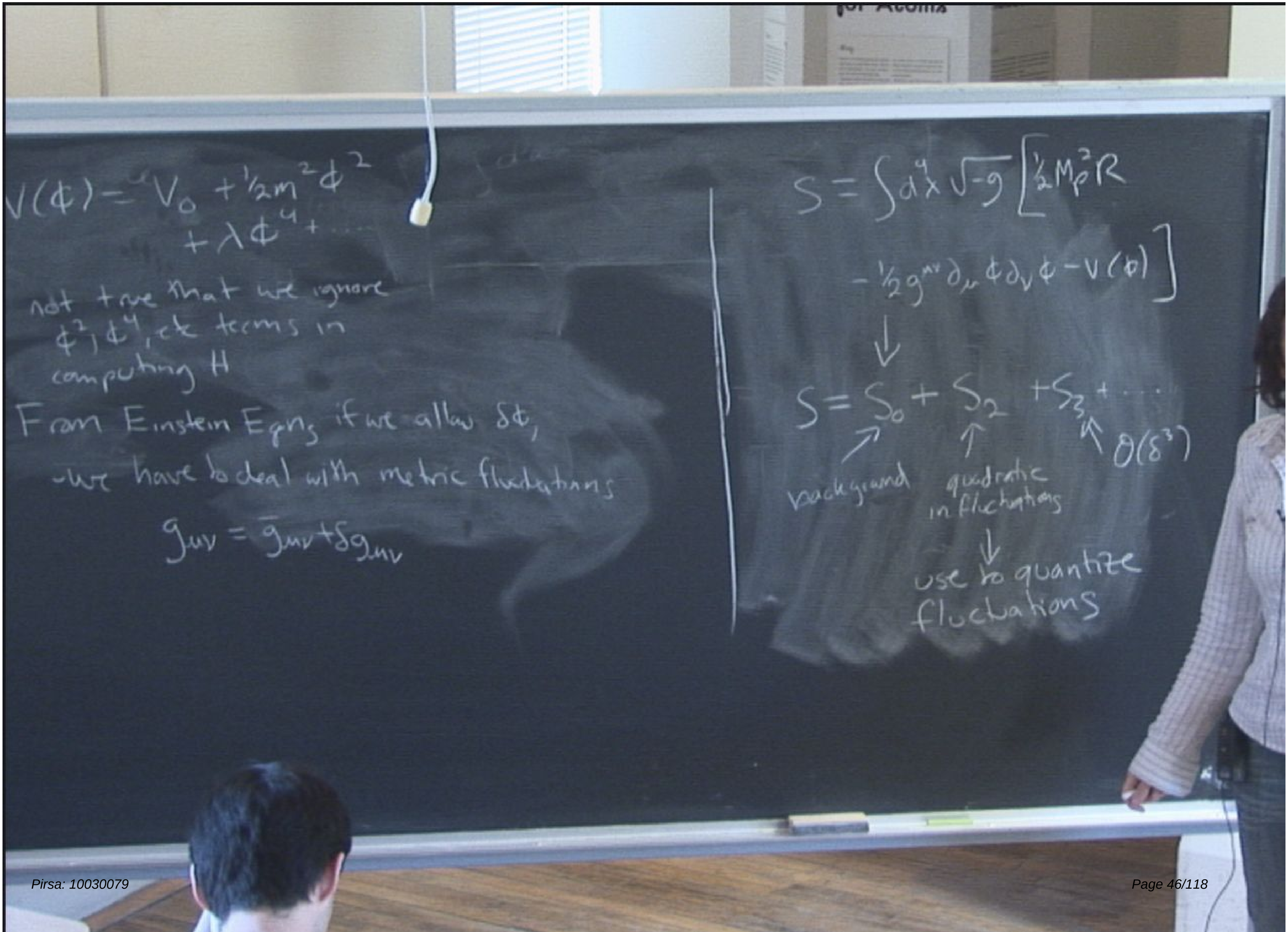

V(ϕ) = V₀ + ½m²ϕ²
+ λϕ⁴ + ...
not true that we ignore
ϕ², ϕ⁴, etc terms in
computing H
From Einstein Eqns if we allow δϕ,
we have to deal with metric fluctuations
gμν = ḡμν + δgμν
S = ∫d⁴x √-g [½Mp²R

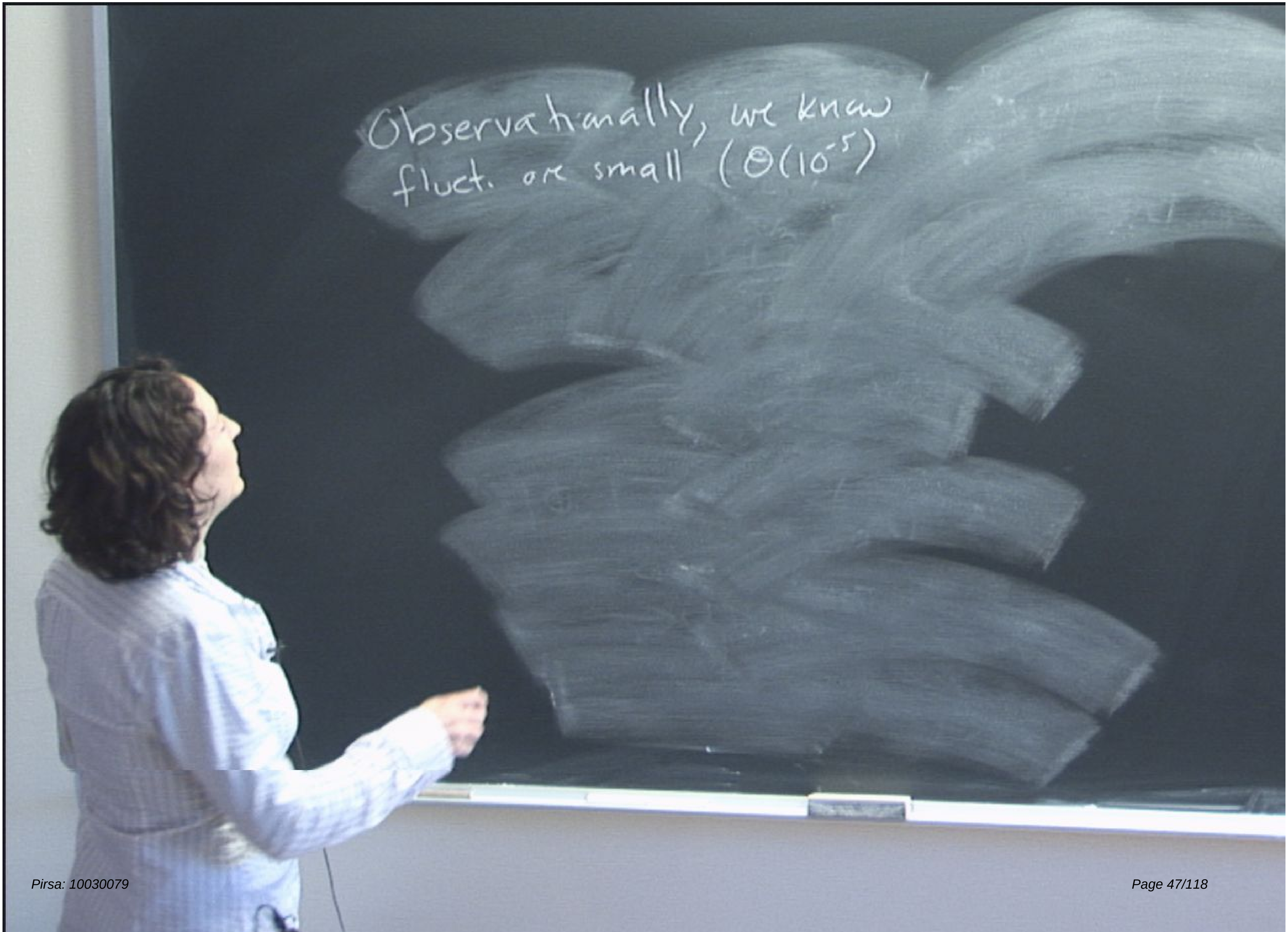

V(ϕ) = V0 + ½m²ϕ² + λϕ⁴ + ...
not true that we ignore ϕ², ϕ⁴, etc terms in computing H
From Einstein Eqns if we allow δϕ,
- we have to deal with metric fluctuations
gμν = ḡμν + δgμν
S = ∫d⁴x √-g [½M²R - ½g^μν ∂μϕ∂νϕ - V(ϕ)]
S = S0 + S2
background
quadratic in fluctuations

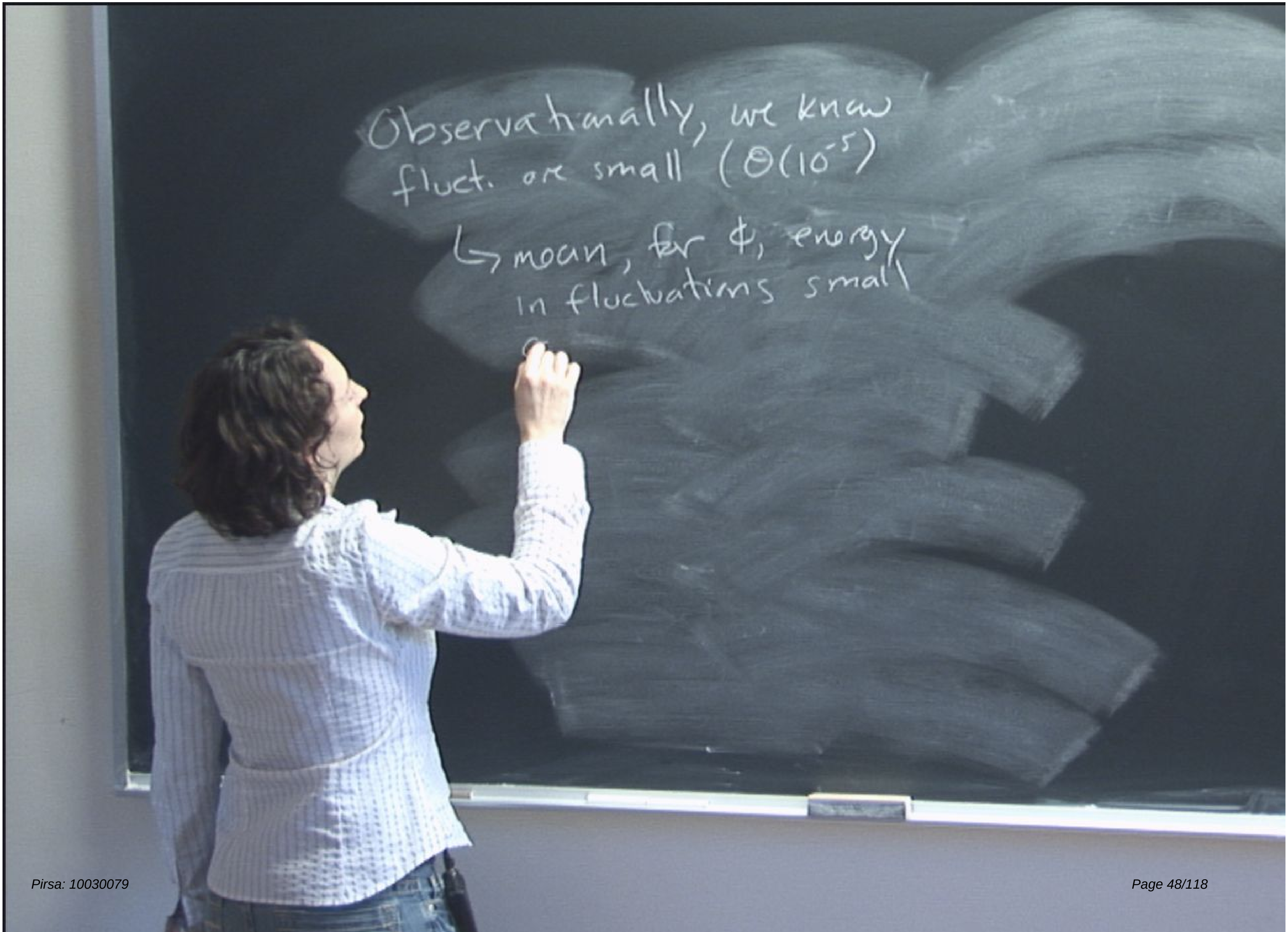

V(ϕ) = V0 + ½m²ϕ² + λϕ⁴ + ...
not true that we ignore ϕ², ϕ⁴, etc terms in computing H
From Einstein Eqns if we allow δϕ, we have to deal with metric fluctuations
gμν = ḡμν + δgμν
S = ∫d⁴x √-g [½Mp²R − ½g^μν ∂μϕ ∂νϕ − V(ϕ)]
S = S0 + S2 + S3 + ...
background
quadratic in fluctuations
O(δ³)

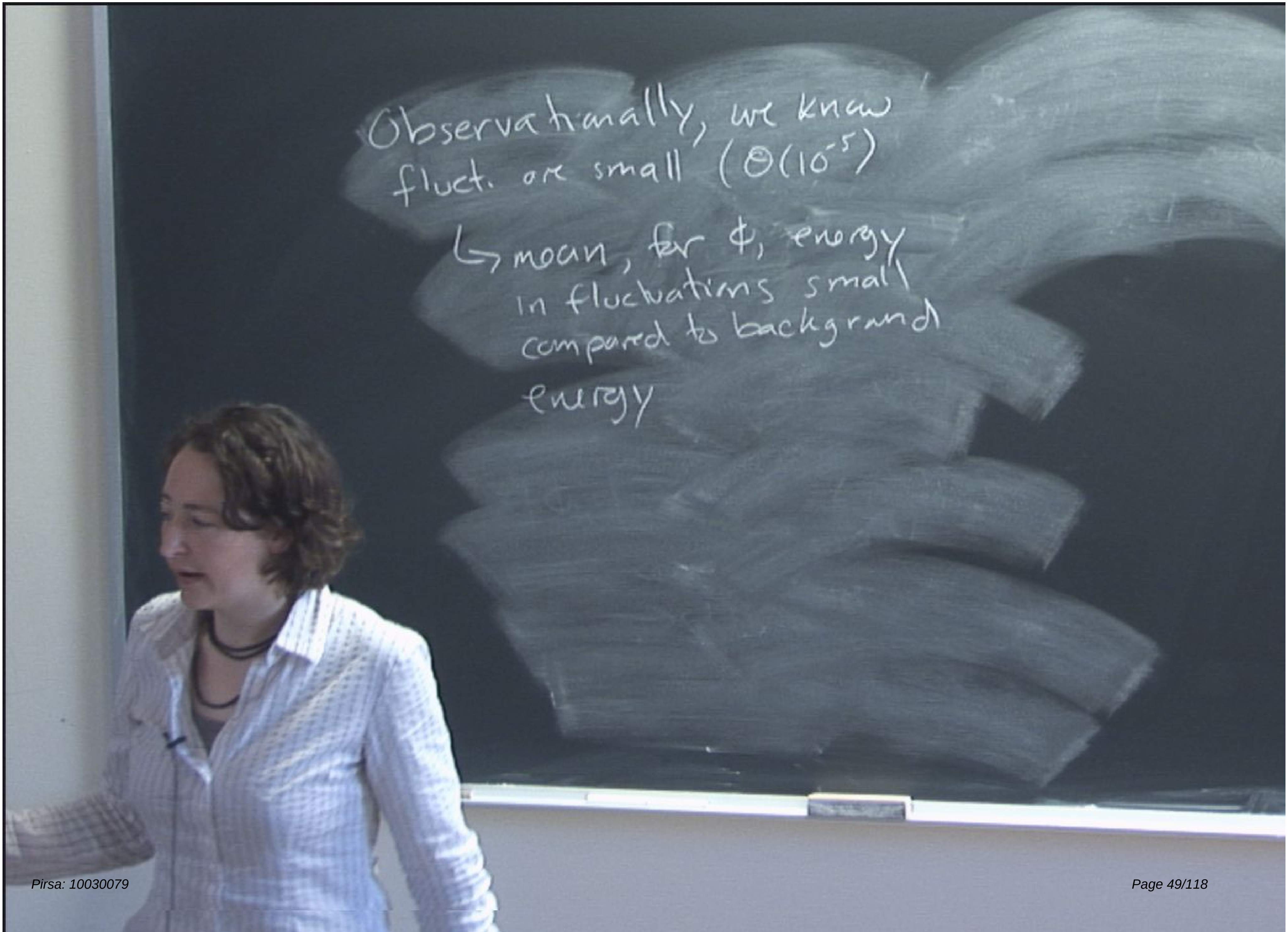


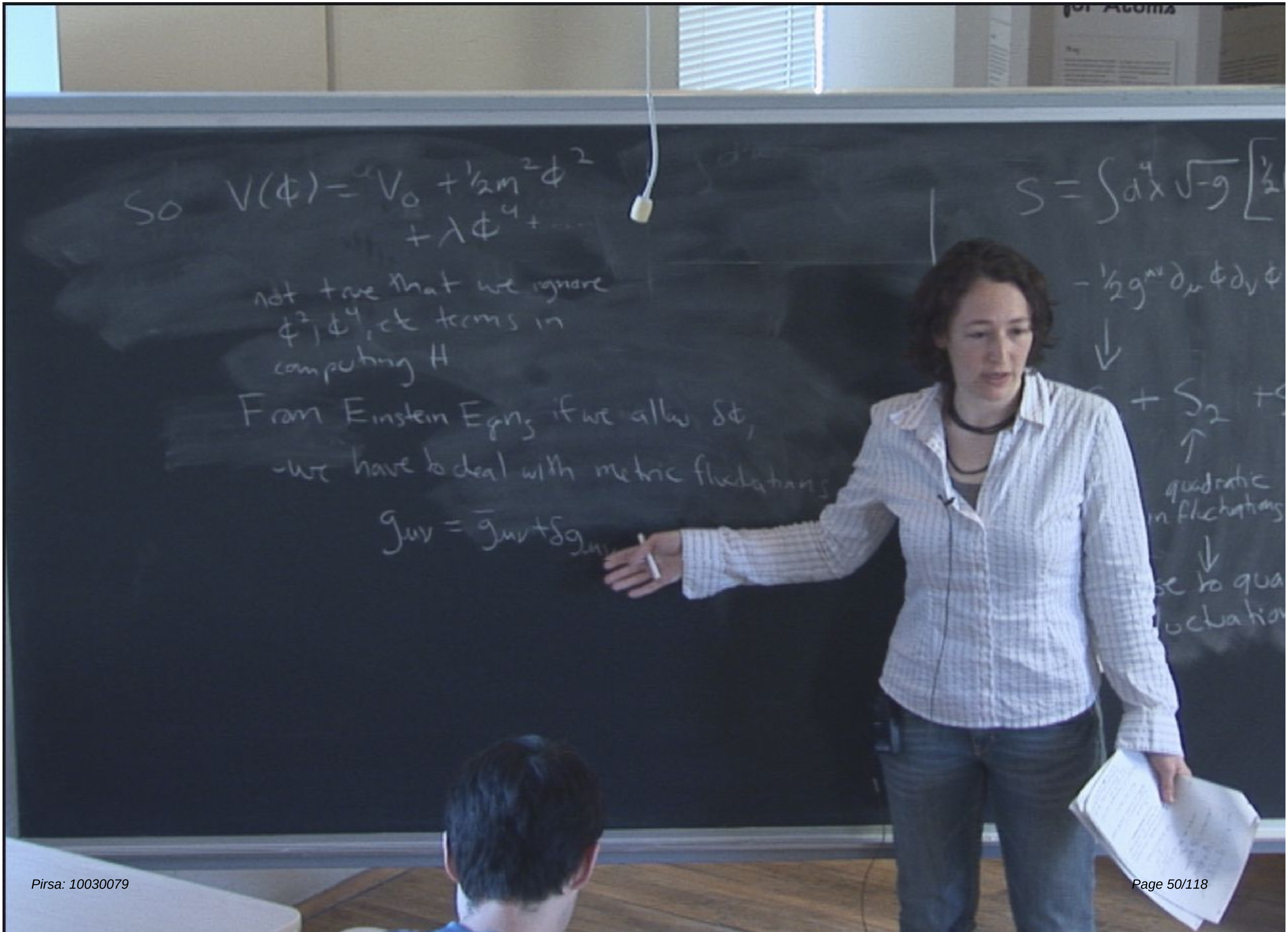
$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \dots$
not true that we ignore ϕ^2, ϕ^4, etc terms in computing H
From Einstein Eqns, if we allow $\delta\phi$,
- we have to deal with metric fluctuations
$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$
$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$
$S = S_0 + S_2 + S_3 + \dots$
background
quadratic in fluctuations
$O(\delta^3)$

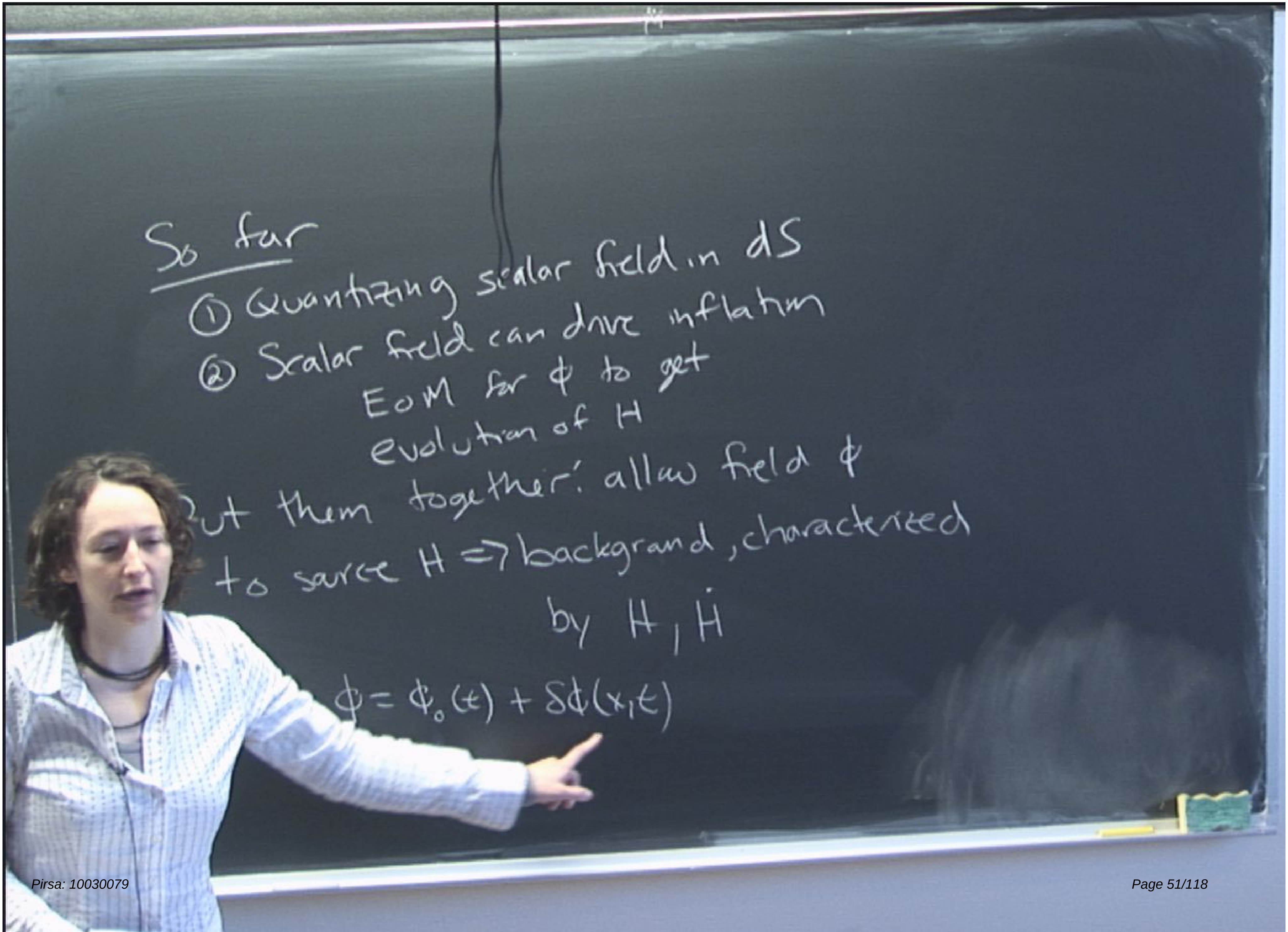

V(ϕ) = V₀ + ½m²ϕ² + λϕ⁴ + ...
not true that we ignore ϕ², ϕ⁴, etc terms in computing H
From Einstein Eqns, if we allow δϕ,
- we have to deal with metric fluctuations
g_μν = ḡ_μν + δg_μν
S = ∫d⁴x √-g [½ M_P² R - ½ g^μν ∂_μ ϕ ∂_ν ϕ - V(ϕ)]
S = S₀ + S₂ + S₃ + ...
background
quadratic in fluctuations
O(δ³)
use to quantize fluctuations

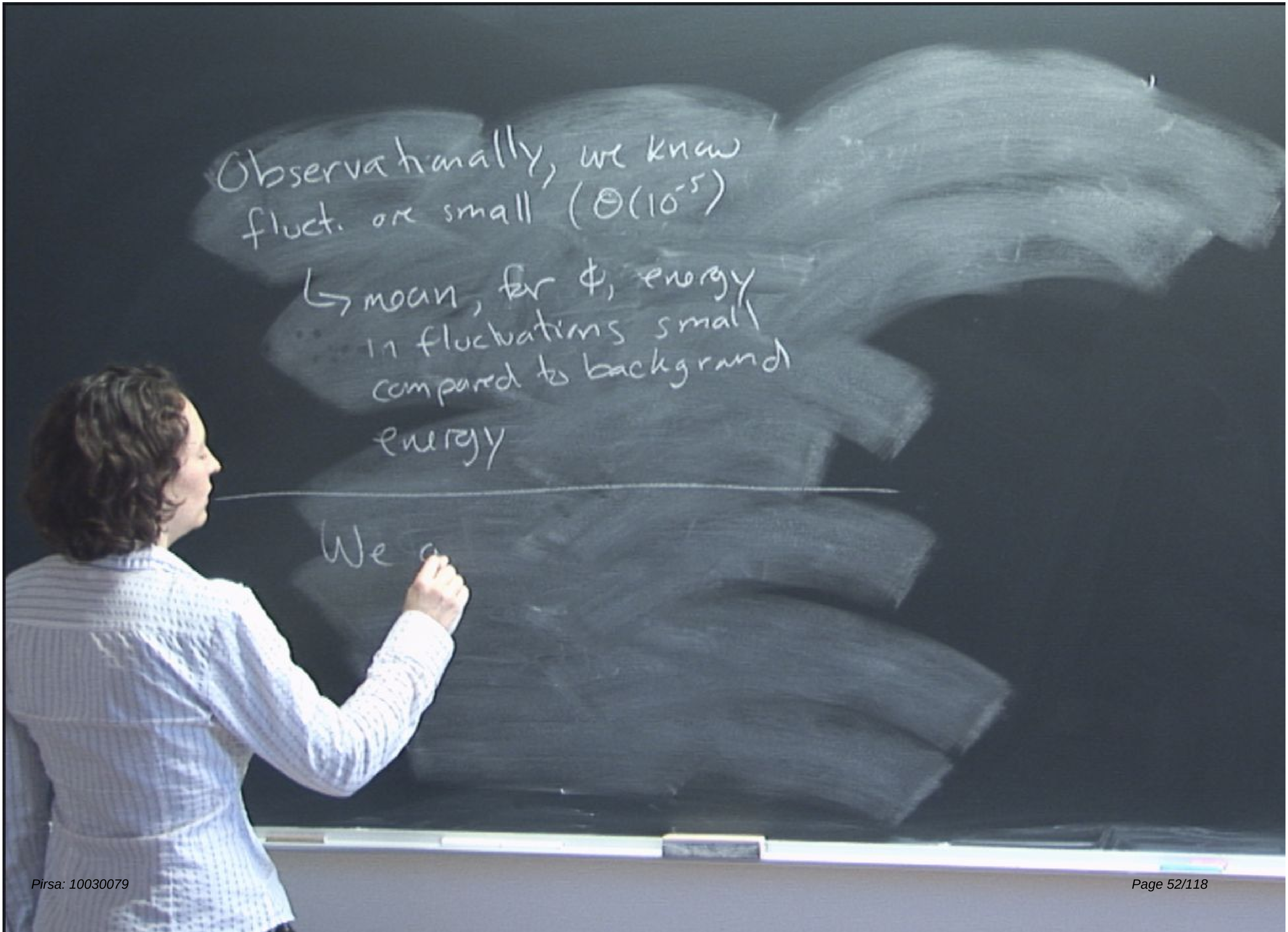

Observationally, we know
fluct. are small (O(10^-5))


Observationally, we know
fluct. are small (O(10^-5))
↳ mean, for φ, energy
in fluctuations small


Observationally, we know
fluct. are small ($\Theta(10^{-5})$
↳ mean, for ϕ, energy
in fluctuations small
compared to background
energy


So V(ϕ) = V_0 + ½m²ϕ²
+ λϕ⁴ + ...
not true that we ignore
ϕ², ϕ⁴, etc terms in
computing H
From Einstein Eqns if we allow δϕ,
we have to deal with metric fluctuations
gμν = ḡμν + δgμν
S = ∫d⁴x √-g [
-½g^μν ∂_μϕ ∂_νϕ
+ S_2
quadratic
in fluctuations


So far
① Quantizing scalar field in dS
② Scalar field can drive inflation
EoM for ϕ to get
evolution of H
Put them together: allow field ϕ
to source H ⟹ background, characterized
by $H, \dot{H}$
$\phi = \phi_0(t) + \delta\phi(x,t)$


Observationally, we know
fluct. are small (O(10^-5))
↳ mean, for ϕ, energy
in fluctuations small
compared to background
energy
We a

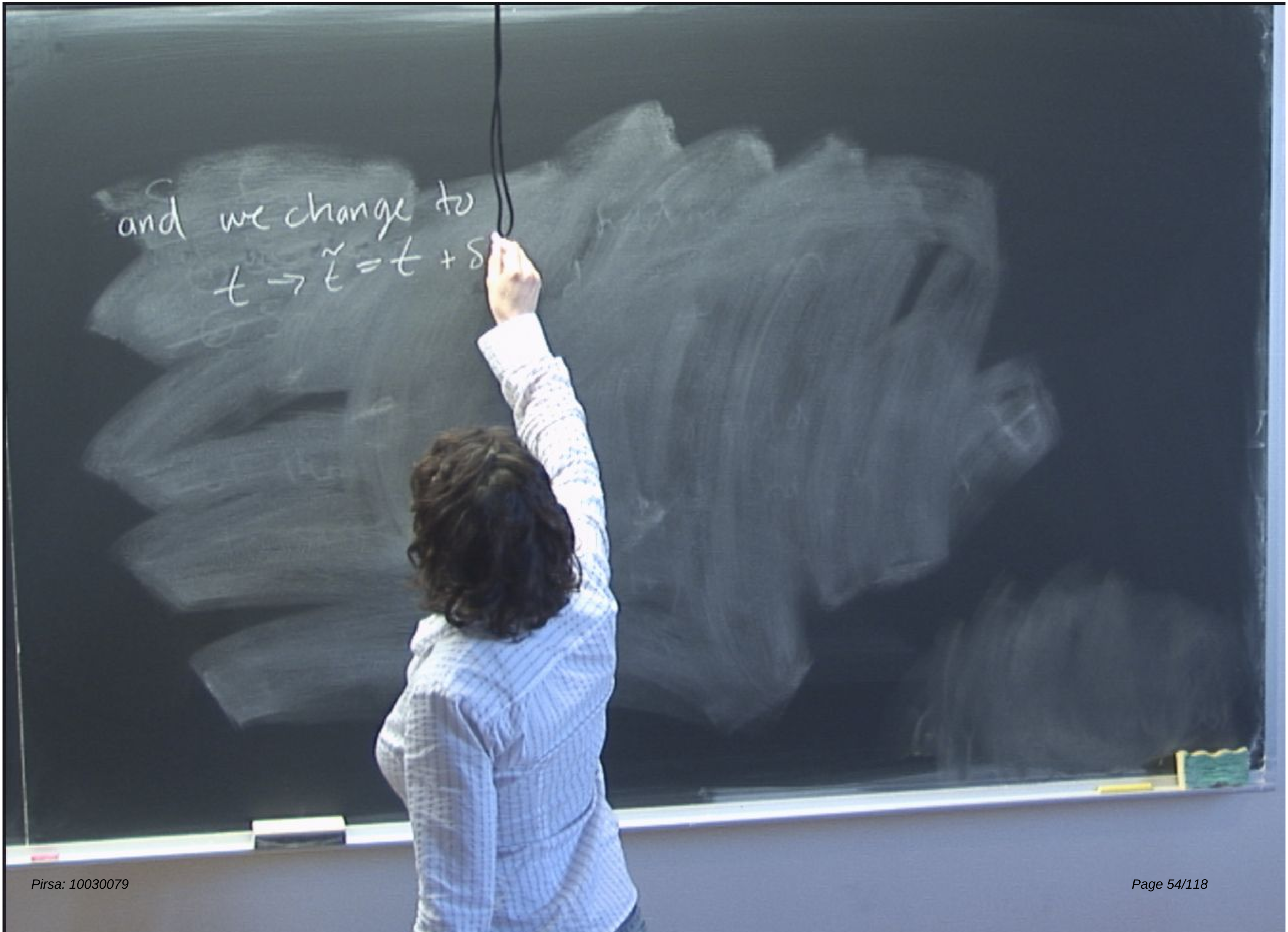
Observationally, we know fluct. are small ($O(10^{-5})$)

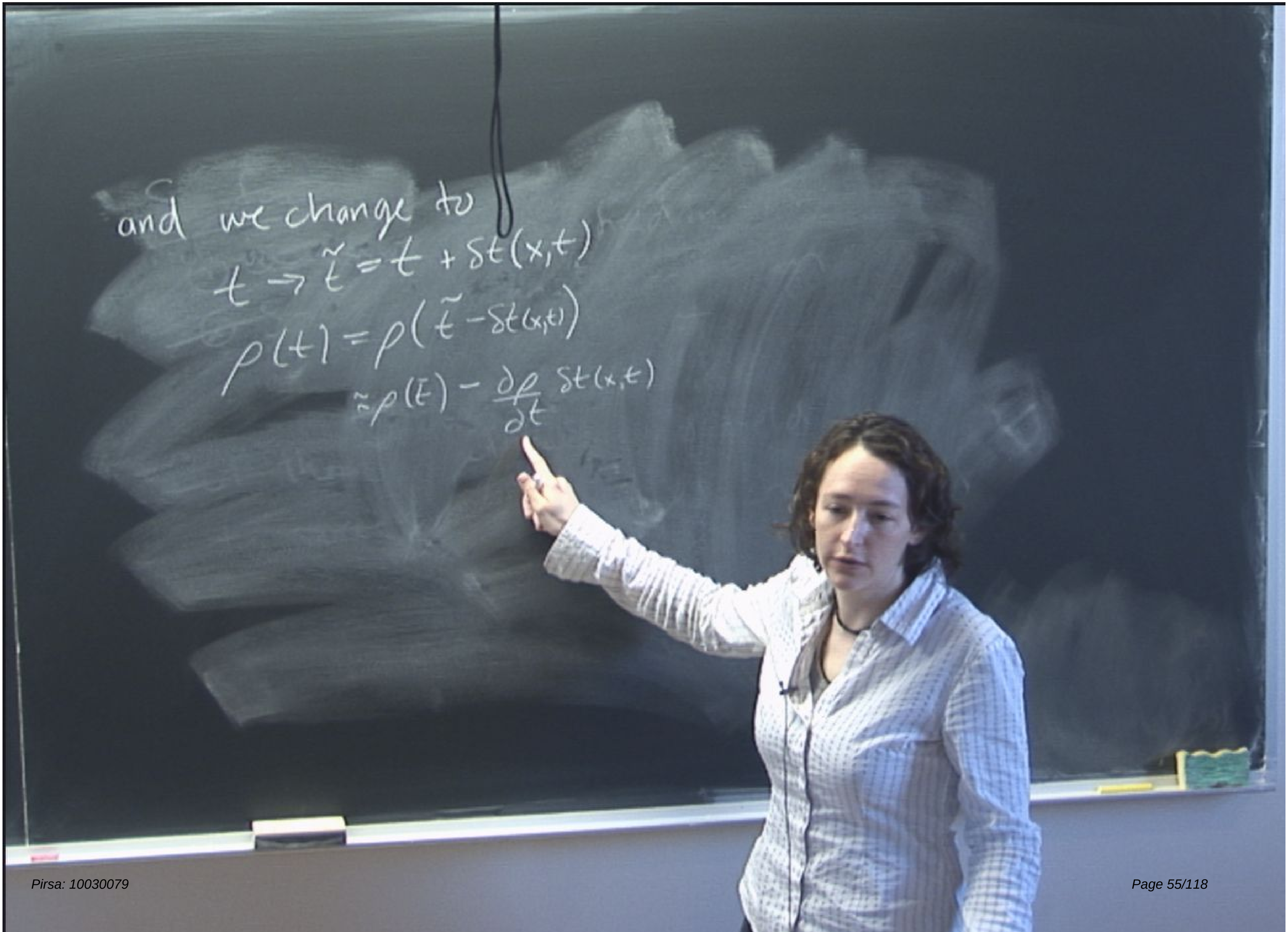
↳ mean, for ϕ, energy in fluctuations small compared to background energy

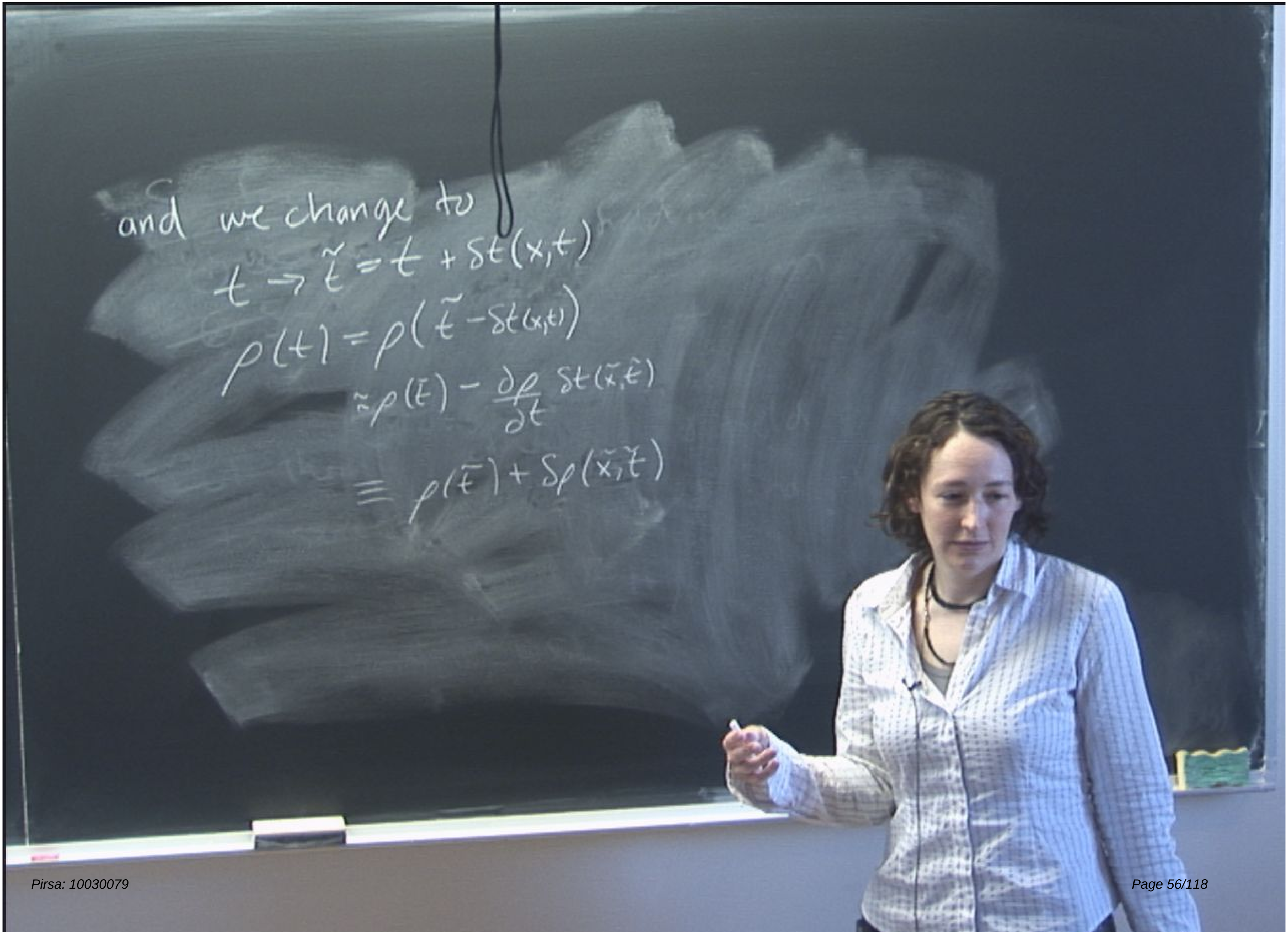
---

We are free to change coordinates

Suppose we have a homogeneous background energy density $\rho=\rho(t)$


and we change to
t → t̃ = t + δ


and we change to
$t \to \tilde{t} = t + \delta t(x,t)$
$\rho(t) = \rho(\tilde{t} - \delta t(x,t))$
$\approx \rho(\tilde{t}) - \frac{\partial \rho}{\partial t} \delta t(x,t)$


and we change to
$t \to \tilde{t} = t + \delta t(x,t)$
$\rho(t) = \rho(\tilde{t} - \delta t(x,t))$
$\approx \rho(\tilde{t}) - \frac{\partial \rho}{\partial t} \delta t(\tilde{x},\tilde{t})$
$\equiv \rho(\tilde{t}) + \delta\rho(\tilde{x},\tilde{t})$

and we change to

$$t \to \tilde{t} = t + \delta t(x,t)$$

$$\rho(t) = \rho(\tilde{t} - \delta t(x,t))$$

$$\simeq \rho(\tilde{t}) - \frac{\partial \rho}{\partial t}\,\delta t(\tilde{x},\tilde{t})$$

$$\equiv \rho(\tilde{t}) + \delta\rho(\tilde{x},\tilde{t})$$

coordinate change has "produced" perturbations

and we change to

$$t \to \tilde{t} = t + \delta t(x,t)$$

$$\rho(t) = \rho(\tilde{t} - \delta t(x,t))$$

$$\simeq \rho(\tilde{t}) - \frac{\partial \rho}{\partial t} \delta t(\tilde{x},\tilde{t})$$

$$\equiv \rho(\tilde{t}) + \delta\rho(\tilde{x},\tilde{t})$$

coordinate change has "produced" perturbations

↳ Best thing is to define variables that are independent of coordinate transformation

and we change to

$$t \to \tilde{t} = t + \delta t(x,t)$$

$$\rho(t) = \rho(\tilde{t} - \delta t(x,t))$$

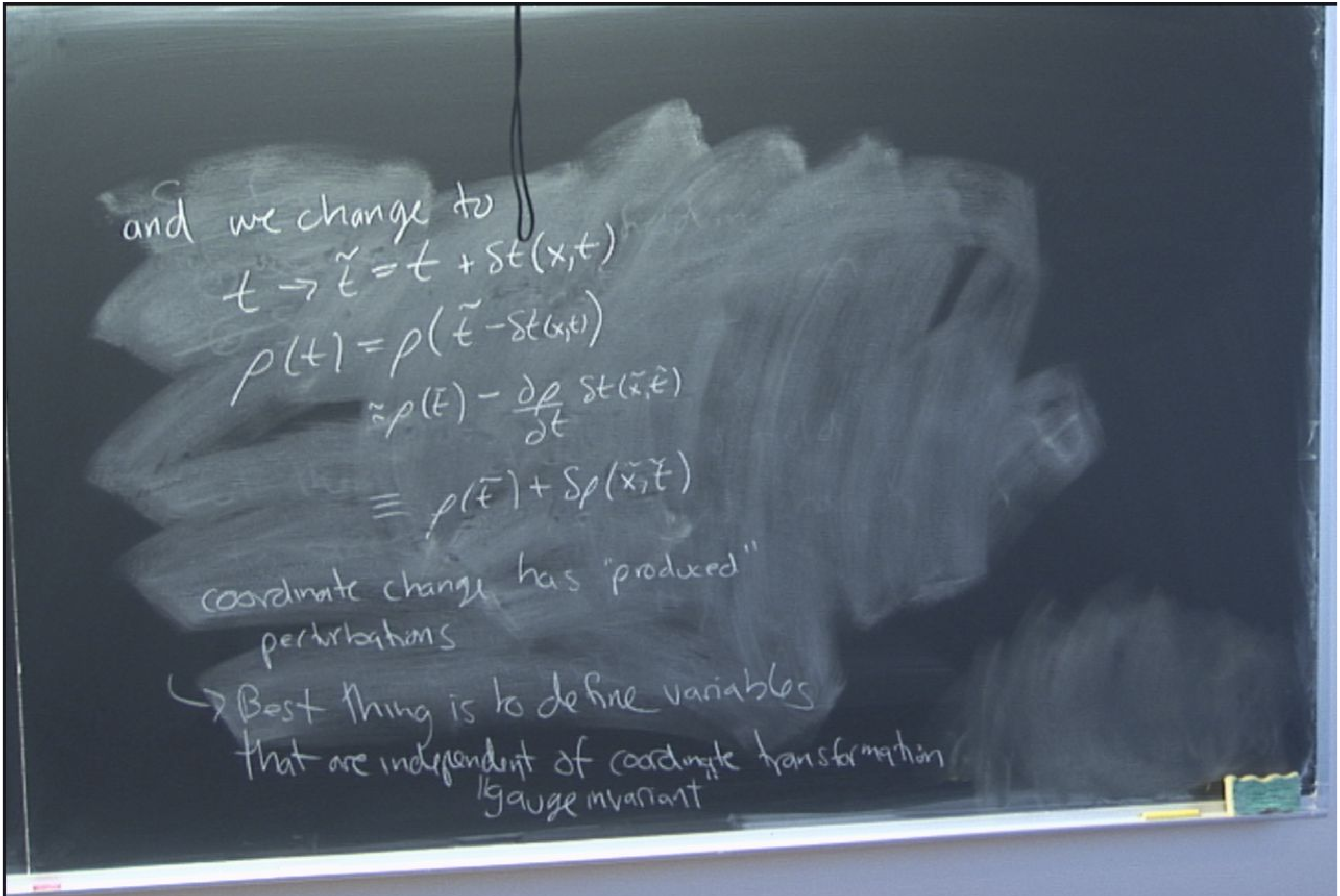
$$\simeq \rho(\tilde{t}) - \frac{\partial \rho}{\partial t}\,\delta t(\tilde{x},\tilde{t})$$

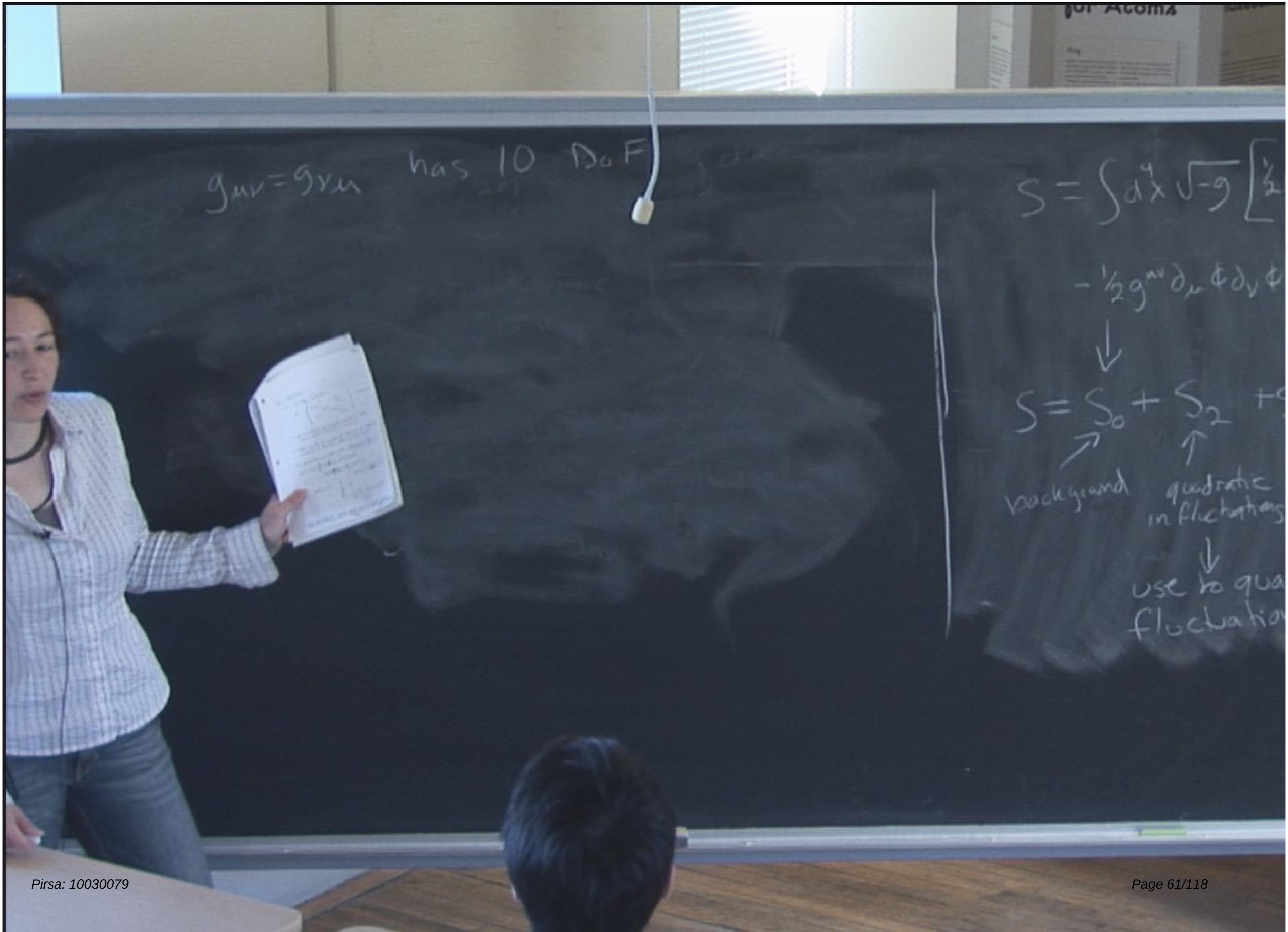
$$\equiv \rho(\tilde{t}) + \delta\rho(\tilde{x},\tilde{t})$$

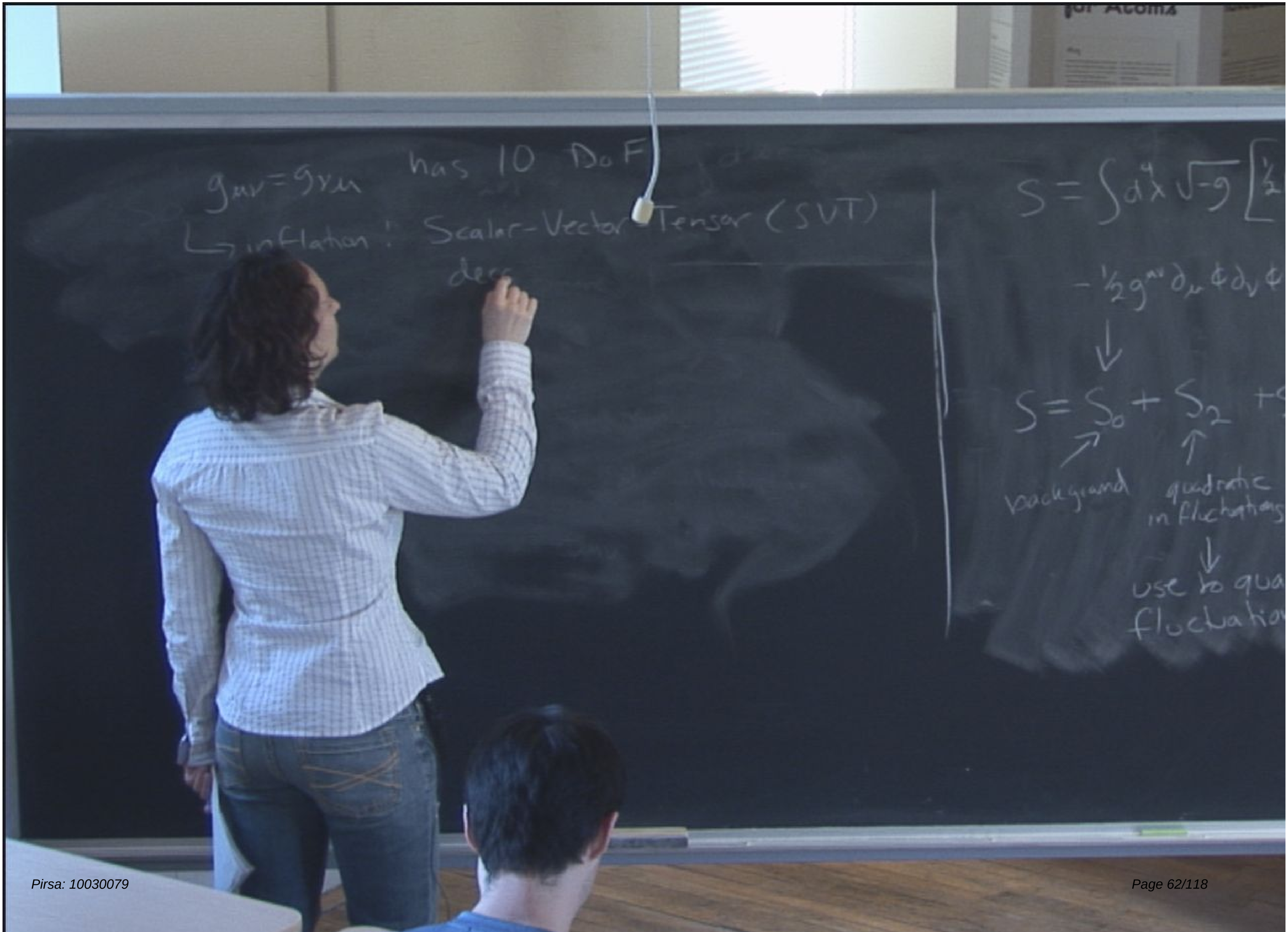
coordinate change has "produced" perturbations

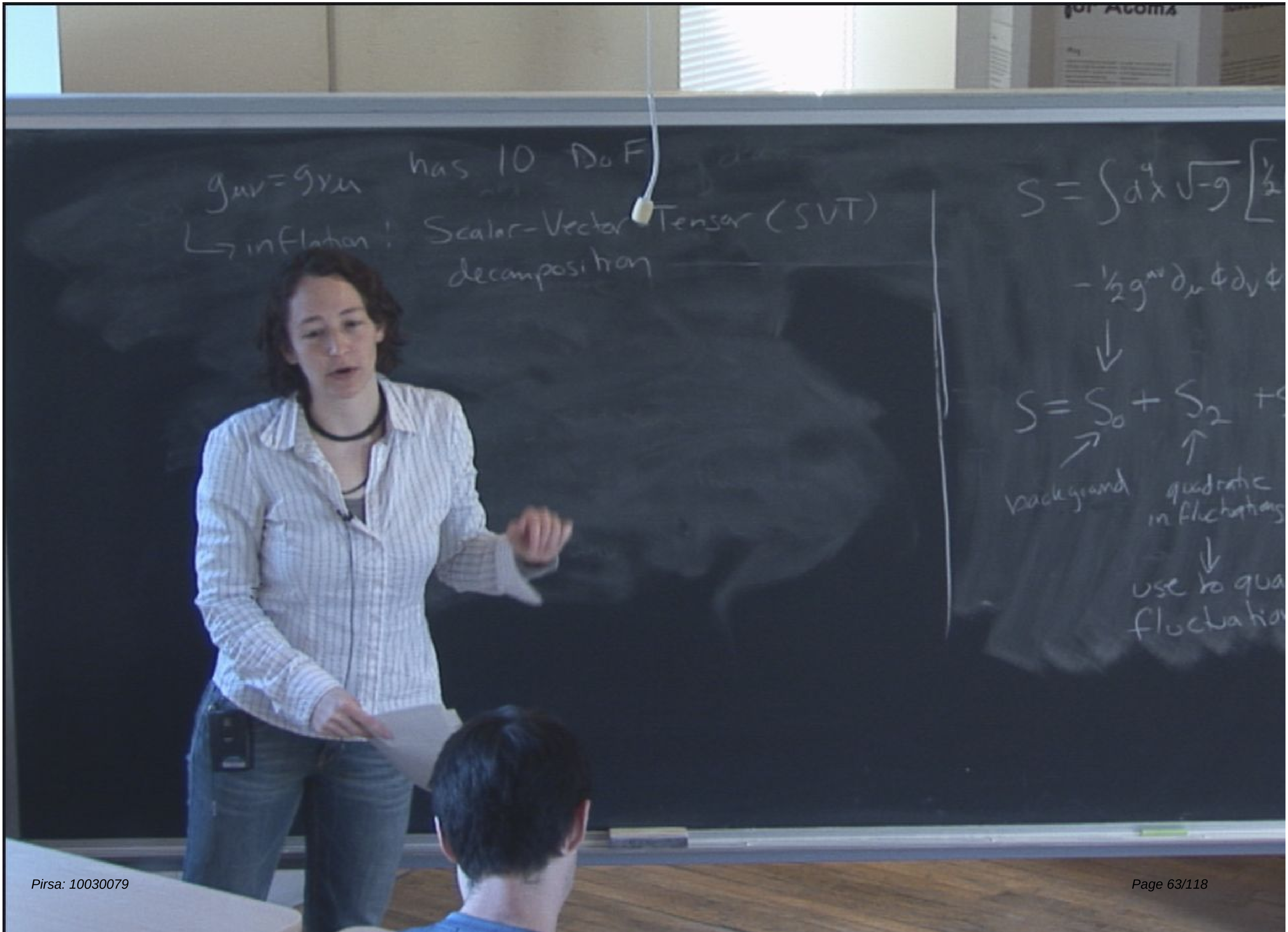
↳ Best thing is to define variables that are independent of coordinate transformation

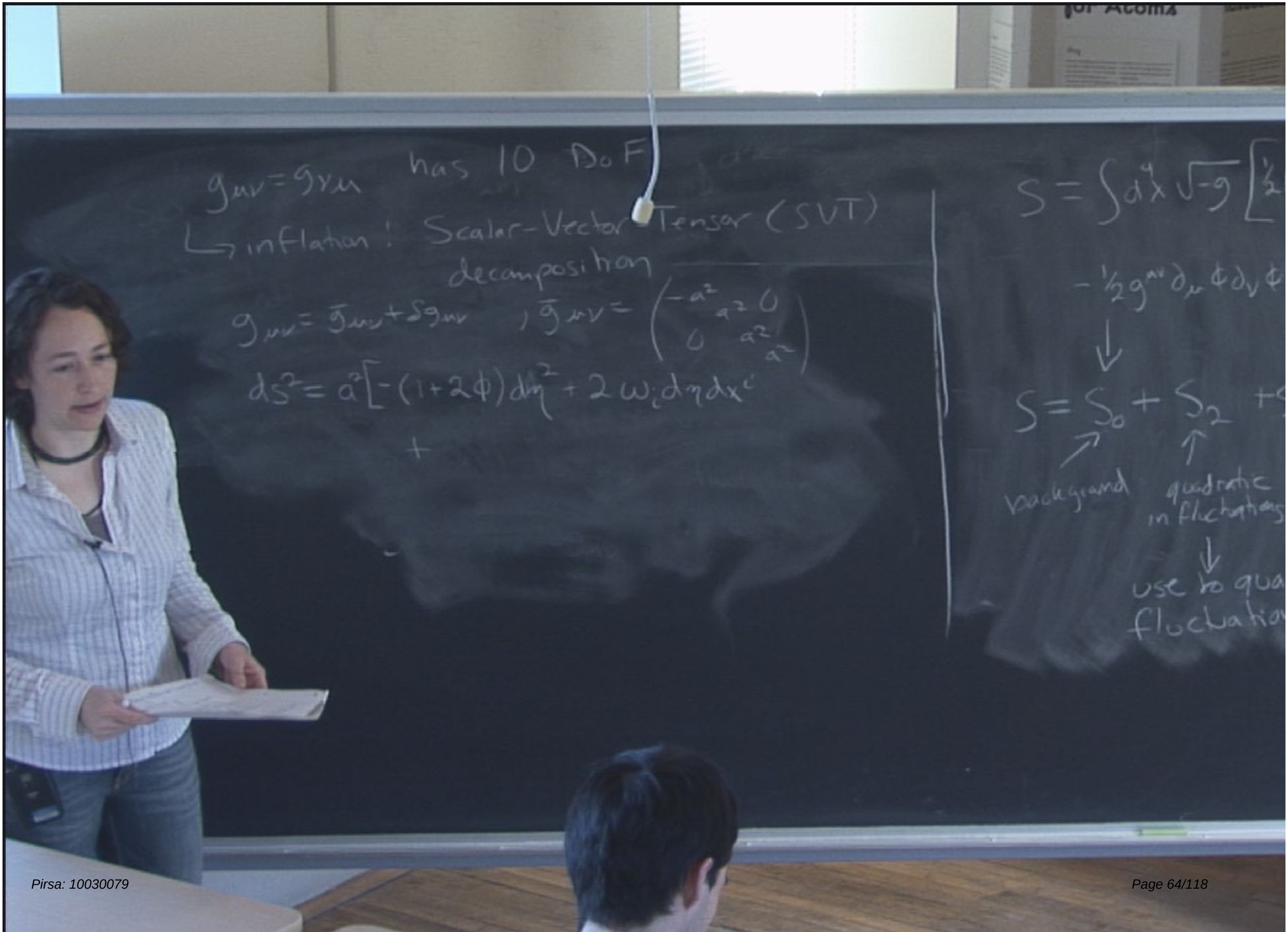
"gauge invariant"

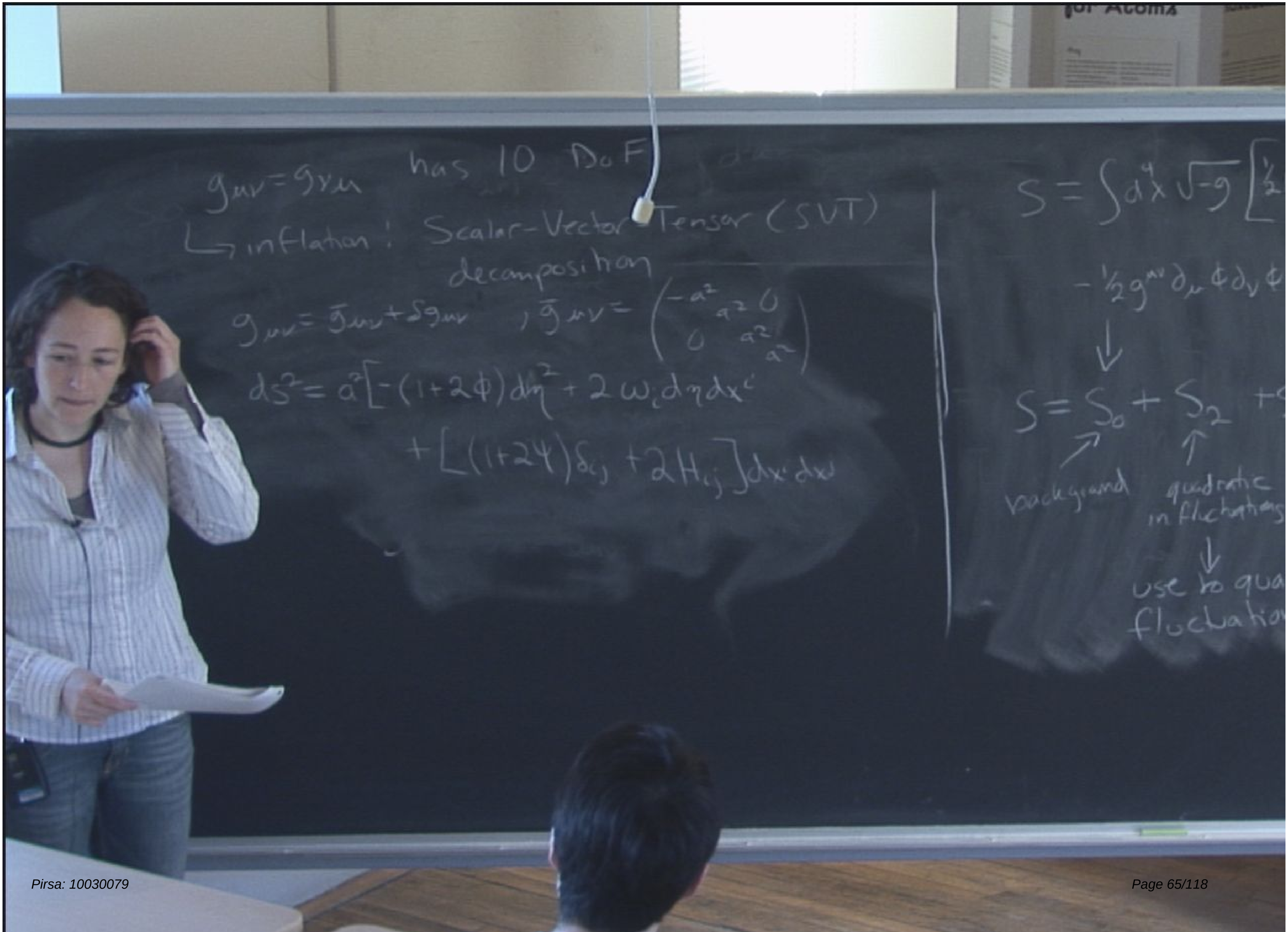

and we change to
$t \to \tilde{t} = t + \delta t(x,t)$
$\rho(t) = \rho(\tilde{t} - \delta t(x,t))$
$\simeq \rho(\tilde{t}) - \frac{\partial \rho}{\partial t} \delta t(\tilde{x},\tilde{t})$
$\equiv \rho(\tilde{t}) + \delta\rho(\tilde{x},\tilde{t})$
coordinate change has "produced" perturbations
↳ Best thing is to define variables that are independent of coordinate transformation
"gauge invariant"

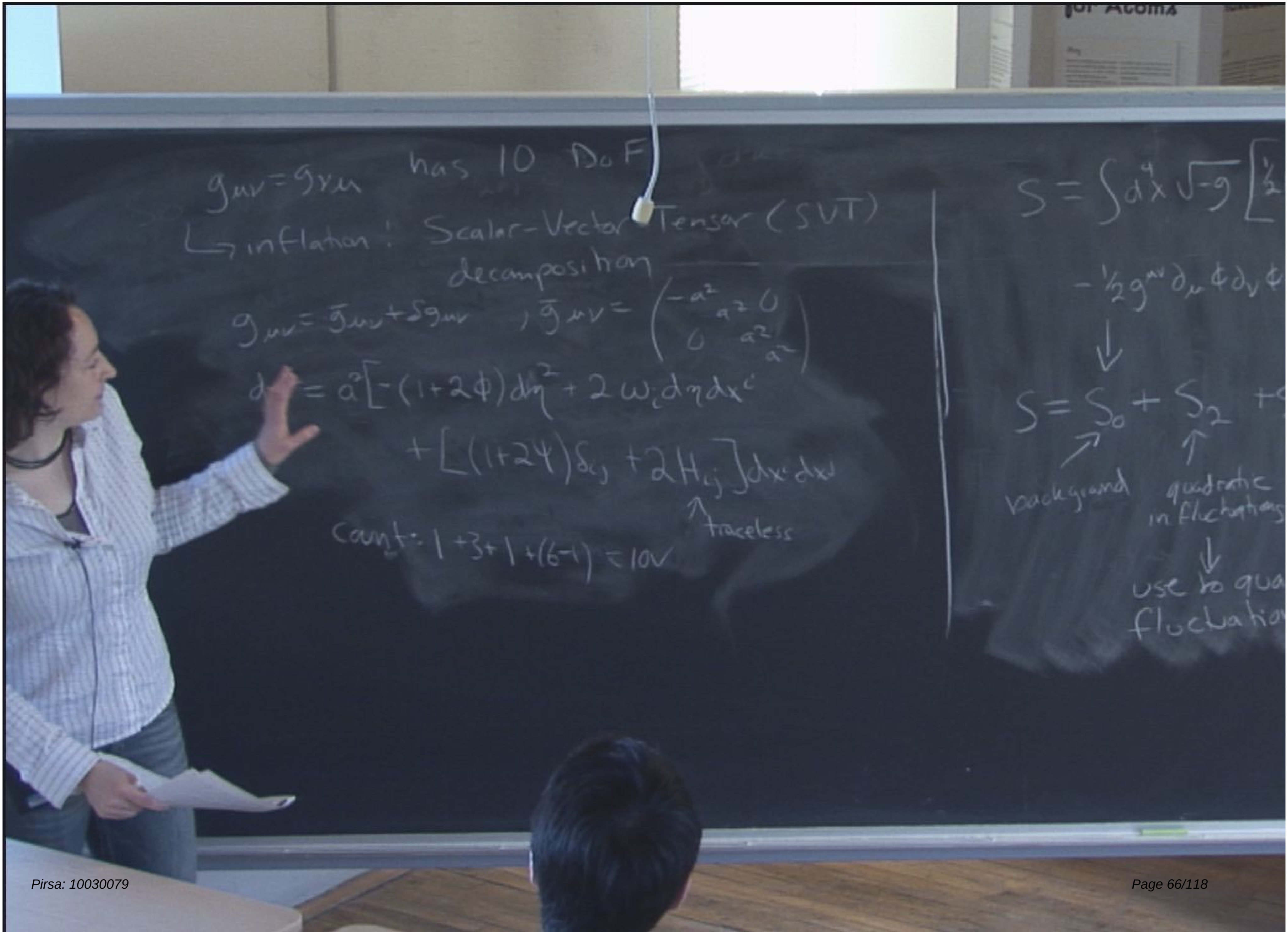

gμν=gνμ has 10 DoF
S = ∫d⁴x √-g [½
-½ g^μν ∂μ φ ∂ν φ
S = S₀ + S₂ +
background
quadratic
in fluctuations
use to qua
fluctation

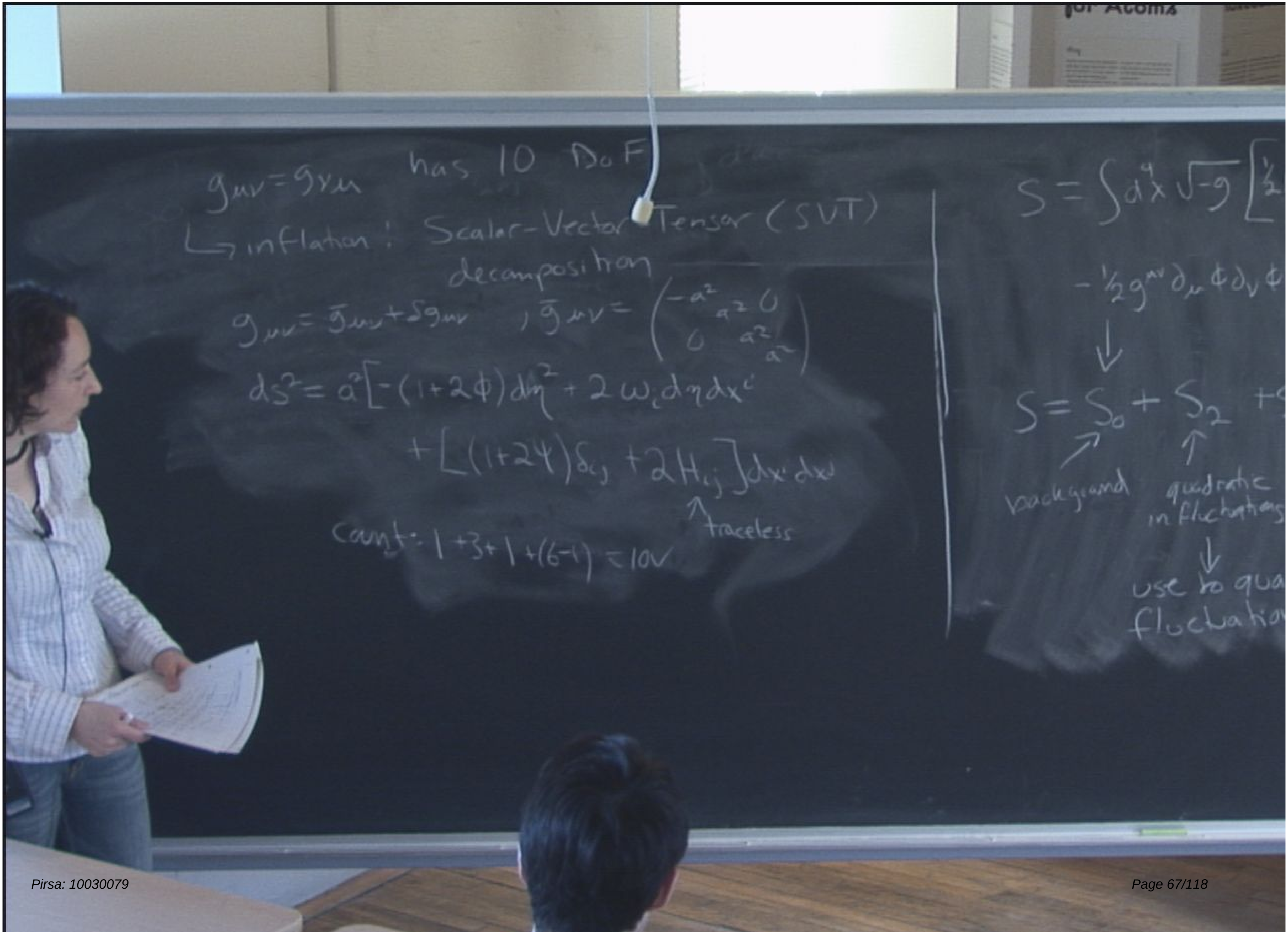

gμν = gνμ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT)
S = ∫d⁴x √-g [
-½ g^μν ∂μ φ ∂ν φ
S = S0 + S2 +
background
quadratic in fluctuations

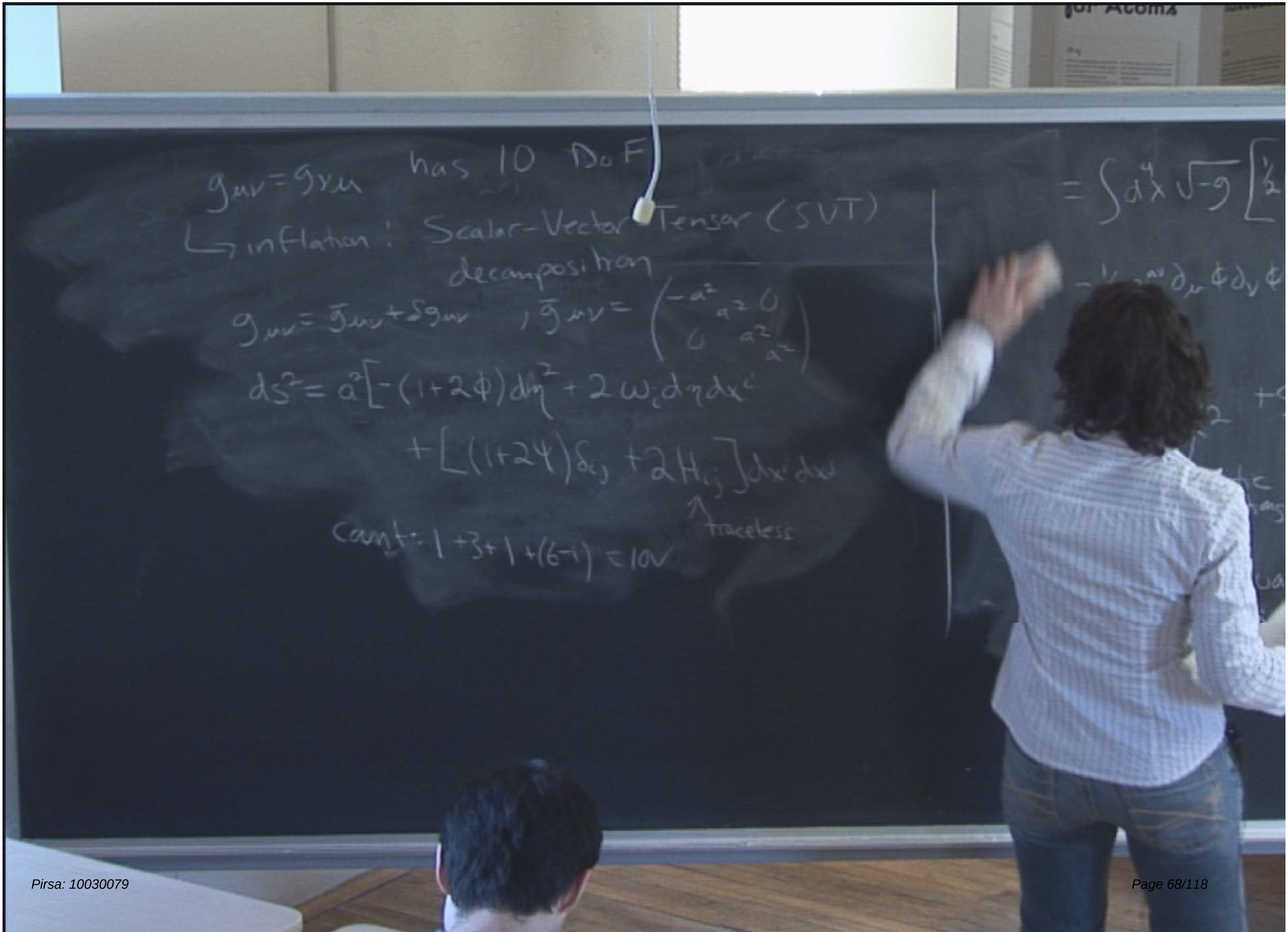

gμν = gνμ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT)
decomposition
S = ∫d⁴x √-g [
- ½ g^μν ∂μ φ ∂ν φ
S = S₀ + S₂
background
quadratic in fluctuations
use to qua
fluctuation

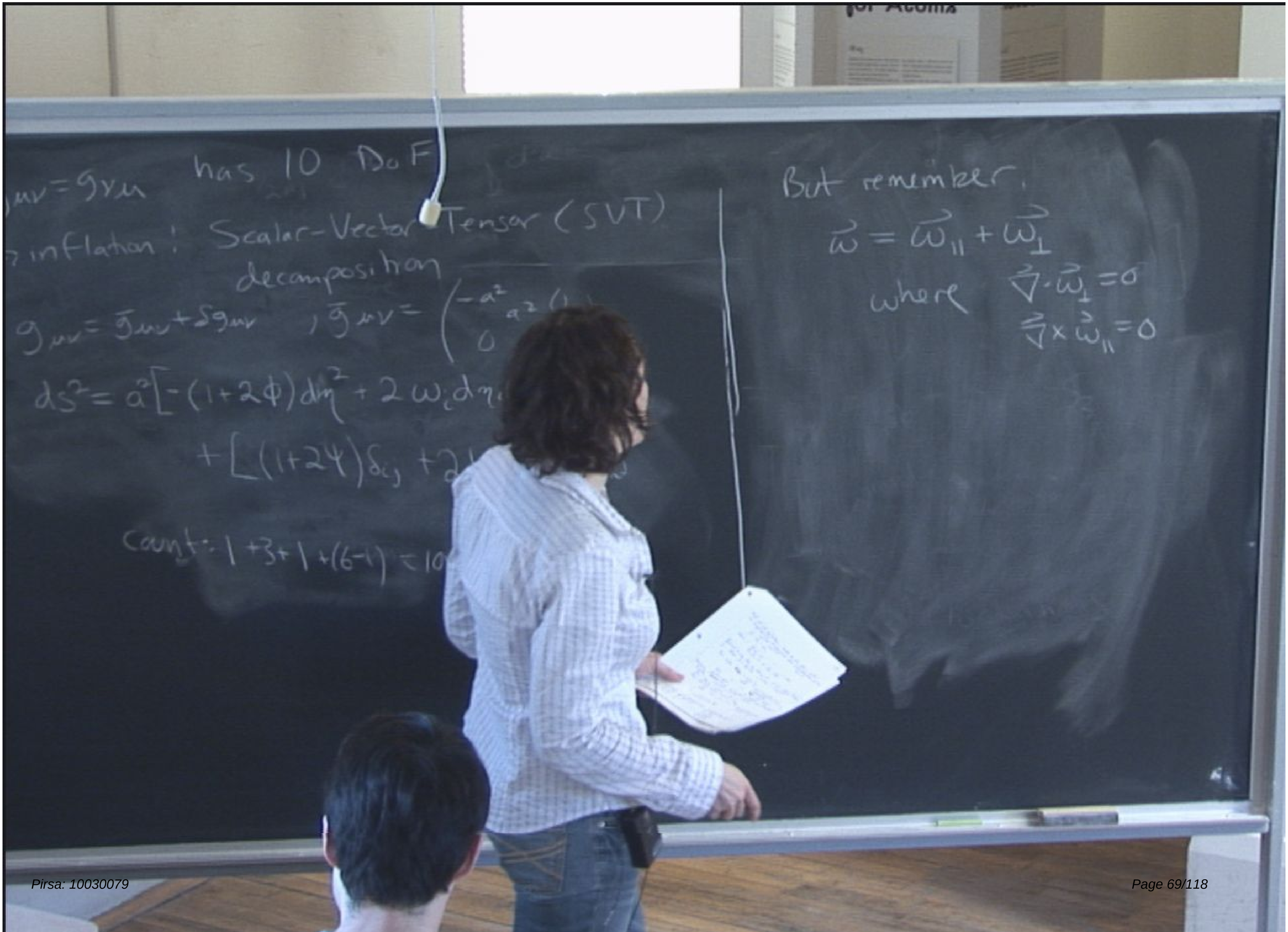

gμν = gνμ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT)
decomposition
gμν = ḡμν + δgμν , ḡμν =
ds² = a²[-(1+2ϕ)dη² + 2ωᵢdηdxⁱ
+
S = ∫d⁴x √-g [
- ½ g^μν ∂μ ϕ ∂ν ϕ
S = S₀ + S₂
background
quadratic
in fluctuations
use to qua
fluctuation


$g_{\mu\nu} = g_{\nu\mu}$ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT) decomposition
$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$, $\bar{g}_{\mu\nu} = \begin{pmatrix} -a^2 & & & 0 \\ & a^2 & & \\ & & a^2 & \\ 0 & & & a^2 \end{pmatrix}$
$ds^2 = a^2[-(1+2\Phi)d\eta^2 + 2\omega_i d\eta dx^i + [(1+2\Psi)\delta_{ij} + 2H_{ij}]dx^i dx^j$
$S = \int d^4x \sqrt{-g} [\frac{1}{2}$
$-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$
$S = S_0 + S_2 +$
background
quadratic in fluctuations
use to qua
fluctuation

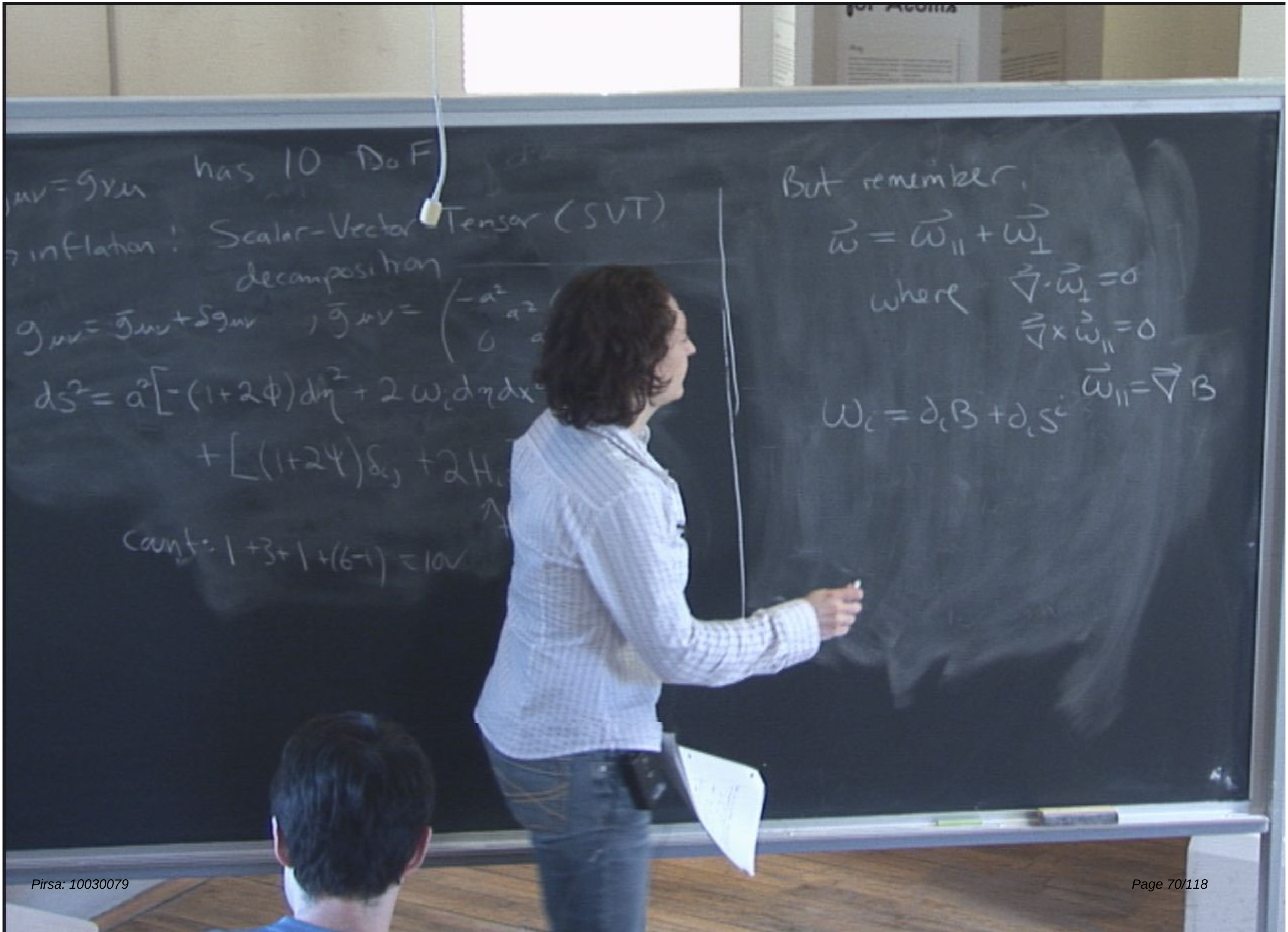

g_{μν} = g_{νμ} has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT) decomposition
g_{μν} = ḡ_{μν} + δg_{μν} , ḡ_{μν} = (−a² 0; a²; 0 a²; a²)
ds² = a²[−(1+2Φ)dη² + 2ω_i dη dx^i
+ [(1+2Ψ)δ_{ij} + 2H_{ij}]dx^i dx^j]
traceless
count: 1 + 3 + 1 + (6−1) = 10 ✓
S = ∫d⁴x √−g [½
− ½ g^{μν} ∂_μ φ ∂_ν φ
S = S_0 + S_2 +
background
quadratic in fluctuations
use to qua
fluctuation

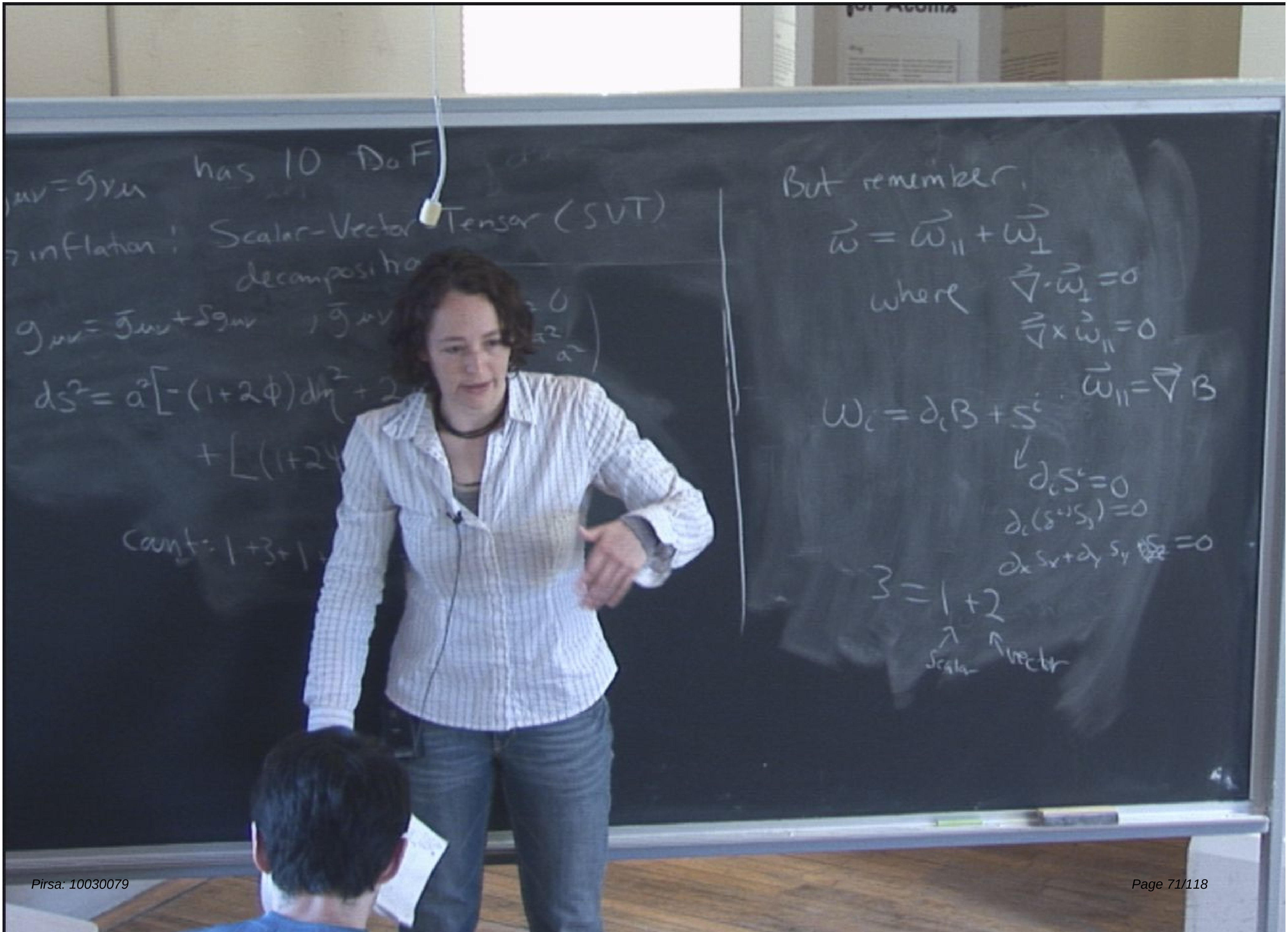

gμν = gνμ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT) decomposition
gμν = ḡμν + δgμν , ḡμν = (−a² a² 0 0 a² a²)
ds² = a²[−(1+2Φ)dη² + 2 ωi dη dx^i + [(1+2Ψ)δij + 2Hij] dx^i dx^j
traceless
count: 1+3+1+(6−1) = 10 ✓
S = ∫d⁴x √−g [
−½ g^μν ∂μΦ ∂νΦ
S = S0 + S2 +
background
quadratic in fluctuations
use to qua
fluctuation

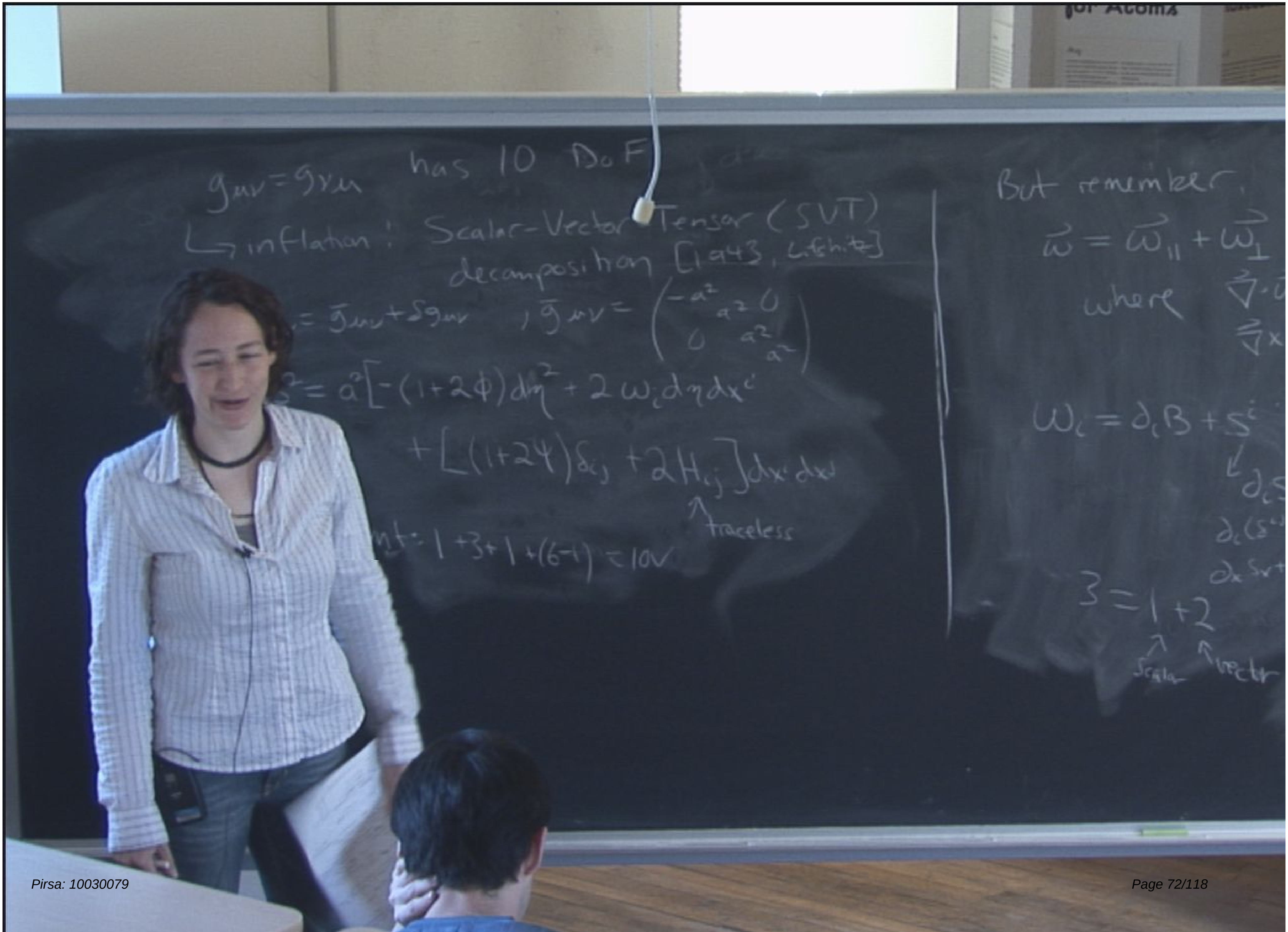

gμν = gνμ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT) decomposition
gμν = ḡμν + δgμν , ḡμν = (−a² a² 0 0 a² a²)
ds² = a²[−(1+2Φ)dη² + 2ωi dη dx^i
+ [(1+2Ψ)δij + 2Hij] dx^i dx^j
↑ traceless
count: 1 + 3 + 1 + (6−1) = 10✓
= ∫d⁴x √−g [½
−½ g^μν ∂μφ ∂νφ

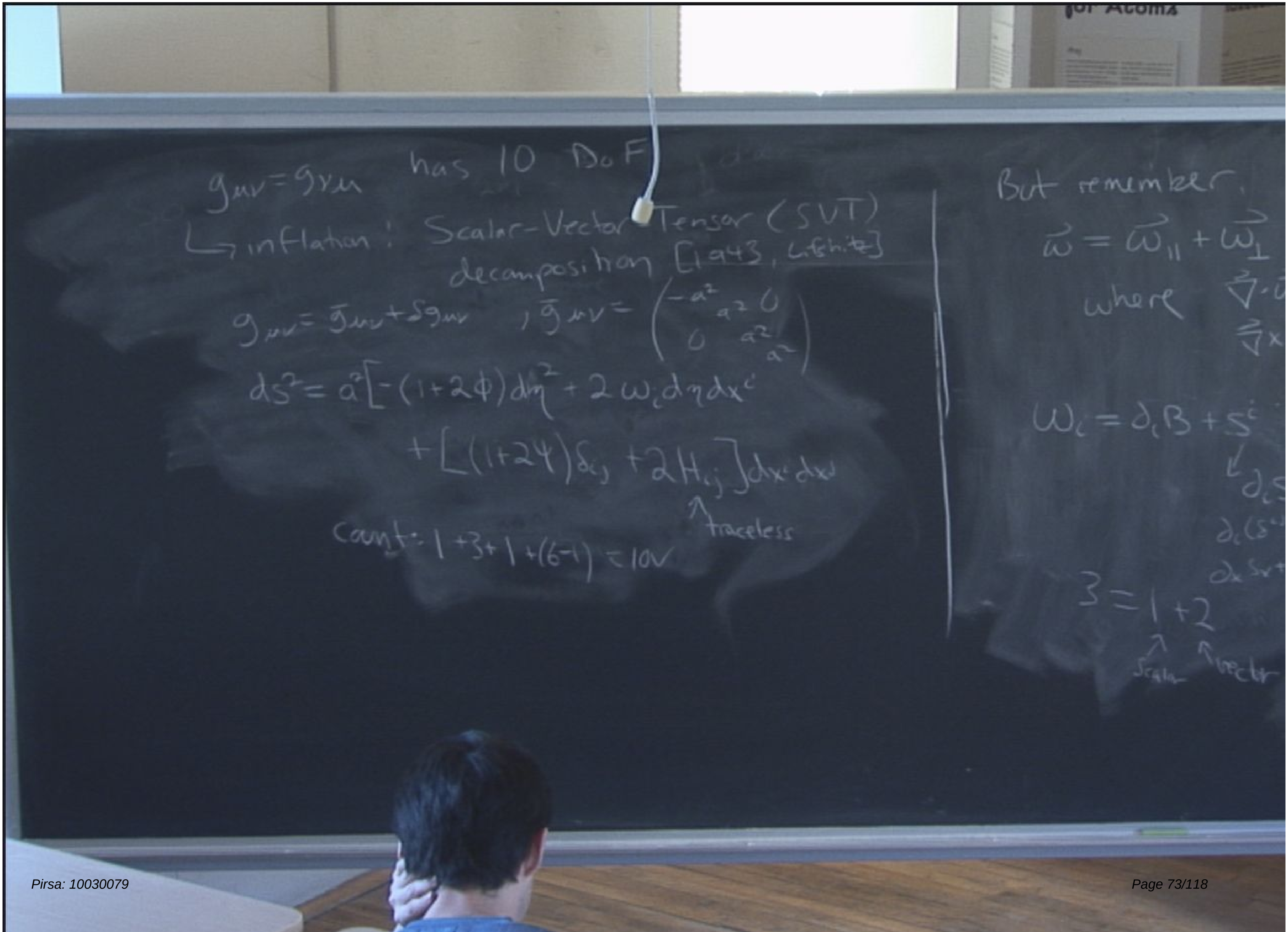


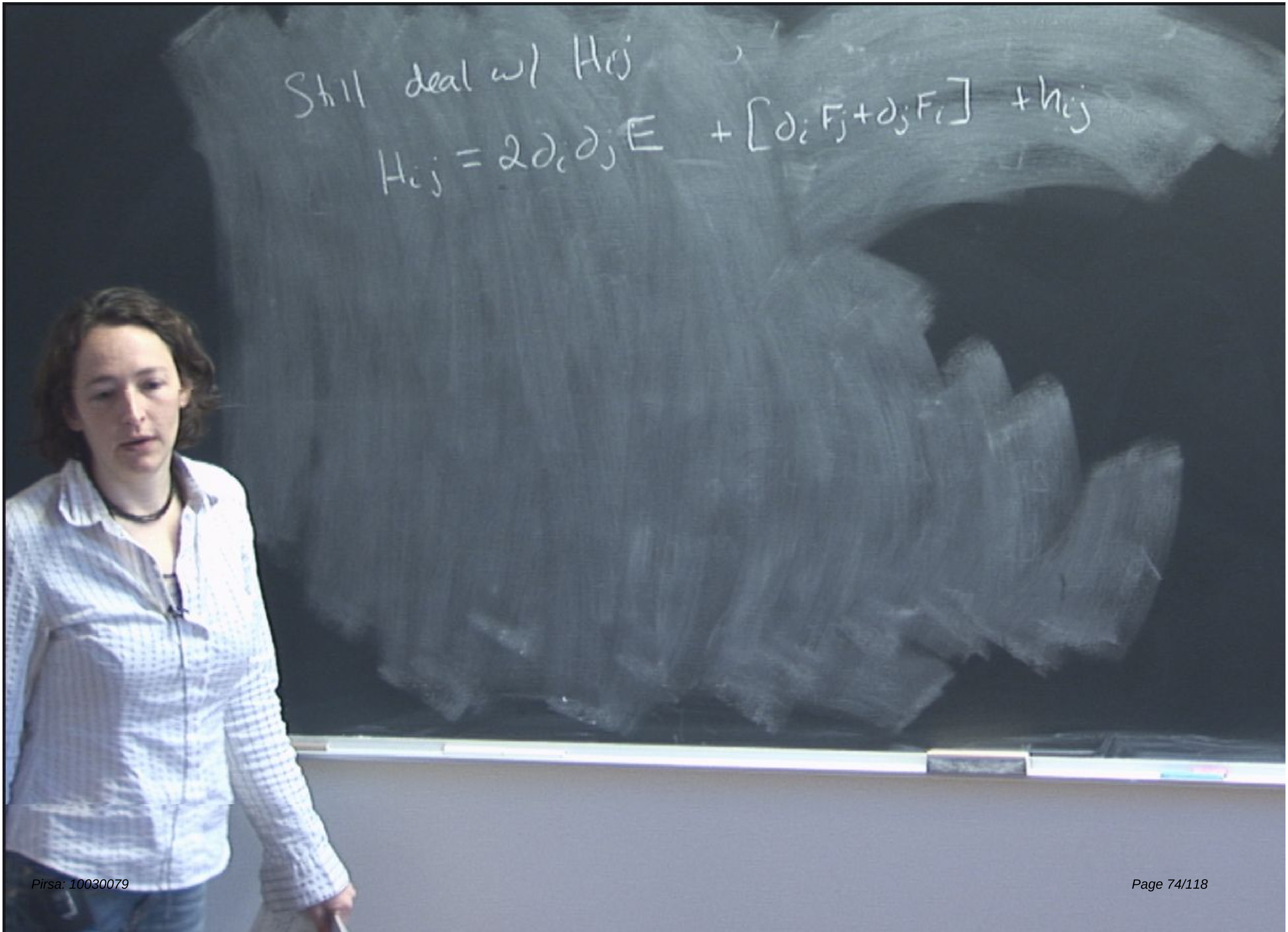
has 10 DoF
But remember,
Scalar-Vector-Tensor (SVT)
decomposition
where
count: 1+3+1+(6-1) = 10

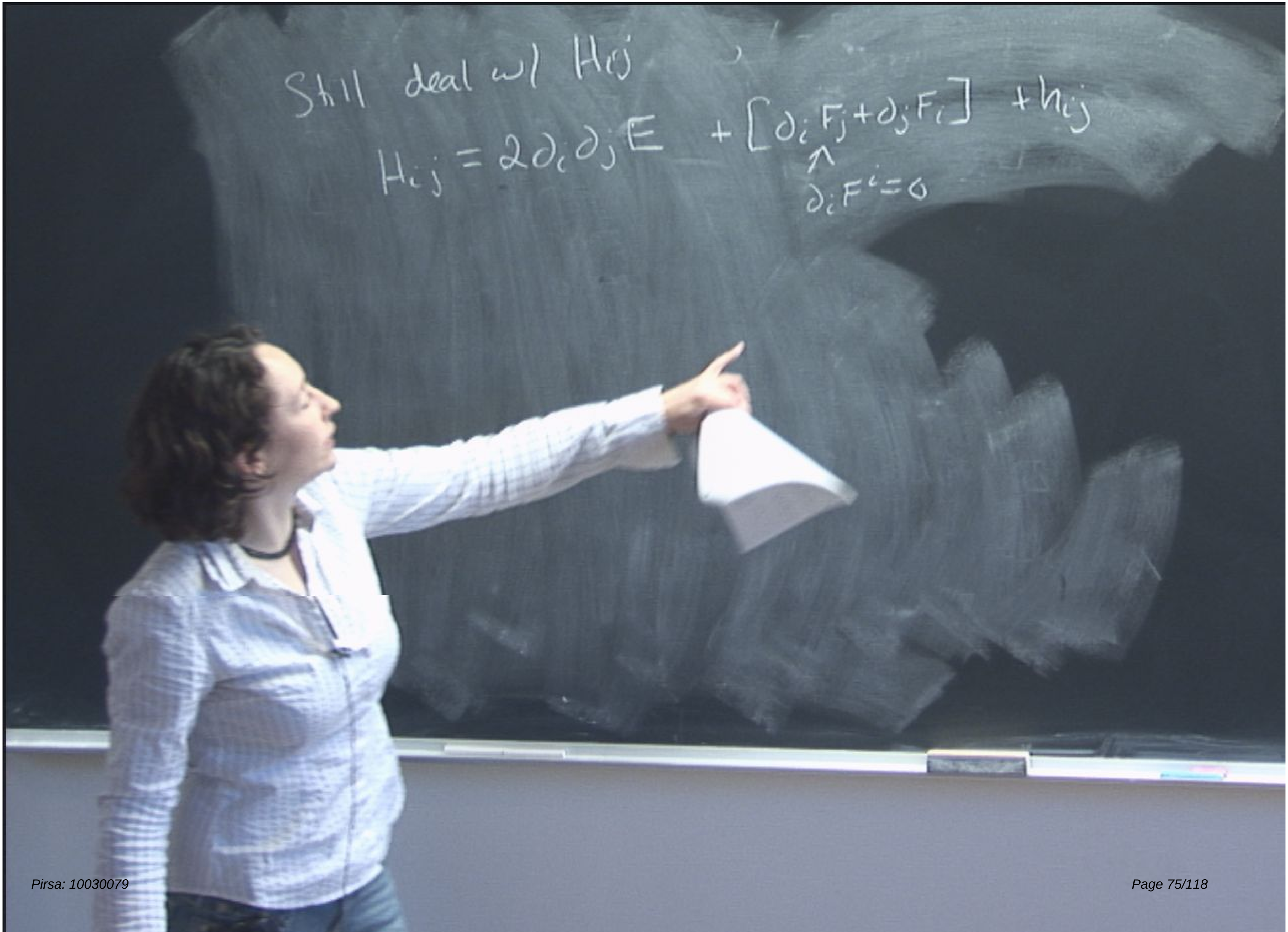

has 10 DoF
inflation: Scalar-Vector-Tensor (SVT)
decomposition
count: 1+3+1+(6-1) = 10
But remember,
where

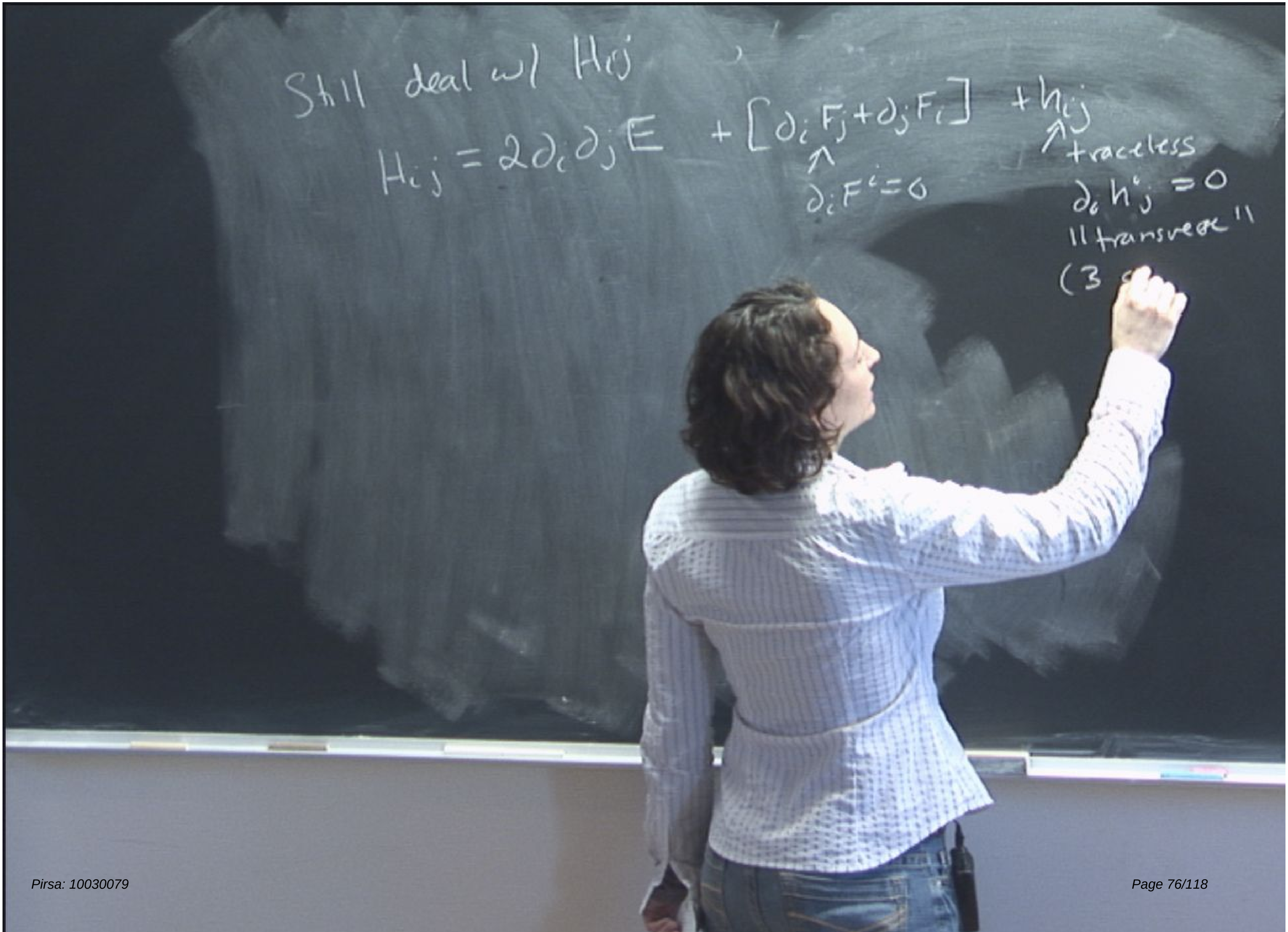

has 10 DoF
Scalar-Vector-Tensor (SVT)
But remember:
where
3 = 1 + 2
scalar
vector

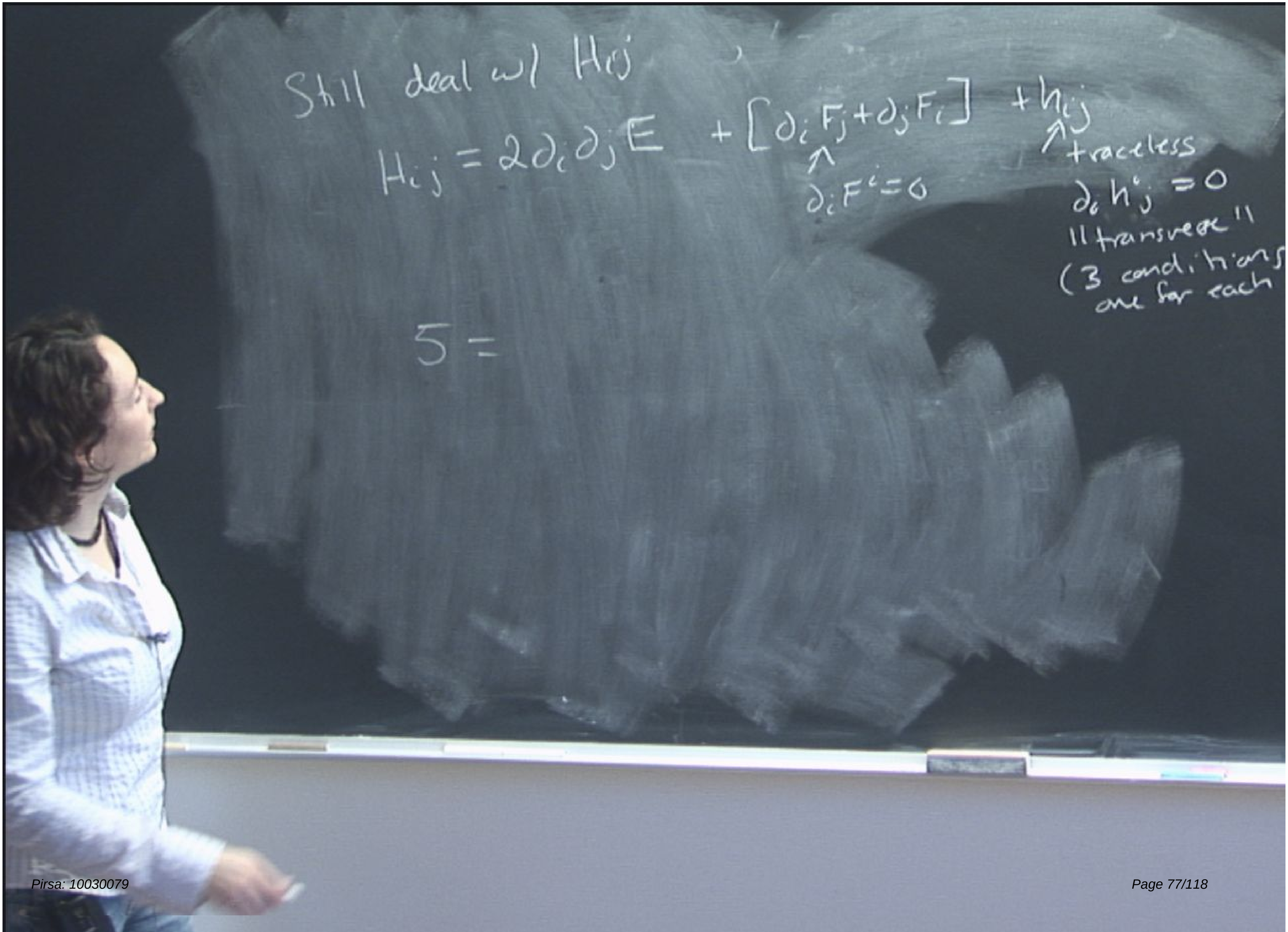

g_{μν} = g_{νμ} has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT) decomposition [1943, Lifshitz]
traceless
But remember,
where
3 = 1 + 2
scalar
vector

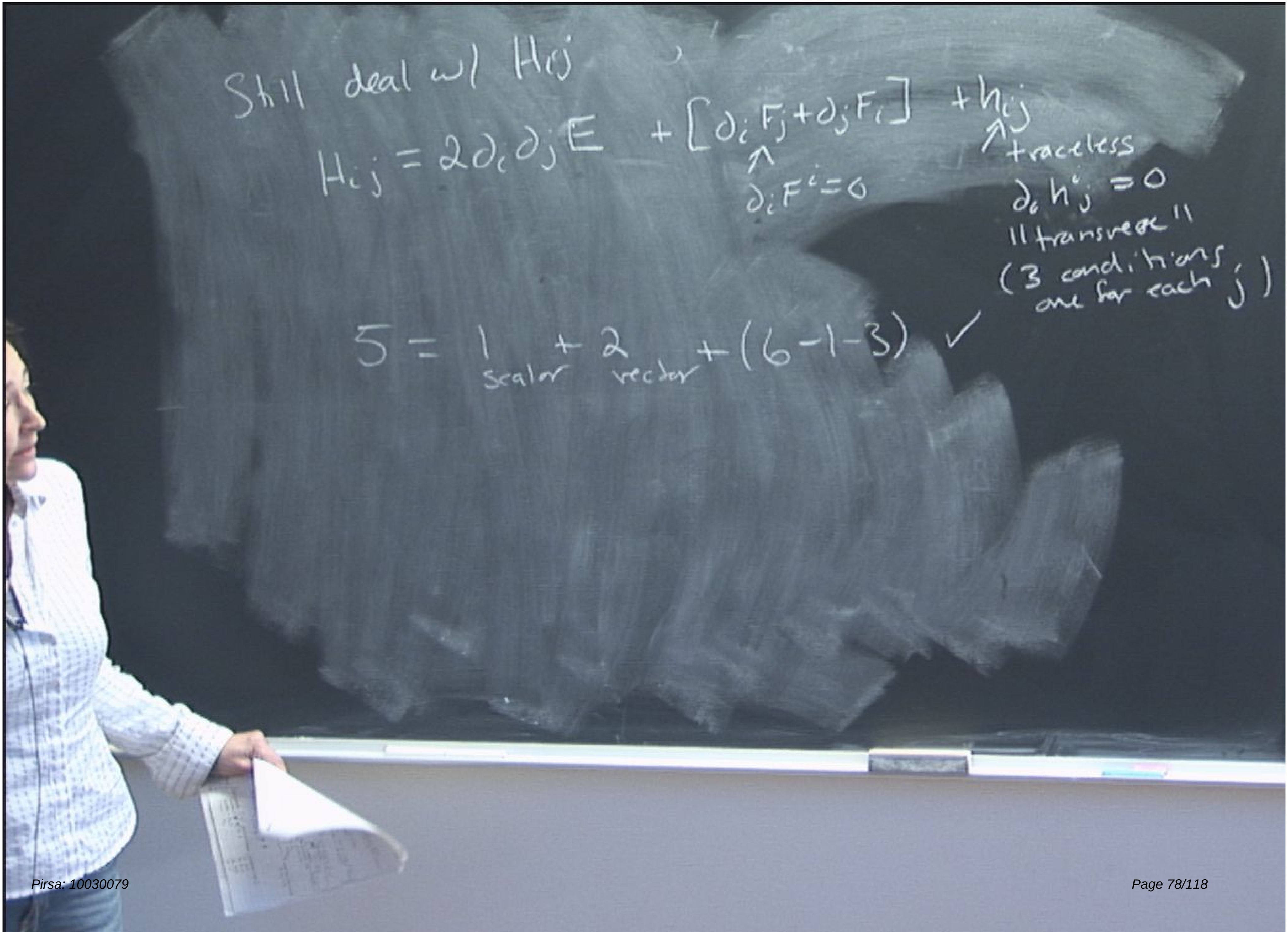

$g_{\mu\nu} = g_{\nu\mu}$ has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT)
decomposition [1943, Lifshitz]
$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$, $\bar{g}_{\mu\nu} = \begin{pmatrix} -a^2 & & & 0 \\ & a^2 & & \\ & & a^2 & \\ 0 & & & a^2 \end{pmatrix}$
$ds^2 = a^2[-(1+2\phi)d\eta^2 + 2\omega_i d\eta dx^i$
$+ [(1+2\psi)\delta_{ij} + 2H_{ij}]dx^i dx^j$
traceless
count: $1+3+1+(6-1) = 10$ ✓
But remember,
$\vec{\omega} = \vec{\omega}_{\parallel} + \vec{\omega}_{\perp}$
where
$\omega_i = \partial_i B + S^i$
$3 = 1 + 2$
scalar
vector

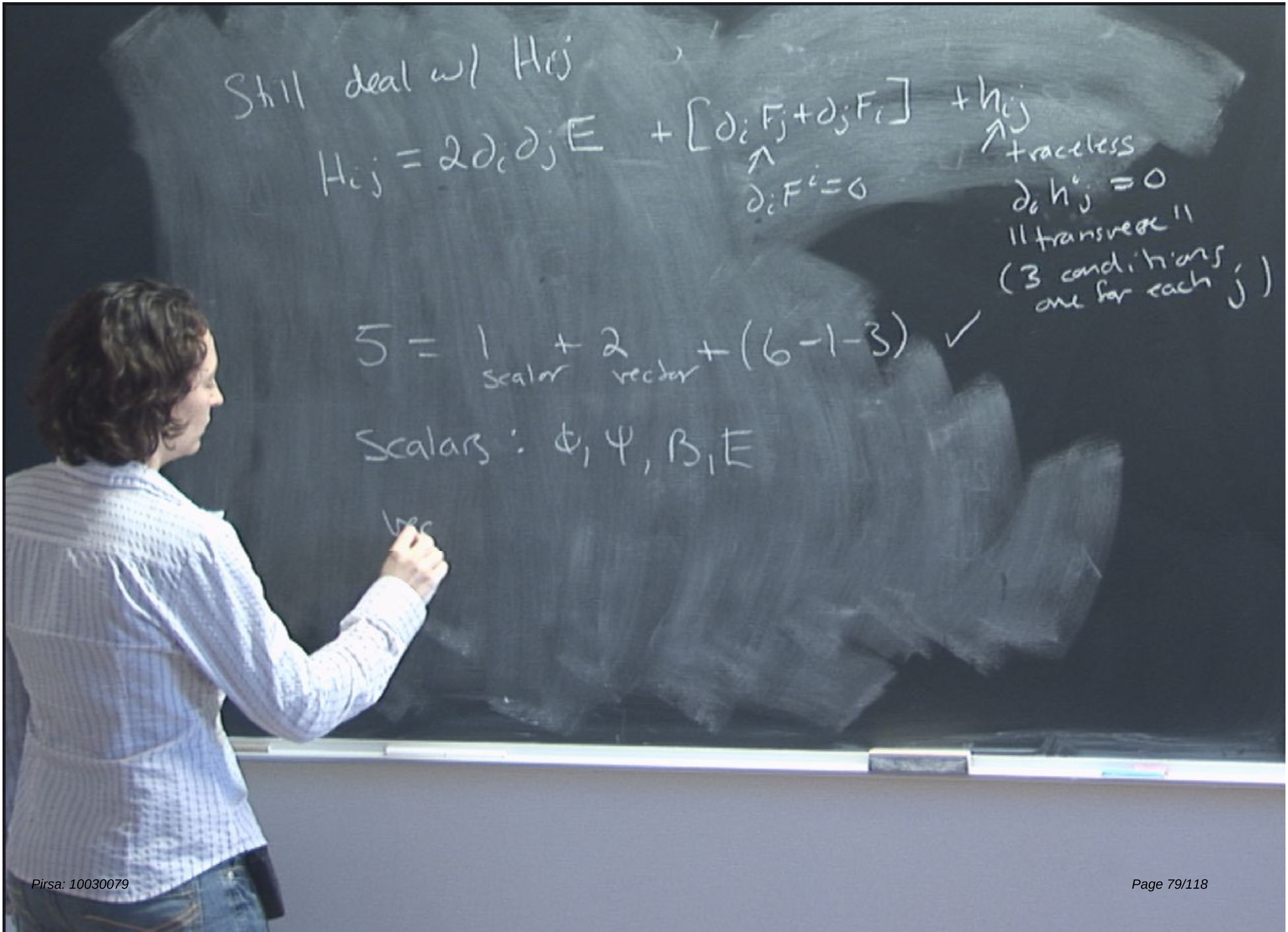

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$

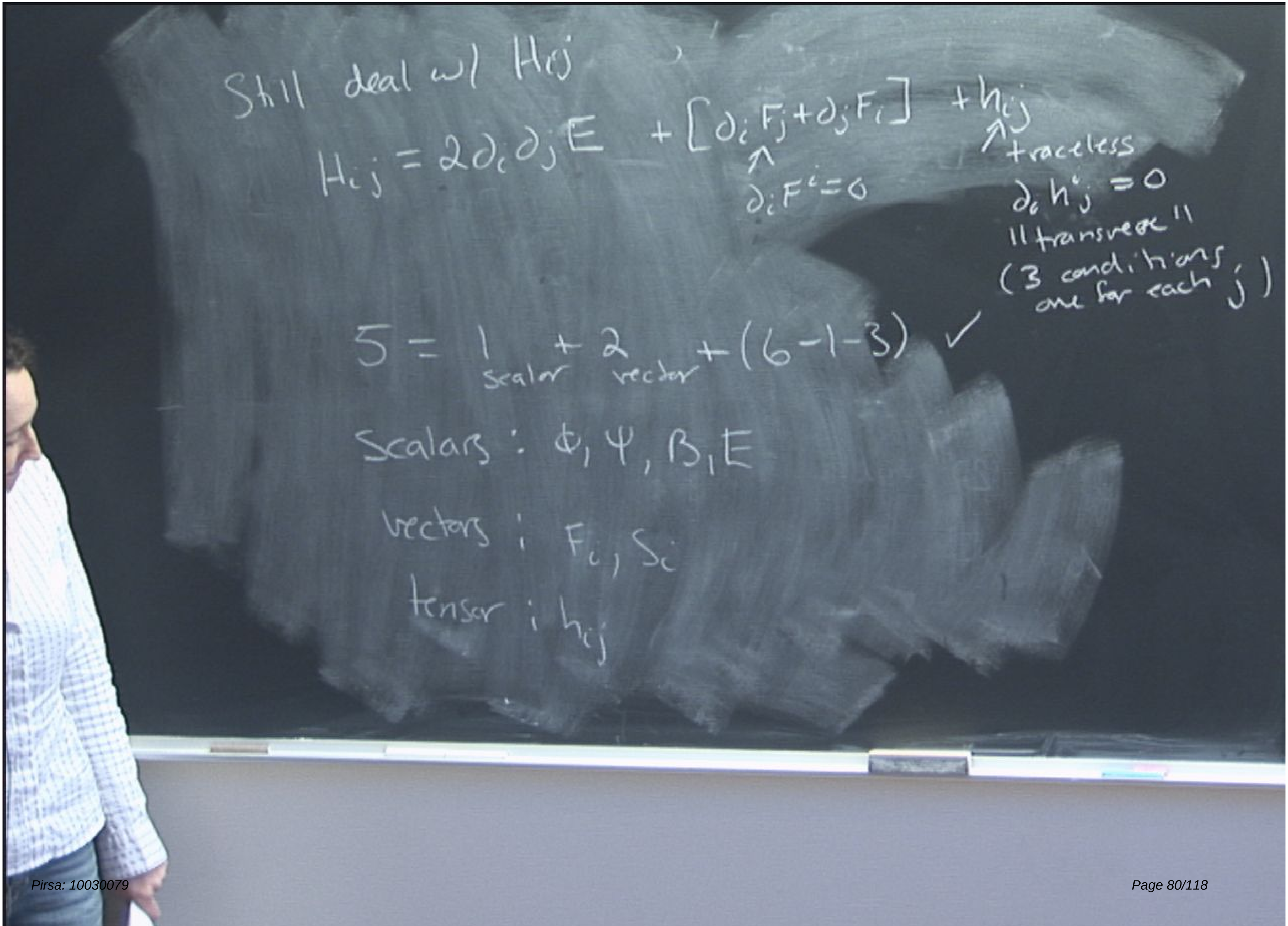

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$

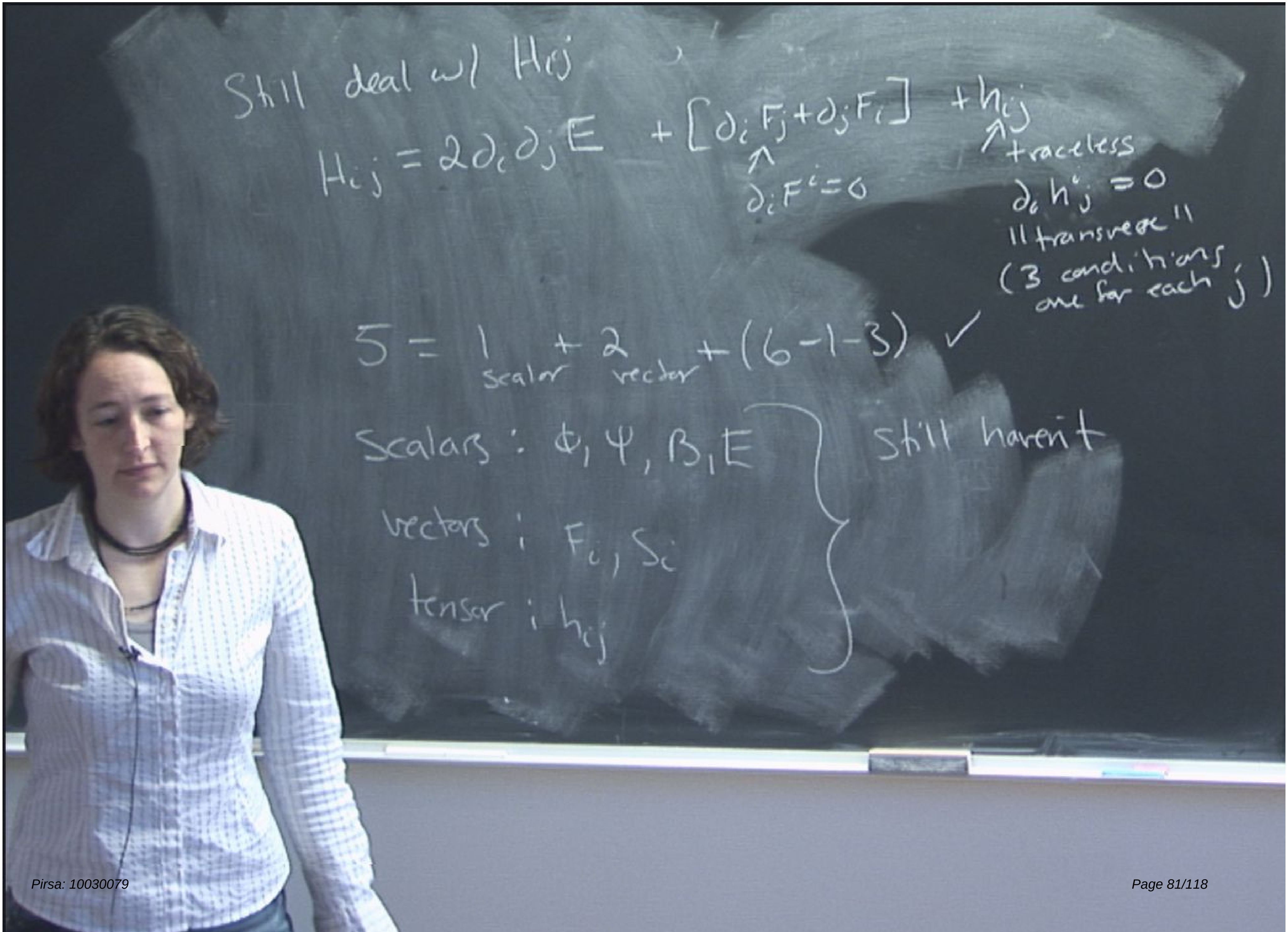

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$
traceless
$\partial_i h^i_j = 0$
"transverse"
(3

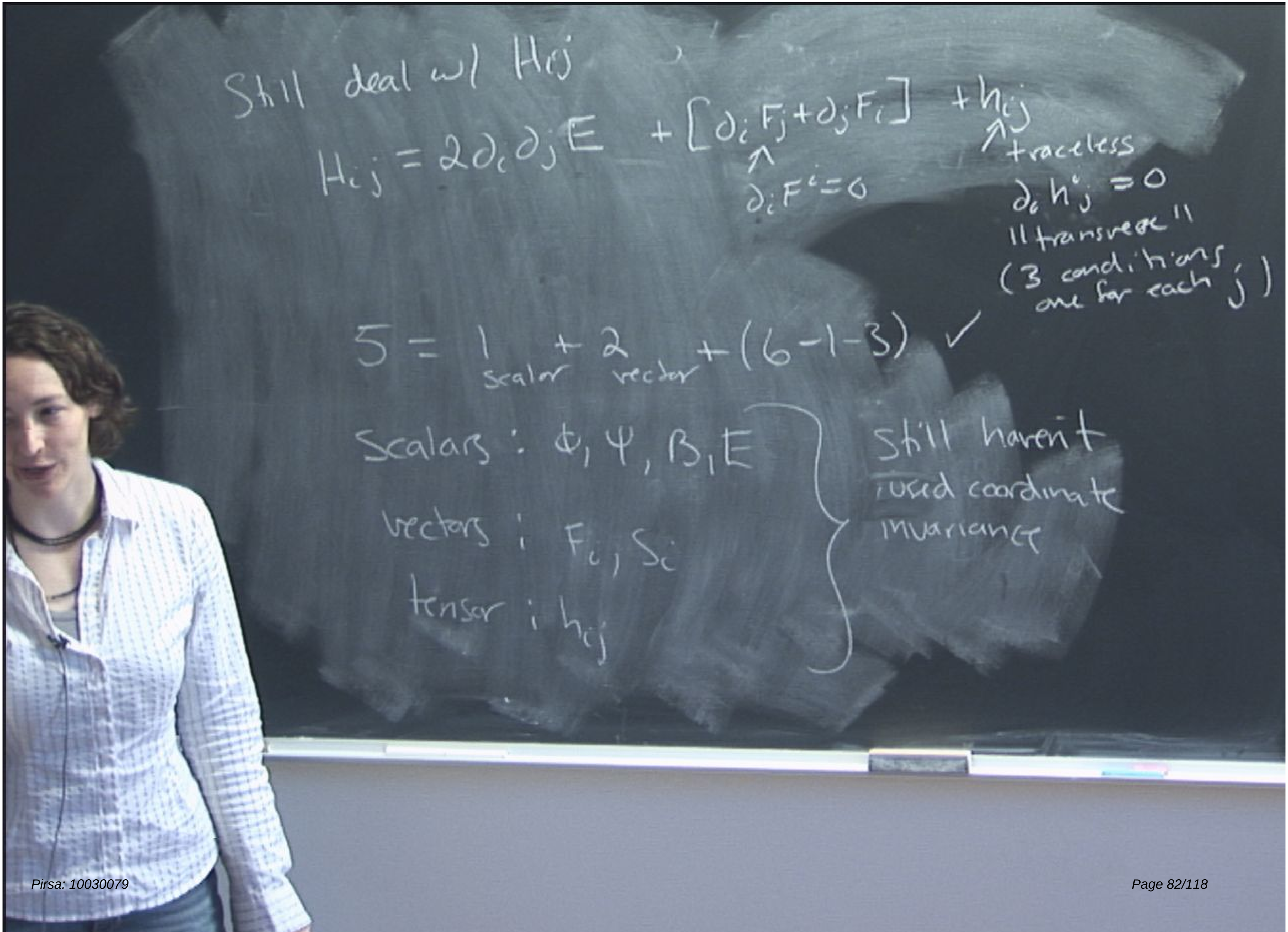

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$
traceless
$\partial_i h^i_j = 0$
"transverse"
(3 conditions
one for each
5 =

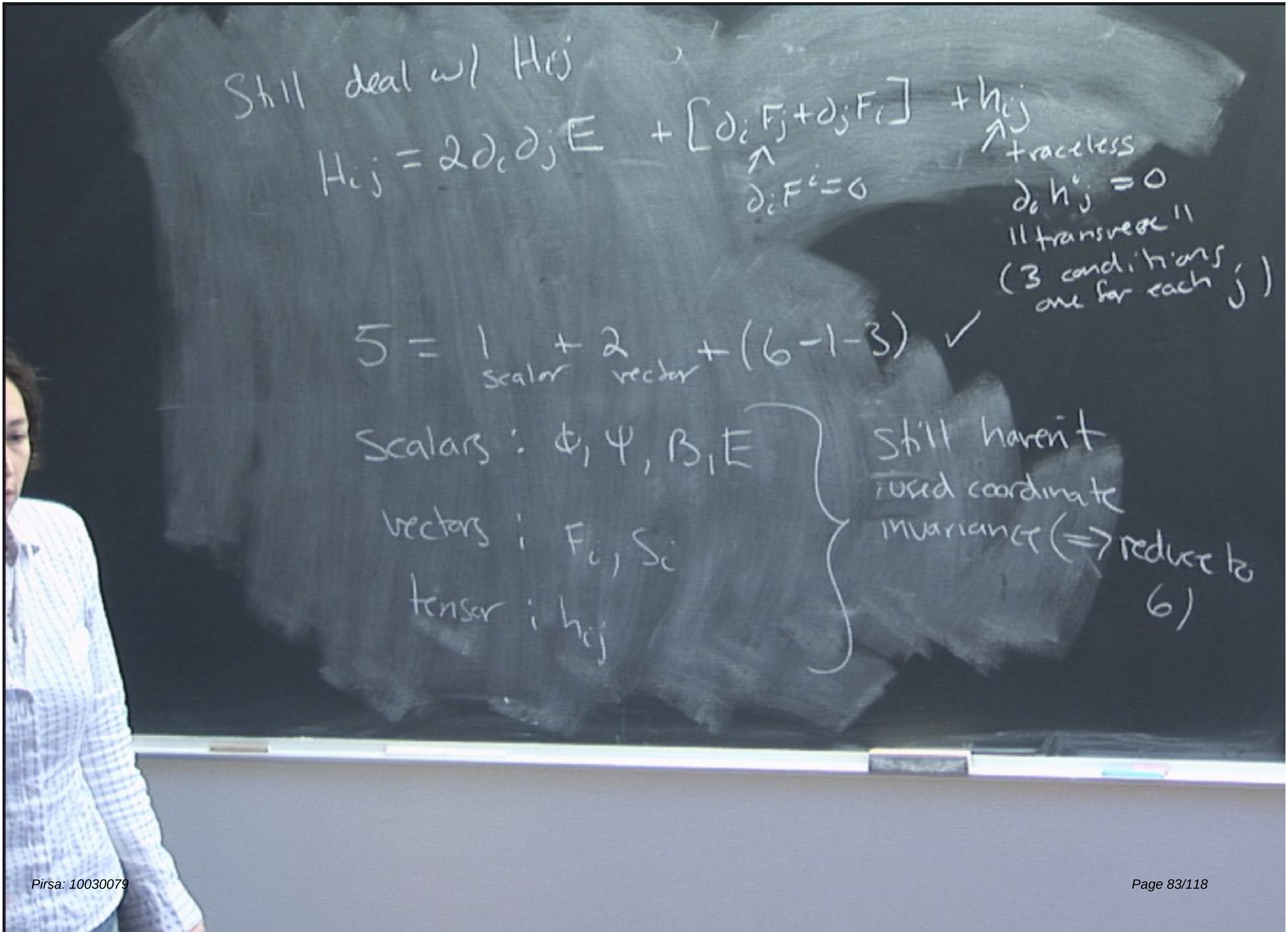

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i \partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$
traceless
$\partial_i h^i_j = 0$
"transverse"
(3 conditions one for each j)
$5 = 1 + 2 + (6-1-3)$ ✓
scalar
vector


Still deal w/ Hij
Hij = 2∂i∂jE + [∂iFj + ∂jFi] + hij
∂iF^i = 0
traceless
∂i h^i j = 0
"transverse"
(3 conditions, one for each j)
5 = 1 + 2 + (6-1-3) ✓
scalar
vector
Scalars : Φ, Ψ, B, E

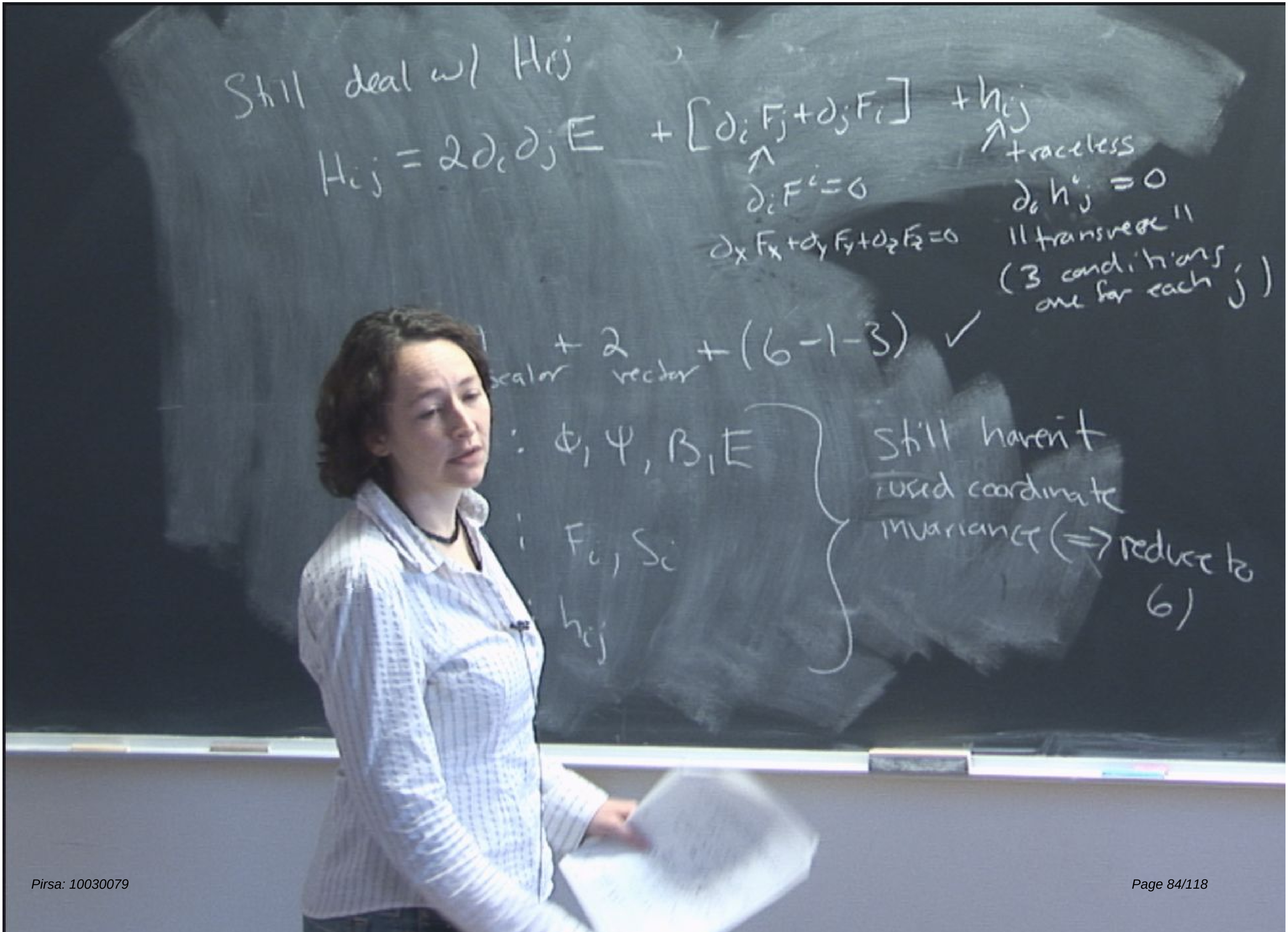

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$
traceless
$\partial_i h^i_j = 0$
"transverse"
(3 conditions one for each j)
$5 = 1 + 2 + (6-1-3)$ ✓
scalar
vector
Scalars: Φ, Ψ, B, E
vectors: F_i, S_i
tensor: h_{ij}

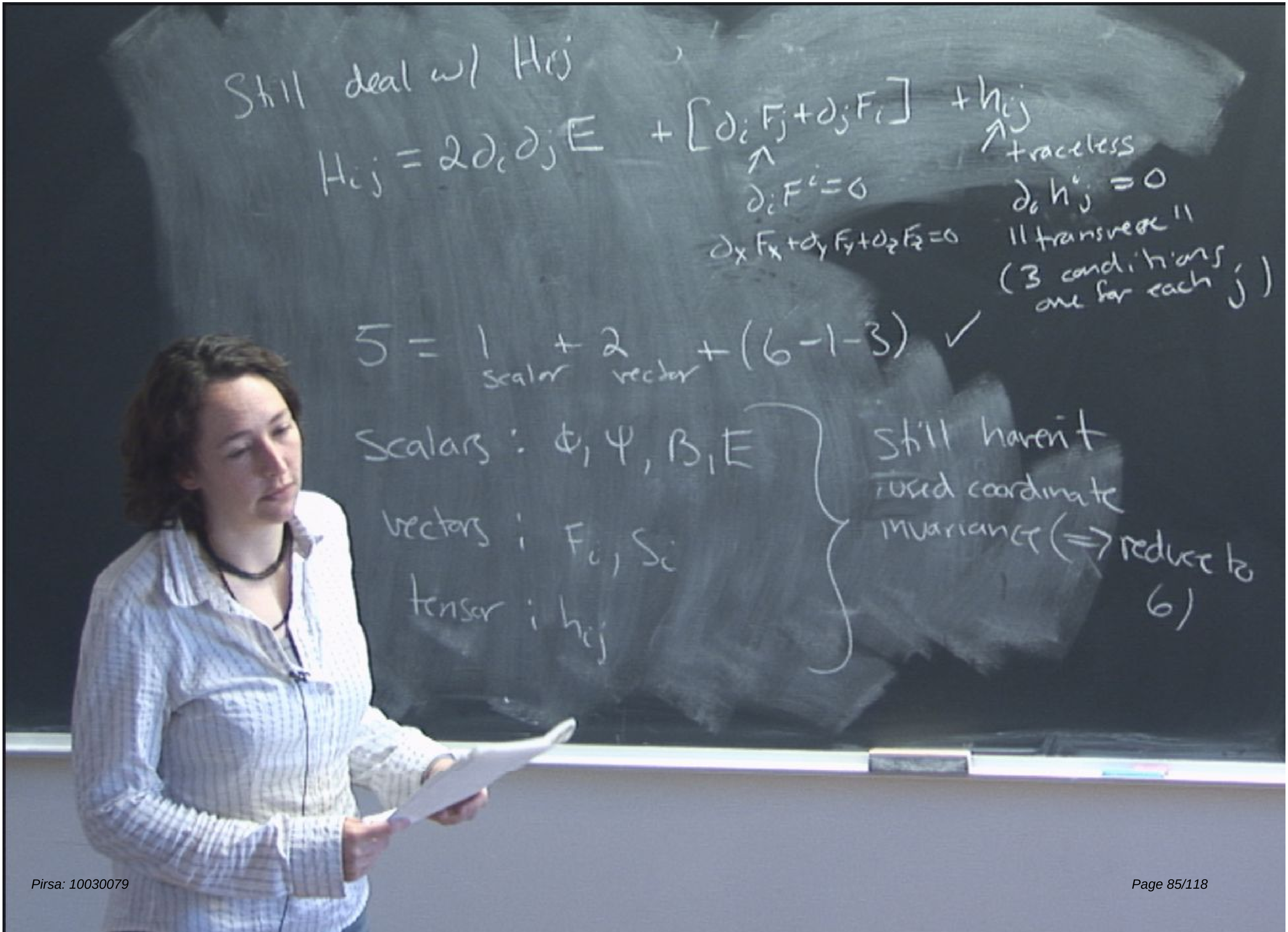

Still deal w/ Hij
Hij = 2∂i∂jE + [∂iFj + ∂jFi] + hij
∂iF^i = 0
traceless
∂i h^i_j = 0
"transverse"
(3 conditions one for each j)
5 = 1 + 2 + (6-1-3) ✓
scalar
vector
Scalars: Φ, Ψ, B, E
vectors: Fi, Si
tensor: hij
still haven't

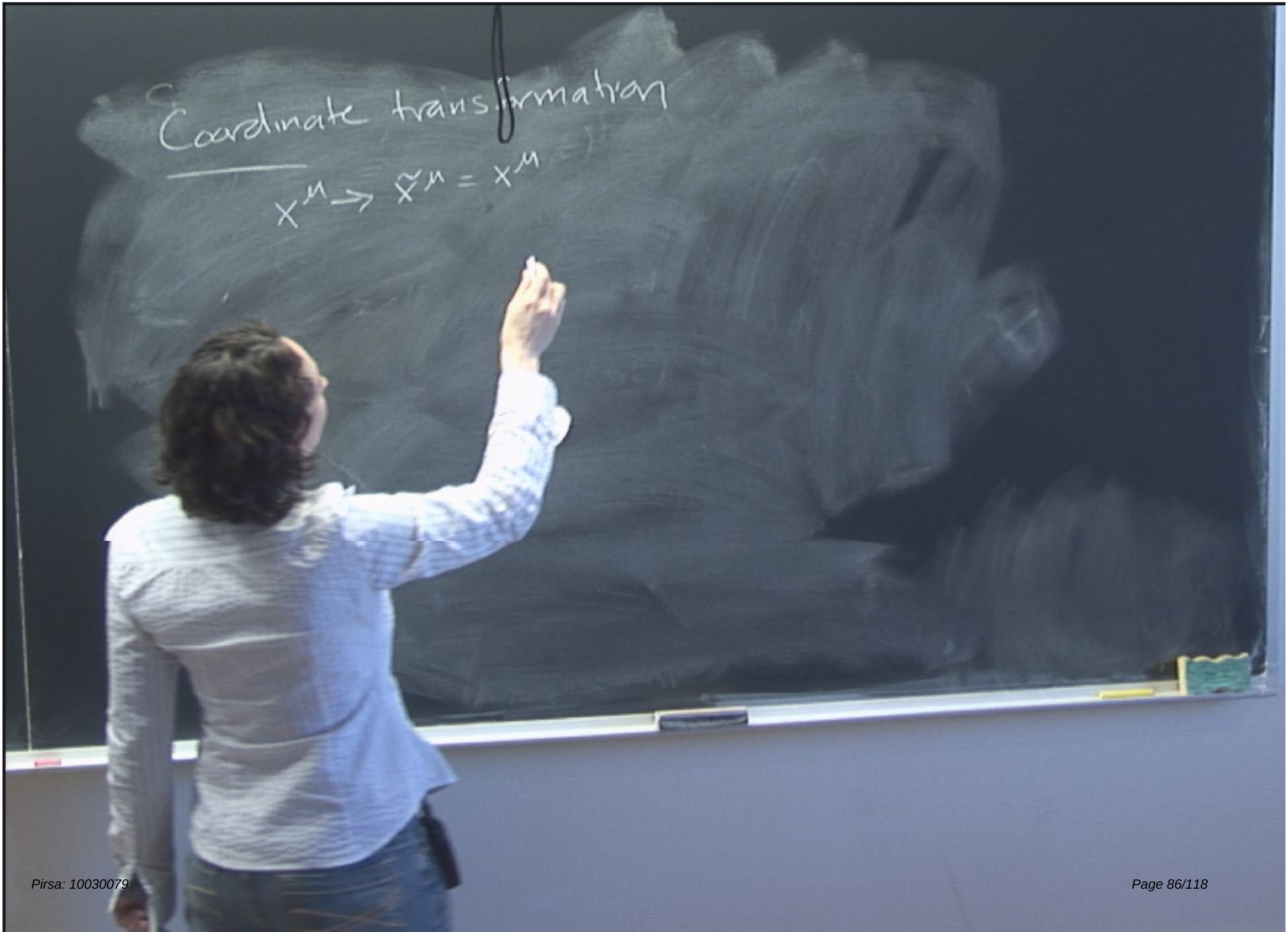

Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$
traceless
$\partial_i h^i_j = 0$
"transverse"
(3 conditions one for each j)
$5 = 1 + 2 + (6-1-3)$ ✓
scalar
vector
Scalars : Φ, Ψ, B, E
vectors : F_i, S_i
tensor : h_{ij}
Still haven't used coordinate invariance

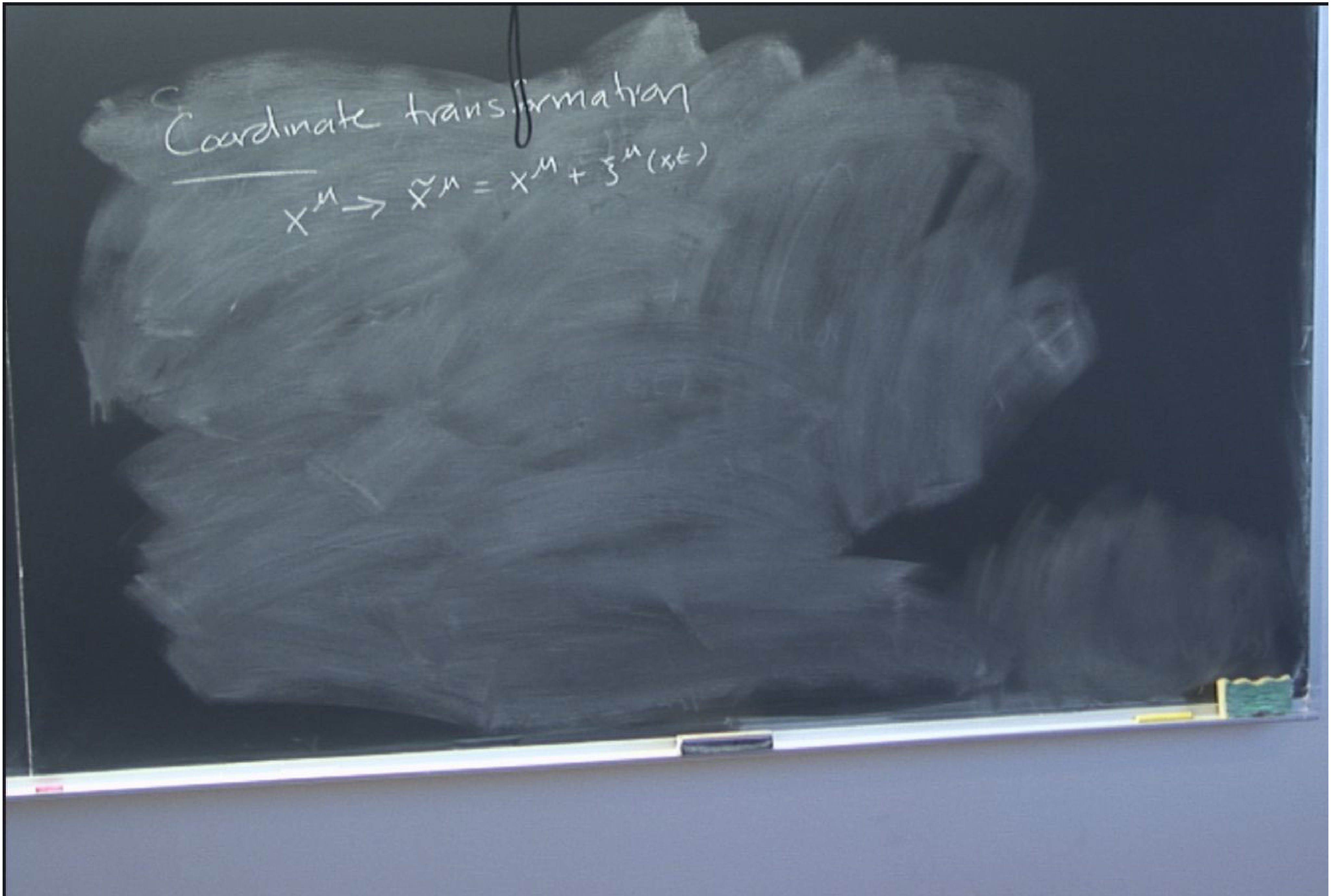


Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
$\partial_i F^i = 0$
traceless
$\partial_i h^i_j = 0$
"transverse"
(3 conditions one for each j)
$5 = 1 + 2 + (6-1-3)$ ✓
scalar vector
Scalars : Φ, Ψ, B, E
vectors : F_i, S_i
tensor : h_{ij}
Still haven't used coordinate invariance (⇒ reduce to 6)


Still deal w/ H_{ij}
$H_{ij} = 2\partial_i\partial_j E + [\partial_i F_j + \partial_j F_i] + h_{ij}$
traceless
$\partial_i F^i = 0$
$\partial_i h^i_j = 0$
$\partial_x F_x + \partial_y F_y + \partial_z F_z = 0$
"transverse"
(3 conditions one for each j)
+ 2 + (6−1−3) ✓
scalar vector
: Φ, Ψ, B, E
: F_i, S_i
h_{ij}
Still haven't used coordinate invariance (⇒ reduce to 6)

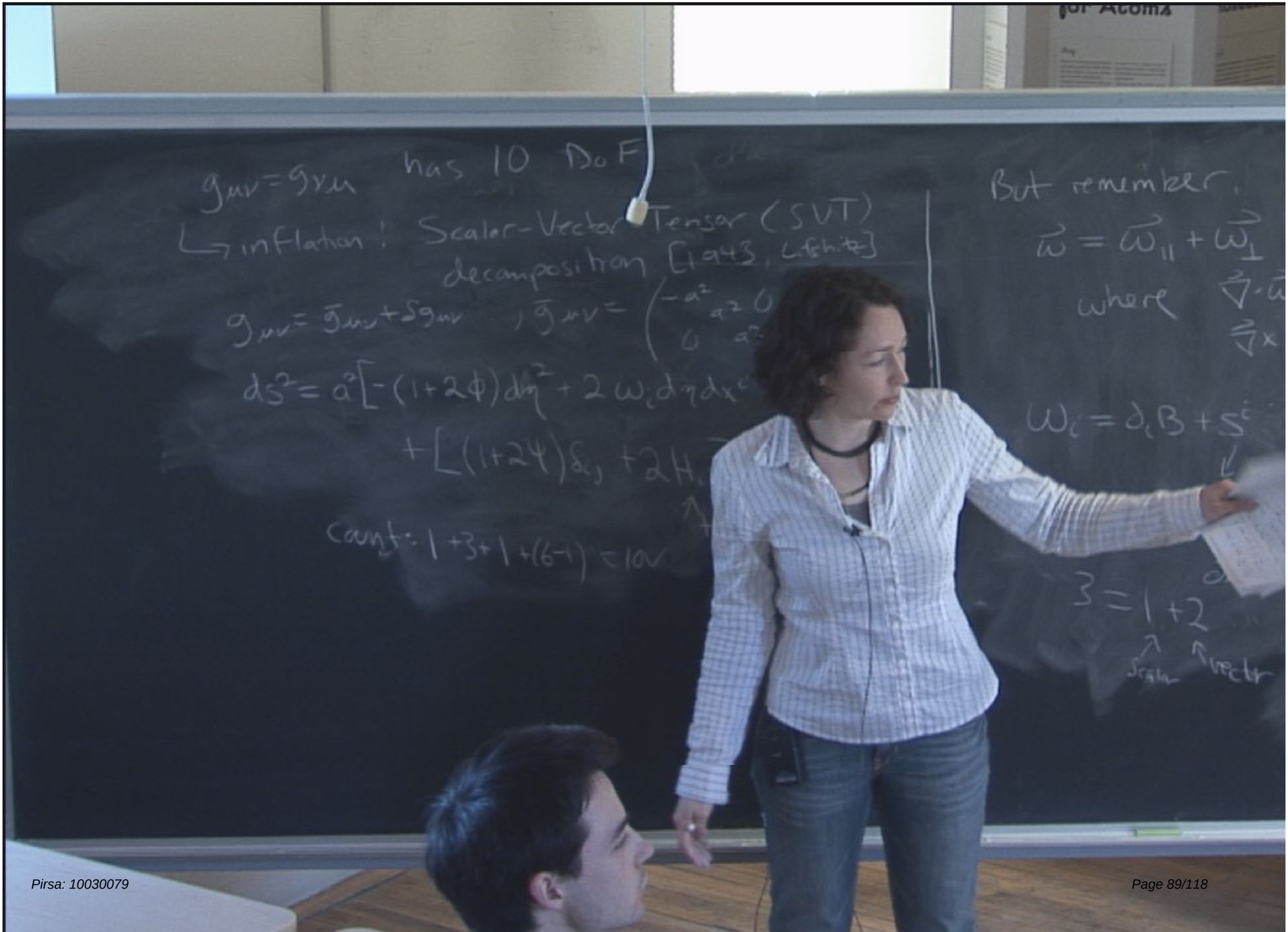

Still deal w/ Hij
Hij = 2∂i∂jE + [∂iFj+∂jFi] +hij
∂iF^i=0
∂xFx+∂yFy+∂zFz=0
traceless
∂i h^i_j = 0
"transverse"
(3 conditions one for each j)
5 = 1 + 2 + (6-1-3) ✓
scalar vector
Scalars: Φ, Ψ, B, E
vectors: Fi, Si
tensor: hij
Still haven't used coordinate invariance (⇒ reduce to 6)

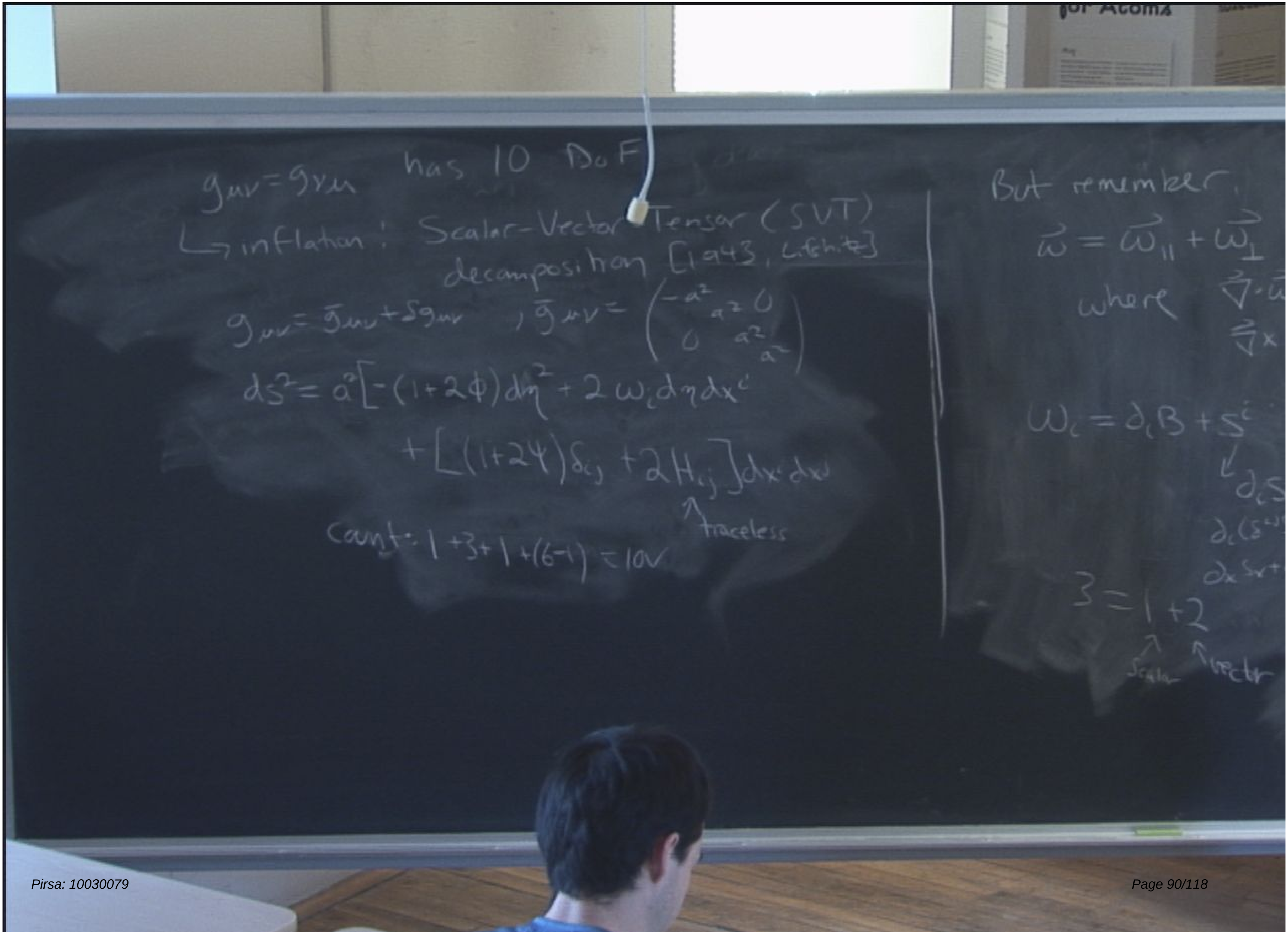

Coordinate transformation
$x^\mu \rightarrow \tilde{x}^\mu = x^\mu$


Coordinate transformation
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$

Coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

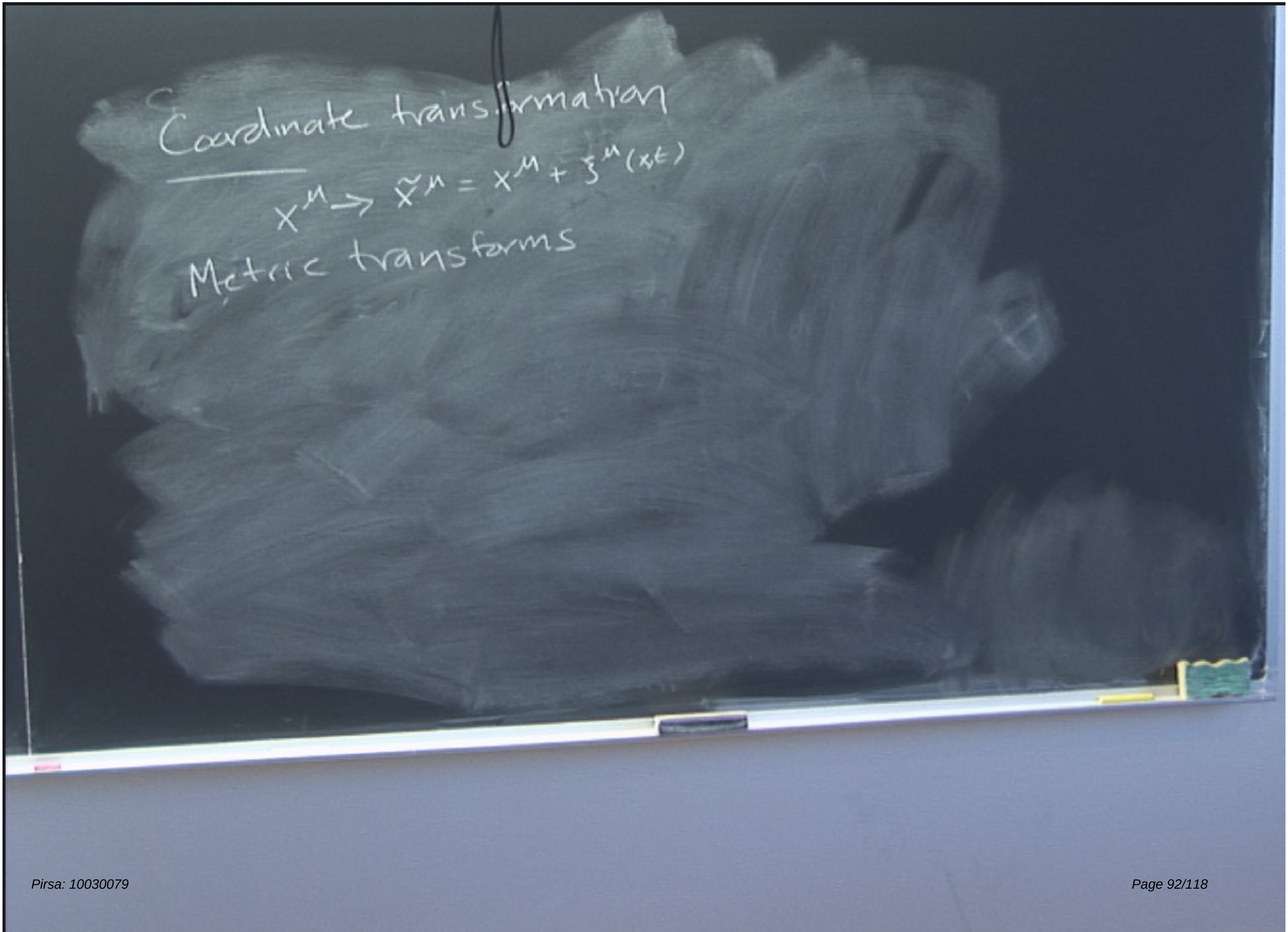

g_uv = g_vu has 10 DoF
inflation! Scalar-Vector-Tensor (SVT)
decomposition [1943, Lifshitz]
ds² = a²[-(1+2Φ)dη² + 2 ω_i dη dx^i
+ [(1+2Ψ)δ_ij + 2H_ij
count: 1+3+1+(6-1) = 10 ✓
But remember,
ω = ω_∥ + ω_⊥
where
ω_i = ∂_i B + S^i
3 = 1 + 2
scalar vector

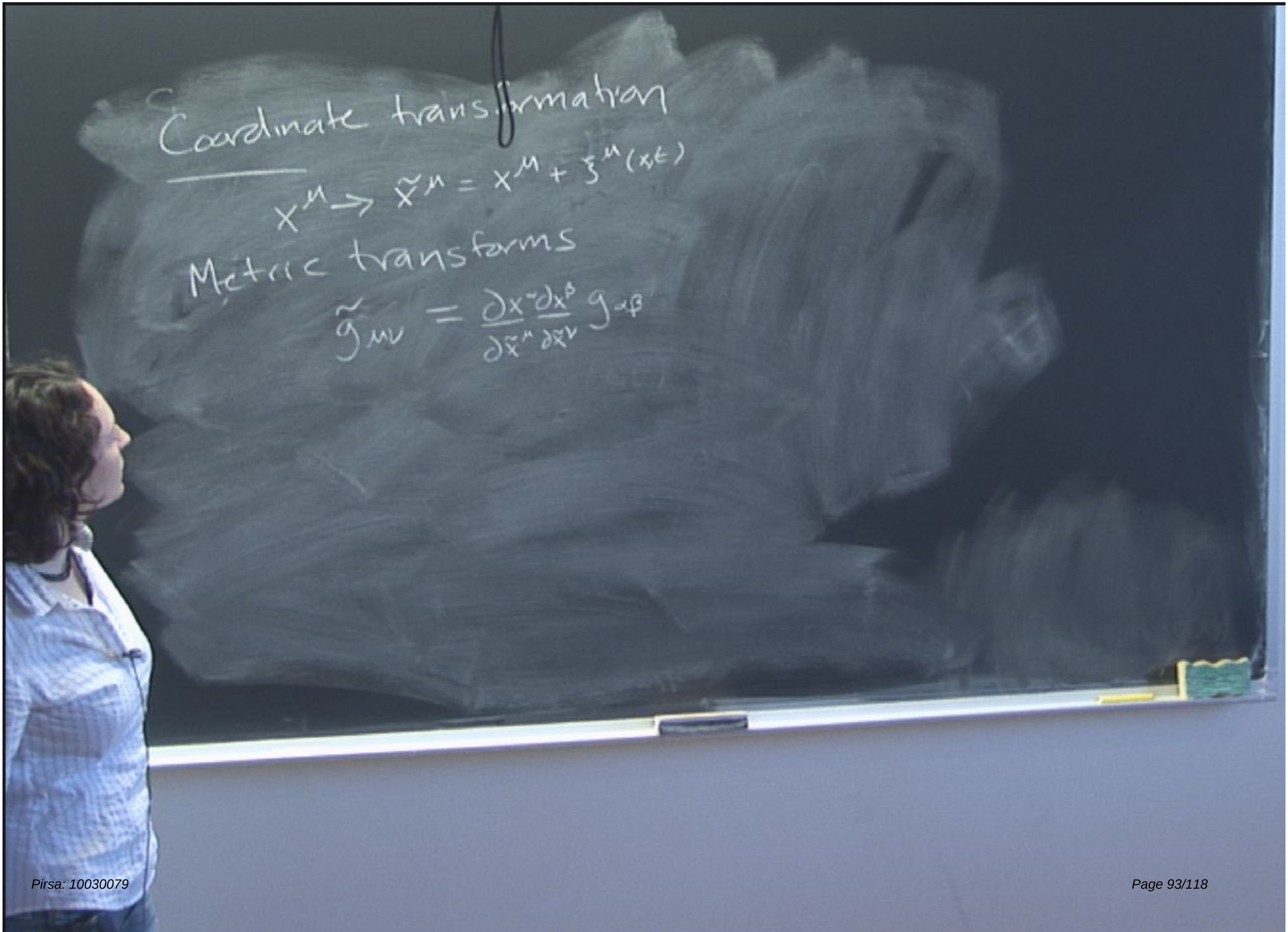

g_{μν} = g_{νμ} has 10 DoF
↳ inflation: Scalar-Vector-Tensor (SVT) decomposition [1943, Lifshitz]
g_{μν} = ḡ_{μν} + δg_{μν} , ḡ_{μν} = (−a² 0; a²; 0 a²; a²)
ds² = a²[−(1+2Φ)dη² + 2 ω_i dη dx^i
+ [(1+2Ψ)δ_{ij} + 2H_{ij}]dx^i dx^j]
traceless
count: 1+3+1+(6−1) = 10 ✓
But remember,
ω = ω_∥ + ω_⊥
where
ω_i = ∂_i B + S^i
3 = 1 + 2
scalar vector

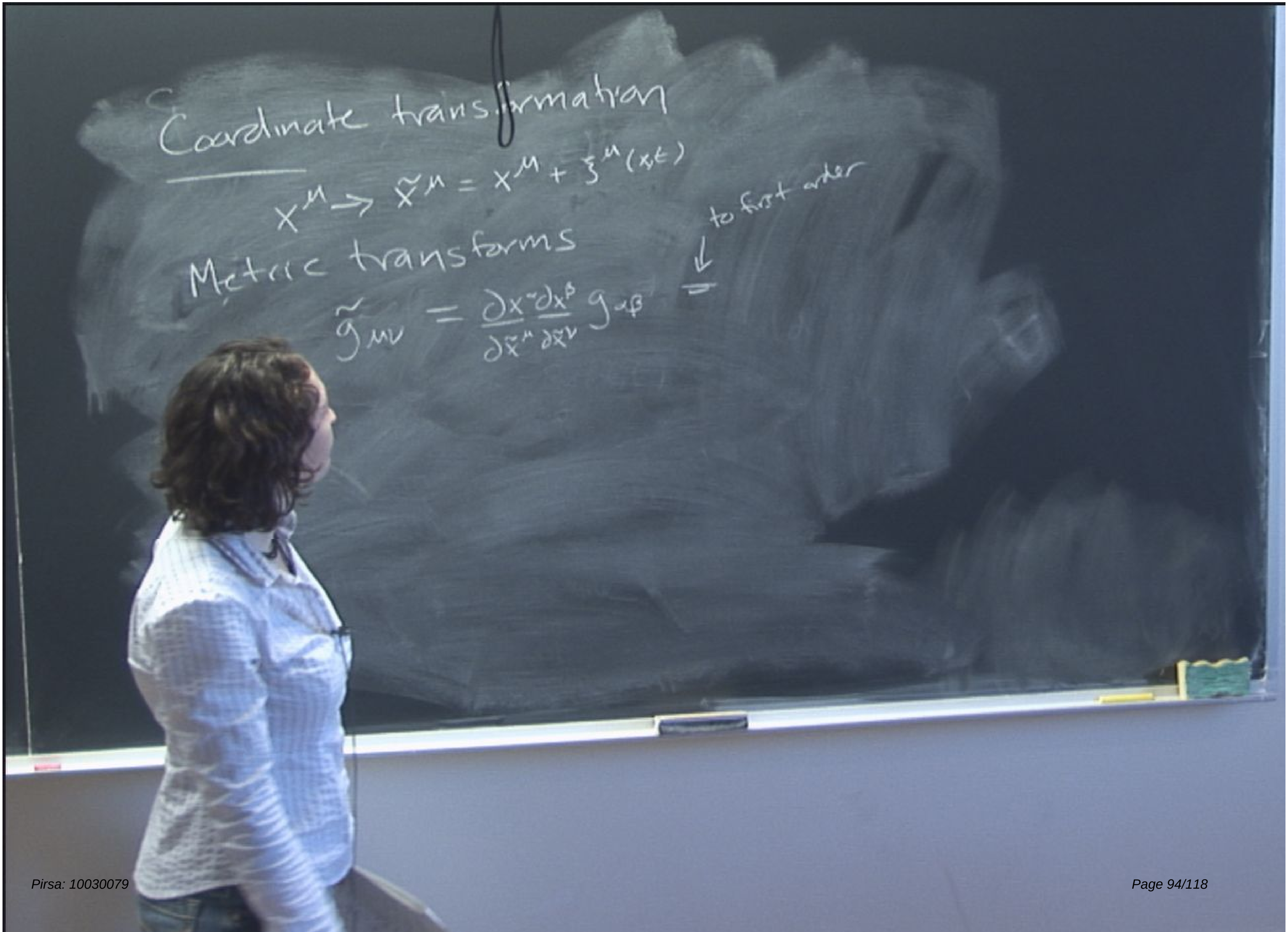
Coordinate transformation

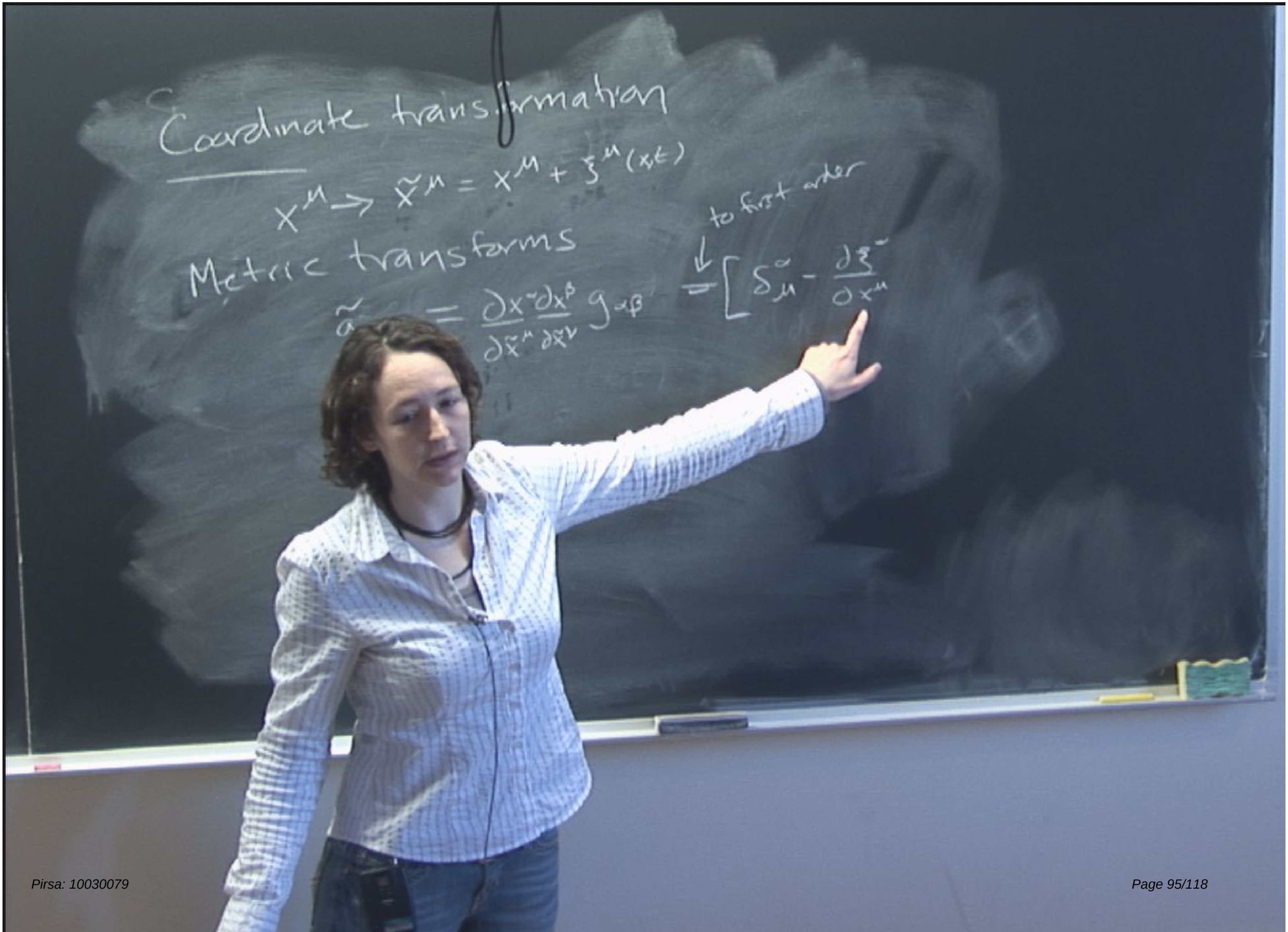
$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

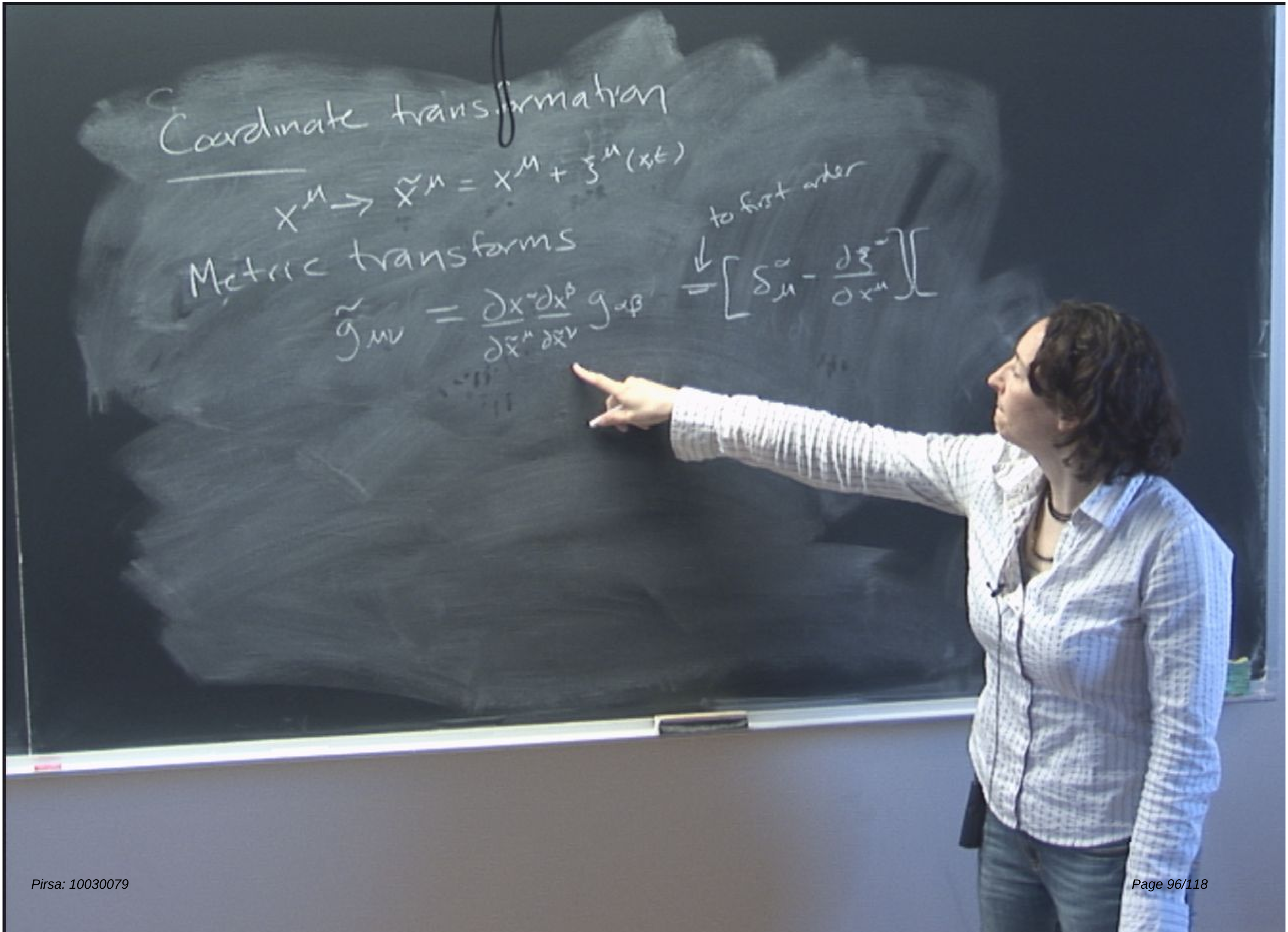
Metric tran

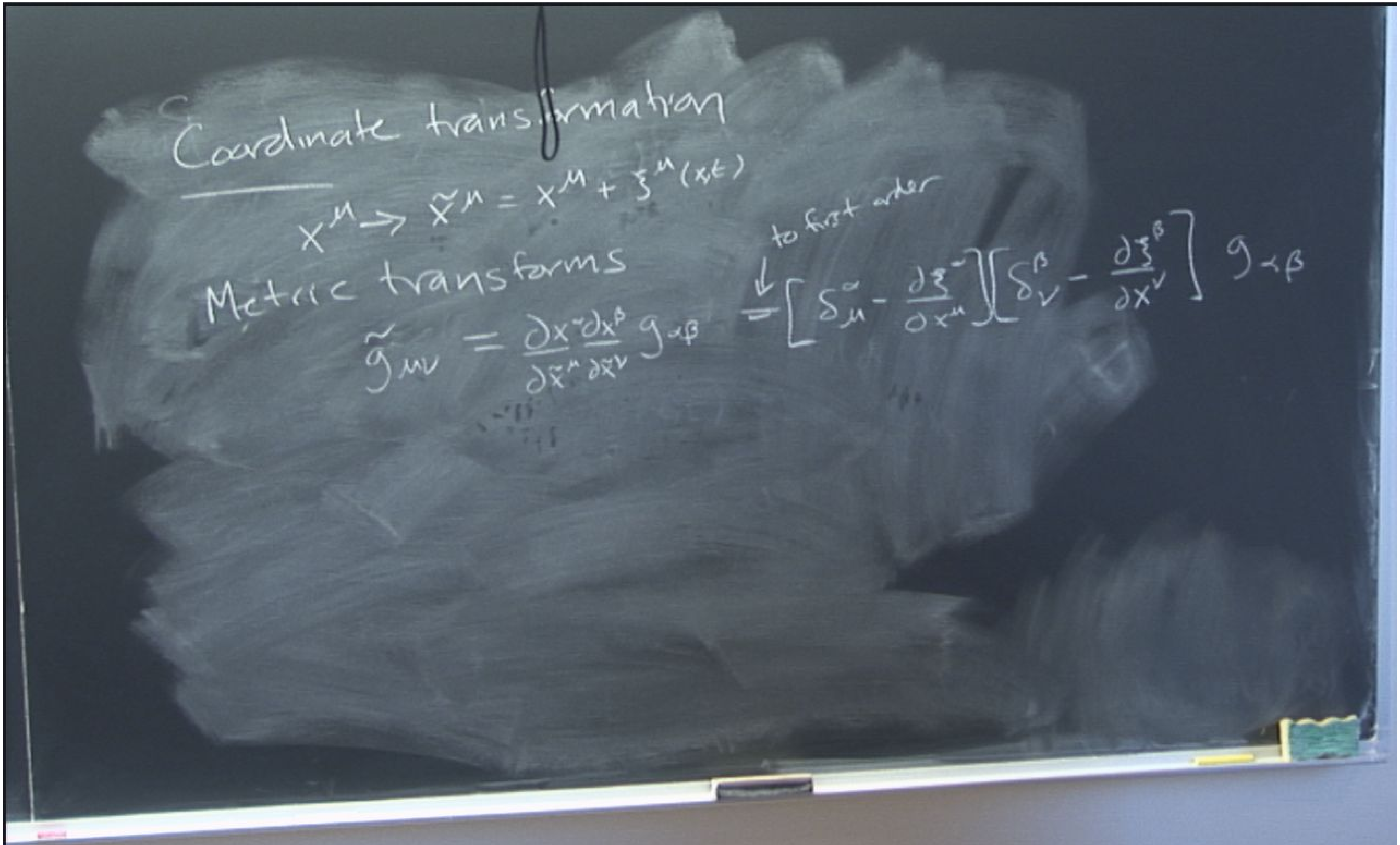

Coordinate transformation
$x^{\mu} \rightarrow \tilde{x}^{\mu} = x^{\mu} + \xi^{\mu}(x,t)$
Metric transforms

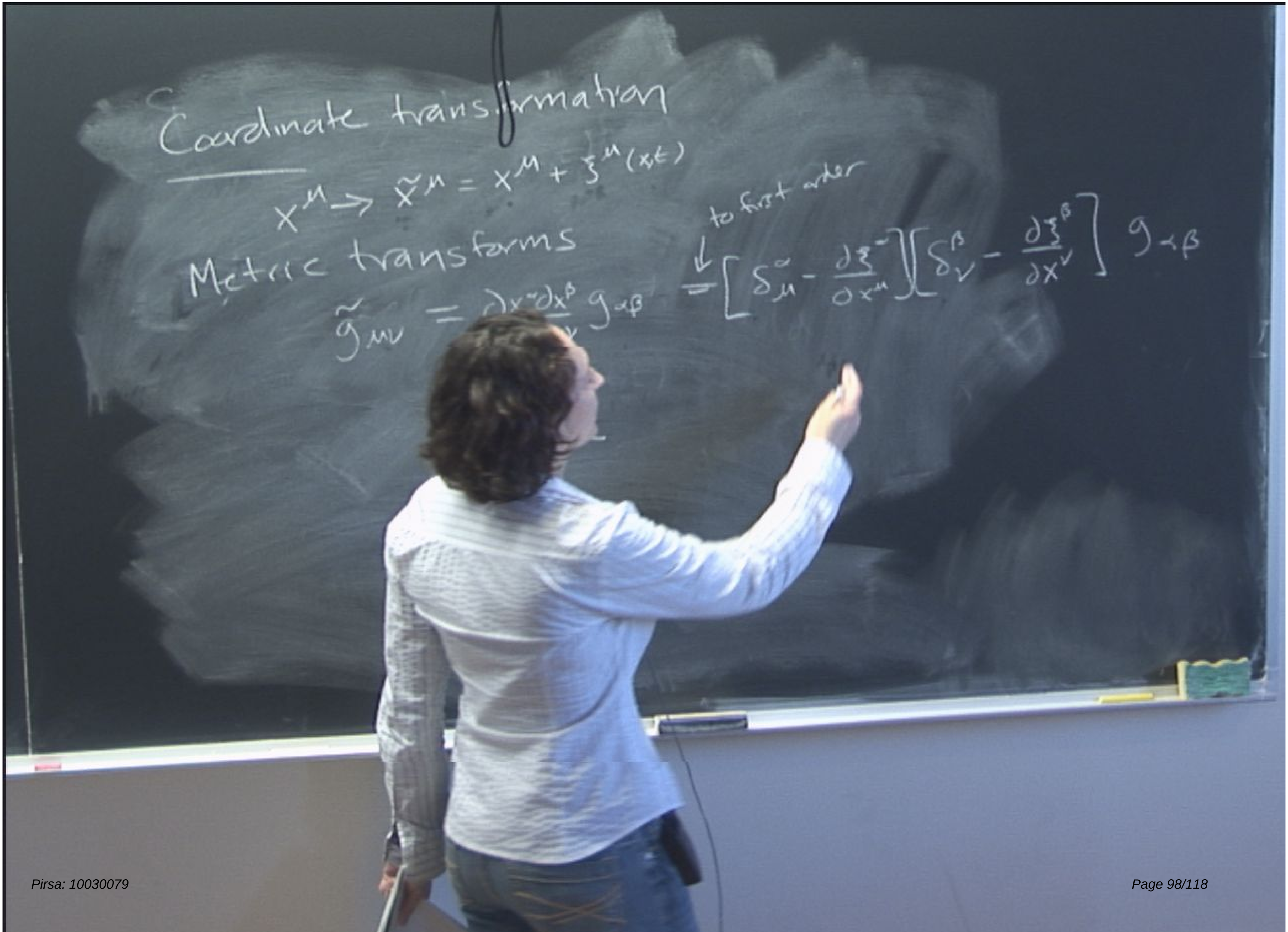

Coordinate transformation
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$
Metric transforms
$\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}$

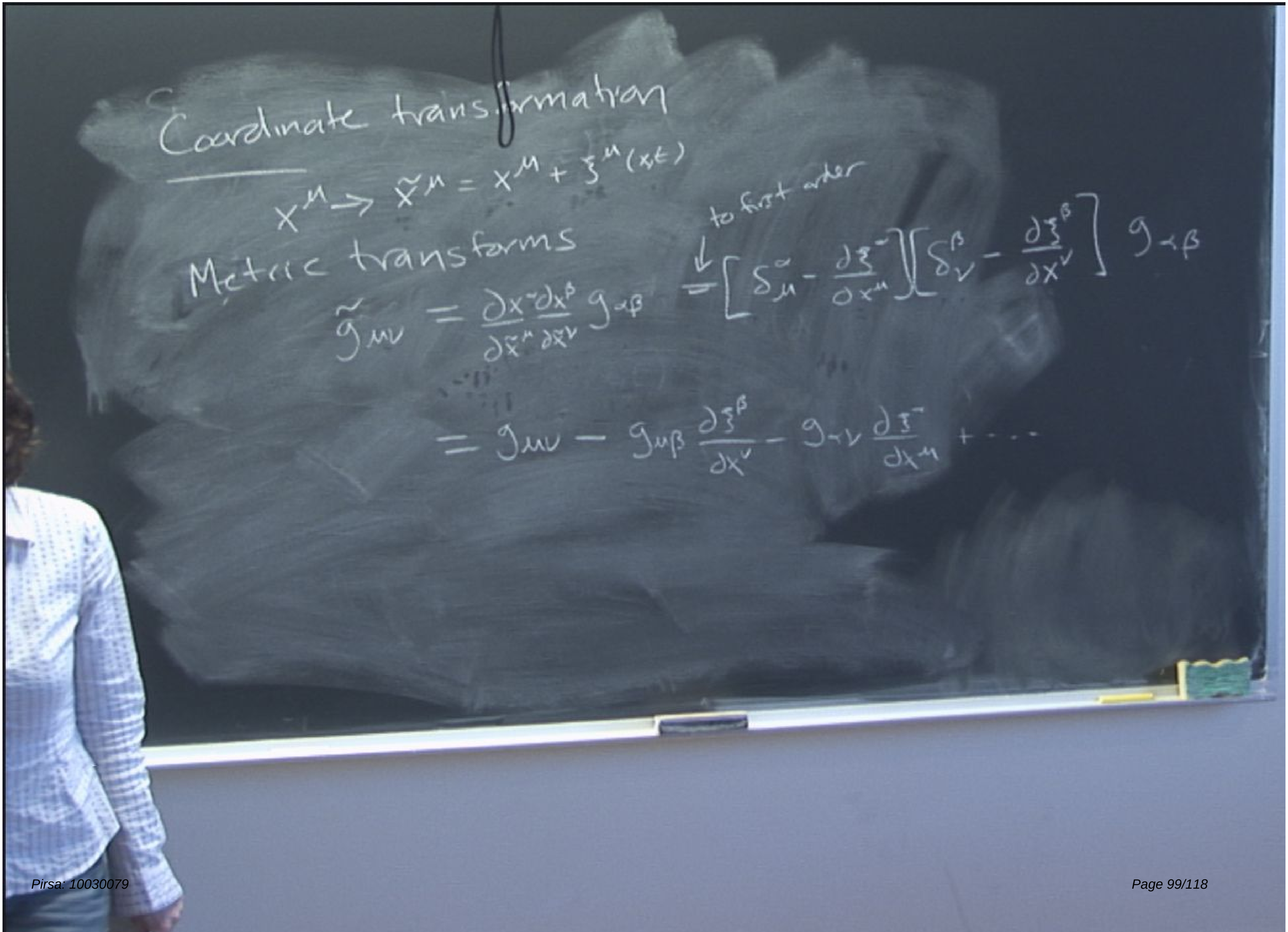

Coordinate transformation
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$
Metric transforms
$\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta} =$
to first order


Coordinate transformation
to first order
Metric transforms


Coordinate transformation
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$
Metric transforms
$\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}$
to first order
$= \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\right.$


Coordinate transformation
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$
Metric transforms
to first order
$\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta} = \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$


Coordinate transformation
x^μ → x̃^μ = x^μ + ξ^μ(x,t)
Metric transforms
g̃_μν = ∂x^α ∂x^β g_αβ
to first order
= [δ^α_μ - ∂ξ^α/∂x^μ][δ^β_ν - ∂ξ^β/∂x^ν] g_αβ


Coordinate transformation
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$
Metric transforms
to first order
$\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha \partial x^\beta}{\partial \tilde{x}^\mu \partial \tilde{x}^\nu} g_{\alpha\beta} = \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$
$= g_{\mu\nu} - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu} + \cdots$

Coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

Metric transforms

$$\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta} = \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$$

(to first order)

$$= g_{\mu\nu} - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu} + \cdots$$

$$= \left[g_{\mu\nu} + \delta g_{\mu\nu}\right]$$

Coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

Metric transforms

$$\tilde{g}_{\mu\nu}(\tilde{x},t) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta} = \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$$

to first order

$$= g_{\mu\nu} - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu} + \cdots$$

FRW in new coordinates

$$\rightarrow \bar{g}_{\mu\nu}(\tilde{x},t) + \delta\tilde{g}_{\mu\nu} = \left[\bar{g}_{\mu\nu}(x,t) + \delta g_{\mu\nu}\right] - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu}$$

Coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

Metric transforms

to first order

$$\tilde{g}_{\mu\nu}(\tilde{x},\tilde{t}) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta} = \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$$

$$= g_{\mu\nu} - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu} + \dots$$

FRW in new coordinates

$$\to \bar{g}_{\mu\nu}(\tilde{x},\tilde{t}) + \delta\tilde{g}_{\mu\nu} = \left[\bar{g}_{\mu\nu}(x,t) + \delta g_{\mu\nu}\right] - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu}$$

Coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

Metric transforms

$$\tilde{g}_{\mu\nu}(\tilde{x},t) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu}\, g_{\alpha\beta}$$

to first order

$$= \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$$

$$= g_{\mu\nu} - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu} + \dots$$

FRW in new coordinates

$$\to \bar{g}_{\mu\nu}(\tilde{x},\tilde{t}) + \delta\tilde{g}_{\mu\nu} = \left[\bar{g}_{\mu\nu}(x,t) + \delta g_{\mu\nu}\right] - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu}$$

$$\downarrow$$

$$\bar{g}_{\mu\nu}(x,t) + \partial_\lambda(\bar{g}_{\mu\nu})\,\xi^\lambda$$

Coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x,t)$$

Metric transforms

$$\tilde{g}_{\mu\nu}(\tilde{x},\tilde{t}) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta} \overset{\text{to first order}}{=} \left[\delta^\alpha_\mu - \frac{\partial \xi^\alpha}{\partial x^\mu}\right]\left[\delta^\beta_\nu - \frac{\partial \xi^\beta}{\partial x^\nu}\right] g_{\alpha\beta}$$

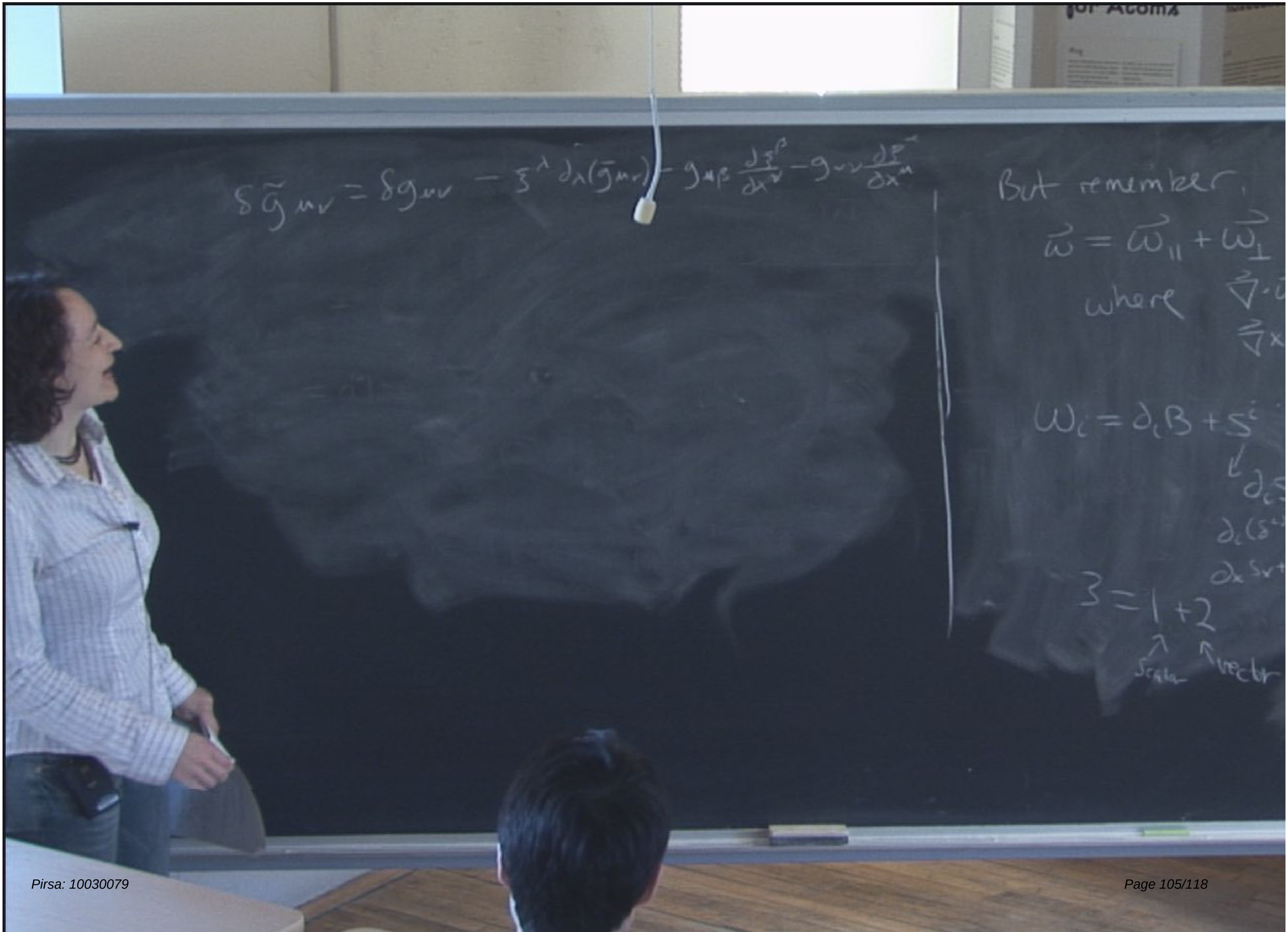
$$= g_{\mu\nu} - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu} + \cdots$$

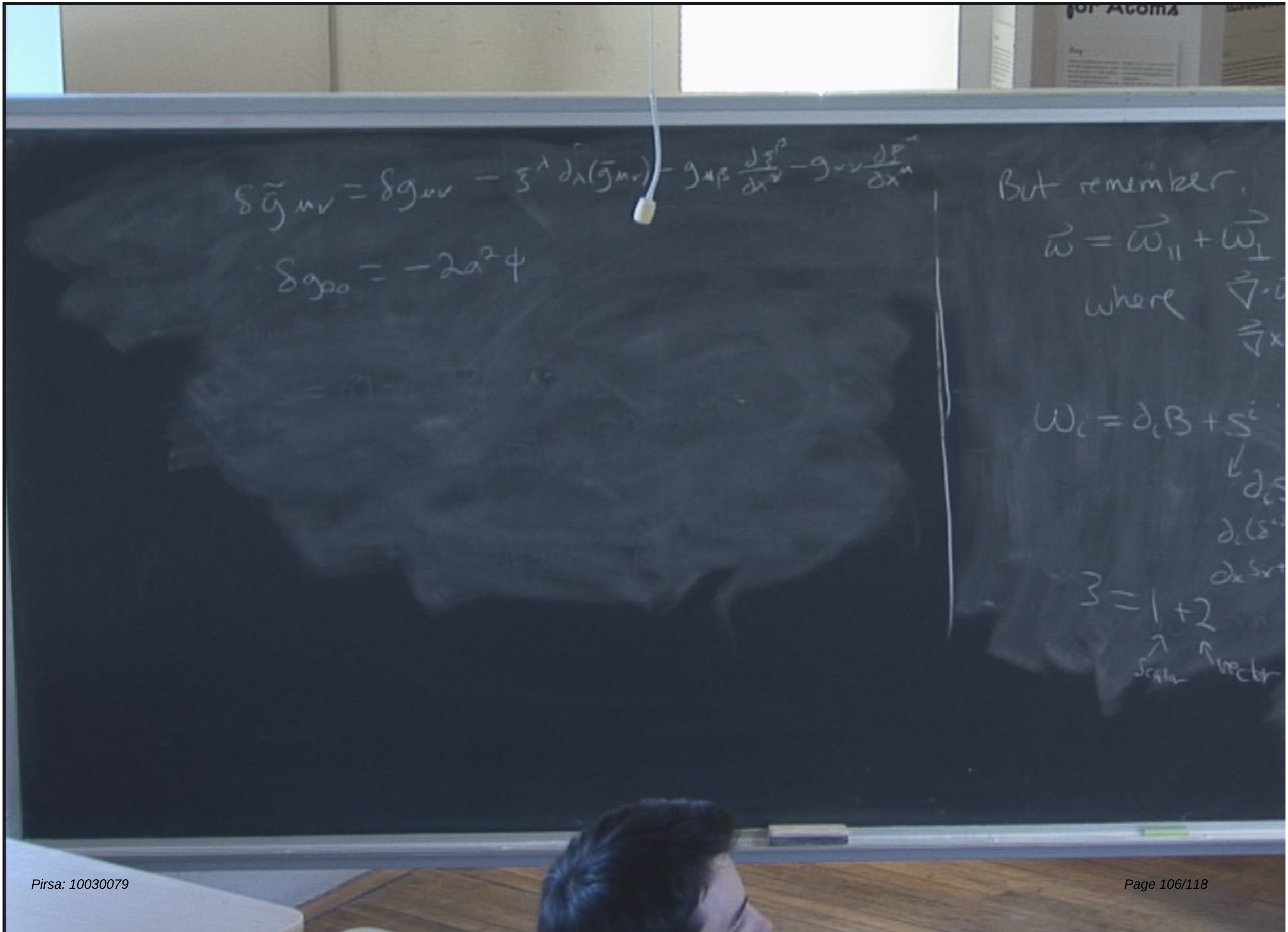
FRW in new coordinates

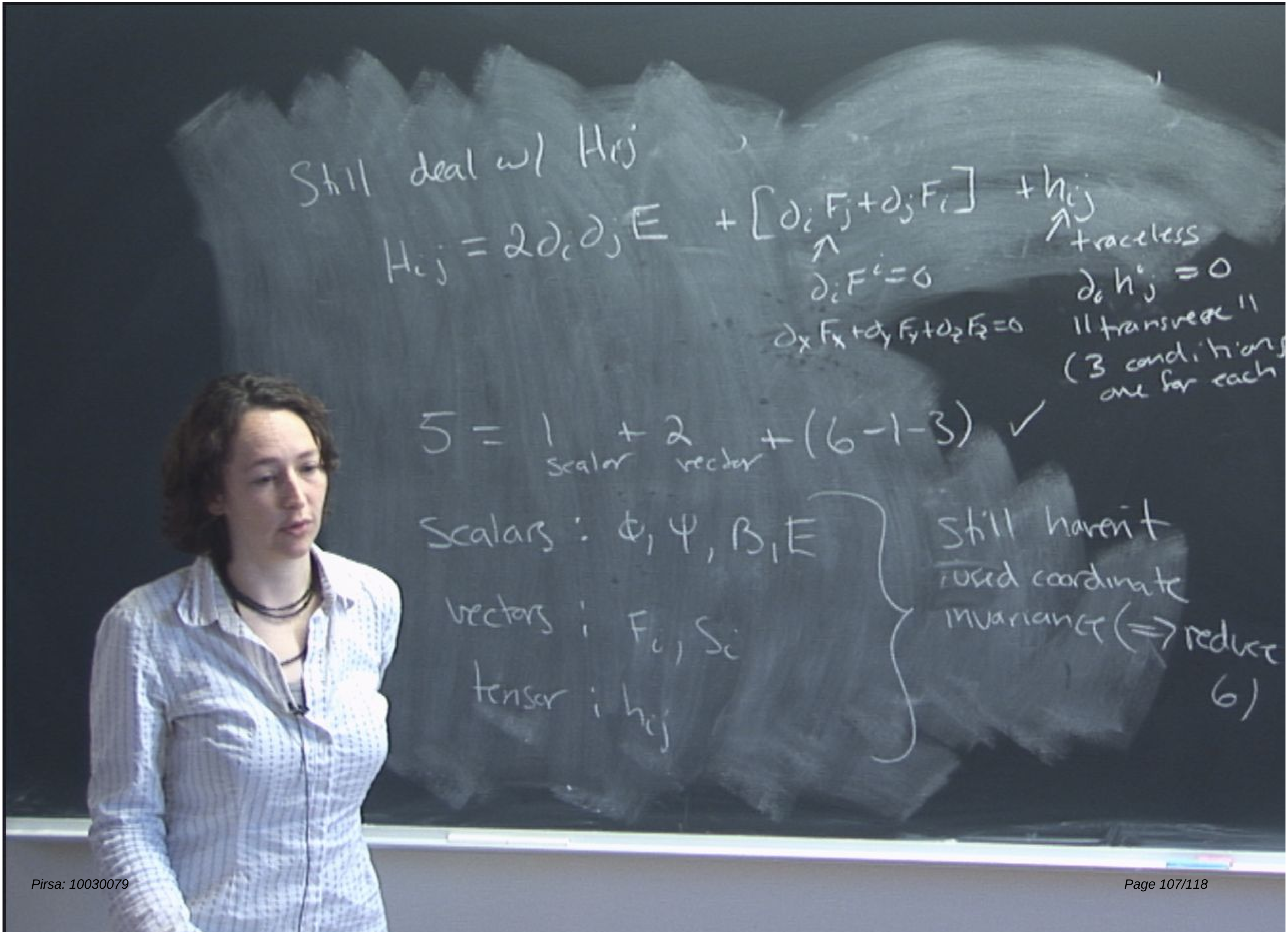
$$\to \bar{g}_{\mu\nu}(\tilde{x},\tilde{t}) + \delta\tilde{g}_{\mu\nu} = \left[\bar{g}_{\mu\nu}(x,t) + \delta g_{\mu\nu}\right] - g_{\mu\beta}\frac{\partial \xi^\beta}{\partial x^\nu} - g_{\alpha\nu}\frac{\partial \xi^\alpha}{\partial x^\mu}$$

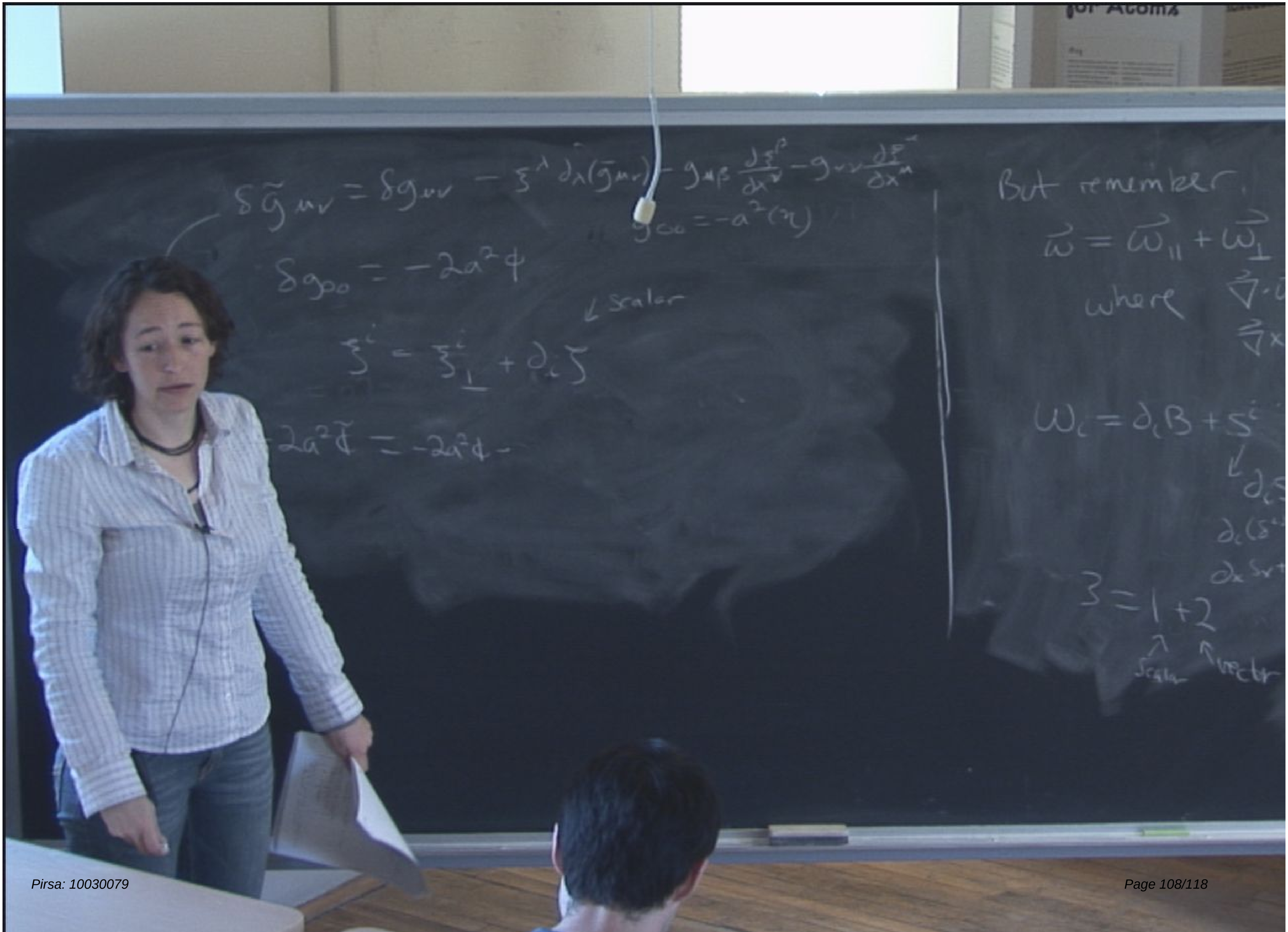
$$\downarrow$$

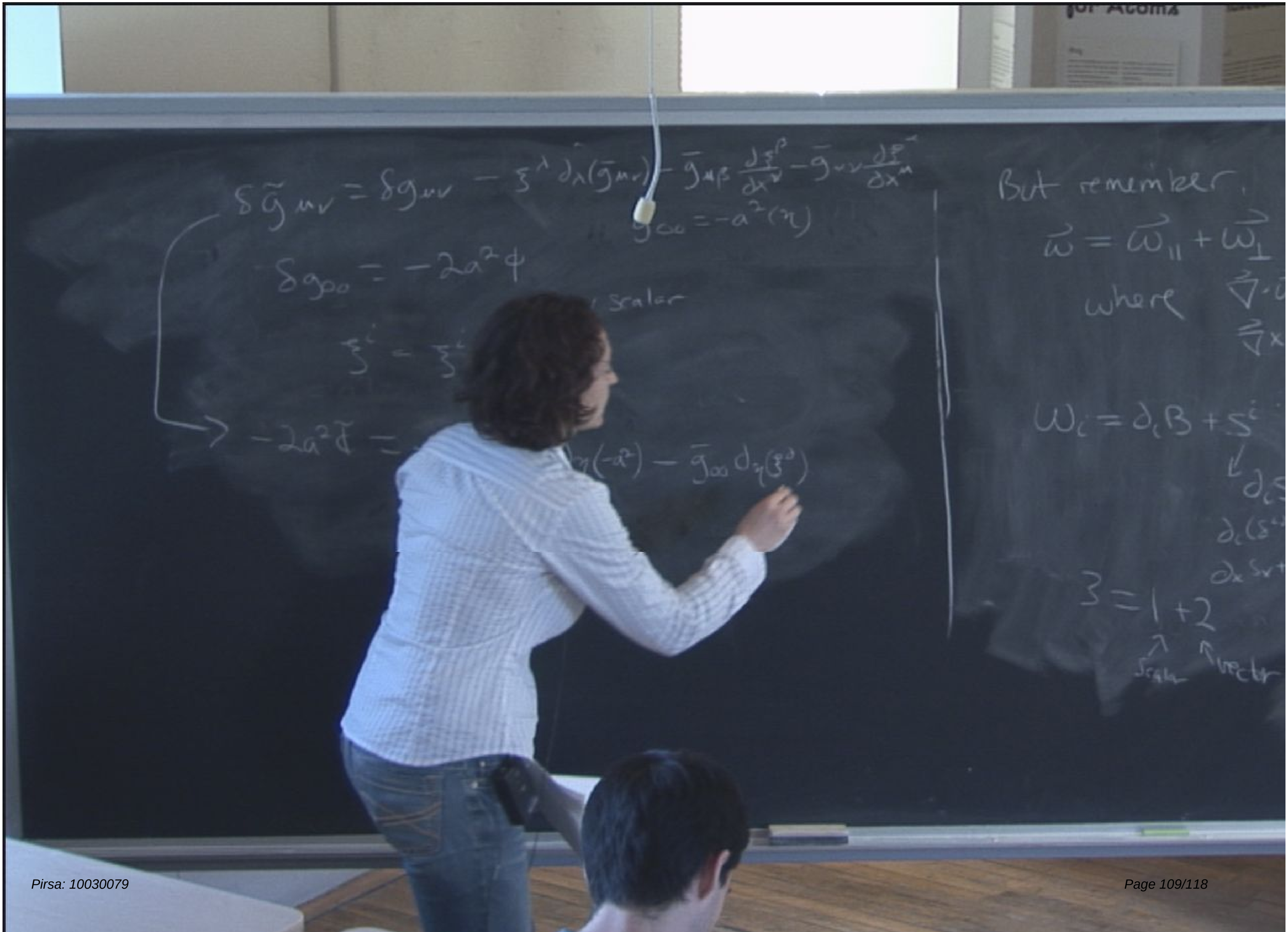
$$\bar{g}_{\mu\nu}(x,t) + \partial_\lambda(\bar{g}_{\mu\nu})\xi^\lambda + \delta\tilde{g}_{\mu\nu} =$$

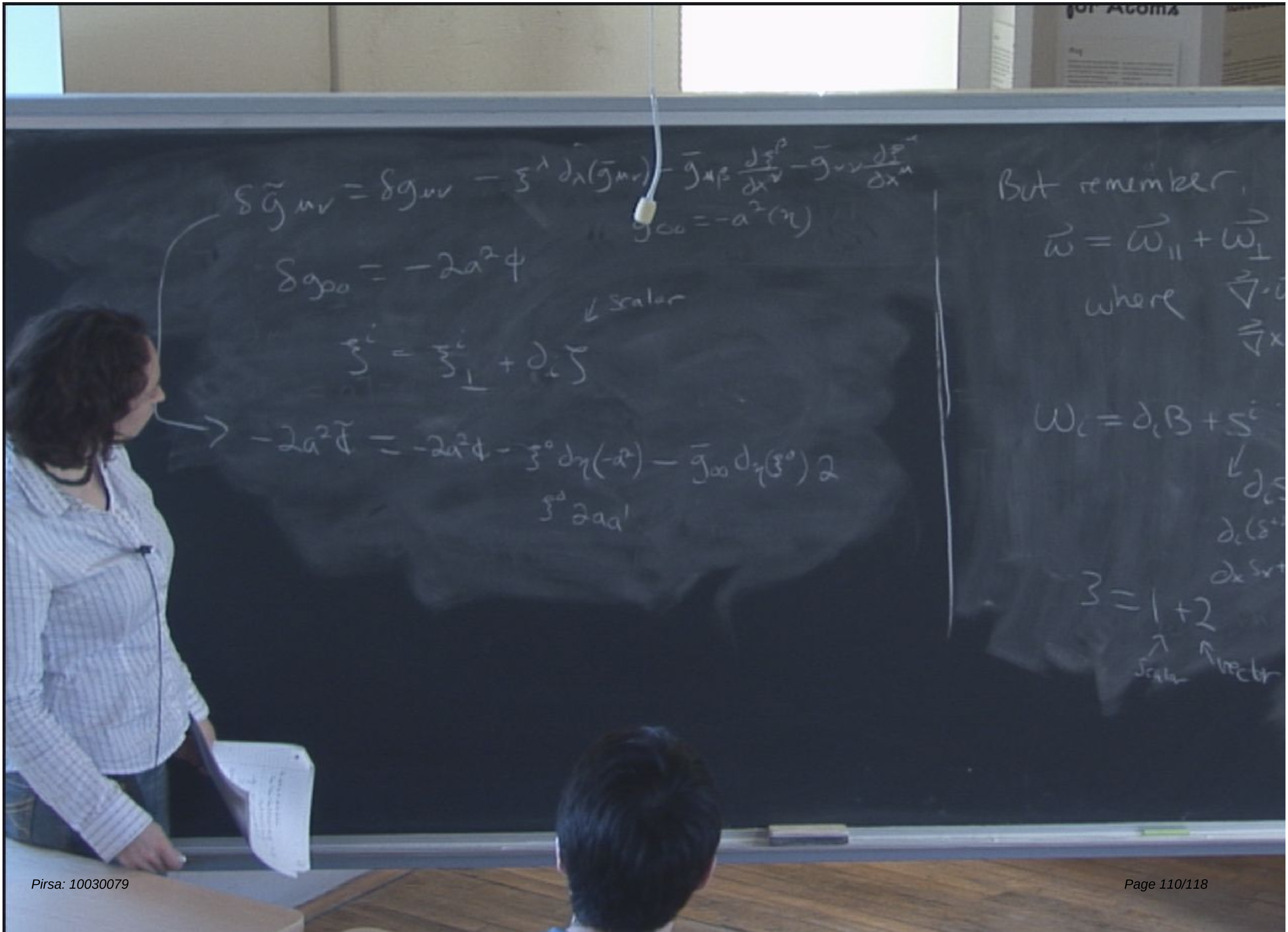

But remember,
where
3 = 1 + 2
scalar
vector

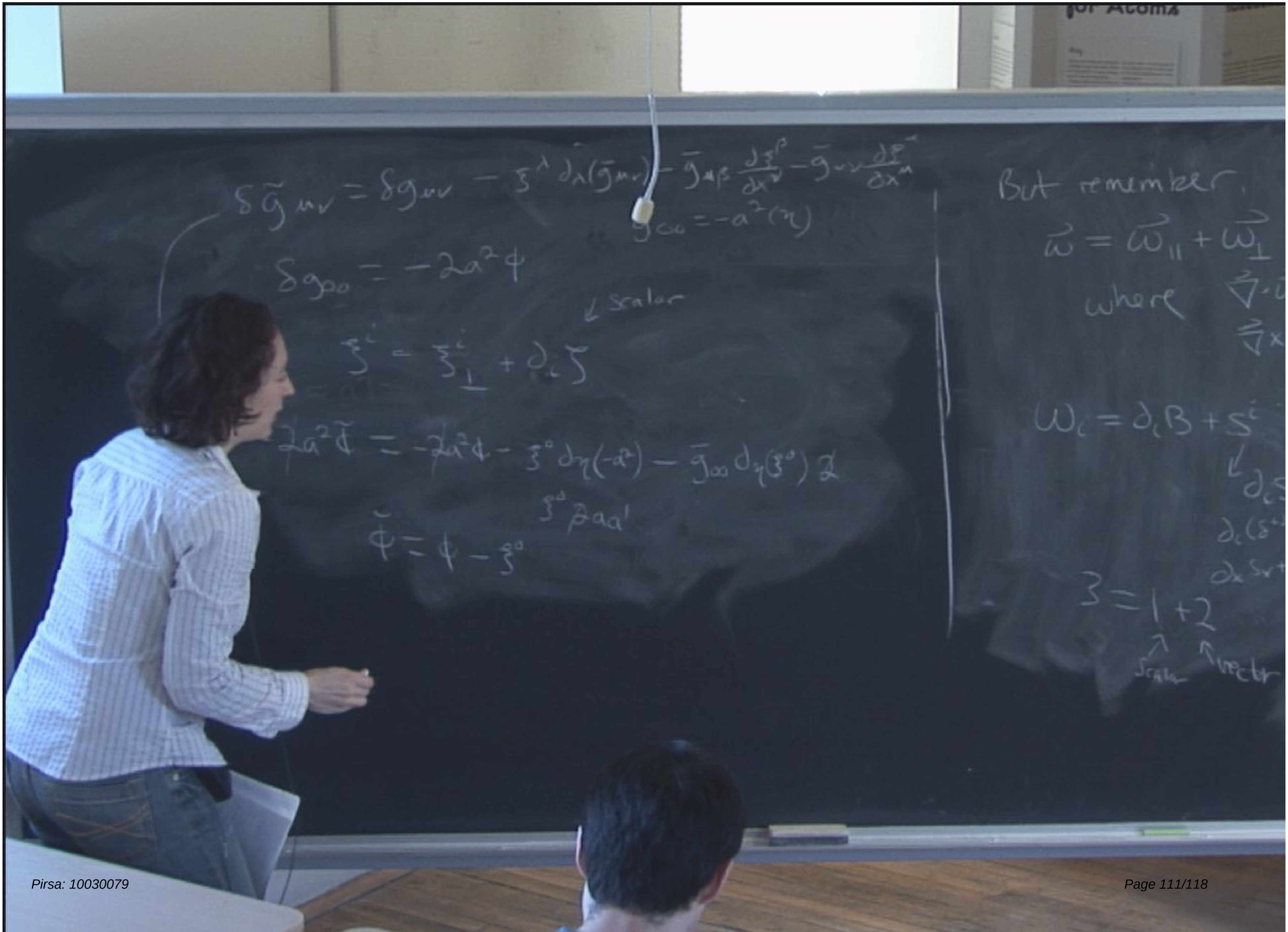

But remember,
where
3 = 1 + 2
scalar
vector

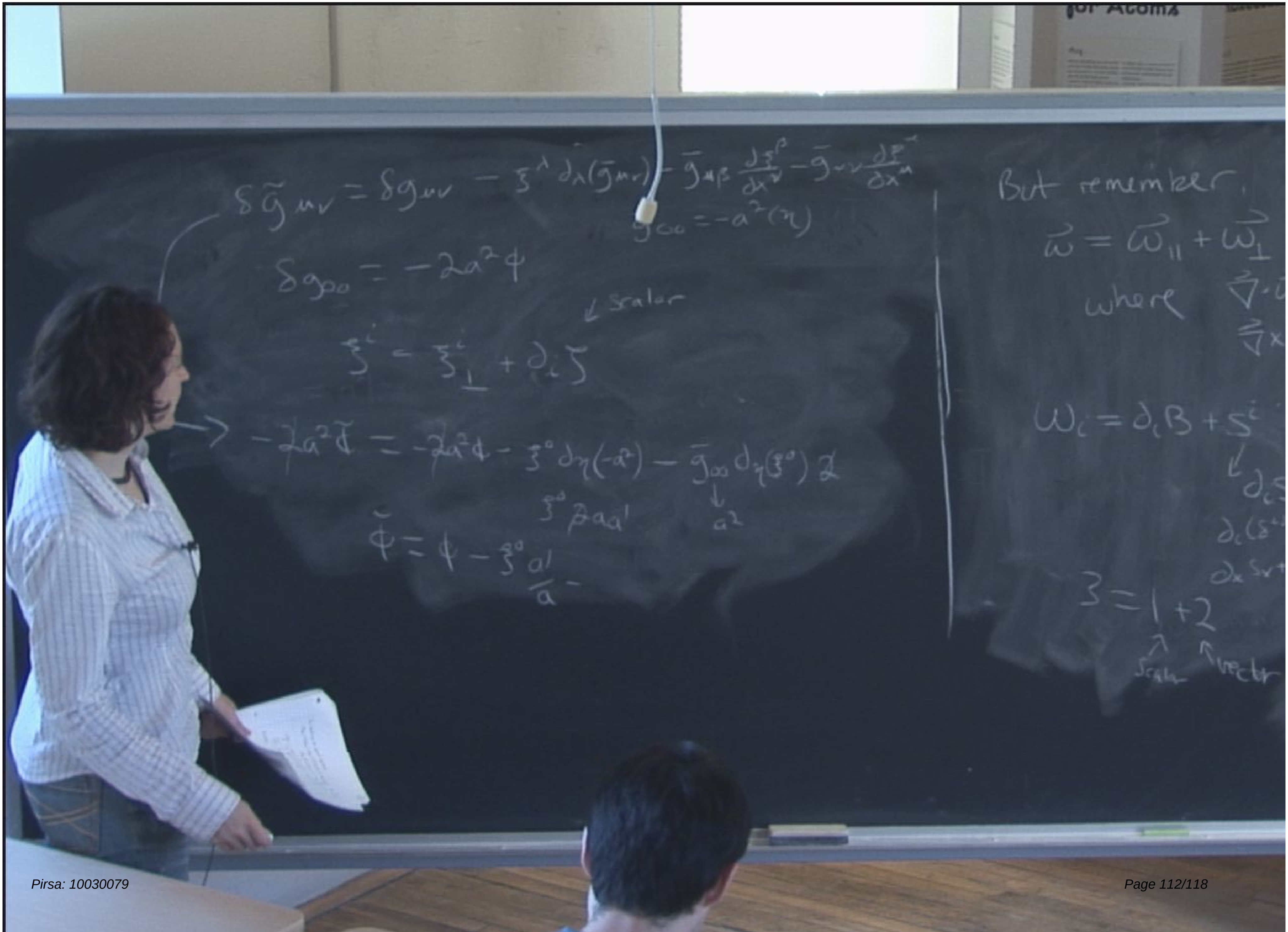

Still deal w/ Hij
Hij = 2∂i∂j E + [∂i Fj + ∂j Fi] + hij
∂i F^i = 0
∂x Fx + ∂y Fy + ∂z Fz = 0
traceless
∂i h^i_j = 0
"transverse"
(3 conditions
one for each
5 = 1 + 2 + (6-1-3) ✓
scalar
vector
Scalars : Φ, Ψ, B, E
vectors : Fi, Si
tensor : hij
Still haven't
used coordinate
invariance (⇒ reduce
6)

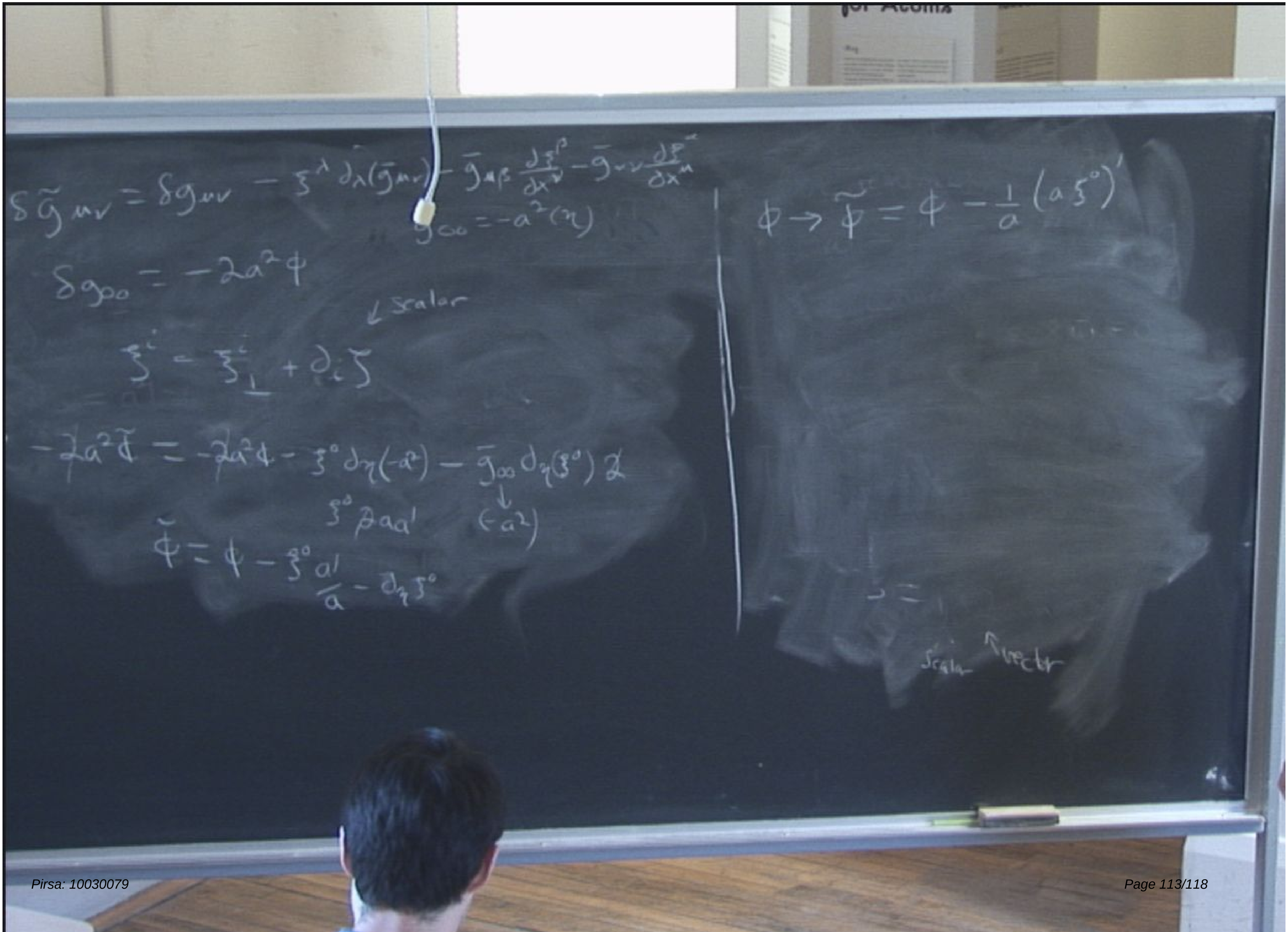

But remember,
scalar
vector
3 = 1 + 2

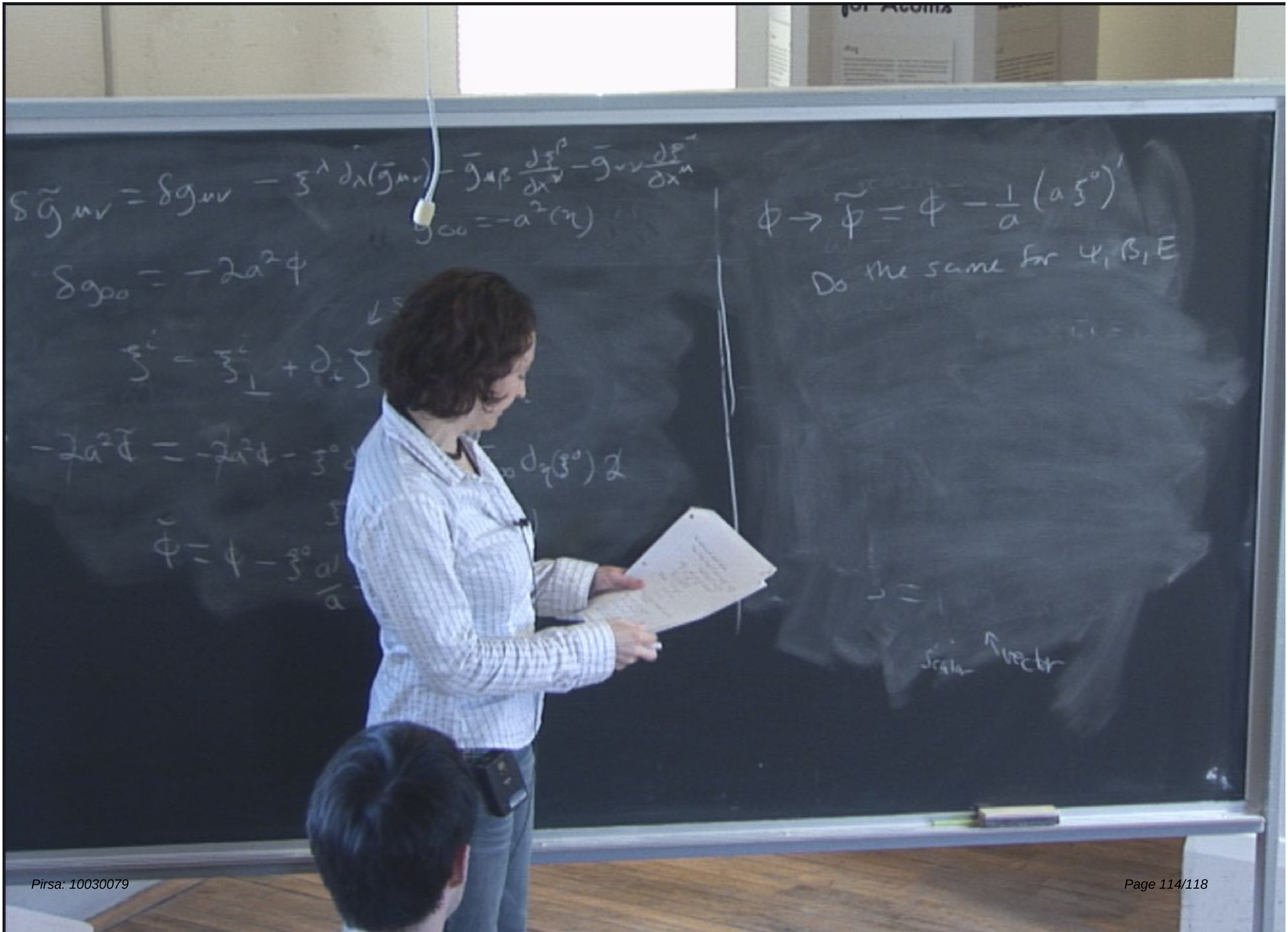

But remember
where
scalar
3 = 1 + 2
scalar
vector

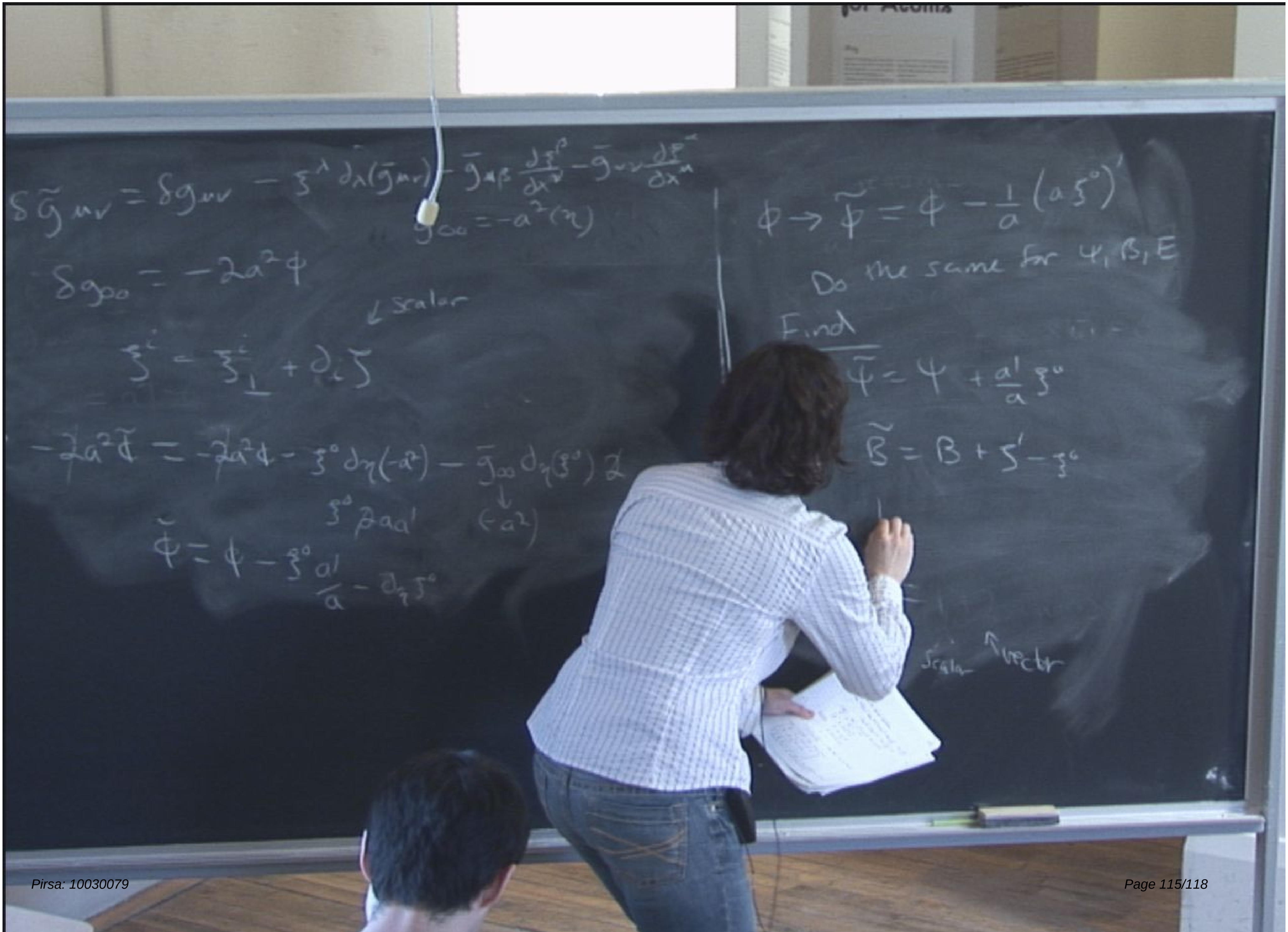

But remember,
where
scalar
scalar
vector

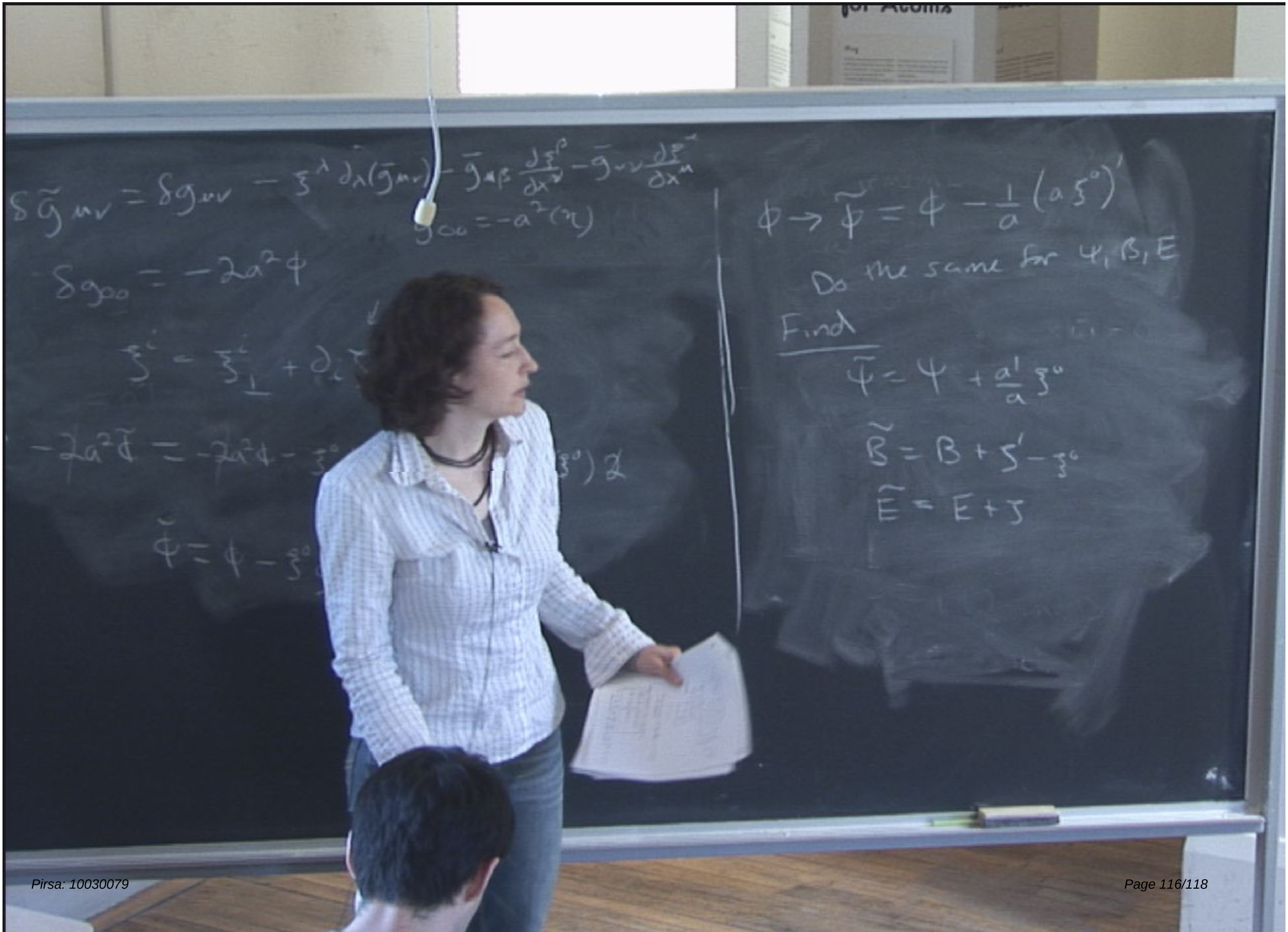

But remember
where
scalar
scalar
vector
3 = 1 + 2

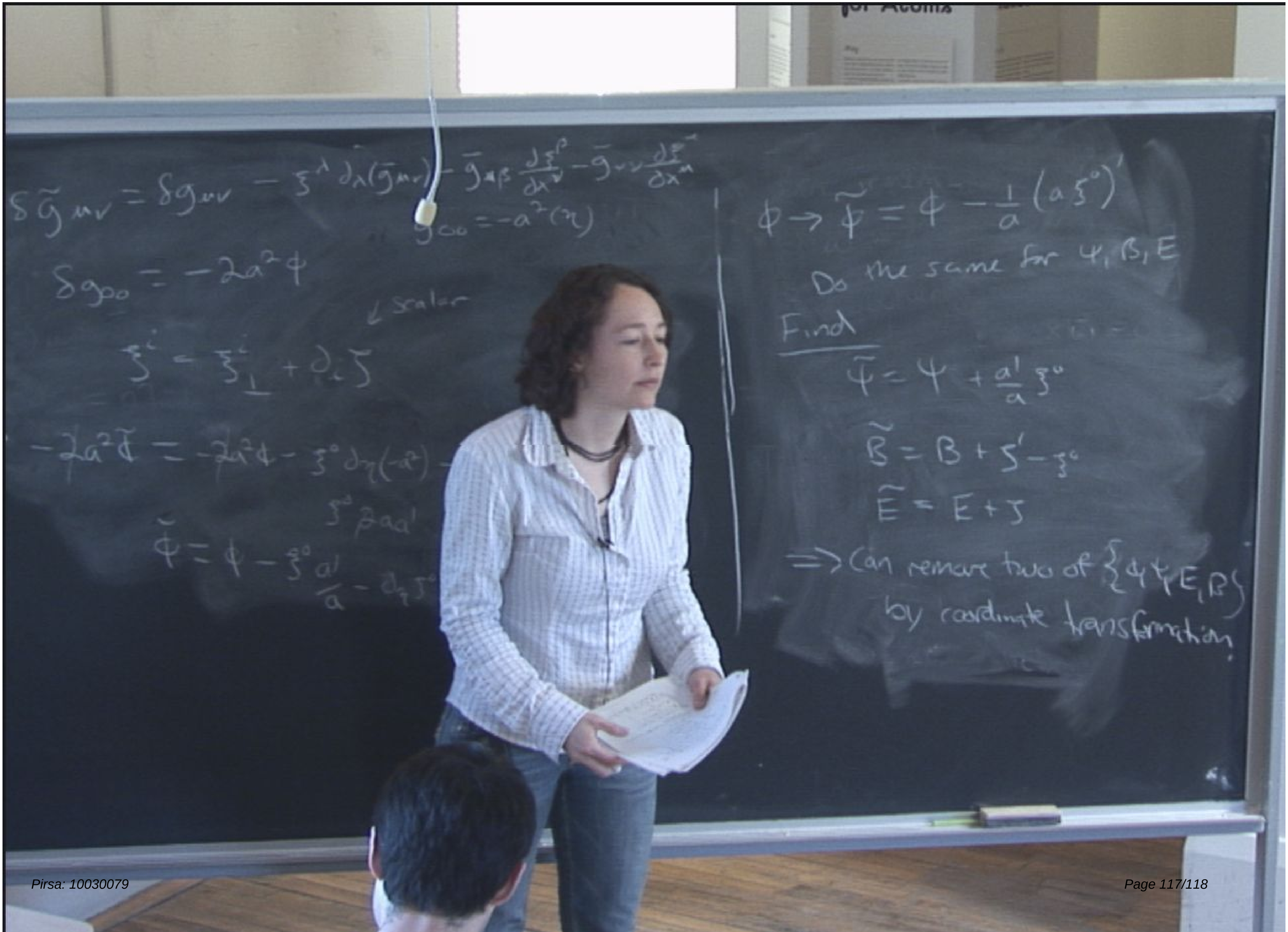

But remember
where
scalar
vector

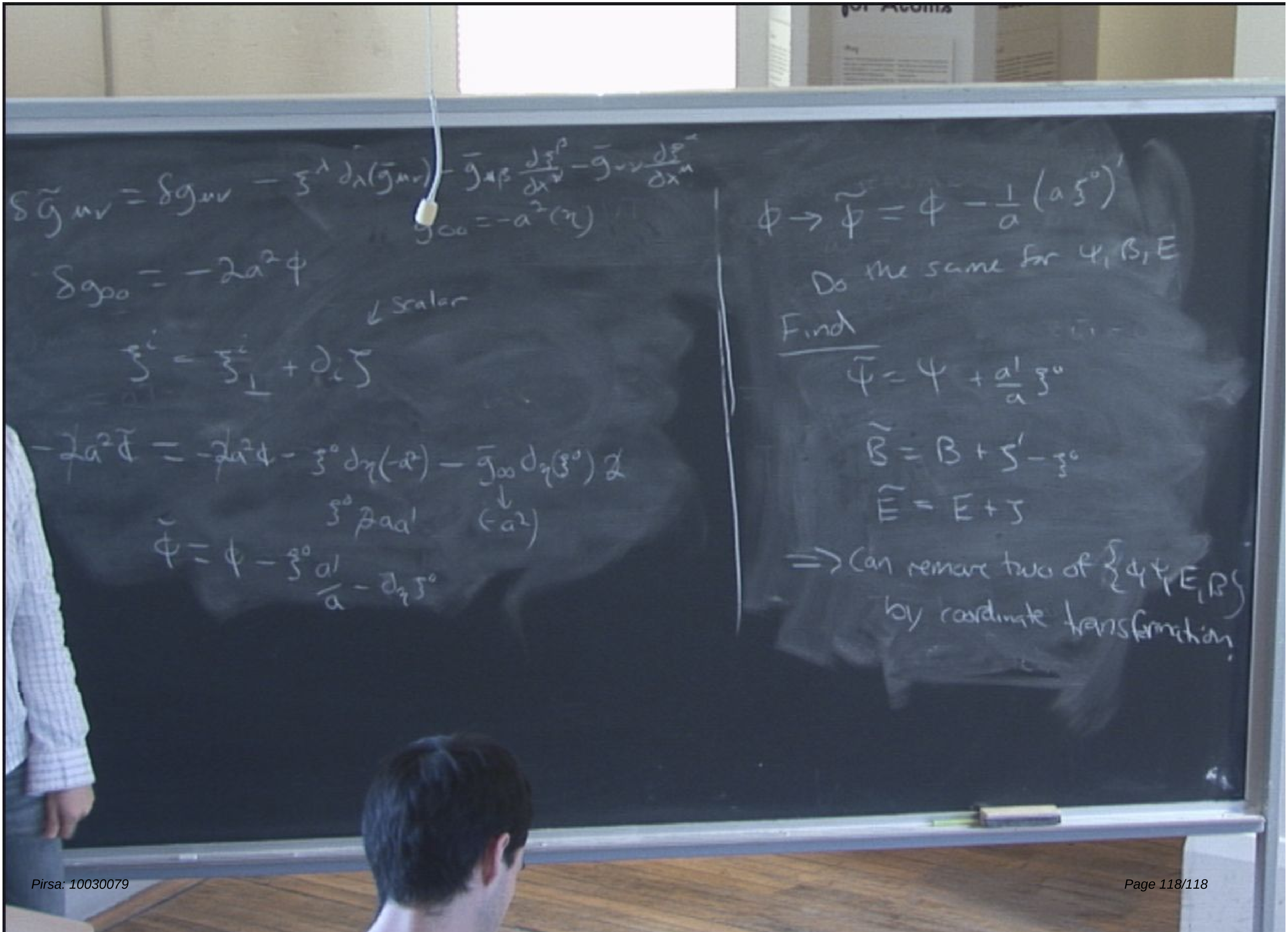

δg̃μν = δgμν − ξ^λ ∂λ(ḡμν) − ḡμβ dξ^β/dx^ν − ḡαν dξ^α/dx^μ
ḡ00 = −a²(η)
δg00 = −2a²φ
scalar
ξ^i = ξ^i⊥ + ∂i ζ
−2a²φ̃ = −2a²φ − ξ^0 ∂η(−a²) − ḡ00 ∂η(ξ^0) 2
ξ^0 2aa′
(−a²)
φ̃ = φ − ξ^0 a′/a − ∂η ξ^0
φ → φ̃ = φ − 1/a (a ξ^0)′
scalar
vector


δg̃μν = δgμν − ξ^λ ∂λ(ḡμν) − ḡαβ dξ^β/dx^ν − ḡαν dξ^α/dx^μ
ḡ00 = −a²(η)
δg00 = −2a²φ
φ → φ̃ = φ − 1/a (a ξ^0)′
Do the same for Ψ, B, E
scalar
vector


Do the same for Ψ, B, E
Find
scalar
vector


$\delta g_{00} = -2a^2\phi$
$g_{00} = -a^2(\eta)$
$\phi \to \tilde{\phi} = \phi - \frac{1}{a}(a\xi^0)'$
Do the same for Ψ, B, E
Find
$\tilde{\Psi} = \Psi + \frac{a'}{a}\xi^0$
$\tilde{B} = B + \xi' - \xi^0$
$\tilde{E} = E + \xi$


scalar
Do the same for Ψ, B, E
Find
⇒ Can remove two of {Φ, Ψ, E, B}
by coordinate transformation


δg̃μν = δgμν − ξ^λ ∂λ(ḡμν) − ḡαβ ∂ξ^β/∂x^ν − ḡαν ∂ξ^α/∂x^μ
ḡ00 = −a²(η)
δg00 = −2a²φ
↙ scalar
ξ^i = ξ^i_⊥ + ∂_i ζ
−2a²φ̃ = −2a²φ − ξ⁰∂η(−a²) − ḡ00 ∂η(ξ⁰) 2
ξ⁰ 2aa′
(−a²)
φ̃ = φ − ξ⁰ a′/a − ∂η ξ⁰
φ → φ̃ = φ − (1/a)(a ξ⁰)′
Do the same for ψ, B, E
Find
ψ̃ = ψ + (a′/a) ξ⁰
B̃ = B + ζ′ − ξ⁰
Ẽ = E + ζ
⇒ Can remove two of {φ, ψ, E, B}
by coordinate transformation