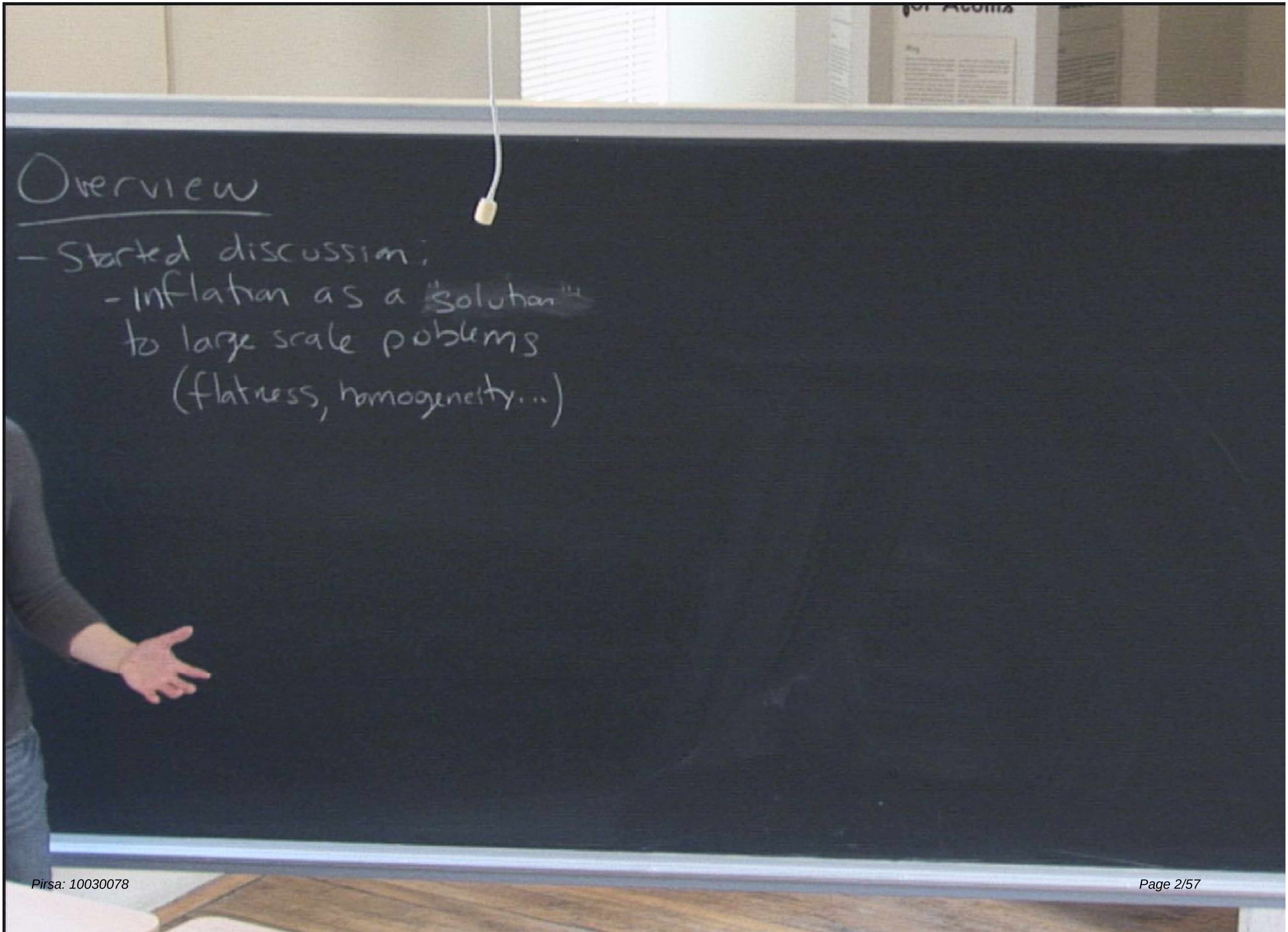
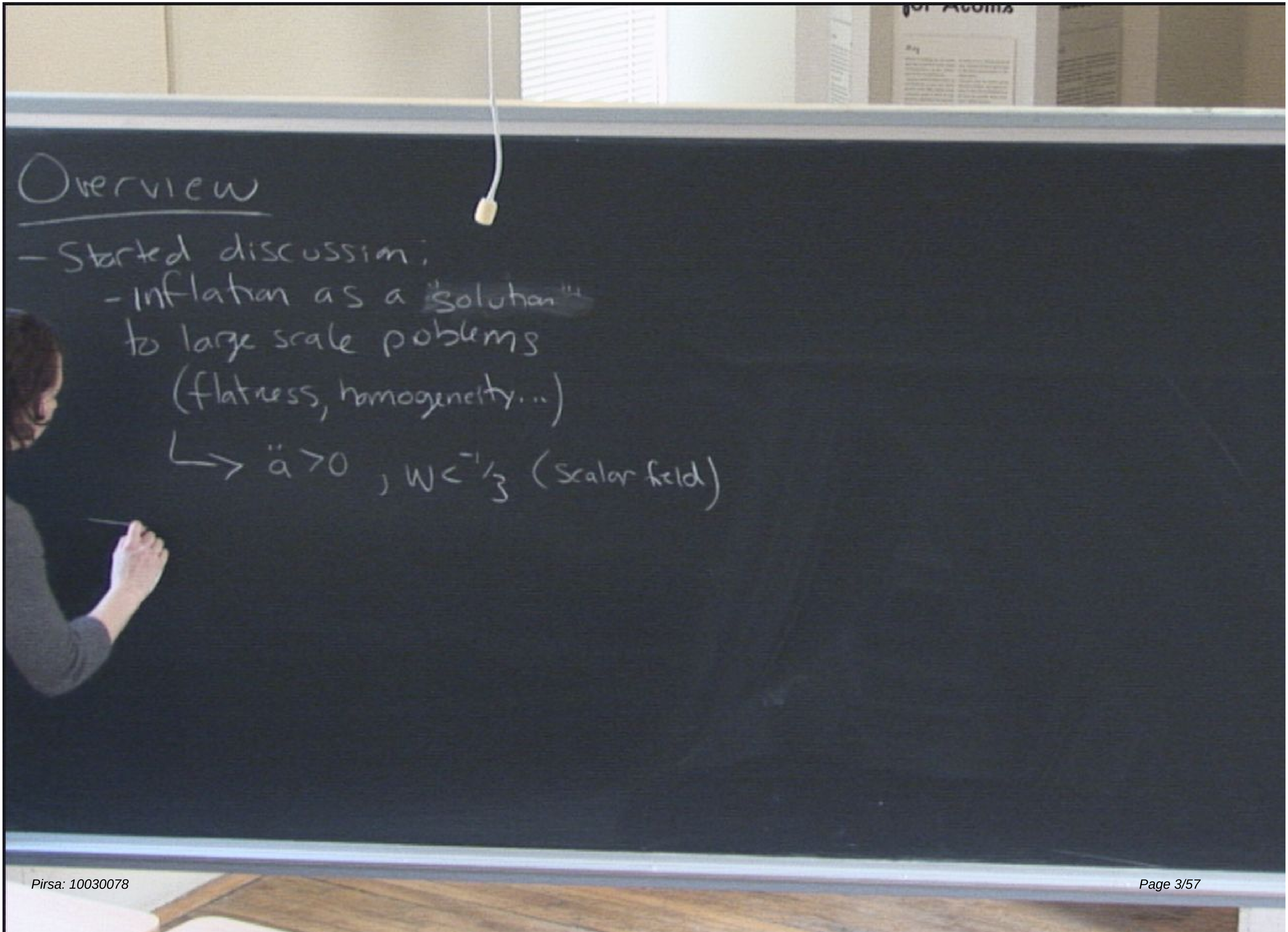
Title: Explorations in Cosmology (PHYS 649) - Lecture 9

Date: Mar 25, 2010 09:00 AM

URL: http://pirsa.org/10030078

Abstract:


Overview
– Started discussion:
- inflation as a "solution"
to large scale problems
(flatness, homogeneity...)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- Started discussion:
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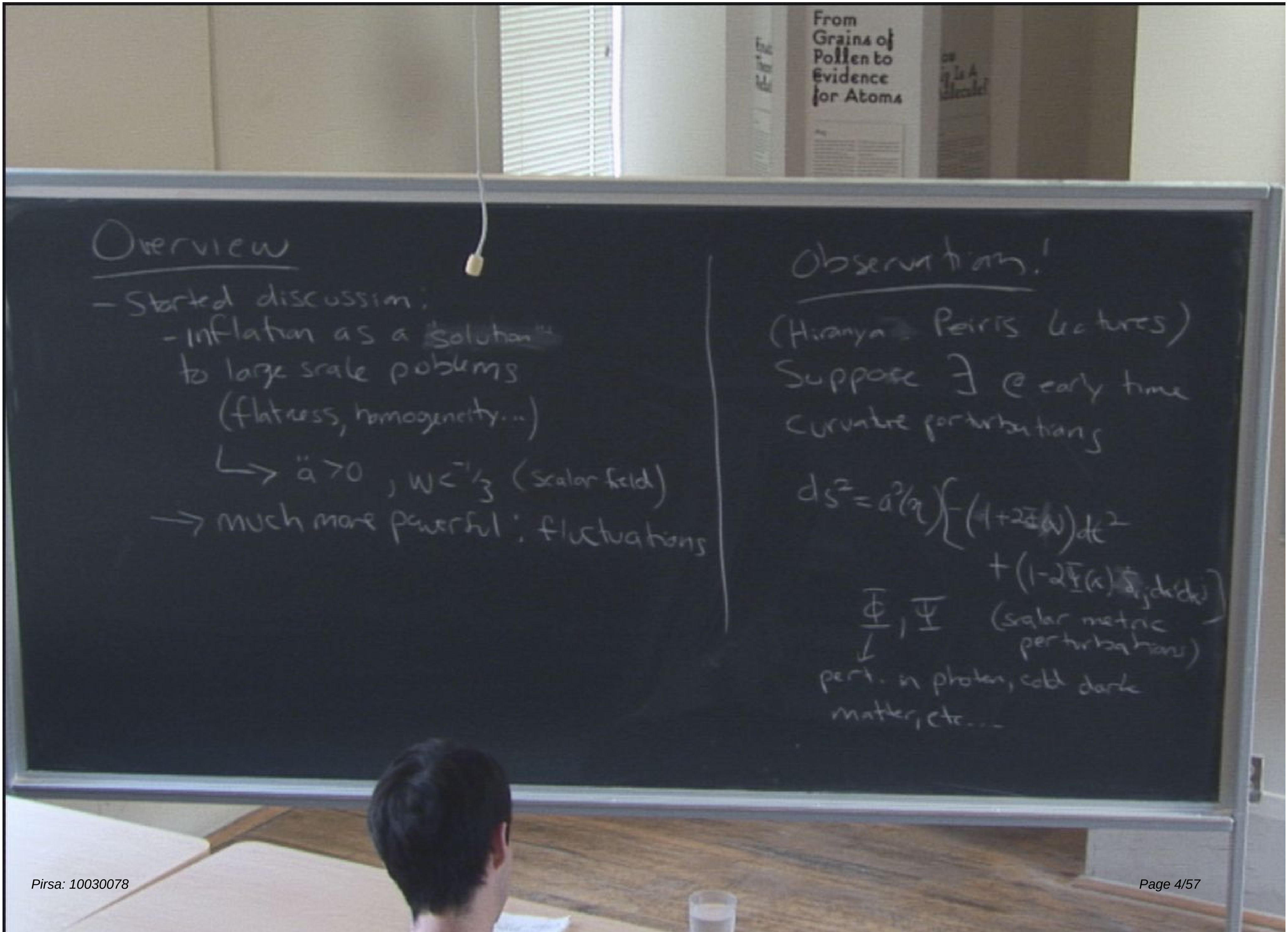
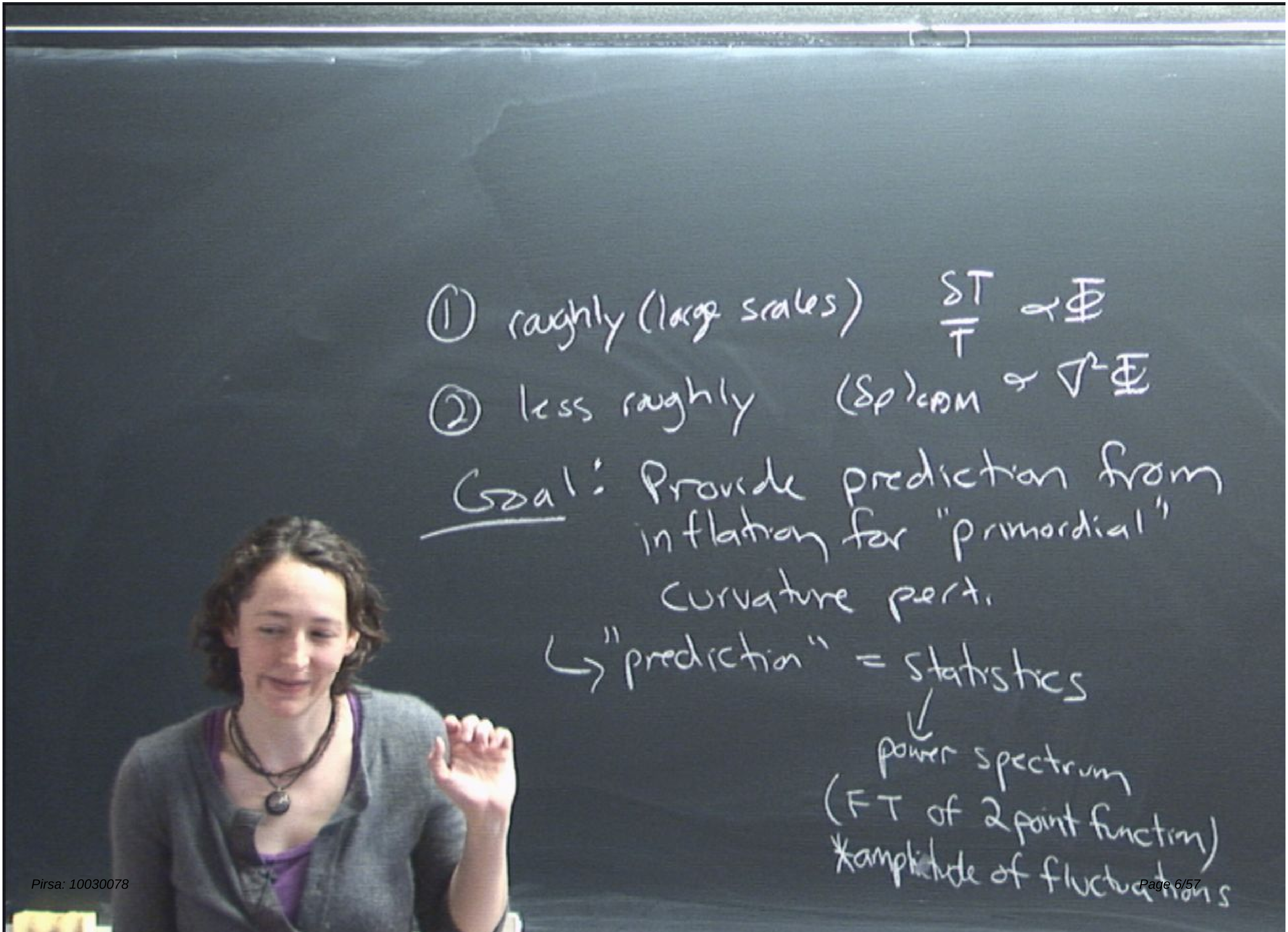
↳ ä>0 , w<-1/3 (scalar field)

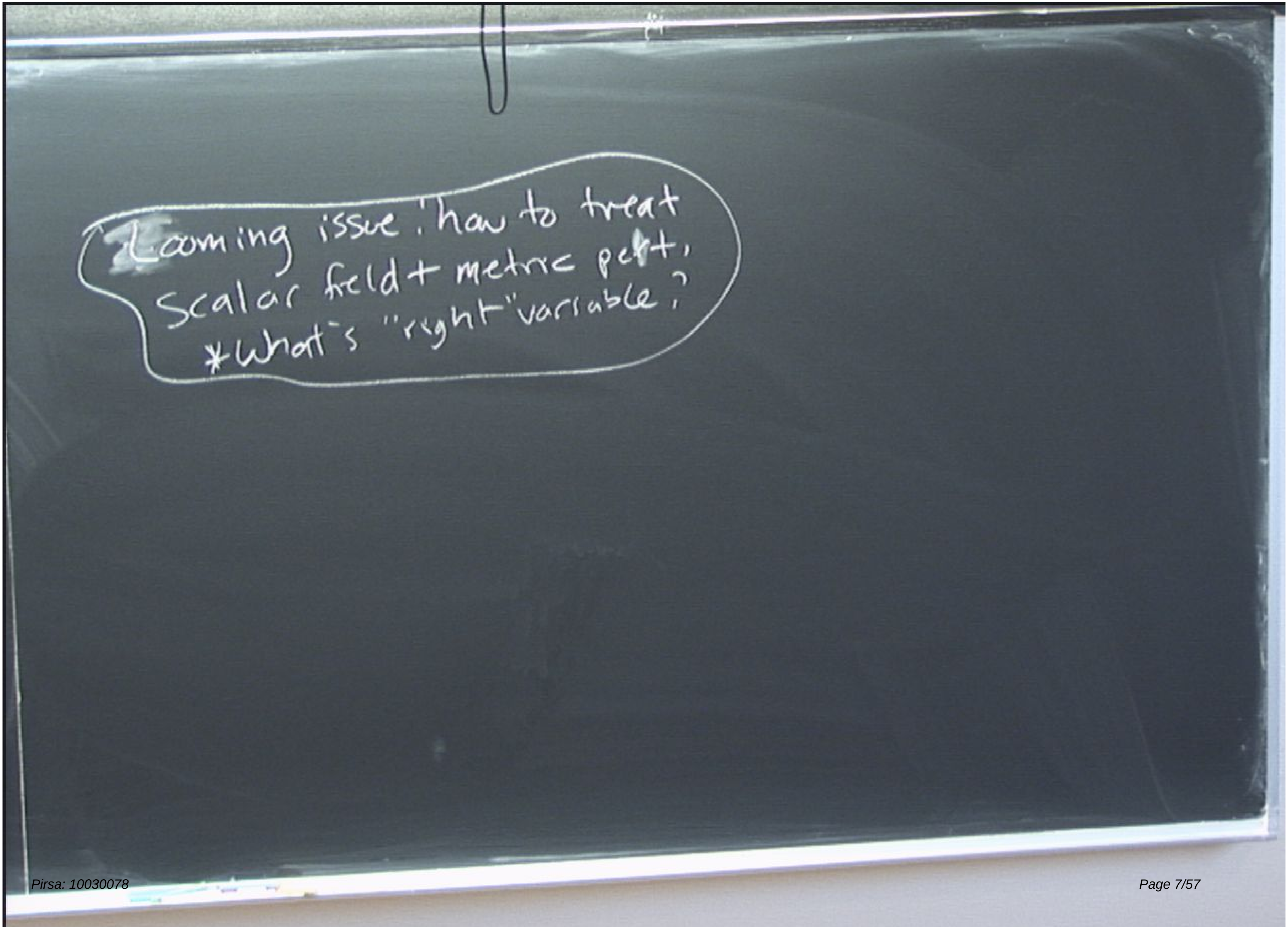

From Grains of Pollen to Evidence for Atoms
Overview
- Started discussion:
- inflation as a "solution" to large scale problems (flatness, homogeneity...)
↳ ä > 0, w < -1/3 (scalar field)
→ much more powerful: fluctuations
Observations!
(Hiranya Peiris lectures)
Suppose ∃ @ early time curvature perturbations
Φ, Ψ (scalar metric perturbations)
pert. in photon, cold dark matter, etc...

① roughly (large scales) $\frac{\delta T}{T} \propto \Phi$

② less roughly $(\delta\rho)_{CDM} \propto \nabla^2 \Phi$

<u>Goal</u>: Provide prediction from inflation for "primordial" curvature pert.


① roughly (large scales) δT/T ∝ Φ
② less roughly (δρ)_CDM ∝ ∇²Φ
Goal: Provide prediction from inflation for "primordial" curvature pert.
↳ "prediction" = statistics
power spectrum
(FT of 2 point function)
*amplitude of fluctuations


Looming issue: how to treat
scalar field + metric pert.,
*What's "right" variable?

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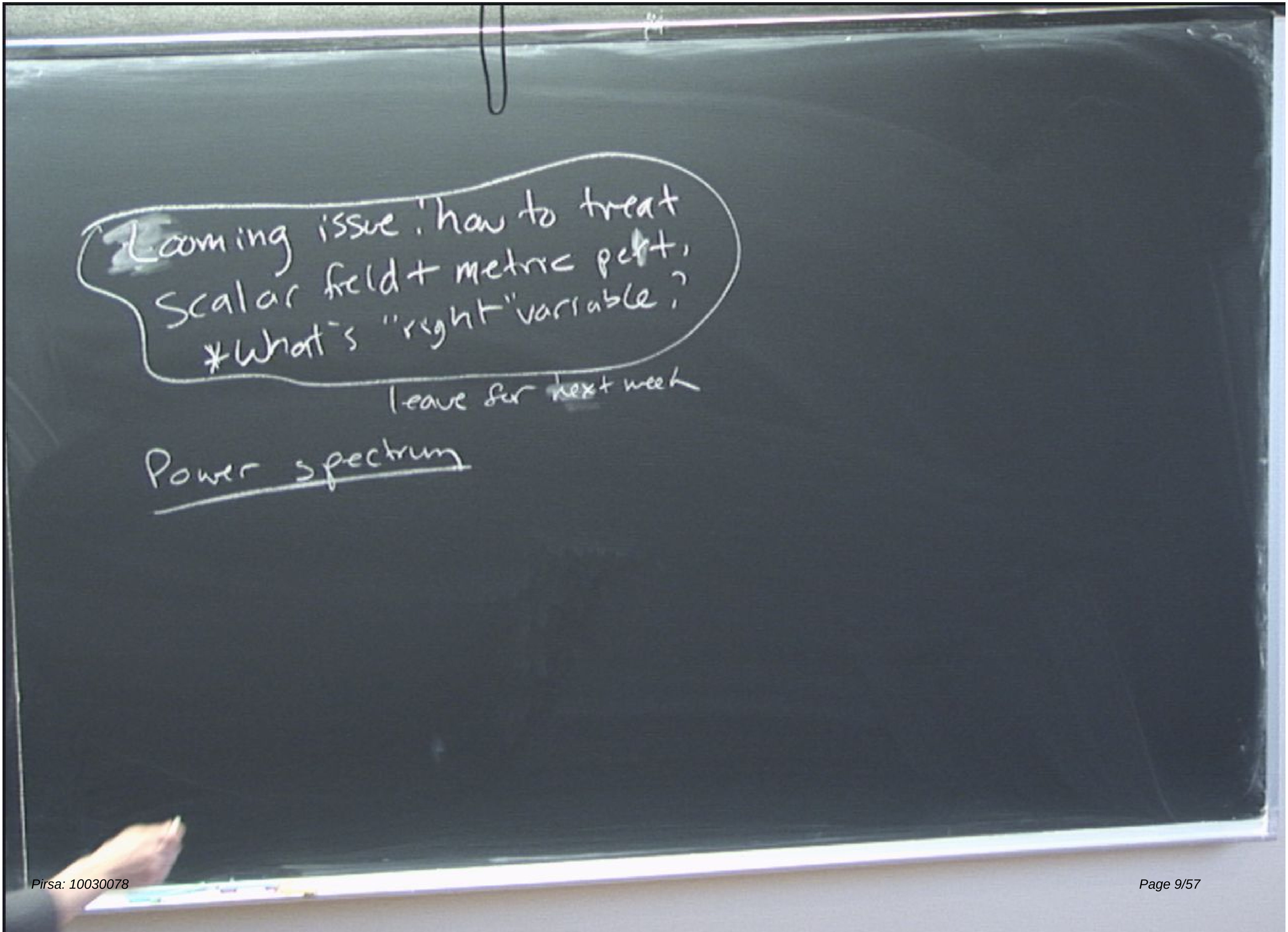
leave for next week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Power spectrum

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Power spectrum

$$\left(\frac{\delta\rho}{\rho}\right)_{CDM} = \delta$$

$$\delta(x,t) = \int \frac{d^3k}{(2\pi)^3}\, \delta_k\, e^{-i\vec{k}\cdot\vec{x}}$$

Looming issue: how to treat scalar field + metric pert.; * What's "right" variable?

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$$\langle \delta(k_1)\delta(k_2) \rangle = (2\pi)^3\, \delta^3(\vec{k}_1+\vec{k}_2)\, \mathcal{P}(k)$$

Looming issue: how to treat scalar field + metric pert.; *what's "right" variable?

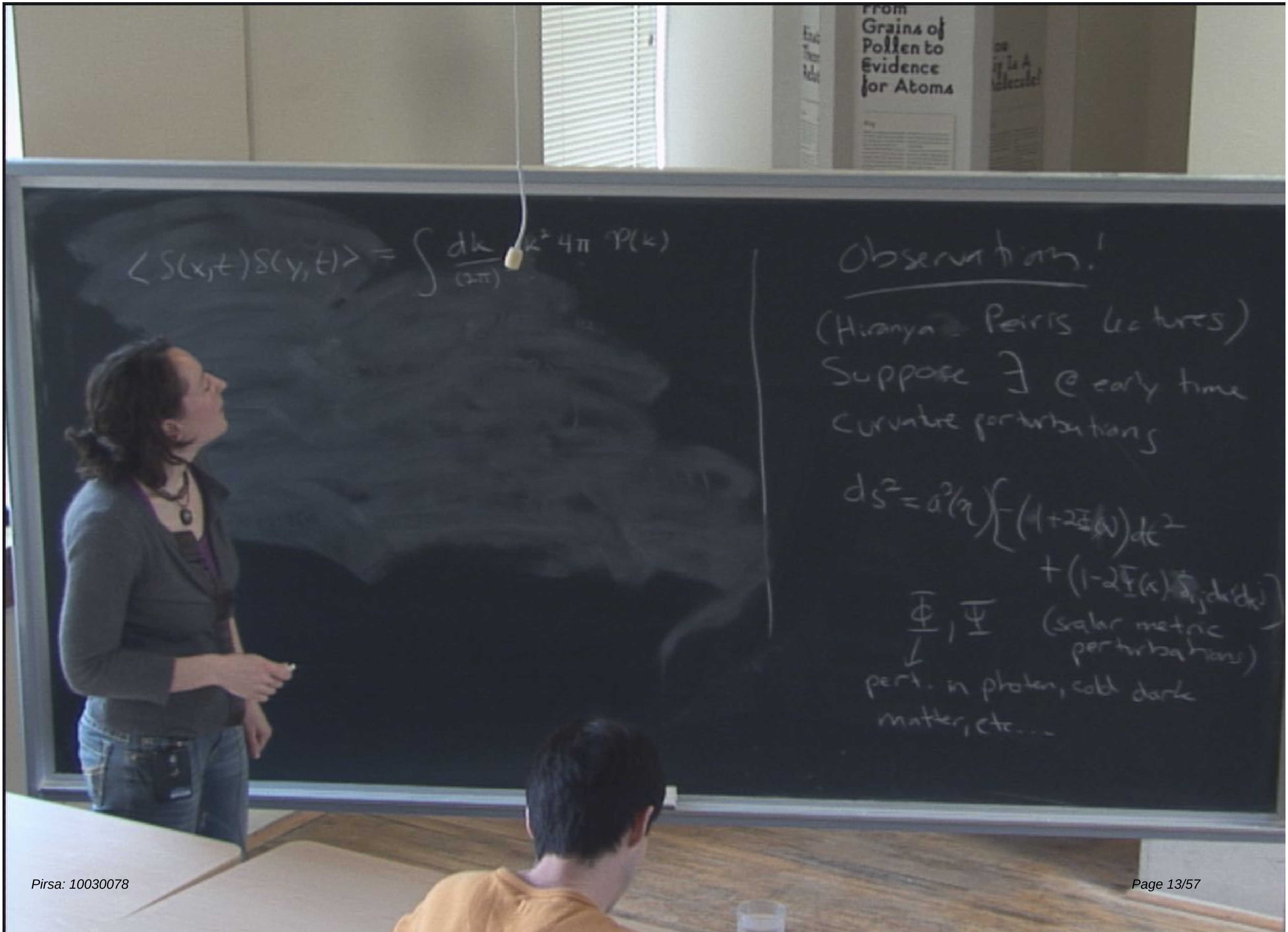
leave for next week

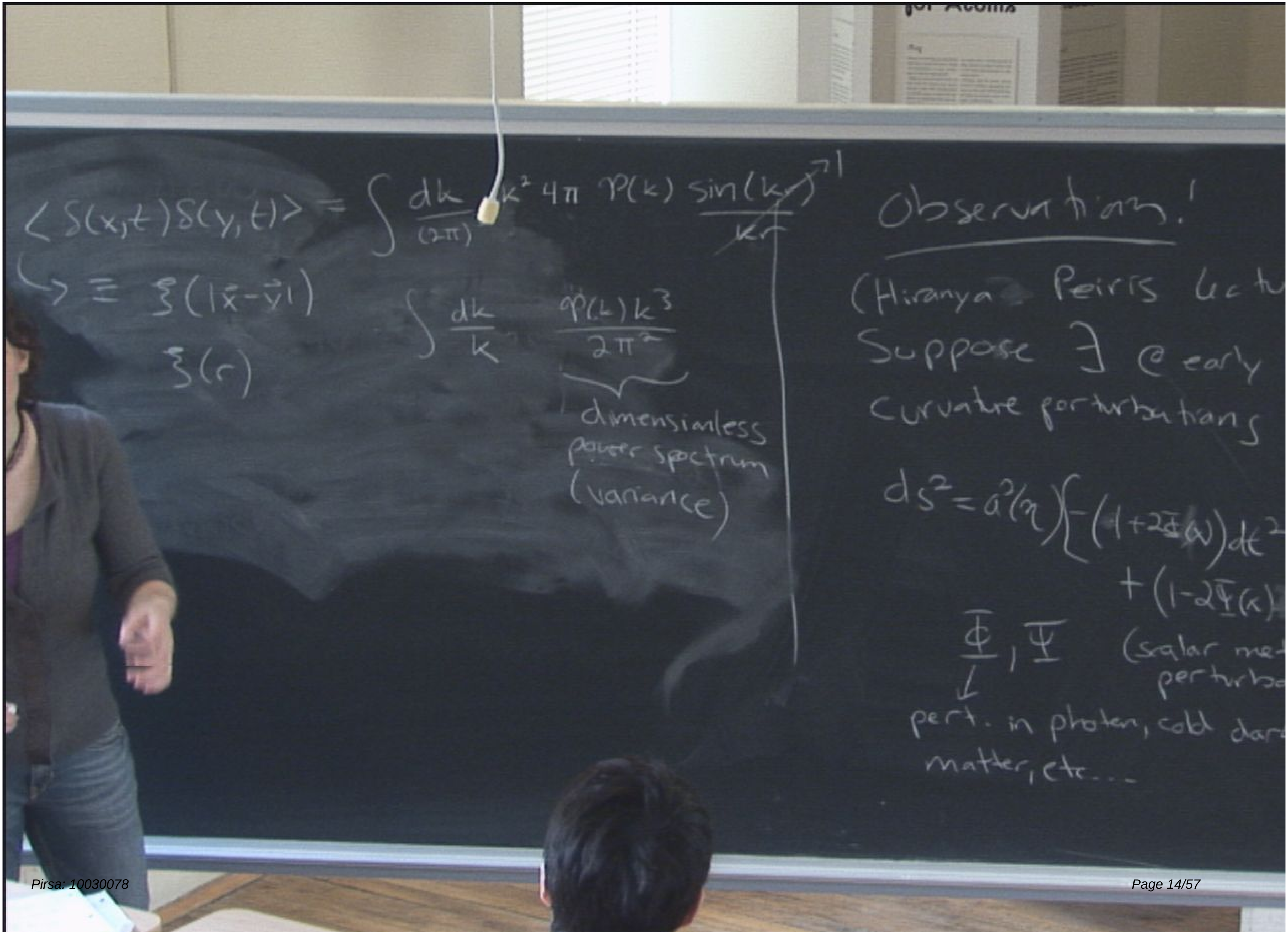
Power spectrum

$$\left(\frac{\delta\rho}{\rho}\right)_{CDM} = \delta \qquad \delta(x,t) = \int \frac{d^3k}{(2\pi)^3}\, \delta_k\, e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \delta(k_1)\delta(k_2) \rangle = (2\pi)^3\, \delta^3(\vec{k}_1+\vec{k}_2)\, \mathcal{P}(k_1)$$

$$\langle \delta(x,t)\,\delta(y,t) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot(x-y)}\, \mathcal{P}(k) \Big\} \text{power spectrum}$$


From Grains of Pollen to Evidence for Atoms
⟨δ(x,t)δ(y,t)⟩ = ∫ dk/(2π) k² 4π P(k)
Observation!
(Hiranya Peiris lectures)
Suppose ∃ @ early time curvature perturbations
ds² = a²(η){-(1+2Φ(x))dt² + (1-2Ψ(x))δij dxi dxj}
Φ, Ψ (scalar metric perturbations)
pert. in photon, cold dark matter, etc...


Observations!
(Hiranya Peiris lectu
Suppose ∃ @ early
curvature perturbations
dimensionless
power spectrum
(variance)
(scalar met
perturba
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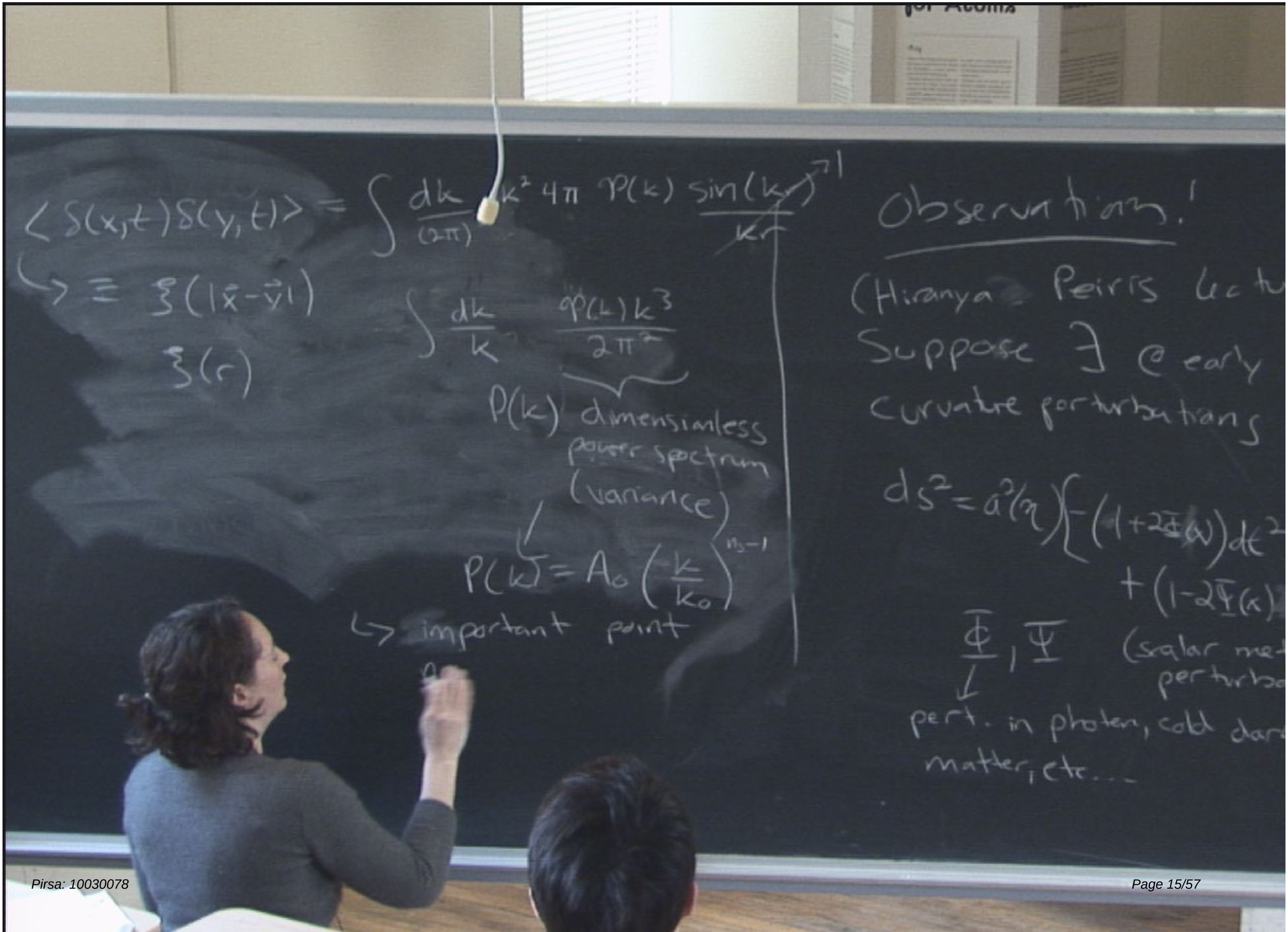

⟨δ(x,t)δ(y,t)⟩ = ∫ dk/(2π) k² 4π P(k) sin(kr)/kr
↳ ≡ ξ(|x − y|)
ξ(r)
∫ dk/k P(k)k³/2π²
P(k) dimensionless power spectrum (variance)
P(k) = A₀ (k/k₀)^(n_s−1)
↳ important point
Observations!
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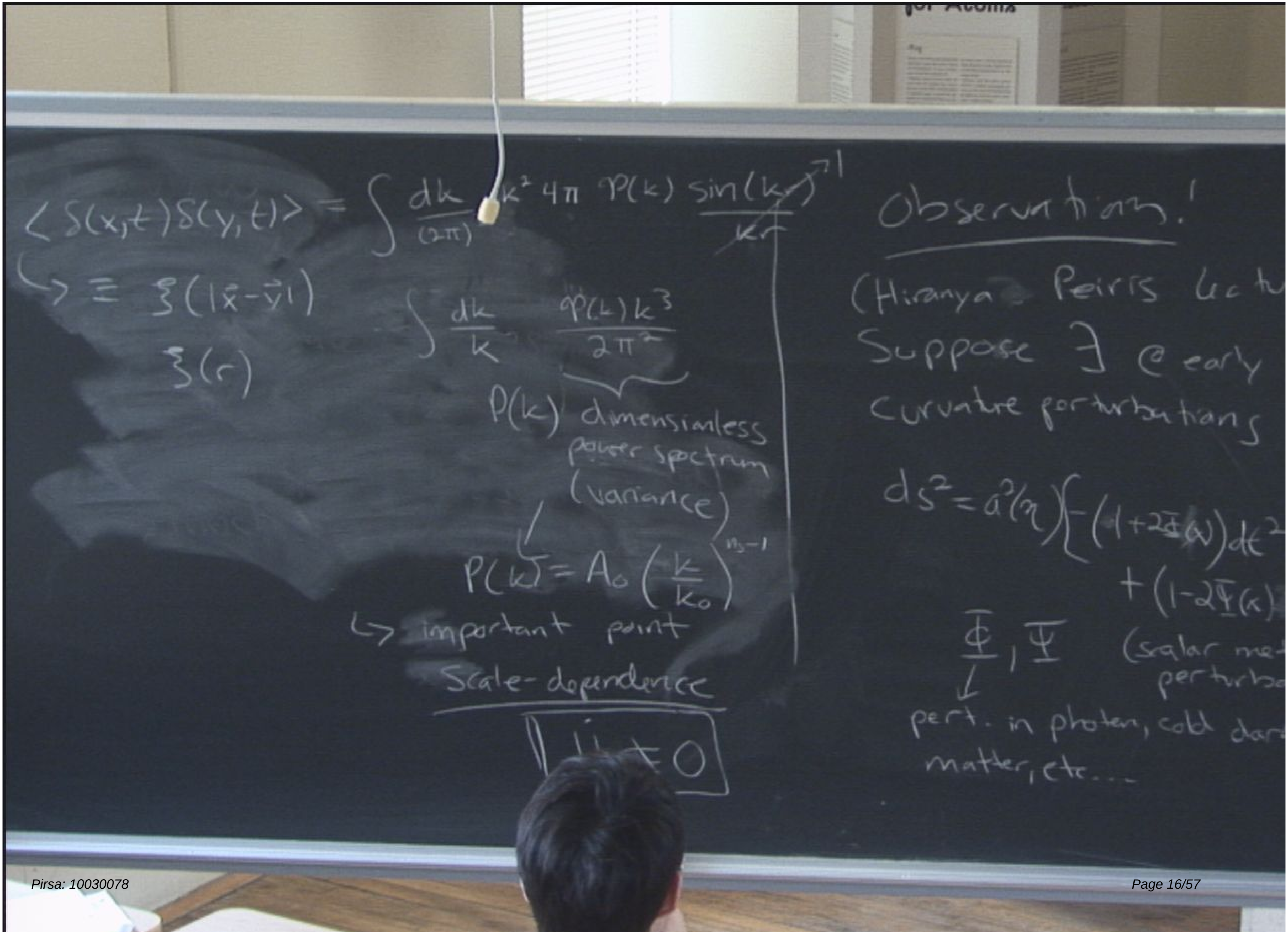
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Scale-dependence
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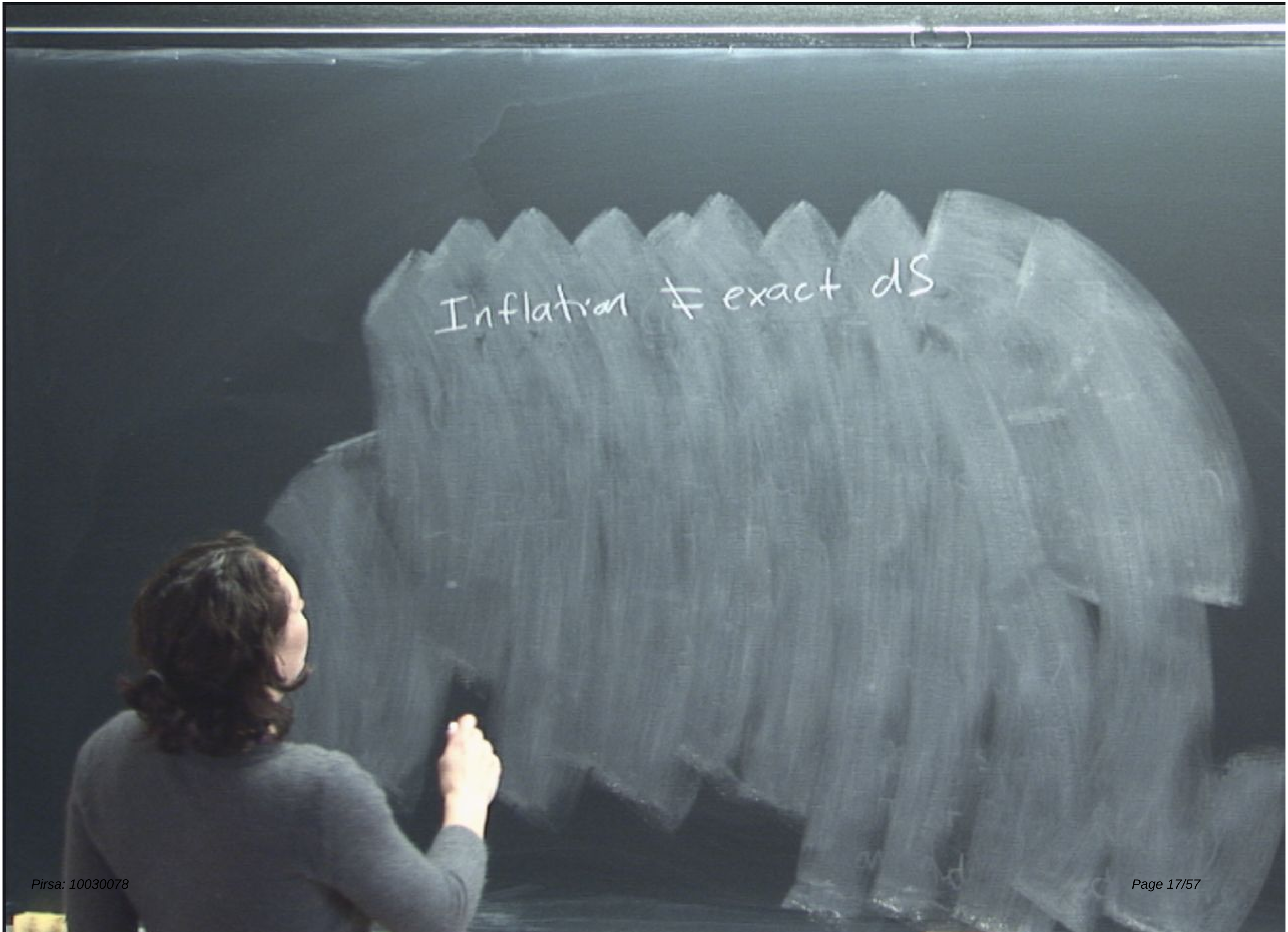
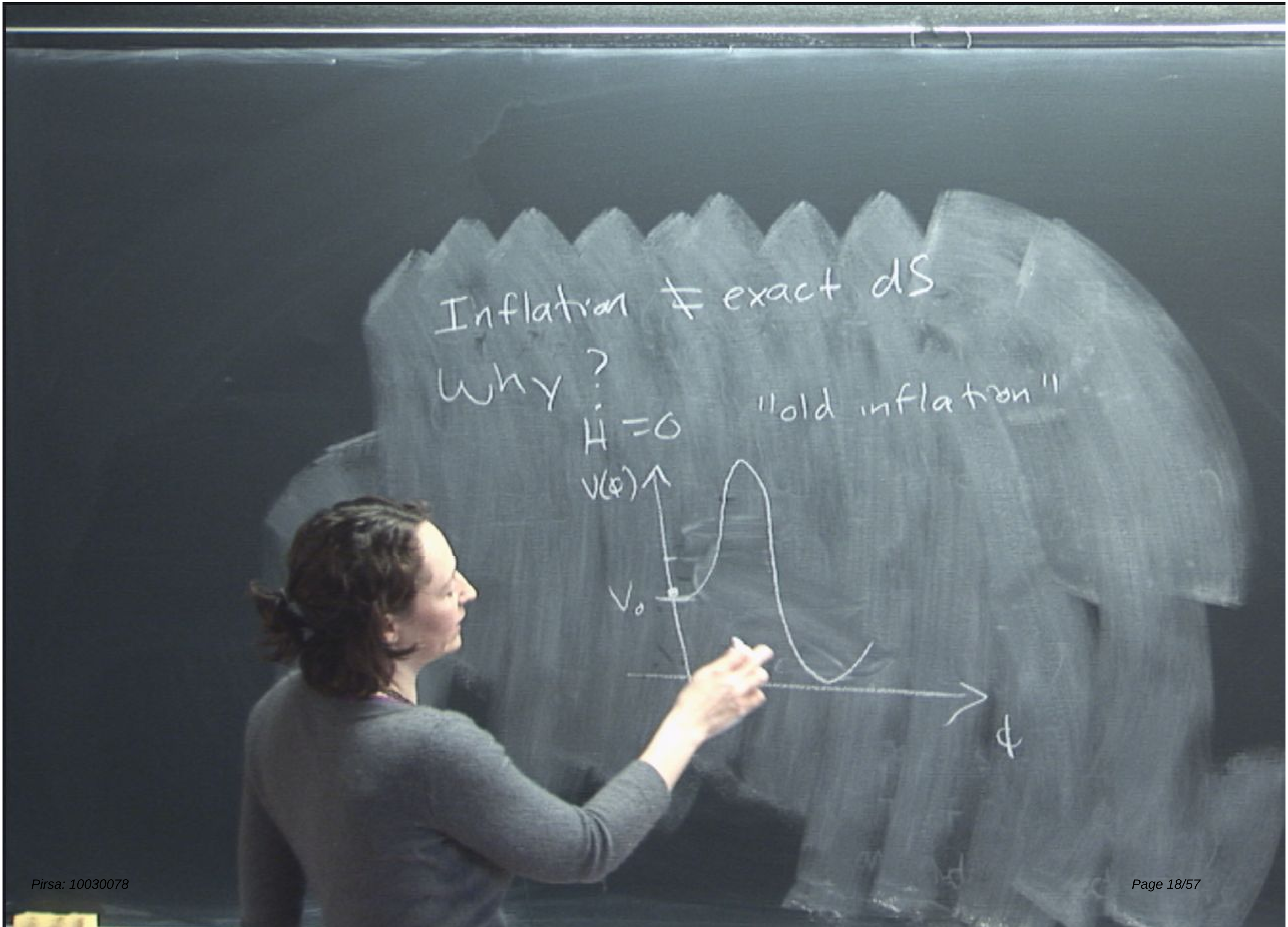
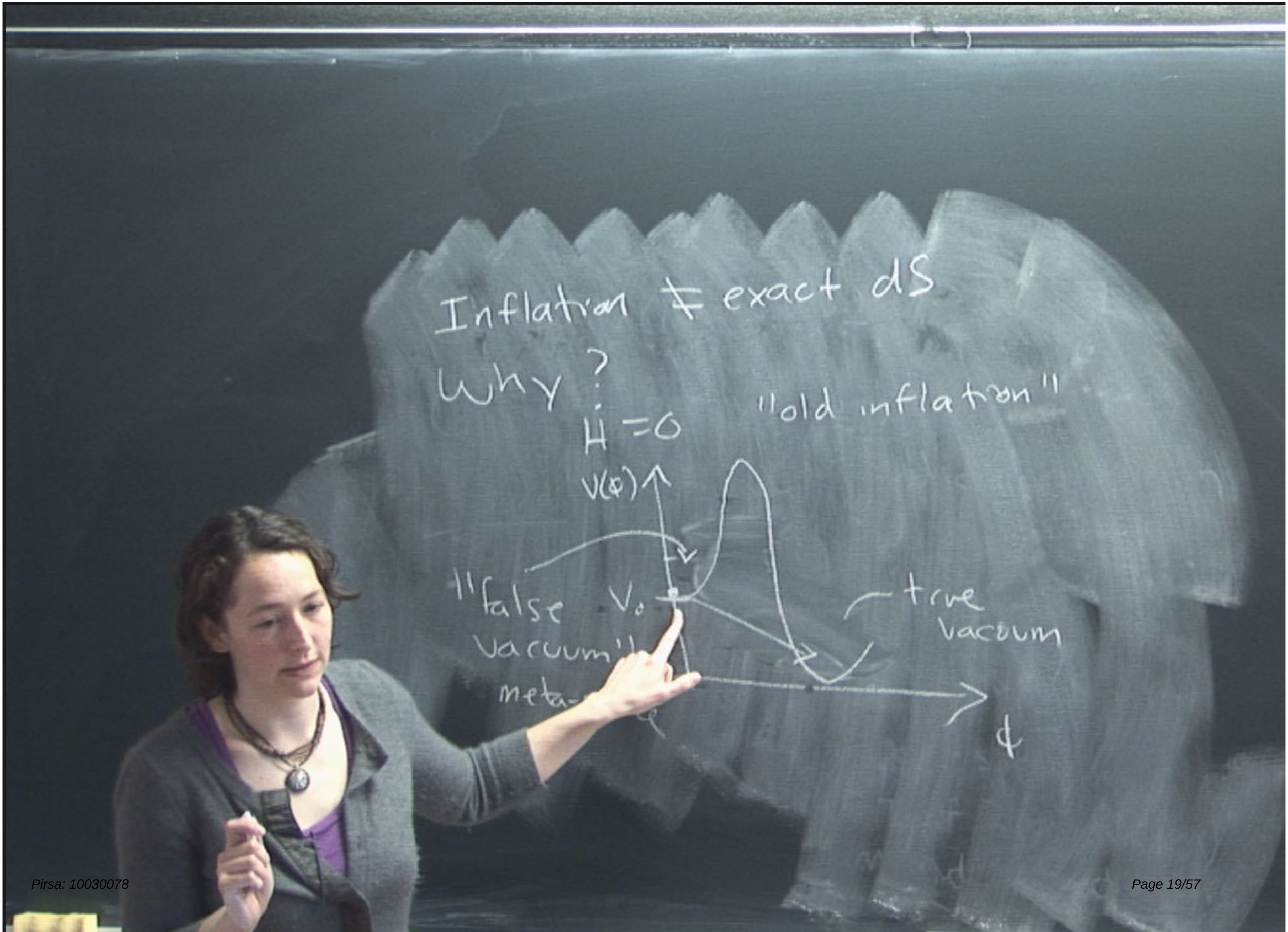

Inflation ≠ exact dS


Inflation ≠ exact dS
Why?
$\dot{H} = 0$
"old inflation"
$V(\phi)$
V_0
ϕ



Inflation ≠ exact dS
Why?
Ḣ = 0
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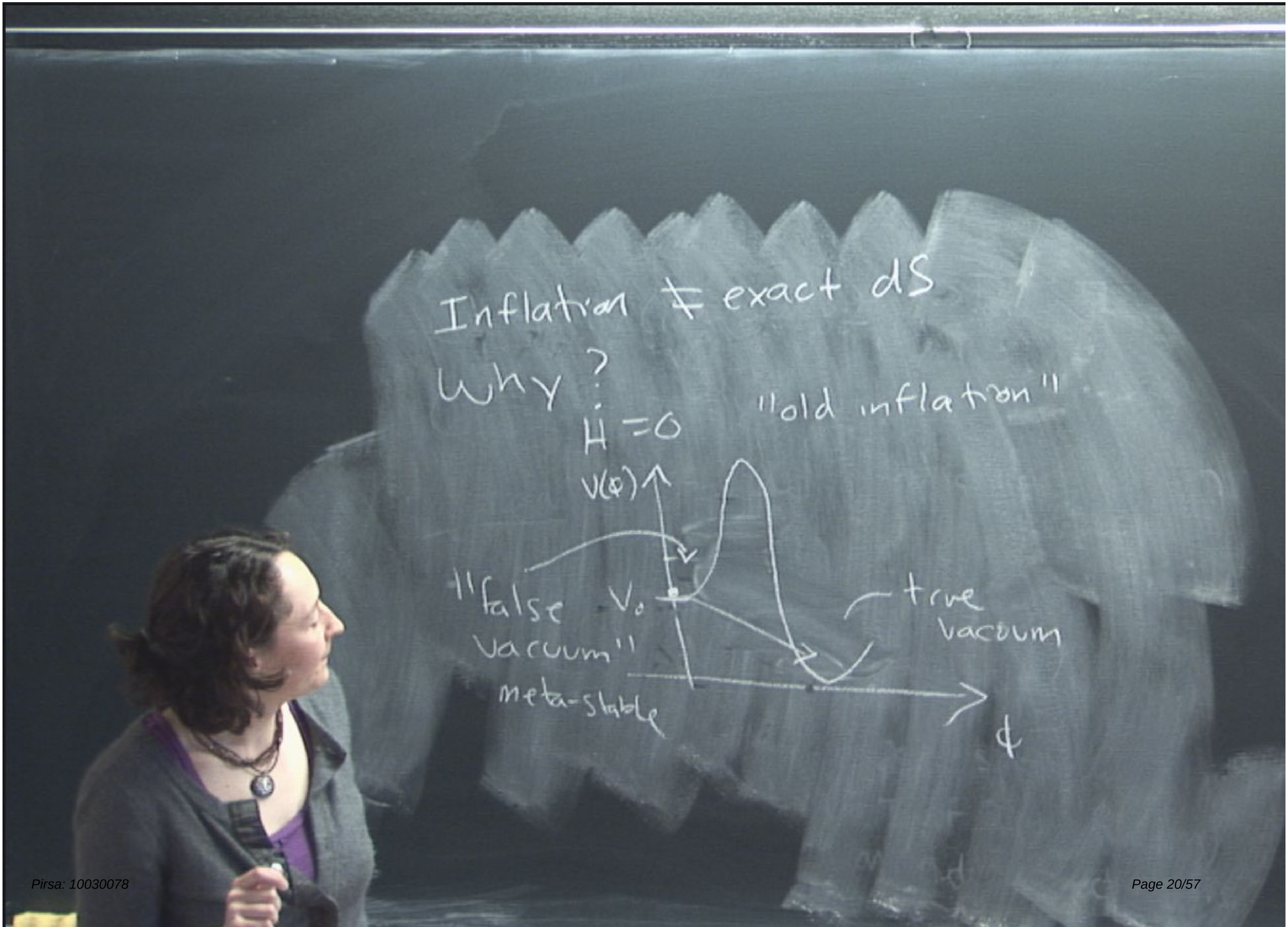
V(ϕ)
"false vacuum"
V₀
true vacuum
ϕ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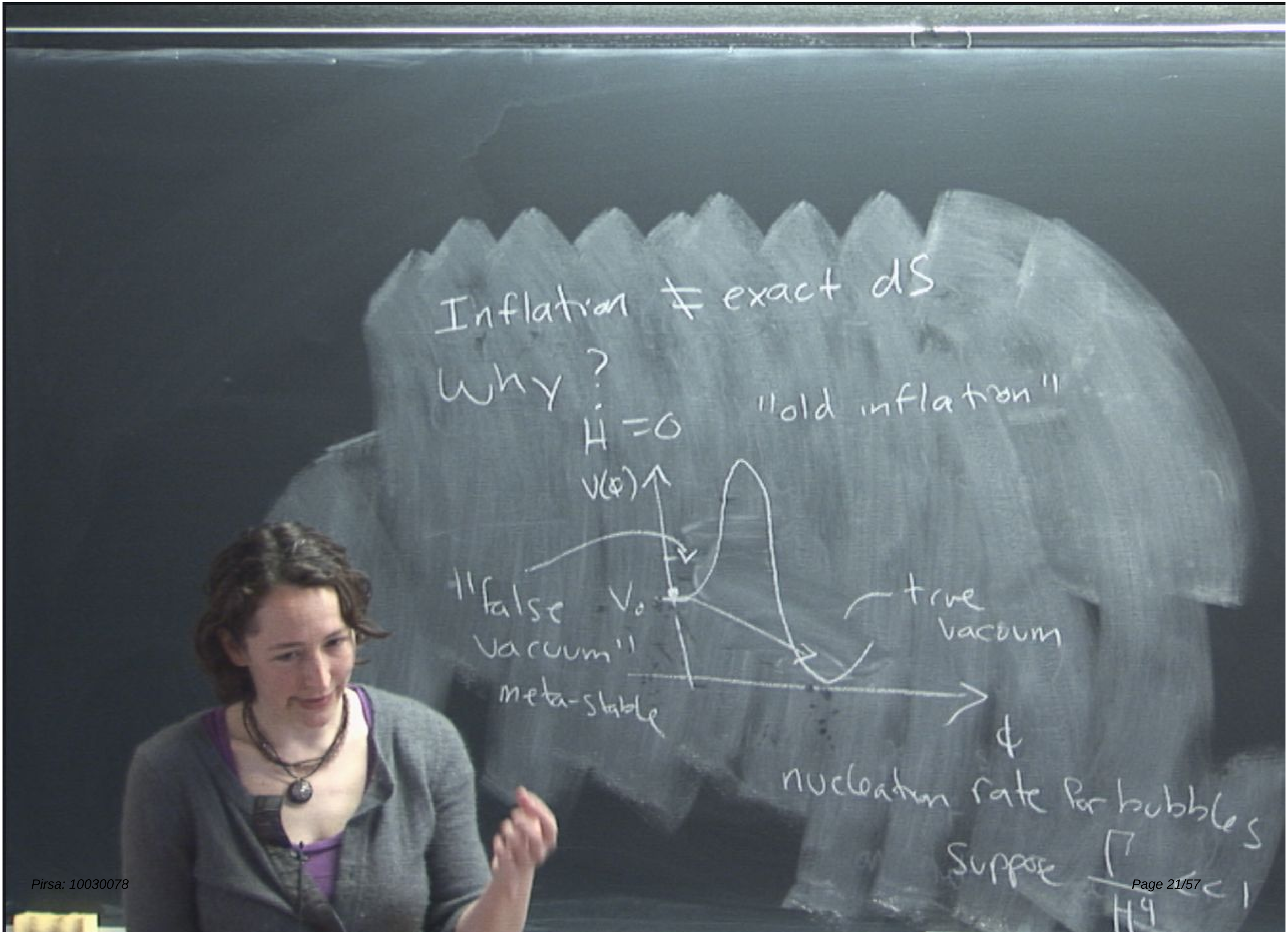

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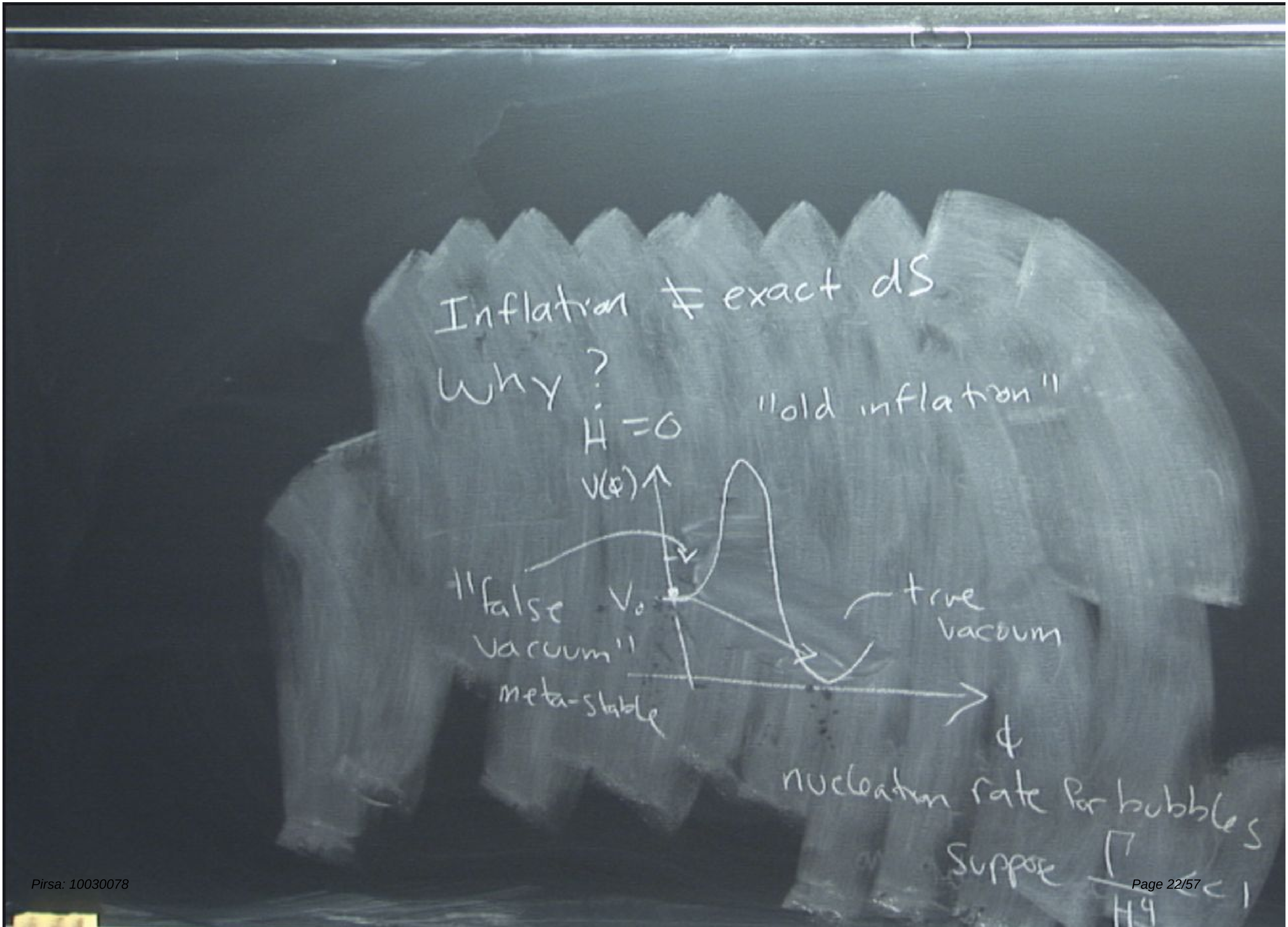
nucleation rate for bubbles
Suppose Γ/H^4 << 1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"old inflation"
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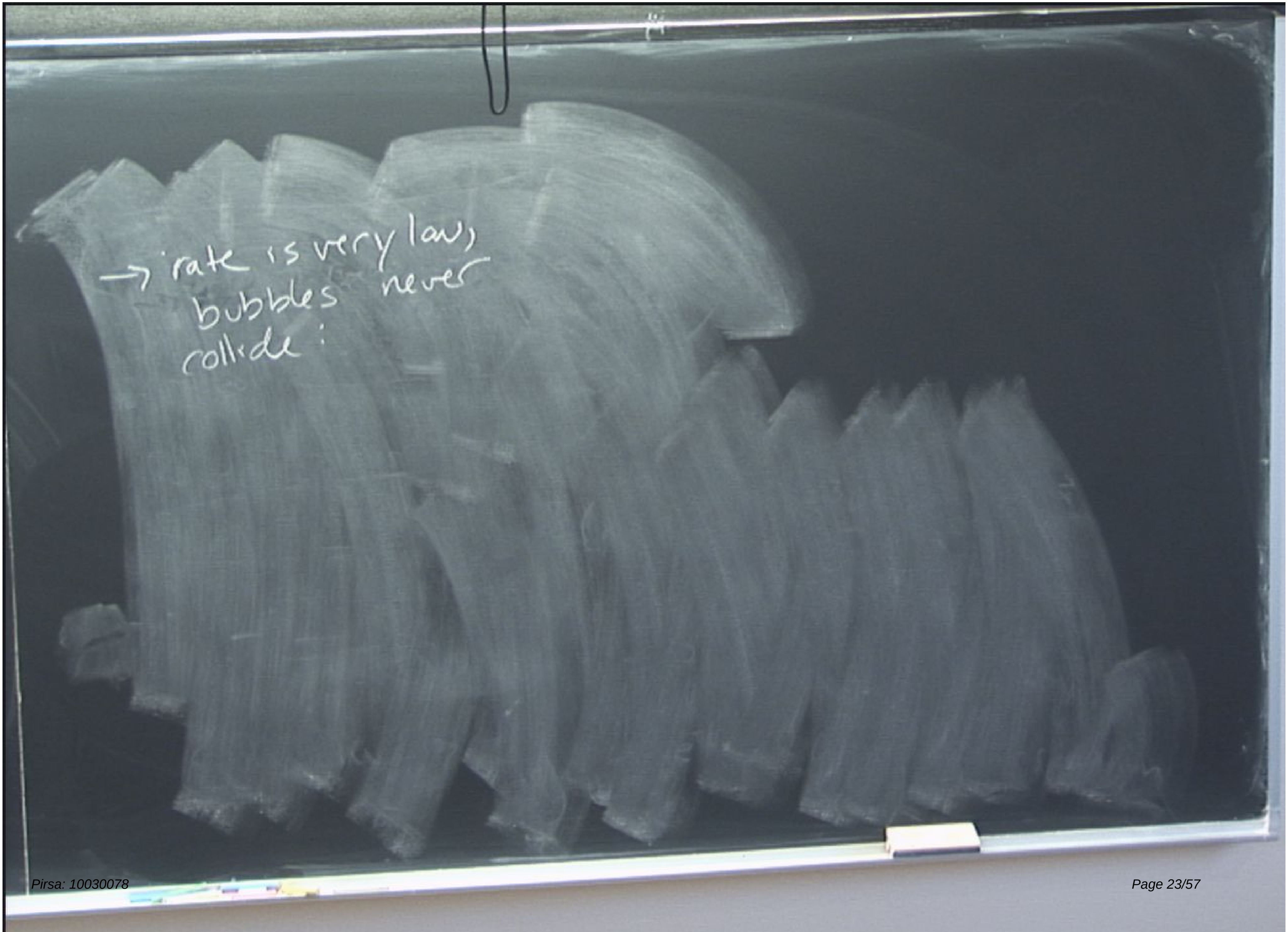
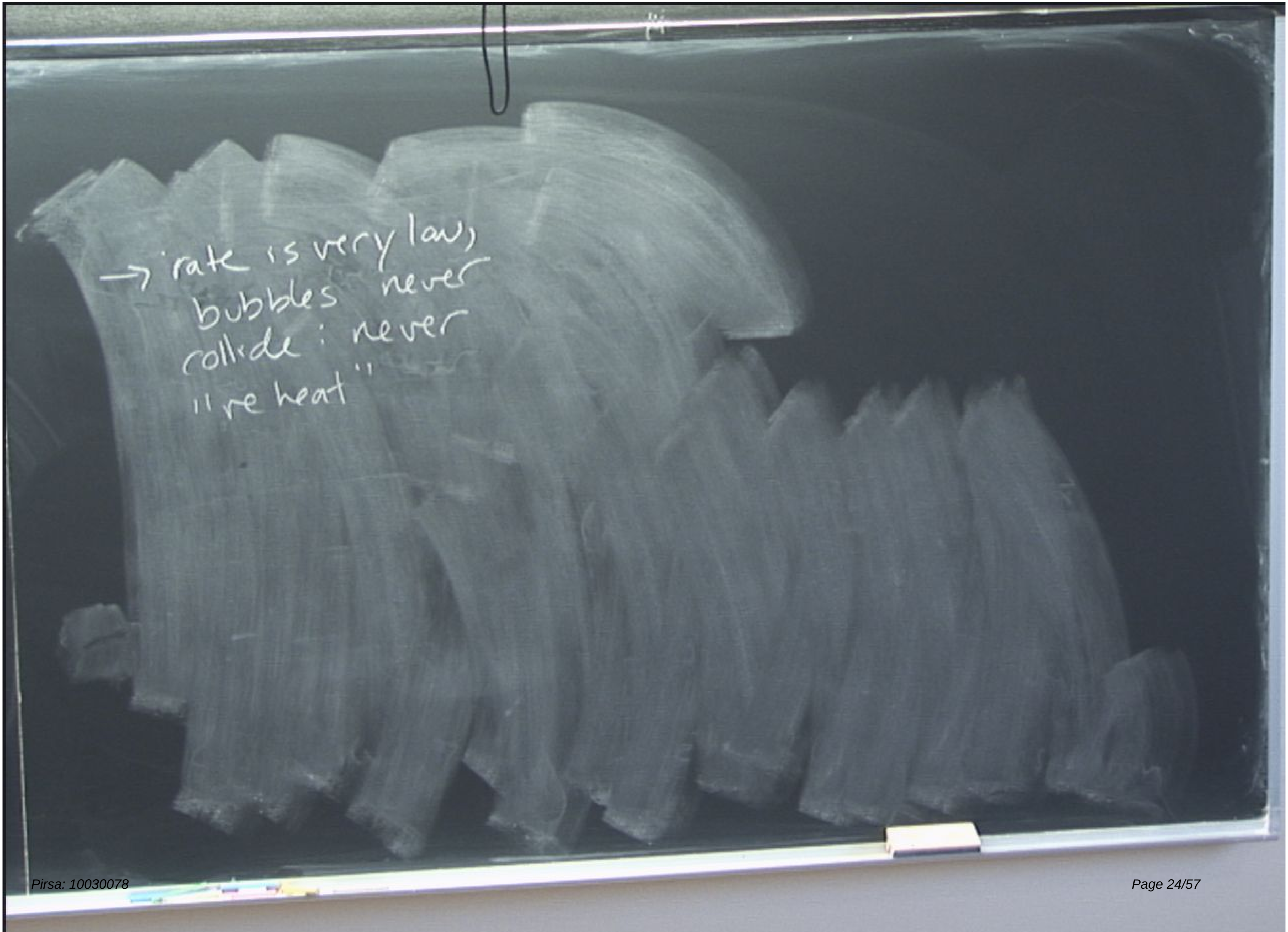
nucleation rate for bubbles
Suppose $\frac{\Gamma}{H^4} < 1$


→ rate is very low,
bubbles never
collide :


→ "rate is very low;
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collide : never
"reheat"

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→ rate is large (so
bubbles collide)
not enough inflation

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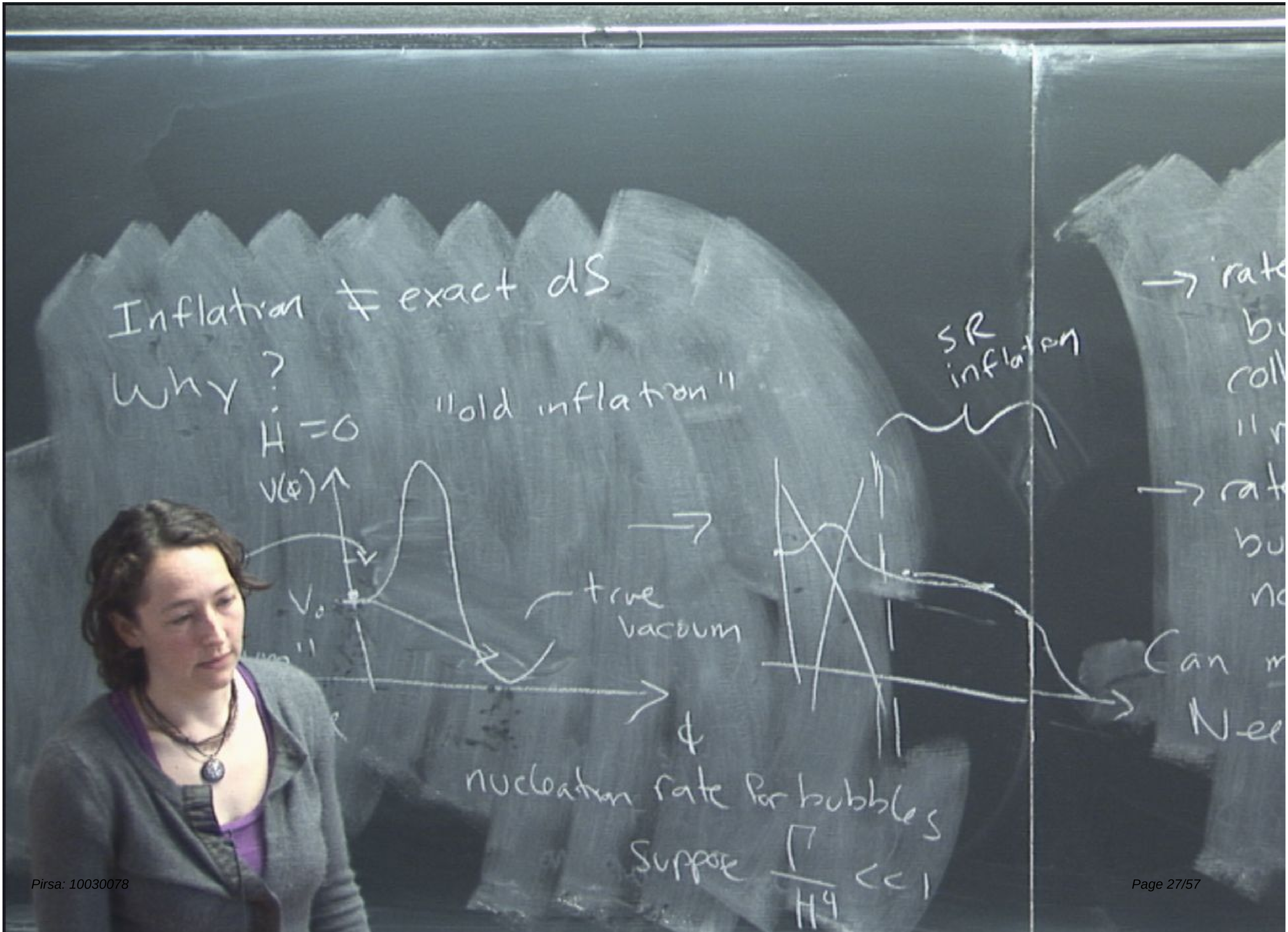
collide: never
"reheat"

→ rate is large (so
bubbles collide)
not enough inflation

Can make precise, doesn't work

Need $\dot{H} \neq 0$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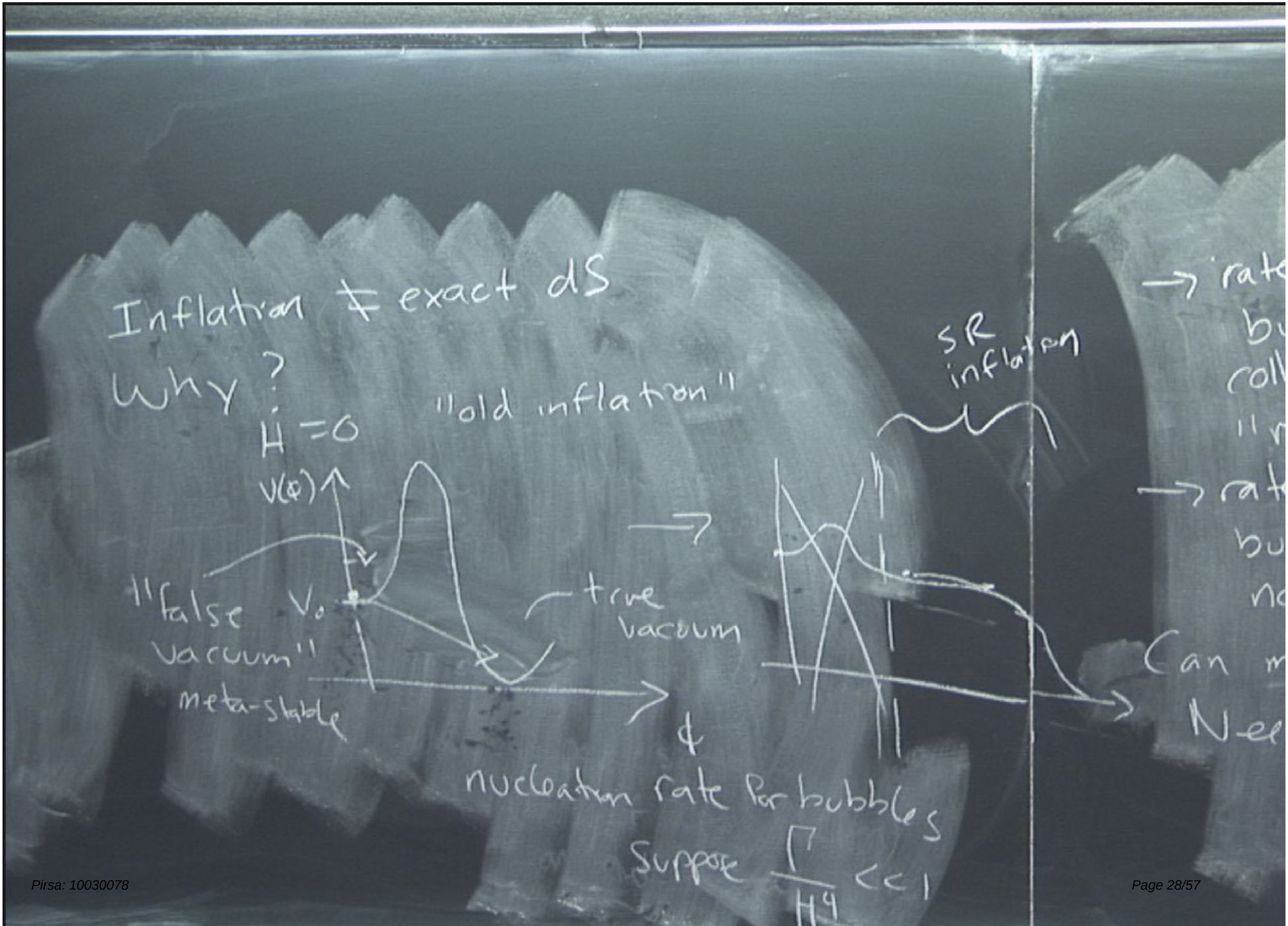

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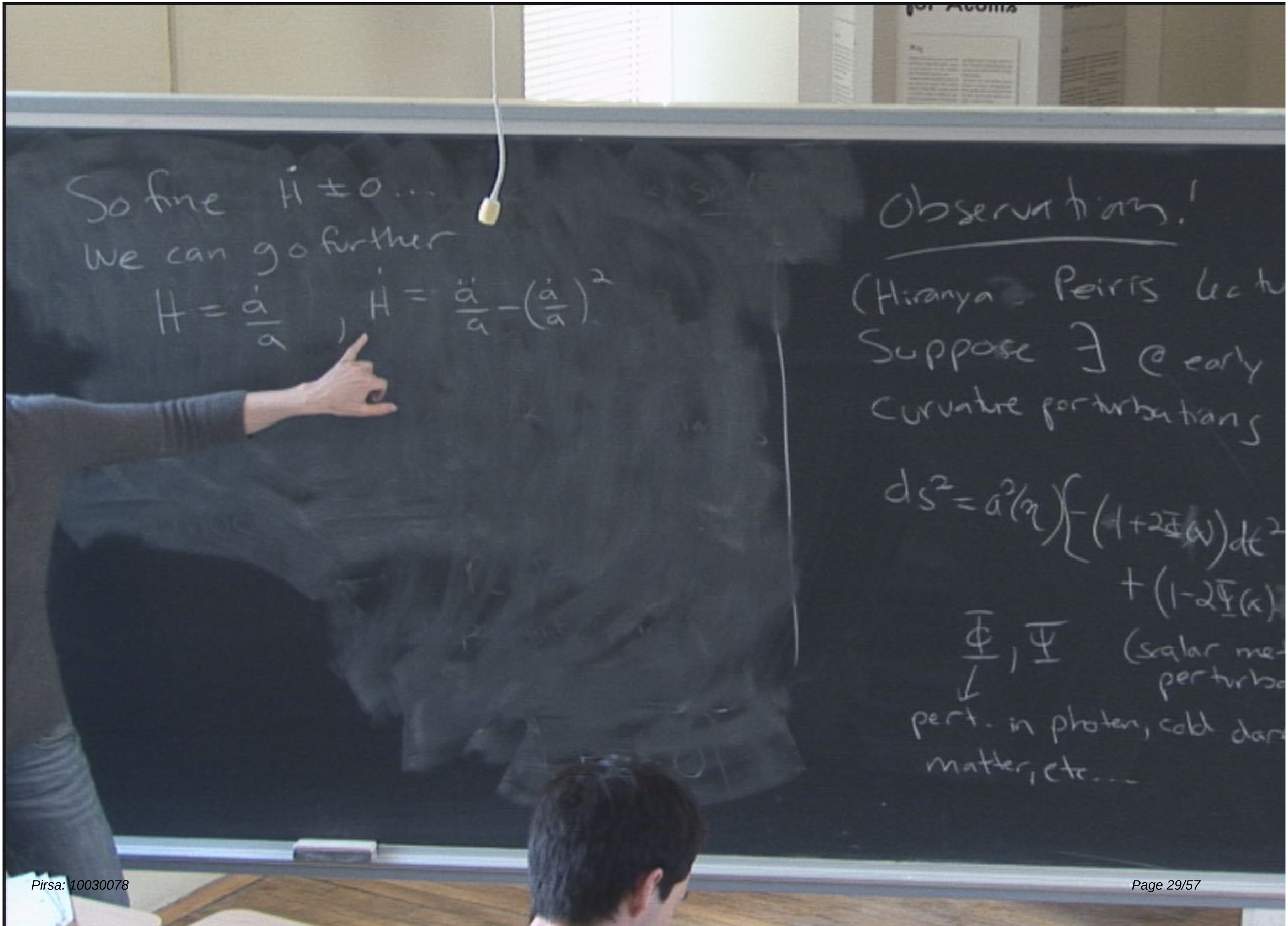
So fine $\dot{H} \neq 0$ ...
We can go further
$H = \frac{\dot{a}}{a}$, $\dot{H} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2$
Observations!
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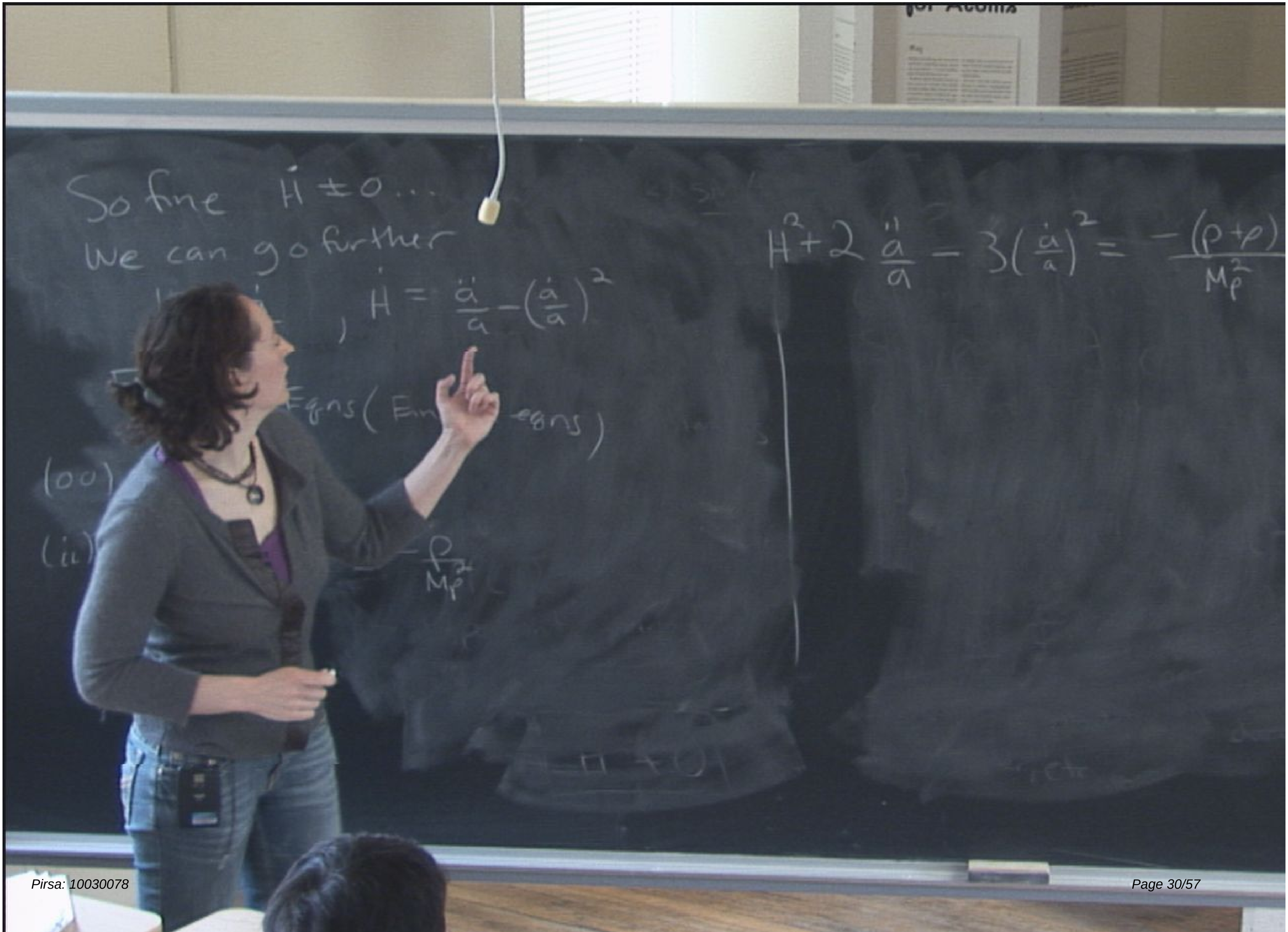
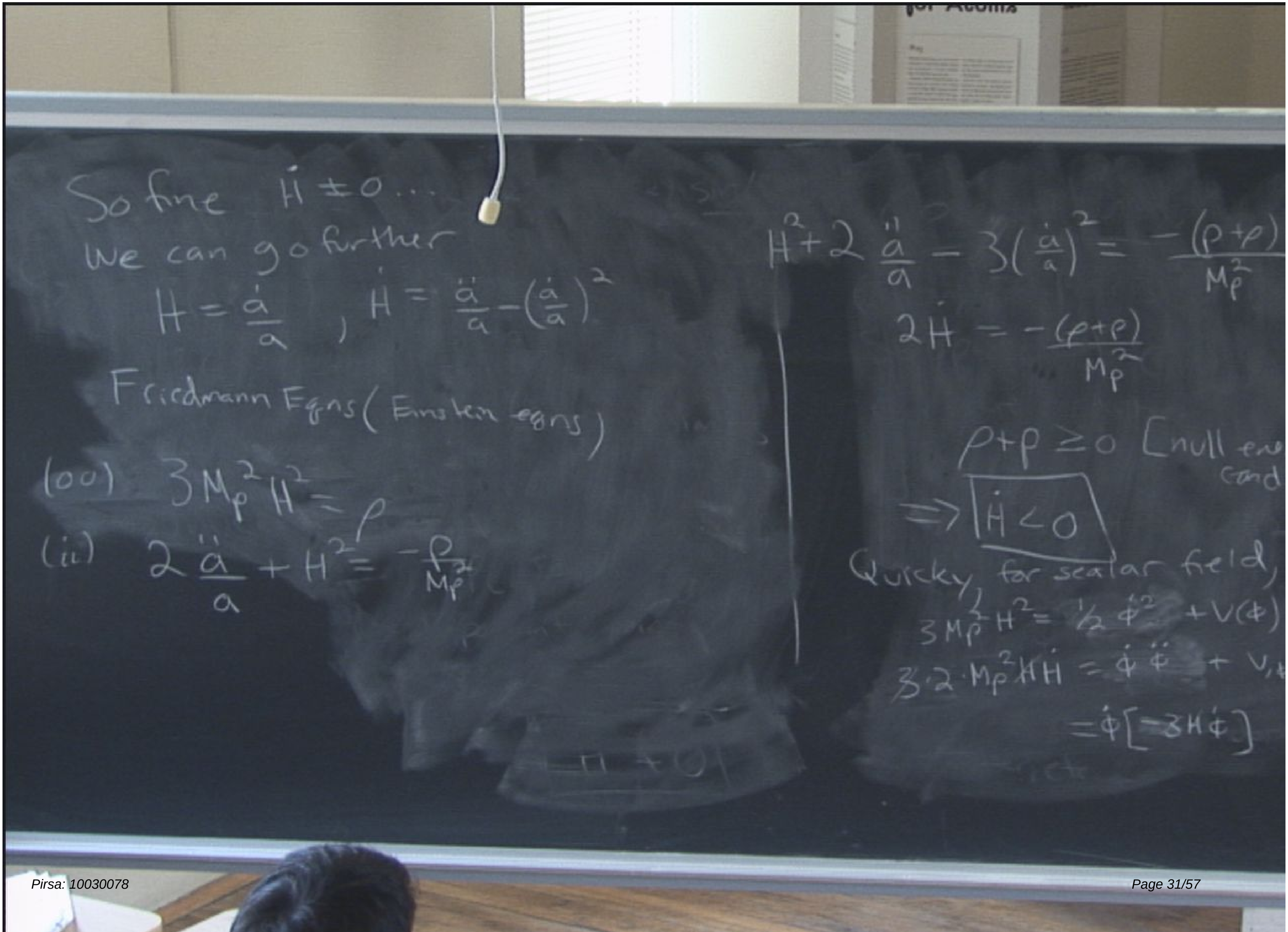
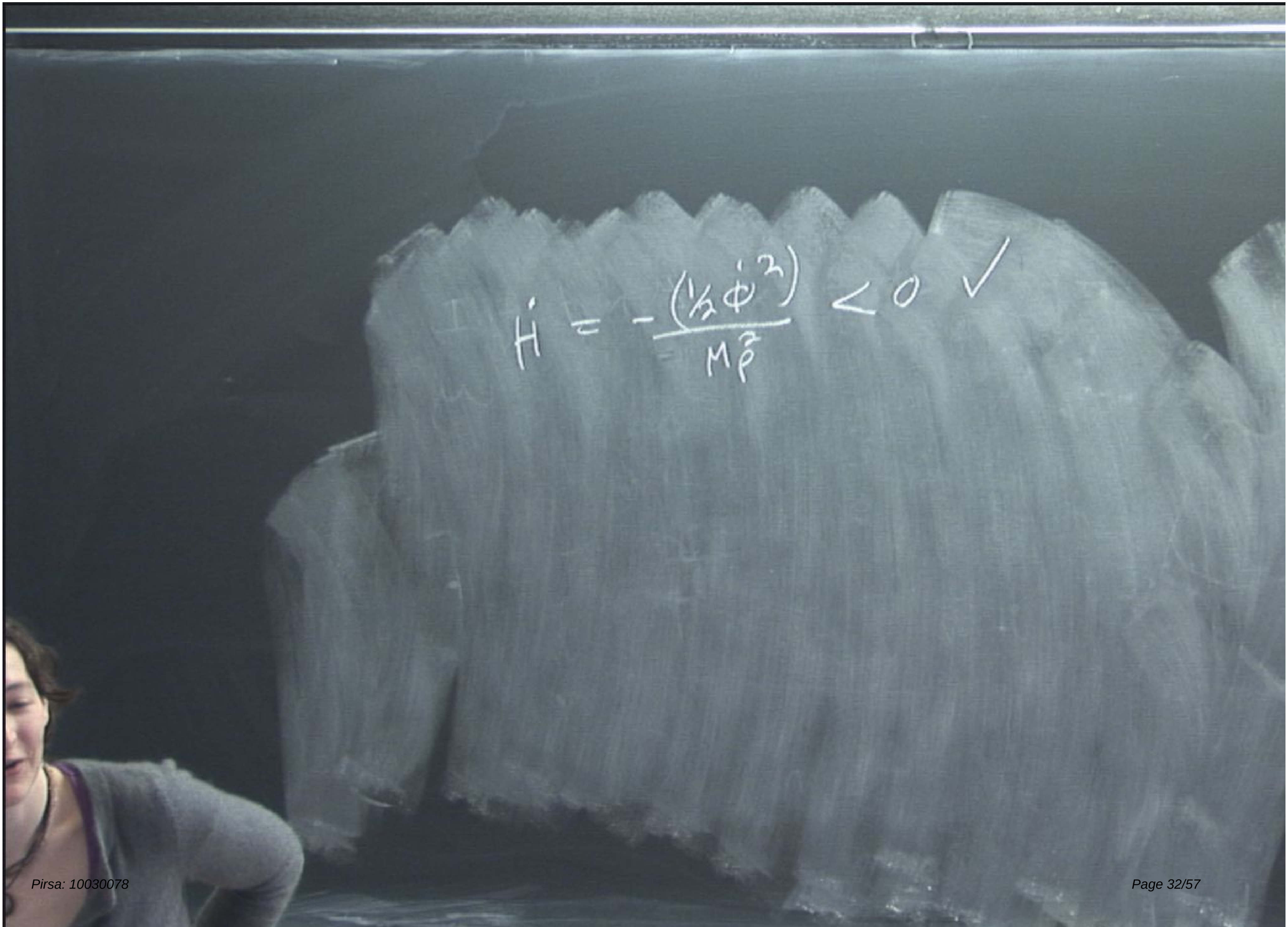
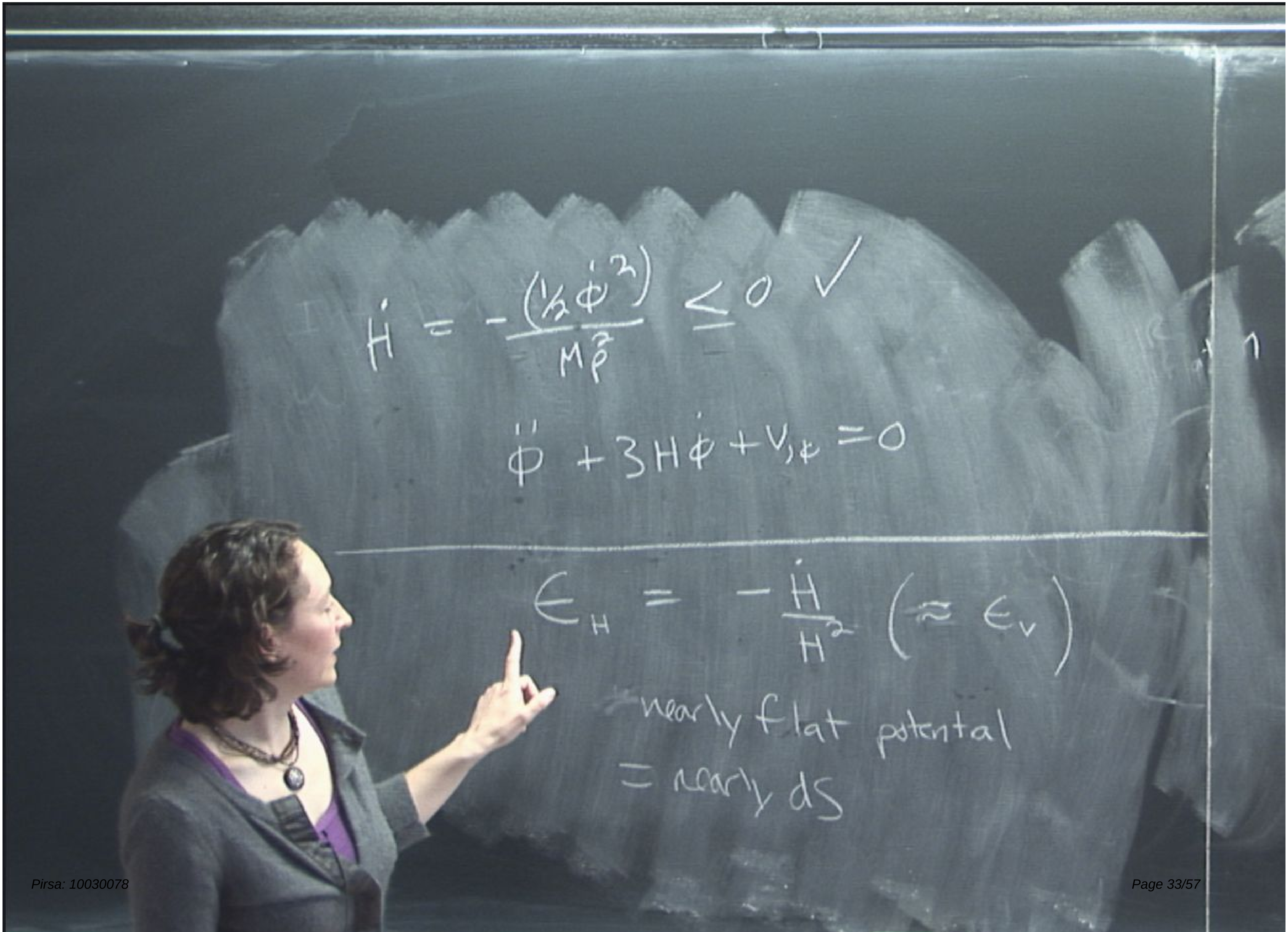
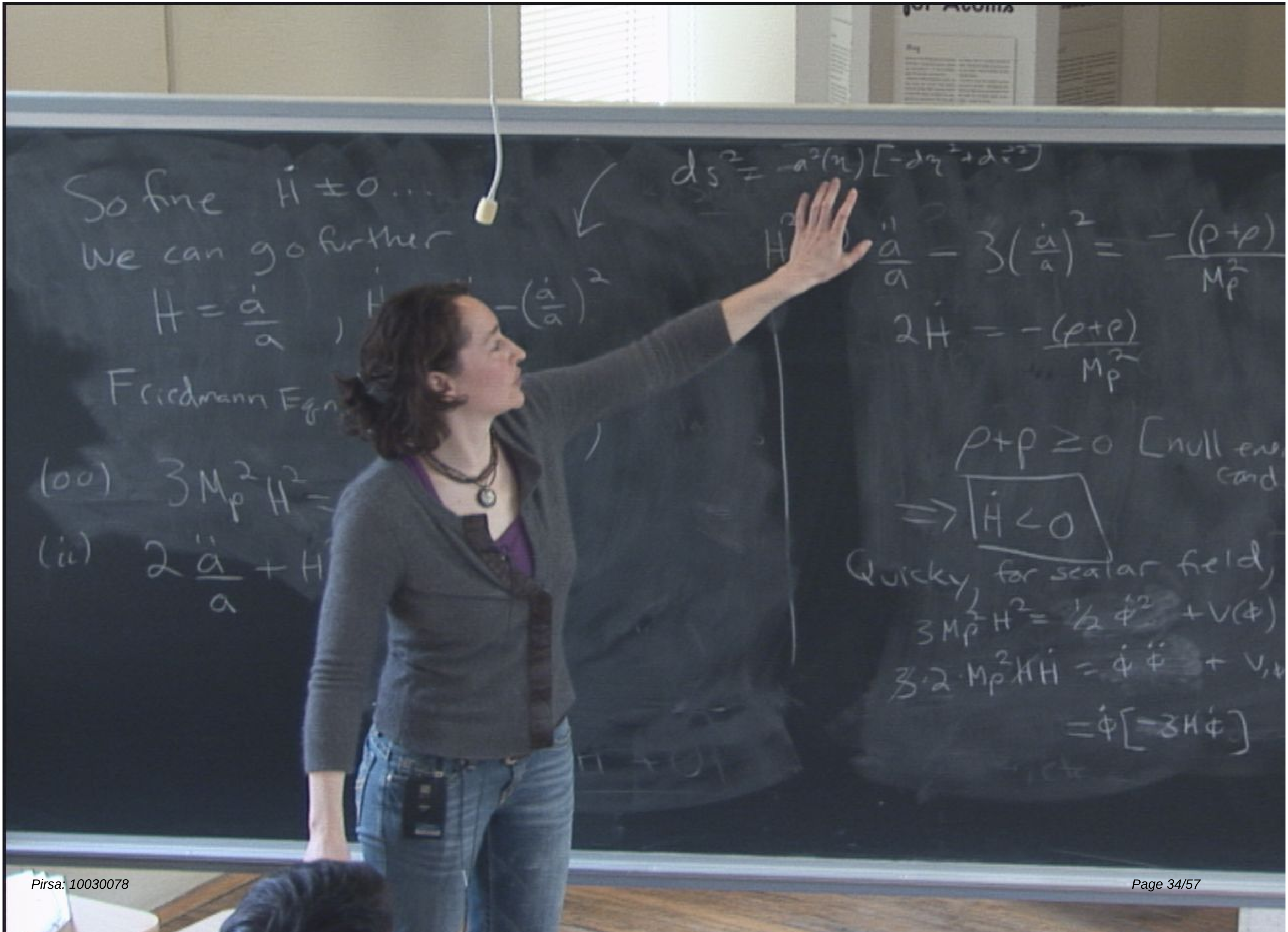


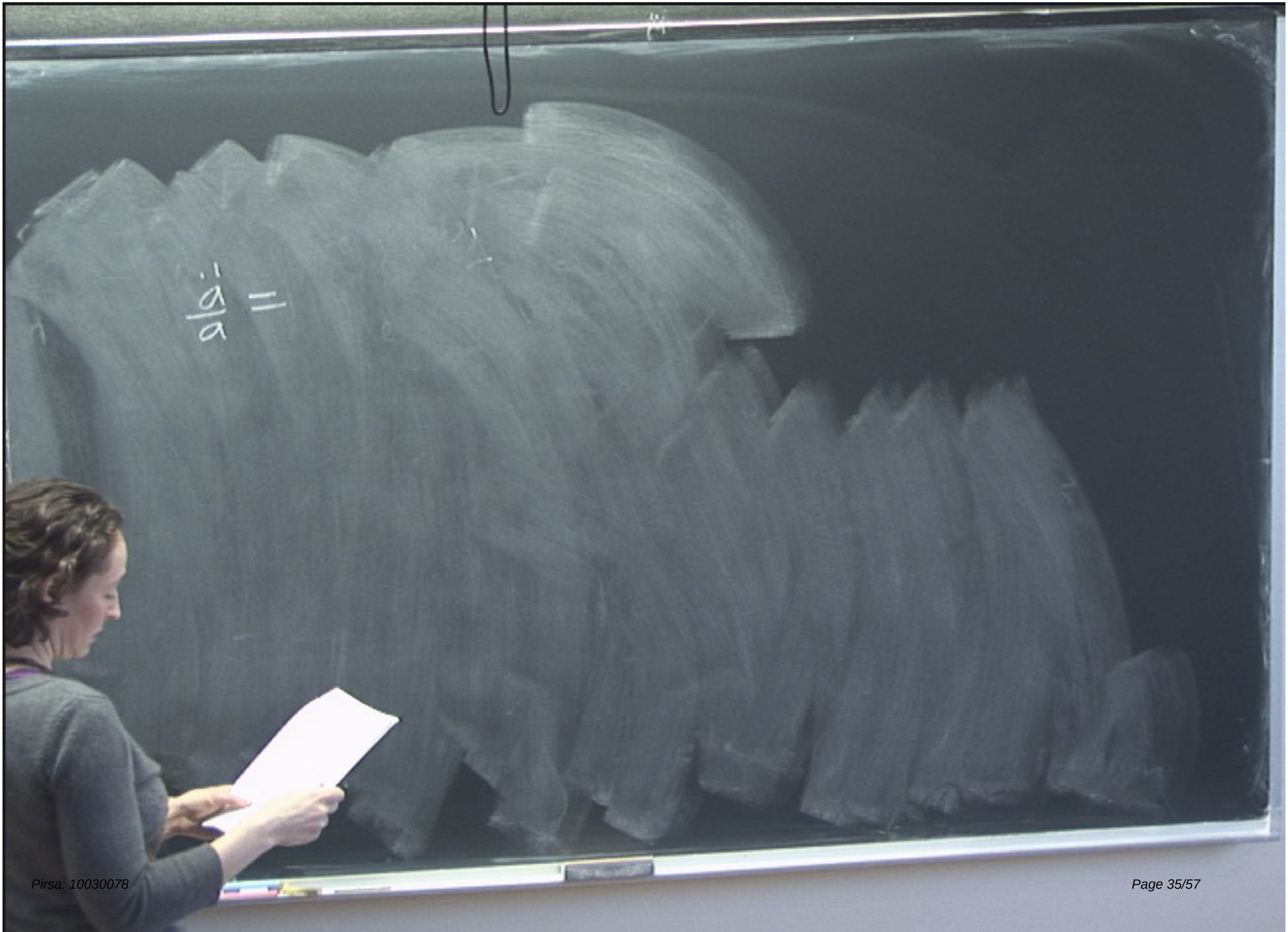
So fine $\dot{H} \neq 0$ ...
We can go further
$\dot{H} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2$
$H^2 + 2\frac{\ddot{a}}{a} - 3\left(\frac{\dot{a}}{a}\right)^2 = -\frac{(p+\rho)}{M_p^2}$

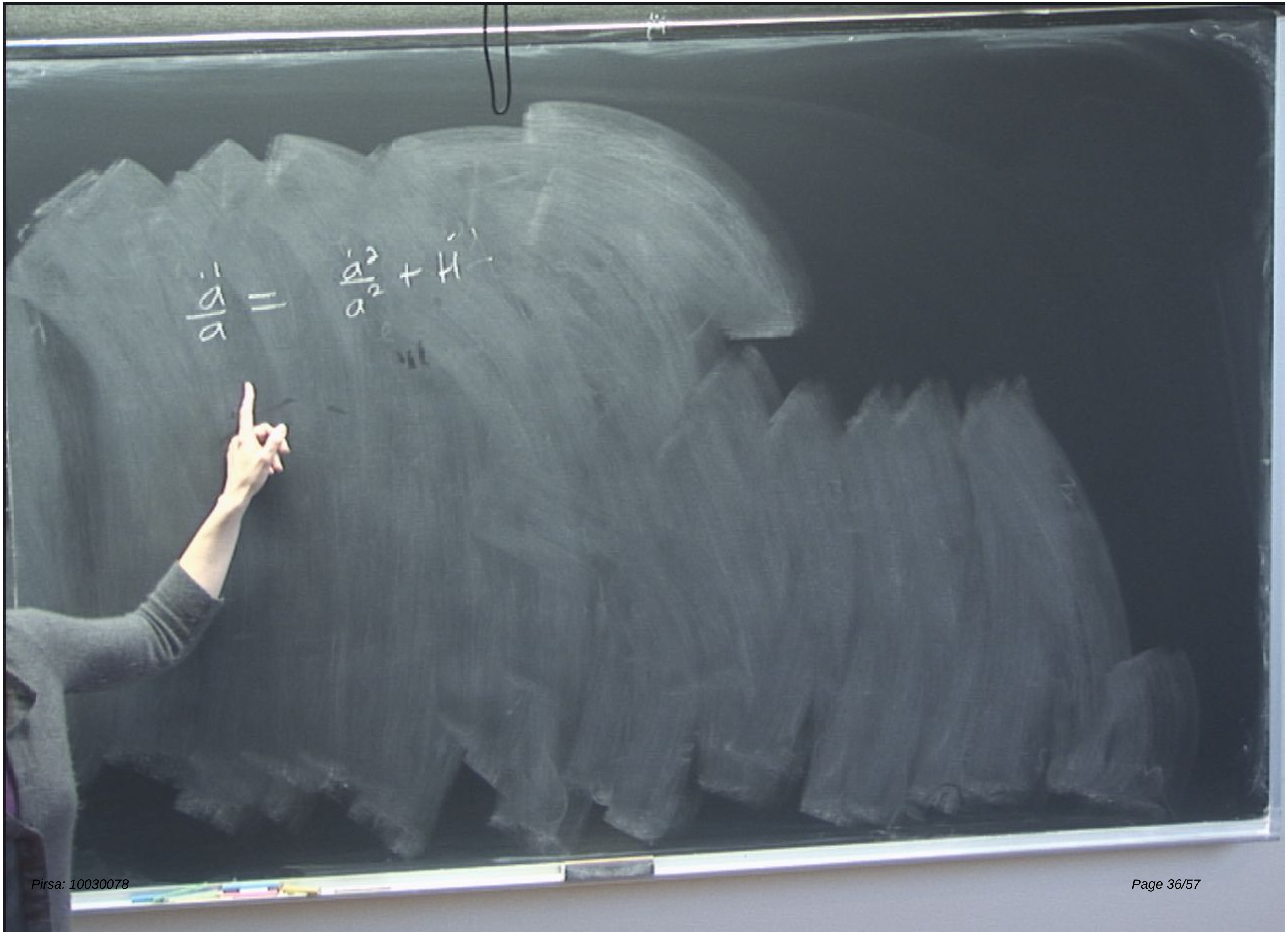

So fine Ḣ ≠ 0 ...
We can go further
H = ȧ/a , Ḣ = ä/a − (ȧ/a)²
Friedmann Eqns (Einstein eqns)
(00) 3Mp² H² = ρ
(ii) 2 ä/a + H² = −p/Mp²
H² + 2 ä/a − 3(ȧ/a)² = −(ρ+p)/Mp²
2Ḣ = −(ρ+p)/Mp²
ρ+p ≥ 0 [null en
cond
⇒ Ḣ < 0
Quicky, for scalar field,
3Mp² H² = ½ φ̇² + V(φ)
3·2·Mp² H Ḣ = φ̇ φ̈ + V,
= φ̇ [−3H φ̇]


$\dot{H} = -\frac{(\frac{1}{2}\dot{\phi}^2)}{M_p^2} < 0$ ✓


$\dot{H} = -\frac{(\frac{1}{2}\dot{\phi}^2)}{M_p^2} \leq 0$ ✓
$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$
$\epsilon_H = -\frac{\dot{H}}{H^2} \ (\approx \epsilon_V)$
nearly flat potential
= nearly dS


So fine
We can go further
Friedmann Eqn
Quicky, for scalar field,
ρ+p ≥ 0 [null energy condition]
⇒ Ḣ < 0


ä/a =


$\frac{\ddot{a}}{a} = \frac{\dot{a}^2}{a^2} + \dot{H}$

$$\frac{\ddot{a}}{a} = \frac{\dot{a}^2}{a^2} + \dot{H}$$

$$= H^2 + (-H^2\epsilon)$$

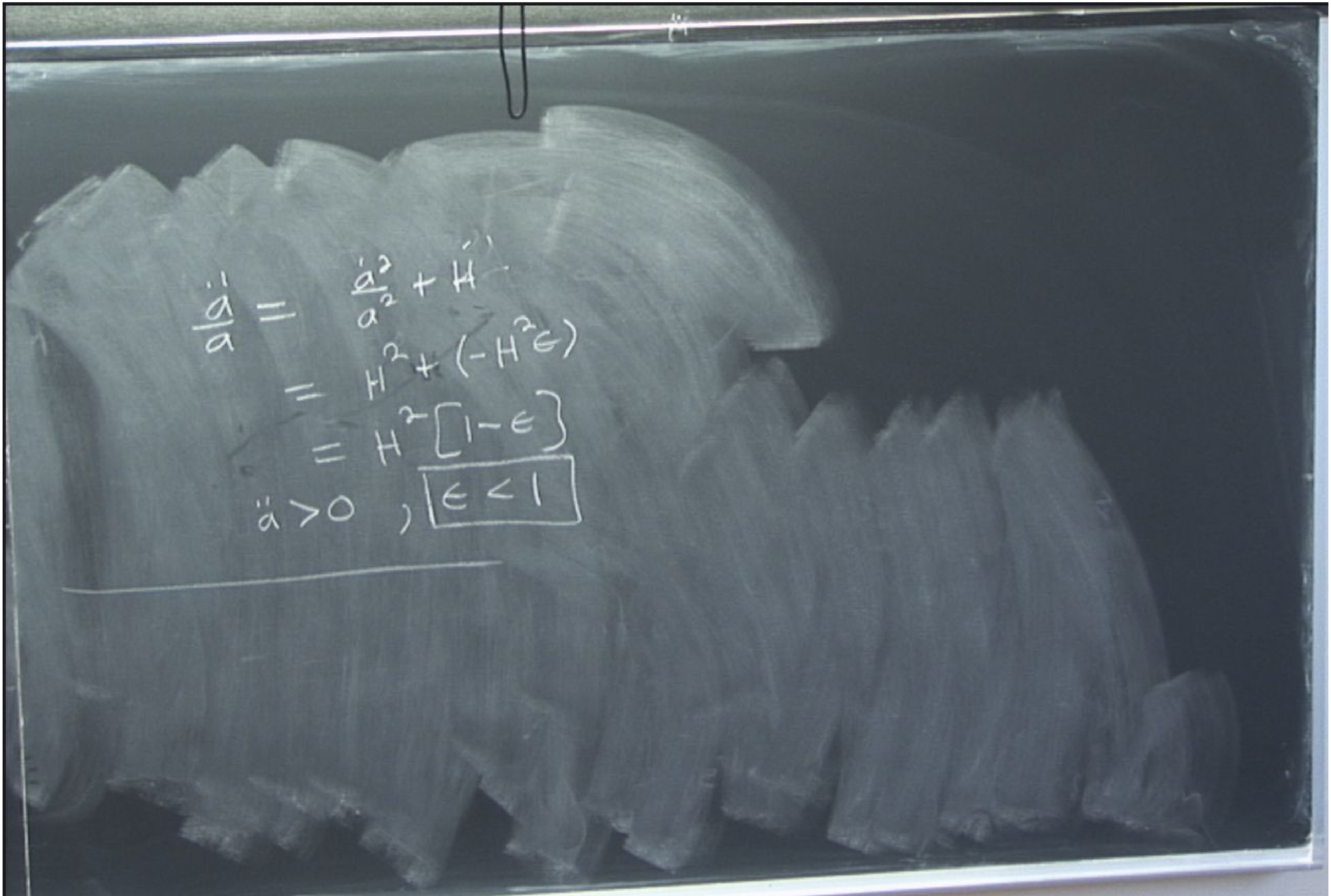
$$= H^2[1-\epsilon]$$

$$\frac{\ddot{a}}{a} = \frac{\dot{a}^2}{a^2} + \dot{H}$$

$$= H^2 + (-H^2\epsilon)$$

$$= H^2[1-\epsilon]$$

$$\ddot{a} > 0 \quad , \quad \boxed{\epsilon < 1}$$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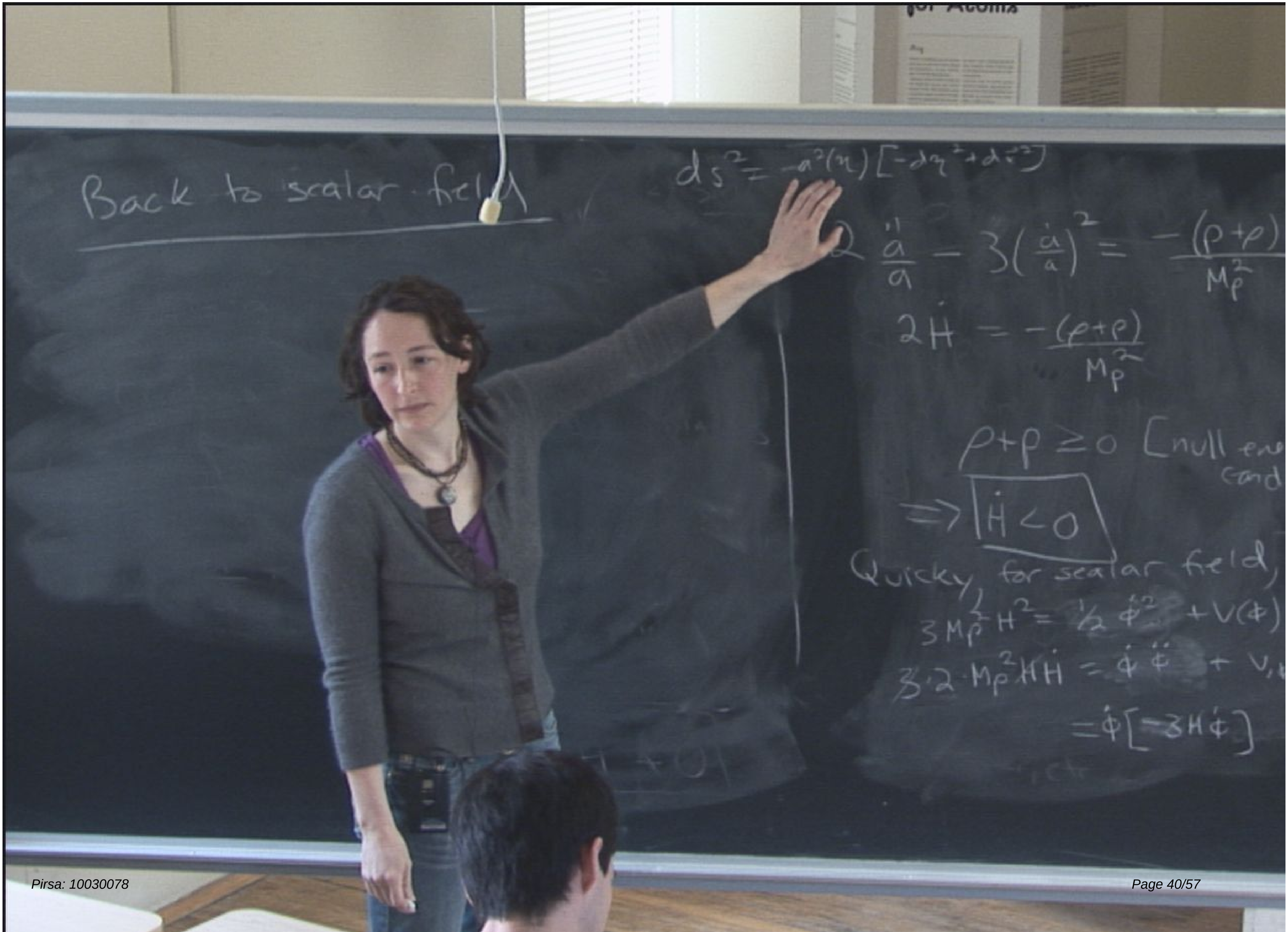
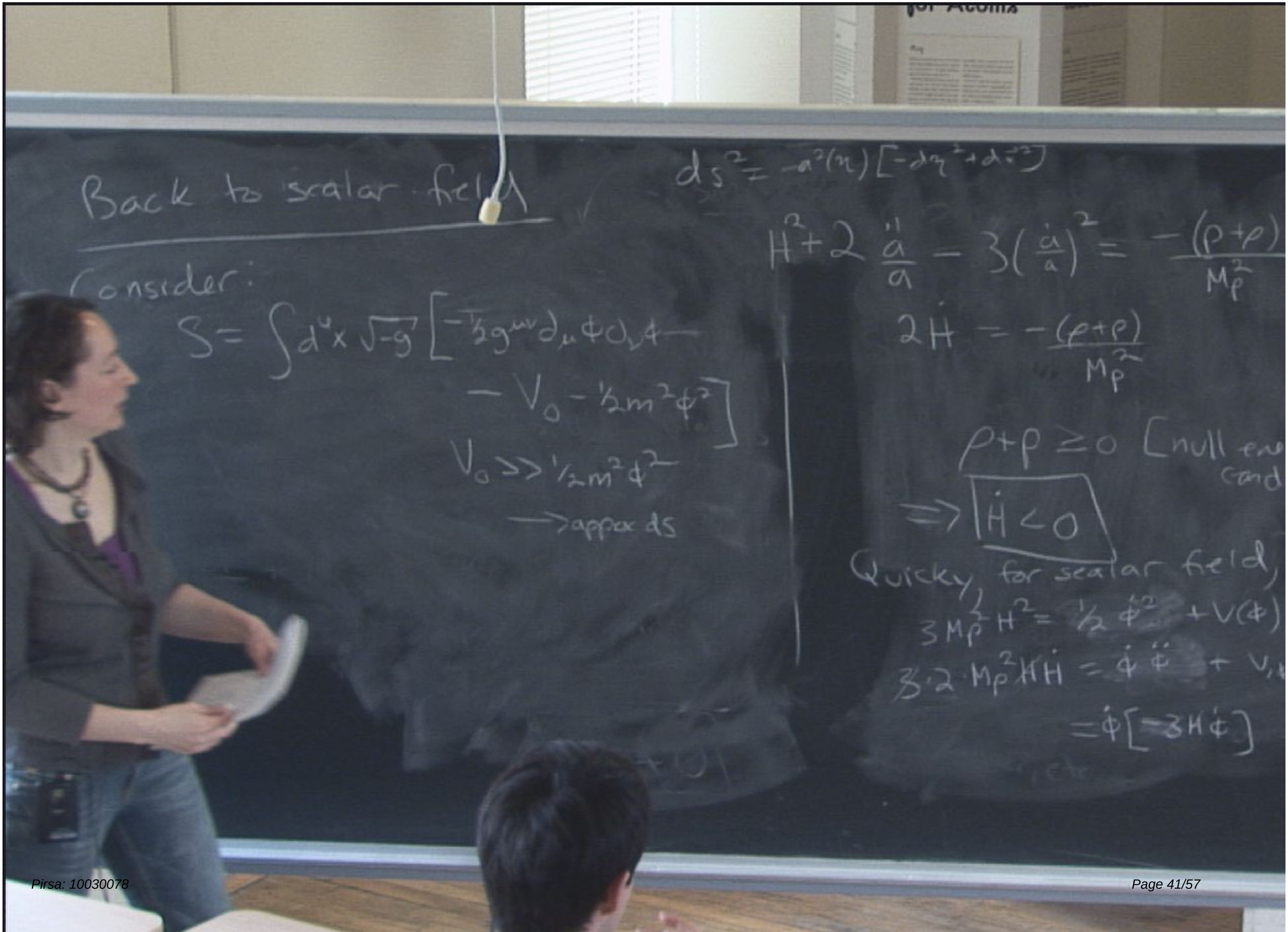
$= H^2 + (-H^2\epsilon)$
$= H^2[1-\epsilon]$
$\ddot{a} > 0$, $\epsilon < 1$


Back to scalar field
2H = −(ρ+p)/Mp²
ρ+p ≥ 0 [null
⇒ H < 0
Quicky, for scalar field



Back to scalar field
Consider:
S= ∫d^4x √-g [-½g^μν ∂_μφ∂_νφ
- V_0 - ½m^2φ^2]
V_0 >> ½m^2φ^2
→ approx dS
ds^2 = a^2(η)[-dη^2 + dx^2]
H^2 + 2 ä/a - 3(ȧ/a)^2 = -(ρ+p)/M_P^2
2Ḣ = -(ρ+p)/M_P^2
ρ+p ≥ 0 [null energy cond
⇒ Ḣ < 0
Quicky for scalar field,
3M_P^2 H^2 = ½φ̇^2 + V(φ)
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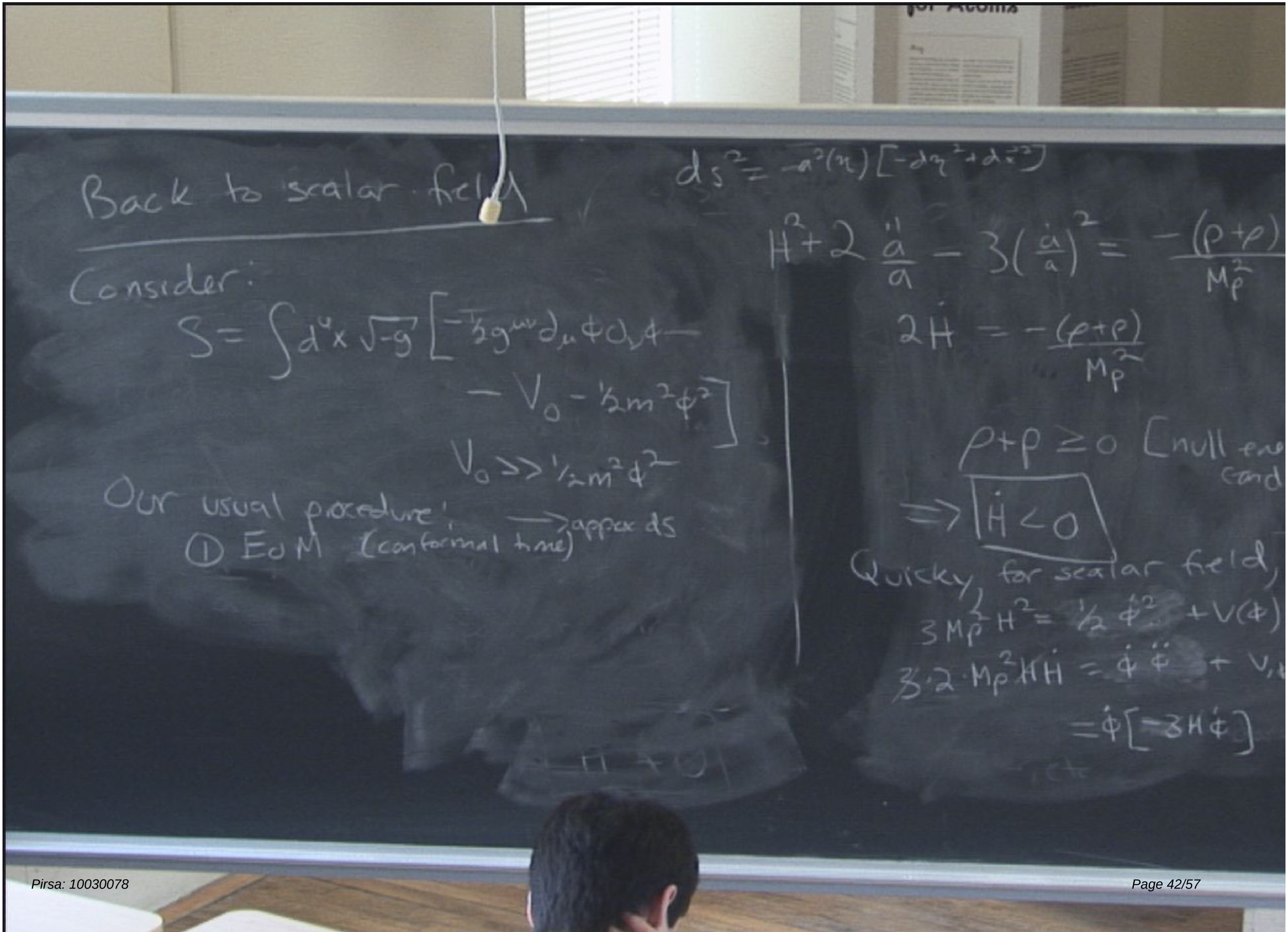
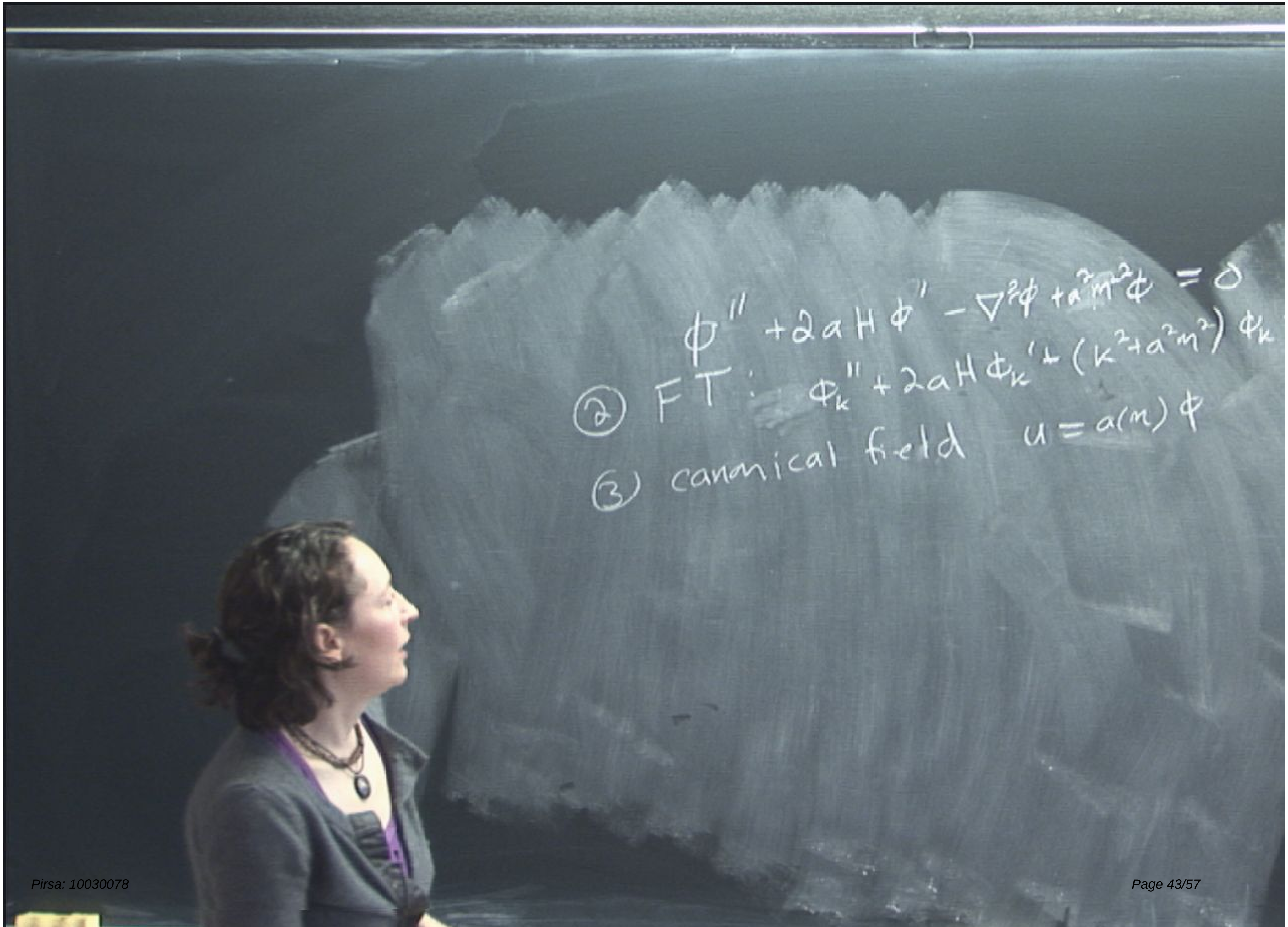

Back to scalar field
Consider:
$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_0 - \frac{1}{2} m^2 \phi^2 \right]$
$V_0 >> \frac{1}{2} m^2 \phi^2$
Our usual procedure: → approx dS
① EoM (conformal time)
$ds^2 = -a^2(\eta)[-d\eta^2 + dx^2]$
$\mathcal{H}^2 + 2\frac{a''}{a} - 3\left(\frac{a'}{a}\right)^2 = -\frac{(\rho+p)}{M_p^2}$
$2\dot{H} = -\frac{(\rho+p)}{M_p^2}$
$\rho + p \geq 0$ [null energy cond
$\Rightarrow \dot{H} < 0$
Quickly, for scalar field,
$3M_p^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
$3 \cdot 2 \cdot M_p^2 H \dot{H} = \dot{\phi}\ddot{\phi} + V_{,\phi}$
$= \dot{\phi}[-3H\dot{\phi}]$


φ'' + 2aHφ' − ∇²φ + a²m²φ = 0
② FT: φ_k'' + 2aHφ_k' + (k² + a²m²) φ_k
③ canonical field u = a(η) φ

$$\phi'' + 2aH\phi' - \nabla^2\phi + a^2m^2\phi = 0$$

② FT: $\phi_k'' + 2aH\phi_k' + (k^2 + a^2m^2)\phi_k$

③ canonical field $u = a(\eta)\phi$

$$u'' + \omega^2 u = 0$$

$$\omega^2 = k^2 + a^2m^2 - \frac{a''}{a}$$

$$= k^2 + a^2m^2 - a^2H^2(1$$

$\frac{m}{H} \to 0, \epsilon \to 0$

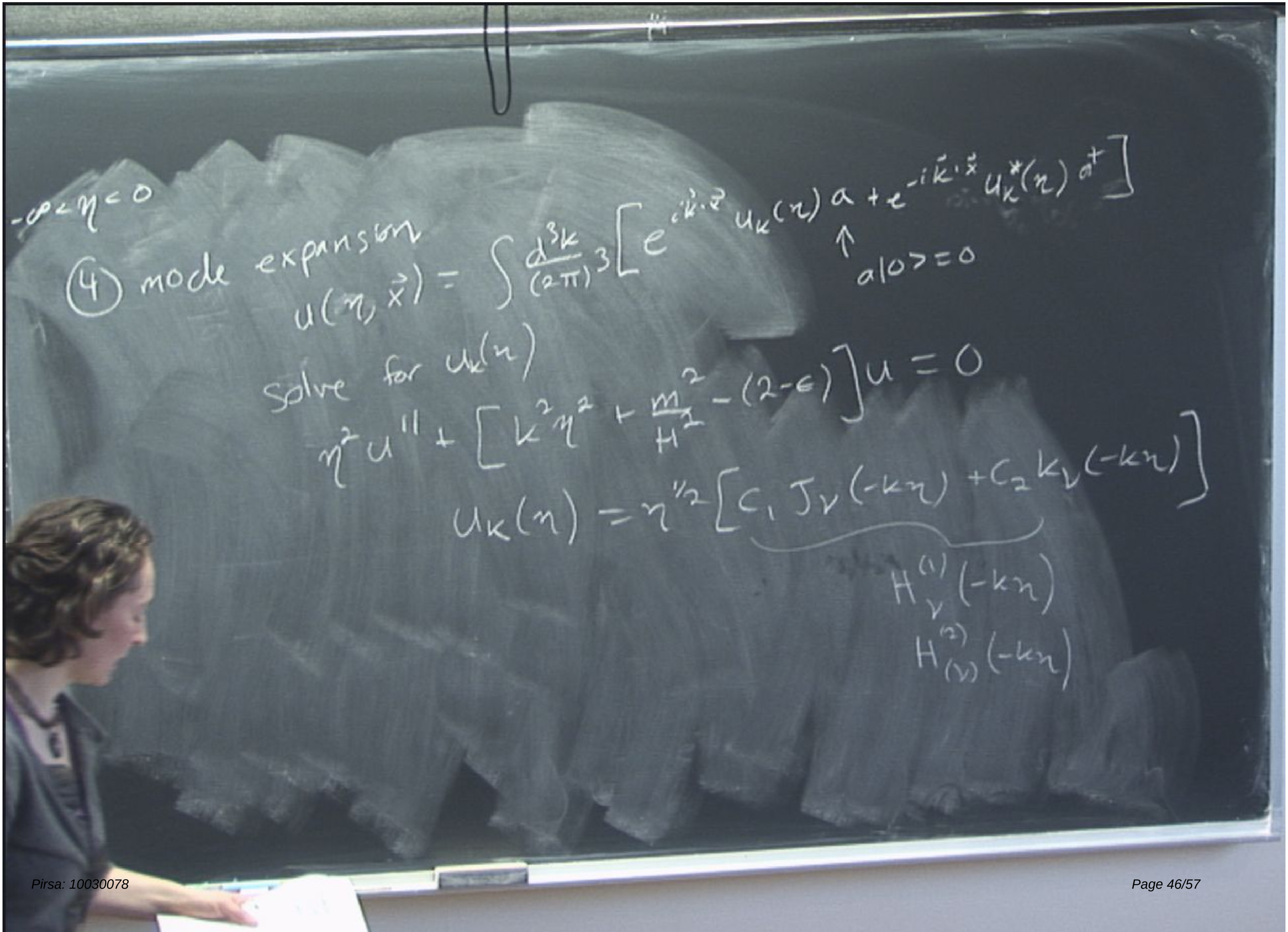
$K < aH$, growing/deca
(super-horizon) solutions

(4) mode expansion

$$u(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[e^{i\vec{k}\cdot\vec{x}} u_k(\eta)\, a + e^{-i\vec{k}\cdot\vec{x}} u_k^*(\eta)\, a^\dagger \right]$$

↑ $a|0\rangle = 0$

solve for $u_k(\eta)$


$-\infty < \eta < 0$
(4) mode expansion
$u(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[e^{i\vec{k}\cdot\vec{x}} u_k(\eta)\, a + e^{-i\vec{k}\cdot\vec{x}} u_k^*(\eta)\, a^\dagger \right]$
$a|0\rangle = 0$
solve for $u_k(\eta)$
$\eta^2 u'' + \left[k^2\eta^2 + \frac{m^2}{H^2} - (2-\epsilon) \right] u = 0$
$u_k(\eta) = \eta^{1/2} \left[c_1 J_\nu(-k\eta) + c_2 K_\nu(-k\eta) \right]$
$H_\nu^{(1)}(-k\eta)$
$H_{(\nu)}^{(2)}(-k\eta)$

$-\infty < \eta < 0$

④ mode expansion

$$u(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[e^{i\vec{k}\cdot\vec{x}} u_k(\eta)\, a + e^{-i\vec{k}\cdot\vec{x}} u_k^*(\eta)\, a^\dagger\right]$$

↑ $a|0\rangle = 0$

solve for $u_k(\eta)$

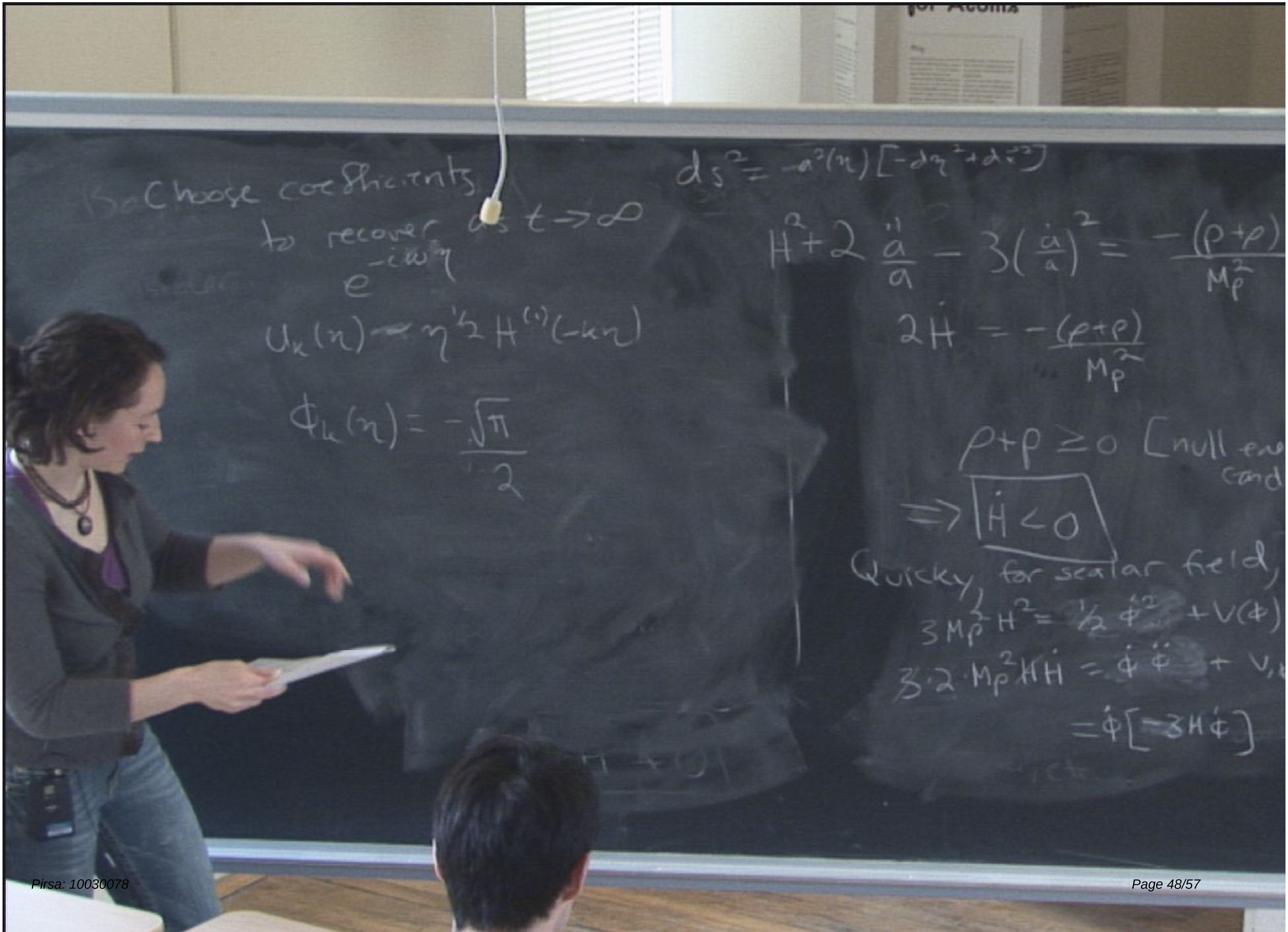
$$\eta^2 u'' + \left[k^2\eta^2 + \frac{m^2}{H^2} - (2-\epsilon)\right]u = 0$$

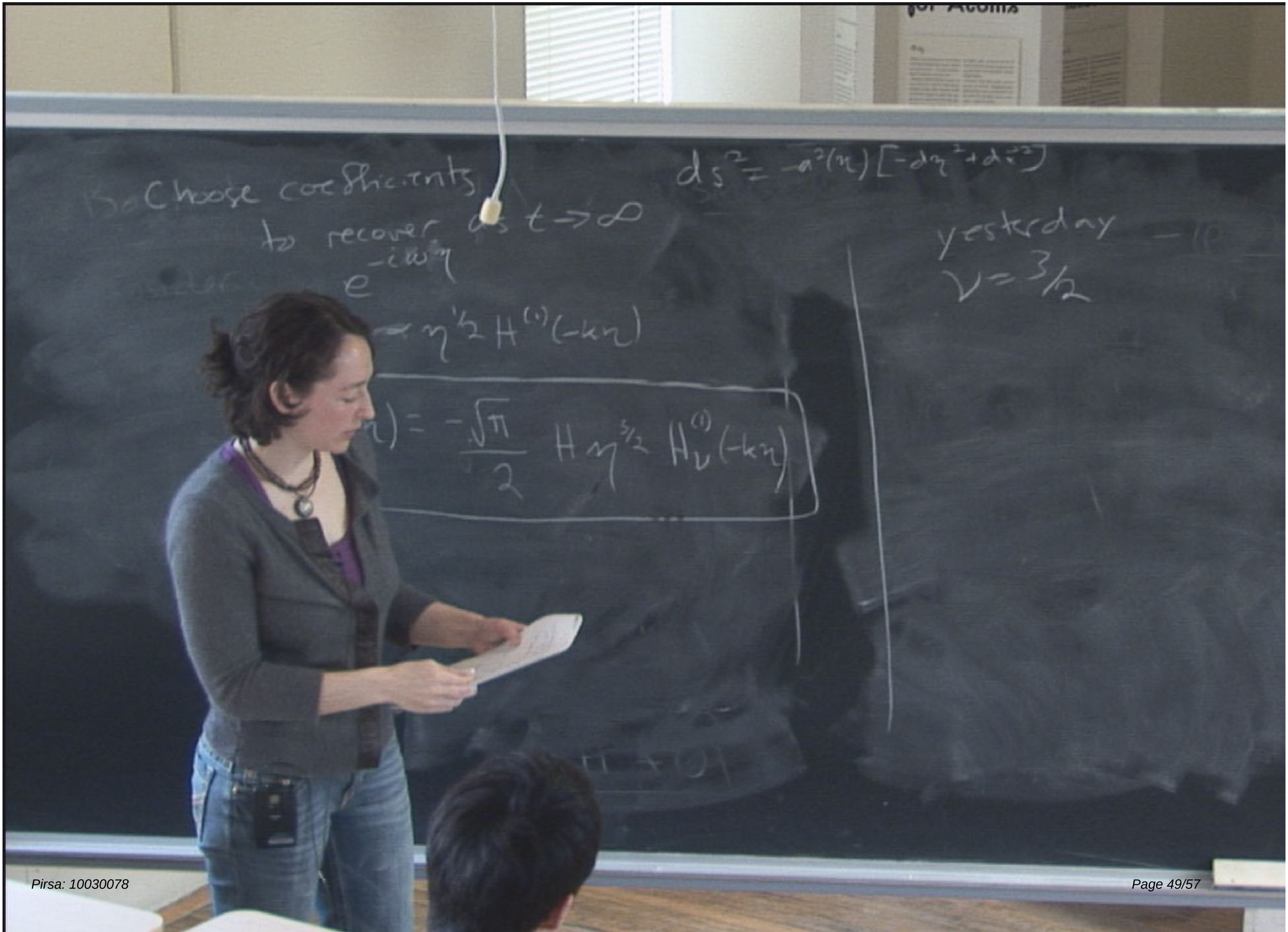
$$u_k(\eta) = \eta^{1/2}\left[c_1 J_\nu(-k\eta) + c_2 K_\nu(-k\eta)\right]$$

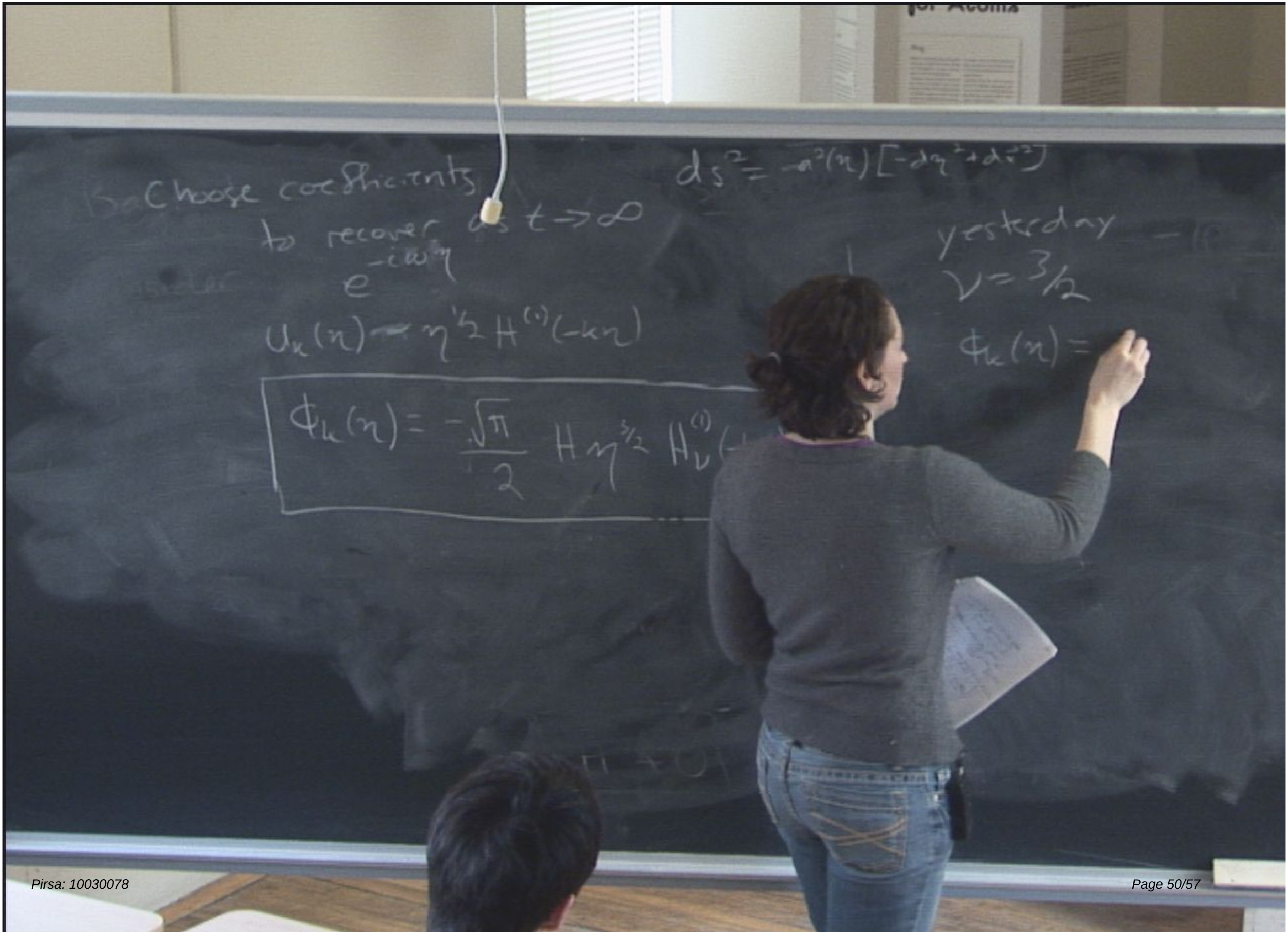
$H_\nu^{(1)}(-k\eta)$

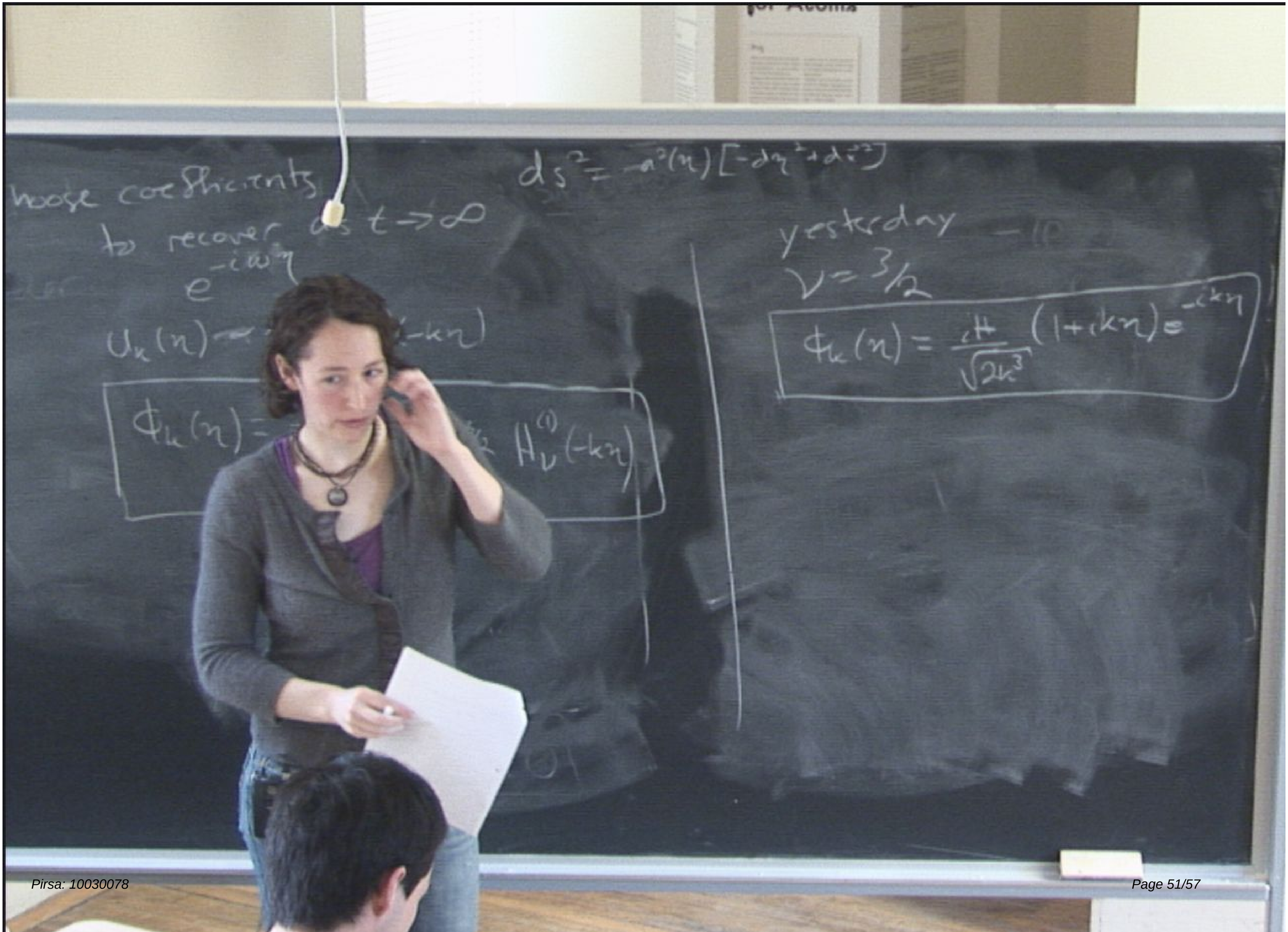
$H_{(\nu)}^{(2)}(-k\eta)$

$$\nu = \sqrt{9/4 - \frac{m^2}{H^2} - \epsilon}$$

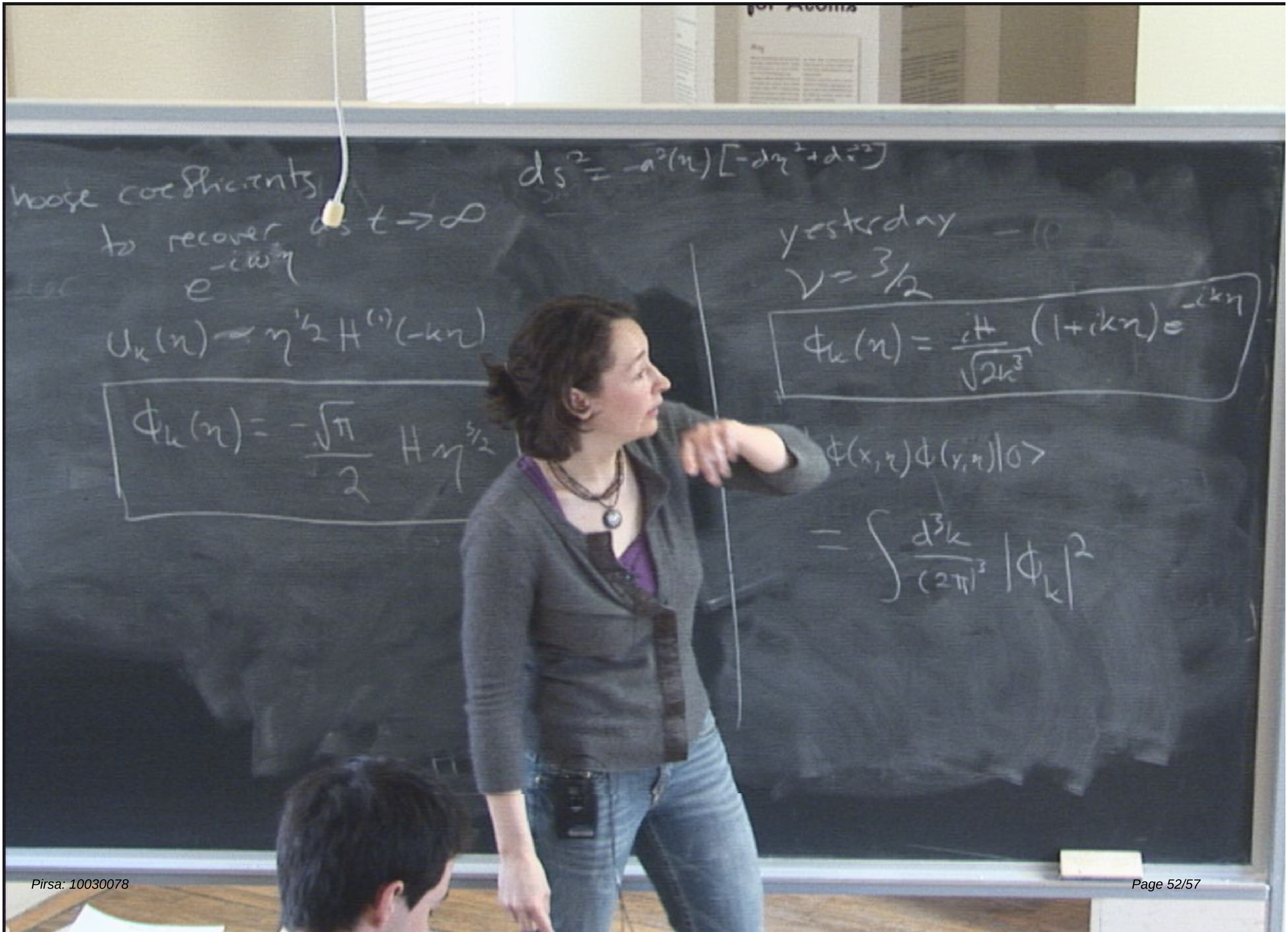

Choose coefficients
to recover as t → ∞
e^{-iωη}
U_k(η) ∝ η^{1/2} H^{(1)}(-kη)
φ_k(η) = -√π / 2
ds² = a²(η)[-dη² + dx²]
H² + 2 ä/a - 3(ȧ/a)² = -(ρ+p)/M_p²
2Ḣ = -(ρ+p)/M_p²
ρ+p ≥ 0 [null energy cond
⇒ Ḣ < 0
Quicky, for scalar field,
3M_p² H² = ½ φ̇² + V(φ)
3·2·M_p² H Ḣ = φ̇ φ̈ + V,
= φ̇[-3Hφ̇]

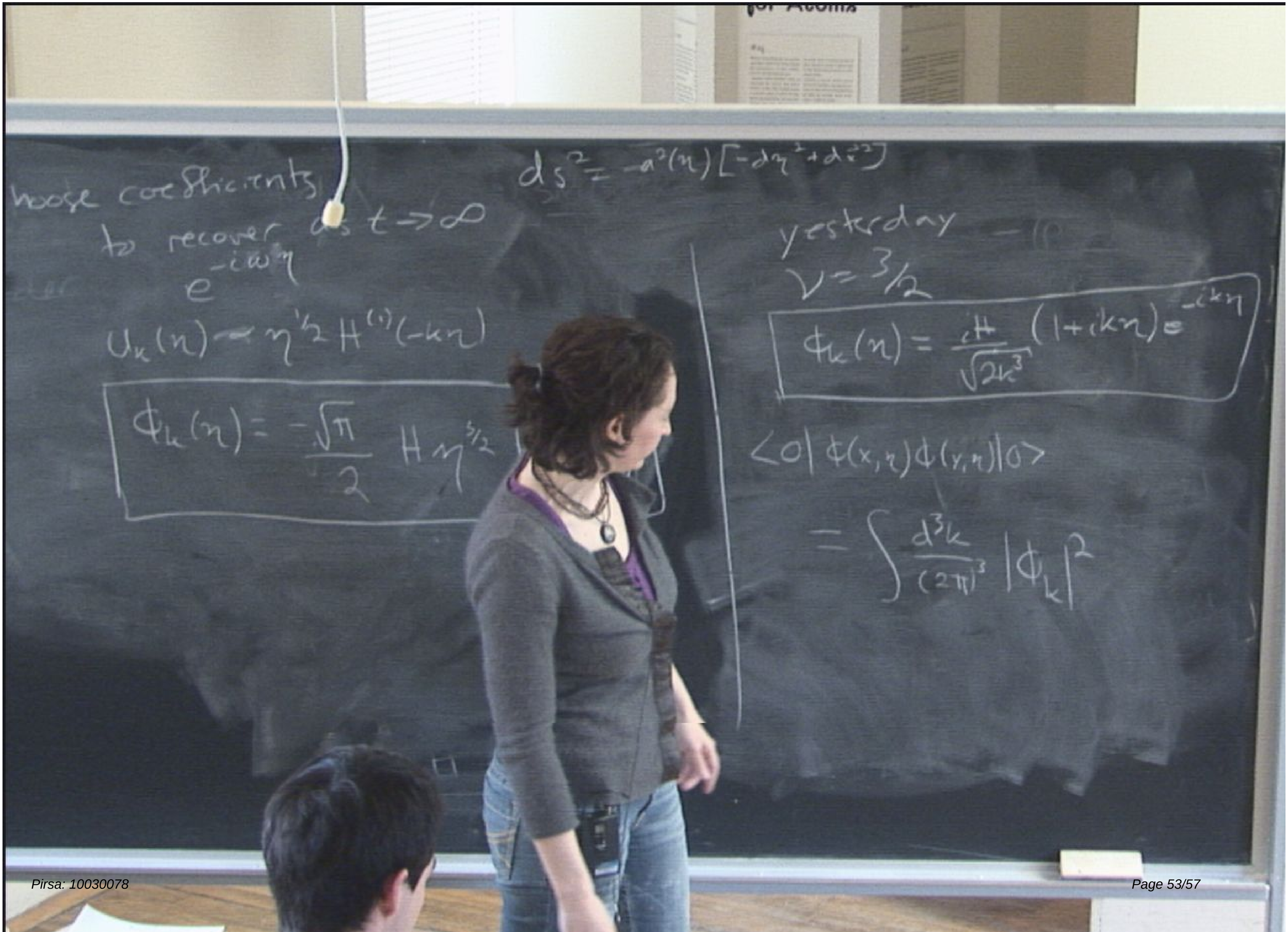

Choose coefficients
to recover as t → ∞
yesterday
ν = 3/2


Choose coefficients
to recover as t → ∞
e^{-iωη}
u_k(η) ∝ η^{1/2} H^{(1)}(-kη)
φ_k(η) = -√π/2 H η^{3/2} H_ν^{(1)}(
ds² = a²(η)[-dη² + dx²]
yesterday
ν = 3/2
φ_k(η) =

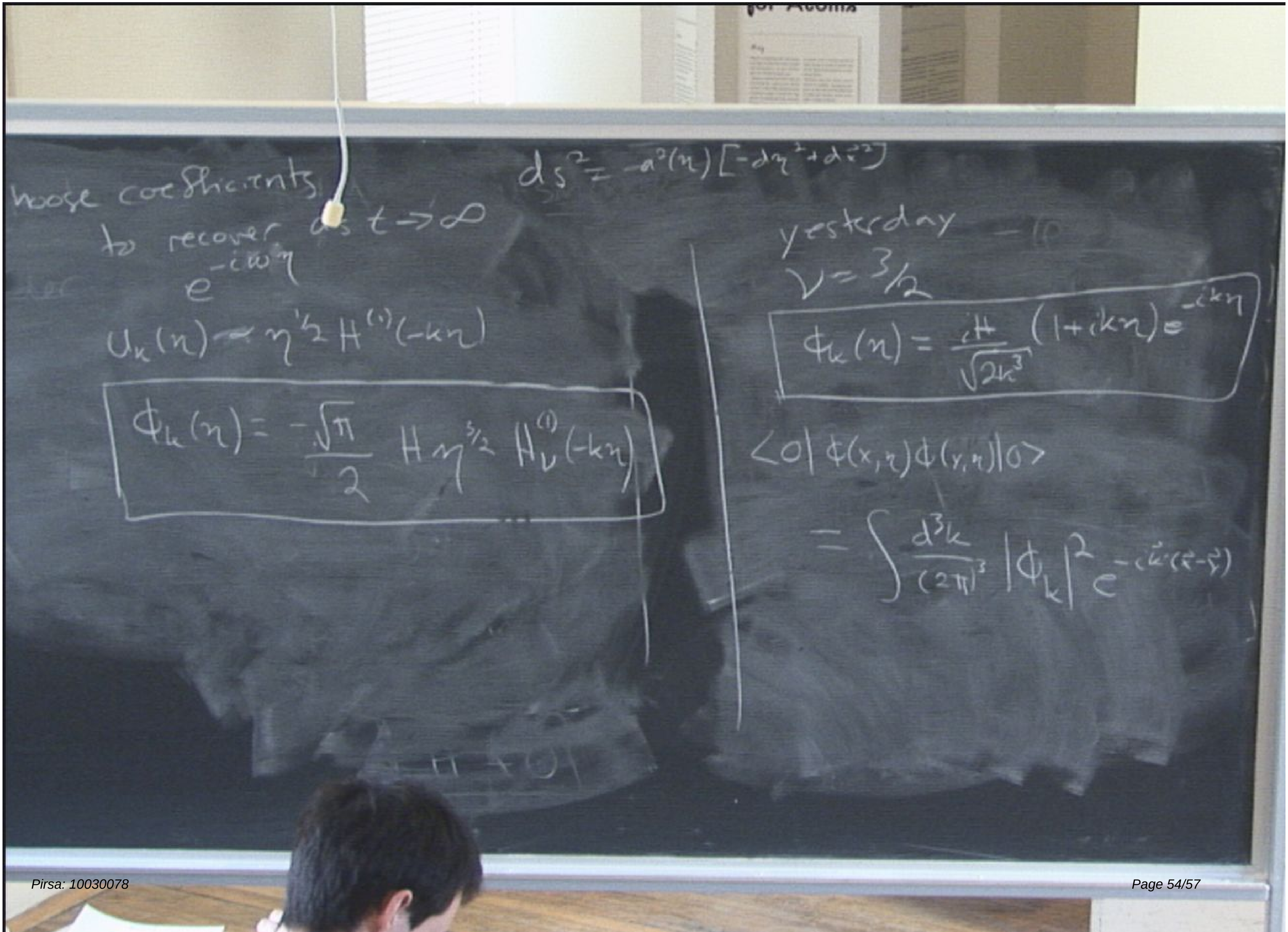


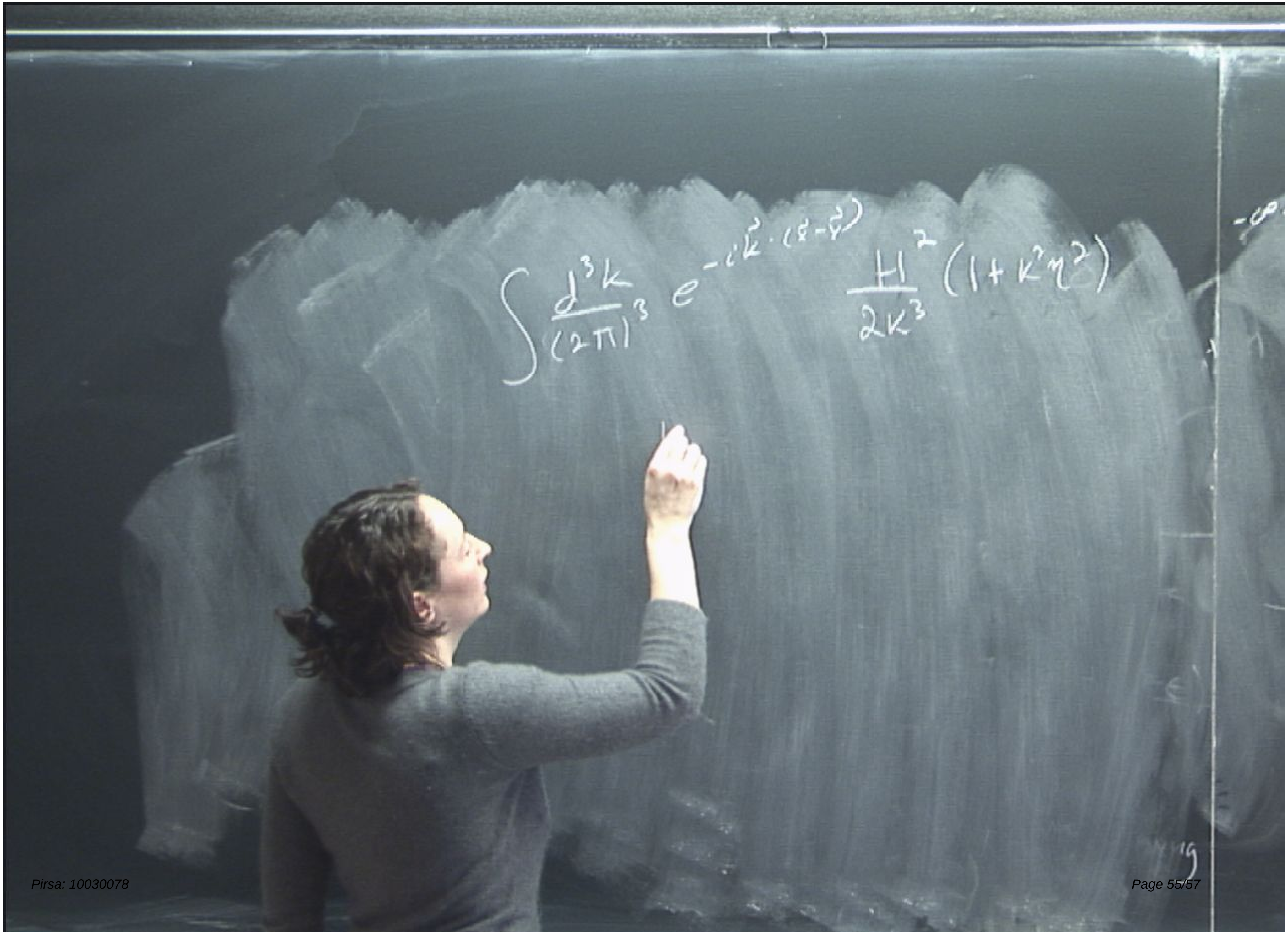
hoose coefficients
to recover as t→∞
ds² = a²(η) [-dη² + dx²]
yesterday
ν = 3/2
φk(η) = iH/√(2k³) (1+ikη) e^(-ikη)

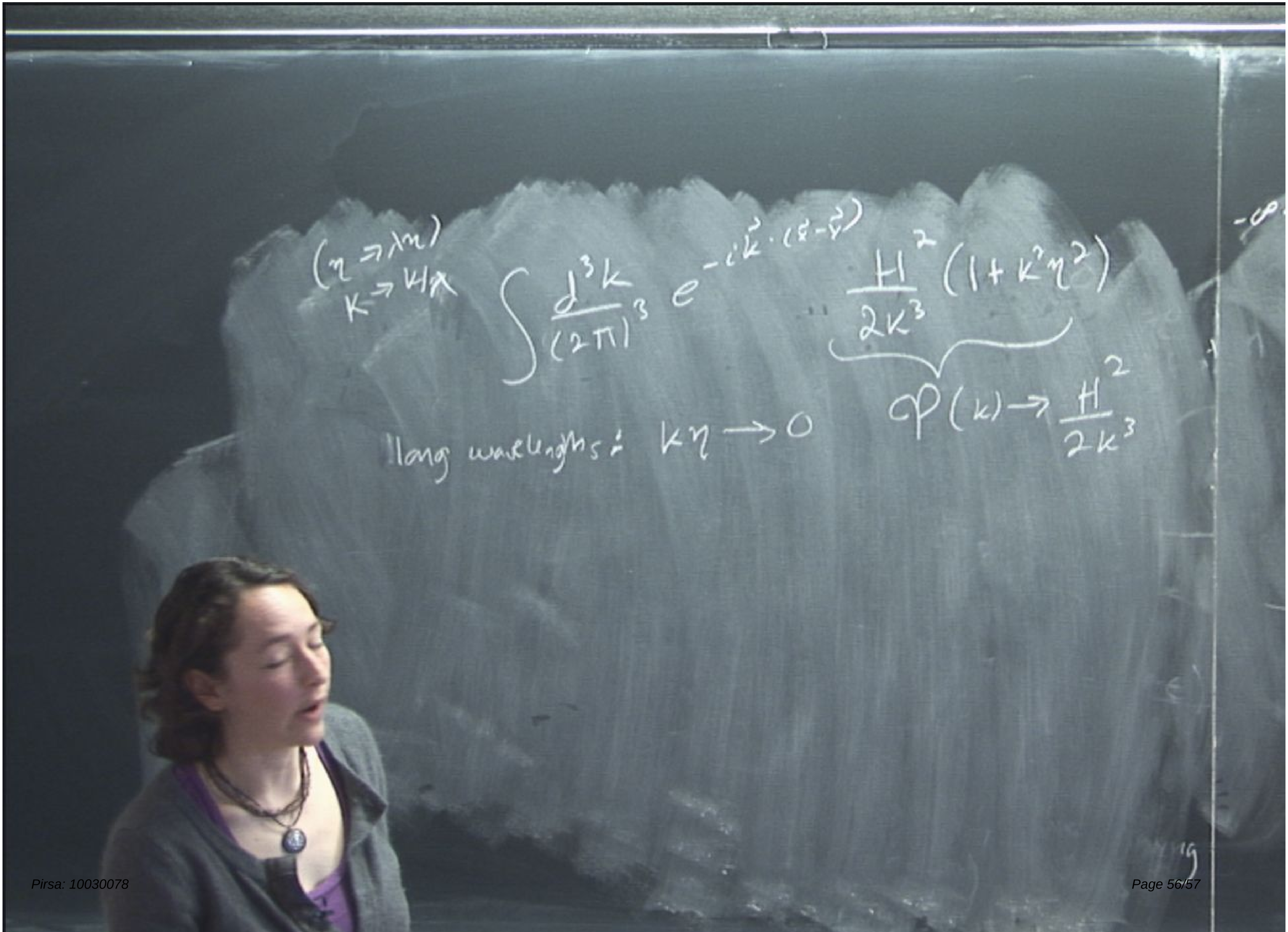

ds² = −a²(η) [−dη² + dx²]
hoose coefficients
to recover as t → ∞
e^{−iωη}
U_k(η) ∝ η^{1/2} H^{(1)}(−kη)
φ_k(η) = −√π/2 H η^{3/2}
yesterday
ν = 3/2
φ_k(η) = iH/√(2k³) (1 + ikη) e^{−ikη}
φ(x,η) φ(y,η)|0>
= ∫ d³k/(2π)³ |φ_k|²



choose coefficients
to recover as t → ∞
e^{-iωη}
yesterday
ν = 3/2


hoose coefficients
to recover as t → ∞
e^{-iωη}
u_k(η) ∝ η^{1/2} H^{(1)}(-kη)
φ_k(η) = -√π/2 H η^{3/2} H_ν^{(1)}(-kη)
ds² = a²(η) [-dη² + dx²]
yesterday
ν = 3/2
φ_k(η) = iH/√(2k³) (1 + ikη) e^{-ikη}
<0| φ(x,η) φ(y,η) |0>
= ∫ d³k/(2π)³ |φ_k|² e^{-ik·(x-y)}


$\int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \frac{H^2}{2k^3}(1+k^2\eta^2)$


$(\eta \to \lambda\eta)$
$k \to k/\lambda$
$\int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \frac{H^2}{2k^3}(1+k^2\eta^2)$
long wavelengths: $k\eta \to 0$
$\varphi(k) \to \frac{H^2}{2k^3}$

$(\eta \to \lambda \eta)$
$k \to k/\lambda$

$$\int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \underbrace{\frac{H^2}{2k^3}(1+k^2\eta^2)}$$

long wavelengths: $k\eta \to 0$ $\quad \mathcal{P}(k) \to \frac{H^2}{2k^3}$

$dS \longleftrightarrow$ scale-invariance