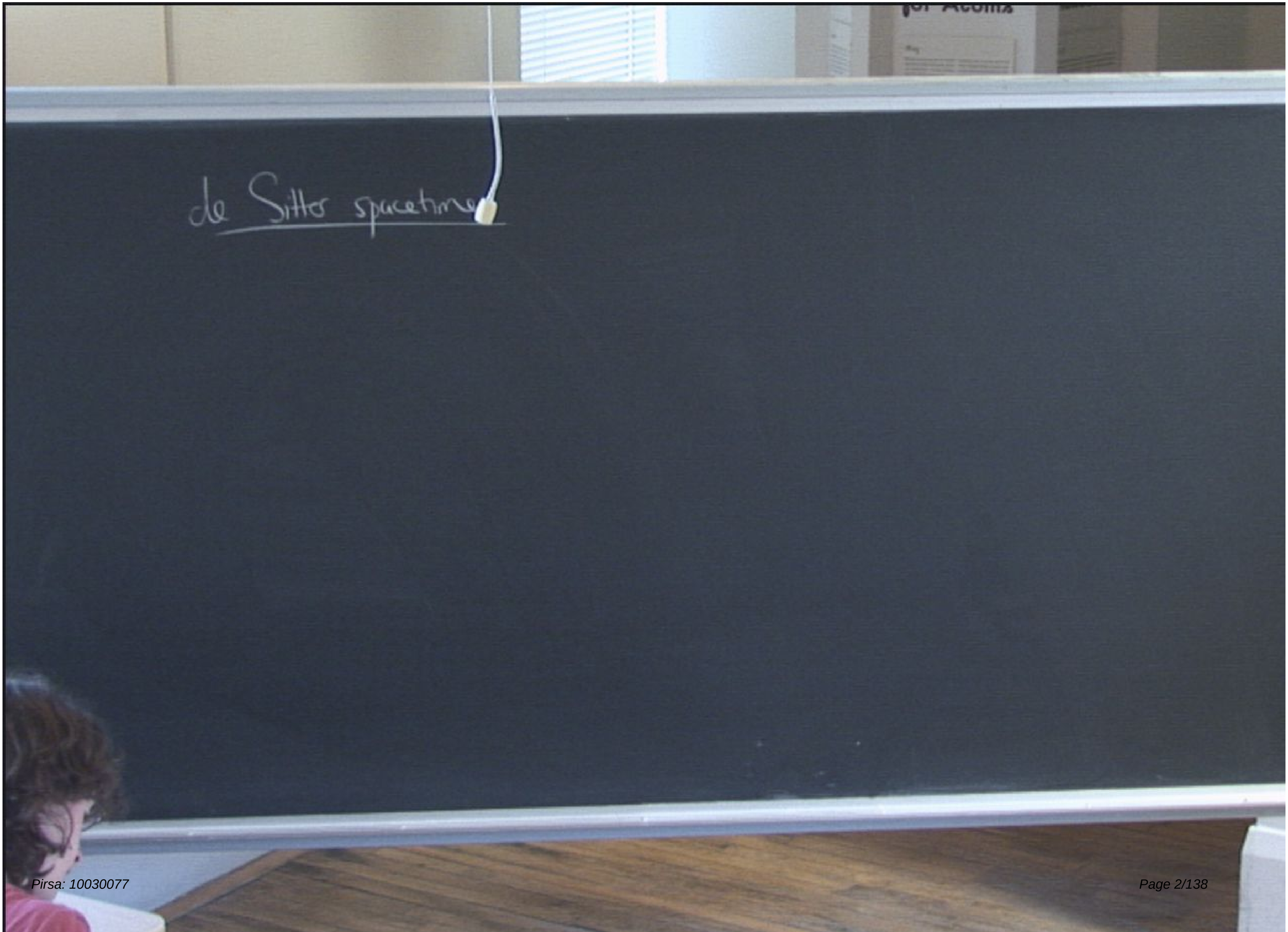
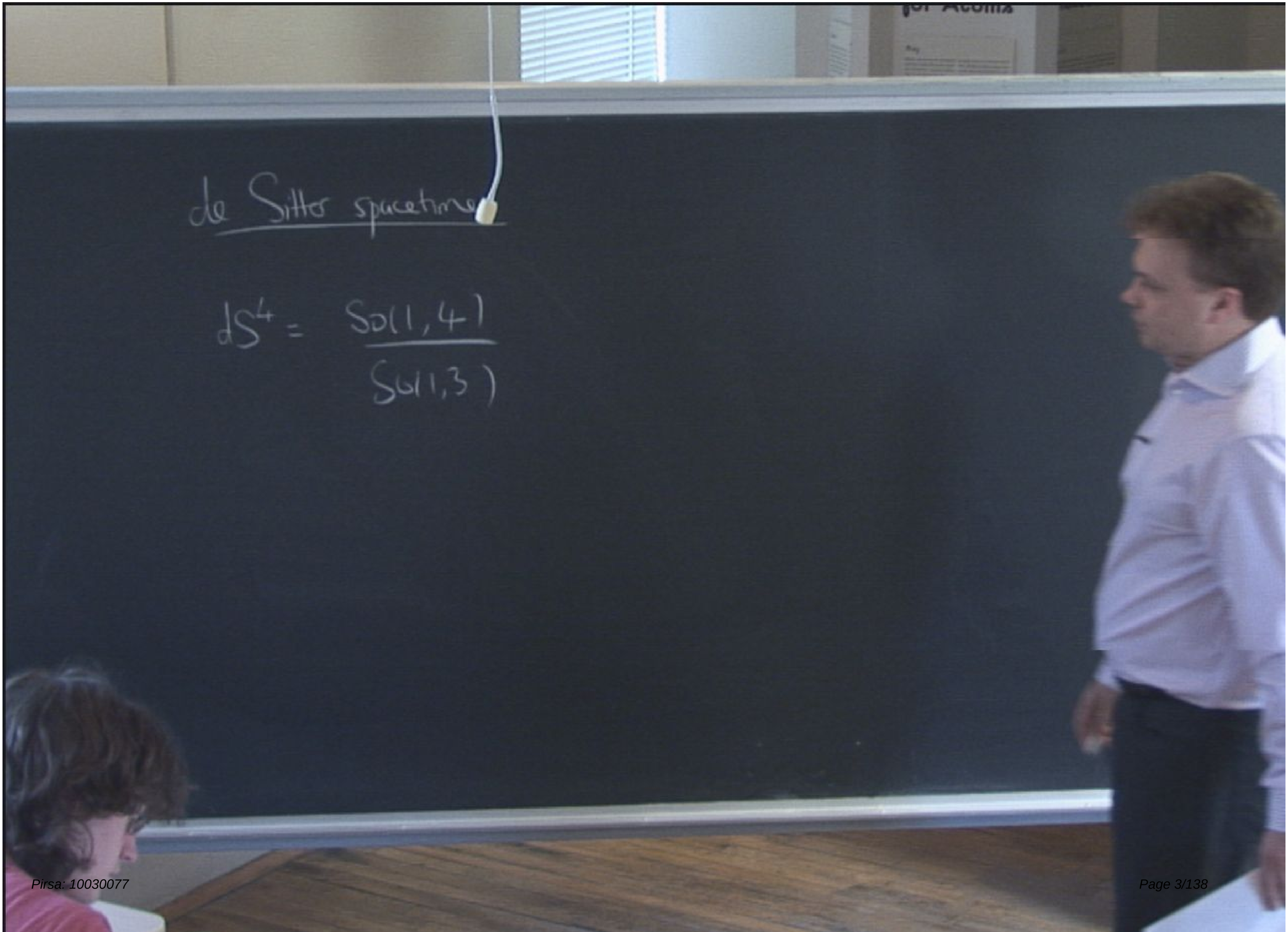
Title: Explorations in Cosmology (PHYS 649) - Lecture 8

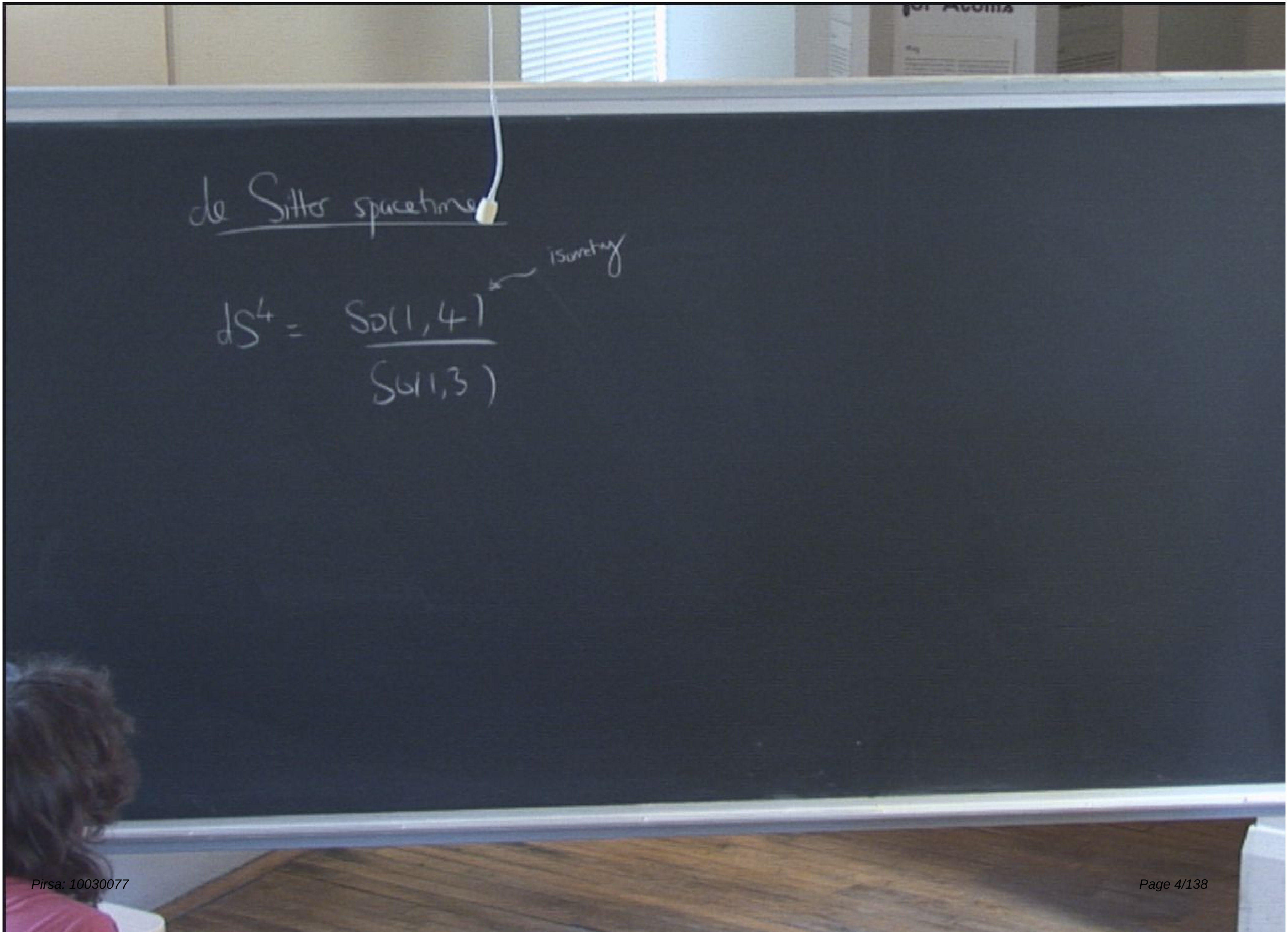
Date: Mar 24, 2010 09:00 AM

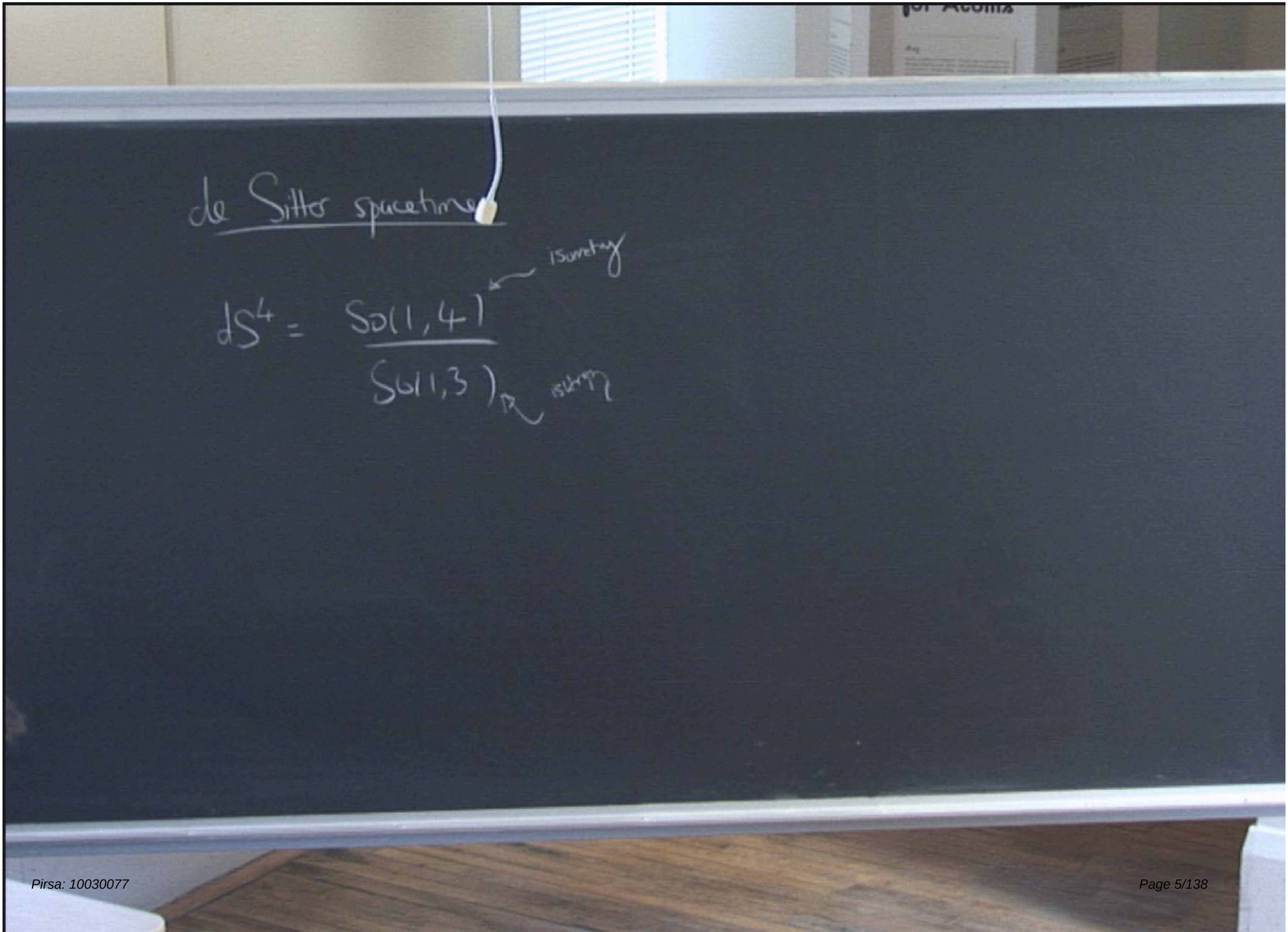
URL: http://pirsa.org/10030077

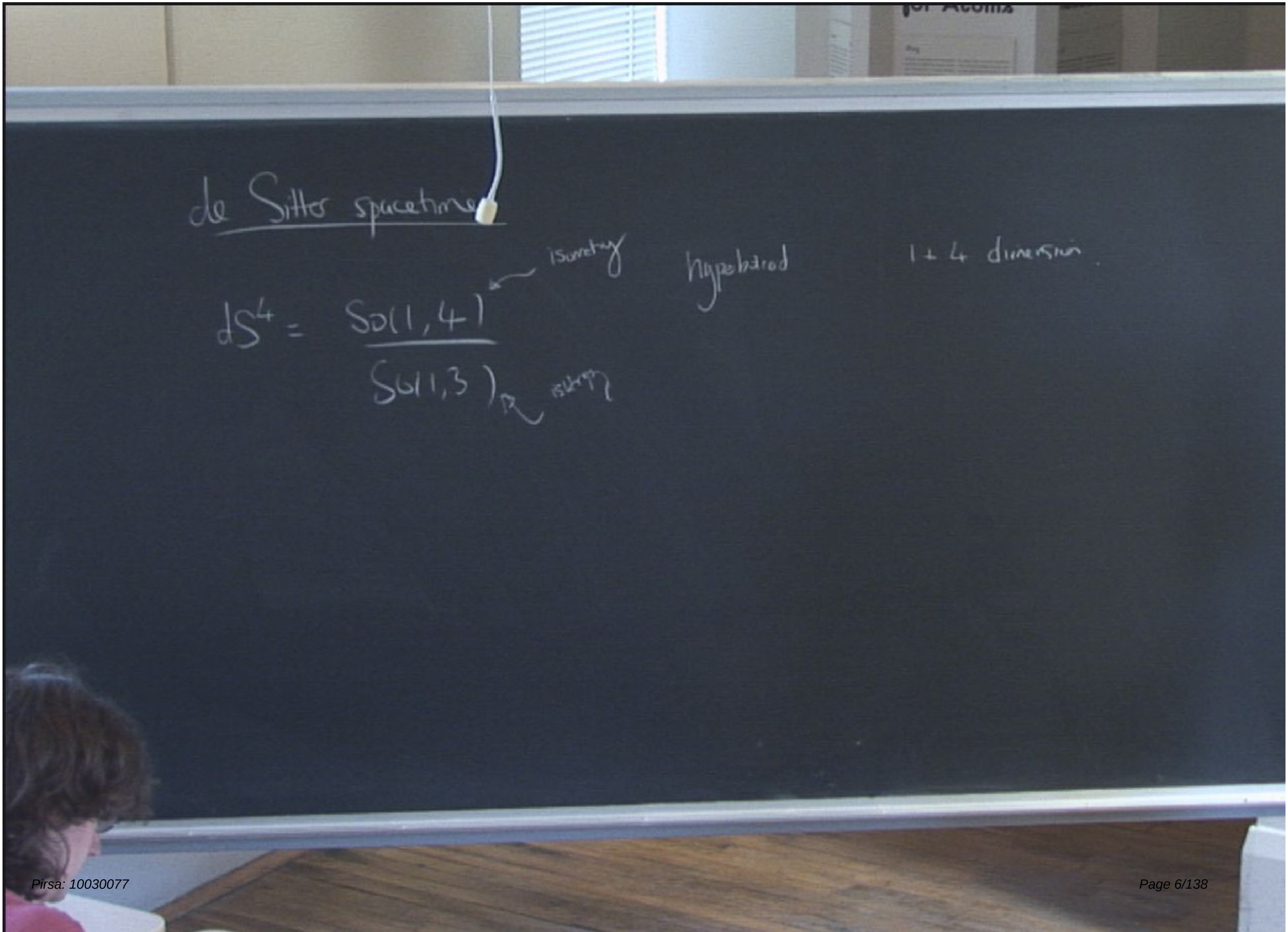
Abstract:

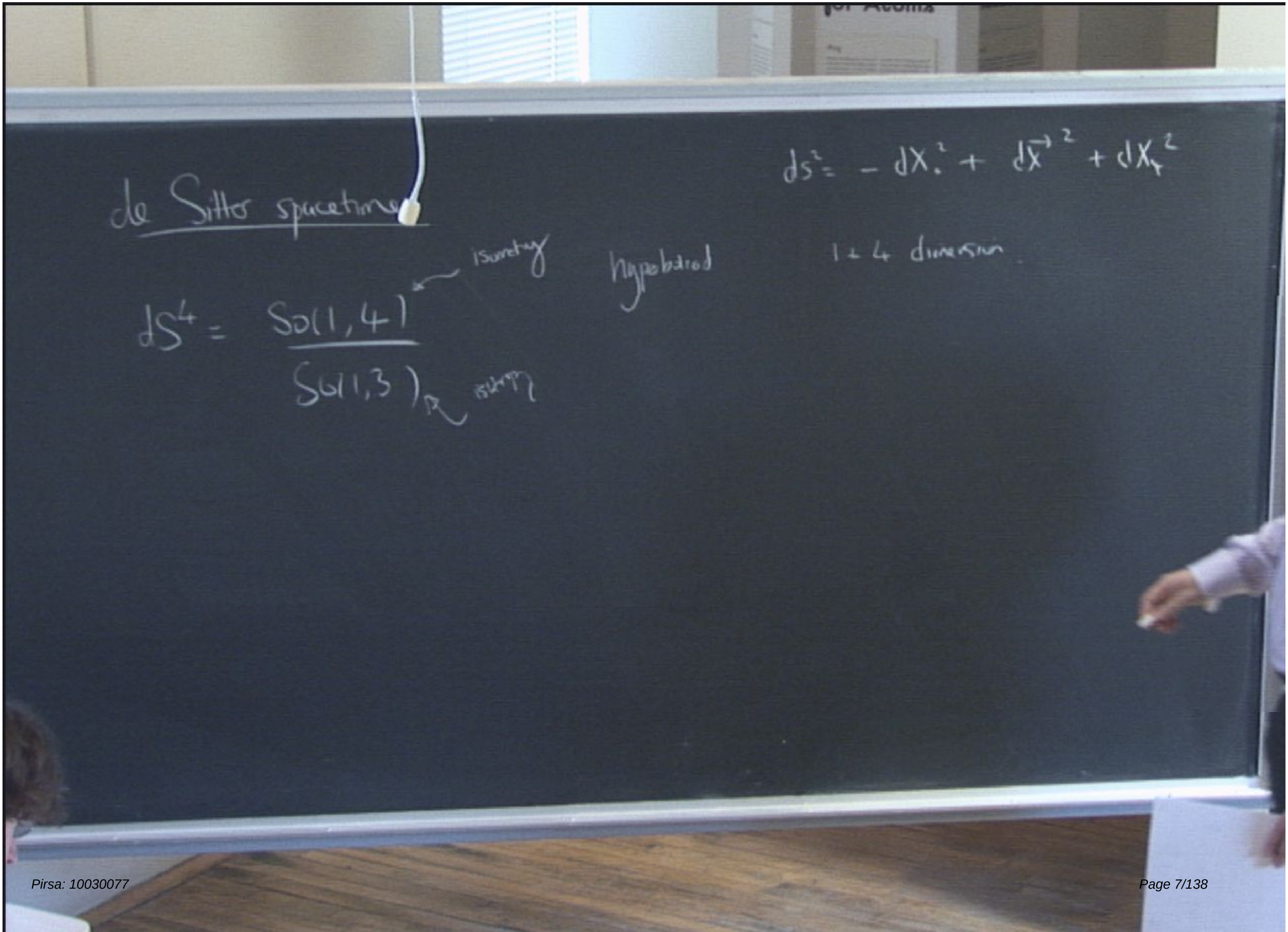

de Sitter spacetime

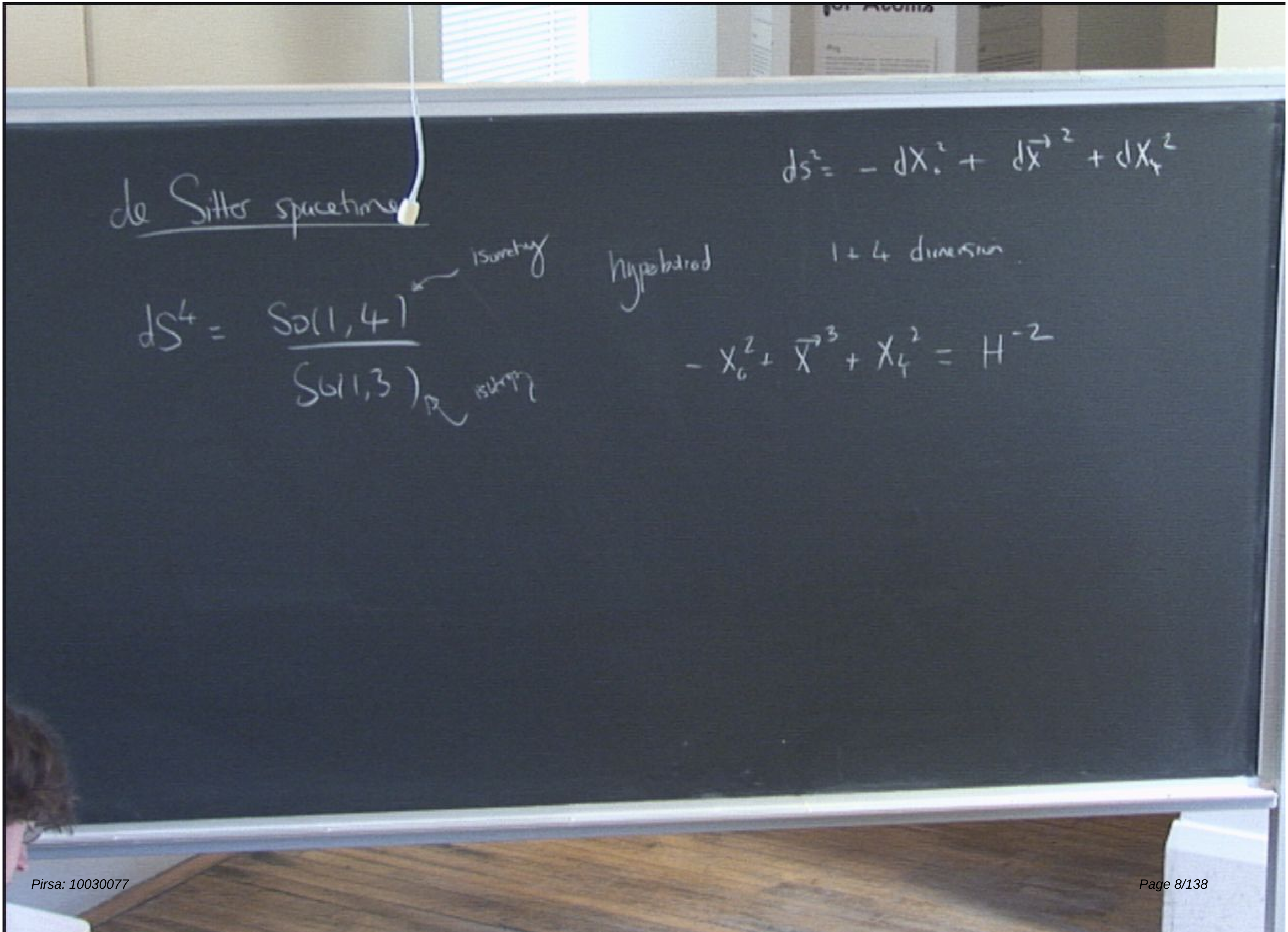

de Sitter spacetime
dS4 = SO(1,4) / SO(1,3)

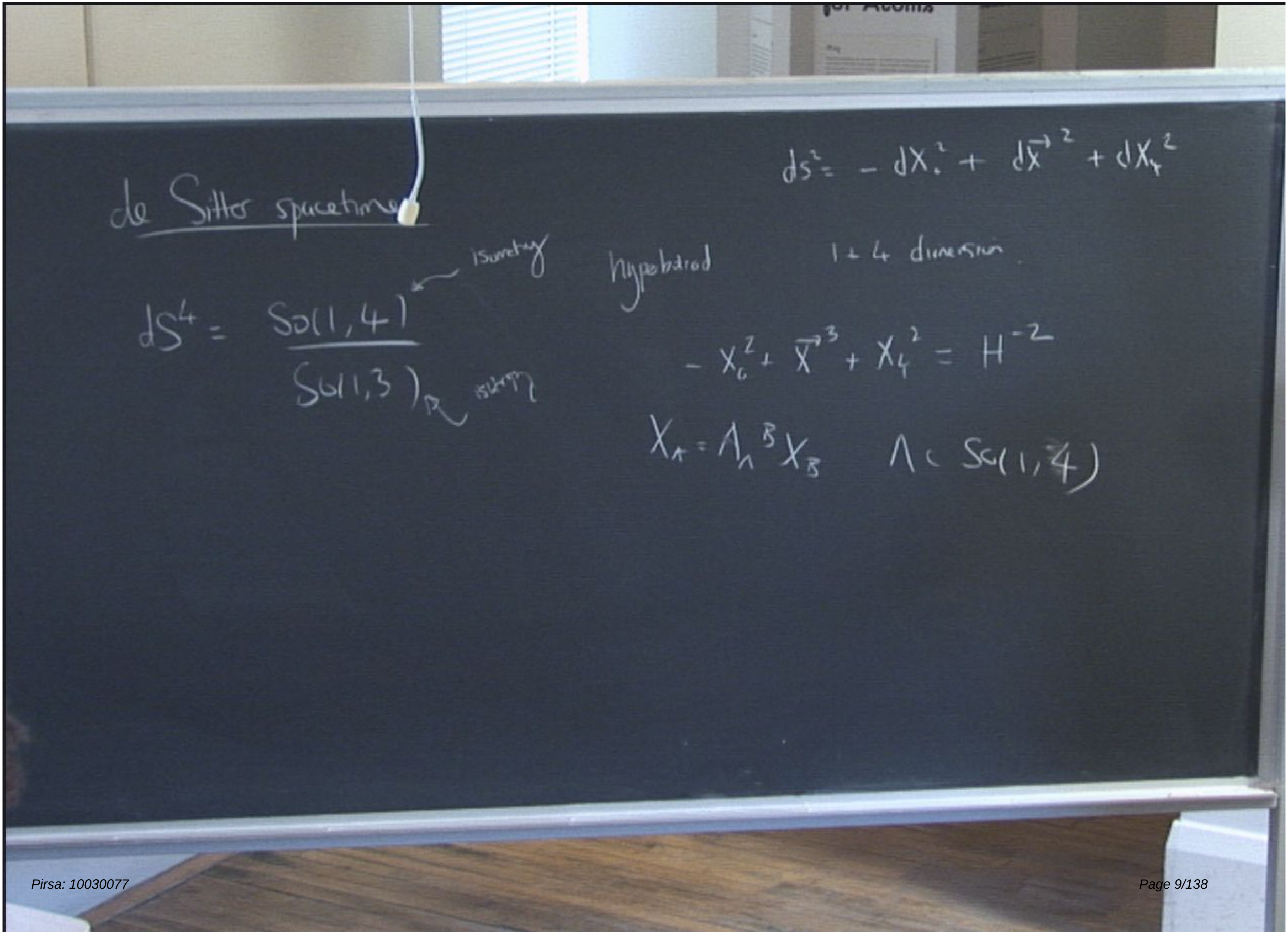

de Sitter spacetime
isometry
dS^4 = SO(1,4) / SO(1,3)

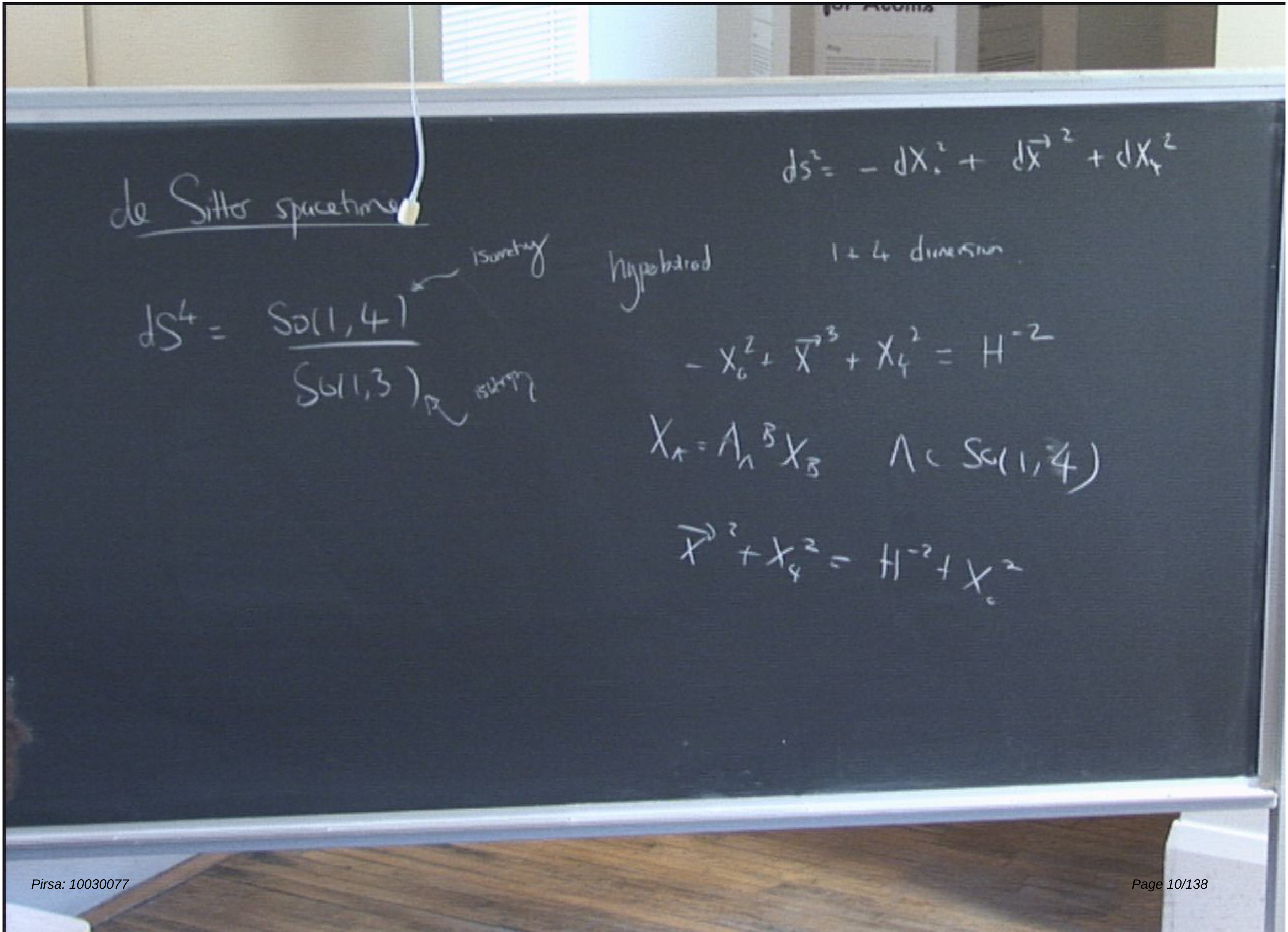

de Sitter spacetime
isometry
$dS^4 = \frac{SO(1,4)}{SO(1,3)}$

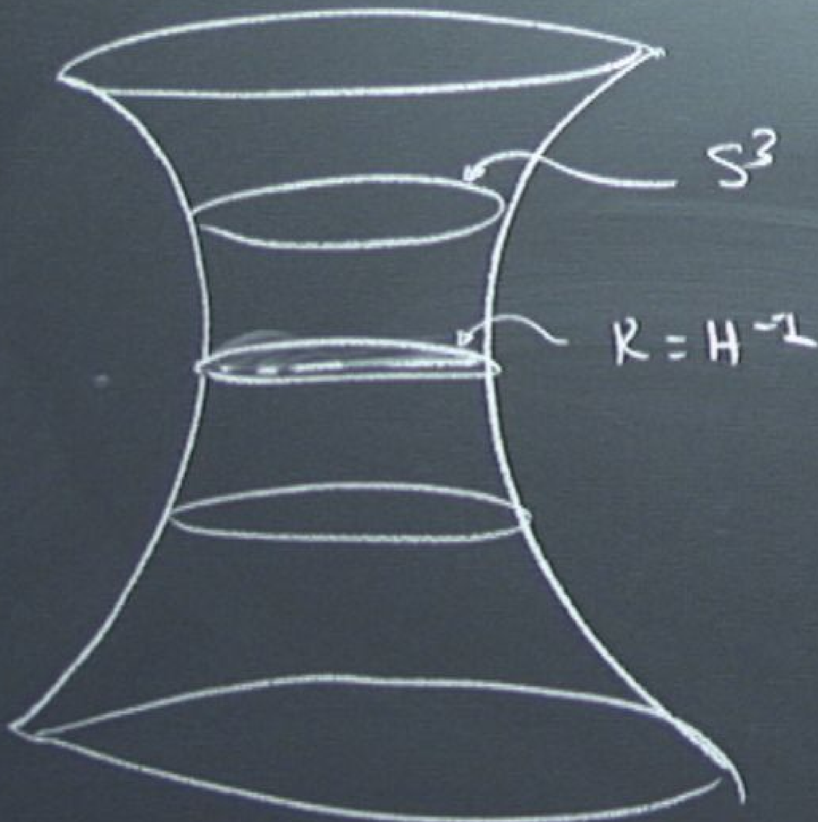

de Sitter spacetime
isometry
1 + 4 dimension
dS4 = SO(1,4) / SO(1,3)


de Sitter spacetime
dS^4 = SO(1,4) / SO(1,3)
isometry
isotropy
hyperboloid
ds^2 = - dX_0^2 + dX^2 + dX_4^2
1+4 dimension

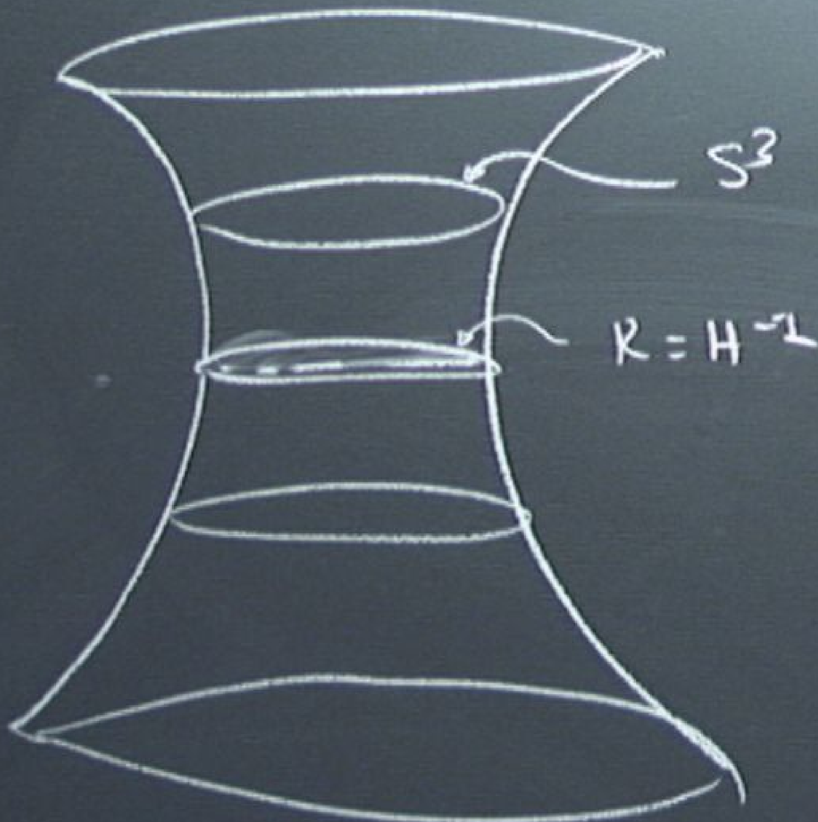

de Sitter spacetime
isometry
hyperboloid
1+4 dimension
ds² = − dX₀² + dX⃗² + dX₄²
dS⁴ = SO(1,4) / SO(1,3)
− X₀² + X⃗³ + X₄² = H⁻²


de Sitter spacetime
$ds^2 = -dX_0^2 + d\vec{X}^2 + dX_4^2$
isometry
hyperboloid
1+4 dimension
$dS^4 = \frac{SO(1,4)}{SO(1,3)}$
$-X_0^2 + \vec{X}^3 + X_4^2 = H^{-2}$
$X_A = \Lambda_A{}^B X_B \quad \Lambda \in SO(1,4)$


de Sitter spacetime
isometry
$dS^4 = \frac{SO(1,4)}{SO(1,3)}$
hyperboloid
1+4 dimension
$ds^2 = -dX_0^2 + d\vec{X}^2 + dX_4^2$
$-X_0^2 + \vec{X}^3 + X_4^2 = H^{-2}$
$X_A = \Lambda_A{}^B X_B \quad \Lambda \in SO(1,4)$
$\vec{X}^2 + X_4^2 = H^{-2} + X_0^2$



$$\vec{X}^2 + X_4^2 = R^2 \qquad R = \sqrt{H^{-2} + X_0^2}$$



$$\vec{X}^2 + X_4^2 = R^2 \qquad R = \sqrt{H^{-2} + X_0^2}$$

SO(1,4)

$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X}$$

$$R = \sqrt{H^{-2} + X_0^2}$$

$$\ldots_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$$

$$X_A = \eta_{AB} X^B \qquad \eta_{AB} = (-1, +1, +1, +1, +1)$$

$S^3 \quad \vec{X}^2 + X_4^2 = R^2 \quad R = \sqrt{H^{-2} + X_0^2}$

$SO(1,4)$

$= H^{-1}$

$A = 0 \to 4$

$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$$

$X_A = \eta_{AB} X^B \quad \eta_{AB} = (-$

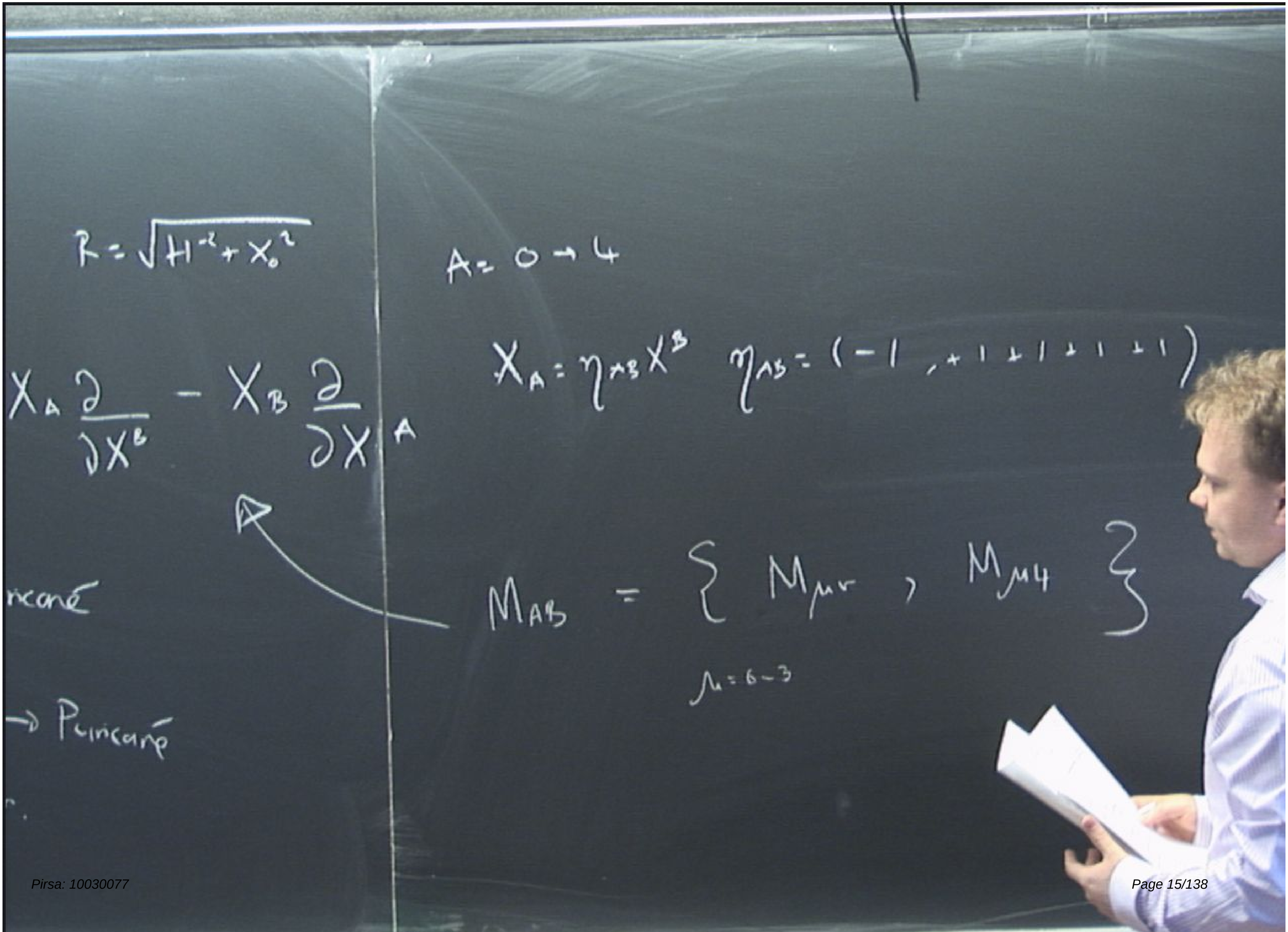
$$\lim_{H^{-1} \to \infty} dS^4 = \text{Poincaré}$$

$M_{AB} = \{ M_{\mu\nu}$

$\mu = 0 \to 3$

$$\lim_{H^{-1} \to \infty} SO(1,4) \to \text{Poincaré}$$

Wigner-Inönü contraction.



$R = \sqrt{H^{-2} + X_0^2}$
$A = 0 \to 4$
$X_A = \eta_{AB} X^B \quad \eta_{AB} = (-1, +1 +1 +1 +1)$
$X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$
$M_{AB} = \{ M_{\mu\nu} , M_{\mu 4} \}$
$\mu = 0 \to 3$
$\to$ Poincaré

$R = \sqrt{H^{-2} + X_0^2}$

$A = 0 \to 4$

$X_A = \eta_{AB} X^B \quad \eta_{AB} = (-1, +1 +1 +1 +1)$

$X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$

...ncaré

$\to$ Poincaré

$so(1,3) \qquad P_\mu$

$M_{AB} = \{ M_{\mu\nu} , M_{\mu 4} \}$

$\mu = 0 - 3$

$A = 0 \to 4$

$X_A = \eta_{AB} X^B \quad \eta_{AB} = (-1, +1 +1 +1 +1)$

A

$$M_{AB} = \{ M_{\mu\nu} , M_{\mu 4} \}$$

$SO(1,3)$ → $M_{\mu\nu}$; P_μ → $M_{\mu 4}$

$\mu = 0 - 3$

$A = 0 \to 4$

$X_A = \eta_{AB} X^B \quad \eta_{AB} = (-1, +1 +1 +1 +1)$

de Sitter $[P_\mu, P_\nu] \sim M_{\mu\nu}$

A

$$M_{AB} = \{ M_{\mu\nu}, M_{\mu 4} \}$$

$M_{\mu\nu} \leftarrow SO(1,3)$, $M_{\mu 4} \leftarrow P_\mu$

$\mu = 0 \to 3$

$A = 0 \rightarrow 4$

$X_A = \eta_{AB} X^B \quad \eta_{AB} = (-1, +1 +1 +1 +1)$

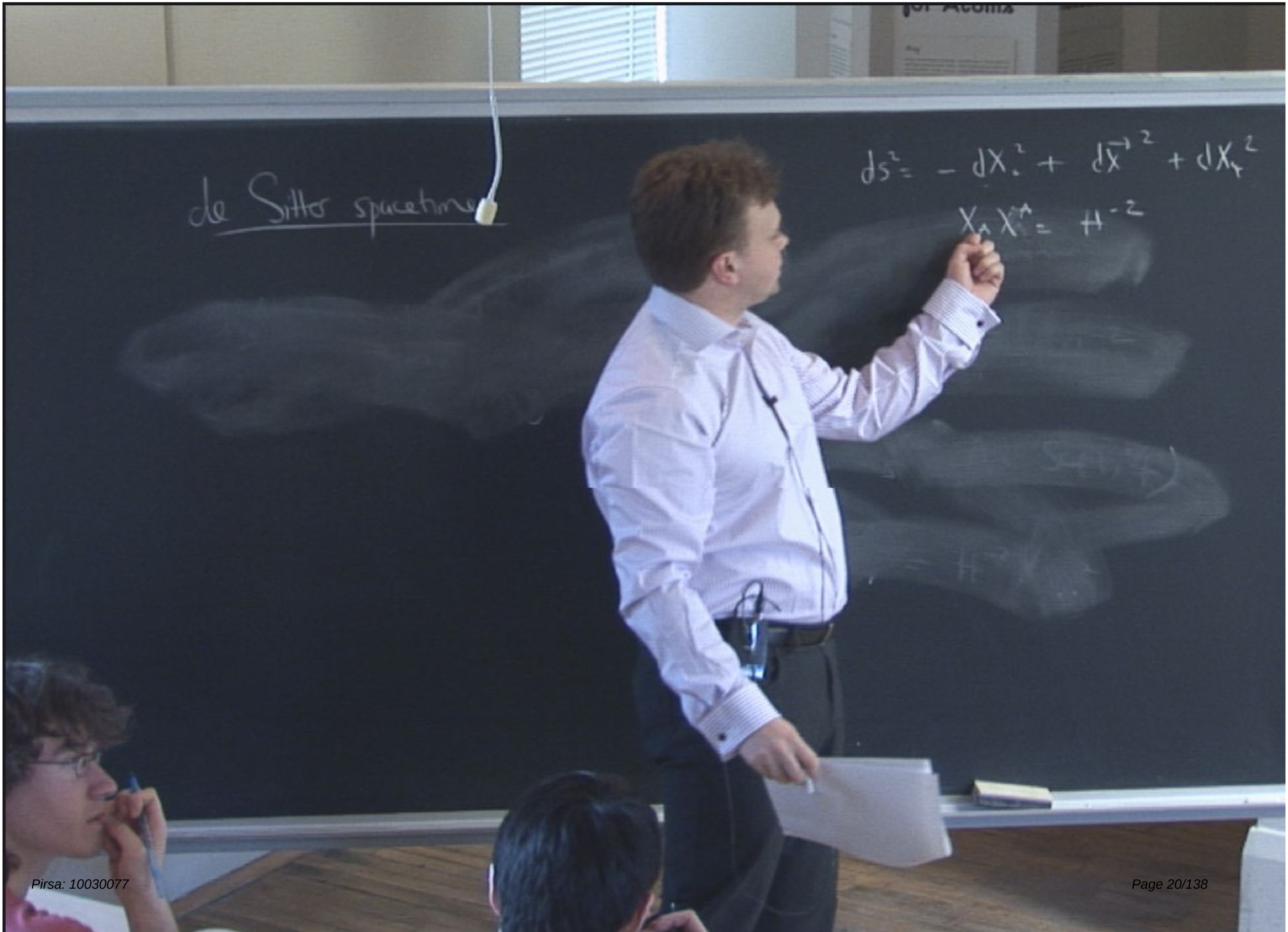
$SO(1,3) \rightarrow M_{\mu\nu}$, $\quad P_\mu \rightarrow M_{\mu 4}$

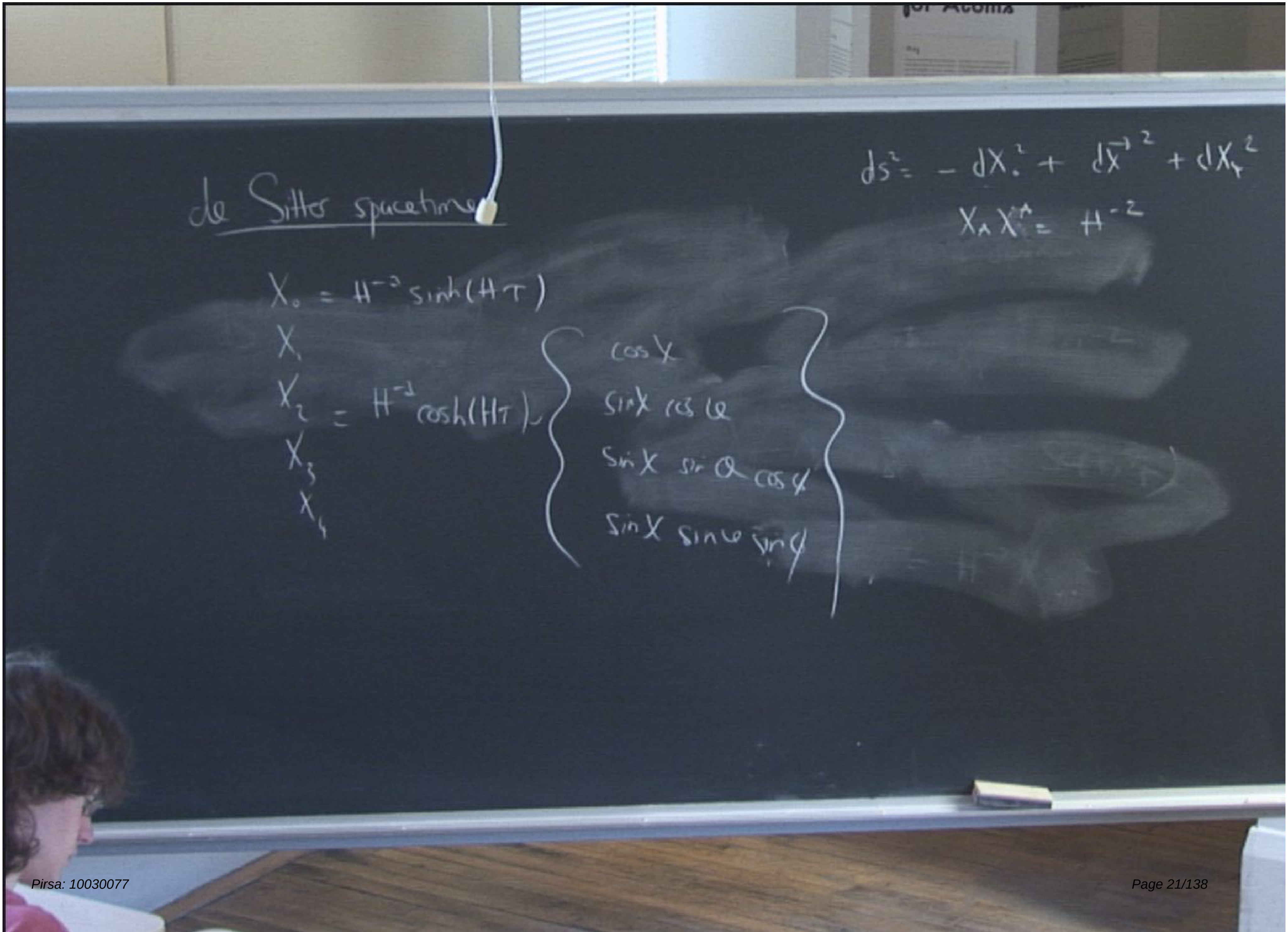
$$M_{AB} = \{ M_{\mu\nu} , M_{\mu 4} \}$$

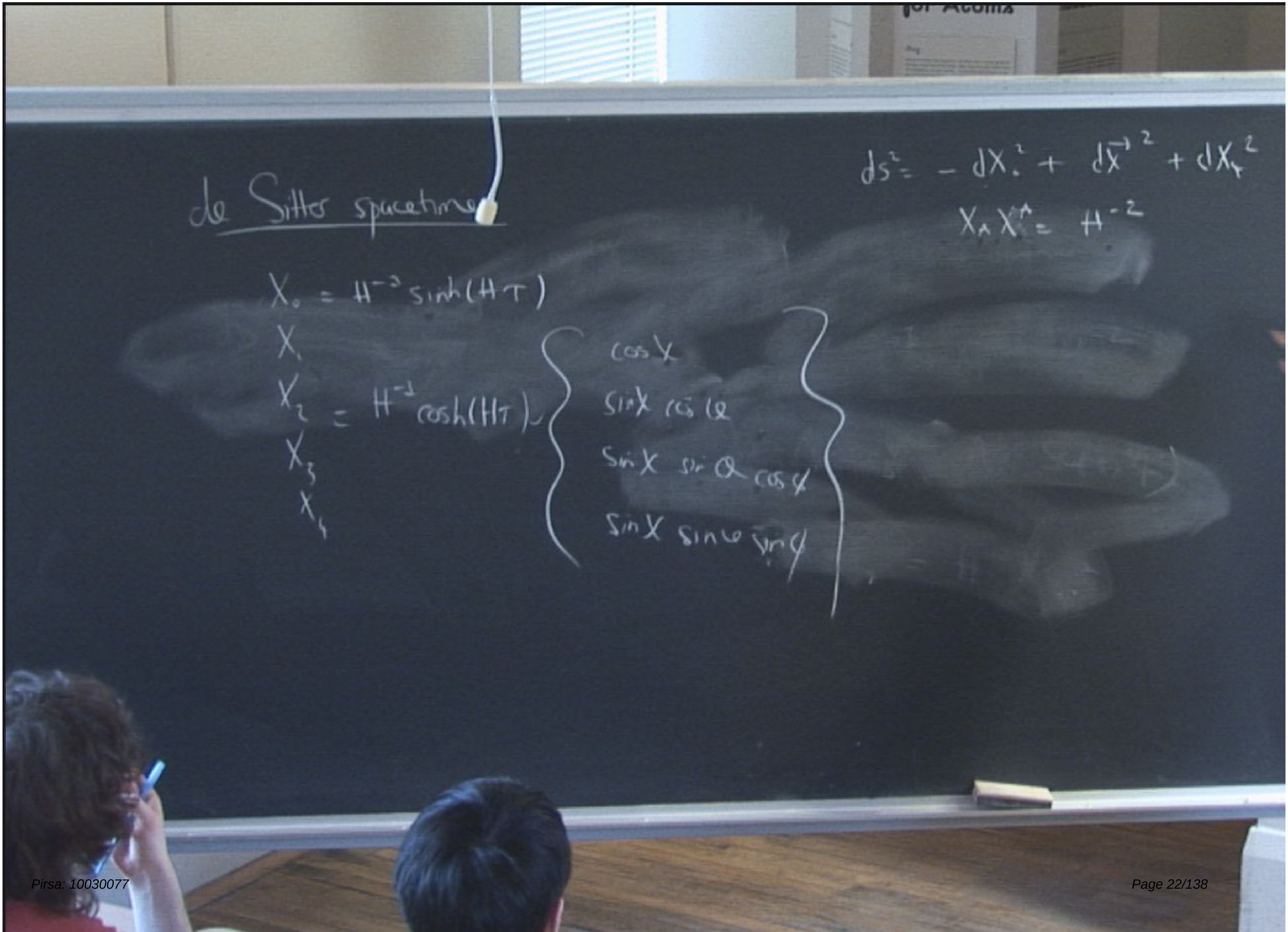
$\mu = 0 \rightarrow 3$

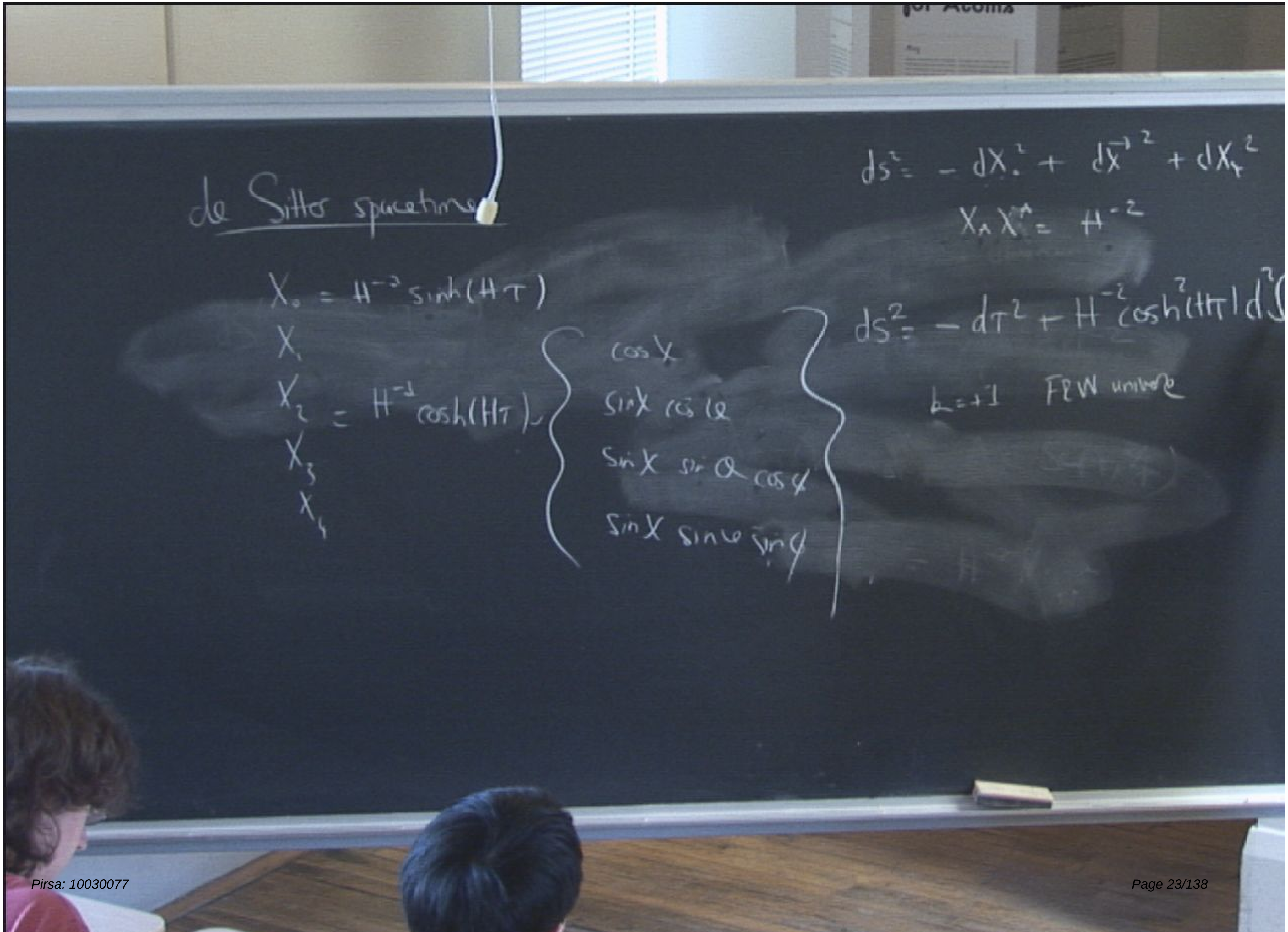
de Sitter $[P_\mu , P_\nu] \sim M_{\mu\nu}$

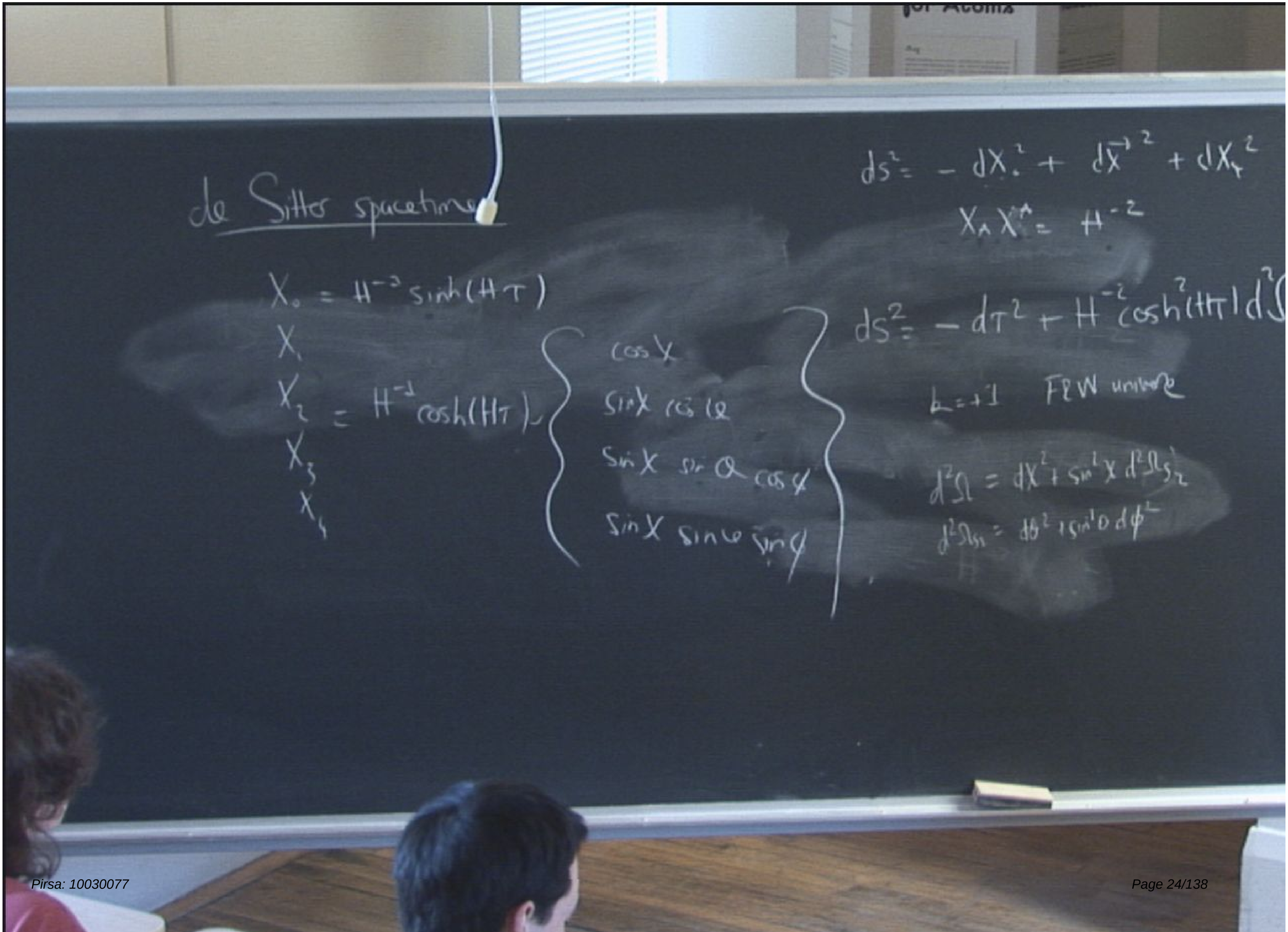
Poincaré $[P_\mu , P_\nu] = 0$

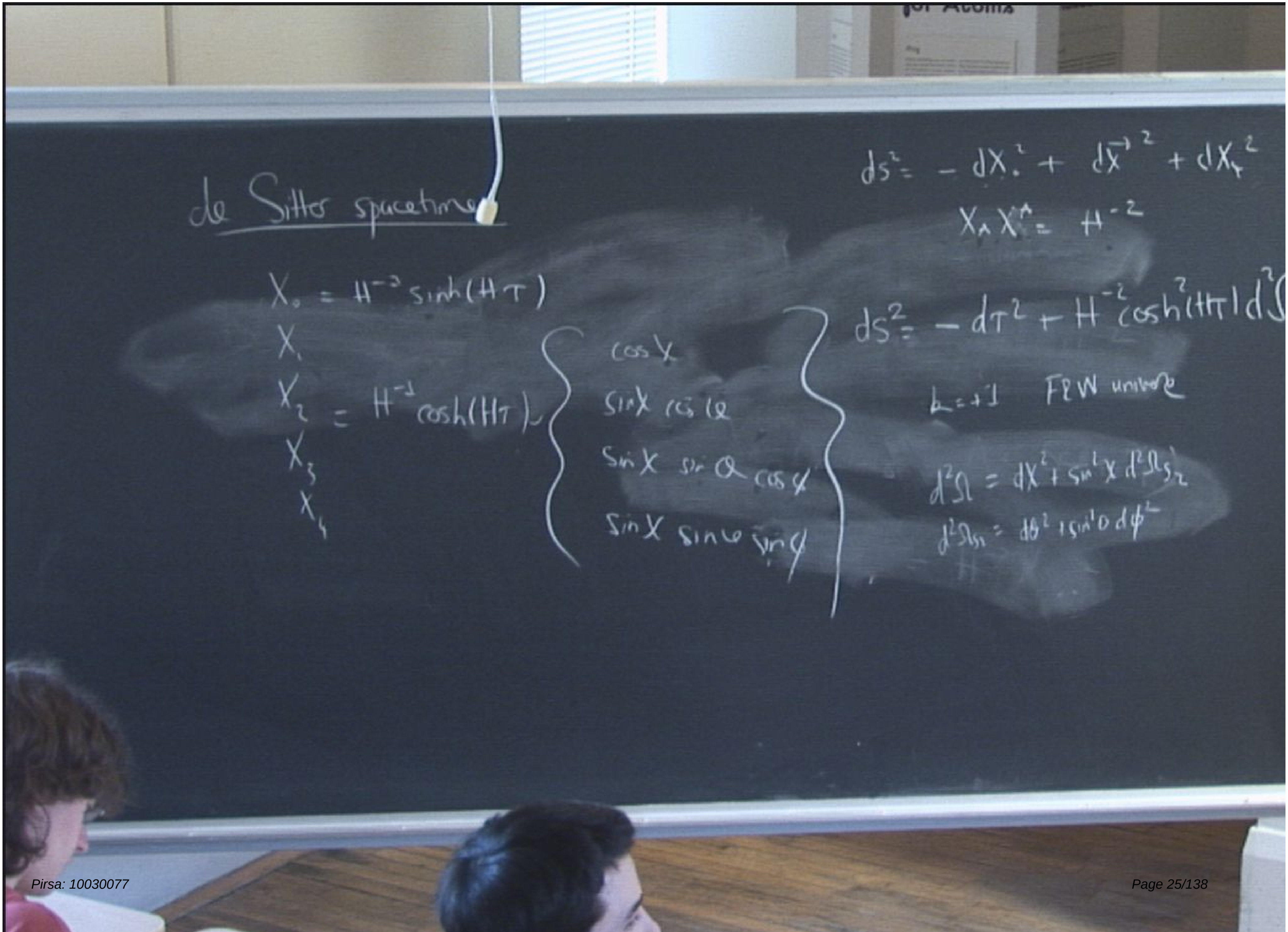

de Sitter spacetime
$ds^2 = -dX_0^2 + d\vec{X}^2 + dX_4^2$
$X_A X^A = H^{-2}$

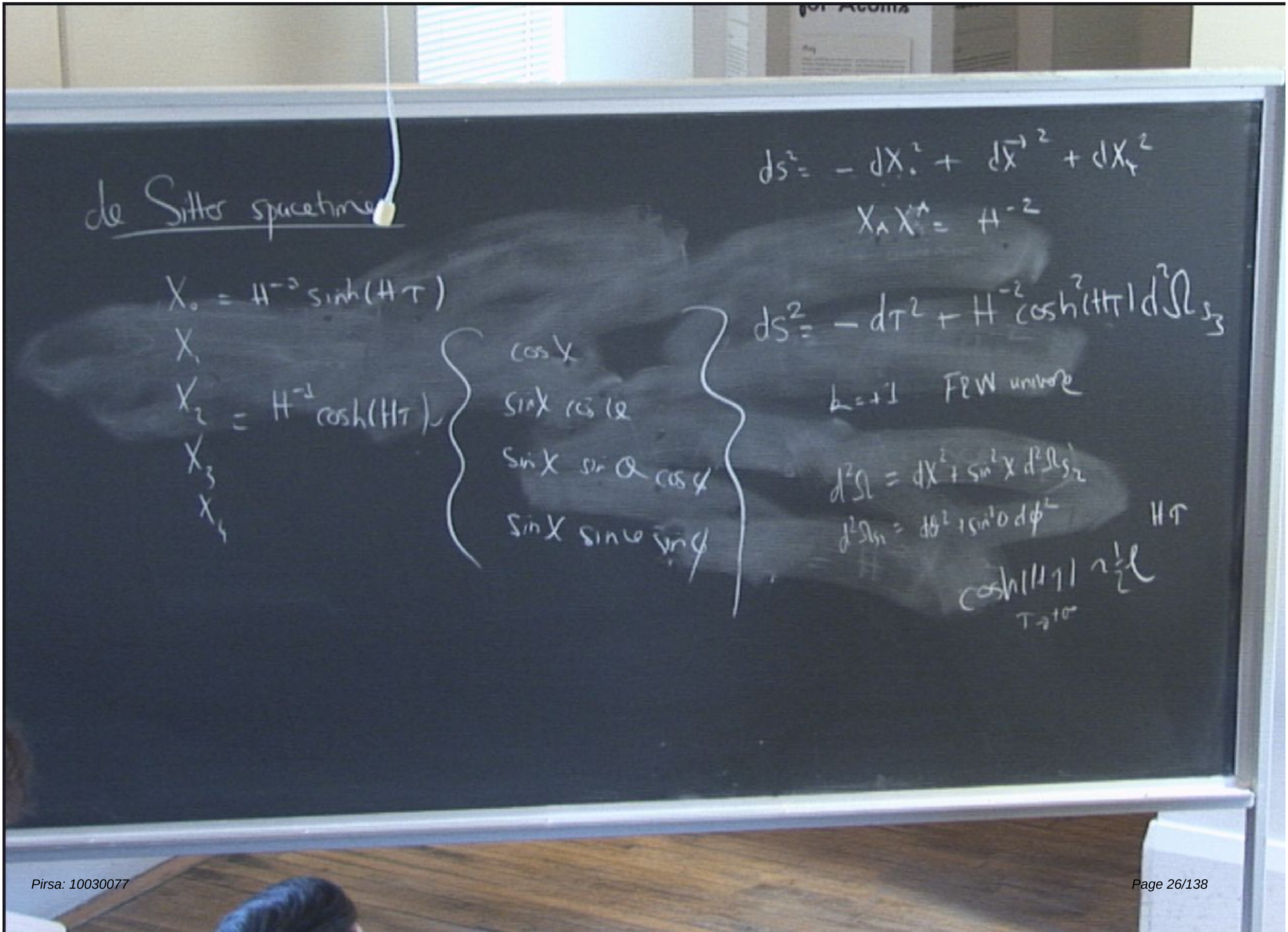

de Sitter spacetime
$ds^2 = -dX_0^2 + d\vec{X}^2 + dX_4^2$
$X_A X^A = H^{-2}$
$X_0 = H^{-1} \sinh(H\tau)$
X_1
$X_2 = H^{-1} \cosh(H\tau)$
X_3
X_4
$\cos\chi$
$\sin\chi \cos\theta$
$\sin\chi \sin\theta \cos\phi$
$\sin\chi \sin\theta \sin\phi$

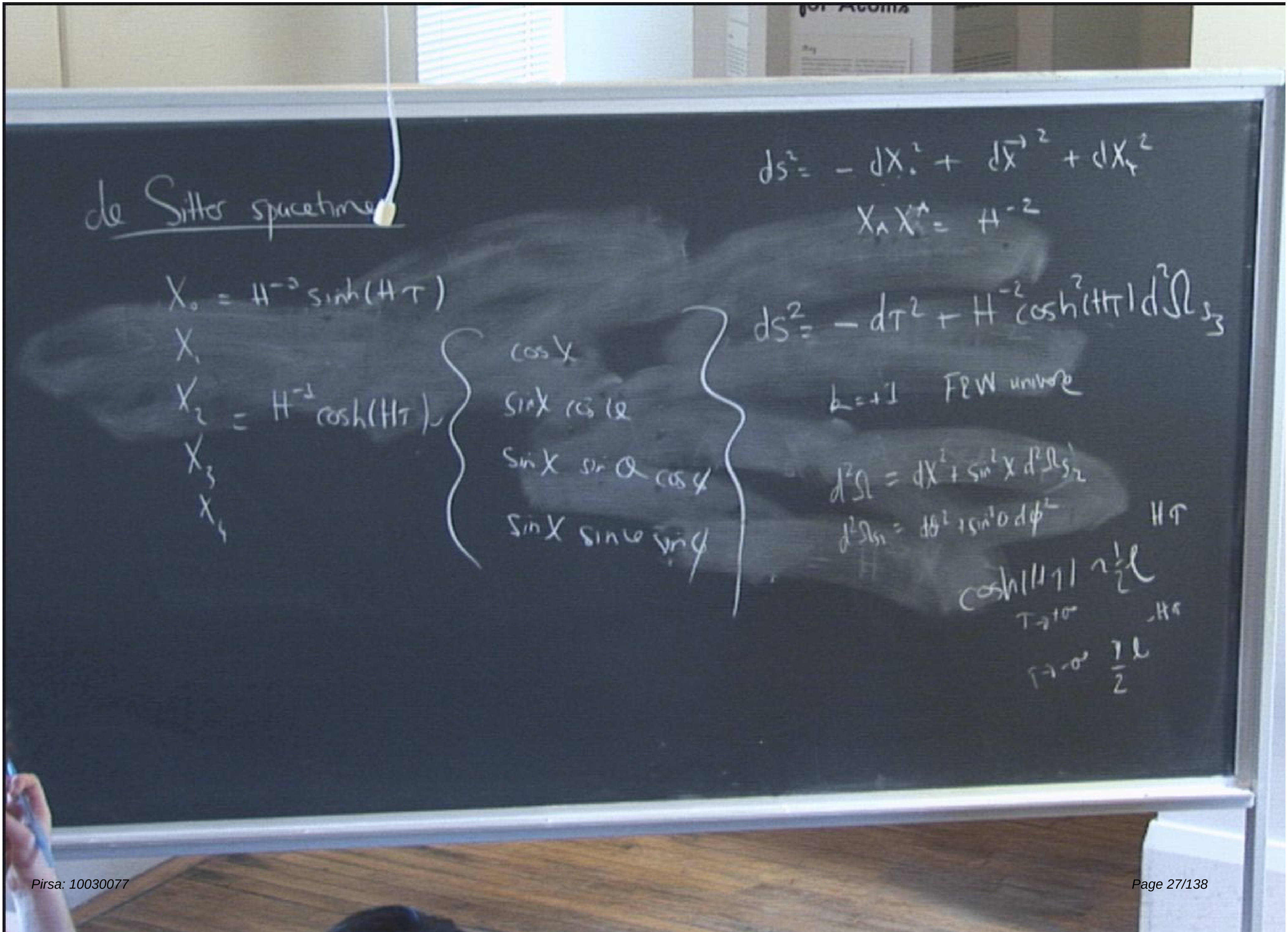

de Sitter spacetime
ds² = − dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
X₀ = H⁻¹ sinh(Hτ)
X₁
X₂ = H⁻¹ cosh(Hτ)
X₃
X₄
cos χ
sin χ cos θ
sin χ sin θ cos φ
sin χ sin θ sin φ

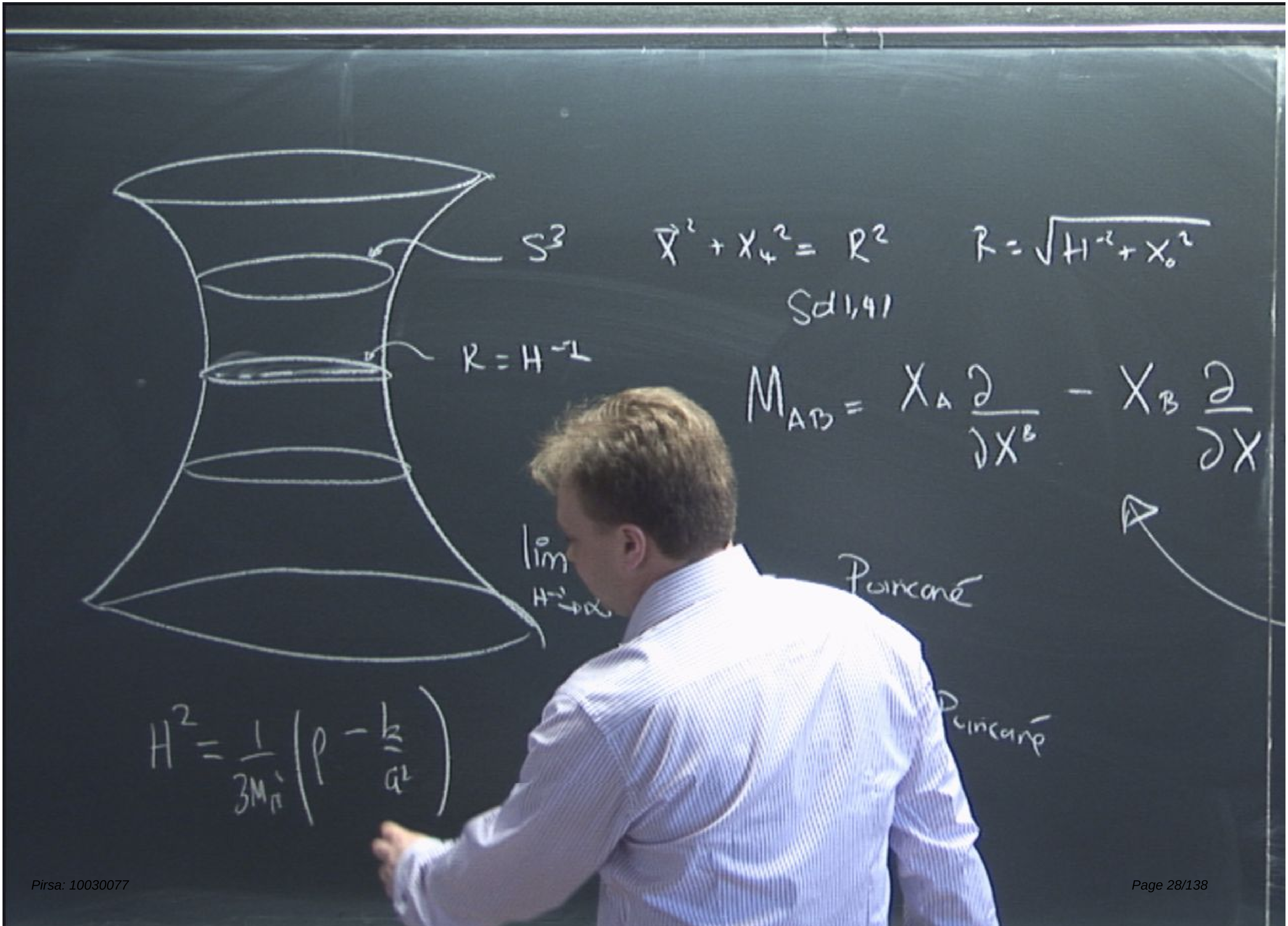

de Sitter spacetime
ds² = − dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
X₀ = H⁻¹ sinh(Hτ)
X₁
X₂ = H⁻¹ cosh(Hτ)
X₃
X₄
cos χ
sin χ cos θ
sin χ sin θ cos φ
sin χ sin θ sin φ
ds² = − dτ² + H⁻² cosh²(Hτ) dΩ²
k = +1 FRW universe

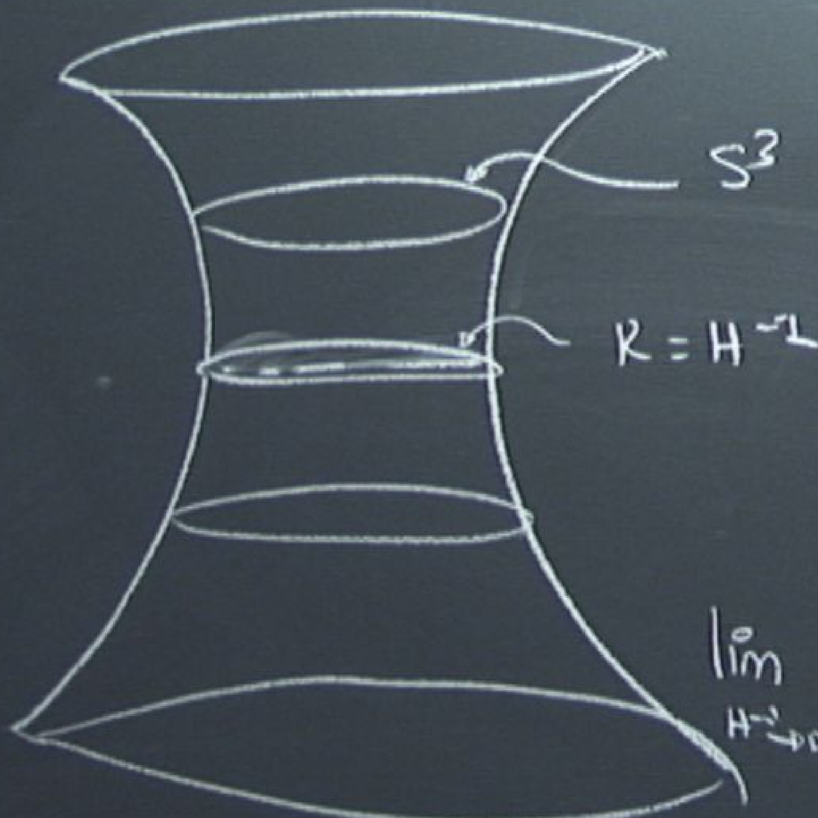

de Sitter spacetime
ds² = − dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
X₀ = H⁻¹ sinh(Hτ)
X₁
X₂ = H⁻¹ cosh(Hτ)
X₃
X₄
cos χ
sin χ cos θ
sin χ sin θ cos φ
sin χ sin θ sin φ
ds² = − dτ² + H⁻² cosh²(Hτ) d²Ω
k = +1 FRW universe
d²Ω = dχ² + sin²χ d²Ω_{S₂}
d²Ω_{S₂} = dθ² + sin²θ dφ²


de Sitter spacetime
ds² = − dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
X₀ = H⁻¹ sinh(Hτ)
X₁
X₂ = H⁻¹ cosh(Hτ)
X₃
X₄
cos χ
sin χ cos θ
sin χ sin θ cos φ
sin χ sin θ sin φ
ds² = − dτ² + H⁻² cosh²(Hτ) d²Ω
k = +1 FRW universe
d²Ω = dχ² + sin²χ d²Ω_{S₂}
d²Ω_{S₂} = dθ² + sin²θ dφ²


de Sitter spacetime
ds² = − dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
X₀ = H⁻¹ sinh(Hτ)
X₁
X₂ = H⁻¹ cosh(Hτ)
X₃
X₄
cos χ
sin χ cos θ
sin χ sin θ cos φ
sin χ sin θ sin φ
ds² = − dτ² + H⁻² cosh²(Hτ) d²Ω_S3
k = +1 FRW universe
d²Ω = dχ² + sin²χ d²Ω_S2
d²Ω_S2 = dθ² + sin²θ dφ²
cosh(Hτ) ~ ½ e^Hτ
τ → +∞


de Sitter spacetime
ds² = − dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
X₀ = H⁻¹ sinh(Hτ)
X₁
X₂ = H⁻¹ cosh(Hτ)
X₃
X₄
cos χ
sin χ cos θ
sin χ sin θ cos φ
sin χ sin θ sin φ
ds² = − dτ² + H⁻² cosh²(Hτ) d²Ω_{S₃}
k = +1 FRW universe
d²Ω = dχ² + sin²χ d²Ω_{S₂}
d²Ω_{S₂} = dθ² + sin²θ dφ²
Hτ
cosh(Hτ) ~ ½ e
τ → +∞
τ → −∞


S³
X⃗² + X₄² = R²
R = √(H⁻² + X₀²)
SO(1,4)
R = H⁻¹
M_AB = X_A ∂/∂X^B − X_B ∂/∂X
lim
H⁻¹→∞
Poincaré
H² = 1/(3M_Pl²) (ρ − k/a²)



$$\vec{X}^2 + X_4^2 = R^2 \qquad R = \sqrt{H^{-2} + X_0^2}$$

$SO(1,4)$

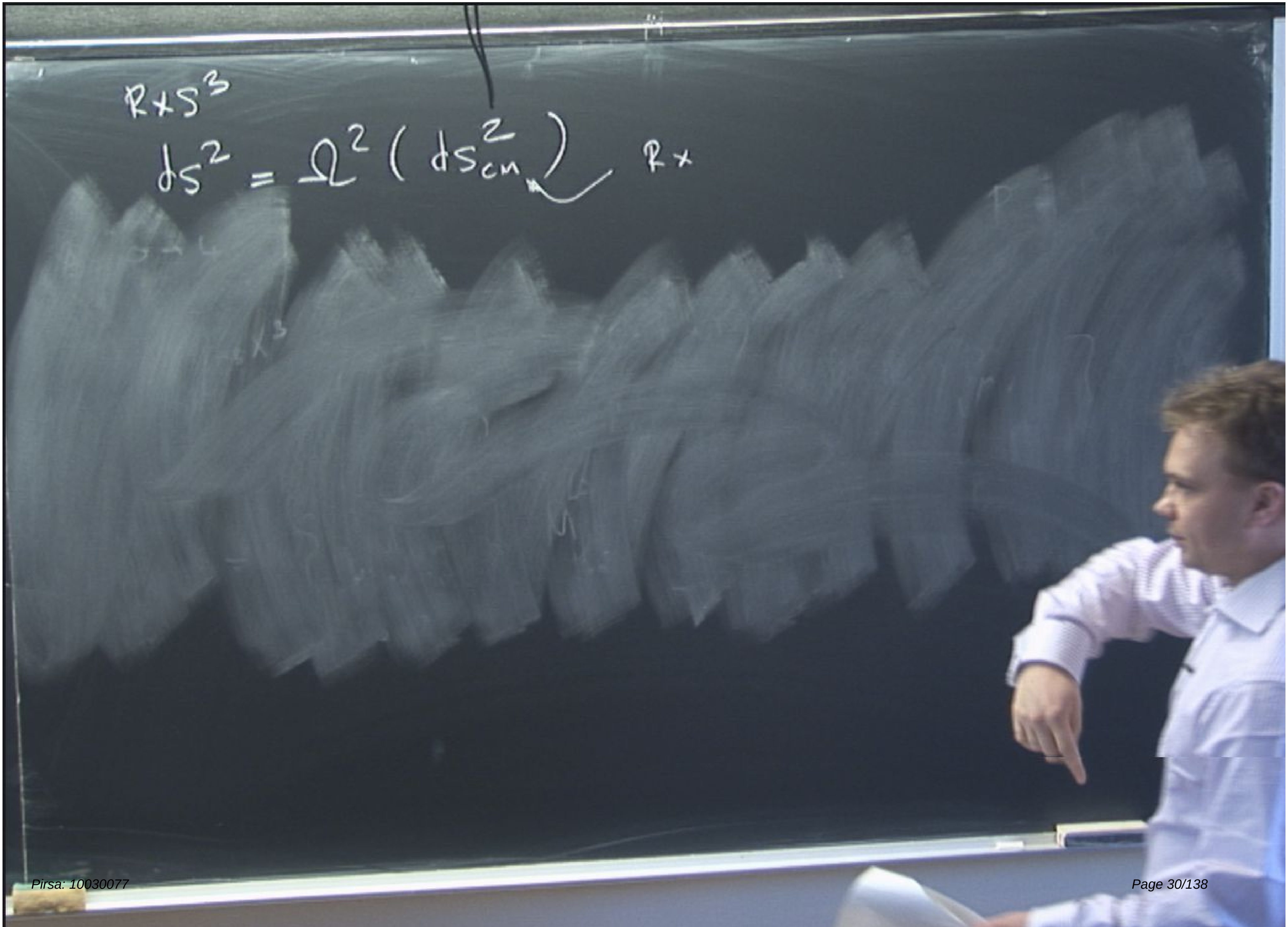
$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$$

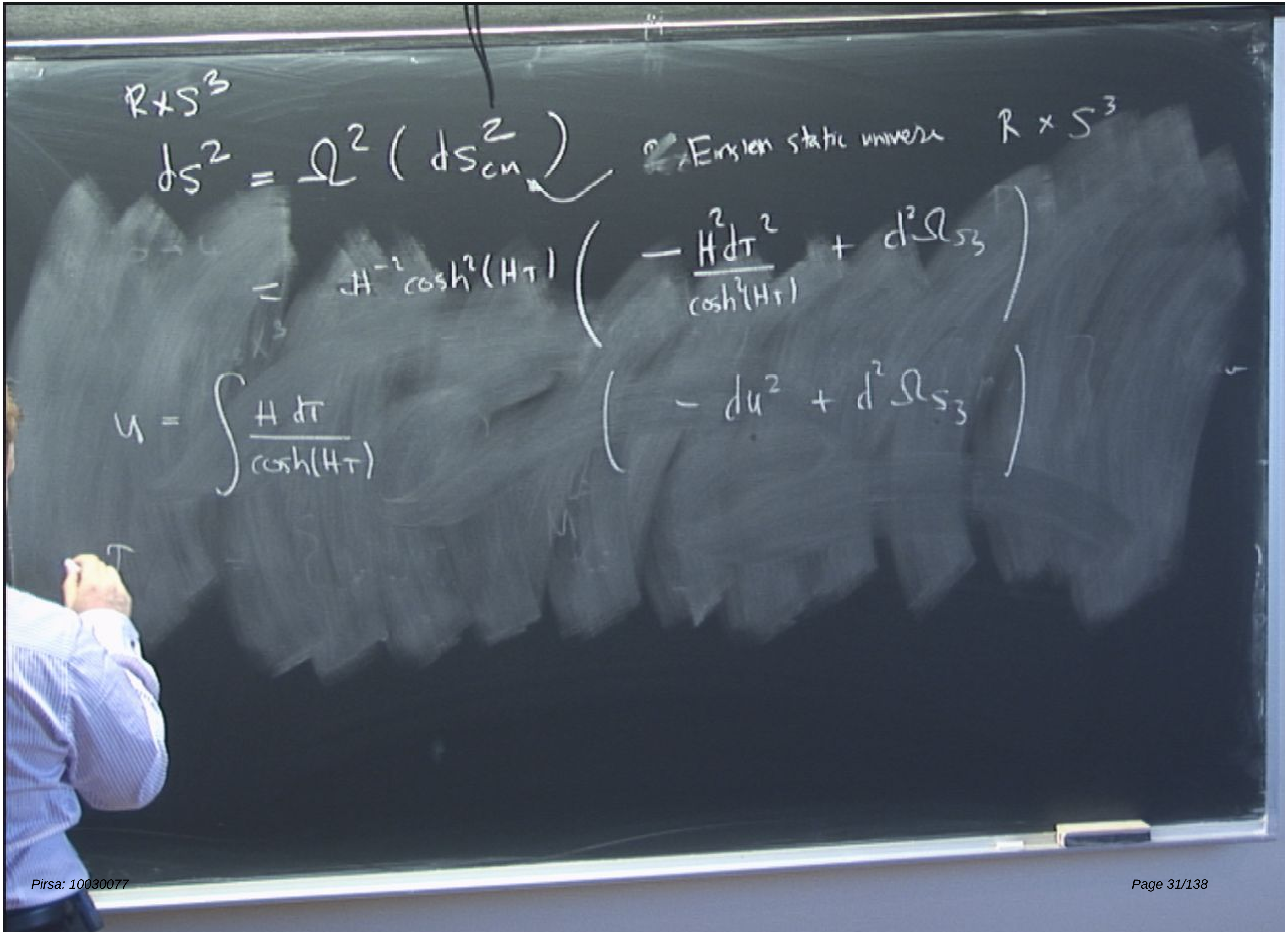
$$\lim_{H^{-1}\to\infty} dS^4 = \text{Poincaré}$$

$$\lim_{H^{-1}\to\infty} SO(1,4) \to \text{Poincaré}$$

Wigner-Inönü contraction.

$$H^2 = \frac{1}{3M_P^2}\left(\rho - \frac{k}{a^2}\right)$$


R×S3
ds2 = Ω2 (ds2cm)
R×


R×S^3
ds^2 = Ω^2 (ds_cm^2)
Einstein static universe R × S^3
= H^-2 cosh^2(Hτ) (− H^2 dτ^2 / cosh^2(Hτ) + d^2Ω_S3)
u = ∫ H dτ / cosh(Hτ)
(− du^2 + d^2Ω_S3)

$R \times S^3$

$$ds^2 = \Omega^2 \left(ds^2_{cm} \right)$$

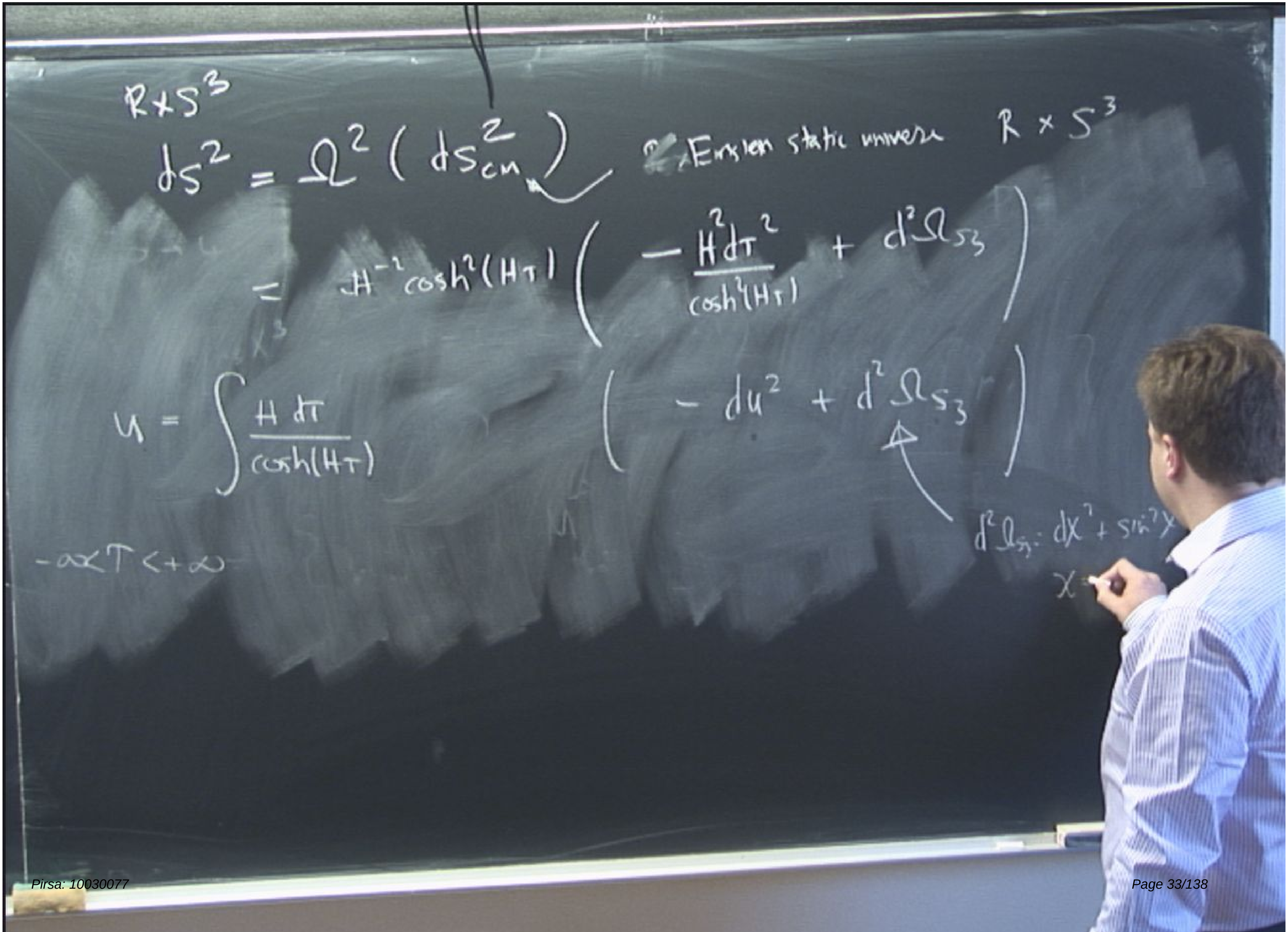
Einstein static universe $R \times S^3$

$$= H^{-2} \cosh^2(H\tau) \left(- \frac{H^2 d\tau^2}{\cosh^2(H\tau)} + d^2\Omega_{S_3} \right)$$

$$\left(- du^2 + d^2\Omega_{S_3} \right)$$

$$u = \int \frac{H\, d\tau}{\cosh(H\tau)}$$

$-\infty < \tau < +\infty$


$R \times S^3$
$ds^2 = \Omega^2 (ds^2_{cm})$
Einstein static universe $R \times S^3$
$= H^{-2} \cosh^2(H\tau) \left(- \frac{H^2 d\tau^2}{\cosh^2(H\tau)} + d^2\Omega_{S_3} \right)$
$\left(- du^2 + d^2\Omega_{S_3} \right)$
$u = \int \frac{H\, d\tau}{\cosh(H\tau)}$
$d^2\Omega_{S_3} = d\chi^2 + \sin^2\chi$
$-\infty < T < +\infty$

$R \times S^3$

$$ds^2 = \Omega^2 \left(ds^2_{cm} \right)$$

Einstein static universe $R \times S^3$

$$= H^{-2} \cosh^2(H\tau) \left(- \frac{H^2 d\tau^2}{\cosh^2(H\tau)} + d^2\Omega_{S_3} \right)$$

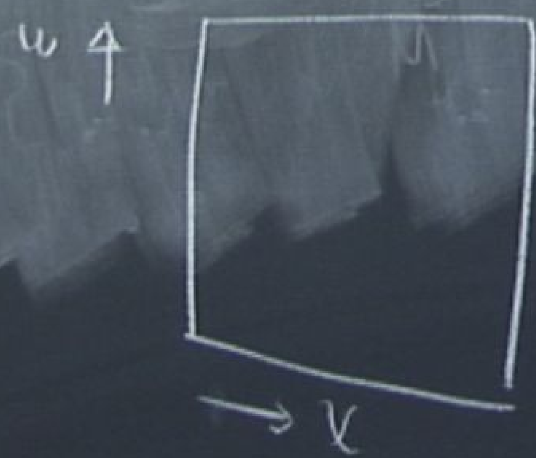
$$\left(- du^2 + d^2\Omega_{S_3} \right)$$

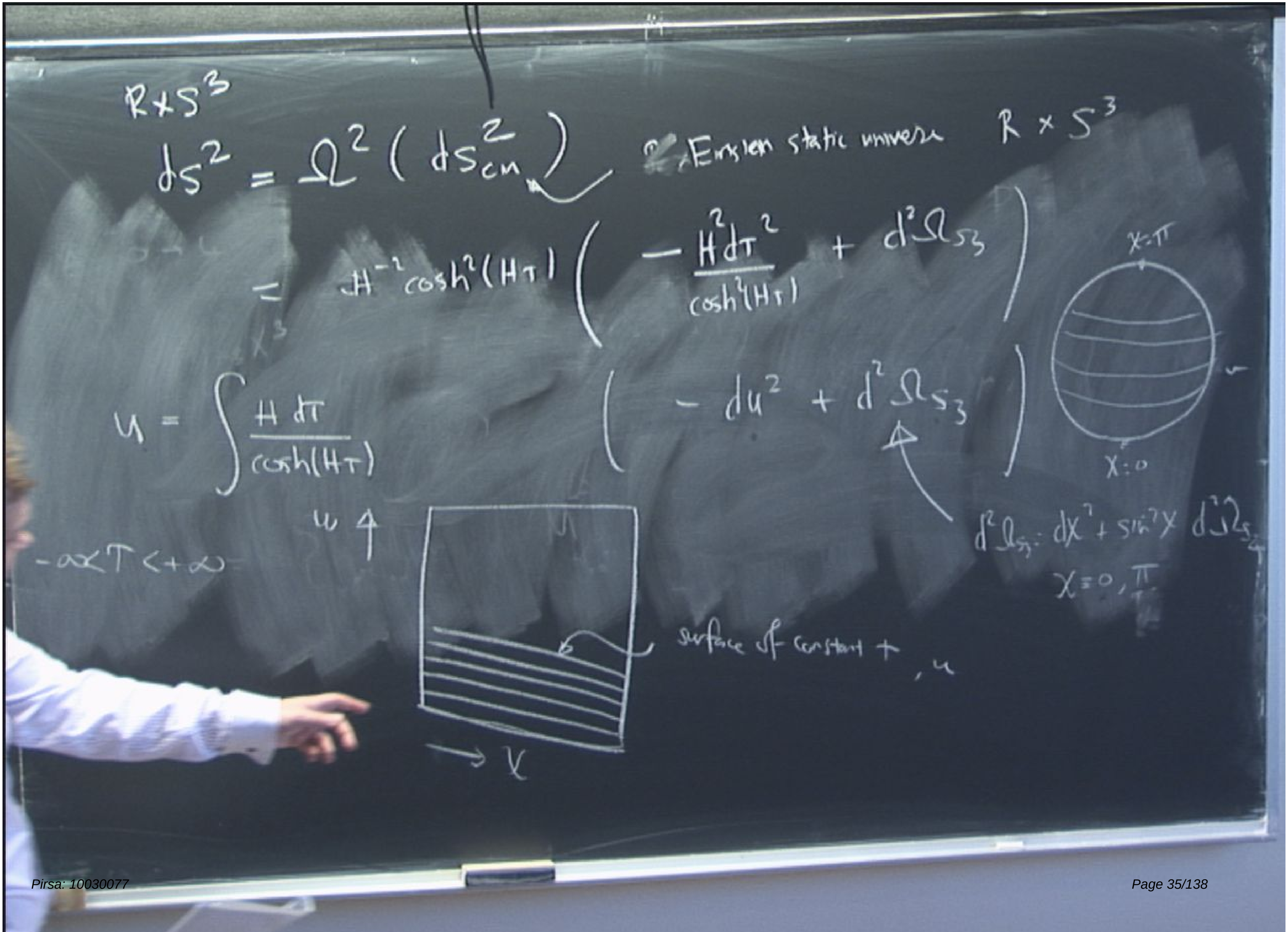
$$u = \int \frac{H\, d\tau}{\cosh(H\tau)}$$

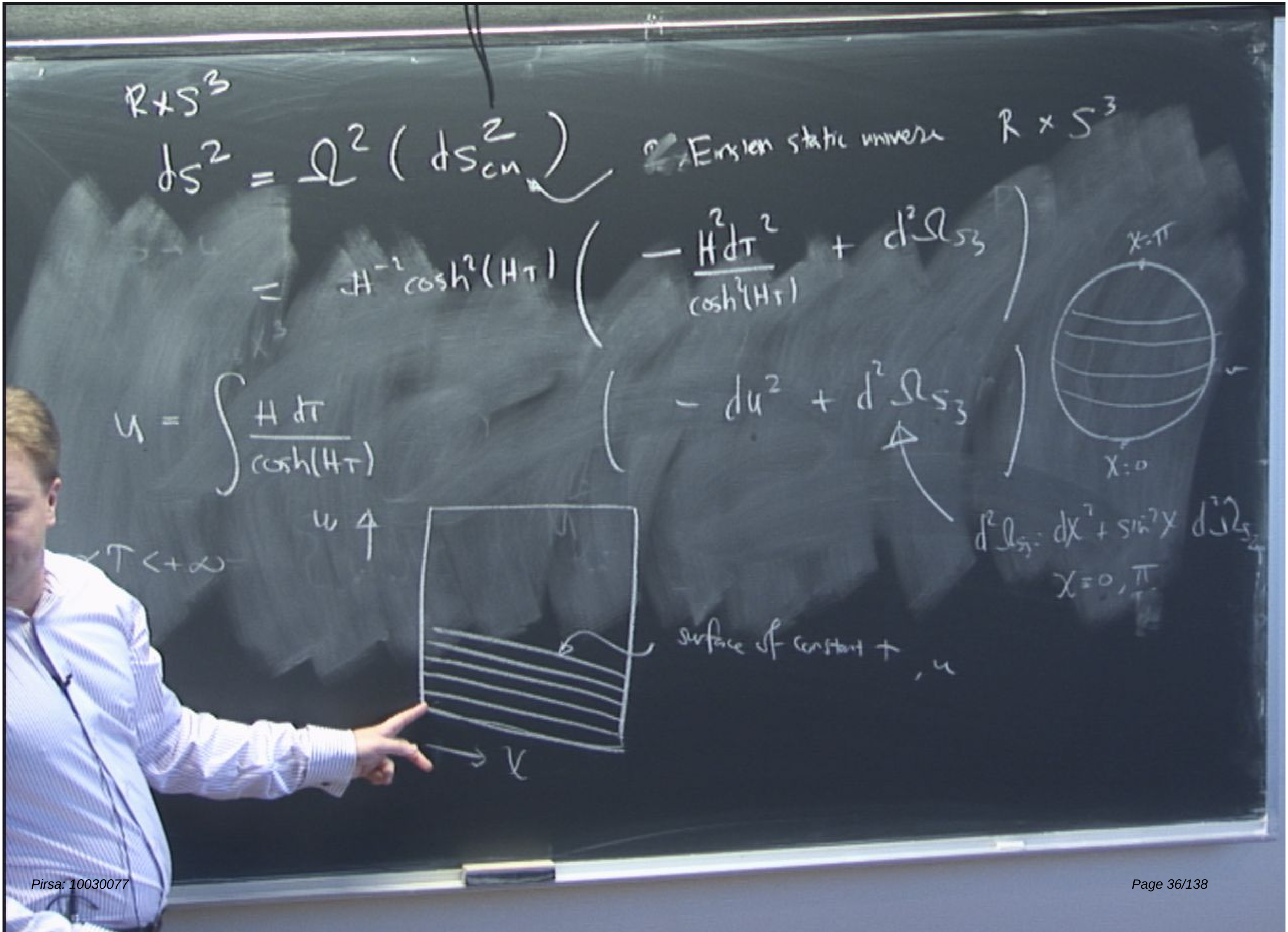
$$-\infty < \tau < +\infty$$

$$d^2\Omega_{S_3} = d\chi^2 + \sin^2\chi \, d^2\Omega_{S_2}$$

$$\chi = 0, \pi$$




R×S³
ds² = Ω² (ds²_cm)
Einstein static universe R × S³
= H⁻² cosh²(Hτ) (− H²dτ² / cosh²(Hτ) + d²Ω_S3)
u = ∫ H dτ / cosh(Hτ)
(− du² + d²Ω_S3)
χ=π
χ=0
d²Ω_S3 = dχ² + sin²χ d²Ω_S2
χ=0, π
−∞<τ<+∞
u
surface of constant τ, u
χ


R×S^3
ds^2 = Ω^2 (ds^2_cm)
Einstein static universe R × S^3
= H^-2 cosh^2(Hτ) (− H^2 dτ^2 / cosh^2(Hτ) + d^2Ω_S3)
u = ∫ H dτ / cosh(Hτ)
(− du^2 + d^2Ω_S3)
χ=π
χ=0
d^2Ω_S3 = dχ^2 + sin^2χ d^2Ω_S2
χ=0, π
T < +∞
u
surface of constant τ, u
χ

$R \times S^3$

$$ds^2 = \Omega^2 \left(ds^2_{cm} \right)$$

Einstein static universe $R \times S^3$

$$= H^{-2} \cosh^2(H\tau) \left(- \frac{H^2 d\tau^2}{\cosh^2(H\tau)} + d^2\Omega_{S_3} \right)$$

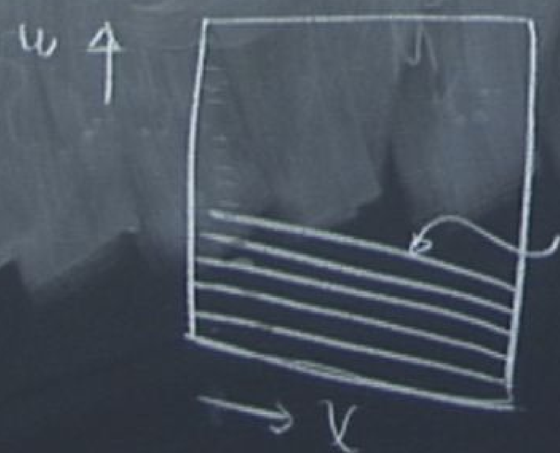
$$\left(- du^2 + d^2\Omega_{S_3} \right)$$

$$u = \int \frac{H \, d\tau}{\cosh(H\tau)}$$

$$-\infty < \tau < +\infty$$

$$d^2\Omega_{S_3} = d\chi^2 + \sin^2\chi \, d^2\Omega_{S_2}$$

$$\chi = 0, \pi$$



surface of constant τ, u

$R \times S^3$

$$ds^2 = \Omega^2 \left(ds_{cm}^2 \right)$$

Einstein static universe $R \times S^3$

$$= H^{-2} \cosh^2(H\tau) \left(-\frac{H^2 d\tau^2}{\cosh^2(H\tau)} + d^2\Omega_{S_3} \right)$$

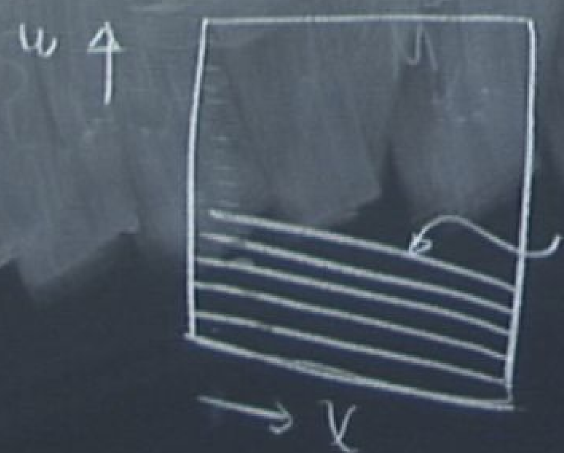
$$\left(-du^2 + d^2\Omega_{S_3} \right)$$

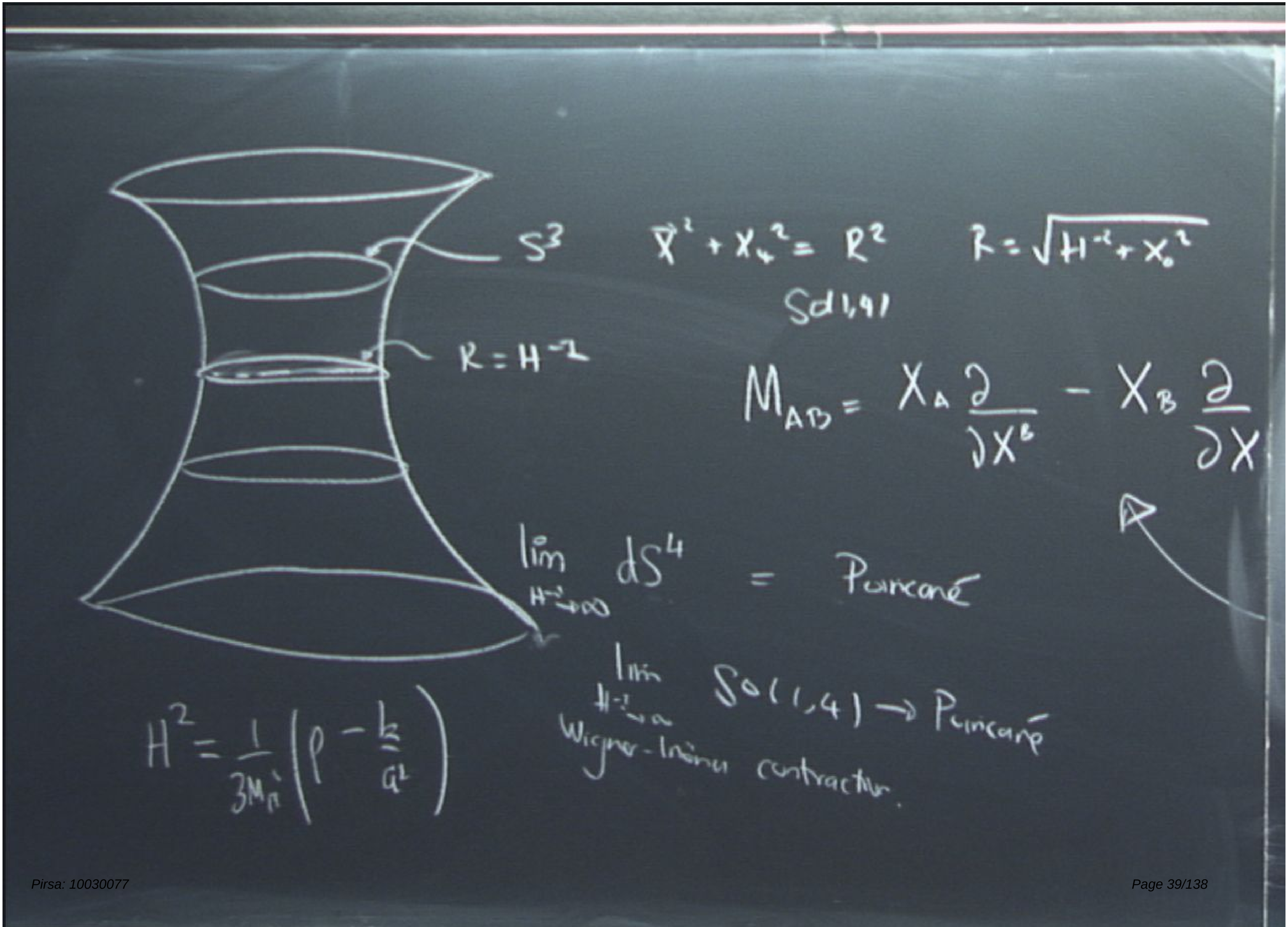
$$u = \int \frac{H\, d\tau}{\cosh(H\tau)}$$

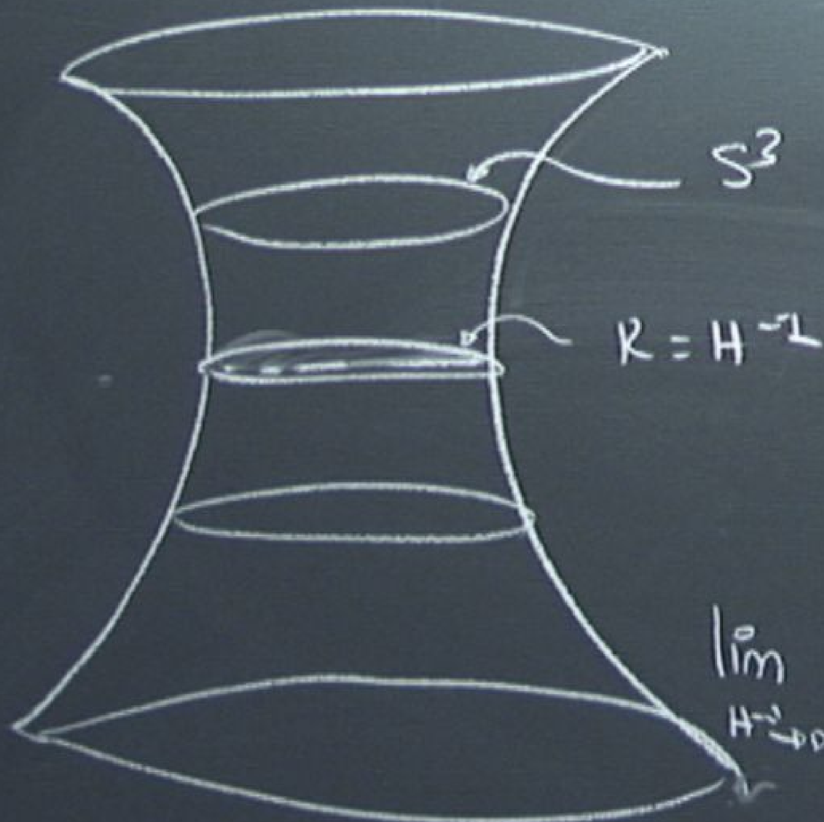
$$-\infty < \tau < +\infty$$

$$d^2\Omega_{S_3} = d\chi^2 + \sin^2\chi \, d^2\Omega_{S_2}$$

$$\chi = 0, \pi$$




S3
X² + X4² = R²
R = √(H⁻² + X0²)
SO(1,4)
R = H⁻²
MAB = XA ∂/∂X^B − XB ∂/∂X
lim dS4 = Poincaré
lim SO(1,4) → Poincaré
Wigner-Inönü contraction.
H² = 1/(3Mpl²) (ρ − k/a²)



$\vec{X}^2 + X_4^2 = R^2$ $\quad R = \sqrt{H^{-2} + X_0^2}$

SO(1,4)

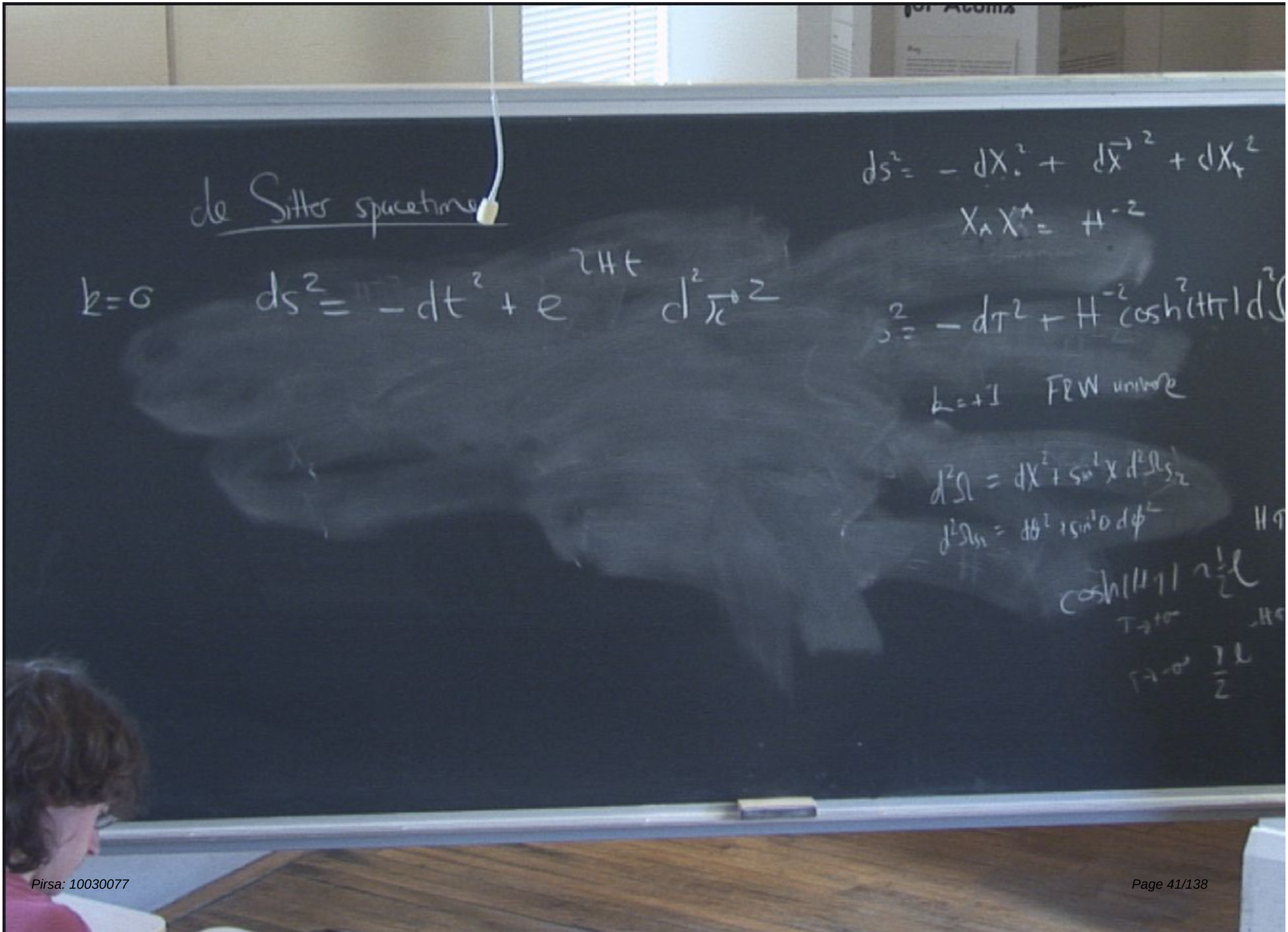
$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X}$$

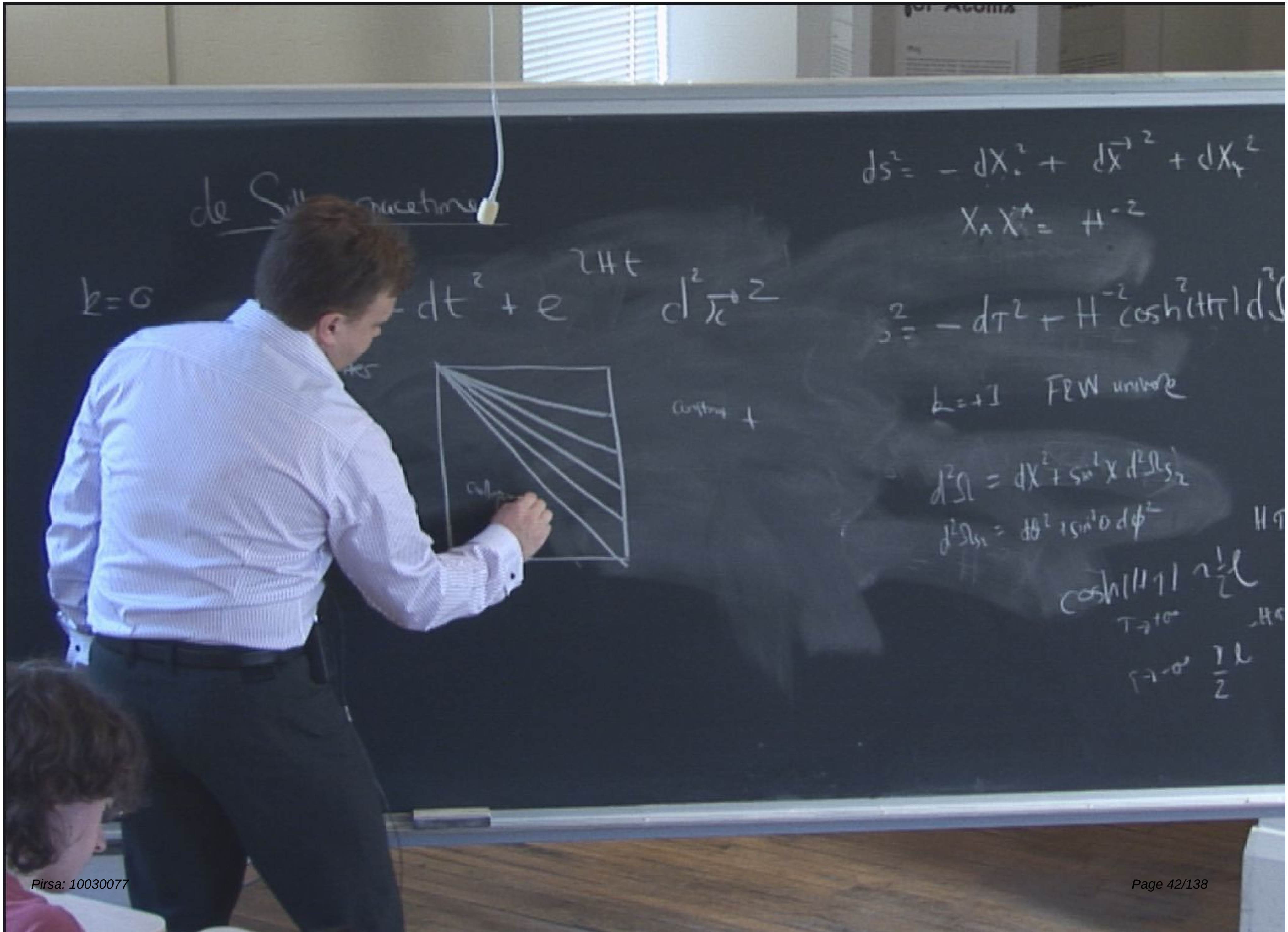
$\lim_{H^{-1} \to \infty} dS^4$ = Poincaré

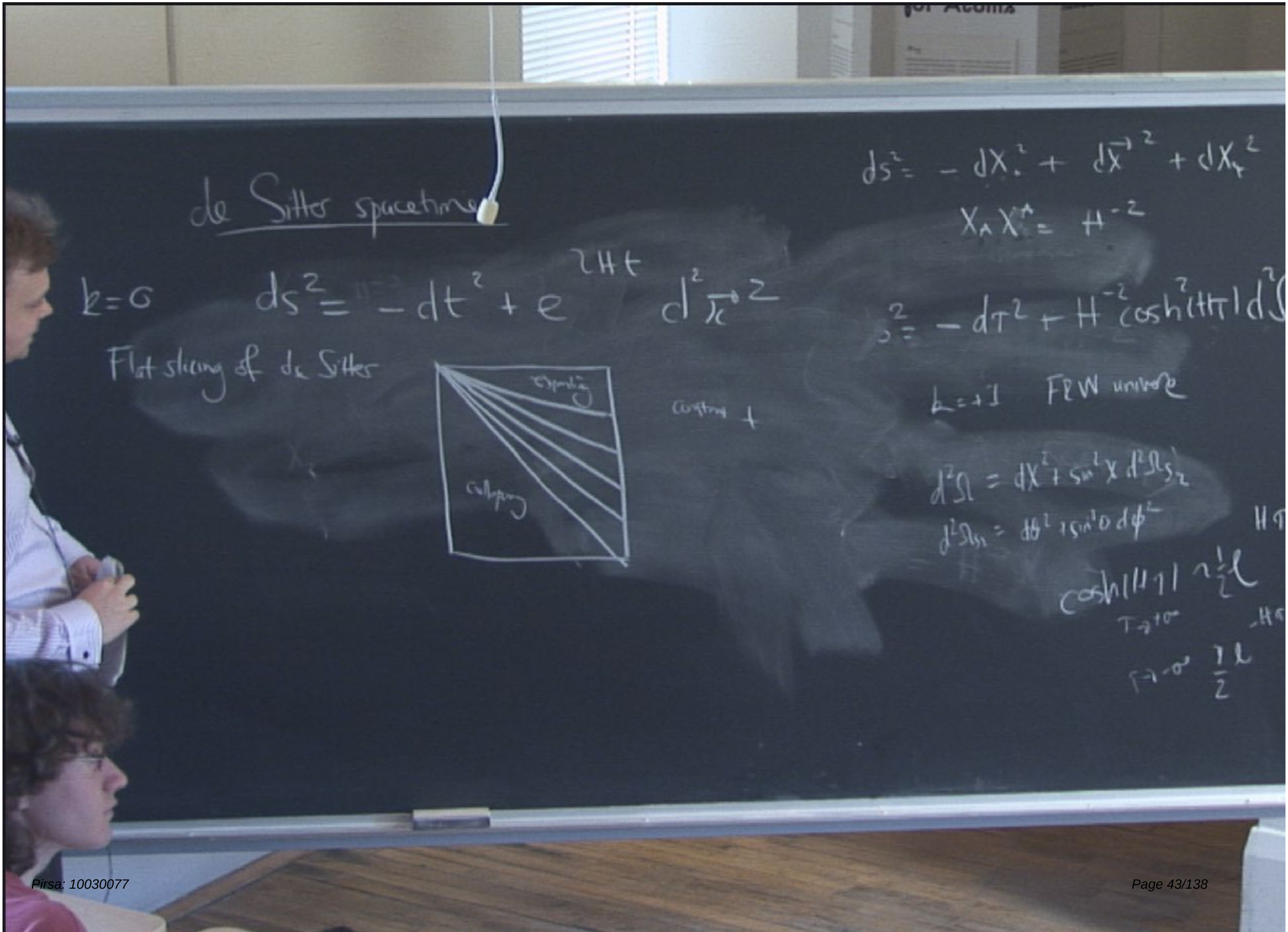
$\lim_{H^{-1} \to \infty}$ SO(1,4) → Poincaré

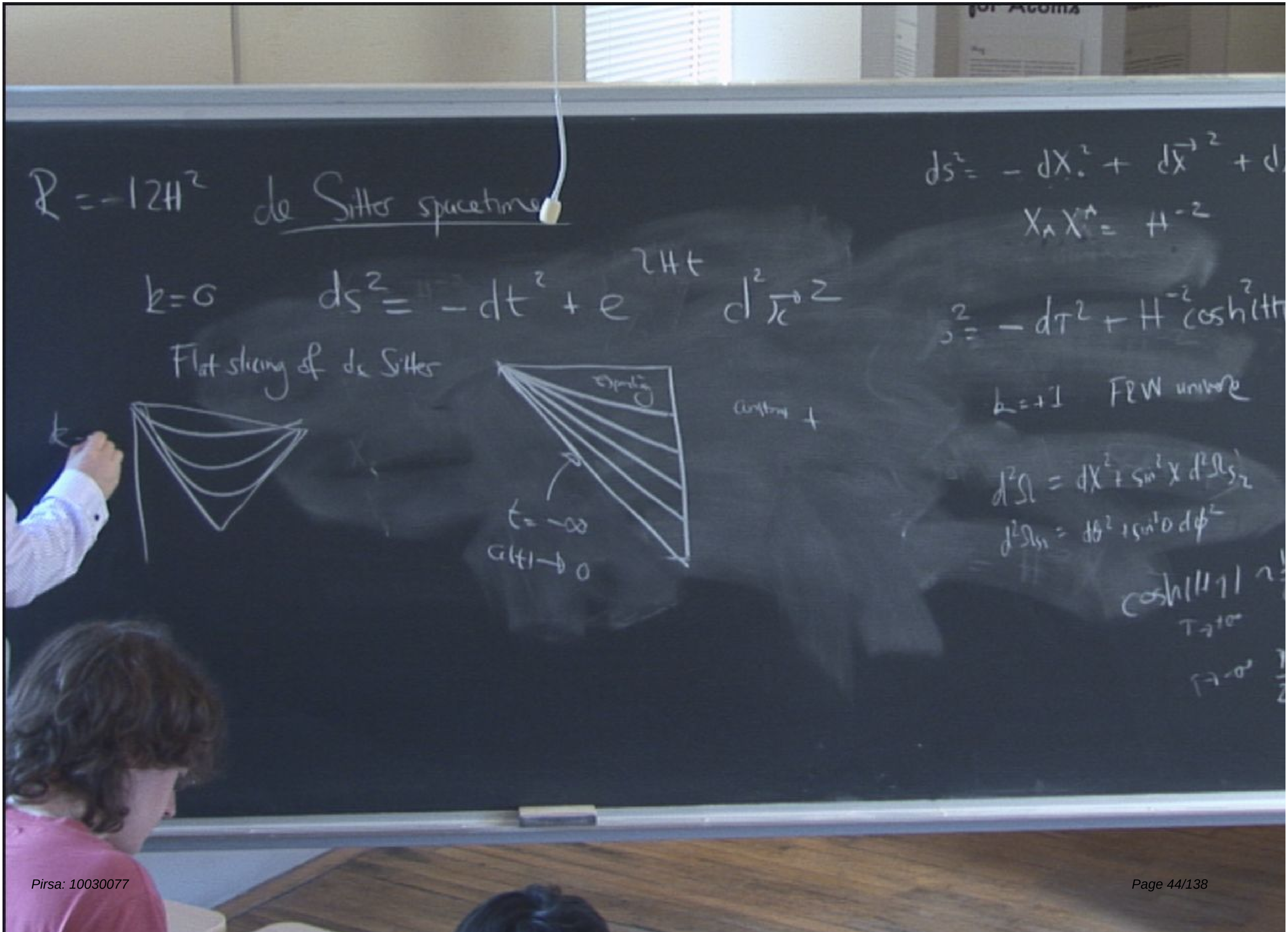
Wigner-Inönü contraction.

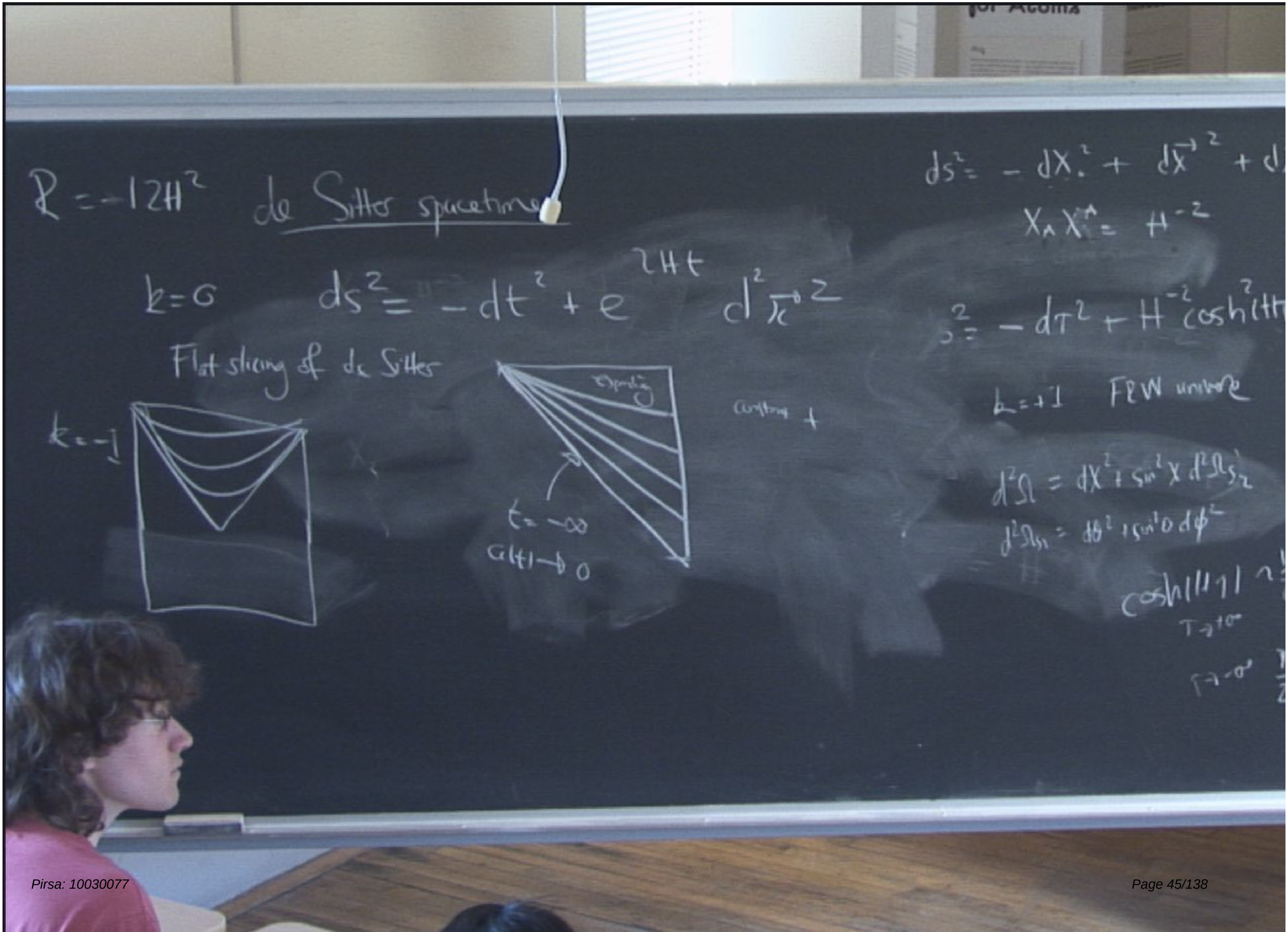
$$H^2 = \frac{1}{3M_{Pl}^2}\left(\rho - \frac{k}{a^2}\right)$$

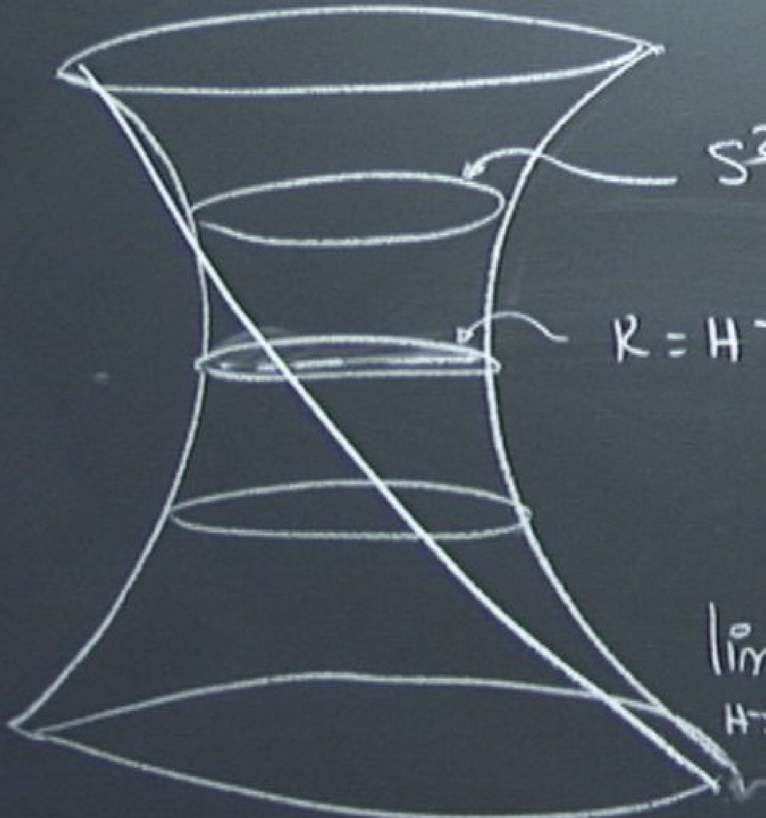

de Sitter spacetime
k=0 ds² = −dt² + e^{2Ht} dx⃗²
ds² = −dX₀² + dX⃗² + dX₄²
X_A X^A = H⁻²
ds² = −dτ² + H⁻² cosh²(Hτ) dΩ²
k=+1 FRW universe
d²Ω = dχ² + sin²χ d²Ω_{S²}
d²Ω_{S²} = dθ² + sin²θ dφ²
cosh(Hτ) ~ ½ e
τ → +∞
τ → −∞


k=0
k=+1 FRW universe


de Sitter spacetime
k=0
ds² = −dt² + e^{2Ht} dx²
Flat slicing of de Sitter
expanding
collapsing
ds² = −dX₀² + dX² + dX₄²
X_A X^A = H⁻²
k=+1 FRW universe
cosh(H|τ|)


de Sitter spacetime
Flat slicing of de Sitter
FRW universe
t = −∞


R = -12H^2
de Sitter spacetime
k=0
ds^2 = -dt^2 + e^{2Ht} d^2x^2
Flat slicing of de Sitter
k=-1
Expanding
t=-∞
a(t)→0
constant t
ds^2 = -dX_0^2 + dX^2 + d
X_A X^A = H^{-2}
ds^2 = -dτ^2 + H^{-2} cosh^2(H
k=+1 FRW universe
d^2Ω = dχ^2 + sin^2χ d^2Ω_{S_2}
d^2Ω_{S_2} = dθ^2 + sin^2θ dφ^2



$$\vec{X}^2 + X_4^2 = R^2 \qquad R = \sqrt{H^{-2} + X_0^2}$$

SO(1,4)

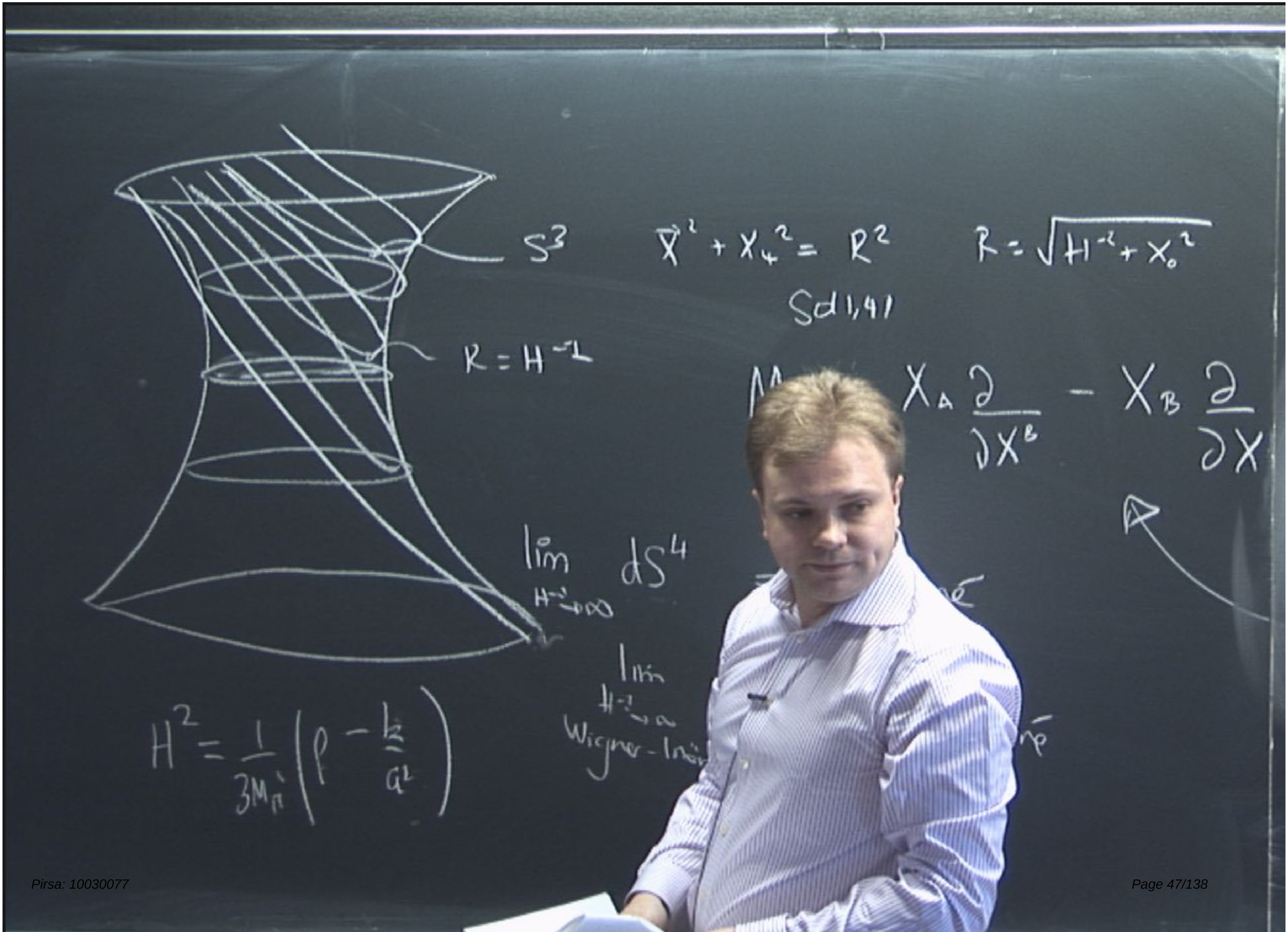
$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$$

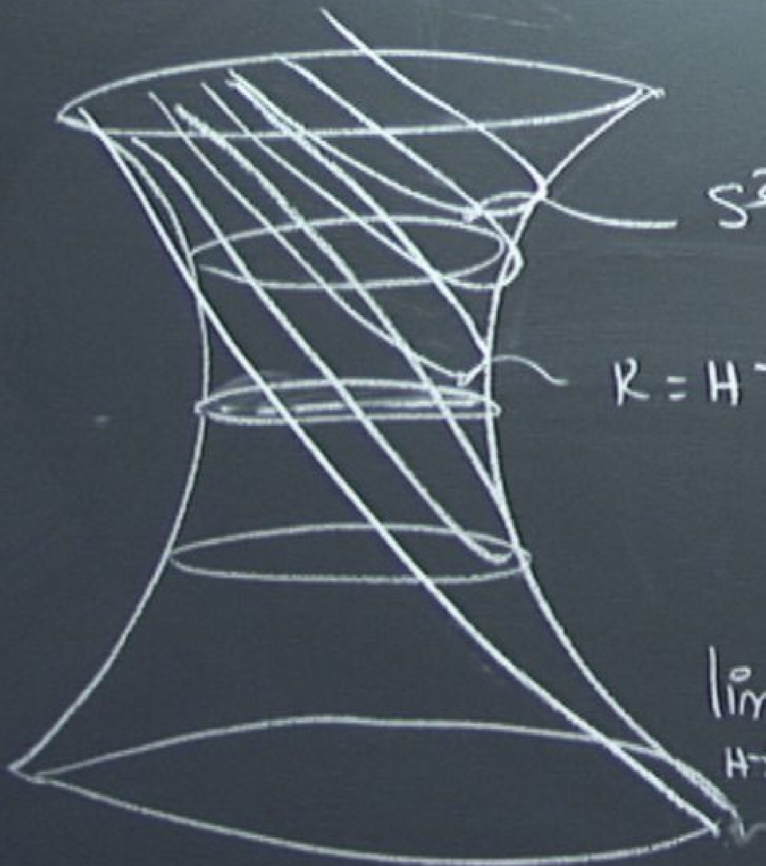
$$\lim_{H^{-1}\to\infty} dS^4 = \text{Poincaré}$$

$$\lim_{H^{-1}\to\infty} SO(1,4) \to \text{Poincaré}$$

Wigner-Inönü contraction.

$$H^2 = \frac{1}{3M_{Pl}^2}\left(\rho - \frac{k}{a^2}\right)$$


S3
R = H-1
lim dS4
lim



$\vec{X}^2 + X_4^2 = R^2 \qquad R = \sqrt{H^{-2} + X_0^2}$

SO(1,4)

$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X}$$

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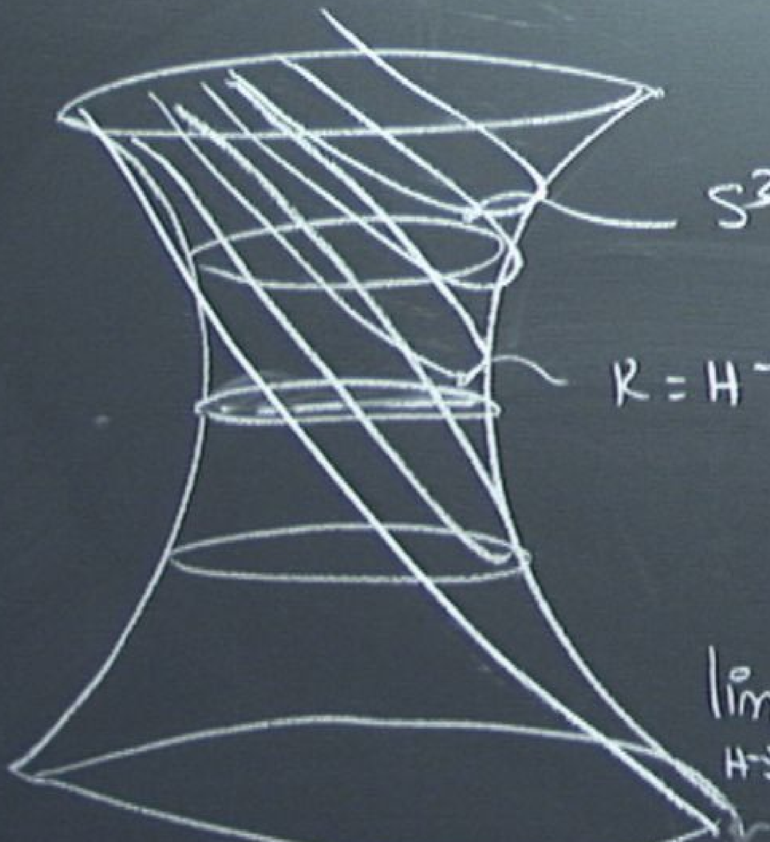

$$\vec{X}^2 + X_4^2 = R^2 \qquad R = \sqrt{H^{-2} + X_0^2}$$

SO(1,4)

$$M_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$$

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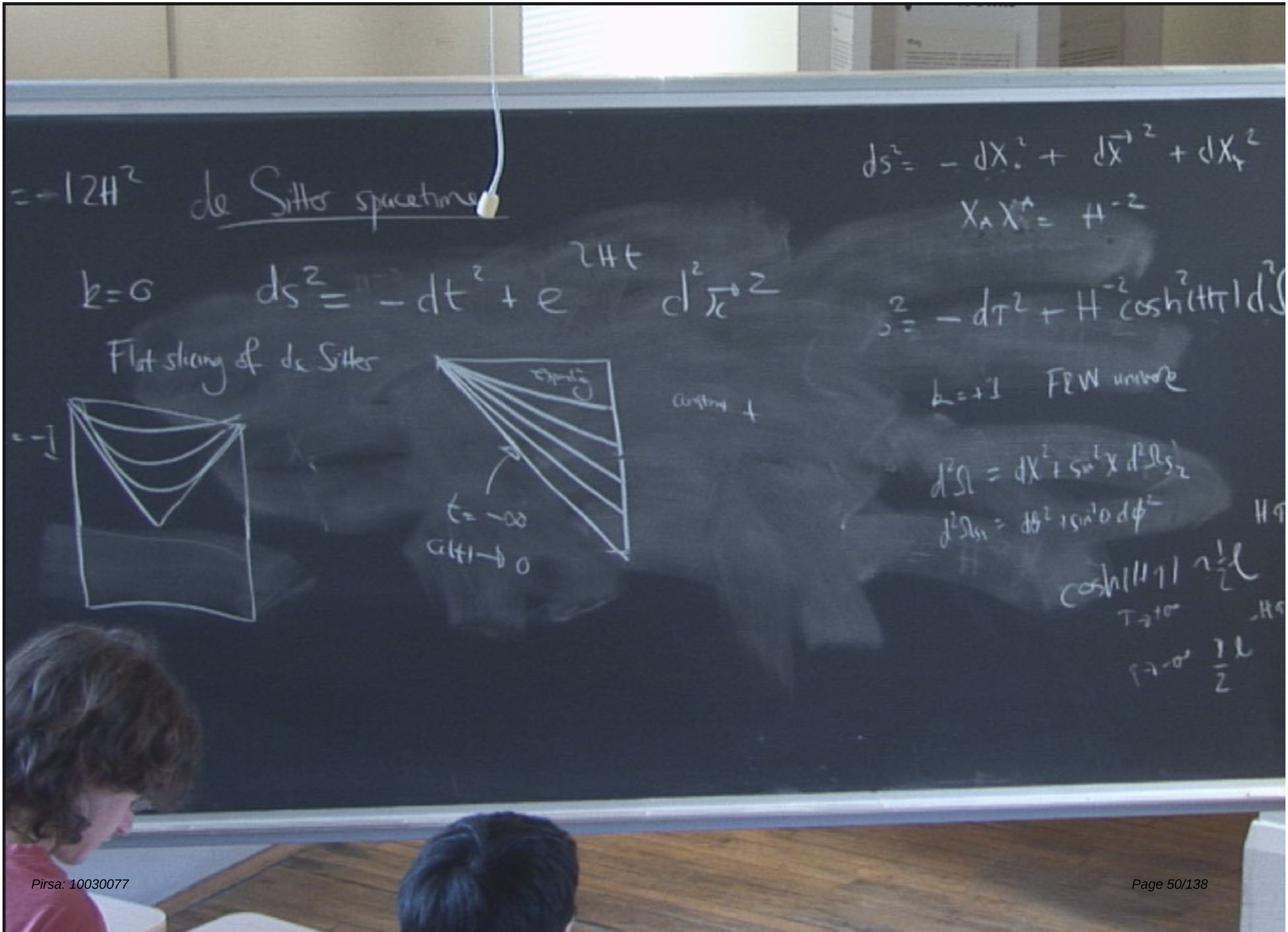
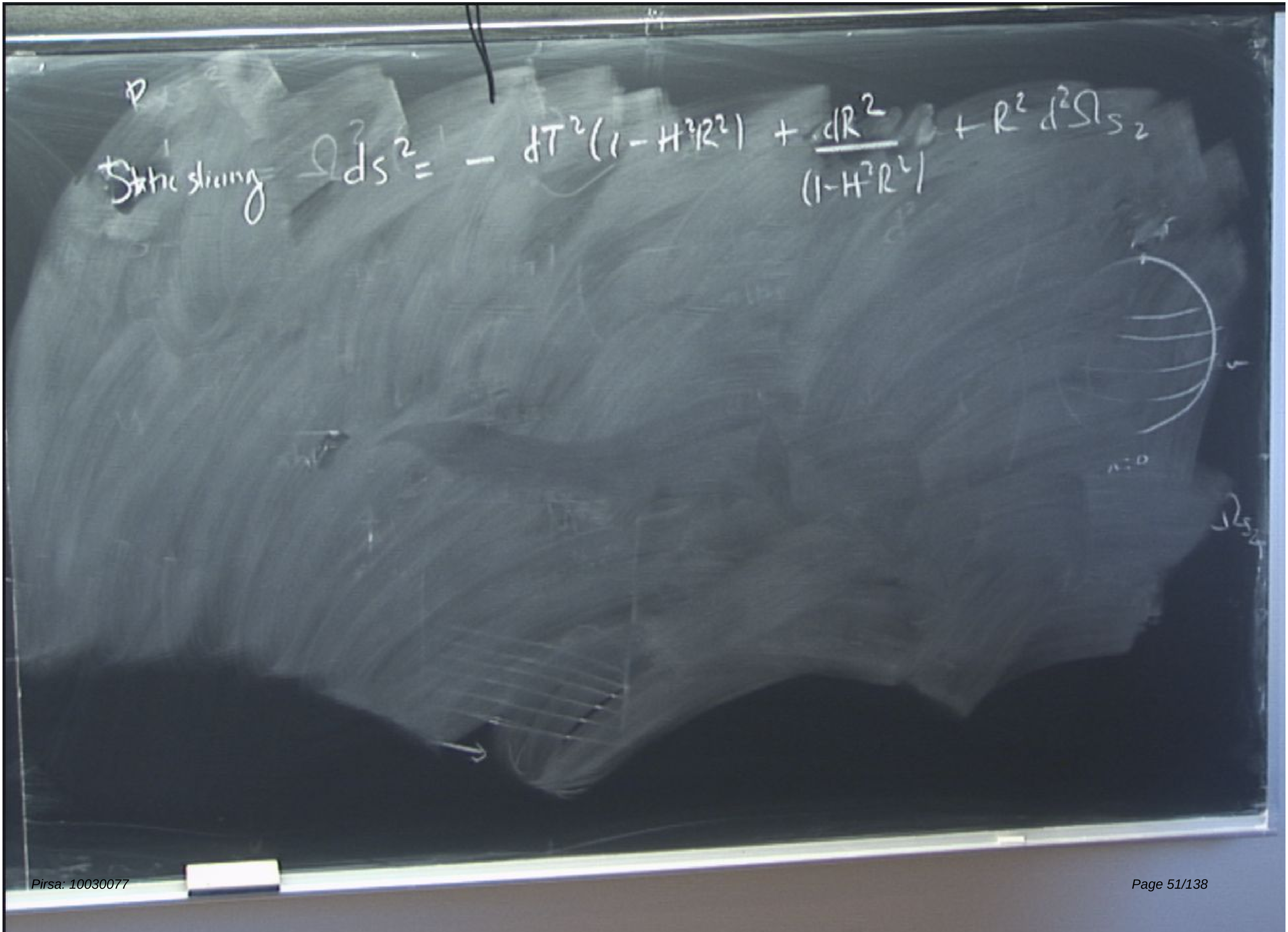
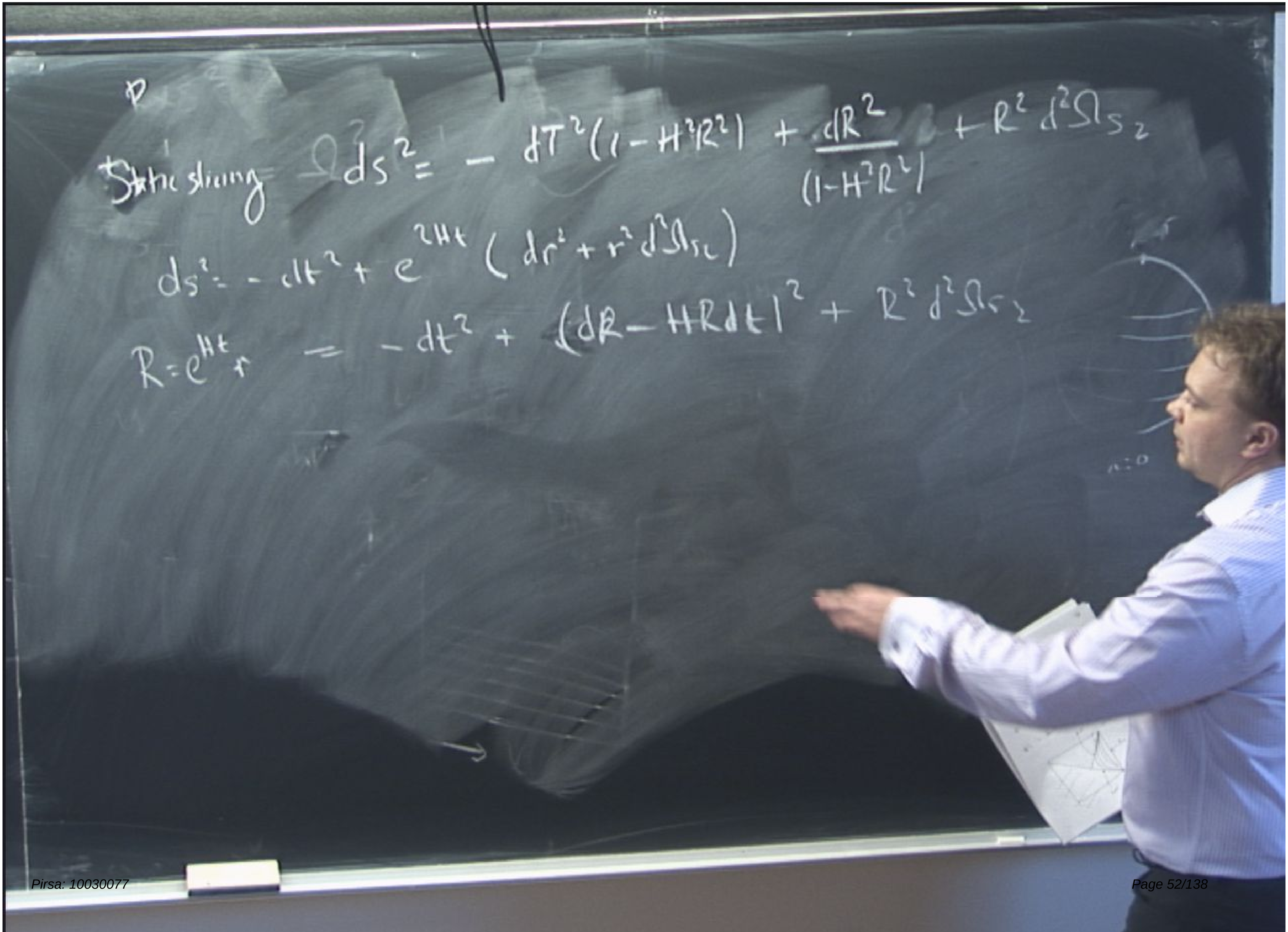
$$\lim_{H^{-1} \to \infty} SO(1,4) \to \text{Poincaré}$$

Wigner-Inönü contraction.

$$H^2 = \frac{1}{3M_{Pl}^2}\left(\rho - \frac{k}{a^2}\right)$$


de Sitter spacetime
k=0
$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$
Flat slicing of de Sitter
t = −∞
$ds^2 = -dX_0^2 + d\vec{X}^2 + dX_4^2$
$X_A X^A = H^{-2}$
k = +1 FRW universe


Static slicing $ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$


Static slicing $ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$
$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$
$R = e^{Ht} r$
$= -dt^2 + (dR - HRdt)^2 + R^2 d^2\Omega_{S_2}$

Static slicing

$$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

$$R = e^{Ht} r \qquad = -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

Static slicing

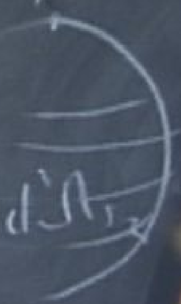
$$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

$R = e^{Ht} r$

$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$



$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$

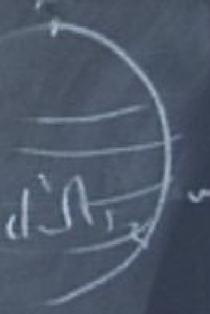
Static slicing $ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

$R = e^{Ht} r$

$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$



$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$

Ω_{S_2}

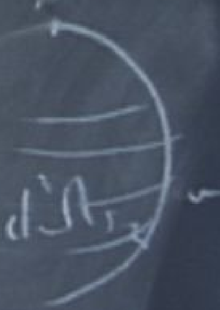
Static slicing $ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

$R = e^{Ht} r$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$



$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$

Ω_{S_2}

Static slicing

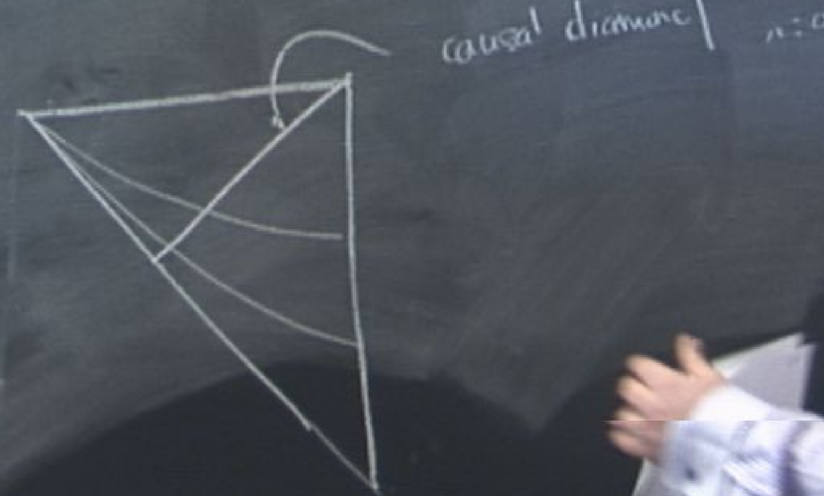
$$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

$$R = e^{Ht} r \qquad = -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$



Static slicing $ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$

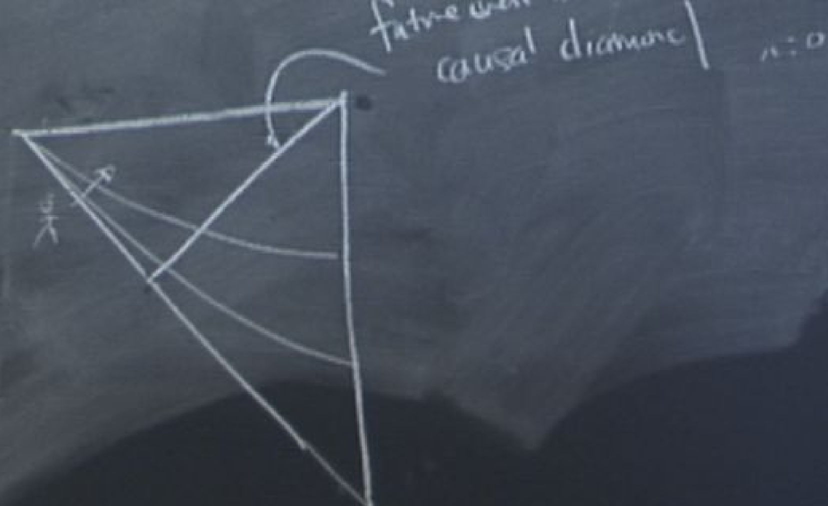
$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

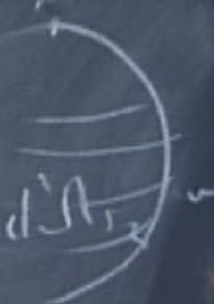
$R = e^{Ht} r$

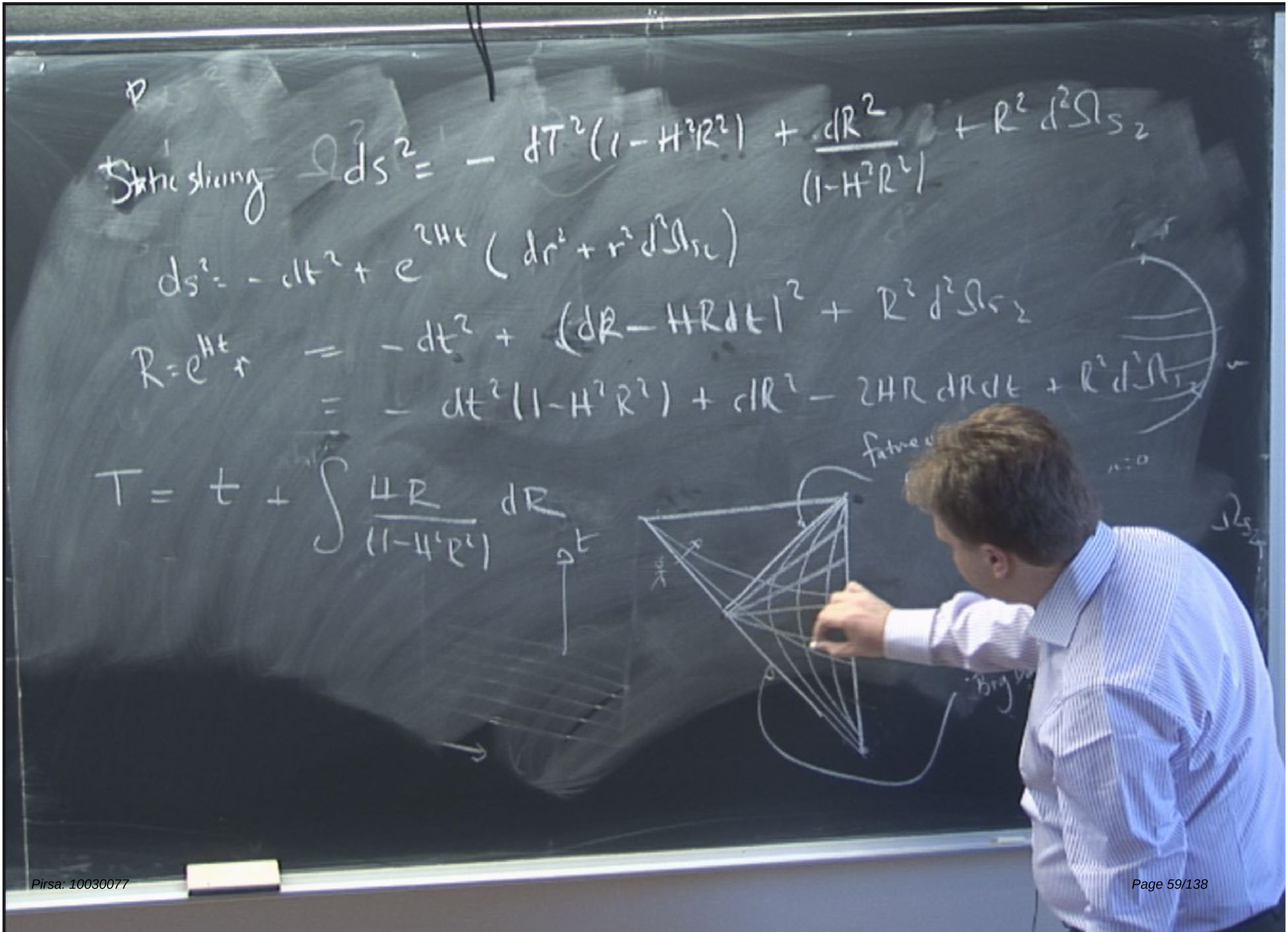
$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR}{(1-H^2R^2)}\,dR$$






Static slicing
ds² = − dT²(1 − H²R²) + dR²/(1 − H²R²) + R² d²Ω_S2
ds² = − dt² + e^2Ht (dr² + r² d²Ω_S2)
R = e^Ht r
= − dt² + (dR − HR dt)² + R² d²Ω_S2
= − dt²(1 − H²R²) + dR² − 2HR dR dt + R² d²Ω_S2
T = t + ∫ HR/(1 − H²R²) dR
t

P

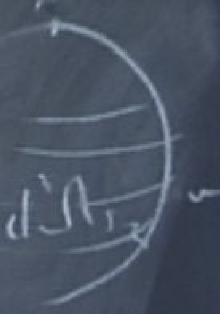
Static slicing $ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

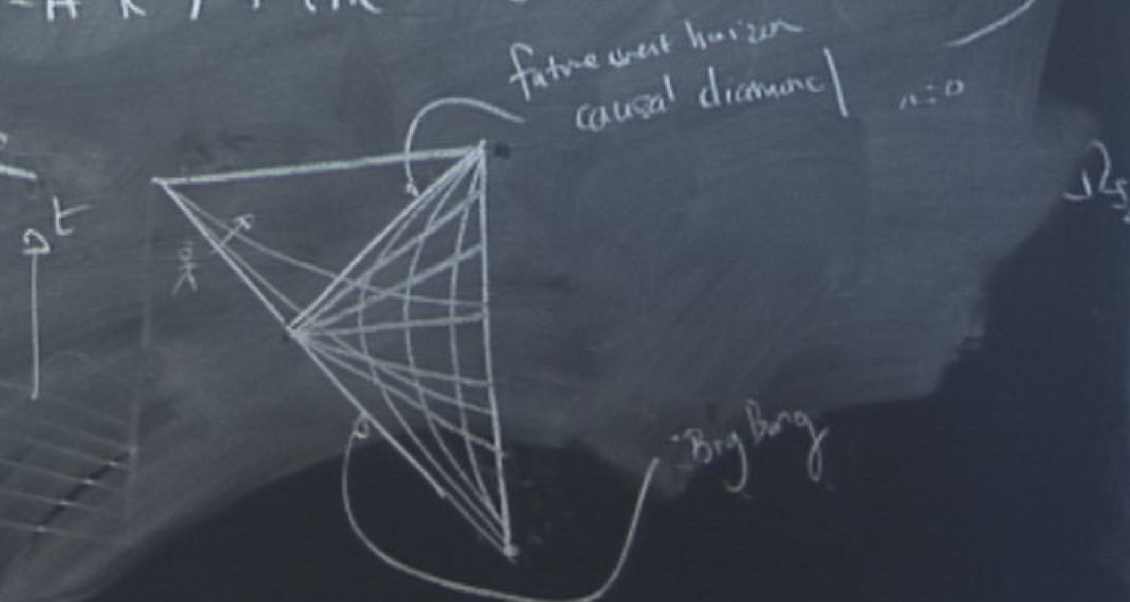
$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

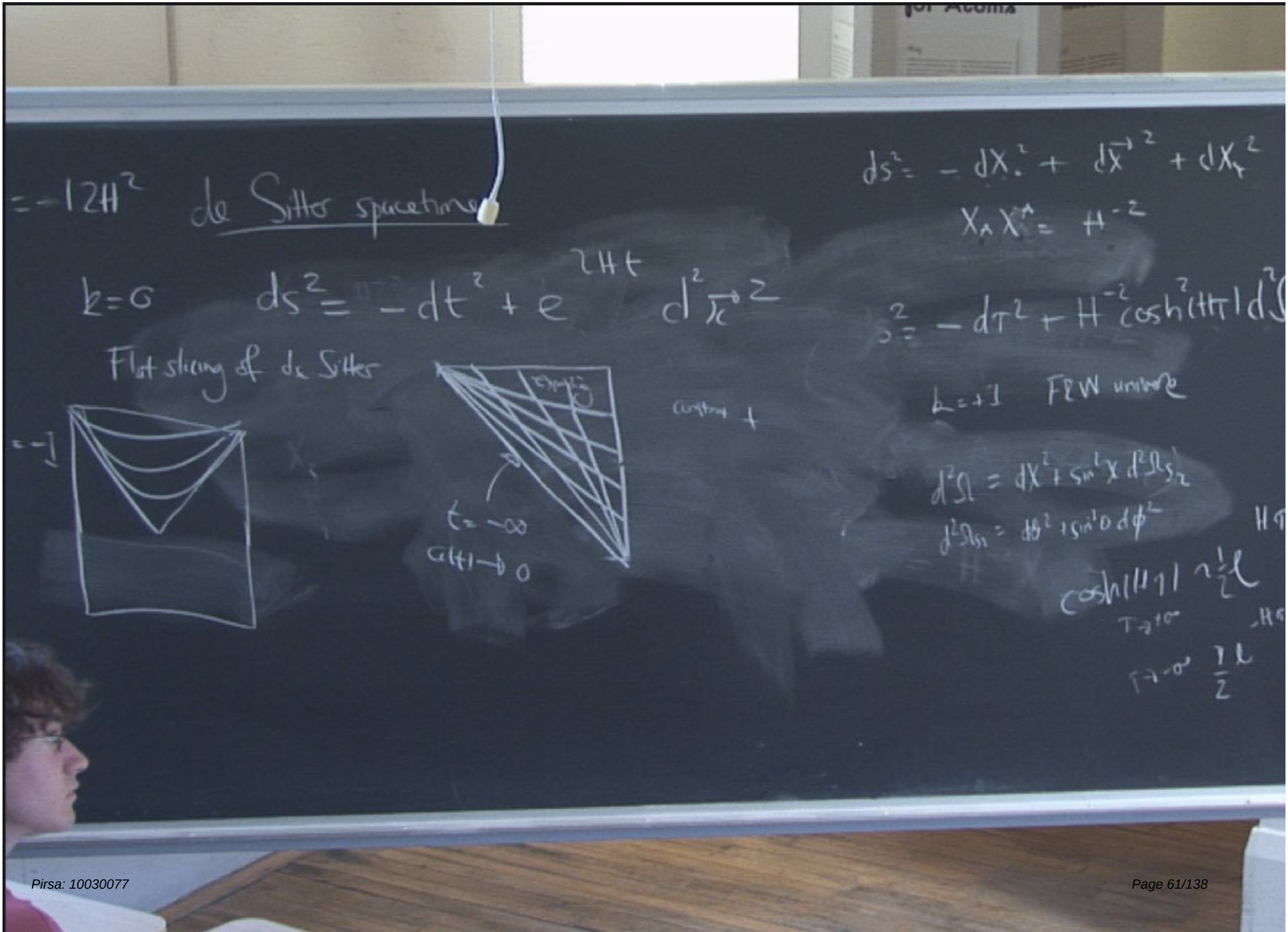
$R = e^{Ht} r$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$



$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$




de Sitter spacetime
k=0
$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$
Flat slicing of de Sitter
t = −∞
FRW universe
$d^2\Omega = d\chi^2 + \sin^2\chi \, d^2\Omega_{S_2}$
$d^2\Omega_{S_2} = d\theta^2 + \sin^2\theta \, d\phi^2$

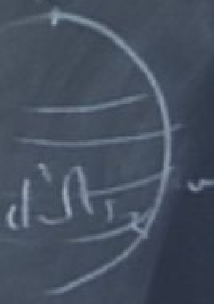
Static slicing

$$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

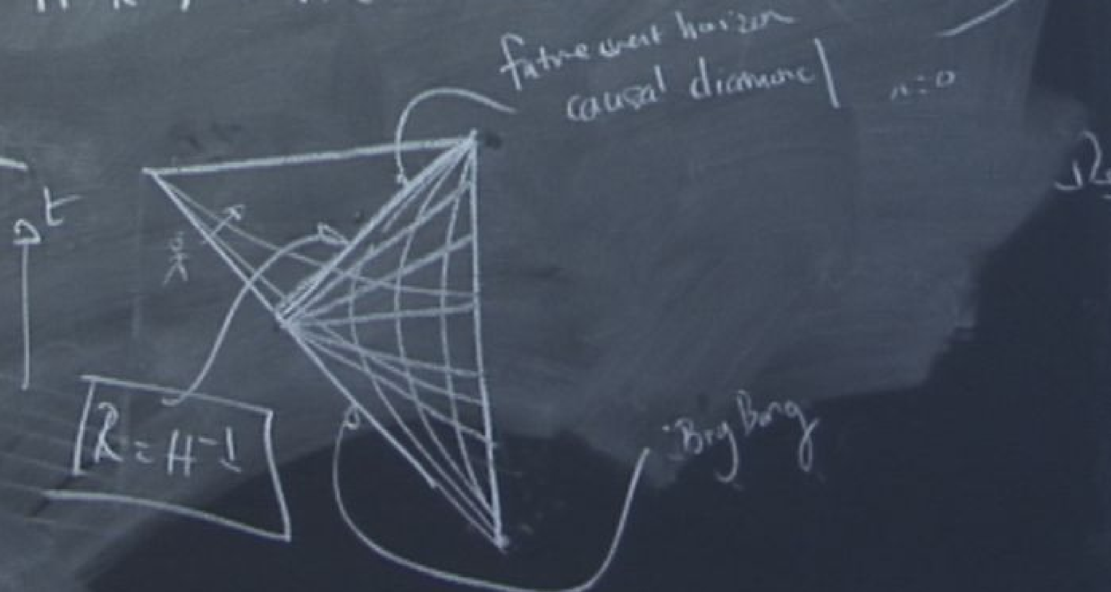
$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

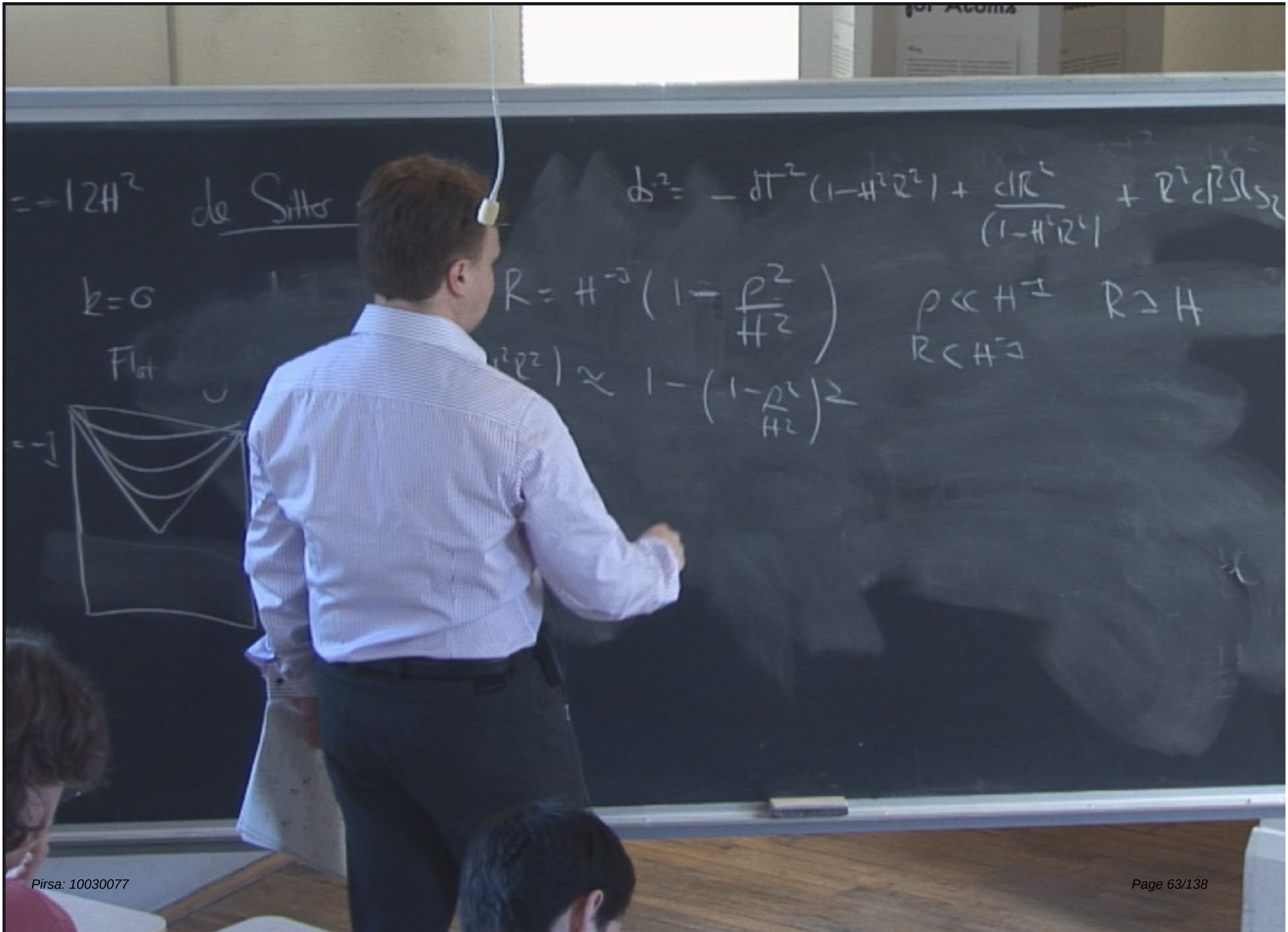
$$R = e^{Ht} r \qquad = -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

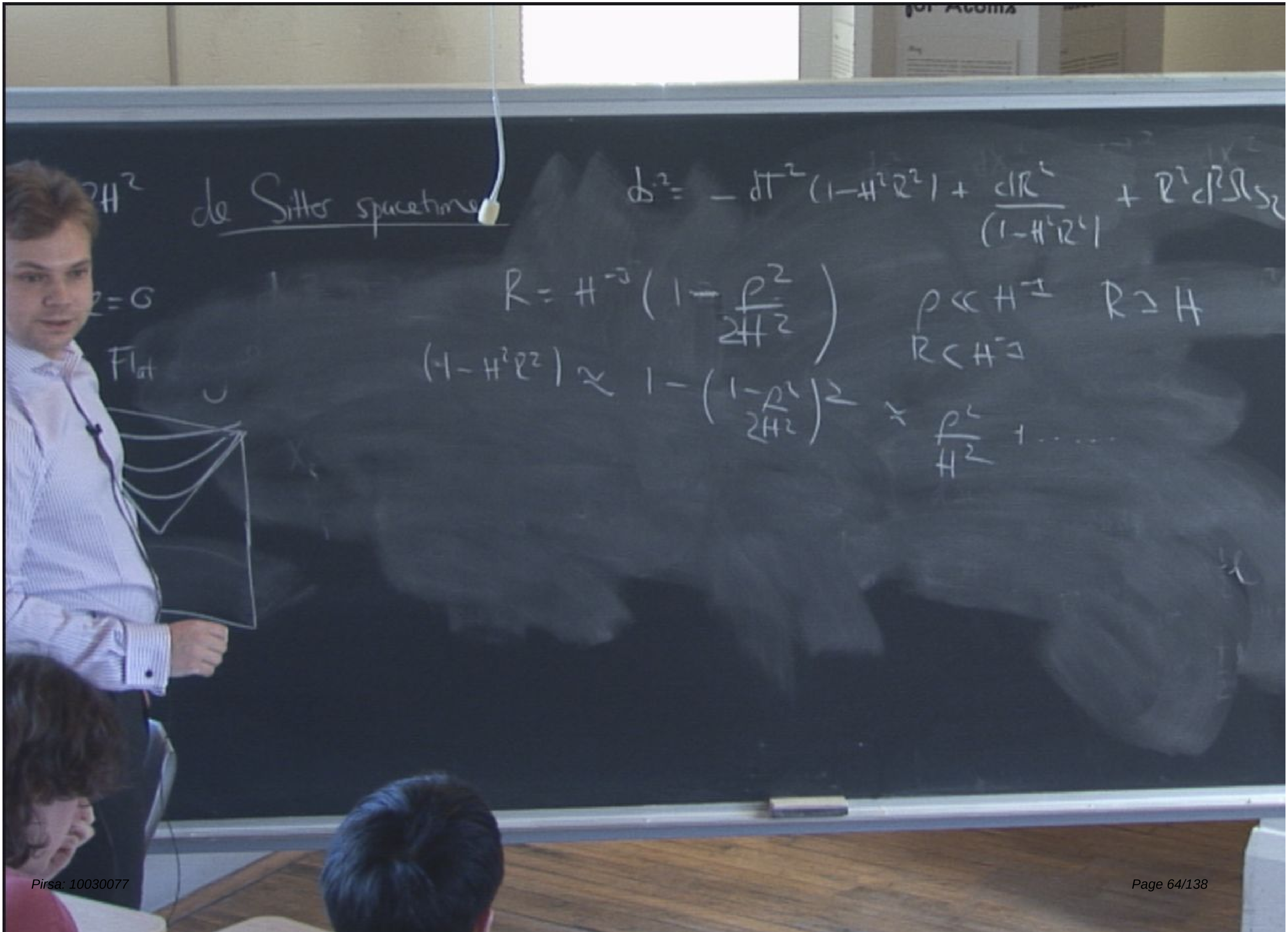
$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$

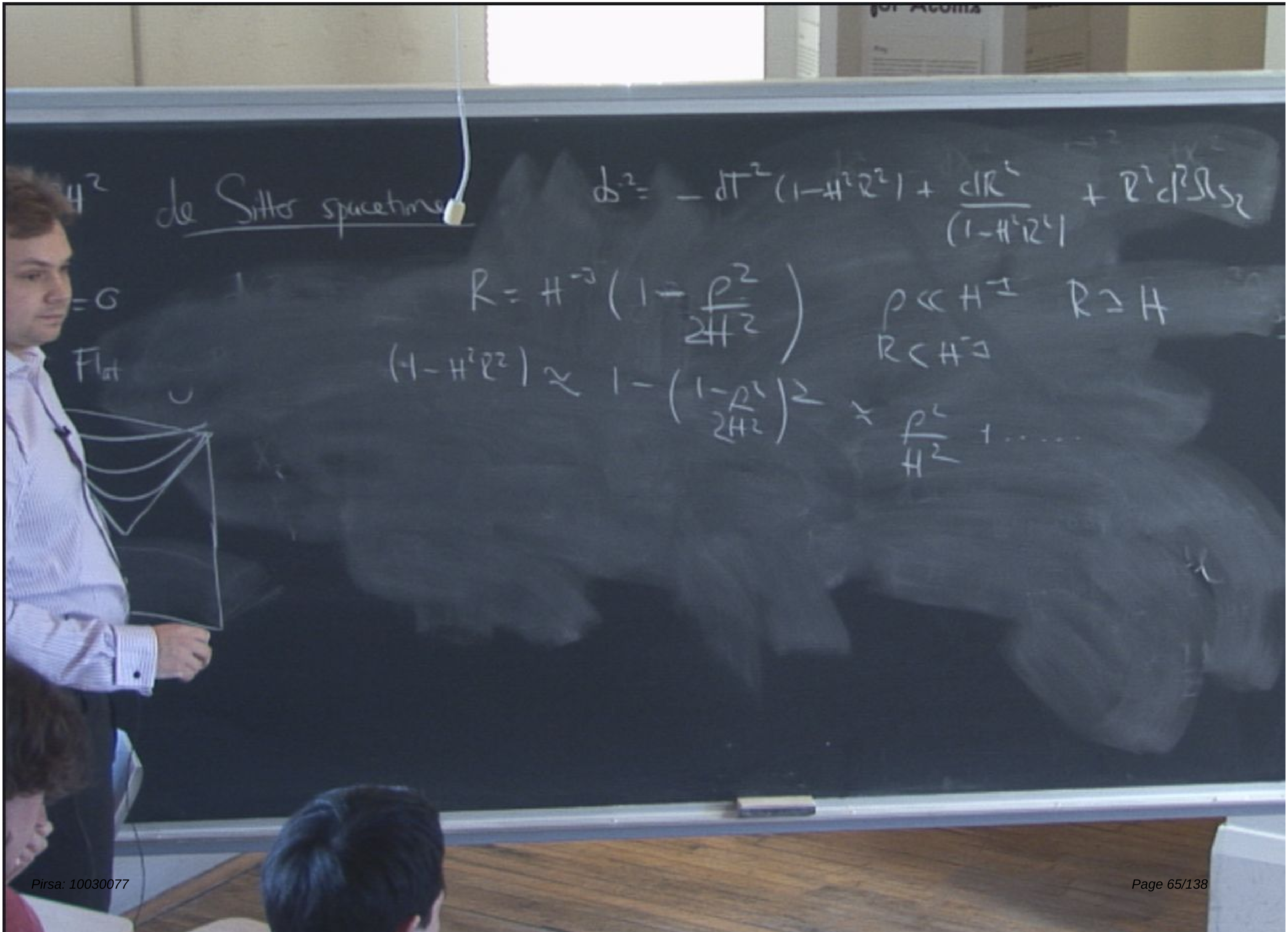


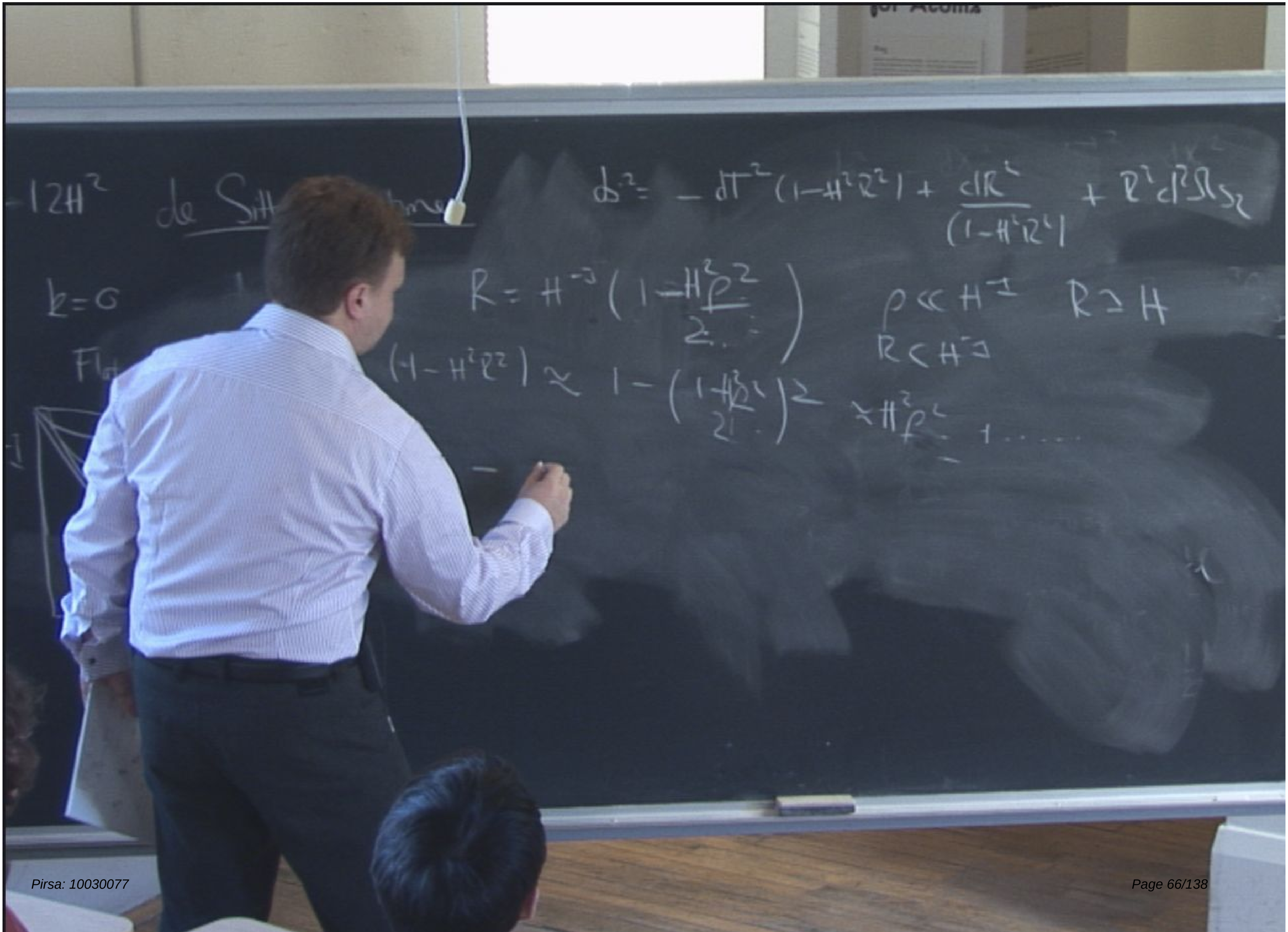
$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$

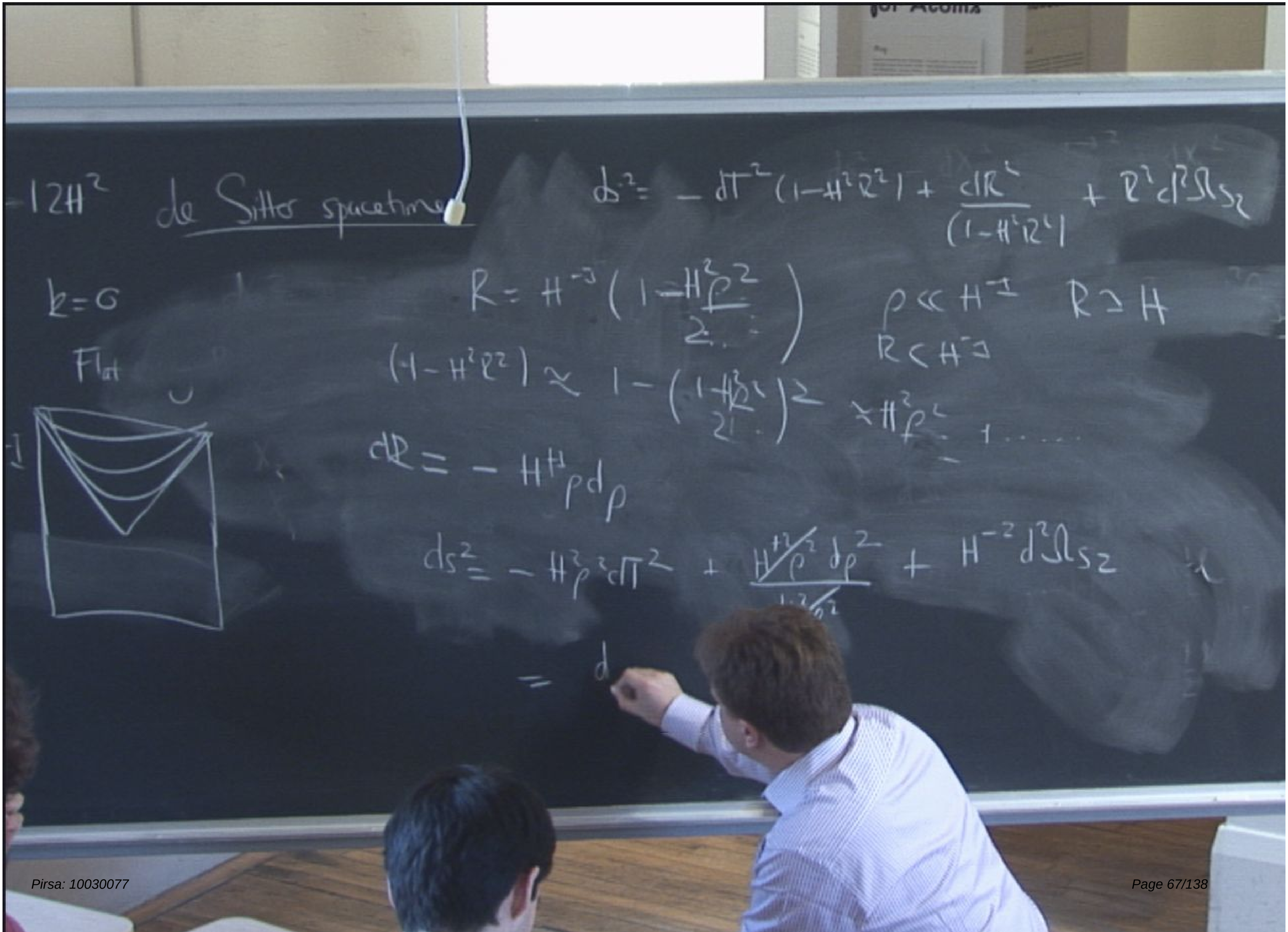


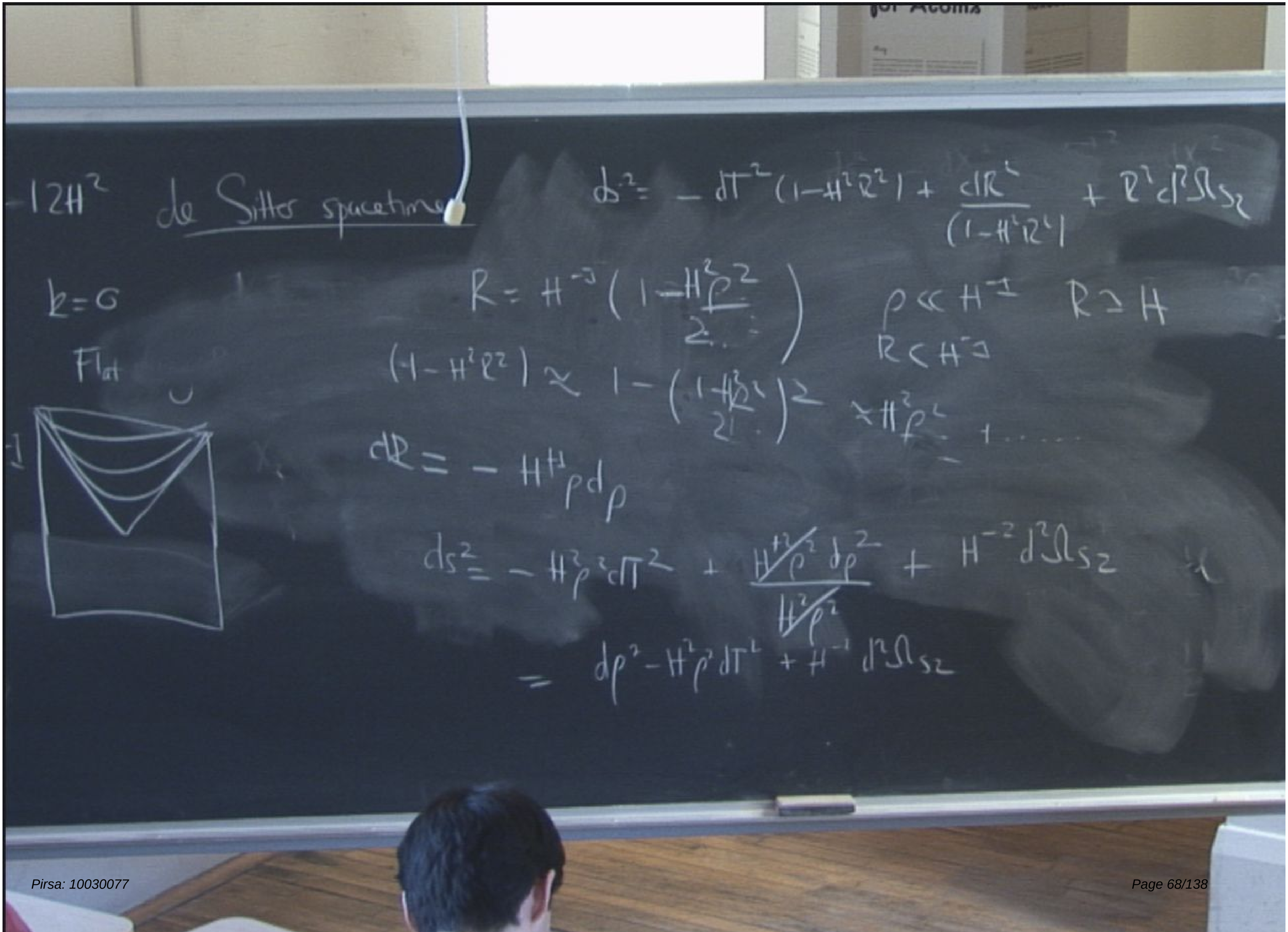

de Sitter
k=0
Flat
R = H^-1 (1 - ρ^2 / H^2)
ρ << H^-1
R ≃ H
R < H^-1

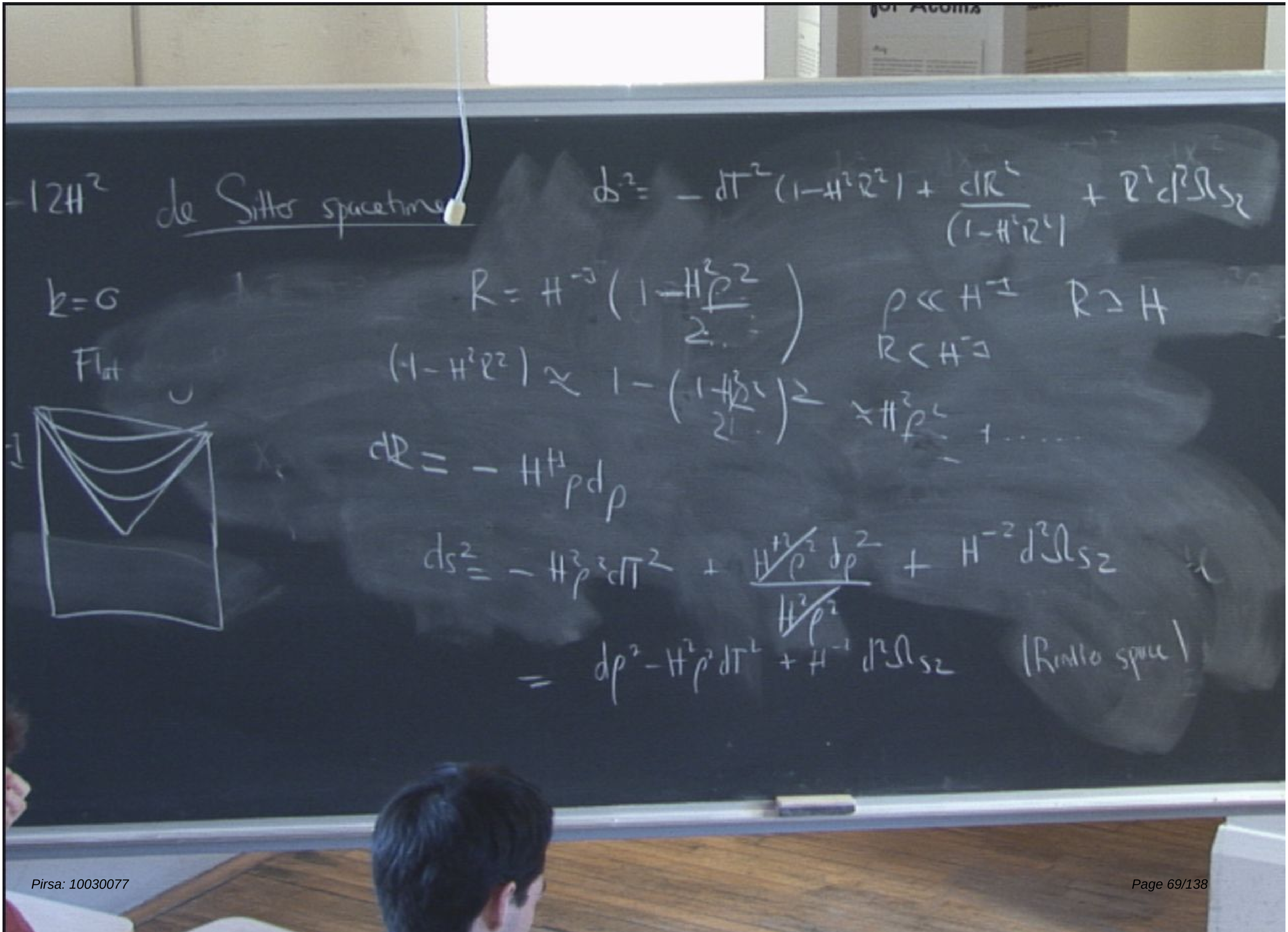

de Sitter spacetime
ds² = − dT²(1−H²R²) + dR²/(1−H²R²) + R²dΩ²S2
R = H⁻¹(1 − ρ²/2H²)
ρ ≪ H⁻¹ R ≥ H
R < H⁻¹
(1 − H²R²) ≃ 1 − (1 − ρ²/2H²)² ≃ ρ²/H² + ...
Flat

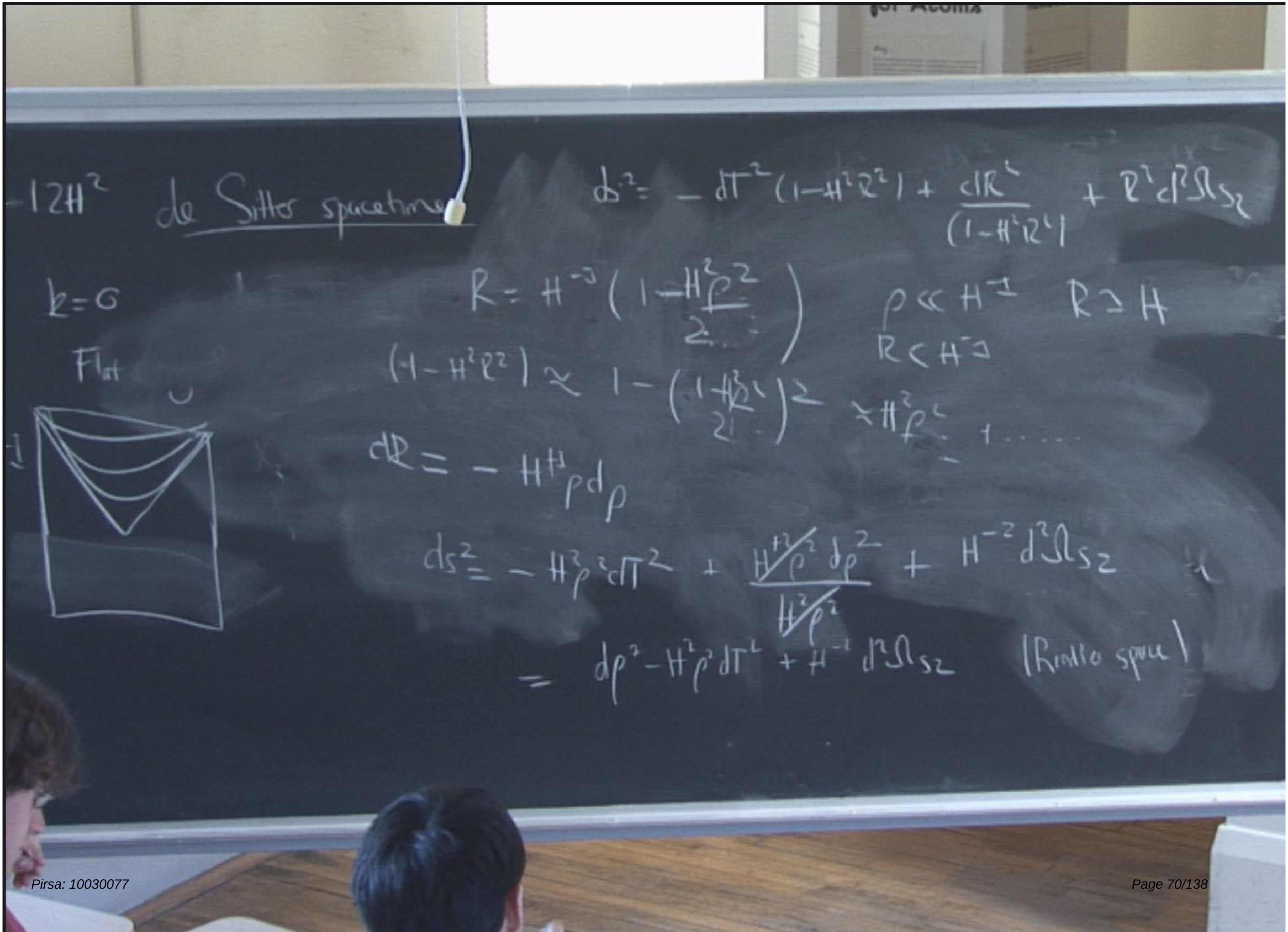

de Sitter spacetime
ds² = − dT² (1 − H²R²) + dR² / (1 − H²R²) + R² dΩ_S2
R = H⁻¹ (1 − ρ² / 2H²)
ρ ≪ H⁻¹
R ≃ H
R < H⁻¹
(1 − H²R²) ≃ 1 − (1 − ρ² / 2H²)² ≃ ρ² / H² + ...
Flat

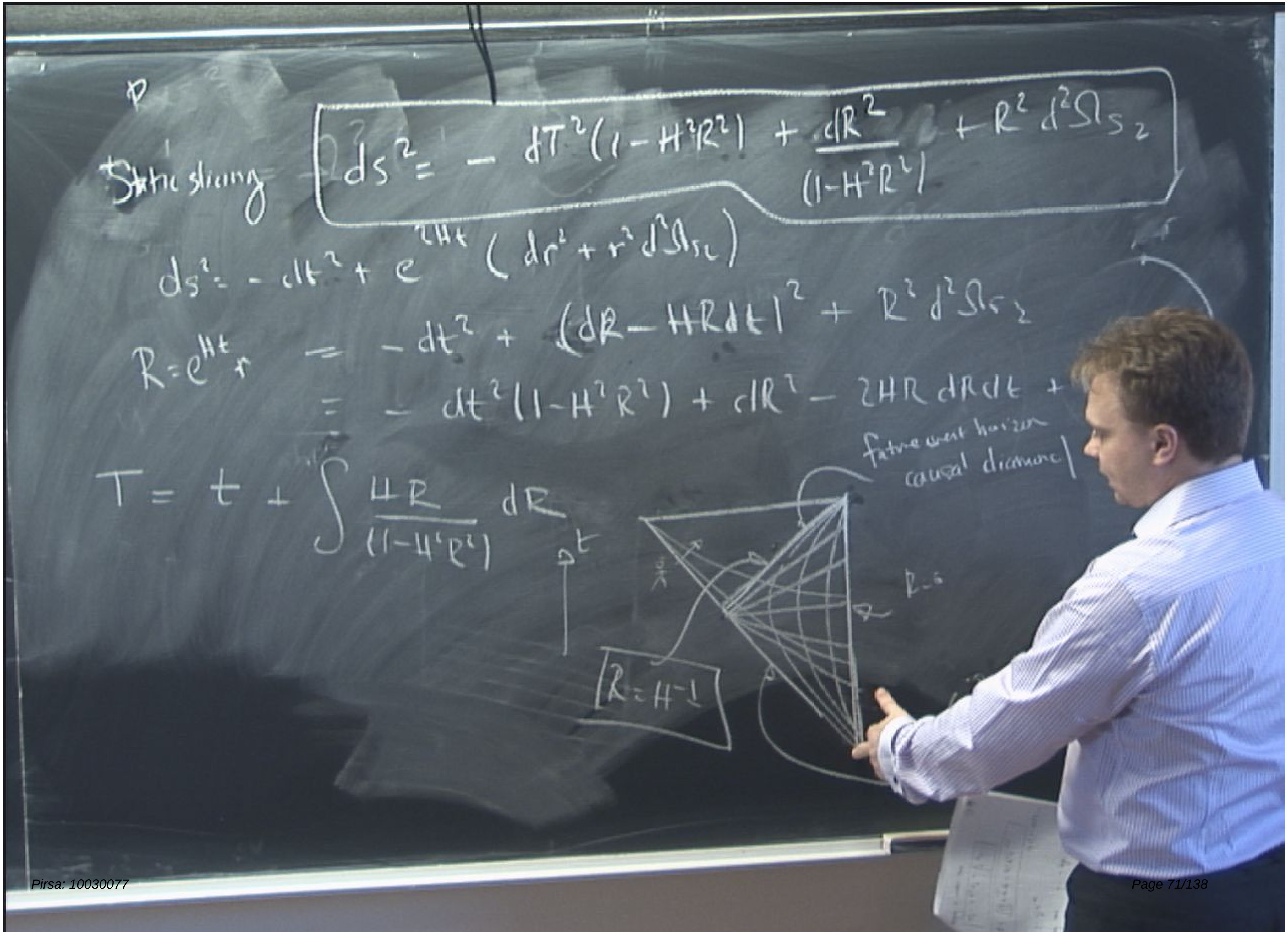

de Sitter
R ≪ H


12H²
de Sitter spacetime
k=0
Flat
ρ ≪ H⁻¹   R ≥ H
R < H⁻¹


12H²
de Sitter spacetime
k=0
Flat
ds² = −dT²(1−H²R²) + dR²/(1−H²R²) + R²d²Ω_{S2}
R = H⁻¹(1 − H²ρ²/2)
ρ ≪ H⁻¹
R < H⁻¹
dR = − H⁺¹ρdρ
ds² = − H²ρ²dT² + H⁺²ρ²dρ²/H²ρ² + H⁻²d²Ω_{S2}
= dρ² − H²ρ²dT² + H⁻¹d²Ω_{S2}


-12H^2
de Sitter spacetime
ds^2 = -dT^2(1-H^2R^2) + dR^2/(1-H^2R^2) + R^2 d^2Ω_S2
k=0
Flat
R = H^-1(1 - H^2ρ^2/2)
ρ << H^-1 R ≥ H
R < H^-1
(1 - H^2R^2) ≃ 1 - (1 + H^2ρ^2/2)^2 ≃ H^2ρ^2 + ...
dR = - H^+1 ρ dρ
ds^2 = - H^2ρ^2 dT^2 + H^+2 ρ^2 dρ^2 / H^2 ρ^2 + H^-2 d^2Ω_S2
= dρ^2 - H^2ρ^2 dT^2 + H^-2 d^2Ω_S2 (Rindler space)


de Sitter spacetime
$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$
$k=0$
Flat
$R = H^{-1}\left(1 - \frac{H^2\rho^2}{2} \ldots\right)$
$\rho \ll H^{-1}$ $R \simeq H$
$R < H^{-1}$
$(1-H^2R^2) \simeq 1 - \left(1 - \frac{H^2\rho^2}{2}\right)^2 \simeq H^2\rho^2 + \ldots$
$dR = -H^{+1}\rho\, d\rho$
$ds^2 = -H^2\rho^2 dT^2 + \frac{H^{+2}\rho^2 d\rho^2}{H^2\rho^2} + H^{-2} d^2\Omega_{S_2}$
$= d\rho^2 - H^2\rho^2 dT^2 + H^{-2} d^2\Omega_{S_2}$
(Rindler space)


Static slicing
$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$
$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$
$R = e^{Ht} r$
$= -dt^2 + (dR - HRdt)^2 + R^2 d^2\Omega_{S_2}$
$= -dt^2(1-H^2R^2) + dR^2 - 2HR\, dR\, dt +$
$T = t + \int \frac{HR}{(1-H^2R^2)} dR$
future event horizon
causal diamond
t
$R=H^{-1}$

Static slicing

$$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

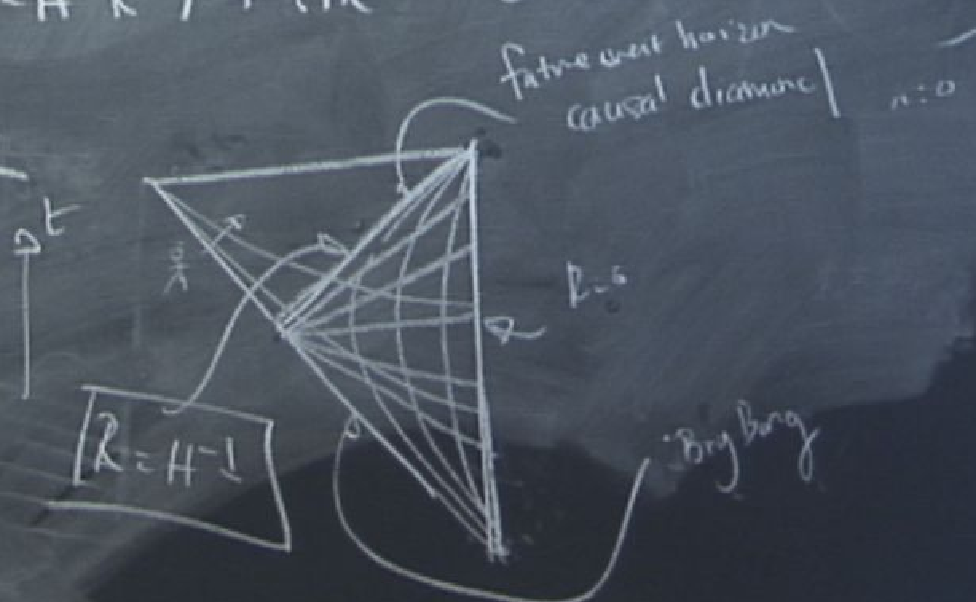
$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

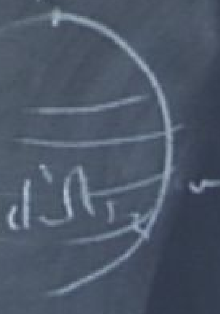
$R = e^{Ht} r$

$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR}{(1-H^2R^2)}\, dR$$





Static slicing

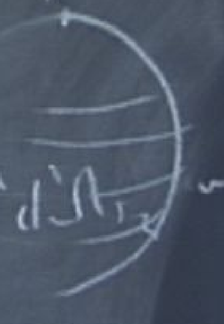
$$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$$

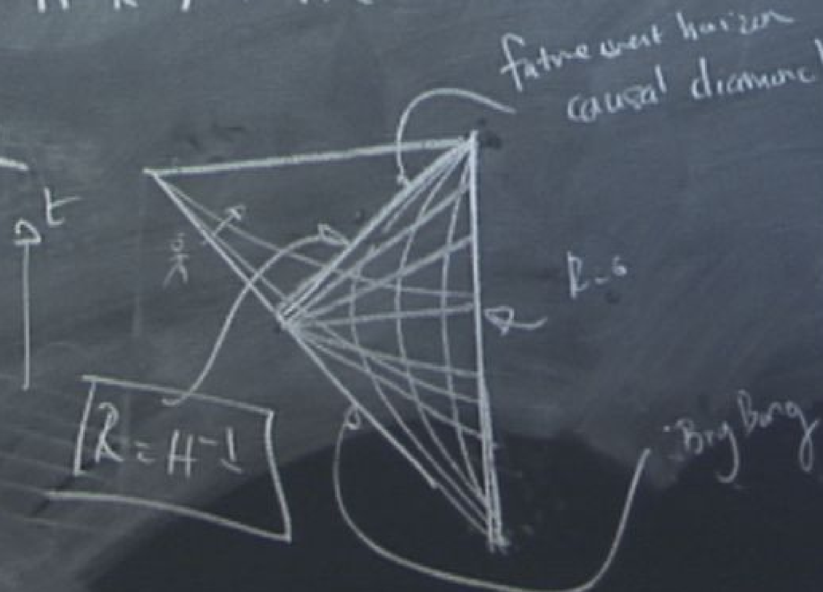
$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S_2}$$

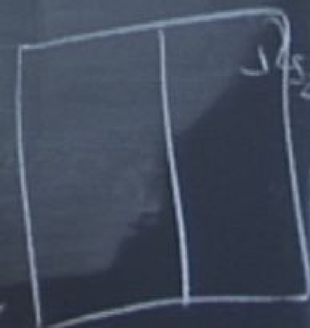
$R = e^{Ht} r$

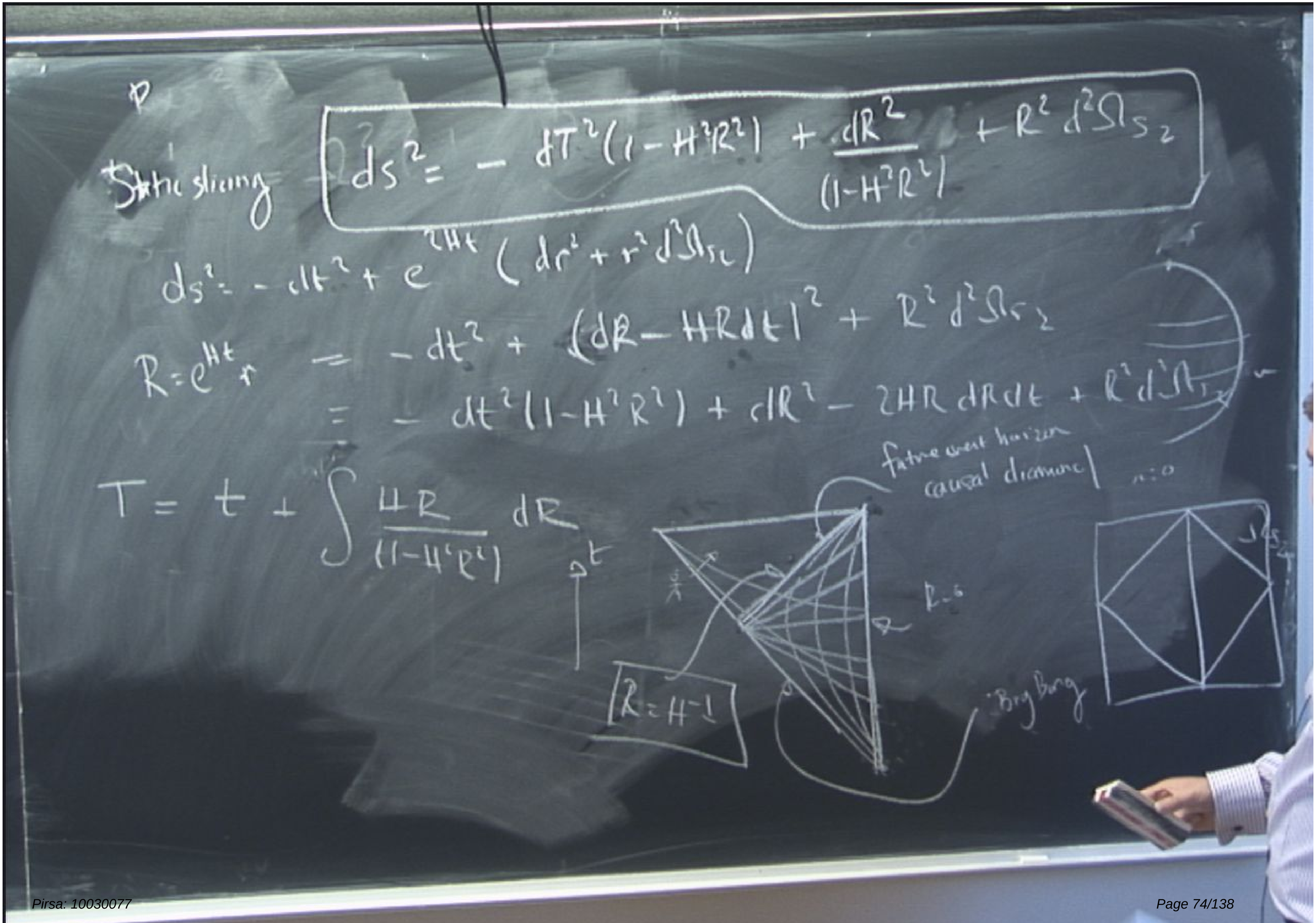
$$= -dt^2(1-H^2R^2) + dR^2 - 2HR\,dR\,dt + R^2 d^2\Omega_{S_2}$$



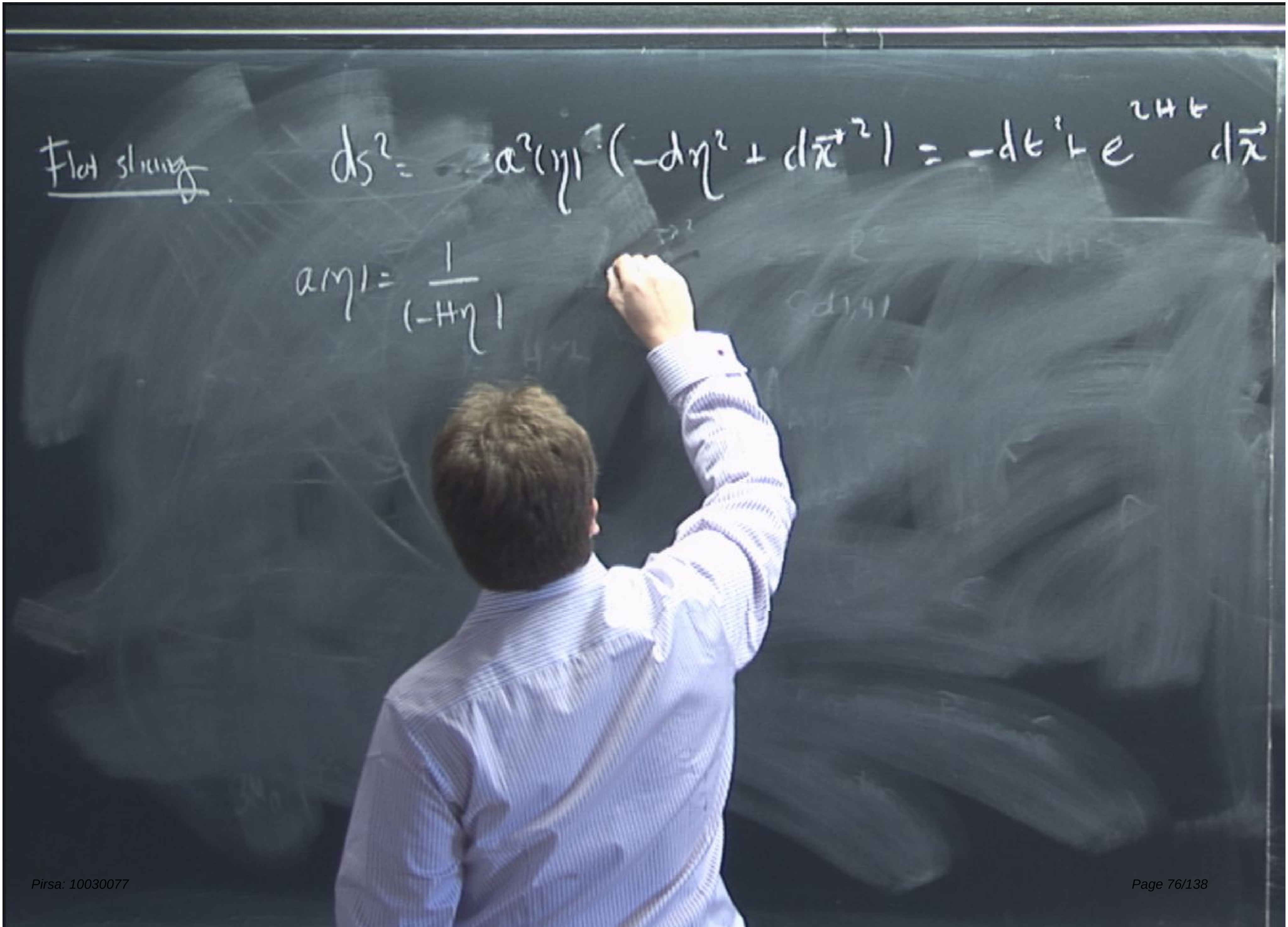
$$T = t + \int \frac{HR}{(1-H^2R^2)}\,dR$$






Static slicing
$ds^2 = -dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$
$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S_2})$
$= -dt^2 + (dR - HRdt)^2 + R^2 d^2\Omega_{S_2}$
$R = e^{Ht} r$
$= -dt^2(1-H^2R^2) + dR^2 - 2HR\, dR\, dt + R^2 d^2\Omega_{S_2}$
$T = t + \int \frac{HR}{(1-H^2R^2)} dR$
future event horizon
causal diamond
$R = H^{-1}$
Big Bang

Flat slicing

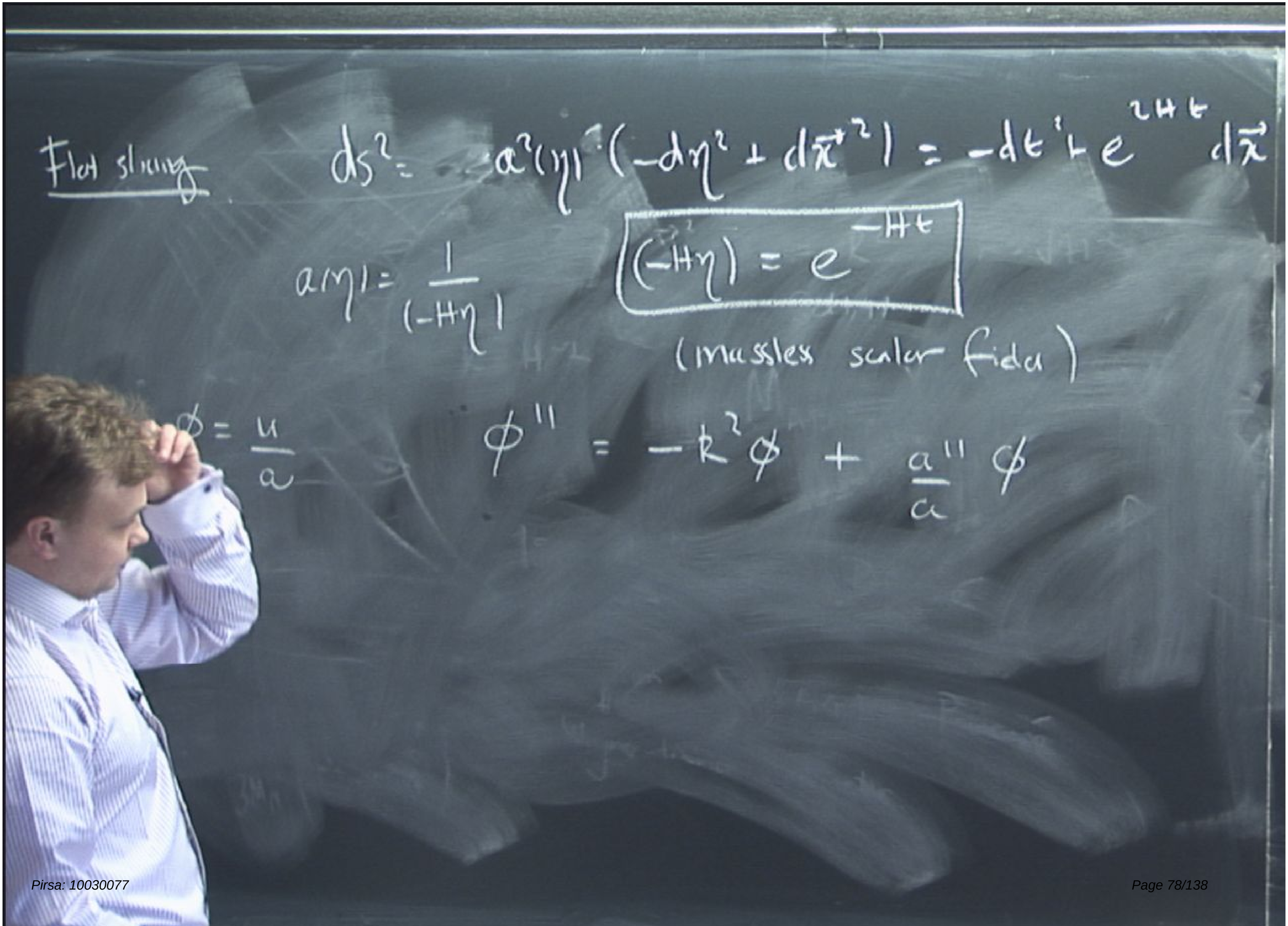

Flat slicing
$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$
$a(\eta) = \frac{1}{(-H\eta)}$

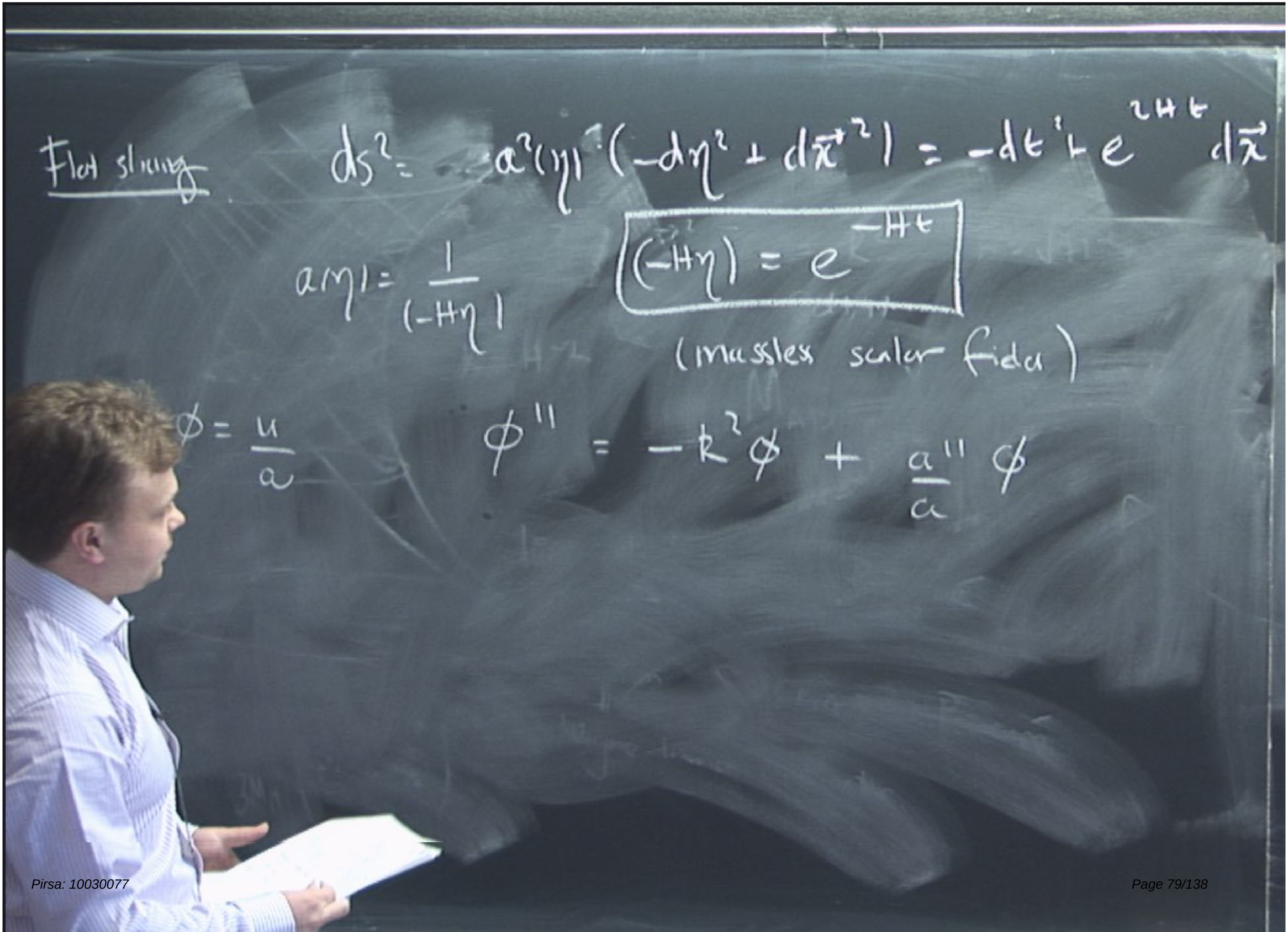
Flat slicing

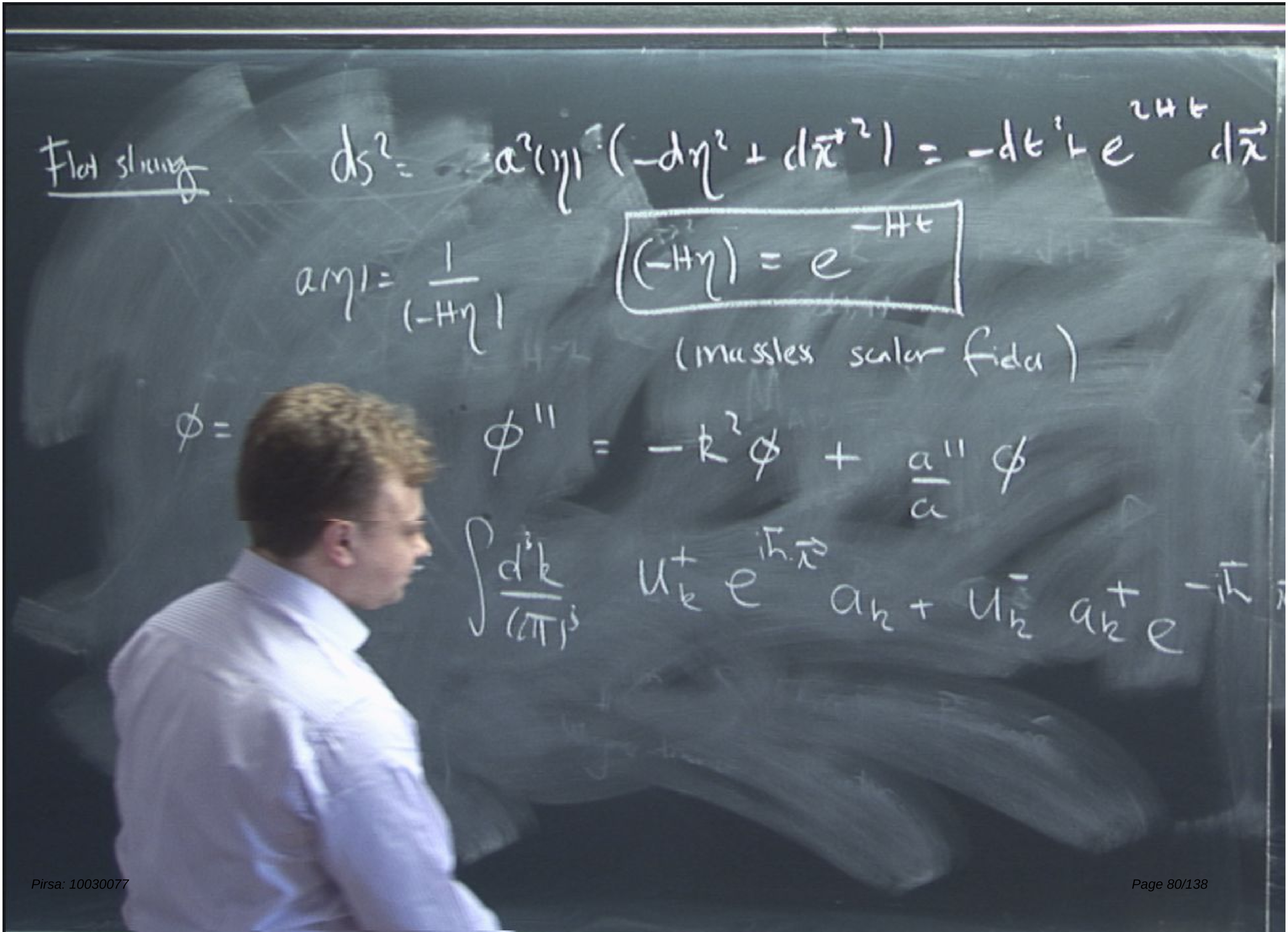
$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right) = -dt^2 + e^{2Ht}\, d\vec{x}^2$$

$$a(\eta) = \frac{1}{(-H\eta)}$$

$$\boxed{(-H\eta) = e^{-Ht}}$$


Flat slicing
ds² = a²(η)(−dη² + dx⃗²) = −dt² + e^{2Ht} dx⃗²
a(η) = 1/(−Hη)
(−Hη) = e^{−Ht}
(massless scalar field)
φ = u/a
φ'' = −k²φ + (a''/a) φ


Flat slicing
$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$
$a(\eta) = \frac{1}{(-H\eta)}$
$(-H\eta) = e^{-Ht}$
(massless scalar field)
$\phi = \frac{u}{a}$
$\phi'' = -k^2\phi + \frac{a''}{a}\phi$


Flat slicing
$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$
$a(\eta) = \frac{1}{(-H\eta)}$
$(-H\eta) = e^{-Ht}$
(massless scalar field)
$\phi =$
$\phi'' = -k^2\phi + \frac{a''}{a}\phi$
$\int \frac{d^3k}{(2\pi)^3} u_k^+ e^{i\vec{k}\cdot\vec{x}} a_k + u_k^- a_k^+ e^{-i\vec{k}\cdot\vec{x}}$

Flat slicing

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht}\, d\vec{x}^2$$

$$a(\eta) = \frac{1}{(-H\eta)}$$

$$\boxed{(-H\eta) = e^{-Ht}}$$

(massless scalar field)

$$\phi = \frac{u}{a}$$

$$\phi'' = -k^2\phi + \frac{a''}{a}\phi$$

$$\phi = \int \frac{d^3k}{(2\pi)^3}\, u_k^+ e^{i\vec{k}\cdot\vec{x}} a_k + u_k^- a_k^+ e^{-i\vec{k}\cdot\vec{x}}$$

string

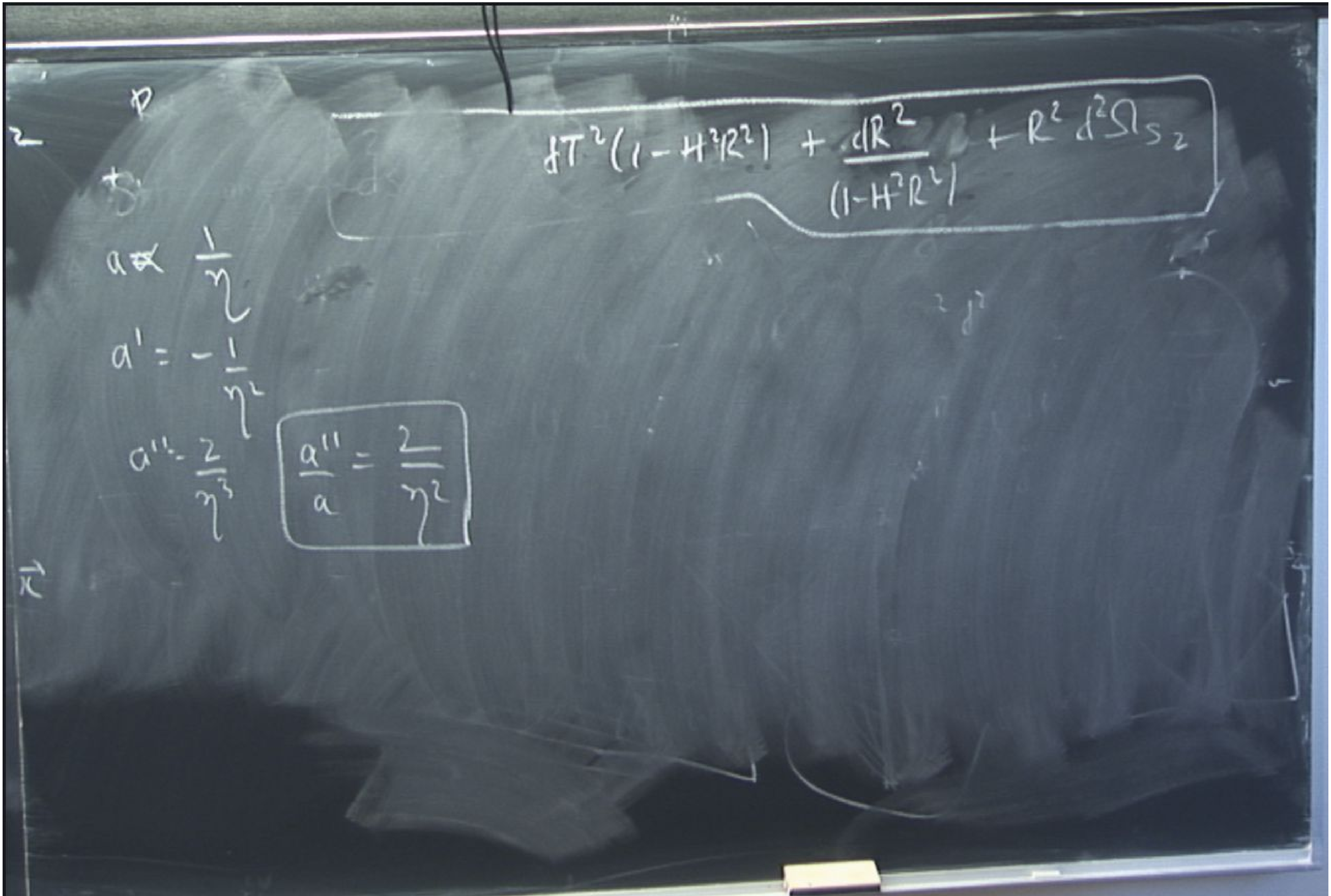
$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$$

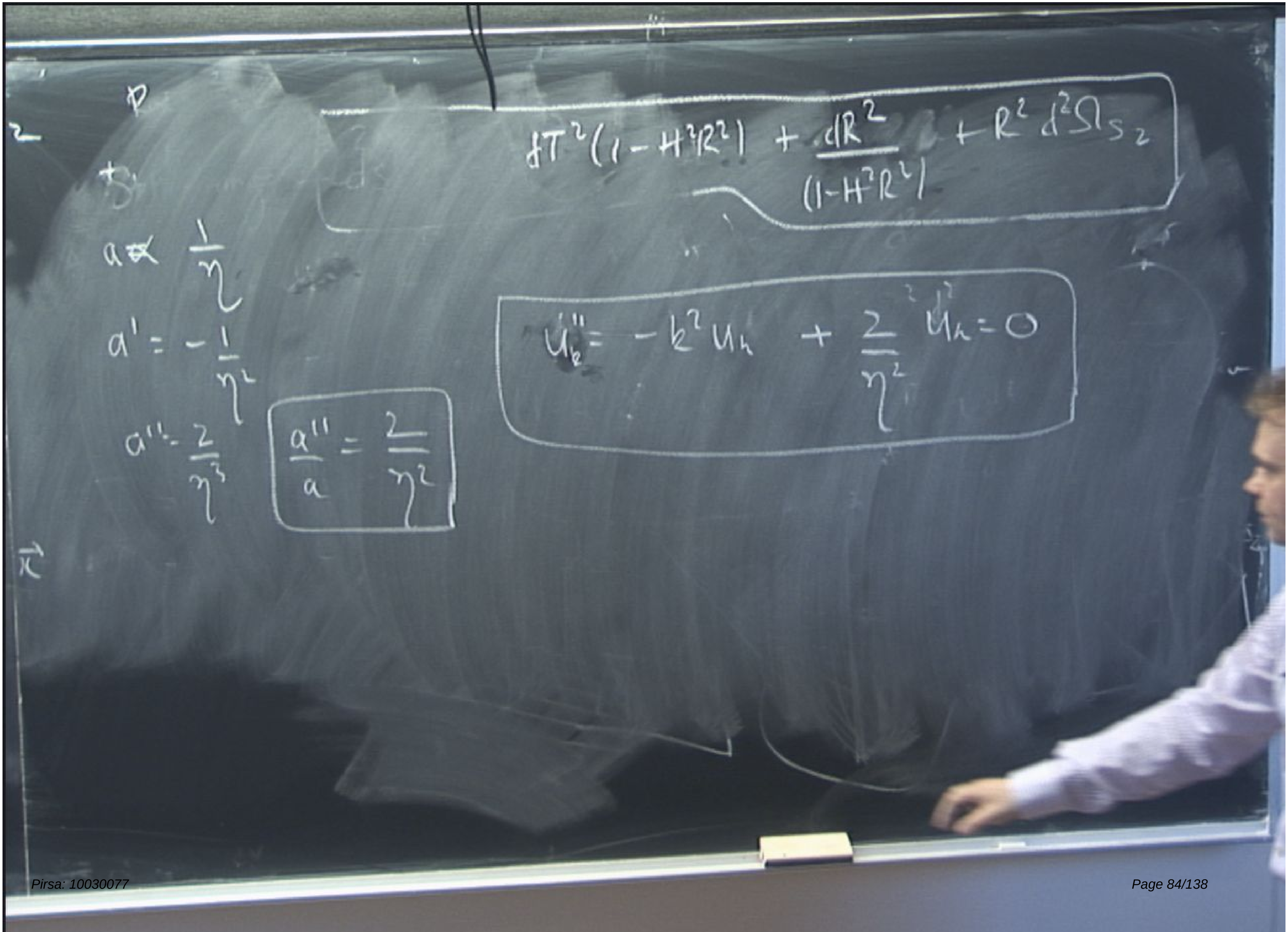
$$a(\eta) = \frac{1}{(-H\eta)} \qquad \boxed{(-H\eta) = e^{-Ht}}$$

(massless scalar field)

$$\phi = \frac{u}{a} \qquad \phi'' = -k^2\phi + \frac{a''}{a}\phi$$

$$\phi = \int \frac{d^3k}{(2\pi)^3}\, u_k^+ e^{i\vec{k}\cdot\vec{x}} a_k + u_k^- a_k^+ e^{-i\vec{k}\cdot\vec{x}}$$


$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$
$a \propto \frac{1}{\eta}$
$a' = -\frac{1}{\eta^2}$
$a'' = \frac{2}{\eta^3}$
$\frac{a''}{a} = \frac{2}{\eta^2}$


dT²(1 − H²R²) + dR² / (1−H²R²) + R² d²Ω_S2
a ∝ 1/η
a' = −1/η²
a'' = 2/η³
a''/a = 2/η²
u_k'' = −k² u_k + 2/η² u_k = 0

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2\, d^2\Omega_{S_2}$$

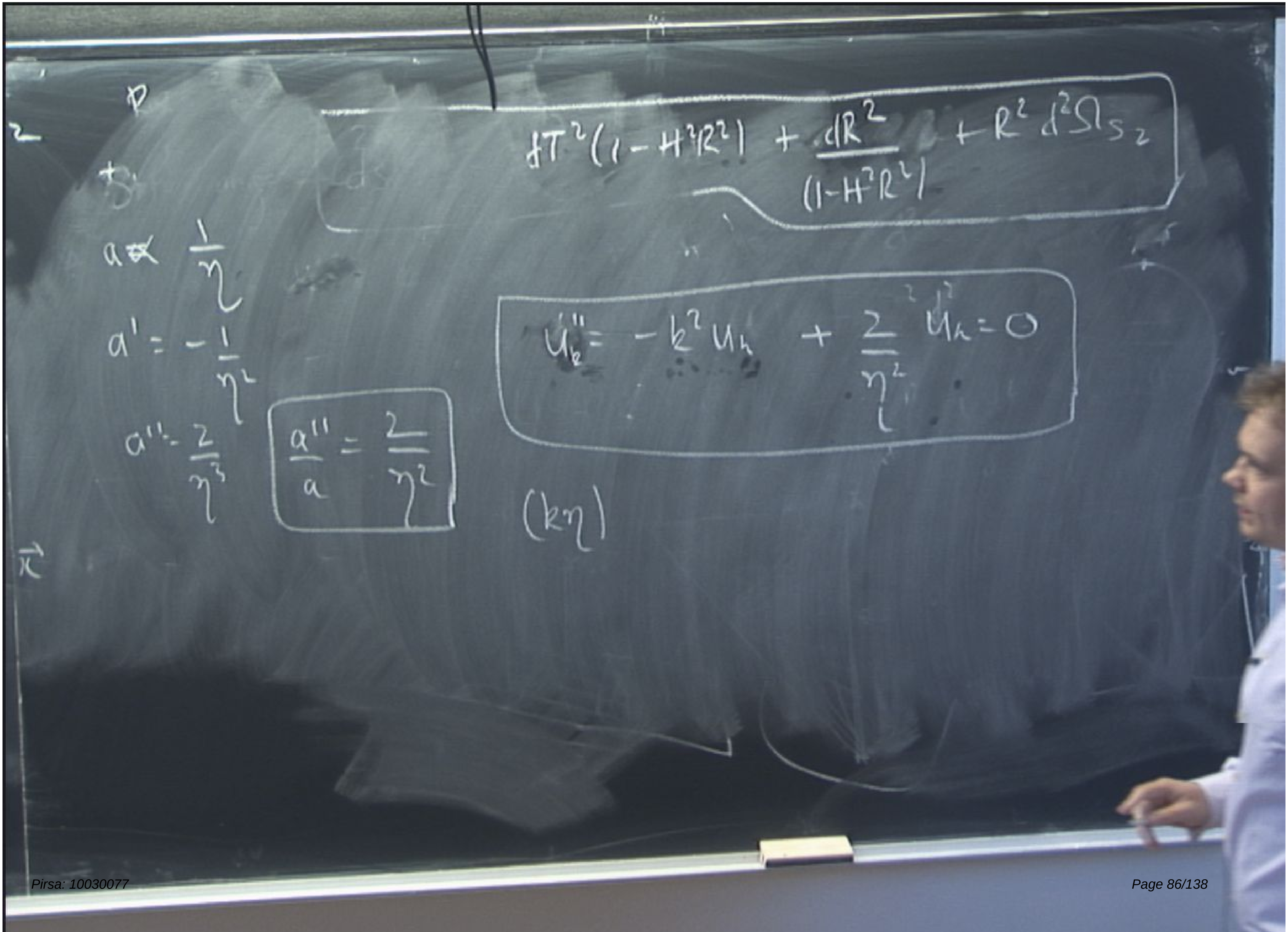
$$a \propto \frac{1}{\eta}$$

$$a' = -\frac{1}{\eta^2}$$

$$a'' = \frac{2}{\eta^3}$$

$$\frac{a''}{a} = \frac{2}{\eta^2}$$

$$u_k'' = -k^2 u_k + \frac{2}{\eta^2} u_k = 0$$


dT^2(1 - H^2R^2) + dR^2/(1 - H^2R^2) + R^2 d^2Ω_{S_2}
a ∝ 1/η
a' = -1/η^2
a'' = 2/η^3
a''/a = 2/η^2
u_k'' = -k^2 u_k + 2/η^2 u_k = 0
(kη)

$t^2 + e^{2Ht} d\vec{x}^{\,2}$

$t\ K + \ldots$

$< \eta < 0$

$da)$

ϕ

$u_k\ a_k^+ e^{-i\vec{k}\cdot\vec{x}}$

ϕ

$+$

$a \propto \frac{1}{\eta}$

$a' = -\frac{1}{\eta^2}$

$a'' = \frac{2}{\eta^3}$

$$\frac{a''}{a} = \frac{2}{\eta^2}$$

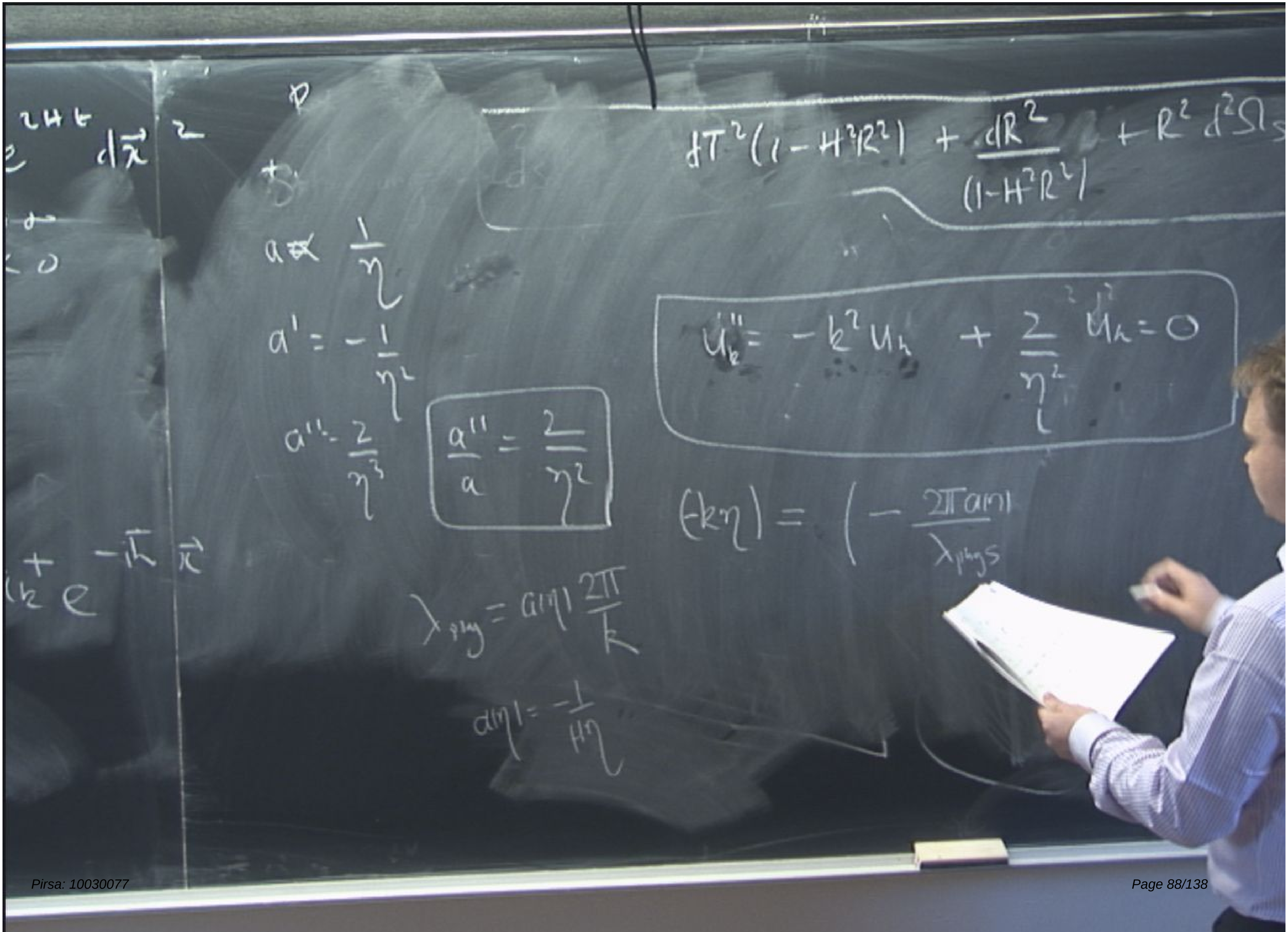
$\lambda_{phy} = a(\eta)\frac{2\pi}{k}$

$a(\eta) = \frac{-1}{H\eta}$

$dT^2(1 - H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2$

$$u_k'' = -k^2 u_k + \frac{2}{\eta^2} u_k = 0$$

$(-k\eta) =$


$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega$
$a \propto \frac{1}{\eta}$
$a' = -\frac{1}{\eta^2}$
$a'' = \frac{2}{\eta^3}$
$\frac{a''}{a} = \frac{2}{\eta^2}$
$u_k'' = -k^2 u_k + \frac{2}{\eta^2} u_k = 0$
$(-k\eta) = \left(-\frac{2\pi a(\eta)}{\lambda_{phys}}\right.$
$\lambda_{phys} = a(\eta)\frac{2\pi}{k}$
$a(\eta) = -\frac{1}{H\eta}$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$a \propto \frac{1}{\eta}$

$a' = -\frac{1}{\eta^2}$

$a'' = \frac{2}{\eta^3}$

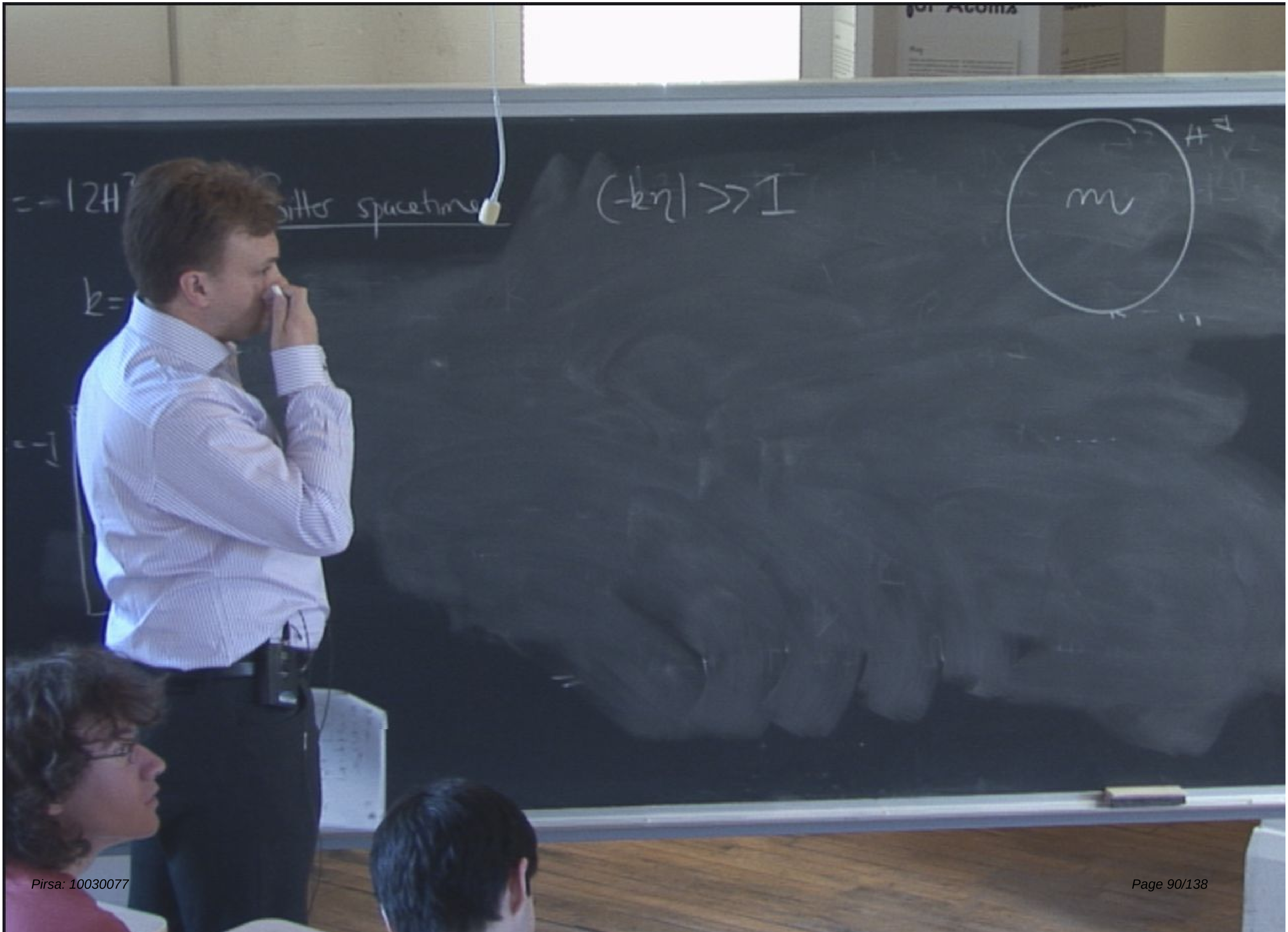
$$\frac{a''}{a} = \frac{2}{\eta^2}$$

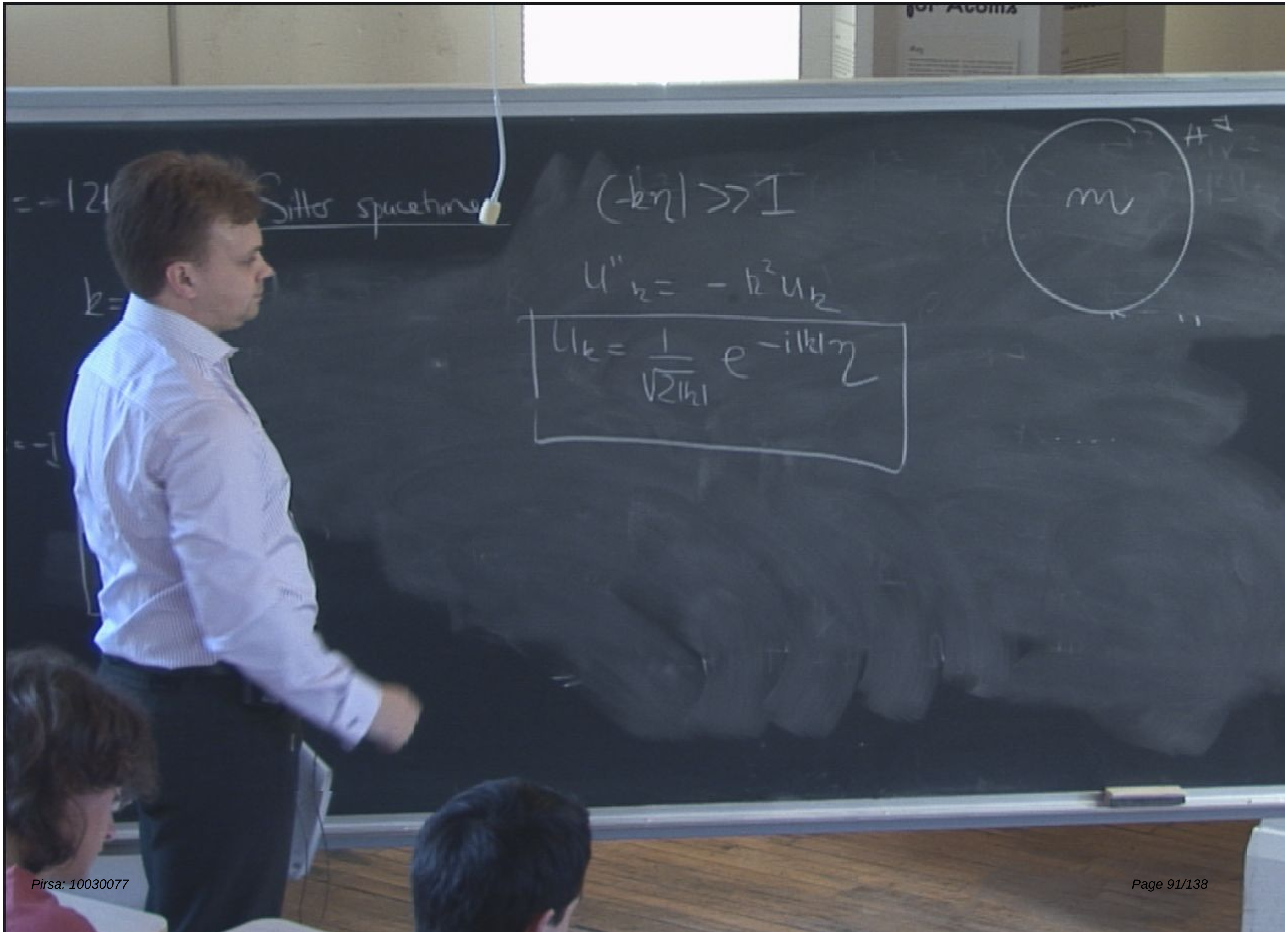
$$u_k'' = -k^2 u_k + \frac{2}{\eta^2} u_k = 0$$

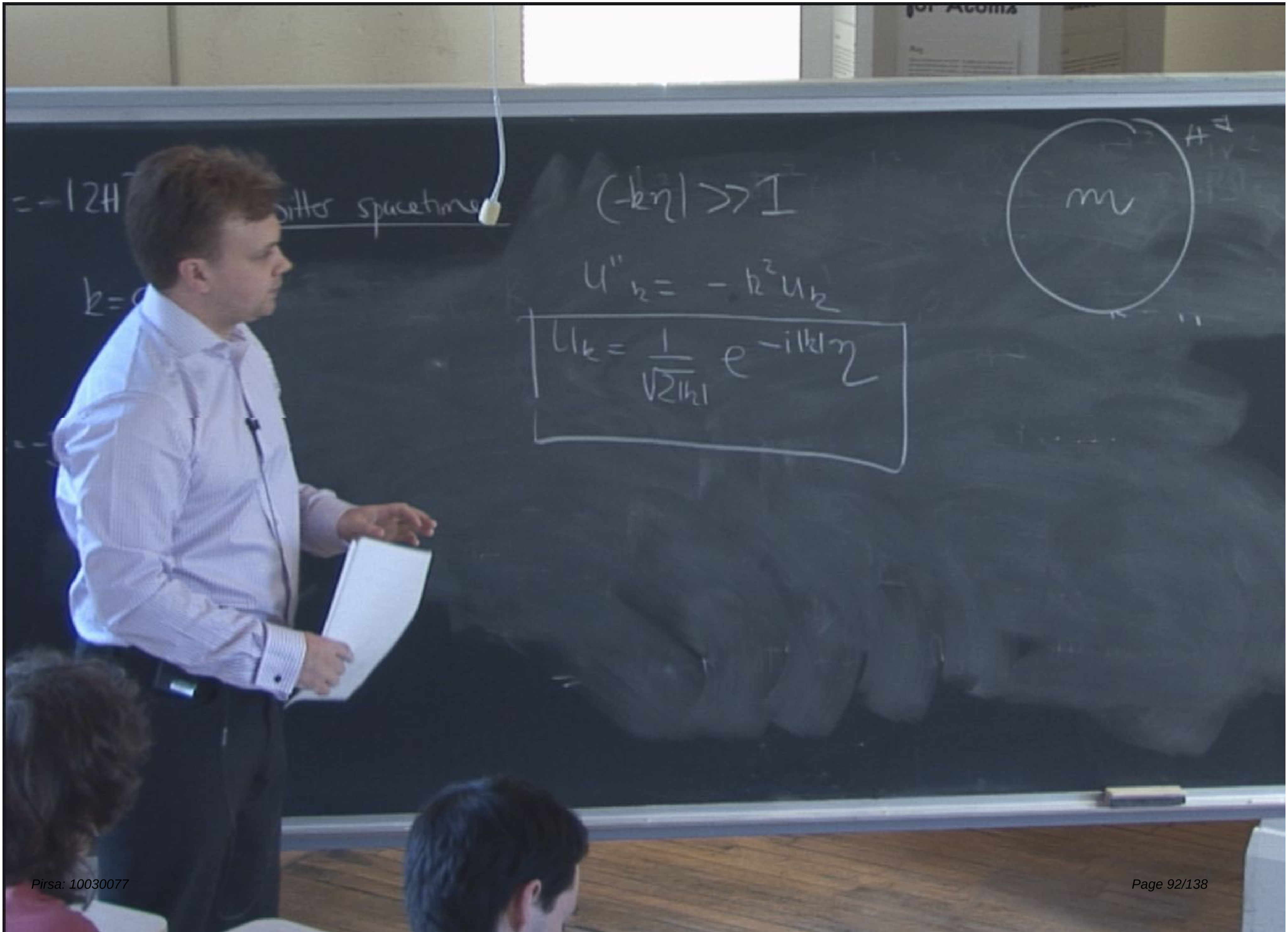
$$(-k\eta) = \left(-\frac{2\pi a(\eta)}{\lambda_{phys}}\left(-\frac{1}{H a(\eta)}\right)\right) = \frac{2\pi H^{-1}}{\lambda_{phys}}$$

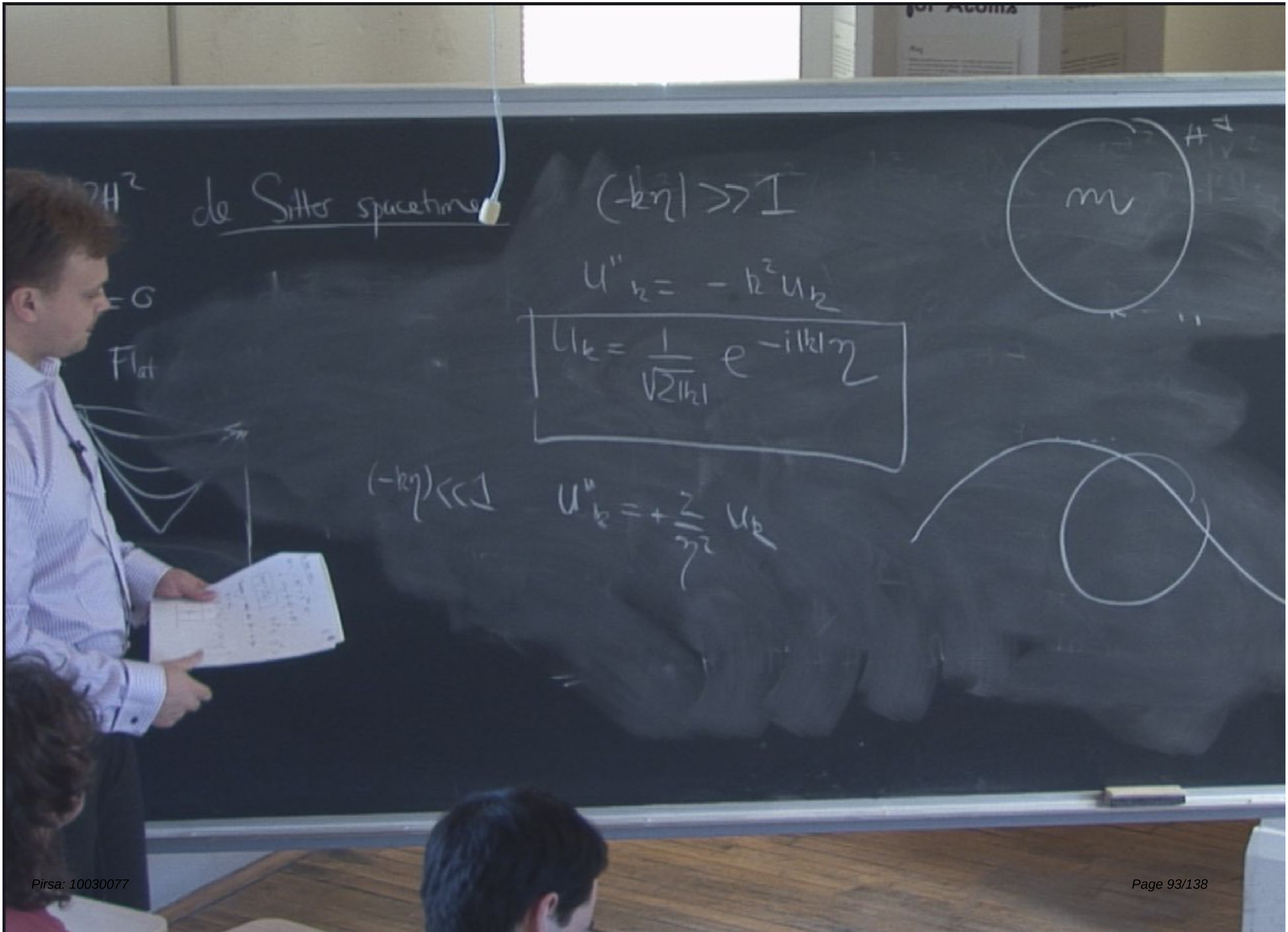
$\lambda_{phys} = a(\eta)\frac{2\pi}{k}$

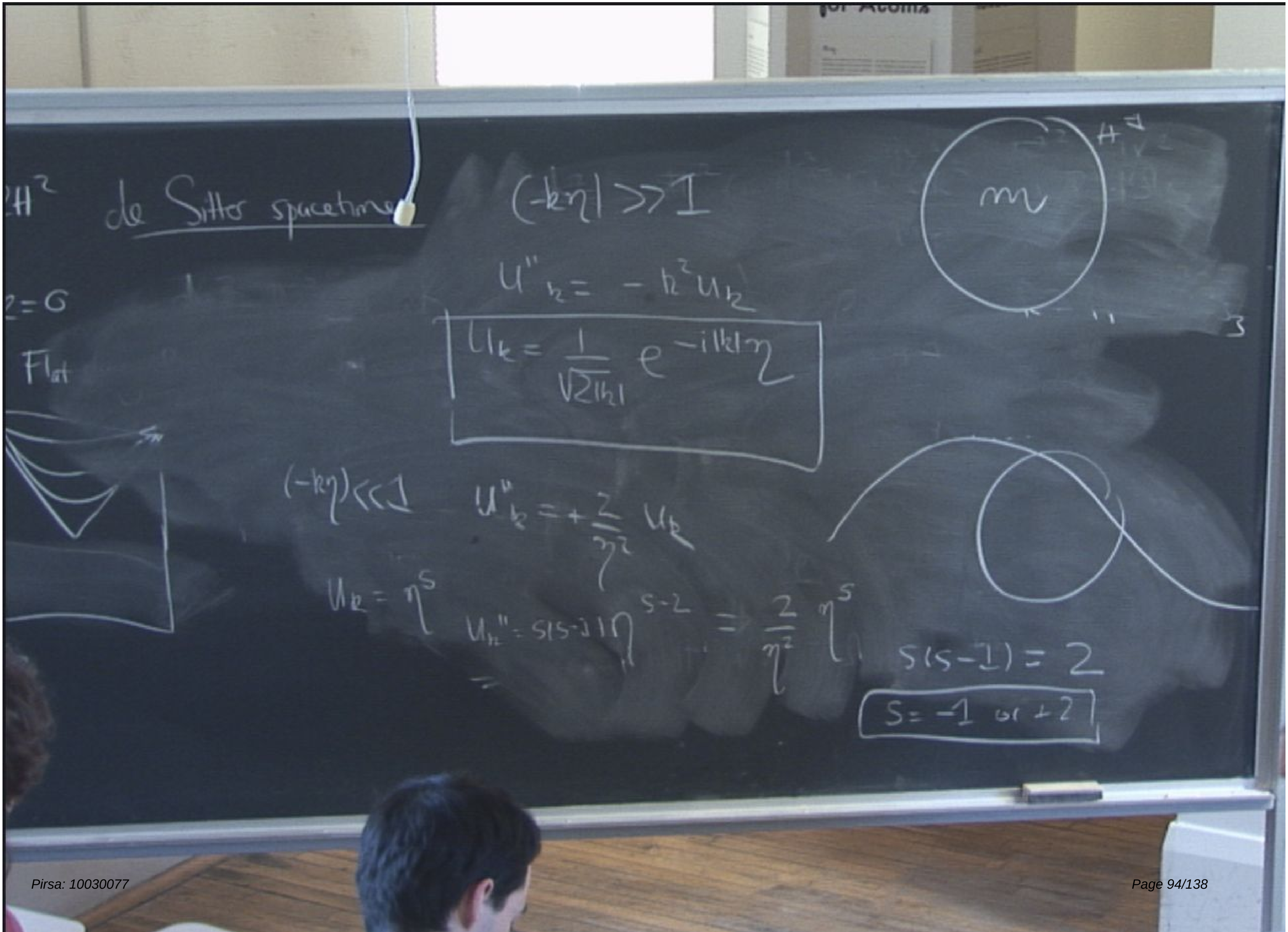
$a(\eta) = \frac{-1}{H\eta}$

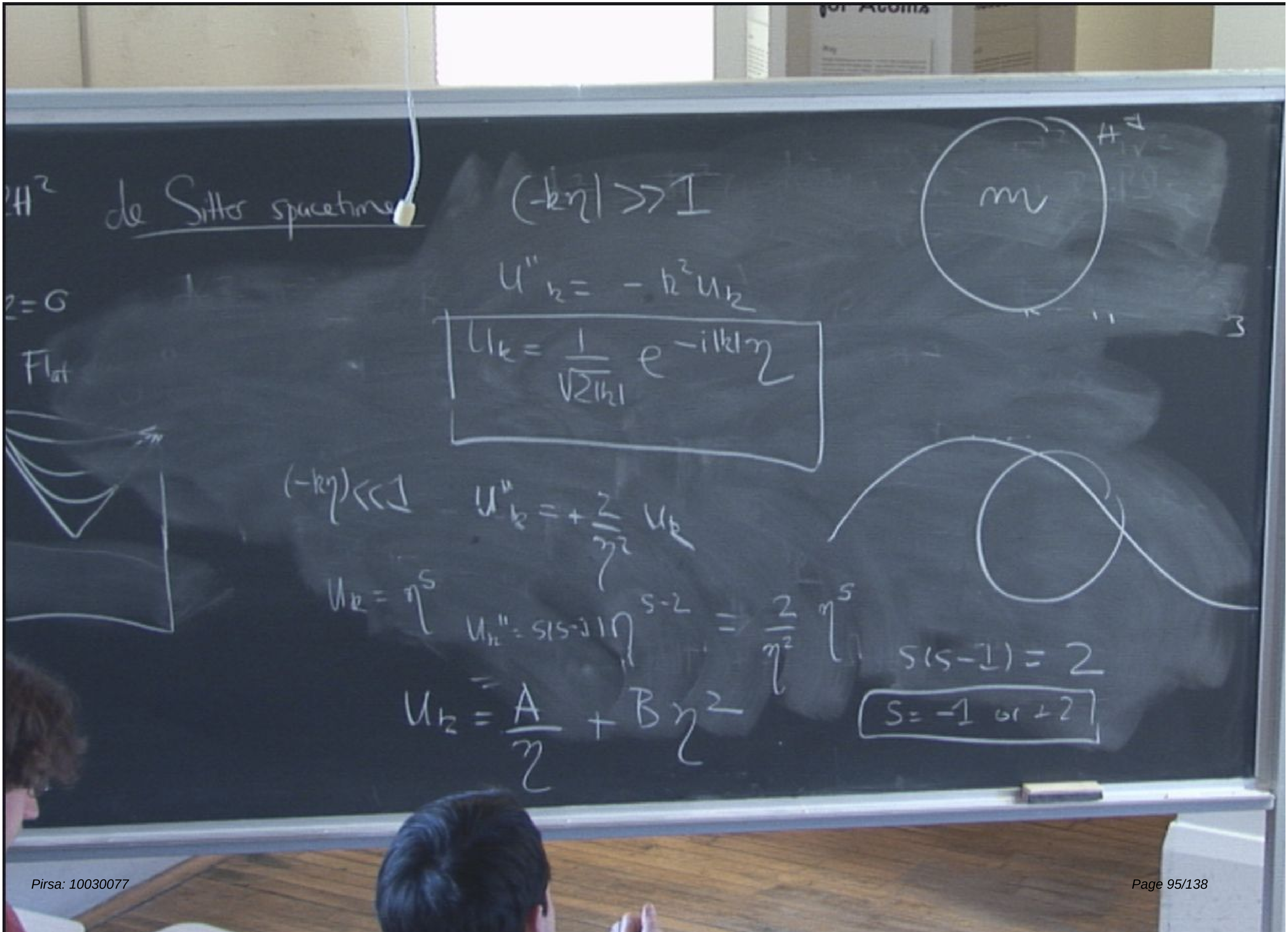

Sitter spacetime
(-kη| >> 1
m

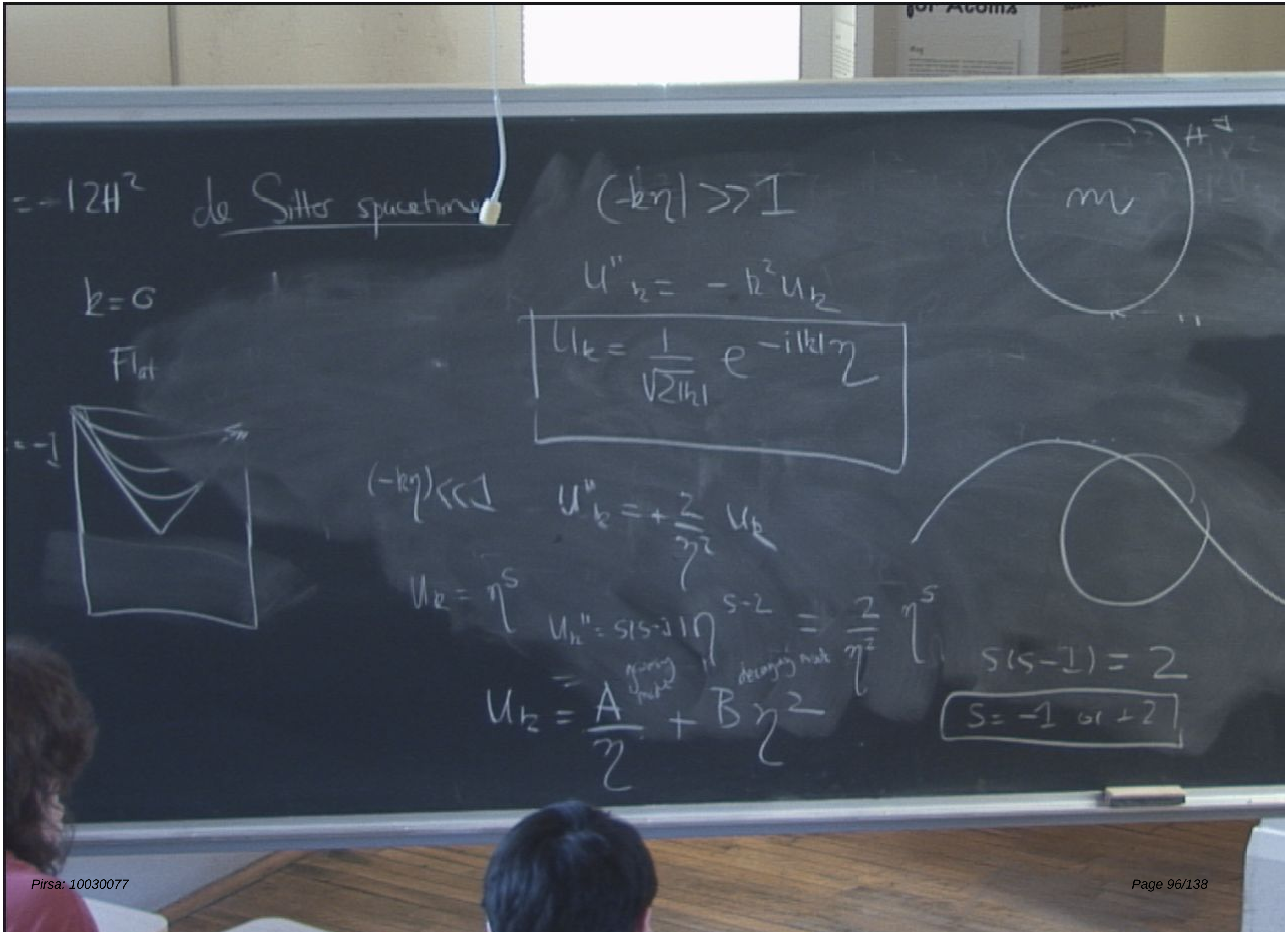

Sitter spacetime
(-kη| >> 1
u''_k = - k² u_k
u_k = 1/√(2|k|) e^{-i|k|η}
m


$= -12H^2$
sitter spacetime
$(-k\eta) \gg 1$
$u''_k = -k^2 u_k$
$u_k = \frac{1}{\sqrt{2|k|}} e^{-i|k|\eta}$
m


de Sitter spacetime
(-kη) >> 1
u''_k = - k^2 u_k
u_k = 1/√(2|k|) e^{-i|k|η}
Flat
(-kη) << 1
u''_k = + 2/η^2 u_k
m
H^{-1}


de Sitter spacetime
Flat
(-kη) >> 1
u''_k = - k^2 u_k
u_k = 1/√(2|k|) e^{-i|k|η}
(-kη) << 1
u''_k = + 2/η^2 u_k
u_k = η^s
u_k'' = s(s-1)η^{s-2} = 2/η^2 η^s
s(s-1) = 2
s = -1 or +2
m

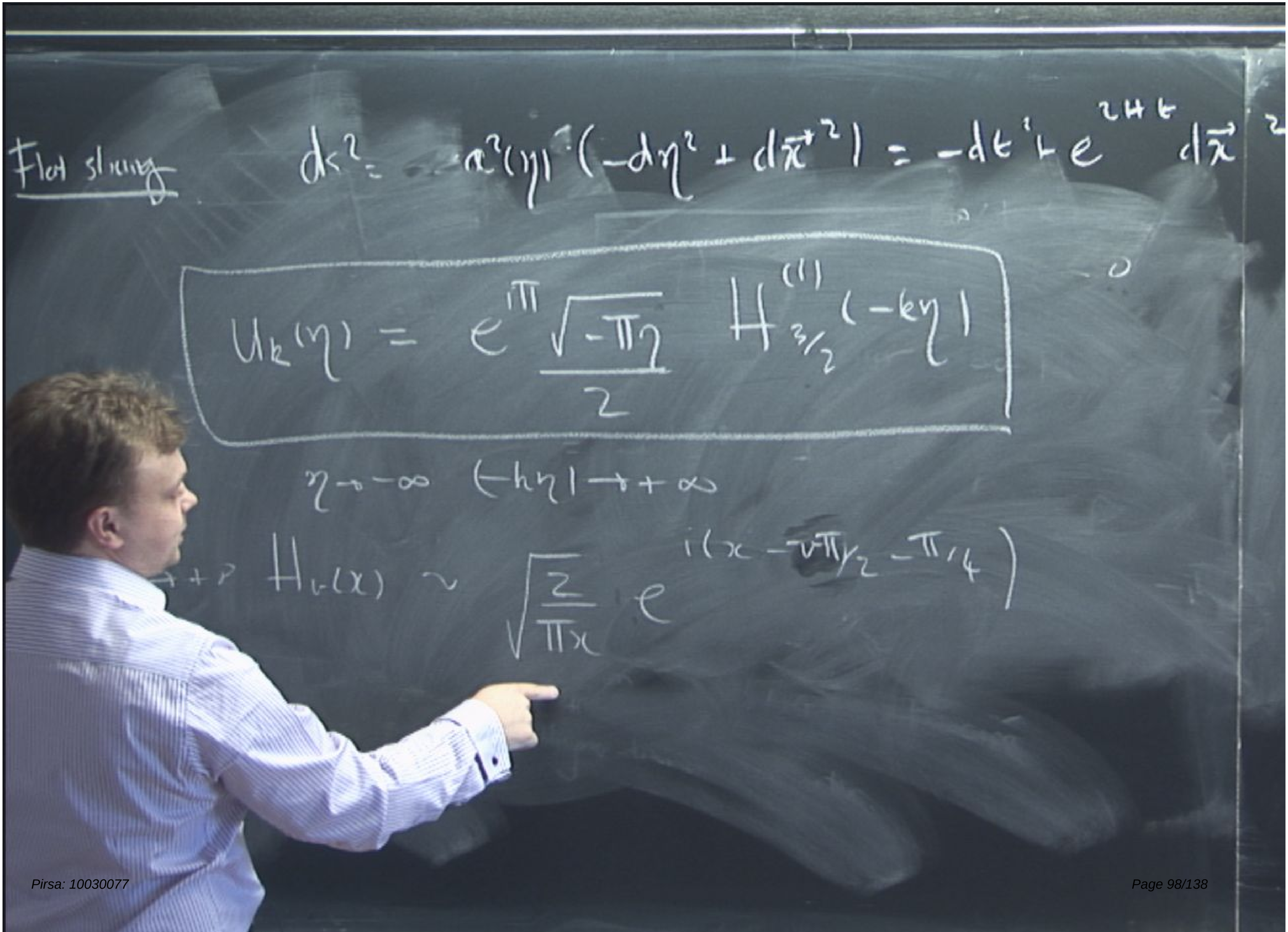

de Sitter spacetime
Flat
(-kη) >> 1
u''_k = - k^2 u_k
u_k = 1/√(2|k|) e^{-i|k|η}
(-kη) << 1
u''_k = + 2/η^2 u_k
u_k = η^s
u''_k = s(s-1) η^{s-2} = 2/η^2 η^s
s(s-1) = 2
s = -1 or +2
u_k = A/η + B η^2
m


= -12H^2
de Sitter spacetime
(-kη) >> 1
u''_k = -k^2 u_k
u_k = 1/√(2|k|) e^{-i|k|η}
k = 0
Flat
(-kη) << 1
u''_k = +2/η^2 u_k
u_k = η^s
u_k'' = s(s-1) η^{s-2} = 2/η^2 η^s
s(s-1) = 2
s = -1 or +2
decaying mode
u_k = A/η + B η^2
m

Flat slicing

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$$


Flat slicing
$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$
$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$
$\eta \to -\infty \quad (-k\eta) \to +\infty$
$H_\nu(x) \sim \sqrt{\frac{2}{\pi x}} e^{i(x - \nu\pi/2 - \pi/4)}$

Flat slicing

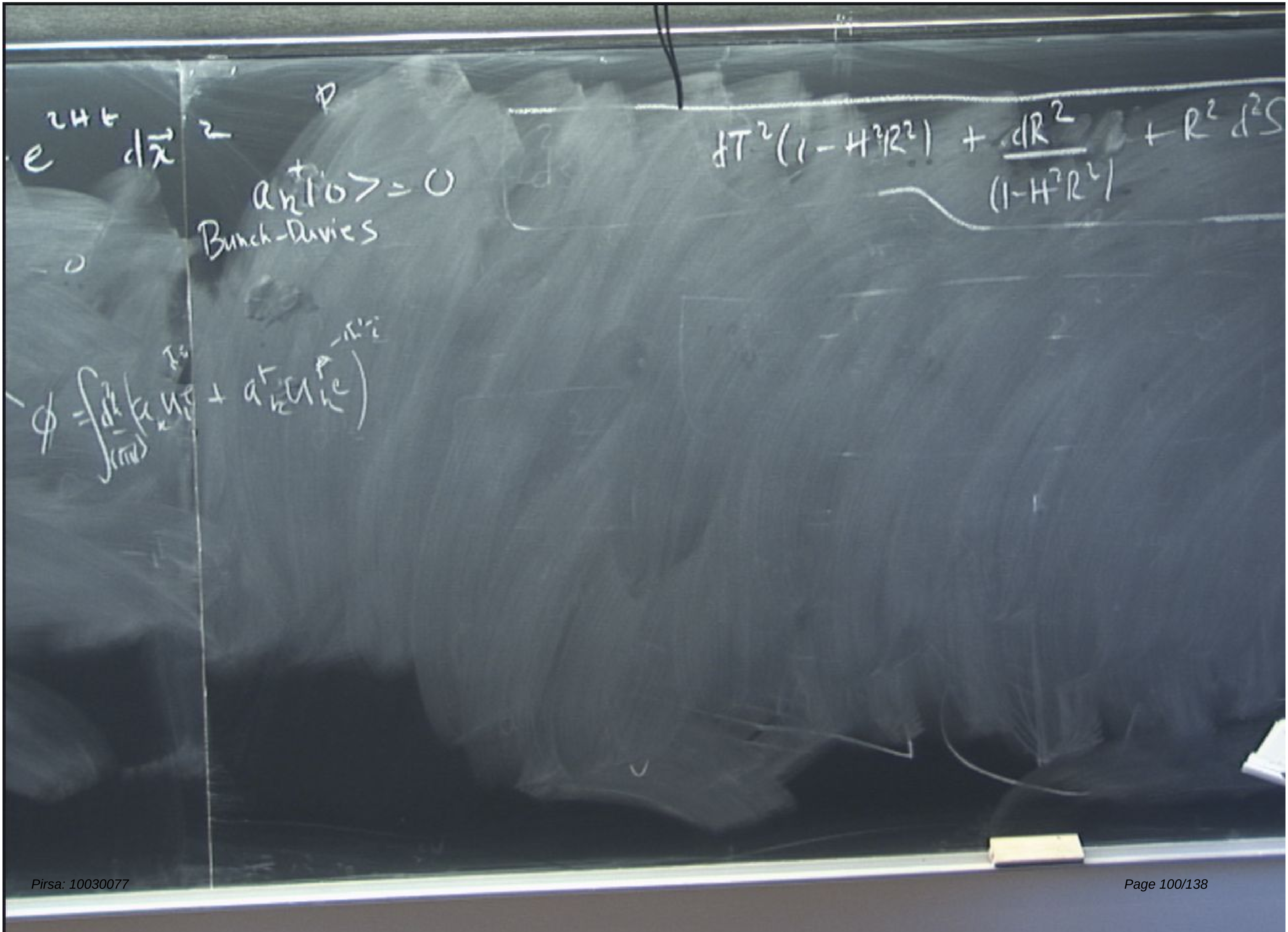
$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$$

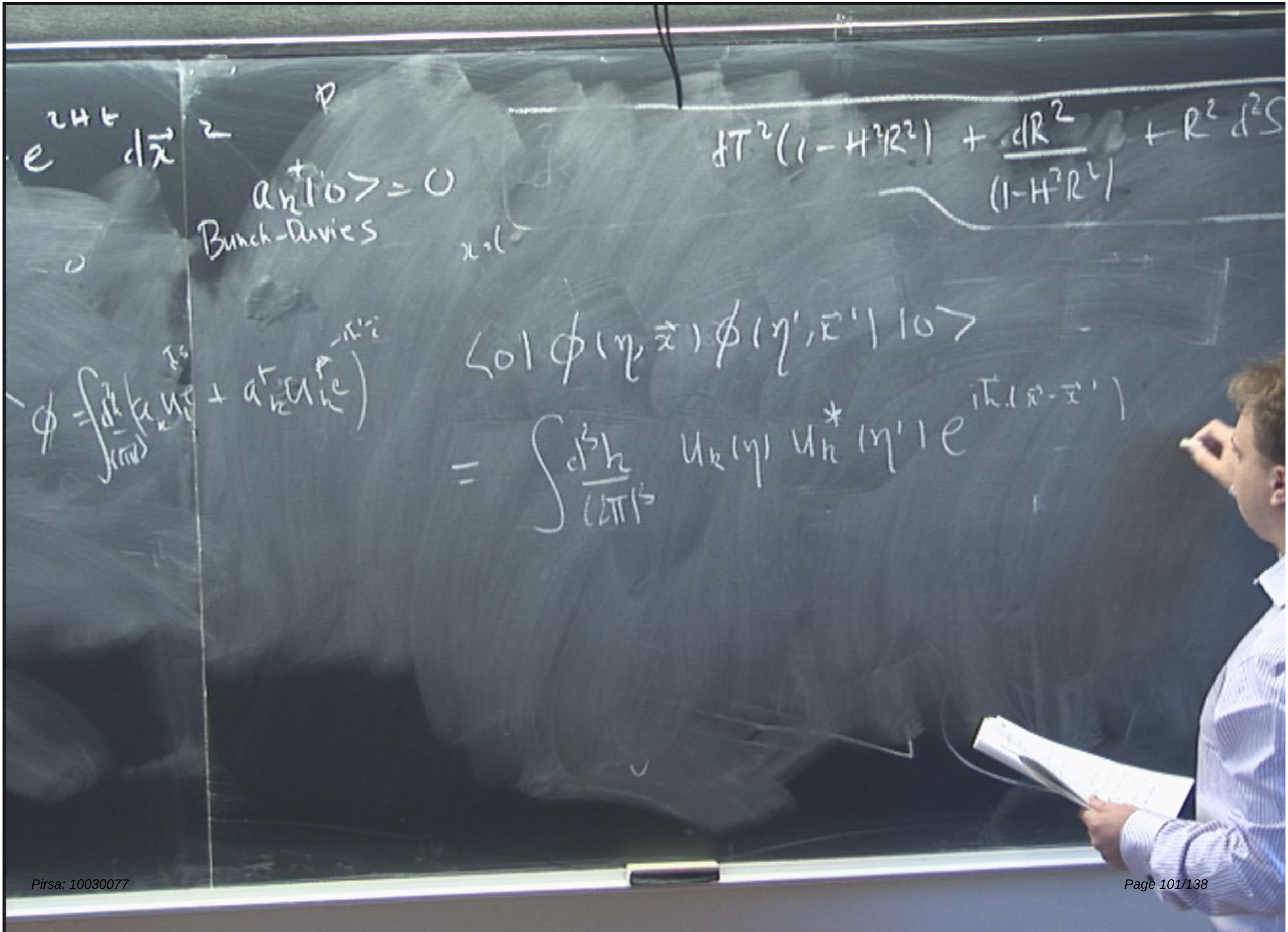
$$\boxed{u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)}$$

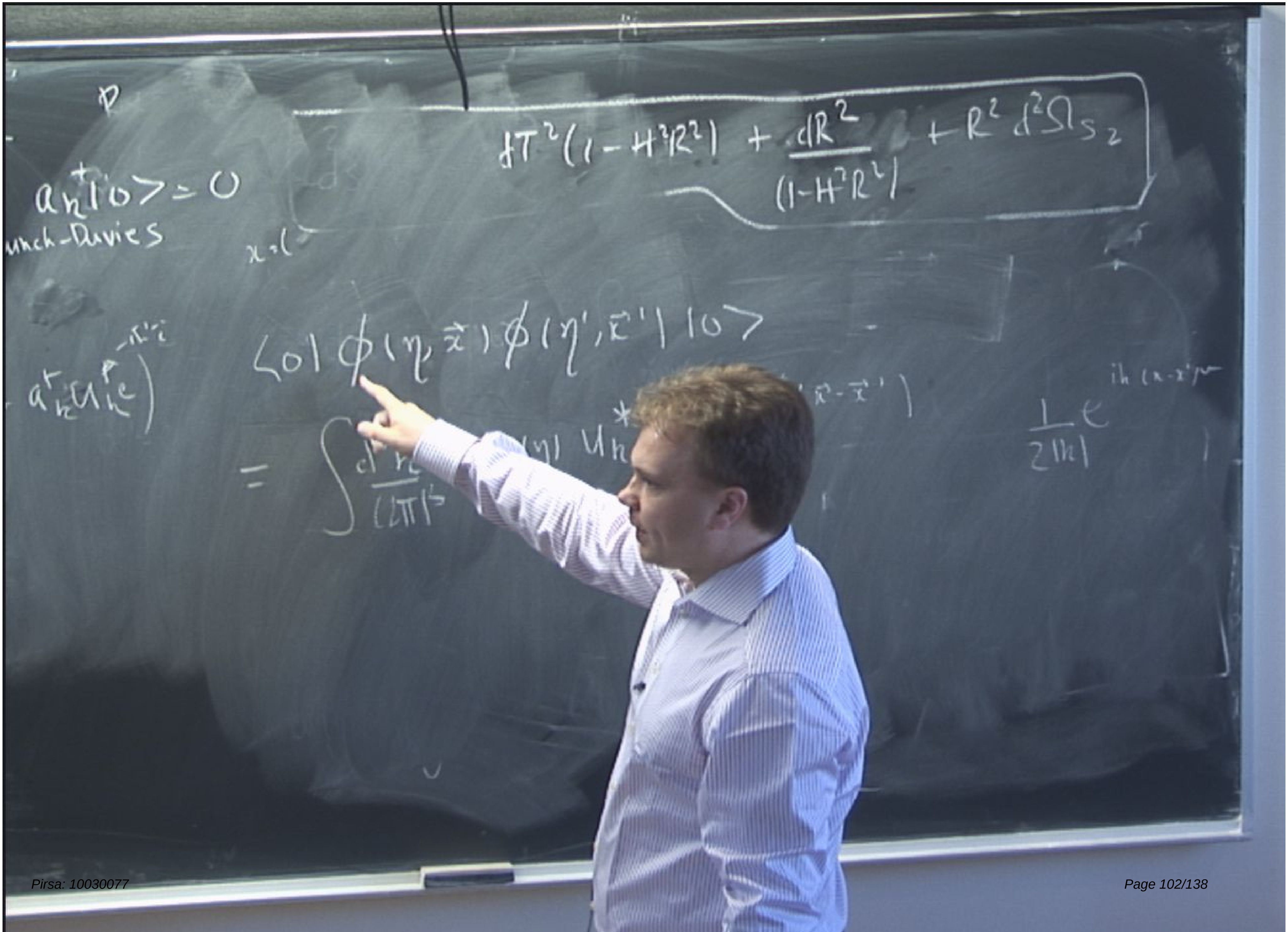
$\eta \to -\infty \quad (-k\eta) \to +\infty$

$$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i(x - \nu\pi/2 - \pi/4)}$$

$$\frac{1}{\sqrt{2|k|}}\, e^{-ik\eta}$$


e^{2Ht} dx^2
a_k |0> = 0
Bunch-Davies
dT^2(1 - H^2R^2) + dR^2/(1-H^2R^2) + R^2 d^2S
φ = ∫ d^3k/(2π)^3 (a_k u_k e^{ikx} + a^†_k u^*_k e^{-ikx})


Bunch-Davies
$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2S$
$\langle 0| \phi(\eta,\vec{x}) \phi(\eta',\vec{x}') |0\rangle$
$= \int \frac{d^3k}{(2\pi)^3} u_k(\eta) u_k^*(\eta') e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$


unch-Davies
dT²(1 − H²R²) + dR²/(1 − H²R²) + R² d²Ω_S2
⟨0| φ(η, x⃗) φ(η′, x⃗′) |0⟩

ϕ

$a_k^{+}|0\rangle = 0$

unch-Davies

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$x = ($

$a_k^{+} u_k^{*} e^{-ik\cdot x}\Big)$

$$\langle 0|\phi(\eta,\vec{x})\,\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^{*}(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')\cdots}$$

$\phi = \frac{u}{a}$

$$a_k |0\rangle = 0$$

Bunch-Davies

$$d\tau^2(1 - H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$$+ a_k^\dagger u_k^* e^{-ik \cdot x}\Big)$$

$$\langle 0 | \phi(\eta, \vec{x}) \phi(\eta', \vec{x}') | 0 \rangle$$

$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')} \int \frac{d^3k}{(2\pi)^3} \, u_k(\eta) \, u_k^*(\eta') \, e^{i\vec{k}\cdot(\vec{x} - \vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')\mu}$$

$$\phi = \frac{u}{a}$$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$a_k|0\rangle = 0$

Bunch-Davies

de Sitter invariant

$+ a_k^\dagger u_k^* e^{-ik\cdot x})$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')\ldots}$$

$\phi = \frac{u}{a}$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$a_k|0\rangle = 0$

Bunch-Davies

$+ a_k^\dagger u_k^* e^{-ik\cdot x})$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3} u_k(\eta) u_k^*(\eta') e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$\frac{1}{2|k|} e^{ik(x-x')}$

$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} = \lambda\vec{x}$ $\eta \to \lambda\eta$ $ds^2 \to ds^2$

$a_k|0\rangle = 0$

Bunch-Davies

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$$+ a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}})$$

$$\langle 0|\phi(\eta,\vec{x})\,\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')}$$

$$\phi = \frac{u}{a}$$

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$$

$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta$ $\quad ds^2 \to ds^2$

$$d\tau^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$a_k|0\rangle = 0$

Bunch-Davies

$+ a_k^\dagger u_k^* e^{-ik\cdot x})$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')}$$

$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta$ $\quad ds^2 \to ds^2$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$a_k|0\rangle = 0$

Bunch-Davies

$+ a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}})$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

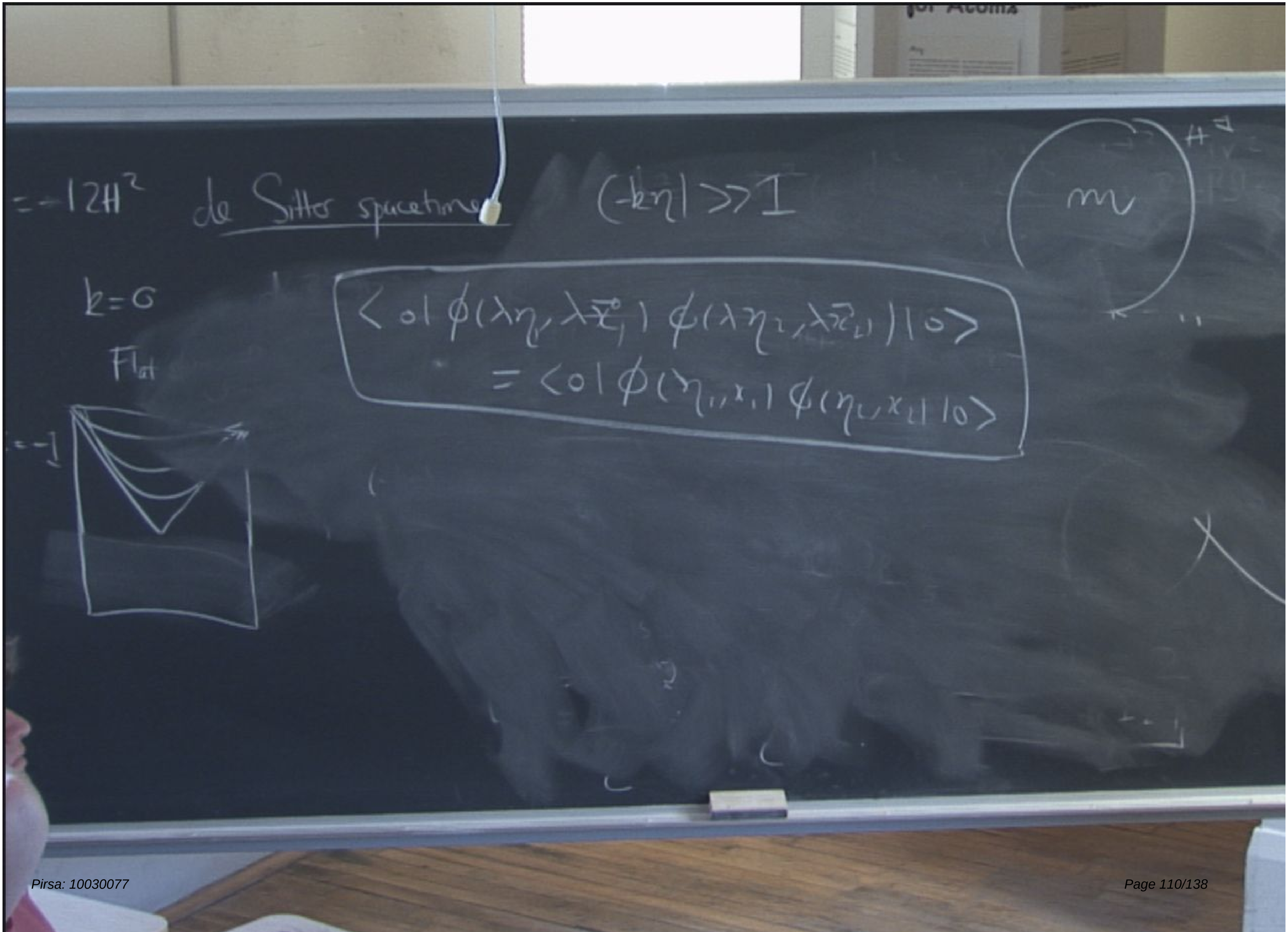
$$= \frac{1}{a(\eta)a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

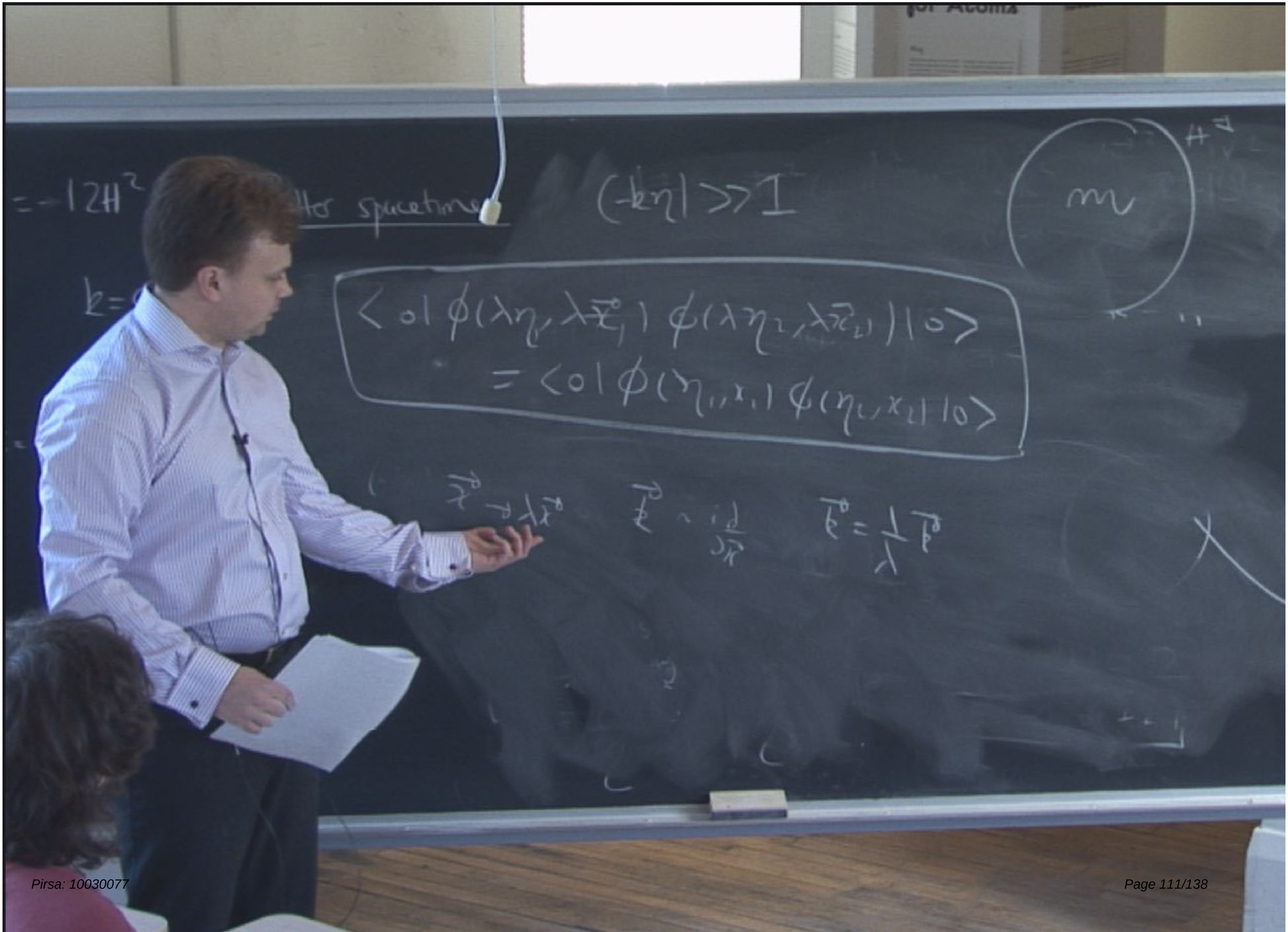
$$\frac{1}{2|k|}e^{ik(x-x')}$$

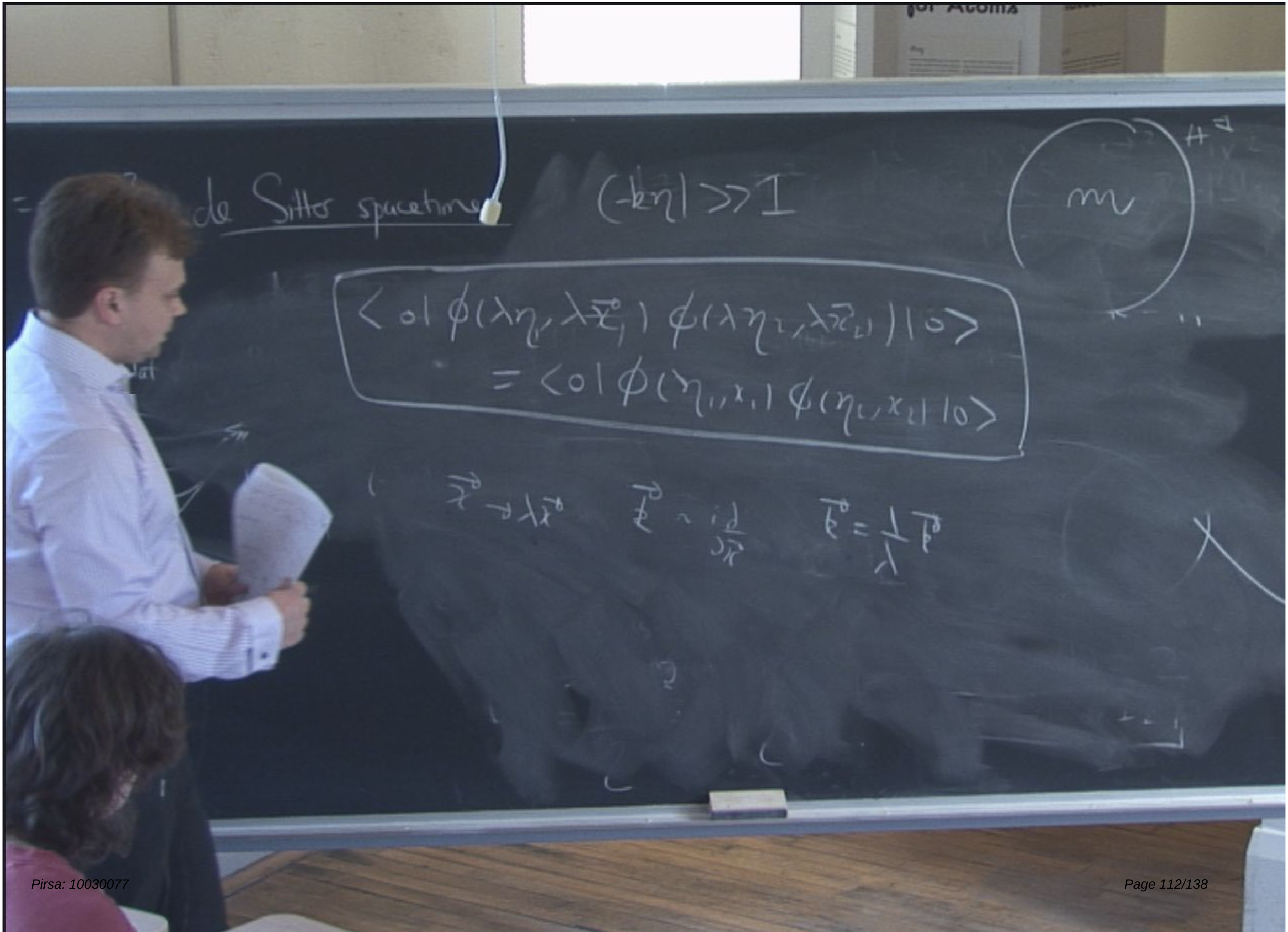
$\phi = \frac{u}{a}$

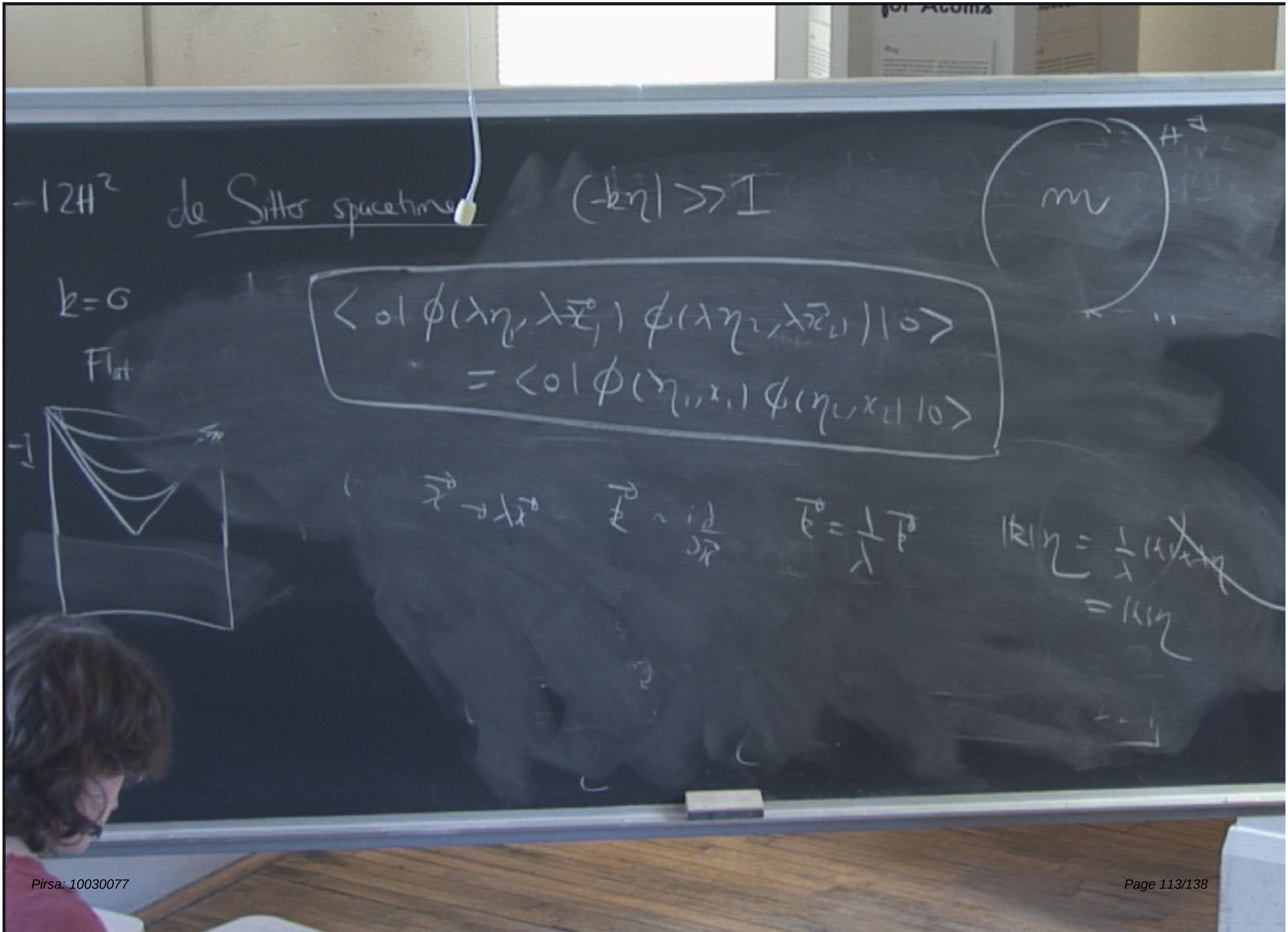
$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta$ $\quad ds^2 \to ds^2$


de Sitter spacetime
$|-k\eta| >> 1$
$k=0$
Flat
m
$\langle 0|\phi(\lambda\eta_1,\lambda\vec{x}_1)\,\phi(\lambda\eta_2,\lambda\vec{x}_2)|0\rangle$
$= \langle 0|\phi(\eta_1,x_1)\,\phi(\eta_2,x_2)|0\rangle$


(-kη| >> 1
< 0 | φ(λη1, λx1) φ(λη2, λx2) | 0 >
= < 0 | φ(η1, x1) φ(η2, x2) | 0 >
m


de Sitter spacetime


-12H^2
de Sitter spacetime
(-kη| >> 1
k = 0
Flat
<0| φ(λη1, λx1) φ(λη2, λx2) |0>
= <0| φ(η1, x1) φ(η2, x2) |0>
x → λx
k ~ i ∂/∂x
k = 1/λ k
|k|η = 1/λ |k| λη
= |k|η

Flat slicing $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$

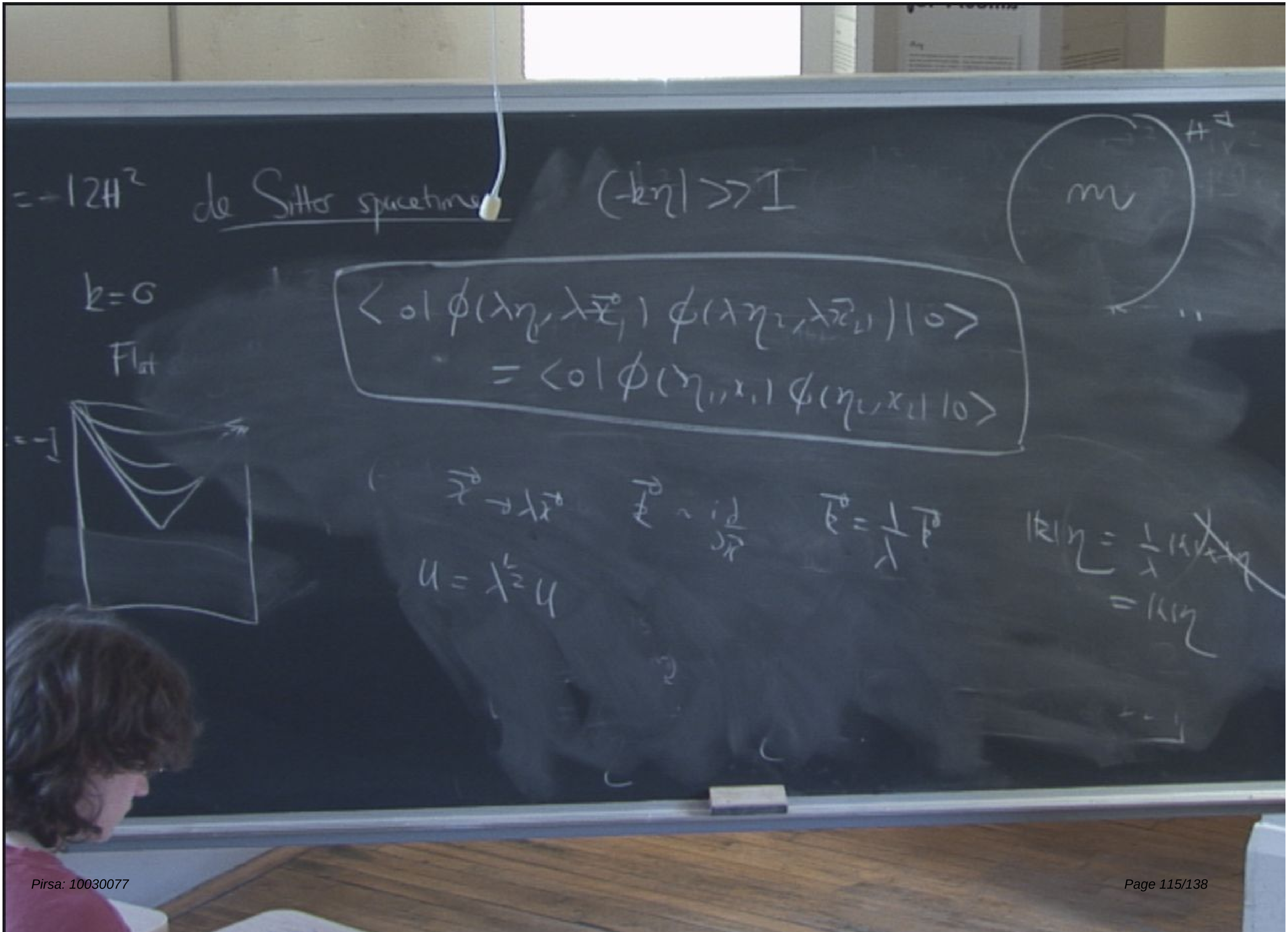
$$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$$

$$\phi = \int \frac{d^3k}{a(\eta)(\pi\nu)} (a_k u_k e^{ik\cdot x} \ldots$$

$\eta \to -\infty \quad (-k\eta) \to +\infty$

$$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i(x - \nu\pi/2 - \pi/4)}$$

$$\frac{1}{\sqrt{2|k|}} e^{-ik\eta}$$


de Sitter spacetime
$= -12H^2$
$|k\eta| \gg 1$
$k=0$
Flat
$\langle 0|\phi(\lambda\eta_1, \lambda\vec{x}_1)\,\phi(\lambda\eta_2, \lambda\vec{x}_2)|0\rangle$
$= \langle 0|\phi(\eta_1, x_1)\,\phi(\eta_2, x_2)|0\rangle$
$\vec{x} \to \lambda\vec{x}$
$\vec{k} = \frac{1}{\lambda}\vec{k}$
$|k|\eta = \frac{1}{\lambda}|k|\lambda\eta$
$= |k|\eta$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$a_k|0\rangle = 0$

Bunch-Davies

de Sitter invariant

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

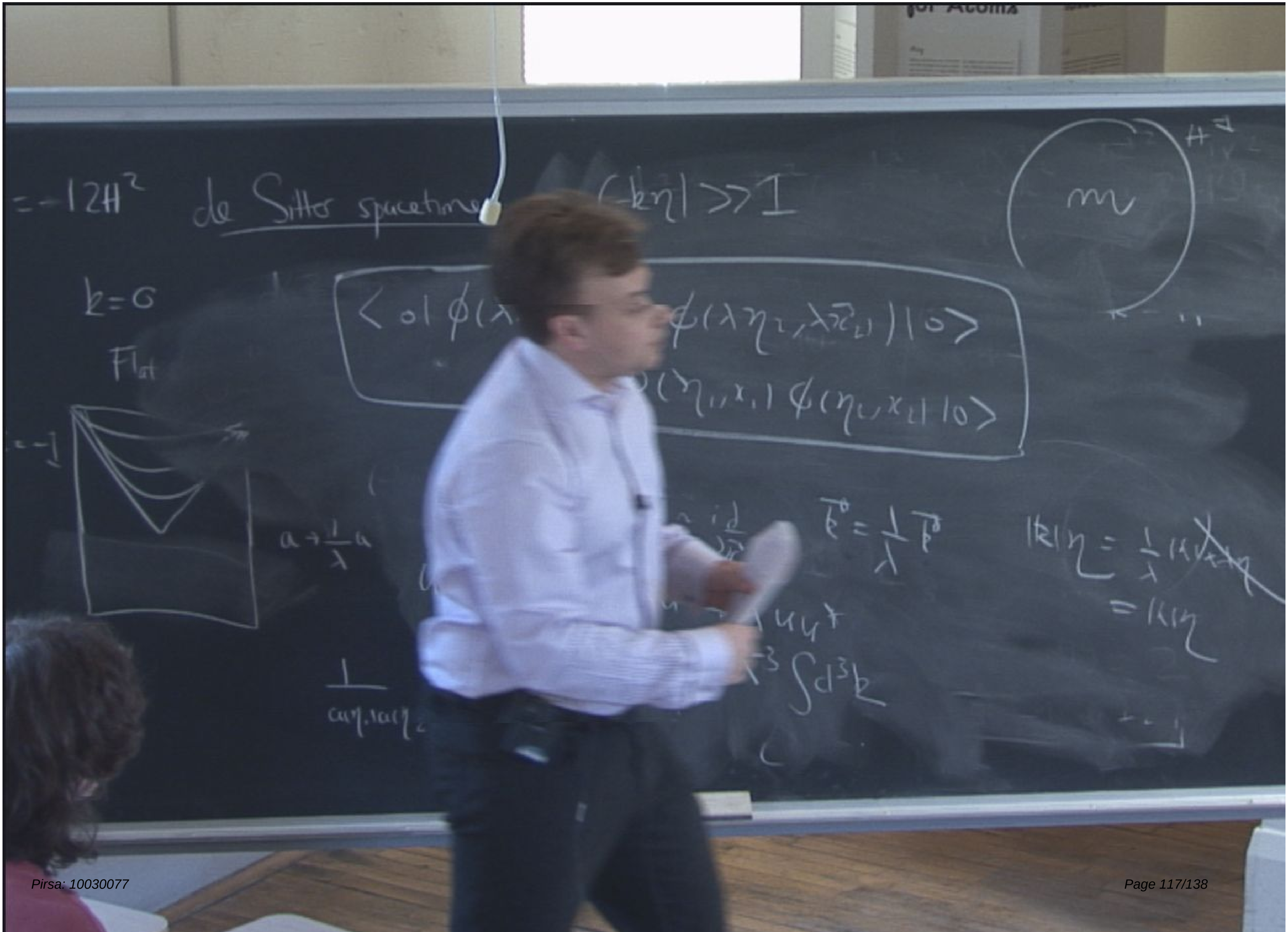
$+ a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}})$

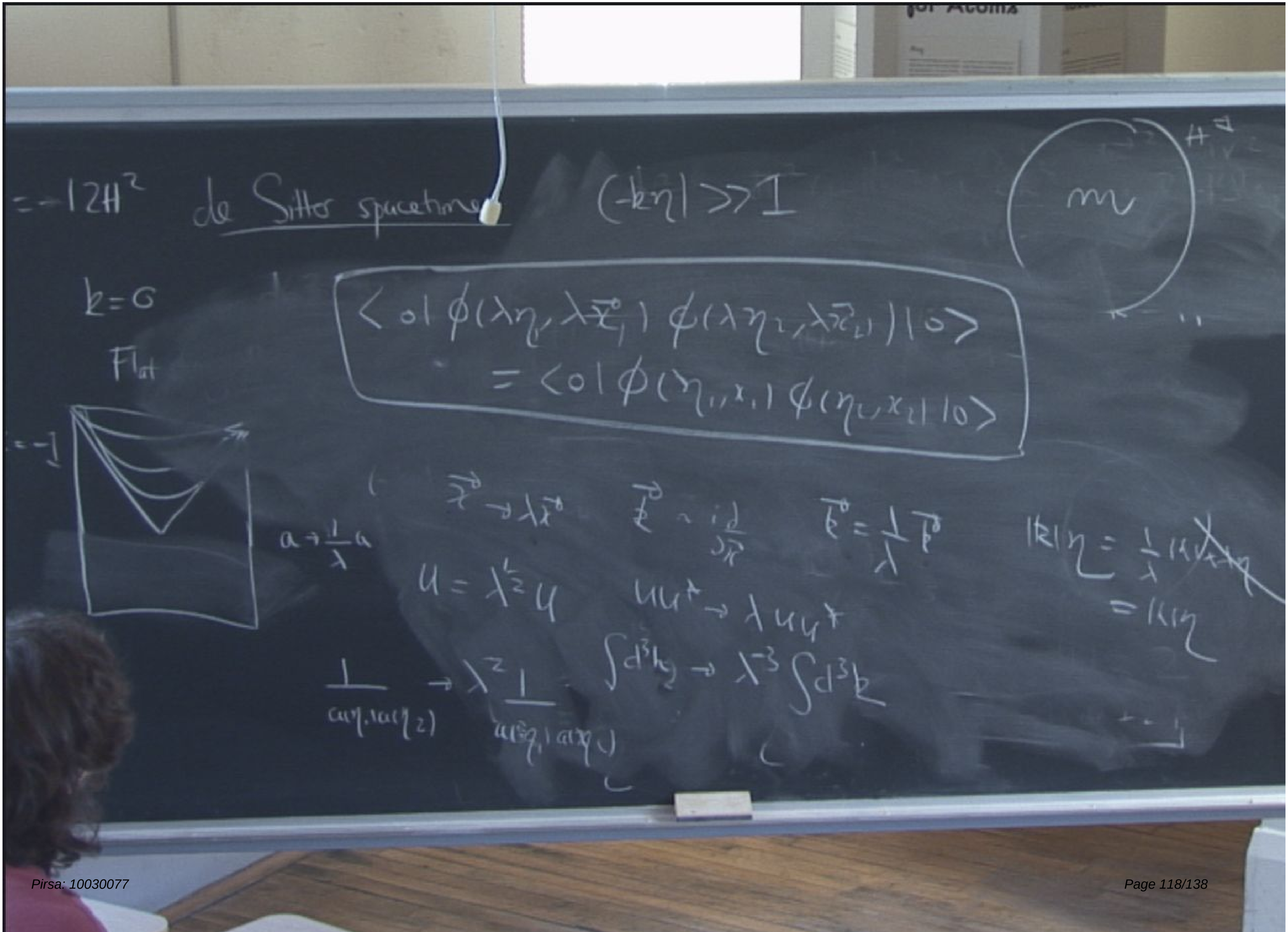
$$= \frac{1}{a(\eta)} \frac{1}{a(\eta')} \int \frac{d^3k}{(2\pi)^3} u_k(\eta) u_k^*(\eta') e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$\frac{1}{2|k|} e^{ik(x-x')}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} = \lambda\vec{x} \quad \eta \to \lambda\eta \quad ds^2 \to ds^2$


de Sitter spacetime
k=0
Flat


de Sitter spacetime
k=0
Flat
m

$a_k|0\rangle = 0$

Bunch-Davies

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$$+ a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}})$$

$$\langle 0|\phi(\eta,\vec{x})\,\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')}$$

$$\phi = \frac{u}{a}$$

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$$

$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta$ $\quad ds^2 \to ds^2$

Flat slicing

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{x}^{\,2}\right) = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

$$\boxed{u_k(\eta) = e^{i\pi}\, \frac{\sqrt{-\pi\eta}}{2}\, H^{(1)}_{3/2}(-k\eta)}$$

$$H^{(1)}_{3/2}(-k\eta)$$

$$\phi = \int \frac{d^3k}{a(\eta)(2\pi)^3} \left(a_k u_k \ldots \right)$$

$$\eta \to -\infty \quad (-k\eta) \to +\infty$$

$$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i(x - \nu\pi/2 - \pi/4)}$$

$$\frac{1}{\sqrt{2|k|}}\, e^{-ik\eta}$$

$a_k|0\rangle = 0$

Bunch-Davies

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$+ a_k^\dagger u_k^* e^{-i\vec k\cdot\vec x})$

$$\langle 0|\phi(\eta,\vec x)\,\phi(\eta',\vec x')|0\rangle$$

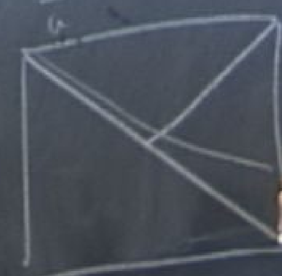
$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec k\cdot(\vec x-\vec x')}$$

$$\frac{1}{2|k|} e^{ik(x-x')}$$

$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec x^{\,2}\right)$$

$\vec x = \lambda\vec x \quad \eta \to \lambda\eta \quad ds^2 \to ds^2$



$a_k|0\rangle = 0$

Bunch-Davies

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

de Sitter invariant

$+ a_k^\dagger u_k^* e^{-ik\cdot x})$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

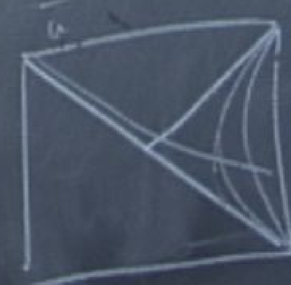
$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

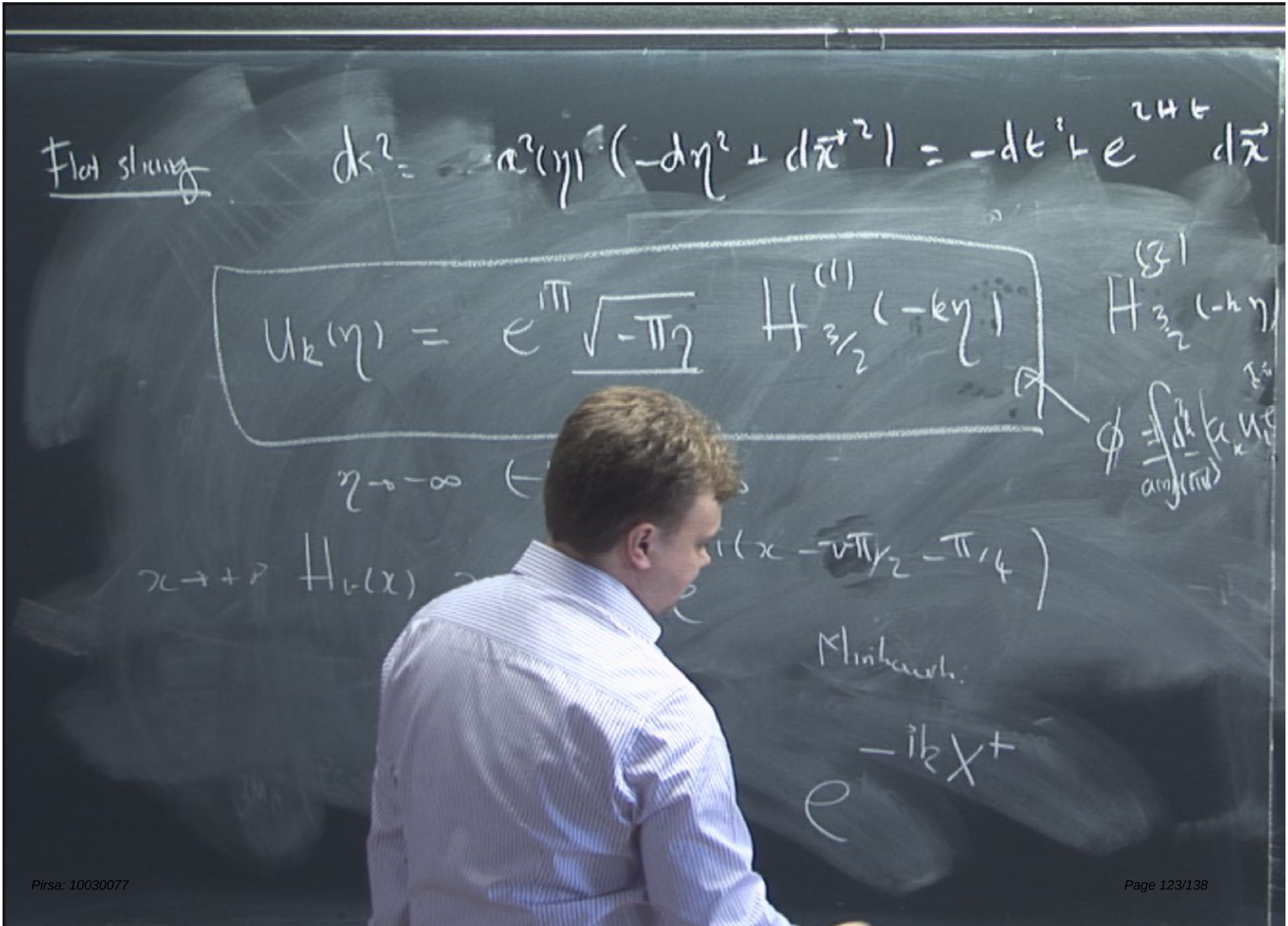
$\frac{1}{2|k|} e$

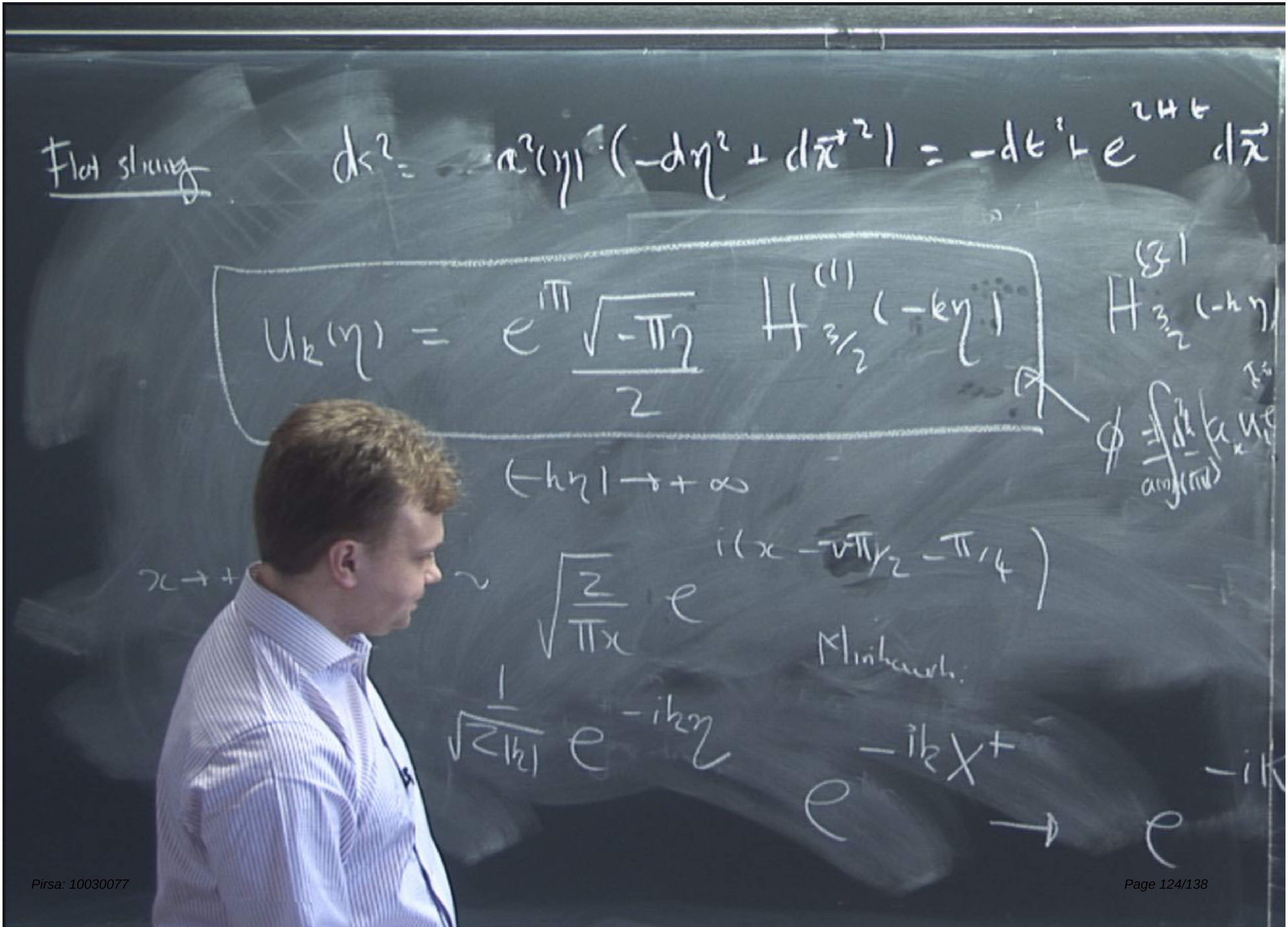
$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta \quad ds^2 \to ds^2$




Flat slicing
ds² = a²(η)(−dη² + dx⃗²) = −dt² + e^{2Ht} dx⃗²
u_k(η) = e^{iπ} √(−πη) H^{(1)}_{3/2}(−kη)
H^{(2)}_{3/2}(−kη)
η → −∞
x → +∞ H_ν(x)
(x − νπ/2 − π/4)
Minkowski
e^{−ikx⁺}


Flat slicing
ds^2 = a^2(η)(-dη^2 + dx^2) = -dt^2 + e^{2Ht} dx^2
u_k(η) = e^{iπ} √(-πη) / 2 H^{(1)}_{3/2}(-kη)
H_{3/2}(-kη)
(-kη) → +∞
x → +
~ √(2/(πx)) e^{i(x - νπ/2 - π/4)}
Minkowski:
1/√(2|k|) e^{-ikη}
e^{-ikX^+}

Flat slicing $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$

$$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$$

$H^{(2)}_{3/2}(-k\eta)$

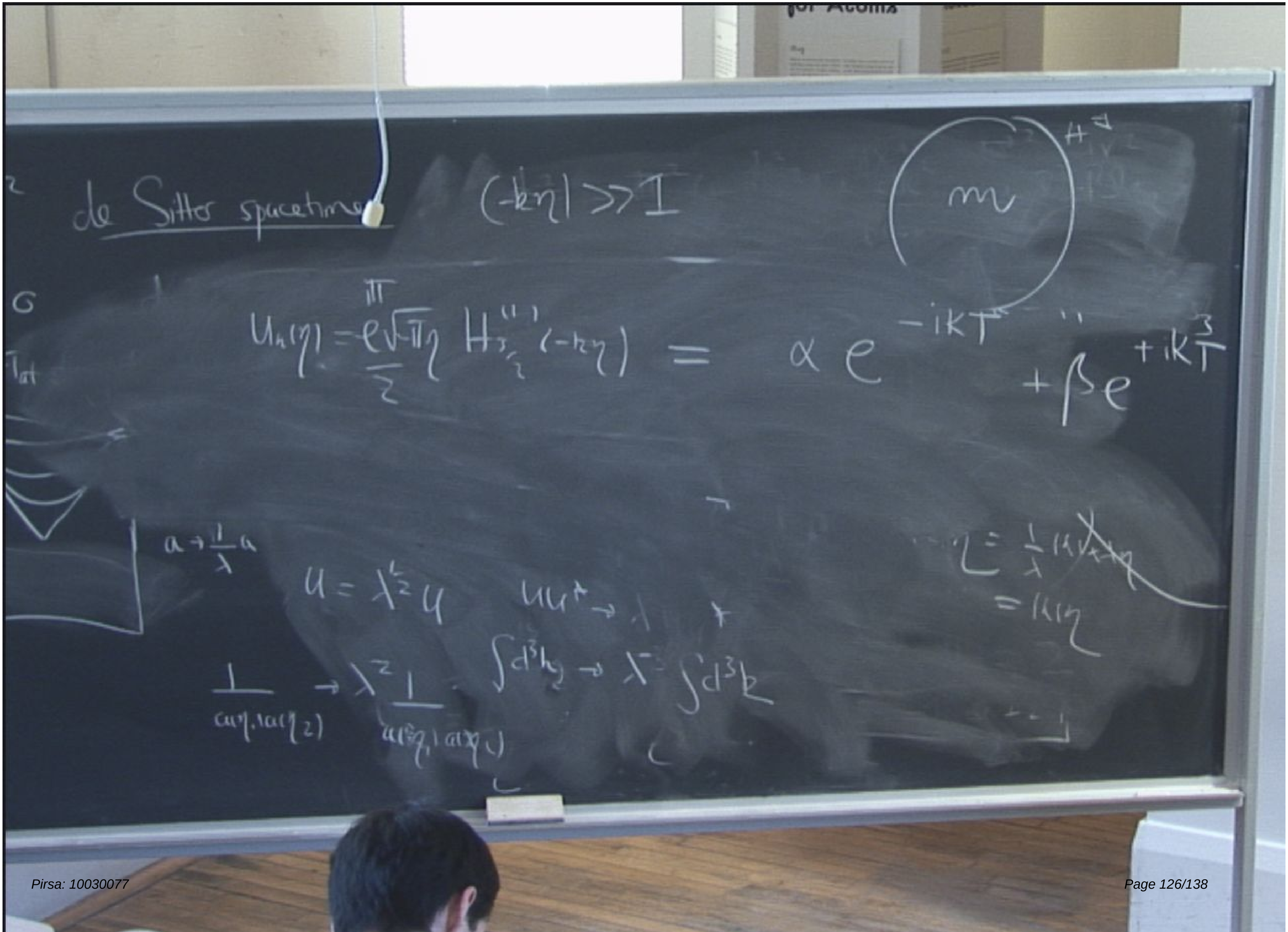
$\phi = \int \frac{d^3k}{a(\eta)(2\pi)^3} (a_k u_k \ldots$

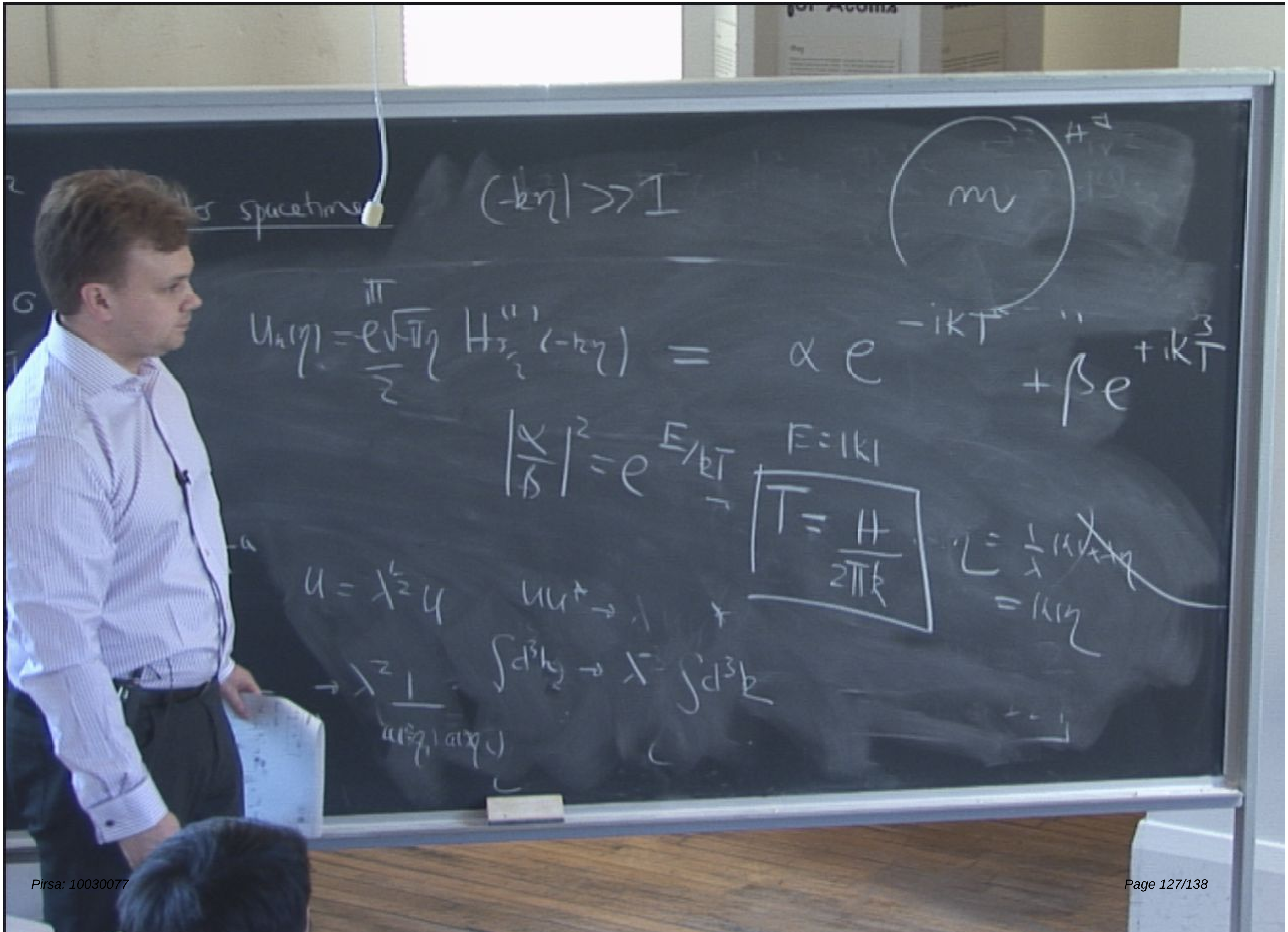
$\eta \to -\infty \quad (-k\eta) \to +\infty$

$$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i(x - \nu\pi/2 - \pi/4)}$$

Minkowski:

$$\frac{1}{\sqrt{2|k|}} e^{-ik\eta} \qquad e^{-ikX^+} \to e^{-ik\ldots}$$


de Sitter spacetime
(-kη| >> 1
m
$u_k(\eta) = \frac{e^{i\pi}\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta) = \alpha e^{-ikT} + \beta e^{+ikT}$
$a \to \frac{1}{\lambda} a$
$u = \lambda^{1/2} u$
$uu^* \to$
$\int d^3k \to \lambda^{-3} \int d^3k$
$= |k|\eta$


spacetime
E = |k|

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$a_k|0\rangle = 0$

Bunch-Davies

de Sitter invariant

$+ a_k^\dagger u_k^* e^{-ik\cdot x})$

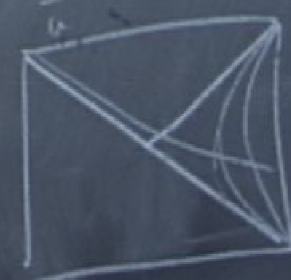
$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

$$= \frac{1}{a(\eta)a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|}e^{ik(x-x')}$$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^{\,2})$$

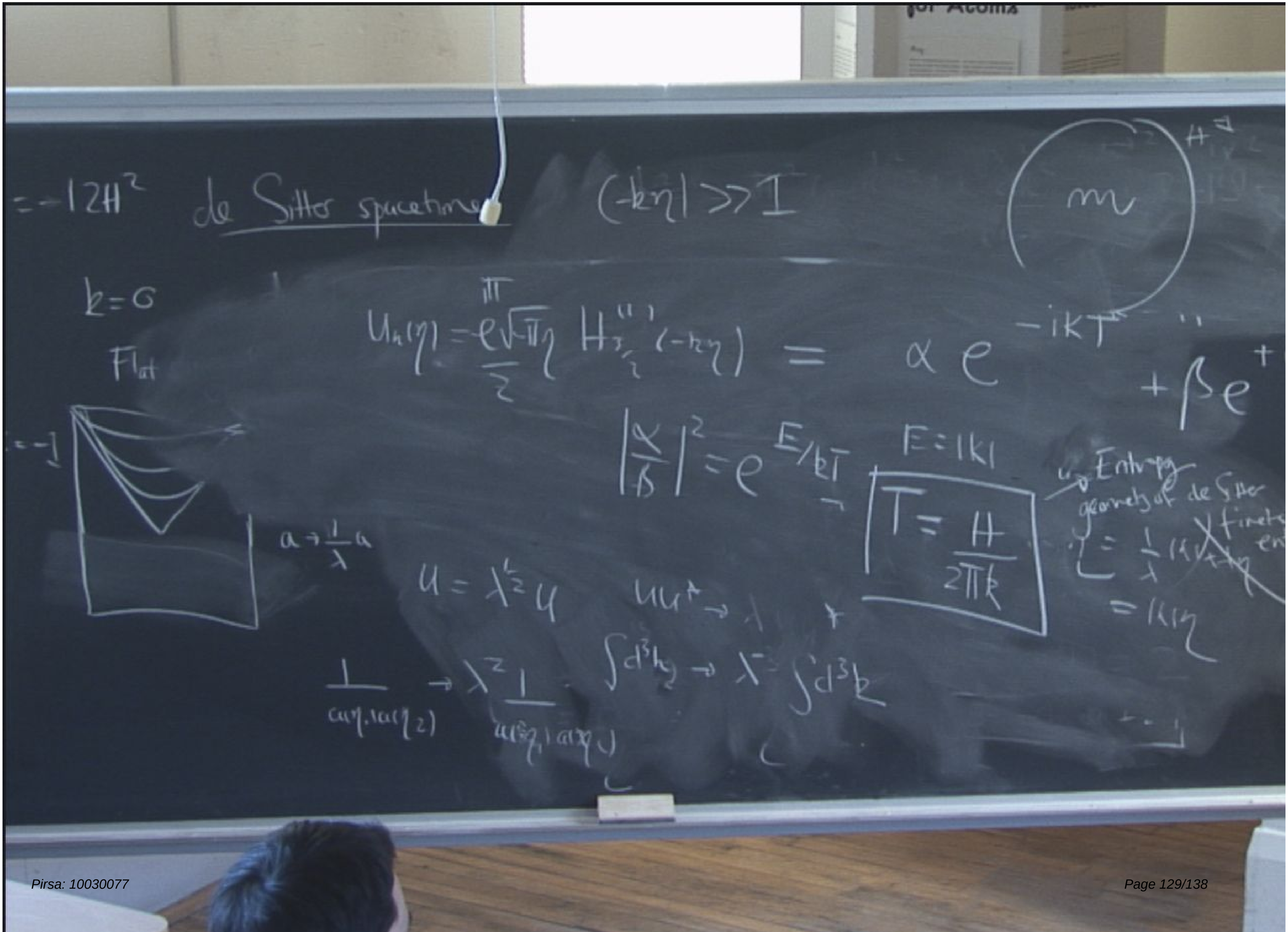
$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta$ $\quad ds^2 \to ds^2$

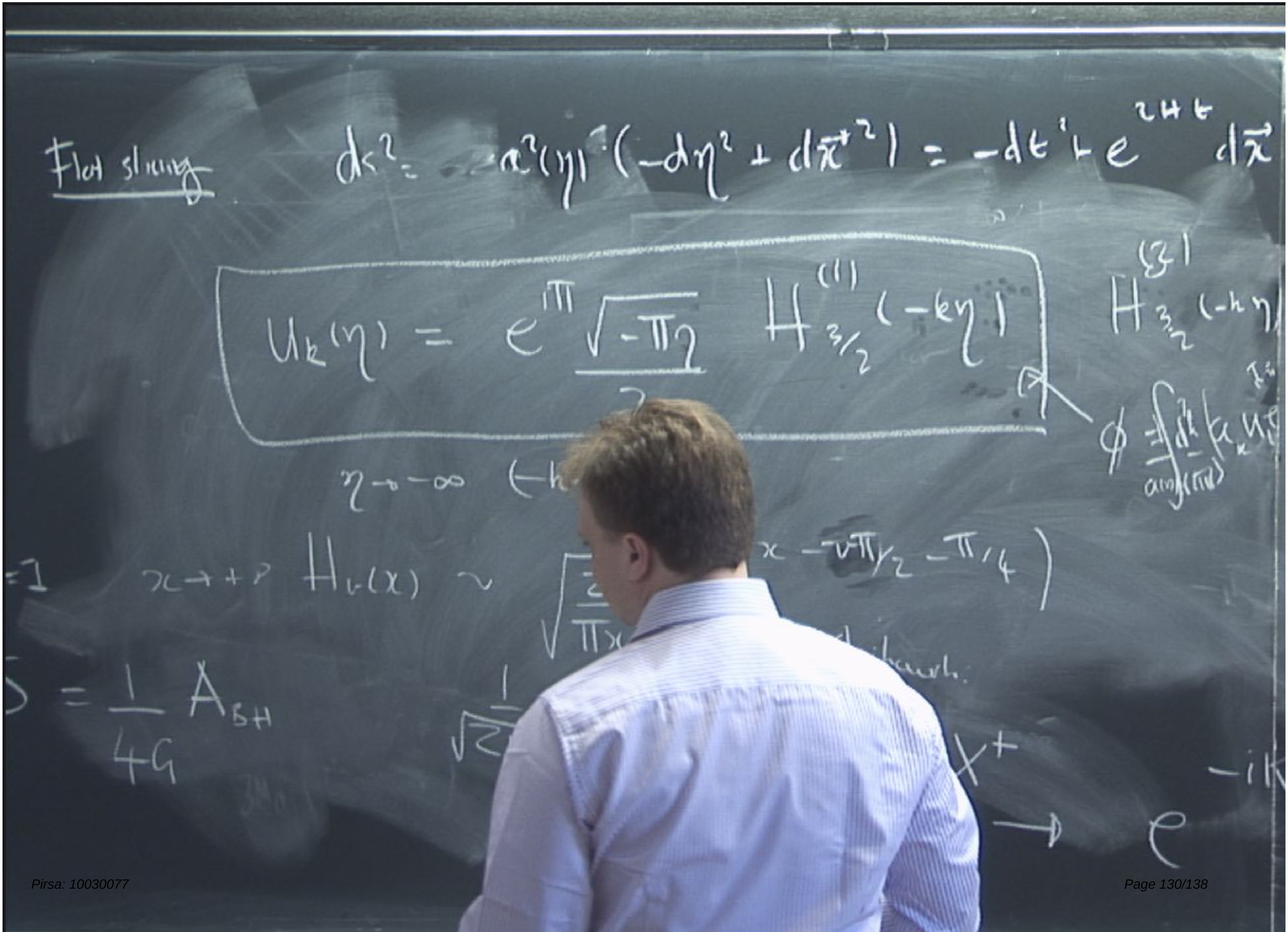


e^{ikx^+}

$+ e^{+ikx^-}$

$$\left|\frac{\alpha}{\beta}\right|^2 = e^{E/kT} \qquad E=|k| \qquad T=\frac{a}{2\pi}$$


de Sitter spacetime
k=0
Flat
$|k\eta| >> 1$
αe^{-ikT}
$+ \beta e^{+}$
$E = |k|$
$T = \frac{H}{2\pi k}$
Entropy


Flat slicing

Flat slicing $\quad ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$

M_{pl}^2

$$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$$

$H^{(1)}_{\frac{3}{2}}(-k\eta)$

$\phi = \int \frac{d^3k}{(2\pi)^3} (a_k u_k \ldots)$

$\eta \to -\infty \quad (-k\eta) \to +\infty$

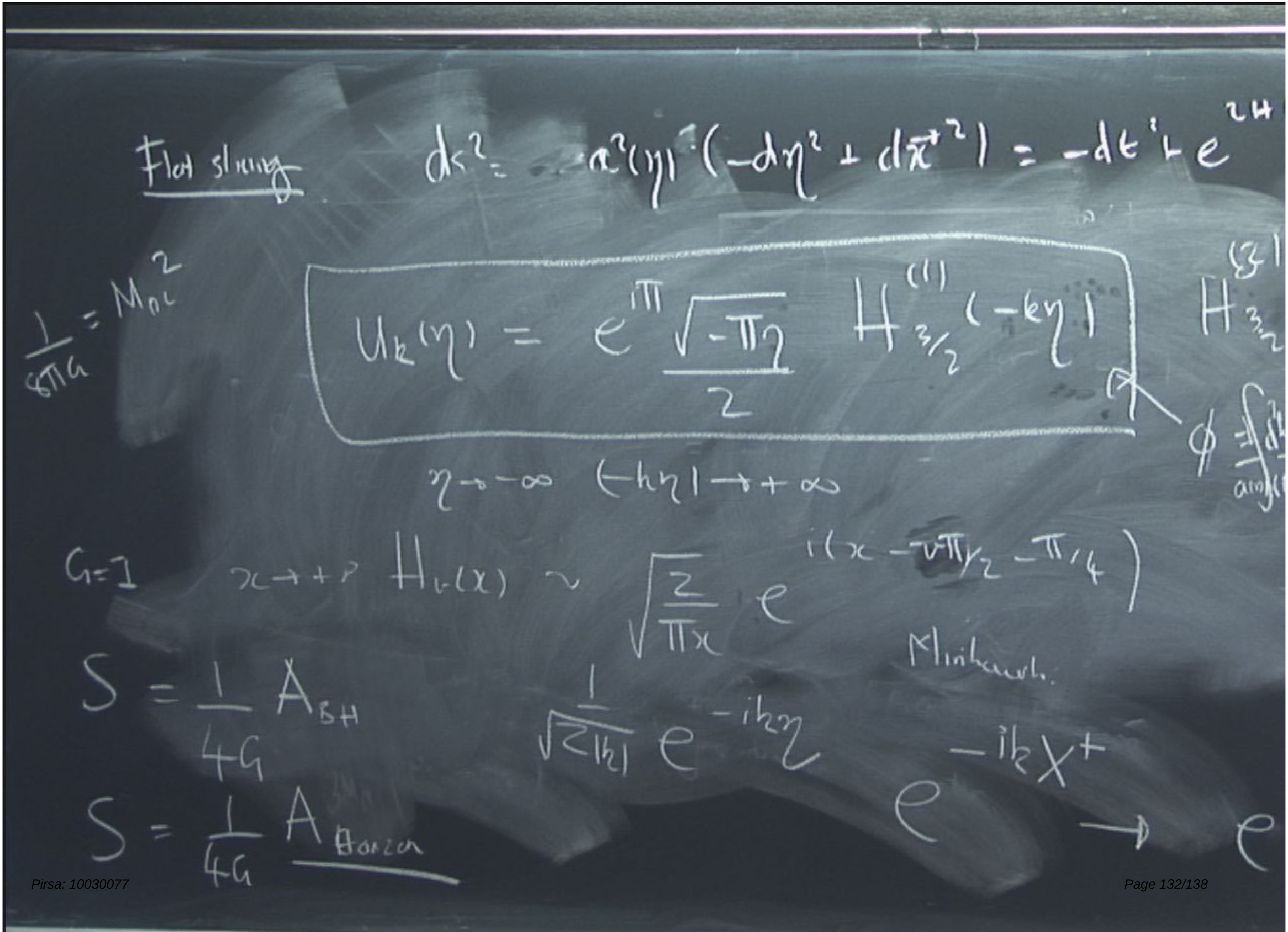
$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i(x - \nu\pi/2 - \pi/4)}$

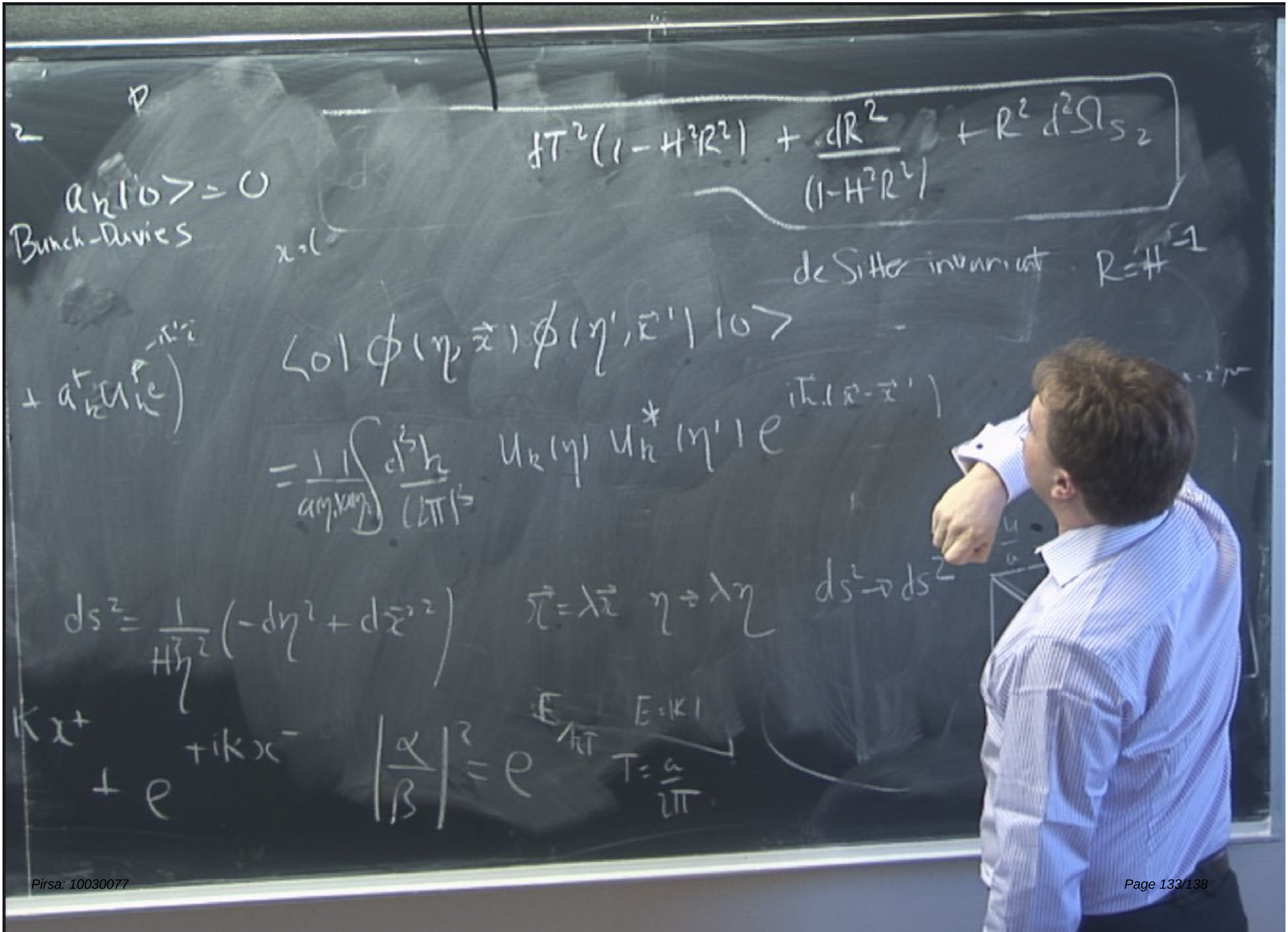
Minkowski

$S = \frac{1}{4G} A_{BH}$

$\frac{1}{\sqrt{2|k|}} e^{-ik\eta}$

$e^{-ikX^+} \to e^{-ik}$


Flat slicing
$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2H}$
$\frac{1}{8\pi G} = M_{pl}^2$
$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$
$\eta \to -\infty \quad (-k\eta) \to +\infty$
$G = 1$
$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}} e^{i(x - \nu\pi/2 - \pi/4)}$
Minkowski
$S = \frac{1}{4G} A_{BH}$
$\frac{1}{\sqrt{2|k|}} e^{-ik\eta}$
e^{-ikX^+}
$S = \frac{1}{4G} A_{Horizon}$
$\to e$


dT^2(1 - H^2R^2) + dR^2/(1-H^2R^2) + R^2 d^2Ω_S2
a_k|0> = 0
Bunch-Davies
deSitter invariant
R = H^-1
<0| φ(η, x) φ(η', x') |0>
u_k(η) u_k*(η') e^(ik·(x - x'))
ds^2 = 1/(H^2η^2) (-dη^2 + dx^2)
x = λx   η → λη
ds^2 → ds^2
|α/β|^2 = e^(-E/kT)
E = |k|
T = a/2π

$a_k|0\rangle = 0$

Bunch-Davies

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S^2})$$

de Sitter invariant $R = H^{-1}$

$+ a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}})$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

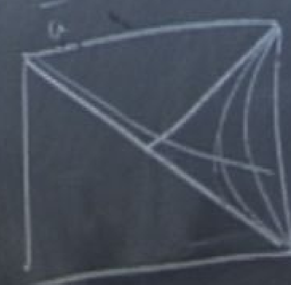
$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')}$$

$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} \to \lambda\vec{x}$ $\quad \eta \to \lambda\eta$ $\quad ds^2 \to ds^2$



Kx^+

$+ e^{+ikx^-}$

$$\left|\frac{\alpha}{\beta}\right|^2 = e^{-E/kT} \quad E = |k| \quad T = \frac{a}{2\pi}$$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S^2})$$

$a_k|0\rangle = 0$

Bunch-Davies

de Sitter invariant $R = H^{-1}$

$A = 4\pi R^2 =$

$+ a_k^\dagger u_k^* e^{-i\vec k\cdot\vec x})$

$$\langle 0|\phi(\eta,\vec x)\phi(\eta',\vec x')|0\rangle$$

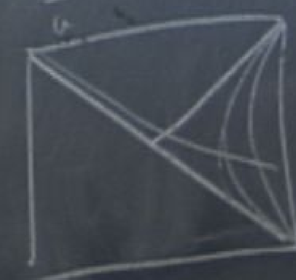
$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec k\cdot(\vec x-\vec x')}$$

$$\frac{1}{2|k|} e^{ik(x-x')}$$

$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec x^2)$$

$\vec x = \lambda\vec x \quad \eta \to \lambda\eta \quad ds^2 \to ds^2$



Kx^+

$+ e^{+ikx^-}$

$$\left|\frac{\alpha}{\beta}\right|^2 = e^{E/kT} \qquad E = |k| \qquad T = \frac{a}{2\pi}$$

$$dT^2(1-H^2R^2) + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

$a_k|0\rangle = 0$

Bunch-Davies

de Sitter invariant $R = H^{-1}$

$$A = 4\pi R^2 = \frac{4\pi}{H^2}$$

$$+ a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}})$$

$$\langle 0|\phi(\eta,\vec{x})\phi(\eta',\vec{x}')|0\rangle$$

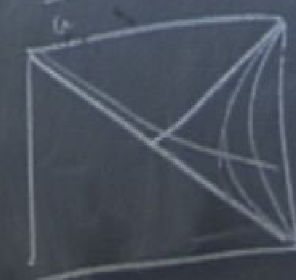
$$= \frac{1}{a(\eta)}\frac{1}{a(\eta')}\int \frac{d^3k}{(2\pi)^3}\, u_k(\eta)\, u_k^*(\eta')\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\frac{1}{2|k|} e^{ik(x-x')/\ldots}$$

$\phi = \frac{u}{a}$

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

$\vec{x} = \lambda\vec{x}$ $\quad \eta \to \lambda\eta \quad ds^2 \to ds^2$



$$kx^+ \qquad + e^{+ikx^-}$$

$$\left|\frac{\alpha}{\beta}\right|^2 = e^{E/kT} \qquad E = |k| \qquad T = \frac{a}{2\pi}$$

Flat slicing

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2$$

M_{pl}^2

$$u_k(\eta) = e^{i\pi} \frac{\sqrt{-\pi\eta}}{2} H^{(1)}_{3/2}(-k\eta)$$

$H^{(2)}_{3/2}(-k\eta)$

$\phi = \int \frac{d^3k}{a(\eta)(2\pi)^3} (a_k u_k \ldots)$

$\eta \to -\infty \quad (-k\eta) \to +\infty$

$$x \to +\infty \quad H_\nu(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i(x - \nu\pi/2 - \pi/4)}$$

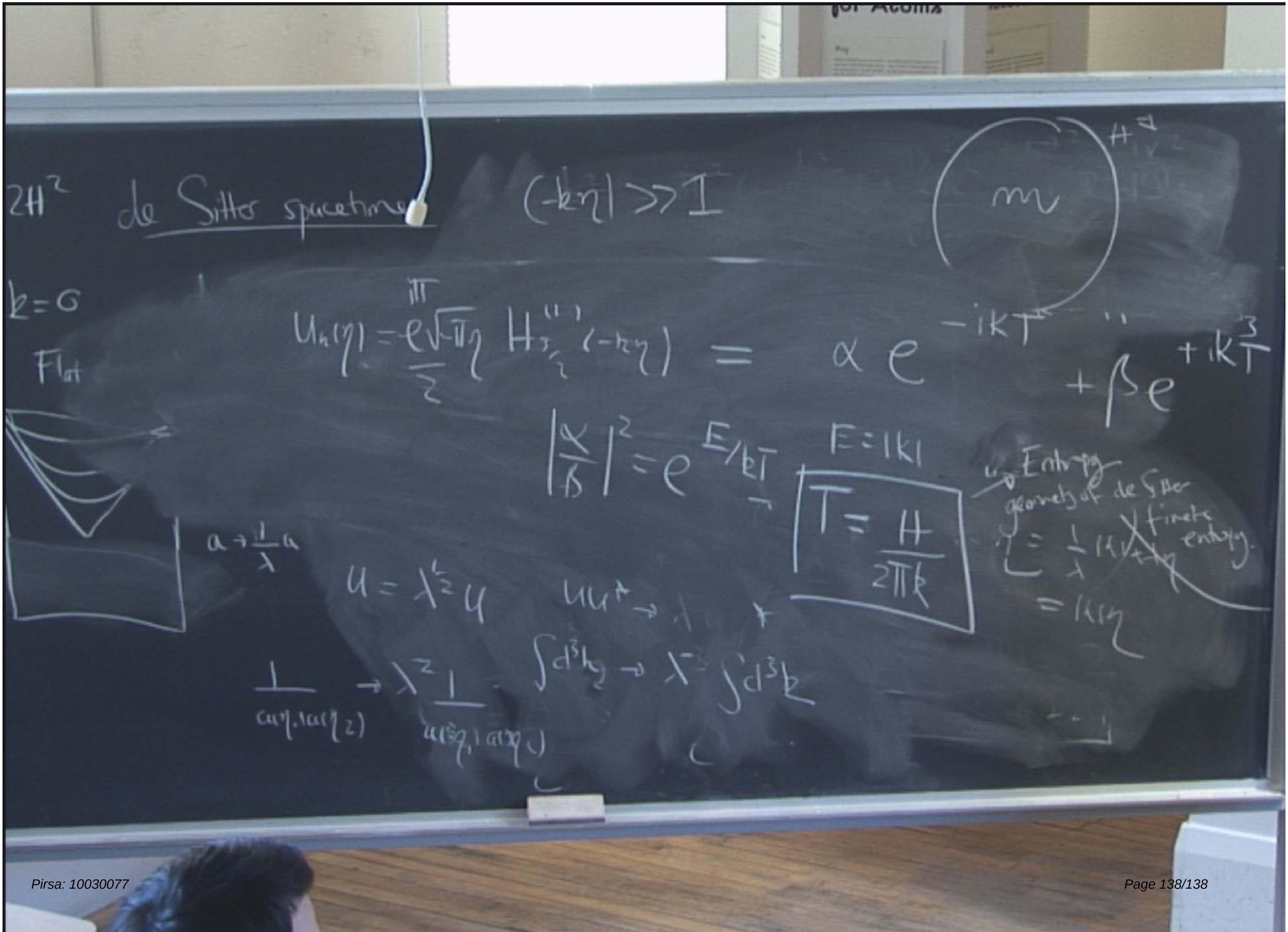
Minkowski:

$$\frac{1}{\sqrt{2|k|}} e^{-ik\eta}$$

$$e^{-ikX^+} \quad e \to e^{-ik}$$

$$S = \frac{1}{4G} A_{BH}$$

$$= \frac{1}{4G} A_{\text{Horizon}} = \frac{\pi H^{-2}}{G}$$


2H^2
de Sitter spacetime
(-kη| >> 1
m
k = 0
Flat
E = |k|
T = H/2πk
Entropy
geometry of de Sitter
finite entropy